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and Scientific Research
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STABILITY OF CONCRETE GRAVITY DAMS WITH INTERMEDIATE FILTER

**A thesis
submitted to the Civil Engineering Department
of the College of Engineering, University of
Babylon in partial fulfillment of the
requirements for the degree of
Master of Science in
Water Resources
Engineering**

**By
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بسم الله الرحمن الرحيم

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ABSTRACT

In this research, a hypothetical concrete gravity dam having a horizontal base floor (**75m**) long, an upstream and downstream water depths of (**70m**) and (**10m**), respectively, has been considered. A stability analysis is made to evaluate the performance of a filter media under the base of the dam with an upstream cutoff or/and downstream cutoff. The stability parameters considered are the factors of safety against overturning, sliding, and piping, and the tension criterion through checking the eccentricity of the resultant vertical force. The **Galerkin** method for finite elements has been adopted and the program **Seep2d** has been used in the analysis of seepage.

An optimization process is used to arrive at the optimum feasible combination of the depths of the upstream and downstream cutoffs and the depth, width, and location of the intermediate filter media through minimizing the total difference between the calculated and adopted safety factors against overturning, sliding, and piping.

The results revealed the following:

A. From the stability analysis, with a filter media of depth (**f**), width (**w**), and distance (**b₁**) or (**b₂**) from the upstream or downstream face of the base, respectively:

1. When there is a cutoff at the upstream side only:

- a.** Increasing the depth and width of the filter media yields increasing the stability of the dam.
- b.** Moving the filter media towards the downstream side will decrease the stability of the dam against overturning and sliding but not piping, which will increase.

- c. Increasing the depth of the upstream cutoff results in increasing the safety of the dam.

2. When there is a cutoff at the downstream side only:

- a. Increasing the depth and width of the filter media results in increasing the safety of the considered dam.
- b. Moving the filter media towards the upstream side yields decreasing the stability of the dam against overturning, sliding, and piping.
- c. Increasing the depth of the downstream cutoff results in increasing the piping safety factor but not overturning and sliding safety factors, which will decrease.

3. When there are cutoffs at the upstream and downstream sides:

- a. Increasing the depth and width of the filter media yields increasing the stability of the dam against overturning, sliding, and piping.
- b. Moving the filter media towards the downstream side results in decreasing the stability of the structure against overturning and sliding but not piping, which will increase.
- c. Increasing the depth of the upstream and downstream cutoffs causes increasing the stability of the dam.

- 4. The best case is when there is a cutoff at the upstream side of the dam of a depth ($d_1 = 7.5\text{m}$), a filter media with a depth ($f = 3\text{m}$), width ($w = (7.5 / 4) \text{ m}$), and starts at a distance from the upstream face of the floor equals to ($b_1 = 15\text{m}$).

B. From the optimization analysis:

- 1. The grand minimum total difference is when there is a filter media with a cutoff at the upstream side of the dam only.
- 2. From the grand minimum total difference:
 - a. The best depth of the upstream cutoff is equal to ($d_1 = 7.5\text{m}$).
 - b. The best depth of the filter media is ($f = 3 \text{ m}$).
 - d. The best location of the filter media from the upstream face of the base is ($b_1 = 15\text{m}$).

c. The best width of the filter media is $[w = (7.5/4) \text{ m}]$.

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LIST OF SYMBOLS AND ABBREVIATIONS

Symbol	Definition	dimensions
A	Area of the problem domain	L^2
a	The area of the triangle element	L^2
b₁ , b₂	Distances from the upstream and downstream faces of the filter media to the upstream and downstream cutoffs, respectively	L
C	Cohesion of the soil under the dam	F/L^2
d₁ , d₂	Depths of the cutoffs in the upstream and downstream sides of the dam, respectively	L
f	Depth of the filter	L
F_{OV}	Calculated factor of safety against overturning	-
F_P	Calculated factor of safety against piping	-
F_{SL}	Calculated factor of safety against sliding	-
h	The field variable	L
h₁	Height of water in the upstream	L
h₂	Height of water in the downstream	L
h₃	Piezometric head at any point in the problem domain	L
H	Ratio of head difference	-
H^e	Approximate solution for piezometric head distribution in the element (e)	L
H_i	Nodal value of head, H, of element (e)	L
{H_i}	The vector matrix of nodal values of the head	-
I_{cr}	Critical exit gradient	-
I_e	Exit gradient	-
I_x , I_y	Direction cosines	-
[K]	The global matrix	-
[K^e]	The element matrix	-

Symbol	Definition	dimensions
K_x , K_y	Hydraulic conductivity in x, y-directions, respectively	L/T
n	Number of nodes in element (e)	-
n_e	The total number of elements in the problem domain	-
N_i	Shape function of the element (e)	-
[N_i]	The matrix of shape function of the element	-
R^e	Element residual	-
S	Surface boundaries of the element	-
S₁^e	Reservoir boundary	-
S₂^e	Impervious boundary	-
T.D.	Total difference between the calculated and allowable safety factors	-
{V}	Vector of velocity in two orthogonal axes X and Y	-
w	Width of the filter	L
W_j	Weighted function	-
X	Total floor length of the dam	L
$\bar{x}$	Distance of the uplift pressure along the base floor of the dam	L
x₂	Triangle base of tail water	L
X₃	Distance of the vertical resultant force with respect to toe	L
X₄	The calculated eccentricity of the vertical resultant force	L
Y	Total height of the dam	L
y₃	Height of water over the upstream inclined line	L
y₄	Value of the uplift pressure at the upstream	L
y₅	Value of the uplift pressure at the downstream	L
y₆	Height of the dam under the soil	L
y₇	Height of a dam over the upstream inclined line	L

Symbol	Definition	dimensions
y_8	Height of a dam over the downstream inclined line	L
γ^-	Effective unit weight between the soil and the water	F/L ³
γ_c	Unit weight of a concrete gravity dam	F/L ³
γ_w	Unit weight of water	F/L ³
α, λ, γ	Penalty coefficients	-
ϕ	Angle of internal earth friction	-
β	Angle of earth slopping over the horizontal surface	-
δ	Angle of friction between the soil, base of the dam, and the concrete cutoff	-

CHAPTER ONE

CHAPTER ONE

INTRODUCTION

1.1 GENERAL

The underground flow of water has significant practical consequences and refers to a wide range of problems. It is usual to divide underground flow into two categories, seepage and groundwater flow. Seepage is generally defined as the percolation of water through dams, river banks, or into excavation. However, the groundwater flow problems are concerned with the natural process of infiltration and subsequent flow of water through pervious strata until discharge occurs at springs, rivers, or abstraction wells [**Rushton and Redshaw, (1979)**].

One of the most important problems that causes damage to the dams is the seepage through the foundation. Such a problem occurs due to difference in water level between the upstream and downstream sides of the structure. The complete safety of big and important hydraulic structures such as dams, barrages, and main head and cross regulators is of foremost priority to the respective authorities. Such structures usually represent keystones in water-resources projects so that their failure may be disastrous through endangering life and property. Besides, the design and construction (and reconstruction after failure) of such structures consumes vast amounts of effort, time, and money.

The basic causes that threaten safety of the dam are settling, overturning, sliding, and piping. The designer usually bases his designs on certain practically-reasonable values of factors of safety. Nevertheless, unexpected severe situations may occur once and again. Consequently, a good system of observing the behavior of the structure after construction in order to monitor the reliable service and operation of the project and improve the conditions of operation for the project is necessary.

Of the different measures in use to improve the safety of a dam, the use of cutoffs with or without an intermediate filter media is the most common. This calls for an “optimum” design that would be intermediate between a costly over-design and a dangerous under-design of the involved safety measures of the respective concrete gravity dam.

1.2 OBJECTIVES OF THE RESEARCH

The objectives of this research are:

1. Finding the appropriate designs (**dimensions and locations**) of an intermediate filter media under the dam that ensure reasonable values of the stability safety factors.
2. Deducing the optimum design of the intermediate filter media through an established optimization model.

CHAPTER TWO

CHAPTER TWO

REVIEW OF LITERATURE

The seepage of water through, around or under a dam is expected in all embankment dams and also even in concrete dams. The quantity of seepage, the seepage path and the velocity of seepage water are of great concern in analyzing the structural behavior of a dam [Encardio-Rite, (2001)]. Therefore, some of the previous studies on seepage will be presented in this chapter. Moreover, some of the previous studies on the stability and on the optimization problem are also presented.

2.1 TYPES OF THE STABILITY

The stability of a dam may be divided into two categories:

1. Structural stability of the dam.
2. Hydraulic stability of the dam.

2.1.1 Structural stability of the dam

The structural stability of the dam is represented by the safety of the structure against the external forces such as the water weight and water pressure, wind pressure, uplift pressure, silt pressure, earthquake, ice pressure, ...etc. These forces would make the dam unstable when they are large and causing an overturning, sliding, and tension effects on the dam.

2.1.2 Hydraulic stability of the dam

The hydraulic stability of the structure is represented by the stability of the dam against the forces that result due to the seepage of water through the foundation soil under the dam. These forces are the uplift pressure and exit force; a large value of the uplift pressure would cause failure of the floor base of

the dam by crushing and a large value of the exit force would cause movement of the soil particles under the considered dam (the phenomenon of piping).

2.2 A REVIEW ON THE SEEPAGE ANALYSIS

In order to do seepage analysis, a general model describing the phenomena of seepage must be available. There are different methods to find the piezometric head distribution under the concrete gravity dams from which the hydraulic gradient is computed. Examples of such methods are graphical, empirical, analytic, and numerical methods. In this research, a numerical method named the finite elements method is used to analyze the seepage problem because it is the most common method in this respect.

The finite elements method was first applied to the boundary value problems [**Zienkiewicz and Cheung, (1965)**] and their method was later extended to obtain a solution for a steady state seepage in an isotropic foundation under a concrete dam [**Zienkiewicz, (1966)**].

Tomlin, (1966) analyzed the seepage problem through porous media comprising zones of different permeability and anisotropic properties by using a numerical procedure with triangular mesh system. His results have been represented by plotting the equipotential lines for different cases from the computer solution to indicate the ability of the method to analyze seepage flow through complex soil sections.

Conner and Brebbia, (1976) solved the seepage problem under a hydraulic structure with two piles resisting on isotropic soil by using finite element method in order to find the distribution of the pressure under the base of the structure.

Desai and Christian, (1977) studied the seepage through a two layered foundation of a dam, within each layer the soil was assumed homogenous. They

used the finite element method. The computed values from the finite element method were compared with those from graphical solutions.

Cechi and Mancino, (1978) and **Zienkiewicz, (1982)** have given application examples of simple cases of seepage under hydraulic structures by using triangular grid of finite element to find the head distribution under the structures.

Nath, (1981) proposed a novel finite element method for the economic and accurate solution of large aspect ratio linear and non-linear seepage problems in two and three dimensions. He applied it for steady state seepage around vertical sheet pile, which was installed in anisotropic and homogeneous soil. His results indicated that the method is capable of giving a high degree of accuracy achieved economically and with efficiency.

Griffiths, (1984) who used the method of the finite element, presented charts for estimating seepage quantities and exit gradients in problems of confined flow. In this rationalized dimensionless charts anisotropic soil properties can be accounted for directly.

Hatab, (1987) and **Ijam and Hatab, (1993)** have applied the finite element method in the analysis of seepage through homogeneous and isotropic porous media below hydraulic structures with horizontal filters. They also studied the stability of the structures with vertical drains. As a result, useful design information was presented. The results indicated the advantage of using a filter in reducing the uplift pressure and hydraulic gradient.

Ijam and Nassir, (1988) used the finite element method to study seepage under hydraulic structures for isotropic soil with two cut-offs. They presented a set of uplift pressure distribution curves, whereas the efficiency of the cut-offs was defined. Another relationship was also obtained to calculate the factor of safety against failure due to heave.

Abdul-Hussain, (1989) studied the use of the finite element method in the analysis of seepage under hydraulic structures in homogeneous and isotropic soils, once with a filter trench and another with pressure relief wells. The results were presented in the form of curves showing the distribution of the piezometric head and hydraulic gradient. The results were good as compared with the analytical and experimental results.

Hillo, (1993) who applied the finite element method to study the effect of anisotropic soil foundation on the seepage under the hydraulic structure concluded that the effect of angle inclination of the major axis of hydraulic conductivity with the horizontal axis is more than the effect of the degree of anisotropy on distribution of piezometric head and the exit gradient.

Ijam and Abdul-Hussain, (1994) obtained finite element solutions for seepage below hydraulic structures having a flat floor with two cutoffs and a trench drain, located between two cutoffs and found on permeable soil of finite depth. The results showed that the uplift pressure decreased considerably along the entire length of the floor with provision of trench drain of even small dimensions. Similar effects were reached for the exit gradients. The uplift pressure decreased on the upstream side and increases on downstream side of the trench as the trench drain is moved from upstream to downstream. Therefore, it is advantageous to provide the trench close to the gateline. The decrease in pressure and exit gradient with subsequent increasing in thickness and width of the trench is small.

Griffiths and Fenton, (1998) combined together a finite element analysis and random field theory to study a simple boundary value problem of steady seepage. The aim of their investigation was to observe the influence of statistically variable soil permeability on the exit gradient at the downstream side of the hydraulic structure with a single sheet pile embedded in a layer of random soil in both two and three dimensions.

Khasaf, (1998) used the finite element method to analyze seepage in isotropic, anisotropic, homogeneous and nonhomogeneous soil foundation underneath hydraulic structures provided with flow control devices. He studied a hydraulic structure with an upstream and/or downstream cut-offs with various permeable-layer thickness and various depths and locations of cut-offs. He found that the location, number and depth of the cut-offs had a noticeable effect in reducing the uplift pressure.

Al-Hashamie, (2002) used the finite element method for analyzing a two-dimensional steady state seepage through a stratified soil with different permeability and anisotropic properties to predict the piezometric head distribution and computing the hydraulic gradient under the base of structure. The results indicated that the distribution of the flow velocity depends on the depth of soil layer and the soil permeability of each layer which increased with increasing the soil permeability. Moreover, the head was found to increase with the decrease of soil permeability due to the transport of fine particles by flowing water through the foundation soil and then deposit elsewhere, resulting in a more condensed soil structure therein.

2.3 A REVIEW ON THE STABILITY STUDIES

Garg and Chawla, (1969) and **Chawla, (1972)** studied an exact solution for the determination of the exit gradient and uplift pressure on the bottom of foundation of a hydraulic structure found on semi-infinite pervious material and pervious material of finite depth with finite or infinite pervious reaches on the upstream and downstream sides by using the method of conformal transformation. The results showed that the uplift pressure increased throughout the floor length of the structure with an increase in the upstream pervious reach while decrease with an increase in the downstream pervious reach; the uplift pressure increase on the upstream side and decrease on the downstream side of a

central cutoff for an equal increase in the length of the pervious reaches on either side of the cutoff and for an increase in the depth of the cutoff . The value of exit gradient increases with an increase in the upstream pervious reach, whereas it decreases with an increase in the downstream pervious reach.

Chawla, (1975) obtained an exact solutions for the problems of two-dimensional seepage flow below a hydraulic structure having a flat floor with two end cutoffs and a horizontal filter of any dimension located anywhere between the two cutoffs found on permeable soil of infinite depth with the help of conformal mapping transformation. The results showed that the uplift pressure decreased considerably along the entire profile of the structure with the provision of filter of even very small length. The uplift pressure decreased at the downstream side and increased at the upstream as the filter is moved from upstream to the downstream side. The filter should be located such that the total uplift on the floor downstream of the gateline is minimum.

Ijam and Hatab, (1993) studied the hydraulic stability of a hydraulic structures found on permeable foundation of finite depth and having two end cutoffs and a vertical drain located between them by using the finite element method. The results showed that an increase in the filter depth will decrease the value of uplift pressure and hydraulic gradient on the exit surface behind the hydraulic structure. The decrease in uplift pressure and exit gradient with subsequent increase in the filter depth is less. The filter should be located such that the total uplift on the floor downstream of the gateline is minimum.

Salem and Ghazaw, (2000) obtained an exact solution for the problem of seepage beneath two consecutive structures with intermediate filter found on infinite pervious soil by using the method of conformal mapping transformation. The floor of the upstream structure is flat with two end cutoffs and the downstream structure has middle inclined apron, upstream and downstream horizontal apron and two end cutoffs. The results showed that the net uplift pressure acting below the floor upstream and downstream were significantly

decreased as the ratio of the effective heads increased, and the net uplift pressure acting on the upstream floor were slightly increased as the relative filter length with respect to upstream apron length was increased while the uplift pressure and exit gradient were significantly decreased at the downstream floor as the relative filter length with respect to the upstream structure length was increased. The increasing of the length of filter beyond five times the length of the upstream floor will have a negligible effect on the uplift pressure acting on both upstream and downstream floors and on the exit gradient. The uplift pressure acting on the upstream floor increases with an increase in depth of the downstream cutoff, and the exit gradient at the downstream end of the upstream structure decreases with an increase in the depth of the downstream cutoff.

2.4 A REVIEW ON THE OPTIMIZATION PROBLEM

The purpose of optimization is to find the best feasible solution among the many potential solutions satisfying the chosen criteria. Designers often base their designs on minimizing (cost, penalty, losses, damage, ...etc) or maximizing (interest, profit, serviceability, ...etc) as an objective, taking into account mainly the safety as a major goal.

An optimization process constitutes formulating the respective optimization mathematical model, and then solving the formulated model by an appropriate solution procedure.

In general, the case to be optimized could be “**static**” (a single-stage problem) or “**dynamic**” (a multi-stage problem). The former is commonly covered by linear programming or non-linear programming, as the case enforces. The latter is covered by dynamic programming.

The case that is studied in this research is to find the stability of a dam when using an intermediate filter media. The stability may be represented by finding the factors of safety against overturning, sliding, and piping. The objective of the optimization process in this research is to find the best factors of

safety that give minimum total difference between the calculated and allowable factors of safety. This process is a single-stage problem or “**static**”.

The available methods of optimization of “**static**” problems, as outlined hereinbefore, are the linear programming, **LP**, and the non-linear programming, **NLP**. These methods are outlined hereinafter.

2.4.1 The linear programming (LP)

The main characteristic of a linear programming (**LP**) problem is that the objective function and all the constraints are linear functions (equalities and/or inequalities) of the decision variables.

Although a relatively small number of engineering problems can be formulated as (**LP**) problems, the method is widely used.

Several algorithms are available to solve **LP**-problems. However, the simplex method, developed by **C.B. Dantzig in 1946**, is the most widely used [**Dantzig, (1963)**]. Some other methods are available such as graphical methods, revised simplex method, and transportation method [**Phillips et al.,(1976)**].

2.4.2 The non-linear programming (NLP)

If the objective function or any of the constraints is non-linear, the optimization problem is termed a non-linear problem. Such a formulation is more important than (**LP**) since most of real-world design problems are in fact non-linear.

There are different algorithms and techniques for solving **NLP**-problems. The most common ones are the analytical approach and the numerical approach. These procedures are outlined hereinafter.

2.4.2.1 The analytical approach

In this approach, the problem is represented by a number of mathematical relationships (equations) from which certain equations can be developed which aid in the search for an optimum. This approach usually requires the use of

differential calculus and the exact optimum solution is theoretically found [Gallagher and Zienkiewicz, (1973)]. The most familiar methods underlying this approach are:

a. Differential calculus

This method is generally used to solve simple unconstrained **NLP**-problems by using the laws of differential calculus to find the optimum solution.

b. The Lagrange multiplier

This method is used to solve constrained non-linear optimization problems.

c. Geometric programming

Geometric programming is one branch of (**NLP**) which aims at obtaining the optimum solution for a non-linear objective function subjected to non-linear constraints. It differs from other methods in that it tries to distribute the total cost among the various terms of the objective function before finding the corresponding design variable [Duffin et al., (1967)].

2.4.2.2 The numerical approach

In this approach, a near optimum solution is automatically generated in an iterative manner. An initial guess is used as a starting point for search of a better solution. The search continues until no further improvement in the objective function is possible or until a certain convergence criterion is satisfied, which indicates that the optimum solution has been achieved within the desired accuracy. The two distinguished methods in this respect are the direct-search method and the gradient-search methods.

a. Direct search

In this method, the search proceeds with the evaluation of the objective function only in an iterative manner until a local or an approximate optimum solution is reached. The step sizes and the direction of moves at each iteration represent the main features of this approach. Many methods are proposed as standard algorithms such as the pattern search of **Hooke and Jeeves, (1961)** the complex method, the **Fletcher and Powell** method, and the **Rosenbrock, (1960)** method [**Bunday, (1984)**].

b. Gradient search

The basic idea here is to evaluate the gradient of the objective function at a point and utilize it to improve and accelerate the search. The acceleration is gained by finding the direction of the steepest descent, following this direction until no further improvement is possible, then the direction is changed again.

The search continues until the gradient becomes zero indicating that the optimum solution is reached.

The most widely used methods following this approach are the Rosen method [**Rosen, (1960)**], the Sequential Unconstrained Minimization Technique (SUMT) [**Flacco and McCormick, (1968)**] and **Fletcher and Powell** method [**Bunday, (1984)**]. These methods require smaller number of iterations than the direct search methods; however, **Carpenter and Smith, (1975)** showed that the high order derivatives of these methods are more expensive, needing longer computer time than the direct search methods.

Many studies used the optimization model in the gravity dams problems. Some of them are given below:

Gill, (1980) used the theoretical ideas presented by **Bennet (1946)** criteria for optimal design of rectangular, triangular and trapezoidal blankets. It is shown that a triangular blanket is more effective, hence more economical than

rectangular and trapezoidal, in reducing under seepage and uplift force. The uplift force is reduced in the same proportion as the seepage.

Hamed, (1996) used Rosen Brock constrained optimization and Sequential Unconstraint Minimization Technique in order to solve the non-linear programming problem of the optimum design of barrage floor (area of concrete and reinforcement).

[Al-Musawi, (2002)] studied the optimum design of control devices for safe seepage under hydraulic structures and used the finite element method to analyze seepage below hydraulic structures. The results indicated that the use of an upstream blanket would reduce the uplift pressure and exit gradient. However, the use a of downstream blanket would increase the uplift pressure and exit gradient. Also, for a hydraulic structure with different control devices, the minimum total cost could be achieved when using a filter trench, while the maximum total cost was when a downstream cut-off is used.

CHAPTER THREE

CHAPTER THREE

THE FINITE ELEMENT THEORETICAL BACKGROUND

Controlling seepage is of great importance. One of the objectives is to prevent, or at least reduce to a small value, the magnitude of the seepage flow. The control of seepage is not the only reason for analyzing seepage problems. Other consequences such as excessive saturation, seepage forces and uplift pressures can lead to failures. Indeed, the large deleterious effects seem out of all proportion to the slow movement of water through pores in the soil. Frequently, seepage forces are only considered when they reach their final steady-state value. However, whenever seepage paths are disturbed the transition from the original to the new conditions can take a significant time. In certain instances critical conditions can occur during this change and it is therefore important to investigate time-variant seepage behavior [**Rushton and Redshaw, (1979)**]. The case study in this research (concrete gravity dam) involves three layers of soil under the dam as well as the layers of filter media (two layers); so, the finite difference method is not useful to analyze this case. The finite element method may be more usable for analyzing such a case and also the mesh of this method could be a variety of element types.

In groundwater flow problems, one could imagine that the region is subdivided into small elements which may be two or three dimensional and joined to each other by those nodes which exist on the element boundaries. For each element, the flow is described in terms of the head in the nodal points, and then a system of equations is obtained from the conditions that the flow must be continuous at each node [**Bear and Verruijt, (1990)**].

In general, flow of water through earth masses is three-dimensional. However, this condition is too complicated for practical analysis and the

fundamentals of flow can best be shown by using the simple two-dimensional case [Taylor, (1948)].

This chapter presents the principles of the finite element method and the finite element formulations for two-dimensional flow through porous media by using **Galerkin** weighted residual method.

In addition, this chapter includes an explanation of the finite elements computer program that has been used in this research, the verification of this program, and its general flow chart.

3.1 THE FUNDAMENTAL CONCEPT OF THE F.E.M

A wide range of techniques is available for the study of seepage problems. Much of the development in recent years in the analysis of seepage have been made by means of mathematical models. They are usually based on analog or digital computers [Rushton and Redshaw, (1979)].

The finite element method is a very powerful and relatively modern computation tool. It requires the use of digital computer because of the large number of computations involved [Stas, (1986)].

The variation and behavior of the field variable (the piezometric head) on each element is represented approximately by a continuous function depending on the nodal values of the field variable, as follows:

$$\mathbf{H}^e(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^n \mathbf{N}_i(\mathbf{x}, \mathbf{y}) \cdot \mathbf{H}_i \quad (3.1)$$

where:

H^e: Approximate solution for piezometric head distribution in the element **e**.

H_i: Nodal value of head, **H**, of element (**e**).

n: Number of nodes in element (**e**).

N_i: Shape function of the element (**e**).

It is possible to write Eq.(3.1) in matrix form as follows [Zienkiewicz, (1966)]:

$$H^e = [N_i] \{H_i\} \quad (3.2)$$

where:

$[N_i]$: The matrix of shape function of the element.

$\{H_i\}$: The vector matrix of nodal values of the head.

The approximate solution of head distribution, $\mathbf{H}$, over the whole domain of the flow is given by:

$$\mathbf{H} = \sum_{e=1}^{ne} H^e = \sum_{e=1}^{ne} [N_i] \{H_i\} \quad (3.3)$$

where:

ne : The total number of elements in the problem domain.

3.2 METHODS OF APPROXIMATE SOLUTION

The approximate solution techniques that are useful for the understanding of how the finite element method works are the following [Stas, (1986)]:

1. Ritz Method.
2. Variational, or Rayleigh-Ritz, Method.
3. Weighted residual method.

The basic principles of these methods are given in the references [Zienkiewicz (1977) and Rao, (1982)].

From these, the weighted residual method is the most widely used in finite element work [Smith, (1998)], and the best of all because it leads frequently to symmetric matrices [Stas, (1986)].

The weighted residual method

The weighted residual method is a technique which can be used to obtain the approximate solutions to linear and non-linear differential equations. In this method the finite element equations can be derived directly from the governing differential equation of the problem [Zienkiewicz, (1982)].

Now, the governing equation can be written as:

$$f(h) = 0 \quad \text{in} \quad A \quad (3.4)$$

where:

A : Area of the problem domain.

h : The field variable.

By substituting the approximate finite element solution (**H**) as given in **Eq.(3.3)** in **Eq.(3.4)**, the equation result would not equal to zero but there is a residual (**R**), as follows:

$$F(H) = R \neq 0 \quad (3.5)$$

The best solution is the one which makes this residual a minimum or maintains it small at all points of the domain. In order to satisfy this aim, the differential relationship is integrated on the problem domain after weighting by a certain function and should equal zero as follows

$$\left. \begin{array}{l} \int_A W_j \cdot R \cdot dA = 0 \\ \text{or} \\ \sum_{n=1}^{ne} \int_{A^e} W_j \cdot R^e \cdot dA = 0 \end{array} \right\} \quad (3.6)$$

where:

W_j: Weighted function.

R^e: Element residual.

Here, the general equation of flow in a porous medium must be defined in order to be applied in the finite element analysis.

3.3 THE GENERAL EQUATION OF FLOW

The equations of flow are written in terms of the total head or the hydraulic head; these heads being the fundamental driving potentials for both saturated

and unsaturated flow. A full mathematical treatment of the whole subject is available in numerous references [e.g., **Harr, (1962)** and **Freeze, and Cherry, (1979)**]. Seepage of water through porous media depends on the properties of the medium in which the flow occurs, the properties of liquid and the hydraulic gradient.

The flow of water through saturated porous medium is generally governed by Darcy's law [**Harr,(1962)**].

The “**Darcy's law of seepage**” establishes a linear relationship between the seepage velocity and the hydraulic gradient. This law, which is a simple consequence of viscous flow neglecting inertia effects, can be obviously generalized to a two dimensional situation.

The velocities in the directions of the two orthogonal axes **x** and **y** are defined by a vector (**V**) or

$$\{\mathbf{V}\} = \begin{Bmatrix} V_x \\ V_y \end{Bmatrix} \quad (3.7)$$

where:

{V}: Vector of velocity in two orthogonal axes **X** and **Y**.

If the head gradient is defined similarly by its two components

$$-\{\mathbf{grad} \ h\} = \begin{Bmatrix} -\frac{\partial h}{\partial x} \\ -\frac{\partial h}{\partial y} \end{Bmatrix} \quad (3.8)$$

then, the most general linear relationship that can exist between the two quantities is of the form:

$$\{\mathbf{V}\} = -[\mathbf{K}]\{\mathbf{grad} \ h\} \quad (3.9)$$

in which **[K]** is a hydraulic conductivity matrix. The components of the hydraulic conductivity vector depend on the choice of the coordinates system. If the coordinated system is parallel to the principle directions [**global coordinated system**], the off-diagonal components of the vector are zero, so it will be:

$$[K] = \begin{vmatrix} k_x & 0 \\ 0 & k_y \end{vmatrix} \quad (3.10)$$

where:

k_x, k_y: Hydraulic conductivity in x, y-directions, respectively.

All equations which define the seepage can be derived from the principle of conservation of mass (the continuity equation). Its form is:

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \quad (3.11)$$

Then, the differential equation of steady state seepage flow can be developed by combining **Darcy's** equation, **Eq.(3.9)**, with the continuity equation, **Eq.(3.11)**, that is:

$$\frac{\partial}{\partial x} \left(K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial h}{\partial y} \right) = 0 \quad (3.12)$$

This equation is the **Laplace's** equation for two-dimensional seepage analysis and for non-homogeneous, anisotropic, and incompressible flow domain. Consequently, for the conditions of steady-state laminar flow, the seepage pattern can be completely determined by solving **Eq.(3.12)**, subject to the boundary conditions of the flow domain.

3.4 BOUNDARY CONDITIONS

The soil that is considered for analysis must be defined by boundaries, permeability of the soil, and heads imposed upon the water. This section considers the types of boundaries which may define a particular porous soil mass considered for analysis [**U.S. Army Corps of Engineers, (1986)**].

The boundary conditions should be specified before starting the solution. For the steady state of a confined flow, the boundary conditions are defined as follows:

3.4.1 Reservoir boundary

Along the boundaries of a reservoir, the pressure distribution may be taken as hydrostatic; that is, the pressure must be as a function of depth only, i.e., constant head. This type of boundary represents equipotential lines.

3.4.2 Impervious boundary

At impervious boundaries, the fluid can neither penetrate the boundary nor leave gaps, thus the velocity component normal to the boundary must be zero, so, this boundary is a streamline. Defining $(\mathbf{n})$ as the perpendicular direction to this line, then $(\partial h / \partial \mathbf{n} = 0)$ at the boundary.

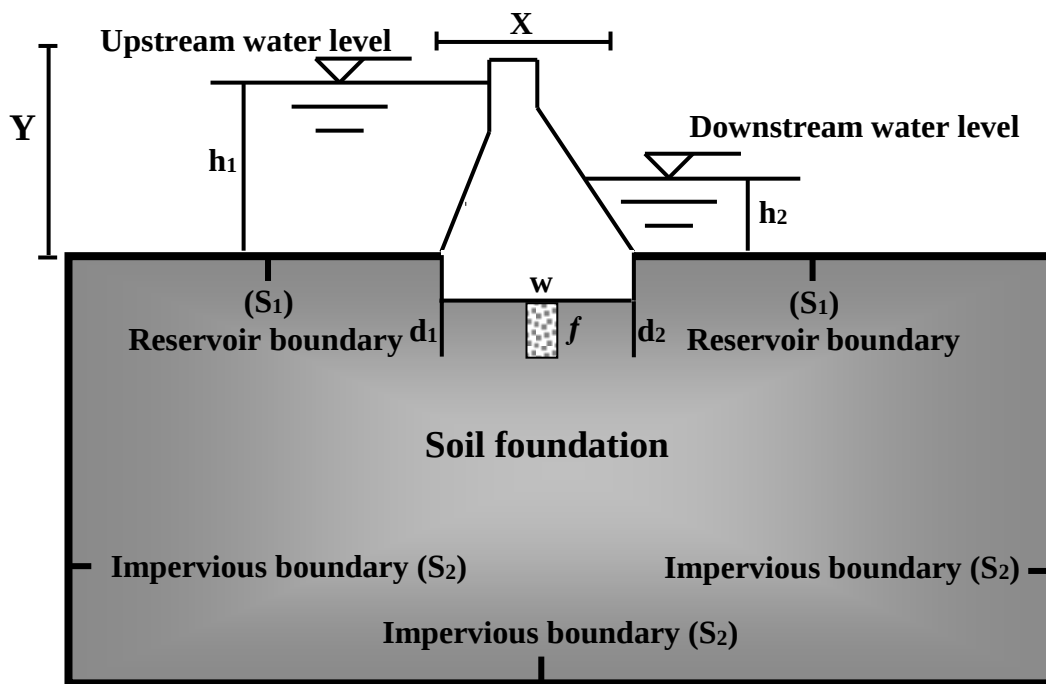


Fig. (3.1): Boundary conditions of a typical problem of seepage under a dam

3.5 THE FINITE ELEMENT FORMULATION

As it has been outlined before, the governing differential equation for two dimensional steady state seepage through anisotropic porous medium is given by Eq.(3.12).

There are several choices for the weighting functions. The best of all is the one which is known as **Galerkin** technique where the weighted function is taken equal to shape function (**i.e., $W_j = N_j$**) [**Segerlind, (1984)**].

The Galerkin principle

The **Galerkin** principle is applied to derive the elements matrix. The finite element approximation is applied to the general equation of seepage in porous media, **Eq.(3.12)**.

Substituting **Eq.(3.1)** in **Eq.(3.12)** gives:

$$\frac{\partial}{\partial x} \left[K_x \frac{\partial}{\partial x} \sum_{i=1}^n N_i \cdot H_i \right] + \frac{\partial}{\partial y} \left[K_y \frac{\partial}{\partial y} \sum_{i=1}^n N_i \cdot H_i \right] = R \neq 0 \quad (3.13)$$

By applying the **Galerkin** weighted residual method and substituting **Eq.(3.13)** in **Eq.(3.6)**, the result would be:

$$\sum_{i=1}^{ne} \int_{A^e} N_j^e \left[\frac{\partial}{\partial x} \left(K_x \frac{\partial}{\partial x} \sum_{i=1}^n N_i \cdot H_i \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial}{\partial y} \sum_{i=1}^n N_i \cdot H_i \right) \right] dA = 0 \quad (3.14)$$

where:

$$dA = dx * dy \quad \text{and } (i, j = 1, 2, \dots, n)$$

k_x, k_y: Hydraulic conductivity in x, y-directions, respectively.

H_i: Nodal value of head, **H**, of element (**e**).

N_i: Shape function of the element (**e**).

n : Number of nodes of element (**e**).

To reduce the continuity requirements for the shape function, **N**, from **C¹**-continuity to **C⁰**-continuity, integration by parts with **Green's** theorem is applied to the second order derivatives terms, where **C¹** and **C⁰** are the continuity for the shape function for the first and zero stage, respectively [**Burnett, (1987)**]. The first term of **Eq.(3.14)** will be:

$$\int_{A^e} N_j^e \frac{\partial}{\partial x} \left[K_x \frac{\partial}{\partial x} \sum_{i=1}^n N_i \cdot H_i \right] dA = \left\{ \int_S N_j^e \cdot K_x \frac{\partial}{\partial x} \sum_{i=1}^n N_i \cdot H_i \cdot dy - \int_{A^e} \frac{\partial N_j^e}{\partial x} K_x \frac{\partial}{\partial x} \sum_{i=1}^n N_i \cdot H_i \cdot dA \right\} \quad (3.15)$$

The second term of **Eq.(3.14)** will be:

$$\int_{A^e} N_j^e \frac{\partial}{\partial y} \left[K_y \frac{\partial}{\partial y} \sum_{i=1}^n N_i \cdot H_i \right] dA = \left\{ \int_S N_j^e \cdot K_y \frac{\partial}{\partial y} \sum_{i=1}^n N_i \cdot H_i \cdot dx - \int_{A^e} \frac{\partial N_j^e}{\partial y} K_y \frac{\partial}{\partial y} \sum_{i=1}^n N_i \cdot H_i \cdot dA \right\} \quad (3.16)$$

By substituting **Eq.(3.15)** and **Eq.(3.16)** in **Eq.(3.14)**, the result will be:

$$\left\{ \sum_{i=1}^{ne} \left[\int_{A^e} - \left[\frac{\partial N_j}{\partial x} K_x \frac{\partial}{\partial x} \sum_{i=1}^n N_i \cdot H_i + \frac{\partial N_j}{\partial y} K_y \frac{\partial}{\partial y} \sum_{i=1}^n N_i \cdot H_i \right] dA \right] + \int_S N_j \cdot K_n \frac{\partial}{\partial n} \sum_{i=1}^n N_i \cdot H_i \cdot dS = 0 \right\} \quad (3.17)$$

where:

$S = S_1^e + S_2^e$ represents the surface boundaries of the element.

The boundary conditions are:

$$1: H = H_0 \quad (3.18a)$$

on S_1 which represents a reservoir boundary, and

$$2: \left[K_x \frac{\partial H}{\partial x} \right] I_x + \left[K_y \frac{\partial H}{\partial y} \right] I_y = 0 \quad (3.18b)$$

on S_2 which represents the impermeable boundary.

By applying the finite element method and using **Galerkin** weighted residual method, **Eq.(3.18b)** will be:

$$\sum_{i=1}^{ne} \left[\int_{S_2^e} N_j \left[K_x \frac{\partial}{\partial x} \sum_{i=1}^n N_i \cdot H_i \cdot I_x + K_y \frac{\partial}{\partial y} \sum_{i=1}^n N_i \cdot H_i \cdot I_y \right] dS \right] = 0 \quad (3.19)$$

where:

$$dx = I_x \cdot ds \quad \text{and} \quad dy = I_y \cdot ds$$

Subtracting **Eq.(3.17)** from **Eq.(3.19)** gives:

$$\sum_{i=1}^{ne} \left[\int_{A^e} \left[\frac{\partial N_j}{\partial x} K_x \frac{\partial}{\partial x} \sum_{i=1}^n N_i \cdot H_i + \frac{\partial N_j}{\partial y} K_y \frac{\partial}{\partial y} \sum_{i=1}^n N_i \cdot H_i \right] dx \cdot dy - \int_{S_1^e} \left[N_j \left(K_x \frac{\partial}{\partial x} \sum_{i=1}^n N_i \cdot H_i \cdot I_x + K_y \frac{\partial}{\partial y} \sum_{i=1}^n N_i \cdot H_i \cdot I_y \right) \right] dS \right] = 0 \quad (3.20)$$

and in matrix form:

$$\sum_{i=1}^{ne} [K^e] \{H_i\} = 0 \quad (3.21)$$

where:

$[k^e]$ represents the element matrix and its typical coefficient is:

$$K_{ij} = \left\{ \int_{A^e} \left[\frac{\partial N_j}{\partial x} K_x \frac{\partial N_i}{\partial x} + \frac{\partial N_j}{\partial y} K_y \frac{\partial N_i}{\partial y} \right] dx \cdot dy - \int_{S_1^e} \left[K_x \cdot I_x \cdot N_j \frac{\partial N_i}{\partial x} + K_y \cdot I_y \cdot N_j \frac{\partial N_i}{\partial y} \right] dS \right\} \quad (3.22)$$

The second term of **Eq.(3.22)** will be deleted from it because of this term not effect to the form of the element matrix. Therefore, **Eq.(3.22)** will be:

$$K_{ij} = \int_{A^e} \left[\frac{\partial N_j}{\partial x} K_x \frac{\partial N_i}{\partial x} + \frac{\partial N_j}{\partial y} K_y \frac{\partial N_i}{\partial y} \right] dx \cdot dy \quad (3.23)$$

To represent $[k^e]$ in matrix form as follows:

$$[K^e] = \int_{A^e} [B^e]^T \cdot [D^e] \cdot [B^e] dx \cdot dy = 0 \quad (3.24)$$

where:

$$[B^e] = \begin{bmatrix} \frac{\partial N_1^e}{\partial x} & \frac{\partial N_2^e}{\partial x} & \dots & \frac{\partial N_n^e}{\partial x} \\ \frac{\partial N_1^e}{\partial y} & \frac{\partial N_2^e}{\partial y} & \dots & \frac{\partial N_n^e}{\partial y} \end{bmatrix}$$

$$[D^e] = \begin{bmatrix} K_x & 0 \\ 0 & K_y \end{bmatrix}$$

Assembling the equations or matrices of the finite element $[k^e]$ to form the global matrix gives:

$$[K]\{H\} = 0 \quad (3.25)$$

where:

$$[k]: \text{The global matrix} = \sum [K^e]$$

The assembled **Eq.(3.25)** is solved by using the frontal solution because of its efficiency in computers storage requirement [**Irons, (1970)**].

3.6 DERIVATION OF ELEMENT MATRIX FOR LINEAR TRIANGULAR ELEMENT

The linear triangular element shown in **Fig. (3.2)** has straight sides and three nodes, one at each corner. A constant labeling of the nodes is a necessity and the labeling here proceeds counterclockwise from node **(1)**, which is arbitrarily specified. The nodal values of **H** are **H₁**, **H₂**, and **H₃**, whereas the nodal coordinates are **(x₁,y₁)**, **(x₂,y₂)**, and **(x₃,y₃)** [**Segerlind, (1984)**].

The interpolation polynomial is:

$$H = a_0 + a_1 \cdot x + a_2 \cdot y \quad (3.26)$$

To find the constant values **a₀**, **a₁** and **a₂**, substitute **x**, **y** and **H** for the three nodes, that results in:

$$H_1 = a_0 + a_1 \cdot x_1 + a_2 \cdot y_1$$

$$H_2 = a_0 + a_1 \cdot x_2 + a_2 \cdot y_2$$

$$H_3 = a_0 + a_1 \cdot x_3 + a_2 \cdot y_3$$

In matrix form:

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \quad (3.27)$$

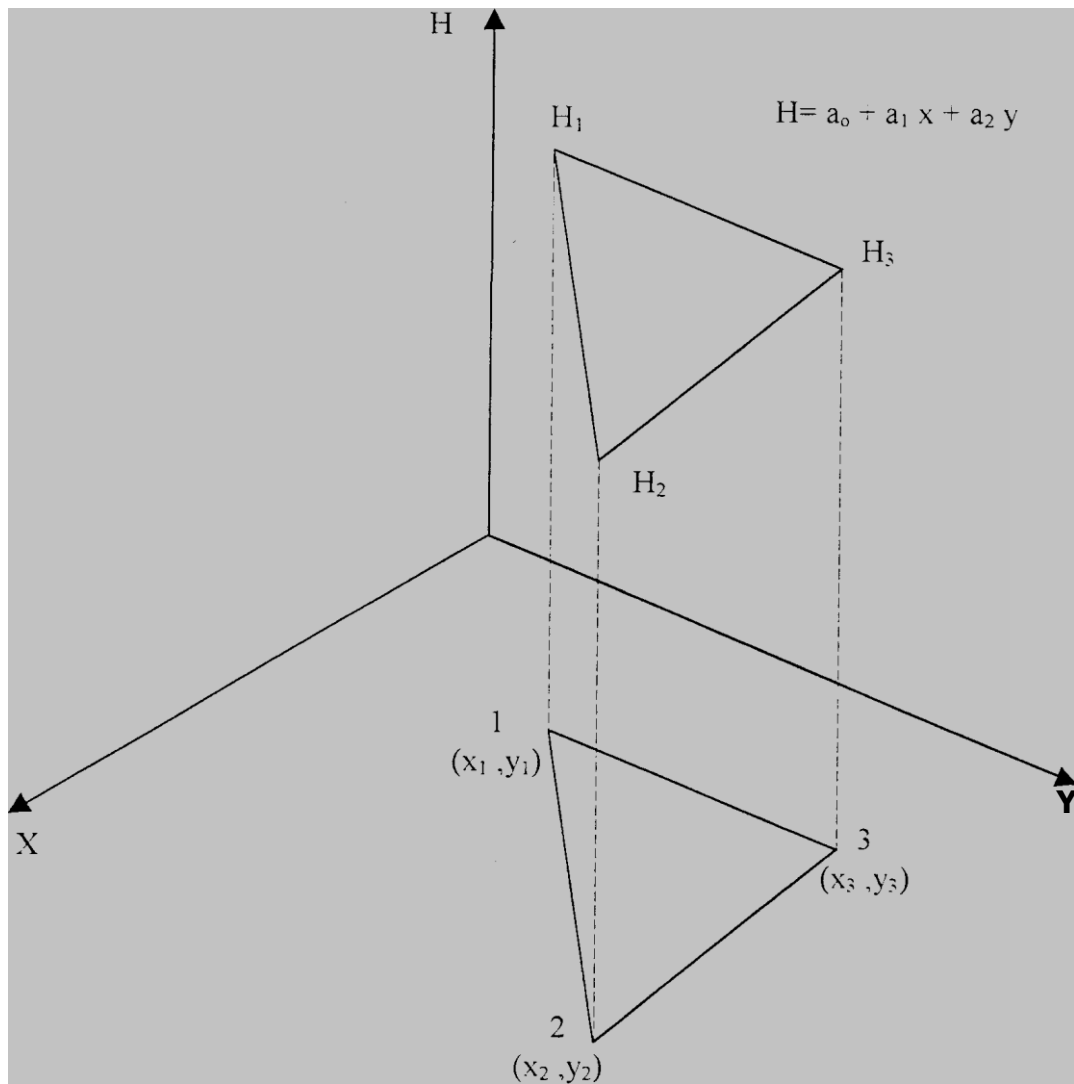


Fig. (3.2): Parameters for the linear triangular element according to [Segerlind,(1984)].

Solving **Eq.(3.27)** for the constants and substituting the constants in **Eq.(3.26)**, the result, in matrix form, will be:

$$H = \begin{bmatrix} 1 & x & y \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}^{-1} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} \quad (3.28)$$

Then, the total head over the element **(e)** is:

$$H = (a_1 + b_1 \cdot x + c_1 \cdot y)H_1 + (a_2 + b_2 \cdot x + c_2 \cdot y)H_2 + (a_3 + b_3 \cdot x + c_3 \cdot y)H_3$$

or, when substituting the shape functions, the total head will be:

$$H = N_1 \cdot H_1 + N_2 \cdot H_2 + N_3 \cdot H_3$$

or,

$$H = \sum_{i=1}^n N_i \cdot H_i$$

where:

n: Number of nodes in the element = 3

The interpolation function is:

$$N_i = a_i + b_i \cdot x + c_i \cdot y \quad (3.29)$$

Hence, the constants at node **(1)** equal:

$$\left. \begin{aligned} a_1 &= \frac{1}{D} (x_2 \cdot y_3 - x_3 \cdot y_2) \\ b_1 &= \frac{1}{D} (y_2 - y_3) \\ c_1 &= \frac{1}{D} (x_3 - x_2) \end{aligned} \right\} \quad (3.30a)$$

The constants at node **(2)** equal:

$$\left. \begin{aligned} a_2 &= \frac{1}{D} (x_3 \cdot y_1 - x_1 \cdot y_3) \\ b_2 &= \frac{1}{D} (y_3 - y_1) \\ c_2 &= \frac{1}{D} (x_1 - x_3) \end{aligned} \right\} \quad (3.30b)$$

And, the constants at node **(3)** equal:

$$\left. \begin{aligned} a_3 &= \frac{1}{D}(x_1 \cdot y_2 - x_2 \cdot y_1) \\ b_3 &= \frac{1}{D}(y_1 - y_2) \\ c_3 &= \frac{1}{D}(x_2 - x_1) \end{aligned} \right\} \quad (3.30c)$$

where:

D: The determinant of the element **(e)**, and will be:

$$\mathbf{D} = \begin{bmatrix} \mathbf{1} & \mathbf{x}_1 & \mathbf{y}_1 \\ \mathbf{1} & \mathbf{x}_2 & \mathbf{y}_2 \\ \mathbf{1} & \mathbf{x}_3 & \mathbf{y}_3 \end{bmatrix} = 2\mathbf{a} \quad (3.30d)$$

where:

a : The area of the triangle element.

The interpolation function derivatives with respect to **x** and **y** would be:

$$\left. \begin{aligned} \frac{\partial N_i}{\partial x} &= b_i \\ \frac{\partial N_i}{\partial y} &= c_i \end{aligned} \right\} \quad (3.31)$$

By substitution **Eq.(3.31)** in **Eq.(3.23)** would be:

$$\mathbf{K}_{ij} = \int_{\mathbf{A}^e} \left(\mathbf{K}_x \cdot \mathbf{b}_i \cdot \mathbf{b}_j + \mathbf{K}_y \cdot \mathbf{c}_i \cdot \mathbf{c}_j \right) d\mathbf{x} \cdot d\mathbf{y} \quad (3.32a)$$

$$\mathbf{K}_{ij} = \mathbf{A} \left(\mathbf{K}_x \cdot \mathbf{b}_i \cdot \mathbf{b}_j + \mathbf{K}_y \cdot \mathbf{c}_i \cdot \mathbf{c}_j \right) \quad (3.32b)$$

Equation (3.32b) represents the element matrix and it is rectangular matrix **(3 * 3)**.

3.7 COMPUTATION OF THE HYDRAULIC GRADIENT

Hydraulic gradient is calculated directly from head distribution over elements as follows:

$$H = \sum_{i=1}^n N_i \cdot H_i$$

Therefore, $\frac{\partial H}{\partial x}$ and $\frac{\partial H}{\partial y}$ can be written as follows:

$$\begin{aligned} \frac{\partial H}{\partial x} &= \frac{\partial [N]}{\partial x} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} \\ \frac{\partial H}{\partial y} &= \frac{\partial [N]}{\partial y} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} \end{aligned} \quad (3.33)$$

From **Eq.(3.33)** the hydraulic gradient in **x** and **y** directions are obtained.

3.8 COMPUTER PROGRAM

The computer program which is used in this research is entitled “**Seep2d**”. This program is in Fortran and it's a two-dimensional finite element flow model designed to compute seepage in earth and concrete dams. **Seep2d** can be used to model confined and unconfined flow situations. Seep2d can model complicated two-dimensional seepage problems involving complex model geometries and soils that are non-homogenous and anisotropic.

Seep2d is a steady state flow model and will compute the piezometric head value at each node of the finite element mesh. From these values, flow lines and equipotential lines are plotted showing the resulting seepage net.

3.8.1 Mesh generation

At first, the region is subdivided into triangular elements by using a computer program “**Seep2d**”. After mesh generation, the nodes are numbered, the elements are numbered, and the different materials of the structure are specified. (**GMS “Ground water Modeling System”** program) is applied for a hypothetical case study presented in **Fig. (3.3)** as shown below.

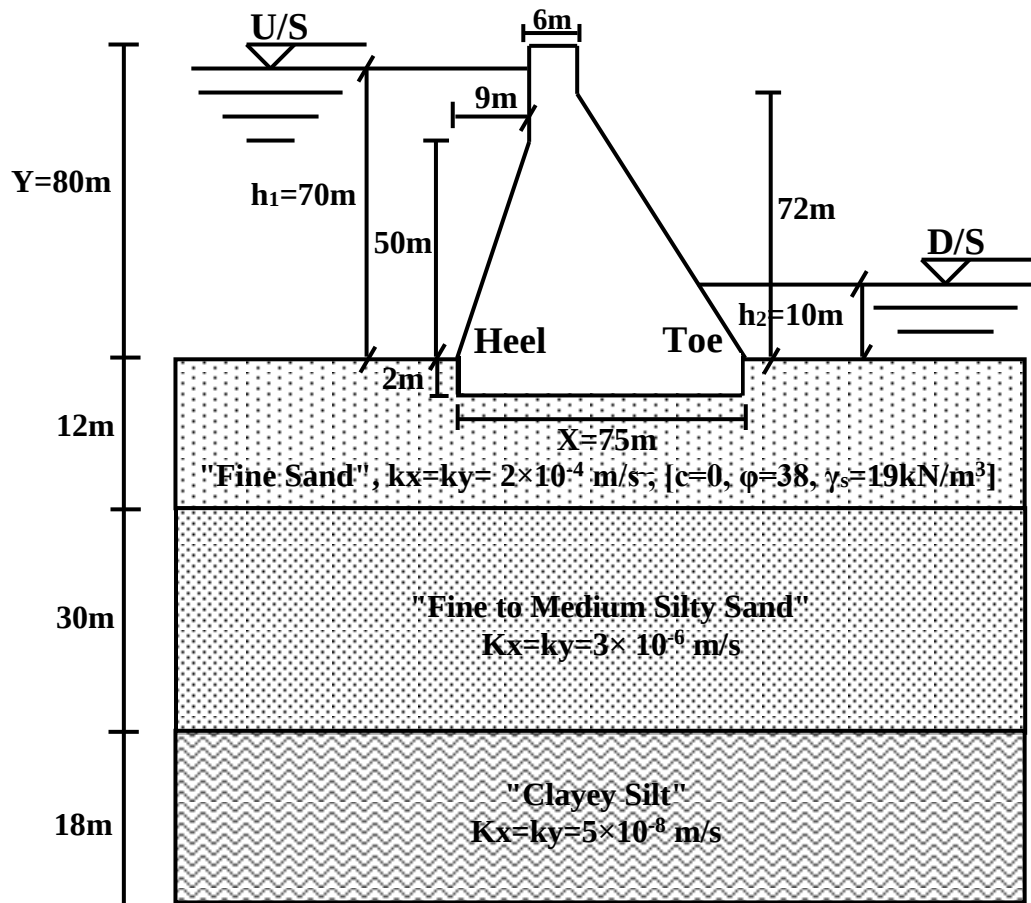


Fig. (3.3): Hypothetical case study
(Concrete gravity dam)

The output of the **GMS** (mesh generation) is shown in the **Fig. (3.4)**.

