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College of Engineering



***ELASTIC-PLASTIC LARGE DEFORMATION
DYNAMIC ANALYSIS OF STEEL PLANE
TRUSSES BY THE METHOD OF BOUNDARY
CHARACTERISTICS***

A thesis

**Submitted to the College of Engineering of the University of
Babylon in Partial Fulfillment of the Requirements for the
Degree of Master of Science in Civil Engineering
(Structures)**

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جمهورية العراق
وزارة التعليم العالي والبحث العلمي
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التحليل الديناميكي المرن -اللدن للمسلمات الحديدية المستوية ذات التشوهات الكبيرة باستخدام طريقة الخواص الحديدية

رسالة

مقدمة إلى كلية الهندسة في جامعة بابل كجزء من متطلبات نيل درجة
الماجستير في علوم الهندسة المدنية (إنشاءات)

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

وَاللَّهُ أَكْبَرُ

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آية من سورة النساء

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

الخلاصة

تم تحري الاستجابة الديناميكية للمسلمات والأعضاء الأحادية البعد في هذا البحث باستعمال طريقة الخواص (Method of Characteristics). وتم تخمين السلوك الخطي (التشوه الصغير) والسلوك اللاخطي (اللاخطية الهندسية والمادية) للأعضاء الأحادية البعد، والجلونات. وكذلك تم اعتماد علاقة الإجهاد والانفعال الثنائية (Bilinear stress-strain relationship) في تنبؤ السلوك اللاخطي للمادة (التحليل المرن-اللدن).

تمت دراسة شروط الحد المختلفة و تنبؤ السلوك لكل واحد منها. وقد أجريت بعض المقارنات لتأكيد صحة التحليل.

تم استعمال برامج الحاسوب (MNLDAPT،GNLDAPT،LDAPT،LDAPM) لتحري السلوك الخطي (التشوه الصغير) و السلوك اللاخطي الهندسي والمادي للأعضاء أحادية البعد و المسلمات، عدة أمثلة حُلّت، و قُورنت بَعْضها بالبحوث المتوفرة، باستعمال طرق مختلفة للحل.

تم اختبار و توضيح تأثير عدة عوامل على الأعضاء الأحادية البعد والمسلمات. من النتائج التي تم الحصول عليها، وُجِدَ بان الشكل الهندسي و أنواع النهايات و خواص مادة العضو بالإضافة إلى اتجاه وشدة الحمل لها تأثيرات كبيرة على استجابة الأعضاء المحملة محورياً و المسلمات. وُجِدَ بأن تأثير التشوهات الكبيرة، والسلوك اللاخطي للمادة له تأثير متغير على السلوك الديناميكي للأعضاء الأحادية البعد والمسلمات، لذا، يجب أن لا يُهمل تأثيرهما. إنَّ الجلونات في طبيعتها مرنة الربط، هذا يؤدي إلى نظام اهتزاز معقد، حيث تتذبذب الأعضاء الداخلية بشكل منفرد و بتردد طبيعي معين، بينما يتذبذب الجلون بأكمله في نظام مختلف. يغير السلوك اللاخطي الهندسي والمادي فترة الوقت الطبيعية (natural time period) للجلون، في النتيجة، التردد الطبيعي للجلون سيتفاوت. أيضاً، تحليل التشوه الصغير لا يعطي نتائج مقبولة عندما يتوقع بان هناك مستويات تحميل عالية ، لأنه يُهمل تأثيرات اللاخطية الهندسية وتأثيرات اللدونة.

ABSTRACT

The dynamic response of truss structures and one-dimensional members are being investigated in this research by using The Method of Characteristics. The linear behavior (small deformation) and the non-linear behavior (geometrical and material non-linearity) for one-dimensional members and trusses are predicted. The bilinear stress-strain relationship has been adopted in the material of the non-linear behavior prediction (elastic-plastic analysis).

Different boundary conditions have been studied and the behavior for each one has been predicted. Some comparisons have been done to validate the analysis.

The computer programs (LDAPM, LDAPT, GNLDAPT, MNLDAPT) are used to investigate the linear (small deformation) of prismatic members and trusses, geometrical and material non-linear behavior of trusses respectively. Several examples have been analyzed, some examples have been compared with available researches by using different methods of solution.

The effect of several parameters on the one-dimensional members and trusses have been tested and explained. From the results obtained, it was found that the geometrical shape, boundary conditions, and material's properties of the member as well as the load direction and intensity have considerable effects on the response of axially loaded members and truss structures. It was found that the effect of large deformation and the material non-linear behavior has variable influence on the dynamic behavior of both one-dimensional members and the truss structures, so, it must not be neglected. The truss structure in its nature is flexibly connected. This leads to a complex vibration system, the local members vibrate individually in a certain natural frequency, while the whole truss structure vibrates in a different history. The geometrical and material non-linear behavior vary the natural period of the truss structure, in the result the natural frequency of the structure will vary. Also, the small deformation analysis does not give an acceptable result when high loading conditions are expected, because it neglects effects of geometrical non-linearity and effects of plasticity.

DEDICATION

TO MY LOVELY COUNTRY

TO MY PARENTS

TO MY HUSBAND

TO MY BROTHERS AND SISTERS

TO MY DAUGHTER (AFNAN)



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"In the name of **Allah**, most Gracious, most merciful"

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Shereen Kassim Abd-Alridha

September, 2007

CERTIFICATION

We certify that this thesis entitled "**Elastic-Plastic Large Deformation Dynamic Analysis of Trusses by The Method of Boundary Characteristics**" was prepared by **Shereen Kassim Abd–Alridha** under our supervision at the University of Babylon in partial fulfillment of the requirements for the degree of **Master of Science in Civil Engineering**.

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NOTATIONS

<i>Symbol</i>	<i>Definition</i>	<i>Units</i>
A	Cross-sectional area at node	m ²
C	Elastic axial wave speed	m/s
[C_e]	Characteristics coefficient matrix in local coordinates (characteristics member matrix)	-
[C_J]	Characteristics coefficient matrix in global coordinates (characteristics joint matrix)	-
C_m	Damping coefficient	N.s/m
D.I.F	Dynamic Intensification Factor	-
E	Modulus of elasticity	Pa
F	Axial force	N
{F}	Internal forces in global coordinates at new time	-
{F'}	Internal forces in local coordinates at new time	-
{f}	Forces coefficient in global coordinates at old time	-
{f'}	Forces coefficient in local coordinates at old time	-
GL	Grid length	m
I	Second moment of area at node	m ⁴
K	Spring stiffness	N/m
L	Length of the member	m
LT	Loading time	sec
MOC	Method of Characteristics	-
ms	Millisecond	-

<i>Symbol</i>	<i>Definition</i>	<i>Units</i>
NG	Number of grids per meter	-
$\{P_{(t)}\}$	External loads	N
SSPJ	Single Step Programming of Joints	-
t	Time	sec
[T]	Transformation matrix	-
T_i	Time of integration	sec
T_n	Fundamental Period	sec
U	Axial velocity	m/s
U_m	Maximum axial velocity	m/s
u	Axial displacement	m
$\ddot{u}$	Axial acceleration	m/s ²
x,y	Cartesian axes	m
Δt	Time incremental	sec
μs	Microsecond	-
ρ	Mass density	kg/m ³
ϵ	Axial strain	m/m
σ	Axial stress	N/m ²
σ_y	Elastic yield limit	N/m ²
ω_n	Natural Frequency	Rad/sec

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CHAPTER ONE

INTRODUCTION

1.1: General:

Truss structures are very practical due to their lightness, easiness in manufacturing, possibility of covering large open areas. They are suitable also for building folding/unfolding systems. These structures are found in power transmission towers, spacecrafts and bridges. Figure (1-1) shows samples of the truss structures.

In such applications, plane trusses would certainly be subject to dynamic loads and all that they entail of vibrations and the effect on internal member forces.

The response of a structure subjected to dynamic load, especially impact load is much more complex than that of static load⁽⁷⁷⁾.

A dynamic load is one whose magnitude, or point of application varies with time. The resulting time varying displacements and stresses contribute to the dynamic response. If the loading is known, the loading is said to be prescribed and the analysis is called a deterministic analysis. If the time history of the loading is not completely known, but is known in a statistical sense, the loading is said to be random.

A structural dynamic problem differs from the corresponding static problem in two important aspects:

1. Time varying nature of the excitation.
2. Effect of acceleration and velocity in dynamic problems (present of inertia and damping forces).

There are generally two approaches for dynamic analysis of trusses. One is to treat the structure as a distributive system of multi-connected bars or beams which

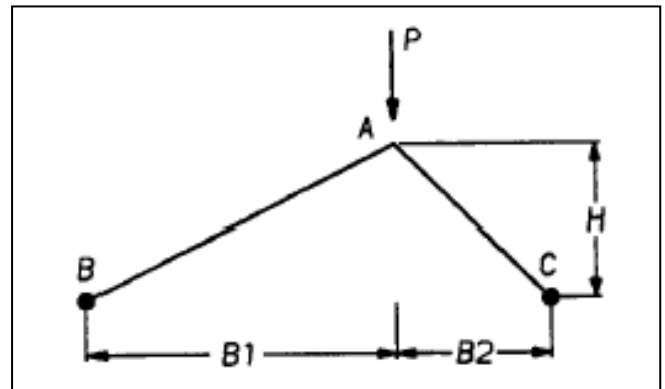


Figure (1-1): Samples of the Truss Structures

can be solved by, for instance, transfer matrix method, direct stiffness method, compliance method etc. The other one is to divide every member of the structure into a number of elements, for example Finite Element Method⁽⁷⁷⁾. However, the transcendental function appeared in the matrix of the former approach often results in difficulty of solution, while the latter needs numerous elements to get relatively accurate result. It is likely applicable for low frequency and long term response⁽²¹⁾.

An alternative approach is using wave propagation theory to obtain the response of the structure. Recently based on the theory of wave propagation proposed The Method of Characteristics. The present study extends The Method of Characteristics to investigate the dynamic response of trusses, including linear and non-linear behaviors.

1.2: Dynamic Analysis:

Structural safety cannot be guaranteed by static analysis alone and account must be taken of dynamic loads due to, for example, winds, earthquakes and impacts.

In dynamic analysis, attention is commonly focused on forces and acceleration in substitute systems, see for example, Clough & Penzien (1993)⁽²¹⁾. Attention is usually focused on the motion of masses rather than on the propagation of waves in structural members. In practice they present computational difficulties, particularly for non-linear systems and for complex structures with multiple degree of freedom. Also these methods may be applicable only to loads with slowly varying amplitudes⁽⁴⁾.

The dynamic behavior of a structure depends on the material properties, the geometry and the nature of the applied loads. The rate of change of a load also influences the structural behavior. In the case of very slowly varying loads, the condition of the structure at any instant closely approximates to static equilibrium and so the dynamic behavior is virtually the same as a series of equilibrium states. When the load changes more rapidly, however, inertial effects become significant, and the instantaneous structural states can differ markedly from static equilibrium.

This is notably the case for many types of periodic loads, and care must be taken to avoid exciting structures with cyclic loads then can induce resonant behavior of the whole structure or of individual components.

With even more rapid load changes (impacts for instance) it is necessary to consider local displacements of individual members. Account must be taken of the detailed behavior of individual waves as they propagate through the structure.

The well known lumped mass and distributed mass method is commonly used for the intermediate rates of change of load that have had greatest attention over the years. The method is extremely powerful and can be used to simulate most types of dynamic behaviors. It is not particularly well suited, however, to the simulation of sudden load changes. For this purpose, wave propagation methods are useful

because they enable the passage of discrete waves to be represented in a simple manner⁽¹⁸⁾.

1.2.1: Linear Dynamic Analysis:

Most linear dynamic analysis in engineering are applied to idealized non-linear systems⁽⁵⁶⁾. When the non-linearity is small, the idealization is justified because it enables the system to be described with sufficient accuracy and relative ease. Strictly, however, linear analysis apply only to structures undergoing small deflection and having linear geometrical characteristics and linear material properties.

The equation of motion for a typical mass can be written according to many concepts such as Hamilton's principals:

$$[M] \left\{ \frac{\partial^2 \delta}{\partial t^2} \right\} + [C] \left\{ \frac{\partial \delta}{\partial t} \right\} + [K] \{ \delta \} = \{ P(t) \} \quad 1.1$$

where : δ : the dynamic component of the displacement vector .

$[M]$: the diagonal mass matrix.

$[C]$: the damping matrix.

$[K]$:the stiffness matrix.

$\{ P(t) \}$:set of time-dependent loads.

1.2.2:Non-Linear Dynamic Analysis:

Neglecting the effects of non-linearity may lead to significant errors in predicting the dynamic behavior of system. For example **jump phenomena** and **sub** or **super harmonic resonance** are found only in non-linear systems⁽⁵⁾.

Several approximate methods are available for the solution of non-linear, differential equations of motion, Nayfeh (1973)⁽⁴⁹⁾, such as :-

$$[M] \left\{ \frac{\partial^2 \delta}{\partial t^2} \right\} + [C] \left\{ \frac{\partial \delta}{\partial t} \right\} + [k] \{\delta\} = f(\{\delta\}, \left\{ \frac{\partial \delta}{\partial t} \right\}) + \{P(t)\} \quad 1.2$$

where f is a non-linear function of $\{\delta\}$ and $\left\{ \frac{\partial \delta}{\partial t} \right\}$. Solutions of the above equation are frequently sought in two different forms. In the first form, the dynamic non-linearity is examined in terms of generalized stability criteria. The second form of solution favours the determination of the steady-state amplitude of vibration, when it exists. When it does not exist, then quantitative as well as qualitative information concerning the expected response is explored through a time history approach.

Some of the approximate methods deal with step function of excitation. These include graphical methods (Jacobson, 1952), Linear approximations (Ergin, 1956) and Ultra spherical Polynomials (Anderson 1974 and Sinha, 1971). Others deal with harmonic excitation– for example the Krylov–BoGoliubov–Mitropolsky (KBM) method and multiple scale and perturbation methods (Nayfeh , 1981), (cited in Al-Sarraj. A. T. (1989)⁽¹⁰⁾.

1.3: Wave Propagation:

When stress waves spread out from the disturbance point (loading point) it will induce the behavior of this structural element. This behavior is a subject of great interest in the structural dynamic analysis. So, When forces are applied to an elastic body under impact or impulse load, the response should be considered in terms of wave propagation theory. During the last three decades the revival of interest in elastic wave propagation has been possible because of the rapid development of computing facilities.

1.4: Method of Analysis of (One–Dimensional) Elastic Members :

In elastic analysis of elements, deformations disappear with the removal of the load. This means that the material properties of the member will not change. The most common methods used in static analysis are the **Flexibility Method** and the **Stiffness Method**. The size of the simultaneous equations used in these methods depends on the degree of indeterminacy of the structure.

For dynamic analysis, **Mass Spring Method** is used to simulate the vibration of structures such as : multistory, multi–bay, sway framed buildings. In this method any system is represented by masses and springs, i.e. the system is replaced with an equivalent system which consists of masses and springs. Damping effect is related to velocity, while the applied forces are idealized to simplify the solution. Refer to Paz (1985)⁽⁵³⁾, this approach enables the solution to be independent of the number of bays in particular storey.

The solution of the equations of motion becomes difficult to handle when structures have many degrees of freedom. This method does not usually take the wave action into account.

The other method, which is the **Finite Element Method (FEM)**– see for instance Weaver and Johnston (1987)⁽⁷¹⁾, combines certain features of both lumped mass and generalized coordinate procedures. The beginning of this method was as a method of static analysis related to the direct stiffness method. In vibration analysis, the method treats every element in the structure as a part of the continuous structural member. By requiring that displacements are compatible and the internal forces are in balance at nodes shared by two or more elements, the entire structure is compelled to act as one entity.

The motion of any point inside each element is expressed in terms of the displacements at nodal points using an interpolation function involving low or high degree polynomials depending upon the degree of accuracy required. The accuracy can be improved by refining the mesh–i.e. by increasing the degree of polynomial

interpolation function. The motion of the overall system is specified by the nodal displacements. The equations of motion for the complete system can be obtained by an assembling process that expresses the kinetic and potential energies and the virtual work as the sum of the contributions of the individual elements. The resulting simultaneous, differential equations of motion may be solved either numerically or analytically. In practice, with complex structures, only numerical solutions are practicable.

1.5: Method of Analysis of Geometrical Non-Linearity :

Geometrical non-linearity in members is due to the change in the geometrical properties of the member such as cross sectional area (A), and length (L).

The analysis of non-linear structure can not be estimated by considering linear methods. Therefore, methods for the non-linear analysis are required to find the solution of the non-linear system. There is no exact method to solve the non-linear equation. Numerical methods, such as **Linear Acceleration Method** are used to solve such cases, see Paz (1985)⁽⁵³⁾. In this method, stiffness is required at each time step.

The Method of Characteristics (MOC) can be used to analyze both static and dynamic behaviors. This method takes into account the wave propagation along the one dimensional (1-D) members and can give more realistic predictions of the behavior at high frequencies⁽²¹⁾.

1.6: Method of Characteristics :

The Method of Characteristics (MOC) is used to reduce the hyperbolic-partial differential equations to ordinary differential equations. It is used to determine forces and velocities in a structure. The natural frequencies of a structure are obtained during and after the application of vibration in conjunction with the Fast Fourier Transform (FFT), Brigham (1975)⁽¹⁶⁾. The Method of Characteristics

(MOC) has been widely used in several engineering applications. It was first observed by Pfgiffer (1947)(cited in Jwad, Z. M. (1992)⁽³⁷⁾), who obtained a general solution of the Timoshenko beam equation as a system of two second order partial differential equations. Closed conduit, open channel and ground–water flows have been analyzed using this technique by Streeter (1979)⁽⁶¹⁾.

In structural dynamic applications , many studies were made in order to apply the method of characteristics (MOC) on structural problems.

The Method of Characteristics (MOC) is considered to be more reliable method of analysis due to its ability to analyze both static and dynamic loads⁽¹⁸⁾. Also, it can be used to analyze highly complex structures, because the method needs relatively small computer storage.

1.7: Objective and Outline of The Study :

The main objective of this thesis is to develop the Method of Characteristics to deal with the structural dynamics. The method is used to deal with the general dynamic behavior of different systems, namely one–dimensional members and trusses. Linear oscillations of the above structures result from the propagation of axial wave are considered .

Different boundary conditions are examined with one–dimensional members. Comparisons with each other are established. Both geometrical and material non–linearity analysis are developed and examined.

Chapter two shows the previous researches which focus on the linear and non–linear dynamic behavior of structures in one and two–dimension.

The analysis of the propagation of waves in one–dimensional structural members is presented in chapter three, in addition, it is applied to single members. Attention is paid to longitudinal waves.

In chapter four geometrical and material non–linearity are developed and examined for one–dimensional members.

Chapter five is devoted to the development of the technique into an efficient method of dynamic analysis which is then applied to trusses.

The main features of the work are summarized in chapter six which contains the most important conclusions. Recommendations for further studies are listed in this chapter also.

CHAPTER TWO

REVIEW OF LITERATURE

2.1: General:

The dependence upon the stress wave propagation in materials was a useful method for the analysis of structural systems. This method has been used widely in the dynamic analysis of one dimensional members and was the subject of many researchers.

This review covers two aspects. The first includes the developments in the analysis of the dynamic response of one-dimensional structure subjected to transient excitations, while the second aspect is concerned with the developments in the analysis of the dynamic response of two dimensional structures of such members.

2.2: Dynamic Response of One-Dimensional Structures:

Leonard and Budiansky (1954)⁽⁴²⁾ utilized the method of characteristics to analyze traveling waves in beams subjected to lateral loading, governed by the Timoshenko's beam theory, but for the physically rather unrealistic assumption where the two wave speeds are equal, which simplified the numerical work involved. This simplification made the waves to propagate in more dispersive characteristics but in stable characters, when the numerical results were compared with an experimental data.

Skalak (1957)⁽⁶⁰⁾ obtained the analytical solution of the exact equations for the semi-infinite circular elastic bar. The results were extended by more general loading conditions by **De Vault and Curtis (1962)**⁽²³⁾, where the solutions were expressed as a double series of Fourier integrals. The integrals

have been accomplished only for large distance of travel and long time after application of the load.

Bertholf (1967)⁽¹⁴⁾ presented the finite difference method to solve the problem of two-dimensional axisymmetric elastic wave propagation in finite bars when it was subjected to continuous and discontinuous axial end impacts. The solution for a step change in stress agrees with experimental data near the end of the bar and exhibits a region that agrees with the one-dimensional strain approximation. Reflections from free and rigid-lubricated ends are studied. The results were compared with experimental data of other research and the maximum probable error in the numerical solution was about 7%. The investigator explained that this error was come from the inexact repotting of the experimental data and the experimental limitations.

Chou and Mortimer (1967)⁽²⁰⁾ treated the Timoshenko's beam equations as a system of second order partial differential equations by the method of characteristics in a unified approach capable of dealing with discontinuities along the characteristics in semi-infinite members, where no reflections were involved. They also compared the results of numerical calculations with Laplace transforms method when the two wave speeds were equal. The comparisons of the numerical results agree within 5% with the Laplace transform technique.

Koenig and David's (1968)⁽⁴⁰⁾ studied the response of a prismatic cantilever beam, subjected to a ramp platform bending moment at its free end, by using finite element method, where the effect of reflected waves in a very short cantilever was examined. Their results were presented for only a very small period, possibly due to stability and round-off error problems. The investigator concluded that the dynamic characteristics are vibrated about the static conditions.

Habberstad (1971)⁽³⁴⁾ approximated the exact equations of motion governing elastic, axisymmetric wave propagation in a cylindrical rod by first-order finite difference scheme. This scheme was based on a displacement rather than a velocity formulation, thereby making it unnecessary to explicitly introduce an artificial viscosity term into the finite difference equations. The resulted difference equations were used in conjunction with the boundary and initial conditions to study a pressure pulse applied to the end of a semi-infinite bar, a bar containing a discontinuity in cross section and a bar composed of two materials joined together at some point along its length. The numerical results so obtained were compared with available experimental data and other analytical solution. The numerical solution showed the same basic configuration as the elementary solution, except that significant deviations were noted in the region near the discontinuity.

Koenig and Berry (1975)⁽³⁹⁾ investigated the transient response of a non-prismatic cantilever beam subjected to end moment and lateral forces separately. The problem was solved numerically by using finite element method. They also studied the effect of non-homogeneity of beam material, caused by two materials fused together at some points along its length, on the response of the beam. The investigators concluded that the dispersion phenomenon was increased when non-homogeneities (geometrical and physical) were increased.

Rao, et al. (1976)⁽⁵⁵⁾ studied the large-amplitude free vibration of beams including shear and rotatory inertia effects. Using their non-linear beam model, they showed that the secondary effects have negligible influence on the response when $\frac{L}{r} > 100$, where L and r denote the beam length and the radius of gyration for the beam section respectively.

Al-Mousawi, et al. (1988)⁽⁷⁾ used the method of characteristics for the numerical solution of the transient flexural wave propagation in finite cylindrical beams with discontinuities in cross-sectional area. Several factors had been found to affect the level of stress and the associated peak values such as the diameter ratio and the position of discontinuity. Comparisons with experimental data and finite element solution were made to demonstrate the validity of the method for one-dimensional problems. He neglect the effect of unstressed zone nearby discontinuity.

Vardy and Al-Sarraj (1989)⁽⁶⁸⁾ applied The Method of Characteristics to simulate axial, torsional and flexural vibrations in single structural members supporting discrete masses. It was shown that the method is capable of predicting natural frequency of vibration in dissipative as well as non-dissipative systems. They used a numerical technique to solve the equation of motion at the masses. They showed that the method has useful advantages over conventional mass-spring analyses, including the ability to simulate wave action in members and to predict true stiffness and the equivalent free masses of the system.

Vardy and Al-Sarraj (1991)⁽⁶⁹⁾ used The Method of Characteristics to model vibrations of one-dimensional members in the presence of coupling between axial and flexural waves. Only small deflections were considered and so the couple $\mathbf{F}_x \delta_y$ is large only when the axial force $\mathbf{F}_x$ is large. Alternative numerical schemes were presented for the integration of terms describing coupling and a suitable explicit scheme was identified. The analysis was used to illustrate the influence of axial loads on frequencies and amplitudes of lateral oscillations of cantilever structures. Strong parametric behavior was reproduced for a single-storey shear structure, including unstable resonance at double the linear natural frequency.

Al-Daami (1992)⁽¹⁾ studied the transient and dynamic elastic response of non-prismatic straight members and arches by modeling axial and flexural waves transmission along such structures, using the method of characteristics. Several numerical examples were presented and favorable agreement with finite element and finite difference methods were determined. The study was restricted to a small deformation analysis that neglected the effect of geometrical non-linearity.

Jwad (1992)⁽³⁷⁾ investigated the non-linear elastic behavior of axially vibrated one-dimensional prismatic beams under high loading conditions by using the method of characteristics. The effect of geometrical non-linearity, caused by the large deformation behavior, was considered. The investigator concluded that this type of non-linearity would make the axial waves to propagate in dispersive form. Several examples of loading conditions of the beam were also considered.

Al-Sarraj and Al-Daami (1993)⁽¹²⁾ investigated the transient and dynamic response of non-prismatic structural members by modeling uncoupled axial and flexural wave transmission along such members, using The Method of Characteristics. It was shown that the method is capable of predicting natural frequencies of vibration. Many numerical examples were presented and favorable agreement with finite element method analysis was determined.

Al-Sarraj, Othman and Jwad (1993)⁽¹¹⁾ introduced a method of analyzing one-dimensional axially loaded bars. The method designed for nonlinear characteristics, was based on The Method of Characteristics that has gained acceptance in the analysis of transient flows in fluid pipelines. The nonlinear characteristics had been shown to be well suited to simulate the dynamic behavior beyond the elastic stage.

Fahmi (1998)⁽²⁹⁾ used The Method of Characteristics to determine the nonlinear transient response of single and continuous beams with internal resonance, then

compare the results with finite element method. The comparison gives good agreement.

Al-Haidar (2005)⁽²⁾ studied the elastic–plastic dynamic response of prismatic and non–prismatic straight members, using The Method of Characteristics to model axial and flexural waves transmission along such structures. The analysis is based on the wave propagation in one–dimensional structural element approach. The effect of coupling between axial and flexural waves due to the unsymmetrical variation of the cross–sectional area about the longitudinal axis was considered. Several examples were investigated and the results are compared with the experimental data and the finite element method that were determined. The investigator concluded that the dispersion phenomenon in flexural waves increased when the plastic waves were propagated in these members.

2.3: Dynamic Response of Two –Dimensional Structures:

Houbt J.C. (1950)⁽³⁵⁾ presented a method for the dynamic analysis using the direct time integration method. In this method, standard finite difference expressions are used to approximate the acceleration and velocity components in terms of the displacement component. He used a backward difference formulation with an error of order $(\Delta t)^2$.

Newmark N. M. (1959)⁽⁵⁰⁾ generalized certain direct numerical integration methods that had been in use up to that time. He presented equations for approximating the velocity and displacement of a single degree of freedom system at time $(t+\Delta t)$. These equations involve two constants (γ and β). The parameter γ controls the velocity within the time step while the parameter β controls the variation of acceleration. Because of that the technique is referred to as Newmark's generalized acceleration method (or *Newmark– β method*).

Gallagher R. H. and Lee C. H. (1970)⁽³⁰⁾ derived approximate formulations of tapered beam element stiffness matrices for dynamic and elastic instability analysis. Displacement function for the uniform beam segment are employed in this development. Moment of inertia and area of the element are prescribed by arbitrary powers of the axial coordinate. Numerical results are obtained and compared with both analytical solutions and numerical solutions based up on stepped representations using uniform section elements. The numerical results have shown that simple modifications to the beam element stiffness and mass matrices to account for non-uniform section properties have produced highly accurate solutions for idealizations consisting of few elements.

Week's (1972)⁽⁷³⁾, used three methods for solving the non-linear dynamic problems. He considered the implicit **Houbt** and the **Newmark** average acceleration methods as well as explicit central **Finite Difference Method (F.D.M)** for one and multiple degree of freedom systems. He found no essential differences in the characteristic of the temporal operators for the linear and non-linear problems provided a small time step.

Wilson .et. al (1973)⁽⁷⁴⁾ extended the linear acceleration method in a manner that makes it numerically stable. The basic assumption of the **Wilson- θ** method is that the acceleration varies over an extended time **($\theta \Delta t$)**. It has been shown that this approach is numerically stable when **($\theta > 1.38$)**.

Atkins and Hunter (1975)⁽¹³⁾ demonstrated experimentally and theoretically the propagation of axial elastic waves around right angled corners. They used **(L)** and **(T)** geometry systems with rods of square cross section, and identical cross-sectional and material properties. With a symmetrical **(T)** geometry, they found that incident axial waves in the central rod failed to produce transmitted axial waves in the other rods.

Oran and Kassimali (1976)⁽⁵¹⁾, presented a nonlinear dynamic analysis of beams by using the **Newmark Average Acceleration Method**. They assumed that

the masses are lumped at joints. The effect of rotary inertia on mass matrix and the effect of damping of the system were neglected. In their work, they used the incremental procedure in the formulation of the equations of motion at time $(t+\Delta t)$. They solved a fixed ended beam with an iteration process and compared the results with others, and they found that their technique gives a more representative solution compared with other technique.

Zienkiewicz (1977)⁽⁷⁸⁾ offered a new look at time integration schemes based on a generalized weighted residual approach. The main advantage of this approach is that it contains most of the existing methods as special cases. On the other hand, this method is a multi-step method, and thus, it suffers from the following computational disadvantages :

1. Changing the time step size during the analysis is not possible.
2. Consistent starting values are not readily available.
3. The application to non-linear problems is cumbersome.

Wood (1977)⁽⁷⁵⁾, investigated the accuracy and stability for the **Zienkiewicz** method and obtained the value of the investigation parameters which minimize the incremental phase error.

Mondkar D. P. and Powell G. H. (1978)⁽⁴⁸⁾ investigated solution schemes, which they implemented in general purpose computer cod for nonlinear finite element analysis. A vary flexible solution strategy is discussed, in which a variety of solution schemes can be implemented by specifying the values of certain solution, **Newton–Raphson** iteration, constant stiffness iteration, and an adaptable schemes in which the structure stiffness is reformed only when needed.

Remseth S. N. (1979)⁽⁵⁷⁾ presented a study concerned with method of nonlinear static and dynamic analysis of structures, particularly for application to geometrically nonlinear space frames. The displacement formulation of the finite element method is adopted. Large displacements are accounted for by using

Lagrangian description of motion from a fixed reference frame, assuming small strains and moderately large rotations.

Zienkiewicz .et at. (1984)⁽⁷⁹⁾ and **Katona** and **Zienkiewicz (1985)⁽³⁸⁾** introduced two similar, but distinct single–step methods separately; one is based on a weight residual formulation, and the other is based on generalization of **Newmark's** scheme. The first one was known as the **SSPJ** Method. The latter was called the **Beta–m** method. The method is specialized by specifying the method order m along with m integration parameters $(\beta_0, \beta_1, \dots, \beta_{m-1})$, for a particular choice of m , the integration parameters provide a subfamily of methods which to be special cases within the beta-m family. Stability and accuracy are presented for method order $m=2,3$ and 4 (cited in Al-Haidar, N. H. M. (2005)⁽²⁾).

Chan (1983)⁽¹⁸⁾ developed a compatible analysis method of skeletal structures. The Method of Characteristics was used to model axial, torsional and flexural waves in prismatic members. By using this technique the static behavior of trusses and frames and the dynamic behavior of single members were described. The study was extended to include large displacement problems in the static analysis of trusses only. Also damping effect was considered. **Vardy and Chan (1988)⁽⁷⁰⁾** used The Method of Characteristics to model waves in skeletal structures comprising prismatic members.

Alsarraj A.T. (1989)⁽¹⁰⁾ studied the transient and dynamic elastic response of prismatic members supporting concentrated masses by modeling axial and flexural waves transmission along one dimensional members by using the **Method of Characteristics (M.O.C)**, and applied the **Method of Characteristics (M.O.C)** to the analysis of trusses and plane frames by reduces the method to the **Boundary Characteristic Method (B.C.M)**. Uncoupled and coupled effects between axial and flexural waves included. All displacement assumed to be small compared to the over all dimension of the structure, natural frequencies of the system were determined from the temporal responses using the **Fast Fourier Transform (FFT)**

routines. Several numerical examples were presented and the results were compared with many other methods of analysis and with experiments.

Dokainish and **Subbaraj (1989)**⁽²⁴⁾, presented a survey on implicit time integration method, viz. *Newmark- β* , *Wilson- θ* and *Houbolt* methods. The formulation and computational step-by step procedures for linear and non-linear system were given stability aspects of these methods were also described.

Carol and **Murcia (1989)**⁽¹⁷⁾ presented an exact formulation for the nonlinear time-dependent analysis of planar frames formed by straight bars presented in three steps. A basic linear formulation, which is equivalent to the classical matrix theory, its generalization to nonlinear and time-dependant material analysis, and the inclusion of second order equilibrium nonlinear effects.

Due to the exact character of the interpolation used by the authors, no intrinsic errors exist in this formulation, and therefore there are no limitations to the size of the elements for this reason.

Hasiao K. M. and **Jang J.Y. (1989)**⁽³⁶⁾ presented a co-rotational formulation and numerical procedure for the non-linear dynamic analysis of plane frames. They defined the incremental displacements and rotations, velocities, accelerations and equations of motion of the system in terms of fixed global coordinates. The element matrices are constructed first in an element coordinate system, which rotates and translates with the element, and are then transformed to the global coordinate system by standard procedure. The beam element used by the authors was an extension of that proposed by **Hasiao** and **Huo**. In their proposed approach the major geometric nonlinearities were shown to be embodied in the coordinate transformation when the element assemblage is formed.

In **(1996)**⁽⁹⁾, (cited in **Al-Mutairee (2000)**⁽⁸⁾) a new implicit time integration method was submitted to **Al-Sarraf** for evaluation to be published. The basis of this method is to assume that the acceleration varies harmonically during the time step, so it was called the harmonic acceleration method. The numerical examples

were used to examine the application of the proposed method for linear system. Each example was solved by the harmonic acceleration method and the linear acceleration method, in addition to the exact solution. The author concluded that the errors with the proposed method are smaller than those with the linear acceleration method for the same time step.

Ali A. A. (1999)⁽³⁾ a theoretical analysis is presented for estimating the in-plane large displacement elastic stability behavior of structures with prismatic or tapered members subjected to static and dynamic loading. The analysis adopts the **Beam–Column (B-C)** approach. The **Beam–Column** element is based on the **Eulerian** approach taking into consideration the influence of the axial force on bending stiffness and the changes in member chord length due to axial strain and flexural bowing.

In the dynamic analysis, four different numerical integration algorithms have been used for solving the equation of motion. System mass properties have been represented using both consistent and lumped mass matrices. System natural frequency is obtained by solving the eigenvalue problem. Some examples which have been investigated. In addition a parametric study is to be made using the large displacement elastic stability analysis to determine the effect of several influencing parametric on the behavior of plane structures with the tapered members subjected to static and dynamic analysis loads.

Al-Mutairee H. M. K (2000)⁽⁸⁾ presented a theoretical analysis for estimating the in plane large displacement elastic behavior with post-buckling of plane steel frames with prismatic and non-prismatic members and subjected to static and dynamic loads. The analysis is based on **Beam–Column (B-C)** approach. The formulation of the **(B.C)** elements is based on the **Eulerian** approach taking in to consideration the influence of the axial force on bending stiffness and the changes in the member chord length due to axial strain and flexural bowing. In the dynamic analysis five different numerical integration algorithms have been used for solving the equation of motion. The consistent mass matrix for non-prismatic member is

derived in this study. System's natural frequencies are obtained by solving eigenvalue problem. Also the study has been presented to test the effect of several parameters on the non-linear elastic behavior of frames tapered member under static and dynamic loads, such as the effect of tapering ratio and the effect of damping and to test the efficiency of the derived consistent mass matrix for tapered members.

Driemeier. L. et al (2005)⁽²⁵⁾ presented theoretical and numerical aspects of the behavior of spatial trusses undergoing large displacements and strains. Nonlinear material behavior is taken into account. Key examples were chosen to explore the various features of the model and to highlight the interplay between plasticity and damage. Finite element approach is used to solve the nonlinear boundary value problem. Both theoretical and algorithmic tangent moduli were derived. The analysis not include the finite plastic strain and the behavior before damage, but the analysis introduced a development for these features.

Ligaro S. S. and Valvo P. S. (2005)⁽⁴³⁾ examined the equilibrium stability problem for a simple class of elastic space trusses in the shape of regular pyramid. Joints located at the vertices of the base polygon are fixed while the joint at the apex is subjected to a proportionally increasing load acting in either the vertical direction, in the horizontal plane, or along a generic oblique direction. A shallow symmetric two bar planar truss subjected to a vertical load at its top joint, which is no doubt the most popular example of a structural system where snap-through is the prevailing form of instability. It is currently associated to the name of **Von Misses (1923-1925)**, who first used this model to explore kinds of structural instability more general than **Euler's** buckling of a single bar. Exact closed-form solution were derived for each common hypotheses of linear material law, small or moderate axial deformation in bars and large nodal displacements.

2.4: Closing Remarks:

The subject of many researches was restricted to use limited methods for solution. This methods no doubt has one disadvantage or more, and may be restricted, such as: with one type of loading, one type of structures, or etc. **Mass Spring Method** does not usually take the wave action into account, while **Method of Characteristics (M.O.C)** takes into account the wave propagation along one-dimensional, two and three dimensional structures and can give more realistic predictions of the behavior at high frequencies⁽²²⁾. In other researches the affect of large deformation is neglected. Here this study is an extension to the **Method of Characteristics** to deal with the small, large amplitude vibration of prismatic one-dimensional structures that based on Euler's theory and two-dimensional structures (trusses), and the plastic behavior of it.

