

Republic of Iraq  
Ministry of Higher Education  
and Scientific Research  
University of Babylon  
College of Education for Pure Sciences



## **Lower and Upper Probability Via Pre- Open and Pre-Closed Sets**

Research Submitted to University of Babylon/College of Education for Pure Sciences/ Mathematic Department as Part of the Requirements for The Degree of B.Sc. in Mathematical Science

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

(وَلَا تَقْفُ مَا لَيْسَ لَكَ بِهِ عِلْمٌ إِنَّ السَّمْعَ وَالْبَصَرَ وَالْفَوَادَ كُلُّ أُولَئِكَ كَانَ عَنْهُ مَسْؤُلًا وَلَا تَمْشِ فِي  
الْأَرْضَ مَرَحًا إِنَّكَ لَنْ تَخْرُقَ الْأَرْضَ وَلَنْ تَبْلُغَ الْجِبَالَ طُولاً)

صدق الله العلي العظيم

(سورة الاسراء : آية ٣٦)

## الاهداء

اهدي ثمرة جهدي الى من تسعد الروح للقائهما ويفيض القلب حبا لهم الى

الاغلى في الوجود عندي امي وابي .

الى اخي واختي وابيهما الاعزاء على قلبي .

الى كل من خصني بالحب والاحترام .

الى كل اصدقائي واحبائي من الدراسة وخارجها .

الى شهدائنا من الحشد الشعبي والجيش .

الى شهداء ثورة تشرين الخالدة .

## شكر وعرفان

نحمد الله عز وجل الذي انار لنا درب العلم والمعرفة ، واعاننا على اداء هذا الواجب ووفقا على انجاز هذا العمل المتواضع ،

اتقدم بجزيل الشكر والتقدير والامتنان الى كل من ساعدني من قريب او بعيد في انجاز هذا العمل وفي تذليل ما واجهته من صعوبات كما اتقدم بجزيل شكري للسيد المشرف (الدكتور مصطفى حسن هادي) كم لا يفوتنـي ان اشكر اساتذـتنا في قسم الرياضيات واعضاء اللجنة التي تكـرمـت بـمناقشة هذا العمل المتواضع .

## Contents

Subject	Page Number
<b>Abstract</b>	<b>Vi</b>
<b>Introduction</b>	<b>1</b>
<b>Chapter One Topologies on The Sets Containing Three Elements</b>	<b>2</b>
<b>References</b>	<b>٣٥</b>

## Abstract

Rather, the aim of my research is the process of calculating all the topologies on the three-point group, and then calculating the upper and lower probability of all the groups within the  $IP(x)$  corresponding to those topologies, by the help of the interior points and closure on the one side, and on the other side, calculating the interior points and closure of the harmonic sets Pre –Open

## **Introduction**

What is the science of topology? A question asked by mathematicians themselves. Some of them imagine that it is a science that has reached the top of abstraction and perfection, while its applications invaded multiple branches such as medicine, engineering and urban planning, in addition to all branches of mathematics. The beginning history of this science back to 1860 when the german mathematician(WeierStrass) analyzed The concept of the aim of regular applications and in the context of this analysis reconstructed the space of real numbers and highlighted their properties, which are called, the properties of the topology

In the context of this, in 1873(Cantor) introduced a new language characterized by generality and accuracy, and in expressing these characteristics, and he called it groups. This helped each of (Ascoli), (Volterra), and (Arzela) to pave the way for (Fre éhet)in 1906 to discover what is called today metric spaces

## **Chapter One**

Topologies on The Sets Containing Three Elements

## Chapter One

In this chapter, we will show set of basic definitions on which our works are based, to a set of questions that we worked on through three elements, which are (a,b,c), where we made 29 topologies and extracted interiors and closures of aR-open at the end of the research ,we studied the probabilities that are (upper and lower) on the topologies that we knew, in addition to the probabilities on topologies for the aR-open set.

