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Recognizing the Reliability of Parallel and Series Systems

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By the student

Ruqaya Majid Batran

Supervision

Dr. Nada Mohammed Abbas

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بسم الله الرحمن الرحيم

وَلَمَّا بَلَغَ أَشُدَّهُ وَاسْتَوَىٰ آتَيْنَاهُ حُكْمًا وَعِلْمًا وَكَذَٰلِكَ

نَجْزِي الْمُحْسِنِينَ

صدق الله العظيم

{القصص (14)}

الإهـــــــــداء

إلى من كان لي سنداً وعوناً عند الشدائد طوال عمري، إلى الرجل الأبرز في حياتي

أبي العزيـز

إلى القلب المعطاء والصدر الحاني

أمي الحبيبة

إلى من شد الله بهم عضدي فكانوا خير معين

أخواتي

الحمد الله حبا وشكرا وامتنانا ما كنت لأفعل هذا لولا فضل الله فالحمد الله على البدء والختام

قوله تعالى: { وَآخِرُ دَعْوَاهُمْ أنِ الْحَمْدُ لِلَّهِ رَبِّ الْعَالَمِينَ }.

والى د. ندى محمد عباس مشرفة هذا البحث التي لم تتوانى في مد يد العون لي.

شكر و تقدير

لم تكن هذه الورقة والبحث الذي وراءها ممكناً لولا الدعم الاستثنائي من مشرفي... لقد كان حماسه ومعرفته واهتمامه الشديد بالتفاصيل مصدر إلهام وأبقى عملي على المسار الصحيح من أول بداية حقيقية لهذا البحث وصولاً إلى قائمة المراجع.

CONTENTS

LIST OF FIGURES

ABSTRACT

In the growing age of advanced technology, the importance of the subject reliability has been felt in almost all manufacturing, engineering and management sectors as a quantitative measure of the adequate functioning of the systems. In fact, the performance of systems depends on the types of components used, their quantities and the structure in which they are arranged. In this work, reliability and mean time to failure (MTTF) of some series and parallel systems have been analyzed by considering arbitrary values of the parameters related to the number of components, operating time and failure rate of the component .

INTRODUCTION

Reliability engineers often need to work with systems having elements connected in parallel or series, and to calculate their reliability.

Many physical and non-physical systems (e.g. bridges, car engines, air-conditioning systems, biological and ecological systems, chains of command in civilian or military organizations, quality control systems in manufacturing plants, etc.) may be viewed as assemblies of many interacting elements. The elements are often arranged in mechanical or logical series or parallel configurations.

This works consists of two chapters , In chapter one The fundamental probability concepts have been studied, like the probability, mutually exclusive events, conditional probability, probability density function p.d.f., Cumulative distribution function c.d.f.

In chapter two study the basic reliability like the failure rate, hazard rate and Mean Time to Failure (MTTF). These concepts are studied to get clear idea on how to calculate the reliability of systems. Then study the simple reliability system as series and parallel system

Chapter one

Introduction and Basic Concepts

1. Basic Concept of probability distribution

1.1 Introduction [1]

Probability is used to quantify the likelihood, or chance, that an outcome of a random experiment will occur. The likelihood of an outcome is quantified by assign a number from the interval [0,1] to the outcome (or a percentage from 0 to 100%).Higher numbers indicate that the outcome is more likely than lower numbers. A zero indicates an outcome will not occur. A probability of one indicates that an outcome will occur with certainty.

Now, whenever a sample space consists of N possible outcomes that are equally likely, the probability of each outcome is $\frac{1}{N}$, while the probability of an event $A \subseteq S$ is

$$P(A) = \frac{\text{Number of outcome in A}}{\text{Total number of outcomes in the sample space}}$$

This probability is denoted by $P(A) = \frac{\text{n (A)}}{\text{n (S)}}$.

If the outcomes are not equally likely, then the probability of an event A, denoted as $P(A)$, equals the sum of the probabilities of the outcome in A.

