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Solving of Partial Differential Equation by ELzaki Transform

A research submitted to the University of Babylon – College of Education of Pure Sciences – Department of Mathematics as part of the requirements for obtaining a Bachelor's degree in Mathematics.

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بِسْمِ ٱللَّهِ ٱلرَّحْمَٰنِ ٱلرَّحِيمِ

﴿ يَرْفَعِ اللَّهُ الَّذِينَ آمَنُوا مِنْكُمْ والَّذِينَ أُوتُوا الْعِلْمَ دَرَجاتٍ ﴾

صدق الله العلي العظيم

﴿سورة المجادلة، اية 11﴾

الاهداء

إلى من كانا النور الذي أضاء دربي، والسند الذي لم يخذلني يومًا...

إلى والديّ العزيزين، أمي وأبي،

إليكما أهدي ثمرة جهدي هذا، تقديرًا لِما قدّمتماه لي من حبٍّ ودعمٍ وتضحيات لا تُنسى. فبفضلكما وصلتُ إلى هذه المرحلة، وبدعائكما تجاوزتُ كل الصعاب.

أسأل الله أن يحفظكما ويجزيكما عني خير الجزاء.

الشكر والتقدير

اتوجه بالشكر الى الاستاذة الدكتورة سحر محسن جبار

لما قدمته لي من جهد ونصح ومعرفة طيلة انجاز هذا البحث

كما اتقدم بالشكر الى جميع أساتذتي الذين تتلمذت على ايديهم طوال مدة دراستي

والى اهلي وأصدقائي

وتحياتي وشكري الى كل الايادي البيضاء التي قدمت لنا يد العون

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Abstract

The ELzaki transform of partial differential equation is derived, and its applicability demonstrated using four different partial differential equations. In this research we find the particular solutions of those equations.

Keywords: ELzaki Transform-Partial Differential Equations.

INTRODUCTION

The term "differential equations" was proposed in 1676 by G. Leibniz. The first studies of these equations were carried out in the late 17th century in the context of certain problems in mechanics and geometry. Ordinary and partial differential equations have important applications and are a powerful tool in the study of many problems in the natural sciences and in technology; they are extensively employed in mechanics, astronomy, physics, and in many problems of chemistry and biology. The reason for this is the fact that objective laws governing certain phenomena (processes) can be written as ordinary and partial differential equations, so that the equations themselves are a quantitative expression of these laws. In physics and engineering, there are many partial differential equations such as Heat Equation, Wave Equation, Laplace Equation and Telegrapher's Equation etc. These equations are quite useful and applicable in engineering, physics and science. In the last few decades, researchers have paid their attention to find the solution of ordinary, partial, linear, nonlinear, homogeneous and non-homogeneous differential equations by using various integral transform, see [1-10]. One of such transforms known as ELzaki transform, introduced by Tarig M. ELzaki in 2011, is also very useful for solving ordinary and partial differential equations in the time domain, see [11-14]. In this paper, we apply ELzaki transform in solving various useful partial differential equations such as Heat Equation, Wave Equation, Laplace Equation and Telegrapher's Equation.

CHAPTER ONE

The ELzaki Transform

1.1. **Introduction**

ELzaki transform which is a modified general Laplace and Sumudu transforms, has been shown to solve effectively, easily and accurately a large Class of linear differential equations. ELzaki transform was successfully applied to integral equations, partial differential equations, ordinary differential equations with variable coefficients and system of all these equations.

1.2. Definition and Derivations the ELzaki Transform of Derivatives

The ELzaki Transform of the function $f(t)$ is defined as

$$E[f(t)] = T(v) = v\int_0^\infty f(t)e^{-t/v}\,dt\,,\ t > 0\,,\ v \in (-k_1, k_2)$$ [5]

To obtain the ELzaki transform of partial derivatives we use integration by parts as follows:

i. $E\left[\frac{\partial f}{\partial t}\right] = \frac{T(x,v)}{v} - v\,f(x,0)$

ii. $E\left[\frac{\partial f}{\partial t}\right] = \frac{dT(x,v)}{dx}$

iii. $E\left[\frac{\partial^2 f}{\partial t^2}\right] = \frac{T(x,v)}{v^2} - f(x,0) - v\frac{\partial f}{\partial t}(x,0)$

