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Aboodh Transform

and linking it to some effective integral Transformations

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بسم الله الرحمن الرحيم

(يَرْفَعِ اللَّهُ الَّذِينَ آمَنُوا مِنكُمْ وَالَّذِينَ أُوتُوا الْعِلْمَ دَرَجَاتٍ)

صدق الله العلي العظيم

(المجادلة: الآية ١١)

الإهداء

بكل فخر وامتنان، أقدم هذا البحث بفخر وامتنان لعائلتي العزيزة،

التي كانت دائماً داعمة لي ومصدر إلهامي وقوتي.

بفضل حبهم وتشجيعهم الدائم، تمكنت من تحقيق هذا الإنجاز العظيم.

إلى أهلي الأعزاء، لقلوبهم الطيبة التي لا تعرف الحدود،

ولروحهم النبيلة التي أضاءت دربي بالأمل والثقة،

أهدي هذا العمل كتعبير عن امتناني وحبي الكبير لهم.

دائماً معي في كل خطوة، شكراً لكم على كل شيء..

الشكر والتقدير

بكل فخر وامتنان، أتقدم بخالص الشكر والتقدير لمشرفي ، م.م. علي حسن عبد الخالق،
الذي بفضل ارشاداته وتفانه ودعمه لي لعب دوراً بارزاً في اتمام هذا العمل.
مما ساهم في تطوير مهاراتي البحثية. أعرب عن امتناني العميق له،
وأشكره من أعماق قلبي.
كما أرغب في تقدير كل أساتذتي الذين شاركوني المعرفة والخبرة خلال رحلتي الأكاديمية،
فلقد كانوا مصدر إلهام وتحفيز لي.
أشكرهم جميعًا على جهودهم الجبارة والمستمرة في دعمي وتوجيهي نحو التميز والنجاح.

Abstract:

In this research, we studied the Aboodh transformation, where we presented its transformations and the inverse of the transformation, in addition to clarifying how it is applied to differential equations, as it can be used to solve some initial value problems in ordinary differential equations, while presenting a group of examples of that, and linking it to some effective integral Transformations.

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Introduction

This conversion has been applied in various fields of science, engineering, and technology [1],[2]. Ordinary linear differential equations with a fixed coefficient and a variable coefficient can be easily solved using the Abudha transform method without finding the general and arbitrary constant solution. Differential equations arising in scientific and engineering problems are usually solved: Laplace transform, convolution method, calculus method, Zaki transform, residuals, mahgoub transform, mohand transform, sumudu transform… In this research, the Abudha transformation technique is presented for analyzing simultaneous differential equations with boundary conditions. It explains how to apply it to solve equations, in addition to linking it to some of the famous integral transformations.

Chapter one

Aboodh Transform

1.1 Introduction to Aboodh Transform

Aboodh Transform is derived from the classical Fourier integral. Based on the mathematical simplicity of the Aboodh transform and its fundamental properties

Aboodh transform was introduced by Khalid Aboodh to facilitate the process of solving ordinary and partial differential equations in the time domain.

Typically, Fourier, Laplace, Elzaki and Sumudu transforms are the convenient mathematical tools for solving differential equations.

Also Aboodh transform and some of its fundamental properties are used to solve differential equations. [2]

Aboodh Transform [1] was introduced by Khalid Suliman Aboodh in 2013, in Omdurman Islamic University to facilitate the process of solving ordinary and partial differential equations in the time domain.

This transformation has deeper connection with the Laplace Transform. Introduced a comparative study of Laplace and Aboodh transform and applied both transforms to solve system of differential equations to see the differences and similarities. The result shows that Laplace and Aboodh transform are closely related.

1..٢ Definition of Aboodh Transformation [2]

A new transform called the Aboodh transform defined for function of exponential order we consider functions in the set A defined by

$$A = \{f(t): \exists M, k_1, k_2 > 0, |f(t)| < Me^{-vt}] \quad ....(1)$$

For a given function in the set A, the constant M must be finite number k_1, k_2 may be finite or infinite.

The Aboodh transform denoted by the operator A (.) Defined by the integral equations

$$A[f(t)] = K(v) = \frac{1}{v}\int_0^{\infty} f(t)e^{-vt}dt \quad t \geq 0, k^1 \leq v \leq k^2 \quad \dots (2)$$

The variable v in this transform is used to factor the variable in the argument of the function f. This transform has deeper Connection with the Laplace and Elzaki transform. We also present many different of properties of this new transform and Sumudu. Elzaki transforms, few properties exptent.

