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Reliability Allocation in Mixed system

A Research Paper Submitted to the Council of the College of Education for Pure Sciences / University of Babylon as Partial Fulfillment of the Requirements for the Degree of Bachelor of Science in Mathematics.

by:

Sakina Hamza Hamid

Supervised by:

Prof.Dr.Zahir Abdul Haddi Hassan

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بِسْمِ اللَّهِ الرَّحْمَٰنِ الرَّحِيمِ

﴿الَّذِي خَلَقَ سَبْعَ سَمَاوَاتٍ طِبَاقًا ۖ مَا تَرَىٰ فِي خَلْقِ الرَّحْمَٰنِ مِن تَفَاوُتٍ ۖ فَارْجِعِ الْبَصَرَ هَلْ تَرَىٰ مِن فُطُورٍ﴾

سورة الملك - الآية 3

صدق الله العلي العظيم

Dedication

To my dearest parents,
the eternal source of love, sacrifice, and prayers,
whose unwavering faith in me has been my shelter in moments of weakness
and my inspiration in moments of hope.
No words could ever express the depth of my gratitude
for all that they have given with silent patience and endless devotion.

To my respected teachers,
whose knowledge illuminated my path
and whose guidance opened before me the doors of learning,
I owe much of what I have become today.

To every soul that stood beside me,
offered me support, encouragement, or sincere prayer,
I dedicate this work with heartfelt appreciation and profound respect.

This humble achievement is not mine alone;
it is the reflection of love, patience, and generosity
granted to me by those who believed in me and never let me walk alone.

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Abstract

Reliability is an Important concept in the design and operation of complex systems. It means that a system can perform its required function without failure for a certain period of time. In many real-life situations, reliability is very important, especially in systems like healthcare, transportation, and industrial machines, where failures can lead to serious problems.

There are several factors that affect system reliability. One of the main factors is good design, because a well-designed system is less likely to fail. Another important factor Is fault tolerance, which helps the system continue working even If some parts stop working. Maintenance is also very Important, since regular checking and repairs can prevent unexpected failures.

Reliable systems help Improve safety and efficiency. When systems work correctly, the chances of accidents decrease and performance becomes better. For this reason, engineers need to focus on reliability during all stages of system development. In general, understanding reliability and applying its principles can help in building stronger and more dependable systems.

Keywords: Reliability, Complex Systems, System Dependability, Fault Tolerance, System Safety, Critical Systems

Introduction

Complex systems are considered fundamental components in many critical fields in the modern world, such as aviation systems, electrical power networks, medical systems, and communication infrastructures. These systems are characterized by consisting of a large number of interconnected components that work together to perform specific functions, which often makes their behavior unpredictable and difficult to control completely.

In such a context of complexity, reliability emerges as one of the most important criteria used to evaluate the performance of these systems. Reliability refers to the ability of a system to perform its required functions efficiently and without failure over a specified period of time. Even a minor fault in one component of the system may lead to significant consequences, potentially resulting in total system failure, financial losses, or even risks to human life.

Therefore, studying reliability in complex systems is essential to understand the factors that Influence system performance and to identify methods and techniques used to enhance reliability and reduce the likelihood of failures. This research aims to highlight the concept of reliability, explain its importance in complex systems, and review the main methods used to measure and Improve it.

The main objective of this research is to study and analyze the reliability of mixed systems that consist of both series and parallel configurations. Mixed systems are widely used in engineering applications, and understanding their reliability is essential for improving system performance and reducing the probability of failure.

Problem Statement

In reliability engineering, many practical systems are composed of several components connected in different configurations. While the reliability of simple configurations such as series systems and parallel systems can be calculated easily using well-known formulas, the analysis becomes more complicated when these configurations are combined within the same system.

Mixed systems consist of both series and parallel structures, which makes their reliability evaluation more challenging. The complexity arises because the failure of certain components may affect only a part of the system, while the failure of other components may lead to the failure of the entire system. Therefore, determining the overall reliability of such systems requires breaking the system into smaller subsystems and analyzing each configuration separately.

Another difficulty in mixed system reliability analysis is identifying the critical components that have the greatest impact on the overall system performance. Without proper analysis, it becomes difficult to Improve system design, predict system failure, or optimize maintenance strategies.

For this reason, studying the reliability of mixed systems is an important research topic in reliability engineering. It helps researchers develop mathematical and statistical methods to accurately evaluate system reliability and improve the performance and safety of complex engineering systems.

Research Objectives

The specific objectives of this research can be summarized as follows:

1. To explain the concept of system reliability and its importance in engineering and statistical applications.
2. to study the reliability of basic system structures, including series systems and parallel systems.
3. to analyze the reliability of mixed systems that combine both series and parallel components within the same structure.
4. to develop or apply mathematical methods for calculating the reliability of mixed systems
5. to provide an illustrative example showing how the reliability of a mixed system can be evaluated in practical situations.

CHAPTER ONE

Some definition and general concepts

Definition (1.1.1) system Reliability[4]:

System reliability can be defined as the probability that a system performs its intended functions correctly without failure under specified conditions for a certain period of time. This concept is widely used in engineering to evaluate how dependable a system is during its operation.

