University of Babylon

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Department of Mathematic

Extract Lower and Upper Probability of β - Open Set

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بسم الله الرحمن الرحيم

﴿ قَالُوا سُبْحَانَكَ لَا عِلْمَ لَنَا إِلَّا مَا عَلَّمْتَنَا إِنَّكَ أَنْتَ الْعَلِيمُ الْحَكِيمُ ﴾

صدق الله العظيم

[البقرة: 32]

الاهداء..

بعد مسيرة دراسة دامت سنوات حملت في طياتها الكثير من الصعوبات ها أنا اليوم اقف على عتبة تخرجي اقطف ثمار تعبي ورفع قبعتي بكل فخر ، فاللهم لك الحمد قبل أن ترضى ولك الحمد اذا رضيت ولك الحمد بعد الرضا ، لأنك وفقتني على اتمام هذا العمل وتحقيق حلمي . . .

اهدي هذا البحث . . .

الى كل من ساندني وضحى من اجلي وبذل الغالي والنفيس من أجل تحقيق هذا النجاح في حياتي فشكرا الى هذا الإنسان .

الشكر والتقدير

قال تعالى ﴿ وَمَنْ يَشْكُرْ فإنَّمَا يَشْكُرُ لنَفْسِهِ ﴾

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نتقدم بالشكر لجامعة كلية العلوم الصرفة قسم الرياضيات .

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واتوجه بالشكر لكل عائلتي والأصدقا لانهم لم يتركوني يوماَ وقدموا الي

الدعم فجزا الله الجميع عني خير الجزا

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Abstract

The main objective of our work is to find some topologies on the set containing four elements, then we extracted the interior and closure and then we extracted weak sets using definition β -open set then we calculated interior and closure of β -open set, and then we extracted the upper and lower probability for some topologies and β -open set.

Introduction

The science of topology extends its roots to the era of the Greek civilization, where the Greeks studied the concept of continuous, but the science of topology did not appear in its current state until the beginning of this century when Frechet published in 1906 -his thesis that dealt with the function of distance and relationship There is a difference between it and the concept of continuity, but the two scholars Ries, Haus, and Dorff later showed that there is no need for this conjunction, and continuity can be studied without referring to the distance coupling, and with this, what is called general topology appeared. In general, any group whose elements fulfill some hypotheses constitutes a mathematical system that is consistent (consistent if its arguments, results, and hypotheses are not contradictory (this method was born in the past in the subject of Euclidean geometry). In recent years, mathematics has developed rapidly after the group theory was known at the beginning of the twentieth century. Whereas, any group whose elements fulfill certain hypotheses is called a mathematical sentence that fulfills the hypotheses, and in this case there is more than one mathematical system such as groups, rings, Euclidean geometry, metric spaces, topological spaces. (topological spaces). Topology is a word translated from the English word topology, and the word topology is divided into two syllables, the first syllable (Topo) which is of Greek origin to topo's (which means "place"), and the second syllable is (logy). Which is of Greek origin (Logos) which means "study", if we make the process of linking the meanings in the word, we find that topology is modern engineering in the study of all the structures and components of the different spaces. I began to study the subject of topology on the set of real numbers and then on the Euclidean level.[2]

Chapter One

Some Topologies on The Sets Containing Four Elements

In this chapter, we will show set of basic definitions on which our solutions are based, to a set of questions that we worked on through the four elements, which are {a, b, c, d, },where made 10 topologies and extracted (interiors and closures) of β - open at the end of the research, we studied the probabilities that are (upper and lower) on the topologies that we knew, in addition to the probabilities on the topologies for that β - open set.

Definition 1.1 [5]

Let X be a nonempty set and τ be a family of subsets X (i.e. $\tau \subseteq$ IP (x)). We say t is topology on X if satisfy the following conditions:

1. X, $\Phi \in \tau$

2. If U,V $\in \tau$, then U$\cap$ V $\in \tau$

The finite intersection of elements from t is again an element of T.

