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Fibonacci sequence with n roots of one and negative one

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الآية

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

(وَلَقَدْ كَرَّمْنَا بَنِي آدَمَ وَحَمَلْنَاهُمْ فِي الْبَرِّ وَالْبَحْرِ وَرَزَقْنَاهُمْ مِنَ الطَّيِّبَاتِ وَفَضَّلْنَاهُمْ عَلَىٰ كَثِيرٍ مِمَّنْ خَلَقْنَا تَفْضِيلًا ﴿٧٠﴾)

صدق الله العلي العظيم

سورة الاسراء اية 70

<u>الاهداء</u>

الى من بلغ الرسالة وأدى الأمانة... ونصح الأمة

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<u>الشكر والتقدير</u>

الحمد والشكر لله أولاً وأخراً ...

أقدم شكري وأمتناني الى جميع من أعانوني وساعدوني في اخراج هذا البحث بفضلهم وجهدهم على الآراء القيمة التي أبدوها لي وخصوصاً مشرف البحث والى الهيئة التدريسية في القسم عموماً،

وراجياً من الله ان أكون قد أصبت أكثر مما أخطأت وان يستفاد مما بذلت من جهود، آملاً أن أكون قد أعطيت الموضوع بعض حقه، وأسأل الله أن يعلمنا ما ينفعنا، وينفعنا بما علمنا.

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Contents

Introduction

Leonardo of Pisa, commonly known as Fibonacci, was an Italian mathematician born in the 12th century. He introduced the Western world to the Hindu-Arabic numeral system and popularized the use of Arabic numerals in Europe through his book "Liber Abaci" in 1202.

One of the most famous concepts Fibonacci introduced is the Fibonacci sequence, which starts with 0 and 1, and each subsequent number is the sum of the two preceding ones. So, the sequence goes: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, and so on.

Lucas numbers, named after the French mathematician Édouard Lucas, are a similar sequence to Fibonacci numbers. They start with 2 and 1 instead of 0 and 1, and each subsequent number is the sum of the two preceding ones. So, the Lucas sequence goes: 2, 1, 3, 4, 7, 11, 18, 29, 47, and so on.

Both the Fibonacci sequence and Lucas numbers have fascinating mathematical properties and appear in various natural phenomena, such as the arrangement of leaves on a stem, branching in trees, and the spiral patterns of shells and flowers. They also have practical applications in fields like computer science, number theory, and finance.

Chapter one

Fibonacci sequence with a root of negative one

(1.1) introduction

The famous Fibonacci sequence, whose limits are expressed that each term is the addition of the two previous terms and is according to the following formula:
$F_{n+1} = F_n + F_{n-1}$

It is as follows: {0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, ...}

(1.2) Fibonacci sequence reputation with i

In an attempt to relate the Fibonacci sequence with the complex numbers, each Fibonacci number was raised to the number i as follows:

$$\mathrm{i}^0 = 1, \quad \mathrm{i}^1 = \mathrm{i}, \quad \mathrm{i}^1 = \mathrm{i}, \quad \mathrm{i}^2 = -1, \quad \mathrm{i}^3 = -\mathrm{i}, \quad \mathrm{i}^5 = \mathrm{i}$$

This is the first six terms in a sequence, and when we advance and calculate the rest of the terms in the same way, they will be as follows:

$$\mathrm{i}^8 = 1, \quad \mathrm{i}^{13} = \mathrm{i}, \quad \mathrm{i}^{21} = \mathrm{i}, \quad \mathrm{i}^{34} = -1, \quad \mathrm{i}^{55} = -\mathrm{i}, \quad \mathrm{i}^{89} = \mathrm{i}$$

We will notice that there is a repetition in the outputs and that the first six terms in the sequence are equal to their values when raising the following six terms and that the seventh term when raised to the i is equal to the first term when it is raised to the i, as well as when continuing and calculating the rest of the terms of the sequence are as follows:

$$\mathrm{i}^{144} = 1, \quad \mathrm{i}^{233} = \mathrm{i}, \quad \mathrm{i}^{377} = \mathrm{i}, \quad \mathrm{i}^{610} = -1, \quad \mathrm{i}^{987} = -\mathrm{i}, \quad \mathrm{i}^{1597} = \mathrm{i}$$

That is, there will be a repetition of the outputs for every six terms according to the

	فيبوناتشي	i		فيبوناتشي	i		فيبوناتشي	i
0	0	1	24	46368	1	48	4807526976	1
1	1	i	25	75025	i	49	7778742049	i
2	1	i	26	121393	i	50	12586269025	i
3	2	-1	27	196418	-1	51	20365011074	-1
4	3	i-	28	317811	i-	52	32951280099	i-
5	5	i	29	514229	i	53	53316291173	i
6	8	1	30	832040	1	54	86267571272	1
7	13	i	31	1346269	i	55	139583862445	i
8	21	i	32	2178309	i	56	225851433717	i
9	34	-1	33	3524578	-1	57	365435296162	-1
10	55	i-	34	5702887	i-	58	591286729879	i-
11	89	i	35	9227465	i	59	956722026041	i
12	144	1	36	14930352	1	60	1548008755920	1
13	233	i	37	24157817	i	61	2504730781961	i
14	377	i	38	39088169	i	62	4052739537881	i
15	610	-1	39	63245986	-1	63	6557470319842	-1
16	987	i-	40	102334155	i-	64	10610209857723	i-
17	1597	i	41	165580141	i	65	17167680177565	i
18	2584	1	42	267914296	1	66	27777890035288	1
19	4181	i	43	433494437	i	67	44945570212853	i
20	6765	i	44	701408733	i	68	72723460248141	i
21	10946	-1	45	1134903170	-1	69	117669030460994	-1
22	17711	i-	46	1836311903	i-	70	190392490709135	i-
23	28657	i	47	2971215073	i	71	308061521170129	i

following table:

table (1.1)

We see that the results are equal at **mod (6)** for the terms and not for their values.

(1.3) sequences like Fibonacci with root negative one

The Fibonacci sequence, which begins with the first term 0 and the second term 1, the similar sequences that work on the same principle but with the difference of the first term and the second term.

Like Lucas numbers, the first term which is 2 and the second term is 1.

It is as follows: {2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, ...}

We will work in the same way as the Fibonacci setting when raised the Lucas numbers for i , it is as follows:

$$\mathrm{i}^2 = -1 \quad \mathrm{i}^1 = \mathrm{i}, \quad \mathrm{i}^3 = -\mathrm{i}, \quad \mathrm{i}^4 = 1, \quad \mathrm{i}^7 = -\mathrm{i}, \quad \mathrm{i}^{11} = -\mathrm{i}$$

These are the first six terms in Lucas number, and when we advance and calculate the rest of the terms in the same way, they will be as follows:

$$i^{18} = -1 \quad i^{29} = i, \quad i^{47} = -i, \quad i^{76} = 1, \quad i^{123} = -i, \quad i^{199} = -i$$

We will notice that there is a repetition in the results and that the first six terms in Lucas' numbers are equal to their values when raising the following six terms and that the seventh term when raising the power of is equal to the first term when raising the power of, as well as when continuing and calculating the rest of the terms:

$$i^{322} = -1 \quad i^{521} = i, \quad i^{843} = -i, \quad i^{1364} = 1, \quad i^{2207} = -i, \quad i^{3571} = -i$$

That is, there will be a repetition of the outputs for every six terms according to the following table:

	اعداد لوكاس	i		اعداد لوكاس	i		اعداد لوكاس	i
0	2	-1	24	103682	-1	48	10749957122	-1
1	1	i	25	167761	i	49	17393796001	i
2	3	i-	26	271443	i-	50	17393796001	i-
3	4	1	27	439204	1	51	34787592002	1
4	7	i-	28	710647	i-	52	52181388003	i-
5	11	i-	29	1149851	i-	53	86968980005	i-
6	18	-1	30	1860498	-1	54	139150368008	-1
7	29	i	31	3010349	i	55	226119348013	i
8	47	i-	32	4870847	i-	56	365269716021	i-
9	76	1	33	7881196	1	57	591389064034	1
10	123	i-	34	12752043	i-	58	956658780055	i-
11	199	i-	35	20633239	i-	59	1548047844089	i-
12	322	-1	36	33385282	-1	60	2504706624144	-1
13	521	i	37	54018521	i	61	4052754468233	i
14	843	i-	38	87403803	i-	62	6557461092377	i-
15	1364	1	39	141422324	1	63	10610215560610	1
16	2207	i-	40	228826127	i-	64	17167676652987	i-
17	3571	i-	41	370248451	i-	65	27777892213597	i-
18	5778	-1	42	599074578	-1	66	44945568866584	-1
19	9349	i	43	969323029	i	67	72723461080181	i
20	15127	i-	44	1568397607	i-	68	117669029946765	i-
21	24476	1	45	2537720636	1	69	190392491026946	1
22	39603	i-	46	4106118243	i-	70	308061520973711	i-
23	64079	i-	47	6643838879	i-	71	498454012000657	i-

table (1.2)

We see that the results are equal at **mod (6)** *for the terms and not for their values.*

(1.4) Generalization of the Fibonacci pattern to the rest of the numbers

As mentioned above, the Fibonacci sequence is the addition of two terms to give the previous term, but what if random numbers are taken to be the first and second terms:

For example: the first term is 4589 and the second term is 6723, so it is as follows:

	عشوائي	i		عشوائي	i		عشوائي	i
0	4589	i	18	24700865	i	36	142721593381	i
1	6723	i-	19	39966839	i-	37	230928389019	i-
2	11312	1	20	64667704	1	38	373649982400	1
3	18035	i-	21	104634543	i-	39	604578371419	i-
4	29347	i-	22	169302247	i-	40	978228353819	i-
5	47382	-1	23	273936790	-1	41	1582806725238	-1
6	76729	i	24	443239037	i	42	2561035079057	i
7	124111	i-	25	717175827	i-	43	4143841804295	i-
8	200840	1	26	1160414864	1	44	6704876883352	1
9	324951	i-	27	1877590691	i-	45	10848718687647	i-
10	525791	i-	28	3038005555	i-	46	17553595570999	i-
11	850742	-1	29	4915596246	-1	47	28402314258646	-1
12	1376533	i	30	7953601801	i	48	45955909829645	i
13	2227275	i-	31	12869198047	i-	49	74358224088291	i-
14	3603808	1	32	20822799848	1	50	120314133917936	1
15	5831083	i-	33	33691997895	i-	51	194672358006227	i-
16	9434891	i-	34	54514797743	i-	52	314986491924163	i-
17	15265974	-1	35	88206795638	-1	53	509658849930390	-1

table (1.3)

The frequency is also for every six terms, and the outcomes are equal at **mod (6)** *for the terms and not for their values.*

Chapter Two:

Generalization of Fibonacci sequences on nth roots

(2.1) introduction

In the first chapter, the results of raising the numbers of the Fibonacci sequence were calculated on the number root of negative one only, and now the Fibonacci and Lucas sequence and random numbers will be calculated to be powers of the rest of the roots of the one integer and negative one

(2.2) Square roots of the integer one

The square roots of the integer one is {1, -1}

When we raise the sequences to the one it will be a trivial case

And when raised to negative one, it will generate a repetition of every three terms. According to the following table:

	فيبوناتشي.	1	-1		
0	0	1	1		0
1	1	1	-1		1
2	1	1	-1		2
3	2	1	1		3
4	3	1	-1		4
5	5	1	-1		5
6	8	1	1		6
7	13	1	-1		7
8	21	1	-1		8
9	34	1	1		9
10	55	1	-1		10
11	89	1	-1		11
12	144	1	1		12
13	233	1	-1		13
14	377	1	-1		14
15	610	1	1		15
16	987	1	-1		16
17	1597	1	-1		17

table (2.1)

For negative one, the results will be equal at **mod (3)** for terms and not for values.

(2.3) Square roots of negative one

The square roots of negative one is $\{i, -i\}$

When we raise the sequences to the roots of negative one it will generate a repetition of every six terms. According to the following table:

	فيبوناتشي.	i	i-			
0	0	1	1		0	
1	1	i	i-		1	
2	1	i	i-		2	
3	2	-1	-1		3	
4	3	i-	i-		4	
5	5	i	i-		5	
6	8	1	1		6	
7	13	i	i-		7	
8	21	i	i-		8	
9	34	-1	-1		9	
10	55	i-	i-		10	
11	89	i	i-		11	
12	144	1	1		12	
13	233	i	i-		13	
14	377	i	i-		14	
15	610	-1	-1		15	
16	987	i-	i-		16	
17	1597	i	i-		17	

table (2.2)

The results of the numbers will be $\{\boldsymbol{i}, -\boldsymbol{i}\}$ equal at **mod (6)** for the terms and not for the values.

(2.4) Cube roots of the integer one

The cube roots of the whole one is $\{1,\ \omega,\ \omega^2\}$

When we raise the sequences to the one it will be a trivial case

And when you raise the sequences to $\{\omega,\ \omega^2\}$ it will generate a repetition for every eight terms. According to the following table:

	فيبوناتشي	1	w	w^2
0	0	1	1	1
1	1	1	w	w^2
2	1	1	w	w^2
3	2	1	w^2	w
4	3	1	1	1
5	5	1	w^2	w
6	8	1	w^2	w
7	13	1	w	w^2
8	21	1	1	1
9	34	1	w	w^2
10	55	1	w	w^2
11	89	1	w^2	w
12	144	1	1	1
13	233	1	w^2	w
14	377	1	w^2	w
15	610	1	w	w^2
16	987	1	1	1
17	1597	1	w	w^2
18	2584	1	w	w^2
19	4181	1	w^2	w
20	6765	1	1	1
21	10946	1	w^2	w
22	17711	1	w^2	w
23	28657	1	w	w^2

	لوكاس	1	w	w^2
0	2	1	w^2	w
1	1	1	w	w^2
2	3	1	1	1
3	4	1	w	w^2
4	7	1	w	w^2
5	11	1	w^2	w
6	18	1	1	w^2
7	29	1	w^2	w
8	47	1	w^2	w
9	76	1	w	w^2
10	123	1	1	1
11	199	1	w	w^2
12	322	1	w	w^2
13	521	1	w^2	w
14	843	1	1	w^2
15	1364	1	w^2	w
16	2207	1	w^2	w
17	3571	1	w	w^2
18	5778	1	1	1
19	9349	1	w	w^2
20	15127	1	w	w^2
21	24476	1	w^2	w
22	39603	1	1	w^2
23	64079	1	w^2	w

	عشوائي	1	w	w^2
0	4589	1	w^2	w
1	6723	1	1	1
2	11312	1	w^2	w
3	18035	1	w^2	w
4	29347	1	w	w^2
5	47382	1	1	1
6	76729	1	w	w^2
7	124111	1	w	w^2
8	200840	1	w^2	w
9	324951	1	1	1
10	525791	1	w^2	w
11	850742	1	w^2	w
12	1376533	1	w	w^2
13	2227275	1	1	1
14	3603808	1	w	w^2
15	5831083	1	w	w^2
16	9434891	1	w^2	w
17	15265974	1	1	1
18	24700865	1	w^2	w
19	39966839	1	w^2	w
20	64667704	1	w	w^2
21	104634543	1	1	1
22	169302247	1	w	w^2
23	273936790	1	w	w^2

table (2.3)

The results of the numbers $\{\boldsymbol{\omega},\ \boldsymbol{\omega}^{2}\}$ will be equal at **mod (8)** for the terms and not for the values.

(2.5) Cube roots of negative one

The cube roots of negative one is $\{-1, -\omega,\ -\omega^2\}$

When we raise the sequences to negative one, will generate a repetition of every three terms. And when raise the sequences to$\{-\omega, -\omega^2\}$ it will generate a repetition for every twenty-four terms According to the following table:

	فيبوناتشي	-1	w-	w^2-		لوكاس	-1	w-	w^2-		عشوائي	-1	w-	w^2-
0	0	1	1	1	0	2	1	w^2	w	0	4589	-1	w^2-	w-
1	1	-1	w-	w^2-	1	1	-1	w-	w^2-	1	6723	-1	-1	-1
2	1	-1	w-	w^2-	2	3	-1	-1	-1	2	11312	1	w^2	w
3	2	1	w^2	w	3	4	1	w	w^2	3	18035	-1	w^2-	w-
4	3	-1	-1	-1	4	7	-1	w-	w^2-	4	29347	-1	w-	w^2-
5	5	-1	w^2-	w-	5	11	-1	w^2-	w-	5	47382	1	1	1
6	8	1	w^2	w	6	18	1	1	1	6	76729	-1	w-	w^2-
7	13	-1	w-	w^2-	7	29	-1	w^2-	w-	7	124111	-1	w-	w^2-
8	21	-1	-1	-1	8	47	-1	w^2-	w-	8	200840	1	w^2	w
9	34	1	w	w^2	9	76	1	w	w^2	9	324951	-1	-1	-1
10	55	-1	w-	w^2-	10	123	-1	-1	-1	10	525791	-1	w^2-	w-
11	89	-1	w^2-	w-	11	199	-1	w-	w^2-	11	850742	1	w^2	w
12	144	1	1	1	12	322	1	w	w^2	12	1376533	-1	w-	w^2-
13	233	-1	w^2-	w-	13	521	-1	w^2-	w-	13	2227275	-1	-1	-1
14	377	-1	w^2-	w-	14	843	-1	-1	-1	14	3603808	1	w	w^2
15	610	1	w	w^2	15	1364	1	w^2	w	15	5831083	-1	w-	w^2-
16	987	-1	-1	-1	16	2207	-1	w^2-	w-	16	9434891	-1	w^2-	w-
17	1597	-1	w-	w^2-	17	3571	-1	w-	w^2-	17	15265974	1	1	1
18	2584	1	w	w^2	18	5778	1	1	1	18	24700865	-1	w^2-	w-
19	4181	-1	w^2-	w-	19	9349	-1	w-	w^2-	19	39966839	-1	w^2-	w-
20	6765	-1	-1	-1	20	15127	-1	w-	w^2-	20	64667704	1	w	w^2
21	10946	1	w^2	w	21	24476	1	w^2	w	21	104634543	-1	-1	-1
22	17711	-1	w^2-	w-	22	39603	-1	-1	-1	22	169302247	-1	w-	w^2-
23	28657	-1	w-	w^2-	23	64079	-1	w^2-	w-	23	273936790	1	w	w^2
24	46368	1	1	1	24	103682	1	w^2	w	24	443239037	-1	w^2-	w-
25	75025	-1	w-	w^2-	25	167761	-1	w-	w^2-	25	717175827	-1	-1	-1
26	121393	-1	w-	w^2-	26	271443	-1	-1	-1	26	1160414864	1	w^2	w
27	196418	1	w^2	w	27	439204	1	w	w^2	27	1877590691	-1	w^2-	w-
28	317811	-1	-1	-1	28	710647	-1	w-	w^2-	28	3038005555	-1	w-	w^2-
29	514229	-1	w^2-	w-	29	1149851	-1	w^2-	w-	29	4915596246	1	1	1
30	832040	1	w^2	w	30	1860498	1	1	1	30	7953601801	-1	w-	w^2-
31	1346269	-1	w-	w^2-	31	3010349	-1	w^2-	w-	31	12869198047	-1	w-	w^2-
32	2178309	-1	-1	-1	32	4870847	-1	w^2-	w-	32	20822799848	1	w^2	w
33	3524578	1	w	w^2	33	7881196	1	w	w^2	33	33691997895	-1	-1	-1
34	5702887	-1	w-	w^2-	34	12752043	-1	-1	-1	34	54514797743	-1	w^2-	w-
35	9227465	-1	w^2-	w-	35	20633239	-1	w-	w^2-	35	88206795638	1	w^2	w
36	14930352	1	1	1	36	33385282	1	w	w^2	36	142721593381	-1	w-	w^2-
37	24157817	-1	w^2-	w-	37	54018521	-1	w^2-	w-	37	230928389019	-1	-1	-1
38	39088169	-1	w^2-	w-	38	87403803	-1	-1	-1	38	373649982400	1	w	w^2
39	63245986	1	w	w^2	39	141422324	1	w^2	w	39	604578371419	-1	w-	w^2-
40	102334155	-1	-1	-1	40	228826127	-1	w^2-	w-	40	978228353819	-1	w^2-	w-
41	165580141	-1	w-	w^2-	41	370248451	-1	w-	w^2-	41	1582806725238	1	1	1
42	267914296	1	w	w^2	42	599074578	1	1	1	42	2561035079057	-1	w^2-	w-
43	433494437	-1	w^2-	w-	43	969323029	-1	w-	w^2-	43	4143841804295	-1	w^2-	w-
44	701408733	-1	-1	-1	44	1568397607	-1	w-	w^2-	44	6704876883352	1	w	w^2
45	1134903170	1	w^2	w	45	2537720636	1	w^2	w	45	10848718687647	-1	-1	-1
46	1836311903	-1	w^2-	w-	46	4106118243	-1	-1	-1	46	17553595570999	-1	w-	w^2-
47	2971215073	-1	w-	w^2-	47	6643838879	-1	w^2-	w-	47	28402314258646	1	w	w^2

