وزارة التعليم العالي والبحث العلمي

جامعة بابل

كلية التربية للعلوم الصرفة

قسم الرياضيات

Rough approximation probability of Regular open sets

بحث مقدم من قبل الطالب (منتظر سلام جاسم)

الى عمادة كلية التربية للعلوم الصرفة – قسم الرياضيات

كجزء من متطلبات نيل شهادة البكالوريوس في علوم الرياضيات

اشراف الأستاذ الدكتور

لؤي عبد الهاني السويدي

1447هـ 2026م

الإهداء

إلى من كان لهم الفضل الأول في مسيرتي، إلى من تعبوا وسهروا لأجلي...

إلى أبي الغالي وأمي الحبيبة، أهدي ثمرة جهدي وتعبي، فأنتم سرّ النجاح وسبب الوصول.

إلى إخوتي وأخواتي، سندي وقوتي في هذه الحياة...

إلى كل من دعمني ووقف بجانبي في لحظات التعب واليأس.

إلى أساتذتي الكرام، الذين أناروا لي طريق العلم...

إلى أصدقائي الذين شاركوني هذه الرحلة بكل تفاصيلها.

إلى كل من علّمني حرفًا، وإلى كل قلب تمنى لي الخير...

أهدي هذا العمل المتواضع .

الشكر والتقدير

الحمدُ لله الذي وفقني لإتمام هذا العمل، ونسأله أن يكون خالصًا لوجهه الكريم.

أتقدم بجزيل الشكر وعظيم الامتنان إلى أستاذي المشرف(د لؤي عبد الهاني السويدي) لما قدمه من توجيهات قيّمة ودعم مستمر كان له الأثر الكبير في إنجاز هذا البحث.

كما أتقدم بالشكر إلى أساتذتي الكرام في قسم الرياضيات، لما بذلوه من جهود مباركة في بناء مسيرتي العلمية.

ولا يفوتني أن أعبّر عن بالغ شكري وامتناني لعائلتي الكريمة، التي كانت السند الحقيقي والداعم الأول لي في كل خطوة.

وفي الختام، أشكر كل من ساهم وساعد في إنجاح هذا العمل، وأسأل الله التوفيق للجميع .

المحتويات

المستخلص:

احتساب احتمالية التقريب الاولي Rough approximation probability

لثمان تبولجيات معرفة على المجموعة X التي تحتوي على ثمان عناصر بعد استخراج interior وclosure لجميع المجاميع الجزئية الى X والتي تكون من 16 مجموعة واحتساب $\underline{P}(A)$ و $\overline{P}(A)$ و كذلك استخراج عائله جميع المجاميع R-open و R-closed ثم بعد ذلك استخراج R-interior و R-closure لمجاميع الجزئية الى X واحتساب R.$\underline{\mathrm{P}}(\mathrm{A})$ و R.$\overline{P}(A)$ ، حيث

$$R.\underline{P}(A) = \frac{|R.\underline{R}(A)|}{|X|} = \frac{|R.int(A)|}{|X|}$$

$$R.\overline{P}(A) = \frac{|R.\overline{R}(A)|}{|X|} = \frac{|R.cl(A)|}{|X|}$$

Introduction

Pawlak introducedion approximation spaces during the early 1980 as part of his research on classifying objects by means of their featurs [4]. Rough set theory introduced by Pawlak in 1982, as an extension of set theary, mainly in the domain of intelligent systems [3] -Rough Set theory is a mathematical tool to deal is with vagueness and incomplete information or imprecise by dividing these data into equivalenc classes using equivalence relations which resul from the same data [5, 6]. Therefore it is relate to the concept of approximation. This .theory wa applied successfully in several application

e.g. information analysis, data analysis and Knowledge discovery [3].

Definition 1-1 [7] let X be any set the family T of subsets of X is called topology if satisfy Thy following properties

1- ø,X ∈ T

2- If A,B ∈ $T \rightarrow A \cap B \in T$

3-for any index Λ, and $A_\lambda \in T \rightarrow \bigcup_{\lambda\varepsilon} A_\lambda \epsilon T$

The pair (X,T) is called topological space.

Definition 1-2 [7]

Let (X,T) be topological space, x ∈ X is called interior of A iff .∃ G∈T ∋ x ∈G ⊆A and The set A is called nbd of x

The set of all interior points of A is denoted by int(A) or A° = ∪ $\{\mathrm{G} \in \mathrm{T}; \mathrm{G} \subseteq A\}$

Definition 1-3 [7]

let (X,T) be topological space , x∈ X is called adherent point of a set A iff $\forall G \in T, x \in G$ Such that $G \cap A \neq \emptyset$. The set of all adherent points of A is called closure of A and is denoted by $\overline{A}$ or cl(A)

- Also cl(A) = ∩ $\{F \text{ } is \text{ } ciosed \text{ }, A \subseteq F\}$

- $\overline{A} = A^{c^{\circ c}}$ and $A^\circ = A^{c^{-c}}$

Regular-open

Definition 1-4 [1]. let (X,T) be a topological space and A⊆X . A is said to be regular open (R-open) iff A=$\overline{A}^{\circ}$

-The family of all R. open sets in X is denoted by R.0. (X)

-The complement of R-open is called R-closed and the family of all R-closed sets in X is denoted by R.C.(X)

Note 1-5 [1]

1- Each R-open is open but the converse may be not true

2-The intersection of any two R-open is R-open

3. The family R.O.(X) is a base to generated topology.

Definition 1-6 [1]. let (X,T) be a topological space, A⊆X.

A point x∈X is called R-adherent point of A iff $\forall G \in R.O.(X)$, x∈ G such that G∩A≠ ∅ The set of all

R-adherent points is called R-closure of A and is denoted by R-cl(A)

Theonem 1-7 [2] let (X.T) be a topological space and A⊆X. Then the following are true

1- R-cl(A) =∩ {F ∈ R.C.(X); A ⊆F }

2-If B ∈ R.C.(X), then B = R.cl(B)

3-Cl(B) ⊆ R.cl(B).

Definition 1-8 [2]. let (X,T) be a topological space and A⊆X. A point x ∈A is called R-interior point of A iff ∃ G ∈ R.O.(X) Such that x∈G⊆A and A is called R-nhd of X.

-The set of all R-interior points of A is called the R-interior of A and denoted by R.int (A)

Theorem 1-9 [2]. let (X.T) be a topological space, then the .following properties are true.

1- R.int(B)=U{G∈R.O.(X); G⊆ A}

2- R.int(B) ⊆ int(B)

3- If B∈R.O.(X), then B=R.int(B).

Note 1- 10 [1] let (X.T) be topological space And A⊆X

-The closure of A is $\overline{A}$=cl (A) = ∩ {F is closed ;A⊆F}

- The interior of A is A°=int(A)=U{G∈T; G⊆A}

The intersection of any two R-open sets is not necessary to be R.open

The union of any s-open sets is R-open because

$$\bigcup_{\lambda\in\Lambda} A_\lambda \subseteq \bigcup_{\lambda\in\Lambda} \bar{A}_\lambda^{\circ} \subseteq (\bigcup_{\lambda\epsilon} A_\lambda)^{\bar{\circ}}$$

The family of R-open sets R.O.(X) is Subbase.