Definition 1.2 [9] : Let P_r be a function that assigns a nonnegative real number to each event E of a sample space S i.e. $P_r: S \longrightarrow [0,1]$ We call P_r a **probability** if ,

Axiom 1: Non-negative

$$0 \leq P_r(E) \leq 1$$

Axiom 2: Total one

$$P_r(S) = 1 \text{ and } P_r(\emptyset) = 0$$

Axiom 3: : $P_r(E^c) = 1 - P_r(E)$, here E^c represent to complement of E

Axiom 4: For any pair of event $E_1, E_2 \subseteq S$,we have:

$$P_r(E_1 \cup E_2) = P_r(E_1)+P_r(E_2) - P_r(E_1 \cap E_2).$$

A probability distribution is a description of how likely a random variable or set of random variable is to take on each of its possible states. It is described by

- A probability mass function (p.m.f.)in the case of discrete variables.
- A probability density function (p.d.f.)in the case of continuous variables

Definition 1.3 [8] : Two events E_1 and E_2 are **mutually exclusive or disjoint** if $E_1 \cap E_2 = \emptyset$ that is, if A and E_2 have no elements in common.

So , for any two disjoint E_1 and E_2 we have ;

$$P_r(E_1 \cup E_2) = P_r(E_1)+P_r(E_2$$

Definition1.4[1]: The probability of an event E_2 under the knowledge that the outcome will be in event E_1 is denoted as P($E_2 \backslash E_1$), and this is called the **conditional probability** of E_2 given E_1 . So, P($E_2 \backslash E_1$) is given by

$$P_r(E_2 \backslash E_1) = \frac{P_r(E_1 \cap E_2)}{P_r(E_1)} \text{ , } for \ P(E_1) > 0$$

In case The two event E_1 and E_2 are equally likely outcomes then

$$P_r(E_2 \backslash E_1) = \frac{n(E_1 \cap E_2)}{n(E_1)}$$

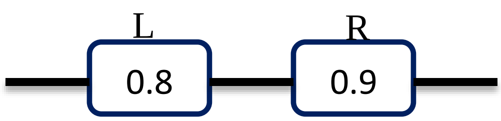
The conditional probability of E_2 given E_1 equals to zero if two events are mutually exclusive.

Definition1.5 [1] :Two event E_1 and E_2 are **independent** if any one of the following equivalent statements is true:

1) $P_r(E_2 \backslash E_1) = P_r(E_2)$
2) $P_r(E_1 \backslash E_2) = P_r(E_1)$
3) $P_r(E_1 \cap E_2) = P_r(E_1)P_r(E_2)$.

Example: (1)[9]

Circuit contained two functional device connected in series. The probability that each device functions is shown on the graph. Assume that devices fail independently.

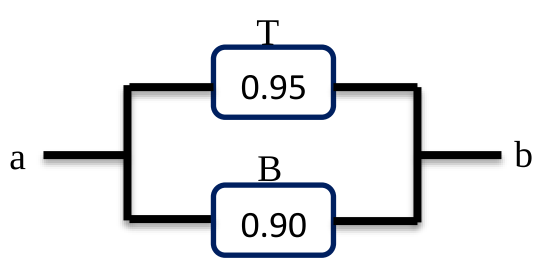


Then the probability that the circuit operates is

$P(land\ R) = P(L \cap R) = P(L)P(R) = (0.8)(0.9) = 0.72$

Example: (2)[9]

Circuit connected two functional devices connected in parallel. The probability that each device functions is shown on the graph and assume that devices fail independently.



Then the probability that the circuit operates is:

$$P(T or\ B) = P(T \cup B) = 1 - P(T \cup B)^c$$
$$= 1 - P(T^c \cap B^c)$$
$$= 1 - P(T^c)\ P(B^c)$$
$$= 1 - (1 - P(T)\ (1 - P(B))$$
$$= 1 - (1 - 0.95)\ (1 - 0.90)$$
$$= 1 - (0.05)(0.10)$$
$$= 0.995$$

Definition 1.6.[8]: The set of ordered pairs $(x, p(x))$ is a probability function, **probability mass function** (p.m.f) probability distributes of the discrete random variable X , if for each possible outcome x,

1) $p(x) \geq 0$
2) $\sum_x p(x) = 1$
3) $P_r(X = x) = p(x)$.

The cumulative distribution function (cdf) F(x) of discrete random variable X probability distribution $P_r(x)$ is ,

$$F(x) = P_r(X \leq x) = \sum_{y \leq x} p(y), for\ -\infty < x < \infty.$$

Definition 1.7.[1]: For a continuous random variable X a **probability density function** (pdf) is a function such that :

1. $f(x) \geq 0$
2. $\int_{-\infty}^{\infty} f(x)\ dx = 1$
3. $P_r(a \leq x \leq b) = \int_a^b f(x)\ dx$

The cumulative distribute function of a continuous random variable X is

$$F(x) = P_r(X \leq x) = \int_{-\infty}^{x} f(u)\ du$$

And $P_r(a < x < b) = F(b) - F(a)$

From (2.35) and (2.36) a relationship can be deduced between pdf and cdf as follows: $f(x) = \frac{dF(x)}{dx}$ if the derivative exists .

