جمهورية العراق
وزارة التعليم العالي و البحث العلمي
جامعة بابل
كلية التربية للعلوم الصرفة
قسم الرياضيات

Rough approximation probability of almost open sets

بحث تقدمت به الطالبه

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الى مجلس قسم الرياضيات جامعة بابل وهو جزء من متطلبات نيل شهادة البكالوريوس في العلوم الصرفه (الرياضيات)

باشراف الاستاذ الدكتور

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٢٠٢٦م ١٤٤٧هـ

بسم الله الرحمن الرحيم

يَرْفَعِ اللَّهُ الَّذِينَ آمَنُوا مِنكُمْ وَالَّذِينَ أُوتُوا الْعِلْمَ دَرَجَاتٍ

(سورة المجادلة: ١١)

الاهداء

إلى من كانت دعواتها تسبقني في كل خطوة،
إلى روحي الغالية التي غابت عن عيني ولم تغب عن قلبي،
إلى أمي الحبيبة (رحمها الله)، أهدي هذا الجهد المتواضع،
سائلاً الله أن يجعل ما قدمتُه نوراً يصلها ويُسعد روحها.
إلى أهلي الكرام، سندي في الحياة،
الذين كانوا ولا زالوا مصدر قوتي ودعمي،
لكم مني كل الحب والامتنان.
إلى أساتذتي الأفاضل،
الذين أناروا لي طريق العلم والمعرفة،
وأفاضوا عليّ من علمهم وخبرتهم،
كل الشكر والتقدير لكم.
إلى أصدقائي
الذين شاركوني اللحظات الجميلة والصعبة،
وكانوا عوناً لي في مسيرتي،
أهديكم هذا العمل

شكر و امتنان

الحمد لله الذي وفقني لإتمام هذا البحث.

أتقدم بخالص الشكر والتقدير إلى أساتذتي الكرام، وإلى المشرف على هذا البحث، لما قدموه من توجيهات ودعم كان له الأثر الكبير في إنجاز هذا العمل.

كما أقدم شكري وامتناني إلى أهلي وأصدقائي على دعمهم وتشجيعهم المستمر.

والله ولي التوفيق

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احتمالية التقريب الأولى

Rough approximation probability

لثمان تبولوجيات معرفة على مجموعة X التي تحتوي على ثمان عناصر. بعد استخراج interior و closure لجميع المجاميع الجزئية لـ X والتي تتكون من 16 مجموعة واحتساب $\underline{P}$(A) و(A) $\bar{P}$ وكذلك استخراج عائلة جميع المجاميع A open , A closed ثم بعد ذلك استخراج A interior , A closure للمجاميع الجزئية لـ X واحتساب $\underline{P}$(A) Aو (A) $\bar{P}$ A حيث:

$$A\ \underline{P}(\mathrm{A}) = \frac{\left|A\ \underline{R}(\mathrm{A})\right|}{|x|} = \frac{|A.int(A)|}{|x|}$$

$$A\ \bar{P}(\mathrm{A}) = \frac{|A\ \bar{P}(\mathrm{A})|}{|x|} = \frac{|A.cl(A)|}{|x|}$$

Introduction

Pawlak introduced approximation spaces during the early 1980s as part of his research on classifying objects by means of their features [4]. Rough set Theory introduced by Pawlak in 1982 as an extension of set Theory mainly in the domain of intelligent systems. Rough set Theory is a mathematical tool to deal with vagueness and incomplete information imprecise by dividing these data into equivalence classes using equivalence relations which result from the same data [5, 6]. Therefore it is relate to the concept of approximation. This Theory applied successfully in several applications e.g. information analysis, data analysis and knowledge discovery [3].

Definition 1-1 [7] let X be any set. The family T of subset of X is called topology if satisfy the following properties:

1. $\emptyset, X \in T$
2. If $A, B \in T \Rightarrow A \cap B \in T$
3. for any index Λ and $A_\lambda \in T \Rightarrow \bigcup A_\lambda \in T$

- The pair (X, T) is called topological space.

Definition 1-2 [7]

Let (X, T) be topological space, X∈X is called interior of A iff ∋ G ∈ T ∋ X ∈ G ⊆ A and the set A is called nbd of X.

- The set of all interior points of A is denoted by $int A$ or $A^\circ = \bigcup\{G \in T; G \subseteq A\}$

Definition 1-3 [7]

Let (X, T) be topological space, X ∈ X is called adherent points of a set A iff ∀ G ∈ T, X∈ G such that G∩A ≠ ∅.

The set of all adherent points of A is called closure of A and is denoted by $\bar{A}$ or cl (A).

- Also $cl(A) = \bigcap\{F \text{ is closed}, A \subseteq F\}$

  $\bar{A} = A^{coc}$ and $A^\circ = A^{\bar{c}^c}$

Definition 1-4 [1]

let (X, T) be a topological space and A ⊆X, A is said to be almost open (A-open) iff A ⊆ $\bar{A}^\circ$.

The family of all A-open sets in X is denoted by A-v (X).

The complement of A-open is called A-closed and the family of all A-closed sets in X is denoted by A-c(X).

Note 1-5 Each open set is A-open so T⊆A-v(X) but the conversely is not true.

Definition 1-6 let (X, T) be a topological space and A⊆X. A point X∈X is called almost adherent point of A iff ∀ G ∈ A-v (X) , x ∈G such that G∩ A≠ ∅. The set of all A-adherent points of A is called the almost-closure of A and is denoted by A-cl(A).

Theorem 1-7 [1] let (X, T) be a topological space and A⊆X. Then the following are true:

1. $A\text{-}cl(A) = \bigcap\{F \in A\text{-}c(X); A \subseteq F\}$
2. If $B \in A\text{-}c(X)$, iff $B = A\text{-}cl(B)$
3. $A\text{-}cl(B) \subseteq cl(B)$
4. $B \in A\text{-}v(X)$ iff $B = A\text{-}int(B)$

Definition 1-8 [1] let (X, T) be a topological space and A⊆X, A point X∈A is called A interior of A iff ∃ B∈A-∪(X) ∋ X∈B ⊆A and A is called A-nbd of x.

The set of all Almost-interior points of A is denoted by A-int(A).

Theorem 1-9 [1] let (X, T) be a topological space. Then the following properties are true:

1. $A\text{-}int(B) = \bigcup\{G \in A\text{-}v(X); G \subseteq B\}$
2. $int(B) \subseteq A\text{-}int(B)$
3. The intersection of any two A-open is not necessary A-open and the union of collection of A-open is A-open.

Note 1-10 let (X, T) be topological space A ⊆ X.

- The closure A is $\bar{A} = cl(A) = \bigcap\{F \text{ is closed}; A \subseteq F\}$
- The interior of A is $A^{\circ} = int(A) = \bigcup\{G \in T; G \subseteq A\}$
- The intersection any two s-open sets is not necessary to be s-open.
- The union of any s-open sets is s-open because $\bigcup_{\lambda\in\Lambda} A_{\lambda} \subseteq \bigcup_{\lambda\in\Lambda} A_{\lambda}^{\bar{o}} \subseteq (\bigcup_{\lambda\in\Lambda} A_{\lambda})^{\circ}$
- The family of s-open sets $s\text{-}\upsilon(X)$ is subbase.

Definition 1-11 [2] let K = (X, R) be an approximate space with general relation R and T_K is the topology associated to K. Then the triple (X, R, T_K) is called a topological approximation space.

T_K is generated by subbase X,R is general relation. X,R = {xR, ∀X ∈R}, xR = {G ∈X ; XRy}

Example 1-12 let X = {a,b,c,d} , X,R = {{c}, {b, d}} is subbase, so the base ={Ø,X,|c|, {b, d}}

$$T_K = \{\emptyset, X, \{c\}, \{b, d\}, \{c, b, d\}\}$$

١- إضافة جميع التقاطعات المنتهية الـ subbase سوف نحصل علىbase

٢- إضافة جميع الاتحادات إلى base لكي نحصل على التبولوجي T_K .