The purpose of this study is to show the applicability of this interesting new transform and its efficiency in solving the linear differential equation. [2]

1.3. Linearity Property:

A Linearity Property of Aboodh Transform: If Aboodh transform of functions $F_1(t)$ and $F_2(t)$ are $K_1(v)$ and $K_2(v)$ respectively.

then Aboodh transform of $[aF_1\,(t) + b\,F_2(t)]$ is given by $[a\,K_1(v) + \,b\,K_2(v)]$, where a and b are arbitrary constants and mathematically:

$$A[aF_1(t) + bF_2(t)] = aA\,[F_1(t)] + bA\,[F_2(t)]$$

$$= a\,K_1(v) + \,b\,K_2(v)$$

1.4. Aboodh Transform of the Some Functions:

For any function $f(t)$, we assume that the integral equation (2) exists. The Sufficient Conditions for the existence of Aboodh transform are that $f(t)\ for\ t \geq\ 0$ be piecewise continuous and of exponential order. Otherwise Aboodh transform may or may not exist.

In this section we find Aboodh transform of simple functions

1) let $f(t) = 1$,

then

$$A\,[1] = \frac{1}{v^2}$$

Proof:

$$A[1] = \frac{1}{v}\int_0^{\infty} e^{-vt}\,dt \; = \frac{1}{-v^2}[0-1] = \frac{1}{v^2}$$

2) let $f(t) \; = \; t$.

then

$$t] = \frac{1}{v^3} A\,[$$

Proof:

$$A[t] = \frac{1}{v}\int_0^{\infty} te^{-vt}\,dt$$

Integrating by parts to find that: A [t] $= \frac{1}{v^3}$

3) let $f(t) \; = t^n$

then

$$A[t^n] = \frac{n!}{v^{n+2}}$$

:

Proof

$$A[t^n] = \frac{1}{v}\int_0^{\infty} t^n\, e^{-vt}\,dt$$

Integrating by parts to find that:

$$A[t^n] = \frac{n!}{v^{n+2}}$$

Substitution $n = \; 0,1,2,3, \dots, n$ is an Integer numbers, $n > 0$.

$$n = 0, A\,(t^0) = \; A[1]$$

$$= \frac{0!}{v^{0+2}} = \frac{1}{v^2}$$

$$n = 1, \rightarrow A(t^1) = \; A[t]$$

$$= \frac{1!}{v^{1+2}} = \frac{1}{v^3}$$

$$n = 2, \rightarrow A(t^2) = A(t^2)$$

$$= \frac{2!}{v^{2+2}} = \frac{2}{v^4}$$

So $A[t^n] = \frac{n!}{v^{n+2}}$

4) Let $f(t) = e^{at}$,

Then

$$\text{A}\ [\ e^{at}] = \frac{1}{v^2 - av}$$

Proof:

$$\text{A}\ [\ e^{at}] = \frac{1}{v}\int_0^\infty e^{at}\, e^{-vt}\, dt$$

$$= \frac{1}{v}\int_0^\infty e^{at-vt}\ dt$$

$$= \frac{1}{v}\int_0^\infty e^{(a-v)t}\ dt$$

$$= \frac{1}{v}\int_0^\infty e^{-(v-a)t}\ dt$$

$$= \frac{1}{v}\frac{-1}{v-a}\int_0^\infty -(v-a)e^{-(v-a)t}\ dt$$

$$= \frac{-1}{v^2 - av}(0-1)$$

$$= \frac{1}{v^2 - av}$$

5) Let $f(t) = sin\ at$,

then

$$A\ [sin\ at] = \frac{a}{v(v^2 + a^2)}$$

Proof:

$$A\,[\sin at] = \frac{1}{v}\int_0^\infty \; e^{-vt}\left(\frac{e^{iat} - e^{-iat}}{2i}\right)dt$$

$$= \frac{1}{2i\;v}\int_0^\infty e^{(ia-v)t} - e^{(-ia-v)t}\;dt$$

$$= \frac{1}{2i\;v}\int_0^\infty e^{-(v-ia)t} - e^{-(ia+v)t}\;dt$$

$$= \frac{1}{2i\;v}\left[\frac{1}{v-ia} - \frac{1}{v+ia}\right]$$

$$= \frac{1}{2i\;v}\left[\frac{v-ia-v+ia}{v^2+a^2}\right]$$

$$= \frac{a}{v(v^2+a^2)}$$

٦) $Let\; f(t) \;=\; \cos\; at$.

