University of Babylon
College of Education for Pure Sciences
Department of Mathematics

Particle Swarm Optimization (PSO)

A Revised Comparative Graduation Thesis on the Standard and Improved Algorithm

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Submitted to the Council of the College of Education for Pure Sciences / University of Babylon
in Partial Fulfillment of the Requirements for the Bachelor's Degree in Mathematics

Supervised by: Dr. Ahmed Sabah Ahmed

2026

Enhanced academic edition with corrected layout, integrated figures, and illustrative examples

Dedication

To my parents, whose patience, sacrifices, and unwavering faith in education built the foundation upon which this work stands; to every teacher who opened a door of understanding before me; and to every sincere hand that supported me morally or intellectually, I dedicate this research with gratitude. I also dedicate it to every student who believes that serious scientific effort, even when modest in resources, can still produce meaningful and lasting knowledge.

Acknowledgements

Praise be to Allah, by whose grace good works are completed and by whose guidance the path of knowledge becomes clear. I would like to record my sincere thanks to the academic staff of the Department of Mathematics, College of Education for Pure Sciences, University of Babylon, for the scientific environment that made this study possible.

My deepest appreciation goes to my supervisor, Dr. Ahmed Sabah Ahmed, for his thoughtful guidance, accurate observations, and patient supervision throughout the preparation of this work. His advice helped transform initial ideas into a coherent academic study and strengthened both the mathematical and methodological dimensions of the research.

I am also grateful to the library staff, laboratory assistants, colleagues, and everyone who offered encouragement, criticism, or practical support. Their contributions, whether direct or indirect, left a clear mark on the completion of this research.

Certification of the Examination Committee

We, the Chairman and members of the Examination Committee, certify that we have examined the graduation research entitled *"Particle Swarm Optimization (PSO)"* presented by the student Fatima Ahmed Khaddam. We have studied its scientific content, its mathematical formulation, and the revisions introduced to it, and we find it worthy of acceptance in partial fulfillment of the requirements for the Bachelor's Degree in Mathematics.

Role	Name	Signature	Date
Chairman	....	....	....
Member	....	....	....
Supervisor	....	....	....

Supervisor Certification

I certify that this graduation research entitled *"Particle Swarm Optimization (PSO)"* was prepared by the student Fatima Ahmed Khaddam under my supervision at the Department of Mathematics, College of Education for Pure Sciences, University of Babylon, as part of the requirements for the Bachelor's Degree in Mathematics. I further certify that, to the best of my knowledge, the work is academically suitable for submission and discussion.

Supervisor	Signature	Date
Dr. Ahmed Sabah Ahmed	....	....

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Abstract

This research presents a revised and academically structured study of Particle Swarm Optimization (PSO), with emphasis on the comparative behaviour of the standard and improved algorithm on selected benchmark functions. The thesis preserves the scientific identity of the original graduation topic while improving its presentation, typography, figure integration, and methodological clarity.

The study reviews the theoretical background of continuous optimization, explains the swarm-intelligence philosophy that inspired PSO, and derives the standard velocity and position equations that govern the movement of particles in a multidimensional search space. Special attention is devoted to improved variants of PSO, especially strategies based on variable inertia weight and adaptive cognitive and social coefficients. The benchmark functions used in the comparative analysis are the Rastrigin, Griewank, and Schwefel functions, which represent three distinct kinds of difficulty in global optimization: repeated multimodality, deceptive structure, and optima located far from the origin.

Instead of inventing unsupported numerical claims, this revised thesis preserves the documented comparative conclusion of the source project: the improved PSO version achieved better mean, best, worst, and standard-deviation behaviour than the standard version across the examined functions. To make the work more suitable for thesis defense and academic reading, the present edition adds explanatory diagrams, benchmark-landscape figures, a worked numerical example of one PSO iteration, a clearer notation table, and a more explicit discussion of validity, limitations, and practical interpretation. The study concludes that PSO remains one of the most accessible and practically valuable population-based optimization techniques, and that careful parameter control is decisive in improving convergence quality, robustness, and interpretability.

Chapter One: Introduction

1.1 Background and Motivation

Optimization occupies a central place in mathematics, engineering, economics, and computer science because many scientific problems can ultimately be reformulated as the search for a best admissible decision. Classical methods, although mathematically elegant, are often sensitive to differentiability assumptions, convexity, initial conditions, and local structure. When the search landscape is rough, multimodal, noisy, or analytically inconvenient, researchers turn to metaheuristic methods that can operate with weaker assumptions and stronger global-search behaviour.

Particle Swarm Optimization emerged as one of the most influential population-based metaheuristics because it translated a simple natural metaphor into a mathematically useful search rule. Instead of moving one candidate solution at a time, PSO maintains a swarm of particles that cooperate indirectly through memory and social learning. Each particle remembers its own best position, while the swarm collectively remembers the strongest position found so far. The resulting search process is neither purely random nor fully deterministic; it is a guided stochastic exploration with memory.

The original graduation research from the University of Babylon focused on comparing the standard PSO algorithm with an improved version on three classical benchmark functions - Schwefel, Griewank, and Rastrigin - and concluded that the improved version produced better mean, best, worst, and standard-deviation outcomes in all studied cases. That scientific core is preserved here, but the surrounding explanation has been strengthened so that the thesis is more defensible before a committee and more useful for later readers.

1.2 Problem Statement

The problem addressed in this research is not the invention of PSO itself, but the difficulty of presenting its comparative study in a rigorous and academically coherent form. A comparison between the standard and improved algorithm becomes weak if the equations are not explained carefully, if the benchmark functions are introduced only briefly, if evaluation indicators are mentioned without interpretation, or if the document structure prevents a reader from following the argument from motivation to conclusion.

