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Solving the Ordinary Differential Equations by using Elzaki Transform

A Graduation Research

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بِسْمِ اللهُ الرَّحْمَنِ الرَّحِيم

(يَرْفَعِ اللهُ الَّذِينَ آمَنُوا مِنكُمْ وَالَّذِينَ أُوتُوا الْعِلْمَ دَرَجَاتٍ)

صدق الله العلي العظيم

سورة المجادلة الآية (11)

الشكر والتقدير

الحمد لله رب العالمين والصلَّاة والسلَّام على سيد المرسلين وعلى اله اجمعين

لابد لنا ونحن نخطوا خطواتنا الاخيرة في الحياة الجامعية

من وقفة نعود إلى أعوام قضيناها في رحاب الجامعة مع أساتذتنا الكرام الذين قدموا لنا الكثير باذلين بذلك جهودا كبيرة في بناء جيل الغد تبعث الأمة من جديد

وقبل أن نمضي أقدم أسمى آيات الشكر والامتنان والتقدير والمحبة الذين حملوا أقدس رسالة في الحياة. . .

إلى الذين مهدوا لنا طريق العلم والمعرفة.

إلى أساتذتنا الأفاضل وبالأخص الى الأستاذة (هدى عامر هادي)

الإهداء

الى خاتم النبيين محمد الصادق الامين (ص) معلم البشرية الاول

الى من عينهما على دربي أمي وأبي العظيمان حفظهما الله ورعاهما برعايته.. .. ..

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أهدي جهدي المتواضع هذا. . .

CONTENTS

Abstract

In this research a new integral transformation was applied, called the Elzaki transform method, and it is still not widely known, but it was found that this is a more efficient and easy way to get an exact solution of ordinary linear differential equations . In the chapter one we will talk about the definition of differential equations and the properties. and in Chapter tow we will introduce the basic principles about the Elzaki transform that we will apply to solve ordinary differential equation examples.

Introduction

The differential equations represent the most important phenomena occurring in the world. This phenomenon is importance in applied mathematics, physics, and issues related to engineering. The importance of obtaining the exact solution of differential equations is still a big problem that needs new methods to discover new exact or approximate solutions.

Several techniques such as A domain decomposition method [1], Variational iteration method [2, 3], Homotopy perturbation method [4], Laplace decomposition method [5, 6, 8], Sumudu decomposition method [7], have been used Such that is the original function and to solve linear and nonlinear partial differential equations. The main aim of this paper is to solve the differential equations by using of ELzaki transform.

Chapter one

Chapter one

1.1 Introductionp

In this chapter we present the definition of DE differential equations in Section 1.2 and the properties such as order , linearity and homogeneous and discuss in the sections 1.3 , 1.4 and 1.5 respectively , in section 1.6 we will display the definition of the solution of a DE. Boundary and initial conditions will introduce in 1.7

1.2 Definition of Differential Equations

- **Ordinary differential equations**: containing dependent variables with one independent variable and derivatives of this variable.
- **Partial differential equations**: contain mathematical functions of more than one independent variable with their partial derivatives.

Examples of the DEs are given by

$u_t = Ku_{xx},$ (1.1)

$u_t = K\left(u_{xx} + u_{yy}\right),$ (1.2)

$x = 5y - 4y'$ (1.3)

These equations describe the heat flow in one-dimensional space, two-dimensional space, and three-dimensional space respectively. In (1.1), the dependent variable u = u(x, t) depends on the position x and on the time variable t. However, in (1.2), u = u(x, y, t) depends on three independent variables, the space variables x, y and the time variable t. Other examples of DEs are given by

$u_{tt} = c2u_{xx}$ (1.4)

$u_{tt} = c2\left(u_{xx} + u_{yy}\right)$ (1.5)

$u_{tt} = c2\left(u_{xx} + u_{yy\_}u_{yy}\right)$ (1.6)

These equations describe the wave propagation in one-dimensional space, two-dimensional space, and three-dimensional space respectively. Moreover, the unknown functions in (1.4), (1.5), and (1.6) are defined by u = u(x, t), u = u(x, y, t) , and u = u(x, y, z, t) respectively.

