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Lower and Upper Probability Via αR-Open and αR-Closed Sets

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بِسْمِ اللَّـهِ الرَّحْمَـٰنِ الرَّحِيم

(هُوَ الَّذِي أَنْزَلَ عَلَيْكَ الْكِتَابَ مِنْهُ آيَاتٌ مُحْكَمَاتٌ هُنَّ أُمُّ الْكِتَابِ وَأُخَرُ مُتَشَابِهَاتٌ فَأَمَّا الَّذِينَ فِي قُلُوبِهِمْ زَيْغٌ فَيَتَّبِعُونَ مَا تَشَابَهَ مِنْهُ ابْتِغَاءَ الْفِتْنَةِ وَابْتِغَاءَ تَأْوِيلِهِ وَمَا يَعْلَمُ تَأْوِيلَهُ إِلَّا اللَّهُ وَالرَّاسِخُونَ فِي الْعِلْمِ يَقُولُونَ آمَنَّا بِهِ كُلٌّ مِنْ عِنْدِ رَبِّنَا وَمَا يَذَّكَّرُ إِلَّا أُوْلُوا)

صدق اللَّهُ العظيم

[آل عمران:٧]

الأهداء

اهدي هذا البحث الى كل من ساندني وضحى من اجلي

وبذل الغالي والنفيس من اجل تحقيق النجاح في حياتي

فشكرا الى هذا الانسان

الشكر والتقدير

اتقدم بجزيل الشكر إلى عائلتي لتشجيعهم ومساندتهم لي. ثم أتوجه بجزيل الشكر وعظيم الامتنان (الدكتور مصطفى حسن هادي) على ما بذله من جهد في توجيهي ومتابعتي والإشراف على بحثي. كما أتقدم بخالص الشكر والتقدير إلى جميع الاساتذة المحترمين. شكرا إلى كل شخص ساهم ولو بكلمة تشجيع ومسانده التي اعطتني الدافع لإكمال الدراسه.

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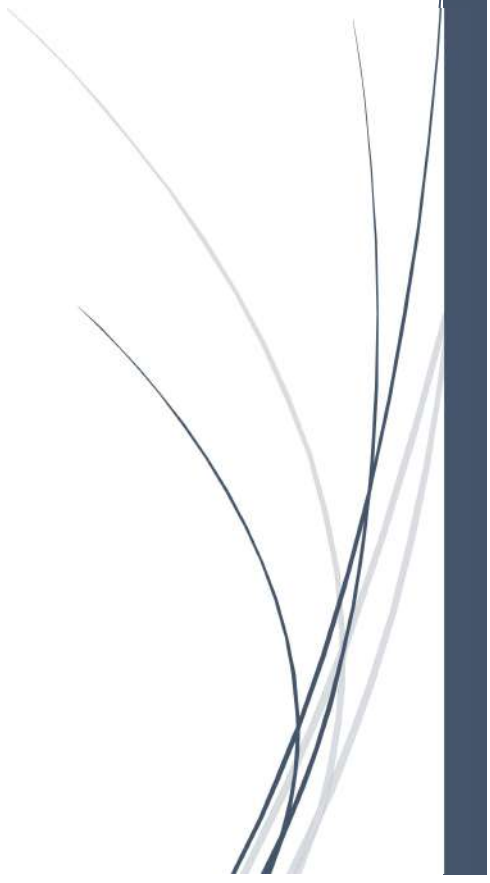
Abstract.

Rather, the aim of my research is the process of calculating all the topologies on the three-point group, and then calculating the upper and lower probability of all the groups within the IP(x) corresponding to those topologies, by the help of the interior points and closure on the one side, and on the other side, calculating the interior points and closure of the harmonic sets αR -Open.

Introduction.

