

Republic of Iraq

Ministry of Higher Education and Scientific Research

University of Babylon

College of Education for Pure Sciences

Department of Mathematics



On Use of Mathematical Models in Systems Reliability

A Thesis

submitted to the College of Education for Pure Sciences, University of Babylon in
Partial Fulfillment of the Requirements for the Degree of
Master in Education / Mathematics.

By

Noor Sabah Hassan

Supervised by

Prof.Dr. Zahir Abdul Haddi Hassan

2023 A.D.

1445 A.H.



يَرْفَعُ اللَّهُ الَّذِينَ آمَنُوا مِنْكُمْ وَالَّذِينَ أُوتُوا الْعِلْمَ

دَرَجَاتٍ وَاللَّهُ بِمَا تَعْمَلُونَ خَبِيرٌ

صَلَّى اللَّهُ عَلَيْهِ وَسَلَّمَ
العظيم

سورة المجادلة: من الآية ١١

Dedication

To my parents.

*Who were the reason of what I become today,
thanks for your great support and continuous care.*

To my dear husband.

*Thank you for standing by my side and for your continued
support.*

*To my brother, sisters and my dear daughter
and to everyone who stood beside me to reach this success. I
am very grateful for your love and care.*

Acknowledgements

First and foremost, all thanks are due to Allah for giving me the ability to complete this study. Many people have provided me, in a way or another, with assistance throughout the process of writing this thesis. To each of them, I owe sincere thanks and gratitude. I am specifically and sincerely grateful to my supervisor.

Prof. **Dr. Zahir Abdul Haddi Hassan** for his continual support and patience. His suggestions and comments greatly helped me to improve my work. My heartfelt gratitude goes to him.

I would like to thank **Hatem Kareem Sulaiman Ali Aljobory** , **Haider Saleh Howeidi** , both Master Mathematics, University of Babylon, College of Pure Sciences, Mathematics Department, and **Dr. Fouad Hamza**, College of Basic Education, University of Babylon.

My thanks also extend to all members of the Mathematics Department (University of Babylon, College of Education for Pure Sciences) for their help and encouragement. I can never express appropriately and adequately my appreciation towards them.

Finally, I would like to express my deepest appreciation to my family and friends for supporting and encouraging me throughout the years of my study and writing this thesis. This work would not have been possible without them.

The Supervisor Certificate

I certify that this thesis entitled ” [On Use of Mathematical Models in Systems Reliability](#) ” was prepared by the student ” [Noor Sabah Hassan](#) ” under my supervision at the University of Babylon, College of Education for Pure Sciences as a partial fulfillment of the requirement the Degree of Master in Education /Mathematics.

Signature:

Name: Prof.Dr. Zahir Abdul Haddi Hassan

Scientific Grade: Professor

Date: / / 2023

According to the available recommendation, I forward this thesis for debate by the examining committee.

Signature:

Name: Dr. Azal Jaafar Musa

Head of Mathematices Department

Title: Assistant Professor

Date: / / 2023

Scientific Supervisor's Certification

This is to certify that I have read this thesis entitled ” [On Use of Mathematical Models in Systems Reliability](#) ” and I found it is qualified for debate.

Signature:

Name:

Title:

Date: / / 2023

Linguistic Supervisor's Certification

This is to certify that I have read this thesis entitled ” [On Use of Mathematical Models in Systems Reliability](#) ” and I found it is qualified for debate.

Signature:

Name:

Title:

Date: / / 2023

Examining Committee Certificate

We certify that we have read this thesis entitled ” [On use of Mathematical Models in Systems Reliability](#) ” as an examination committee, examined the student ” [Noor Sabah Hassan](#) ” in its contents and that in our opinion it meets the standard of a dissertation for the Degree of Master in Education/ Mathematics.

Signature:

Name: Dr.Hussam A.A. Mohammed

Title: Assist. Professor

Date : / / 2023

Chairman

Signature:

Name : Dr. Kawthar Fawzi Hamza

Title: Assist. Professor

Date : / / 2023

Member

Signature:

Name: Dr. Ghazi Abdullah Madloul

Title: Lecture

Date : / / 2023

Member

Signature:

Name :Dr. Zahir Abdul Haddi Hassan

Title: Professor

Date : / / 2023

Member / Advisor

Approved by the Dean of College.

Signature:

Name: Dr. Bahaa Hussein Salih Rabee

Scientific grade : Professor

Address: Dean of the College of Education for Pure Sciences.

Date : / / 2023

Contents

Dedication	ii
Acknowledgements	iii
List of Figures	xiii
List of Tables	xv
List of abbreviations	xvi
Abstract	xvii
Introduction	1
1 Some Definitions and Basic Concepts	5
1.1 Introduction	6
1.2 Basic for Graph Theory	6
1.2.1 Graph Theory as a Tool for Reliability Evaluation	6
1.2.2 Main Reasons to Choose Reliability System in Design Optimal System	7
1.3 Matrix Representation of Graphs	14
1.4 Structure Function	15
1.5 Reliability Systems	17
1.5.1 Series System	17
1.5.2 Parallel System	17
1.5.3 Series-Parallel System	18
1.5.4 Parallel - Series System	19
1.5.5 Complex System	19
2 Some Methods for Finding Minimal (path / cut) Sets and Exact Methods to Evaluating the Reliability of Complex Systems	22

2.1	Introduction	23
2.2	Generation of Minimal Path Sets	23
2.2.1	Connection Matrix	24
2.2.2	Delimiter's Method	24
2.2.3	Node Removal Method	26
2.2.4	Powers of Adjacency Matrix	29
2.3	Generation of Minimal Cut Sets	32
2.3.1	Create the Minimal Cut Sets from Minimal Paths	33
2.3.2	Enumerate the Minimal Cut Sets Using the Connection Matrix	34
2.4	General Methods for Evaluating Reliability of the Complex System	38
2.4.1	Pivotal Decomposition Method	39
2.4.2	Sum-of-Disjoint-Products (S.D.P) Method	42
2.5	Summary chapter two	44
3	Some Applications to Improve Reliability of Systems	45
3.1	Introduction	46
3.2	Redundancy Technique to Improve Reliability Systems	46
3.2.1	Element redundancy method	46
3.2.2	Unit Redundancy Method	47
3.3	Application of Redundancy Methods in Reliability to Systems	48
3.3.1	Application of Redundancy Methods to Series Systems	48
3.3.2	Application of Redundancy Methods to Parallel Systems	50
3.3.3	Application of Redundancy Methods to Complex Systems	53
3.4	Allocation Technique to Improve Reliability Systems	56
3.4.1	Application of Reliability Allocation on a Series System	57
3.4.2	Application of Reliability Allocation on a Parallel System	57
3.4.3	Application of Reliability Allocation on a Complex System	59
3.5	Summary chapter three	60

4	The Importance of Reliability Systems	63
4.1	Introduction	64
4.2	Evaluating the Reliability Importance of each Units in Some Types of Systems	64
4.2.1	For Series Systems	65
4.2.2	For Parallel Systems	66
4.2.3	For Complex Systems	69
4.3	Evaluating the Reliability Importance of each Units in Minimal Path Sets for Some Types of Systems	73
4.3.1	For Series Systems	74
4.3.2	For Parallel Systems	75
4.3.3	For Complex Systems	75
4.4	Evaluating the Reliability Importance of each Minimal Path Sets for Some Types of Systems	80
4.4.1	First Method (F.M)	81
4.4.2	Second Method (S.M)	86
4.5	Evaluating the Reliability Importance of each Units in Minimal Cut Sets for Some Types of Systems	91
4.5.1	For Series Systems	91
4.5.2	For Parallel Systems	91
4.5.3	For Complex Systems	93
4.6	A New Measure to Evaluate the Reliability Importance of Minimal Cuts for Complex System	97
4.7	Evaluating the Reliability Importance of each Minimal Cut Sets for Some Types of Systems	98
4.7.1	First Method (F.M)	98
4.7.2	Second Method (S.M)	104
4.8	Summary chapter four	109

5	Conclusions and Future Works	111
5.1	Conclusions	112
5.2	Future works	113
	References	114

List of Figures

1.1	Graph.	7
1.2	Simple graph.	8
1.3	System.	8
1.4	Graph with loop and multiple edge.	9
1.5	Undirected graph.	10
1.6	Mixed graph G.	11
1.7	Deletion of edge.	11
1.8	The contraction of edge.	12
1.9	Deletion of vertex 3.	12
1.10	Partial graph H of G.	13
1.11	Sub graph.	13
1.12	Reliability block diagram of a series structure.	16
1.13	Series system.	17
1.14	Parallel system.	18
1.15	Series-Parallel system.	18
1.16	Parallel - Series System.	19
1.17	Complex system and with - terminal reliability.	20
2.1	Complex system.	23
2.2	All minimal path sets of complex system.	32
2.3	All minimal cuts of complex system.	38
2.4	Complex system with eight components.	39
2.5	Unit R_3 work.	40
2.6	Unit R_3 fail.	40

2.7	Unit R_7 work.	41
2.8	Unit R_7 fail.	41
3.1	Element redundancy.	47
3.2	Unit redundancy.	47
3.3	Series system.	48
3.4	Element redundancy for series system with four components.	49
3.5	Unit redundancy for series system.	49
3.6	Parallel system with four components.	50
3.7	Element redundancy for parallel system with four components.	51
3.8	Unit redundancy for parallel system.	52
3.9	Element redundancy for complex system with eight components.	53
3.10	Unit redundancy for complex system.	54
3.11	Parallel System with three elements.	58
4.1	Series system with three components.	65
4.2	Representing the results of the reliability importance levels of a table (4.1).	66
4.3	Parallel system with three components.	67
4.4	Representing the results of the reliability importance levels of a table (4.2)	68
4.5	Representing the results of the reliability importance levels of a table (4.3)	71
4.6	Representing the results of the reliability importance levels of a table (4.4)	73
4.7	Representing the results of the reliability importance levels of a table (4.5)	74
4.8	Representing the results of the reliability importance levels of a table (4.6)	77
4.9	Representing the results of the reliability importance levels of a table (4.7)	80
4.10	Representing the results of the reliability importance levels of a table (4.8)	82
4.11	Representing the results of the reliability importance levels of a table (4.9)	84
4.12	Representing the results of the reliability importance levels of a table (4.10)	85
4.13	Representing the results of the reliability importance levels of a table (4.11)	88

4.14	Representing the results of the reliability importance levels of a table (4.12)	89
4.15	Representing the results of the reliability importance levels of a table (4.13)	90
4.16	Representing the results of the reliability importance levels of a table (4.14)	92
4.17	Representing the results of the reliability importance levels of a table (4.15)	94
4.18	Representing the results of the reliability importance levels of a table (4.16)	97
4.19	Representing the results of the reliability importance levels of a table (4.17)	99
4.20	Representing the results of the reliability importance levels of a table (4.18)	102
4.21	Representing the results of the reliability importance levels of a table (4.19)	104
4.22	Representing the results of the reliability importance levels of a table (4.20)	105
4.23	Representing the results of the reliability importance levels of a table (4.21)	107
4.24	Representing the results of the reliability importance levels of a table (4.22)	108

List of Tables

1	List of Abbreviations	xvi
4.1	Determine the importance of reliability and its levels for each units of series system.	66
4.2	Determine the importance of reliability and its levels for each units of parallel system.	68
4.3	Determine the importance of reliability and its levels for identical units of complex system.	71
4.4	Determine the importance of reliability and its levels for different units of complex system.	72
4.5	Determine the importance of reliability and its levels for minimal path set of series system.	74
4.6	Determine the importance of reliability and its levels for identical units in minimal path set of complex system.	76
4.7	Determine the importance of reliability and its levels for different units in minimal path set of complex system.	79
4.8	Determine the reliability importance and its level of the minimal path sets of parallel system by using F.M.	81
4.9	Determine the importance of reliability and its levels for identical values of a minimal path set of complex system by using F.M.	84
4.10	Determine the importance of reliability and its levels for different values of a minimal path set of complex system by using F.M.	85
4.11	Determine the reliability importance and its level of the minimal path sets of parallel system by using S.M.	87

4.12	Determine the importance of reliability and its levels for identical values of a minimal path set of complex system by using S.M.	89
4.13	Determine the importance of reliability and its levels for different values of a minimal path set of complex system by using S.M.	90
4.14	Determine the importance of reliability and its levels for minimal cut set for parallel system.	92
4.15	Determine the importance of reliability and its levels for identical units in minimal cut set of complex system.	93
4.16	Determine the importance of reliability and its levels for different units in minimal cut set of complex system.	96
4.17	Determine the reliability importance and its level of the minimal cut sets of parallel system by using F.M.	99
4.18	Determine the importance of reliability and its levels for identical values of a minimal cut set of complex system by using F.M.	102
4.19	Determine the importance of reliability and its levels for different values of a minimal cut set of complex system by using F.M.	103
4.20	Determine the reliability importance and its level of the minimal cut sets of series system by using S.M.	105
4.21	Determine the importance of reliability and its levels for identical values of a minimal cut set of complex system by using S.M.	106
4.22	Determine the importance of reliability and its levels for different values of a minimal cut set of complex system by using S.M.	108

Table 1: List of Abbreviations

Symbol	Description
G	A graph defined by an ordered pair (V,E)
V	vertex set
E	edge set
$P_r(x)$	Probability of element x
R_s	Reliability of the system
R_i	Reliability of the component i
$R_s(0_i, R_i)$	Reliability of the system when $R_i=0$
$R_s(1_i, R_i)$	Reliability of the system when $R_i=1$
CM	Connection matrix of the system
a_{ij}^k	New element k of new matrix k
IM	Incidence matrix of all minimal paths
R_{sub}	Reliability of subsystem
$MaxR(MP_i)$	Maximum reliability of minimal path set.
$MinR(MC_i)$	Minimum reliability of minimal cut set.
$R_S^\#$	Objective function for system reliability.
$R_i^\#$	Objective function for component reliability.
$R_{E_S}^*$	Element redundancy.
$R_{U_S}^*$	Unit redundancy.
R_i^*	Redundancy of components.
$F.M.$	First Method.
$S.M.$	Second Method.

Abstract

This thesis aims to using some applications to improve the reliability of the systems that we will adopt in this thesis. These are series, parallel and complex systems. The complex system will be the focus of attention in this thesis. Finding a simple, direct, and mathematically, in addition to two other methods for constructing minimal path sets for complex system, and two methods for constructing minimal cut sets for a complex system based on graph science. Then, two methods were used to calculate the reliability function for the complex system. Applying two techniques (redundancy and allocation) to increase value the reliability of the three systems (series, parallel and complex). The redundancy technique will be in two methods, namely redundancy of elements and redundancy of the system, the two methods are applied to the three systems. And applying the method of redundancy the element to one of the minimum path set and the minimum cut set for complex system. Then applying the allocation technique to increase the reliability value of the three systems. Study the reliability importance of in simple and complex systems in multiple and different situations. The reliability importance of each component in the three systems will be evaluated, the importance of each component in the minimal paths and minimal cuts for the three systems will be assessed, the reliability importance for each minimal paths and cuts for the three systems will be assessed, then a new measure will be studied to evaluate the reliability importance of the minimal cuts for more than one component.

Introduction

Samuel Taylor Coleridge who first attested the term **”Reliability”** in 1816 [6]. In daily life, reliability is an ephemeral but desirable of a product or service that is usually evaluated in a highly subjective method. Hardware and software systems tends to be enormous, complex, and frequently have unique features and systems.

One needs to access these systems’ reliability as well as other pertinent metrics in order to increase their reliability[40]. Engineering systems in use today are more powerful than ever thanks to recent advancements in science and technology[56]. High-tech industrial trial procedures are becoming more sophisticated, which suggests that reliability problems will not only continue but also likely require to ever-complicated solutions[28]. Additionally, reliability problems are having more of an effect on society generally than ever before, for instance, the breakdown of a nuclear power plant or the improper management of a major city’s power distribution system, or the malfunction of an air traffic control system at an international airport, or the failure of a system. It is difficult to overestimate the importance of reliability in modern engineering processes at all steps, including design, manufacture, distribution, and operation[25].

Reliability theory describes a system’s capacity to carry out the task for which it is responsible at a specific time[10]. It is one of the pillars of engineering. Systems like those in aircraft, linear accelerators, and other items, function better as a result, and the probability of failure is reduced. Due to the consequences of failures in today’s complex systems , which can result in operational inefficiency [32].

Shannon and Moore, initially discussed network reliability in 1950 during their examination of the reliability of relay circuits [14]. According to their definition of network dependability, a circuit will remain closed if all of its constituent contacts are closed with a particular probability [7]. Many studies have been conducted on the topic since their work was first published more than 50 years ago. Up until 1970, the evaluation of network reliability and the use of graph theory have grown more and more connected[6]. The

factoring theorem was described by Page, L. B. and Perry, J. E. In 1989, as an impressive conceptual tool for carefully evaluating the system's reliability[17]. Since 1969, researchers have created multiple evaluations of the importance of various components[5]. These scales provide a numerical ranking to assess which system reliability-improving elements are most essential.

The researcher Hoang Pham discusses in detail problems with improving the reliability of systems insecure about two forms of failure in 2006[16]. The number of components that will be used is unlimited, and it is expected that the system's constituent countries are statistically independent and identical. Improvement in series, parallels, parallel-series, and series-parallel reliability. In order to preserve or improve circuit reliability[17]. Engineering methods that are easily understood demonstrate that components of reliability must have a linear relationship with time when evaluating the reliability of electrical systems used in aviation[20]. Reliability equivalence factors for a series-parallel system of components with exponentiated Weibull lifetimes were published in 2015 by Alghamdi, S. M., and Percy, D. F., Et al. [8].

In 2017, Emad introduced a thesis entitled(On the geometry of the reliability polynomials)[46]. In 2018, saw the introduction of (Reliability of electric vehicle with wind turbine based on particle swarm optimization) [18].

In 2019, Hatem introduced a thesis entitled (A study of mathematical models in reliability of networks)[52], also in the same year, he was published (Reliability allocation and optimization for (ROSS) of a spacecraft by using genetic algorithm). In 2023, the same researchers introduced a paper which is; Computational models for allocation and optimization of reliability for ROSS network [53].

In 2021, Ghazi introduced a dissertation entitled (Some reliability optimization techniques for networks)[43]. In the same year, Ghufran introduced a thesis entitled (Using of some techniques to improve reliability networks)[47].

In 2022, Haider introduced a thesis entitled (Use innovative methods to

increase the reliability of complex and mixed networks)[27].

In our thesis we presented accurate methods for calculating the minimal path sets and the minimal cut sets for reliability to calculate the reliability function of complex systems. Study techniques to improve the reliability of the systems has been interested. Such of them redundancy, allocation, and importance.

The aims of this thesis

The aims of this study, it is to provide an accurate and direct method (the Delimiter's method) to calculate the minimal path set for a complex system and calculate the reliability function of the complex system, also we try to study some techniques to improve reliability the systems: Series, parallel and complex, in addition to studying the reliability importance the components behavior in the systems, minimal paths and minimal cuts and study the reliability importance the minimal paths and cuts of in the systems.

Contributions

The thesis included a set of contributions that can be summarized as follows:

1. In this thesis, all the minimal path sets of the complex system are found in a specific method to succeed for the complex system.
2. Apply two methods to generate a minimal cut sets for complex systems.
3. The reliability function of complex system is found in two methods, both of which depend on the laws of probability.
4. Study two techniques to improve for the reliability of the three systems(series, parallel, and complex): redundancy, are two types redundancy for components and redundancy for systems, in addition to improve of the minimal path set by elements redundancy. Then we study allocation technique to improve the systems' based on

greatest minimal path set reliability, thus the reliability of the whole system was improved.

5. Applying the reliability importance for the three systems in terms the behavior of all components in their different states of existence in the three systems.
6. Finding a new measure (the reliability importance absolute) to evaluate the importance of reliability in the case minimal cut set for parallel systems and complex systems.
7. Generalization two methods (F.M and S.M) from Birnbaum's measure in the case studying the importance of more than one component for the systems.

Outline of Thesis

This thesis consists of five chapters. **Chapter one**, contains some definitions and determined concepts, including reliability polynomials and basic graph theory.

Chapter two, contains methods for finding minimal (path /cut) sets and exact techniques to evaluating the reliability for complex systems.

Chapter three, focused on two methods to improve the reliability of a three systems series, parallel and complex, where the first technique is the Redundancy technique and the second is allocation.

Chapter four, includes a study of the behavior of the reliability importance of components in relation to the three systems, series, parallel and complex, in addition to the importance of components in relation to each minimal path set and for each minimal cut set in the three systems.

Chapter five, includes conclusions and future works.

Chapter 1

Some Definitions and Basic Concepts

1.1 Introduction

This chapter introduces the main definitions and basic concepts and focuses on the basic information of the thesis. This chapter is concerned with two parts, the first is the graph theory and its impact and linking it with the reliability of systems of all kinds, and the second is reliability and its basic definitions and concepts.

1.2 Basic for Graph Theory

In this section several definitions, basic concepts types of direct and indirect graphs, simple, sub graphs, mixed graphs, partial graphs, etc.[35, 36, 42], and also matrix representations of graphs have all been discussed.

1.2.1 Graph Theory as a Tool for Reliability Evaluation

In recent times, scientists and engineers have developed a greater interest in graph theory, this increased interest in graph theory is largely due to its proven capacity to address issues from a wide range of fields[14]. Graphs have been proven to be particularly helpful in modeling systems developing in physical science, engineering, social science, and economic problems because of their straightforward diagrammatic form, and reliability engineering has not been an exception[9]. The application of graph theory in reliability studies did not acquire much be seen to several studies have been published in the literature since (Misra & Rao, 1970), proposed the application of graph theory for evaluating network reliability[4]. Created node - link graphs, which become identified as a fundamental innovation in the evaluation of network reliability[14]. In reality, the application of graph theory and network reliability evaluation have become inseparable [12].

1.2.2 Main Reasons to Choose Reliability System in Design Optimal System

- Reliability systems can easily represent systems with complex structure.
- In the reliability networks approach we can connect directly components between them (point to point connection).
- Once the reliability network of the system is obtained. The corresponding adjacency matrix of the system is also formed[40].

Definition 1.1 [16] **A graph** $G = (V, E)$ is defined by an ordered pair (V, E) , where V is a nonempty set whose components are referred to as vertices (nodes) and E is a set of elements of $V (G)$. The edges or (lines, arcs) of the graph G are also called are E 's components.

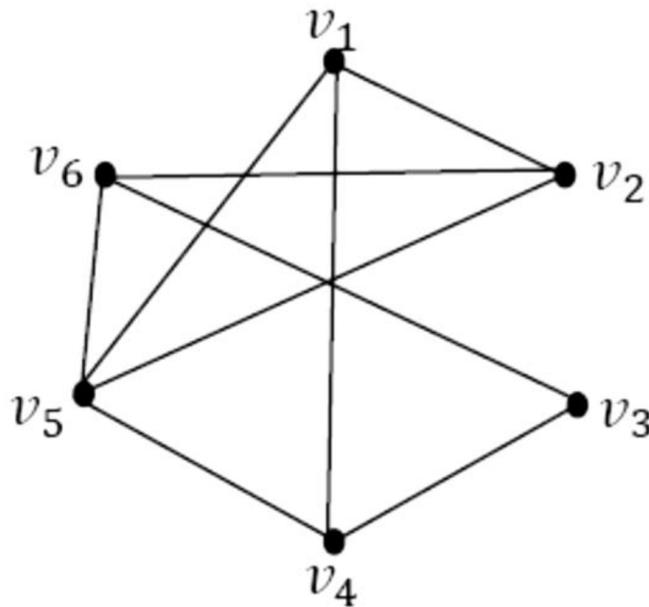


Figure 1.1: Graph.

Definition 1.2 [10] **A simple graph** is a graph that does not include loops or multiple edges, as an example, see fig. (1.2).

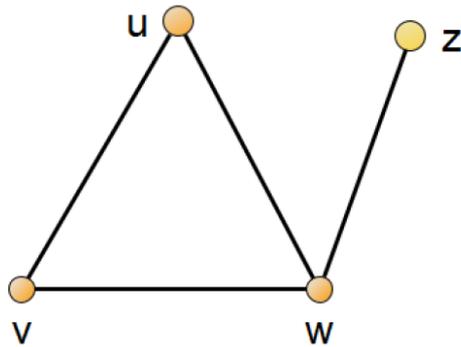


Figure 1.2: Simple graph.

Definition 1.3 [9] *System* is a set of components, subsystems, or assemblies arranged to a specific design in order to achieve required functions with acceptable performance and reliability fig.(1.3) shows system. The arcs represent the system components e_1, e_2, e_3, e_4 and e_5 . The nodes (node 1, node 2, node 3, node 4) represent the connections between the system components.

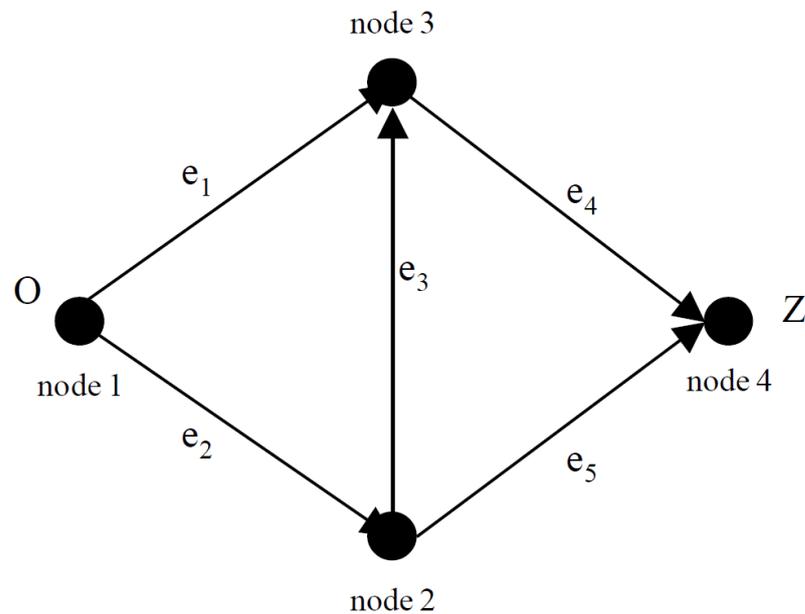


Figure 1.3: System.

Definition 1.4 [1] Vertex set and edge set must both be finite for a graph to be called *finite*

Definition 1.5 [18] **A loop** It is possible to have an edge from a vertex to itself. As an example, see fig. (1.4) (e_4).

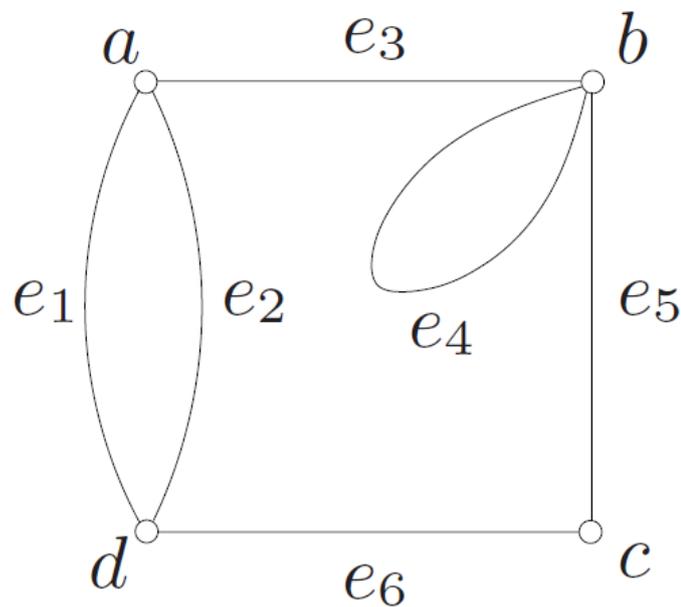


Figure 1.4: Graph with loop and multiple edge.

Definition 1.6 [31] If two or more edges in a graph G have the same endpoints (end vertices), they are said to be **parallel** or **multiple edge**, see fig.(1.4) (e_1, e_2).

Definition 1.7 [9] Vertices u and v are said to be **incident** with each other and to be connected by e if they are the endpoints of edge e in graph G . For example, e_2 is incident to 1 and 2 in fig.(1.3).

Definition 1.8 [38] A **directed edge** is an edge connected to the ordered pair $(u, v) \in E$, and it is thought to have a direction from node u to node v .

Definition 1.9 [37] *Undirected edge* is an edge (no arrow shown) without a node or edge in a graph, undirected edges are used to denote connections that allow for two-way communication between nodes.