The well-known Laplace equation is given by:
$u_{xx} + u_{yy} = 0,$ (1.7)
$u_{xx} + u_{yy} + u_{zz} = 0,$ (1.8)
where the function u does not depend on the time variable t. Moreover, the Burgers equation and the KdV equation are given by
$u_t + uu_x - vu_{xx} = 0,$ (1.9)
$u_t + 6uu_x + u_{xxx} = 0,$ (1.10)
Respectively, where the function u depends on x and t

1.3 Order of DE

he order of a DE is the order of the highest derivative that appears in the equation. For example, the following equations

$$u_x - u_y = 0 ,$$

$$u_{xx} - u_t = 0 , \qquad (1.11)$$

$$u_y - uu_{xxx} = 0$$

Example 1. The order of the following DEs:

$$(a) u_t = u_{xx} + u_{yy} \qquad (b) u_x + u_y = 0$$

$$(c) u\ 4u_{xx} + u_{xxx} = 2 \qquad (d) u_{xx} + u_{yxxy} = 1$$

(a)The highest derivative contained in this equation is u_{xx} or u_{yy}. The DE is therefore of order two.

(b) The highest derivative contained in this equation is $u_x$or u_y . The DE is therefore of order one

(c) The highest derivative contained in this equation is u_{xxx} . The DE is therefore of order three.

(d) The highest derivative contained in this equation is u_{yxxy} . The DE is therefore of order four.

1.4. Linear and Nonlinear DEs

Differential equations are classified as linear or nonlinear. A differential equation is called linear if:

1. The power of the dependent variable and each derivative contained in the equation is one, and
2. The coefficients of the dependent variable and the coefficients of each derivative are constants or independent variables. However, if any of these conditions is not satisfied, the equation is called nonlinear.

Example 1. To classify the following DEs as linear or nonlinear

$$(a)\ xu_{xx} + yu_{yy} = 0$$

$$(b) uu_t + xu_x = 2$$

$$(c) u_x + \sqrt{u} \ = x$$

$$(d) u_{rr} + r\,u_r + r_2 u_{\theta\theta} = 0$$

a) The power of each derivative u_{xx} and u_{yy} is one. In addition, the coefficients of the derivatives are the independent variables x and y respectively. Hence, the DE is linear
b) Although the power of each derivative is one, but u_t has the dependent variable u as its coefficient. Therefore, the DE is nonlinear.
c) The equation is nonlinear because of the term $\sqrt{u}$.
d) The equation is linear because it satisfies the two necessary conditions.

1.4.1. Some Linear Differential Equations

As stated before, linear differential equations arise in many areas of scientific applications, such as the diffusion equation and the wave equation. In what follows, we list some of the well-known models that are of important concern:

1. The heat equation in one-dimensional space is given by

$$u_t = ku_{xx} \qquad (1.12)$$

Where k is a constant

2. The wave equation in one-dimensional space is given by

$$u_{tt} = c2u_{xx} \qquad (1.13)$$

where c is a constant.

3. The Laplace equation is given by

$$u_{xx} + u_{yy} = 0 \qquad (1.14)$$

4. The Linear Schrodinger's equation is given by

$$iu_t + u_{xx} = 0. ,i = \sqrt{-1} \qquad (1.15)$$

5. The Telegraph equation is given by

$$u_{xx} = au_{tt} + bu_t + \text{cu} \qquad (1.16)$$

where a, b and c are constants. It is to be noted that these linear models and others will be studied in details in the forthcoming chapters.

