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Department of Mathematics

Development The Linear Programming Model Based On The Simplex Method

A research

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By:

Aqeel Jabbar Abood

Supervised by:

Asst. Prof. Ahmed Sabah Al-Jilawi

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بسم الله الرحمن الرحيم

﴿ وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا ۝ ﴾

الاسراء (من الآية 85)

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Signature:

Name : Dr. Ahmed Sabah Al jilawi

Scientific grade :

Date: / / 2023

According to the available recommendation , I forward this project for debate by the examining committee .

Signature:

Name: Dr. Azal Jafar Musa

Head of mathematics Department

Scientific grade: Asst. Prof.

Date: / / 2023

Examining Committee Certification

We certify that we have read this research entitled "**Development The Linear Programming Model Based On The Simplex Method**" as examining committee examined the student "**Aqeel Jabbar Abood**" in its contents and that in our opinion it is adequate for the partial fulfillment of the requirement for the Degree of Higher Diploma Education / Mathematics .

Chairman	Member
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Name: Dr. Bushra Hussein Aliwi	Name: Dr. Hassanein Qassem Zaidan
Title:	Title:
Date: / /2023	Date: / /2023

Member and supervisor

Signature

Name: Dr. Ahmed Sabah Al jilawi

Title :

Date: / /2023

Approved by the dean of college of education for pure science:

Signature

Name: Dr. Bahaa Hussien Salih Rabee

Title: Professor

Date: / /2023

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Name: Dr. Janan Hamza Farhoud

Scientific grade:

Date: / / 2023

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Signature:

Name: Dr. Hasna Hassan Shaheed

Scientific grade:

Date: /   /2023

Dedication

To the lady of the women of the universe Fatima Al -Zahra, peace be upon her

Man was found on the face of the simple, and he did not live in isolation from the rest of the people

At all stages of life, there are people who deserve thanks

The first of the people is the parents; Because they have the credit for the sky;

Their existence is a reason for survival and the farmer in our religion and the hereafter.

To the companion of the struggle in the march of life. . my wife

To my friends, who I bear witness to them, are companions in all matters.

I dedicate this simple effort
Development The Linear Programming Model
Based On The Simplex Method

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In the name of Allah, the Most Gracious, the Most Merciful First and above all, Praise be to Allah who gave me the health and patience to conduct this work.

And then, I would like to thank prophet Mohammad and Ahel Al-Bate (peace be upon them).

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Abstract

The study provided a good description of linear programming problems and how to solve them using the Simplex method, which is an effective method for solving linear programming problems with more than two variables and any number of constraints. The Big-M and two-stage methods were also reviewed. The researcher developed the Simplex algorithm using the Python language, which is a widely used language used in software development, machine learning, and mathematical algorithms. It is also a high-level programming language that is easy to learn.

Contents

Introduction :

The first beginning of the development of the emergence of operations research was in 1909, when the English operator Erlag noticed the problem of crowding on the telephone booth by phone call seekers and then tried to establish the Queing Theory. In 1918, the scientific management movement appeared when the scientist Frederick presented his book (Scientific Management), in which he called for the use of the scientific research method in management.

The diet problem was one of the first optimization problems studied in the 1930s and 1940s. The problem was motivated by the Army's desire to minimize the cost of feeding in the field while still providing a healthy diet. One of the early researchers to study the problem was Georg. In the late 1930's and early 1940's the diet problem was formulated. It was motivated by the desire of the U.S. Army to feed its troops in World War II at a minimal cost, while maintaining the basic nutritional guidelines. Using a heuristic method, George Stigler, an early researcher of the problem, made an educated guess of $39.93 per year[4].

The real progress in operations research was in 1939 when the urgent need for the leadership of the British Air Force appeared to the contribution of scientists in the various branches of science to develop a scientific method to repel the German air attack, through the optimal exploitation of the limited resources available from manpower and equipment for the British forces, which made progress in this field at the time [2] .

After the end of World War II, this science was developed and benefited from through its application in the various fields of life (economic, agricultural, service), which prompted the rest of the countries to pay attention to this science. Optimization is central to any problem involving decision making, whether in engineering or in economics. The task of decision making entails choosing among various alternatives. This choice is governed by our desire to make the "best" decision. The measure of goodness of the alternatives is described by an objective function or performance index. Optimization theory and methods deal with selecting the best alternative in the sense of the given objective function [5] .

In general, the problem of reducing or increasing is the problem of improvement and is subject to the number of restrictions. Everywhere we find improvement problems. All Chief Executive Officers faces the problem of increasing profits with limited resources. This is a general problem that has no explicit or alone solution; But decisions can be successfully addressed using improvement methods. This includes, for example, production, inventory, transportation and processing problems. Most companies actually depend on improvement techniques. However, the problems of improvement are not limited to companies only. When using GPS, it improves arrival time, that is, how to reduce travel time between two different sites. City designers may need to choose the best place to build new fire extinguishing stations in order to serve its citizens efficiently, and how to build a flexible remote network and cheaper as much as possible; How to organize flights in an effective manner in terms of cost while meeting the demand for passengers or using a few classrooms how to set a schedule for final exams.[6]

Chapter One

Concepts about Linear Programming

1. Chapter One : Concepts about Linear Programming

1.1 The Concept of Linear Programming :

The term linear programming was presented in the fifties of the last century, where computers were often very confidential, and the word programming was a military term at the time, and it was referring to plans, the publication of men, logistical shows, or training tables. The term sin indicates that possible plans are restricted by linear restrictions (inequality), in addition to the efficiency of the plan (for example, costs or time) is measured by a written function of the studied quantities. Soon, linear programming began to solve problems for all types of economic activities, such as transporting products between factories, transporting crude oil, or transporting various crops to factories. The phrase "planning with linear restrictions" is to get this meaning for linear programming better. However, the term linear programming remained firmly for many years, and at the same time, it has gained a much broader meaning that does not play a role only in mathematical economics, but it appears frequently in computer science and in many other areas. [7]

Linear programming is an important sports applications in making good decisions and solving problems such as allocating limited materials and human resources to achieve lower costs and possible high profits. Linear programming can determine a method or method used to analyze problems and its real goal is the best solution to the problem.

The linear programming method is an elaborate way to find better solutions to economic, industrial and other problems that include many variables, it depends on a set of independent variables that are a group of restrictions such as profit or cost. The linear programming model is also used to distribute limited resources customization, as the model depends on a simple idea of the scarcity of the content corresponding to the content. Some researchers considered linear programming a mathematical method to distribute a set of limited number of tools and resources, which includes a set of restrictions and fixed factors to achieve the best possible distribution, which is an ideal or perfect distribution [8]. Because of the importance of using the programming model, in 1947 (G.Dantzig) Simplex method to solve linear programming problems [9].

1.2 Conditions for Using Linear Programming [2]

In order to apply linear programming methods, the following conditions must be met in the problem to be solved .

- It should be used in the case of scarcity of resources. If the resources were completely available, there would be no problem. This scarcity represents one of the most important constraints that management is subject to in its pursuit of the goal, and it constitutes constraints that link the variables involved in the objective function with each other. In the form of inequalities and equations, these are called structural constraints.
- There must be a specific goal expressed in a quantitative way, and the goal must be clear and precise so that it can take the form of a mathematical equation, and usually the goal is to achieve the maximum possible profits or reduce costs to the lowest possible extent.
- It is assumed that there are different alternatives to achieve the goal, so there must be scientific methods for mixing resources to reach the goal where each alternative has an expected return, so the task becomes choosing the alternative that gives the highest return within the limits of the imposed restrictions.
- It is assumed that the relationships between the variables that make up the problem are linear, and this means that any change in one of

the variables causes a completely appropriate change with the other variable.

- That there are constraints on the variables included in the objective function and structural constraints from which negative values are excluded.

1.3 The Useful of Linear Programming

In the field of optimization linear programming is widely applied. Many functional problems in process analysis can be represented as linear programming problems. Linear programming problems such as network flow queries, multiple commodity flow queries, and determining the highest profit and lowest cost are important for producing a lot of functional success for organizations, companies, institutions, factories, and others.

Linear programming also provides an important benefit for business problems. It helps in solving variable dimension problems. According to the change of capabilities and resources, as well as its benefit in making adjustments and improvements.

By calculating the cost and profit of different outputs, linear programming has the benefit of showing solutions and highlighting the best ones.

It can be said that the usefulness of linear programming is summarized in the following uses:

1- Organizing production processes to obtain the largest possible output within the available conditions.

2- Reducing production wastes to the lowest possible extent.

3- Choosing the best ways to distribute products from production areas to their use sites.

4- Production planning and control.

5- Access to the best utilization of the energies of machinery and equipment.

6- Make maximum use of raw materials.

The benefits of linear programming appear in many areas of life, including:

- Engineering: It solves design and manufacturing problems because it is useful in improving the shape.
- Effective manufacturing: to maximize the profits of companies and institutions.
- Power industry: to improve the electric power system, etc.
- Transport optimization: for less cost and less time [19].

1.4 The Specification of Linear Programming Problem

To initialize the linear programming model the following should be done.

1- The objective must be clear, precise and specific

2- Decision variables, the problem of selecting the optimal value. Each has a desired goal. The variants may be different products, areas, or distribution company.

3- There are restrictions on the desire of companies to obtain energies for work. values that achieve the desired goal.

4- Continuous decision variables. Any decision variables may be fractional values and not necessarily integers.

5- A linear relationship between the variables contained in the problem [1] .

1.5 Formulation of Linear Programming Model

The importance of the linear programming method is due to the importance of the problems that can be solved. However, not every problem can be solved by linear programming method, as solving the problem by linear programming method requires that the following conditions be met:

1.5.1 Objective function:

It is the desired goal that we want to achieve and the possibility of expressing this goal in the form of a linear function and obtaining a numeric value for it and trying to maximize this value and find a maximum point for it if the desired goal is profit, or reduce the value and find a minimum point if the goal is the lowest possible cost. The objective function consists of variables and the coefficient of each variable is the profit of one unit in the case of maximizing the objective function, or it is the cost of one unit in the case of reducing the objective function.