Reliability becomes especially important when dealing with complex systems. Such systems are usually composed of many interconnected components, where the failure of a single part may lead to the failure of the entire system. For example, in power grids, transportation networks, and industrial automation systems, even a minor fault can cause serious disruptions and affect overall performance.

Improving system reliability plays a key role in reducing the likelihood of unexpected failures. It also helps in lowering maintenance costs by minimizing the need for frequent repairs and system downtime. In addition, reliable systems contribute significantly to safety, as they reduce the risk of accidents and operational hazards. As a result, users tend to have greater confidence in systems that consistently perform as expected.

Several studies have highlighted the importance of reliability in modern engineering systems (MDPI, n.d.; ScienceDirect, 2025). These studies emphasize that focusing on reliability during the design and operation stages can greatly enhance system performance and long-term stability.

Factors Affecting System Reliability[3]:

1. System reliability depends on several important factors that determine how likely a system is to perform its intended functions correctly over time. Understanding these factors helps engineers design and operate systems that are more dependable and safer.
2. System Design Quality: The quality of a system's design has a major impact on its reliability. A well-thought-out design, which includes proper component selection and system architecture, can greatly reduce the chances of failure. On the other hand, poor design increases the likelihood of system malfunctions.
3. Component Reliability: Each component or subsystem contributes to the overall reliability of the system. If even one critical component is prone to failure, it can affect the entire system's performance.
4. Operational Environment: External conditions such as temperature, humidity, vibration, and other environmental factors can significantly influence how a system performs. Systems exposed to harsh or variable conditions may experience a higher rate of failures if not properly designed.
5. Maintenance Practices: Regular maintenance, especially preventive maintenance, plays a key role in improving reliability. By addressing potential problems before they lead to failure, maintenance reduces unexpected downtime and extends system life.
6. Human Factors: Mistakes during operation, configuration, or maintenance can reduce system reliability. Training, clear procedures, and automation where possible can help minimize human errors.

Definition (1.1.2) Reliability Allocation[5]:

Reliability allocation is the process of dividing the required overall system reliability among individual components or subsystems. This ensures that the entire system meets its reliability targets. By using this approach, engineers can identify critical components that need extra attention and guide design improvements to achieve the desire system performance. Reliability allocation is an essential step in designing complex systems where each part contributes to the overall dependability.

Definition (1.1.3) Failure Rate[4]:

The failure rate is a measure of how often a system or one of its components fails, typically expressed as the number of failures per unit of time. It is a key concept in reliability engineering, as it helps engineers quantify and predict the likelihood of system failures over time.

Mathematical Definition:

The failure rate at a given time is defined as the instantaneous rate at which a system fails, assuming it has survived up to that point. Mathematically, it can be expressed as:

$$\lambda(t) = \frac{f(t)}{\mathcal{R}(t)} \qquad \ldots(1.1)$$

Where:

$f(t)$ is the probability density function (PDF) of the time to failure.

$\mathcal{R}(t)$ is the reliability function, which represents the probability that the system will survive beyond time ($\mathcal{R}(t) = P(T > t)$) .

Relationship to Reliability[4]:

The reliability function can also be expressed in terms of the failure rate as:

$$\mathcal{R}(t) = e^{\left(-\int_0^t \lambda(T)dT\right)} \qquad \ldots(1.2)$$

This relationship shows that the failure rate directly affects the probability of system survival over time. In other words, higher failure rates correspond to a lower likelihood that the system will continue functioning as time progresses.

(2.1) Mathematical Relationship between the PDF, CDF and R(t)[4]:

1.ProbabIlity Density Function (PDF)[4]

The probability density function f(t) describes how likely an event is to occur at a specific time t.

In other words:

$$f(t) = \frac{dFt}{dt} \qquad ...(1.3)$$

2. Cumulative Distribution Function (CDF)

$$F(t) = P(T \leq t) = \int_0^t f(T)\, dT \qquad ...\ (1.4)$$

The cumulative distribution function F(t) gives the probability that the event occurs at or before time t:

3. Reliability Function (Survival Function)

The reliability function R(t) gives the probability that the system survives beyond time t:

$$R(t) = p(T > t) = 1 - F(t) = \int_t^\infty f(T)\ dt \qquad ...\ (1.5)$$

4. Summary of Relationships

PDF, CDF and R(t):

