Repuplic of Iraq

Ministry of Higher Education

and Scientific Research

University of Babylon

College of Education for Pure Sciences

# Computation Lower and Upper Probability of Semi Open Set.

Research Submitted to University of Babylon / College of Education for Pure Sciences / Mathematic Department as Part of the Requirements for The Degree of B.Sc. in Mathematical Science.

By

**Saadoun Rajab Garimat Al-Ghazali**

Supervised By

**Prof.Dr. Luay Abd-Al-Hani Al-Swidi**

**1445** **2024**

بِسْمِ اللَّهِ الرِحْمَٰنَّ الرَحْيم

﴿لَّٰكِنِ الرَّاسِخُونَ فِي الْعِلْمِ مِنْهُمْ وَالْمُؤْمِنُونَ يُؤْمِنُونَ بِمَا أُنزِلَ إِلَيْكَ وَمَا أُنزِلَ مِن قَبْلِكَ

وَالْمُقِيمِينَ الصَّلَاةَ وَالْمُؤْتُونَ الزَّكَاةَ وَالْمُؤْمِنُونَ بِاللَّهِ وَالْيَوْمِ الْآخِرِ أُولَٰئِكَ سَنُؤْتِيهِمْ أَجْرًا

عَظِيمًا﴾

**صدق الله العظيم**

**[162 ]سورة النساء :**

# الاهداء

الحمدلله حباً وشكراً وامتنان، ما كنت افعل هذا لولا فضل الله فالحمدلله على البدء وعلى الختام....

إلى ملاكي العزيز.... الغائبه عن عيني والحاضره في قلبي، إلى من بها انتصرت والدتي (رحمها الله واسكنها فسيح جناته) .

إلى سندي وقوتي وملاذي بعد الله، إلى بهجة حياتي، والى من اثروني على أنفسهم ابي واخوتي....

والى جميع استاذتي الكرام ممن لم يتوانوا في مد يد العون لي فقد على الدوام ملهمي فعلى خطاكم اسير وبعلمكم اقتدي....

أخيراً اهدي هذا البحث إلى كل من يتكبد عناء قرائته سواء لتقييمه او لنقده او لزيادة علمه، فجزاكم الله كل خير واثابكم خير الجزاء.

# الشكر والتقدير

بعد شكري الله عز وجل على اعانتي لانجاز هذا البحث المتواضع.

أتقدم بجزيل الشكر والامتنان الى ا لدكتور الفاضل لؤي عبد الهاني السويدي وأتقدم بجزيل الشكر والامتنان الى الدكتور الفاضل مصطفى حسن هادي، على تفضيلهما بقبول الاشراف على بحثي واسدادهما لى النصائح والارشادات التي كانت بمثابة النبراس المنير في حل خطواتي.

# Contents

# Abstract.

The main objective of our work is to calculate some topologies on the four-point set, then calculate the upper and lower Probability of all sets within the IP(x) corresponding to those topologies, using the internal points and closing points on one side within this path, and the second path calculates the set of internal points and the set of closing points for that Set Semi.

# Introduction.

Topology is a word translated from the english word topology, and the word topology is divided into two syllables, the first syllable (topo) which is of greek origin to topos (which means “place”), and the second syllable is (logy). it is the study of variable groups in which the nature of their contents does not change. this has led some mathematicians and engineers to call it rubber engineering. topology is concerned with the study of spatial properties that are preserved according to the two-continuity distortions (tension without tearing), these properties are usually known as topological inequalities. the foundations of this branch of mathematics were established at the beginning of the twentieth century, and it continued to develop from 1925 to 1975 , when it witnessed its maturation and its formation as an integrated specialty. it can also be said that topology is the science that is concerned with the study of topological properties that move from one place to another through morphology. to understand the meaning of the word isomorphism, it is said of a function that it forms symmetry if it is a continuous function and its inverse form is also continuous, comprehensive and divergent. a topology of modern theories (compositions) in mathematics that originated in the nineteenth century and crystallized during the twentieth century. although his roots extend in geometry and mathematical analysis, he grew independent of them and has now become a tool that serves all mathematics [3].

# Chapter one

# Some Topologies on The Sets Containing Four Elements.

In this chapter, we will show set of basic definitions on which our solutions are based, to a set of questions that we worked on through the four elements, which are {a,b,c,d}, where we made 10 topologies and extracted (interiors and closures ) of Semi open at the end of the research ,we studied the probabilities that are (upper and lower) on the topologies that we knew, in addition to the probabilities on topologies for the Semi open set.

