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**Ministry of Higher Education**

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**University of Babylon**

**College of Education for Pure Sciences**

**Department of Mathematics**



# **Some Kinds of Domination in Graphs**

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of B.A. in Mathematics

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

﴿وَقُلْ رَبِّ زِدْنِي عِلْمًا﴾

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صَدَقَ اللهُ الْعَظِيمُ

## **Dedication**

With deepest gratitude and appreciation,

I dedicate this humble work to:

My beloved parents,

for their endless support, patience, and sacrifices,  
and for being the light that guided my path toward knowledge.

To my family,

who stood by me with encouragement and faith throughout my  
journey.

To my teachers,

whose knowledge and guidance inspired me to pursue excellence.

And to everyone who believes in the value of education and  
knowledge.

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## **Abstract**

This research studies some kinds of domination in graph theory. Domination is an important concept that deals with selecting a subset of vertices so that every vertex in the graph is either selected or adjacent to a selected vertex. The research explains the basic concepts of graph theory, including vertices, edges, paths, cycles, degree, complete graphs, connected graphs, and bipartite graphs.

The study also discusses several types of domination, such as domination, total domination, independent domination, and connected domination. It shows the differences between these types according to the conditions imposed on the dominating set. In addition, the research presents solved examples for special graphs such as complete graphs, path graphs, cycle graphs, and complete bipartite graphs.

The results show that domination numbers depend strongly on the structure of the graph and the way its vertices are connected. The research also highlights the importance of domination in practical applications, especially in communication networks, sensor networks, monitoring systems, and optimization problems.

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## **Introduction**

Graph theory is an important branch of discrete mathematics that studies the relationships between objects by representing them as vertices and edges. The vertices represent the objects, while the edges represent the connections or relationships between them. Because of this simple and effective structure, graph theory has become a useful mathematical tool in many fields, including computer science, communication networks, engineering, optimization, transportation systems, and social sciences.

One of the fundamental concepts in graph theory is domination. A dominating set is a subset of vertices chosen so that every vertex in the graph is either a member of this subset or adjacent to at least one vertex in it. This concept is important because it provides a mathematical way to study coverage, control, monitoring, and influence within different types of networks.

Over the years, several kinds of domination have been introduced according to different conditions imposed on the dominating set. These include total domination, independent domination, and connected domination. Each type has its own definition, properties, and applications, which makes the study of domination a rich and important topic in graph theory.

Therefore, this research aims to study some kinds of domination in graphs, explain their basic concepts, present suitable examples, and show their importance in both theoretical and practical applications.

## **Problem Statement**

The problem of this research lies in the fact that determining the smallest dominating set, total dominating set, or independent dominating set in a graph is not always simple. It mainly depends on the structure of the graph, the number of vertices and edges, and the way in which the vertices are connected. Moreover, some domination problems are considered computationally difficult, which makes their study important from both theoretical and practical perspectives.

## **Research Objectives**

This research aims to:

1. Study the concept of domination in graphs.
2. Explain some important kinds of domination in graph theory.
3. Clarify the differences between domination, total domination, independent domination, and connected domination.
4. Illustrate domination numbers for some special graphs such as  $K_n$ ,  $P_n$ ,  $C_n$ , and  $K_{m,n}$ .
5. Highlight the practical applications of domination concepts in networks, communication systems, and computer systems.

## **Research Importance**

The importance of this research comes from the fact that domination in graphs is one of the significant topics in graph theory. It has wide applications in network design, monitoring systems, sensor networks, communication systems, and social network modeling. In addition, studying different kinds of domination helps in understanding how complete coverage can be achieved by using the minimum possible number of vertices.

# Chapter One

## Basic Concepts in Graph Theory

### 1.1 Introduction to Graph Theory [5]

Graph theory is a fundamental branch of discrete mathematics concerned with the study of graphs as mathematical structures used to represent relationships among objects. In a graph, objects are represented by vertices, while the relationships or connections between these objects are represented by edges. This simple structure gives graph theory a powerful ability to model many real-world systems in a clear and organized mathematical form.

