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Some Methods for Solving Transportation Problems

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Dedication

To my dear country, Iraq
to my family
to my teacher
To all my loyal friends.

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I would like to express my thanks to Allah, the Most Gracious, and the Most Merciful and after that; I thank my “supervisors” Prof**. Dr. Mushtak A.K. Shiker Al-Jenabi** for his continuous support in making this work possible. I am also pleased to express my thanks and gratitude to all of my professors in the Department of Mathematics, College of Education for Pure Sciences, University of Babylon.

Abstract

In this research, different and varied techniques were presented to find the initial solution and the optimal solution to transportation problems . The focus was on finding an initial basic feasible solution and conducting optimality tests to improve the solution .Three methods were employed in order to obtain an initial basic feasible solution. These methods were: the North West Corner Method (NWCR), Vogle's Approximation Method (VAM), and the Least Cost Method (LCM). Each of these methods provided a systematic approach to allocate quantities from sources to destinations, considering cost constraints and availability .Once an initial solution was obtained, we proceeded with the optimality test phase. The two methods used for this purpose were the Modified Distribution Method and the Stepping Stone Method. These techniques aimed to refine the initial solution by iteratively evaluating alternative routes and reallocating quantities based on the associated costs .The results of these methods were efficient in initial solution and achieving optimal transportation plans. The optimality tests recognized the most efficient routes and allocate quantities optimally, including both cost minimization and resource allocation. Finally, this research highlighted the importance of employing appropriate methods for solving transportation problems. The combination of initial solution methods and optimality tests provided a comprehensive framework for addressing complex transportation scenarios. By utilizing these techniques, decision-makers can optimize their resource allocation, minimize costs, and enhance the overall efficiency of their transportation systems.

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North West Corner Method	NWCM
Vogle’s Approximation Method	VAM
Least Cost Method	LCM
Basic feasible solution	BFS
Transportation Problems	TP
Linear Programming Problem	LPP
Supply	S
Demand	D

Chapter 1

Introduction and Important Concept

1.1 Introduction

The transportation problem is a classical optimization problem that has a long history in operations research. The problem involves allocating a set of goods from a set of suppliers to a set of demanders subject to capacity constraints. The objective is to minimize the total cost of transportation. The problem was first formulated in the late 1940s and has since then been the focus of intensive research. In this research, we will explore the history of solving transportation problems. Transportation is a fundamental aspect of modern society, enabling people and goods to move from one place to another. However, transportation also poses several challenges, including traffic congestion, energy consumption, and environmental impact, [1,2]. Over the years, optimization techniques have been applied to transportation systems to improve efficiency, reduce costs, and minimize negative effects on the environment. The earliest optimization techniques for transportation problems can be traced back to the mid-20th century, when linear programming was first introduced. Linear programming is a mathematical modeling technique that involves representing a problem as a set of linear equations, with a linear objective function to be optimized subject to linear constraints. In transportation, linear programming was used to optimize the allocation of resources such as time, fuel, and energy, while minimizing costs and maximizing profits, [3]. One of the earliest examples of transportation optimization using linear programming was the development of the transportation algorithm by George Dantzig in 1951. The transportation algorithm is a method for solving linear programming problems that involve transportation costs, such as the transportation of goods between warehouses and retail locations. The transportation algorithm involves a two-phase process, where the first phase involves finding an initial feasible solution and the second phase involves optimizing the solution using the simplex algorithm. In the 1960 and 1970, optimization techniques for transportation problems continued to evolve, with the introduction of network flow models and integer programming, [4]. Network flow models involve representing the transportation system as a network of nodes and arcs, with flows of goods or people moving along the arcs. Integer programming involves modeling problems that require integer solutions, such as the allocation of resources to discrete units. One of developments of transportation optimization was in 1980s, with the introduction of heuristic algorithms. Heuristic algorithms depend on simple rules of thumb or probabilistic methods to quickly identify good solutions, even if they may not be the best possible solution. Heuristic algorithms were particularly useful for transportation problems that were too complex for linear programming or that had multiple objectives that needed to be optimized. One of the most popular heuristic algorithms for transportation optimization is the genetic algorithm. In recent years, machine learning techniques have been applied to transportation optimization, particularly when there is a large amount of data available,[5]. Machine learning algorithms can be trained on historical transportation data to identify patterns and predict future demand or optimal routes. Transportation problems have been a significant challenge in optimization for decades. The history of transportation optimization can be traced back to the mid-20th century, with the introduction of linear programming. Over the years, optimization techniques have continued to evolve, with the introduction of heuristic algorithms and machine learning. As transportation continues to play a vital role in modern society, optimization techniques will continue to be essential for improving efficiency, reducing costs, and minimizing negative impacts on the environment. In addition to the optimization techniques mentioned above, there have been several other notable developments in the history of transportation optimization. One of these developments is the introduction of nonlinear programming. Nonlinear programming involves modeling problems that require nonlinear equations to be optimized. Nonlinear programming has been used in transportation optimization to model problems such as traffic flow, where the relationship between traffic volume and speed is nonlinear. Simulation modeling is a significant development in transportation optimization. Simulation modeling involves creating a computer model of a transportation system and then running simulations to evaluate the performance of the system under different scenarios. Simulation modeling has been used in transportation optimization to identify bottlenecks, test new policies or interventions, and evaluate the impact of changes to the transportation system, [6]. Additionally, there has been a growing focus on sustainability in transportation optimization. As concerns over climate change and environmental impact have become more prominent, optimization techniques have been used to reduce energy consumption, minimize greenhouse gas emissions, and promote the use of alternative transportation modes such as walking, cycling, and public transportation. Finally, transportation optimization has also been applied to other areas beyond traditional transportation systems. For example, optimization techniques have been used in logistics to optimize the routing and scheduling of delivery trucks or in emergency response systems to optimize the deployment of resources during disasters or crises. In conclusion, the history of transportation optimization has been marked by a steady evolution of techniques and approaches, from linear programming to machine learning. As transportation continues to be a vital aspect of modern society, optimization techniques will continue to play an essential role in improving efficiency, reducing costs, and minimizing negative impacts on the environment. With the rapid advancements in technology, it is likely that new and innovative approaches to transportation optimization will continue to emerge in the future. Transportation optimization solved the problem through finding the best way to fulfill the demand of n demand points using the capacities of m supply points. While trying to find the best way, generally a variable cost of shipping the product from one supply point to a demand point or a similar constraint should be taken into consideration, [7]. To sum up, a transportation problem basically deals with the problem, which aims to find the best way to fulfill the demand of n demand points using the capacities of m supply points. While trying to find the best way, generally a variable cost of shipping the product from one supply point to a demand point or a similar constraint should be taken into consideration.

1.2 The Concept of Transportation

A transportation problem is a linear programming problem that deals with identifying an optimal solution for transportation and allocating resources to various destinations and from one site to another while keeping the expenditure to a minimum, [8]. In simple words, the main objective of the transportation problem is to deliver (from the source to the destination) the resources at the minimum cost. It involves transporting a single product from 'm' source (origin) to 'n' destinations.

Assumptions: The supply level of each source and the demand at each destination are known.

Objective: To minimize the total transportation cost.

Variables: Quantity of goods shipped from each supply point to each demand point.

Restrictions : Non negative shipments, supply availability at each supply location and demand need at each demand location.

1.3 Basic Definitions

In this section, some important definitions of transportation problem are introduced [1, 11, 17].

Definition 1.3.1 (Feasible solution): A set of non-negative values x_{ij}, $i = 1, 2 \ldots m$; $j = 1, 2 \ldots n$ that satisfies the constraints (rim conditions) is called a feasible solution.

Definition 1.3.2 (Basic feasible solution (BFS)): A feasible solution to a ($m \times n$) transportation problem that contains no more than $m + n - 1$ non-negative independent allocations is called a basic feasible solution (BFS) to the transportation problem.

Definition 1.3.3 (Non-degenerate basic feasible solution): A basic feasible solution to a (*m x n*) transportation problem is said to be a non-degenerate basic feasible solution if it contains exactly $m + n - 1$ non-negative allocations in independent positions.

Definition 1.3.4: A basic feasible solution that contains less than $m + n - 1$ non-negative allocations is said to be a degenerate basic feasible solution.

Definition 1.3.5: A feasible solution (not necessarily basic) is said to be an optimal solution if it minimizes the total transportation cost.