Proof:

1. $E\left[\frac{\partial f}{\partial t}(x,t)\right] = \int_0^\infty v\frac{\partial f}{\partial t}e^{\frac{-t}{v}}\,dt = \lim_{p\to\infty}\int_0^p ve^{\frac{-t}{v}}\frac{\partial f}{\partial t}\,dt$

$$= \lim_{p\to\infty}\left\{\left[ve^{\frac{-t}{v}}f(x,t)\right]_0^p - \int_0^p e^{\frac{-t}{v}}f(x,t)dt\right\}$$

$$= \frac{T(x,v)}{v} - vf(x,0)$$

2. $E\left[\frac{\partial f}{\partial x}\right] = \int_0^\infty v e^{\frac{-t}{v}} \frac{\partial f(x,t)}{\partial x} dt$

$$= \frac{\partial}{\partial x}\int_0^\infty v e^{\frac{-t}{v}} f(x,t)\, dt \ \ (\,u\sin g \ \ the\ Leibnitz\ rule)$$

$$= \frac{\partial}{\partial x}[\,T(x,v)] \qquad and \qquad E\left[\frac{\partial f}{\partial x}\right] = \frac{d}{dx}[T(x,v)]$$

3. To find

$$E\left[\frac{\partial^2 f}{\partial t^2}\,(x,t)\right]$$

Let

$$\frac{\partial f}{\partial t} = g\,, \qquad then$$

$$E\left[\frac{\partial^2 f}{\partial t^2}\,(x,t)\right] = E\left[\frac{\partial g(x,t)}{\partial t}\right] = E\,\frac{[g(x,t)]}{v} - vg(x,0)$$

$$E\left[\frac{\partial^2 f}{\partial t^2}\,(x,t)\right] = \frac{1}{v^2}T(x,v) - f(x,0) - v\frac{\partial f}{\partial t}(x,0)$$

We can easily extend this result to the nth partial derivative by using mathematical induction.

1.3. ELzaki Transform for some functions

In this section we will find ELzaki transform of some functions [4]

1. $f(t) = 1$

$$E[f(t)] = T(v) = v\int_0^\infty f(t)e^{\frac{-t}{v}}dt \qquad\qquad , f(t) = 1$$

$$= v\int_0^\infty 1.\, e^{\frac{-t}{v}}\, dt$$

$$= -v^2\left(e^{\frac{-t}{v}}\Big|_0^\infty\right) \quad (\,using\ Tabular\ Integration)$$

$$= -v^2\left[e^{\frac{-\infty}{v}} - \ e^{\frac{0}{v}}\right]$$

$$= \ -v^2\left[\frac{1}{e^{\frac{\infty}{v}}} - e^0\right] = \ -v^2[0-1] = \ v^2$$

2. f(t) = t

$$E[f(t)] = T(v) = v\int_0^\infty f(t)e^{\frac{-t}{v}}\,dt$$

$$= v\int_0^\infty t\,e^{\frac{-t}{v}}\,dt$$

$$= v\left[-v\,t\,e^{\frac{-t}{v}} - v^2e^{\frac{-t}{v}}\right]_0^\infty \quad \text{(using Tabular Integration)}$$

$$= v\left[-v\,\infty\,e^{\frac{-\infty}{v}} - v^2e^{\frac{-\infty}{v}} + v(0)e^{\frac{0}{v}} + v^2e^{\frac{0}{v}}\right]$$

$$= v(v^2) = v^3$$

3. f(t) = t^n

$$E[f(t)] = T(v) = v\int_0^\infty f(t)e^{\frac{-t}{v}}dt$$

$$= v\int_0^\infty t\,e^{\frac{-t}{v}}\,dt$$

$$= v\left[-vt^ne^{\frac{-t}{v}} - v^2nt^{n-1}e^{\frac{-t}{v}} - v^3n(n-1)t^{n-2}e^{\frac{-t}{v}} - \cdots - n!\ v^{n+1}e^{\frac{-t}{v}}\right]_0^\infty$$