Definition 1-11 [2] let K=(X, R) be an approximation Space, with general relation R and T_k is the topology associated to K. Then the triph (X, R.T_K) is called a topological approximation space

-T_K is generated by sub base X/R is general relation.
X/R={xR;∀x∈R} xR={y∈x,xRy}

Example 1-12 let X = {a,b,c,d} X/R = {{c},{b,d}} is sub base, so the base = {Ø,X,{c},{b,d}}

T_K={Ø,X,{c},{b,d},{c,b,d}}

١- اضافة لجميع التقاطعات المنتهية الى Sub base سوف تحصل على base

٢- اضافه جميع الاتحادات الى base لكي تحصل على التبولوجي T_K

Definition 1.13 [2]

The Lower approximation is $\underline{R}$A= int(A)

R-$\underline{R}$A=R.int (A)

-The upper approximation is $\overline{R}$A = cl(A)

R.$\overline{R}A$ = R.Cl(A) .

-The Rough Probability of P^*(A)=($\underline{P}$(A),$\overline{P}$(A))

$$\underline{P}(A) = \frac{The\ cardinal\ of\ \underline{R}(A)}{|X|} = \frac{|\underline{R}(A)|}{|X|}$$

$$\overline{P}(A) = \frac{The\ cardinal\ of\ \overline{R}(A)}{|X|} = \frac{|\overline{R}(A)|}{|X|}$$

X={1,2,3,4}

P(X)={ø,X,{1},{2},{3},{4},{1,2},{1,3},{1,4},{2,3},{2,4},

{3,4},{1,3,4},{1,2,4},{1,2,3},{2,3,4}}

T_1={ø,X,{1},{2},{1,2}}

C(x)={X,ø,{2,3,4},{1,3,4},{3,4}}

A={1}→ $\{1\}^{\circ} = \{1\}$

$\overline{\{1\}} = \{1,3,4\} \rightarrow \overline{\{1\}}^{\circ} = \{1,3,4\}^{\circ} = \{1\} \rightarrow \mathrm{A} = \overline{A}^{\circ}$

A={2}→ $\{2\}^{\circ} = \{2\}$

$\overline{\{2\}} = \{2,3,4\} \rightarrow \overline{\{2\}}^{\circ} = \{2,3,4\}^{\circ} = \{2\} \rightarrow A = \overline{A}^{\circ}$

A={3}→ $\{3\}^{\circ} = \emptyset$

$\overline{\{3\}} = \{3,4\} \rightarrow \overline{\{3\}}^{\circ} = \{3,4\}^{\circ} = \emptyset \rightarrow \mathrm{A} \neq \overline{A}^{\circ}$

A={4}→ $\{4\}^{\circ} = \emptyset$

$\overline{\{4\}} = \{3,4\} \rightarrow \overline{\{4\}}^{\circ} = \{3,4\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

A={1,2}→ $\{1,2\}^{\circ} = \{1,2\}$

$\overline{\{1,2\}} = X \rightarrow \overline{\{1,2\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={1,3}→ $\{1,3\}^{\circ} = \{1\}$

$\overline{\{1,3\}} = \{1,3,4\} \rightarrow \overline{\{1,3\}}^{\circ} = \{1,3,4\}^{\circ} = \{1\} \rightarrow A \neq \overline{A}^{\circ}$

A={1,4}→ $\{1,4\}^{\circ} = \{1\}$

$\overline{\{1,4\}} = \{1,3,4\} \rightarrow \overline{\{1,4\}}^{\circ} = \{1,3,4\}^{\circ} = \{1\} \rightarrow A \neq \overline{A}^{\circ}$

A={2,3}→ $\{2,3\}^{\circ} = \{2\}$

$\overline{\{2,3\}} = \{2,3,4\} = \overline{\{2,3\}}^{\circ} = \{2,3,4\}^{\circ} = \{2\} \rightarrow A \neq \overline{A}^{\circ}$

A={2,4}→ $\{2,4\}^{\circ} = \{2\}$

$\overline{\{2,4\}} = \{2,3,4\} = \overline{\{2,4\}}^{\circ} = \{2,3,4\}^{\circ} = \{2\} \rightarrow A \neq \overline{A}^{\circ}$

A={3,4}→ $\{3,4\}^{\circ} = \emptyset$

$\overline{\{3,4\}} = \{3,4\} \rightarrow \overline{\{3,4\}}^{\circ} = \{3,4\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

A={1,3,4}→ $\{1,3,4\}^{\circ} = \{1\}$

$\overline{\{1,3,4\}} = \{1,3,4\} = \overline{\{1,3,4\}}^{\circ} = \{1,3,4\}^{\circ} = \{1\} \rightarrow A \neq \overline{A}^{\circ}$

A={1,2,4}→ $\{1,2,4\}^{\circ} = \{1,2\}$

$\overline{\{1,2,4\}} = X \rightarrow \overline{\{1,2,4\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={1,2,3}→ $\{1,2,3\}^{\circ} = \{1,2\}$

$\overline{\{1,2,3\}} = X = \overline{\{1,2,3\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={2,3,4}→ $\{2,3,4\}^{\circ} = \{2\}$

$\overline{\{2,3,4\}} = \{2,3,4\} \rightarrow \overline{\{2,3,4\}}^{\circ} = \{2,3,4\}^{\circ} = \{2\} \rightarrow A \neq \overline{A}^{\circ}$

$R.O_{1}.(X)=\{\emptyset,X,\{1\},\{2\}\}$

$R.C_{1.}(X)=\{X,\emptyset,\{2,3,4\},\{1,3,4\}\}$

$Int_R(\{1\})=\{1\}$	$Cl_R(\{1\})=\{1,3,4\}$
$Int_R(\{2\})=\{2\}$	$Cl_R(\{2\})=\{2,3,4\}$
$Int_R(\{3\})=\emptyset$	$Cl_R(\{3\})=\{3,4\}$
$Int_R(\{4\})=\emptyset$	$Cl_R(\{4\})=\{3,4\}$
$Int_R(\{1,2\})=\{1,2\}$	$Cl_R(\{1,2\})=X$
$Int_R(\{1,3\})=\{1\}$	$Cl_R(\{1,3\})=\{1,3,4\}$
$Int_R(\{1,4\})=\{1\}$	$Cl_R(\{1,4\})=\{1,3,4\}$
$Int_R(\{2,3\})=\{2\}$	$Cl_R(\{2,3\})=\{2,3,4\}$
$Int_R(\{2,4\})=\{2\}$	$Cl_R(\{2,4\}=\{2,3,4\}$
$Int_R(\{3,4\})=\emptyset$	$Cl_R(\{3,4\})=\{3,4\}$
$Int_R(\{1,3,4\})=\{1\}$	$Cl_R(\{1,3,4\})=\{1,3,4\}$
$Int_R(\{1,2,4\})=\{1,2\}$	$Cl_R(\{1,2,4\})=X$
$Int_R(\{1,2,3\})=\{1,2\}$	$Cl_R(\{1,2,3\})=X$
$Int_R(\{2,3,4\})=\{2\}$	$Cl_R(\{2,3,4\})=\{2,3,4\}$
$Int_R(X)=X$	$Cl_R(X)=x$
$Int_R(\emptyset)=\emptyset$	$Cl_R(\emptyset)=\emptyset$