### Definition 1.1 [1]

Let  $X$  be a nonempty set and  $\tau$  be a family of subsets of  $X$ (i.e. , $\tau \subseteq \text{IP}(X)$ ). we say  $\tau$  is a topology on  $X$  satisfy the following conditions:

1.  $X, \Phi \in \tau$
2. If  $U, V \in \tau$ , then  $U \cap V \in \tau$  the finite intersection of elements from  $\tau$
3. If  $U_\alpha \in \tau$ ;  $\alpha \in \Lambda$ , then  $\bigcup_{\alpha \in \Lambda} U_\alpha \in \tau$   $\forall \alpha \in \Lambda$  the arbitrary (finite or infinite )union of element of  $\tau$  is again an element of  $\tau$ .

### Definition 1.2 [2]

Let  $(X, \tau)$  be a topological space.the subset of  $X$  belonging to  $\tau$  are called open sets in the space

If  $A \subseteq X$  and  $A \in \tau$  then  $A$  open set

## Definition 1.3 [2]

The subset of  $X$  is called set in the space  $x$  if its complement  $X/A$  is open set we will denote the family of closed sets

if  $A \subseteq X$  and  $A \in F$  then  $A$  closed set

## Definition 1.4 [3]

Let  $(X, \tau)$  be a topological space and let  $A \subseteq X$ . A point  $X \in A$  is called an interior point of  $A$  iff there exists an open set  $U \in \tau$  containing  $X$  such that  $X \in U \subseteq A$ . The set of all interior points of  $A$  is called the interior of  $A$  and is denoted by  $A^\circ$  or  $\text{Int}(A)$  i.e

$$A^\circ = \{x \in A : \exists U \in \tau ; x \in U \subseteq A\}$$
$$x \in A^\circ \leftrightarrow \exists U \in \tau ; x \in U \subseteq A$$

## Definition 1.5

Let  $(X, \tau)$  be a topological space and let  $A$  be a subset of  $X$ . Then the intersection of all  $\tau$ -closed containing the set  $A$  is called the closure of  $A$  and denoted by  $\bar{A}$  or  $C_A$  or  $\text{cl}(A)$ . i.e

$$\text{cl}(A) = \cap \{F : F \text{ is closed} , A \subseteq F\}$$

## Definition 1.6

A subset  $A$  of a space  $X$  is said to be Pre-open if  $A = \text{int}(\text{cl}(\text{int}(A)))$ . And the complement Pre-open is called Pre-closed set. And the complement  $A = \text{cl}(\text{int}(\text{cl}(A)))$ .

## Definition 1.7 [4]

Let  $(X, \tau)$  be a topological space and let  $A \subseteq X$ . A point  $x \in A$  is called an pre- interior point of  $A$  iff there exists an pre-open set  $U \in \tau$  containing  $x$  such that  $x \in U \subseteq A$ . the set of all pre-interior point of  $A$  is called the pre-interior of  $A$  and is denoted by pre-  $A^\circ$  or pre-Int( $A$ ) i.e

$$\begin{aligned} \text{Pre-}A^\circ &= \{x \in A : \exists U \in \tau ; x \in U \subseteq A\} \\ x \in \text{pre- } A^\circ &\iff \exists U \in \tau ; x \in U \subseteq A \end{aligned}$$

## Definition 1.8 [4]

Let  $(X, \tau)$  be a topological space and let  $A$  be a subset of  $X$ . then the intersection of all pre-closed containing the set  $A$  is called the pre-closure of  $A$  and denoted by pre-  $\bar{A}$  or pre- C A or pre-cl( $A$ ).i.e

$$\text{Pre-cl}(A) = \cap \{F : F \text{ is closed , } A \subseteq F\}$$

## Definition 1.9

- $p(A) = p(\text{In}(A)) = \frac{\text{number element of } A^\circ}{\text{number element of } X}$
- $p(A) = p(\text{cl}(A)) = \frac{\text{number element of } \bar{A}}{\text{number element of } X}$
- $p(\text{Pre-} (A^\circ)) = \frac{\text{number element of Pre-} \text{int} (A)}{\text{number element of } X}$
- $p(\text{Pre-} (A)) = \frac{\text{number element of Pre-} \text{cl}(A)}{\text{number element of } X}$

$$T_1 = \{\emptyset, X\}$$

$$T_1^c = \{\emptyset, X\}$$

$$Pre \cdot T_1(X) = \{X, \emptyset\}, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}$$

$$Pre \cdot T_1^c(X) = \{\emptyset, X\}, \{b,c\}, \{a,c\}, \{a,b\}, \{c\}, \{b\}, \{a\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{a,c\}$	$\{b,c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{P}(\overline{A})$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

$$T_2 = \{X, \emptyset, \{a\}\}$$

$$T_2^c = \{\emptyset, X, \{b, c\}\}$$

$$Pre \cdot T_2 \cdot (X) = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$Pre \cdot T_2^c \cdot (X) = \{\emptyset, X, \{b, c\}, \{c\}, \{b\}\}$$

