وزاره التعليم العالي والبحث العلمي

جامعة بابل / كليه التربية للعلوم الصرفة

قسم الرياضيات

Digital Topology

بحث تخرج مقدم الى رئاسة قسم الرياضيات كلية التربية

للعلوم الصرفة كجزء من متطلبات نيل شهادة البكالوريوس

في قسم الرياضيات

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بسم الله الرحمن الرحيم

إنما يخشى الله من عباده العلماء

صدق الله العظيم

الاهداء

إلى صاحب السيرة العطرة، والفكر المُستنير؛

فلقد كان له الفضل الأوَّل في بلوغي التعليم العالي

(والدي الحبيب)، أطال الله في عُمره.

إلى من وضعتني على طريق الحياة، وجعلتني رابط الجأش،

وراعتني حتى صرت كبيرًا

(أمي الغالية)، طيَّب الله ثراها.

إلى إخوتي؛ من كان لهم بالغ الأثر في كثير من العقبات والصعاب.

إلى جميع أساتذتي الكرام؛ ممن لم يتوانوا في مد يد العون لي

أُهدي إليكم بحثي

الشكر والامتنان

الحمد لله رب العالمين والصلاة والسلام على أشرف الأنبياء والمرسلين سيدنا محمَّد وعلى آله وسلم ومن تبعهم بإحسان إلى يوم الدين، وبعد ..
فإني أشكر الله تعالى على فضله حيث أتاح لي إنجاز هذا العمل بفضله، فله الحمد أولاً وآخرًا.

ثم أشكر أولئك الأخيار الذين مدوا لي يدَ المساعدة، خلال هذه الفترة، وفي مقدمتهم أستاذي المشرف على الرسالة فضيلة الأستاذ الدكتور/ حسناء حسن شهيد الذي لم يدَّخر جهدًا في مساعدتي، فقد فتح لي بيته، كما هي عادته مع كل طلبة العلم، وكنت أجلس معه بالساعات الطوال أقرأ عليه ولا يجد في ذلك حرجًا، وكان يحثني على البحث، ويرغِّبني فيه، ويقوّي عزيمتي عليه فله من الله الأجر ومني كل تقدير حفظه الله ومتّعه بالصحة والعافية ونفع بعلومه.

Conclusion

Digital topology studies geometric and topological properties of digital images arising in many areas of science including fluid dynamics, geoscience, neuroscience and medical imaging. In particular, integrating geometric and topological constraints into discretization schemes in order to generate geometrically and topologically correct digital models of anatomical structures is critical for many clinical and research applications. Although many clinical and research applications require accurate segmentations, only a few techniques have been proposed to achieve accurate and geometrically and topologically correct segmentations. Discretization schemes preserving geometry and topology of the object of interest were introduced in These schemes enable to obtain more detailed geometric and topological information about the specific regions of the object. The shape and the size of individual elements can be arbitrary within the framework of the proposed scheme that allows representing objects with fine anatomical details. Another feature is that the digital model of a 2-dimensional continuous object is necessarily a digital 2-surface preserving the topology of the object. The approach studied in this paper is based on locally centered lump (LCL) discretization of n-dimensional objects

الفهرس

Chapter One

definition 1.1 : topology or topological space

Definition 1.2 : open set and close set

Definition 1.3 : continuous topology

Definition 1.4:- Homeomorphism

Definition1.5:- intrinsic topology

Definition1.6 interior point and exterior point

Definition1.7 : locally centered lump

Digital topology

1. introduction

digital topology studies geometric and topological properties of digital images arising in many areas of science including fluid dynamics, geoscience, neuroscience and medical imaging. In particular, integrating geometric and topological constraints into discretization schemes in order to generate geometrically and topologically correct digital models of anatomical structures is critical for many clinical and research .

definition 1.1 : topology or topological space [1]

let X is a non-empty set and T is a family of subsets of X such that T if satisfy the following condition :-

1- X , $\emptyset \in T$

2- if U,V $\in$ T , then U$\cap$ V $\in$ T

(the finite intersection of elements from T is a gain an element of T .

3- If $U_z \in T$:- $Z \in \Lambda$, then $U_{z \in \Lambda} \in T$ $\forall_z \in \Lambda$

Example:- let X = {a,b,c} T_1= { X, Ø ,{a}} T_2={x, Ø,{a,c}} T_3={x, Ø ,{a,b}} T_4={x, Ø,{a},{a,c}} T_5={x,{a},{b},{a,b}}
is T_1,T_2,T_3,T_4,T_5 topology on X ?