The importance of graph theory appears in its wide range of applications. It is used in computer science, communication networks, transportation systems, electrical circuits, social networks, and optimization problems. For example, in a communication network, devices may be represented as vertices and communication links as edges. Similarly, in a social network, people can be represented as vertices, while friendships or interactions are represented as edges.[2]

In this research, graph theory represents the main theoretical foundation for studying domination and its different kinds. Since domination depends mainly on adjacency and relationships between vertices, it is necessary to understand the basic concepts of graphs before discussing dominating sets, total domination, independent domination, and connected domination. Therefore, this chapter prepares the reader for the later chapters and supports the main aim of the research, which is to study some kinds of domination in graphs.

### 1.2 Basic Elements of Graphs: Vertices and Edges [15]

The two essential elements of any graph are vertices and edges. A graph is usually denoted by

$$G = (V, E)$$

where  $V$  is the set of vertices and  $E$  is the set of edges. The vertices represent the objects or points in the graph, while the edges represent the connections between pairs of vertices. Without vertices and edges, the structure of a graph cannot be formed.

If two vertices are connected by an edge, they are called adjacent vertices. This concept is one of the most important ideas in graph theory because many other definitions depend on it. For instance, the degree of a vertex, paths, cycles, connectedness, and domination are all based on the way vertices are joined by edges.

Adjacency is particularly important in domination theory. A vertex is considered dominated if it belongs to the dominating set or is adjacent to at least one vertex in that set. Therefore, the relation between vertices and edges is not merely a basic definition, but it is also the main condition for understanding domination. This makes the study of vertices and edges necessary for achieving the research objective related to explaining the concept of domination and its main types.[4]

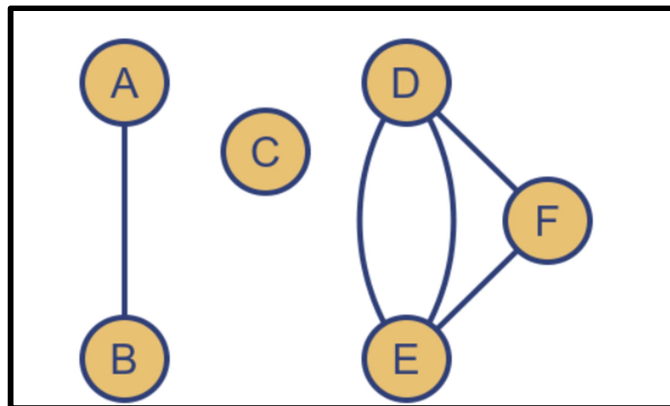


Figure (1.1): Vertices and Edges in a Graph

### **1.3 Types of Graphs: Simple, Complete, Connected, and Bipartite Graphs [3]**

Graphs can be classified into different types according to their structure and the nature of their edges. A simple graph is a graph that contains no loops and no multiple edges between the same pair of vertices. This type is commonly used in most theoretical studies of domination because it provides a clear and standard graph structure.

A complete graph, denoted by  $K_n$ , is a graph in which every pair of distinct vertices is connected by an edge. Complete graphs are important because they represent the highest possible level of connectivity among vertices. In domination theory, this high connectivity often leads to smaller domination numbers, since one vertex may be adjacent to many or all other vertices.

A connected graph is a graph in which there exists a path between every pair of vertices. This means that all vertices belong to one connected structure. Connected graphs are especially important when studying connected domination, where the selected dominating set must dominate the graph and also form a connected subgraph.[11]

A bipartite graph is a graph whose vertex set can be divided into two disjoint subsets such that every edge connects a vertex from one subset to a vertex from the other. A complete bipartite graph is denoted by  $K_{m,n}$ , where each vertex in the first part is adjacent to every vertex in the second part. These graphs are useful in studying relationships between two different groups and are important in applications such as matching problems, scheduling, and network design.

