جمهورية العراق
وزارة التعليم العالي والبحث والعلمي
جامعـــة بــابــل
كليـــة التربية للعلوم الصرفة
قسم الرياضيات

Random Graph

بحث مقدم من قبل الطالبة

زهراء صالح سلطان

الى مجلس كلية التربية للعلوم الصرفة/ قسم الرياضيات/ جامعة بابل و هو
كجزء من متطلبات نيل درجة البكالوريوس في الرياضيات

اشراف الدكتورة

د.رومى كريم خضر عجينه

1444 هـ　　　　2023م

بسم الله الرحمن الرحيم

ن وَالْقَلَمِ وَمَا يَسْطُرُونَ

صدق الله العظيم

(سورة القلم - الآية 1)

الإهداء

لوجه الله تبارك وتعالى والحمد لله الذي هداني لهذا وأعانني فيه..

إلى من أوصاني بطلب العلم من المهد إلى اللحد.

نبينا محمد صلى الله عليه وآله وسلم

إلى من بنعمه تربيت وبمائه ارتويت وبأرضه سعيت

وطني الجريح

الى من بذكره عطر دربي ودفعني الى تقديم المزيد من العطاء...

والدي

إلى من رفعت حاجبها الى السماء وأغدقت علي ببركات دعائها...

والدتي العزيزة

الى الذي ساندني ودفعني إلى أعلى درجات التقدم والنجاح

والعلم اساتذتي الاعزاء

الى زملائي في دراستي لكم ثمرة تعبي

تقديراً واحتراماً

وأسال الله سبحانه حسن القبول

بسم الله الرحمن الرحيم

(نِّعْمَةً مِّنْ عِندِنَا كَذَٰلِكَ نَجْزِي مَن شَكَرَ)

صدق الله العلي العظيم القمر: (35)

شكر وامتنان

باسمه تعالى ابتدأ وافتتح ، و باسمه إن شاء الله أختم ما به ابتدأت ، باسمه الجليل مالك الملك الذي استخلفنا وكلفنا في الأرض لننوب عن جلالته عز شأنه، في كل أمر به.

ثم الحمد الله ذي المنة والفضل ، إذ شرفني وسخر لي من أعانني من الأساتذة الأكفاء الادلاء والعلماء الأجلاء لجمع ما تيسر جمعه من المصادر والأقوال.

وأفضل الصلاة وأتم التسليم على صاحب الوحي الأمين محمد (ص) الذي اقتدينا بهديه عبر العصور قائدا ، وتأسينا به معلما وسرنا على نهجه لنعلم الكتاب والحكمة فكان لنا على الدوام داعيا إلى الله وسراجا منيرا.

وعلى آله الطيبين الأطهار الذين ما اقتد بهم مقتد حق الاقتداء إلا اهتدى واستقام. وما أنكر منكر عملهم وفضلهم بظلم إلا ضاع في الظلام.

وعلى صحبه المنتجبين الأخيار الذين كانوا حوله كالنجوم والأقمار. وعلى التابعين وتابعي التابعين ومن تبعهم بإحسان من العلماء والمجاهدين والعاملين إلى يوم الدين. وبعد حمد الله وشكره الذي هداني ووفقني لانجاز هذا العمل وألهمني الصبر والتحمل لما واجهتني من صعوبات واعترافا بالفضل لأهله.

وأتوجه بجزيل الشكر والتقدير إلى الدكتوره (رومى كريم خضر عجينه) لما بذلت من جهود كبيرة من خلال آرائها وتوجيهاتها القيمة أثناء مدة إعداد البحث. وفقها الله وأدام عليها نعمة التواضع.

لمده يد العون والمساعدة في كتابة واتمام هذا البحث .

Content

Abstract

The random graph has been studied in this research. Several examples are built based on the definition of a random graph with different values of the vertices. The random networks are discussed in two models.

First one which is G (N,L) that is connected through using random placed edege.

Second one that is G(N,P) has been connected with a probability p.

The connectivity of the random graph is studied as well, some examples are discussed. On the other hand, the random subgraphs are presented.

Introduction

In mathematics, a random graph is a mathematical model that represents a collection of nodes or vertices connected by edges according to a random process. The concept of random graphs was first introduced by mathematician Paul Erdős and mathematician Alfréd Rényi in the 1960s as a way to study the properties of large, complex networks.

In a random graph model, the graph's structure is not predetermined but is generated probabilistically. The model typically involves specifying a set of parameters that determine the probability of each possible edge existing between any pair of vertices. The resulting graph is then constructed by randomly selecting edges based on these probabilities.

Random graphs have become a fundamental tool for studying various phenomena and properties of networks in fields such as computer science, physics, sociology, and biology. They provide a simplified representation of real-world complex systems and help researchers understand the behavior of these systems under different conditions.

Random graph models have been used to analyze a wide range of network characteristics, including connectivity, clustering, degree distributions, shortest paths, and resilience to random failures or attacks. They have also been instrumental in studying phase transitions and critical phenomena in network theory.

Different types of random graph models exist, each with its own set of assumptions and characteristics. Some well-known models include the Erdős-Rényi model, the Watts-Strogatz model, and the Barabási-Albert model. These models exhibit distinct structural properties and have been extensively studied to uncover insights into network behavior.

The study of random graphs is an active area of research, with ongoing investigations into more sophisticated models that capture additional features observed in real-world networks. Random graph theory continues to contribute to our understanding of network phenomena, providing valuable insights into the structure and dynamics of complex systems[1].

Chapter One

Random Graph

1.1 Random Graph

In this section, the random graph is discussed as follows.

Definition 1.1.1. (Random Graph). A random graph is a graph in which properties such as the number of graph vertices, graph edges, and connections between them are determined in some random way. The graphs illustrated above are random graphs on 10 vertices with edge probabilities distributed uniformly in [0,1]. Some kinds of random graphs are shown in Figure (2.1).

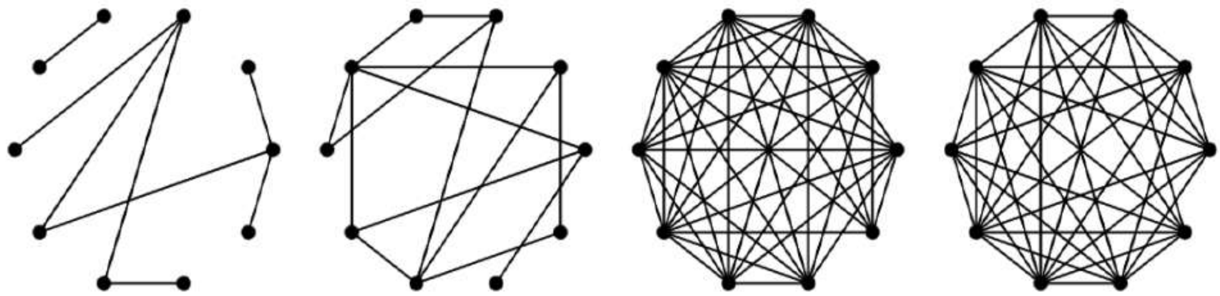


Figure 2.1. Random graphs.

Example 1.1.2. Let n = 5 be the vertices. Then, the random graphs are generated as shown in Figure (2.2).

