***Republic of Iraq***
***Ministry of Higher Education***
***and Scientific Research***
***University of Babylon***
***College of Education for Pure***
***Department of Mathematics***

*Research Title:*

# Strong and Weak Domination for various Graphs and some operations

***The research is submitted to the Council***

***of the College of Education for Pure Science University of Babylon***

***Bachelor's degree in the Department of Mathematics as part of the***

***Prepared by the student***

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**بِسْمِ اللهِ الرَّحْمنِ الرَّحِيمِ**

(وَأَن لَّيْسَ لِلإِنسَانِ إِلاَّ مَا سَعَى، وَأَنَّ سَعْيَهُ سَوْفَ يُرَى ، ثُمَّ يُجْزَاهُ الْجَزَاء الأَوْفَى)

صَدَقَ اللهُ العَلِيُّ العَظِيمُ

سورة النجم: الآيات ٢٩–٣١

## الاهداء

الحمد لله حباً ورضا وامتنانا على البدء والختام

(( واخر دعواهم الحمدلله رب العالمين))

بداية لم تكن الرحله قصيره ولا طريقاً مليئاً بالتسهيلات ولكنني فعلتها فالحمدلله حمداً كثيراً

في البداية اهدي نفسي الطموحه التي صبرت وجاهدت من اجل الوصول الى هذا النجاح الكثير.

اهدي نجاحي وتخرجي الى من علمني ان الدنيا كفاح وسلاحها العلم والمعرفه. الى ذلك الرجل العظيم الذي شجعني دائماً للوصول الى طموحاتي وأحلامي والذي بذل كل مابوسعه من اجلنا ولم يبخل علينا يوماً من ألايام، والدي العزيز وملهمي وسندي الاول الذي دعمني بلا حدود واعطاني بلا مقابل ولا تفكير، ها انا اليوم أتممت وعدي لك واهديته لك

الى من جعل الله الجنه تحت اقدامها، الى معلمتي الاولى الى من غرست فيني حب العلم والمعرفه إلى ملاكي في الحياة الى معنى الحب والحنان إلى سر الحياة إلى من أضاءت دربي في الليالي المظلمه ألى من سهرت وكافحت من أجلي إلى من كان دعاؤها سر نجاحي والى داعمي الاول والمكان الذي استمد منهُ قوتي .. (أمي الغاليه)

إلى من راهنوا على نجاحاتي والذين دائماً يذكروني بمدى قوتي واللذين يؤمنون بشجاعتي مهمى ضعفت إلى خيرة ايامي إلى قرة عيني وإلى من كانوا جزءاً من هذا الانتصار الكبير.. ( اخواني، اخواتي)

الى منساندني بكل حب عند ضعفي وأزاح عني طريق المتاعب ممهداً لي الطريق الخالي من العثرات إلى رفيق دربي الذي شاركتهُ الفرح والحزن والنجاح والفشل إلى من انتضر هذهِ اللحظات ليفتخر لي إلى من أحلوت الحياة بقربه إلى من أرهُ خالداً وسط قلبي إلى الذي اغمرني بالحب إلى سندي الثاني وشريك حياتي.. (زوجي)

إلى رفقاء دربي إلى أصدقاء الرحله والنجاح إلى الذين امدوني بالقوه والذين دعموني في ألاوقات الصعبه إلى الذين احسست بجانبهم بطعم الصداقه والاخوه الحقيقين ( صديقاتي)

إلى كل من ساهم وله الفضل بالمساعده بطريقة او بأخرى في مسيرتي شكراً لكم.

# الشكر و التقدير

الشكر والثناء لله عز وجل اولاً الذي بفضله تم هذا العمل ،فالله الحمد من قبل ومن بعد .

اود ان اتوجه بجزيل الشكر والتقدير "إلى عائلتي التي كانت السند، وأصدقائي الذين كانوا النور في كل عتمة، وزوجي الذي كان القلب والدعم في كل لحظة... شكرًا لأنكم كنتم القوة التي استندت إليها، والدفء الذي احتواني. لولاكم، ما كنت كما أنا اليوم".

اتقدم بجزيل الشكر والامتنان لمشرف هذا البحث الاستاذ (احسان عبد الرحمن) على ما قدمه من دعم علمي وتوجيه مستمر طوال فترة اعداد البحث. كانت ارشاداته القيمة وملاحظاته البناءة مصدرا اساسيا في تحسين جودة العمل، وبدونه لما كان هذا الانجاز ممكناً. فلهُ مني كل الشكر والتقدير على جهوده المثمره .

ثم الشكر لمن كانوا خيراً عوناً لنا، شكراً لجميع دكاترتنا واساتذتنا على ما قدموا لنا ولن نفيكم حقكم كنتم خير معين لنا طوال عامنا الدراسي .

الى من كانت لها بصمه جميله في مساعدتي دوماً لها مني كل الاحترام والشكر والتقدير اسأل الله لها التوفيق ( فاطمه ساجت)

***Contents***

## 1. Introduction

A graph consists of a set of vertices and set of edges, each joining two vertices.