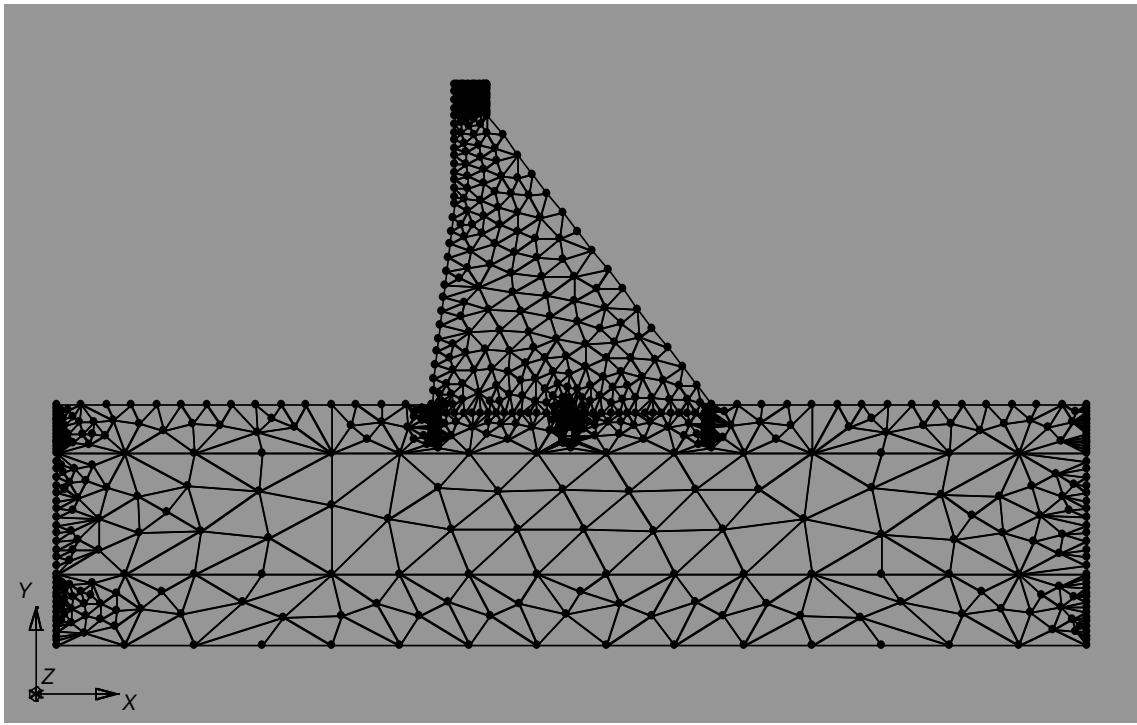


Fig. (3.4): Finite element mesh of the hypothetical case study

3.8.2 Inputs file

The inputs for “**Seep2d**” program are:

1. The number of nodes, elements, and different materials of the region.
2. The values of soil permeabilities (k_x and k_y) for each material.
3. Placing the (x, y) coordinates of each node and the total number of elements of the mesh.
4. Indicating the boundary conditions (known head) at some nodes, (upstream and downstream nodes) of concrete dam.

3.8.3 Outputs of the program

The outputs of the “**Seep2d**” program are:

1. Flow lines, flow rate, pressure head, pore pressure, velocity, and total head for each element.
2. Indicating the [adjusting mesh].

3.9 VERIFICATION OF THE GMS PROGRAM

The computation procedure for the water seepage under the dam has been previously presented in this chapter. The verification of the procedure is very important, thus it should be done before applying the respective approach to any practical problem. The main aims of the verification are to check the reliability of the theoretical aspects utilized in the derivation of the considered approach, and then examine the proper working of the computer program that has been prepared to execute the computations. The verification is performed by applying the developed computation procedure to some problems that had been previously analyzed by other researchers.

Special cases are investigated herein for the purposes of discussion with the results of **Chawla, (1975)** [analytical solution] and **Chawla, (1976)** [experimental results] as shown in **Figs. (3.5) and (3.6)**, respectively, by the line curves in **black**. These cases involved upstream and downstream cutoffs of lengths [**$d_1 = d_2 = 7.5\text{m}$**] with a filter media of depth and width of [**$f = 0.0$ and $w = d_1$**] or with a filter media of depth and width of [**$f = 0.5 d_1$ and $w = 0.0$**]. The finite element method and the package (**Seep2d**) program are used for analyzing these cases.

For the purposes of the analysis, a dimensionless parameter (**H**), representing the difference ratio of the piezometric head [**as defined in Eq. (3.34)**], has been considered as below:

$$H = \frac{h_3 - h_2}{h_1 - h_2} \quad (3.34)$$

where:

H : ratio of head difference;

h_1 : Height of water in the upstream, (L);

h_2 : Height of water in the downstream, (L);

h_3 : piezometric head at any point in the problem domain, (L).

Consequently, (H) equals (one) for the upstream and (zero) for the downstream.

The results of running the (GMS-program) are shown in Figs. (3.5) and (3.6). The results when there is a filter media for the case [$d_1 = d_2 = 7.5\text{m}$, $f = 0.0$, and $w = d_1$] and without a filter media, and when there is a filter media for the case [$d_1 = d_2 = 7.5\text{m}$, $f = 0.5 d_1$, and $w = 0.0$] and without a filter media are represented in the figures by the red curves when there is a filter media and the blue curves when without a filter media for each case, respectively.

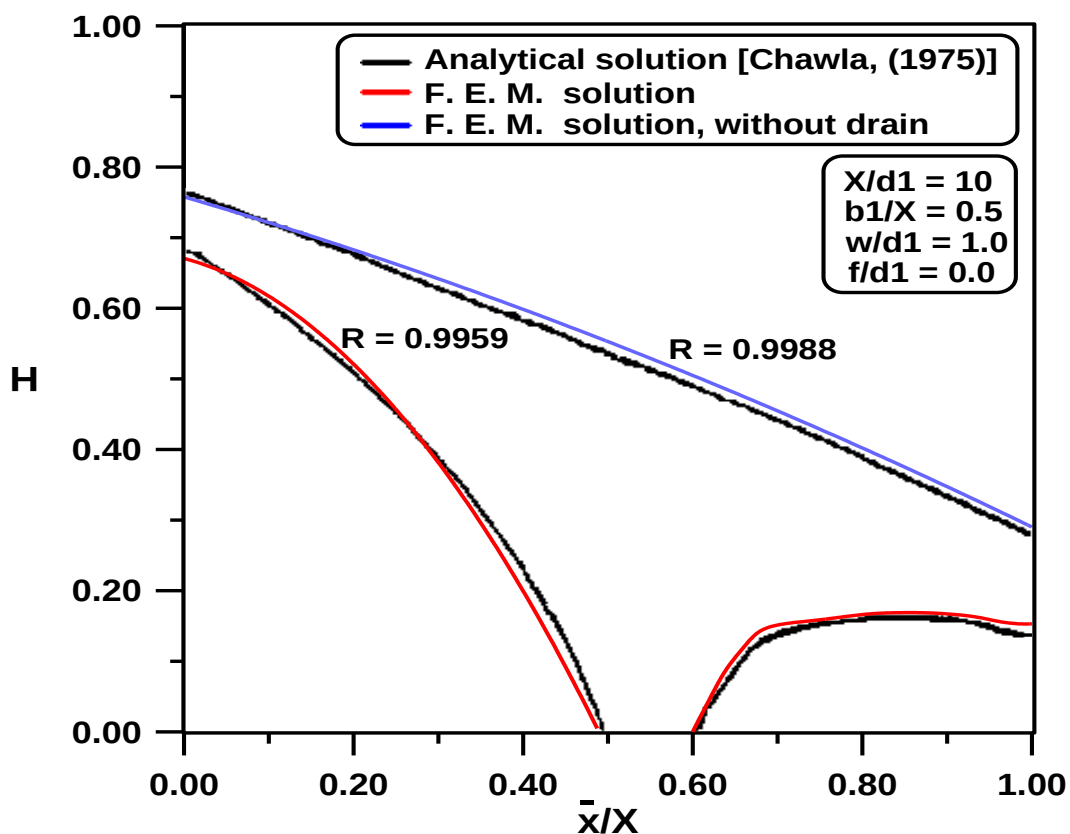


Fig. (3.5): Comparison between the F. E. M. solution and the analytical solution

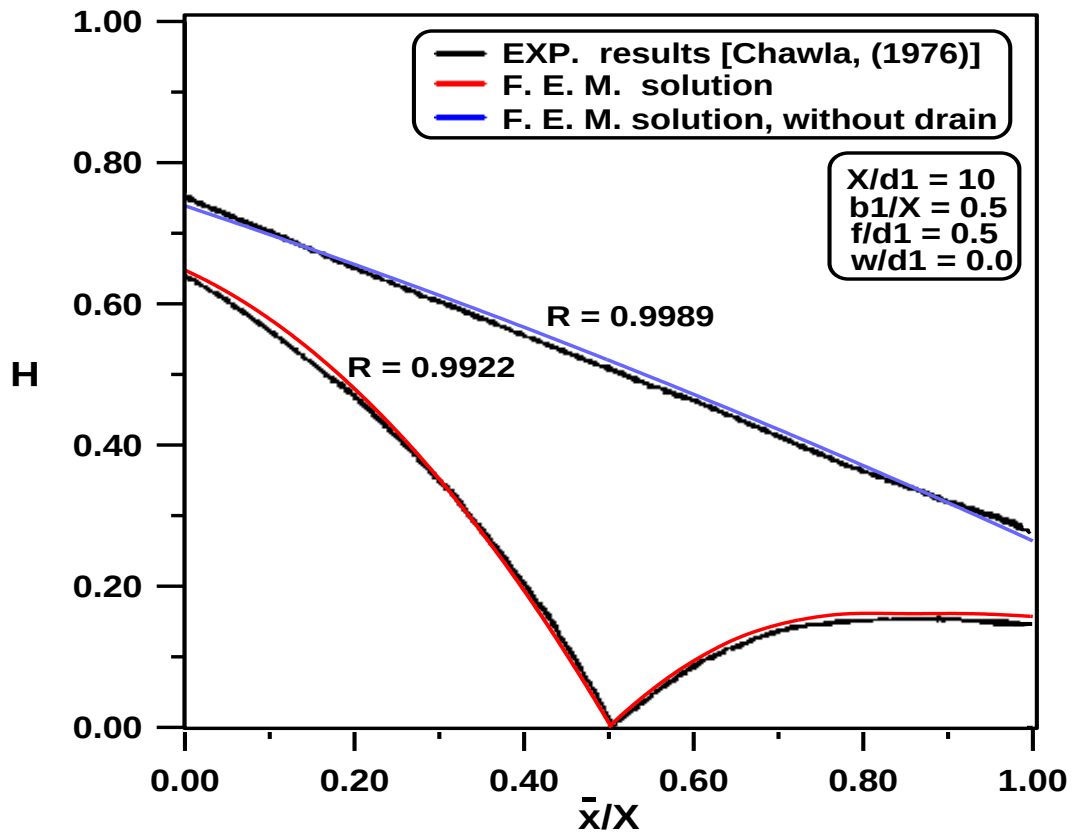


Fig. (3.6): Comparison between the F. E. M. solution and the experimental results

When the results of (**GMS-Seep2d**) program shown in **Figs. (3.5) and (3.6)** are compared with the results of **Chawla, (1975)** and **Chawla, (1976)** that are shown in the same figures, it can be seen that the effect of the width and depth of the filter media as computed by the former is almost identical to that arrived at by the analytical solution and experimental results. Therefore, the aforementioned program is considered acceptable as it gave results that agree well with those of comparable researches.

The overall structure chart of the (**GMS**) program is shown in **Fig. (3.7)**.

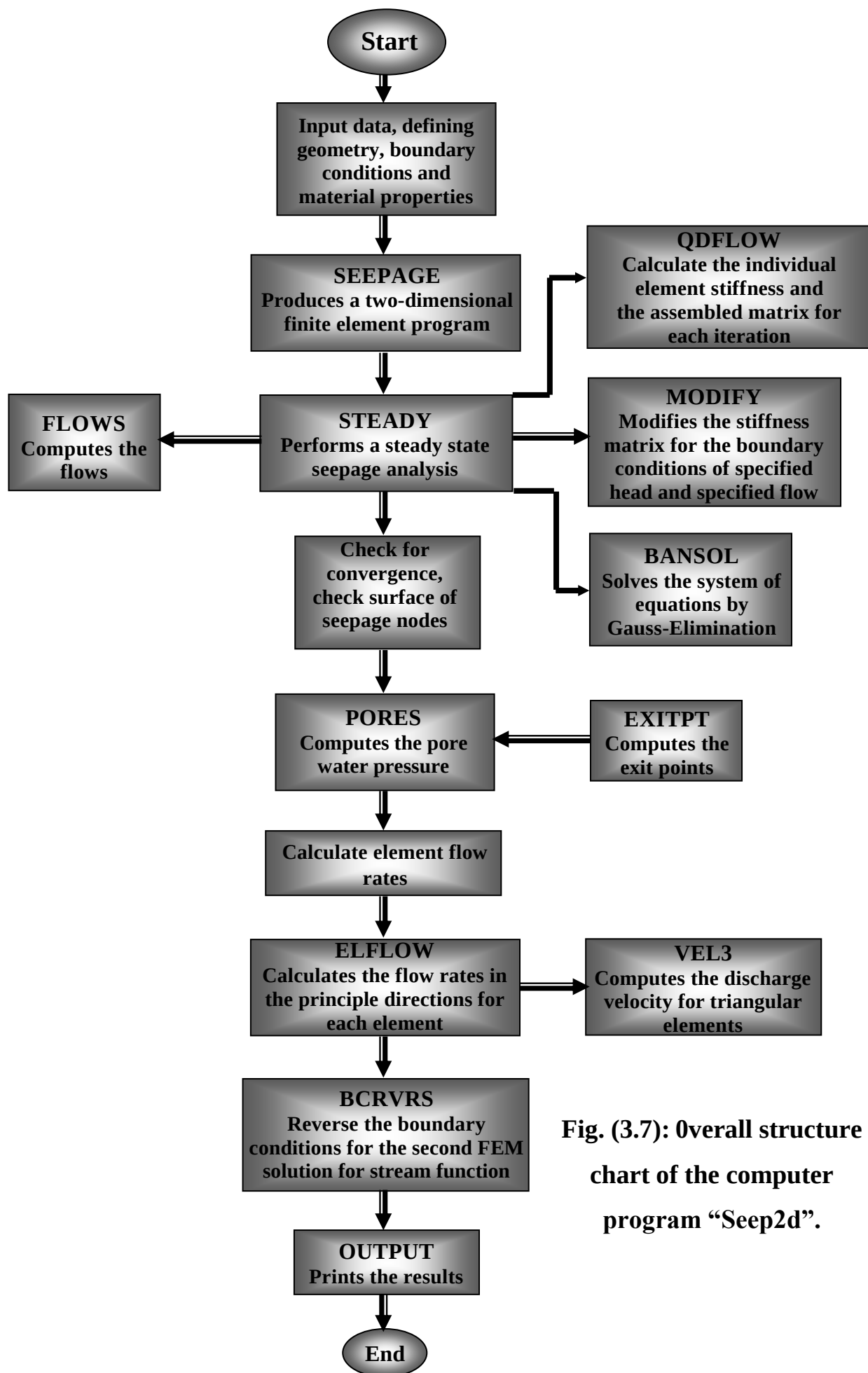


Fig. (3.7): Overall structure chart of the computer program "Seep2d".

CHAPTER FOUR

CHAPTER FOUR

STABILITY ANALYSES

Specific stability criteria for a particular loading combination are dependent upon the degree of understanding of the soil structure interaction and site geology, and to some extent, on the method of analysis. Assumptions used in the analysis should be used upon construction records and the performance of the structures under historical loading conditions. In the absence of available design data and records, site investigations may be required to verify assumptions. Safety factors are intended to reflect uncertainty associated with the analysis. Uncertainty resides in the knowledge of the loading conditions and the material parameters that define the dam and the foundation. Uncertainty can also be introduced by simplifying assumptions made in analysis. When sources of uncertainty are removed, safety factors can be lowered. The basic requirement for stability of a gravity dam subjected to static loads is that force and moment equilibrium be maintained without exceeding the limits of concrete, foundation or concrete-foundation interface strength [FERC, (2002)].

This chapter presents the analysis of the factors of safety against overturning, sliding, and piping as well as the eccentricity of the vertical resultant force to make the floor of the dam safe against tension under the foundation. The stability of the dam considered in this research may be studied in two types as follows:

4.1 TYPES OF THE STABILITY ANALYSIS

The stability of the concrete gravity dam (the hypothetical case study) shown in **Fig. (3.3)** presented in this research as mentioned previously may be divided into two types:

1. Soil stability analysis (hydraulic analysis).
2. Structure stability analysis.

4.1.1 Soil stability analysis

One of the familiar failure of hydraulic structures is the failure caused by soil piping downstream of the structures especially in cases where loose soils are involved [Al-Oubaidy et al., (1988)]. Therefore, the seepage of water under the structure must be limited in order to avoid failure of the structure by the phenomenon of piping.

To safeguard the structure from this phenomenon, the factor of safety against piping may be found as follows:

$$\text{Factor of safety against piping} = \frac{\text{Critical Gradient (I}_{cr}\text{)}}{\text{Exit Gradient (I}_e\text{)}} \quad (4.1)$$

The critical gradient is determined by the place unit weight of the soil and is the gradient at which upward forces on the soil particles equal the submerged weight of the soil; therefore, it can be considered ($I_{cr} = 1.0$). The escape or exit gradient, I_e , is the rate of dissipation of head per unit length in the area where seepage is exiting the porous media. For confined flow, the area of concern is usually along the uppermost flow line near the flow exit [U.S. Army Corps of Engineers, (1986)].

4.1.2 Structure stability analysis

It is impossible to quantify the overall safety of a dam. The approach to achieve maximum dam safety is well understood from viewpoints of design, construction, operation, and maintenance. Therefore, the most important prerequisite for dam safety is the professional competence of people associated with the dam over its life span. Because of the undetected deficiencies, a dam that is observed and stated as safe may still fail. Dam safety must take precedence over all other consideration. Therefore, the concepts have been continuously developed through time for better understanding and for more logical approaches to the design of dams [Beser, (2005)].

The stability of a gravity dam can be approximately and easily analyzed by two dimensional gravity method and can be precisely analyzed by three dimensional methods as slab analogy method, trail load twist method, or by experimental studies on models [**Garg, (1978)**].

Selection of the method of analysis should be governed by the type and configuration of the structure being considered. The gravity method will generally be sufficient for the analysis of most structure, however, more sophisticated methods may be required for the structures that are curved in plan, or structures with unusual configurations. The gravity method assumes that the dam is a two-dimensional rigid block. The foundation pressure distribution is assumed to be linear. It is usually prudent to perform gravity analysis before doing rigorous studies [**FERC, (2002)**]. Therefore, it is used in this research, and according to that, the forces acting on the concrete gravity dam are:

4.2 FORCES ACTING ON THE DAM

A gravity dam derives its stability from the force of gravity of the materials in its section, and hence the name. The gravity dam has sufficient weight so as to withstand the forces and the overturning moment caused by the water impounded in the lake behind it. It transfers the loads to the foundations by cantilever action and hence good foundations are prerequisite for the gravity dam [**Varshney et al., (1982)**].

The following basic loadings are generally used in concrete gravity dams:

1. Weight of the dam.
2. Weight of the water at upstream and downstream of the dam.
3. Water pressure.
4. Uplift pressure.
5. Ice pressure.
6. Silt pressure.

7. Seismic forces (earth quake forces) [U.S. Army Corps of Engineers, (1995)].

In addition to these forces, there are forces acting on the cutoffs due to the soil pressure and these forces are named as:

8. Active and passive earth pressures (Forces due to earth pressure).

Each of the forces listed above, may be considered as follows:

4.2.1 Weight of the dam

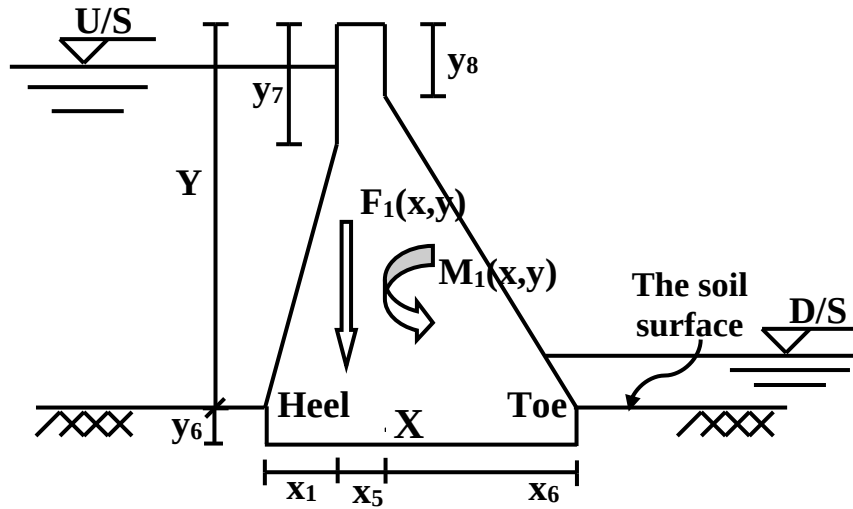


Fig. (4.1): Forces of weight of the dam

where:

$x_1 + x_5 + x_6 = X$ = Total floor length of the dam (m).

Y : Total height of the dam (m).

y_6 : Height of the dam under the soil.

y_7 : Height of a dam over the upstream inclined line (m).

y_8 : Height of a dam over the downstream inclined line (m).

γ_c : Unit weight of a concrete gravity dam (kN/m^3).

4.2.2 Weight of the water at upstream and downstream of the dam

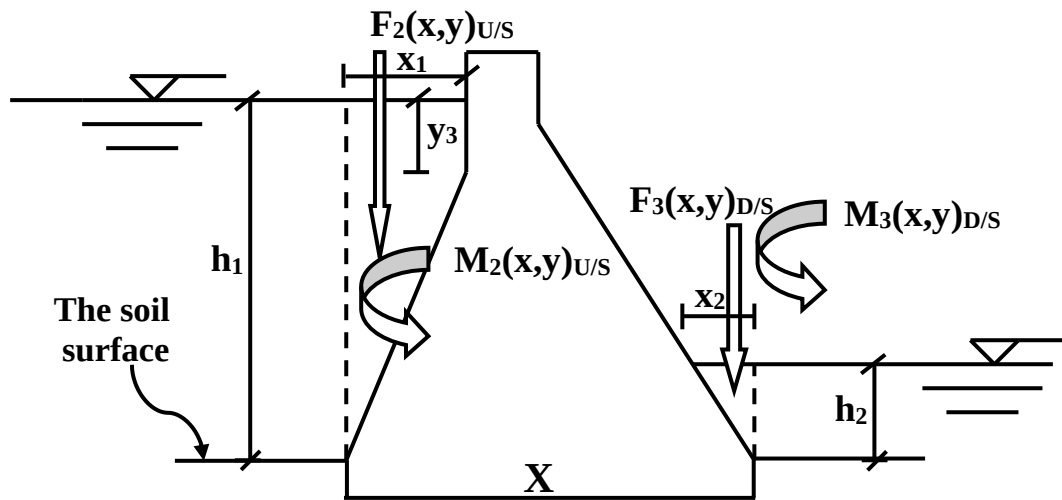


Fig. (4.2): Forces of weight of the water

where:

γ_w : Unit weight of the water (10 kN/m³).

x_2 : Triangle base of tail water (m).

h_1 : Height of water in the upstream (m).

h_2 : Height of water in the downstream (m).

y_3 : Height of water over the upstream inclined line (m).

4.2.3 Water pressure

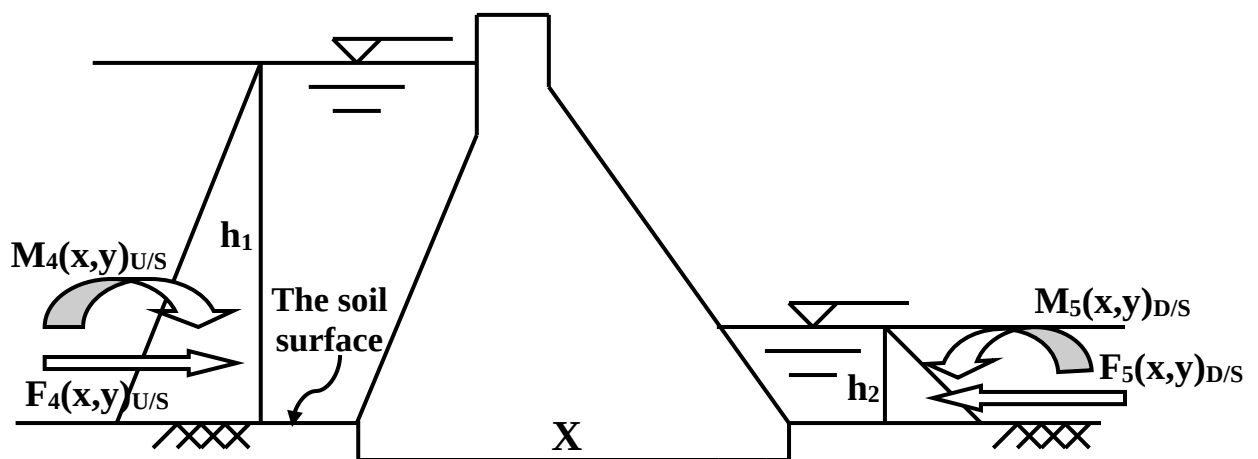


Fig.(4.3): Forces due to water pressures

4.2.4 Uplift pressure

The distribution of the uplift pressure that result from the (GMS-program) when there is an intermediate filter media under the hypothetical concrete gravity dam is shown in **Fig. (4.4)**:

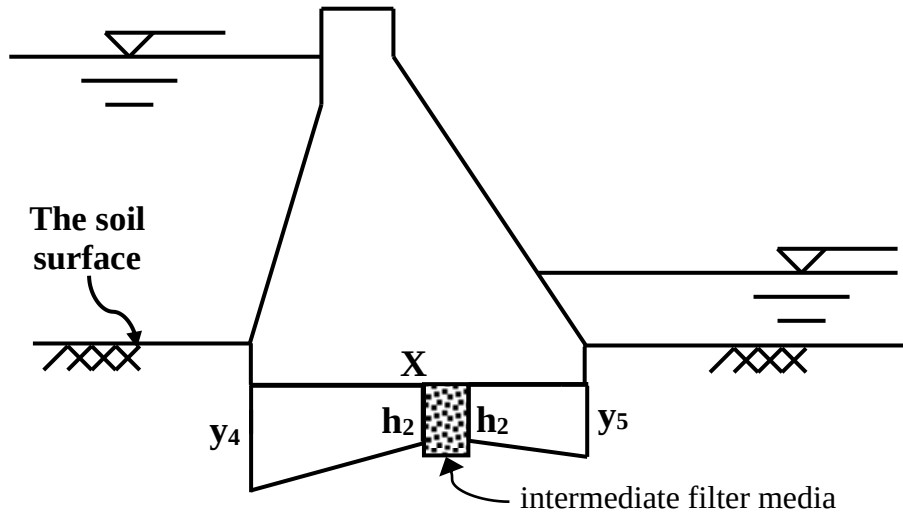


Fig.(4.4): Distribution of the uplift pressure due to the effect of the filter media where:

y_4 : Value of the uplift pressure at the upstream (m).

y_5 : Value of the uplift pressure at the downstream (m).

In this research, for easy of the calculation at the stability program to find the stability criterions, using the distribution of the uplift pressure shown in **Fig. (4.5)** below because of the area of the trapezoidal in this figure is approximately the same area of the two trapezoidal shown in **Fig. (4.4)**.

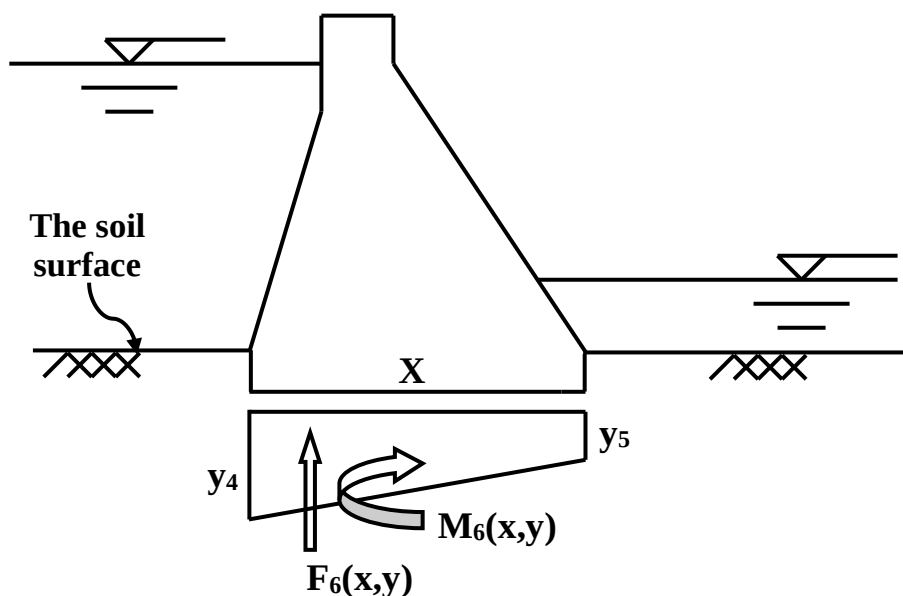


Fig.(4.5): Forces due to uplift pressures

4.2.5 Ice pressure

The ice pressure which may be formed on the water surface of the reservoir in cold countries, may sometimes melt and expand. The dam face has then to resist the thrust exerted by the expanding ice. This force acts linearly along the length of the dam and at the reservoir level. The magnitude of this force depends upon temperature variations [**Garg, (1978)**]. This force is not important except for reservoirs at very high elevations [**Varshney et al., (1982)**].

4.2.6 Silt pressure

Many gravity dams have been designed without regard to silt load. In general, the silt load against storage dams will be a small factor but against diversion dams it is likely to be more important. In either case there is some basis neglecting the silt load [**Bureau of Reclamation, (1987)**]. So in this research, the silt pressure is ignored.

4.2.7 Seismic forces

Earth-quakes impart acceleration to the dam which usually increase the effective loading on the dam. Some allowance for earthquake effects must therefore be made in the design of concrete gravity dams [**Bureau of Reclamation, (1987)**]. In areas of no earth quakes or very few earthquakes, these forces are neglected [**Garg, (1978)**]. In Iraq the earthquakes are not considered because they rarely happen, therefore, they are not considered in this research.

4.2.8 Forces due to earth pressure

When there are concrete cutoffs or sheet piles drilled in the soil under a dam and joining with it, there will be active and passive earth pressures that affect in the cutoffs or sheet piles and participate in the stability calculations of the structure.

Earth pressure is the force per unit area exerted by the soil on a structure. Its magnitude depends on the physical properties of the soil, the nature of the soil-structure interface, and possible modes of deformation of the structural system. In the case of cohesive soils, the earth pressure is influenced by the time-dependent nature of soil properties [Al-Khafaji and Anderslan, (1992)]. The quantitative measure of the lateral earth pressure is required to estimate a structural member for either a design or stability analysis [Bowles, (1996)].

Theories of earth pressure

There are two theories for calculating earth pressure:

1. Rankine's Theory.
2. Coulumb's Theory [Syal and Goel, (2004)].

4.2.8.1 Rankine's theory

In this theory the friction between retaining wall and earth is neglected [Syal and Goel, (2004)]. But when there are concrete cutoffs existing at the upstream or/and downstream side of the dam to safeguard the dam from the movement of the soil and finally safeguard the dam against the phenomenon of piping; therefore, the friction between the soil and the concrete cutoffs for active and passive earth pressures is must be considered in the calculations of the stability analysis, as follows:

A. Active pressures

If the soil surface subtends an angle (β) with the horizontal, then, according to Rankine, the coefficient for active earth pressure is: [Nilson and Winter, (1986)].

$$K_a = \cos\beta \left[\frac{\cos\beta - \sqrt{\cos^2\delta\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \right] \quad (4.2)$$

where:

ka : Coefficient of active earth pressure.

ϕ : Angle of internal earth friction.

β : Angle of earth slopping over the horizontal surface.

The forces of active pressure acting on the cutoffs at the upstream and downstream of the dam are shown in **Fig. (4.5)** as below:

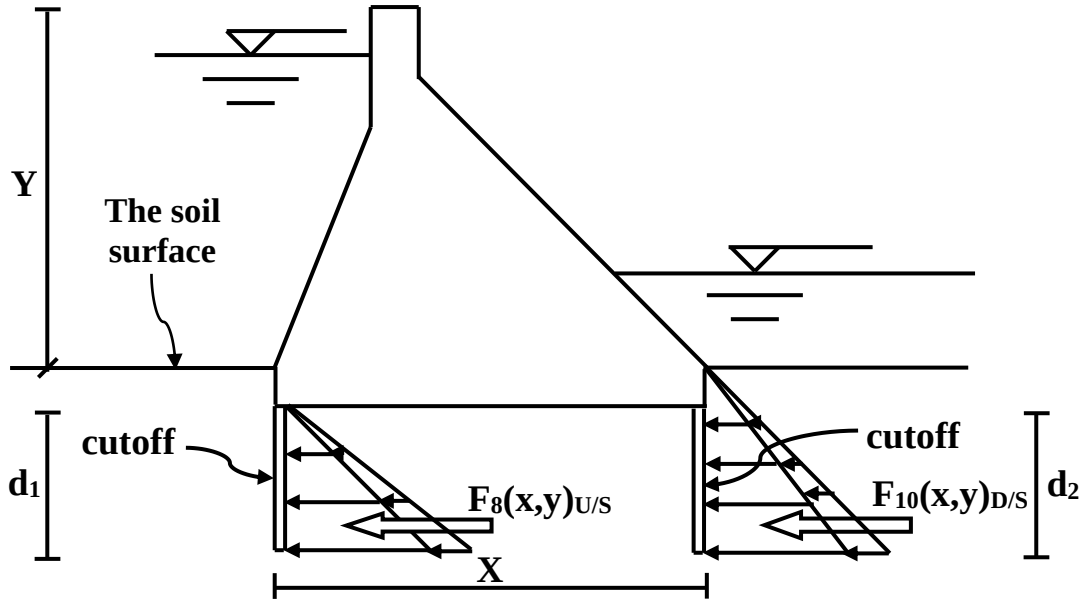


Fig. (4.6): Distribution of active forces on the cutoffs

$$F_{8(x,y)U/S} = \left[\left(\frac{1}{2} \gamma^- \cdot ka \cdot d_1^2 \right) - (2C \cdot d_1 \cdot \sqrt{ka}) + (2C^2 / \gamma^-) + \left(\frac{1}{2} \gamma_w \cdot d_1^2 \right) \right] \quad (4.3)$$

$$F_{10(x,y)D/S} = \left[\left(\frac{1}{2} \gamma^- \cdot ka \cdot (d_2 + y_6)^2 \right) - (2C \cdot (d_2 + y_6) \cdot \sqrt{ka}) + (2C^2 / \gamma^-) + \left(\frac{1}{2} \gamma_w \cdot (d_2 + y_6)^2 \right) \right] \quad (4.4)$$

where:

C : Cohesion of the soil under the dam (kN/m²).

γ^- : Effective unit weight between the soil and water ($\gamma_s - \gamma_w$) (kN/m³).

γ_s : Saturated unit weight of the soil (kN/m³).

d1 : Depth of upstream cutoff (m).

d2 : Depth of downstream cutoff (m).

B. Passive pressures

According to the **Rankine**, the coefficient of passive earth pressure is:[Nilson and Winter, (1986)].

$$K_p = \cos\beta \left[\frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}} \right] \quad (4.5)$$

where:

kp : Coefficient of active earth pressure.

The forces of passive pressure acting on the cutoffs at the upstream and downstream sides of the dam are as shown in **Fig. (4.6)** below:

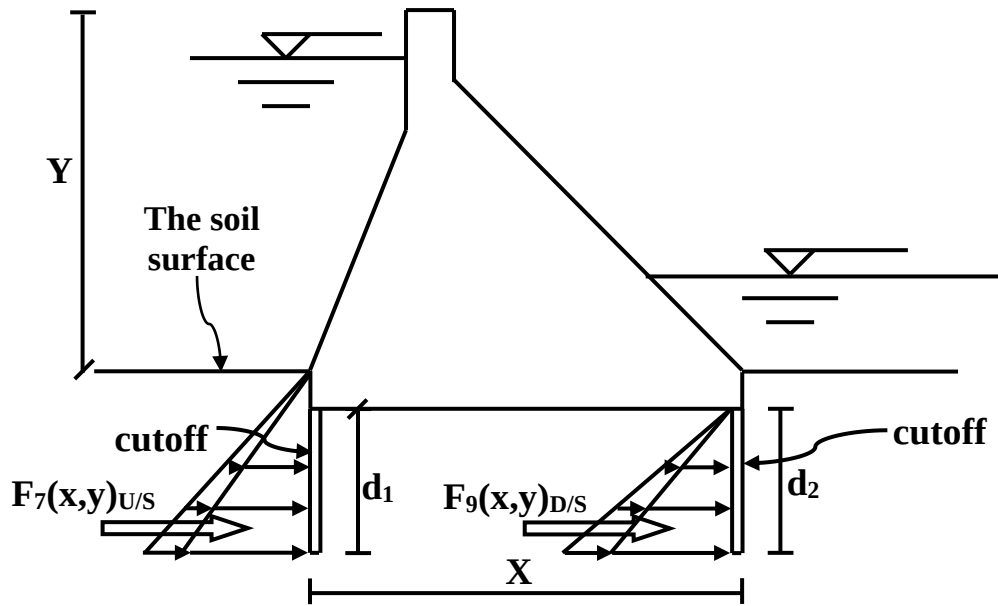


Fig. (4.7): Distribution of passive forces on the cutoffs

The value of the forces of the passive pressure acting on the cutoffs at the upstream and downstream of the dam will be:

$$F_{7(x,y)U/S} = \left[\begin{aligned} & \left(\frac{1}{2} \gamma^- \cdot kp \cdot (d_1 + y_6)^2 + (2C \cdot (d_1 + y_6) \cdot \sqrt{kp}) \right. \\ & \left. + \left(\frac{1}{2} \gamma_w \cdot (d_1 + y_6)^2 \right) \right) \end{aligned} \right] \quad (4.6)$$

$$F_{9(x,y)D/S} = \left[\left(\frac{1}{2} \gamma^- \cdot kp \cdot d_2^2 \right) + (2C \cdot d_2 \cdot \sqrt{kp}) + \left(\frac{1}{2} \gamma_w \cdot d_2^2 \right) \right] \quad (4.7)$$

For the frequent case of horizontal surface, that is, $\beta=0$, the coefficients of active and passive earth pressures are:

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} \quad (4.8)$$

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi} \quad (4.9)$$

Rankine's theory is valid only for non-cohesive soils as sand and gravel but, with corresponding adjustment, can also be used successfully for cohesive clay soils [Nilson and Winter, (1986)].

The forces of the earth pressure and the hydrostatic pressure that are calculated on the upstream and downstream cutoffs shall be participate in the stability of the dam with respect to the overturning and sliding of the dam as follows:

1. Overturning

The algebraic summation of the forces acting on each cutoff will form a resultant force; this force will produce a moment effect on the stability analysis as follows:

A. For upstream cutoff

$$F_a(x,y) = F_{7(x,y)}_{U/S} \text{ (passive)} - F_{8(x,y)}_{U/S} \text{ (active)} \quad (4.10)$$

The force (F_a) represents the resultant of the horizontal forces acting on the upstream cutoff. This force will be attribute to find the friction force that acting on the cutoff as shown in Fig. (4.7):

$$F_{11}(x,y) = F_a(x,y) \cdot \tan \delta + C \cdot (2d_1 + y_6) \quad (4.11)$$

where:

δ : Angle of friction between the soil, base of the dam, and the concrete cutoff.

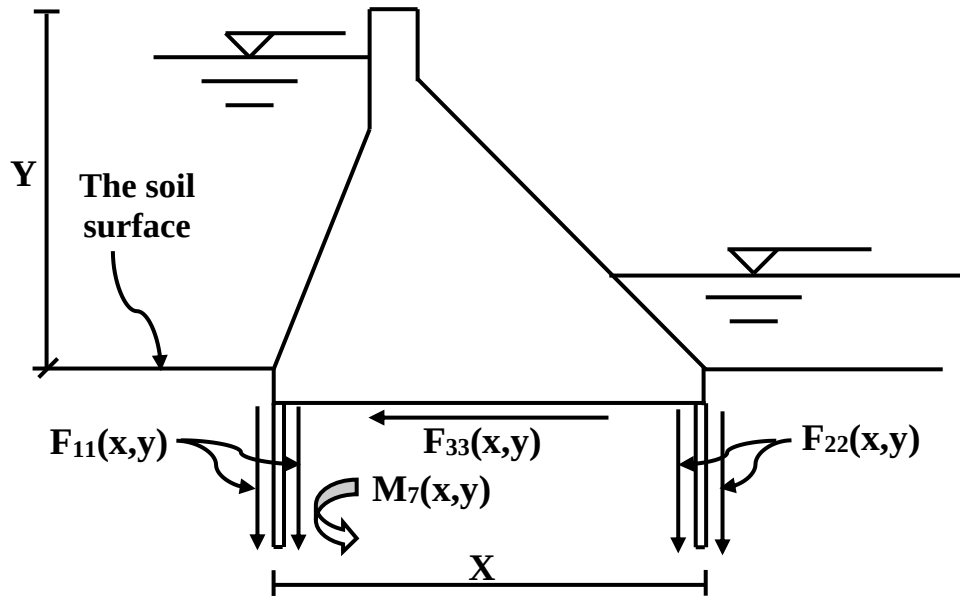


Fig. (4.8): Forces due to sliding that acting on the dam

The friction angle (δ) can be taken equal to (ϕ) where the concrete is poured directly onto the compacted base soil [Bowles, (1996)].

The force (F_{11}) results in a moment about the toe; this moment will be:

$$M_7(x,y) = F_{11}(x,y) \cdot X \quad (4.12)$$

B. For downstream cutoff

Similarly, the friction force resulting from the cutoff at the downstream side of the dam is:

$$F_{22}(x,y) = F_b(x,y) \cdot \tan \delta + C \cdot (2d_2 + y_6) \quad (4.13)$$

where:

$$F_b(x,y) = F_9(x,y)_{D/S} \text{ (passive)} - F_{10}(x,y)_{D/S} \text{ (active)} \quad (4.14)$$

The moment of the force (F_{22}) is neglected because it is passing through the moment centre at **toe**.

2. Sliding

The horizontal forces due to active and passive earth pressure acting on the upstream or/and downstream cutoffs may have effect on the stability of the dam against sliding because of these forces participate in the calculation process of the factor of safety against sliding.

There is a horizontal force due to the friction between the soil and the base floor of the dam must be considered in the calculations of the sliding safety factor, this force shown in **Fig. (4.7)**, and may be illustrated as follows:

$$F_{33}(x,y) = F_c(x,y) \cdot \tan \delta + C \cdot X \quad (4.15)$$

where:

$F_c(x,y)$ or $(\sum v)$: Summation of all vertical forces (kN).

C : Cohesion of the soil under the dam (kN/m²).

X : Total floor length of the dam (m).

4.2.8.2 Coulumb's theory

Coulumb's theory is one of the earliest methods for estimating earth pressures against walls. **Coulumb's** theory considers the friction between the retaining wall and earth filling, because the back of the structure is never so smooth to neglect the friction [**Syal and Goel, (2004)**].

The principle deficiencies in the **Conlumb** theory are the assumptions that the soil is ideal and that the rupture zone is a plane [**Bowles, (1996)**]. Therefore, it is not considered in the analysis of earth pressure in this research.

4.3 STABILITY FACTORS

Engineering is based on predicting the performance of structures before they are actually built. This requires an assessment of how well the system performance can be predicted for the intended materials, expected use, foreseeable abuse, the expected service environment, and the expected life of the system. The transition from engineering model to reality is usually facilitated by

including a factor of safety in the design to accommodate uncertainty in material properties and the design process, the consequences of failure, risk to people, and degree of characterization of and control over the service environment [Anderson, (2001)].

4.3.1 Factor of safety against overturning ($F_{ov.}$)

If the crack lengths are limited such that the allowable compressive stress is not exceeded, the overturning stability could be obtained. The factor of safety is:

$$F_{ov.} = \frac{\sum \mathbf{M}_s}{\sum \mathbf{M}_o} \quad (4.16)$$

where:

$\sum M_s$ = Is the summation of stabilizing moments about the downstream toe.

$\sum M_o$ = Is the summation of destabilizing (overturning) moments [Beser, (2005)].

The dam usually has a factor of safety against overturning between (1.5 to 2.5) [Sharma and Sharma, (2003)].

4.3.2 Factor of safety against sliding ($F_{SL.}$)

The sliding stability is based on a measure of determining the resistance of the dam against sliding [U.S. Army Corps of Engineers, (1995)].

The basic formula of the sliding safety factor for horizontal sliding plane including all loads is:

$$F_{SL.} = \frac{\sum F_r}{\sum F_s} \quad (4.17)$$

where:

$\sum F_r$ = Summation of forces that resist the sliding.

$\sum F_s$ = Summation of forces that causing sliding [Beser, (2005)].

A factor of safety is required in sliding analyses to provide a suitable margin of safety between the loads that can cause instability and the strength of

the materials along potential failure planes that can be mobilized to prevent instability. The factor of safety for sliding is **(2)** for ordinary site information-usual load condition-critical structures [U.S. Army Corps of Engineers, (2005)].

4.3.3 Factor of safety against piping (F_p)

The stability of the sand particles on the exit face is essential for the safety of the dam [Hammad, (1963)]. The factor of safety against piping is as follows:

$$F_p = \frac{I_{cr.}}{I_e} \quad (4.18)$$

where:

$I_{cr.}$: The critical gradient = **(1.0)**.

I_e : The exit gradient.

The value of (F_p) is generally in the range of **(4 to 5)** [Harr, (1962)] or **(2.5 to 3)** [Cedergren, (1977)] have been proposed [U.S. Army Corps of Engineers, (1986)]. In this research used factor of safety against piping according to Harr, (1962) that mentioned hereinbefore.

4.3.4 Checking the calculated eccentricity of the vertical resultant force (X_4)

The dam may fail by the failure of its material, **i.e.**, the compressive stresses produced may exceed the allowable stresses and the dam material may get crushed. The distance of the vertical resultant force from the toe is:

$$X_3 = \frac{\sum M_{Toe}}{\sum V} \quad (4.19)$$

where:

X_3 : Distance of the vertical resultant force with respect to toe.

$\sum M_{Toe}$: Algebraic summation of the moments around the toe.

$\sum V$: Algebraic summation of the vertical forces.

The calculated eccentricity of the vertical resultant force is:

$$\left. \begin{array}{l} X_4 = \frac{X}{2} - X_3 \quad \text{where } X_3 \leq \frac{X}{2} \\ \text{or} \\ X_4 = X_3 - \frac{X}{2} \quad \text{where } X_3 > \frac{X}{2} \end{array} \right\} \leq e \quad (4.20)$$

Hence, the maximum value of eccentricity that can be permitted on either side of the center is equal to **(X/6 = 12.5m) [Garg, (1978)]**. Therefore, the value of **(X₄)** (the calculated eccentricity of the vertical resultant force in this research) must be less than or equal to the value of **(X/6)**.

This means that the resultant must lie within the middle third of the base of the structure in order to avoid the failure of the hydraulic structure by the tension stress **[Garg, (1978)]**.

The overall flow chart of the stability analysis program is shown in **Fig. (4.9)**.

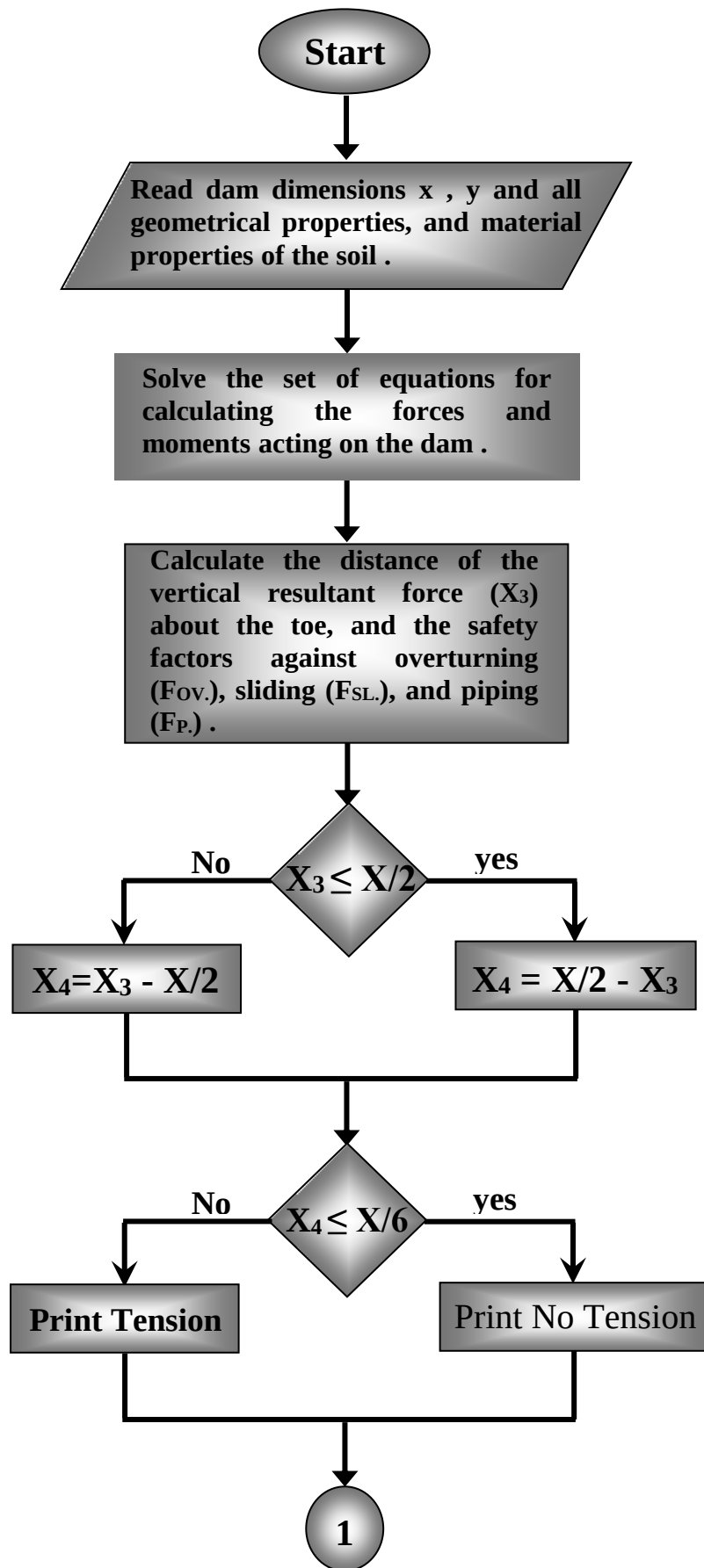


Fig. (4.9): Overall flow chart of the stability analysis program

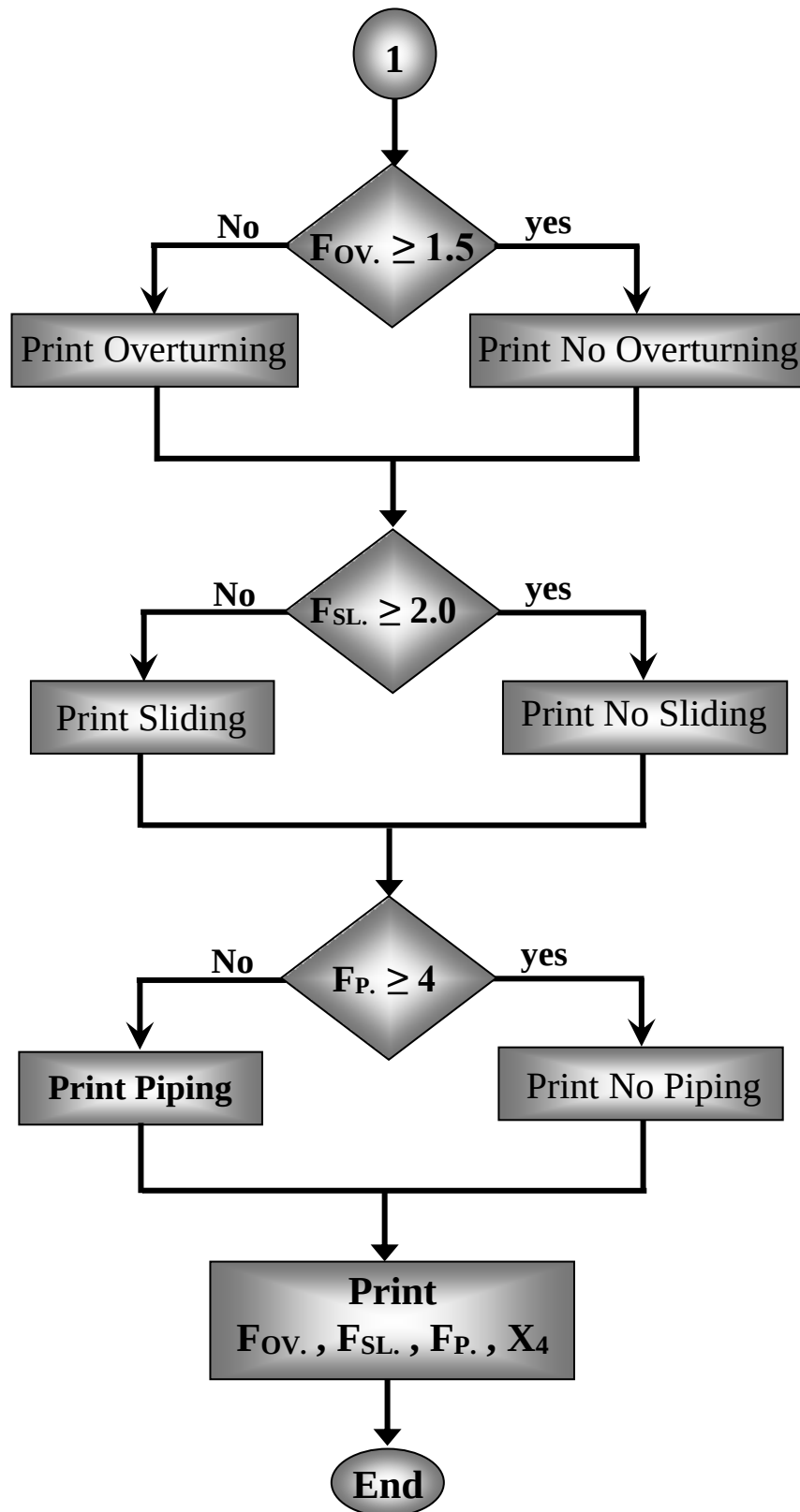


Fig. (4.9): Continued

CHAPTER FIVE

CHAPTER FIVE

THE OPTIMIZATION PROBLEM

5.1 FORMULATION OF THE OPTIMIZATION PROBLEM

As mentioned in **Sec. (2.4)**, the formulation of an optimization problem implies identifying the objective of the optimization process, defining the controlling decision variables, setting the objective in a mathematical form ,and setting the constraints imposed on the system as function (equalities and/or inequalities) of the decision variables. The following items discuss the optimization process for the chosen case study:

5.1.1 The objective

The objective of the optimization process in this research is to minimize the total difference between the calculated and the allowable factors of safety against overturning, sliding, and piping, denoted as “**T.D.**”.

