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Special matrices

Research submitted by the student

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to

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بسم الله الرحمن الرحيم

(مُتَّكِئِينَ عَلَىٰ سُرُرٍ مَّصْفُوفَةٍ
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صدق الله العلي العظيم

- الطور **20** -

الإهداء

بعض التواريخ يبقى شعورها عمراً

إنها نهايات التعب وبدايات الفرح

إلى من علمني الدراسة كفاح وجهد

إلى من سعى معي إلى تحقيق ما أريد

أهدي تخرجي إلى عائلتي الكبيرة

إلى أبي وأمي وزوجي وأولادي أعظم عائلة في الكون

شكراً لكم بحجم السماء على كل ماقدمتموه لي

الباحثة

نور كاظم

شكر وامتنان

بسم الله الرحمن الرحيم

الحمد لله رب العالمين باعث الأنبياء والمرسلين لهداية الناس أجمعين وأفضل الصلاة والسلام على خير الأنام صاحب الشريعة ومنقذ البشرية من الضلال إلى نور محمد المصطفى وعلى آله المعصومين الميامين وأصحابه المنتجبين ومن دعا بدعوتهم إلى يوم الدين.

أما بعد ..

يشرفني أن أقدم الشكر الجزيل و الامتنان إلى الدكتور (أحمد عبد علي) لما أداه من توجيهات علمية والمصداقية في ملاحظاته الدقيقة

كان لها الأثر المباشر في التوجيه إلى الوجهة العلمية الصحيحة ، فجزاه الله عني خير الجزاء.

وأتقدم بالشكر والامتنان إلى أساتذة قسم الرياضيات في جامعة بابل للعلوم الصرفة قسم الرياضيات لما أبدوا من مشورات علمية وتعليمات دقيقة ساهمت في إنجاز البحث.

Summary of the research

This research talked about matrices and their importance, especially matrices in mathematics and in public life.

The first chapter dealt with an introduction to arrays in general and the benefits and types of arrays, while the second chapter dealt with operations on the array .

Contents

Chapter One: The concept of matrices & Matrix theory

Introduction

There is no doubt that matrices are one of the important and basic matters in mathematical operations, as they resemble in their shape a rectangle with numbers inside it in rows and columns, and it is an important theory in mathematics that is used in many mathematical applications, in addition to its importance in daily life applications as it helps us to minimize errors and reach the correct results, matrices are that science that is related to electrical circuits that calculate electric current and are also used in mechanical applications with the aim of calculating forces and are also used in encryption processes and sending messages that include ciphers.

Matrices appeared in 1800 as arrays and then spread in China, European countries, and the countries of the whole world through scientists, and many researches on matrices were published in 1983, a research on matrices was published by the Japanese mathematician Seki Takazao In 1693, research related to matrices was published by the German scientist Jofried Leitens, and in 1848 the term matrix was coined by GG for an ordered set of numbers in 1855, Arthur Cayley introduced the matrix as a representation of linear elements.

A matrix is a rectangular collection of numbers, symbols, or expressions organized in columns and rows. Each element of this set is called an element or entry of the matrix.

A matrix can be defined as a specific arrangement of numbers in the form of columns and rows, and matrices are usually written in the form of a square or rectangular box, and the vertical line inside the matrix is called a column, while the horizontal line is called a row, and the size of the matrix can be expressed through the number of rows and columns it contains as follows:

Matrix size: Number of rows x Number of columns

For example, if the number of rows in a matrix is 2 and the number of columns is 3, its size is expressed as follows: 2x3 and the rows and columns are the dimensions of the matrix.

If the number of rows and columns of one matrix is equal to the number of rows and columns of another matrix, then these two matrices are considered equal in size, and the matrix can be named with any letter of the Arabic language, while in the English language, it is expressed using a capital letter, and what is inside the matrix, i.e. the elements, is expressed by writing the letter that expresses the name of the matrix, and writing the number of each row and column of that element respectively below that letter; i.e. the name of the matrix, row, column, and to illustrate this, here is the following example:

An example of entries in a matrix is 1, 9, 13, 20, 55. Any entry in a matrix is usually denoted by a small Latin letter and two small numbers below it so that the first number represents the row number and the second the column number as in the attached figure. The number of rows times the number of columns is known as the rank of the matrix or the size of the matrix. For example, a matrix with 4 rows and 3

Columns of size 4*3 and addition and subtraction can be performed on isometric matrices.