Usually an object is represented by a vertex and a relationship between two objects is represented by an edge. Thus a graph may be used to represent any information that can be modeled as objects and relationships between those objects. Graph theory deals with study of graphs. The foundation stone of graph theory was laid by Euler in 1736 by solving a puzzle called Königsberg seven-bridge problem . Königsberg is an old city in Eastern Prussia lies on the Pregel River. The Pregel River surrounds an island called Kneiphof and separates into two branches where four land areas are created: the island a, two river banks b and c, and the land d between two branches. Seven bridges connect the four land areas of the city. It is said that the people of Königsberg used to entertain themselves by trying to devise a walk around the city which would cross each of the seven bridges just once. Since their attempts had always failed, many of them believed that the task was impossible, but there was no proof until 1736. In that year, one of the leading mathematicians of that time, Leonhard Euler published a solution to the problem that no such walk is possible. He not only dealt with this particular problem, but also gave a general method for other problems of the same type. Euler constructed a mathematical model for the problem in which each of the four lands a, b, c and d is represented by a point and each of the seven bridges is represented by a curve or a line segment The problem can now be stated as follows: Beginning at one of the points a, b, c and d, is it possible to trace the figure without traversing the same edge twice? The mathematical model constructed for the problem is known as a graph model of the problem. The points a, b, c and d are called vertices, the line segments are called edges, and the whole diagram is called a graph. , we need to know some terminologies. A graph G is a tuple (V, E) which consists of a finite set V of vertices and a finite set E of edges; each edge is an unordered pair of vertices. The two vertices associated with an edge e are called the end-vertices of e. We oftendenote by (u, v), an edge between two vertices u and v. We also denote the set of vertices of a graph G by V(G) and the set of edges of G by E(G)..dominating set of a graph G = (V, E) is any subset D of V such that every vertex not in D is adjacent to at least one member of D. The minimum cardinality of all dominating sets of G is called the domination number of G and is denoted by $\gamma(G)$. This parameter has been extensively studied in the literature and there are hundreds of papers concerned with domination. concept of domination and related invariants have been generalized in many ways. The corona product G ◦ H of two graphs G and H is defined as the graph obtained by taking one copy of G and |V (G)| copies of H and joining the i-th vertex of G to every vertex in the i-th copy of H. A set D ⊆ V (G) is a strong dominating set of G, if for every vertex x ∈ V (G) \ D there is a vertex y ∈ D with xy ∈ E(G) and deg(x) ≤ deg(y). The strong domination number γst(G) is defined as the minimum cardinality of a strong dominating set. A strong dominating set with cardinality γst(G) is called a γst-set. a set D ⊂ V is a weak dominating set of G if every vertex v ∈ V \ S is adjacent to a vertex u ∈ D such that deg(v) ≥ deg(u).

***1.1.Graph:***[**1**]

A graph is an ordered triplet G = {v(G), E(G ،(IG) Where V (G) is a non-empty set. E(G) is a set disjoined from V (G) and Io is an incident map that associates with each element of E(G (unordered pair of elements of V (G).V (G) is the vertex set and E(G) is the edge set.

***1.2.Order:*** [2]

of a graph is the number of vertices in the graph.

***1.3.Degree*** [**3**]***:***

of a vertex is the number of edges falling on it . It tells as how many other vertices are adjacent to that vertex.

***1.4**. **Maximum and Minimum Degree***

*1.4.1.* ***Maximum Degree***

The maximum degree of a graph G, denoted by Delta(G) is the maximum value among the degrees of all the vertices of G, i.e., Delta(G)=max v \in V(G) deg(v) . Similarly, we define

*1.4.2* ***Minimum Degree***

the minimum degree of a graph G and denote it by delta(G) i.e., delta(G)=min v \in V(G) deg(v).

***1.5. Size :***

of a graph is the number of edges in the graph.

***1.6.path***

a path of length $n-1$ denoted by $P_n$ is a sequence of distinct edges $v_1v_2, v_2v, v_n-1v_n$

***1.7. A cycle:***

is one that is obtained by joining the two end-vertices of a path graph Thus, the degree of each vertex of a cycle graph is two. a cycle graph with n vertices is often denoted by $C_n$.

***1.8. wheel:***

with n vertices, denoted by $W_n$ is obtained from a cycle graph $C_n - 1$ with $n - 1$ vertices by adding a new vertex w and joining an edge from w to each vertex of $C_n - 1$.

***1.9. Null***

Thus the null graph is a regular graph of degree zero. Some authors exclude K0 from consideration as a graph (either by definition, or more simply as a matter of convenience). Whether including $K_0$ as a valid graph is useful depends on context. On the positive side, K0 follows naturally from the usual set-theoretic definitions of a graph (it is the ordered pair (V, E) for which the vertex and edge sets, V and E, are both empty),

***1.10.Star***

Star graph is a special type of graph in which n-1 vertices have degree 1 and a single vertex have degree n – 1. This looks like n – 1 vertex is connected to a single central vertex. A star graph with total n – vertex is termed as $S_n$.

***1.11.Complete***

A graph in which each pair of distinct vertices are adjacent is called a complete graph. A complete graph with n vertices is denoted by Kn. It is trivial to see that $K_n$ contains n(n − 1)/2 edges. Any graph is a sub graph of the complete graph with the same number of vertices and thus the number of edges in a graph with n vertices is at most n(n−1)/2

***1.12.Complete bipartite***

If the vertices set of a graph G can be partitioned into two sets $V_1$ and $V_2$ such that any edge of G joins one vertex in $V_1$ to one vertex in $V_2$ then G is called a bipartite graph having bipartite($V_1$, $V_2$).

***1.13.Fan***

The fan $F_n$ is defined to be the graph $P_n + K1$

***1.14.Double fan***

The double fan is defined to be the graph $P_n + \overline{K2}$

***1.15.Corona***

. Let $G_1$ and $G_2$ be two disjoint graph. The Corona $C_n \odot C_m$ Corona G2 of two graphs G1 and $G_2$ is the graph obtained by taking one copy of $G_1$ (which has $P_1$ vertices) and $P_1$ copies of $G_2$, and then joining the $j^{th}$ vertex of $G_1$ to every vertex in the j copy of $G_2$.

***1.16.Ccomplement***

The complement of a graph $G = (V, E)$ is another graph $\bar{G} = (V, \bar{E})$ with the same vertex set such that for any pair of distinct vertices $u, v \in V, (u, v) \in \overline{E}$.

***1.17.Dominating set***

A set $S \subseteq V$ of vertices in a graph G = (V, E) is called a dominating set if every vertex $v \in V$ is either an element of *S* or is adjacent to an element of *S*.