$$= v\left[\left(-vt^ne^{\frac{-\infty}{v}} - v^2n\infty^{n-1}e^{\frac{-\infty}{v}} - v^3n(n-1)\infty^{n-2}e^{\frac{-\infty}{v}} - ..-n!\ v^{n+1}e^{\frac{-\infty}{v}}\right)\right.$$
$$\left. - \left(-v0^ne^{\frac{0}{v}} - v^2n0^{n-1}e^{\frac{0}{v}} - v^3n(n-1)0^{n-2}e^{\frac{0}{v}} - ..-n!\ v^{n+1}e^{\frac{0}{v}}\right)\right]$$

$$= v[0 - n!\,v^{n+1}]$$
$$= n!\,v^{n+2}$$

And so on …

1.4. ELzaki Transform table for some functions [3]:

F(t)	E[f(t)] =T(v)
1	v^2
t	v^3
t^n	$n!\,v^{n+1}$
$\frac{t^{a-1}}{\Gamma(a)}$	v^{a+1}
e^{at}	$\frac{v^2}{1-av}$
te^{at}	$\frac{v^3}{(1-av)^2}$
$\frac{t^{n-1}e^{at}}{(n-1)!}$, $n = 1,2,3 \ldots$	$\frac{v^{n+1}}{(1-av)^n}$
$\sin at$	$\frac{av^3}{1+a^2v^2}$
$\cos at$	$\frac{v^2}{1+a^2v^2}$
$\sinh at$	$\frac{av^3}{1-a^2v^2}$
$\cosh at$	$\frac{av^2}{1-a^2v^2}$
$e^{at}\sin bt$	$\frac{bv^3}{(1-av)^2+b^2v^2}$
$e^{at}\cos bt$	$\frac{(1-av)v^2}{(1-av)^2+b^2v^2}$
$t\ \sin at$	$\frac{2av^4}{1+a^2v^2}$
$J_0(at)$	$\frac{v^2}{\sqrt{1+a^2v^2}}$
$H(t-a)$	$v^2e^{\frac{-a}{v}}$
$\delta(t-a)$	$ve^{\frac{-a}{v}}$

1.5. The Inverse of ELzaki Transform [2]

Definition: Let the functions $\bar{f}(u) = E\{f\}$ is the ELzaki transform of the function f(t), then f(t) called the inverse transform of the function $\bar{f}(u)$ and we will write it as:

$$f(t) = E^{-1}\{\bar{f}(u)\}$$

Remark: The inverse transform has the linear combination property, i.e.

$$E^{-1}\left\{\sum_{k=1}^{n} a_k\, \bar{f}_k(u)\right\} = \sum_{k=1}^{n} a_k\, E^{-1}\{\bar{f}(u)\}$$

1.6. Solution of Partial Differential Equations [5]

In this section we solve first order Partial differential Equations and Second order partial differential equation, wave equation, heat equation, Laplace's and Telegraphers equation which are known as four.

Fundamental equations in mathematical physics and occur in many branches of physics, in applied mathematics as well as in engineering.

Example 1:

Find the solution of the first order initial value problem:

$$\frac{\partial y}{\partial x} = 2\frac{\partial y}{\partial x} + y$$

$y(x,0) = 6e^{-3x}$ and y is bounded for x > 0, t > 0.

Sol: Let Y be the ELzaki transform of y. then, taking the ELzaki transform of y(x,0) we have

$$\frac{dY(x,v)}{dx} - \left(\frac{2}{v} + 1\right)Y(x,v) = -12ve^{-3x}$$

This is the linear ordinary differential equation

The integration factor is $p = \exp\left(\int -\left(\frac{2}{v} + 1\right)dx\right) = e^{-\left(\frac{2}{v}+1\right)x}$

Therefore

$$Y(x,v) = \frac{12v^2}{2+4v}e^{-3x} + cve^{\left(\frac{2}{v}+1\right)x}$$

Since Y is bounded, c should be zero. Taking the inverse ELzaki transform we have:

$$y(x,t) = 6e^{-2t} . e^{-3x} = 6e^{-2t-3x}$$

Example 2:

