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بسم الله الرحمن الرحيم

اللّه نور السموات والأرض مثل نوره كمشكاة فيها مصباح المصباح في زجاجة الزجاجة كأنها كوكب دري يوقد من شجرة مباركة زيتونة لا شرقية ولا غربية يكاد زيتها يضيء ولو لم تمسسه نار نور على نور يهدي الله لنوره من يشاء ويضرب الله الأمثال للناس والله بكل شيء عليم

سورة النور أيه (35)   صدق الله العلي العظيم

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that God will Accept this work with good acceptance

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Dedication

To those who enlightened my mind others ....

my instruction

To the light that illuminates my path of success ....

my parents

To my companions and loved ones ...

my brother and sister

Give this humble effort and hope that it will be

accepted

Kawther Hatem Alwan

List of symbols

Symbol	Description	page
Max f(x)	The maximum of objective function	11
Min f(x)	The minimum of objective function	11
F(x)	Objective function	11

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Contents

Abstract

Linear programming (LP, or linear optimization) is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. Linear programming is a special case of Mathematical programming (mathematical optimization) More formally, linear programming is a technique for the Optimization of a linear objective function, In this work, we studied some method, to solve linear programming such as, graphical method, simplex method and the dual method and we gave some numerical examples . The graphical method is useful if we have two variables, where the simplex method is useful for two variables or more, where use the dual method if we face a difficult to find the solution by using the simplex method.

Chapter One

Introduction

Linear programming is a basic and important technique that helps decision makers make sound decisions in a scientific way . linear programming problems are part of the mathematical programming problems that include linear and nonlinear ones. Then mathematical programming is in turn, part of a more comprehensive subject, called operations Research or operational Research which all relate to issues of organization and management, issues of tran sportation, agriculture industry, and so forth

Linear mathematical programming is a matter of preference,and here preference problems are meant those mathematical problems that seek to maximize or minimize alinear function. Set to linear mathematical constraints as well .

A historical perspective:-

During World War II, as a result of limited military resources, the British government commissioned a team of senior scientists to study Issues of how to distribute its resources. military and commensurate with the best air and land defense situation. The studies of this team have been called Operations Research or Operational Research. Then this designation was applied to all research and studies that deal with software issues and decision-making issues. The encouraging results of the British Operations Research Team urged the US Air Force to form a team to carry out the necessary studies in this field. This team found that the traditional methods of preference issues, such as the speculative method Lagrange, for example, is not of great use in solving linear programming problems, which necessitated finding more effective methods in 1947 while George Dantzig, a member of

the American team for operations research, developed the simplified method (the Simplex) to solve the problem of linear programming, but the details of this method were not published until 1956, and after the publication of the simple method (the Simplex), a great acceleration occurred in the use and development of linear programming [1].While the French mathematician Jean-Baptiste Fourier was alerted to Its potential contributions in 1923. The first use or application of linear programming was by the economist George Stilger In the forties, where he aimed to determine the components of the daily food (Diet), which will provide the body with the minimum needs of vitamins, iron and other substances, and at the lowest possible cost[2].

Advantages of using linear programming

 The most important and prominent advantages of using linear programming are as follows [3] :

●Logical thinking and giving the full comprehensive vision of issues

●Determine the best solutions by evaluating costs and profits

●**The database provided by linear programming is optimized for resources**

●**Apply different modifications to solutions according to changing conditions.**

Disadvantages of using linear programming

In contrast to the advantages of linear programming, there are many disadvantages and drawbacks, which are as follows

•**Difficulty defining the objective function**

•**Assuming linear programming that it Is a linea relationship between inputs and outputs, but in reality It is difficult to do so, because some real-world problems such as commercial and industrial ones can be non-line**

•**Linear programming assumes that it Is based on the principle of perfect market competition, but in fact it is not**

•**Linear programming assumes constant returns, but in reality returns are variable, either by increase or decrease**

•The linear programming technique is a complex technique. Solving the problem requires enlarging or minimizing a specific variable in difficult and complex ways of expression because it includes a large number of large calculations, such as the simplex method

•Providing experimental solutions that may include error, which makes the process of finding solutions difficult