***1.18. Domination number***

Definition. A minimum dominating set in a graph G is a dominating set of minimum cardinality. The cardinality of minimum dominating set is called the domination number of G and is denoted by y(G).The minimum dominating set is called y set.

**Inverse Dominating set:** If V-D contains a dominating set say D ' of G, then D ' is called an inverse dominating set with respect to D.

**Inverse Domination number:** The inverse domination number γ ' (G) of G is the order of a smallest inverse dominating set of G

***1.19. Strong domination number***

A set $D \subset V(G)$ is a strong dominating set of G, if for every vertex $x \in V(G) \setminus D$ there is a vertex $y \in D$ with $xy \in E(G)$ and deg $(x) \leq deg(y)$. The strong domination number $\gamma_{st}(G)$ is defined as the minimum cardinality of a strong dominating set.

***1.20. Weak domination number***

set D ⊂ V is a weak dominating set of G if every vertex v ∈ V \ S is adjacent to a vertex u ∈ D such that deg(v) ≥ deg(u) The minimum cardinality of a weak dominating set of G is denoted by $\gamma_w(G)$.

## *2. Strong Domination.*

***Theorem 2.1:***

*Let $P_n$ be a path graph of order n, then:*

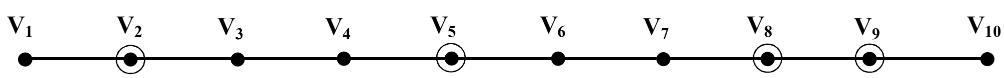
$$\gamma_s(P_n) = \begin{Bmatrix} \frac{n}{3} \; if \; n \equiv 0 \; (mod \; 3) \\ \left\lceil \frac{n}{3} \right\rceil \; if \; n \equiv 1, 2 (mod \; 3) \end{Bmatrix}$$

*Ex: Find the strong domination number of* $P_6$


*n ≡ 0 (mod 3)*

$$\gamma_s(P_6) = \frac{n}{3} = \frac{6}{3} = 2$$

*Ex: Find the strong domination number of* $P_{10}$




*n ≡ 1 (mod 3)*

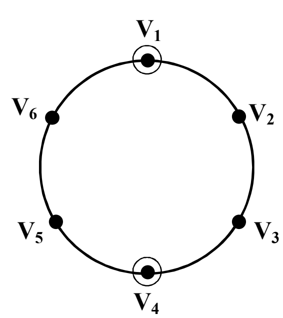
$$\gamma_s(P_{10}) = \left\lceil \frac{n}{3} \right\rceil = \left\lceil \frac{10}{3} \right\rceil = 4$$

***Theorem 2.2:***

*Let $C_n$ be a cycle graph of order n then*:

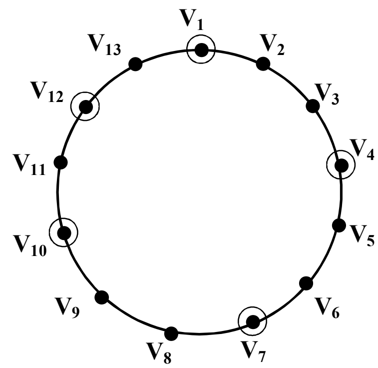
$$\gamma_s(C_n) = \begin{Bmatrix} \frac{n}{3} \; if \; n \equiv 0 \; (mod \; 3) \\ \left\lceil \frac{n}{3} \right\rceil \; if \; n \equiv 1, 2 (mod \; 3) \end{Bmatrix}$$

*Ex: Find the strong domination number of* $C_6$




*if n ≡ 0 (mod 3)* $\Rightarrow \gamma_s(C_6) = \frac{n}{3} = \frac{6}{3} = 2$

*Ex: Find the strong domination number of $C_{13}$*




*$n \equiv 1 \pmod 3$*

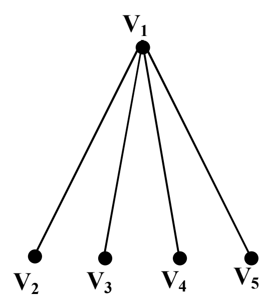
$$\gamma_s(C_{13}) = \left\lceil \frac{n}{3} \right\rceil = \left\lceil \frac{13}{3} \right\rceil = 5$$

***Theorem 2.3:***

*Let $S_n$ be a star graph of order n then:*

$$\gamma_s(S_n) = 1$$

*Ex: Find the strong domination number of $S_5$*



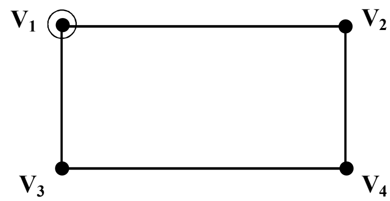

$$\gamma_s(S_5) = 1$$

***Theorem 2.4:***

*Let $K_n$ be a complete graph of order n then:*

$$\gamma_s(K_n) = 1$$

*Ex: Find the strong domination number of* $K_4$



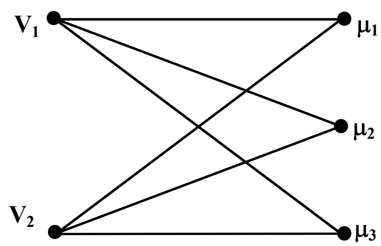

$\gamma_s(K_4) = 1$

***Theorem 2.5:***

*Let* $K_{(n,m)}$ *be a complete Bipartite graph of order* $n + m$ *then:*

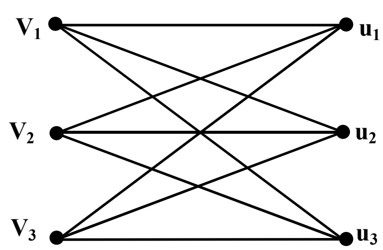
$$\gamma_s(K_{n,m}) = \begin{Bmatrix} n \text{ if } n < m \\ m \text{ if } m < n \\ n \text{ or } m \text{ if } n = m \end{Bmatrix}$$