5.1.2 The decision variables

The decision variables are those parameters in the studied process which are controllable or partially controllable .

In this research, the decision variables are:

d₁: Depth of the upstream cutoff.

d₂: Depth of the downstream cutoff.

y₄: Upstream uplift pressure (which implicitly involves **f**, **w**, **b₁**, **b₂**).

y₅: Downstream uplift pressure (which implicitly involves **f**, **w**, **b₁**, **b₂**).

Ie: Value of the exit gradient (which implicitly involves **f**, **w**, **b₁**, **b₂**).

where:

f: The depth of the filter.

w: The width of the filter.

b₁: The distance of the upstream face of the filter media from the upstream cutoff.

b₂: The distance of the downstream face of the filter media from the downstream cutoff, [**b₂** = (**X** – **b₁** – **w**)].

5.1.3 The objective function

The objective function is the objective, set in a mathematical form as function of the specified decision variables and of the involved state variables, if any. It reflects the effectiveness of the taken decisions on the performance of the system. The objective function in this research is formulated as follows:

$$\text{Min. T.D.} = \left[\alpha \left(F_{\text{OV.}} - F_{\text{OV.}}^{\text{L}} \right)^2 + \lambda \left(F_{\text{SL.}} - F_{\text{SL.}}^{\text{L}} \right)^2 + \gamma \left(F_{\text{P.}} - F_{\text{P.}}^{\text{L}} \right)^2 \right] \quad (5.1)$$

where:

T.D.: Total difference between the calculated and allowable safety factors.

(α , λ , γ): Penalty coefficients. For comparison purposes, the values of the penalty coefficients in this research have been assumed **(1.0)**.

F_{OV.}, F_{SL.}, F_{P.}: The calculated factors of safety against overturning, sliding, and piping, respectively.

F_{OV.}^L, F_{SL.}^L, F_{P.}^L: The minimum allowable factors of safety; the typical adopted values in this research are **(1.5 to 2.5)**, **(2)**, and **(4 to 5)** for safety against overturning [Sharma and Sharma, (2003)], sliding [U.S. Army Corps of Engineers, (2005)], and piping [Harr, (1962)], respectively. Consequently, the details of the three terms of the right side of the objective function, **Eq.(5.1)**, are as follows [All symbols are previously defined in **chapter (4)**]:

A. The first term:

$$\left[\alpha \left(F_{OV.} - F_{OV.}^L \right)^2 \right]$$

With ($\alpha = 1$), the first term may be written as:

$$\left(\frac{\sum \mathbf{Mr}}{\sum \mathbf{Mo}} - 1.5 \right)^2 \quad (5.2)$$

where:

$\sum \mathbf{Mr}$: Summation of the resistance moments around the toe.

$\sum \mathbf{Mo}$: Summation of the overturning moments around the toe.

$$\sum \mathbf{Mr} = \mathbf{M}_1(\mathbf{x}, \mathbf{y}) + \mathbf{M}_2(\mathbf{x}, \mathbf{y}) + \mathbf{M}_3(\mathbf{x}, \mathbf{y}) + \mathbf{M}_5(\mathbf{x}, \mathbf{y}) + \mathbf{M}_7(\mathbf{x}, \mathbf{y}) \quad (5.3)$$

$$\sum \mathbf{Mo} = \mathbf{M}_4(\mathbf{x}, \mathbf{y}) + \mathbf{M}_6(\mathbf{x}, \mathbf{y}) \quad (5.4)$$

where:

$\mathbf{M}_1(\mathbf{x}, \mathbf{y})$ = Moment due to the weight of the considered dam.

$\mathbf{M}_2(\mathbf{x}, \mathbf{y})$ = Moment due to the weight of the water on the upstream side.

$\mathbf{M}_3(\mathbf{x}, \mathbf{y})$ = Moment due to the weight of the water on the downstream side.

$\mathbf{M}_4(\mathbf{x}, \mathbf{y})$ = Moment due to the upstream head of the considered dam.

$\mathbf{M}_5(\mathbf{x}, \mathbf{y})$ = Moment due to the downstream head of the considered dam.

$\mathbf{M}_6(\mathbf{x}, \mathbf{y})$ = Moment due to the uplift pressure under the considered dam.

$\mathbf{M}_7(\mathbf{x}, \mathbf{y})$ = Moment due to the earth pressure that affects the stability of the considered dam.

The aforementioned moments are shown schematically in **Fig. (5.1)**:

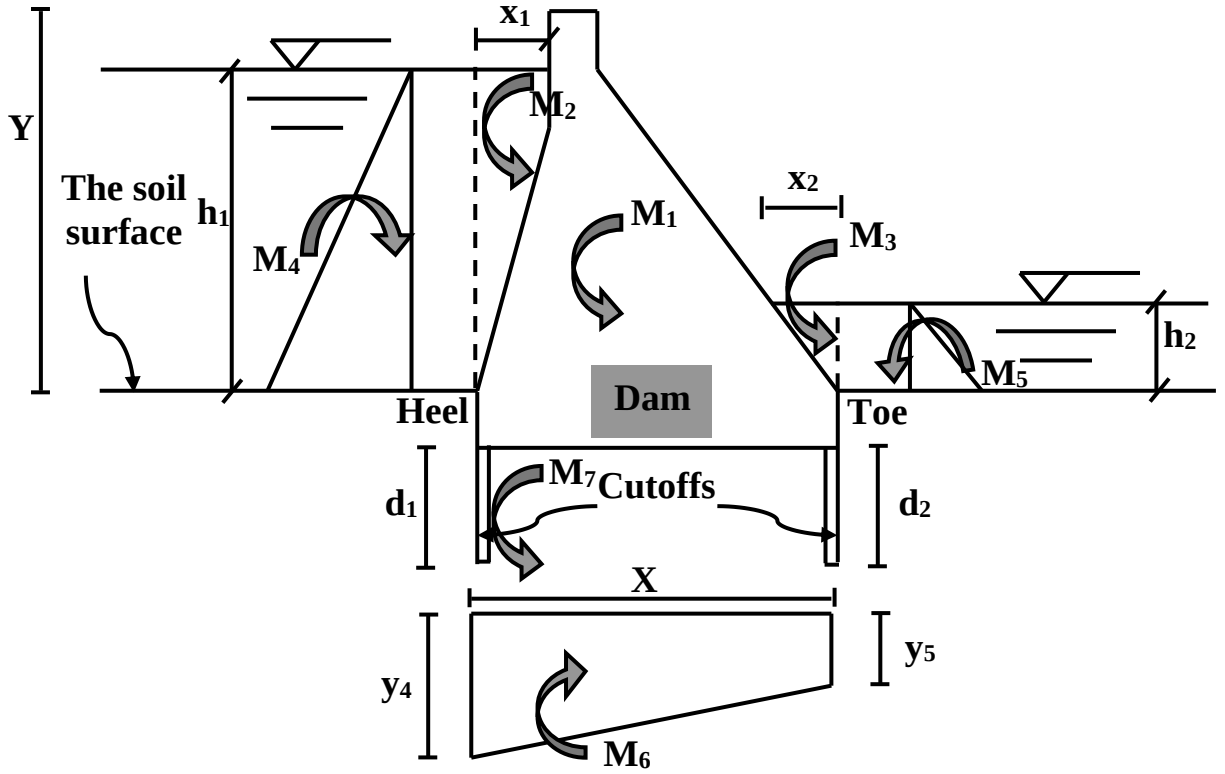


Fig. (5.1): Moments acting on the dam

$$M_1(x,y) = \gamma_c \left[(0.5X \cdot x_1 \cdot Y) - (0.5X \cdot x_1 \cdot y_7) - \left(\frac{1}{3}x_1^2 \cdot Y\right) + \left(\frac{1}{3}x_1^2 \cdot y_7\right) + \left(\frac{1}{2}x_5^2 \cdot Y\right) + (x_5 \cdot x_6 \cdot Y) + \left(\frac{1}{3}x_6^2 \cdot Y\right) - \left(\frac{1}{3}x_6^2 \cdot y_8\right) + (0.5X^2 \cdot y_6) \right] \quad (5.5)$$

$$M_2(x,y)_{U/S} = \gamma_w \left[(0.5X \cdot x_1 \cdot h_1) - (0.5X \cdot x_1 \cdot y_3) - \left(\frac{1}{6}x_1^2 \cdot h_1\right) + (X \cdot x_1 \cdot y_3) - \left(\frac{1}{3}x_1^2 \cdot y_3\right) \right] \quad (5.6)$$

$$M_3(x,y)_{D/S} = \gamma_w (x_2^2 \cdot h_2 / 6) \quad (5.7)$$

$$M_4(x,y)_{U/S} = \gamma_w (h_1^3 / 6) \quad (5.8)$$

$$M_5(x,y)_{D/S} = \gamma_w (h_2^3 / 6) \quad (5.9)$$

$$M_6(x,y) = \gamma_w \left[(X^2 \cdot y_4 / 3) + (X^2 \cdot y_5 / 6) \right] \quad (5.10)$$

$$M_7 = \left[\begin{aligned} & \left(0.5X \cdot \gamma^- \cdot K_p \cdot (d_1 + y_6)^2 \cdot \tan \delta \right) + \left(2X \cdot C \cdot (d_1 + y_6) \cdot \tan \delta \cdot \sqrt{K_p} \right) \\ & + \left(0.5X \cdot \gamma_w \cdot (d_1 + y_6)^2 \cdot \tan \delta \right) - \left(0.5X \cdot \gamma^- \cdot K_a \cdot (d_1 + y_6)^2 \cdot \tan \delta \right) \\ & + \left(2X \cdot C \cdot (d_1 + y_6) \cdot \tan \delta \cdot \sqrt{K_a} \right) - \left(2X \cdot C^2 \cdot \tan \delta / \gamma^- \right) \\ & - \left(0.5X \cdot \gamma_w \cdot (d_1 + y_6)^2 \cdot \tan \delta \right) + \left(2C \cdot X \cdot (d_1 + y_6) \right) \end{aligned} \right] \quad (5.11)$$

B. The second term:

$$\left[\lambda \left(F_{SL} - F_{SL}^L \right)^2 \right]$$

With ($\lambda = 1$), the second term may be written as:

$$\left(\frac{\sum F_r}{\sum F_s} - 2 \right)^2 \quad (5.12)$$

where:

$\sum F_r$: Summation of the forces that resist sliding of the considered dam.

$\sum F_s$: Summation of the forces that make sliding of the considered dam.

$$\sum F_r = F_5(x,y) + F_8(x,y) + F_{10}(x,y) + F_{33}(x,y) \quad (5.13)$$

$$\sum F_s = F_4(x,y) + F_7(x,y) + F_9(x,y) \quad (5.14)$$

where:

$F_4(x,y)$ = Force due to the upstream head.

$F_5(x,y)$ = Force due to the downstream head.

$F_7(x,y)$ = Force due to the passive earth pressure on the upstream cutoff.

$F_8(x,y)$ = Force due to the active earth pressure on the upstream cutoff.

$F_9(x,y)$ = Force due to the passive earth pressure on the downstream cutoff.

$F_{10}(x,y)$ = Force due to the active earth pressure on the downstream cutoff.

$F_{33}(x,y)$ = Force due to the friction between the base and the soil under it.

The aforementioned forces are shown in **Fig. (5.2)**:

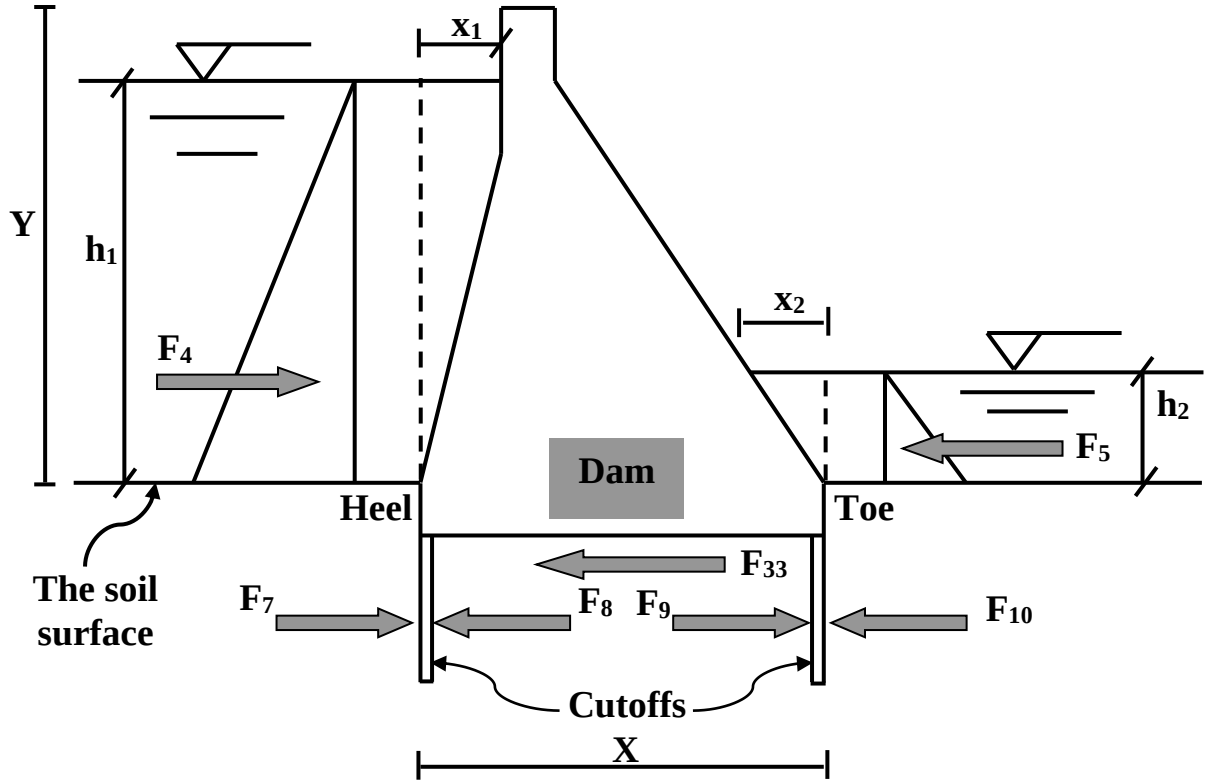


Fig. (5.2): Forces acting on the dam

$$F_4(x,y)_{U/S} = \frac{1}{2} \gamma_w \cdot h_1^2 \quad (5.15)$$

$$F_5(x,y)_{D/S} = \frac{1}{2} \gamma_w \cdot h_2^2 \quad (5.16)$$

$$F_7(x,y)_{U/S} = \left[\begin{aligned} & \left(\frac{1}{2} \gamma^- \cdot kp \cdot (d_1 + y_6)^2 + (2C \cdot (d_1 + y_6) \cdot \sqrt{kp}) \right) \\ & + \left(\frac{1}{2} \gamma_w \cdot (d_1 + y_6)^2 \right) \end{aligned} \right] \quad (5.17)$$

$$F_8(x,y)_{U/S} = \left[\begin{aligned} & \left(\frac{1}{2} \gamma^- \cdot ka \cdot d_1^2 \right) - (2C \cdot d_1 \cdot \sqrt{ka}) + (2C^2 / \gamma^-) + \left(\frac{1}{2} \gamma_w \cdot d_1^2 \right) \end{aligned} \right] \quad (5.18)$$

$$F_9(x,y)_{D/S} = \left[\begin{aligned} & \left(\frac{1}{2} \gamma^- \cdot kp \cdot d_2^2 \right) + (2C \cdot d_2 \cdot \sqrt{kp}) + \left(\frac{1}{2} \gamma_w \cdot d_2^2 \right) \end{aligned} \right] \quad (5.19)$$

$$F_{10}(x,y)_{D/S} = \left[\begin{aligned} & \left(\frac{1}{2} \gamma^- \cdot ka \cdot (d_2 + y_6)^2 - (2C \cdot (d_2 + y_6) \cdot \sqrt{ka}) \right) \\ & + (2C^2 / \gamma^-) + \left(\frac{1}{2} \gamma_w \cdot (d_2 + y_6)^2 \right) \end{aligned} \right] \quad (5.20)$$

$$F_{33} = \left\{ \begin{aligned} & \gamma_c \cdot \tan \delta \cdot [0.5x_1 \cdot (Y - y_7) + x_5 \cdot Y + 0.5x_6 \cdot (Y - y_8) + (X \cdot y_6)] \\ & + \gamma_w \cdot \tan \delta \cdot [0.5x_1 \cdot h_1 + 0.5x_1 \cdot y_3] + \gamma_w \cdot \tan \delta \cdot [0.5x_2 \cdot h_2] \\ & + (0.5\gamma^- \cdot Kp \cdot (d_1 + y_6)^2 \cdot \tan^2 \delta) + (2C \cdot (d_1 + y_6) \cdot \tan^2 \delta \cdot \sqrt{Kp}) \\ & + (0.5\gamma_w \cdot (d_1 + y_6)^2 \cdot \tan^2 \delta) - (0.5\gamma^- \cdot Ka \cdot (d_1 + y_6)^2 \cdot \tan^2 \delta) \\ & + (2C \cdot (d_1 + y_6) \cdot \tan^2 \delta \cdot \sqrt{Ka}) - (2C^2 \cdot \tan^2 \delta / \gamma^-) \\ & - (0.5\gamma_w \cdot (d_1 + y_6)^2 \cdot \tan^2 \delta) + (2C \cdot (d_1 + y_6) \cdot \tan \delta) + \\ & (0.5\gamma^- \cdot Kp \cdot (d_2 + y_6)^2 \cdot \tan^2 \delta) + (2C \cdot (d_2 + y_6) \cdot \tan^2 \delta \cdot \sqrt{Kp}) \\ & + (0.5\gamma_w \cdot (d_2 + y_6)^2 \cdot \tan^2 \delta) - (0.5\gamma^- \cdot Ka \cdot (d_2 + y_6)^2 \cdot \tan^2 \delta) \\ & + (2C \cdot (d_2 + y_6) \cdot \tan^2 \delta \cdot \sqrt{Ka}) - (2C^2 \cdot \tan^2 \delta / \gamma^-) \\ & - (0.5\gamma_w \cdot (d_2 + y_6)^2 \cdot \tan^2 \delta) + (2C \cdot (d_2 + y_6) \cdot \tan \delta) \\ & - (0.5\gamma_w \cdot X \cdot y_4 \cdot \tan \delta) + (0.5\gamma_w \cdot X \cdot y_5 \cdot \tan \delta) + (C \cdot X) \end{aligned} \right\} \quad (5.21)$$

C. The third term:

$$\gamma(F_{P.} - F_{P.}^L)^2$$

With $(\gamma=1)$, the third term may be written as:

$$\left(\frac{1}{I_e} - 4 \right)^2 \quad (5.22)$$

5.1.4 The constraints

The objective function is minimized subject to a set of constraints. These constraints are the uplift pressure in the upstream and downstream sides and the exit gradient as follows:

1. $y_4 \leq h_1$
2. $y_5 \geq h_2$
3. $0 \leq I_e \leq \left(\frac{1}{F_{P.}} \right)$

5.2 SOLUTION OF THE FORMULATED OPTIMIZATION PROBLEM

It is clear from noticing **Eq.(5.1)** and its complementaries that the problem is a non-linear one. There are many procedures to solve such a model as outlined in **Sec. (2.4.2)**.

The formulated non-linear objective function of this research is so complex that its differentiation with respect to the design variables would be an arduous task and the gradient method for solving non-linear optimization, such as **SUMT** method, cannot be used. Moreover, the large number of constraints will make the use of the Lagrange Multiplier method very difficult. However, the modified **Hooke** and **Jeeves** of direct search method [**Bunday, (1984)**] has been found to be the appropriate choice in this regard.

The general flow chart of the optimization model is shown in **Fig. (5.3)**.

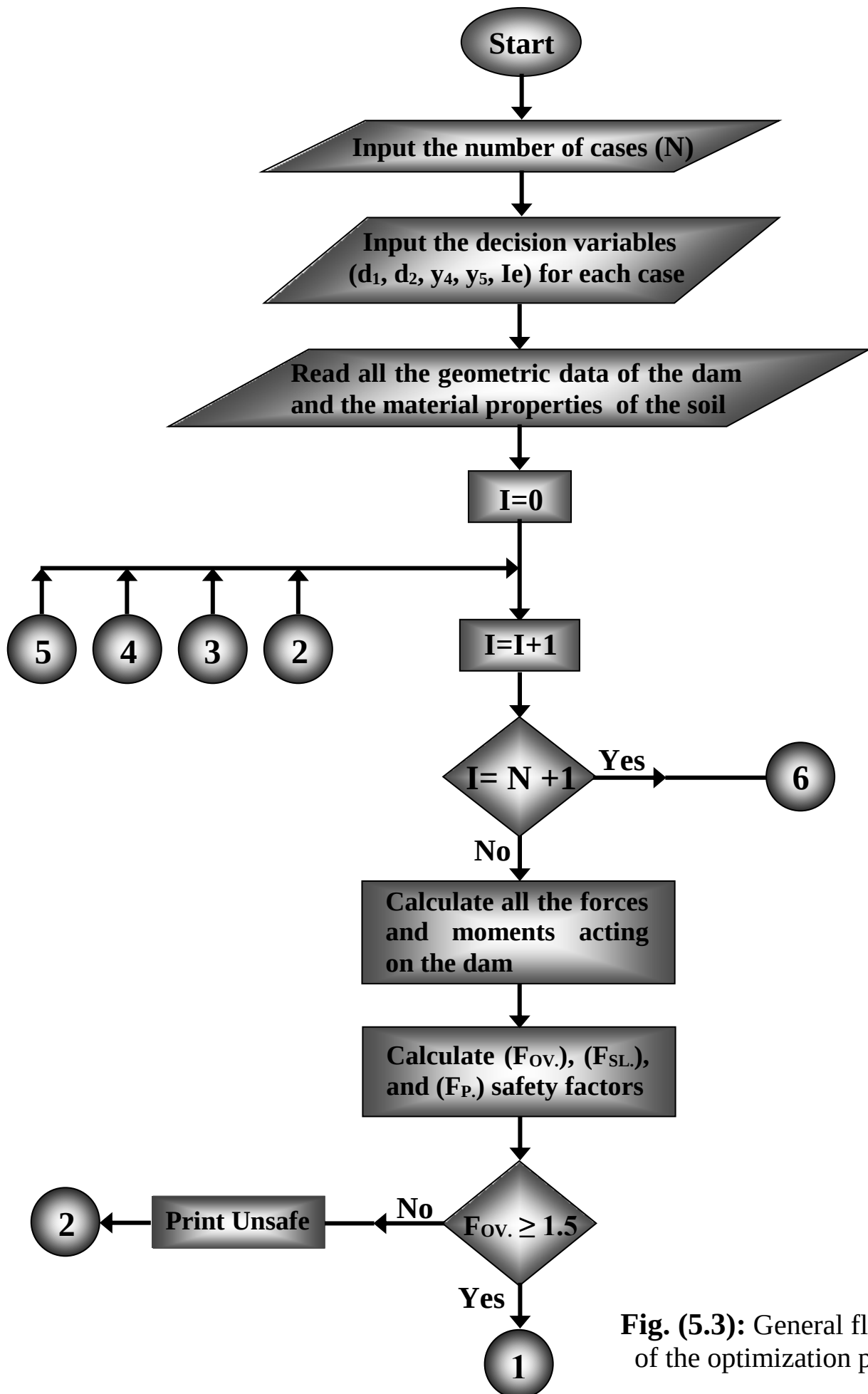
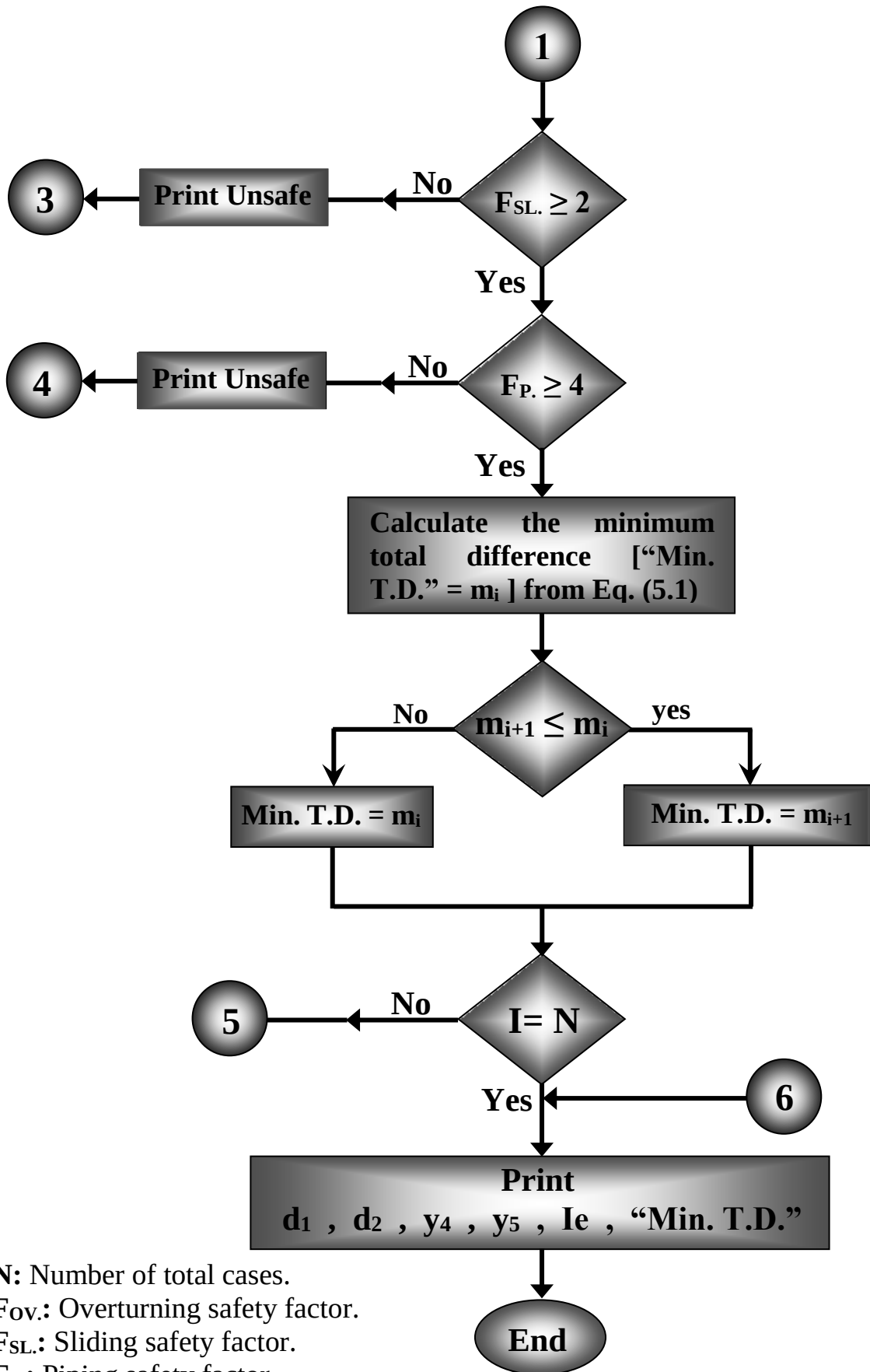


Fig. (5.3): General flow chart of the optimization problem



N: Number of total cases.
F_{OV}. Overturning safety factor.
F_{SL}. Sliding safety factor.
F_P. Piping safety factor.
Min. T.D.: Minimum total difference.

Fig. (5.3): Continued

CHAPTER SIX

CHAPTER SIX

RESULTS AND ANALYSES

In this research, different cases are analyzed to investigate the effect of an intermediate filter media of different depths, widths, and locations on the safety factors against overturning, sliding, and piping when there is a cutoff of different depths at the upstream or/and downstream side of the investigated dam as well as to study the location of the vertical resultant force under the dam for safety against tension.

An optimization model is formulated and run in order to calculate the minimum total different between the calculated and the allowable safety factors, denoted as “**T.D.**” for the feasible cases of the considered alternatives.

6.1 RUNNING THE STABILITY PROGRAM

The program for stability analysis is run for selected depths of cutoffs in the upstream or/and downstream side of the considered hypothetical case study when there is a filter media of different dimensions and locations.

Four values of cutoff depth have been chosen in this respect for each of the cutoffs [**i.e., (d_1)** for the upstream cutoff and **(d_2)** for the downstream cutoff]. These values are **(3, 5, 7.5, and 10m)**. The **(3m)** value is thought to be the minimum practical value that worth the effort of installation. The **(10m)** value has been found sufficient to insure full safety of the considered hypothetical case study.

The aforementioned cutoffs depths are used for the hypothetical case study shown in **Fig. (3.3)** in order to calculate the factors of safety against overturning, sliding, and piping, as well as to find the eccentricity of the vertical resultant force under the base of the considered dam for safety of the dam against tension.

6.2 THE RESULTS OF THE STABILITY PROGRAM

Table (6.1) shows the calculated safety factors against overturning, (F_{ov}), sliding, (F_{sl}), and piping, (F_p) for different depths and locations of the intermediate filter media when the width of the filter media is equal to ($w=0.25 d_1$) and when there is a cutoff at the upstream side having a depth equal to (3m). Table (6.2) shows the same aforementioned results but for a width of the filter media ($w = 0.50 d_1$).

Results of the other runs are given in Tables (A.1) through (A.22) in Appendix (A).

Tables (A.1) through (A.6) show the aforementioned results for the depths of upstream cutoff equal to (5m), (7.5m), and (10m), respectively.

Table (A.7) shows the calculated safety factors against overturning, sliding, and piping for different depths and locations of the intermediate filter media when the width of the filter media ($w = 0.25 d_2$) with a cutoff at the downstream side of the hypothetical case study having a depth equal to (3m). Table (A.8) illustrates the same aforementioned results but for a width of the filter media ($w = 0.50 d_2$).

Tables (A.9) through (A.14) illustrate the aforementioned results but for depths of downstream cutoff of (5m), (7.5m), and (10m), respectively.

Table (A.15) lists the calculated safety factors against overturning, sliding, and piping for different depths and locations of the intermediate filter media when the width of the filter media is ($w = 0.25 d_1$) and when there are cutoffs at the upstream and downstream sides of the hypothetical case study having depths equal to (3m). Table (A.16) shows the same aforementioned results but when the width of the filter media is ($w = 0.50 d_1$).

Tables (A.17) through (A.22) show all the aforementioned results for the depths of upstream and downstream cutoffs equal to (5m), (7.5m), and (10m), respectively.

Table (6.1)
Calculated values of the safety factors
 $(d_1 = 3\text{m} \text{ , } w = 0.25 d_1)$

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_1$	$0.2 d_1$	2.09	1.71	3.84
2		$0.4 d_1$	2.12	1.73	3.95
3		$0.6 d_1$	2.18	1.77	4.13
4		$0.8 d_1$	2.19	1.78	4.21
5		$1.0 d_1$	2.20	1.79	4.24
6	$3 d_1$	$0.2 d_1$	2.05	1.68	3.97
7		$0.4 d_1$	2.08	1.70	4.11
8		$0.6 d_1$	2.14	1.74	4.27
9		$0.8 d_1$	2.15	1.75	4.33
10		$1.0 d_1$	2.16	1.76	4.37
11	$4 d_1$	$0.2 d_1$	2.01	1.65	4.10
12		$0.4 d_1$	2.04	1.67	4.24
13		$0.6 d_1$	2.10	1.71	4.42
14		$0.8 d_1$	2.11	1.72	4.50
15		$1.0 d_1$	2.12	1.73	4.55
16	$5 d_1$	$0.2 d_1$	1.97	1.62	4.26
17		$0.4 d_1$	2.00	1.64	4.40
18		$0.6 d_1$	2.06	1.68	4.58
19		$0.8 d_1$	2.07	1.69	4.65
20		$1.0 d_1$	2.08	1.70	4.68
21	$6 d_1$	$0.2 d_1$	1.93	1.59	4.40
22		$0.4 d_1$	1.96	1.61	4.54
23		$0.6 d_1$	2.02	1.65	4.71
24		$0.8 d_1$	2.03	1.66	4.78
25		$1.0 d_1$	2.04	1.67	4.83

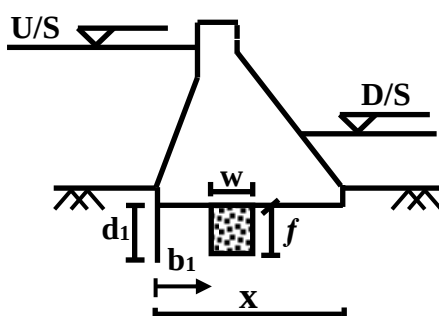


Table (6.2)
Calculated values of the safety factors
 $(d_1 = 3\text{m} \text{ , } w = 0.50 d_1)$

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	2 d_1	0.2 d_1	2.13	1.74	4.01
2		0.4 d_1	2.16	1.76	4.11
3		0.6 d_1	2.22	1.80	4.27
4		0.8 d_1	2.23	1.81	4.35
5		1.0 d_1	2.24	1.82	4.40
6	3 d_1	0.2 d_1	2.09	1.71	4.10
7		0.4 d_1	2.12	1.73	4.23
8		0.6 d_1	2.18	1.77	4.40
9		0.8 d_1	2.19	1.78	4.48
10		1.0 d_1	2.20	1.79	4.52
11	4 d_1	0.2 d_1	2.05	1.68	4.27
12		0.4 d_1	2.08	1.70	4.40
13		0.6 d_1	2.14	1.74	4.58
14		0.8 d_1	2.15	1.75	4.67
15		1.0 d_1	2.16	1.76	4.70
16	5 d_1	0.2 d_1	2.01	1.65	4.40
17		0.4 d_1	2.04	1.67	4.54
18		0.6 d_1	2.10	1.71	4.71
19		0.8 d_1	2.11	1.72	4.82
20		1.0 d_1	2.12	1.73	4.85
21	6 d_1	0.2 d_1	1.97	1.62	4.55
22		0.4 d_1	2.00	1.64	4.71
23		0.6 d_1	2.06	1.68	4.91
24		0.8 d_1	2.07	1.69	5.00
25		1.0 d_1	2.08	1.70	5.05

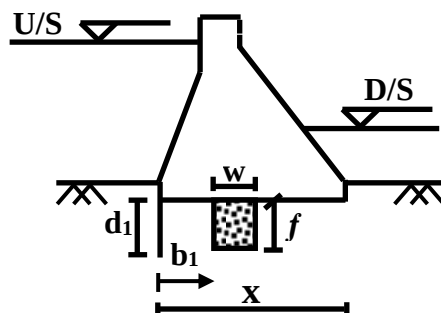


Table **(6.3)** shows the calculated eccentricity, **(X_4)**, of the vertical resultant force acting on the base of the dam (the hypothetical case) for different dimensions and locations of the filter media when there is a cutoff at the upstream side **(d_1)** of a depth equal to **(3m)**.

Results of the other runs of the eccentricity of the vertical resultant force are given in Tables **(B.1)** to **(B.11)** in Appendix **(B)**.

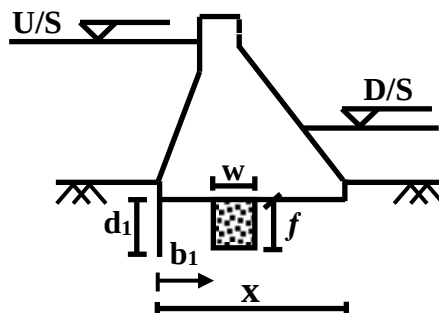
Tables **(B.1)** through **(B.3)** show the aforementioned results for the depths of the upstream cutoff equal to **(5m)**, **(7.5m)**, and **(10m)**, respectively.

Tables **(B.4)** through **(B.7)** contain the same aforementioned information about the eccentricity of the vertical resultant force acting on the base of the hypothetical case study when a cutoff is placed at the downstream end of the base with a depth equal to **(3m)**, **(5m)**, **(7.5m)**, and **(10m)**, respectively.

Tables **(B.8)** through **(B.11)** summarize the results of the same aforementioned subject for the case when cutoffs of equal lengths are placed at both the upstream and downstream ends of the base of the considered dam and for lengths of **(3m)**, **(5m)**, **(7.5m)**, and **(10m)**, respectively.

Table (6.3)
Calculated values of the eccentricity (X_4) of the resultant force
($d_1 = 3\text{m}$)

A: $w = 0.25 d_1$				B: $w = 0.50 d_1$			
NO.	b_1	f	X_4	NO.	b_1	f	X_4
1	$2 d_1$	$0.2 d_1$	1.271	1	$2 d_1$	$0.2 d_1$	1.159
2		$0.4 d_1$	1.199	2		$0.4 d_1$	1.091
3		$0.6 d_1$	1.055	3		$0.6 d_1$	0.942
4		$0.8 d_1$	1.011	4		$0.8 d_1$	0.901
5		$1.0 d_1$	0.975	5		$1.0 d_1$	0.881
6	$3 d_1$	$0.2 d_1$	1.370	6	$3 d_1$	$0.2 d_1$	1.273
7		$0.4 d_1$	1.293	7		$0.4 d_1$	1.199
8		$0.6 d_1$	1.147	8		$0.6 d_1$	1.056
9		$0.8 d_1$	1.104	9		$0.8 d_1$	1.010
10		$1.0 d_1$	1.075	10		$1.0 d_1$	0.984
11	$4 d_1$	$0.2 d_1$	1.485	11	$4 d_1$	$0.2 d_1$	1.369
12		$0.4 d_1$	1.407	12		$0.4 d_1$	1.249
13		$0.6 d_1$	1.251	13		$0.6 d_1$	1.146
14		$0.8 d_1$	1.209	14		$0.8 d_1$	1.105
15		$1.0 d_1$	1.187	15		$1.0 d_1$	2.078
16	$5 d_1$	$0.2 d_1$	1.598	16	$5 d_1$	$0.2 d_1$	1.477
17		$0.4 d_1$	1.519	17		$0.4 d_1$	1.400
18		$0.6 d_1$	1.361	18		$0.6 d_1$	1.252
19		$0.8 d_1$	1.316	19		$0.8 d_1$	1.210
20		$1.0 d_1$	1.293	20		$1.0 d_1$	1.190
21	$6 d_1$	$0.2 d_1$	1.701	21	$6 d_1$	$0.2 d_1$	1.591
22		$0.4 d_1$	1.631	22		$0.4 d_1$	1.520
23		$0.6 d_1$	1.475	23		$0.6 d_1$	1.364
24		$0.8 d_1$	1.431	24		$0.8 d_1$	1.314
25		$1.0 d_1$	1.411	25		$1.0 d_1$	1.293



6.3 ANALYSIS OF THE STABILITY RESULTS

In this study, the stability of the considered dam (structurally and hydraulically) will be analyzed together.

The analysis of the stability results with respect to overturning, sliding, and piping safety factors for different dimensions [depth (f), width (w)] and locations [(b_1) from the upstream floor face or (b_2) from the downstream floor face] of the intermediate filter media, and the analysis of the calculated eccentricity results are discussed hereinafter.

6.3.1 Factor of safety against overturning

The effect of the depth, width, and location of the filter media on the overturning safety factor when there is a cutoff at the upstream side or/and downstream side of the dam is as follows:

A. Depth and width of the filter media

Depth and width of the filter media are two of the factors that affect the stability of the dam. Table (6.4) shows the summary of results of checking the overturning safety factor with respect to the depth (f) of the filter media. This table summarizes the results given in Tables (6.1) and (6.2) and Tables (A.1) through (A.22) given in Appendix (A) when there is a cutoff at the upstream side or/and downstream side of the dam of different depths.

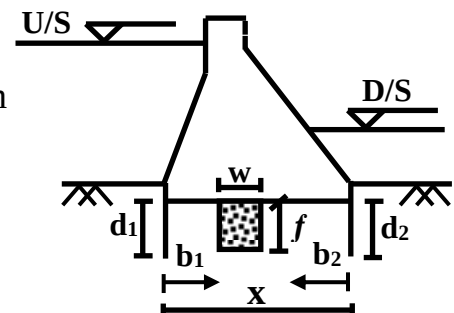
With the minimum value of the safety factor against overturning, ($F_{ov.}$), of (1.5), the results considered in Table (6.4) indicate the cases where this criterion has been fulfilled or not.

Figures (6.1) through (6.6) show the trend of the variation of ($F_{ov.}$) with the depth of the filter media, (f), for the whole cases considered, which sum to (120) case. The results of each case have been regressed as a polynomial of second degree with the value of regression coefficient (R) as indicated on each individual curve.

Table (6.4): Summary of results of checking the overturning safety factor with respect to the depth (f) of the filter media

NO.	Depth of the cutoffs	Overturning safety factor $F_{OV} \geq 1.5^{(1)}$	Width of the filter media (w)	f					
				0.2d ₁	0.4d ₁	0.6d ₁	0.8d ₁	1.0d ₁	
1.	d ₁ = 3m d ₂ = 0	Fov.	Min.	w = 0.25 d ₁	1.93	1.96	2.02	2.03	2.04
			Min.	w = 0.50 d ₁	1.97	2.00	2.06	2.07	2.08
	d ₁ = 5m d ₂ = 0	Fov.	Min.	w = 0.25 d ₁	2.13	2.16	2.22	2.23	2.24
			Min.	w = 0.50 d ₁	2.17	2.20	2.26	2.27	2.28
	d ₁ = 7.5m d ₂ = 0	Fov.	Min.	w = 0.25 d ₁	2.29	2.32	2.38	2.39	2.40
			Min.	w = 0.50 d ₁	2.33	2.36	2.42	2.43	2.44
	d ₁ = 10m d ₂ = 0	Fov.	Min.	w = 0.25 d ₁	2.37	2.40	2.46	2.47	2.48
			Min.	w = 0.50 d ₁	2.41	2.44	2.50	2.51	2.52
2.	d ₁ =d ₂ = 3m	Fov.	Min.	w = 0.25 d ₁	1.37	1.43	1.54	1.58	1.61
			Min.	w = 0.50 d ₁	1.44	1.50	1.60	1.64	1.67
	d ₁ =d ₂ = 5m	Fov.	Min.	w = 0.25 d ₁	1.45	1.51	1.61	1.65	1.68
			Min.	w = 0.50 d ₁	1.52	1.58	1.68	1.72	1.75
	d ₁ =d ₂ =7.5m	Fov.	Min.	w = 0.25 d ₁	1.50	1.55	1.65	1.69	1.72
			Min.	w = 0.50 d ₁	1.56	1.62	1.72	1.76	1.79
	d ₁ =d ₂ = 10m	Fov.	Min.	w = 0.25 d ₁	1.51	1.57	1.68	1.72	1.75
			Min.	w = 0.50 d ₁	1.58	1.64	1.75	1.79	1.82
	Depth of the cutoffs	$F_{OV} \geq 1.5^{(1)}$	(w)	f					
				0.2d ₂	0.4d ₂	0.6d ₂	0.8d ₂	1.0d ₂	
3.	d ₂ = 3m d ₁ = 0	Fov.	Max.	w = 0.25 d ₂	1.08	1.11	1.17	1.18	1.19
			Max.	w = 0.50 d ₂	1.12	1.15	1.21	1.22	1.23
	d ₂ = 5m d ₁ = 0	Fov.	Max.	w = 0.25 d ₂	0.81	0.84	0.90	0.91	0.92
			Max.	w = 0.50 d ₂	0.85	0.88	0.94	0.95	0.96
	d ₂ = 7.5m d ₁ = 0	Fov.	Max.	w = 0.25 d ₂	0.69	0.72	0.78	0.79	0.80
			Max.	w = 0.50 d ₂	0.73	0.76	0.82	0.83	0.84
	d ₂ = 10m d ₁ = 0	Fov.	Max.	w = 0.25 d ₂	0.61	0.64	0.70	0.71	0.72
			Max.	w = 0.50 d ₂	0.65	0.68	0.74	0.75	0.76

⁽¹⁾ : The minimum and maximum denote the minimum and maximum values of (F_{OV}) for the values of the location (2, 3, 4, 5, and 6) of (b_1 or b_2).