CHAPTER THREE

AXIAL WAVE PROPAGATION IN ONE-DIMENSIONAL MEMBER

3.1: Introduction:

Wave propagation means a transmission of stress and velocity waves from the point of application of sudden applicant force. These waves strike boundaries such as joints, foundation, masses or discontinuities inside the system which exposed to this sudden force. The waves may be wholly or partially transmitted to the surroundings or reflected back into the system.

In this study, ideal case of an elastic member is considered, in which the damping is assumed to be negligible (for transient response). The most important assumption in the case of wave propagating in one dimensional prismatic members is usually that 'plane sections remain plane'. This assumption can be made with confidence in the case of longitudinal waves which is the objective of the present study. The analysis begins with a simple study of the propagation of longitudinal waves.

This study involves the influences of sudden excitation on a prismatic one-dimensional member with different boundary conditions and a brief discussion and comparison with each other.

3.2: Longitudinal Waves:

According to Euler's theory which assumes that each plane cross section remains plane, the longitudinal wave propagates in a one-dimensional member, that means the particles in the cross-sections

move only in the axial direction of the member. When one end of the member is subjected to a force, it will be disturbed, this disturbance will move along the member to the other end where it is reflected back again when strike the boundary.

Axial waves resemble pressure waves in a fluid pipe–line, see Streeter (1979)⁽⁶¹⁾. Consider a small element of a prismatic elastic one–dimensional straight member with cross–section area (A) to axial load at both ends as shown in Figure (3-1).

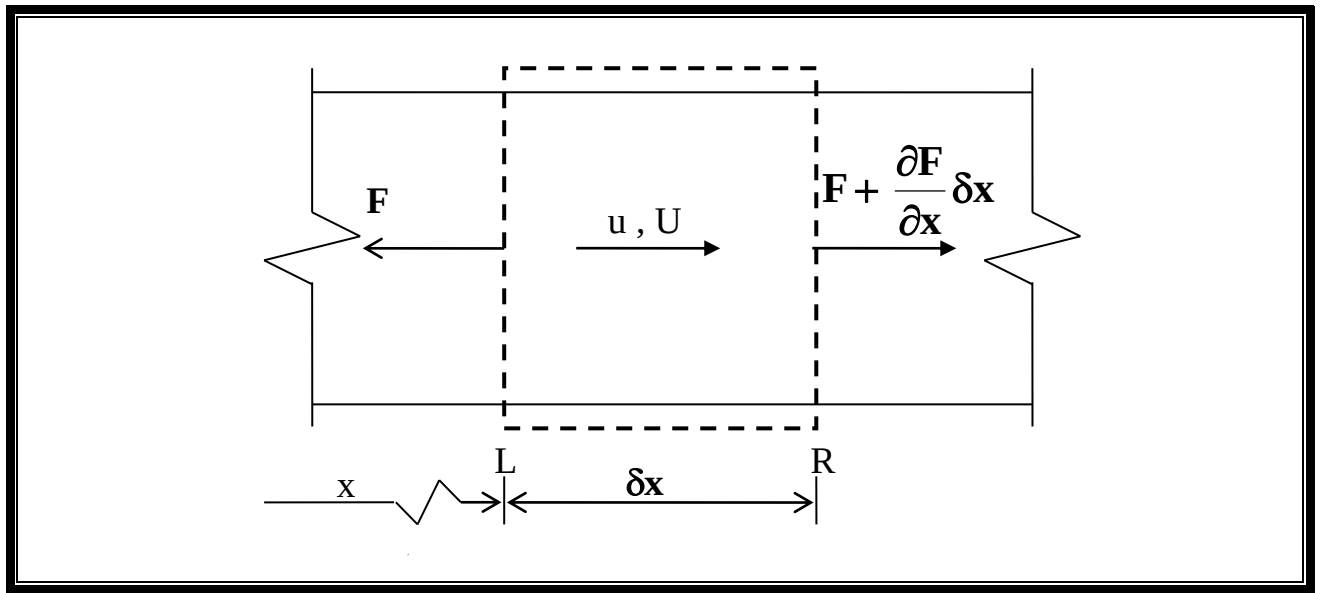


Figure (3-1): One–Dimensional Prismatic Element under Axial Load

From Newton's second law of motion, the net axial force (F) is related to the acceleration $\frac{\partial^2 \mathbf{u}}{\partial \mathbf{t}^2}$:

The resultant force = mass $\times$ linear acceleration

$$\frac{\partial \mathbf{F}}{\partial \mathbf{x}} \delta \mathbf{x} = \rho \ A \ \delta \mathbf{x} \ \frac{\partial^2 \mathbf{u}}{\partial \mathbf{t}^2} \quad (3.1)$$

where ρ = material density of the member(kg/m³)

u = the axial displacement of the cross–section.

This equation can be simplified to:-

$$\frac{\partial \mathbf{F}}{\partial \mathbf{x}} = \rho A \frac{\partial \mathbf{U}}{\partial \mathbf{t}} \quad (3.2)$$

where $\mathbf{U}$: Axial velocity of the cross-section (m/sec)

From material property relationship which is obtained from Hook's law:

The axial normal stress(σ_x) is related to the strain(ϵ_x) by:

$$\sigma_x = E \epsilon_x \quad (3.3)$$

where E : modulus of Elasticity, Pa

$$\epsilon : \text{Axial strain } (\epsilon = \frac{\partial \mathbf{u}}{\partial \mathbf{x}})$$

u : Axial displacement (m).

The above equation can be written as :

$$\mathbf{F} = EA \frac{\partial \mathbf{u}}{\partial \mathbf{x}} \quad (3.4)$$

Which can be differentiate with respect to time as:

$$\frac{\partial \mathbf{F}}{\partial \mathbf{t}} = EA \frac{\partial \mathbf{U}}{\partial \mathbf{x}} \quad (3.5)$$

Equations (3.2)&(3.5) are pair of liner hyperbolic partial differential equations concerning force and velocity. To solve the above equation, it is required to use the Method of Characteristics.

3.3 : Method of Characteristics Equations:

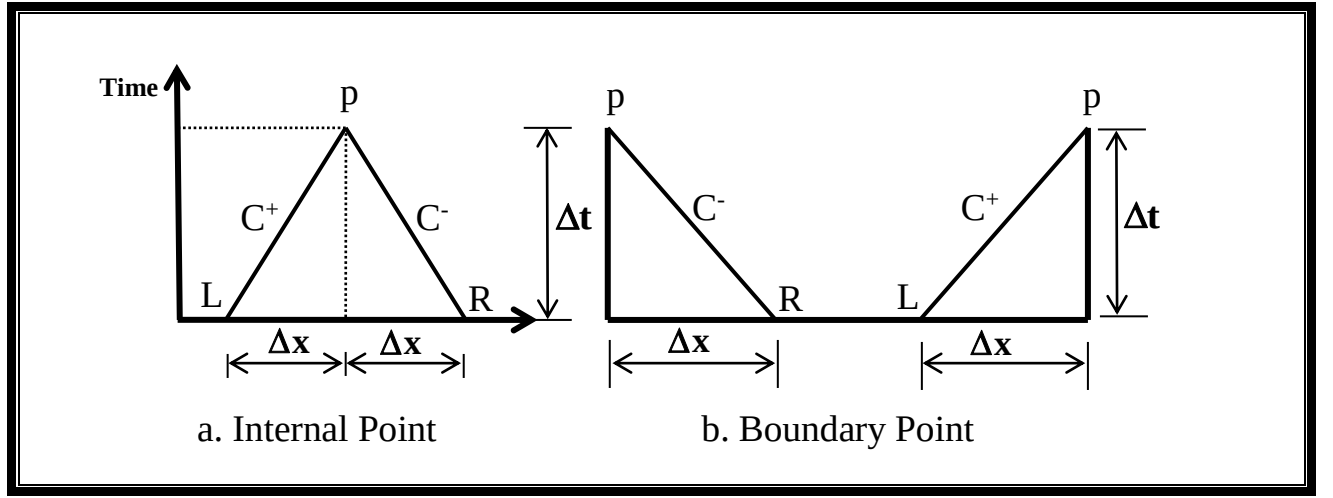
Equations (3.2)&(3.5) are a pair of hyperbolic partial differential equations, which can be combined and expressed in characteristics form as:

$$\text{along } C^+: \frac{d\mathbf{F}}{d\mathbf{t}} - \rho AC \frac{d\mathbf{U}}{d\mathbf{t}} = 0 \quad (3.6.a)$$

$$\text{along } C^-: \frac{dF}{dt} + \rho AC \frac{dU}{dt} = 0 \quad (3.6.b)$$

where (C) is the speed of transmission of axial waves along the member, (m/sec), $C = \sqrt{\frac{E}{\rho}}$

Equations (3.6.a, 3.6.b) can be integrated along the characteristics lines LP & RP in Figure (3-2) accurately since the product (ρAC) is constant.



Figure(3-2): Characteristics Line

$$\text{along } \frac{dx}{dt} = C^+: (F_P - F_L) - \rho AC (U_P - U_L) = 0 \quad (3.7)$$

$$\text{along } \frac{dx}{dt} = C^-: (F_P - F_R) + \rho AC (U_P - U_R) = 0 \quad (3.8)$$

Equations (3.7)&(3.8) are the characteristics equations used to produce force and velocity at any internal grid point, one of them is used at the boundaries.

3.4: Numerical Formulation :

The previous pair of equations is solved to calculate the forces and the velocities at successive points. Only one of the characteristics lines either C^+ or C^- is available at boundaries as shown in Figure (3-2.b).

An additional equation must be provided to describe the boundary condition. This additional equation is based on the restraining case at this boundary, as it will be explained later. This pair of equations to be solved simultaneously to describe this boundary.

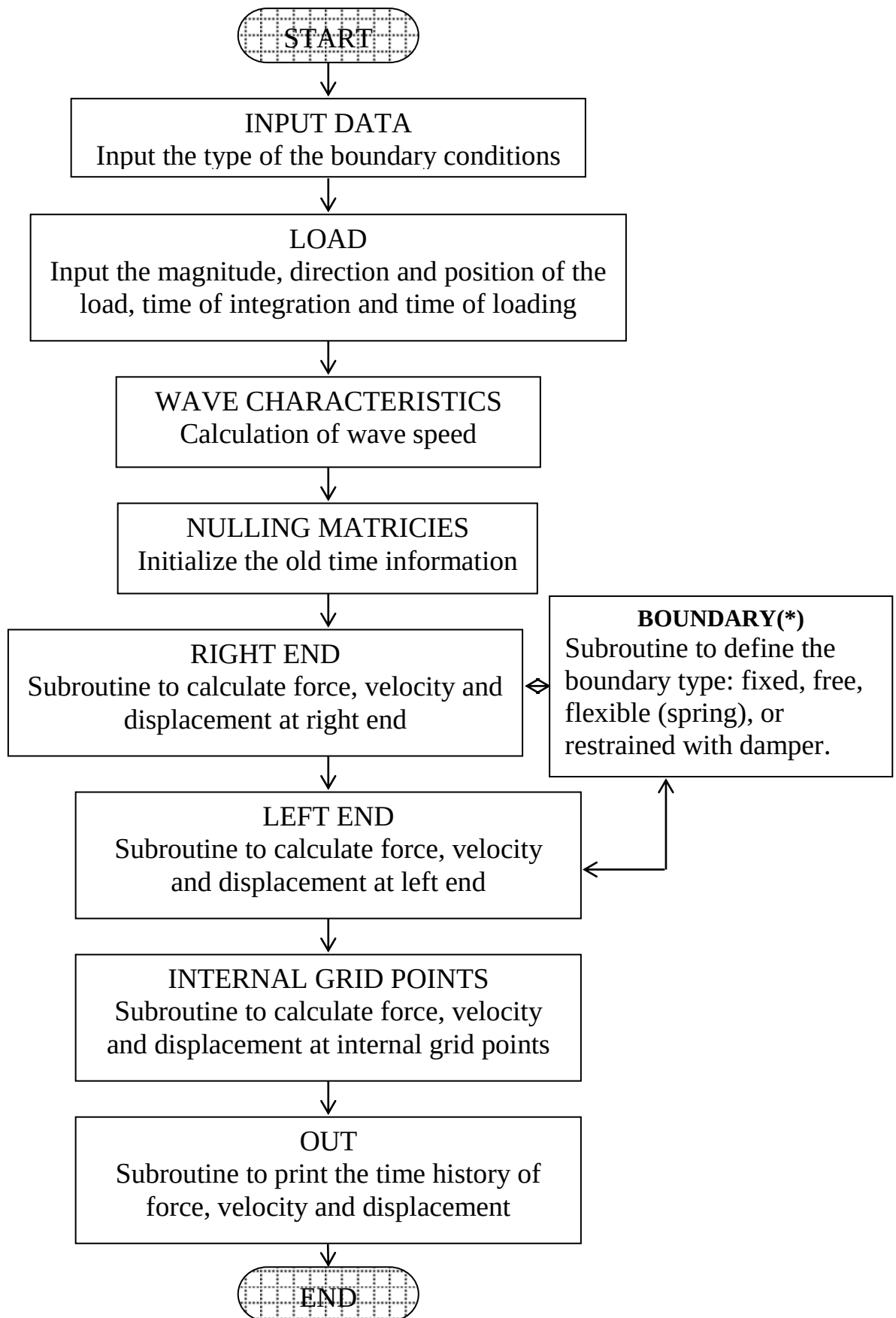
Some boundary conditions such as rigid joints, unrestrained joints, and partially restrained joints included "spring and damper" are discussed with examples in this chapter.

3.5: Computer Program:

The computer Program (**LDAPM**) *Linear Dynamic Analysis of Prismatic Members* was designed by Al-Daami⁽¹⁾ for analyzing the (fixed-free) one-dimensional members. In the present study this computer program has developed to carryout another case of restraining boundary such as: free ends and restrained with spring and damper. This computer program deals with the transient response of prismatic members, and can be used with any function of loading (i.e. step, terminated ramp, triangular and harmonic loading).

The input data of the program consists of the type of the transient loading and the overall member geometry including length, cross sectional area and the type of boundary conditions.

The output of the computer program includes the time history distributions of the axial force, axial velocity and axial displacement. The flow-chart of the computer program (**LDAPM**) is shown in Figure (3-3).



(*): Developed in the present study

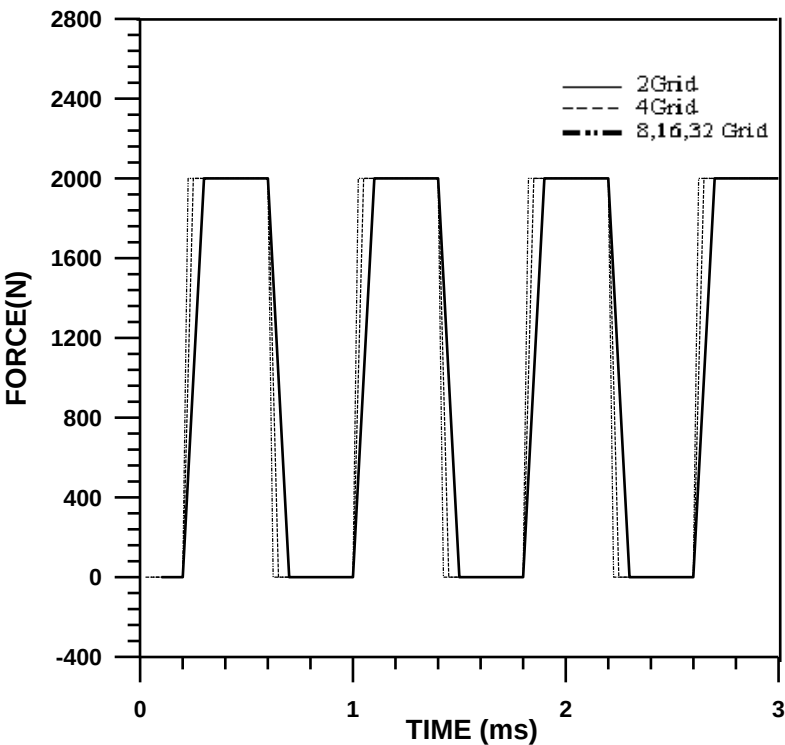
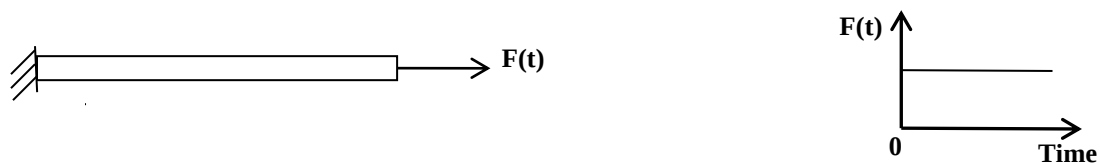
Figure (3-3): The Flow Chart of **(LDAPM)**

3.6: Grid Size Dependence:

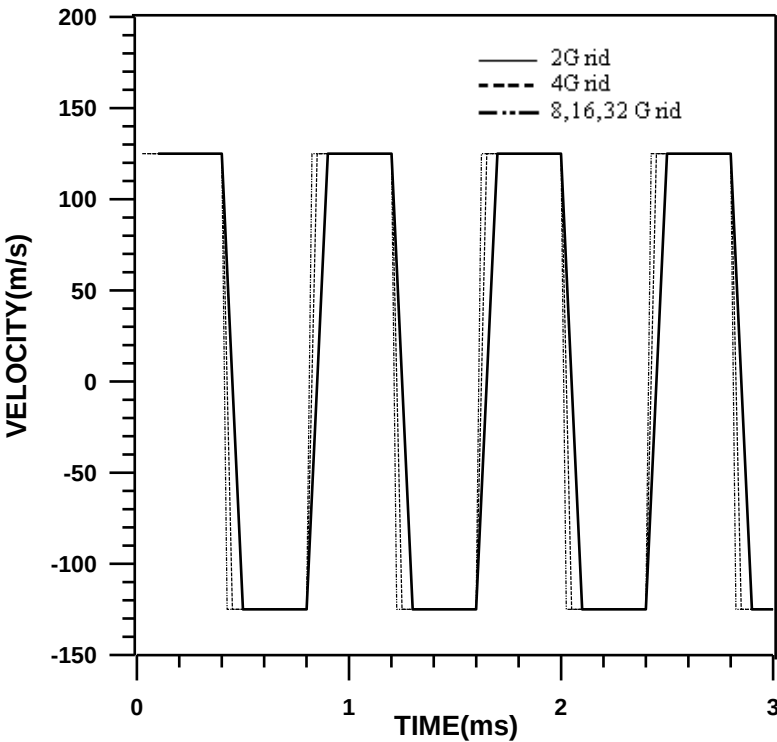
Several grid lengths are needed to ensure good accuracy. By inspection, sufficient accuracy is obtained with (4) grid length for a member exposed suddenly axial force, with one meter length.

Figure (3-4) shows the internal axial force history at the fixed end. Also The velocity history and displacement history at the tip end of the member with various grid lengths are depicted in these figures. It is noticed that the amplitude of the oscillating axial force obtained with a single grid length is the same as that obtained with 16 grid length.

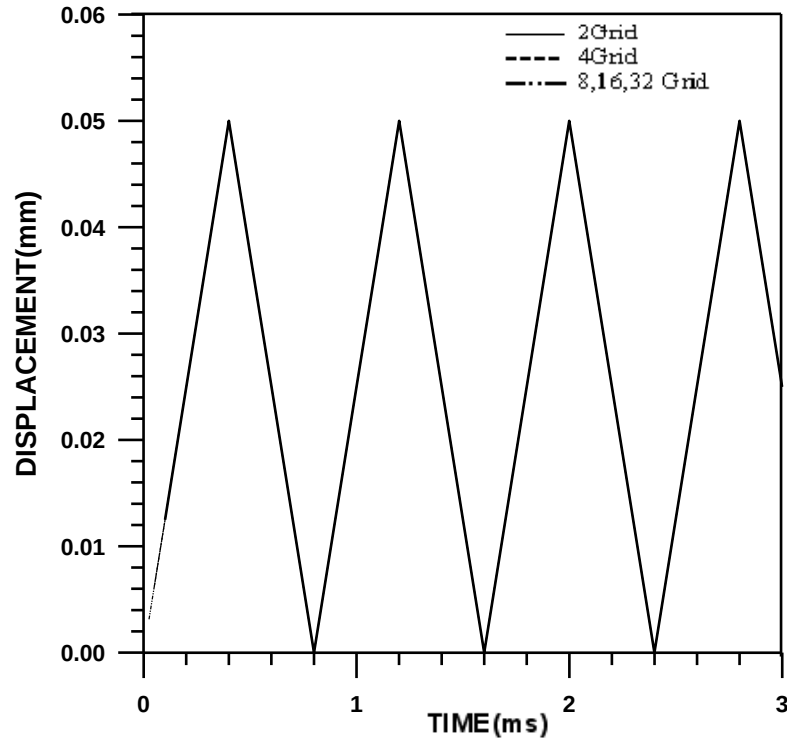
There is no difference in the amplitude of force if it uses one grid length or n-grid length. That is attributed to the validity of equilibrium equation at all cases, but the effect of dividing will be on contribution of dividing mass in actual excitation for the member which is represented by natural frequency.



(3-4A): Force Response at Fixed End



(3-4B): Velocity Response at Tip End



(3-4C): Displacement at Tip End

Figure (3-4): Grid Size Dependence
 $E=2 \times 10^{11}$ Pa , $A= 2 \times 10^{-4}$ m² , $\rho=8000$ kg / m³ , $L=1.0$ m
 $F=1000$ N , $LT=3$ ms

3.7: Displacement and Acceleration:

From the observation of Characteristic equations the derivative of forces are related to the velocities, not to the displacements or accelerations. Displacements can be obtained by integration as :

$$u = \int_0^{T_i} \text{velocity} \cdot \Delta T \quad (3.9)$$

where (T_i) is the time during which the displacement required.

In finite difference form, this become :

$$u \approx \sum U \cdot \Delta T \quad (3.10)$$

where U is the velocity, (m/sec)

Such as :

$$\ddot{u} \approx \frac{1}{\Delta T} (U_P - U_0) \quad (3.11)$$

where ($\ddot{u}$) is the acceleration, (m/sec²), U_P : the velocity of the point at this time, U_0 : the velocity at the old time.

It is noticeable that the displacement cannot be obtained as accurately as the velocity because the finite difference expression (3.10) can be used only after the velocity has been obtained from the general solution. The approximation of equation (3.10) does not affect the solution of forces and velocities, in the case of small displacement problems, where the change in geometry of the structure is assumed to be small and insufficient to alter the forces and velocities in the structure. This approximation may lead to large errors in the case of large displacement problems, unless small time intervals of integration are chosen, compared with time of stress wave propagation.

3.8: Boundary Conditions :

At internal point of a member such as point (A) as shown in Figure (3-5a) both characteristic equations must be used. But at boundary points such as point (B) as shown in Figure (3-5b), only one of the characteristic equations can be used, either forward or backward, for each type of waves.

There are many types of boundary condition, such as rigid end, free end, flexible (spring), restrained with damper and their combination. In this chapter, four of these boundary conditions have been discussed. The Boundary Characteristic Method (BCM) pays large attention to the ends of the member, so the behavior of waves is different from one boundary to another. This study explains those differences.

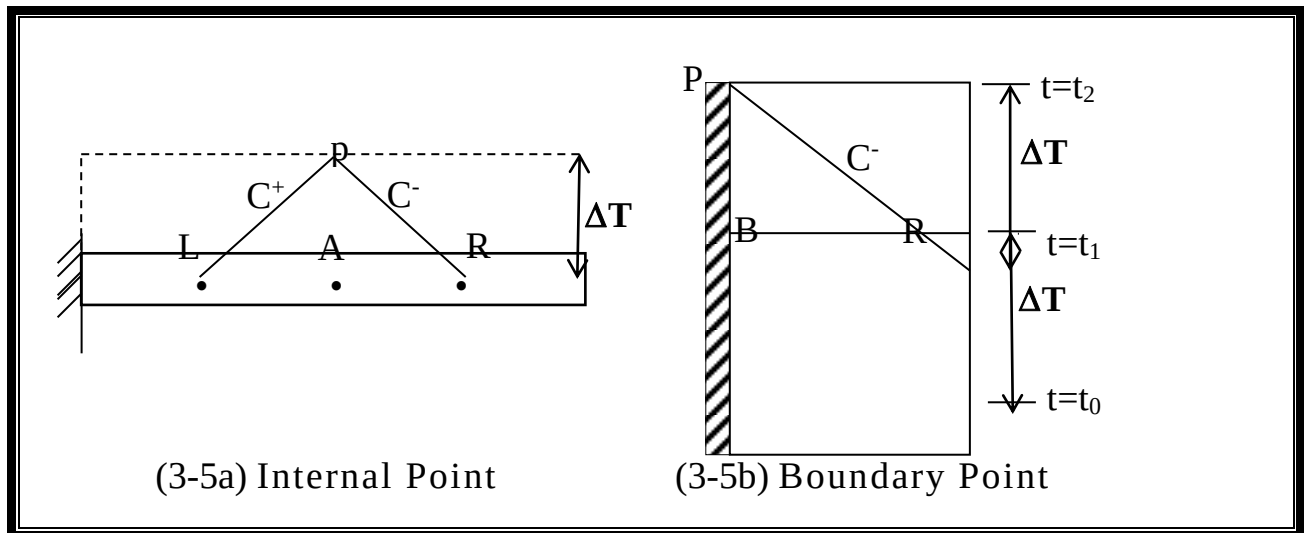


Figure (3-5): Internal Point and Boundary Point

3.9: Numerical Applications :

The behavior of a prismatic member have been shown when a longitudinal wave propagate through it. This member is examined with various boundary conditions such as "fixed, free and restrained with spring, and damper".

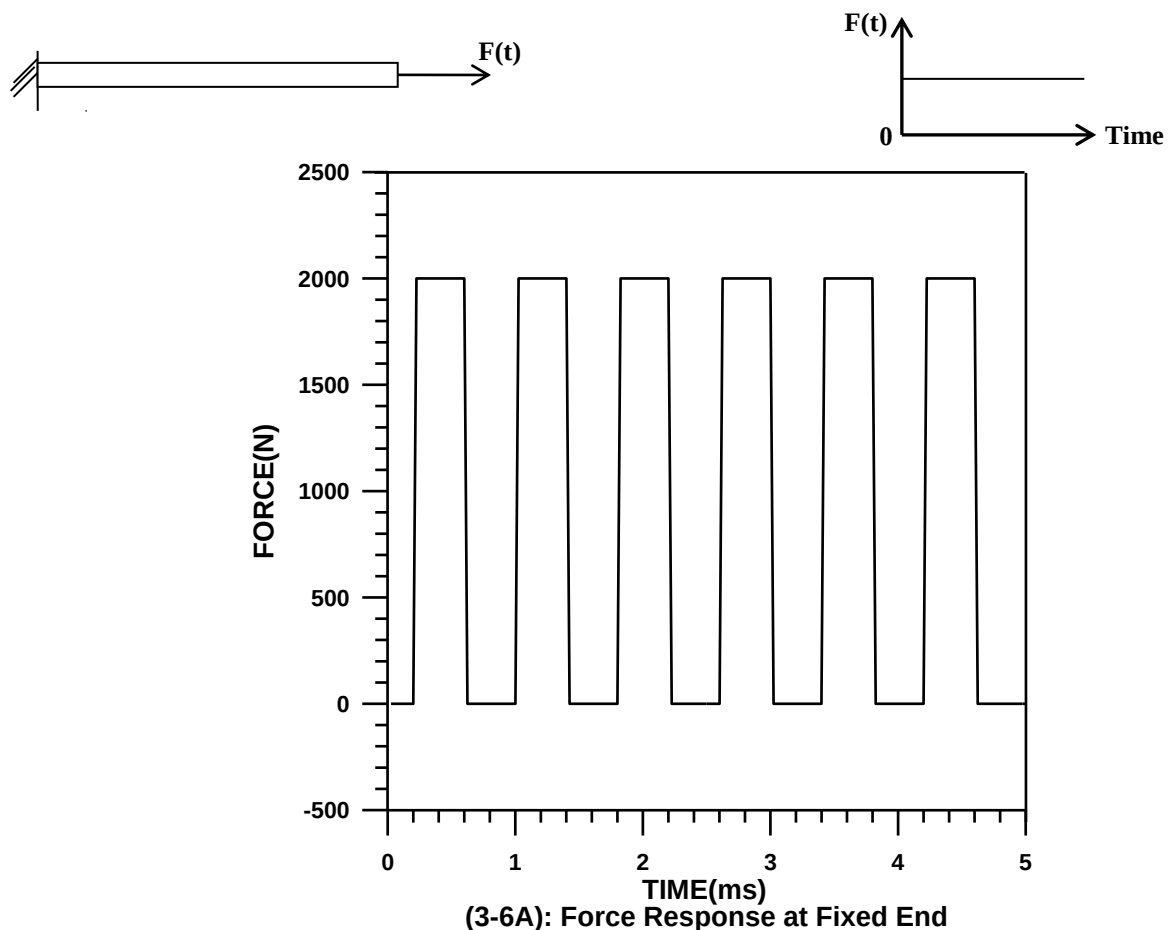
EXAMPLE NO.1: CANTILEVER MEMBER:

The fixed end can be considered as fully restrained. At the restrained end the velocity in the restrained direction is zero because the joint cannot move in that direction, while the reaction (force) on the joint is unknown. At the free end the converse is true, where the joint is completely unrestrained, so the reaction force is zero. However, the velocity of the joint is unknown. The unknown velocities and reactions can be solved simultaneously.

Many cases of loading condition which have been studied include: "continuous loading, impulse loading and harmonic loading", see Figure (3-6), (3-7), (3-8). When the member is subjected to both

continuous and impulse forcing function, the amount of the reflected force wave is (two times) the subjected forcing function, (i.e. the dynamic intensification factor (D.I.F) is two), but when the member is subjected to harmonic loading, the amount of the reflected force wave is about (3.25 times) the subjected forcing function, that is attributed to the nature of the applied forcing function.

It can be concluded that the effect of cyclic forcing functions, like harmonic forcing function, is more than the effect of other types of dynamic loading and has a dangerous effect on the structures, where the amplitude of force, velocity and displacement wave is large.



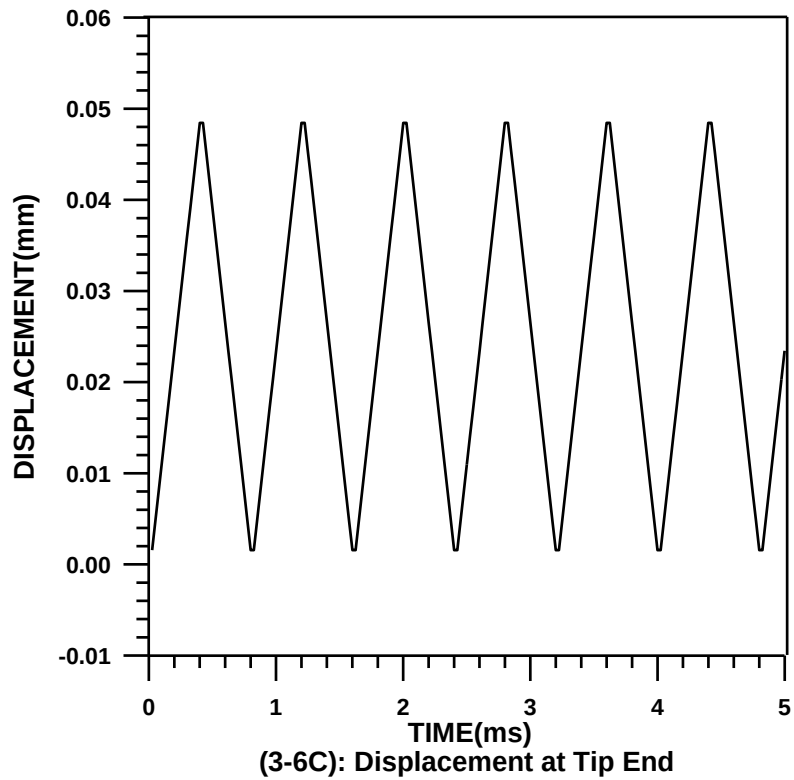
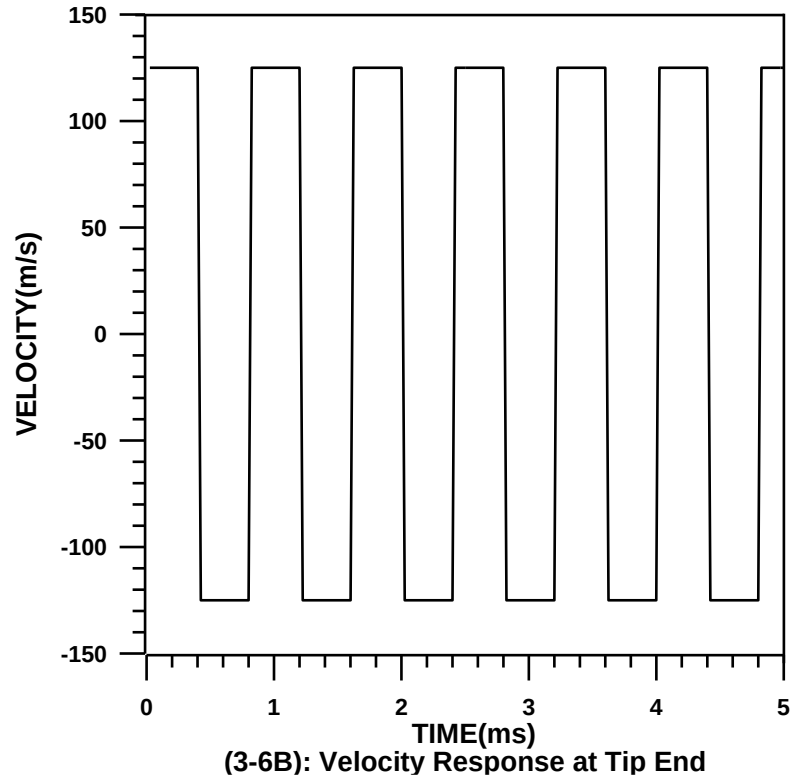
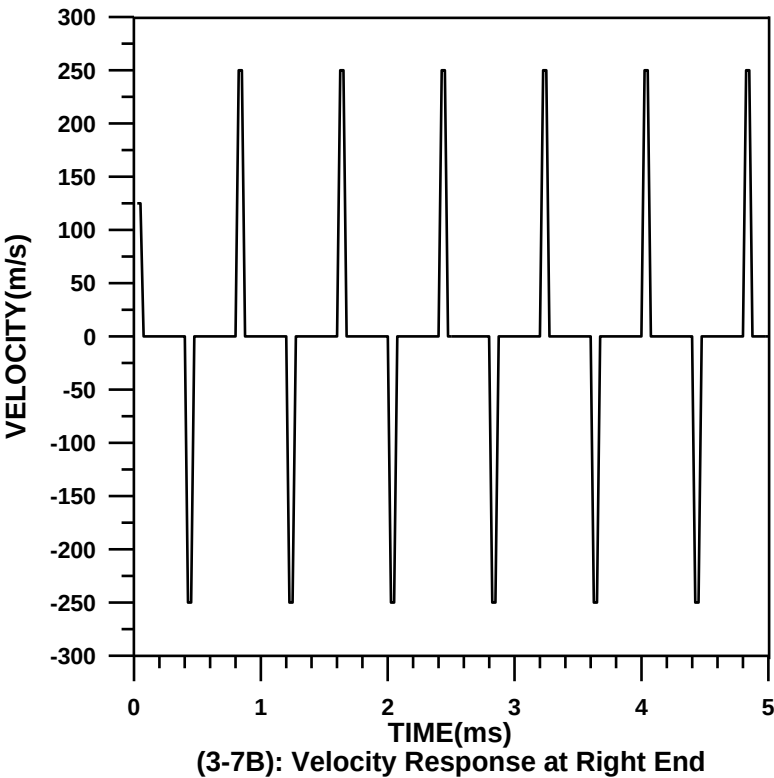
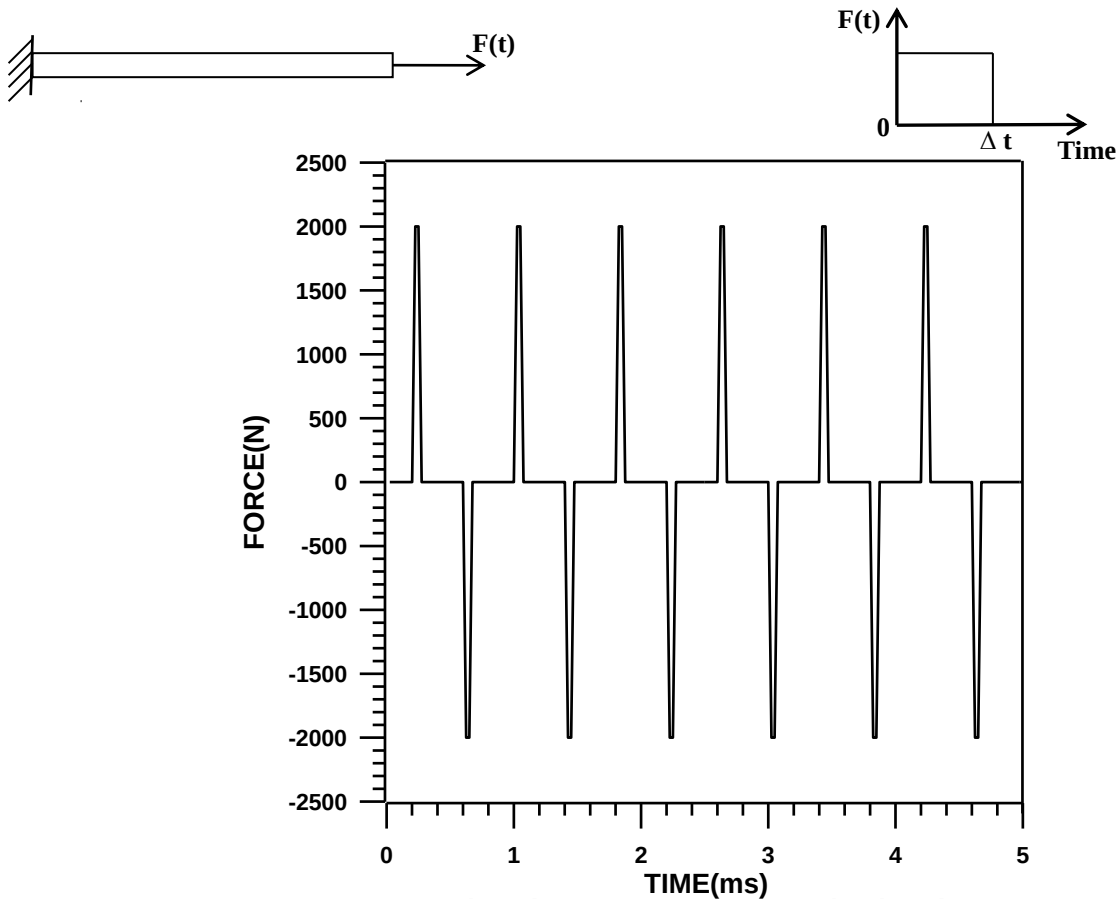
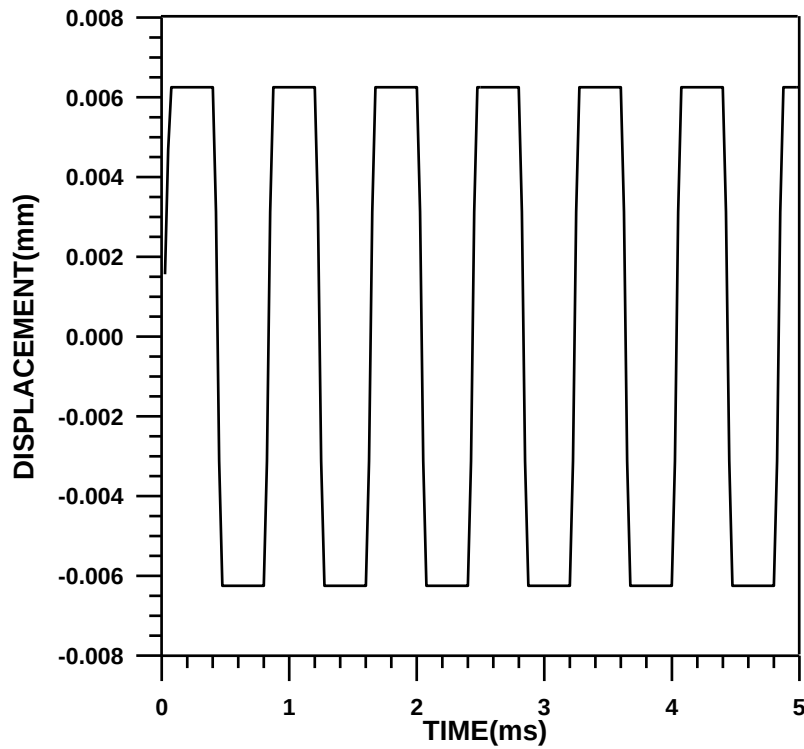