1.4.2. Some Nonlinear Differential Equations

It was mentioned earlier that differential equations arise in different areas of mathematical physics and engineering, including fluid dynamics, plasma physics, quantum field theory, nonlinear wave propagation and nonlinear fiber optics [8]. In what follows we list some of the well-known nonlinear models that are of great interest:

1. The Advection equation is given by

$$u_t + uu_x = f(x,t). \qquad (1.17)$$

2. The Burgers equation is given by

$$u_t + uu_x = \alpha u_{xx} \qquad (1.18)$$

3. The Korteweg de-Vries (KdV) equation is given by

$$u_t + auu_x + bu_{xxx} = 0. \qquad (1.19)$$

4. The modified KdV equation (mKdV) is given by

$$u_t - 6u2\, u_x + u_{xxx} = 0 \qquad (1.20)$$

5. The Boussinesq equation is given by

$$u_{tt} - u_{xx} + 3(u2)xx - u_{xxxx} = 0 \qquad (1.21)$$

6. The sine-Gordon equation is given by

$$u_{tt}\ \ u_{xx} = \alpha \sin u \qquad (1.22)$$

7.The sinh-Gordon equation is given by

$$u_{tt} - u_{xx} = \alpha \sinh u \qquad (1.23)$$

8. The Liouville equation is given by

$$u_{tt} - u_{xx} = e \pm u \qquad (1.24)$$

9. The Fisher equation is

$$u_t = Du_{xx} + u(1-u). \quad (1.25)$$

10. The Kadomtsev-Petviashvili (KP)equation is given by

$$(u_t + auu_x + bu_{xxx})x + u_{yy} = 0 \quad (1.26)$$

11. The K(n,n)equation is given by

$$u_t + a(un)x + b(un)xx = 0 \quad ,n > 1 \quad (1.27)$$

12. The Nonlinear Schrodinger (NLS) equation is

$$iu_t + u_{xx} + \gamma|u|2u = 0. \quad (1.28)$$

13. The Camassa-Holm(CH)equation is given by

$$u_t - u_{xxt} + au_x + 3uu_x = 2u_x u_{xx} + uu_{xxx}. \quad (1.29)$$

14. The Degasperis-Procesi (DP) equation is given by

$$u_t - u_{xxt} + au_x + 4uu_x = 3u_x u_{xx} + uu_{xxx}. \quad (1.30)$$

The above-mentioned nonlinear differential equations and many others will be examined in the forthcoming chapters. These equations are important and many give rise to solitary wave solutions.

1.5. Homogeneous and Inhomogeneous DEs

Differential equations are also classified as homogeneous or inhomogeneous.

Adifferential equation of any order is called homogeneous if every term of the DE contains the dependent variable u or one of its derivatives, other- wise, it is called an inhomogeneous DE. This can be illustrated by the following example.

Example 3. To classify the following differential equations as homogeneous or inhomogeneous

$(a)\ u_t = 4u_{xx}$
$(b)\ u_t\ = u_{xx} + x$

$(c)\ u_{xx} + u_{yy} = 0$

$(d)\ u_x + u_y = u + 4$

Have we:

a) The terms of the equation contain derivatives of u only, therefore it is a homogeneous DE.
b) The equation is an inhomogeneous DE, because one term contains the in-dependent variable x.
c) The equation is a homogeneous DE.
d) d) The equation is an inhomogeneous DE.

1.6. Solution of a DE

A solution of a DE is a function u such that it satisfies the equation under discussion and satisfies the given conditions as well. In other words, for u to satisfy the equation, the left hand side of the DE and the right hand side should be the same upon substituting the resulting solution. This concept will be illustrated by examining the following examples. Examples of differential equations subject to specific conditions will be examined in the coming chapters.

Example 4. The function $(x, y) = \sin x\ e - 4t$ is a solution of the following DE

$$u_t = 4u_{xx} \qquad (1.31)$$

Since

Right Hand Side (RHS)= $4u_{xx} = -4\sin x\, e - 4t =$ LHS

Example 5. The function $u(x, y) = \sin x \sin y + x2$ is a solution of the following DE