3. If U $\in\tau$; a$\in\Lambda$ then Uren $U_{a\epsilon\wedge}U_a \in \tau\forall\, a \in\wedge$.

The arbitrary (finite or infinite) uinion of elements of t is again an element of τ.

We called a pair (X, τ) topological space.

Definition 1.2 [5]

Let (X, τ) be a topological space .The subsets of X belonging to τ are called open set in the space X .i.e.,

If A⊆ X and A∈ τ then A open set

Definition 1.3 [5]

Let (X , τ) be a topological space. The subset of X is called closed set in the space X if its complement X \ A is open set. We will denoted the family of closed sets by $\mathcal{F}$.i.e.,

IfA ⊆ X and A ∈ $\mathcal{F}$ then A closed set

Definition 1.4 [1]

Let (X , τ) be a topological space and let A⊆X. A point x ∈ A is called an interior point of A if there exists an open set U ∈ τ containing x such that x ∈ U ⊆A. The set of all interior points of A is called the interior of A and is denoted by A° or Int(A)I. e,.

$$A^\circ = \{x \in A : \exists U \epsilon \tau : x \ \epsilon U \subseteq A\}$$

$$x \ \epsilon A^\circ \leftrightarrow \ \exists U \epsilon \tau : \epsilon U \subseteq A$$

Definition 1.5 [1]

Let (X , τ) be a topological space and let A be a subset of X. Then the intersection of all τ -closed containing the set A is called the closure of A and denoted by $\bar{A}$ or CI(A). i.e. CI(A)=$A^{C^{\circ}C}$.

Definition 1.6[2]

A subset A of a space X is said to be β -open if A ⊆ $\bar{A}^{\circ -}$ and the complacent β-open is called β -closed set.

.

Definition 1.7 [4]

Let (X, τ) be a topological space and let A⊆X. A point x∈A is called an β - interior point of A iff there exists an β - open set U∈ τ containing x such that $X \epsilon U \subseteq A$ The set of all β -interior point of A is called the β -interior of A and is denoted by β -A∘ or β –Int (A) i.e..

$$\beta\, A^{\circ} \; = \{x \in \; A\colon \exists \; U \in \; \tau; X \in U \subseteq A\}$$

$$X \in \beta \leftrightarrow A^{\circ} \; \leftrightarrow \; \exists U \epsilon \tau\colon X \epsilon U \subseteq A$$

Definition 1.8 [4]

Let (X , τ) be a topological space and let A be a subset of X. Then the intersection of all β -closed containing the set A is called the β -closure of A and denoted - by β -$\bar{A}$ or β -CI(A). i .e., β -CI(A=$A)^{C^{\circ}C}$

Definition 1.9

- $\underline{\rho}(A^{\circ}) = \underline{P}\,(\text{int(A)}) = \dfrac{\text{number elements of } A^{\circ}}{\text{number elements of X}}$
- $\bar{\rho}(\bar{A}) = \bar{p}(\text{c}L(\bar{A})) = \dfrac{\text{number elements of } \bar{A}}{\text{number elements of X}}$
- $\beta\;.\;\underline{\rho}\text{-}(A^{\circ})=\underline{P}(\;\beta.\;\text{in (A)}) = \dfrac{\text{number elements of } \beta\text{.int (A)}}{\text{number elements of X}}$
- $\beta.\;\bar{\rho}\;(\bar{A})= \bar{\rho}\;((\;\beta.\text{cl(A)}) = \dfrac{\text{number elements of } \beta\text{.cl(A)}}{\text{number elements of X}}$

Example 1

$\tau_1 = \{ X, \phi \} \Rightarrow \tau_1^C = \{X, \phi \}$

$\beta.\boldsymbol{\tau_1}(X)$

$= \{ X , \phi , \{ a\}, \{b\}, \{c\}, \{d\} , \{a, b\}, \{a, c\}, \{ b, d\}, \{a, d\}, \{b, c\}, \{c, d\},$