$$f(t) = \frac{dF(t)}{dt} \qquad ...(1.6)$$

$$R(t) = 1 - F(t) \qquad ...(1.7)$$

$$f(t) = \frac{-dR(t)}{d(t)} \qquad ...(1.8)$$

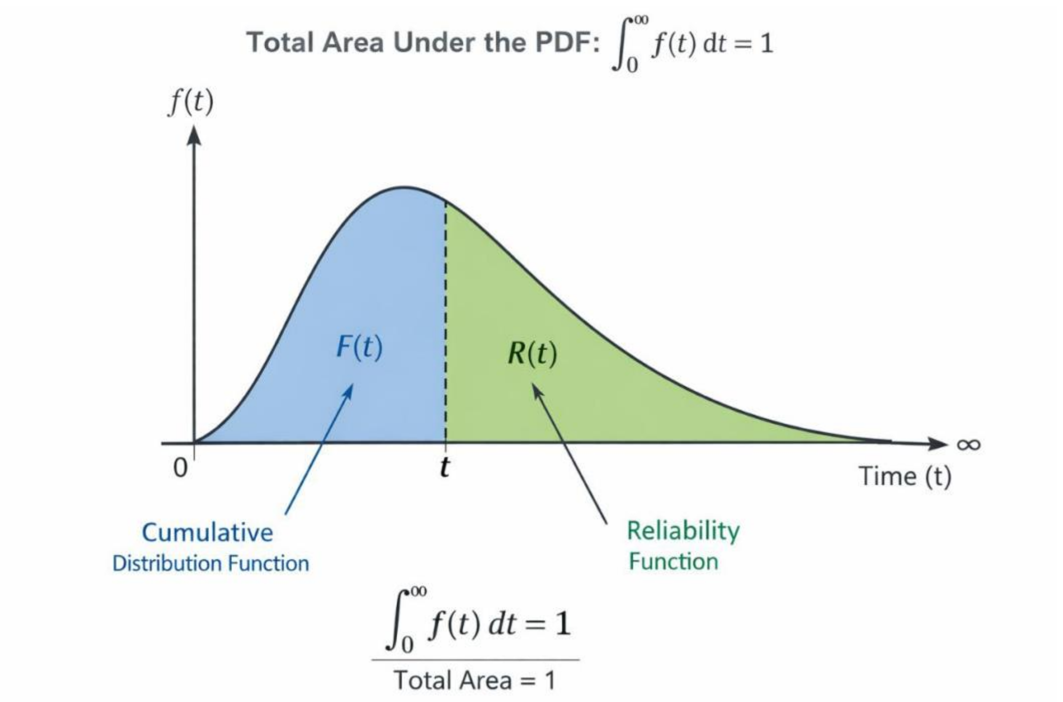


Figure (1): Total area under pdf

Figure (1) We can understand the reliability function through the familiar concept of the cumulative distribution function (CDF). The CDF gives the probability that an event occurs by a certain time t. In the context of a device or component, this means:

$$F(t) = \int_0^t f(s)\,ds \qquad ...1.9)$$

Represents the probability that the unit fails before time t where f(s) is the probability density function (PDF). The reliability function R(t), on the other hand, represents the probability that the unit survives beyond time t, i.e. it has not failed yet. Since total probability is 1, we have:

$$R(t) = p(T > t) = 1 - F(t) = 1 - \int_0^t f(s)\ ds = \int_t^\infty f(s)\, ds \quad \ldots(1.10)$$

In simple terms, the reliability function is just the complement of the cumulative probability of failure. So, if we know the PDF or CDF of failure, we can directly calculate the reliability.

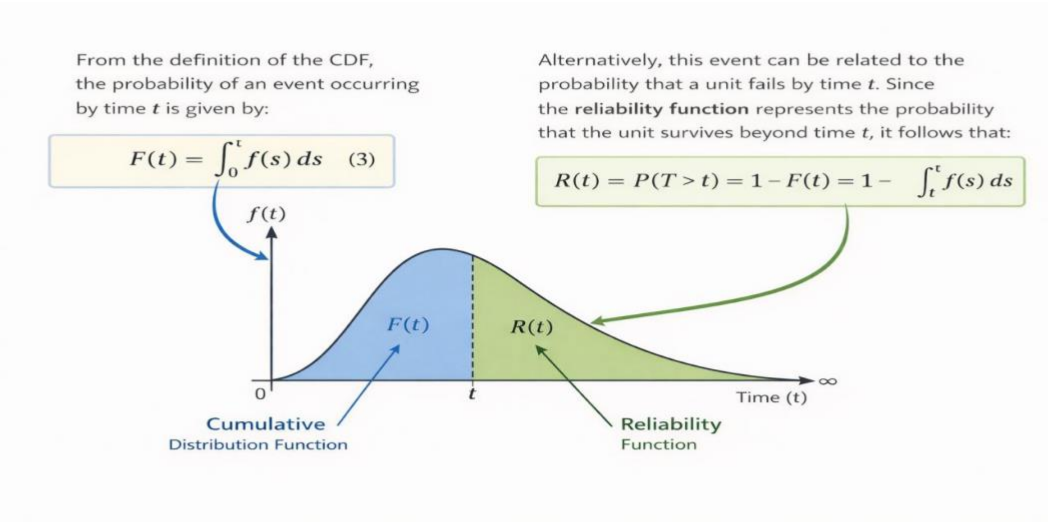


Figure (2): relationship between CDF and R (t)

Figure (2) illustrates the concept of reliability as the area under the probability density function (PDF). Since the PDF represents the likelihood of failure occurring at a specific time, the cumulative probability of failure up to a certain time t can be obtained by integrating the PDF over the interval {0,t}.

Consequently, the reliability function R(t) represents the probability that a system continues to operate successfully beyond time t. It can be obtained by subtracting the cumulative probability of failure from one. For this reason, the reliability function plays a fundamental role in lifetime data analysis, as it provides a quantitative measure of the likelihood that a system will keep functioning over time.

When considering a mission with a specified duration, Figure (2) illustrates this concept mathematically. First, we define the unreliability function Q(t), which represents the probability of failure. In other words, it is the probability that the time-to-failure occurs within the Interval from 0 to t. This is equivalent to the cumulative distribution function (CDF), and it is given by:

$$Q(t) = F(t) = \int_0^t f(s)\,ds \qquad \ldots(1.11)$$

Where f(s) is the probability density function (PDF).