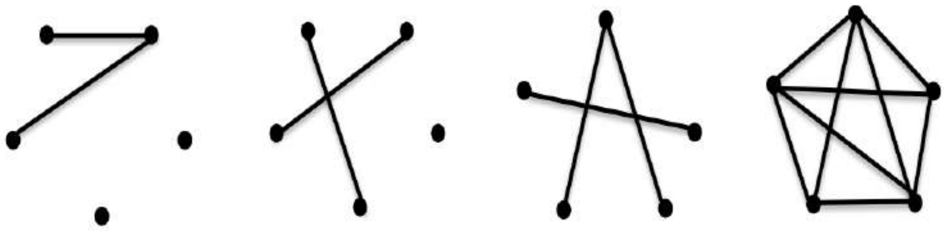


Figure 2.2. Random graphs with 5 vertices.

Example 2.2.2. Let n = 12 be the vertices. Then, the random graphs are generated as shown in Figure (2.2).

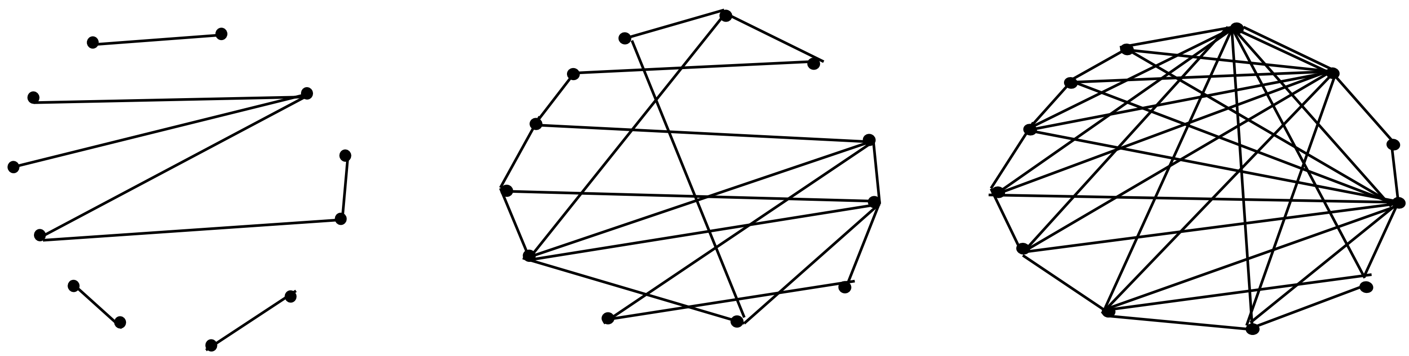


Figure 2.2. Random graphs with 12 vertices

The graphs in the above figure are random diagrams overlying 12 vertices with the probability of a distributed edge as shown in the[o,1]

Example 1.2.3. Let n = 9 be the vertices. Then, the random graphs are generated as shown in Figure (2.3).

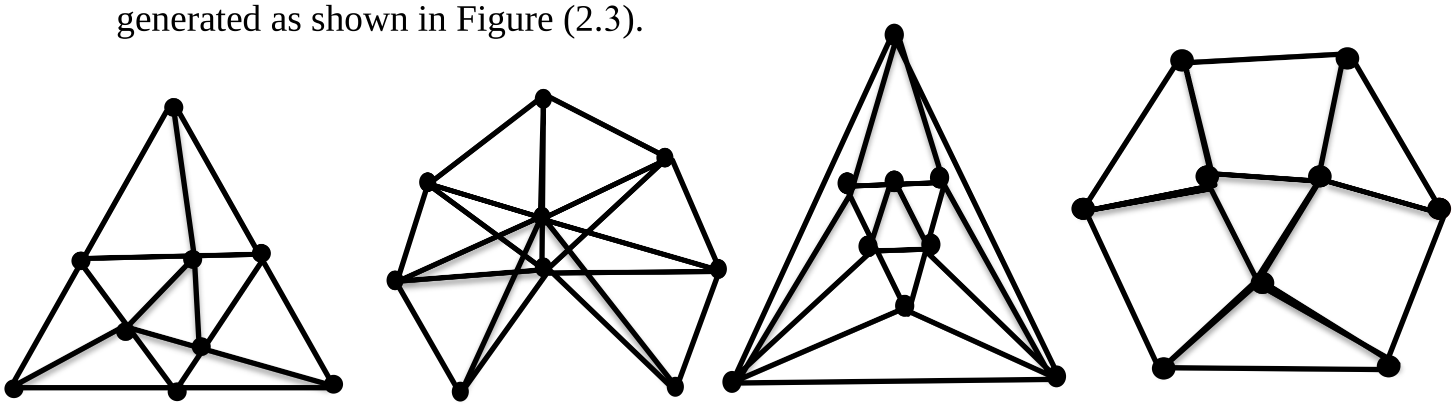


Figure 2.3. Random graphs with 9 vertices

The graph above contains a high number of vertices(9). By sharing several edges, more than one vertex can be obtained differently, depending on the distribution of the vertices in the figure.

Example 1.1.4. Let n = 10 be the vertices. Then, the random graphs are generated as shown in Figure (2.4).

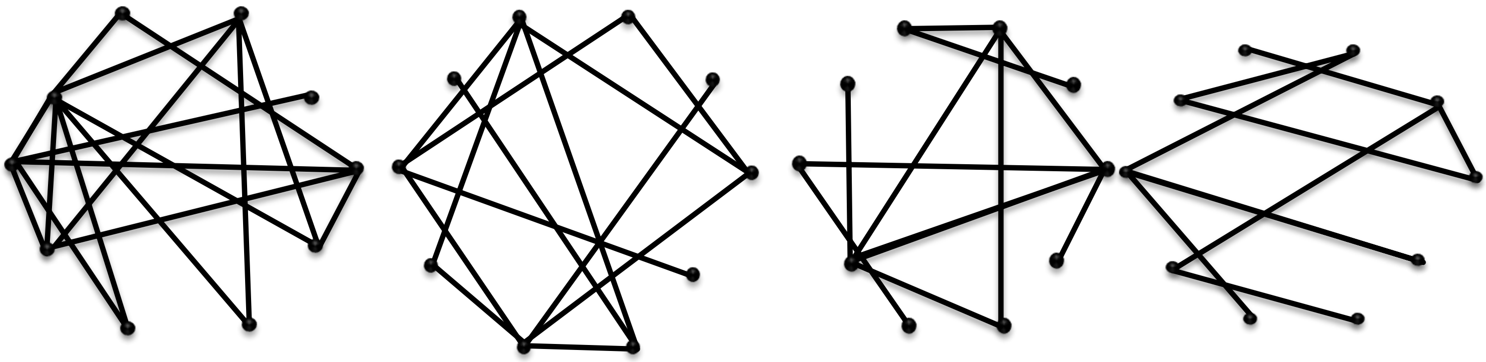


Figure 2.4. Random graphs with 10 vertices

The graph above contains a high number of vertices(10). By sharing several edges, more than one vertex can be obtained differently, depending on the distribution of the vertices in the figure.

Example 1.1.5. Let n = 30 be the vertices. Then, the random graphs are generated as shown in Figure (2.5).

