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Numerical Optimization Concepts and Applications in Engineering

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By

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بسم الله الرحمن الرحيم

(اللَّـهُ الَّذي رَفَعَ السَّماواتِ بِغَيرِ عَمَدٍ تَرَونَها ثُمَّ استَوى عَلَى العَرشِ وَسَخَّرَ الشَّمسَ وَالقَمَرَ كُلٌّ يَجري لِأَجَلٍ مُسَمًّى يُدَبِّرُ الأَمرَ يُفَصِّلُ الآياتِ لَعَلَّكُم بِلِقاءِ رَبِّكُم توقِنونَ). [الرعد، آية: ٢]

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Dedication

To my family

Acknowledgments

I thank God Almighty first, and I thank all my professors in the College of Education for Pure Sciences - University of Babylon, and I would like to extend my sincere thanks to Professor Dr. Ahmed Sabah Al-Jilawi for his continuous support.

Abstract

This research explores the basic concepts and applications of numerical optimization in the field of engineering. The study begins with an introduction to the topic, highlighting the importance of numerical optimization in solving complex engineering problems. It delves into various aspects of optimization, including linear programming, nonlinear programming, numerical optimization, and approximate methods. In fact, the first chapter included an introduction and related works, including numerical optimization within applied mathematics, in addition to linear and nonlinear programming and approximation methods in brief. In the second chapter, the research provides a comprehensive overview of the key concepts of optimization. It discusses convexity, optimal solution, feasible area, writing and diagramming of a function, the rule, and bound and unrestricted formulas. This chapter also presents optimal conditions that help determine the most effective solution to optimization problems. The third chapter focuses on practical applications of optimization in engineering. It studies how to apply numerical optimization techniques in various engineering fields, such as Global Positioning System (GPS), structural optimization, process optimization, control optimization, product design optimization, and energy optimization as well as other applications. The fourth chapter includes numerical optimization methods for linear programming. Finally, the conclusion and future work are included in chapter five.

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Chapter 1

Exploring Numerical Optimization

1.1 Introduction

Numerical optimization is the process of finding the optimal solution to a mathematical problem by minimizing or maximizing a given function. The history of numerical optimization dates back to ancient times when mathematicians and scientists used optimization methods to solve problems related to geometry, physics, and astronomy. However, the development of modern optimization algorithms and techniques began in the 20th century with the advent of computers and advanced mathematical theories .The earliest optimization techniques were based on trial and error methods, where mathematicians would guess the solution to a problem and then refine it through successive iterations. One of the earliest documented optimization problems is the method of exhaustion used by ancient Greek mathematicians to approximate the area of a circle. This method involved inscribing and circumscribing polygons around the circle to obtain increasingly accurate approximations of its area. During the Renaissance, mathematicians such as Leonardo da Vinci, Niccolo Tartaglia, and Gerolamo Cardano developed methods for solving optimization problems using algebraic equations, [1,2,3]. However, it wasn't until the 18th century that the calculus-based optimization methods were developed. One of the earliest optimization algorithms based on calculus was the steepest descent method, which was proposed by the French mathematician Joseph Louis Lagrange in 1762 .In the 19th century, mathematicians and scientists began to apply optimization techniques to a wide range of practical problems. One notable example was the development of the simplex method for linear programming by the American mathematician George Dantzig in the 1940. This algorithm revolutionized optimization by enabling the solution of large-scale optimization problems that were previously considered intractable .In the mid-20th century, the development of digital computers enabled the use of numerical optimization algorithms for solving complex problems in engineering, physics, and economics. One of the most important optimization algorithms developed during this time was the gradient descent method, which was first proposed by Richard Bellman and later refined by David Rumelhart and James McClelland in the 1980. The field of numerical optimization continued to evolve throughout the latter half of the 20th century with the development of new optimization algorithms and techniques such as simulated annealing, genetic algorithms, and particle swarm optimization. These algorithms were developed to address the limitations of traditional optimization methods and to handle complex, multi-objective optimization problems.

In recent years, advances in machine learning and artificial intelligence have led to the development of new optimization techniques such as stochastic gradient descent and reinforcement learning. These algorithms are used extensively in deep learning and neural network training, as well as other machine learning applications .In conclusion, the history of numerical optimization is a long and fascinating journey that has spanned centuries and involved the contributions of many great mathematicians, scientists, and engineers. From the ancient Greeks to the modern-day machine learning algorithms, optimization has played a crucial role in advancing our understanding of the world and solving practical problems. The development of new optimization techniques and algorithms continues to be an active area of research, and it is likely that many more breakthroughs will be made in the years to come, [4,5]. In general the numerical optimization procedure can be summarized by diagram shown in Figure 1:

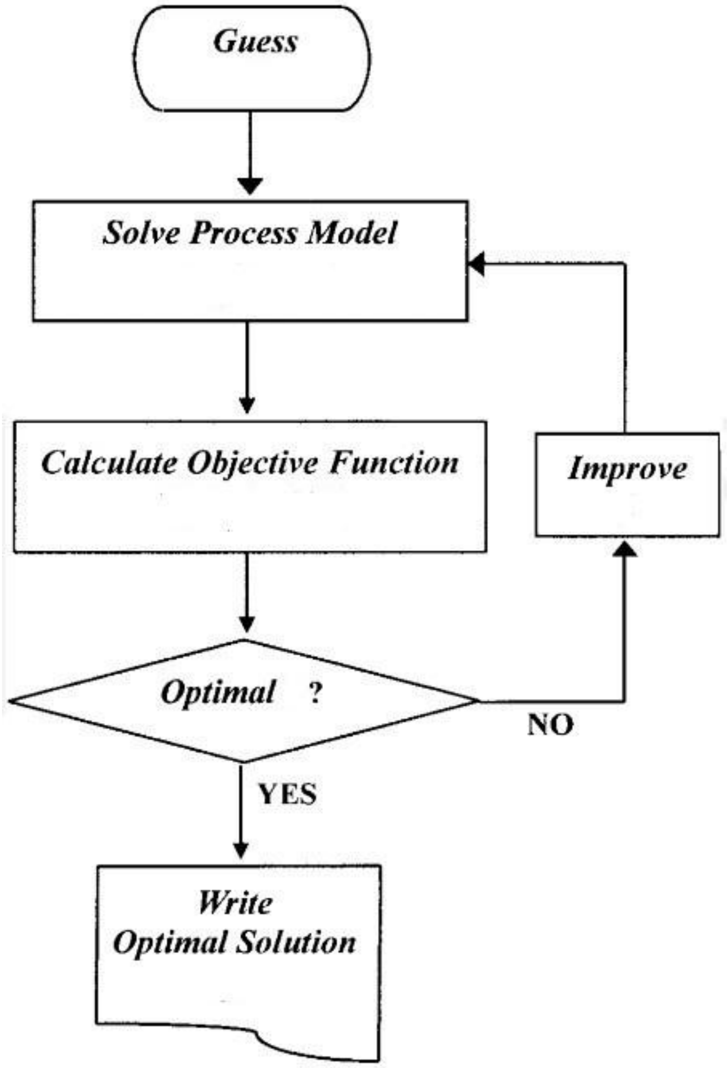


Figure 1: Numerical optimization procedure.

1.2 Numerical Optimization and Applied Math

Numerical optimization and applied mathematics are two closely related fields that play a crucial role in modern science, engineering, and technology. These fields are concerned with the development and application of mathematical models and algorithms to solve complex problems in various domains, such as finance, engineering, physics, and computer science. Numerical optimization refers to the process of finding the best solution to a problem among a set of feasible solutions. This process involves using mathematical models and algorithms to search for an optimal solution that satisfies a given set of constraints and objectives. The goal of numerical optimization is to minimize or maximize an objective function subject to certain constraints. This process is essential in various fields such as finance, engineering, and physics, where optimization problems arise frequently. Applied mathematics, on the other hand, is a field of mathematics that deals with the application of mathematical methods and models to real-world problems. This field involves the development of mathematical models and algorithms to solve practical problems in various domains such as biology, physics, engineering, and finance. Applied mathematics uses various mathematical techniques such as differential equations, linear algebra, probability theory, and numerical analysis to develop models that can be used to solve complex problems. The relationship between numerical optimization and applied mathematics is fundamental. Numerical optimization relies on mathematical models and algorithms developed in applied mathematics to solve complex problems. Applied mathematics, on the other hand, relies on numerical optimization to find the best solution to a given problem. Therefore, the two fields are closely related and work together to solve complex problems in various domains. One of the most important applications of numerical optimization and applied mathematics is in finance. In finance, the goal is to maximize profits while minimizing risks. This requires the development of mathematical models and algorithms that can be used to optimize investment portfolios, calculate risk, and make predictions about future market trends. Numerical optimization is essential in finance, as it allows investors to find the best investment strategy that maximizes profits while minimizing risks. Applied mathematics, on the other hand, is essential in finance as it provides the mathematical tools to develop these models and algorithms. Finally, numerical optimization and applied mathematics are two closely related fields that play a crucial role in modern science, engineering, and technology. These fields provide the mathematical models and algorithms needed to solve complex problems in various domains such as finance, engineering, and physics. The relationship between the two fields is fundamental, as they work together to solve complex problems and find the best solution to a given problem. Numerical optimization and applied mathematics will continue to play a crucial role in solving complex problems and advancing science and technology in the years to come. Numerical optimization is a branch of applied mathematics that deals with finding the optimal solution to a given problem through the use of mathematical algorithms and techniques. The field of numerical optimization is closely related to various other areas of applied mathematics, including calculus, linear algebra, probability theory, and statistics. The application of numerical optimization techniques can be seen in a wide range of fields, including engineering, finance, physics, computer science, and machine learning. One of the primary objectives of numerical optimization is to find the best solution to a given problem with a minimum amount of time and computational resources. This is achieved through the use of mathematical models and algorithms that can efficiently search for the optimal solution. One of the most commonly used optimization algorithms is the gradient descent method, which is widely used in machine learning and optimization problems involving large datasets. Numerical optimization and applied mathematics have a broad range of applications across many fields, including physics, engineering, computer science, biology, and many more. In physics, for example, numerical optimization is used to find the optimal solution to complex problems in areas such as quantum mechanics and relativity. In engineering, numerical optimization is used to design and optimize complex systems such as aircraft, automobiles, and buildings. Applied mathematics is essential in developing the mathematical models and algorithms needed to solve these problems. In computer science, numerical optimization and applied mathematics are used to solve problems related to artificial intelligence, machine learning, and data analysis. For example, machine learning algorithms use numerical optimization to find the best model parameters that minimize the error between predicted and actual outcomes. In data analysis, applied mathematics is used to develop statistical models that can be used to analyze and interpret large datasets. In biology, numerical optimization and applied mathematics are used to model complex biological systems such as gene regulatory networks, metabolic pathways, and population dynamics. For example, numerical optimization is used to find the best parameter values for models of protein interactions and metabolic pathways. Applied mathematics is essential in developing these models and algorithms and in interpreting the results. Overall, numerical optimization and applied mathematics have numerous applications across various fields and are essential in solving complex problems. The development of new numerical optimization methods and mathematical models will continue to be a critical area of research in the future, as the demand for solving complex problems in various domains continues to grow. As new fields and new problems emerge, the relationship between numerical optimization and applied mathematics will remain crucial in advancing science and technology. One of the significant challenges in numerical optimization and applied mathematics is the trade-off between computational efficiency and accuracy. As the complexity of the problem increases, the computational requirements also increase exponentially. Therefore, it is often necessary to develop new and more efficient algorithms to solve these problems .Moreover, there is a significant need to develop robust optimization algorithms that can handle uncertainties and noisy data. In many real-world applications, the data may not be accurate or complete, and the models may be subject to various uncertainties. Therefore, it is essential to develop algorithms that can handle such uncertainties and provide robust and reliable solutions. In recent years, there has been a significant interest in developing optimization algorithms inspired by natural phenomena such as genetic algorithms, swarm intelligence, and ant colony optimization. These algorithms are based on the principles of natural evolution and social behavior and have shown promising results in solving complex optimization problems. Finally, the field of numerical optimization and applied mathematics is constantly evolving, with new methods and techniques being developed regularly. Researchers are continuously working to develop new mathematical models and optimization algorithms that can be used to solve complex problems more efficiently and accurately. As a result, the field is growing rapidly, and there is a significant demand for skilled professionals in this area. Numerical optimization and applied mathematics are two fundamental fields that play a crucial role in solving complex problems in various domains. The relationship between these two fields is fundamental, as they work together to develop mathematical models and algorithms to find the best solution to a given problem. The challenges in this area are significant, and there is a need to develop new and more efficient optimization algorithms to handle uncertainties and provide robust solutions. With the rapid advancement of science and technology, the field of numerical optimization and applied mathematics is expected to continue growing and playing a critical role in solving complex problems, [6,7,8].