Since reliability and unreliability are complementary and mutually exclusive events, their probabilities must sum to one:

$$Q(t) + R(t) = 1 \qquad \ldots(1.12)$$

Thus, the reliability function can be expressed as:

$$R(t) = 1 - F(t) \qquad \ldots(1.13)$$

Alternatively, by differentiation, we obtain:

$$f(t) = -\frac{dR(t)}{d(t)} \qquad \ldots(1.14)$$

Or equivalently:

$$Q(t) = F(t) = 1 - R(t) \qquad \ldots(1.15)$$

CHAPTER TWO

Reliability of systems

2.1 Simple systems

Suppose that we have to calculate the Reliability of a system made up of several components. The total reliability can be calculated by calculating the reliability of each individual component, and combining these individual reliabilities. The way in which they are combined depends on the way in which the components are connected. That is, whether they are connected [1-8]:

1– series

2– parallel

3– series – parallel.

4– parallel – series.

5– Combination of series and parallel.

2. 2 Series Systems [7]

Consider a system of n components connected in series so that the system will only work (i.e. a signal will pass from I to O) if all of the components work.

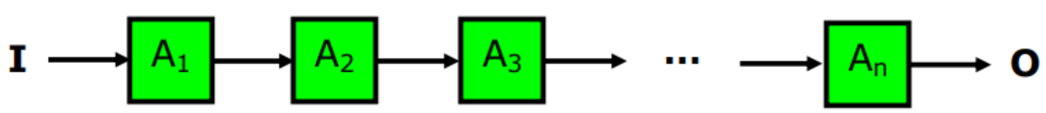


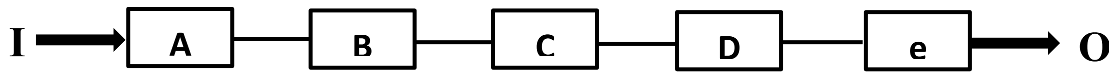
Figure (2.1)

If the components fail independent of each other it is easy to show that if R_1, R_2, …, R_n are the reliabilities of the individual components, then the reliability of the system is given by:

$R_{SYSTEM} = R_1 \times R_2 \times \ldots \times R_n$ …(2.1)

Example (2.1)[8]

Consider a system of 3 components connected in series, each component having a constant failure rate. (In other words, the components have exponential lifetimes). These rates for components A, B and C are 0.3, 0.4 and 0.6 per 10,000 hours respectively. Thus we have



For constant failure rate λ, reliability

$R(t) = e^{-\lambda t}$ . ...(2.2)

Thus for component A, $R_A = e^{-0.3t}$

Similarly, $R_B = e^{-0.4t}$ and $RC = e^{-0.6t}$

Hence the reliability of the system is:

$R(t) = e_{-0.3t} \times e^{-0.4t} \times e^{-0.6t}$ …(2.3)

$= e^{-0.3t - 0.4t - 0.6t}$

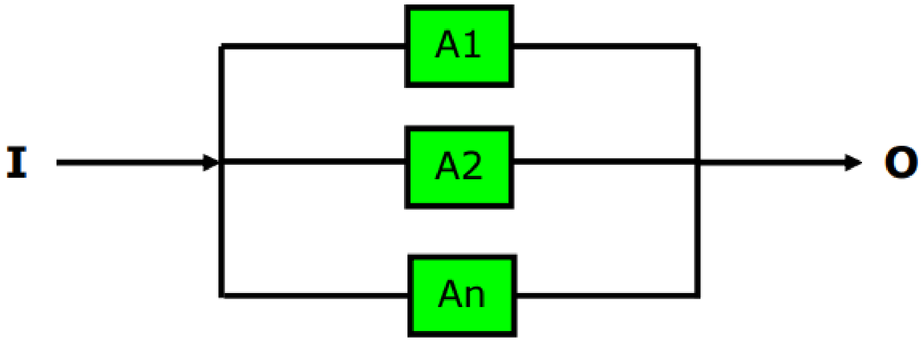
$= e^{-1.3t}$

Then, for example, the probability that the system is still working after 30,000 hours = $R(3) = e^{-3.9}$

= 0.02024191145.

2.3 Parallel Systems [5]

If n components are connected in parallel so that the system works (signal from I to O) as long as at least one of the components works.



The reliability of the system (again assuming independent failures) is then:

R_{SYSTEM} = 1- P (all fail)

$$= 1 - [P(A1 \text{ fails }) \times P(A2 \text{ fails }) \times \ldots \times P(\text{ An fails })]$$
$$= 1 - (1 - R_1)(1 - R_2) \ldots (1 - R_n)$$

$$R_{SYSTEM} = 1 - \prod_{i=1}^{n} (1 - R_i)$$

Example (2.2) [4]

Component A with a constant failure rate of 1.5 per 1000 hrs and component B with a constant failure rate of 2 per 1000 hrs are connected in parallel. Find the overall reliability of this system.(Note that components A and B have exponential lifetimes).