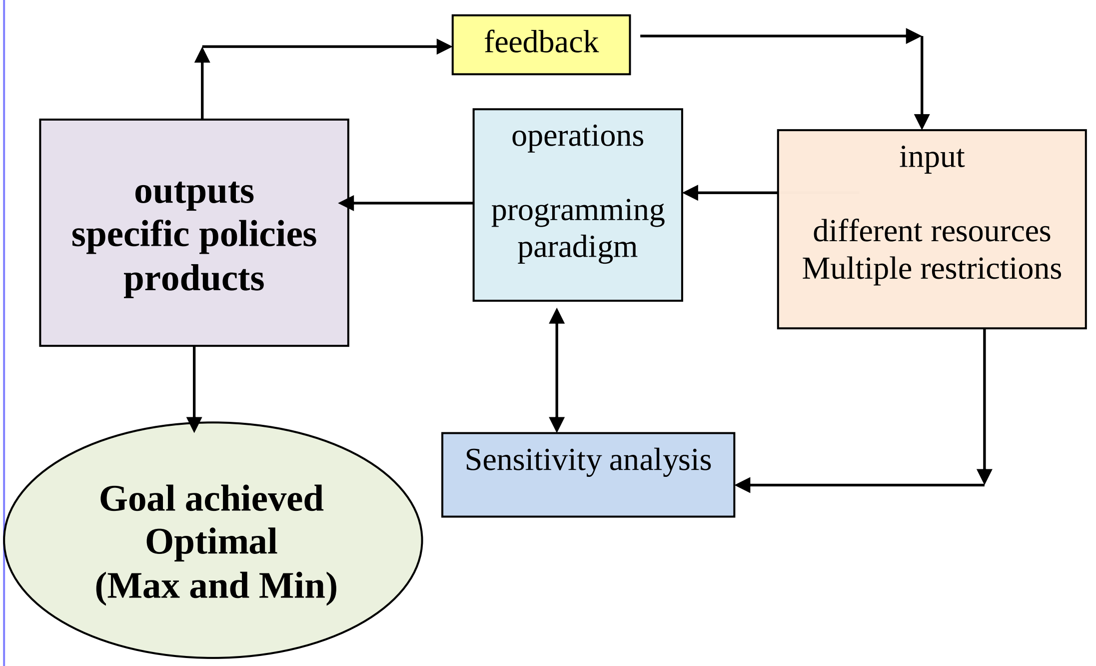
1.5.2 Constraints :

That is, the possibility of expressing the relationship between the decision variables and the capabilities available in the form of Linear constraints, and it shows what each production unit needs from each of the available limited resources in the form of Linear Inequalities or Linear Equations, or a mixture of them, and it is called structural constraints

1.5.3 Negativity-Non Conditions :

The decision variables in the problem under study must be positive, zero or non-negative variables [2] .

figure 1.1



The general location of the linear programming problem within a system [3]

Example 1.1 [3] :

A ready-made clothing company produces three types of sizes: large, medium, and small. These clothes go through three stages of cutting, knitting, and ironing – the large size requires 5 days for cutting, 3 days for sewing, and one day for ironing, while the medium size needs 3 days for cutting, 2 days for sewing, and 1 day for ironing. Small to 4 days for cutting, 3 days for sewing, and 2 days for ironing. If you know that the time available for cutting is 60 days, knitting is 45 days, and ironing is 30 days, and that the large size makes a profit of 50 riyals, the medium size is 100 riyals, and the small size is 200 riyals.

Required :

Formulating the linear programming model that achieves the largest possible profit .

the solution :

Let : Large size = X_1 , Medium size = X_2 , Small size = X_3

1- Putting the data into a table

Operations / Type	Cutting	Knitting	Ironing	The Profit
Large Size	5	3	1	50
Medium Size	3	2	1	100
Small Size	4	3	2	200
Time	60	45	30	

2- Formulation :

Objective function : maximizing profit

$$Max: Z \ = 50x_1 + 100x_2 + 200x_3$$

Constraints :

1- Shearing limit $5x_1 + 3x_2 + 4x_3 \ \leq \ 60$

2-Knitting process $3x_1 + 2x_2 + 3x_3 \ \leq \ 45$

3-Ironing restriction $x_1 + x_2 + 2x_3 \ \leq \ 30$

4- Non-negative constraint $x_1, x_2, x_3 \ \geq \ 0$

1.6 Formulas of Linear Models

Linear programming is considered one of the mathematical programming models that solves issues of allocation or distribution of limited resources or energies to achieve a goal, and this goal is expressed in a function called Objective Function and symbolized by the symbol (Z), and it is of two types, either (Max) and comes when the company or factory intends to maximize Profits or (Min) come when the company or factory aims to reduce costs.

This function is subject to determinants, conditions, or restrictions, which are variations that express raw materials, working hours, machinery capacities, electrical energy, and manpower. . . etc.

As well as non-negative constraints, which express that all variables of the mathematical model are positive or zero values and cannot be negative values.

The linear programming model comes in three forms [2] :

1.6.1 : The general form of linear programming [2] :

If we look at the linear programming model in the previous example, which was formulated and built based on the problem shown in that example, we find that this model is in the general form.

Therefore, the linear programming model generally consists of:

1- Variables

2- Signs: it is

$\leq$ less than or equal to

$\geq$ More than or equal to

$<$ less than

$>$ More than

$=$ equals

3- Variables Parameters

4- The objective function

5- Constraints

6- Non-negativity Constraints

7- Subject to (meaning that the facility, company, or organization seeks to achieve the objective indicated in the linear function based on the conditions or restrictions)

Therefore, the general form of the linear programming model is as follows:

$$\begin{matrix} Max \\ or \\ Min \end{matrix} \quad Z = C_1X_1 + \cdots + C_nX_n$$

$$\begin{matrix} a_{11}X_1 + \cdots + a_{1n}X_n \leq, =, \geq b_1 \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ a_{m1}X_1 + \cdots + a_{mn}X_n \leq, =, \geq b_m \end{matrix}$$

$$X_1, \ldots, X_n \geq 0$$

This formula is the general formula for linear programming, and if we look closely at it, we find that (a, b, c) are constants and that (X_i) are variables.

The general form of the linear programming model can be abbreviated as follows :

$$\begin{matrix} Max \\ or \\ Min \end{matrix} \quad Z = \sum_{i=1}^{n} C_iX_i$$

$$\sum_{\substack{1\leq i\leq n \\ 1\leq j\leq m}} a_{ji}X_i \leq, =, \geq b_j$$

$$X_i \geq 0 \quad \forall\, i$$

1.6.2 Canonical form of linear programming [2] :

The difference between the canonical form of the linear programming model and the general form of the linear programming model is as follows:

1- The objective function (Z) in the general form of the linear programming model is either of type (Max) or of type (Min), while in the canonical form of the linear programming model it is of type (Max) only

2- The signs of constraints in the general form of the linear programming model are $(\leq, =, \geq)$, while in the canonical form of the linear programming model they are less than or equal to $(\leq)$ only.

The components of linear programming are the same in the general and canonical formats.

The canonical form of the linear programming model :

$$Max\ \ Z = C_1X_1 + \cdots + C_nX_n$$

$$\begin{array}{c} a_{11}X_1 + \cdots + a_{1n}X_n \leq b_1 \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ a_{m1}X_1 + \cdots + a_{mn}X_n \leq b_m \end{array}$$

$$X_1, \dots, X_n \geq 0$$

The canonical formula for the linear programming model can be shortened to become the following formula :

$$Max\ \ Z = \sum_{i=1}^{n} C_iX_i$$

$$\sum_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} a_{ji}X_i \leq b_j$$

$$X_i \geq 0 \quad \forall\, i$$

The canonical formula is used in some special cases for linear programming models.

The general form can be converted into the canonical form using the following rules:

1- The objective function (Min) can be converted to (Max) and vice versa by multiplying the objective function (-1)

2- An entry greater than or equal to $(\geq)$ can be converted into an entry less than or equal to $(\leq)$ by multiplying both sides of the inequality by (-1)

3- The equality constraint can be converted into two constraints, the first is less than or equal to $(\leq)$ and the second is greater than or equal to $(\geq)$, then the second constraint is converted to less than or equal to by multiplying both sides of the inequality by (-1)

4- The absolute value constraint can be converted into two constraints of type less than or equal to $(\leq)$.

1.6.3 standard form of linear programming [2] :

The difference between the standard form of the linear programming model and the general form of the linear programming model is as follows:

1- The objective function (Z) in the general form of the linear programming model is either of type (Max) or of type (Min), and in the standard form of the linear programming model it is of type (Max) only.

2- The signs of constraints in the general form of the linear programming model are $(\leq, =, \geq)$, while in the standard form of the linear programming model they are equal to $(=)$ only after adding non-negative Slack Variables and are denoted by the symbol $(S_j \geq 0)$ and are in the form of $(+S)$ when the inequality sign is less than or equal to $(\leq)$, $(-S)$ when the inequality sign is greater than or equal to $(\geq)$, and nothing is equal in the case of equality.

3- The right side of the constraints is non-negative $(b_i \geq 0)$.

In light of this, the standard formula for the linear programming model is as follows:

$$Max\ \ Z = C_1X_1 + \cdots + C_nX_n + 0S_1 + \cdots + 0S_m$$

$$\begin{array}{c} a_{11}X_1 + \cdots + a_{1n}X_n + S_1 = b_1 \\ \vdots \qquad\qquad \vdots \qquad \vdots \quad \vdots \\ a_{m1}X_1 + \cdots + a_{mn}X_n + S_m = b_m \end{array}$$

$$X_1, \dots, X_n \geq 0$$

$$S_1, \dots, S_m \geq 0$$

The standard form of the linear programming model can be abbreviated as follows:

$$Max\ \ Z = \sum_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} C_iX_i + 0S_j$$

$$\sum_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} a_{ji}X_i + S_j = b_j$$

$$X_i \geq 0 \quad \forall\, i$$

$$S_j \geq 0 \quad \forall\, j$$

Chapter Two

The Simplex method

2. Chapter Two : The Simplex method

2.1 Simplex method and Python

The Simplex algorithm provided optimal solutions to large linear programming issues discovered by American mathematician Goerge B. Dantzig (1914-2005). The discovery of the Simplex algorithm led to the development of the previously unexplained areas of improvement.

Dantzig was working as a sports consultant for the Pentagon during World War II, where he was asked to find a way for the logistical training and supply program better. Dantzig has expanded the idea of solving linear programming issues, as it reached a model to determine the space of possible solutions by analyzing the activity that consists mainly of linear equations and inequality. The preliminary model of Dantzig faced important disabled: (1) The solution of the solution was very large, which is difficult to reach the results in the absence of the computer, a problem that cannot be overcome. (2) The goal of the problem was not clear enough on the ground, and Dantzig called it the "inability" model. Dantzig looked at the concept of using an improved function that is credited with discovering its Simplex method [10] .

Simplex method is the general way to solve linear programming issues for more than two variables and for any number of restrictions, and thus is considered a efficient way to use the computer[11] .

The researcher to solve the computers' linear programming issues in the Python programming language is a programming language that is widely used in network applications, software development, data science and automated learning (ML). Python developers are used because they are efficient and easy to learn and can be run on many different platforms.

The Python program is available for free download and integrates well with all types of systems and increases the speed of development [18].