1.4 Formulation of Transportation Problem

The transportation problems deal with the transportation of product manufactured at different plants or factories (supply origins) to a number of different warehouses (demand destination). The objective is to satisfy the destination requirements within the plant's capacity constraints at the minimum transportation cost. Transportation problems thus typically arise in situations involving physical movement of good from plants to warehouses, warehouses to warehouses, wholesalers to retailers and retailers to customers. Solution of the transportation problems requires the determination of how many units should be transported from each supply origin to each demand destination in order to satisfy all the destination demands while minimizing the total associated cost of transportation. The easiest way to recognize a transportation problem is to consider a typical situation as shown in the following figure. Assume that a manufacturer has *n* factories S1, S2 to Sn producing the same product. From these factories, the product is transported to three warehouses D_1, D_2, to D_n. Each factory has a limited supply and each warehouse has specific demand. Each factory can transport to each warehouse but the transportation costs vary for different combinations. The problem is to determine the quantity each factory should transport to each warehouse in order to minimize total transportation costs, [9,10].

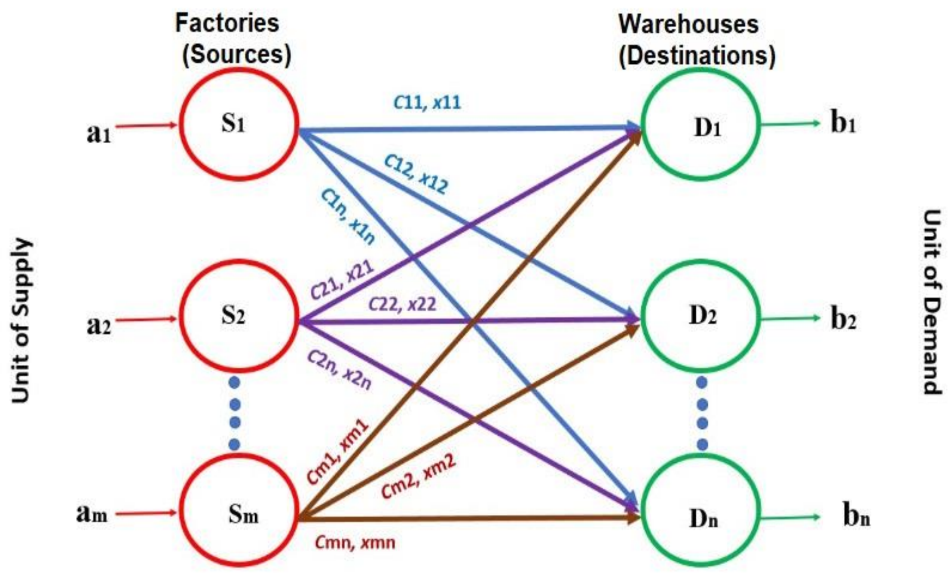


Figure 1: Transportation Problem

It can be seen that a typical transportation problem requires three sets of numbers:

1. **Capacities (or supplies)**: Indicate the most each plant can supply in a given time period.

2. **Demands (or requirements):** They are typically estimated from some type of forecasting model. Often demands are based on historical customer demand data.

3. **Unit shipping (and possibly production) cost**: It is calculated through a transportation cost analysis.

So, the Total Cost of Transposition is:

$$(c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + \ldots\ldots + c_{1n}x_{1n}) + (c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + \ldots\ldots + c_{2n}x_{2n}) + \ldots\ldots + (c_{m1}x_{m1} + c_{m2}x_{m2} + c_{m3}x_{m3} + \ldots\ldots + c_{mn}x_{mn})$$

As we already mentioned, our objective is to minimize the Total Cost:

Min Z $= (c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + \ldots\ldots + c_{1n}x_{1n}) + (c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + \ldots\ldots + c_{2n}x_{2n}) + \ldots\ldots + (c_{m1}x_{m1} + c_{m2}x_{m2} + c_{m3}x_{m3} + \ldots\ldots + c_{mn}x_{mn})$

Subject to: $x_{i1} + x_{i2} + \ldots\ldots + x_{in} = a_i$ & $x_{1j} + x_{2j} + \ldots\ldots + x_{mn} = b_j$

$$x_{ij} >= 0, i = 1, 2, 3, \ldots\ldots, m, j = 1, 2, 3, \ldots\ldots, n.$$

The transportation or shipping problem involves determining the amount of goods or items to be transported from a number of sources to a number of destinations.

Usually, the objective is to minimize total shipping costs or distances. Transportation problem is a specific case of Linear Programming problems. The below-given matrix can also represent the above diagram. Therefore, the standard form of transportation tableau it will be as follows:

Destination

Supply

	i / j	(1)	(2)	…	(n)	a_i
	(1)	x_{11} c_{11}	x_{12} c_{12}	… …	x_{1n} c_{1n}	a_1
Source	(2)	x_{21} c_{21}	x_{22} c_{22}	… …	x_{2n} c_{2n}	a_2
	…	… …	… …	… …	… …	…
	(m)	x_{m1} c_{m1}	x_{m2} c_{m2}	… …	x_{mn} c_{mn}	a_m
Demand	b_j	b_1	b_2		b_n	$\Sigma a_i = \Sigma b_j$

The *mn* squares are called cells. The various as and b's are called the constraints (rim conditions). In short, the problem can be reformulated as follows:

The objective function

$$minimize\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to

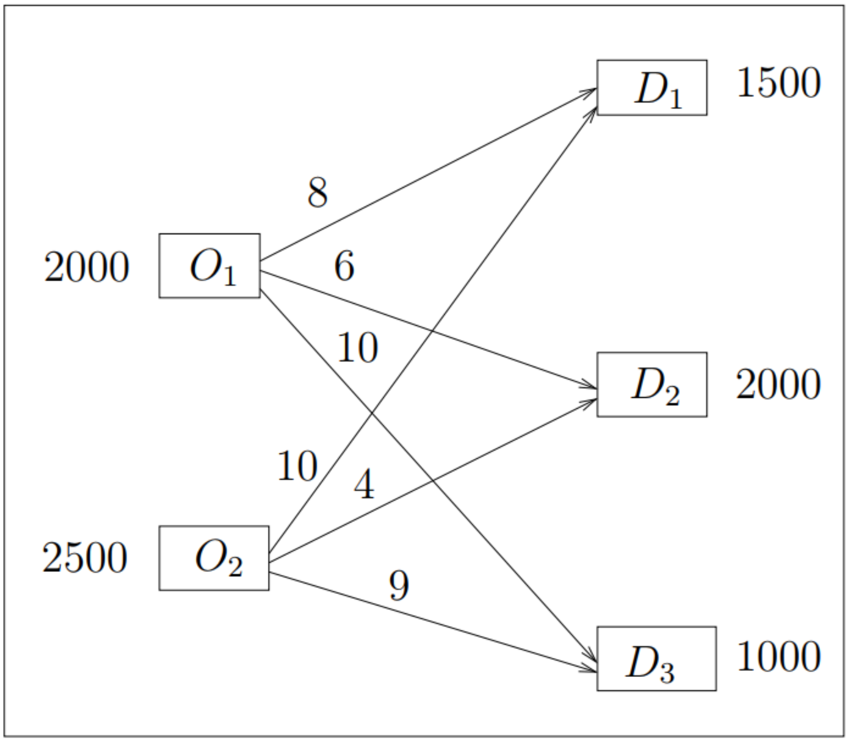
$$\sum_{j=1}^{n} x_{ij} = a_i\ for\ i = 1,2, \dots m.$$

$$\sum_{i=1}^{m} x_{ij} = b_j\ for\ j = 1,2, \dots .n.$$

$$x_{ij} \geq 0 \text{ for all } i \text{ to } j.$$

Example 1.1:

Two pastry factories, O1 and O2, make the town's daily bread. The Bread is delivered to the city's three bakeries: D1, D2 and D3. Bread factory supplies, bakery requirements and transportation costs per unit are shown in the following graph:



We define the following decision variables:

xij**:** the number of loaves of bread to be distributed from the bread factory O_i to the bakery D_j, i = 1, 2, j = 1, 2, 3. The corresponding linear model is the following:

$$\min z = 8x_{11} + 6x_{12} + 10x_{13} + 10x_{21} + 4x_{22} + 9x_{23}$$

Subject to
$$x_{11} + x_{12} + x_{13} = 2000$$
$$x_{21} + x_{22} + x_{23} = 2500$$
$$x_{11} + x_{21} = 1500$$
$$x_{12} + x_{22} = 2000$$
$$x_{13} + x_{23} = 1000$$
$$x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geq 0$$