## Definition 1.1 [1]

Let X be a nonempty set and $\tau$ be a family of subsets of X(i.e.,$\tau \subseteq$ IP(X)) . we say $\tau$ is a topology on X if satisfy the following conditions:

1. X, $\emptyset \in \tau$
2. If U , V $\in \tau$ , then U$\cap V \in \tau$

   The finite intersection of elements from $\tau$ is again an element of $\tau$.

3. If $U_a \in \tau\,;\, a \in \wedge\,, then \cup_{a\in\wedge} U_a \in \tau \;\; \forall\alpha \in\wedge$.

   The arbitrary (finite or infinite) union of elements of $\tau$ is again an element of $\tau$.

   We called a pair $(X, \tau)$ topological space.

## Definition 1.2 [4]

Let (X,$\tau$) be a topological space .The subsets of X belonging to $\tau$ are called open set in the space X.i.e.,

If A $\subseteq$ X and A $\in \tau$ *then* A open set.

## Definition 1.3 [4]

Let (X,$\tau$) be a topological space**.** The subset of X is called closed set in the space X if its complement X \A is open set. We will denoted the family of closed sets by Ƒ.i.e., If A⊆ X and A ∈ Ƒ *then* A closed set.

## Definition 1.4 [2]

Let (X,τ) be a topological space and let A⊆X. A point x∈A is called an interior point of A iff there exists an open set U∈τ containing x such that x∈U⊆A.The set of all interior points of A is called the interior of A and is denoted by $A°$ or Int(A). i.e.,

$$A° = \{x \in A: \exists\ U \in \tau;\ x \in U \subseteq A\}$$

$$x \in A° \leftrightarrow \exists\ U \in \tau;\ x \in U \subseteq A\ .$$

## Definition 1.5 [4]

Let $(X, \tau)$ be a topological space and let A be a subset of X . Then the Intersection of all $\tau$-closed containing the set A is called the closure of A and denoted by $\overline{A}$ or Cl(A). i.e., Cl(A)= $A^{c^{\circ c}}$.

## Definition 1.6 [5]

A subset A of a space X is said to be Semi-open if A⊆ cl (int (A)) . And the complement Semi-open is called Semi-closed set.

## Definition 1.7 [2]

Let (X,τ) be a topological space and let A⊆X. A point x∈A is called a semi-interior point of A iff there exists an semi-open set U∈τ containing x such that x∈U⊆A .The set of all semi-interior point of A is called the semi-interior of A and is denoted by semi-$A°$ or semi-Int(A). i.e.,

$$\text{Semi-}A° = \{x \in A : \exists\ U \in \tau ; x \in U \subseteq A\}$$

$$x \in \text{Semi-}A° \leftrightarrow \exists\ U \in \tau ; x \in U \subseteq A.$$

## Definition 1.8 [2]

Let $(X, \tau)$ be a topological space and let A be a subset of X . Then the intersection of all semi-closed containing the set A is called the semi-closure of A and denoted by Semi- $\bar{A}$ or semi-Cl(A). i.e., Semi-Cl(A)=$A^{c^{\circ c}}$.

## Definition 1.9

- $\underline{\rho}(A°) = \underline{\rho}\,(int(A)) = \frac{number\ elements\ of\ A°}{number\ elements\ of\ X}$
- $\overline{\rho}(\overline{A}) = \overline{\rho}(cl(A)) = \frac{number\ elements\ of\ \overline{A}}{number\ elements\ of\ X}$
- Semi. $\underline{P}$(A°) = $\underline{P}$ (Semi. int(A) ) = $\frac{number\ elements\ of\ Semi.int(A)}{number\ elements\ of\ X}$
- Semi. $\bar{\rho}(\overline{A})$ = $\bar{\rho}$ (Semi. $cl(A)$) = $\frac{number\ elements\ \ of\ Semi.cl(A)}{number\ elements\ of\ X}$

## Example 1

$\tau_1$ ={x,∅,{d},{a,b,d}}

$\tau_1{}^c$={x,∅,{c},{a,b,c}}

Semi. $\tau_1$={x, ,∅,{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d} }

Semi. $\tau_1{}^c$ ={x,∅,{a},{b},{ c},{a, b},{a, c},{b, c},{a, b, c} }

| $\tau_1$ | {a} | {b} | {c} | {d} | {a,b} | {a,c} | {a,d} | {b,c} | {b,d} | {c,d} | {a,b,c} | {a,b,d} | {a,c,d} | {b,c,d} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\underline{P}(A^{o})$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| $\overline{P}(\overline{A})$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ |
| **Semi-** $\underline{P}(A^{o})$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ |
| **Semi-** $\overline{P}(\overline{A})$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ |

## Example 2

$\tau_2 = \{X, \emptyset, \{d\}, \{a, c, d\}\}$

$\tau_2{}^c = \{X, \emptyset, \{b\}, \{a, b, c\}\}$

Semi. $\tau_2 = \{X, \emptyset, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