Consider the Laplace Equation:

$$u_{xx} + u_{tt} = 0, \qquad u(x,0) = 0, \qquad u_t(x,0) = \cos x\,, \qquad x,t > 0 \qquad (1)$$

Sol: Let T(v) be the ELzaki transform of u . Then,

$$\frac{T(x,v)}{v^2} - u(x,0) - vu_t(x,0) + T''(x,v) = 0$$

$$v^2T''(x,v) + T(x,v) = v^3 \cos x$$

This is the second order differential equation have the particular solution in the form:

$$T(x,v) = \frac{v^3 \cos x}{v^2D^2 + 1} = \frac{v^3 \cos x}{1 - v^2} \qquad (2)$$

$$Where \qquad D^2 \equiv \frac{d^2}{dx^2}$$

If we take the inverse ELzaki transform for Eq. (2), we obtain solution of Eq. (1) in the form:

$$u(x,t) = \cos x \; \sinh t$$

Example 3:

Solve the wave equation:

$$u_{tt} - 4u_{xx} = 0, \qquad u(x,0) = \sin \pi x\,, \qquad u_t(x,0) = 0\,, \qquad x,t > 0 \qquad (3)$$

Sol: Taking the ELzaki transform for Eq (3) and making use of Conditions we obtain.

$$4v^2 T''(x,v) - T(x,v) = -v^2 \sin \pi x$$

$$T(x,v) = -\frac{v^2 \sin \pi x}{4v^2 D^2 - 1} = \frac{v^2 \sin \pi x}{1 + (2\pi)^2 v^2}$$

Now we take the Inverse ELzaki transform to find the particular solution of Eq (3) in the form:

$$u(x,t) = \sin \pi x \;\cos 2\pi t$$

Example 4:

Consider the homogeneous heat equation in one dimension in a normalized form:

$$4\,u_t = u_{xx}, \qquad u(x,0) = \sin \frac{\pi}{2} x\,, \qquad x,t > 0 \qquad (4)$$

Sol: By using the ELzaki transform for Eq (4)

We can obtain

$$vT''(x,v) - 4\,T\,(x,v) = \; -4v^2 \sin \frac{\pi}{2} x$$

Solve for *T* (x,v) we find that the particular solution is

$$T(x,v) = \frac{v^2 \sin \frac{\pi}{2} x}{1 + \frac{\pi^2\, v}{16}} \qquad (5)$$

And similarly, if we take the inverse ELzaki transform for Eq (5), we obtain the solution of Eq (4) in the form:

$$u(x,t) = e^{\frac{-\pi^2}{16} t} \sin \frac{\pi}{2} x$$

Example 5:

Consider the telegraphers equation:

$$u_{tt}(x,t) + 2\alpha u_t(x,t) = \alpha^2 u_{xx}(x,t), \qquad 0 < x < 1\,, t > 0 \tag{6}$$

With the initial conditions:

$$u(x,0) = \cos x\,, \qquad u_t(x,0) = 0 \tag{7}$$

Sol:

Take ELzaki transform for Eq (6) we get:

$$\frac{T(v)}{v^2} - u(x,0) - vu'(x,0) + 2\alpha\frac{T(v)}{v} - 2\alpha v u(x,0) = \alpha^2 T''(v) \tag{8}$$

Substituting Eq (7) into Eq (8) we have:

$$\alpha^2 v^2 T''(v) - (1 + 2\alpha v)T(v) = -(2\alpha v^3 + v^2)\cos x$$

$$or \qquad T(v) = \frac{-(2\alpha v^3 + v^2)\cos x}{\alpha^2 v^2 D^2 - (1 + 2\alpha v)} = \frac{\cos x\,(2\alpha v^3 + v^2)}{(1 + \alpha v)^2}$$

Which is the particular solution of (6).