$T_2$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$
$\overline{P}(\overline{A})$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{0}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$

$$T_3 = \{X, \emptyset, \{b\}\}$$

$$T_3^c = \{\emptyset, X, \{a, c\}\}$$

$$Pre \cdot T_3 \cdot (X) = \{X, \emptyset, \{b\}, \{a, b\}, \{b, c\}\}$$

$$Pre \cdot T_3^c \cdot (X) = \{\emptyset, X, \{a, c\}, \{c\}, \{a\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$
$\bar{P}(\overline{A})$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$	$\frac{2}{3}$
$\bar{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

$$T_4 = \{X, \emptyset, \{c\}\}$$

$$T_4^c = \{\emptyset, X, \{b, a\}\}$$

$$Pre \cdot T_4 \cdot (X) = \{X, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$$

$$Pre \cdot T_4^c \cdot (X) = \{\emptyset, X, \{a, b\}, \{b\}, \{a\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\bar{P}(\overline{A})$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$
$\bar{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

$$T_5 = \{X, \emptyset, \{a\}\}, \{a, b\}$$

$$T_5^c = \{\emptyset, X, \{b, c\}, \{c\}\}$$

$$Pre \cdot T_5 \cdot (X) = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$$

$$Pre \cdot T_5^c \cdot (X) = \{\emptyset, X, \{b, c\}, \{c\}, \{b\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{0}{3}$
$\bar{P}(\overline{A})$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{0}{3}$
$\bar{P}(Pre(\overline{A}))$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$

$$T_6 = \{X, \emptyset, \{a\}, \{a, b\}\}$$

$$T_6^c = \{\emptyset, X, \{b, c\}, \{b\}\}$$

$$Pre \cdot T_6 \cdot (X) = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$$

$$Pre \cdot T_6^c \cdot (X) = \{\emptyset, X, \{b, c\}, \{c\}, \{b\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{0}{3}$
$\overline{P}(\overline{A})$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{0}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$

$$T_7 = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$$

$$Pre \cdot T_7 \cdot (X) = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$$

$$Pre \cdot T_7^c \cdot (X) = \{\emptyset, X, \{b, c\}, \{c\}, \{b\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{0}{3}$
$\overline{P}(\overline{A})$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{0}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$

$$T_8 = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$$

$$Pre \cdot T_8 \cdot (X) = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$$

$$Pre \cdot T_8^c \cdot (X) = \{\emptyset, X, \{b, c\}, \{c\}, \{b\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$	$\frac{1}{3}$
$\overline{P}(\overline{A})$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

$$T_9 = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$$

$$T_9^c = \{\emptyset, X, \{b, c\}, \{c\}, \{b\}\}$$

$$Pre \cdot T_9 \cdot (X) = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$$

$$Pre \cdot T_9^c \cdot (X) = \{\emptyset, X, \{b, a\}, \{c\}, \{b\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$	$\frac{1}{3}$
$\overline{P}(\overline{A})$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

$$T_{10} = \{X, \emptyset, \{b\}, \{a, b\}, \{b, c\}\}$$

$$T_{10}^c = \{\emptyset, X, \{a, c\}, \{c\}, \{a\}\}$$

$$Pre \cdot T_{10} \cdot (X) = \{X, \emptyset, \{b\}, \{a, b\}, \{b, c\}\}$$

$$Pre \cdot T_{10}^c \cdot (X) = \{\emptyset, X, \{a, c\}, \{c\}, \{a\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$	$\frac{2}{3}$
$\overline{P(A)}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

$$T_{11} = \{X, \emptyset, \{c\}, \{a, c\}\}$$

$$T_{11}^c = \{\emptyset, X, \{a, b\}, \{b\}\}$$

$$Pre \cdot T_{11} \cdot (X) = \{X, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$$