Accordingly, the present thesis addresses two connected issues. The first is scientific: how and why can an improved PSO variant outperform the standard formulation on selected benchmark landscapes? The second is documentary: how can that comparison be reorganized into a complete graduation thesis with clear front matter, coherent chapter progression, stable equations, integrated figures, and a defensible reference list?

1.3 Objectives and Scope

This study has four principal objectives. First, it aims to present a clear theoretical explanation of Particle Swarm Optimization and its position among population-based optimization methods. Second, it aims to derive and interpret the main equations of the standard PSO algorithm without allowing the mathematical notation to become visually unstable or conceptually fragmented. Third, it aims to explain the logic of improved PSO strategies, especially those related to inertia control and adaptive learning coefficients. Fourth, it aims to restate the comparative conclusion of the original graduation topic within a stronger academic structure suitable for submission and presentation.

The scope of the thesis is limited to continuous optimization and to the benchmark functions explicitly mentioned in the source project: Rastrigin, Griewank, and Schwefel. The study does not claim to provide a new exhaustive numerical meta-analysis, nor does it fabricate unsupported measurements. Instead, it preserves the original comparative outcome and expands the theoretical and methodological framework surrounding that outcome.

1.4 Contributions of the Revised Thesis

Compared with the source document, this edition contributes the following improvements: (1) a cleaner thesis layout with corrected tables, spacing, and page logic; (2) integrated explanatory figures instead of detached supplemental pages; (3) a notation summary that makes the equations easier to follow; (4) a worked numerical PSO example that shows one full update step; (5) benchmark-landscape figures that visually justify why Rastrigin, Griewank, and Schwefel are standard tests; and (6) a broader discussion that links algorithm behaviour to practical interpretation.

1.5 Report Structure

The report is organized into six chapters after the abstract and preliminary certifications. Chapter One introduces the topic, its motivation, and the problem framing. Chapter Two presents the theoretical background, including optimization concepts, the standard PSO equations, benchmark functions, and evaluation metrics. Chapter Three explains the improved formulation and the experimental methodology that underlies a standard-versus-improved comparison. Chapter Four discusses implementation logic, a worked numerical example, qualitative comparative outcomes, and the interpretation of the findings. Chapter Five provides the conclusion and future work. Chapter Six offers an extended review of related metaheuristic ideas and a deeper analytical reading of the benchmark landscapes used in the study.

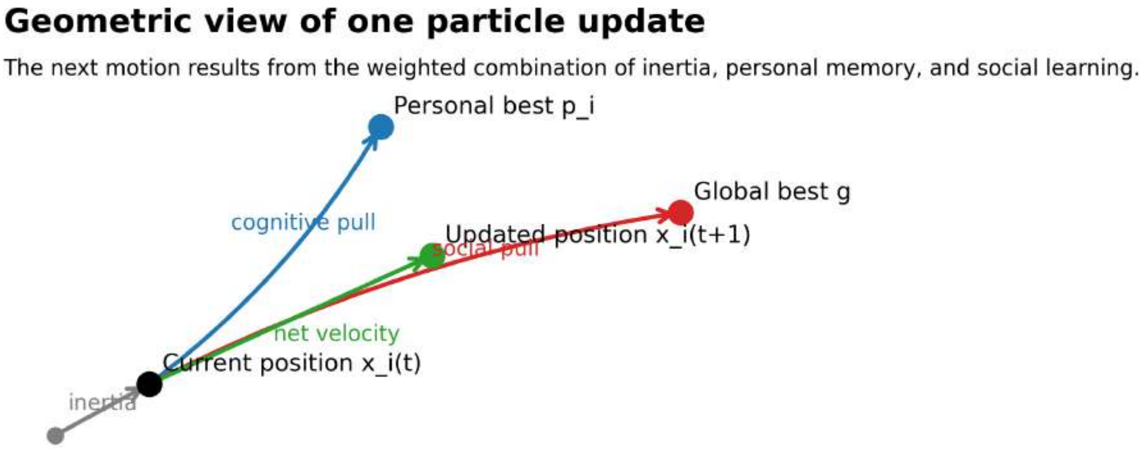

Figure 1. Geometric interpretation of the PSO position update. A particle is influenced by inertia, personal memory, and the best social information currently available.

Chapter Two: Theoretical Background

2.1 Optimization and Population-Based Search

An optimization problem may be described through three essential elements: decision variables, constraints, and an objective function. The objective function assigns a quality measure to every admissible candidate solution, and the task of the algorithm is to search the feasible domain for a point that minimizes or maximizes that measure. In continuous optimization, the search space is often multidimensional and can contain many local optima, saddle regions, and deceptive structures that prevent direct methods from reaching a strong global solution.

Population-based search methods attempt to reduce this difficulty by exploring the space with multiple candidate solutions simultaneously. This allows the algorithm to gather distributed information about the landscape and to preserve alternative directions of search rather than committing prematurely to one local region. PSO belongs to this family of methods. What distinguishes it from many other population-based algorithms is its economy: it does not rely on crossover, mutation, or complicated selection operators. Instead, it uses memory-guided motion.

