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On Neutrosophic Crisp Set

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بِسْمِ اللَّـهِ الرَّحْمَٰنِ الرَّحِيمِ

(وَلَمَّا بَلَغَ أَشُدَّهُ وَاسْتَوَى آتَيْنَاهُ حُكْمًا وَعِلْمًا وَكَذَلِكَ نَجْزِي الْمُحْسِنِينَ)

صَدَقَ اَللَّهُ اَلْعَظِيمُ

سورة القصص

الآية ١٤

الاهداء

إلى صاحب السيرة العطرة، والفكر المُستنير؛

فلقد كان له الفضل الأوَّل في بلوغي التعليم العال

(والدي الحبيب)، أطال الله في عُمره.

إلى من وضعتني على طريق الحياة، وجعلتني رابط الجأش،

وراعتني حتى صرت كبيرًا

(أمي الغالية)، طيَّب الله ثراها.

إلى إخوتي؛ من كان لهم بالغ الأثر في كثير من العقبات والصعاب.

إلى جميع أساتذتي الكرام؛ ممن لم يتوانوا في مد يد العون لي

أُهدي إليكم بحثي

شكر وتقدير

لابد لنا ونحن نخطوا خطوتنا الاخيرة في الحياة الجامعية من وقفه نعود الى أعوام قضيناها في رحاب الجامعة مع اساتذتنا الكرام الذين قدموا لنا الكثير باذلين جهودا كبيره في بناء جيل الغد لتبعث الامة من جديد

ثم أتوجه بجزيل الشكر وعظيم الامتنان الى (أ-مصطفى حسن هادي)

على ما بذله من جهد لغرض الاشراف على بحثي ومتابعته لي بآرائه القيمة ومساعدته لي فجزاه الله خير الجزاء

وقبل ان نمضي نقدم آيات الشكر والامتنان والتقدير والمحبة الى الذين حملوا أقدس رسالة في الحياة الى الذين مهدوا لنا طريق العلم والمعرفة ... الى جميع اساتذتها الافاضل

Subject **page**

Introduction

Since the world is full of indeterminacy, the Neutrosophics found their place into contemporary research. We now introduce for the first time the notions of Neutrosophic Crisp Sets and Neutrosophic Topology on Crisp Sets. We develop the 2012 notion of Neutrosophic Topological Spaces and give many practical examples. Neutrosophic Science means development and applications of Neutrosophic Logic, Set, Measure, Integral, Probability etc., and their applications in any field. It is possible to define the neutrosophic measure and consequently the neutrosophic integral and neutrosophic probability in many ways, because there are various types of indeterminacies, depending on the problem we need to solve.

Indeterminacy is different from randomness. Indeterminacy can be caused by physical space, materials and type of construction, by items involved in the space, or by other factors. In 1965 , Zadeh [1] generalized the concept of crisp set by introducing the concept of fuzzy set, corresponding to the situation in which there is no precisely defined set; there are increasing applications in various fields, including probability, artificial intelligence, control systems, biology and economics. Thus, developments in abstract mathematics using the idea of fuzzy sets possess sound footing. In accordance, fuzzy topological spaces were introduced by Chang [2] and Lowen [3]. After the development of fuzzy sets, much attention has been paid to the generalization of basic concepts of classical topology to fuzzy sets and accordingly developing a theory of fuzzy topology. In 1983, the intuitionistic fuzzy set was introduced by K. Atanassov as a generalization of the fuzzy set, beyond the degree of membership and the degree of non-membership of each element. In 1999 and 2002, Smarandache defined the notion of Neutrosophic Sets, which is a generalization of Zadeh's fuzzy set and Atanassov's intuitionistic fuzzy set. Some neutrosophic concepts have been investigated by Salama et al [4-12]. Forwarding the study of neutrosophic sets, this book consists of seven chapters, targeting to:

- generalize the previous studies in neutrosophic, and so to define the neutrosopic crisp set and neutrosophic set concepts;
- discuss their main properties;
- introduce and study some concepts of neutrosophic crisp and neutrosophic topological spaces and deduce their properties;
- deduce many types of functions and give the relationships between different neutrosophic topological spaces, which helps to build new properties of neutrosophic topological spaces;

- stress once more the importance of Neutrosophic Ideal as a nontrivial extension of neutrosophic set and neutrosophic logic ;
- propose applications on computer sciences by using neutrosophic sets.

