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College of Engineering
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NONLINEAR ANALYSIS AND OPTIMAL DESIGN OF REINFORCED CONCRETE PLATES AND SHELLS

**A THESIS
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ENGINEERING OF BABYLON UNIVERSITY IN
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ENGINEERING
(STRUCTURAL ENGINEERING)**

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

وَيَسْأَلُونَكَ عَنِ الرُّوحِ قُلِ الرُّوحُ
مِنْ أَمْرِ رَبِّي وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ
إِلَّا قَلِيلًا

صدق الله العلي العظيم

(الإسراء الآية: 85)

الخلاصة

هذا البحث معنيّ بالتصميم المرن-اللدن الأمثل للالواح و القشريات الخرسانية المسلحة وبالاتماد على التحليل المرن-اللدن اللاخطي هندسياً، باستخدام طريقة العناصر المحددة. تم استخدام العنصر المتطابق القشري ذي العقد الثمانية مع تحديد خمس درجات للحرية في كل عقدة متعلقة بالازاحات الثلاث ودورانين للمتجه العمودي في كل عقدة. لقد تم استخدام نموذج الطبقات في سلوك الخرسانة المسلحة مع فرض ترابط كلي بين حديد التسليح والخرسانة. تم تمثيل سلوك الخرسانة تحت تأثير إجهادات الانضغاط كمادة مرنة مع انفعالات لدنة متصلة أو مادة مرنة تامة اللدونة بعد الخضوع. تم التعبير عن شروط الخضوع بدلالة المتغيرين الأوليين للإجهاد. تم السيطرة على حركة الأسطح المتعاقبة بقاعدة التصلب التي تستقر من علاقة الإجهاد-الانفعال المحوري والممثل عنها بدالة قطع مكافئ.

إن تهشم الخرسانة هي ظاهرة مسيطر عليها بالانفعال ومنظمة بسطح سحق يشبه سطح الخضوع. استعمل أسلوب الشق الثابت لتمثيل سلوك الخرسانة المتشققة مصاحب بشرط الشد للتنبؤ بحدوث الشق. كما أخذ بنظر الاعتبار تخفيض معامل القص في منطقة الشق. تناقص الترابط التدريجي باستمرار التشقق تم تمثيله من خلال نموذج صلابة شد الخرسانة. لقد مثل سلوك حديد التسليح تحت تأثير قوى الشد والانضغاط كمادة مرنة تامة اللدونة أو مادة مرنة مع تصلب انفعال خطي.

تم استخدام التقنية المعروفة بالتزايديّة – التعاودية والتي تعمل تحت أحمال متزايدة لحل معادلات التوازن الغير خطية. أما طريقة الحل التي استخدمت لمعالجة هذه التقنية هي طريقة محورة لطريقة (نيوتن - رافسون). في هذه الطريقة (نيوتن - رافسون) المحورة تعدل مصفوفة الصلابة في الإعادة الثانية لكل زيادة حمل. كما تم استخدام تقارب كل من الإزاحة والقوة في هذا البحث. تم استعمال طرق متعددة لأجراء التكاملات العددية مثل التكامل الكامل والاختياري والناقص للتطبيقات المستخدمة في هذا البحث. لقد تم الأخذ بنظر الاعتبار في النموذج الرياضي المستخدم في هذا البحث التغير غير الخطي في الشكل مستنداً على أسلوب (الكرانج) الكلي أخذاً بنظر الاعتبار فرضيات (فون كارمن).

في حالة الأمثلة الإنشائية و التي هي أمثلة مقيدة لا خطية، فقد تم استخدام طريقة هوك و جيفس (Hooke and Jeeves) المعدلة وباعتبار حجم المنشأ كدالة الهدف وأبعاده كمتغيرات التصميم مع قيود هندسية. العديد من الأمثلة المتعلقة بالسقوف و القشريات الخرسانية المسلحة، والتي قد تم تحليلها سابقاً من قبل باحثين آخرين، تم تحليلها باستخدام طريقة العناصر المحددة الحالية والنتائج أظهرت تقارب جيد و خاصة مع نتائج التجارب العملية ومع وجود فرق

بحدود (3%-16.8%) من الحمل الأقصى. التصميم الأمثل للسقوف و القشريات الخرسانيه المسلحة قد تم تناوله. لقد أظهرت النتائج بان اقل كلفة للسقف نستطيع الحصول عليها عند استخدام اقل سمك. أما في حالة القشريات الخرسانيه المسلحة فان زيادة السمك و درجة التقوس تعطي اقل كلفة.

ABSTRACT

This research deals with the optimal design of reinforced concrete plate and shell structures based on nonlinear finite element method. The eight-node degenerated curved shell element is used in which five degrees of freedom are specified at each nodal point, which are three displacements and two rotations of the normal at the node. A layered model is considered in the modelling of the reinforced concrete behaviour and a perfect bond between the concrete and reinforcement has been assumed. The compressive behaviour of the concrete has been modelled by employing two approaches both elastic-strain hardening and elastic-perfectly plastic plasticity approach. The yield condition is formulated in terms of the first stress and second deviatoric stress invariants. The motion of the subsequent loading surface is controlled by the hardening rule that is extrapolated from the uniaxial stress-strain relationship given by a parabolic function.

The behaviour of cracked concrete has been modelled using a smeared fixed crack approach, coupled with a tensile criterion to predict crack initiation. Gradual bond deterioration with progressive cracking is simulated by means of a tension stiffening model. A reduced shear modulus is employed in the cracked zone. The behaviour of steel reinforcement is idealized by elastic-perfectly plastic relation with linear strain hardening for tensile and compressive stresses. The nonlinear equations of equilibrium have been solved using an incremental-iterative technique operating under load control. Modified Newton-Raphson method has been employed. In this method, the stiffness matrix is updated in the second iteration of each load increment. A load convergence criterion is adopted in the present work. Various integration rules (full, selective, reduced) have been used in the analysis of the

application presented in this study. A nonlinear geometrical model based on the total Lagrangian approach and taking into account the von Karman assumptions.

For the structural optimization problem, which is dealt with as a constraint nonlinear optimization, the so-called Modified Hooke and Jeeves method, (Bunday 1984) is employed by considering the total cost of the structure as the objective function and the dimensions as the design variables with geometrical constraints.

For the analysis of reinforced concrete plates and shells, several examples are worked out using the present finite element analysis and the results show good agreement with experimental results and the difference at the range of (3%-16.8%) for the ultimate load. The results of optimization for reinforced concrete plates show that the optimal cost occurs when using of minimum thickness of slab. The optimal cost for reinforced concrete cylindrical shell occurs when the thickness and curvature of the shell increases and shell angle decreases.

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Certificate of the Examining Committee

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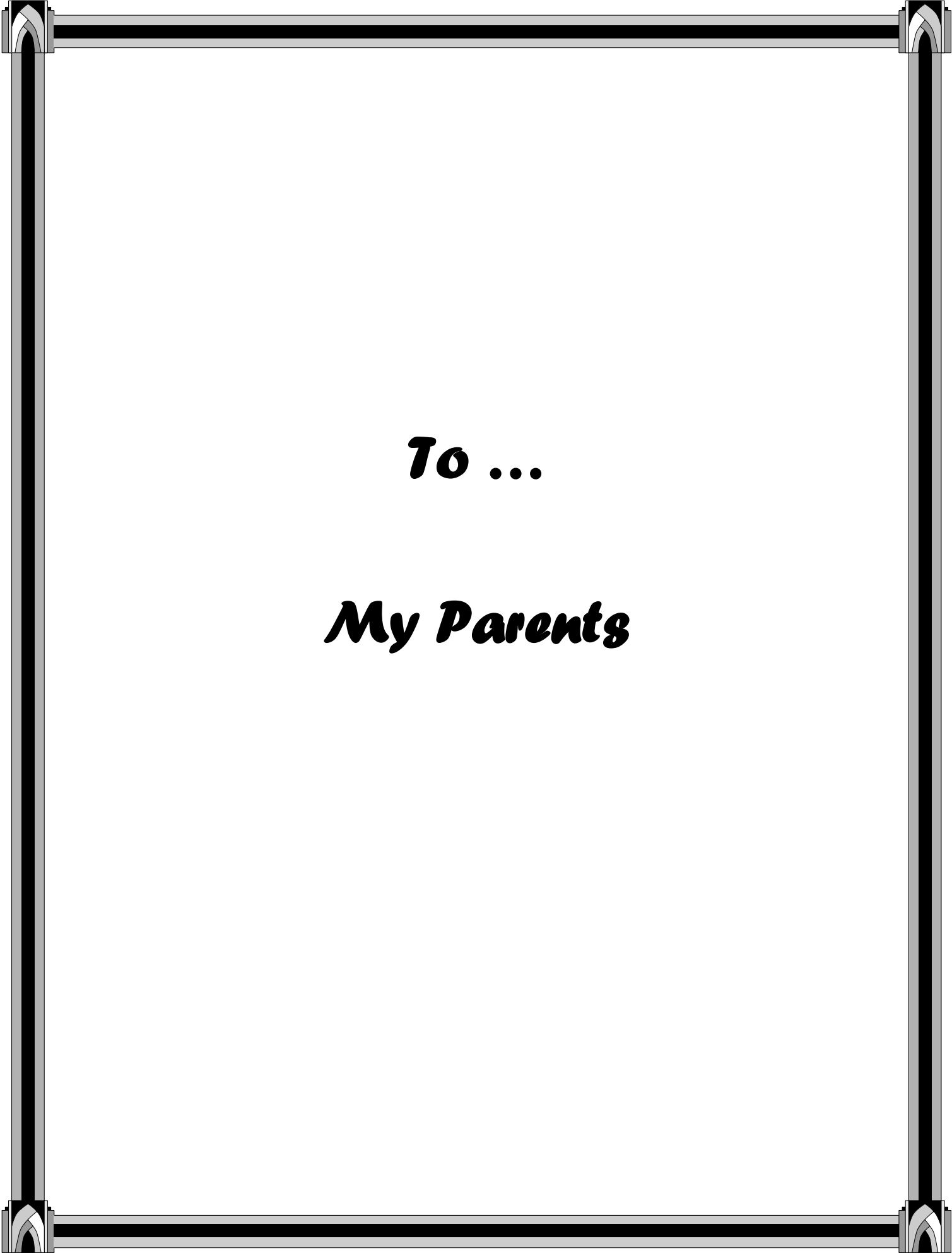
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We certify that the preparation of this thesis titled “**Nonlinear Analysis and Optimal Design of Reinforced Concrete Plates and Shells**” By (**Ali Abdul Hussein M .Alsultany**) was under our supervision at the University of Babylon in partial fulfilment of the requirements for the degree of Master of science in Civil Engineering.

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NOTATIONS

Symbol	Description
$\{a\}$	Unknown nodal displacement vector
a_j^i	The total displacement either the incremental translation or rotational displacement vector at iteration (i)
Δa_j^i	Either the incremental translation or rotational displacement vector at iteration
$[B_j]$	The strain matrix at layer (j)
$[D]$	The elasticity matrix
$d\varepsilon_p$	The plastic strain increment
$d\lambda$	The proportional constant ,which determines the magnitude of the plastic strain increment
$d\{\varepsilon\}$	The total strain increment vector
E	The Young's modulus of elasticity
E_o	The initial Young's modulus
E_{02}	Tensile secant modulus at the point $(\sigma_{02}, \varepsilon_{02})$
E_s	The elastic modulus of steel
$\{f\}$	Nodal applied force vector
$\{f^e\}$	The internal force vector
G	The shear modulus
G^r	The reduced shear modulus
H'	The hardening modulus
h_f	Shell thickness at node f
Δh_j	The thickness jth layer
I_1	The first stress invariant
I_1'	The first strain invariant
$[J]$	The Jacobian matrix
J_2	The second deviator stress invariant
J_2'	The second deviator strain invariant
$ J(\xi, \eta, \zeta) $	The determinant of the Jacobian matrix for layer (j)
$[K]$	Stiffness matrix
K_1, K_2	The shear correction factors
n	Number of nodes per element
nL	The total number of layers
N_f	Shape function matrix of the degenerated
r^n	The out of balance forces
S_f	Transformation matrix of the displacement vector at node f
$[S]$	Transformation matrix
TOLER	The specified convergence tolerance
$\bar{V}_{3f}^i$	The component of the unit normal vector to the middle surface
u_i	The total displacement along the thickness at each nodal point

u_{of}^i	The displacement of f th nodal point in the Cartesian coordinate
$u_{of}^{',i}$	The displacement of the nodal point f on the " normal" resulting from the two rotation α_{1f} and α_{2f}
X_i	Cartesian coordinate of any point in element
α, β	Material parameters
δ_{1f}	Displacement in the direction of $\bar{V}_{1f}$
δ_{2f}	Displacement in the negative direction of $\bar{V}_{2f}$
$\{\sigma_j\}$	Current stress vector
σ_o	The equivalent effective stress taken from uniaxial tests
σ_2	Tensile stress
$\{\varepsilon\}$	The strain vector and details of its formulation are explained in the next section
ε	The current total strain
ε_o	The strain at peak stress $f_c^{'}$
ε_p	The plastic strain component
ε_e	The elastic strain componect
ε_{cu}	The ultimate total strain extrapolated from the Uniaxial Compressive test results
ε_2	Corresponding strain
ε_2^{**}	Ultimate tensile
ν	Poisson`s ratio
$[\vartheta]$	The transformation

Note:/Any other notation may be explained where it appears.

Chapter One

INTRODUCTION

1.1 General

Reinforced concrete is one of the most commonly used construction materials which consist of two materials (Concrete and Steel) with different mechanical properties. The subject of optimization has received increasing attention in the field of design in recent years due to the economic return, the development of improved analytical and computational method.

1.2 Nonlinear Behavior of Reinforced Concrete Plates and Shells

Reinforced concrete Plates and shells are efficient and economic structures under conditions of heavy loads to be transmitted and to cover large areas. The economy of plates and shells is mainly the results of saving in materials they use.

Several discretisation processes have been previously employed in the finite element analysis of reinforced concrete plates and shells. The rigidity of a cracked concrete region has been modelled by a bilinear moment-curvature relationship and used in conjunction with simple quadrilateral element. A layered approach has been used by several investigators to represent the concrete and steel reinforcement and also to simulate progressive cracking through the concrete layers .Such a process has been utilized with triangular element, shallow shell parallelogram elements and Reissner-Mindlin plate element. In the present study, degenerated thick shell elements employing a layered discretization through the thickness are adopted. The concrete characteristics are specified for each layer through the thickness and the reinforcing steel is

represented as a smeared layer of equivalent thickness with (anisotropic) uniaxial strength and rigidity properties (Hinton and Owen 1984).

1.3 Optimization

Reinforced concrete structural problem has numerous solutions. The purpose of optimization is to find the best possible solution among the many alternative solutions satisfying the prechosen criteria. The objective function is often the minimum weight especially for steel structures, or minimum cost taking into account function, safety and serviceability. In the present work, the objective function is the minimum amount of reinforcement for reinforced concrete plates and shells since reinforcement cost, nowadays, represents the major portion in the total cost of construction.

A general mathematical model of the structural optimization problem can be represented in the following form:

A certain function (z) called an objective function,

$$Z=f(x_i) \quad i=1, 2 \dots n$$

which is usually the weight or the cost or amount of reinforcement of the structure involves (n) design variables{x}, and is minimized subject to inequality constraints

$$q_j(x_i) \geq 0 \quad j=1,2,\dots,m$$

and/or equality constraints

$$g_k(x_i) = 0 \quad k=1, 2,\dots,s$$

The vector{x} of design variables will have optimum values when the objective function reaches its minimum value.

The constraints reflect the design and functional requirements.

More detailed definition and explanation of the optimization process will be given in chapter four.

1.4 Scope and Aims

The present work has main aims, namely:

- 1- The use of appropriate mathematical model, which is based on degenerated thick shell element in order to improve the simulation of the nonlinear behaviour of reinforced concrete plates and shells structures.
- 2- A geometric nonlinearity based on total Lagrangian approach taking into account von-Karman assumption is implemented in order to predict the realistic behaviour of structures up to the ultimate load level.
- 3- An efficient and appropriate material constitutive relation has been used to represent the reinforced concrete properties during the analysis. Such relation is based on the elastio- plastic, plastic flow or crushing failure criterion, strain hardening of concrete and yield condition of reinforcement. The adopted criterion for crack initiation is the maximum tensile strength. The post-cracking behaviour of concrete is described using the tension – stiffening concept.
- 4- The validity of the model is checked by the analysis of different examples. The analytical results have been compared with the some available experimental results.
- 5- To perform an optimal design of reinforced concrete plate and shell structures based on the nonlinear analysis and to carry out the structural optimization, the modified Hooke and Jeeves method is used.

1.5 Thesis Layout

The thesis consists of seven chapters. The present introductory chapter is being the first. Chapter two is concerned with the review of literature; outlines of some previous research work on optimal design and nonlinear analysis of reinforced concrete plates and shells are presented. Chapter three is concerned with the material constitutive relationships based on the considered plasticity model for concrete in compression and smeared cracked model for concrete in tension in addition to an elastic-plastic model for steel reinforcement. The formulation of the degenerated thick shell element is reviewed in chapter four. Also, the geometric nonlinear model is described in chapter four and the solution techniques for the nonlinear behaviour reinforced concrete plate and shell is presented.

Chapter five is devoted to the optimal design formulation. In chapter six, several numerical examples are presented for both nonlinear analysis and optimal design and verified with previous analytical and experimental studies. Finally, chapter seven gives the conclusions and suggestions for future studies.

Chapter Two

REVIEW OF LITERATURE

2.1 Introduction

This review covers two aspects. The first is the developments in the nonlinear analysis of reinforced concrete plates and shells using the finite element method. A review is also presented on the use of the degenerated shell element, which is used in this investigation for the nonlinear analysis of reinforced concrete members. The second is the developments in the optimal design of reinforced concrete plates and shells.

2.2 Nonlinear Analysis of Reinforced Concrete Plates and Shells

The finite element method has become a powerful tool for the numerical solution of the reinforced concrete structure, such as slabs and shells due to its efficiency when appropriate models are adopted. It gives flexibility more than other methods that are provided. Adopting this method in a computer program provides time saving and reasonable accuracy for designers and users of such computer program.

During the last four decades, the finite element method has been used extensively for the nonlinear analysis of reinforced concrete members, such as slabs and shells. This is due to the wide use of plain, reinforced and prestressed concrete as structural material.

The degenerated shell element was originally introduced by (Ahmad and Zienkiewicz 1970) to the linear finite element analysis of thick and thin shell structures. The three dimensional stress and strain conditions were degenerated to shell behaviour by employing isoparametric element with independent rotation and displacement degrees of freedom. The Mindlin assumptions were used in place of Kirchhoff

hypothesis. In the formulation of this element only the nodes on the middle of the element were retained. Five degrees of freedom were used at each node (three displacements and two independent rotations).

After that, (Zienkiewicz and Taylor 1971) achieved a great improvement of this element by using a reduced integration technique in general analysis of plate and shells problem.

(Jofriet 1971) used the modified stiffness approach to analyze reinforced concrete slabs. This approach is based on an empirical moment-curvature relationship, where in different flexural rigidities were assumed for different material states. A bilinear moment curvature relation was used to study the reinforced concrete plate bending. The flexural rigidity after cracking was based on an empirical effective moment of inertia of the cracked section which attempts to account for the tension stiffening effect of the concrete between cracks and the effect on rigidity due to steel orientation.

(Hand and Schnobrich 1973) used doubly curved shallow shell layered finite element to analyze reinforced concrete plates and shells subjected to short time loading. The element was rectangular in shape with only corner nodes; each node had five degrees of freedom. Elasto-plastic behavior of concrete and limiting tension of concrete was taken into account.

(Mawenya and Davies 1974) used the finite element technique to account the effect of membrane action in orthotropically reinforced concrete slabs as cited by (Usama 1997) The analysis included the effects of inplane stresses. The results obtained in this study were compared with the classical thin plate theory solutions.

(Scanlon and Murry 1974) Presented a finite element model which was capable of simulating the behavior of reinforced concrete slabs at working loads, including the effects of nonuniform reinforcing and tensile Cracking. That study concentrates on technique for including the effects of creep and shrinkage.

(Klein 1975) used a finite element model to investigate the influence of lateral shear on the behaviour of reinforced concrete slabs. In order to model the nonlinear distribution of shear strains across the slab thickness a multi-layered plate element was used with independent rotations in each layer. Material nonlinearity due to cracking of concrete in the tensile range and crushing under compression including elastic perfectly plastic behaviour was used to model the reinforcement which was idealized by a smeared layer element.

(Lin and Scordelis 1975) used triangular finite element for the nonlinear analysis of reinforced concrete shells of general form. The analysis includes tracing the load-deflection response and the crack propagation through the elastic, inelastic and ultimate ranges. A layered approach was used to represent both the concrete and steel. A bilinear state of stress was assumed for both the concrete and steel, the tension stiffening effect of the concrete between cracks was included in such study and was found to have significant influence on the post cracking load-deflection response of under reinforced concrete structures. The importance of failure criteria and flow rules were investigated in this study.

(Kabir 1976) extended the work of Lin and Scordelis(1975) to include the time dependent effect of creep, shrinkage and load history for the non-linear analysis of reinforced concrete shells as cited by (Nesreen 1998)

(Mueller 1977) used the procedure developed by (Kabir1976) to analyze and study several types of reinforced concrete hyperbolic parabolic shells.

(Bathe and Bolourhi 1980) adopted the finite element techniques for analysis of linear, geometric and material nonlinearity of plates and shells. The element was formulated by interpolating the element geometry using the mid-surface nodal point coordinates and mid-surface nodal point normal. The element was implemented a variable-number-nodes element and could be employed as a fully compatible transition element to model shell intersections and shell-solid regions. A total and an updated Lagrangian formulation was presented, that allowed very large displacements and rotations.

(Mang and Floegl 1981) used curved triangular shell finite element to analyze reinforced concrete shells under short time loading. Both material and geometrical nonlinearities were included. Tension stiffening model was based upon bond slip between the reinforcement and the surrounding concrete.

(Ghoneim and Ghali 1982) used thin constant thickness plate elements to analyze reinforced and prestressed concrete structures subjected to monotonic or cycling loading and initial stress or initial strain. The element has six degrees of freedom at each of four corner nodes situated on a reference surface (chosen for conveniences as the middle surface). Each element is assumed to be composed of a number of layers. The reinforcement is idealized either by smearing a mesh or bars into a steel layer or, with heavy concentrated reinforcement or a prestressed tendon, each reinforcing element is accounted for separately. Each finite element layer was treated as an orthotropic material that can assume any

state; elastic yielded, cracked or crushed, depending on the stress level reached.

(Chan 1982) extended the work of (Kabir 1976) to study the nonlinear behaviour of reinforced concrete shells with edge members. An improved layered composite 9-node Lagrangian shell element with equivalent smeared steel layers was developed to represent the reinforced concrete shell in which the material properties could vary within the element. A filament reinforced concrete beam element was used to model either concentric or eccentric edge beam.

(Owen and Figueiras 1983) applied the finite element techniques for the nonlinear analysis of reinforced concrete thick plate and shells. Degenerated thick shell elements employing a layered approach through the thickness were adopted. Both reduced and selectively integrated Heterosis and Serendipity elements were considered. Also in the analysis, thermal loading of presented concrete structures was considered. A comparison of the numerical results obtained from the finite element analysis with available experimental results showed a good agreement.

(Figueiras 1986) used the finite element method for the nonlinear analysis of reinforced concrete plates and shells. Degenerated quadratic thick shell elements using a layered discretization through the thickness were used. The compression behaviour of the concrete was modeled using two approaches an elastic perfectly plastic and strain hardening plasticity approach. Material and geometric nonlinearities were considered in the analysis.

(Cervera and Hinton 1986) developed fully three dimensional finite element models for nonlinear structural analysis of reinforced concrete plates and shells structures subjected to short-term static loading. 20-noded

isoparametric brick elements with 15-integration points were used. Concrete and reinforcement were represented with a single element. Perfect bond was assumed between the reinforcement and the surrounding concrete. The steel reinforcement was smeared into equivalent steel layer with uniaxial stresses in the bar direction. Both an elastic perfectly plastic and strain-hardening models were adopted in the finite element analysis. A smeared crack model of the concrete was used and post cracking behaviour of the concrete based on fracture energy (strain-softening), tension stiffening rule and a model for shear transfer was taken into account.

(Abdel-Rahman and Hinton 1986) presented linear and nonlinear finite element analysis of reinforced and prestressed concrete cellular slabs based on slab-beam model. Mindlin/Timoshenko assumptions are adopted in the slab-beam model which thus allows for transverse shear deformation. A layered approach was used to represent the concrete and steel. The slab-beam model consists of Mindlin plate elements to represent the slab and Timoshenko beam elements to represent eccentric beam stiffeners. A reference plane at which all degrees of freedom are defined is usually chosen to coincide with plate mid-surface. It is assumed that the stiffeners are monolithically connected to the plate. This assumption implies that identical displacement field must be produced at the junction between the plate and the beams.

(Huang 1987) improved the degenerated shell element by assuming transverse shear and membrane strains. An explanation of the locking behaviour and also the location of sampling points for the assumed strain fields were given. In order to overcome the shear locking problem of the new element, transverse shear strains in the natural coordinate system were assumed and membrane strains in the orthogonal curvilinear coordinate

system were applied to this element to avoid membrane locking behaviour. Many examples were considered for both linear and nonlinear analysis.

(Thannon 1988) applied the finite element method for the nonlinear analysis of reinforced concrete stiffened shells and folded slabs. Degenerated quadratic thick shell elements were employed for the shell and for beams. A curved element formulation was adopted based on the beam theory and incorporating transverse shear deformation. For the stiffened shell the modification of the degenerated shell element was made which was needed for the connection with curved beam element. A layered model was employed with special treatment of the combined torque and bending about the weak axis of the beam. Various reinforced concrete beams, stiffened shells and folded slabs were analyzed. The numerical results of the structures analyzed show a fair agreement compared with the experimental results.

(Guice and Rhomberg 1989) presented a truss-element model which incorporates nonlinear geometric theory to reinforced concrete slabs response by relating the behavior of the slab to that of shallow shell. A simple truss element model, which is typically used in the stability analysis of thin shells is used to predict the total load-deflection curve for restrained slabs. The comparison of the results with experimental data suggests that it is a reasonable approach to predict the total load-deflection behavior especially when a single degree of freedom model is considered.

(Vecchio and Collins 1990) investigated the ultimate load carrying capacity of uniformly loaded orthotropically reinforced concrete flat slabs by using finite element method. The material and geometric non-linearities and membrane action effect were taken into account. A comparison of the

numerical results obtained from finite element analysis with the experimental results showed a good agreement.

(Ghalib1990) used layered 8-node quadrilateral flat shell elements with a total of six degrees of freedom per node. Prestressed concrete was considered as an elastic-plastic material with work hardening in compression and as elastic brittle fracture in tension. Reinforcement was represented by either a smeared, fully bonded steel layer of orthotropic properties to represent equally spaced reinforcement arrangements, or as a discrete, generally curved steel strand element to model special heavy reinforcement concentrations.

(Abbasi and Abdel-Rahaman 1992) employed a layered degenerated quadratic plate element with five degrees of freedom in the nonlinear finite element for the analysis of reinforced concrete slabs which were subjected to central patch loads. The components considered in their work were; perfectly plastic models versus hardening models, role of on punching capacity, parametric variation of tension stiffening parameters, parametric variation of degraded shear modulus and the role of yield criterion. Also a smeared crack model with orthogonal cracking was adopted.

(Scordelis 1993) presented status of nonlinear analysis in the design of concrete shell structures. Material and geometric nonlinearities were included in the analysis. A 9-noded isoparametric shell element was employed in the analysis of hyperbolic paraboloid roofs.

(Usama 1997) used layered degenerated shell element for the nonlinear finite element analysis of reinforced concrete partially restrained slabs. Perfect bond between the concrete and reinforcement is assumed. The compressive behavior of reinforced concrete is simulated by an elasto-plastic work harding model. Heterosis isoparametric element has been used

to model the concrete. In tension, a smeared crack model with fixed orthogonal cracks has been used with the inclusion of models to account for the retained post cracking stress and shear modulus. The nonlinear equation of equilibrium has been solved using an incremental-iterative technique operating under load or displacement control, standard and modified Newton-Raphson solver technique is adopted. Displacement control incrementation scheme is used to trace the falling branch of the load-deflection curve. Good agreement with experimental results for slabs analyzed was gained.

(Maria 1998) used the degenerated isoparametric shell elements in the thermal finite element analysis of reinforced concrete shells. It incorporates the nonlinear material model for concrete, which takes into account cracking of concrete, tension stiffening, softening of concrete in compression, and yielding of reinforcement. The finite element formulation used was originally presented in detail by (Maria 1998). Parametric studies that include the influence of reinforcement ratio, cracking strength, and material models are presented.

(Maria 1998) employed a formulation of degenerated shell finite element, which takes into account three-dimensional states of stress and strain throughout the element in the shear analysis of reinforced concrete shells. The formulation was developed with the aim of modelling reinforced concrete shell structures subjected to high transverse shear stresses. A three-dimensional constitutive model for concrete was implemented and it allowed using out-of-plane (transverse) reinforcement in the elements. A layered technique was used to represent the concrete and steel. The rotating crack model is adopted where the transverse shear strains influence the principal directions of strains and stress. The formulation was verified against experimental results. The specimens

analyzed were shell panels subjected to out-of- plane (transverse) and in-plane shear and reinforced concrete shells subjected to punching shear.

(Baka 2002) applied the nonlinear finite element for the analysis of flat slab and reinforced concrete funicular shell. A layered degenerated quadratic shell elements with five degrees of freedom per node were employed. Both the elastic-perfectly plastic and strain hardening plasticity approach had been employed to model the compressive behavior of the concrete. A smeared fixed cracked approach was used; the reinforcement is represented by equivalent thickness with uniaxial properties. Perfect bond between the concrete and reinforcement has been assumed. The geometric and material nonlinearity were considered in the analysis. Instead of modeling the edge beam in these structures, it was considered as boundary conditions only. Numerical results obtained from the finite element analysis showed good agreement with experimental results.

2.3 Optimal Design of Plates and Shells

A multi-level optimal design approach for reinforced concrete structures was suggested by (Kirsch 1983). The cost was considered as the objective function. In the third (system) level, the design moments are found from an elastic analysis depending on the update concrete dimensions for each cycle. In the second (element) level, the concrete dimensions for each element are optimized successively and only a single independent variable (depth) is involved considering the width to be constant for all members. Hence, the problem is reduced to a set of one-dimensional explicit optimization problems subjected to side constraints (deflection, shear, moment, upper bound and lower bound of depth). In the first (cross section) level, the amount of reinforcement in each critical

section is found for the corresponding updated concrete dimensions and design moment as found in the third level. The proposed multi-level approach was found to be most suitable for microcomputer applications.

(Gotsis 1992) worked out the structural optimization of thin shell structures that are subjected to stress and displacement constraints. In the static finite element stage, a triangular element was developed by modifying the membrane stiffness matrix in the local coordinates system and adding a fictitious rotational stiffness matrix. In the optimization part, and instead of the iterative solution, a stress ratio formula, which redesigns the thickness of each finite element of the structure, was solved by an analytical method. The modified method was used to design a thin, cylindrical shell structure with minimum weight.

(Bletzinger 1993) demonstrated the use of optimization techniques for the form-finding of shells. Stress distributions in shell structures have been shown to be extremely parameter sensitive, especially where the parameter mid-surface is shape. A method was established on the synthesis of design modeling, structural analysis and mathematical optimization techniques.

(Zhiming 1995) presented the optimal design, nonlinear bending and stability of shallow shells of revolution with variable thickness. The problem was investigated by means of a modified iterative method proposed earlier by the same author. Analytical solutions for nonlinear bending shallow shells of revolution with variable thickness, such as spherical and conical shells were presented. The optimal design of plates and shells, in which the volume is minimized or the critical load of the shell is maximized, was investigated. When the volume of the shell and the arch height of the shell are given, the variable thickness problem was

solved. In addition, the paper gave the constraints of optimization for circular plates under nonlinear bending.

(Ostwald 1996) presented the multi-objective optimization of thin-walled sandwich cylindrical shells subjected to axial compression. The optimization problem was considered as bicriterial one, with the weight of the shell as the first objective function, and the flexibility as second (the flexibility was defined as the susceptibility of the shell to deflection). The optimization was carried out using the so-called “Pareto optimality”, in which a set of optimal comprise solutions is generated and then, the best optimal solution is chosen from this set. The thicknesses of the layers were considered as design variables. The linear theory of shell stability was employed to calculate the critical load. The constraints were imposed on stability, stresses, deflection and technological requirements.