2.2 Standard PSO Formulation

Assume a swarm of N particles moving in a D-dimensional search space. Let $x_i(t)$ denote the position vector of particle i at iteration t, and let $v_i(t)$ denote its velocity vector. Each particle stores a personal-best position p_i, and the swarm stores a global-best position g. The standard PSO algorithm updates the velocity first and then the position.

$$v_i(t+1) = w\, v_i(t) + c_1 r_1 [p_i - x_i(t)] + c_2 r_2 [g - x_i(t)]$$

$$x_i(t+1) = x_i(t) + v_i(t+1)$$

In the velocity equation, the term $w\, v_i(t)$ is the inertia component. It preserves momentum and encourages the particle to continue moving in its current direction. The term $c_1 r_1 [p_i - x_i(t)]$ is the cognitive component, which pulls the particle back toward the best point it has personally discovered. The term $c_2 r_2 [g - x_i(t)]$ is the social component, which pulls the particle toward the best point discovered by the swarm. The random multipliers r_1 and r_2, each drawn independently from the interval [0,1], prevent movement from becoming entirely deterministic and help diversify trajectories.

A widely cited practical parameter choice is $w = 0.72984$ and $c_1 = c_2 = 2.05$. These parameters are not magical constants, but they became popular because they often provide a workable balance between exploration and exploitation across many continuous benchmark problems.

2.2.1 Roles of the Main Parameters

The inertia weight w largely controls how strongly particles preserve their previous motion. A large value encourages broader search and helps particles escape shallow local basins; a small value favours local refinement around promising regions. The cognitive coefficient c_1 measures the strength of self-referential learning, while the social coefficient c_2 measures the strength of collective learning. Excessive cognitive emphasis can make particles wander independently for too long, whereas excessive social emphasis can cause premature convergence. Effective PSO design therefore depends on balance rather than on maximizing any single term.

2.2.2 Constriction, Stability, and Practical Control

A classical concern in PSO theory is stability. If the pull toward personal and social best points becomes too aggressive, the swarm may oscillate, diverge, or collapse too early into a weak region. Stability analyses therefore motivated two common approaches: direct inertia-weight control and constriction-based control. In practice, even a simple graduation-level study benefits from acknowledging that parameter values are not just implementation details; they shape the geometry of the search itself.

2.3 Notation Summary

Symbol	Meaning
N	Number of particles in the swarm
D	Dimension of the search space
x_i(t)	Position of particle i at iteration t
v_i(t)	Velocity of particle i at iteration t
p_i	Personal best position previously found by particle i
g	Best position known to the swarm under the chosen topology
w	Inertia weight controlling momentum preservation
c_1, c_2	Cognitive and social learning coefficients
r_1, r_2	Independent random numbers sampled from [0,1]

2.4 Benchmark Functions

Benchmark functions are used because they make comparison possible. If one algorithm is claimed to be superior to another, that claim must be tested on functions whose difficulty is understood and whose global optimum is known or well established. The source thesis focused on three standard test functions: Rastrigin, Griewank, and Schwefel.