The science of topology extends its roots to the era of the Greek civilization, where the Greeks studied the concept of continuous, but the science of topology did not appear in its current state until the beginning of this century when Frechet published in 1906 his thesis that dealt with the function of distance and relationship There is a difference between it and the concept of continuity, but the two scholars Ries, Haus, and Dorff [1] later showed that there is no need for this conjunction, and continuity can be studied without referring to the distance coupling, and with this, what is called general topology appeared. In general, any group whose elements fulfill some hypotheses constitutes a mathematical system that is consistent (consistent if its arguments, results, and hypotheses are not contradictory (this method was born in the past in the subject of Euclidean geometry). In recent years, mathematics has developed rapidly after the group theory was known at the beginning of the twentieth century. Whereas, any group whose elements fulfill certain hypotheses is called a mathematical sentence that fulfills the hypotheses, and in this case there is more than one mathematical system such as groups, rings, Euclidean geometry, metric spaces, topological spaces. (topological spaces) [2]. Topology is a word translated from the English word topology, and the word topology is divided into two syllables, the first syllable (Topo) which is of Greek origin to topo's (which means “place”), and the second syllable is (logy). Which is of Greek origin (Logos) which means "study", if we make the process of linking the meanings in the word, we find that topology is modern engineering in the study of all the structures and components of the different spaces. I began to study the subject of topology on the set of real numbers and then on the Euclidean level.

In this chapter, we will show set of basic definitions on which our works are based, to a set of questions that we worked on through three elements, which are {a,b,c}, where we made 29 topologies and extracted (interiors and closures) of αR-open at the end of the research ,we studied the probabilities that are (upper and lower) on the topologies that we knew, in addition to the probabilities on topologies for the αR-open set.

Definition 1.1 [1]

Let X be a nonempty set and τ be a family of subsets of X(i.e.,$\tau \subseteq$ IP(X)) . we say τ is a topology on X satisfy the following conditions:

1. X, $\emptyset \in \tau$
2. If U , V $\emptyset \in \tau$, then U$\cap$ $v \in \tau$ the finite intersection of elements from τ
3. If $U_a \in \tau ; a \in \wedge , then\ U_{a\in\wedge}U_a \in \tau\ \forall\alpha \in\wedge$ the arbitrary (finite or infinite)union of element of τ is again an element of τ .

Definition 1.2 [2]

Let (X,τ) be a topological space .The subsets of X belonging to τ are called open set in the space X.i.e.,

If A $\subseteq$ X and A $\in$ τ *then* A open set

Definition 1.3 [2]

Let (X,τ) be a topological space. The subset of X is called closed set in the space X if its complement X \A is open set. We will denoted the family of closed sets by □.i.e.,

If A⊆ X and A ∈ □ *then* A closed set

Definition 1.4 [3]

Let (X,τ) be a topological space and let A⊆X. A point x∈A is called an interior point of A iff there exists an open set U∈τ containing x such that x∈U⊆A.The set of all interior points of A is called the interior of A and is denoted by $A°$ or Int(A) i.e,.

$$A° = \{x \in A: \exists\ U \in \tau; x \epsilon\ U \subseteq A\}$$

$$x \epsilon\ A° \leftrightarrow \exists\ U \epsilon\ \tau; x \epsilon\ U \subseteq A$$

Definition 1.5 [3]

Let (X, τ) be a topological space and let A be a subset of X . Then the intersection of all τ-closed containing the set A is called the closure of A and denoted by $\overline{A}$ or c(A) or cl(A). i.e cl(A)=∩ $\{F: F\ is\ closed, A \subseteq F\}$.

Definition 1.6

A subset A of a space X is said to be αR-open if A= int (cl(int(A))). And the complement αR-open is called αR-closed set. And the complement A=cl(int(cl(A))).

Definition 1.7 [4]

Let (X,τ) be a topological space and let A⊆X. A point x∈A is called an αR-interior point of A iff there exists an αR-open set U∈τ containing x such that x∈U⊆A .The set of all αR-interior point of A is called the αR-interior of A and is denoted by αR-A° or αR-Int(A) i.e,.

$$\alpha R\text{-}A^{\circ} = \{x \in A: \exists\, U \in \tau; x \in U \subseteq A\}$$

$$x \in \alpha R\text{-}A^{\circ} \leftrightarrow \exists\, U \in \tau; x \in U \subseteq A$$

Definition 1.8 [4]

Let (X, τ) be a topological space and let A be a subset of X . Then the intersection of all αR-closed containing the set A is called the αR-closure of A and denoted by αR-$\overline{A}$ or αR-c(A) or αR-cl(A). i.e αR-cl(A)=∩ $\{F: F\ is\ closed, A \subseteq F\}$.