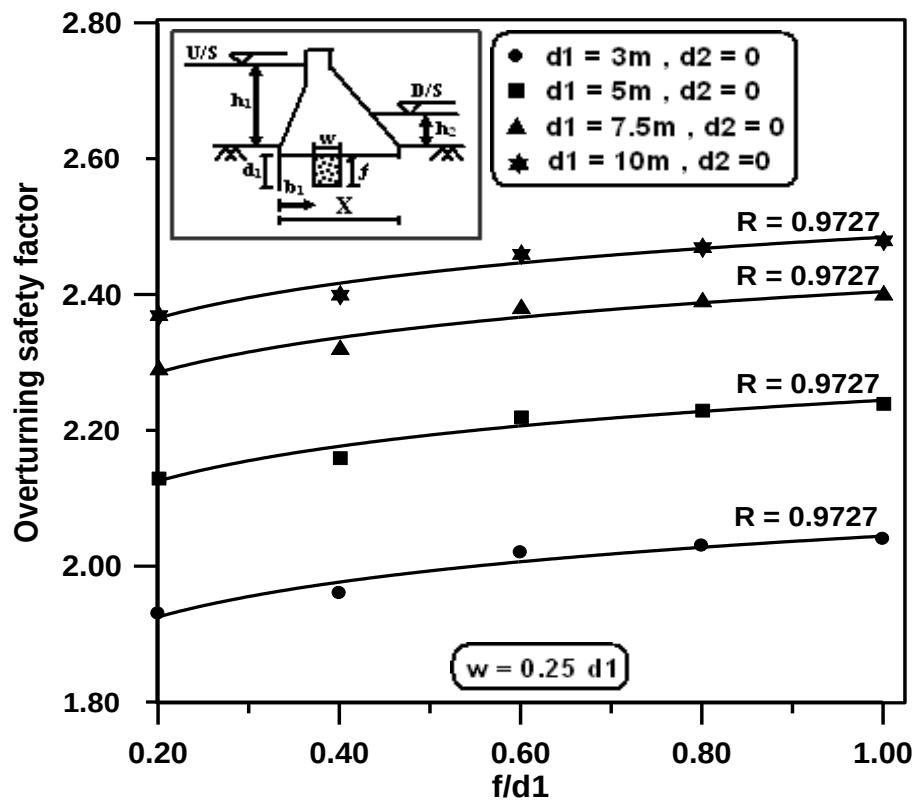


Fig. (6.1): The relation between the overturning safety factor and the depth of the filter media when ($w = 0.25 d_1$) and ($d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

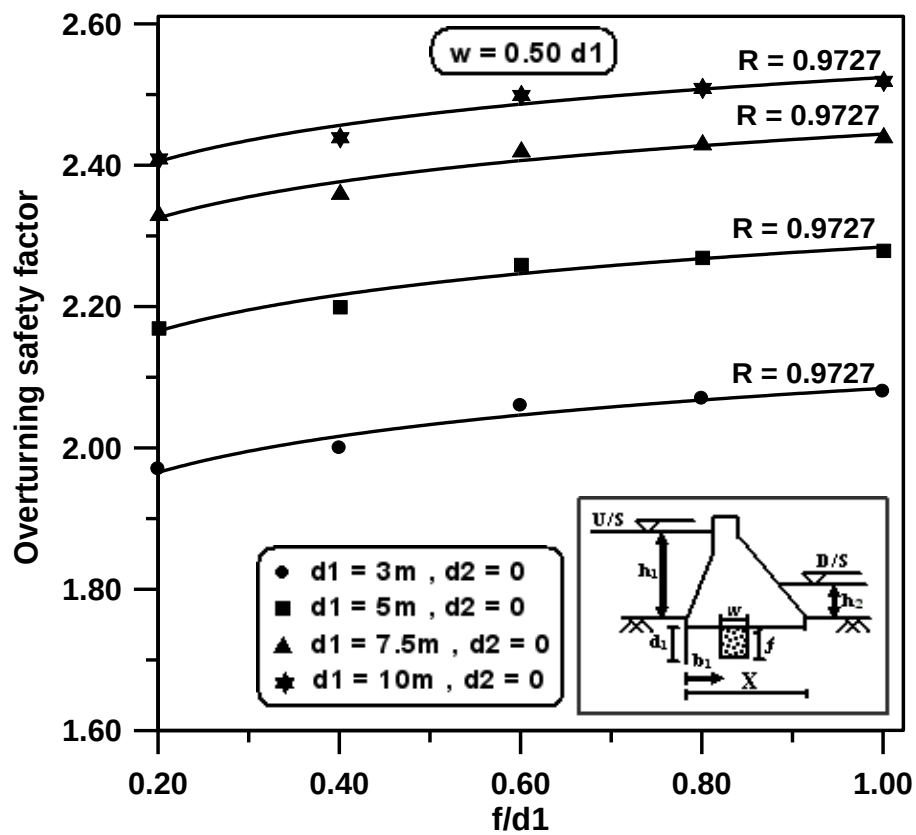


Fig. (6.2): The relation between the overturning safety factor and the depth of the filter media when ($w = 0.50 d_1$) and ($d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

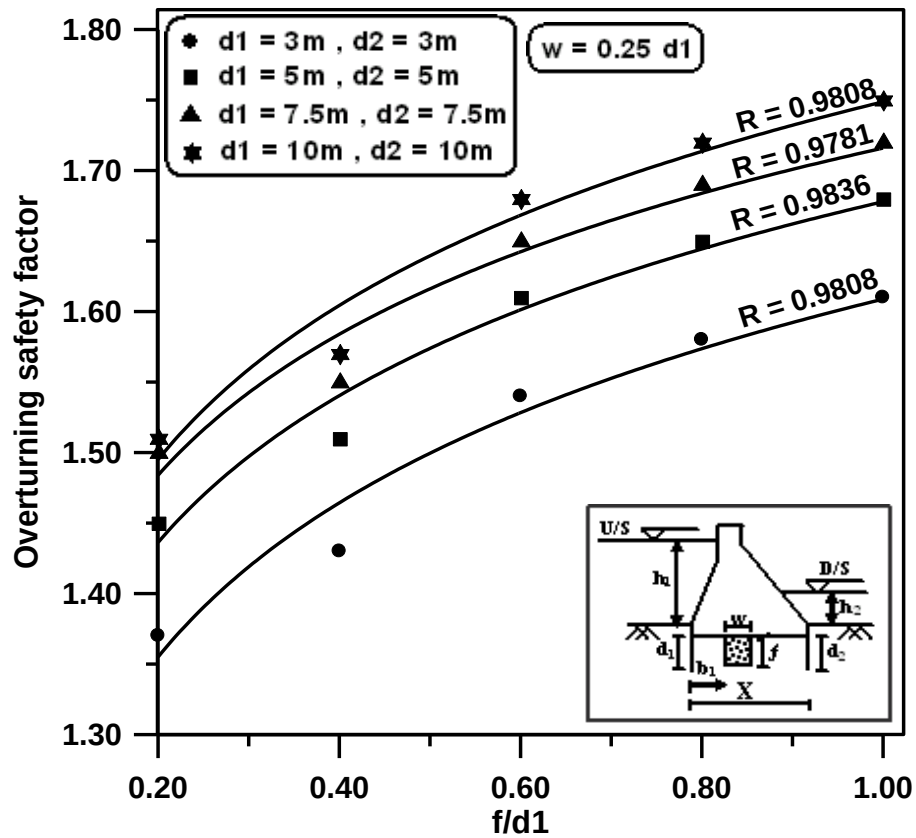


Fig. (6.3): The relation between the overturning safety factor and the depth of the filter media when ($w = 0.25 d_1$) and ($d_1 = d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

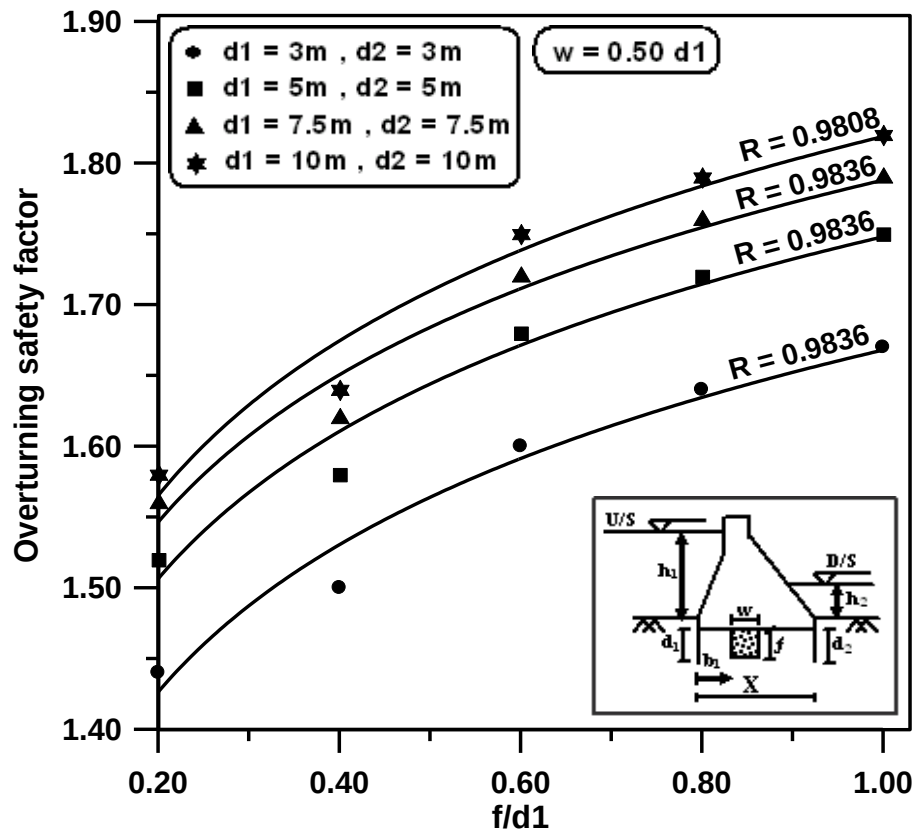


Fig. (6.4): The relation between the overturning safety factor and the depth of the filter media when ($w = 0.50 d_1$) and ($d_1 = d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

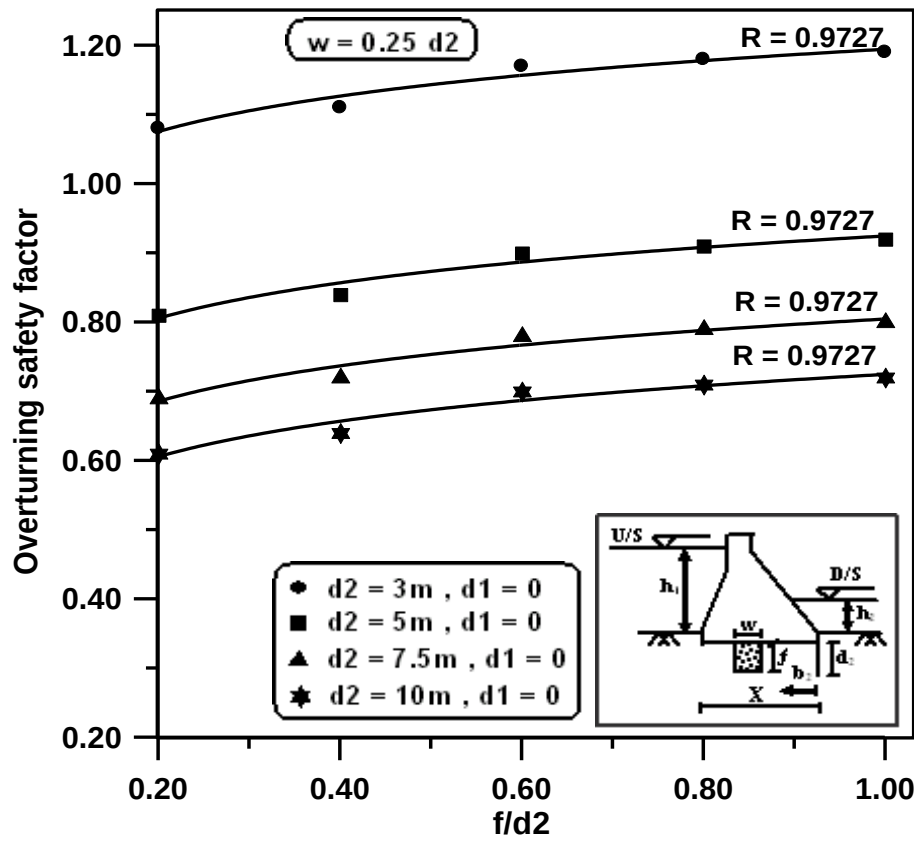


Fig. (6.5): The relation between the overturning safety factor and the depth of the filter media when ($w = 0.25 d_2$) and ($d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

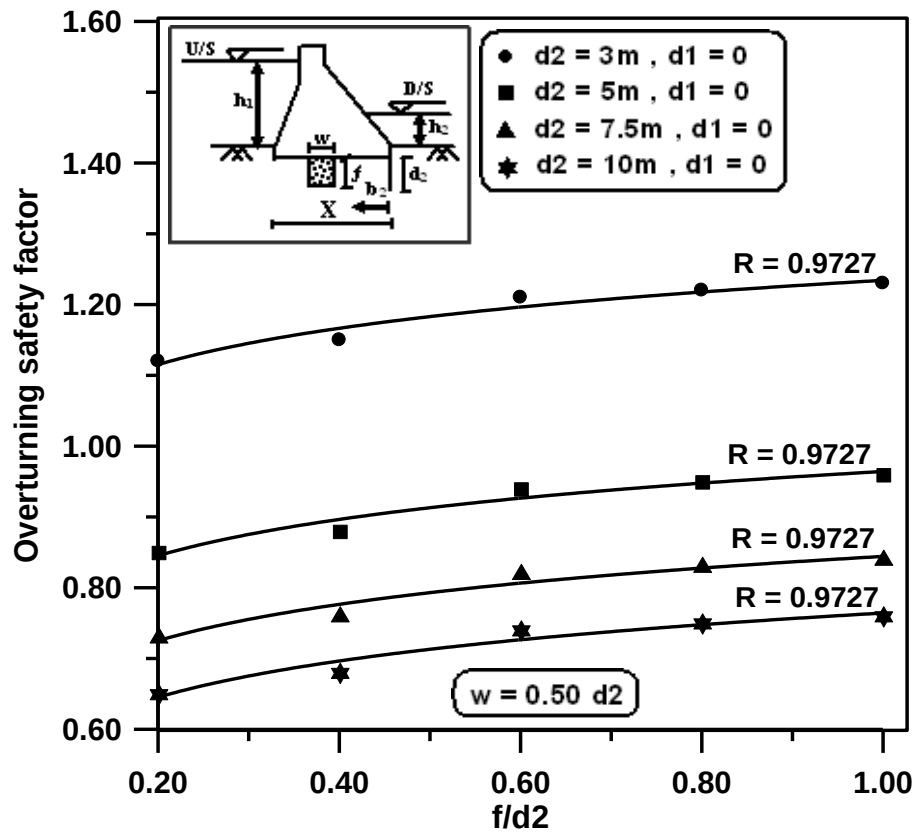


Fig. (6.6): The relation between the overturning safety factor and the depth of the filter media when ($w = 0.50 d_2$) and ($d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

Figures (6.1) and (6.2) show the relation between the overturning safety factor and the depth of the filter media for the widths of the filter media of ($w = 0.25 d_1$) and ($w = 0.50 d_1$), respectively, when there is a cutoff at the upstream side only of the dam of depth equal [$d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, \text{and } 10\text{m}$]. It can be seen from these figures that increasing the filter media depth will increase the overturning safety factor; the same effect is noticed when increasing the filter media width. The increase of this factor of safety is because of the respective decrease of the uplift pressure due to the increasing of the filter medium's depth and width. The figures also show that the same aforementioned goal is attained through increasing the depth of the upstream cutoff, (d_1).

The use of (**10m**) for (d_1), or (d_2), or (d_1) and (d_2) indicated less improvement in ($F_{ov.}$) from that with (**7.5m**) relative to the other values of increase [say from (**3m**) to (**5m**)].

Figures (6.3) and (6.4) present the relation between the overturning safety factor and the depth of the filter media for the widths of the filter media of ($w = 0.25 d_1$) and ($w = 0.50 d_1$), respectively, when there are cutoffs at the upstream and downstream sides of the dam of depths equal [$d_1 = d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, \text{and } 10\text{m}$]. The figures show that the effect of increasing the depth and width of the filter media when there are cutoffs at the upstream and downstream sides of the dam will increase the factor of safety against overturning. Comparison of the trends of variation of ($F_{ov.}$) with the depth of the filter media, (f), as illustrated in **Figs. (6.1) and (6.2)** with those in **Figs. (6.3) and (6.4)** indicates that the magnitude of increasing of the overturning safety factor is more when there are cutoffs at the upstream and downstream sides of the dam as compared with the same factor but when there is a cutoff at the upstream side of the dam only. This is because the decreasing of the uplift pressure when there are cutoffs at the two ends of the dam is greater than that when there is a cutoff at the upstream side of the dam only.

Figures (6.5) and (6.6) show the relation between the overturning safety factor and the depth of the filter media for the widths of the filter media of $(w = 0.25 d_2)$ and $(w = 0.50 d_2)$, respectively, when there is a cutoff at the downstream side only of the dam of depth equal $[d_2 = 3m, 5m, 7.5m, \text{ and } 10m]$. These figures show that increasing the filter media depth and width will increase the overturning safety factor. This is because of the decreasing of the value of the uplift pressure.

These figures show that increasing the downstream cutoff depth yields decreasing the overturning safety factor because of increasing the uplift pressure.

Moreover, the results in Table (6.4) indicate that the depths $[(f=0.4 d_1), \text{ and } (f=0.6 d_1)]$ and width $[(w=0.50 d_1)]$ of the filter media and with a cutoff at the upstream side or at both the upstream and downstream sides play a distinguished role in increasing the safety factor against overturning. When there is a cutoff at the downstream side, the depths $[(f=0.4 d_2), \text{ and } (f=0.6 d_2)]$ and width $[(w=0.50 d_2)]$ of the filter media have the same aforementioned effect as compared with the other depths and widths of the filter media.

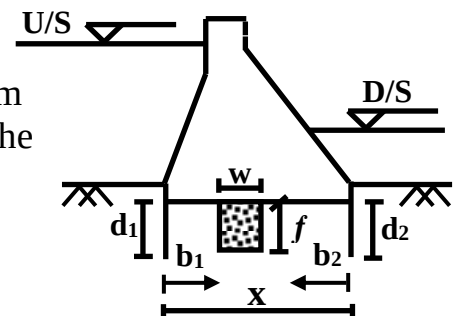
B. Location of the filter media

Moving the filter media from the upstream side to the downstream side of the considered dam and vice versa causes a noticeable effect on the stability of the dam. Table (6.5) summarizes the results of checking the factor of safety against overturning with respect to the location (b_1) or (b_2) of the filter media.

Table (6.5): Summary of results of checking the overturning safety factor with respect to the location (b_1) or (b_2) of the filter media

NO.	Depth of the cutoffs	Overturning safety factor $F_{ov} \geq 1.5^{(2)}$		Width of the filter media (w)	b_1				
					2 d_1	3 d_1	4 d_1	5 d_1	6 d_1
1.	$d_1 = 3m$ $d_2 = 0$	Fov.	Min.	$w = 0.25 d_1$	2.09	2.05	2.01	1.97	1.93
			Min.	$w = 0.50 d_1$	2.13	2.09	2.05	2.01	1.97
	$d_1 = 5m$ $d_2 = 0$	Fov..	Min.	$w = 0.25 d_1$	2.29	2.25	2.21	2.17	2.13
			Min.	$w = 0.50 d_1$	2.33	2.29	2.25	2.21	2.17
	$d_1 = 7.5m$ $d_2 = 0$	Fov.	Min.	$w = 0.25 d_1$	2.45	2.41	2.37	2.33	2.29
			Min.	$w = 0.50 d_1$	2.49	2.45	2.41	2.37	2.33
	$d_1 = 10m$ $d_2 = 0$	Fov.	Min.	$w = 0.25 d_1$	2.53	2.49	2.45	2.41	2.37
			Min.	$w = 0.50 d_1$	2.57	2.53	2.49	2.45	2.41
2.	$d_1=d_2= 3m$	Fov..	Min.	$w = 0.25 d_1$	1.57	1.52	1.47	1.42	1.37
			Min.	$w = 0.50 d_1$	1.64	1.59	1.54	1.49	1.44
	$d_1=d_2= 5m$	Fov.	Min.	$w = 0.25 d_1$	1.65	1.60	1.55	1.50	1.45
			Min.	$w = 0.50 d_1$	1.72	1.67	1.62	1.57	1.52
	$d_1=d_2=7.5m$	Fov..	Min.	$w = 0.25 d_1$	1.69	1.64	1.59	1.54	1.50
			Min.	$w = 0.50 d_1$	1.76	1.71	1.66	1.61	1.56
	$d_1=d_2= 10m$	Fov.	Min.	$w = 0.25 d_1$	1.71	1.66	1.61	1.56	1.51
			Min.	$w = 0.50 d_1$	1.78	1.73	1.68	1.63	1.58
	Depth of the cutoffs	$F_{ov} \geq 1.5^{(2)}$		(w)	b_2				
					2 d_2	3 d_2	4 d_2	5 d_2	6 d_2
3.	$d_2 = 3m$ $d_1 = 0$	Fov..	Max.	$w = 0.25 d_2$	1.19	1.15	1.11	1.07	1.03
			Max.	$w = 0.50 d_2$	1.23	1.19	1.15	1.11	1.07
	$d_2 = 5m$ $d_1 = 0$	Fov.	Max.	$w = 0.25 d_2$	0.92	0.88	0.84	0.80	0.76
			Max.	$w = 0.50 d_2$	0.96	0.92	0.88	0.84	0.80
	$d_2 = 7.5m$ $d_1 = 0$	Fov..	Max.	$w = 0.25 d_2$	0.80	0.76	0.72	0.68	0.64
			Max.	$w = 0.50 d_2$	0.84	0.80	0.76	0.72	0.68
	$d_2 = 10m$ $d_1 = 0$	Fov.	Max.	$w = 0.25 d_2$	0.72	0.68	0.64	0.60	0.57
			Max.	$w = 0.50 d_2$	0.76	0.72	0.68	0.64	0.61

⁽²⁾ : The minimum and maximum denote the minimum and maximum values of (F_{ov}) for the values of the depth (0.2, 0.4, 0.6, 0.8, and 1.0) of (f).



These summarized results represent the results given in Tables (6.1) and (6.2) and Tables (A.1) through (A.22) given in Appendix (A) when there is a cutoff at the upstream side or/and downstream side of the dam.

With the minimum value of the safety factor against overturning, ($F_{ov.}$), of (1.5), the results contains in Table (6.5) indicate the cases where this criterion has been fulfilled or not.

Figures (6.7) through (6.12) show the trend of the variation of ($F_{ov.}$) with the location of the filter media, (b_1) or (b_2), for the whole cases considered, which sum to (120) case. The results of each case have been regressed logarithmic with the value of regression coefficient (R) as indicated on each individual curve.

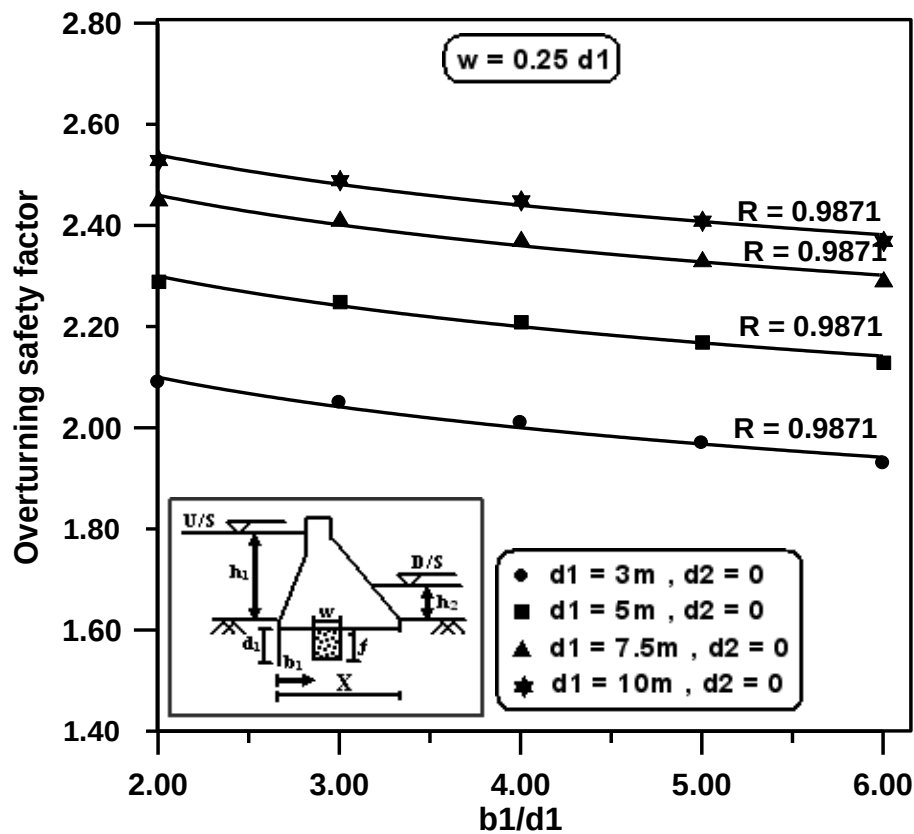


Fig. (6.7): The relation between the overturning safety factor and the location of the filter media when ($w = 0.25 d_1$) and ($d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

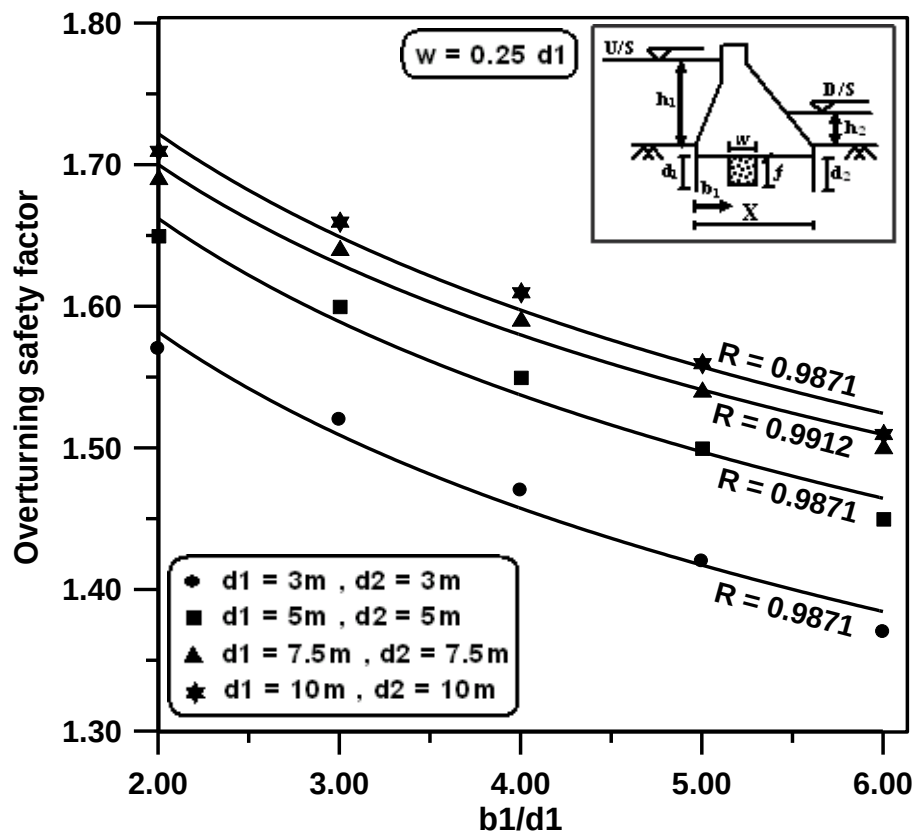


Fig. (6.8): The relation between the overturning safety factor and the location of the filter media when ($w = 0.25 d_1$) and ($d_1 = d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

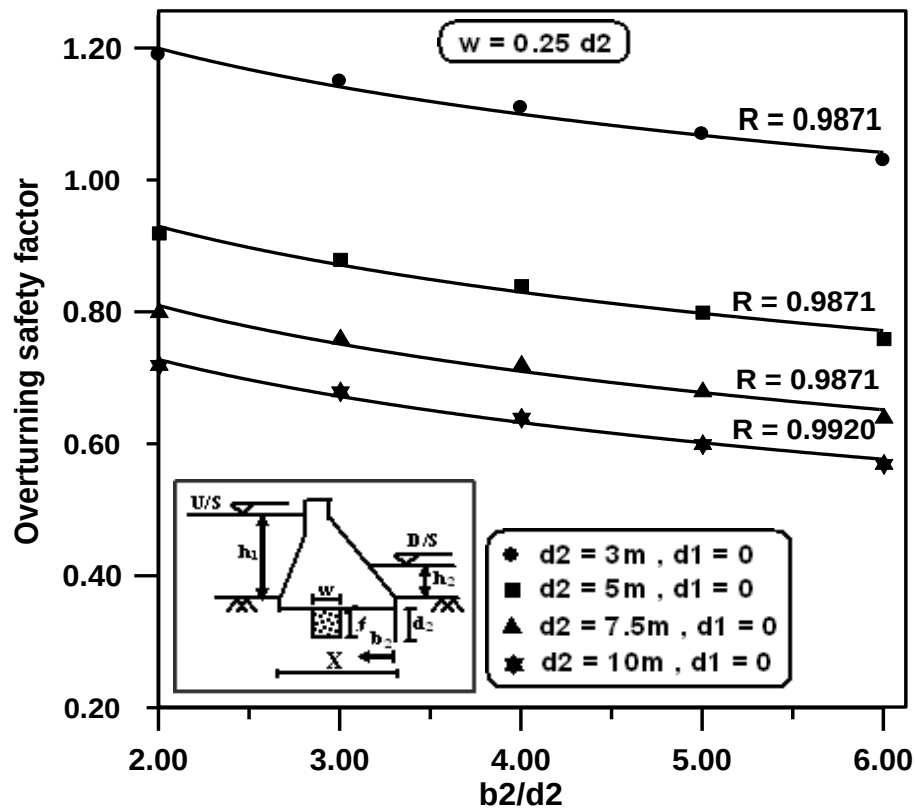


Fig. (6.9): The relation between the overturning safety factor and the location of the filter media when ($w = 0.25 d_2$) and ($d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

Figures (6.7), (6.8) and (6.9) show the relation between the overturning safety factor and the location of the filter media for the width of the filter media of ($w = 0.25 d_1$) when there is a cutoff at the upstream side only of depth equal [$d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, \text{ and } 10\text{m}$] or when there are cutoffs at the two ends of the dam of equal depths [$d_1 = d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, \text{ and } 10\text{m}$], and for the width of the filter media ($w = 0.25 d_2$) when there is a cutoff at the downstream side of the dam only of depth equal [$d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, \text{ and } 10\text{m}$], respectively. Figures (6.7) and (6.8) show that moving the filter media from the upstream side toward the downstream side of the considered dam causes a decrease in the overturning safety. The same aforementioned effect could be noticed in **Fig. (6.9)** when moving the filter media from the downstream side toward the upstream side of the dam.

Moreover, **Figs. (6.7) and (6.8)** show that increasing the cutoff depth when there is a cutoff at the upstream side only or at the upstream and downstream sides of the dam, respectively, will increase the overturning safety factor. However, this factor decreases through increasing the depth of the cutoff when it is at the downstream side only as shown in **Fig. (6.9)**.

Comparison of the trends of variation of (F_{ov}) with the location of the filter media, (b_1), as illustrated in **Fig. (6.8)** with those in **Figs. (6.7) and (6.9)** indicates that the magnitude of increasing of the overturning safety factor is more when there are cutoffs at the upstream and downstream sides of the dam as compared with the same factor but when there is a cutoff at the upstream side of the dam only or at the downstream side only, respectively. This is because the decreasing of the uplift pressure when there are cutoffs at the two ends of the dam is greater than that when there is a cutoff at the upstream side of the dam only or at the downstream side only.

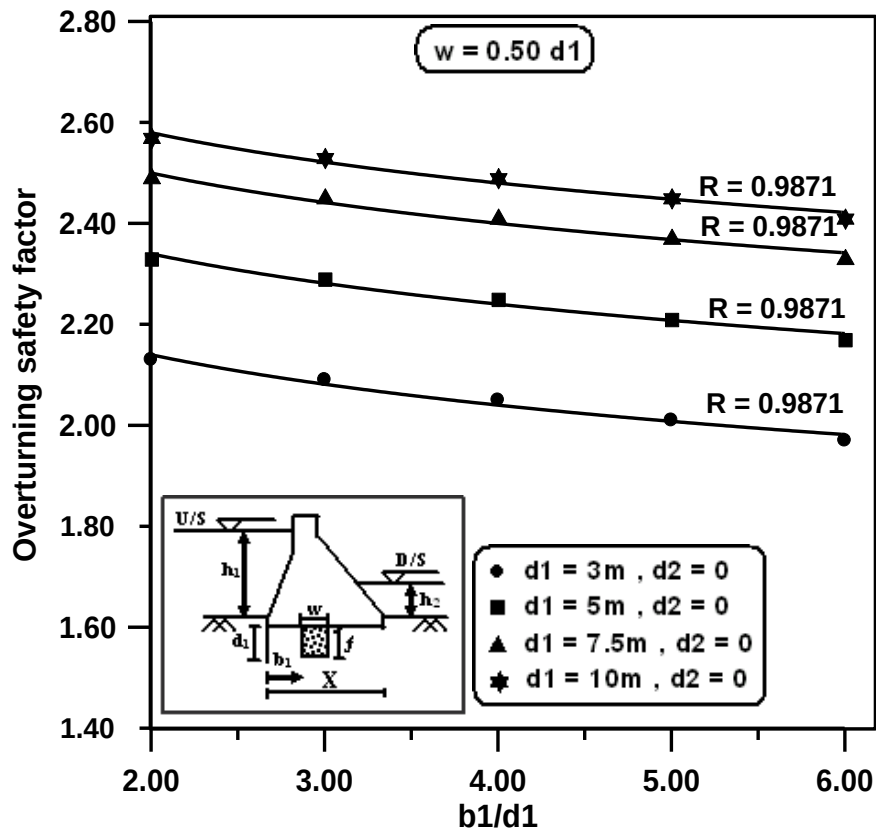


Fig. (6.10): The relation between the overturning safety factor and the location of the filter media when ($w = 0.50 d_1$) and ($d_1 = 3\text{ m}, 5\text{ m}, 7.5\text{ m}, 10\text{ m}$)

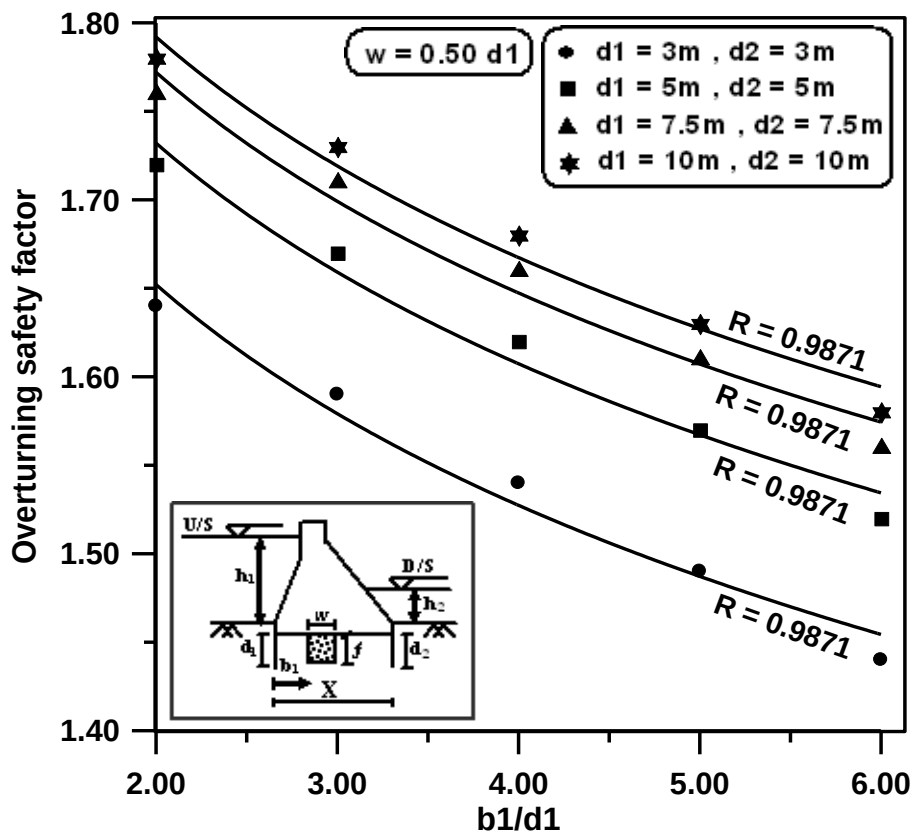


Fig. (6.11): The relation between the overturning safety factor and the location of the filter media when ($w = 0.50 d_1$) and ($d_1 = d_2 = 3\text{ m}, 5\text{ m}, 7.5\text{ m}, 10\text{ m}$)

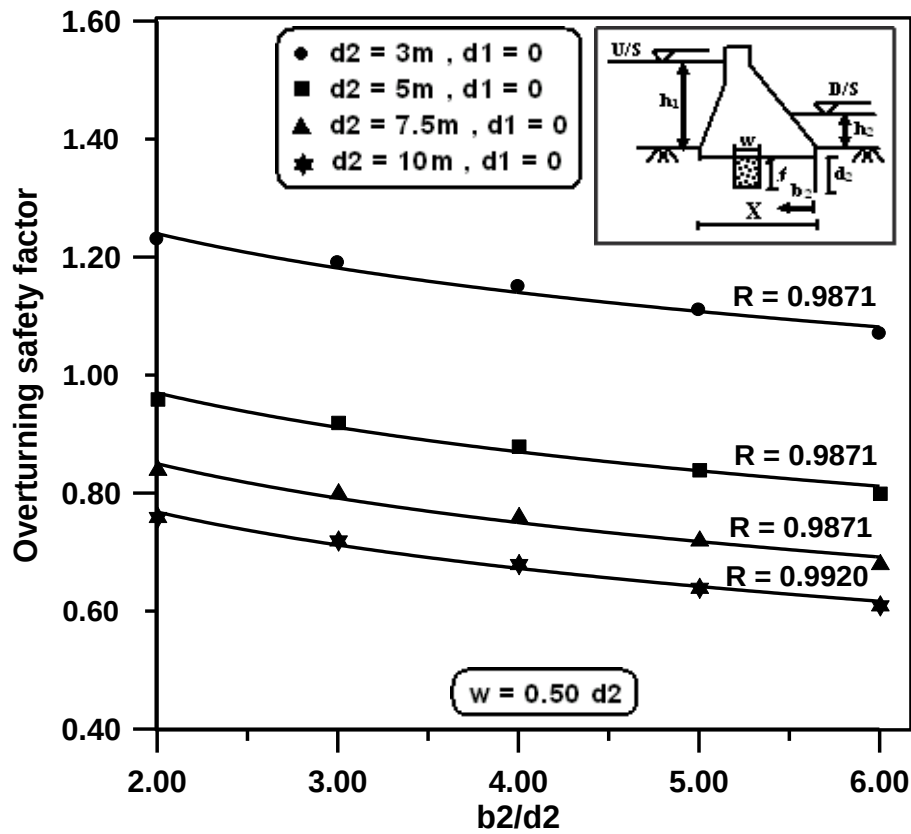


Fig. (6.12): The relation between the overturning safety factor and the location of the filter media when ($w = 0.50 d_2$) and ($d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

Figures (6.10), (6.11) and (6.12) present the relation between the overturning safety factor and the location of the filter media for the width of the filter media of ($w = 0.50 d_1$) when there is a cutoff at the upstream side only of depth equal [$d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, \text{and } 10\text{m}$] or when there are cutoffs at the two ends of the dam of depths equal [$d_1 = d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, \text{and } 10\text{m}$] and for the width of the filter media ($w = 0.50 d_2$) when there is a cutoff at the downstream side only of depth equal [$d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, \text{and } 10\text{m}$], respectively.

Figures (6.10) and (6.11) show that moving the filter media from the upstream side toward the downstream side of the considered dam causes a decrease in the overturning safety. The same aforementioned effect could be noticed in Fig. (6.12) when moving the filter media from the downstream side toward the upstream side of the dam. Increasing the width of the filter media yields increasing the overturning safety factor as shown in Figs. (6.10), (6.11) and (6.12) when compared with Figs. (6.7), (6.8) and (6.9), respectively .

The overturning safety factor increases as increasing the depth of the cutoff at the upstream side only or at the two ends of the dam as shown in **Figs. (6.10)** and **(6.11)**, while decreases as increasing the depth of the downstream cutoff as shown in **Fig. (6.12)**. The increasing and decreasing of this factor of safety are because of the decreasing and increasing the uplift pressure, respectively.

Table **(6.5)** shows that the location ($b_1=2 d_1$) of the filter media when there is a cutoff at the upstream side or at the upstream and downstream sides gives an overturning safety factor greater than that given by other locations of the filter media for different widths of the filter media and depths of the cutoffs. The location ($b_2=2 d_2$) of the filter media acts similarly but when there is a cutoff at the downstream side of the dam.

6.3.2 Factor of safety against sliding

The effect of the depth, width, and location of the filter media on the sliding safety factor when there is a cutoff at the upstream side or/and downstream side of the dam of different depths is as follows:

A. Depth and width of the filter media

Table **(6.6)** contains the summary of results of checking the sliding safety factor with respect to the depth (f) of the filter media. This table summarizes the results given in Tables **(6.1)** and **(6.2)** and Tables **(A.1)** through **(A.22)** given in Appendix **(A)** when there is a cutoff at the upstream side or/and downstream side of the dam of different depths.

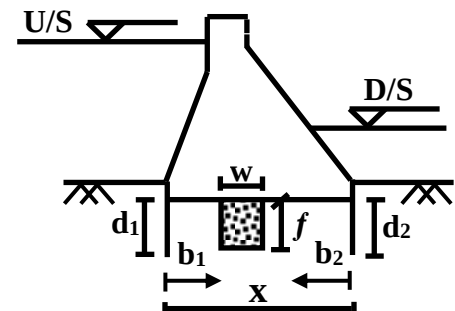
With the minimum value of the safety factor against sliding, (F_{SL}), of **(2)**, the results considered in Table **(6.6)** indicate the cases where this criterion has been fulfilled or not.

Figures **(6.13)** through **(6.18)** show the trend of the variation of (F_{SL}) with the depth of the filter media, (f), for the whole cases considered, which sum to **(120)** case. The results of each case have been regressed as a polynomial of second degree with the value of regression coefficient (**R**) as indicated on each individual curve.

Table (6.6): Summary of results of checking the sliding safety factor with respect to the depth (f) of the filter media

NO.	Depth of the cutoffs	Sliding safety factor $F_{SL} \geq 2^{(3)}$		Width of the filter media (w)	f				
					0.2d ₁	0.4d ₁	0.6d ₁	0.8d ₁	1.0d ₁
1.	$d_1 = 3m$ $d_2 = 0$	F_{SL}	Max.	$w = 0.25 d_1$	1.71	1.73	1.77	1.78	1.79
			Max.	$w = 0.50 d_1$	1.74	1.76	1.80	1.81	1.82
	$d_1 = 5m$ $d_2 = 0$	F_{SL}	Max.	$w = 0.25 d_1$	1.86	1.88	1.92	1.93	1.94
			Max.	$w = 0.50 d_1$	1.89	1.91	1.95	1.96	1.97
	$d_1 = 7.5m$ $d_2 = 0$	F_{SL}	Max.	$w = 0.25 d_1$	1.98	2.00	2.04	2.05	2.06
			Max.	$w = 0.50 d_1$	2.01	2.03	2.07	2.08	2.09
	$d_1 = 10m$ $d_2 = 0$	F_{SL}	Max.	$w = 0.25 d_1$	2.04	2.06	2.10	2.11	2.12
			Max.	$w = 0.50 d_1$	2.07	2.09	2.13	2.14	2.15
2.	$d_1=d_2= 3m$	F_{SL}	Max.	$w = 0.25 d_1$	1.32	1.37	1.45	1.48	1.50
			Max.	$w = 0.50 d_1$	1.37	1.42	1.50	1.53	1.55
	$d_1=d_2= 5m$	F_{SL}	Max.	$w = 0.25 d_1$	1.38	1.43	1.51	1.54	1.56
			Max.	$w = 0.50 d_1$	1.43	1.48	1.56	1.59	1.61
	$d_1=d_2=7.5m$	F_{SL}	Max.	$w = 0.25 d_1$	1.41	1.46	1.54	1.57	1.59
			Max.	$w = 0.50 d_1$	1.46	1.51	1.59	1.62	1.64
	$d_1=d_2= 10m$	F_{SL}	Max.	$w = 0.25 d_1$	1.42	1.47	1.55	1.58	1.60
			Max.	$w = 0.50 d_1$	1.47	1.52	1.60	1.63	1.65
	Depth of the cutoffs	$F_{SL} \geq 2^{(3)}$		(w)	f				
					0.2d ₂	0.4d ₂	0.6d ₂	0.8d ₂	1.0d ₂
3.	$d_2 = 3m$ $d_1 = 0$	F_{SL}	Max.	$w = 0.25 d_2$	0.96	0.98	1.02	1.03	1.04
			Max.	$w = 0.50 d_2$	0.99	1.01	1.05	1.06	1.07
	$d_2 = 5m$ $d_1 = 0$	F_{SL}	Max.	$w = 0.25 d_2$	0.75	0.77	0.81	0.82	0.83
			Max.	$w = 0.50 d_2$	0.78	0.80	0.84	0.85	0.86
	$d_2 = 7.5m$ $d_1 = 0$	F_{SL}	Max.	$w = 0.25 d_2$	0.66	0.68	0.72	0.73	0.74
			Max.	$w = 0.50 d_2$	0.69	0.71	0.75	0.76	0.77
	$d_2 = 10m$ $d_1 = 0$	F_{SL}	Max.	$w = 0.25 d_2$	0.60	0.62	0.66	0.67	0.68
			Max.	$w = 0.50 d_2$	0.63	0.65	0.69	0.70	0.71

⁽³⁾ : The maximum denote the maximum values of (F_{SL}) for the values of the location (2, 3, 4, 5, and 6) of (b_1 or b_2).



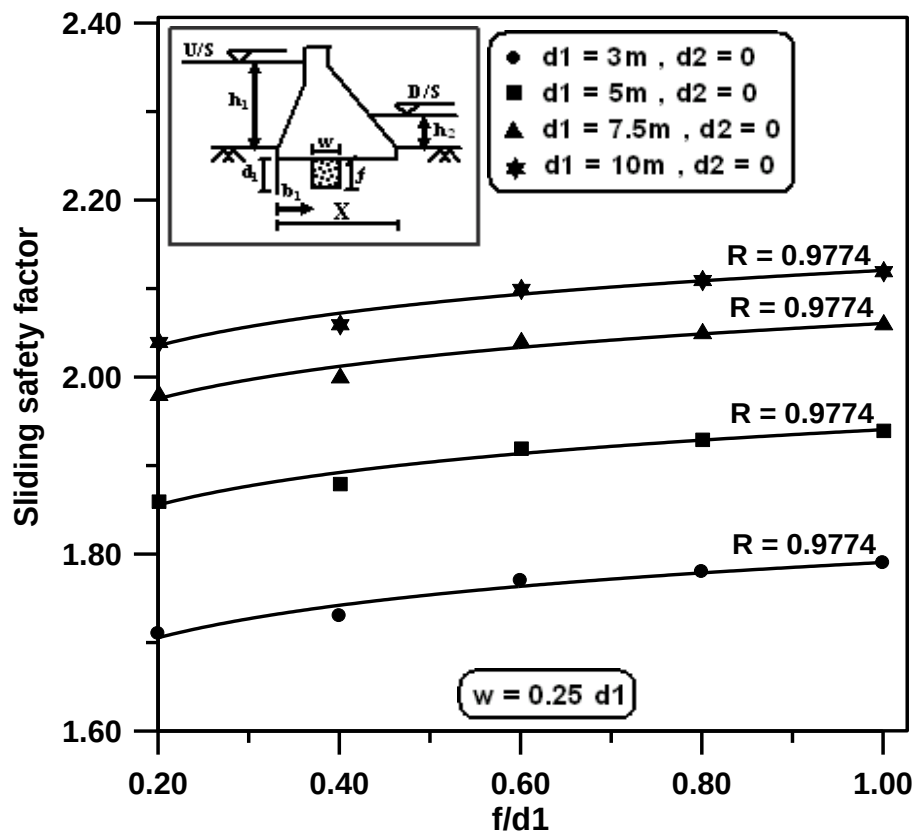


Fig. (6.13): The relation between the sliding safety factor and the depth of the filter media when ($w = 0.25 d1$) and ($d1 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

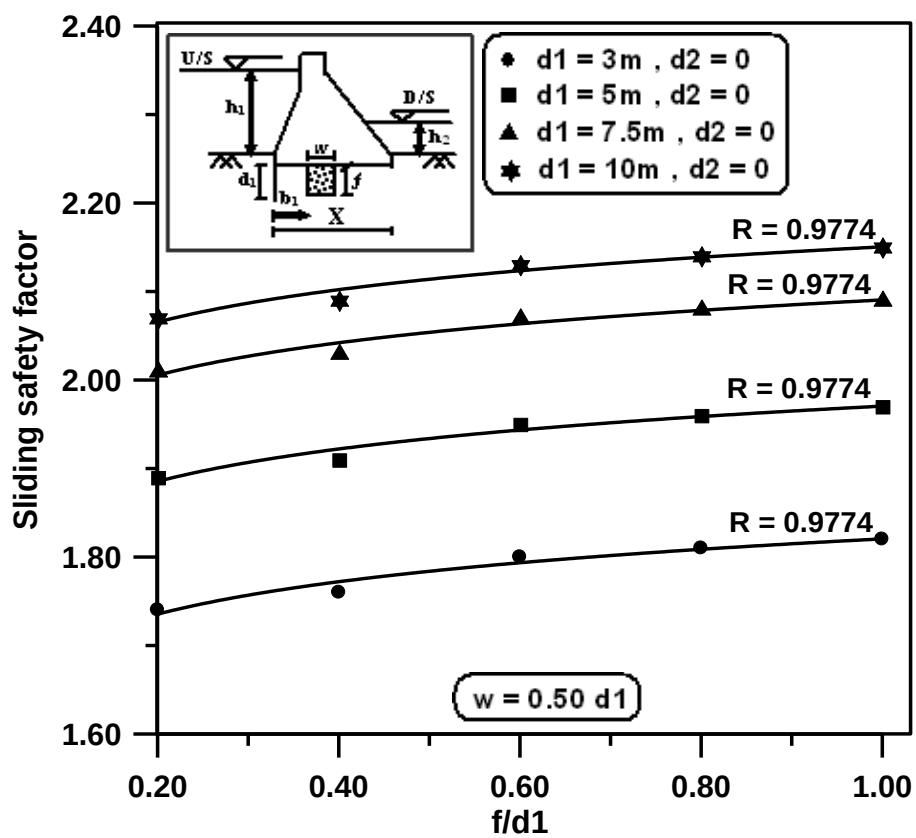


Fig. (6.14): The relation between the sliding safety factor and the depth of the filter media when ($w = 0.50 d1$) and ($d1 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

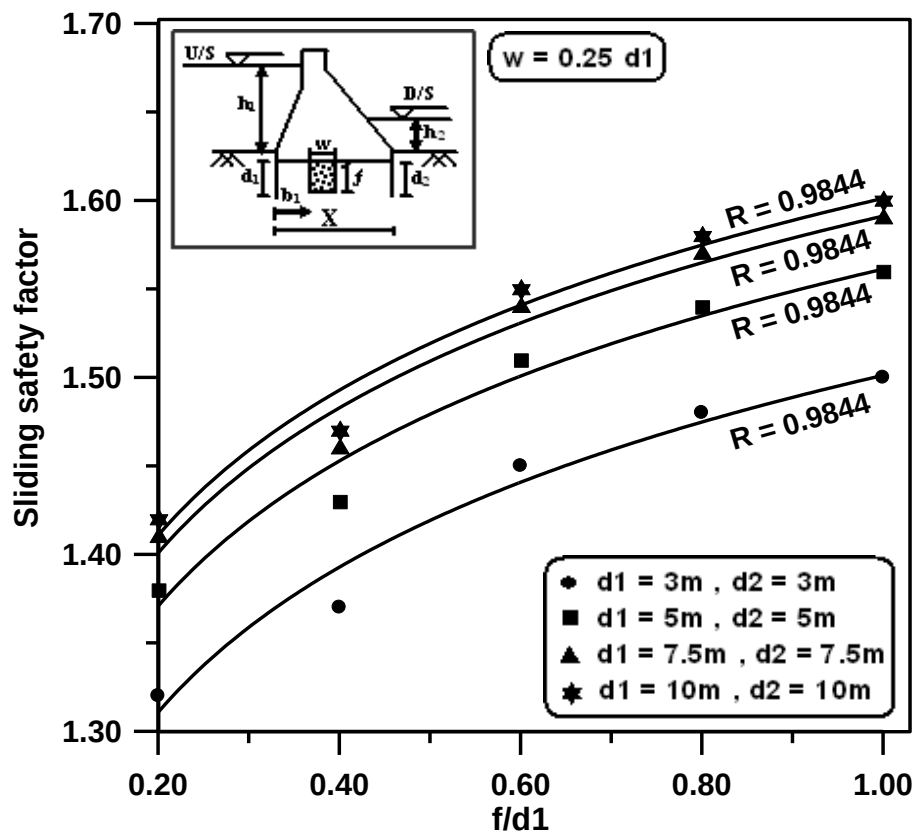


Fig. (6.15): The relation between the sliding safety factor and the depth of the filter media when ($w = 0.25 d_1$) and ($d_1 = d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

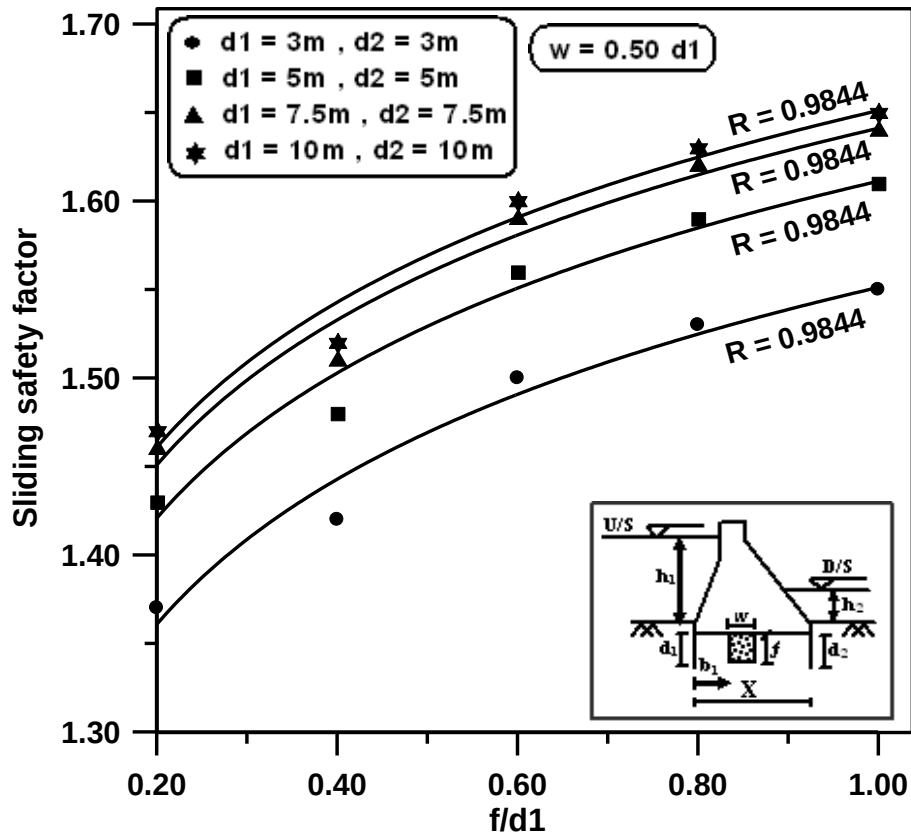


Fig. (6.16): The relation between the sliding safety factor and the depth of the filter media when ($w = 0.50 d_1$) and ($d_1 = d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

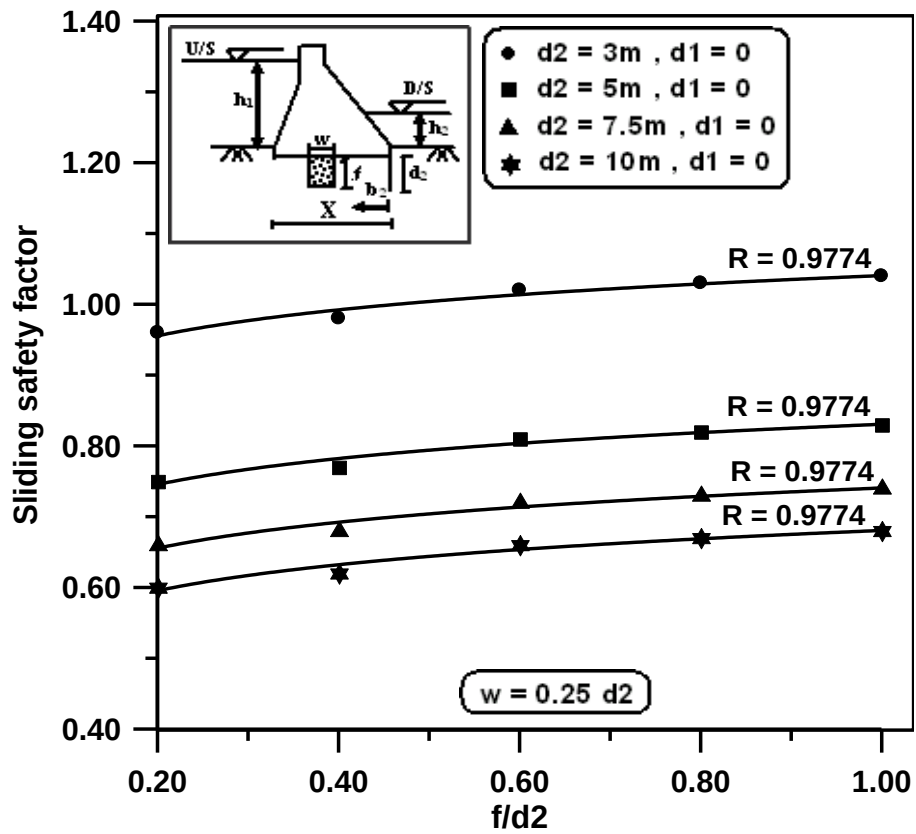


Fig. (6.17): The relation between the sliding safety factor and the depth of the filter media when ($w = 0.25 d_2$) and ($d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

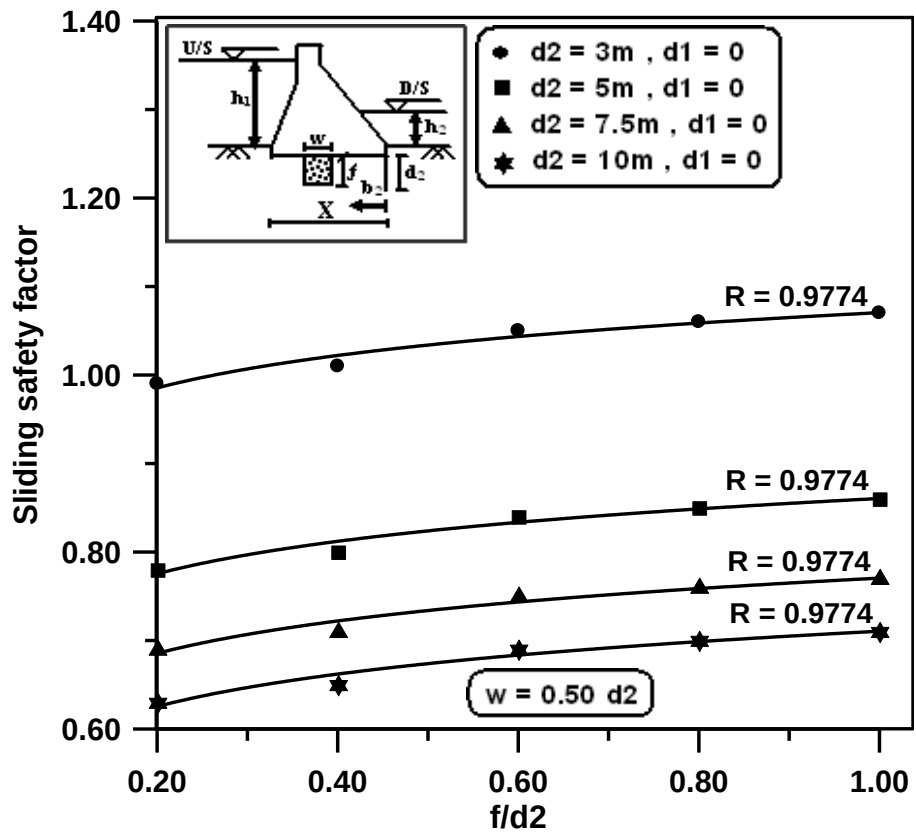


Fig. (6.18): The relation between the sliding safety factor and the depth of the filter media when ($w = 0.50 d_2$) and ($d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

Figures (6.13) and (6.14) show the relation between the sliding safety factor and the depth of the filter media for the widths of the filter media of ($w = 0.25 d_1$) and ($w = 0.50 d_1$), respectively, when there is a cutoff at the upstream side only of the dam of depth equal [$d_1 = 3m, 5m, 7.5m, \text{ and } 10m$]. These figures show that increasing the depth of the filter media yields increasing the sliding safety factor. Increasing the width of the filter media to ($w = 0.50 d_1$) results in increasing the sliding safety factor by a large value as compared with the width ($w = 0.25 d_1$) of the filter media. The increasing of this factor of safety due to the increasing of the depth and width of the filter media is because of decreasing the value of the uplift pressure.