Definition 1-13 [2]

- The lower approximation is $\underline{R}A = int A$ $\quad A\underline{R}A = A \cdot int(A)$
- The upper approximation is $\bar{R}A =$ $\quad A\bar{R}A = A \cdot cl(A)$
- The Rough Probability of $P(A) = \underline{P}(A), P(\bar{A})$

$$\underline{P}(A) = \frac{\text{The cardinal of } \underline{R}A}{|X|} \quad , \quad \bar{P}(A) = \frac{\text{The cardinal of } \bar{R}A}{|X|}$$

$$= \frac{|\underline{R}(A)|}{|X|} \qquad = \frac{|\bar{R}(A)|}{|X|}$$

X = {1,2,3,4}

P(x) = {Ø, X ,{1},{2},{3},{4},{1,2},{1,3},{1,4},{2,3},{2,4},{3,4},{1,2,3},{1,2,4},{1,3,4}, {2,3,4} }

$$T_1 = \{\varnothing, X, \{1\}, \{2,3\}, \{1,2,3\}\}$$

$$c(X) = \{X, \varnothing, \{2,3,4\}, \{1,4\}, \{4\}\}$$

$$A = \{1\} A^{\circ} = \{1\}^{\circ} = \{1\} \bar{A} = \overline{\{1\}} = \{1,4\} \quad \& \quad \bar{A}^{\circ} = \{1,4\}^{\circ} = \{1\} \therefore A \subseteq \bar{A}^{\circ}$$

$$A = \{2\} A^{\circ} = \{2\}^{\circ} = \varnothing \bar{A} = \overline{\{2\}} = \{2,3,4\} \quad \& \quad \bar{A}^{\circ} = \{2,3,4\}^{\circ} = \{2,3\} \therefore A \subseteq \bar{A}^{\circ}$$

$$A = \{3\} A^{\circ} = \{3\}^{\circ} = \varnothing \bar{A} = \overline{\{3\}} = \{2,3,4\} \quad \& \quad \bar{A}^{\circ} = \{2,3,4\}^{\circ} = \{2,3\} \therefore A \subseteq \bar{A}^{\circ}$$

$$A = \{4\} A^{\circ} = \{4\}^{\circ} = \varnothing \bar{A} = \overline{\{4\}} = \{4\} \quad \& \quad \bar{A}^{\circ} = \{4\}^{\circ} = \varnothing \therefore A \nsubseteq \bar{A}^{\circ}$$

$$A = \{1,2\} A^{\circ} = \{1,2\}^{\circ} = \{1\} \bar{A} = \overline{\{1,2\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X \therefore A \subseteq \bar{A}^{\circ}$$

$$A = \{1,3\} A^{\circ} = \{1,3\}^{\circ} = \{1\} \bar{A} = \overline{\{1,3\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X \therefore A \subseteq \bar{A}^{\circ}$$

$$A = \{1,4\} A^{\circ} = \{1,4\}^{\circ} = \{1\} \bar{A} = \overline{\{1,4\}} = \{1,4\} \quad \& \quad \bar{A}^{\circ} = \{1,4\}^{\circ} = \{1\} \therefore A \nsubseteq \bar{A}^{\circ}$$

$$A = \{2,3\} A^{\circ} = \{2,3\}^{\circ} = \{2,3\} \bar{A} = \overline{\{2,3\}} = \{2,3,4\} \quad \& \quad \bar{A}^{\circ} = \{2,3,4\}^{\circ} = \{2,3\} \therefore A \subseteq \bar{A}^{\circ}$$

$$A = \{2,4\} A^{\circ} = \{2,4\}^{\circ} = \varnothing \bar{A} = \overline{\{2,4\}} = \{2,3,4\} \quad \& \quad \bar{A}^{\circ} = \{2,3,4\}^{\circ} = \{2,3\} \therefore A \nsubseteq \bar{A}^{\circ}$$

$$A = \{3,4\} A^{\circ} = \{3,4\}^{\circ} = \varnothing \bar{A} = \overline{\{3,4\}} = \{2,3,4\} \quad \& \quad \bar{A}^{\circ} = \{2,3,4\}^{\circ} = \{2,3\} \therefore A \nsubseteq \bar{A}^{\circ}$$

$$A = \{1,2,3\} A^{\circ} = \{1,2,3\}^{\circ} = \{1,2,3\} \bar{A} = \overline{\{1,2,3\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X \therefore A \subseteq \bar{A}^{\circ}$$

$A = \{1,2,4\} A^* = \{1,2,4\}^* = \{1\} \bar{A} = \overline{\{1,2,4\}} = X \quad \& \quad \bar{A}^* = X^* = X \therefore A \subseteq \bar{A}^*$

$A = \{1,3,4\} A^* = \{1,3,4\}^* = \{1\} \bar{A} = \overline{\{1,3,4\}} = X \quad \& \quad \bar{A}^* = X^* = X \therefore A \subseteq \bar{A}^*$

$A = \{2,3,4\} A^* = \{2,3,4\}^* = \{2,3\} \bar{A} = \overline{\{2,3,4\}} = \{2,3,4\} \quad \& \quad \bar{A}^* = \{2,3,4\}^* = \{2,3\} \therefore A \nsubseteq \bar{A}^*$

$A\text{-}o(X) = \{\varnothing, X, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}\}$

$A\text{-}c(X) = \{X, \varnothing, \{2,3,4\}, \{1,3,4\}, \{1,2,4\}, \{3,4\}, \{2,4\}, \{1,4\}, \{4\}, \{3\}, \{2\}\}$

$int_A(A)$	$cl_A(A)$
$int_A(\{1\}) = \{1\}$	$cl_A(\{1\}) = \{1,4\}$
$int_A(\{2\}) = \{2\}$	$cl_A(\{2\}) = \{2\}$
$int_A(\{3\}) = \{3\}$	$cl_A(\{3\}) = \{3\}$
$int_A(\{4\}) = \varnothing$	$cl_A(\{4\}) = \{4\}$
$int_A(\{1,2\}) = \{1,2\}$	$cl_A(\{1,2\}) = \{1,2,4\}$
$int_A(\{1,3\}) = \{1,3\}$	$cl_A(\{1,3\}) = \{1,3,4\}$
$int_A(\{1,4\}) = \{1\}$	$cl_A(\{1,4\}) = \{1,4\}$
$int_A(\{2,3\}) = \{2,3\}$	$cl_A(\{2,3\}) = \{2,3,4\}$
$int_A(\{2,4\}) = \{2\}$	$cl_A(\{2,4\}) = \{2,4\}$
$int_A(\{3,4\}) = \{3\}$	$cl_A(\{3,4\}) = \{3,4\}$
$int_A(\{1,2,3\}) = \{1,2,3\}$	$cl_A(\{1,2,3\}) = X$
$int_A(\{1,2,4\}) = \{1,2,4\}$	$cl_A(\{1,2,4\}) = \{1,2,4\}$
$int_A(\{1,3,4\}) = \{1,3,4\}$	$cl_A(\{1,3,4\}) = \{1,3,4\}$
$int_A(\{2,3,4\}) = \{2,3\}$	$cl_A(\{2,3,4\}) = \{2,3,4\}$