Semi. $\tau_2{}^c = \{x, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

| $\tau_2$ | {a} | {b} | {c} | {d} | {a,b} | {a,c} | {a,d} | {b,c} | {b,d} | {c,d} | {a,b,c} | {a,b,d} | {a,c,d} | {b,c,d} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\underline{P}(A^{o})$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ |
| $\overline{P}(\overline{A})$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ |
| **Semi-** $\underline{P}(A^{o})$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ |
| **Semi-** $\overline{P}(\overline{A})$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ |

## Example 3

$\tau_3 = \{X, \emptyset, \{d\}, \{b, c, d\}\}$

$\tau_3{}^c = \{X, \emptyset, \{a\}, \{a, b, c\}\}$

Semi. $\tau_3$={X,,∅,{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d} }

Semi. $\tau_3{}^c$ ={x,∅,{a},{b},{ c},{a, b},{a, c},{b, c},{a, b, c}}

| $\tau_3$ | {a} | {b} | {c} | {d} | {a,b} | {a,c} | {a,d} | {b,c} | {b,d} | {c,d} | {a,b,c} | {a,b,d} | {a,c,d} | {b,c,d} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\underline{P}(A^o)$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ |
| $\overline{P}(\overline{A})$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ |
| **Semi-** $\underline{P}(A^o)$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ |
| **Semi-** $\overline{P}(\overline{A})$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ |

## Example 4

$\tau_4 = \{X, \emptyset, \{a\}, \{d\}, \{a, d\}\}$

$\tau_4{}^c = \{X, \emptyset, \{b, c\}, \{a, b, c,\}, \{b, c, d\}\}$

Semi. $\tau_4$={X,,$\emptyset$,{a},{d},{a, d},{a, b, d},{a, c, d}}

Semi. $\tau_4{}^c$={x,$\emptyset$,{b},{c},{b, c},{a, b, c},{b, c, d}}

| $\tau_4$ | {a} | {b} | {c} | {d} | {a,b} | {a,c} | {a,d} | {b,c} | {b,d} | {c,d} | {a,b,c} | {a,b,d} | {a,c,d} | {b,c,d} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\underline{P}(A^{o})$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ |
| $\overline{P}(\overline{A})$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ |
| **Semi-** $\underline{P}(A^{o})$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ |
| **Semi-** $\overline{P}(\overline{A})$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ |

## Example 5

$\tau_5 = \{X, \emptyset$,{b}, {d}, {b, d}}

$\tau_5{}^c = \{X, \emptyset$, {a, c}, {a,b, c}, {a,c, d}}

Semi. $\tau_5$={X,,∅,{b}, {d}, {b, d}, {a, b, d}, {b, c, d}}

Semi. $\tau_5{}^c$ ={x,∅,{a}, {c},{a, c},{a, b, c},{a, c, d}}

| $\tau_5$ | {a} | {b} | {c} | {d} | {a,b} | {a,c} | {a,d} | {b,c} | {b,d} | {c,d} | {a,b,c} | {a,b,d} | {a,c,d} | {b,c,d} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\underline{P}(A^{o})$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ |
| $\overline{P(A)}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ |
| **Semi-** $\underline{P}(A^{o})$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ |
| **Semi-** $\overline{P(A)}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ |

## Example 6

$\tau_6=\{X,\emptyset$,{c}, {d}, {c, d}}

$\tau_6{}^c=\{X,\emptyset$, {a, b,}, {a, b,c}, {a, b, d}}

Semi. $\tau_6$={X,,$\emptyset$,{c}, {d}, {c, d}, {a, c, d}, {b, c, d}}

Semi. $\tau_6{}^c$ ={x,$\emptyset$,{a}, {b},{a, b},{a, b, c},{a, b, d}}

| $\tau_6$ | {a} | {b} | {c} | {d} | {a,b} | {a,c} | {a,d} | {b,c} | {b,d} | {c,d} | {a,b,c} | {a,b,d} | {a,c,d} | {b,c,d} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\underline{P}(A^{o})$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ |
| $\overline{P(A)}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ |
| **Semi-** $\underline{P}(A^{o})$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ |
| **Semi-** $\overline{P(A)}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ |

## Example 7

$\tau_7$={X,∅,{d}, {a,b}, {a,b, d}}

$\tau_7{}^c$ ={X,∅, {c}, {c, d}, {a, b, c } }

Semi. $\tau_7$ ={X,,∅,{a},{b},{d}, {a, b} ,{a, d} , {b,d},{a, b, d} ,{a, c, d},{b, c, d}}