1.3 Optimization

Optimization, also known as mathematical programming, collection of mathematical principles and methods used for solving quantitative problems in many disciplines, including physics, biology, engineering, economics, and business. The subject grew from a realization that quantitative problems in manifestly different disciplines have important mathematical elements in common. Because of this commonality, many problems can be formulated and solved by using the unified set of ideas and methods that make up the field of optimization. The historical term mathematical programming, broadly synonymous with optimization, was coined in the 1940s before programming became equated with computer programming. Mathematical programming includes the study of the mathematical structure of optimization problems, the invention of methods for solving these problems, the study of the mathematical properties of these methods, and the implementation of these methods on computers. Faster computers have greatly expanded the size and complexity of optimization problems that can be solved. The development of optimization techniques has paralleled advances not only in computer science but also in operations research, numerical analysis, game theory, mathematical economics, control theory, and combinatorics. Optimization problems typically have three fundamental elements. The first is a single numerical quantity, or objective function, that is to be maximized or minimized. The objective may be the expected return on a stock portfolio, a company's production costs or profits, the time of arrival of a vehicle at a specified destination, or the vote share of a political candidate. The second element is a collection of variables, which are quantities whose values can be manipulated in order to optimize the objective. Examples include the quantities of stock to be bought or sold, the amounts of various resources to be allocated to different production activities, the route to be followed by a vehicle through a traffic network, or the policies to be advocated by a candidate. The third element of an optimization problem is a set of constraints, which are restrictions on the values that the variables can take. For instance, a manufacturing process cannot require more resources than are available, nor can it employ less than zero resources. Within this broad framework, optimization problems can have different mathematical properties. Problems in which the variables are continuous quantities (as in the resource allocation example) require a different approach from problems in which the variables are discrete or combinatorial quantities (as in the selection of a vehicle route from among a predefined set of possibilities.(An important class of optimization is known as linear programming. Linear indicates that no variables are raised to higher powers, such as squares. For this class, the problems involve minimizing (or maximizing) a linear objective function whose variables are real numbers that are constrained to satisfy a system of linear equalities and inequalities. Another important class of optimization is known as nonlinear programming. In nonlinear programming the variables are real numbers, and the objective or some of the constraints are nonlinear functions (possibly involving squares, square roots, trigonometric functions, or products of the variables). Both linear and nonlinear programming are discussed in this chapter. Other important classes of optimization problems not covered in this chapter include stochastic programming, in which the objective function or the constraints depend on random variables, so that the optimum is found in some "expected," or probabilistic, sense; network optimization, which involves optimization of some property of a flow through a network, such as the maximization of the amount of material that can be transported between two given locations in the network; and combinatorial optimization, in which the solution must be found among a finite but very large set of possible values, such as the many possible ways to assign 20 manufacturing plants to 20 locations, [1,3,5]. In general terms, the diagram in Figure 2 summarizes optimization classification techniques:

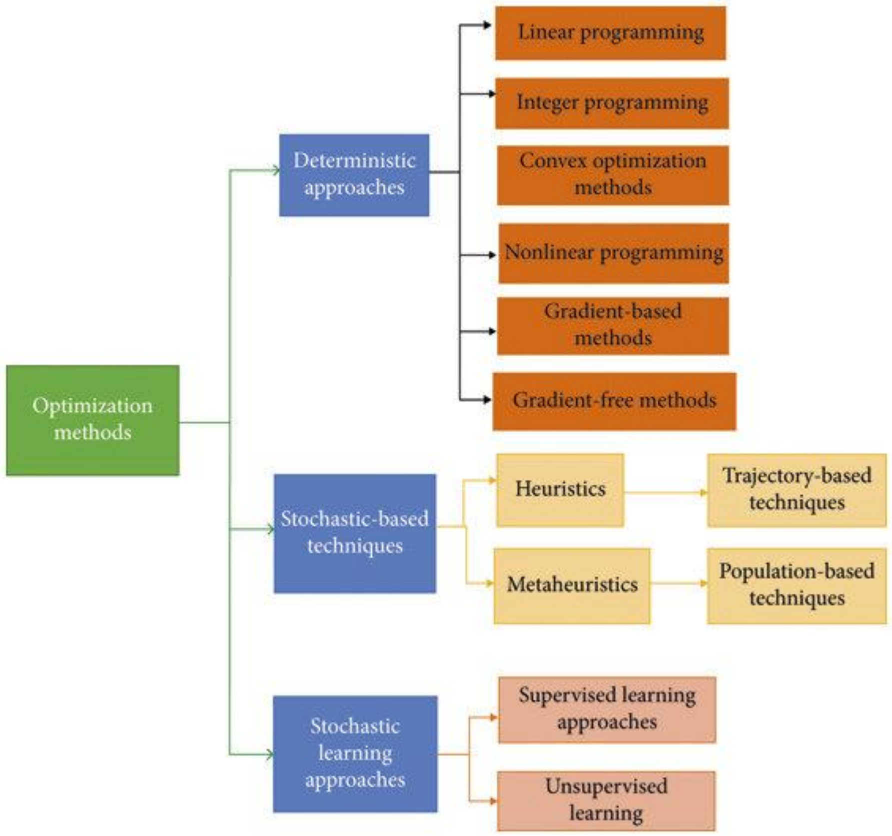


Figure 2: Optimization classification techniques.

1.4 Linear programming

Although widely used now to solve everyday decision problems, linear programming was comparatively unknown before 1947. No work of any significance was carried out before this date, even though the French mathematician Joseph Fourier seemed to be aware of the subject's potential as early as 1823. In 1939 a Russian mathematician, Leonid Vitalyevich Kantorovich, published an extensive monograph, Matematicheskie metody organizatsi i planirovaniya proizvodstva ("Mathematical Methods for Organization and Planning of Production"), which is now credited with being the first treatise to recognize that certain important broad classes of scheduling problems had well-defined mathematical structures. Unfortunately, Kantorovich's proposals remained mostly unknown both in the Soviet Union and elsewhere for nearly two decades. Meanwhile, linear programming had developed considerably in the United States and Western Europe. In the period following World War II, officials in the United States government came to believe that efficient coordination of the energies and resources of a whole nation in the event of nuclear war would require the use of scientific planning techniques. The advent of the computer made such an approach feasible. Intensive work began in 1947 in the U.S. Air Force. The linear programming model was proposed because it was relatively simple from a mathematical viewpoint, and yet it provided a sufficiently general and practical framework for representing interdependent activities that share scarce resources. In the linear programming model, the modeler views the system to be optimized as being made up of various activities that are assumed to require a flow of inputs (e.g., labour and raw materials) and outputs (e.g., finished goods and services) of various types proportional to the level of the activity. Activity levels are assumed to be representable by nonnegative numbers. The revolutionary feature of the approach lies in expressing the goal of the decision process in terms of minimizing or maximizing a linear objective function—for example, maximizing possible sorties in the case of the air force, or maximizing profits in industry. Before 1947 all practical planning was characterized by a series of authoritatively imposed rules of procedure and priorities. General objectives were never stated, probably because of the impossibility of performing the calculations necessary to minimize an objective function under constraints. In 1947 a method (described in the section The simplex method) was introduced that turned out to solve practical problems efficiently. Interest in linear programming grew rapidly, and by 1951 its use spread to industry. Today it is almost impossible to name an industry that is not using mathematical programming in some form, although the applications and the extent to which it is used vary greatly, even within the same industry. A simple problem in linear programming is one in which it is necessary to find the maximum (or minimum) value of a simple function subject to certain constraints. An example might be that of a factory producing two commodities. In any production run, the factory produces x1 of the first type and x_2 of the second. If the profit on the second type is twice that on the first, then $x1 + 2x_2$ represents the total profit. The function $x1 + 2x_2$ is known as the objective function.

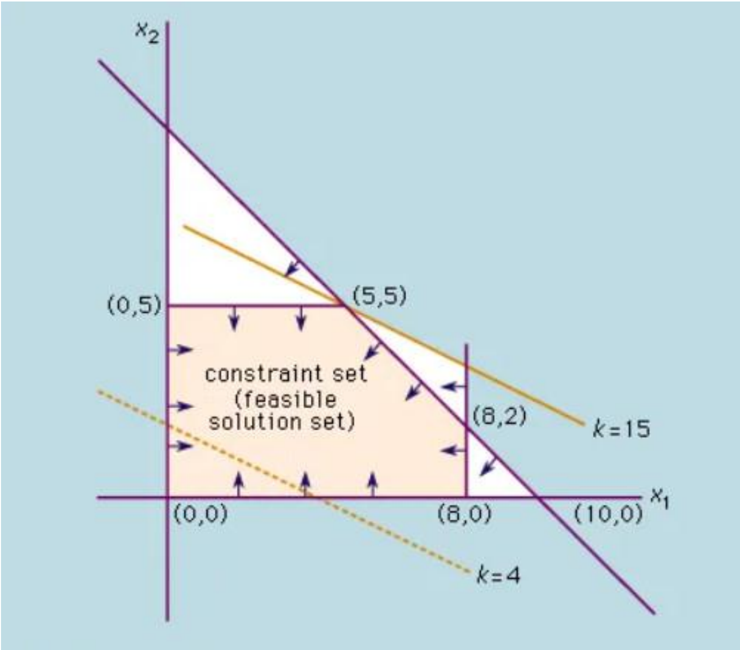


Figure (3) : Optimization problem.

Clearly the profit will be highest if the factory devotes its entire production capacity to making the second type of commodity. In a practical situation, however, this may not be possible; a set of constraints is introduced by such factors as availability of machine time, labour, and raw materials. For example, if the second type of commodity requires a raw material that is limited so that no more than five can be made in any batch, then x_2 must be less than or equal to five; i.e., $x_2 \leq 5$. If the first commodity requires another type of material limiting it to eight per batch, then $x_1 \leq 8$. If x_1 and x_2 take equal time to make and the machine time available allows a maximum of 10 to be made in a batch, then $x_1 + x_2$ must be less than or equal to 10; i.e., $x_1 + x_2 \leq 10$.

Two other constraints are that x_1 and x_2 must each be greater than or equal to zero, because it is impossible to make a negative number of either; i.e., $x_1 \geq 0$ and $x_2 \geq 0$.

The problem is to find the values of x_1 and x_2 for which the profit is a maximum. Any solution can be denoted by a pair of numbers (x_1, x_2); for example, if $x_1 = 3$ and $x_2 = 6$, the solution is (3, 6). These numbers can be represented by points plotted on two axes, as shown in the figure. On this graph the distance along the horizontal axis represents x_1 and that along the vertical represents x_2. Because of the constraints given above, the feasible solutions must lie within a certain well-defined region of the graph. For example in figure 3 , the constraint $x_1 \geq 0$ means that points representing feasible solutions lie on or to the right of the x_2 axis. Similarly, the constraint $x_2 \geq 0$ means that they also lie on or above the x_1 axis. Application of the entire set of constraints gives the feasible solution set, which is bounded by a polygon formed by the intersection of the lines $x_1 = 0$, $x_2 = 0$, $x_1 = 8$, $x_2 = 5$, and $x_1 + x_2 = 10$. For example, production of three items of commodity x_1 and four of x_2 is a feasible solution since the point (3, 4) lies in this region. To find the best solution, however, the objective function $x_1 + 2x_2 = k$ is plotted on the graph for some value of k, say k = 4. This value is indicated by the broken line in figure. As *k* is increased, a family of parallel lines are produced and the line for k = 15 just touches the constraint set at the point (5, 5). If k is increased further, the values of x_1 and x_2 will lie outside the set of feasible solutions. Thus, the best solution is that in which equal quantities of each commodity are made. It is no coincidence that an optimal solution occurs at a vertex, or "extreme point," of the region. This will always be true for linear problems, although an optimal solution may not be unique. Thus, the solution of such problems reduces to finding which extreme point (or points) yields the largest value for the objective function, [9,10].

1.5 Non-Linear Programming

Non-Linear problems (NLP) take the general shape of linear programming problems, except that the objective function and/or the constraint involve non-linear terms. Typically, seeking an exact solution to NLP problems is quite challenging. The different algorithms usually find an estimated answer within a permissible optimum error. In a few NLP problems, there is no accurate way to calculate the global maximum. A general optimization problem is to select n decision variables $x_1, x_2, \ldots, x_n$ from a given feasible region in such a way as to optimize (minimize or maximize) a given objective function

$$f\,(x_1, x_2, \ldots, x_n)$$

of the decision variables. The problem is called a nonlinear programming problem (NLP) if the objective function is nonlinear and/or thefeasible region is determined by nonlinear constraints, [10,11]. Thus, in maximization form, the general nonlinear program is stated as:

$$Maximize\ f\,(x_1, x_2, \ldots, x_n)،$$

subject to:

$$g1(x_1, x_2, \ldots, x_n) \leq b_1،$$

$$\ldots$$

$$\ldots$$

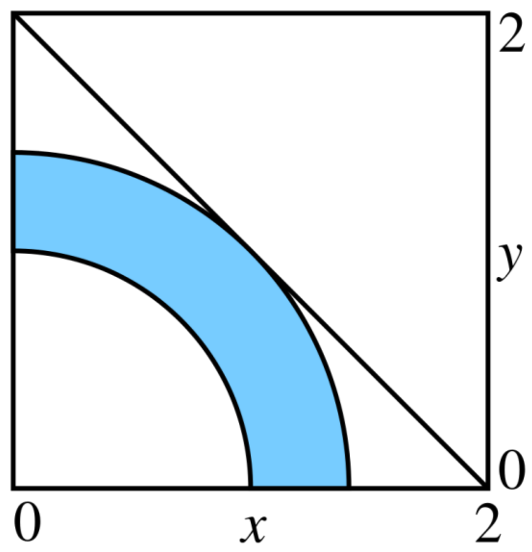
$$gm(x_1, x_2, \ldots, x_n) \leq b_m,$$

where each of the constraint functions g1 through g_m is given. A special case is the linear program that has been treated previously. The obvious association for this case is

$$f(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{n} c_j x_j,$$

And

$$g_i(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{n} a_{ij} x_j \qquad (i = 1, 2, \ldots, m).$$



Figure(4): Non-Linear programming (The blue region is the feasible region. The tangency of the line with the feasible region represents the solution. The line is the best achievable contour line (locus with a given value of the objective function).

