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The Optimal Dietary Decision to Treat Some Health Problems by Using a Mathematical Model

A Dissertation
Submitted to the Council of College of Education for Pure Sciences,
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the Degree of Doctor of Philosophy in Education / Mathematics.

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بِسْمِ اللهِ الرَّحْمَنِ الرَّحِيم

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فَيَمْكُثُ فِي الْأَرْضِ كَذَلِكَ يَضْرِبُ اللهُ الْأَمْثَالَ﴾

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DEDICATION

To the savior of humanity, who is the best example for us

the Prophet Muhammad

(peace and blessings of Allah be upon him and his household).

To my family (my parents, my brothers, my wife and my children).

To my friends.

To my country.

I offer this humble work ...

Karrar Habeeb

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LIST OF SYMBOLS

List of Symbols

$f(x)$	The Objective Function
$Min f(x)$	The Minimum of Objective Function
$Max f(x)$	The Maximum of Objective Function
$g(x)$	The Inequality Constraint
$h(x)$	The Equality Constraint
f_i	The Non-linear Real Functions
$\nabla f(x)$	The Gradient of f
d	The Descent Direction of f
$f^{'}(x)$	The First Derivative of f for x

$f^{"}(x)$	The Second Derivative of f for x
x^*	The Minimizer (Local, Global)
x	The Design Variable Vector
$F(x)$	The Vector of Multi-Objective Functions
A	The $n \times m$ Matrix
P	Optimization Problem
S	Slack Variable, Surplus Variable
R	Real Numbers
R^n	Dimensional Space

ABBREVIATIONS LIST

Abbreviations List

LP	Linear Programming
NLP	Non-Linear Programming
$GLPP$	General Linear Programming Problem
$GNLPP$	General Non-Linear Programming Problems
NP	Non Deterministic Polynomial Time
GCM	Global Criterion Method

Abstract

The right diet and the selection of appropriate food has a significant impact on health by maintaining a balanced body weight and strengthening the immune system. Healthy diets that help the body maintain its health and fitness are essential in our daily life.

By using the mathematical model of linear programming, the diet problem is formulated as a linear program with a linear objective function and linear constraints.

This thesis discusses four food systems, each of which has an effective importance in maintaining the health and fitness of the human body. Each of these systems has been discussed on a specific gender with specific ages.

The simplex method had the largest role in solving these systems, in addition to using some methods such as weighted-sums method, global criterion method, and the augmented lagrangian method. The main goal is to choose a group of foods that meet a set of daily nutritional requirements at the lowest cost.

INTRODUCTION

Introduction

Healthy diets are important in our daily lives, especially if they are accompanied by some exercise that can help the body maintain its health [1, 2]. There are many diets that meet a wide range of goals, such as dieting to lose weight [3] or control to some diseases such as coronary heart disease [4], kidney disease [5] and diabetes [6].

Many people suffer from some chronic diseases such as high blood pressure and other diseases that are directly related to weight gain and obesity that is caused by the accumulation of fat in the body due to the lack of sport activity in addition to unhealthy diets [7].

Also, in recent decades, the prevalence of type 2 diabetes has increased dramatically, and this has side effects on patients in terms of their normal lifestyle, as it can significantly increase some of the diseases that accompany it, such as cardiovascular diseases, as well as its effect on the eyes and kidneys [8, 9].

Linear programming plays an important role in solving optimization problems through the use of linear objective functions with a set of linear constraints that can appear in the case of equality or inequality for the purpose of meeting the constraints by finding the best solutions [10, 11].

It is known that the linear programming model of the diet has its limits, as is the case in any model, but the important goal remains at the lowest cost to meet the daily nutritional need of people [12, 13].

At the same time, it needs to be noted that there can be negative side effects from excessive intake of nutrients, such as increased fiber, which can cause some health problems such as diarrhea and cramps, as well as excess magnesium can cause tachycardia and slow breathing, in addition to some negatives that can to infect people by increasing the rest of the elements [14].

From this standpoint comes the need to create a healthy and balanced food model that meets the daily nutritional need of the human being as a theoretical way to deal with decision-making problems with multiple goals [15, 16]. Many researchers have presented plans for balanced diets to reduce eating disorders and meet nutritional requirements as well as the need to avoid some chronic diseases such as diabetes, heart attacks and high blood pressure [17].

This work focuses on the diet problem and how to use some healthy food models at the lowest cost and contains eight chapters.

Chapter one, discusses some important definitions that deal with constrained optimization and some diets.

Chapter two, discusses the linear and non-linear Programming and some of the methods used to solve them.

Chapter three, discusses the diet problem

Chapter four, discusses a healthy dietary model that helps reduce the risk of developing high blood pressure for males aged 31-50 years.

Chapter five, discusses a healthy and balanced diet model for type 2 diabetics that can regulate blood glucose and insulin secretion for females aged 31-50 years.

Chapter six, a healthy and balanced keto diet model for males aged 31-50 years, the model has multiple goals of finding the best solutions that include the lowest cost, the lowest amount of carbohydrates and the highest percentage of fat, accompanied by several linear restrictions to meet the individual needs of nutrients.

Chapter seven, discusses a healthy, low-fat diet model for females aged 19-50 years, the model has multiple objectives including lower cost and lower fat content with a number of attendant limitations.

Chapter eight, includes conclusions and future work.

Note:

All the calculations and the experiments are made in this thesis have been done in a **COMPUTER**,[1] with writing some codes in the Python program to reach the solution.

[1] All the experiments are running on a computer with 4 GB of RAM, Core(TM) i5-2410M and CPU 2.30 GHz processor.

CHAPTER 1

SOME IMPORTANT DEFINITIONS

In this chapter, some important definitions will be presented that can be used in the next chapters.

Definition 1.0.1. Optimization [18, 19].

Optimization means finding the perfect solution to a particular problem. Mathematically, it means finding the minimum or maximum value of the objective function of n variables, $f(x_1, x_2, ..., x_n)$ where $n > 0$.

Definition 1.0.2. The Objective Function [20].

The objective function expresses the main aim of the model which is either to be minimized or maximized.

Definition 1.0.3. Differentiable Function [19, 21].

Let f be any function, we say that f is a differentiable function if its derivatives exist at every point of its domain.

Definition 1.0.4. Smooth Function [19, 21].

A function f is said to be a smooth function if its derivatives are continuous over some domain.

Definition 1.0.5. Non-Smooth Function [19, 21].

Non-smooth functions include discontinuous and non-differentiable functions.

Definition 1.0.6. A Multi-Objective Optimization Problem [22].

Consider an optimization problem with equality or inequality constraints :

$$(P)\begin{cases} \text{Minimize, Maxmize } F(x) = f_1(x), f_2(x), ..., f_k(x) \\ \quad \text{Subject to :} \\ \qquad g_j(x) \geq 0, & j = 1, 2, ..., m \\ \qquad h_i(x) = 0, & i = 1, 2, ..., n \\ \qquad\; x \in Q. \end{cases} \tag{1.1}$$

Where x denotes the design variable vector, $F(x)$ denotes to the vector of multi-objective functions, k represents the number of objective functions, $g_j(x)$ and $h_i(x)$ are the inequality and equality constraints, respectively and Q represents the design domain.

The above problem represents a multi-objective optimization problem, through which it is possible to find the best possible solutions to the decision variables that achieve the minimum or maximum of the multi-objective optimization problem simultaneously.

Definition 1.0.7. Constrained Optimization [21, 23].

Consider the following generally constrained problem of optimization :

$$(P)\begin{cases} \text{Maximize } f(x_1, x_2, ..., x_n) \\ \text{Subject to :} \\ g_1(x_1, x_2, ..., x_n) \leq b_1, ..., g_j(x_1, x_2, ..., x_n) \leq b_j & j = 1, 2, ..., m \\ h_1(x_1, x_2, ..., x_n) = c_1, ..., h_i(x_1, x_2, ..., x_n) = c_i & i = 1, 2, ..., n \end{cases} \tag{1.2}$$

where $f(x)$ is the objective function, $g(x)$ represents a constraint of inequality, and $h(x)$ represent a constraint of equality, such that f, g and h are differentiable functions and their derivatives are continuous.

Definition 1.0.8. Unconstrained Optimization [21, 23].
Consider the problem of minimizing $f(x)$ that relies on real variables without limiting their values, where $f(x)$ is an objective function. Mathematically, let x be a real vector such that $x \in R^n$ with $n \geq 1$ components and let f be a smooth function, where $f : R^n \to R$. Then, the problem of unconstrained optimization is indicated by:

$$min_x \quad f(x) \tag{1.3}$$

In some applications, unconstrained optimization problems can arise directly and can also arise indirectly through rework processes of constrained optimization problems.

Definition 1.0.9. The Gradient Operator [19, 21].
Suppose that $f : R^n \to R$ be a differentiable function, then we denote to the gradient operator of f by ∇f and defined by:

$$\bigtriangledown f(x) = \begin{bmatrix} \dfrac{\partial f}{\partial x_1} \\ \dfrac{\partial f}{\partial x_2} \\ \vdots \\ \dfrac{\partial f}{\partial x_n} \end{bmatrix} \tag{1.4}$$

Definition 1.0.10. Descent Direction [18, 19, 21].
Suppose that $f : R^n \to R$ be a differentiable at $x \in R^n$, and let $d \in R^n$ be a vector, then we say that d is a descent direction of f at x if its satisfy $\nabla f(x)^T d < 0$.

Definition 1.0.11. Iterative Methods [19].

The iterative method is a mathematical procedure repeated over and over again in order to get the solution to the system of equations or to find the root of the equation.

Definition 1.0.12. The Neighbourhood [19] .

A neighbourhood of a point $x \in R^n$ is the set:

$$\{y \in R^n : \| y - x \| < \epsilon\} \tag{1.5}$$

where ϵ is positive number.

Definition 1.0.13. Global Minimizer [18, 19, 21].

Let F be a feasible set, we say that a point $x^* \in F$ is a global minimizer if $f(x^*) \leq f(x)$, for all $x \in F$.

Also x^* is said to be a strict global minimizer if $f(x^*) < f(x)$, for all $x \in F$, and $x \neq x^*$.

Definition 1.0.14. Local Minimizer [18, 19, 21].

Let F be a feasible set, we say that a point $x^* \in F$ is local minimizer if there is an open neighbourhood U of x^* such that $f(x^*) \leq f(x)$, for $x \in U$.

Also x^* is said to be a strict local minimizer if $f(x^*) < f(x)$, for all $x \in U$, and $x \neq x^*$.

Definition 1.0.15. Solution [18, 24, 25].

A set of values $x_1, x_2, \ldots, x_n$ which satisfies the constraints in the linear programming problem is called solution.

Definition 1.0.16. Feasible Solution [20, 24, 25].

Any solution to the linear programming problem which satisfies the non-negativity constraints of the linear programming problem is called feasible solution.

Definition 1.0.17. Optimum Solution [18, 24].

Any feasible solution which optimizes (Minimizes or Maximizes) the objective function of the linear programming problem is called optimum solution.

Definition 1.0.18. Unbounded Solutions [18, 24, 25].

When the value of the objective function increased or decreased indefinitely, then the solutions are called unbounded solutions.

Definition 1.0.19. Basic Variables and Non-Basic Variables [18, 24, 25] .

Given a system of m linear equations with n variables $(m < n)$, then any solution which is obtained by solving m variables keeping the remaining $n-m$ variables zero is called a basic solution. Such m variables are called basic variables and the remaining variables are called non-basic variables.

Definition 1.0.20. Basic Feasible Solution [18, 24, 25].

A basic feasible solution is a basic solution which also satisfies the non-negativity constraints of the linear programming problem, that is all basic variables are non-negative.

Definition 1.0.21. Basic Variables [18, 24, 25].

The variables which coefficients one in the equations and zero in the other equations are called basic variables.

Definition 1.0.22. Non-Basic Variables [18, 24, 25].

The variables that take any value from the coefficients, whether they are positive, negative, or zero are called non-basic variables

Definition 1.0.23. Slack Variables [18, 23, 24].

If the constraints of a general linear programming problem in the form:

$$\sum_{j=1}^{n} a_{ij}x_i \leq b_i \ (i = 1, 2, ..., m), \tag{1.6}$$

Then the variables S_i which are introduced to convert the inequalities $(\leq)$ to the equalities

$$\sum_{j=1}^{n} a_{ij}x_i + S_i = b_i \ (i = 1, 2, ..., m), S_i \geq 0 \tag{1.7}$$

are called slack variables.

Definition 1.0.24. Surplus Variables [18, 23, 24].

If the constraints of a general linear programming problem in the form :

$$\sum_{j=1}^{n} a_{ij}x_i \geq b_i \ (i = 1, 2, ..., m), \tag{1.8}$$

Then the variables S_i which are introduced to convert the inequalities ($\geq$) to the equalities

$$\sum_{j=1}^{n} a_{ij}x_i - S_i = b_i \ (i = 1, 2, ..., m), S_i \geq 0 \tag{1.9}$$

are called surplus variables.

Definition 1.0.25. Artificial Variables [18, 23, 24].

The variables which are add to the left side of each of the equations corresponding to constraints of the type $\geq or =$ are called artificial variables.