We take the inverse of ELzaki transform we find that:

$$u = \cos x\, T^{-1}\left[\frac{2\alpha v^3}{(1 + \alpha v)^2} + \frac{v^2}{1 + \alpha v}\right] = \cos x\,[2\alpha t e^{-\alpha\, t} + e^{-\alpha\, t}]$$

$$= (1 + 2\alpha t)e^{-\alpha t}\cos x$$

CHAPTER TWO

Application of ELzaki Transform to Partial Differential Equation

2.1. Introduction

In this section we solve partial differential equations such as Heat Equation, Wave Equation. Laplace Equation and Telegrapher's Equation which are known as four fundamental equations in mathematical physics and occur in many branches of physics, in applied mathematics as well as in engineering.

2.2. Heat Conduction Equation [1]

The most important physical phenomenon of conduction in a metallic rod of finite length can explicitly be described by using the partial differential equation.

Let u(x, t) be the temperature distribution in a rod at any distance 'x' measured from initial point and at any time 't' then the event is governed by the following PDF

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial t^2}$$

Where $k = \frac{K}{cp}; K = Thermal\ Conductivity\ ,\ c = Specific\ Heat\ of\ meta$

$and\ \rho\ is\ density\ (mass\ per\ unit\ volume)$.

Various types of boundary conditions can be imposed to solve this equation.

For illustration.

Example 1: Consider the heat equation in one dimension:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}; u(x,0) = \sin 2\pi x \,, u(0,t) = 0, u(1,t) = 0, 0 < x < 1, t > 0.$$

Solution: Given PDF is an example of heat conduction equation in a metallic rod of unit length. Taking ELzaki transform on both sides of given equation and making use of conditions, we get

$$\frac{T(x,t)}{v} - v\,u(x,0) = \frac{d^2T(x,v)}{dx^2}$$

$$\Rightarrow \frac{d^2T(x,v)}{dx^2} - \frac{T(x,v)}{v} = -v\,sin\,2\pi x \qquad (1)$$

This is the linear differential equation of second order, therefore solution of (1)

$$T(x,v) = c_1 e^{\frac{x}{v}} + c_2 e^{\frac{-x}{v}} + \frac{v^2}{(1+4\pi^2 v)} \sin 2\pi x$$

Using boundary conditions, we get $c_1 = c_2 = 0$

$$\Rightarrow E[u(x,t)] = T(x,v) = \frac{v^2}{(1+4\pi^2 v)} \sin 2\pi x \qquad (2)$$

If we take the inverse ELzaki transform of (2), we obtain solution of PDF, an given in example 1

$$\Rightarrow u(x,t) = e^{-4\pi^2 t} \sin 2\pi x$$

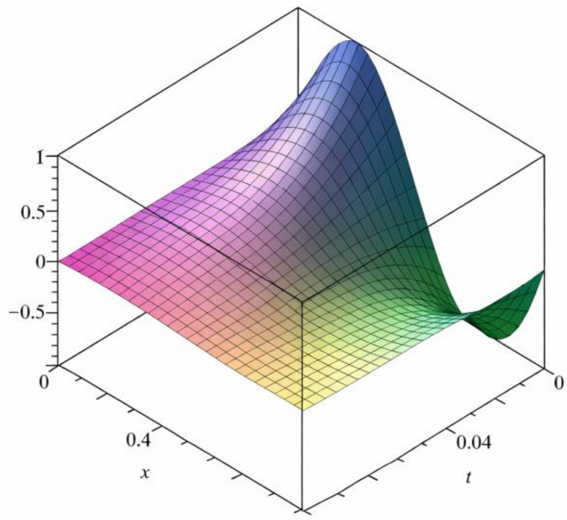

Figure-1: Graph of $u(x,t) = e^{-4\pi^2 t} \sin 2\pi x \,, t > 0 \; and \; 0 \leq x \leq 1.$

2.3. Wave Equation [1]

Yet another important physical phenomenon of vibrations in tightly stretched string refers to wave motion. The small transverse vibrations of a flexible string are governed by one-dimensional wave equation expressed as

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Where $u(x,t)$ denotes the displacement of any point of the string at any time t. The constant $c^2 = \frac{T}{\rho}$ is described as the ratio of Tension T in the string to the mass per unit length of the string which is supposed to be uniform. Various types of boundary conditions can be imposed to solve this equation. For illustration,