*Ex: Find the strong domination number of* $K_{(2,3)}$




$\gamma_s(K_{2,3}) = 2$

*Ex: Find the strong domination number of* $K_{(3,3)}$



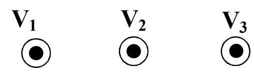

$\gamma_s(K_{3,3}) = 2$

***Theorem 2.6:***

*Let $N_n$ be a null graph of order n then:*

$\gamma_s(N_n) = n$

*Ex: Find the strong domination number of $N_3$*



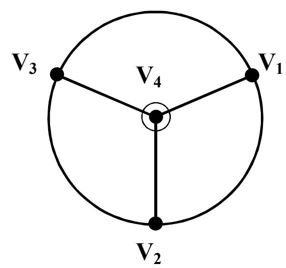

$\gamma_s(N_3) = 3$

***Theorem 2.7:***

*Let $W_n$ be a wheel graph of order n then:*

$\gamma_s(W_n) = 1$

*Ex: Find the domination number of $W_4$*



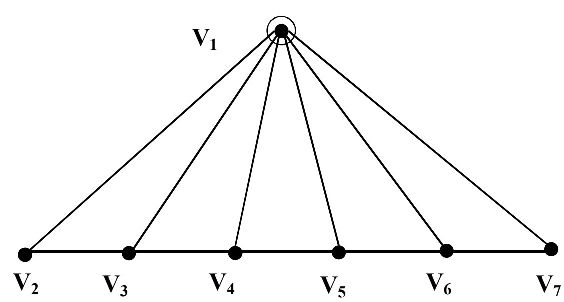

$\gamma_s(W_4) = 1$

***Theorem 2.8:***

*Let $F_n = P_n + K_1$ be a fan graph of order n then:*

$\gamma_w(P_n + k_1) = 1$

*Ex: Find the strong domination number of $P_6+K_1$*



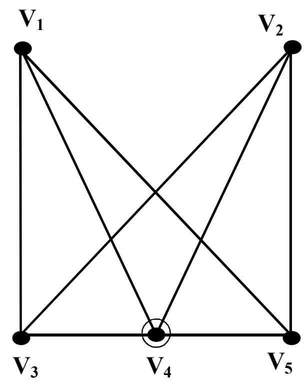

$\gamma_s(P_6 + k_1) = 1$

***Theorem 2.9:***

*Let $P_n + \overline{K_2}$ be a double fan graph of order n then:*

$$\gamma_s(P_n + \overline{K_2}) = \begin{Bmatrix} 1 \; if \; n = 3 \\ 2 \; if \; n \geq 4 \end{Bmatrix}$$

*Ex: Find the strong domination number of $P_3 + \overline{K_2}$*




$$\gamma_s\left(P_3 + \overline{k_2}\right) = 1$$

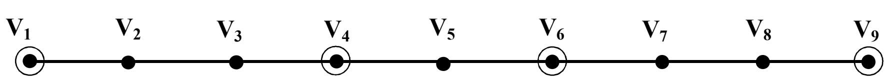
## *3. Weak Domination:*

***Theorem 3.1 :***

*Let $P_n$ be a path graph of order n then:*

$$\gamma_w(P_n) = \begin{Bmatrix} \frac{n}{3} + 1 if \; n \equiv 0(mod3) \\ \left\lceil \frac{n}{3} \right\rceil \; if \; n \equiv 1(mod3) \\ \left\lfloor \frac{n}{3} \right\rfloor + 2or \left\lceil \frac{n}{3} \right\rceil + 1 \; if \; n \; \equiv 2(mod3) \end{Bmatrix}$$

*Ex: Find the strong domination number of $P_9$*




$$n \equiv 0(mod3)$$

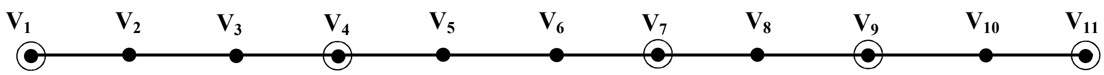
$$\gamma_w(P_9) = \frac{n}{3} + 1 = \frac{9}{3} + 1 = 3 + 1 = 4$$

*Ex: Find the weak domination number of* $P_{10}$


$$n \equiv 1(mod3)$$

$$\gamma_w(P_{10}) = \left\lceil \frac{n}{3} \right\rceil = \left\lceil \frac{10}{3} \right\rceil = 4$$

*Ex: Find the weak domination number of* $P_{11}$




$$n \equiv 2(mod3)$$

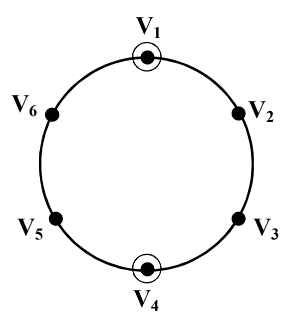
$$\gamma_w(P_{11}) = \left\lfloor \frac{n}{3} \right\rfloor + 2 = \left\lfloor \frac{11}{3} \right\rfloor + 2 = 3 + 2 = 5$$

***Theorem 3.2:***

*Let* $C_n$ *be a cycle graph of order n then:*

$$\gamma_w(C_n) = \begin{Bmatrix} \frac{n}{3} \ if\ n \equiv 0(mod3) \\ \left\lceil \frac{n}{3} \right\rceil \ if\ n \equiv 1,2(mod3) \end{Bmatrix}$$