We can write the constraints in equation form, because the total supply is equal to the total demand. In order to analyze the structure of matrix A, we write the model in matrix form.

$$\min z = (8,6,10,10,4,9)\begin{pmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{21} \\ x_{22} \\ x_{23} \end{pmatrix}$$

$$\text{subject to } \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{21} \\ x_{22} \\ x_{23} \end{pmatrix} = \begin{pmatrix} 2000 \\ 2500 \\ 1500 \\ 2000 \\ 1000 \end{pmatrix}$$

In this example, we have two origins, $m = 2$, and three destinations, $n = 3$. Matrix A has $2 + 3$ rows and 2×3 columns. It can be verified that the rank of the matrix is 4. Note that each column of the matrix has only two 1 's and 0 's elsewhere. We use two subscripts to denote each one of the columns of matrix A, as we did when we defined the decision variables for the transportation problem. Thus, in this example we denote by $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$ the six column vectors of the matrix. If we observe the positions at which the 1's can be found, we can easily see that, vector a_{11} for instance has a 1 in the 1st row and the other in the $m + 1$ th row, vector a_{21} has the 1 's in the 2 nd and $m + 1$ th rows and vector a_{23} has the 1 's in the 2 nd and $m + 3$ th rows. In general, we can state that the two 1 's of vector a_{ij} of matrix A are in rows i and $m + j$. Therefore, the structure of matrix A depends on the number of origins and the number of destinations of the problem; any transportation problem having m origins and n destinations has the same matrix A. This matrix has $m + n$ rows and $m \times n$ columns. The rank of matrix A is $m + n - 1$, that is, any basis consists of $m + n - 1$ vectors. There are only two 1 's in each column vector a_{ij} of matrix A, and they are located in rows i and $m + j$, being 0 the rest of the values of the column vector. Thus, a transportation problem is specified by the number of origins, the number of destinations, the supplies, the demands and the per-unit transportation costs. All this information can be abbreviated in the form of a rectangular array.

Theorem 1.1: The necessary and sufficient condition for a transportation problem to have a solution is that the total demand equals the total supply, [10].

Proof. According to the standard form of the transportation problem, each of the supplies a_i satisfies one of the following constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i, \ i = 1, \dots, m$$

If we sum all the supplies, we get the total supply, which is:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} a_i \ \dots\dots\dots\dots\dots\dots (1)$$

In a similar way, demands b_j satisfy the following constraints:

$$\sum_{i=1}^{m} x_{ij} = b_j, \ j = 1, \dots, n$$

The total demand is the following:

$$\sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} b_j \ \dots\dots\dots\dots\dots\dots (2)$$

The left-hand sides of equations (1) and (2) are equal. It follows that the equations mentioned are satisfied if and only if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

1.5 Types of Transportation Problems

Transportation problems are broadly classified into balanced and unbalanced, depending on the source's supply and the requirement at the destination. A transportation problem said to be balanced if the supply from all sources equals the total demand in all destinations,[11].

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Otherwise, it is called unbalanced. Therefore, transportation problem is said to be balanced if the total supply from all sources equals the total demand in all destinations, otherwise it is called unbalanced. In such cases, we add a dummy source giving dummy supply with each cost as zero (0) but dummy supply for the dummy destination as total demand-total supply as shown in the table below.

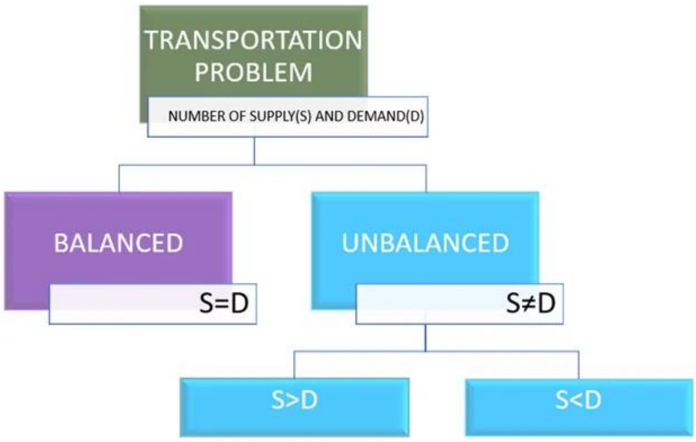


Figure (2): Classification of transportation problem

Table (1): An example balanced transportation problem with Source = {A, B, C} with total supply=75 and Destinations= {1,2,3} with total demand=75

		DESTINATIONS			SUPPLY
		1	2	3	
SOURCE	A	2	3	1	20
	B	5	4	8	15
	C	5	6	8	40
DEMAND		20	30	25	75 / 75

Table (2):An example unbalanced transportation problem with Source ={A,B,C} with total supply=65 and Destinations={1,2,3} with total demand=60

		DESTINATIONS			SUPPLY
		1	2	3	
SOURCE	A	2	3	1	20
	B	5	4	8	15
	C	5	6	8	30
DEMAND		20	15	25	60 / 65

As previously mentioned in such cases and for balance we add a dummy destination which gives a dummy demand with each cost as zero (0) but the dummy destination dummy demand as total supply.

Table (3): Achieving the balance by adding a dummy destination that gives a dummy demand with each cost as zero (0) but the dummy demand of the dummy destination as the total supply

		DESTINATIONS					SUPPLY
		1	2	3	DUMMY DESTINATION		
SOURCE	A	2	3	1	0		20
	B	5	4	8	0		15
	C	5	6	8	0		30
DEMAND		20	15	25	65-60= 5	65	65

Chapter 2

Methods for finding the initial solution and the optimal solution for transportation problems

Transportation problems arise in a wide range of industries, from manufacturing and logistics to healthcare and agriculture. These problems involve allocating resources, such as goods, people, or equipment, from their sources to their destinations, while minimizing transportation costs or maximizing profits. To solve these problems, various methods have been developed over the years. In this chapter, the most important stages and methods for solving the transportation problem will be mentioned [12,13]. Actually, there are three stages of solving the transportation problem:

Stage 1: Formulation the transportation model in LPP

Stage 2: Obtains the initial basic feasible solution.

Stage 3: Obtain the optimal solution.

2.1 **Formulation of the Transportation Model in LPP**

For transportation problem usually, the problem will be given in a tabular form or matrix form called transportation table or cost-effective matrix, [14]**.** Let check an example below:

Table (4): An example transportation problem with Source = {A, B, C} with total supply=70 and Destinations= {1,2,3} with total demand=70

		DESTINATIONS			SUPPLY
		1	2	3	
SOURCE	A	2	3	1	10
	B	5	4	8	35
	C	5	6	8	25
DEMAND		20	25	25	

From the above table, the cost incurred in moving 1 unit of the commodity from source A to destination 1= 2, as shown in the table below:

Table (5): The cost incurred in moving 1 unit of the commodity from source A to destination 1= 2 Likewise, the cost incurred in moving one unit of the commodity from source B to destination 2= 4 and so on.

		DESTINATIONS			SUPPLY
		1	2	3	
SOURCE	A	2	3	1	10
	B	5	4	8	35
	C	5	6	8	25
DEMAND		20	25	25	

2.2 Initial Basic Feasible Solution

There are different methods available to obtain the initial basic feasible solution of transportation problems, [15,16,17]:

1- North West Corner Method (NWCM).

2- Vogle's Approximation Method (VAM).

3- Least Cost Method (LCM).

The work of each method will be detailed as follows:

2.2.1 The North west Corner Method

The north west corner method is a simple method used to solve transportation problems, especially when the cost of transportation is the only factor considered. This method starts by allocating the maximum possible amount of the resource from the northwest corner of the table and proceeds to allocate resources column-wise and row-wise until all resources are allocated. This method is useful in industries where transportation costs are a significant factor, such as the transportation of raw materials to a manufacturing facility, [18]. Here's how the Northwest Corner Method works:

Step 1: Identify the north west corner cell of the transportation table and allocate as much as possible from it.

Step 2: Cross out the row or column that has been fully allocated.

Step 3: Identify the next north west corner cell in the remaining unallocated rows or columns and allocate as much as possible from it.

Step 4: Repeat Step 2 and Step 3 until all resources have been allocated.

Step 5: Calculate the total transportation cost by multiplying the amount of resources allocated in each cell by the respective transportation cost and summing up all the results. While the North west Corner Method is simple and easy to use, it may not always result in the optimal solution, especially when the transportation costs are not the only consideration.