$$u_{xx} = u_{yy} + 2 \qquad (1.32)$$

Since

Left Hand Side (LHS)= $u_{xx} = -\sin x \sin y + 2$

Right Hand Side (RHS)= $u_{yy} + 2 = -\sin x \sin y + 2 = LHS$

Example 6. Show that $u(x, y) = \cos x \cos t$ is a solution of the following DE

$$u_{tt} = u_{xx} \qquad (1.33)$$

Since

Left Hand Side (LHS)= $u_{tt} = -\cos x \cos t$

Right Hand Side (RHS)= $u_{xx} = -\cos x \cos t =$ LHS

1.7. Initial Conditions

It was indicated before that the DEs mostly arise to govern physical phenomenon such as heat distribution, wave propagation phenomena and phenomena of quantum mechanics. Most of the DEs, such as the diffusion equation and the wave equation, depend on the time t . Accordingly, the initial values of the dependent variable u at the starting time t = 0 should be prescribed. It will be discussed later that for the heat case, the initial value u(t = 0), that defines the temperature at the starting time, should be prescribed. For the wave equation, the initial conditions u(t = 0) and ut(t = 0) should also be prescribed.

Chapter two

Chapter tow :

2.1 Introduction

In this chapter, we will introduce the basic properties for the ELzaki Transform in sections 2.2, 2.3 and 2.4, then we will apply this transform to solve the ordinary differential equations in section 2.5.

2.2 ELzaki Transform

Elzaki Transform is derived from the classical Fourier integral[**1**]. Based on the mathematical simplicity of the Elzaki transform and its fundamental properties. Elzaki transform was introduced by Tarig Elzaki to facilitate the process of solving ordinary and partial differential equations in the time domain.[**2**, **3**]

Typically, Fourier, Laplace and Sumudu transforms are the convenient mathematical tools for solving differential equations,[**5**, **6**, **7**, **8**] also Elzaki transform and some of its fundamental properties are used to solve differential equations.

A new transform called the Elzaki transform defined for function of exponential order we consider functions in the set A defined by:

$$A = \left\{ f(t) : \exists M, k_1, k_2 > 0, |f(t)| < Me^{t/k_j}, if \ \ t \in (-1)^j \ \ X[0, \infty) \right\} \qquad (2.1)$$

For a given function in the set A, the constant M must be finite number, k_1, k_2 may be finite or infinite. The ELzaki transform denoted by the operator E (.) defined by the integral equations

$$E[f(t)] = T(v) = v \int_0^\infty f(t) e^{\frac{-t}{v}} \, dt \ , \quad t \geq 0, \ \ k_1 \leq v \leq k_2 \qquad (2.2)$$

The variable v in this transform is used to factor the variable t in the argument of the function f. This transform has deeper Connection with the Laplace transform. We also present many different of properties of this new transform and Sumudu transform, few properties exptent The purpose of this study is to show the applicability of this interesting new transform and its efficiency in solving the linear differential equations.

2.3 Elzaki Transform of the Some Functions.

For any function $f(t)$.$we\ assume\ that\ the$ integral equation (2) exist. The Sufficient Conditions for the existence of ELzaki transform are that $f(t)$ for $t \geq 0$ be piecewise continuous and of exponential order, Otherwise ELzaki transform may or maynot exist In this section we find ELzaki transform of simple functions.

In this section we find ELzaki transform of simple functions.

(i) $let\ f(t) = 1$, $then$:

$$E(1) = v\int_0^\infty e^{\frac{-t}{v}}\,dt = v\left[-ve^{\frac{-t}{v}}\right]_0^\infty = v^2$$

(ii) $let\ f(t) = t$,$then$:

$$E(t) = v\int_0^\infty te^{\frac{-t}{v}}\,dt$$

Integrating by parts to find that:$E(t) = v^3$

In the general case if $n \geq 0$ is integer number, then.

$\boldsymbol{E}(t^n) = n!\,v^{n+2}$

$$(iii)\ E[e^{at}] = v\int_0^\infty e^{\frac{-t}{v}}\,e^{at}dt = \frac{v^2}{1-av}$$

This result will be useful, to find ELzaki transform of:

$E[\sin at] = \frac{av^3}{1+a^2\,v^2}$, $E[\cos at] = \frac{v^2}{1+a^2v^2}$

$E[\sinh at] = \frac{av^3}{1-a^2v^2}$, $E[\cosh at] = \frac{av^3}{1-a^2v^2}$

<u>Theorem:</u>

Let T(v) is the Elzaki transform of $\left[E\big(f(t)\big) = T(v)\right]$.then:

(i) $E[f'(t)] = \frac{T(v)}{v} - v\,f(0)$

$(ii)\ E[f''^{(t)}] = \frac{T(v)}{v^2} - f(0) - vf'(0)$

$(iii)\ E[f^{(n)}(t)] = \frac{T(v)}{v^n} - \sum_{k=0}^{n-1} v^{2-n+k}\, f^{(k)}(0)$

Proof:

$(i)\ E[f'(t)] = v\int_0^\infty f'(t)\, e^{\frac{-t}{v}}\, dt$

Integrating by parts to find that :

$$E[f'(t)] = \frac{T(v)}{v} - v\, f(0)$$

(ii) *let* $g(t) = f'(t),$ *then*:

$E[g'(t)] = \frac{1}{v}\, E[g(t)] - v\, g(0)$

we find that by using (i),

$E[f''(t)] = \frac{T(v)}{v^2} - f(0) - vf'(0)$

(iii) *Can be proof by mathmatical induction*

2.4 The Inverse of Elzaki Transform

Definition: Let the functions $f(u) = E\{f\}$ is the Elzaki transform of the function $f(t)$, then$f(u)$ called the inverse transform of the function $f(u)$ and we will write it as :

$$f(t) = E^{-1}\{f(u)\}$$

Remark: The inverse transform has the linear combination property, i.e.

$$E^{-1}\{\sum_{k=1}^{n} a_k f_k(u)\} = \sum_{k=1}^{n} a_k E^{-1}\{f_k(u)\}$$

2.5 Application of ELzaki Transform of Ordinary Differential Equations.

As stated in the 2.1 of this paper, the ELzaki transform can be used as an effective tool. For analyzing the basic characteristics of a linear system governed by the differential equation in response to initial data. The following examples illustrate the use of the ELzaki transform in solving certain initial value problems described by ordinary differential equations.

Consider the first-order ordinary differential equation.

$$\frac{dx}{dt} + p(x) = f(t)\,,\ t > 0 \qquad (2.3)$$

With the initial condition

$$x(0) = a \qquad (2.4)$$

Where p and a are constants and $f(t)$ is an external input function so that its ELzaki transform exists.

Applying ELzaki transform of the equation (3) we have :

$$\frac{1}{v}\,\bar{x}\,(v) - vx(0) + p\bar{x}\,(v) = \bar{f}\,(v)$$

$$\bar{x}(v) = \frac{v\bar{f}\,(v)}{1+pv} + \frac{av^2}{1+pv}$$

The inverse ELzaki transform leads to the solution. The second order linear ordinary differential equation has the general form

$$\frac{d^2y}{dx^2} + 2p\frac{dy}{dx} + qy = f(x)\ \,,x > 0 \qquad (2.5)$$

The initial conditions are

$$y(0) = a\,,\ \frac{dy}{dx}(0) = b$$

Are constants. Application of the ELzaki transforms b and p, q, a where to this general initial value problem gives

$$\frac{1}{v^2}\bar{y}(v) - y(0) - vy'(0) + 2p\left[\frac{1}{v}\bar{y}\,(v) - vy(0)\right] + g\bar{y}(v) = \bar{f}(v) \qquad (2.6)$$

The use of (6) leads to the solution for $\bar{y}(v)$ as

$$\bar{y}(v) = \frac{v^2 \bar{f}(v)}{qv^2+2pv+1} + \frac{av^2}{qv^2+2pv+1} + \frac{(b+2pa)v^3}{qv^2+2pv+1}$$

The inverse transform gives the solution.

Example (1): Consider the first order differential equation

$$\frac{dy}{dx} + y = 0 \quad , \qquad y(0) = 1$$

Take ELzaki transform to this equation gives:

$$\frac{1}{v}E(y) - vy(0) + E(y) = 0$$

$$E(y) = \frac{v^3}{1+v} \quad and \;\; y(x) = e^{-x}$$

$The\ New\ Integral\ Transform\ ''ELzaki\ Transform''$

Where $E(y)$ is the ELzaki transform of the function $y(x)$.