Matrices can also be multiplied in a certain order of magnitude. These operations have many of the properties of regular arithmetic, except that matrix multiplication is not commutative, and in general we can say that A.B is not equal to B.A , The one-row or one-column matrix is known as a vector.

The concept of matrices

Matrices are one of the most important keys to linear algebra. Matrices can be used to solve for linear transportation. Matrix multiplication corresponds to linear transfer of a composite function. Matrices can also track the coefficients in a system of linear equations.

A matrix can be broadly defined as a linear mathematical function that transforms a starting set (domain) into an ending set (range). The starting and ending sets can be made up of integers, decimals, or rays of numbers, and these sets can also be made up of mathematical functions or rays of mathematical functions.

Horizontal lines in a matrix are called rows and vertical lines are called columns. As for numbers are called matrix entries or matrix elements. A matrix is symbolized by a Latin letter is symbolized by a capital letter with two natural numbers below it, m and n, where m is the number of rows and n is the number of columns. Thus, a matrix is defined by the number of rows and columns of an $n \times m$ matrix, and m and n are the dimensions of the matrix. The dimensions of the matrix above are 4*3, which is 4 rows and 3 columns.

A one-column matrix is defined as an $m \times 1$ matrix and is known as a column vector. A matrix consisting of a single row and n columns is defined as n x 1 matrices and is known as a row vector.

A matrix is a table of elements, which may be real numbers or complex numbers, and may be functions, which is a mathematical form of putting numbers in a table.

Matrix theory

is a branch of mathematics that focuses on the actual study of matrices. It is considered one of the branches of linear algebra. Matrices can be defined as a two-dimensional tunnel consisting of real or decimal numbers or is an expression of a value or information by a set of columns or rows, and a table of elements that the matrix contains, which are real numbers or complex numbers, and may be functions of a rectangular set of numbers or symbols organized in columns or rows written in parentheses.

The matrix can be in square brackets or crescent brackets, and the matrix is symbolized by one of the capital English letters. A linear mathematical function that converts the starting set of any starting range into an arrival set or end range, and the starting set can be made up of integers, nodal numbers, or rays of numbers, and these two sets can also be made up of mathematical functions and can be integers or complex numbers.

The most important benefits of arrays

1- Minimizing time and effort on the programmer

2- Speed in performance

3- Reducing the size of the code

4- The ability to access values in a very quick and easy way

Types of matrices

There are several types of matrices:

- **Rectangular matrix:** It is characterized by the fact that the number of rows is different from the number of columns so that its size is equal to (number of rows * number of columns), so a matrix of order 4*3 consists of four rows and three columns, and this matrix does not have a determinant but only a reciprocal on one side

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & -2 \\ -3 & 5 & 6 \\ -4 & -5 & 7 \end{bmatrix}_{4*3}$$

$$B = \begin{bmatrix} 5 & 8 & 11 & 14 \\ 7 & 10 & 13 & 16 \\ 9 & 12 & 18 & 18 \end{bmatrix}_{3*4}$$

$$C = \begin{bmatrix} 1 & -1 \\ 5 & 2 \\ 3 & 0 \end{bmatrix}_{3*2}$$

- **Square matrix :** It is characterized by having the same number of rows and columns, which is used in computer graphics transformations and the determinant can be used for this type of matrix and its reciprocal can be calculated and its determinant can be equal to zero so that the matrix with no reciprocal is called a single matrix and its half and quarter terms are equal to the number of its rows

$$A = \begin{bmatrix} 5 & 4 & 3 \\ -4 & 0 & 4 \\ 7 & 10 & 3 \end{bmatrix}_{3*3}$$

$$B = \begin{bmatrix} 5 & 0 \\ 9 & -2 \end{bmatrix}_{2*2}$$

$$C = \begin{bmatrix} 9 & 13 & 5 & 2 \\ 1 & 11 & 7 & 6 \\ 3 & 7 & 4 & 1 \\ 6 & 0 & 7 & 10 \end{bmatrix}_{4*4}$$

- **Transpose of matrix:** for a matrix A of size m*n, if the rows of the matrix are placed in columns in order, i.e., the first row takes the position of the first column, the new matrix of size m*n resulting from this process is called the transpose of matrix A and is denoted by A' That is, the element at position (ij) in A' Element a_{ij}