•There is no single global goal that all organizations strive for, and this contradicts the aspiration to reach linear programming, as it focuses on achieving only one goal, for example, reducing costs paid

•It is difficult to apply linear programming to the various set of problems, where the values of the coefficients are not probable

•There is room for variables to take a non-negative integer as well as rational values, yet It contains variables that have an integer value

Areas of application of linear programming

Linear programming technology is involved in some areas of life, the most prominent of which are [4]

•Food and agriculture

Farmers apply the linear programming technique in their work by specifying all the details of the crops that need to be grown, such as how much, and how to use them efficiently so that they can increase their financialreturns.

In the field of nutrition, this technology is an important way to help plan nutritional needs, such as providing healthy food baskets at reasonable costs to needy families, or identifying nutritious foods in order to prevent disease. In addition to related data such as prices and other, with restrictions or limitations such as culture, or proper nutrition guidelines.

•Engineering

This technology is used to help solve design and manufacturing problems, as linear programming is adopted as a basic tool in optimizing aerodynamic shape, as in air foil networks to create a wing free of impacts and defects, with high accuracy, basedn on foundations and constraints.

•Transport sector

Increases cost and time efficiency, as linear programming takes routes and times into account, airlines use it to optimize their profits according to seat prices, customer demand, fligh scheduling androutes.

•Efficient manufacturing

Every step of the manufacturing process must work efficiently to make profits, so companies use linear

programming to determine how much raw material to use, how much time each machine needs in the manufacturing process, and other things related to the production process.

•Energy field

Linear programming provides a way to improve both traditional and modern electrical energy systems represented by renewable energy sources such as wind energy. And solar photovoltaic energy, as this technology improves electrical load requirements by taking into account generators

and transmission lines; distribution and warehousing, while keeping costs sustainable for profit.

Chapter Two

Introduction

The science of linear programming is one of the applied sciences that has achieved a wide spread, as it is one of the scientific methods Help give different alternatives to the decision-making process in a more accurate way to rely on information

Appropriateness in choosing the best alternative to solve the problems facing the decision-maker and enabling him to make the best decision

By using mathematical models to address and analyze the influencing factors and then make the appropriate decision

Linear programming has applications in engineering, economics and management sciences .

Basic concepts

Linear programming:

They are defined as linear (first-order) mathematical expressions that are used to solve mathematical models that indicate to the objective function with basic variables and certain constraints and determinants , provided that the variables are not negative[5] .

Linear programming components:

1-Existence of an Objective function (Profit maximization or cost minimization)

Max or Min Z= $\sum_{j=1}^{n} C_j X_j$

2-Existence of a certain number of basic variables

It requires only two variables to be expressed in the basic variables (X1, X2).

3-Existence of constraints they are expressed differently the relationship is less than or equal to <= or greater than or equal to >=.

$$\sum_{j=1}^{n} a_{ij} x_j (\leq, \geq, =) b_i$$

4-Non-negative.   Xj >= 0

Forms of Linear programming Models [6] .

1-General form of linear programming models

1-Objective function

Min or Max Z=$c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$

2-Constraint:

Subject to (s. t.)

$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \geq \text{ or } = \text{ or } \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq or = or \geq b_2.$$

.

.

.

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq or = or \geq b_m$$

$$x_1, x_2, \ldots, x_n \geq 0$$

Where:

(n):Number of variables.

(m):The number of restrictions.

(c1,c2,......,cn) and (b1,b2,......,bn) and matrix a[i,j] and (i=1,2,.......,m and j=1,2,......n) its values and nature are determined according to the problem to be solved.

2 -*Canonical form of linear programming models*

The difference between the canonical form of the linear programming model and the general form of the linear programming model is as follows:

i-The objective function (Z) in the general form of the linear programming model is either of type (Max) or of type (Min) , While in the canonical form of the linear programming model it is of type (Max) only.[8]

ii-The constraints signs in the general form of the linear programming model are (>= ,= , <=) while in the canonical form of the linear programming model they are smaller and equal to only the components of the linear programming model are the same in both the general and canonical forms , as follows

Canonical form of the linear programming model :

Max Z=$c_1x_1 + c_2x_2 + \cdots + c_nx_n$

S.t.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$
$$\cdot$$
$$\cdot$$
$$\cdot$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$
$$x_1, x_2, \ldots., x_n \geq 0$$

The canonical form of the linear programming model

can be abbreviated as follows .