Definition 1.0.26. DASH Plan (Dietary Approaches to Stop Hypertension) [26, 27]

It is a plan developed to eat foods that help reduce blood pressure and maintain heart health, as this plan has proven successful in reducing blood pressure and levels of harmful cholesterol, which led to a reduction in the risk of cardiovascular disease. This line has specific calorie levels, is low in saturated fat, and is rich in potassium, calcium, and magnesium, as well as fiber and protein.

Definition 1.0.27. Diabetes [26, 27].

Diabetes occurs as a result of a metabolic disorder that is considered the way that the body uses digested food (specifically carbohydrate) for growth and energy.In diabetes, the pancreas either produces little or no insulin (a hormone that helps glucose, the body's main source of fuel, get into cells), or the cells do not respond appropriately to the insulin that is produced, which causes too much glucose to be released in the blood. The three main types of diabetes are type 1, type 2, and gestational diabetes.If not controlled, diabetes can lead to serious complications.

Definition 1.0.28. Nutrient [26, 27].

Some of the requirements that help the natural body to perform its functions, such as fatty acids, an amino acid, a vitamin, or a mineral, which must be obtained from a food source because the body cannot manufacture them at all, or the amount of their manufacture is insufficient for good health.

Definition 1.0.29. Health [26, 27].

A state of complete physical, mental, and social well-being and not merely the absence of disease or infirmity.

Definition 1.0.30. Hypertension [26, 27].

A condition, also known as high blood pressure. Hypertension makes the heart work too hard, and the high force of the blood flow can harm arteries and organs, such as the heart, kidneys, brain, and eyes. High blood pressure is usually caused by low potassium levels, High percentage of saturated fats as well as eating large amounts of salts.

Definition 1.0.31. Carbohydrates [26, 27].

One of the macro-nutrients and a source of energy. They include sugars, starches, and fiber.

Definition 1.0.32. Calorie [26, 27].

It is a unit through which the energy content of some foods and beverages is measured, in addition to the body's use of the energy required to maintain the various functions of the body, including metabolic processes and physical activity. Carbohydrate, fat, protein, and alcohol provide all of the energy supplied by foods and beverages.

Definition 1.0.33. Starches [26, 27].

It is the association of a number of glucose units in long chains, which is represented by foods that contain starch for vegetables such as carrots and potatoes, some grains such as rice brown, oats, wheat, barley, corn, and some legumes such as beans and peas.

Definition 1.0.34. Blood Cholesterol [26, 27].

Distinct particles containing both lipids and proteins (lipoproteins). Also referred to as serum cholesterol.

Definition 1.0.35. Fats [26, 27].

One of the macro-nutrients and a source of energy.

Definition 1.0.36. Protein [26, 27].

It is one of the macro-nutrients of the body that can be considered as a major function and structural component of animal cells. Where proteins are made up of amino-acids, which humans cannot synthesize and therefore must be obtained from the diet.

CHAPTER 2

LINEAR AND NON-LINEAR PROGRAMMING

2.1 Linear Programming (LP)

linear programming is a model that used for optimum allocation of scarce or limited resources to competing products or activities under such assumptions as certainty, linearity, fixed technology, and constant profit per unit [25].

Linear programming is considered one of the most versatile and powerful techniques in terms of its use as a decision-making tool, which has proven its value in many areas such as finance, production, marketing and others. It can also be used to solve many problems that arise in some commercial or industrial businesses, transportation schedules and allocation problems [24, 25].

Linear programming deals with the optimization (maximization or minimization) of a function of variables known as objective functions. It is subject to a set of linear equalities and/or inequalities known as constraints [24].

Definition 2.1.1. Linear programming [23].
Linear programming is a mathematical technique which involves the allocation of limited resources in an optimal manner on the basis of a given criterion of optimality.

2.1.1 Properties of a Linear Programming Model

The properties of a linear programming model are [23, 25]:

1- The relationship between variables and constraints must be linear.

2- The model must have an objective function.

3- The model must have structural constraints.

4- The model must have non-negativity constraint.

2.1.2 General Linear Programming Problem (GLPP)

The general form of a linear programming problem is as given below [25]:

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \quad \rightarrow \quad \textit{Objective Function}$$

Subject to the Conditions :

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \ (\geq, =, \leq) \ b_1; \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \ (\geq, =, \leq) \ b_2; \quad \rightarrow \quad \textit{Structural Constraints} \\ \qquad\qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \ (\geq, =, \leq) \ b_m; \\ \qquad x_1, x_2, \ldots, x_n \geq 0 \qquad \rightarrow \quad \textit{Non-Negative Constraints.} \end{cases} \tag{2.1}$$

Where all c_i, a_{ji}, b_j are constants and x_i is are decision variable, such that $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

2.1.3 The Standard Form of a Linear Programming

The standard form of a linear programming problem is as given below [18, 20]:

$$\begin{cases} \quad Z = \ cx \quad \rightarrow \ Objective\ Function \\ Subject\ to\ the\ Conditions: \\ \quad Ax = b; \quad \rightarrow \ Structural\ Constraints \\ \quad x \geq 0 \quad \rightarrow \ Non-Negative\ Constraints. \end{cases} \tag{2.2}$$

Where A is $n \times m$ matrix, with $n > m$, $b \in R^m$, $c \in R^n$.

2.2 Duality in Linear Programming Model

For every linear programming problem there (called the primal) is a corresponding dual linear programming problem and this problem arises from the cost and constraints function of the original problem as it is a linear problem, and therefore it can be solved by simplex method [23].

The need for duality comes to the point that the solution is easier in most cases such that when the primal contains a smaller number of variables and a large number of constraints, then the computation can be considerably reduced by converting it in to the dual problem and then solving it, and also solving the problem by finding a bound [18].

2.2.1 Formulation of Dual Problem

The formulation of the dual problem is [23]:

1- The minimization of the objective function in the primal is changed to the maximization in the dual and vice versa.

2- The number of variables in the primal is the number of constraints in the dual and vice versa.

3- The values of $C_1, C_2, \ldots, C_n$ which represent the cost coefficients in the objective function of the primal is the RHS constant of constraints in the dual and vice versa.

4- We consider the transpose of the body matrix of the primal problem when forming the constraints for the dual.

5- For both problems the variables are non-negative.

Notes [20, 23] :

1- If the objective function in minimization form in the primal then When we want to convert to dual, all constraints must be of type $\geq$, if one or more of constraints of type $\leq$ then we multiply it by (-1) to make it $\geq$.

2- If the objective function in maximization form in the primal then When we want to convert to dual, all constraints must be of type $\leq$, if one or more of constraints of type $\geq$ then we multiply it by (-1) to make it $\leq$.

3- If one or more of constraints of type $=$ then we convert it into two constraints one of type $\geq$ and the other of type $\leq$.

4- Primal of a primal is primal.

5- Dual of a dual is primal.

6- Primal of a dual is primal.

7- Dual of a primal is dual.

8- Dual of a dual of a dual is primal.

2.2.2 The Definition of Dual Problem

The definition of the dual problem is as follow [20]:

Let the primal problem be:

$$\begin{cases} \textit{Minimize } Z \;=\; c_1x_1 + c_2x_2 + \ldots + c_nx_n \\ \textit{Subject to the Conditions :} \\ \qquad a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \geq b_1; \\ \qquad a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2; \\ \qquad \vdots \\ \qquad a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \geq b_m; \end{cases} \tag{2.3}$$

$$x_1, x_2, \ldots, x_n \geq 0$$

Then we define the dual problem by :

$$\begin{cases} \textit{Maximize } Z' \;=\; b_1y_1 + b_2y_2 + \ldots + b_ny_n \\ \textit{Subject to the Conditions :} \\ \qquad a_{11}y_1 + a_{21}y_2 + \ldots + a_{m1}y_m \leq c_1; \\ \qquad a_{21}y_1 + a_{22}y_2 + \ldots + a_{m2}y_m \leq c_2; \\ \qquad \vdots \\ \qquad a_{1n}y_1 + a_{2n}y_2 + \ldots + a_{mn}y_m \leq c_n; \end{cases} \tag{2.4}$$

$$y_1, y_2, \ldots, y_m \geq 0$$

Example 2.2.1. For the following primal LP problem, write the dual problem:

$$Min\ Z = 3x_1 + x_2 + 5x_3$$

Subjects to the constraints,

$$x_1 + 2x_2 \geq 5$$

$$2x_1 + x_2 + 4x_3 \leq 9$$

$$3x_1 + x_2 + 2x_3 \geq 6$$

$$x_1, x_2, x_3 \geq 0$$

Solution: The problem above is a minimization problem, so all constraints must be of type $\geq$ when converting to the dual, so we multiply the second constraint by (-1), then the problem will be as follows:

$$Min\ Z = 3x_1 + x_2 + 5x_3$$

Subjects to the constraints,

$$x_1 + 2x_2 \qquad\qquad \geq 5$$

$$-2x_1 - x_2 - 4x_3 \geq -9$$

$$3x_1 + x_2 + 2x_3 \geq 6$$

$$x_1, x_2, x_3 \geq 0$$

The Dual: Let y_1, y_2, y_3 be the dual variables, then the dual of the problem is:

$$Max\ Z^{'} = 5y_1 - 9y_2 + 6y_3$$

Subjects to the constraints,

$$y_1 - 2y_2 + 3y_3 \leq 3$$

$$2y_1 - y_2 + y_3 \leq 1$$

$$-4y_2 + 2y_3 \geq 5$$

$$y_1, y_2, y_3 \geq 0$$

2.3 Some Methods for Solving Linear Programming Problem

The goal for solve linear programming Problem may be either to reduce the cost or maximize the return, and one of the most important methods that used to solve linear programming Problem are:

1- Graphical Method

2- Simplex Method

2.3.1 Graphical Method

The graphical method is usually used when we have two decision variables where straight lines are drawn in a two-dimensional plane [24]. In addition, all decision variables must always have positive values because there are non-negative constraints in the problem where the solution is limited to the first quadrant of the graph with note that it is possible that the value of the variables will be in the rest of the quarters, and here this case is treated by extending the line connecting the values of the variables to reach the first quadrant [25].

The Steps for Solving LPP by Graphical Method [24, 25].

1- Convert each constraint of the inequality into an equation.

2- Each equation is drawn in a straight line in the graph by choosing the value of the first variable zero and finding the value of the second variable and vice versa.

3- Mark the region as follows:

i- If the inequality constraint corresponding to the line is in the form $\leq$, then the area below the line in the first quadrant is shaded.

ii- If the inequality constraint corresponding to the line is in the form $\geq$, then the area above the line in the first quadrant is shaded.
The obtained common area is called the feasible region and the points in it will satisfy all constraints simultaneously.

4- The optimal values will always be at the corner points of the feasible region, and therefore we will be limited to calculating the value of the objective function for the corner points of the region and determining the value of the optimal solution in the case of maximum or minimum value of Z.

2.3.2 Simplex Method

Simplex method is an iterative procedure and the most successful method for finding the optimal solution to the linear programming problem models in a finite number of steps which contain two or more decision variables [18, 24].

Simplex Algorithm [23–25].

The existence of an initial basic feasible solution is always assumed for the solution of any linear programming problem by using simplex algorithm. The steps for the computation of an optimum solution of the simplex method are as follows:

1- Check that each value of b_i $(i = 1, 2, \ldots, m)$ are positive. If any one of b_i is negative then multiply the in-equation of the constraint by (-1) so as to get all bi to be positive.

2- Express the problem in standard form by converting the inequality constraints into equality by introducing slack/surplus variables.

3- Obtain an initial basic feasible solution X_B to the problem in the form $X_B = B^{-1}b$, where B is the basis matrix formed by the column of basic variables.

4- Put the initial simplex table as follows:

Table 2.1: Initial Simplex Table

C_B	S_B	C_j X_B	C_1 X_1	C_2 X_2	$\ldots C_n$ $\ldots X_n$	$0\ 0 \ldots 0$ $S_1 S_2 \ldots S_n$
C_{B1}	S_1	b_1	a_{11}	a_{12}	$\ldots a_{1n}$	$1\ 0 \ldots 0$
C_{B2}	S_2	b_2	a_{21}	a_{22}	$\ldots a_{2n}$	$0\ 0 \ldots 1$

Where C_B is the cost vector associated with the basic feasible solution X_B, C_j is the cost vector associated with the objective function variables.

5- Compute $Z_j - C_j$ that represents the net evaluations row by:

$$Z_j - C_j = C_B a_j - c_j,$$

and then we check the following:

i- In the case of a maximization objective function, if all $Z_j - C_j \geq 0$ then the initial basic feasible solution X_B to the problem is an optimum basic feasible solution, and if at least one $Z_j - C_j < 0$ then proceed to the next step as the solution is not optimal.

ii- In the case of a minimization objective function, if all $Z_j - C_j \leq 0$ then the initial basic feasible solution X_B to the problem is an optimum basic feasible solution, and if at least one $Z_j - C_j > 0$ then proceed to the next step as the solution is not optimal.

6- Determined the entering variable (key column) by using the optimality condition which states:

i- The entering variable in a minimization (maximization) problem is the non-basic variable having the most positive (negative) coefficient in the net evaluations row.

ii- When all the net evaluations row coefficient of the non-basic variables are non-positive (non-negative) then the optimum is reached.

Stop if there is no entering variable.