Example 1 :

$$A = \begin{bmatrix} 5 & 6 \\ 4 & 3 \end{bmatrix} = A^T = \begin{bmatrix} 5 & 4 \\ 6 & 3 \end{bmatrix}_{2*2}$$

Example 2 :

$$B = \begin{bmatrix} 1 & 4 & 0 \\ 2 & 5 & 1 \\ 3 & 3 & 2 \end{bmatrix} = B^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 3 \\ 0 & 1 & 2 \end{bmatrix}_{3*3}$$

Example 3 :

$$[AB] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 12+32 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix} \quad (AB)^T = \begin{bmatrix} 19 & 43 \\ 22 & 50 \end{bmatrix}$$

- **Upper Triangular matrix:** The upper triangular matrix is defined as any square matrix whose entries of the main diagonal and above are equal to any value and is always the product of two upper triangular matrices with an upper triangular matrix, and the reciprocal of this type of matrices also produces an upper triangular matrix, and these matrices are not unique if they do not have zeros in their diagonal entries, and examples of them are

Example 1 :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \\ 0 & 0 & 2 \end{bmatrix}_{3*3}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}_{3*3}$$

Example 2 :

$$A = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & -1 \end{bmatrix}_{4*4}$$

- **Lower Triangular Matrix:** The lower triangular matrix is defined as any square matrix whose entries above the main diagonal are equal to zero while the entries of the main diagonal and below it is equal to any value, and always the product of two lower triangular matrices is always a lower triangular matrix, and the reciprocal of this type of matrices also produces a lower triangular matrix.

Example 1 :

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 2 \end{bmatrix}_{4*4}$$

Example 2 :

$$B = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 5 & 3 & 0 & 0 \\ 1 & 2 & 2 & 1 \\ 5 & 6 & 8 & 4 \end{bmatrix}_{4*4}$$

- **Identity matrix:** It is defined as a square matrix so that all its entries without falling on the diagonal are equal to zero while all the data on its diagonal extending from the extreme left to the extreme right is equal to the number 1 and it is worth noting that the product of multiplying a matrix with the identity matrix is equal to the matrix itself:

Example 1 :

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2*2} , \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3*3}$$

Example 2 :

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4*4}$$

Example 3:

$$A * I = I * A = A$$

$$A * I = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -3 & 4 \\ 1 & 5 & 2 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -3 & 4 \\ 1 & 5 & 2 \end{bmatrix}$$

$$I * A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 0 \\ 0 & -3 & 4 \\ 1 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -3 & 4 \\ 1 & 5 & 2 \end{bmatrix}$$

- **Monotone matrix:** A monotone matrix of order **n** is an **n*n** matrix in which each element is either zero or contains a number from the set {n,1}

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

- **Bimatrix** :is the union of two matrices A1 and A2 arranged in rows and columns and is written :

$$AB = A_1 \cup A_2 \qquad where \quad A_1 \neq A_2$$

With

$$A_1 = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 3 & -1 \end{bmatrix} \quad and \quad A_2 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

Band matrix:

$$A = \begin{bmatrix} x & x & 0 & 0 & 0 & 0 \\ x & x & x & 0 & 0 & 0 \\ x & x & x & x & 0 & 0 \\ x & x & x & x & x & 0 \\ 0 & x & x & x & x & x \\ 0 & 0 & x & x & x & x \\ 0 & 0 & 0 & x & x & x \\ 0 & 0 & 0 & 0 & x & x \end{bmatrix}_{8*6}$$

$$B = \begin{bmatrix} x & x & x & x & x & x & x \\ 0 & x & x & x & x & x & x \\ 0 & 0 & x & x & x & x & x \\ 0 & 0 & 0 & x & x & x & x \\ 0 & 0 & 0 & 0 & x & x & x \end{bmatrix}_{5*7}$$

$$C = \begin{bmatrix} x & x & 0 & 0 & 0 & 0 \\ x & x & x & 0 & 0 & 0 \\ x & x & x & x & 0 & 0 \\ x & x & x & x & x & 0 \end{bmatrix}_{4*6}$$

Chapter Two : Operations on Matrices

Operations on Matrices

In mathematics, there is a rectangular set of numbers, symbols, or expressions organized in columns and rows, and each element of this set is called an element or entry of the matrix, and we will talk here about a matrix with two rows or three columns.

Any entry in a matrix is usually denoted by the name of the matrix with a lowercase Latin letter and two numbers below it

so that the first number represents the row number and the second the column number like the attached figure.