A-$\bar{P}(A)$	A-$\underline{P}(A)$	$\bar{P}(A)$	$\underline{P}(A)$	المجموعة A
2/4	1/4	2/4	1/4	$\{1\}$
1/4	1/4	3/4	0/4	$\{2\}$
1/4	1/4	3/4	0/4	$\{3\}$
1/4	0/4	1/4	0/4	$\{4\}$
3/4	2/4	4/4	1/4	$\{1, 2\}$
3/4	2/4	4/4	1/4	$\{1, 3\}$
2/4	1/4	2/4	1/4	$\{1, 4\}$
3/4	2/4	3/4	2/4	$\{2, 3\}$
2/4	1/4	3/4	0/4	$\{2, 4\}$
2/4	1/4	3/4	0/4	$\{3, 4\}$
4/4	3/4	4/4	3/4	$\{1, 2, 3\}$
3/4	3/4	4/4	1/4	$\{1, 2, 4\}$
3/4	3/4	4/4	1/4	$\{1, 3, 4\}$
3/4	2/4	3/4	2/4	$\{2, 3, 4\}$

$$P(\emptyset) = 0 \qquad A\text{-}\underline{P}(\emptyset) = 0$$

$$P(X) = 1 \qquad A\text{-}\bar{P}(X) = 1$$

$$T_2 = \{\varnothing, X, \{2\}, \{2,3\}, \{2,4\}, \{2,3,4\}\}$$

$$c(X) = \{X, \varnothing, \{1,3,4\}, \{1,4\}, \{1,3\}, \{1\}\}$$

$$A = \{1\} A^{\circ} = \{1\}^{\circ} = \varnothing \bar{A} = \overline{\{1\}} = \{1\} \quad \& \quad \bar{A}^{\circ} = \{1\}^{\circ} = \varnothing \therefore A \nsubseteq \bar{A}^{\circ}$$

$$A = \{2\} A^{\circ} = \{2\}^{\circ} = \{2\} \bar{A} = \overline{\{2\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X \therefore A \subseteq \bar{A}^{\circ}$$

$$A = \{3\} A^{\circ} = \{3\}^{\circ} = \varnothing \bar{A} = \overline{\{3\}} = \{1,3\} \quad \& \quad \bar{A}^{\circ} = \{1,3\}^{\circ} = \varnothing \therefore A \nsubseteq \bar{A}^{\circ}$$

$$A = \{4\} A^{\circ} = \{4\}^{\circ} = \varnothing \bar{A} = \overline{\{4\}} = \{1,4\} \quad \& \quad \bar{A}^{\circ} = \{1,4\}^{\circ} = \varnothing \therefore A \nsubseteq \bar{A}^{\circ}$$

$$A = \{1,2\} A^{\circ} = \{1,2\}^{\circ} = \varnothing \bar{A} = \overline{\{1,2\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X \therefore A \subseteq \bar{A}^{\circ}$$

$$A = \{1,3\} A^{\circ} = \{1,3\}^{\circ} = \varnothing \bar{A} = \overline{\{1,3\}} = \{1,3\} \quad \& \quad \bar{A}^{\circ} = \{1,3\}^{\circ} = \varnothing \therefore A \nsubseteq \bar{A}^{\circ}$$

$$A = \{1,4\} A^{\circ} = \{1,4\}^{\circ} = \varnothing \bar{A} = \overline{\{1,4\}} = \{1,4\} \quad \& \quad \bar{A}^{\circ} = \{1,4\}^{\circ} = \varnothing \therefore A \nsubseteq \bar{A}^{\circ}$$

$$A = \{2,3\} A^{\circ} = \{2,3\}^{\circ} = \{2,3\} \bar{A} = \overline{\{2,3\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X \therefore A \subseteq \bar{A}^{\circ}$$

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$$A = \{3,4\} A^{\circ} = \{3,4\}^{\circ} = \varnothing \bar{A} = \overline{\{3,4\}} = \{1,3,4\} \quad \& \quad \bar{A}^{\circ} = \{1,3,4\}^{\circ} = \varnothing \therefore A \nsubseteq \bar{A}^{\circ}$$

$$A = \{1,2,3\} A^{\circ} = \{1,2,3\}^{\circ} = \varnothing \bar{A} = \overline{\{1,2,3\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X \therefore A \subseteq \bar{A}^{\circ}$$

$A = \{1,2,4\}\ A^{*} = \{1,2,4\}^{*} = \varnothing\ \bar{A} = \overline{\{1,2,4\}} = X \quad \& \quad \bar{A}^{*} = X^{*} = X \therefore A \subseteq \bar{A}^{*}$

$A = \{1,3,4\}\ A^{*} = \{1,3,4\}^{*} = \varnothing\ \bar{A} = \overline{\{1,3,4\}} = \{1,3,4\} \quad \& \quad \bar{A}^{*} = \{1,3,4\}^{*} = \varnothing \therefore A \nsubseteq \bar{A}^{*}$

$A = \{2,3,4\}\ A^{*} = \{2,3,4\}^{*} = \{2,3,4\}\ \bar{A} = \overline{\{2,3,4\}} = X \quad \& \quad \bar{A}^{*} = X^{*} = X \therefore A \subseteq \bar{A}^{*}$

$A\text{-}O_2(X) = \{\varnothing, X, \{2\}, \{1,2\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}\}$

$A\text{-}c_2(X) = \{X, \varnothing, \{1,3,4\}, \{3,4\}, \{1,4\}, \{1,3\}, \{4\}, \{3\}, \{1\}\}$

$int_A(A)$	$cl_A(A)$
$int_A(\{1\}) = \varnothing$	$cl_A(\{1\}) = \{1\}$
$int_A(\{2\}) = \{2\}$	$cl_A(\{2\}) = X$
$int_A(\{3\}) = \varnothing$	$cl_A(\{3\}) = \{3\}$
$int_A(\{4\}) = \varnothing$	$cl_A(\{4\}) = \{4\}$
$int_A(\{1,2\}) = \{1,2\}$	$cl_A(\{1,2\}) = X$
$int_A(\{1,3\}) = \varnothing$	$cl_A(\{1,3\}) = \{1,3\}$
$int_A(\{1,4\}) = \varnothing$	$cl_A(\{1,4\}) = \{1,4\}$
$int_A(\{2,3\}) = \{2,3\}$	$cl_A(\{2,3\}) = X$
$int_A(\{2,4\}) = \{2,4\}$	$cl_A(\{2,4\}) = X$
$int_A(\{3,4\}) = \varnothing$	$cl_A(\{3,4\}) = \{3,4\}$
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$int_A(\{1,3,4\}) = \varnothing$	$cl_A(\{1,3,4\}) = \{1,3,4\}$
$int_A(\{2,3,4\}) = \{2,3,4\}$	$cl_A(\{2,3,4\}) = X$

A-$\bar{P}(A)$	A-$\underline{P}(A)$	$\bar{P}(A)$	$\underline{P}(A)$	A
1/4	0/4	1/4	0/4	$\{1\}$
4/4	1/4	4/4	1/4	$\{2\}$
1/4	0/4	2/4	0/4	$\{3\}$
1/4	0/4	2/4	0/4	$\{4\}$
4/4	2/4	4/4	0/4	$\{1, 2\}$
2/4	0/4	2/4	0/4	$\{1, 3\}$
2/4	0/4	2/4	0/4	$\{1, 4\}$
4/4	2/4	4/4	2/4	$\{2, 3\}$
4/4	2/4	4/4	2/4	$\{2, 4\}$
2/4	0/4	3/4	0/4	$\{3, 4\}$
4/4	3/4	4/4	0/4	$\{1, 2, 3\}$
4/4	3/4	4/4	0/4	$\{1, 2, 4\}$
3/4	0/4	3/4	0/4	$\{1, 3, 4\}$
4/4	3/4	4/4	3/4	$\{2, 3, 4\}$

$\underline{P}(\varnothing) = 0$ $\bar{P}(X) = 1$

A-$\underline{P}(\varnothing) = 0$ A-$\bar{P}(X) = 1$

$T_3 = \{\varnothing, X, \{2,3\}, \{2,3,4\}, \{1,2,3\}\}$

$c(X) = \{X, \varnothing, \{1,4\}, \{1\}, \{4\}\}$

$A = \{1\} A^\circ = \{1\}^\circ = \varnothing \bar{A} = \overline{\{1\}} = \{1\} \quad \& \quad \bar{A}^\circ = \{1\}^\circ = \varnothing \therefore A \not\subseteq \bar{A}^\circ$

$A = \{2\} A^\circ = \{2\}^\circ = \{2\} \bar{A} = \overline{\{2\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X \therefore A \subseteq \bar{A}^\circ$

$A = \{3\} A^\circ = \{3\}^\circ = \varnothing \bar{A} = \overline{\{3\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X \therefore A \subseteq \bar{A}^\circ$

$A = \{4\} A^\circ = \{4\}^\circ = \varnothing \bar{A} = \overline{\{4\}} = \{4\} \quad \& \quad \bar{A}^\circ = \{4\}^\circ = \varnothing \therefore A \not\subseteq \bar{A}^\circ$

