

Republic of Iraq
Ministry of Higher Education and Scientific Research
University of Babylon
College of Education for Pure Sciences



Lower and Upper Probability By β -Open and β -Closed Sets

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By

Zaid Zohair Naji Abood

Supervised By

Prof : Dr.Luay Abd-Al-Hani Al-Swidi

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

((وَأَنْزَلَ اللَّهُ عَلَيْكَ الْكِتَابَ وَالْحِكْمَةَ وَعَلَّمَكَ مَا لَمْ تَكُنْ تَعْلَمُ ۗ وَكَانَ فَضْلُ اللَّهِ عَلَيْكَ عَظِيمًا))

صدق الله العلي العظيم

سورة النساء اية (١١٣)

الإهداء

أهدي هذا العمل المتواضع إلى أبي الذي لم يبخل علي يوماً بشيء وإلى
أمي التي زودتني بالحنان والمحبة

الشكر والتقدير

قال تعالى (وَمَنْ يَشْكُرْ فَإِنَّمَا يَشْكُرُ لِنَفْسِهِ) (لقمان : ١٣)

وقال رسوله الكريم (ص): «لا يَشْكُرُ اللّٰهَ مَنْ لا يَشْكُرُ النَّاسَ»

احمد الله تعالى حمد كثير طيبا مباركا ملئ السموات والارض
على ما اكرمني به من اتمام هذه الدراسة التي ارجو ان تنال
رضاه

ثم اتوجه بجزيل الشكر وعظيم الامتنان الى كل من

*الدكتور لؤي عبد الهادي السويدي *والدكتور مصطفى
حسن هادي حفظهم الله واطال في اعمارهم لتفضلهم
الكريم بالإشراف على هذه الدراسة وتكرمهم بنصحي
وتوجيهي حتى إتمام هذه الدراسة

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Abstract

Rather, the aim of my research is the process of calculating all the topologies on the three-point group, and then calculating the upper and lower probability of all the groups within the $IP(x)$ corresponding to those topologies, by the help of the interior points and closure on the one side, and on the other side, calculating the interior points and closure of the harmonic Sets β -Open.

Introduction:

Brief history of topology:

Topological thinking began with Euler's problem in the famous "Seven Bridges of Königsberg", and Euler's 1736 paper was the first result on topological elimination.

The term topology was first introduced by the Germans as "topologie" in 1847 by Johann Benedict, and then the specialists in the English language showed that the word topologist is every person who specializes in topology. In the nineteenth century, several scholars developed specific definitions for it. Ascoli and others developed the first definition of the metric rule, which

It is considered a special case in the current topology in the year 1906

After that, the world Hausdorff and Sierpinski put a definition of it, which is currently known as the very famous Hausdorff and Sierpinski in the year

1914 But the world came as Kazimierz Kuratowski. The year 1922 put the definition known to us now.

Chapter One

Topologies on The Sets Containing Three Elements

In this chapter, we will show set of basic definitions on which our works are based, to a set of questions that we worked on through three elements, which are $\{a,b,c\}$, where we made 29 topologies and extracted (interiors and closures) of β -open at the end of the research ,we studied the probabilities that are (upper and lower) on the topologies that we knew, in addition to the probabilities on topologies for the β -open set.

Definition 1.1 [1]

Let X be a nonempty set and τ be a family of subsets of X (i.e., $\tau \subseteq IP(X)$) . we say τ is a topology on X satisfy the following conditions:

1. $X, \emptyset \in \tau$
2. If $U, V \in \tau$, then $U \cap V \in \tau$ the finite intersection of elements from τ
3. If $U_\alpha \in \tau ; \alpha \in \Lambda$, then $\bigcup_{\alpha \in \Lambda} U_\alpha \in \tau \forall \alpha \in \Lambda$ the arbitrary (finite or infinite)union of element of τ is again an element of τ .