2.2.2 The Vogel's Approximation Method

The vogel's approximation method is another basic method used to solve transportation problems, but it takes into consideration both the cost of transportation and the availability of resources. This method involves finding the difference between the two smallest costs for each row and column and selecting the one with the largest difference. This method is useful in industries where the availability of resources is limited, [19]. Here's how the Vogel's Approximation Method works:

Step 1: Find the difference between the two smallest costs in each row and column.

Step 2: Identify the row or column with the largest difference, which is called the "penalty."

Step 3: Allocate as much as possible from the cell with the lowest cost in the penalty row or column.

Step 4: Cross out the row or column that has been fully allocated.

Step 5: Repeat Steps 1 to 4 until all resources have been allocated.

Step 6: Calculate the total transportation cost by multiplying the amount of resources allocated in each cell by the respective transportation cost and summing up all the results.

The Vogel's Approximation Method can provide a better solution than the Northwest Corner Method, but it can be time-consuming to calculate the differences for each row and column.

2.2.3 The Least Cost Method

The least cost method is a technique used in linear programming to find the initial basic feasible solution to a transportation problem. This method is used to allocate scarce resources (such as goods or materials) from sources (such as factories) to destinations (such as warehouses) at the lowest possible cost. In the least cost method, we start by identifying the cell with the lowest transportation cost in the matrix of source-destination costs. This cell represents the initial allocation of the resource. The amount allocated is equal to the smaller of the available supply or demand for the corresponding row or column. Next, we subtract the allocated amount from the supply or demand for the corresponding row or column, and we eliminate any row or column that has been completely satisfied (i.e., its supply or demand has been reduced to zero). We then repeat the process by identifying the cell with the next lowest cost, and we continue until all supply and demand requirements are met. If at any point in the process, there are multiple cells with the same minimum cost, we can choose any of them to allocate the resource. The Least Cost Method is relatively easy to apply and is often used as a starting point for finding the optimal solution to a transportation problem. However, it may not always provide the best solution, and other methods such as the vogel's approximation method or the northwest corner method may need to be used to improve the results, [20]. Now, the steps involved in the least cost method in more detail:

Step 1: Set up the transportation cost matrix with the costs of transporting from each source to each destination.

Step 2: Identify the cell with the lowest cost in the matrix. If there are multiple cells with the same minimum cost, choose any one of them.

Step 3: Allocate the maximum possible amount to this cell. This will be the smaller of the available supply or demand for the corresponding row or column.

Step 4: Update the supply and demand values for the corresponding row and column by subtracting the allocated amount.

Step 5: If the supply or demand for any row or column becomes zero, eliminate it from the matrix.

Step 6: Repeat steps 2-5 until all supply and demand requirements are met.

Step 7: Calculate the total cost of the allocation by multiplying the quantity allocated in each cell with its respective transportation cost.

Step 8: Verify the solution by checking that the total supply and total demand match, and that all constraints are satisfied.

2.3 Optimality Test

The optimal basic solution is verified by the following methods:

2.3.1 The Modified Distribution Method

The modified distribution method solved the transportation problems when there are restrictions on the number of resources that can be transported. This method involves finding an initial feasible solution and then testing it against the restrictions to ensure that the solution meets the constraints. This method is useful in industries where there are limitations on the quantity of resources that can be transported, [21]. Here's how the modified distribution method works:

Step 1: Find an initial feasible solution using any of the basic methods, such as the Northwest corner method, the least cost method, the Vogel's Approximation Method.

Step 2: Test the initial feasible solution against the constraints to ensure that the solution meets the limitations on the number of resources that can be transported.

Step 3: If the initial feasible solution meets the constraints, then it is the optimal solution. If not, then find the cell that violates the constraint the most and adjust the allocation in that cell.

Step 4: Repeat Step 2 and Step 3 until the solution meets the constraints.

Step 5: Calculate the total transportation cost by multiplying the amount of resources allocated in each cell by the respective transportation cost and summing up all the results.

The modified distribution method can provide a feasible solution that meets the constraints, but it can be time-consuming to test and adjust the solution.

2.3.2 The Stepping Stone Method

The stepping stone method is a method used to improve an existing optimal solution. This method involves testing different routes and identifying the one that minimizes the cost of transportation. This method is useful in industries where the cost of transportation varies depending on the route taken, [22,23]. Here is how the Stepping Stone Method works:

Step 1: Start with an initial feasible solution.

Step 2: Test alternative routes by moving resources from one cell to another and recalculating the transportation cost.

Step 3: Identify the route that results in the smallest decrease in transportation cost.

Step 4: Update the allocation by moving resources along the identified route.

Step 5: Repeat Steps 2 to 4 until there are no more routes that result in a decrease in transportation cost.

Step 6: Calculate the total transportation cost by multiplying the amount of resources allocated in each cell by the respective transportation cost and summing up all the results.

Chapter 3

Applications and Discussion the Results

In this chapter, applications for these methods will be presented and how to deal with transport problems according to the techniques and procedure for those methods. The next transportation problem will be addressed and all the methods that were dealt with in the previous chapter will be applied in finding the solution until a complete conception of each method is taken in a clear and simplified manner.

Example 3.1

Three factories for children's clothing - A, B, and C with production capacity 70, 30, and 50 units per week respectively. These units are to be shipped to four warehouses W1, W2, and W3 with requirement of 65, 42, and 43 units per week respectively. The transportation costs (in dollar) per unit between factories and warehouses are given below.

Factory	**Warehouse**			**Supply**
	W_1	**W_2**	**W_3**	
A	5	7	8	70
B	4	4	6	30
C	6	7	7	50
Demand	65	42	43	

3.1 Application of North West Corner Method (NWCM)

We seek to Find an initial basic feasible solution of the given transportation problem using northwest corner rule

Solution:

The first step is to make it a standard transportation problem. For that check whether it is a balanced or unbalanced transportation problem.

Factory	Warehouse			Supply	
	W_1	**W_2**	**W_3**		
A	5	7	8	70	
B	4	4	6	30	
C	6	7	7	50	
Demand	65	42	43	65+42+43 **=150**	70+30+50 **=150**

The given problem is a balanced transportation problem. So, we can start. Select the northwest corner cell. That is, the cost of the intersection of the first row and the first column, whose value here is equal to 5, and we compare between demand and supply, i.e., between (65,70). Then allocate the cell with the lowest value which is here, 65 (in yellow) and subtract the excluded cell with the lowest value i.e., 70–65 = 5

Factory	Warehouse				Supply	
	W_1		**W_2**	**W_3**		
A	5	65	7	8	70-65= 5	
B	4		4	6	30	
C	6		7	7	50	
Demand	65		42	43	**150**	**150**

Eliminate the column or row accordingly by striking it off. [Here, the column with destination 1 (marked with red line). Now we continue the process with the remaining cells. Again, find the northwest (northwest) cell and do the same steps as above.

Factory	Warehouse		Supply	
	W_2	**W_3**		
A	7 5	8	70-65= 5	
B	4	6	30	
C	7	7	50	
Demand	42-5=37	43	**150**	**150**

Factory	Warehouse		Supply	
	W_2	**W_3**		
B	4 30	6	30	
C	6	7	50	
Demand	37-30=7	43	**150**	**150**

Factory	Warehouse		Supply	
	W_2	**W_3**		
C	7 7	7	50-7=43	
Demand	7	43	**150**	**150**

Factory	Warehouse	Supply	
	W_3		
C	7 43	43	
Demand	43	**150**	**150**

As shown in last table, both the demand and supply will be the same which will be further allocated in the remaining single cell and which is here 43. Therefore, this is a way to check whether all the above steps were correct or not. Thus, we give the initial feasible solution by the N-W Corner method. The final table with all allocated cell as follows:

Factory	Warehouse			Supply
	W_1	**W_2**	**W_3**	
A	5 / **65**	7 / **5**	8	70
B	4	4 / **30**	6	30
C	6	7 / **7**	7 / **43**	50
Demand	65	42	43	65+42+43 **=150** / 70+30+50 **=150**

To calculate the cost associated with these customizations, we follow the following procedure:

Total cost= (65*5) +(5*7) +(30*4) +(7*7) +(43*7) = 301+49+120+35+325

=830$

It was found that 830$represents the total cost involved in moving the commodities. The path followed is represented by the red arrows as we found by the N-W Corner method.