$\{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$

$\beta.\boldsymbol{\tau}_1^C =$

$\{ X, \phi , \{a\}, \{ b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, d\}, \{b, c\}, \{c, d\}$,{a,b,c},

{a,b,d},{a,c,d},{b,c,d}}

$\boldsymbol{\tau_1}$	**{a}**	**{b}**	**{c}**	**{d}**	**{a,b}**	**{a ,c}**	**{a,d}**	**{b,c}**	**{b,d}**	**{c,d}**	**{a,b,c}**	**{a,b,d}**	**{a,c,d}**	**{b,c,d}**
$\underline{\boldsymbol{p}}(\boldsymbol{A}^{\circ})$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$
$\overline{\boldsymbol{p}}\ (\overline{\boldsymbol{A}})$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{0}{4}$
$\beta\_\ \underline{\boldsymbol{P}}(\boldsymbol{A}^{\circ})$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
$\beta\_\overline{\boldsymbol{p}}(\overline{\mathbf{A}})$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$

Example 2

$\boldsymbol{\tau_2} = \{ X, \phi, \{a\}\}$

$\boldsymbol{\tau_2^C} = \{ \phi , X , \{ b, c, d\}\}$

$\beta.\boldsymbol{\tau_2} = \{ X, \phi , \{a\}, \{ a, b\}, \{ a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\beta.\boldsymbol{\tau_2^C} = \{X, \phi \{b, c, d\}, \{ c, d\}, \{b, d\}, \{ b, c\}, \{d\}, \{ c\}, \{b\}\}$

$\boldsymbol{\tau_2}$	**{a}**	**{b}**	**{c}**	**{d}**	**{a,b}**	**{a ,c}**	**{a,d}**	**{b,c}**	**{b,d}**	**{c,d}**	**{a,b,c}**	**{a,b,d}**	**{a,c,d}**	**{b,c,d}**
$\underline{\boldsymbol{p}}(\boldsymbol{A}^\circ)$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$
$\overline{\boldsymbol{p}}\ (\overline{\boldsymbol{A}})$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$
$\beta\_\boldsymbol{p}(\boldsymbol{A}^\circ)$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{6}{4}$	$\frac{0}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{0}{4}$
$\beta\_\overline{\boldsymbol{p}}\ (\overline{\boldsymbol{A}})$	$\frac{4}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$

Example 3

$\boldsymbol{\tau}_3 = \{ X, \phi, \{b\}\}$

$\boldsymbol{\tau}_3^C = \{ \phi , X , \{ a, c, d\}\}$

$\beta . \boldsymbol{\tau}_3 = \{ X, \phi , \{a\}, \{ a, b\}, \{ b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$

$\beta . \boldsymbol{\tau}_3^C = \{ X, \phi \{b, c, d\}, \{ c, d\}, \{a, d\}, \{ a, c\}, \{d\}, \{ c\}, \{a\}\}$

$\boldsymbol{\tau}_3$	**{a}**	**{b}**	**{c}**	**{d}**	**{a,b}**	**{a ,c}**	**{a,d}**	**{b,c}**	**{b,d}**	**{c,d}**	**{a,b,c}**	**{a,b,d}**	**{a,c,d}**	**{b,c,d}**
$\underline{\boldsymbol{p}}(\boldsymbol{A}^\circ)$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$
$\overline{\boldsymbol{p}}\ (\overline{\boldsymbol{A}})$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
$\beta\_\underline{\boldsymbol{P}}(\boldsymbol{A}^\circ)$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$
$\beta\_\overline{\boldsymbol{p}}(\overline{\boldsymbol{A}})$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$

Example 4

$\boldsymbol{\tau}_4 = \{ X, \phi, \{d\}\}$

$\boldsymbol{\tau}_4^C = \left\{ \phi , X , \{ a, b, c\}\right\}$

$\beta.\boldsymbol{\tau}_4 = \{ X, \phi , \{d\}, \{ b, d\}, \{a, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