(McGowan and Anderson 1997) prepared a design-optimization study of composite cylindrical shells subjected to uniform axial compression. The design was based on nonlinear analysis including material nonlinearity with prismatic flat or curved-plate elements that are rigidly connected along their longitudinal edges. The analysis included the effects of anisotropy and through-the-thickness or transverse shear deformation. The objective function was the mass of the structure. The design variables were the thicknesses of the face sheets and the core. The design constraints were considered as stresses and strains.

(Rich 1997) presented an optimization strategy for the optimal design of composite shells. The study focused on the optimal design of balanced symmetric cylinders. The design was based on linear static analysis with large rotations. The implemented element was a quadrilateral flat four-node element with six degrees of freedom at each node using linear and cubic-linear shape functions. The constraints of the nonlinear

optimization problem were solved by sequential linear programming (SLP). The objective function was the weight of the cylinder and the design variables were related to shell wall thickness, in-plane stiffness and bending stiffness. The design constraints were introduced to prevent buckling.

(Tong and Agrawal 1998) performed an analytical model, finite element solutions and optimal shape control of composite thin plates. Two dimensional finite element approaches were developed by assuming that the in-plane displacements are negligible. The four-node twelve d.o.f. DKQ (Discrete Kirchhoff Quadrilateral) element was implemented in the elastic analysis. The nodal variables were the transverse displacement w and its derivatives at the four corner nodes. The optimization is constrained to obtain a prescribed deflection shape.

(Kruzelecki and Trzeciak) investigated the problem of optimal design of rotationally symmetric inelastic shells loaded by hydrostatic pressure. The objective of the study was to obtain such geometry of a shell and a shape of a middle surface which lead to the minimum volume of shell material (or weight) under a constant given critical pressure. Additionally, and as an equality constraint, the internal capacity of the shell was considered. The hypothesis of the locality of buckling was utilized and the optimal structure was sought in the class of the shell of uniform stability. The inelastic material behaviour was defined by a tangent modulus. A parametrical optimization was performed.

(Dipperry and Srivastava 1999) studied the optimal design of pressure vessels to predict the stresses in the vessel. The design was based on a linear analysis with stress limits as allowable stress magnitudes. The objective function was the weight of the vessel as a measure of fabrication costs. The constraints were established to satisfy stress limits of the

American Society of Mechanical Engineering (1998). The study also included notes about locating the weld in an area of lower bending stress or in an area where it may require less weld material in a two-dimensional problem.

(Rozvany and Olhoff 2000) prepared a study about topology optimization of structures and composite continua. The study included a formulation for optimal structural design of plate-like structures via plate theory and 3-D theory of elasticity, based on finite elements. The influence of geometrical nonlinearity was examined. The objective function and design variables were related to sizes and shapes under displacements, buckling and stress constraints.

(Rigo 2001) presented the optimal design of marine structures (ships and naval structures). To perform a rational analysis, the structures were modelled by using stiffened plate and stiffened cylindrical shell elements in the linear elastic analysis of orthotropic structures using the so-called “stiffened plate” method to provide a fast and reliable assessment of the stress pattern existing in the stiffened structure. The nonlinear constrained optimization problem was solved by iterative approach. The objective of the design was the minimum cost including raw material, labour and overhead costs. The design variables were the plate thickness and dimensions of the longitudinal stiffeners. Three types of constraints were distinguished: structural, technological and geometrical.

(Hayder 2005) presented the structural optimization of plates and shells problem, which is dealt with as a constrained nonlinear optimization, the so-called Modified Hooke and Jeeves method is employed by considering the volume of the structure as the objective function and dimensions as the design variables with geometrical constraints. A plastic

design is carried out using von-Mises yield criterion in terms of stress resultants with the effect of axial force being incorporated.

The present work deals with the optimal design of reinforced concrete plates and shells. In the analysis a layered degenerated quadratic shell elements with five degrees of freedom per node were employed. Both the geometric and material nonlinearity were considered in the analysis. the elasto-plastic and strain hardening plasticity approaches had been employed to model the compressive behavior of the concrete. A smeared fixed cracked approach was used; the reinforcement is represented by equivalent thickness with uniaxial properties. Perfect bond between the concrete and reinforcement has been assumed. In the optimization problem the so-called Modified Hooke and Jeeves method is employed by considering the volume of the structures the objective function and dimensions as the design variables with geometrical constraints.

NONLINEAR MATERIALS MODELING

3.1 Introduction

In this chapter the constitutive behaviour of concrete is described in a form suitable for numerical computation. Essentially the same constitutive relationships are employed for the elasto-plastic plate and shell analysis. For reasons of space detailed expressions are only provided for the relatively more intricate plate and shell formulation. (Owen and Figueiras 1983)

3.2 Experimental Behavior of Concrete

It is well known that the experimental data on the properties of concrete are essential to construct the constitutive relations required for finite element analysis, but it is also well known that there are many factors which affect the mechanical properties of concrete. The most important of these factors is the presence of fine micro-cracks in the internal structure of concrete even prior to application of the external load (Hsu 1963). This is due to the structure composition of concrete, which is a composite material consisting of paste made from Portland cement and water, which binds the particles of aggregate. Some times the incomplete hydration of the Portland cement (which consists mainly of lime, silica, alumina and iron oxide and these compounds interact with one another in the kiln to form a series of complex products), usually produces a concrete which contains capillary pores filled with water or air. This may be a source of weakness of concrete structure (Neville 1981) because of the presence of gaps in the bond between the paste and aggregate which represent a project of micro-cracks. Other source of micro-cracks is the circumferences of mixing, placing and curing which may cause segregation, shrinkage and thermal movements in the

mortar; more micro-crack may develop during loading. All these developed micro-cracks contribute to the nonlinear behavior of concrete when these micro-cracks gradually grow with the increasing of loading.

Concrete can behave as either a linear or nonlinear material depending on the level and the nature of the stresses to which it is subjected. Under low levels of stress, concrete behaves as “a linear elastic material”, while for higher values of stress and for sustained loading it exhibit is highly nonlinear properties that have a considerable effect on the behavior of reinforced concrete structures. Under multiaxial loading, concrete exhibits its strength and stiffness properties, which differ from those, displayed under uniaxial compression (Gerstle 1980). Many experimental studies have been performed on the behavior of concrete under uniaxial and biaxial behavior (Kupfer and Rusch 1969), The object of such studies is to develop efficient and accurate numerical models after obtaining the necessary experimental data of the complex response of concrete under various stress conditions and to use such numerical model for the nonlinear analysis of concrete structures by finite element analysis.

Some typical response of concrete and its experimental stress-strain relations under uniaxial and abiaxial states of stresses are summarized in the following sections.

3.2.1 Uniaxial and Biaxial Behavior of Concrete

A typical stress-strain curve is shown in Figure (3.1) for concrete subjected to uniaxial compression. Concrete behaves as a linear elastic material up to a stress level of about 30% of its uniaxial compressive strength f_c' . The stress level of $0.3f_c'$ has been onset of cracking and proposed as a limit of elasticity (Kotsovos and Newman 1977). After this limit, the curve shows a gradual increase in curvature up to about $0.75f_c'$. For compression stresses above this value, the rate of crack propagation increases rapidly and

the stress-strains curve bends sharply until the peak stress level is reached (Chen 1982). Beyond this peak stress, the stress-strain curve has a descending part until crushing failure occurs (crushing of the concrete occurs when maximum compressive strain is reached as a result of formation of many cracks parallel to the load direction) at ultimate compressive strain ϵ_u .

The initial modulus of elasticity of concrete can be expressed as a function of the compressive strength. Poisson's ratio ν for concrete under uniaxial compressive loading ranges from about (0.15 to 0.22). Under uniaxial loading the ratio ν remains constant until approximately $(0.8-0.85) f'_c$ after which the ratio is increased (Chen 1982).

Under tensile stress, concrete exhibit a stress-strain curve of similar shape to that under compressive loading as shown in Figure (3.2) and the strength of concrete, f'_t , is approximately a one-tenth of the compressive strength (Neville 1981).

Concrete is an elastic-softening material, which means that the stress beyond the peak load decreases with increasing deformation is a fact which makes the investigation of concrete behavior rather difficult. The steep decay in the stress-strain curve beyond the peak load is due to the development of a single crack in the specimen (Thannon 1988).

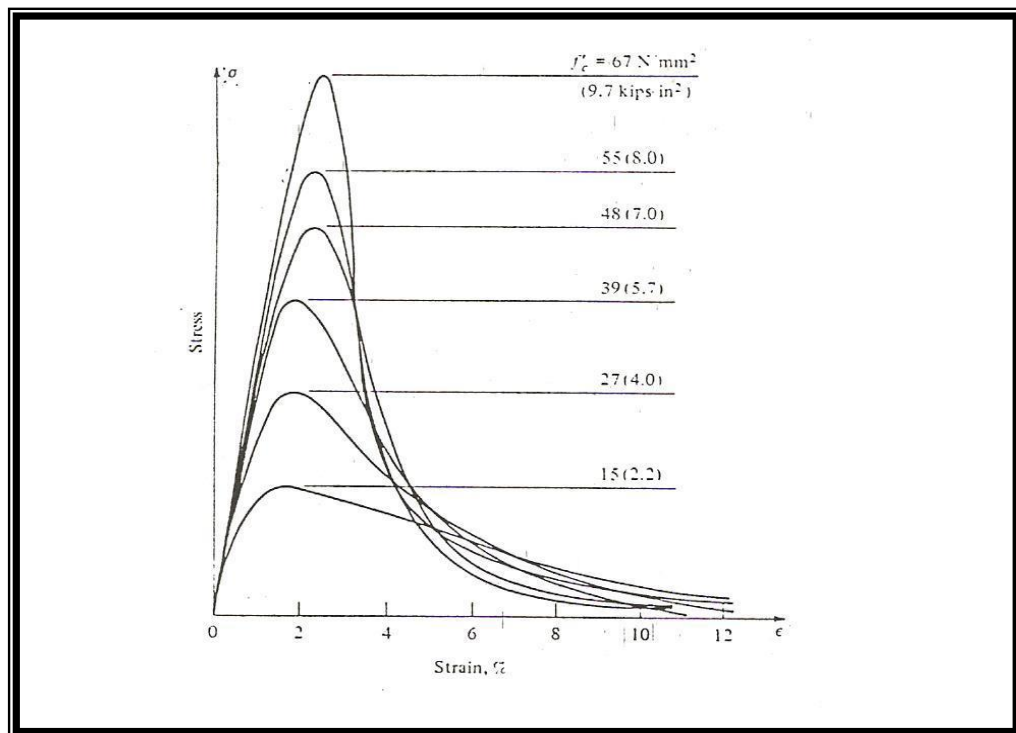


Fig. (3.1) Uniaxial compression stress-strain curves for concrete(Chen 1982).

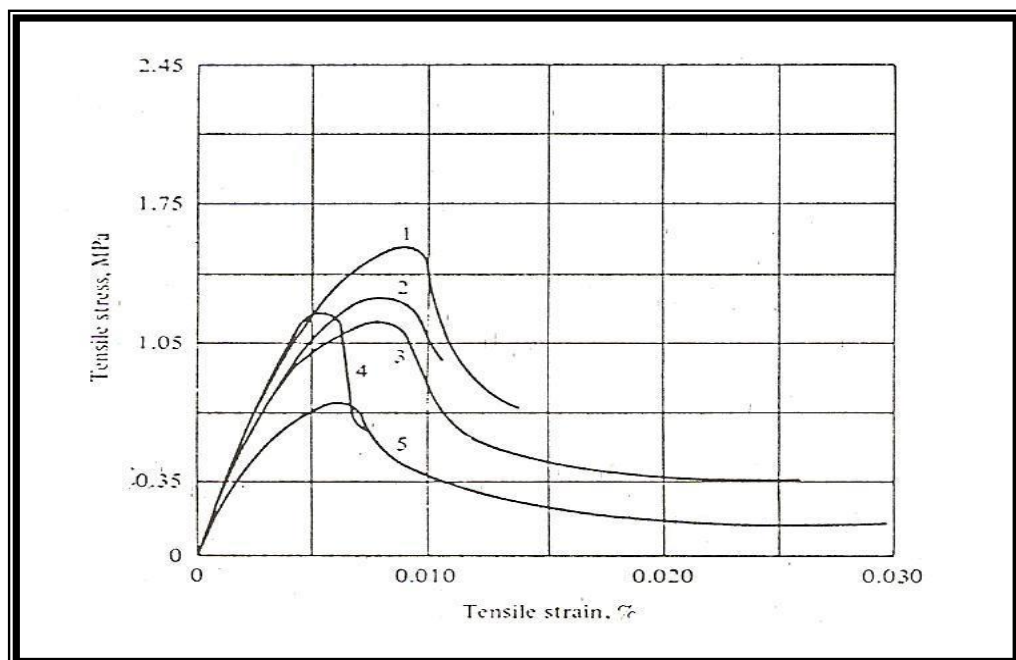


Fig. (3.2): Uniaxial tensile stress-strain curves for concrete (Chen 1982).

The initial modulus of elasticity of concrete in tension is slightly greater than that in uniaxial compression, while Poisson's ratio in tension is

some what smaller than that in compression (Chen 1982). Under biaxial states of stress, concrete exhibits strength, stiffness and stress-strain behavior somewhat different from those of uniaxial state. The stress-strain relationships are shown in Figure (3.3). It can be seen that the maximum compressive strength and strain at the peak stress increases with lateral compression stress. At a stress ratio ($\sigma_2/\sigma_1=0.5$), an increase of about 25% of the maximum strength is obtained, and about 16% increase at stress ratio ($\sigma_2/\sigma_1=1.0$). In the compression-tension state, the compressive strength decreases almost linearly as the applied tensile stress is increased. Under biaxial tension, the strength is almost the same as the uniaxial tensile strength. Two dimensional failure of concrete occurs by tensile splitting. The fracture surface is orthogonal to direction of maximum tensile stress or strain.

The state of stresses in many practical structures is a biaxial one, which led to the use of a biaxial strength envelope, directly extrapolated from experimental results. The biaxial envelope proposed (Kupfer and Gerstle 1973) is shown in Figure (3.4) and compared with experimental results. The curve can be divided into three regions, compression-compression, compression-tension and tension-tension.



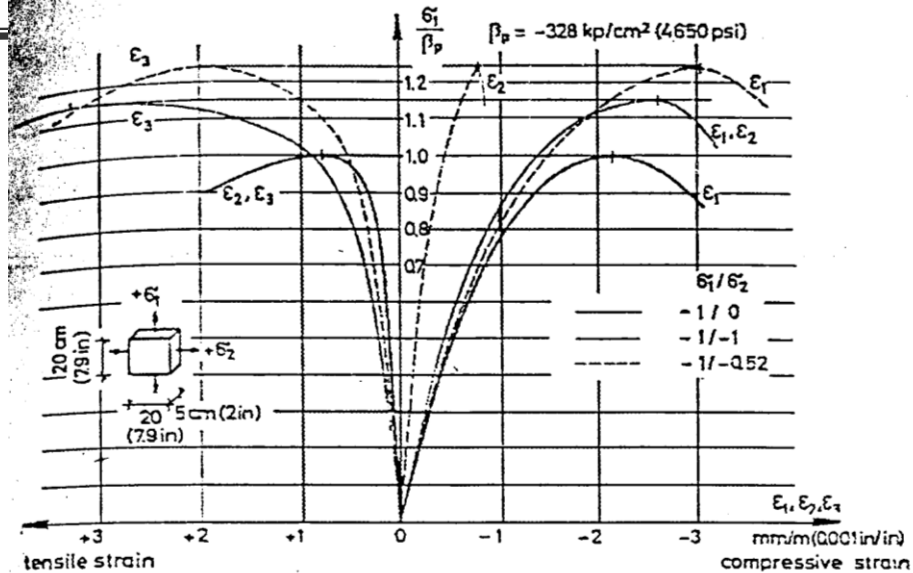


Fig. (3.3): Typical stress-strain relation for concrete under biaxial for concrete under biaxial compression (Kupfer and Hilsdorf 1969).

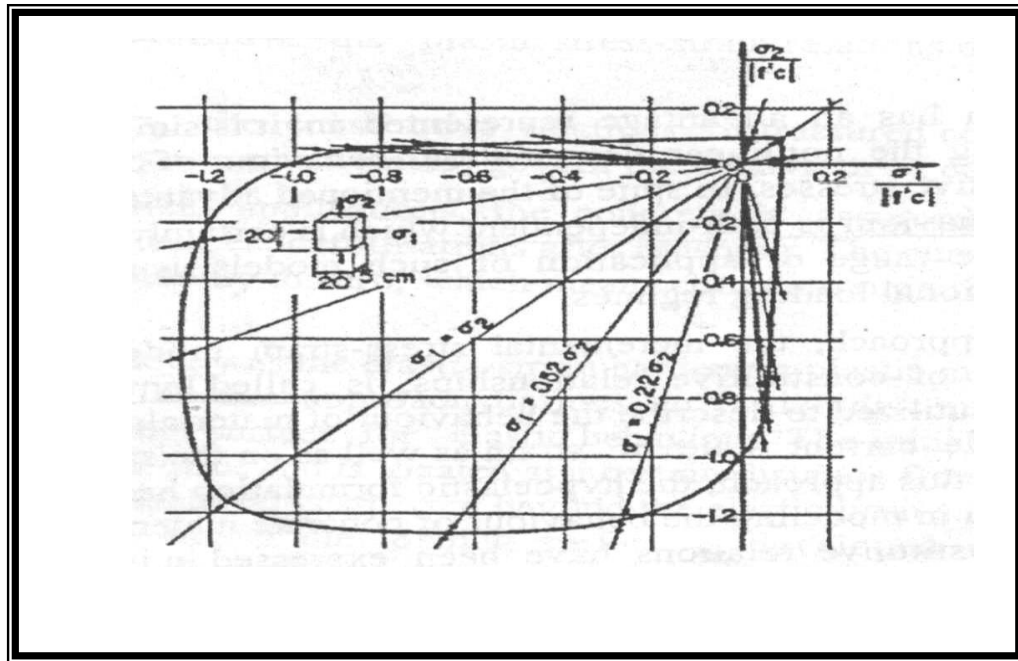


Fig. (3.4): Biaxial strength envelopes (Kupfer and Gerstle 1973).

3.3 Numerical Models for Concrete

Numerical representation of concrete properties in connection with the nonlinear finite element modeling of reinforced concrete members is one of the most important aspects of any realistic analysis.

Some numerical models of concrete are summarized here with some of the failure theories and criteria, which have been employed.

3.3.1 Failure Theories

In general, concrete failure can be divided into tensile (cracking) and compressive (crushing) models. The tensile failure of concrete is a brittle failure characterized by a gradual growth of micro-cracks, which join together and eventually disconnect larger parts of the structures. The compression failure shows relative ductility and appearance of many small cracks.

The failure theories are analytic formulations expressed in terms of stresses or strains (or their invariants) and a varying numbers of material constants are used to define failure under multiaxial stress state. The failure theories applied to concrete are discussed in reference (Chen 1982).

3.3.2 Constitutive Relationships

Mathematical description of the mechanical properties of concrete is established by the constitutive equation. Several approaches have been proposed to model the observed stress-strain relationships under multiaxial stress states. Such approaches can be classified into; Elasticity-based model, Plasticity-based model. These approaches will be briefly discussed in the following sections; more details are given in references (Chen 1982).

3.3.2.1 Linear and Nonlinear Elasticity Models

These models are based on elasticity theory to represent the behavior of concrete under general type of loading. Linear elastic models have been used

in cases where the cracking behavior is the most important factor of nonlinearity.

In general, two different approaches are employed in the formulation of nonlinear elastic models. These are the **total (secant)** and the **incremental (tangent)** formulation. In the total (secant) stress-strain models, the current state of stress is assumed to be uniquely determined as a function of the current state of strain. This type of formulation is path –independent which is not true for concrete in general. Therefore, the range of application of such models is only suitable for monotonic or proportional loading regimes. The total stress-strain models have been utilized in describing the nonlinear deformation behaviour of concrete under biaxial and triaxial compressive stresses, mainly because of its simplicity.

The incremental stress-strain models are used to describe the behaviour of materials in which the state of stress depends on the current state of strain and the stress path followed to reach that state. Due to path-dependent behaviour, incremental models provide more realistic description of concrete behaviour under general loading condition (non-monotonic and non-proportional loading regimes) than the total stress-strain formulation.

In general most of the secant models or tangent models for concrete have been formulated basically as a simple extension of the isotropic linear elastic stress-strain relations with secant or tangential modular (e.g. bulk and shear module) which are assumed to be functions of the stress or strain invariants.

3.3.2.2 Perfect and Strain Hardening Plasticity Models

Nonlinear deformations of concrete under compressive stress are basically inelastic since upon unloading only a portion of the total strain can be recovered. Therefore, the stress-strain behavior of concrete materials may

be separated into recoverable and irrecoverable components. The recoverable behavior is treated within the framework of elasticity theory, while the irrecoverable part is based on the theory of plasticity.

In triaxial compression regimes, concrete behaves as a ductile material on the yield of failure surface before reaching its crushing strain. To account for this plastic flow, various plasticity formulations are adopted as an elastic-perfectly plastic material (perfect plasticity), or as an elastic-plastic hardening (strain-hardening plasticity) behavior of concrete up to the ultimate strength.

In the perfectly plastic model, the complete stress-strain relationships include three parts: before yielding; during plastic flow; and after fracture (failure). Concrete behavior before yielding is assumed to be elastic until the combined state of stress reaches the yield surface. During plastic flow, the behavior of concrete is described by the plastic stress-strain relations. Post-fracture behavior is governed by the constitutive relations of fractured concrete (crushing under compressive stress state). After crushing, the concrete is assumed to lose its resistance completely against further deformations and the current stresses drop suddenly to zero.

The formulation of the constitutive relations for a strain hardening plastic material is based on three fundamental assumptions. These assumptions are the shape of an initial yield surface, the formulation of a suitable hardening rule that describes the evolution of subsequent loading surfaces and the formulation of an appropriate flow rule that specifies the stress-strain relation in the plastic range.

The initial yield surface is required to mark the stress level at the onset of plastic deformations. The evolution of subsequent loading surfaces during plastic deformation is described by specifying an appropriate hardening rule. Three types of hardening rule are frequently used in strain-hardening

plasticity models. These are isotropic, kinematics, and mixed hardening rule. In an isotropic hardening model, the initial yield surface is assumed to expand uniformly without distortion as plastic flow occurs. The kinematics hardening rule assumes that during plastic flow the loading surface translates as a rigid body in the stress space, maintaining the size and shape of the initial yield surface. In the mixed hardening rule, the loading surface experiences both translation and uniform expansion in all directions (Owen and Hinton 1980).

3.4 Modeling of Concrete Cracking

Due to its low tensile strength, concrete cracks at very low stress compared with its ultimate compressive stress. The tensile crack reduces the stiffness of concrete and it is usually considered as the major source of nonlinearity in the behavior of plane and reinforced concrete structures (Maria 1998). In general, each model which has been developed to represent cracking during a finite element analysis of a reinforced concrete member is composed of the following three basic components. These are criteria for crack initiation, a method of crack representation and criteria for crack propagation. Two criteria for crack initiation have been commonly used, the maximum principal stress criterion and the maximum principle strain criterion. When a principal stress or strain exceeds its limiting value a crack is assumed to occur in a plane normal to the direction of the offending principal stress or strain. The initiation of cracking is not of a primary importance in most reinforced concrete structures, but the extension of the cracked zone and post cracking behavior represent a significant portion in the structural response.

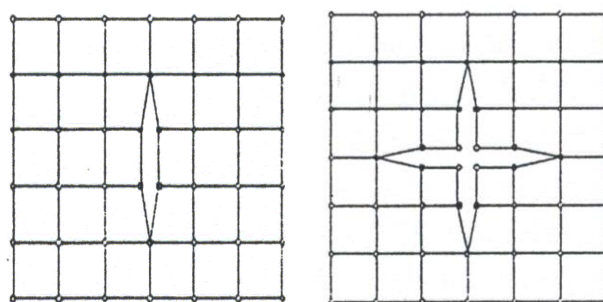
In general, two main approaches have been used to represent the cracking of concrete, the discrete and smeared crack representation. The select of discrete or smeared crack representation depends primarily on the purpose of the finite element analysis. Both these models use a strength concept for cracking initiation and propagation by defining tensile cracking in

terms of maximum stress or strain. These methods will be discussed as follows:

3.4.1 Discrete Crack Model

In this model, a crack is produced by the physical separation into two sides across the crack. The first finite element model of reinforced concrete to include the effect of cracking was developed (Ngo and Scordelis 1967). The crack pattern was previously assumed and different nodal point numbers were assigned to each side of the crack (Figure 3.5).

Generally, the location and orientation of the cracks are not known in advance, and hence the difficulty of applying such an approach is obvious. The improvement in applying this model may be achieved by changing of the topology and the change of the finite element mesh with progressive crack propagation, is computationally complex and time consuming. The discrete crack model is convenient for cases in which the local behavior is of interest, and the bond slip and shear transfer across the crack can be better accounted for. If the overall behavior of the structure is desired, the smeared approach seems to be a better choice.



a) One-directional cracking

b) Two-directional cracking

Fig. (3.5) discrete crack model

3.4.2 Smeared Crack Model

Material continuity is maintained in this model during cracking. The material properties are modified to account for the damage included in the material (Thannon 1988). When the principal tensile stress in an element exceeds the tensile strength of concrete, the element assumed a crack in the direction normal to the direction of the principal stress. Rather than representing a single crack, this procedure has the effect of representing many finely spread cracks in the cracking direction as shown in Figure (3.6). The smeared crack is more popular than the discrete crack model because it allows for automatic generation of cracks without redefining of the finite element topology and offers complete generality in representing possible crack directions.

Two different models are commonly used for defining the crack direction. The first is the fixed-orthogonal crack model. In this model the direction of the crack is fixed normal to the direction of the first principal tensile stress that exceeds the cracking stress. After a crack is formed the concrete behaves orthotropically with local material axes being defined by the directions normal and tangential to the crack plane as shown in Figure(3.6). By fixing the direction of the cracks, the subsequent rotation of the principal stress is ignored which leads to a stiffer response and higher collapse loads (Gupta and Akbar). The second model is the rotating or swinging crack. In this model, the crack direction is assumed to be normal to the principal tensile

strain direction when the strains reach a specified limiting value. With further loading and changing of the principal strain direction, the crack is assumed to rotate and orthotropic material axes are set in the new crack direction. In the present study the fixed-orthogonal smeared crack model has been used.

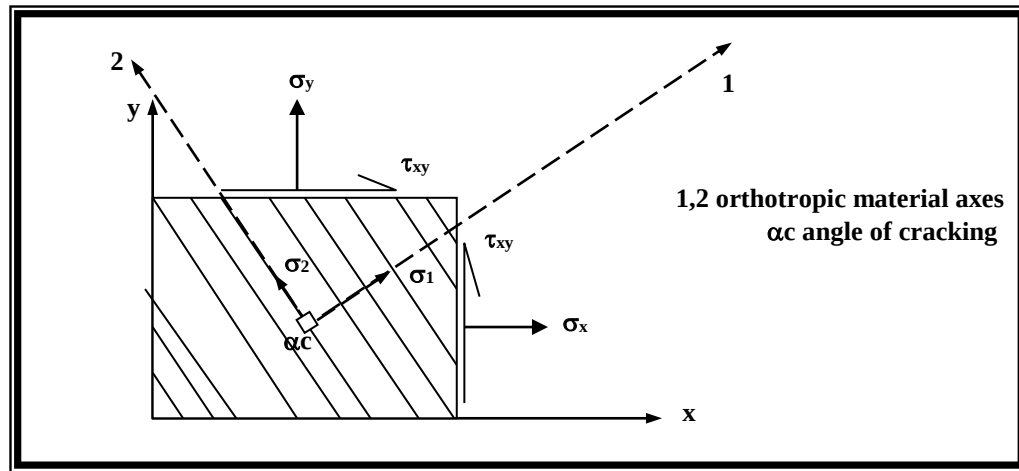


Fig. (3.6) Smeared crack model and corresponding material axes for concrete cracked in one direction.

3.5 Post Cracking Models

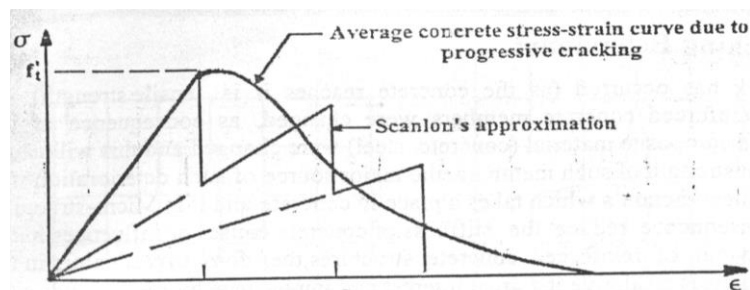
After crack has occurred (as the concrete reaches its tensile strength), the characteristics of reinforced concrete members were changed, and all the additional deformation due to micro-cracks will localize within a fracture zone. In this zone the stress gradually decreases as the strain increases. However, in the vicinity of the fracture zone a softening zone is formed where the strain decreases with crack opening and some of the micro-cracks at this zone are arrested or even closed. These changes in the internal microstructure of concrete contribute to the residual strength of plain concrete after cracking. When reinforcement exists, the nature of the stress release is further complicated by the restraining effects of the reinforcing steel. After cracking, the concrete stress drops to zero and the steel carries the full load. The concrete between cracks, however, still carries some tensile stresses. This

tensile stress drops as the load increases and this dropping is associated primarily with bond deterioration between steel and concrete. This ability of concrete to share the tensile load with the reinforcement is termed as tension stiffening. The tension stiffening effect of concrete has been represented in finite element analysis using two procedures (Scanlon and Murry 1974). In the first method, the tension portion of the concrete stress-strain curve has been given a descending branch, Figure (3.7a and b). This form of tension stiffening effect was first introduced (Scanlon and Murry 1974).

The second method of representing the stiffening effects is to increase the steel stiffness (Scanlon and Murry 1974). As illustrated in the modified stress-strain diagram in Figure (3.7c), the additional stress in the steel represents the total tensile force carried by both the steel and concrete between the cracks.

Experimental evidence reveals that cracked reinforced concrete member retains significant shear stiffness. This is mainly due to both the frictional effect at the rough crack interface, and the restraining contribution of the reinforcing steel. These processes are often termed as aggregate interlock and dowel action respectively. The effect of shear stress carrying capacity of cracked concrete in reinforced concrete members has been investigated by many experimental workers (Taylor 1959).

The numerical representation of interface shear transfer in finite element models depends on the type of crack representation. In the discrete crack model, special linkage elements with fictitious springs such as those introduced (Ngo and Scordelis 1967) are usually used when smeared cracking is employed, reduced shear stiffness may be used in the constitutive relation of cracked concrete.



(a): Scanlon's stepped model.

Fig. (3.7): Models to account for tension stiffening in concrete after cracking (Thannon 1988).

3.6 Present Model for Concrete

The simplest and modest characteristic should offer by any numerical material models, it is accurate trace of the overall behaviour of member within an acceptable degree of engineering accuracy, and also such models is preferred to be simple and can be easily implemented in finite element programs, this will make such models practical and widely used to present the behavior of material property. The material model adopted in this study is

suitable for the nonlinear analysis of reinforced concrete structures under monotonically increasing load.

An elasto-plastic work hardening model has been used to simulate the behavior of concrete in compression with limited ductility, which is terminated at the onset of crushing. The model will be described in terms of the yield criterion, the hardening rule, the flow rule, and the crushing condition.

In tension, the response is assumed to be elastic until cracking occurs. The onset of cracking is controlled by a maximum principal stress criterion. A smeared crack model is chosen to represent the cracked concrete.