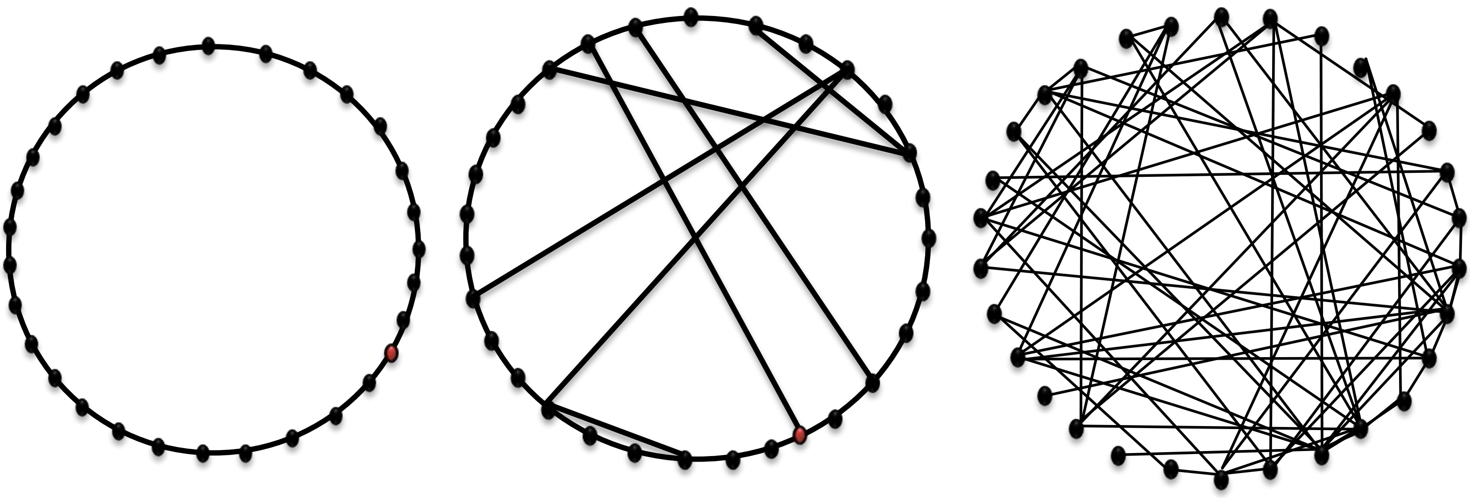


Figure 2.5. Random graphs with 30 vertices

Example 1.1.6. Let n = 17 be the vertices. Then, the random graphs are generated as shown in Figure (2.6).

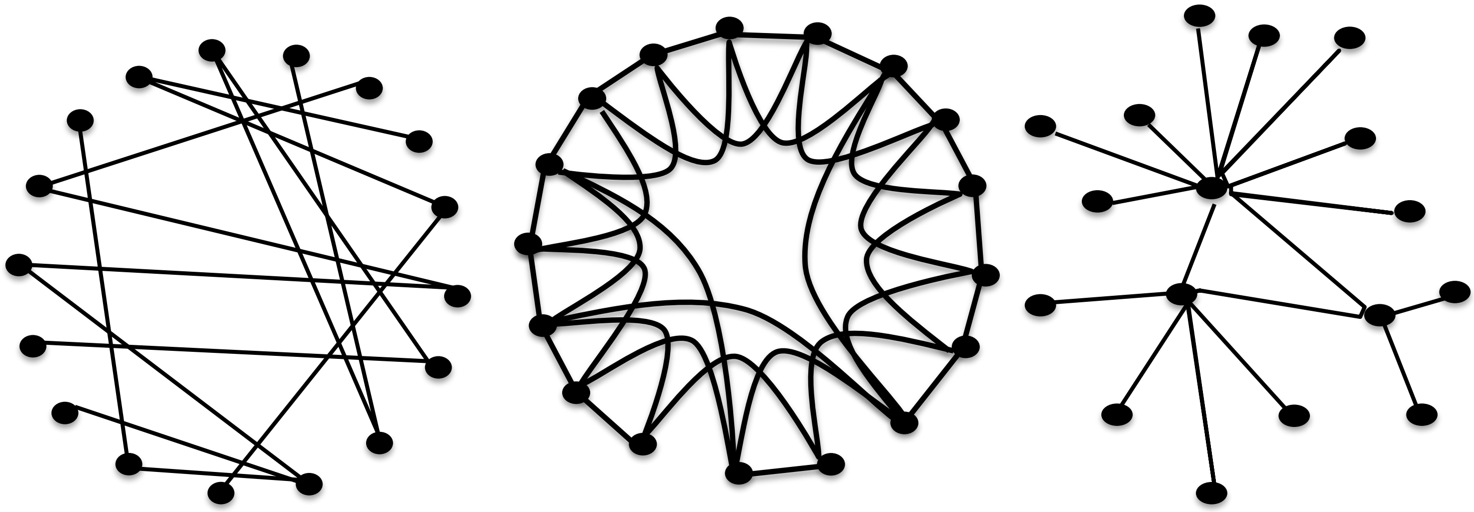


Figure 2.6. Random graphs with 17 vertices

Example 1.1.7. Let n = 20 be the vertices. Then, the random graphs are generated as shown in Figure (2.7).

➢

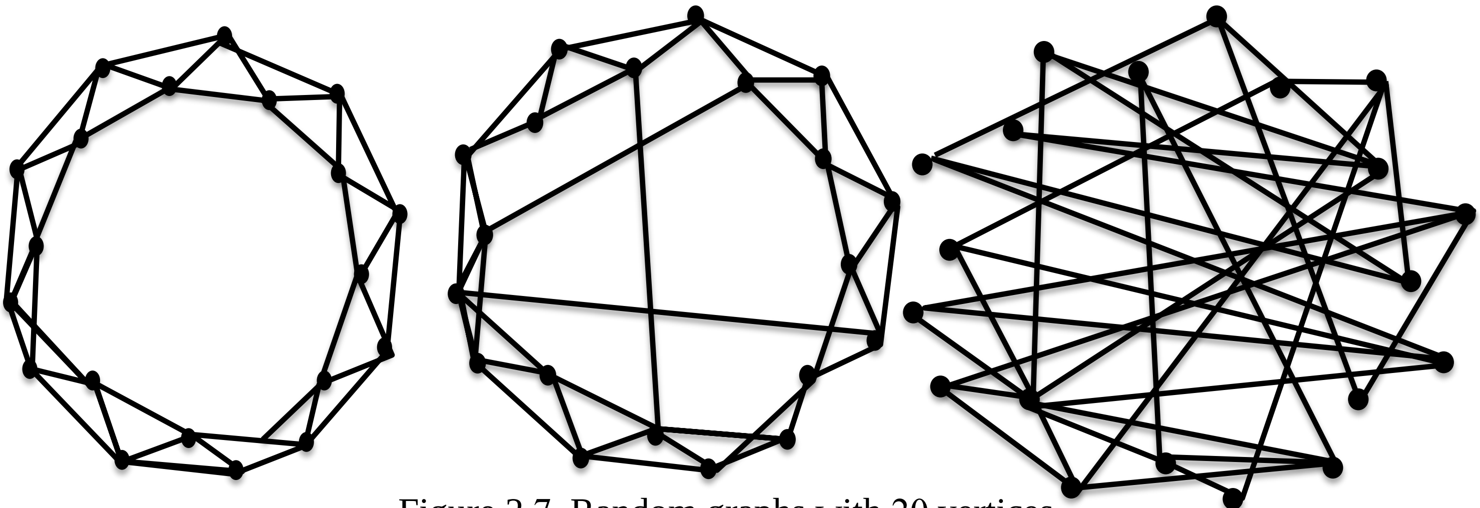


Figure 2.7. Random graphs with 20 vertices

A random graph is obtained by starting with a set of n isolated vertices and randomly adding successive edges between them. The goal of the study in this area is to identify the stage at which a particular characteristic of the graph is likely to appear. Different random graph models produce different probability distributions on graphs. The most commonly studied are those proposed by Edgar Gilbert, referred to as G (n, p), where each possible edge occurs independently with the probability $0 < p < 1$.