Sol :- Note that T_1 and T_2 is topology on X since is satisfy the three conditions of topology

T_3 is not topology on X since {a,b}∩{a,c}={a} $\notin T_3$

T_4 is not topology on X since {a}∪{b}={a,b} $\notin T_4$

T_5 is not topology on X since $\emptyset \notin T_5$

Definition 1.2 : open set and close set [2]

Let (x,T) be a topological space the subsets of X belonging to T are called “ open set” in space X

If A ⊆ X و A∈T= A open set

Let (X ,T) and F be closed set if X / A is open set we will denoted the family of closed set "

If A ⊆X و A ∈ F = A closed set

Definition 1.3 : continuous topology [3]

The concept of continuous arose when defining true coupling in the subject of calculus so we will touch on this subject using this concept

Let R—> R true coupling the basic condition that the coupling in order for it to be continuous at a point

Let it be (a) , is for every X ∈ R must be number f(x) and must be close the number f(a) by an amount proportional to the proximity of the point X from point(a) that is approach of two points f(x) and f(a)

Remark: let R—>R be associated the(f) function (f) is called continuous at point a ∈ R if and only if for every positive real number there ε is real number “σ”

$|f(x) - f (a)|< \varepsilon$ that mean $|x-a|< \sigma$ that called coupling continuity in all points

Definition 1.4:- Homeomorphism [4]

Let f: (x,T_1)—>(y,T_2) function of topological spas (x,T_1) to spas(y,T_2) that called f is homeomorphism or topological function since f satisfy following

1- F one-sided symmetry function
2- F Continuous function
3- X —> Y :F^{-1} Continuous function

At that time, it is said that the two spaces are equivalent (homeomorphism) and this is symbolized by the symbol. From this definition it becomes clear to us that an equivalence relationship can be defined on the family of topological spaces.

Definition1.5:- intrinsic topology[5]

Let Y be the set of isolated points of X, and for each finite subset F of Y, let CF = {/ G G: PC Z(f)}. Now take the ideals of the form Cf as a base for the neighborhood system of the zero element in C(X). Clearly C with this topology is a topological ring (not necessarily Hausdor fT). Since every Cp is a finite intersection of isolated maximal ideals, this topology coincides with the intrinsic topology introduced by Goldman [6] for arbitrary rings. From now on we call it the In-topology on C(X). We denote the intersection of the isolated maximal ideals by Jo. So Jo = G if Y — 0. The next result relates the density of the isolated points in X to some algebraic properties of C(X).

Definition1.6 interior point and exterior point [6]

let A be a subset of space (X,T) the point $p \in A$ is called an **(interior point)** of set A if and only if there is an open set $U \in T$ such that $P \in U \subseteq A$

It is denoted by a A^{o}

$P \in A^{o} \Longleftrightarrow \exists\, U \in T, P \in U$ S.t $P \in U \subseteq A$

Let A be a subset of space(X,T) a point ($P \in X$) is called an

external point of set A if and only if there is an open set $U \in T$

It is denoted by A^{C}

$P \in U \subseteq A^{C} = X - A$ And read externally. The sum of all the external points of set A

$P \in A^{C} \Longleftrightarrow \exists\, U \in T, P \in U$ S.t $P \in U \subseteq A^{C}$

Definition1.7 : locally centered lump

Let $W=\{D_1,D_2,\ldots\}$ be a collection of n-tiles, n=1, 2.

- W is called a locally centered collection (LC collection) if from condition $Di(k) \cap Di(m) \neq \emptyset$, $m \neq k$, m, k=1,2,...s, it follows that $Di(1) \cap Di(2) \cap \ldots Di(s) \neq \emptyset$.

- W is called a locally lump collection (LL collection) if any nonempty intersection of s distinct n-tiles is an (n-s+1)-tile: $Di(1) \cap Di(2) \cap \ldots Di(s) = \partial Di(1) \cap \partial Di(2) \cap \ldots \partial Di(s) = Dn\text{-}s\text{+}1$.

- W is called a locally centered lump collection (LCL collection) if W is a locally centered collection and a locally lump collection at the same time

Chapter Two

Definition : 2.1

•if C is a circle and D is a 1-cell contained in C then C-IntD is a 1-cell.

• Let C1 and C2 be 1-cells such that $C1\cap C2=\partial C1\cap\partial C2=v$ is an endpoint of C1 and C2. Then $C1\cup C2=E$ is a 1-cell.