3.6.1 Modeling of Concrete in Compression

3.6.1.1 The Yield Criterion

The yield criterion determines the stress level at which plastic deformation begins. For isotropic material, the initial yield criterion should be independent of the orientation of the coordinate system in which the stress state is defined and therefore it should be a function of the stress invariants only (Owen and Hinton 1980) . The yield criterion in this work is written as:

$$f(I_1, J_2) = (\alpha I_1 + 3\beta J_2)^{1/2} = \sigma_o \quad \dots\dots\dots (3.1)$$

where α , β are material parameters, I_1 is the first stress invariant, J_2 is the second deviatoric stress invariant which is defined in Table (3.1) and σ_o is the equivalent effective stress taken from uniaxial tests. For plane stress problem, I_1 and J_2 can be written as:

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$J_2 = \frac{1}{3} \left\{ (\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - (\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_x \sigma_z) \right\} + (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) \dots (3.2)$$

Since the stress normal to the middle plane (σ_z) is neglected in the degenerated element formulation, Therefore equation (3.2) can be written as:

$$I_1 = \sigma_x + \sigma_y$$

$$J_2 = \frac{1}{3} (\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y) + (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) \dots (3.3)$$

and yield function as:

$$f(\sigma) = \left[\alpha (\sigma_x + \sigma_y) + \beta \left\{ (\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y) + 3 (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) \right\} \right]^{1/2} = \sigma_o \dots (3.4)$$

It should be noticed that the Huber-Misses yield condition can be recovered by assuming $\alpha = 0.0$ and $\beta = 1.0$.

The material parameters can be obtaining by fitting biaxial test results, using the uniaxial compression test and the biaxial test under equal compression stresses (f_{cb}) are used to determine the material parametric α and β . A relation between the biaxial compression stresses (f_{cb}) and the uniaxial compression yield stress (f_c') can be assumed as:

$$f_{cb} = (1.15 \text{ to } 1.2) f_c' \dots (3.5)$$

In the result obtained (Kupfer and Rusch 1969), ($f_{cb} = 1.16 f_c'$) for a failure envelope is employed for initial yield, then the value of the constants are:

$$\alpha = 0.355 \sigma_o \quad \text{and} \quad \beta = 1.355 \dots (3.6)$$

The resulting yield criterion, equation (3.4) is compared with the experimental results presented. (Kupfer and Rusch 1969) In the biaxial stress space for $\sigma_o = f_c'$, as shown in Figure (3.8).

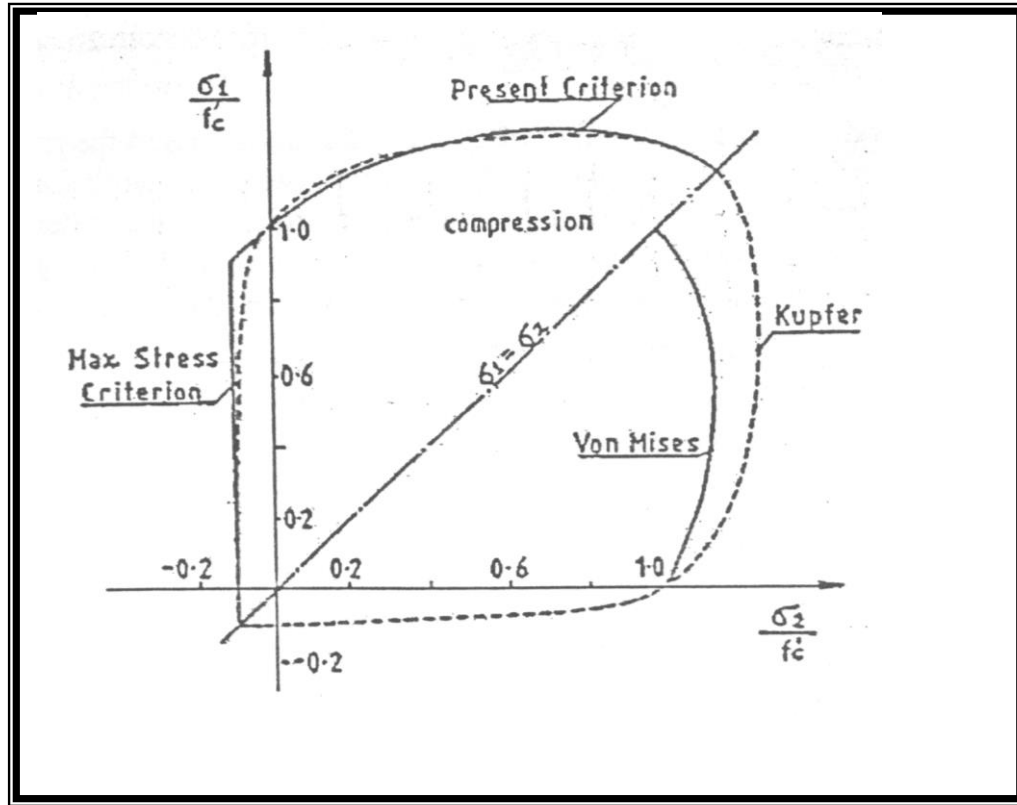


Fig. (3.8) Comparison yield and tensile cracking criterion for concrete (biaxial stress space), (Hinton and Owen 1984).

In elastic perfectly plastic model, the yield criterion can be used when the effective stress (σ_o) is taken to be equal to the ultimate compressive strength f'_c . In this model, an elastic response is assumed up to the level where the effective stress (σ_o) reaches f'_c after which a perfectly plastic response follows until the crushing surface is reached. The initial yield surface in the strain hardening model is attained when the effective stress reaches 30% of the ultimate stress f'_c . The subsequent loading surface depends on the hardening parameter H . Strain hardening will continue until the effective stress (σ_o) reaches the ultimate stress f'_c , after which a perfectly plastic response is assumed again until the crushing surface is encountered.

The one-dimensional representation of both the perfectly plastic and strain hardening model is shown in Figure (3.9).

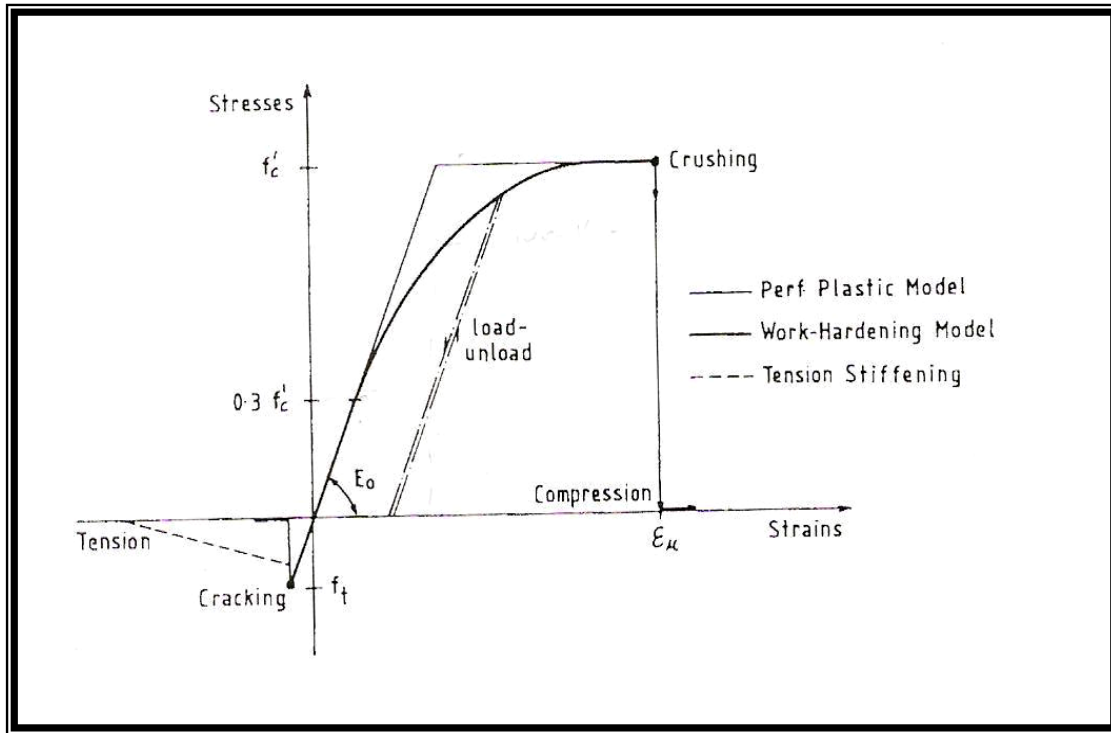


Fig. (3.9):One-dimensional representation of the concrete constitutive model, (Hinton and Owen 1984).

3.6.1.2 The Hardening Rule

The hardening rule is necessary to define the change of position of loading (yielding) surface during plastic deformation, when a material is stressed beyond its initial yielding surface. In the current study an isotropic hardening rule is adopted. Using this rule the initial yield surface is assumed to expand uniformly without distortion as plastic deformation occurs. A relationship between the accumulated (effective) plastic strain (ϵ_p) and

effective (equivalent) stress σ' is required to control the size of expansion. The subsequent yielding surface may be written as:

$$f(\sigma_{ij}) = [\alpha I_1 + \beta(3 J_2)]^{1/2} = \sigma' \quad \dots\dots\dots (3.7)$$

The concepts of effective plastic strain and effective stress are used to allow for extrapolation from the results of a uniaxial test to the multiaxial situation. The effective stress may be defined as the stress level at which further plastic deformation will occur. The effective plastic strain is an integrated increasing function of the plastic strain increments.

The equivalent stress-strain relation can be obtained from the uniaxial compressive stress-strain curve beyond the limit of elasticity (the tangent at any point of this curve is given the instantaneous modulus for all points lying on the corresponding loading surface). In the present study a proposed parabolic stress-strain relationship has been used, in which the zero magnitude is assumed for plastic strain component at the initial yield point equal to $0.3f'_c$, whereas at the peak compressive stress f'_c , the total strain has a value of ε_o . The stress-strain relationship that is adopted in the present study as shown in Figure (3.9) is as follows (Hinton and Owen 1984):

$$\sigma_o = f'_c \left[\frac{2\varepsilon}{\varepsilon_o} - \left(\frac{\varepsilon}{\varepsilon_o} \right)^2 \right] \quad \dots\dots\dots (3.8)$$

where σ_o is the effective stress, ε is the current total strain and ε_o is the strain at peak stress f'_c given by:

$$\varepsilon_o = \frac{2f'_c}{E_o} \quad \dots\dots\dots (3.9)$$

where E_o is the initial young's modulus. Substitution of equation (3.9) into equation (3.8) yields:

$$\sigma_o = E_o \varepsilon - \frac{E_o}{2\varepsilon_o} \varepsilon^2 \quad \text{..... (3.10)}$$

It is well known that the total strain ε can be separated into elastic and plastic components as follows:

$$\varepsilon = \varepsilon_e + \varepsilon_p \quad \text{..... (3.11)}$$

Where:

ε_p is the plastic strain component

ε_e is the elastic strain component given by,

$$\varepsilon_e = \frac{\sigma_o}{E_o} \quad \text{..... (3.12)}$$

To derive the relationship between the effective stress and the effective plastic strain, equation (3.11) and (3.12) are substituted into equation (3.10) and the following expression is obtained:

$$\sigma' = -E_o \varepsilon_p + E_o (2\varepsilon_o \varepsilon_p)^{1/2} \quad \text{for} \quad 0.3f'_c \leq \sigma' < f'_c \quad \text{..... (3.13)}$$

The slope of the effective stress-plastic strain curve, the hardening parameter, is used in the formulation of the elasto-plastic incremental stress-strain relationship. By the differentiation of equation (3.13) with respect to effective plastic strain, the hardening parameter can be expressed as:

$$H' = \frac{d\sigma'}{d\varepsilon_p} = E_o \left\{ \left[\frac{\varepsilon_o}{2\varepsilon_p} \right]^{1/2} - 1 \right\} \quad \text{..... (3.14)}$$

3.6.1.3 The Flow Rule

The flow rule is considered to construct the stress-strain relationship in the plastic range, so that the plastic strain increment can be determined for a given stress increment. Such definition is associated with the assumption, which makes normality of the plastic rate vector to the yield surface. The flow

rule is characterized by its simplicity, and it is widely used for concrete models. The plastic strain increment is expressed as follows:

$$d\epsilon_p = d\lambda \frac{\partial f(\sigma)}{\partial \sigma} \quad \dots\dots\dots (3.15)$$

where;

$d\epsilon_p$ is a plastic strain increment vector

$d\lambda$ is a proportional constant, which determines the magnitude of the plastic strain increment.

$\frac{\partial f(\sigma)}{\partial \sigma}$ is termed as the flow vector which defines the direction perpendicular to the yield surface.

Equation (3.7) can be rewritten as:

$$f(\sigma) = [\beta(3J_2) + 2C\sigma_o I_1]^{1/2} = \sigma_o \quad \dots\dots\dots (3.16)$$

which $C = \alpha / 2\sigma_o$ This can be solved for σ_o , giving:

$$f(\sigma) = CI_1 + [(CI_1)^2 + \beta(3J_2)]^{1/2} = \sigma_o \quad \dots\dots\dots (3.17)$$

The yield function derivatives with respect to the stress components define the flow vector {a} as:

$$\{a\} = \left[\frac{\partial f}{\partial \sigma_x}, \frac{\partial f}{\partial \sigma_y}, \frac{\partial f}{\partial \tau_{xy}}, \frac{\partial f}{\partial \tau_{yz}}, \frac{\partial f}{\partial \tau_{zx}} \right]^T \quad \dots\dots\dots (3.18)$$

in which,

$$\left. \begin{aligned} a_1 &= \frac{\partial f}{\partial \sigma_x} = C + [2(C^2 + \beta)\sigma_x + (2C^2 - \beta)\sigma_y] / Q \\ a_2 &= \frac{\partial f}{\partial \sigma_y} = C + [2(C^2 + \beta)\sigma_y + (2C^2 - \beta)\sigma_x] / Q \\ a_3 &= \frac{\partial f}{\partial \tau_{xy}} = 6\beta\tau_{xy} / Q \\ a_4 &= \frac{\partial f}{\partial \tau_{yz}} = 6\beta\tau_{yz} / Q \\ a_5 &= \frac{\partial f}{\partial \tau_{zx}} = 6\beta\tau_{zx} / Q \end{aligned} \right] \quad \dots\dots (3.19)$$

where, Q is:

$$Q = 2[(C^2 + \beta)\sigma_x^2 + (C^2 + \beta)\sigma_y^2 + (2C^2 - \beta)\sigma_x\sigma_y + 3\beta(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]^{1/2} \quad \dots (3.20)$$

The plastic multiplier can be found as:

$$d\lambda = \left[\frac{\{a\}^T [D]}{H' + \{a\}^T [D] \{a\}} \right] d\epsilon \quad \dots\dots (3.21)$$

where;

[D] is the elastic constitutive matrix

$d\{\epsilon\}$ is the total strain increment vector and H' is the hardening modulus

$$(H' = \frac{d\sigma}{d\epsilon_p})$$

The elasto-plastic increment of total strain is calculated by summing up the elastic and plastic strain components as:

$$d\{\epsilon\} = d\{\epsilon_e\} + d\{\epsilon_p\} \quad \dots\dots\dots(3.22)$$

The elastic strain increment is related to the stress increment by the elastic constitutive relation given by:

$$d\{\sigma\}=[D]d\{\varepsilon_e\} \quad \dots\dots\dots(3.23)$$

Substituting equation (3.15) and (3.23) into equation (3.22) yields:

$$d\{\varepsilon\}=[D]^{-1}d\{\sigma\}+d\lambda\frac{\partial f}{\partial \sigma} \quad \dots\dots\dots(3.24)$$

$$=[D]^{-1}d\{\sigma\}+\frac{\{a\}\{a\}^T[D]}{H'+\{a\}\{a\}^T[D]}d\{\varepsilon\} \quad \dots\dots\dots(3.25)$$

The complete elasto-plastic incremental stress-strain relationship can be expressed as:

$$d\{\varepsilon\}=[D]_{ep}d\{\varepsilon\} \quad \dots\dots\dots(3.26)$$

where,

$$[D]_{ep}=[D]-\frac{[D]\{a\}\{a\}^T[D]}{H'+[D]\{a\}\{a\}^T[D]} \quad \dots\dots\dots(3.27)$$

where the second term in the bracket represents the stiffness degradation due to the plastic deformations.

3.6.1.4 The Crushing Condition

The crushing failure of concrete is a strain-controlled phenomenon. A failure surface in the strain space must be defined so that this kind of fracture can be taken into account. Lack of available experimental data has resulted in the failure surface being developed by simply converting the yield criterion, which described in terms of stresses directly into strains (Chen 1982). Thus,

$$[\alpha I_1' + \beta (3J_2')^{1/2}] = \varepsilon_{cu} \quad \dots\dots\dots(3.28)$$

where;

ε_{cu} is the ultimate total strain extrapolated from the uniaxial compressive test results

I_1' is the first strain invariant .

J_2' is the second deviatoric strain invariant .

If the material parameters (α and β) obtained (Kupfer and Rusch 1969) is substituted in equation (3.28), the crushing condition is expressed in terms of the total strain components as follows:

$$0.355\varepsilon_{cu}(\varepsilon_x + \varepsilon_y) + 1.355[(\varepsilon_x^2 + \varepsilon_y^2 - \varepsilon_x \varepsilon_y) + 0.75(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)] = \varepsilon_{cu}^2 . \quad (3.29)$$

The above equation is controlled by the ultimate strain ε_{cu} , therefore when such value is reached or exceeded, concrete is assumed to lose all its characteristics of strength and rigidity.

3.6.2 Concrete in Tension

The tensile strength of concrete is relatively small and its influence on the reinforced concrete structure comes from the cracked zones. Therefore, a simplified averaging procedure is more convenient for finite element formulation. A smeared representation for cracks is assumed, which implies that cracks are distributed across a region of the finite element. A linear elastic response of concrete is assumed until the fracture surface is reached. This tensile type of cracking is governed by a maximum tensile stress criterion. The fixed orthogonal crack model is used here, in which cracks are assumed to form in a plane perpendicular to the direction of the maximum principle stress. A sudden or gradual relaxation of the normal stress is adopted after cracking has occurred, according to the tension stiffening diagram. The modulus of elasticity and Poisson's ratio are reduced to zero in the direction perpendicular to the crack and a reduced shear modulus is employed.

Cracked concrete is treated as an orthotropic material with axes parallel and normal to the crack direction. Let the stress in principal direction (1) be the major stress for both directions (1) and (2) in the plane of the

structure. The cracking is presumed to occur in the orthogonal direction (2), when the principal stress in direction (1) exceeds the limiting tensile strength of concrete and the stress-strain relationship in this case is

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \\ \tau_{13} \\ \tau_{23} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & E & 0 & 0 & 0 \\ 0 & 0 & G_{12}^r & 0 & 0 \\ 0 & 0 & 0 & G_{13}^r & 0 \\ 0 & 0 & 0 & 0 & 5/6 G \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} \dots\dots\dots (3.34)$$

where;

E is the current modulus in direction (2)

G^r is the reduced shear modulus to be defined later

A second crack plane perpendicular to the first one is formed when the value of tensile stress reaches **f_t'** (see Figure (3.10)). The stress-strain relationship for cracked in two directions becomes (Chen 1982)

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \\ \tau_{13} \\ \tau_{23} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 G_{12}^r & 0 & 0 \\ 0 & 0 & 0 & G_{13}^r & 0 \\ 0 & 0 & 0 & 0 & G_{23}^r \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} \dots\dots\dots (3.35)$$

These relations must be transformed to the local axes system using the following transformations (Lin and Scordelis 1975).

$$\{\sigma\}_{1,2,3} = [T] \{\sigma\}_{x,y,z} \dots\dots\dots (3.36)$$

$$\{\epsilon\}_{1,2,3} = [T'] \{\epsilon\}_{x,y,z} \dots\dots\dots (3.37)$$

where;

$$\{\sigma\}_{x,y,z} = [\sigma_x, \sigma_y, \tau_{xy}, \tau_{xz}, \tau_{yz}] \dots\dots\dots (3.38)$$

$$\{\epsilon\}_{x,y,z} = [\epsilon_x, \epsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}] \dots\dots\dots (3.39)$$

$$[T] = \begin{bmatrix} \cos^2 \theta & \sin^2 \theta & 2\sin\theta\cos\theta & 0 & 0 \\ \sin^2 \theta & \cos^2 \theta & -2\sin\theta\cos\theta & 0 & 0 \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2 \theta - \sin^2 \theta & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix} \dots (3.40)$$

$$[T'] = \begin{bmatrix} \cos^2 \theta & \sin^2 \theta & \sin\theta\cos\theta & 0 & 0 \\ \sin^2 \theta & \cos^2 \theta & -2\sin\theta\cos\theta & 0 & 0 \\ \sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2 \theta - \sin^2 \theta & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix} \dots (3.41)$$

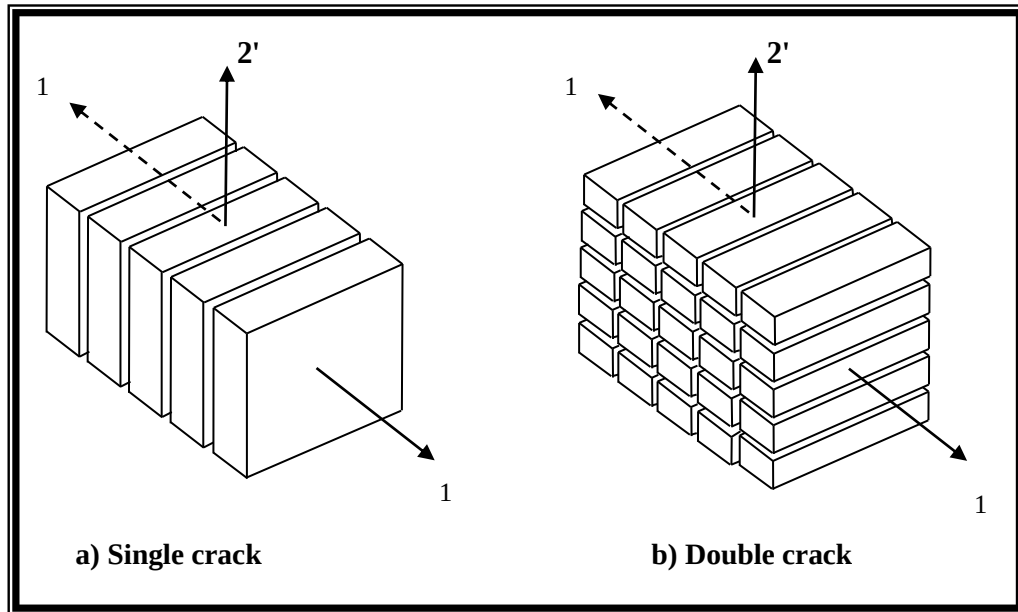


Fig. (3.10) Crack patterns in load material axes.

3.6.2.1 Tension Stiffening

Cracked concrete can still initially carry some tensile stress in the direction normal to the crack. The behavior of tension stiffening is considered as indicated in Figure (3.11) by assuming a gradual release of the concrete stress component normal to the cracked plane for the layers of concrete after cracking has occurred. Also, the linear softening process during the loading and unloading of the cracked concrete has been assumed. Two constant parameters α_m and ϵ_m are used to represent and determine the shape of such a model. The range of the value of first parameter α_m is taken between (0.5-0.7), and the value of the second parameter (ϵ_m) is taken between the ranges (0.0015-0.002). However, it is found that the change of the structural behavior with this variation of constants is small and a constant value of $\alpha_m = 0.6$ and $\epsilon_m = 0.002$ could be used (Thannon 1988).

Clearly, the crack direction and maximum tensile strain at any stage of loading are important in reinforced concrete analysis. Therefore, during the computation process these values are stored to be used in subsequent iterations. The unloading and reloading behavior (shown in Figure (3.11)) can be obtained by using a fictitious modulus of elasticity E as,

$$E_i = \frac{\alpha f'_t (1 - \epsilon_i / \epsilon_m)}{\epsilon_i} \quad \text{for } \epsilon_t < \epsilon_i < \epsilon_m \dots\dots\dots (3.42)$$

where;

α_m, ϵ_m are tension stiffening parameters as shown in Figure(3.11).

ϵ_i is the maximum value reached by the tensile strain at the point considered.

When the strain component normal to the crack becomes negative (i.e. compressive), this implies that the crack is closed and the concrete is treated again as an uncracked material in the corresponding direction, but the crack direction and the maximum tensile strain continue to be stored. The normal stress can be obtained from the following expression:

$$\sigma_1 = \alpha f_t'(1-\epsilon_1/\epsilon_m) \quad \text{for } \epsilon_t < \epsilon_1 < \epsilon_m \quad \dots\dots\dots (3.43)$$

or,

$$\sigma_1 = \sigma_i \epsilon_1 / \epsilon_i \quad \text{for } \epsilon_1 < \epsilon_i \quad \dots\dots\dots (3.44)$$

where;

ϵ_1 is the current tensile in direction 1.

f_t' is uniaxial tensile strength of concrete.

Since the sliding between the steel and concrete is represented by the strain normal to the crack plane, only when the reinforcement is perpendicular to the crack plane, otherwise these quantities are not directly related. For problems with an identical crack direction, the angle between the reinforcement and crack direction should be taken into account for defining the constant ϵ_m (Thannon 1988).

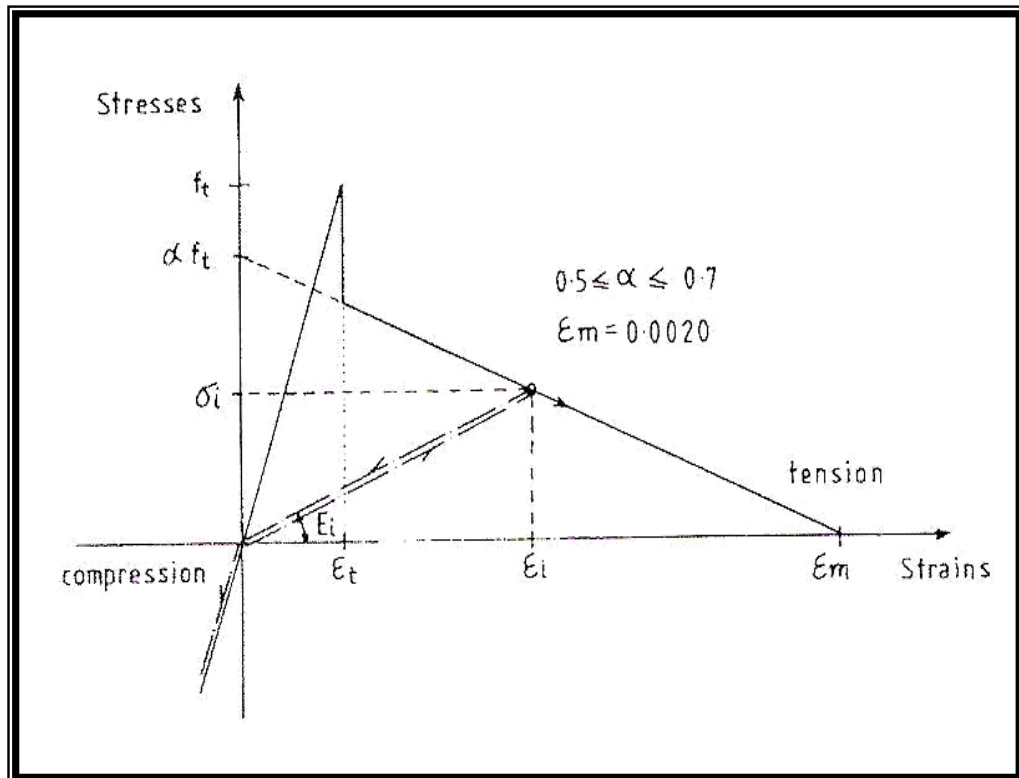


Fig. (3.11): Loading and unloading behavior of cracking concrete illustrating tension – stiffening behavior (Thannon 1988).

3.6.2.2 Cracked Shear Modulus

In finite element analysis of reinforced concrete plates and shells, a simplified approach is usually used to take into account the shear transfer capacity of cracked concrete. A simple and effective way for use with the smeared crack model is given by assigning a reduced value G^r to the shear modulus.

In the present study, a value of G^r is assumed to be linearly decreasing with the fictitious strain normal to the crack plane as shown in Figure (3.12). This approach was shown to give a better approximation to the experimental results (Thannon 1988). This leads to express the cracked shear modulus as an assumed function of the current tensile strain. According to this approach, for concrete cracked in the 1-direction,

$$\begin{aligned} G_{12}^r &= 0.25 G (1 - \varepsilon_1 / 0.0045) & \text{for } \varepsilon_1 < 0.0045 \\ &= 0 & \text{for } \varepsilon_1 \geq 0.0045 \end{aligned} \quad \dots\dots\dots (3.45)$$

$$G_{13}^r = G_{12}^r$$

For concrete cracked in both direction

$$\begin{aligned} G_{13}^r &= 0.25 G (1 - \varepsilon_1 / 0.0045) & \text{for } \varepsilon_1 < 0.0045 \\ &= 0 & \text{for } \varepsilon_1 \geq 0.0045 \\ G_{23}^r &= 0.25 G (1 - \varepsilon_2 / 0.0045) & \text{for } \varepsilon_2 < 0.045 \\ &= 0 & \text{for } \varepsilon_2 \geq 0.0045 \end{aligned} \quad \dots\dots\dots (3.46)$$

$$G_{12}^r = 0.5 G_{13}^r$$

or

$$G_{12}^r = 0.5 G_{23}^r \quad \text{for } G_{23}^r < G_{13}^r$$

where,

G^r is the cracked concrete shear modulus.

G is the uncracked concrete shear modulus

ε_1 is the tensile strain in the direction 1

ε_2 is the tensile strain in the direction 2

It is good to recall that when the crack is closed the uncracked shear modulus G is again assumed in the corresponding direction.

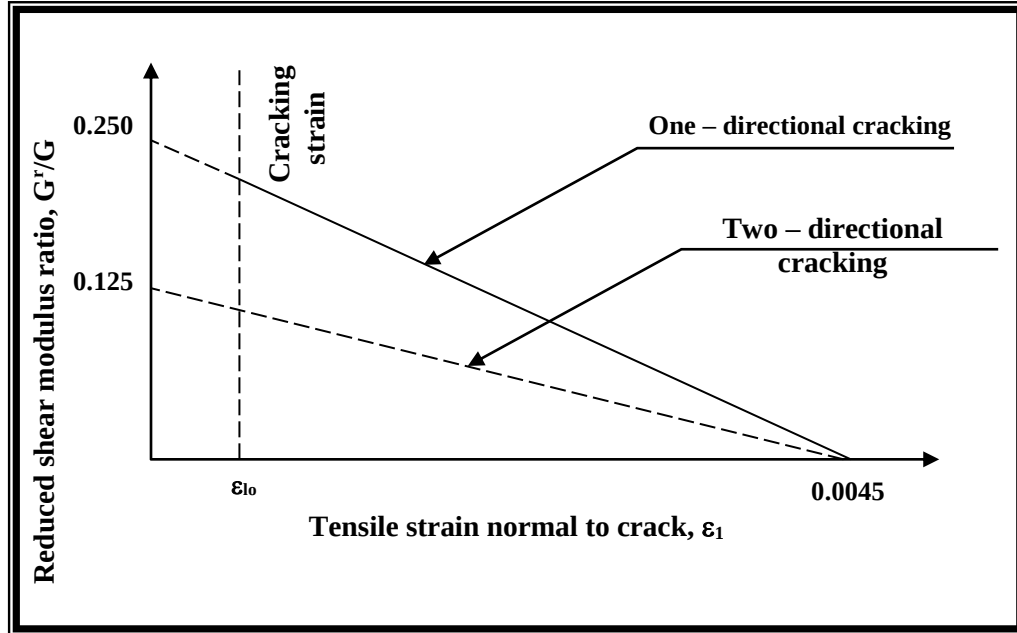


Fig. (3.12) Reduced shear modulus for cracked concrete (Chen 1982).

3.6.3 Steel Reinforcement Modeling

The mechanical properties of steel in comparison to the properties of concrete are well-known and better understood. Steel is a homogenous material and has usually the same yield strength in tension and compression. Typical stress-strain curves for steel reinforcement are presented in Figure (3.13). These curves can be divided into four distinct stages; initial linear elastic, yielding, strain hardening, and descending stages.

In the present study, the steel reinforcement is smeared into equivalent steel layers with uniaxial properties. An elasto-plastic behavior with possible strain hardening assumed and elastic unloading and reloading in the plastic range are allowed to follow a path parallel to the initial elastic modulus E_s as

shown in Figure(3.14). When the elasto-plastic Young's modulus E_s' equals zero, an elastic perfectly plastic model is obtained.