2.2 Solving linear programming issues of type maximization

The Simplex method is good in solving linear programming issues, as it addresses issues that contain two or more variables and in linear programming matters in which the function is intended to be the best solution if all the transactions in the goal function are negative[12]. In the next item, we will look at the steps of the solution to the maximum issues. To solve linear programming models in a Simplex method, it should take place in three serial stages, which are the following:

1- The feasible solution: It is to find a possible basic solution.

2- The best solution: is to improve the feasible solution.

3- The optimum solution: is to improve the best solution [2] .

2.2.1 The steps of simplex method

To find an optimal solution in the Simplex method for linear programming models, the following steps must be followed :

Step 1 : Converting the written programming model from the canonical formula to the standard formula And made the goal function equal to zero[2] .

Step 2 : Organizing the basic or primary solution schedule, as it will be explained in the example of the end of the item [13] .

Step 3 : Determine the highest negative value in the target function row, and the column that contains this value is called the pivotal column [14] . The variable that corresponds to this value in the target function is called the Entering variable [2] .

Step 4 : We divide the components of the column each on its positive counterpart only from the elements of the pivotal column, so that the leaving variable is the corresponding to the row with the lowest percentage [12] . The row in which the exterior variable is called the pivotal row [2] . The number that is located in the intersection of the pivotal row with the pivotal column is called the pivotal number [13] .

Step 5 : By dividing the values in the row of the leaving variable on the pivot element, we get the central equation [2] .

Step 6 : Finding new target function transactions through New Z row = Current Z row – [pivot column coefficient that in Z * pivot Equation][10]

Step 7 : Finding new restrictions (S_i) transactions through new (S_i) transactions = old (S_i) transactions - [pivot column coefficient that in (S_i) row * pivot equation] [10] .

Step 8 : We get the best solution when all Z transactions are greater than or equal to zero and if one value of transactions is smaller than zero, this means that we have not reached the best solution [15] . Therefore, the solution is returned from the step 4 .

Example (2.1) [10] : $Max\ Z = 5x_1 + 4x_2$

Subject to :

$$6x_1 + 4x_2 \le 24$$
$$x_1 + 2x_2 \le 6$$
$$-x_1 + x_2 \le 1$$
$$x_2 \le 2$$
$$x_1, x_2 \ge 0$$

Solution :

Step 1 : $Max\ Z - 5x_1 - 4x_2 - 0S_1 - 0S_2 - 0S_3 = 0$

Subject to :

$$6x_1 + 4x_2 + S_1 = 24$$
$$x_1 + 2x_2 + S_2 = 6$$
$$-x_1 + x_2 + S_3 = 1$$
$$x_2 + S_4 = 2$$
$$x_1, x_2, S_1, S_2, S_3, S_4 \geq 0$$

Step 2 : **Table 2.1**

Basic	Z	x_1	x_2	S_1	S_2	S_3	S_4	b_i	Ratio
Z	1	-5	-4	0	0	0	0	0	
S_1	0	6	4	1	0	0	0	24	
S_2	0	1	2	0	1	0	0	6	
S_3	0	-1	1	0	0	1	0	1	
S_4	0	0	1	0	0	0	1	2	

We note that (-5) the largest negative number in objective function .

Step 3 : **Table 2.2**

Basic	Z	x_1	x_2	S_1	S_2	S_3	S_4	b_i	Ratio
Z	1	-5	-4	0	0	0	0	0	
S_1	0	6	4	1	0	0	0	24	
S_2	0	1	2	0	1	0	0	6	
S_3	0	-1	1	0	0	1	0	1	
S_4	0	0	1	0	0	0	1	2	

Now , we must find the (Ratio) to choose the pivot row .

Step 4 : **Table 2.3**

Basic	Z	x_1	x_2	S_1	S_2	S_3	S_4	b_i	Ratio
Z	1	-5	-4	0	0	0	0	0	
S_1	0	6	4	1	0	0	0	24	4
S_2	0	1	2	0	1	0	0	6	6
S_3	0	-1	1	0	0	1	0	1	-1
S_4	0	0	1	0	0	0	1	2	∞

x_1 is entering variable . S_1 is the leaving . Replace S_1 in the Basic column with x_1

Step 5 :

New x_1 row = Current S_1 row ÷ 6

$= \frac{1}{6}(0 \quad 6 \quad 4 \quad 1 \quad 0 \quad 0 \quad 0 \quad 24)$

$= (0 \quad 1 \quad 4 \quad 1 \quad 0 \quad 0 \quad 0 \quad 24)$

Step 6 :

New Z row = Current Z row $-(-5) \times$ New x_1 row

$= (1 \quad -5 \quad -4 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0) - (-5) \times \left(0 \quad 1 \quad \frac{2}{3} \quad \frac{1}{6} \quad 0 \quad 0 \quad 0 \quad 4\right)$

$= \left(1 \quad 0 \quad -\frac{2}{3} \quad \frac{5}{9} \quad 0 \quad 0 \quad 0 \quad 20\right)$

Step 7 :

New S_2 row = Current S_2 row $-(1) \times$ New x_1 row

$= (0 \quad 1 \quad 2 \quad 0 \quad 1 \quad 0 \quad 0 \quad 6) - (1) \times \left(0 \quad 1 \quad \frac{2}{3} \quad \frac{1}{6} \quad 0 \quad 0 \quad 0 \quad 4\right)$

$= \left(0 \quad 0 \quad \frac{4}{3} \quad -\frac{1}{6} \quad 1 \quad 0 \quad 0 \quad 2\right)$

New S_3 row = Current S_3 row $-(-1) \times$ New x_1 row

$= (0 \quad -1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1) - (-1) \times \left(0 \quad 1 \quad \frac{2}{3} \quad \frac{1}{6} \quad 0 \quad 0 \quad 0 \quad 4\right)$

$= \left(0 \quad 0 \quad \frac{5}{3} \quad \frac{1}{6} \quad 0 \quad 1 \quad 0 \quad 5\right)$

New S_4 row = Current S_4 row $-(-1)\times$ New x_1 row

$$= (0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 2) - (0) \times \left(0\ \ 1\ \ \tfrac{2}{3}\ \ \tfrac{1}{6}\ \ 0\ \ 0\ \ 0\ \ 4\right)$$

$$= (0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 2)$$

The new basic variables are (x_1, S_2, S_3, S_4) , and the new table becomes

Table 2.4

Basic	Z	x_1	x_2	S_1	S_2	S_3	S_4	b_i	Ratio
Z	1	0	$-\frac{2}{3}$	$\frac{5}{6}$	0	0	0	20	
x_1	0	1	$\frac{2}{3}$	$\frac{1}{6}$	0	0	0	4	6
S_2	0	0	$\frac{4}{3}$	$-\frac{1}{6}$	1	0	0	2	1.5
S_3	0	0	$\frac{5}{3}$	$\frac{1}{6}$	0	1	0	5	3
S_4	0	0	1	0	0	0	1	2	2

In Table 2.4 , we have reached a better solution that the value of $(Z = 20)$ is when the values $(x_1 = 4, S_2 = 2, S_3 = 5, S_4 = 2)$ but there is a negative value in the Z row $\left(-\frac{2}{3}\right)$.

Step 8 :

it is possible to improve this solution to an optimal solution where it appears to us through note the smallest ratio (1.5) that the entering variable is x_2 and the leaving variable is S_2 .

We will restore the steps of the solution starting from the step 4 due to the emergence of a negative value in the Z row .

Step 4 : **Table 2.5**

Basic	Z	x_1	x_2	S_1	S_2	S_3	S_4	b_i	Ratio
Z	1	0	$-\frac{2}{3}$	$\frac{5}{6}$	0	0	0	20	
x_1	0	1	$\frac{2}{3}$	$\frac{1}{6}$	0	0	0	4	6
S_2	0	0	$\frac{4}{3}$	$-\frac{1}{6}$	1	0	0	2	1.5
S_3	0	0	$\frac{5}{3}$	$\frac{1}{6}$	0	1	0	5	3
S_4	0	0	1	0	0	0	1	2	2

x_2 is entering variable . S_2 is the leaving . Replace S_2 in the Basic column with x_2

Step 5 :

New x_2 row = Current S_2 row $\div \frac{4}{3}$

$$= \frac{3}{4}\left(0 \quad 0 \quad \frac{4}{3} \quad -\frac{1}{6} \quad 1 \quad 0 \quad 0 \quad 2\right)$$

$$= \left(0 \quad 0 \quad 1 \quad -\frac{1}{8} \quad \frac{3}{4} \quad 0 \quad 0 \quad \frac{3}{2}\right)$$

Step 6 :

New Z row = Current Z row $-(-5)\times$ New x_2 row

$$= \left(1 \;\; 0 \;\; -\frac{2}{3} \;\; \frac{5}{9} \;\; 0 \;\; 0 \;\; 0 \;\; 20\right) - \left(-\frac{2}{3}\right) \times \left(0 \;\; 0 \;\; 1 \;\; -\frac{1}{8} \;\; \frac{3}{4} \;\; 0 \;\; 0 \;\; \frac{3}{2}\right)$$

$$= \left(1 \quad 0 \quad 0 \quad \frac{3}{4} \quad \frac{1}{2} \quad 0 \quad 0 \quad 21\right)$$

Step 7 :

New x_1 row = Current x_1 row $-\left(\frac{2}{3}\right)\times$ New x_2 row

$$=\left(0\ \ 1\ \ \frac{2}{3}\ \ \frac{1}{6}\ \ 0\ \ 0\ \ 0\ \ 4\right)-\left(\frac{2}{3}\right)\times\left(0\ \ 0\ \ 1\ \ -\frac{1}{8}\ \ \frac{3}{4}\ \ 0\ \ 0\ \ \frac{3}{2}\right)$$

$$=\left(0\ \ 1\ \ 0\ \ \frac{1}{4}\ \ -\frac{1}{2}\ \ 0\ \ 0\ \ 3\right)$$

New S_3 row = Current S_3 row $-\left(\frac{5}{3}\right)\times$ New x_2 row

$$=\left(0\ \ 0\ \ \frac{5}{3}\ \ \frac{1}{6}\ \ 0\ \ 1\ \ 0\ \ 5\right)-\left(\frac{5}{3}\right)\times\left(0\ \ 0\ \ 1\ \ -\frac{1}{8}\ \ \frac{3}{4}\ \ 0\ \ 0\ \ \frac{3}{2}\right)$$

$$=\left(0\ \ 0\ \ 0\ \ \frac{3}{8}\ \ -\frac{5}{4}\ \ 1\ \ 0\ \ \frac{5}{2}\right)$$

New S_4 row = Current S_4 row $-(1)\times$ New x_2 row

$$=(0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 2)-(1)\times\left(0\ \ 0\ \ 1\ \ -\frac{1}{8}\ \ \frac{3}{4}\ \ 0\ \ 0\ \ \frac{3}{2}\right)$$

$$=\left(0\ \ 0\ \ 0\ \ \frac{1}{8}\ \ -\frac{3}{4}\ \ 0\ \ 1\ \ \frac{1}{2}\right)$$