Max Z=$\sum_{j=1}^{n}$ Cj Xj

S.t.

$$\sum_{j=1}^{n} aij\ xj \leq bi$$

i=1,2,3,.......,m

$$j=1,2,3,\ldots\ldots,n$$

$$X_j \geq 0$$

The canonical form is used in some special cases of linear programming models

The general form can be converted to the canonical form using the following rules :

i-The minimization of the objective function can be converted in to a maximization and vice versa by multiplying the objective function by (-1)

ii-A constraint greater than or equal to is converted to less than or equal to by multiplying both sides of the inequality by (-1)

iii-The equality constraints , the first is less than or equal to ($\leq$) and the second is greater than or equal to ($\geq$) , than converting the second using rule (2) a bove .

iv-An absolute value constraint can be converted to two constraints of type less than or equal to ($\leq$) .

3-Standard form of linear programming model :

The difference between the standard form of the linear programming model and the general form of the linear programming model is as follows :

i-The objective function in the general form of the linear programming model is either of type (max) or of type (min), as well as the standard form of the linear programming model

ii- The signs of constraints in the general form of the linear programming model are ($\geq$, $=$, $\leq$) while in the standard form of the linear programming model they are equal to (=) only after adding the non- negative dummy variables (slack variable) and are symbolized by (Si $\geq$ 0) and are in the form of (+S) when the inequality sign is less than or equal to and (-S) when the inequality sign is greater than or equal to and we do not add anything in the case of equality

iii- The right side of the constraints is non-negative (bi $\geq$ 0) .

In light of this, the standard formula for the linear programming model is as follows :

Max Z=$c_1x_1 + c_2x_2 + \cdots + c_nx_n + 0S_1 + 0S_2 + \cdots + 0S_m$

S.t.

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + S_1 = b_1$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + S_2 = b_2$$

.

.

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + S_m = b_m$$

$$x_1, x_2, \ldots \ldots, x_n \geq 0$$

$$S_1, S_2, \ldots \ldots, S_m \geq 0$$

The standard form of the linear programming model can be abbreviated as follows :

The abbreviated form of the standard form of the linear programming model :

Max Z=$\sum_{j=1}^{n}$ Cj Xj + 0Si

S.t. $\sum_{j=1}^{n}$ aij xj + 0Si = bi

j=1,2,......, n   i=1,2,......,m

Si ≥0

<u>Types the solution to the linear programming problems: [7]</u>

1-Feasible solution: is the solution X=(x1,x2,....,xn) which does not conflict with one or more of the actual constraints and satisfies all constraints to extract the values of Xj (regardless of whether the values of Xj are positive, negative, or zero)

2- Infeasible Solution: A solution that contradicts one or more of the actual constraints of the problem.

3- Basic feasible solution: The basic solution is called feasible if the number of positive variables in it does not exceed the number of constraints m (the values of X_j are positive or zero as much as the number of constraints).

4-Optimal Solution : It is the best feasible solution that achieves all constraints. In addition, it makes the value of the objective function at its maximum or minimum end. The issue may have a unique optimal solution or multiple optimal solutions. Multiple optimal solutions usually give greater flexibility to decision makers when they implement one of them.

Method to solve a linear programming problem :

(1) Graphical Method

(2) Simplex Method

(3) The Dual Method

1-Graphical Method

This method is one of the easiest methods used to solve the linear programming method, and this method is characterized by the fact

that it deals with simple problems, that is, in the case of the presence of only two variables, and its usefulness is to clarify possible solutions and how to obtain the optimal solution.

This method depends on the graph of the variables of the problem in the frame of horizontal and vertical coordinates to determine the area of the feasible solution (which is a closed area with all the restrictions contained in the problem) and then the extreme points are determined (which are the points that are on the borders of the area of the feasible solution) that maximize or reduce the objective function .