Studying these types of graphs is necessary in this research because domination numbers and domination properties depend strongly on the structure of the graph. This section therefore supports the objective of illustrating domination in special graphs such as  $K_n$ ,  $P_n$ ,  $C_n$ , and  $K_{m,n}$ . [11]

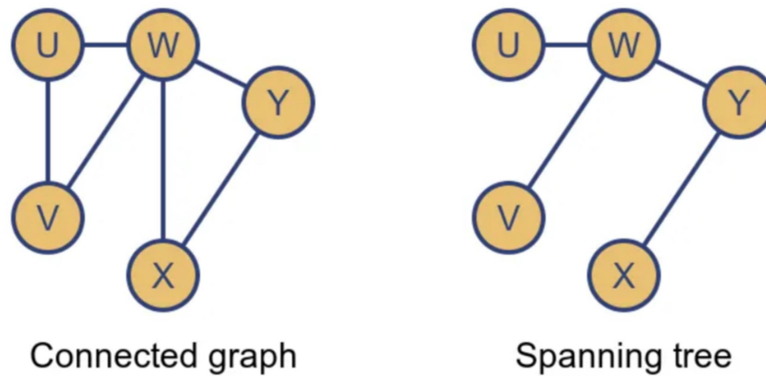


Figure (1.2): Example of a Simple Graph

#### 1.4 Paths, Cycles, Degree, and Isolated Vertices [1]

A path in a graph is a sequence of vertices in which each consecutive pair of vertices is joined by an edge. Paths are important because they describe how movement, communication, or influence can pass from one vertex to another within a graph. A path graph, usually denoted by  $P_n$ , is one of the basic graph families used in domination studies.

A cycle is a closed path that starts and ends at the same vertex, with no repeated vertices except the first and last one. Cycle graphs, denoted by  $C_n$ , are also important in graph theory because they represent circular structures. Such structures appear in communication rings, circular routes, and cyclic networks.

The degree of a vertex is the number of edges incident with that vertex. It measures how many direct connections the vertex has with other vertices. A vertex with a high degree may play an important role in domination because it can dominate or influence several adjacent vertices. On the other hand, a vertex

with degree zero is called an isolated vertex, meaning that it has no connection with any other vertex.[6]

Isolated vertices are highly significant in total domination. In a total dominating set, every vertex must be adjacent to at least one vertex in the set. Therefore, if a graph contains an isolated vertex, total domination cannot be achieved. This explains why the study of degree and isolated vertices is essential for understanding the difference between ordinary domination and total domination.[8]

### **1.5 Minimal and Minimum Sets in Graph Theory [10]**

The concepts of minimal and minimum sets are very important in graph theory, especially in the study of domination. Although these two terms may seem similar, they have different mathematical meanings.

A set is called minimal with respect to a certain property if no element can be removed from the set without losing that property. In this sense, every element of a minimal set is necessary. For example, a minimal dominating set is a dominating set in which removing any vertex causes the set to stop being dominating.

On the other hand, a set is called minimum if it has the smallest possible cardinality among all sets satisfying the same property. Thus, a minimum dominating set is a dominating set with the least possible number of vertices. The size of such a set is called the domination number of the graph.

This distinction is essential because a graph may have several minimal dominating sets with different sizes, but the minimum dominating set has the smallest size among them. Therefore, every minimum set is minimal, but not every minimal set is minimum.[14]

Understanding this difference is necessary for the later chapters of this research, especially when studying domination number, total domination number, independent domination number, and connected domination number. It also supports the research objective related to finding or explaining the smallest possible dominating sets in special graphs.

## Chapter Two

### Domination and Its Main Types

#### 2.1 Dominating Set and Domination Number [10]

Domination is one of the most important concepts in graph theory. It is mainly concerned with selecting a subset of vertices that can cover or control all vertices of a graph through adjacency. Let  $G = (V, E)$  be a graph. A subset  $D \subseteq V(G)$  is called a dominating set if every vertex not belonging to  $D$  is adjacent to at least one vertex in  $D$ . In other words, every vertex in the graph is either selected in the set  $D$  or is directly connected to a selected vertex.

