University of Babylon

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Department of Mathematics

Extract Lower and Upper Probability of Pre Open Set

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بسم الله الرحمن الرحيم

" يرفع الله الذين آمنوا منكم والذين أوتوا العلم

درجات والله خبير بما تعملون "

صدق الله العلي العظيم

سورة المجادلة: الآية (11)

الاهداء

الى من علمني كيف اقف بكل ثبات وأتخطى الصعاب,وعلمني ان الدنيا كفاح

الى من كلله الله بالهيبة والوقار,الى من احمل اسمه بكل فخر, الى من سعى لأجل راحتي ونجاحي

أبي الغالي

الى من اسقتني الحب والحنان , الى مصدر الأمان والملجأ الابدي التي ما زالت تمدني

بالقوة والثقة بالنفس , الى الداعم الاكبر بدعائها

الى امي الحبيبة

الى من يجعل حياتي مفعمة بالدفئ , الى اليد الداعمة والحنونة

الى من يؤنسني في كل حياتي

اخواتي العزيزات

الى من قيل فيه سنشد عضدك بأخيك , الى من مد يده لي في اوقات ضعفي

الي اخي وسندي

الى من اشاركهم لحظاتي وتفاصيل ايامي , الى من يفرحون لنجاحي

الى كل الذين غمروني بالحب ورزقني الله بهم لأعرف من خلالهم طعم الحياة

صديقاتي

الشكر والتقدير

الحمد لله عز وجل الذي الهمنا الصحة والعافية والعزيمة والذي وفقنا لاتمام هذا البحث العلمي.

فالحمد لله حمدا كثيرا.

اتقدم بجزيل الشكر والأمتنان الى الدكتور الفاضل لؤي عبد الهاني السويدي

واتقدم بجزيل الشكر الى الدكتور الفاضل مصطفى حسن هادي

على كل ما قدماه لي من توجيهات ومعلومات قيمة ساهمت في اثراء موضوع دراستي في جوانبه المختلفة .

Contents

Abstract

The main objective of our work is to find some topologies on the set containing four elements, then we extracted the interior and closure and then we extracted weak sets using definition pre open set and we calculate the interior and closure, then we calculated upper and lower probability for the some topologies and pre open set.

Introduction

In mathematics, general topology (or point set topology) is the branch of topology that deals with the basic set-theoretic definitions and constructions used in topology. It is the foundation of most other branches of topology, including differential topology, geometric topology, and algebraic topology.

The fundamental concepts in point-set topology are continuity, compactness, and connectedness

The terms 'nearby', 'arbitrarily small', and 'far apart' can all be made precise by using the concept of open sets. If we change the definition of 'open set', we change what continuous functions, compact sets, and connected sets are. Each choice of definition for 'open set' is called a topology. A set with a topology is called a topological space.[5]

Chapter One

Some Topologies on The Sets Containing Four Elements

Chapter One

In this chapter , we will show set of basic definitions on which our solutions are based, to a set of questions that we worked on through four elements, which are {a, b, c, d }, where we made 10 topologies and extracted (interior and closure) of pre open at the end of research, we studied the probabilities that are (upper and lower) on the topologies that we knew, in addition to the probabilities on the topologies for that pre open set.

Definition 1.1 [2]

Let X be a nonempty set and τ be a family of subsets X(i.e. $\tau \subseteq IP(x)$). We say τ is topology on X if satisfy the following conditions :

1. $X , \emptyset \in \tau$
2. If $U,V \in \tau$, then $U \cap V \in \tau$

   The finite intersection of elements from τ is again an element of τ .

3. If $U_a \in \tau$; $a \in \Lambda$ then $U_{a \in \Lambda} U_a \in \tau \ \forall a \in \Lambda$.

   The arbitrary (finite or infinite) uinion of elements of τ is again an element of τ .

   We called a pair (X , τ) topological space.

Definition 1.2 [4]

Let (X ,τ) be a topological space . The subset of X belong to τ are called open set in the space X i.e. ,

If $A \sqsubseteq X$ and $A \in \tau$ then A open set .

Definition 1.3 [4]

Let (X,) be a topological space. The subset of X is called closed set in the space X if its complement X \A is open set. We will denoted the family of closed sets by Ƒ.i.e.,

If A⊆ X and A ∈ Ƒ *then* A closed set

Definition 1.4 [3]

Let (X,τ) be a topological space and let A⊆X. A point x∈A is called an interior point of A iff there exists an open set U∈τ containing x such that x∈U⊆A.The set of all interior points of A is called the interior of A and is denoted by A° or Int(A) i.e,.