Definition 1.8.[1]: The expectation or expected value of a function $f(x)$ with respect to a probability distribution $p(x)$ is the average of $f(x)$ for $X \sim p(X)$.

For discrete variables this can be computed with a summation

$$E_{X\sim p}[f(x)] = \sum_x p(x)\, f(x)$$

For continuous variable , it is computed with an integral :

$$E_{X\sim p}[f(x)] = \int_{-\infty}^{\infty} p(x) f(x)\ dx$$

Observation 1.9.[8] :

1- An important special case is the one where $f(x) = x$, in which case we obtain the expectation of random variable X .

- For discrete variable we have : $E_{X\sim p}[x] = \sum_x p(x)\, x$
- For continuous variable we obtain: $E_{X\sim p}[x] = \int_{-\infty}^{\infty} p(x)\, x\ dx.$

2- Properties of Expectations

1) Expectations are linear :

$$E[\propto f(x) + \beta g(x)] = \alpha E[f(x)] + \beta E[g(x)],\ \ \alpha, \beta \in R$$

2) $E[\propto f(x) + \beta] = \alpha\, E[f(x)] + \beta.$

Chapter two

Reliability of Series and Parallel Systems

2.1 Reliability

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Definition 2.1.1.[2]: **A component** is a chunk of equipment of system, that it is evaluated as a separated existence, that's mean the reliability of any component does not effect by another one .

Definition 2.1.2.[4]: A system is a configuration of components that react with each other, foreign components of another systems, and operators to implement of any intentional function

Definition 2.1.3.[3] The reliability of a component (or a system) is understood as its capability to function without breakdown for a specified period of time t under a given set of operating conditions.

In other words, reliability is the probability that a component does not fail during the interval (0, t).Let a component be put into operation at some specified time t=0. Let T be its life length (i.e. the time until it fails). Then T is a continuous random variable with some probability density function f(t). Therefore the reliability or reliability function of the component at time t denoted by R(t) is defined as:

$$R(t) = P(T > t) = 1 - P(T \leq t)$$
$$= 1 - F(t)$$

Therefore, we can write the reliability function in terms of pdf as follows:

$$R(t) = 1 - \int_0^t f(x)\,dx = \int_t^\infty f(x)\,dx \qquad)$$

Conversely, we can write the pdf in terms of R(t) as follows:

$$f(t) = -\frac{d(R(t))}{dt}$$

Where F (t) $= P(T < t) = \int_0^t f(x)\,dx$ is the failure distribution function or Failure Function.

The reliability R(t) at time t , has the following properties:

1- $R(t) \in [0,1]$

2- Since $F(0) = 0, F(\infty) = 1,$ therefore

$R(0) = 1$ *and* $R(\infty) = 0$ *this implies that* $0 \le R(t) \le 1$

3- R(t) is a decreasing function of time t.

The probability of failure of a given system in a particular time interval $[t_1, t_2]$ can be written in terms of the reliability function as:

$$\int_{t_1}^{t_2} f(x)\ \ dx = \int_{t_1}^{\infty} f(x)\ \ dx - \int_{t_2}^{\infty} f(x)\ \ dx$$

$$= R(t_1) - R(t_2)$$

Using the exponential distribution, the pdf can be written in the form:

$$f(t) = \lambda e^{-\lambda t}$$

here λ is a parameter of the exponential distribution.

Therefore, the reliability function of the exponential distribution can be derived as follows:

$$R(t) = 1 - \int_0^t \lambda e^{-\lambda x} dx$$

So, the reliability function becomes as follows:

$$R(t) = e^{-\lambda t}$$

here λ is a failure rate.