Two definitions of random networks

- <u>**Two definitions of random networks**</u>
- *G(N, L) model*: N labeled vertices are connected with L randomly placed edges
- ***G(N, p) model***: Each pair of N labeled vertices are connected with a probability p.
- Though the average degree for a vertices is simply 2L/N in a G(N, L) model, the other key network characteristics are easier to calculate in the G(N, p) model.

Constructing a G(N, p) Network

Algorithm

• Step 1: Start with N isolated vertices

• Step 2: For a particular vertices pair (u, v), generate a random number r. If r≤p, then, add the edges (u, v) to the network.

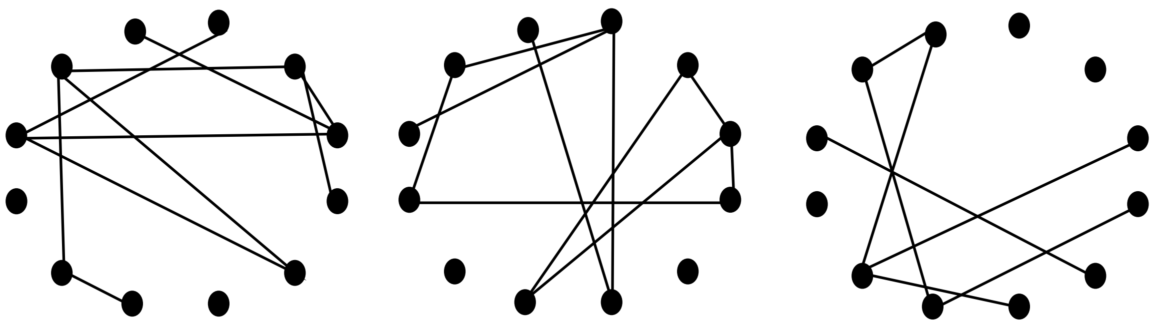
• Repeat Step 2 for each of the N(N-1)/2 vertices pairs.

• Each random network we generate with the same parameters (N, p) will look slightly different.

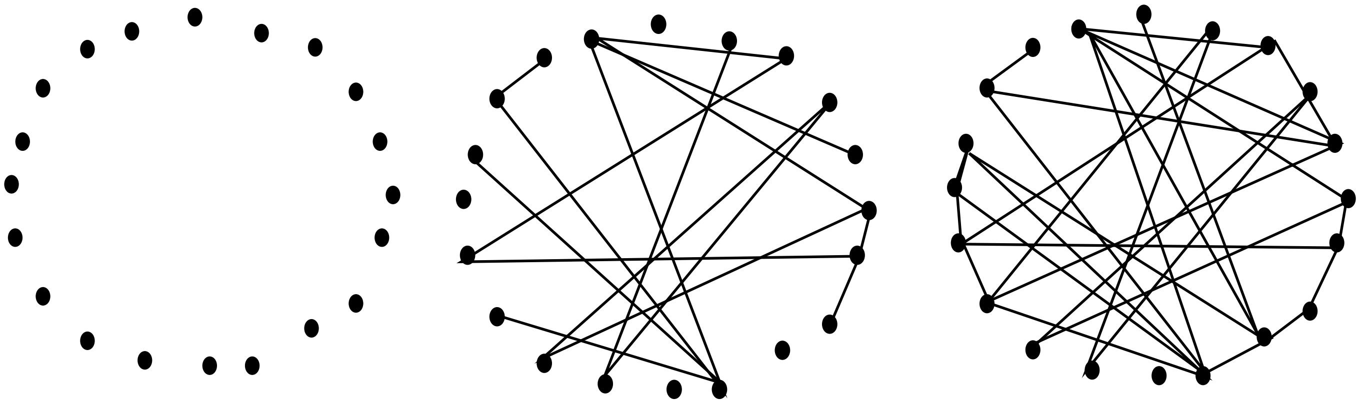
– The number of edges L is likely to be different.

➢ **N=12 vertices, P=1/6**

L 8 L 10 L 7

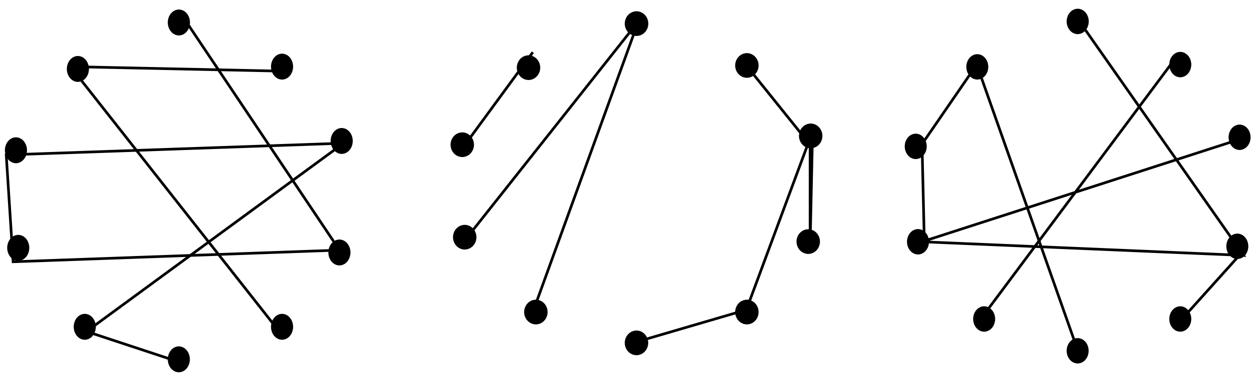


- **For Example. N=20**



P=0 **p=0.1** **p=0.2**

- **For Example. N=10**



Binomial Distribution

Binomial distribution is a common discrete distribution used in statistics, as opposed to a continuous distribution, such as normal distribution. This is because binomial distribution only counts two states, typically represented as 1 (for a success) or 0 (for a failure), given a number of trials in the data.

Binomial distribution thus represents the probability for x successes in n trials, given a success probability and the possibility of failure(p-1)for each. Trial

The binomial distribution has the form $$p_x = \binom{N}{x}\, p^x (1-p)^{N-x}$$ The mean of the distribution (first moment) is $$\langle x \rangle = \sum_{x=0}^{N} x\, p_x = Np$$	Its second moment is $$\langle x^2 \rangle = \sum_{x=0}^{N} x^2\, p_x = p(1-p)\, N + p^2 N^2$$ providing its standard deviation as $$\sigma_x = (\langle x^2 \rangle - \langle x^2 \rangle)^{\frac{1}{2}} = [p(1-p)\, N]^{\frac{1}{2}}$$

$C(N, x) = \binom{N}{x}$ is the different combinations of the results of the N experiments in x which there will be X successes and N-X failures.