• Let D1 and D2 be 2-cells such that $D1\cap D2=\partial D1\cap\partial D2=C$ is a 1-cell. Then $D1\cup D2=B$ is a 2- cell

Definition 2.2

Let W={X1,…Xs} be a collection of sets. W is called a locally centered collection (LC collection) if for all m,k Q , from condition $Xk\cap Xm\neq\emptyset$, $m\neq k$, it follows that Xk : k Q}$\neq\emptyset$. In topology, this property is often called Helly's property. In application to digital topology, collections of sets with similar properties were studied in a number of works. Definition 2.2 1 . Let W={C1,C2,… } be a collection of 1-cells. W is called a locally lump collection (LL collection) if: a) From condition $Ck\cap Cm \neq\emptyset$, $m\neq k$, it follows that $Ck\cap Cm=\partial Ck\cap\partial Cm=v$ is a point. b) The intersection of any three distinct 1-cells is empty. 2 . W is called a locally centered lump collection (LCL collection) if W is a locally centered collection and a locally lump collection at the same time (fig. 1)

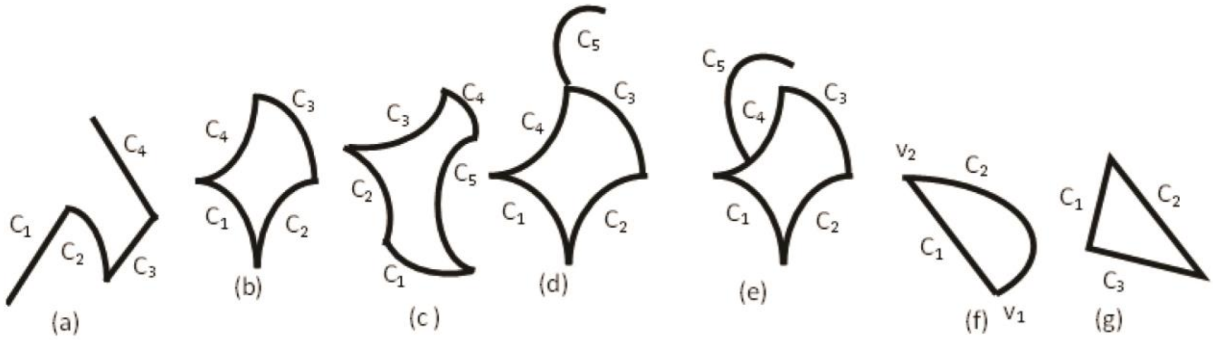

Figure 1. Collections of 1-cells. (a)-(c) are LCL collections. (d)-(f) are not LCL collections.

. Collections of 1-cells are shown in fig. 1. Collections (a)-(c) are LCL collections. (d) is not an LCL collection because $C_3 \cap C_4 \cap C_5 \neq \emptyset$. (e) is not an LL collection because $C_4 \cap C_5 \neq \partial C_4 \cap \partial C_5$. (f) is not an LL collection because $C_1 \cap C_2 = \{v_1, v_2\}$. (g) is not an LCL collection because $C_1 \cap C_2 \cap C_3 = \emptyset$. Clearly, any subcollection of an LCL collection of 1- cells is an LCL collection.

Definition 2.3

Let $W=\{D_1,D_2,\dots\}$ be a collection of 2-cells. W is called a locally lump collection (LL collection) if: a) From condition $D_k \cap D_m \neq \emptyset$, $m \neq k$, it follows that $D_k \cap D_m = \partial D_k \cap \partial D_m = Ck_m$ is a 1-cell. b) From condition $D_k \cap D_m \cap D_p \neq \emptyset$, $m \neq k$, $m \neq p$, $p \neq k$, it follows that $D_k \cap D_m \cap D_p = \partial D_k \cap \partial D_m \cap \partial D_p = Ck_m \cap Ck_p = \partial Ck_m \cap \partial Ck_p = v$ is a point. c) The intersection of any four distinct 2-cells is empty. 2 . W is called a locally centered lump collection (LCL collection) if W is a locally centered collection and a locally lump collection at the same time (fig. 2). LCL collections of 1- and 2-cells were defined and studied. Collections of 2-cells are depicted in fig. 2. Collection (a) is not an LCL collection, because $D_1 \cap D_3$ and