Factory	Warehouse			Supply
	W_1	**W_2**	**W_3**	
A	5 / **65**	7 / **5**	8	70
B	4	4 / **30**	6	30
C	6	7 / **7**	7 / **43**	50
Demand	65	42	43	65+42+43 **=150** / 70+30+50 **=150**

These paths can be summarized by a simplified table as follows"

From	To	Quantity (units)
A	W_1	65
A	W2	5
B	W2	30
C	W2	7
C	W_3	43
Demand	**Warehouse**	Total=150

3.2 **Application of least cost method**

The least cost method is a technique used in linear programming to find the optimal solution for a transportation problem by selecting the lowest cost route. We will apply the same data of previous example to solve a transportation problem.

Factory	**Warehouse**			**Supply**	
	W_1	**W_2**	**W_3**		
A	5	7	8	70	
B	4	4	6	30	
C	6	7	7	50	
Demand	65	42	43	150	150

The first step is choose the lowest value or the smallest cost, from the list of costs in white which is [4 (written in blue)]. There are two cells with the lowest cost here. It is entirely up to the user to determine which one to utilize.

Factory	**Warehouse**			**Supply**
	W_1	**W_2**	**W_3**	
A	5	7	8	70
B	4	4	6	30
C	6	7	7	50

Demand	65	42	43	150	150

Compare the cell's demand and supply. [Here, 30 and 65 are shown in red], choose the cell with the lowest value. [Here's 30 (in yellow)]. Subtract the value from the omitted cell with the lowest value. Specifically, the assigned cell value. [Here, 65–30=35]. After that, strike through the column or row to remove it. [Here is the row with source B (denoted by a red line)]:

Factory	Warehouse			Supply	
	W_1	W_2	W_3		
A	5	7	8	70	
B	4 30	4	6	30	
C	6	7	7	50	
Demand	65-30=35	42	43	35+42+43+30 =150	70+50+30 =150

We continue the procedure with the remaining cells and find the lowest-cost cell and repeat the preceding procedures.

Factory	Warehouse			Supply	
	W_1	W_2	W_3		
A	5 35	7	8	70-35=35	
C	6	7	7	50	
Demand	35	42	43	35+42+43+30 =150	50+35+30+35 =150

Factory	Warehouse		Supply	
	W_2	W_3		
A	7 35	8	35	
C	7	7	50	
Demand	42-35=7	43	7+43+30+35+35 =150	50+35+30+35 =150

Factory	Warehouse		Supply	
	W_2	W_3		
C	7 7	7	50-7=43	
Demand	7	43	7+43+30+35+35 =150	43+7+35+30+35 =150

Factory	Warehouse	Supply	
	W_3		
C	7 43	43	
Demand	43	7+43+30+35+35 =150	43+7+35+30+35 =150

We note that both demand and supply will be the same in this case, which will be further assigned in the remaining single cell. [Here, 43] (This is another approach for determining whether all of the preceding procedures were followed correctly.). This is the final table with all of the allotted cells:

Factory	Warehouse			Supply
	W1	W2	W3	
A	5 **35**	7 **35**	8	70
B	4 **30**	4	6	30
C	6	7 **7**	7 **43**	50
Demand	65	42	43	**150** **150**

Now we will calculate the cost associated with these allocations.

Total cost = (35*5) +(30*4) +(35*7) +(7*7) +(43*7)

= 175+120+245+49+301

=890$

The red arrows reprint the path taken, as determined by the least-cost approach.

Factory	Warehouse			Supply
	W1	W2	W3	
A	5 **35**	7 **35**	8	70
B	4 **30**	4	6	30
C	6	7 **7**	7 **43**	50
Demand	65	42	43	**150** **150**

We will represent the overall results by the following table:

From	To	Quantity (units)
A	W_1	30
A	W1	35
B	W2	35
C	W2	7
C	W_3	43
Demand	**Warehouse**	Total=150

3.3 **Application of vogle's approximation method (VAM)**

As we mentioned in the previous chapter, vogel's approximation method is a technique used to solve transportation problems, which involve transporting goods from sources to destinations at minimum cost. It is an iterative procedure that involves finding the two least-cost allocations in each row and column of the transportation table and selecting the one with the highest opportunity cost for the next iteration. We will apply this method to solve the same transfer problem in the previous example. The example data is:

Factory	**Warehouse**			**Supply**	
	W_1	**W_2**	**W_3**		
A	5	7	8	70	
B	4	4	6	30	
C	6	7	7	50	
Demand	65	42	43	65+42+43 **=150**	70+30+50 **=150**

Deciding which cost cell to allocate is not so easy in VAM as if we have discussed in N-W Corner method and Least Cost Cell method. In VAM, we first have to identify the differences between the lowest costs in each row and column. These are known as penalties/additional costs, and the difference between the smaller of the two values is taken into account.

Factory	**Warehouse**			**Supply**		**Row Difference**
	W_1	**W_2**	**W_3**			
A	5	7	8	70		7-5=2
B	4	4	6	30		4-4=0
C	6	7	7	50		7-6=1
Demand	65	42	43	65+42+43 **=150**	70+30+50 **=150**	
Column Difference	5-4=1	7-4=3	7-6=1			

Therefore, the penalties= {2,0,1,1,3,1}. Now we find the maximum value among the penalties irrespective of row or column. As shown, max (Penalties)=3 which has been marked green:

Factory	**Warehouse**			**Supply**		**Row Difference**
	W_1	**W_2**	**W_3**			
A	5	7	8	70		7-5=2
B	4	4	6	30		4-4=0
C	6	7	7	50		7-6=1
Demand	65	42	43	65+42+43 **=150**	70+30+50 **=150**	
Column Difference	5-4=1	7-4=3	7-6=1			

Now, we look into the respective row or column accordingly. Here, the column given in green. Then we select the least value among all the costs (given in green), i.e., minimum cost. Here, 4 given in blue in the Table below:

Factory	**Warehouse**			**Supply**	**Row Difference**
	W_1	**W_2**	**W_3**		

A	5	7	8	70		7-5=2
B	4	4	6	**30**		4-4=0
C	6	7	7	50		7-6=1
Demand	65	**42**	43	65+42+43 **=150**	70+30+50 **=150**	
Column Difference	5-4=1	7-4=3	7-6=1			

Compare the demand and supply of that cell (30 and 42) given in red.

Factory	**Warehouse**			**Supply**		**Row Difference**
	W_1	**W_2**	**W_3**			
A	5	7	8	70		7-5=2
B	4	4 30	6	**30**		4-4=0
C	6	7	7	50		7-6=1
Demand	65	**42-30=12**	43	65+12+42+30 **=150**	70+30+50 **=150**	
Column Difference	5-4=1	7-4=3	7-6=1			

Then allocate the cell with the least value [Here, 30 (given in yellow). After that we subtract the excluded cell with the least value, the allocated cell vale here is 42–30=12. The next step is eliminating the column or row accordingly by striking it off, as shown the column with source B which marked with red line. It is worth noting that the total demand and supply will always remain the same, and this matter is considered a verification of whether we are moving in the right direction or not. Now we continue the process with the remaining cells. Again, we find the penalty and doing the same steps as above.

Factory	**Warehouse**			**Supply**	**Row Difference**
	W_1	**W_2**	**W_3**		

A	5 65	7	8	70-65=5		7-5=2
C	6	7	7	50		7-6=1
Demand	65	12	43	12+43+30+65 **=150**	5+50+30+65 **=150**	
Column Difference	6-5=1	7-7=0	8-7=1			

Factory	**Warehouse**		**Supply**		**Row Difference**
	W2	**W3**			
A	7 5	8	5		8-7=1
C	7	7	50		7-7=0
Demand	12-5=7	43	7+43+30+65+5 **=150**	50+30+65+5 **=150**	
Column Difference	7-7=0	8-7=1			

Factory	**Warehouse**		**Supply**		**Row Difference**
	W2	**W3**			
C	7 7	7	50-7-43		7-7=0
Demand	7	43	43+30+65+5+7 **=150**	43+30+65+5+7 **=150**	
Column Difference	7	7			

Factory	**Warehouse**	**Supply**	**Row Difference**
	W3		

C	7 43	43		7
Demand	43	43+30+65+5+7 **=150**	43+30+65+5+7 **=150**	
Column Difference	7			

To verify the correctness of the process, we note the total demand and supply will remain the same through the steps. Also, in the last step, the single the cell will be allocated with the value in either with demand or supply as both will have the same values. If the demand and supply have the same values, we you can choose any one of them to allocate the cell making other value zero. The final table with all the allocated cell will be as follow:

Factory	**Warehouse**			**Supply**
	W_1	**W_2**	**W_3**	
A	5 **65**	7 **5**	8	70
B	4	4 **30**	6	30
C	6	7 **7**	7 **43**	50
Demand	65	42	43	**150** **150**

Now we have to calculate the cost associated with these allocations. we add all the products of all allocated cell values (given in yellow) and the cost of the respective cell (given in blue).