And can. Find Bernoulli distributions using the following law

The formula

If we consider the probability that in n number of trials, with r successes and n - r failures by $P_r(x = r)$ then,

$$P_r(x = r) = (x^n) p^x q^{n-x}$$

$$P_r(x = r) = {}^n C_r\, P^r q^{n-r}$$

Where p is probability of success and q is the probability of failure.

Recall: ${}^n C_r = \frac{n!}{(n-r)! \times r!}$

Note that the events are independent of one another for the number of trials.

The Properties

If we denote the mean and standard deviation of the binomial distribution as μ and σ respectively, then:

(i) mean (μ) = np

(ii) standard deviation (σ)= $\sqrt{npq}$

(iii) variance (σ^2) = npq

Example 1

A fair coin is tossed 6 times. Find the probability of obtaining:

$$\boxed{\boldsymbol{P_r(x=r) \;=\; {}^{n}C_r\, P^r q^{n-r}}}$$

(a) exactly 4 heads;

(b) at least 5 heads;

(c) at most 2 heads

$$p = \frac{1}{2} \quad q = \frac{1}{2} \quad n = 6$$

n=6

(a) $P_r(x=4) \;=\; {}^{6}C_4\,(\frac{1}{2})^4(\frac{1}{2})^{6-4}$

$$=\; 15 \times \frac{1}{16} \times \frac{1}{4}$$

$=\frac{15}{64}$

(b) $P_r(x \geq 5) = \quad P_r(x=5)\;\; or\;\; P_r(x=6)$

$$=\; {}^{6}C_5 \;\times (\frac{1}{2})^5 \times (\frac{1}{2})^1 + {}^{6}C_6 \;\times (\frac{1}{2})^6 \times (\frac{1}{2})^0$$

$$= 6 \times \frac{1}{32} \times \frac{1}{2} + 1 \times \frac{1}{64} \times 1$$

$$= \frac{6}{64} \times \frac{1}{64} = \frac{7}{64}$$

(c) $P_r(x \leq 2) = \quad P_r(x = 0)\ or\ P_r(x = 1) or\ P_r(x = 2)$

$$= {}^6C_0 \times (\frac{1}{2})^0 \times (\frac{1}{2})^6 + {}^6C_1 \times (\frac{1}{2})^1 \times (\frac{1}{2})^5 + {}^6C_2 \times (\frac{1}{2})^2 \times (\frac{1}{2})^4$$

$$= 1 \times 1 \times \frac{1}{64} + 6 \times \frac{1}{2} \times \frac{1}{32} + 15 \times \frac{1}{4} \times \frac{1}{16}$$

$$= \frac{1}{64} + \frac{6}{64} + \frac{15}{64}$$

$$= \frac{22}{64} = \frac{11}{32}$$

Example 2

An unbiased die with 6 faces is thrown 5 times. Find the probability that a:

$$\boldsymbol{P_r(x = r) = {}^n C_r\, P^r q^{n-r}}$$

(a) factor of 6 appears exactly 3 times;

(b) perfect square appears at most 4 times.

(a) S = {1, 2, 3, 4, 5, 6} and n(S) = 6

F = {1, 2, 3, 6} and n(F) = 4

$p = \frac{4}{6} = \frac{2}{3}, q = \frac{1}{3}, n = 5\ and\ r = 3$

$$P_r(x = 3) = {}^5C_3 \times (\frac{2}{3})^3 \times (\frac{1}{3})^2$$

$$= 10 \times \frac{8}{27} \times \frac{1}{9}$$

$$= \frac{80}{243}$$

(b) S = {1, 2, 3, 4, 5, 6} and n(S) = 6

E = {1, 4) and n(E) = 2

$p = \frac{4}{6} = \frac{1}{3}, q = \frac{2}{3}, n = 5\ and\ r = 0, 1,2,3\ and\ 4$

$$P_r(x \leq 4) = 1 - P_r(x = 5) = 1 - {}^5C_5 \times \left(\frac{1}{3}\right)^5 \times \left(\frac{2}{3}\right)^0$$

$$= 1 - 1 \times \frac{1}{243} \times 1$$

$$= \frac{242}{243}$$

Example 3

A test contains 10 multiple choice questions comprising of 4 options in which only one option is correct. Find the probability that a candidate can guess 7 out of the 10 questions correctly. Then extract the mean and standard deviation and variance.

$$\boxed{\boldsymbol{P_r(x = r) = {}^nC_r\, P^r q^{n-r}}}$$

$$p = \frac{1}{4} \quad q = \frac{3}{4} \quad n = 10 \quad r = 7$$

$$P_r(x = 7) = {}^{10}C_7 \times \left(\frac{1}{4}\right)^7 \times \left(\frac{3}{4}\right)^3$$

$$= 120 \times \frac{1}{16384} \times \frac{27}{64}$$

$$= 0.0030899$$

$$-\ \boldsymbol{mean\ (\mu) = \eta p}$$

$$= 10.\frac{1}{4}$$

$$= \frac{10}{4}$$

$$= 2.5$$

$$-\ \boldsymbol{standard\ deviation}\ \ (\boldsymbol{\sigma}) = \sqrt{\boldsymbol{\eta pq}}$$

$$= \sqrt{10\ \frac{1}{4}.\frac{3}{4}}$$

$$= \sqrt{\frac{30}{16}} \Longrightarrow \sqrt{1.875}$$

- $\boldsymbol{Variance}\ \ (\boldsymbol{\sigma^2}) = \boldsymbol{\eta pq}$

$$= 10\ \frac{1}{4}.\frac{3}{4}$$

$$= \frac{10}{4} \times \frac{3}{4}$$

$$= \frac{30}{16}$$

$$= 1.875$$

Example 4

The probability that a patient will be cured of corona virus when injected with the new vaccine is 0.8. Find the probability that exactly 3 out of the 8 corona virus patients will be cured on being injected with the vaccine. Then extract the mean and standard deviation and variance

$$p = 0.8\ \ q = 0.2\ \ n = 8\ \ \ r = 3$$

$$P_r(x\ =\ 3)\ =\ {}^8C_3\ \times\ (0.8)^3\ \times\ (0.2)^5$$

$$=\ 56\ \times\ 0.512\ \times\ 0.00032$$

$$= 0.00917504$$

$$-\ \boldsymbol{mean}\ (\boldsymbol{\mu}) = \boldsymbol{\eta p}$$

$$= 8\ \ \times 0.8$$

$$= 6.4$$

$$- \boldsymbol{standard\ deviation\ \ (\sigma) = \sqrt{\eta pq}}$$

$$= \sqrt{8 \ \times 0.8 \ \times 0.2}$$

$$= \ \sqrt{1.28}$$

$$- \boldsymbol{Variance\ (\sigma^2) = \eta pq}$$

$$= \ 8 \ \ \times 0.8 \ \times 0.2$$

$$= \ 1.28$$

How is Binomial Distribution Used?

This distribution pattern is used in statistics but has implications in finance and other fields. Banks may use it to estimate the likelihood of a particular borrower defaulting, how much money to lend, and the amount to keep in reserve. It's also used in the insurance industry to determine policy pricing and assess risk.

Why is Binomial Distribution Important?

Binomial distribution is used to figure the probability of a pass or fail outcome in a survey, or experiment replicated numerous times. There are only two potential outcomes for this type of distribution. More broadly, distribution is an important part of analyzing data sets to estimate all the potential outcomes of the data and how frequently they occur. Forecasting and understanding the success or failure of outcomes is essential to business development.