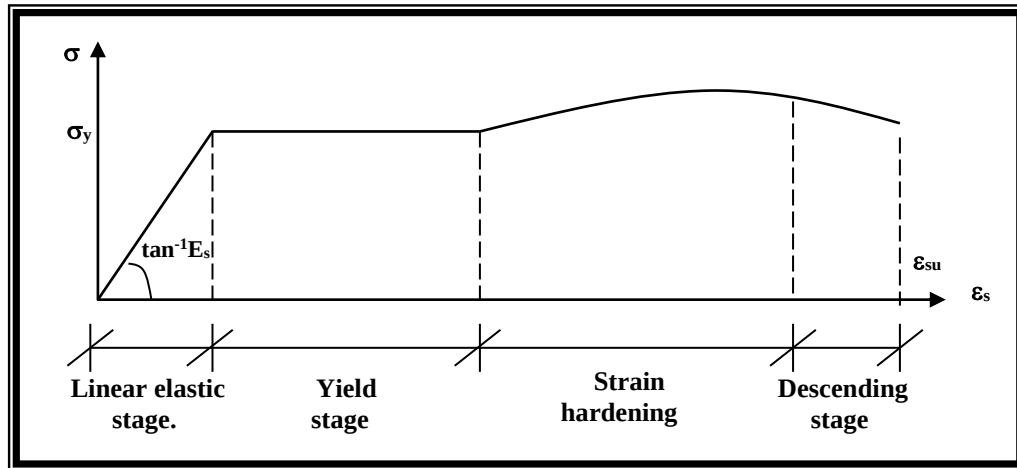


Fig. (3.13): Typical stress-strain curves for reinforcing steel (Lin and Scordelis 1975).

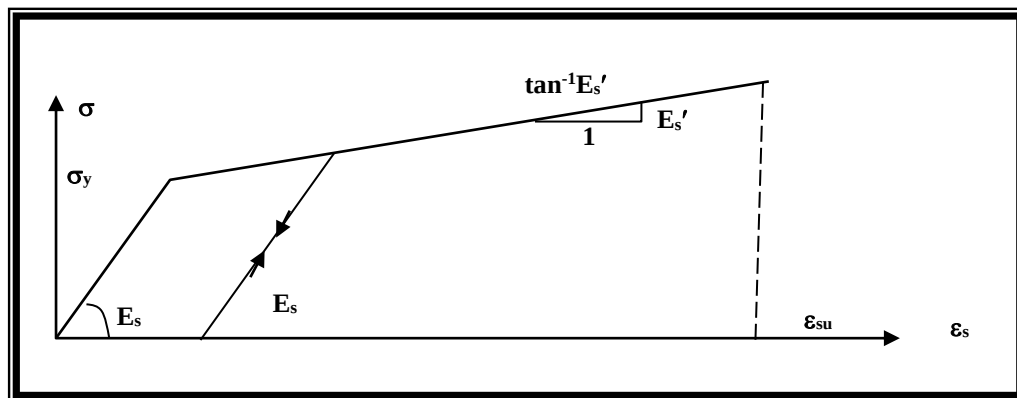


Fig. (3.14): Idealized elasto-plastic stress-strain relation for reinforcing steel (Lin and Scordelis 1975).

FINITE ELEMENT FORMULATION

4.1 Introduction

The analysis of plate or shell with an arbitrary defined shape presents intractable analytical problems. At present, the finite element method is the most powerful numerical technique, which offers an approximate solution to realistic types of structure such as shells. In the development of analysis procedures for plates and shells, accurate and efficient elements are required. The search for such suitable element which is readily applicable to general shell structures was and has been the target of researchers for the past forty years. Three distinct approaches to the finite element representation of plates and shell structures have traditionally been used, these approaches are:

1. Flat triangular and quadrilateral elements.
2. Curved elements, formulated the basis of various shell theories.
3. Elements derived from three – dimensional elements by the use of degeneration method.
4. Three dimensional brick elements.

(Ahmad and Zienkiewicz 1970) appeared to open a possible and promising avenue by giving fundamental idea of formulation of degenerated elements. The suggested process by Ahmed uses a full quadratic three-dimensional (20-noded) isoparametric for deriving (or degenerating) the formulation necessary to give the basic assumption of degenerated shell element. This approach employs a reference surface translations and rotations (mid-surface), which represent a replacement of the independent top and bottom node displacements in three dimension brick element and variation of displacements across the thickness is prescribed only linear. This is a great improvement of the element in the formulation to overcome some difficulties,

which arise in satisfying the necessary continuity of slopes at interfaces and the inability of such formulation to account for shear deformation. The strain energy corresponding to the stresses perpendicular to the middle surface is ignored but those surfaces normal to mid-surface before deformation remain straight but not necessarily normal to mid-surface after deformation as shown in Fig. (4.1).

Some difficulties appeared, due to degeneration processes. The thickness of the element was reduced, but also a great improvement of the model was achieved by the application of the so-called reduced integration technique. Since then the element has become applicable to thin as well as thick shells, because of some advantages:

First in describing the rotations independent of the displacement, the element avoids the normality (Kirchhoff hypothesis); therefore the result should be in agreement with Reissner's theory for thick shells. The sides of this element can be curved to match more general boundaries without any obstacles.

Many investigators applied the degenerated concept and its formulation to non-linear analysis of shells, as well as to combined geometric and material nonlinearities (Owen and Hinton 1984).

4.2 Degenerated Shell Element Formulations

4.2.1 Definition of Element Coordinate System

In the typical shell element as shown in Fig. (4.1), the external faces (surfaces) of the element are curved, while the thickness is generated by straight lines. Many coordinate sets are employed in the formulation of

degenerated shell element. These coordinate systems are described in the following section.

1. Global Coordinate System - (X_i)

This is a Cartesian coordinate system, freely chosen, in relation to which the geometry of the structure is defined in space. Nodal coordinates and displacements, as well as the global stiffness matrix and applied force vector are referred to in this system. The following notation is used (see Fig. (3.15-a)):

$$X_i \text{ (I=1,3) and } X_1 = X, X_2 = Y, X_3 = Z$$

$$U_i \text{ (I=1,3) and } U_1 = U, U_2 = V, U_3 = W$$

$$\bar{X}_i \text{ (I=1,3) is a unit vector in the } X_i \text{ direction}$$

This system is used to define global stiffness matrix and applied force vector and also in the structure geometry, as well as nodal coordinates and displacements refer to this system (Owen and Hinton 1984).

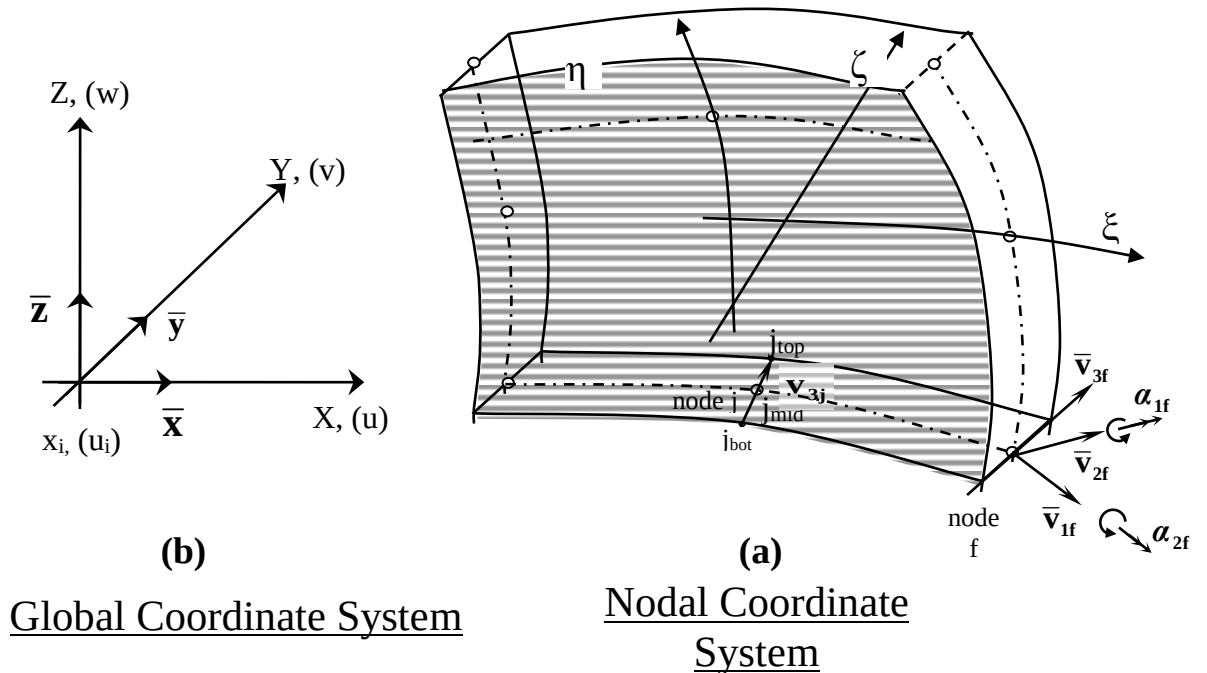


Fig. (4.1) Coordinate systems: Nodal and Global Coordinate System

2. Nodal Coordinate System (V_{1f} , V_{2f} , V_{3f})

This kind of system can be established at each nodal point as shown in Fig. (4.1-a) with origin at reference surface (shell or slab mid-surface), V_{3f} is a vector constructed from nodal coordinates of top and bottom surfaces at node f,

$$V_{3f} = X_f^{\text{top}} - X_f^{\text{bot}}$$

$$V_{3f} = [X_f \ Y_f \ Z_f]_{\text{top}}^T - [X_f \ Y_f \ Z_f]_{\text{bot}}^T \quad \dots (4.1)$$

The unit vector $\bar{V}_{3f}$ can be normalize from the vector V_{3f} , thus:

$$\bar{V}_{3f} = \frac{V_{3f}}{h_f} [V_{3f}^x \ V_{3f}^y \ V_{3f}^z]^T \quad \dots (4.2)$$

Where,

$$h_f = \left((\Delta X_{1f})^2 + (\Delta X_{2f})^2 + (\Delta X_{3f})^2 \right)^{1/2} \quad \dots (4.3)$$

and

$$\Delta X_{if} = X_{if}^{\text{top}} - X_{if}^{\text{bot}} \quad (i = 1, 2, 3) \quad \dots (4.4)$$

The vector $\bar{V}_{3f}$ defines the direction of the “normal” at the node f, which is not necessarily perpendicular to the mid-surface at f.

The vector V_{2f} is perpendicular to the plane defined by V_{1f} and V_{3f} :

$$V_{2f} = V_{1f} \times V_{3f} \quad \dots (4.5)$$

As shown in Fig. (4.1-a), each unit vector in the directions of V_{1f} , V_{2f} and V_{3f} is represented by $\bar{V}_{1f}$ and $\bar{V}_{2f}$ defines the rotations (α_{2f} and α_{1f}) respectively of the corresponding “normal”.

The mathematical characteristic of vector V_{1f} is perpendicular to V_{3f} and parallel to the global X – Z plane, thus:

$$V_{1f}^x = V_{3f}^z, \quad V_{1f}^y = 0.0, \quad V_{1f}^z = -V_{3f}^x$$

Or, if the vector is in the Y-direction ($V_{3f}^x = V_{3f}^z = 0.0$)

$$V_{1f}^x = -V_{3f}^z, \quad V_{1f}^y = 0.0, \quad V_{1f}^z = -V_{3f}^x$$

From the above formulation and description of the vectors, two advantages are obtained. The first one is the definition of the vector $\bar{V}_{3f}$ (not necessarily perpendicular to shell mid-surface) in which as a consequence there are no gaps or overlaps along element boundaries. The second advantage is the definition used for $\bar{V}_{1f}$ provides an easy identification of its direction in general curved structures like shells (Owen and Hinton 1984).

3. Curvilinear Coordinate System (ξ, η, ζ)

The description of this system is established by the definition of three coordinates each one has its characteristic as shown in Fig. (4.1-b). The ζ coordinate is a linear coordinate in the thickness direction and since ζ coordinate is defined as a function of $\bar{V}_{3f}$, the ζ -direction is only approximately perpendicular to the shell mid-surface. It is assumed that curvilinear coordinate vary between (-1 and +1), which represent the top and bottom surfaces of the element respectively. The two other coordinates are ξ, η which are curvilinear coordinates in the middle plane of the shell element and are also assumed to vary between (-1 and +1) which represent the faces of the element.

4. Local Coordinate System (X'_i or X' , Y' , Z')

This is a Cartesian coordinate system defined at the sampling points wherein stresses and strains are to be calculated. The direction Z' is taken perpendicular to the mid-surface ($\zeta = \text{constant}$), X' and Y' tangent to it as shown in Fig. (4.1-b). The vectors V'_1, V'_2, V'_3 define X', Y' and Z' direction of mid-surface respectively, and it should be noted that,

$$\xi = \begin{bmatrix} \frac{\partial x}{\partial \xi} \\ \frac{\partial y}{\partial \xi} \\ \frac{\partial z}{\partial \xi} \end{bmatrix} \quad \text{and,} \quad \eta = \begin{bmatrix} \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \eta} \\ \frac{\partial z}{\partial \eta} \end{bmatrix} \quad \dots\dots(4.6)$$

The vector V'_1 can be taken tangent to ξ direction at the sampling point, thus:

$$V'_1 = \xi = \begin{bmatrix} \frac{\partial x}{\partial \xi} \\ \frac{\partial y}{\partial \xi} \\ \frac{\partial z}{\partial \xi} \end{bmatrix} \quad \dots\dots (4.7)$$

The vector V'_2 can be defined by the cross product of the vectors V'_3 and V'_1 as follows:

$$V'_2 = V'_1 \times V'_3 \quad \dots\dots (4.8)$$

V'_3 can be also obtained by the cross product, but this time between ξ and η direction as follows:

$$\mathbf{V}'_3 = \xi \times \eta = \begin{bmatrix} \frac{\partial x}{\partial \xi} \\ \frac{\partial y}{\partial \xi} \\ \frac{\partial z}{\partial \xi} \end{bmatrix} \times \begin{bmatrix} \frac{\partial X}{\partial \eta} \\ \frac{\partial Y}{\partial \eta} \\ \frac{\partial Z}{\partial \eta} \end{bmatrix} \quad \dots\dots (4.9)$$

It can be noted that, the above local coordinate system needs to establish a direction cosine matrix $[\vartheta]$ which is useful to relate the transformation between the local and the global coordinate system. This is essential knowing that the local coordinate system varies along the thickness of the shells. Such variation depends on the variable thickness and shell curvature, so the direction cosine matrix $[\vartheta]$ can be defined as (Owen and Hinton 1984)

$$[\vartheta] = [\bar{\mathbf{V}}'_1, \bar{\mathbf{V}}'_2, \bar{\mathbf{V}}'_3] \quad \dots\dots(4.10)$$

where $\bar{\mathbf{V}}'_1, \bar{\mathbf{V}}'_2, \bar{\mathbf{V}}'_3$ are unit vectors in the direction of X', Y' and Z' axes respectively.

4.2.2 Geometry of the Element

To define the element geometry it is necessary to have two points at each node, one at the bottom surface and the other at top surface of the respective node. The global coordinates of those points are usually input to furnish the mid-surface nodal coordinate and the corresponding direction thickness.

To obtain the formulation of the coordinates of any point within the element, it is simpler to divide the general formulation of coordinate in two parts. The first part is to establish the term of formulation, which represents the intercept of the “normal” with the mid-surface. If there is a node (f) at the

“normal” at $\zeta=0$ surfaces and this “normal” have a top and bottom nodes at $\zeta=+1$ and $\zeta=-1$ surface respectively, the coordinates of node (f) can be obtained as follows:

$$X'_{if} = \frac{1}{2} \left\{ [X'_{if}]_{top} + [X'_{if}]_{bot} \right\}$$

or

.....(4.11)

$$\begin{bmatrix} X_f \\ Y_f \\ Z_f \end{bmatrix} = \frac{1}{2} \begin{bmatrix} X_f \\ Y_f \\ Z_f \end{bmatrix}_{top} + \begin{bmatrix} X_f \\ Y_f \\ Z_f \end{bmatrix}_{bot}$$

And since node (f) is in the mid-surface (at $\zeta=0$ surface), the two dimensional interpolation functions $N_f(\xi, \eta)$ can be applied from Table (4.1) , to obtain relationship between the Cartesian point and the curvilinear coordinate as follows(Owen and Hinton 1984).

$$X_i = \sum_{f=1}^n N_f(\xi, \eta) X_{if} \quad \text{..... (4.12)}$$

The second part is to establish the second term of the general formulation, which defines the position of the point along this “normal” . But to establish that, it is necessary to define arbitrary point (p) on the “normal” at node (f) see Fig. (4.1-a), therefore the vector V_{3f} which represents the thickness of the shell h_f , can be written as equation (4.1):

$$h_f = V_{3f} = \begin{bmatrix} X_f \\ Y_f \\ Z_f \end{bmatrix}_{top} - \begin{bmatrix} X_f \\ Y_f \\ Z_f \end{bmatrix}_{bot} \quad \text{..... (4.13)}$$

And the distance between the arbitrary point (p) and the node f (at mid-surface) can be written as follows:

$$\text{Unit Distance} = \frac{h_f \zeta}{2} \quad \dots (4.14)$$

And since the $h_f = V_{3f}^i$ and distance means the coordinates of the arbitrary point, equation (4.14) can be written with the application of the two dimensional interpolation function $N_f(\xi, \eta)$ from Table (4.1) so that:

$$X_i = \sum_{f=1}^n N_f(\xi, \eta) \frac{\zeta}{2} V_{3f}^i \quad \dots\dots\dots (4.15)$$

The general formulation of the coordinates defines the geometry of the shell element which represents the rotations between the curvilinear coordinates (ξ, η, ζ) and global coordinates (X, Y, Z) . It is obtained by the summation of the previous two parts (equation (4.12) & equation (4.15) so that:

$$X_i = \sum_{f=1}^n N_f(\xi, \eta) \frac{1+\zeta}{2} [X_{if}]_{top} + \sum_{f=1}^n N_f(\xi, \eta) \frac{1-\zeta}{2} [X_{if}]_{bot}$$

$$\text{or} \quad \dots\dots\dots (4.16)$$

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \sum_{f=1}^n N_f(\xi, \eta) \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}_{mid} + \sum_{f=1}^n N_f(\xi, \eta) \frac{\zeta h_f}{2} \begin{Bmatrix} \bar{V}_{3f}^x \\ \bar{V}_{3f}^y \\ \bar{V}_{3f}^z \end{Bmatrix}$$

where,

X_i is the Cartesian coordinate of any point in element,
 $(X_1 = X, X_2 = Y, X_3 = Z)$

n is the number of nodes per element,

h_f is the shell thickness at node f, i.e. the respective “normal” length,

x_{if} is the Cartesian coordinate of nodal point f, and

$N_f(\xi, \eta)$ is the two dimensional interpolation functions corresponding to the surface. ($\zeta = \text{constant}$) at nodal point f. (see Table (4.1)).

$\bar{V}_{3f}^i$ is the component of the unit normal vector to the middle surface.

Table (4.1) Shape Functions and Their Derivatives

Shape element function for 8-node Serendipity, 9-node Lagrangian and Heterosis				
	Function corner nodes (1,3,5,7)	Edge nodes (2,6)	Edge nodes (4,8)	9-node
N_k	$\frac{1}{4}(1+\xi_o)(1+\eta_o)(\xi_o+\eta_o-1)$	$\frac{1}{2}(1-\xi^2)(1+\eta_o)$	$\frac{1}{2}(1-\eta^2)(1+\xi_o)$	$(1-\xi^2)(1-\eta^2)$
$N_{k,\xi}$	$\frac{\xi_k}{4}(1+\eta_o)(2\xi_o+\eta_o)$	$-\xi(1+\eta_o)$	$\frac{\xi_k}{2}(1+\eta^2)$	$-2\xi(1-\eta^2)$
$N_{k,\eta}$	$\frac{\eta_k}{4}(1+\xi_o)(2\eta_o+\xi_o)$	$\frac{\eta_k}{2}(1-\xi^2)$	$-\eta(1+\xi_o)$	$-2\eta(1-\xi^2)$

4.2.3 Displacement Field

Five degrees of freedom at each node of shell element specify the displacement field, as the strain in the directions to the mid-surface is assumed to be negligible. The displacement throughout the element can be

defined by the three displacements (u, v and w) for its mid-point (u_{of}^i) and two rotations of the nodal vector V_{3f} about orthogonal directions normal to it. One of the two orthogonal directions is represented by unit vector $\bar{V}_{1f}$ and the corresponding rotation (α_{1f}). The other direction is $\bar{V}_{2f}$ and the corresponding rotation is (α_{2f}), and it is worth recalling that the unit vector $\bar{V}_{3f}$ in the direction of V_{3f} is used to define the direction of the normal at node f as shown in Fig. (4.2).

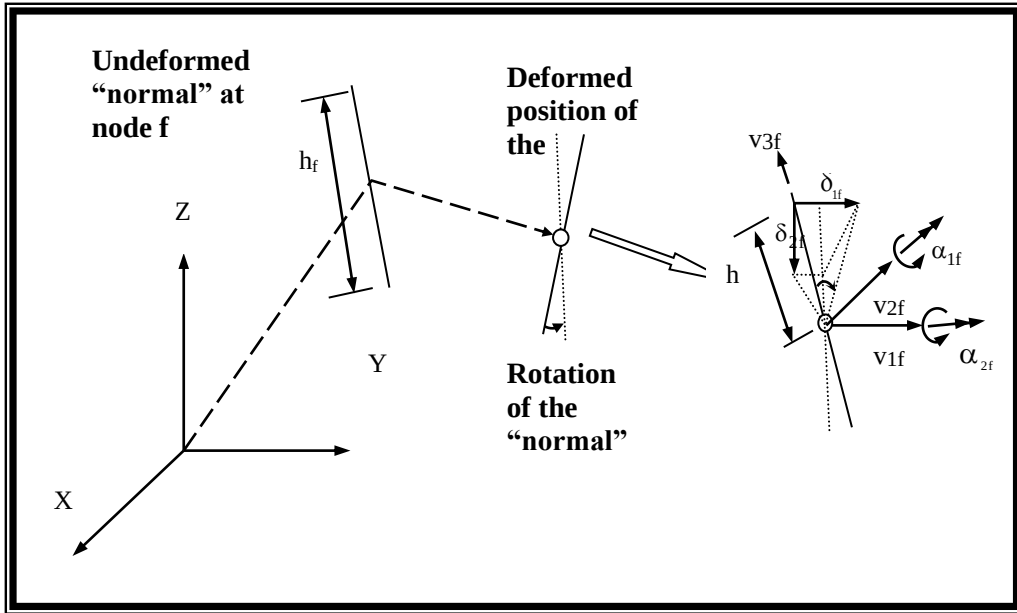


Fig. (4.2) Displacements of a point on the “normal” at node f.

The displacements (δ_{1f}, δ_{2f}) of a point at unit distance (h) from node f on the “normal” resulting from the two rotations (α_{1f}, α_{2f}) are calculated as follows:

$$\alpha_{1f} = \frac{\delta_{1f}}{h} \quad \text{or} \quad \delta_{1f} = h \alpha_{1f}$$

and

$$\dots\dots(4.17)$$

$$\alpha_{2f} = \frac{\delta_{2f}}{h} \quad \text{OR} \quad \delta_{2f} = h \alpha_{2f}$$

where $h = \frac{h_f \zeta}{2}$ and ,

δ_{1f} is the displacement in the direction of $\bar{V}_{1f}$

δ_{2f} is the displacement in the negative direction of $\bar{V}_{2f}$

Equation (4.17) can be written as follows:

$$\begin{Bmatrix} \delta_{1f} \\ \delta_{2f} \end{Bmatrix} = h \begin{Bmatrix} \alpha_{1f} \\ \alpha_{2f} \end{Bmatrix} \quad \dots (4.18)$$

The global displacement can be found from:

$$\{u_i\} = \sum_{f=1}^n N_f \{u_{of}^i\}_{mid} + \sum N_f \{u_{of}'^i\} \quad \dots (4.19)$$

where,

u_i is the total displacement along the thickness at each nodal point,

u_{of}^i is the displacement of the nodal point in the Cartesian coordinate, and

$u_{of}'^i$ is the displacement of the nodal point f on the “normal” resulting from the two rotation α_{1f} and α_{2f} .

Since;

$$\bar{u}_{of}^i = \delta_{1f}' + \delta_{2f}' \quad \dots (4.20)$$

where the corresponding displacement components $(\delta_{1f}', \delta_{2f}')$ of $\bar{u}_{of}'$ can be calculated as follows(Owen and Hinton 1984).

$$\delta'_{1f} = \frac{h_f \zeta}{2} \alpha_{1f} \bar{V}_{1f}^i \quad \dots (4.21)$$

$$\delta'_{1f} = \frac{h_f \zeta}{2} \alpha_{1f} \bar{V}_{1f}^i$$

and since the global displacement can be found as shown before in equation (4.19) as the simulation of the mid-surface nodal displacement (u_{of}^i) and the relative displacements caused by the rotations of the normal (u_{of}^i), then the element displacement field can be expressed by:

$$u_{if} = u_{of}^i + \frac{\zeta \cdot h_f}{2} (\bar{V}_{1f}^i \alpha_{1f} - \bar{V}_{2f}^i \alpha_{2f}) \quad \dots (4.22)$$

OR

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{f=1}^n N_f(\xi, \eta) \begin{Bmatrix} u_f \\ v_f \\ w_f \end{Bmatrix} + \sum_{f=1}^n N_f(\xi, \eta) \frac{\zeta h_f}{2} \begin{Bmatrix} \bar{V}_{1f}^x & \bar{V}_{2f}^x \\ \bar{V}_{1f}^y & \bar{V}_{2f}^y \\ \bar{V}_{1f}^z & \bar{V}_{2f}^z \end{Bmatrix} \begin{Bmatrix} \alpha_{1f} \\ \alpha_{2f} \end{Bmatrix} \dots (4.23)$$

The contribution to the global displacement from a given node f in the general form and for complete element is:

$$u_i = \sum_{f=1}^n N_f(\xi, \eta, \zeta) S_f \quad \dots (4.24)$$

OR

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} N_f & 0 & 0 & N_f \frac{\zeta h_f}{2} V_{1f}^x & -N_f \frac{\zeta h_f}{2} V_{2f}^x \\ 0 & N_f & 0 & N_f \frac{\zeta h_f}{2} V_{1f}^y & -N_f \frac{\zeta h_f}{2} V_{2f}^y \\ 0 & 0 & N_f & N_f \frac{\zeta h_f}{2} V_{1f}^z & -N_f \frac{\zeta h_f}{2} V_{2f}^z \end{bmatrix} \begin{Bmatrix} u_f \\ v_f \\ w_f \\ \alpha_{1f} \\ \alpha_{2f} \end{Bmatrix} \quad \dots (4.25)$$

where,

N_f is the shape function matrix of the degenerated shell element.

S_f is the transformation matrix of the displacement vector at node f of shell element $\{u_{of}, v_{of}, w_{of}, \alpha_{1f}, \alpha_{2f}\}^T$

It is good to recall that the above equation (4.25) gives an admissible result when the rotations (the increment angle) are small.

4.2.4 Definition of Stresses

The earlier definition of the local coordinate system in the previous sections is essential in order to deal with the degeneration assumption of the stresses and strains, because the local coordinate system is more appropriate system for expressing the stress and strain components for the shell assumption of zero local stress in the direction normal to the shell or slab mid-surface in Z' -direction ($\sigma'_z = 0$) enables the stresses vector to be reduced to the following five stress components (Ahmad and Zienkiewicz 1970)

$$\sigma = \begin{bmatrix} \sigma_{x'} \\ \sigma_{y'} \\ \tau_{x'y'} \\ \tau_{x'z'} \\ \tau_{y'z'} \end{bmatrix} = [D] \{ \{\varepsilon\} - \{\varepsilon_o\} \} \quad \dots\dots (4.26)$$

where,

$\{\varepsilon_o\}$ is the initial strain vector and may also represent the expansion due to thermal load,

$\{\varepsilon\}$ is the strain vector and details of its formulation are explained in the next section, and

$[D]$ is the elasticity matrix given by,

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & K_1 G & 0 \\ 0 & 0 & 0 & 0 & K_2 G \end{bmatrix} \quad \dots\dots (4.27)$$

where,

G is the shear modulus,

E is the Young's modulus of elasticity,

ν is Poisson's ratio, and

K_1, K_2 are the shear correction factors.

In the degenerated thick shell element, it will be seen that the transverse shear distribution is assumed to be constant through the thickness. Whereas in reality the shear distribution is approximately parabolic, so the shear modulus needs to be multiplied by a shear correction factor whose values $K_1 = K_2 = 5/6$.

4.2.5 Definition of Strains

The normal strain in the Z' -direction (ϵ'_z) is neglected. Therefore the general vector of Green strains will be reduced to the following five components,

$$\{\varepsilon\} = \begin{bmatrix} \varepsilon_{x'} \\ \varepsilon_{y'} \\ \gamma_{x'y'} \\ \gamma_{x'z'} \\ \gamma_{y'z'} \end{bmatrix} = \begin{bmatrix} \frac{\partial u'}{\partial x'} \\ \frac{\partial v'}{\partial y'} \\ \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} \\ \frac{\partial u'}{\partial z'} + \frac{\partial w'}{\partial x'} \\ \frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'} \end{bmatrix} \quad \dots (4.28)$$

The local derivatives above of the displacement components u', v' and w' in the local coordinates system (X_1') can be obtained from the global derivatives of the displacements u, v and w by the following operation:

$$\begin{bmatrix} \frac{\partial u'}{\partial x'} & \frac{\partial v'}{\partial x'} & \frac{\partial w'}{\partial x'} \\ \frac{\partial u'}{\partial y'} & \frac{\partial v'}{\partial y'} & \frac{\partial w'}{\partial y'} \\ \frac{\partial u'}{\partial z'} & \frac{\partial v'}{\partial z'} & \frac{\partial w'}{\partial z'} \end{bmatrix} = [g]^T \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{bmatrix} [g] \quad \dots (4.29)$$

where $[g]$ is the transformation matrix given by ,

$$[g] = \begin{bmatrix} \frac{\partial x}{\partial x'} & \frac{\partial x}{\partial y'} & \frac{\partial x}{\partial z'} \\ \frac{\partial y}{\partial x'} & \frac{\partial y}{\partial y'} & \frac{\partial y}{\partial z'} \\ \frac{\partial z}{\partial x'} & \frac{\partial z}{\partial y'} & \frac{\partial z}{\partial z'} \end{bmatrix} \quad \dots (4.30)$$

The global derivatives of the displacement u, v and w are given by:

$$\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \frac{\partial u}{\partial \xi} & \frac{\partial v}{\partial \xi} & \frac{\partial w}{\partial \xi} \\ \frac{\partial u}{\partial \eta} & \frac{\partial v}{\partial \eta} & \frac{\partial w}{\partial \eta} \\ \frac{\partial u}{\partial \zeta} & \frac{\partial v}{\partial \zeta} & \frac{\partial w}{\partial \zeta} \end{bmatrix} \quad \dots (4.31)$$

Where $[J]$ is the Jacobian matrix as:

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix} \quad \dots (4.32)$$

The transpose of the Jacobian matrix results from equation (4.16) can be expressed as follow:

$$[J]^T = \sum_{f=1}^n \left\{ \begin{bmatrix} X_f \\ Y_f \\ Z_f \end{bmatrix} \right\} [N_{f,\xi} \ N_{f,\eta} \ 0] + \frac{h_f}{2} \left\{ \begin{bmatrix} V_{3f}^x \\ V_{3f}^y \\ V_{3f}^z \end{bmatrix} \right\} \begin{bmatrix} 0 & 0 & N_f \end{bmatrix} + \frac{h_f \zeta}{2} \left\{ \begin{bmatrix} V_{3f}^x \\ V_{3f}^y \\ V_{3f}^z \end{bmatrix} \right\} \begin{bmatrix} N_{f,\xi} & N_{f,\eta} & 0 \end{bmatrix} \dots (4.33)$$

where:

n is the number of node per element,

h_f is the element thickness at node f, and

$$N_{f,\xi} = \frac{\partial N_f(\xi, \eta)}{\partial \xi}; \quad N_{f,\eta} = \frac{\partial N_f(\xi, \eta)}{\partial \eta} \quad \dots (4.34)$$

are the derivatives of the global displacements referred to the curvilinear coordinates are obtained from equation (4.25), such as $\frac{\partial u}{\partial \xi}$ etc.

$$\frac{\partial u}{\partial \xi} = \sum_{f=1}^n \left(N_{f,\zeta} u_f + \frac{h_f \zeta}{2} V_{1f}^x N_{f,\zeta} \alpha_{1f} - \frac{h_f \zeta}{2} V_{2f}^x N_{f,\zeta} \alpha_{2f} \right) \quad \dots (4.35)$$

The strain matrix **B** (of order 5 by the total d.o.f. of the element), relating the strain components in the local system to the element nodal variables, can be constructed from equations (4.24),(4.28) and (4.29) as:

$$\{\varepsilon\} = [B] \{\delta\} \quad \dots (4.36)$$

where,

$$\{\delta\} = [u, v, w, \alpha_1, \alpha_2]^T \quad \dots (4.37)$$

4.2.6 The Layered Model

The layering technique has been used in many methods of analysis, especially in general nonlinear finite element analysis of reinforced concrete structures such as plates and shells. This is due to the characteristics of this technique of accommodated simplicity and affecting manner concerning the nonlinear behavior of concrete in compression. Cracking and reinforcement are represented across the element (Ahmad and Zienkiewicz 1970). The basic idea of this approach consists of the shell element cross section being divided into concrete and steel layers. Each layer may have a different or the same material properties (and stresses) and layers may have a different thickness, and different numbers of layers per element can be employed. The steel can be represented by a number of smeared layers with an equivalent thickness. This approach specifies the layer thickness in terms of natural curvilinear coordinate, ζ which permits the variation of the layer thickness as the shell or slab thickness varies, so the element of thickness h has its reference coordinate in the mid of element thickness and the natural coordinate ζ varies

from -1 at $-h/2$ to $+1$ at $+h/2$, and is determined at the middle point of each layer (Thomas 1998).