Solution:

For the individual components, assuming a constant failure rate

$R_A(t) = e_{-1.5t}$

$R_B(t) = e_{-2t}$

Then if these components are connected in parallel,

R system (t) = $1 - [(1 - R_A(t)) \times (1 - R_B(t))]$

$= 1 - [(1 - e^{-1.5t})(1 - e^{-2t})]$

$= 1 - [1 - e^{-1.5t} - e^{-2t} + e^{-1.5t} e^{-2t}]$

$= e^{-1.5t} + e^{-2t} - e^{-3.5t}$

Then, for example, the probability that the system is still working after 1000 hours is

$R(1) = e^{-1.5(1)} + e^{-2(1)} - e^{-3.5(1)}$

$= 0.3283$

i.e. there is a 33% chance that such a system will still be working after 1000 hours. …(2.5)

2. 4 Series - Parallel systems [5]

The reliability of a system comprising n component connected in parallel redundancy, is

$R_P = 1 - (1 - R)^n$ …(2.6)

Where R is the reliability of an individual component.

If we place n sets in parallel, where each set has m components connected in series, we get the reliability of such a system as

$R_S = 1 - (1 - R)^n$

Where $R = \prod_{i=1}^{m} r_i$, and r_i is the reliability of the i th component in series. Thus,

$$R_S = 1 - \left(1 - \prod_{i=1}^{m} r_i\right)^n \quad \ldots(2.7)$$

The series – parallel configuration is shown in fig (2.3)

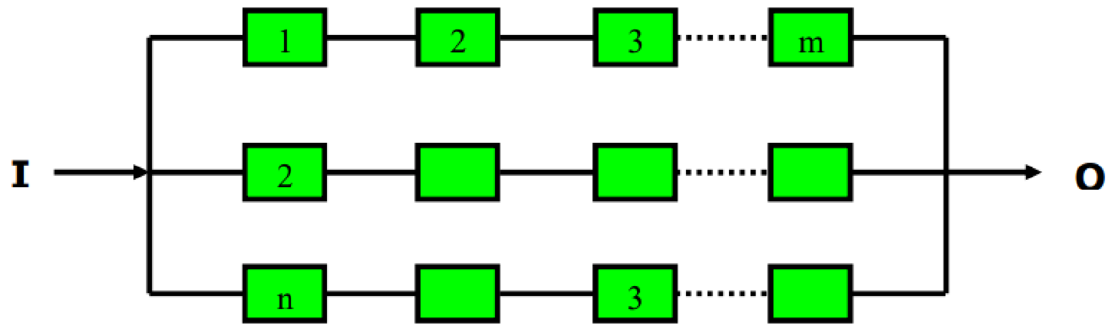
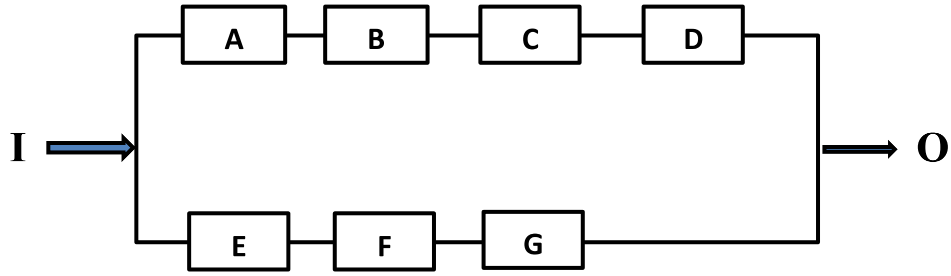


Fig (2.3) series – parallel system

Example (3)[9]

Compute the reliability of the system for the connection given in



The reliability of A, B, C and D are 0.95, 0.99 0.90 and 0.96 respectively.

Making use of equation (2.7), we get the system reliability for m = 4 and n = 2.

RS = 1- (1- ∏ ri)²

= 1- [1 – (0.95 × 0.99×0.90×0.96)]²

= 1- (1-0.813)²

= 0.9650 or 96.50%

$$R_S = 1 - \left(1 - \prod_{i=1}^{4} r_i\right)^2$$
$$= 1 - [1 - (0.95 \times 0.99 \times 0.90 \times 0.96)]^2$$
$$= 1 - (1 - 0.813)^2$$
$$= 0.9650 \text{ or } 96.50\%$$

2.5 Parallel – Series systems [3]

The reliability of n components connected in parallel is given by R_P

RP = 1- (1- R) n …(2.8)

Where R is the reliability of an individual component.

If m such sets are connected in series, where each set consists of n component in parallel, then the reliability of the system is given by

$R_S = [1- (1- R)^{n}]^{m}$ (2.9)