The domination number of a graph  $G$ , denoted by  $\gamma(G)$ , is the minimum cardinality of a dominating set in  $G$ . This number represents the smallest possible number of vertices needed to dominate the whole graph. Therefore, domination number is an important parameter because it measures how efficiently a graph can be covered using selected vertices.

This concept is closely related to many practical problems. For example, in a communication network, selected vertices may represent control stations, and all other vertices must be connected to at least one of these stations. Thus, domination provides a mathematical model for monitoring, coverage, and resource allocation. This section supports the research objective of studying the basic concept of domination in graphs.[13]

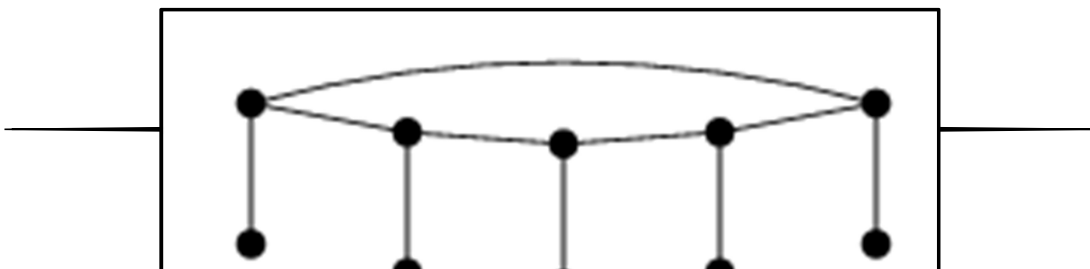


Figure (2.1): Example of a Dominating Set in a Graph

## 2.2 Minimal and Minimum Dominating Sets [9]

In domination theory, it is necessary to distinguish between minimal dominating sets and minimum dominating sets. A dominating set  $D$  is called minimal if no proper subset of  $D$  is also a dominating set. This means that each vertex in  $D$  is necessary to preserve the domination property. If any vertex is removed from a minimal dominating set, the remaining set will no longer dominate the graph.

On the other hand, a minimum dominating set is a dominating set with the smallest possible number of vertices among all dominating sets of the graph. The cardinality of a minimum dominating set is equal to the domination number  $\gamma(G)$ . Thus, the term minimal refers to the necessity of elements, while the term minimum refers to the smallest possible size.

This distinction is important because a graph may have several minimal dominating sets with different cardinalities, but the minimum dominating set has the least size among them. Every minimum dominating set is minimal, but not every minimal dominating set is minimum. Understanding this difference is essential for studying domination numbers and comparing different domination parameters in special graphs.[7]

## 2.3 Total Domination and Total Domination Number [11]

Total domination is an important variation of ordinary domination. Let  $G$  be a graph with no isolated vertices. A subset  $T \subseteq V(G)$  is called a total dominating set if every vertex of  $G$  is adjacent to at least one vertex in  $T$ . Unlike ordinary

domination, a vertex in the set  $T$  cannot dominate itself; it must have a neighbor in  $T$ .

The total domination number of a graph  $G$ , denoted by  $\gamma_t(G)$ , is the minimum cardinality of a total dominating set in  $G$ . This means that  $\gamma_t(G)$  represents the smallest number of vertices required so that every vertex in the graph has a neighbor in the selected set. The condition that the graph has no isolated vertices is very important, because an isolated vertex has no adjacent vertex and therefore cannot be totally dominated.

Total domination is stronger than ordinary domination because it requires all vertices, including the selected vertices, to be dominated by adjacency. This property makes it useful in systems where every active node must be supported, monitored, or connected to another active node. For this reason, total domination has important applications in communication networks, sensor networks, and monitoring systems.