Definition 1.10 [45] If all edges of the graph G are directed edges, then G is called a *directed graphs*, also known as (**digraphs**), as an example, see fig.(1.3)

Definition 1.11 [51] An *undirected graph* is a graph in which all of the edges are not directed in any one direction, as an example, see fig. (1.5)

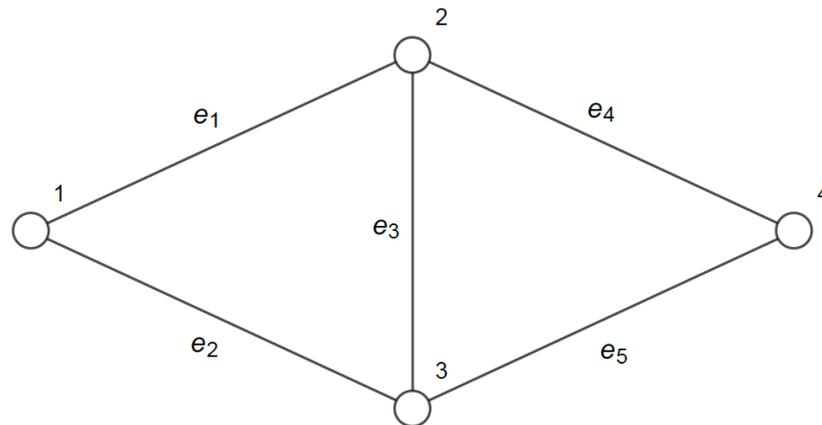
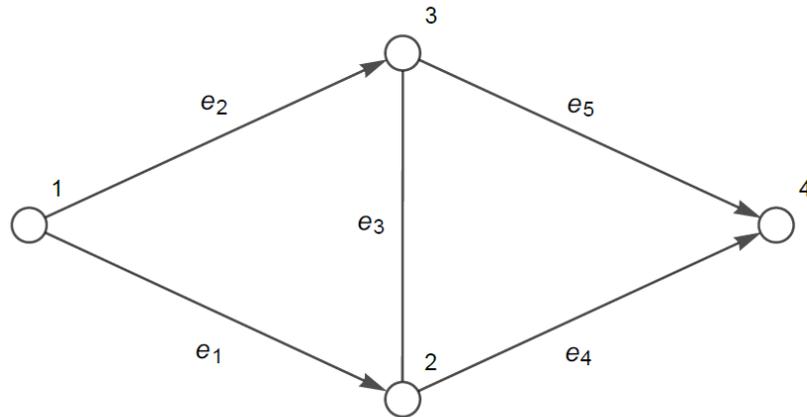


Figure 1.5: Undirected graph.

Definition 1.12 [52] The graph or digraph is only referred to as **connected** if there is at least one minimal path set connecting every pair of nodes; otherwise, it is referred to as *be disconnected*.

Definition 1.13 [46] A *mixed graph* consists of a graph that has both directed and undirected edges. as an example, see fig.(1.6)

Figure 1.6: Mixed graph G .

Definition 1.14 [45] If e is an edge of a graph G , then G_e (obtained graph) refers to the graph that was created by **deleting** e from G , as an example, see fig.(1.7).

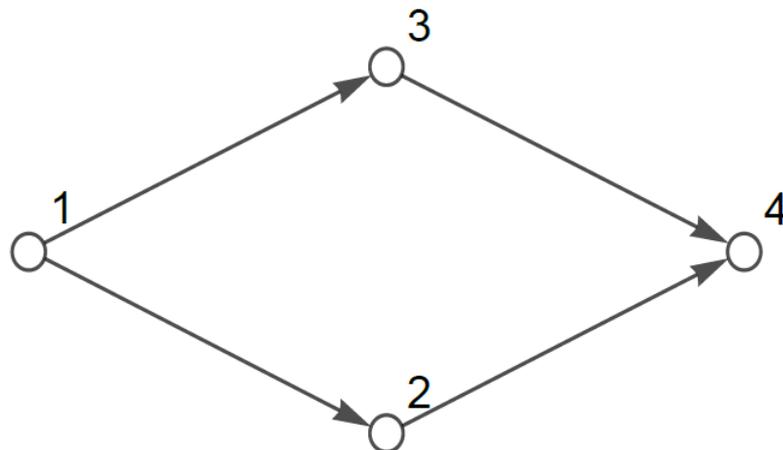


Figure 1.7: Deletion of edge.

Definition 1.15 [30] Let G be a graph, given that $e = u, v \in E$. The two steps within constitute the contraction of edge e .

1. Removing e from E and identifying vertices u and v in a new vertex called $u-v$.
2. All G edges with one end at either u or v will have this end at $u-v$ with the other end remaining unchanged, as an example, see fig.(1.8).

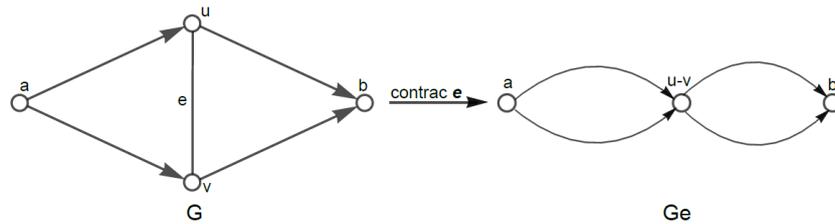


Figure 1.8: The contraction of edge.

Definition 1.16 [46] If v is a vertex in G , then G_v represents the graph created from G by **deleting** vertex and all edges that create it, as an example, see fig.(1.9)

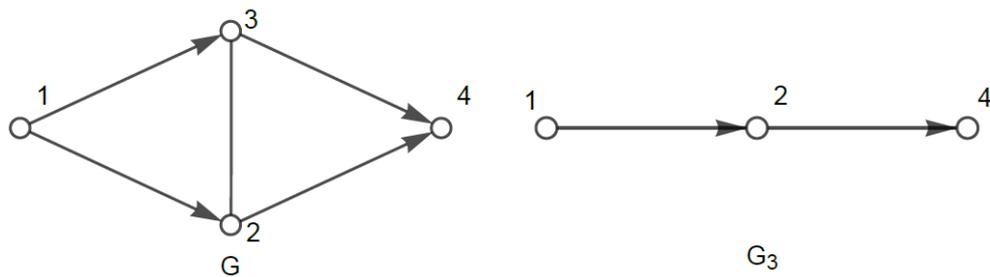


Figure 1.9: Deletion of vertex 3.

Definition 1.17 [25] A graph $H = (V, E')$ that has the properties $E' \subseteq E$, is **apartial graph** of a given graph $G = (V, E)$, as an example, see fig.(1.10).

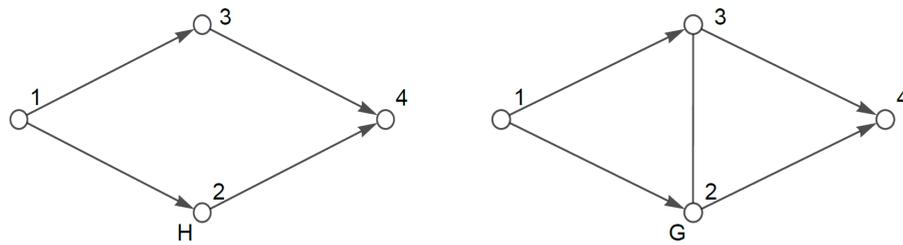


Figure 1.10: Partial graph H of G.

Definition 1.18 [27] *H is a **sub graph** of G if its vertex set and edge set are subsets of G each vertex set and edge set, respectively. In other words, $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$, as an example, fig.(1.11).*

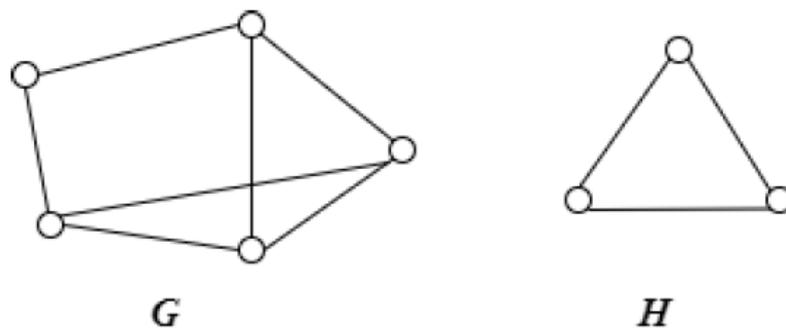


Figure 1.11: Sub graph.

Definition 1.19 [47] *A **path** is a set of components that, when working together, ensure that the system is working.*

Definition 1.20 [47, 52] *A path set is called **minimal path set** if it cannot be reduced without losing its status as a path set.*

Definition 1.21 [11] *A **cut** is a set of components that by failing causes the system to fail.*

Definition 1.22 [11, 45] A cut set is **minimal cut set** if it cannot be reduced without losing its status as a cut set .

Example 1.1 Consider the system of a bridge shown in fig.(1.3), by definition (1.19 and 1.20), we get all minimal path sets as follows: $H_1 = e_1, e_4$; $H_2 = e_2, e_3, e_4$; $H_3 = e_2, e_5$. Then, by definition (1.21 and 1.22), we get all minimal cut sets as follows: $K_1 = e_1, e_2$; $K_2 = e_2, e_4$; $K_3 = e_4, e_5$; $K_4 = e_1, e_3, e_5$; $K_5 = e_2, e_3, e_4$

1.3 Matrix Representation of Graphs

A method for presenting and evaluating system data is the use of matrices [43]. Although a matrix and a graph both contain the same information, the former is more advantageous for computation and data analysis [39].

Definition 1.23 [27] **Adjacency matrix (A)** of a simple graph is a matrix with rows and columns by graph vertices, with a 1 or 0 in position (u,v) according to whether u and v are adjacent or not.

Example 1.2 The adjacency matrix of the graph in fig.(1.3) is

$$[A] = \begin{bmatrix} 0 & x_2 & x_1 & 0 \\ 0 & 0 & x_3 & x_5 \\ 0 & 0 & 0 & x_4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Definition 1.24 [46] A **Connection Matrix (CM)** is constructed by adding an adjacency matrix and the identity matrix $[I_m]$, or by changing the zeros in an adjacency matrix's diagonal to ones, as in $CM = A + I_m$.

Example 1.3 The connection matrix of network in fig.(1.3) is

$$[CM] = \begin{bmatrix} 1 & x_2 & x_1 & 0 \\ 0 & 1 & x_3 & x_5 \\ 0 & 0 & 1 & x_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.4 Structure Function

A structure function $\varphi(x)$ is a binary property that indicates whether or not a network is working. It gives each component's state [22].

Consider a network with m components, each of which has a probability to be present in one of two states: success or failure. The binary variable x_k gives details on the component's state [21].

$$x_k = \begin{cases} 1 & \text{if component } k \text{ succeeds,} \\ 0 & \text{if component } k \text{ fails.} \end{cases}$$

From the formula x_k above one have for all i and n .

$$x_i^n = x_i. \quad (1.1)$$

And, the structure function of system as follows:

$$\varphi(x_1, x_2, \dots, x_m) = \begin{cases} 1 & \text{if the system succeeds,} \\ 0 & \text{if the system fails.} \end{cases}$$

Definition 1.25 Reliability Function [49] denoted $R(t)$, is the probability that a device will perform out its intended functions satisfactorily for a specified amount of time under defined operating conditions. The time to system failure T , can be used to express this [52]:

$$R(t) = Pr\{T > t\}. \quad (1.2)$$

Considered the reliability of a system is a function of the reliability of its components since the performance of a still depends on the performance of its components.

- **Some properties of $R(t)$** [52]

1. $0 \leq R(t) \leq 1$.
2. $R(0) = 1$; the device is assumed to be working properly at time $t = 0$, and $R(\infty) = 0$; no device can work forever without failure.
3. $R(t)$ in general is decreasing function of time t . i.e., if $t_1 < t_2$ then $R(t_1) > R(t_2)$ this means that the probability failure takes place at a time less than or equal to t .
Mathematically speaking, let

$$F(t) = Pr\{T \leq t\}, \quad (1.3)$$

be the failure function (cumulative distribution function, (c. d. f), or unreliability) [45].
It follows that the reliability function (non-failure function) is:

$$R(t) = 1 - F(t) \quad (1.4)$$

Eq.(1.4) is equivalent to

$$R(t) = \int_t^{\infty} f(t) dt. \quad (1.5)$$

Definition 1.26 [17, 24, 32] **Reliability Block Diagram (RBD)** of a system is a graph with the system components as its edges, in a reliability block diagram, a circle or rectangle is often used to represent a component. It only indicates how the components work to ensure the system works. Each reliability block diagram should include the reliability parameters and assumptions or simplifications used to develop the diagram, as an example, see fig.(1.12).

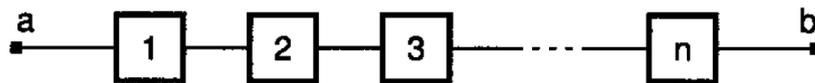


Figure 1.12: Reliability block diagram of a series structure.

Definition 1.27 [8] *Simple Reliability* suppose we need to determine a system's reliability that consists of a number of components, it is possible to determine the reliability of each individual component and sum these individual reliabilities based on how they are connected in order to obtain the whole reliability.

1.5 Reliability Systems

A set of components, a device or system. The structural function determines the system's status based on the status of its main components [54].

1.5.1 Series System

The foremost common and most basic design is the series system. A series system works satisfactorily if and only if all of its components perform satisfactorily. The system fails if any one of its components fails. The formula for a series system's structural function is [17].

$$R_s = \prod_{i=1}^n R_i. \quad (1.6)$$

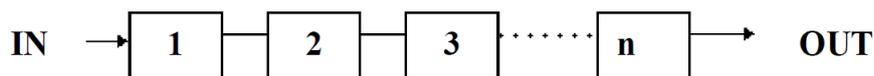


Figure 1.13: Series system.

1.5.2 Parallel System

A parallel system with n components is one that succeeds if one that at least one of the n components succeeds (with all n components operating simultaneously) [45].

$$R_s = 1 - \prod_{i=1}^n (1 - R_i) \quad (1.7)$$

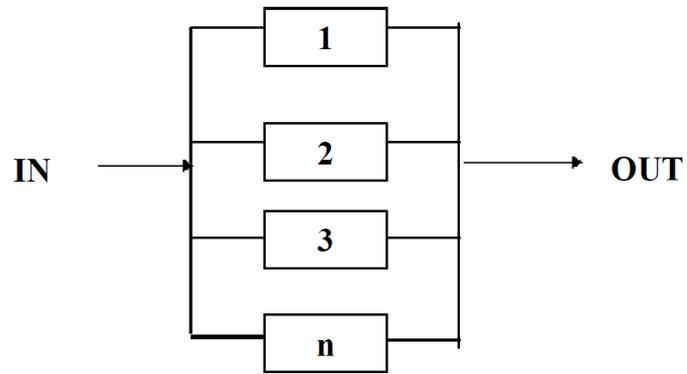


Figure 1.14: Parallel system.

1.5.3 Series-Parallel System

Series-Parallel system is dependent on its m series of redundantly connected components.

$$R_s = R_1.R_2.R_3 \dots R_m = \prod_{i=1}^m R_i$$

R is a reliability of series system. If we arrange a set in parallel each one with n components in series[33].

$$R_s = 1 - \prod_{i=1}^n (1 - \prod_{i=1}^m R_i) \quad (1.8)$$

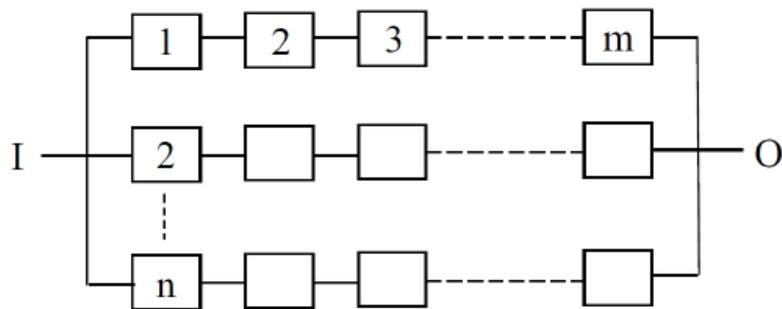


Figure 1.15: Series-Parallel system.

1.5.4 Parallel - Series System

The reliability of parallel - series elements can be description as:

$$R_s = \prod_{j=1}^m (1 - \prod_{i=1}^n (1 - R_i)) \quad (1.9)$$

where R_i is the reliability of a single component. Each made up of n parallel components, and m sets are connected in series the system's reliability is evaluated [17].

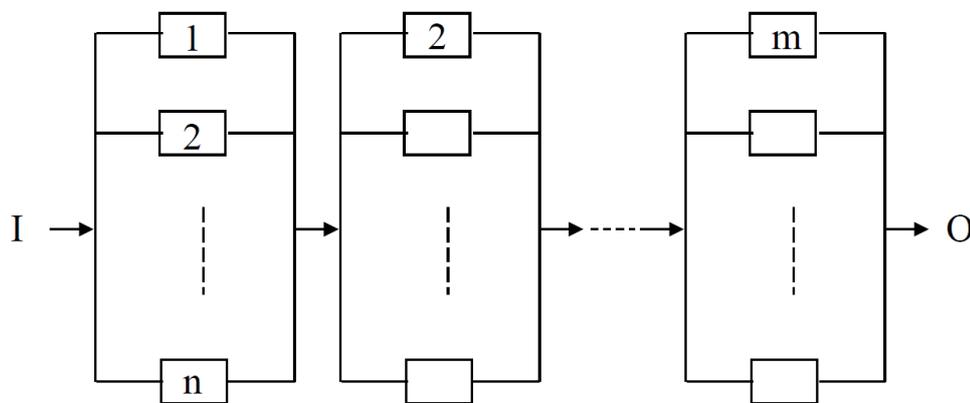


Figure 1.16: Parallel - Series System.

1.5.5 Complex System

When the reliability network of a system cannot be decomposed into parallel subsystems with independent elements or group series, the system is referred to as complex [1, 34, 45]. Where a complex system one that has interconnected or interwoven parts (components) and is difficult to evaluate in terms of reliability or a problem because of the limits imposed on it by the techniques, algorithms, and software presently in use (such as operating systems and programming languages), as an example, see fig.(1.17).

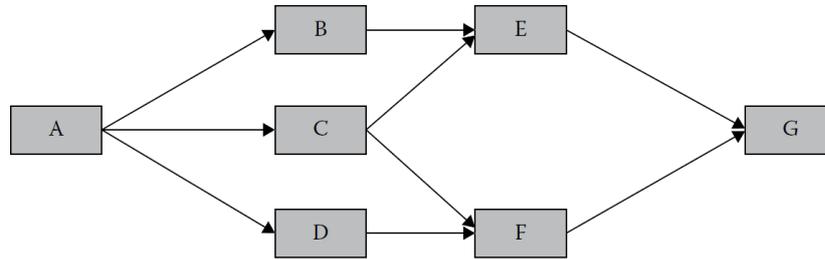


Figure 1.17: Complex system and with - terminal reliability.

There are several methods for evaluating a complex system's reliability, include: Path tracing method, minimal cut method, sum of disjoint product method and pivotal decomposition [23, 29, 50].

Definition 1.28 [28] ***Terminal Reliability*** which is defined as the probability that a path exists between each pair of nodes in the specified set of nodes of the network, is a need that a set of nodes of the network be able to communicate with one another, as an example, see fig.(1.17).

Definition 1.29 [37] ***Reliability Allocation*** used to allocate the failure allowance between each network, its sub networks, and its constituent parts. The basic goal of reliability allocation is for creating a value or aim for the reliability of each component of the product that will be made so that the producers may determine the performance required by that product.

Definition 1.30 [19] ***Redundant Components or Reliability Redundancy***, redundancy is commonly used in systems design to improve system reliability, especially when it is difficult to increase the reliability of the component itself.

Definition 1.31 [26] *Redundant system consisting of two or more components connected in parallel and both components were operating simultaneously is called **active redundancy***. In active redundancy all the redundant units are operated simultaneously instead of switching on only when need arises.

Definition 1.32 [5, 15, 28] *The reliability importance aims to determine the contribution of components in measuring system performance. Engineers are frequently given the responsibility of identifying the system's least reliable component(s) in order to improve the design after the reliability of a system has been evaluated.*

Chapter 2

Some Methods for Finding Minimal
(path / cut) Sets and Exact Methods
to Evaluating the Reliability of
Complex Systems

2.1 Introduction

In order to calculate network reliability using minimal path sets or minimal cut sets [37, 50]. There are several different techniques for finding the minimal path sets and minimal cut sets. In this chapter, we studied three methods to find the minimal path sets and two methods to find the minimal cut sets, we discussed methods for evaluating the reliability of systems. The following methods are based on basic concepts in network theory as in the first chapter in order to simplify the reliability of the system [44].

2.2 Generation of Minimal Path Sets

In evaluating the reliability of a system, minimal path sets are important [4]. The minimal path sets of a system must be known in order to use many of the methods to be discussed in this thesis. There are many different methods to find the minimal path set enumeration [57], we study three techniques to find the minimal path sets enumeration of a complex system shown in fig.(2.1).

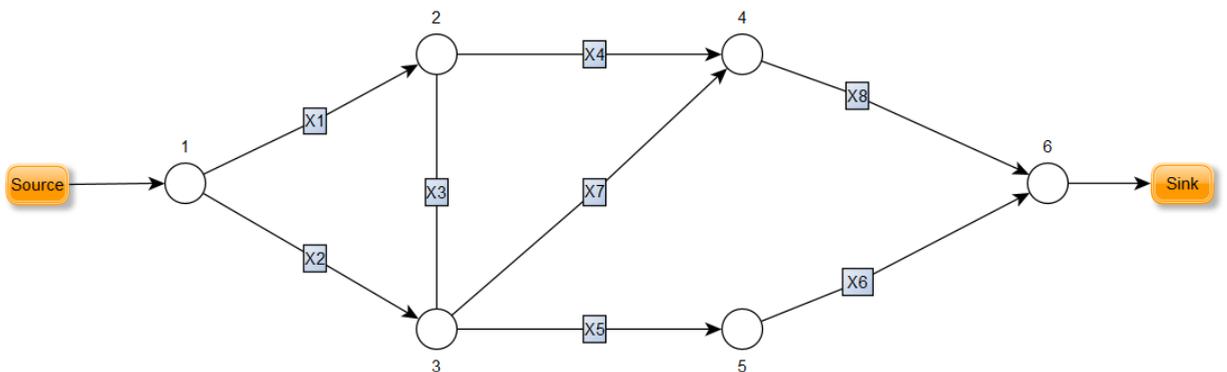


Figure 2.1: Complex system.

2.2.1 Connection Matrix

The direct connections between each pair of signal nodes going from the source to the sink are described by a connection matrix[12], as shown in the matrix's (2.1). A node is always connected to itself, hence the matrix's main diagonal elements are (1)[33]. The entries at positions (i, j) and (j, i) in the connection matrix are x_k if component k . Connects node i and j , as an example, the connection matrix for fig.(2.1) that edges from 1 to 8 and nodes 1, 2, 3, 4, 5 and 6 respectively as follows:

$$[C] = \begin{array}{c|cccccc} \text{nodes} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & x_1 & x_2 & 0 & 0 & 0 \\ 2 & 0 & 1 & x_3 & 0 & x_4 & 0 \\ 3 & 0 & x_3 & 1 & x_5 & x_7 & 0 \\ 4 & 0 & 0 & 0 & 1 & 0 & x_6 \\ 5 & 0 & 0 & 0 & 0 & 1 & x_8 \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \quad (2.1)$$

2.2.2 Delimiter's Method

For reliability expression generally of the system in two steps are [12, 55]:

- i. All minimal path sets are determined.
- ii. The problem of minimal path sets enumeration in a general system can be solved using a variety of methods, have chosen the minimal paths using the connection matrix's. Using the connection matrix $[C]$ knowledge to compute the system-success determinant $|S|$.

2.2.2.1 Delimiter's Method Steps to Find the Minimal Path Sets

Step 1. Construct the logic graph's connection matrix.

Step 2. To the connection matrix $[C]$, add a diagonal unity matrix $[U]$ of dimension $p \times p$.

Step 3. This step include the options as:

a. Remove the column corresponding to source 1 and the row corresponding to sink 6 in the matrix generated at *step2* .

b. Take remaining rows and columns and define the system success determinant $|S|$ of size $(p-1)$. All of the algebraic variables are converted to their equivalent Boolean variables in this stage.

Step 4. Expand the determinant $|S|$ using Boolean sum and product operations[62].

Let's take an example to illustrate this method.

Example 2.1 Consider the complex system shown in fig.(2.1), to find the determinant of success for that system. Based on connection matrix (2.1).

$$[C] = \begin{bmatrix} 0 & x_1 & x_2 & 0 & 0 & 0 \\ 0 & 0 & x_3 & 0 & x_4 & 0 \\ 0 & x_3 & 0 & x_5 & x_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & x_6 \\ 0 & 0 & 0 & 0 & 0 & x_8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_2 & 0 & 0 & 0 \\ 0 & 1 & x_3 & 0 & x_4 & 0 \\ 0 & x_3 & 1 & x_5 & x_7 & 0 \\ 0 & 0 & 0 & 1 & 0 & x_6 \\ 0 & 0 & 0 & 0 & 1 & x_8 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

We delete the first column and the last row of the matrix, then we find the determinant of the matrix as shown below:

$$|S| = \begin{vmatrix} x_1 & x_2 & 0 & 0 & 0 \\ 1 & x_3 & 0 & x_4 & 0 \\ x_3 & 1 & x_5 & x_7 & 0 \\ 0 & 0 & 1 & 0 & x_6 \\ 0 & 0 & 0 & 1 & x_8 \end{vmatrix} = x_1 \begin{vmatrix} x_3 & 0 & x_4 & 0 \\ 1 & x_5 & x_7 & 0 \\ 0 & 1 & 0 & x_6 \\ 0 & 0 & 1 & x_8 \end{vmatrix} - x_2 \begin{vmatrix} 1 & 0 & x_4 & 0 \\ x_3 & x_5 & x_7 & 0 \\ 0 & 1 & 0 & x_6 \\ 0 & 0 & 1 & x_8 \end{vmatrix}$$

$$= x_1 \cdot x_3 \begin{vmatrix} x_5 & x_7 & 0 \\ 1 & 0 & x_6 \\ 0 & 1 & x_8 \end{vmatrix} + x_1 \cdot x_4 \begin{vmatrix} 1 & x_5 & 0 \\ 0 & 1 & x_6 \\ 0 & 0 & x_8 \end{vmatrix} - x_2 \cdot 1 \begin{vmatrix} x_5 & x_7 & 0 \\ 1 & 0 & x_6 \\ 0 & 1 & x_8 \end{vmatrix} - x_2 \cdot x_4 \begin{vmatrix} x_3 & x_5 & 0 \\ 0 & 1 & x_6 \\ 0 & 0 & x_8 \end{vmatrix}$$

$$S = x_1x_4x_8 + x_2x_5x_6 + x_2x_7x_8 - x_1x_3x_5x_6 - x_1x_3x_7x_8 - x_2x_3x_4x_8$$

By Boolean sum and product operations.

$$S = x_1x_4x_8 \cup x_2x_5x_6 \cup x_2x_7x_8 \cup x_1x_3x_5x_6 \cup x_1x_3x_7x_8 \cup x_2x_3x_4x_8$$

In this method, we obtained six minimal path sets for complex system fig. (2.1).

$$S = \{MP_1 = \{x_2x_5x_6\}, MP_2 = \{x_1x_4x_8\}, MP_3 = \{x_2x_7x_8\}, MP_4 = \{x_1x_3x_5x_6\}, MP_5 = \{x_1x_3x_7x_8\}, MP_6 = \{x_2x_3x_4x_8\}\} \quad (2.2)$$

2.2.2.2 Advantages of Delimiter Method

1. An efficient minimal path sets enumeration method that is simple and directly.
2. This method directly neglects the minimal paths of cyclic type.
3. It does not require time and effort for calculations unlike other methods.
4. It does not require a recursive determinant of the operation x of the multiples of the array, but only requires the expansion of one element to size (p-1).

2.2.3 Node Removal Method

Aggarwal et al [6] describe a technique to generate minimal path sets in a network link matrix by removing ideal nodes. The direct connection from node i to node j is represented by the symbol c_{ij} in the connection matrix M of order $m \times m$. This process removes each node from the link matrix that is neither the source nor the sink one at a time until just the source node and the sink node remain left[52]. To change the link matrix entries with the remaining nodes, according to equation shown below:

$$C' = C_{ij} + C_{il}C_{lj} \quad (2.3)$$

If node l removed, where $i \neq j$, $i \neq l$, $1 < m$, $1 < j \leq m$, for $i = 1, 2, \dots, m$. Otherwise $C'_{ii} = 1$, if and only if $i = j$.

When using this method, it is important to label the source node as the first node and the

sink node as the last node, each intermediate node is removed one by one until a 2×2 matrix is left. Let's take an example to illustrate this method.