Figure (3-6): Dynamic Response for Cantilever Member with Continuous Forcing Function

$E=2 \times 10^{11}$ Pa, $A= 2 \times 10^{-4}$ m², $\rho=8000$ kg / m³, $L=1.0$ m, $F=1000$ N

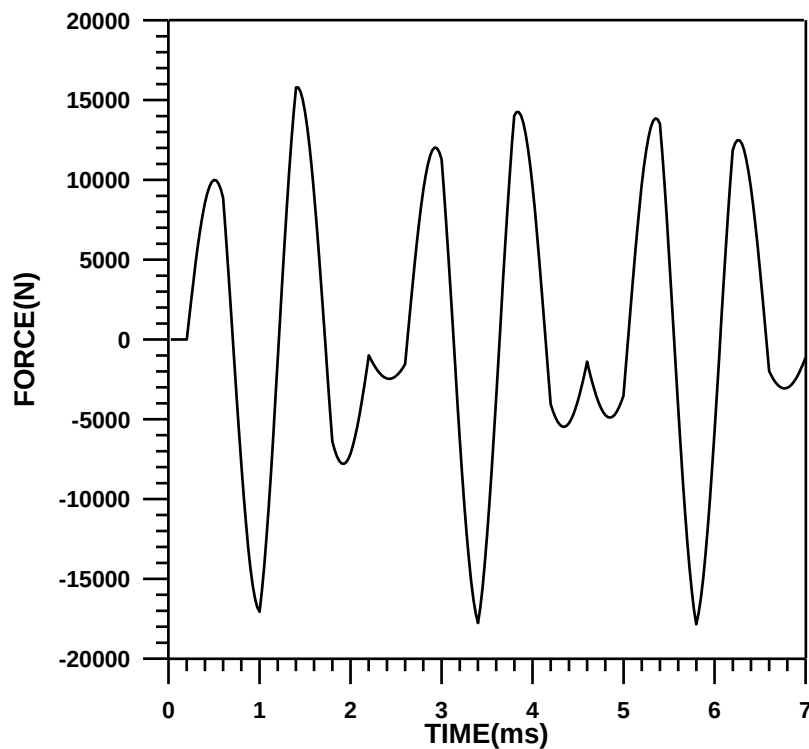




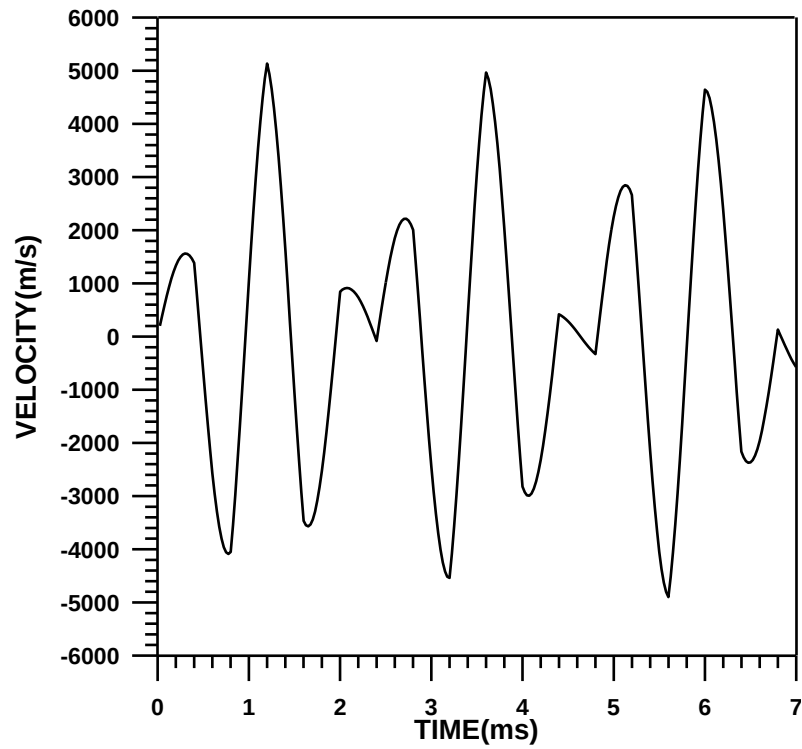
(3-7C): Displacement at Right End

Figure (3-7): Dynamic Response for Cantilever Member with Impulse Loading

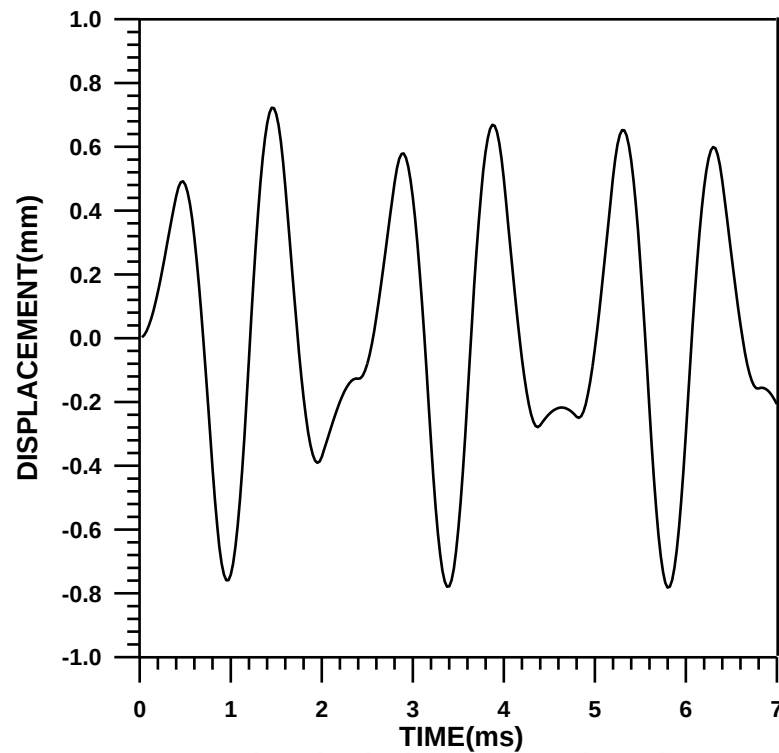
$$E=2 \times 10^{11} \text{ Pa}, A=2 \times 10^{-4} \text{ m}^2, \rho=8000 \text{ kg/m}^3, L=1.0 \text{ m}, F=1000 \text{ N}, LT=0.05 \text{ ms}$$



(3-8A): Force Response at Fixed End



(3-8B): Velocity Response at Tip End



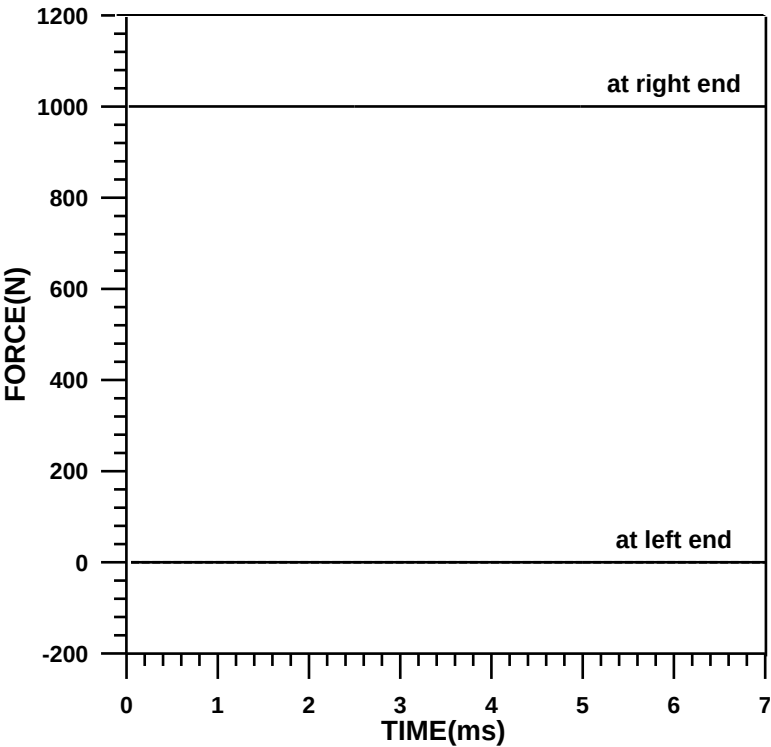
(3-8C): Displacement at Tip End

Figure (3-8): Dynamic Response for Cantilever Member with Harmonic Forcing Function

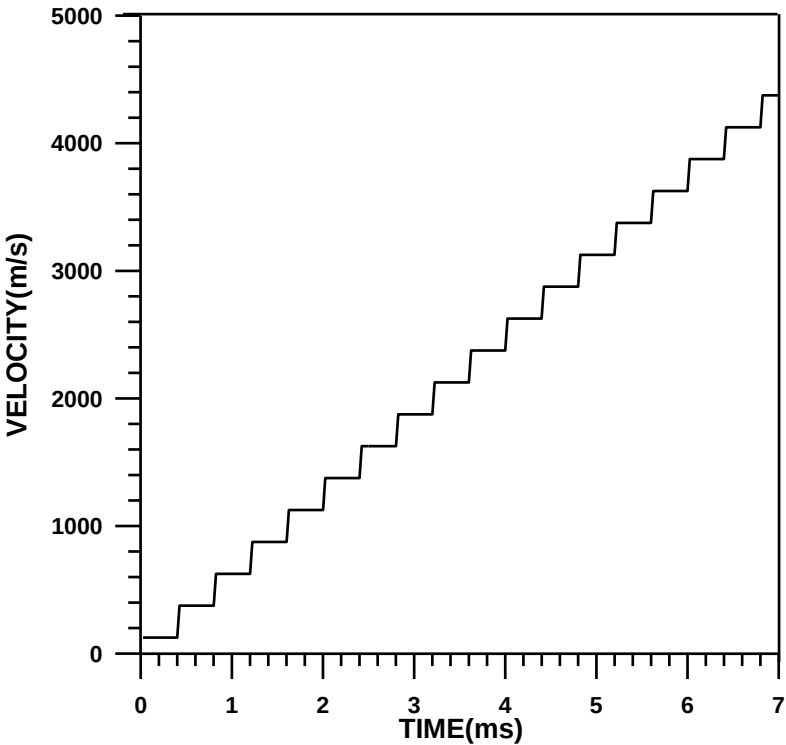
$$E=2 \times 10^{11} \text{ Pa}, A=0.8 \times 10^{-4} \text{ m}^2, \rho=8000 \text{ kg / m}^3, L=1.0 \text{ m}, F(t)=5000 \sin(\omega t), \\ LT = 7 \text{ ms}, \omega=5129.047 \text{ rad/sec.}$$

EXAMPLE NO.2: MEMBER FREE AT BOTH ENDS:

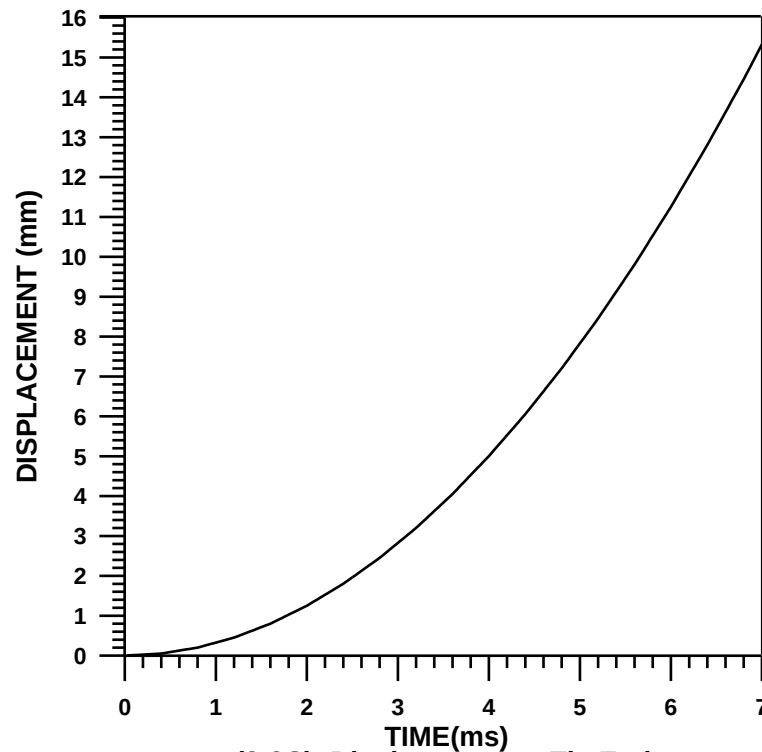
As explained in example number one, about the unrestrained joint, where the reaction force is zero, however, the velocity of the joint is unknown. Figure (3-9) represents some relations with time, for prismatic cross-sectional member, imposed to impulse forcing function of 1000N at the right end. The properties of this member are detailed in Figure (3-9). These figures represent the following relations with time: axial force at both ends, the velocity at right end and the displacement at right end. The amount of the amplitude of force wave is exactly equal to the subjected forcing function, where, there is no reflection for waves (i.e. the force wave does not find a boundary which enables it to reflect and return back). On the other hand, the amplitude of velocity and displacement wave is very large, when it is compared with other types of boundaries.



(3-9A): Force Response



(3-9B): Velocity Response at Right End



(3-9C): Displacement at Tip End

Figure (3-9): Dynamic Response for Member Free at Both Ends

$E=2 \times 10^{11}$ Pa , $A= 2 \times 10^{-4}$ m² , $\rho=8000$ kg / m³ , $L=1.0$ m, $F=1000$ N, $LT=7$ ms

EXAMPLE NO.3: MEMBER RESTRAINED WITH SPRING :

Many engineering applications in the analysis stages present the flexible boundaries as a spring with a linear or nonlinear stiffnesses. It is convenient to replace any structural element by a spring or springs its stiffness is the element stiffness, also the behavior of soil in many cases can be represented by springs. Therefore, it is significant to study this case. Its elastic behavior is approximated by additional grid (imaginary or fictitious). The ratio of ($\frac{EA}{L}$) of this additional grid are made equal to the elastic coefficients (stiffness) of the spring.

Figure (3-10) shows the behavior of member (column) supported by a beam of flexural stiffness $K = \frac{48EI}{L^3}$

where E = modulus of elasticity, I = moment of inertia($\frac{bh^3}{12}$, where: b = width of section, h = height of section), L_1 = length of the member.

Here : $E = 200 \text{ GPa}$, $I = 4 \times 10^{-8} \text{ m}^4$, $L_1 = 1.566 \text{ m}$, $\therefore K = 1 \text{ MN/m}$

It can be noticed that the amplitude of force wave is more than (two times) the subjected forcing function, (i.e. the dynamic intensification factor is more than (2)). Also the natural time period is large, so the spring may reduce the wave speed; as a result, it can affect the natural frequency of the member.

In order to understand the influence of the elastic supports at boundaries, a case study with different elastic stiffness of the spring at the fixed end is simulated and considered herein, see Figure (3-11). For the cases when the stiffness is very small (approaches from zero) the member behaves like free ends (rocket member), while the case with large stiffness magnitudes (or approaches from infinity) shows a behavior like a fixed end member.

From that, it can be concluded that the representation of the fictitious grid length, which is explained above is suitable for the Method of Characteristics (M.O.C).

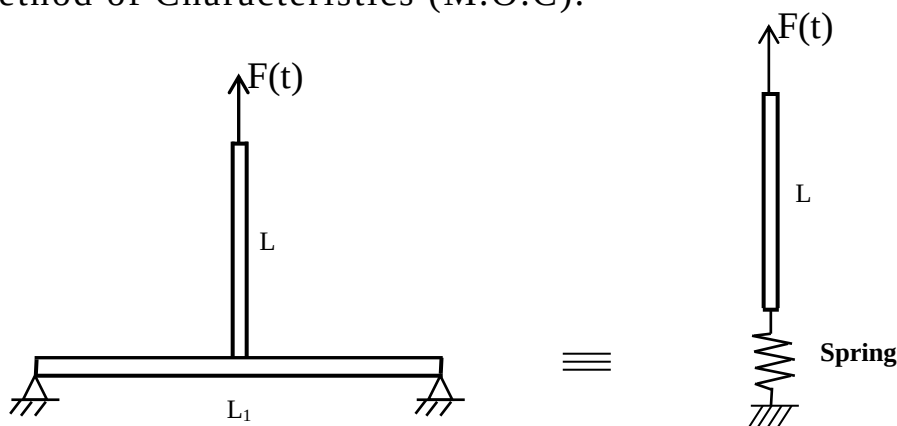
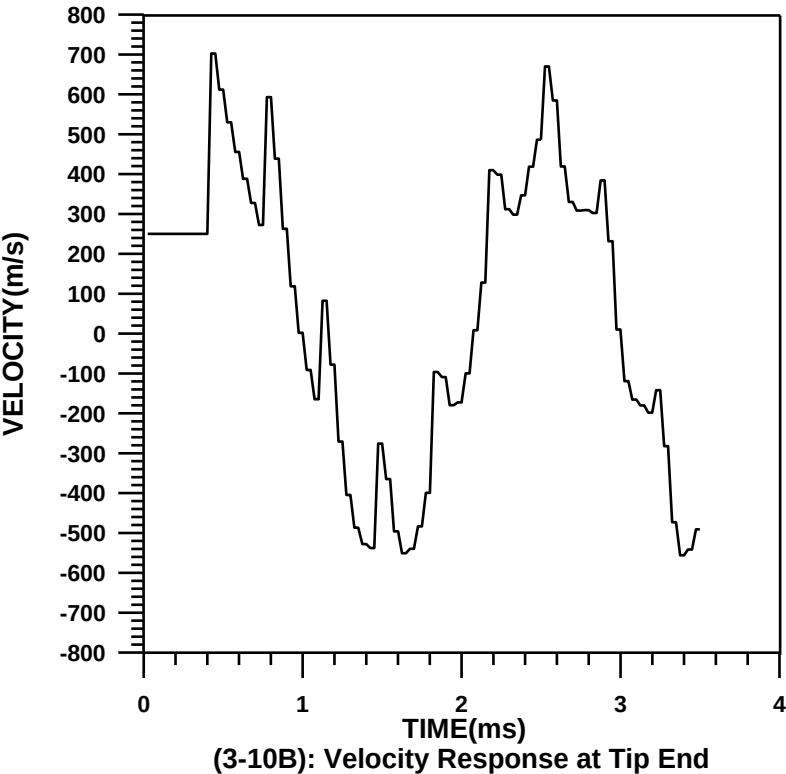
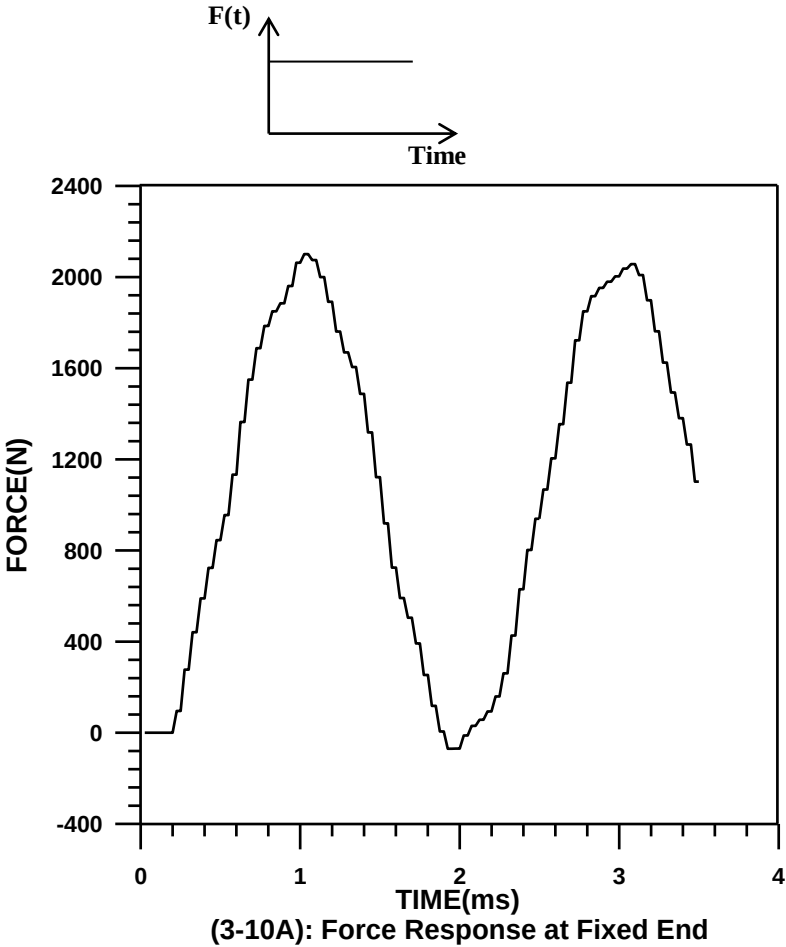


Figure (3-10): Continued



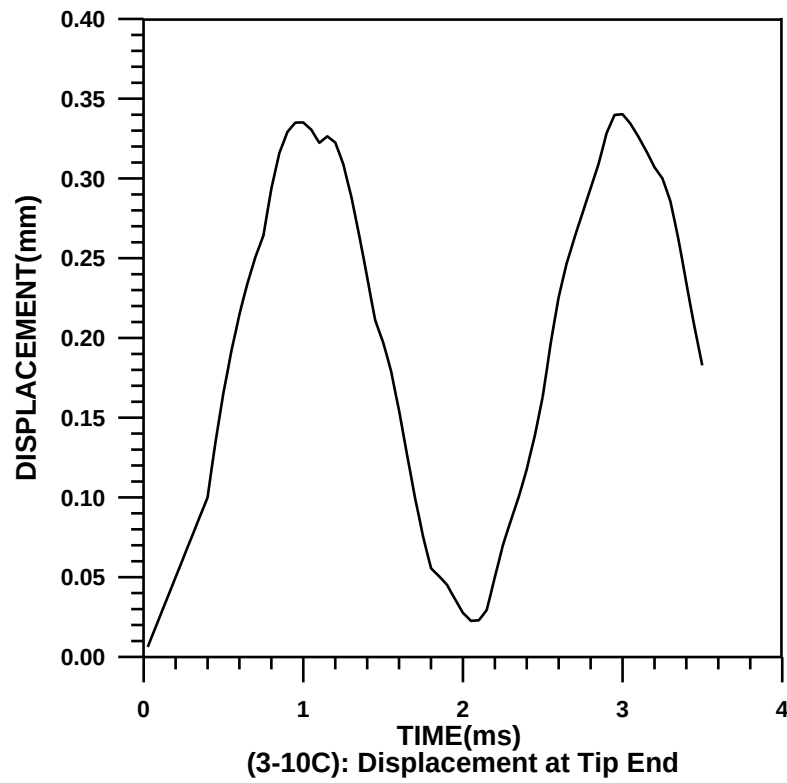
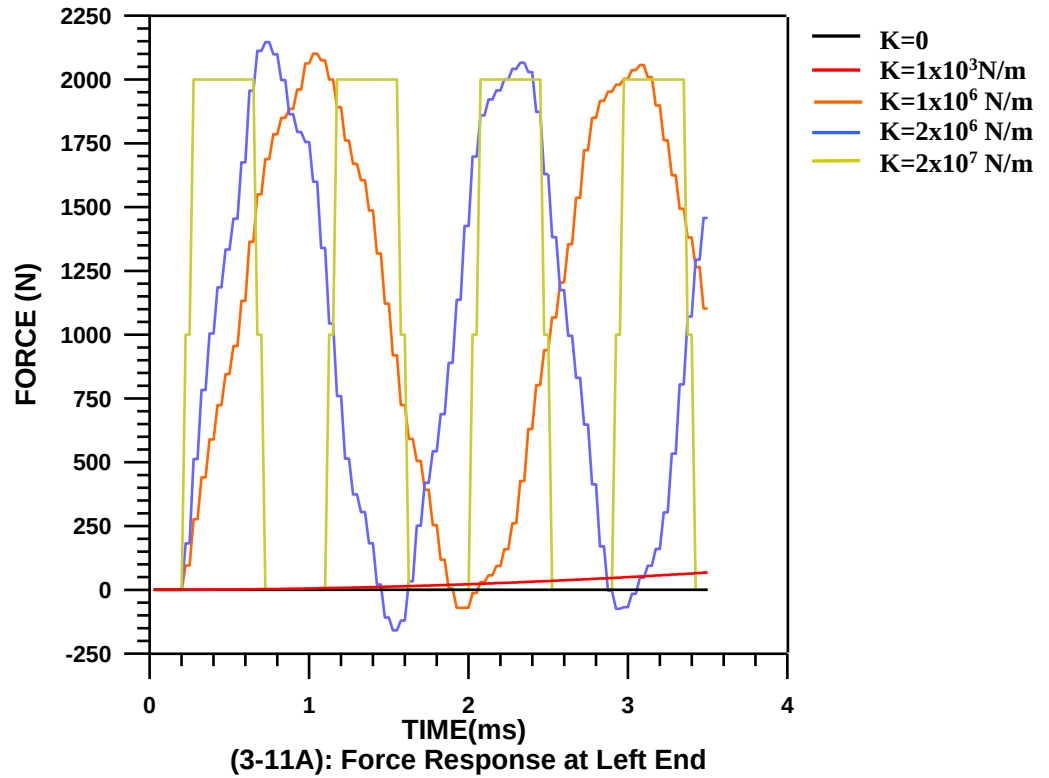


Figure (3-10): Dynamic Response for Cantilever Member Restrained with Spring

$E=2 \times 10^{11}$ Pa , $A=2 \times 10^{-4}$ m² , $\rho=8000$ kg/m³ , $L=1$ m , $F=1000$ N
 $LT=3.5 \times 10^{-3}$ sec , $K=1 \times 10^6$ N/m



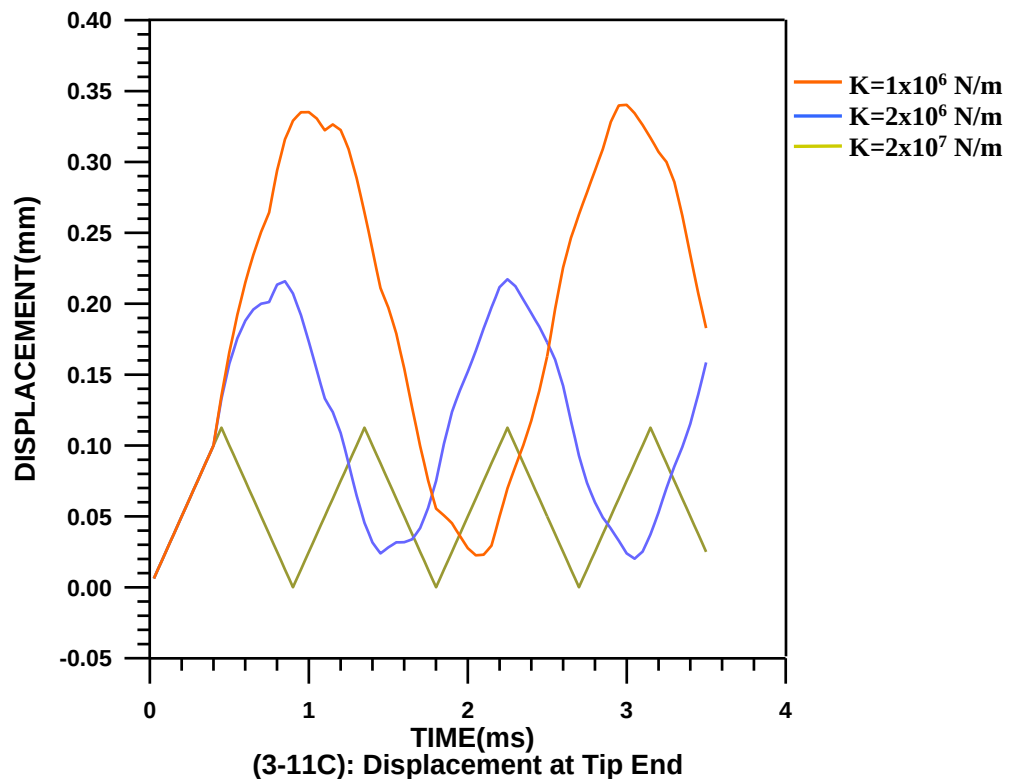
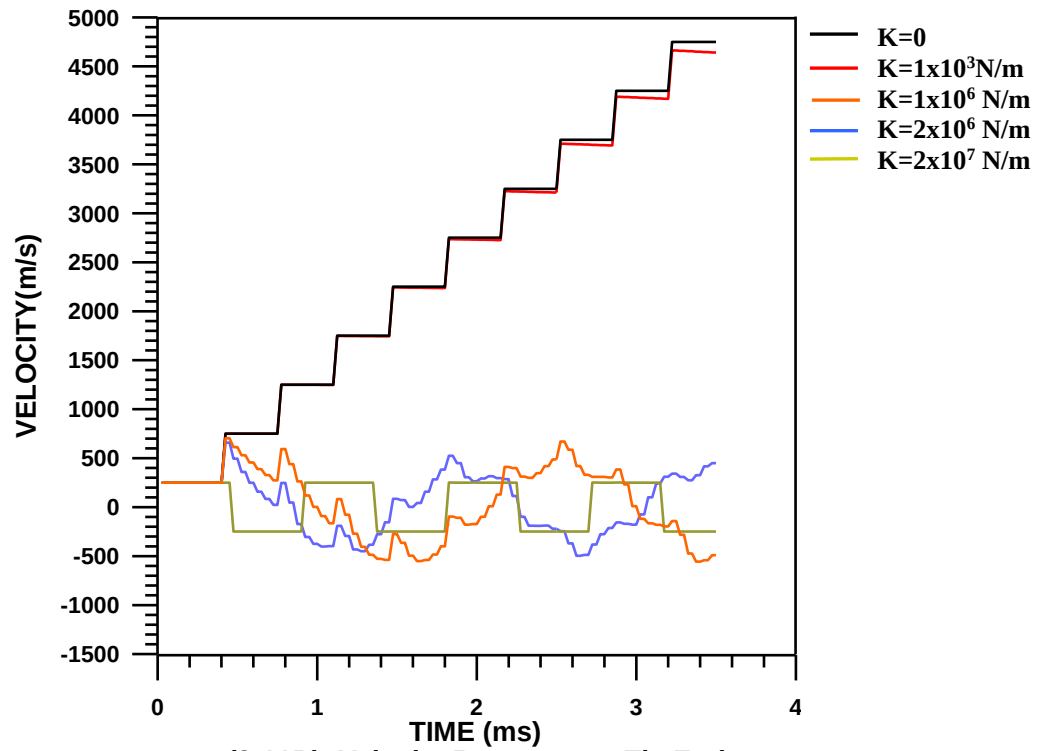


Figure (3-11): Dynamic Response for Cantilever Member Restrained with Spring and Various Modulus of Elasticity

$E=2 \times 10^{11}$ Pa, $A= 2 \times 10^{-4}$ m², $\rho=8000$ kg/m³, $L=1.0$ m, $F=1000$ N, $LT=3.5$ ms

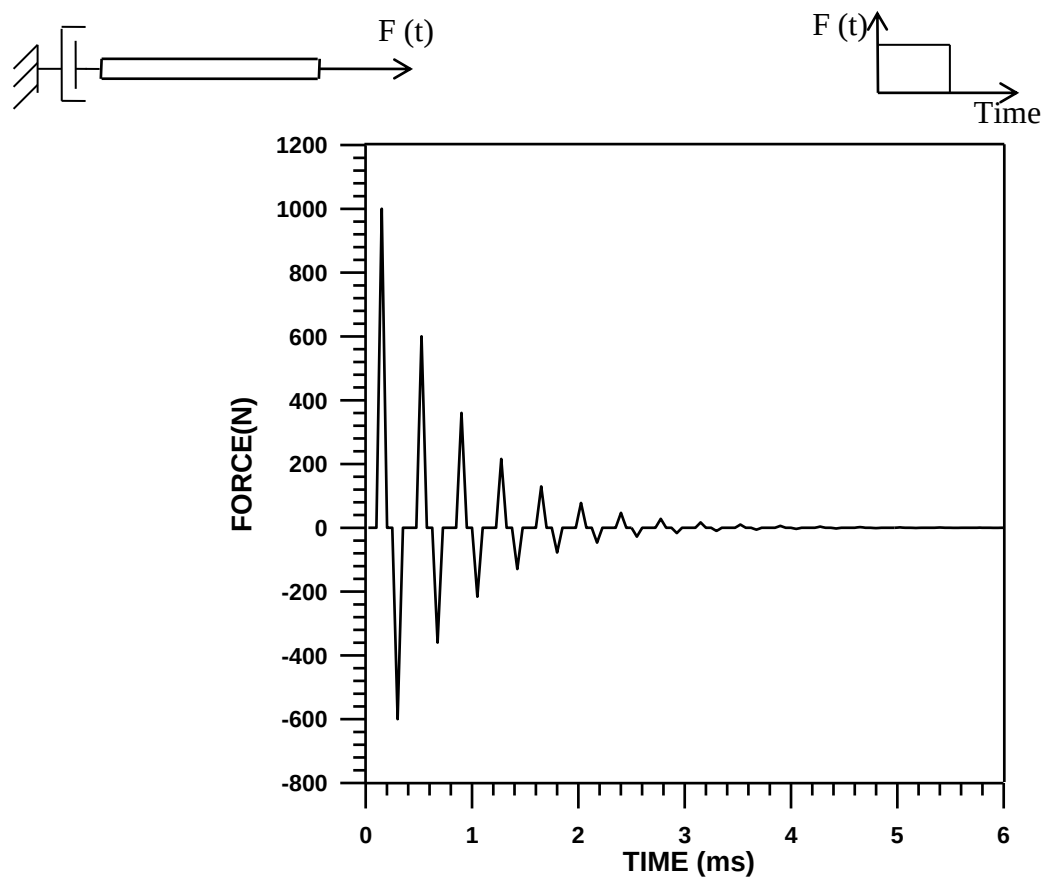
EXAMPLE NO.4: MEMBER RESTRAINED WITH DAMPER:

As it is known; there are two types of damping: viscous damping and artificial damping. In general, for purposes of analysis, structural damping may be assumed to be viscous type, i.e., the damping force is opposite but proportional to the velocity.