7- Determined the leaving variable (key raw) by using the feasibility condition which states:

(For both minimization and maximization problems, the leaving variable is the basic variable that is associated with the smallest non-negative ratio (with strictly positive denominator)), that is:

$$leaving\ variable = min\ \frac{X_{Bi}}{X_{ir}}, for\ some\ j = r,\ (i = 1, 2, \ldots, m), and\ X_{ir} > 0$$

8- pivot row calculation:

i- In the basic column, the leaving variable is replaced by the entering variable.

ii- $New\ pivot\ row = \dfrac{the\ current\ pivot\ row}{the\ pivot\ element}$

iii- The all other rows:

$New\ row = current\ row - (pivot\ column\ coefficient)\ *\ new\ pivot\ row$

9- The procedure is repeated by moving to step 5 in order to obtain the optimal solution or there is an indication that the solution is unbounded.

Notes [18, 23, 24]:

1- If all $X_{ir} \leq 0$, $(i = 1, 2 \ldots m)$ then the given problem having unbounded solution.

2- If at least $X_{ir} \geq 0$,$(i = 1, 2 \ldots m)$ then the corresponding vector X_r enters the basis.

3- In minimization (maximization) problem if there are more than one variables having the same most positive (negative) of $Z_j - C_j$ then any one of the variable can be selected arbitrarily as the entering variable.

Example 2.3.1. Use the simplex method to solve linear programming model:

$$Min\ Z = 3x_1 + 7x_2 - 4x_3$$

Subjects to the constraints,

$$x_1 + 2x_2 + x_3 \leq 100$$

$$x_1 + x_2 + 2x_3 \leq 50$$

$$2x_1 + x_2 \qquad \leq 70$$

$$x_1, x_2, x_3 \geq 0$$

Solution: convert the problem into the standard form by introducing the slack variables S_1, S_2, S_3

$$Min\ Z = 3x_1 + 7x_2 - 4x_3 + 0S_1 + 0S_2 + 0S_3$$

Subjects to the constraints,

$$x_1 + 2x_2 + x_3 + S_1 = 100$$

$$x_1 + x_2 + 2x_3 + S_2 = 50$$

$$2x_1 + x_2 \qquad + S_3 = 70$$

$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$$

An initial basic feasible solution is given by:

$$X_B = B^{-1}b \text{ , where } B = I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , X_B = (S_1, S_2, S_3)$$

i.e., $(S_1, S_2, S_3) = I_3\ (100, 50, 70) = (100, 50, 70)$.

Table 1:

C_B	S_B	C_j / X_B	3 / x_1	7 / x_2	-4 / x_3	$0 \quad 0 \quad 0$ / $S_1 \quad S_2 \quad S_3$	$\min \frac{X_B}{X_3}$
0	S_1	100	1	2	1	1 0 0	$\frac{100}{1} = 100$
0	S_2	50	1	1	(2)	0 1 0	$\frac{50}{2} = 25$ →
0	S_3	70	2	1	0	0 0 1	...
	$Z = 0$	Z_j	0	0	0	0 0 0	
		$Z_j - C_j$	-3	-7	4 ↑	0 0 0	

Since the positive maximum of $Z_j - C_j$ is (4) and its column index is (3). So, the entering variable is x_3, and the minimum ratio is 25 and its row index is 2. So, the leaving basis variable is S_2.

Then the pivot element is (2).

Now:

$$New\ x_3\ row = \frac{old\ S_2\ row}{2}.$$

$$New\ S_1\ row = old\ S_1\ row - (1)\ new\ x_3\ row.$$

$$New\ S_3\ row = old\ S_3\ row - (0)\ new\ x_3\ row.$$

Table 2:

C_B	S_B	C_j X_B	3 x_1	7 x_2	-4 x_3	0 0 0 S_1 S_2 S_3	**Min Ratio**
0	S_1	75	0.5	1.5	0	1 -0.5 0	
-4	x_3	25	0.5	0.5	**1**	0 0.5 0	
0	S_3	70	2	1	0	0 0 1	
	$Z = -100$	Z_j	-2	-2	-4	0 -2 0	
		$Z_j - C_j$	-5	-9	0	0 -2 0	

Since all $Z_j - C_j \leq 0$

Hence, optimal solution is arrived with value of variables as :

$x_1 = 0,\ x_2 = 0,\ x_3 = 25,\ S_1 = 75,\ S_2 = 0,\ S_3 = 70,\ min\ Z = -100.$

Flow Chart for the Simplex Method [25]:

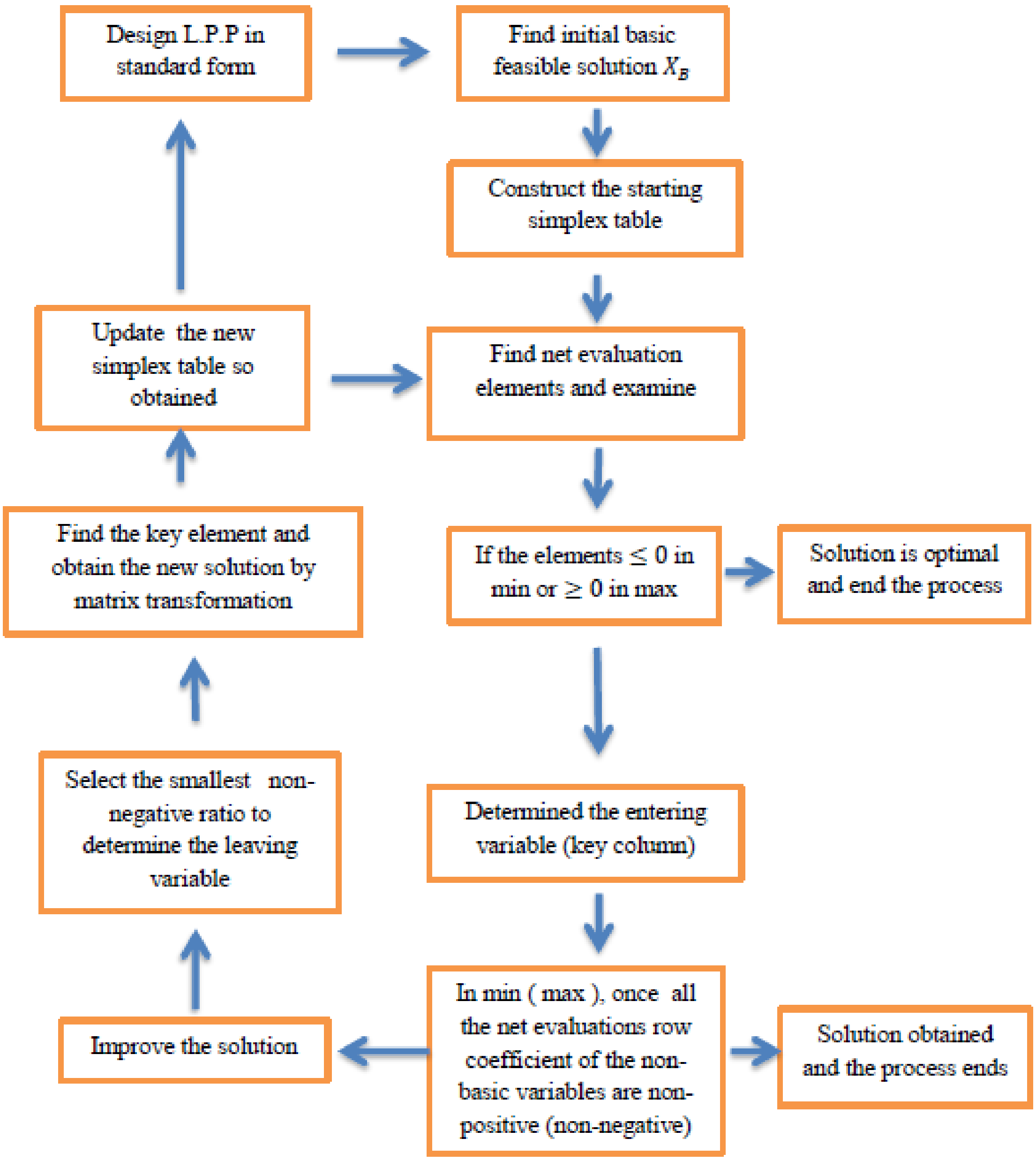

Figure 2.1: Flow Chart for the Simplex Method

Big M Method

Big M method is a method used to solve a linear programming problem by using the simplex algorithm. This method extends the simplex algorithm to problems that contain ($\geq$ *or* $=$) constraints by linking the constraints to large constants (artificial variables) that should be eliminated in the optimal solution if exists by assigning a very large penalty ($-M$ for maximization and M for minimization) in the objective function [20, 24].

Steps of Using the Big M Method [18, 20]:

1- Express the problem in the standard form.

2- For each constraints of the type ($\geq$ *or* $=$) add non-negative artificial variables to the left side.

3- Use the simplex method to solve the modified linear programming problem, then the following is noticed:

i- In the basis and the optimality, if no artificial variable appears and the conditions are satisfied, then it can be considered that the current solution is an optimal basic feasible solution.

ii- At zero level, if at least one artificial variable in the basis and the optimality condition is satisfied, then it can be considered that the current solution is an optimal basic feasible solution.

iii- At positive level, if at least one artificial variable appears in the basis and the optimality condition is satisfied, then there is no feasible solution to the original problem.

Example 2.3.2. Use the big M method to solve linear programming model:

$$Min\ Z = 2x_1 + 5x_2$$

Subjects to the constraints,

$$x_1 + 2x_2 \geq 3$$

$$x_1 + x_2 \leq 7$$

$$x_1, x_2, \geq 0$$

Solution:

convert the problem into the standard form by introducing surplus variable S_1 the slack variables S_2 and artificial variable A_1.

$$Min\ Z = 2x_1 + 5x_2 + 0S_1 + 0S_2 + MA_1$$

Subjects to the constraints,

$$x_1 + 2x_2 - S_1 + A_1 = 3$$

$$x_1 + x_2 + S_2 = 7$$

$$x_1, x_2, , S_1, S_2, A_1 \geq 0$$

An initial basic feasible solution is $A_1 = 3, S_2 = 7$.

Table 1:

C_B	B	C_j / X_B	2 / x_1	5 / x_2	0 0 M / S_1 S_2 A_1	$\min \frac{X_B}{X_2}$
M	A_1	3	1	(2)	-1 0 1	$\frac{3}{2} = 1.5 \rightarrow$
0	S_2	7	1	1	0 1 0	$\frac{7}{1} = 7$
	$Z = 3M$	Z_j	M	$2M$	$-M$ 0 M	
		$Z_j - C_j$	$M - 2$	$2M$-5 $\uparrow$	$-M$ 0 0	

Since the positive maximum of $Z_j - C_j$ is $(2M - 5)$ and its column index is (2). So, the entering variable is x_2, and the minimum ratio is (1.5) and its row index is 1. So, the leaving basis variable is A_1.

Then the pivot element is (2).

Now:

$$New\ x_2\ row = \frac{old\ A_1\ row}{2}.$$

$$New\ S_2\ row = old\ S_2\ row - (1)\ new\ x_2\ row.$$

Table 2:

C_B	B	C_j X_B	2 x_1	5 x_2	0 S_1	0 S_2	$\min \frac{X_B}{X_1}$
5	x_2	1.5	(0.5)	**1**	-0.5	0	$\frac{1.5}{0.5} = 3 \rightarrow$
0	S_2	5.5	0.5	0	0.5	1	$\frac{5.5}{0.5} = 11$
	$Z = 7.5$	Z_j	2.5	5	-2.5	0	
		$Z_j - C_j$	0.5 $\uparrow$	0	-2.5	0	

Since the positive maximum of $Z_j - C_j$ is (0.5) and its column index is (1). So, the entering variable is x_1, and the minimum ratio is (3) and its row index is 1. So, the leaving basis variable is x_1.

Then the pivot element is (0.5).

Now:

$$New\ x_1\ row = \frac{old\ x_2\ row}{0.5}.$$

$$New\ S_2\ row = old\ S_2\ row - (0.5)\ new\ x_1\ row.$$

Table 3:

C_B	B	C_j X_B	2 x_1	5 x_2	0 S_1	0 S_2	**Min Ratio**
2	x_1	3	**1**	**2**	**-1**	0	
0	S_2	4	0	-1	1	1	
	$Z=6$	Z_j	2	4	-2	0	
		$Z_j - C_j$	0	**-1**	-2	0	

Since all $Z_j - C_j \leq 0$

Hence, optimal solution is arrived with value of variables as :

$x_1 = 3,\ x_2 = 0,\ min\ Z = 6.$

2.4 Non-Linear Programming (NLP)

It is a mathematical technique used to determine the optimal solution to many problems. Where this name indicates that the non-linear programming problem can be solved through a series of solutions to the linear programming problem [19].

Noting that the optimal solution to the linear programming problem will be at a corner point, while the optimal solution to the non-linear programming problem will be anywhere within or along the boundaries of the feasible region [22].

2.4.1 General Non-Linear Programming Problem (GNLPP)

[19, 28].

Let Z be a real valued function of n variables defined by :

$$Z \;=\; c_1x_1 + c_2x_2 + \ldots + c_nx_n \quad \rightarrow \quad Objective\ Function$$

With Subjects to the Conditions :

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \ (\geq, =, \leq)\ b_1; \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \ (\geq, =, \leq)\ b_2; \quad \rightarrow \quad Structural\ Constraints \\ \qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \ (\geq, =, \leq)\ b_m; \\ \qquad x_1, x_2, \ldots, x_n \geq 0 \qquad \rightarrow \quad Non-Negative\ Constraints. \end{cases} \tag{2.5}$$

Where all c_i, a_{ji}, b_j are constants and x_i is are decision variable, such that $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

If either Z or some of constraints or both are non-linear, then the problem of determining the value of $(x_1, x_2, \ldots, x_n)$ which makes Z a maximum or minimum and that satisfies both constraints and non-negative constraints is called a general non-linear programming problem $(GNLPP)$.