$A^{\circ} = \{x \in A \colon \exists\ U \in \tau;\ x \epsilon\ U \subseteq A\}$

$x \epsilon A^{\circ} \leftrightarrow \exists\ U \epsilon\ \tau;\ x \epsilon\ U \subseteq A.$

Definition 1.5 [4]

Let (X, τ) be a topological space and let A be a subset of X . Then the intersection of all τ-closed containing the set A is called the closure of A and denoted by $\overline{\text{A}}$ or Cl(A). i.e. Cl(A)=$A^{c\circ c}$.

Definition 1.6 [1]

A subset A of a space X is said to be pre-open if A⊆ int (cl(A)) And the complement pre-open is called pre-closed set.

Definition 1.7 [1]

Let (X,τ) be a topological space and let A⊆X. A point x∈A is called an pre-interior point of A iff there exists an pre-open set U∈τ containing x such that x∈U⊆A .

The set of all Pre-interior point of A is called the pre-interior of A and is denoted by pre-A° or pre-Int(A) i.e.

Pre-$A^{\circ} = \{x \in A: \exists\ U \in \tau; x \epsilon U \subseteq A\}$

$x \epsilon$ Pre-$A^{\circ} \leftrightarrow \exists\ U \epsilon \tau; x \epsilon U \subseteq$ A

Definition 1.8 [1]

Let (X, τ) be a topological space and let A be a subset of X . Then the intersection of all pre-closed containing the set A is called the pre-closure of A and denoted by

pre- $\bar{A}$ or pre-Cl(A). i.e pre-Cl(A)=$A^{c \circ c}$.

Definition 1.9

- $(A^{\circ}) = \underline{P}\,((A)) = \frac{number\ element\ of\ A^{\circ}}{number\ element\ of\ X}$

- $\bar{\rho}(\ \bar{A}) = \bar{\rho}\ (cl(\bar{A})) = \frac{number\ element\ of\ \bar{A}}{numer\ element\ of\ X}$

- pre. $(A^{\circ}) = \underline{P}$(pre. $int(A)) = \frac{number\ element\ of\ Pre-int(A)}{number\ element\ of\ X}$

- pre. $\bar{\rho}(\ A) = \bar{\rho}$(pre. $cl(A)) = \frac{number\ element\ of\ Pre-cl(A)}{number\ element\ of\ X}$

Example 1

τ_1={X,∅, {a},{c},{a,c}}

τ_1^c={X, ∅, {b,c,d} ,{a,b,d}, {b,d}}

pre. $\boldsymbol{\tau_1}$={X, ∅, {a} ,{c} ,{a,b} ,{a,c} ,{a,d} ,{b,c} ,{c,d} ,{a,b,c} ,{a,b,d} ,{a,c,d},{b,c,d}}

pre . $\boldsymbol{\tau_1^c}$**=** {X,∅, {b,c,d},{a,b,d},{c,d},{b,d},{b,c},{a,d},{a,b},{d},{c},{b} ,{a}}

τ_1	{a}	{b}	{c}	{d}	{a,b}	{a,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{P}(A^o)$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{1}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$
$\overline{P}(A)$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$
pre-$\underline{P}(A^o)$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{1}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
pre-$\overline{P}(A)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$

Example 2

τ_2= {X, ∅, {b},{b,c}}

τ_2^c={X, ∅, {a,c,d},{a,d}}

pre. τ_2={X, ∅, {b},{c},{a,b},{a,c},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d} ,{b,c,d}}

pre .τ_2^c= {X,∅,{a}, {b},{c},{d},{a,b},{a,c},{a,d],{b,d},{c,d},{a,b,d},{a,c,d}}

τ_2	{a}	{b}	{c}	{d}	{a,b}	{a,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{P}(A^o)$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$
$\overline{P}(\overline{A})$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
pre-$\underline{P}(A^o)$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
pre-$\overline{P}(\overline{A})$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$

Example 3

τ_3={X,∅, {c},{d},{c,d}}

τ_3^c={X,∅, {a,b,d},{a,b,c},{a,b}}

pre .τ_3={X,∅,{c},{d},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

pre .τ_3^c={X,∅, {a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,c},{a,b,d}}