A	{1}	{2}	{3}	{4}	{1,2}	{1,3}	{1,4}	{2,3}
$\underline{P}(A)$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\overline{P}(A)$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	1	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
R.$\underline{P}(A)$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
R.$\overline{P}(A)$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	1	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$

A	{2,4}	{3,4}	{1,3,4}	{1,2,4}	{1,2,3}	{2,3,4}	X	ø
$\underline{P}(A)$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	1	0
$\overline{P}(A)$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	1	$\frac{3}{4}$	1	0
R.$\underline{P}(A)$	$\frac{2}{4}$	0	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	1	0
R.$\overline{P}(A)$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	1	$\frac{3}{4}$	1	0

$T_2 = \{\emptyset, X, \{1\}, \{1,2\}, \{1,3,4\}\}$

$C_2(X) = \{X, \emptyset, \{2,3,4\}, \{3,4\}, \{2\}\}$

$A = \{1\} \rightarrow \{1\}^{\circ} = \{1\}$

$\overline{\{1\}} = X \rightarrow \overline{\{1\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

$A = \{2\} \rightarrow \{2\}^{\circ} = \emptyset$

$\overline{\{2\}} = \{2\} \rightarrow \overline{\{2\}}^{\circ} = \{2\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

$A = \{3\} \rightarrow \{3\}^{\circ} = \emptyset$

$\overline{\{3\}} = \{3,4\} \rightarrow \overline{\{3\}}^{\circ} = \{3,4\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

$A=\{4\} \rightarrow \{4\}^{\circ} = \emptyset$

$\overline{\{4\}} = \{3,4\} \rightarrow \overline{\{4\}}^{\circ} = \{3,4\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

$A = \{1,2\} \rightarrow \{1,2\}^{\circ} = \{1,2\}$

$\overline{\{1,2\}} = X \rightarrow \overline{\{1,2\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

$A=\{1,3\} \rightarrow \{1,3\}^{\circ} = \{1\}$

$\overline{\{1,3\}} = X \rightarrow \overline{\{1,3\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

$A = \{1,4\} \rightarrow \{1,4\}^{\circ} = \{1\}$

$\overline{\{1,4\}} = X \rightarrow \overline{\{1,4\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

$A = \{2,3\} \rightarrow \{2,3\}^{\circ} = \emptyset$

$\overline{\{2,3\}} = \{2,3,4\} \to \overline{\{2,3\}}^{\circ} = \{2,3,4\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={2,4}→ $\{2,4\}^{\circ} = \emptyset$

$\overline{\{2,4\}} = \{2,3,4\} \to \overline{\{2,4\}}^{\circ} = \{2,3,4\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={3,4}→ $\{3,4\}^{\circ} = \emptyset$

$\overline{\{3,4\}} = \{3,4\} \to \overline{\{3,4\}}^{\circ} = \{3,4\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

$\mathrm{A} = \{1,3,4\} \to \{1,3,4\}^{\circ} = \{1,3,4\}$

$\overline{\{1,3,4\}} = \mathrm{X} \to \overline{\{1,3,4\}}^{\circ} = \mathrm{X}^{\circ} = \mathrm{X} \to \mathrm{A} \neq \overline{\mathrm{A}}^{\circ}$

A={1,2,4}→ $\{1,2,4\}^{\circ} = \{1,2\}$

$\overline{\{1,2,4\}}$=X→ $\overline{\{1,2,4\}}^{\circ} = X^{\circ} = X \to A \neq \overline{A}^{\circ}$

A={1,2,3}→ $\{1,2,3\}^{\circ} = \{1,2\}$

$\overline{\{1,2,3\}} = X \to \overline{\{1,2,3\}}^{\circ} = X^{\circ} = X \to A \neq \overline{A}^{\circ}$

$A = \{2,3,4\} \to \{2,3,4\}^{\circ} = \emptyset$

$\overline{\{2,3,4\}} = \{2,3,4\} \to \overline{\{2,3,4\}}^{\circ} = \{2,3.4\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

R.O_2 (x)={$\emptyset, X$}

R. $C_2(x) = \{X, \emptyset\}$

$\text{Int}_R(\{1\}) = \emptyset$	Cl_R ({1})=X
Int_R ({2})=ø	Cl_R ({2})=X
Int_R ({3})= ø	Cl_R ({3})=X
Int_R ({4})=ø	Cl_R ({4})=X
Int_R ({1,2})=ø	Cl_R ({1,2})=X
Int_R ({1,3})=ø	Cl_R ({1,3})=X
Int_R ({1,4})=ø	Cl_R ({1,4})=X
Int_R ({2,3})=ø	Cl_R ({2,3})=x
Int_R ({2,4})=ø	Cl_R ({2,4})=X
Int_R ({3,4})=ø	Cl_R ({3,4})=X
Int_R ({1,3,4})=ø	Cl_R ({1,3,4})=X
Int_R ({1,2,4})=ø	Cl_R ({1,2,4})=X
Int_R ({1,2,3})=ø	Cl_R ({1,2,3})=x
Int_R ({2,3,4})=ø	Cl_R ({2,3,4})=x
Int_R (X)=X	Cl_R (x)=X
Int_R (ø)=ø	Cl_R (ø)=ø

A	{1}	{2}	{3}	{4}	{1,2}	{1,3}	{1,4}	{2,3}
$\underline{P}(A)$	$\frac{1}{4}$	0	0	0	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0
$\overline{P}(A)$	1	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	1	1	1	$\frac{3}{4}$
R.$\underline{P}(A)$	0	0	0	0	0	0	0	0
R.$\overline{P}(A)$	1	1	1	1	1	1	1	1

A	{2,4}	{3,4}	{1,3,4}	{1,2,4}	{1,2,3}	{2,3,4}	X	ø
$\underline{P}(A)$	0	0	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	0	1	0
$\overline{P}(A)$	$\frac{3}{4}$	$\frac{2}{4}$	1	1	1	$\frac{3}{4}$	1	0
R.$\underline{P}(A)$	0	0	0	0	0	0	1	0
R.$\overline{P}(A)$	1	1	1	1	1	1	1	0

$T_3=\{ø,X,\{1\},\{1,4\},\{1,2,4\},\{1,3,4\}\}$

$C_3(x)=\{ø,X,\{2,3,4\},\{2,3\},\{3\},\{2\}\}$

$A=\{1\} \rightarrow \{1\}^{\circ} = \{1\}$

$\overline{\{1\}} = X \rightarrow \overline{\{1\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

$A=\{2\} \rightarrow \{2\}^{\circ} = \emptyset$

$\overline{\{2\}} = \{2\} \rightarrow \overline{\{2\}}^{\circ} = \{2\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

$A=\{3\} \rightarrow \{3\}^{\circ} = \emptyset$

$\overline{\{3\}} = \{3\} \rightarrow \overline{\{3\}}^{\circ} = \{3\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

$A=\{4\} \rightarrow \{4\}^{\circ} = \emptyset$

$\overline{\{4\}} = \{2,3,4\} \rightarrow \overline{\{4\}}^{\circ} = \{2,3,4\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