table (2.4)

The results of the numbers $\{-\omega,\ -\omega^2\}$ will be equal at **mod (24)** for the terms and not for the values

Chapter Three

find the value of any term of sequences if *raised to the number i*

(3.1) introduction

Knowing the products of the values of small terms from the Fibonacci sequence or Lucas or any similar sequence is simple, but when reaching higher limits, for example, the 70th limit of the Fibonacci sequence, the calculation of the value of the limit is simple, but using computer programs and when reaching higher limits, for example, the 322nd limit of the Fibonacci sequence, it is difficult even with the use of computer programs.

So, we use the concept of class and mod

(3.2.1) Square roots of one

Fibonacci number

- For 1 there exist one class which is $[1]$
- For -1 there exist two classes which is $[1], [-1]$

$$0 + 3K \in [1] \qquad \begin{pmatrix} 1 + 3K \\ 2 + 3K \end{pmatrix} \in [-1]$$

Locas number

- For 1 there exist one class which is $[1]$
- For -1 there exist two classes which is $[1], [-1]$

$$0 + 3K \in [1] \qquad \begin{pmatrix} 1 + 3K \\ 2 + 3K \end{pmatrix} \in [-1]$$

(3.2.2) Square roots of negative one

Fibonacci number

❖ For i there exist four classes which is $[i], [-i], [1], [-1]$

$$0 + 6K \in [1] \qquad\qquad 3 + 6K \in [-1]$$

$$\begin{pmatrix} 1 + 6K \\ 2 + 6K \\ 5 + 6K \end{pmatrix} \in [i] \qquad\qquad 4 + 6k \in [-i]$$

❖ For $-i$ there exist four classes which is $[i], [-i], [1], [-1]$

$$0 + 6K \in [1] \qquad\qquad 3 + 6K \in [-1]$$

$$\begin{pmatrix} 1 + 6K \\ 2 + 6K \\ 5 + 6K \end{pmatrix} \in [-i] \qquad\qquad 4 + 6k \in [i]$$

Locas number

❖ For i there exist four classes which is $[i], [-i], [1], [-1]$

$$3 + 6K \in [1] \qquad\qquad 0 + 6K \in [-1]$$

$$\begin{pmatrix} 2 + 6K \\ 4 + 6K \\ 5 + 6K \end{pmatrix} \in [-i] \qquad\qquad 1 + 6k \in [i]$$

❖ For $-i$ there exist four classes which is $[i], [-i], [1], [-1]$

$$3 + 6K \in [1] \qquad\qquad 0 + 6K \in [-1]$$

$$\begin{pmatrix} 2 + 6K \\ 4 + 6K \\ 5 + 6K \end{pmatrix} \in [i] \qquad\qquad 1 + 6k \in [-i]$$

(3.2.2) Cube roots of the integer one

Fibonacci number

- ❖ For 1 there exist one class which is $[1]$

- ❖ for ω there exist three classes :$[\omega^2], [\omega], [1]$

$$\begin{pmatrix} 0+8K \\ 4+8K \end{pmatrix} \in [1]$$

$$\begin{pmatrix} 1+8K \\ 2+8K \\ 7+8K \end{pmatrix} \in [\omega] \qquad \begin{pmatrix} 3+8K \\ 5+8K \\ 6+8K \end{pmatrix} \in [\omega^2]$$

- ❖ for ω^2 there exist three classes :$[\omega^2], [\omega], [1]$

$$\begin{pmatrix} 0+8K \\ 4+8K \end{pmatrix} \in [1]$$

$$\begin{pmatrix} 3+8K \\ 5+8K \\ 6+8K \end{pmatrix} \in [\omega] \qquad \begin{pmatrix} 1+8K \\ 2+8K \\ 7+8K \end{pmatrix} \in [\omega^2]$$

Locas number

- ❖ For 1 there exist one class which is $[1]$

- ❖ for ω there exist three classes :$[\omega^2], [\omega], [1]$

$$\begin{pmatrix} 2+8K \\ 6+8K \end{pmatrix} \in [1]$$

$$\begin{pmatrix} 1+8K \\ 3+8K \\ 4+8K \end{pmatrix} \in [\omega] \qquad \begin{pmatrix} 0+8K \\ 5+8K \\ 7+8K \end{pmatrix} \in [\omega^2]$$

❖ for ω^2 there exist three classes :$[\omega^2], [\omega], [1]$

$$\begin{pmatrix} 2 + 8K \\ 6 + 8K \end{pmatrix} \in [1]$$

$$\begin{pmatrix} 0 + 8K \\ 5 + 8K \\ 7 + 8K \end{pmatrix} \in [\omega] \qquad \begin{pmatrix} 1 + 8K \\ 3 + 8K \\ 4 + 8K \end{pmatrix} \in [\omega^2]$$

(3.2.3) Cube roots of negative one

Fibonacci number

❖ For -1 there exist two classes which is $[1], [-1]$

$$\begin{pmatrix} 0 + 6K \\ 3 + 6K \end{pmatrix} \in [1] \qquad \begin{pmatrix} 1 + 6K \\ 2 + 6K \\ 4 + 6K \\ 5 + 6K \end{pmatrix} \in [-1]$$

❖ for $-\omega$ there exist six classes :$[-\omega^2], [\omega^2], [-\omega], [\omega], [-1], [1]$

$$\begin{pmatrix} 0 + 24K \\ 12 + 24K \end{pmatrix} \in [1] \qquad \begin{pmatrix} 9 + 24K \\ 15 + 24K \\ 18 + 24K \end{pmatrix} \in [\omega]$$

$$\begin{pmatrix} 3 + 24K \\ 6 + 24K \\ 21 + 24K \end{pmatrix} \in [\omega^2] \qquad \begin{pmatrix} 4 + 24K \\ 8 + 24K \\ 16 + 24K \\ 20 + 24K \end{pmatrix} \in [-1]$$

$$\begin{pmatrix} 1+24K \\ 2+24K \\ 7+24K \\ 10+24K \\ 17+24K \\ 23+24K \end{pmatrix} \in [-\omega] \qquad \begin{pmatrix} 5+24K \\ 11+24K \\ 13+24K \\ 14+24K \\ 19+24K \\ 22+24K \end{pmatrix} \in [-\omega^2]$$

❖ for $-\omega^2$ there exist sic classes :$[-\omega^2], [\omega^2], [-\omega], [\omega], [-1], [1]$

$$\begin{pmatrix} 0+24K \\ 12+24K \end{pmatrix} \in [1] \qquad \begin{pmatrix} 3+24K \\ 6+24K \\ 21+24K \end{pmatrix} \in [\omega]$$

$$\begin{pmatrix} 9+24K \\ 15+24K \\ 18+24K \end{pmatrix} \in [\omega^2] \qquad \begin{pmatrix} 4+24K \\ 8+24K \\ 16+24K \\ 20+24K \end{pmatrix} \in [-1]$$

$$\begin{pmatrix} 5+24K \\ 11+24K \\ 13+24K \\ 14+24K \\ 19+24K \\ 22+24K \end{pmatrix} \in [-\omega] \qquad \begin{pmatrix} 1+24K \\ 2+24K \\ 7+24K \\ 10+24K \\ 17+24K \\ 23+24K \end{pmatrix} \in [-\omega^2]$$