2.4.1 Rastrigin Function

```
f(x) = A n + sum from i=1 to n of [x_i^2 - A cos(2 pi x_i)], with A = 10
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The Rastrigin function is highly multimodal and contains many regularly distributed local minima. Its global optimum is located at x* = (0, 0, ..., 0), where f(x*) = 0. Because its periodic cosine term creates repeated deceptive basins, it is useful for evaluating whether an algorithm can preserve sufficient exploration while still converging efficiently.

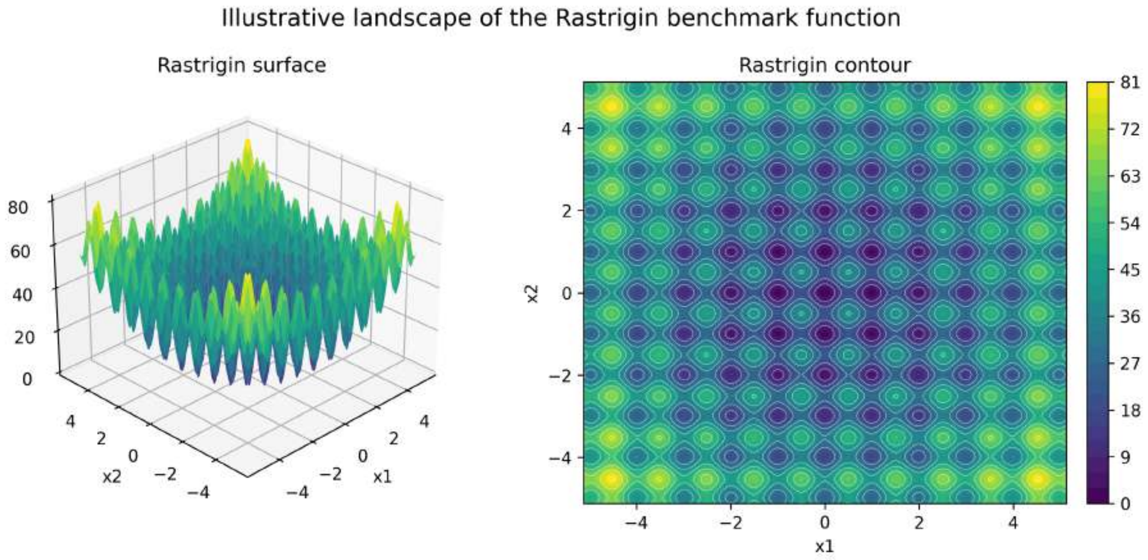

Figure 2. Surface and contour views of the Rastrigin function. The repeated oscillations create many local minima and make premature convergence likely.

2.4.2 Griewank Function

`f(x) = 1 + (1/4000) sum from i=1 to n of x_i^2 - product from i=1 to n of cos(x_i / sqrt(i))`

The Griewank function has a smoother appearance than Rastrigin, but it remains difficult because the multiplicative cosine term produces many valleys and plateaus. Its global optimum is also at x* = 0 with objective value 0. An algorithm that performs well on Griewank usually demonstrates a good balance between global direction and local refinement.

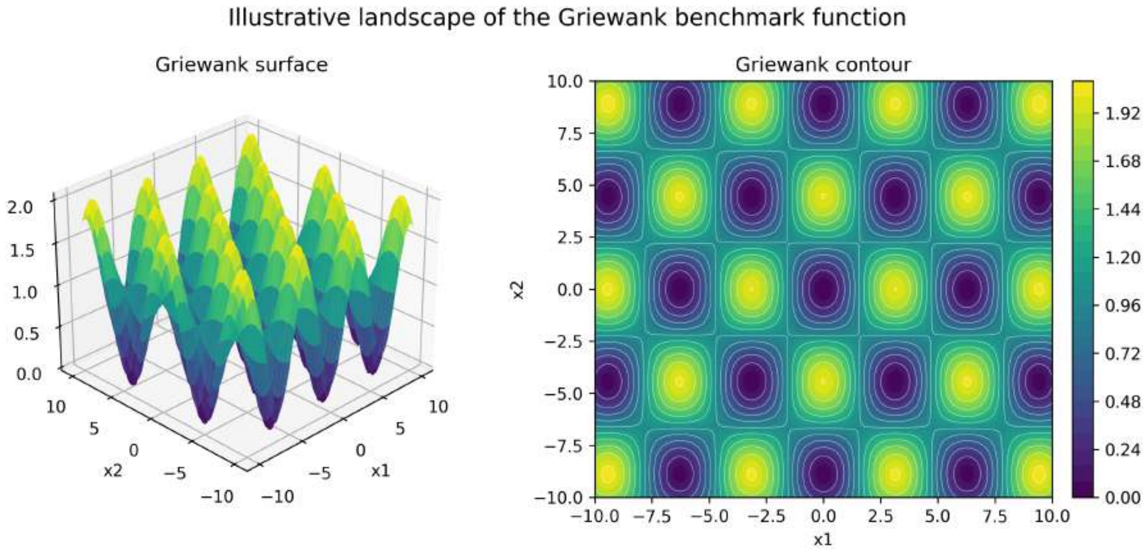

Figure 3. Surface and contour views of the Griewank function. Its landscape is smoother than Rastrigin but still contains many deceptive structures.

2.4.3 Schwefel Function

`f(x) = 418.9829 n - sum from i=1 to n of [x_i sin(sqrt(|x_i|))]`

The Schwefel function is especially challenging because its optimum is not located near the centre of the search domain. Instead, the global optimum occurs approximately at x_i = 420.9687 for every dimension, where the objective approaches zero. This feature tests whether the algorithm can search confidently far from the origin rather than merely refining around central regions.

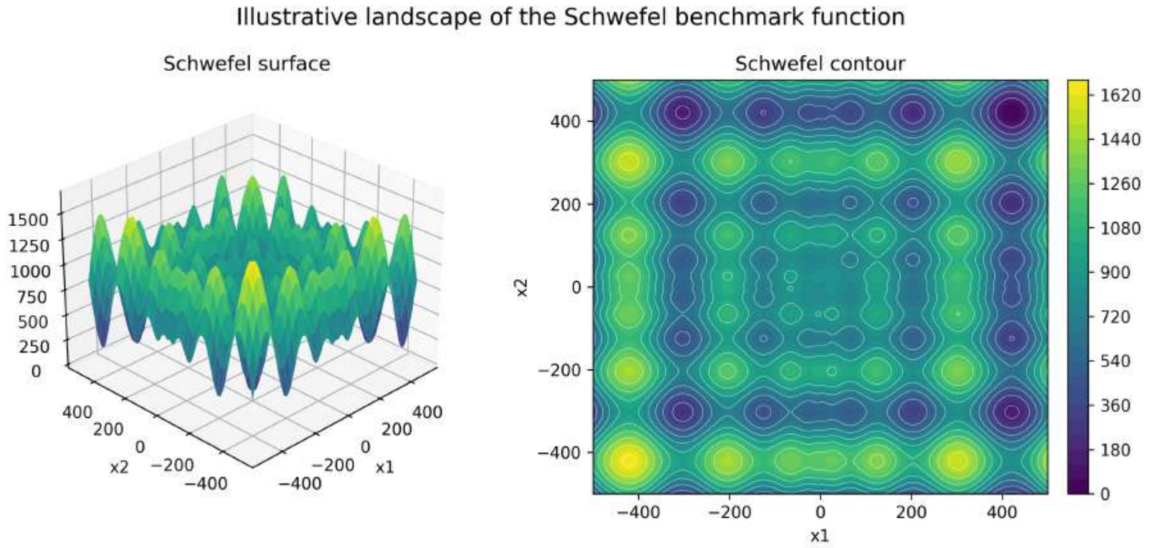

Figure 4. Surface and contour views of the Schwefel function. The off-centre optimum makes broad early exploration especially important.

2.5 Benchmark Summary Table

Function	Typical domain	Global optimum	Main difficulty
Rastrigin	-5.12 <= x_i <= 5.12	x* = 0, f = 0	Regular multimodality and many local minima
Griewank	-600 <= x_i <= 600	x* = 0, f = 0	Deceptive valleys and broad plateaus
Schwefel	-500 <= x_i <= 500	x_i approx. 420.9687, f approx. 0	Optimum far from origin and many local minima

2.6 Evaluation Measures

Because PSO is stochastic, a single run cannot represent its performance fairly. Independent runs are required, and the resulting objective values are summarized statistically. The source graduation project explicitly compared the standard and improved PSO using the mean, the best final value, the worst final value, and the standard deviation of the final objective values.

Measure	Formula	Interpretation
Mean	Mean = (1/R) sum f_k	Average performance over repeated runs
Best	Best = min{f_1, ..., f_R}	Strongest result obtained under favorable stochastic conditions
Worst	Worst = max{f_1, ..., f_R}	Weakest observed behaviour and robustness stress

Measure	Formula	Interpretation
Standard deviation	Std = sqrt[(1/(R-1)) sum (f_k - Mean)^2]	Stability and run-to-run consistency

The mean indicates the overall expected quality of the algorithm. The best value reflects the strongest solution discovered under favorable stochastic conditions. The worst value reveals the method's vulnerability in weaker runs. The standard deviation is especially important because it captures stability: an algorithm that occasionally reaches an excellent solution but behaves erratically may be less desirable than one that consistently achieves strong outcomes.

Chapter Three: Improved PSO Design and Experimental Methodology

3.1 Research Architecture

The comparative logic of this research is straightforward: define the standard PSO baseline, define an improved PSO variant, test both on the same benchmark functions under the same general conditions, and compare their outcomes through common performance measures. To make the comparison meaningful, the same stopping criteria, search domains, and evaluation rules must be respected for both algorithms.