$$Pre \cdot T_{11}^c \cdot (X) = \{\emptyset, X, \{a, b\}, \{b\}, \{a\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{P(A)}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

$$T_{12} = \{X, \emptyset, \{c\}, \{b, c\}\}$$

$$T_{12}^c = \{\emptyset, X, \{a, b\}, \{a\}\}$$

$$Pre \cdot T_{12} \cdot (X) = \{X, \emptyset, \{c\}, \{b, c\}, \{a, c\}\}$$

$$Pre \cdot T_{12}^c \cdot (X) = \{\emptyset, X, \{a, b\}, \{a\}, \{b\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{P(A)}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

$$T_{13} = \{X, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$$

$$T_{13}^c = \{\emptyset, X, \{a, b\}, \{b\}, \{a\}\}$$

$$Pre \cdot T_{13} \cdot (X) = \{X, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$$

$$Pre \cdot T_{13}^c \cdot (X) = \{\emptyset, X, \{a, b\}, \{b\}, \{a\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{P}(\overline{A})$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

$$T_{14} = \{X, \emptyset, \{a, b\}\}$$

$$T_{14}^c = \{\emptyset, X, \{c\}\}$$

$$Pre \cdot T_{14} \cdot (X) = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$Pre \cdot T_{14}^c \cdot (X) = \{\emptyset, X, \{a, b\}, \{a\}, \{b\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{P(A)}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

$$T_{15} = \{X, \emptyset, \{b, c\}\}$$

$$T_{15}^c = \{X, \emptyset, \{a\}\}$$

$$Pre \cdot T_{15} \cdot (X) = \{X, \emptyset, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$$

$$Pre \cdot T_{15}^c \cdot (X) = \{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$
$\overline{P}(\overline{A})$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

$$T_{16} = \{X, \emptyset, \{a, c\}\}$$

$$T_{16}^c = \{\emptyset, X, \{b\}\}$$

$$Pre \cdot T_{16} \cdot (X) = \{X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$Pre \cdot T_{16}^c \cdot (X) = \{\emptyset, X, \{b, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$
$\overline{P}(\overline{A})$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$

$$T_{17} = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$$

$$T_{17}^c = \{X, \emptyset, \{b, c\}, \{a, c\}, \{c\}\}$$

$$Pre \cdot T_{17} \cdot (X) = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$$

$$Pre \cdot T_{17}^c \cdot (X) = \{\emptyset, X, \{b, c\}, \{a, c\}, \{c\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\overline{P(A)}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

$$T_{18} = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$$

$$T_{18}^c = \{\emptyset, X, \{b, c\}, \{a, b\}, \{b\}\}$$

$$Pre \cdot T_{18} \cdot (X) = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$$

$$Pre \cdot T_{18}^c \cdot (X) = \{\emptyset, X, \{b, c\}, \{a, b\}, \{b\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{P}(\overline{A})$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$

$$T_{19} = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$$

$$T_{19}^c = \{X, \emptyset, \{a, c\}, \{a, b\}, \{a\}\}$$

$$Pre \cdot T_{19} \cdot (X) = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$$

$$Pre \cdot T_{19}^c \cdot (X) = \{\emptyset, X, \{a, c\}, \{a, b\}, \{a\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{P}(\overline{A})$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

$$T_{20} = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$$

$$T_{20}^c = \{\emptyset, X, \{b, c\}, \{a, c\}, \{c\}, \{a\}\}$$

$$Pre \cdot T_{20} \cdot (X) = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$$

$$Pre \cdot T_{20}^c \cdot (X) = \{\emptyset, X, \{b, c\}, \{a, c\}, \{c\}, \{a\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{P}(\overline{A})$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

$$T_{21} = \{X, \emptyset, \{a\}, \{c\}, \{a,c\}, \{b,c\}\}$$

$$T_{21}^c = \{X, \emptyset, \{b,c\}, \{a,b\}, \{b\}, \{a\}\}$$

$$Pre \cdot T_{21} \cdot (X) = \{X, \emptyset, \{a\}, \{c\}, \{a,c\}, \{b,c\}\}$$