$D_2 \cap D_4$ are not 1-cells. Collection (b) is not an LCL collection because $D_1 \cap D_2 \neq \varnothing$, $D_1 \cap D_3 \neq \varnothing$, $D_2 \cap D_3 \neq \varnothing$ but $D_1 \cap D_2 \cap D_3 = \varnothing$. Collections (c)-(e) are LCL collections of 2-cells. Obviously, any subcollection of an LCL collection of 2-cells is an LCL collection. In paper , a locally centered collection is called continuous and it is shown that for a given object, the intersection graphs of all continuous, regular and contractible covers are homotopy equivalent to each other, a normal set W of convex nongenerate polygons (intersection of any two of them is an edge, a vertex, or empty) is called strongly normal (SN) if for all P, $P_1, \ldots P_n$ $(n>0) \in W$, if each Pi intersects P and $I = P_1 \cap \ldots \cap P_n$ is nonempty, then I intersects, extended basic results about strong normality to collections of polyhedra in Rn . There are obvious differences between SN collections of polygons and LCL collections. For example, elements of an SN collection are convex sets whereas any 2-cell in an LCL collection can be of an arbitrary shape and size. Let us remind the definition of isomorphic sets. A collection $W=\{A_0,A_1,\ldots\}$ of sets is isomorphic to a collection $V=\{B_0,B_1,\ldots\}$ of sets, if there exists one-one onto correspondence $f: W \rightarrow V$ such that $Ai \cap \ldots Ak \cap \ldots Ap \neq \varnothing$ if and only if $f(Ai) \cap \ldots f(A_k) \cap \ldots f(A_p) \neq \varnothing$. The following assertion was proven in =

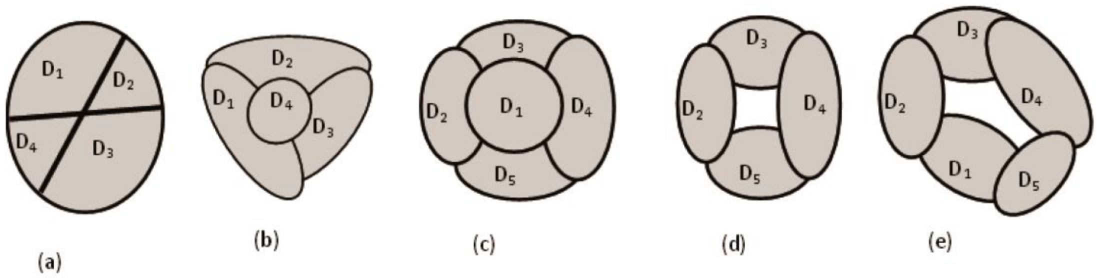

Figure. 2. (a)-(b) are not LCL collections. (c)-(e) are LCL collections of 2-cells.

Proposition :

Let $W=\{D_0,D_1,\ldots\}$ be an LCL collection of 2-cells, $U=\{D_1,D_2,\ldots D_s\}$ be a collection of all 2-cells intersecting D0, and $V=\{C_1,C_2,\ldots C_s\}$ be the collection of 1-cells $C_i=D_0\cap D_i\neq\emptyset$, i=1,…s. Then: • V is an LCL collection. • Collections U and V are isomorphic, and f: $U\to V$, $f(D_i)=C_i$, i=1,…s, is an isomorphism

Definition 2.4

Let M be a circle, $W(M)=\{C_0,C_1,C_2,...,C_n\}$ be an LCL cover of $M=C_0 \cup ...C_n$ by 1-cells, and $G(W(M))=\{x_0,x_1,x_2,...,x_n\}$ be the intersection graph of W(M). Then G(W(M)) is called the digital model of M in regard to W(M). In fig 7, W_1, W_2, W_3 are LCL covers of a circle C, G_1, G_2, G_3 are the corresponding digital models of C. Obviously, G_1, G_2, G_3 are digital 1-spheres. The following theorem was proven in

Theorem 2.4.1

Let M be a circle, and $W(M)=\{C_0,C_1,...,C_n\}$ be an LCL cover of M by 1-cells. Then the digital model G(W(M)) of M is a digital 1-sphere. Remark 1 Suppose that D is a 2-cell in Euclidean space E_2 . Divide E_2 into a set $V=\{u_1,u_2,\ldots\}$ of closed squares with edge length L, and vertex coordinates equal to nL. V is a cover of E_2 . Pick out the family $V(D)=\{u_1,\ldots us\}$ of squares intersecting D. V(D) is a cover of D. Then build the intersection graph G(V(D)) of V(D). Since D is a topological closed 2-disk then for small enough L, the union E=u1 …us is a 2-cell, and the intersection graph G(V(D))={x1,…xs} of V(D) is contractible.

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