1.6 Numerical Optimization

Numerical optimization is a mathematical technique used to find the values of one or more variables that minimize or maximize a given objective function. In other words, it is a process of finding the best possible solution to a problem, given a set of constraints. The objective function is a mathematical expression that maps the values of the variables to a scalar value that represents the performance or quality of the solution. The variables are typically subject to constraints, which are conditions that limit the values they can take. Numerical optimization methods use iterative algorithms to search for the values of the variables that minimize or maximize the objective function, while satisfying the constraints. These methods may be divided into two main categories: gradient-based methods and gradient-free methods. Gradient-based methods use information about the gradient of the objective function to guide the search for the optimal solution. Gradient-free methods, on the other hand, do not rely on gradient information and may be used for non-smooth or non-differentiable objective functions. Numerical optimization has numerous applications in various fields, including engineering, finance, economics, and machine learning, to name a few, [2,3,6].

1.7 Approximate Method

The approximate method of optimization is a technique used to find an approximate solution to an optimization problem when an exact solution is not feasible or is computationally expensive. It is often used in cases where the objective function is complex or expensive to evaluate, or when there are many constraints that make it difficult to find an exact solution. In approximate optimization, the objective function is approximated using a simpler or more computationally efficient function that closely approximates the original function. This approximation is then used to find the optimal solution to the problem using traditional optimization techniques, [12]. There are several types of approximate methods of optimization, including:

1- Linear programming relaxation: This method approximates a non-linear optimization problem as a linear programming problem, which is easier to solve.

2- Quasi-Newton methods: These methods approximate the Hessian matrix, which is a matrix of second-order partial derivatives of the objective function, using an iterative update. This approximation is used to guide the search for the optimal solution.

3- Randomized methods: These methods use randomization to find an approximate solution to the optimization problem. Examples include simulated annealing and genetic algorithms.

4- Gradient-free methods: These methods do not require gradient information and instead use search strategies such as random search or Nelder-Mead algorithm to find the optimal solution.

While approximate methods of optimization may not provide an exact solution to the problem, they can often provide a good solution that is close to the optimal solution. Additionally, they are often much faster and more computationally efficient than exact optimization methods, making them useful in many real-world applications.

Chapter 2

Basic Concept of Optimization

Introduction

Optimization is the process of finding the best solution among a set of possible solutions to a problem. The basic concept of optimization involves identifying the objective or goal of the problem, defining the decision variables or parameters that can be controlled, and determining the constraints or limitations that must be considered. In mathematical terms, optimization involves finding the values of the decision variables that maximize or minimize the objective function, subject to the constraints. The objective function is a mathematical expression that defines the goal of the problem, while the decision variables are the values that can be changed to achieve the goal. Optimization can be used in a wide range of fields, including engineering, finance, operations research, and management. In engineering, optimization is used to design and improve systems and processes to achieve desired performance while minimizing cost, time, and energy consumption. Optimization techniques include linear programming, nonlinear programming, integer programming, and dynamic programming, among others. These techniques involve mathematical algorithms that systematically search for the best solution among a large set of possible solutions. In this chapter we will review the most basic concepts of mathematical optimization, [13,14,15].

2.1 Convexity

Convexity refers to a property of certain mathematical functions or sets where a straight line segment connecting any two points on the function or set lies entirely above or on the function or set. In simpler terms, a convex shape is one that doesn't have any indentations or "dips" in it. Convexity has various applications in engineering, particularly in optimization and control systems.

1- **Convex Set:**

A set S is said to be convex if, for any two points x and y in S, the line segment connecting x and y lies entirely within S, see Fig.5. Mathematically, this can be expressed as:

$$\forall x, y \in S,\ \forall \theta \in [0, 1]: \theta x + (1-\theta)y \in S$$

1- Convex Function:

A real-valued function f: $\mathbb{R}^n \to \mathbb{R}$ is said to be convex if, for any two points x and y in the domain of f, the function value along the line segment connecting x and y lies below or on the line connecting the corresponding function values, see Fig.6. Mathematically, this can be expressed as:

$$\forall x, y \in dom\ (f),\ \forall \theta \in [0, 1]: f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta)f(y)$$

Example 1:

Consider the function $f(x) = 2x - 1$.To show that $f(x)$ is convex, we can use the definition of convexity. For any two points x_1 and x_2 and any $\theta \in [0, 1]$, we need to prove the following inequality:

$$f(\theta x_1 + (1-\theta)x_2) \leq \theta f(x_1) + (1-\theta)f(x_2)$$

Step 1: Choose two points, x_1 and x_2. Let's choose $x_1 = 1$ and $x_2 = 3$.

Step 2: Compute the function values at x_1 and x_2.

$f(x_1) = 2(1) - 1 = 1$, $f(x_2) = 2(3) - 1 = 5$

Step 3: Choose a value for θ between 0 and 1. Let's choose θ = 0.5.

Step 4: Compute the convex combination of x_1 and x_2 using θ.

$$\theta x_1 + (1-\theta)x_2 = 0.5(1) + (1-0.5)(3) = 0.5 + 0.5(3) = 0.5 + 1.5 = 2$$

Step 5: Compare the function value at the convex combination to the convex combination of the function values. $f(\theta x_1 + (1-\theta)x_2) = f(2) = 2(2) - 1 = \mathbf{3}$

and $\theta f(x_1) + (1-\theta)f(x_2) = 0.5(1) + (1-0.5)(5) = 0.5 + 0.5(5) = 0.5 + 2.5 = \mathbf{3}$

Now, we have $f(\theta x_1 + (1-\theta)x_2) = 3 = \theta f(x_1) + (1-\theta)f(x_2)$, satisfying the convexity condition.

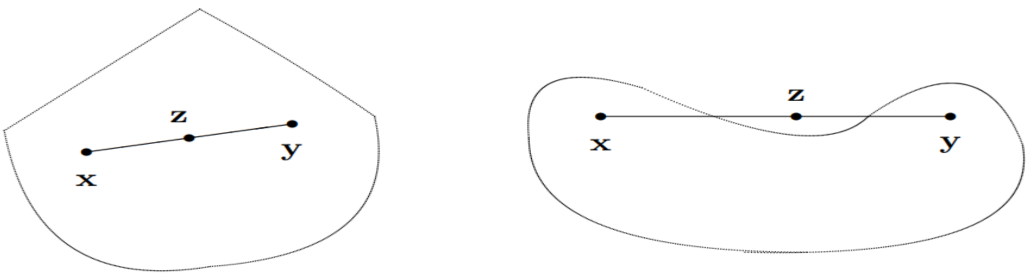


Figure ٥: A convex set (on the left) and a non-convex set (on the right).

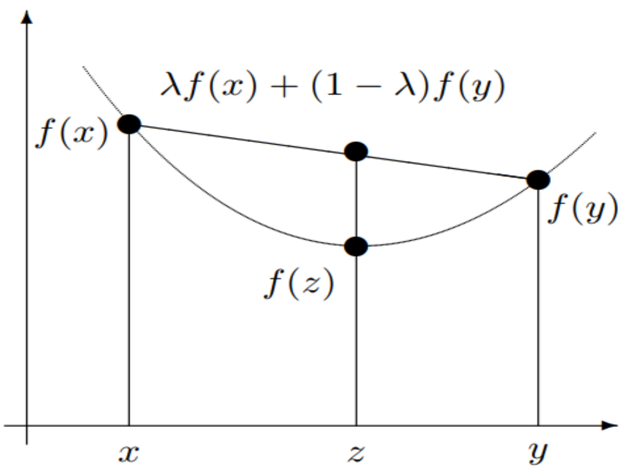


Figure 6: A convex function *f(x)*.

2.2 Optimal Solution

Optimal Solution is a solution to an optimization problem which minimizes (or maximizes) the objective function.

2.3 Objective Function

An objective function (Figure 7) is part of a linear programming optimization strategy, which finds the minimum or maximum of a linear function.

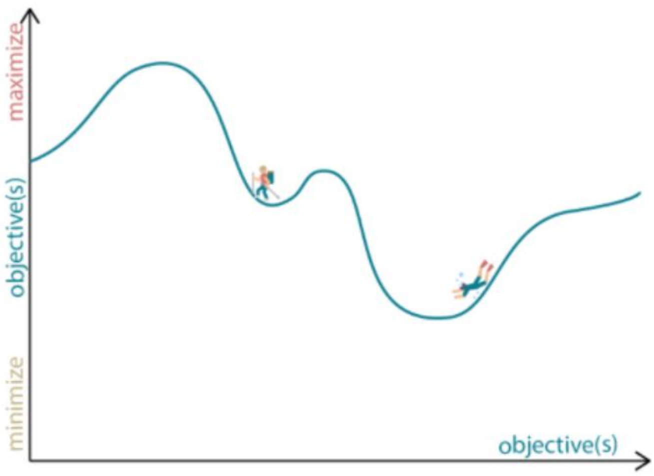


Figure 7: An example of objective function

2.4 Feasible Region

The feasible region (Figure 8) definition is the set of points that satisfies every inequality in a system of inequalities. The feasible region is so named because only points within that region are feasible solutions to the system of inequalities. The most common case in which a feasible region is needed is for linear programming problems.

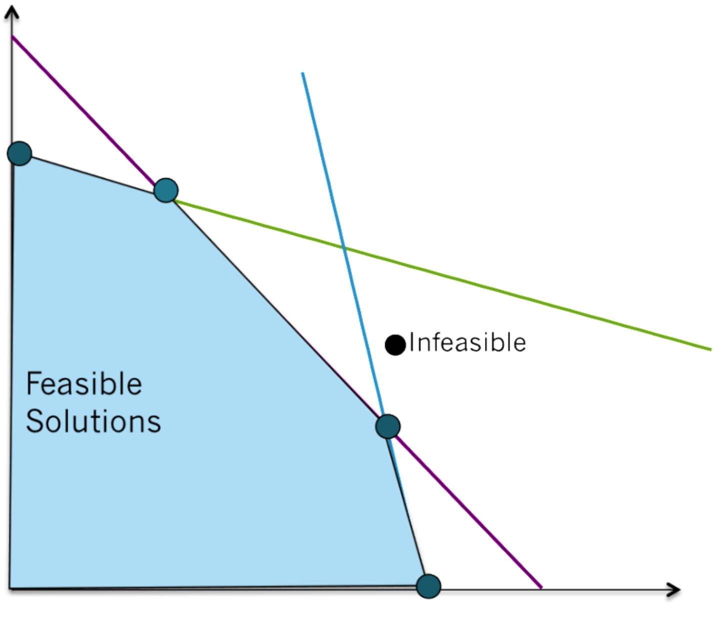


Figure 8: Feasible region definition.

Proposition 2.1 The intersection of two convex sets A; B ⊆ R^n is still a convex set.

Definition 2.4 : Let $S \subseteq \mathbb{R}^n$ be a nonempty set. If $f: S \rightarrow \mathbb{R}$, the argument of the minimum is the set of elements in S that achieve the global minimum in S, which is defined by :

1 $x^* \in S$ is called a **global minimum** point of f over S if $f(x) \geq f(x^*)$ for any $x \in S$

2 $x^* \in S$ is called **astrict global minimum** point of f over S if $f(x) > f(x^*)$ for any $x^* \neq x \in S$

3 $x^* \in S$ is called a **global maximum point** of f over S if $f(x) \leq f(x^*)$ for any $x \in S$

4 $x^* \in S$ is called a **strict global maximum** point of f over S if $f(x) < f(x^*)$ for any $x^* \neq x \in S$. See Figure 9.