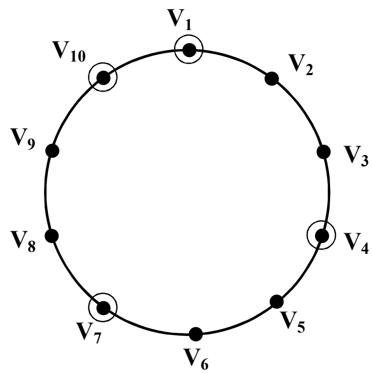
*Ex: Find the weak domination number of* $C_6$




$$n \equiv 0(mod3)$$

$$\gamma_w(C_6) = \frac{n}{3} = \frac{6}{3} = 2$$

*Ex: Find the weak domination number of* $C_{10}$




$n \equiv 1(mod3)$

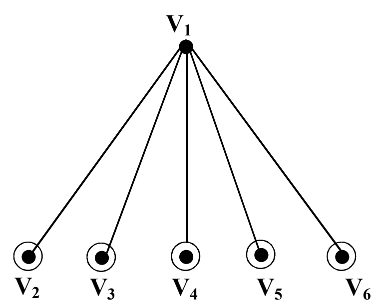
$\gamma_w(C_{10}) = \left\lceil \frac{n}{3} \right\rceil = \left\lceil \frac{10}{3} \right\rceil = 4$

***Theorem 3.3:***

*Let* $S_n$ *be a star graph of order n then:*

$\gamma_w(S_n) = n - 1$

*Ex: Find the weak domination number of* $S_6$



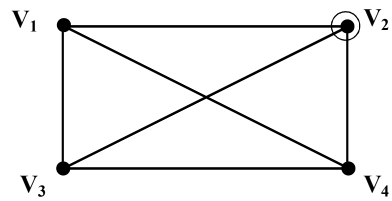

$\gamma_w(S_6) = n - 1 = 6 - 1 = 5$

***Theorem 3.4:***

*Let* $K_n$ *be a complete graph of order n then:*

$\gamma_w(K_n) = 1$

*Ex: Find the weak domination number of* $K_4$



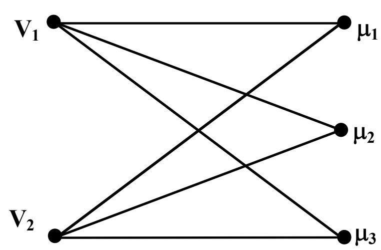

$\gamma_w(K_4) = 1$

***Theorem 3.5:***

*Let* $K_{(n, m)}$ *be a complete Bipartite graph of order nm then:*

$$\gamma_w(K_{n,m}) = \left\{ \begin{matrix} n \ if \ n > m \\ m \ if \ m > n \\ n \ or \ m \ if \ n = m \end{matrix} \right\}$$

*Ex: Find the weak domination number of* $K_{2,3}$



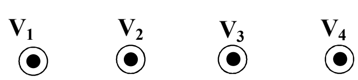

$\gamma_w(K_{2,3}) = 3$

***Theorem 3.6:***

*Let* $N_n$ *be a null graph of order n then:*

$\gamma_w(N_n) = n$

*Ex: Find the weak domination number of* $N_4$



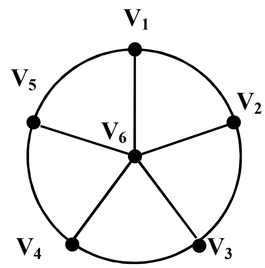

$\gamma_w(N_4) = 4$

***Theorem 3.7:***

*Let* $W_n$ *be a wheel graph of order of n then:*

$$\gamma_w(W_n) = \gamma_w(C_n) = \left\lceil \frac{n}{3} \right\rceil$$

*Ex: Find the weak domination number of $W_6$*



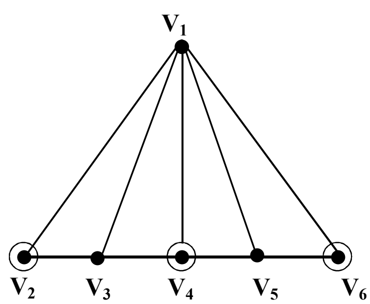

$$\gamma_w(W_n) = \gamma_w(C_n) = \left\lceil \frac{n}{3} \right\rceil = \left\lceil \frac{5}{3} \right\rceil = 2$$

***Theorem 3.8:***

*Let $F_n = P_n + K_1$ be a fan graph of order n then:*

$$\gamma_w(F_n) = \gamma_w(P_n) = \left\{ \begin{array}{c} \frac{n}{3} + 1 if\ n \equiv 0(mod3) \\ \left\lceil \frac{n}{3} \right\rceil\ if\ n \equiv 1(mod3) \\ \left\lfloor \frac{n}{3} \right\rfloor + 2 or \left\lceil \frac{n}{3} \right\rceil + 1\ if\ n\ \equiv 2(mod3) \end{array} \right\}$$

*Ex: Find the domination number of $P_5+K_1$*



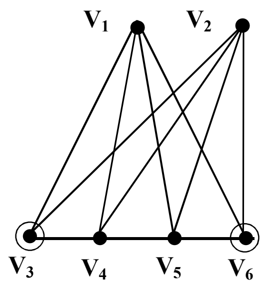

$$\gamma_w(P_5 + K_1) = \gamma_w(P_5) = \left\lceil \frac{n}{3} \right\rceil + 1 = \left\lceil \frac{5}{3} \right\rceil + 1 = 2 + 1 = 3$$

***Theorem 3.9:***

*Let $(P_n + \overline{K_2})$ be a double fan graph of order n then:*

$$\gamma_w(P_n + \overline{K_2}) = \gamma_w(P_n) = \left\{ \begin{array}{c} \frac{n}{3} + 1 if\ n \equiv 0(mod3) \\ \left\lceil \frac{n}{3} \right\rceil\ if\ n \equiv 1(mod3) \\ \left\lfloor \frac{n}{3} \right\rfloor + 2 or \left\lceil \frac{n}{3} \right\rceil + 1\ if\ n\ \equiv 2(mod3) \end{array} \right\}$$