The new basic variables are (x_1, x_2, S_3, S_4) , and the new table becomes

Table 2.6

Basic	Z	x_1	x_2	S_1	S_2	S_3	S_4	b_i	Ratio
Z	1	0	0	$\frac{3}{4}$	$\frac{1}{2}$	0	0	21	
x_1	0	1	0	$\frac{1}{4}$	$-\frac{1}{2}$	0	0	3	
x_2	0	0	1	-1	$\frac{3}{4}$	0	0	$\frac{3}{2}$	
S_3	0	0	0	$\frac{3}{8}$	$-\frac{5}{4}$	1	0	$\frac{5}{2}$	
S_4	0	0	0	$\frac{1}{8}$	$-\frac{3}{4}$	0	1	$\frac{1}{2}$	

From the table 2.6 it becomes clear to us that all the values of Z row are positive and there is no negative value, and thus represents a table 2.6 the best solution and can be explained as follows :

$$Max\ Z = 21$$

$$x_1 = 3\ ,\ \ x_2 = \frac{3}{2}$$

2.2.2 Algorithm Simplex in python [of the researcher's work]

```
objective = [-5, -4, 0, 0, 0, 0, 0]

cond1 = [6, 4, 1, 0, 0, 0, 24]
cond2 = [1, 2, 0, 1, 0, 0, 6]
cond3 = [-1, 1, 0, 0, 1, 0, 1]
cond4 = [0, 1, 0, 0, 0, 1, 2]
RHS = [objective[-1], cond1[-1], cond2[-1], cond3[-1], cond4[-1]]
all_module = [objective, cond1, cond2, cond3, cond4]

print ("objective = ", objective)
print ("cond1 = ", cond1)
print ("cond2 = ", cond2)
print ("cond3 = ", cond3)
print ("cond4 = ", cond4)

import numpy as np

negative_in_objective = [x for x in objective if x < 0]

while negative_in_objective != [] :

  max_negative = min([y for y in negative_in_objective])
  print ("max negative = "  , max_negative)

  num_max1 = objective.index(max_negative)
  num_max2 = [num_max1]
  num_max = num_max2[0]

  Pivot_column = [objective[num_max], cond1[num_max], cond2[num_max],
cond3[num_max], cond4[num_max]]
  print ("Pivot column = ", Pivot_column)

  Ratio = np.array(RHS[0:]) / np.array(Pivot_column[0:])
  print ("Ratio = ", Ratio)

  positive_in_Ratio = np.array(Ratio)
  positive_in_Ratio = [b for b in Ratio if b > 0]
  smallest_positive = np.min(positive_in_Ratio)
  print ("smallest positive in Ratio = ", smallest_positive)
```

```
num_small1 = np.where(Ratio == smallest_positive)
num_small2 = num_small1[0]
num_small = num_small2[0]

Pivot_row = all_module[num_small]
print ("Pivot raw = ", Pivot_row)

Pivot_elemant = Pivot_row[num_max]
print ("Pivot element = ", Pivot_elemant)

Pivot_Equation = [c / Pivot_elemant for c in Pivot_row]
print ("Pivot equation = ", Pivot_Equation)

new_objective1 = [Pivot_column[0] * z for z in Pivot_Equation]
objective = [d - f for d , f in zip (objective , new_objective1)]
print ("objective = ", objective)

if 1 == num_small:
  cond1 = Pivot_Equation
else :
  new_cond11 = [Pivot_column[1] * a for a in Pivot_Equation]
  cond1 = [a - b for a , b in zip (cond1 , new_cond11)]
print ("cond1 = ", cond2)
if 2 == num_small:
  cond2 = Pivot_Equation
else :
  new_cond21 = [Pivot_column[2] * a for a in Pivot_Equation]
  cond2 = [a - b for a , b in zip (cond2 , new_cond21)]
print ("cond2 = ", cond2)
if 3 == num_small:
  cond3 = Pivot_Equation
else :
  new_cond31 = [Pivot_column[3] * a for a in Pivot_Equation]
  cond3 = [a - b for a , b in zip (cond3 , new_cond31)]
print ("cond3 = ", cond3)
if 4 == num_small:
  cond4 = Pivot_Equation
else :
  new_cond41 = [Pivot_column[4] * a for a in Pivot_Equation]
  cond4 = [a - b for a , b in zip (cond4 , new_cond41)]
print ("cond4 = ", cond4)
RHS = [objective[-1], cond1[-1], cond2[-1], cond3[-1], cond4[-1]]
all_module = [objective, cond1, cond2, cond3, cond4]
negative_in_objective = [x for x in objective if x < 0]
print ("——————————————————")

print ("the last objective and last conds are obtimum solutions")
```

The solution from python is

```
objective =  [-5, -4, 0, 0, 0, 0, 0]
cond1 =  [6, 4, 1, 0, 0, 0, 24]
cond2 =  [1, 2, 0, 1, 0, 0, 6]
cond3 =  [-1, 1, 0, 0, 1, 0, 1]
cond4 =  [0, 1, 0, 0, 0, 1, 2]
max negative =  -5
Pivot column =  [-5, 6, 1, -1, 0]
main.py:34: RuntimeWarning: divide by zero encountered in divide
  Ratio = np.array(RHS[0:]) / np.array(Pivot_column[0:])
Ratio =  [-0.  4.  6. -1. inf]
smallest positive in Ratio =  4.0
Pivot raw =  [6, 4, 1, 0, 0, 0, 24]
Pivot element =  6
Pivot equation =  [1.0, 0.6666666666666666, 0.16666666666666666, 0.0, 0.0, 0.0, 4.0]
objective =  [0.0, -0.666666666666667, 0.8333333333333333, 0.0, 0.0, 0.0, 20.0]
cond1 =  [1, 2, 0, 1, 0, 0, 6]
cond2 =  [0.0, 1.3333333333333335, -0.16666666666666666, 1.0, 0.0, 0.0, 2.0]
cond3 =  [0.0, 1.6666666666666665, 0.16666666666666666, 0.0, 1.0, 0.0, 5.0]
cond4 =  [0.0, 1.0, 0.0, 0.0, 0.0, 1.0, 2.0]
________________________________
max negative =  -0.666666666666667
Pivot column = [-0.666666666666667, 0.6666666666666666, 1.3333333333333335, 1.6666666666666665, 1.0]
Ratio =  [-30.    6.    1.5   3.    2. ]
smallest positive in Ratio =  1.4999999999999998
Pivot raw =  [0.0, 1.3333333333333335, -0.16666666666666666, 1.0, 0.0, 0.0, 2.0]
Pivot element =  1.3333333333333335
Pivot equation = [0.0, 1.0, -0.12499999999999999, 0.7499999999999999, 0.0, 0.0, 1.4999999999999998]
objective =  [0.0, 0.0, 0.7499999999999999, 0.5000000000000001, 0.0, 0.0, 21.0]
cond1 =  [0.0, 1.3333333333333335, -0.16666666666666666, 1.0, 0.0, 0.0, 2.0]
cond2 =  [0.0, 1.0, -0.12499999999999999, 0.7499999999999999, 0.0, 0.0, 1.4999999999999998]
cond3 =  [0.0, 0.0, 0.37499999999999994, -1.2499999999999998, 1.0, 0.0, 2.5000000000000004]
cond4 =  [0.0, 0.0, 0.12499999999999999, -0.7499999999999999, 0.0, 1.0, 0.5000000000000002]
________________________________
the last objective and last conds are obtimum solutions
```

2.3 Solving linear programming issues of type minimization

In the case of the $min\ Z$, that is, when the signs of restrictions are of more than or equal $\geq$, equal $=$, or both, then the Simplex model is solved. We use one of the following two styles :

- The (Big – M) method
- Two – phase method [2] .

2.3.1 The (Big – M) method

The method (Big – M) depends on the addition of artificial variables alongside the slack variables to the restrictions of the linear programming model and the objective function that the variables artificial in the objective function are associated with very large transactions called (M) and these laboratories bear a positive signal in the case (Min) and a negative signal in (Max) [10] .

2.3.1.1 The steps of the simplex by The (Big – M) method [2] :

Step 1 : Transforming the linear programming model from the canonical formula to the standard formula, after adding slack variables (S_i) to the restrictions of the model and objective function. Then the addition of artificial variables (R_i) to restrictions and target function also requires.

Step 2 : Drafting a new objective function (Z), in terms of variables (x_i) and (S_i), taking into account the function is equal to the value of (M) only.

Step 3 : Design the possible basic solution schedule, depending on all variable transactions (R_i, S_i, x_i) found in model restrictions and objective function

Step 4 : Determining the entering variable, on the basis of the largest positive value in the objective function Z row .

Step 5 : Adoption of the rest of the previous and mentioned steps in the case of maximization, when all C_i transactions are a new objective function in the solution schedule, less or equal to zero, i.e. $(C_i \leq 0)$, which means the optimal solution was obtained.

Example 2.2 [10] :

$$Min\ Z = 4x_1 + x_2$$

Subject to

$$3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2, \geq 0$$

Solution :

Step 1 :

$$Min\ Z = 4x_1 + x_2 + MR_1 + MR_2 + 0S_1 + 0S_2$$

Subject to

$$3x_1 + x_2 + R_1 = 3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$4x_1 + 3x_2 - S_1 + R_2 = 6 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$x_1 + 2x_2 + S_2 = 4$$

$$x_1, x_2, R_1, S_1, S_2, R_1, R_2 \geq 0$$

Step 2 :

From (1) and (2) we obtain :

$$R_1 = 3 - 3x_1 - x_2 \quad \text{and} \quad R_2 = 6 - 4x_1 - 3x_2 + S_1$$

Now but R_1 and R_2 in objective function , we obtain :

$$Min\ Z = 4x_1 + x_2 + M(3 - 3x_1 - x_2) + M(6 - 4x_1 - 3x_2 + S_1) + 0S_2$$

$$Min\ Z = 4x_1 + x_2 + 3M - 3Mx_1 - Mx_2 + 6M - 4Mx_1 - 3Mx_2 + MS_1 + 0S_2$$

$$Min\ Z = 4x_1 - 3Mx_1 - 4Mx_1 + x_2 - Mx_2 - 3Mx_2 + MS_1 + 0S_2 + 3M + 6M$$

$$Min\ Z = 4x_1 - 7Mx_1 + x_2 - 4Mx_2 + MS_1 + 0S_2 + 9M$$

$$Min\ Z = (4 - 7M)x_1 + (1 - 4M)x_2 + MS_1 + 0S_2 + 9M$$

$$Min\ Z - (4 - 7M)x_1 - (1 - 4M)x_2 - MS_1 - 0S_2 = 9M$$

Step 3 :