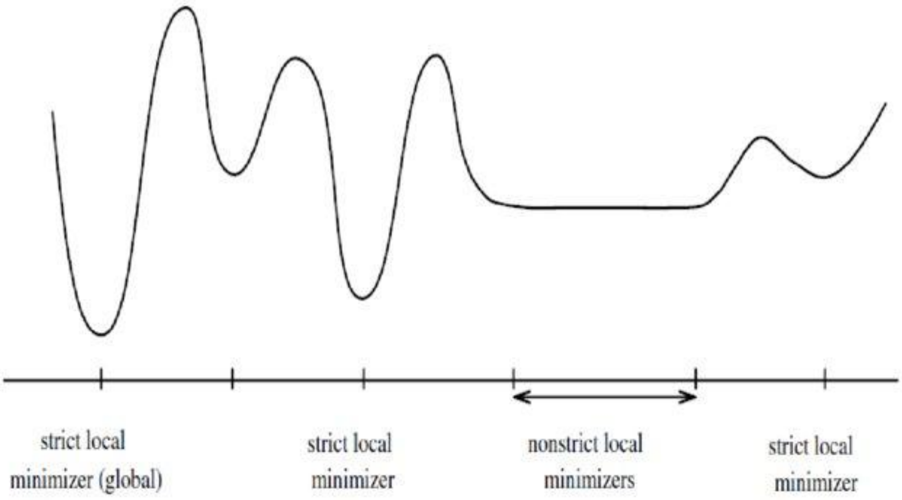


Figure ٩: An examples of local-minimizers

2.5 Epigraph and Hypograph of a Function

Let $(S, \mathcal{O})$ be a topological space and let $f: S \longrightarrow \hat{\mathbb{R}}$ be a function. Its epigraph is the set

$$\mathrm{epi}(f) = \{(x, y) \in S \times \mathbb{R} \mid f(x) \leqslant y\}.$$

The hypograph of f is the set

$$\mathrm{hyp}(f) = \{(x, y) \in S \times \mathbb{R} \mid y \leqslant f(x)\}$$

The epigraph of a function f : R ⟶ R is the dotted area in R^n located above the graph of the function f ; the hypograph of f is the dotted area below the graph. As indicated in Figure 10.

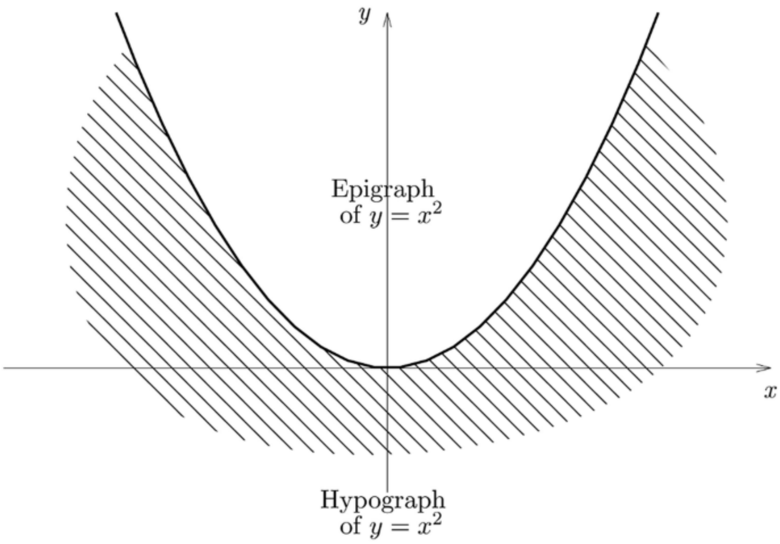


Figure ١٠ : Epigraph and hypograph for function *f*

2.6 Norm

A function $\|\cdot\|: R^n \rightarrow R$ is said to be a norm, if it satisfies the following properties:

(1) $\|x\| \geq 0, \forall x \in R^n$; $\|x\| = 0$ if and only if $x = 0$.

(2) $\|\alpha x\| = |\alpha|\|x\|, \forall \alpha \in R, \forall x \in R^n$.

(3) $\|x + y\| \leq \|x\| + \|y\|, \forall x, y \in R^n$.

2.7 Constrained and Unconstrained Formulations

Optimization refers to the process of finding the best solution to a problem from all possible solutions. Optimization problems can be classified into two main categories: constrained and unconstrained.

2.7.1 Unconstrained Optimization

Unconstrained optimization involves finding the best solution to a problem without any constraints or limitations. In other words, the problem is to optimize a function without any restrictions. Examples of unconstrained optimization problems include finding the maximum or minimum value of a function or finding the roots of an equation. An unconstrained optimization problem can be written as follows:

$$minimize\ f(x)$$

where x = $[x_1, \cdots, x_N]^T \in R^N$ is the vector we are trying to optimize.

2.7.2 **Constrained Optimization**

Constrained optimization involves finding the best solution to a problem subject to certain constraints or limitations. In other words, the problem is to optimize a function while satisfying a set of constraints. Examples of constrained optimization problems include finding the highest profit while minimizing costs, or maximizing crop yields while limiting the use of water. A constrained optimization problem can be written as follows:

$$minimize\ f(x)$$

$$such\ that\ g_k(x) \leq b_k,\ for\ k = 1, \ldots, K$$

$$such\ that\ h_l(x) = c_l,\ for\ l = 1, \ldots, L$$

where x = $[x_1, \cdots, x_N]^T \in R^N$ is the vector we are trying to optimize.

2.8 Optimality Conditions

Optimality conditions are a set of conditions that must be satisfied for a solution to be optimal in an optimization problem. There are two types of optimality conditions: necessary conditions and sufficient conditions.

Necessary conditions are conditions that must be satisfied for a solution to be optimal, but they do not guarantee that the solution is optimal. In other words, if these conditions are not satisfied, then the solution is definitely not optimal. The most well-known necessary condition for optimization problems is the first-order condition, which states that the gradient of the objective function must be equal to zero at the optimal solution. In other words, the optimal solution must be a critical point of the objective function.

Sufficient conditions, on the other hand, are conditions that, if satisfied, guarantee that the solution is optimal. The most well-known sufficient condition for optimization problems is the second-order condition, which states that the Hessian matrix of the objective function evaluated at the optimal solution must be positive definite. In other words, the optimal solution must be a local minimum of the objective function.

However, it is important to note that not all optimization problems satisfy the second-order condition, and in such cases, other methods such as convexity or global optimization methods may be required to ensure optimality.

Chapter 3

Engineering Applications within the Concept of Optimization

Optimization is a powerful tool that engineers use to design and improve systems, processes, and products. From designing stronger and lighter structures to improving energy efficiency, optimization can have a significant impact on the performance and efficiency of engineering systems. In this chapter, we will explore the various applications of optimization within engineering, and provide an example of each application.

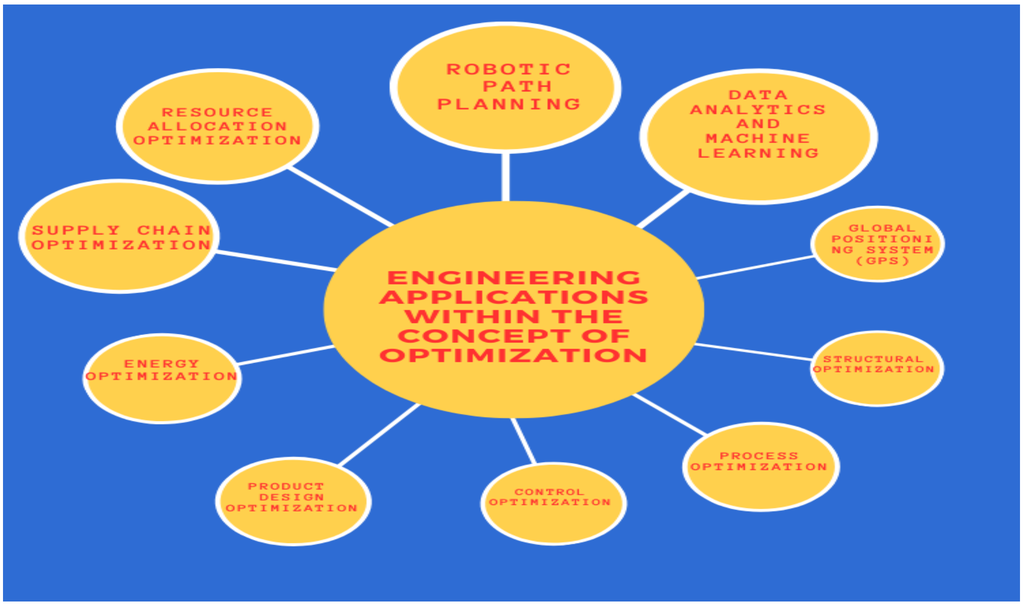


Figure (11): Flow chart of the most important applications of optimization in engineering

3.1 Global Positioning System (GPS):

GPS is considered one of the applications of optimization within engineering. GPS uses optimization algorithms to determine the position of a receiver on the Earth's surface based on signals received from satellites orbiting the Earth. The receiver uses optimization techniques to calculate its position by solving a system of equations based on the travel time of the signals from multiple satellites. In GPS, optimization is used to minimize the errors and uncertainties in the calculation of the receiver's position. These errors can arise due to various factors such as atmospheric conditions, signal reflections, and clock errors. By using optimization techniques, GPS can improve the accuracy and reliability of the receiver's position calculation. Furthermore, optimization is also used in GPS for route optimization and navigation. GPS can use optimization algorithms to find the shortest or fastest route between two locations based on various factors such as traffic congestion, road conditions, and speed limits. This can help drivers save time and fuel, and reduce traffic congestion. In summary, GPS is a powerful application of optimization within engineering, and it has revolutionized the way we navigate and locate ourselves on the Earth's surface, [16]. As shown in Figure12.

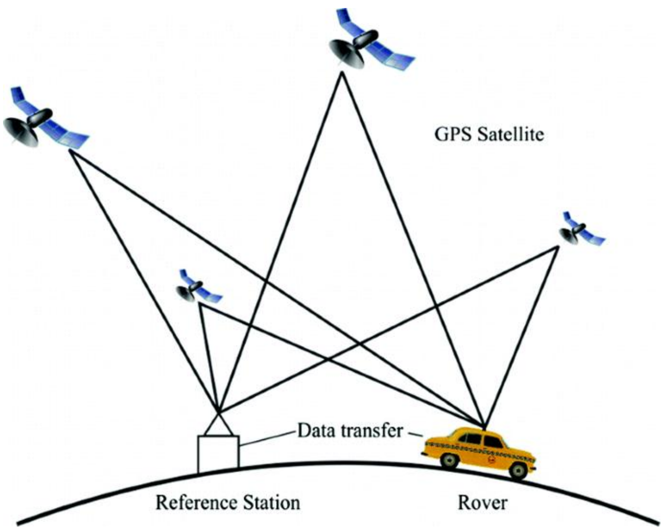
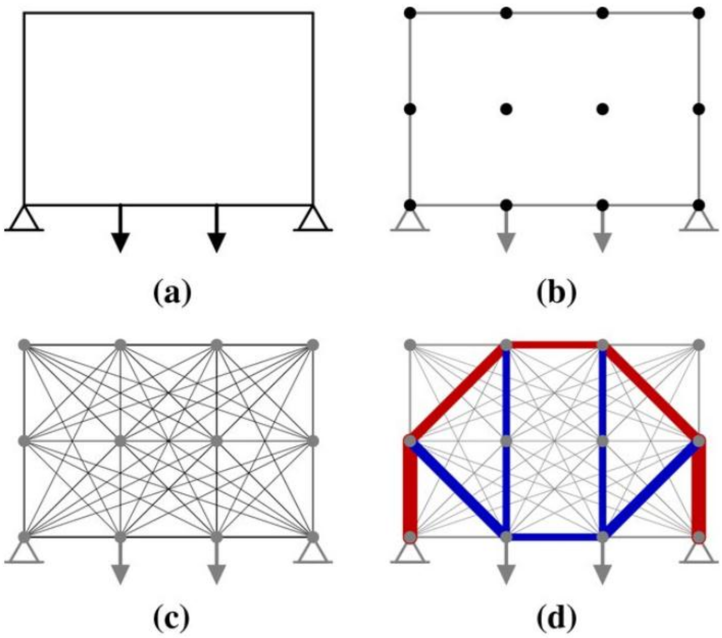


Figure (12): Simplified figure showing GPS technology.

3.2 Structural Optimization:

Structural optimization is a process of designing and optimizing the structural components of a system or product. The main goal of structural optimization is to design structures that can withstand loads while minimizing weight or material usage. In this way, engineers can improve the performance, safety, and cost-effectiveness of structures. Structural optimization can be performed using various optimization techniques such as gradient-based optimization, genetic algorithms, and simulated annealing. These techniques can help engineers find the optimal design parameters that meet the required performance and safety criteria. An example of structural optimization is the design of truss structures.