τ_3	{a}	{b}	{c}	{d}	{a,b}	{a,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{P}(A^o)$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$
$\overline{P}(\overline{A})$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$
pre-$\underline{P}(A^o)$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
pre-$\overline{P}(\overline{A})$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$

Example 4

τ_4={X,∅, {c},{a,b},{a,b,c}}

τ_4^c={X,∅ ,{d},{c,d},{a,b.d}}

Pre .τ_4={X, ∅,{c},{a,b},{c,d},{a,b,c} {a,b,d}}

Pre .τ_4^c={X,∅,{c},{d},{a,b},{c,d},{a,b,d}}

τ_4	{a}	{b}	{c}	{d}	{a,b}	{a,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{P}(A^{o})$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\overline{P}(\overline{A})$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$
pre-$\underline{P}(A^{o})$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{2}{4}$
pre-$\overline{P}(\overline{A})$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$

Example 5

τ_5={X,∅ ,{c},{a,d},{a,c,d}}

τ_5^c={X,∅, {b},{b,c},{a,b,d}}

pre .τ_5={X, ∅, {c},{a,d},{b,c},{a,b,d},{a,c,d}}

pre .τ_5^c={X,∅, {b},{c},{a,d},{b,c},{a,b,d}}

τ_5	{a}	{b}	{c}	{d}	{a,b}	{a,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{P}(A^o)$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{1}{4}$
$\overline{P}(\overline{A})$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$
pre-$\underline{P}(A^o)$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$
pre-$\overline{P}(\overline{A})$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$

Example 6

τ_6={X,∅, {c},{b,d},{b,c,d}}

τ_6^c={X,∅,{a},{a,c},{a,b,d}}

pre . τ_6={X,∅, {c},{a,c},{b,d},{a,b,d},{b,c,d}}

pre . τ_6^c={X, ∅, {a},{c},{a,c},{b,d},{a,b,d}}

τ_6	{a}	{b}	{c}	{d}	{a,b}	{a,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{P}(A^{o})$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{3}{4}$
$\overline{P}(\overline{A})$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$
pre-$\underline{P}(A^{o})$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$
pre-$\overline{P}(\overline{A})$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$

Example 7

τ_7={X, ∅, {d},{a,d}}

τ_7^c={X,∅, {b,c},{a,b,c}}

pre .τ_7={X,∅,{d},{a,d},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}

pre .τ_7^c={X,∅ ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}

τ_7	{a}	{b}	{c}	{d}	{a,b}	{a,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{P}(A^o)$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{1}{4}$
$\overline{P}(\overline{A})$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
pre-$\underline{P}(A^o)$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
pre-$\overline{P}(\overline{A})$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$

Example 8

τ_8={X,∅,{d},{b,d}}

τ_8^c={X,∅, {a,c}{a,b,c}}

pre .τ_8={X, ∅ ,{d},{a,d},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}

pre .τ_8^c={X,∅,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}

τ_8	{a}	{b}	{c}	{d}	{a,b}	{a,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{P}(A^o)$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{2}{4}$
$\overline{P}(\overline{A})$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$
pre-$\underline{P}(A^o)$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
pre-$\overline{P}(\overline{A})$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$

Example 9

τ_9={X,∅ ,{d},{c,d}}

τ_9^c={X, ∅, {a,b},{a,b,c}}

pre .τ_9={X, ∅, {d},{a,d},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}

pre .τ_9^c={X,∅, {a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}

τ_9	{a}	{b}	{c}	{d}	{a,b}	{a,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{P}(A^o)$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$
$\overline{P}(\overline{A})$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$
pre-$\underline{P}(A^o)$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{0}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
pre-$\overline{P}(\overline{A})$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{2}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$

Example 10

τ_{10}={X, ∅, {d},{a,b,c}}

τ_{10}^{c}={X,∅, {d},{a,b,c}}

pre .τ_{10}={X,∅,{d},{a,b,c}}

pre .τ_{10}^{c}={X, ∅, {d},{a,b,c}}

τ_{10}	{a}	{b}	{c}	{d}	{a,b}	{a,c}	{a,d}	{b,c}	{b,d}	{c,d}	{a,b,c}	{a,b,d}	{a,c,d}	{b,c,d}
$\underline{P}(A^{o})$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\overline{P}(\overline{A})$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$
pre-$\underline{P}(A^{o})$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
pre-$\overline{P}(\overline{A})$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$

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