Definition 1.2 [2]

Let (X, τ) be a topological space.the subset of X belonging to τ are called open sets in the space

If $A \subseteq X$ and $A \in \tau$ then A open set

Definition 1.3 [2]

The subset of X is called closed set in the space X if its complement X/A is open set we will denote the family of closed sets

if $A \subseteq X$ and $A \in \mathcal{C}$ then A closed set

Definition 1.4 [3]

Let (X, τ) be a topological space and let $A \subseteq X$. A point $a \in A$ is called an Interior Point of A if there exists an open neighbourhood U ($U \in \tau$) of a such that $a \in U \subseteq A$. The set of all interior points of A is called the Interior of A and is denoted $\text{int}(A)$.

Definition 1.5 [3]

Definition : Let X be a space and $A \subseteq X$. The set $A \subseteq \bigcap \{F \mid F \text{ is closed in } X \text{ and } A \subseteq F\}$ is the smallest closed set containing A . This is called the closure of A , sometimes denoted by $\text{cl}(A)$.

Definition 1.6

A subset A of a space X is said to be β -open if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$. And the complement β -open is called β -closed set. And the complement $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$.

Definition 1.7 [4]

Let (X, τ) be a topological space and let $A \subseteq X$. A point $x \in A$ is called an αR -interior point of A iff there exists an β -open set $U \in \tau$ containing x such that $x \in U \subseteq A$. The set of all β -interior point of A is called the β -interior of A and is denoted by $\beta - A^\circ$ or $\beta - \text{Int}(A)$ i.e.,

$$\beta - A^\circ = \{x \in A: \exists U \in \tau; x \in U \subseteq A\}$$

$$x \in \beta - A^\circ \leftrightarrow \exists U \in \tau; x \in U \subseteq A$$

Let (X, τ) be a topological space and let $A \subseteq X$. A point $x \in A$ is called an interior point of A iff there exists an open set $U \in \tau$ containing x such that $x \in U \subseteq A$. The set of all interior points of A is called the interior of A and is denoted by A° or $\text{Int}(A)$ i.e

$$A^\circ = \{x \in A: \exists U \in \tau; x \in U \subseteq A\}$$

$$x \in A^\circ \leftrightarrow \exists U \in \tau; x \in U \subseteq A$$

Definition 1.8 [4]

Let (X, τ) be a topological space and let A be a subset of X . Then the intersection

of all β -closed containing the set A is called the β -closure of A and denoted by $\beta - A$ or $\beta - c(A)$ or $\beta - \text{cl}(A)$. i.e $\beta - \text{cl}(A) = \bigcap \{F: F \text{ is closed}, A \subseteq F\}$.

Definition 1.9

$$\rho(A^\circ) = \rho(\text{int}(A)) = \frac{\text{number elements of } A}{\text{number elements of } X}$$

$$\rho(A) = \rho(\text{cl}(A)) = \frac{\text{number elements of } \bar{A}}{\text{number elements of } X}$$

$$\beta \cdot \rho(A^\circ) = \rho(\beta \cdot \text{int}(A)) = \frac{\text{number elements of } \beta \cdot \text{int}(A)}{\text{number elements of } X}$$

$$\beta \cdot \rho(A) = \rho(\beta \cdot \text{cl}(A)) = \frac{\text{number elements of } \beta \cdot \text{cl}(A)}{\text{number elements of } X}$$

$$T_1 = \{x, \emptyset\}$$

$$T^{c1} = \{x, \emptyset\}$$

$$\beta - T_1 = \{x, \emptyset, \{\alpha\}, \{b\}, \{c\}, \{\alpha, b\}, \{\alpha, c\}, \{b, c\}\}$$

$$\beta - T_{c1} = \{x, \emptyset, \{b, c\}, \{\alpha, b\}, \{\alpha, b\}, \{c\}, \{b\}, \{c\}\}$$

| T_1 | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a,b\}$ | $\{a,c\}$ | $\{b,c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |

$$T_2 = \{x, \emptyset, \{a\}\}$$

$$T_2^c = \{x, \emptyset, \{b, c\}\}$$

$$\beta\text{-}T_2 = \{x, \emptyset, \{a\}, \{a, b\}, \{a, b\}, \{a, c\}\}$$

$$\beta\text{-}T_2^c = \{x, \emptyset, \{b, c\}, \{c\}, \{b\}\}$$

| T_2 | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |

$$T_3 = \{x, \emptyset, \{b\}\}$$

$$T^{c_3} = \{x, \emptyset, \{a, c\}\}$$

$$\beta\text{-}T_3 = \{x, \emptyset, \{b\}, \{a, b\}, \{b, c\}\}$$

$$\beta\text{-}T^{c_3} = \{x, \emptyset, \{a, c\}, \{c\}, \{a\}\}$$

| T_3 | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|---------------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ |
| $\overline{\rho}(\overline{A})$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ |
| $\overline{\rho}(\beta \overline{A})$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |

$$T_4 = \{x, \emptyset, \{c\}\}$$

$$T^{c_4} = \{x, \emptyset, \{a, b\}\}$$

$$\beta\text{-}T_4 = \{x, \emptyset, \{c\}, \{b, c\}\}$$

$$\beta\text{-}T^{c_4} = \{x, \emptyset, \{a, b\}, \{a\}\}$$

| T_4 | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |

$$T_5 = \{x, \emptyset, \{a\}, \{a, b\}\}$$

$$T^{c_5} = \{x, \emptyset, \{b, c\}, \{c\}\}$$

$$\beta\text{-}T_5 = \{x, \emptyset, \{a, b\}, \{a, c\}\}$$

$$\beta\text{-}T^{c_5} = \{x, \emptyset, \{b, c\}, \{c\}, \{b\}\}$$

| T_5 | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |

$$T_6 = \{x, \emptyset, \{a\}, \{a,c\}\}$$

$$T^c_6 = \{x, \emptyset, \{a\}, \{a,c\}\}$$

$$\beta\text{-}T_6 = \{x, \emptyset, \{a\}, \{a,b\}, \{a,c\}\}$$

$$\beta\text{-}T^c_6 = \{x, \emptyset, \{b,c\}, \{c\}, \{b\}\}$$

| T_6 | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a,b\}$ | $\{a,c\}$ | $\{b,c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |

$$T_7 = \{x, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$$

$$T^{c_7} = \{x, \emptyset, \{b, c\}, \{c\}, \{b\}\}$$

$$\beta\text{-}T_7 = \{x, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$$

$$\beta\text{-}T^{c_7} = \{x, \emptyset, \{b, c\}, \{c\}, \{b\}\}$$

| T_7 | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |

$$T_8 = \{x, \emptyset, \{b\}, \{a, b\}\}$$

$$T^c_8 = \{x, \emptyset, \{a, c\}, \{c\}\}$$

$$\beta-T_8 = \{x, \emptyset, \{b\}, \{a, b\}, \{b, c\}\}$$

$$\beta-T^c_8 = \{x, \emptyset, \{a, c\}, \{c\}, \{a\}\}$$

| T_8 | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ |

$$T_9 = \{x, \emptyset, \{b\}, \{b, c\}\}$$

$$T^{c_9} = \{x, \emptyset, \{a, c\}, \{a\}\}$$

$$\beta - T_9 = \{x, \emptyset, \{b\}, \{a, b\}, \{b, c\}\}$$

$$\beta - T^{c_9} = \{x, \emptyset, \{a, c\}, \{c\}, \{a\}\}$$

| T_9 | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ |

$$T_{10} = \{x, \emptyset, \{b\}, \{a, b\}, \{b, c\}\}$$

$$T^c_{10} = \{x, \emptyset, \{a, c\}, \{c\}, \{a\}\}$$

$$\beta - T_{10} = \{x, \emptyset, \{b\}, \{a, b\}, \{b, c\}\}$$

$$\beta - T^c_{10} = \{x, \emptyset, \{a, c\}, \{c\}, \{a\}\}$$

| T_{10} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |

$$T_{11} = \{x, \emptyset, \{C\}, \{a, c\}\}$$

$$T_{11}^c = \{x, \emptyset, \{a, b\}, \{b\}\}$$

$$\beta-T_{11} = \{x, \emptyset, \{b\}, \{a, c\}, \{b, c\}\}$$

$$\beta-T_{11}^c = \{x, \emptyset, \{a, b\}, \{b\}, \{c\}\}$$

| T_{11} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |

$$T_{12} = \{x, \{c\}, \{b, c\}, \emptyset\}$$

$$T_{12}^c = \{x, \emptyset, \{a, b\}, \{a\}\}$$

$$\beta - T_{12} = \{x, \emptyset, \{c\}, \{a, b\}, \{b, c\}\}$$

$$\beta - T_{12}^c = \{x, \emptyset, \{a, b\}, \{b\}, \{a\}\}$$

| T_{12} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |

$$T_{13} = \{x, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$$

$$T_{13}^c = \{X \emptyset, \{a, b\}, \{b\}, \{a\}\}$$

$$\beta. T_{13}(x) = \{x, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$$

$$\beta. T_{13}^c(x) = \{x, \emptyset, \{a, b\}, \{b\}, \{a\}\}$$

| T_{13} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |

$$T_{14} = \{x, \emptyset, \{a, b\}\}$$

$$T^{c_{14}} = \{x, \emptyset, \{c\}\}$$

$$\beta\text{-}T_{14} = \{x, \emptyset, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$\beta\text{-}T^{c_{14}} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{c\}, \{b\}, \{a\}\}$$

| T_{14} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |

$$T_{15} = \{x, \emptyset, \{b, c\}\}$$

$$T_{15}^c = \{x, \emptyset, \{a\}\}$$

$$\beta\text{-}T_{15} = \{x, \emptyset, \{b\}, \{c\}, \{a, d\}, \{a, c\}, \{a, b\}\}$$

$$\beta\text{-}T_{15}^c = \{x, \emptyset, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$$

| T_{15} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |

$$T_{16} = \{x, \emptyset, \{a, c\}\}$$

$$T^c_{16} = \{x, \emptyset, \{b\}\}$$

$$\beta\text{-}T_{16} = \{x, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$\beta\text{-}T^c_{16} = \{x, \emptyset, \{b, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$$

| T_{16} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{0}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{3}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |

$$T_{17} = \{x, \emptyset, \{a\}, \{b\}, \{a, b\}\}$$

$$T^c_{17} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{c\}\}$$

$$\beta\text{-}T_{17} = \{x, \emptyset, \{a, c\}, \{b, c\}\}$$

$$\beta\text{-}T^c_{17} = \{x, \emptyset, \{b\}, \{a\}\}$$

| T_{17} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |

$$T_{18} = \{x, \emptyset, \{b\}, \{c\}, \{b, c\}\}$$

$$T^c_{18} = \{x, \emptyset, \{b, c\}, \{a, b\}, \{a\}\}$$

$$\beta\text{-}T_{18} = \{x, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$$

$$\beta\text{-}T^c_{18} = \{x, \emptyset, \{b, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$$

| T_{18} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |

$$T_{19} = \{x, \emptyset, \{b\}, \{c\}, \{b, c\}\}$$

$$T^{c_{19}} = \{x, \emptyset, \{b, c\}, \{a, b\}, \{a\}\}$$

$$\beta\text{-}T_{19} = \{x, \emptyset, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$\beta\text{-}T^{c_{19}} = \{x, \emptyset, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$$

| T_{19} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |

$$T_{20} = \{x, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$T^c_{20} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{c\}, \{b\}, \{a\}\}$$

$$\beta\text{-}T_{20} = \{x, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$$

$$\beta\text{-}T^c_{20} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{c\}, \{a\}\}$$