Definition 1.9

- $\underline{\rho}(A^{\circ}) = \underline{\rho}\,(int(A)) = \frac{number\ elements\ of\ A^{\circ}}{number\ elements\ of\ X}$
- $\overline{\rho}(\overline{A}) = \overline{\rho}(cl(A)) = \frac{number\ elements\ of\ \overline{A}}{number\ elements\ of\ X}$
- $\alpha R.\underline{\rho}(A^{\circ}) = \underline{\rho}(\alpha R.int(A)) = \frac{number\ elements\ of \alpha R.int(A)}{number\ elements\ of\ X}$
- $\alpha R.\overline{\rho}(\overline{A}) = \overline{\rho}((\alpha R.cl(A)) = \frac{number\ elements\ of \alpha R.cl(A)}{number\ elements\ of\ X}$

T_1={$x, \emptyset$}

$T_1^c = \{x, \emptyset\}$

$\alpha R.T_1(x) = \{x, \emptyset\}$

$\alpha R.T_1^c(x) = \{x, \emptyset\}$

T_1	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\overline{A})$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_2=$\{x, \emptyset, \{a\}\}$

$T_2^c = \{x, \emptyset, \{b, c\}\}$

$\alpha R.T_2(x) = \{x, \emptyset\}$

$\alpha R.T_2^c(x) = \{x, \emptyset\}$

T_2	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$
$\overline{\rho}(\overline{A})$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_3=$\{x, \emptyset, \{b\}\}$

$T_3^c = \{x, \emptyset, \{a, c\}\}$

$\alpha R.T_3(x) = \{x, \emptyset\}$

$\alpha R.T_3^c(x) = \{x, \emptyset\}$

T_3	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$
$\overline{\rho}(\overline{A})$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_4=$\{x, \emptyset, \{c\}\}$

$T_4^c = \{x, \emptyset, \{a, b\}\}$

$\alpha R.T_4(x) = \{x, \emptyset\}$

$\alpha R.T_4^c(x) = \{x, \emptyset\}$

T_4	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\overline{\rho}(\overline{A})$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_5=$\{x, \emptyset, \{a\}, \{a, b\}\}$

$T_5^c = \{x, \emptyset, \{b, c\}, \{c\}\}$

$\alpha R.T_5(x) = \{x, \emptyset\}$

$\alpha R.T_5^c(x) = \{x, \emptyset\}$

T_5	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\overline{A})$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_6=$\{x, \emptyset, \{a\}, \{a, c\}\}$

$T_6^c = \{x, \emptyset, \{b, c\}, \{b\}\}$

$\alpha R.T_6(x) = \{x, \emptyset\}$

$\alpha R.T_6^c(x) = \{x, \emptyset\}$

T_6	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{0}{3}$
$\overline{\rho}(\overline{A})$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_7=$\{x, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$

$T_7^c = \{x, \emptyset, \{b, c\}, \{c\}, \{b\}\}$

$\alpha R.T_7(x) = \{x, \emptyset\}$

$\alpha R.T_7^c(x) = \{x, \emptyset\}$

T_7	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{0}{3}$
$\overline{\rho}(\overline{A})$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_8=$\{x, \emptyset, \{b\}, \{a, b\}, \}$

$T_8^c = \{x, \emptyset, \{a, c\}, \{c\}\}$

$\alpha R.T_8(x) = \{x, \emptyset\}$

$\alpha R.T_8^c(x) = \{x, \emptyset\}$

T_8	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$	$\frac{1}{3}$
$\overline{\rho}(\overline{A})$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_9=$\{x, \emptyset, \{b\}, \{b, c\}\}$

$T_9^c = \{x, \emptyset, \{a, c\}, \{a\}\}$

$\alpha R.T_9(x) = \{x, \emptyset\}$

$\alpha R.T_9^c(x) = \{x, \emptyset\}$

T_9	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_{10}=$\{x, \emptyset, \{b\}, \{a, b\}, \{b, c\}\}$