Example 2: Consider the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}; \quad u(x,0) = 20\sin 2\pi x - 10\sin 5\pi x, \quad u_t(x,0) = 0, u(0,t) = 0,$$
$$u(2,t) = 0, \quad 0 < x < 2, \quad t < 0.$$

Solution: Given PDF is an example of wave equation. Taking ELzaki transform on both sides of given equation and making use of conditions, we get

$$\frac{T(x,v)}{v^2} - u(x,0) - vu_t(x,0) = 9\frac{d^2T(x,v)}{dx^2}$$

$$\Longrightarrow E[u(x,t)] = T(x,v) = \frac{v^2}{(1+4\pi^2 v)}\sin 2\pi x \qquad (3)$$

This is linear differential equation of second order, therefor solution of (5)

$$T(x,v) = c_1 e^{\frac{x}{3v}} + c_2 e^{\frac{-x}{3v}} + \frac{20v^2}{(1+36\pi^2 v^2)}\sin 2\pi x - \frac{10v^2}{(1+22\pi^2 v^2)}\sin 5\pi x \qquad (4)$$

If we take the inverse ELzaki transform of (4), we obtain solution of PDF, as given in example 2 $\Longrightarrow u(x,t) = 20\sin 2\pi x\cos 6\pi t - 10\sin 5\pi x\cos 15\pi t$

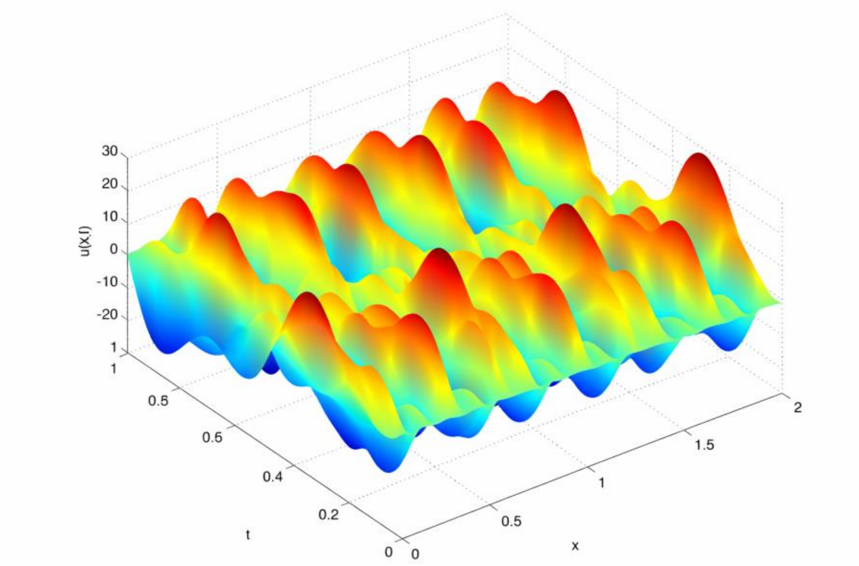

Figure-2: Graph of $u(x,t) = 20\sin 2\pi x \cos 6\pi t - 10\sin 5\pi x \cos 15\pi t\,, t > 0 and\ 0 \le x \le 2.$

2.4. Laplace Equation (1)

We want to study the steady-state temperature distribution in a thin, flat, rectangular plat. Without any loss of generality.

let the boundaries of the plate be $x = 0, x = a, y = 0\ and\ y = b.$

The differential equation modeling the steady-state temperature distribution is given by the Laplace equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 < x < a, \qquad 0 < y < b.$$

Various types of boundary conditions can be imposed to solve this equation. For illustration,

Example 3: Consider the Laplace 's equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 0, \qquad \text{which statisfies the conditions}$$

$$u(0,t) = 0 = u(l,t), u(x,0) = 0, u(x,a) = \frac{n\pi}{l}\sin\frac{n\pi x}{l}, u_t(x,0)$$
$$= \frac{n\pi}{l} cosech\ \frac{n\pi a}{l}\sin\frac{n\pi x}{l}, \qquad 0 < x < l, \qquad 0 < t < a.$$