$A=\{1,2\} \rightarrow \{1,2\}^{\circ} = \{1\}$

$\overline{\{1,2\}} = X \rightarrow \overline{\{1,2\}}^{\circ} = X^{\circ} = x \rightarrow A \neq \overline{A}^{\circ}$

$A=\{1,3\} \rightarrow \{1,3\}^{\circ} = \{1\}$

$\overline{\{1,3\}} = X \rightarrow \overline{\{1,3\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

$A=\{1,4\} \rightarrow \{1,4\}^{\circ} = \{1,4\}$

$\overline{\{1,4\}} = X \rightarrow \overline{\{1,4\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={2,3}→ $\{2,3\}^{\circ} = \emptyset$

$\overline{\{2,3\}} = \{2,3\} \rightarrow \overline{\{2,3\}}^{\circ} = \{2,3\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

A={2,4}→ $\{2,4\}^{\circ} = \emptyset$

$\overline{\{2,4\}} = \{2,3,4\} \rightarrow \overline{\{2,4\}}^{\circ} = \{2,3,4\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

A={3,4}→ $\{3,4\}^{\circ} = \emptyset$

$\overline{\{3,4\}} = \{2,3,4\} \rightarrow \overline{\{3,4\}}^{\circ} = \{2,3,4\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

A={1,2,3}→ $\{1,2,3\}^{\circ} = \{1\}$

$\overline{\{1,2,3\}} = X \rightarrow \overline{\{1,2,3\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={1,3,4}→ $\{1,3,4\}^{\circ} = \{1,3,4\}$

$\overline{\{1,3,4\}} = X \rightarrow \overline{\{1,3,4\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={1,2,4}→ $\{1,2,4\}^{\circ} = \{1,2,4\}$

$\overline{\{1,2,4\}} = X \rightarrow \overline{\{1,2,4\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={2,3,4}→ $\{2,3,4\}^{\circ} = \emptyset$

$\overline{\{2,3,4\}} = \{2,3,4\} \rightarrow \overline{\{2,3,4\}}^{\circ} = \{2,3,4\}^{\circ} = \emptyset \rightarrow A = \overline{A}^{\circ}$

$R.O_3(x)=\{\emptyset,X\}$

$R.C_3(X)=\{\emptyset,X\}$

$int_R(\{1\})=ø$	$cl_R(\{1\})=X$
$int_R(\{2\})=ø$	$cl_R(\{2\})=X$
$int_R(\{3\})=ø$	$cl_R(\{3\})=X$
$int_R(\{4\})=ø$	$cl_R(\{4\})=X$
$int_R(\{1,2\})=ø$	$cl_R(\{1,2\})=X$
$int_R(\{1,3\})=\emptyset$	$cl_R(\{1,3\})=X$
$int_R(\{1,4\})=ø$	$cl_R(\{1,4\})=X$
$int_R(\{2,3\})=ø$	$cl_R(\{2,3\})=X$
$int_R(\{2,4\})=ø$	$cl_R(\{2,4\})=X$
$int_R(\{3,4\})=ø$	$cl_R(\{3,4\})=X$
$int_R(\{1,3,4\})=ø$	$cl_R(\{1,3,4\})=X$
$int_R(\{1,2,4\})=ø$	$cl_R(\{1,2,4\})=X$
$int_R(\{1,2,3\})=ø$	$cl_R(\{1,2,3\})=X$
$int_R(\{2,3,4\})=ø$	$cl_R(\{2,3,4\})=X$
$int_R(x)=X$	$cl_R(X)=X$
$int_R(ø)=ø$	$cl_R(ø)=ø$

A	{1}	{2}	{3}	{4}	{1,2}	{1,3}	{1,4}	{2,3}
$\underline{P}(A)$	$\frac{1}{4}$	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	0
$\overline{P}(A)$	1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	1	1	1	$\frac{2}{4}$
R.$\underline{P}(A)$	0	0	0	0	0	0	0	0
R.$\overline{P}(A)$	1	1	1	1	1	1	1	1

A	{2,4}	{3,4}	{1,3,4}	{1,2,4}	{1,2,3}	{2,3,4}	x	Ø
$\underline{P}(A)$	0	0	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	0	1	0
$\overline{P}(A)$	$\frac{3}{4}$	$\frac{3}{4}$	1	1	1	$\frac{3}{4}$	1	0
R.$\underline{P}(A)$	0	0	0	0	0	0	1	0
R.$\overline{P}(A)$	1	1	1	1	1	1	1	0

T_4={ø,X,{1},{1,4},{1,2,4}}

C_4={ø,X,{2,3,4},{2,3},{3}}

A={1}→ $\{1\}^\circ = \{1\}$

$\overline{\{1\}} = X \rightarrow \overline{\{1\}}^\circ = X^\circ = X \rightarrow A \neq \overline{A}^\circ$

A={2}→ $\{2\}^\circ = \emptyset$

$\overline{\{2\}} = \{2,3\} \rightarrow \overline{\{2\}}^\circ = \{2,3\}^\circ = \emptyset \rightarrow A \neq \overline{A}^\circ$

A={3}→ $\{3\}^\circ = \emptyset$

$\overline{\{3\}} = \{3\} \rightarrow \overline{\{3\}}^\circ = \{3\}^\circ = \emptyset \rightarrow A \neq \overline{A}^\circ$

A={4}→ $\{4\}^\circ = \emptyset$

$\overline{\{4\}} = \{2,3,4\} \rightarrow \overline{\{4\}}^\circ = \{2,3,4\}^\circ = \emptyset \rightarrow A \neq \overline{A}^\circ$

A={1,2}→ $\{1,2\}^\circ = \{1\}$

$\overline{\{1,2\}} = X \rightarrow \overline{\{1,2\}}^\circ = X^\circ = X \rightarrow A \neq \overline{A}^\circ$

A={1,3}→ $\{1,3\}^\circ = \{1\}$

$\overline{\{1,3\}} = X \rightarrow \overline{\{1,3\}}^\circ = X^\circ = X \rightarrow A \neq \overline{A}^\circ$

A={1,4}→ $\{1,4\}^\circ = \{1,4\}$

$\overline{\{1,4\}} = X \rightarrow \overline{\{1,4\}}^\circ = X^\circ = X \rightarrow A \neq \overline{A}^\circ$

A={2,3}→ $\{2,3\}^{\circ} = \emptyset$

$\overline{\{2,3\}} = \{2,3\} \to \overline{\{2,3\}}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={2,4}→ $\{2,4\}^{\circ} = \emptyset$

$\overline{\{2,4\}} = \{2,3,4\} \to \overline{\{2,4\}}^{\circ} = \{2,3,4\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={3,4}→ $\{3,4\}^{\circ} = \emptyset$

$\overline{\{3,4\}} = \{2,3,4\} \to \overline{\{3,4\}}^{\circ} = \{2,3,4\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={1,3,4}→ $\{1,3,4\}^{\circ} = \{1,4\}$

$\overline{\{1,3,4\}} = X \to \overline{\{1,3,4\}}^{\circ} = X^{\circ} = X \to A \neq \overline{A}^{\circ}$

A={1,2,4}→ $\{1,2,4\}^{\circ} = \{1,2,4\}$

$\overline{\{1,2,4\}} = X \to \overline{\{1,2,4\}}^{\circ} = X^{\circ} = X \to A \neq \overline{A}^{\circ}$