$\beta.\boldsymbol{\tau}_4^C = \left\{ \phi, X, \{a, b, c\}, \{ a, c\}, \{b, c\}, \{ a, d\}, \{c\}, \{b\}, \{a\}\right\}$

$\boldsymbol{\tau}_4$	{a}	{b}	{c}	{d}	{a,b}	{a ,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{\boldsymbol{p}}(\boldsymbol{A}^{\circ})$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\overline{\boldsymbol{p}}\ (\overline{\boldsymbol{A}})$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$
$\beta\_\underline{\boldsymbol{p}}(\boldsymbol{A}^{\circ})$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
$\beta\_\boldsymbol{p}(\overline{\boldsymbol{A}})$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$

Example 5

$\boldsymbol{\tau}_5 = \{ X, \phi, \{a\}, \{a, b\}\}$

$\boldsymbol{\tau}_5^C = \{ \phi , X , \{ b, c, d\}, \{c, d\}\}$

$\beta.(\boldsymbol{\tau}_5^C) = \{ X, \phi , \{a\}, \{ a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\beta.\left(\boldsymbol{\tau}_5^C\right) = \left\{ \phi, X, \{b, c, d\}, \{ c, d\}, \{b, d\}, \{ b, c\}, \{d\}, \{c\}, \{b\}\right\}$

$\boldsymbol{\tau}_5$	**{a}**	**{b}**	**{c}**	**{d}**	**{a,b}**	**{a ,c}**	**{a,d}**	**{b,c}**	**{b,d}**	**{c,d}**	**{a,b,c}**	**{a,b,d}**	**{a,c,d}**	**{b,c,d}**
$\underline{\boldsymbol{p}}(\boldsymbol{A}^\circ)$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{0}{4}$
$\overline{\boldsymbol{p}}\ (\overline{\boldsymbol{A}})$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$
$\beta\_\underline{\boldsymbol{p}}(\boldsymbol{A}^\circ)$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{0}{4}$
$\beta\_\overline{\boldsymbol{p}}(\overline{\mathbf{A}})$	$\frac{4}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$

Example 6

$\boldsymbol{\tau}_6 = \{ X, \phi, \{a\}, \{a, C\}\}$

$\boldsymbol{\tau}_6^C = \{ \phi , X , \{ b, c, d\}, \{b, d\}\}$

$\beta.(\boldsymbol{\tau}_6) = \{ X, \phi , \{a\}, \{ a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\beta.(\boldsymbol{\tau}_6^C) = \{ \phi, X, \{b, c, d\}, \{ c, d\}, \{b, d\}, \{ b, c\}, \{d\}, \{c\}, \{b\}\}$

$\boldsymbol{\tau}_6$	{a}	{b}	{c}	{d}	{a,b}	{a ,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{\boldsymbol{p}}(\boldsymbol{A}^{\circ})$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{0}{4}$
$\overline{\boldsymbol{p}}\,(\overline{\boldsymbol{A}})$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$
$\beta\_\underline{\boldsymbol{p}}(\boldsymbol{A}^{\circ})$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{0}{4}$
$\beta\_\overline{\boldsymbol{p}}(\overline{\mathbf{A}})$	$\frac{4}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$

Example 7

$\boldsymbol{\tau}_7 = \{ X, \phi, \{a\}, \{a, d\}\}$

$\boldsymbol{\tau}_7^C = \{ \phi , X , \{ b, c, d\}, \{b, c\}\}$

$\beta.(\boldsymbol{\tau}_7) = \{ X, \phi , \{a\}, \{ a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\beta.(\boldsymbol{\tau}_7^C) = \{ \phi, X, \{b, c, d\}, \{ c, d\}, \{b, d\}, \{ b, c\}, \{d\}, \{c\}, \{b\}\}$