Definition 2.1.4.[3]:The expected life or (expectation concept of probability theory), that is the expected time during which a component will survive and perform successfully, can be expressed as :

$$E(T) = \int_0^{\infty} t\, f(t)\, dt$$

$$= -\int_0^{\infty} t \frac{d\, R(t)}{dt}\, dt$$

$$= -t\, R(t)|_0^{\infty} + \int_0^{\infty} R(t)\, dt$$

Since $R(t = 0) = 1$ *and* $R(t = \infty) = 0$, therefore the expected life can be expressed as: $E(T) = \int_0^{\infty} R(t)\, dt$, when there is a constant failure rate,

$$E(T) = \int_0^{\infty} e^{-\lambda t}\, dt = \frac{1}{\lambda}.$$

If the system is simply replaced by a good system (i.e no maintenance required or non-repairable), the E(T) useful life is also defined as the mean time to failure which is denoted by (MTTF) and represented by $MTTF = E(T) = \frac{1}{\lambda}$

In general, $E(T) = MTTF = \int_0^{t} t\, f(t)\ dt$

$$= \int_0^{\infty} R(t)\ dt$$

2.2 Simple reliability systems

Definition 2.2.1[6,7]: A Reliability Block Diagram is often used to depict the relationship between the functioning of a system and the functioning of its components. In a reliability block diagram ,a rectangle or a circle is often used to represent a component. A reliability block diagram does not necessarily represent how the components are physically connected in the system. It only indicates how the functioning of the components will ensure the functioning of the system. That is why a reliability block diagram represents the logic relationship between the functioning of the system and the functioning of its components . Reliability block diagram have been used to represent series structures, parallel structures, series-parallel structures, and parallel- series structures. The diagrams of these structures will be given when they are introduced. However, not all systems can be represented by reliability block diagram. For example, the k-out-f-m system cannot be represented by a reliability block diagram without duplicating components. In discussions of system structures, we often use "n" to indicate the number of components in the system and each components is given a unique label from set {1,2,…,n}. The set of components in a system is denoted by C.

2.2.2 Reliability of Series Systems of “n” Identical and Independent Components [4,5]

In series system all components must be connected serially in order to make the system to perform flawlessly. Series system fails if any one of its component fails. Therefore, any weak (unreliable) component leads to the complete breakdown of the whole system[12,45,58,72] . The block diagram of series system is given below in figure 2.3

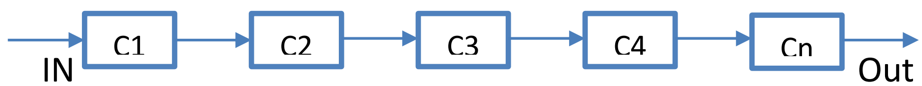

Figure 1: Block diagram for components in series

Suppose a system consists of total "n" components connected in series model. Further suppose that the ith component failure time follows the p.d.f $f(t, \lambda_i), i = 1,2, \dots, n$. The distribution of the entire system is defined by $F(t) = p(T < t)$ where T denotes the failure time of the system with $T = \min(T_1, T_2, T_3, \dots, T_n)$ (i.e) the lifetime of a series system is equal to the smallest lifetime among the lifetimes of all components, where T_i represents the failure time of the ith component. Then the cdf corresponding to the system lifetime is :

$$F_s(t) = P[T \leq t] = P[\min(T_1, T_2, T_3, \dots, T_n) \leq t]$$
$$= 1 - P[\min(T_1, T_2, T_3, \dots, T_n) > t]$$
$$= 1 - P[T_1 > t,\ T_2 > t\ , \dots, T_n > t]$$

$$= 1 - \prod_{i=1}^{n} (1 - P[T_i \leq t])$$

$$= 1 - \prod_{i=1}^{n} (1 - F_{T_i}(t))$$

on assuming the series system with" n" i.i.d. components, then

$$F_s(t) = 1 - \{1 - F(t, \lambda)\}^n$$

Similarly the system reliability

$$R_s(t) = P[T_s > t] = P[\min(T_1, T_2, T_3, \dots, T_n) > t]$$

$$= P[T_1 > t,\ T_2 > t\ , \dots, T_n > t]$$

$$= \prod_{i=1}^{n} P[T_i > t] = \prod_{i=1}^{n} R_i(t)$$

on assuming the series system with "n" i.i.d. components then we have

$$R_s(t) = \{R(t)\}^n$$

Example: [4]

Let a computer system be composed of five identical terminals in series. Let the required system reliability, for unit mission time (T = 1) be R(1) = 0.999.