A={1,2,3}→ $\{1,2,3\}^{\circ} = \{1\}$

$\overline{\{1,2,3\}} = X \to \overline{\{1,2,3\}}^{\circ} = X^{\circ} = X \to A \neq \overline{A}^{\circ}$

A={2,3,4}→ $\{2,3,4\}^{\circ} = \emptyset$

$\overline{\{2,3,4\}} = \{2,3,4\} \to \overline{\{2,3,4\}}^{\circ} = \{2,3,4\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

R.O$_4$(x)={ø,X}

R.C$_4$(x)={ø,X}

$int_R(\{1\})=ø$	$cl_R(\{1\})=X$
$int_R(\{2\})=ø$	$Cl_R(\{2\})=X$
$int_R(\{3\})=ø$	$Cl_R(\{3\})=X$
$int_R(\{4\})=ø$	$Cl_R(\{4\})=X$
$int_R(\{1,2\})=ø$	$Cl_R(\{1,2\})=X$
$int_R(\{1,3\})=ø$	$Cl_R(\{1,3\})=X$
$Int_R(\{1,4\})=ø$	$Cl_R(\{1,4\})=X$
$Int_R(\{2,3\})=ø$	$Cl_R(\{2,3\})=X$
$Int_R(\{2,4\})=ø$	$Cl_R(\{2,4\})=X$
$Int_R(\{3,4\})=ø$	$Cl_R(\{3,4\})=X$
$Int_R(\{1,3,4\})=ø$	$Cl_R(\{1,3,4\})=X$
$Int_R(\{1,2,4\}=ø$	$Cl_R(\{1,2,4\})=X$
$Int_R(\{1,2,3\})=ø$	$Cl_R(\{1,2,3\})=X$
$Int_R(\{2,3,4\})=ø$	$Cl_R(\{2,3,4\})=X$
$Int_R(X)=X$	$Cl_R(x)=X$
$Int_R(ø)=ø$	$Cl_R(ø)=ø$

A	{1}	{2}	{3}	{4}	{1,2}	{1,3}	{1,4}	{2,3}
$\underline{P}(A)$	$\frac{1}{4}$	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	0
$\overline{P}(A)$	1	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	1	1	1	$\frac{2}{4}$
R.$\underline{P}(A)$	0	0	0	0	0	0	0	0
R.$\overline{P}(A)$	1	1	1	1	1	1	1	1

A	{2,4}	{3,4}	{1,3,4}	{1,2,4}	{1,2,3}	{2,3,4}	x	ø
$\underline{P}(A)$	0	0	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	0	1	0
$\overline{P}(X)$	$\frac{3}{4}$	$\frac{3}{4}$	1	1	1	$\frac{3}{4}$	1	0
R.$\underline{P}(A)$	0	0	0	0	0	0	1	0
R.$\overline{P}(A)$	1	1	1	1	1	1	1	0

T_5={ø,X,{2},{1,2},{1,2,3}}

C_5={ø,X,{1,3,4},{3,4},{4}}

A={1}→ $\{1\}^{\circ} = \emptyset$

$\overline{\{1\}} = \{1,3,4\} \to \overline{\{1\}}^{\circ} = \{1,3,4\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={2}→ $\{2\}^{\circ} = \{2\}$

$\overline{\{2\}} = X \to \overline{\{2\}}^{\circ} = X^{\circ} = X \to A \neq \overline{A}^{\circ}$

A={3}→ $\{3\}^{\circ} = \emptyset$

$\overline{\{3\}} = \{3,4\} \to \overline{\{3\}}^{\circ} = \{3,4\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={4}→ $\{4\}^{\circ} = \emptyset$

$\overline{\{4\}} = \{4\} \to \overline{\{4\}}^{\circ} = \{4\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={1,2}→ $\{1,2\}^{\circ} = \{1,2\}$

$\overline{\{1,2\}} = X \to \overline{\{1,2\}}^{\circ} = X^{\circ} = X \to A \neq \overline{A}^{\circ}$

A={1,3}→ $\{1,3\}^{\circ} = \emptyset$

$\overline{\{1,3\}} = \{1,3,4\} \to \overline{\{1,3\}}^{\circ} = \{1,3,4\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={1,4}→ $\{1,4\}^{\circ} = \emptyset$

$\overline{\{1,4\}} = \{1,3,4\} \to \overline{\{1,4\}}^{\circ} = \{1,3,4\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={2,3}→ $\{2,3\}^{\circ} = \{2\}$

$\overline{\{2,3\}} = X \rightarrow \overline{\{2,3\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={2,4}→ $\{2,4\}^{\circ} = \{2\}$

$\overline{\{2,4\}} = X \rightarrow \overline{\{2,4\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={3,4}→ $\{3,4\}^{\circ} = \emptyset$

$\overline{\{3,4\}} = \{3,4\} \rightarrow \overline{\{3,4\}}^{\circ} = \{3,4\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

A={1,3,4}=$\{1,3,4\}^{\circ} = \emptyset$

$\overline{\{1,3,4\}} = \{1,3,4\} \rightarrow \overline{\{1,3,4\}}^{\circ} = \{1,3,4\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

A={1,2,4}→ $\{1,2,4\}^{\circ} = \{1,2\}$

$\overline{\{1,2,4\}} = X \rightarrow \overline{\{1,2,4\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={1,2,3}→ $\{1,2,3\}^{\circ} = \{1,2,3\}$

$\overline{\{1,2,3\}} = X \rightarrow \overline{\{1,2,3\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={2,3,4}→ $\{2,3,4\}^{\circ} = \{2\}$

$\overline{\{2,3,4\}} = X \rightarrow \overline{\{2,3,4\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

$R.O_5.(X)=\{\emptyset,X\}$

$R.C_5.(X)=\{\emptyset,X\}$

$Int_R(\{1\})=ø$	$Cl_R(\{1\})=X$
$Int_R(\{2\})=ø$	$Cl_R(\{2\})=X$
$Int_R(\{3\})=ø$	$Cl_R(\{3\})=X$
$Int_R(\{4\})=ø$	$Cl_R(\{4\})=X$
$Int_R(\{1,2\})=ø$	$Cl_R(\{1,2\})=X$
$Int_R(\{1,3\})=ø$	$Cl_R(\{1,3\})=X$
$Int_R([1,4\})=ø$	$Cl_R(\{1,4\})=X$
$Int_R(\{2,3\})=ø$	$Cl_R(\{2,3\})=X$
$Int_R(\{2,4\})=ø$	$Cl_R(\{2,4\})=X$
$Int_R(\{3,4\})=ø$	$Cl_R(\{3,4\})=X$
$Int_R(\{1,3,4\})=ø$	$Cl_R(\{1,3,4\})=X$
$Int_R(\{1,2,4\})=ø$	$Cl_R(\{1,2,4\})=X$
$Int_R(\{1,2,3\})=ø$	$Cl_R(\{1,2,3\})=X$
$Int_R(\{2,3,4\})=ø$	$Cl_R(\{2,3,4\})=X$
$Int_R(X)=X$	$Cl_R(X)=X$
$Int_R(ø)=ø$	$Cl_R(X)=X$