Chapter One

Neutrosophic Crisp Set

1.1 Neutrosophic Crisp Set

Let us consider some possible definitions for various types of neutrosophic crisp sets.

Definition 1.1.1

Let X be a non-empty fixed sample space. A neutrosophic crisp set (NCS) A is an object having the form $A = \langle A_1, A_2, A_3 \rangle$ where A_1, A_2 and A_3 are subsets of X.

Definition 1.1.2

The object having the form $A = \langle A_1, A_2, A_3 \rangle$ is called:

a) A neutrosophic crisp set of Type 1 (NCS-Type1) if satisfying

$A_1 \cap A_2 = \phi, A_1 \cap A_3 = \phi \text{ and } A_2 \cap A_3 = \phi$.

b) A neutrosophic crisp set of Type2 (NCS-Type2) if satisfying

$A_1 \cap A_2 = \phi, A_1 \cap A_3 = \phi, A_2 \cap A_3 = \phi, A_1 \cup A_2 = X$

c) A neutrosophic crisp set of Type 3 (NCS-Type3) if satisfying

$A_1 \cap A_2 \cap A_3 = \phi \text{ and } A_1 \cup A_2 \cup A_3 = X$

Remark 1.1.1

A neutrosophic crisp set $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$, subsets in X, and one can define several relations and operations between NCSs.

Since our purpose is to construct the tools for developing neutrosophic crisp set, we must introduce types of CNS φ_N, X_N X.

1) N may be defined as the following four types:
   (a) Tape 1: $\varphi_N = \langle \varphi, \varphi, X \rangle$.
   (b) Tape 2: $\varphi_N = \langle \varphi, X, X \rangle$.
   (c) Tape 3: $\varphi_N = \langle \varphi, X, \varphi \rangle$.
   (d) Tape 4: $\varphi_N = \langle \varphi, \varphi, \varphi \rangle$.
2) X_N may be defined as the following four types:
   (a) Tape 1: $X_N = \langle X, \varphi, \varphi \rangle$.

(b) Tape 2: $X_N = \langle X , X, \varphi \rangle$.

(c) Tape 3: $X_N = \langle X , \varphi, X \rangle$.

(d) Tape 4: $X_N = \langle X , X, X \rangle$.

Every neutrosophic crisp set A on a non-empty set X is obviously NCS having the form $A = \langle A_1 , A_2 , A_3 \rangle$.

Definition 1.1.3

Let $A = \langle A_1 , A_2 , A_3 \rangle$ be a NCS in X, then the complement of the set A (A^c for short) may be defined as three kinds of complements:

$(C_1) A^c = \langle A_1^c , A_2^c , A_3^c \rangle , or$

$(C_2) A^c = \langle A_3 , A_2 , A_1 \rangle$

$(C_3) A^c = \langle A_3 , A_2^c , A_1 \rangle$.

One can define several relations and operations between NCS as it follows:

Definition 1.1.4

Let X be a non-empty set, and the NCSS **A** and **B** be in the form $A = \langle A_1 , A_2 , A_3 \rangle$, $B = \langle B_1 , B_2 , B_3 \rangle$. We consider two possible definitions for subsets ($A \subseteq B$). So ($A \subseteq B$) may be defined as two types:

Type 1. $A \subseteq B \iff A_1 \subseteq B_1 , A_2 \subseteq B_2 \; and \; A_3 \supseteq B_3$

Type 2. $A \subseteq B \iff A_1 \subseteq B_1 , A_2 \supseteq B_2 \; and \; A_3 \supseteq B_3$.