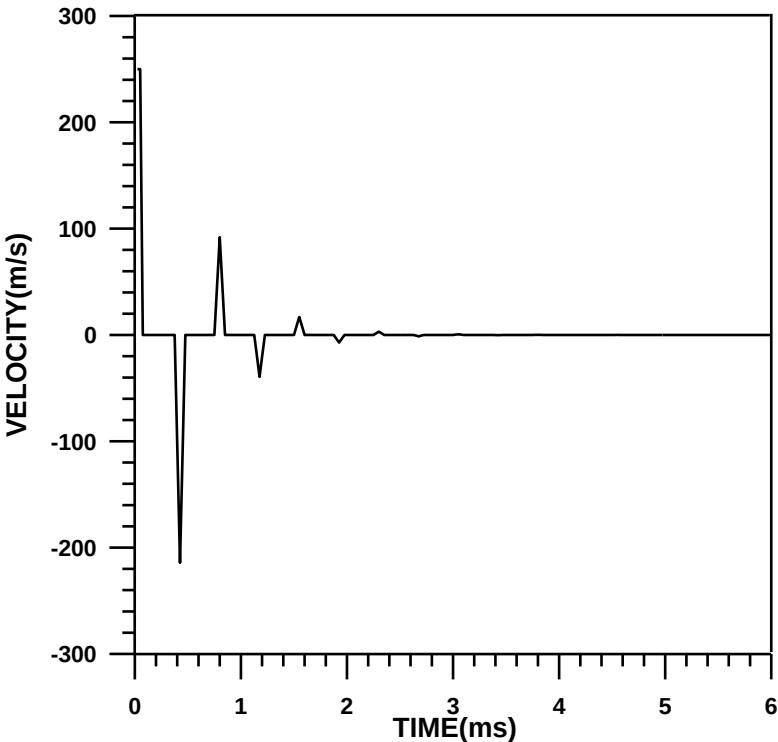
All structural dynamic systems contain damping, but the effect may not be significant if the load duration is short⁽¹⁵⁾. As in the present study, so the interest will be on artificial damping. This type of damping represents any material in the structure trend to damp the structure vibration. In some cases the gusset plate and bolted connection which are used to connect some structures trend to damp the excitations, which may be considered as artificial damping.

Figure (3-12) represents the behavior of member with damper at one end. It can be noticed that the reflected force wave exactly equals the subjected forcing function, (i.e. D.I.F =1), but the amplitude of displacement has increased gradually until reaching the stability on the magnitude (0.05mm), which is (two times) the static solution

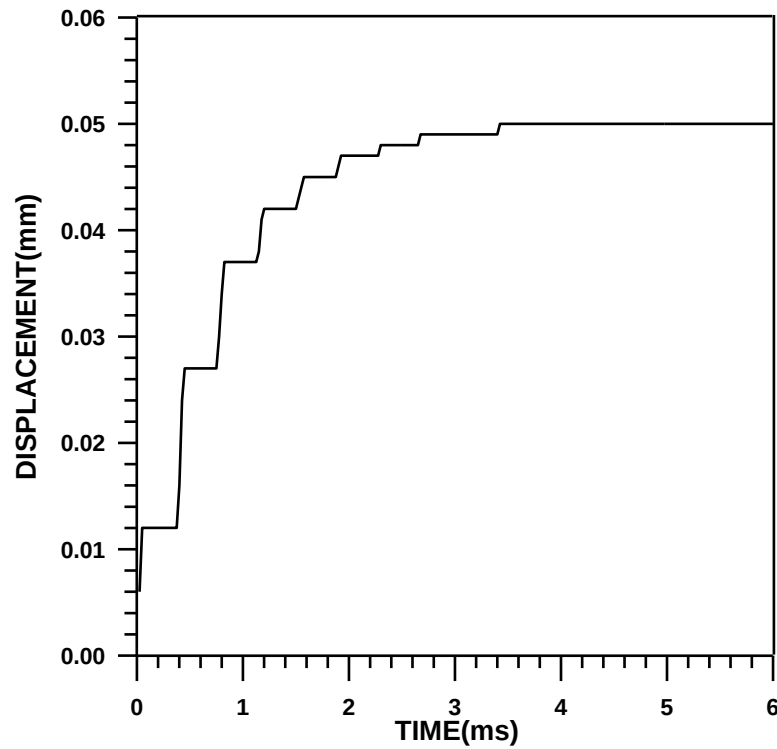
$\left(\Delta_{\text{static}} = \frac{PL}{AE} = 0.025 \right)$. The analysis of this case resembles that for member restrained with spring. As the damper coefficient is increased the member behavior converges to fixed end member. On the other hand, when the damper coefficient is decreased, the member behavior converges to free ended member, as noticed in Figure (3-13).



(3-12A): Force Response at Mid Point



(3-12B): Velocity Response at Tip End

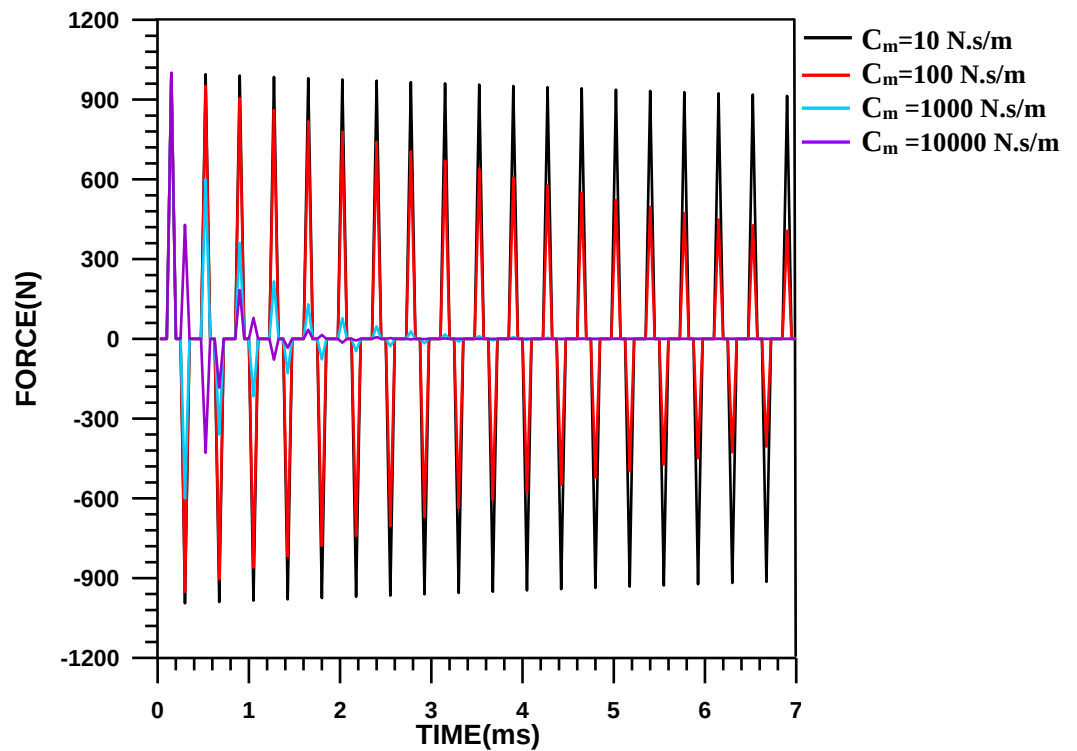


(3-12C): Displacement Response at Tip End

Figure (3-12): Dynamic Response for Member Restrained with Damper

$E=2 \times 10^{11}$ Pa , $A=2 \times 10^{-4}$ m² , $\rho=8000$ kg/m³ , $L=1$ m , $F=1000$ N

$LT=0.05$ ms , Damping Coefficient = 10000 N.s/m



(3-13A): Force Response at Mid Point

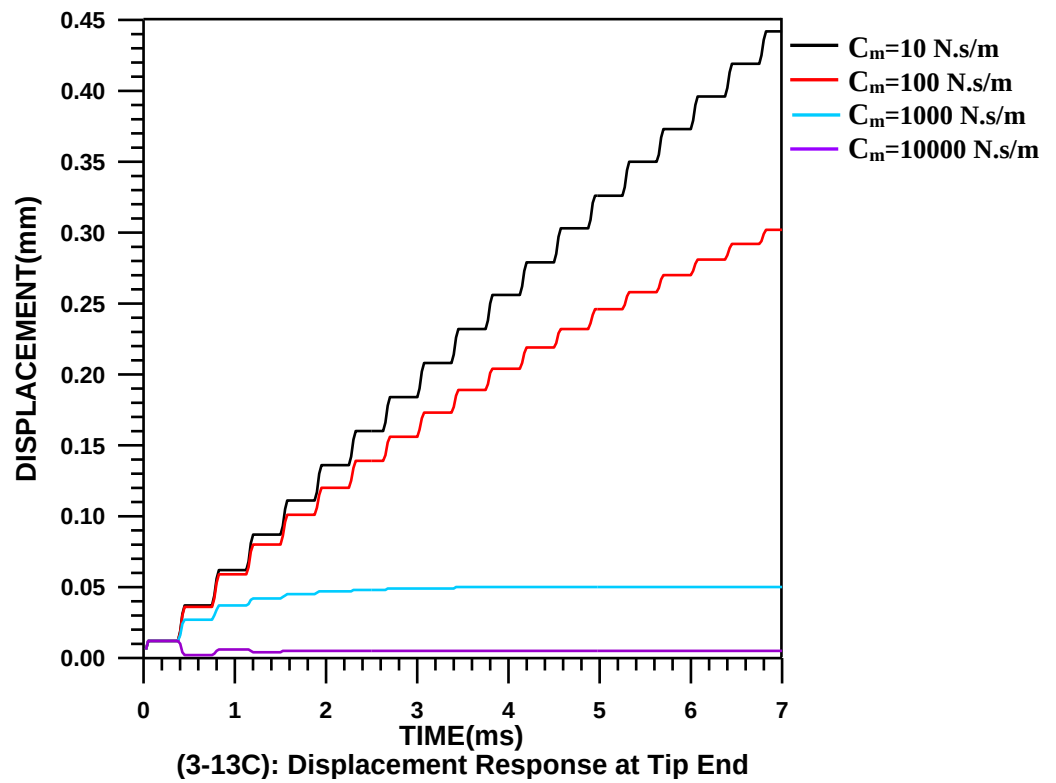
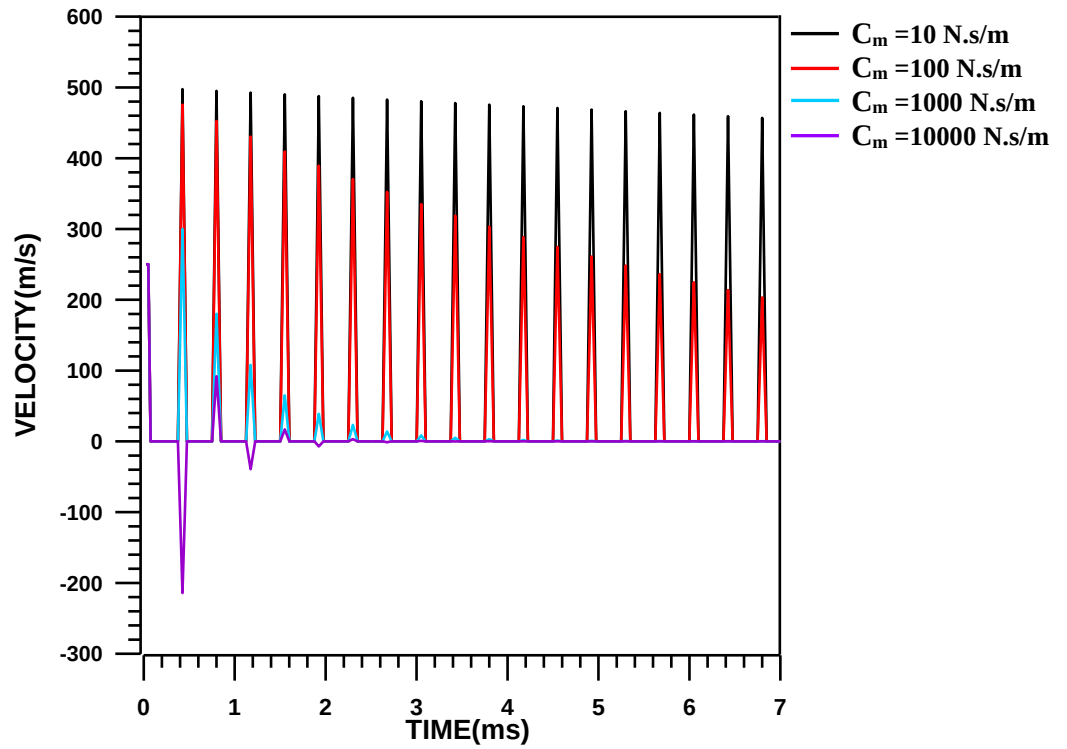


Figure (3-13): Dynamic Response for Member Restrained with Damper and Various Damping Coefficient

$E=2 \times 10^{11}$ Pa, $A=2 \times 10^{-4}$ m², $\rho=8000$ kg/m³, $L=1.0$ m, $F=1000$ N,
 $L T=0.05$ ms

CHAPTER FOUR

NON-LINEAR ANALYSIS

OF ONE-DIMENSIONAL MEMBER

4.1: Introduction:

Structures are usually designed on a small deflection assumption and that the material remain within elastic limit when subjected to any dynamic loading. However, there are situations in which the structure has to be designed for an excitation of large amplitude, such as severe earthquake motion. In these cases, it is not realistic to assume linear behavior for such a structure. So, it is necessary to design the structure under the assumption of large deformation.

When the member is subjected to large external excitation, material and geometrical properties will change. The linear case is considered when these changes are small. When the changes are large, the case is called non-linear behavior. It can also occur in the case of resonance loading. So, the changes in properties will be noticeable and linear system cannot give the reasonable solution.

4.2: Non-Linear Behavior :

Uncertainties in the analysis are introduced due to the limited ability of analytical models to describe the linear and non-linear responses of complex structures subjected to dynamic loads. Non-linearity can be classified into two types according to the reason which may lead to non-linearity: geometrical non-linearity and material non-linearity. Both geometrical and material non-linearity will be considered in this study.

4.2.1: Geometrical Non-linearity:

In general, non-linearity is due to the changes in material and geometrical properties of the member as discussed in the preceding section. In this study, the geometrical non-linearity due to the change in length is considered only. The effect of the change in area of the cross-section will be neglected to simplify the analysis. This will produce unreal case, but it is required as a first step to study the geometrical non-linearity in the (one-dimensional) members.

In linear analysis, the change in length is assumed to be negligible. Therefore, the length of the member remains constant during the analysis. In the present development of the analysis, a new length of the structure is calculated every time step. This could lead to a change in natural time period and in amplitude of vibration, thus producing non-linear behavior of the member.

4.2.2: Material Non-linearity:

In principle, the extension of the analysis to include material non-linearity in members is straightforward. This type of non-linearity is introduced when the structures material is loaded above its yield point, the structure will then tend to be less stiff and permanent displacement will exist. In this condition the material properties are varied with the loading level, i.e. the constitutive laws are functions of the current load.

In each time step an appropriate modulus of elasticity must be chosen for the member, based upon the instantaneous strain in the member and the specified stress-strain characteristics⁽¹⁸⁾.

When non-linear elastic behavior is assumed for one or more members in a structure, the stability of the structure needs to be carefully considered. Consider for example the non-linear stress-strain relationship shown in Figure (4-1), for materials which may be represented by bilinear stress-strain relationship. The

elastic constant (E) is linear within the central region (elastic zone) and (αE) outside this region (plastic zone), when the material is going into the reduction in elastic constant the value of (α) is less than one and approaches from zero for some materials.

The implication of a zero (αE) is that the member converted to a mass, i.e. the member has lost its stiffness, and only its mass is considered. It is reasonable to imagine the member to have been removed from the structure and replaced by lumped masses at its supporting joints. When sufficient members are removed from a structure then it may become unstable as the loads on the remaining members will increase. To guarantee a static equilibrium condition, a stable structure must be ensured. It is therefore necessary for the designer to ensure that only redundant members can operate in the zero modulus state.

In determining the modulus of elasticity, a practical difficulty arises. It is that the elastic condition needs to be known before the strain condition is obtained. The magnitude of (α) has taken (0.1), which approaches the behavior of the steel structures⁽²¹⁾.

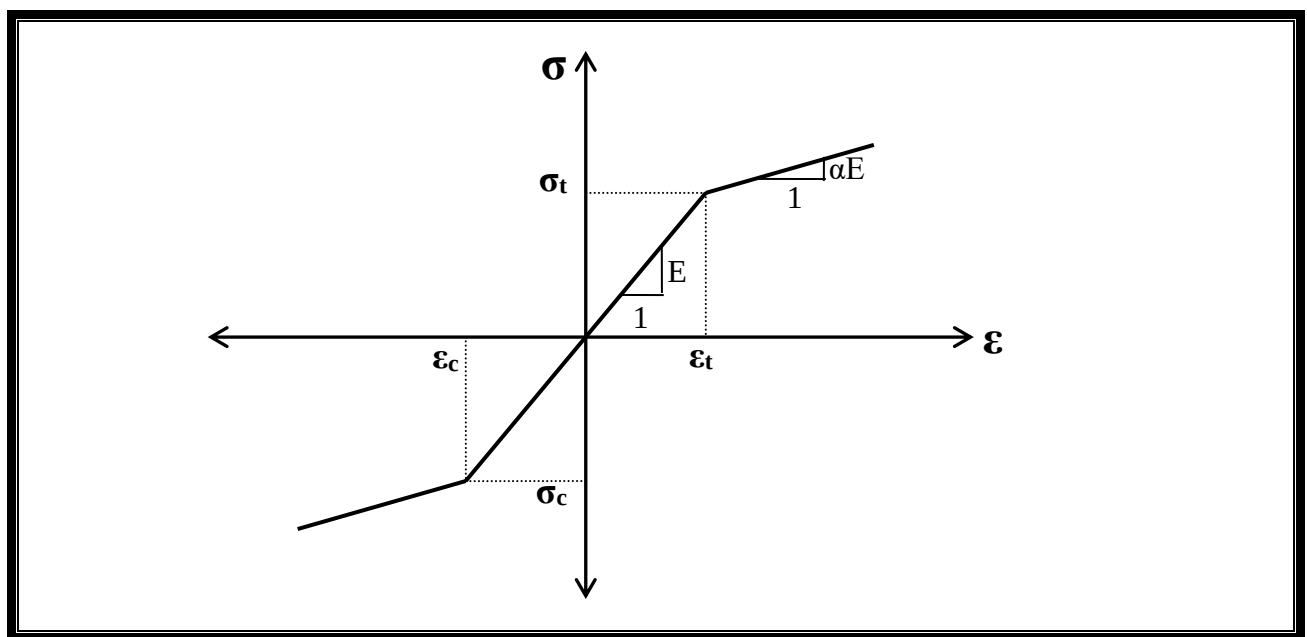


Figure (4-1): Bilinear Stress–Strain Relationship

4.3: **Method of Characteristics:**

The two characteristic equations, previously derived, are assumed fixed grids (small deflection at each grid). When deflection is large, the grid points positions are changed considerably. This leads to a change in the grid lengths. Thus, the time required for the wave to propagate between two grid points is changed. It will be more or less than the original time depending on the sign of deflection (i.e. compression or tension strain).

In elastic material, the wave speed is constant, while the grid length (Δx) will be changed with time. Therefore, the time increment (Δt) for every grid length will be changed. In the solution, the lowest (Δt) is taken as the basic time interval. In order to obtain the structure information for this situation, an interpolation technique must be used along the time intervals (refer to article 4.5).

The deflection is calculated for each grid by integrating their velocities and adding it to the old length to obtain the new grid length for the next step.

Generally, the following assumptions are adopted :

- 1- The length of each grid is calculated at the beginning of the time interval, and remains constant throughout the time interval.
- 2- The lowest time interval is adopted.
- 3- Linear characteristic equations are used for every time interval.

This is because the **Method of Characteristics (MOC)** is considered as a step-by-step method, and the summation of linear behavior at each time step with updated nodes will lead to non-linear behavior.

4.4: **Wave Behavior:**

If the large deformation in a member subjected to an impulsive load is considered, the wave will be reflected at each interface between two grids. Thus, the wave will move in a changing form (as will be discussed later), where the grid

points act as points of boundaries, this is due to the change in the geometry in each grid length. Thus the behavior of the wave will change with time. This can be called as the dispersive phenomenon. It is well known that in linear analysis the axial wave is non-dispersive.

4.5: Interpolation Methods :

In Characteristic equations, all values of relevant parameters at the old time level are already known. This is true only with characteristic lines which proceed from grid points such as left and right (L & R) as shown in Figure (3-2). It is untrue for all other lines. Therefore, interpolations are necessary between grid points at which the conditions are known. Interpolation is required in many cases, such as flexural wave or in non-linear axial or torsional vibration. Interpolation are usually sources of numerical errors in dynamic analysis of members. Therefore, care must be taken to choose the type and order of interpolation.

As shown in Figure (4-2), there are three different methods of interpolation within a fixed grid :

1. x-wise interpolation:

It is the simplest and most commonly used method. In this method, the interpolation is between adjacent grid points at the same time level. For example, see Alsarraj (1989)⁽¹⁰⁾.

2. t-wise interpolation:

This method needs more space to store old time information. The interpolation is between present and previous time levels at the same grid point. See Goldberg & Wylie (1983)⁽³³⁾.

3. x-t wise interpolation:

This method is more complicated and rarely used. Interpolation in the x-t wise interpolation is done from adjacent grid points at different time levels. See Almousawi (1983)⁽⁶⁾.

In this study, t-wise interpolation is used with a double precision system of computer digits to overcome the numerical errors. Linear interpolations are made and gave acceptable results.

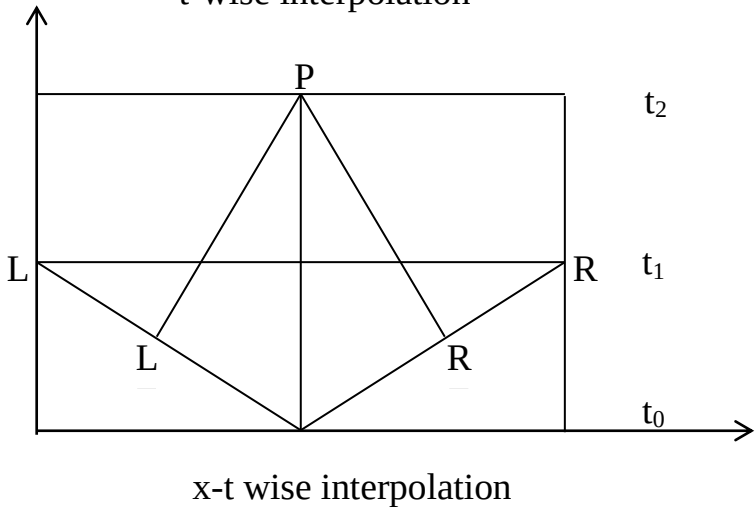
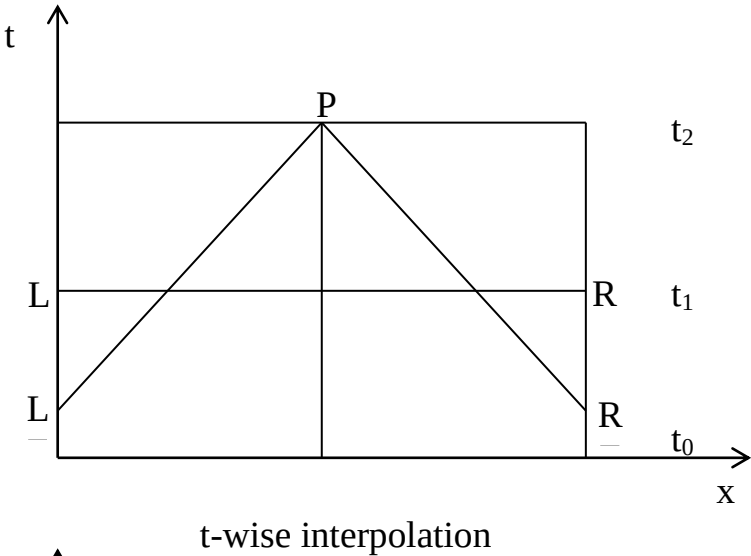
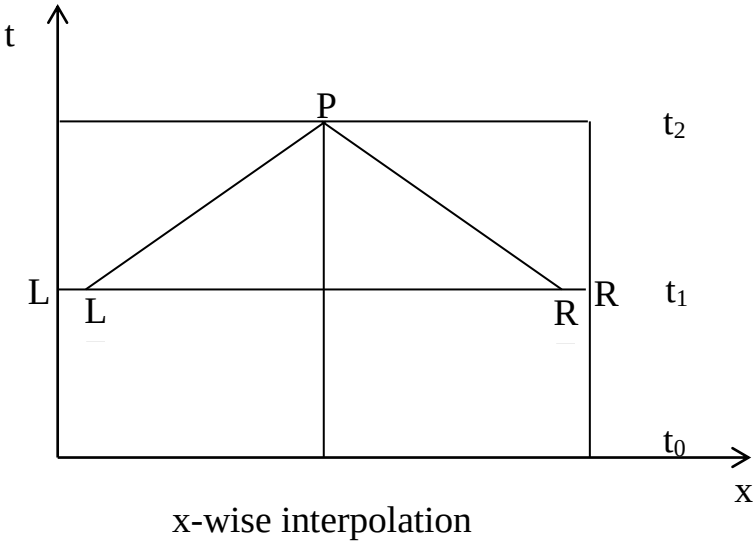


Figure (4-2): Interpolation Methods

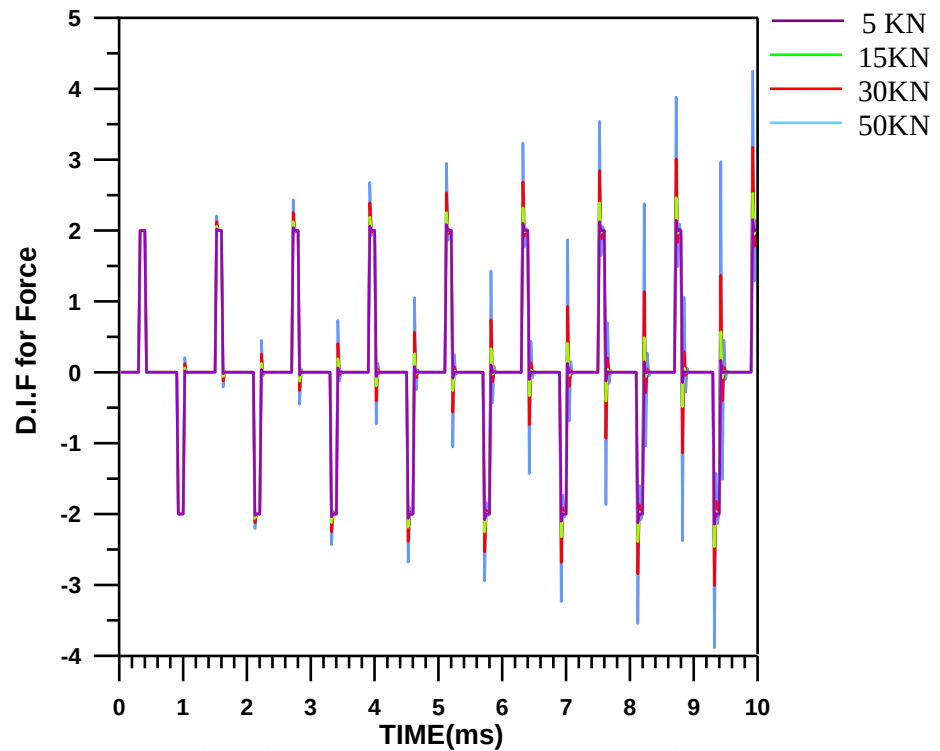
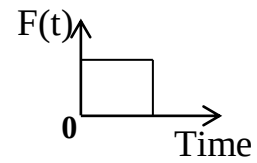
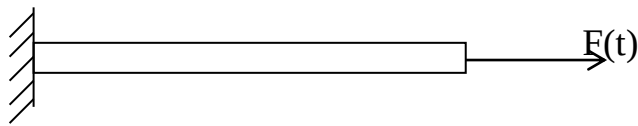
4.6: Dynamic Behavior of One-Dimensional Members :

There are many parameters affecting the non-linear behavior of one-dimensional member namely: the loading type, loading direction, and material properties.

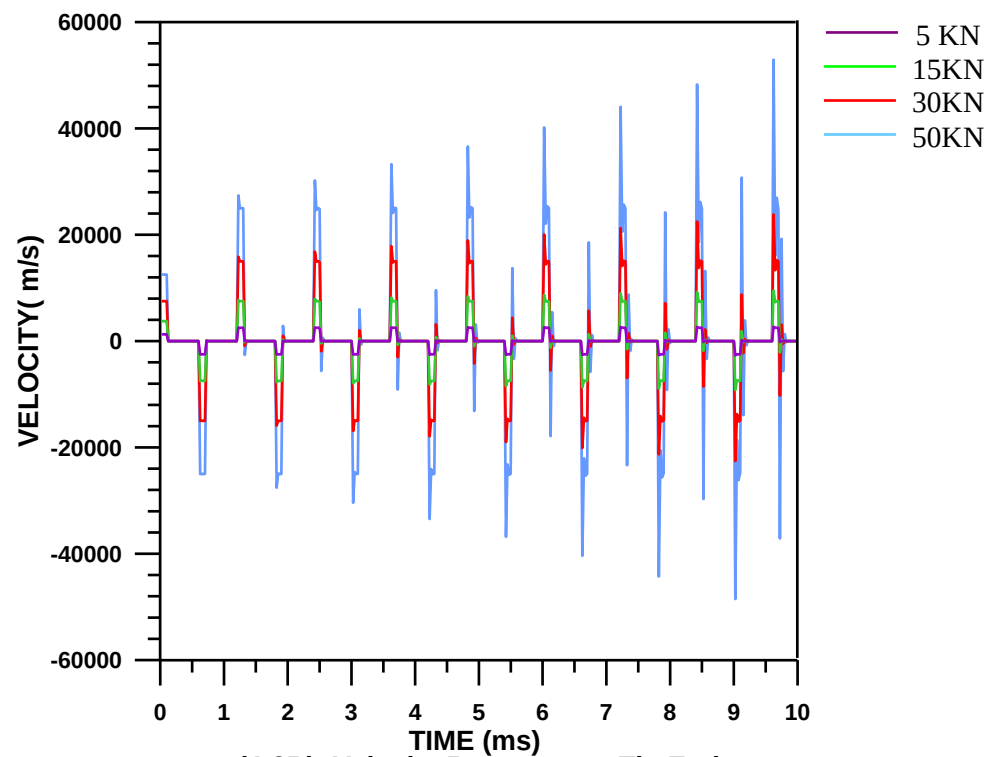
The t-wise interpolation is used in this study to avoid the unuseful difficulty of the moving grids in interpolation. One-dimensional member has been examined and different impulse loads magnitudes are considered, as shown in Figure (4-3). Non-linear analysis causes the change in natural time period and amplitude of vibration. When the member is subjected to impulse forcing function of (5KN), the dynamic behavior of the member is linear and the effect of geometrical non-linearity does not appear, but when the subjected forcing function has increased, the effect of geometrical non-linearity can be noticed, where the amplitude of waves has increased gradually when the forcing function has increased.

Figure (4-3) includes a comparison not only with different magnitudes of impulse loads but also with static displacement. The ratio between displacement at dynamic loading and displacement at static loading is denoted by Dynamic Intensification Factor (D.I.F).

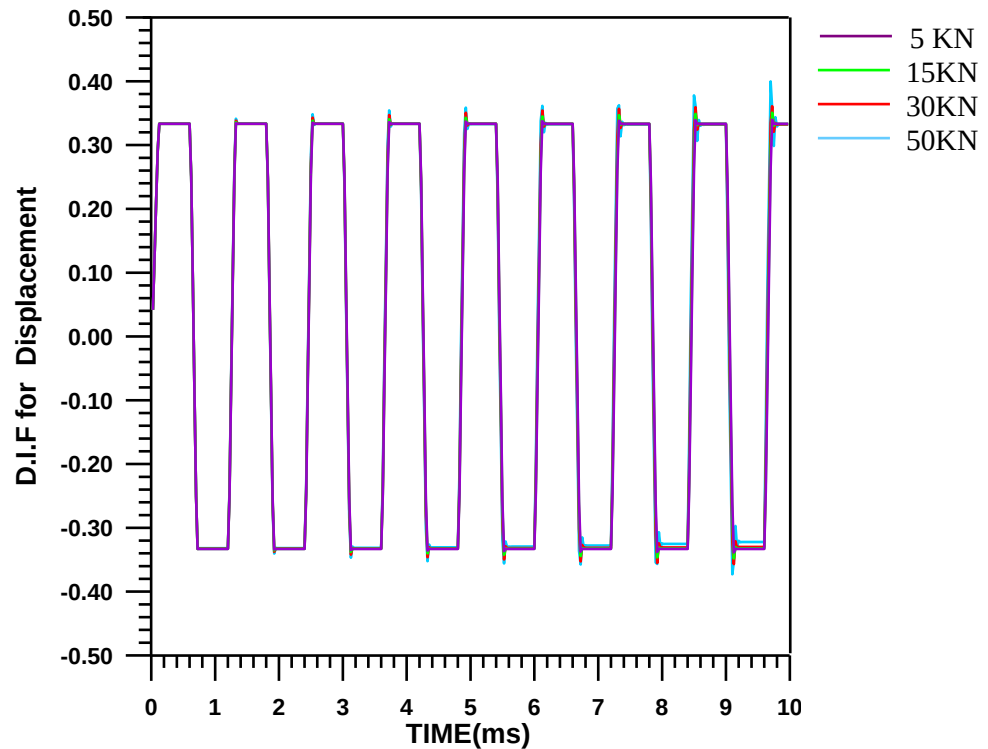
To assess effect of the load direction on one-dimensional member, the same cantilever member is subjected once to tension impulse forcing function, and then to compression impulse forcing function. The results showed that the tension axial load has large effect than the compression axial load and the geometrical non-linearity clearly appears. That is due to the relative large elongation of each grid, which may decrease the stiffness of the member, see Figure (4-4).