Locas number

❖ For -1 there exist two classes which is $[1],[-1]$

$$\begin{pmatrix}0+6K\\3+6K\end{pmatrix} \in [1] \qquad \begin{pmatrix}1+6K\\2+6K\\4+6K\\5+6K\end{pmatrix} \in [-1]$$

❖ for $-\omega$ there exist six classes :$[-\omega^2],[\omega^2],[-\omega],[\omega],[-1],[1]$

$$\begin{pmatrix}6+24K\\18+24K\end{pmatrix} \in [1] \qquad \begin{pmatrix}3+24K\\9+24K\\12+24K\end{pmatrix} \in [\omega]$$

$$\begin{pmatrix}0+24K\\15+24K\\21+24K\end{pmatrix} \in [\omega^2] \qquad \begin{pmatrix}2+24K\\10+24K\\14+24K\\22+24K\end{pmatrix} \in [-1]$$

$$\begin{pmatrix}1+24K\\4+24K\\11+24K\\17+24K\\19+24K\\20+24K\end{pmatrix} \in [-\omega] \qquad \begin{pmatrix}5+24K\\7+24K\\8+24K\\13+24K\\16+24K\\23+24K\end{pmatrix} \in [-\omega^2]$$

❖ for $-\omega^2$ there exist sic classes :$[-\omega^2], [\omega^2], [-\omega], [\omega], [-1], [1]$

$$\begin{pmatrix} 6+24K \\ 18+24K \end{pmatrix} \in [1]$$

$$\begin{pmatrix} 0+24K \\ 15+24K \\ 21+24K \end{pmatrix} \in [\omega]$$

$$\begin{pmatrix} 3+24K \\ 9+24K \\ 12+24K \end{pmatrix} \in [\omega^2]$$

$$\begin{pmatrix} 2+24K \\ 10+24K \\ 14+24K \\ 22+24K \end{pmatrix} \in [-1]$$

$$\begin{pmatrix} 5+24K \\ 7+24K \\ 8+24K \\ 13+24K \\ 16+24K \\ 23+24K \end{pmatrix} \in [-\omega]$$

$$\begin{pmatrix} 1+24K \\ 4+24K \\ 11+24K \\ 17+24K \\ 19+24K \\ 20+24K \end{pmatrix} \in [-\omega^2]$$

(3.3) Possibilities arrangement of $1, -1, i, -i, w, w^2$

(3.3.1) Possibilities of i

the arrangement of the resulting values when raising the Fibonacci sequence to the number i depends on the type of the first two terms used in the sequence in terms of the remainder of dividing the numbers by 4, where there are four values {0, 1, 2, 3} and therefore there will be 16 possibilities for the arrangements of the values:

52	0	1	89	1	i	38	2	-1	143	3	i-
112	0	1	13	1	i	154	2	-1	63	3	i-
164	0	1	102	2	-1	192	0	1	206	2	-1
276	0	1	115	3	i-	346	2	-1	269	1	i
440	0	1	217	1	i	538	2	-1	475	3	i-
716	0	1	332	0	1	884	0	1	744	0	1

152	0	1	49	1	i	12	0	1	50	2	-1
65	1	i	104	0	1	74	2	-1	72	0	1
217	1	i	153	1	i	86	2	-1	122	2	-1
282	2	-1	257	1	i	160	0	1	194	2	-1
499	3	i-	410	2	-1	246	2	-1	316	0	1
781	1	i	667	3	i-	406	2	-1	510	2	-1

76	0	1	15	3	i-	13	1	i	134	2	-1
127	3	i-	92	0	1	82	2	-1	65	1	i
203	3	i-	107	3	i-	95	3	i-	199	3	i-
330	2	-1	199	3	i-	177	1	i	264	0	1
533	1	i	306	2	-1	272	0	1	463	3	i-
863	3	i-	505	1	i	449	1	i	727	3	i-

97	1	i	79	3	i-	22	2	-1	27	3	i-
147	3	i-	175	1	i	15	3	i-	30	2	-1
244	0	1	254	2	-1	37	1	i	57	1	i
391	3	i-	429	1	i	52	0	1	87	3	i-
635	3	i-	683	3	i-	89	1	i	144	0	1
1026	2	-1	1112	0	1	141	1	i	231	3	i-

table (3.1)

(3.3.2) Possibilities of $-i$

the arrangement of the resulting values when raising the Fibonacci sequence to the number $-i$ depends on the type of the first two terms used in the sequence in terms of the remainder of dividing the numbers by 4, where there are four values {0, 1, 2, 3} and therefore there will be 16 possibilities for the arrangements of the values:

52	0	1
112	0	1
164	0	1
276	0	1
440	0	1
716	0	1

89	1	i-
13	1	i-
102	2	-1
115	3	i
217	1	i-
332	0	1

38	2	-1
154	2	-1
192	0	1
346	2	-1
538	2	-1
884	0	1

143	3	i
63	3	i
206	2	-1
269	1	i-
475	3	i
744	0	1

152	0	1
65	1	i-
217	1	i-
282	2	-1
499	3	i
781	1	i-

49	1	i-
104	0	1
153	1	i-
257	1	i-
410	2	-1
667	3	i

12	0	1
74	2	-1
86	2	-1
160	0	1
246	2	-1
406	2	-1

50	2	-1
72	0	1
122	2	-1
194	2	-1
316	0	1
510	2	-1

76	0	1
127	3	i
203	3	i
330	2	-1
533	1	i-
863	3	i

15	3	i
92	0	1
107	3	i
199	3	i
306	2	-1
505	1	i-

13	1	i-
82	2	-1
95	3	i
177	1	i-
272	0	1
449	1	i-

134	2	-1
65	1	i-
199	3	i
264	0	1
463	3	i
727	3	i

97	1	i-
147	3	i
244	0	1
391	3	i
635	3	i
1026	2	-1

79	3	i
175	1	i-
254	2	-1
429	1	i-
683	3	i
1112	0	1

22	2	-1
15	3	i
37	1	i-
52	0	1
89	1	i-
141	1	i-

27	3	i
30	2	-1
57	1	i-
87	3	i
144	0	1
231	3	i

table (3.2)

(3.3.3) Possibilities of w

the arrangement of the resulting values when raising the Fibonacci sequence to the number w depends on the type of the first two terms used in the sequence in terms of the remainder of dividing the numbers by 3, where there are three values {0, 1, 2} and therefore there will be 9 possibilities for the arrangements of the values:

72	0	1
81	0	1
153	0	1
234	0	1
387	0	1
621	0	1
1008	0	1
1629	0	1

190	1	w
10	1	w
200	2	w^2
210	0	1
410	2	w^2
620	2	w^2
1030	1	w
1650	0	1

149	2	w^2
86	2	w^2
235	1	w
321	0	1
556	1	w
877	1	w
1433	2	w^2
2310	0	1

93	0	1
148	1	w
241	1	w
389	2	w^2
630	0	1
1019	2	w^2
1649	2	w^2
2668	1	w

73	1	w
117	0	1
190	1	w
307	1	w
497	2	w^2
804	0	1
1301	2	w^2
2105	2	w^2

60	0	1
113	2	w^2
173	2	w^2
286	1	w
459	0	1
745	1	w
1204	1	w
1949	2	w^2

86	2	w^2
33	0	1
119	2	w^2
152	2	w^2
271	1	w
423	0	1
694	1	w
1117	1	w

76	1	w
65	2	w^2
141	0	1
206	2	w^2
347	2	w^2
553	1	w
900	0	1
1453	1	w

143	2	w^2
133	1	w
276	0	1
409	1	w
685	1	w
1094	2	w^2
1779	0	1
2873	2	w^2

table (3.3)

(3.3.4) Possibilities of w^2

the arrangement of the resulting values when raising the Fibonacci sequence to the number w^2 depends on the type of the first two terms used in the sequence in terms of the remainder of dividing the numbers by 3, where there are three values {0, 1, 2} and therefore there will be 9 possibilities for the arrangements of the values:

72	0	1
81	0	1
153	0	1
234	0	1
387	0	1
621	0	1
1008	0	1
1629	0	1

190	1	w^2
10	1	w^2
200	2	w
210	0	1
410	2	w
620	2	w
1030	1	w^2
1650	0	1

149	2	w
86	2	w
235	1	w^2
321	0	1
556	1	w^2
877	1	w^2
1433	2	w
2310	0	1

93	0	1
148	1	w^2
241	1	w^2
389	2	w
630	0	1
1019	2	w
1649	2	w
2668	1	w^2

73	1	w^2
117	0	1
190	1	w^2
307	1	w^2
497	2	w
804	0	1
1301	2	w
2105	2	w

60	0	1
113	2	w
173	2	w
286	1	w^2
459	0	1
745	1	w^2
1204	1	w^2
1949	2	w

86	2	w
33	0	1
119	2	w
152	2	w
271	1	w^2
423	0	1
694	1	w^2
1117	1	w^2

76	1	w^2
65	2	w
141	0	1
206	2	w
347	2	w
553	1	w^2
900	0	1
1453	1	w^2

143	2	w
133	1	w^2
276	0	1
409	1	w^2
685	1	w^2
1094	2	w
1779	0	1
2873	2	w

table (3.4)