$$Pre \cdot T_{21}^c \cdot (X) = \{\emptyset, X, \{b,c\}, \{a,b\}, \{b\}, \{a\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{a,c\}$	$\{b,c\}$
$\underline{P}(A^\circ)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(\overline{A})$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$

$$T_{22} = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$$

$$T_{22}^c = \{\emptyset, X, \{a, c\}, \{a, b\}, \{a\}, \{b\}\}$$

$$Pre \cdot T_{22} \cdot (X) = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$$

$$Pre \cdot T_{22}^c \cdot (X) = \{\emptyset, X, \{a, c\}, \{a, b\}, \{a\}, \{b\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(\overline{A})$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

$$T_{23} = \{X, \emptyset, \{a\}, \{b, c\}\}$$

$$T_{23}^c = \{X, \emptyset, \{b, c\}, \{a\}\}$$

$$Pre \cdot T_{23} \cdot (X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$Pre \cdot T_{23}^c \cdot (X) = \{\emptyset, X, \{b, c\}, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{P}(\overline{A})$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

$$T_{24} = \{X, \emptyset, \{b\}, \{a, c\}\}$$

$$T_{24}^c = \{X, \emptyset, \{a, c\}, \{b\}\}$$

$$Pre \cdot T_{24} \cdot (X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$Pre \cdot T_{24}^c \cdot (X) = \{\emptyset, X, \{b, c\}, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{P}(\overline{A})$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

$$T_{25} = \{X, \emptyset, \{c\}, \{a, c\}\}$$

$$T_{25}^c = \{X, \emptyset, \{a, b\}, \{b\}\}$$

$$Pre \cdot T_{25} \cdot (X) = \{X, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$$

$$Pre \cdot T_{25}^c \cdot (X) = \{\emptyset, X, \{a, b\}, \{b\}, \{a\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{P}(\overline{A})$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

$$T_{25} = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$$

$$T_{25}^c = \{X, \emptyset, \{a,c\}, \{b\}\}$$

$$Pre \cdot T_{25} \cdot (X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}\}$$

$$Pre \cdot T_{25}^c \cdot (X) = \{\emptyset, X, \{b,c\}, \{a,c\}, \{c\}, \{b\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a,b\}$	$\{a,c\}$	$\{b,c\}$
$\underline{P}(A^\circ)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{P(A)}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

$$T_{27} = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$$

$$T_{27}^c = \{X, \emptyset, \{b, c\}, \{a, b\}, \{b\}, \{c\}\}$$

$$Pre \cdot T_{27} \cdot (X) = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$$

$$Pre \cdot T_{27}^c \cdot (X) = \{\emptyset, X, \{b, c\}, \{a, b\}, \{b\}, \{c\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{P}(\overline{A})$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$

$$T_{28} = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$$

$$T_{25}^c = \{X, \emptyset, \{a, c\}, \{a, b\}, \{a\}, \{c\}\}$$

$$Pre \cdot T_{28} \cdot (X) = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$$

$$Pre \cdot T_{28}^c \cdot (X) = \{\emptyset, X, \{a, c\}, \{a, b\}, \{a\}, \{b\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{P}(\overline{A})$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

$$T_{29} = \{X, \emptyset, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$T_{29}^c = \{X, \emptyset, \{b, c\}, \{a, c\}, \{a, b\}, \{a\}, \{c\}\}$$

$$Pre \cdot T_{29} \cdot (X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$Pre \cdot T_{29}^c \cdot (X) = \{X, \emptyset, \{b, c\}, \{a, c\}, \{a, b\}, \{a\}, \{c\}\}$$

$T_1$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
$\underline{P}(A^\circ)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(\overline{A})$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\underline{P}(Pre(A^\circ))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{P}(Pre(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

## References

- [1] A.S. Mashhour, M. E. Abd El-Monsef and S.N. El-Deeb, on precontinuous and weak precontinuous mappings, Proc. Math. Phy. Soc. Egypt, 53 (1982), 47-53.
- [2] M. Ganster, Kyungpook Math. J. 27(2) (1987), 135-43.
- [3] Sidney A. MORRI 1 Topology Without Tears 11Version of February 20.2012
- [4] "University of Mosul-College of eduction" Introduction to General Eduction, Samir Bashir Hadid, Dar Al-Laith for Printing and Publishing 1988.