Proposition1.1.1

For any neutrosophic crisp set A, we hold the following:

a) $\phi_N \subseteq A , \phi_N \subseteq \phi_N$

b) $A \subseteq A_X , X_N \subseteq X_N$

Definition 1.1.5

Let X be a non-empty set, and the NCSS A and B be of the form $A = \langle A_1 , A_2 , A_3 \rangle$, $B = \langle B_1 , B_2 , B_3 \rangle$ be NCSS. Then:

1. $A \cap B$ may be defined as two types:

   Type 1. $A \cap B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$,

   Type 1. $A \cap B = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$,

2. $A \cup B$ may be defined as two types:

   Type 1. $A \cup B = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$,

   Type 1. $A \cup B = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle$,

3. $[\ \ ]A = \langle A_1, A_2, A_1^c \rangle$

4. $\langle\ \rangle A = \langle A_3^c, A_2, A_3 \rangle$

Proposition 1.1.2

For all two neutrosophic crisp sets A and B in X, the following assertions are true:

$(A \cap B)^c = A^c \cup B^c$;

$(A \cup B)^c = A^c \cap B^c$;

We can easily generalize the operations of intersection and union in Definition 1.1.2 to an arbitrary family of neutrosophic crisp subsets as it follows:

Proposition 1.1.3

Let $\{A_j : j \in J\}$ be an arbitrary family of neutrosophic crisp subsets in X, then:

1) $\cap A_j$ may be defined as the following two types:

   (a) Type 1. $\cap A_j = \langle \cap Aj_1, \cap Aj_2, \cup Aj_3 \rangle$

   (b) Type 1. $\cap A_j = \langle \cap Aj_1, \cup Aj_2, \cup Aj_3 \rangle$

2) $\cap A_j$ may be defined as the following types:

   (a) Type 1. $\cup A_j = \langle \cup Aj_1, \cap Aj_2, \cap Aj_3 \rangle$

(b) Type 1. $\cup A_j = \langle \cup Aj_1 , \cup Aj_2 , \cap Aj_3 \rangle$

Definition 1.1.6

The product of two neutrosophic crisp sets A and B is a neutrosophic crisp set $A \times B$ given by $A \times B = A = \langle A_1 \times B_1 , A_2 \times B_2 , A_3 \times B_3 \rangle$.

Definition 1.1.7

A NCS-Type1 ϕ_{N_1} , X_{N_1} in X may be defined as it follows:

1. ϕ_{N_1} may be defined as three types:
   (a) Tape 1: $\phi_{N_1} = \langle \phi , \phi, X \rangle$.
   (b) Tape 2: $\phi_{N_1} = \langle \phi , X, \phi \rangle$.
   (c) Tape 3: $\phi_{N} = \langle \phi , \phi, \phi \rangle$.
2. X_{N_1} may be defined as one type:
   (a) Tape 1: $X_{N_1} = \langle X , \phi, \phi \rangle$.

Definition 1.1.8

A NCS-Type2 , ϕ_{N_2} , X_{N_2} in X may be defined it as follows:

1. ϕ_{N_2} may be defined as three types:
   (a) Tape 1: $\phi_{N_2} = \langle \phi , \phi, X \rangle$.
   (b) Tape 2: $\phi_{N_2} = \langle \phi , X, \phi \rangle$.
2. X_{N_2} may be defined as one type:
   (a) Tape 1: $X_{N_2} = \langle X , \phi, \phi \rangle$.

Definition 1.1.9

A NCS-Type 3, ϕ_{N_3} , X_{N_3} in X may be defined as it follows:

1. ϕ_{N_3} may be defined as three types:
   (a) Tape 1: $\phi_{N_3} = \langle \phi , \phi, X \rangle$.

(b) Tape 2: $\phi_{N_3} = \langle \phi , X, \phi \rangle$.

(c) Tape 3: $\phi_{N_3} = \langle \phi , X, X \rangle$.

2. X_{N_3} may be defined as one types:

(a) Tape 1: $X_{N_3} = \langle X , \phi, \phi \rangle$.

(b) Tape 2: $X_{N_3} = \langle X , X, \phi \rangle$.

(c) Tape 3: $X_{N_3} = \langle \phi , \varphi, X \rangle$.

Corollary 1.1.1

In genera ,

(a) Every NCS-Type 1, 2, 3 is NCS.

(b) Every NCS-Type 1 is not NCS-Type2, 3.

(c) Every NCS-Type 2 is not NCS-Type1, 3.

(d) Every NCS-Type 3 is not NCS-Type2, 1, 2.

(e) Every crisp set is NCS.