Table 2.7

Basic	Z	x_1	x_2	S_1	S_2	R_1	R_2	b_i	Ratio
Z	1	$-4+7M$	$-1+4M$	$-M$	0	0	0	$9M$	
R_1	0	3	1	0	0	1	0	3	
R_2	0	4	3	-1	0	0	1	6	
S_2	0	1	2	0	1	0	0	4	

Step 4 : **Table 2.8**

Basic	Z	x_1	x_2	S_1	S_2	R_1	R_2	b_i	Ratio
Z	1	$-4+7M$	$-1+4M$	$-M$	0	0	0	$9M$	
R_1	0	3	1	0	0	1	0	3	1
R_2	0	4	3	-1	0	0	1	6	$\frac{3}{2}$
S_2	0	1	2	0	1	0	0	4	4

Step 5 :

New x_1 row = Current R_1 row ÷ 3

$$=\tfrac{1}{3}(0 \quad 3 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 3)$$

$$=\left(0 \quad 1 \quad \tfrac{1}{3} \quad 0 \quad 0 \quad \tfrac{1}{3} \quad 0 \quad 1\right)$$

New Z row = Current Z row $-(-4+7M)$ New x_1 row

$$=(1 \quad -4+7M \quad -1+4M \quad -M \quad 0 \quad 0 \quad 0 \quad 9M)$$

$$-(-4+7M)\times\left(0 \quad 1 \quad \frac{1}{3} \quad 0 \quad 0 \quad \frac{1}{3} \quad 0 \quad 1\right)$$

$$=(1 \quad -4+7M \quad -1+4M \quad -M \quad 0 \quad 0 \quad 0 \quad 9M)$$

$$+\left(0 \quad 4-7M \quad \frac{4}{3}-\frac{7M}{3} \quad 0 \quad 0 \quad \frac{4}{3}-\frac{7M}{3} \quad 0 \quad 4-7M\right)$$

$$=\left(1 \quad 0 \quad \tfrac{1}{3}+\tfrac{5}{3}M \quad -M \quad 0 \quad \tfrac{4}{3}-\tfrac{7}{3}M \quad 0 \quad 4+2M\right)$$

New R_2 row = Current R_2 row $-(4)\times$ New x_1 row

$$= (0 \quad 4 \quad 3 \quad -1 \quad 0 \quad 0 \quad 1 \quad 6) - (4)\times\left(0 \quad 1 \quad \tfrac{1}{3} \quad 0 \quad 0 \quad \tfrac{1}{3} \quad 0 \quad 1\right)$$

$$= \left(0 \quad 0 \quad \tfrac{5}{3} \quad -1 \quad 0 \quad -\tfrac{1}{3} \quad 1 \quad 2\right)$$

New S_2 row = Current S_2 row $-(1)\times$ New x_1 row

$$= (0 \quad 1 \quad 2 \quad 0 \quad 1 \quad 0 \quad 0 \quad 4) - (1)\times\left(0 \quad 1 \quad \tfrac{1}{3} \quad 0 \quad 0 \quad \tfrac{1}{3} \quad 0 \quad 1\right)$$

$$= \left(0 \quad 0 \quad \tfrac{5}{3} \quad 0 \quad 1 \quad -\tfrac{1}{3} \quad 0 \quad 3\right)$$

Table 2.9

Basic	Z	x_1	x_2	S_1	S_2	R_1	R_2	b_i	Ratio
Z	1	0	$\frac{1}{3}+\frac{5}{3}M$	$-M$	0	$\frac{4}{3}-\frac{7}{3}M$	0	$4+2M$	
x_1	0	1	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0	1	3
R_2	0	0	$\frac{5}{3}$	-1	0	$-\frac{1}{3}$	1	2	$\frac{6}{5}$
S_2	0	0	$\frac{5}{3}$	0	1	$-\frac{1}{3}$	0	3	$\frac{9}{5}$

New x_2 row = Current R_2 row $\div \frac{5}{3}$

$$= \tfrac{3}{5}\left(0 \quad 0 \quad \tfrac{5}{3} \quad -1 \quad 0 \quad -\tfrac{1}{3} \quad 1 \quad 2\right)$$

$$= \left(0 \quad 0 \quad 1 \quad -\tfrac{3}{5} \quad 0 \quad -\tfrac{1}{5} \quad \tfrac{3}{5} \quad \tfrac{6}{5}\right)$$

New Z row = Current Z row $-\left(\frac{1}{3}+\frac{5}{3}M\right)$ New x_2 row

$$= \left(1 \quad 0 \quad \tfrac{1}{3}+\tfrac{5}{3}M \quad -M \quad 0 \quad \tfrac{4}{3}-\tfrac{7}{3}M \quad 0 \quad 4+2M\right)$$

$$-\left(\tfrac{1}{3}+\tfrac{5}{3}M\right)\times\left(0 \quad 0 \quad 1 \quad -\tfrac{3}{5} \quad 0 \quad -\tfrac{1}{5} \quad \tfrac{3}{5} \quad \tfrac{6}{5}\right)$$

$$= \left(1 \quad 0 \quad \tfrac{1}{3}+\tfrac{5}{3}M \quad -M \quad 0 \quad \tfrac{4}{3}-\tfrac{7}{3}M \quad 0 \quad 4+2M\right)$$

$$+\left(0 \quad 0 \quad -\tfrac{1}{3}-\tfrac{5}{3}M \quad \tfrac{1}{5}+M \quad 0 \quad \tfrac{1}{15}+\tfrac{1}{3}M \quad -\tfrac{1}{5}-M \quad -\tfrac{2}{5}-2M\right)$$

$$= \left(1 \quad 0 \quad 0 \quad \tfrac{1}{5} \quad 0 \quad \tfrac{21}{15}-2M \quad -\tfrac{1}{5}-M \quad \tfrac{18}{5}\right)$$

New x_1 row = Current x_1 row $-\left(\frac{1}{3}\right)\times$ New x_2 row

$$=\begin{pmatrix}0 & 1 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 1\end{pmatrix}-\left(\frac{1}{3}\right)\times\begin{pmatrix}0 & 0 & 1 & -\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} & \frac{6}{5}\end{pmatrix}$$

$$=\begin{pmatrix}0 & 1 & 0 & \frac{1}{5} & 0 & \frac{2}{5} & -\frac{1}{5} & \frac{3}{5}\end{pmatrix}$$

New S_2 row = Current S_2 row $-(1)\times$ New x_2 row

$$=\begin{pmatrix}0 & 0 & \frac{5}{3} & 0 & 1 & -\frac{1}{3} & 0 & 3\end{pmatrix}-\left(\frac{5}{3}\right)\times\begin{pmatrix}0 & 0 & 1 & -\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} & \frac{6}{5}\end{pmatrix}$$

$$=\begin{pmatrix}0 & 0 & 0 & -1 & 1 & -\frac{2}{3} & -1 & 1\end{pmatrix}$$

Table 2.10

Basic	Z	x_1	x_2	S_1	S_2	R_1	R_2	b_i	Ratio
Z	1	0	0	$\frac{1}{5}$	0	$\frac{21}{15}-2M$	$-\frac{1}{5}-M$	$\frac{18}{5}$	
x_1	0	1	0	$\frac{1}{5}$	0	$\frac{2}{5}$	$-\frac{1}{5}$	$\frac{3}{5}$	3
x_2	0	0	1	$-\frac{3}{5}$	0	$-\frac{1}{5}$	$\frac{3}{5}$	$\frac{6}{5}$	-2
S_2	0	0	0	-1	1	$-\frac{2}{3}$	-1	1	-1

New S_1 row = Current x_1 row $\div \frac{1}{5}$

$$=5\begin{pmatrix}0 & 1 & 0 & \frac{1}{5} & 0 & \frac{2}{5} & -\frac{1}{5} & \frac{3}{5}\end{pmatrix}$$

$$=(0 \quad 5 \quad 0 \quad 1 \quad 0 \quad 2 \quad -1 \quad 3)$$

New Z row = Current Z row $-\left(\frac{1}{5}\right)$ New S_1 row

$$=\begin{pmatrix}1 & 0 & 0 & \frac{1}{5} & 0 & \frac{21}{15}-2M & -\frac{1}{5}-M & \frac{18}{5}\end{pmatrix}$$

$$-\left(\frac{1}{5}\right)\times(0 \quad 5 \quad 0 \quad 1 \quad 0 \quad 2 \quad -1 \quad 3)$$

$$=\begin{pmatrix}1 & 0 & 0 & \frac{1}{5} & 0 & \frac{21}{15}-2M & -\frac{1}{5}-M & \frac{18}{5}\end{pmatrix}$$

$$+\begin{pmatrix}0 & -1 & 0 & -\frac{1}{5} & 0 & -\frac{2}{5} & \frac{1}{5} & -\frac{3}{5}\end{pmatrix}$$

$$=(1 \quad -1 \quad 0 \quad 0 \quad 0 \quad 1-2M \quad -M \quad 3)$$

New x_2 row = Current x_2 row $-\left(-\frac{5}{3}\right)\times$ New S_1 row

$$=\left(0\quad 0\quad 1\quad -\frac{5}{3}\quad 0\quad \frac{5}{2}\quad -\frac{1}{5}\quad \frac{5}{3}\right)-\left(-\frac{5}{3}\right)\times(0\quad 5\quad 0\quad 1\quad 0\quad 2\quad -1\quad 3)$$

$$=\left(0\quad \frac{25}{3}\quad 1\quad 0\quad 0\quad \frac{35}{6}\quad -\frac{28}{15}\quad \frac{20}{3}\right)$$

New S_2 row = Current S_2 row $-(1)\times$ New S_1 row

$$=\left(0\ 0\ 0\ -1\ 1\ -\frac{2}{3}\ -1\ 1\right)-(-1)\times(0\ \ 5\ 0\ 1\ \ 0\ 2\ -1\ 3)$$

$$=\left(0\ \ 5\ \ 0\ \ 0\ 1\ \frac{4}{3}\ -2\ \ 4\right)$$

Table 2.11

Basic	Z	x_1	x_2	S_1	S_2	R_1	R_2	b_i	Ratio
Z	1	-1	0	0	0	$1-2M$	$-M$	3	
S_1	0	5	0	1	0	2	-1	3	
x_2	0	$\frac{25}{3}$	1	0	0	$\frac{35}{6}$	$-\frac{28}{15}$	$\frac{20}{3}$	
S_2	0	5	0	0	1	$\frac{4}{3}$	-2	4	

From table 2.11 we obtain the Z row is less than or equal zero then it is the optimal solution where $x_1 = 0$, $x_2 = \frac{20}{3}$, $Z = 3$