There are many methods that have been developed and used to get the solution to the non-linear programming problem, which is similar to the simplex algorithm, one of these methods is augmented Lagrange multiplier method.

2.4.2 Augmented Lagrange Multiplier Method

The augmented Lagrange multiplier method is one of the transformation methods that works to convert constrained optimization problems into unconstrained problems, the purpose of this transformation is to solve constrained optimization problems through the use of algorithms for unconstrained problems in an efficient and reliable manner [29, 30].

Augmented Lagrangian Multiplier Method for Inequality Constraints [19, 29, 31, 32].

Consider the following problem:

$$Minimize\ f(x)$$

Subjects to ,

$$g_j\ (x) \leq 0, \quad j = 1, 2, ..., m$$

$$x \in R^n$$

The Lagrangian function for this problem is:

$$L(x, \lambda) = f(x) + \sum_{j=1}^{m} \lambda_j [g_j(x) + S_j]^2$$

Where $(\lambda_j, j = 1, 2, \ldots, m)$ are Lagrange multipliers and $S_j \geq 0$ are the slack variables.

The augmented Lagrangian function denotes by:

$$AL(x, \lambda, \mu) = f(x) + \sum_{j=1}^{m} \lambda_j [g_j(x) + S_j] + \mu \sum_{j=i}^{m} [g_j(x) + S_j]^2$$

Where μ is the penalty parameter.

Now :

The augmented Lagrangian should satisfy the optimality condition for slack variable S_j, that is :

$$\nabla_{Sj} AL(x, \lambda, \mu) = \sum_{j=1}^{m} \lambda_j + 2\mu \sum_{j=i}^{m} [g_j(x) + S_j] = 0$$

Hence, an optimum of slack variable S_j is:

$$S_j^* = max\{0, \ -g_j(x) - \frac{\lambda_j}{2\mu}\}$$

Now,

$$g_j(x) + S_j^* = g_j(x) + max\{0, \ -g_j(x) - \frac{\lambda_j}{2\mu}\} = max\{g_j(x), \ -\frac{\lambda_j}{2\mu}\}$$

Hence, the augmented Lagrangian for inequality constraints is:

$$AL(x, \lambda, \mu) = f(x) + \sum_{j=1}^{m} \lambda_j \alpha_j + \mu \sum_{j=i}^{m} {\alpha_j}^2$$

Where $\alpha_j = max\{g_j(x), \ -\frac{\lambda_j}{2\mu}\}$, μ is the penalty parameter.

The Lagrange multiplier update rule for λ_j is defined as:

$$\lambda_j^{k+1} = \lambda_j^k + 2\mu_k \ max\{g_j(x), \ -\frac{\lambda_j^k}{2\mu_k}\}$$

Where the superscript k is the iteration of the augmented Lagrangian method algorithm.

Augmented Lagrangian Multiplier Method for Equality Constraints [19, 29, 31, 32].

Consider the following problem:

$$Minimize\ f(x)$$

Subjects to ,

$$h_i\ (x) = 0, \quad i = 1, 2, ..., n$$

$$x \in R^n$$

The Lagrangian function for this problem is:

$$L(x, \lambda) = f(x) + \sum_{j=1}^{n} \lambda_i h_i(x)$$

Where $(\lambda_i, i = 1, 2, \ldots, n)$ are Lagrange multipliers .

The augmented Lagrangian function denotes by:

$$AL(x, \lambda, \mu) = f(x) + \sum_{i=1}^{n} \lambda_i h_i(x) + \mu \sum_{i=j}^{n} [h_i(x)]^2$$

Where μ is the penalty parameter.

The Lagrange multiplier update rule for λ_i is defined as:

$$\lambda_i^{k+1} = \lambda_i^k + 2\mu_k h_i(x^k)$$

Where the superscript k is the iteration and x^k is the starting vector of the augmented Lagrangian method algorithm.

The value of μ_k is updated by:

$$\mu_{k+1} = c\ \mu_k, with\ c \geq 1$$

The advantages of the augmented Lagrangian multiplier method [29, 30]:

1- The value of penalty parameter need be increased to infinity for convergence.

2- The starting vector x^0 do not need to be feasible.

3- It is possible to achieve $h(x) = 0$ and $g(x) = 0$ precisely and non-zero values of Lagrange multipliers for the active constraints.

2.5 Optimality in Unconstrained Optimization

Theorem 2.1. First Order Necessary Condition [22, 28].

If x^* is a local minimizer of $f(x)$, and $f(x)$ is contentiously differentiable in an open neighbourhood of $x^*(f \in C^1)$ then:

$$\nabla f(x^*) = 0$$

Theorem 2.2. Second Order Necessary Conditions [22, 28].

If x^* is a local minimizer of $f(x)$, and $\nabla^2 f(x)$ is contentiously differentiable in an open neighbourhood of $x^*(f \in C^2)$ then:

$$\nabla f(x^*) = 0.$$

$$d\ \nabla^2 f(x^*)\ d^T \geq 0.$$

Theorem 2.3. Second Order Sufficient Condition [22, 28].

Suppose that $f(x)$ and $\nabla^2 f(x)$ is contentiously differentiable in an open neighbourhood of x^*, if the two following conditions are satisfied:

$$\nabla f(x^*) = 0.$$

$$d\ \nabla^2 f(x^*)\ d^T > 0.$$

Then x^* is a strict local minimum of f .

CHAPTER 3

THE DIET PROBLEM

3.1 Introduction

Figure 3.1: Some Healthy Eating Patterns

The diet problem is one of the first optimization problems that were studied in the early twentieth century by Jerry Cornfield, as the motivation behind the problem was the army's desire to reduce the cost of feeding soldiers in the field while continuing to provide a healthy diet, so in general, the main goal is to choose a group of foods that meet a set of daily nutritional requirements at the lowest cost [33].

The diet problem is formulated as a linear program where the objective is to reduce cost and constraints are to meet specific nutritional requirements. diet Problem restrictions usually regulate the number of calories and the amount of vitamins, minerals, fats, sodium and cholesterol in the diet [33, 34].

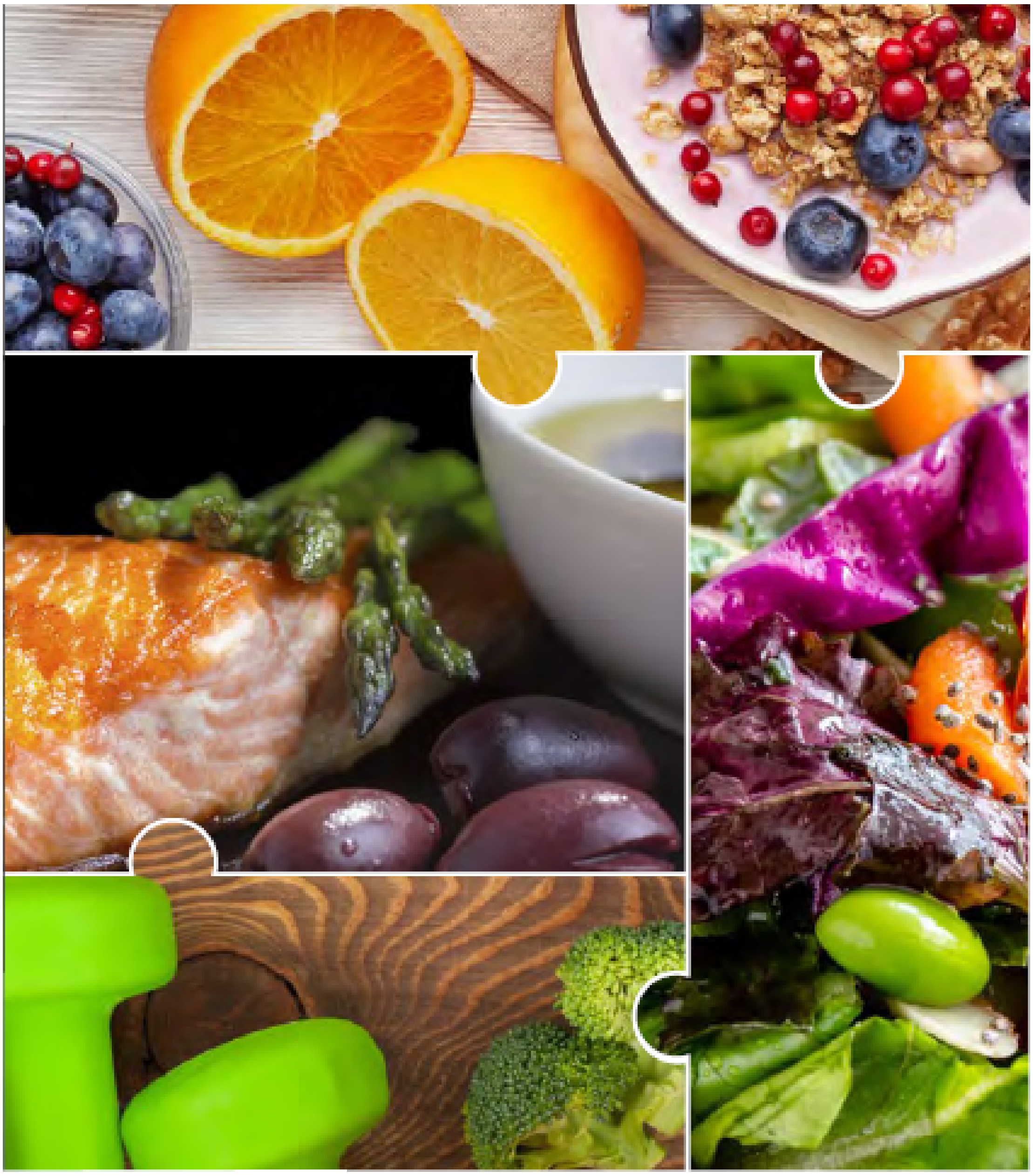

Figure 3.2: Application for a Balanced Diet

The diet problem is one of the most important problems facing a person during his life, and the important question is what we have to deal with to achieve daily health goals, and here came the role of scientists in using mathematical programming to answer this question through the use of operational research techniques through which we can find a list that includes Foods in certain quantities can help provide health recommendations during the day [34,35].

Diets differ from each other, as some require certain percentages of nutrients, others restrict the percentage of fats and carbohydrates, and some depend on liquids only. The list of foods continues in unlimited quantities that can be used by people all over the world [35].

There are many environmental and societal reasons that clearly affect healthy nutrition. Despite the importance of the diet during World War II, its importance has become more important at the present time [36].

Many mathematicians were hired by the Army and Air Force during the World War II period to solve the problem of diet and good planning to propose healthy meals at an appropriate price. Among the participants in solving this problem was the researcher George who was the first to provide a correct mathematical result for an algorithm he proposed as he took this It's up until 1947 [34,37].

3.2 Mathematical Formula for the Diet Problem

A balanced diet is a multi-dimensional problem in terms of its formation of many components that are necessary for the growth of the body and maintaining its fitness and health, the nutritional model consists of a group of foods that contain nutritional components represented by vitamins, minerals, fiber and fats, where this model is summarized in a table that includes the quantity of each nutrient available in foods and their daily requirements that provide the individual with good health conditions, in addition to the standard cost of these foods, as shown in the following table [38]:

Table 3.1: General Model of Diet Problem

Nutritional Requirement						
Nutrient	Food Items					Daily Requirement
	Food 1 x_1	Food 2 x_2	Food 3 x_3	...	Food n x_n	
Nutrient 1	a_{11}	a_{12}	a_{13}	...	a_{1n}	b_1
Nutrient 2	a_{21}	a_{22}	a_{23}	...	a_{2n}	b_2
Nutrient 3	a_{31}	a_{32}	a_{33}	...	a_{3n}	b_3
. . .	. . .	. . .	. . .	... ...	. . .	. . .
Nutrient m	a_{m1}	a_{m2}	a_{m3}	...	a_{mn}	b_m
Cost	c_1	c_2	c_3	...	c_n	Minimization

The mathematical formula for the diet problem for the above table will be in the following:

$$Min\ Z \ = \ c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

Subject to the nutritional constraints :

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \geq b_1;$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2;$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \geq b_m;$$

$$x_1, x_2, \ldots, x_n \geq 0$$

Where,

c_j = cost of food j.

x_j = the number of units of food j in the diet.

a_{ij} = amount of i^{th} nutrient in food type j.

b_i = the required daily amount of nutrient i.

m = the number of nutrients.

n = the number of food items.

Where this Model contains z of variables and g of equations, which can be solved and reach the optimal solution through some mathematical methods such as the simplex method.

3.3 The Duality of Diet Problem

The diet problem is a linear programming problem and therefore there will be a corresponding dual linear programming problem that arises from the objective function and the constraints of the original problem and through the mathematical model of the diet problem, suppose that a health food store that sells nutrient pills (all types of nutrients are available), and let y_i be the price of a unit of the i^{th} nutrient, note that since prices are non-negative, we have $y > 0$ [18].