We will now calculate each component's reliability, unreliability, and failure rate values. From the data and formulas just given, each terminal reliability Ri(T) can be obtained by inverting the system reliability R(T) equation for unit mission time (T = 1):.

$$R(1) = e^{-\lambda_s} = (e^{-5\lambda}) = (e^{-\lambda})^5 = [R_i(1)]^5 = 0.999$$

$$\Rightarrow R_i(1) = [R(1)]^{1/5} = (0.999)^{1/5} = 0.9998$$

Component unreliability is : Ui(1) = 1 - Ri(1) = 1 - 0.9998 = 0.0002. Component FR is obtained by solving for λ in the equation for component reliability:

$$\lambda = -\frac{\ln(R_i(T))}{T} = \frac{-\ln(0.9998)}{1} = 0.0002$$

Now, assume, that component reliability for mission time T = 1 is given: Ri(1) = 0.999. Now, we are asked to obtain total system reliability, unreliability, and FR, for the (computer) system and mission time T = 10 hours. First, for unit time:

$$R(1) = e^{-\lambda_s} = (e^{-5\lambda}) = (e^{-\lambda})^5 = [R_i(1)]^5 = (0.999)^5 = 0.995$$

Hence, system FR is:

$$\lambda_s = -\frac{\ln(R(T))}{T} = \frac{-\ln(0.995)}{1} = 0.005013$$

If we require system reliability for mission time T = 10 hours, R(10), and the unit time reliability is R(1) = 0.995, we can use either the 10th power or the FR λs:

$R(10) = e^{-10\lambda s} = (e^{-\lambda s})^{10} = [R(1)]^{10} = (0.995)^{10}$

$= e^{-10\lambda s} = (e^{-10x0.00501}) = e^{-0.05} = 0.9512$

If mission time T is arbitrary, then R(T) is called "Survival Function" (of T). R(T) can then be used to find mission time "T" that accomplishes a pre-specified reliability. Assume that R(T) = 0.98 is required and we need to find out maximum time T:

$$R(T) = e^{-\lambda_s T} = e^{-n\lambda T} = 0.98; \Rightarrow T = -\frac{\ln R(T)}{\lambda_s} = -\frac{\ln(0.98)}{0.005013} = 4.03$$

Hence, a Mission Time of T = 4.03 hours (or less) meets the requirement of reliability 0.98 (or more). Let's now assume that a new system, a ship, will be propelled by five identical engines. The system must meet a reliability requirement R(T) = 0.9048 for a mission time T = 10. We need to allocate reliability by engine (component reliability), for the required

$$R(10) = e^{-10\lambda_s} = (e^{-10x5\lambda}) = (e^{-10\lambda})^5 = [R|_i(10)]^5 = 0.9048$$

$$\Rightarrow R_i(10) = [R(10)]^{1/5} = (0.9048)^{0.2} = 0.9802$$

mission time T. We invert the formula for system reliability R(10), expressed as a function of component reliability. Then, we solve for component reliability Ri(10):

We now calculate system FR (λs) and MTTF (μ) for the fiveengine system. These are obtained for mission time T = 10 hours and required system reliability R(10) = 0.9048:

$$\lambda_s = -\frac{\ln(R(T))}{T} = \frac{-\ln(0.9048)}{10} = \frac{0.1001}{10}$$

$$= 0.010005 \Rightarrow MTTF = \mu = \frac{1}{\lambda_s} = 99.96$$

FR and MTTF values, equivalently, can be obtained using FR per component, yielding the same results:

$$\lambda = -\frac{\ln(R_i(T))}{T} = \frac{-\ln(0.9802)}{10} = \frac{0.019999}{10} = 0.0019999$$

$$\Rightarrow \lambda_s = \sum \lambda_i = 5 \text{ x } \lambda = 5 \text{ x } 0.0019999 = 0.009999 \approx 0.01$$

$$\Rightarrow MTTF = \int_0^\infty R(T)dT = \int_0^\infty e^{-\lambda_s T}\, dT = \mu = \frac{1}{\lambda_s} = 99.96$$

Finally, assume that the required ship FR $\lambda s = 5 \times \lambda = 0.010005$ is given. We now need component reliability, Unreliability and FR, by unit mission time (T = 1):

$$R(1) = \mathrm{Exp}\{-\lambda_s\} = \mathrm{Exp}\ \{-0.010005\} = 0.99$$
$$= \mathrm{Exp}\{-5 \times \lambda\} = [\mathrm{Exp}(-\lambda)]^5 = [R_i(1)]^5$$

Component reliability: $R_i(1) = [R(1)]^{1/5} = [0.99]^{0.2}$
$= 0.998$

Component unreliability: $U_i(1) = 1 - R_i(1) = 1 - 0.998$
$= 0.002$

Component FR: $\lambda = [-\ln (R(1))]/n \times 1 = [-\ln(0.99)]/5$
$= 0.002$

2.2.3. Reliability of Parallel Systems:[2,3]

A system is referred to as parallel system if its components are connected in such logic that system fails when all components have failed . Thus only one of the components is enough to operate the system satisfactorily, the block diagram is given in figure 2.4.