(4-3A): Force Response at Fixed End

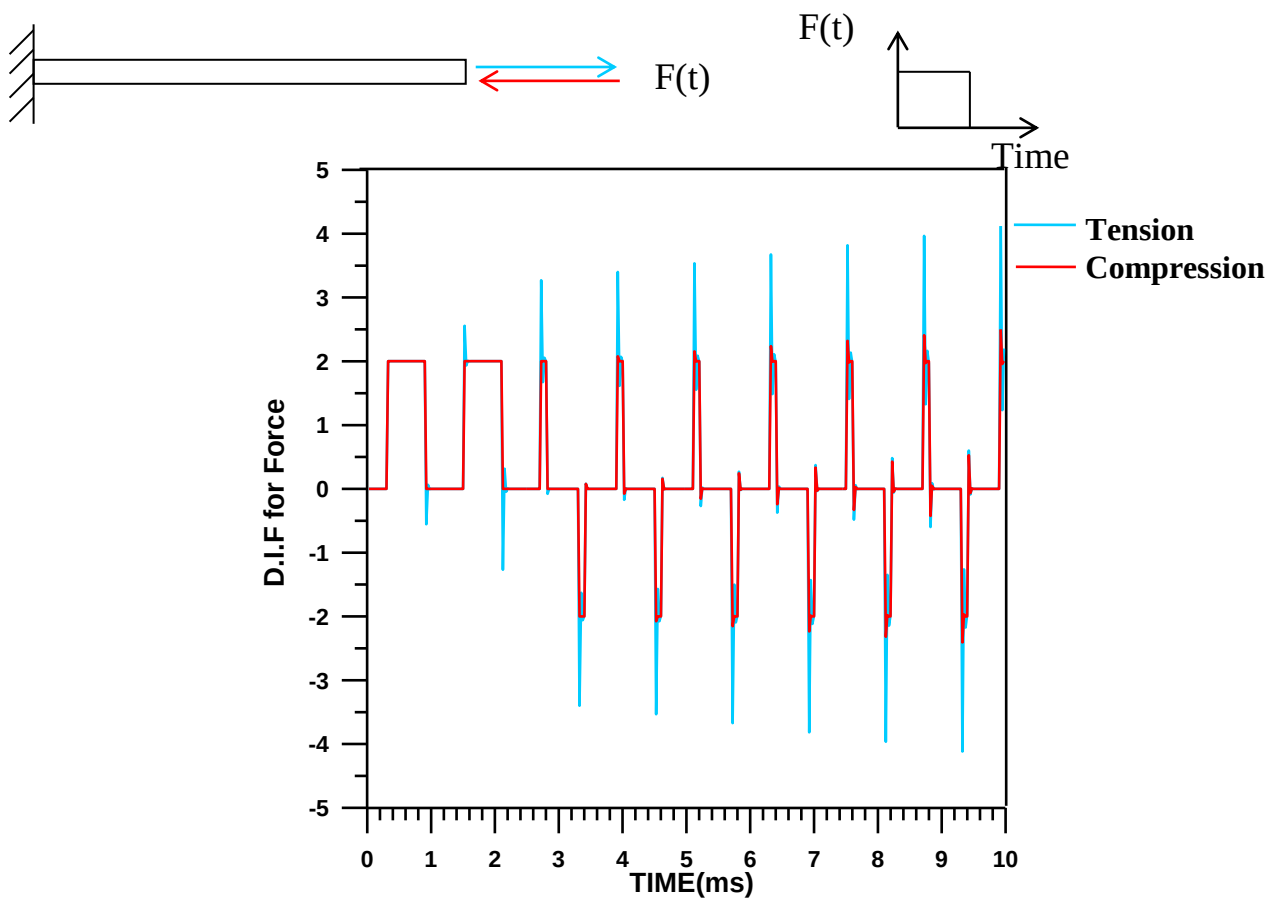


(4-3B): Velocity Response at Tip End



(4-3C): Displacement at Tip End

Figure (4-3): The Effect of Load Magnitude on Cantilever Member
 $E=200 \text{ GPa}$, $A=1 \times 10^{-4} \text{ m}^2$, $\rho=8000 \text{ kg/m}^3$, $L=1.5 \text{ m}$, $L_T=1 \text{ ms}$



(4-4A): Force Response at Fixed End

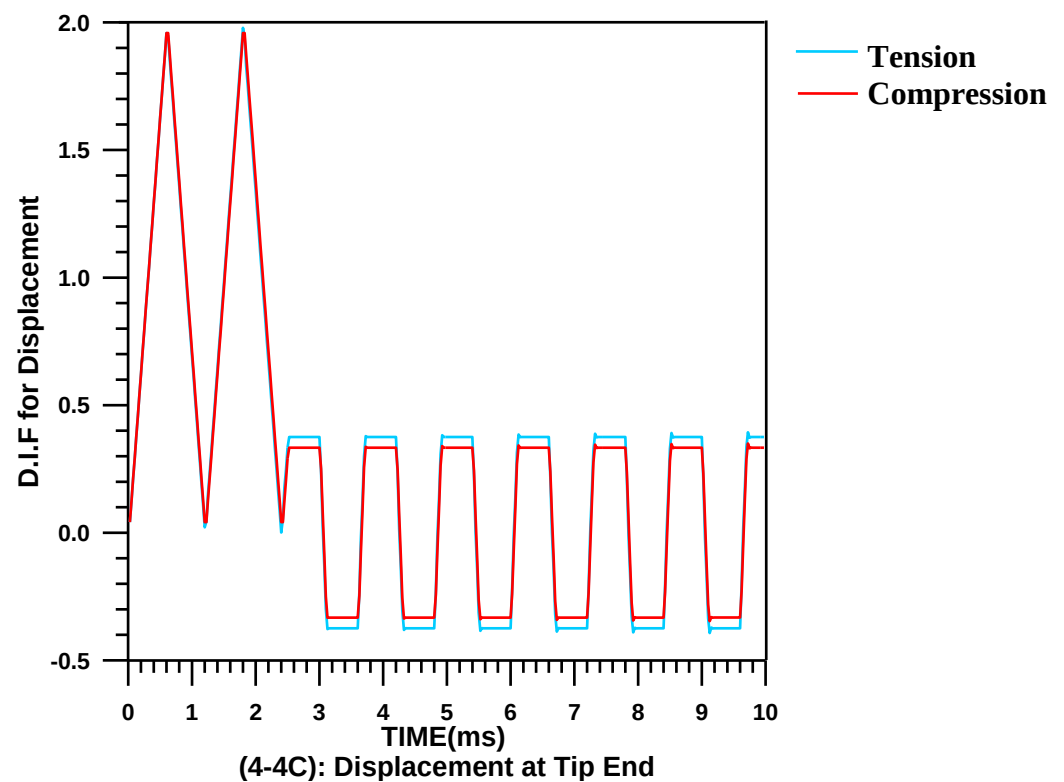
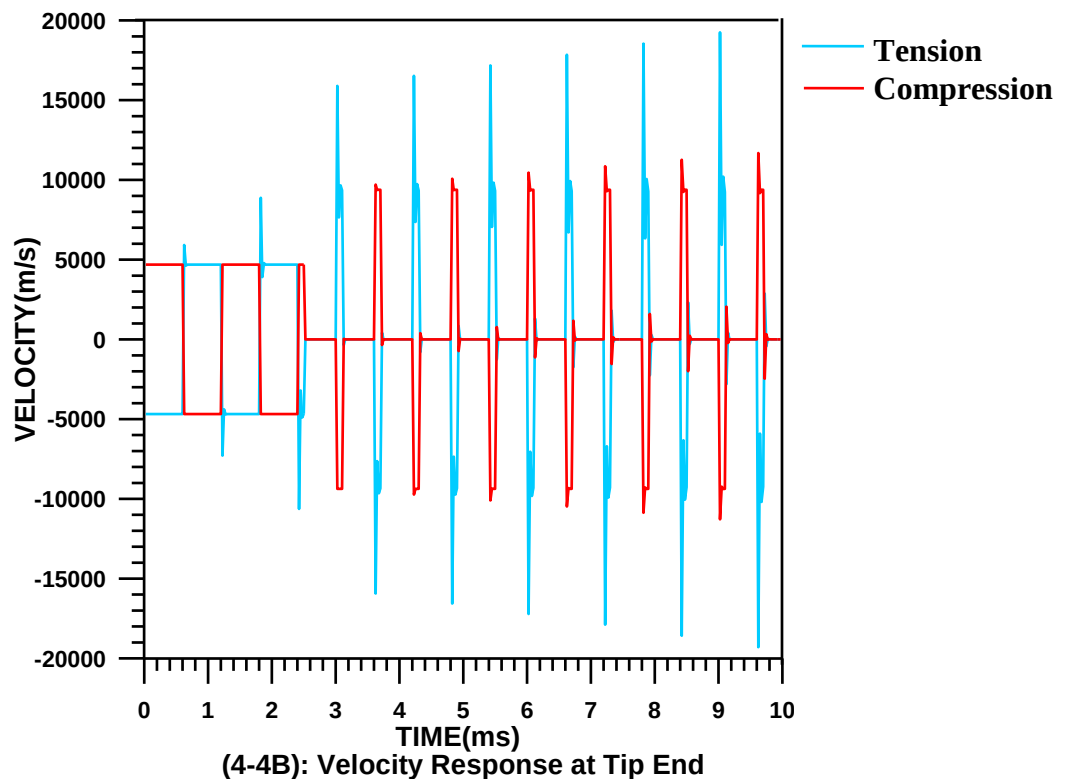
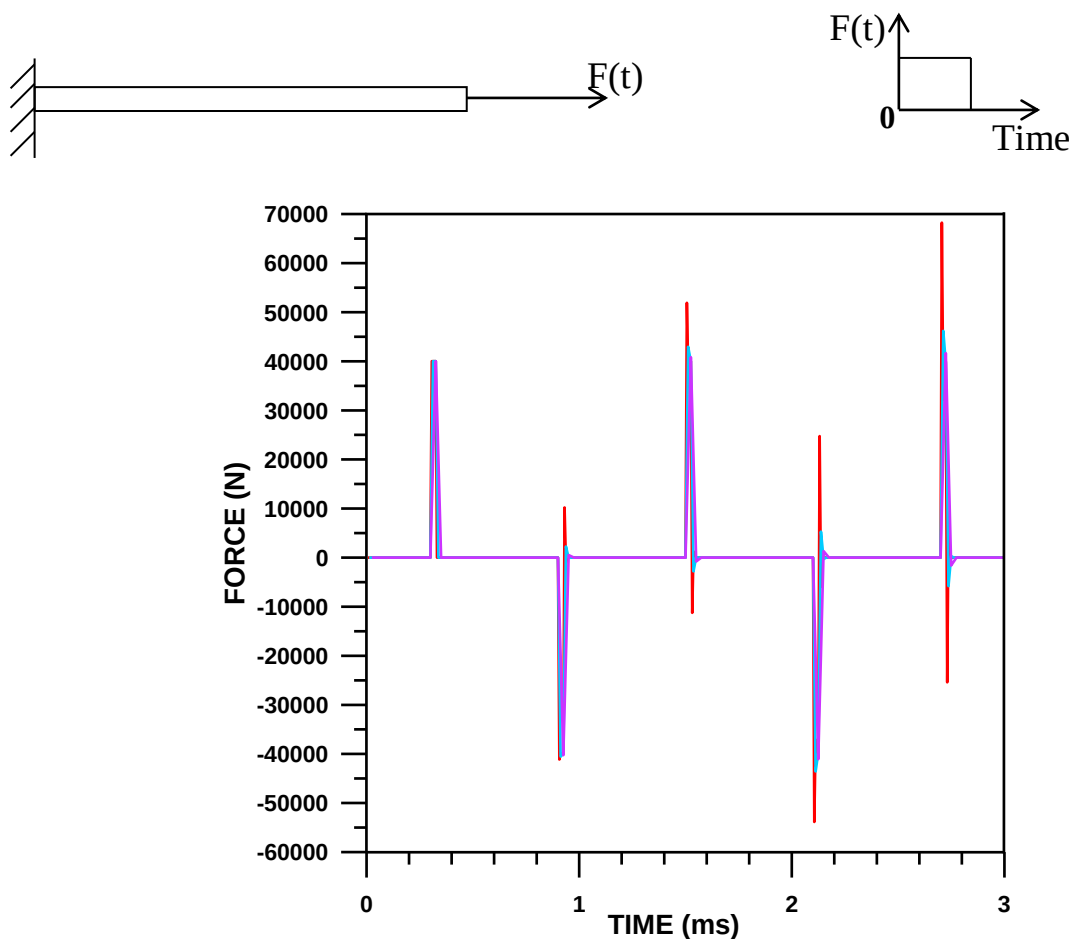


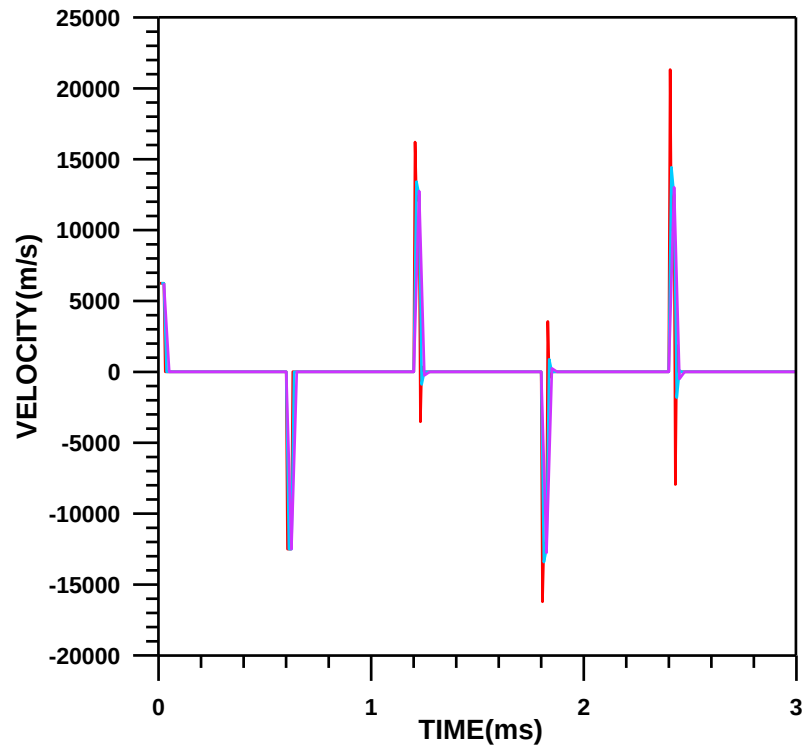
Figure (4-4): Effect of Load Direction on Cantilever Member
 $E=200 \text{ GPa}$, $A=0.8 \times 10^{-4} \text{ m}^2$, $\rho=8000 \text{ kg/m}^3$, $F=15000 \text{ N}$,
 $L=1.5 \text{ m}$, $LT=2.5 \text{ ms}$

4.7: Grid Size Dependence:

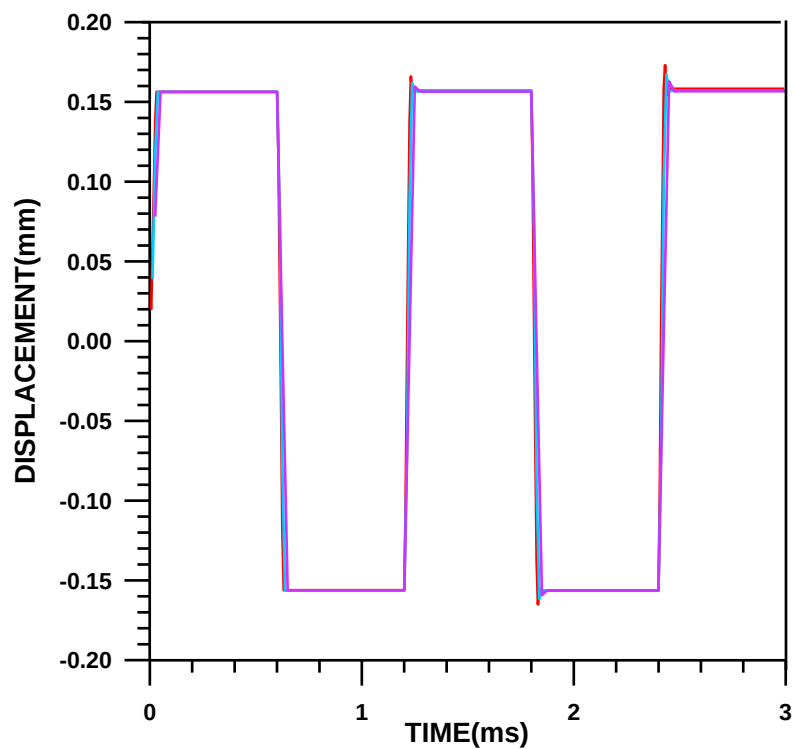
Figure (4-5) represents effect of the grid size on the solution for a cantilever member subjected to an impulse load. From the comparison of member behavior, it can be concluded that the four grid length is not suitable to represent the non-linear behavior as for the linear behavior which has been depicted in article (3-6). So, four grid length cannot track the non-linear behavior of the member, therefore eight grid length must be chosen for analyzing the one dimensional member.



(4-5A): Force Response at Fixed End



(4-5B): Velocity Response at Tip End



(4-5C): Displacement at Tip End

— 4 Grids per meter
 — 8 Grids per meter
 — 16 Grids per meter

Figure (4-5): Grid Size Dependence
 $E=200\text{GPa}$, $A=1\times 10^{-4}\text{m}^2$, $\rho=8000\text{kg/m}^3$, $L=1.5\text{m}$, $F=20000\text{N}$,
 $LT=25\times 10^{-6}\text{sec}$

4.8: Wave Front in Longitudinal Waves :

Longitudinal waves in a solid member resemble pressure waves in a fluid pipeline. When one end of the member is disturbed, the disturbance is sent to the other end where it is reflected back again.

In small deformation, the form of the disturbance does not change as it travels along the member. While for large deformation the form of the disturbance changes as it travels along the member. This is due to the change in the path of the traveled wave and also the arrival of stress wave in different grids that will be changed so, some inflected and reflected wave may be expected. This depends on the stress wave sign (i.e. compression or tension wave).

Figure (4-6) shows the propagation of a longitudinal wave under a constant applied axial force, so Figure (4-6A) shows the force wave before their arrival to the fixed end, and the magnitude of force wave has been still constant and equal to the subjected impulse forcing function for both linear and non-linear analysis. Figure (4-6B) show the force waves when they strike the fixed boundary and return back, so the magnitude of the reflected force waves are (two times) the magnitude of the subjected forcing function at the fixed boundary for both linear and non-linear. Also these figures show a comparison between small deformation, large deformation and material non-linearity. The comparison shows that the wave in small deformation does not suffer from energy dispersive, while in large deformation and in material non-linearity the energy of the wave is dispersive. Also the stress wave for the member in elastic behavior is faster than for member in plastic behavior.

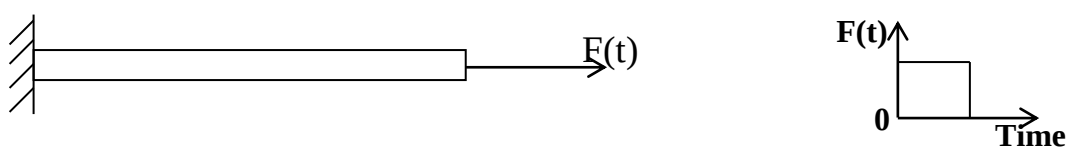
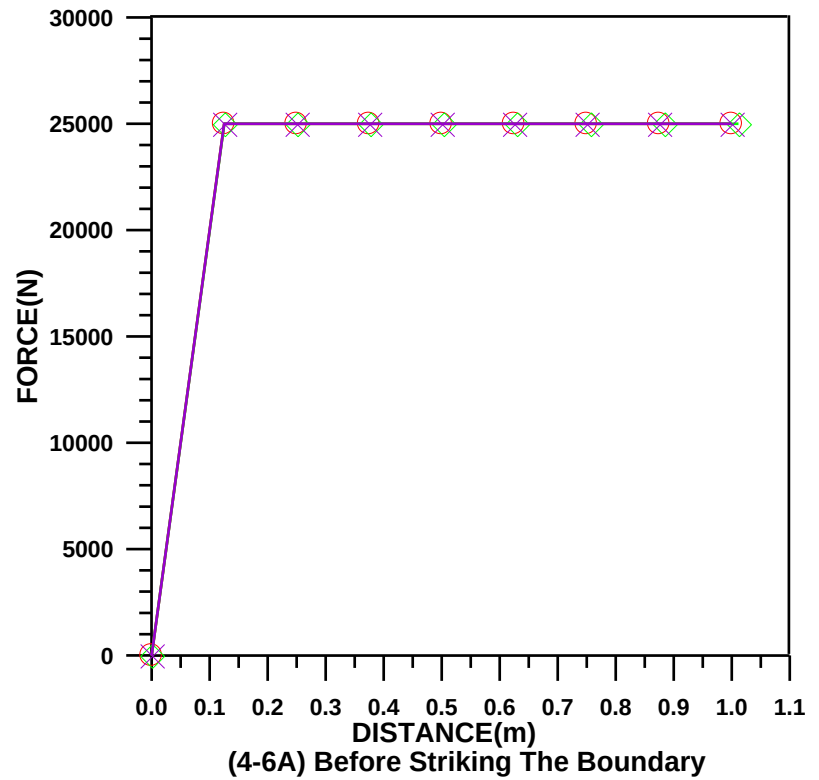
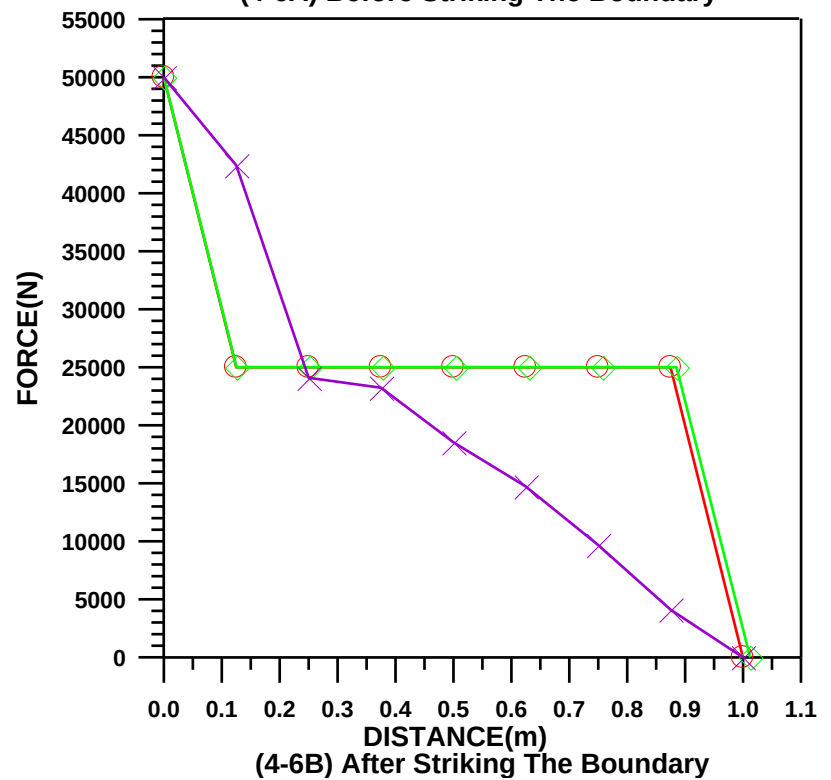


Figure (4-6): Continued

Integration Time
=200ms



Integration Time
=300ms



- small deformation
- ◇-◇ large deformation (geometrical non-linearity)
- ×-× material non-linearity

Figure (4-6): Propagation of Axial waves
 $E=200\text{GPa}$, $A=1\times 10^{-4}\text{m}^2$, $\rho=8000\text{kg/m}^3$, $L=1.0\text{m}$, $F=25000\text{N}$, $LT=0.025\text{ms}$

4.9: Numerical Applications :

One can conclude that when the load is increased until exceeding its elastic ultimate strength, the linear solution can not give the correct solution. The non-linear behavior is required when the deflection exceeds a certain limit depending on the loading value and the material properties.

To investigate the effect of material non-linearity, a cantilever member is subjected to a constant and impulse forcing function respectively. Figures (4-7) and (4-8) show the general behavior of this member when material non-linearity is considered. The general conclusion is that the decreasing behavior is noticed clearly, that is due to the loss in the elastic energy of the vibrated structure.

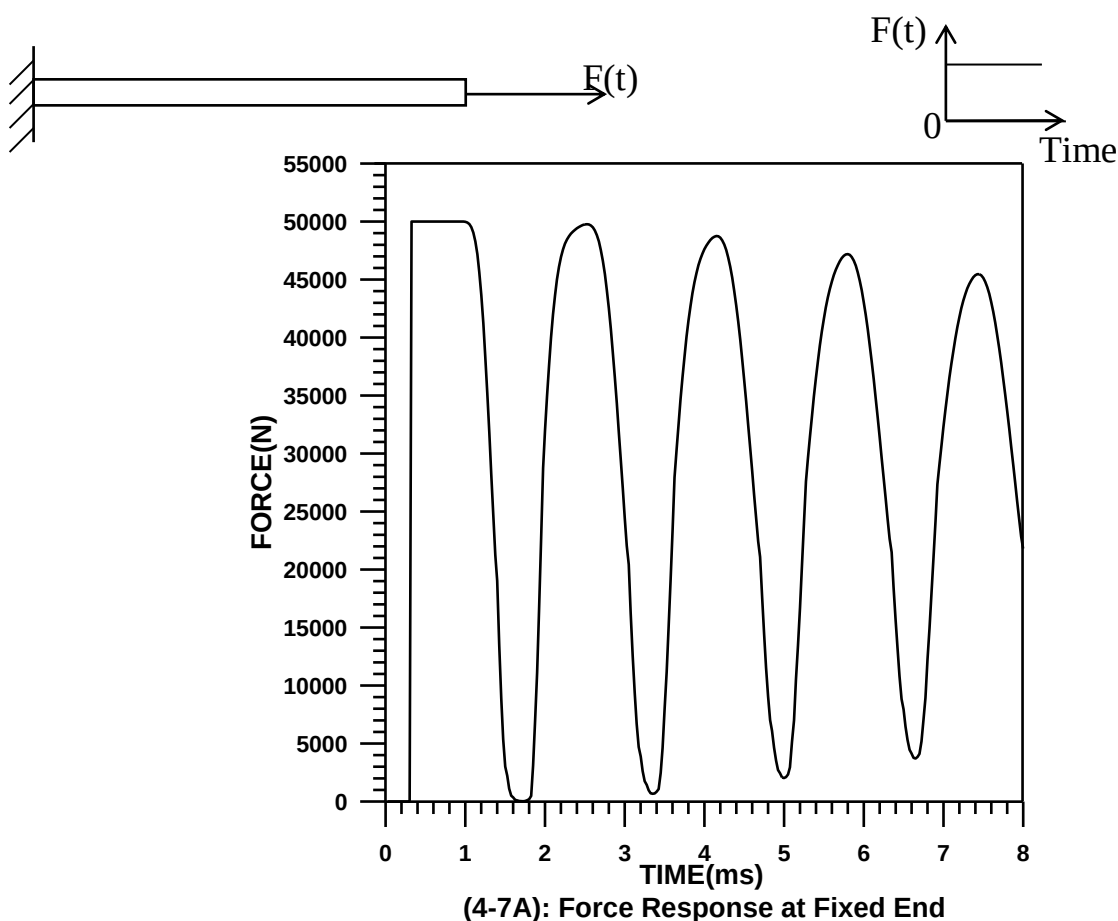
When the geometrical non-linearity is considered, the general behavior of a cantilever beam subjected to an impulse forcing function is shown in Figure (4-9). The free vibration response governs the behavior of the structure after the removal of the force. The increasing behavior is obvious when the geometrical non-linearity is considered for sensitive members, thus an added elastic energy due to the geometrical non-linearity will change the amount of deformations according to the loading sign and magnitude⁽³⁷⁾.

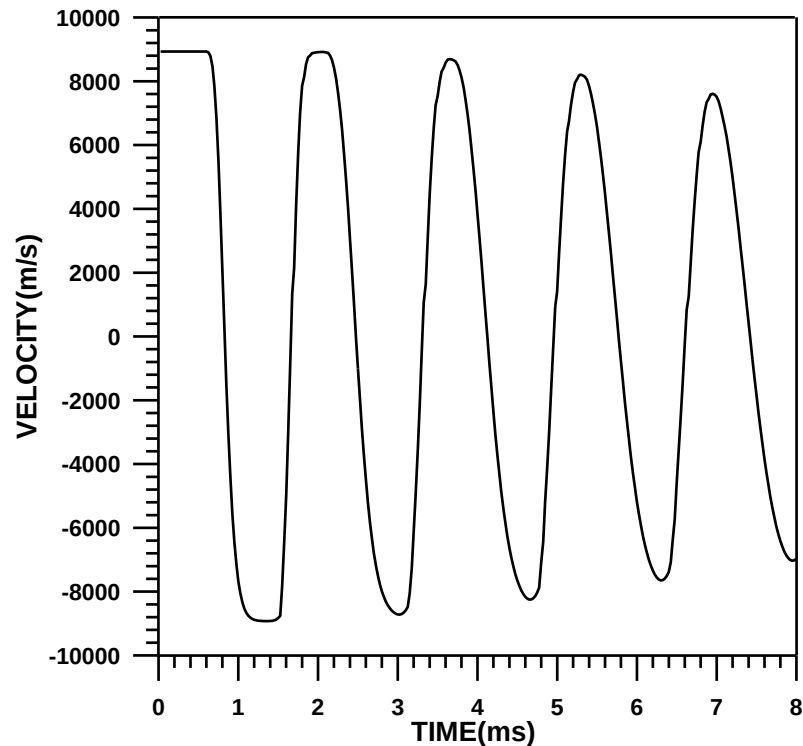
A simple one-dimensional member is examined with different loading types, such as constant loading, harmonic loading, triangular loading. Also the resonance phenomenon is studied. Figure (4-10) represents the behavior of a cantilever member subjected to harmonic loading when both geometrical and material non-linearities are studied. For the cases studied here, the average increase in the displacement at the geometrical non-linear analysis is about (6%).

The resonance phenomenon is studied when one-dimensional member is subjected to harmonic loading, when the loading frequency equal or nearly equal to the natural frequency, as shown in Figure (4-11).

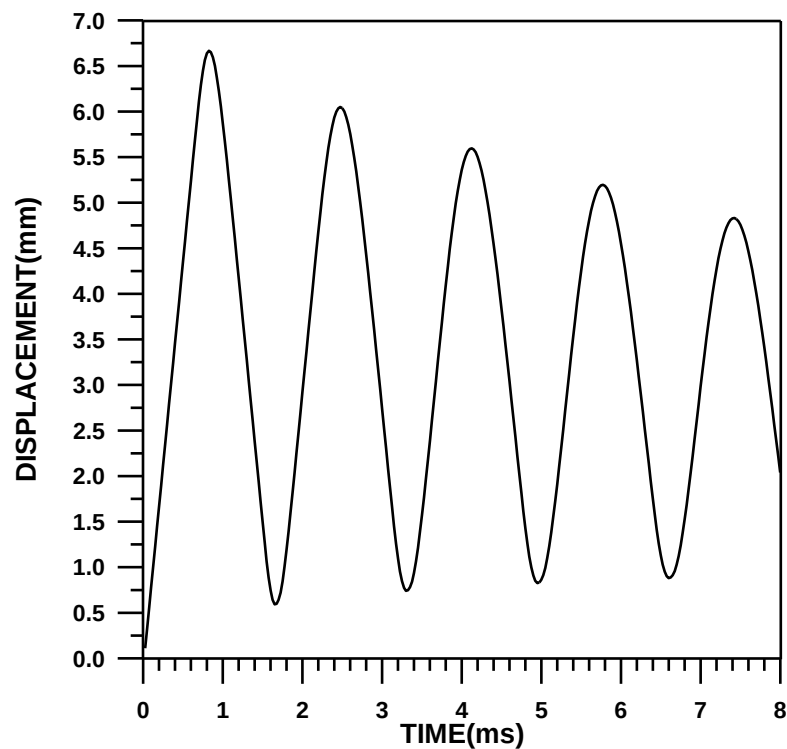
Another loading type is considered which represents the behavior of the same member subjected to triangular loading, as shown in Figure (4-12).

A comparison between the linear and non-linear solutions for the impulse forcing function type is investigated, see Figure (4-13), which includes both types of non-linear analysis. The largest amplitude of displacement is noticed when the material non-linearity is taken into account in the analysis. Where at the latest time stages, the increase in the amplitude of displacement in large deformation than linear analysis is about (15%), but the increase in the amplitude of displacement in material non-linearity than in linear analysis is about (84%), that is because the structure becomes of less stiffness, with non-dispersive masses.





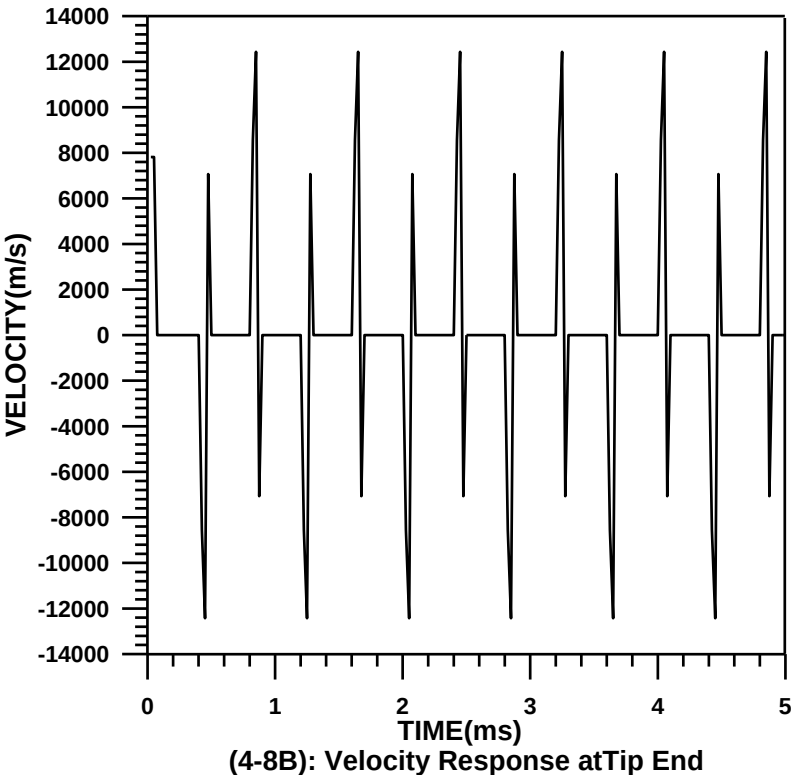
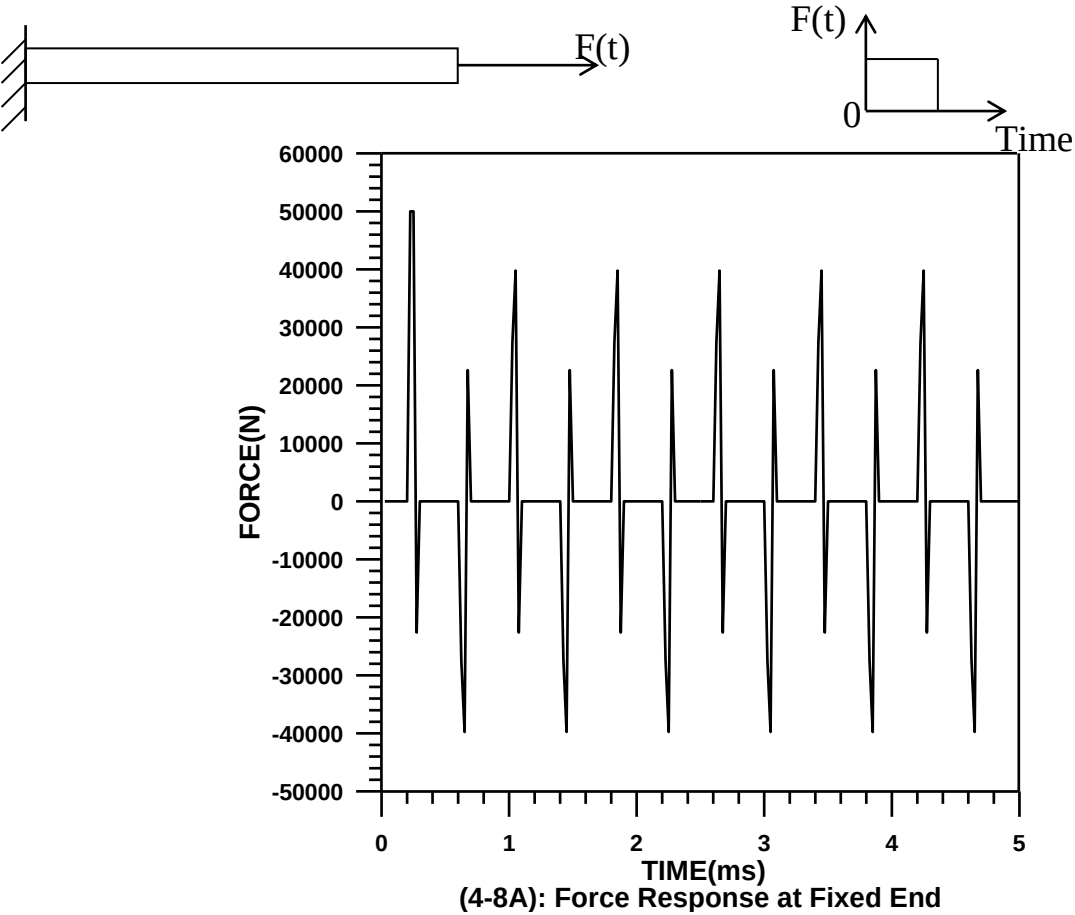
(4-7B): Velocity Response at Tip End



(4-7C): Displacement at Tip End

Figure (4-7):Material Non-Linearity of Member Subjected to Constant Forcing Function

$E=200\text{GPa}$, $A=70\text{mm}^2$, $\rho=8000\text{kg/m}^3$, $L=1.5\text{m}$, $F=25000\text{N}$, $LT=8\text{ms}$, $F_y=350\text{MPa}$



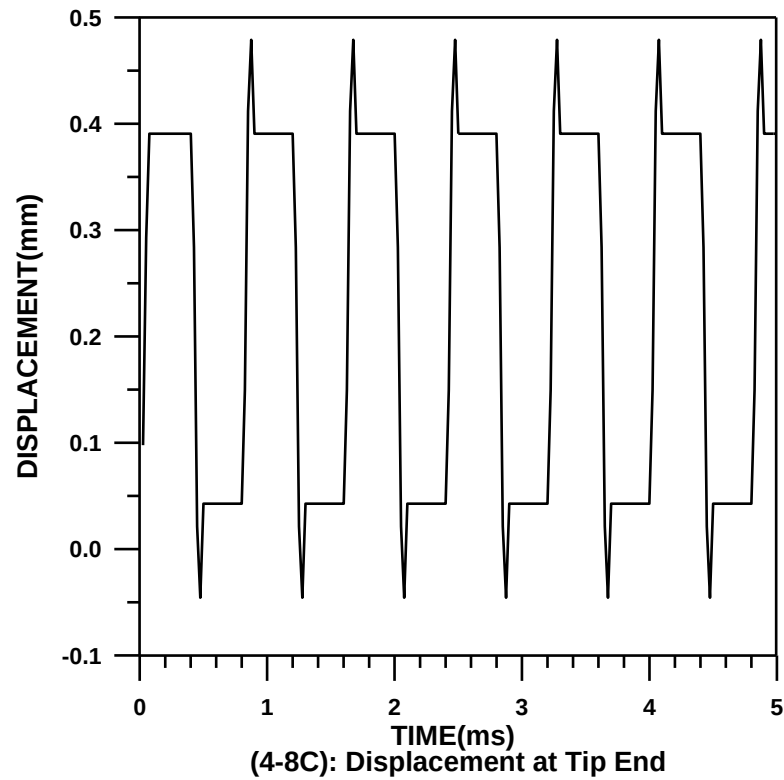
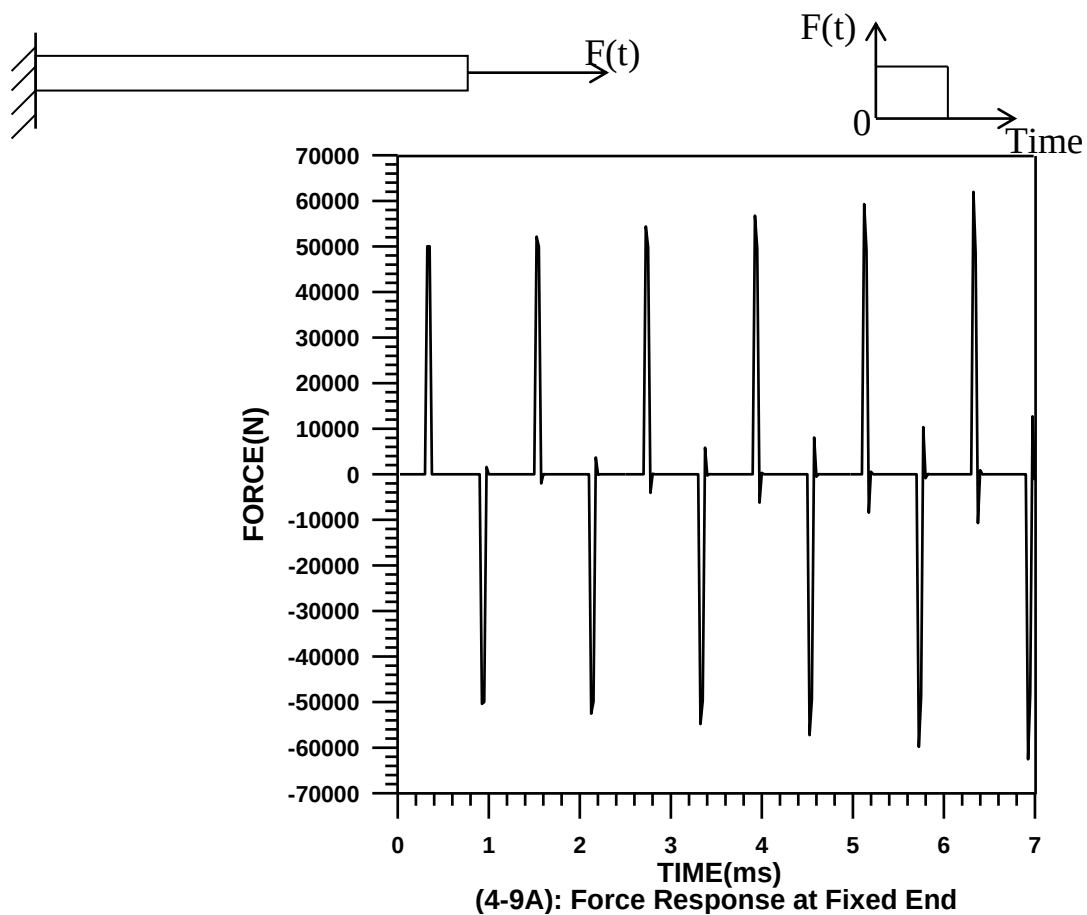