Semi. $\tau_7{}^c$ ={x,∅,{a} ,{b} ,{ c} ,{a, c} ,{ b, c} ,{c, d},{a, b, c} ,{a, c, d} {b, c, d} }

| $\tau_7$ | {a} | {b} | {c} | {d} | {a,b} | {a,c} | {a,d} | {b,c} | {b,d} | {c,d} | {a,b,c} | {a,b,d} | {a,c,d} | {b,c,d} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\underline{P}(A^o)$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| $\overline{P}(A)$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ |
| **Semi-** $\underline{P}(A^o)$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ |
| **Semi-** $\overline{P}(A)$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ |

## Example 8

$\tau_8 = \{X, \emptyset$, {d}, {a,c}, {a,c, d}}

$\tau_8{}^c = \{X, \emptyset$, {b}, {b, d}, {a, b, c }}

Semi. $\tau_8$ ={X,,$\emptyset$,{a},{c}, {d} , {a, c} ,{a, d} , {c, d}, {a, b, d} ,{a, c, d},{b, c, d}}

Semi. $\tau_8{}^c$ ={x,$\emptyset$,{a} ,{b} ,{ c} ,{a, b} ,{b, c} ,{b, d},{a, b, c} ,{a, b, d} ,{b, c, d} }

| $\tau_8$ | {a} | {b} | {c} | {d} | {a,b} | {a,c} | {a,d} | {b,c} | {b,d} | {c,d} | {a,b,c} | {a,b,d} | {a,c,d} | {b,c,d} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\underline{P}(A^{o})$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ |
| $\overline{P(A)}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ |
| **Semi-** $\underline{P}(A^{o})$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ |
| **Semi-** $\overline{P(A)}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ |

## Example 9

$\tau_9 = \{X, \emptyset$,{d}, {b,c}, {b,c, d}}

$\tau_9{}^c = \{X, \emptyset$, {a}, {a, d}, {a, b, c }}

Semi. $\tau_9$ ={X,,$\emptyset$,{b},{c}, {d} , {b, c} ,{b, d} , {c, d}, {a, b, d} ,{a, c, d},{b, c, d}}

Semi. $\tau_9{}^c$ ={x,$\emptyset$,{a} ,{b} ,{ c} ,{a, b} ,{a, c} ,{a, d},{a, b, c} ,{a, b, d} ,{b, c, d} }

| $\tau_9$ | {a} | {b} | {c} | {d} | {a,b} | {a,c} | {a,d} | {b,c} | {b,d} | {c,d} | {a,b,c} | {a,b,d} | {a,c,d} | {b,c,d} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\underline{P}(A^{o})$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ |
| $\overline{P(A)}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ |
| **Semi-** $\underline{P}(A^{o})$ | $\frac{0}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ |
| **Semi-** $\overline{P(A)}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ |

## Example 10

$\tau_{10} = \{X, \emptyset$,{a}, {a,b},{a,d}, {a,b, d}}

$\tau_{10}{}^{c} = \{X, \emptyset$, {c}, {b,c}, {c, d}, {b, c,d }}

Semi. $\tau_{10}$ = {X,,$\emptyset$,{a},{a,b}, {a,c} , {a, d} ,{b, d} , {a, b, c} ,{a, b, d},{a, c, d}}

Semi. $\tau_{10}{}^{c}$ ={x,$\emptyset$, {b} ,{c} ,{d} ,{a, c} ,{b, c} ,{b, d},{c, d} ,{b, c, d} }

| $\tau_{10}$ | {a} | {b} | {c} | {d} | {a,b} | {a,c} | {a,d} | {b,c} | {b,d} | {c,d} | {a,b,c} | {a,b,d} | {a,c,d} | {b,c,d} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\underline{P}(A^{o})$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ |
| $P(\overline{A})$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ |
| Semi-$\underline{P}(A^{o})$ | $\frac{1}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{0}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ | $\frac{2}{4}$ | $\frac{0}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{2}{4}$ |
| Semi-$P(\overline{A})$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{4}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{2}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{4}{4}$ | $\frac{3}{4}$ |

# References


1. M.Ganster, Kyungpook Math.J.27(2) (1987).135-43.
2. Margaret H. Wright. The Interior-Point Revolution in Optimization: History, Recent Developments, And Lasting Consequences. Bull. Amer. Math. Soc. (N.S, 42:39,56, 2005.
3. S. Willard, General Topology, Addison Wesley Publishing Company, Inc, Usa, 1970.
4. Sidney A. Morrl"Topology Without Tears" Verion of Febrouary. 20,2012.
5. T. Noiri and V. Popa, Almost weakly continuous multifunctions, Demonstrati Math., 26 (1993), 363–380.