Truss structures are widely used in engineering to support loads such as bridges, towers, and roofs. Truss structures are made up of a series of interconnected triangles that distribute the loads evenly and efficiently. By optimizing the design of a truss structure, engineers can minimize weight and material usage while still meeting the required strength and stiffness. Let's consider the example of designing a truss bridge. A truss bridge is a type of bridge that uses a truss structure to support the weight of the bridge deck and the vehicles crossing it. The design of a truss bridge involves optimizing the shape and size of the truss members to minimize weight while still meeting the required strength and stiffness. The optimization process begins with defining the design parameters such as the length and cross-sectional area of the truss members, the number of members, and the shape of the truss structure. The engineer then sets the performance criteria such as the maximum stress and deflection limits, and the safety factors. The optimization algorithm is then used to find the optimal values of the design parameters that meet the performance criteria while minimizing weight. For example, let's assume we are designing a truss bridge with a span of 50 meters. The optimization algorithm can be used to find the optimal design parameters such as the length and cross-sectional area of the truss members. The algorithm will consider the performance criteria such as the maximum stress and deflection limits, and the safety factors. After several iterations, the optimization algorithm will converge to the optimal design parameters that meet the performance criteria while minimizing weight. The final design will consist of a truss structure with the optimal size and shape of the members, resulting in a cost-effective and safe truss bridge. Finally, structural optimization is an essential application of optimization within engineering that allows engineers to design and optimize structures that can withstand loads while minimizing weight or material usage. The example of designing a truss bridge illustrates how optimization techniques can be used to design cost-effective and safe structures that meet the required performance and safety criteria, [17].



Figure(13):Simplified figure showing Structural optimization.

3.3 Process Optimization:

Process optimization is a branch of engineering that involves the improvement of industrial processes to enhance their efficiency, quality, and productivity. The main goal of process optimization is to identify and eliminate inefficiencies in a process, resulting in reduced costs and improved output. Process optimization can be performed using various techniques such as statistical process control, Lean Six Sigma, and design of experiments. These techniques can help engineers analyze and improve various aspects of a process, such as material flow, process control, and resource utilization. An example of process optimization is the optimization of a manufacturing process for a product. Let's consider the example of a manufacturing company that produces electronic components. The manufacturing process involves several steps such as assembly, testing, and packaging. The goal of process optimization in this case would be to identify and eliminate inefficiencies in each step of the manufacturing process to improve the quality and productivity of the process. The optimization process begins with a detailed analysis of the manufacturing process. Engineers collect data on various parameters such as cycle time, scrap rate, and downtime. They use this data to identify the bottlenecks in the process and areas where the process can be improved. For example, let's assume that the testing step in the manufacturing process is taking longer than expected due to equipment downtime. Engineers can use statistical process control techniques to monitor the equipment's performance and identify the root cause of the downtime. Once the root cause is identified, engineers can develop a plan to eliminate the downtime by optimizing the equipment maintenance schedule or by upgrading the equipment. In another example, the packaging step in the manufacturing process may be causing delays due to manual labeling. Engineers can use Lean Six Sigma techniques to analyze the packaging process and identify opportunities to reduce waste and improve efficiency. They may introduce automation or use a different labeling technique to eliminate the delays. In both examples, the optimization process involves identifying inefficiencies in the manufacturing process and implementing changes to eliminate them. The end result is a manufacturing process that is more efficient, cost-effective, and produces higher quality products. Another example, in chemical engineering and manufacturing, optimization is used to improve production processes and reduce costs. One example of process optimization is the optimization of a chemical reaction. Chemical engineers use optimization techniques to find the optimal reaction conditions that result in maximum yield and minimal waste. For example, the reaction time, temperature, and concentration can be optimized to improve the efficiency of the reaction and reduce the amount of waste produced. Therefore, process optimization is a critical application of optimization within engineering that can improve the efficiency, quality, and productivity of industrial processes. The example of optimizing a manufacturing process illustrates how optimization techniques can be used to identify and eliminate inefficiencies in a process, resulting in improved output and reduced costs, [18].

Figure (14):Simplified figure showing Process optimization.

3.4 Control Optimization:

Control optimization is a field of engineering that involves the improvement of control systems to enhance their performance and stability. Control systems are used in various engineering applications such as aerospace, automotive, and industrial control to regulate and control a system's behavior. The main goal of control optimization is to design control systems that can efficiently and accurately regulate a system's behavior while minimizing energy consumption and minimizing the system's response time. This is accomplished by optimizing the control parameters such as gain, time constant, and bandwidth. An example of control optimization is the optimization of a cruise control system in a car. The cruise control system is a control system that automatically regulates the car's speed to maintain a constant speed without requiring the driver to continuously adjust the accelerator pedal. The control optimization process begins with modeling the car's dynamics and developing a mathematical model of the cruise control system. The mathematical model is then used to design the control system's parameters such as the gain, time constant, and bandwidth. For example, let's assume that the car's cruise control system is not providing a smooth ride due to abrupt changes in speed. Engineers can use control optimization techniques to improve the system's response time and stability. They may adjust the control parameters such as the gain and time constant to provide a smoother ride and minimize energy consumption. In another example, let's assume that the car's cruise control system is consuming too much fuel due to frequent changes in speed. Engineers can use control optimization techniques to optimize the control parameters to minimize energy consumption while maintaining the required performance criteria. In both examples, the optimization process involves adjusting the control system's parameters to improve performance, stability, and energy efficiency. The end result is a cruise control system that provides a smoother ride and minimizes energy consumption. In conclusion, control optimization is an essential application of optimization within engineering that allows engineers to design and optimize control systems that regulate a system's behavior while minimizing energy consumption and maximizing performance. The example of optimizing a cruise control system illustrates how optimization techniques can be used to improve the performance, stability, and energy efficiency of a control system, [19].

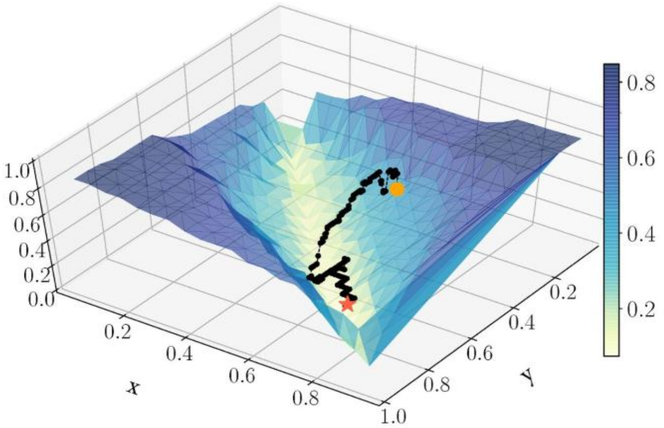


Figure (15):Simplified figure showing Control Optimization.

3.5 Product Design Optimization:

Product design optimization is a field of engineering that involves optimizing the design of a product to improve its performance, reduce costs, and enhance its manufacturability. The optimization process involves analyzing various design parameters and identifying opportunities to improve the product's performance, reduce costs, and improve manufacturability. An example of product design optimization is the design of a bicycle frame. The bicycle frame is an essential component of the bicycle that determines its strength, stiffness, weight, and other performance characteristics. The optimization process involves analyzing various design parameters such as frame geometry, material selection, and manufacturing process to improve the frame's performance, reduce costs, and improve its manufacturability. The optimization process begins with defining the design objectives and constraints. For example, the design objective may be to minimize the weight of the frame while maintaining its strength, stiffness, and durability. The design constraints may include material cost, manufacturing complexity, and production volume. Once the design objectives and constraints have been defined, engineers can use various optimization techniques such as computer-aided design (CAD), finite element analysis (FEA), and topology optimization to optimize the design of the bicycle frame. For example, engineers can use CAD software to design a frame with a specific geometry that meets the design objectives and constraints. They can use FEA software to simulate the frame's behavior under different loading conditions and identify areas of high stress or deformation. Engineers can then use topology optimization techniques to redesign the frame's geometry to reduce weight while maintaining its strength and stiffness. Topology optimization involves systematically removing material from the frame while maintaining its performance characteristics. Finally, engineers can use their knowledge of the manufacturing process to optimize the frame's manufacturability. For example, they may adjust the frame's geometry to reduce the number of manufacturing steps, simplify the tooling, or minimize material waste. Product design optimization is an important application of optimization within engineering that can improve the performance, reduce costs, and enhance the manufacturability of a product. The example of optimizing the design of a bicycle frame illustrates how optimization techniques can be used to analyze various design parameters and identify opportunities to improve the product's performance, reduce costs, and enhance its manufacturability, [20].

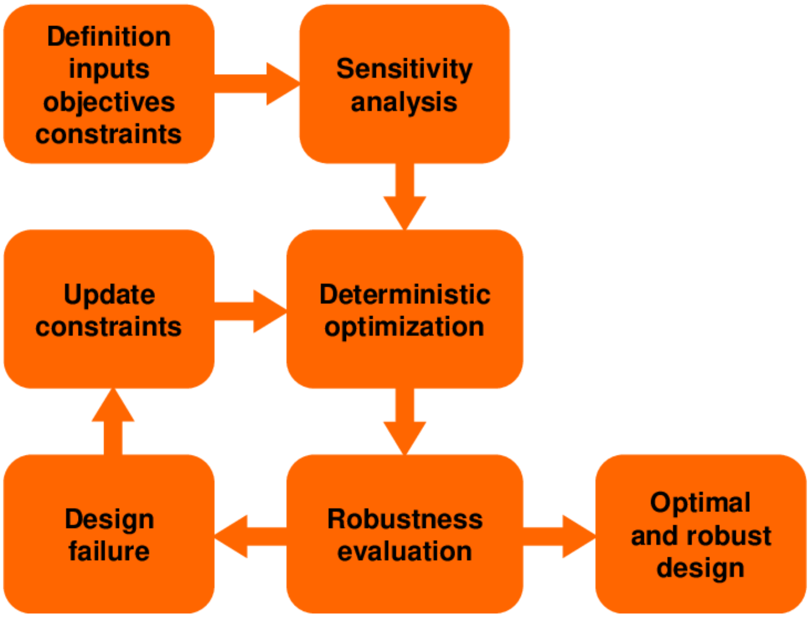


Figure (16):Simplified figure showing Product design and optimization concept.

3.6 Energy Optimization:

Energy optimization is a field of engineering that involves the optimization of energy systems and processes to minimize energy consumption while maintaining the desired level of performance. The main goal of energy optimization is to reduce energy consumption, minimize environmental impact, and reduce operating costs.

Energy optimization can be performed using various techniques such as energy audits, energy modeling, and energy-efficient design. These techniques can help engineers analyze and improve various aspects of an energy system, such as energy efficiency, energy management, and energy storage .An example of energy optimization is the optimization of a building's heating, ventilation, and air conditioning (HVAC) system. The HVAC system is responsible for maintaining the desired temperature and air quality within a building. The optimization process involves identifying inefficiencies in the HVAC system and implementing changes to improve its energy efficiency .The optimization process begins with an energy audit of the building, which involves collecting data on various parameters such as energy consumption, system performance, and operating conditions. Engineers use this data to identify areas where the HVAC system can be optimized .For example, let's assume that the HVAC system in a building is consuming more energy than necessary due to inefficient air circulation. Engineers can use energy modeling techniques to analyze the air circulation patterns and identify opportunities to improve energy efficiency. They may install variable speed fans, adjust the air duct layout, or add occupancy sensors to reduce energy consumption .In another example, let's assume that the HVAC system is consuming more energy than necessary due to inefficient heating or cooling. Engineers can use energy-efficient design techniques to optimize the system's design and improve energy efficiency. They may install high-efficiency heat pumps, use insulation materials with a high R-value, or install a building automation system to optimize the HVAC system's performance .In both examples, the optimization process involves identifying inefficiencies in the HVAC system and implementing changes to improve its energy efficiency. The end result is an HVAC system that consumes less energy, reduces operating costs, and minimizes environmental impact. Therefore, energy optimization is an important application of optimization within engineering that can improve the energy efficiency of various systems and processes. The example of optimizing a building's HVAC system illustrates how optimization techniques can be used to identify and eliminate inefficiencies, resulting in reduced energy consumption, operating costs, and environmental impact. In electrical engineering and energy systems, optimization is used to design energy systems that maximize efficiency and reduce energy consumption. One example of energy optimization is the design of a solar panel system. Solar panel systems are designed to capture and convert sunlight into electrical energy, and optimization techniques can be used to maximize the efficiency of the system. For example, the orientation and tilt angle of the solar panels can be optimized to capture the maximum amount of sunlight, while minimizing shading and other losses. In conclusion, optimization is a powerful tool that engineers use to design and improve systems, processes, and products. Whether it is designing a stronger and lighter structure or improving the efficiency of a chemical reaction, optimization can have a significant impact on the performance and efficiency of engineering systems. By using optimization techniques, engineers can design more efficient, cost-effective, and sustainable solutions to the challenges faced by our modern world, [21].