| T_{20} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |

$$T_{21} = \{x, \emptyset, \{a\}, \{c\}, \{a,c\}, \{b,c\}\}$$

$$T^c_{21} = \{x, \emptyset, \{b,c\}, \{a,b\}, \{b\}, \{a\}\}$$

$$\beta-T_{21} = \{x, \emptyset, \{a\}, \{c\}, \{a,c\}, \{b,c\}\}$$

$$\beta-T^c_{21} = \{x, \emptyset, \{b,c\}, \{a,b\}, \{b\}, \{a\}\}$$

| T_{21} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a,b\}$ | $\{a,c\}$ | $\{b,c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |

$$T_{22} = \{x, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$$

$$T^c_{22} = \{x, \emptyset, \{a, c\}, \{a, b\}, \{a\}, \{b\}\}$$

$$\beta\text{-}T_{22} = \{x, \emptyset, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$$

$$\beta\text{-}T^c_{22} = \{x, \emptyset, \{a, c\}, \{a, b\}, \{b\}, \{a\}\}$$

| T_{22} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |

$$T_{23} = \{x, \emptyset, \{a\}, \{b, c\}\}$$

$$T^c_{23} = \{x, \emptyset, \{b, c\}, \{a\}\}$$

$$\beta\text{-}T_{23} = \{x, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}\}$$

$$\beta\text{-}T^c_{23} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$$

| T_{23} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |

$$T_{24} = \{x, \emptyset\{b\}, \{a,c\}\}$$

$$T^c_{24} = \{x, \emptyset, \{a, c\}, \{b\}\}$$

$$\beta\text{-}T_{24} = \{x, \emptyset, \{a\}, \{b\}\{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$$

$$\beta\text{-}T^c_{24} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{a,b\}, \{c\}, \{b\}, \{a\}\}$$

| T_{24} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a,b\}$ | $\{a,c\}$ | $\{b,c\}$ |
|---------------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ |
| $\overline{\rho}(\overline{A})$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\overline{\rho}(\beta \overline{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |

$$T_{25} = \{x, \emptyset, \{c\}, \{a, b\}\}$$

$$T^c_{25} = \{x, \emptyset, \{a, b\}, \{c\}\}$$

$$\beta\text{-}T_{25}(x) = \{x, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$\beta\text{-}T^c_{25} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$$

| T_{25} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |

$$T_{26} = \{x, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$$

$$T^c_{26} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{c\}, \{b\}\}$$

$$\beta\text{-}T_{26} = \{x, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$$

$$\beta\text{-}T^c_{26} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{c\}, \{b\}\}$$

| T_{26} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |

$$T_{27} = \{x, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$$

$$T^c_{27} = \{x, \emptyset, \{b, c\}, \{a, b\}, \{b\}, \{c\}\}$$

$$\beta\text{-}T_{27} = \{x, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$$

$$\beta\text{-}T^c_{27} = \{x, \emptyset, \{b, c\}, \{a, b\}, \{c\}, \{b\}\}$$

| T_{27} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{2}{3}$ |

$$T_{28} = \{x, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$$

$$T^c_{28} = \{x, \emptyset, \{a, c\}, \{a, b\}, \{a\}, \{c\}\}$$

$$\beta\text{-}T_{28} = \{x, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$$

$$\beta\text{-}T^c_{28} = \{x, \emptyset, \{a, c\}, \{a, b\}, \{a\}, \{c\}\}$$

| T_{28} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{b, c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{0}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ |

$$T_{29} = \{x, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$T^c_{29} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$$

$$\beta\text{-}T_{29} = \{x, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$\beta\text{-}T^c_{29} = \{x, \emptyset, \{b, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$$

| T_{29} | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a,b\}$ | $\{a,c\}$ | $\{b,c\}$ |
|-----------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\underline{\rho}(A^\circ)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\underline{\rho}(\beta A^\circ)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| $\bar{\rho}(\beta \bar{A})$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |

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