Example 2.2 *Reconsider the complex system shown in fig. (2.1). The nodes are labeled as 1, 2, 3, 4, 5, and 6, and the edges are numbered from x_1 through x_8 . The connection matrix of the complex system is (2.1), and according the above descriptions: Now, we remove node 2. The modified input of the original matrix becomes as follows according to eq. (2.3)*

$$\begin{aligned}
C'_{13} &= x_2 + x_1x_3, & C'_{15} &= 0 + x_1x_4 = x_1x_4, & C'_{16} &= 0 + x_10 = 0 \\
C'_{31} &= 0 + x_30 = 0, & C'_{34} &= x_5 + x_30 = x_5, & C'_{35} &= x_7 + x_3x_4, \\
C'_{36} &= 0 + x_30 = 0, & C'_{41} &= 0 + 0 = 0, & C'_{43} &= 0, \\
C'_{45} &= 0, & C'_{46} &= x_6 + 0 = x_6, & C'_{51} &= 0 + 0 = 0, \\
C'_{53} &= 0, & C'_{54} &= 0, & C'_{56} &= x_8 + 0 = x_8, \\
C'_{61} &= C'_{63} = C'_{64} = C'_{65} = 0.
\end{aligned}$$

The modified connection matrix has become 5×5 matrix as shown:

$$C' = \begin{bmatrix} 1 & x_2 + x_1x_3 & 0 & x_1x_4 & 0 \\ 0 & 1 & x_5 & x_7 + x_3x_4 & 0 \\ 0 & 0 & 1 & 0 & x_6 \\ 0 & 0 & 0 & 1 & x_8 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, we remove node 2, the modified input of the original matrix becomes as follows according to the eq (2.3):

$$\begin{aligned}
C''_{13} &= x_2x_5 + x_1x_3x_5 \\
C''_{14} &= x_1x_4 + (x_2 + x_1x_3)(x_7 + x_3x_4) = x_1x_4 + x_2x_7 + x_2x_3x_4 + x_1x_3x_7 \\
C''_{15} &= 0 + (x_2 + x_1x_3)0 = 0 \\
C''_{31} &= 0, & C''_{34} &= 0, & C''_{35} &= x_6 + 0 = x_6 \\
C''_{41} &= 0, & C''_{43} &= 0, & C''_{45} &= x_8 + 0 = x_8, & C''_{51} &= 0, & C''_{53} &= 0, & C''_{54} &= 0
\end{aligned}$$

$$C'' = \begin{bmatrix} 1 & x_2x_5 + x_1x_3x_5 & x_1x_4 + x_2x_7 + x_2x_3x_4 + x_1x_3x_7 & 0 \\ 0 & 1 & 0 & x_6 \\ 0 & 0 & 1 & x_8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, we remove node 2. The modified input of the original matrix becomes as follows according to the eq.(2.3):

$$C'''_{13} = x_1x_4 + x_2x_7 + x_2x_3x_4 + x_1x_3x_7$$

$$C'''_{14} = x_2x_5x_6 + x_1x_3x_5x_6$$

$$C'''_{31} = 0$$

$$C'''_{34} = x_8 + 0 = x_8$$

$$C'''_{41} = 0$$

$$C'''_{43} = 0$$

connection matrix becomes the following 3×3 matrix:

$$C'''' = \begin{bmatrix} 1 & x_1x_4 + x_2x_7 + x_2x_3x_4 + x_1x_3x_7 & x_2x_5x_6 + x_1x_3x_5x_6 \\ 0 & 1 & x_8 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, we remove node 2. The modified input of the original matrix becomes as follows according to the eq (2.3):

$$C''''_{13} = x_2x_5x_6 + x_1x_3x_5x_6 + (x_1x_4 + x_2x_7 + x_2x_3x_4 + x_1x_3x_7)x_8$$

$$C''''_{13} = x_2x_5x_6 + x_1x_3x_5x_6 + x_1x_4x_8 + x_2x_7x_8 + x_2x_3x_4x_8 + x_1x_3x_7x_8$$

Connection matrix becomes the following 2×2 matrix:

$$C'''' = \begin{bmatrix} 1 & x_2x_5x_6 + x_1x_3x_5x_6 + x_1x_4x_8 + x_2x_7x_8 + x_2x_3x_4x_8 + x_1x_3x_7x_8 \\ 0 & 1 \end{bmatrix}$$

All minimal path sets are determined in the entry, C''''_{13} which is located in the

matrix's top right corner, of the resulting 2×2 matrix. The system diagram from fig.(2.1) shows the minimal path sets for communication between the source node and sink node are: $x_2x_5x_6 + x_1x_3x_5x_6 + x_1x_4x_8 + x_2x_7x_8 + x_2x_3x_4x_8 + x_1x_3x_7x_8$

Hence C_{13}'''' represent the sum of all the minimum path sets of the complex system.

So the minimal path sets:

$$MP_1 = x_2x_5x_6, MP_2 = x_1x_4x_8, MP_3 = x_2x_7x_8, MP_4 = x_1x_3x_5x_6$$

$$MP_5 = x_1x_3x_7x_8, MP_6 = x_2x_3x_4x_8.$$

In this method, we also obtained the same result as in eq (2.2).

2.2.4 Powers of Adjacency Matrix

The network graph's connection matrix representation works as the method's base. This matrix is iteratively multiplied by itself algebraically (remember, not numerically)[38, 47]. The number of times this multiplication will proceed depends on the maximum cardinality path. When performing a multiplication, elements are discarded if the number of elements is less than the order of the forming matrix [43]. Let's take an example to illustrate this method.

Example 2.3 Consider the complex system in fig.(2.1). Enumerate the minimal path sets of the system using the powers of its adjacency matrix.

$$M = \begin{bmatrix} 0 & x_1 & x_2 & 0 & 0 & 0 \\ 0 & 0 & x_3 & 0 & x_4 & 0 \\ 0 & x_3 & 0 & x_5 & x_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & x_6 \\ 0 & 0 & 0 & 0 & 0 & x_8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.4)$$

To find the first row of the adjacency matrix (2.4), $M^2 = M \times M$, we taking the first row and multiply it by each column of the adjacency matrix M as follows:

$$\begin{bmatrix} 0 & x_1 & x_2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0.$$

To find the second row of the adjacency matrix M^2 , we take the first row and multiply by the second column of the e matrix M as follows:

$$\begin{bmatrix} 0 & x_1 & x_2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ 0 \\ x_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = x_2x_3,$$

$$\begin{bmatrix} 0 & x_1 & x_2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = x_1x_3.$$

We continue this procedure by multiplying the rest of the rows by the columns to get the following matrix M^2 :

$$M^2 = \begin{bmatrix} 0 & x_2x_3 & x_1x_3 & x_2x_5 & x_1x_4 + x_2x_7 & 0 \\ 0 & 0 & 0 & x_3x_5 & x_3x_7 & x_4x_8 \\ 0 & 0 & 0 & 0 & x_3x_4 & x_5x_6 + x_7x_8 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

we continue to apply the same steps above to find a matrix of the third $M^3 = M \times M^2$ and fourth $M^4 = M \times M^3$ degrees.

$$M^3 = \begin{bmatrix} 0 & x_1x_3 & x_2x_3 & x_1x_3x_5 & x_2x_3x_4 + x_1x_3x_7 & x_2x_5x_6 + x_1x_4x_8 + x_2x_7x_8 \\ 0 & 0 & 0 & x_5 & 0 & x_3x_5x_6 + x_3x_7x_8 \\ 0 & 0 & 0 & 0 & 0 & x_3x_4x_8 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M^4 = \begin{bmatrix} 0 & x_2x_3 & x_1x_3 & x_2x_3x_5 & x_1x_3x_4 + x_2x_3x_7 & x_1x_3x_5x_6 + x_2x_3x_4x_8 + x_1x_3x_7x_8 \\ 0 & 0 & 0 & x_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

So the minimal path sets between the node pair (n_1, n_6) are:

- (0) from element $M(n_1, n_6)$ in M^2 .
- $(x_2x_5x_6, x_1x_4x_8, x_2x_7x_8)$ from element $M(n_1, n_6)$ in M^3 .
- $(x_1x_3x_5x_6, x_2x_3x_4x_8, x_1x_3x_7x_8)$ from element $M(n_1, n_6)$ in M^4 .

In this method, we also obtained the same result as in eq (2.2).

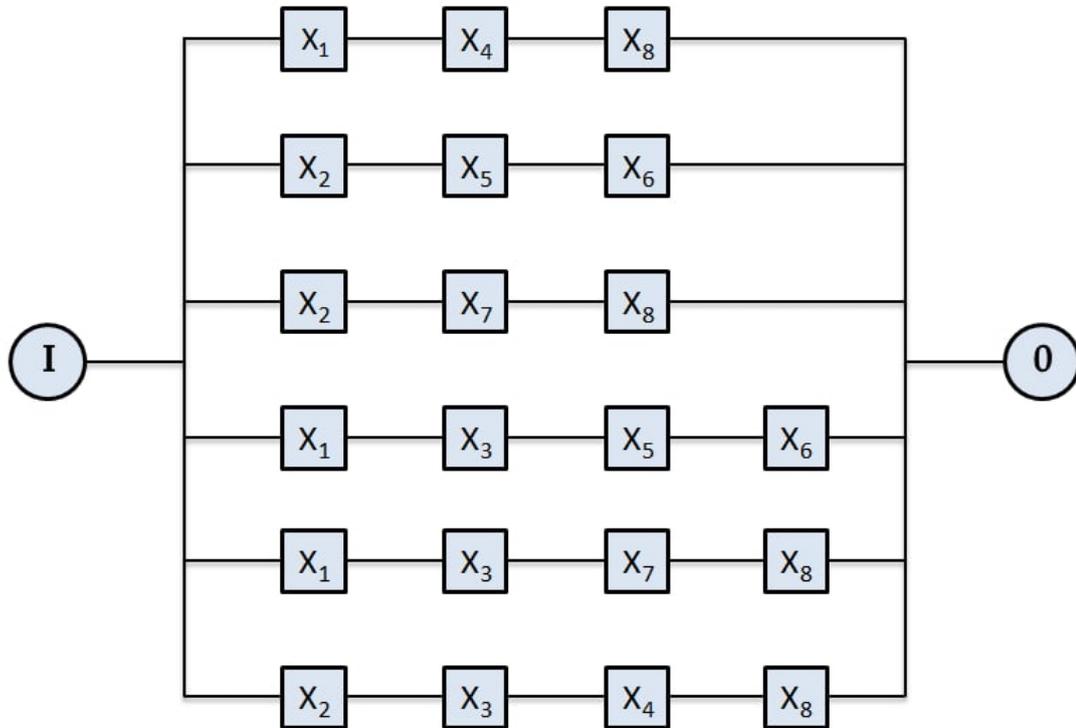


Figure 2.2: All minimal path sets of complex system.

2.3 Generation of Minimal Cut Sets

Enumerating minimal cut sets of a graph has many kinds of applications. Although methods with path sets being generated, many algorithms to evaluate the reliability of systems begin by finding all minimal cut sets separating a given node pair. The number of cut sets usually much lower than the number of path sets in most systems, particularly in well-connected networks. Working with minimal cut sets rather than minimal path sets in these instances may be preferable. The methods for enumerating cut sets can be generally divided into [27, 37]:

- (i) Direct methods that make use of the network graph's structure and its representation

3. To generate the third order reductions, repeat step 2 with three columns at once. This time, remove any first or second order cuts, and continue until the highest cut order is achieved.

Example 2.4 *The complex system in fig.(2.1) has the minimal path sets*

$$MP_1 = R_2R_5 R_6, \quad MP_2 = R_1R_4 R_8, MP_3 = R_2R_7 R_8, MP_4 = R_1R_3 R_5R_6, \quad MP_5 = R_2R_3 R_4R_8, \quad MP_6 = R_1R_3 R_7R_8$$

Based on the descriptions above, IM of the complex system as follows,

$$IM = \begin{matrix} & \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \end{matrix} \\ \begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

Hence, no single column in IM exists in which all elements are non-zero, then there are no first order minimal cuts. Then, applying Step(2)we found second order minimal cuts. The resultant minimal cut sets are:

$$\begin{aligned} C = \{ & MC_1 = \{x_1x_2\}, \quad MC_2 = \{x_5x_8\}, MC_3 = \{x_6x_8\}, \\ & MC_4 = \{x_2x_3x_4\}, MC_5 = \{x_2x_3x_8\}, MC_6 = \{x_4x_6x_7\}, \\ & MC_7 = \{x_4x_5x_7\}, \quad MC_8 = \{x_1x_3x_6x_7\}, MC_9 = \{x_1x_3x_5x_7\} \} \end{aligned} \quad (2.5)$$

2.3.2 Enumerate the Minimal Cut Sets Using the Connection Matrix

This method to presenting minimal cut sets is explained in the steps which make up the method are as follows[47, 50]:

Step 1: Generate the directed graph's connection matrix, in which the matrix's elements

stand in for the link labels connecting the nodes.

Step 2: Collect all the entries that exist in the first row, which represents a source minimal cut, and in the last column, which represents a destination minimal cut.

Step 3: Using only column numbers 2 to $(n-1)$, generate a set S that includes all combinations of columns of order (1) to $(n-3)$, $n \geq 4$, ($n < 4$ products the simple an instance). This set is formed to generate all possible cuts that differ from the cut sets obtained in step 2.

Step 4: Delete those combinations of set S that result in redundant or non-minimal cut sets following the rules the following:

a. If the combination only consists of columns with zero entries in the first row, step 1 will result in a cut set that includes the cut set created in step 3. For instance, in equation (2.5), the columns x_4, x_5 and x_4, x_5 are removed. Additionally, since it does not constitute a minimal cut set, we remove all combinations that they contain.

b. If the combination only includes rows with non-zero entries in the last column, as this creates a cut set that includes the cut set created in step 3. These rules help to reduce the number of combinations that must be evaluated in order generate minimal cut sets as well as deleting combinations that eventually generate non-minimal cut sets

Step 5: Take one combination and, without taking note of the columns it represents, collect all of the links-labels that exist in the row(s) that correspond to row 1 plus this combination in the group created in Step 3. Another cut set will be provided by this combination.

Step 6: Go through the remaining combinations by repeat step 5.

Let's take an example to illustrate the method.

Example 2.5 Consider the complex system fig.(2.1) to find the minimal cut sets by applying above steps:

1. Applying the method enumerate the minimal cut sets using the connection matrix described on the connected matrix in (2.1) to extract the connection matrix.

2. Collecting terms from the first row of the connection matrix C, step 2. Here, we have $\{x_1, x_2\}$, which isolates the source node and is a minimal cut set. Aside from that, collect terms for the final column n from the connection matrix C. Here it is $\{x_6, x_8\}$, that it is a minimal cut set as isolates the sink node.
3. Construct a set S of all the column combinations of order 1 to n-3 only, using only column numbers 2 to n-1. Here, orders of combinations are 1, 2, and 3 because n = 6. These combinations are now formed using x_2, x_3, x_4, x_5 . The set is:

$$S = \{\{x_2\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_2, x_5\}, \{x_3, x_4\}, \{x_3, x_5\}, \{x_4, x_5\}, \{x_2, x_3, x_4\}, \{x_2, x_3, x_5\}, \{x_3, x_4, x_5\}, \{x_2, x_4, x_5\}\} \quad (2.6)$$

4. Delete the combinations $\{x_4\}, \{x_5\}$, and $\{x_4, x_5\}$ from eq (2.6) because they both have zero entries in the first row, as well remove all combinations that they contain, $\{x_3, x_4, x_5\}, \{x_2, x_4, x_5\}$. The updated set is as follows:

$$S^* = \{\{x_2\}, \{x_3\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_2, x_5\}, \{x_3, x_4\}, \{x_3, x_5\}, \{x_2, x_3, x_4\}, \{x_2, x_3, x_5\}\} \quad (2.7)$$

5. We take the first combination $\{x_2\}$ of the set (2.7), plus row 1, here the rows $\{x_1, x_2\}$ are consider the entries in these rows, from the connection matrix M and we delete the columns corresponding to it, after deleting the first and second columns from the combination, we generate a minimal cut set consisting of the entries $\{x_2, x_3, x_4\}$. The minimal cut set is formed by taking the combination $\{x_3\}$ of the set (2.7) plus row 1, where rows $\{x_1, x_3\}$ are considered the entries in these rows, of the connection matrix M, and deleting the columns that correspond to it, i.e., we delete the first and third columns from the combination, we form a minimal cut set that is $\{x_1, x_3, x_5, x_7\}$.

We take combination $\{x_2, x_3\}$ of the set (2.7) plus row 1, and remove the columns that correspond to it, i.e., we remove the first, second, and third columns from the combination. From the remaining entries after deletion, we form a minimal cut set

that is $\{x_4, x_5, x_7\}$.

We take combination x_2, x_4 of the set (2.7) plus row 1, where rows $\{x_1, x_2, x_4\}$ are considered the entries in these rows, and we remove the first, second, and fourth columns from the combination. The remaining entries after deletion form the set $\{x_2, x_3, x_4, x_6\}$, but this set is not a minimal cut set because it also contains a minimal cut set is $\{x_2, x_3, x_4\}$. Therefore, it is deleted according to the deletion rules.

We take combination $\{x_2, x_5\}$ of the set (2.7) plus row 1, where rows $\{x_1, x_2, x_5\}$ are considered the entries in these rows, and we remove the first, second, and fifth columns from the combination. From the combination's remaining entries, we then form a minimal cut set that is $\{x_2, x_3, x_8\}$.

We take the connection matrix C's combination $\{x_3, x_4\}$ of the set (2.7) plus row 1, where rows $\{x_1, x_3, x_4\}$ are considered the entries in these rows, and we remove the first, third, and fourth columns from the combination. From the combination's remaining entries after deletion, we form a minimal cut set that is $\{x_1, x_3, x_6, x_7\}$.

We take the connection matrix C's combination $\{x_3, x_5\}$ of the set (2.7) plus row 1, where rows $\{x_1, x_3, x_5\}$ are considered the entries in these rows, and delete the columns that correspond to it. The remaining entries after deletion form a set $\{x_1, x_3, x_5, x_8\}$, but this set is not a minimal cut set because it contains a minimal cut set is x_5, x_8 . Therefore, it is deleted according to the deletion rules.

We take the combination $\{x_2, x_3, x_4\}$ of the set (2.7) plus row 1 of the connection matrix C, where rows $\{x_1, x_2, x_3, x_4\}$, and remove the columns that correspond to it, i.e., we remove the first, second, third, and fourth columns from the combination. From the remaining entries after deletion, we form a minimal cut set that is x_4, x_6, x_7 .

We take the combination x_2, x_3, x_5 of the set (2.7) plus row 1 of the connection matrix C and remove the columns that correspond to it, i.e., we remove the first, second, third, and fifth columns from the combination. From the remaining entries

after deletion, we form a minimal cut set that is x_5, x_8 . As a result, the following nine minimal cut sets are generated by this method, it's the same in eq.(2.5).

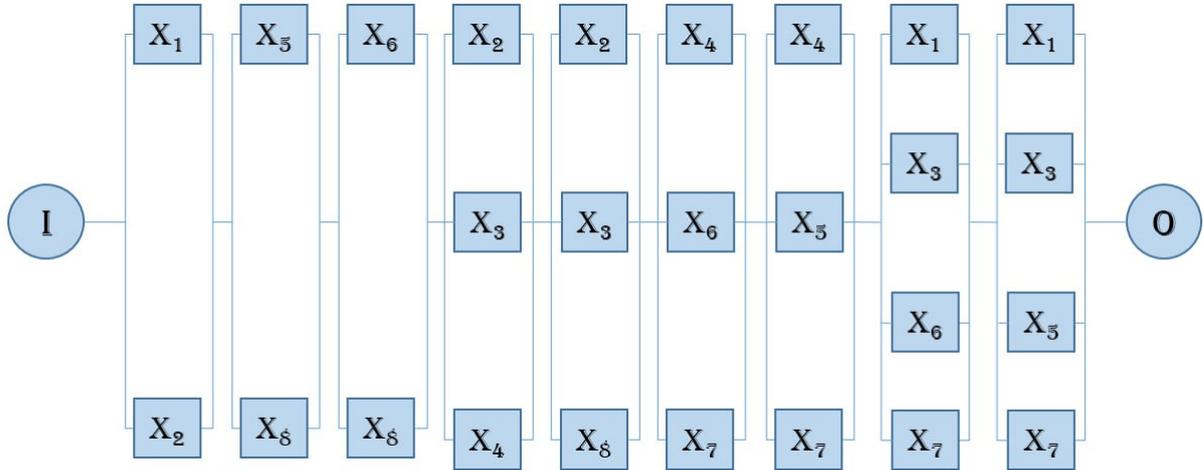


Figure 2.3: All minimal cuts of complex system.

2.4 General Methods for Evaluating Reliability of the Complex System

To compute the system reliability based on the components reliability, it can be used to have knowledge of the system structure as a function of its components [19, 37, 49]. There are many methods for evaluating reliability the systems are path tracing, parallel and series reductions, pivotal decomposition, inclusion-exclusion, sum-of-disjoint-products, delta-star transformation and others. In this section, we discuss methods for evaluating reliability the complex system shown in fig.(2.4), they are pivotal decomposition and sum-of-disjoint-products. Therefore, we studied in the above section about methods to generate minimal paths and minimal cuts because it requires knowledge the minimal path sets or minimal cut sets for many of these methods.

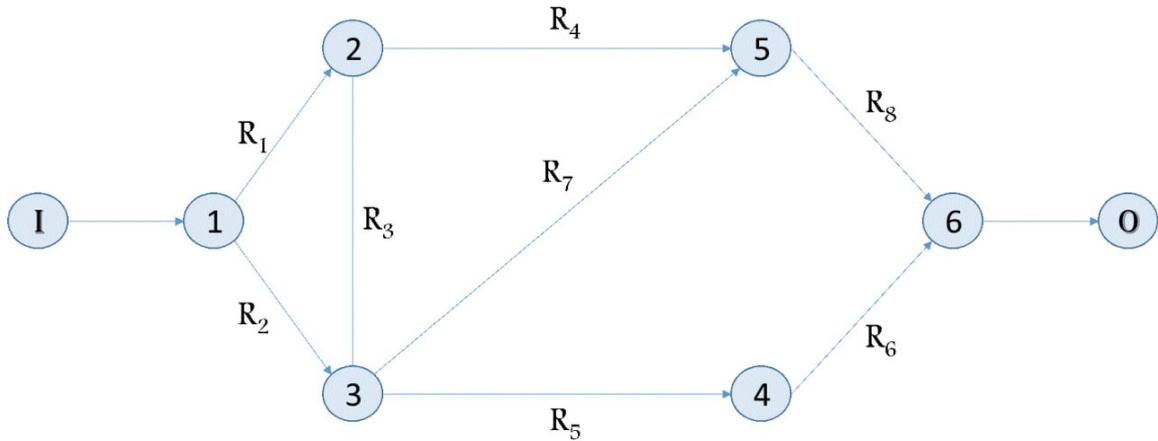


Figure 2.4: Complex system with eight components.

2.4.1 Pivotal Decomposition Method

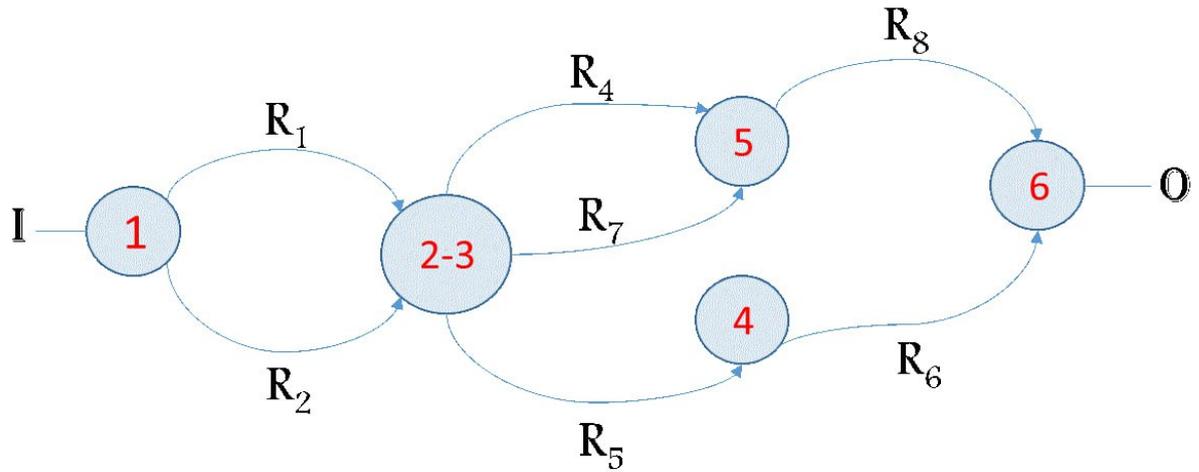
Conditional probability is used the foundation for the pivotal decomposition method. The idea behind this technique is shown by the equation below [37].

$$\begin{aligned}
 P_r(\text{system works}) &= P_r(\text{unit } i \text{ works}) P_r(\text{system works} \mid \text{unit } i \text{ works}) \\
 &+ P_r(\text{unit } i \text{ fails}) P_r(\text{system works} \mid \text{unit } i \text{ fails})
 \end{aligned}
 \tag{2.8}$$

The efficiency of this method depends on the ease of evaluating conditional probabilities.[37, 49] This means that choosing the component to be decomposed plays an important role in the efficiency of this method. If decomposition of the identified component leads to two system structures parallel and/or sequential cuts can be applied again, the efficiency of system reliability assessment will be improved. Let's take an example to illustrate the method.

Example 2.6 *Reconsider the complex system in fig (2.4). Calculate the reliability of a complex system based on eq (2.8).*

When choose edge R_3 from fig (2.4), and its contraction, we get the fig. (2.5):

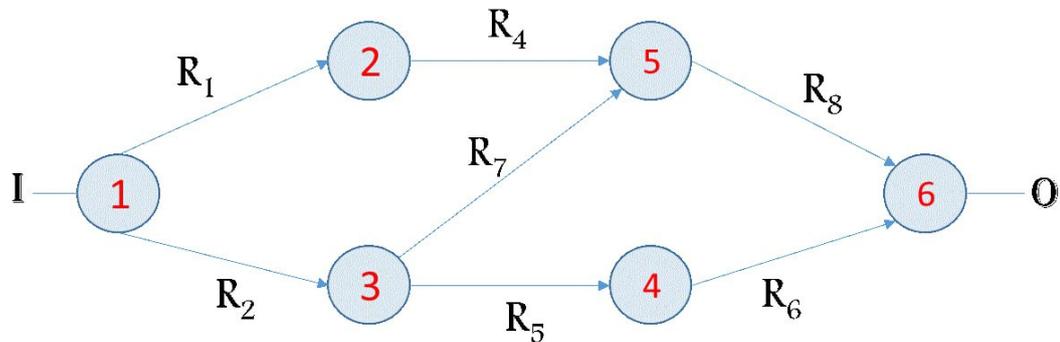
Figure 2.5: Unit R_3 work.

$$R_D = [1 - (1 - R_4)(1 - R_7)]R_8 = [1 - (1 - R_7 - R_4 + R_4R_7)]R_8 = [R_4 + R_7 - R_4R_7]R_8$$

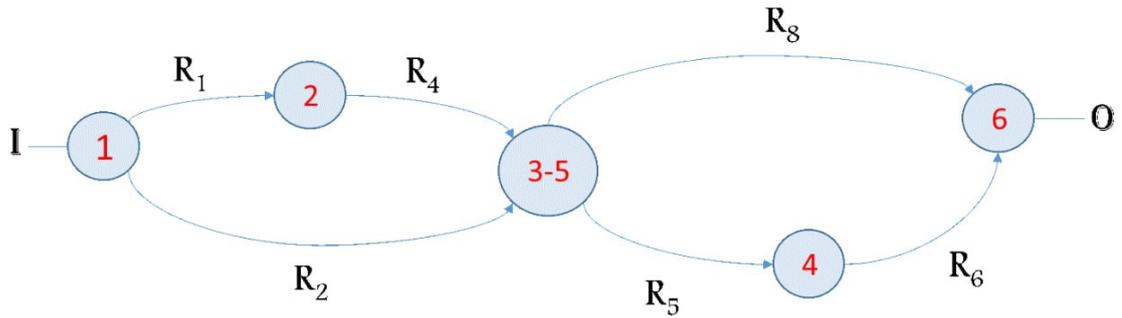
$$R_D = R_4R_8 + R_7R_8 - R_4R_7R_8$$

$$R_A = [1 - (1 - R_1)(1 - R_2)][1 - (1 - R_D)(1 - R_5R_6)]$$

When choose edge R_3 from fig (2.4), and its remove, we get fig. (2.6):

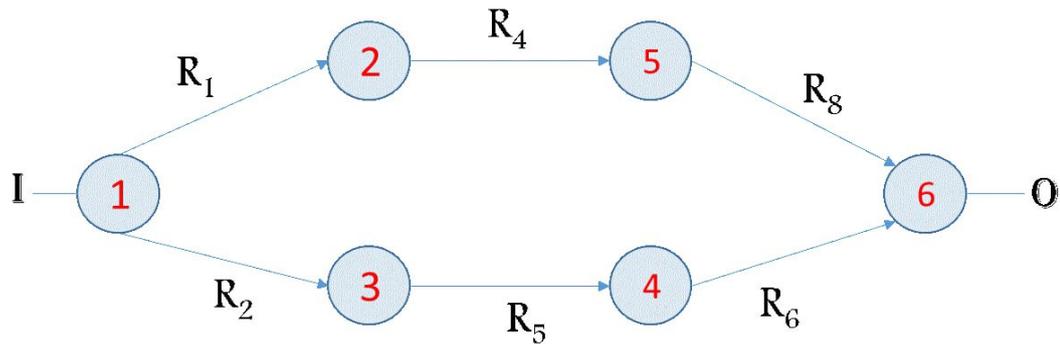
Figure 2.6: Unit R_3 fail.