Solution: Given PDF is an example of Laplace 's equation. Taking ELzaki transform on both sides of given equation and making use of conditions, we get

$$\frac{d^2T(x,v)}{dx^2}+\frac{T(x,v)}{v^2}-u(x,0)-v\,u_t(x,v)=0 \tag{5}$$

$$\Rightarrow \frac{d^2T(x,v)}{dx^2}+\frac{T(x,v)}{v^2}=v\frac{n\pi}{l}cosech\frac{n\pi a}{l}\sin\frac{n\pi x}{l}$$

This is linear differential equation of second order, therefor solution of (5)

$$T(x,v)=c_1\cos\frac{x}{v}+c_2\sin\frac{x}{v}+\frac{v^3}{1-\frac{n^2\pi^2}{i^2}v^2}\frac{n\pi}{l}cosech\frac{n\pi a}{l}\sin\frac{n\pi x}{l}$$

Using boundary conditions, we get $c_1=c_2=0$,

$$E[u(x,t)]=T(x,v)=\frac{n\pi}{l}cosech\frac{n\pi a}{l}\sin\frac{n\pi x}{l}\ \frac{v^3}{1-\frac{n^2\pi^2}{i^2}v^2} \tag{6}$$

If we take the inverse ELzaki transform of (8), we obtain solution of PDF, as given in example 3

$$\Rightarrow u(x,t)=\frac{n\pi}{l}cosech\frac{n\pi a}{l}\sin\frac{n\pi x}{l}\sinh\frac{n\pi t}{l}$$

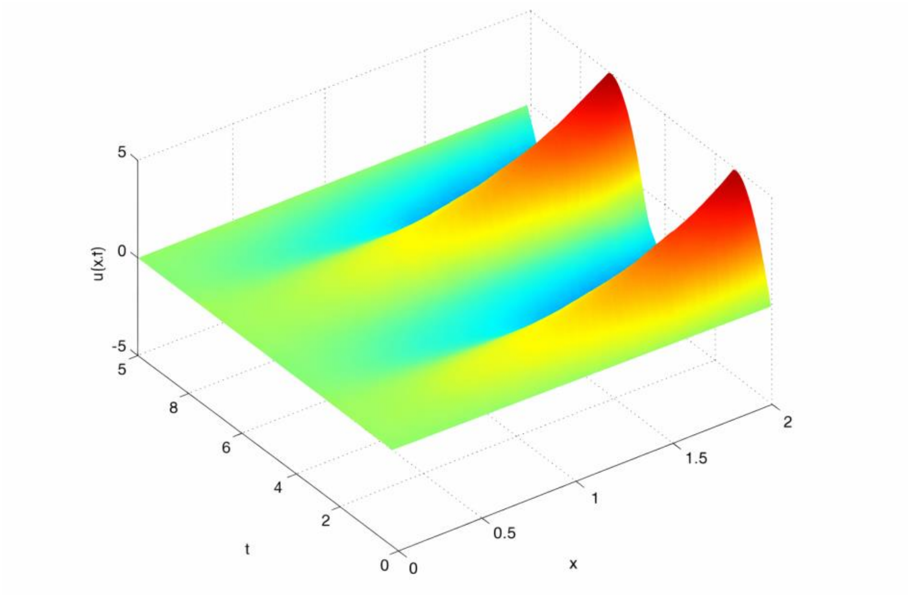

Figure-3: Graph of $u(x,t)=\frac{n\pi}{l}cosech\frac{n\pi a}{l}\sin\frac{n\pi x}{l}\sinh\frac{n\pi t}{l}, t>0\ and\ 0\le x\le 1, l=5, n=2, a=1.$

2.5. Telegrapher 's Equation [1]

Telegrapher equation is generally used to describe electrical phenomenon in a more practical approach by using short segments of an electrical system. For analysis of distributed electrical parameters of any electrical system, Telegrapher equation is being used because analysis of electrical parameters in lumped form is very difficult. Various types of boundary conditions can be imposed to solve this equation. For illustration.