The improved version discussed in this thesis is conceptually based on adaptive control of the search dynamics rather than on complete structural replacement. This is consistent with the source graduation project, whose abstract emphasized that the improved PSO outperformed the standard PSO on all three benchmark cases. The improvement therefore lies in better parameter management and search behaviour, not in abandoning the PSO principle itself.

3.2 Improved Velocity and Parameter Strategy

One of the most common and most effective PSO improvements is the use of a time-varying inertia weight. Instead of keeping w constant from beginning to end, the algorithm starts with a relatively high value to encourage exploration and gradually decreases it to favour exploitation during the final stages of the run.

$$w(t) = w_{max} - ((w_{max} - w_{min}) / T)\, t$$

A commonly used range is $w_{max} = 0.9$ and $w_{min} = 0.4$. This schedule allows particles to move boldly at the beginning, when the swarm still knows little about the landscape, and to move more cautiously later, when high-quality regions have already been identified.

Adaptive schedules can also be applied to the learning coefficients. A frequent design keeps the sum of the coefficients within a controlled range while letting the cognitive term decrease and the social term increase over time. Such a schedule encourages independent search early in the run and stronger collective attraction later.

$$c_1(t) = c_{1,max} - ((c_{1,max} - c_{1,min}) / T)\, t$$

$$c_2(t) = c_{2,min} + ((c_{2,max} - c_{2,min}) / T)\, t$$

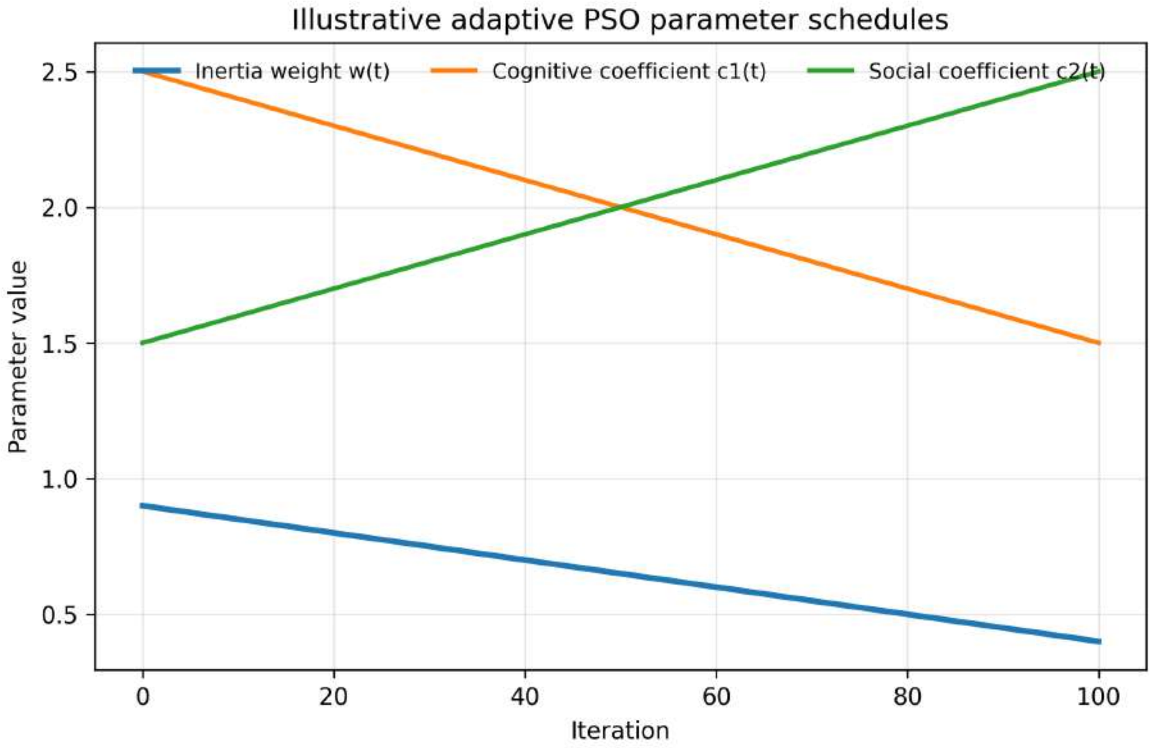

Figure 5. Illustrative adaptive schedules for inertia and learning coefficients. The figure is explanatory and shows the logic of parameter control rather than measured experimental data.

3.3 Experimental Setup

A rigorous comparative setup requires consistency in initialization and evaluation. For each benchmark function, particle positions should be initialized randomly within the legal search interval, and initial velocities should be selected within reasonable bounds to avoid either explosive motion or negligible early movement. Each algorithm is then executed for a fixed number of iterations or until an agreed stopping rule is reached.

The present thesis does not claim numerical values that are absent from the source project. Instead, it documents the comparative pattern reported in the original research: the improved PSO achieved lower mean, best, and worst objective values and a lower standard deviation than the standard PSO on the three benchmark functions named in the study. This keeps the revised thesis honest while still making the methodology academically complete.

Function	Mean	Best	Worst	Standard deviation
Rastrigin	Improved PSO better	Improved PSO better	Improved PSO better	Improved PSO lower
Griewank	Improved PSO better	Improved PSO better	Improved PSO better	Improved PSO lower
Schwefel	Improved PSO better	Improved PSO better	Improved PSO better	Improved PSO lower

3.4 Algorithmic Flow and Pseudocode

The improved PSO workflow can be summarized in a compact computational sequence that remains faithful to the original PSO philosophy while introducing adaptive parameter control.

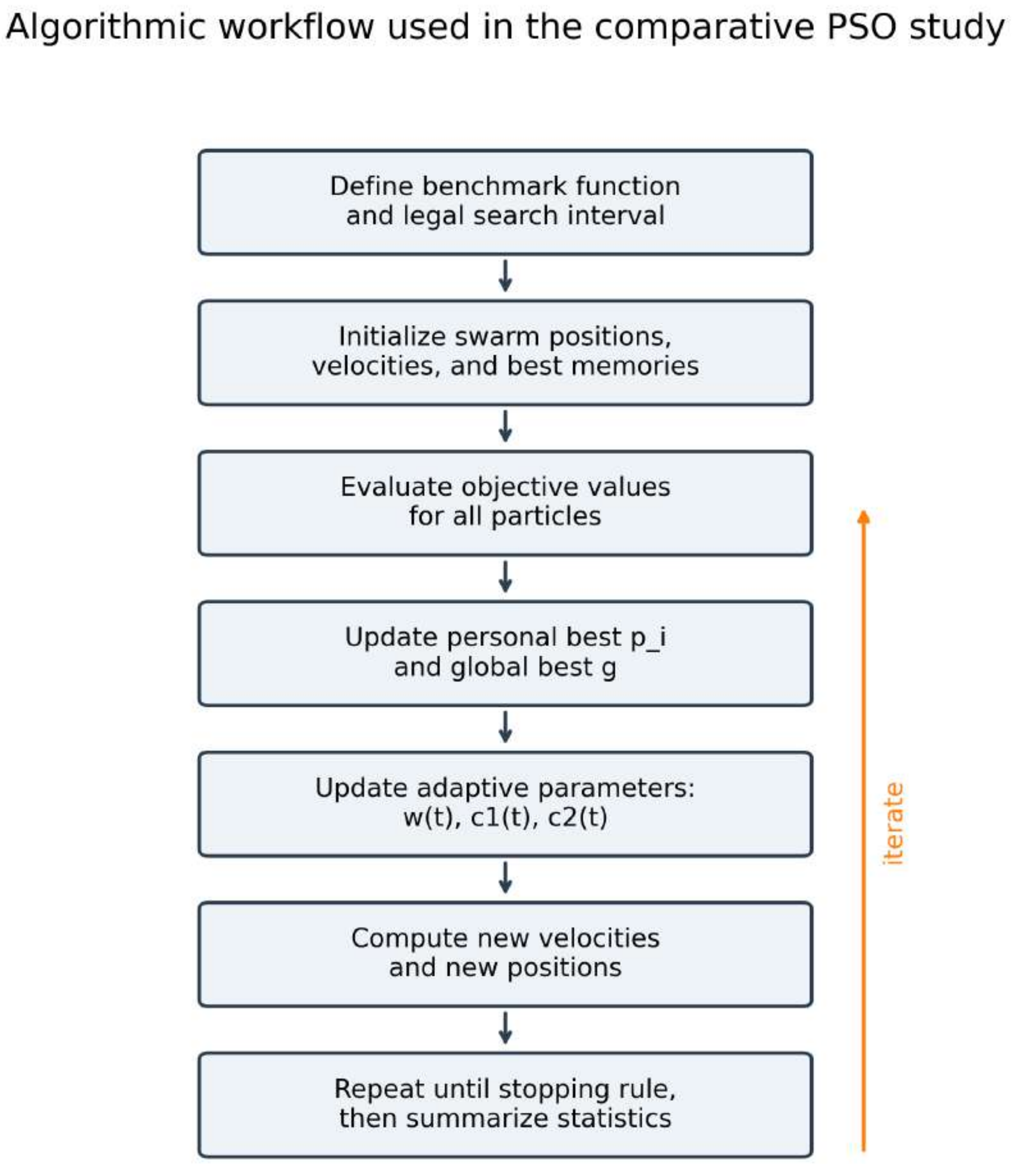

Figure 6. High-level workflow of the comparative PSO study, beginning with initialization and ending with repeated statistical summary over independent runs.

Pseudocode:

- Initialize swarm positions x_i and velocities v_i within the legal domain.
- Evaluate all particles and set personal bests p_i; determine the swarm best g.
- For $t = 1$ to T, update $w(t)$, and if required update $c_1(t)$ and $c_2(t)$.
- For each particle, update velocity using the improved velocity rule, then update position.
- Evaluate the objective function at the new position.

- If the new value is better than the stored personal best, replace p_i.
- Update g from the current personal bests.