Example (2): Solve the differential equation

$$y' + 2y = x \quad , \qquad y(0) = 1$$

Take ELzaki transform to this equation is

$$\frac{1}{v}E(y) - vy(0) + 2E(y) = v^3$$

$$E(y) = \frac{(v^3+v)v}{1+2v}$$

$$E(y) = \frac{1}{2}v^3 + \frac{5}{4}\left(\frac{v^2}{1+2v}\right) - \frac{1}{4}v^2$$

The inverse transform of this equation gives the Solution:

$$y(x) = \frac{1}{2}x + \frac{5}{4}e^{-2x} - \frac{1}{4}$$

Example (3): Let us consider the second–order differential equation

$$y'' + y = 0 \ \ , \ \ y(0) = y'(0) = 1$$

we take ELzaki transform to this equation gives

$\frac{1}{v^2}E(y) - 1 + E(y) - v = 0$

We solve this equation for $E(y)$ to get

$E(y) = \frac{v^2}{v^2+1} + \frac{v^3}{v^2+1}$

The inverse ELzaki transform of this equation is simply obtained as

$y(x) = \sin x + \cos x$

Example (4): Consider the following equation

$y'' - 3y' + 2y = 0 \ \ , \ \ y(0) = 1 \ , \ y'(0) = 4$Take ELzaki transform of this equation we find that:

$E(y) = \frac{v^2(v+1)}{(2v-1)(v-1)} = v^2\left[\frac{2}{v-1} - \frac{3}{2v-1}\right]$

$E(y) = \frac{-2v^2}{1-v} + \frac{3v^2}{1-2v}$

Then the solution is $y(x) = -2e^x + 3e^{2x}$

Example (5): Let the second order differential equation:

$$y'' + 9y = \cos 2t \ \ if \ \ y(0) = 1 \ , \qquad y\left(\frac{\pi}{2}\right) = -1$$

Since y'(0) is not know, let y'(0) =c .

Take Elzaki transform of this equation and using the conditions, we have

$T(v) = v^2\left[\frac{v^2}{(1+4v^2)(1+9v^2)}\right] + \frac{cv^3}{1+9v^2} + \frac{v^2}{1+9v^2} = v^2\left[\frac{4}{5(1+9v^2)} + \frac{cv}{3(1+9v^2)} + \frac{1}{5(1+4v^2)}\right]$

And invert to find the solution.

$y = \frac{4}{5}\cos 3t + \frac{c}{3}\sin 3t + \frac{1}{5}\cos 2t$

To determine c not that $y\left(\frac{\pi}{2}\right) = -1$ thin we find $c = \frac{12}{5}$ then,

$$y = \frac{4}{5}\cos 3t + \frac{4}{5}\sin 3t + \frac{1}{5}\cos 2t$$

Example (6): Solve the differential equation:

$$y'' - 3y' + 2y = 4e^{3t} \ , \ y(0) = -3 \ , \ y'(0) = 5$$

Taking the Elzaki transforms both side of the differential equation and using the given conditions we have,

$$\frac{T(v)}{v^2} + 3 - 5v - 3\left[\frac{T(v)}{v} + 3v\right] + 2T(v) = \frac{4v^2}{1-3v}$$

$$\left[\frac{1}{v^2} - \frac{3}{v} + 2\right] T(v) = \frac{4v^2}{1-3v} + 14v - 3$$

$The\ New\ Integral\ Transform''ELzaki\ Transform''$

Or

$$T(v) = v^2\left[\frac{4}{1-2v} + \frac{2}{1-3v} - \frac{9}{1-v}\right]$$

Inverting to find the solution in the form.

$$y(t) = 4e^{2t} + 2e^{3t} - 9e^{t}$$

Conclusion

The definition and application of the new transform " Elzaki transform" to the solution of ordinary differential equations has been demonstrated we have proven that the Elzaki transformation is a very effective and convenient method for solving differential equations in time domain. The purpose of this study is to show the applicability and efficiency of this new interesting transformation to solve linear differential equations.

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