Total cost=(65*5) +(5*7) +(30*4) +(7*7) +(43*7)

=325+35+120+49+301

=830 $

The path followed is represented by the red arrows as we found as the following table:

Factory	Warehouse			Supply	
	W_1	W_2	W_3		
A	5 **65**	7 **5**	8	70	
B	4	4 **30**	6	30	
C	6	7 **7**	7 **43**	50	
Demand	65	42	43	**150**	**150**

We will represent the overall results by the following table:

From	To	Quantity (units)
A	W_1	65
A	W2	5
B	W2	30
C	W2	7
C	W_3	43
Demand	**Warehouse**	Total=150

3.4 **Optimality test**

Optimality tests are used in transportation and assignment problems to determine whether a given solution is optimal or if further improvements can be made. Two commonly used optimality tests are the Modified Distribution Method and the Stepping Stone Method.

3.4.1 **The modified distribution method**

The Modified Distribution Method is an optimality test used in transportation problems. It evaluates the optimality of a solution by examining the opportunity costs associated with allocating units from sources to destinations. The method involves calculating the difference between the lowest and next lowest costs in each row and column of the transportation table. If all these differences are non-negative, the solution is considered optimal. If any of the differences are negative, the Modified Distribution Method identifies a cell with the most negative difference, and a distribution path is established by starting from that cell. The units are then redistributed along this path until a new solution is obtained. The process continues until an optimal solution is achieved, where all differences are non-negative.

3.4.2 **The stepping stone method**

The Stepping Stone Method is an optimality test used in transportation and assignment problems. It evaluates the optimality of a solution by examining the improvement potential associated with moving units from one cell to another. The method involves calculating the improvement values associated with each empty cell in the transportation table. To apply the Stepping Stone Method, the improvement value of each empty cell is calculated by following a closed path known as a stepping stone. The stepping stone starts and ends at the same unoccupied cell, and it traverses occupied cells along an alternating pattern of horizontal and vertical moves. The improvement value is determined by summing the unit cost of the cells visited by the stepping stone. If the improvement value of a cell is positive, it indicates that moving units into that cell will improve the overall cost of the solution. The Stepping Stone Method identifies the cell with the highest improvement value and performs the necessary unit reallocation. This process continues until no further improvements can be made, indicating an optimal solution.

Both the Modified Distribution Method and the Stepping Stone Method are iterative procedures that help optimize transportation and assignment problems by iteratively improving the initial solution until an optimal solution is obtained. These optimality tests play a crucial role in evaluating and refining solutions to achieve the most cost-effective allocations in transportation and assignment scenarios. Before we start applying this methods, we need to find a basic, practical solution using any of the three methods NWCM, LCM, or VAM that were previously reviewed. Therefore, we will take an example, and then find the basic initial solution for it, and then apply the two methods of finding the optimal solution to it.

Example 3.2

Let's we find a Solution to the following transportation problem by using Vogel's Approximation method, also we find an optimal solution using modified distribution method and the stepping stone method:

Factory	**Warehouse**				**Supply**	
	D_1	**D_2**	**D_3**	**D_4**		
S_1	19	30	50	10	7	
S_2	70	30	40	60	9	
S_3	40	8	70	20	18	
Demand	5	8	7	14	**34**	**34**

Solution:

TOTAL number of supply constraints: 3

TOTAL number of demand constraints: 4

Factory	**Warehouse**				**Supply**		**Row Penalty**
	D_1	**D_2**	**D_3**	**D_4**			
S_1	19	30	50	10	7		9=19-10
S_2	70	30	40	60	9		10=40-30
S_3	40	**8**	70	20	18		12=20-8
Demand	5	8	7	14	**34**	**34**	

The maximum penalty, 22, occurs in column D_2.

The minimum c_{ij} in this column is c32 = **8**.

The maximum allocation in this cell is min(18,8) = 8.

It satisfy demand of D_2 and adjust the supply of S_3 from 18 to 10 (18 - 8 = 10).

Factory	**Warehouse**				**Supply**		**Row Penalty**
	D_1	**D_2**	**D_3**	**D_4**			
S_1	**19**	30	50	10	7		9=19-10
S_2	70	30	40	60	9		20=60-40
S_3	40	**8(8)**	70	20	10		20=40-20
Demand	5	0	7	14	**34**	**34**	
Column Penalty	**21**=40-19	**--**	10=50-40	10=20-10			

The maximum penalty, 21, occurs in column D_1.

The minimum c_{ij} in this column is c_{11} = 19.

The maximum allocation in this cell is min(7,5) = 5.

It satisfy demand of D_1 and adjust the supply of S_1 from 7 to 2 (7 - 5 = 2).

Factory	**Warehouse**				**Supply**		**Row Penalty**
	D_1	**D_2**	**D_3**	**D_4**			
S_1	19**(5)**	30	50	10	2		40=50-10
S_2	70	30	40	60	9		20=60-40
S_3	40	**8(8)**	70	20	10		50=70-20
Demand	0	0	7	14	**34**	**34**	
Column Penalty	**--**	**--**	10=50-40	10=20-10			

The maximum penalty, 50, occurs in row S_3.

The minimum c_{ij} in this row is c_{34} = 20.

The maximum allocation in this cell is min (10,14) = 10.

It satisfies supply of S_3 and adjust the demand of D_4 from 14 to 4 (14 - 10 = 4).

Factory	**Warehouse**	**Supply**	**Row**

	D_1	D_2	D_3	D_4			Penalty
S_1	19**(5)**	30	50	10	2		40=50-10
S_2	70	30	40	60	9		20=60-40
~~S_3~~	~~40~~	~~**8(8)**~~	~~70~~	~~20**(10)**~~	0		--
Demand	0	0	7	4	**34**	**34**	
Column Penalty	**--**	**--**	10=50-40	**50**=60-10			

The maximum penalty, 50, occurs in column D_4.

The minimum cij in this column is c14 = 10.

The maximum allocation in this cell is min(2,4) = 2.

It satisfy supply of S_1 and adjust the demand of D_4 from 4 to 2 (4 - 2 = 2).

Factory	**Warehouse**				**Supply**		**Row Penalty**
	D_1	D_2	D_3	D_4			
S_1	19**(5)**	30	50	10**(2)**	0		--
S_2	70	30	40	**60**	9		20=60-40
~~S_3~~	~~40~~	~~**8(8)**~~	~~70~~	~~20**(10)**~~	0		--
Demand	0	0	7	2	**34**	**34**	
Column Penalty	**--**	**--**	40	60			

The maximum penalty, 60, occurs in column D4.

The minimum c_{ij} in this column is c_{24} = 60.

The maximum allocation in this cell is min (9,2) = 2.

It satisfies demand of D_4 and adjust the supply of S_2 from 9 to 7 (9 - 2 = 7).

Factory	**Warehouse**				**Supply**	**Row Penalty**
	D_1	D_2	D_3	D_4		

~~S_1~~	~~19**(5)**~~	~~30~~	~~50~~	~~10**(2)**~~	0		--
S_2	70	30	40	**60(2)**	7		40
~~S_3~~	~~40~~	~~**8(8)**~~	~~70~~	~~20**(10)**~~	0		--
Demand	0	0	7	0	**34**	**34**	
Column Penalty	**--**	**--**	40	**--**			

The maximum penalty, 40, occurs in row S_2.

The minimum cij in this row is $c_{23} = 40$.

The maximum allocation in this cell is min (7,7) = 7.

It satisfies supply of S_2 and demand of D_3.

So, the initial feasible solution is:

Factory	**Warehouse**				**Supply**		**Row Penalty**
	D_1	**D_2**	**D_3**	**D_4**			
S_1	19**(5)**	30	50	10**(2)**	7		9|9 |40 |40 |-- |--|
S_2	70	30	40**(7)**	60**(2)**	9		10|20|20|20|20|40|
S_3	40	8**(8)**	70	20**(10)**	18		12|20|50|--|--|--|
Demand	5	8	7	14	**34**	**34**	
Column Penalty	21 21 -- -- -- --	22 -- -- -- -- --	10 10 10 10 40 40	10 10 10 50 60 --			

The minimum total transportation cost =19×5+10×2+40×7+60×2+8×8+20×10=779

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6

Optimality test using Modi method.