The basic steps of this method can be summarized in the following points .

1- The sign of the inequalities is converted to equality, and then we draw the straight lines representing these constraints

2- We determine the solution area for each constraint, where we substitute the origin point in the constraint. If the constraint relationship is achieved, then the solution area is towards the origin point. Otherwise, the solution area moves away from the origin point. In the case of equality, the solution area is only the points located on .that straight line .

3- All straight lines must fall in the first quadrant now Xj≥0 otherwise the line is extended to reach the first quadrant unless the non-negativity constraint indicates otherwise.-

4- The polygon formed by the intersection of the solution regions of all lines represents the area of the feasible solution .

5- We define a point or extreme points in the region of the feasible solution that achieve the maximum or minimum value of the objective function (called the angle points method) .

6- We draw the line representing the objective function (profit lines) as starting from the origin point when (Z = 0) as an additional way to confirm the point or points of the extreme optimal solution.
Some criticisms are taken on this method, as if the problem to be solved contains three variables or more, it is difficult to represent them graphically, because it requires the use of special engineering theories, as well as accuracy in drawing to obtain accurate results, which leads to considering that this method is of limited interest, so we resort to the simplex method to solve problems related to more than two variables in the issue [8].

(2) Simplex Method

The simplex method is one of the most important methods adopted for solving linear programming problems because it addresses the most complex problems that cannot be solved using the graphical method, i.e. those problems that include a large number of variables. This method has the following characteristics .[9]

i-This method is used to solve any type of linear programming problem

ii-This method is based on following sequential and sequential steps that are determined in advance and are implemented at each stage of the solution until the optimal solution is reached.

iii-This method provides solutions that fall within the scope and limits of the facility's capabilities until it reaches the optimal solution.

iv-This method discloses the optimal solution once it is reached.

In order to reach the optimal solution using the simplex method, the following steps must be followed

1-The first step : Is to convert the problem from the legal formula to the standard formula by adding stagnant variables with the problems of maximization and positive sign and with the problems of decreasing the excess variables are subtracted with the addition of positive artificial variables to the negative sign equation, in order to achieve the non-negativity condition

2 - The second step: converting the problem from the standard formula to the tabular form in the form of a table called the simplified table divided into a number of rows and columns, as the rows are the coefficients of the variables in the objective function and the constraints, while the columns are represented by the basic variables of the problem and the surplus and artificial variables .

3- The Third step : The pivot column that corresponds to the largest absolute value in the target function is chosen with in the simplified table .

4- The fourth step: The pivot value is determined by dividing each value in the constants column (b) by the corresponding value in the pivot column, and the pivot value represents the lowest positive division.

5- The fifth step: The pivot row that contains the pivot value is selected

6- The sixth step: converting the pivot value into one, and then converting the rest of the pivot column values into zeros .

7- The seventh step: test the optimization of the solution .

(3) The Dual Method :

The original problem (the primal) may require lengthy solution procedures. Conversely , solving the corresponding problem (the dual

) may be difficult , and the solution may be easier if the dual is converted to the original model. One of the most important developments of linear programming is in the duality theory.

**The steps of converting the primal problems to dual problems **

1-The objective function is converted from Min to Max and Max to Min .

2-The constraints must be ($\leq$ in the case of Max) and ($\geq$ in the case of Min) .

3- The number of variables in the primal model = the number constraints in the dual model , and vice versa .

4- The Coefficients of the objective function (Cj) in the primal model = the right-hand said (bi) in the constraints of the dual model , and vice versa.

5- Changing the Coefficients matrix , so that the row becomes column and the column becomes row .

6- Changing the variables symbols , for example , (X) to (Y) and vice versa .

7-The objective functions symbol in the dual model must be different from the symbol in the primal model .

8- The optimal solution of the objective function in the primal problem and the dual problem are always identical .

9- We can obtain the values of the optimal solution to the primal problem from the values of the optimal solution of the dual problem , and vice versa.