*Ex: Find the weak domination number of* $P_4 + \overline{K_2}$




$$\gamma_w(P_4 + \overline{K_2}) = \gamma_w(P_4) = \left\lceil \frac{n}{3} \right\rceil$$

$$= \left\lceil \frac{4}{3} \right\rceil = 2$$

## *4.Inverse Strong Domination Number*

***Theorem4.1:***

*A path has no inverse strong domination number .*

***Theorem4.2:***

*A star has no inverse strong domination number.*

***Theorem4.3:***

*Inverse strong domination number of wheel if n=4 then*

$\gamma_s^{-1}(W_4) = 1$,

*but has no inverse strong domination number if* $n \geq 5$

***Theorem4.4:***

*Inverse strong domination number of fan if n=3 then*

$\gamma_s^{-1}(F_4) = 1$,

*but has no inverse strong domination number if* $n > 3$

***Theorem4.5:***

*Inverse strong domination number of double fan if n=4 then*

$\gamma_s^{-1}(P_4 + \overline{K_2}) = 2$,

*but has no inverse strong domination number if* $n = 3$ *and* $n \geq 5$

***Theorem4.6:***

*Inverse strong domination number for complete Bipartite if n=m then*

$\gamma_s^{-1}(K_{3,3}) = 2$,

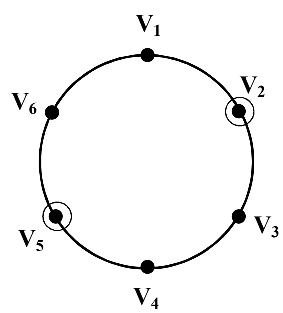
*but has no inverse strong domination number if n < m or n >m*

***Theorem4.7:***

*A cycle has inverse then*

$$\gamma_s^{-1}(C_n) = \begin{cases} \frac{n}{3} & if\ n \equiv 0(mod3) \\ \left\lceil \frac{n}{3} \right\rceil & if\ n \equiv 1,2(mod3) \end{cases}$$

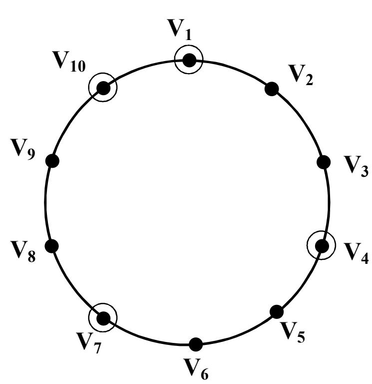
*Ex: Find the inverse strong domination number of* $C_6$




$n \equiv 0(mod3)$

$$\gamma_s^{-1}(C_6) = \frac{n}{3} = \frac{6}{3} = 2$$

*Ex: Find the inverse strong domination number of* $C_{10}$



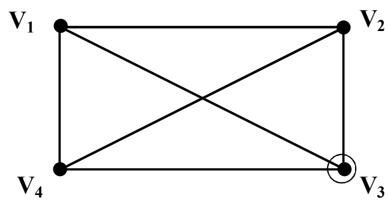

$$n \equiv 1(mod3) \Rightarrow \gamma_s^{-1}(C_{10}) = \left\lceil \frac{n}{3} \right\rceil = \left\lceil \frac{10}{3} \right\rceil = 4$$

***Theorem 4.8:***

*A complete has inverse then:*

$\gamma_s^{-1}(K_n) = 1$

*Ex: Find the inverse strong domination number of $K_4$*




$\gamma_s^{-1}(K_4) = 1$

## *5. Inverse weak domination number*

***Theorem 5.1:***

*A Path has no inverse weak domination number .*

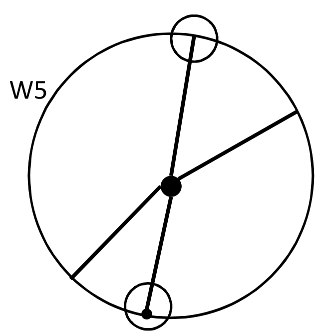
***Theorem 5.2 :***

*A star has no inverse weak domination number.*

***Theorem 5.3:***

*A wheel has inverse ,Then* $y_w^{-1}(wn) = y_w^{-1}(cn) = \left\lceil \frac{n}{3} \right\rceil$

*Ex: Find the inverse weak domination number of $w_5$*




$\gamma_w^{-1}(w4) = \gamma_w^{-1}(c4) = \left\lceil \frac{n}{3} \right\rceil = \left\lceil \frac{4}{3} \right\rceil = 2$

***Theorem 5.4:***

*A fan has no inverse weak domination number.*

***Theorem 5.5:***

*Inverse weak domination number for double fan if n = 3 then*

$\gamma_s^{-1}(P_n + \overline{K_2}) = 2$,

*but has no inverse weak domination number fan if n ≥ 4*

***Theorem 5.6:***

*Inverse weak domination number for complete Bipartite if n = m then*

$\gamma_w^{-1}(P_{3,3}) = 2$,

*but has no inverse weak domination number if n > m or n <m*

***Theorem 5.7:***

*A cycle has inverse weak domination then*

$$\gamma_s^{-1}(C_n) = \begin{cases} \frac{n}{3} \; if\; n \equiv 0 (mod 3) \\ \left\lceil \frac{n}{3} \right\rceil \; if\; n \equiv 1,2 (mod 3) \end{cases}$$

***Theorem 5.8:***

*A complete has inverse weak domination number then*

$\gamma_w^{-1}(K_n) = 1$

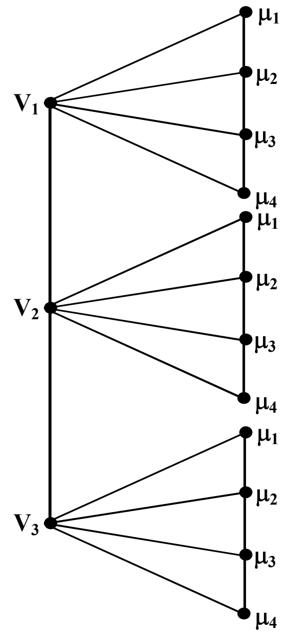
## *6. Strong Domination of the operation Corona*