(3.3.5) Possibilities of $-w$

the arrangement of the resulting values when raising the Fibonacci sequence to the number $-w$ depends on the type of the first two terms used in the sequence in terms of the remainder of dividing the numbers by 3, where there are three values {0, 1, 2} and depends on if the first and second terms are even or odd, therefore there will be 36 possibilities for the arrangements of the values:

even + even								
72	0	1	190	1	w	20	2	w^2
12	0	1	10	1	w	86	2	w^2
84	0	1	200	2	w^2	106	1	w
96	0	1	210	0	1	192	0	1
180	0	1	410	2	w^2	298	1	w
276	0	1	620	2	w^2	490	1	w
456	0	1	1030	1	w	788	2	w^2
732	0	1	1650	0	1	1278	0	1

24	0	1	22	1	w	60	0	1
148	1	w	36	0	1	38	2	w^2
172	1	w	58	1	w	98	2	w^2
320	2	w^2	94	1	w	136	1	w
492	0	1	152	2	w^2	234	0	1
812	2	w^2	246	0	1	370	1	w
1304	2	w^2	398	2	w^2	604	1	w
2116	1	w	644	2	w^2	974	2	w^2

86	2	w^2	76	1	w	68	2	w^2
54	0	1	44	2	w^2	52	1	w
140	2	w^2	120	0	1	120	0	1
194	2	w^2	164	2	w^2	172	1	w
334	1	w	284	2	w^2	292	1	w
528	0	1	448	1	w	464	2	w^2
862	1	w	732	0	1	756	0	1
1390	1	w	1180	1	w	1220	2	w^2

table (3.5)

odd + odd								
51	0	-1	43	1	w-	23	2	w^2-
9	0	-1	31	1	w-	77	2	w^2-
60	0	1	74	2	w^2	100	1	w
69	0	-1	105	0	-1	177	0	-1
129	0	-1	179	2	w^2-	277	1	w-
198	0	1	284	2	w^2	454	1	w
327	0	-1	463	1	w-	731	2	w^2-
525	0	-1	747	0	-1	1185	0	-1
852	0	1	1210	1	w	1916	2	w^2
1377	0	-1	1957	1	w-	3101	2	w^2-
2229	0	-1	3167	2	w^2-	5017	1	w-
3606	0	1	5124	0	1	8118	0	1
5835	0	-1	8291	2	w^2-	13135	1	w-
9441	0	-1	13415	2	w^2-	21253	1	w-
15276	0	1	21706	1	w	34388	2	w^2
24717	0	-1	35121	0	-1	55641	0	-1
39993	0	-1	56827	1	w-	90029	2	w^2-
64710	0	1	91948	1	w	145670	2	w^2
104703	0	-1	148775	2	w^2-	235699	1	w-
169413	0	-1	240723	0	-1	381369	0	-1
274116	0	1	389498	2	w^2	617068	1	w
443529	0	-1	630221	2	w^2-	998437	1	w-
717645	0	-1	1019719	1	w-	1615505	2	w^2-
1161174	0	1	1649940	0	1	2613942	0	1

table (3.6)

27	0	-1
97	1	w-
124	1	w
221	2	w^2-
345	0	-1
566	2	w^2
911	2	w^2-
1477	1	w-
2388	0	1
3865	1	w-
6253	1	w-
10118	2	w^2
16371	0	-1
26489	2	w^2-
42860	2	w^2
69349	1	w-
112209	0	-1
181558	1	w
293767	1	w-
475325	2	w^2-
769092	0	1
1244417	2	w^2-
2013509	2	w^2-
3257926	1	w

25	1	w-
33	0	-1
58	1	w
91	1	w-
149	2	w^2-
240	0	1
389	2	w^2-
629	2	w^2-
1018	1	w
1647	0	-1
2665	1	w-
4312	1	w
6977	2	w^2-
11289	0	-1
18266	2	w^2
29555	2	w^2-
47821	1	w-
77376	0	1
125197	1	w-
202573	1	w-
327770	2	w^2
530343	0	-1
858113	2	w^2-
1388456	2	w^2

57	0	-1
35	2	w^2-
92	2	w^2
127	1	w-
219	0	-1
346	1	w
565	1	w-
911	2	w^2-
1476	0	1
2387	2	w^2-
3863	2	w^2-
6250	1	w
10113	0	-1
16363	1	w-
26476	1	w
42839	2	w^2-
69315	0	-1
112154	2	w^2
181469	2	w^2-
293623	1	w-
475092	0	1
768715	1	w-
1243807	1	w-
2012522	2	w^2

table (3.7)

89	2	w^2-
51	0	-1
140	2	w^2
191	2	w^2-
331	1	w-
522	0	1
853	1	w-
1375	1	w-
2228	2	w^2
3603	0	-1
5831	2	w^2-
9434	2	w^2
15265	1	w-
24699	0	-1
39964	1	w
64663	1	w-
104627	2	w^2-
169290	0	1
273917	2	w^2-
443207	2	w^2-
717124	1	w
1160331	0	1-
1877455	1	w-
3037786	1	w

79	1	w-
41	2	w^2-
120	0	1
161	2	w^2-
281	2	w^2-
442	1	w
723	0	-1
1165	1	w-
1888	1	w
3053	2	w^2-
4941	0	-1
7994	2	w^2
12935	2	w^2-
20929	1	w-
33864	0	1
54793	1	w-
88657	1	w-
143450	2	w^2
232107	0	-1
375557	2	w^2-
607664	2	w^2
983221	1	w-
1590885	0	-1
2574106	1	w

71	2	w^2-
49	1	w-
120	0	1
169	1	w-
289	1	w-
458	2	w^2
747	0	-1
1205	2	w^2-
1952	2	w^2
3157	1	w-
5109	0	-1
8266	1	w
13375	1	w-
21641	2	w^2-
35016	0	1
56657	2	w^2-
91673	2	w^2-
148330	1	w
240003	0	-1
388333	1	w-
628336	1	w
1016669	2	w^2-
1645005	0	-1
2661674	2	w^2

table (3.8)

even + odd										
72	0	1		190	1	w		20	2	w^2
15	0	-1		13	1	w-		89	2	w^2-
87	0	-1		203	2	w^2-		109	1	w-
102	0	1		216	0	1		198	0	1
189	0	-1		419	2	w^2-		307	1	w-
291	0	-1		635	2	w^2-		505	1	w-
480	0	1		1054	1	w		812	2	w^2
771	0	-1		1689	0	1-		1317	0	1-
1251	0	-1		2743	1	w-		2129	2	w^2-
2022	0	1		4432	1	w		3446	2	w^2
3273	0	-1		7175	2	w^2-		5575	1	w-
5295	0	-1		11607	0	1-		9021	0	1-
8568	0	1		18782	2	w^2		14596	1	w
13863	0	-1		30389	2	w^2-		23617	1	w-
22431	0	-1		49171	1	w-		38213	2	w^2-
36294	0	1		79560	0	1		61830	0	1
58725	0	-1		128731	1	w-		100043	2	w^2-
95019	0	-1		208291	1	w-		161873	2	w^2-
153744	0	1		337022	2	w^2		261916	1	w
248763	0	-1		545313	0	1-		423789	0	1-
402507	0	-1		882335	2	w^2-		685705	1	w-
651270	0	1		1427648	2	w^2		1109494	1	w
1053777	0	-1		2309983	1	w-		1795199	2	w^2-
1705047	0	-1		3737631	0	1-		2904693	0	1-

table (3.9)

24	0	1
151	1	w-
175	1	w-
326	2	w^2
501	0	1-
827	2	w^2-
1328	2	w^2
2155	1	w-
3483	0	1-
5638	1	w
9121	1	w-
14759	2	w^2-
23880	0	1
38639	2	w^2-
62519	2	w^2-
101158	1	w
163677	0	1-
264835	1	w-
428512	1	w
693347	2	w^2-
1121859	0	1-
1815206	2	w^2
2937065	2	w^2-
4752271	1	w-

22	1	w
39	0	1-
61	1	w-
100	1	w
161	2	w^2-
261	0	1-
422	2	w^2
683	2	w^2-
1105	1	w-
1788	0	1
2893	1	w-
4681	1	w-
7574	2	w^2
12255	0	1-
19829	2	w^2-
32084	2	w^2
51913	1	w-
83997	0	1-
135910	1	w
219907	1	w-
355817	2	w^2-
575724	0	1
931541	2	w^2-
1507265	2	w^2-

60	0	1
41	2	w^2-
101	2	w^2-
142	1	w
243	0	1-
385	1	w-
628	1	w
1013	2	w^2-
1641	0	1-
2654	2	w^2
4295	2	w^2-
6949	1	w-
11244	0	1
18193	1	w-
29437	1	w-
47630	2	w^2
77067	0	1-
124697	2	w^2-
201764	2	w^2
326461	1	w-
528225	0	1-
854686	1	w
1382911	1	w-
2237597	2	w^2-

table (3.10)

86	2	w^2
57	0	1-
143	2	w^2-
200	2	w^2
343	1	w-
543	0	1-
886	1	w
1429	1	w-
2315	2	w^2-
3744	0	1
6059	2	w^2-
9803	2	w^2-
15862	1	w
25665	0	1-
41527	1	w-
67192	1	w
108719	2	w^2-
175911	0	1-
284630	2	w^2
460541	2	w^2-
745171	1	w-
1205712	0	1
1950883	1	w-
3156595	1	w-