The number of rows times the number of columns is known as the order of the matrix or the size of the matrix, for example a matrix with 4 rows or 3 columns of size 3*4 can be added and subtracted to isometric matrices.

Matrices can be multiplied in a certain order of magnitude, and these operations have many of the properties of the normal, except that matrix multiplication is not a commutative operation, and in general it is safe to say that A.B is not equal to B.A

Matrices consisting of one row and one column are known as one-row and one-column matrices.

- **Square matrix:**

Addition:

1- $A = \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ -2 & 5 \end{bmatrix}$

$A + B = \begin{bmatrix} 7 & 2 \\ 0 & 8 \end{bmatrix}$

2- $A = \begin{bmatrix} 1 & 4 & 3 \\ -2 & 1 & 0 \\ 6 & 7 & 10 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 & 0 \\ 6 & -1 & 2 \\ 4 & 1 & 8 \end{bmatrix}$

$A + B = \begin{bmatrix} 4 & 6 & 3 \\ 4 & 0 & 2 \\ 10 & 8 & 18 \end{bmatrix}$

Subtracting:

1- $A = \begin{bmatrix} -3 & 4 \\ -9 & -5 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 12 \\ 8 & -7 \end{bmatrix}$ $A - B = \begin{bmatrix} 1 & -8 \\ -17 & 2 \end{bmatrix}$

2- $A = \begin{bmatrix} -4 & 12 & 2 \\ 4 & -2 & 3 \\ 1 & 8 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 8 & 2 \\ 5 & 3 & 1 \\ 1 & -2 & 4 \end{bmatrix}$

$A - B = \begin{bmatrix} -10 & 4 & 0 \\ -1 & -5 & 2 \\ 0 & 6 & -6 \end{bmatrix}$

Multiplication:

1- $A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 4 \\ 2 & 5 \end{bmatrix}$ $A * B = \begin{bmatrix} -1 & 4 \\ 1 & 22 \end{bmatrix}$

2- $A = \begin{bmatrix} 5 & 9 \\ -3 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 6 & -5 \end{bmatrix}$ $A * B = \begin{bmatrix} 64 & -50 \\ -18 & 13 \end{bmatrix}$

- **Rectangular matrix**

Addition

1- $A = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$

$A + B = \begin{bmatrix} 3 & 6 & 2 \\ 6 & 9 & 3 \end{bmatrix}$

2- $A = \begin{bmatrix} 1 & 5 \\ -2 & -6 \\ 3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -5 \\ 3 & 3 \\ 4 & 7 \end{bmatrix}$

$A + B = \begin{bmatrix} 3 & 0 \\ 1 & -3 \\ 7 & 8 \end{bmatrix}$

Subtracting

1- $A = \begin{bmatrix} 2 & 4 \\ 5 & 1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 5 & 7 \end{bmatrix}$

$A - B = \begin{bmatrix} -1 & 3 \\ 3 & -3 \\ -3 & -7 \end{bmatrix}$

2- $A = \begin{bmatrix} 5 & -1 & 0 \\ 4 & 6 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 9 & 8 & -6 \\ -3 & 2 & 1 \end{bmatrix}$

$A - B = \begin{bmatrix} -4 & -9 & -6 \\ 7 & 4 & 6 \end{bmatrix}$

1- $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 & 2 \\ -4 & 1 & 0 \end{bmatrix}$

Multiplication

$$A*B=\begin{bmatrix}-8 & 1 & 2\\ 0 & 1 & -2\\ -12 & 3 & 0\end{bmatrix}$$

$$2\text{-}\ A=\begin{bmatrix}1 & 3 & -5\\ 4 & -2 & 0\end{bmatrix},B=\begin{bmatrix}2 & 3\\ -2 & -8\\ 1 & 7\end{bmatrix}$$

$$A*B=\begin{bmatrix}-9 & -56\\ 12 & 28\end{bmatrix}$$

- **Identity matrix**

Addition

Ex1:

$$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},A=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$$

$$I+A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}=\begin{bmatrix}3 & 3\\ 4 & 6\end{bmatrix}$$

Ex2:

$$I=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},\quad A=\begin{bmatrix}2 & 3 & 0\\ 1 & 4 & 2\\ 5 & 0 & 1\end{bmatrix}$$

$$I+A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}+\begin{bmatrix}2 & 3 & 0\\ 1 & 4 & 2\\ 5 & 0 & 1\end{bmatrix}=\begin{bmatrix}3 & 3 & 0\\ 1 & 5 & 2\\ 5 & 0 & 2\end{bmatrix}$$