4.2.7 Stress Resultants

As mentioned in the previous section, the layered approach is built up from series of layers over the thickness of the shell element. Each layer contains stress points (Gauss points) on its mid surface where in a mid-point rule integration scheme is adopted from each layer to compute the stress components at these stress points. For each layer and according to this approach they are assumed to be constant over the thickness (Ahmad and Zienkiewicz 1970).

The following formulation represents the stress resultants at each Gauss point obtained by integrating the corresponding stress components with respect to the thickness coordinate and its sign convention for stress resultants is shown in Fig. (4.3).

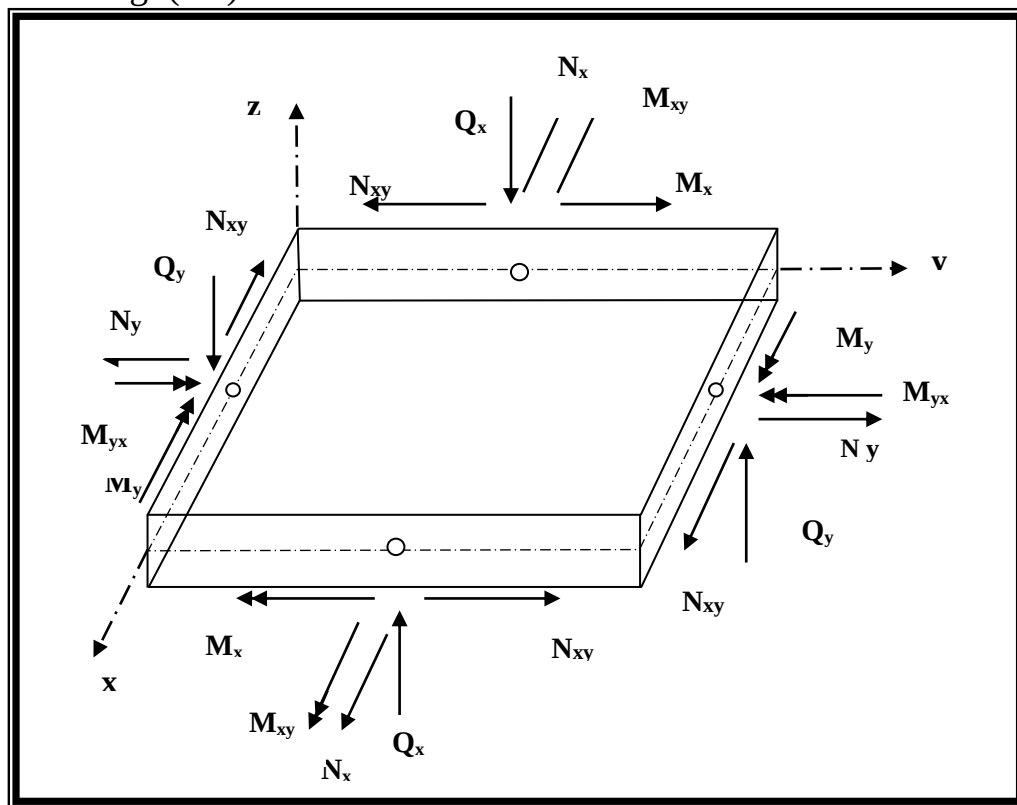


Fig. (4.3) sign convention for the stress resultants.

I. Membrane or normal force:

$$N_x = \int_{-h/2}^{+h/2} \sigma_x dz = \frac{h}{2} \sum_{i=1}^{nL} \sigma_x^i \Delta \zeta^i \quad \dots (4.38)$$

$$N_y = \int_{-h/2}^{+h/2} \sigma_y dz = \frac{h}{2} \sum_{i=1}^{nL} \sigma_y^i \Delta \zeta^i \quad \dots (4.39)$$

$$N_{xy} = \int_{-h/2}^{+h/2} \tau_{xy} dz = \frac{h}{2} \sum_{i=1}^{nL} \tau_{xy}^i \Delta \zeta^i \quad \dots (4.40)$$

II. Bending moments:

$$M_x = \int_{-h/2}^{+h/2} \sigma_x z dz = \frac{-h^2}{4} \sum_{i=1}^{nL} \sigma_x^i \Delta \zeta^i \zeta^i \quad \dots (4.41)$$

$$M_y = \int_{-h/2}^{+h/2} \sigma_y z dz = \frac{-h^2}{4} \sum_{i=1}^{nL} \sigma_y^i \Delta \zeta^i \zeta^i \quad \dots (4.42)$$

$$M_{xy} = \int_{-h/2}^{+h/2} \tau_{xy} z dz = \frac{-h^2}{4} \sum_{i=1}^{nL} \tau_{xy}^i \Delta \zeta^i \zeta^i \quad \dots (4.43)$$

III. Shearing forces:

$$Q_{xz} = \int_{-h/2}^{+h/2} \tau_{xy} dz = \frac{h}{2} \sum_{i=1}^{nL} \tau_{xy}^i \Delta \zeta^i \quad \dots (4.44)$$

$$Q_{yz} = \int_{-h/2}^{+h/2} \tau_{yz} dz = \frac{h}{2} \sum_{i=1}^{nL} \tau_{yz}^i \Delta \zeta^i \quad \dots (4.45)$$

where, nL is the total number of layers

4.2.8 The Element Stiffness Matrix

The stiffness matrix of degenerated shell element is computed at the mid-section of each layer. The Jacobian matrix through the shell thickness must be taken into account. It is more appropriate to use an integration process, which may split the volume integral into integrals over the area of the shell mid-surface and through the thickness (**h**). Therefore, the process consists of the calculation of strain matrix **[B_j]** at the mid-surface of each layer. Consequently it is used in the calculation of the stiffness matrix **[K]** using the mid-ordinate rule. Thus, the stiffness matrix is computed by summing up the contribution of each layer at the Gauss points and may be written as follows (Owen and Hinton 1980):

$$[K] = \int_V [B]^T [D] [B] dV = \int_S \int_{-h/2}^{h/2} ([B]^T [D] [B] dz) dS \dots\dots\dots (4.46)$$

$$[K] = \int_{-1}^{+1} \int_{-1}^{+1} \left[\int_{-1}^{+1} [B]^T [D] [B] J(\xi, \eta, \zeta) |d\zeta| d\xi d\eta \dots\dots\dots (4.47) \right.$$

Then, **[k]** can be written as summing up the contribution of each layer at the Gauss points,

$$[K] = \int_{-1}^{+1} \int_{-1}^{+1} \left[\sum_{j=1}^n [B_j]^T [D_j] [B_j] J(\xi, \eta, \zeta_j) \left| \frac{2\Delta h_j}{h} \right| d\xi d\eta \dots\dots\dots (4.48) \right.$$

where;

[k] = is the stiffness matrix.

[D]= is the elasticity matrix modified to account for tensile cracking, nonlinear behavior in compression and material matrix of steel layer.

[B_j]= is the strain matrix calculated at the mid section of each layer.

|J (ξ,η, ζ_j)|= is the determinant of the Jacobian matrix for layer (j).

Δh_j= is the thickness of the jth layer.

n= is the total number of layers.

4.2.9 Internal Force Vector

It is essential in the nonlinear analysis to determine the internal force vector (or equivalent nodal forces) at the end of each iteration. These forces can be determined by summing up the contribution of each layer at each Gauss point.

$$\{f^e\} = \int_{-1}^{+1} \int_{-1}^{+1} \left[\sum_{j=1}^n [B_j]^T \{\sigma_j\} J(\xi, \eta, \zeta_j) \frac{2\Delta h_j}{h} \right] d\xi d\eta \dots\dots\dots (4.49)$$

$\{f^e\}$ = is the internal force vector.

$\{\sigma\}$ = is the current stress vector.

4.2.10 Types of Quadratic Element

A brief description of the quadratic element used in the present model is given below. The behavior of the elements discussed here is mainly based on studies of linear beam, plates and shells problems (Pugh and Zienkiewicz 1978). The elements considered are the 8-node serendipity, 9-node Lagrangian and Heterosis element as shown in Figure (4.4).

1. Eight-node Serendipity element

This element as shown in Figure (4.4a) is the simplest element which is considered in the original work of (Ahmad and Zienkiewicz 1970) in the degenerated shell concept. The results obtained by using full integration rule, show that as the shell thickness is reduced the solution becomes stiff. This locking phenomenon is exhibited by this element even in moderately thin situations. A great improvement of the results is achieved by using a reduced integration rule. In spite of the improvement, the locking may still occur. Despite the above noted drawbacks, the eight node Serendipity element remains one of the most popular of the isoparametric elements.

2. Nine-node Lagrangian element

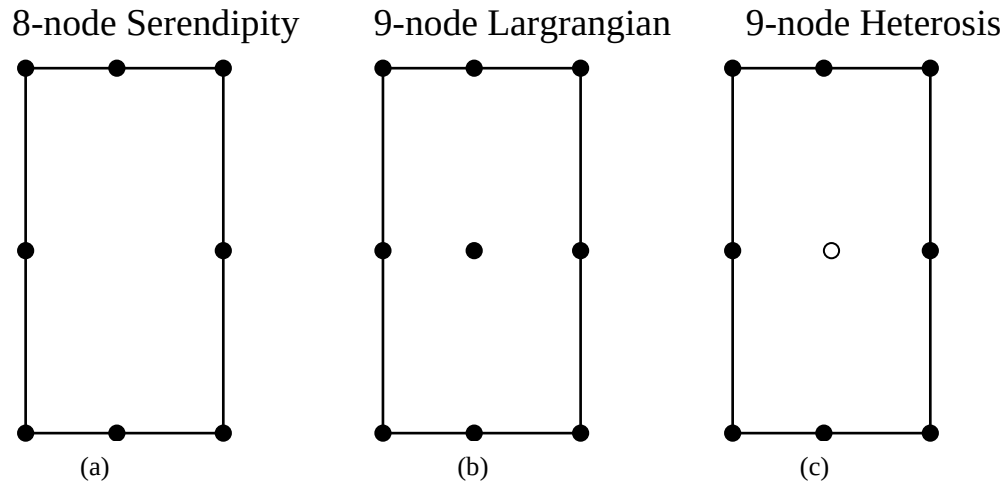
Investigation carried out (Pugh and Zienkiewicz 1978) as shown in Figure(4.4b) is more optimal as general plate element. The performance exhibited by Largangian element appears to be superior to Serendipity element when reduced or selective integration is adopted. However some drawbacks may arise when reduced integration is employed. The stiffness matrix exhibits rank deficiency, which causes the appearance of spurious mechanisms (Pugh and Zienkiewicz 1978). This drawback is not exhibited by eight nodes Serendipity with reduced integration.

3. Nine-node Heterosis element

The reasons which led to the development of this nine node quadrilateral element shown in Figure (4.4c) are to overcome the shortcomings mentioned above (the locking and spurious mechanisms). Because this element has improved characteristics when it is compared with nine node Lagrangian and eight node Serendipity element .The locking problem is overcome by using selective integration, which leads to obtain a correct rank of stiffness matrix of this element, and does not lock in very thin situation. The other problem (spurious mechanisms) can be overcome by using reduced integration. The element gives a better behavior compared with nine-node Lagrangian element which makes the appearance of spurious mechanisms improbable.

It is good to recall that this element is initially developed for plates (Pugh and Zienkiewicz 1978) and employs Serendipity shape functions for vertical displacement w , and Lagrangian shape function for rotations α_1 and α_2 . Then, the Heterosis concept has been extended to shell analysis (. Hughes 1981) using the Serendipity shape functions for the lateral displacement, only).





- Node with $u, v, w, \beta_1, \beta_2$
- Node with β_1, β_2 degrees of freedom

Fig. (4.4) Nodal Configuration of the three quadratic shell elements

4.2.11 Numerical Integration of Isoparametric Shell

It has become clear that it is important to choose a suitable integration rule that is both accurate and computationally efficient because numerical integration of shell elements effects the characteristic of the element efficiency.

The rules adopted in this study, and their advantages and disadvantages are briefly described as follows:

1. Full integration rule

The full integration rule consists of $m \times m$ Gauss points, where “ m ” is the number of nodes along each element side. This rule exhibits some disadvantages which appear in the quadrilateral plate and shell element applications. These disadvantages represented are shear and membranes locking phenomenon which consists of the bending and shear results in stiffening of the solution when the thickness is small compared with other dimensions, such phenomenon takes place with applications of plate for

different reasons. For shell results from the inability of the full integrated shell element to undergo in extensional bending, which is more complicated than the same situation in plate, which is caused by the transverse shear energy. It does not diminish as desired when the plate becomes thin.

2. Reduced integration

Reduced integration consists of $(m-1) \times (m-1)$ Gauss point. By using such integration rule, the solution accuracy for thin plates and shells applications may be significantly improved. On the other hand, uniform application of reduced integration tends to diminish the rank of the stiffness matrix resulting in the occurrence of mechanisms in addition to the rigidity body modes (Hinton and Owen 1984).

3. Selective integration rule

In the selective integration technique, bending (or bending and membrane) energies are integrated using full rule, while the shear and membrane (or just shear) terms are computed using the reduced integration rule. The process involves the following three steps, which are particularized for quadratic element.

- The shear and membrane strains (corresponding terms in the strain matrix **B**), are computed at the four Gauss points (I, II, III, IV) as shown in Figure(4.5) using 2×2 reduced integration rule.
- Using the shape functions $\bar{N}_r(\xi, \eta)$, the terms of the **B** matrix that previously computed are extrapolated to nine Gauss points as shown in Figure(3.19) of the 3×3 full integration rule,

$$B(\xi, \eta)_f = \sum_{r=1}^4 N_r(\xi, \eta) B(\xi, \eta)_r \dots\dots\dots (4.50)$$

where $(\xi, \eta)_f$ and $(\xi, \eta)_r$ correspond to Gauss points position in the full and reduced integration rules respectively.

The four shape functions $\bar{N}_r(\xi, \eta)$ have been defined by,

$$\bar{N}_r(\xi, \eta) = \frac{1}{4} (1 + \xi \bar{\xi}_r)(1 + \eta \bar{\eta}_r) \dots\dots\dots (4.51)$$

where, $\bar{\xi}_r$ and $\bar{\eta}_r$ (as shown in Table 4.2) are determined, so that each of the shape functions N_r take unit value at the corresponding Gauss point (2×2) rule and zero at the three positions (Hinton and Owen 1984).

- The bending (or bending and membrane) strains (corresponding B matrix terms) are computed at the nine Gauss points of the full (3×3) integration rule.

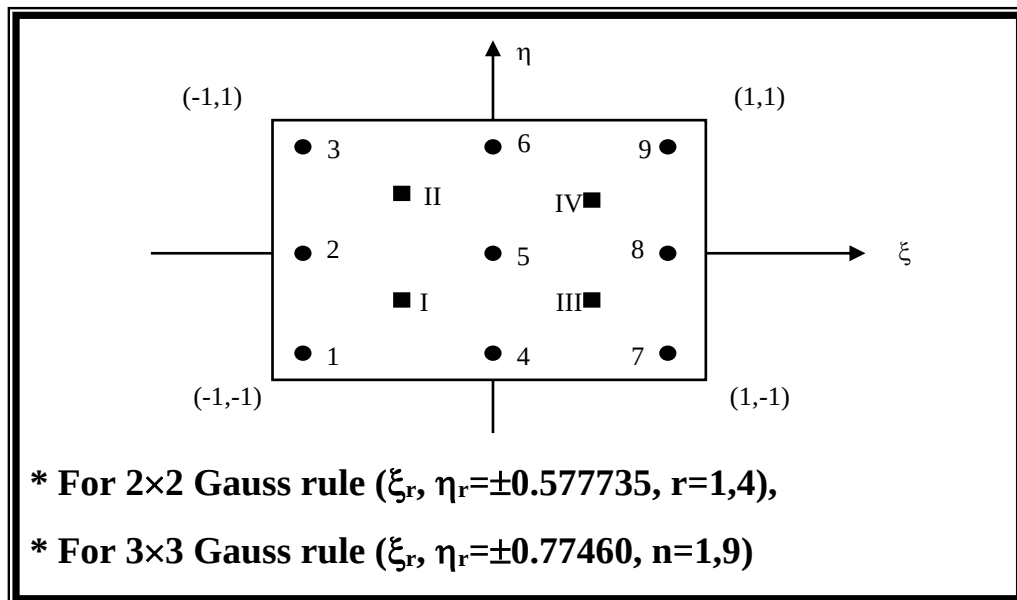


Fig. (4.5) Four and nine Gauss point position for selective integration

Table (4.2) Constants for the shape functions used in the selective integration rule.

Point number	I	II	III	IV
Shape function	$\bar{N}_1$	$\bar{N}_2$	$\bar{N}_3$	$\bar{N}_4$
ξ_r	-1.0/0.57735	-1.0/0.57735	+1.0/0.57735	+1.0/0.57735
$\bar{\eta}_r$	-1.0/0.57735	+1.0/0.57735	-1.0/0.57735	+1.0/0.57735

It is good to recall that the sampling points for all components of strains and stresses are the nine positions of the normal integration rule.

4.3 Nonlinear Solution Techniques

The solution of finite element equations for linear problems can be accomplished in a direct manner without major difficulties. However, this is no longer the case for nonlinear problems where certain sophisticated solution strategies must be employed. In the analysis of reinforced concrete structures, the material nonlinear behavior, including sudden changes in the element stiffness due to cracking and crushing of concrete and yielding of reinforcement, represents the main source of nonlinearity. The influence of geometry changes on the behavior is also significant in many cases. An analysis based on large deformations must be considered for those cases. An incremental finite element formulation with corrective iteration cycles is usually used for nonlinear finite element analysis to trace out the entire structural response of reinforced concrete structures and to guarantee the convergence towards the correct solution path.

4.3.1 General Solution Procedures

The solution for linear elastic structure problems can be obtained directly from solving a set of algebraic equations in the following form:

$$[K]\{a\} = \{f\} \quad \dots\dots(4.52)$$

where;

$\{a\}$ =unknown nodal displacement vector

$\{f\}$ =nodal applied force vector

$[K]$ =stiffness matrix

This cannot be achieved in the case of nonlinear problems, where the stiffness matrix $[K]$ is a function of material properties and structural displacement.

For the solution of a nonlinear structure problem the state of equilibrium of the structural system corresponding to the applied load must be found. In finite element terms these equilibrium equations can be written as:

$$\{f\}-\{P\}=\{O\} \quad \dots\dots(4.53)$$

The equilibrium equation (4.53) will not be exactly satisfied and a system of residual force $\{r\}$ will exist such that:

$$\{r\}=\{f\}-\{P\}\{a\}\neq\{O\} \quad \dots\dots(4.54)$$

where;

$\{a\}$ is the nodal displacement vector and

$$\{P\}\{a\} = \int_v [B]^T \{\sigma\} dv \quad \dots\dots (4.55)$$

The basic solution techniques of the above nonlinear of system equations are the iterative, incremental and combined incremental-iterative approaches. These procedures are shown in Figure (4.6)

The iterative technique, as shown in Figure (4.6-a), consists of the application of total load in a single increment. And then an initial estimate of the vector of unknown nodal displacement $\{a_1\}$ is obtained. The stresses corresponding to the initial solution are calculated using the relevant constitutive relations. These stresses are used to calculate the internal force vector $\{p(\{a\})\}$ and then the out of balance forces is successively applied to the structure in order to obtain progressively improved solutions $\{a_2\}$, $\{a_3\}$. The total displacements are taken as the sum of the accumulated displacements from each iteration. The iteration corrections continue until the out of balance forces becomes negligible small. This type of technique has a disadvantage. It is not suitable for tracing the complete nonlinear equilibrium path because it fails to produce information about the intermediate stages of loading. For such cases the incremental techniques are essential (Nesreen 1998).

The incremental technique, Figure (4.6-b), consists of the application of external loads as a sequence of sufficiently small increments. Within each increment of loading, linear constitutive relationships are generally assumed. The disadvantage is that it does not take into account the redistribution of forces during the application of increments. This technique suffers from a progressive and uncorrected tendency to drift from the true equilibrium path. The drift depends on the degree of nonlinearity and the size of loading increment.

The combined incremental-iterative technique, Figure (4.6-c), implies the subdivision of the total external load into smaller increments. Within each increment of loading iterative cycles are performed in order to obtain a converged solution corresponding to the stage of loading under consideration.

Nowadays, the incremental-iterative method is the most commonly used technique for solving nonlinear finite element problems. The standard and the modified **Newton-Raphson** methods are usually used in connection with this above technique.

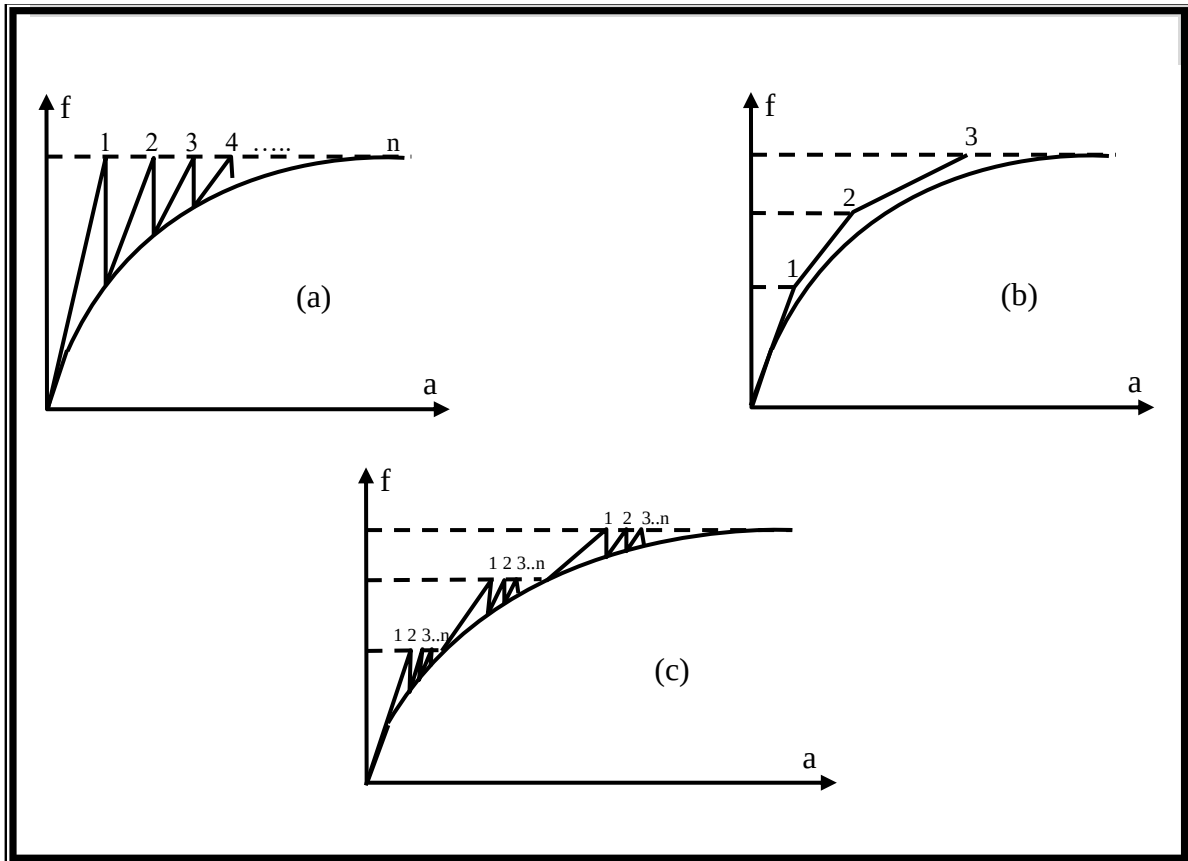


Fig. (4.6). Basic techniques used for the solution of nonlinear equations.

4.3.2 Solution Techniques

Different solution algorithms are used for calculating the incremental displacement (Δa) at i^{th} iteration during the n^{th} load increment in the nonlinear solution system. The computation time and the convergence to the correct solution is the measure of their effectiveness. The most important iteration schemes are some form of the *Newton-Raphson method*.

4.3.2.1 Tangential Stiffness Method (Standard Newton-Raphson Method)

In this method, the **standard Newton-Raphson method** technique is used. The element stiffness is recalculated and a new system of equations is solved for every iteration of each load increment as shown in Figure (4.7-a). This is an expensive process since it requires a long computation time.

4.3.2.2 Modified Newton-Raphson Method

This algorithm is a modification of the standard **Newton-Raphson** method it involves updating the stiffness matrix only once for each increment loading. One dimensional case is illustrated in Figure (4.7-b and c). The nonlinear effects are more accurately represented in the stiffness matrix. The modified Newton-Raphson method has disadvantages represented in the fact that convergence is slower than the tangential stiffness method and requires a great number of iterations to achieve solution within each increment of loading. This is especially true for an increment of loading at which a sudden softening may occur due to cracking, yielding or substantial nonlinear behavior of concrete in compression.

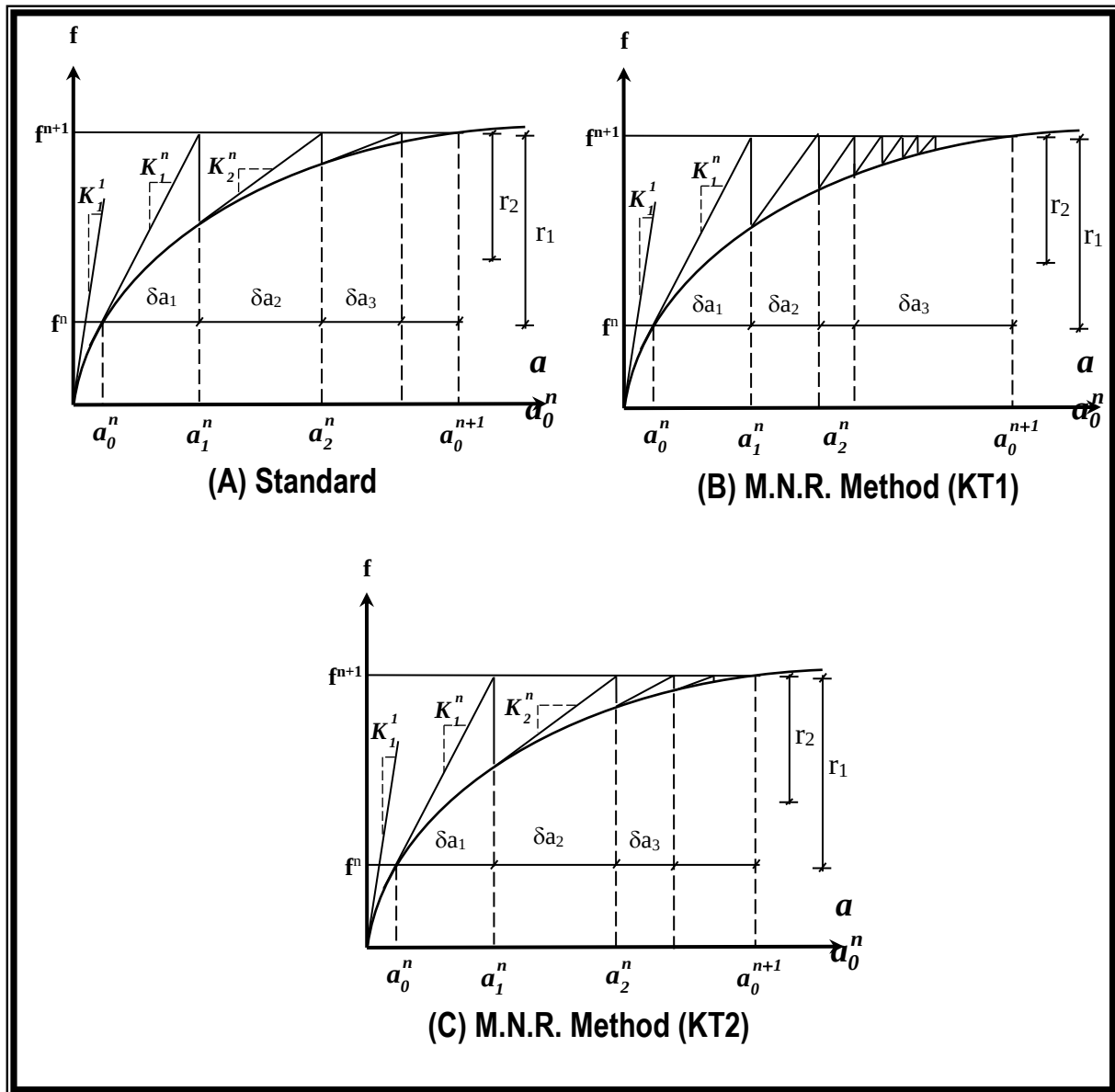


Fig. (4.7) Standard and modified Newton-Raphson procedures, one Degree of freedom case

4.3.2.3 Initial Stiffness Method

The initial stiffness algorithm is an extreme modification of the **Newton-Raphson method**. In this algorithm, the element stiffness is calculated at the beginning of the analysis (at the first iteration of the first increment) and used throughout the entire nonlinear analysis. The

computational cost per iteration is significantly reduced. But without loss in accuracy the solution requires many more of iterations with reduce the convergence rate of the process. The initial stiffness algorithm can be used only if problems with slight nonlinear effects take place.

The **modified Newton-Raphson method** in which the stiffness matrix is updated at first or second iterations of each increment of loading is considered in the computer program used in the present investigation.

4.4 Convergence Criteria

The term of convergence criteria is usually used in nonlinear structural analysis. The term is essential to terminate the progress of the iterative procedure when the solution is considered to be accurate enough compared with specified convergence tolerance. Several convergence criteria based on internal energy, displacement and out of balance forces are usually used in the nonlinear analysis of the structural members. Two of the convergence criteria mentioned above are used in the present work these are the out of balance forces and displacement criterion.

- **The Force Convergence Criteria**

$$\frac{\sqrt{\left(\sum_{i=1}^n \Delta r_i^n \Delta r_i^n\right)}}{\sqrt{\left(\sum_{j=1}^n f_j^n f_j^n\right)}} \leq \text{TOLER} \quad \dots\dots\dots (4.56)$$

where;

r^n is the out of balance forces.

f^n is the current applied forces.

n is the total number of degrees of freedom per element.

TOLER is the specified convergence tolerance

- **The Displacement Convergence Criteria**

The criterion can be written as follows:

$$\frac{\sqrt{\left(\sum_{j=1}^n \Delta \mathbf{a}_j^i \Delta \mathbf{a}_j^i \right)}}{\sqrt{\left(\sum_{j=1}^n \mathbf{a}_j^i \mathbf{a}_j^i \right)}} \leq \text{TOLER} \quad \dots\dots\dots (4.57)$$

where;

$\mathbf{a}_j^i$ is the total displacement either the incremental translation or rotational displacement vector at iteration (i).