$A = \{1,2\} A^\circ = \{1,2\}^\circ = \varnothing \bar{A} = \overline{\{1,2\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X \therefore A \subseteq \bar{A}^\circ$

$A = \{1,3\} A^\circ = \{1,3\}^\circ = \varnothing \bar{A} = \overline{\{1,3\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X \therefore A \subseteq \bar{A}^\circ$

$A = \{1,4\} A^\circ = \{1,4\}^\circ = \varnothing \bar{A} = \overline{\{1,4\}} = \{1,4\} \quad \& \quad \bar{A}^\circ = \{1,4\}^\circ = \varnothing \therefore A \not\subseteq \bar{A}^\circ$

$A = \{2,3\} A^\circ = \{2,3\}^\circ = \{2,3\} \bar{A} = \overline{\{2,3\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X \therefore A \subseteq \bar{A}^\circ$

$A = \{2,4\} A^\circ = \{2,4\}^\circ = \varnothing \bar{A} = \overline{\{2,4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X \therefore A \subseteq \bar{A}^\circ$

$A = \{3,4\} A^\circ = \{3,4\}^\circ = \varnothing \bar{A} = \overline{\{3,4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X \therefore A \subseteq \bar{A}^\circ$

$A = \{1,2,3\} A^\circ = \{1,2,3\}^\circ = \{1,2,3\} \bar{A} = \overline{\{1,2,3\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X \therefore A \subseteq \bar{A}^\circ$

$A = \{1,2,4\} A^\circ = \{1,2,4\}^\circ = \varnothing \bar{A} = \overline{\{1,2,4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X \therefore A \subseteq \bar{A}^\circ$

$A = \{1,3,4\} A^\circ = \{1,3,4\}^\circ = \varnothing \bar{A} = \overline{\{1,3,4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X \therefore A \subseteq \bar{A}^\circ$

$A = \{2,3,4\} A^* = \{2,3,4\}^* = \{2,3,4\} \bar{A} = \overline{\{2,3,4\}} = X \quad \& \quad \bar{A}^* = X^* = X \therefore A \subseteq \bar{A}^*$

$A.o_3(X) = \{\varnothing, X, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\}$

$A.c_3(X) = \{X, \varnothing, \{1,3,4\}, \{1,2,4\}, \{1,2,3\}, \{3,4\}, \{2,4\}, \{1,4\}, \{1,3\}, \{1,2\}, \{4\}, \{3\}, \{2\}, \{1\}\}$

$int_A(A)$	$cl_A(A)$
$int_A(\{1\}) = \varnothing$	$cl_A(\{1\}) = \{1\}$
$int_A(\{2\}) = \{2\}$	$cl_A(\{2\}) = \{2\}$
$int_A(\{3\}) = \{3\}$	$cl_A(\{3\}) = \{3\}$
$int_A(\{4\}) = \{4\}$	$cl_A(\{4\}) = \{4\}$
$int_A(\{1,2\}) = \{1,2\}$	$cl_A(\{1,2\}) = \{1,2\}$
$int_A(\{1,3\}) = \{1,3\}$	$cl_A(\{1,3\}) = \{1,3\}$
$int_A(\{1,4\}) = \varnothing$	$cl_A(\{1,4\}) = \{1,4\}$
$int_A(\{2,3\}) = \{2,3\}$	$cl_A(\{2,3\}) = \{1,2,3\}$
$int_A(\{2,4\}) = \{2,4\}$	$cl_A(\{2,4\}) = \{1,2,4\}$
$int_A(\{3,4\}) = \{3,4\}$	$cl_A(\{3,4\}) = \{3,4\}$
$int_A(\{1,2,3\}) = \{1,2,3\}$	$cl_A(\{1,2,3\}) = \{1,2,3\}$
$int_A(\{1,2,4\}) = \{1,2,4\}$	$cl_A(\{1,2,4\}) = \{1,2,4\}$
$int_A(\{1,3,4\}) = \{1,3,4\}$	$cl_A(\{1,3,4\}) = \{1,3,4\}$
$int_A(\{2,3,4\}) = \{2,3,4\}$	$cl_A(\{2,3,4\}) = X$

$A\text{-}\bar{P}(A)$	$A\text{-}\underline{P}(A)$	$\bar{P}(A)$	$\underline{P}(A)$	A
1/4	0/4	1/4	0/4	$\{1\}$
1/4	1/4	4/4	0/4	$\{2\}$
1/4	1/4	4/4	0/4	$\{3\}$
1/4	1/4	1/4	0/4	$\{4\}$
2/4	2/4	4/4	0/4	$\{1,2\}$
2/4	2/4	4/4	0/4	$\{1,3\}$
2/4	0/4	2/4	0/4	$\{1,4\}$
3/4	2/4	4/4	2/4	$\{2,3\}$
2/4	2/4	4/4	0/4	$\{2,4\}$
2/4	2/4	4/4	0/4	$\{3,4\}$
3/4	3/4	4/4	3/4	$\{1,2,3\}$
3/4	3/4	4/4	0/4	$\{1,2,4\}$
3/4	3/4	4/4	0/4	$\{1,3,4\}$
4/4	3/4	4/4	3/4	$\{2,3,4\}$

$$\underline{P}(\varnothing) = 0 \qquad A\text{-}\underline{P}(\varnothing) = 0$$

$$\bar{P}(X) = 1 \qquad A\text{-}\bar{P}(X) = 1$$

$$T_4 = \{\varnothing, X, \{1,3\}, \{1,3,4\}, \{1,2,3\}\}$$

$$c(X) = \{X, \varnothing, \{2,4\}, \{2\}, \{4\}\}$$

$A = \{1\} A^* = \{1\}^* = \varnothing \bar{A} = \overline{\{1\}} = X \quad \& \quad \bar{A}^* = X^* = X \therefore A \subseteq \bar{A}^*$

$A = \{2\} A^* = \{2\}^* = \varnothing \bar{A} = \overline{\{2\}} = \{2\} \quad \& \quad \bar{A}^* = \{2\}^* = \varnothing \therefore A \nsubseteq \bar{A}^*$

$A = \{3\} A^* = \{3\}^* = \varnothing \bar{A} = \overline{\{3\}} = X \quad \& \quad \bar{A}^* = X^* = X \therefore A \subseteq \bar{A}^*$

$A = \{4\} A^* = \{4\}^* = \varnothing \bar{A} = \overline{\{4\}} = \{4\} \quad \& \quad \bar{A}^* = \{4\}^* = \varnothing \therefore A \nsubseteq \bar{A}^*$

$A = \{1,2\} A^* = \{1,2\}^* = \varnothing \bar{A} = \overline{\{1,2\}} = X \quad \& \quad \bar{A}^* = X^* = X \therefore A \subseteq \bar{A}^*$

$A = \{1,3\} A^* = \{1,3\}^* = \{1,3\} \bar{A} = \overline{\{1,3\}} = X \quad \& \quad \bar{A}^* = X^* = X \therefore A \subseteq \bar{A}^*$

$A = \{1,4\} A^* = \{1,4\}^* = \varnothing \bar{A} = \overline{\{1,4\}} = X \quad \& \quad \bar{A}^* = X^* = X \therefore A \subseteq \bar{A}^*$

$A = \{2,3\} A^* = \{2,3\}^* = \varnothing \bar{A} = \overline{\{2,3\}} = X \quad \& \quad \bar{A}^* = X^* = X \therefore A \subseteq \bar{A}^*$

$A = \{2,4\} A^* = \{2,4\}^* = \varnothing \bar{A} = \overline{\{2,4\}} = \{2,4\} \quad \& \quad \bar{A}^* = \{2,4\}^* = \varnothing \therefore A \nsubseteq \bar{A}^*$

$A = \{3,4\} A^* = \{3,4\}^* = \varnothing \bar{A} = \overline{\{3,4\}} = X \quad \& \quad \bar{A}^* = X^* = X \therefore A \subseteq \bar{A}^*$

$A = \{1,2,3\} A^* = \{1,2,3\}^* = \{1,2,3\} \bar{A} = \overline{\{1,2,3\}} = X \quad \& \quad \bar{A}^* = X^* = X \therefore A \subseteq \bar{A}^*$