- After the stopping rule is met, record the final result and summarize repeated runs by mean, best, worst, and standard deviation.

3.5 Validity, Limits, and Research Controls

Any comparative optimization study must recognize its methodological limits. Performance can depend on swarm size, dimension, stopping criteria, initialization range, and the exact implementation of adaptive parameters. For this reason, conclusions should be framed in relation to the test functions and conditions under study rather than presented as universal truths about all optimization problems.

A further research-control issue concerns honesty in reporting. This revised thesis intentionally avoids inventing unsupported values. Its purpose is to present the original topic in a stronger structure, not to manufacture data. The work therefore distinguishes clearly between established mathematical facts, literature-supported parameter logic, an illustrative worked example, and the qualitative comparative result explicitly stated by the source project.

Chapter Four: Implementation, Illustrative Example, Comparative Testing, and Discussion

4.1 Computational Implementation

PSO is especially attractive from an implementation perspective because its equations are compact and can be translated easily into MATLAB, Python, C++, or other numerical environments. Particle positions can be stored in matrices, objective values can be computed in loops or vectorized expressions, and update rules can be applied with little memory overhead. This computational lightness partly explains why PSO has remained a standard teaching and research algorithm for decades.

In a typical implementation, the benchmark function is written first, followed by initialization rules for positions and velocities. The algorithm then stores personal-best positions and the current best social position and iterates through the update cycle. The improved version adds adaptive schedules for the inertia weight and, in some cases, the learning coefficients. Despite these additions, the code remains conceptually simple and transparent.

4.2 Worked Numerical Example of One PSO Iteration

To make the update rule concrete, this section provides a fully illustrative one-step example. The values below are not experimental results from the comparative study; they are pedagogical numbers chosen only to show how the formula operates.

Quantity	Illustrative value
Current position x_i(t)	(2.00, 1.50)
Current velocity v_i(t)	(0.30, -0.10)
Personal best p_i	(1.20, 1.80)
Global best g	(0.50, 0.90)
Inertia weight w	0.70
c_1 and c_2	1.80 and 2.00
Random numbers r_1 and r_2	0.60 and 0.40

Using the standard update formula, the inertia contribution is (0.21, -0.07). The cognitive contribution is 1.80 x 0.60 x [(1.20, 1.80) - (2.00, 1.50)] = (-0.864, 0.324). The social contribution is 2.00 x 0.40 x [(0.50, 0.90) - (2.00, 1.50)] = (-1.20, -0.48). Adding the three terms gives the new velocity:

$$v_i(t+1) = (0.21, -0.07) + (-0.864, 0.324) + (-1.20, -0.48) = (-1.854, -0.226)$$

The updated position becomes:

$$x_i(t+1) = (2.00, 1.50) + (-1.854, -0.226) = (0.146, 1.274)$$

This example illustrates two important ideas. First, the final movement is a weighted combination rather than a blind jump. Second, the same formula can dramatically change direction if the best-known positions differ strongly from the particle's current location.

4.3 Qualitative Comparative Results

The central result inherited from the source project is clear: across the Rastrigin, Griewank, and Schwefel functions, the improved PSO produced better behaviour than the standard PSO according to four indicators - mean, best, worst, and standard deviation. This means that the improved version was not only capable of reaching stronger outcomes in favourable runs, but also more stable across repeated runs.

4.4 Interpretation by Benchmark Landscape