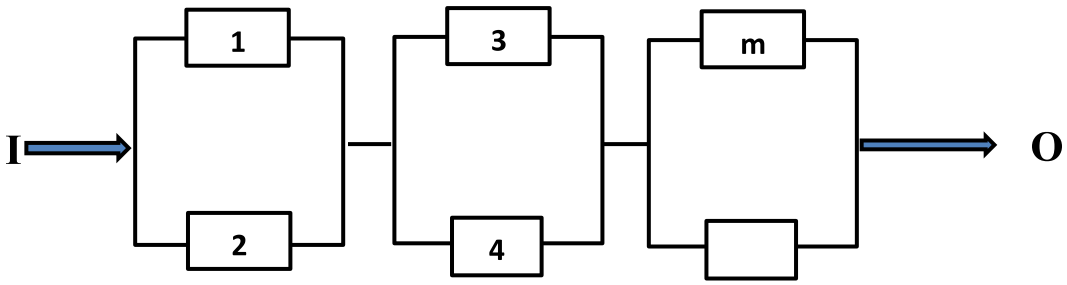
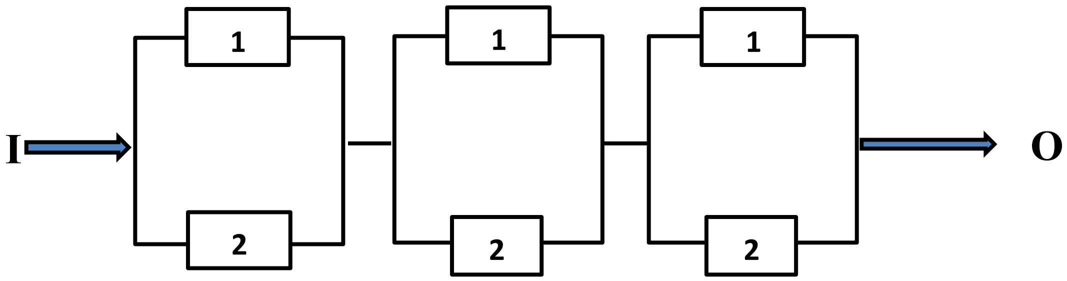


Figure (2.5) parallel – series system

Example (4)[4]

Compute the reliability of the system for the connections given in fig (2.6):



Assuming the reliability of each component is 0.95

Here, we are given n=2, m= 4, using eq. (2.9), We get

$RS = [1- (1- 0.95)^{2}]^{4}$

=(0.9975) 4

= 0.9900 = 99%

CHAPTER THREE

Reliability Allocation Methods in Series and Mixed Systems

3.1 Reliability allocation

Reliability allocation is the process by which the failure allowance for a system is allocated in some logical manner it, sub-systems and elements. The purpose of reliability allocation is to establish a goal or objective for the reliability of each component so that the manufacturers can have an idea of the performance required of this product.

The reliability factor of a system is known or is specified on the basis of the overall mission required. if the system comprises many elements and units, we must have method to determine the reliability factor for each of them.

Consider a system consisting of n components (dependent or independent) with individual reliability factors $R_1, R_2, R_3, \dots R_n$. The system reliability $R(t)$is a function of these components reliability. Thus, $R(t) = f(R_1, R2, R3, \dots, Rn)$

The problem now is to determine the values of R_1,R_2,R_3,…,R_n for a given value of R(t) .this problem will not have a unique answer , at the same time , the value of R_1, R_2, R_3,…, R_n cannot be altogether arbitrary . The problem can be viewed from two aspects. First, if we allocate the reliability requirement in some logical manner among the n component, a goal will be set for the manufacturer to product components with these reliability factors. Second, if the state of art is such that the reliability Ri of a particular component I cannot possibly be improved upon, and if the allocated reliability factor is higher.

In our project we shall discuss two methods by which the reliability can be logically apportioned among the consistent element of a system. The problem will not be treated in its most general form as stated in eq. (4.1), but in a slightly different manner. The reliability or the predicted failure probability of each component is obtained from its failure – data analysis. If the failure allowance specified for the system is apportioned among the components in some equitable manner [8].

3.2 Reliability allocation for a series system.

We discuss the reliability allocation for series system by using two

methods. 1. First method

The system consisting of n components connected in series .the principle adopted in this method for subdividing the system failure allowance is that the failure allowance of each component is directly proportional to the predicted probability of failure. This rule based on the assumption that the components exhibit a constant failure rate, and if λi is the failure rate of component i, i= 1,…. , n then we can use the approximation :

$R_s(t) = e^{-\lambda} s^t \approx 1 - \lambda st$

Where $\lambda s =$, $\sum^{n} \lambda i \lambda s$ is termed the predicted system failure

rate. $i = 1$

*s be the specified system failure rate which is assumed to be less than

λ

λs.to compute the required failure rate for component I which is denoted

by λ

*

i , i = 1,2,…,n , we use the following equation :

λ*it = (λit/λst). λ^*st

So the reliability goal for the component i is

$R*_i(t) = e - \lambda *_{it} \approx 1 - \lambda *_{it}$

Example (3.1):-

A system is composed of 4 units connected in series the failure rate for these units are as follows:- $\lambda 1 = 0.05$, $\lambda 2 = 0.2$

$\lambda 3 = 0.25$, $\lambda^{4=0.04}$

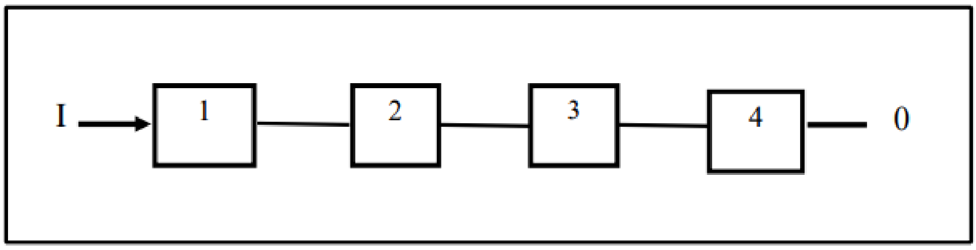


Figure (4.1)

it is desired that the maximum failure rate for the system be λ $*S=0.5$,

determine the reliability goal of each component.