"Chart junk" is a term coined by data visualization expert Edward Tufte to refer to elements of a chart or graph that do not add value to the data being presented, but rather serve only to distract or confuse the viewer. These elements might include unnecessary grid lines, 3D effects, or decorative flourishes, among others.

According to Tufte, the use of chart junk can detract from the clarity and effectiveness of a visualization, making it more difficult for the viewer to understand the data being presented. He advocates for a more minimalistic approach to data visualization, with a focus on presenting the data in a clear and concise manner.

The concept of chart junk has been widely adopted in the field of data visualization, and is often cited as a best practice when it comes to designing effective charts and graphs. Many experts argue that the use of chart junk can lead to "chart clutter," making it more difficult for the viewer to discern the key takeaways from the data.

While Tufte's ideas on chart junk are not universally accepted, his work has had a significant influence on the field of data visualization and has helped to shape best practices for designing effective charts and graphs.

Presenting data to clients: When presenting data to clients, a management consultant might use the concept of chart junk to avoid cluttering the visualization with unnecessary elements. By focusing on presenting the data in a clear and concise manner, the consultant can help the client understand the key takeaways more easily.

Analyzing data: When analyzing data, a consultant might use the concept of chart junk to help identify the most important or relevant information By eliminating unnecessary elements, the consultant can focus on the data that is most relevant to the problem at hand.

Communicating findings: When communicating findings or recommendations to a client, a consultant might use the concept of chart junk to help simplify complex information and present it in a way that is easy for the client to understand.

Designing charts and graphs: When designing charts and graphs, a consultant might use the concept of chart junk to guide their design. choices and ensure that the visualizations are clear and effective.

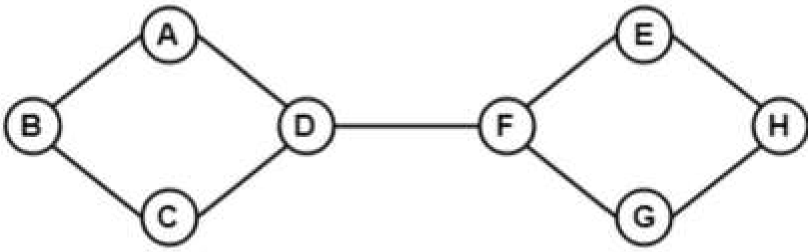
Training clients: A consultant might also use the concept of chart junk to teach clients how to design effective charts and graphs, helping them to avoid common pitfalls and create visualizations that are clear and easy to understand.

Chapter Two

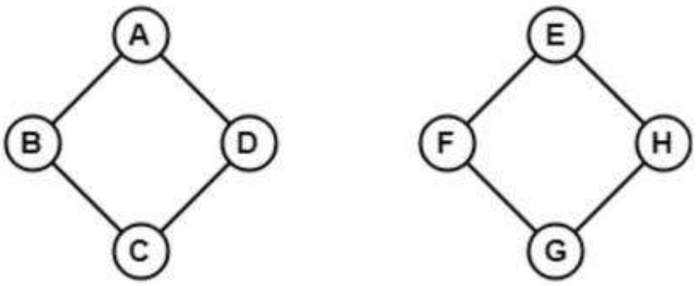
Connectivity

Connectivity is a basic concept of graph theory. It defines whether a graph is connected or disconnected. Without connectivity, it is not possible to traverse a graph from one vertex to another vertex.

- A graph is said to be **connected graph if there is a path between every pair of vertex.** From every vertex to any other vertex there must be some path to traverse. This is called the connectivity of a graph.
- A graph is said to be **disconnected, if there exists multiple disconnected vertices and edges**.
- Graph connectivity theories are essential in network applications, routing transportation networks, network tolerance etc.



In the above example, it is possible to travel from one vertex to another vertex. Here, we can traverse from vertex B to H using the path B -> A->D -> F->E -> H. Hence it is a connected graph.



In the above example, it is not possible to traverse from vertex B to H because there is no path between them directly or indirectly. Hence, it is a disconnected graph.

Let's see some basic concepts of Connectivity.

1. Cut Vertex

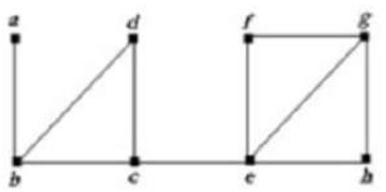
A single vertex whose removal disconnects a graph is called a cut-vertex. Let G be a connected graph. A vertex v of G is called a cut vertex of G, if G-v (Remove v from G) results a disconnected graph.

When we remove a vertex from a graph then graph will break into two or more graphs. This vertex is called a cut vertex.

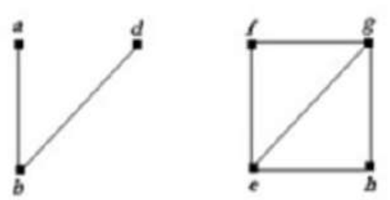
- A connected graph G may have maximum (n- 2) cut vertices.
- Removing a cut vertex may leave a graph disconnected.
- Removing a vertex may increase the number of components in a graph by at least one.
- Every non-pendant vertex of a tree is a cut vertex.

Example 1

Original graph:

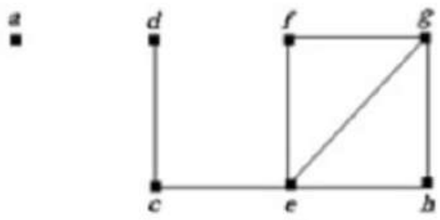


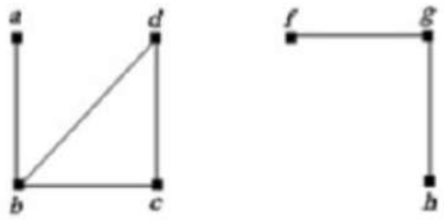
Vertex c is a cut vertex:



Vertex b is a cut vertex:

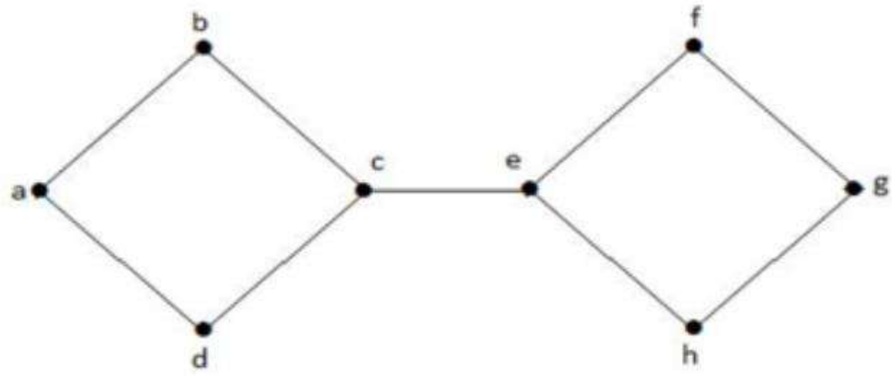
Vertex e is a cut vertex:

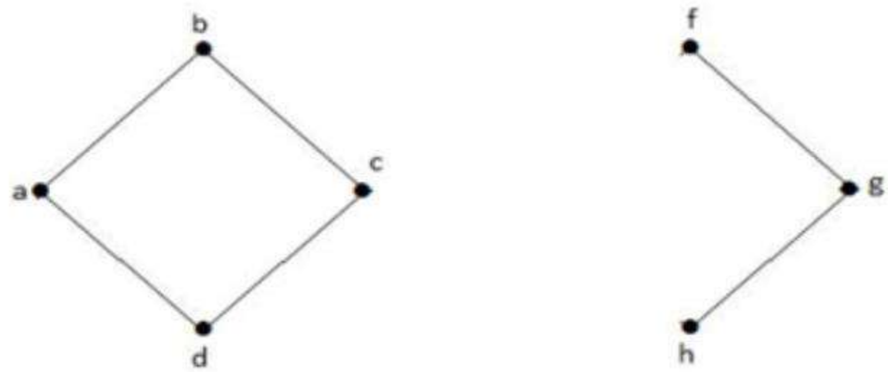




In the above graph, vertex 'e' is a cut-vertex. After removing vertex 'e' from the above graph the graph will become a disconnected graph.