Increasing the depth of the upstream cutoff yields decreasing the uplift pressure and results in increasing the sliding safety factor.

Figures (6.15) and (6.16) present the relation between the sliding safety factor and the depth of the filter media for the widths of the filter media of ($w = 0.25 d_1$) and ($w = 0.50 d_1$), respectively, when there are cutoff at the upstream and downstream sides of the dam of depths equal [$d_1 = d_2 = 3m, 5m, 7.5m, \text{ and } 10m$]. These figures show that increasing the filter media depth and width results in increasing the sliding safety factor because of decreasing the uplift pressure, also, noted that the correlation factor, (**R**); of the curves in **Figs. (6.15) and (6.16)** is greater than that shown in **Figs. (6.13) and (6.14)** when there is a cutoff at the upstream side of the dam only. This is because the increasing in the sliding safety factor when there are cutoffs at the two ends of the dam is greater than that when there is a cutoff at the upstream side only. This is because of the large decreasing in the uplift pressure happened when using two ends cutoffs.

Increasing the depth of the two end cutoffs yields increasing the sliding safety factor due to the decreasing of the value of the uplift pressure.

Figures (6.17) and (6.18) present the relation between the sliding safety factor and the depth of the filter media for the widths of the filter media of

($w = 0.25 d_2$) and ($w = 0.50 d_2$), respectively, when there is a cutoff at the downstream side of the dam only of depth equal [$d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, \text{and } 10\text{m}$].

Figures (6.17) and (6.18) show that increasing the filter media depth and width yields increasing the sliding safety factor due to the decreasing in the uplift pressure.

Increasing the depth of the downstream cutoff results in decreasing the sliding safety factor as shown in **Figs. (6.17) and (6.18)**. This decreasing is because of increasing the value of the uplift pressure.

Although the depths [$(f=0.8 d_1)$, and $(f= d_1)$] of the filter media when there is a cutoff at the upstream side or at the upstream and downstream sides of the dam increase the sliding safety factor, but, the magnitude of increasing is less than that with the depths [$(f=0.4 d_1)$, and $(f=0.6 d_1)$] of the filter media. The depths [$(f=0.4 d_2)$, and $(f=0.6 d_2)$] of the filter media when there is a cutoff at the downstream side increase the sliding safety factor more than the other depths of the filter media.

B. Location of the filter media

The summarized of results of checking the sliding safety factor with respect to the location (b_1) or (b_2) of the filter media are given in Table (6.7). This table is abstracted from Tables (6.1) and (6.2) and Tables (A.1) through (A.22) given in Appendix (A) when there is a cutoff at the upstream side or/and downstream side of the dam of different depths.

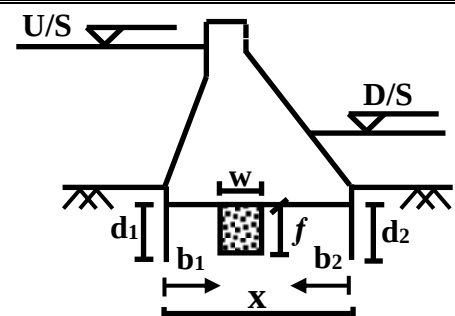
With the minimum value of the safety factor against sliding, ($F_{SL.}$), of (2), the results considered in Table (6.7) indicate the cases where this criterion has been fulfilled or not.

Figures (6.19) through (6.24) show the trend of the variation of ($F_{SL.}$) with the location of the filter media, (b_1) or (b_2), for the whole cases considered, which sum to (120) case. The results of each case have been regressed logarithmic with the value of regression coefficient (R) as indicated on each individual curve.

Table (6.7): Summary of results of checking the sliding safety factor with respect to the location (b_1) or (b_2) of the filter media

NO.	Depth of the cutoffs	Sliding safety factor $F_{SL} \geq 2^{(4)}$		Width of the filter media (w)	b_1				
					2 d_1	3 d_1	4 d_1	5 d_1	6 d_1
1.	$d_1 = 3m$ $d_2 = 0$	F_{SL}	Max.	$w = 0.25 d_1$	1.79	1.76	1.73	1.70	1.67
			Max.	$w = 0.50 d_1$	1.82	1.79	1.76	1.73	1.70
	$d_1 = 5m$ $d_2 = 0$	F_{SL}	Max.	$w = 0.25 d_1$	1.94	1.91	1.88	1.85	1.82
			Max.	$w = 0.50 d_1$	1.97	1.94	1.91	1.88	1.85
	$d_1 = 7.5m$ $d_2 = 0$	F_{SL}	Max.	$w = 0.25 d_1$	2.06	2.03	2.00	1.97	1.94
			Max.	$w = 0.50 d_1$	2.09	2.06	2.03	2.00	1.97
	$d_1 = 10m$ $d_2 = 0$	F_{SL}	Max.	$w = 0.25 d_1$	2.12	2.09	2.06	2.03	2.00
			Max.	$w = 0.50 d_1$	2.15	2.12	2.09	2.06	2.03
2.	$d_1=d_2= 3m$	F_{SL}	Max.	$w = 0.25 d_1$	1.50	1.44	1.42	1.38	1.34
			Max.	$w = 0.50 d_1$	1.55	1.51	1.47	1.43	1.39
	$d_1=d_2= 5m$	F_{SL}	Max.	$w = 0.25 d_1$	1.56	1.52	1.48	1.44	1.40
			Max.	$w = 0.50 d_1$	1.61	1.57	1.53	1.49	1.45
	$d_1=d_2=7.5m$	F_{SL}	Max.	$w = 0.25 d_1$	1.59	1.55	1.51	1.47	1.43
			Max.	$w = 0.50 d_1$	1.64	1.60	1.56	1.52	1.48
	$d_1=d_2= 10m$	F_{SL}	Max.	$w = 0.25 d_1$	1.60	1.56	1.52	1.48	1.44
			Max.	$w = 0.50 d_1$	1.65	1.61	1.57	1.53	1.49
	Depth of the cutoffs	$F_{SL} \geq 2^{(4)}$		(w)	b_2				
					2 d_2	3 d_2	4 d_2	5 d_2	6 d_2
3.	$d_2 = 3m$ $d_1 = 0$	F_{SL}	Max.	$w = 0.25 d_2$	1.04	1.01	0.98	0.95	0.92
			Max.	$w = 0.50 d_2$	1.07	1.04	1.01	0.98	0.95
	$d_2 = 5m$ $d_1 = 0$	F_{SL}	Max.	$w = 0.25 d_2$	0.83	0.80	0.77	0.74	0.71
			Max.	$w = 0.50 d_2$	0.86	0.83	0.80	0.77	0.74
	$d_2 = 7.5m$ $d_1 = 0$	F_{SL}	Max.	$w = 0.25 d_2$	0.74	0.71	0.68	0.65	0.62
			Max.	$w = 0.50 d_2$	0.77	0.74	0.71	0.68	0.65
	$d_2 = 10m$ $d_1 = 0$	F_{SL}	Max.	$w = 0.25 d_2$	0.68	0.65	0.62	0.59	0.56
			Max.	$w = 0.50 d_2$	0.71	0.68	0.65	0.62	0.59

⁽⁴⁾ : The maximum denote the maximum values of (F_{SL}) for the values of the depth (0.2, 0.4, 0.6, 0.8, and 1.0) of (f).



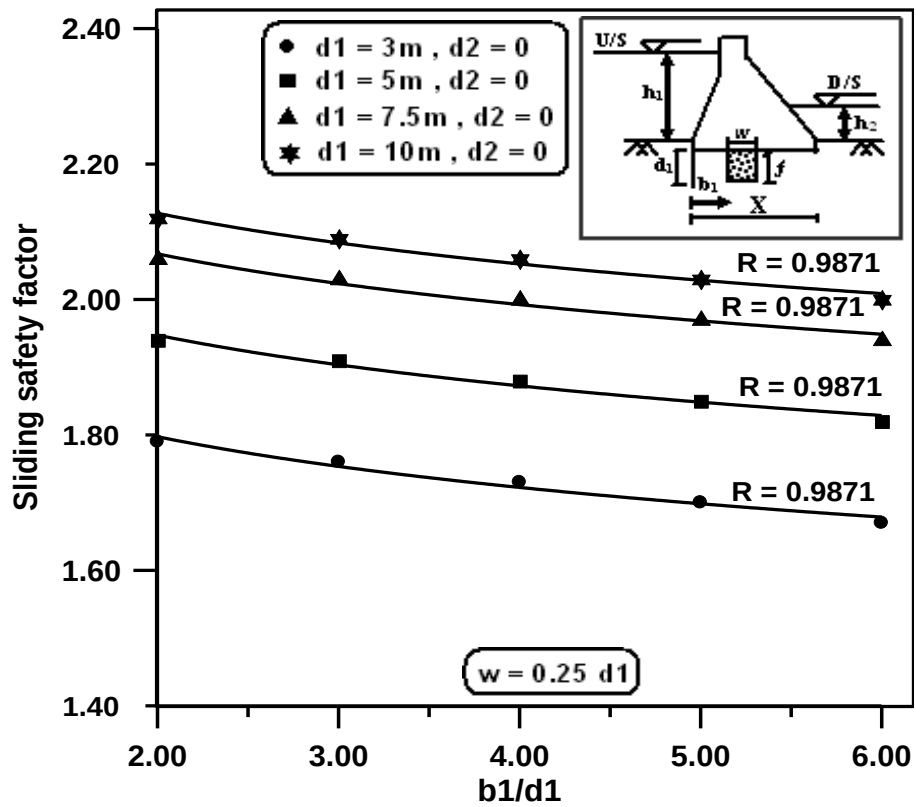


Fig. (6.19): The relation between the sliding safety factor and the location of the filter media when ($w = 0.25 d_1$) and ($d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

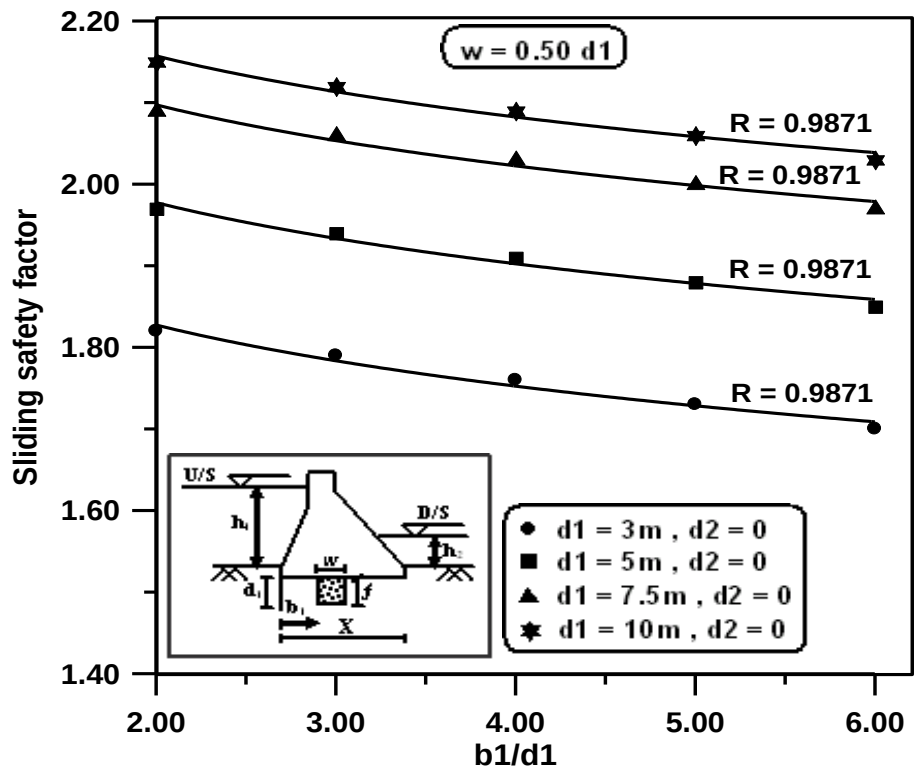


Fig. (6.20): The relation between the sliding safety factor and the location of the filter media when ($w = 0.25 d_1$) and ($d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

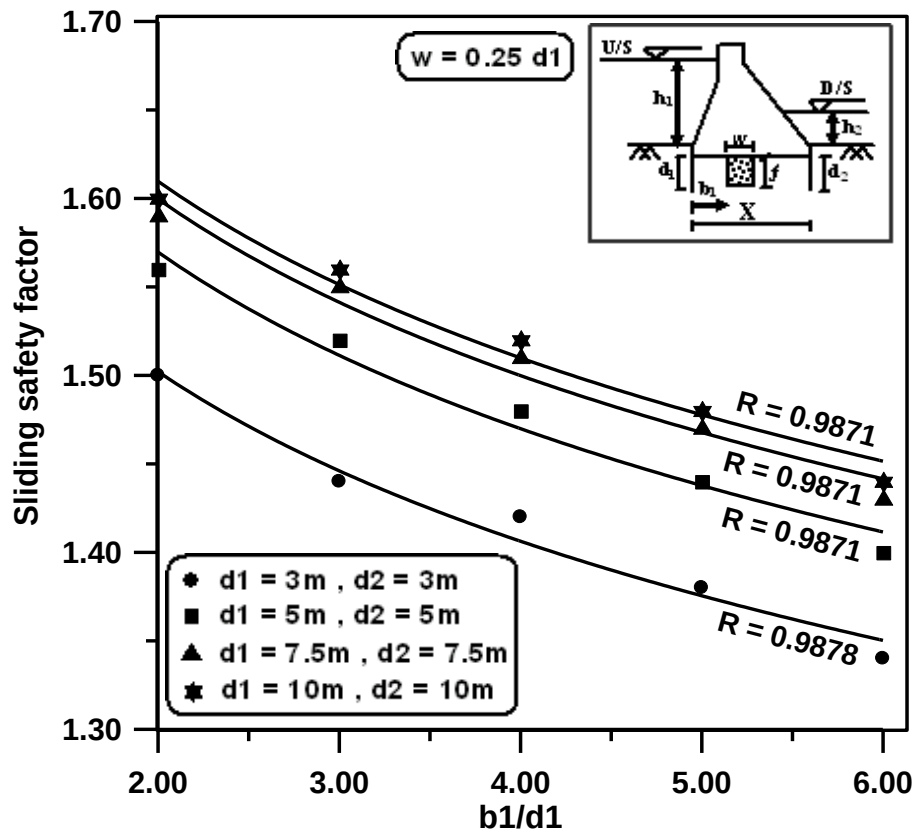


Fig. (6.21): The relation between the sliding safety factor and the location of the filter media when ($w = 0.25 d_1$) and ($d_1 = d_2 = 3\text{ m}, 5\text{ m}, 7.5\text{ m}, 10\text{ m}$)

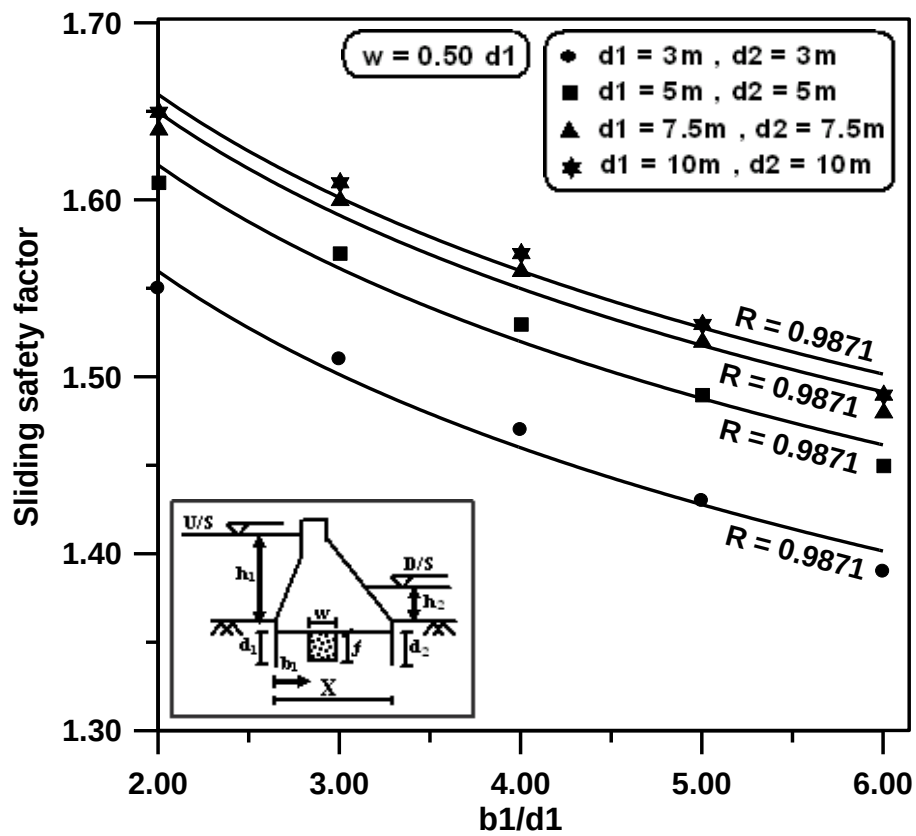


Fig. (6.22): The relation between the sliding safety factor and the location of the filter media when ($w = 0.25 d_1$) and ($d_1 = d_2 = 3\text{ m}, 5\text{ m}, 7.5\text{ m}, 10\text{ m}$)

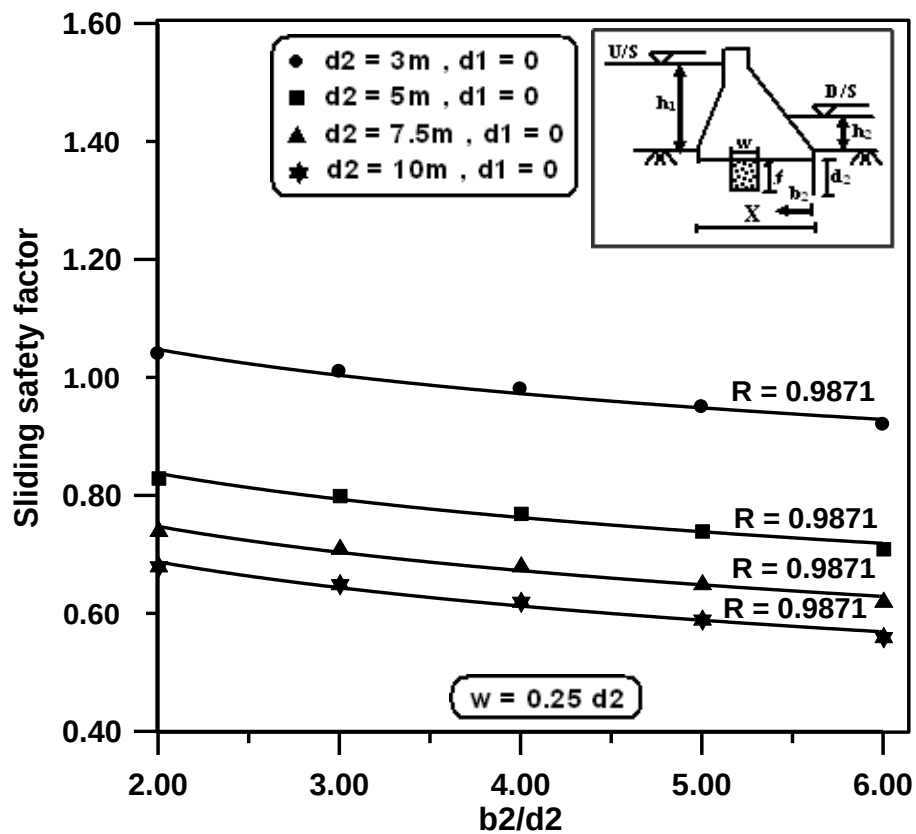


Fig. (6.23): The relation between the sliding safety factor and the location of the filter media when ($w = 0.25 d_2$) and ($d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

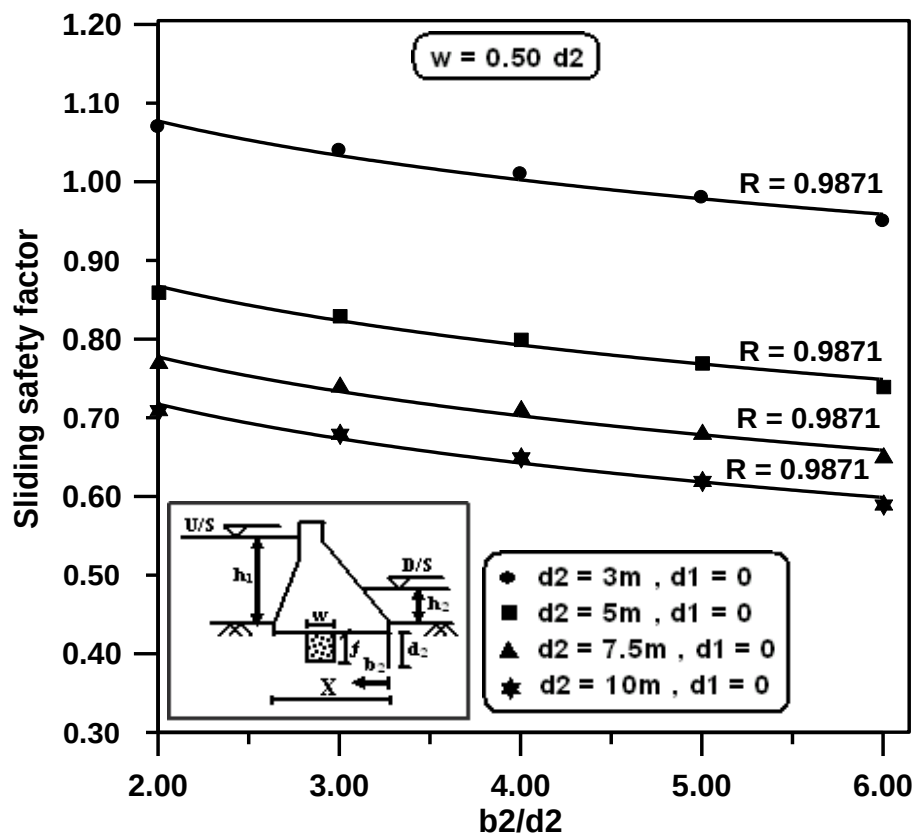


Fig. (6.24): The relation between the sliding safety factor and the location of the filter media when ($w = 0.50 d_2$) and ($d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

Figures (6.19) and (6.20) present the relation between the sliding safety factor and the location of the filter media for the widths of the filter media of ($w = 0.25 d_1$) and ($w = 0.50 d_1$), respectively, when there is a cutoff at the upstream side of the dam only of depth equal [$d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, \text{and } 10\text{m}$]. **Figures (6.21), (6.22), and (6.23), (6.24)** represent the same aforementioned relation of **Figs. (6.19) and (6.20)**, but when there are cutoffs at the upstream and downstream sides together and at the downstream side of the dam only, respectively.

Moving the filter media from the upstream side toward the downstream side yields decreasing the sliding safety factor as shown in **Figs. (6.19), (6.20), and Figs. (6.21), (6.22)** when there is a cutoff at the upstream side only and at the upstream and downstream sides of the dam, respectively.

Increasing the filter media width to ($w = 0.50 d_1$) as shown in **Figs. (6.20) and (6.22)** when there is a cutoff at the upstream side only and at the two ends of the dam, respectively, will increase the sliding safety factor as compared with the same factor of safety for the width of the filter media ($w = 0.25 d_1$) that is shown in **Figs. (6.19) and (6.21)**. The increasing of the sliding safety factor is because of decreasing the value of the uplift pressure.

The increasing of the cutoff depth at the upstream side only or at the upstream and downstream sides of the dam results in increasing the sliding safety factor due to decreasing the uplift pressure as shown in **Figs. (6.19), (6.20), and Figs. (6.21), (6.22)**, respectively.

When there is a cutoff at the downstream side of the dam only, moving the filter media from the downstream side toward the upstream side of the dam decreases the sliding safety factor as shown in **Figs. (6.23) and (6.24)**. This decreasing is due to the increasing of the uplift pressure. Increasing the width of the filter media will increase the sliding safety factor as shown in **Fig. (6.24)** when compared with **Fig. (6.23)**. This increasing is because of the decreasing of the uplift pressure.

Increasing the depth of the downstream cutoff yields decreasing the sliding safety factor as shown in **Figs. (6.23) and (6.24)**. This is because of increasing the value of the uplift pressure.

Table **(6.7)** shows that the location **($b_1=2\ d_1$)** of the filter media when there is a cutoff at the upstream side or at the upstream and downstream sides of the dam gives a sliding safety factor greater than the other locations of the filter media for different depths of the cutoffs. The location **($b_2=2\ d_2$)** acts similarly when there is a cutoff at the downstream side. The uplift pressure at the aforementioned locations of the filter media is less than the other locations of the filter media.

6.3.3 Factor of safety against piping

The effect of the depth, width, and location of the filter media on the piping safety factor when there is a cutoff at the upstream side or/and downstream side of the considered dam of different depths is as follows:

A. Depth and width of the filter media

Table **(6.8)** summarizes the results of checking the piping safety factor with respect to the depth (**f**) of the filter media. This table is summarized from the results given in Tables **(6.1)** and **(6.2)** and Tables **(A.1)** through **(A.22)** given in Appendix **(A)** when there is a cutoff at the upstream side or/and downstream side of the dam of different depths.

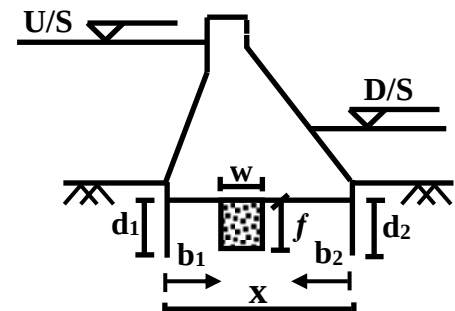
With the minimum value of the safety factor against sliding, **(F_P)**, of **(4)**, the results considered in Table **(6.8)** indicate the cases where this criterion has been fulfilled or not.

Figures **(6.25)** through **(6.30)** show the trend of the variation of **(F_P)** with the depth of the filter media, (**f**), for the whole cases considered, which sum to **(120)** case. The results of each case have been regressed as a polynomial of second degree with the value of regression coefficient **(R)** as indicated on each individual curve.

Table (6.8): Summary of results of checking the piping safety factor with respect to the depth (f) of the filter media

NO.	Depth of the cutoffs	Piping safety factor $F_P \geq 4^{(5)}$		Width of the filter media (w)	f				
					0.2d ₁	0.4d ₁	0.6d ₁	0.8d ₁	1.0d ₁
1.	$d_1 = 3\text{m}$ $d_2 = 0$	F_P	Min.	$w = 0.25 d_1$	3.84	3.95	4.13	4.21	4.24
			Min.	$w = 0.50 d_1$	4.01	4.11	4.27	4.35	4.40
	$d_1 = 5\text{m}$ $d_2 = 0$	F_P	Min.	$w = 0.25 d_1$	4.08	4.21	4.36	4.42	4.45
			Min.	$w = 0.50 d_1$	4.27	4.38	4.53	4.56	4.60
	$d_1 = 7.5\text{m}$ $d_2 = 0$	F_P	Min.	$w = 0.25 d_1$	4.22	4.36	4.53	4.62	4.65
			Min.	$w = 0.50 d_1$	4.37	4.55	4.75	4.81	4.84
	$d_1 = 10\text{m}$ $d_2 = 0$	F_P	Min.	$w = 0.25 d_1$	4.36	4.50	4.66	4.74	4.76
			Min.	$w = 0.50 d_1$	4.52	4.68	4.90	4.98	5.01
2.	$d_1=d_2= 3\text{m}$	F_P	Min.	$w = 0.25 d_1$	4.92	5.08	5.29	5.41	5.45
			Min.	$w = 0.50 d_1$	5.12	5.31	5.53	5.67	5.70
	$d_1=d_2= 5\text{m}$	F_P	Min.	$w = 0.25 d_1$	5.30	5.53	5.83	5.94	5.97
			Min.	$w = 0.50 d_1$	5.52	5.75	6.07	6.19	6.23
	$d_1=d_2=7.5\text{m}$	F_P	Min.	$w = 0.25 d_1$	5.55	5.83	6.17	6.29	6.32
			Min.	$w = 0.50 d_1$	5.88	6.24	6.67	6.80	6.83
	$d_1=d_2= 10\text{m}$	F_P	Min.	$w = 0.25 d_1$	5.82	6.10	6.49	6.66	6.69
			Min.	$w = 0.50 d_1$	6.18	6.50	6.90	7.10	7.14
	Depth of the cutoffs	$F_P \geq 4^{(5)}$		(w)	f				
					0.2d ₂	0.4d ₂	0.6d ₂	0.8d ₂	1.0d ₂
3.	$d_2 = 3\text{m}$ $d_1 = 0$	F_P	Min.	$w = 0.25 d_2$	4.41	4.53	4.71	4.78	4.81
			Min.	$w = 0.50 d_2$	4.56	4.71	4.88	4.95	4.98
	$d_2 = 5\text{m}$ $d_1 = 0$	F_P	Min.	$w = 0.25 d_2$	4.45	4.60	4.80	4.89	4.92
			Min.	$w = 0.50 d_2$	4.65	4.81	5.04	5.12	5.15
	$d_2 = 7.5\text{m}$ $d_1 = 0$	F_P	Min.	$w = 0.25 d_2$	4.75	4.91	5.11	5.23	5.26
			Min.	$w = 0.50 d_2$	4.98	5.19	5.46	5.58	5.60
	$d_2 = 10\text{m}$ $d_1 = 0$	F_P	Min.	$w = 0.25 d_2$	4.95	5.15	5.41	5.51	5.54
			Min.	$w = 0.50 d_2$	5.23	5.46	5.73	5.83	5.85

⁽⁵⁾ : The minimum denote the minimum values of (F_P) for the values of the location (2, 3, 4, 5, and 6) of (b_1 or b_2).



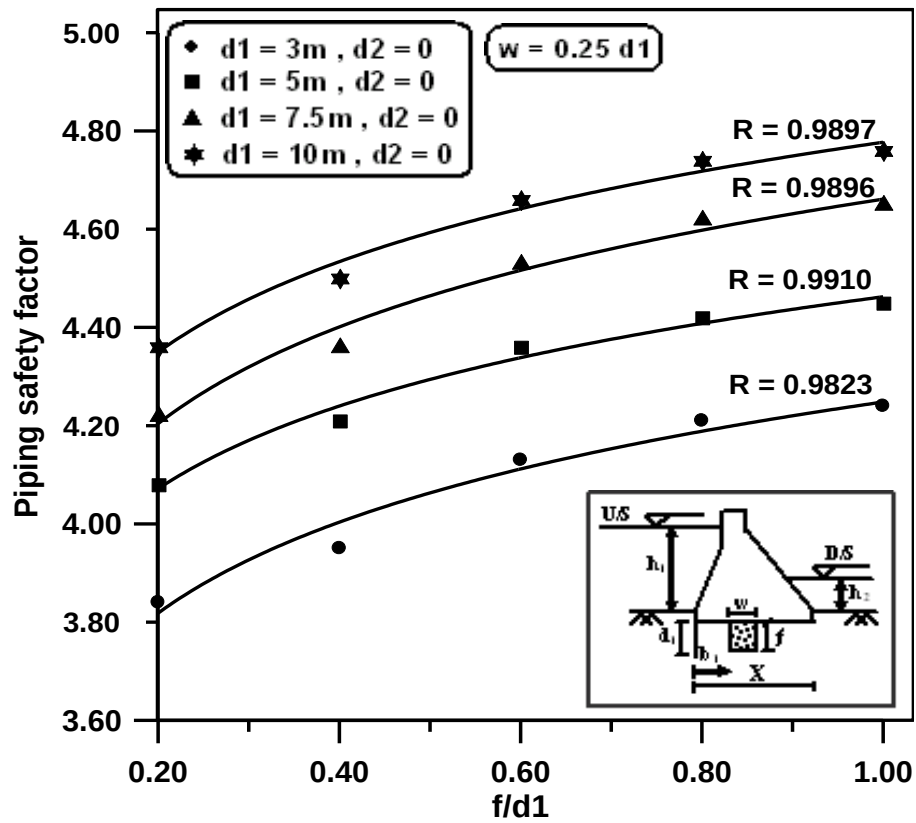


Fig. (6.25): The relation between the piping safety factor and the depth of the filter media when ($w = 0.25 d_1$) and ($d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

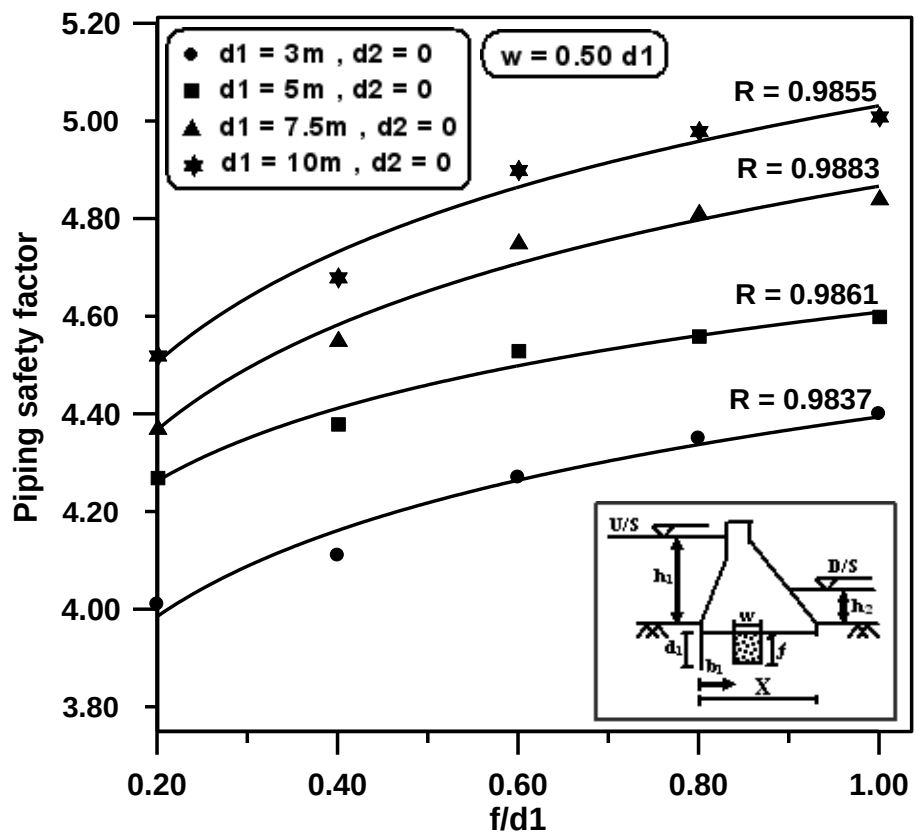


Fig. (6.26): The relation between the piping safety factor and the depth of the filter media when ($w = 0.50 d_1$) and ($d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

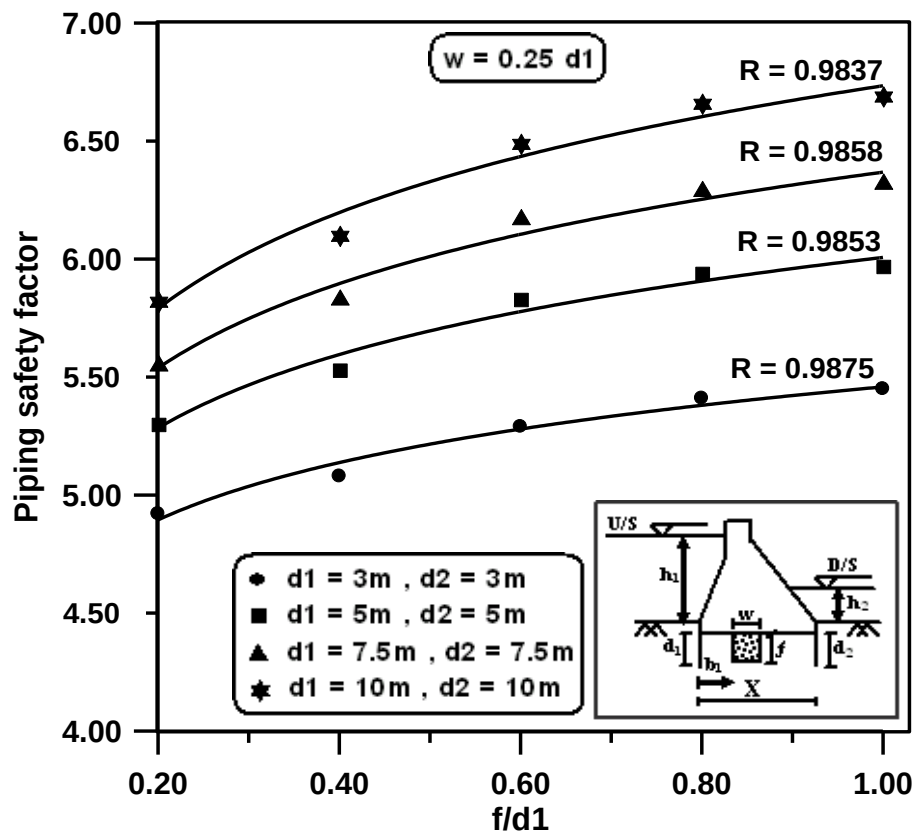


Fig. (6.27): The relation between the piping safety factor and the depth of the filter media when ($w = 0.25 d_1$) and ($d_1 = d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

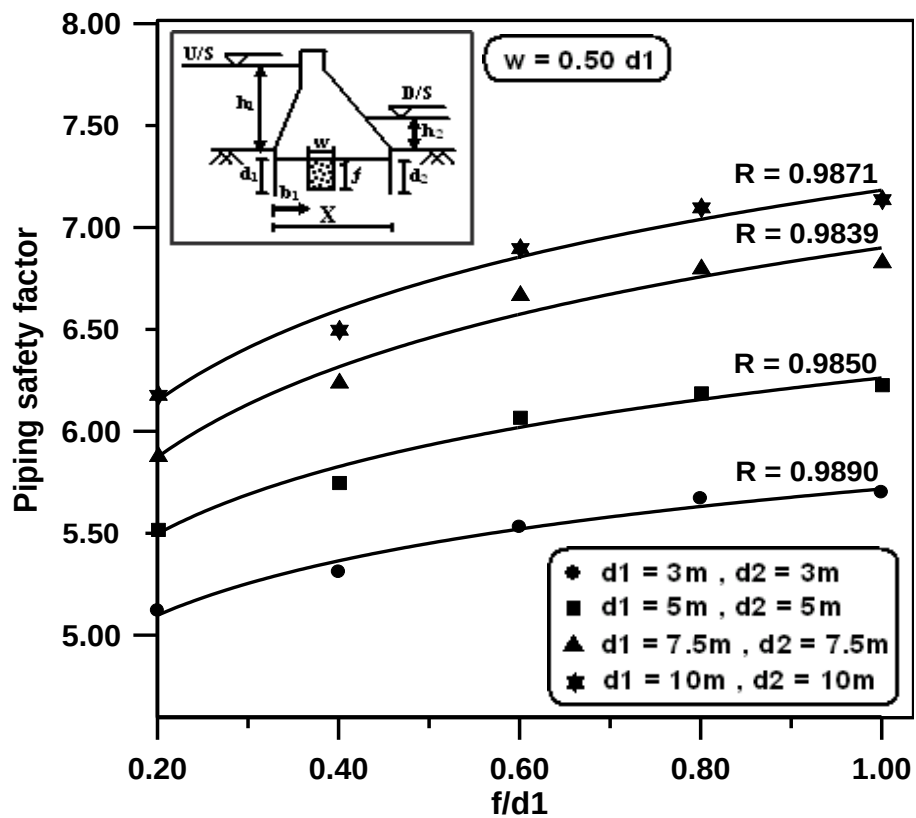


Fig. (6.28): The relation between the piping safety factor and the depth of the filter media when ($w = 0.50 d_1$) and ($d_1 = d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

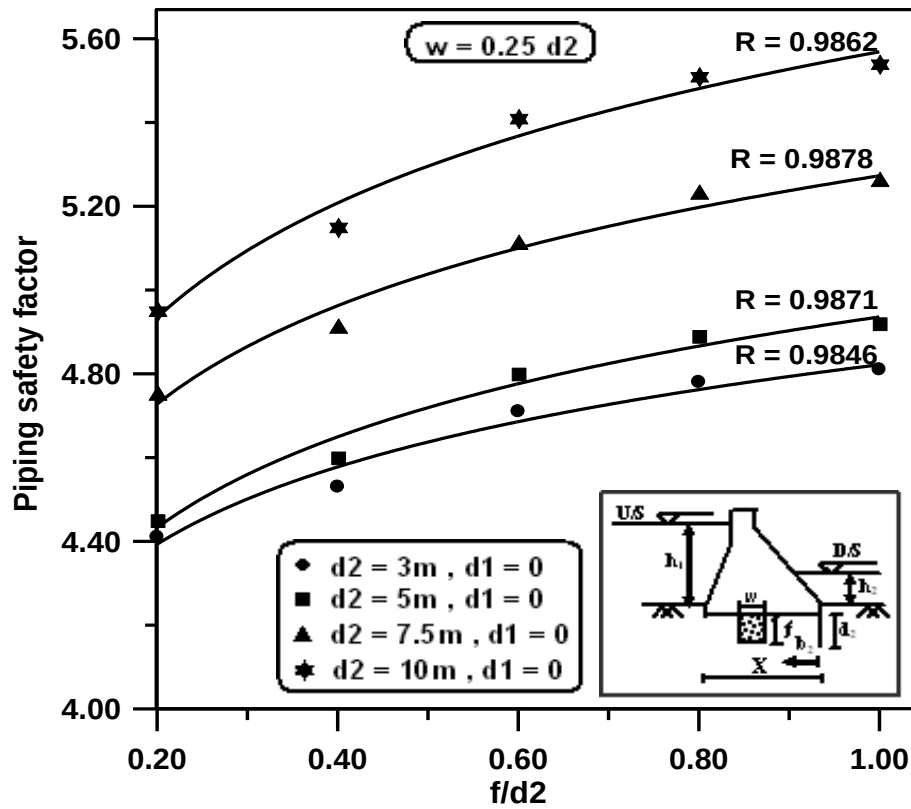


Fig. (6.29): The relation between the piping safety factor and the depth of the filter media when ($w = 0.25 d_2$) and ($d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

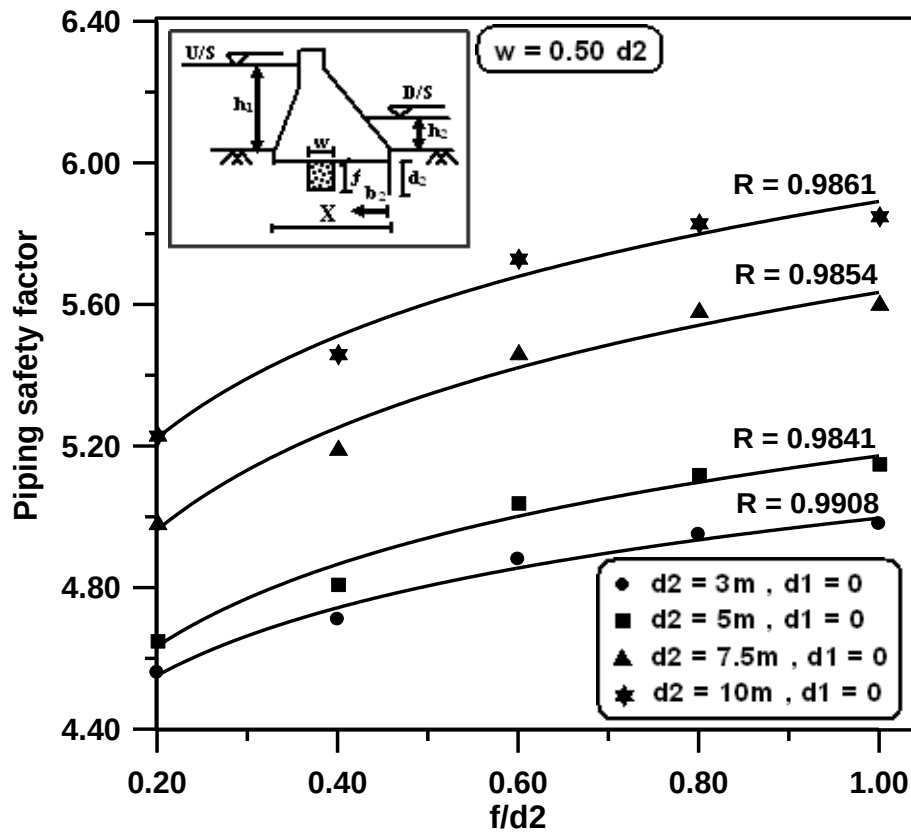


Fig. (6.30): The relation between the piping safety factor and the depth of the filter media when ($w = 0.50 d_2$) and ($d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

Figures (6.25) and (6.26) show the relation between the piping safety factor and the depth of the filter media for the widths of the filter media of ($w = 0.25 d_1$) and ($w = 0.50 d_1$), respectively, when there is a cutoff at the upstream side of the dam only of depth equal [$d_1 = 3m, 5m, 7.5m, \text{ and } 10m$]. **Figures (6.27), (6.28), and (6.29), (6.30)** present the same aforementioned relation of the **Figs. (6.25) and (6.26)**, but when there are cutoffs at the upstream and downstream sides together and at the downstream side of the dam only, respectively.

The **Figures (6.25) through (6.30)** show that increasing the filter media depth will increase the factor of safety against piping. Increasing of this factor of safety is because of decreasing the value of the exit gradient. Increasing the width of the filter media for ($w = 0.50 d_1$) as shown in **Figs. (6.26) and (6.28)** when there is a cutoff at the upstream side only and when there are cutoffs at the two ends of the dam, respectively, or for the width ($w = 0.50 d_2$) of the filter media when there is a cutoff at the downstream side only as shown in **Fig. (6.30)**, will increase the piping safety factor to a value greater than the same factor of safety, but when the width of the filter media equal ($w = 0.25 d_1$) and ($w = 0.25 d_2$) when there is a cutoff at the upstream side only or at the two ends of the dam and at the downstream side of the dam only, respectively, as shown in **Figs. (6.25), (6.27) and (6.29)**, respectively.

Increasing of the depth of the cutoff when it is there at the upstream side of the dam only, at the upstream and downstream sides of the dam, and at the downstream side of the dam only, will increase the piping safety factor as shown in **Figs. (6.25) through (6.30)**. This means that increasing the cutoff depth at the upstream side or/and downstream side of the considered dam result in increasing the piping safety factor because of decreasing the value of the exit gradient.

Figures (6.27) and (6.28) show that the value of the piping safety factor when there are cutoffs at the upstream and downstream sides of the dam is greater than that when there is a cutoff at the upstream side only or at the

downstream side only. This means that the two ends cutoffs having more effect on increasing the piping safety factor as compared with the case of upstream cutoff only or downstream cutoff only as shown in **Figs. (6.25), (6.26), and (6.29), (6.30)**, respectively.

It is noted from Table **(6.8)** that increasing the depth and width of the filter media causes increasing the piping safety factor as mentioned hereinbefore, but, the depths $[(f=0.4 \ d_1), \text{ and } (f=0.6 \ d_1)]$ and width $[(w=0.50 \ d_1)]$ of the filter media when there is a cutoff at the upstream side or at the upstream and downstream sides, have high effect on increasing this factor as compared with the other depths and widths of the filter media and for different depths of the cutoffs. Moreover, for the case when there is a cutoff at the downstream side only, the depths $[(f=0.4 \ d_2), \text{ and } (f=0.6 \ d_2)]$ of the filter media and the width $[(w=0.50 \ d_2)]$ of the filter media result in a similar effect.

B. Location of the filter media

The summary of results of checking the piping safety factor with respect to the location **(b₁)** or **(b₂)** of the filter media [summarized from the results that are given in Tables **(6.1)** and **(6.2)** and Tables **(A.1)** through **(A.22)** given in Appendix **(A)**] are given in Table **(6.9)**. The Table shows the summarize results when there is a cutoff at the upstream side or/and downstream side of the dam of different depths.