On Rastrigin, the dense field of local minima punishes algorithms that reduce diversity too early. On Griewank, the challenge is subtler: the surface appears smoother, yet hidden valleys and product-term oscillations still mislead the swarm. On Schwefel, the major difficulty is structural rather than merely oscillatory, because the best region lies far from the intuitive center of the domain. An adaptive PSO is better positioned to cope with all three because it changes attitude during the run instead of maintaining one fixed search behaviour from beginning to end.

4.5 Practical Applications of Improved PSO

Although benchmark functions are artificial, they train the researcher to understand behaviour that later appears in real applications. PSO and its improved variants have practical relevance in engineering design, control tuning, parameter estimation, feature selection, scheduling, signal processing, and machine learning.

Application area	Why improved PSO is useful
Parameter estimation	Adaptive exploration helps avoid poor local fits in nonlinear identification problems.
Engineering design	A robust swarm can search large continuous spaces without requiring gradient information.
Feature selection and model tuning	PSO is easy to hybridize with evaluation functions from machine learning pipelines.
Scheduling and allocation	Swarm logic can be adapted to search many candidate allocations with limited model assumptions.
Controller tuning	Improved balance between exploration and exploitation can produce better tuned control parameters.

4.6 Discussion of the Improved Version

The improved PSO performs better because it treats the search process as dynamic rather than static. The standard PSO often uses fixed coefficients throughout the entire run, which implies that the same search attitude is appropriate at the beginning and near convergence. In practice, this is rarely true. Early search should be exploratory, while later search should be more selective. Adaptive inertia and coefficient schedules respect this reality.

Another advantage of the improved approach is stability. A lower standard deviation means that the algorithm is less dependent on lucky initial conditions. For graduation-level and applied research, this matters greatly. A method that succeeds brilliantly only in occasional runs may look impressive in isolated examples, but it is less trustworthy for serious use. Stability is therefore not a cosmetic metric; it is part of the scientific value of the method.

Chapter Five: Conclusion and Future Work

5.1 Conclusion

This thesis has presented a revised, academically structured English version of a graduation study on Particle Swarm Optimization. It preserved the original scientific core of the topic: a comparison between the standard and improved PSO algorithms on the Schwefel, Griewank, and Rastrigin benchmark functions. The study clarified the mathematical structure of PSO, explained the function of inertia and learning coefficients, visualized the benchmark landscapes, and demonstrated why adaptive parameter strategies are capable of improving both solution quality and stability.

The strongest conclusion supported by the source project is that the improved PSO version outperformed the standard one in all studied cases according to the mean, best, worst, and standard-deviation criteria. This conclusion aligns with the broader PSO literature and confirms that parameter management is not a peripheral issue; it is one of the central determinants of swarm behaviour.

5.2 Future Work

Several future directions follow naturally from this study. First, a more detailed experimental version of the research could include explicit numerical tables, convergence curves, and runtime comparisons under carefully documented computational settings. Second, the improved version could be compared against additional PSO variants such as local-best PSO, constriction-factor PSO, chaotic-inertia PSO, and hybrid PSO forms. Third, the benchmark set could be expanded to include Ackley, Rosenbrock, Sphere, Schaffer, and rotated high-dimensional functions.