Our goal is to create the most economical diet and at the same time meet or exceed nutritional requirements.

Now Consider the task of substituting the natural food in the form of nutrient pills, then the duality of the mathematical formula for the diet problem will be in the following form [18, 20]:

$$Max\ Z \ = \ b_1y_1 + b_2y_2 + \ldots + b_my_m$$

Subject to the Conditions :

$$a_{11}y_1 + a_{21}y_2 + \ldots + a_{m1}y_m \leq c_1;$$

$$a_{12}y_1 + a_{22}y_2 + \ldots + a_{m2}y_m \leq c_2;$$

$$\vdots$$

$$a_{1n}y_1 + a_{2n}y_2 + \ldots + a_{mn}y_m \leq c_n;$$

$$y_1, y_2, \ldots, y_m \geq 0$$

3.4 Some Methods for Solving Multiple Objective Functions

Some diets may contain multiple objective functions in addition to dietary restrictions, and therefore we need some ways to convert these systems with multiple functions into a single target function diet. Among the important methods used for conversion are:

1- Weighted-sums Method .

2- Global Criterion Method (GCM).

3.4.1 Weighted-Sums Method

consider a multi-objective optimization problem with equality or inequality constraints:

$$Minimize\ f_1(x), f_2(x), ..., f_k(x)$$

Subjects to ,

$$0 \leq g_j\ (x) \leq 0, \quad j = 1, 2, ..., m$$

$$h_i\ (x) = 0, \quad i = 1, 2, ..., n$$

$$x \in R^n$$

Where $f_p(x),\ p = 1, 2, \ldots, k$, are the objective functions, and $g_j\ (x), h_i\ (x)$ are the constraints.

In this method, a weight is assigned to each objective function, and then the functions are combined to build one objective function, such that the sum of the weights assigned to the functions equals one, where these weights represent the importance of each goal to decision makers [39–42]. Therefore, the above optimization model will be in the following form:

$$Minimize\ W_1 f_1(x) + W_2 f_2(x) + ... + W_k f_k(x)$$

Subjects to ,

$$\sum_{k=1}^{p} W_k = 1, \quad 0 \leq W_k \leq 1$$

$$0 \leq g_j\ (x) \leq 0, \quad j = 1, 2, ..., m$$

$$h_i\ (x) = 0, \quad i = 1, 2, ..., n$$

$$x \in R^n$$

Example 3.4.1. [41].

$$Min\ Z_1 = \sum_t \sum_h \sum_i A_{iht} N_{iht}$$

$$Min\ Z_2 = \sum_t \sum_h \sum_i a_{ih} K_{ih} N_{iht}$$

$$Min\ Z_3 = \sum_t \sum_h \sum_i a_{ih} L_{ih} N_{iht}$$

$$Max\ Z_4 = \sum_t \sum_h \sum_i a_{ih} M_{ih} N_{iht}$$

Subjects to,

$$B_t \leq \sum_h \sum_i a_{ih} C_{ih} N_{iht} \leq D_t, \quad \forall t$$

$$E_{it} \leq \sum_h \sum_i a_{ih} F_{jih} N_{iht} \leq G_{jt}, \quad \forall j, t$$

$$O_t \leq \sum_h \sum_i a_{ih} P_{ih} N_{iht} \leq Q_t, \quad \forall t$$

$$R_t \leq \sum_h \sum_i a_{ih} S_{ih} N_{iht} \leq V_t, \quad \forall t$$

$$X_t \leq \sum_h \sum_i a_{ih} Y_{ih} N_{iht} \leq Z_t, \quad \forall t$$

$$\beta_t \leq \sum_h \sum_i a_{ih} \gamma_{ih} N_{iht} \leq \alpha_t, \quad \forall t$$

$$\rho_t \leq \sum_h \sum_i a_{ih} \theta_{ih} N_{iht} \leq \sigma_t, \quad \forall t$$

$$\sum_i a_{ih} N_{iht} = \psi_{ht}, \quad \forall h, t$$

$$N_{iht} \geq 0, \quad \forall i, h, t$$

Where:

$i =$Type of food $(1, 2, ..., I)$.

$h =$Food group $(1, 2, ..., H)$.

$t =$Period $(1, 2, ..., T)$.

$j =$Vitamin $(1, 2, ..., J)$.

$a_{ih} =$ Size of food i in food group h.

$A_{iht} =$ Cost of food i in food group h and period t.

$B_t =$ Minimum total calories in period t.

C_{ih} = Calories of food i (in each unit) in food group h.

D_t = Maximum total calories in period t.

E_{jt} = Minimum total vitamin j in period t.

F_{jih} = Vitamin j in food i (in each unit) and food group h.

G_{jt} = Maximum total vitamin j in period t.

K_{ih} = Saturated and trans fats of food i (in each unit) in food group h.

L_{ih} = Sugar of food i (in each unit) in food group h.

M_{ih} = Fiber of food i (in each unit) in food group h.

O_t = Minimum total cholesterol in period t.

P_{ih} = Cholesterol of food i (in each unit) in food group h.

Q_t = Maximum total cholesterol in period t.

R_t = Minimum total sodium in period t.

S_{ih} = Sodium of food i (in each unit) in food group h.

V_t = Maximum total sodium in period t.

X_t = Minimum total protein in period t.

Y_{ih} = Protein of food i (in each unit) in food group h.

Z_t = Maximum total protein in period t.

β_t = Minimum total calcium in period t.

γ_{ih} = Calcium of food i (in each unit) in food group h.

α_t = Maximum total calcium in period t.

ρ_t = Minimum total iron in period t.

θ_{ih} = Iron of food i (in each unit) in food group h.

σ_t = Maximum total iron in period t.

ψ_{ht} = Amount of food guide servings per month in food group h in period t.

N_{iht} = Number of food i in food group h and period t.

Now :

By using weighted- sums method, the proposed improvement model will be converted to the following optimization formula:

$$Minimize\ Z_5 = W_1Z_1 + W_2Z_2 + W_3Z_3 - W_4Z_4$$

Subjects to ,

$$\sum_{u=1}^{4} W_u = 1, \quad 0 \leq W_u \leq 1$$

With the above constraints.

It is noted that the above model is a linear programming model with a single objective function, where the model can be solved using the big M method after assigning an appropriate weight to each of the above functions.

3.4.2 Global Criterion Method (GCM)

consider a multi-objective optimization problem with equality or inequality constraints:

$$Minimize\ f_1(x), f_2(x), ..., f_k(x)$$

Subjects to ,

$$0 \leq g_j\ (x) \leq 0, \quad j = 1, 2, ..., m$$

$$h_i\ (x) = 0, \quad i = 1, 2, ..., n$$

$$x \in R^n$$

where $f_p(x),\ p = 1, 2, \ldots, k$, are the objective functions, and $g_j\ (x), h_i\ (x)$ are the constraints.

There are many ways to solve this type of problem, but one of them is the global criterion method, which Rao [43] considers a strategic method through which the optimal solution is found by reducing a predetermined global criterion F represented by the square of the sum of the relative deviations of the differences between the results of the optimal solutions $f_p(x^*)$ and the original functions $f_p(x)$, and it is given by the following formula [44]:

$$min\ F = \sum_{p=1}^{k} \left[\frac{f_p(x^*) - f_p(x)}{f_p(x^*)} \right]^2$$

Subjects to ,

$$0 \leq g_j\ (x) \leq 0, \quad j = 1, 2, ..., m$$

$$h_i\ (x) = 0, \quad i = 1, 2, ..., n$$

Where F is the global criterion, $f_p(x^*)$ is the optimal solutions for the p^{th} objective which is found by dividing the original problem into k of sub-problems ,$f_p(x)$ are the original functions, $g_j\ (x), h_i\ (x)$ are constraints, and k is number of objectives.

Thus, by defining the objectives for each sub-problems, the multi-objective functions are combined into one objective function, which represents global optimization function through which the optimal solution can be obtained by applying several optimization algorithms [42].

Example 3.4.2. [42]

$$Max\ f_1 = 120x_1 + 130x_2$$

$$Max\ f_2 = 65x_1 + 40x_2$$

Subjects to,

$$2x_1 + 1.5x_2 \qquad\qquad \leq 35$$

$$1.8x_3 + 1.3x_4 \leq 50$$

$$0.5x_1 + 0.75x_2 + 1.5x_3 + 1.5x_4 \leq 55$$

$$2x_1 + 1.5x_2 + 1.8x_3 + 1.3x_4 \quad \leq 40$$

$$x_1, x_2, x_3, x_4 \; Integer \; Number$$

Divide the main problem into K of sub-problems:

1- When i = 1 $\Rightarrow$ Sub-Problem 1 :

$$Max \; f_1 = 120x_1 + 130x_2$$

Subjects to,

$$2x_1 + 1.5x_2 \leq 35$$

$$1.8x_3 + 1.3x_4 \leq 50$$

$$0.5x_1 + 0.75x_2 + 1.5x_3 + 1.5x_4 \leq 55$$

$$2x_1 + 1.5x_2 + 1.8x_3 + 1.3x_4 \quad \leq 40$$

$$x_1, x_2, x_3, x_4 \; Integer \; Number$$

by using the simplex method, a solution to this problem was obtained, where:

$$Max f_1 = 2290 \; \$, \;\; x_1^* = \; x_3^* = \; x_4^* = \; 0, \;\; x_2^* = 23$$

2- When i = 2 $\Rightarrow$ Sub-Problem 2 :

$$Max \; f_2 = 65x_1 + 40x_2$$

Subjects to,

$$2x_1 + 1.5x_2 \leq 35$$

$$1.8x_3 + 1.3x_4 \le 50$$

$$0.5x_1 + 0.75x_2 + 1.5x_3 + 1.5x_4 \le 55$$

$$2x_1 + 1.5x_2 + 1.8x_3 + 1.3x_4 \quad \le 40$$

$$x_1, x_2, x_3, x_4 \; Integer\ Number$$

by using the simplex method, a solution to this problem was obtained, where:

$$Max f_2 = 1430\ \$,\ x_1^* = \ x_2^* = \ x_4^* = \ 0,\ x_3^* = 22$$

Create a table of optimal final solutions for sub-problems:

$$Optimal\ Solutions\ for\ Sub - Problems$$

$f_i(x)$ \ $f_i(x^*)$	$f_1(x^*)$	$f_2(x^*)$	x_1^*	x_2^*	x_3^*	x_4^*
$f_1(x)$	2290$	0	0	23	0	0
$f_2(x)$	0	1430$	0	0	2	0

The global criterion method formulation is given by:

$$min\ F = \sum_{i=1}^{k} \left[\frac{f_i(x^*) - f_i(x)}{f_i(x^*)} \right]^2$$

$$= \sum_{i=1}^{2} \left[\frac{f_i(x^*) - f_i(x)}{f_i(x^*)} \right]^2$$

$$= \left[\frac{2290 - (120x_1 + 130x_2)}{2290} \right]^2 + \left[\frac{1430 - (65x_1 + 40x_2)}{1430} \right]^2$$

Subjects to,

$$2x_1 + 1.5x_2 \leq 35$$

$$1.8x_3 + 1.3x_4 \leq 50$$

$$0.5x_1 + 0.75x_2 + 1.5x_3 + 1.5x_4 \leq 55$$

$$2x_1 + 1.5x_2 + 1.8x_3 + 1.3x_4 \leq 40$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$x_1, x_2, x_3, x_4 \; Integer \; Number$$

It can be seen that the above model represents a non-linear (quadratic) programming problem that can be solved by relying on some well-known methods or algorithms.

CHAPTER 4

DIET FOR HYPERTENSION

4.1 Introduction

Most research has shown that the DASH diet plan to reduce hypertension requires some foods that must be eaten on a daily basis to maintain a healthy heart, and these foods are rich in calcium, potassium, magnesium, protein and fiber, taking into account the reduction of saturated fat and sodium in it [45].

The increase in some nutrients can cause some health problems, such as diarrhoea or cramps that result from increased fiber, as well as slow breathing that results from increased magnesium, hence the need to create a food model that meets the daily human nutritional need at the lowest cost, and it can also be considered a common theoretical method for dealing with multiple objective decision-making problems [15, 16].

To model diet problems, many researchers have introduced a balanced diet plan to reduce eating disorders and meet the nutritional requirements of their bodies as well as help them avoid certain chronic diseases such as diabetes and heart attacks [17].

In order to obtain a balanced diet, we present in this chapter a model for a diet consisting of eight types of food which contain seven nutrients. These types of food contain a low percentage of fat and in the same time rich in potassium, calcium, magnesium, iron, and vitamin E together with fiber and protein which in turn can help reduce the risk of developing hypertension and high harmful cholesterol percentage for people (males) from 31-50 years old.

The best solution was obtained at the lowest cost by developing the model in the form of a linear programming problem and solving it using the big M method.

4.2 Methodology

The linear programming model of diet problem is given as follows:

$$Min\ Z \ = \ c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

Subject to the nutritional constraints :

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \geq b_1;$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2;$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \geq b_m;$$

$$x_1, x_2, \ldots, x_n \geq 0$$

Where,

c_j = cost of food j.

x_j = the number of units of food j in the diet.

a_{ij} = amount of i^{th} nutrient in food type j.

b_i = the required daily amount of nutrient i.

m = the number of nutrients.

n = the number of food items.