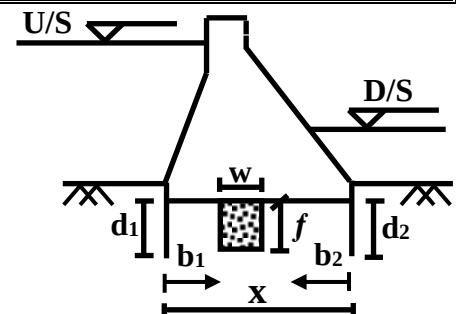
With the minimum value of the safety factor against sliding, **(F_P)**, of **(4)**, the results considered in Table **(6.9)** indicate the cases where this criterion has been fulfilled or not.

Figures **(6.31)** through **(6.36)** show the trend of the variation of **(F_P)** with the location of the filter media, **(b₁)** or **(b₂)**, for the whole cases considered, which sum to **(120)** case. The results of each case have been regressed as a polynomial of second degree as shown in **Figs. (6.31)** through **(6.34)** and logarithmic as shown in **Figs. (6.35)** and **(6.36)** with the value of regression coefficient **(R)** as indicated on each individual curve.

Table (6.9): Summary of results of checking the piping safety factor with respect to the location (b_1) or (b_2) of the filter media

NO.	Depth of the cutoffs	Piping safety factor $F_P \geq 4^{(6)}$		Width of the filter media (w)	b_1				
					2 d_1	3 d_1	4 d_1	5 d_1	6 d_1
1.	$d_1 = 3m$ $d_2 = 0$	F_P	Min.	$w = 0.25 d_1$	3.84	3.97	4.10	4.26	4.40
			Min.	$w = 0.50 d_1$	4.01	4.10	4.27	4.40	4.55
	$d_1 = 5m$ $d_2 = 0$	F_P	Min.	$w = 0.25 d_1$	4.08	4.24	4.39	4.55	4.71
			Min.	$w = 0.50 d_1$	4.27	4.38	4.53	4.75	4.93
	$d_1 = 7.5m$ $d_2 = 0$	F_P	Min.	$w = 0.25 d_1$	4.22	4.36	4.52	4.72	4.92
			Min.	$w = 0.50 d_1$	4.37	4.55	4.73	4.91	5.10
	$d_1 = 10m$ $d_2 = 0$	F_P	Min.	$w = 0.25 d_1$	4.36	4.52	4.70	4.93	5.16
			Min.	$w = 0.50 d_1$	4.52	4.73	4.95	5.10	5.39
2.	$d_1=d_2= 3m$	F_P	Min.	$w = 0.25 d_1$	4.92	5.09	5.30	5.52	5.67
			Min.	$w = 0.50 d_1$	5.12	5.38	5.55	5.82	6.07
	$d_1=d_2= 5m$	F_P	Min.	$w = 0.25 d_1$	5.30	5.52	5.79	6.09	6.36
			Min.	$w = 0.50 d_1$	5.52	5.81	6.17	6.48	6.76
	$d_1=d_2=7.5m$	F_P	Min.	$w = 0.25 d_1$	5.55	5.82	6.17	6.50	6.91
			Min.	$w = 0.50 d_1$	5.88	6.23	6.61	6.95	7.32
	$d_1=d_2= 10m$	F_P	Min.	$w = 0.25 d_1$	5.82	6.19	6.50	6.91	7.36
			Min.	$w = 0.50 d_1$	6.18	6.56	6.94	7.34	7.94
	Depth of the cutoffs	$F_P \geq 4^{(6)}$		(w)	b_2				
					2 d_2	3 d_2	4 d_2	5 d_2	6 d_2
3.	$d_2 = 3m$ $d_1 = 0$	F_P	Min.	$w = 0.25 d_2$	4.98	4.81	4.66	4.56	4.41
			Min.	$w = 0.50 d_2$	5.12	4.99	4.80	4.67	4.56
	$d_2 = 5m$ $d_1 = 0$	F_P	Min.	$w = 0.25 d_2$	5.36	5.11	4.87	4.66	4.45
			Min.	$w = 0.50 d_2$	5.54	5.37	5.12	4.84	4.65
	$d_2 = 7.5m$ $d_1 = 0$	F_P	Min.	$w = 0.25 d_2$	5.65	5.37	5.19	4.99	4.75
			Min.	$w = 0.50 d_2$	5.84	5.60	5.36	5.20	4.98
	$d_2 = 10m$ $d_1 = 0$	F_P	Min.	$w = 0.25 d_2$	6.03	5.75	5.46	5.23	4.95
			Min.	$w = 0.50 d_2$	6.36	6.01	5.67	5.45	5.23

⁽⁶⁾ : The minimum denote the minimum values of (F_P) for the values of the depth (0.2, 0.4, 0.6, 0.8, and 1.0) of (f).



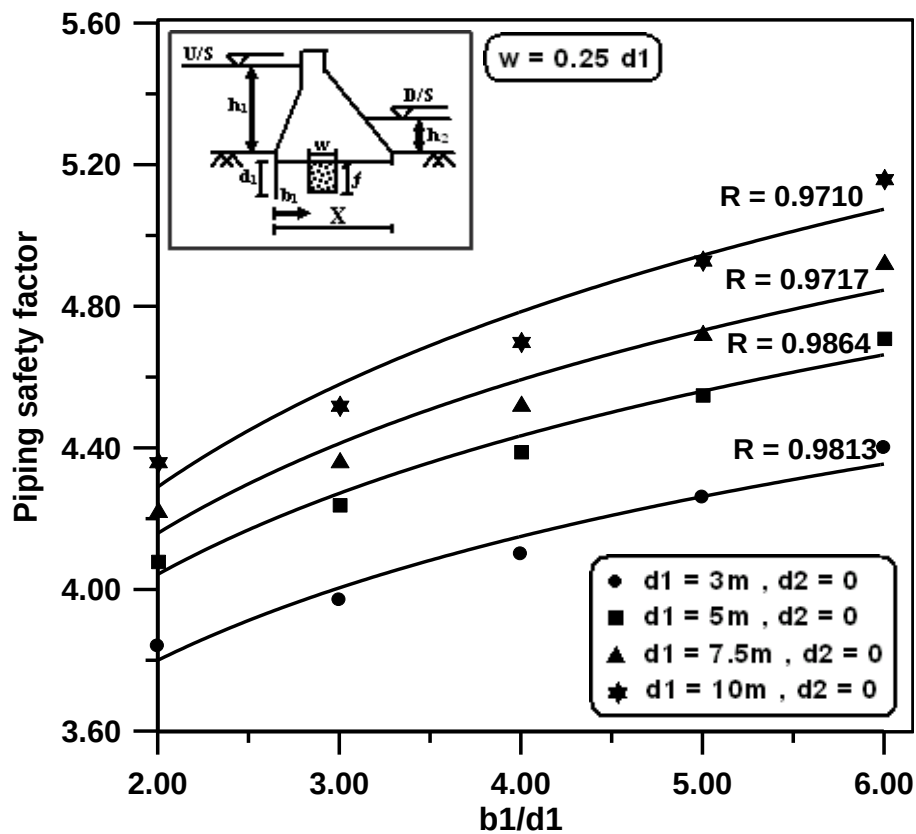


Fig. (6.31): The relation between the piping safety factor and the location of the filter media when ($w = 0.25 d_1$) and ($d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

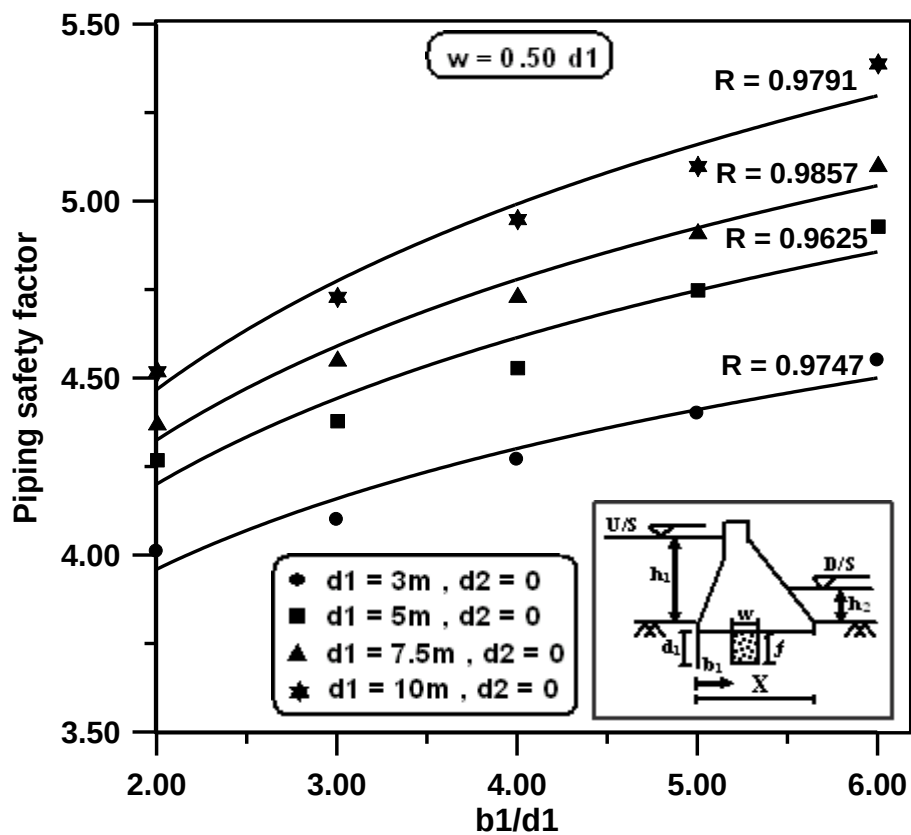


Fig. (6.32): The relation between the piping safety factor and the location of the filter media when ($w = 0.50 d_1$) and ($d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

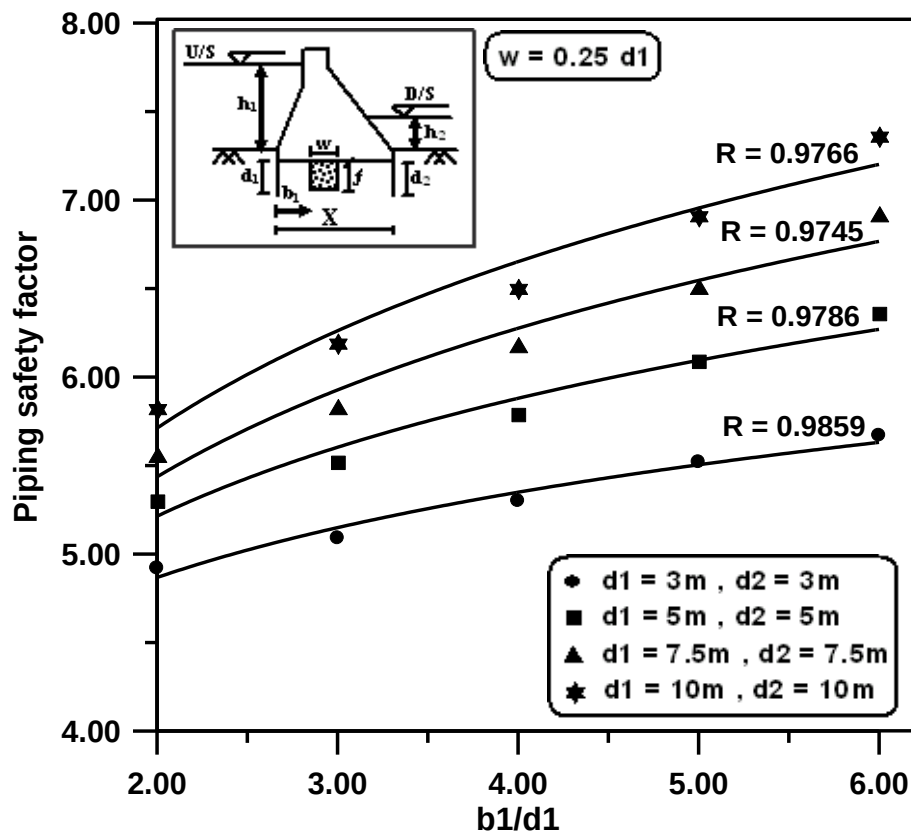


Fig. (6.33): The relation between the piping safety factor and the location of the filter media when ($w = 0.25 d_1$) and ($d_1 = d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

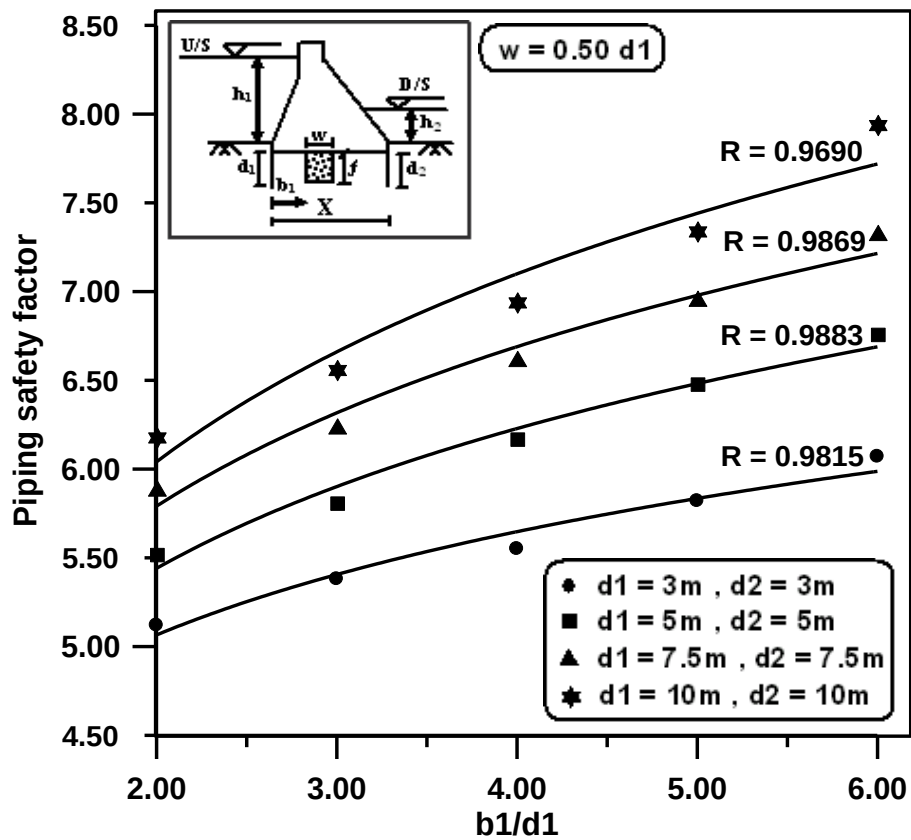


Fig. (6.34): The relation between the piping safety factor and the location of the filter media when ($w = 0.50 d_1$) and ($d_1 = d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

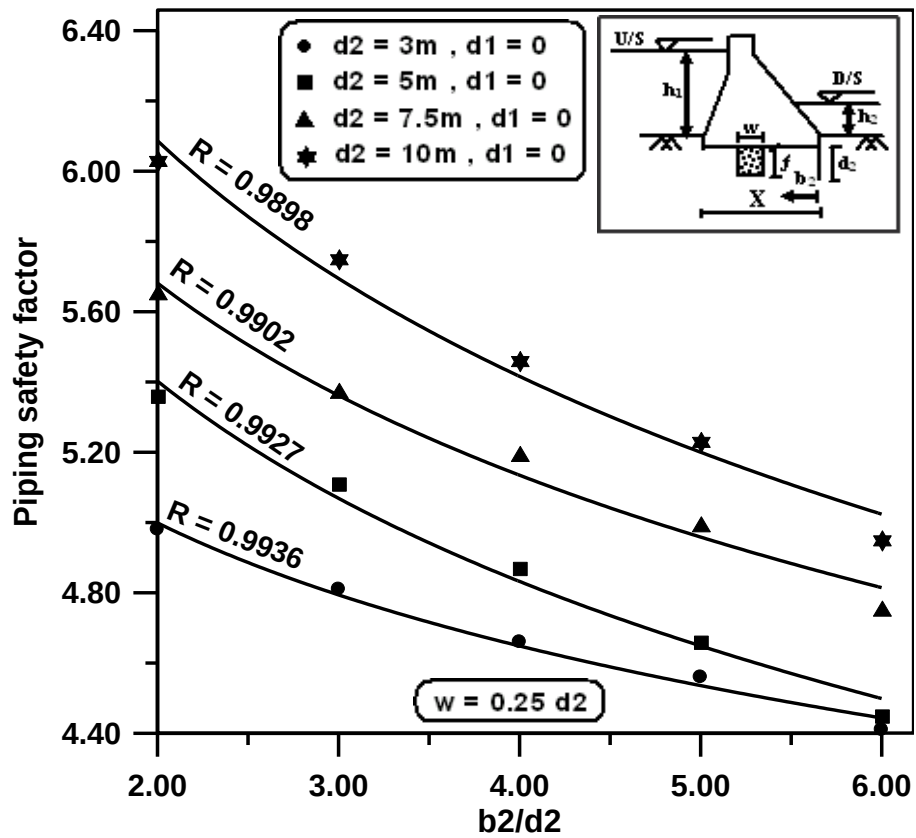


Fig. (6.35): The relation between the piping safety factor and the location of the filter media when ($w = 0.25 d_2$) and ($d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

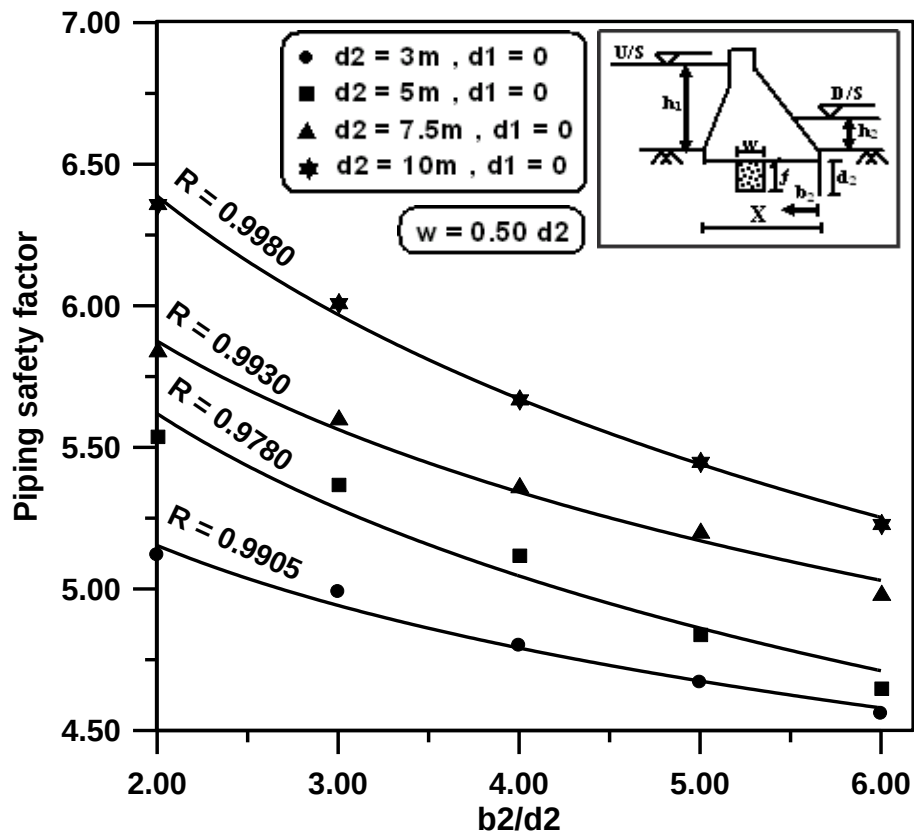


Fig. (6.36): The relation between the piping safety factor and the location of the filter media when ($w = 0.50 d_2$) and ($d_2 = 3\text{m}, 5\text{m}, 7.5\text{m}, 10\text{m}$)

Figures (6.31) and (6.32) show the relation between the piping safety factor and the location of the filter media for the widths of the filter media of ($w = 0.25 d_1$) and ($w = 0.50 d_1$), respectively, when there is a cutoff at the upstream side of the dam only of depth equal [$d_1 = 3\text{m}, 5\text{m}, 7.5\text{m}, \text{and } 10\text{m}$]. **Figures (6.33) through (6.36)** present the same aforementioned relation of **Figs. (6.31) and (6.32)**, but when there are cutoffs at the upstream and downstream sides together and at the downstream side of the dam only, respectively.

Moving the filter media from the upstream side toward the downstream side of the dam results in increasing the piping safety factor when there is a cutoff at the upstream side only and at the upstream and downstream sides together of the dam as shown in **Figs. (6.31) and (6.32)**, and **(6.33) and (6.34)**, respectively. However, this factor of safety decreases as the filter media is moved from the downstream side toward the upstream side of the dam when there is a cutoff at the downstream side of the dam only as shown in **Figs. (6.35)**, and **(6.36)**. The increasing and decreasing of the piping safety factor are because of the decreasing and increasing of the value of the exit gradient, respectively.

Increasing the width of the filter media yields increasing the piping safety factor as shown in **Figs. (6.32), (6.34), and (6.36)** when compared with **Figs. (6.31), (6.33), and (6.35)**, respectively. This increasing in this factor of safety due to the increasing of the width of the filter media is because of decreasing of the value of the exit gradient.

The increase of the cutoff depth when there is one at the upstream side and/or the downstream side of the dam results in increasing the piping safety factor. This is because of decreasing the value of the exit gradient as shown in **Figs. (6.31) through (6.36)**.

From Table (6.9), it can be seen that the locations [$(b_1=6 d_1)$] and [$(b_2=2 d_2)$] of the filter media when there is a cutoff at the upstream side, at the

upstream and downstream sides, or at the downstream side of the dam only, have a piping safety factor greater than the other locations of the filter media for different depths of the cutoffs. This is because of the minimum values of exit gradient which occur at these locations.

6.3.4 Analysis of the calculated eccentricity results

The resulting values of the calculated eccentricity of the vertical resultant force are shown in Table (6.3) and Tables (B.1) through (B.11) given in Appendix (B). These tables show that the magnitude of the calculated eccentricity (X_4) is lower than the adopted value of the eccentricity ($X/6$). According to that, the vertical resultant force lies in the middle third of the floor base under the considered concrete gravity dam (the hypothetical case study). This means that the dam is safe against the tension failure.

6.4 RESULTS OF THE OPTIMIZATION PROBLEM

The formulated optimization model was run for the safety factors that resulted from the stability analysis in order to find the best minimum total difference, denoted as “T.D.”, as given in Eq. (5.1). The results are shown in Tables (6.10) and (6.11) and the other results are given in Tables (C.1) through (C.22) in Appendix (C).

Table (6.10) shows the calculated total difference for different depths and locations of the intermediate filter media and shows too the upstream and downstream uplift pressure when the width of the filter media is equal to ($w = 0.25 d_1$) and when there is a cutoff at the upstream side having a depth equal to (3m). Table (6.11) shows the same aforementioned results but when the width of the filter media is equal to ($w = 0.50 d_1$).

Tables (C.1) through (C.6) show the aforementioned results for depths of the upstream cutoff equal to (5m), (7.5m), and (10m), respectively.

Table (C.7) shows the calculated total difference for different depths and locations of the intermediate filter media and shows too the upstream and downstream uplift pressures when the width of the filter media is equal to ($w = 0.25 d_2$) and when there is a cutoff at the downstream side having a depth equal to (3m). Table (C.8) shows the same aforementioned results but when the width of the filter media is equal to ($w = 0.50 d_2$).

Tables (C.9) through (C.14) show the aforementioned results but for depths of the downstream cutoff equal to (5m), (7.5m), and (10m), respectively.

Table (C.15) shows the calculated total difference for various depths and locations of the intermediate filter media and shows also the upstream and downstream uplift pressures when the width of the filter media is equal to ($w = 0.25 d_1$) and when there are cutoffs at the upstream and downstream sides having depths equal to (3m). Table (C.16) shows the same aforementioned results but when the width of the filter media is equal to ($w = 0.50 d_1$).

Tables (C.17) through (C.22) show the aforementioned results for depths of the upstream and downstream cutoffs equal to (5m), (7.5m), and (10m), respectively.

Table (6.10)
Calculated values of the total difference
 $(d_1 = 3\text{m} , d_2 = 0 , w = 0.25 d_1 , y_5 = 10\text{m})$

NO.	b_1	f	y_4	I_e	T.D.
1	$2 d_1$	$0.2 d_1$			Unsafe
2		$0.4 d_1$			Unsafe
3		$0.6 d_1$			Unsafe
4		$0.8 d_1$			Unsafe
5		$1.0 d_1$			Unsafe
6	$3 d_1$	$0.2 d_1$			Unsafe
7		$0.4 d_1$			Unsafe
8		$0.6 d_1$			Unsafe
9		$0.8 d_1$			Unsafe
10		$1.0 d_1$			Unsafe
11	$4 d_1$	$0.2 d_1$			Unsafe
12		$0.4 d_1$			Unsafe
13		$0.6 d_1$			Unsafe
14		$0.8 d_1$			Unsafe
15		$1.0 d_1$			Unsafe
16	$5 d_1$	$0.2 d_1$			Unsafe
17		$0.4 d_1$			Unsafe
18		$0.6 d_1$			Unsafe
19		$0.8 d_1$			Unsafe
20		$1.0 d_1$			Unsafe
21	$6 d_1$	$0.2 d_1$			Unsafe
22		$0.4 d_1$			Unsafe
23		$0.6 d_1$			Unsafe
24		$0.8 d_1$			Unsafe
25		$1.0 d_1$			Unsafe

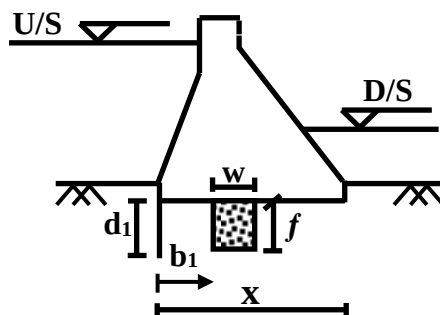
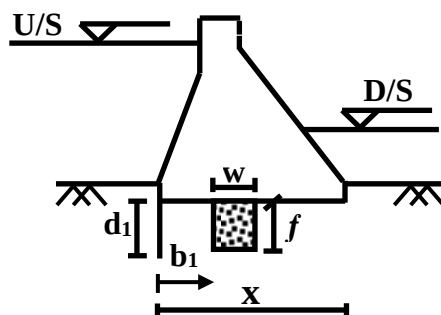


Table (6.11)
Calculated values of the total difference
 $(d_1 = 3\text{m} , d_2 = 0 , w = 0.50 d_1 , y_5 = 10\text{m})$

NO.	b_1	f	y_4	I_e	T.D.
1	$2 d_1$	$0.2 d_1$			Unsafe
2		$0.4 d_1$			Unsafe
3		$0.6 d_1$			Unsafe
4		$0.8 d_1$			Unsafe
5		$1.0 d_1$			Unsafe
6	$3 d_1$	$0.2 d_1$			Unsafe
7		$0.4 d_1$			Unsafe
8		$0.6 d_1$			Unsafe
9		$0.8 d_1$			Unsafe
10		$1.0 d_1$			Unsafe
11	$4 d_1$	$0.2 d_1$			Unsafe
12		$0.4 d_1$			Unsafe
13		$0.6 d_1$			Unsafe
14		$0.8 d_1$			Unsafe
15		$1.0 d_1$			Unsafe
16	$5 d_1$	$0.2 d_1$			Unsafe
17		$0.4 d_1$			Unsafe
18		$0.6 d_1$			Unsafe
19		$0.8 d_1$			Unsafe
20		$1.0 d_1$			Unsafe
21	$6 d_1$	$0.2 d_1$			Unsafe
22		$0.4 d_1$			Unsafe
23		$0.6 d_1$			Unsafe
24		$0.8 d_1$			Unsafe
25		$1.0 d_1$			Unsafe



6.5 ANALYSIS OF THE OPTIMIZATION RESULTS

When a certain design practice specifies that the respective factor of safety should be ($F.S. \geq 2$), for example, then a design yielding a factor of safety of ($F.S. = 10$) would obviously be a safe design. However, such a design would be infeasible in the engineering sense because it also obviously indicates an overdesign.

The cases considered in this research could be set into (24) sub-classes, namely, $\{[(4) \text{ values of upstream cutoff depth } (d_1) \text{ only} + (4) \text{ values of downstream cutoff depth } (d_2) \text{ only} + (4) \text{ values of both upstream cutoff depth } (d_1) \text{ and downstream cutoff depth } (d_2) \text{ together}] \times [(2) \text{ values of the width of the filter media } (w)]\}$. Each sub-class is run for (5) values for the depth of the filter media (f). Moreover, each of the aforementioned cases was run with (5) values of the distance of the filter media from the leading end of the base, (b_1) or (b_2), making a total of (600) runs.

The results of the (600) considered cases showed that the majority were reasonably acceptable, few were unsafe and few were too safe. Table (6.12) summarizes the information and the results concerning the considered safety factors.

Table (6.12): Summary of information and results concerning the considered factors of safety

NO.	Name and designation of the factor of safety		Limit	Obtained results	
				Min.	Max.
1	Factor of safety against overturning	$F_{ov.}$	≥ 1.5	0.45	2.68
2	Factor of safety against sliding	$F_{SL.}$	≥ 2.0	0.48	2.15
3	Factor of safety against piping	$F_P.$	≥ 4.0	3.84	9.51

The optimization model has been established to identify the sub-optimal and then the grand optimal case through evaluating the divergence of the calculated values of the factors of safety from the recommended ones. The established optimization model was run for the feasible cases of stability as identified by the stability model.

As mentioned in **Sec. (6.4)**, the results of running the optimization model are contained in **(24)** tables [Tables **(6.10)**, **(6.11)**, and Tables **(C.1)** through **(C.22)** in Appendix **(C)**]; each table concerns a sub-class of the considered cases. The sub-optimal value of the calculated total difference **(T.D.)** for each sub-class is indicated on the table as shaded.

The optimization results indicate the following:

A. For the case of a filter media with an upstream cutoff (d_1) only:

The minimum total difference is **(1.090)** as shown in Table **(C.3)** in Appendix **(C)**. This value of minimum total difference corresponds to values of the factors of safety against overturning, sliding, and piping of **(2.48)**, **(2.00)**, and **(4.36)**, respectively; this concerns the case when [**$d_1 = 7.5\text{m}$; $d_2 = 0$; $f = 0.4 d_1$; $w = 0.25 d_1$; $b_1 = 2 d_1$; $y_5 = 10\text{m}$**].

The aforementioned value of the minimum total difference corresponds to reasonably safe values of overturning, sliding and piping safety factors.

B. The grand optimum total difference is arrived at when there is a filter media with a cutoff at the upstream side of the considered dam, which is given in Item (A).

CHAPTER SEVEN

CHAPTER SEVEN

CONCLUSIONS AND RECOMMENDATIONS

7.1 CONCLUSIONS

Denoting (d_1) as the depth of the cutoff at the upstream end of the considered concrete gravity dam, (d_2) as of that at the downstream, and (f) , (w) , and (b_1) or (b_2) as the depth and width of the filter media and the location of its upstream or downstream side from the upstream side of the base, respectively, and $(F_{ov.})$, $(F_{sl.})$, $(F_p.)$, and (X_4) to denote the factors of safety against overturning, sliding, and piping, and the eccentricity of the vertical resultant force.

Based on seepage and stability analyses, and through an optimization process, the following main conclusions have been drawn.

1. With a cutoff at the upstream end of the base of the considered dam, the best case which gave satisfying safety factors against overturning, sliding and piping is when: $[d_1 = 7.5\text{m} ; d_2 = 0 ; f = 0.4 d_1 ; w = 0.25 d_1 ; b_1 = 2 d_1]$, which corresponds to safety factors of (2.48), (2.00), and (4.36) against overturning, sliding, and piping, respectively.
2. With a cutoff at the downstream end of the base of the considered dam, the case which gave maximum safety factors against overturning and sliding, is when: $[d_1 = 0 ; d_2 = 3\text{m} ; f = 1.0 d_2 ; w = 0.50 d_2 ; b_2 = 2 d_2]$, which corresponds to unsatisfying safety factors of (1.23) and (1.07) against overturning and sliding, but a satisfying piping safety factor of (5.77).
3. With cutoffs at the upstream and downstream ends of the base of the considered dam, the case which gave maximum safety factors against

overturning and sliding is when: [$d_1 = 10 \text{ m}$; $d_2 = 10 \text{ m}$; $f = 1.0 d_1$; $w = 0.50 d_1$; $b_1 = 2 d_1$], which corresponds to safety factors of (2.02), (1.65), and (7.14) against overturning, sliding, and piping, respectively. However, the sliding safety factor is unsafe.

4. The eccentricity of the vertical resultant force (to defend the dam against the failure by tension) is within the safe range for all the aforementioned cases.

5. The overall best safe case is:

[$d_1 = 7.5 \text{ m}$; $d_2 = 0$; $f = 3 \text{ m}$; $w = (7.5 / 4) \text{ m}$; $b_1 = 15 \text{ m}$], which gave: [$F_{OV.} = 2.48$; $F_{SL.} = 2.00$; and $F_P. = 4.36$] against overturning, sliding, and piping, respectively, which is given in **Item (1)**.

A candidate conclusions through the optimization analysis are achieved as follows:

6. With a cutoff at the upstream end of the base of the considered dam, the best case is when: [$d_1 = 7.5 \text{ m}$; $d_2 = 0$; $f = 0.4 d_1$; $w = 0.25 d_1$; $b_1 = 2 d_1$], which corresponds to satisfying safety factors against overturning, sliding, and piping of (2.48), (2.00), and (4.36), respectively.

7. The overall best case is:

[$d_1 = 7.5 \text{ m}$; $d_2 = 0$; $f = 3 \text{ m}$; $w = (7.5/4) \text{ m}$; $b_1 = 15 \text{ m}$], which is given in **Item (1)**.

7.2 RECOMMENDATIONS

Further development dealing with the subject can be imposed on the present work. In this respect, the following points are suggested:

1. The analysis can be modified further if seepage problem under transient flow is added.
2. Anisotropic and non-homogeneous soil conditions can be added in the formulation by specifying different permeabilities in the **X** and **Y** directions.
3. Extending the present study to analyze the effect of earthquake on the stability of the gravity dam.
4. A reasonable dynamic analysis is essential. By such an analysis the actual dynamic response of a gravity dam due to realistic dynamic excitements is to be investigated. The pore water pressure generation due to such excitements area is a very important factor that governs the stability of the whole dam.

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APPENDICES

APPENDIX

(A)

Table (A.1)
Calculated values of the safety factors
 $(d_1 = 5\text{m} \text{ , } w = 0.25 d_1)$

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_1$	$0.2 d_1$	2.29	1.86	4.08
2		$0.4 d_1$	2.32	1.88	4.21
3		$0.6 d_1$	2.38	1.92	4.36
4		$0.8 d_1$	2.39	1.93	4.42
5		$1.0 d_1$	2.40	1.94	4.45
6	$3 d_1$	$0.2 d_1$	2.25	1.83	4.24
7		$0.4 d_1$	2.28	1.85	4.35
8		$0.6 d_1$	2.34	1.89	4.52
9		$0.8 d_1$	2.35	1.90	4.60
10		$1.0 d_1$	2.36	1.91	4.64
11	$4 d_1$	$0.2 d_1$	2.21	1.80	4.39
12		$0.4 d_1$	2.24	1.82	4.54
13		$0.6 d_1$	2.30	1.86	4.74
14		$0.8 d_1$	2.31	1.87	4.81
15		$1.0 d_1$	2.32	1.88	4.84
16	$5 d_1$	$0.2 d_1$	2.17	1.77	4.55
17		$0.4 d_1$	2.20	1.79	4.71
18		$0.6 d_1$	2.26	1.83	4.93
19		$0.8 d_1$	2.28	1.84	5.02
20		$1.0 d_1$	2.29	1.85	5.06
21	$6 d_1$	$0.2 d_1$	2.13	1.74	4.71
22		$0.4 d_1$	2.16	1.76	4.90
23		$0.6 d_1$	2.22	1.80	5.12
24		$0.8 d_1$	2.23	1.81	5.20
25		$1.0 d_1$	2.24	1.82	5.24

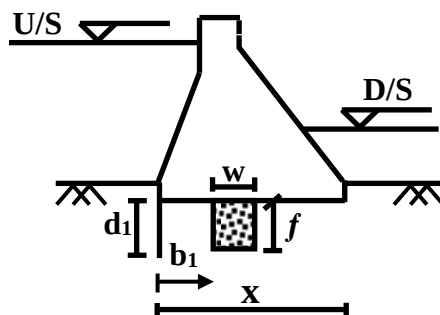


Table (A.2)
Calculated values of the safety factors
 $(d_1 = 5\text{m} \text{ , } w = 0.50 d_1)$

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_1$	$0.2 d_1$	2.33	1.89	4.27
2		$0.4 d_1$	2.36	1.91	4.38
3		$0.6 d_1$	2.42	1.95	4.53
4		$0.8 d_1$	2.43	1.96	4.56
5		$1.0 d_1$	2.44	1.97	4.60
6	$3 d_1$	$0.2 d_1$	2.29	1.86	4.38
7		$0.4 d_1$	2.32	1.88	4.52
8		$0.6 d_1$	2.38	1.92	4.68
9		$0.8 d_1$	2.39	1.93	4.75
10		$1.0 d_1$	2.40	1.94	4.78
11	$4 d_1$	$0.2 d_1$	2.25	1.83	4.53
12		$0.4 d_1$	2.28	1.85	4.70
13		$0.6 d_1$	2.34	1.89	4.91
14		$0.8 d_1$	2.35	1.90	5.00
15		$1.0 d_1$	2.36	1.91	5.03
16	$5 d_1$	$0.2 d_1$	2.21	1.80	4.75
17		$0.4 d_1$	2.24	1.82	4.91
18		$0.6 d_1$	2.30	1.86	5.11
19		$0.8 d_1$	2.31	1.87	5.19
20		$1.0 d_1$	2.32	1.88	5.22
21	$6 d_1$	$0.2 d_1$	2.17	1.77	4.93
22		$0.4 d_1$	2.20	1.79	5.10
23		$0.6 d_1$	2.26	1.83	5.36
24		$0.8 d_1$	2.27	1.84	5.46
25		$1.0 d_1$	2.28	1.85	5.49

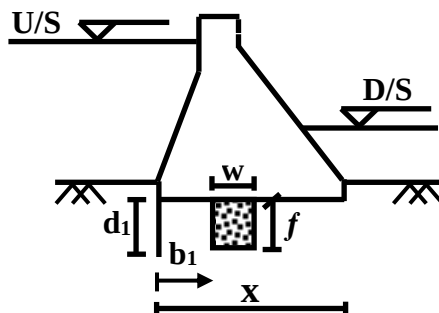


Table (A.3)
Calculated values of the safety factors
 $(d_1 = 7.5\text{m} \text{ , } w = 0.25 d_1)$

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_1$	$0.2 d_1$	2.45	1.98	4.22
2		$0.4 d_1$	2.48	2.00	4.36
3		$0.6 d_1$	2.54	2.04	4.53
4		$0.8 d_1$	2.55	2.05	4.62
5		$1.0 d_1$	2.56	2.06	4.65
6	$3 d_1$	$0.2 d_1$	2.41	1.95	4.36
7		$0.4 d_1$	2.44	1.97	4.48
8		$0.6 d_1$	2.50	2.01	4.65
9		$0.8 d_1$	2.51	2.02	4.71
10		$1.0 d_1$	2.52	2.03	4.74
11	$4 d_1$	$0.2 d_1$	2.37	1.92	4.52
12		$0.4 d_1$	2.40	1.94	4.70
13		$0.6 d_1$	2.46	1.98	4.94
14		$0.8 d_1$	2.47	1.99	5.01
15		$1.0 d_1$	2.48	2.00	5.05
16	$5 d_1$	$0.2 d_1$	2.33	1.89	4.72
17		$0.4 d_1$	2.36	1.91	4.88
18		$0.6 d_1$	2.42	1.95	5.11
19		$0.8 d_1$	2.43	1.96	5.21
20		$1.0 d_1$	2.44	1.97	5.24
21	$6 d_1$	$0.2 d_1$	2.29	1.86	4.92
22		$0.4 d_1$	2.32	1.88	5.11
23		$0.6 d_1$	2.38	1.92	5.35
24		$0.8 d_1$	2.39	1.93	5.45
25		$1.0 d_1$	2.40	1.94	5.49

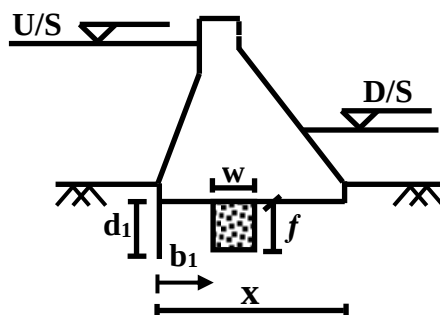


Table (A.4)
Calculated values of the safety factors
 $(d_1 = 7.5\text{m} \text{ , } w = 0.50 d_1)$

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_1$	$0.2 d_1$	2.49	2.01	4.37
2		$0.4 d_1$	2.52	2.03	4.55
3		$0.6 d_1$	2.58	2.07	4.75
4		$0.8 d_1$	2.59	2.08	4.81
5		$1.0 d_1$	2.60	2.09	4.84
6	$3 d_1$	$0.2 d_1$	2.45	1.98	4.55
7		$0.4 d_1$	2.48	2.00	4.75
8		$0.6 d_1$	2.54	2.04	4.99
9		$0.8 d_1$	2.55	2.05	5.05
10		$1.0 d_1$	2.56	2.06	5.08
11	$4 d_1$	$0.2 d_1$	2.41	1.95	4.73
12		$0.4 d_1$	2.44	1.97	4.87
13		$0.6 d_1$	2.50	2.01	5.09
14		$0.8 d_1$	2.51	2.02	5.19
15		$1.0 d_1$	2.52	2.03	5.22
16	$5 d_1$	$0.2 d_1$	2.37	1.92	4.91
17		$0.4 d_1$	2.40	1.94	5.12
18		$0.6 d_1$	2.46	1.98	5.43
19		$0.8 d_1$	2.47	1.99	5.47
20		$1.0 d_1$	2.48	2.00	5.49
21	$6 d_1$	$0.2 d_1$	2.33	1.89	5.10
22		$0.4 d_1$	2.36	1.91	5.34
23		$0.6 d_1$	2.42	1.95	5.61
24		$0.8 d_1$	2.43	1.96	5.74
25		$1.0 d_1$	2.44	1.97	5.77

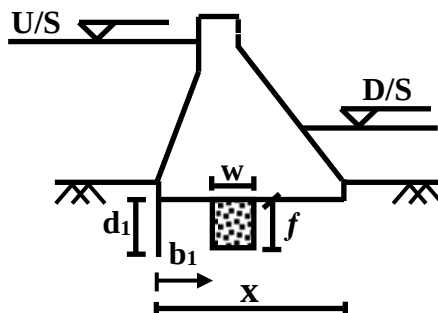


Table (A.5)
Calculated values of the safety factors
($d_1 = 10\text{m}$, $w = 0.25 d_1$)

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_1$	$0.2 d_1$	2.53	2.04	4.36
2		$0.4 d_1$	2.56	2.06	4.50
3		$0.6 d_1$	2.62	2.10	4.66
4		$0.8 d_1$	2.63	2.11	4.74
5		$1.0 d_1$	2.64	2.12	4.76
6	$3 d_1$	$0.2 d_1$	2.49	2.01	4.52
7		$0.4 d_1$	2.52	2.03	4.69
8		$0.6 d_1$	2.58	2.07	4.90
9		$0.8 d_1$	2.59	2.08	4.99
10		$1.0 d_1$	2.60	2.09	5.01
11	$4 d_1$	$0.2 d_1$	2.45	1.98	4.70
12		$0.4 d_1$	2.48	2.00	4.87
13		$0.6 d_1$	2.54	2.04	5.07
14		$0.8 d_1$	2.55	2.05	5.12
15		$1.0 d_1$	2.56	2.06	5.15
16	$5 d_1$	$0.2 d_1$	2.41	1.95	4.93
17		$0.4 d_1$	2.44	1.97	5.12
18		$0.6 d_1$	2.50	2.01	5.39
19		$0.8 d_1$	2.51	2.02	5.52
20		$1.0 d_1$	2.52	2.03	5.55
21	$6 d_1$	$0.2 d_1$	2.37	1.92	5.16
22		$0.4 d_1$	2.40	1.94	5.41
23		$0.6 d_1$	2.46	1.98	5.73
24		$0.8 d_1$	2.47	1.99	5.86
25		$1.0 d_1$	2.48	2.00	5.89

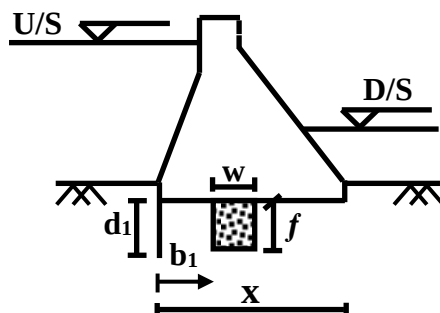


Table (A.6)
Calculated values of the safety factors
($d_1 = 10\text{m}$, $w = 0.50 d_1$)

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_1$	$0.2 d_1$	2.57	2.07	4.52
2		$0.4 d_1$	2.60	2.09	4.68
3		$0.6 d_1$	2.66	2.13	4.90
4		$0.8 d_1$	2.67	2.14	4.98
5		$1.0 d_1$	2.68	2.15	5.01
6	$3 d_1$	$0.2 d_1$	2.53	2.04	4.73
7		$0.4 d_1$	2.56	2.06	4.87
8		$0.6 d_1$	2.62	2.10	5.10
9		$0.8 d_1$	2.63	2.11	5.13
10		$1.0 d_1$	2.64	2.12	5.15
11	$4 d_1$	$0.2 d_1$	2.49	2.01	4.95
12		$0.4 d_1$	2.52	2.03	5.13
13		$0.6 d_1$	2.58	2.07	5.40
14		$0.8 d_1$	2.59	2.08	5.51
15		$1.0 d_1$	2.60	2.09	5.54
16	$5 d_1$	$0.2 d_1$	2.45	1.98	5.10
17		$0.4 d_1$	2.48	2.00	5.32
18		$0.6 d_1$	2.54	2.04	5.58
19		$0.8 d_1$	2.55	2.05	5.70
20		$1.0 d_1$	2.56	2.06	5.72
21	$6 d_1$	$0.2 d_1$	2.41	1.95	5.39
22		$0.4 d_1$	2.44	1.97	5.70
23		$0.6 d_1$	2.50	2.01	6.07
24		$0.8 d_1$	2.51	2.02	6.21
25		$1.0 d_1$	2.52	2.03	6.24

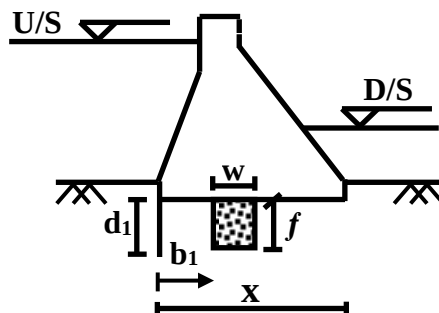


Table (A.7)
Calculated values of the safety factors
 $(d_2 = 3m \text{ , } w = 0.25 d_2)$

NO.	b_2	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_2$	$0.2 d_2$	1.08	0.96	4.98
2		$0.4 d_2$	1.11	0.98	5.14
3		$0.6 d_2$	1.17	1.02	5.32
4		$0.8 d_2$	1.18	1.03	5.46
5		$1.0 d_2$	1.19	1.04	5.50
6	$3 d_2$	$0.2 d_2$	1.04	0.93	4.81
7		$0.4 d_2$	1.07	0.95	4.94
8		$0.6 d_2$	1.13	0.99	5.12
9		$0.8 d_2$	1.14	1.00	5.22
10		$1.0 d_2$	1.15	1.01	5.25
11	$4 d_1$	$0.2 d_2$	1.00	0.90	4.66
12		$0.4 d_2$	1.03	0.92	4.80
13		$0.6 d_2$	1.09	0.96	5.01
14		$0.8 d_2$	1.10	0.97	5.13
15		$1.0 d_2$	1.11	0.98	5.17
16	$5 d_2$	$0.2 d_2$	0.96	0.87	4.56
17		$0.4 d_2$	0.99	0.89	4.67
18		$0.6 d_2$	1.05	0.93	4.86
19		$0.8 d_2$	1.06	0.94	4.95
20		$1.0 d_2$	1.07	0.95	4.98
21	$6 d_2$	$0.2 d_2$	0.92	0.84	4.41
22		$0.4 d_2$	0.95	0.86	4.53
23		$0.6 d_2$	1.01	0.90	4.71
24		$0.8 d_2$	1.02	0.91	4.78
25		$1.0 d_2$	1.03	0.92	4.81

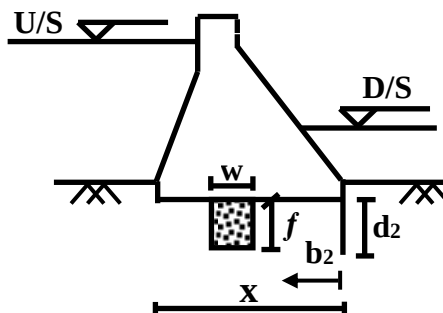


Table (A.8)
Calculated values of the safety factors
 $(d_2 = 3\text{m} \text{ , } w = 0.50 d_2)$

NO.	b_2	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_2$	$0.2 d_2$	1.12	0.99	5.12
2		$0.4 d_2$	1.15	1.01	5.34
3		$0.6 d_2$	1.21	1.05	5.60
4		$0.8 d_2$	1.22	1.06	5.73
5		$1.0 d_2$	1.23	1.07	5.77
6	$3 d_2$	$0.2 d_2$	1.08	0.96	4.99
7		$0.4 d_2$	1.11	0.98	5.15
8		$0.6 d_2$	1.17	1.02	5.34
9		$0.8 d_2$	1.18	1.03	5.46
10		$1.0 d_2$	1.19	1.04	5.50
11	$4 d_1$	$0.2 d_2$	1.04	0.93	4.80
12		$0.4 d_2$	1.07	0.95	4.94
13		$0.6 d_2$	1.13	0.99	5.13
14		$0.8 d_2$	1.14	1.00	5.25
15		$1.0 d_2$	1.15	1.01	5.31
16	$5 d_2$	$0.2 d_2$	1.00	0.90	4.67
17		$0.4 d_2$	1.03	0.92	4.82
18		$0.6 d_2$	1.09	0.96	5.01
19		$0.8 d_2$	1.10	0.97	5.12
20		$1.0 d_2$	1.11	0.98	5.16
21	$6 d_2$	$0.2 d_2$	0.96	0.87	4.56
22		$0.4 d_2$	0.99	0.89	4.71
23		$0.6 d_2$	1.05	0.93	4.88
24		$0.8 d_2$	1.06	0.94	4.95
25		$1.0 d_2$	1.07	0.95	4.98

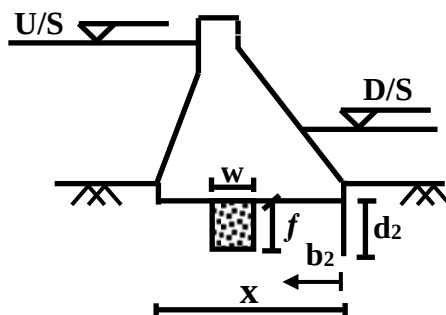


Table (A.9)
Calculated values of the safety factors
 $(d_2 = 5\text{m} \text{ , } w = 0.25 d_2)$

NO.	b_2	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_2$	$0.2 d_2$	0.81	0.75	5.36
2		$0.4 d_2$	0.84	0.77	5.57
3		$0.6 d_2$	0.90	0.81	5.84
4		$0.8 d_2$	0.91	0.82	5.99
5		$1.0 d_2$	0.92	0.83	6.02
6	$3 d_2$	$0.2 d_2$	0.77	0.72	5.11
7		$0.4 d_2$	0.80	0.74	5.32
8		$0.6 d_2$	0.86	0.78	5.58
9		$0.8 d_2$	0.87	0.79	5.68
10		$1.0 d_2$	0.88	0.80	5.71
11	$4 d_1$	$0.2 d_2$	0.73	0.69	4.87
12		$0.4 d_2$	0.76	0.71	5.04
13		$0.6 d_2$	0.82	0.75	5.27
14		$0.8 d_2$	0.83	0.76	5.37
15		$1.0 d_2$	0.84	0.77	5.40
16	$5 d_2$	$0.2 d_2$	0.69	0.66	4.66
17		$0.4 d_2$	0.72	0.68	4.83
18		$0.6 d_2$	0.78	0.72	5.07
19		$0.8 d_2$	0.79	0.73	5.20
20		$1.0 d_2$	0.80	0.74	5.25
21	$6 d_2$	$0.2 d_2$	0.65	0.63	4.45
22		$0.4 d_2$	0.68	0.65	4.60
23		$0.6 d_2$	0.74	0.69	4.80
24		$0.8 d_2$	0.75	0.70	4.89
25		$1.0 d_2$	0.76	0.71	4.92