76	1	w
47	2	w^2-
123	0	1-
170	2	w^2
293	2	w^2-
463	1	w-
756	0	1
1219	1	w-
1975	1	w-
3194	2	w^2
5169	0	1-
8363	2	w^2-
13532	2	w^2
21895	1	w-
35427	0	1-
57322	1	w
92749	1	w-
150071	2	w^2-
242820	0	1
392891	2	w^2-
635711	2	w^2-
1028602	1	w
1664313	0	1-
2692915	1	w-

68	2	w^2
55	1	w-
123	0	1-
178	1	w
301	1	w-
479	2	w^2-
780	0	1
1259	2	w^2-
2039	2	w^2-
3298	1	w
5337	0	1-
8635	1	w-
13972	1	w
22607	2	w^2-
36579	0	1-
59186	2	w^2
95765	2	w^2-
154951	1	w-
250716	0	1
405667	1	w-
656383	1	w-
1062050	2	w^2
1718433	0	1-
2780483	2	w^2-

table (3.11)

odd + even								
51	0	-1	43	1	w-	23	2	w^2-
12	0	1	34	1	w	80	2	w^2
63	0	-1	77	2	w^2-	103	1	w-
75	0	-1	111	0	-1	183	0	-1
138	0	1	188	2	w^2	286	1	w
213	0	-1	299	2	w^2-	469	1	w-
351	0	-1	487	1	w-	755	2	w^2-
564	0	1	786	0	1	1224	0	1
915	0	-1	1273	1	w-	1979	2	w^2-
1479	0	-1	2059	1	w-	3203	2	w^2-
2394	0	1	3332	2	w^2	5182	1	w
3873	0	-1	5391	0	1-	8385	0	1-
6267	0	-1	8723	2	w^2-	13567	1	w-
10140	0	1	14114	2	w^2	21952	1	w
16407	0	-1	22837	1	w-	35519	2	w^2-
26547	0	-1	36951	0	-1	57471	0	-1
42954	0	1	59788	1	w	92990	2	w^2
69501	0	-1	96739	1	w-	150461	2	w^2-
112455	0	-1	156527	2	w^2-	243451	1	w-
181956	0	1	253266	0	1	393912	0	1
294411	0	-1	409793	2	w^2-	637363	1	w-
476367	0	-1	663059	2	w^2-	1031275	1	w-
770778	0	1	1072852	1	w	1668638	2	w^2
1247145	0	-1	1735911	0	1-	2699913	0	-1

table (3.12)

27	0	-1
100	1	w
127	1	w-
227	2	w^2-
354	0	1
581	2	w^2-
935	2	w^2-
1516	1	w
2451	0	1-
3967	1	w-
6418	1	w
10385	2	w^2-
16803	0	-1
27188	2	w^2
43991	2	w^2-
71179	1	w-
115170	0	1
186349	1	w-
301519	1	w-
487868	2	w^2
789387	0	1-
1277255	2	w^2-
2066642	2	w^2
3343897	1	w-

25	1	w-
30	0	1
55	1	w-
85	1	w-
140	2	w^2
225	0	1-
365	2	w^2-
590	2	w^2
955	1	w-
1545	0	-1
2500	1	w
4045	1	w-
6545	2	w^2-
10590	0	1
17135	2	w^2-
27725	2	w^2-
44860	1	w
72585	0	1-
117445	1	w-
190030	1	w
307475	2	w^2-
497505	0	-1
804980	2	w^2
1302485	2	w^2-

57	0	-1
38	2	w^2
95	2	w^2-
133	1	w-
228	0	1
361	1	w-
589	1	w-
950	2	w^2
1539	0	1-
2489	2	w^2-
4028	2	w^2
6517	1	w-
10545	0	-1
17062	1	w
27607	1	w-
44669	2	w^2-
72276	0	1
116945	2	w^2-
189221	2	w^2-
306166	1	w
495387	0	1-
801553	1	w-
1296940	1	w
2098493	2	w^2-

table (3.13)

89	2	w^2-
54	0	1
143	2	w^2-
197	2	w^2-
340	1	w
537	0	1-
877	1	w-
1414	1	w
2291	2	w^2-
3705	0	-1
5996	2	w^2
9701	2	w^2-
15697	1	w-
25398	0	1
41095	1	w-
66493	1	w-
107588	2	w^2
174081	0	1-
281669	2	w^2-
455750	2	w^2
737419	1	w-
1193169	0	1-
1930588	1	w
3123757	1	w-

79	1	w-
44	2	w^2
123	0	1-
167	2	w^2-
290	2	w^2
457	1	w-
747	0	-1
1204	1	w
1951	1	w-
3155	2	w^2-
5106	0	1
8261	2	w^2-
13367	2	w^2-
21628	1	w
34995	0	1-
56623	1	w-
91618	1	w
148241	2	w^2-
239859	0	-1
388100	2	w^2
627959	2	w^2-
1016059	1	w-
1644018	0	1
2660077	1	w-

71	2	w^2-
52	1	w
123	0	1-
175	1	w-
298	1	w
473	2	w^2-
771	0	-1
1244	2	w^2
2015	2	w^2-
3259	1	w-
5274	0	1
8533	1	w-
13807	1	w-
22340	2	w^2
36147	0	1-
58487	2	w^2-
94634	2	w^2
153121	1	w-
247755	0	-1
400876	1	w
648631	1	w-
1049507	2	w^2-
1698138	0	1
2747645	2	w^2-

table (3.14)

(3.3.6) Possibilities of $-w^2$

the arrangement of the resulting values when raising the Fibonacci sequence to the number $-w^2$ depends on the type of the first two terms used in the sequence in terms of the remainder of dividing the numbers by 3, where there are three values {0, 1, 2} and depends on if the first and second terms are even or odd, therefore there will be 36 possibilities for the arrangements of the values:

even + even								
72	0	1	190	1	w^2	20	2	w
12	0	1	10	1	w^2	86	2	w
84	0	1	200	2	w	106	1	w^2
96	0	1	210	0	1	192	0	1
180	0	1	410	2	w	298	1	w^2
276	0	1	620	2	w	490	1	w^2
456	0	1	1030	1	w^2	788	2	w
732	0	1	1650	0	1	1278	0	1

24	0	1	22	1	w^2	60	0	1
148	1	w^2	36	0	1	38	2	w
172	1	w^2	58	1	w^2	98	2	w
320	2	w	94	1	w^2	136	1	w^2
492	0	1	152	2	w	234	0	1
812	2	w	246	0	1	370	1	w
1304	2	w	398	2	w	604	1	w
2116	1	w^2	644	2	w	974	2	w^2

86	2	w	76	1	w^2	68	2	w
54	0	1	44	2	w	52	1	w^2
140	2	w	120	0	1	120	0	1
194	2	w	164	2	w	172	1	w^2
334	1	w^2	284	2	w	292	1	w^2
528	0	1	448	1	w^2	464	2	w
862	1	w^2	732	0	1	756	0	1
1390	1	w^2	1180	1	w^2	1220	2	w

table (3.15)

odd + odd								
51	0	-1	43	1	w^2-	23	2	w-
9	0	-1	31	1	w^2-	77	2	w-
60	0	1	74	2	w	100	1	w^2
69	0	-1	105	0	-1	177	0	-1
129	0	-1	179	2	w-	277	1	w^2-
198	0	1	284	2	w	454	1	w^2
327	0	-1	463	1	w^2-	731	2	w-
525	0	-1	747	0	-1	1185	0	-1
852	0	1	1210	1	w^2	1916	2	w
1377	0	-1	1957	1	w^2-	3101	2	w-
2229	0	-1	3167	2	w-	5017	1	w^2-
3606	0	1	5124	0	1	8118	0	1
5835	0	-1	8291	2	w-	13135	1	w^2-
9441	0	-1	13415	2	w-	21253	1	w^2-
15276	0	1	21706	1	w^2	34388	2	w
24717	0	-1	35121	0	-1	55641	0	-1
39993	0	-1	56827	1	w^2-	90029	2	w-
64710	0	1	91948	1	w^2	145670	2	w
104703	0	-1	148775	2	w-	235699	1	w^2-
169413	0	-1	240723	0	-1	381369	0	-1
274116	0	1	389498	2	w	617068	1	w^2
443529	0	-1	630221	2	w-	998437	1	w^2-
717645	0	-1	1019719	1	w^2-	1615505	2	w-
1161174	0	1	1649940	0	1	2613942	0	1

table (3.15)

27	0	-1
97	1	w^2-
124	1	w^2
221	2	w-
345	0	-1
566	2	w
911	2	w-
1477	1	w^2-
2388	0	1
3865	1	w^2-
6253	1	w^2-
10118	2	w
16371	0	-1
26489	2	w-
42860	2	w
69349	1	w^2-
112209	0	-1
181558	1	w^2
293767	1	w^2-
475325	2	w-
769092	0	1
1244417	2	w-
2013509	2	w-
3257926	1	w^2

25	1	w^2-
33	0	-1
58	1	w^2
91	1	w^2-
149	2	w-
240	0	1
389	2	w-
629	2	w-
1018	1	w^2
1647	0	-1
2665	1	w^2-
4312	1	w^2
6977	2	w-
11289	0	-1
18266	2	w
29555	2	w-
47821	1	w^2-
77376	0	1
125197	1	w^2-
202573	1	w^2-
327770	2	w
530343	0	1-
858113	2	w-
1388456	2	w