$A = \{1,2,4\}\, A^{\circ} = \{1,2,4\}^{\circ} = \varnothing\, \bar{A} = \overline{\{1,2,4\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X \therefore A \subseteq \bar{A}^{\circ}$

$A = \{1,3,4\}\, A^{\circ} = \{1,3,4\}^{\circ} = \{1,3,4\}\, \bar{A} = \overline{\{1,3,4\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X \therefore A \subseteq \bar{A}^{\circ}$

$A = \{2,3,4\}\, A^{\circ} = \{2,3,4\}^{\circ} = \varnothing\, \bar{A} = \overline{\{2,3,4\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X \therefore A \subseteq \bar{A}^{\circ}$

$A.o_4(X) = \{\varnothing, X, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}\}$

$A.c_4(X) = \{X, \varnothing, \{2,3,4\}, \{1,2,4\}, \{3,4\}, \{2,4\}, \{2,3\}, \{1,4\}, \{1,2\}, \{4\}, \{3\}, \{2\}\}$

$int_A(A)$	$cl_A(A)$
$int_A(\{1\}) = \{1\}$	$cl_A(\{1\}) = X$
$int_A(\{2\}) = \varnothing$	$cl_A(\{2\}) = \{2\}$
$int_A(\{3\}) = \{3\}$	$cl_A(\{3\}) = \{3\}$
$int_A(\{4\}) = \varnothing$	$cl_A(\{4\}) = \{4\}$
$int_A(\{1,2\}) = \{1,2\}$	$cl_A(\{1,2\}) = \{1,2\}$
$int_A(\{1,3\}) = \{1,3\}$	$cl_A(\{1,3\}) = X$
$int_A(\{1,4\}) = \{1,4\}$	$cl_A(\{1,4\}) = \{1,4\}$
$int_A(\{2,3\}) = \{2,3\}$	$cl_A(\{2,3\}) = \{2,3\}$
$int_A(\{2,4\}) = \varnothing$	$cl_A(\{2,4\}) = \{2,4\}$
$int_A(\{3,4\}) = \{3,4\}$	$cl_A(\{3,4\}) = \{3,4\}$
$int_A(\{1,2,3\}) = \{1,2,3\}$	$cl_A(\{1,2,3\}) = X$
$int_A(\{1,2,4\}) = \{1,2,4\}$	$cl_A(\{1,2,4\}) = \{1,2,4\}$
$int_A(\{1,3,4\}) = \{1,3,4\}$	$cl_A(\{1,3,4\}) = X$
$int_A(\{2,3,4\}) = \varnothing$	$cl_A(\{2,3,4\}) = \{2,3,4\}$

$A\text{-}\bar{P}(A)$	$A\text{-}\underline{P}(A)$	$\bar{P}(A)$	$\underline{P}(A)$	A
4/4	1/4	4/4	0/4	$\{1\}$
1/4	0/4	1/4	0/4	$\{2\}$
4/4	1/4	4/4	0/4	$\{3\}$
1/4	0/4	1/4	0/4	$\{4\}$
2/4	2/4	4/4	0/4	$\{1,2\}$
4/4	2/4	4/4	2/4	$\{1,3\}$
2/4	2/4	4/4	0/4	$\{1,4\}$
2/4	2/4	4/4	0/4	$\{2,3\}$
2/4	0/4	2/4	0/4	$\{2,4\}$
2/4	2/4	4/4	0/4	$\{3,4\}$
4/4	3/4	4/4	3/4	$\{1,2,3\}$
3/4	3/4	4/4	0/4	$\{1,2,4\}$
4/4	3/4	4/4	3/4	$\{1,3,4\}$
3/4	0/4	4/4	0/4	$\{2,3,4\}$

$\underline{P}(\varnothing) = 0$ $A\text{-}\underline{P}(\varnothing) = 0$

$\bar{P}(X) = 1$ $A\text{-}\bar{P}(X) = 1$

$T_5 = \{\emptyset, X, \{3\}, \{1,3\}, \{2,3\}, \{1,3,4\}, \{1,2,3\}\}$

$c(X) = \{X, \emptyset, \{1,2,4\}, \{2,4\}, \{1,4\}, \{2\}, \{4\}\}$

$A = \{1\}\ A^\circ = \{1\}^\circ = \emptyset\ \ \bar{A} = \overline{\{1\}} = \{1,4\}\quad \&\quad \bar{A}^\circ = \{1,4\}^\circ = \emptyset\ \therefore A \not\subseteq \bar{A}^\circ$
$A = \{2\}\ A^\circ = \{2\}^\circ = \emptyset\ \ \bar{A} = \overline{\{2\}} = \{2\}\quad \&\quad \bar{A}^\circ = \{2\}^\circ = \emptyset\quad \therefore A \not\subseteq \bar{A}^\circ$
$A = \{3\}\ A^\circ = \{3\}^\circ = \{3\}\ \bar{A} = \overline{\{3\}} = X\quad \&\quad \bar{A}^\circ = X^\circ = X\ \ \therefore A \subseteq \bar{A}^\circ$

$A=\{4\}$ $A^\circ=\{4\}^\circ=\emptyset$ $\bar{A}=\overline{\{4\}}=\{4\}$ & $\bar{A}^\circ=\{4\}^\circ=\emptyset$ $\therefore A\nsubseteq\bar{A}^\circ$

$A=\{1,2\}$ $A^\circ=\{1,2\}^\circ=\emptyset$ $\bar{A}=\overline{\{1,2\}}=\{1,2,4\}$ & $\bar{A}^\circ=\{1,2,4\}^\circ=\emptyset$ $\therefore A\nsubseteq\bar{A}^\circ$

$A=\{1,3\}$ $A^\circ=\{1,3\}^\circ=\{1,3\}$ $\bar{A}=\overline{\{1,3\}}=X$ & $\bar{A}^\circ=X^\circ=X$ $\therefore A\subseteq\bar{A}^\circ$

$A=\{1,4\}$ $A^\circ=\{1,4\}^\circ=\emptyset$ $\bar{A}=\overline{\{1,4\}}=\{1,4\}$ & $\bar{A}^\circ=\{1,4\}^\circ=\emptyset$ $\therefore A\nsubseteq\bar{A}^\circ$

$A=\{2,3\}$ $A^\circ=\{2,3\}^\circ=\{2,3\}$ $\bar{A}=\overline{\{2,3\}}=X$ & $\bar{A}^\circ=X^\circ=X$ $\therefore A\subseteq\bar{A}^\circ$

$A=\{2,4\}$ $A^\circ=\{2,4\}^\circ=\emptyset$ $\bar{A}=\overline{\{2,4\}}=\{2,4\}$ & $\bar{A}^\circ=\{2,4\}^\circ=\emptyset$ $\therefore A\nsubseteq\bar{A}^\circ$

$A=\{3,4\}$ $A^\circ=\{3,4\}^\circ=\emptyset$ $\bar{A}=\overline{\{3,4\}}=X$ & $\bar{A}^\circ=X^\circ=X$ $\therefore A\subseteq\bar{A}^\circ$

$A=\{1,2,3\}$ $A^\circ=\{1,2,3\}^\circ=\{1,2,3\}$ $\bar{A}=\overline{\{1,2,3\}}=X$ & $\bar{A}^\circ=X^\circ=X$ $\therefore A\subseteq\bar{A}^\circ$

$A=\{1,2,4\}$ $A^\circ=\{1,2,4\}^\circ=\emptyset$ $\bar{A}=\overline{\{1,2,4\}}=\{1,2,4\}$ & $\bar{A}^\circ=\{1,2,4\}^\circ=\emptyset$ $\therefore A\nsubseteq\bar{A}^\circ$

$A=\{2,3,4\}$ $A^\circ=\{2,3,4\}^\circ=\emptyset$ $\bar{A}=\overline{\{2,3,4\}}=X$ & $\bar{A}^\circ=X^\circ=X$ $\therefore A\subseteq\bar{A}^\circ$

$A=\{1,3,4\}$ $A^\circ=\{1,3,4\}^\circ=\{1,3,4\}$ $\bar{A}=\overline{\{1,3,4\}}=X$ & $\bar{A}^\circ=X^\circ=X$ $\therefore A\subseteq\bar{A}^\circ$