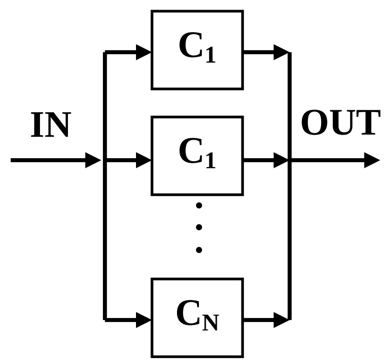

Figure 2: Block diagram for components in Parallel

Suppose, a parallel system consists of total "n" components and failure time of the ith component follows the pdf $f(t, \lambda_i), i = 1,2,3, \dots, n$. The distribution of the entire system is defined by $F(t) = P[T \le t]$, where T denotes the failure time of the system with $T = \max(T_1, T_2, \dots, T_n)$, while T_i represents the failure time of the ith component. Then the C.D.F. corresponding to the system lifetime is,

$$F_s(t) = P[T_s \le t] = P[\max(T_1, T_2, \dots, T_n) \le t]$$

$$= P[T_1 \le t\ , T_2 \le t\ , \dots, T_n \le t]$$

$$= \prod_{i=1}^{n} F_{T_i}(t)$$

On assuming the system with "n" i.i.d. components then,

$$F_s(t) = \{F_T(t)\}^n$$

Similarly, the system reliability

$$R_s(t) = P[T_s > t] = P[\max(T_1, T_2, \dots, T_n) > t]$$

$$= 1 - P[\max(T_1, T_2, \dots, T_n) \leq t]$$

$$= 1 - P[T_1 \leq t, T_2 \leq t, \dots, T_n \leq t]$$

$$= 1 - \prod_{i=1}^{n} P[T_i \leq t]$$

$$= 1 - \prod_{i=1}^{n} (1 - R_i(t))$$

On assuming the system with "n" i.i.d. components then we have

$$R_s(t) = 1 - \{1 - R(t)\}^n$$

Example: [2]

Let a parallel system be composed of n = 2 identical components, each with FR λ = 0.01 and mission time T = 10 hours, only one of which is needed for system success. Then, total system reliability, by both calculations, is:

$$R_i(10) = P\{X > 10\} = e^{-10\lambda} = e^{-0.1} = 0.9048; i = 1,2$$

$$R(10) = 1 - [1 - R_1(10)][1 - R_2(T)] = 1 - [1 - R_i(10)]^2$$

$$= 1 - (1 - 0.9048)^2 = 0.9909$$

$$R(T) = e^{-\lambda_1 T} + e^{-\lambda_2 T} - e^{-(\lambda_1 + \lambda_2)T} = 2e^{-\lambda T} - e^{-2\lambda T}$$

$$R(10) = 2e^{-10\lambda} - e^{-20\lambda} = 2e^{0.1} - e^{-0.2} = 0.9909; \text{ for } T = 10;$$

Mean Time to Failure (in hours):

$$MTTF = \mu = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2} = \frac{2}{0.01} - \frac{1}{0.02} = 150$$

The failure (hazard) rate for the two-component parallel system is now a function of T:

$$\lambda_s(T) = \frac{\lambda_1 e^{-\lambda_1 T} + \lambda_2 e^{-\lambda_2 T} - (\lambda_1 + \lambda_2) e^{-(\lambda_1 + \lambda_2)T}}{e^{-\lambda_1 T} + e^{-\lambda_2 T} - e^{-(\lambda_1 + \lambda_2)T}}$$

$$\equiv \frac{0.02e^{-0.01T} - 0.02e^{-0.02T}}{2e^{-0.01T} - e^{-0.02T}}$$

This system hazard rate $\lambda s(T)$ can be calculated as a function of any mission time T .

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