A	{1}	{2}	{3}	{4}	{1,2}	{1,3}	{1,4}	{2,3}
$\underline{P}(A)$	0	$\frac{1}{4}$	0	0	$\frac{2}{4}$	0	0	$\frac{1}{4}$
$\overline{P}(A)$	$\frac{3}{4}$	1	$\frac{2}{4}$	$\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{3}{4}$	1
R.$\underline{P}(A)$	0	0	0	0	0	0	0	0
R.$\overline{P}(A)$	1	1	1	1	1	1	1	1

A	{2,4}	{3,4}	{1,3,4}	{1,2,4}	{1,2,3}	{2,3,4}	X	ø
$\underline{P}(A)$	$\frac{1}{4}$	0	0	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	1	0
$\overline{P}(A)$	1	$\frac{2}{4}$	$\frac{3}{4}$	1	1	1	1	0
R.$\underline{P}(A)$	0	0	0	0	0	0	1	0
R.$\overline{P}(A)$	1	1	1	1	1	1	1	0

T_6={ø,x,{3},{4},{3,4}}

C_6={ø,x,{1,2,4},{1,2,3},{1,2}}

A={1}→ $\{1\}^{\circ} = \emptyset$

$\overline{\{1\}} = \{1,2\} \to \{1\}^{\circ} = \{1,2\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={2}→ $\{2\}^{\circ} = \emptyset$

$\overline{\{2\}} = \{1,2\} \to \overline{\{2\}}^{\circ} = \{1,2\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={3}→ $\{3\}^{\circ} = \{3\}$

$\overline{\{3\}} = \{1,2,3\} \to \overline{\{3\}}^{\circ} = \{1,2,3\}^{\circ} = \{3\} \to A = \overline{A}^{\circ}$

A={4}→ $\{4\}^{\circ} = \{4\}$

$\overline{\{4\}} = \{1,2,4\} \to \overline{\{4\}}^{\circ} = \{1,2,4\}^{\circ} = \{4\} \to A = \overline{A}^{\circ}$

A={1,2}→ $\{1,2\}^{\circ} = \emptyset$

$\overline{\{1,2\}} = \{1,2\} \to \overline{\{1,2\}}^{\circ} = \{1,2\}^{\circ} = \emptyset \to A \neq \overline{A}^{\circ}$

A={1,3}→ $\{1,3\}^{\circ} = \{3\}$

$\overline{\{1,3\}} = \{1,2,3\} \to \overline{\{1,3\}}^{\circ} = \{1,2,3\}^{\circ} = \{3\} \to A \neq \overline{A}^{\circ}$

A={1,4}→ $\{1,4\}^{\circ} = \{4\}$

$\overline{\{1,4\}} = \{1,2,4\} \to \overline{\{1,4\}}^{\circ} = \{1,2,4\}^{\circ} = \{4\} \to A \neq \overline{A}^{\circ}$

A={2,3}→ $\{2,3\}^{\circ} = \{3\}$

$\overline{\{2,3\}} = \{1,2,3\} \to \overline{\{2,3\}}^{\circ} = \{1,2,3\}^{\circ} = \{3\} \to A \neq \overline{A}^{\circ}$

A={2,4}→ $\{2,4\}^{\circ} = \{4\}$

$\overline{\{2,4\}} = \{1,2,4\} \to \overline{\{2,4\}}^{\circ} = \{1,2,4\}^{\circ} = \{4\} \to A \neq \overline{A}^{\circ}$

A={3,4}→ $\{3,4\}^{\circ} = \{3,4\}$

$\overline{\{3,4\}} = X \to \overline{\{3,4\}}^{\circ} = X^{\circ} = X \to A \neq \overline{A}^{\circ}$

A={1,3,4}→ $\{1,3,4\}^{\circ} = \{3,4\}$

$\overline{\{1,3,4\}} = X \to \overline{\{1,3,4\}}^{\circ} = X^{\circ} = X \to A \neq \overline{A}^{\circ}$

A={1,2,4}→ $\{1,2,4\}^{\circ} = \{4\}$

$\overline{\{1,2,4\}} = \{1,2,4\} \to \overline{\{1,2,4\}}^{\circ} = \{1,2,4\}^{\circ} = \{4\} \to A \neq \overline{A}^{\circ}$

A={1,2,3}→ $\{1,2,3\}^{\circ} = \{3\}$

$\overline{\{1,2,3\}} = \{1,2,3\} \to \overline{\{1,2,3\}}^{\circ} = \{1,2,3\}^{\circ} = \{3\} \to A \neq \overline{A}^{\circ}$

A={2,3,4}→ $\{2,3,4\}^{\circ} = \{3,4\}$

$\overline{\{2,3,4\}} = X \to \overline{\{2,3,4\}}^{\circ} = X^{\circ} = X \to A \neq \overline{A}^{\circ}$

$R.O_6.(X)$={ø,X,{3},{4}}

$R.C_6.(X)$={ø,X,{1,2,4},{1,2,3}}

$Int_R(\{1\})=ø$	$Cl_R(\{1\})=\{1,2\}$
$Int_R(\{2\})=ø$	$Cl_R(\{2\})=\{1,2\}$
$Int_R(\{3\})=\{3\}$	$Cl_R(\{3\})=\{1,2,3\}$
$Int_R(\{4\})=\{4\}$	$Cl_R(\{4\})=\{1,2,4\}$
$Int_R(\{1,2\})=ø$	$Cl_R(\{1,2\})=\{1,2\}$
$Int_R(\{1,3\})=\{3\}$	$Cl_R(\{1,3\})=\{1,2,3\}$
$int_R(\{1,4\})=\{4\}$	$Cl_R(\{1,4\})=\{1,2,4\}$
$Int_R(\{2,3\})=\{3\}$	$Cl_R(\{2,3\})=\{1,2,3\}$
$Int_R(\{2,4\})=\{4\}$	$Cl_R(\{2,4\})=\{1,2,4\}$
$Int_R(\{3,4\})=\{3,4\}$	$Cl_R(\{3,4\})=X$
$Int_R(\{1,3,4\}=\{3,4\}$	$Cl_R(\{1,3,4\})=X$
$Int_R(\{1,2,4\})=\{4\}$	$Cl_R(\{1,2,4\})=\{1,2,4\}$
$Int_R(\{1,2,3\})=\{3\}$	$Cl_R(\{1,2,3\})=\{1,2,3\}$
$Int_R(\{2,3,4\})=\{3,4\}$	$Cl_R(\{2,3,4\})=X$
$Int_R(X)=x$	$Cl_R(X)=X$
$Int_R(ø)=ø$	$Cl_R(ø)=ø$

A	{1}	{2}	{3}	{4}	{1,2}	{1,3}	{1,4}	{2,3}
$\underline{P}(A)$	0	0	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\overline{P}(A)$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
R.$\underline{P}(A)$	0	0	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
R.$\overline{P}(A)$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$

A	{2,4}	{3,4}	{1,3,4}	{1,2,4}	{1,2,3}	{2,3,4}	x	ø
$\underline{P}(A)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	1	0
$\overline{P}(A)$	$\frac{3}{4}$	1	1	$\frac{3}{4}$	$\frac{3}{4}$	1	1	0
R.$\underline{P}(A)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	1	0
R.$\overline{P}(A)$	$\frac{3}{4}$	1	1	$\frac{3}{4}$	$\frac{3}{4}$	1	1	0