$\Delta \mathbf{a}_j^i$ is either the incremental translation or rotational displacement vector at iteration (i).

n is the total number of translation or rotational degrees of freedom

4.5 Nonlinear Geometric Analysis

General theoretical formulations and computational techniques for the geometrically nonlinear analysis of reinforced concrete structures have been used in recent years (Abdel-Rahman and Hinton 1986). One of the most popular descriptions is still the Lagrangian approach within a finite element analysis. A Cartesian coordinate system is used to describe the deformation of the structure during loading. Two different formulations can be applied in the incremental finite element used in geometrically nonlinear analysis of large deformation problems. The first is the total Lagrangian approach, in which,

current fields refer to the initial configuration of the structural system, so that the displacement field defines the current configuration of the system with reference to its initial position. The second is the updated Lagrangian approach, in which the current configuration of the system is used to define the current stress and strain state. The geometry of the structure is successively updated during the incremental process and the stress and strain fields refer to the last evaluated configuration.

The physical meaning of Piola-Kirchhoff stresses may be lost in large strain problems. However, in most large deformation structural problems, it is sufficient to deal with large displacements and small strains and in this case Piola-Kirchhoff stress has the usual physical meaning.

In the present work, a specific and appropriate formulation for the nonlinear analysis of reinforced plates and shells has been employed. In this formulation, large deflections and moderate rotations are taken into account with the simplified Von-Karman assumptions. Large strains are not considered and the effects of deformation dependent loading are neglected (Bathe and Bolourhi 1980). The use of the total Lagrangian approach is more advantageous for degenerated shell elements. The strain matrix according to this formulation needs to be calculated only once during the nonlinear process and its nonlinear part can be calculated and added by using the current displacements. The relations defined in terms of strains and stresses are considered valid for the new quantities measured in the original configuration (Stolarski, and Belytschko 1982). The details of this approach will be explained in the next section.

4.5.1 Nonlinear Geometric Formulation in Degenerated Shell Element

a. Von – Karman assumptions

The Von-Karman assumptions for large deflection of plates should take the following forms when applied to shells(Chen 1982) .

1. The thickness (**h**) is much smaller than the length (**L**), (this hypothesis is not restrictive since otherwise the displacements are not large).
2. The magnitude of the deflection (**w**) is of the order of the thickness (**h**).
3. The slope is small everywhere, $\partial w/\partial x \ll 1$, $\partial w/\partial y \ll 1$.
4. The tangential displacements, **u** and **v**, are small. Only nonlinear terms which depend on $\partial w/\partial x$, $\partial w/\partial y$ have to be retained in the strain – displacement relations.
5. All strain components are small.

Applying these assumptions, the strain vector component may be expressed in terms of local derivatives of the displacements for the degenerate shell element and can be written as:

$$\begin{bmatrix} \epsilon_{x'} \\ \epsilon_{y'} \\ \gamma_{x'y'} \\ \gamma_{x'z'} \\ \gamma_{y'z'} \end{bmatrix} = \begin{bmatrix} \frac{\partial u'}{\partial x'} + \frac{1}{2} \left(\frac{\partial w'}{\partial x'} \right)^2 \\ \frac{\partial v'}{\partial y'} + \frac{1}{2} \left(\frac{\partial w'}{\partial y'} \right)^2 \\ \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} + \frac{\partial w'}{\partial x'} \cdot \frac{\partial w'}{\partial y'} \\ \frac{\partial u'}{\partial z'} + \frac{\partial w'}{\partial x'} \\ \frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'} \end{bmatrix} \quad \text{..... (4.58)}$$

Separating the strain components into a linear part $\{\epsilon_o\}$ and a nonlinear part $\{\epsilon_L\}$ which can be expressed as,

$$\{\epsilon\} = \{\epsilon_o\} + \{\epsilon_L\} \quad \text{..... (4.59)}$$

where,

$$\{\epsilon_o\} = \begin{bmatrix} \frac{\partial u'}{\partial x'} \\ \frac{\partial v'}{\partial y'} \\ \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} \\ \frac{\partial u'}{\partial z'} + \frac{\partial w'}{\partial x'} \\ \frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'} \end{bmatrix}, \quad \{\epsilon_L\} = \begin{bmatrix} \frac{1}{2} \left(\frac{\partial w'}{\partial x'} \right)^2 \\ \frac{1}{2} \left(\frac{\partial w'}{\partial y'} \right)^2 \\ \frac{\partial w'}{\partial x'} \frac{\partial w'}{\partial y'} \\ 0 \\ 0 \end{bmatrix} \quad \text{.....(4.60)}$$

and

$$d\{\epsilon\} = d\{\epsilon_o\} + d\{\epsilon_L\} \quad \text{.....(4.61)}$$

b. Strain-displacement matrix [B]

The strain displacement matrix [B] may be separated into two parts,

$$[B] = [B_o] + [B_L] \quad \text{..... (4.62)}$$

where;

[B_o] is the linear part,

[B_L] is the nonlinear part.

The strain displacement matrix [B_o] was calculated for shell element. The large strain matrix [B_L] is related to the nonlinear components of the strain vector and can be defined for use in the nonlinear process. In modeling reinforced concrete, the total strain has been used and the total strain-displacement relation should be defined.

For the degenerated shell elements, the nonlinear components of the strain vector can be expressed by (Zienkiewicz 1977),

$$\{\varepsilon_L\} = \begin{Bmatrix} \frac{1}{2} \left(\frac{\partial w'}{\partial x'} \right)^2 \\ \frac{1}{2} \left(\frac{\partial w'}{\partial y'} \right)^2 \\ \frac{\partial w'}{\partial x'} \cdot \frac{\partial w'}{\partial y'} \\ 0 \\ 0 \end{Bmatrix} = \frac{1}{2} \begin{Bmatrix} \frac{\partial w'}{\partial x'} & 0 \\ 0 & \frac{\partial w'}{\partial y'} \\ \frac{\partial w'}{\partial y'} & \frac{\partial w'}{\partial x'} \\ 0 & 0 \\ 0 & 0 \end{Bmatrix} \begin{Bmatrix} \frac{\partial w'}{\partial x'} \\ \frac{\partial w'}{\partial y'} \end{Bmatrix} \dots\dots$$

(4.63)

The nonlinear contribution to the strain vector can be written as:

$$\{\varepsilon_L\} = \frac{1}{2} [A][R] \dots\dots\dots (4.64)$$

where;

$$[A]^T = \begin{bmatrix} \frac{\partial w'}{\partial x'} & 0 & \frac{\partial w'}{\partial y'} & 0 & 0 \\ 0 & \frac{\partial w'}{\partial y'} & \frac{\partial w'}{\partial x'} & 0 & 0 \end{bmatrix} \dots\dots\dots (4.65)$$

By summing up the contribution of each nodal variable, the derivatives of w' can be calculated and the vector $\{R\}$ can be expressed as:

$$\{R\} = \begin{Bmatrix} \frac{\partial w'}{\partial x'} \\ \frac{\partial w'}{\partial y'} \end{Bmatrix} = [G]\{a\} \dots\dots\dots (4.66)$$

where;

[G] : a matrix with two rows and a number of columns equal to the total number of element nodal variables. The first row contains the contribution of each nodal variable to the local derivatives $\partial w'/\partial x'$ (corresponding shape function derivative) and the second row contains similar contributions for $\partial w'/\partial y'$.

By taking the variation of Equation (4.64) gives:

$$d\{\epsilon_L\} = (1/2) d[A] \{R\} + (1/2)[A]d\{R\} = [A]d\{R\} = [A] [G] \{a\} \dots (4.67)$$

Then from the definition of the strain matrix:

$$\{B_L\} = [A] [G] \dots\dots\dots (4.68)$$

In the present study, the incremental strain-displacement matrix $[B]$ is evaluated by summing the matrix $[B_o]$ to the current value of matrix $[B_L]$. In the present finite element program, matrices $[B_o]$ and $[G]$ are initially calculated and stored for linear elastic conditions. The summation of the strain-displacement matrix $[B]$ is defined in Equation (4.68).

c. Tangential stiffness matrix $[k]$

The tangential stiffness matrix for the current configuration $[k]$ can be derived from the variation of Equation (4.55) with respect to a displacement variation $\{a\}$.

$$d\{P\} = \int_V [B]^T d\{\sigma\} dV + \int_V [dB]^T \{\sigma\} dV = [K] d\{a\} \dots\dots (4.69)$$

where;

$$d\{\sigma\} = [D] d\{\epsilon\} = [D][B] d\{a\} \dots\dots\dots (4.70)$$

and,

$$d[B]^T = d[B_L]^T \dots\dots\dots (4.71)$$

The initial stress or geometric stiffness matrix $[K]_\sigma$, for the definition of the second term of Equation (4.69) can be expressed as,

$$[k]_\sigma d\{a\} = \int_V d[B]^T [\sigma] dV = \int_V d[B_L]^T [\sigma] dV \dots\dots\dots (4.72)$$

Then, the tangential stiffness matrix can be written as,

$$[K] = [\bar{K}] + [K]_\sigma \dots\dots\dots (4.73)$$

in which $[\bar{K}]$ is given by the usual expression as,

$$[\bar{K}] = \int_V [B]^T [D][B] dV \dots\dots\dots (4.74)$$

The geometric stiffness matrix $[K]_6$ must be defined explicitly in order to determine the tangential stiffness $[K]$. Substituting of Equation (4.68) into Equation (4.72), then

$$[K]_6 d\{a\} = \int_V [G]^T d[A]^T [\sigma] dV \quad \dots\dots\dots (4.75)$$

where $d[A]^T[\sigma]$ can be written as,

$$d[A]^T[\sigma] = \begin{bmatrix} \frac{\partial w'}{\partial x'} & 0 & \frac{\partial w'}{\partial y'} & 0 & 0 \\ 0 & \frac{\partial w'}{\partial y'} & \frac{\partial w'}{\partial x'} & 0 & 0 \end{bmatrix} \begin{Bmatrix} \sigma_{x'} \\ \sigma_{y'} \\ \tau_{x'y'} \\ \tau_{x'z'} \\ \tau_{y'z'} \end{Bmatrix} \quad \dots\dots\dots (4.76)$$

This vector can be arrange as,

$$d[A]^T[\sigma] = \begin{bmatrix} \sigma_{x'} & \tau_{x'y'} \\ \tau_{x'y'} & \sigma_{y'} \end{bmatrix} \begin{bmatrix} d\left(\frac{\partial w'}{\partial x'}\right) \\ d\left(\frac{\partial w'}{\partial y'}\right) \end{bmatrix} \quad \dots\dots\dots (4.77)$$

The term $d[A]^T[\sigma]$ can be written, with the definition of $[G]$ from Equation (4.66),

$$d[A]^T[\sigma] = \begin{bmatrix} \sigma_{x'} & \tau_{x'y'} \\ \tau_{x'y'} & \sigma_{y'} \end{bmatrix} [G] d\{a\} \quad \dots\dots\dots (4.78)$$

Substituting Equation (4.78) into Equation (4.75) gives the geometric stiffness matrix to be a symmetric matrix and can be written, as

$$[K]_6 = \int_V [G]^T [\sigma] [G] dV \quad \dots\dots\dots (4.79)$$

4.6 The Employed Computer Program

In the present study, a computer program has been used for nonlinear analysis of reinforced concrete plates and shells structures. The program's subroutines are taken from Reference (Hinton and Owen 1984).

In this program a layered approach is adopted with material and geometric nonlinear effects may also be considered. The reinforcement is represented by a smeared layer of equivalent thickness. The nonlinear solution technique includes standard and modified Newton-Raphson and the initial stiffness methods may optionally be performed, The program is coded in FORTRAN-77 Language.

Chapter Five

Formulation of Optimal Design Problem

5-1 General

This chapter presents the formulation of the optimal design problem of reinforced concrete plates and shells. Optimization is defined as finding the optimum value (maximum or minimum) of a function called the objective function $f(x_1, x_2, x_3, \dots, x_n)$ of n real variables called the design variables. The optimization is constrained when certain limitations are imposed on the design variables, while the optimization is nonlinear when the mathematical expressions relating the design variables in nonlinear constraints or in the objective function are of nonlinear nature.

5-2 Design variables and Constraints

The design variables of plates and shells are classified into independent variables and dependent variables. In case of plates the independent variables include the depth of plate, while the dependent variables, considered here, are the longitudinal and lateral reinforcements at critical sections. For shells the independent variables include the depth and the radius of curvature of the shell, while the dependent variables, considered here, are the longitudinal and lateral reinforcement. The total cost, involving cost of steel reinforcement, concrete and formwork, are considered as the objective function. Several design constraints, which include strength constraints and side constraints are related to functional, architectural and constructional requirements and are taken into account in the formulation of the optimization problem. At last,

the optimization method used in this work is explained with the aid of flow charts.

The objective function is minimized subject to a set of constraints. Some of these constraints can be explicitly defined in terms of the design variables (i.e. side constraints), while others are implicitly related to design variables (i.e. flexural, shear ...etc). The equations according to ACI 318-83 Code.

$$M_u = \phi b d^2 \rho f_y \left(1 - 0.59 \rho \left(\frac{f_y}{f_c'}\right)\right)$$

$$\phi = 0.9$$

$$\text{Min thickness (t)} = \frac{\ln}{20} \text{ For simply supported one way reinforced}$$

concrete slab

$$\text{Min cover of slab} = 20 \text{ mm}$$

$$d_b \text{ use } \phi 8 \text{ mm}, \phi 12 \text{ mm} \leq \phi 6 \text{ mm}$$

$$d = t - \text{cover} - 0.5 d_b$$

$$A_{s \max} = \rho_{\max} \cdot b \cdot d$$

$$\rho_{\max} = 0.75 \rho_b$$

$$\rho_b = 0.85 \beta_1 \left(\frac{f_c'}{f_y}\right) \cdot (600 / (600 + f_y))$$

$$\beta_1 = 0.85 \quad f_c' \leq 30 \text{ MPa}$$

$$\beta_1 = 0.85 - (f_c' - 30) / 10 \cdot 0.08 \geq 0.65 \text{ MPa} \quad f_c' > 30 \text{ MPa}$$

$$A_{s \min} = 0.0018 \cdot b \cdot t \text{ for } f_y = 400 \text{ MPa}$$

$$A_{s \min} = 0.002 \cdot b \cdot t \text{ for } f_y = 300, 350 \text{ MPa} < 400 \text{ MPa}$$

$A_{s \min} = 0.0018.(4000 / f_y).b.t$ for $f_y \geq 400\text{MPa}$ and must
 $0.0018(400 / f_y) > 0.0014.$

$$A_{s \min} \leq A_s \leq A_{s \max}$$

$$(V_{ud} / \phi) \leq V_c$$

$$\phi = 0.85$$

$$V_c = 0.17 \text{ squar}(f_c').b.d$$

Min thickness (t) = perimeter / 180 for two way reinforced concrete slab
(According to ACI 318-63 Code)

$$d_{\text{shor}} = t - \text{cover} - 0.5d_b$$

$$d_{\text{long}} = t - \text{cover} - 1.5d_b$$

For the reinforced concrete cylindrical shell

The equations according to ACI 318-63 Code.

$$M = f_s.A_s.j.d$$

$$f_s < 0.5.f_y$$

$$f_c < 0.45.f_c'$$

$$n = E_s / E_c$$

$$k = 1 / (1 + (f_s / n.f_c))$$

$$j = 1 - (k / 3)$$

5-3 Objective Function

5.3.1 One- way reinforced concrete slab

The non – linear cost objective function (Z) involving the cost of steel reinforcement, concrete and formwork is used in the optimization problem of this study.

where:

Cost of steel =

$$C_s \left(\left(\frac{4}{3} \right) * A_s * \left(\frac{b}{5} \right) * L + \left(\frac{1}{3} \right) * A_s * L * \left(b - \left(\frac{2 * b}{5} \right) \right) + A_{s_{min}} * b * L \right) * w_s.$$

Cost of concrete = $C_c (b * t * L)$

Cost of formwork = $C_f (b * L + 2 * L * t + 2 * b * t)$

where:

C_s = unit price of steel reinforcement involving material and labour cost

A_s = Area of tension reinforcement

L = Length of slab

b = width of slab

t = Thickness of slab

w_s = Unit weight of steel

C_c = Unit price of concrete involving material and labour cost

C_f = Unit price of formwork

5.3.2 Simply supported Square slab

The non – linear cost objective function (Z) involving the cost of steel reinforcement, concrete and formwork is used in the optimization problem of this study.

where:

$$\text{Cost of Steel} = C_s ((4/3)A_{s1} * b * (L/5) + (1/3) * A_{s1} * b * (L - (2L/5)) + (4/3)A_{s2} * L * (b/5) + (1/3) * A_{s2} * L * (b - 2b/5))) * w_s$$

$A_{s1} \neq A_{s2}$ because the steel reinforcement in the Example of analysis is not equal.

$$\text{Cost of concrete} = C_c (b * t * L)$$

$$\text{Cost of formwork} = C_f (b * L + 2 * L * t + 2 * b * t)$$

where:

C_s = Unit price of steel reinforcement involving material and labour cost

A_s = Area of tension reinforcement

L = Length of slab

b = width of slab

t = Thickness of slab

w_s = unit weight of steel

C_c = Unit price of concrete involving material and labour cost

C_f = Unit price of formwork

5.3.3 Reinforced concrete cylindrical shell

The non – linear cost objective function (Z) involving the cost of steel reinforcement, concrete and formwork is used in the optimization problem of this study.

where:

Cost of Steel

$$C_s (2A_{s1} * \beta * R * L + 2A_{s2} * \beta * R * L + 2A_{s3} * \beta * R * L + 2A_{s4} * \beta * R * L) * w_s$$

$$\text{Cost of concrete} = C_c (2\beta * R * t * L)$$

$$\text{Cost of formwork} = C_f (2\beta * R * L + 4\beta * R * t + 4t * L)$$

where:

C_s = Unit price of steel reinforcement involving material and labour cost

A_s = Area of reinforcement

L = Length of shell

R = Radius of shell

t = Thickness of shell

β = The shell angle

w_s = Unit weight of steel

C_c = Unit price of concrete involving material and labour cost

C_f = Unit price of formwork

5.4 Optimization Process

5.4.1 Decomposition and Multilevel Solution

The problem that is formulated in the preceding sections can be solved as an integrated unit using any suitable method of non –linear optimization, but this involves real computational difficulties. Some of these may be summarized here (Kirsch 1983).

- 1- The problem size (number of variables and constraints) is relatively large even for simple structures. This will greatly increase the computational time which is usually an exponential function of the problem size. In addition, such problem needs a computer of high capacity.
- 2- The different natures of design variables may produce numerical problem in the solution.
- 3- The non –linear analysis of structures, being of a repetitive nature and involving a solution of set of simultaneous equations, will increase the total computational time (**CPU time**).

These difficulties are largely resolved by decomposing the problem into smaller sub problems. The (multilevel approach) developed by (Bunday 1984) is adopted here. The approach is more suitable for micro –computer applications and its effectiveness has been already tested and verified in recent works for such type of problems (Bunday 1984). The problem is subdivided into three levels as given below.

Third Level (Outer Level)

In this level, the design moments and forces are obtained from structural analysis (elastic or inelastic as the case may be). The analysis is repeated after each optimization process(in second and third levels). To satisfy the optimization at this level, the convergence criterion on the objective function, which is the total cost of the structure (Z), is

$$\left| \frac{Z^i - Z^{i-1}}{Z^i} \right| \leq e \quad \text{.....(4.1)}$$

Where (Z^i) is the total cost of the structure in the *ith* iteration and (e) is a prechosen tolerance for the total cost, which is selected depending on a trial test on different types of problems. The value selected and used here is ($e = .001$).

Second Level

In the level, the independent variables of each member are optimized independently for the computed forces (from third level) using a non –linear optimization method as will be explained later. After each modification in the independent variable, the solution enters the first level to determine the values of the dependent variables.

First Level

At this level, the value of the dependent variables are calculated directly using the value of the independent variables and design forces from the 2nd and 3rd levels respectively.

5.4.2 Method of Optimization

Since the minimization of the objective function depends on the section resistance, which is an implicit function of the independent design variables and cannot be expressed directly as a function of these variables, it cannot be derived with respect to the independent design variables. Hence the gradient method of non –linear optimization such as **Hooke and Jeeves** is simpler than (SUMT Method) (sequential unconstrained minimization technique) therefore can be used in this study. So the modified direct search method of **Hooke and Jeeves** (Bunday 1984) will be used which uses the function values only. The search consists of a sequence of exploration steps about a base point which if successful are followed by pattern moves. The modification was made on this method to take account of constraints. Indeed it has been suggested that merely giving the objective function a very large value (in a minimization problem) whenever the constraints are violated will suffice (usually $z = 1 * 10^{30}$). Certainly this idea has an obvious intuitive appeal and it is easy to program.

The procedure is as follows.

- A) Choose an initial base point $\mathbf{b}_1$ and a step length $\mathbf{h}_j$ for each variable $\mathbf{X}_{j, j} = 1, 2, \dots, n$,

B) Carry out an exploration about $\mathbf{b}_1$. The purpose of this is to acquire knowledge about the local behavior about the function. This knowledge is used to find a likely direction for the pattern move by which it is hoped to obtain an even greater reduction in the value of the function. The exploration about $\mathbf{b}_1$ proceeds as indicated.

1- Evaluation of $(\mathbf{b}_1)$.

2- Each variable is now changed in turn, by adding the step length.

Thus, the value of $f(\mathbf{b}_1 + h_1 \mathbf{e}_1)$ where $\mathbf{e}_1$ is a unit vector in the direction of the x –axis is then calculated. If this reduces the function replace $\mathbf{b}_1$ by $\mathbf{b}_1 + h_1 \mathbf{e}_1$. if not find $f(\mathbf{b}_1 - h_1 \mathbf{e}_1)$ and replace $\mathbf{b}_1$ by $\mathbf{b}_1 - h_1 \mathbf{e}_1$ if the function is reduced. If neither step gives a reduction leave $\mathbf{b}_1$ unchanged and consider changes in $\mathbf{X}_2$, i.e. find $f(\mathbf{b}_1 + h_2 \mathbf{e}_2)$ etc. When all n variables are considered, a new base point $\mathbf{b}_2$ is obtained.

3- If $\mathbf{b}_2 = \mathbf{b}_1$ i.e. no function reduction has been achieved, the exploration is repeated about the same point $\mathbf{b}_1$ but with a reduced step length. Reducing the step length (s) to one tenth of its former value appears to be satisfactory in practice.

4- If $\mathbf{b}_2 \neq \mathbf{b}_1$ a pattern move can be made.

C) Pattern moves utilize the information acquired by exploration and accomplish the function minimization by moving in the direction of the established “pattern “. The procedure is as follows.

1- It seems sensible to move further from the base point $\mathbf{b}_2$ in the direction $\mathbf{b}_2 - \mathbf{b}_1$ since that move already led to a reduction in the function value. So, the function value is evaluated at the next pattern point

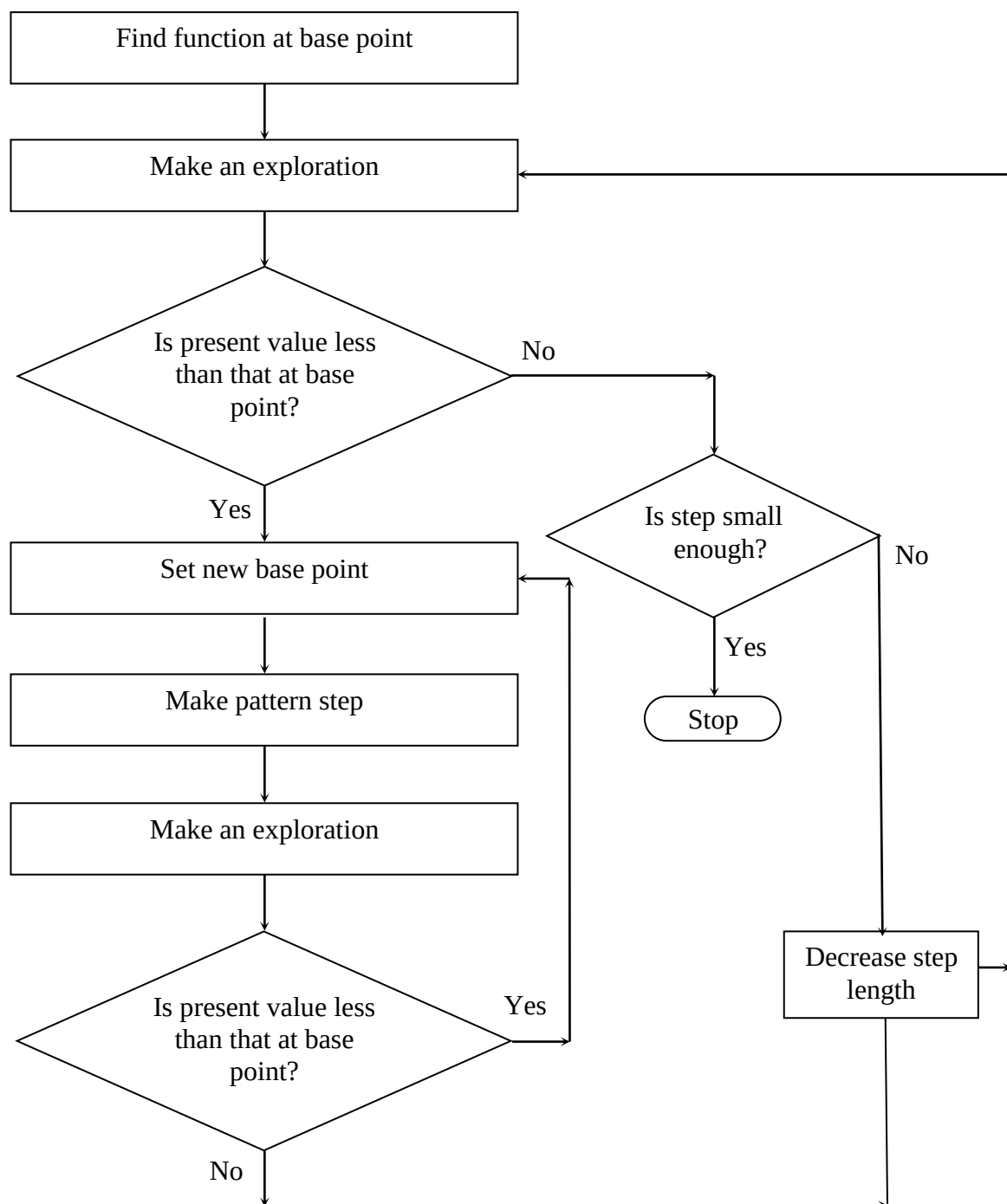
$$\mathbf{P}_1 = \mathbf{b}_1 + 2(\mathbf{b}_2 - \mathbf{b}_1) \quad \dots\dots\dots(4.2)$$

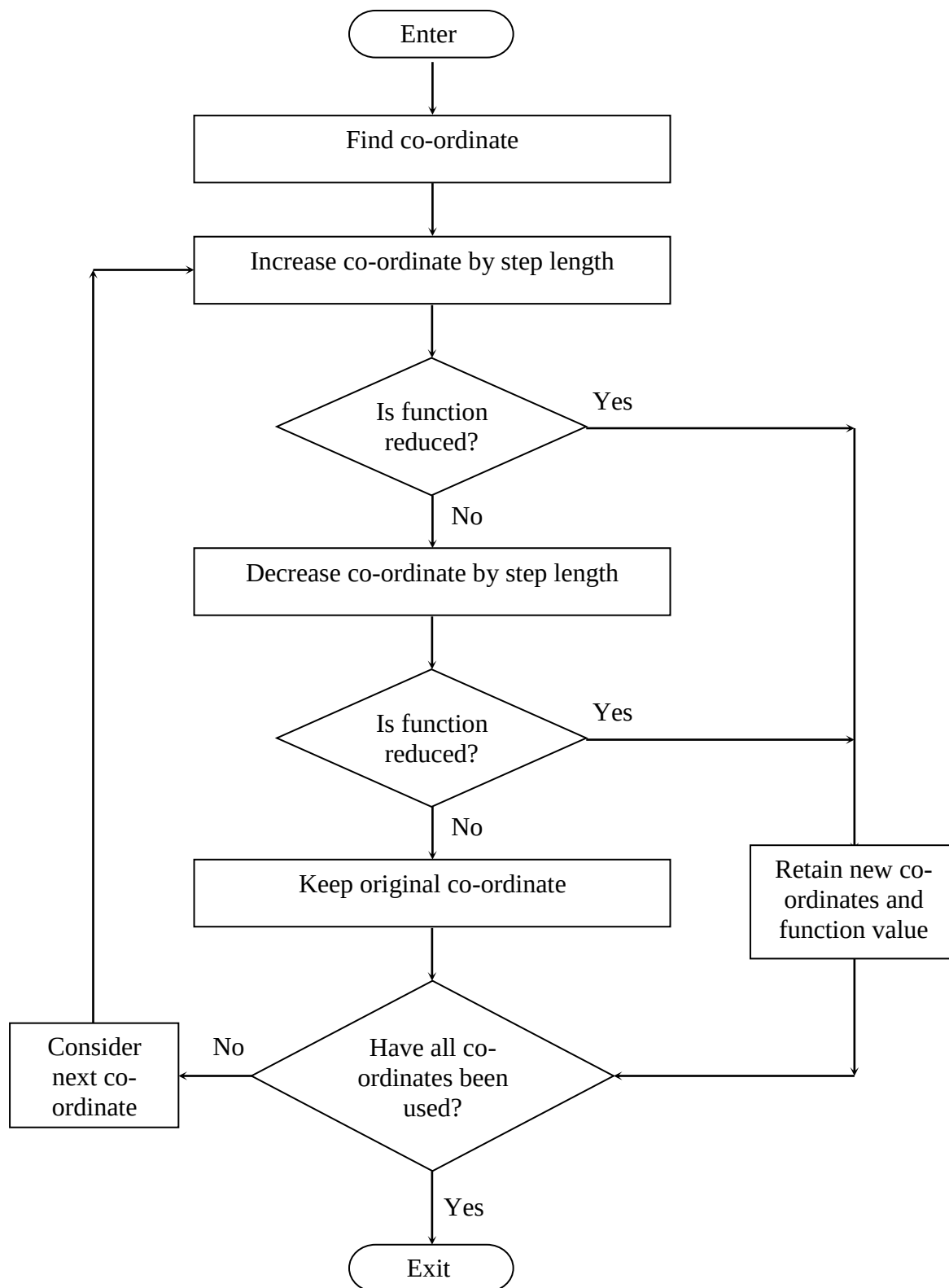
In general

$$P_i = b_i + 2 (b_{i+1} - b_i) \quad \dots\dots\dots (4.3)$$

- 2- Then continue with exploratory move about $p_1 (p_i)$.
- 3- If the lowest value at step **C (2)** is less than the value at the base point $b_2 (b_{i+1}$ in general) than a new base point $b_3 (b_{i+2})$ has been reached. In this case, repeat **C (1)**. Otherwise abandon the pattern move from $b_2 (b_{i+1})$ and continue with an exploration about $b_2 (b_{i+1})$.
- D) Terminate the process when the step length (s) has been reduced to a predetermined small value, (Usually less than $1*10^{-8}$).

The following flow charts Fig. (5.1) a, b represent the method.

**Fig. (5.1), a: Flow chart for Hooke and Jeeves Method**

**Fig. (5.1), b: Flow chart for an Exploration**

Chapter Six

APPLICATION AND DISCUSSIONS

6.1 General

In this chapter, some numerical examples are worked out to compare results obtained by finite element analysis to those obtained from experimental tests. A number of numerical examples has been analyzed using the computer program. The examples also serve as means to check the validity of the materials models used. It is important to mention that this theoretical study is approximate in nature due to different factors mainly;

1. Approximations in the material modeling of concrete and steel.
2. Approximation inherent in the finite element modeling technique.
3. Approximation in the integration functions used in this numerical analysis.
4. Approximation introduced due to the type of procedure used in solving the nonlinear system of equations.

Most of the numerical examples considered in this chapter were previously tested experimentally by other researchers. The main results obtained by the solution introduced through this study are the load-deflection responses.

6.2.1 Corner Supported Square Slab

Three slabs tested experimentally by Duddeck et al. and theoretically by (Hinton and Hassan 1987) are studied. These slabs have various degrees of orthotropy of the steel reinforcement. It had the advantage of well defined support conditions. Only transverse deflections

are restrained at the corner supports .The material properties are rather poorly defined. The values assumed here summarized in Table (6.1).