***Theorem 6.1:***

*Let $G = P_n \odot P_m$ be a graph of order (nm+n) then*

$\gamma_s(P_n \odot P_m) = n$

*Ex: Find the strong domination number of $P_3 \odot P_4$*



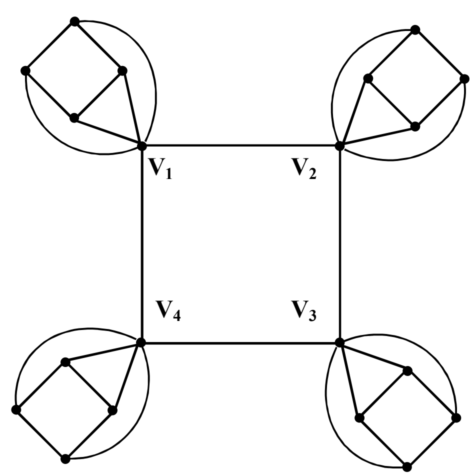

$\gamma_s(P_3 \odot P_4) = 3$

***Theorem 6.2:***

*Let* $G = C_n \odot C_m$ *be a corona cycle of order nm+n then*

$\gamma_s(C_n \odot C_m) = n$

*Ex: Find the inverse strong domination number of* $(C_4 \odot C_4)$




$\gamma_s(C_4 \odot C_4) = 4$

***Theorem 6.3:***

*Let* $G = N_n \odot N_m$ *be a corona null of order nm+n then*

$\gamma_s(N_n \odot N_m) = n$

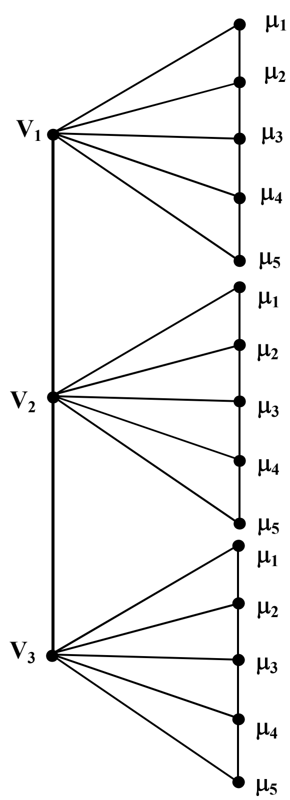
## *7. Weak Domination of the operation Corona*

***Theorem 7.1:***

*Let* $G = P_n \odot P_m$ *be a corona path of order nm+n then*

$\gamma_w(P_n \odot P_m) = \gamma_w(P_m).n$

*Ex: Find the inverse weak domination number of* $(P_3 \odot P_5)$




$\gamma_w(P_3 \odot P_5) = \gamma_w(P_5).n$

*= (3).3*

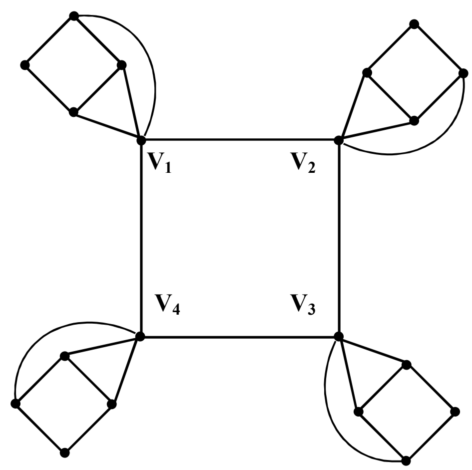
*= 9*

***Theorem 7.2:***

*Let* $G = C_n \odot C_m$ *be a corona cycle of order nm+n then*

$\gamma_w(C_n \odot C_m) = \gamma_w(C_m).n$

*Ex: Find the inverse weak domination number of* $(C_4 \odot C_3)$




$\gamma_w(C_4 \odot C_3) = \gamma_w(C_3).n$

*= (1).4*

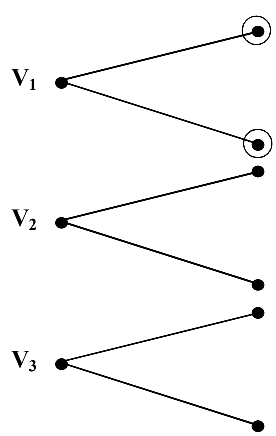
*= 4*

***Theorem 7.3:***

*Let* $G = N_n \odot N_m$ *be a corona null of order nm+n then*

$\gamma_w(N_n \odot N_m) = \gamma_w(N_m).n = m \times n$

*Ex: Find the inverse weak domination number of* $(N_3 \odot N_2)$




$\gamma_w(N_3 \odot N_2) = mn = 2 \times 3 = 6$

## 8. *Inversa strong domination number of the operation Corona*

***Theorem 8.1:***

*Let $G = P_n \odot P_m$ be a graph of order nm+n then G has no inverse strong domination.*

***Theorem 8.2:***

*Let $G = C_n \odot C_m$ be a graph of order nm+n then G no inverse strong domination.*

***Theorem 8.3:***

*Let $G = N_n \odot N_m$ be a graph of order nm+n then G no inverse strong domination.*

### 9.Inversa Weak domination number of the operation Corona

***Theorem 9.1:***

*a corona path strong has inverse if m=2, but has no inverse if m ≥ 3*

***Theorem 9.2:***

*A corona cycle strong has inverse weak domination then:*

$$\gamma_s^{-1}(C_n \odot C_m) = \gamma_s^{-1}(C_n) \,.\, n$$

***Theorem 9.3:***

*Let $G = N_n \odot N_m$ be a graph of order nm+n then G no inverse weak domination.*