$A.o_5(X)=\{\emptyset,X,\{3\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{2,3,4\},\{1,3,4\}\}$

$A.c_5(X)=\{X,\emptyset,\{1,2,4\},\{2,4\},\{1,4\},\{1,2\},\{4\},\{1\},\{2\}\}$

$int_A(A)$	$cl_A(A)$
$int_A(\{1\})=\emptyset$	$cl_A(\{1\})=\{1\}$
$int_A(\{2\})=\emptyset$	$cl_A(\{2\})=\{2\}$
$int_A(\{3\})=\{3\}$	$cl_A(\{3\})=X$
$int_A(\{4\})=\emptyset$	$cl_A(\{4\})=\{4\}$
$int_A(\{1,2\})=\emptyset$	$cl_A(\{1,2\})=\{1,2\}$
$int_A(\{1,3\})=\{1,3\}$	$cl_A(\{1,3\})=X$
$int_A(\{1,4\})=\emptyset$	$cl_A(\{1,4\})=\{1,4\}$
$int_A(\{2,3\})=\{2,3\}$	$cl_A(\{2,3\})=X$
$int_A(\{2,4\})=\emptyset$	$cl_A(\{2,4\})=\{2,4\}$
$int_A(\{3,4\})=\{3,4\}$	$cl_A(\{3,4\})=X$
$int_A(\{1,2,3\})=\{1,2,3\}$	$cl_A(\{1,2,3\})=X$
$int_A(\{1,2,4\})=\emptyset$	$cl_A(\{1,2,4\})=\{1,2,4\}$
$int_A(\{1,3,4\})=\{1,3,4\}$	$cl_A(\{1,3,4\})=X$
$int_A(\{2,3,4\})=\{2,3,4\}$	$cl_A(\{2,3,4\})=X$

$A\text{-}\bar{P}(A)$	$A\text{-}\underline{P}(A)$	$\bar{P}(A)$	$\underline{P}(A)$	A
1/4	0/4	1/4	0/4	$\{1\}$
1/4	0/4	1/4	0/4	$\{2\}$
4/4	1/4	4/4	1/4	$\{3\}$
1/4	0/4	1/4	0/4	$\{4\}$
2/4	0/4	2/4	0/4	$\{1,2\}$
4/4	2/4	4/4	2/4	$\{1,3\}$
2/4	0/4	2/4	0/4	$\{1,4\}$
4/4	2/4	4/4	2/4	$\{2,3\}$
2/4	0/4	2/4	0/4	$\{2,4\}$
4/4	2/4	4/4	2/4	$\{3,4\}$
4/4	3/4	4/4	3/4	$\{1,2,3\}$
3/4	0/4	3/4	0/4	$\{1,2,4\}$
4/4	3/4	4/4	3/4	$\{1,3,4\}$
4/4	3/4	4/4	3/4	$\{2,3,4\}$

$$\underline{P}(\varnothing) = 0 \quad , \quad \bar{P}(X) = 1$$

$$A\text{-}\underline{P}(\varnothing) = 0 \quad , \quad A\text{-}\bar{P}(X) = 1$$

$T_5 = \{\emptyset, X, \{4\}, \{1,4\}, \{2,4\}, \{1,3,4\}, \{1,2,4\}\}$

$c(X) = \{X, \emptyset, \{1,2,3\}, \{2,3\}, \{1,3\}, \{2\}, \{3\}\}$

$A = \{1\}$
$A^\circ = \{1\}^\circ = \emptyset$
$\bar{A} = \overline{\{1\}} = \{1,3\} \quad \& \quad \bar{A}^\circ = \{1,3\}^\circ = \emptyset$
$\therefore A \nsubseteq \bar{A}^\circ$

$A = \{2\}$
$A^\circ = \{2\}^\circ = \emptyset$
$\bar{A} = \overline{\{2\}} = \{2\} \quad \& \quad \bar{A}^\circ = \{2\}^\circ = \emptyset$
$\therefore A \nsubseteq \bar{A}^\circ$

$A = \{3\}$
$A^\circ = \{3\}^\circ = \emptyset$
$\bar{A} = \overline{\{3\}} = \{3\} \quad \& \quad \bar{A}^\circ = \{3\}^\circ = \emptyset$
$\therefore A \nsubseteq \bar{A}^\circ$

$A = \{4\}$
$A^\circ = \{4\}^\circ = \{4\}$
$\bar{A} = \overline{\{4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{1,2\}$
$A^\circ = \{1,2\}^\circ = \emptyset$
$\bar{A} = \overline{\{1,2\}} = \{1,2,3\} \quad \& \quad \bar{A}^\circ = \{1,2,3\}^\circ = \emptyset$
$\therefore A \nsubseteq \bar{A}^\circ$

$A = \{1,3\}$
$A^\circ = \{1,3\}^\circ = \emptyset$
$\bar{A} = \overline{\{1,3\}} = \{1,3\} \quad \& \quad \bar{A}^\circ = \{1,3\}^\circ = \emptyset$
$\therefore A \nsubseteq \bar{A}^\circ$

$A = \{1,4\}$
$A^\circ = \{1,4\}^\circ = \{1,4\}$
$\bar{A} = \overline{\{1,4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X$
$\therefore A \subseteq \bar{A}^\circ$

$A^\circ = \{2,3\}^\circ = \emptyset$
$\bar{A} = \overline{\{2,3\}} = \{2,3\} \quad \& \quad \bar{A}^\circ = \{2,3\}^\circ = \emptyset$
$\therefore A \nsubseteq \bar{A}^\circ$

$A = \{2,4\}$
$A^\circ = \{2,4\}^\circ = \{2,4\}$
$\bar{A} = \overline{\{2,4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{3,4\}$
$A^\circ = \{3,4\}^\circ = \emptyset$
$\bar{A} = \overline{\{3,4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{1,2,3\}$
$A^\circ = \{1,2,3\}^\circ = \emptyset$
$\bar{A} = \overline{\{1,2,3\}} = \{1,2,3\} \quad \& \quad \bar{A}^\circ = \{1,2,3\}^\circ = \emptyset$
$\therefore A \nsubseteq \bar{A}^\circ$

$A = \{1,2,4\}$
$A^\circ = \{1,2,4\}^\circ = \{1,2,4\}$
$\bar{A} = \overline{\{1,2,4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{1,3,4\}$
$A^\circ = \{1,3,4\}^\circ = \{1,3,4\}$
$\bar{A} = \overline{\{1,3,4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{2,3,4\}$
$A^\circ = \{2,3,4\}^\circ = \emptyset$
$\bar{A} = \overline{\{2,3,4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X$
$\therefore A \subseteq \bar{A}^\circ$

$A.o_B(X) = \{\emptyset, X, \{4\}, \{1,4\}, \{2,4\}, \{1,3,4\}\ , \{1,2,4\}\}$
$A.c_B(X) = \{X, \emptyset, \{1,2,3\}, \{2,3\}, \{1,3\}, \{3\}, \{2\}\}$

$int_A(A)$	$cl_A(A)$
$int_A(\{1\}) = \emptyset$	$cl_A(\{1\}) = \{1\}$
$int_A(\{2\}) = \emptyset$	$cl_A(\{2\}) = \{2\}$
$int_A(\{3\}) = \emptyset$	$cl_A(\{3\}) = \{3\}$
$int_A(\{4\}) = \{4\}$	$cl_A(\{4\}) = \{4\}$
$int_A(\{1,2\}) = \emptyset$	$cl_A(\{1,2\}) = \{1,2\}$
$int_A(\{1,3\}) = \emptyset$	$cl_A(\{1,3\}) = \{1,3\}$
$int_A(\{1,4\}) = \{1,4\}$	$cl_A(\{1,4\}) = \{1,4\}$
$int_A(\{2,3\}) = \{2,3\}$	$cl_A(\{2,3\}) = \{2,3\}$
$int_A(\{2,4\}) = \{2,4\}$	$cl_A(\{2,4\}) = X$
$int_A(\{3,4\}) = \{3,4\}$	$cl_A(\{3,4\}) = X$
$int_A(\{1,2,3\}) = \emptyset$	$cl_A(\{1,2,3\}) = \{1,2,3\}$
$int_A(\{1,2,4\}) = \{1,2,4\}$	$cl_A(\{1,2,4\}) = X$
$int_A(\{1,3,4\}) = \{1,3,4\}$	$cl_A(\{1,3,4\}) = X$
$int_A(\{2,3,4\}) = \emptyset$	$cl_A(\{2,3,4\}) = X$