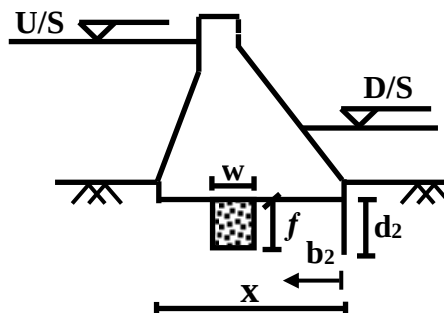


Table (A.10)
Calculated values of the safety factors
 $(d_2 = 5\text{m} \text{ , } w = 0.50 d_2)$

NO.	b_2	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	2 d_2	0.2 d_2	0.85	0.78	5.54
2		0.4 d_2	0.88	0.80	5.75
3		0.6 d_2	0.94	0.84	6.04
4		0.8 d_2	0.95	0.85	6.18
5		1.0 d_2	0.96	0.86	6.21
6	3 d_2	0.2 d_2	0.81	0.75	5.37
7		0.4 d_2	0.84	0.77	5.57
8		0.6 d_2	0.90	0.81	5.81
9		0.8 d_2	0.91	0.82	5.93
10		1.0 d_2	0.92	0.83	5.95
11	4 d_1	0.2 d_2	0.77	0.72	5.12
12		0.4 d_2	0.80	0.74	5.31
13		0.6 d_2	0.86	0.78	5.53
14		0.8 d_2	0.87	0.79	5.65
15		1.0 d_2	0.88	0.80	5.68
16	5 d_2	0.2 d_2	0.73	0.69	4.84
17		0.4 d_2	0.76	0.71	5.06
18		0.6 d_2	0.82	0.75	5.32
19		0.8 d_2	0.83	0.76	5.43
20		1.0 d_2	0.84	0.77	5.46
21	6 d_2	0.2 d_2	0.69	0.66	4.65
22		0.4 d_2	0.72	0.68	4.81
23		0.6 d_2	0.78	0.72	5.04
24		0.8 d_2	0.79	0.73	5.12
25		1.0 d_2	0.80	0.74	5.15

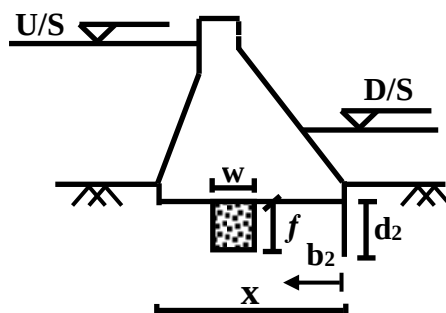


Table (A.11)
Calculated values of the safety factors
 $(d_2 = 7.5\text{m} \text{ , } w = 0.25 d_2)$

NO.	b_2	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_2$	$0.2 d_2$	0.69	0.66	5.65
2		$0.4 d_2$	0.72	0.68	5.89
3		$0.6 d_2$	0.78	0.72	6.17
4		$0.8 d_2$	0.79	0.73	6.29
5		$1.0 d_2$	0.80	0.74	6.32
6	$3 d_2$	$0.2 d_2$	0.65	0.63	5.37
7		$0.4 d_2$	0.68	0.65	5.60
8		$0.6 d_2$	0.74	0.69	5.86
9		$0.8 d_2$	0.75	0.70	5.99
10		$1.0 d_2$	0.76	0.71	6.02
11	$4 d_1$	$0.2 d_2$	0.61	0.60	5.19
12		$0.4 d_2$	0.64	0.62	5.43
13		$0.6 d_2$	0.70	0.66	5.76
14		$0.8 d_2$	0.71	0.67	5.90
15		$1.0 d_2$	0.72	0.68	5.93
16	$5 d_2$	$0.2 d_2$	0.57	0.57	4.99
17		$0.4 d_2$	0.60	0.59	5.17
18		$0.6 d_2$	0.66	0.63	5.41
19		$0.8 d_2$	0.67	0.64	5.46
20		$1.0 d_2$	0.68	0.65	5.48
21	$6 d_2$	$0.2 d_2$	0.53	0.54	4.75
22		$0.4 d_2$	0.56	0.56	4.91
23		$0.6 d_2$	0.62	0.60	5.11
24		$0.8 d_2$	0.63	0.61	5.23
25		$1.0 d_2$	0.64	0.62	5.26

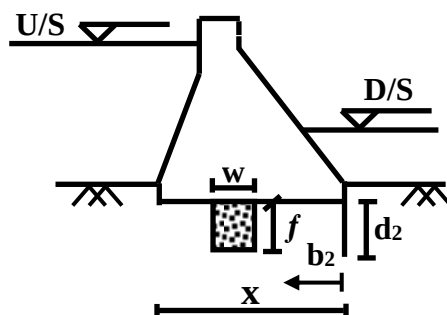


Table (A.12)
Calculated values of the safety factors
 $(d_2 = 7.5\text{m} \text{ , } w = 0.50 d_2)$

NO.	b_2	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	2 d_2	0.2 d_2	0.73	0.69	5.84
2		0.4 d_2	0.76	0.71	6.13
3		0.6 d_2	0.82	0.75	6.47
4		0.8 d_2	0.83	0.76	6.61
5		1.0 d_2	0.84	0.77	6.64
6	3 d_2	0.2 d_2	0.69	0.66	5.60
7		0.4 d_2	0.72	0.68	5.82
8		0.6 d_2	0.78	0.72	6.11
9		0.8 d_2	0.79	0.73	6.26
10		1.0 d_2	0.80	0.74	6.29
11	4 d_1	0.2 d_2	0.65	0.63	5.36
12		0.4 d_2	0.68	0.65	5.57
13		0.6 d_2	0.74	0.69	5.83
14		0.8 d_2	0.75	0.70	5.96
15		1.0 d_2	0.76	0.71	5.99
16	5 d_2	0.2 d_2	0.61	0.60	5.20
17		0.4 d_2	0.64	0.62	5.41
18		0.6 d_2	0.70	0.66	5.68
19		0.8 d_2	0.71	0.67	5.80
20		1.0 d_2	0.72	0.68	5.83
21	6 d_2	0.2 d_2	0.57	0.57	4.98
22		0.4 d_2	0.60	0.59	5.19
23		0.6 d_2	0.66	0.63	5.46
24		0.8 d_2	0.67	0.64	5.58
25		1.0 d_2	0.68	0.65	5.60

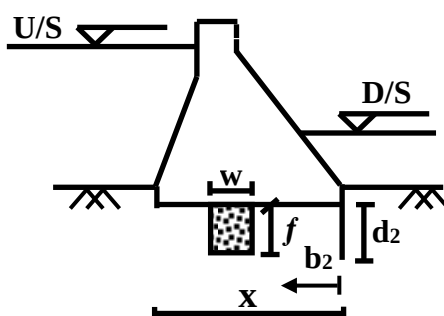


Table (A.13)
Calculated values of the safety factors
($d_2 = 10\text{m}$, $w = 0.25 d_2$)

NO.	b_2	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_2$	$0.2 d_2$	0.61	0.60	6.03
2		$0.4 d_2$	0.64	0.62	6.32
3		$0.6 d_2$	0.70	0.66	6.66
4		$0.8 d_2$	0.71	0.67	6.83
5		$1.0 d_2$	0.72	0.68	6.85
6	$3 d_2$	$0.2 d_2$	0.57	0.57	5.75
7		$0.4 d_2$	0.60	0.59	6.00
8		$0.6 d_2$	0.66	0.63	6.30
9		$0.8 d_2$	0.67	0.64	6.45
10		$1.0 d_2$	0.68	0.65	6.48
11	$4 d_1$	$0.2 d_2$	0.53	0.54	5.46
12		$0.4 d_2$	0.56	0.56	5.70
13		$0.6 d_2$	0.62	0.60	6.01
14		$0.8 d_2$	0.63	0.61	6.12
15		$1.0 d_2$	0.64	0.62	6.15
16	$5 d_2$	$0.2 d_2$	0.49	0.51	5.23
17		$0.4 d_2$	0.52	0.53	5.45
18		$0.6 d_2$	0.58	0.57	5.70
19		$0.8 d_2$	0.59	0.58	5.83
20		$1.0 d_2$	0.60	0.59	5.85
21	$6 d_2$	$0.2 d_2$	0.45	0.48	4.95
22		$0.4 d_2$	0.48	0.50	5.15
23		$0.6 d_2$	0.54	0.54	5.41
24		$0.8 d_2$	0.56	0.55	5.51
25		$1.0 d_2$	0.57	0.56	5.54

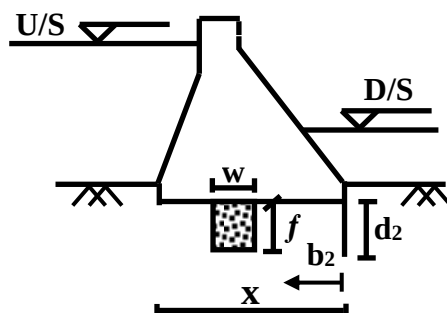


Table (A.14)
Calculated values of the safety factors
($d_2 = 10\text{m}$, $w = 0.50 d_2$)

NO.	b_2	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_2$	$0.2 d_2$	0.65	0.63	6.36
2		$0.4 d_2$	0.68	0.65	6.66
3		$0.6 d_2$	0.74	0.69	7.00
4		$0.8 d_2$	0.75	0.70	7.14
5		$1.0 d_2$	0.76	0.71	7.17
6	$3 d_2$	$0.2 d_2$	0.61	0.60	6.01
7		$0.4 d_2$	0.64	0.62	6.31
8		$0.6 d_2$	0.70	0.66	6.69
9		$0.8 d_2$	0.71	0.67	6.86
10		$1.0 d_2$	0.72	0.68	6.88
11	$4 d_1$	$0.2 d_2$	0.57	0.57	5.67
12		$0.4 d_2$	0.60	0.59	5.90
13		$0.6 d_2$	0.66	0.63	6.17
14		$0.8 d_2$	0.67	0.64	6.30
15		$1.0 d_2$	0.68	0.65	6.33
16	$5 d_2$	$0.2 d_2$	0.53	0.54	5.45
17		$0.4 d_2$	0.56	0.56	5.70
18		$0.6 d_2$	0.62	0.60	5.99
19		$0.8 d_2$	0.63	0.61	6.12
20		$1.0 d_2$	0.64	0.62	6.14
21	$6 d_2$	$0.2 d_2$	0.49	0.51	5.23
22		$0.4 d_2$	0.52	0.53	5.46
23		$0.6 d_2$	0.58	0.57	5.73
24		$0.8 d_2$	0.60	0.58	5.83
25		$1.0 d_2$	0.61	0.59	5.85

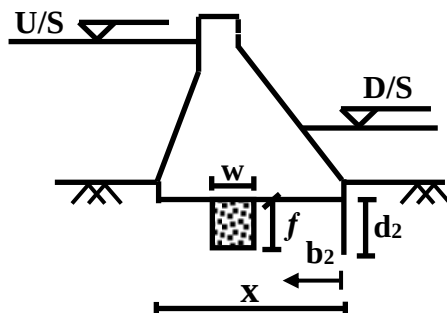


Table (A.15)
Calculated values of the safety factors
 $(d_1 = d_2 = 3\text{m} \text{ , } w = 0.25 d_1)$

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_1$	$0.2 d_1$	1.57	1.32	4.92
2		$0.4 d_1$	1.63	1.37	5.08
3		$0.6 d_1$	1.73	1.45	5.29
4		$0.8 d_1$	1.77	1.48	5.41
5		$1.0 d_1$	1.80	1.50	5.45
6	$3 d_1$	$0.2 d_1$	1.52	1.28	5.09
7		$0.4 d_1$	1.58	1.33	5.29
8		$0.6 d_1$	1.68	1.39	5.53
9		$0.8 d_1$	1.72	1.42	5.66
10		$1.0 d_1$	1.75	1.44	5.69
11	$4 d_1$	$0.2 d_1$	1.47	1.24	5.30
12		$0.4 d_1$	1.53	1.29	5.52
13		$0.6 d_1$	1.64	1.37	5.78
14		$0.8 d_1$	1.68	1.40	5.92
15		$1.0 d_1$	1.71	1.42	5.96
16	$5 d_1$	$0.2 d_1$	1.42	1.20	5.52
17		$0.4 d_1$	1.48	1.25	5.75
18		$0.6 d_1$	1.58	1.33	6.03
19		$0.8 d_1$	1.62	1.36	6.18
20		$1.0 d_1$	1.65	1.38	6.21
21	$6 d_1$	$0.2 d_1$	1.37	1.16	5.67
22		$0.4 d_1$	1.43	1.21	5.92
23		$0.6 d_1$	1.54	1.29	6.21
24		$0.8 d_1$	1.58	1.32	6.36
25		$1.0 d_1$	1.61	1.34	6.40

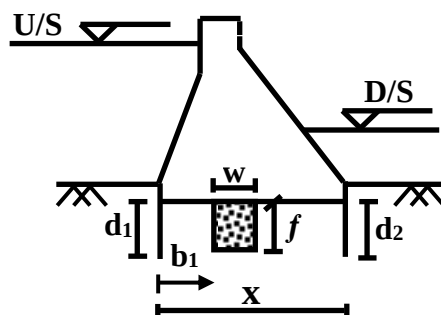


Table (A.16)
Calculated values of the safety factors
 $(d_1 = d_2 = 3\text{m} \text{ , } w = 0.50 d_1)$

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_1$	$0.2 d_1$	1.64	1.37	5.12
2		$0.4 d_1$	1.70	1.42	5.31
3		$0.6 d_1$	1.80	1.50	5.53
4		$0.8 d_1$	1.84	1.53	5.67
5		$1.0 d_1$	1.87	1.55	5.70
6	$3 d_1$	$0.2 d_1$	1.59	1.33	5.38
7		$0.4 d_1$	1.65	1.38	5.57
8		$0.6 d_1$	1.76	1.46	5.80
9		$0.8 d_1$	1.80	1.49	5.94
10		$1.0 d_1$	1.83	1.51	5.98
11	$4 d_1$	$0.2 d_1$	1.54	1.29	5.55
12		$0.4 d_1$	1.60	1.34	5.78
13		$0.6 d_1$	1.71	1.42	6.08
14		$0.8 d_1$	1.75	1.45	6.23
15		$1.0 d_1$	1.78	1.47	6.27
16	$5 d_1$	$0.2 d_1$	1.49	1.25	5.82
17		$0.4 d_1$	1.55	1.30	6.03
18		$0.6 d_1$	1.65	1.38	6.35
19		$0.8 d_1$	1.69	1.41	6.51
20		$1.0 d_1$	1.72	1.43	6.55
21	$6 d_1$	$0.2 d_1$	1.44	1.21	6.07
22		$0.4 d_1$	1.50	1.26	6.35
23		$0.6 d_1$	1.60	1.34	6.68
24		$0.8 d_1$	1.64	1.37	6.83
25		$1.0 d_1$	1.67	1.39	6.86

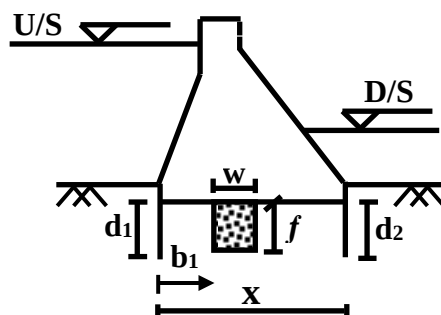


Table (A.17)
Calculated values of the safety factors
 $(d_1 = d_2 = 5\text{m} \text{ , } w = 0.25 d_1)$

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	2 d_1	0.2 d_1	1.65	1.38	5.30
2		0.4 d_1	1.71	1.43	5.53
3		0.6 d_1	1.81	1.51	5.83
4		0.8 d_1	1.85	1.54	5.94
5		1.0 d_1	1.88	1.56	5.97
6	3 d_1	0.2 d_1	1.60	1.34	5.52
7		0.4 d_1	1.66	1.39	5.78
8		0.6 d_1	1.77	1.47	6.09
9		0.8 d_1	1.81	1.50	6.19
10		1.0 d_1	1.84	1.52	6.22
11	4 d_1	0.2 d_1	1.55	1.30	5.79
12		0.4 d_1	1.61	1.35	6.05
13		0.6 d_1	1.71	1.43	6.39
14		0.8 d_1	1.75	1.46	6.54
15		1.0 d_1	1.78	1.48	6.57
16	5 d_1	0.2 d_1	1.50	1.26	6.09
17		0.4 d_1	1.56	1.31	6.35
18		0.6 d_1	1.67	1.39	6.71
19		0.8 d_1	1.71	1.42	6.90
20		1.0 d_1	1.74	1.44	6.93
21	6 d_1	0.2 d_1	1.45	1.22	6.36
22		0.4 d_1	1.51	1.27	6.70
23		0.6 d_1	1.61	1.35	7.18
24		0.8 d_1	1.65	1.38	7.41
25		1.0 d_1	1.68	1.40	7.44

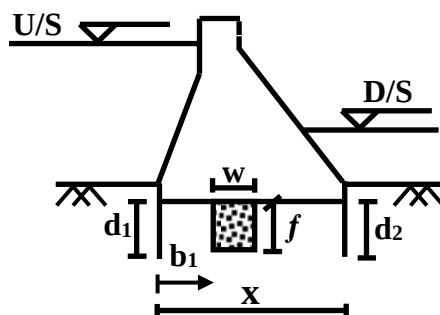


Table (A.18)
Calculated values of the safety factors
 $(d_1 = d_2 = 5\text{m} \text{ , } w = 0.50 d_1)$

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	2 d_1	0.2 d_1	1.72	1.43	5.52
2		0.4 d_1	1.78	1.48	5.75
3		0.6 d_1	1.88	1.56	6.07
4		0.8 d_1	1.92	1.59	6.19
5		1.0 d_1	1.95	1.61	6.23
6	3 d_1	0.2 d_1	1.67	1.39	5.81
7		0.4 d_1	1.73	1.44	6.11
8		0.6 d_1	1.84	1.52	6.45
9		0.8 d_1	1.88	1.55	6.57
10		1.0 d_1	1.91	1.57	6.61
11	4 d_1	0.2 d_1	1.62	1.35	6.17
12		0.4 d_1	1.68	1.40	6.44
13		0.6 d_1	1.78	1.48	6.79
14		0.8 d_1	1.82	1.51	6.91
15		1.0 d_1	1.85	1.53	6.94
16	5 d_1	0.2 d_1	1.57	1.31	6.48
17		0.4 d_1	1.63	1.36	6.79
18		0.6 d_1	1.73	1.44	7.17
19		0.8 d_1	1.77	1.47	7.41
20		1.0 d_1	1.80	1.49	7.44
21	6 d_1	0.2 d_1	1.52	1.27	6.76
22		0.4 d_1	1.58	1.32	7.08
23		0.6 d_1	1.68	1.40	7.57
24		0.8 d_1	1.72	1.43	7.84
25		1.0 d_1	1.75	1.45	7.87

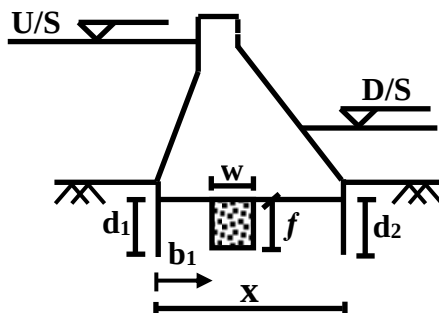


Table (A.19)
Calculated values of the safety factors
 $(d_1 = d_2 = 7.5\text{m} \text{ , } w = 0.25 d_1)$

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	2 d_1	0.2 d_1	1.69	1.41	5.55
2		0.4 d_1	1.75	1.46	5.83
3		0.6 d_1	1.85	1.54	6.17
4		0.8 d_1	1.89	1.57	6.29
5		1.0 d_1	1.92	1.59	6.32
6	3 d_1	0.2 d_1	1.64	1.37	5.82
7		0.4 d_1	1.70	1.42	6.13
8		0.6 d_1	1.80	1.50	6.48
9		0.8 d_1	1.84	1.53	6.63
10		1.0 d_1	1.87	1.55	6.67
11	4 d_1	0.2 d_1	1.59	1.33	6.17
12		0.4 d_1	1.65	1.38	6.56
13		0.6 d_1	1.76	1.46	7.00
14		0.8 d_1	1.80	1.49	7.14
15		1.0 d_1	1.83	1.51	7.17
16	5 d_1	0.2 d_1	1.54	1.29	6.50
17		0.4 d_1	1.60	1.34	6.88
18		0.6 d_1	1.70	1.42	7.38
19		0.8 d_1	1.74	1.45	7.57
20		1.0 d_1	1.77	1.47	7.60
21	6 d_1	0.2 d_1	1.50	1.25	6.91
22		0.4 d_1	1.55	1.30	7.38
23		0.6 d_1	1.65	1.38	7.91
24		0.8 d_1	1.69	1.41	8.14
25		1.0 d_1	1.72	1.43	8.17

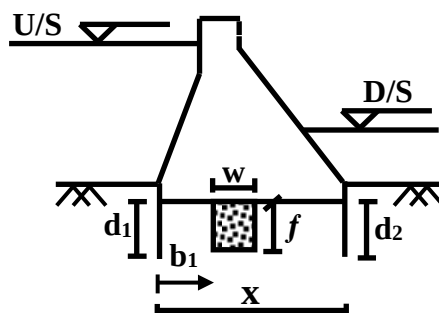


Table (A.20)
Calculated values of the safety factors
 $(d_1 = d_2 = 7.5\text{m} \text{ , } w = 0.50 d_1)$

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_1$	$0.2 d_1$	1.76	1.46	5.88
2		$0.4 d_1$	1.82	1.51	6.24
3		$0.6 d_1$	1.92	1.59	6.67
4		$0.8 d_1$	1.96	1.62	6.80
5		$1.0 d_1$	1.99	1.64	6.83
6	$3 d_1$	$0.2 d_1$	1.71	1.42	6.23
7		$0.4 d_1$	1.77	1.47	6.60
8		$0.6 d_1$	1.87	1.55	7.06
9		$0.8 d_1$	1.91	1.58	7.22
10		$1.0 d_1$	1.94	1.60	7.25
11	$4 d_1$	$0.2 d_1$	1.66	1.38	6.61
12		$0.4 d_1$	1.72	1.43	6.97
13		$0.6 d_1$	1.82	1.51	7.39
14		$0.8 d_1$	1.86	1.54	7.54
15		$1.0 d_1$	1.89	1.56	7.58
16	$5 d_1$	$0.2 d_1$	1.61	1.34	6.95
17		$0.4 d_1$	1.67	1.39	7.36
18		$0.6 d_1$	1.78	1.47	7.87
19		$0.8 d_1$	1.82	1.50	8.13
20		$1.0 d_1$	1.85	1.52	8.16
21	$6 d_1$	$0.2 d_1$	1.56	1.30	7.32
22		$0.4 d_1$	1.62	1.35	7.82
23		$0.6 d_1$	1.72	1.43	8.42
24		$0.8 d_1$	1.76	1.46	8.64
25		$1.0 d_1$	1.79	1.48	8.67

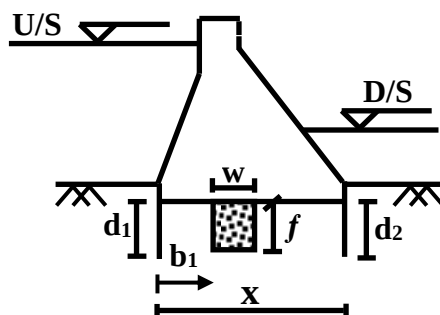


Table (A.21)
Calculated values of the safety factors
 ($d_1 = d_2 = 10\text{m}$, $w = 0.25 d_1$)

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_1$	$0.2 d_1$	1.71	1.42	5.82
2		$0.4 d_1$	1.77	1.47	6.10
3		$0.6 d_1$	1.87	1.55	6.49
4		$0.8 d_1$	1.91	1.58	6.66
5		$1.0 d_1$	1.94	1.60	6.69
6	$3 d_1$	$0.2 d_1$	1.66	1.38	6.19
7		$0.4 d_1$	1.72	1.43	6.47
8		$0.6 d_1$	1.82	1.51	6.82
9		$0.8 d_1$	1.86	1.54	7.05
10		$1.0 d_1$	1.89	1.56	7.09
11	$4 d_1$	$0.2 d_1$	1.61	1.34	6.50
12		$0.4 d_1$	1.67	1.39	6.88
13		$0.6 d_1$	1.78	1.47	7.35
14		$0.8 d_1$	1.82	1.50	7.54
15		$1.0 d_1$	1.85	1.52	7.58
16	$5 d_1$	$0.2 d_1$	1.56	1.30	6.91
17		$0.4 d_1$	1.62	1.35	7.38
18		$0.6 d_1$	1.73	1.43	7.89
19		$0.8 d_1$	1.77	1.46	8.09
20		$1.0 d_1$	1.80	1.48	8.14
21	$6 d_1$	$0.2 d_1$	1.51	1.26	7.36
22		$0.4 d_1$	1.57	1.31	7.85
23		$0.6 d_1$	1.68	1.39	8.45
24		$0.8 d_1$	1.72	1.42	8.70
25		$1.0 d_1$	1.75	1.44	8.74

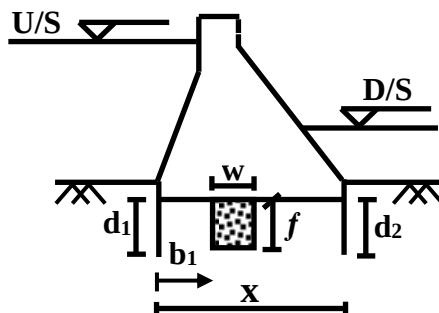
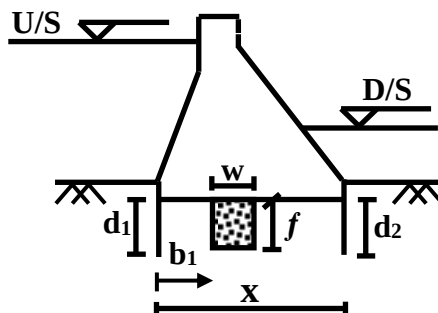


Table (A.22)
Calculated values of the safety factors
 ($d_1 = d_2 = 10\text{m}$, $w = 0.50 d_1$)

NO.	b_1	f	$F_{OV.}$	$F_{SL.}$	$F_P.$
1	$2 d_1$	$0.2 d_1$	1.78	1.47	6.18
2		$0.4 d_1$	1.84	1.52	6.50
3		$0.6 d_1$	1.95	1.60	6.90
4		$0.8 d_1$	1.99	1.63	7.10
5		$1.0 d_1$	2.02	1.65	7.14
6	$3 d_1$	$0.2 d_1$	1.73	1.43	6.56
7		$0.4 d_1$	1.79	1.48	6.94
8		$0.6 d_1$	1.89	1.56	7.39
9		$0.8 d_1$	1.93	1.59	7.57
10		$1.0 d_1$	1.96	1.61	7.60
11	$4 d_1$	$0.2 d_1$	1.68	1.39	6.94
12		$0.4 d_1$	1.74	1.44	7.47
13		$0.6 d_1$	1.85	1.52	8.06
14		$0.8 d_1$	1.89	1.55	8.31
15		$1.0 d_1$	1.92	1.57	8.35
16	$5 d_1$	$0.2 d_1$	1.63	1.35	7.34
17		$0.4 d_1$	1.69	1.40	7.86
18		$0.6 d_1$	1.80	1.48	8.47
19		$0.8 d_1$	1.84	1.51	8.78
20		$1.0 d_1$	1.87	1.53	8.81
21	$6 d_1$	$0.2 d_1$	1.58	1.31	7.94
22		$0.4 d_1$	1.64	1.36	8.48
23		$0.6 d_1$	1.75	1.44	9.16
24		$0.8 d_1$	1.79	1.47	9.48
25		$1.0 d_1$	1.82	1.49	9.51



APPENDIX

(B)

Table (B.1)
Calculated values of the eccentricity (X_4) of the resultant force
($d_1 = 5\text{m}$)

A: $w = 0.25 d_1$				B: $w = 0.50 d_1$			
NO.	b_1	f	X_4	NO.	b_1	f	X_4
1	$2 d_1$	$0.2 d_1$	0.791	1	$2 d_1$	$0.2 d_1$	0.684
2		$0.4 d_1$	0.685	2		$0.4 d_1$	0.579
3		$0.6 d_1$	0.586	3		$0.6 d_1$	0.479
4		$0.8 d_1$	0.528	4		$0.8 d_1$	0.417
5		$1.0 d_1$	0.506	5		$1.0 d_1$	0.386
6	$3 d_1$	$0.2 d_1$	0.883	6	$3 d_1$	$0.2 d_1$	0.797
7		$0.4 d_1$	0.772	7		$0.4 d_1$	0.688
8		$0.6 d_1$	0.666	8		$0.6 d_1$	0.576
9		$0.8 d_1$	0.611	9		$0.8 d_1$	0.519
10		$1.0 d_1$	0.580	10		$1.0 d_1$	0.487
11	$4 d_1$	$0.2 d_1$	1.007	11	$4 d_1$	$0.2 d_1$	0.888
12		$0.4 d_1$	0.896	12		$0.4 d_1$	0.763
13		$0.6 d_1$	0.782	13		$0.6 d_1$	0.682
14		$0.8 d_1$	0.702	14		$0.8 d_1$	0.607
15		$1.0 d_1$	0.685	15		$1.0 d_1$	0.585
16	$5 d_1$	$0.2 d_1$	1.105	16	$5 d_1$	$0.2 d_1$	0.975
17		$0.4 d_1$	1.003	17		$0.4 d_1$	0.885
18		$0.6 d_1$	0.891	18		$0.6 d_1$	0.773
19		$0.8 d_1$	0.817	19		$0.8 d_1$	0.705
20		$1.0 d_1$	0.793	20		$1.0 d_1$	0.684
21	$6 d_1$	$0.2 d_1$	1.198	21	$6 d_1$	$0.2 d_1$	1.085
22		$0.4 d_1$	1.078	22		$0.4 d_1$	0.979
23		$0.6 d_1$	0.989	23		$0.6 d_1$	0.953
24		$0.8 d_1$	0.892	24		$0.8 d_1$	0.822
25		$1.0 d_1$	0.872	25		$1.0 d_1$	0.808

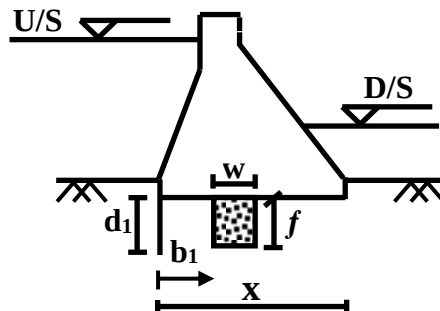


Table (B.2)
Calculated values of the eccentricity (X_4) of the resultant force
($d_1 = 7.5\text{m}$)

A: $w = 0.25 d_1$				B: $w = 0.50 d_1$			
NO.	b_1	f	X_4	NO.	b_1	f	X_4
1	$2 d_1$	$0.2 d_1$	0.200	1	$2 d_1$	$0.2 d_1$	0.104
2		$0.4 d_1$	0.105	2		$0.4 d_1$	0.009
3		$0.6 d_1$	0.005	3		$0.6 d_1$	0.106
4		$0.8 d_1$	0.055	4		$0.8 d_1$	0.155
5		$1.0 d_1$	0.080	5		$1.0 d_1$	0.187
6	$3 d_1$	$0.2 d_1$	0.287	6	$3 d_1$	$0.2 d_1$	0.189
7		$0.4 d_1$	0.180	7		$0.4 d_1$	0.090
8		$0.6 d_1$	0.084	8		$0.6 d_1$	0.009
9		$0.8 d_1$	0.023	9		$0.8 d_1$	0.066
10		$1.0 d_1$	0.011	10		$1.0 d_1$	0.102
11	$4 d_1$	$0.2 d_1$	0.366	11	$4 d_1$	$0.2 d_1$	0.265
12		$0.4 d_1$	0.264	12		$0.4 d_1$	0.180
13		$0.6 d_1$	0.159	13		$0.6 d_1$	0.088
14		$0.8 d_1$	0.081	14		$0.8 d_1$	0.024
15		$1.0 d_1$	0.066	15		$1.0 d_1$	0.014
16	$5 d_1$	$0.2 d_1$	0.457	16	$5 d_1$	$0.2 d_1$	0.383
17		$0.4 d_1$	0.366	17		$0.4 d_1$	0.270
18		$0.6 d_1$	0.257	18		$0.6 d_1$	0.161
19		$0.8 d_1$	0.182	19		$0.8 d_1$	0.107
20		$1.0 d_1$	0.162	20		$1.0 d_1$	0.088
21	$6 d_1$	$0.2 d_1$	0.558	21	$6 d_1$	$0.2 d_1$	0.480
22		$0.4 d_1$	0.453	22		$0.4 d_1$	0.369
23		$0.6 d_1$	0.357	23		$0.6 d_1$	0.267
24		$0.8 d_1$	0.296	24		$0.8 d_1$	0.197
25		$1.0 d_1$	0.273	25		$1.0 d_1$	0.179

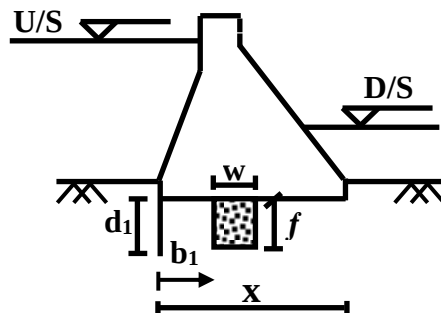


Table (B.3)
Calculated values of the eccentricity (X_4) of the resultant force
($d_1 = 10\text{m}$)

A: $w = 0.25 d_1$				B: $w = 0.50 d_1$			
NO.	b_1	f	X_4	NO.	b_1	f	X_4
1	$2 d_1$	$0.2 d_1$	0.507	1	$2 d_1$	$0.2 d_1$	0.583
2		$0.4 d_1$	0.598	2		$0.4 d_1$	0.674
3		$0.6 d_1$	0.685	3		$0.6 d_1$	0.760
4		$0.8 d_1$	0.740	4		$0.8 d_1$	0.809
5		$1.0 d_1$	0.771	5		$1.0 d_1$	0.835
6	$3 d_1$	$0.2 d_1$	0.428	6	$3 d_1$	$0.2 d_1$	0.517
7		$0.4 d_1$	0.524	7		$0.4 d_1$	0.601
8		$0.6 d_1$	0.609	8		$0.6 d_1$	0.698
9		$0.8 d_1$	0.660	9		$0.8 d_1$	0.740
10		$1.0 d_1$	0.693	10		$1.0 d_1$	0.773
11	$4 d_1$	$0.2 d_1$	0.337	11	$4 d_1$	$0.2 d_1$	0.433
12		$0.4 d_1$	0.430	12		$0.4 d_1$	0.510
13		$0.6 d_1$	0.516	13		$0.6 d_1$	0.596
14		$0.8 d_1$	0.574	14		$0.8 d_1$	0.649
15		$1.0 d_1$	0.584	15		$1.0 d_1$	0.664
16	$5 d_1$	$0.2 d_1$	0.252	16	$5 d_1$	$0.2 d_1$	0.345
17		$0.4 d_1$	0.360	17		$0.4 d_1$	0.431
18		$0.6 d_1$	0.434	18		$0.6 d_1$	0.519
19		$0.8 d_1$	0.490	19		$0.8 d_1$	0.569
20		$1.0 d_1$	0.499	20		$1.0 d_1$	0.584
21	$6 d_1$	$0.2 d_1$	0.163	21	$6 d_1$	$0.2 d_1$	0.249
22		$0.4 d_1$	0.260	22		$0.4 d_1$	0.343
23		$0.6 d_1$	0.344	23		$0.6 d_1$	0.437
24		$0.8 d_1$	0.395	24		$0.8 d_1$	0.492
25		$1.0 d_1$	0.411	25		$1.0 d_1$	0.510

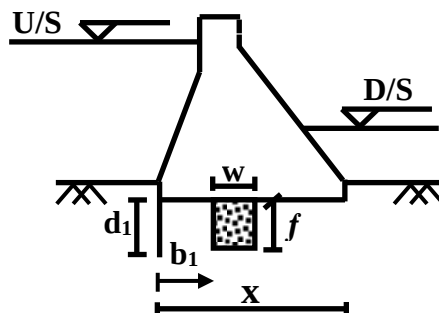


Table (B.4)
Calculated values of the eccentricity (X_4) of the resultant force
($d_2 = 3\text{m}$)

A: $w = 0.25 d_2$				B: $w = 0.50 d_2$			
NO.	b_2	f	X_4	NO.	b_2	f	X_4
1	$2 d_2$	$0.2 d_2$	2.320	1	$2 d_2$	$0.2 d_2$	2.416
2		$0.4 d_2$	2.385	2		$0.4 d_2$	2.481
3		$0.6 d_2$	2.519	3		$0.6 d_2$	2.610
4		$0.8 d_2$	2.557	4		$0.8 d_2$	2.647
5		$1.0 d_2$	2.573	5		$1.0 d_2$	2.666
6	$3 d_2$	$0.2 d_2$	2.223	6	$3 d_2$	$0.2 d_2$	2.310
7		$0.4 d_2$	2.290	7		$0.4 d_2$	2.378
8		$0.6 d_2$	2.425	8		$0.6 d_2$	2.508
9		$0.8 d_2$	2.465	9		$0.8 d_2$	2.548
10		$1.0 d_2$	2.483	10		$1.0 d_2$	2.570
11	$4 d_2$	$0.2 d_2$	2.112	11	$4 d_2$	$0.2 d_2$	2.220
12		$0.4 d_2$	2.182	12		$0.4 d_2$	2.291
13		$0.6 d_2$	2.319	13		$0.6 d_2$	2.419
14		$0.8 d_2$	2.361	14		$0.8 d_2$	2.462
15		$1.0 d_2$	2.387	15		$1.0 d_2$	2.481
16	$5 d_2$	$0.2 d_2$	2.019	16	$5 d_2$	$0.2 d_2$	2.122
17		$0.4 d_2$	2.091	17		$0.4 d_2$	2.191
18		$0.6 d_2$	2.231	18		$0.6 d_2$	2.326
19		$0.8 d_2$	2.271	19		$0.8 d_2$	2.367
20		$1.0 d_2$	2.291	20		$1.0 d_2$	2.387
21	$6 d_2$	$0.2 d_2$	2.919	21	$6 d_2$	$0.2 d_2$	2.012
22		$0.4 d_2$	1.980	22		$0.4 d_2$	2.082
23		$0.6 d_2$	2.124	23		$0.6 d_2$	2.223
24		$0.8 d_2$	2.172	24		$0.8 d_2$	2.261
25		$1.0 d_2$	2.192	25		$1.0 d_2$	2.288

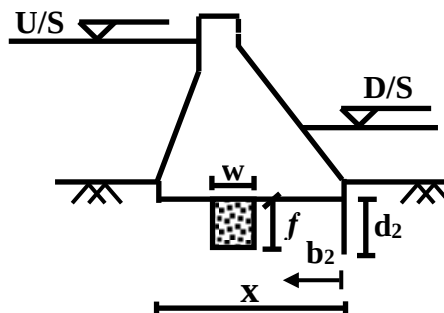


Table (B.5)
Calculated values of the eccentricity (X_4) of the resultant force
($d_2 = 5\text{m}$)

A: $w = 0.25 d_2$				B: $w = 0.50 d_2$			
NO.	b_2	f	X_4	NO.	b_2	f	X_4
1	2 d_2	0.2 d_2	2.684	1	2 d_2	0.2 d_2	2.774
2		0.4 d_2	2.776	2		0.4 d_2	2.868
3		0.6 d_2	2.885	3		0.6 d_2	2.968
4		0.8 d_2	2.936	4		0.8 d_2	3.022
5		1.0 d_2	2.969	5		1.0 d_2	3.055
6	3 d_2	0.2 d_2	2.601	6	3 d_2	0.2 d_2	2.669
7		0.4 d_2	2.699	7		0.4 d_2	2.773
8		0.6 d_2	2.802	8		0.6 d_2	2.871
9		0.8 d_2	2.856	9		0.8 d_2	2.918
10		1.0 d_2	2.888	10		1.0 d_2	2.952
11	4 d_2	0.2 d_2	2.515	11	4 d_2	0.2 d_2	2.591
12		0.4 d_2	2.610	12		0.4 d_2	2.701
13		0.6 d_2	2.712	13		0.6 d_2	2.783
14		0.8 d_2	2.775	14		0.8 d_2	2.846
15		1.0 d_2	2.791	15		1.0 d_2	2.862
16	5 d_2	0.2 d_2	2.422	16	5 d_2	0.2 d_2	2.508
17		0.4 d_2	2.526	17		0.4 d_2	2.610
18		0.6 d_2	2.630	18		0.6 d_2	2.711
19		0.8 d_2	2.693	19		0.8 d_2	2.760
20		1.0 d_2	2.709	20		1.0 d_2	2.775
21	6 d_2	0.2 d_2	2.308	21	6 d_2	0.2 d_2	2.402
22		0.4 d_2	2.417	22		0.4 d_2	2.507
23		0.6 d_2	2.509	23		0.6 d_2	2.604
24		0.8 d_2	2.581	24		0.8 d_2	2.666
25		1.0 d_2	2.599	25		1.0 d_2	2.679

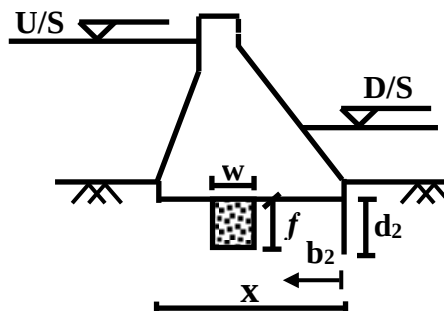


Table (B.6)
Calculated values of the eccentricity (X_4) of the resultant force
($d_2 = 7.5\text{m}$)

A: $w = 0.25 d_2$				B: $w = 0.50 d_2$			
NO.	b_2	f	X_4	NO.	b_2	f	X_4
1	$2 d_2$	$0.2 d_2$	2.779	1	$2 d_2$	$0.2 d_2$	2.873
2		$0.4 d_2$	2.881	2		$0.4 d_2$	2.975
3		$0.6 d_2$	2.987	3		$0.6 d_2$	3.071
4		$0.8 d_2$	3.038	4		$0.8 d_2$	3.127
5		$1.0 d_2$	3.071	5		$1.0 d_2$	3.162
6	$3 d_2$	$0.2 d_2$	2.687	6	$3 d_2$	$0.2 d_2$	2.771
7		$0.4 d_2$	2.788	7		$0.4 d_2$	2.877
8		$0.6 d_2$	2.881	8		$0.6 d_2$	2.983
9		$0.8 d_2$	2.945	9		$0.8 d_2$	3.039
10		$1.0 d_2$	2.979	10		$1.0 d_2$	3.072
11	$4 d_2$	$0.2 d_2$	2.596	11	$4 d_2$	$0.2 d_2$	2.708
12		$0.4 d_2$	2.698	12		$0.4 d_2$	2.808
13		$0.6 d_2$	2.807	13		$0.6 d_2$	2.887
14		$0.8 d_2$	2.859	14		$0.8 d_2$	2.936
15		$1.0 d_2$	2.874	15		$1.0 d_2$	2.956
16	$5 d_2$	$0.2 d_2$	2.520	16	$5 d_2$	$0.2 d_2$	2.611
17		$0.4 d_2$	2.613	17		$0.4 d_2$	2.719
18		$0.6 d_2$	2.717	18		$0.6 d_2$	2.807
19		$0.8 d_2$	2.770	19		$0.8 d_2$	2.859
20		$1.0 d_2$	2.787	20		$1.0 d_2$	2.879
21	$6 d_2$	$0.2 d_2$	2.409	21	$6 d_2$	$0.2 d_2$	2.505
22		$0.4 d_2$	2.517	22		$0.4 d_2$	2.613
23		$0.6 d_2$	2.621	23		$0.6 d_2$	2.706
24		$0.8 d_2$	2.680	24		$0.8 d_2$	2.771
25		$1.0 d_2$	2.703	25		$1.0 d_2$	2.791

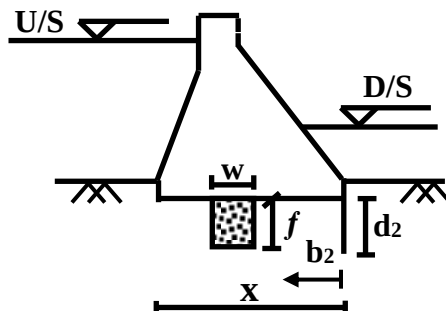


Table (B.7)
Calculated values of the eccentricity (X_4) of the resultant force
($d_2 = 10\text{m}$)

A: $w = 0.25 d_2$				B: $w = 0.50 d_2$			
NO.	b_2	f	X_4	NO.	b_2	f	X_4
1	$2 d_2$	$0.2 d_2$	3.038	1	$2 d_2$	$0.2 d_2$	3.142
2		$0.4 d_2$	3.136	2		$0.4 d_2$	3.242
3		$0.6 d_2$	3.237	3		$0.6 d_2$	3.337
4		$0.8 d_2$	3.292	4		$0.8 d_2$	3.382
5		$1.0 d_2$	3.327	5		$1.0 d_2$	3.414
6	$3 d_2$	$0.2 d_2$	2.954	6	$3 d_2$	$0.2 d_2$	3.050
7		$0.4 d_2$	3.058	7		$0.4 d_2$	3.148
8		$0.6 d_2$	3.154	8		$0.6 d_2$	3.241
9		$0.8 d_2$	3.210	9		$0.8 d_2$	3.297
10		$1.0 d_2$	3.243	10		$1.0 d_2$	3.333
11	$4 d_2$	$0.2 d_2$	2.885	11	$4 d_2$	$0.2 d_2$	2.965
12		$0.4 d_2$	2.965	12		$0.4 d_2$	3.067
13		$0.6 d_2$	3.077	13		$0.6 d_2$	3.159
14		$0.8 d_2$	3.127	14		$0.8 d_2$	3.216
15		$1.0 d_2$	3.146	15		$1.0 d_2$	3.232
16	$5 d_2$	$0.2 d_2$	2.792	16	$5 d_2$	$0.2 d_2$	2.877
17		$0.4 d_2$	2.878	17		$0.4 d_2$	2.966
18		$0.6 d_2$	2.991	18		$0.6 d_2$	2.077
19		$0.8 d_2$	3.050	19		$0.8 d_2$	3.123
20		$1.0 d_2$	3.067	20		$1.0 d_2$	3.140
21	$6 d_2$	$0.2 d_2$	2.675	21	$6 d_2$	$0.2 d_2$	2.773
22		$0.4 d_2$	2.779	22		$0.4 d_2$	2.857
23		$0.6 d_2$	2.878	23		$0.6 d_2$	2.976
24		$0.8 d_2$	2.944	24		$0.8 d_2$	3.032
25		$1.0 d_2$	2.964	25		$1.0 d_2$	3.047

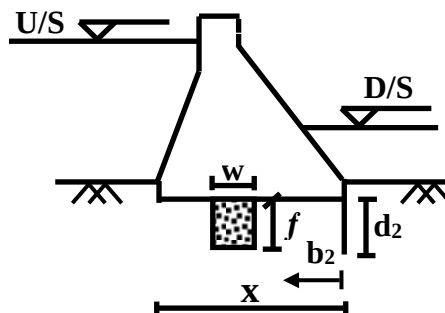


Table (B.8)
Calculated values of the eccentricity (X_4) of the resultant force
($d_1 = d_2 = 3\text{m}$)

A: $w = 0.25 d_1$				B: $w = 0.50 d_1$			
NO.	b_1	f	X_4	NO.	b_1	f	X_4
1	$2 d_1$	$0.2 d_1$	0.530	1	$2 d_1$	$0.2 d_1$	0.531
2		$0.4 d_1$	0.514	2		$0.4 d_1$	0.526
3		$0.6 d_1$	0.513	3		$0.6 d_1$	0.512
4		$0.8 d_1$	0.515	4		$0.8 d_1$	0.511
5		$1.0 d_1$	0.517	5		$1.0 d_1$	0.514
6	$3 d_1$	$0.2 d_1$	0.834	6	$3 d_1$	$0.2 d_1$	0.831
7		$0.4 d_1$	0.821	7		$0.4 d_1$	0.820
8		$0.6 d_1$	0.808	8		$0.6 d_1$	0.802
9		$0.8 d_1$	0.807	9		$0.8 d_1$	0.795
10		$1.0 d_1$	0.808	10		$1.0 d_1$	0.792
11	$4 d_1$	$0.2 d_1$	1.156	11	$4 d_1$	$0.2 d_1$	1.171
12		$0.4 d_1$	1.147	12		$0.4 d_1$	1.146
13		$0.6 d_1$	1.123	13		$0.6 d_1$	1.126
14		$0.8 d_1$	1.185	14		$0.8 d_1$	1.116
15		$1.0 d_1$	1.171	15		$1.0 d_1$	1.104
16	$5 d_1$	$0.2 d_1$	1.500	16	$5 d_1$	$0.2 d_1$	1.499
17		$0.4 d_1$	1.486	17		$0.4 d_1$	1.474
18		$0.6 d_1$	1.444	18		$0.6 d_1$	1.428
19		$0.8 d_1$	1.516	19		$0.8 d_1$	1.424
20		$1.0 d_1$	1.501	20		$1.0 d_1$	1.423
21	$6 d_1$	$0.2 d_1$	1.855	21	$6 d_1$	$0.2 d_1$	1.841
22		$0.4 d_1$	1.826	22		$0.4 d_1$	1.812
23		$0.6 d_1$	1.796	23		$0.6 d_1$	1.765
24		$0.8 d_1$	0.528	24		$0.8 d_1$	0.520
25		$1.0 d_1$	1.771	25		$1.0 d_1$	1.743

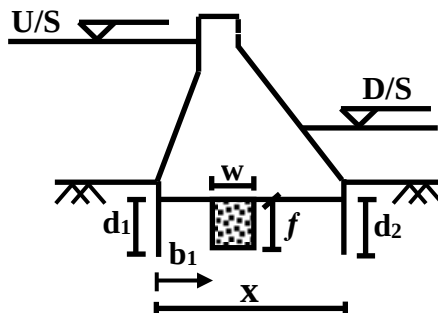


Table (B.9)
Calculated values of the eccentricity (X_4) of the resultant force
($d_1 = d_2 = 5\text{m}$)

A: $w = 0.25 d_1$				B: $w = 0.50 d_1$			
NO.	b_1	f	X_4	NO.	b_1	f	X_4
1	$2 d_1$	$0.2 d_1$	0.889	1	$2 d_1$	$0.2 d_1$	0.853
2		$0.4 d_1$	0.869	2		$0.4 d_1$	0.841
3		$0.6 d_1$	0.854	3		$0.6 d_1$	0.840
4		$0.8 d_1$	0.839	4		$0.8 d_1$	0.823
5		$1.0 d_1$	0.827	5		$1.0 d_1$	0.815
6	$3 d_1$	$0.2 d_1$	0.659	6	$3 d_1$	$0.2 d_1$	0.642
7		$0.4 d_1$	0.653	7		$0.4 d_1$	1.541
8		$0.6 d_1$	0.635	8		$0.6 d_1$	0.623
9		$0.8 d_1$	0.626	9		$0.8 d_1$	0.618
10		$1.0 d_1$	0.622	10		$1.0 d_1$	0.622
11	$4 d_1$	$0.2 d_1$	0.345	11	$4 d_1$	$0.2 d_1$	0.339
12		$0.4 d_1$	0.330	12		$0.4 d_1$	0.335
13		$0.6 d_1$	0.338	13		$0.6 d_1$	0.341
14		$0.8 d_1$	0.335	14		$0.8 d_1$	0.339
15		$1.0 d_1$	0.334	15		$1.0 d_1$	0.337
16	$5 d_1$	$0.2 d_1$	0.082	16	$5 d_1$	$0.2 d_1$	0.096
17		$0.4 d_1$	0.060	17		$0.4 d_1$	0.058
18		$0.6 d_1$	0.058	18		$0.6 d_1$	0.074
19		$0.8 d_1$	0.052	19		$0.8 d_1$	0.075
20		$1.0 d_1$	0.049	20		$1.0 d_1$	0.076
21	$6 d_1$	$0.2 d_1$	0.193	21	$6 d_1$	$0.2 d_1$	0.184
22		$0.4 d_1$	0.177	22		$0.4 d_1$	0.183
23		$0.6 d_1$	0.199	23		$0.6 d_1$	0.179
24		$0.8 d_1$	0.196	24		$0.8 d_1$	0.181
25		$1.0 d_1$	0.190	25		$1.0 d_1$	0.180