Table (6.1) Material Properties are rather poorly defiend, the values assumed here are summarized (Hinton and Hassan 1987) for Duddeck *et al.* Slab considered in the analysis

Material properties and parameters		Value
Concrete	Young's Modulus, E_c , MPa	16400.
	Compressive Strength, f_c' , MPa	43.0
	Tensile Strength, f_t , MPa	3.0
	Poisson's Ratio, ν	0.15
	Uniaxial crushing strain ϵ_{cu}	0.0035
Steel	Young's Modulus, E_s , MPa	201000
	Yield Strength, f_y , MPa	670
Tension-Stiffening Parameters	α_m	0.70
	ϵ_m	0.002

Table (6.2) the equivalent reinforcement layers for the three slabs are given below in millimetres.

	Top layers		Bottom layers	
	x-dir	y-dir	x-dir	y-dir
s1	0.193	0.193	0.397	0.397
s2	0.252	0.133	0.520	0.273
s3	0.283	0.103	0.582	0.212

All three slabs were tested under the same loading conditions of a concentrated or patch load at the slab center. In the finite element analysis, by taking advantage of summetry,it is only necessary to consider one-quarter of the slab. The slab geometry, and reinforcement details are shown in Fig. (6.1). Two different meshes for the finite element analysis

are used to study the effect of mesh size for the three slabs as shown in Fig. (6.2). In the present study 4-elements with eight nodes for each element and the second was 9-elements with eight node for each element, it can be seen from Fig. (6.3.a to 6.3.c) that the second mesh was closer to the experimental results for both the overall load-deflection behaviour and ultimate load ,the collapse load for two meshes are 70 kN and 60 kN respectively, for the first slab and 51 kN, 50 kN for the second slab. Finally 50 kN, 40 kN for the third slab.

agreement with the experiments is observed for the failure loads and it is also found though out the whole loading process as can be seen from the load-central deflection curves of Fig (6.4 .a to 6.4. c).

Crushed gauss points were initiated at a load of 0.98, 1.136 and 1.168 of ultimate load. The presence of these crushed points at the top layer began when the compressive strain of the gauss point increased the ultimate crushing strain. When the gauss point is considered crushed, zero stresses and stiffness are assigned to them.

A number of previous theoretical results is plotted in Fig.(6.4.a to 6.4.c) , (Hinton and Hassan 1987) using 3-D finite element analysis ,the 20-node isoparametric brick element. Hierarchical, rather than standard, interpolation functions are used.Trilinear shape functions are used for eight corner node and quadratic hierarchical shape functions are employed for 12 mid-side nodes.. A hardening-plasticity model is used. Crushed gauss points were initiated at a load of 0.94, 1.045, and 1.091 of ultimate load. For comparison purposes, the present model seems to provide a good representation of deformation path.

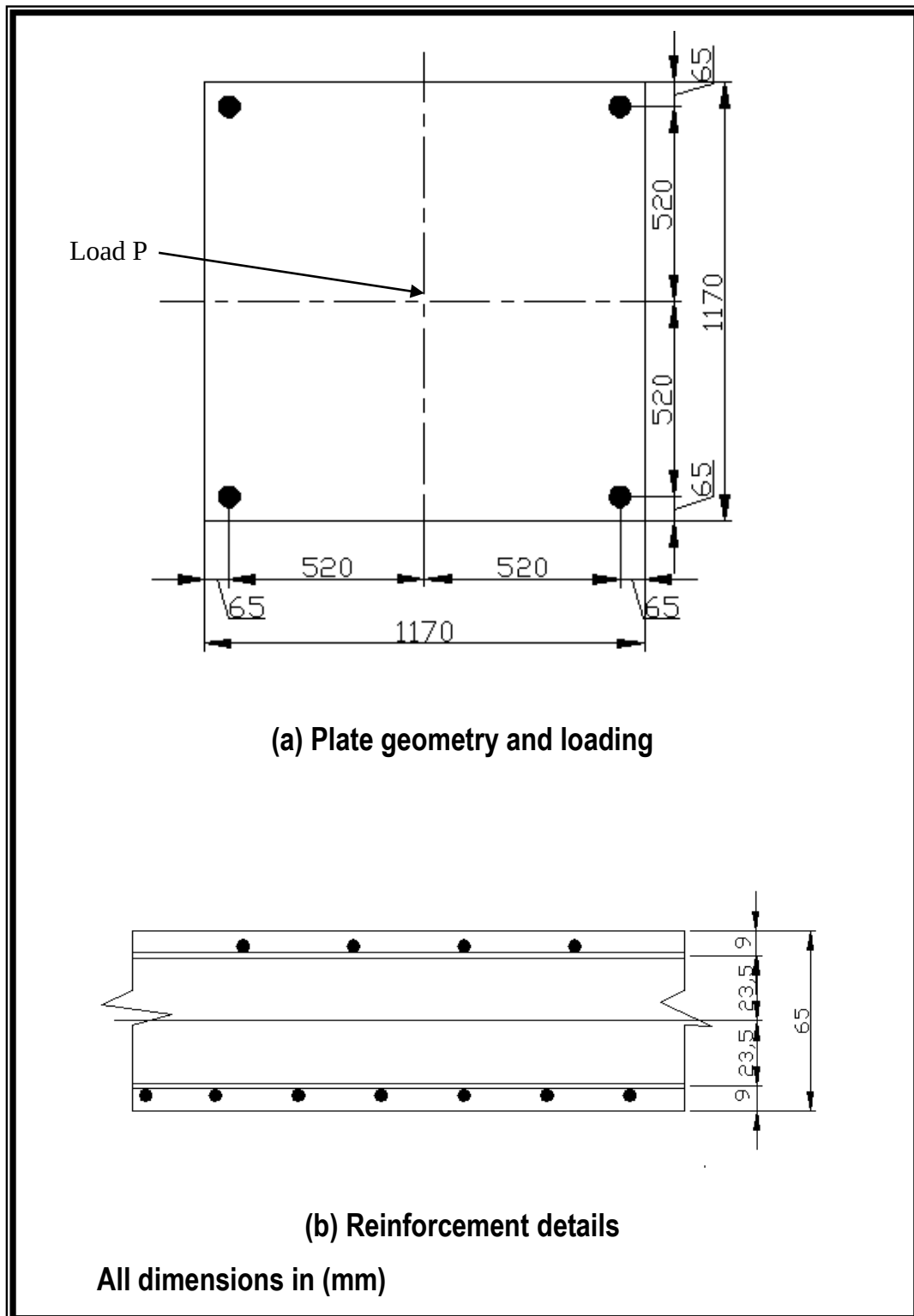


Fig. (6.1): Plate geometry and reinforcement details for Corner supported square slab

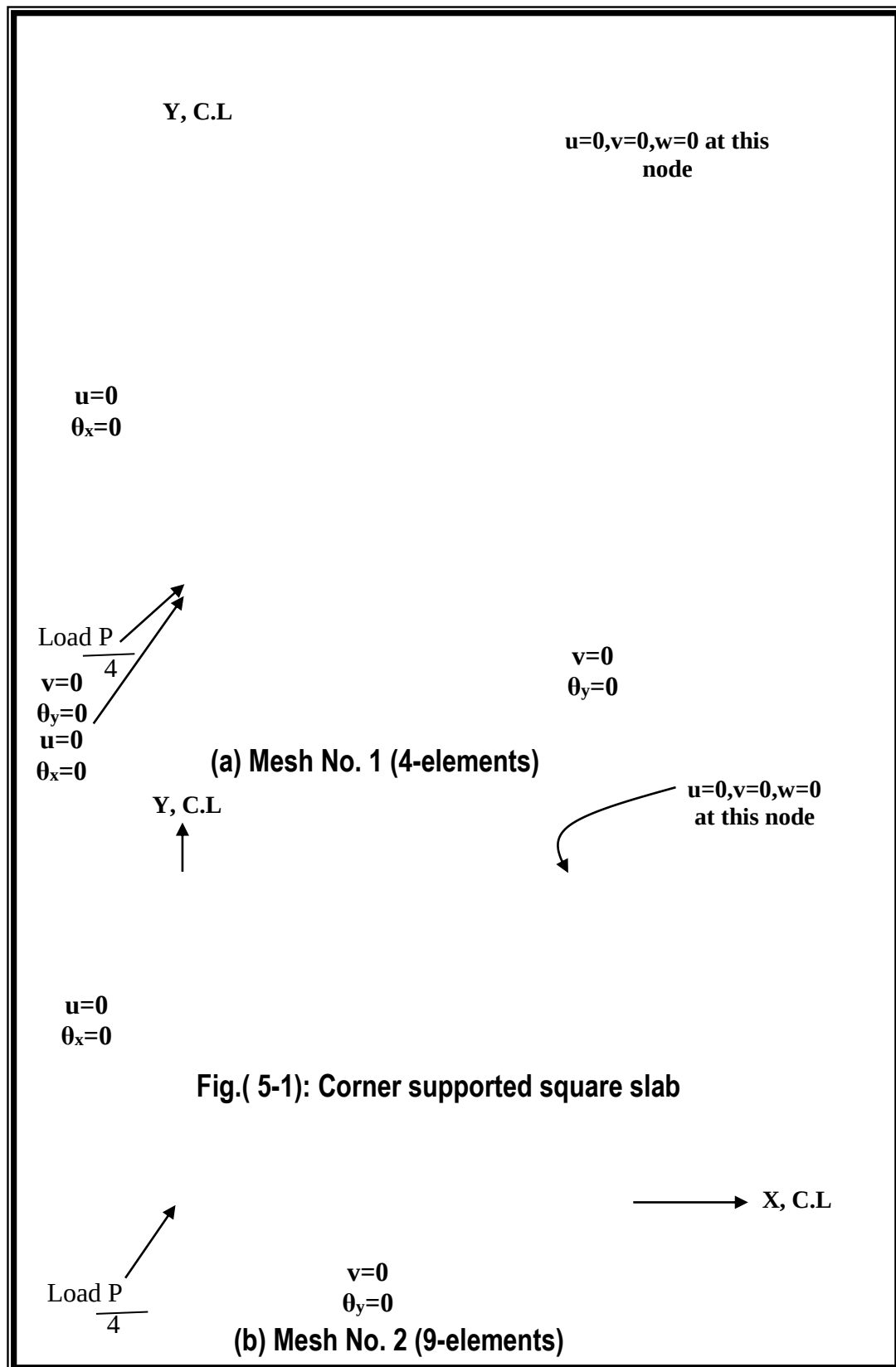


Fig. (6.2): Finite element mesh for Corner supported square slab

(a) Load deflection curve for Duddeck's Slab1

(b) Load deflection curve for Duddeck's Slab

(c) Load deflection curve for Duddeck's Slab

Fig (6.3). Comparison of load-deflection curves for Duddeck's slabs (a) slab 1, (b) slab 2, (c) slab 3

Exp.

(a) Load deflection curve for Duddeck's Slab

(b) Load deflection curve for Duddeck's Slab

Fig. (6.4). Central load-deflection curves for Duddeck's slabs(a) slab 1, (b) slab 2 , (c) slab 3

6.2.2 One-Way Reinforced Concrete Slab

A one-way slab supported at two edges was tested experimentally by (McNiece and Jofriet 1971). The geometrical details, reinforcement layout, loading are shown in Fig. (6.5). Utilizing symmetry of loading and geometry, only a quarter of the slab is modelled by the finite elements method. Two mesh of four and nine eight-node shell elements are used for this quarter structure are shown in Fig. (6.6). Very small difference in load deflection behaviour for the two mesh at shown in Fig(6.7) .The steel reinforcement is represented by a layer with a thickness equals to 0.356 mm. Material properties of concrete and steel are given in Table (6.2).

In Fig. (6.8), the load-deflection curve at midspan of the slab is shown. Good agreement with experimental results are obtained through most loading level.

Table (6.2) Material Properties for McNiece *et al.* Slab considered in the analysis

Material properties and parameters		Value
Concrete	Young's Modulus, E_c , MPa	31000
	Compressive Strength, f_c' , MPa	35.0
	Tensile Strength, f_t , MPa	3.0
	Poisson's Ratio, ν	0.15
	Uniaxial crushing strain ϵ_{cu}	0.0035
Steel	Young's Modulus, E_s , MPa	210000
	Yield Strength, f_y , MPa	345
Tension-Stiffening Parameters	α_m	0.50
	ϵ_m	0.002

Crushed gauss points were initiated at a load of 0.97 of ultimate load. The presence of these crushed points at the top layer began when the compressive strain of the gauss point increased the ultimate crushing strain. The consideration of geometrical nonlinearity in the finite element analysis has been found to be the major parameter in the analysis of reinforced concrete plate structures, which exhibit relatively large deformations before failure as shown in Fig (6.8.a).

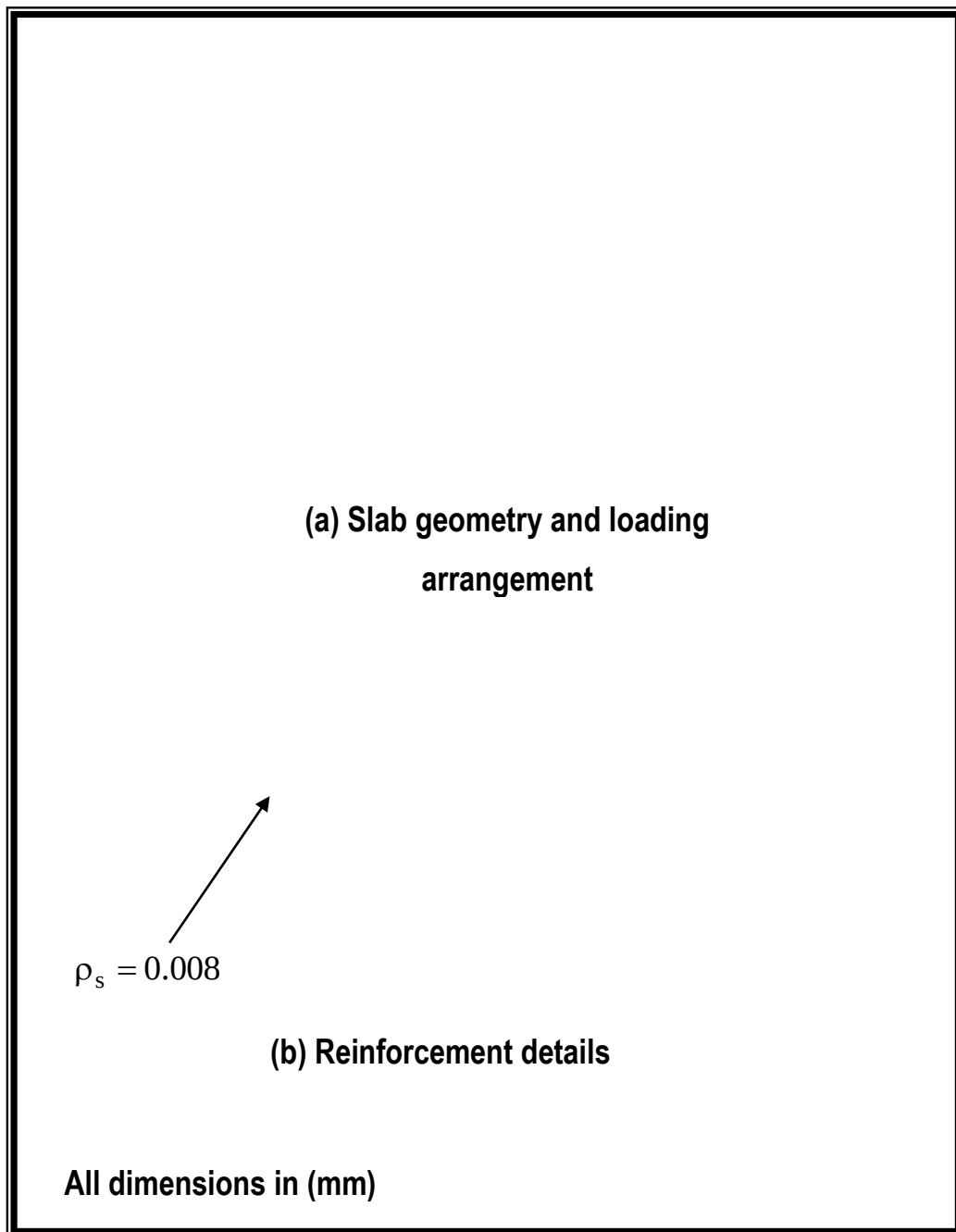


Fig. (6.5). Slab geometry and reinforcement details for One-Way reinforced concrete

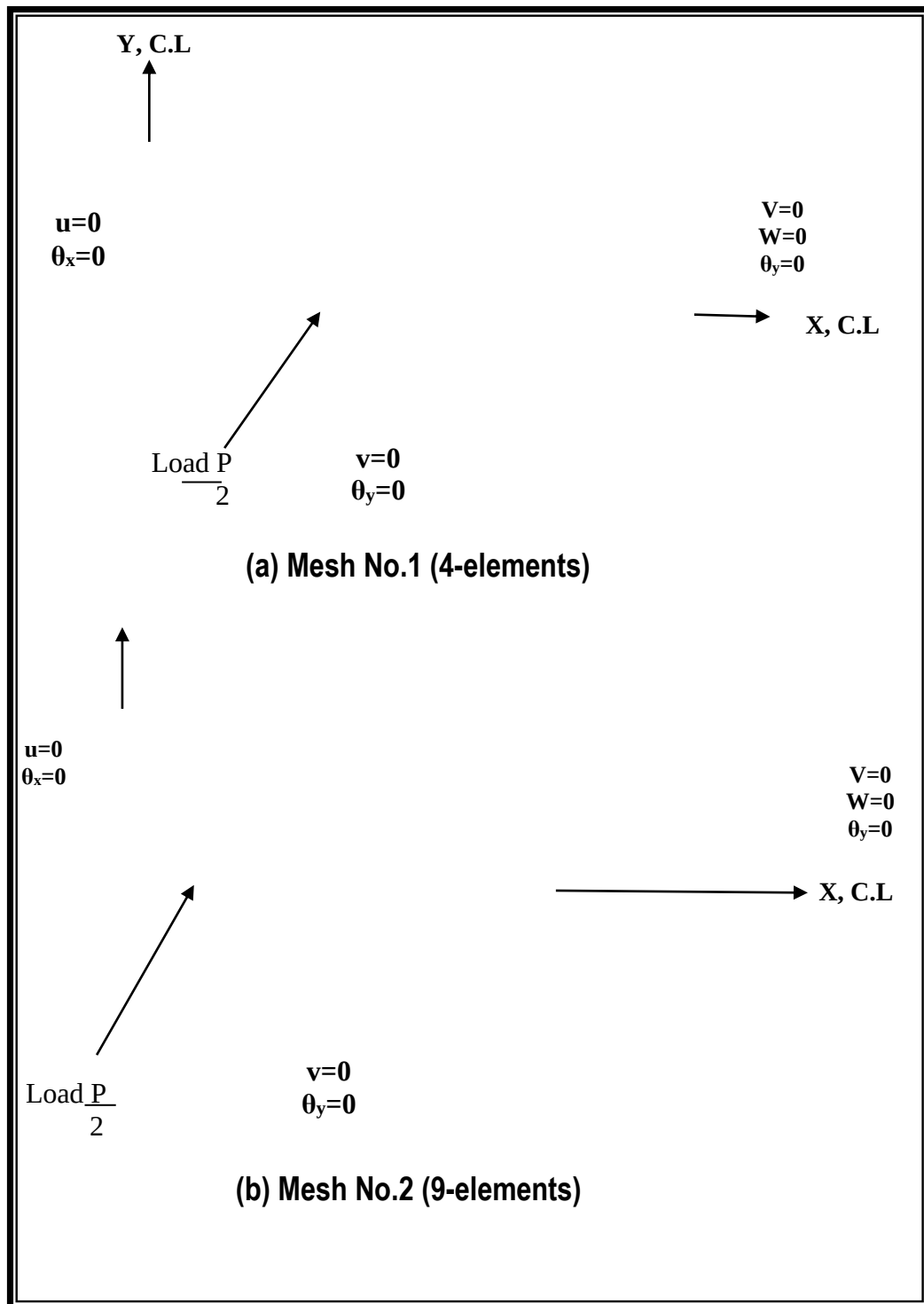


Fig. (6.6). Finite element mesh for One –way reinforced concrete slab

Fig. (6.7). Comparison of load-deflection curves for One-way slab

**Fig. (6.8 .a).Load-deflection curves at midspan of one- way
Simply supported slab**

The distribution of the concrete stress a long length quarter Of the one way reinforced concrete slab is shown in Fig. (6.8.b) at the concentrated load Of 3.84 kN. At the length direction, higher stresses are record near the application of concentrated load while the stress decreases at the support. The Fig (6.8, c) has shown the stresses through the thickness near the concentrated load.

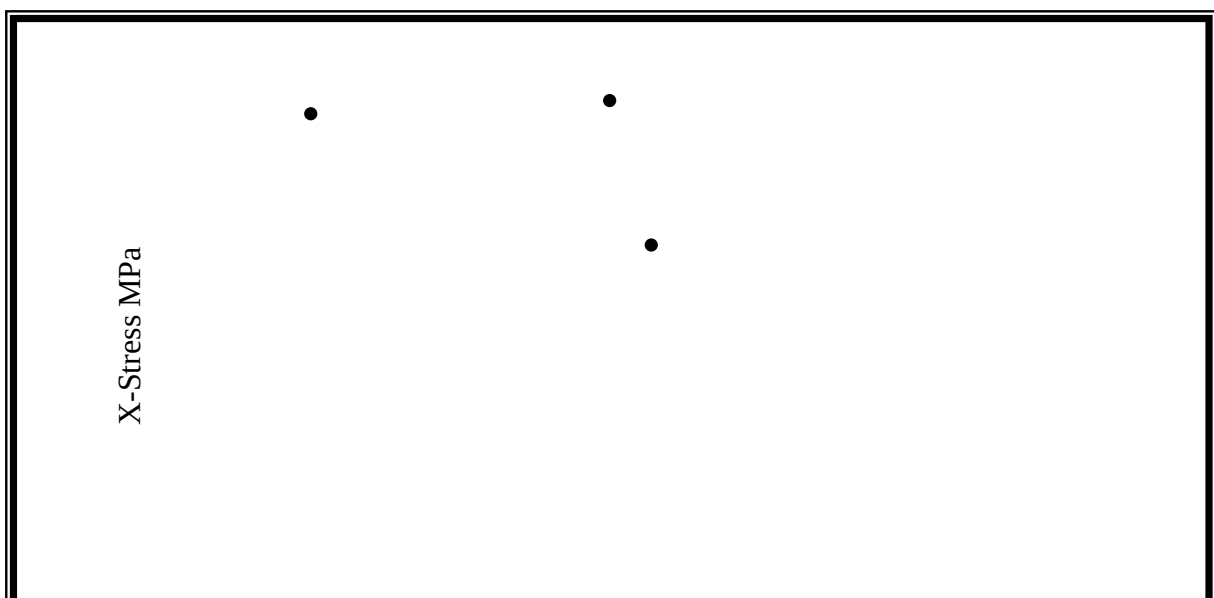


Fig. (6.8 .b) Distribution of stress along length of the quarter of the one way Reinforced concrete slab.

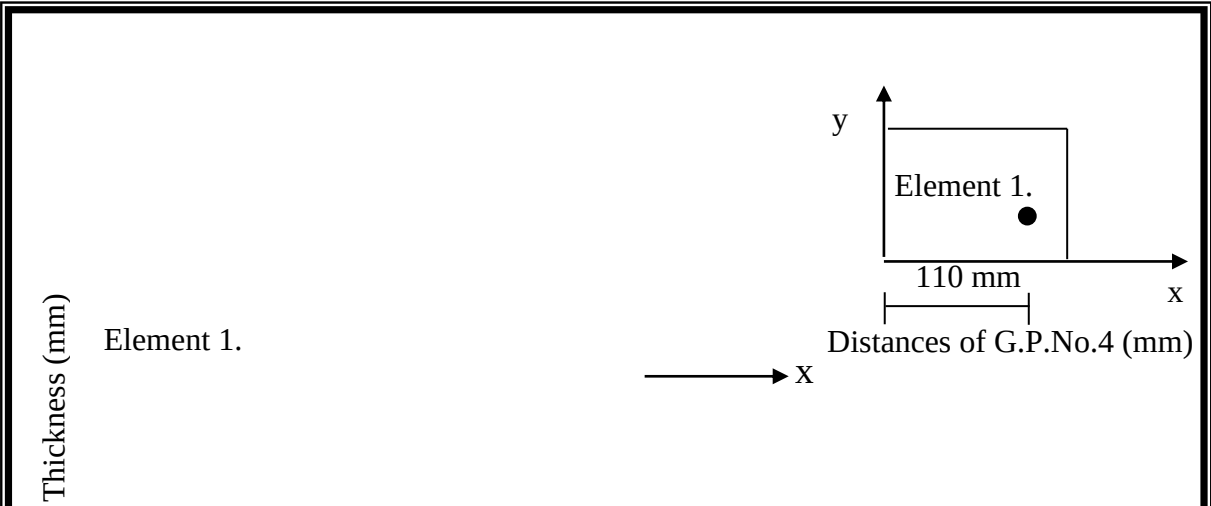
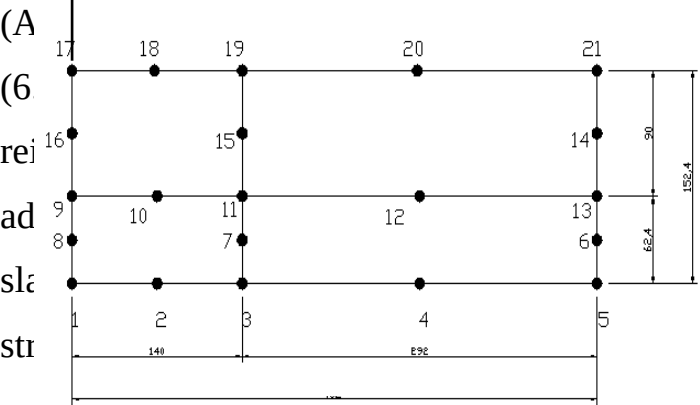


Fig. (6.8. c) Variation of stress at mid span of the quarter of the one way Reinforced concrete slab.

6.2.3 Simply Supported Square Slab

A simply supported square slab subjected to a uniformly distributed load is analyzed experimentally by Taylor et al. and theoretically by



properties are specified in Table and the arrangements of the element analysis, by taking consider one-quarter of the t is used for this quarter

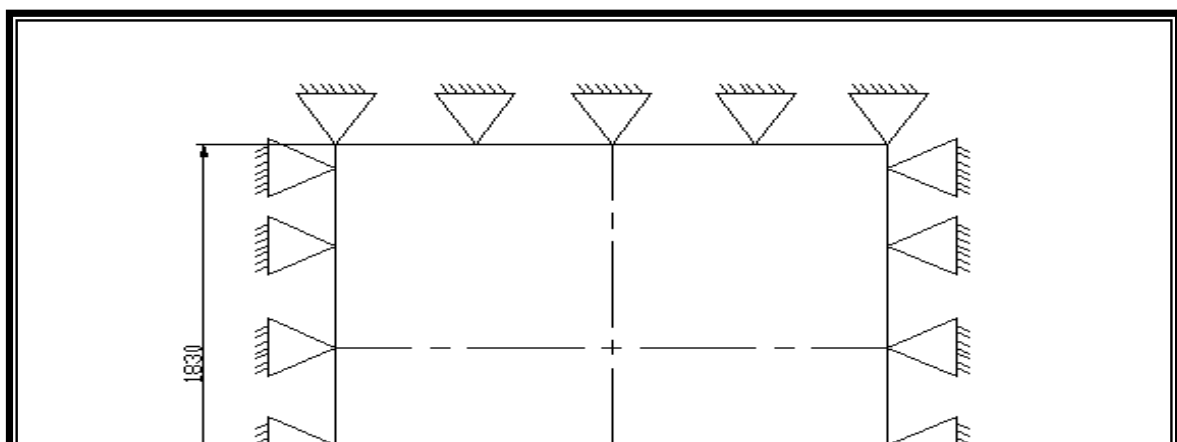
Slab considered in the

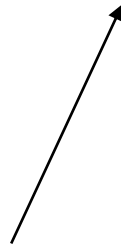
analysis

Material properties and parameters	Value
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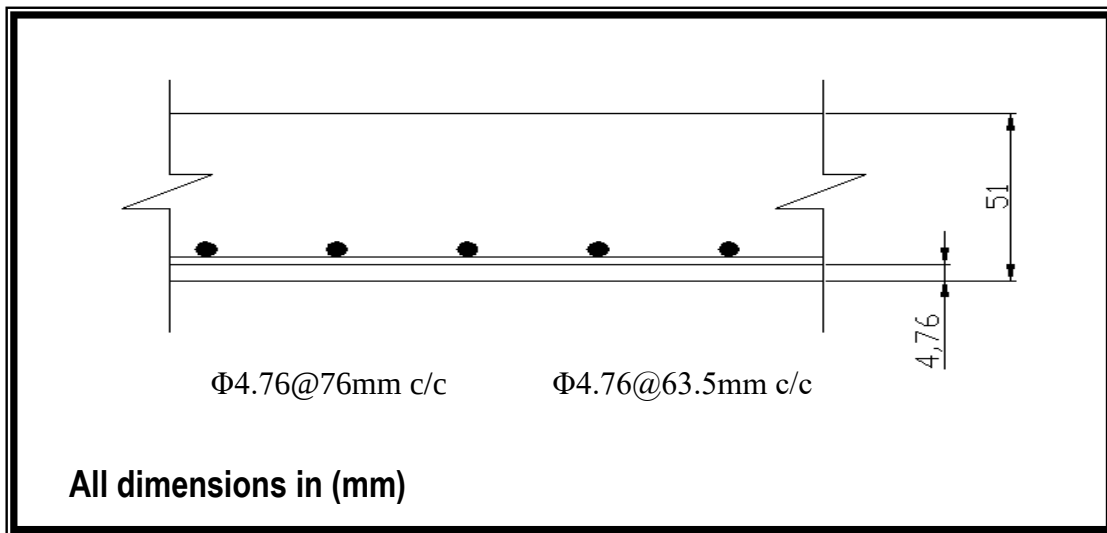
Concrete	Young's Modulus, E_c , MPa	30000
	Compressive Strength, f'_c , MPa	35.0
	Tensile Strength, f_t , MPa	3.0
	Poisson's Ratio, ν	0.15
	Uniaxial crushing strain ϵ_{cu}	0.0035
Steel	Young's Modulus, E_s , MPa	210000
	Yield Strength, f_y , MPa	375
Tension-Stiffening Parameters	α_m	0.60
	ϵ_m	0.002

At load of about 0.97 of the ultimate load, the numerical model recorded Crushing of gauss points at the bottom face of the central point. A number of previous theoretical results is plotted in Fig. (6.10), (Arnesen and Bergan 1980) employed a triangular shell element with numerical integration through thickness. They used end chronic theory for the concrete and a trilinear stress-strain law for the steel. Nonlinear geometry was included using an updated Lagrange approach. Cyclic loading was also considered. Crushed gauss points were initiated at a load of 1.007 of ultimate load. From the comparasion between the present model and previous methods, the finite element method can be proved as good representation to the overall behaviour and load carrying capacity of flat slab.