The mathematical formula for the primal problem and the corresponding (the dual) problem

The primal problem :

$$\text{Max } Z = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

Subject to

$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \leq b_1$$

$$a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n \leq b_2$$

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$$am_1 X_1 + am_2 X_2 + \cdots + a_{mn} X_n \leq bm$$

$$X_1, X_2, X_3, \ldots \ldots, X_n \geq 0$$

The dual problem :

$$\text{Min } Z = b_1 Y_1 + b_2 Y_2 + \ldots.. + b_m Y_m$$

Subject to

$$a_{11} Y_1 + a_{12} Y_2 + \cdots + a_{m1} Y_m \geq C_1$$

$$a_{21} Y_1 + a_{22} Y_2 + \cdots + a_{m2} Y_m \geq C_2$$

.

.

.

$$a_{1n} Y_1 + a_{2n} Y_2 + \cdots + a_{mn} Y_m \geq C_n$$

$$Y_1, Y_2, \ldots\ldots\ldots, Y_m \geq 0$$

The primal problem and the corresponding (the dual) problem can

be summarized as follows :

The primal problem	|The dual problem
Max Z= Cn Xn	Min Z'= bm Ym
Subject to AX ≤ b	Subject to A'Y ≥ C
And X ≥ 0	And Y ≥ 0

Chapter Three

Example (3-1)

Convert the following linear programming model into matrix canonical form [10].

$$\text{Max } Z = 30X_1 + 20X_2$$

Sub. to

$$5X_1 + X_2 \geq 60$$
$$5X_1 + 3X_2 = 20$$
$$4X_1 + 3X_2 \leq 10$$
$$X_1, X_2 \geq 0$$

Solution :

We must change the inequality (≥) into inequality (≤) by multiplying it by (-1) .

We must change the inequality (=) into inequality (≥) and inequality (≤) and we must change the inequality (≥) into inequality (≤) by multiplying it by (-1)

$$\text{Max } Z = 30X_1 + 20 \quad _2$$

Sub. to

$$-5X_1 - X_2 \leq -60$$

$$5X_1 + 3X_2 \leq 20$$

$$-5X_1 - 3X_2 \leq -20$$

$$4X_1 + 3X_2 \leq 10$$

$$X_1, X_2 \geq 0$$

Example (3-2)

Convert the linear programming problem to standard form :

Max $Z = 3X_1 + 2X_2$

Sub. to

$$X_1 - 2X_2 \geq 3$$

$$3X_1 + 4X_2 \leq 2$$

$$X_1 + 3X_2 = 5$$

Solution:

We must first convert the inequalities to equalities by the introduction

of slack variable S1 $\geq$ 0 and S2 $\geq$ 0 to give :

The objective function Thus :

Max $Z = 3X_1 + 2X_2 + 0S_1 + 0S_2$

Sub. to

$$X_1 - 2X_2 - S_1 = 30$$

$$3X_1 + 4X_2 + S_2 = 20$$

X1	X2

$$X_1 + 3X_2 = 50$$

$$X1, X2, S1, S2 \geq 0$$

$$b1 = 30 > 0, b2 = 20 > 0, b3 = 50 > 0$$

Example (3-3)

Find the optimal solution for the following linear programming by using graphical method [11]

$$\text{Max } Z = 8X_1 + 6X_2$$

$$4X_1 + 2x_2 \leq 60$$

$$2X_1 + 4X_2 \leq 48$$

$$X_1, X_2 \geq 0$$

Solution:

(1)Transfer restrictions to equation as follows

The straight 1 $4X_1 + 2X_2 = 60$

The straight 2 $2X_1 + 4X_2 = 48$

(2)Determine of two points for each straight :

$$4X_1 + 2X_2 = 60 \ldots\ldots\ldots (1)$$

0	30
15	0

-→ (0 , 30)

-→ (15 , 0)

$2X_1 + 4X_2 = 48$ .... (2)

X1	X2
0	12
24	0

--→ (0 , 12)