T_7={ø,X,{1},{2},{1,2},{3,4},{1,3,4},{2,3,4}}

C_7={ø,X,{2,3,4},{1,3,4},{3,4},{1,2},{2},{1}}

A={1}→ $\{1\}^{\circ} = \{1\}$

$\overline{\{1\}} = \{1\} \to \overline{\{1\}}^{\circ} = \{1\}^{\circ} = \{1\} \to A = \overline{A}^{\circ}$

A={2}→ $\{2\}^{\circ} = \{2\}$

$\overline{\{2\}} = \{2\} \to \overline{\{2\}}^{\circ} = \{2\}^{\circ} = \{2\} \to A = \overline{A}^{\circ}$

A={3}→ $\{3\}^{\circ} = \emptyset$

$\overline{\{3\}} = \{3,4\} \to \overline{\{3\}}^{\circ} = \{3,4\}^{\circ} = \{3,4\} \to A \neq \overline{A}^{\circ}$

A={4}→ $\{4\}^{\circ} = \emptyset$

$\overline{\{4\}} = \{3,4\} \to \overline{\{4\}}^{\circ} = \{3,4\}^{\circ} = \{3,4\} \to A \neq \overline{A}^{\circ}$

A={1,2}→ $\{1,2\}^{\circ} = \{1,2\}$

$\overline{\{1,2\}} = \{1,2\} \to \overline{\{1,2\}}^{\circ} = \{1,2\}^{\circ} = \{1,2\} \to A = \overline{A}^{\circ}$

A={1,3}→ $\{1,3\}^{\circ} = \{1\}$

$\overline{\{1,3\}} = \{1,3,4\} \to \overline{\{1,3\}}^{\circ} = \{1,3,4\}^{\circ} = \{1,3,4\} \to A \neq \overline{A}^{\circ}$

A={1,4}→ $\{1,4\}^{\circ} = \{1\}$

$\overline{\{1,4\}} = \{1,3,4\} \to \overline{\{1,4\}}^{\circ} = \{1,3,4\}^{\circ} = \{1,3,4\} \to A \neq \overline{A}^{\circ}$

A={2,3}$\rightarrow \{2,3\}^{\circ} = \{2\}$

$\overline{\{2,3\}} = \{2,3,4\} \rightarrow \overline{\{2,3\}}^{\circ} = \{2,3,4\}^{\circ} = \{2,3,4\} \rightarrow A \neq \overline{A}^{\circ}$

A={2,4}$\rightarrow \{2,4\}^{\circ} = \{2\}$

$\overline{\{2,4\}} = \{2,3,4\} \rightarrow \overline{\{2,4\}}^{\circ} = \{2,3,4\}^{\circ} = \{2,3,4\} \rightarrow A \neq \overline{A}^{\circ}$

A={3,4}$\rightarrow \{3,4\}^{\circ} = \{3,4\}$

$\overline{\{3,4\}} = \{3,4\} \rightarrow \overline{\{3,4\}}^{\circ} = \{3,4\}^{\circ} = \{3,4\} \rightarrow A = \overline{A}^{\circ}$

A={1,3,4}$\rightarrow \{1,3,4\}^{\circ} = \{1,3,4\}$

$\overline{\{1,3,4\}} = \{1,3,4\} \rightarrow \overline{\{1,3,4\}}^{\circ} = \{1,3,4\}^{\circ} = \{1,3,4\} \rightarrow A = \overline{A}^{\circ}$

A={1,2,4}$\rightarrow \{1,2,4\}^{\circ} = \{1,2\}$

$\overline{\{1,2,4\}} = X \rightarrow \overline{\{1,2,4\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={1,2,3}$\rightarrow \{1,2,3\}^{\circ} = \{1,2\}$

$\overline{\{1,2,3\}} = X \rightarrow \overline{\{1,2,3\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={2,3,4}$\rightarrow \{2,3,4\}^{\circ} = \{2,3,4\}$

$\overline{\{2,3,4\}} = \{2,3,4\} \rightarrow \overline{\{2,3,4\}}^{\circ} = \{2,3,4\}^{\circ} = \{2,3,4\} \rightarrow A = \overline{A}^{\circ}$

$R.O_7.(x)=\{ø,X,\{1\},\{2\},\{1,2\},\{3,4\},\{1,3,4\},\{2,3,4\}$

$R.C_7.(X)=\{ø,X,\{2,3,4\},\{1,3,4\},\{3,4\},\{1,2\},\{2\},\{1\}\}$

$Int_R(\{1\})=\{1\}$	$Cl_R(\{1\})=\{1\}$
$Int_R(\{2\})=\{2\}$	$Cl_R(\{2\})=\{2\}$
$Int_R(\{3\})=\emptyset$	$Cl_R(\{3\})=\{3,4\}$
$Int_R(\{4\})=\emptyset$	$Cl_R(\{4\})=\{3,4\}$
$Int_R(\{1,2\})=\{1,2\}$	$Cl_R(\{1,2\})=\{1,2\}$
$Int_R(\{1,3\})=\{1\}$	$Cl_R(\{1,3\})=\{1,3,4\}$
$Int_R(\{1,4\})=\{1\}$	$Cl_R(\{1,4\})=\{1,3,4\}$
$Int_R(\{2,3\})=\{2\}$	$Cl_R(\{2,3\})=\{2,3,4\}$
$Int_R(\{2,4\})=\{2\}$	$Cl_R(\{2,4\})=\{2,3,4\}$
$Int_R(\{3,4\})=\{3,4\}$	$Cl_R(\{3,4\})=\{3,4\}$
$Int_R(\{1,3,4\})=\{1,3,4\}$	$Cl_R(\{1,3,4\})=\{1,3,4\}$
$Int_R(\{1,2,4\})=\{1,2\}$	$Cl_R(\{1,2,4\})=X$
$Int_R(\{1,2,3\})=\{1,2\}$	$Cl_R(\{1,2,3\})=X$
$Int_R(\{2,3,4\})=\{2,3,4\}$	$Cl_R(\{2,3,4\})=\{2,3,4\}$
$Int_R(X)=X$	$Cl_R(X)=X$
$Int_R(\emptyset)=\emptyset$	$Cl_R(\emptyset)=\emptyset$

A	{1}	{2}	{3}	{4}	{1,2}	{1,3}	{1,4}	{2,3}
$\underline{P}(A)$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\overline{P}(A)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
R.$\underline{P}(A)$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
R.$\overline{P}(A)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$

A	{2,4}	{3,4}	{1,3,4}	{1,2,4}	{1,2,3}	{2,3,4}	X	Ø
$\underline{P}(A)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	0
$\overline{P}(A)$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	1	$\frac{3}{4}$	1	0
R.$\underline{P}(A)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	0
R.$\overline{P}(A)$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	1	$\frac{3}{4}$	1	0

T_8={ø,X,{1},{2},{1,2},{1,3,4}}

C_8(X)={ø,X,{2,3,4},{1,3,4},{3,4},{2}}

A={1}→ $\{1\}^{\circ} = \{1\}$

$\overline{\{1\}} = \{1,3,4\} \rightarrow \overline{\{1\}}^{\circ} = \{1,3,4\}^{\circ} = \{1,3,4\} \rightarrow A \neq \overline{A}^{\circ}$