$A\text{-}\bar{P}(A)$	$A\text{-}\underline{P}(A)$	$\bar{P}(A)$	$\underline{P}(A)$	A
1/4	0/4	1/4	0/4	$\{1\}$
1/4	0/4	2/4	0/4	$\{2\}$
1/4	0/4	1/4	0/4	$\{3\}$
2/4	1/4	4/4	1/4	$\{4\}$
2/4	0/4	3/4	0/4	$\{1,2\}$
2/4	0/4	2/4	0/4	$\{1,3\}$
2/4	2/4	4/4	2/4	$\{1,4\}$
2/4	2/4	4/4	0/4	$\{2,3\}$
4/4	2/4	4/4	2/4	$\{2,4\}$
4/4	2/4	4/4	0/4	$\{3,4\}$
3/4	0/4	3/4	0/4	$\{1,2,3\}$
4/4	3/4	4/4	3/4	$\{1,2,4\}$
4/4	3/4	4/4	3/4	$\{1,3,4\}$
4/4	0/4	4/4	0/4	$\{2,3,4\}$

$$\underline{P}(\emptyset) = 0 \quad , \quad \bar{P}(X) = 1$$
$$A\text{-}\underline{P}(\emptyset) = 0 \quad , \quad A\text{-}\bar{P}(X) = 1$$

$T_7 = \{\emptyset, X, \{2,4\}, \{1,3\}\}$

$c(X) = \{X, \emptyset, \{1,3\}, \{2,4\}\}$

$A = \{1\}$
$A^\circ = \{1\}^\circ = \emptyset$
$\bar{A} = \overline{\{1\}} = \{1,3\} \quad \& \quad \bar{A}^\circ = \{1,3\}^\circ = \{1,3\}$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{2\}$
$A^\circ = \{2\}^\circ = \emptyset$
$\bar{A} = \overline{\{2\}} = \{2,4\} \quad \& \quad \bar{A}^\circ = \{2,4\}^\circ = \{2,4\}$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{3\}$
$A^\circ = \{3\}^\circ = \emptyset$
$\bar{A} = \overline{\{3\}} = \{1,3\} \quad \& \quad \bar{A}^\circ = \{1,3\}^\circ = \{1,3\}$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{4\}$
$A^\circ = \{4\}^\circ = \emptyset$
$\bar{A} = \overline{\{4\}} = \{2,4\} \quad \& \quad \bar{A}^\circ = \{2,4\}^\circ = \{2,4\}$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{1,2\}$
$A^\circ = \{1,2\}^\circ = \emptyset$
$\bar{A} = \overline{\{1,2\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{1,3\}$
$A^\circ = \{1,3\}^\circ = \{1,3\}$
$\bar{A} = \overline{\{1,3\}} = \{1,3\} \quad \& \quad \bar{A}^\circ = \{1,3\}^\circ = \{1,3\}$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{1,4\}$
$A^\circ = \{1,4\}^\circ = \emptyset$
$\bar{A} = \overline{\{1,4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{2,3\}$
$A^{\circ} = \{2,3\}^{\circ} = \emptyset$
$\bar{A} = \overline{\{2,3\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X$
$\therefore A \subseteq \bar{A}^{\circ}$

$A = \{2,4\}$
$A^{\circ} = \{2,4\}^{\circ} = \{2,4\}$
$\bar{A} = \overline{\{2,4\}} = \{2,4\} \quad \& \quad \bar{A}^{\circ} = \{2,4\}^{\circ} = \{2,4\}$
$\therefore A \subseteq \bar{A}^{\circ}$

$A = \{3,4\}$
$A^{\circ} = \{3,4\}^{\circ} = \emptyset$
$\bar{A} = \overline{\{3,4\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X$
$\therefore A \subseteq \bar{A}^{\circ}$

$A = \{1,2,3\}$
$A^{\circ} = \{1,2,3\}^{\circ} = \emptyset$
$\bar{A} = \overline{\{1,2,3\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X$
$\therefore A \subseteq \bar{A}^{\circ}$

$A = \{1,2,4\}$
$A^{\circ} = \{1,2,4\}^{\circ} = \emptyset$
$\bar{A} = \overline{\{1,2,4\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X$
$\therefore A \subseteq \bar{A}^{\circ}$

$A = \{1,3,4\}$
$A^{\circ} = \{1,3,4\}^{\circ} = \emptyset$
$\bar{A} = \overline{\{1,3,4\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X$
$\therefore A \subseteq \bar{A}^{\circ}$

$A = \{2,3,4\}$
$A^{\circ} = \{2,3,4\}^{\circ} = \emptyset$
$\bar{A} = \overline{\{2,3,4\}} = X \quad \& \quad \bar{A}^{\circ} = X^{\circ} = X$
$\therefore A \subseteq \bar{A}^{\circ}$

$A.o_7(X) =$
$\{\emptyset, X, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\}$
$A.c_7(X) =$
$\{X, \emptyset, \{2,3,4\}, \{1,3,4\}, \{1,2,4\}, \{1,2,3\}, \{3,4\}, \{2,4\}, \{2,3\}, \{1,4\}, \{1,3\}, \{1,2\}, \{4\}, \{3\}, \{2\}, \{1\}\}$

$int_A(A)$	$cl_A(A)$
$int_A(\{1\}) = \{1\}$	$cl_A(\{1\}) = \{1\}$
$int_A(\{2\}) = \{2\}$	$cl_A(\{2\}) = \{2\}$
$int_A(\{3\}) = \{3\}$	$cl_A(\{3\}) = X$
$int_A(\{4\}) = \{4\}$	$cl_A(\{4\}) = \{4\}$
$int_A(\{1,2\}) = \{1,2\}$	$cl_A(\{1,2\}) = \{1,2\}$
$int_A(\{1,3\}) = \{1,3\}$	$cl_A(\{1,3\}) = \{1,3\}$
$int_A(\{1,4\}) = \{1,4\}$	$cl_A(\{1,4\}) = \{1,4\}$
$int_A(\{2,3\}) = \{2,3\}$	$cl_A(\{2,3\}) = \{2,3\}$
$int_A(\{2,4\}) = \{2,4\}$	$cl_A(\{2,4\}) = \{2,4\}$
$int_A(\{3,4\}) = \{3,4\}$	$cl_A(\{3,4\}) = \{3,4\}$
$int_A(\{1,2,3\}) = \{1,2,3\}$	$cl_A(\{1,2,3\}) = \{1,2,3\}$
$int_A(\{1,2,4\}) = \{1,2,4\}$	$cl_A(\{1,2,4\}) = \{1,2,4\}$
$int_A(\{1,3,4\}) = \{1,3,4\}$	$cl_A(\{1,3,4\}) = \{1,3,4\}$
$int_A(\{2,3,4\}) = \{2,3,4\}$	$cl_A(\{2,3,4\}) = \{2,3,4\}$

$A\text{-}\bar{P}(A)$	$A\text{-}\underline{P}(A)$	$\bar{P}(A)$	$\underline{P}(A)$	المجموعة A
1/4	1/4	2/4	0/4	$\{1\}$
1/4	1/4	2/4	0/4	$\{2\}$
1/4	1/4	2/4	0/4	$\{3\}$
1/4	1/4	2/4	0/4	$\{4\}$
2/4	2/4	4/4	0/4	$\{1, 2\}$
2/4	2/4	2/4	2/4	$\{1, 3\}$
2/4	2/4	4/4	0/4	$\{1, 4\}$
2/4	2/4	4/4	0/4	$\{2, 3\}$
2/4	2/4	2/4	2/4	$\{2, 4\}$
2/4	2/4	4/4	0/4	$\{3, 4\}$
3/4	3/4	4/4	0/4	$\{1, 2, 3\}$
3/4	3/4	4/4	0/4	$\{1, 2, 4\}$
3/4	3/4	4/4	0/4	$\{1, 3, 4\}$
3/4	3/4	4/4	0/4	$\{2, 3, 4\}$