2.3.1.2 Algorithm (Big-M) in python [of the researcher's work]

If M = 100

```
objective = [696, 399, -100, 0, 0, 0, 900]
cond1 = [3, 1, 0, 0, 1, 0, 3]
cond2 = [4, 3, -1, 0, 0, 1, 6]
cond3 = [1, 2, 0, 1, 0, 0, 4]

RHS = [objective[-1], cond1[-1], cond2[-1], cond3[-1]]
all_module = [objective, cond1, cond2, cond3]

print ("objective = ", objective)
print ("cond1 = ", cond1)
print ("cond2 = ", cond2)
print ("cond3 = ", cond3)
import numpy as np
positive_in_objective = [x for x in objective[:-1] if x > 0]
```

```
while positive_in_objective != [] :
  max_positive = max([y for y in positive_in_objective])
  print ("max positive = " , max_positive)
  num_max1 = objective.index(max_positive)
  num_max2 = [num_max1]
  num_max = num_max2[0]

  Pivot_column = [objective[num_max], cond1[num_max], cond2[num_max],
cond3[num_max]]
  print ("Pivot column = ", Pivot_column)

  Ratio = np.array(RHS[0:]) / np.array(Pivot_column[0:])
  print ("Ratio = ", Ratio)

  positive_in_Ratio = np.array(Ratio)
  positive_in_Ratio = [b for b in Ratio if b > 0]
  smallest_positive = np.min(positive_in_Ratio)
  print ("smallest positive in Ratio = ", smallest_positive)

  num_small1 = np.where(Ratio == smallest_positive)
  num_small2 = num_small1[0]
  num_small = num_small2[0]

  Pivot_row = all_module[num_small]
  print ("Pivot raw = ", Pivot_row)

  Pivot_elemant = Pivot_row[num_max]
  print ("Pivot element = ", Pivot_elemant)

  Pivot_Equation = [c / Pivot_elemant for c in Pivot_row]
  print ("Pivot equation = ", Pivot_Equation)

  new_objective1 = [Pivot_column[0] * z for z in Pivot_Equation]
  objective = [d - f for d , f in zip (objective , new_objective1)]
  print ("objective = ", objective)

  if 1 == num_small:
    cond1 = Pivot_Equation
  else :
    new_cond11 = [Pivot_column[1] * a for a in Pivot_Equation]
    cond1 = [a - b for a , b in zip (cond1 , new_cond11)]
  print ("cond1 = ", cond2)
  if 2 == num_small:
    cond2 = Pivot_Equation
  else :
    new_cond21 = [Pivot_column[2] * a for a in Pivot_Equation]
    cond2 = [a - b for a , b in zip (cond2 , new_cond21)]
  print ("cond2 = ", cond2)
  if 3 == num_small:
    cond3 = Pivot_Equation
  else :
    new_cond31 = [Pivot_column[3] * a for a in Pivot_Equation]
```

```
  cond3 = [a - b for a , b in zip (cond3 , new_cond31)]
 print ("cond3 = ", cond3)

 RHS = [objective[-1], cond1[-1], cond2[-1], cond3[-1]]
 all_module = [objective, cond1, cond2, cond3]
 positive_in_objective = [x for x in objective[:-1] if x > 0]
 print ("————————————————————")

print ("the last objective and last conds are obtimum solutions")
```

The solution from python is

```
objective =  [696, 399, -100, 0, 0, 0, 900]
cond1 =  [3, 1, 0, 0, 1, 0, 3]
cond2 =  [4, 3, -1, 0, 0, 1, 6]
cond3 =  [1, 2, 0, 1, 0, 0, 4]
max positive =  696
Pivot column =  [696, 3, 4, 1]
Ratio =  [1.29310345 1.         1.5        4.        ]
smallest positive in Ratio =  1.0
Pivot raw =  [3, 1, 0, 0, 1, 0, 3]
Pivot element =  3
Pivot equation =  [1.0, 0.3333333333333333, 0.0, 0.0, 0.3333333333333333, 0.0, 1.0]
objective =  [0.0, 167.0, -100.0, 0.0, -232.0, 0.0, 204.0]
cond1 =  [4, 3, -1, 0, 0, 1, 6]
cond2 =  [0.0, 1.6666666666666667, -1.0, 0.0, -1.3333333333333333, 1.0, 2.0]
cond3 =  [0.0, 1.6666666666666667, 0.0, 1.0, -0.3333333333333333, 0.0, 3.0]
————————————————————
max positive =  167.0
Pivot column =  [167.0, 0.3333333333333333, 1.6666666666666667, 1.6666666666666667]
Ratio =  [1.22155689 3.         1.2        1.8       ]
smallest positive in Ratio =  1.2
Pivot raw =  [0.0, 1.6666666666666667, -1.0, 0.0, -1.3333333333333333, 1.0, 2.0]
Pivot element =  1.6666666666666667
Pivot equation =  [0.0, 1.0, -0.6, 0.0, -0.7999999999999999, 0.6, 1.2]
objective =  [0.0, 0.0, 0.20000000000000284, 0.0, -98.4, -100.2, 3.5999999999999943]
cond1 =  [0.0, 1.6666666666666667, -1.0, 0.0, -1.3333333333333333, 1.0, 2.0]
cond2 =  [0.0, 1.0, -0.6, 0.0, -0.7999999999999999, 0.6, 1.2]
cond3 =  [0.0, 0.0, 1.0, 1.0, 1.0, -1.0, 1.0]
————————————————————
max positive =  0.20000000000000284
Pivot column =  [0.20000000000000284, 0.19999999999999998, -0.6, 1.0]
Ratio =  [18.  3. -2.  1.]
smallest positive in Ratio =  1.0
Pivot raw =  [0.0, 0.0, 1.0, 1.0, 1.0, -1.0, 1.0]
Pivot element =  1.0
Pivot equation =  [0.0, 0.0, 1.0, 1.0, 1.0, -1.0, 1.0]
objective =  [0.0, 0.0, 0.0, -0.20000000000000284, -98.60000000000001, -100.0,
3.3999999999999915]
cond1 =  [0.0, 1.0, -0.6, 0.0, -0.7999999999999999, 0.6, 1.2]
cond2 =  [0.0, 1.0, 0.0, 0.6, -0.19999999999999996, 0.0, 1.7999999999999998]
cond3 =  [0.0, 0.0, 1.0, 1.0, 1.0, -1.0, 1.0]
————————————————————
the last objective and last conds are obtimum solutions
```

2.3.2 Two – phase method

Artificial variables in Method M may lead to a round computer error. The dual -phase method removes the use of the fix completely, and this is done in two phases: the first stage tries to find a possible basic solution and if the basic solution is found, the second stage is called to solve the original problem [10] .

2.3.2.1 The first phase [16] :

Here the goal function appears only with artificial variables and with one factor and excludes all other variables from the goal function (whether the basic variables or stagnant variables), and the MAX target function appears with artificial variables and transactions (-1) .

$$Min\ Z = R_1 + R_2 + \cdots + R_m$$

$$Max\ Z = -R_1 - R_2 - \cdots - R_m$$

The first phase ends in Max and Min when the objective function is equal to zero, we move to the second stage, but if the objective function is not zero then there is no solution to the question of written programming .

2.3.2.2 The steps of first phase [17] :

Step 1 :

Add Slack variables and artificial variables to the restrictions of the model .

Step 2 :

Create a new objective function in terms of artificial variables .

Step 3 :

Considering the artificial variables as basic variables, and the basic and Slack variables as non-essential variables, then finding the artificial variables in terms of the basic and stagnant variables and replacing them in the new objective function.

Step 4 :

Solve the model using the previous Simplex method (following the Simplex steps), if we find that the value of the objective function is equal to zero, then this means that we have reached an initial solution, and thus we move to the second phase, but if the value of the objective function is greater than zero, then this means that there is no solution to the model and thus ends the solution .

2.3.2.3 The second phase :

To solve the original problem, we start from the possible solution that we reached in the first stage[10].

2.3.2.4 The steps of second phase [17] :

Step 1 :

We express the essential variables in terms of the non-essential variables, depending on the data of the last table that we reached in the first phase.

Step 2 :

We replace the row of the objective function in the last table that we reached in the first phase with the original objective function and exclude the data of the artificial variables.

Step 3 :

We use the same steps in the Simplex method to find the optimal solution

Example 2.3 [17] :

$$Max\ Z = 3x_1 + 2x_2$$

Subject :

$$3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Solution :

First phase

Step 1 :

$$Max\ Z = 3x_1 + 2x_2$$

$$3x_1 + x_2 + R_1 = 3$$

$$4x_1 + 3x_2 - S_1 + R_2 = 6$$

$$x_1 + 2x_2 + S_2 = 3$$

$$x_1, x_2, S_1, S_2, R_1, R_2 \geq 0$$

Step 2 :

$$Min\ Z = R_1 + R_2$$

$$3x_1 + x_2 + R_1 = 3 \ldots\ldots (1)$$

$$4x_1 + 3x_2 - S_1 + R_2 = 6 \ldots\ldots (2)$$

$$x_1 + 2x_2 + S_2 = 3$$

$$x_1, x_2, x_3, S_1, S_2, R_1, R_2 \geq 0$$

Step 3 :

From (1) and (2) we obtain

$$R_1 = 3 - 3x_1 - x_2$$

$$R_2 = 6 - 4x_1 - 3x_2 + S_1$$

Now we put R_1 and R_2 in objective function

$$Min\ Z = 3 - 3x_1 - x_2 + 6 - 4x_1 - 3x_2 + S_1$$

$$Min\ Z = -3x_1 - 4x_1 - x_2 - 3x_2 + S_1 + 3 + 6$$

$$Min\ Z = -7x_1 - 4x_2 + S_1 + 9$$

$$Min\ Z + 7x_1 + 4x_2 - S_1 = 9$$

Table 2.12

Basic	Z	x_1	x_2	S_1	S_2	R_1	R_2	b_i	Ratio
Z	1	7	4	-1	0	0	0	9	
R_1	0	3	1	0	0	1	0	3	
R_2	0	4	3	-1	0	0	1	6	
S_2	0	1	2	0	1	0	0	3	

Step 4 :

Table 2.13

Basic	Z	x_1	x_2	S_1	S_2	R_1	R_2	b_i	Ratio
Z	1	7	4	-1	0	0	0	9	
R_1	0	3	1	0	0	1	0	3	1
R_2	0	4	3	-1	0	0	1	6	$\frac{3}{2}$
S_2	0	1	2	0	1	0	0	3	3

New x_1 row = Current R_1 row ÷ 3

$$= \frac{1}{3}(0 \quad 3 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 3)$$

$$= \left(0 \quad 1 \quad \frac{1}{3} \quad 0 \quad 0 \quad \frac{1}{3} \quad 0 \quad 1\right)$$