Figure (4-8): Material Non-Linear of Member Subjected to Impulse Loading
 $E=200\text{GPa}$, $A=80\text{mm}^2$, $\rho=8000\text{kg/m}^3$, $L=1.5\text{m}$, $F=25000\text{N}$, $LT=0.05\text{ms}$, $F_y=350\text{MPa}$



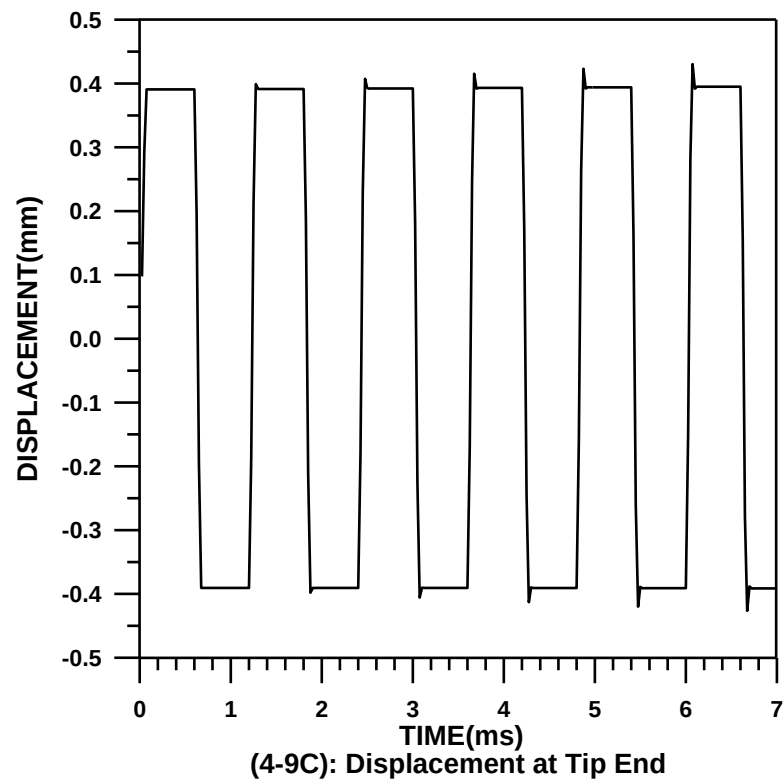
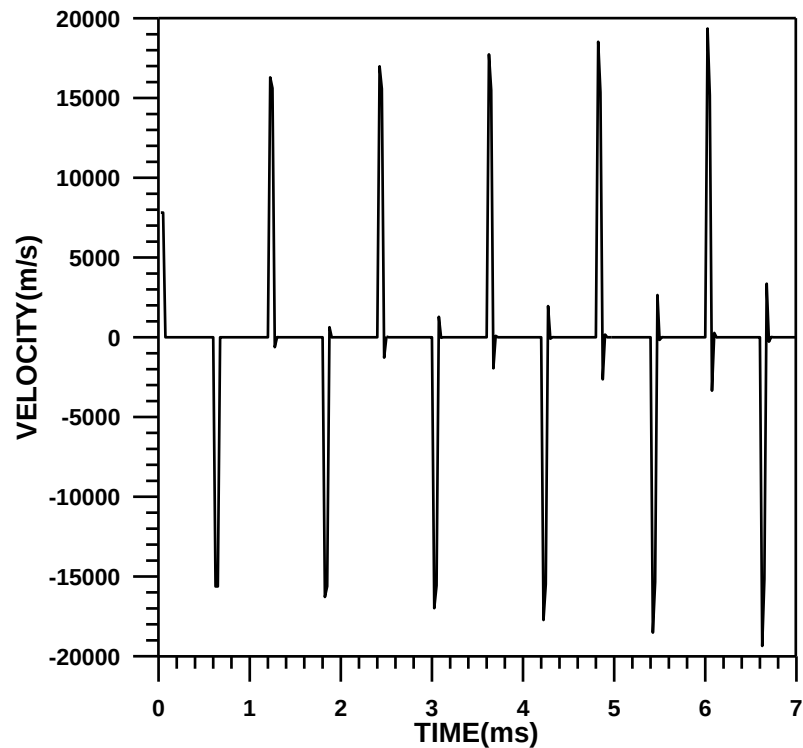
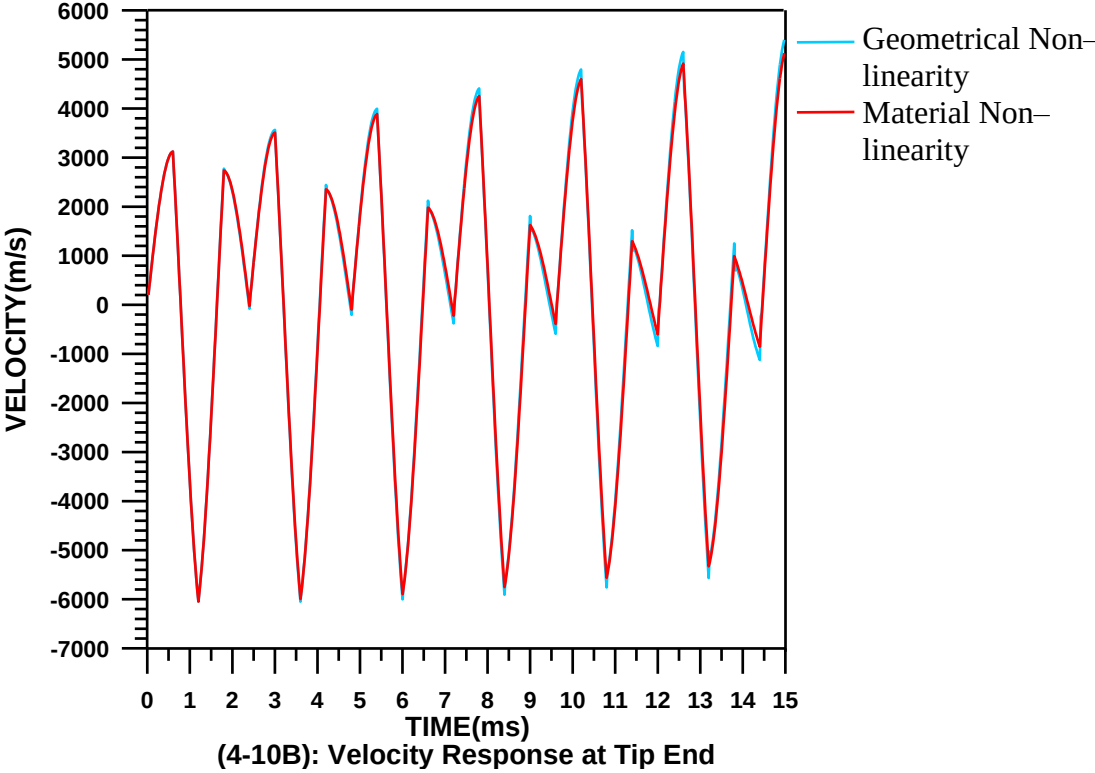
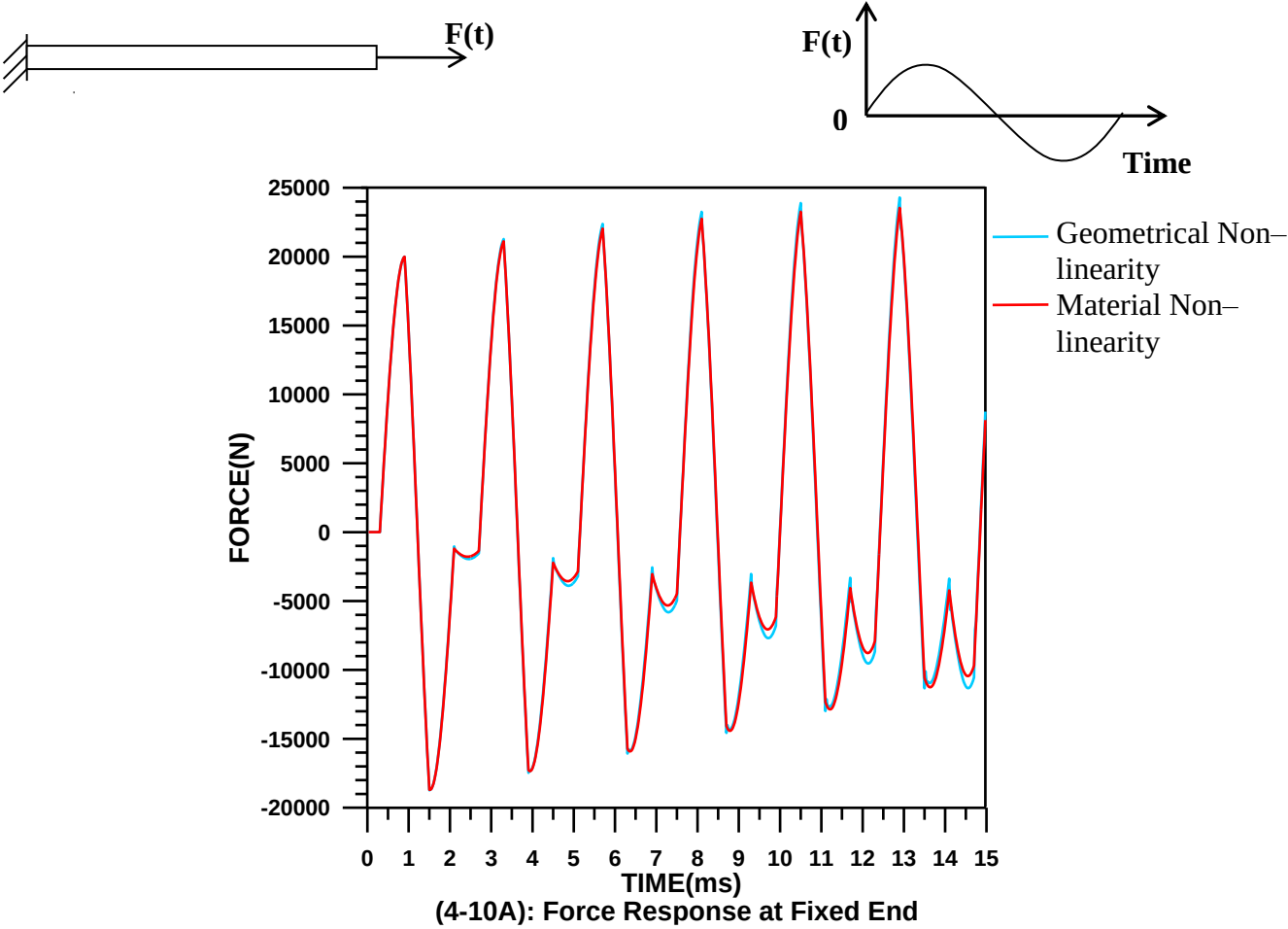
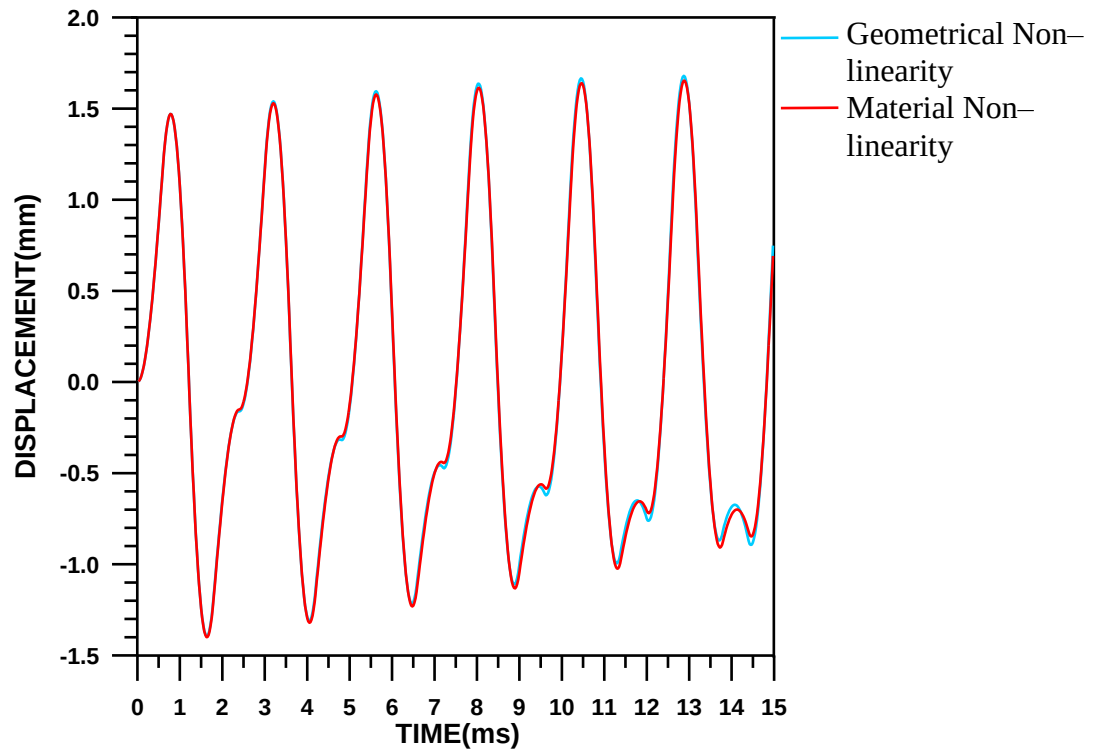


Figure (4-9): Geometrical Non-Linearities of Member Subjected to Impulse Forcing Function

$E=200\text{GPa}$, $A=80\text{mm}^2$, $\rho=8000\text{kg/m}^3$, $L=1.5\text{m}$, $F=25000\text{N}$, $LT=0.05\text{ms}$

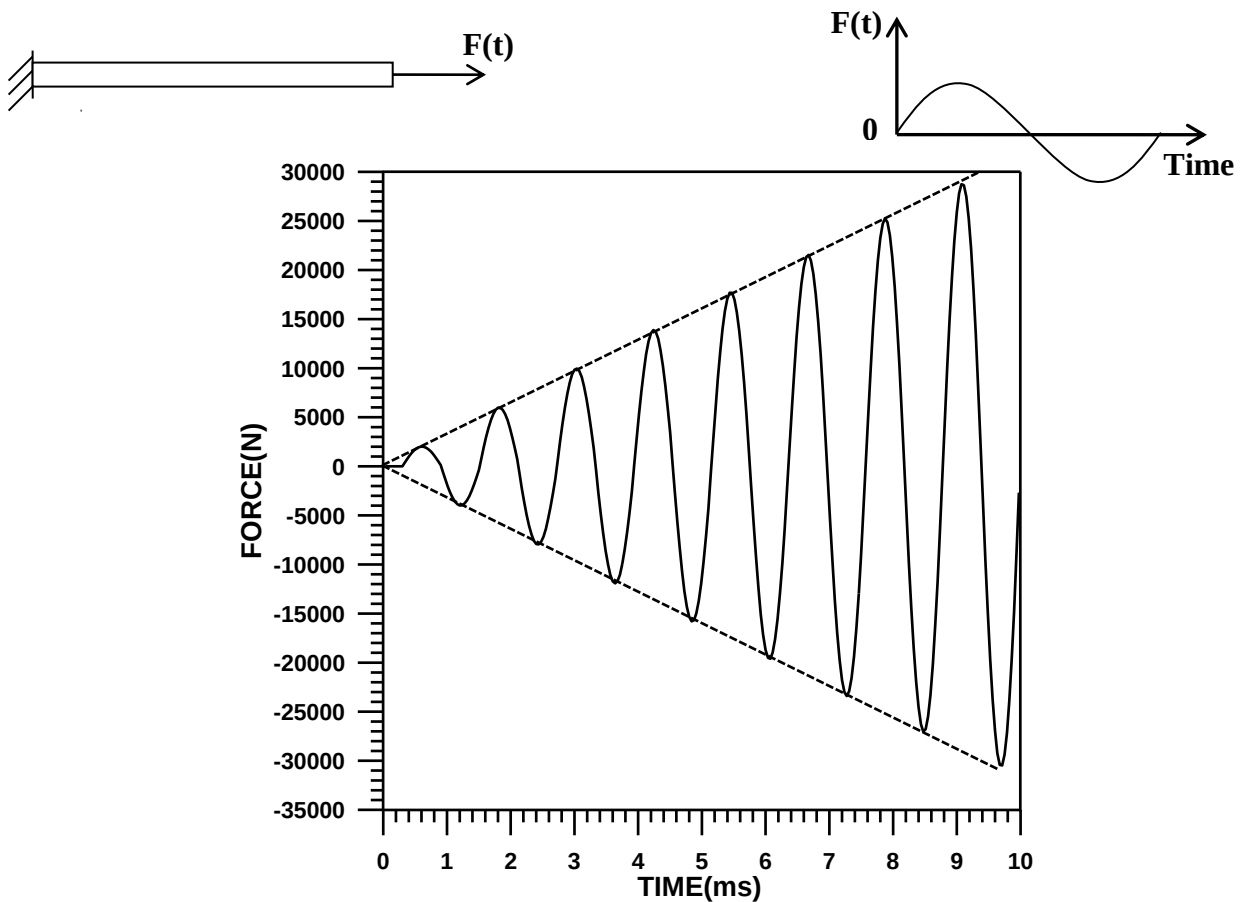




(4-10C): Displacement at Tip End

Figure (4-10): Harmonic Loading

$E=200\text{GPa}$, $A=80\text{mm}^2$, $\rho=8000\text{kg/m}^3$, $L=1.5\text{m}$, $F(t)=10000 \sin(\omega)t$, $\omega=0.5 \omega_n$
 ($\omega_n=5129.047\text{rad/sec.}$)



(4-11A): Force Response at Fixed End

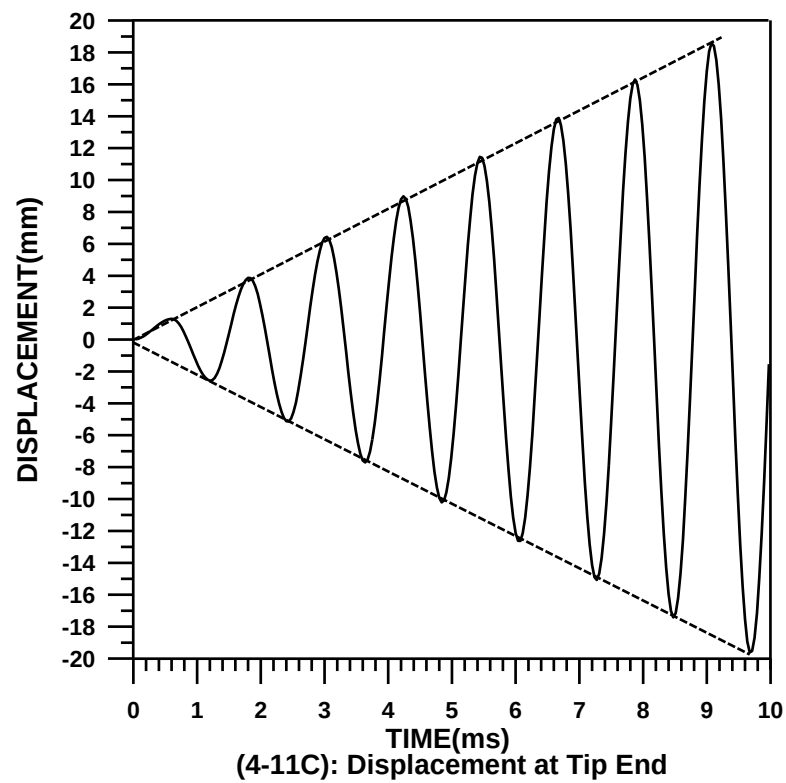
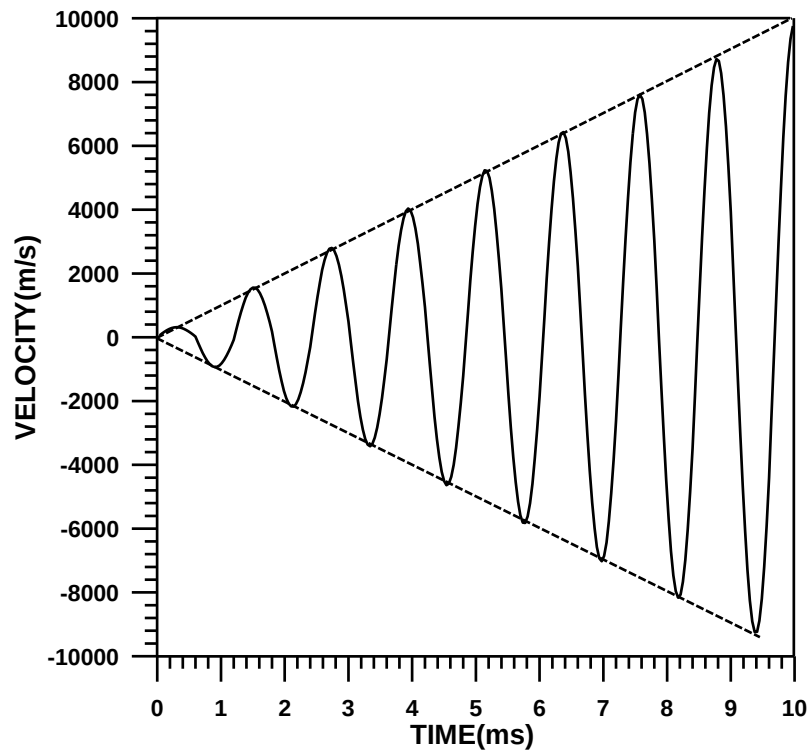
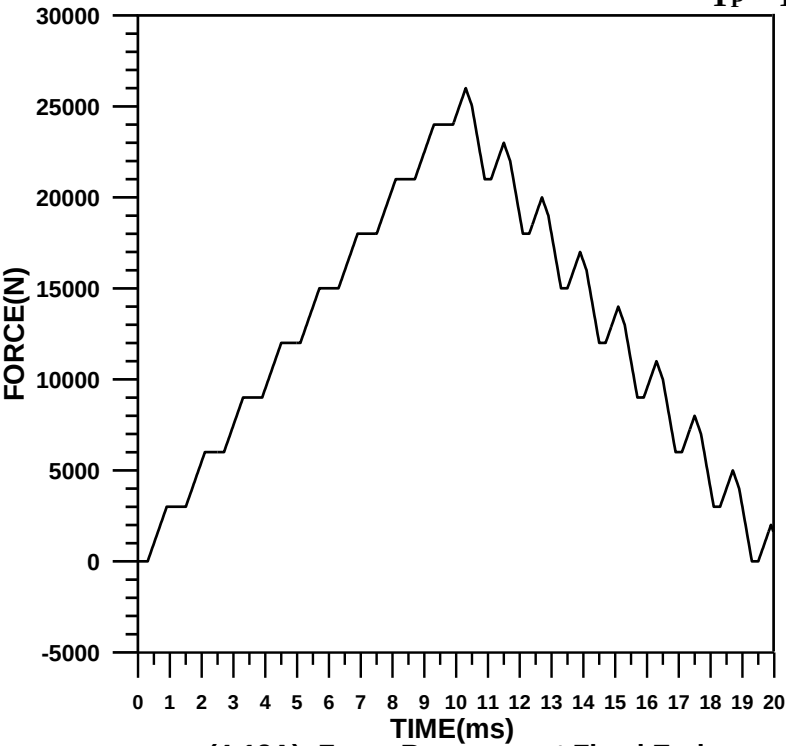
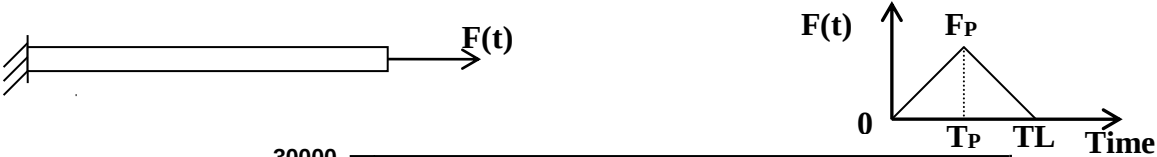
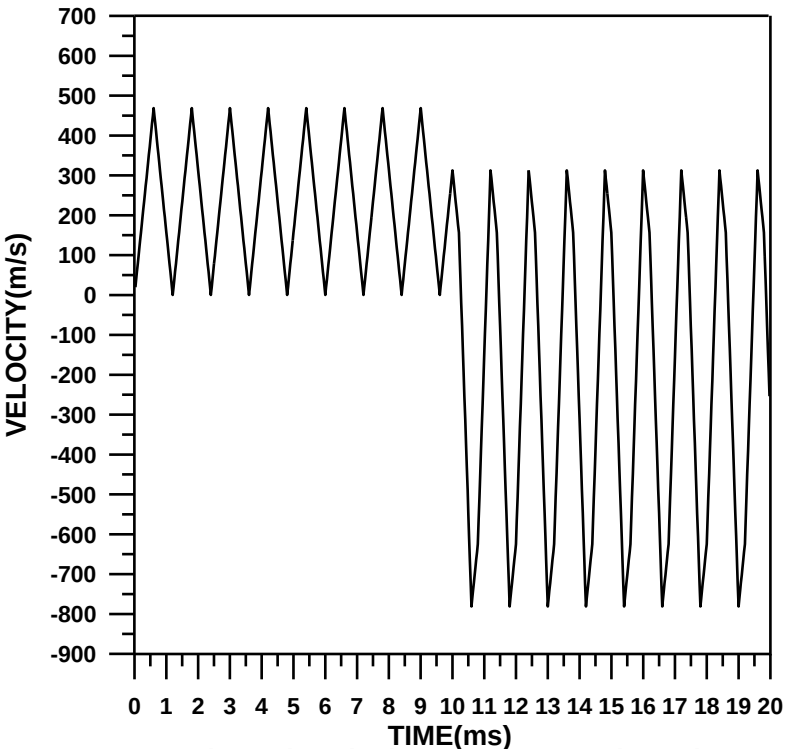


Figure (4-11): Resonance Phenomenon in Harmonic Forcing Function

$E=200\text{GPa}$, $A=80\text{mm}^2$, $\rho=8000\text{kg/m}^3$, $L=1.5\text{m}$, $F(t)=10000 \sin(\omega_n t)$, $\omega_n=5129.047\text{rad/sec}$.



(4-12A): Force Response at Fixed End



(4-12B): Velocity Response at Tip End

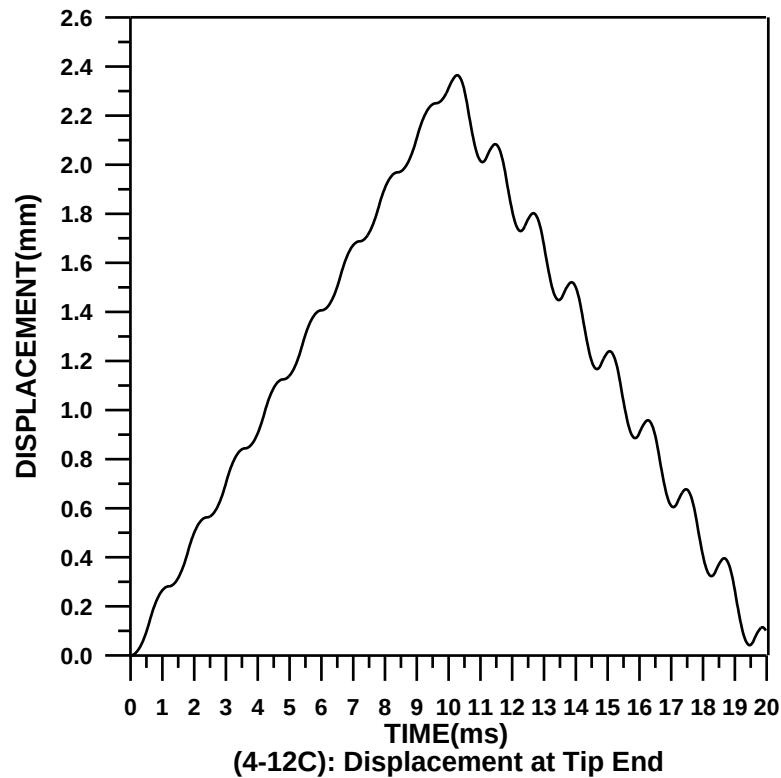
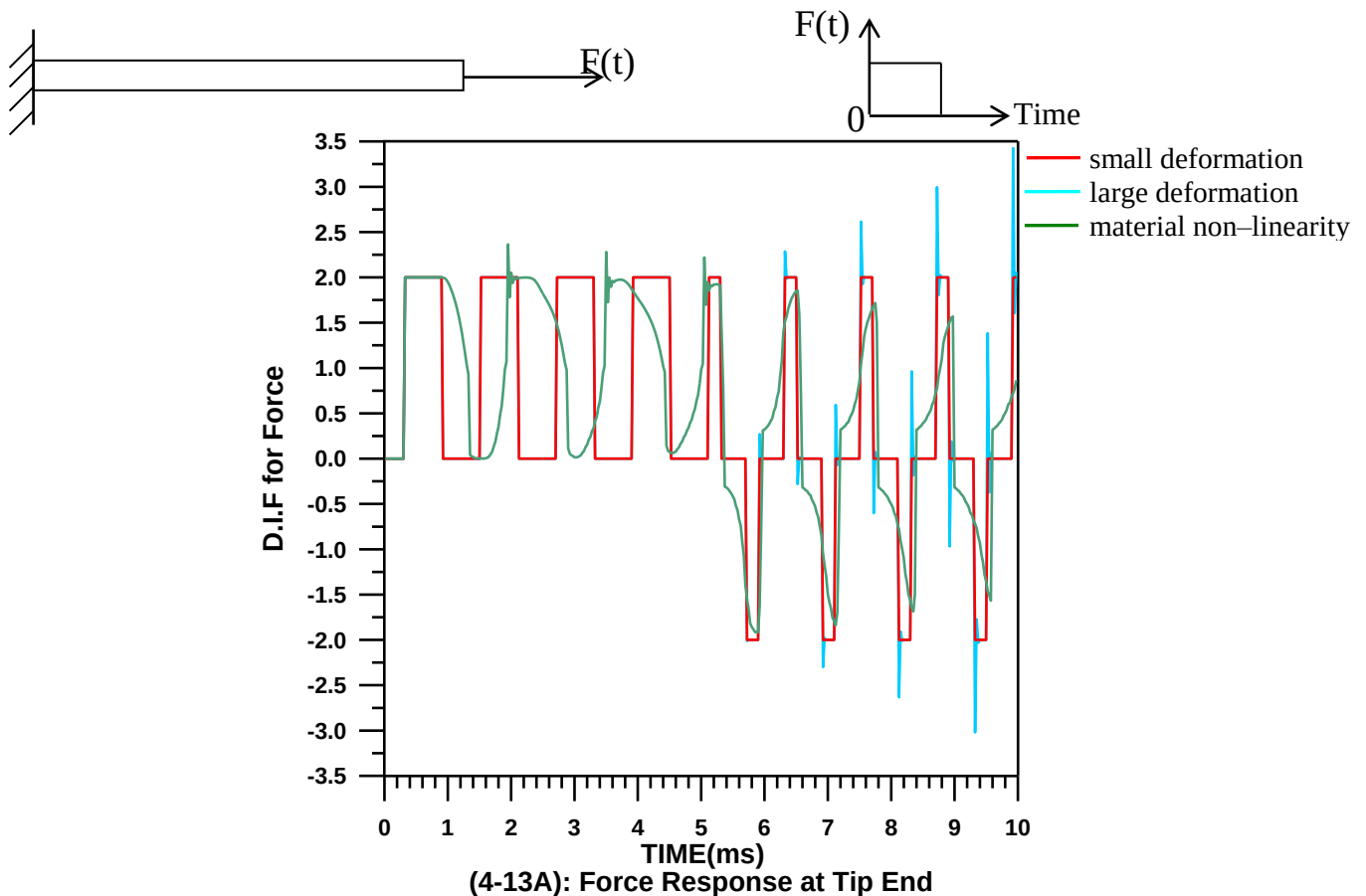


Figure (4-12) :Geometrical Non-Linear of Member Subjected to Triangular Loading

$E=200\text{GPa}$, $A=80\text{mm}^2$, $\rho=8000\text{kg/m}^3$, $L=1.5\text{m}$, $T_L=20\text{ms}$, $F_P=25000\text{N}$, $T_P= T_L/2$



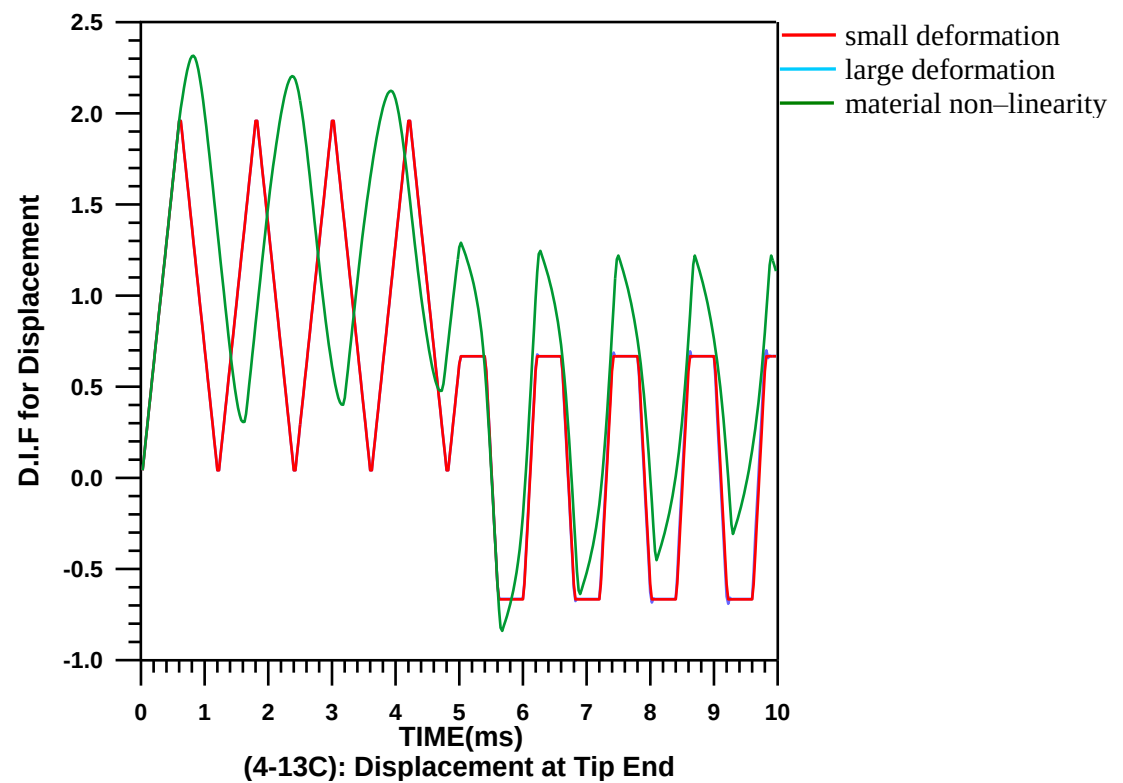
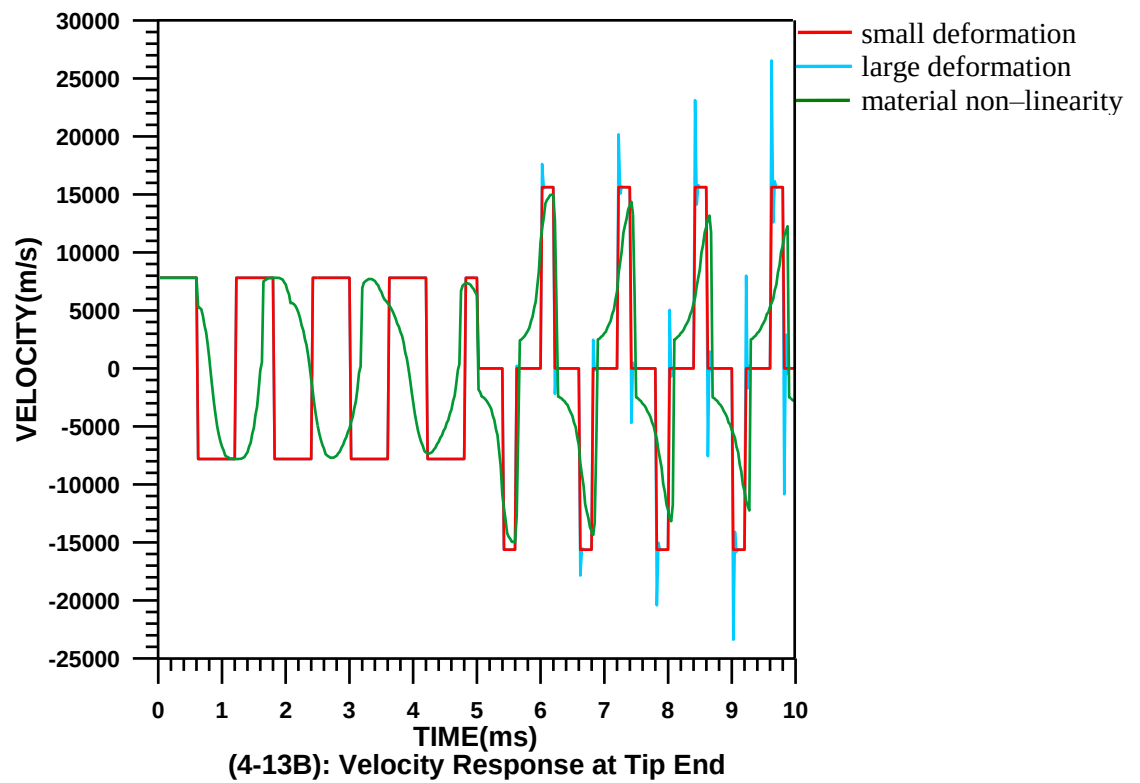


Figure (4-13): Comparison Between Linear and Non-Linear Analysis (Impulse Loading)

$E=200\text{GPa}$, $A=80\text{mm}^2$, $\rho= 8000 \text{ kg/m}^3$, $L=1.5 \text{ m}$, $F= 25000 \text{ N}$, $LT=5\text{ms}$

CHAPTER FIVE

DYNAMIC ANALYSIS OF TRUSSES

5.1: Introduction:

In this chapter, plane truss structures undergoing small and large deflection when subjected to dynamic loadings are studied by using the Method of Boundary Characteristics. For this purpose, a truss is regarded as an assemblage of members.