Example 2





2. Cut Edge (Bridge)

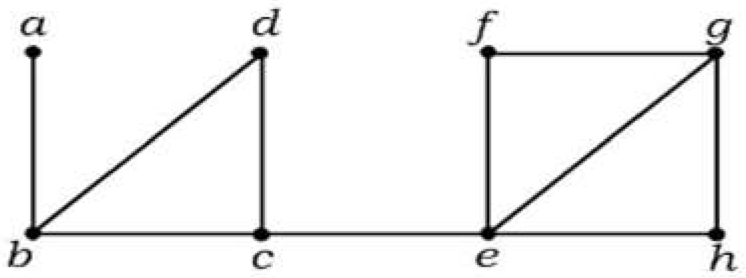
A **cut- Edge or bridge** is a single edge whose removal disconnects a graph.

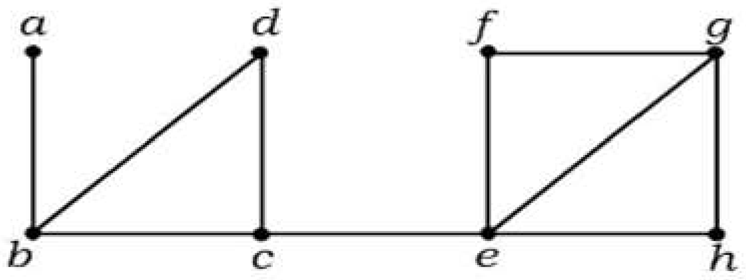
Let G be a connected graph. An edge e of G is called a cut edge of G, if G-e (Remove e from G) results a disconnected graph.

When we remove an edge from a graph then graph will break into two or more graphs. This removal edge is called a cut edge or bridge.

- A connected graph G may have at most (n-1) cut edges.
- Removing a cut edge may leave a graph disconnected.
- Removal of an edge may increase the number of components in a graph by at most one.
- A cut edge 'e' must not be the part of any cycle in G.
- If a cut edge exists, then a cut vertex must also exist because at least one vertex of a cut edge is a cut vertex.
- If a cut vertex exists, then the existence of any cut edge is not necessary.

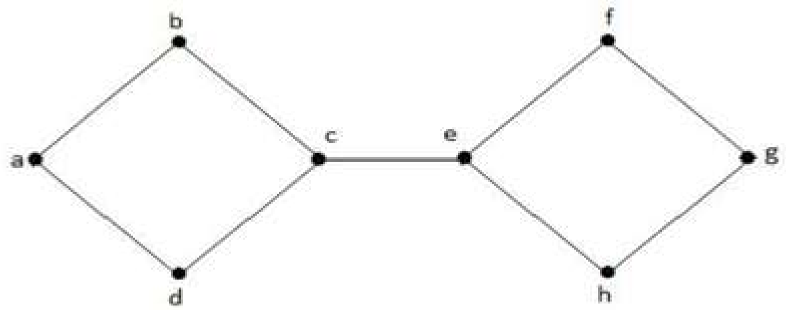
Example 1

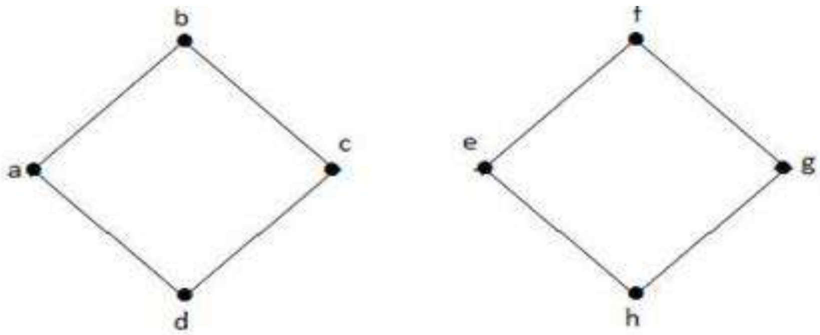




In the above graph, edge (c, e) is a cut-edge. After removing this edge from the above graph the graph will become a disconnected graph.

Example 2





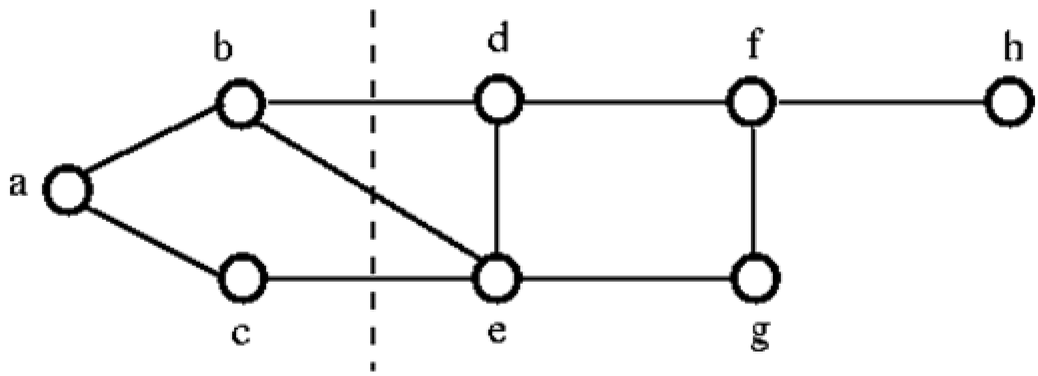
In the above graph, edge (c, e) is a cut-edge. After removing this edge from the above graph the graph will become a disconnected graph.

3. Cut Set

In a connected graph G, a cut set is a set S of edges with the following properties:

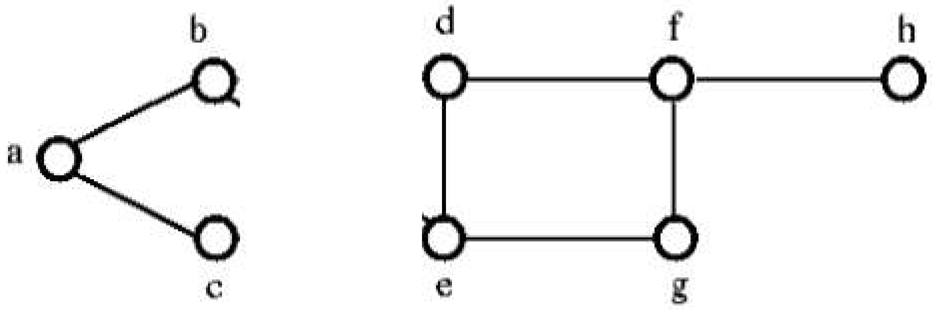
- The removal of all the edges in S disconnects G.
- The removal of some of edges (but not all) in S does not disconnect G.