When choose edge R_7 from fig (2.6), and its contraction, we get fig. (2.7):

Figure 2.7: Unit R_7 work.

$$R_B = [1 - (1 - R_1 R_4) (1 - R_2)] [1 - (1 - R_8) (1 - R_5 R_6)]$$

When choose edge R_7 from fig (2.6), and its remove, we get the fig. (2.8):

Figure 2.8: Unit R_7 fail.

$$R_C = 1 - (1 - R_1 R_4 R_8) (1 - R_2 R_4 R_6)$$

$$R_S = R_3 R_A + (1 - R_3) [R_7 R_B + (1 - R_7) R_C].$$

The enumeration of the states of a chosen component presents a basis for the

decomposition method. So, the complex system reliability polynomial is:

$$\begin{aligned}
R_S = & R_1 R_4 R_8 + R_2 R_5 R_6 + R_2 R_7 R_8 + R_1 R_3 R_5 R_6 + R_2 R_3 R_4 R_8 \\
& + R_1 R_3 R_7 R_8 - R_1 R_2 R_3 R_5 R_6 - R_1 R_2 R_3 R_4 R_8 - R_1 R_2 R_3 R_7 R_8 \\
& - R_1 R_2 R_4 R_7 R_8 - R_1 R_3 R_4 R_7 R_8 - R_2 R_3 R_4 R_7 R_8 - R_2 R_5 R_6 R_7 R_8 \\
& + 2R_1 R_2 R_3 R_4 R_7 R_8 - R_1 R_2 R_4 R_5 R_6 R_8 - R_1 R_3 R_4 R_5 R_6 R_8 - R_2 R_3 R_4 R_5 R_6 R_8 \\
& - R_1 R_3 R_5 R_6 R_7 R_8 + 2R_1 R_2 R_3 R_4 R_5 R_6 R_8 + R_1 R_2 R_3 R_5 R_6 R_7 R_8 + R_1 R_2 R_4 R_5 R_6 R_7 \\
& + R_1 R_3 R_4 R_5 R_6 R_7 R_8 + R_2 R_3 R_4 R_5 R_6 R_7 R_8 - 2R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8
\end{aligned} \tag{2.9}$$

2.4.2 Sum-of-Disjoint-Products (S.D.P) Method

Sum-of-disjoint-products (S.D.P) method was first introduced by Fratta and Montanari in 1973[3, 16]. Where S.D.P method evaluates the probability of the union of several events using minimal paths or minimal cuts, by depends on the equation [37]:

$$\cup_{i=1}^n P_i = P_1 \cup (\overline{P_1} P_2) \cup \dots \cup (\overline{P_1} \overline{P_2} \dots P_n) \tag{2.10}$$

Example 2.7 Looking for complex system fig .(2.4), we have the following eq.(2.10) is called the disjoint process. Compute the polynomial reliability depending on all minimal path sets given in eq (2.2).

$$\begin{aligned}
R_S = & P(P_1) + P(\overline{P_1} P_2) + P(\overline{P_1} \overline{P_2} P_3) + P(\overline{P_1} \overline{P_2} \overline{P_3} P_4) + P(\overline{P_1} \overline{P_2} \overline{P_3} \overline{P_4} P_5) \\
& + P(\overline{P_1} \overline{P_2} \overline{P_3} \overline{P_4} \overline{P_5} P_6)
\end{aligned} \tag{2.11}$$

By applying the eq.(2.11) and compensate for the reliability in each minimal path set.

$$\begin{aligned}
R_S = & R_1 R_4 R_8 + (1 - R_1 R_4 R_8)(R_2 R_5 R_6) + (1 - R_1 R_4 R_8)(1 - R_2 R_5 R_6)(R_2 R_7 R_8) \\
& + (1 - R_1 R_4 R_8)(1 - R_2 R_5 R_6)(1 - R_2 R_7 R_8)(R_1 R_3 R_5 R_6) \\
& + (1 - R_1 R_4 R_8)(1 - R_2 R_5 R_6)(1 - R_2 R_7 R_8)(1 - R_1 R_3 R_5 R_6)(R_1 R_3 R_7 R_8) \\
& + (1 - R_1 R_4 R_8)(1 - R_2 R_5 R_6)(1 - R_2 R_7 R_8)(1 - R_1 R_3 R_5 R_6)(1 - R_1 R_3 R_7 R_8)(R_2 R_3 R_4 R_8)
\end{aligned} \tag{2.12}$$

In this method, we also obtained the same polynomial in eq.(2.9).

2.5 Summary chapter two

- Three methods were used to find the minimal path sets of a complex system (Delimiter's method, Node removal method, and Powers method for the adjacency matrix) for finding the minimal path sets because they all give the same minimal path sets as eq (2.2), but the difference in steps is that Delimiter's method is better than two methods other in terms of time, performance, ease, and direct solution.
- Two methods were used to find the minimal cut sets for a complex system (create the minimal cut sets from minimal paths, enumerate the minimal cut sets using the connection matrix). The first method depends on the minimal paths and edges of the complex system, and the second method depends on the connection matrix for the complex system. The sets of minimal cut sets for the complex system were equal in the two methods as eq.(2.5).
- The reliability function of the complex system was calculated in two methods (Pivotal Decomposition method and Sum-of-Disjoint-Products method). The first method depends on choosing the component that analyzes the system and converts it from a complex into a parallel series, and the second method depends on the probability union of several events using the minimal path sets, and the reliability function of the two methods was equal as eq.(2.9).

Chapter 3

Some Applications to Improve Reliability of Systems

3.1 Introduction

There are many different methods to improve the reliability of systems and devices. In this chapter, we discuss two techniques to improve the reliability of three types of systems: the series system, the parallel system, and the complex system [26, 58]. Techniques are: redundancy, and be in two cases, namely redundancy for the element and redundancy for the system. In addition, we study the improvement of the reliability of one of the combinations of minimal path set and minimal cut set for the complex system by the method of redundancy for the component[59]. Then we discuss the allocation technique to improve the reliability of the three systems, by based on reliability for greatest minimal path set of the system is increased Howei's Theorem [27].

3.2 Redundancy Technique to Improve Reliability Systems

Used a techniques to improve the systems reliability, which is the redundancy technique when one piece of equipment or component may replace another piece of equipment or component that has failed, the general term "Redundancy" is used [6, 37]. More specifically, redundancy is a technique for increasing reliability that includes putting two or more components, substructures, or complex devices in parallel. Were two methods for this technique.

3.2.1 Element redundancy method

Each component of the system has its own path. The series components E_1 and E_2 , that constitute the system's individual components, shown in fig.(3.1), the system as a whole has improved, as has the reliability of each component[5]. The reliability of this system is $R_S = R_{E_1}R_{E_2}$, the formula for redundancy the element be:

$$R_{E_1}^* = 1 - (1 - R_{E_1})^2, \quad R_{E_2}^* = 1 - (1 - R_{E_2})^2,$$

we get, $R_{E_S}^* = R_{E_1}^* R_{E_2}^*$, where general formula is:

$$R_i^* = 1 - (1 - R_i)^2, \quad i = 1, \dots, n \quad (3.1)$$

Then,

$$R_{E_S}^* = R_1^* R_2^* \dots R_i^* \dots R_n^* \quad , i = 1, 2, \dots, n \quad (3.2)$$

Where $R_{E_S}^*$ represent reliability of system is after adding backup components for each component in the system.

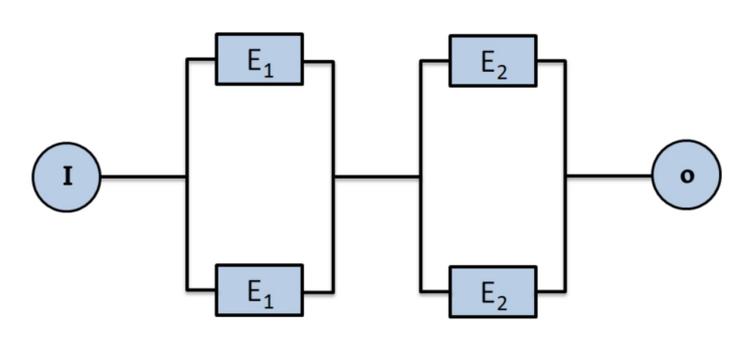


Figure 3.1: Element redundancy.

3.2.2 Unit Redundancy Method

The entire system has an additional path E_1 and E_2 are system components in fig. (3.2), and two components are provided in parallel resulting in improving the reliability of the entire system. [37].

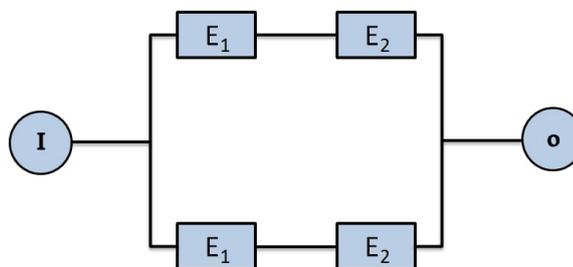


Figure 3.2: Unit redundancy.

Unit redundancy can be computed using the equation :

$$R_{U_S}^* = 1 - (1 - R_S)^2 \quad (3.3)$$

$R_{U_S}^*$ represent the reliability in unit redundancy and R_S represent the reliability in system.

3.3 Application of Redundancy Methods in Reliability to Systems

The redundancy technique can be applied when it was not possible to produce highly reliable components that are used to improve the reliability of the system, especially when from difficult increase the reliability of the components [46].

3.3.1 Application of Redundancy Methods to Series Systems

In this section, study the series connected systems to apply redundancy methods and how to improve the reliability of these systems.

Example 3.1 *The series system shown in Fig.(3.3). We take reliability values for the four components are 0.6, 0.7, 0.8 and 0.9 respectively.*



Figure 3.3: Series system.

Where system reliability:

$$R_S = R_1 R_2 R_3 R_4 \quad (3.4)$$

Its reliability value, $R_S = 0.3024$.

Study element redundancy method on Fig.(3.3). Applying eq.(3.1) to the components of the series system, we get Fig.(3.4).

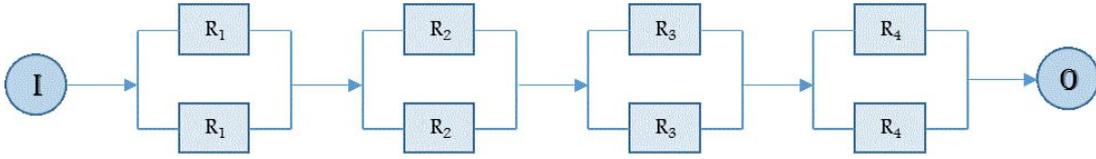


Figure 3.4: Element redundancy for series system with four components.

Then, the element redundancy reliability of each component for the series system:

$$R_1^* = 1 - (1 - 0.6)^2 = 0.84, \quad R_2^* = 1 - (1 - 0.7)^2 = 0.91$$

$$R_3^* = 1 - (1 - 0.8)^2 = 0.96, \quad R_4^* = 1 - (1 - 0.9)^2 = 0.99$$

The components reliability set by the element redundancy method is:

$$S_E = \{0.84, 0.91, 0.96, 0.99\}$$

After substituting S_E in eq. (3.2) we get reliability the series system up by the redundancy element method, $R_{E_S}^* = 0.7265$.

Study unit redundancy method on Fig.(3.3). Applying eq.(3.3) to the series system, we get fig.(3.5).

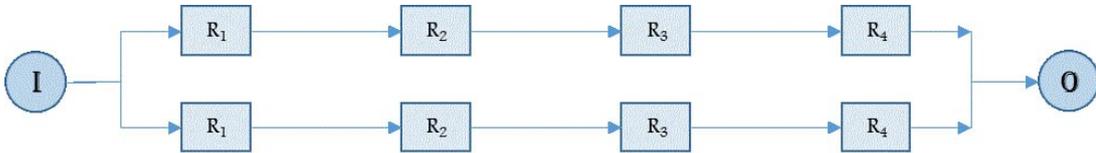


Figure 3.5: Unit redundancy for series system.

The unit redundancy reliability for the series system:

$$R_{U_S}^* = 1 - (1 - 0.3024)^2 = 0.5134$$

From the study of two redundancy methods, the reliability of the series system was increased from (0.3024) to (0.7265) by applying the redundancy for the elements, and increase to (0.5134) by applying the system redundancy.

3.3.2 Application of Redundancy Methods to Parallel Systems

In this section we study the parallel connected systems to apply redundancy methods and how to improve the reliability of these systems, as shown in the example.

Example 3.2 *The parallel system shown in Fig.(3.6). We take reliability values for the four components are 0.6, 0.7, 0.8 and 0.9 respectively.*

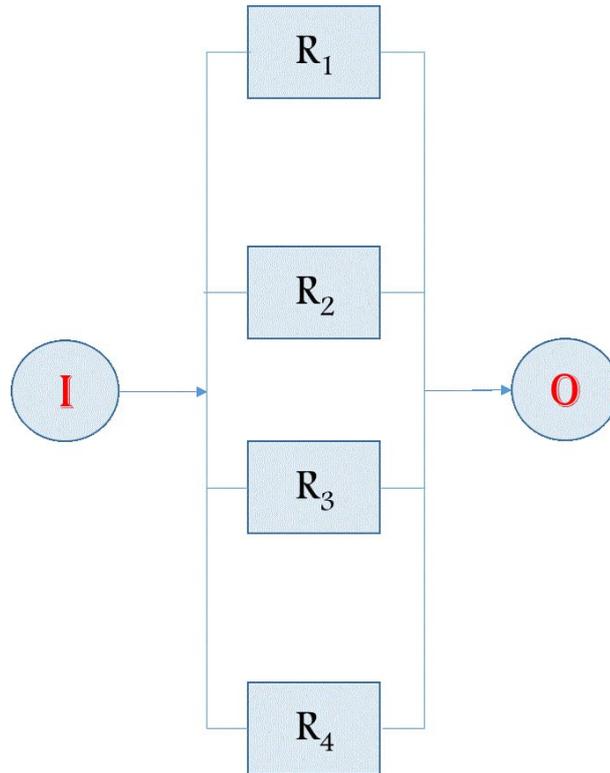


Figure 3.6: Parallel system with four components.

Where system reliability:

$$R_S = 1 - [(1 - R_1)(1 - R_2)(1 - R_3)(1 - R_4)]$$

$$\begin{aligned} R_S = & R_1 + R_2 + R_3 + R_4 - R_3R_4 - R_2R_4 - R_2R_3 + R_2R_3R_4 - R_1R_4 - R_1R_3 \\ & + R_1R_3R_4 - R_1R_2 + R_1R_2R_4 + R_1R_2R_3 - R_1R_2R_3R_4 \end{aligned} \quad (3.5)$$

We get the reliability of the parallel system, $R_S = 0.9976$.

Study element redundancy method on Fig. (3.6). Applying eq.(3.1) to the components of the parallel system, we get Fig.(3.7).

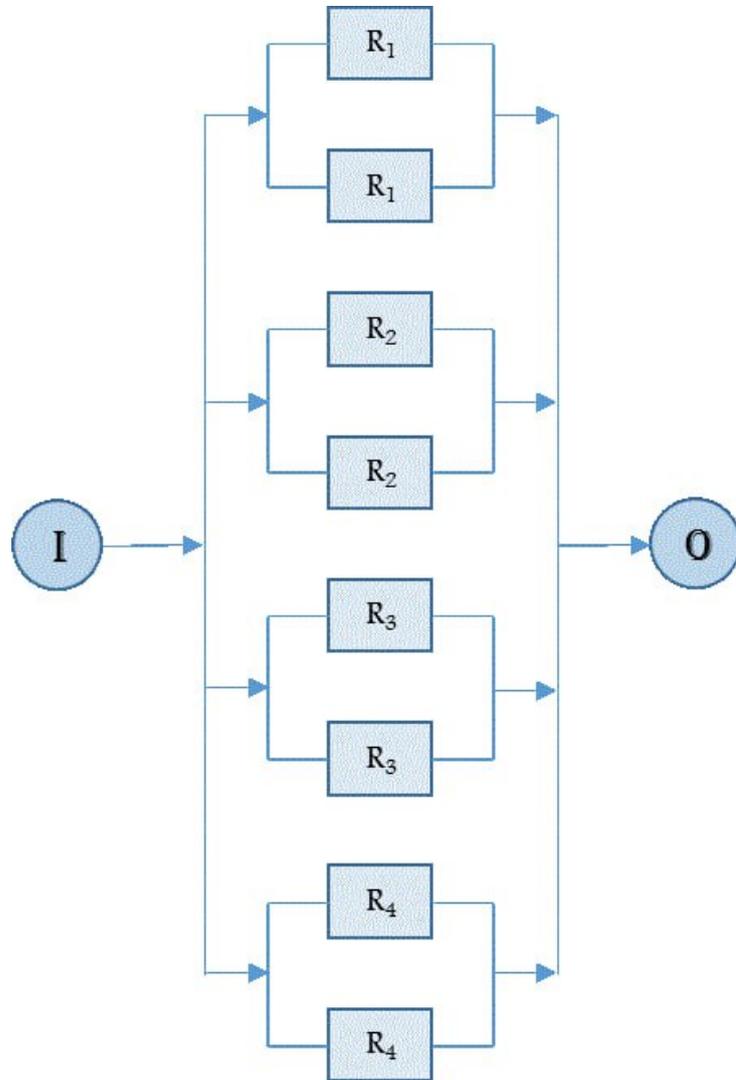


Figure 3.7: Element redundancy for parallel system with four components.

Then, the element redundancy reliability of each component for the parallel system. Its similar to set the components reliability values for the series system S_E . Then we substitute the set of values of S_E in the parallel system reliability polynomial mentioned in relation (3.5). We get the system reliability using the element of redundancy the system, $R_{E_S}^* = 0.9999$.

Study unit redundancy method on Fig. (3.6). Applying eq.(3.3) to the parallel system, we get Fig.(3.8).

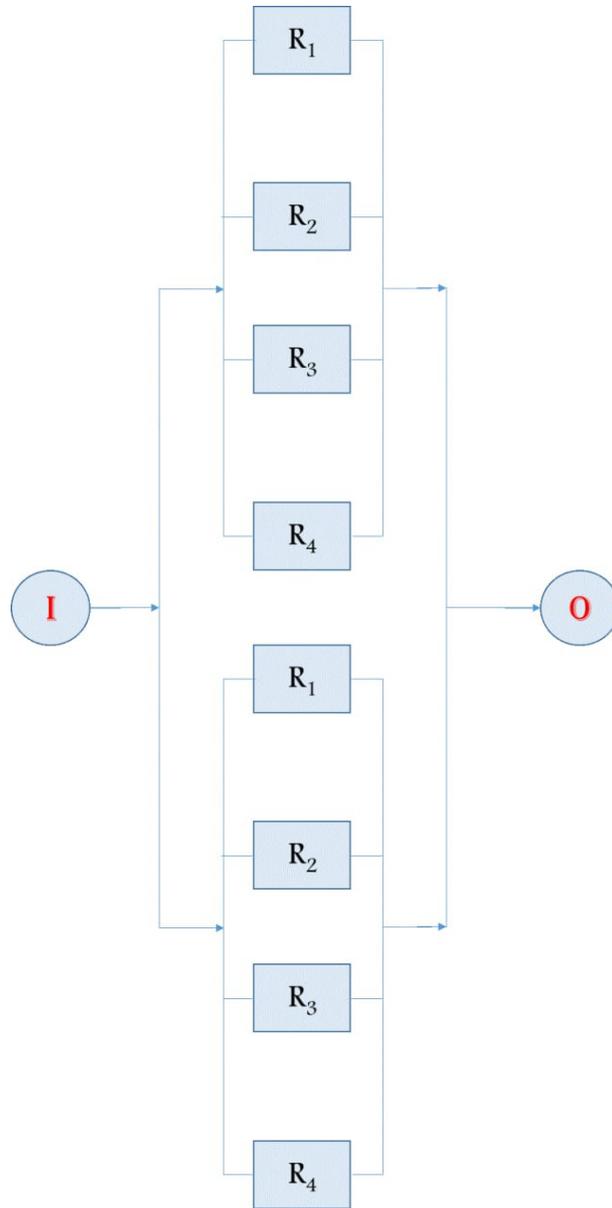


Figure 3.8: Unit redundancy for parallel system.

Then, the unit redundancy reliability for the parallel system:

$$R_{U_s}^* = 1 - (1 - 0.9976)^2 = 0.9999$$

From our study to improve reliability of the parallel system by the two redundancy

methods, the reliability of the system was increased from 0.9976 to 0.9999.

3.3.3 Application of Redundancy Methods to Complex Systems

In this section, we apply redundancy Methods to complex systems which contain a bridge in its system structure and how to improve the reliability of these systems, as shown in the example.

Example 3.3 *The complex system shown in fig.(2.4). Where complex system reliability in eq.(2.9). The reliability values for the eight components are 0.5, 0.7, 0.6, 0.7, 0.8 and 0.9 respectively (where $R_5 = R_6$ and $R_7 = R_8$). Reliability value for complex system $R_S = 0.7885$.*

Study element redundancy method on fig.(2.4). Applying eq. (3.1) to the components of the complex system, we get Fig. (3.9).

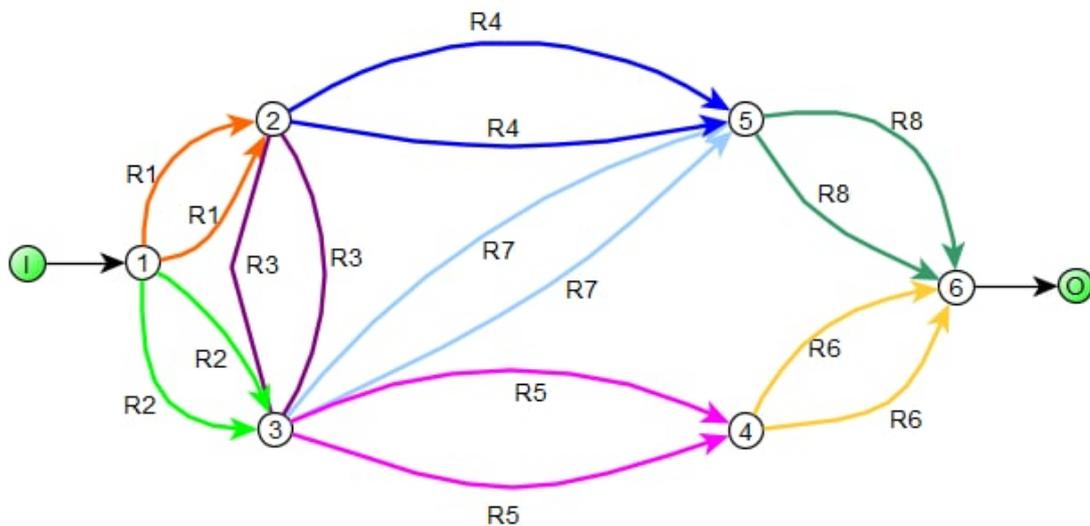


Figure 3.9: Element redundancy for complex system with eight components.

Then, the element redundancy reliability values of each component for the complex system:

$$R_1^* = 1 - (1 - 0.5)^2 = 0.75, \quad R_2^* = 1 - (1 - 0.7)^2 = 0.91$$

$$R_3^* = 1 - (1 - 0.6)^2 = 0.84, \quad R_4^* = 1 - (1 - 0.7)^2 = 0.91$$

$$R_5^* = R_6^* = 1 - (1 - 0.8)^2 = 0.96, \quad R_7^* = R_8^* = 1 - (1 - 0.9)^2 = 0.99$$

We put values $R_1^*, R_2^*, R_3^*, R_4^*, R_5^*, R_6^*, R_7^*$ and R_8^* in eq.(2.9) , we get the reliability value by element redundancy method for complex system, $R_{E_S}^* = 0.9756$.

Study unit redundancy method on Fig.(2.4). Applying eq.(3.3) to the complex system, we get Fig.(3.10).

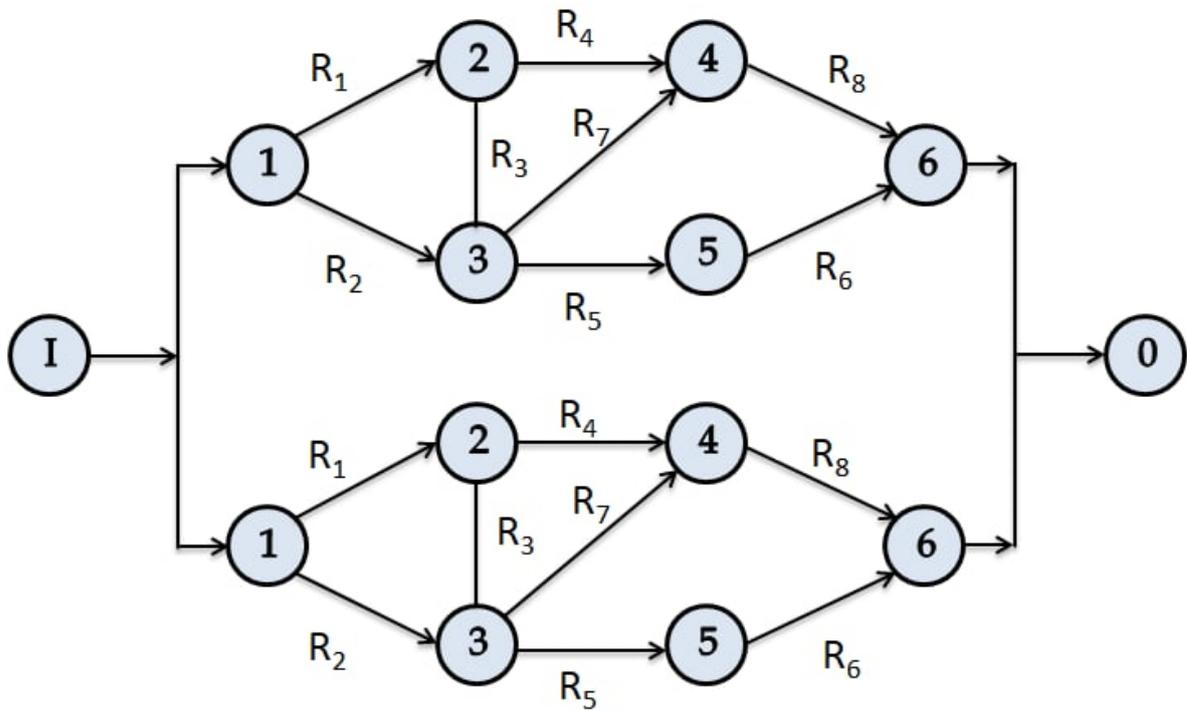


Figure 3.10: Unit redundancy for complex system.

The reliability value for a complex system after a system redundancy is,

$$R_{U_S}^* = 1 - (1 - 0.7885)^2 = 0.9552$$

We note from our study to improve reliability of the complex system shown in fig.(2.4) using the redundancy two methods, the reliability of the system increases in both methods.

3.3.3.1 Applying element redundancy to minimal path sets of the given complex system

In this section, study addition components redundancy to one minimal path sets of the complex system to improve minimal path set reliability mentioned in eq. (2.2).

Example 3.4 Consider the complex system (2.4) that contains six minimal path sets. We take one of these minimal path sets, it was sixth minimal path set (P_6), four components reliability values are 0.6, 0.7, 0.9 respectively, where $R_2=R_4$.

$$P_6 = R_2R_3R_4R_8 \quad (3.6)$$

Where $P_6 = 0.2646$.

Applying eq.(3.1) to calculate element redundancy for components minimal path set,

$$R_2^* = R_4^* = 1 - (1 - 0.7)^2 = 0.91, \quad R_3^* = 1 - (1 - 0.6)^2 = 0.84$$

$$R_8^* = 1 - (1 - 0.9)^2 = 0.99$$

We put values $R_2^*, R_3^*, R_4^*, R_8^*$ in sixth minimal path sets (3.6) depending on the eq. (3.2), we get value reliability by element redundancy method for sixth minimal path set. $P_6^* = R_2^*R_3^*R_4^*R_8^* = 0.0.68865$.