By requiring that velocities are compatible and that forces are in equilibrium in all members meeting at a massless joint, the entire structure is compelled to allow waves to pass through the joints.

The analytical model considered in this chapter includes axial stress wave propagation through trusses. The members are assumed to be prismatic and are connected together by massless joints. The joints are assumed to be frictionless. Therefore, there is no resistance to rotation and so moments at the ends of the members are equal to zero.

Both linear and non-linear analysis of trusses are predicted. Sufficient grid lengths are used to ensure good accuracy in the predictions of responses history.

In order to obtain the dynamic response of linear and non-linear plane truss structures, the following hypotheses are considered⁽⁴⁶⁾:

- a. the truss element is a straight bar with only axial deformation;
- b. the transversal cross section remains plane, normal to the axis and with no rotation;
- c. the properties of the material are constant in the cross section; but may vary in longitudinal direction.

Weaver (1985) & Loh (1984)⁽⁷²⁾ used the Component Mode Method (CMM) to analyze plane and space trusses. They treated the members as simply supported beams and used a limited number of vibrational mode shapes to represent the

displacement shape function. They found that three modes per member provided sufficient accuracy for practical purposes. They also found that the overall influence of flexural vibrations of members is not significant in trusses with many members.

In this chapter Boundary Characteristic Method is illustrated for the analysis of trusses. It is broadly analogous to dynamic relaxation, and requires even less computer memory to the analysis, and its dynamic formulation is capable of simulating transient forms of dynamic behavior more realistically ⁽¹⁸⁾.

To ensure the accuracy and the efficiency of the solutions obtained by the Method of Characteristics, many examples are compared with other researches.

5.2: Formulation:

In order to formulate the equations required to simulate the dynamic behavior of truss member required for the method adopted here, the following aspects are followed.

5.2.1: Characteristic Matrix:

In practice, the member axes (local axes) and the joint axes (global axes) are not necessarily to be the same. Therefore, before applying the boundary conditions, a transformation from the member axes to the joint axes for the end forces and the end velocities of each member is needed.

At each end of a member, the characteristics equations (3.7 & 3.8) for longitudinal waves can be written as ⁽¹⁰⁾:-

$$F'_m = f'_m + C_{um} U'_m \quad (5.1)$$

where:

$$f'_m = (-s F'_A - \rho A C U_A) \quad (5.2)$$

and

$$C_{um} = (\rho A C)_m \quad (5.3)$$

Here, $s=-1$ for forward Characteristic lines (C^+ equations) and $s=+1$ for backward Characteristic lines (C^- equations). The subscript (A) denotes the old time information at the remote grid and the subscript (m) denotes the particular member (m). The primes denote the use of axis chosen relative to a member (local axis). F'_m and U'_m are the unknown end force and velocity at the new time level. See Figure (5-1).

The relationships between the end forces F'_m and the end velocities U'_m in the local axis and F_1 & U_1 in the global axis for plane truss are :

$$\left. \begin{aligned} F_1 &= F'_m \cos(\alpha) \\ F_2 &= F'_m \cos(\beta) \end{aligned} \right\} \quad (5.4)$$

and

$$U'_m = U_1 \cos(\alpha) + U_2 \cos(\beta) \quad (5.5)$$

where α and β are the angles between the local axis of the member (m) and the global axis of the structure. The subscript 1 and 2 denote the x and y directions respectively.

After transformation into global axis, equation (5.1) can be written in matrix form as :

$$\{F\} = \{f_0\} + [C_e] \{U\} \quad (5.6)$$

where :

$\{F\}$: is the joint forces vector at the new time,

$\{f_0\} = [T]^T f'_m$: is the information vector at the old time,

$\{U\}$: is the joint velocities vector at the new time,

$[C_e] = [T]^T C_{um} [T]$: is the characteristic matrix,

and

$[T]$: is the transformation matrix.

For plane trusses, the transformation matrix can be written as :

$$[T] = \{ \cos(\alpha) \quad \cos(\beta) \}^T \quad (5.7)$$

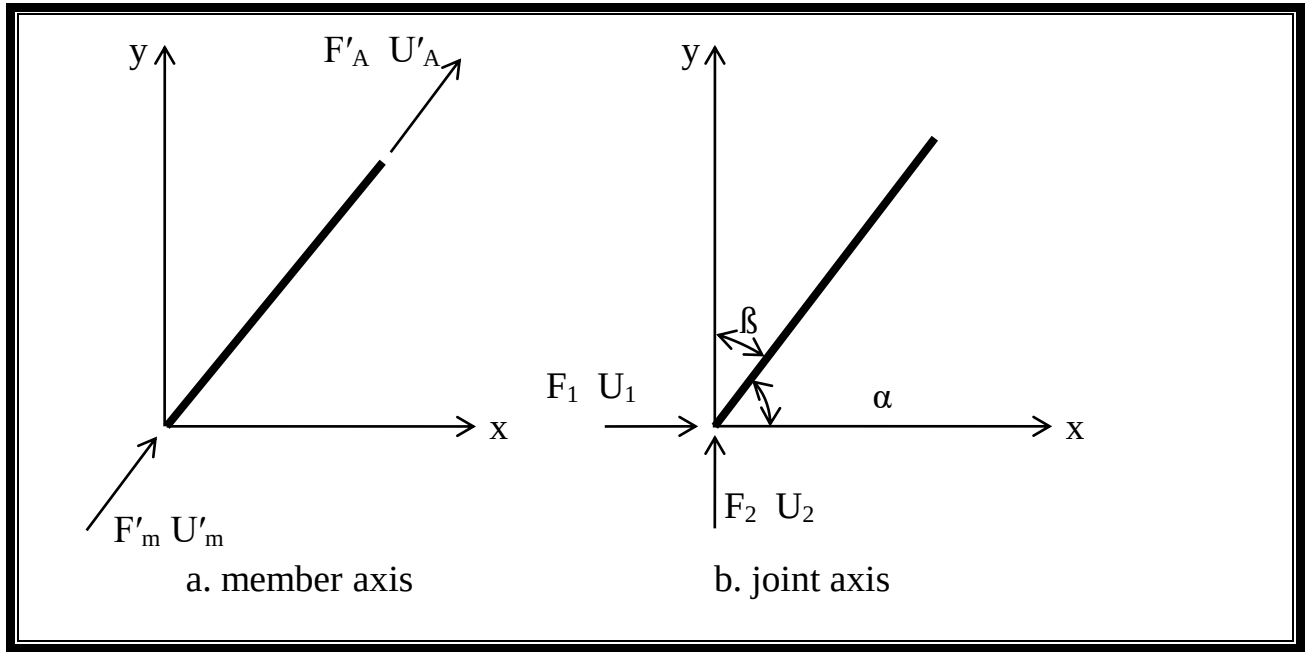


Figure (5-1): Forces and Velocities

5.2.2: Equilibrium and Compatibility Conditions :

After transformation into global axes, it is necessary to satisfy the conditions of equilibrium and compatibility at the joint. The ends of all members need to have the same velocity components as the joint at which they meet. Also, the sum of the internal forces in the members at a massless joint needs to balance the external load $\{P(t)\}$. The equilibrium equation for a joint of (N) members is :

$$\{P(t)\} = \sum_{i=1}^N \{F\}_i \quad (5.8)$$

Substituting equation (5.6) into (5.8) gives:

$$\{P(t)\} = \sum_{i=1}^N \{f_0\}_i + \left(\sum_{i=1}^N [C_e]_i \right) \{U\} \quad (5.9)$$

Equation (5.9) may be written as :

$$\{P(t)\} = \{f\} + [C_J] \{U\} \quad (5.10)$$

In the above equilibrium equation, the joint characteristic matrix $[C_J]$ is symmetric and dependent upon the geometrical and mechanical properties of the structure. The old time information matrix comprises the forces and velocities in members at the old time level.

5.3: Boundary Conditions:

The most common boundary in a truss is a joint, at which one or more members meet. At a joint, the condition of equilibrium needs to be satisfied between the external loads and the internal forces at the ends of the members, and the condition of compatibility needs to be applied to the velocity of the joint and the end velocities of the members.

For a joint at which number of members (N) meet, each one of the (N) unknown axial velocities at the ends of the members is identical to the velocity of the joint (compatibility). If the loading condition at the joint is given, the (N) unknown axial forces at the ends of the members and two components of velocity of the joint can be solved using (N) characteristic equations and two force equilibrium equations.

5.4: Massless Joints:

There are two types of massless joints; namely, the free (or non-restrained) joint and the restrained joint in one or two directions, as described below.

5.4.1: Free Joints:

At the typical unrestrained joint (free joint) shown in Figure (5-2), with (N) members, the external loads and internal forces at the ends of the members must be in equilibrium. Also the velocity at the joint can be calculated from the equilibrium

equation (5.10). Also the velocity at the end of each member must be identical to the velocity of the joint (compatibility).

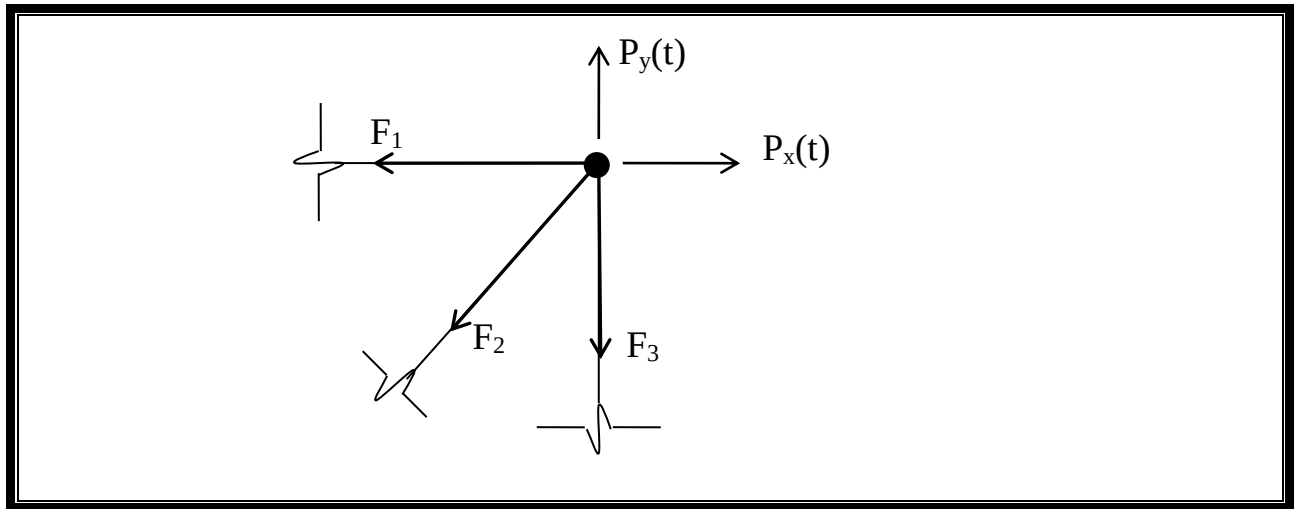


Figure (5-2) : Free Joint in Truss

5.4.2: Restrained Joints:

The velocity in the restrained direction is zero because the joint cannot move in that direction. So the reactions on the joint are unknown. The unknown velocities and reactions can be solved simultaneously. Fully and partially restrained support are now considered, see Figure (5-3).

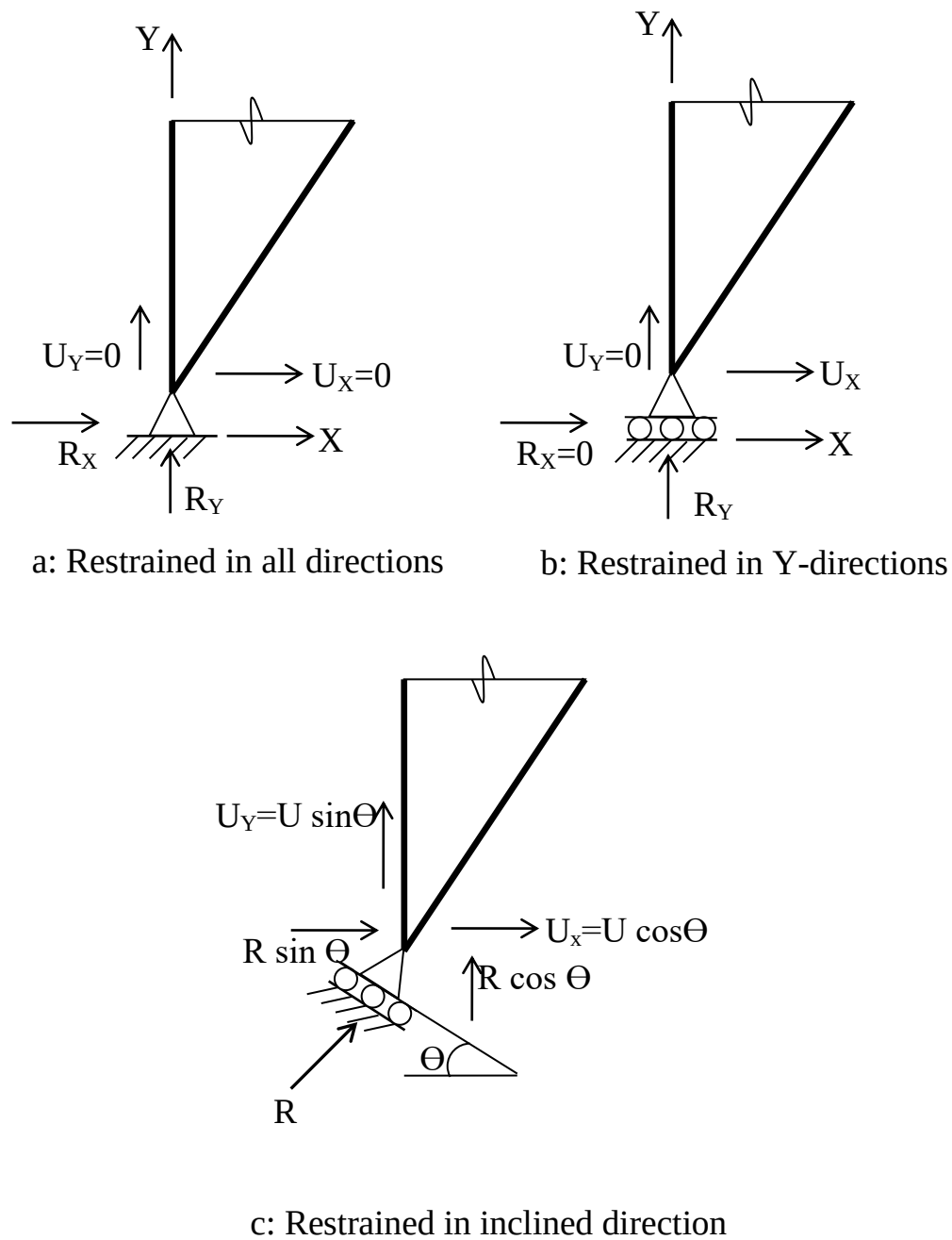


Figure (5-3): Restrained Supports in Plane Trusses

5.5: Dynamic Behavior of Trusses :

The response of a structure subjected to dynamic load, especially impact load is much more complex than that of static load. The solution of initial response of trusses is worth attention for engineering application. Nowadays, buildings are becoming taller and slender, and machines, automobiles, planes are made with stronger and lighter new materials. The dynamic effects are getting much more significant. In most cases, it is essential that a vibration analysis be used in the structural design.

There are generally two approaches for dynamic analysis of trusses. One is to treat the structure as a distributive system of multi connected bars or beams which can be solved by, for instance, Transfer Matrix Method, Direct Stiffness Method, Compliance Method etc⁽²¹⁾. The other one is to deal with every member of the structure as discrete member and divide it into a number of elements, for example: Finite Element Method⁽⁷⁸⁾.

An alternative approach is using the wave propagation theory to obtain the response of the structure. There are two concepts to understand the wave propagation, these concepts are: dispersion and attenuation⁽²¹⁾. Dispersion is the change in wave speed in a material, which is due to reflection of waves or damping. Attenuation is the change in amplitude of a traveling wave over a given distance. While propagating through a solid medium, transferred back energy is not perfect, where attenuation occurs. This loss in energy can be due to heat being generated, waves leaking into sideband frequencies or spreading into different paths, restraints such as a bonded core, or in the case of composite materials⁽⁴⁶⁾.

The present study extends The Method of Characteristics to investigate the dynamic response of trusses, including small deformations, large deformations (geometrically non-linear behavior) and plastic behavior.

5.6: Large Deformation Analysis :

To understand the influence of non-linear behavior of a structure, it will be helpful to review first the characteristics of non-linear behavior.

In a linear analysis, the kinematics, compatibility and constitutive relationships are all linear, also displacements and strains are small. The equilibrium relationship is linear, meaning that equilibrium is satisfied in the undeformed configuration of the structure. Boundary conditions do not change during linear analysis. If one of the above is violated or cannot be considered, then non-linear analysis must be used. In addition, if the non-linear analysis is performed, the use of certain characteristics of linear analysis cannot be applied (such as the principle of superposition).

There are many characteristics of large deformation analysis which are⁽³¹⁾:

- ◆ Large deformation analysis is more accurate than linear (small deformation) analysis.
- ◆ It can be useful in predicting the buckling potential of structures.
- ◆ In linear (small deformation) analysis, equilibrium is computed for the undeformed, original configuration of the truss. In large deformation analysis, on the other hand, equilibrium is computed about the deformed shape of the truss.

This leads to some difference, one needs to know the deformed shape in order to apply loads and carry out the analysis, but one does not know the deformed shape until the analysis is complete. The solution of this problem, as in many non-linear problems, is to proceed incrementally and apply deformations in small steps until equilibrium is achieved about the deformed configuration⁽⁴¹⁾.

The geometrical non-linearity is accounted for through continuous updating of the structure geometry. At the end of each time increment (Δt) the displacement, velocity, and force are updated. The damping effects of the truss has been ignored.

5.7: Characteristics of The Computer Programs :

Three computer programs (**LDAPT**) *Linear Dynamic Analysis of Plane Trusses*, (**GNLDAPT**) *Geometric Non-Linear Dynamic Analysis of Plane Trusses* and (**MNLDAPT**) *Material Non-Linear Dynamic Analysis of Plane Trusses* are used in the present study. The first program was designed by Al-Daami and developed in the present study, while the second and third programs are written in the present study. These three computer programs are designed to carry out the linear (small deformation), large deformation (geometrically non-linear) and material non-linear analysis of plane trusses respectively. The truss structures consist of straight members under dynamic load by using the boundary characteristic method.

Any function of loading can be applied in these computer programs, i.e. step, terminated ramp, and impact. The input data of the programs consists of the overall member geometry, material properties, type of the boundary conditions, type of the transient loading, and the members connectivity. The output of computer programs includes the time history distributions of the axial force, axial velocity and displacements in both degrees of freedom. The result of several examples will be checked with other researches.

The computer programs is coded in Fortran-77 language. They consist of a main routine and many subroutines. Each one of these subroutines has been designed to deal with a part of the analysis and that part may be repeated more than once.

The main difference between (**LDAPT**) and (**GNLDAPT**) computer programs is that in (**LDAPT**) *Linear Dynamic Analysis of Plane Trusses* the grid length remain constant during the loading time, but in (**GNLDAPT**) *Geometric Non-Linear Dynamic Analysis of Plane Trusses* updating to the grid length is done every time step and new length of the member is calculated every time step, and hence the direction cosine of each member will be calculated for each time step.

In **(MNLDAPT)** *Material Non-Linear Dynamic Analysis of Plane Trusses* the bilinear stress-strain relationship is being dependent. Testing of the stress every time step is done, when the stress below yield stress the modulus of elasticity (E) is taken without reduction factor; on the other hand when the stress above the stress limit the modulus of elasticity is taken with reduction factor (α). The reduction factor is taken (0.1), because it closely approaches the behavior of the steel structure⁽²¹⁾.

The flow-chart of the **(GNLDAPT)** computer program is given in Figure (5-4).

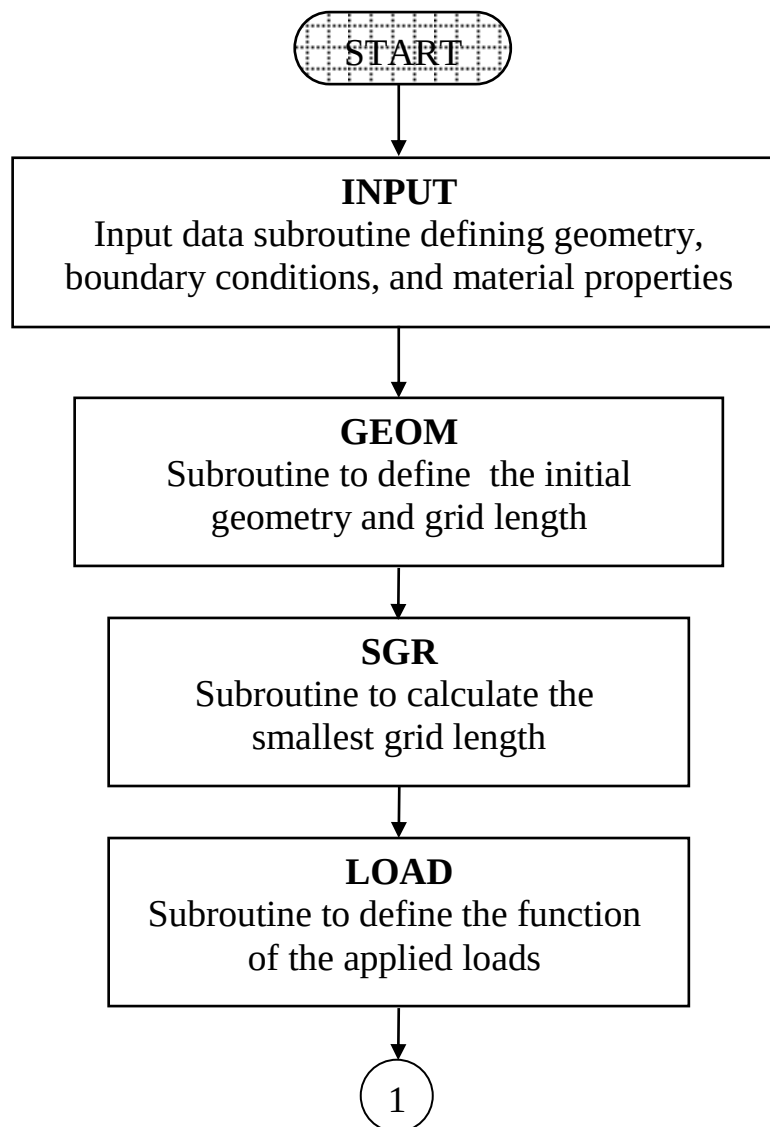


Figure (5-4): The Flow-Chart of The **(GNLDAPT)** Computer Program

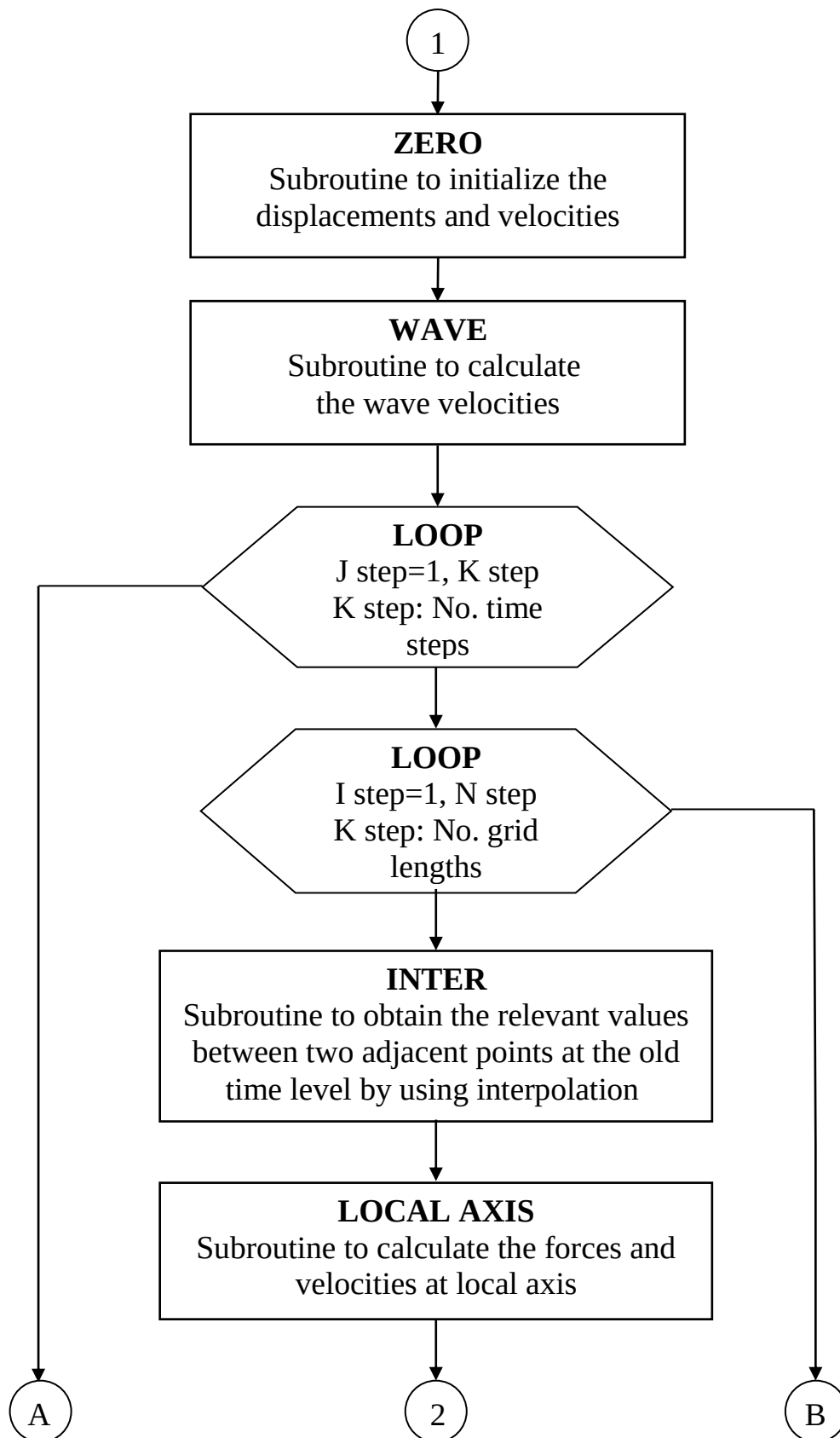


Figure (5-4): Continued

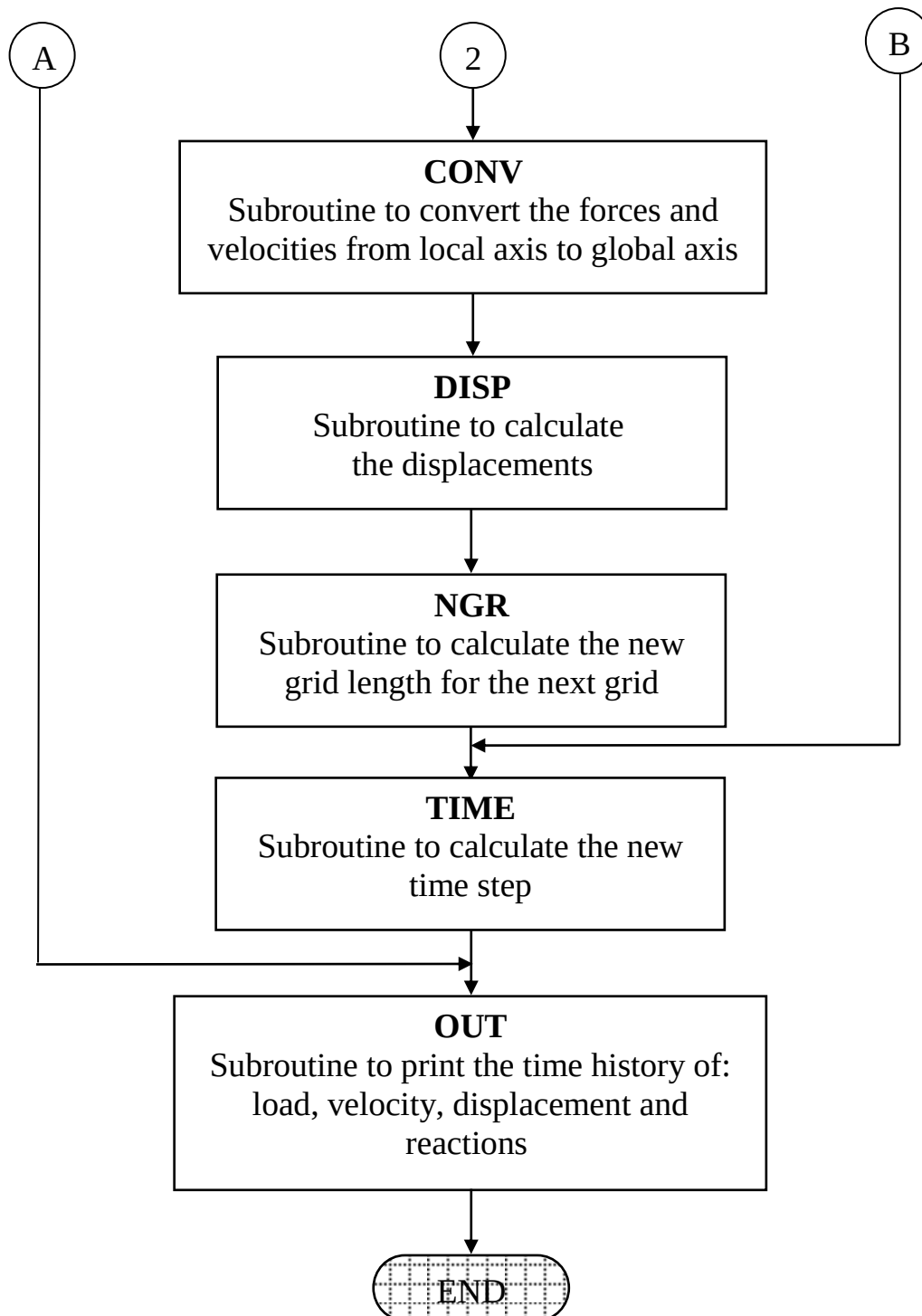


Figure (5-4): Continued

5.8: Verification of The Method of Characteristics :

In order to verify the proposed method and the reliability of the computer programs, results for several examples of truss structures under various types of

dynamic loading are presented and compared with available numerical or analytical results wherever possible.

EXAMPLE NO.1 :

To validate the computer program (**LDAPT**) *Linear Dynamic Analysis of Plane Trusses* and The Method of Boundary Characteristics, the dynamic response of truss structure is presented and compared with the results of Finite Element Method by Al-Sarraj⁽¹⁰⁾. The details of the truss structure geometry and dynamic characteristics are given in Figure (5-5). The displacement histories in two degrees of freedom (X,Y) are shown in Figure (5-6), and good agreement between the results of these two methods of solution can be noticed where the two curves coincide well. At the initial and the latest time the lag time in horizontal displacement is about (0.05ms) and the difference between displacements at initial and latest time is about (4.9%), in vertical displacement there is no lag time at all time of analysis, also there is no difference in displacement at initial time but at the latest time there is a difference about (8.2%).

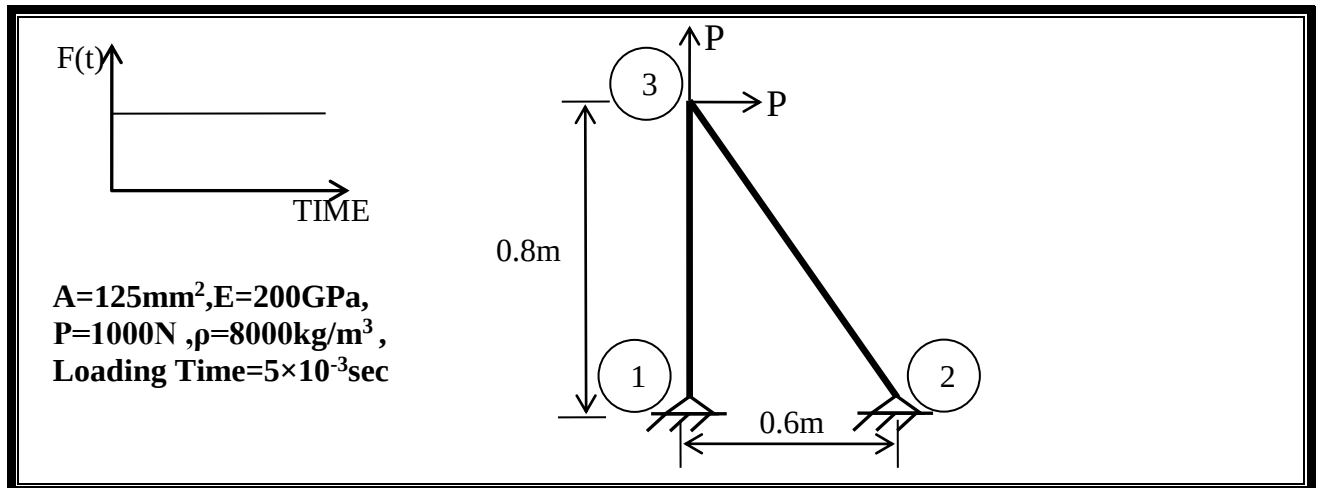


Figure (5-5): Details of Example No.1

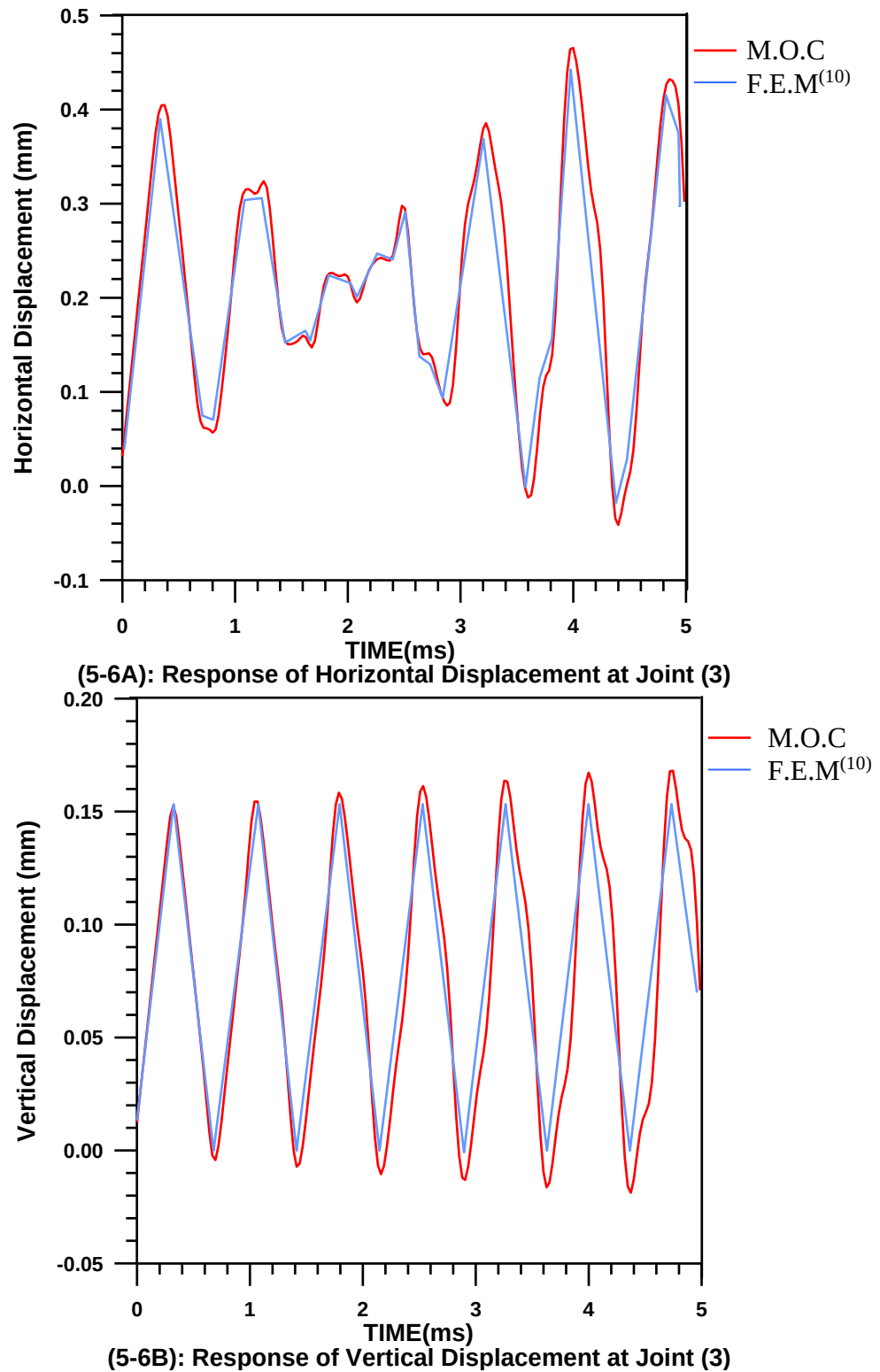


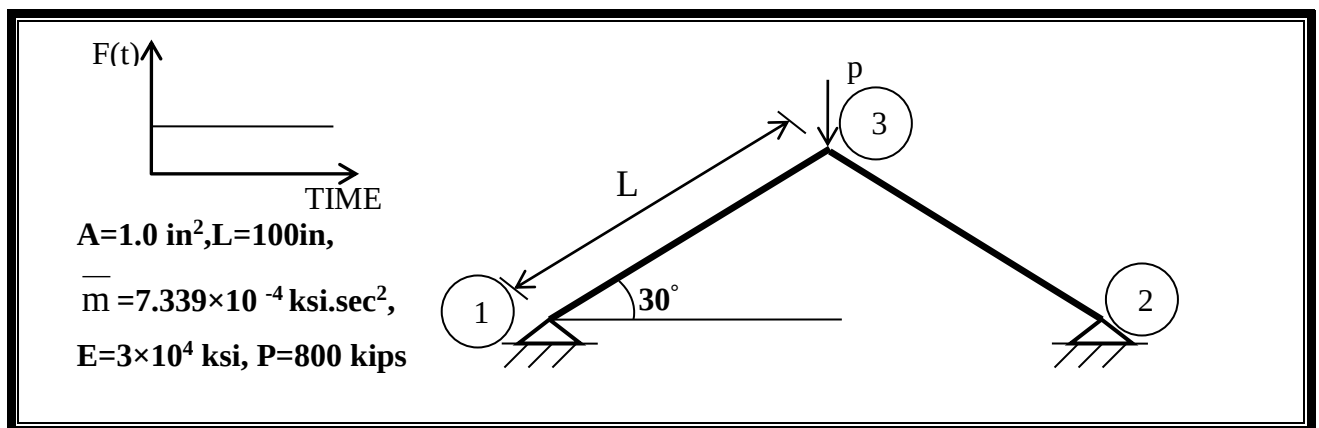
Figure (5-6): Dynamic Response for Truss structure of Example No.1

EXAMPLE NO.2 :

The geometry and dynamic characteristics of toggle truss structure are shown in Figure (5-7). The accuracy and the efficiency of the computer program **(GNLDAPT) Geometric Non-Linear Dynamic Analysis of Plane Trusses** is checked by comparing its results with the results of the same example solved by K. ZHU, and et. al. (1994)⁽⁴¹⁾ by using an updated Lagrangian method based on the incremental formulation of the equation of motion.