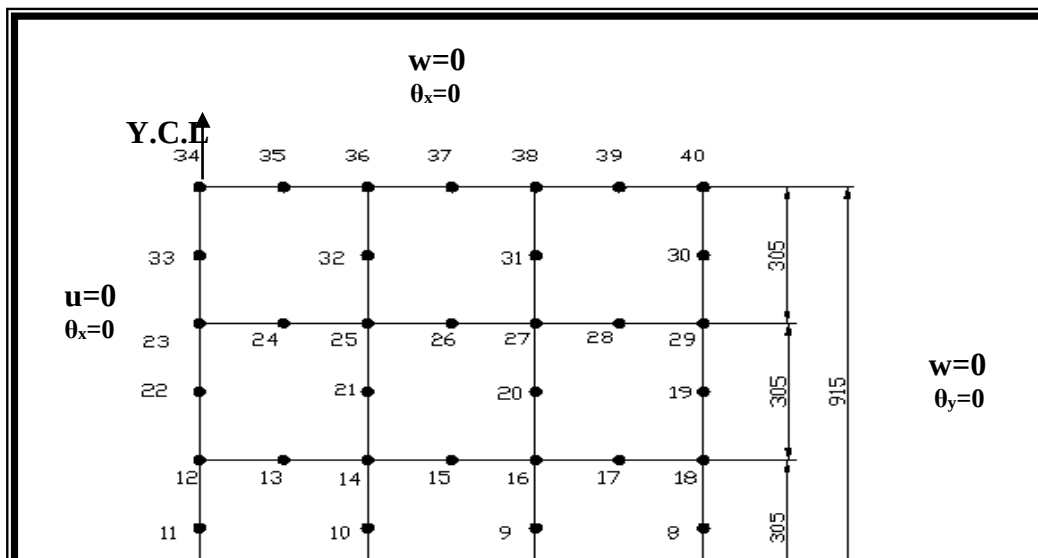




(a) Plate geometry and loading



(b) Steel arrangement



(c) Finite element mesh
Fig. (6.9). Simple supported square slab



Fig. (6.10). Central load-deflection curves for Simple supported square slab

6.2.4 Two-Way Reinforced Concrete Slab

An isotropically reinforced concrete square slab, simply supported at four corners was tested experimentally under central point load by (McNiece and Jofriet 1971). Several investigators proved their theoretical work by comparing their analytical results with that of two-way McNiece slab. Geometric details, reinforcement layout and finite element

idealizations are shown in Fig.(6.11). Taking the symmetry of loading and geometry, only a quarter of the slab is modelled by the finite element method. A mesh of four eight-node shell elements is used for this quarter structure is shown in Fig. (6.12)

The steel reinforcement is represented by two layers with a thickness equals to 0.378 mm each. One in the y-direction. Material properties of concrete and steel are the same given in Table (6.4).

In Fig.(6.12), the load-deflection curve of point (b) at the slab edge is also studied giving good results when compared with the experimental curve.

Table (6.4) Material Properties for McNiece *et al.* Slab considered in the analysis

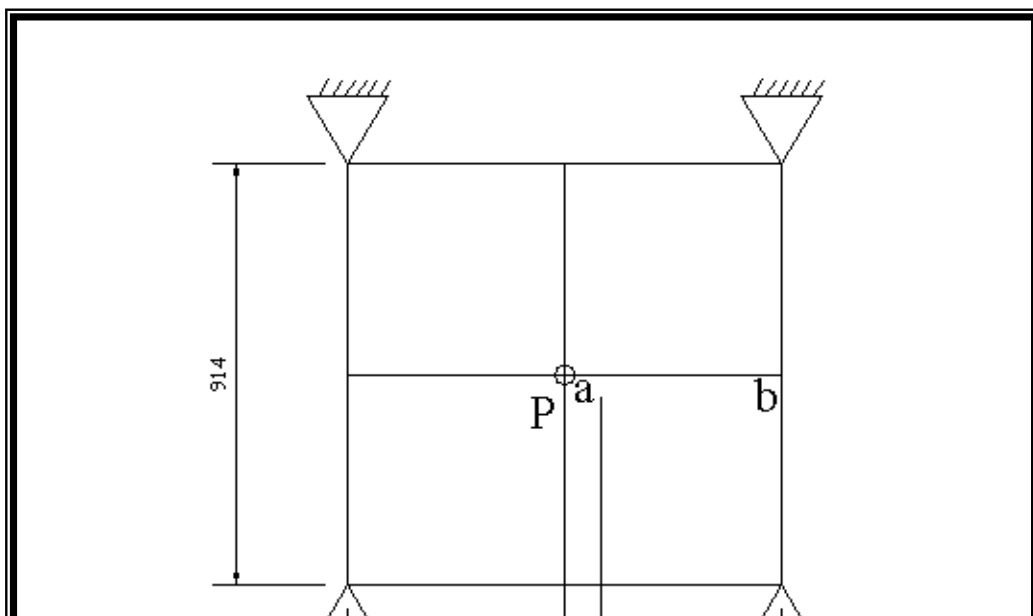
Material properties and parameters		Value
Concrete	Young's Modulus, E_c , MPa	31000.
	Compressive Strength, f'_c , MPa	35.0
	Tensile Strength, f_t , MPa	3.0
	Poisson's Ratio, ν	0.15
	Uniaxial crushing strain ϵ_{cu}	0.0035
Steel	Young's Modulus, E_s , MPa	210000
	Yield Strength, f_y , MPa	345
Tension-Stiffening Parameters	α_m	0.50
	ϵ_m	0.002

Crushed gauss points were initiated at a load of 1.016 of ultimate load.

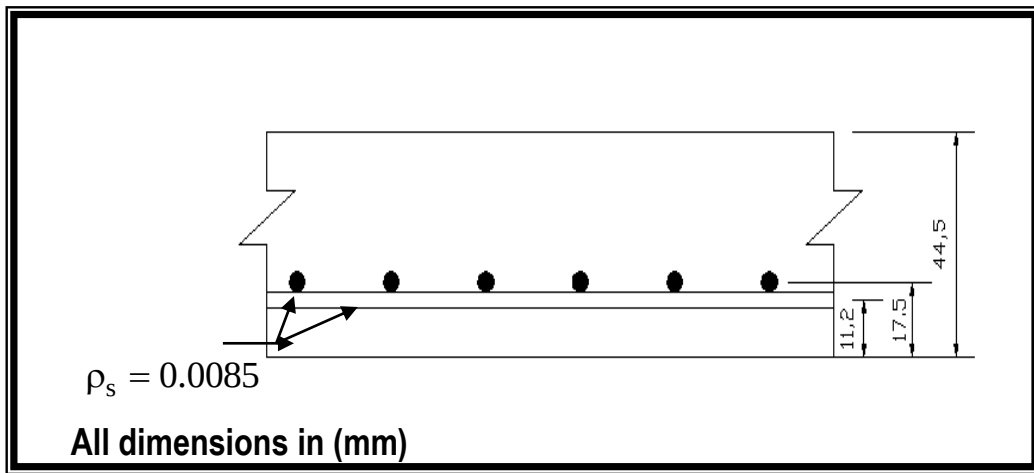
A number of previous theoretical results for point (a) are plotted in Fig. (6.13), (Al-Naimi 1996) studied the behaviour of three-dimensional reinforced concrete structures subjected to static and dynamic loadings. Nonlinear material and time dependent effects were included in the finite element analysis. Material nonlinearities as a result of tension cracking,

strain softening after cracking, the nonlinear response of concrete in compression, crushing of concrete and the yielding of reinforcement were considered. Concrete was considered as a time dependent material

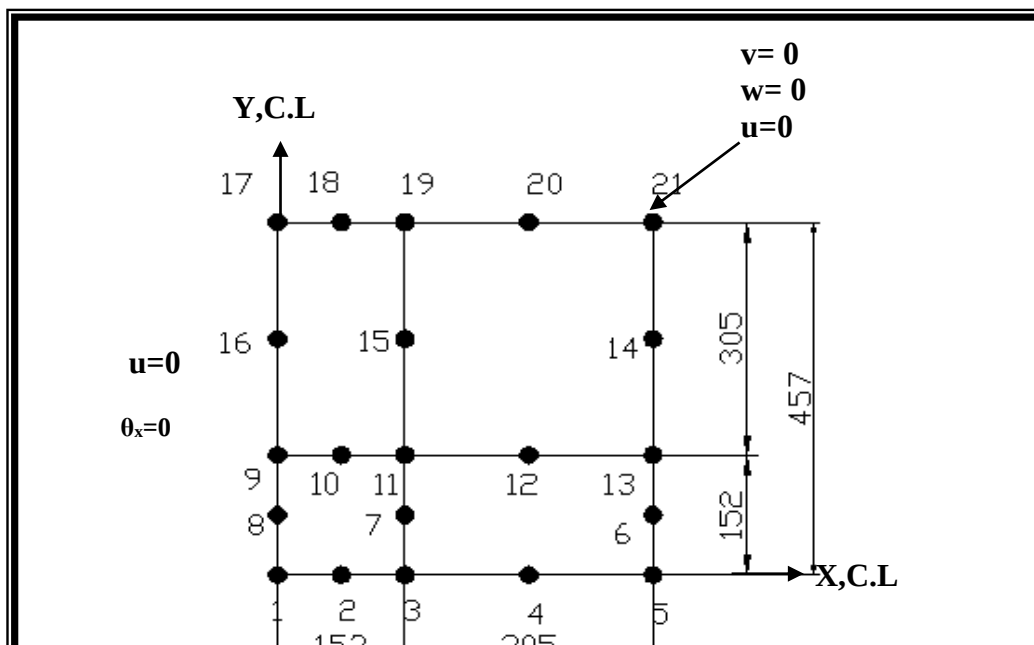
The three-dimensional computational model was adopted with eight and twenty hexahedral isoparametric element. In the idealization of the reinforced concrete structure, the steel reinforcement was incorporated in the concrete brick element by assuming perfect bond. Failure accruing at a load of 0.93 of ultimate load. (Al-Imam 1992) used a lyered mindlin flat shell 8-node isoparametric element for slab discretization. The reinforcement was treated as composite smeared layer. An element with 21-node was developed to model either concentric or eccentric edge beams. Perfect bond between the concrete and steel was assumed. The nonlinearities included were the cracking of concrete in tension, the nonlinear response of concrete in compression, concrete crushing, and yielding of the reinforcement. Failure accruing at a load of 0.98 of ultimate load. Together with the results of the present analysis for comparison purposes. The present model seems to provide good representation of deformation path.



All dimensions in (mm)



(b) Steel arrangement



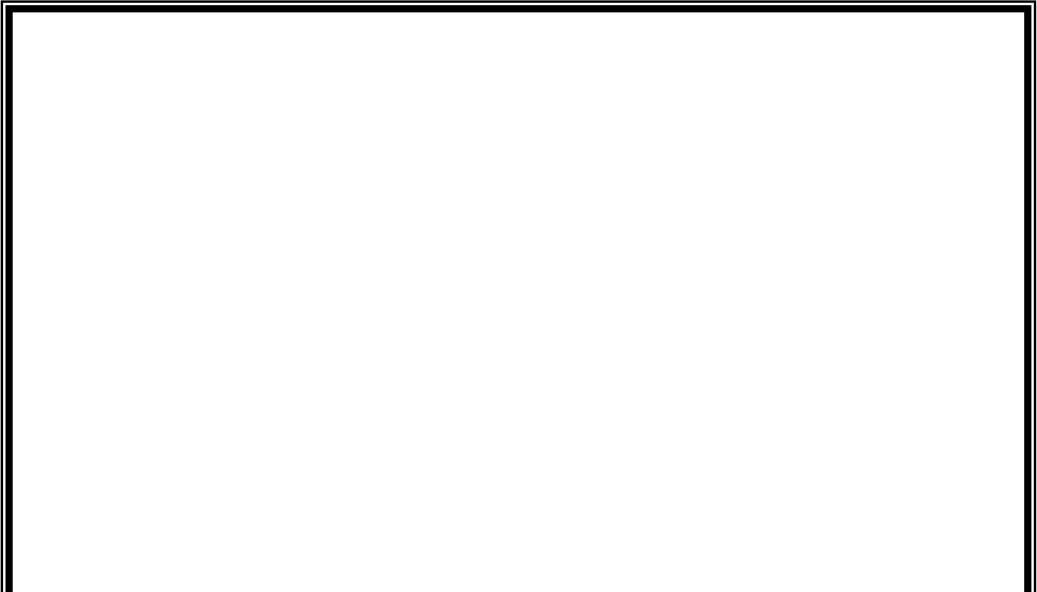


Fig.(6.13).Load-deflection curves at point (a) for two way reinforced concrete slab

6.2.5 Reinforced Concrete Cylindrical Shell

The reinforced concrete cylindrical shell, simply supported in the circumferential direction at the curved edges was tested experimentally under pressure load by Van Riel et.al. (1957) and theoretical by (Arnesen and Bergan 1980). Several investigators proved their theoretical work by comparing their analytical results with that of Van Riel shell. Geometric details, reinforcement layout and finite element idealization are shown in Fig.(6.14). Taking the symmetry of loading and geometry, only a quarter of the shell-beam system is modeled by the finite element method. A mesh of nine eight-node shell elements is used for this quarter structure is shown in Fig.(6.15). The number of element used in the solution gives the best result from another number.

The steel reinforcement for the shell is represented by four layers with a thickness equals to 0.04 mm each. While, the reinforcement for the beam is represented by a layer with thickness equals to 5.6 mm.. Material properties of concrete and steel are given in Table (6.5).

Table (6.5): Concrete and steel material properties for the shell.

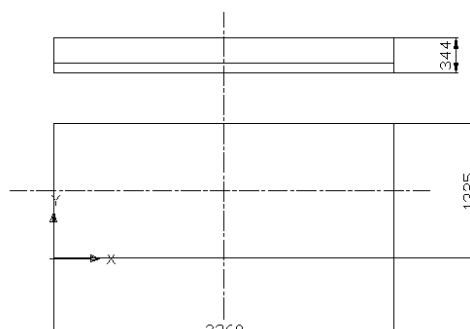
Material properties and parameters		Value
Concrete	Young's Modulus, E_c , MPa	30000.
	Compressive Strength, f_c' , MPa	30.0
	Tensile Strength, f_t , MPa	4.91
	Poisson's Ratio, ν	0.15
	Uniaxial crushing strain ϵ_{cu}	0.0035
Steel	Young's Modulus, E_s , MPa	210000
	Yield Strength, f_y , MPa	295
Tension-Stiffening Parameters	α_m	0.50
	ϵ_m	0.002

Crushed gauss points were initiated at a load of 0.978 of ultimate load. The presence of these crushed points at the top layer began when the compressive strain of the gauss point increased the ultimate crushing strain. When the gauss points are considered crushed, zero stresses and stiffness are assigned to them.

A number of previous theoretical results are plotted in Fig. (6.15), (Arnesen and Bergan 1980). employed a triangular shell element with numerical integration through thickness. They used end chronic theory for the concrete and a trilinear stress-strain law for the steel. Nonlinear geometry was included using an updated Lagrange approach. Cyclic loading was also considered. Crushed gauss points were initiated at a load of 0.91 of ultimate load. (Chan 1982), analyzed reinforced concrete shell finite element with edge beams using a layered curved shell finite element. He also used a filament line reinforced concrete beam element to

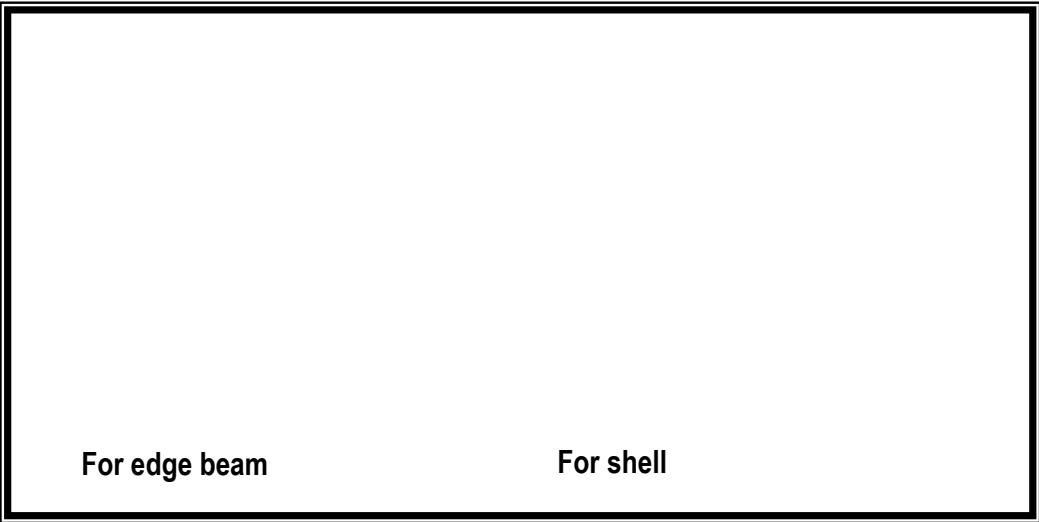
model the edge beams. Each filament was assumed to be in a uniaxial stress state, i.e., cracks in the beam filaments were formed perpendicular to the axis of the beam. Hence tensional cracking could not be modelled with such an assumption. The beam elements were connected to shell elements at discrete points by means of rigid links to model eccentric beams. Concrete was modelled as a nonlinear orthotropic hypo elastic material with biaxial state of stress in shell and uniaxial state of stress in beams. Steel was considered in uniaxial state of stress with a bilinear stress-strain model. Crushed gauss points were initiated at a load of 0.84 of ultimate load together with the results of the present analysis for comparison purposes. The present model seems to provide good representation of deformation path.

Cross section

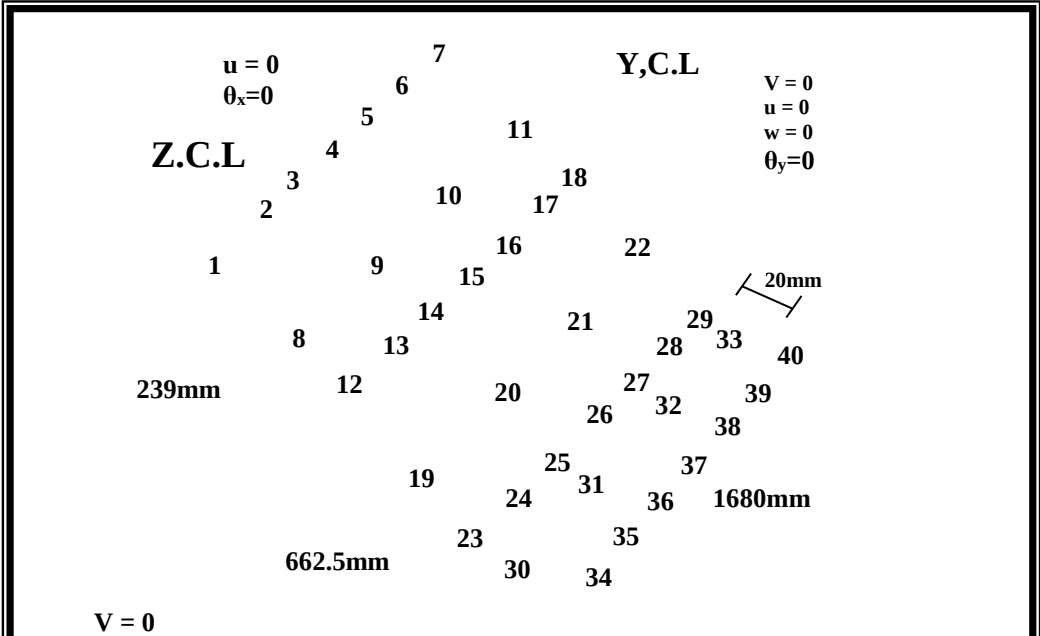


All dimensions in (mm)

(a) Shell geometry and loading



(b) Steel arrangement



X,C.L

(c) Finite element mesh
Fig. (6.14). Reinforced concrete shell

6.3 Optimal Design Application

6.3.1 One-Way Reinforced Concrete Slab

The optimal design of One –way reinforced concrete slab is investigated. The slab is exposed to concentrated load of 2.0 kN. Nine elements and six equal concrete and one steel layers through the thickness discredited a quarter of the slab. The design variable is thickness t . The initial values used are $t=0.0445$ and the step length is 0.01. The constraints

are and $0.0432 \leq t \leq 0.2055$. The material properties and dimensions are shown in Example (2).

The non – linear cost objective function (Z) involving the cost of steel reinforcement, concrete and formwork is used in the optimization problem of this study.

where :

Cost of steel =

$$C_s \left(\left(\frac{4}{3} \right) * A_s * \left(\frac{b}{5} \right) * L + \left(\frac{1}{3} \right) * A_s * L * \left(b - \left(\frac{2 * b}{5} \right) \right) + A_{s_{min}} * b * L \right) * w_s.$$

$$\text{Cost of concrete} = C_c (b * t * L)$$

$$\text{Cost of formwork} = C_f (b * L + 2 * L * t + 2 * b * t)$$

where:

C_s = unit price of steel reinforcement involving material and labour cost

A_s = Area of tension reinforcement

L = Length of slab

b = width of slab

t = Thickness of slab

w_s = Unit weight of steel

C_c = Unit price of concrete involving material and labour cost

C_f = Unit price of formwork

$C_s = 800$ units/ton, $C_c = 400$ units/m³, $C_f = 10$ units/m².

Table (6.6) Optimization results

Cycle No.	Thickness (mm)	Main Reinforcement (mm ²)	Total Cost (Units)
1	145.5	116	21.9
2	135.5	127	20.1

3	125.5	142	18.7
4	111.5	159	16.2
5	105.5	183	15.1
6	95.5	201	13.2
7	85.5	250	10.2
8	75.5	310	9.4
9	55.5	435	9.27
10	43.2	465	9.2

Fig.(6.16) .Variation of Total Cost with No. of Analysis

The result is presented in Tables (6.6) and Figure (6.16). In Fig (6.16), the variation of total cost with the number of optimization cycles is presented for the slab. It can be seen from these figure that the total cost is reduced when the thickness is reduced. The minimum thickness gives the minimum cost. In the example we ignore the coupling between bending moment, shear and axial forces. The shear force and deflection were in the allowable limited.

6.3.2 Simply supported squar Slab

The optimal design of Simple supported square slab is investigated. The slab is exposed to uniform distributed load including self weight. Nine elements and eight equal concrete and two steel layers through the thickness discredited a quarter of the slab. The design variable is thickness t . The initial values used are $t=0.051$ and the step length is

0.01. The Constraints are. $0.0406 \leq t \leq 0.081$, the material properties and the dimension of the slab are shown in Example (3).

The non – linear cost objective function (Z) involving the cost of steel reinforcement, concrete and formwork is used in the optimization problem of this study.

where:

$$\text{Cost of Steel} = C_s ((4/3)A_{s1} * b * (L/5) + (1/3) * A_{s1} * b * (L - (2L/5)) + (4/3)A_{s2} * L * (b/5) + (1/3) * A_{s2} * L * (b - 2b/5))) * w_s$$

$A_{s1} \neq A_{s2}$ because the steel reinforcement in the Example of analysis is not equal.

$$\text{Cost of concrete} = C_c (b * t * L)$$

$$\text{Cost of formwork} = C_f (b * L + 2 * L * t + 2 * b * t)$$

where:

C_s = Unit price of steel reinforcement involving material and labour cost

A_s = Area of tension reinforcement

L = Length of slab

b = width of slab

t = Thickness of slab

w_s = unit weight of steel

C_c = Unit price of concrete involving material and labour cost

C_f = Unit price of formwork

$C_s = 800$ unites/ton, $C_c = 400$ unites/m³, $C_f = 10$ unites/m².

Fig.(6.17) .Variation of Total Cost with No. of Analysis

Fig (6.17) Shown the variation of total cost with the number of optimization cycles is presented for the slab. It can be seen from these figure that the total cost is reduced when the thickness is reduced .The minimum thickness gives the minimum cost. In the example we ignore the coupling between bending moment, shear and axial forces. The shear force and deflection were in the allowable limited.

6.3.3 Reinforced Concrete Cylindrical Shell

The optimal design of reinforced concrete cylindrical shell is investigated. The shell is exposed to uniform distributed load including self weight. Nine elements and eight equal concrete and five steel layers through the thickness discretized a quarter of the shell. The design variable is thickness t and shell angle. The Initial values used are $t=0.005$ and the step length is 0.005. The Constraints are, $0.005 \leq t \leq 0.02$ and $35^\circ \leq \beta \leq 45^\circ$ The material properties and the dimension of the slab are shown in Example (5).

The non – linear cost objective function (Z) involving the cost of steel reinforcement, concrete and formwork is used in the optimization problem of this study.

where :

Cost of steel=

$$C_s (2A_{s1} * \beta * R * L + 2A_{s2} * \beta * R * L + 2A_{s3} * \beta * R * L + 2A_{s4} * \beta * R * L) * w_s$$

$$\text{Cost of concrete} = C_c (2\beta * R * t * L)$$

$$\text{Cost of formwork} = C_f (2\beta * R * L + 4\beta * R * t + 4t * L)$$

where:

C_s = Unit price of steel reinforcement involving material and labour cost

A_s = Area of reinforcement

L = Length of shell

R = Radius of shell

t = Thickness of shell

β = The shell angle

w_s = Unit weight of steel

C_c = Unit price of concrete involving material and labour cost

C_f = Unit price of formwork

C_s = 800 units/ton, C_c = 400 units/m³, C_f = 10 units/m².

Fig. (6.18): Variation of Total Cost with No. of Analysis

The figure (6.18) shows the variation of total cost with the number of optimization cycles is presented for the shell. It can be seen from these figures that the total cost is reduced when the thickness, curvature of the

shell increases and the shell angle decreases. In the example we ignore the coupling between bending moment, shear and axial forces.

6.3.4 Two Way Reinforced Concrete Slab

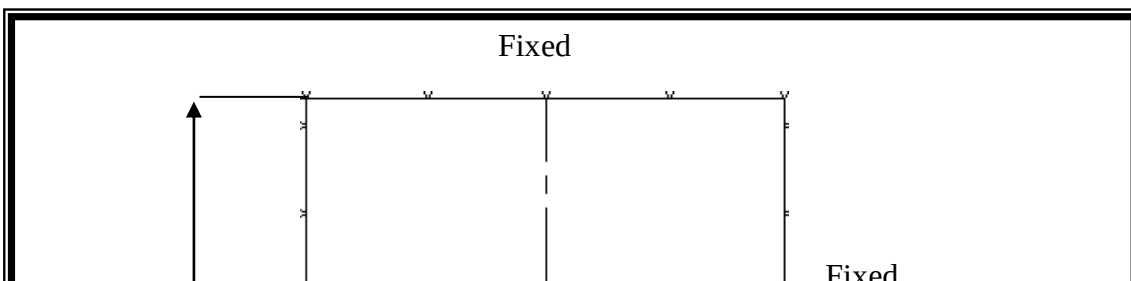
The optimal design of two way reinforced concrete slab is investigated. The slab is exposed to uniform distributed load. Nine elements and eight equal concrete and four steel layers through the thickness discretized a quarter of the slab. Geometric details, reinforcement layout and finite element idealizations are shown in Fig. (6.19). Taking the symmetry of loading and geometry, only a quarter of the slab is modelled by the finite element method.

The design variable is thickness t . The initial values used are $t=0.14\text{m}$ and the step length is 0.01. The Constraints are. $0.10 \leq t \leq 0.18$, the material properties and the dimension of the slab are shown:

Live load = 5 kN/m^2 , super imposed dead load = 0.8 kN/m^2 , diameter of bar = 12mm .

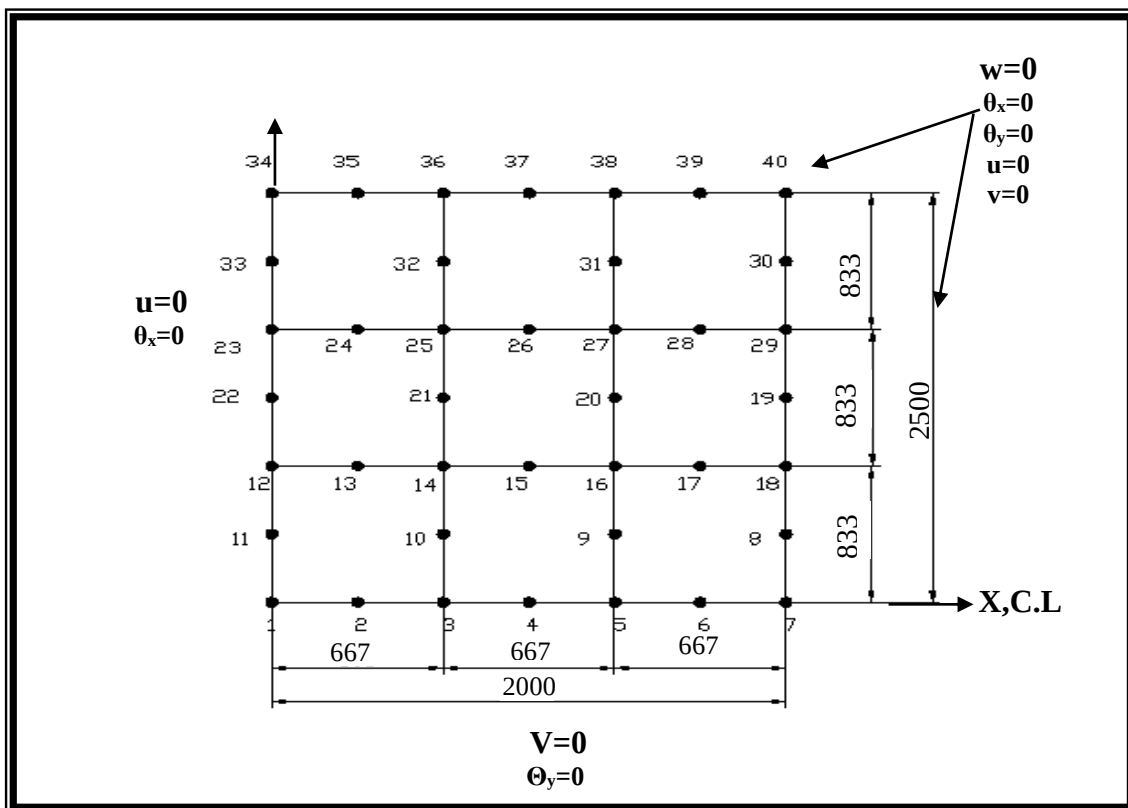
Table (6.7) Material Properties considered in the analysis

Material properties and parameters		Value
Concrete	Young's Modulus, E_c , MPa	21538.00
	Compressive Strength, f'_c , MPa	21.0
	Tensile Strength, f_t , MPa	2.0
	Poisson's Ratio, ν	0.15
	Uniaxial crushing strain ϵ_{cu}	0.0035
Steel	Young's Modulus, E_s , MPa	201000
	Yield Strength, f_y , MPa	350
Tension-Stiffening Parameters	α_m	0.70
	ϵ_m	0.002

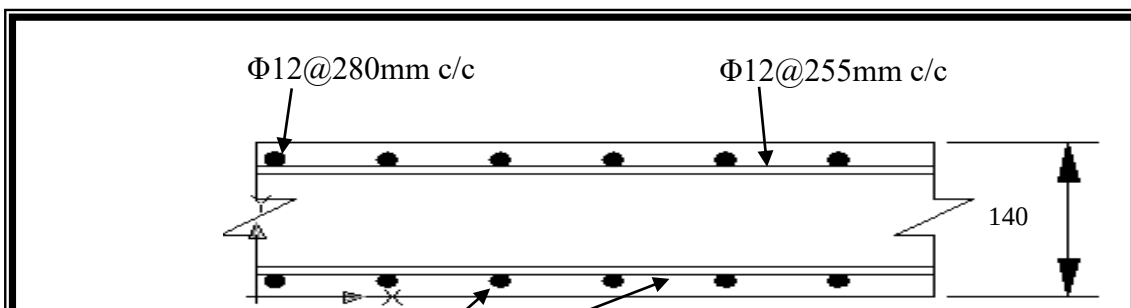


All dimensions in (mm) Uniform Distributed Load

(a) Plate geometry and loading



(b) Finite element mesh



(c) Steel arrangement
Fig. (6.19). Two Way Reinforced Concrete Slab

The non – linear cost objective function (Z) involving the cost of steel reinforcement, concrete and formwork is used in the optimization problem of this study.

where :

Cost of steel =

$$C_s (A_{snw} * (\frac{b}{2}) * L + A_{spw} * b * L + A_{snl} * b * L + (A_{spl} * b * L).w_s$$

Cost of concrete= $C_c (b*t*L)$

Cost of formwork = $C_f (b * L + 2 * L * t + 2 * b * t)$

where :

C_s = unit price of steel reinforcement involving material and labour cost

A_s = area reinforcement

L = length slab

b = width of slab

t = thickness of slab

w_s = unit weight of steel

C_c = unit price of concrete involving material and labour cost

C_f = unit price of formwork

$C_s=1000$ unites/ton, $C_c=400$ unites/m³, $C_f=10$ unites/m².



Fig.(6.20) .Variation of Total Cost with No. of Analysis

Fig (6.20) Shown the variation of total cost with the number of optimization cycles is presented for the slab. It can be seen from these

figure that the total cost is reduced when the thickness is reduced .The minimum thickness gives the minimum cost. In the example we ignore the coupling between bending moment, shear and axial forces.

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