3.3.3.2 Applying element redundancy to minimal cut sets of the given complex system

In this section, study addition components redundancy to one minimal cut set for complex system to improve minimal cut set reliability mentioned in eq.(2.5).

Example 3.5 Reconsider the complex system fig. (2.4) that contains 9 minimal cut sets. We take one of these minimal cut set, its ninth minimal cut set (C_9), four components reliability values are 0.5, 0.6, 0.8, 0.9 respectively.

$$C_9 = 1 - [(1 - R_1)(1 - R_3)(1 - R_5)(1 - R_7)].$$

$$\begin{aligned}
C_9 = & R_1 + R_3 + R_5 + R_7 - R_5R_7 - R_3R_7 - R_3R_5 + R_3R_5R_7 - R_1R_7 - R_1R_5 \\
& + R_1R_5R_7 - R_1R_3 + R_1R_3R_7 + R_1R_3R_5 - R_1R_3R_5R_7
\end{aligned} \tag{3.7}$$

Then, $C_9 = 0.024$.

Applying eq.(3.1) to calculate element redundancy for components minimal cut set,

$$R_1^* = 1 - (1 - 0.5)^2 = 0.75, \quad R_3^* = 1 - (1 - 0.6)^2 = 0.84$$

$$R_5^* = 1 - (1 - 0.8)^2 = 0.96, \quad R_7^* = 1 - (1 - 0.9)^2 = 0.99.$$

We put values $R_1^*, R_3^*, R_5^*, R_7^*$ in ninth minimal cut set (3.7), we get value reliability minimal cut set by element redundancy , $C_9^* = 0.999984$.

3.4 Allocation Technique to Improve Reliability Systems

In this section, allocation problem because it is of great practical importance [26]. Reliability engineers are often asked to make decisions to improve a particular component or components in order to achieve the minimum required system reliability. The allocation technique is approximate, and design feasibility is determined using the system effectiveness elements, such as reliability and maintainability allocated to the subsystems. The purpose of reliability allocation determining a goal or aim for reliability of each component, which gives the producers a concept of the performance needed. To explain allocation Technique depending on Howaidi's Theorem [27], that states:

$$\text{Max } R(MP_i) \leq R_S \leq \text{Min } R(MC_i) \tag{3.8}$$

Where the reliability system R_S is limited between values, $\text{Max } R(MP_i)$ it represents the highest reliability value of the minimal path, and $\text{Min } R(MC_i)$ it represents the lowest reliability value of the minimal cut.

3.4.1 Application of Reliability Allocation on a Series System

In this section, take the series connected systems to apply allocation technique and how to improve the reliability of these systems.

Example 3.6 Consider the Example (3.1) the series system and the system reliability in eq.(3.4).

Let S_1 is a set of all minimal path sets for series system.

$$S_1 = \{MP = (R_1, R_2, R_3, R_4)\}, R(MP) = 0.3024$$

C_1 is a set of all minimal cut sets for series system.

$$C_1 = \{MC_1 = (R_1), MC_2 = (R_2), MC_3 = (R_3), MC_4 = (R_4)\}$$

$$R(MC_1) = 0.6, \quad R(MC_2) = 0.7, \quad R(MC_3) = 0.8, \quad R(MC_4) = 0.9$$

$$\text{Max } R(MP_i) = R(MP) = 0.3024, \quad \text{Min } R(MC_i) = R(MC_1) = 0.6$$

According to eq. (3.8) the highest reliability value of the minimal path set will be (0.3024) and the least reliability value of the minimal cut set (0.6).

$$0.3024 \leq R_S \leq 0.6$$

$$0.3024 \leq 0.3024 \leq 0.6$$

If we want to increase the reliability of the system from (0.30) to (0.90). We make

$$\text{Max}\{R(MP)\} = R_1^\# R_2^\# R_3^\# R_4^\# = 0.9$$

If $R_1^\# = R_2^\# = R_3^\# = R_4^\#$, then $R(Mp) = R^4 = 0.9$, $R = (0.9)^{\frac{1}{4}} = 0.974$, and put $R_1^\# = R_2^\# = R_3^\# = R_4^\# = 0.974$ in R_S we get $R_S^\# = 0.90$

3.4.2 Application of Reliability Allocation on a Parallel System

In this section, study the parallel connected systems to allocation technology apply and how to improve the reliability of these systems.

Example 3.7 Consider parallel system in Fig.(3.11) with three elements A, B, and C with reliability values 0.3, 0.4, 0.5, respectively. The reliability of this system:

$$R_S = 1 - [(1 - R_A)(1 - R_B)(1 - R_C)]$$

$$R_S = R_B + R_A - R_A R_B + R_C - R_B R_C - R_A R_C + R_A R_B R_C \quad (3.9)$$

Where, $R_S = 0.79$

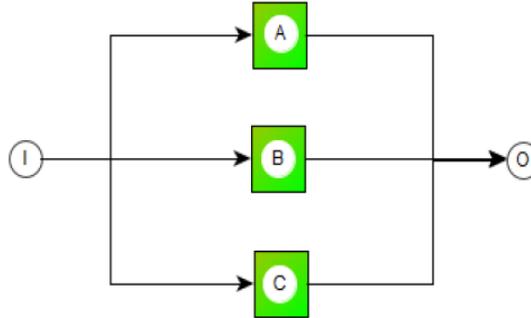


Figure 3.11: Parallel System with three elements.

If we want to increase the reliability of the system from (0.79) to (0.95).

Case one: Howaidi's Theorem [27]

Put ($R_A = R_B = R_C$) in eq. (3.9)

$$R_S = 3R - 3R^2 + R^3 \quad (3.10)$$

$$0.95 = 3R - 3R^2 + R^3.$$

Solve eq. (3.10), we get: $R = 0.631$, put ($R_A = R_B = R_C = 0.631$) in eq. (3.9), the reliability of the system (0.95).

Case two: Howaidi's Theorem [27]

The allocation for one unit, choose unit R_C , and put ($R_A = 0.3, R_B = 0.4$) in eq. (3.9), we get:

$$0.95 = 0.3 + 0.4 + R_C - 0.3 \times 0.4 - 0.3 \times R_C - 0.4 \times R_C + 0.3 \times 0.4 \times R_C$$

$$0.95 = 0.58 - 0.42 \times R_C, \quad \text{we get: } R_C = 0.88$$

put ($R_C = 0.88$) in eq.(3.9), we get the reliability of the system (0.95).

3.4.3 Application of Reliability Allocation on a Complex System

In this section, we study the reliability allocation of the components of systems which contain a bridge in its system structure, as shown in the example.

Example 3.8 *Reconsider in Example (3.3), take the complex system and reliability values for the eight components. S_2 is a set of all minimal path sets for complex system.*

$$S_2 = \{MP_1 = (R_1, R_4, R_8), MP_2 = (R_2, R_5, R_6), MP_3 = (R_2, R_7, R_8), MP_4 = (R_1, R_3, R_5, R_6), MP_5 = (R_2, R_3, R_4, R_8), MP_6 = (R_1, R_3, R_7, R_8)\}$$

C_2 is a set of all minimal cut sets for complex system.

$$C_2 = \{MC_1 = (R_1, R_2), MC_2 = (R_5, R_8), MC_3 = (R_6, R_8), MC_4 = (R_2, R_3, R_4), MC_5 = (R_2, R_3, R_8), MC_6 = (R_4, R_6, R_7), MC_7 = (R_4, R_5, R_7), MC_8 = (R_1, R_3, R_6, R_7), MC_9 = (R_1, R_3, R_5, R_7)\}$$

$$R(MP_1) = 0.315, \quad R(MP_2) = 0.448, \quad R(MP_3) = 0.567, \quad R(MP_4) = 0.192,$$

$$R(MP_5) = 0.2646, \quad R(MP_6) = 0.243$$

$$R(MC_1) = 0.85, \quad R(MC_2) = 0.98, \quad R(MC_3) = 0.98, \quad R(MC_4) = 0.964,$$

$$R(MC_5) = 0.988, \quad R(MC_6) = 0.994, \quad R(MC_7) = 0.994, \quad R(MC_8) = 0.996,$$

$$R(MC_9) = 0.996$$

$$\text{Max } R(MP_i) = R(MP_3) = 0.567, \quad \text{Min } R(MC_i) = R(MC_1) = 0.85$$

According to equation (3.8) the highest reliability value of the minimal path (P_3) will be (0.569) and the least reliability value of the minimal cut C_1 (0.85).

$$0.567 \leq R_S \leq 0.85$$

$$0.567 \leq 0.7885 \leq 0.85.$$

If we want to increase the reliability of the system from (0.79) to (0.96). We make,

$$\text{Max } \{R(MP_5)\} = R_2^\# R_7^\# R_8^\# = 0.9$$

If $R_2^\# = R_7^\# = R_8^\#$, then $R(MP_3) = R^3 = 0.9$, $R = (0.9)^{\frac{1}{3}} = 0.965$ and put

$R_2^\# = R_7^\# = R_8^\# = 0.965$ in R_S we get $R_S^\# = 0.96$.

3.5 Summary chapter three

1. The redundancy technique (element redundancy and system redundancy) was used to improve the reliability of the three systems (series system, parallel system and complex system).
 - Applying the two redundancy methods on a series system that consists of four components, where the reliability of the system before improvement was 0.30, and the system reliability value was increased by the element redundancy method to 0.73 and by the system redundancy method to 0.51. This means, in the two methods, the reliability of the series system was improved, and that the element redundancy method is better than the unit redundancy method because the element redundancy method as shown in fig. (3.4) shows that get a parallel-series system, while in the method of redundancy of the series system get series - parallel as shown in the fig. (3.5).
 - The application of the two redundancy methods on the parallel system, which consists of four components, where the reliability of the system before improvement was 0.9976, and the value of the reliability of the system in the two methods was increased to 0.9999, that the reliability of the system after improvement is equal in both methods because we made the improvement on a parallel system, which is the best in all methods because if at least one component works, the whole system will still work.
 - The application of the two redundancy methods on a complex system consisting of eight components, where the reliability of the system before improvement was 0.7885, its increases by the method of redundancy each element to 0.9756, and by the redundancy of the system increases to 0.9552, also that the improvement by the method redundancy of the element is better than the method redundancy system because in the redundancy of the element, the linkage for each element will be

parallel, as shown in fig (3.9), and thus it be an improvement for each element in the system.

- Applying the element redundancy method to the sixth minimal path, which includes four components of a complex system, We note from our study to improve the reliability of minimal path set using element redundancy method, increases value sixth minimal path set reliability from 0.2646 to 0.68865 meaning its has been improved to double.
 - Applying the element redundancy method to the ninth minimal cut, which includes four components of a complex system, the reliability of minimal cut sets has been increased from 0.024 to 0.999984. This type of improve by redundancy method is considered active redundancy because the elements are inserted in parallel to minimal cut components.
2. The allocation technique was used to improve the reliability of the three systems (series system, parallel system and complex system).
- The allocation technique was used on series system, from our study of the reliability allocation of the series system using Howaidi's theorem, where improved minimal path set for series system, whole system is improved, as the reliability of the system improved from 0.30 to 0.90.
 - The allocation technique was used on parallel system, Howaidi's theorem eq.(3.8) was used in two cases, we noticed that the reliability of the system was improved, where in the first case we made an allocation of the three components A, B, C ($R_A = R_B = R_C$) we notice an increase in the value of the system reliability from 0.79 to 0.95, and in the second case the allocation was made to one component, which is C, we note Also, the system reliability value was increased from 0.79 to 0.95
 - The allocation technique was used on complex system, from study allocate the

reliability of the complex system using Howeydi's Theorem, where improved the greatest minimal path set of the complex system is third minimal path set and by increasing the reliability from 0.569 to 0.965, whole system was improved by increasing its reliability from 0.79 to 0.96.

Chapter 4

The Importance of Reliability Systems

4.1 Introduction

Measures of reliability importance depend on both the systems design and the reliability of each component over courses of action and defined mission period [37]. It compares the change in component reliability to the change in system reliability when evaluating system reliability. The purpose of an importance measure is to help the design in determining the importance of the components that need to be improved[48].

In this chapter, study the reliability importance and its impact of systems in general and in particular of complex systems. Where study three types of systems are: Series system, parallel system and complex system. Where evaluate the reliability importance of the three system components, then the importance of the minimal path sets relation to the systems, the reliability importance of minimal cut sets for the systems, then study the importance of each component relation to each minimal path set and for each minimal cut set in the systems.

4.2 Evaluating the Reliability Importance of each Units in Some Types of Systems

The reliability importance of the components must be studied. Regarding the systems design or suggestions for maintaining and operating it optimally[13]. Identification identifying weaknesses critical system components, determining the effects of failure at these components, are the main objectives of a reliability study. Lambert (1975) according to a components importance should be determined by two reasons [37]:

- The components location in the system.
- The components reliability is in explanation.

The reliability of a component can be increased by choosing higher quality components, adding redundant components, reducing the operational and environmental conditions

imposed on the component, or improving the components main portability [41]. In this part, we discuss the reliability importance of each component in relation to the systems, and find the reliability importance of the components depending on the first method of Birnbaum measure(1969)[5], according to equation:

$$\mathbf{I}(i) = \frac{\partial \mathbf{R}_s}{\partial \mathbf{R}_i} \quad (4.1)$$

which depends on the derivative of the system for each component present in the systems.

4.2.1 For Series Systems

In this section, study the reliability importance of the components of systems that are linked by a series.

Example 4.1 Consider the series system shown in Fig.(4.1). We take reliability values for the three components are 0.6, 0.7 and 0.8 respectively.

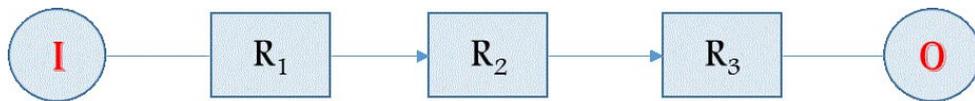


Figure 4.1: Series system with three components.

Where the reliability system:

$$R_s = R_1 R_2 R_3. \quad (4.2)$$

finding the reliability importance of each component in the Fig.(4.1) according to eq.(4.1), as follows:

$$I(R_1) = \frac{\partial R_s}{\partial R_1} = R_2 R_3, \quad I(R_2) = \frac{\partial R_s}{\partial R_2} = R_1 R_3, \quad I(R_3) = \frac{\partial R_s}{\partial R_3} = R_1 R_2.$$

Table 4.1: Determine the importance of reliability and its levels for each units of series system.

R_i	$I(R_i)$	Level
R_1	0.56	1
R_2	0.48	2
R_3	0.42	3

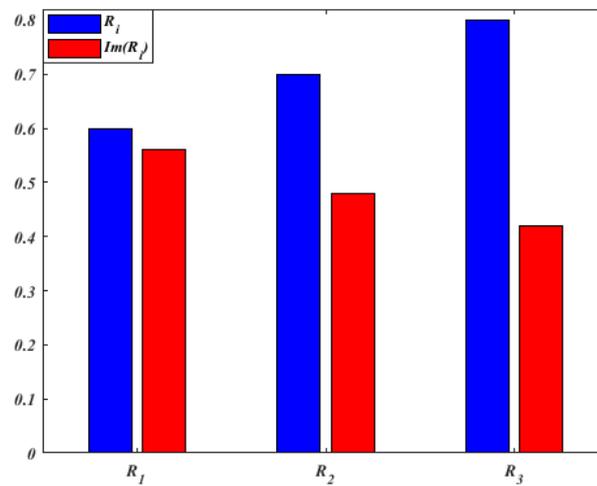


Figure 4.2: Representing the results of the reliability importance levels of a table (4.1).

According to Birnbaum's measure, the least reliable component of a sequence structure is the most important. From the table (4.1) that the first component (R_1), whose reliability (0.6) is less reliable than the rest of the components, is the most important and is at the first level, then the second and third components.

4.2.2 For Parallel Systems

study the reliability importance of the components of systems that are linked by a parallel.

Example 4.2 Consider the parallel system shown in Fig.(4.3). We take reliability values for the three components are 0.6, 0.7 and 0.8 respectively.

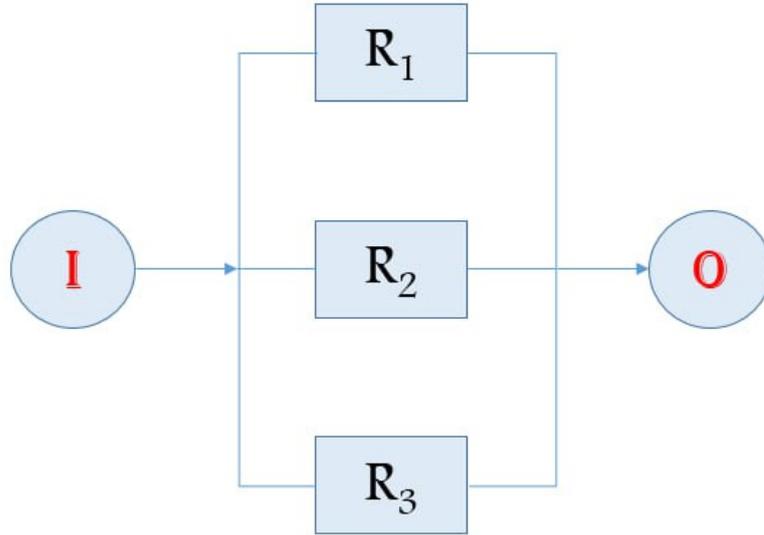


Figure 4.3: Parallel system with three components.

Where the reliability system:

$$R_S = 1 - [(1 - R_1)(1 - R_2)(1 - R_3)]$$

$$R_S = R_2 + R_1 - R_1R_2 + R_3 - R_2R_3 - R_1R_3 + R_1R_2R_3 \quad (4.3)$$

Finding the reliability importance of each component in the Fig.(4.3) according to equation (4.1), as follows:

$$I(R_1) = \frac{\partial R_S}{\partial R_1} = 1 - R_2 - R_3 + R_2R_3,$$

$$I(R_2) = \frac{\partial R_S}{\partial R_2} = 1 - R_1 - R_3 + R_1R_3,$$

$$I(R_3) = \frac{\partial R_S}{\partial R_3} = 1 - R_1 - R_2 + R_1R_2.$$

Table 4.2: Determine the importance of reliability and its levels for each units of parallel system.

R_i	$I(R_i)$	Level
R_3	0.12	1
R_2	0.08	2
R_1	0.06	3

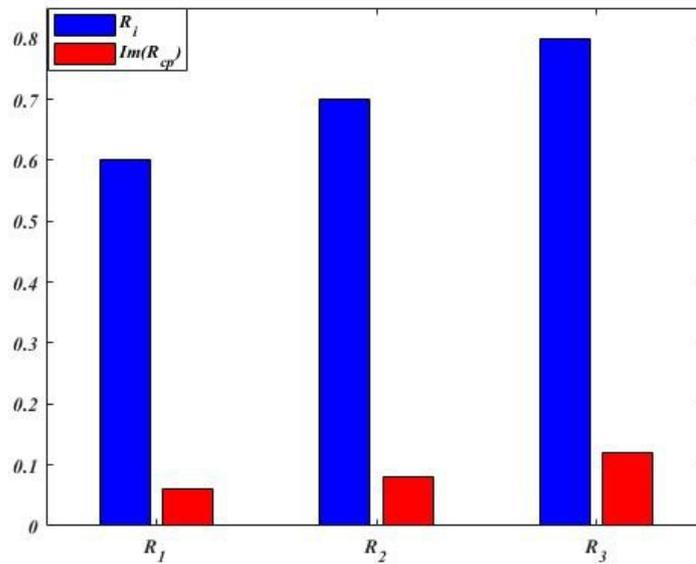


Figure 4.4: Representing the results of the reliability importance levels of a table (4.2)

According to Birnbaum's measure, the component with the highest reliability is the most important in a parallel structure. A parallel structure will work as long as at least one of its components is working. Therefore, it is logical to say that the most reliable component is the most important. We note from the table (4.2) that the third component in the parallel link (R_3) whose reliability (0.8) is the largest reliable of the rest of the components is the most important and its the first level, then the second and first component, respectively.

4.2.3 For Complex Systems

In this section, the reliability importance of components of systems which contain a bridge in its system structure. Where the reliability importance of the components of a complex system in two cases if the reliability values are equal and different, as following:

Example 4.3 Consider the complex system shown in Fig.(2.4). Where reliability complex system in eq.(2.9), according to eq.(4.1), as follows:

$$\begin{aligned}
 I(R_1) &= \frac{\partial R_s}{\partial R_1} = R_4R_8 + R_3R_5R_6 + R_3R_7R_8 - R_2R_3R_5R_6 - R_2R_3R_4R_8 \\
 &\quad - R_2R_3R_7R_8 - R_2R_4R_7R_8 - R_3R_4R_7R_8 + 2R_2R_3R_4R_7R_8 \\
 &\quad - R_2R_4R_5R_6R_8 - R_3R_4R_5R_6R_8 - R_3R_5R_6R_7R_8 + 2R_2R_3R_4R_5R_6R_8 \\
 &\quad + R_2R_3R_5R_6R_7R_8 + R_2R_4R_5R_6R_7R_8 + R_3R_4R_5R_6R_7R_8 - 2R_2R_3R_4R_5R_6R_7R_8
 \end{aligned}$$

$$\begin{aligned}
 I(R_2) &= \frac{\partial R_s}{\partial R_2} = R_5R_6 + R_7R_8 + R_3R_4R_8 - R_1R_3R_5R_6 - R_1R_3R_4R_8 - R_1R_3R_7R_8 \\
 &\quad - R_1R_4R_7R_8 - R_3R_4R_7R_8 - R_5R_6R_7R_8 + 2R_1R_3R_4R_7R_8 \\
 &\quad - R_1R_4R_5R_6R_8 - R_3R_4R_5R_6R_8 + 2R_1R_3R_4R_5R_6R_8 + R_1R_3R_5R_6R_7R_8 \\
 &\quad + R_1R_4R_5R_6R_7 + R_3R_4R_5R_6R_7R_8 - 2R_1R_3R_4R_5R_6R_7R_8
 \end{aligned}$$

$$\begin{aligned}
 I(R_3) &= \frac{\partial R_s}{\partial R_3} = R_1R_5R_6 + R_2R_4R_8 + R_1R_7R_8 - R_1R_2R_5R_6 - R_1R_2R_4R_8 \\
 &\quad - R_1R_2R_7R_8 - R_1R_4R_7R_8 - R_2R_4R_7R_8 + 2R_1R_2R_4R_7R_8 \\
 &\quad - R_1R_4R_5R_6R_8 - R_2R_4R_5R_6R_8 - R_1R_5R_6R_7R_8 + 2R_1R_2R_4R_5R_6R_8 \\
 &\quad + R_1R_2R_5R_6R_7R_8 + R_1R_4R_5R_6R_7R_8 + R_2R_4R_5R_6R_7R_8 \\
 &\quad - 2R_1R_2R_4R_5R_6R_7R_8
 \end{aligned}$$

$$\begin{aligned}
 I(R_4) &= \frac{\partial R_s}{\partial R_4} = R_1R_8 + R_2R_3R_8 - R_1R_2R_3R_8 - R_1R_2R_7R_8 - R_1R_3R_7R_8 \\
 &\quad - R_2R_3R_7R_8 + 2R_1R_2R_3R_7R_8 - R_1R_2R_5R_6R_8 - R_1R_3R_5R_6R_8 \\
 &\quad - R_2R_3R_5R_6R_8 + 2R_1R_2R_3R_5R_6R_8 + R_1R_2R_5R_6R_7 + R_1R_3R_5R_6R_7R_8 \\
 &\quad + R_2R_3R_5R_6R_7R_8 - 2R_1R_2R_3R_5R_6R_7R_8
 \end{aligned}$$

$$\begin{aligned}
I(R_5) = \frac{\partial R_s}{\partial R_5} &= R_2R_6 + R_1R_3R_6 - R_1R_2R_3R_6 - R_2R_6R_7R_8 - R_1R_2R_4R_6R_8 \\
&- R_1R_3R_4R_6R_8 - R_2R_3R_4R_6R_8 - R_1R_3R_6R_7R_8 + 2R_1R_2R_3R_4R_6R_8 \\
&+ R_1R_2R_3R_6R_7R_8 + R_1R_2R_4R_6R_7 + R_1R_3R_4R_6R_7R_8 + R_2R_3R_4R_6R_7R_8 \\
&- 2R_1R_2R_3R_4R_6R_7R_8
\end{aligned}$$

$$\begin{aligned}
I(R_6) = \frac{\partial R_s}{\partial R_6} &= R_2R_5 + R_1R_3R_5 - R_1R_2R_3R_5 - R_2R_5R_7R_8 - R_1R_2R_4R_5R_8 \\
&- R_1R_3R_4R_5R_8 - R_2R_3R_4R_5R_8 - R_1R_3R_5R_7R_8 + 2R_1R_2R_3R_4R_5R_8 \\
&+ R_1R_2R_3R_5R_7R_8 + R_1R_2R_4R_5R_7 + R_1R_3R_4R_5R_7R_8 + R_2R_3R_4R_5R_7R_8 \\
&- 2R_1R_2R_3R_4R_5R_7R_8
\end{aligned}$$

$$\begin{aligned}
I(R_7) = \frac{\partial R_s}{\partial R_7} &= R_2R_8 + R_1R_3R_8 - R_1R_2R_3R_8 - R_1R_2R_4R_8 - R_1R_3R_4R_8 - R_2R_3R_4R_8 \\
&- R_2R_5R_6R_8 + 2R_1R_2R_3R_4R_8 - R_1R_3R_5R_6R_8 + R_1R_2R_3R_5R_6R_8 \\
&+ R_1R_2R_4R_5R_6 + R_1R_3R_4R_5R_6R_8 + R_2R_3R_4R_5R_6R_8 \\
&- 2R_1R_2R_3R_4R_5R_6R_8
\end{aligned}$$

$$\begin{aligned}
I(R_8) = \frac{\partial R_s}{\partial R_8} &= R_1R + R_2R_7 + R_2R_3R + R_1R_3R_7 - R_1R_2R_3R - R_1R_2R_3R_7 \\
&- R_1R_2R_4R_7 - R_1R_3R_4R_7 - R_2R_3R_4R_7 - R_2R_5R_6R_7 + 2R_1R_2R_3R_4R_7 \\
&- R_1R_2R_4R_5R_6 - R_1R_3R_4R_5R_6 - R_2R_3R_4R_5R_6 - R_1R_3R_5R_6R_7 \\
&+ 2R_1R_2R_3R_4R_5R_6 + R_1R_2R_3R_5R_6R_7 + R_1R_2R_4R_5R_6R_7 \\
&+ R_1R_3R_4R_5R_6R_7 + R_2R_3R_4R_5R_6R_7 - 2R_1R_2R_3R_4R_5R_6R_7
\end{aligned}$$

Case 1. Where components reliability values are equal ($R_1 = R_2, \dots = R_8 = 0.9$)

Table 4.3: Determine the importance of reliability and its levels for identical units of complex system.

R_i	$I(R_i)$	Level
R_8	0.1925	1
R_2	0.113	2
R_1	0.0976	3
R_5, R_6	0.0969	4
R_4	0.0247	6
R_7	0.0182	7
R_3	0.0166	5

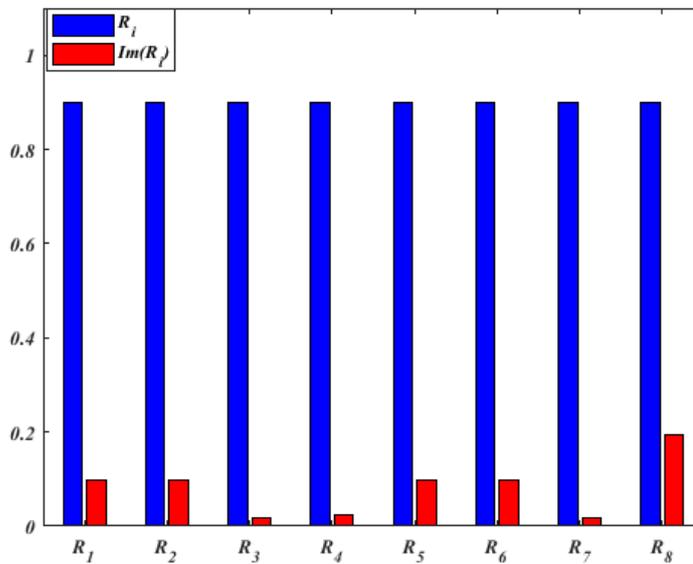


Figure 4.5: Representing the results of the reliability importance levels of a table (4.3)

From the table (4.3), the reliability importance of the components of the complex system shown in (2.4), the component (R_8) is the most important and its in the

first level, then (R_2) , (R_1) , and then the two components (R_5) and (R_6) will have same value of importance and level, which is the fourth level, then the lower level components R_4, R_7, R_3 respectively .