then

$$A\;[cos\;at] \;=\; \frac{1}{(v^2+a^2)}$$

Proof:

$$A\,[\cos at] = \frac{1}{v}\int_0^\infty \; e^{-vt}\left(\frac{e^{iat} + e^{-iat}}{2i}\right)dt$$

$$= \frac{1}{2i\;v}\int_0^\infty e^{(ia-v)t} + e^{(-ia-v)t}\;dt$$

$$= \frac{1}{2i\;v}\int_0^\infty e^{-(v-ia)t} + e^{-(ia+v)t}\;dt$$

$$= \frac{1}{2i\;v}\left[\frac{1}{v-ia} + \frac{1}{v+ia}\right]$$

$$= \frac{1}{2i\;v}\left[\frac{v-ia+v-ia}{v^2+a^2}\right]$$

$$= \frac{1}{(v^2 + a^2)}$$

٧) $Let\ f(t)\ =\ \sinh\ at,$

then

$$A\ [sinh\ at]\ = \frac{a}{v(v^2 - a^2)}$$

Proof:

$$A\ [\sin at] = \frac{1}{v}\int_0^\infty\ e^{-vt}\left(\frac{e^{at} - e^{-at}}{2}\right) dt$$

$$= \frac{1}{2\ v}\int_0^\infty\ e^{(a-v)t} - e^{(-a-v)t}\ dt$$

$$= \frac{1}{2\ v}\int_0^\infty\ e^{-(v-a)t} - e^{-(a+v)t}\ dt$$

$$= \frac{1}{2i\ v}\left[\frac{1}{v - ia} - \frac{1}{v + ia}\right]$$

$$= \frac{1}{2i\ v}\left[\frac{v - a - v + a}{v^2 + a^2}\right]$$

$$= \frac{a}{v(v^2 - a^2)}$$

٨) $Let\ f(t)\ =\ \cosh\ at$

then

$$A\ [cosh\ at]\ = \frac{1}{(v^2 - a^2)}$$

Proof:

$$A\ [\cos at] = \frac{1}{v}\int_0^\infty\ e^{-vt}\left(\frac{e^{at} + e^{-at}}{2}\right) dt$$

$$= \frac{1}{2\ v}\int_0^\infty\ e^{(a-v)t} + e^{(-a-v)t}\ dt$$

$$= \frac{1}{2\ v}\int_0^\infty\ e^{-(v-a)t} + e^{-(a+v)t}\ dt$$

$$= \frac{1}{2\,v}\left[\frac{1}{v-a}+\frac{1}{v+a}\right]$$

$$= \frac{1}{2v}\left[\frac{v-a+v-a}{v^2-a^2}\right]$$

$$= \frac{1}{(v^2-a^2)}$$

1.5. The invers Aboodh Transform[3]

If $A\{f(t)\} \; = \; K(v)$ then $F(t)$ is called the inverse Aboodh transform of $H(v)$ and mathematically it is defined as

$$F(t) = \; \mathrm{A}^{-1}\{\mathrm{H(v)}\}.$$

Where A^{-1} is the inverse Aboodh transform operator.

The integral equation of the inverse Aboodh transform Defined as

$$\mathrm{A}^{-1}(K(v)) \; = \; f(t) = \frac{1}{2\pi i}\int_{w-i\infty}^{w+i\infty} ve^{-vt}\,K(v)\,dv, \;\; w \geq 0$$

	$f(t)$	$A[f(t)] \; = \; K(v)$	$f(t) \; = \; \mathrm{A}^{-1}\,\{K(v)\}$
1	1	$\frac{1}{v^2}$	1
2	t	$\frac{1}{v^3}$	t
3	t^2	$\frac{2!}{v^4}$	t^2
4	$t^n, \quad n > 0$	$\frac{n!}{v^{n+2}}$	t^n
5	e^{at}	$\frac{1}{v^2-av}$	e^{at}
6	$sin\;at$	$\frac{a}{\mathrm{v}(\mathrm{a}^2+\;\mathrm{v}^2)}$	$sin\;at$
7	$cos\;at$	$\frac{1}{(\mathrm{a}^2+\;\mathrm{v}^2)}$	$cos\;at$
8	$sinh\;at$	$\frac{a}{\mathrm{v}(\mathrm{a}^2-\;\mathrm{v}^2)}$	$sinh\;at$
9	$cosh\;at$	$\frac{1}{(\mathrm{a}^2-\;\mathrm{v}^2)}$	$cosh\;at$