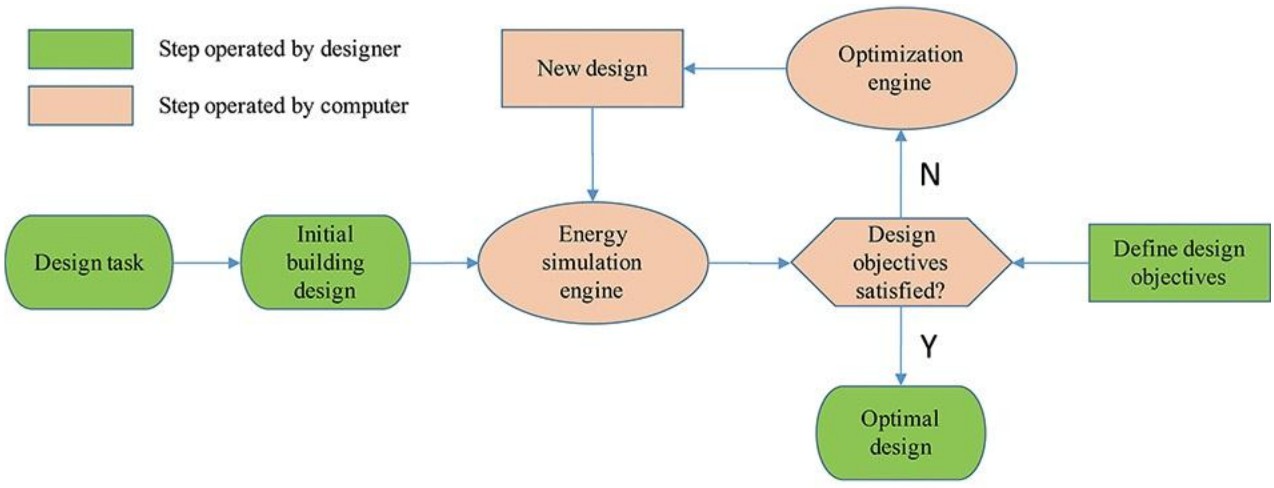


Figure (17):Simplified figure showing Energy optimization.

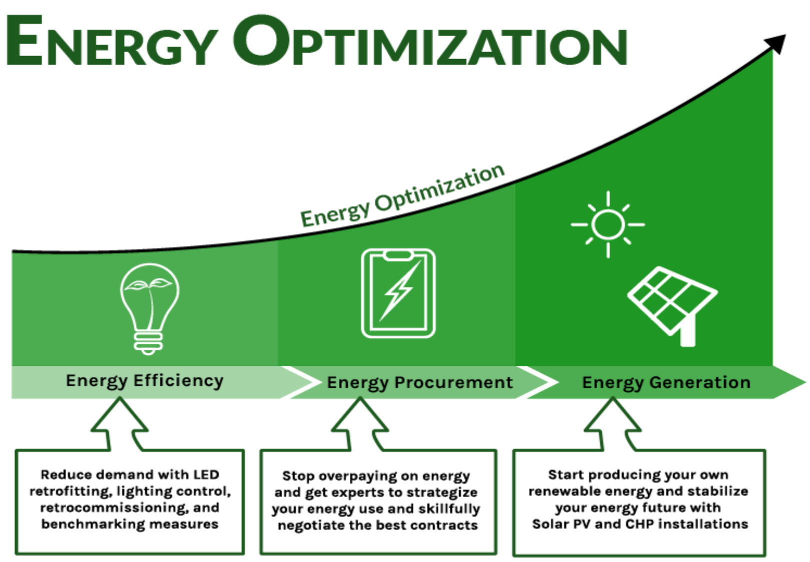


Figure (18): A simplified figure showing energy optimization and keeping pace with modern times.

3.7 Supply Chain Optimization

Supply Chain Optimization is applied in supply chain management to optimize inventory levels, transportation routes, and distribution networks. By finding the most efficient allocation of resources, companies can minimize costs, reduce delivery times, optimize warehouse locations, and improve overall supply chain performance, [22]. There are some key areas where optimization is applied in supply chain management:

1. **Inventory Optimization:** Optimization is used to determine the optimal inventory levels and reorder points for different products in the supply chain. By considering factors such as demand variability, lead times, storage costs, and service level requirements, companies can minimize inventory holding costs while ensuring an adequate level of stock to meet customer demand.

2. **Network Design and Facility Location:** Optimization techniques are employed to determine the optimal location and configuration of distribution centers, warehouses, manufacturing plants, and retail outlets within a supply chain network. By considering factors such as transportation costs, customer demand patterns, production capacities, and facility costs, companies can design an efficient network that minimizes transportation costs and improves customer service.

3. **Transportation and Routing Optimization:** Optimization is used to optimize transportation and routing decisions within a supply chain. This involves determining the most efficient routes for product deliveries, selecting

the appropriate modes of transportation, optimizing truckloads and container assignments, and scheduling shipments to minimize transportation costs and improve delivery timeframes.

4. **Demand Planning and Forecasting:** Optimization techniques are applied to forecast demand patterns accurately and improve demand planning. By analyzing historical data, market trends, and other relevant factors, companies can optimize their demand forecasting models to reduce forecast errors, minimize stock-outs, and improve overall supply chain efficiency.

5. **Production Planning and Scheduling:** Optimization is used to optimize production planning and scheduling processes. This involves determining the optimal allocation of resources, such as labor, machines, and raw materials, to meet customer demand while minimizing production costs, inventory levels, and lead times.

6. **Supplier Selection and Vendor Managed Inventory:** Optimization techniques are employed to select the most suitable suppliers and determine the optimal inventory levels for vendor-managed inventory (VMI) programs. By considering factors such as supplier performance, pricing, lead times, and transportation costs, companies can optimize their supplier selection process and establish effective VMI partnerships that improve supply chain efficiency.

7. **Risk Management and Resilience:** Optimization is used to identify and mitigate risks within the supply chain. Companies can apply optimization techniques to develop risk models, analyze potential disruptions, and determine

the optimal allocation of resources to ensure supply chain resilience. This includes optimizing safety stock levels, developing alternative sourcing strategies, and implementing contingency plans.

By applying optimization techniques in these areas, companies can streamline their supply chain operations, reduce costs, improve customer service, and enhance overall supply chain performance. Optimization enables companies to make data-driven decisions and achieve an optimal balance between various factors and constraints within the supply chain network.

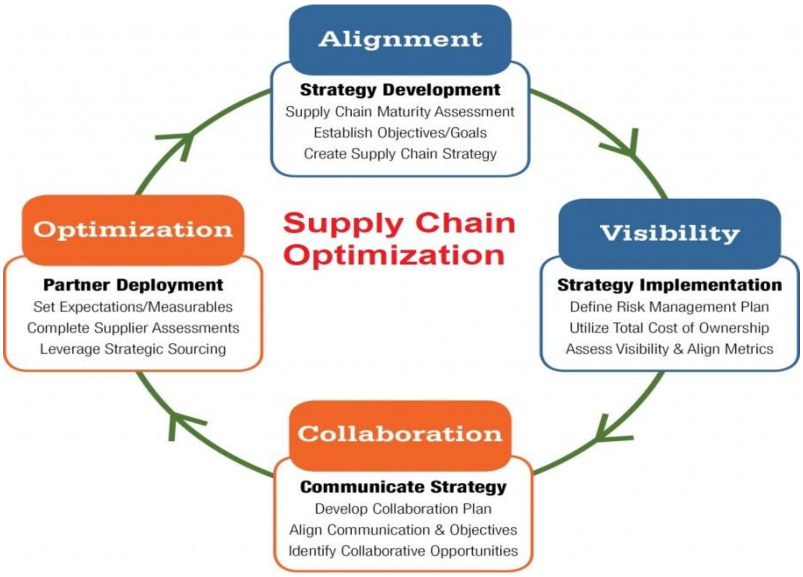


Figure (19): Supply Chain Optimization

3.8 Resource Allocation Optimization

Resource allocation optimization is a process that involves determining the optimal allocation of resources within a system or organization to achieve specific objectives. It is widely applied in various industries and domains where efficient utilization of resources is crucial, [23]. Here are some key aspects and applications of resource allocation optimization:

1. **Resource Constraints:** Resource allocation optimization takes into account the constraints and limitations associated with the available resources. These constraints may include factors such as budget constraints, time constraints,

capacity constraints, labor availability, material availability, and other resource-specific limitations.

2. **Objective Function:** An objective function is formulated to represent the desired outcome or goal of the resource allocation process. The objective function could be maximizing efficiency, minimizing costs, optimizing productivity, maximizing revenue, or achieving a specific performance target.

3. **Decision Variables:** Decision variables are identified to represent the allocation decisions that need to be made. These variables could represent the quantities, locations, timing, or other relevant aspects of resource allocation. Examples of decision variables include the allocation of funds, personnel assignments, equipment usage, inventory levels, and project scheduling.

4. **Mathematical Model:** A mathematical model is developed to represent the relationship between the decision variables, the objective function, and the constraints. The model can be expressed as a set of equations or inequalities that capture the resource allocation problem.

5. **Optimization Algorithm:** An appropriate optimization algorithm is selected to solve the mathematical model and find the optimal resource allocation solution. Different optimization techniques can be employed, such as linear programming, integer programming, dynamic programming, or metaheuristic algorithms like genetic algorithms or simulated annealing.

6. **Resource Allocation Scenarios:** Resource allocation optimization can be applied in various scenarios, depending on the specific domain and objectives. Some common applications include:

a. Project Management: Optimizing the allocation of resources (e.g., time, budget, personnel) to different project activities to minimize project duration or costs while meeting project constraints.

b. Personnel Scheduling: Optimizing the allocation of employees to shifts or tasks to meet operational requirements while considering factors such as skills, availability, and labor regulations.

c. Portfolio Optimization: Optimizing the allocation of financial resources across a portfolio of investments to maximize returns while managing risk.

d. Supply Chain Management: Optimizing the allocation of inventory, transportation resources, and production capacity to meet customer demand efficiently and minimize costs.

e. Energy Resource Allocation: Optimizing the allocation of energy resources (e.g., power generation, storage, distribution) to meet demand while minimizing costs and ensuring reliability.

7. **Sensitivity Analysis:** Sensitivity analysis is often performed to assess the impact of changes in variables, constraints, or objectives on the optimal resource allocation solution. This analysis helps in understanding the robustness of the solution and making informed decisions under different scenarios or uncertainties.

8. **Continuous Improvement:** Resource allocation optimization is often an ongoing process, as resource availability, demand patterns, and business conditions change over time. Regular reviews and updates are necessary to ensure that the resource allocation remains optimal and aligned with the changing requirements and objectives.

By applying resource allocation optimization techniques, organizations can make efficient and effective use of their resources, improve operational efficiency, reduce costs, and enhance overall performance and competitiveness.

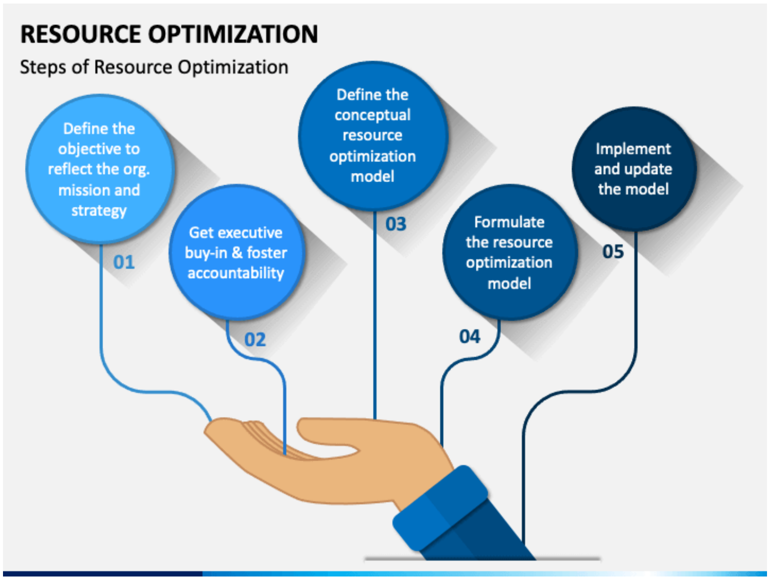


Figure (20): An example resource allocation optimization

3.9 Robotic Path Planning

Optimization is utilized in robotics to optimize the path planning of robotic systems. By optimizing the movement of robots in complex environments, engineers can minimize travel time, avoid obstacles, and improve overall operational efficiency, [24,25]. There are the key aspects and applications of robotic path planning:

1. **Map Representation:** The environment is typically represented as a map or a grid, where each cell or pixel represents a specific area or location. The map can be generated from sensor data or pre-defined based on the known environment.

2. **Obstacle Detection and Representation:** The obstacles in the environment are detected and represented in the map. This can be done using various sensing techniques, such as LiDAR, cameras, or proximity sensors. The obstacles are typically represented as occupied cells or regions in the map.

3. **Path Planning Algorithms:** Various optimization algorithms are used to find the optimal path or trajectory for the robot. These algorithms explore the map and search for the path that minimizes a certain cost function, such as distance, time, energy consumption, or risk. Common path planning algorithms include A*, Dijkstra's algorithm, Rapidly-exploring Random Trees (RRT), and Probabilistic Roadmaps (PRM).

4. **Collision Avoidance:** One of the key objectives of path planning is to avoid collisions with obstacles or other robots in the environment. Collision avoidance techniques, such as potential fields, velocity obstacles, or reciprocal velocity obstacles, are integrated into the path planning algorithms to ensure safe robot navigation.

5. **Constraints and Optimization Criteria:** The path planning process takes into account various constraints and optimization criteria specific to the robotic system. These constraints could include robot kinematics and dynamics, sensor limitations, actuator constraints, or specific task

requirements. The optimization criteria depend on the application and can include factors such as minimizing energy consumption, maximizing speed, or optimizing for smoothness of the robot's motion.