Case 2. Where components reliability values are different ($R_1 = 0.6, R_2 = 0.5, R_3 = 0.7, R_4 = 0.8, R_5 = 0.4, R_6 = 0.9, R_7 = 0.3, R_8 = 0.9$)

Table 4.4: Determine the importance of reliability and its levels for different units of complex system.

R_i	$I(R_i)$	Level
R_1	0.4558	1
R_8	0.4413	2
R_4	0.3431	3
R_2	0.3272	4
R_5	0.1716	6
R_7	0.1094	7
R_3	0.1051	8
R_6	0.0763	6

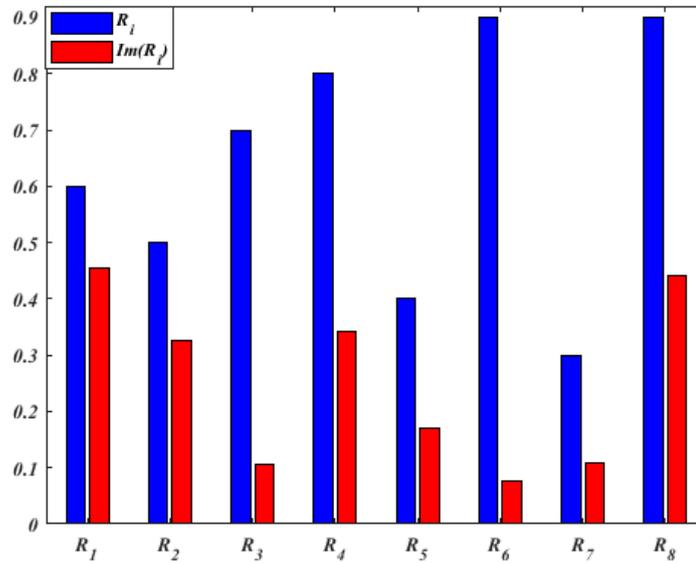


Figure 4.6: Representing the results of the reliability importance levels of a table (4.4)

According to table (4.4), the values of reliability importance and level of the components of the complex system will be $R_1, R_8, R_4, R_2, R_5, R_7, R_3, R_6$ respectively, where the first component (R_1) with reliability (0.6) is in the first level in relation to the rest of the components of the complex system shown in fig.(2.4). Notice from the two tables to measure the reliability importance of the reliability of the components of the complex system that the level of importance for each component is different.

4.3 Evaluating the Reliability Importance of each Units in Minimal Path Sets for Some Types of Systems

In this section, study the reliability importance of each component present in every minimal path sets for all systems, which is the derivation method, depending on the eq.(4.1).

4.3.1 For Series Systems

The reliability importance of minimal path components for systems that are linked by a series.

Example 4.4 Reconsider example (4.1), where fig. (4.1). Minimal path sets for the series system that contains three independent units is:

$$P_s = \{R_1 R_2 R_3\} \quad (4.4)$$

$$\frac{\partial P_s}{\partial R_1} = R_2 R_3, \quad \frac{\partial P_s}{\partial R_2} = R_1 R_3, \quad \frac{\partial P_s}{\partial R_3} = R_1 R_2$$

Table 4.5: Determine the importance of reliability and its levels for minimal path set of series system.

R_i	$I(R_{P_s})$	Level
R_1	0.56	1
R_2	0.48	2
R_3	0.42	3

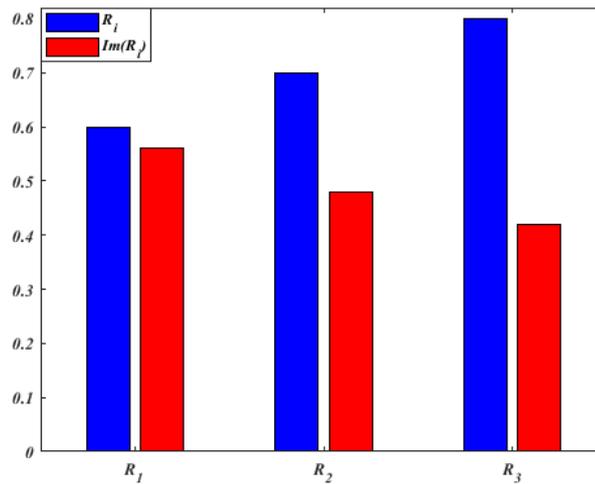


Figure 4.7: Representing the results of the reliability importance levels of a table (4.5)

Notice from the table (4.5) that the components in the series system shown in fig.(4.1) when derived minimal path sets for the system with respect to each component it contains, found that the first component that is less reliable is the most important and in the first level for the two components (R_2, R_3) respectively. Then, notice the table (4.5) completely similar to the table (4.1) in terms of importance and level, and this indicates that the reliability importance of the components of minimal path sets is similar to the reliability importance of the components for the system, and the reason behind this is because the equation of the system (4.2) is same as eq. Ps (4.4) for the series system.

4.3.2 For Parallel Systems

In this section, study the reliability importance minimal path components for systems that are linked by a parallel.

Example 4.5 Looking for the example (4.2) that shown fig. (4.3),it contains three minimal path sets:

$$S = \{P_1 = R_1, P_2 = R_2, P_3 = R_3\} \quad (4.5)$$

$$\frac{\partial P_1}{\partial R_1} = 1, \quad \frac{\partial P_2}{\partial R_2} = 1, \quad \frac{\partial P_3}{\partial R_3} = 1$$

We note the reliability importance of the components all minimal path sets for parallel system are equal.

4.3.3 For Complex Systems

In this section, study the reliability importance minimal path components for systems which contain a bridge in its system structure. Where the reliability importance minimal path components for a complex system in two cases if the reliability values are equal and different.

Example 4.6 Consider the complex system shown in fig.(2.4). Which consists of 6 minimal path sets as mentioned in eq.(2.2). Finding the reliability importance of each

component present for each minimal path sets it contains according to Birnbaum measure eq.(4.1). Uses same two cases for the components reliability values in example (4.3), after substituting values case 1:

$$\begin{aligned} \frac{\partial P_1}{\partial R_1} &= \frac{\partial P_1}{\partial R_4} = \frac{\partial P_1}{\partial R_8} = \frac{\partial P_2}{\partial R_2} = \frac{\partial P_2}{\partial R_5} = \frac{\partial P_2}{\partial R_6} = \frac{\partial P_3}{\partial R_2} = \frac{\partial P_3}{\partial R_7} = \frac{\partial P_3}{\partial R_8} = 0.81 \\ \frac{\partial P_4}{\partial R_1} &= \frac{\partial P_4}{\partial R_3} = \frac{\partial P_4}{\partial R_5} = \frac{\partial P_4}{\partial R_6} = \frac{\partial P_5}{\partial R_1} = \frac{\partial P_5}{\partial R_3} = \frac{\partial P_5}{\partial R_7} = \frac{\partial P_5}{\partial R_8} = \frac{\partial P_6}{\partial R_2} = \frac{\partial P_6}{\partial R_3} = \frac{\partial P_6}{\partial R_4} \\ &= \frac{\partial P_6}{\partial R_8} = 0.729 \end{aligned}$$

Table 4.6: Determine the importance of reliability and its levels for identical units in minimal path set of complex system.

R_i	P_i	$I(R_{P_i})$	Biggest value of $I(R_{P_i})$	Level
$R_1, R_2, R_4,$ R_5	$P_1 = P_2 = P_3$	0.81	0.81	1
R_6, R_7, R_8	$P_4 = P_5 = P_6$	0.729		
R_3	$P_4 = P_5 = P_6$	0.729	0.729	2

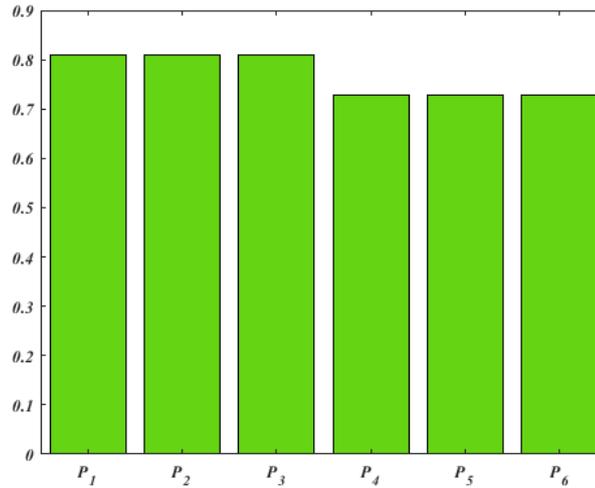


Figure 4.8: Representing the results of the reliability importance levels of a table (4.6)

We note from the table (4.6) finding the reliability importance components of minimal path sets for complex system is that some components are present in more than one minimal path set, meaning we studied the reliability importance of each component in more than one minimal path set, for example, the component (R_1) is present in the first minimal path set, and its also present in the fourth minimal path set, which is the second level, so (R_1) is in the first level because its in most important minimal path set.

After substituting values case 2:

$$\frac{\partial P_1}{\partial R_1} = R_4 R_8 = (0.8)(0.9) = 0.72$$

$$\frac{\partial P_1}{\partial R_4} = R_1 R_8 = (0.6)(0.9) = 0.54$$

$$\frac{\partial P_1}{\partial R_8} = R_1 R_4 = (0.6)(0.8) = 0.48$$

$$\frac{\partial P_2}{\partial R_2} = R_5 R_6 = (0.4)(0.9) = 0.36$$

$$\frac{\partial P_2}{\partial R_5} = R_2 R_6 = (0.5)(0.9) = 0.45$$

$$\frac{\partial P_2}{\partial R_6} = R_2 R_5 = (0.5)(0.4) = 0.2$$

$$\frac{\partial P_3}{\partial R_2} = R_7 R_8 = (0.3)(0.9) = 0.27$$

$$\frac{\partial P_3}{\partial R_7} = R_2 R_8 = (0.5)(0.9) = 0.45$$

$$\frac{\partial P_3}{\partial R_8} = R_2 R_7 = (0.5)(0.3) = 0.15$$

$$\frac{\partial P_4}{\partial R_1} = R_3 R_5 R_6 = (0.7)(0.4)(0.9) = 0.252$$

$$\frac{\partial P_4}{\partial R_3} = R_1 R_5 R_6 = (0.6)(0.4)(0.9) = 0.216$$

$$\frac{\partial P_4}{\partial R_5} = R_1 R_3 R_6 = (0.6)(0.7)(0.9) = 0.378$$

$$\frac{\partial P_4}{\partial R_6} = R_1 R_3 R_5 = (0.6)(0.7)(0.4) = 0.168$$

$$\frac{\partial P_5}{\partial R_1} = R_3 R_7 R_8 = (0.7)(0.3)(0.9) = 0.189$$

$$\frac{\partial P_5}{\partial R_3} = R_1 R_7 R_8 = (0.6)(0.3)(0.9) = 0.162$$

$$\frac{\partial P_5}{\partial R_7} = R_1 R_3 R_8 = (0.6)(0.7)(0.9) = 0.378$$

$$\frac{\partial P_5}{\partial R_8} = R_1 R_3 R_7 = (0.6)(0.7)(0.3) = 0.126$$

$$\frac{\partial P_6}{\partial R_2} = R_3 R_4 R_8 = (0.7)(0.8)(0.9) = 0.504$$

$$\frac{\partial P_6}{\partial R_3} = R_2 R_4 R_8 = (0.5)(0.8)(0.9) = 0.36$$

$$\frac{\partial P_6}{\partial R_4} = R_2 R_3 R_8 = (0.5)(0.7)(0.9) = 0.315$$

$$\frac{\partial P_6}{\partial R_8} = R_2 R_3 R_8 = (0.5)(0.7)(0.9) = 0.28$$

Table 4.7: Determine the importance of reliability and its levels for different units in minimal path set of complex system.

R_i	P_i	$I(R_{P_i})$	Biggest value of $I(R_{P_i})$	Leve
R_1	P_1	0.72	0.72	1
	P_4	0.252		
	P_5	0.189		
R_4	P_1	0.54	0.54	2
	P_6	0.315		
R_2	P_2	0.36	0.504	3
	P_3	0.27		
	P_6	0.504		
R_8	P_1	0.48	0.48	4
	P_3	0.15		
	P_5	0.126		
	P_6	0.28		
R_5, R_7	P_2	0.45	0.45	5
	P_4	0.378		
R_3	P_4	0.216	0.36	6
	P_5	0.162		
	P_6	0.36		
R_6	P_2	0.2	0.2	7
	P_4	0.168		

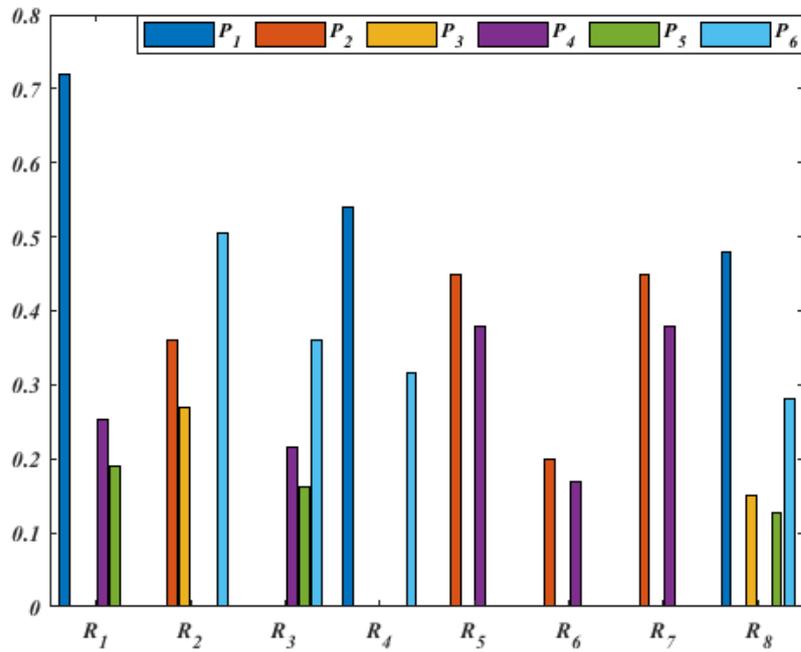


Figure 4.9: Representing the results of the reliability importance levels of a table (4.7)

We note table (4.7) is similar to work table (4.6) in terms finding the importance of the components of minimal path set for complex system, but in this table we used different reliability values. We notice the two components (R_5, R_7) have the same value of importance and level, and therefore they will have the same effect on the system, some of the components in tables (4.7) and (4.4) have same the level.

4.4 Evaluating the Reliability Importance of each Minimal Path Sets for Some Types of Systems

In this section, the reliability importance of minimal path sets for the systems, we discuss two methods to evaluated importance reliability the systems.

4.4.1 First Method (F.M)

Considered F.M. a measure to evaluating reliability importance minimal path sets reliability of systems components general which is a generalization for Birnbaum measure [37, 5].

4.4.1.1 Using F.M. to evaluate reliability importance and its level of the minimal path sets of series system

Looking for the series system in fig.(4.1), where the system reliability (R_s) in eq.(4.2), and also minimal path sets for a system in eq.(4.4) so, $I(p) = \frac{\partial R_s}{\partial P} = 1$. Notice by F.M to evaluate the reliability importance for minimal path sets of series system that importance is(1) because in general the series system has only one minimal path set, when derived the reliability system to minimal path sets was reliability importance (1).

4.4.1.2 Using F.M. to evaluate reliability importance and its level of the minimal path sets of parallel system

Consider the parallel system fig.(4.2), where the system reliability (R_s) in eq.(4.3), and also minimal path sets for a system in eq.(4.5), applying eq.(4.1):

$$I(P_1) = \frac{\partial R_s}{\partial P_1} = 1 - R_2 - R_3 + R_2 R_3, \quad I(P_2) = \frac{\partial R_s}{\partial P_2} = 1 - R_1 - R_3 + R_1 R_3$$

$$I(P_3) = \frac{\partial R_s}{\partial P_3} = 1 - R_1 - R_2 + R_1 R_2.$$

Table 4.8: Determine the reliability importance and its level of the minimal path sets of parallel system by using F.M.

P_i	$I(P_{R_s})$	Level
P_3	0.12	1
P_2	0.08	2
P_1	0.06	3

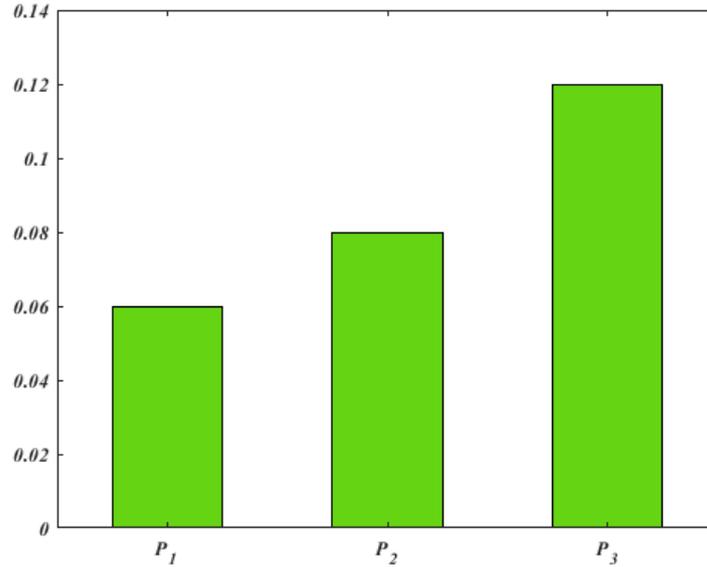


Figure 4.10: Representing the results of the reliability importance levels of a table (4.8)

The table (4.8), which illustrates the study reliability importance minimal path sets for the parallel system shown in fig.(4.2), where the third minimal path set in the first level that contains one component (R_3) is largest reliable in relation to the rest of the system components, then minimal path set 2 in level 2, and finally path 1 in level 3. Also, table (4.8) is similar to the table (4.2) in terms values importance and level of the parallel system because in both cases derive the reliability system mentioned in eq.(4.3) once for components the system and we got the table (4.2) and once for minimal path sets the system and we got table (4.8).

4.4.1.3 Using F.M. to evaluate reliability importance and its level of the minimal path sets of complex system

The reliability importance for each minimal path sets relation to the complex system shown in fig(2.4), where minimal path sets of the system in eq.(4.2). Derived (R_s) the

polynomial of the system (2.9) for each minimal path sets in complex system.

$$I(P_1) = \frac{\partial^3 R_s}{\partial R_1 \partial R_4 \partial R_8} = 1 - R_2 R_3 - R_2 R_7 - R_3 R_7 + 2R_2 R_3 R_7 - R_2 R_5 R_6 - R_3 R_5 R_6 \\ + 2R_2 R_3 R_5 R_6 + R_2 R_5 R_6 R_7 + R_3 R_5 R_6 R_7 - 2R_2 R_3 R_5 R_6 R_7$$

$$I(P_2) = \frac{\partial^3 R_s}{\partial R_2 \partial R_5 \partial R_6} = 1 - R_1 R_3 - R_7 R_8 - R_1 R_4 R_8 - R_3 R_4 R_8 + 2R_1 R_3 R_4 R_8 \\ + R_1 R_3 R_7 R_8 + R_1 R_4 R_7 + R_3 R_4 R_7 R_8 - 2R_1 R_3 R_4 R_7 R_8$$

$$I(P_3) = \frac{\partial^3 R_s}{\partial R_2 \partial R_7 \partial R_8} = 1 - R_1 R_3 - R_1 R_4 - R_3 R_4 - R_5 R_6 + 2R_1 R_3 R_4 + R_1 R_3 R_5 R_6 \\ + R_1 R_4 R_5 R_6 + R_3 R_4 R_5 R_6 - 2R_1 R_3 R_4 R_5 R_6$$

$$I(P_4) = \frac{\partial^4 R_s}{\partial R_1 \partial R_3 \partial R_5 \partial R_6} = 1 - R_2 - R_4 R_8 - R_7 R_8 + 2R_2 R_4 R_8 + R_2 R_7 R_8 + R_4 R_7 R_8 \\ - 2R_2 R_4 R_7 R_8$$

$$I(P_5) = \frac{\partial^4 R_s}{\partial R_1 \partial R_3 \partial R_7 \partial R_8} = 1 - R_2 - R_4 + 2R_2 R_4 - R_5 R_6 + R_2 R_5 R_6 + R_4 R_5 R_6 \\ - 2R_2 R_4 R_5 R_6$$

$$I(P_6) = \frac{\partial^4 R_s}{\partial R_2 \partial R_3 \partial R_4 \partial R_8} = 1 - R_1 - R_7 + 2R_1 R_7 - R_5 R_6 + 2R_1 R_5 R_6 + R_5 R_6 R_7 \\ - 2R_1 R_5 R_6 R_7$$

Applying all partial derivatives and substitute the reliability values as in two cases example (4.3).

Table 4.9: Determine the importance of reliability and its levels for identical values of a minimal path set of complex system by using F.M.

P_i	$I(P_{Rs})$	Level
P_6	0.8848	1
P_5	0.1558	2
P_2	0.09442	3
P_4	0.0838	4
P_1	0.01342	5
P_3	0.00532	6

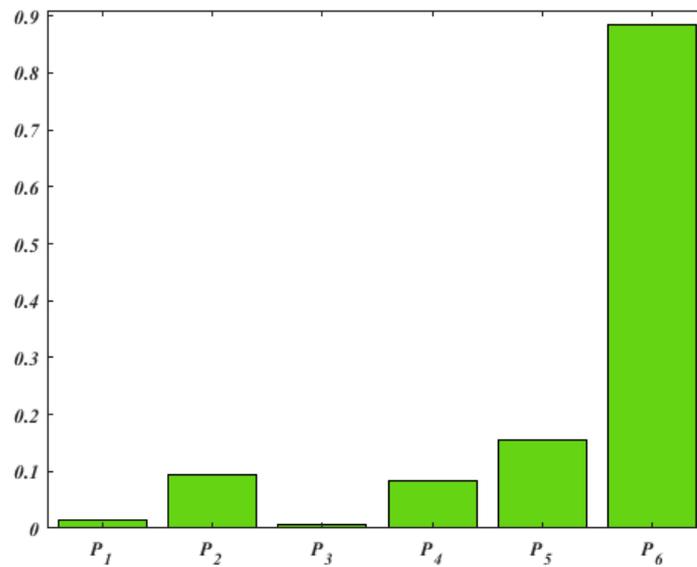


Figure 4.11: Representing the results of the reliability importance levels of a table (4.9)

Table (4.9) shows the reliability importance minimal path sets for complex system using (F.M.) When the components of the system have the same reliability values (0.9), the reliability importance minimal path sets for the complex system be ($P_6, P_5, P_2, P_4, P_1, P_3$) respectively.

Table 4.10: Determine the importance of reliability and its levels for different values of a minimal path set of complex system by using F.M.

P_i	$I(P_{Rs})$	Level
P_6	0.56768	1
P_3	0.5104	2
P_1	0.374	3
P_4	0.365	4
P_5	0.32	5
P_2	0.20596	6

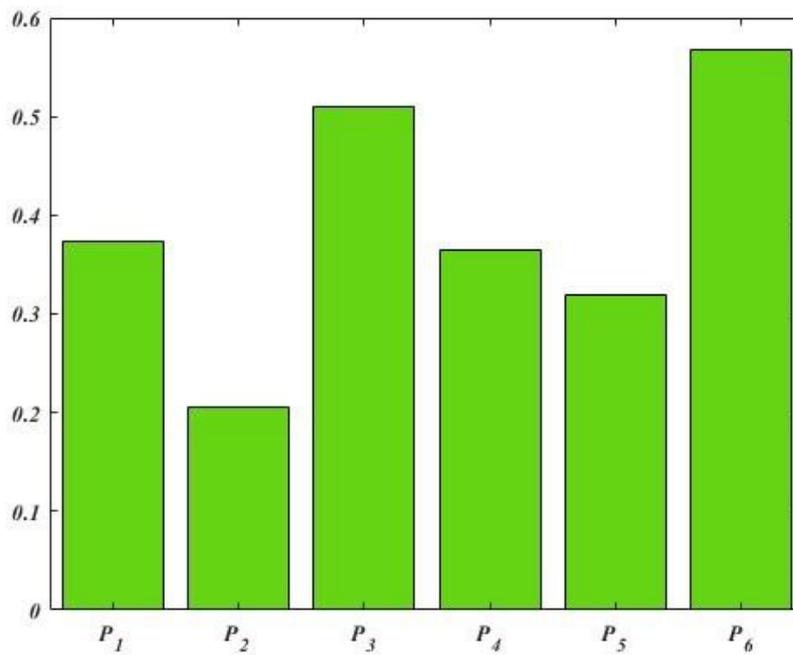


Figure 4.12: Representing the results of the reliability importance levels of a table (4.10)

Table (4.10) shows our study of the importance minimal path sets of the complex

system using (F.M) When the components of the system have the different reliability values, importance minimal path sets of the complex system will be $(P_6, P_3, P_1, P_4, P_5, P_2)$ respectively.

4.4.2 Second Method (S.M)

In this method, we generalize what the scientist Zygmund William Birnbaum [5, 37], when he relied on the pivotal decomposition technique to evaluate the reliability importance of the components. Where the scientists work was on the components of the systems only, and we generalized the work on minimal path sets and minimal cut sets for the systems. Thus, the basic equation to illustrate this method:

$$I(P_i) = Rs(1i, P_i) - Rs(0i, P_i) \quad (4.6)$$

$Rs(1i, P_i)$ minimal path sets is working, $Rs(0i, P_i)$ minimal path sets is failure . Some theories also explained the scientific explanation for the reasons for the increase and decrease in the reliability of the systems.

Theorem 4.1 [27] *If $\forall r_i = 1, i = 1, \dots, n$ in a minimal path, then the reliability of this minimal path is equal to 1. Or $\lim R(M_{P_i}) = 1$.*

Theorem 4.2 [27] *If the system contains one minimal path and its reliability is equal to one, then the reliability of the system is one, if $\exists MP_i \in S$ (where S is a set of all minimal path sets) $\ni R(M_{P_i}) = 1$ iff $R_s = 1, i = 1, 2 \dots, n$, or $\lim(R_s) = 1$.*

4.4.2.1 Using S.M. to evaluate reliability importance and its level of the minimal path sets of series system

Based on two theorems(4.1 and 4.2) and their application to eq. (4.6). Using same systems existing in Part (4.4.1). According to this method, importance minimal path sets in series systems is,

$$I(P) = Rs(1i, 1) - Rs(0i, 0) = 1$$

Reliability importance minimal path sets for the series system is (1) as in F.M.

4.4.2.2 Using S.M. to evaluate reliability importance and its level of the minimal path sets of parallel system

Looking for the section (4.4.1.2) in case parallel system, with reliability values for three components are 0.6, 0.7 and 0.8 respectively. According to this method, was compensate for each minimal path sets in Rs mentioned in eq. (4.3) by (1) in case the minimal path components are working $-(0)$ in case the minimal path components fail. Where the reliability importance of the minimal path sets according to eq.(4.6) for parallel system is,

$$I(P_1) = 1 - R_2 - R_3 + R_2R_3, \quad I(P_2) = 1 - R_1 - R_3 + R_1R_3,$$

$$I(P_3) = 1 - R_1 - R_2 + R_1R_2.$$

Table 4.11: Determine the reliability importance and its level of the minimal path sets of parallel system by using S.M.

P_i	$I(P_i)$	Level
P_3	0.12	1
P_2	0.08	2
P_1	0.06	3

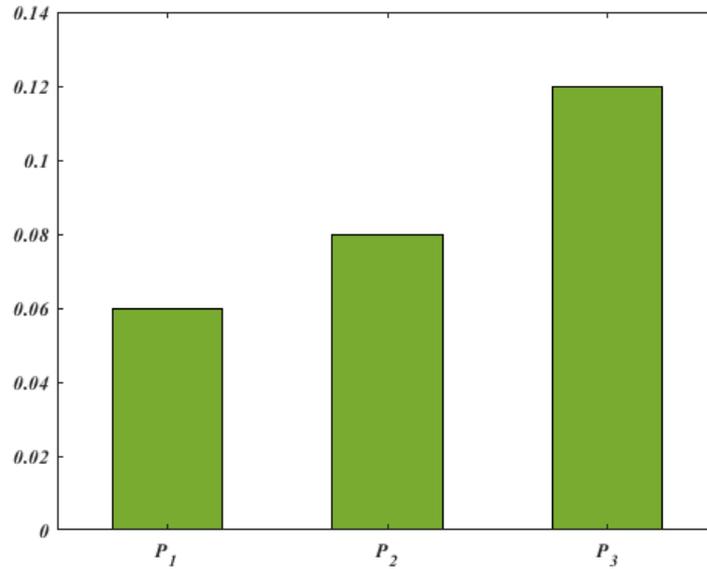


Figure 4.13: Representing the results of the reliability importance levels of a table (4.11)

We note from the table (4.11) that shows results importance and level of the minimal path sets for the parallel system using S.M (application of eq. (4.6)). The results of table (4.11) will be similar to the two tables (4.2) and (4.8), we conclude from this that every minimal path set of the parallel system has same only one component, for example, minimal path 3 contains component 3 only, and so for all of the minimal path sets.