$T_{10}^c = \{x, \emptyset, \{a, c\}, \{c\}, \{a\}\}$

$\alpha R.T_{10}(x) = \{x, \emptyset\}$

$\alpha R.T_{10}^{c}(x) = \{x, \emptyset\}$

T_{10}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_{11}=$\{x, \emptyset, \{c\}, \{a, c\}\}$

$T_{11}^{c} = \{x, \emptyset, \{a, b\}, \{b\}\}$

$\alpha R.T_{11}(x) = \{x, \emptyset\}$

$\alpha R.T_{11}^{c}(x) = \{x, \emptyset\}$

T_{11}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{\rho}(\overline{A})$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_{12}=$\{x, \emptyset, \{c\}, \{b, c\}\}$

$T_{12}^{c} = \{x, \emptyset, \{a, b\}, \{a\}\}$

$\alpha R. T_{12}(x) = \{x, \emptyset\}$

$\alpha R. T_{12}^{c}(x) = \{x, \emptyset\}$

T_{12}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_{13}=$\{x, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$

$T_{13}^{c} = \{x, \emptyset, \{a, b\}, \{b\}, \{a\}\}$

$\alpha R. T_{13}(x) = \{x, \emptyset\}$

$\alpha R. T_{13}^{c}(x) = \{x, \emptyset\}$

T_{13}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_{14}=$\{x, \emptyset, \{a, b\}\}$

$T_{14}^{c} = \{x, \emptyset, \{c\}\}$

$\alpha R.T_{14}(x) = \{x, \emptyset\}$

$\alpha R.T_{14}^{c}(x) = \{x, \emptyset\}$

T_{14}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\overline{A})$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_{15}=$\{x, \emptyset, \{b, c\}\}$

$T_{15}^{c} = \{x, \emptyset, \{a\}\}$

$\alpha R. T_{15}(x) = \{x, \emptyset\}$

$\alpha R. T_{15}^{c}(x) = \{x, \emptyset\}$

T_{15}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_{16}=$\{x, \emptyset, \{a, c\}\}$

$T_{16}^{c} = \{x, \emptyset, \{b\}\}$

$\alpha R. T_{16}(x) = \{x, \emptyset\}$

$\alpha R. T_{16}^{c}(x) = \{x, \emptyset\}$

T_{16}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{0}{3}$
$\overline{\rho}(\overline{A})$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_{17}=$\{x, \emptyset, \{a\}, \{a, b\}, \{b\}\}$

$T_{17}^{c} = \{x, \emptyset, \{b, c\}, \{c\}, \{a, c\}\}$

$\alpha R. T_{17}(x) = \{x, \emptyset, \{a\}, \{b\}\}$

$\alpha R. T_{17}^{c}(x) = \{x, \emptyset, \{b, c\}, \{a, c\}\}$

T_{17}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\overline{\rho}(\overline{A})$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

T_{18}=$\{x, \emptyset, \{b\}, \{c\}, \{b, c\}\}$

$T_{18}^{c} = \{x, \emptyset, \{a, c\}, \{a, b\}, \{a\}\}$

$\alpha R. T_{18}(x) = \{x, \emptyset, \{b\}, \{b, c\}\}$

$\alpha R. T_{18}^{c}(x) = \{x, \emptyset, \{a, c\}, \{a\}\}$

T_{18}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

T_{19}=$\{x, \emptyset, \{a\}, \{c\}, \{a, c\}\}$

$T_{19}^{c} = \{x, \emptyset, \{b, c\}, \{a, b\}, \{b\}\}$

$\alpha R. T_{19}(x) = \{x, \emptyset, \{a\}, \{a, c\}\}$

$\alpha R. T_{19}^{c}(x) = \{x, \emptyset, \{b, c\}, \{b\}\}$

T_{19}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{0}{3}$	$\frac{3}{3}$

T_{20}=$\{x, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$T_{20}^{c} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{c\}, \{b\}, \{a\}\}$