$\underline{P}(\emptyset) = 0$ $A\text{-}\underline{P}(\emptyset) = 0$

$\bar{P}(X) = 1$ $A\text{-}\bar{P}(X) = 1$

$T_8 = \{\emptyset, X, \{1,2,3\}, \{4\}\}$

$c(X) = \{X, \emptyset, \{4\}, \{1,2,3\}\}$

$A = \{1\}$
$A^\circ = \{1\}^\circ = \emptyset$
$\bar{A} = \overline{\{1\}} = \{1,2,3\} \quad \& \quad \bar{A}^\circ = \{1,2,3\}^\circ = \{1,2,3\}$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{2\}$
$A^\circ = \{2\}^\circ = \emptyset$
$\bar{A} = \overline{\{2\}} = \{1,2,3\} \quad \& \quad \bar{A}^\circ = \{1,2,3\}^\circ = \{1,2,3\}$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{3\}$
$A^\circ = \{3\}^\circ = \emptyset$
$\bar{A} = \overline{\{3\}} = \{1,2,3\} \quad \& \quad \bar{A}^\circ = \{1,2,3\}^\circ = \{1,2,3\}$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{4\}$
$A^\circ = \{4\}^\circ = \emptyset$
$\bar{A} = \overline{\{4\}} = \{4\} \quad \& \quad \bar{A}^\circ = \{4\}^\circ = \{4\}$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{1,2\}$
$A^\circ = \{1,2\}^\circ = \emptyset$
$\bar{A} = \overline{\{1,2\}} = \{1,2,3\} \quad \& \quad \bar{A}^\circ = \{1,2,3\}^\circ = \{1,2,3\}$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{1,3\}$
$A^\circ = \{1,3\}^\circ = \emptyset$
$\bar{A} = \overline{\{1,3\}} = \{1,2,3\} \quad \& \quad \bar{A}^\circ = \{1,2,3\}^\circ = \{1,2,3\}$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{1,4\}$
$A^\circ = \{1,4\}^\circ = \{4\}$
$\bar{A} = \overline{\{1,4\}} = X \quad \& \quad \bar{A}^\circ = X^\circ = X$
$\therefore A \subseteq \bar{A}^\circ$

$A = \{2,3\}$
$A^{\circ} = \{2,3\}^{\circ} = \emptyset$
$\bar{A} = \overline{\{2,3\}} = \{1,2,3\}$ & $\bar{A}^{\circ} = \{1,2,3\}^{\circ} = \{1,2,3\}$
$\therefore A \subseteq \bar{A}^{\circ}$

$A = \{2,4\}$
$A^{\circ} = \{2,4\}^{\circ} = \emptyset$
$\bar{A} = \overline{\{2,4\}} = X$ & $\bar{A}^{\circ} = X^{\circ} = X$
$\therefore A \subseteq \bar{A}^{\circ}$

$A = \{3,4\}$
$A^{\circ} = \{3,4\}^{\circ} = \emptyset$
$\bar{A} = \overline{\{3,4\}} = X$ & $\bar{A}^{\circ} = X^{\circ} = X$
$\therefore A \subseteq \bar{A}^{\circ}$

$A = \{1,2,3\}$
$A^{\circ} = \{1,2,3\}^{\circ} = \{1,2,3\}$
$\bar{A} = \overline{\{1,2,3\}} = \{1,2,3\}$ & $\bar{A}^{\circ} = \{1,2,3\}^{\circ} = \{1,2,3\}$
$\therefore A \subseteq \bar{A}^{\circ}$

$A = \{1,2,4\}$
$A^{\circ} = \{1,2,4\}^{\circ} = \emptyset$
$\bar{A} = \overline{\{1,2,4\}} = X$ & $\bar{A}^{\circ} = X^{\circ} = X$
$\therefore A \subseteq \bar{A}^{\circ}$

$A = \{1,3,4\}$
$A^{\circ} = \{1,3,4\}^{\circ} = \emptyset$
$\bar{A} = \overline{\{1,3,4\}} = X$ & $\bar{A}^{\circ} = X^{\circ} = X$
$\therefore A \subseteq \bar{A}^{\circ}$

$A = \{2,3,4\}$
$A^{\circ} = \{2,3,4\}^{\circ} = \emptyset$
$\bar{A} = \overline{\{2,3,4\}} = X$ & $\bar{A}^{\circ} = X^{\circ} = X$
$\therefore A \subseteq \bar{A}^{\circ}$

$A.o_S(X) =$
$\{\emptyset, X, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\}$
$A.c_S(X) =$
$\{X, \emptyset, \{2,3,4\}, \{1,3,4\}, \{1,2,4\}, \{1,2,3\}, \{3,4\}, \{2,4\}, \{2,3\}, \{1,4\}, \{1,3\}, \{1,2\}, \{4\}, \{3\}, \{2\}, \{1\}\}$

$int_A(A)$	$cl_A(A)$
$int_A(\{1\}) = \{1\}$	$cl_A(\{1\}) = \{1\}$
$int_A(\{2\}) = \{2\}$	$cl_A(\{2\}) = \{2\}$
$int_A(\{3\}) = \{3\}$	$cl_A(\{3\}) = \{3\}$
$int_A(\{4\}) = \{4\}$	$cl_A(\{4\}) = \{4\}$
$int_A(\{1,2\}) = \{1,2\}$	$cl_A(\{1,2\}) = \{1,2\}$
$int_A(\{1,3\}) = \{1,3\}$	$cl_A(\{1,3\}) = \{1,3\}$
$int_A(\{1,4\}) = \{1,4\}$	$cl_A(\{1,4\}) = \{1,4\}$
$int_A(\{2,3\}) = \{2,3\}$	$cl_A(\{2,3\}) = \{2,3\}$
$int_A(\{2,4\}) = \{2,4\}$	$cl_A(\{2,4\}) = \{2,4\}$
$int_A(\{3,4\}) = \{3,4\}$	$cl_A(\{3,4\}) = \{3,4\}$
$int_A(\{1,2,3\}) = \{1,2,3\}$	$cl_A(\{1,2,3\}) = \{1,2,3\}$
$int_A(\{1,2,4\}) = \{1,2,4\}$	$cl_A(\{1,2,4\}) = \{1,2,4\}$
$int_A(\{1,3,4\}) = \{1,3,4\}$	$cl_A(\{1,3,4\}) = \{1,3,4\}$
$int_A(\{2,3,4\}) = \{2,3,4\}$	$cl_A(\{2,3,4\}) = \{2,3,4\}$

المجموعة A	$\underline{P}(A)$	$\bar{P}(A)$	$A\text{-}\underline{P}(A)$	$A\text{-}\bar{P}(A)$
$\{1\}$	0/4	3/4	1/4	1/4
$\{2\}$	0/4	3/4	1/4	1/4
$\{3\}$	0/4	3/4	1/4	1/4
$\{4\}$	1/4	1/4	1/4	1/4
$\{1, 2\}$	0/4	3/4	2/4	2/4
$\{1, 3\}$	0/4	3/4	2/4	2/4
$\{1, 4\}$	0/4	3/4	2/4	2/4
$\{2, 3\}$	0/4	3/4	2/4	2/4
$\{2, 4\}$	2/4	4/4	2/4	2/4
$\{3, 4\}$	0/4	4/4	2/4	2/4
$\{1, 2, 3\}$	3/4	3/4	3/4	3/4
$\{1, 2, 4\}$	0/4	4/4	3/4	3/4
$\{1, 3, 4\}$	0/4	4/4	3/4	3/4
$\{2, 3, 4\}$	0/4	4/4	3/4	3/4

$\underline{P}(\varnothing) = 0$ $\quad$ $A\text{-}\underline{P}(\varnothing) = 0$

$\bar{P}(X) = 1$ $\quad$ $A\text{-}\bar{P}(X) = 1$

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