Subtracting

$$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},B=\begin{bmatrix}2 & 4\\ 3 & 5\end{bmatrix}$$

$$I-B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}2 & 4\\ 3 & 5\end{bmatrix}=\begin{bmatrix}-1 & 4\\ 3 & -4\end{bmatrix}$$

Ex2:

$$I=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},\quad B=\begin{bmatrix}2 & 1 & 3\\ 4 & 5 & 2\\ 3 & 0 & 1\end{bmatrix}$$

$$I - B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 2 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 3 \\ 4 & -4 & 2 \\ 3 & 0 & 0 \end{bmatrix}$$

Multiplication

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$I * A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Ex2:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & 3 \\ 4 & 1 & 3 \end{bmatrix}$$

$$I * B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & 3 \\ 4 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & 3 \\ 4 & 1 & 3 \end{bmatrix}$$

- **Upper triangular matrix**

Addition

Ex1:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 2 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & -6 & 8 \\ 7 & 2 & 5 \end{bmatrix}$$

Ex2:

$$A = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 0 \\ 5 & 1 & 2 & 1 \\ 0 & 1 & 3 & 4 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 2 & 4 & 4 \\ 2 & 4 & 5 & 1 \\ 5 & 1 & 2 & 4 \\ 0 & 1 & 4 & -3 \end{bmatrix}$$

Subtracting

$$A=\begin{bmatrix}1 & 2 & 3\\0 & -1 & 4\\0 & 0 & 2\end{bmatrix},B=\begin{bmatrix}3 & 4 & 1\\5 & 0 & 1\\2 & 3 & 4\end{bmatrix}$$

$$A-B=\begin{bmatrix}1 & 2 & 3\\0 & -1 & 4\\0 & 0 & 2\end{bmatrix}-\begin{bmatrix}3 & 4 & 1\\5 & 0 & 1\\2 & 3 & 4\end{bmatrix}=\begin{bmatrix}-2 & -2 & 2\\5 & -1 & 3\\2 & 3 & -2\end{bmatrix}$$

Ex2:

$$A=\begin{bmatrix}2 & 0 & 1 & 0\\0 & 1 & 1 & 1\\0 & 0 & 0 & 3\\0 & 0 & 3 & -1\end{bmatrix},B=\begin{bmatrix}3 & 4 & 0 & 1\\2 & 3 & 1 & 2\\3 & 0 & 4 & 5\\2 & 6 & 3 & 4\end{bmatrix}$$

$$A-B=\begin{bmatrix}-1 & 4 & 1 & 1\\2 & -2 & 0 & -1\\3 & 0 & 4 & -2\\2 & 6 & 0 & -3\end{bmatrix}$$

Multiplication

Ex :

$$A=\begin{bmatrix}-1 & 2 & 3\\0 & -1 & 4\\0 & 0 & 3\end{bmatrix},B=\begin{bmatrix}1 & 2 & 3\\4 & 5 & 0\\1 & 2 & 3\end{bmatrix}$$

$$A*B=\begin{bmatrix}12 & 18 & 12\\0 & -3 & 12\\3 & 6 & 9\end{bmatrix}$$

- **Lower triangular matrix**

Addition

Ex1:

$$A=\begin{bmatrix}1 & 0 & 0\\2 & 3 & 0\\4 & 5 & 2\end{bmatrix},B=\begin{bmatrix}1 & 2 & 3\\1 & 4 & 0\\5 & 2 & 1\end{bmatrix}$$

$$A+B=\begin{bmatrix}1&0&0\\2&3&0\\4&5&2\end{bmatrix}+\begin{bmatrix}1&2&3\\1&4&0\\5&2&1\end{bmatrix}=\begin{bmatrix}2&2&3\\3&7&0\\9&7&3\end{bmatrix}$$