4.4.2.3 Using S.M. to evaluate reliability importance and its level of the minimal path sets of complex system

Study S.M. on the complex system shown in fig.(2.4) that is focus of our interest. Applying eq.(4.6) on minimal path sets for a complex system mentioned in eq.(2.2), when the reliability system polynomial in eq.(2.9). We take same two cases for the reliability values in example (4.3), after substituting values case 1:

$$I(P_1) = 1 - 0.7290 = 0.271, \quad I(P_2) = 1 - 0.7290 = 0.271,$$

$$I(P_3) = 1 - 0.7290 = 0.271, \quad I(P_4) = 1 - 0.7946 = 0.2054,$$

$$I(P_5) = 1 - 0.6561 = 0.3439, \quad I(P_6) = 1 - 0 = 1$$

Table 4.12: Determine the importance of reliability and its levels for identical values of a minimal path set of complex system by using S.M.

P_i	$I(P_i)$	Level
P_6	1	1
P_5	0.3439	2
P_1, P_2, P_3	0.271	3
P_4	0.2054	4

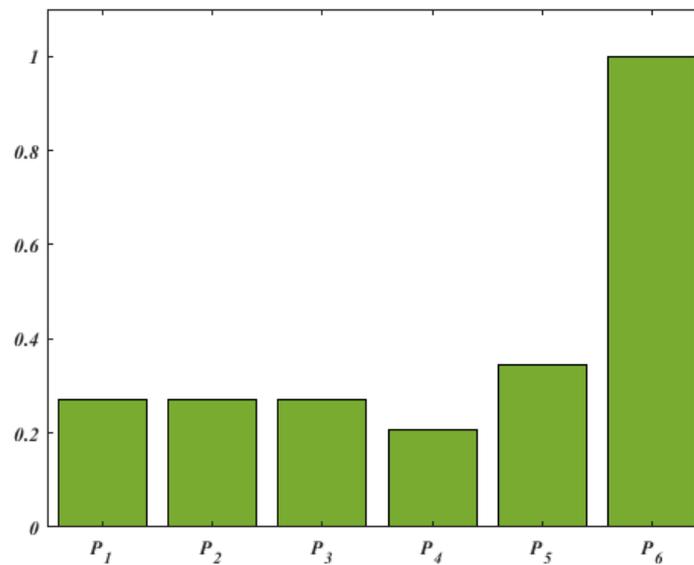


Figure 4.14: Representing the results of the reliability importance levels of a table (4.12)

Table (4.12) shows the results of the reliability importance of minimal path sets for a complex system when we took equal reliability values using (the application of eq.(4.6)). Notice minimal path 6 at the first level, then minimal path 5, and after that

minimal path sets 1, 2 and 3 have the same level because each minimal path set contains 3 components, they have the same effect on the system.

After substituting values case 2:

$$I(P_1) = 1 - 0.18 = 0.82, \quad I(P_2) = 1 - 0.18 = 0.82,$$

$$I(P_3) = 1 - 0.1350 = 0.865, \quad I(P_4) = 1 - 0.4547 = 0.5453,$$

$$I(P_5) = 1 - 0.1512 = 0.8488, \quad I(P_6) = 1 - 0 = 1.$$

Remark 4.1 *Since minimal path set P_6 contains minimal cut set of the system, its zero at $Rs(0i, P_i)$.*

Table 4.13: Determine the importance of reliability and its levels for different values of a minimal path set of complex system by using S.M.

P_i	$I(P_i)$	Level
P_6	1	1
P_3	0.865	2
P_5	0.8488	3
P_1, P_2	0.82	4
P_4	0.5453	5

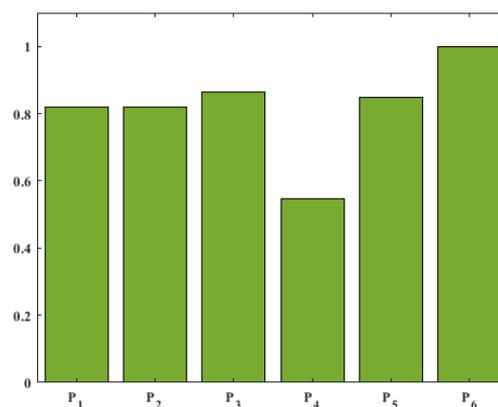


Figure 4.15: Representing the results of the reliability importance levels of a table (4.13)

Table (4.13) shows the results of the reliability importance of minimal path sets for a complex system. When we took different reliability values using (applying eq.(4.6)), notice minimal Path 6 is the first level and then P_3, P_5, P_1, P_2, P_4 respectively.

4.5 Evaluating the Reliability Importance of each Units in Minimal Cut Sets for Some Types of Systems

In this part, we study the reliability importance of each component present in every minimal cut sets for all systems, we adopt a Birnbaum measure [5] which is derivation method depending on eq.(4.1).

4.5.1 For Series Systems

In this section, study the reliability importance of the minimal cut components for systems that are linked by a series.

Example 4.7 Consider the series system shown in fig.(4.1). Where the minimal cut sets for the series system that contains three independent units is:

$$Cs = \{C_1 = R_1, C_2 = R_2, C_3 = R_3\} \quad (4.7)$$

$$\frac{\partial C_1}{\partial R_1} = 1, \quad \frac{\partial C_2}{\partial R_2} = 1, \quad \frac{\partial C_3}{\partial R_3} = 1.$$

We note the importance of the components of all minimal cut sets for the series system are equal.

4.5.2 For Parallel Systems

Study the reliability importance of the cut components of systems that are linked by a parallel.

Example 4.8 Consider the parallel system shown in fig.(4.2). Where the minimal cut sets for the parallel system that contains three independent units their values 0.6, 0.7 and

0.8 respectively.

$$Cp = \{R_1 R_2 R_3\} \quad (4.8)$$

$$I(R_1) = \frac{\partial Cp}{\partial R_1} = R_2 R_3, \quad I(R_2) = \frac{\partial Cp}{\partial R_2} = R_1 R_3, \quad I(R_3) = \frac{\partial Cp}{\partial R_3} = R_1 R_2.$$

Table 4.14: Determine the importance of reliability and its levels for minimal cut set for parallel system.

R_i	$I(R_{Cp})$	Level
R_1	0.56	1
R_2	0.48	2
R_3	0.42	3

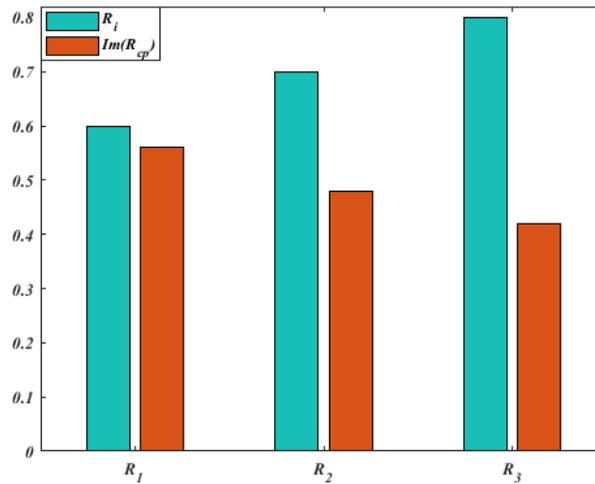


Figure 4.16: Representing the results of the reliability importance levels of a table (4.14)

From the table (4.14), which shows the results of importance and level of the minimal cut sets components of the series system, when derive the minimal cut sets eq. (4.8) for the minimal cut components, that R_1 is the least reliable in the first level and then R_2 and R_3 in the second level.

4.5.3 For Complex Systems

The reliability importance of the minimal cut components of systems which contain a bridge in its system structure. Where study the importance in two cases of a complex system if the reliability values are equal and different.

Example 4.9 Looking for the complex system shown in fig. (2.4). Where the minimal cut sets for the complex system in eq. (2.5), some components are present in more than one minimal cut set. Finding the reliability importance of components the minimal cut sets according to the Birnbaum measure (the partial derivatives). Take same two cases for the reliability values in example (4.3), after substituting values case 1:

$$\begin{aligned} \frac{\partial C_1}{\partial R_1} = \frac{\partial C_1}{\partial R_2} = \frac{\partial C_2}{\partial R_5} = \frac{\partial C_2}{\partial R_8} = \frac{\partial C_3}{\partial R_6} = \frac{\partial C_3}{\partial R_8} &= 0.1, \\ \frac{\partial C_4}{\partial R_2} = \frac{\partial C_4}{\partial R_3} = \frac{\partial C_4}{\partial R_4} = \frac{\partial C_5}{\partial R_2} = \frac{\partial C_5}{\partial R_3} = \frac{\partial C_5}{\partial R_8} = \frac{\partial C_6}{\partial R_4} = \frac{\partial C_6}{\partial R_6} = \frac{\partial C_6}{\partial R_7} = \frac{\partial C_7}{\partial R_4} = \frac{\partial C_7}{\partial R_5} \\ &= \frac{\partial C_7}{\partial R_7} = 0.01, \\ \frac{\partial C_8}{\partial R_1} = \frac{\partial C_8}{\partial R_3} = \frac{\partial C_8}{\partial R_6} = \frac{\partial C_8}{\partial R_7} = \frac{\partial C_9}{\partial R_1} = \frac{\partial C_9}{\partial R_3} = \frac{\partial C_9}{\partial R_5} = \frac{\partial C_9}{\partial R_7} &= 0.001 \end{aligned}$$

Table 4.15: Determine the importance of reliability and its levels for identical units in minimal cut set of complex system.

R_i	C_i	$I(R_{C_i})$	Biggest value of $I(R_{C_i})$	Level
$R_1, R_2,$ R_5, R_6	$C_1 = C_2 = C_3$ $C_4 = C_5 = C_6 = C_7$	0.1 0.01	0.1	1
R_8	$C_8 = C_9$	0.001		
R_3, R_4 R_7	$C_4 = C_5 = C_6 = C_7$ $C_8 = C_9$	0.01 0.001	0.01	2

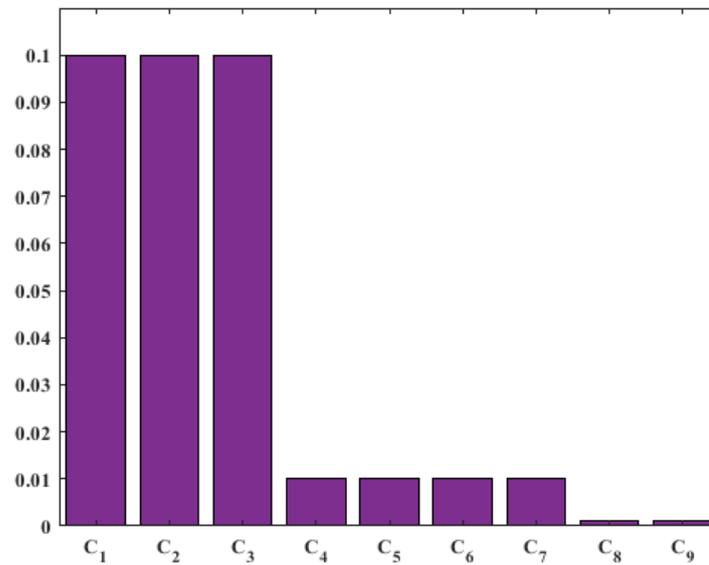


Figure 4.17: Representing the results of the reliability importance levels of a table (4.15)

Table (4.15) that shows the reliability importance components of the minimal cut sets for the complex system is that some components are present in more than one minimal cut sets, meaning we studied the reliability importance of each component in more than one minimal cut sets, for example, the component (R_1) is present in the first minimal cut (C_1), and its also present in (C_8, C_9), which is the third level, so (R_1) is in the first level because its in the most important minimal cut set.

After substituting values case 2:

$$\begin{aligned} \frac{\partial C_1}{\partial R_1} &= 1 - R_2 = 1 - 0.5 = 0.5, \\ \frac{\partial C_1}{\partial R_2} &= 1 - R_1 = 1 - 0.6 = 0.4, \\ \frac{\partial C_2}{\partial R_5} &= 1 - R_8 = 1 - 0.9 = 0.1, \\ \frac{\partial C_2}{\partial R_8} &= 1 - R_5 = 1 - 0.4 = 0.6, \\ \frac{\partial C_3}{\partial R_6} &= 1 - R_8 = 1 - 0.9 = 0.1, \\ \frac{\partial C_3}{\partial R_8} &= 1 - R_6 = 1 - 0.9 = 0.1, \\ \frac{\partial C_4}{\partial R_2} &= 1 - R_3 - R_4 + R_3R_4 = 1 - 0.7 - 0.8 + (0.56) = 0.06, \\ \frac{\partial C_5}{\partial R_8} &= 1 - R_2 - R_3 + R_2R_3 = 0.15, \\ \frac{\partial C_6}{\partial R_4} &= 1 - R_6 - R_7 + R_6R_7 = 0.07, \\ \frac{\partial C_6}{\partial R_6} &= 1 - R_4 - R_7 + R_4R_7 = 0.14, \\ \frac{\partial C_6}{\partial R_7} &= 1 - R_6 - R_4 + R_4R_6 = 0.02, \\ \frac{\partial C_7}{\partial R_4} &= 1 - R_5 - R_7 + R_5R_7 = 0.42, \\ \frac{\partial C_7}{\partial R_5} &= 1 - R_4 - R_7 + R_4R_7 = 0.14, \\ \frac{\partial C_7}{\partial R_7} &= 1 - R_5 - R_4 + R_4R_5 = 0.12, \\ \frac{\partial C_8}{\partial R_1} &= 1 - R_3 - R_6 + R_3R_6 - R_7 + R_3R_7 + R_6R_7 - R_3R_6R_7 = 0.021, \\ \frac{\partial C_8}{\partial R_3} &= 1 - R_1 - R_6 + R_1R_6 - R_7 + R_1R_7 + R_6R_7 - R_1R_6R_7 = 0.028, \\ \frac{\partial C_8}{\partial R_6} &= 1 - R_3 - R_1 + R_1R_3 - R_7 + R_3R_7 + R_1R_7 - R_1R_3R_7 = 0.084, \\ \frac{\partial C_8}{\partial R_7} &= 1 - R_3 - R_1 + R_1R_3 - R_6 + R_3R_6 + R_1R_6 - R_1R_3R_6 = 0.264, \\ \frac{\partial C_9}{\partial R_1} &= 1 - R_3 - R_5 + R_3R_5 - R_7 + R_3R_7 + R_5R_7 - R_3R_5R_7 = 0.126, \\ \frac{\partial C_9}{\partial R_3} &= 1 - R_1 - R_5 + R_1R_5 - R_7 + R_1R_7 + R_5R_7 - R_1R_5R_7 = 0.168, \\ \frac{\partial C_9}{\partial R_5} &= 1 - R_3 - R_1 + R_1R_3 - R_7 + R_1R_7 + R_3R_7 - R_1R_3R_7 = 0.084, \\ \frac{\partial C_9}{\partial R_7} &= 1 - R_3 - R_1 + R_1R_3 - R_5 + R_3R_5 + R_1R_5 - R_1R_3R_5 = 0.072, \end{aligned}$$

Table 4.16: Determine the importance of reliability and its levels for different units in minimal cut set of complex system.

R_i	C_i	$I(R_{C_i})$	Biggest value of $I(R_{C_i})$	Leve
R_8	C_2	0.6	0.6	1
	C_3	0.1		
	C_5	0.15		
R_1	C_1	0.5	0.5	2
	C_8	0.021		
	C_9	0.126		
R_4	C_4	0.15	0.42	3
	C_6	0.07		
	C_7	0.42		
R_2	C_1	0.4	0.4	4
	C_4	0.06		
	C_5	0.03		
R_3	C_4	0.26	0.26	5
	C_5	0.05		
	C_8	0.028		
	C_9	0.168		
R_7	C_6	0.02	0.264	6
	C_7	0.12		
	C_8	0.264		
	C_9	0.072		
R_5, R_6	C_2, C_3	0.1	0.14	7
	C_6, C_7	0.14		
	C_8, C_9	0.084		

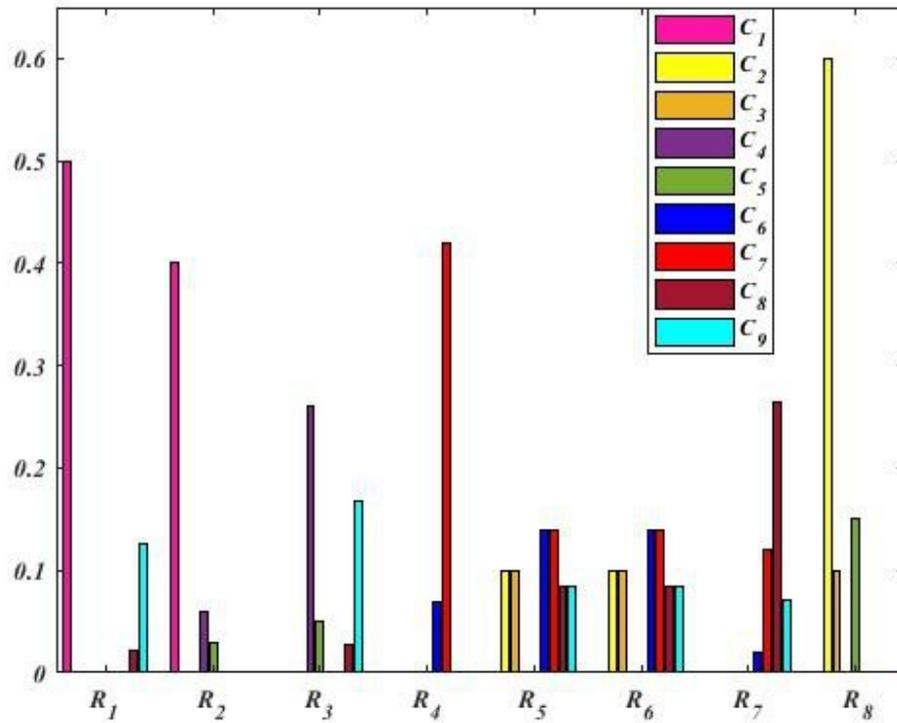


Figure 4.18: Representing the results of the reliability importance levels of a table (4.16)

Table (4.16) is similar to work of table (4.15) in terms study the importance of the components of the minimal cut sets for the complex system, but in this table we used different reliability values, the two components(R_5, R_6) have same the level, and therefore they have the same effect on the system.

4.6 A New Measure to Evaluate the Reliability Importance of Minimal Cuts for Complex System

In this section, derived the reliability of the parallel system shown in fig.(4.2) for more than one component. The binary derivative of the parallel system be as follows:

$$\frac{\partial^2 R_s}{\partial R_1 \partial R_2} = -1 + R_3, \quad \frac{\partial^2 R_s}{\partial R_1 \partial R_3} = -1 + R_2, \quad \frac{\partial^2 R_s}{\partial R_2 \partial R_3} = -1 + R_1$$

Take reliability values for the three components are 0.6, 0.7 and 0.8 respectively.

$$\frac{\partial^2 R_s}{\partial R_1 \partial R_2} = |-0.2| = 0.2, \quad \frac{\partial^2 R_s}{\partial R_1 \partial R_3} = |-0.3| = 0.3, \quad \frac{\partial^2 R_s}{\partial R_2 \partial R_3} = |-0.4| = 0.4$$

By using the absolute value measure of importance and according to the single derivative (the derivative of one component) and according to table (4.2), we note that the biggest reliable component is the most important in the parallel system, and noticed in this example that whenever we derive the biggest reliable binary components, they are the most important components, as is the case in the single derivative.

Then, we generalize the formula to calculate the reliability importance of minimal cut set in complex systems.

$$\mathbf{I}(\mathbf{C}_i) = \left| \frac{\partial^n R_s}{\partial R_1 \partial R_2 \dots \partial R_n} \right| \quad n = 1, 2, 3, \dots, i \quad (4.9)$$

It is an absolute measure of negative importance in order to maintain the level of importance of the components.

4.7 Evaluating the Reliability Importance of each Minimal Cut Sets for Some Types of Systems

In this section, study the reliability importance of minimal cut sets for the systems, we will adopt the same two methods in the section (4.4) to evaluated the reliability importance of systems.

4.7.1 First Method (F.M)

In this part, applying F.M. to three types of systems, consider the first measure to evaluating the reliability importance of systems components in general.

4.7.1.1 Using F.M. to evaluate the reliability importance of minimal cut sets for series systems

Looking for the part (4.4.1.1), the series system in fig.(4.1), where the reliability system (R_s) in eq. (4.2), the minimal cut sets in eq. (4.7) that contains three independent units their values 0.6, 0.7 and 0.8 respectively.

$$I(C_1) = \frac{\partial R_s}{\partial C_1} = R_2 R_3, \quad I(C_2) = \frac{\partial R_s}{\partial C_2} = R_1 R_3, \quad I(C_3) = \frac{\partial R_s}{\partial C_3} = R_1 R_2$$

Table 4.17: Determine the reliability importance and its level of the minimal cut sets of parallel system by using F.M.

C_i	$I(C_i)$	Level
C_1	0.56	1
C_2	0.48	2
C_3	0.42	3

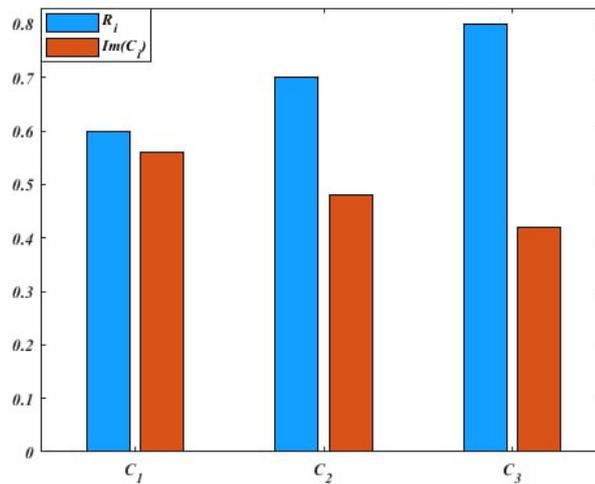


Figure 4.19: Representing the results of the reliability importance levels of a table (4.17)

Table (4.17) shows the results of significance of minimal cut sets for the series

system using (F.M.). When derived the system reliability equation for each minimal cut set in the system, its be C_1 in level 1, then C_2 in level 2, and finally C_3 in level 3. Since each minimal cut set in the series system contains only one component, this table is similar to tables (4.1),(4.5) and (4.14) from in terms of importance and level.

4.7.1.2 Using F.M to evaluate the reliability importance of minimal cut sets for parallel systems

Reconsider in fig. (4.2) whose reliability is in eq. (4.3). Where the minimal cut sets in eq. (4.8) that contains three independent units.

$I(C) = \frac{\partial R_s}{\partial C} = 1$. In general, when derive R_S relation to the minimal cut sets of the parallel system, the importance is (1), because the parallel system contains only one minimal cut set.

4.7.1.3 Using F.M to evaluate the reliability importance of minimal cut sets for complex systems

We study importance of each minimal cut sets for the complex system, where the minimal cut sets of the system in eq.(2.5), derive(R_s)indicated by (2.9) for each minimal cut set in complex system, be:

$$\begin{aligned} \mathbf{I}(C_1) &= \frac{\partial^2 R_s}{\partial R_1 \partial R_2} \\ &= -R_3 R_5 R_6 - R_3 R_4 R_8 - R_3 R_7 R_8 - R_4 R_7 R_8 + 2R_3 R_4 R_7 R_8 \\ &\quad - R_4 R_5 R_6 R_8 + 2R_3 R_4 R_5 R_6 R_8 + R_3 R_5 R_6 R_7 R_8 + R_4 R_5 R_6 R_7 \\ &\quad - 2R_3 R_4 R_5 R_6 R_7 R_8 \end{aligned}$$

$$\begin{aligned}
\mathbf{I}(\mathbf{C}_2) &= \frac{\partial^2 R_s}{\partial R_5 \partial R_8} \\
&= -R_2 R_6 R_7 - R_1 R_2 R_4 R_6 - R_1 R_3 R_4 R_6 - R_2 R_3 R_4 R_6 - R_1 R_3 R_6 R_7 \\
&\quad + 2R_1 R_2 R_3 R_4 R_6 + R_1 R_2 R_3 R_6 R_7 + R_1 R_3 R_4 R_6 R_7 + R_2 R_3 R_4 R_6 R_7 \\
&\quad - 2R_1 R_2 R_3 R_4 R_6 R_7
\end{aligned}$$

$$\begin{aligned}
\mathbf{I}(\mathbf{C}_3) &= \frac{\partial^2 R_s}{\partial R_6 \partial R_8} \\
&= -R_2 R_5 R_7 - R_1 R_2 R_4 R_5 - R_1 R_3 R_4 R_5 - R_2 R_3 R_4 R_5 - R_1 R_3 R_5 R_7 \\
&\quad + 2R_1 R_2 R_3 R_4 R_5 + R_1 R_2 R_3 R_5 R_7 + R_1 R_3 R_4 R_5 R_7 + R_2 R_3 R_4 R_5 R_7 \\
&\quad - 2R_1 R_2 R_3 R_4 R_5 R_7
\end{aligned}$$

$$\begin{aligned}
\mathbf{I}(\mathbf{C}_4) &= \frac{\partial^3 R_s}{\partial R_2 \partial R_3 \partial R_4} \\
&= R_8 - R_1 R_8 - R_7 R_8 + 2R_1 R_7 R_8 - R_5 R_6 R_8 + 2R_1 R_5 R_6 R_8 \\
&\quad + R_5 R_6 R_7 R_8 - 2R_1 R_5 R_6 R_7 R_8
\end{aligned}$$

$$\begin{aligned}
\mathbf{I}(\mathbf{C}_5) &= \frac{\partial^3 R_s}{\partial R_2 \partial R_3 \partial R_8} \\
&= R_4 - R_1 R_4 - R_1 R_7 - R_4 R_7 + 2R_1 R_4 R_7 - R_4 R_5 R_6 + 2R_1 R_4 R_5 R_6 \\
&\quad + R_1 R_5 R_6 R_7 + R_4 R_5 R_6 R_7 - 2R_1 R_4 R_5 R_6 R_7
\end{aligned}$$

$$\mathbf{I}(\mathbf{C}_6) = \frac{\partial^3 R_s}{\partial R_4 \partial R_6 \partial R_7} = R_1 R_2 R_5 + R_1 R_3 R_5 R_8 + R_2 R_3 R_5 R_8 - 2R_1 R_2 R_3 R_5 R_8$$

$$\mathbf{I}(\mathbf{C}_7) = \frac{\partial^3 R_s}{\partial R_4 \partial R_5 \partial R_7} = R_1 R_2 R_6 + R_1 R_3 R_6 R_8 + R_2 R_3 R_6 R_8 - 2R_1 R_2 R_3 R_6 R_8$$

$$\mathbf{I}(\mathbf{C}_8) = \frac{\partial^4 R_s}{\partial R_1 \partial R_3 \partial R_6 \partial R_7} = -R_5 R_8 + R_2 R_5 R_8 + R_4 R_5 R_8 - 2R_2 R_4 R_5 R_8$$

$$\mathbf{I}(\mathbf{C}_9) = \frac{\partial^4 R_s}{\partial R_1 \partial R_3 \partial R_5 \partial R_7} = -R_6 R_8 + R_2 R_6 R_8 + R_4 R_6 R_8 - 2R_2 R_4 R_6 R_8$$

Uses same two cases for the components reliability values in example (4.3), applying eq.(4.9) to calculate the reliability importance of the minimal cut sets for the system, as follows:

$$\frac{\partial R_s}{\partial C_1} = |-0.895212|, \quad \frac{\partial R_s}{\partial C_2} = \frac{\partial R_s}{\partial C_3} = |-0.493832|, \quad \frac{\partial R_s}{\partial C_4} = 0.79632, \quad \frac{\partial R_s}{\partial C_5} = 0.64242,$$

$$\frac{\partial R_s}{\partial C_6} = \frac{\partial R_s}{\partial C_7} = 0.86022, \quad \frac{\partial R_s}{\partial C_8} = \frac{\partial R_s}{\partial C_9} = |-0.66421|$$

Table 4.18: Determine the importance of reliability and its levels for identical values of a minimal cut set of complex system by using F.M.