Example 1



To disconnect the above graph G, we have to remove the three edges. i.e. bd, be and ce. We cannot disconnect it by removing just two of three edges. Hence, {bd, be, ce} is a cut set.

After removing the cut set from the above graph, it would look like as follows:



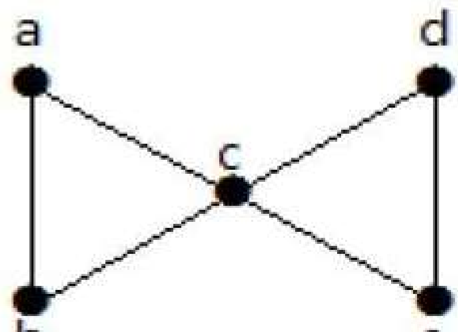
4. Edge Connectivity

The **edge connectivity** of a connected graph G is the minimum number of edges whose removal makes G disconnected. It is denoted by **λ(G)**.

When λ(G) ≥ k, then graph G is said to be **k-edge-connected**.

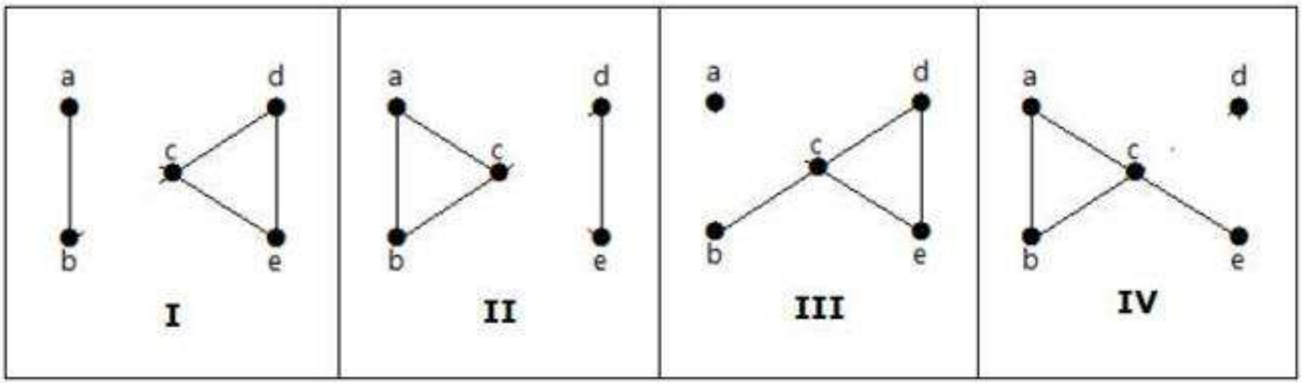
Example

Let's see an example,



From the above graph, by removing two minimum edges, the connected graph becomes disconnected graph. Hence, its edge connectivity is 2. Therefore the above graph is a **2-edge-connected graph**.

Here are the following four ways to disconnect the graph by removing two edges:



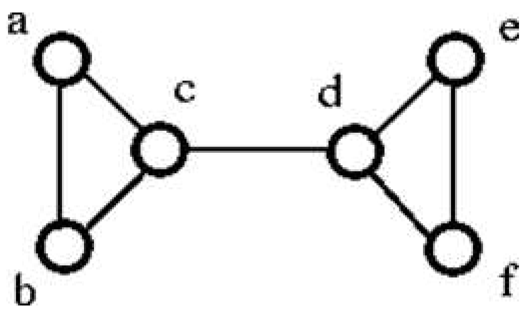
5. Vertex Connectivity

The connectivity (or vertex connectivity) of a connected graph G is the minimum number of vertices whose removal makes G disconnects or reduces to a trivial graph. It is denoted by K(G).

The graph is said to be k- connected or k-vertex connected when K(G) ≥ k. To remove a vertex we must also remove the edges incident to it.

Example

Let's see an example:



The above graph G can be disconnected by removal of the single vertex either 'c' or 'd'. Hence, its vertex connectivity is 1. Therefore, it is a 1-connected graph.

Random subgraph

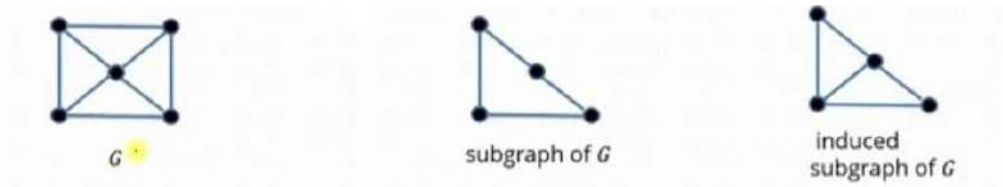
We say that $G' = (V', E')$ is a **subgraph** of $G = (V, E)$, and write $G' \subseteq G$, provided $V' \subseteq CV$ and $E' \subseteq E$.

We say that $G' = (V', E')$ is an **induced subgraph** of $G = (V, E)$ provided $V' \subseteq V$ and every edge in E whose vertices are still in V' is also an edge in E'.

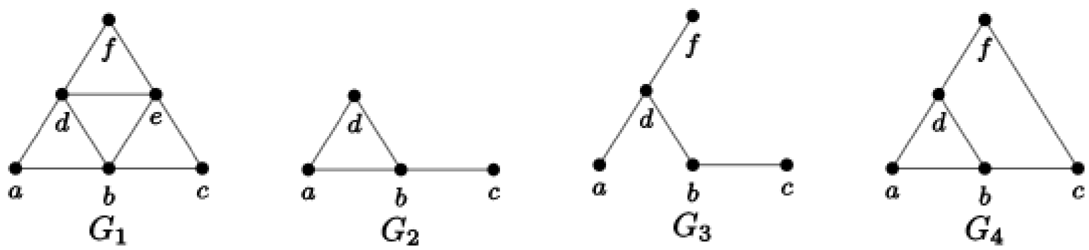
Think of a **subgraph** as the result of deleting some vertices and edges from the larger graph.

For the subgraph to be an **induced subgraph**, we can still delete vertices, but now we only delete those edges that included the deleted vertices.

Notice that every induced subgraph is also an ordinary subgraph, but not conversely.



Example 4.1.4 Consider the graphs. Determine if G_2, G_3, and G_4 are subgraphs and induced subgraphs of G_1.



G_2 is a subgraph of G_1. G_2 is also an induced subgraph of G_1. Every edge in G, that connects vertices in G_2 is also an edge in G_2.

G_3 is a subgraph of G_1. G_3 is not an induced subgraph of G_1, In G_3, the edge (a, b) is in E, but not E_3, even though vertices a and b are in V_3.

The graph G_4 is NOT a subgraph of G_1, even though it looks like all we did is remove vertex e. The reason is that in E_4, we have the edge *{c,f}* but this is not an element of E_1, so we don't have the required $E_4 \subseteq E_1$. If G4 is NOT a subgraph, then it can't be an induced subgraph.

cycle of length at least $|P|- 2b \geq n-24n^{3/4}$, completing the proof.

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