The following Venn diagram represents the relation between NCSS:

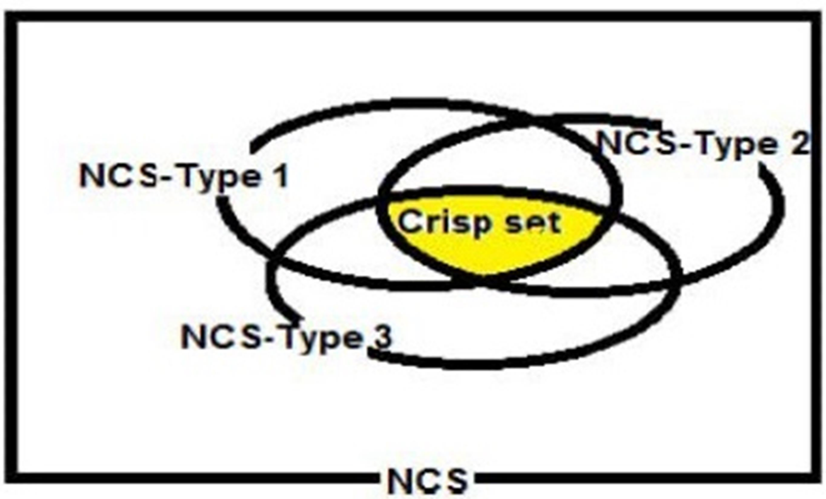

Figure 1. Venn diagram representing the relation between NCSS.

Example 1.1.1

Let $X = \{a,b,c,d,e,f\}$, $A\langle = \{a,b,c,d\},\{e\},\{f\}\rangle$, $D = \langle\{a,b\},\{e,c\},\{f,d\}\rangle$ be a NCS-Type 2, $B = \langle\{a,b,c\},\{d\},\{e\}\rangle$ be a NCT-Type1, but not NCS-Type2, 3, $C = \langle\{a,b\},\{c,d\},\{e,f,a\}\rangle$ be a NCS-Type 3, but not NCS-Type1,2.

Definition 1.1.10

Let X be a non-empty set, $A = \langle A_1 , A_2 , A_3 \rangle$.

1) If A is a NCS-Type1 in X, then the complement of the set $A(A^c)$ may be defined as one kind of complement Type1: $A^c = \langle A_3 , A_2 , A_1 \rangle$.
2) If A is a NCS-Type 2 in X, then the complement of the set $A(A^c)$ may be defined as one kind of complement $A^c = \langle A_3 , A_2 , A_1 \rangle$.
3) If A is NCS-Type3 in X, then the complement of the set $A(A^c)$ may be defined as one kind of complement defined as three kinds of complements:

$$(C_1)\text{Type1: } A^c = \langle A_1 , A_2 , A_1^c \rangle$$

$$(C_2)\text{Type2: } A^c = \langle A_3 , A_2 , A_3 \rangle$$

$$(C_3)\text{Type3: } A^c = \langle A_3 , A_2^c , A_1 \rangle$$

Example 1.1.2

Let $X = \{a, b, c, d, e, f\}$, $A = \langle \{a, b, c, d\}, \{e\}, \{f\} \rangle$ be a NCS-Type 2, $B = \langle \{a, b, c\}, \{4\}, \{d, e\} \rangle$ be a NCS-Type1, $C = \langle \{ab\}, \{c, d\}, \{e, f\} \rangle$ be a NCS-Type 3, then

1) the complement $A = \langle \{a, b, c, d\}, \{e\}, \{f\} \rangle$,
   $A = \langle \{f\}, \{e\}, \{a, b, c, d\} \rangle$ NCS-Type 2;
2) the complement of $B = \langle \{a, b, c\}, \{o\}, \{d, e\} \rangle$,
   $B = \langle \{d, e\}, \{o\}, \{a, b, c\} \rangle$ NCS-Type1;
3) the complement of $C = \langle \{a, b), (c, d\}, \{e, f\} \rangle$ may be defined as three types:

$$Type\ 1: C^c = \langle \{c, d, e, f\}, \{a, b, e, f\}, \{a, b, c, d\} \rangle.$$

$$Type\ 2: C^c = \langle \{e, f\}, \{a, b, e, f\}, \{a, b\} \rangle,$$

$$Type\ 3: C^c = \langle \{e, f\}, \{c, d\}, \{a, b\} \rangle.$$

Proposition 1.1.4

Let $\{A_j : j \in J\}$ be an arbitrary family of neutrosophic crisp subsets in X, then:

1) $\cap A_j$ may be defined as two types:
   (a) Type1: $\cap A_j, = \langle \cap A_{j1}, \cap A_{j2}, \cup A_{j3} \rangle$
   (b) Type2: $\cap A_j, = \langle \cap A_{j1}, \cup A_{j2}, \cup A_{j3} \rangle$
2) $\cup A_j$, may be defined as two types:
   (a) Type1: $\cup A_j = \langle \cup A_{j1}, \cap A_{j2}, \cap A_{j3} \rangle$
   (b) Type2: $\cup A_j = \langle \cup A_{j1}, \cup A_{j2}, \cap A_{j3} \rangle$

Definition 1.1.11

If $B = \langle B_1, B_2, B_3 \rangle$ is a NCS in Y, then the preimage of B under f. denoted by $f^{-1}(B)$, is a NCS in X defined by $f^{-1}(B) = \langle f^{-1}(B_1), f^{-1}(B_2), f^{-1}(B_3) \rangle$.

If $A = \langle A_1, A_2, A_3 \rangle$ is a NCS in X, then the image of A under f, denoted by $f(A)$, is the a NCS in Y defined by $f^{-1}(B) = \langle f^{-1}(A_1), f^{-1}(A_2), f^{-1}(A_3) \rangle$.

Here we introduce the properties of images and preimages, some of which we frequently use in the following chapters.

Corollary 1.1.2

Let $A, \{A_j : i \in J\}$ be a family of NCS in X, and $B, \{B_j : i \in J\}$ NCS in Y, and f: X→Y a function. Then:

(a) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2), B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
(b) $A \subseteq f^{-1}(f(A))$ *and if t is injective, then* $A = f^{-1}(f(A))$,
(c) $f^{-1}(f(B)) \subseteq B$ *and if t is surjective, then* $f^{-1}(f(A)) = B$,
(d) $f^{-1}(\cup B_1) = f^{-1}(B_1), f^{-1}(\cap B_1) = \cap f^{-1}(B_1)$,
(e) $f(\cup A_1) = \cup f(A); f(\cap A_1) \subseteq \cap f(A_1)$, *and if t is injective, then* $f(\cap A_1) = \cap f(A_1)$;
(f) $f^{-1}(Y_N) = X_N, f^{-1}(\phi_N) = \phi_N$

(g) $f(\phi_N) = \phi_N$, $f(X_N) = Y_N$, $if\ t\ is\ subjective.$

Proof: Obvious.

Chapter Two

Neutrosophic Crisp Points

1.2 Neutrosophic Crisp Points

One can easily define the nature of neutrosophic crisp set in X, called neutrosophic crisp point in X, corresponding to an element X.

Now we present some types of inclusion of a neutrosophic crisp point to a neutrosophic crisp set.

Definition 1.2.1

Let $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set on a set X, then $p = \langle \{p_1\}\{p_2\}\{p_3\} \rangle. p_1 \neq p_2 \neq p_3 \in X$ is called a neutrosophic crisp point.

An NCP $p = \langle \{p_1\}\{p_2\}\{p_3\} \rangle$ belongs to a neutrosophic crisp set $A = \langle A_1, A_2, A_3 \rangle$, of X, denoted by pe A, if it may be defined by two types:

(a) Type1: $\{p_1\} \subseteq A_1$, $\{p_2\} \subseteq A_2$ and $\{p_3\} \subseteq A_3$

(b) Type1: $\{p_1\} \subseteq A_1$, $\{p_2\} \supseteq A_2$ and $\{p_3\} \subseteq A_3$

Theorem 1.2.1

Let $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$ be neutrosophic crisp subsets of X. Then $A \subseteq B$ if $p \in A$ and $p \in B$ for any neutrosophic crisp point p in X.

Proof

Let $A \subseteq B$ and $p \in A$. Then we have:

(a) Type1: $\{p_1\} \subseteq A_1$,$\{p_2\} \subseteq A_2$ and $\{p_3\} \subseteq A_3$,or

(b) Type1: $\{p_1\} \subseteq A_1$,$\{p_2\} \supseteq A_2$ and $\{p_3\} \subseteq A_3$

Thus, $p \in B$. Conversely, take any x in X. Let $p_1 \in A_1$, and $p_2 \in A_2$ and $p_3 \in A_3$. Then p is a neutrosophic crisp point in X, and $p \in A$. By the hypothesis, $p \in B$. Thus $p_1 \in B$, or Type 1: $\{p_1\} \subseteq B_1$,$\{p_2\} \subseteq B_2$ and $\{p_3\} \subseteq B_3$, or Type 2: $\{p_1\} \subseteq B_1$,$\{p_2\} \subseteq B_2$, and $\{p_3\} \subseteq B_3$,. Hence, $A \subseteq B$.