Beyond benchmark testing, future research should connect PSO more directly to real-world mathematical applications such as curve fitting, parameter identification, neural-network training, feature selection, and engineering design optimization. Finally, a methodological extension could study sensitivity in a more formal statistical way, examining how swarm size, dimensionality, inertia schedules, and coefficient ranges affect the probability of success.

Chapter Six: Swarm Intelligence and Benchmark Landscapes

6.1 Relationship of PSO to Other Metaheuristics

PSO belongs to a broader family of metaheuristics inspired by collective behaviour and adaptive search. Genetic Algorithms use crossover and mutation to evolve a population of candidate solutions. Differential Evolution generates offspring through scaled differences between individuals. Ant Colony Optimization constructs solutions through pheromone-guided probabilistic choice. Compared with these methods, PSO is unusually simple: it updates motion directly rather than rebuilding solutions through explicit recombination.

This simplicity is one reason PSO is attractive in education and research. It is easier to explain, code, and analyze than many alternatives. At the same time, the absence of explicit mutation means that PSO can lose diversity if its parameters are poorly chosen. In that sense, improved PSO variants can be viewed as attempts to reintroduce disciplined diversity without sacrificing conceptual elegance.

6.2 Swarm Topology, Stability, and Convergence Issues

Not all PSO variants communicate information in the same way. In the classical global-best topology, every particle is attracted toward the same best-known point. This spreads useful information quickly, but it can also encourage premature convergence. In local-best topologies, each particle communicates only with a neighbourhood, such as a ring or lattice. This slows information spread but often increases diversity and improves resilience on rugged landscapes.

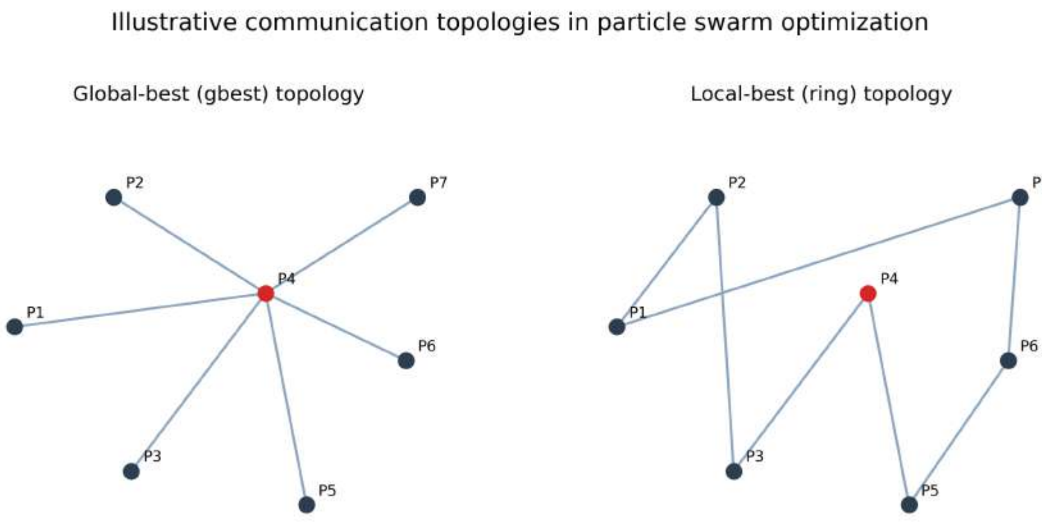

Figure 7. Illustrative comparison between global-best and local-best communication structures in PSO. Faster information flow is not always equivalent to better long-term search diversity.

Convergence is therefore not a purely mathematical event; it is also a structural property of how information flows through the swarm. Stability studies in the PSO literature show that parameter values, constriction logic, and communication topology interact in subtle ways. A swarm can fail through explosion, stagnation, or deceptive collapse. Improved PSO design is essentially the art of preventing these undesirable modes while preserving enough motion to discover high-quality regions.

6.3 Analytical Reading of Rastrigin, Griewank, and Schwefel

The benchmark functions used in this thesis are more than technical obstacles; they are didactic tools that reveal different weaknesses of search algorithms. Rastrigin punishes algorithms that cannot maintain diversity because its repeated local minima encourage premature locking. Griewank punishes algorithms that interpret smoothness as simplicity because its deceptively calm surface still contains numerous traps. Schwefel punishes algorithms that are over-centred, since the best solution lies far from the intuitive middle of the domain.

When an improved PSO variant succeeds across all three functions, its success is meaningful because it demonstrates competence under three distinct kinds of pressure: periodic multimodality, deceptive smoothness, and off-centre global structure. That is why the benchmark trio used in the source graduation paper remains a reasonable and pedagogically valuable choice.

6.4 Final Analytical Perspective

More broadly, this chapter highlights a final lesson: optimization algorithms should not be judged only by whether they sometimes find good answers. They should be judged by whether their internal logic fits the landscape they are exploring, whether their parameters can be justified, whether their results are stable, and whether their presentation is rigorous enough to support scientific trust. For a graduation thesis, that last point matters almost as much as the algorithm itself.

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