$\boldsymbol{\tau_7}$	**{a}**	**{b}**	**{c}**	**{d}**	**{a,b}**	**{a ,c}**	**{a,d}**	**{b,c}**	**{b,d}**	**{c,d}**	**{a,b,c}**	**{a,b,d}**	**{a,c,d}**	**{b,c,d}**
$\underline{\boldsymbol{p}}(\boldsymbol{A}^{\circ})$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$
$\overline{\boldsymbol{p}}\ (\overline{\boldsymbol{A}})$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$
$\beta\_\underline{\boldsymbol{p}}(\boldsymbol{A}^{\circ})$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{0}{4}$
$\beta\_\overline{\boldsymbol{p}}(\overline{\mathbf{A}})$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$

Example 8

$\boldsymbol{\tau}_8 = \{ X, \phi, \{a\}, \{a, b, C\}\}$

$\boldsymbol{\tau}_8^C = \{ \phi , X , \{ b, c, d\}, \{d\}\}$

$\beta.(\boldsymbol{\tau}_8) = \{ X, \phi , \{a\}, \{ a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\beta.(\boldsymbol{\tau}_8^C) = \{ \phi, X, \{a, c, d\}, \{ c, d\}, \{b, d\}, \{ b, c\}, \{d\}, \{c\}, \{b\}\}$

$\boldsymbol{\tau}_8$	{a}	{b}	{c}	{d}	{a,b}	{a ,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{p}(A^\circ)$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$
$\overline{p}\,(\overline{A})$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$
$\beta\_\underline{p}(A^\circ)$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{0}{4}$
$\beta\_\overline{p}\,(\overline{A})$	$\frac{4}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$

Example 9

$\boldsymbol{\tau}_9 = \{ X, \phi, \{a\}, \{a, b, d\}\}$

$\boldsymbol{\tau}_9^C = \{ \phi , X , \{ b, c, d\}, \{c\}\}$

$\beta.(\boldsymbol{\tau}_9) = \{ X, \phi , \{a\}, \{ a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\beta.(\boldsymbol{\tau}_9^C) = \{ \phi, X, \{b, c, d\}, \{ c, d\}, \{b, d\}, \{ b, c\}, \{d\}, \{c\}, \{b\}\}$

$\boldsymbol{\tau}_9$	{a}	{b}	{c}	{d}	{a,b}	{a ,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{\boldsymbol{p}}(\boldsymbol{A}^{\circ})$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{0}{4}$
$\overline{\boldsymbol{P}}\,(\overline{\boldsymbol{A}})$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$
$\beta\_\,\underline{\boldsymbol{P}}(\boldsymbol{A}^{\circ})$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{0}{4}$
$\beta\_\,\overline{\boldsymbol{P}}(\overline{\mathbf{A}})$	$\frac{4}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$

Example 10

$\boldsymbol{\tau}_{10} = \{ X, \phi, \{a\}, \{b, c, d\}\}$

$\boldsymbol{\tau}_{10}^{C} = \{ \phi , X , \{ b, c, d\}, \{a\}\}$

$\beta.(\boldsymbol{\tau}_{10})$
$= \{ X, \phi , \{a\}, \{ b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}$

$. \{a, c, d\}, \{b, c, d\}\}$

$\beta.(\boldsymbol{\tau}_{10}^{C})$
$= \{ \phi, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{ a, b, c\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{a, b\}$

{a,b},{d},{c},{b},{a}}

$\boldsymbol{\tau}_{10}$	**{a}**	**{b}**	**{c}**	**{d}**	**{a,b}**	**{a ,c}**	**{a,d}**	**{b,c}**	**{b,d}**	**{c,d}**	**{a,b,d}**	**{a,b,c}**	**{a,c,d}**	**{b,c,d}**
$\underline{\boldsymbol{P}}(\boldsymbol{A}^{\circ})$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$
$\overline{\boldsymbol{p}}\ (\overline{\boldsymbol{A}})$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$
$\beta\_\underline{\boldsymbol{P}}(\boldsymbol{A}^{\circ})$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
$\beta\_\overline{\boldsymbol{p}}(\overline{\mathbf{A}})$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$

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