4.3 Data Analysis

The number of samples that are used in the diet is eight types and the number of nutrients in it is seven. Food prices were collected from grocery stores and in dollar terms, as dietary requirements were specific to males aged 31-50 years [46].

Table (4.1) shows the amount of foods in grams, the amount of nutrients per unit of food (www.webteb.com), and the minimum daily nutritional requirements. The objective is to select the x_j to minimize the total cost of the diet.

Table 4.1: Nutrient Requirement Per Day (Males, 31-50 Years Old)

Nutritional Requirement									
Nutrient	Food Items								Daily Requirement
	100gms of chicken	100gms of rice	100gms soya beans	100gms wheat	100gms of spinach	1 class milk	100gms of Orange	100gms of patato	
Calcium	16	28	37	34	99	350.35	43	8	1000 mg
Potassium	267	115	332	405	558	445.9	166	328	4700 mg
Iron	1.14	0.8	2.57	3.88	2.71	0.15	0.13	0.31	8 mg
Protein	33.44	7.1	13.05	13.7	2.86	9.68	0.91	1.71	56 g
Fiber	0	1.3	0	12.2	2.2	0	2.2	1.8	30.8 g
Magnesium	31	25	130	138	79	36.75	11	20	420 mg
Vitamin E	0.42	0.11	0.9	0.82	2.03	0	0.15	0.01	15 mg
Cost in $	0.25	0.15	0.20	0.15	0.15	0.25	0.20	0.08	Minimization

4.3.1 Objective Function

The total formulation of the above problem is :

$$Minz = 0.25x_1 + 0.15x_2 + 0.20x_3 + 0.15x_4 + 0.15x_5 + 0.25x_6 + 0.20x_7 + 0.08x_8$$

Subject to,

$$16x_1 + 28x_2 + 37x_3 + 34x_4 + 99x_5 + 350.35x_6 + 43x_7 + 8x_8 \geq 1000;$$

$$267x_1 + 115x_2 + 332x_3 + 405x_4 + 558x_5 + 445.9x_6 + 166x_7 + 328x_8 \geq 4700;$$

$$1.14x_1 + 0.8x_2 + 2.57x_3 + 3.88x_4 + 2.71x_5 + 0.15x_6 + 0.13x_7 + 0.31x_8 \geq 8;$$

$$33.44x_1 + 7.1x_2 + 13.05x_3 + 13.7x_4 + 2.86x_5 + 9.68x_6 + 0.91x_7 + 1.71x_8 \geq 56;$$

$$0x_1 + 1.3x_2 + 0x_3 + 12.2x_4 + 2.2x_5 + 0x_6 + 2.2x_7 + 1.8x_8 \geq 30.8;$$

$$31x_1 + 25x_2 + 130x_3 + 138x_4 + 79x_5 + 36.75x_6 + 11x_7 + 20x_8 \geq 420;$$

$$0.42x_1 + 0.11x_2 + 0.9x_3 + 0.82x_4 + 2.03x_5 + 0x_6 + 0.15x_7 + 0.01x_8 \geq 15;$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0$$

4.3.2 Results and Discussion

The above model contains 8 variables, 7 constraints, and by using the big M method, a solution to the problem was obtained, where the minimum costs involved is 1.495 $ with a balanced diet consisting of:

1- 33 gms of chicken per day.

2- 130 gms of wheat per day.

3- 680 gms of spinach per day.

4- 0.79 class of milk per day.

Conclusion

The presented mathematical model, which in turn reduces the daily cost according to the diet problem, was formulated in a linear programming model that was illustrated by using some of the food samples that were used from the DASH food chart.

Through the presented form, we note that the minimum eating plan is obtained, in addition to the main objective of reducing the daily cost of the diet, knowing that it is possible to obtain adjustments in the solution by replacing some foods.

CHAPTER 5
DIET FOR PATIENTS WITH TYPE 2 DIABETES

5.1 Introduction

Diabetes is a chronic metabolic disease that can occur in the body that is unable to produce enough insulin or its inability to use it properly, which in turn leads to high blood sugar. The second type of this disease is the most common in about 90% of diabetes cases. Existing world, which affects adults due to the amount of insulin produced and insufficient [47].

There are some factors that can be considered as changes accompanying the human lifestyle, which play an important role in controlling type 2 diabetes or avoiding its complications in the long term. These factors include a balanced diet and exercise [48].

The daily consumption of foods high in blood sugar, fats and carbohydrates is reduced, this will help the body in lowering the level of glucose in the blood, which in turn can reduce the amount of insulin, and this reduction is considered a healthy food in addition to the important role of exercise in reducing the proportion of insulin through reducing the level of glucose in the blood [49, 50].

The treatment of type 2 diabetes is widespread and variable over time, and the reason is mostly due to the lack of control over a healthy diet, and here it can be said that a healthy diet in addition to the use of drugs can increase the quality of health care for patients with type 2 diabetes [51, 52].

In order to have a balanced diet that helps diabetic patients, especially females between the ages of 31-50 years, we present in this paper a model for a diet that consists of seven foods important for this disease, which contain six nutrients that in turn can regulate the level of glucose in the blood and regulate the secreted insulin where a solution was obtained at the lowest possible cost by developing the model into a linear programming problem and the importance of the dual in most systems. It was used for the proposed model and both models were solved using the big M method.

5.2 Methodology

The linear programming model of diet problem is given as follows:

$$Minimize \quad \sum_{j=1}^{n} c_j x_j$$

Subject to the nutritional constraints :

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i, \quad \sum_{j=1}^{n} a_{ij} x_j \leq d_i, i = 1, 2, ..., m.$$

$$x_j \geq 0$$

Where,

c_j = cost of food j.

x_j = the number of units of food j in the diet.

a_{ij} = amount of i^{th} nutrient in food type j.

b_i = the required daily amount of nutrient i.

d_i = the maximum daily amount of nutrient i.

m = the number of nutrients.

n = the number of food items.

5.3 Data Analysis

In the proposed diet, seven types of foods and six nutrients were used. Food prices were collected from grocery stores at a dollar value, in addition to the nutritional requirements for females whose ages ranged between 31 and 50 years [46].

Table (5.1) shows the amount of foods in grams, the amount of nutrients per unit of food, and the minimum daily nutritional requirements. The aim is to select the x_j to minimize the total cost of the diet.

Table 5.1: Nutrient Requirement Per Day (Females, 31-50 Years Old)

Nutritional Requirement									
Nutrient	Food Items								Daily Requirement
	100gms of tuna fish	100gms of almonds	100gms beans	100gms wheat	1 class of milk	100gms Strawberry	100gms of spinach	100gms of Orange	
Magnesium	34	268	192	138	13.45	13	79	11	320 mg
Iron	1.25	3.72	6.7	3.88	0.15	0.41	2.71	0.13	18 mg
Protein	22	21.22	26.12	13.7	9.68	0.67	2.86	0.91	46 g
Fiber	0	12.2	25	12.2	0	2	2.2	2.2	25.2 g
Potassium	407	705	1062	405	445.9	153	558	166	4700 mg
Vitamin C	1	0	1.4	0	2.7	58.8	28.1	59.1	75 mg
Vitamin B12	1.9	0	0	0	0.6	0	0	0	2.4 mcg
Cost in $	1	1	0.20	0.15	0.25	0.40	0.15	0.20	Minimization

5.3.1 Objective Function

The total formulation of the above problem is :

$$Min\ z = 1x_1 + 1x_2 + 0.20x_3 + 0.15x_4 + 0.25x_5 + 0.40x_6 + 0.15x_7 + 0.20x_8$$

Subject to,

$$34x_1 + 268x_2 + 192x_3 + 138x_4 + 13.45x_5 + 13x_6 + 79x_7 + 11x_8 \geq 320;$$

$$34x_1 + 268x_2 + 192x_3 + 138x_4 + 13.45x_5 + 13x_6 + 79x_7 + 11x_8 \leq 1200;$$

$$1.25x_1 + 3.72x_2 + 6.7x_3 + 3.88x_4 + 0.15x_5 + 0.41x_6 + 2.71x_7 + 0.13x_8 \geq 18;$$

$$
\begin{aligned}
&1.25x_1 + 3.72x_2 + 6.7x_3 + 3.88x_4 + 0.15x_5 + 0.41x_6 + 2.71x_7 + 0.13x_8 && \leq 45;\\
&22x_1 + 21.22x_2 + 26.12x_3 + 13.7x_4 + 9.68x_5 + 0.67x_6 + 2.86x_7 + 0.91x_8 && \geq 46;\\
&22x_1 + 21.22x_2 + 26.12x_3 + 13.7x_4 + 9.68x_5 + 0.67x_6 + 2.86x_7 + 0.91x_8 && \leq 172;\\
&0x_1 + 12.2x_2 + 25x_3 + 12.2x_4 + 0x_5 + 2x_6 + 2.2x_7 + 2.2x_8 && \geq 25.2;\\
&407x_1 + 705x_2 + 1062x_3 + 405x_4 + 445.9x_5 + 153x_6 + 558x_7 + 166x_8 && \geq 4700;\\
&1x_1 + 0x_2 + 1.4x_3 + 0x_4 + 2.7x_5 + 58.8x_6 + 28.1x_7 + 59.1x_8 && \geq 75;\\
&1x_1 + 0x_2 + 1.4x_3 + 0x_4 + 2.7x_5 + 58.8x_6 + 28.1x_7 + 59.1x_8 && \leq 2000;\\
&1.9x_1 + 0x_2 + 0x_3 + 0x_4 + 0.6x_5 + 0x_6 + 0x_7 + 0x_8 && \geq 2.4;
\end{aligned}
$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_7, x_8 \geq 0$$

5.3.2 Results and Discussion

The above model contains 8 variables, 11 constraints, we solved it by using the simplex method and get the following results:
The minimum cost is 1.67 $ with a balanced diet consisting of:

1- 170 gms of beans per day.

2- 4 class of milk per day.

3- 220 gms of spinach per day.

5.4 Duality

Consider now the task of substituting the natural food in the form of nutrient pills, then the duality of the above system will be in the following form:

$$Max\ z = 320y_1 - 1200y_2 + 18y_3 - 45y_4 + 46y_5 - 172y_6 + 25.2y_7 + 4700y_8 + 75y_9 - 2000y_{10} + 2.4y_{11}$$

Subject to,

$$34y_1 - 34y_2 + 1.25y_3 - 1.25y_4 + 22y_5 - 22y_6 + 0y_7 + 407y_8 + 1y_9 - 1y_{10} + 1.9y_{11} \leq 1;$$

$$268y_1 - 268y_2 + 3.72y_3 - 3.72y_4 + 21.22y_5 - 21.22y_6 + 12.2y_7 + 705y_8 + 0y_9 - 0y_{10} + 0y_{11} \leq 1;$$

$$192y_1 - 192y_2 + 6.7y_3 - 6.7y_4 + 26.12y_5 - 26.12y_6 + 25y_7 + 1062y_8 + 1.4y_9 - 1.4y_{10} + 0y_{11} \leq 0.20;$$

$$138y_1 - 138y_2 + 3.88y_3 - 3.88y_4 + 13.7y_5 - 13.7y_6 + 12.2y_7 + 405y_8 + 0y_9 - 0y_{10} + 0y_{11} \leq 0.15;$$

$$13.45y_1 - 13.45y_2 + 0.15y_3 - 0.15y_4 + 9.68y_5 - 9.68y_6 + 0y_7 + 445.9y_8 + 2.7y_9 - 2.7y_{10} + 0.6y_{11} \leq 0.25;$$

$$13y_1 - 13y_2 + 0.41y_3 - 0.41y_4 + 0.67y_5 - 0.67y_6 + 2y_7 + 153y_8 + 58.8y_9 - 58.8y_{10} + 0y_{11} \leq 0.40;$$

$$79y_1 - 79y_2 + 2.71y_3 - 2.71y_4 + 2.86y_5 - 2.86y_6 + 2.2y_7 + 558y_8 + 28.1y_9 - 28.1y_{10} + 0y_{11} \leq 0.15;$$

$$11y_1 - 11y_2 + 0.13y_3 - 0.13y_4 + 0.91y_5 - 0.91y_6 + 2.2y_7 + 166y_8 + 59.1y_9 - 59.1y_{10} + 0y_{11} \leq 0.20;$$

$$y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}, +y_{11} \geq 0.$$

By applying the simplex method to solve the above system, we found that the total cost is 1.67 $.

We conclude that the duality gap at optimality is zero, that is, the sufficient conditions Theorem for optimality is satisfy.

Theorem 5.1. Sufficient Conditions for Optimality [19].

If x, z are feasible for the primal and y is feasible for the dual, and x, z, y satisfy $(c^T - y^T A)x = y^T z = 0$ (complementary slackness), then x is optimal for the primal and y is optimal for the dual.

Furthermore $(c^T x = y^T b)$.

Conclusion

The presented mathematical model was formulated for the purpose of reducing the daily cost according to the problem of the diet presented in the form of a linear programming model using samples of foods that can help patients with type 2 diabetes in controlling glucose and insulin levels in the blood. And by using the duality of the presented model, we found that the total cost for both is equal, and from this we conclude that the duality gap at optimality is zero (satisfying the strong duality), in addition to the primary goal of reducing the daily cost of the diet, knowing that it is possible to obtain adjustments in the solution by substituting certain foods.