Ex2:

$$A=\begin{bmatrix}2&1&0&0\\5&3&0&0\\1&2&2&1\\5&6&8&7\end{bmatrix},\quad B=\begin{bmatrix}1&3&4&1\\2&0&1&3\\5&2&3&1\\0&1&2&3\end{bmatrix}$$

$$A+B=\begin{bmatrix}3&3&4&1\\7&3&1&3\\6&4&5&2\\5&7&10&10\end{bmatrix}$$

Subtracting

Ex1:

$$A=\begin{bmatrix}2&0&0\\3&1&0\\4&5&2\end{bmatrix},\quad B=\begin{bmatrix}1&3&2\\5&0&1\\3&4&2\end{bmatrix}$$

$$A-B=\begin{bmatrix}1&3&2\\-2&1&1\\1&1&0\end{bmatrix}$$

Ex2:

$$A=\begin{bmatrix}2&0&0&0\\5&3&0&0\\1&2&2&1\\5&6&8&7\end{bmatrix},B=\begin{bmatrix}1&2&3&4\\5&0&1&2\\3&1&2&4\\5&3&4&1\end{bmatrix}$$

$$A-B=\begin{bmatrix}1&2&3&4\\0&3&1&2\\-2&1&0&-3\\0&3&4&6\end{bmatrix}$$

Multiplication

Ex:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 2 \end{bmatrix} , \qquad B = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$$A * B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 21 & 7 \\ 8 & 52 & 15 \end{bmatrix}$$

- **Transpose matrix**

Addition

Ex1:

$$A = \begin{bmatrix} 5 & 6 \\ 4 & 3 \end{bmatrix} , \qquad B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 5 & 4 \\ 6 & 3 \end{bmatrix} , \qquad B^T = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$(A + B)^T = \begin{bmatrix} 7 & 5 \\ 9 & 7 \end{bmatrix}_{2*2}$$

Ex2:

$$A = \begin{bmatrix} 1 & 4 & 0 \\ 2 & 5 & 1 \\ 3 & 3 & 2 \end{bmatrix} , B = \begin{bmatrix} 3 & 4 & 0 \\ 1 & 2 & 3 \\ 5 & 2 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 3 \\ 0 & 1 & 2 \end{bmatrix} , \qquad B^T = \begin{bmatrix} 3 & 1 & 5 \\ 4 & 2 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$(A + B)^T = \begin{bmatrix} 4 & 3 & 8 \\ 8 & 7 & 5 \\ 0 & 4 & 3 \end{bmatrix}$$

Subtracting

Ex1:

$$A = \begin{bmatrix} 5 & 6 \\ 4 & 3 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 5 & 4 \\ 6 & 3 \end{bmatrix} \;,\; B^T = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

$$(A - B)^T = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}_{2*2}$$

Ex2:

$$A = \begin{bmatrix} 3 & 5 & 6 \\ 1 & 0 & 3 \\ 4 & 2 & 1 \end{bmatrix} ,B = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & 0 \\ 2 & 3 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 3 & 1 & 4 \\ 5 & 0 & 2 \\ 6 & 3 & 1 \end{bmatrix} ,$$

$$B^T = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 3 \\ 1 & 0 & 1 \end{bmatrix}$$

$$(A - B)^T = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 5 & -1 \\ 5 & 3 & 0 \end{bmatrix}_{3*3}$$

Multiplication

Ex:

$$(AB) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$(A * B)^T = \begin{bmatrix} 19 & 43 \\ 22 & 50 \end{bmatrix}$$

The Union :

$$A_B = A_1 \cup A_2 \quad ,C_B = C_1 \cup C_2$$

Two square matrices

A_B,C_B are defined as follows:

$$A_B + C_B = (A_1 \cup A_2) + (C_1 \cup C_2) = [A_1 + C_1] \cup [A_2 + C_2]$$

Wherein

$$A_1 + C_1 \ , \ A_2 + C_2$$

is Matrix addition

Ex1:

$$let\ A_B = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 3 & -1 \end{bmatrix} \cup \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

And

$$C_B = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 2 & -1 \end{bmatrix} \cup \begin{bmatrix} 3 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad \text{Find } A_B, C_B$$

Sol : $A_B + C_B = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 3 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 2 & 2 & -1 \end{bmatrix} \cup \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix}$

$$A_B + C_B = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 5 & -2 \end{bmatrix} \cup \begin{bmatrix} 3 & 4 & 0 \\ 2 & 3 & -1 \end{bmatrix}$$

Ex2:

Let $A_B = [3 \quad 2 \quad 1 \quad -4 \quad 0] \cup [0 \quad 1 \quad -1 \quad 0 \quad 1]$, $C_B = [1 \quad 1 \quad 1 \quad 1 \quad 1]$

$$\cup\, [5 \quad -1 \quad 2 \quad 0 \quad 3]$$

Sol:

$A_B + C_B =$
$[3 \quad 2 \quad 1 \quad -4 \quad 0] + [1 \quad 1 \quad 1 \quad 1 \quad 1] \cup [0 \quad 1 \quad -1 \quad 0 \quad 1] + [5 \quad -1 \quad 2 \quad 0 \quad 3]$

$$A_B + C_B = [4 \quad 3 \quad 2 \quad 3 \quad 1] \cup [5 \quad 0 \quad 1 \quad 0 \quad 4]$$

Subtracting

Consider :

$$A_B = A_1 \cup A_2 \quad , C_B = C_1 \cup C_2$$

Two square matrices

A_B,C_B are defined as follows:

$$A_B - C_B = (A_1 \cup A_2) - (C_1 \cup C_2) = [A_1 - C_1] \cup [A_2 - C_2]$$

Ex:

Let $A_B = [1 \quad 2 \quad 3 \quad -1 \quad 2 \quad 1] \cup [3 \quad -1 \quad 2 \quad 0 \quad 3 \quad 1]$

$$C_B = [-1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0] \cup [2 \quad 0 \quad -2 \quad 0 \quad 3 \quad 0]$$

Sol :

$$A_B - C_B = [1 \quad 2 \quad 3 \quad -1 \quad 2 \quad 1] - [-1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0] \\ \cup [3 \quad -1 \quad 2 \quad 0 \quad 3 \quad 1] - [2 \quad 0 \quad -2 \quad 0 \quad 3 \quad 0]$$

$$A_B - C_B = [2 \quad 1 \quad 2 \quad -2 \quad 1 \quad 1] \cup [1 \quad -1 \quad 4 \quad 0 \quad 0 \quad 1]$$

Multiplication

Consider :

$$A_B = A_1 \cup A_2 \quad , C_B = C_1 \cup C_2$$

Two square matrices

A_B,C_B are defined as follows:

$$A_B * C_B = (A_1 \cup A_2) * (C_1 \cup C_2) = [A_1 * C_1] \cup [A_2 * C_2]$$

Ex1:

$$A_B = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} \cup \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \quad , \quad C_B = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} \cup \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$$

Sol :

$$A_B * C_B = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} * \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} * \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$$

$$A_B * C_B = \begin{bmatrix} -6 & 0 \\ 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$$

$$A_B = \begin{bmatrix} 3 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} \cup \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$C_B = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix} \cup \begin{bmatrix} 2 & 0 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}$$

$$A_B * C_B = \begin{bmatrix} 3 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix} \cup \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}$$

$$A_B * C_B = \begin{bmatrix} 4 & 3 & 1 \\ 8 & -1 & -5 \\ 6 & 0 & -3 \end{bmatrix} \cup \begin{bmatrix} 5 & -1 \\ 1 & -1 \end{bmatrix}$$

- **Band matrix:**

Addition

Ex1 :

$$A = \begin{bmatrix} 1 & 3 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 0 & 0 \\ 7 & 5 \\ 2 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 3 \\ 8 & 5 \\ 3 & 3 \end{bmatrix}$$

Ex2:

$$A = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} \ , \ B = \begin{bmatrix} 4 & 0 \\ 7 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 5 & 6 \\ 7 & 4 \end{bmatrix}$$

Subtracting

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 & 4 \\ 6 & 2 \\ 0 & 7 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 0 & -4 \\ -6 & 0 \\ 0 & -7 \end{bmatrix}$$

Ex2:

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & 1 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 0 & 4 \end{bmatrix} \quad A - B = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 3 & 0 \end{bmatrix}$$

Multiplication

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A * B = \begin{bmatrix} 7 & 0 & 0 \\ -7 & 0 & 3 \end{bmatrix}$$

Conclusions

1- Explain the basic concepts of special matrices and their role in solving mathematical problems.

2- The importance of special matrices in solving mathematical problems and other fields

3- Continuous practice in solving problems facilitates the selection of the appropriate method for solving equations

4- Special matrices are characterized in solving equations, including minimizing the time and effort on the programmer, speed in performance, and the possibility of accessing values in a very quick and easy way.

Recommendations

1- Expanding matrices and equations from the higher grades, explaining the importance of linear equations

2- Explain that solving linear equations and matrices progresses from easy to difficult

3- Indicate the applications in which the equations are solved to emphasize the importance and role of mathematics, especially special matrices

List of sources ل

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