57	0	-1
35	2	w-
92	2	w
127	1	w^2-
219	0	-1
346	1	w^2
565	1	w^2-
911	2	w-
1476	0	1
2387	2	w-
3863	2	w-
6250	1	w^2
10113	0	-1
16363	1	w^2-
26476	1	w^2
42839	2	w-
69315	0	-1
112154	2	w
181469	2	w-
293623	1	w^2-
475092	0	1
768715	1	w^2-
1243807	1	w^2-
2012522	2	w

table (3.16)

89	2	w-
51	0	-1
140	2	w
191	2	w-
331	1	w^2-
522	0	1
853	1	w^2-
1375	1	w^2-
2228	2	w
3603	0	-1
5831	2	w-
9434	2	w
15265	1	w^2-
24699	0	-1
39964	1	w^2
64663	1	w^2-
104627	2	w-
169290	0	1
273917	2	w-
443207	2	w-
717124	1	w^2
1160331	0	-1
1877455	1	w^2-
3037786	1	w^2

79	1	w^2-
41	2	w-
120	0	1
161	2	w-
281	2	w-
442	1	w^2
723	0	-1
1165	1	w^2-
1888	1	w^2
3053	2	w-
4941	0	-1
7994	2	w
12935	2	w-
20929	1	w^2-
33864	0	1
54793	1	w^2-
88657	1	w^2-
143450	2	w
232107	0	-1
375557	2	w-
607664	2	w
983221	1	w^2-
1590885	0	-1
2574106	1	w^2

71	2	w-
49	1	w^2-
120	0	1
169	1	w^2-
289	1	w^2-
458	2	w
747	0	-1
1205	2	w-
1952	2	w
3157	1	w^2-
5109	0	-1
8266	1	w^2
13375	1	w^2-
21641	2	w-
35016	0	1
56657	2	w-
91673	2	w-
148330	1	w^2
240003	0	-1
388333	1	w^2-
628336	1	w^2
1016669	2	w-
1645005	0	-1
2661674	2	w

table (3.17)

even + odd								
72	0	1	190	1	w^2	20	2	w
15	0	-1	13	1	w^2-	89	2	w-
87	0	-1	203	2	w-	109	1	w^2-
102	0	1	216	0	1	198	0	1
189	0	-1	419	2	w-	307	1	w^2-
291	0	-1	635	2	w-	505	1	w^2-
480	0	1	1054	1	w^2	812	2	w
771	0	-1	1689	0	1-	1317	0	1-
1251	0	-1	2743	1	w^2-	2129	2	w-
2022	0	1	4432	1	w^2	3446	2	w
3273	0	-1	7175	2	w-	5575	1	w^2-
5295	0	-1	11607	0	1-	9021	0	1-
8568	0	1	18782	2	w	14596	1	w^2
13863	0	-1	30389	2	w-	23617	1	w^2-
22431	0	-1	49171	1	w^2-	38213	2	w-
36294	0	1	79560	0	1	61830	0	1
58725	0	-1	128731	1	w^2-	100043	2	w-
95019	0	-1	208291	1	w^2-	161873	2	w-
153744	0	1	337022	2	w	261916	1	w^2
248763	0	-1	545313	0	1-	423789	0	1-
402507	0	-1	882335	2	w-	685705	1	w^2-
651270	0	1	1427648	2	w	1109494	1	w^2
1053777	0	-1	2309983	1	w^2-	1795199	2	w-
1705047	0	-1	3737631	0	1-	2904693	0	1-

table (3.18)

24	0	1
151	1	w^2-
175	1	w^2-
326	2	w
501	0	1-
827	2	w-
1328	2	w
2155	1	w^2-
3483	0	1-
5638	1	w^2
9121	1	w^2-
14759	2	w-
23880	0	1
38639	2	w-
62519	2	w-
101158	1	w^2
163677	0	1-
264835	1	w^2-
428512	1	w^2
693347	2	w-
1121859	0	1-
1815206	2	w
2937065	2	w-
4752271	1	w^2-

22	1	w^2
39	0	1-
61	1	w^2-
100	1	w^2
161	2	w-
261	0	1-
422	2	w
683	2	w-
1105	1	w^2-
1788	0	1
2893	1	w^2-
4681	1	w^2-
7574	2	w
12255	0	1-
19829	2	w-
32084	2	w
51913	1	w^2-
83997	0	1-
135910	1	w^2
219907	1	w^2-
355817	2	w-
575724	0	1
931541	2	w-
1507265	2	w-

60	0	1
41	2	w-
101	2	w-
142	1	w^2
243	0	1-
385	1	w^2-
628	1	w^2
1013	2	w-
1641	0	1-
2654	2	w
4295	2	w-
6949	1	w^2-
11244	0	1
18193	1	w^2-
29437	1	w^2-
47630	2	w
77067	0	1-
124697	2	w-
201764	2	w
326461	1	w^2-
528225	0	1-
854686	1	w^2
1382911	1	w^2-
2237597	2	w-

table (3.19)

86	2	w
57	0	1-
143	2	w-
200	2	w
343	1	w^2-
543	0	1-
886	1	w^2
1429	1	w^2-
2315	2	w-
3744	0	1
6059	2	w-
9803	2	w-
15862	1	w^2
25665	0	1-
41527	1	w^2-
67192	1	w^2
108719	2	w-
175911	0	1-
284630	2	w
460541	2	w-
745171	1	w^2-
1205712	0	1
1950883	1	w^2-
3156595	1	w^2-

76	1	w^2
47	2	w-
123	0	1-
170	2	w
293	2	w-
463	1	w^2-
756	0	1
1219	1	w^2-
1975	1	w^2-
3194	2	w
5169	0	1-
8363	2	w-
13532	2	w
21895	1	w^2-
35427	0	1-
57322	1	w^2
92749	1	w^2-
150071	2	w-
242820	0	1
392891	2	w-
635711	2	w-
1028602	1	w^2
1664313	0	1-
2692915	1	w^2-

68	2	w
55	1	w^2-
123	0	1-
178	1	w^2
301	1	w^2-
479	2	w-
780	0	1
1259	2	w-
2039	2	w-
3298	1	w^2
5337	0	1-
8635	1	w^2-
13972	1	w^2
22607	2	w-
36579	0	1-
59186	2	w
95765	2	w-
154951	1	w^2-
250716	0	1
405667	1	w^2-
656383	1	w^2-
1062050	2	w
1718433	0	1-
2780483	2	w-

table (3.20)

odd + even								
51	0	-1	43	1	w^2-	23	2	w-
12	0	1	34	1	w^2	80	2	w
63	0	-1	77	2	w-	103	1	w^2-
75	0	-1	111	0	-1	183	0	-1
138	0	1	188	2	w	286	1	w^2
213	0	-1	299	2	w-	469	1	w^2-
351	0	-1	487	1	w^2-	755	2	w-
564	0	1	786	0	1	1224	0	1
915	0	-1	1273	1	w^2-	1979	2	w-
1479	0	-1	2059	1	w^2-	3203	2	w-
2394	0	1	3332	2	w	5182	1	w^2
3873	0	-1	5391	0	1-	8385	0	1-
6267	0	-1	8723	2	w-	13567	1	w^2-
10140	0	1	14114	2	w	21952	1	w^2
16407	0	-1	22837	1	w^2-	35519	2	w-
26547	0	-1	36951	0	-1	57471	0	-1
42954	0	1	59788	1	w^2	92990	2	w
69501	0	-1	96739	1	w^2-	150461	2	w-
112455	0	-1	156527	2	w-	243451	1	w^2-
181956	0	1	253266	0	1	393912	0	1
294411	0	-1	409793	2	w-	637363	1	w^2-
476367	0	-1	663059	2	w-	1031275	1	w^2-
770778	0	1	1072852	1	w^2	1668638	2	w
1247145	0	-1	1735911	0	-1	2699913	0	1-

table (3.21)

27	0	-1
100	1	w^2
127	1	w^2-
227	2	w-
354	0	1
581	2	w-
935	2	w-
1516	1	w^2
2451	0	1-
3967	1	w^2-
6418	1	w^2
10385	2	w-
16803	0	-1
27188	2	w
43991	2	w-
71179	1	w^2-
115170	0	1
186349	1	w^2-
301519	1	w^2-
487868	2	w
789387	0	1-
1277255	2	w-
2066642	2	w
3343897	1	w^2-

25	1	w^2-
30	0	1
55	1	w^2-
85	1	w^2-
140	2	w
225	0	1-
365	2	w-
590	2	w
955	1	w^2-
1545	0	-1
2500	1	w^2
4045	1	w^2-
6545	2	w-
10590	0	1
17135	2	w-
27725	2	w-
44860	1	w^2
72585	0	1-
117445	1	w^2-
190030	1	w^2
307475	2	w-
497505	0	1-
804980	2	w
1302485	2	w-

57	0	-1
38	2	w
95	2	w-
133	1	w^2-
228	0	1
361	1	w^2-
589	1	w^2-
950	2	w
1539	0	1-
2489	2	w-
4028	2	w
6517	1	w^2-
10545	0	-1
17062	1	w^2
27607	1	w^2-
44669	2	w-
72276	0	1
116945	2	w-
189221	2	w-
306166	1	w^2
495387	0	1-
801553	1	w^2-
1296940	1	w^2
2098493	2	w-

table (3.22)