Chapter Two

Applications of Aboodh transformation and linking it to some effective integral transformations

2.1. Application of Aboodh T ary Differential Equations

the Aboodh transform can be used as an effective tool. For analyzing the basic characteristics of a linear system governed by the differential equation in response to initial data. The following examples illustrate the use of the Aboodh transform in solving certain initial value problems described by ordinary differential equations.

2.2.Consider the first-order ordinary differential equation[2].

$$\frac{dy}{dx} + py \;=\; f(t), \quad t > 0 \quad ....(*)$$

With the initial Condition.

$$y\,(0) = \;a$$

Where p and a are constants and $f(t)$ is an external input function so that its Aboodh transform exists.

Applying Aboodh transform of the equation (*) we have

$$A\bar{y} \;+\; p\bar{y}v \;=\; \bar{f}(v)$$

$$vk(v)\; - \frac{f(0)}{v} + \;p\bar{y}v \;= \bar{f}(v)$$

$$v\bar{y}(v) - \frac{y(0)}{v} + p\bar{y}v \;=\; \bar{f}(v)$$

$$\bar{y}\,(v)\,[v + p] \;=\; \frac{a}{v} + \bar{f}(v)$$

$$\bar{y}\,(v) = \frac{a + \;v\bar{f}(v)}{v(v + p)}$$

The inverse Aboodh transform leads to the solution. The second order linear ordinary differential equation has the general form

$$\frac{d^2y}{dx^2} + ٢p\frac{dy}{dx} + \;qy \;=\; f(t), t > 0$$

The initial conditions are

$$y\,(0) = a\,, \qquad \frac{dy}{dx}(0) = b$$

Where pa and b are constants. Applications of Aboodh transforms to general initial value problem gives

$$v^2\bar{y}(v) - \frac{\acute{y}(0)}{v} - y(0) + 2pv\bar{y}(v) - \frac{2py(0)}{v} + q\bar{y}(v) = \bar{f}(v)$$

$$\bar{y}(v)\,[v^2 + 2pv + q] = \bar{f}(v) + \frac{b}{v} + a + \frac{2pa}{v}$$

$$\bar{y}(v) = \frac{v\bar{f}(v) + b + av + 2pa}{(v^2 + 2pv + q)}$$

$$\bar{y}(v) = \frac{(\bar{f}(v) + a)}{(v^2 + 2pv + q)} + \frac{b + 2pa}{v(v^2 + 2pv + q)}$$

The inverse Aboodh transform gives the solution.

Example (2.2.1)

$$y" + y = 1 \quad y(0) = 0, y'(0) = 0$$

Solution: take Aboodh transform to this equation gives:

$$A(y") + A(y) = A(1)$$

$$V^2K(V) - \frac{\acute{y}(0)}{V} - y(0) + K(V) = \frac{1}{V^2}$$

$$V^2K(V) - \frac{0}{V} - 0 + K(V) = \frac{1}{V^2}$$

$$V^2K(V) + K(V) = \frac{1}{V^2}$$

$$K(V)\,[V^2 + 1] = \frac{1}{V^2}$$

$$\because K(V) = \frac{1}{V^2} * \frac{1}{V^2 + 1}$$

$$K(V) = \frac{1}{V^2(V^2 + 1)}$$

The invers Aboodh transform of this equation is simply obtained as

$$K(V)=\frac{1}{V^2(V^2+1)}=\frac{AV+B}{V^2}+\frac{CV+D}{(V^2+1)} \qquad (**)$$

$$=\frac{(AV+B)(V^2+1)+(CV+D)V^2}{V^2(V^2+1)}$$

$$K(V)=\frac{1}{V^2(V^2+1)}=\frac{AV^3+AV+BV^2+B+DV^2+CV^3}{V^2(V^2+1)}$$

$$\therefore\ AV^3+CV^3=0\ \rightarrow A+C+0$$

$$BV^2+DV^2=0\ \rightarrow B+D=0$$

$$AV\ =\ 0\ \rightarrow A=0$$

$$B=1$$

$$C=0$$

$$D=-1$$

$$But\ A\ =\ 0\ ,B=1,C=0,D=-1\ \ in(**)we\ get:$$

$$K(V)=\frac{0+1}{V^2}+\frac{-1}{V^2+1}$$

$$K(V)=\frac{1}{V^2}-\frac{1}{V^2+1}$$

By using Inverse of Aboodh transform

$$y(t)=1-\cos t$$

Example (2.2.2).