--→ (24 , 0)
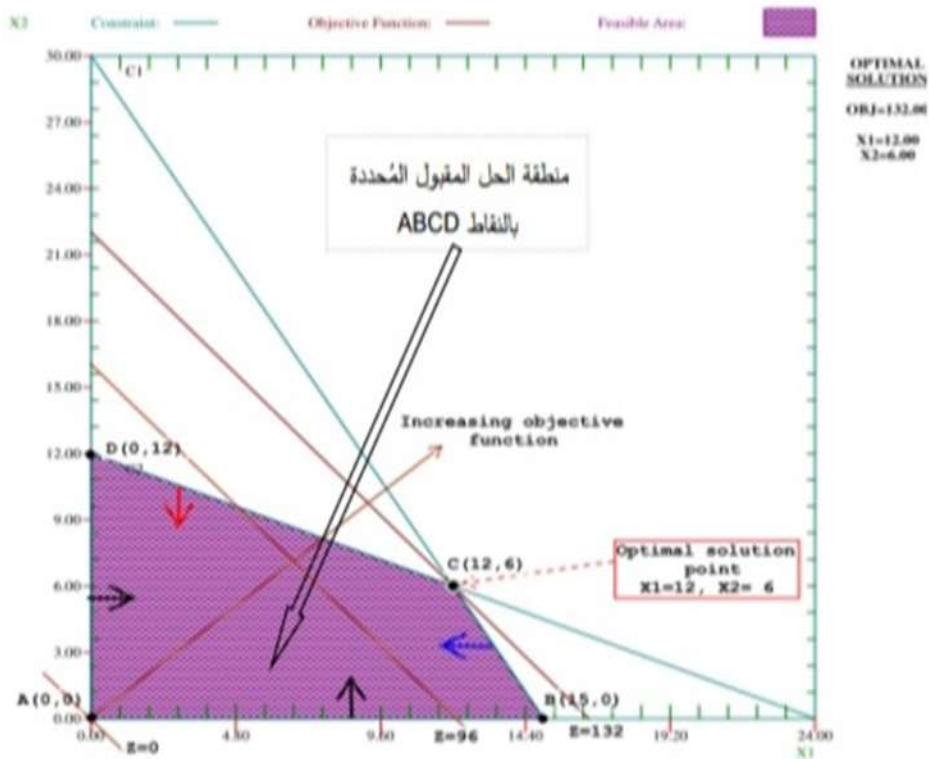
Figure [1] Graph of model limitations and area of acceptable possible

solutions

(3)The joint solution , an area (A B C D) shaded .

points	(X1 , X2)	Z $8X_1 + 6X_2$
A	(0, 0)	0
B	(15 , 0)	120
C	(12 , 6)	132
D	(0 , 12)	72

Max Z = 132

Example (3-4)

Find the optimal solution for the following linear programming by using graphical method:

Min $Z = 2X_1 + 3X_2$

Sub. to

$$X_1 + X_2 \geq 350$$
$$X_1 \geq 125$$
$$X_1 + X_2 \leq 600$$
$$X_1, X_2 \geq 0$$

Solution :

(1)Transfer restrictions to equation as follows

The straight 1 $X_1 + X_2 = 350$

The straight 2 $X_1 = 125$

The straight 3 $X_1 + X_2 = 600$

(2)Determine of two points for each straight :

$X_1 + X_2 = 350$ ....(1)

X1	X2	
0	350	...... (0 , 350)
350	0	.... (350 , 0)

$X_1 = 125$ ....(2)

$X_1 + X_2 = 600$ ....(3)

X1	X2	
0	600	.......(0 , 600)
600	0	.......(600 , 0)