CHAPTER 6

OPTIMAL SOLUTION FOR KETO DIET

6.1 Introduction

The keto diet, which was introduced in the early twenties of the last century, is considered one of the effective alternatives to drugs used in the treatment of some diseases such as obesity or epilepsy, where this diet is represented by the amount of low carbohydrates as well as the percentage of high fats [53], where ketone bodies serve as an important source of energy instead of tissues, fats are converted by the liver into fatty acids and ketone bodies to provide the body with energy [54].

Hence the role and the need to create models for healthy, balanced diets through which to meet the daily human need for food, which is a practical and theoretical way to deal with decision-making problems with multiple objectives [15, 16].

In order to reduce eating disorders and meet nutritional requirements, many researchers have proposed balanced diet plans in order to avoid some chronic diseases such as high blood pressure, epilepsy, diabetes and heart attacks [17].

In this chapter, we present a model of a ketogenic diet for males between the ages of 31-50 years, using nine foods from among the foods that can be used in the keto diet with nutrients that help meet the individual's need, as the presented model has multiple objectives represented in finding the best solutions which includes the lowest cost, the lowest amount of carbohydrates and the highest percentage of fat, accompanied by several linear restrictions to meet the individual's need for nutrients.

The multi-objective system was solved by converting it to a single objective function by suggesting a weight for each function by diet specialists in a manner similar to the interactive methods of multi-linear programming objectives and results were obtained by the simplex method.

6.2 Methodology

The linear programming model of diet problem is given as follows:

$$Minimize \quad \sum_{j=1}^{n} c_j x_j$$

Subject to the nutritional constraints :

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i, \qquad \sum_{j=1}^{n} a_{ij} x_j \leq d_i, i = 1, 2, ..., m.$$

$$x_j \geq 0$$

Where,

c_j = cost of food j.

x_j = the number of units of food j in the diet.

a_{ij} = amount of i^{th} nutrient in food type j.

b_i = the required daily amount of nutrient i.

d_i = the maximum daily amount of nutrient i.

m = the number of nutrients.

n = the number of food items.

6.3 Data Analysis

The proposed diet consists of nine foods whose prices were collected from grocery stores at a dollar value, in addition to nine nutrients and daily requirements for males between the ages of 31-50 years [46].

The table below shows the amount of foods in grams, the amount of nutrients per food unit, as well as the daily nutritional requirements of the individual with the cost of foods.

The goal is to find the value of x_j that achieves the minimize the total cost of the diet in addition to the lowest amount of carbohydrates and the highest percentage of fat.

Table 6.1: Nutrient Requirement Per Day (Males, 19-50 Years Old)

Nutritional Requirement										
Nutrient	Food Items									Daily Requirement
	100gms of meat	100gms of chicken	100gms of tuna fish	egg 1	100gms of almonds	100gms of avocado	100gms broccoli	100gms berries	100gms of spinach	
Carbohydrate	0	0	0	0.78	21.67	8.53	6.64	9.61	3.63	
Fats	6.77	5.66	1.01	9.94	49.42	14.66	0.37	0.49	0.39	
Vitamin D	0	5	0	50	0	0	0	0	0	600 IU
Protein	19.35	28.29	22	12.57	21.22	2	2.82	1.39	2.86	56 mg
Fiber	0	0	0	0	12.2	6.7	2.6	5.3	2.2	33.6 mg
magnesium	24	24	34	12	268	29	21	20	79	400 mg
Potassium	315	246	407	134	705	485	316	162	558	4700 mg
Calcium	15	12	29	53	264	12	47	29	99	1000 mg
Sodium	82	95	37	140	1	7	33	1	79	1500 mg
Cost in $	1	0.25	1	0.15	1	0.40	0.20	0.40	0.15	Minimization

6.3.1 Objective Function

The total formulation of the problem is :

$$Min\ z_1 = 1x_1 + 0.25x_2 + 1x_3 + 0.15x_4 + 1x_5 + 0.40x_6 + 0.20x_7 + 0.40x_8 + 0.15x_9$$

$$Min_c\ z_2 = 0x_1 + 0x_2 + 0x_3 + 0.78x_4 + 21.67x_5 + 8.53x_6 + 6.64x_7 + 9.61x_8 + 3.63x_9$$

$$Max_f\ z_3 = 6.77x_1 + 5.66x_2 + 1.01x_3 + 9.94x_4 + 49.42x_5 + 14.66x_6 + 0.37x_7 + 0.49x_8 + 0.39x_9$$

Subject to,

$$0x_1 + 5x_2 + 0x_3 + 50x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 \geq 600;$$

$$0x_1 + 5x_2 + 0x_3 + 50x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 \leq 4000;$$

$$19.35x_1 + 28.22x_2 + 22x_3 + 12.57x_4 + 21.22x_5 + 2x_6 + 2.82x_7 + 1.39x_8 + 2.86x_9 \geq 56;$$

$$19.35x_1 + 28.22x_2 + 22x_3 + 12.57x_4 + 21.22x_5 + 2x_6 + 2.82x_7 + 1.39x_8 + 2.86x_9 \leq 172;$$

$$0x_1 + 0x_2 + 0x_3 + 0x_4 + 12.2x_5 + 6.7x_6 + 2.6x_7 + 5.3x_8 + 2.2x_9 \geq 33.6;$$

$$0x_1 + 0x_2 + 0x_3 + 0x_4 + 12.2x_5 + 6.7x_6 + 2.6x_7 + 5.3x_8 + 2.2x_9 \leq 70;$$

$$24x_1 + 24x_2 + 34x_3 + 12x_4 + 268x_5 + 29x_6 + 21x_7 + 20x_8 + 79x_9 \geq 400;$$

$$24x_1 + 24x_2 + 34x_3 + 12x_4 + 268x_5 + 29x_6 + 21x_7 + 20x_8 + 79x_9 \leq 1200;$$

$$315x_1 + 246x_2 + 407x_3 + 134x_4 + 705x_5 + 485x_6 + 316x_7 + 162x_8 + 558x_9 \geq 4700;$$

$$15x_1 + 12x_2 + 29x_3 + 53x_4 + 264x_5 + 12x_6 + 47x_7 + 29x_8 + 99x_9 \geq 1000;$$

$$15x_1 + 12x_2 + 29x_3 + 53x_4 + 264x_5 + 12x_6 + 47x_7 + 29x_8 + 99x_9 \leq 2500;$$

$$82x_1 + 95x_2 + 37x_3 + 140x_4 + 1x_5 + 7x_6 + 33x_7 + 1x_8 + 79x_9 \geq 1500;$$

$$82x_1 + 95x_2 + 37x_3 + 140x_4 + 1x_5 + 7x_6 + 33x_7 + 1x_8 + 79x_9 \leq 2300;$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \geq 0$$

Where :

$Minz_1 =$ Represents the lowest cost.

$Min_c z_2 =$ Represents the least amount of carbohydrates.

$Max_f z_3 =$ Represents the largest amount of fat.

Now :

By using weaknesses weighted - sums method (interactive method), a weight is assigned to each objective function, where the functions are combined to build one objective function, knowing that the sum of the weights is equal to one, and therefore the proposed improvement model will be converted to the following optimization formula:

$$Min\ z_4 = W_1 z_1 + W_2 z_2 - W_3 z_3$$

Subjects to ,

$$\sum_{i=1}^{3} W_i = 1, \quad 0 \leq W_i \leq 1.$$

With above constraints.

Where:

W_i = the weight of objective function $z_i,\ i = 1, 2, 3$.

By suggesting a weight for each function by diet specialists where $W_1 = 0.4,\ W_2 = 0.3,\ W_3 = 0.3,$ so the mathematical model will be as follows:

$$Min\ z_4 = 0.4(1x_1 + 0.25x_2 + 1x_3 + 0.15x_4 + 1x_5 + 0.40x_6 + 0.20x_7 + 0.40x_8 + 0.15x_9)+$$

$$0.3(0x_1 + 0x_2 + 0x_3 + 0.78x_4 + 21.67x_5 + 8.53x_6 + 6.64x_7 + 9.61x_8 + 3.63x_9)-$$

$$0.3(6.77x_1+5.66x_2+1.01x_3+9.94x_4+49.42x_5+14.66x_6+0.37x_7+0.49x_8+0.39x_9)$$

$$= -1.631x_1 - 1.598x_2 + 0.097x_3 - 2.688x_4 - 7.925x_5 - 1.679x_6 + 1.961x_7 + 2.896x_8 + 1.032x_9$$

Subject to,

$$0x_1 + 5x_2 + 0x_3 + 50x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 \geq 600;$$

$$0x_1 + 5x_2 + 0x_3 + 50x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 \leq 4000;$$

$$19.35x_1 + 28.22x_2 + 22x_3 + 12.57x_4 + 21.22x_5 + 2x_6 + 2.82x_7 + 1.39x_8 + 2.86x_9 \geq 56;$$

$$19.35x_1+28.22x_2+22x_3+12.57x_4+21.22x_5+2x_6+2.82x_7+1.39x_8+2.86x_9 \leq 172;$$

$$0x_1+0x_2+0x_3+0x_4+12.2x_5+6.7x_6+2.6x_7+5.3x_8+2.2x_9 \geq 33.6;$$

$$0x_1+0x_2+0x_3+0x_4+12.2x_5+6.7x_6+2.6x_7+5.3x_8+2.2x_9 \leq 70;$$

$$24x_1+24x_2+34x_3+12x_4+268x_5+29x_6+21x_7+20x_8+79x_9 \geq 400;$$

$$24x_1+24x_2+34x_3+12x_4+268x_5+29x_6+21x_7+20x_8+79x_9 \leq 1200;$$

$$315x_1+246x_2+407x_3+134x_4+705x_5+485x_6+316x_7+162x_8+558x_9 \geq 4700;$$

$$15x_1+12x_2+29x_3+53x_4+264x_5+12x_6+47x_7+29x_8+99x_9 \geq 1000;$$

$$15x_1+12x_2+29x_3+53x_4+264x_5+12x_6+47x_7+29x_8+99x_9 \leq 2500;$$

$$82x_1+95x_2+37x_3+140x_4+1x_5+7x_6+33x_7+1x_8+79x_9 \geq 1500;$$

$$82x_1+95x_2+37x_3+140x_4+1x_5+7x_6+33x_7+1x_8+79x_9 \leq 2300;$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \geq 0$$

6.3.2 Results and Discussion

The above model contains 9 variables, 13 constraints, and by using the simplex method, a solution to this problem was obtained, where:

$Min\ z_4 = 40.076,\ Min\ z_1 = 4.811\ \$,\ Min_c\ z_2 = 74.82\ g,\ Max_f\ z_3 = 214.82\ g$, with balanced diet consisting of:

1- 12 of eggs per day.

2- 644 gms of avocado per day.

3- 290 gms of spinach per day.

Conclusion

The presented mathematical model was formulated in order to reduce the total cost and obtain the least amount of carbohydrates and the highest percentage of healthy fat according to the diet problem presented in the form of the multi-objective linear programming model by using the samples of important foods in the keto diet that helping in losing the weight as well as treating seizures epilepsy and other diseases, knowing that it is possible to make some modifications in the model by replacing some types of food and obtaining the desired goals.

CHAPTER 7

GLOBAL CRITERION METHOD AND DIET PROBLEM

7.1 Introduction

Many people suffer from some chronic diseases such as high blood pressure and other diseases that are directly related to weight gain and obesity that is caused by the accumulation of fat in the body due to the lack of sports activity in addition to unhealthy diets [7].

In order to reduce these health problems, many researchers presented plans for balanced diets that meet their daily needs by reducing daily costs and also to reduce eating disorders. Therefore, these plans can be considered as a theoretical method that can deal with decision-making problems for multiple objective [15, 16].

There are many diets with multiple objective that want to be improved at the same time when the individual objective functions are combined in one function only using some mathematical methods such as weighted-sum method and the global criterion method, which is an important and interesting way through which the multiple linear functions are combined to non-linear objective function [42, 43].

In this chapter, we present a model for a low-fat diet for females between the ages of 19-50 years, using some nutritional foods that can meet the needs of the proposed diet with nutrients that meet the individual's need, and therefore the model presented has multiple goals that include less cost and less the percentage of fat with a number of restrictions accompanying them. By using the global criterion method, the two linear objective functions were combined and transformed into one non-linear objective function, and with the same constraints, the augmented Lagrangian method was used as a tool to find the best solutions for the proposed system.

7.2 Methodology

The linear programming model of diet problem is given as follows:

$$Minimize \quad \sum_{j=1}^{n} c_j x_j$$

Subject to the nutritional constraints :

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i, \; i = 1, 2, ..., m.$$

$$x_j \geq 0$$

Where,

c_j = cost of food j.

x_j = the number of units of food j in the diet.

a_{ij} = amount of i^{th} nutrient in food type j.

b_i = the required daily amount of nutrient i.

m = the number of nutrients.

n = the number of food items.

7.3 Data Analysis

Eight types of food are used in the proposed diet and six nutrients are utilized. The prices of food samples were collected from grocery stores in dollars. In addition, the nutritional requirements were for females aged between 19-50 years [46].

Table (7.1) shows the group of foods used in grams and the amount of nutrients per food unit in addition to the daily nutritional requirements of the individual with the cost of foods, as the goal is to find the value of x_j that achieves the least cost and the least amount of fat for the proposed diet.