Allocation Table is:

Factory	**Warehouse**				**Supply**
	D_1	**D_2**	**D_3**	**D_4**	

S_1	19**(5)**	30	50	10**(2)**	7	
S_2	70	30	40**(7)**	60**(2)**	9	
S_3	40	8**(8)**	70	20**(10)**	18	
Demand	5	8	7	14	**34**	**34**

Iteration-1 of optimality test

Firstly,

we Find u_i and v_j for all occupied cells (i, j), where $c_{ij} = u_i + v_j$

1. Substituting, $v_4 = 0$, we get

2. $c_{14} = u_1 + v_4 \Rightarrow u_1 = c_{14} - v_4 \Rightarrow u_1 = 10 - 0 \Rightarrow u_1 = 10$

4 $c_{11} = u_1 + v_1 \Rightarrow v_1 = c_{11} - u_1 \Rightarrow v_1 = 19 - 10 \Rightarrow v_1 = 9$

5 $c_{24} = u_2 + v_4 \Rightarrow u_2 = c_{24} - v_4 \Rightarrow u_2 = 60 - 0 \Rightarrow u_2 = 60$

6 $c_{23} = u_2 + v_3 \Rightarrow v_3 = c_{23} - u_2 \Rightarrow v_3 = 40 - 60 \Rightarrow v_3 = -20$

7 $c_{34} = u_3 + v_4 \Rightarrow u_3 = c_{34} - v_4 \Rightarrow u_3 = 20 - 0 \Rightarrow u_3 = 20$

8 $c_{32} = u_3 + v_2 \Rightarrow v_2 = c_{32} - u_3 \Rightarrow v_2 = 8 - 20 \Rightarrow v_2 = -12$

Factory	**Warehouse**				**Supply**		***u_i***
	D_1	**D_2**	**D_3**	**D_4**			
S_1	19**(5)**	30	50	10**(2)**	7		u_1=10
S_2	70	30	40**(7)**	60**(2)**	9		u_2=60
S_3	40	8**(8)**	70	20**(10)**	18		u_3=20
Demand	5	8	7	14			
v_j	V_1=9	V_2=-12	V_3=-20	V_4=0			

Secondly,

We find d_{ij} for all unoccupied cells(i, j), where $d_{ij} = c_{ij} - (u_i + v_j)$

1. $d_{12} = c_{12} - (u_1 + v_2) = 30 - (10 - 12) = 32$
2. $d_{13} = c_{13} - (u_1 + v_3) = 50 - (10 - 20) = 60$
3. $d_{21} = c_{21} - (u_2 + v_1) = 70 - (60 + 9) = 1$

4. $d_{22} = c_{22} - (u_2 + v_2) = 30 - (60 - 12) = -18$

5. $d_{31} = c_{31} - (u_3 + v_1) = 40 - (20 + 9) = 11$

6. $d_{33} = c_{33} - (u_3 + v_3) = 70 - (20 - 20) = 70$

Factory	**Warehouse**				**Supply**	u_i
	D_1	**D_2**	**D_3**	**D_4**		
S_1	19**(5)**	30[32]	50[60]	10**(2)**	7	u_1=10
S_2	70[1]	30[-18]	40**(7)**	60**(2)**	9	u_2=60
S_3	40[11]	8**(8)**	70[70]	20**(10)**	18	u_3=20
Demand	5	8	7	14		
v_j	V_1=9	V_2=-12	V_3=-20	V_4=0		

Thirdly,

Now choose the minimum negative value from all d_{ij} (opportunity cost) = d_{22} = [-18], and draw a closed path from S_2D_2.

Closed path is $S_2D_2 \rightarrow S_2D_4 \rightarrow S_3D_4 \rightarrow S_3D_2$

Closed path and plus/minus sign allocation.

Factory	**Warehouse**				**Supply**	u_i
	D_1	**D_2**	**D_3**	**D_4**		
S_1	19**(5)**	30[32]	50[60]	10**(2)**	7	u_1=10
S_2	70[1]	30[-18]**(+)**	40**(7)**	60**(2) (-)**	9	u_2=60
S_3	40[11]	8**(8) (-)**	70[70]	20**(10) (+)**	18	u_3=20
Demand	5	8	7	14		
v_j	V_1=9	V_2=-12	V_3=-20	V_4=0		

Fourthly,

Minimum allocated value among all negative position **(-)** on closed path = 2

Substract 2 from all **(-)** and Add it to all **(+)**

Factory	**Warehouse**	**Supply**

	D1	D2	D3	D4	
S_1	19**(5)**	30	50	10**(2)**	7
S_2	70	30**(2)**	40**(7)**	60	9
S_3	40	8**(6)**	70	20**(12)**	18
Demand	5	8	7	14	

Fifth, we repeat the step 1 to 4, until an optimal solution is obtained.

Iteration-2 of optimality test

Firstly,

Find u_i and v_j for all occupied cells(i,j), where $c_{ij} = u_i + v_j$

1. Substituting, $u_1 = 0$, we get
2. $c_{11} = u_1 + v_1 \Rightarrow v_1 = c_{11} - u_1 \Rightarrow v_1 = 19 - 0 \Rightarrow v_1 = 19$
3. $c_{14} = u_1 + v_4 \Rightarrow v_4 = c_{14} - u_1 \Rightarrow v_4 = 10 - 0 \Rightarrow v_4 = 10$
4. $c_{34} = u_3 + v_4 \Rightarrow u_3 = c_{34} - v_4 \Rightarrow u_3 = 20 - 10 \Rightarrow u_3 = 10$
5. $c_{32} = u_3 + v_2 \Rightarrow v_2 = c_{32} - u_3 \Rightarrow v_2 = 8 - 10 \Rightarrow v_2 = -2$
6. $c_{22} = u_2 + v_2 \Rightarrow u_2 = c_{22} - v_2 \Rightarrow u_2 = 30 + 2 \Rightarrow u_2 = 32$
7. $c_{23} = u_2 + v_3 \Rightarrow v_3 = c_{23} - u_2 \Rightarrow v_3 = 40 - 32 \Rightarrow v_3 = 8$
8.

Factory	**Warehouse**				**Supply**	u_i
	D1	**D2**	**D3**	**D4**		
S_1	19**(5)**	30	50	10**(2)**	7	u_1=0
S_2	70	30 **(2)**	40**(7)**	60	9	u_2=32
S_3	40	8**(6)**	70	20**(12)**	18	u_3=10
Demand	5	8	7	14		
v_j	V_1=19	V_2=-2	V_3=8	V_4=10		

Secondly, Find d_{ij} for all unoccupied cells (i, j), where $d_{ij} = c_{ij} - (u_i + v_j)$

1. $d_{12} = c_{12} - (u_1 + v_2) = 30 - (0 - 2) = 32$
2. $d_{13} = c_{13} - (u_1 + v_3) = 50 - (0 + 8) = 42$

3. $d_{21} = c_{21} - (u_2 + v_1) = 70 - (32 + 19) = 19$

4. $d_{24} = c_{24} - (u_2 + v_4) = 60 - (32 + 10) = 18$

5. $d_{31} = c_{31} - (u_3 + v_1) = 40 - (10 + 19) = 11$

6. $d_{33} = c_{33} - (u_3 + v_3) = 70 - (10 + 8) = 52$

Factory	Warehouse				Supply	u_i
	D_1	D_2	D_3	D_4		
S_1	19**(5)**	30 [32]	50 [42]	10**(2)**	7	$u_1=0$
S_2	70 [19]	30 **(2)**	40**(7)**	60 [18]	9	$u_2=32$
S_3	40 [11]	8**(6)**	70 [52]	20**(12)**	18	$u_3=10$
Demand	5	8	7	14		
v_j	$V_1=19$	$V_2=-2$	$V_3=8$	$V_4=10$		

Since all $d_{ij} \geq 0$.

So final optimal solution is arrived.