C_i	$I(C_i)$	Level
C_1	0.895212	1
$C_6 = C_7$	0.86022	2
C_4	0.79632	3
$C_8 = C_9$	0.66421	4
C_5	0.64242	5
$C_2 = C_3$	0.493832	6

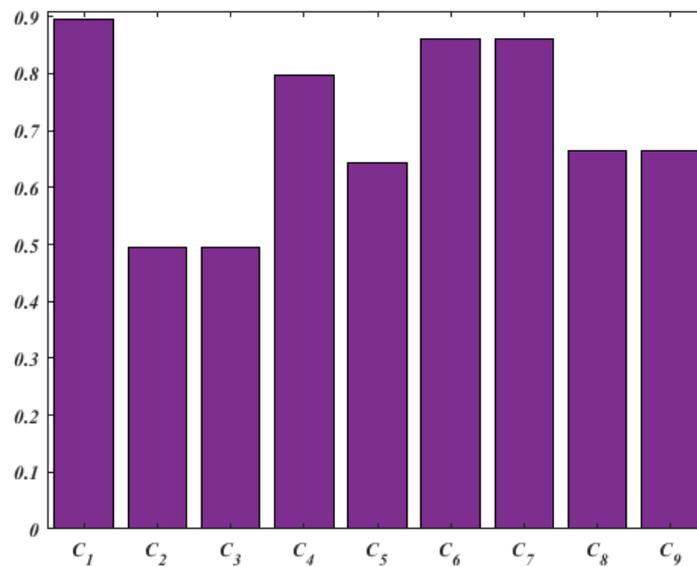


Figure 4.20: Representing the results of the reliability importance levels of a table (4.18)

Table (4.18) gives results for the significance of the minimal cut sets in the case equal reliability values for the complex system and according to eq. (4.9) to calculate the

importance of the negative minimal cut sets.

case 2. After substituting values:

$$\begin{aligned} \frac{\partial R_s}{\partial C_1} &= |-0.641304|, & \frac{\partial R_s}{\partial C_2} &= |-0.5841|, & \frac{\partial R_s}{\partial C_3} &= |-0.2596|, & \frac{\partial R_s}{\partial C_4} &= 0.45936, \\ \frac{\partial R_s}{\partial C_5} &= 0.17672, & \frac{\partial R_s}{\partial C_6} &= 0.246, & \frac{\partial R_s}{\partial C_7} &= 0.5535, & \frac{\partial R_s}{\partial C_8} &= |-0.18|, & \frac{\partial R_s}{\partial C_9} &= |-0.405| \end{aligned}$$

Table 4.19: Determine the importance of reliability and its levels for different values of a minimal cut set of complex system by using F.M.

C_i	$I(C_i)$	Level
C_1	0.641304	1
C_2	0.5841	2
C_7	0.5535	3
C_4	0.45936	4
C_9	0.405	5
C_3	0.2596	6
C_6	0.246	7
C_8	0.18	8
C_5	0.17672	9

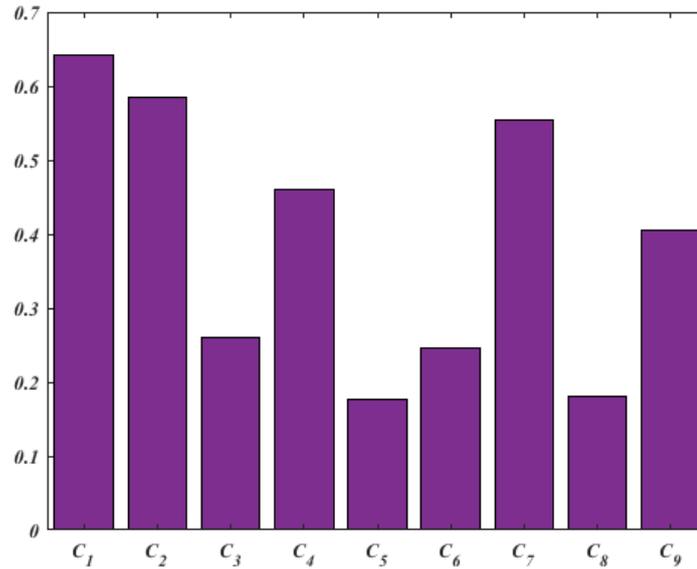


Figure 4.21: Representing the results of the reliability importance levels of a table (4.19)

Table (4.19) is similar to work of table (4.18), but in this table we have taken different values.

4.7.2 Second Method (S.M)

This section is similar to section (4.4.2) but on minimal cut sets, use S.M. to evaluate the reliability importance of the minimal cut sets for systems according to equation:

$$I(C_i) = Rs(1i, C_i) - Rs(0i, C_i) \quad (4.10)$$

$Rs(1i, C_i)$ minimal cut sets is working, $Rs(0i, C_i)$ minimal cut sets is failure .

4.7.2.1 Using S.M. to evaluate the reliability importance of minimal cut sets for series systems

To evaluate the reliability importance of minimal cut sets depend on eq.(4.10), this meaning (calculated R when the minimal cut components are 1 – calculated R when

the minimal cut components are 0). So get importance minimal cut sets for the series system. Where the minimal cut sets in eq. (4.7) that contains three independent units their values 0.6, 0.7 and 0.8 respectively.

$$I(C_1) = Rs(1i, 1R_2R_3) - Rs(0i, 0R_2R_3) = R_2R_3$$

$$I(C_2) = Rs(1i, R_11R_3) - Rs(0i, R_10R_3) = R_1R_3$$

$$I(C_3) = Rs(1i, R_1R_21) - Rs(0i, R_1R_20) = R_1R_2$$

Table 4.20: Determine the reliability importance and its level of the minimal cut sets of series system by using S.M.

C_i	$Im(C_i)$	Level
C_1	0.56	1
C_2	0.48	2
C_3	0.42	3

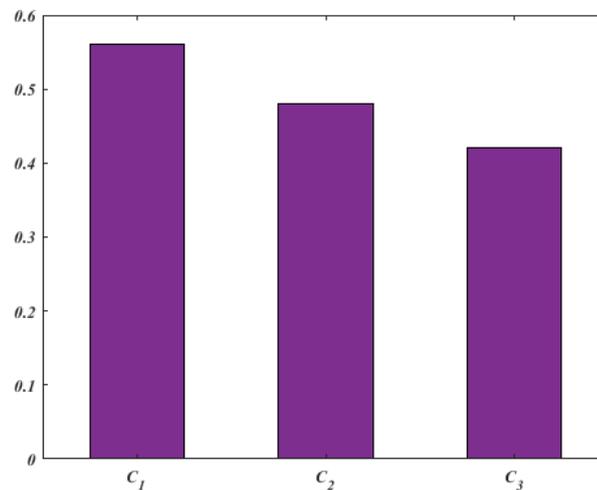


Figure 4.22: Representing the results of the reliability importance levels of a table (4.20)

Table (4.20) shows the results of the importance values and level of the series system, using eq. (4.10), where the first minimal cut set that contains only one component

is R_1 is in the first level, then R_2, R_3 respectively. We also note that table (4.20) is similar to tables (4.1), (4.5), (4.14) and (4.17) in terms the importance values and level.

4.7.2.2 Using S.M. to evaluate the reliability importance of minimal cut sets for parallel systems

Looking for the section (4.7.1.2), where use same system and equations, but applying eq.(4.10), we get: $I(C) = 1$. In general, as mentioned earlier, when derive with respect to minimal cut set of parallel system is the importance (1).

4.7.2.3 Using S.M. to evaluate the reliability importance of minimal cut sets for complex systems

Reconsider in section (4.7.1.3), where use same system and equations, but applying eq.(4.10). Take same two cases for the reliability values in example (4.3).

$$I(C_1) = 0.9793 - 0 = 0.9793, \quad I(C_2) = I(C_3) = 0.9880 - 0 = 0.988$$

$$I(C_4) = 0.9810 - 0 = 0.981, \quad I(C_5) = 0.9981 - 0 = 0.9981$$

$$I(C_6) = I(C_7) = 0.9793 - 0 = 0.9793, \quad I(C_8) = I(C_9) = 0.9900 - 0 = 0.99$$

Table 4.21: Determine the importance of reliability and its levels for identical values of a minimal cut set of complex system by using S.M.

C_i	$\text{Im}(C_i)$	Level
C_5	0.9981	1
$C_8 = C_9$	0.99	2
$C_2 = C_3$	0.988	3
C_4	0.981	4
$C_1 = C_6 = C_7$	0.9793	5

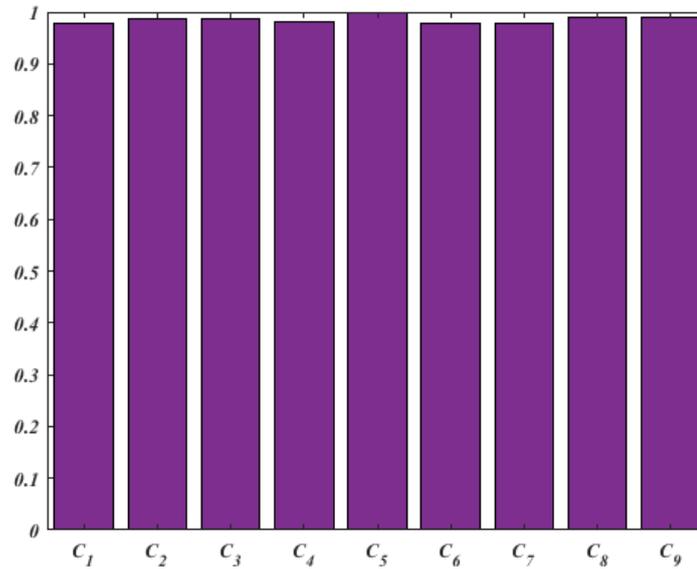


Figure 4.23: Representing the results of the reliability importance levels of a table (4.21)

Table (4.21) shows results of importance of minimal cut sets for complex system in the case of equal reliability values using eq. (4.10). We got C_5 in the first level and note that C_8 and C_9 have the same level, which is level 2, then C_2 and C_4 respectively, and finally minimal cut sets 1,6 and 7 are in the last level.

After substituting values case 2:

$$I(C_1) = 0.8554 - 0 = 0.8554, \quad I(C_2) = 0.7687 - 0 = 0.7687$$

$$I(C_3) = 0.7022 - 0 = 0.7022, \quad I(C_4) = 0.9360 - 0 = 0.936$$

$$I(C_5) = 0.9104 - 0 = 0.9104, \quad I(C_6) = 0.7484 - 0 = 0.7484$$

$$I(C_7) = 0.7839 - 0 = 0.7839, \quad I(C_8) = 0.94 - 0 = 0.94$$

$$I(C_9) = 0.99 - 0 = 0.99$$

Table 4.22: Determine the importance of reliability and its levels for different values of a minimal cut set of complex system by using S.M.

C_i	$\text{Im}(C_i)$	Level
C_9	0.99	1
C_8	0.94	2
C_4	0.936	3
C_5	0.9104	4
C_1	0.8554	5
C_7	0.7839	6
C_2	0.7687	7
C_6	0.7484	8
C_3	0.7022	9

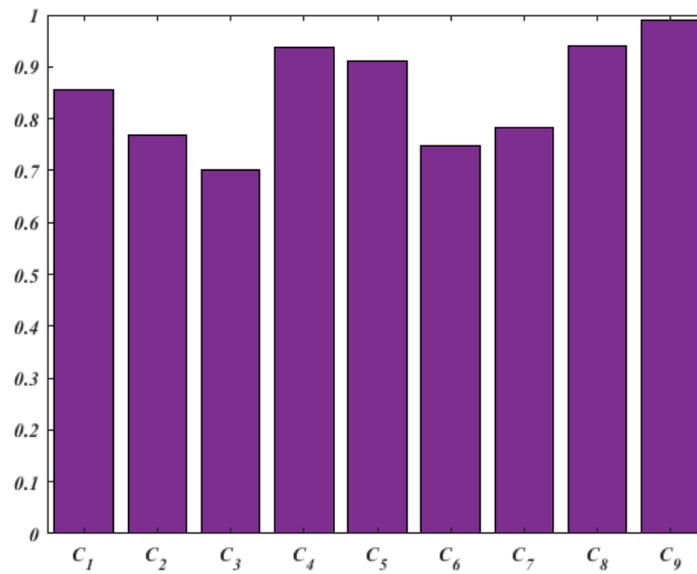


Figure 4.24: Representing the results of the reliability importance levels of a table (4.22)

4.8 Summary chapter four

- In section (4.2), it has been calculated that the single importance according to Birnbaum measure for the components of the three systems (series, parallel and complex) as in tables (4.1), (4.2), (4.3) and (4.4).
- In part (4.3), it has been calculated that the singular importance of the reliability of the three system minimal path components (series, parallel and complex), according to F.M (partial derivative for minimal paths), as shown in tables (4.5), (4.6) and (4.7), the reliability importance value and level of the system minimal path components.
- From parts (4.2) and (4.3), show that tables (4.1) and (4.5) are similar in terms of the reliability importance and level of the series system, in other words, the components of the system are the same as the components of the minimal path.
- In part (4.4), importance was studied according to two methods and for the three systems, the results of importance in the two methods for the series system and the parallel system are similar in terms of importance and level, but in the complex system, the results of the two methods are different in terms of importance and level.
- In part (4.5) is similar to work section (4.3), but on the cut sets of the three systems.
- In section (4.6), it has been calculated the importance of the negative minimal cut set (binary derivative) for the parallel system, where we found a new measure, which is the measure of the absolute value of importance according to eq.(4.10).
- In the section (4.7), from our study the importance of minimal cut sets for the systems, and we found work this section is similar to work sec. (4.4). We also studied two methods for the importance of minimal cut sets for the three systems,

and the results of the two methods for the series and parallel systems are similar in terms of importance and level, but in the complex system, the results of the two methods are different.

- We found from our study for importance of the reliability of the minimal path set and minimal cut set for systems connected in series and parallel, and if they contain only one minimal set path or minimal cut set, the importance will always be (1).

Chapter 5

Conclusions and Future Works

5.1 Conclusions

1. *The minimal path sets for the complex system were calculated in three different methods, and the results of the three methods were equal in finding the minimal path sets, and the minimal cut sets of the complex system were calculated in two different methods, and we got the same sets of minimal cuts for the two methods. Then, the reliability function for the complex system was found in two methods:(the pivotal decomposition method and the sum of disjoint products method), we got the same polynomial for the complex system.*
2. *Applying two techniques (redundancy and allocation) to increase the reliability value of the three systems (series, parallel and complex). Where the redundancy technique was in two methods, through the study in this thesis, we found that the method of redundancy of the elements is better than the method of redundancy of the system, and to increase the reliability one of the minimal path sets and minimal cut sets for complex system. Then, study the allocation technique according to Howaidi's Theorem, we note the reliability of the three systems was increased. The allocation method using Howaidi's Theorem is better than the known methods of repeat reliability, although the number of units added is less than the previous methods.*
3. *Some cases and methods have been studied to calculate the reliability importance of component reliability in simple and complex systems. Where the reliability importance was evaluated for all components in minimal path sets and minimal cut sets for systems (series, parallel and complex). We found a new measure(Absolute reliability importance value) to evaluate the reliability importance in case the minimal cuts for parallel and complex systems is to calculate the negative reliability importance of the minimal cut sets of the systems, then two methods of Birnbaum scale were generalized (F.M.and S.M.), both of which are applied to*

measure the reliability importance of reliability for more than one component, and these two methods apply of minimum paths or cuts.

5.2 Future works

1. The possibility of studying systems (electrical devices, computer network and road network) by two methods F.M and S.M for measure the importance of reliability.
2. It is possible to calculate the best allocation by linking the reliability with the optimization using algorithm (genetic algorithm and particle swarm optimization).
3. Possibility of studying the SDP technique based on sets of minimal cuts of the complex system shown in Figure (2.1).

References

- [1] Abd, F. H. and Hassan, Z. A. H., On Some Approaches to Optimize the Reliability of Complex Networks , Thesis, University of Babylon, (2022).
- [2] Abed, S. A. and Udriste, C., Optimization Techniques and Methods in Reliability Allocation, Ph. D. Thesis, University Politehnica of Bucharest,(2017)
- [3] Abed, S. A., sulaiman, H. K., and Hassan, Z. A. H., Reliability Allocation and Optimization for ROSS of a Spacecraft by using Genetic Algorithm, in Journal of Physics: Conference Series Iop Publishing, Vol. 1294, (2019, September), No. 3, p. 032034.
- [4] Abed, S. A., Udriste, C. and Tevy, I., Optimal reliability allocation for redundancy series-parallel systems, European Journal of Pure and Applied Mathematics, 10(4), (2017), pp. 877-889.
- [5] Abraham, J. A., An improved algorithm for network reliability. IEEE Transactions on Reliability, April (1969), R-28: pp. 58-61.
- [6] Aggarwal, K.K., Reliability engineering, Center for Excellence in Reliability engineering, Regional engineering College ,Kurukshehra, India,(1993).
- [7] Allan, R. N., Billinton, R. and De Oliveira, M. F., An efficient algorithm for deducing the minimal cuts and reliability indices of a general network configuration, IEEE Transactions on Reliability, vol. 25.4 (1976), pp. 226-233.
- [8] Alghamdi, S. M. and Percy, D. F., Reliability equivalence factors for a series-parallel system of components with exponentiated Weibull lifetimes, IMA Journal of Management Mathematics, (2015), vol. 28.3, pp. 339-358.

- [9] Ansell, J. I. and Phillips M. J., Practical Methods for Reliability Data Analysis, Oxford University Press, New York, (1994).
- [10] Beichelt, F. and Tittmann, P., Reliability and maintenance: networks and systems, CRC Press, (2012).
- [11] Biegel, J. E., Determination of tie sets and cut sets for a system without feedback, IEEE Transactions on Reliability, vol. 26.1 (1977), pp. 39-42.
- [12] Billinton, R. and Allan, R. N., Reliability evaluation of engineering systems: concepts and techniques, Springer Science and Business Media, (2013).
- [13] Chelson, P. O. and Eckstein, E., Reliability computation from reliability block diagrams, IEEE Transactions on Reliability, vol. 25(1975), pp. 283-283.
- [14] Chen, W. K., Graph theory and its engineering applications, vol. 5, World Scientific, (2016).
- [15] Chithra, M. and Vijayakumar, A, The diameter variability of the Cartesian product of graphs”, Discrete Math, vol. Algorithms Appl. 6 (2014), pp. 289 -301.
- [16] Diestel, R., Graph Theory. Electronic library of mathematics. Springer Science, (2006).
- [17] Faraci, V., Calculating failure rates of series/parallel networks, The Journal of Alions, System Reliability Center, vol. First Quarter (2006), pp. 1-3.
- [18] Feng, X., Zhu, X., Zhao, W. and Li, X., Reliability of Electric Vehicle with Wind Turbine Based on Particle Swarm Optimization, Chemical Engineering Transactions, (2018), 66, pp. 1291-1296.
- [19] Gertsbakh, I. and Shpungin, Y., Network Reliability and Resilience, 1st ed., Springer-Verlag Berlin Heidelberg, (2011).

- [20] Gilbert, G. T., Positive definite matrices and Sylvester's criterion, *The American Mathematical Monthly*, vol. 98.1 (1991), pp. 44-46.
- [21] Hassan, Z. A. H. and Mutar, E. K., Evaluation the reliability of a high-pressure oxygen supply system for a spacecraft by using GPD method, *Al-Mustansiriyah Journal of college of education*, vol. special issue. 2 (2017), pp. 993 -1004.
- [22] Hassan, Z. A. H. and Mutar, E. K., Geometry of reliability models of electrical system used inside spacecraft, 2017 Second Al-Sadiq International Conference on Multidisciplinary in IT and Communication Science and Applications (AIC-MITCSA). IEEE, (2017), pp. 301-306.
- [23] Hassan, Z. A. H. and Udriste, C., Equivalent reliability polynomials modeling EAS and their geometries, *Annals of West University of Timisoara Mathematics and Computer Science*, vol. 53.1 (2015), pp. 177-195.
- [24] Hassan, Z. A. H. and Udriste, C., *Geometry of Reliability Models*, Ph.D. Thesis, University Politehnica of Bucharest,(2016).
- [25] Haynes, T. W., Hedetniemi, S. T. and Slater, P. J., *Fundamentals of Domination in Graphs*, Marcel Dekker, Inc., New York, (1998).
- [26] Huang, Chia-Ling, and Wei-Chang Yeh. Simplified Swarm Optimization Algorithm for reliability redundancy allocation problems. *International Telecommunication Networks and Applications Conference (ITNAC)*. IEEE, (2015).
- [27] Howeidi, H. S. and Hassan, Z. A. H., Use innovative methods to increase the reliability of complex and mixed networks, Thesis, University of Babylon, (2022).
- [28] Ireson, W. G., Coombs, C. R. and Moss, R. Y., *Handbook of Reliability Engineering and Management*, 2nd ed., McGraw-Hill Comp., U.S.A., (1995).

- [29] Jula, Nicolae, and Cepisca Costin. Methods for analyzing the reliability of electrical systems used inside aircrafts. *Recent Advances in Aircraft Technology*. Intech open, (2012).
- [30] Karim, S. A. and Hassan, Z. A. H., Study of Some Techniques to Support Reliability of Mixed Systems, Thesis, University of Babylon, (2021).
- [31] Katiyar, Sapna. A Comparative Study of Genetic Algorithm and the Particle Swarm Optimization. *International Journal of Technology*, (2010), 2.2: pp. 21-24
- [32] Kim, M. C., Reliability block diagram with general gates and its application to system reliability analysis, *Annals of Nuclear Energy*, vol. 38.11 (2011), pp. 2456-2461.
- [33] Kim, Y. H., Case, K. E. and Ghare, P., A method for computing complex system reliability, *IEEE Transactions on Reliability*, vol. 21.4 (1972), pp. 215-219.
- [34] Ko lowrocki, K., *Reliability of large systems and Complex Systems*, Wiley Online Library, (2008).
- [35] Kulli, V. R. and Soner, N.D., Complementary Edge Domination in Graphs, *Indian J Pure Appl.Math.* 28, 1997, pp. 917-920.
- [36] Kuo, S. Y., Lu, S. K. and Yeh, F. M., Determining terminal-pair reliability based on edge expansion diagrams using OBDD, *IEEE Transactions on Reliability*, vol. 48.3 (1999), pp. 234-246.
- [37] Kuo, Way, and Ming J. Zuo. *Optimal reliability modeling: principles and applications*. John Wiley, Sons, 2003.
- [38] Kumar, A., Khosla, A., Saini, J.S. and Singh, S., Meta-heuristic range based node localization algorithm for wireless sensor networks, in *localization and GNSS (ICLGNSS)*, 2012 International Conference on IEEE, (2012) , pp. 1-7.

- [39] Lakey, P. B. and Neufelder, A. M. , System and Software Reliability Assurance Notebook, Rome Laboratory, Rome, (1996), pp. 6.1- 6.24.
- [40] Lazzaroni, M., Cristaldi, L., Peretto, L., Rinaldi, P. and Catelani, M., Reliability Engineering Basic Concepts and Applications in ICT, Springer Verlag Berlin Heidelberg, Germany, (2011).
- [41] Leemis, L.M, Reliability: Probabilistic Models and Statistical Methods, Prentice-Hall, Englewood Cliffs, NJ, (1995).
- [42] Leitch, R.D., Reliability Analysis for Engineers: An Introduction, Oxford University Press, New York, (1995).
- [43] Madloul, G. A. and Hassan, Z. A. H., Some reliability optimization techniques for networks, Dissertation, University of Babylon, (2021).
- [44] Meeker, W. Q. and L. A. Escobar, Statistical Methods for Reliability Data, John Wiley, Sons, New York, (1998).
- [45] Mettas, A., Reliability allocation and optimization for complex systems, Proceedings Annual Reliability and Maintainability Symposium, Los Angeles, CA, January (2000), pp. 216-221.
- [46] Mutar, E. K. and Hassan, Z. A. H., On the Geometry of the Reliability Polynomials, Thesis, University of Babylon, (2017).
- [47] Mudhar, G. A. and Hassan, Z. A. H., Using of Some Techniques to Improve Reliability Networks, Thesis, University of Babylon, (2021).
- [48] Nedjah, N., dos Santos, L., Coelho, and de- MacedoMourelle, L., Multiobjective swarm intelligent systems: theory experiences. Springer Science Business Media, (2009), vol. 261.

- [49] Page, L. B. and Perry, J. E., Reliability of directed networks using the factoring theorem, *IEEE Transactions on Reliability*, vol. 38.5 (1989), pp. 556-562.
- [50] Rebaiaia, M.L. and AitKadi, D., A new technique for generating minimal cut sets in nontrivial network, *AASRI Procedia*, vol. 5 (2013), pp. 67-76.
- [51] Saleh, A. H. and Hassan, Z. A. H., Study of Some Problems in Reliability Models, Thesis, University of Babylon, (2022).
- [52] Sulaiman, H. K. and Hassan, Z. A. H., A Study of Mathematical Models in Reliability of Networks, Thesis, University of Babylon, (2019).
- [53] Sulaiman, Hatem Kareem, Faez Hassan Ali, and Zahir Abdul Haddi Hassan. Computational models for allocation and optimization of reliability for ROSS network. *AIP Conference Proceedings*. Vol. 2591.No.1. AIP Publishing,
- [54] Sandler, G. H., System Reliability Engineering, Prentice-Hall Int. Series In Space Technology, Prentice Hall Inc., Englewood Cliffs N.J., (1963).
- [55] Shaghghi, Saba, et al. "Comparative analysis of GMDH neural network based on genetic algorithm and particle swarm optimization in stable channel design." *Applied Mathematics and Computation* 313 (2017): pp. 271-286.
- [56] Singh, S., Shivangna, S., Mittal, E., Range Based Wireless Sensor Node Localization Using PSO and BBO and Its Variants. 2013 International Conference on Communication Systems and Network Technologies,(2013), pp. 309-315.
- [57] Srinath, L. S., Concepts in Reliability Engineering, East-West Press Private Ltd., (1985).
- [58] Thoft-Cristensen, P. and Baker, M. J., Structural reliability theory and its applications, Springer Science and Business Media, (2012).

- [59] Todinov, M. T., Risk-based reliability analysis and generic principles for risk reduction, Elsevier, (2006).
- [60] Venter, G. and Sobieski, J., Particle Swarm Optimization, AIAA 2002-1235, 43rd IAAA /ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Denver, CO., April (2002).
- [61] Xu, L., Chen, Y., Briand, F., Zhou, F. and Givanni, M., Reliability Measurement for Multistate Manufacturing Systems with Failure Interaction, Procedia CIRP, vol. 63 (2017), pp. 242-247.
- [62] Yamuna, M. and Karthika, K., Medicine Names as a DNA Sequence using Graph Domination, Scholars Research Library Journals, USA, 5(6), (2014), pp. 2747-2756.

الملخص

تهدف هذه الرسالة إلى استخدام بعض التطبيقات لتحسين موثوقية الأنظمة التي سنعتمدها في هذه الرسالة، أنظمة متسلسلة ومتوازية ومعقدة. سيكون النظام المعقد هو محور الاهتمام في هذه الرسالة. إيجاد طريقة بسيطة ومباشرة ورياضية بالإضافة إلى طريقتين أخرى لإنشاء مجموعات الحد الأدنى لمسارات النظام المعقد ، وطريقتان لإنشاء مجموعات الحد الأدنى لقطع النظام المعقد يعتمد على علم الرسم البياني. ثم، تم استخدام طريقتين لحساب دالة الموثوقية للنظام المعقد. تطبيق تقنيتين (التكرار والتخصيص) لزيادة قيمة الموثوقية للأنظمة الثلاثة (سلسلة ، متوازية ومعقدة). سوف تكون تقنية التكرار بطريقتين ، وهما التكرار في العناصر والتكرار في النظام ، يتم تطبيق الطريقتين على الأنظمة الثلاثة. وتطبيق طريقة تكرار العنصر لحدى مجموعات الحد الأدنى لمسار و قطع النظام المعقد. ثم تطبيق تقنية التخصيص لزيادة قيمة الموثوقية للأنظمة الثلاثة. دراسة أهمية الموثوقية في الأنظمة البسيطة والمعقدة في الثلاثة ، حالات متعددة ومختلفة . سيتم تقييم أهمية الموثوقية لكل مكون في الأنظمة وسيتم تقييم أهمية كل مكون في الحد الأدنى من المسارات والحد الأدنى من القطع للأنظمة الثلاثة. تقييم أهمية الموثوقية لكل حد الأدنى لمسارات والقطع للأنظمة الثلاثة. ثم دراسة مقياس جديد لتقييم أهمية موثوقية الحد الأدنى من القطع لأكثر من مكون.



وزارة التعليم العالي والبحث العلمي

جامعة بابل

كلية التربية للعلوم الصرفة

قسم الرياضيات

حول استخدام النماذج الرياضية في موثوقية الانظمة

رسالة مقدمة الى

مجلس كلية التربية للعلوم الصرفة – قسم الرياضيات

وهي جزء من متطلبات نيل درجة الماجستير

في التربية/الرياضيات

من قبل

نور صباح حسن نوفل

بإشراف

أ.د. زاهر عبدالهادي حسن