## **2.4 Independent Domination and Independent Domination Number [1]**

Independent domination combines two concepts: domination and independence. A subset  $D \subseteq V(G)$  is called an independent dominating set if it satisfies two conditions. First, it is a dominating set, meaning that every vertex outside  $D$  is adjacent to at least one vertex in  $D$ . Second, it is an independent set, meaning that no two vertices in  $D$  are adjacent to each other.

The independent domination number of a graph  $G$ , denoted by  $i(G)$ , is the minimum cardinality of an independent dominating set in  $G$ . This number measures the smallest number of non-adjacent vertices needed to dominate the whole graph. Therefore, independent domination adds an important restriction to ordinary domination by preventing direct connections among the selected vertices.

This concept is useful in applications where selected vertices must not interfere with one another. For example, in wireless networks, selected stations may need to be separated to avoid signal conflict while still covering all other points. Similarly, in scheduling and resource allocation problems, independent domination can represent a set of non-conflicting choices that still cover the whole system. This section supports the research objective of explaining important types of domination and distinguishing them from ordinary domination.[12]

## **2.5 Connected Domination and Connected Domination Number [6]**

Connected domination is another significant type of domination in graph theory. A subset  $D \subseteq V(G)$  is called a connected dominating set if it is a dominating set and the subgraph induced by  $D$ , denoted by  $G[D]$ , is connected. This means that the selected vertices must dominate the whole graph and, at the same time, form a connected structure among themselves.

The connected domination number of a graph  $G$ , denoted by  $\gamma_c(G)$ , is the minimum cardinality of a connected dominating set in  $G$ . This parameter is important because it combines coverage with connectivity. In ordinary domination, the selected vertices may be disconnected, but in connected domination they must remain linked through paths within the selected subgraph.

Connected domination has important applications in network design and communication systems. For example, in a wireless or computer network, selected vertices may represent backbone nodes that cover all other nodes while maintaining communication among themselves. Thus, connected domination is useful for designing reliable and efficient networks.[8]

This chapter shows that domination has several important forms. Ordinary domination focuses on coverage, total domination requires adjacency for every vertex, independent domination adds a non-adjacency condition among selected

vertices, and connected domination requires the selected vertices to form a connected subgraph. These concepts directly support the objectives of this research by clarifying the main kinds of domination and explaining their importance in both theory and applications.

## Chapter Three

### Applied Analytical Examples of Domination in Graphs

#### Example 3.1: Domination Number of the Complete Graph $K_5$ [3]

$$\gamma(K_5) = 1$$

#### Solution:

Let  $G = K_5$  be a complete graph with vertex set

$$V(G) = \{v_1, v_2, v_3, v_4, v_5\}.$$

Since  $G$  is complete, every vertex is adjacent to all other vertices. Consider the set

$$D = \{v_1\}$$

Then every vertex in  $V(G) - D$ , namely  $v_2, v_3, v_4, v_5$ , is adjacent to  $v_1$ . Hence, every vertex of  $G$  is either in  $D$  or adjacent to a vertex in  $D$ . Therefore,  $D$  is a dominating set of  $K_5$ .

Since no dominating set can have cardinality less than one, it follows that

$$\gamma(K_5) =$$

#### Example 3.2: Total Domination Number of the Complete Graph $K_4$ [11]

$$\gamma_t(K_4) = 2$$

**Solution:**

Let  $G = K_4$  be a complete graph with vertex set

$$V(G) = \{v_1, v_2, v_3, v_4\}$$

In total domination, every vertex must be adjacent to a vertex in the total dominating set. A single vertex cannot form a total dominating set, because it has no neighbor inside the set.

Consider

$$T = \{v_1, v_2\}$$

Since  $K_4$  is complete,  $v_1$  is adjacent to  $v_2$ , and  $v_2$  is adjacent to  $v_1$ . Also,  $v_3$  and  $v_4$  are adjacent to both  $v_1$  and  $v_2$ . Therefore, every vertex of  $K_4$  has at least one neighbor in  $T$ .