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المُحددة بالنقاط ABCD
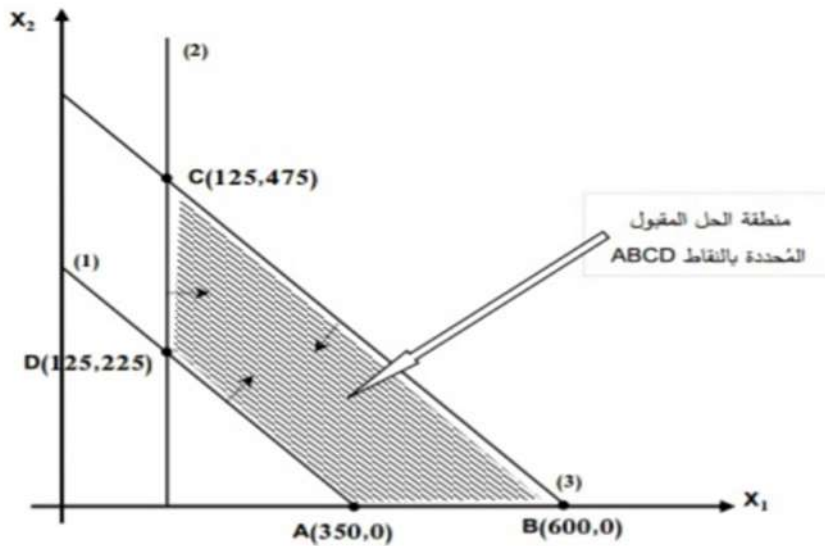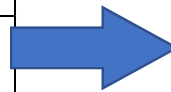
Figure (2) The graph of the constraints of the linear programming model, which shows the area of possible solutions

(3)The joint solution , an area (A B C D) shaded .

points	(X1 , X2)	Z = 2X1 + 3X2
A	(350 , 0)	700
B	(600 , 0)	1200
C	(125 , 475)	1675
D	(125 , 225)	925

Min Z = 700

Example (3-5)

Use the simplex to solve the following problem [12]:

Max $Z = 6X_1 + 8X_2 + 2X_3$

Sub. to

$$X_1 + X_2 \leq 2$$
$$X_1 + + 3X_3 \leq 6$$
$$X_2 \leq 1$$
$$X_1, X_2, X_3 \geq 0$$

Solution:

Step (1) inequalities transfer to the standard form as follows :

Max $Z - 6X_1 - 8X_2 - 2X_3 + 0S_1 + 0S_2 + 0S_3$

Sub. to

$$X_1 + X_2 + + S_1 = 2$$
$$X_1 + + 3X_3 + S_2 = 6$$
$$X_2 + + S_3 = 1$$
$$X_1, X_2, X_3, S_1, S_2, S_3 \geq 0$$

Step (2) Make a table simplex

First stage :

	X1	X2	X3	S1	S2	S3	b
Z	-6	-8	-2	0	0	0	0
S1	1	1	0	1	0	0	2
S2	1	0	3	0	1	0	6

S3	0	1	0	0	0	1	1

Second stage :

	X1	X2	X3	S1	S2	S3	b
Z	-6	0	-2	0	0	8	8
S1	1	0	0	1	0	-1	1
S2	0	0	3	0	1	0	6
X2	0	1	0	0	0	1	0

Third stage :

	X1	X2	X3	S1	S2	S3	b
Z	0	0	-2	6	0	2	14
X1	1	0	0	1	0	-1	1
S2	0	0	3	-1	1	1	5
X2	0	1	0	0	0	1	1

Fourth stage :

(Optimal solution)

	X1	X2	X3	S1	S2	S3	b
Z	0	0	0	16/3	2/3	8/3	52/3
X1	1	0	0	1	0	-1	1

X3	0	0	1	-1/3	1/3	1/3	5/3
X2	0	1	0	0	0	1	1

Because the objective function values are positive values and zero , it indicates that we've come to the optimal solution as follows :

$$X_1 = 1 , X_2 = 1 , X_3 = 5/3 , S_1 = S_2 = S_3 = 0$$

The objective function = optimal profit

Max Z = $6X_1 + 8X_2 + 2X_3$

Max Z = 6 (1) + 8 (1) + 2 (5/3)

Max Z = 52/3 = 17.33

Example (3-6)

Find the optimal solution for (L . P) model using simplex method

Max Z = $3X_1 + 5X_2$

  Sub . to

$$2X_1 + 3X_2 \leq 30$$
$$5X_1 + 4X_2 \leq 60$$
$$X_1 , X_2 \geq 0$$

Solution :

Step (1) inequalities transfer to the standard form as follows :

Max Z – $3X_1 – 5X_2 + 0S_1 + 0S_2 = 0$

Sub. to

$$2X_1 + 3X_2 + S_1 = 30$$

$$5X_1 + 4X_2 + S_2 = 60$$

$$X_1, X_2, S_1, S_2 \geq 0$$

Step (2) Make a table simplex :