Table 7.1: Nutrient Requirement Per Day (Females, 19-50 Years Old)

Nutritional Requirement									
Nutrient	Food Items								Daily Requirement
	100gms of chicken	100gms of almonds	100gms soya beans	100gms wheat	egg 1	1 class milk	100gms of Orange	100gms of spinach	
Fat	3.57	49.42	2.09	1.87	9.94	0.2	0.15	0.39	
Calcium	15	264	37	34	53	298.9	43	99	1000 mg
Potassium	256	705	332	405	134	382.2	166	558	4700 mg
Iron	1.04	3.72	2.57	3.88	1.83	0.07	0.13	2.71	18 mg
Protein	31.02	21.22	13.05	13.7	12.57	8.26	0.91	2.86	46 mg
Fiber	0	12.2	0	12.2	0	0	2.2	2.2	28 mg
Magnesium	29	268	130	138	12	26.95	11	79	310 mg
Cost in $	0.25	1	0.20	0.15	0.15	0.25	0.20	0.15	Minimization

7.3.1 Objective Function

The total formulation of the problem is:

$$Min\ z_1 = 0.25x_1 + 1x_2 + 0.20x_3 + 0.15x_4 + 0.15x_5 + 0.25x_6 + 0.20x_7 + 0.15x_8$$

$$Min_f\ z_2 = 3.57x_1 + 49.42x_2 + 2.09x_3 + 1.87x_4 + 9.94x_5 + 0.2x_6 + 0.15x_7 + 0.39x_8$$

Subject to,

$$15x_1 + 264x_2 + 37x_3 + 34x_4 + 53x_5 + 298.9x_6 + 43x_7 + 99x_8 \geq 1000;$$

$$256x_1 + 705x_2 + 332x_3 + 405x_4 + 134x_5 + 382.2x_6 + 166x_7 + 558x_8 \geq 4700;$$

$$1.04x_1 + 3.72x_2 + 2.57x_3 + 3.88x_4 + 1.83x_5 + 0.07x_6 + 0.13x_7 + 2.71x_8 \geq 18;$$

$$31.02x_1 + 21.22x_2 + 13.05x_3 + 13.7x_4 + 12.57x_5 + 8.26x_6 + 0.91x_7 + 2.86x_8 \geq 46;$$

$$0x_1 + 12.2x_2 + 0x_3 + 12.2x_4 + 0x_5 + 0x_6 + 2.2x_7 + 2.2x_8 \geq 28;$$

$$29x_1 + 268x_2 + 130x_3 + 138x_4 + 12x_5 + 26.95x_6 + 11x_7 + 79x_8 \geq 310;$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0$$

Where :

$Min\ z_1$ = Represent the lowest cost.

$Min_f\ z_2$ = Represents the least amount of fat.

7.3.2 Using Global Criterion Technique

Divide the main problem into K of sub-problems:

1- When i = 1 $\Rightarrow$ Sub-Problem 1 :

$$Min\ z_1 = 0.25x_1 + 1x_2 + 0.20x_3 + 0.15x_4 + 0.15x_5 + 0.25x_6 + 0.20x_7 + 0.15x_8$$

Subject to,

$$15x_1 + 264x_2 + 37x_3 + 34x_4 + 53x_5 + 298.9x_6 + 43x_7 + 99x_8 \geq 1000;$$

$$256x_1 + 705x_2 + 332x_3 + 405x_4 + 134x_5 + 382.2x_6 + 166x_7 + 558x_8 \geq 4700;$$

$$1.04x_1 + 3.72x_2 + 2.57x_3 + 3.88x_4 + 1.83x_5 + 0.07x_6 + 0.13x_7 + 2.71x_8 \geq 18;$$

$$31.02x_1 + 21.22x_2 + 13.05x_3 + 13.7x_4 + 12.57x_5 + 8.26x_6 + 0.91x_7 + 2.86x_8 \geq 46;$$

$$0x_1 + 12.2x_2 + 0x_3 + 12.2x_4 + 0x_5 + 0x_6 + 2.2x_7 + 2.2x_8 \geq 28;$$

$$29x_1 + 268x_2 + 130x_3 + 138x_4 + 12x_5 + 26.95x_6 + 11x_7 + 79x_8 \geq 310;$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0$$

By using the simplex method, a solution to this problem was obtained, where:

$Min\ z_1 = 1.4589,\ x_1^* = x_2^* = x_3^* = x_5^* = x_7^* = 0,\ x_4^* = 1.3733,\ x_6^* = 0.9438,\ x_8^* = 6.7797.$

2- When i = 2 $\Rightarrow$ Sub-Problem 2 :

$$Min_f\ z_2 = 3.57x_1 + 49.42x_2 + 2.09x_3 + 1.87x_4 + 9.94x_5 + 0.2x_6 + 0.15x_7 + 0.39x_8$$

Subject to,

$$15x_1 + 264x_2 + 37x_3 + 34x_4 + 53x_5 + 298.9x_6 + 43x_7 + 99x_8 \geq 1000;$$

$$256x_1 + 705x_2 + 332x_3 + 405x_4 + 134x_5 + 382.2x_6 + 166x_7 + 558x_8 \geq 4700;$$

$$1.04x_1 + 3.72x_2 + 2.57x_3 + 3.88x_4 + 1.83x_5 + 0.07x_6 + 0.13x_7 + 2.71x_8 \geq 18;$$

$$31.02x_1 + 21.22x_2 + 13.05x_3 + 13.7x_4 + 12.57x_5 + 8.26x_6 + 0.91x_7 + 2.86x_8 \geq 46;$$

$$0x_1 + 12.2x_2 + 0x_3 + 12.2x_4 + 0x_5 + 0x_6 + 2.2x_7 + 2.2x_8 \geq 28;$$

$$29x_1 + 268x_2 + 130x_3 + 138x_4 + 12x_5 + 26.95x_6 + 11x_7 + 79x_8 \geq 310;$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0$$

By using the simplex method, a solution to this problem was obtained, where:

$Min_f\ z_2 = 3.9498,\ x_1^* = x_2^* = x_3^* = x_4^* = x_5^* = 0,\ x_6^* = 2.6884, x_7^* = 6.4648, x_8^* =$

6.2625.

Create a table of optimal final solutions for sub-problems

Optimal Solutions for Sub − Problems

$z_i(x)$ \ $z_i(x^*)$	$z_1(x^*)$	$z_2(x^*)$	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	x_8^*
$z_1(x)$	1.4589	0	0	0	0	1.3733	0	0.9438	0	6.7797
$z_2(x)$	0	3.9498	0	0	0	0	0	2.6884	6.4648	6.2625

The global criterion method formulation is given by:

$$Min\ Z = \sum_{i=1}^{k} \left[\frac{z_i(x^*) - z_i(x)}{z_i(x^*)} \right]^2$$

$$= \sum_{i=1}^{2} \left[\frac{z_i(x^*) - z_i(x)}{z_i(x^*)} \right]^2$$

Subject to the constraints.

So our system will be as follows:

$$Min\ Z = \left[\frac{1.4589 - (0.25x_1 + 1x_2 + 0.20x_3 + 0.15x_4 + 0.15x_5 + 0.25x_6 + 0.20x_7 + 0.15x_8)}{1.4589}\right]^2$$

$$+\left[\frac{3.9498 - (3.57x_1 + 49.42x_2 + 2.09x_3 + 1.87x_4 + 9.94x_5 + 0.2x_6 + 0.15x_7 + 0.39x_8)}{3.9498}\right]^2$$

Subjects to,

$$15x_1 + 264x_2 + 37x_3 + 34x_4 + 53x_5 + 298.9x_6 + 43x_7 + 99x_8 \geq 1000;$$

$$256x_1 + 705x_2 + 332x_3 + 405x_4 + 134x_5 + 382.2x_6 + 166x_7 + 558x_8 \geq 4700;$$

$$1.04x_1 + 3.72x_2 + 2.57x_3 + 3.88x_4 + 1.83x_5 + 0.07x_6 + 0.13x_7 + 2.71x_8 \geq 18;$$

$$31.02x_1 + 21.22x_2 + 13.05x_3 + 13.7x_4 + 12.57x_5 + 8.26x_6 + 0.91x_7 + 2.86x_8 \geq 46;$$

$$0x_1 + 12.2x_2 + 0x_3 + 12.2x_4 + 0x_5 + 0x_6 + 2.2x_7 + 2.2x_8 \geq 28;$$

$$29x_1 + 268x_2 + 130x_3 + 138x_4 + 12x_5 + 26.95x_6 + 11x_7 + 79x_8 \geq 310;$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0$$

It can be noticed that the above model represents a non-linear (quadratic) programming problem that can be solved by relying on some well-known methods or algorithms. Here, the augmented Lagrangian method will be used as a tool to find the best solutions by applying it in the Python program to perform all calculations.

Despite the difficulty of determining the initial basic variables to reach the final optimal solution, and by making many input attempts, it has been the 3 best results are listed in the following table:

The Best Possible Solutions with Decimals

No.	Decision Variables								Min z	Min z_1	$\text{Min}_f\ z_2$
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8			
1	0.231	0.440	0.598	1.663	1.804	1.115	0.039	6	121.53	2.323	47.429
2	0.181	0.391	0.169	1.523	1.678	1	0.375	6.421	96.09	2.238	42.61
3	0.229	0.393	0.152	1.619	1.661	1.035	0.3	6.357	97.168	2.244	42.826

The final solutions that were identified in the previous table using the global criterion method are non-dominant solutions, with the attention that it is possible to adopt solution (No. 2) is an optimal solution among the solutions that have been recorded in the table, taking into account that the mentioned numbers can be approximated in accordance with the elements of the problem under the condition realizing the limitations of the problem.

Conclusion

The presented mathematical model was formulated according to the diet problem intended to reduce the daily cost and obtain the lowest possible percentage of fat by using some of the food samples in a linear mathematical model that is restricted and multi-objective. Also, the global criterion method has many non-dominant solutions that the decision maker can choose one of them is to be the best solution so that this solution meets the needs of the problem in terms of reducing the daily cost and reducing the percentage of fat in the proposed diet.

CHAPTER 8

CONCLUSIONS AND FUTURE WORKS

8.1 Conclusions

The diet problem is an NP problems, which is formulated in the form of optimization models whose main objective was to reduce the total cost of foods.

In this work, four optimization models for the diet problem are presented, each with its own objectives. These models were formulated in the form of linear programming using samples of some foods.

In the first food model, we note that a minimum eating plan has been obtained, which in turn can reduce high blood pressure and high levels of harmful cholesterol, in addition to the primary goal of reducing the daily cost of the diet.

In the second model that was presented to patients with type 2 diabetes, the aim was to control glucose and insulin levels in the blood. We also proved that the total cost of the model and its duality are equal (satisfying the strong duality), in addition to reducing the daily cost of the diet.

the third model, which includes one of the important diets, which is the keto diet, which was presented in the form of a multi-objective linear programming model by using important food samples, which aims to obtain the lowest amount of carbohydrates and the highest percentage of healthy fats, which helps in weight loss as well as treatment of epileptic seizures and many more Among other diseases, in addition to reducing the daily cost of this system, because it is considered one of the high-cost diets.

Finally, a multi-objective diet was presented, which was solved by using the global criterion method, with the aim of reducing the daily cost and obtaining the lowest possible percentage of fat, which helps people lose weight.

8.2 Future Works

1- Employing the genetic algorithm to solve a multi-objective diet represented in finding the lowest cost and the lowest percentage of carbohydrates.

2- Using ε -constraint method to solve a multi-objective diet represented in finding the lowest cost and the lowest fat percentage.

3- Using the weighted-sums method to solve the multi-objective keto diet by finding a weight for each function using a specific algorithm that will be worked on in the future.

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الملخص :

النظام الغذائي الصحيح واختيار الأطعمة المناسبة له تأثير كبير على الصحة من خلال الحفاظ على وزن الجسم وتقوية جهاز المناعة.

النظم الغذائية الصحية التي تساعد الجسم في الحفاظ على صحته ولياقته ، ضرورية في حياتنا اليومية.

ومن خلال استخدام النموذج الرياضي للبرمجة الخطية ، تمت صياغة مشكلة النظام الغذائي كبرنامج خطي مع دالة هدف خطية وقيود خطية.

تناقش هذه الأطروحة أربعة أنظمة غذائية ، لكل منها أهمية فعالة في الحفاظ على صحة ولياقة جسم الإنسان ، وقد تمت مناقشتها على جنس معين بأعمار محددة.

في هذا العمل، تم استخدام طريقة (*Simplex*) لحل هذه الأنظمة، بالإضافة إلى استخدام الطرق التالية:

1- *Weighted- Sums Method.*
2- *Global Criterion Method.*
3- *The Augmented Lagrangian Method*

الهدف الرئيسي هو اختيار مجموعة من الأطعمة التي تلبي مجموعة من المتطلبات الغذائية اليومية للإنسان بأقل تكلفة.

جمهورية العراق
وزارة التعليم العالي والبحث العلمي
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قسم الرياضيات

القرار الغذائي الأمثل لعلاج بعض المشكلات الصحية باستعمال النموذج الرياضي

أطروحة مقدمة الى مجلس كلية التربية للعلوم الصرفة – جامعة بابل
كجزء من متطلبات نيل درجة الدكتوراه فلسفة في التربية/ الرياضيات

من قبل

كرار حبيب هاشم خضير

بإشراف

أ. د. مشتاق عبد الغني الجنابي

٢٠٢٣ م **١٤٤٤ هـ**