solution :-

the sum of the unite failure rate

$\lambda s= \lambda 1 + \lambda 2+ \lambda 3 + \lambda 4$ …(4.5)

$= 0.05 + 0.2 + 0.25+ 0.04$

$=0.54$

Hence , the allocated unite failure rate are

$$\lambda^{*}1 = \left(\frac{0.5}{0.54}\right) \times 0.05 = 0.0462963$$

$$\lambda^{*}2 = \left(\frac{0.5}{0.54}\right) \times 0.2 = 0.1851851852$$

$$\lambda^{*}3 = \left(\frac{0.5}{0.54}\right) \times 0.25 = 0.2314814815$$

$$\lambda^{*}4=\left(\frac{0.5}{0.54}\right)\times 0.04=0.037037037$$

(ii) If $\lambda^{*}{}_{1},\lambda^{*}2,\lambda^{*}{}_{3},\lambda^{*}4$ are allocated failure rate, we get

$$\mathrm{R}*\mathrm{s(t)}=e^{-\lambda_{*}1\mathrm{t}}e^{-\lambda_{*}2\mathrm{t}}e^{-\lambda_{*}3\mathrm{t}}e^{-\lambda_{*}4\mathrm{t}}$$

$$\lambda^{*}1=\left(\frac{\lambda^{*}\mathrm{s}}{\mathrm{s}\lambda\mathrm{s}}\right)\lambda1\cdot\lambda^{*}2=\left(\frac{\lambda^{*}\mathrm{s}}{}\right)\lambda2_{\lambda}$$

$$\lambda*\frac{\lambda\mathrm{s}}{\lambda\mathrm{s}}*\frac{\lambda\mathrm{s}}{\lambda\mathrm{s}}3=(\quad)\lambda3\cdot\lambda4=(\ )\lambda4$$

Hence ,the allocated reliabilities are

$$R_{1}^{*}(t)=e^{-\lambda_{1}^{i}}$$

$$=e^{-0.04}=0.955997$$

$$R_{2}^{*}(t)=e^{-\lambda_{2}^{*}}$$

$$=e^{-0.18}=0.835270$$

$$R_{3}^{*}(t)=e^{-\lambda_{3}^{*}}$$

$$=e^{-0.23}=0.798516$$

$$R_{4}^{*}(t)=e^{-\lambda_{4}^{*}}$$

$$=e^{-0.037}=0.964640$$

$$R_{*}S=R_{*1}\times R_{*2}\times R_{*3}\times R_{*4}$$

$$=0.95\times 0.83\times 0.79\times 0.96$$
$$\cong 0.59$$

3.3 Reliability allocation for a mixed system

Similarly, we can find the reliability allocation for complex system by using previous methods

Example (3.4):-

A system consists of seven element connected as shown in figure the predicated reliabilities of the components for a 10 hour period are also shown.

1- Convert the given system into an equivalent system consisting of only series element.
2- Calculate the system reliability
3- If the system reliability is to be improved to value of 0.95 for a 10-hour period , determine the reliability goal of each component assume constant failure rate of all element without using the approximation R(t) 1-λt

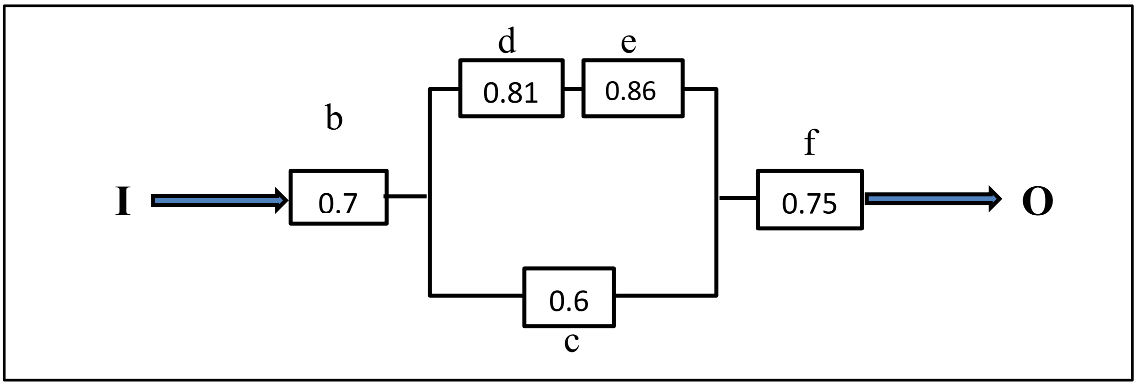


Figure (4.3)

Solution:-

1- For calculating the system reliability, the given system converted into an equivalent system consisting of only series components c, d ,e mentioned in the reduction method

$Rm(t)=Rc+ R_dR_e - R_cR_dR_e =(0.6)+(0.81)(0.86)-(0.6)(0.81)(0.86)$

(by probability rules) =0.878

Now the given system has been reduced to a series system as shown in figure.