Thus,  $T$  is a total dominating set. Since no one-vertex set can be a total dominating set,

$$\gamma_t(K_4) = 2.$$

**Example 3.3: Domination Number of the Path Graph  $P_6$  [2]**

$$\gamma(P_6) = 2$$

**Solution:**

Let  $G = P_6$  be the path graph

$$v_1 - v_2 - v_3 - v_4 - v_5 - v_6$$

Consider the set

$$D = \{v_2, v_5\}$$

The vertex  $v_1$  is adjacent to  $v_2$ ,  $v_2 \in D$ , and  $v_3$  is adjacent to  $v_2$ . Similarly,  $v_4$  is adjacent to  $v_5$ ,  $v_5 \in D$ , and  $v_6$  is adjacent to  $v_5$ . Hence, every vertex of  $P_6$  is either in  $D$  or adjacent to a vertex in  $D$ .

Therefore,  $D$  is a dominating set of  $P_6$ . Since one vertex can dominate at most three consecutive vertices in a path, two vertices are necessary to dominate six vertices. Thus,

$$\gamma(P_6) = 2.$$

**Example 3.4: Domination Number of the Cycle Graph  $C_6$  [13]**

$$\gamma(C_6) = 2$$

**Solution:**

Let  $G = C_6$  be the cycle graph with vertices

$$V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$$

Consider the set

$$D = \{v_2, v_5\}$$

The vertices  $v_1$  and  $v_3$  are adjacent to  $v_2$ , while  $v_4$  and  $v_6$  are adjacent to  $v_5$ . Also,  $v_2$  and  $v_5$  belong to  $D$ . Therefore, every vertex of  $C_6$  is dominated by  $D$ .

Thus,  $D$  is a dominating set of  $C_6$ . Since one vertex can dominate at most three vertices in a cycle, at least two vertices are required. Hence,

$$\gamma(C_6) = 2.$$

**Example 3.5: Independent Domination Number of a Graph  $G$  [1]**

$$i(G) = 2$$

**Solution:**

Let  $G$  be a graph with vertex set

$$V(G) = \{v_1, v_2, v_3, v_4\}$$

and edge set

$$E(G) = \{v_1v_2, v_1v_3, v_2v_4\}$$

Consider the set

$$D = \{v_2, v_3\}$$

First,  $v_2$  and  $v_3$  are not adjacent, so  $D$  is an independent set. Second,  $v_1$  is adjacent to both  $v_2$  and  $v_3$ , while  $v_4$  is adjacent to  $v_2$ . Also,  $v_2$  and  $v_3$  are in  $D$ . Hence, every vertex of  $G$  is either in  $D$  or adjacent to a vertex in  $D$ .

Therefore,  $D$  is an independent dominating set. Since no single vertex can satisfy both domination and independence for this graph,

$$i(G) = 2.$$

### **Example 3.6: Connected Domination Number of the Path Graph $P_4$ [6]**

$$\gamma_c(P_4) = 2$$

#### **Solution:**

Let  $G = P_4$  be the path graph

$$v_1 - v_2 - v_3 - v_4$$

Consider the set

$$D = \{v_2, v_3\}$$

The vertex  $v_1$  is adjacent to  $v_2$ , and  $v_4$  is adjacent to  $v_3$ . Also,  $v_2$  and  $v_3$  belong to  $D$ . Thus,  $D$  is a dominating set of  $P_4$ .

Moreover, the subgraph induced by  $D$ , denoted by  $G[D]$ , contains the edge  $v_2v_3$ . Therefore,  $G[D]$  is connected. Hence,  $D$  is a connected dominating set.

Since no one-vertex set can dominate  $P_4$ , it follows that

$$\gamma_c(P_4) = 2$$

## **Conclusion**

In conclusion, this research presented a study of some kinds of domination in graph theory. The study began with the basic concepts of graph theory, such as vertices, edges, paths, cycles, degree, connected graphs, complete graphs, and bipartite graphs. These concepts provided the necessary foundation for understanding domination and its different forms.