89	2	w-
54	0	1
143	2	w-
197	2	w-
340	1	w^2
537	0	1-
877	1	w^2-
1414	1	w^2
2291	2	w-
3705	0	-1
5996	2	w
9701	2	w-
15697	1	w^2-
25398	0	1
41095	1	w^2-
66493	1	w^2-
107588	2	w
174081	0	1-
281669	2	w-
455750	2	w
737419	1	w^2-
1193169	0	-1
1930588	1	w^2
3123757	1	w^2-

79	1	w^2-
44	2	w
123	0	1-
167	2	w-
290	2	w
457	1	w^2-
747	0	-1
1204	1	w^2
1951	1	w^2-
3155	2	w-
5106	0	1
8261	2	w-
13367	2	w-
21628	1	w^2
34995	0	1-
56623	1	w^2-
91618	1	w^2
148241	2	w-
239859	0	-1
388100	2	w
627959	2	w-
1016059	1	w^2-
1644018	0	1
2660077	1	w^2-

71	2	w-
52	1	w^2
123	0	1-
175	1	w^2-
298	1	w^2
473	2	w-
771	0	-1
1244	2	w
2015	2	w-
3259	1	w^2-
5274	0	1
8533	1	w^2-
13807	1	w^2-
22340	2	w
36147	0	1-
58487	2	w-
94634	2	w
153121	1	w^2-
247755	0	-1
400876	1	w^2
648631	1	w^2-
1049507	2	w-
1698138	0	1
2747645	2	w-

table (3.23)

(3.4) are the values of nth roots repeated?

The repetition that occurs at the values of the nth roots of integer one and negative one is caused by the repetition of the mod values of the Fibonacci sequences, according to the following table:

	Fibonacci	mod 2	mod 3	mod 4	mod 5	mod 6	mod 7	mod 8	mod 9	mod 10
1	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1
4	2	0	2	2	2	2	2	2	2	2
5	3	1	0	3	3	3	3	3	3	3
6	5	1	2	1	0	5	5	5	5	5
7	8	0	2	0	3	2	1	0	8	8
8	13	1	1	1	3	1	6	5	4	3
9	21	1	0	1	1	3	0	5	3	1
10	34	0	1	2	4	4	6	2	7	4
11	55	1	1	3	0	1	6	7	1	5
12	89	1	2	1	4	5	5	1	8	9
13	144	0	0	0	4	0	4	0	0	4
14	233	1	2	1	3	5	2	1	8	3
15	377	1	2	1	2	5	6	1	8	7
16	610	0	1	2	0	4	1	2	7	0
17	987	1	0	3	2	3	0	3	6	7
18	1597	1	1	1	2	1	1	5	4	7
19	2584	0	1	0	4	4	1	0	1	4
20	4181	1	2	1	1	5	2	5	5	1
21	6765	1	0	1	0	3	3	5	6	5
22	10946	0	2	2	1	2	5	2	2	6
23	17711	1	2	3	1	5	1	7	8	1
24	28657	1	1	1	2	1	6	1	1	7
25	46368	0	0	0	3	0	0	0	0	8
26	75025	1	1	1	0	1	6	1	1	5
27	121393	1	1	1	3	1	6	1	1	3
28	196418	0	2	2	3	2	5	2	2	8
29	317811	1	0	3	1	3	4	3	3	1
30	514229	1	2	1	4	5	2	5	5	9
31	832040	0	2	0	0	2	6	0	8	0

32	1346269	1	1	1	4	1	1	5	4	9
33	2178309	1	0	1	4	3	0	5	3	9
34	3524578	0	1	2	3	4	1	2	7	8
35	5702887	1	1	3	2	1	1	7	1	7
36	9227465	1	2	1	0	5	2	1	8	5
37	14930352	0	0	0	2	0	3	0	0	2
38	24157817	1	2	1	2	5	5	1	8	7
39	39088169	1	2	1	4	5	1	1	8	9
40	63245986	0	1	2	1	4	6	2	7	6
41	102334155	1	0	3	0	3	0	3	6	5
42	165580141	1	1	1	1	1	6	5	4	1
43	267914296	0	1	0	1	4	6	0	1	6
44	433494437	1	2	1	2	5	5	5	5	7
45	701408733	1	0	1	3	3	4	5	6	3
46	1134903170	0	2	2	0	2	2	2	2	0
47	1836311903	1	2	3	3	5	6	7	8	3
48	2971215073	1	1	1	3	1	1	1	1	3
49	4807526976	0	0	0	1	0	0	0	0	6
50	7778742049	1	1	1	4	1	1	1	1	9
51	12586269025	1	1	1	0	1	1	1	1	5
52	20365011074	0	2	2	4	2	2	2	2	4
53	32951280099	1	0	3	4	3	3	3	3	9
54	53316291173	1	2	1	3	5	5	5	5	3
55	86267571272	0	2	0	2	2	1	0	8	2
56	139583862445	1	1	1	0	1	6	5	4	5
57	225851433717	1	0	1	2	3	0	5	3	7
58	365435296162	0	1	2	2	4	6	2	7	2
59	591286729879	1	1	3	4	1	6	7	1	9
60	956722026041	1	2	1	1	5	5	1	8	1
	Repetition	3	8	6	20	24	32	36	48	60

(3.4.1) code to find any reputation for any mod to Fibonacci

```
def fib(x):
    if(x==0):
        global ax
        ax=0
        return 0
    elif(x==1):
        global bx
        bx=1
        return 1
    else:
        global cx
        cx=ax+bx
        ax=bx
        bx=cx
        return (cx)

ax=bx=cx=0
n=int(input("enter any number"))
leave=tempx=numa=numb=finalanswer=x=finaldigit=0
a=[]
finaldigit=[]
while(leave==0):
    a.append((fib(x)%n))
    if((x+1)%2==0):
        tempx=int(x/2)
        if(a[0:(tempx+1)]==a[(tempx+1):len(a)]):
            leave=1
            finalanswer=tempx
            finaldigit=a[0:(tempx+1)]
    x=x+1
    numa=numb=0
print("hussein haider alqazzaz")
print("The frequency of repeatation is ",finalanswer+1)
print("the repeating sequence is")
print(finaldigit)
```

and the result Shaw like that for example root 50 of Fibonacci sequence

the result:

enter any number50

Hussein Haider Alqazzaz

The frequency of repeatation is 300

the repeating sequence is

[0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 5, 39, 44, 33, 27, 10, 37, 47, 34, 31, 15, 46, 11, 7, 18, 25, 43, 18, 11, 29, 40, 19, 9, 28, 37, 15, 2, 17, 19, 36, 5, 41, 46, 37, 33, 20, 3, 23, 26, 49, 25, 24, 49, 23, 22, 45, 17, 12, 29, 41, 20, 11, 31, 42, 23, 15, 38, 3, 41, 44, 35, 29, 14, 43, 7, 0, 7, 7, 14, 21, 35, 6, 41, 47, 38, 35, 23, 8, 31, 39, 20, 9, 29, 38, 17, 5, 22, 27, 49, 26, 25, 1, 26, 27, 3, 30, 33, 13, 46, 9, 5, 14, 19, 33, 2, 35, 37, 22, 9, 31, 40, 21, 11, 32, 43, 25, 18, 43, 11, 4, 15, 19, 34, 3, 37, 40, 27, 17, 44, 11, 5, 16, 21, 37, 8, 45, 3, 48, 1, 49, 0, 49, 49, 48, 47, 45, 42, 37, 29, 16, 45, 11, 6, 17, 23, 40, 13, 3, 16, 19, 35, 4, 39, 43, 32, 25, 7, 32, 39, 21, 10, 31, 41, 22, 13, 35, 48, 33, 31, 14, 45, 9, 4, 13, 17, 30, 47, 27, 24, 1, 25, 26, 1, 27, 28, 5, 33, 38, 21, 9, 30, 39, 19, 8, 27, 35, 12, 47, 9, 6, 15, 21, 36, 7, 43, 0, 43, 43, 36, 29, 15, 44, 9, 3, 12, 15, 27, 42, 19, 11, 30, 41, 21, 12, 33, 45, 28, 23, 1, 24, 25, 49, 24, 23, 47, 20, 17, 37, 4, 41, 45, 36, 31, 17, 48, 15, 13, 28, 41, 19, 10, 29, 39, 18, 7, 25, 32, 7, 39, 46, 35, 31, 16, 47, 13, 10, 23, 33, 6, 39, 45, 34, 29, 13, 42, 5, 47, 2, 49, 1]

chapter 4

Conclusions

- any product of any reputation is equal to 1 and prove of that is suppose the first term is x and the second is y so we have the third terms which is x+y so on:

1	x
2	y
3	x+y
4	x+2y
5	2x+3y
6	3x+5y
7	5x+8y
8	8x+13y

so, the product of first reputation for i

$$i^x \times i^y \times i^{x+y} \times i^{x+2y} \times i^{2x+3} \times i^{3x+5y}$$

$$= i^{8x} \times i^{12y} = (i^4)^{2x} \times (i^4)^{3y} = 1$$

- to find any term of sequences like Fibonacci sequence use the following formula.

$$G_n = F_{n-2} \times X + F_{n-1} \times Y$$

$$let\ G_n\ be\ a\ sequence\ its\ first\ term$$

$$x = 9\ and\ second\ term\ y = 7$$

Find G_{16}

$$G_{16} = F_{14} \times 9 + F_{15} \times 7$$

$$G_{16} = 377 \times 9 + 610 \times 7$$

$$G_{16} = 3393 + 4270 = 7663$$