$$\ddot{y}+y=0$$

With the initial condition $y(0)\ =\ y'(0)\ =\ 1$

Solution:

take Aboodh transform of both sides

$$A\{\ddot{y}\}\ +\ A\{y\}\ =\ A\{0\}$$

$$V^2K(V)-\frac{\acute{y}(0)}{V}-y(0)\ +\ K(V)\ =\ 0$$

By using initial condition

$$V^2K(V) - \frac{1}{V} - 1 + K(V) = 0$$

$$[V^2 + 1]K(V) = \frac{1}{V} + 1$$

$$K(V) = \frac{1+V}{V} * \frac{1}{V^2 + 1}$$

$$K(V) = \frac{1+V}{V(V^2 + 1)}$$

$$= \frac{1}{V(V^2 + 1)} + \frac{V}{V(V^2 + 1)}$$

$$= \frac{1}{V(V^2 + 1)} + \frac{1}{(V^2 + 1)}$$

By take the inverse of Aboodh transform

$$y(t) = sint + cost$$

2.3. **Connection Between Aboodh Transform and Some Effective Integral Transforms.**

2.3.1. Laplace Transform[5]

The Laplace transform of the function Z(γ), $\gamma \geq 0$ is

$$L\{Z(\gamma)\} = \int_0^\infty Z(\gamma)e^{-\epsilon y}\, dy = B(\epsilon)$$

Table-I: Some typical functions with their Laplace transform

S.N.	$Z(\gamma)$	$A\{Z(\gamma)\} = J(\epsilon)$	$L\{Z(\gamma)\} = B(\epsilon)$
1.	1	$\frac{1}{\epsilon^2}$	$\frac{1}{\epsilon}$
2.	γ	$\frac{1}{\epsilon^3}$	$\frac{1}{\epsilon^2}$
3.	γ^2	$\frac{2!}{\epsilon^4}$	$\frac{2!}{\epsilon^3}$
4.	γ^n, $n \in N$	$\frac{n!}{\epsilon^{n+2}}$	$\frac{n!}{\epsilon^{n+1}}$
5.	γ^n, $n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+2}}$	$\frac{\Gamma(n+1)}{\epsilon^{n+1}}$
6.	$e^{a\gamma}$	$\frac{1}{\epsilon(\epsilon - a)}$	$\frac{1}{(\epsilon - a)}$
7.	$sina\gamma$	$\frac{a}{\epsilon(\epsilon^2 + a^2)}$	$\frac{a}{(\epsilon^2 + a^2)}$
8.	$cosa\gamma$	$\frac{1}{(\epsilon^2 + a^2)}$	$\frac{\epsilon}{(\epsilon^2 + a^2)}$
9.	$sinha\gamma$	$\frac{a}{\epsilon(\epsilon^2 - a^2)}$	$\frac{a}{(\epsilon^2 - a^2)}$
10.	$cosha\gamma$	$\frac{1}{(\epsilon^2 - a^2)}$	$\frac{\epsilon}{(\epsilon^2 - a^2)}$

2.3.2. Kamal Transform.[5]

Kamal transform of the function Z(γ), $\gamma \geq 0$ is

$$K\{Z(\gamma)\} = \int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}\,dy = C(\epsilon), \qquad 0 < k_1 \le \epsilon \le k2$$