6. **Real-Time Adaptation:** In dynamic environments or scenarios where the environment changes over time, real-time adaptation is necessary. Path planning algorithms can be modified to adapt to dynamic obstacles, changing goals, or updated maps. Online replanning techniques are used to adjust the robot's path based on the latest information.

7. **Simulation and Validation:** Before executing the planned path in the real world, simulations are often used to validate the path planning algorithm and evaluate the robot's behavior. Simulations help in identifying potential issues, optimizing parameters, and ensuring the effectiveness of the planned path.

8. **Implementation and Execution:** Once the optimal path is planned and validated, the robot can execute the planned trajectory using its control system and actuators. The robot follows the planned path while continuously monitoring its surroundings to ensure obstacle avoidance and adapt to any changes.

Robotic path planning plays a vital role in various domains, including autonomous vehicles, industrial robots, unmanned aerial vehicles (UAVs), and mobile robots in logistics or healthcare applications. By applying optimization techniques in path planning, robots can navigate efficiently, avoid collisions, and accomplish tasks effectively in complex and dynamic environments.

3.10 Data Analytics and Machine Learning

Optimization is integrated with data analytics and machine learning algorithms to solve complex engineering problems. For example, optimization techniques are used in machine learning models to optimize the weights and parameters of neural networks, improve predictive accuracy, and enhance decision-making processes. Here's how these four concepts are interconnected:

1. **Data Analytics:** Data analytics involves extracting insights, patterns, and valuable information from large datasets. It includes techniques such as data preprocessing, exploratory data analysis, statistical modeling, and data visualization. Data analytics helps in understanding the characteristics and trends in data, identifying important variables, and gaining insights that can drive decision-making.

2. **Machine Learning:** Machine learning is a subset of artificial intelligence that focuses on developing algorithms and models that enable computers to learn and make predictions or decisions without being explicitly programmed. Machine learning algorithms learn from historical data, identify patterns, and make predictions or take actions based on new, unseen data. This process involves training a model on labeled data and optimizing its parameters to minimize prediction errors or maximize certain performance metrics.

3. **Optimization:** Optimization is the process of finding the best possible solution among a set of alternatives that satisfies specific objectives and constraints. Optimization techniques involve formulating a mathematical model, defining an objective function to be maximized or minimized, and specifying constraints. The optimization algorithm searches for the optimal solution by iteratively exploring different combinations of variables,

evaluating the objective function, and adjusting the variables until an optimal solution is found.

4. **Feature Engineering:** Data analytics is used to preprocess and analyze data, identify relevant features, and transform the raw data into meaningful inputs for machine learning models. Feature engineering involves selecting, transforming, or creating features that have a significant impact on the model's predictive performance.

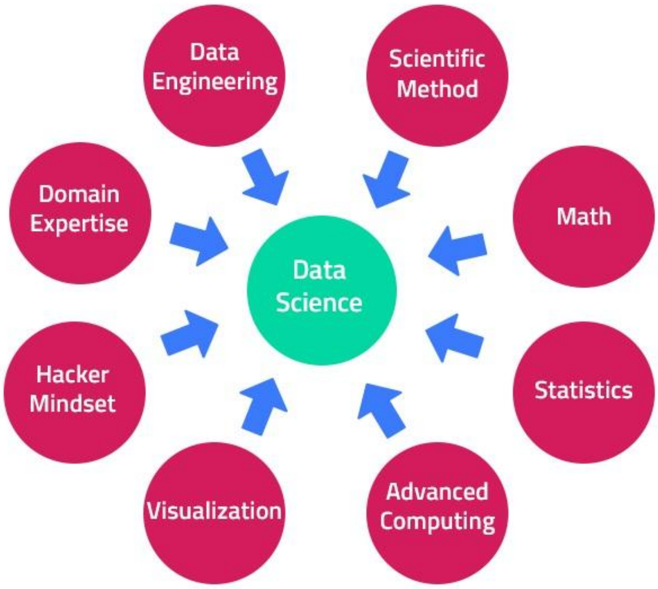


Figure (21):Data analytics and machine learning

Chapter 4

Numerical Optimization Methods for Linear Programming

Linear programming is a mathematical technique used to optimize a linear objective function subject to linear constraints. It has a wide range of applications in engineering, such as production planning, resource allocation, and network optimization. In this chapter, we will discuss some numerical optimization methods commonly used for linear programming problems**:**

4.1 Graphical Method

This method involves representing the constraints and objective function graphically and finding the optimal solution by analyzing the intersections of the constraint lines. The Algebraic Method (Graphical Method) finds its applications in engineering, where it is used for solving optimization problems in areas such as resource allocation, production planning, and project scheduling. Numerous research studies have explored the use of this method in engineering applications, providing practical insights and efficient solutions. The graphical method is used to optimize the two-variable linear programming. If the problem has two decision variables, a graphical method is the best method to find the optimal solution. In this method, the set of inequalities are subjected to constraints. Then the inequalities are plotted in the XY plane. Once, all the inequalities are plotted in the XY graph, the intersecting region will help to decide the feasible region. The feasible region will provide the optimal solution as well as explains what all values our model can take. Now, let's we illustrate the Algebraic Method (Graphical Method) with a simple Example, [26,27]. Now we introduce the steps for finding the optimal solution for linear programming using the Graphical Method:

1- Identify the decision variables: Determine the variables that represent the quantities you want to optimize.
2- Formulate the objective function: Define the objective function that you want to maximize or minimize based on the decision variables.
3- Set up the constraints: Establish the constraints that represent the limitations or requirements of the problem. Each constraint should be in the form of a linear equation or inequality involving the decision variables.
4- Convert inequalities to equalities (if necessary): If any constraints are in the form of inequalities (e.g., "≤" or "≥"), convert them to equalities .
5- Graph the feasible region: Plot the constraints on a graph to identify the feasible region. Each constraint will define a boundary line, and the feasible region is the intersection of all these lines.
6- Determine the vertices of the feasible region: Find the coordinates of the vertices (corner points) of the feasible region. These points are the intersections of the constraint lines.
7- Evaluate the objective function at each vertex: Substitute the coordinates of each vertex into the objective function to determine the corresponding objective function value.
8- Identify the optimal solution: If you are maximizing the objective function, the optimal solution is the vertex with the highest objective function value. If you are minimizing the objective function, the optimal solution is the vertex with the lowest objective function value.

It's important to note that the Graphical Method is suitable for linear programming problems with two decision variables, as it allows for visual representation on a two-dimensional graph. For problems with more than two variables, graphical methods become impractical, and alternative methods such as the Simplex Method or other optimization algorithms are used.

Example 4.1

Consider the following linear programming problem:

$$\max z = 4x_1 + 3x_2$$

Subject to :

$$5x_1 + 3x_2 \leq 30$$

$$2x_1 + 3x_2 \leq 21$$

$$x_1, x_2 \geq 0$$

We will find the optimal solution by Graphical Method:

Solution:

We will convert inequalities constraints to equalities:

1- The first constraint

$$\mathbf{5x_1 + 3x_2 = 30}$$

If $x_1 = 0,\ x_2 = 10 \quad \rightarrow (0,10)$

If $x_2 = 0,\ x_1 = 6 \quad \rightarrow (6,0)$

2- The second constraint

$$\mathbf{2x_1 + 3x_2 = 21}$$

If $x_1 = 0,\ x_2 = 7 \quad \rightarrow (0,7)$

If $x_2 = 0,\ x_1 = 10.5 \quad \rightarrow (10.5,0)$

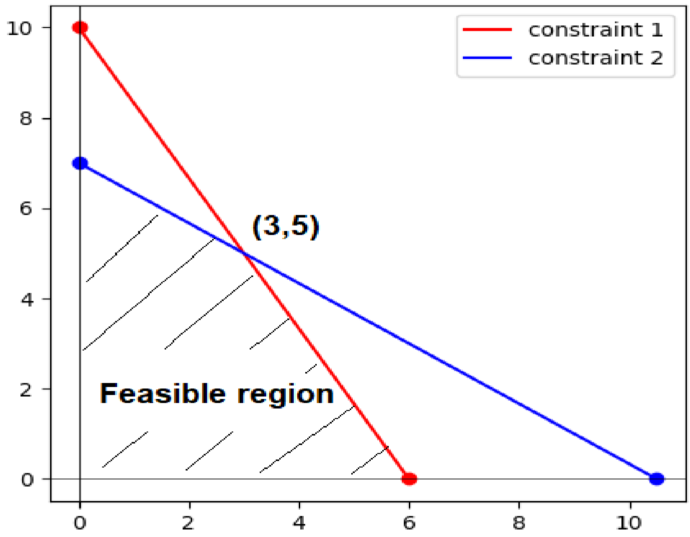
According to the fifth step, we will graph the feasible region :

$$\mathbf{5x_1 + 3x_2 = 30}$$

$$\mathbf{2x_1 + 3x_2 = 21}$$

$$3x_1 = 9 \rightarrow \mathbf{x_1 = 3}$$

$$5(3) + 3x_2 = 30, \qquad 15 + 3x_2 = 30, \qquad 3x_2 = 15, \qquad x_2 = 5$$



Points	Objective function	Max Z
(0,0)	Z=4(0)+3(0)=0	
(6,0)	Z=4(6)+3(0)=24	
(0,7)	Z=4(0)+3(7)=21	
(3,5)	Z=4(3)+3(5)=27	27

Since the objective function is maximizing, then the value of the optimal solution is 27 at point (3,5).

Example 4.2

Consider the following linear programming problem:

$$\max z = 40x_1 + 36x_2$$

Subject to :

$$5x_1 + 3x_2 \geq 45$$
$$x_1 \leq 8$$
$$x_2 \leq 10$$
$$x_1, x_2 \geq 0$$

We will find the optimal solution by Graphical Method

Solution:

We will convert inequalities constraints to equalities:

1- The first constraint

$$\mathbf{5x_1 + 3x_2 = 45}$$

If $x_1 = 0,\ x_2 = 15$ → (0,15)

If $x_2 = 0,\ x_1 = 9$ → (9,0)

2- The second constraint

$$\mathbf{x_1 = 8} \rightarrow (8,0)$$

3- The third constraint

$$\mathbf{x_2 = 10} \rightarrow (0,10)$$

According to the fifth step, we will graph the feasible region by finding the points of intersection of the constraints:

$$\mathbf{5x_1 + 3x_2 = 45}$$
$$\mathbf{x_1 = 8}$$

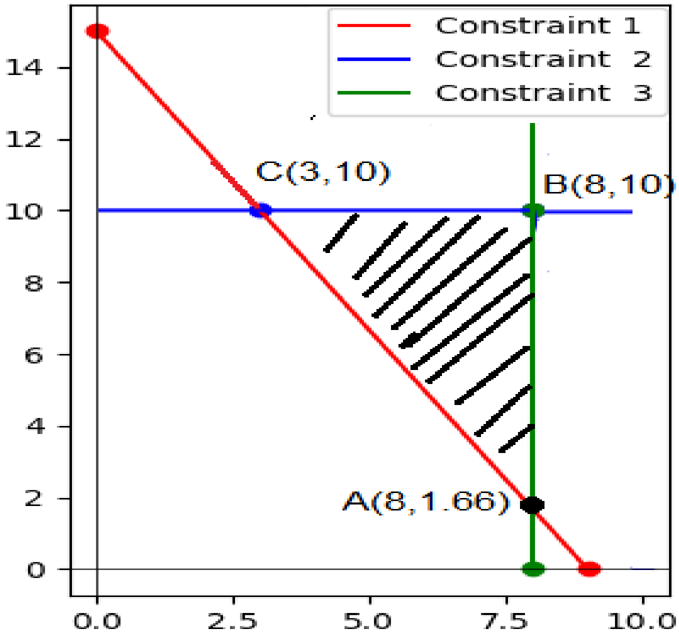
We substitute the value of $\mathbf{x_1 = 8}$ into $\mathbf{5x_1 + 3x_2 = 45}$ to get the value of $\mathbf{x_2}$**,** $\mathbf{x_2 = 1.66}$ hence **A(8,1.66).**

From the second constraint and the third constraint, we get directly to point **B(8,10).**

From the intersection of the first constraint and the third constraint, we go directly to point **C**, as follows:

$$\mathbf{5x_1 + 3x_2 = 45}$$
$$\mathbf{x_2 = 10}$$

We substitute the value of $\boldsymbol{x_2 = 10}$ into $\boldsymbol{5x_1 + 3x_2 = 45}$ to get the value of $\boldsymbol{x_2}$, $\boldsymbol{x_1 = 3}$ hence **C(٣,١٠).**



Points	**Objective function**	**Max Z**
(8,1.66)	**Z=40(8)+36(1.66)=379.76**	
(8,10)	**Z=40(8)+36(10)=680**	**680**
(3,10)	**Z=40(3)+36(10)=480**	

Since the objective function is maximizing, then the value of the optimal solution is 680 at point (8,10).