Example 4: Consider the Telegrapher 's Equation:

$$\frac{\partial^2 u}{\partial t^2} + 2\alpha \frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0 \text{ whith the initial conditions:}$$

$$u(x,0) = \cos x, \qquad u_t(x,0) = 0.$$

Solution: Given PDF is an example of Telegrapher 's Equation. Taking ELzaki transform to both sides of given equation and making use of conditions, we get

$$\frac{T(x,v)}{v^2} - u(x,0) - v\,u_t(x,0) + 2\alpha \frac{T(x,v)}{v} - 2\alpha\, v\, u(x,0) = \alpha^2 \frac{d^2 T(x,v)}{dx^2}$$

$$\Rightarrow \alpha^2 v^2 \frac{d^2 T(x,v)}{dx^2} - (1 + 2\alpha v) T(x,v) = -(2\alpha v^3 + v^2)\cos x \tag{7}$$

This is a linear differential equation of second order, therefor solution of (9)

$$\Rightarrow E[u(x,t)] = T(x,v) = \frac{-(2\alpha v^3 + v^2)\cos x}{\alpha^2 v^2 D^2 - (1 + 2\alpha v)}, \qquad \text{where } D^2 \equiv \frac{d^2}{dx^2}$$

$$\Rightarrow E[u(x,t)] = T(x,v) = \frac{\cos x\,(2\alpha v^3 + v^2)}{(1 + \alpha v)^2} \tag{8}$$

(8) is particular solution of (7).

If we take the inverse ELzaki transform of (8), we obtain solution of PDF, as given in example 4

$$u(x,t) = \cos x\, E^{-1}\left[\frac{2\alpha v^3}{(1 + \alpha v)^2} + \frac{v^2}{(1 + 2\alpha v)}\right]$$

$$\Rightarrow u(x,t) = [2\alpha t e^{-at} + e^{-at}]\cos x.$$

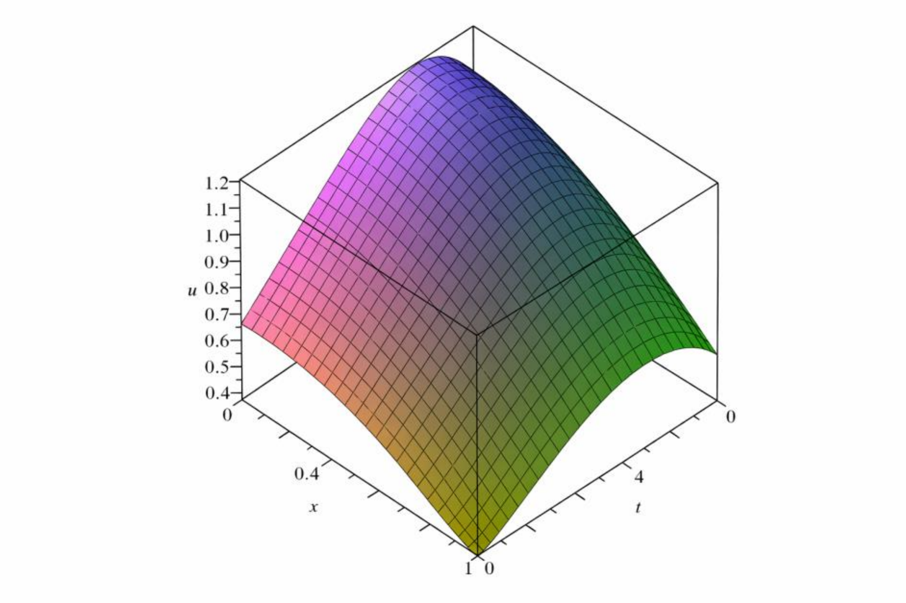

Figure-4: Graph of $u(x,t) = \left[2\alpha t e^{-at} + e^{-at}\right]\cos x\,,\ t > 0\ and\ \ 0 \le x \le 1, \alpha = 0.2$

CONCLUSION

In this paper, we apply interesting new transform "ELzaki transform" in solving various useful partial differential equations such as Heat Equation, Wave Equation, Laplace Equation and Telegrapher's Equation. Our purpose here is to show the applicability of this interesting new transform.

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