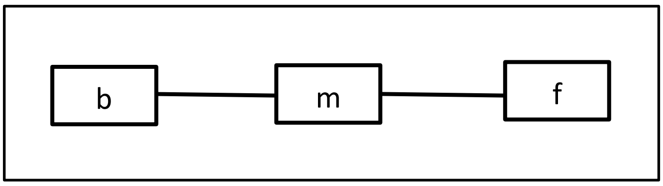


Figure (4.3)

We have, the reliability of the systems

$\text{Ra}(t) = \exp\,(-\lambda at) = 0.5$
$\text{Rb}(t) = \exp\,(-\lambda bt) = 0.7$
$\text{Rm}(t) = \exp\,(-\lambda mt) = 0.87$
$\text{Rf}(t) = \exp\,(-\lambda ft) = 0.75$
$\text{Rg}(t) = \exp\,(-\lambda gt) = 0.94$ The value

of λt would have been λat=0.5

λbt =0.3 λmt =0.12 λft =0.25 and

λgt=0.06

2- Thus the system reliability is given by Rs(t)=

Ra(t)×Rb(t)×Rm(t) ×Rf(t)×Rg(t)

=exp [-(λat+ λbt+ λmt +λft+ λgt)]

=exp(- λst)

Where λst=0.5+0.3+0.122+0.25+0.06

=1.232 (from eq(4.1))

$Rs(t) = \exp\,(-1.232) = 0.29$

3- The desired reliability R∗s(t)=0.95

For the system, therefore R∗s(t)=e

$-\lambda^{*st}$=0.95

$\text{R} * \text{s(t)} = e - {}^{\lambda_*}\text{at}e - {}^{\lambda_*}\text{bt}e - {}^{\lambda_*}\text{mt}e - {}^{\lambda_*}\text{ft}e - {}^{\lambda_*}\text{gt}$
$\lambda^*\text{a} = (\lambda^*\text{s}/\lambda\text{s})\lambda\text{a}, \lambda^*\text{b} = (\lambda^*\text{s}/\lambda\text{s})\lambda\text{b}\lambda^*$
$\text{m} = (\lambda^*\text{s}/\lambda\text{s})\lambda\text{m}, \lambda^*\text{f} = (\lambda^*\text{s}/\lambda\text{s})\lambda\text{f}\lambda^*\text{g}$
$= (\lambda^*\text{s}/\lambda\text{s})\lambda\text{g}$ Hence the allocated

reliabilities are

$R * a(t) = e - (\lambda^* s/\lambda s)\lambda a$
$R * b(t) = e - (\lambda^* s/\lambda s)\lambda b$
$R * m(t) = e - (\lambda^* s/\lambda s)\lambda m$
$R * f(t) = e - (\lambda^* s/\lambda s)\lambda f$
$R * g(t) = e - (\lambda^* s/\lambda s)\lambda g$
$R * a(t) = e^{-\lambda} = 0.96$
$R * b(t) = e - \lambda_i^* = 0.95$
$R^* m(t) = e - \lambda_m^* = 0.94$
$R^* f(t) = e^{-\lambda_j^*} = 0.95$
$R^* g(t) = e^{-\lambda_s^*} = 0.99$

Conclusions:

1. Reliability depends strongly on system configuration. Series systems are more vulnerable to failure, while parallel and mixed systems provide higher reliability due to redundancy and strategic component arrangement.
2. Mixed systems offer an effective balance between system complexity and reliability. By combining series and parallel components, engineers can design systems that are both efficient and dependable.
3. Component reliability is critical. The overall reliability of a system is significantly influenced by the weakest components. Identifying and improving low-reliability components enhances the performance and safety of the entire system.
4. Reliability analysis supports better engineering decisions. Understanding system behavior allows engineers to plan maintenance, optimize designs, and minimize operational risks.
5. Applications are widespread. Mixed systems are extensively used in electrical networks, computer systems, communication networks, and industrial systems, making reliability analysis essential for ensuring continuous operation and reducing downtime.
6. In conclusion, studying the reliability of mixed systems is essential for designing robust engineering systems, improving performance, and minimizing the probability of failure in real-world applications.

References

[1] A. Birolini, *Reliability Engineering: Theory and Practice*, 7th ed. Berlin, Germany: Springer, 2017.

[2] R. Billinton and R. N. Allan, *Reliability Evaluation of Engineering Systems*. New York, NY, USA: Springer, 1996.

[3] B. S. Dhillon, *Engineering Reliability: New Techniques and Applications*. Boca Raton, FL, USA: Taylor & Francis, 2006.

[4] C. E. Ebeling, *An Introduction to Reliability and Maintainability Engineering*. Long Grove, IL, USA: Waveland Press, 2019.

[5] W. Kuo and M. J. Zuo, *Optimal Reliability Design: Fundamentals and Applications*. Hoboken, NJ, USA: John Wiley & Sons, 2003.

[6] E. E. Lewis, *Introduction to Reliability Engineering*, 2nd ed. Hoboken, NJ, USA: Wiley, 1996.

[7] P. D. T. O'Connor and A. Kleyner, *Practical Reliability Engineering*, 5th ed. Hoboken, NJ, USA: Wiley, 2012.