Example 4.3

Consider the following linear programming problem:

$$\min z = 100x_1 + 80x_2$$

Subject to :

$$6x_1 + 2x_2 \geq 12$$

$$2x_1 + 2x_2 \geq 8$$

$$6x_1 + 4x_2 \geq 18$$

$$x_1, x_2 \geq 0$$

We will find the optimal solution by Graphical Method

Solution:

We will convert inequalities constraints to equalities:

1- The first constraint

$$\boldsymbol{6x_1 + 2x_2 = 12}$$

If $x_1 = 0,\ x_2 = 6$ $\rightarrow (0,6)$

If $x_2 = 0,\ x_1 = 2$ $\rightarrow (2,0)$

2- The second constraint

$$\boldsymbol{2x_1 + 2x_2 = 8}$$

If $x_1 = 0,\ x_2 = 4$ $\rightarrow (0,4)$

If $x_2 = 0,\ x_1 = 4$ $\rightarrow (4,0)$

3- The third constraint

$$\boldsymbol{6x_1 + 4x_2 = 18}$$

If $x_1 = 0,\ x_2 = 4.5$ $\rightarrow (0,4.5)$

If $x_2 = 0,\ x_1 = 3$ $\rightarrow (3,0)$

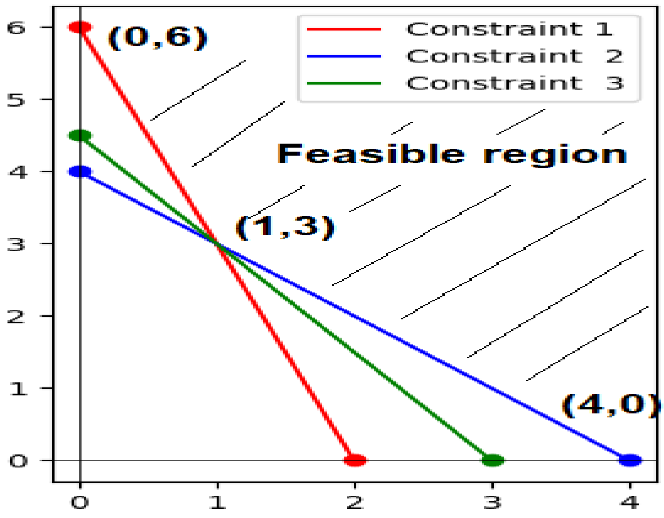
By representing the constraints on the coordinate axes, it has been shown that they all intersect at one point. In this case, we will find the value of this point by the simultaneous solution to the equation of any two constraints that we choose from among the three constraints. Here we chose to find the point of intersection between the first and second constraints, as follows:

$$\boldsymbol{6x_1 + 2x_2 = 12}$$

$$\boldsymbol{2x_1 + 2x_2 = 8}$$

$$4x_1 = 4 \rightarrow \boldsymbol{x_1 = 1}$$

$\boldsymbol{2x_1 + 2x_2 = 8},\ \boldsymbol{2(1) + 2x_2 = 8},\ \boldsymbol{2x_2 = 6},\ \boldsymbol{x_2 = 3}$ that is the point is **(1,3)**



Points	Objective function	Min Z
(4,0)	Z=100(4)+80(0)=400	
(0,6)	Z=100(0)+80(6)=480	
(1,3)	Z=100(1)+80(3)=340	340

Since the objective function is minimizing, then the value of the optimal solution is 340 at point (1,3).

4.2 Simplex Method

The simplex method is an iterative algorithm used to solve linear programming problems. It starts with an initial feasible solution and then iteratively improves it to find the optimal solution. The method operates on a simplex tableau, which represents the linear programming problem in a matrix form. It consists of a set of equations that define the constraints and an objective function equation. At each iteration, the Simplex method identifies a pivot element, which is used to perform row operations to update the tableau. The row operations aim to improve the objective function value while maintaining feasibility with respect to the constraints. The algorithm continues iterating until an optimal solution is reached. The optimal solution is found when the objective function can no longer be further improved, and all the constraints are satisfied. Now we will give some examples of using the simplex method to solve linear programming:

Example 4.٤

Consider the following linear programming problem:

$$\text{Max}\, z = 30x_1 + 18x_2$$

Subject to :

$$x_1 + 2x_2 \leq 200$$
$$3x_1 + 2x_2 \leq 300$$
$$x_1 \leq 150$$
$$x_1, x_2 \geq 0$$

We will find the optimal solution by simplex method

Solution:

Firstly, we convert the constraints into equality by adding the dummy variable (S_i) to each constraint, i.e. converting the model into the standard formula:

$$\text{Max}\, z = 30x_1 + 18x_2 + 0S_1 + 0S_2 + 0S_3$$

Subject to:

$$x_1 + 2x_2 + S_1 = 200$$
$$3x_1 + 2x_2 + S_2 = 300$$
$$x_1 + S_3 = 150$$
$$x_1, x_{2,} S_1, S_2, S_3 \geq 0$$

Secondly, we convert the values of the model into the initial basic solution table, which will include the basic and non-basic variables, in addition to the coefficients of the variables in the objective function.

Note:

The values corresponding to the variable (S_1) in the table are the coefficients of the first constraint.

The values corresponding to the variable (S_2) in the table are the coefficients of the second constraint.

The values corresponding to variable (S_3) in the table are the coefficients of the third constraint.

Basic Variable	X_1	X_2	S_1	S_2	S_3	$R.H.S$
S_1	1	2	1	0	0	200
S_2	3	2	0	1	0	300
S_3	1	0	0	0	1	150
$Z-C_j$	-30	-18	0	0	0	0

Thirdly, we choose (the internal variable), which is the variable that corresponds to the largest value with a negative sign in the ($Z-C_j$) row. Through the table, we notice that the variable (X_1) is the internal variable.

Fourthly, we choose the (output variable), which is the variable that represents the least positive value in column ($R.H.S$) after dividing it by the elements of the input variable (X_1), as follows:

$$\left(S_1=\frac{200}{1}=200\right),\left(S_2=\frac{300}{3}=100\right),\left(S_3=\frac{150}{1}=150\right)$$

Through the quotient, we notice that the variable (S_2) is the external variable because it is the least positive value and is equal to (100).

Fifthly, Elements of the new row = Elements of the old row - (intercept element * axis function)

Row elements of S_1	Row elements of S_3
1-(1*1)=0	1-(1*1)=0
2-(1*2/3)=4/3	0-(1*2/3)=-2/3
1-(1*0)=1	0-(1*0)=1
0-(1*1/3)=-1/3	0-(1*1/3)=-1/3
0-(1*0)=0	1-(1*0)=1
200-(1*100)=100	150-(1*100)=50

Therefore, the following table represents the optimal solution because all values in row ($\boldsymbol{Z - C_j}$) are positive and zero:

$$\boldsymbol{X_1 = 100}, \qquad \boldsymbol{Z = 3000}$$

Basic Variable	$\boldsymbol{X_1}$	$\boldsymbol{X_2}$	$\boldsymbol{S_1}$	$\boldsymbol{S_2}$	$\boldsymbol{S_3}$	$\boldsymbol{R.H.S}$
$\boldsymbol{S_1}$	0	4/3	1	-1/3	0	100
$\boldsymbol{X_1}$	1	2/3	0	1/3	0	100
$\boldsymbol{S_3}$	0	-2/3	0	-1/3	1	50
$\boldsymbol{Z - C_j}$	0	2	0	10	0	3000

Example 4.5

Consider the following linear programming problem:

$$\text{Max}\, z = 8x_1 + 6x_2$$

Subject to :

$$4x_1 + 2x_2 \leq 60$$

$$2x_1 + 4x_2 \leq 48$$

$$x_1, x_2 \geq 0$$

We will find the optimal solution by simplex method

Solution:

Firstly, we convert the constraints into equality by adding the dummy variable (S_i) to each constraint, i.e. converting the model into the standard formula:

$$\text{Max}\, z = 8x_1 + 6x_2 + 0S_1 + 0S_2$$

Subject to :

$$4x_1 + 2x_2 + S_1 = 60$$

$$2x_1 + 4x_2 + S_2 = 48$$

$$x_1, x_{2,} S_1, S_2 \geq 0$$

Secondly, we convert the values of the model into the initial basic solution table, which will include the basic and non-basic variables, in addition to the coefficients of the variables in the objective function.

The values corresponding to the variable (S_1) in the table are the coefficients of the first constraint.

The values corresponding to the variable (S_2) in the table are the coefficients of the second constraint.

	Basic Variable	X_1	X_2	S_1	S_2	$R.H.S$
First Table	S_1	4	2	1	0	60
	S_2	2	4	0	1	48
	$Z - C_j$	-8	-6	0	0	0
Second Table	X_1	1	1/2	1/4	0	15
	S_2	0	3	-1/2	1	18
	$Z - C_j$	0	-2	2	0	120
Third Table	X_1	1	0	1/3	-1/6	12
	X_2	0	1	-1/6	1/3	6
	$Z - C_j$	0	0	5/3	2/3	132

Since all values in last row ($Z - C_j$) are positive and zero:

$$X_1 = 12, \quad X_2 = 6, \quad Z = 132$$

Row elements of S_2 in the second table

2-(2*1)=0

4-(2*1/2)=3

0-(2*1/4)=-1/2

1-(2*0)=1

48- (2*15)=18

Row elements of X_1 in the third table

1-(1/2*0)=1

1/2-(1/2*1)=0

1/4-(1/2*1/3)= 1/3

0-(1/2*1/3)=-1/6

15-(1/2*6)=12

Chapter 5

Conclusion and Future Work

5.1 Conclusion

This research provides a comprehensive understanding of numerical optimization concepts and their applications in engineering. The study highlights the importance of optimization in solving complex engineering problems, enabling engineers to achieve optimal solutions for various design, control, and operational challenges. The research explores different optimization techniques, including linear programming, non-linear programming, and numerical optimization, providing insights into their advantages and limitations. Through the examination of real-world engineering applications, it becomes evident that numerical optimization plays a crucial role in improving the efficiency, reliability, and cost-effectiveness of engineering systems. The optimization methods discussed in this research have proven effective in optimizing the design of structures, processes, and products, as well as in enhancing energy efficiency and performance.

5.2 Future Work

While this research has provided a comprehensive overview of numerical optimization concepts and their applications in engineering, there are several avenues for future work that can further advance the field. The following areas can be explored to enhance the effectiveness and efficiency of numerical optimization:

1. Advanced Optimization Algorithms: Future research can focus on developing and refining advanced optimization algorithms that can handle complex engineering problems more efficiently .

2. Uncertainty and Stochastic Optimization: Many engineering problems involve uncertainties and stochastic variables .

3. Multi-objective Optimization: Engineering problems often involve conflicting objectives, and finding a single optimal solution may not be feasible.

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الخلاصة

يستكشف هذا البحث المفاهيم الأساسية وتطبيقات التحسين العددي في مجال الهندسة. تبدأ الدراسة بمقدمة للموضوع، تسلط الضوء على أهمية التحسين العددي في حل المشكلات الهندسية المعقدة. يتعمق في جوانب مختلفة من التحسين، بما في ذلك البرمجة الخطية، والبرمجة غير الخطية، والتحسين العددي، والطرق التقريبية. في الواقع، تضمن الفصل الأول مقدمة وأعمال ذات صلة، بما في ذلك التحسين العددي في الرياضيات التطبيقية، بالإضافة إلى البرمجة الخطية وغير الخطية وطرق التقريب باختصار. في الفصل الثاني، يقدم البحث نظرة عامة شاملة للمفاهيم الأساسية للتحسين. يناقش التحدب، والحل الأمثل، والمنطقة الممكنة، والكتابة والرسم التخطيطي للدالة، والقاعدة، والصيغ المقيدة وغير المقيدة. يقدم هذا الفصل أيضًا الظروف المثلى التي تساعد في تحديد الحل الأكثر فعالية لمشاكل التحسين. يركز الفصل الثالث على التطبيقات العملية للتحسين في الهندسة. يدرس كيفية تطبيق تقنيات التحسين العددي في مختلف المجالات الهندسية، مثل نظام تحديد المواقع العالمي (GPS)، والتحسين الهيكلي، وتحسين العمليات، وتحسين التحكم، وتحسين تصميم المنتج، وتحسين الطاقة، بالإضافة إلى التطبيقات الأخرى. تضمن الفصل الرابع طرق التحسين العددي للبرمجة الخطية. أخيرًا، تم تضمين الخاتمة والعمل المستقبلي في الفصل الخامس.

جمهورية العراق

وزارة التعليم العالي

والبحث العلمي

جامعة بابل

كلية التربية للعلوم الصرفة

قسم الرياضيات

مفاهيم التحسين العددي وتطبيقاته في الهندسة

بحث مقدم

الى مجلس كلية التربية للعلوم الصرفة / جامعة بابل كجزء
من متطلبات نيل درجة الدبلوم العالي تربية / رياضيات

مقدم من قبل الطالب

قاسم كاظم خريبط

بأشراف الدكتور

أ. م. د. احمد صباح الجيلاوي

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