The research discussed domination, total domination, independent domination, and connected domination. It showed that each type has its own conditions and importance. Ordinary domination focuses on covering all vertices, total domination requires every vertex to be adjacent to a selected vertex, independent domination requires the selected vertices to be non-adjacent, and connected domination requires the selected vertices to form a connected subgraph.

The study also showed that domination numbers depend strongly on the structure of the graph. For example, complete graphs have small domination numbers because of their high connectivity, while path and cycle graphs require more careful selection of vertices. Moreover, domination has important practical applications in communication networks, sensor networks, monitoring systems, and optimization problems.

Therefore, domination remains an important topic in graph theory because it connects theoretical mathematical concepts with practical applications. It

provides useful methods for studying coverage, control, and efficiency in different types of networks.

## **Results**

1. Domination is a fundamental concept in graph theory, as it provides a mathematical method for studying coverage, control, and monitoring in graphs.
2. The domination number depends strongly on the structure of the graph, especially the number of vertices, edges, and adjacency relations.
3. In complete graphs  $K_n$ , the domination number is small because every vertex is adjacent to all other vertices, where  $\gamma(K_n)=1$ .
4. Total domination is stronger than ordinary domination because every vertex must be adjacent to a vertex in the total dominating set, including the selected vertices themselves.
5. Independent domination and connected domination add extra conditions to ordinary domination, making them useful in applications such as communication networks, sensor networks, and optimization problems.

## **Recommendations**

1. Future studies should examine other types of domination, such as Roman domination, restrained domination, and signed domination.

2. It is recommended to study domination in more complex graphs, such as weighted graphs, directed graphs, fuzzy graphs, and dynamic graphs.
3. Researchers should give more attention to computational methods and algorithms for finding minimum dominating sets in large graphs.
4. Future research should connect domination theory with practical applications in communication networks, sensor systems, and resource allocation.

## References

- [1] A. Behzad, *Domination in Graphs*, M.Sc. thesis, Simon Fraser University, Canada, 2006.
- [2] A. Behzad, M. Behzad, and C. E. Praeger, “Fundamental Dominations in Graphs,” arXiv, 2008.
- [3] J. A. Bondy and U. S. R. Murty, *Graph Theory with Applications*. New York, NY, USA: Elsevier, 1976.
- [4] C.-C. Chang, *Algorithmic Aspects of Domination in Graphs*, National Taiwan University, 2011.
- [5] R. Diestel, *Graph Theory*, 3rd ed. Berlin, Germany: Springer, 2005.
- [6] P. A. Dreyer, Jr., *Applications and Variations of Domination in Graphs*, Ph.D. dissertation, Rutgers University, New Brunswick, NJ, USA, 2000.
- [7] A. Hansberg Pastor, *Multiple Domination in Graphs*, Ph.D. dissertation, RWTH Aachen University, Aachen, Germany, 2009.
- [8] T. W. Haynes, S. T. Hedetniemi, and M. A. Henning, *Domination in Graphs: Core Concepts*. Cham, Switzerland: Springer, 2023.

- [9] T. W. Haynes, S. T. Hedetniemi, and M. A. Henning, Eds., *Topics in Domination in Graphs*. Cham, Switzerland: Springer, 2020.
- [10] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, *Fundamentals of Domination in Graphs*. New York, NY, USA: Marcel Dekker, 1998.
- [11] M. A. Henning and A. Yeo, *Total Domination in Graphs*. New York, NY, USA: Springer, 2013.
- [12] L. Lekha, *Domination in Graphs and Fuzzy Graphs*, Ph.D. thesis, University of Calicut, India, 2023.
- [13] J. M. Lewis, *Vertex-Edge and Edge-Vertex Parameters in Graphs*, Ph.D. dissertation, Clemson University, Clemson, SC, USA, 2007.
- [14] Ø. Ore, *Theory of Graphs*. Providence, RI, USA: American Mathematical Society, 1962.
- [15] D. B. West, *Introduction to Graph Theory*, 2nd ed. Upper Saddle River, NJ, USA: Prentice Hall, 2001.