Table-II: Some typical functions with their Kamal transform

S.N.	$Z(\gamma)$	$A\{Z(\gamma)\} = J(\epsilon)$	$K\{Z(\gamma)\} = C(\epsilon)$
1.	1	$\frac{1}{\epsilon^2}$	ϵ
2.	γ	$\frac{1}{\epsilon^3}$	ϵ^2
3.	γ^2	$\frac{2!}{\epsilon^4}$	$2!\,\epsilon^3$
4.	γ^n, $n \in N$	$\frac{n!}{\epsilon^{n+2}}$	$n!\,\epsilon^{n+1}$
5.	γ^n, $n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+2}}$	$\Gamma(n+1)\epsilon^{n+1}$
6.	$e^{a\gamma}$	$\frac{1}{\epsilon(\epsilon - a)}$	$\frac{\epsilon}{(1 - a\epsilon)}$
7.	$sina\gamma$	$\frac{a}{\epsilon(\epsilon^2 + a^2)}$	$\frac{a\epsilon^2}{(1 + a^2\epsilon^2)}$
8.	$cosa\gamma$	$\frac{1}{(\epsilon^2 + a^2)}$	$\frac{\epsilon}{(1 + a^2\epsilon^2)}$
9.	$sinha\gamma$	$\frac{a}{\epsilon(\epsilon^2 - a^2)}$	$\frac{a\epsilon^2}{(1 - a^2\epsilon^2)}$
10.	$cosha\gamma$	$\frac{1}{(\epsilon^2 - a^2)}$	$\frac{\epsilon}{(1 - a^2\epsilon^2)}$

2.3.3. Elzaki Transform[4]

Elzaki transform of the function $Z(\gamma)$, $\gamma \geq 0$ is

$$E\{Z(\gamma)\} = \epsilon \int_0^\infty Z(\gamma)\, e^{\frac{-\gamma}{\epsilon}}\, dy \;=\; D(\epsilon), \qquad 0 < k_1 \le \epsilon \le k2$$

Table-III: Some typical functions with their Elzaki transform

S.N.	$Z(\gamma)$	$A\{Z(\gamma)\} = J(\epsilon)$	$E\{Z(\gamma)\} = D(\epsilon)$
1.	1	$\frac{1}{\epsilon^2}$	ϵ^2
2.	γ	$\frac{1}{\epsilon^3}$	ϵ^3
3.	γ^2	$\frac{2!}{\epsilon^4}$	$2!\,\epsilon^4$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^{n+2}}$	$n!\,\epsilon^{n+2}$
5.	$\gamma^n, n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+2}}$	$\Gamma(n+1)\epsilon^{n+2}$
6.	$e^{a\gamma}$	$\frac{1}{\epsilon(\epsilon - a)}$	$\frac{\epsilon^2}{(1 - a\epsilon)}$
7.	$sina\gamma$	$\frac{a}{\epsilon(\epsilon^2 + a^2)}$	$\frac{a\epsilon^3}{(1 + a^2\epsilon^2)}$
8.	$cosa\gamma$	$\frac{1}{(\epsilon^2 + a^2)}$	$\frac{\epsilon^2}{(1 + a^2\epsilon^2)}$
9.	$sinha\gamma$	$\frac{a}{\epsilon(\epsilon^2 - a^2)}$	$\frac{a\epsilon^3}{(1 - a^2\epsilon^2)}$
10.	$cosha\gamma$	$\frac{1}{(\epsilon^2 - a^2)}$	$\frac{\epsilon^2}{(1 - a^2\epsilon^2)}$

2.3.4 Sumudu Transform[5]

Sumudu transform of the function $Z(\gamma)$, $\gamma \ge 0$

$$S\{Z(\gamma)\} = \int_0^\infty \int Z(\epsilon\gamma)e^{-\gamma}dy = F(\epsilon), \qquad 0 < k_1 \leq \epsilon \leq k2$$

Table-IV: Some typical functions with their Sumudu transform

S.N.	$Z(\gamma)$	$A\{Z(\gamma)\} = J(\epsilon)$	$S\{Z(\gamma)\} = F(\epsilon)$
1.	1	$\frac{1}{\epsilon^2}$	1
2.	γ	$\frac{1}{\epsilon^3}$	ϵ
3.	γ^2	$\frac{2!}{\epsilon^4}$	$2!\,\epsilon^2$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^{n+2}}$	$n!\,\epsilon^n$
5.	γ^n, $n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+2}}$	$\Gamma(n+1)\epsilon^n$
6.	$e^{a\gamma}$	$\frac{1}{\epsilon(\epsilon-a)}$	$\frac{1}{(1-a\epsilon)}$
7.	$sina\gamma$	$\frac{a}{\epsilon(\epsilon^2+a^2)}$	$\frac{a\epsilon}{(1+a^2\epsilon^2)}$
8.	$cosa\gamma$	$\frac{1}{(\epsilon^2+a^2)}$	$\frac{1}{(1+a^2\epsilon^2)}$
9.	$sinha\gamma$	$\frac{a}{\epsilon(\epsilon^2-a^2)}$	$\frac{a\epsilon}{(1-a^2\epsilon^2)}$
10.	$cosha\gamma$	$\frac{1}{(\epsilon^2-a^2)}$	$\frac{1}{(1-a^2\epsilon^2)}$