First stage :

	X1	X2	S1	S2	b
Z	-3	-5	0	0	0
S1	2	3	1	0	30
S2	5	4	0	1	60

Second stage :

(Optimal solution)

	X1	X2	S1	S2	b
Z	1/3	0	5/3	0	50
X2	2/3	1	1/3	0	10
S2	7/3	0	-4/3	1	20

Because the objective function values are positive values and zero , it indicates that we've come to the optimal solution as follows :

$$X_1 = 0, X_2 = 10, S_1 = 0, S_2 = 20$$

The objective function = optimal profit

Max Z= $3X_1 + 5X_2$

Max Z = 3 (0) + 5 (10)

Max Z = 50

Example (3-7)

Convert the primal problem to the dual problem [13]:

Max Z = $5X_1 + 12X_2 + 4X_3$

Sub. to

$$X_1 + 2X_2 + 3X_3 \leq 5$$

$$2X_1 - X_2 + 3X_3 = 2$$

$$X_1, X_2, X_3 \geq 0$$

Solution :

Primal problem	Dual problem
Max Z = $5X_1 + 12X_2$ $+ 4X_3$	Min Z$'$ = $5Y_1 + 2Y_2 - 2Y_3$
S.t $X_1 + 2X_2 + 3X_3 \leq 5$ $2X_1 - X_2 + 3X_3 \leq 2$ $-2X_1 + X_2 - 3X_3 \leq -2$	S.t $Y_1 + 2Y_2 - 2Y_3 \geq 5$ $2Y1 - Y2 + Y3 \geq 12$ $3Y_1 + 3Y_2 - 3Y_3 \geq$ 4
And X1 , X2 , X3 >= 0	And Y1 , Y2 , Y3 >= 0

Example (3-8)

Convert the primal problem to the dual problem

Max Z $= 3X_1 + 5X_2$

S. to:

$$2X_1 + 6X_2 \leq 50$$
$$3X_1 + 2X_2 \leq 35$$
$$5X_1 - 3X_2 \leq 10$$
$$X_2 \leq 20$$
$$X_1, X_2 \geq 0$$

Solution : Dual problem

Min Z' $= 50Y_1 + 35Y_2 + 10Y_3 + 20Y_4$

S. to $2Y_1 + 3Y_2 + 5Y_3 \geq 3$

$$6Y_1 + 2Y_2 - 3Y_3 + Y_4 \geq 5$$
$$Y_1, Y_2, Y_3, Y_4 \geq 0$$

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الملخص

البرمجــة الخطيـــة (تحســين الخطــي) هـو أسـلوب لتحقيـق أفضـل النتـائج (مثـل أقصـى قـدر مـن الأربـاح أو بأقـل تكلفـه) فـي النمـوذج الرياضـي الـذي يـتم تمثيـل العلاقـات الخطيـة المتطلبـات . البرمجـة الخطيـة هـي حالـه خاصـه مـن البرمجـة الرياضية (الحسـابية الأمثـل) . أكثـر رسـميا ، البرمجـة الخطيـة هــي تقنيــة لاسـتمثال الاسـتفادة مــن وظيفــة الخطيـة الموضـوعية

فـي هذا العمـل درسـنا بعـض الطـرق لحـل البرمجـة الخطيـة مثـل الطريقـة الرسـم البيـاني والطريقـة البسـيطة والطريقـة المزدوجـة وأعطينـا بعـض الأمثلـة التطبيقيـة لها.طريقـة الرسـم البيـاني مفيـدة إذا كـان لـدينا متغيـرين ، حيـث تكـون الطريقـة البسـيطة مفيـدة ا لمتغيـرين أو أكثـر ، حيـث تسـتخدم الطريقـة المزدوجـة إذا واجهنـا صـعوبة فـي إيجـاد الحـل الأمثـل بواسـطة اسـتخدام الطريقـة المبسـطة

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بحث مقدم الى

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من قبل الطالبة

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١٤٤٤هـ   ٢٠٢٣م