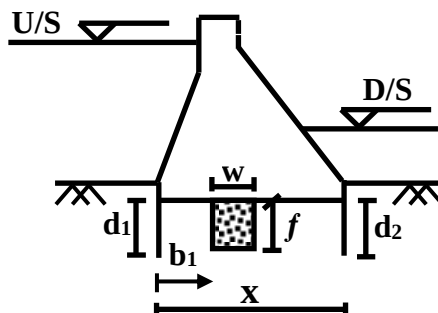


Table (B.10)
Calculated values of the eccentricity (X_4) of the resultant force
($d_1 = d_2 = 7.5\text{m}$)

A: $w = 0.25 d_1$				B: $w = 0.50 d_1$			
NO.	b_1	f	X_4	NO.	b_1	f	X_4
1	$2 d_1$	$0.2 d_1$	0.911	1	$2 d_1$	$0.2 d_1$	0.921
2		$0.4 d_1$	0.902	2		$0.4 d_1$	0.906
3		$0.6 d_1$	0.881	3		$0.6 d_1$	0.895
4		$0.8 d_1$	0.870	4		$0.8 d_1$	0.888
5		$1.0 d_1$	0.865	5		$1.0 d_1$	0.884
6	$3 d_1$	$0.2 d_1$	0.722	6	$3 d_1$	$0.2 d_1$	0.705
7		$0.4 d_1$	0.705	7		$0.4 d_1$	0.694
8		$0.6 d_1$	0.694	8		$0.6 d_1$	0.686
9		$0.8 d_1$	0.687	9		$0.8 d_1$	0.684
10		$1.0 d_1$	0.681	10		$1.0 d_1$	0.688
11	$4 d_1$	$0.2 d_1$	0.431	11	$4 d_1$	$0.2 d_1$	0.406
12		$0.4 d_1$	0.400	12		$0.4 d_1$	0.404
13		$0.6 d_1$	0.414	13		$0.6 d_1$	0.409
14		$0.8 d_1$	0.403	14		$0.8 d_1$	0.393
15		$1.0 d_1$	0.402	15		$1.0 d_1$	0.386
16	$5 d_1$	$0.2 d_1$	0.175	16	$5 d_1$	$0.2 d_1$	0.157
17		$0.4 d_1$	0.156	17		$0.4 d_1$	0.165
18		$0.6 d_1$	0.183	18		$0.6 d_1$	0.179
19		$0.8 d_1$	0.154	19		$0.8 d_1$	0.164
20		$1.0 d_1$	0.156	20		$1.0 d_1$	0.165
21	$6 d_1$	$0.2 d_1$	0.082	21	$6 d_1$	$0.2 d_1$	0.067
22		$0.4 d_1$	0.065	22		$0.4 d_1$	0.054
23		$0.6 d_1$	0.057	23		$0.6 d_1$	0.069
24		$0.8 d_1$	0.066	24		$0.8 d_1$	0.084
25		$1.0 d_1$	0.068	25		$1.0 d_1$	0.083

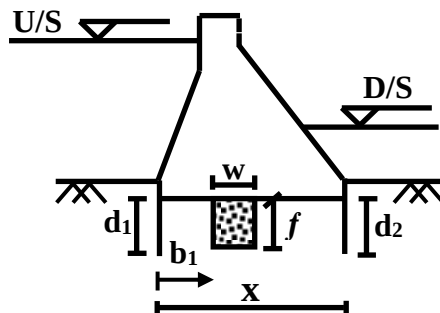
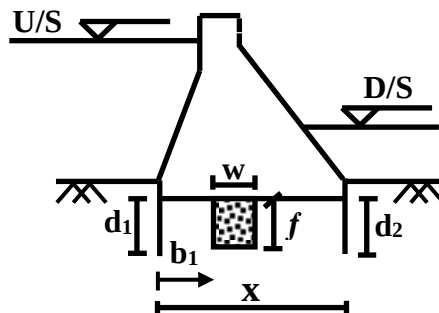


Table (B.11)
Calculated values of the eccentricity (X_4) of the resultant force
($d_1 = d_2 = 10\text{m}$)

A: $w = 0.25 d_1$				B: $w = 0.50 d_1$			
NO.	b_1	f	X_4	NO.	b_1	f	X_4
1	$2 d_1$	$0.2 d_1$	0.953	1	$2 d_1$	$0.2 d_1$	0.954
2		$0.4 d_1$	0.942	2		$0.4 d_1$	0.938
3		$0.6 d_1$	0.924	3		$0.6 d_1$	0.928
4		$0.8 d_1$	0.919	4		$0.8 d_1$	0.923
5		$1.0 d_1$	0.930	5		$1.0 d_1$	0.923
6	$3 d_1$	$0.2 d_1$	0.773	6	$3 d_1$	$0.2 d_1$	0.753
7		$0.4 d_1$	0.739	7		$0.4 d_1$	0.746
8		$0.6 d_1$	0.730	8		$0.6 d_1$	0.726
9		$0.8 d_1$	0.724	9		$0.8 d_1$	0.725
10		$1.0 d_1$	0.727	10		$1.0 d_1$	0.724
11	$4 d_1$	$0.2 d_1$	0.480	11	$4 d_1$	$0.2 d_1$	0.471
12		$0.4 d_1$	0.484	12		$0.4 d_1$	0.469
13		$0.6 d_1$	0.483	13		$0.6 d_1$	0.453
14		$0.8 d_1$	0.461	14		$0.8 d_1$	0.443
15		$1.0 d_1$	0.459	15		$1.0 d_1$	0.437
16	$5 d_1$	$0.2 d_1$	0.252	16	$5 d_1$	$0.2 d_1$	0.242
17		$0.4 d_1$	0.243	17		$0.4 d_1$	0.235
18		$0.6 d_1$	0.248	18		$0.6 d_1$	0.244
19		$0.8 d_1$	0.246	19		$0.8 d_1$	0.240
20		$1.0 d_1$	0.245	20		$1.0 d_1$	0.238
21	$6 d_1$	$0.2 d_1$	0.013	21	$6 d_1$	$0.2 d_1$	0.004
22		$0.4 d_1$	0.016	22		$0.4 d_1$	0.017
23		$0.6 d_1$	0.026	23		$0.6 d_1$	0.027
24		$0.8 d_1$	0.011	24		$0.8 d_1$	0.015
25		$1.0 d_1$	0.002	25		$1.0 d_1$	0.010



APPENDIX

(C)

Table (C.1)
Calculated values of the total difference
 $(d_1 = 5\text{m} , d_2 = 0 , w = 0.25 d_1 , y_5 = 10\text{m})$

NO.	b_1	f	y_4	I_e	T.D.
1	$2 d_1$	$0.2 d_1$			Unsafe
2		$0.4 d_1$			Unsafe
3		$0.6 d_1$			Unsafe
4		$0.8 d_1$			Unsafe
5		$1.0 d_1$			Unsafe
6	$3 d_1$	$0.2 d_1$			Unsafe
7		$0.4 d_1$			Unsafe
8		$0.6 d_1$			Unsafe
9		$0.8 d_1$			Unsafe
10		$1.0 d_1$			Unsafe
11	$4 d_1$	$0.2 d_1$			Unsafe
12		$0.4 d_1$			Unsafe
13		$0.6 d_1$			Unsafe
14		$0.8 d_1$			Unsafe
15		$1.0 d_1$			Unsafe
16	$5 d_1$	$0.2 d_1$			Unsafe
17		$0.4 d_1$			Unsafe
18		$0.6 d_1$			Unsafe
19		$0.8 d_1$			Unsafe
20		$1.0 d_1$			Unsafe
21	$6 d_1$	$0.2 d_1$			Unsafe
22		$0.4 d_1$			Unsafe
23		$0.6 d_1$			Unsafe
24		$0.8 d_1$			Unsafe
25		$1.0 d_1$			Unsafe

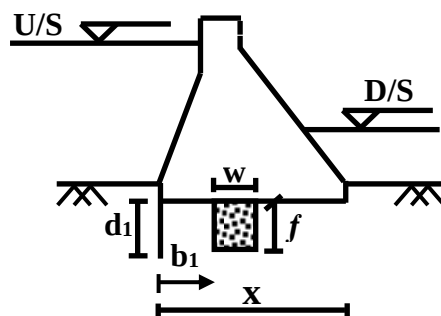


Table (C.2)
Calculated values of the total difference
 $(d_1 = 5\text{m} , d_2 = 0 , w = 0.50 d_1 , y_5 = 10\text{m})$

NO.	b_1	f	y_4	Ie	T.D.
1	$2 d_1$	$0.2 d_1$			Unsafe
2		$0.4 d_1$			Unsafe
3		$0.6 d_1$			Unsafe
4		$0.8 d_1$			Unsafe
5		$1.0 d_1$			Unsafe
6	$3 d_1$	$0.2 d_1$			Unsafe
7		$0.4 d_1$			Unsafe
8		$0.6 d_1$			Unsafe
9		$0.8 d_1$			Unsafe
10		$1.0 d_1$			Unsafe
11	$4 d_1$	$0.2 d_1$			Unsafe
12		$0.4 d_1$			Unsafe
13		$0.6 d_1$			Unsafe
14		$0.8 d_1$			Unsafe
15		$1.0 d_1$			Unsafe
16	$5 d_1$	$0.2 d_1$			Unsafe
17		$0.4 d_1$			Unsafe
18		$0.6 d_1$			Unsafe
19		$0.8 d_1$			Unsafe
20		$1.0 d_1$			Unsafe
21	$6 d_1$	$0.2 d_1$			Unsafe
22		$0.4 d_1$			Unsafe
23		$0.6 d_1$			Unsafe
24		$0.8 d_1$			Unsafe
25		$1.0 d_1$			Unsafe

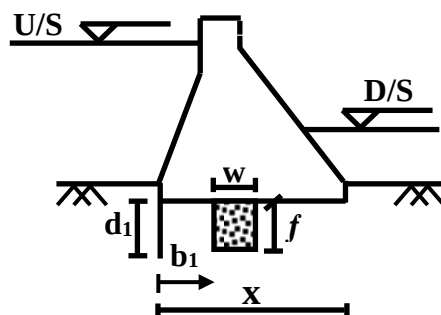


Table (C.3)
Calculated values of the total difference
 $(d_1 = 7.5\text{m} , d_2 = 0 , w = 0.25 d_1 , y_5 = 10\text{m})$

NO.	b_1	f	y_4	I_e	T.D.
1	$2 d_1$	$0.2 d_1$			Unsafe
2		$0.4 d_1$	45.891	0.229	1.090
3		$0.6 d_1$	44.427	0.220	1.364
4		$0.8 d_1$	43.801	0.216	1.489
5		$1.0 d_1$	43.519	0.215	1.549
6	$3 d_1$	$0.2 d_1$			Unsafe
7		$0.4 d_1$			Unsafe
8		$0.6 d_1$	45.360	0.215	1.422
9		$0.8 d_1$	44.752	0.212	1.524
10		$1.0 d_1$	44.399	0.210	1.588
11	$4 d_1$	$0.2 d_1$			Unsafe
12		$0.4 d_1$			Unsafe
13		$0.6 d_1$			Unsafe
14		$0.8 d_1$			Unsafe
15		$1.0 d_1$	45.258	0.198	2.062
16	$5 d_1$	$0.2 d_1$			Unsafe
17		$0.4 d_1$			Unsafe
18		$0.6 d_1$			Unsafe
19		$0.8 d_1$			Unsafe
20		$1.0 d_1$			Unsafe
21	$6 d_1$	$0.2 d_1$			Unsafe
22		$0.4 d_1$			Unsafe
23		$0.6 d_1$			Unsafe
24		$0.8 d_1$			Unsafe
25		$1.0 d_1$			Unsafe

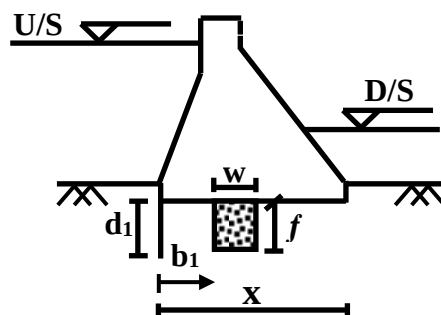


Table (C.4)
Calculated values of the total difference
 $(d_1 = 7.5\text{m} , d_2 = 0 , w = 0.50 d_1 , y_5 = 10\text{m})$

NO.	b_1	f	y_4	I_e	T.D.
1	$2 d_1$	$0.2 d_1$	45.586	0.228	1.117
2		$0.4 d_1$	44.715	0.219	1.343
3		$0.6 d_1$	43.220	0.210	1.733
4		$0.8 d_1$	42.662	0.207	1.850
5		$1.0 d_1$	42.293	0.206	1.923
6	$3 d_1$	$0.2 d_1$			Unsafe
7		$0.4 d_1$	45.730	0.210	1.522
8		$0.6 d_1$	44.317	0.200	2.063
9		$0.8 d_1$	43.671	0.198	2.207
10		$1.0 d_1$	43.260	0.196	2.293
11	$4 d_1$	$0.2 d_1$			Unsafe
12		$0.4 d_1$			Unsafe
13		$0.6 d_1$	45.289	0.196	2.188
14		$0.8 d_1$	44.701	0.192	2.436
15		$1.0 d_1$	44.584	0.191	2.529
16	$5 d_1$	$0.2 d_1$			Unsafe
17		$0.4 d_1$			Unsafe
18		$0.6 d_1$			Unsafe
19		$0.8 d_1$			Unsafe
20		$1.0 d_1$	45.411	0.182	3.180
21	$6 d_1$	$0.2 d_1$			Unsafe
22		$0.4 d_1$			Unsafe
23		$0.6 d_1$			Unsafe
24		$0.8 d_1$			Unsafe
25		$1.0 d_1$			Unsafe

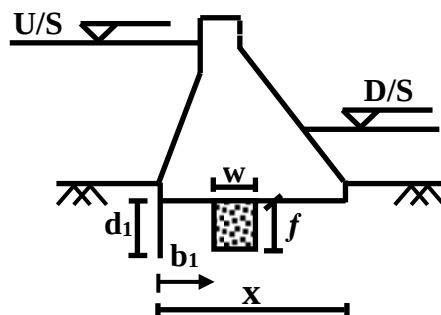


Table (C.5)
Calculated values of the total difference
 $(d_1 = 10\text{m} , d_2 = 0 , w = 0.25 d_1 , y_5 = 10\text{m})$

NO.	b_1	f	y_4	I_e	T.D.
1	$2 d_1$	$0.2 d_1$	44.518	0.229	1.192
2		$0.4 d_1$	43.726	0.222	1.377
3		$0.6 d_1$	42.370	0.214	1.700
4		$0.8 d_1$	41.699	0.211	1.836
5		$1.0 d_1$	41.312	0.210	1.891
6	$3 d_1$	$0.2 d_1$	45.452	0.221	1.250
7		$0.4 d_1$	44.570	0.213	1.517
8		$0.6 d_1$	43.292	0.204	1.981
9		$0.8 d_1$	42.681	0.200	2.174
10		$1.0 d_1$	42.277	0.199	2.238
11	$4 d_1$	$0.2 d_1$			Unsafe
12		$0.4 d_1$	45.624	0.205	1.717
13		$0.6 d_1$	44.328	0.197	2.228
14		$0.8 d_1$	43.719	0.195	2.359
15		$1.0 d_1$	43.599	0.194	2.449
16	$5 d_1$	$0.2 d_1$			Unsafe
17		$0.4 d_1$			Unsafe
18		$0.6 d_1$	45.284	0.185	2.932
19		$0.8 d_1$	44.715	0.181	3.330
20		$1.0 d_1$	44.509	0.180	3.443
21	$6 d_1$	$0.2 d_1$			Unsafe
22		$0.4 d_1$			Unsafe
23		$0.6 d_1$			Unsafe
24		$0.8 d_1$			Unsafe
25		$1.0 d_1$	45.319	0.169	4.532

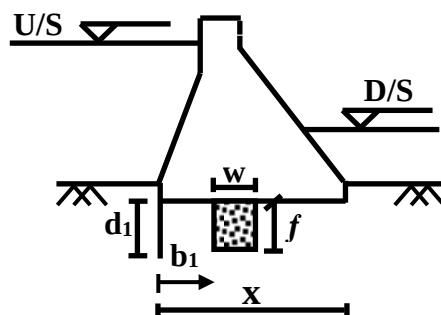


Table (C.6)
Calculated values of the total difference
 $(d_1 = 10\text{m} , d_2 = 0 , w = 0.50 d_1 , y_5 = 10\text{m})$

NO.	b_1	f	y_4	I_e	T.D.
1	$2 d_1$	$0.2 d_1$	43.610	0.221	1.420
2		$0.4 d_1$	42.804	0.213	1.680
3		$0.6 d_1$	41.451	0.204	2.172
4		$0.8 d_1$	40.841	0.200	2.348
5		$1.0 d_1$	40.510	0.199	2.435
6	$3 d_1$	$0.2 d_1$	44.403	0.211	1.595
7		$0.4 d_1$	43.598	0.205	1.884
8		$0.6 d_1$	42.211	0.196	2.474
9		$0.8 d_1$	41.695	0.195	2.565
10		$1.0 d_1$	41.285	0.194	2.636
11	$4 d_1$	$0.2 d_1$	45.390	0.202	1.882
12		$0.4 d_1$	44.683	0.187	2.318
13		$0.6 d_1$	43.356	0.185	3.131
14		$0.8 d_1$	42.813	0.181	3.474
15		$1.0 d_1$	42.533	0.180	3.589
16	$5 d_1$	$0.2 d_1$			Unsafe
17		$0.4 d_1$	45.617	0.187	2.702
18		$0.6 d_1$	44.371	0.179	3.579
19		$0.8 d_1$	43.780	0.175	3.995
20		$1.0 d_1$	43.401	0.174	4.085
21	$6 d_1$	$0.2 d_1$			Unsafe
22		$0.4 d_1$			Unsafe
23		$0.6 d_1$	45.341	0.164	5.285
24		$0.8 d_1$	44.712	0.161	5.904
25		$1.0 d_1$	44.488	0.160	6.058

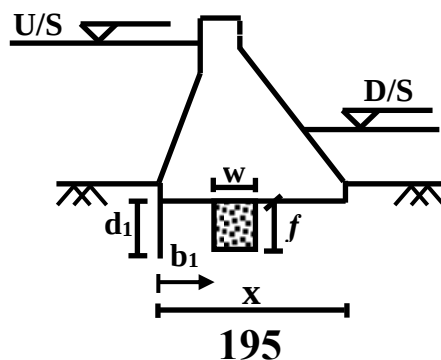


Table (C.7)
Calculated values of the total difference
 $(d_1 = 0 \text{ , } d_2 = 3\text{m} \text{ , } w = 0.25 d_2 \text{ , } y_4 = 70\text{m})$

NO.	b_2	f	y_5	Ie	T.D.
1	$2 d_2$	$0.2 d_2$			Unsafe
2		$0.4 d_2$			Unsafe
3		$0.6 d_2$			Unsafe
4		$0.8 d_2$			Unsafe
5		$1.0 d_2$			Unsafe
6	$3 d_2$	$0.2 d_2$			Unsafe
7		$0.4 d_2$			Unsafe
8		$0.6 d_2$			Unsafe
9		$0.8 d_2$			Unsafe
10		$1.0 d_2$			Unsafe
11	$4 d_2$	$0.2 d_2$			Unsafe
12		$0.4 d_2$			Unsafe
13		$0.6 d_2$			Unsafe
14		$0.8 d_2$			Unsafe
15		$1.0 d_2$			Unsafe
16	$5 d_2$	$0.2 d_2$			Unsafe
17		$0.4 d_2$			Unsafe
18		$0.6 d_2$			Unsafe
19		$0.8 d_2$			Unsafe
20		$1.0 d_2$			Unsafe
21	$6 d_2$	$0.2 d_2$			Unsafe
22		$0.4 d_2$			Unsafe
23		$0.6 d_2$			Unsafe
24		$0.8 d_2$			Unsafe
25		$1.0 d_2$			Unsafe

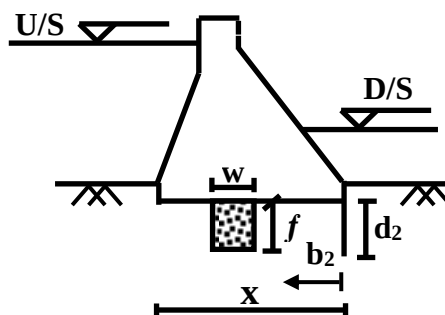


Table (C.8)
Calculated values of the total difference
 $(d_1 = 0 \text{ , } d_2 = 3\text{m} \text{ , } w = 0.50 d_2 \text{ , } y_4 = 70\text{m})$

NO.	b_2	f	y_5	Ie	T.D.
1	$2 d_2$	$0.2 d_2$			Unsafe
2		$0.4 d_2$			Unsafe
3		$0.6 d_2$			Unsafe
4		$0.8 d_2$			Unsafe
5		$1.0 d_2$			Unsafe
6	$3 d_2$	$0.2 d_2$			Unsafe
7		$0.4 d_2$			Unsafe
8		$0.6 d_2$			Unsafe
9		$0.8 d_2$			Unsafe
10		$1.0 d_2$			Unsafe
11	$4 d_2$	$0.2 d_2$			Unsafe
12		$0.4 d_2$			Unsafe
13		$0.6 d_2$			Unsafe
14		$0.8 d_2$			Unsafe
15		$1.0 d_2$			Unsafe
16	$5 d_2$	$0.2 d_2$			Unsafe
17		$0.4 d_2$			Unsafe
18		$0.6 d_2$			Unsafe
19		$0.8 d_2$			Unsafe
20		$1.0 d_2$			Unsafe
21	$6 d_2$	$0.2 d_2$			Unsafe
22		$0.4 d_2$			Unsafe
23		$0.6 d_2$			Unsafe
24		$0.8 d_2$			Unsafe
25		$1.0 d_2$			Unsafe

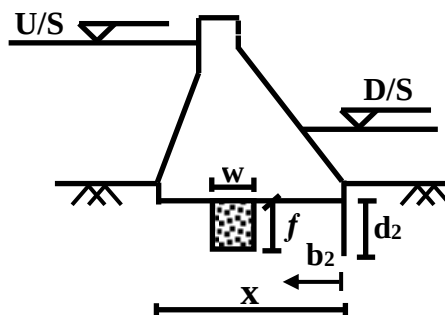


Table (C.9)
Calculated values of the total difference
 $(d_1 = 0 \text{ , } d_2 = 5\text{m} \text{ , } w = 0.25 d_2 \text{ , } y_4 = 70\text{m})$

NO.	b_2	f	y_5	Ie	T.D.
1	$2 d_2$	$0.2 d_2$			Unsafe
2		$0.4 d_2$			Unsafe
3		$0.6 d_2$			Unsafe
4		$0.8 d_2$			Unsafe
5		$1.0 d_2$			Unsafe
6	$3 d_2$	$0.2 d_2$			Unsafe
7		$0.4 d_2$			Unsafe
8		$0.6 d_2$			Unsafe
9		$0.8 d_2$			Unsafe
10		$1.0 d_2$			Unsafe
11	$4 d_2$	$0.2 d_2$			Unsafe
12		$0.4 d_2$			Unsafe
13		$0.6 d_2$			Unsafe
14		$0.8 d_2$			Unsafe
15		$1.0 d_2$			Unsafe
16	$5 d_2$	$0.2 d_2$			Unsafe
17		$0.4 d_2$			Unsafe
18		$0.6 d_2$			Unsafe
19		$0.8 d_2$			Unsafe
20		$1.0 d_2$			Unsafe
21	$6 d_2$	$0.2 d_2$			Unsafe
22		$0.4 d_2$			Unsafe
23		$0.6 d_2$			Unsafe
24		$0.8 d_2$			Unsafe
25		$1.0 d_2$			Unsafe

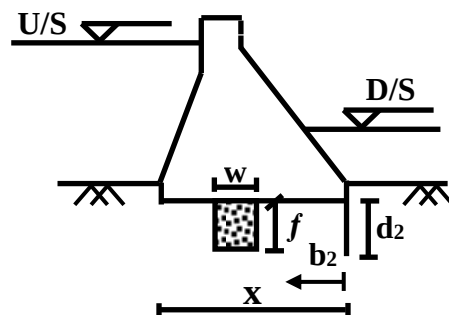


Table (C.10)
Calculated values of the total difference
 $(d_1 = 0 \text{ , } d_2 = 5\text{m} \text{ , } w = 0.50 d_2 \text{ , } y_4 = 70\text{m})$

NO.	b_2	f	y_5	Ie	T.D.
1	$2 d_2$	$0.2 d_2$			Unsafe
2		$0.4 d_2$			Unsafe
3		$0.6 d_2$			Unsafe
4		$0.8 d_2$			Unsafe
5		$1.0 d_2$			Unsafe
6	$3 d_2$	$0.2 d_2$			Unsafe
7		$0.4 d_2$			Unsafe
8		$0.6 d_2$			Unsafe
9		$0.8 d_2$			Unsafe
10		$1.0 d_2$			Unsafe
11	$4 d_2$	$0.2 d_2$			Unsafe
12		$0.4 d_2$			Unsafe
13		$0.6 d_2$			Unsafe
14		$0.8 d_2$			Unsafe
15		$1.0 d_2$			Unsafe
16	$5 d_2$	$0.2 d_2$			Unsafe
17		$0.4 d_2$			Unsafe
18		$0.6 d_2$			Unsafe
19		$0.8 d_2$			Unsafe
20		$1.0 d_2$			Unsafe
21	$6 d_2$	$0.2 d_2$			Unsafe
22		$0.4 d_2$			Unsafe
23		$0.6 d_2$			Unsafe
24		$0.8 d_2$			Unsafe
25		$1.0 d_2$			Unsafe

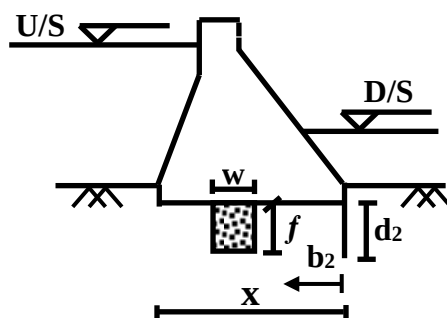


Table (C.11)
Calculated values of the total difference
 $(d_1 = 0 \text{ , } d_2 = 7.5\text{m} \text{ , } w = 0.25 d_2 \text{ , } y_4 = 70\text{m})$

NO.	b_2	f	Y_5	I_e	T.D.
1	$2 d_2$	$0.2 d_2$			Unsafe
2		$0.4 d_2$			Unsafe
3		$0.6 d_2$			Unsafe
4		$0.8 d_2$			Unsafe
5		$1.0 d_2$			Unsafe
6	$3 d_2$	$0.2 d_2$			Unsafe
7		$0.4 d_2$			Unsafe
8		$0.6 d_2$			Unsafe
9		$0.8 d_2$			Unsafe
10		$1.0 d_2$			Unsafe
11	$4 d_2$	$0.2 d_2$			Unsafe
12		$0.4 d_2$			Unsafe
13		$0.6 d_2$			Unsafe
14		$0.8 d_2$			Unsafe
15		$1.0 d_2$			Unsafe
16	$5 d_2$	$0.2 d_2$			Unsafe
17		$0.4 d_2$			Unsafe
18		$0.6 d_2$			Unsafe
19		$0.8 d_2$			Unsafe
20		$1.0 d_2$			Unsafe
21	$6 d_2$	$0.2 d_2$			Unsafe
22		$0.4 d_2$			Unsafe
23		$0.6 d_2$			Unsafe
24		$0.8 d_2$			Unsafe
25		$1.0 d_2$			Unsafe

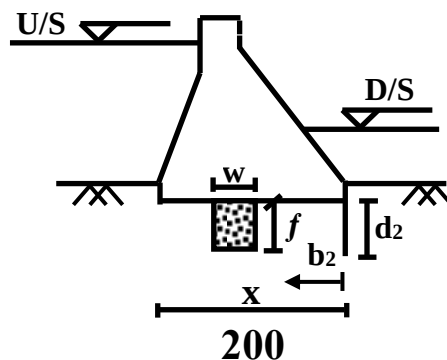


Table (C.12)
Calculated values of the total difference
 ($d_1 = 0$, $d_2 = 7.5\text{m}$, $w = 0.50 d_2$, $y_4 = 70\text{m}$)

NO.	b_2	f	Y_5	I_e	T.D.
1	$2 d_2$	$0.2 d_2$			Unsafe
2		$0.4 d_2$			Unsafe
3		$0.6 d_2$			Unsafe
4		$0.8 d_2$			Unsafe
5		$1.0 d_2$			Unsafe
6	$3 d_2$	$0.2 d_2$			Unsafe
7		$0.4 d_2$			Unsafe
8		$0.6 d_2$			Unsafe
9		$0.8 d_2$			Unsafe
10		$1.0 d_2$			Unsafe
11	$4 d_2$	$0.2 d_2$			Unsafe
12		$0.4 d_2$			Unsafe
13		$0.6 d_2$			Unsafe
14		$0.8 d_2$			Unsafe
15		$1.0 d_2$			Unsafe
16	$5 d_2$	$0.2 d_2$			Unsafe
17		$0.4 d_2$			Unsafe
18		$0.6 d_2$			Unsafe
19		$0.8 d_2$			Unsafe
20		$1.0 d_2$			Unsafe
21	$6 d_2$	$0.2 d_2$			Unsafe
22		$0.4 d_2$			Unsafe
23		$0.6 d_2$			Unsafe
24		$0.8 d_2$			Unsafe
25		$1.0 d_2$			Unsafe

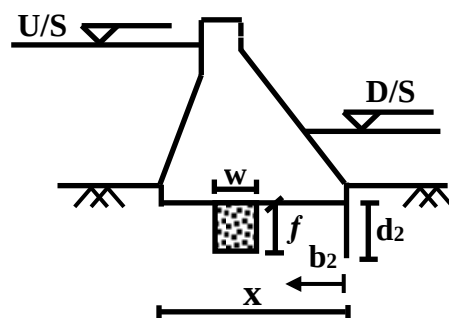


Table (C.13)
Calculated values of the total difference
 $(d_1 = 0 \text{ , } d_2 = 10\text{m} \text{ , } w = 0.25 d_2 \text{ , } y_4 = 70\text{m})$

NO.	b_2	f	Y_5	I_e	T.D.
1	$2 d_2$	$0.2 d_2$			Unsafe
2		$0.4 d_2$			Unsafe
3		$0.6 d_2$			Unsafe
4		$0.8 d_2$			Unsafe
5		$1.0 d_2$			Unsafe
6	$3 d_2$	$0.2 d_2$			Unsafe
7		$0.4 d_2$			Unsafe
8		$0.6 d_2$			Unsafe
9		$0.8 d_2$			Unsafe
10		$1.0 d_2$			Unsafe
11	$4 d_2$	$0.2 d_2$			Unsafe
12		$0.4 d_2$			Unsafe
13		$0.6 d_2$			Unsafe
14		$0.8 d_2$			Unsafe
15		$1.0 d_2$			Unsafe
16	$5 d_2$	$0.2 d_2$			Unsafe
17		$0.4 d_2$			Unsafe
18		$0.6 d_2$			Unsafe
19		$0.8 d_2$			Unsafe
20		$1.0 d_2$			Unsafe
21	$6 d_2$	$0.2 d_2$			Unsafe
22		$0.4 d_2$			Unsafe
23		$0.6 d_2$			Unsafe
24		$0.8 d_2$			Unsafe
25		$1.0 d_2$			Unsafe

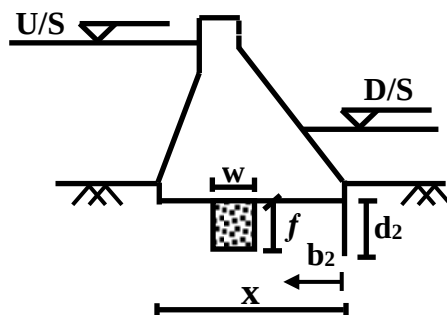


Table (C.14)
Calculated values of the total difference
 $(d_1 = 0 \text{ , } d_2 = 10\text{m} \text{ , } w = 0.50 d_2 \text{ , } y_4 = 70\text{m})$

NO.	b_2	f	Y_5	I_e	T.D.
1	$2 d_2$	$0.2 d_2$			Unsafe
2		$0.4 d_2$			Unsafe
3		$0.6 d_2$			Unsafe
4		$0.8 d_2$			Unsafe
5		$1.0 d_2$			Unsafe
6	$3 d_2$	$0.2 d_2$			Unsafe
7		$0.4 d_2$			Unsafe
8		$0.6 d_2$			Unsafe
9		$0.8 d_2$			Unsafe
10		$1.0 d_2$			Unsafe
11	$4 d_2$	$0.2 d_2$			Unsafe
12		$0.4 d_2$			Unsafe
13		$0.6 d_2$			Unsafe
14		$0.8 d_2$			Unsafe
15		$1.0 d_2$			Unsafe
16	$5 d_2$	$0.2 d_2$			Unsafe
17		$0.4 d_2$			Unsafe
18		$0.6 d_2$			Unsafe
19		$0.8 d_2$			Unsafe
20		$1.0 d_2$			Unsafe
21	$6 d_2$	$0.2 d_2$			Unsafe
22		$0.4 d_2$			Unsafe
23		$0.6 d_2$			Unsafe
24		$0.8 d_2$			Unsafe
25		$1.0 d_2$			Unsafe

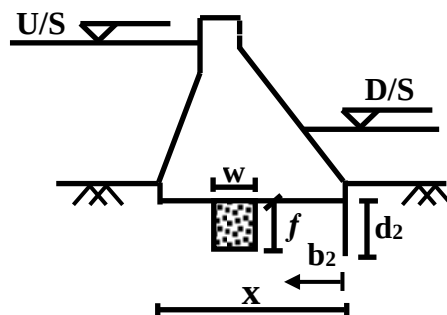


Table (C.15)
Calculated values of the total difference
 $(d_1 = d_2 = 3\text{m} \quad , \quad w = 0.25 d_1)$

NO.	b_1	f	y_4	y_5	I_e	T.D.
1	$2 d_1$	$0.2 d_1$				Unsafe
2		$0.4 d_1$				Unsafe
3		$0.6 d_1$				Unsafe
4		$0.8 d_1$				Unsafe
5		$1.0 d_1$				Unsafe
6	$3 d_1$	$0.2 d_1$				Unsafe
7		$0.4 d_1$				Unsafe
8		$0.6 d_1$				Unsafe
9		$0.8 d_1$				Unsafe
10		$1.0 d_1$				Unsafe
11	$4 d_1$	$0.2 d_1$				Unsafe
12		$0.4 d_1$				Unsafe
13		$0.6 d_1$				Unsafe
14		$0.8 d_1$				Unsafe
15		$1.0 d_1$				Unsafe
16	$5 d_1$	$0.2 d_1$				Unsafe
17		$0.4 d_1$				Unsafe
18		$0.6 d_1$				Unsafe
19		$0.8 d_1$				Unsafe
20		$1.0 d_1$				Unsafe
21	$6 d_1$	$0.2 d_1$				Unsafe
22		$0.4 d_1$				Unsafe
23		$0.6 d_1$				Unsafe
24		$0.8 d_1$				Unsafe
25		$1.0 d_1$				Unsafe

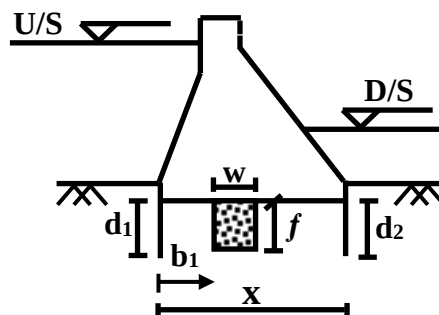


Table (C.16)
Calculated values of the total difference
 $(d_1 = d_2 = 3\text{m} \text{ , } w = 0.50 d_1)$

NO.	b_1	f	y_4	y_5	Ie	T.D.
1	$2 d_1$	$0.2 d_1$				Unsafe
2		$0.4 d_1$				Unsafe
3		$0.6 d_1$				Unsafe
4		$0.8 d_1$				Unsafe
5		$1.0 d_1$				Unsafe
6	$3 d_1$	$0.2 d_1$				Unsafe
7		$0.4 d_1$				Unsafe
8		$0.6 d_1$				Unsafe
9		$0.8 d_1$				Unsafe
10		$1.0 d_1$				Unsafe
11	$4 d_1$	$0.2 d_1$				Unsafe
12		$0.4 d_1$				Unsafe
13		$0.6 d_1$				Unsafe
14		$0.8 d_1$				Unsafe
15		$1.0 d_1$				Unsafe
16	$5 d_1$	$0.2 d_1$				Unsafe
17		$0.4 d_1$				Unsafe
18		$0.6 d_1$				Unsafe
19		$0.8 d_1$				Unsafe
20		$1.0 d_1$				Unsafe
21	$6 d_1$	$0.2 d_1$				Unsafe
22		$0.4 d_1$				Unsafe
23		$0.6 d_1$				Unsafe
24		$0.8 d_1$				Unsafe
25		$1.0 d_1$				Unsafe

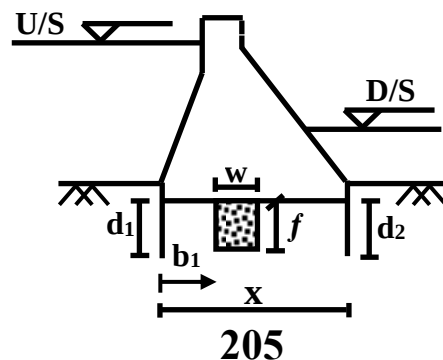


Table (C.17)
Calculated values of the total difference
 $(d_1 = d_2 = 5\text{m} \quad , \quad w = 0.25 d_1)$

NO.	b_1	f	y_4	y_5	Ie	T.D.
1	$2 d_1$	$0.2 d_1$				Unsafe
2		$0.4 d_1$				Unsafe
3		$0.6 d_1$				Unsafe
4		$0.8 d_1$				Unsafe
5		$1.0 d_1$				Unsafe
6	$3 d_1$	$0.2 d_1$				Unsafe
7		$0.4 d_1$				Unsafe
8		$0.6 d_1$				Unsafe
9		$0.8 d_1$				Unsafe
10		$1.0 d_1$				Unsafe
11	$4 d_1$	$0.2 d_1$				Unsafe
12		$0.4 d_1$				Unsafe
13		$0.6 d_1$				Unsafe
14		$0.8 d_1$				Unsafe
15		$1.0 d_1$				Unsafe
16	$5 d_1$	$0.2 d_1$				Unsafe
17		$0.4 d_1$				Unsafe
18		$0.6 d_1$				Unsafe
19		$0.8 d_1$				Unsafe
20		$1.0 d_1$				Unsafe
21	$6 d_1$	$0.2 d_1$				Unsafe
22		$0.4 d_1$				Unsafe
23		$0.6 d_1$				Unsafe
24		$0.8 d_1$				Unsafe
25		$1.0 d_1$				Unsafe

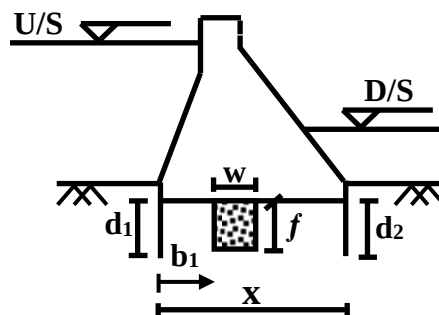


Table (C.18)
Calculated values of the total difference
 $(d_1 = d_2 = 5\text{m} \text{ , } w = 0.50 d_1)$

NO.	b_1	f	y_4	y_5	I_e	T.D.
1	$2 d_1$	$0.2 d_1$				Unsafe
2		$0.4 d_1$				Unsafe
3		$0.6 d_1$				Unsafe
4		$0.8 d_1$				Unsafe
5		$1.0 d_1$				Unsafe
6	$3 d_1$	$0.2 d_1$				Unsafe
7		$0.4 d_1$				Unsafe
8		$0.6 d_1$				Unsafe
9		$0.8 d_1$				Unsafe
10		$1.0 d_1$				Unsafe
11	$4 d_1$	$0.2 d_1$				Unsafe
12		$0.4 d_1$				Unsafe
13		$0.6 d_1$				Unsafe
14		$0.8 d_1$				Unsafe
15		$1.0 d_1$				Unsafe
16	$5 d_1$	$0.2 d_1$				Unsafe
17		$0.4 d_1$				Unsafe
18		$0.6 d_1$				Unsafe
19		$0.8 d_1$				Unsafe
20		$1.0 d_1$				Unsafe
21	$6 d_1$	$0.2 d_1$				Unsafe
22		$0.4 d_1$				Unsafe
23		$0.6 d_1$				Unsafe
24		$0.8 d_1$				Unsafe
25		$1.0 d_1$				Unsafe

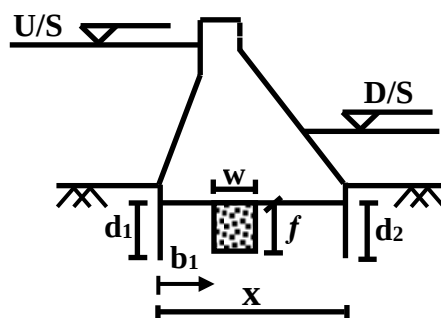


Table (C.19)
Calculated values of the total difference
 $(d_1 = d_2 = 7.5\text{m} \quad , \quad w = 0.25 d_1)$

NO.	b_1	f	y_4	y_5	I_e	T.D.
1	$2 d_1$	$0.2 d_1$				Unsafe
2		$0.4 d_1$				Unsafe
3		$0.6 d_1$				Unsafe
4		$0.8 d_1$				Unsafe
5		$1.0 d_1$				Unsafe
6	$3 d_1$	$0.2 d_1$				Unsafe
7		$0.4 d_1$				Unsafe
8		$0.6 d_1$				Unsafe
9		$0.8 d_1$				Unsafe
10		$1.0 d_1$				Unsafe
11	$4 d_1$	$0.2 d_1$				Unsafe
12		$0.4 d_1$				Unsafe
13		$0.6 d_1$				Unsafe
14		$0.8 d_1$				Unsafe
15		$1.0 d_1$				Unsafe
16	$5 d_1$	$0.2 d_1$				Unsafe
17		$0.4 d_1$				Unsafe
18		$0.6 d_1$				Unsafe
19		$0.8 d_1$				Unsafe
20		$1.0 d_1$				Unsafe
21	$6 d_1$	$0.2 d_1$				Unsafe
22		$0.4 d_1$				Unsafe
23		$0.6 d_1$				Unsafe
24		$0.8 d_1$				Unsafe
25		$1.0 d_1$				Unsafe

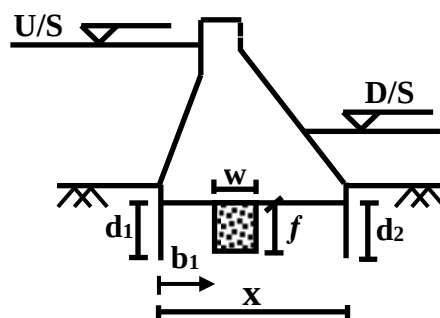


Table (C.20)
Calculated values of the total difference
 $(d_1 = d_2 = 7.5\text{m} \text{ , } w = 0.50 d_1)$

NO.	b_1	f	y_4	y_5	I_e	T.D.
1	$2 d_1$	$0.2 d_1$				Unsafe
2		$0.4 d_1$				Unsafe
3		$0.6 d_1$				Unsafe
4		$0.8 d_1$				Unsafe
5		$1.0 d_1$				Unsafe
6	$3 d_1$	$0.2 d_1$				Unsafe
7		$0.4 d_1$				Unsafe
8		$0.6 d_1$				Unsafe
9		$0.8 d_1$				Unsafe
10		$1.0 d_1$				Unsafe
11	$4 d_1$	$0.2 d_1$				Unsafe
12		$0.4 d_1$				Unsafe
13		$0.6 d_1$				Unsafe
14		$0.8 d_1$				Unsafe
15		$1.0 d_1$				Unsafe
16	$5 d_1$	$0.2 d_1$				Unsafe
17		$0.4 d_1$				Unsafe
18		$0.6 d_1$				Unsafe
19		$0.8 d_1$				Unsafe
20		$1.0 d_1$				Unsafe
21	$6 d_1$	$0.2 d_1$				Unsafe
22		$0.4 d_1$				Unsafe
23		$0.6 d_1$				Unsafe
24		$0.8 d_1$				Unsafe
25		$1.0 d_1$				Unsafe

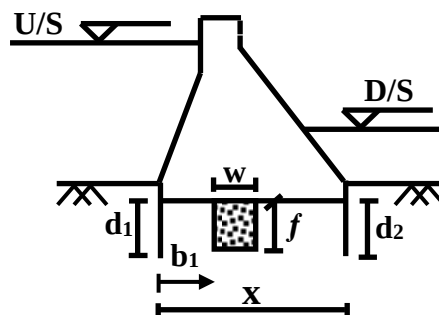


Table (C.21)
Calculated values of the total difference
 $(d_1 = d_2 = 10\text{m} \text{ , } w = 0.25 d_1)$

NO.	b_1	f	y_4	y_5	I_e	T.D.
1	$2 d_1$	$0.2 d_1$				Unsafe
2		$0.4 d_1$				Unsafe
3		$0.6 d_1$				Unsafe
4		$0.8 d_1$				Unsafe
5		$1.0 d_1$				Unsafe
6	$3 d_1$	$0.2 d_1$				Unsafe
7		$0.4 d_1$				Unsafe
8		$0.6 d_1$				Unsafe
9		$0.8 d_1$				Unsafe
10		$1.0 d_1$				Unsafe
11	$4 d_1$	$0.2 d_1$				Unsafe
12		$0.4 d_1$				Unsafe
13		$0.6 d_1$				Unsafe
14		$0.8 d_1$				Unsafe
15		$1.0 d_1$				Unsafe
16	$5 d_1$	$0.2 d_1$				Unsafe
17		$0.4 d_1$				Unsafe
18		$0.6 d_1$				Unsafe
19		$0.8 d_1$				Unsafe
20		$1.0 d_1$				Unsafe
21	$6 d_1$	$0.2 d_1$				Unsafe
22		$0.4 d_1$				Unsafe
23		$0.6 d_1$				Unsafe
24		$0.8 d_1$				Unsafe
25		$1.0 d_1$				Unsafe

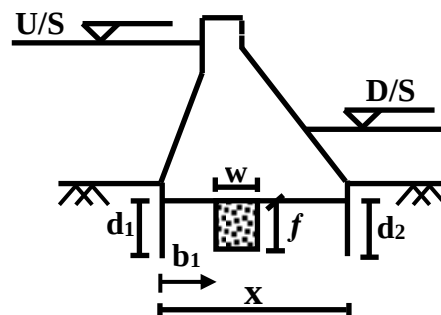
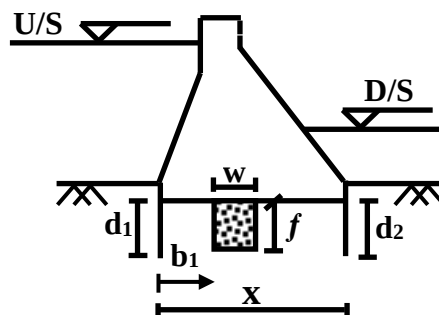


Table (C.22)
Calculated values of the total difference
 $(d_1 = d_2 = 10\text{m} \text{ , } w = 0.50 d_1)$

NO.	b_1	f	y_4	y_5	Ie	T.D.
1	2 d_1	0.2 d_1				Unsafe
2		0.4 d_1				Unsafe
3		0.6 d_1				Unsafe
4		0.8 d_1				Unsafe
5		1.0 d_1				Unsafe
6	3 d_1	0.2 d_1				Unsafe
7		0.4 d_1				Unsafe
8		0.6 d_1				Unsafe
9		0.8 d_1				Unsafe
10		1.0 d_1				Unsafe
11	4 d_1	0.2 d_1				Unsafe
12		0.4 d_1				Unsafe
13		0.6 d_1				Unsafe
14		0.8 d_1				Unsafe
15		1.0 d_1				Unsafe
16	5 d_1	0.2 d_1				Unsafe
17		0.4 d_1				Unsafe
18		0.6 d_1				Unsafe
19		0.8 d_1				Unsafe
20		1.0 d_1				Unsafe
21	6 d_1	0.2 d_1				Unsafe
22		0.4 d_1				Unsafe
23		0.6 d_1				Unsafe
24		0.8 d_1				Unsafe
25		1.0 d_1				Unsafe



الخلاصة

جرى في هذا البحث اعتبار سد ثقالي خرساني افتراضي له أرضية أفقية طولها (75 م)، وعمق الماء في المقدم والمؤخر (70 م) و(10 م) على التوالي. طُبّق تحليل الاستقرار للسد لإيجاد أداء الوسط المرشح تحت قاعدة السد بوجود قاطع رأسي في المقدم أو/و المؤخر. معايير الاستقرار التي تم أخذها بالاعتبار هي معاملات الأمان ضد الانقلاب، الانزلاق، والانجراف الداخلي، بالإضافة إلى معيار الشد من خلال التحقق من اللامركزية لمحصلة القوة العمودية. استخدمت طريقة (Galerkin) للعناصر المحددة وبرنامج (Seep2d) في تحليل التسرب.

تم استخدام عملية إيجاد الامتلية للوصول إلى التراكب الملائم الأمثل لأعماق القواطع الرأسية في المقدم والمؤخر مع (عمق ، عرض ، وموقع) الوسط المرشح الوسطي من خلال التوصل إلى الفرق الكلي الأدنى بين معاملات الأمان المسموحة والمحسوبة ضد الانقلاب، الانزلاق، والانجراف الداخلي مع المحافظة على استقرار السد.

النتائج أظهرت ما يلي:

A. من تحليل الاستقرار، بوجود وسط مرشح عمقه (f) ، عرضه (w) ، ومسافته

(b_1) أو (b_2) من مقدم أو مؤخر القاعدة على التوالي:

1. عند وجود قاطع رأسي في المقدم فقط:

- a. زيادة عمق وعرض الوسط المرشح تؤدي إلى زيادة استقرار السد.
- b. تغيير موقع بداية الوسط المرشح باتجاه المؤخر سوف يقلل استقرار السد ضد الانقلاب والانزلاق ما عدا الانجراف الداخلي، الذي سوف يزداد.
- c. زيادة عمق القاطع الراسي تؤدي إلى زيادة استقرار السد.

2. عند وجود قاطع رأسي في المؤخر فقط:

- a. زيادة عمق وعرض الوسط المرشح تنتج زيادة في استقرار السد.
- b. تغيير موقع بداية الوسط المرشح باتجاه المقدم يؤدي إلى نقصان استقرار السد ضد الانقلاب، الانزلاق، والانجراف الداخلي.

c. زيادة عمق القاطع الرأسي تؤدي إلى زيادة معامل الأمان للانجراف الداخلي بينما ستقلل معاملي الأمان للانقلاب والانزلاق.

3. عند وجود قواطع رأسية في المقدم والمؤخر:

a. زيادة عمق وعرض الوسط المرشح تؤدي إلى زيادة استقرارية السد ضد الانقلاب، الانزلاق، والانجراف الداخلي.

b. تغيير موقع بداية الوسط المرشح باتجاه المؤخر تؤدي إلى تقليل استقرارية السد ضد الانقلاب والانزلاق ولكن ليس الانجراف الداخلي، الذي سوف يزداد.

c. زيادة عمق القواطع الرأسية في المقدم والمؤخر تسبب زيادة استقرارية السد.

4. أفضل حالة هي عندما يوجد قاطع رأسي في مقدم السد بعمق ($d_1 = 7.5 \text{ m}$) مع وسط

مرشح بعمق ($f = 3 \text{ m}$) وعرض ($w = (7.5/4) \text{ m}$) وتقع بدايته على مسافة

($b_1 = 15 \text{ m}$) من جهة مقدم قاعدة السد.

B. من تحليل نتائج الامثلية:

1. أقل فرق كلي مناسب هو عند وجود وسط مرشح مع قاطع رأسي في مقدم السد فقط.

2. من خلال قيمة الفرق الكلي الأدنى:

a. أفضل عمق للقواطع الراسية في المقدم هو ($d_1 = 7.5 \text{ m}$).

b. أفضل عمق للوسط المرشح هو ($f = 3 \text{ m}$).

c. أفضل عرض للوسط المرشح هو ($w = (7.5/4) \text{ m}$).

d. أفضل موقع للوسط المرشح من جهة المقدم للأرضية هو ($b_1 = 15 \text{ m}$).