New Z row = Current Z row $-(7)$ New x_1 row

$$= (1 \quad 7 \quad 4 \quad -1 \quad 0 \quad 0 \quad 0 \quad 9) - (7) \times \left(0 \quad 1 \quad \tfrac{1}{3} \quad 0 \quad 0 \quad \tfrac{1}{3} \quad 0 \quad 1\right)$$

$$= \left(1 \quad 0 \quad \tfrac{5}{3} \quad -1 \quad 0 \quad -\tfrac{7}{3} \quad 0 \quad 2\right)$$

New R_2 row = Current R_2 row $-(4) \times$ New x_1 row

$$= (0 \quad 4 \quad 3 \quad -1 \quad 0 \quad 0 \quad 1 \quad 6) - (4) \times \left(0 \quad 1 \quad \tfrac{1}{3} \quad 0 \quad 0 \quad \tfrac{1}{3} \quad 0 \quad 1\right)$$

$$= \left(0 \quad 0 \quad \tfrac{5}{3} \quad -1 \quad 0 \quad -\tfrac{4}{3} \quad 1 \quad 2\right)$$

New S_2 row = Current S_2 row $-(1) \times$ New x_1 row

$$= (0 \quad 1 \quad 2 \quad 0 \quad 1 \quad 0 \quad 0 \quad 3) - (1) \times \left(0 \quad 1 \quad \tfrac{1}{3} \quad 0 \quad 0 \quad \tfrac{1}{3} \quad 0 \quad 1\right)$$

$$= \left(0 \quad 0 \quad \tfrac{5}{3} \quad 0 \quad 1 \quad -\tfrac{1}{3} \quad 0 \quad 2\right)$$

Table 2.14

Basic	Z	x_1	x_2	S_1	S_2	R_1	R_2	b_i	Ratio
Z	1	0	$\frac{5}{3}$	-1	0	$-\frac{7}{3}$	0	2	
x_1	0	1	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0	1	3
R_2	0	0	$\frac{5}{3}$	-1	0	$-\frac{4}{3}$	1	2	$\frac{6}{5}$
S_2	0	0	$\frac{5}{3}$	0	1	$-\frac{1}{3}$	0	2	$\frac{6}{5}$

New x_2 row = Current R_2 row $\div \frac{5}{3}$

$$= \tfrac{3}{5}\left(0 \quad 0 \quad \tfrac{5}{3} \quad -1 \quad 0 \quad -\tfrac{4}{3} \quad 1 \quad 2\right)$$

$$= \left(0 \quad 0 \quad 1 \quad -\tfrac{3}{5} \quad 0 \quad -\tfrac{4}{5} \quad \tfrac{3}{5} \quad \tfrac{6}{5}\right)$$

New Z row = Current Z row $-\left(\frac{5}{3}\right)$ New x_2 row

$$=\left(1 \quad 0 \quad \frac{5}{3} \quad -1 \quad 0 \quad -\frac{7}{3} \quad 0 \quad 2\right)-\left(\frac{5}{3}\right)\times\left(0 \quad 0 \quad 1 \quad -\frac{3}{5} \quad 0 \quad -\frac{4}{5} \quad \frac{3}{5} \quad \frac{6}{5}\right)$$

$$=(1 \quad 0 \quad 0 \quad 0 \quad 0 \quad -1 \quad -1 \quad 0)$$

New x_1 row = Current x_1 row $-\left(\frac{1}{3}\right)\times$ New x_2 row

$$=\left(0 \quad 1 \quad \frac{1}{3} \quad 0 \quad 0 \quad \frac{1}{3} \quad 0 \quad 1\right)-\left(\frac{1}{3}\right)\times\left(0 \quad 0 \quad 1 \quad -\frac{3}{5} \quad 0 \quad -\frac{4}{5} \quad \frac{3}{5} \quad \frac{6}{5}\right)$$

$$=\left(0 \quad 1 \quad 0 \quad \frac{1}{5} \quad 0 \quad \frac{3}{5} \quad -\frac{1}{5} \quad \frac{3}{5}\right)$$

New S_2 row = Current S_2 row $-(1)\times$ New x_2 row

$$=\left(0 \quad 0 \quad \frac{5}{3} \quad 0 \quad 1 \quad -\frac{1}{3} \quad 0 \quad 2\right)-\left(\frac{5}{3}\right)\times\left(0 \quad 0 \quad 1 \quad -\frac{3}{5} \quad 0 \quad -\frac{4}{5} \quad \frac{3}{5} \quad \frac{6}{5}\right)$$

$$=(0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad -1 \quad 0)$$

Table 2.15

Basic	Z	x_1	x_2	S_1	S_2	R_1	R_2	b_i	Ratio
Z	1	0	0	0	0	-1	-1	0	
x_1	0	1	0	$\frac{1}{5}$	0	$\frac{3}{5}$	$-\frac{1}{5}$	$\frac{3}{5}$	
x_2	0	0	1	$-\frac{3}{5}$	0	$-\frac{4}{5}$	$\frac{3}{5}$	$\frac{6}{5}$	
S_2	0	0	0	1	1	1	-1	0	

From table 2.15 we show $Z = 0$ this mean $R_1 = 0$, $R_2 = 0$ and there exist optimum solution for objective function . now we use steps of second phase .

Second phase

Step 1 :

From data of table 2.15 we obtain

$$x_1 + \frac{1}{5}S_1 + \frac{3}{5}R_1 - \frac{1}{5}R_2 = \frac{3}{5}$$

$$x_1 = \frac{3}{5} - \frac{1}{5}S_1 - \frac{3}{5}R_1 + \frac{1}{5}R_2$$

$$x_1 = \frac{1}{5}(3 - S_1 - 3R_1 + R_2)$$

Where $R_1 = 0$, $R_2 = 0$ then

$$x_1 = \frac{1}{5}(3 - S_1)$$

And

$$x_2 - \frac{3}{5}S_1 - \frac{4}{5}R_1 + \frac{3}{5}R_2 = \frac{6}{5}$$

$$x_2 = \frac{6}{5} + \frac{3}{5}S_1 + \frac{4}{5}R_1 - \frac{3}{5}R_2$$

$$x_2 = \frac{1}{5}(6 + 3S_1 + 4R_1 - 3R_2)$$

Where $R_1 = 0$, $R_2 = 0$ then

$$x_2 = \frac{1}{5}(6 + 3S_1)$$

Step 2 :

$$Max\ Z = 3x_1 + 2x_2$$

$$Max\ Z = 3\left(\frac{1}{5}(3 - S_1)\right) + 2\left(\frac{1}{5}(6 + 3S_1)\right)$$

$$Max\ Z = \frac{3}{5}(3 - S_1) + \frac{2}{5}(6 + 3S_1)$$

$$Max\ Z = \frac{9}{5} - \frac{3}{5}S_1 + \frac{12}{5} + \frac{6}{5}S_1$$

$$Max\ Z = \frac{21}{5} + \frac{3}{5}S_1$$

$$Max\ Z - \frac{3}{5}S_1 = \frac{21}{5}$$

Table 2.16

Basic	Z	x_1	x_2	S_1	S_2	b_i	Ratio
Z	1	0	0	$-\frac{3}{5}$	0	$\frac{21}{5}$	
x_1	0	1	0	$\frac{1}{5}$	0	$\frac{3}{5}$	
x_2	0	0	1	$-\frac{3}{5}$	0	$\frac{6}{5}$	
S_2	0	0	0	1	1	0	

Step 3 :

Table 2.17

Basic	Z	x_1	x_2	S_1	S_2	b_i	Ratio
Z	1	0	0	$-\frac{3}{5}$	0	$\frac{21}{5}$	
x_1	0	1	0	$\frac{1}{5}$	0	$\frac{3}{5}$	3
x_2	0	0	1	$-\frac{3}{5}$	0	$\frac{6}{5}$	-2
S_2	0	0	0	1	1	0	0

New S_1 row = Current S_2 row ÷ 1

$= 1(0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0)$

$= (0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0)$

New Z row = Current Z row $-\left(-\frac{3}{5}\right)$ New S_1 row

$$= \left(1 \quad 0 \quad 0 \quad -\frac{3}{5} \quad 0 \quad \frac{21}{5}\right) - \left(-\frac{3}{5}\right) \times (0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0)$$

$$= \left(1 \quad 0 \quad 0 \quad 0 \quad \frac{3}{5} \quad \frac{21}{5}\right)$$

New x_1 row = Current x_1 row $-\left(\frac{1}{5}\right)\times$ New S_1 row

$$=\begin{pmatrix}0 & 1 & 0 & \frac{1}{5} & 0 & \frac{3}{5}\end{pmatrix}-\left(\frac{1}{5}\right)\times(0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0)$$

$$=\begin{pmatrix}0 & 1 & 0 & 0 & -\frac{1}{5} & \frac{3}{5}\end{pmatrix}$$

New x_2 row = Current x_2 row $-\left(-\frac{3}{5}\right)\times$ New S_1 row

$$=\begin{pmatrix}0 & 0 & 1 & -\frac{3}{5} & 0 & \frac{6}{5}\end{pmatrix}-\left(-\frac{3}{5}\right)\times(0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0)$$

$$=\begin{pmatrix}0 & 0 & 1 & 0 & \frac{3}{5} & \frac{6}{5}\end{pmatrix}$$

Table 2.18

Basic	Z	x_1	x_2	S_1	S_2	b_i	Ratio
Z	1	0	0	0	$\frac{3}{5}$	$\frac{21}{5}$	
x_1	0	1	0	0	$-\frac{1}{5}$	$\frac{3}{5}$	
x_2	0	0	1	0	$\frac{3}{5}$	$\frac{6}{5}$	
S_1	0	0	0	1	1	0	