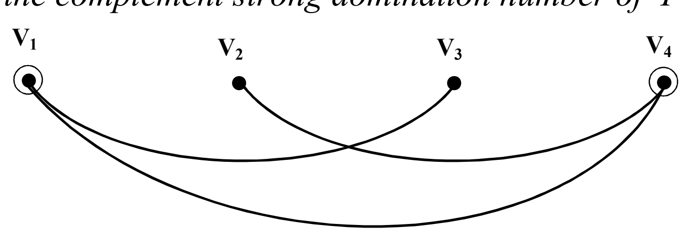
## *10. Complement* of the strong

***Theorem 10.1:***

*Let $\overline{P_n}$ be a complement path of order n, then:*

$$\gamma_s(\overline{P_n}) = \left\{ \begin{array}{c} 2 \; if \; n = 2 \\ has \; no \; strong \; domintion \; if \; n = 3 \\ 2 \; if \; n \geq 4 \end{array} \right\}$$

*Ex: Find the complement strong domination number of $P_4$*



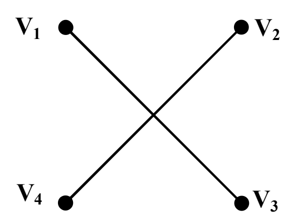
$\gamma_s(\overline{P_n}) = 2$

***Theorem 10.2:***

*Let $\overline{C_n}$ be a complement cycle of order n, then:*

$$\gamma_s(\overline{C_n}) = \begin{Bmatrix} 3 \ if \ n = 3 \\ 2 \ if \ n \geq 4 \end{Bmatrix}$$

*Ex: Find the complement strong domination number of $C_4$*




$\gamma_s(\overline{C_4}) = 2$

***Theorem 10.3:***

*Let $\overline{S_n}$ be a complement star of order n, then:*

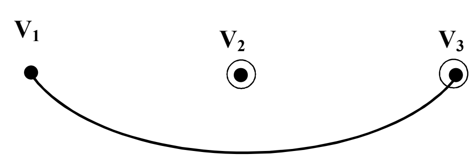
$$\gamma_s(\overline{S_n}) = \begin{Bmatrix} 3 \ if \ n = 3 \\ has \ no \ strongdomination \ number \ if \ n \geq 4 \end{Bmatrix}$$

***11. Complement* of the .** ***Theorem 11.1:***

*Let $\overline{P_n}$ be a complement path of order n, then:*

$$\gamma_w(\overline{P_n}) = \begin{Bmatrix} 2 \ if \ n = 2 \\ 2 \ if \ n \geq 3 \end{Bmatrix}$$

*Ex: Find the complement weak domination number of $P_3$*



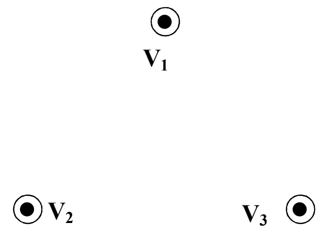

$\gamma_w(\overline{P_3}) = 2$

***Theorem 11.2:***

*Let $\overline{C_n}$ be a complement cycle of order n, then:*

$$\gamma_w(\overline{C_n}) = \begin{Bmatrix} 3 \ if \ n = 3 \\ 2 \ if \ n \geq 4 \end{Bmatrix}$$

*Ex: Find the complement weak domination number of* $C_3$



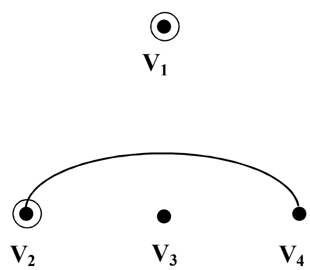

$\gamma_w(\overline{C_3}) = 3$

***Theorem 11.3:***

*Let* $\overline{S_n}$ *be a complement star of order n, then:*

$\gamma_w(\overline{S_n}) = \{2 \; if \; n \geq 3\}$

*Ex: Find the complement weak domination number of* $S_4$



## *12.References* :


1. Gary Chartrand, Linda Lesniak , Ping Zhang (AuthorGraphs & Digraphs (Textbooks in Mathematics) 6th Edition).
2. S. Arumugam and S. Ramachandran, (2015), Invitation to graph theory, Scitech Publ., Kolkata, India .
3. G. S. Singh, (2013). Graph theory, Prentice Hall of India, New Delhi .
4. R. Balakrishnan and K. Ranganathan, (2012). A textbook of graph theory, Springer, New York .
5. J.A. Bondy and U.S.R Murty, (2008). Graph theory, Springer .
6. G. Agnarsson and R. Greenlaw, (2007). Graph theory: Modeling, applications & algorithms, Pearson Education, New Delhi .
7. G. Chartrand and P. Zhang, (2005). Introduction to graph theory, McGraw-Hill Inc .
8. G. Sethuraman, R. Balakrishnan, and R.J. Wilson, (2004). Graph theory and its applications, Narosa Pub. House, New Delhi .
9. Saeid Alikhani, Nima Ghanbari& Hassan Zaherifar,(2002) Strong domination number of some operations on a graph Department of Mathematical Sciences, Yazd University, 89195-741, Yazd, Iran 2Department of Informatics, University of Bergen, P.O. Box 7803, 5020 Bergen, Norway
10. D.B. West, (2001). Introduction to graph theory, Pearson Education Inc., Delhi .
11. V.K. Balakrishnan, (1997). Graph theory, McGrawhill, New York .
12. Akers, S.; Harel, D.; and Krishnamurthy, B,(2019). "The Star Graph: An Attractive Alternative to the n-Cube." In Proc. International Conference of Parallel Processing.
13. G. Chartrand and L. Lesniak, (1996). Graphs and digraphs, CRC Press .
14. J.A. Bondy and U.S.R Murty, (1976). Graph theory with applications, North-Holland, New York.