2.3.5. Mahgoub Transform[6]

Mahgoub (Laplace-Carson) transform of the function $Z(\gamma)$, $\gamma \geq 0$ is

$$M_*\{Z(\gamma)\} = \epsilon \int_0^\infty Z(\gamma)e^{-\epsilon\gamma}\, dy = G(\epsilon), \quad 0 < k_1 \leq \epsilon \leq k2$$

Table-V: Some typical functions with their Mahgoub (Laplace – Carson) transform

S.N.	$Z(\gamma)$	$A\{Z(\gamma)\} = J(\epsilon)$	$M_*\{Z(\gamma)\} = G(\epsilon)$
1.	1	$\frac{1}{\epsilon^2}$	1
2.	γ	$\frac{1}{\epsilon^3}$	$\frac{1}{\epsilon}$
3.	γ^2	$\frac{2!}{\epsilon^4}$	$\frac{2!}{\epsilon^2}$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^{n+2}}$	$\frac{n!}{\epsilon^n}$
5.	$\gamma^n, n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+2}}$	$\frac{\Gamma(n+1)}{\epsilon^n}$
6.	$e^{a\gamma}$	$\frac{1}{\epsilon(\epsilon - a)}$	$\frac{\epsilon}{(\epsilon - a)}$
7.	$sina\gamma$	$\frac{a}{\epsilon(\epsilon^2 + a^2)}$	$\frac{a\epsilon}{(\epsilon^2 + a^2)}$
8.	$cosa\gamma$	$\frac{1}{(\epsilon^2 + a^2)}$	$\frac{\epsilon^2}{(\epsilon^2 + a^2)}$
9.	$sinha\gamma$	$\frac{a}{\epsilon(\epsilon^2 - a^2)}$	$\frac{a\epsilon}{(\epsilon^2 - a^2)}$
10.	$cosha\gamma$	$\frac{1}{(\epsilon^2 - a^2)}$	$\frac{\epsilon^2}{(\epsilon^2 - a^2)}$

2.3.6. Mohand Transform[7]

Mohand transform of the function Z(y), y ≥ 0 is

$$M\{Z(\gamma)\} = \epsilon^2 \int_0^\infty Z(\gamma) e^{-\gamma\epsilon}\, dy = H(\epsilon), \quad 0 < k_1 \leq \epsilon \leq k2$$

Table-VI: Some typical functions with their Mohand transform

S.N.	$Z(\gamma)$	$A\{Z(\gamma)\} = J(\epsilon)$	$M\{Z(\gamma)\} = H(\epsilon)$
1.	1	$\frac{1}{\epsilon^2}$	ϵ
2.	γ	$\frac{1}{\epsilon^3}$	1
3.	γ^2	$\frac{2!}{\epsilon^4}$	$\frac{2!}{\epsilon}$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^{n+2}}$	$\frac{n!}{\epsilon^{n-1}}$
5.	$\gamma^n, n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+2}}$	$\frac{\Gamma(n+1)}{\epsilon^{n-1}}$
6.	$e^{a\gamma}$	$\frac{1}{\epsilon(\epsilon - a)}$	$\frac{\epsilon^2}{(\epsilon - a)}$
7.	$sina\gamma$	$\frac{a}{\epsilon(\epsilon^2 + a^2)}$	$\frac{a\epsilon^2}{(\epsilon^2 + a^2)}$
8.	$cosa\gamma$	$\frac{1}{(\epsilon^2 + a^2)}$	$\frac{\epsilon^3}{(\epsilon^2 + a^2)}$
9.	$sinha\gamma$	$\frac{a}{\epsilon(\epsilon^2 - a^2)}$	$\frac{a\epsilon^2}{(\epsilon^2 - a^2)}$
10.	$cosha\gamma$	$\frac{1}{(\epsilon^2 - a^2)}$	$\frac{\epsilon^3}{(\epsilon^2 - a^2)}$

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