From the table 2.18 it becomes clear to us that all the values of Z row are positive and there is no negative value, and thus represents a table 2.18 the best solution and can be explained as follows :

$$Max\ Z=\frac{21}{5}$$

$$x_1=\frac{3}{5}$$

$$x_2=\frac{6}{5}$$

2.3.2.5 Algorithm of Tow-phase [of the researcher's work]

```
objective = [7, 4, -1, 0, 0, 0, 9]
cond1 = [3, 1, 0, 0, 1, 0, 3]
cond2 = [4, 3, -1, 0, 0, 1, 6]
cond3 = [1, 2, 0, 1, 0, 0, 3]

RHS = [objective[-1], cond1[-1], cond2[-1], cond3[-1]]
all_module = [objective, cond1, cond2, cond3]

print ("objective = ", objective)
print ("cond1 = ", cond1)
print ("cond2 = ", cond2)
print ("cond3 = ", cond3)

import numpy as np
positive_in_objective = [x for x in objective[:-1] if x > 0]

while positive_in_objective != [] :

  max_positive = max([y for y in positive_in_objective])
  print ("max positive = " , max_positive)

  num_max1 = objective.index(max_positive)
  num_max2 = [num_max1]
  num_max = num_max2[0]

  Pivot_column = [objective[num_max], cond1[num_max], cond2[num_max],
cond3[num_max]]
  print ("Pivot column = ", Pivot_column)

  Ratio = np.array(RHS[0:]) / np.array(Pivot_column[0:])
  print ("Ratio = ", Ratio)

  positive_in_Ratio = np.array(Ratio)
  positive_in_Ratio = [b for b in Ratio if b > 0]
  smallest_positive = np.min(positive_in_Ratio)
  print ("smallest positive in Ratio = ", smallest_positive)

  num_small1 = np.where(Ratio == smallest_positive)
  num_small2 = num_small1[0]
  num_small = num_small2[0]

  print ("index smallest positive = ", num_small)

  Pivot_row = all_module[num_small]
  print ("Pivot raw = ", Pivot_row)

  Pivot_elemant = Pivot_row[num_max]
  print ("Pivot element = ", Pivot_elemant)
```

```
Pivot_Equation = [c / Pivot_elemant for c in Pivot_row]
print ("Pivot equation = ", Pivot_Equation)

new_objective1 = [Pivot_column[0] * z for z in Pivot_Equation]
objective = [d - f for d , f in zip (objective , new_objective1)]
print ("objective = ", objective)

if 1 == num_small:
  cond1 = Pivot_Equation
else :
  new_cond11 = [Pivot_column[1] * a for a in Pivot_Equation]
  cond1 = [a - b for a , b in zip (cond1 , new_cond11)]
print ("cond1 = ", cond2)
if 2 == num_small:
  cond2 = Pivot_Equation
else :
  new_cond21 = [Pivot_column[2] * a for a in Pivot_Equation]
  cond2 = [a - b for a , b in zip (cond2 , new_cond21)]
print ("cond2 = ", cond2)
if 3 == num_small:
  cond3 = Pivot_Equation
else :
  new_cond31 = [Pivot_column[3] * a for a in Pivot_Equation]
  cond3 = [a - b for a , b in zip (cond3 , new_cond31)]
print ("cond3 = ", cond3)

RHS = [objective[-1], cond1[-1], cond2[-1], cond3[-1]]
all_module = [objective, cond1, cond2, cond3]
positive_in_objective = [x for x in objective[:-1] if x > 0]
print ("——————————————————")

if objective[-1] != 0 :
  print ("no solution")
else :
  print ("there exist solution")
  print ("——————————————————")
  objective = [0, 0, -3/5, 0, 21/5]
  cond1 = [1, 0, 1/5, 0, 3/5]
  cond2 = [0, 1, -3/5, 0, 6/5]
  cond3 = [0, 0, 1, 1, 0]

  RHS = [objective[-1], cond1[-1], cond2[-1], cond3[-1]]
  all_module = [objective, cond1, cond2, cond3]

  print ("objective = ", objective)
  print ("cond1 = ", cond1)
  print ("cond2 = ", cond2)
  print ("cond3 = ", cond3)

  import numpy as np
  negative_in_objective = [x for x in objective if x < 0]
```

```
while negative_in_objective != [] :
  max_negative = min([y for y in negative_in_objective])
  print ("max negative = " , max_negative)

  num_max1 = objective.index(max_negative)
  num_max2 = [num_max1]
  num_max = num_max2[0]

  Pivot_column = [objective[num_max], cond1[num_max], cond2[num_max],
cond3[num_max]]
  print ("Pivot column = ", Pivot_column)

  Ratio = np.array(RHS[0:]) / np.array(Pivot_column[0:])
  print ("Ratio = ", Ratio)

  positive_in_Ratio = np.array(Ratio)
  positive_in_Ratio = [b for b in Ratio if b > 0]
  smallest_positive = np.min(positive_in_Ratio)
  print ("smallest positive in Ratio = ", smallest_positive)

  num_small1 = np.where(Ratio == smallest_positive)
  num_small2 = num_small1[0]
  num_small = num_small2[0]

  Pivot_row = all_module[num_small]
  print ("Pivot raw = ", Pivot_row)

  Pivot_elemant = Pivot_row[num_max]
  print ("Pivot element = ", Pivot_elemant)

  Pivot_Equation = [c / Pivot_elemant for c in Pivot_row]
  print ("Pivot equation = ", Pivot_Equation)

  new_objective1 = [Pivot_column[0] * z for z in Pivot_Equation]
  objective = [d - f for d , f in zip (objective , new_objective1)]
  print ("objective = ", objective)

  if 1 == num_small:
    cond1 = Pivot_Equation
  else :
    new_cond11 = [Pivot_column[1] * a for a in Pivot_Equation]
    cond1 = [a - b for a , b in zip (cond1 , new_cond11)]
  print ("cond1 = ", cond2)
  if 2 == num_small:
    cond2 = Pivot_Equation
  else :
    new_cond21 = [Pivot_column[2] * a for a in Pivot_Equation]
    cond2 = [a - b for a , b in zip (cond2 , new_cond21)]
  print ("cond2 = ", cond2)
  if 3 == num_small:
    cond3 = Pivot_Equation
  else :
```

```
    new_cond31 = [Pivot_column[3] * a for a in Pivot_Equation]
    cond3 = [a - b for a , b in zip (cond3 , new_cond31)]
  print ("cond3 = ", cond3)
  RHS = [objective[-1], cond1[-1], cond2[-1], cond3[-1]]
  all_module = [objective, cond1, cond2, cond3]
  negative_in_objective = [x for x in objective if x < 0]
  print ("——————————————————")

print ("the last objective and last conds are obtimum solutions")
```

The solution from python is

```
objective =  [7, 4, -1, 0, 0, 0, 9]
cond1 =  [3, 1, 0, 0, 1, 0, 3]
cond2 =  [4, 3, -1, 0, 0, 1, 6]
cond3 =  [1, 2, 0, 1, 0, 0, 3]
max positive =  7
Pivot column =  [7, 3, 4, 1]
Ratio =  [1.28571429 1.         1.5        3.        ]
smallest positive in Ratio =  1.0
index smallest positive =  1
Pivot raw =  [3, 1, 0, 0, 1, 0, 3]
Pivot element =  3
Pivot equation =  [1.0, 0.3333333333333333, 0.0, 0.0, 0.3333333333333333, 0.0, 1.0]
objective =  [0.0, 1.666666666666667, -1.0, 0.0, -2.333333333333333, 0.0, 2.0]
cond1 =  [4, 3, -1, 0, 0, 1, 6]
cond2 =  [0.0, 1.6666666666666667, -1.0, 0.0, -1.3333333333333333, 1.0, 2.0]
cond3 =  [0.0, 1.6666666666666667, 0.0, 1.0, -0.3333333333333333, 0.0, 2.0]
————————————————
max positive =  1.666666666666667
Pivot column =  [1.666666666666667, 0.3333333333333333, 1.6666666666666667,
1.6666666666666667]
Ratio =  [1.2 3.  1.2 1.2]
smallest positive in Ratio =  1.1999999999999997
index smallest positive =  0
Pivot raw =  [0.0, 1.666666666666667, -1.0, 0.0, -2.333333333333333, 0.0, 2.0]
Pivot element =  1.666666666666667
Pivot equation =  [0.0, 1.0, -0.5999999999999999, 0.0, -1.3999999999999995, 0.0,
1.1999999999999997]
objective =  [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
cond1 =  [0.0, 1.6666666666666667, -1.0, 0.0, -1.3333333333333333, 1.0, 2.0]
cond2 =  [0.0, 0.0, -2.220446049250313e-16, 0.0, 0.9999999999999993, 1.0, 4.440892098500626e-
16]
cond3 =  [0.0, 0.0, 0.9999999999999998, 1.0, 1.9999999999999993, 0.0, 4.440892098500626e-16]
————————————————
there exist solution
————————————————
objective =  [0, 0, -0.6, 0, 4.2]
cond1 =  [1, 0, 0.2, 0, 0.6]
cond2 =  [0, 1, -0.6, 0, 1.2]
cond3 =  [0, 0, 1, 1, 0]
max negative =  -0.6
Pivot column =  [-0.6, 0.2, -0.6, 1]
Ratio =  [-7.  3. -2.  0.]
smallest positive in Ratio =  2.9999999999999996
```

```
Pivot raw =  [1, 0, 0.2, 0, 0.6]
Pivot element =  0.2
Pivot equation =  [5.0, 0.0, 1.0, 0.0, 2.9999999999999996]
objective =  [3.0, 0.0, 0.0, 0.0, 6.0]
cond1 =  [0, 1, -0.6, 0, 1.2]
cond2 =  [3.0, 1.0, 0.0, 0.0, 2.9999999999999996]
cond3 =  [-5.0, 0.0, 0.0, 1.0, -2.9999999999999996]
______________________________
the last objective and last conds are obtimum solutions
```

Conclusions and Future work :

Conclusions :

1- Linear programming is very effective in describing many problems in life, especially those related to increasing profit or reducing cost.

2- There are many ways to solve linear programming problems, including the algebraic method, the graph method, and the Simplex method

3- The Simplex method is an effective way to solve linear programming problems for two or more variables and for any number of constraints.

4- An algorithm for the Simplex method can be written in Python as it is a high-level programming language that is free and easy to learn.

Future work :

1- Conducting studies of problems in the field of economics and others, using linear programming to describe these problems.

2- Conducting studies of other methods such as the algebraic method and graphing and developing their algorithms in Python.

3- Conducting studies of the Simplex method and developing its algorithm in Python and other programming languages such as C++.

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جمهورية العراق
وزارة التعليم العالي والبحث العلمي
جامعة بابل
كلية التربية للعلوم الصرفة
قسم الرياضيات

تطوير نموذج البرمجة الخطية على أساس طريقة مبسطه

بحث تقدم به الطالب

عقيل جبار عبود سعيد

الى مجلس كلية التربية للعلوم الصرفة في جامعة بابل
وهو جزء من متطلبات نيل درجة الدبلوم العالي / الرياضيات

بإشراف

أ.د. أحمد صباح الجيلاوي

2023 م **1444 هـ**

الملخص

قدمت الدراسة وصف جيد لمسائل البرمجة الخطية وكيفية حلها بطريقة Simplex وهي طريقة فعالة لحل مسائل البرمجة الخطية بأكثر من متغيرين ولأي عدد من القيود . كما تم استعراض اسلوبي Big-M وذات المرحلتين . وقام الباحث بتطوير خوارزمية Simplex باستخدام لغة Python وهي لغة واسعة الانتشار وتستخدم في تطوير البرمجيات والتعلم الالي والخوارزميات الرياضية كما انها لغة برمجة عالية المستوى وسهلة التعلم .