Factory	Warehouse				Supply
	D_1	D_2	D_3	D_4	
S_1	19**(5)**	30	50	10**(2)**	7
S_2	70	30**(2)**	40**(7)**	60	9
S_3	40	8**(6)**	70	20**(12)**	18
Demand	5	8	7	14	

The minimum total transportation cost $=19\times5+10\times2+30\times2+40\times7+8\times6+20\times12=743$

The stepping stone method will applied to find the optimal solution based on the initial feasible solution (example 3.2) as in the following:

Factory	Warehouse				Supply
	D_1	D_2	D_3	D_4	
S_1	19**(5)**	30	50	10**(2)**	7
S_2	70	30	40**(7)**	60**(2)**	9
S_3	40	8**(8)**	70	20**(10)**	18

Demand	5	8	7	14		

Iteration-1 of optimality test

Firstly,

Create closed loop for unoccupied cells, we get

Unoccupied cell	Closed path	Net cost change
S_1D_2	$S_1D_2 \rightarrow S_1D_4 \rightarrow S_3D_4 \rightarrow S_3D_2$	30 - 10 + 20 - 8 = 32
S_1D_3	$S_1D_3 \rightarrow S_1D_4 \rightarrow S_2D_4 \rightarrow S_2D_3$	50 - 10 + 60 - 40 = 60
S_2D_1	$S_2D_1 \rightarrow S_2D_4 \rightarrow S_1D_4 \rightarrow S_1D_1$	70 - 60 + 10 - 19 = 1
S_2D_2	$S_2D_2 \rightarrow S_2D_4 \rightarrow S_3D_4 \rightarrow S_3D_2$	30 - 60 + 20 - 8 = -18
S_3D_1	$S_3D_1 \rightarrow S_3D_4 \rightarrow S_1D_4 \rightarrow S_1D_1$	40 - 20 + 10 - 19 = 11
S_3D_3	$S_3D_3 \rightarrow S_3D_4 \rightarrow S_2D_4 \rightarrow S_2D_3$	70 - 20 + 60 - 40 = 70

Secondly,

Select the unoccupied cell having the highest negative net cost change i.e. cell S_2D_2=-18.

and draw a closed path from S_2D_2.

Closed path is $S_2D_2 \rightarrow S_2D_4 \rightarrow S_3D_4 \rightarrow S_3D_2$

Closed path and plus/minus allocation for current unoccupied cell S_2D_2

Factory	Warehouse				Supply
	D_1	D_2	D_3	D_4	
S_1	19(5)	30 [32]	50 [60]	10(2)	7
S_2	70 [1]	30 [-18] (+)	40(7)	60(2) (-)	9
S_3	40 [11]	8(8) (-)	70 [70]	20(10) (+)	18
Demand	5	8	7	14	

Third,

Minimum allocated value among all negative position (-) on closed path = 2

Substract 2 from all (-) and Add it to all (+)

Factory	Warehouse				Supply
	D_1	D_2	D_3	D_4	

S_1	19**(5)**	30	50	10**(2)**	7
S_2	70	30**(2)**	40**(7)**	60	9
S_3	40	8**(6)**	70	20**(12)**	18
Demand	5	8	7	14	

Fourthly,

Repeat the step 1 to 3, until an optimal solution is obtained.

Iteration-2 of optimality test

Unoccupied cell	Closed path	Net cost change
S_1D_2	$S_1D_2 \rightarrow S_1D_4 \rightarrow S_3D_4 \rightarrow S_3D_2$	30 - 10 + 20 - 8 = 32
S_1D_3	$S_1D_3 \rightarrow S_1D_4 \rightarrow S_3D_4 \rightarrow S_3D_2 \rightarrow S_2D_2 \rightarrow S_2D_3$	50 - 10 + 20 – 8+30-40 = 42
S_2D_1	$S_2D_1 \rightarrow S_2D_2 \rightarrow S_3D_2 \rightarrow S_3D_4 \rightarrow S_1D_4 \rightarrow S_1D_1$	70 - 30 + 8 – 20+10-19 = 19
S_2D_2	$S_2D_4 \rightarrow S_2D_2 \rightarrow S_3D_2 \rightarrow S_3D_4$	60 - 30 + 8 - 20 = 18
S_3D_1	$S_3D_1 \rightarrow S_3D_4 \rightarrow S_1D_4 \rightarrow S_1D_1$	40 - 20 + 10 - 19 = 11
S_3D_3	$S_3D_3 \rightarrow S_3D_2 \rightarrow S_2D_2 \rightarrow S_2D_3$	70 - 8 + 30 - 40 = 52

Since all net cost change ≥0, So final optimal solution is arrived.

Factory	**Warehouse**				**Supply**
	D_1	**D_2**	**D_3**	**D_4**	
S_1	19**(5)**	30	50	10**(2)**	7
S_2	70	30**(2)**	40**(7)**	60	9
S_3	40	8**(6)**	70	20**(12)**	18
Demand	5	8	7	14	

The minimum total transportation cost =19×5+10×2+30×2+40×7+8×6+20×12=743

Chapter 4

Conclusion and Future Works

4.1 Conclusion

1- This research focused on various methods for solving transportation problems, specifically addressing the initial basic feasible solution and the optimality test phases. Through the utilization of the North West Corner Rule, Vogle's Approximation Method, and the Least Cost Method, we obtained initial solutions that considered cost constraints and resource availability.
2- The Modified Distribution Method and the Stepping Stone Method were employed as optimality tests to refine the initial solutions. These tests allowed for the identification of alternative routes and the reallocation of quantities based on the associated costs. The iterative nature of these methods enabled us to converge towards optimal transportation plans.
3- The findings of this research demonstrated the significance of selecting appropriate solution methods and conducting optimality tests in transportation problem-solving. By applying these techniques, decision-makers can make informed decisions regarding resource allocation, minimize transportation costs, and enhance overall system efficiency.

4.2 Future Works

While this research paper has explored several methods for solving transportation problems and demonstrated their effectiveness, there are still avenues for further investigation and improvement. One area of future work could focus on the development and integration of advanced optimization algorithms, such as genetic algorithms or simulated annealing, to tackle larger and more complex transportation problems. These algorithms have the potential to provide more efficient and robust solutions by exploring a wider range of possible solutions and overcoming local optima. Additionally, incorporating real-time data and dynamic modeling techniques into transportation problem-solving can enhance the responsiveness and adaptability of transportation systems. This could involve integrating real-time traffic information, demand fluctuations, and changing constraints into the solution methods to optimize transportation plans in dynamic environments.

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الخلاصة

في هذا البحث تم تقديم تقنيات مختلفة ومتنوعة لأيجاد الحل الأولي والحل الأمثل لمشاكل النقل. كان التركيز على إيجاد حل عملي أساسي أولي وإجراء اختبارات الأمثل لتحسين الحل. تم استخدام ثلاث طرق من أجل الحصول على حل عملي أساسي أولي. هذه الطرق هي: طريقة الزاوية الشمالية الغربية (NWCR)، طريقة التقريب الخاصة بـ Vogel (VAM) ، وطريقة التكلفة الأقل (LCM). قدمت كل من هذه الطرق نهجًا منهجيًا لتخصيص الكميات من المصادر إلى الوجهات، مع مراعاة قيود التكلفة والتوافر. بمجرد الحصول على حل أولي، شرعنا في مرحلة اختبار الأمثل. الطريقتان المستخدمتان لهذا الغرض هما طريقة التوزيع المعدلة وطريقة خطوة الحجر. تهدف هذه التقنيات إلى تحسين الحل الأولي من خلال التقييم المتكرر للطرق البديلة وإعادة تخصيص الكميات بناءً على التكاليف المرتبطة. كانت نتائج هذه الطرق فعالة في الحل الأولي وتحقيق خطط النقل المثلى. تعرفت اختبارات التحسين على المسارات الأكثر كفاءة وخصصت الكميات على النحو الأمثل، بما في ذلك تقليل التكلفة وتخصيص الموارد. وأخيرًا، سلط هذا البحث الضوء على أهمية استخدام الأساليب المناسبة لحل مشاكل النقل. قدم الجمع بين طرق الحل الأولية واختبارات التحسين إطارًا شاملاً لمعالجة سيناريوهات النقل المعقدة. من خلال استخدام هذه التقنيات، يمكن لصانعي القرار تحسين تخصيص مواردهم وتقليل التكاليف وتعزيز الكفاءة العامة لأنظمة النقل الخاصة بهم.

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بعض الطرق لحل مشاكل النقل

بحث مقدم

الى مجلس كلية التربية للعلوم الصرفة جامعة بابل كجزء
من متطلبات نيل درجة الدبلوم العالي تربية / رياضيات

مقدم من قبل الطالبة

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