$\alpha R. T_{20}(x) = \{x, \emptyset, \{a\}, \{b, c\}\}$

$\alpha R. T_{20}^{c}(x) = \{x, \emptyset, \{b, c\}, \{a\}\}$

T_{20}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$

T_{21}=$\{x, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$

$T_{21}^{c} = \{x, \emptyset, \{b, c\}, \{a, b\}, \{b\}, \{a\}\}$

$\alpha R. T_{21}(x) = \{x, \emptyset, \{a\}, \{b, c\}\}$

$\alpha R. T_{21}^{c}(x) = \{x, \emptyset, \{b, c\}, \{a\}\}$

T_{21}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$

T_{22}=$\{x, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$

$T_{22}^{c} = \{x, \emptyset, \{a, c\}, \{a, b\}, \{a\}, \{b\}\}$

$\alpha R.T_{22}(x) = \{x, \emptyset, \{b\}, \{a, c\}\}$

$\alpha R.T_{22}^{c}(x) = \{x, \emptyset, \{a, c\}, \{b\}\}$

T_{22}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

T_{23}=$\{x, \emptyset, \{a\}, \{b, c\}\}$

$T_{23}^{c} = \{x, \emptyset, \{a\}, \{b, c\}\}$

$\alpha R. T_{23}(x) = \{x, \emptyset, \{a\}, \{b, c\}\}$

$\alpha R. T_{23}^{c}(x) = \{x, \emptyset, \{b, c\}, \{a\}\}$

T_{23}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{2}{3}$

T_{24}=$\{x, \emptyset, \{b\}, \{a, c\}\}$

$T_{24}^{c} = \{x, \emptyset, \{a, c\}, \{b\}\}$

$\alpha R.T_{24}(x) = \{x, \emptyset, \{b\}, \{a, c\}\}$

$\alpha R.T_{24}^{c}(x) = \{x, \emptyset, \{a, c\}, \{b\}\}$

T_{24}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{\rho}(\overline{A})$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

T_{25}=$\{x, \emptyset, \{c\}, \{a, c\}\}$

$T_{25}^{c} = \{x, \emptyset, \{a, b\}, \{b\}\}$

$\alpha R. T_{25}(x) = \{x, \emptyset\}$

$\alpha R. T_{25}^{c}(x) = \{x, \emptyset\}$

T_{25}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{\rho}(\overline{A})$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{0}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_{26}=$\{x, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$

$T_{26}^{c} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{c\}, \{b\}\}$

$\alpha R. T_{26}(x) = \{x, \emptyset, \{a, c\}, \{b\}\}$

$\alpha R. T_{26}^{c}(x) = \{x, \emptyset, \{b\}, \{a, c\}\}$

T_{26}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{\rho}(\overline{A})$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

T_{27}=$\{x, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$

$T_{27}^{c} = \{x, \emptyset, \{b, c\}, \{a, b\}, \{b\}, \{c\}\}$

$\alpha R. T_{27}(x) = \{x, \emptyset, \{a, b\}, \{c\}\}$

$\alpha R. T_{27}^{c}(x) = \{x, \emptyset, \{c\}, \{a, b\}\}$

T_{27}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{1}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{2}{3}$
$\underline{\rho}(\alpha R(A°)$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\overline{\rho}(\alpha R(\overline{A})$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_{28}=$\{x, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$

$T_{28}^{c} = \{x, \emptyset, \{a, c\}, \{a, b\}, \{a\}, \{c\}\}$

$\alpha R. T_{28}(x) = \{x, \emptyset, \{a, b\}, \{c\}\}$

$\alpha R. T_{28}^{c}(x) = \{x, \emptyset, \{c\}, \{a, b\}\}$

T_{28}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{3}$

T_{29}=$\{x, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$T_{29}^{c} = \{x, \emptyset, \{b, c\}, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$

$\alpha R.T_{29}(x) = \{x, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$\alpha R.T_{29}^{c}(x) = \{x, \emptyset, \{b, c\}, \{a, b\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$

T_{29}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}
$\underline{\rho}(A°)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{\rho}(\overline{A})$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\underline{\rho}(\alpha R(A°))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\overline{\rho}(\alpha R(\overline{A}))$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

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