Theorem 1.2.2

Let $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp subset of X.

Then $A = \cup \{p : p \in A\}$.

Proof

Since $\cup \{p : p \in A\}$, we get the following two types:

(a) Type1: $\langle \cup \{p_1 : p_1 \in A_1\}$,$\cup \{p_2 : p_2 \in A_2\}$,$\cap \{p_3 : p_3 \in A_3\} \rangle .or$

(b) Type2: $\langle \cup \{p_1 : p_1 \in A_1\}$,$\cap \{p_2 : p_2 \in A_2\}$,$\cap \{p_3 : p_3 \in A_3\} \rangle$.

$hence\ A = \langle A_1, A_2, A_3 \rangle$

Proposition 1.2.1

Let $\{A_j : i \in J\}$ be a family of NCSS in X. Then:

(a_1) $p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle = \epsilon \bigcap_{j \epsilon J} A_j$ if $p \in A_j$ for each $i \in J$.

(a_2) $p \ \epsilon \bigcap_{j \epsilon J} A_j$ if $\exists j \in J$ such that $p \in A_j$.

Proposition 1.2.2

Let $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$ be two neutrosophic crisp sets in X. Then $A \subseteq B$ if for each p we have $p \in A \iff p \in B$ and for each p we have $p \in A \implies p \in B$. If $A = B$ for each p we have $p \in A \implies p \in B$ and for each p we have $p \in A \implies p \in B$.

Proposition 1.2.3

Let $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then:

$$A = \cup < p_1 : p_2 \in A_1\}, p_2 : p_2 \in A_2\}, \{p_3 : p_3 \in A_3\}.$$

Definition 1.2.2

Let $f: X \rightarrow Y$ be a function and p be a neutrosophic crisp point in X. Then the image of p under f, denoted $f(p)$, is defined by:

$f(p) = \langle \{q_1\}, \{q_2\}, \{q_3\} \rangle$, where $q_1 = f(p_1), q_2 = f(p_1)$ and $q_1 = f(p_1)$.

It is easy to see that $f(p)$ is indeed a NCP in Y, namely $f(p) = q$,

where $q = f(p)$, and it has exactly the same meaning of the image of a NCP under the function f.

Definition 1.2.3

Let X be a non-empty set and $p \in X$. Then the neutrosophic crisp point p_N defined by $p_N = \langle \{p\}, \phi, \{p\}^c \rangle$ is called a neutrosophic crisp point (NCP) in X, where NCP is a triple ({only element in X}, empty set, {the complement of the same element in X}).

The neutrosophic crisp points in X can sometimes be inconvenient when expressing the neutrosophic crisp set in X in terms of neutrosophic crisp points. This situation occurs if $A = \langle A_1, A_2, A_3 \rangle$, $p \notin A_1$, where A_1, A_2, A_3 , are three subsets such that $A_1 \cap A_2 = \phi, A_1 \cap A_3 = \phi, A_{21} \cap A_3 = \phi$.

Therefore, we have to define "vanishing" neutrosophic crisp points.

Definition 1.2.4

Let X be a non-empty set and $p \in X$ be a fixed element in X. The neutrosophic crisp set $p_{N_N} = \langle \phi . \{p\} \{p\}^c \rangle$ is called "vanishing" neutrosophic crisp point (VNCP) in X, where VNCP is a triple (empty set, {only element in X}, {the complement of the same element in X}).

Example 1.2.1

Let $X = \{a, b, c, d\}$ and $p = b \in X$. Then:

$p_N = \langle \{b\}, \phi, \{a, c, d\} \rangle,$

$p_{N_N} = \langle \phi, \{b\}, \{a, c, d\} \rangle,$

$p = \langle \{b\}, \{a\}, \{d\} \rangle,$

Definition 1.2.5

Let $p_{N_N} = \langle \phi . \{p\} \, \{p\}^c \rangle$ be a NCP in X and $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X.