A={2}→ $\{2\}^{\circ} = \{2\}$

$\overline{\{2\}} = \{2\} \rightarrow \overline{\{2\}}^{\circ} = \{2\}^{\circ} = \{2\} \rightarrow A = \overline{A}^{\circ}$

A={3}→ $\{3\}^{\circ} = \emptyset$

$\overline{\{3\}} = \{3,4\} \rightarrow \overline{\{3\}}^{\circ} = \{3,4\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

A={4}→ $\{4\}^{\circ} = \emptyset$

$\overline{\{4\}} = \{3,4\} \rightarrow \overline{\{4\}}^{\circ} = \{3,4\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

A={1,2}→ $\{1,2\}^{\circ} = \{1,2\}$

$\overline{\{1,2\}} = X \rightarrow \overline{\{1,2\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={1,3}→ $\{1,3\}^{\circ} = \{1\}$

$\overline{\{1,3\}} = \{1,3,4\} \rightarrow \overline{\{1,3\}}^{\circ} = \{1,3,4\}^{\circ} = \{1,3,4\} \rightarrow A \neq \overline{A}^{\circ}$

A={1,4}→ $\{1,4\}^{\circ} = \{1\}$

$\overline{\{1,4\}} = \{1,3,4\} \rightarrow \overline{\{1,4\}}^{\circ} = \{1,3,4\}^{\circ} = \{1,3,4\} \rightarrow A \neq \overline{A}^{\circ}$

A={2,3}→ $\{2,3\}^{\circ} = \{2\}$

$\overline{\{2,3\}} = \{2,3,4\} \rightarrow \overline{\{2,3\}}^{\circ} = \{2,3,4\}^{\circ} = \{2\} \rightarrow A \neq \overline{A}^{\circ}$

A={2,4}→ $\{2,4\}^{\circ} = \{2\}$

$\overline{\{2,4\}} = \{2,3,4\} \rightarrow \overline{\{2,4\}}^{\circ} = \{2,3,4\}^{\circ} = \{2\} \rightarrow A \neq \overline{A}^{\circ}$

A={3,4}→ $\{3,4\}^{\circ} = \emptyset$

$\overline{\{3,4\}} = \{3,4\} \rightarrow \overline{\{3,4\}}^{\circ} = \{3,4\}^{\circ} = \emptyset \rightarrow A \neq \overline{A}^{\circ}$

A={1,3,4}→ $\{1,3,4\}^{\circ} = \{1,3,4\}$

$\overline{\{1,3,4\}} = \{1,3,4\} \rightarrow \overline{\{1,3,4\}}^{\circ} = \{1,3,4\}^{\circ} = \{1,3,4\} \rightarrow A = \overline{A}^{\circ}$

A={1,2,4}→ $\{1,2,4\}^{\circ} = \{1,2\}$

$\overline{\{1,2,4\}} = X \rightarrow \overline{\{1,2,4\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={1,2,3}→ $\{1,2,3\}^{\circ} = \{1,2\}$

$\overline{\{1,2,3\}} = X \rightarrow \overline{\{1,2,3\}}^{\circ} = X^{\circ} = X \rightarrow A \neq \overline{A}^{\circ}$

A={2,3,4}→ $\{2,3,4\}^{\circ} = \{2\}$

$\overline{\{2,3,4\}} = \{2,3,4\} \rightarrow \overline{\{2,3,4\}}^{\circ} = \{2,3,4\}^{\circ} = \{2\} \rightarrow A \neq \overline{A}^{\circ}$

$R.O_8(x)$={ø,X,{2},{1,3,4}}

$R.C_8.(X)$={ø,X,{1,3,4},{2}}

$Int_R(\{1\})=ø$	$Cl_R(\{1\})=\{1,3,4\}$
$Int_R(\{2\})=\{2\}$	$Cl_R(\{2\})=\{2\}$
$Int_R(\{3\})=ø$	$Cl_R(\{3\})=\{1,3,4\}$
$Int_R(\{4\})=ø$	$Cl_R(\{4\})=\{1,3,4\}$
$Int_R(\{1,2\})=\{2\}$	$Cl_R(\{1,2\})=X$
$Int_R(\{1,3\})=ø$	$Cl_R(\{1,3\})=\{1,3,4\}$
$Int_R(\{1,4\})=ø$	$Cl_R(\{1,4\})=\{1,3,4\}$
$Int_R(\{2,3\})=\{2\}$	$Cl_R(\{2,3\})=X$
$Int_R(\{2,4\})=\{2\}$	$Cl_R(\{2,4\})=X$
$Int_R(\{3,4\})=ø$	$Cl_R(\{3,4\})=\{1,3,4\}$
$Int_R(\{1,3,4\})=\{1,3,4\}$	$Cl_R(\{1,3,4\})=\{1,3,4\}$
$Int_R(\{1,2,4\})=\{2\}$	$Cl_R(\{1,2,4\})=X$
$Int_R(\{1,2,3\})=\{2\}$	$Cl_R(\{1,2,3\})=X$
$Int_R(\{2,3,4\})=\{2\}$	$Cl_R(\{2,3,4\})=X$
$Int_R(X)=X$	$Cl_R(X)=X$
$Int_R(ø)=ø$	$Cl_R(ø)=ø$

A	{1}	{2}	{3}	{4}	{1,2}	{1,3}	{1,4}	{2,3}
$\underline{P}(A)$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\overline{P}(A)$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	1	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
R.$\underline{P}(A)$	0	$\frac{1}{4}$	0	0	$\frac{1}{4}$	0	0	$\frac{1}{4}$
R.$\overline{P}(A)$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{3}{4}$	$\frac{3}{4}$	1

A	{2,4}	{3,4}	{1,3,4}	{1,2,4}	{1,2,3}	{2,3,4}	X	Ø
$\underline{P}(A)$	$\frac{1}{4}$	0	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	1	0
$\overline{P}(A)$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	1	$\frac{3}{4}$	1	0
R.$\underline{P}(A)$	$\frac{1}{4}$	0	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	1	0
R.$\overline{P}(A)$	1	$\frac{3}{4}$	$\frac{3}{4}$	1	1	1	1	0

المصادر

[1] Long P. E. and Carnahan, D. A., Comparing almost continuous functions, Proc. Amer Math .Soc. 38(1973).pp.413-448-

[2] M. Jamal, on Topological structures and Uncertainty, Tanta university Egypt, phd, 2010

[3] I. Bloch, on links between mathematical morphology and rough sets, pattern Recognition 33, 2000, pp. 1147-1196

[4] J. Peters, A. Skowron and J. Stepaniuk, Nearness in Approximation spaces, proc. CS&P, 06, 2006

[5] M. Abd El-Monsef, A. Kozae and R. Abu-Gdairi, Generalized Near Rough probability in Topological space, Int. J. Contemp. Math Sciences, Vol.6, no. 23, 2011, pp. 1099-1110

[6] M. Jamal, on Topological structures and Uncertainty, Tanta university Egypt, phd, 2010.

[7] Willard I.S , General Topology, Addison wesley publishing Co. 1970