(a) p_N is said to be contained in $A(p_N \in A)$ if $p \in A_1$.

(b) p_{N_N} is VNCP in X and $A = \langle A_1, A_2, A_3 \rangle$ a neutrosophic crisp set in X. Then p_{N_N} is said to be contained in $A(p_N \in A)$ if $p \in A_3$.

Proposition 1.2.4

Let $\{A_j, j \in J\}$ be a family of NCSS in X. Then:

(a_1) $p_N \in \underset{j \epsilon J}{\cap} A_j$ if $p_N \in A_j$ for such $i \in J$.

(a_2) $p_{N_N} \in \underset{j \epsilon J}{\cap} A_j$ if $p_{N_N} \in A_j$ for such $i \in J$.

(b_1) $p_N \in \underset{j \epsilon J}{\cap} A_j$ if $\exists_j \in A_j$ for such $p_N \in A_j$.

(b_2) $p_{N_N} \in \underset{j \epsilon J}{\cap} A_j$ if $\exists_j \in A_j$ for such $p_{N_N} \in A_j$.

Proof

Straightforward.

Proposition 1.2.5

Let $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$ be two neutrosophic crisp sets in X. Then:

(a) $A \subseteq B$ if for each p_N we have $p_N \in A \Leftrightarrow p_N \in B$ and for each p_{N_N} we have $p_N \in A \Longrightarrow p_{N_N} \in B$.

(b) $A = B$ if for each p_N we have $p_N \in A \Leftrightarrow p_N \in B$ and for each p_{N_N} we have $p_{N_N} \in A \Leftrightarrow p_{N_N} \in B$.

Proof: Obvious.

Proposition 1.2.6

Let $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then:

$$A = (\cup \{p_N : p_N \in A\}) \cup (\cup \{p_{NN} : p_{NN} \in A\}).$$

Proof

It is sufficient to show the following equalities:

$A_1 = \left(\cup \left\{\{p\} : p_N \in A\right\}\right) \cup (\cup \{\phi : p_{NN} \in A\})\, , A_3 = \phi\, , and$

$A_1 = (\cap \{\{p\}^c : p_N \in A\}) \cap (\cap \{\{p\}^c : p_{NN} \in A\})$, which are fairly obvious.

Definition 1.2.6

Let $f : X \rightarrow Y$ be a function and pn be a neutrosophic crisp point in X. Then the image of p_N under t denoted by $f(p_N)$ is defined by $f(p_N) = \langle \{q\}. \phi\, , \{q\}^c \rangle$where $q = f(p)$. Let PNN be a VNCP in X. Then the image of p_{NN} under f denoted by $f(p_{NN})$ is defined by $f(p_{NN}) = \langle\, \phi, \{q\}, \{q\}^c \rangle$ where $q = f(p)$. It is easy to observe that $f(p_N)$ is indeed a NCP in Y, namely $f(p_N) = qN$ where $q = f(p)$, and it has exactly the same meaning of the image of a NCP under the function t f. $f(p_{NN})$ is also a VNCP in Y, namely $f(p_{NN}) = q_{NN}$, where $q = f(p)$.

Proposition 1.2.7

We state that any NCS A in X can be written in the form:

$$A = \frac{A}{N} \cup \frac{A}{NN} \cup \frac{A}{NNN}$$

Where $\frac{A}{N} = \cup \{p_N : p_N \in A\}$

$$\frac{A}{N} = \phi_N$$

$$\frac{A}{NNN} = \cup \{p_{NN} : p_{NN} \in A\}$$

It is easy to show that, if $A = \langle A_1, A_2, A_3 \rangle$, then:

$$\frac{A}{N} = \langle A_1, \phi, A_1^c \rangle$$

and

$$\frac{A}{NN} = \langle \varphi, A_2, A_3 \rangle$$

Proposition 1.2.8

Let f:X→Y be a function and $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then we have $f(A) = f(\frac{A}{N}) \cup f(\frac{A}{NN}) \cup f(\frac{A}{NNN})$.

Proof

This is obvious from $A = \frac{A}{N} \cup \frac{A}{NN} \cup \frac{A}{NNN}$

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