Figure (5-8) shows the non-linear transient response under a step forcing function of (800 kips), also this Figure shows good agreement between these two different methods of solution. It can be noticed that the adopted method in this study (Boundary Characteristics Method) gives less results about (2.27%) at the initial time stages, but at the latest time the reverse is true, where the adopted method gives larger results about (5.8%). This difference can be attributed to the varying stiffness of the truss structure with time. At the initial time stages the lag time between two curves is about (0.1sec), and at the latest time stages it becomes (0.05sec), it can be attributed to the natural frequency.

There is a slightly amplifying effect in both methods, it may be due to the numerical updating methods. Also the stiffness of the truss will be more flexible in the adopted method (BCM).



Figure(5-7): Details of Example No.2

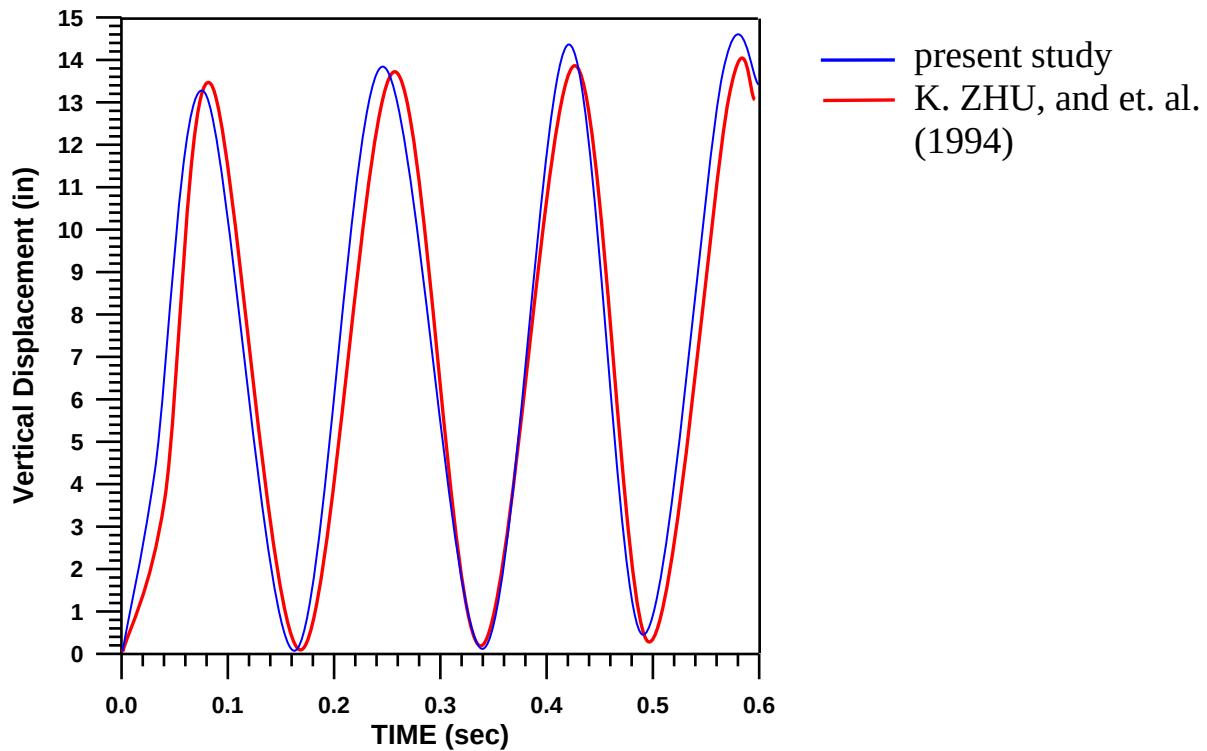


Figure (5-8): Dynamic Response of Toggle Truss under a Step Forcing function

EXAMPLE NO.3 :

The same toggle truss structure of example No.2 is analyzed but under triangular forcing function, see Figure (5-9). To check the adequacy of the computer program **(GNLDAPT) Geometric Non-Linear Dynamic Analysis of Plane Trusses** in predicting the geometrically non-linear (large deformation) behavior of trusses, the results was compared with the those obtained by K. ZHU, and et. al. (1994)⁽⁴¹⁾.

Figure (5-10) shows the non-linear transient response under triangular forcing function. The response during the impulse duration only is shown.

The Method of Boundary Characteristics gives larger results about (4%) at the initial and latest time of analysis. From example number (2) & (3) it can be noticed that there is good agreement between the two methods of solution, where the difference in deflection does not exceed (6%) and the Method of Boundary Characteristics can be used to simulate the dynamic behavior of real complex structures.

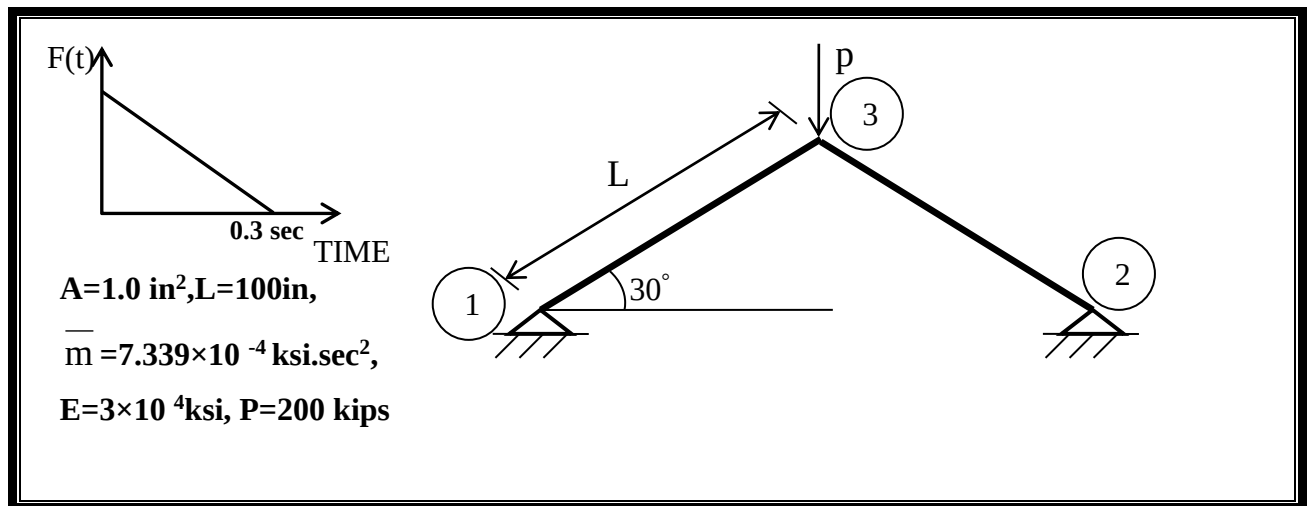


Figure (5-9): Details of Example No.3

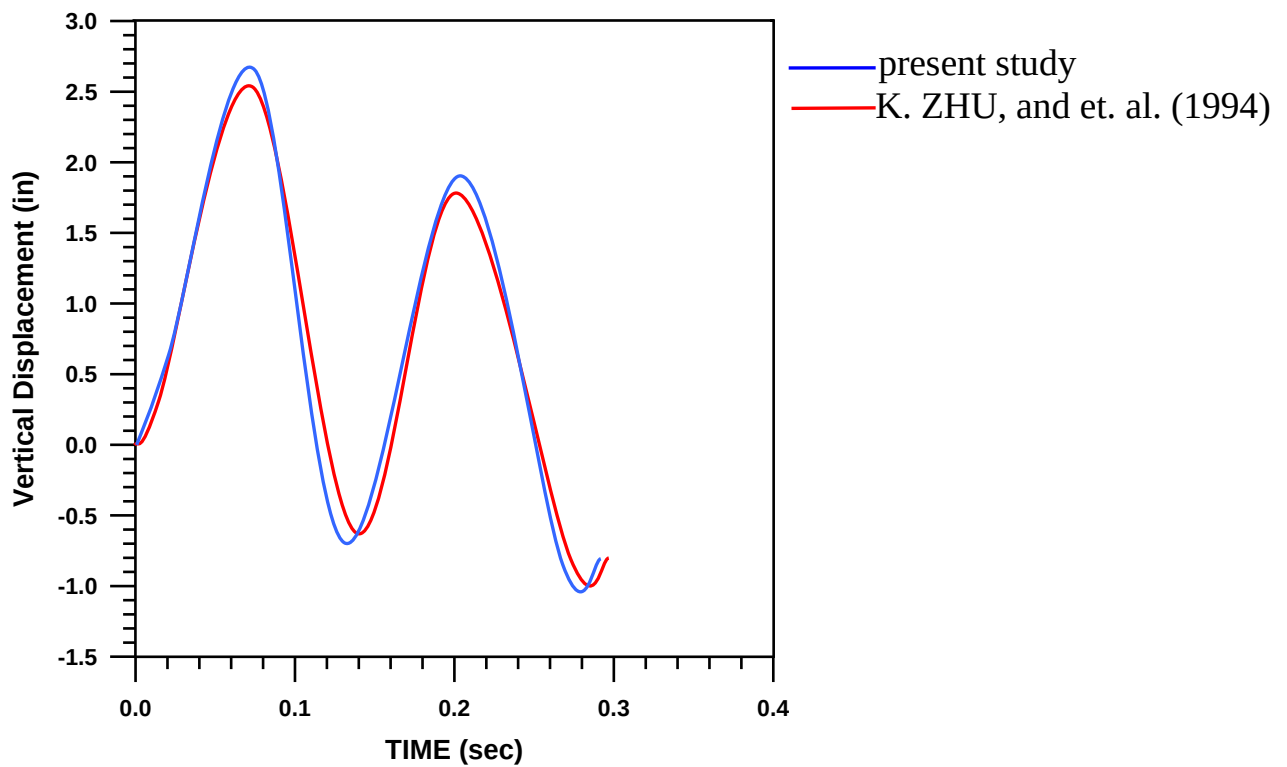


Figure (5-10): Dynamic Response of Toggle Truss under a Triangular Forcing Function

5.9: Numerical Applications :

Several examples of truss structures under dynamic loads are presented in this section to show the ability of The Boundary Characteristics Method in predicting the dynamic behavior of the truss structures. The first five examples are analyzed as linear or small deformation analysis and the other examples are analyzed as large deformation or geometrically non-linear, also several examples are analyzed as material non-linearity.

EXAMPLE NO.4 :

Starting from a most simple truss by considering a two member truss (toggle truss) which has the geometry and the dynamic characteristics as shown in Figure (5-11).

The structure is subjected to both horizontal and vertical step forcing function. The horizontal and vertical movement histories for joint number (3) are depicted in Figure (5-12). From these figures it is concluded that the response of such truss is simple with constant natural time period about (0.9ms), but the amplitude has little change about (6.2%) due to a hidden effect of the fundamental frequency of the truss.

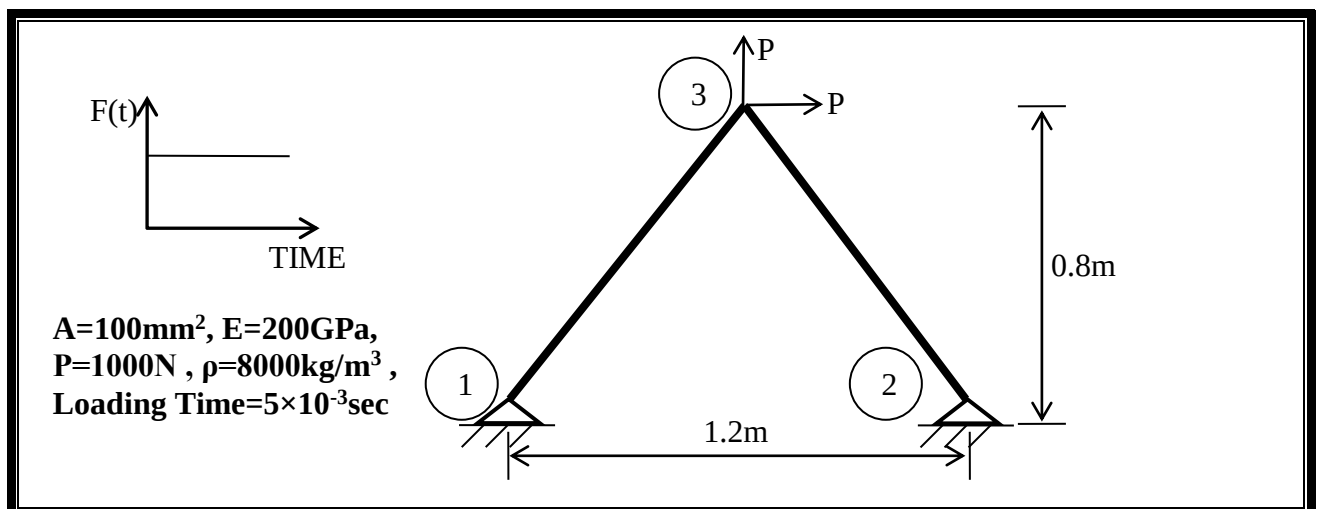
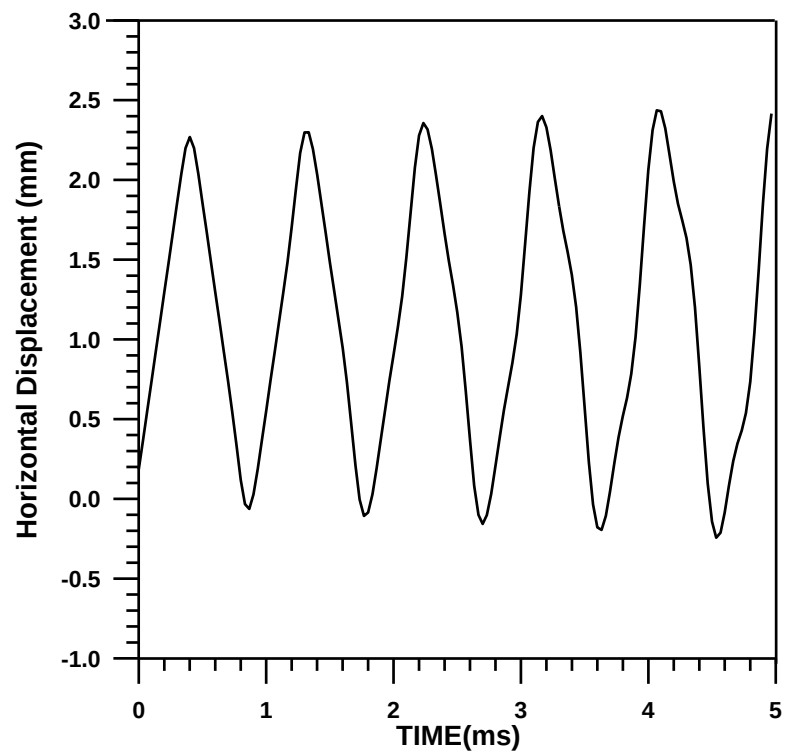
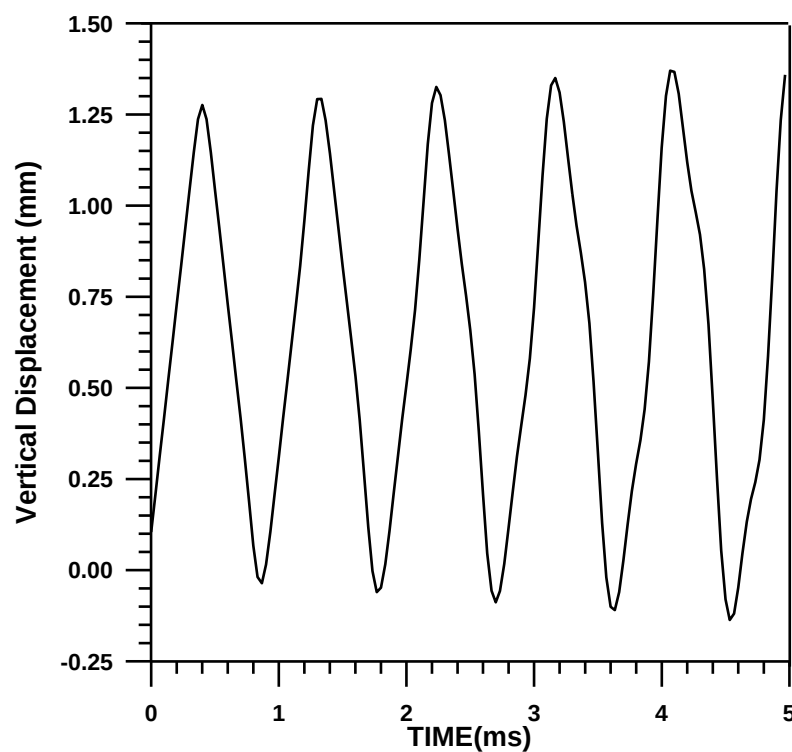


Figure (5-11): Details of Example No.4



(5-12A): Response of Horizontal Displacement at Joint (3)



(5-12B): Response of Vertical Displacement at Joint (3)

Figure (5-12): Dynamic Response for Truss Structure of Example No.4

EXAMPLE NO.5 :

Now a truss structure of three members jointed together at a one end are considered to show the ability of the method to simulate the wave propagation at multi connected members. The geometry and the dynamic characteristics are given in Figure (5-13). The step forces function shown in Figure (5-13) are subjected to the free joint and the linear displacement response of that joint are shown in Figure (5-14). The general vibration of horizontal displacement is happened after (0.9ms), (i.e. after the wave reaches to the boundary).

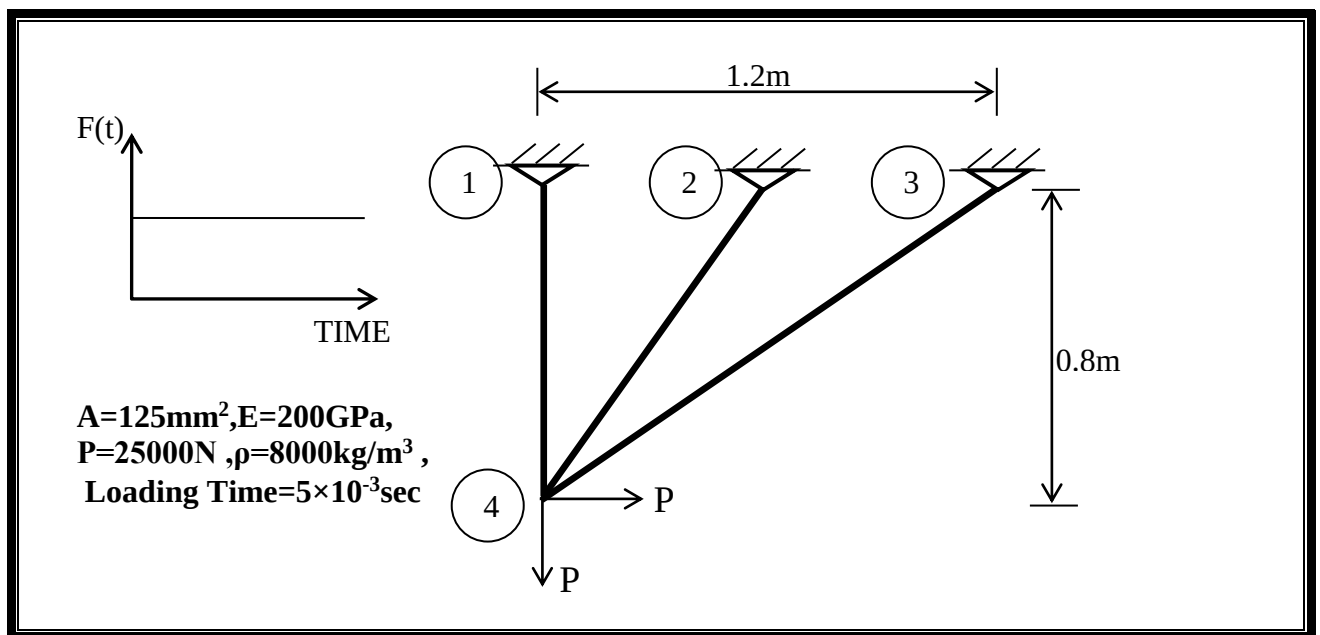
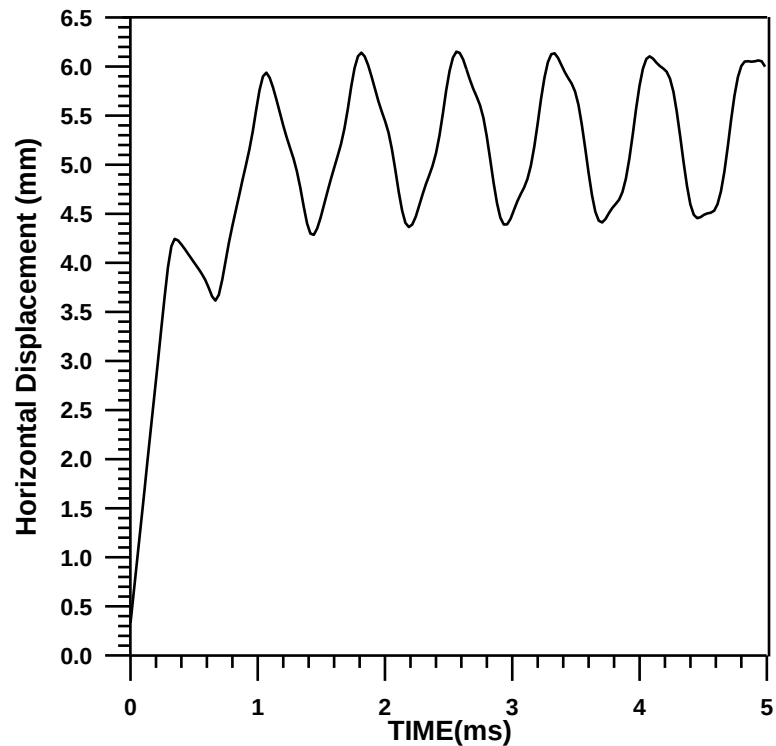
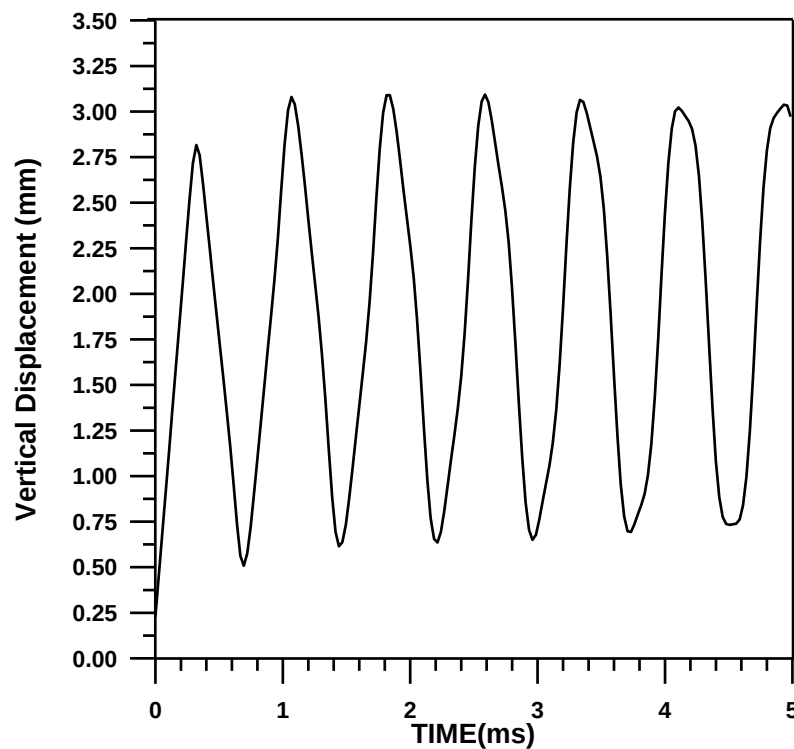


Figure (5-13): Details of Example No.5



(5-14A): Response of Horizontal Displacement at Joint(4)



(5-14B): Response of Vertical Displacement at Joint (4)

Figure (5-14): Dynamic Response for Truss structure of Example No.5

EXAMPLE NO.6 :

A box truss structure of five members is presented in this example. The details of the geometry and dynamic characteristics of this truss are given in Figure (5-15). The step forces function shown in Figure (5-15) are subjected to the truss structure at joint (3). The dynamic behavior of this example including the displacements response is shown in Figure (5-16). It can be noticed that the amplitude of horizontal displacement is larger than the amplitude of vertical displacement. This is attributed to the nature of the boundary condition of the members (restrained or joints), where the horizontal forcing function affected directly on a member has joints at both ends, but the vertical forcing function affected directly on a member has a joint at one end and the other is fully restrained.

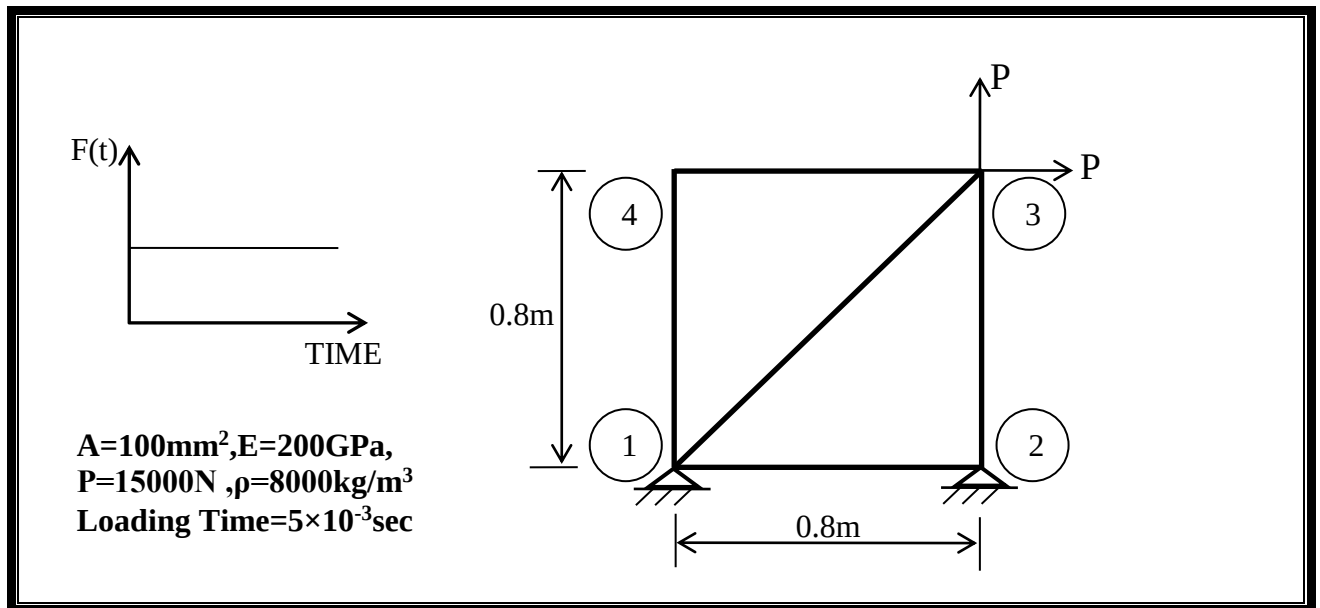
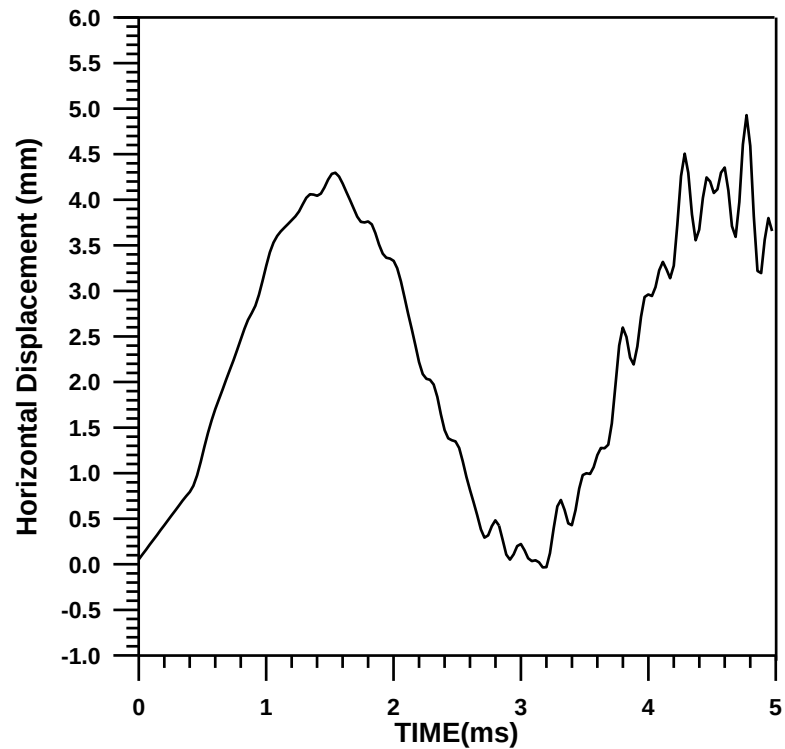
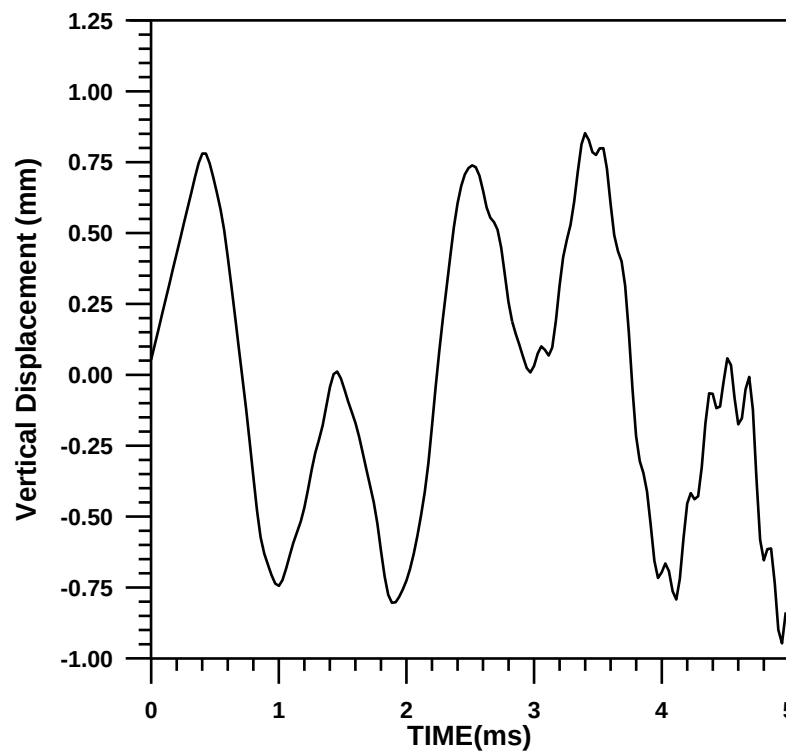


Figure (5-15): Details of Example No.6



(5-16A): Response of Horizontal Displacement at Joint (3)



(5-16B): Response of Vertical Displacement at Joint (3)

Figure (5-16): Dynamic Response for Truss structure of Example No.6

EXAMPLE NO.7 :

A truss structure of six members is considered in this example. The geometry and dynamic characteristics of this truss are given in Figure (5-17). The displacement histories in both directions (X–direction and Y–direction) are predicted and plotted in Figure (5-18). It can be noticed from displacement response of truss in example seven is less than the displacement response of truss in example six. That's caused by the additional member which has increased the stiffness of the structure, also the wave speed has decreased (i.e. number of cycles has increased).

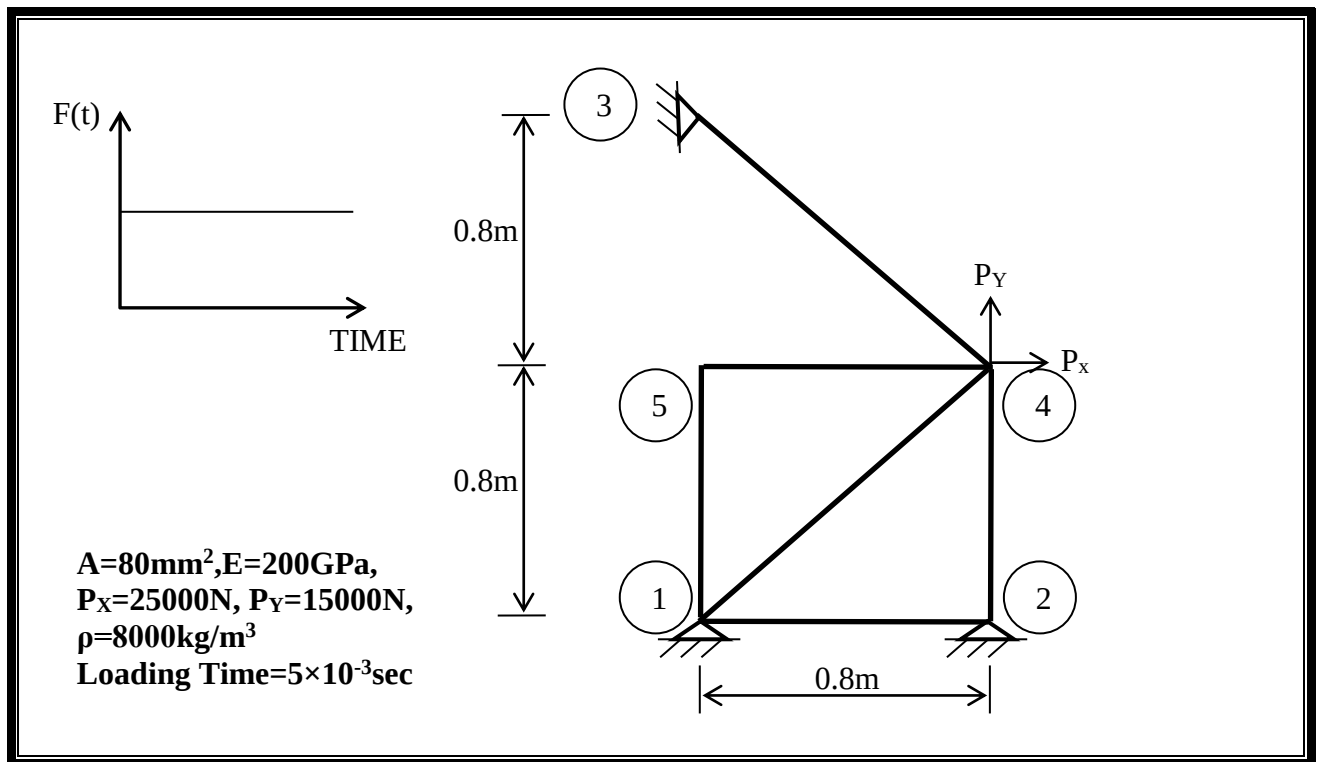
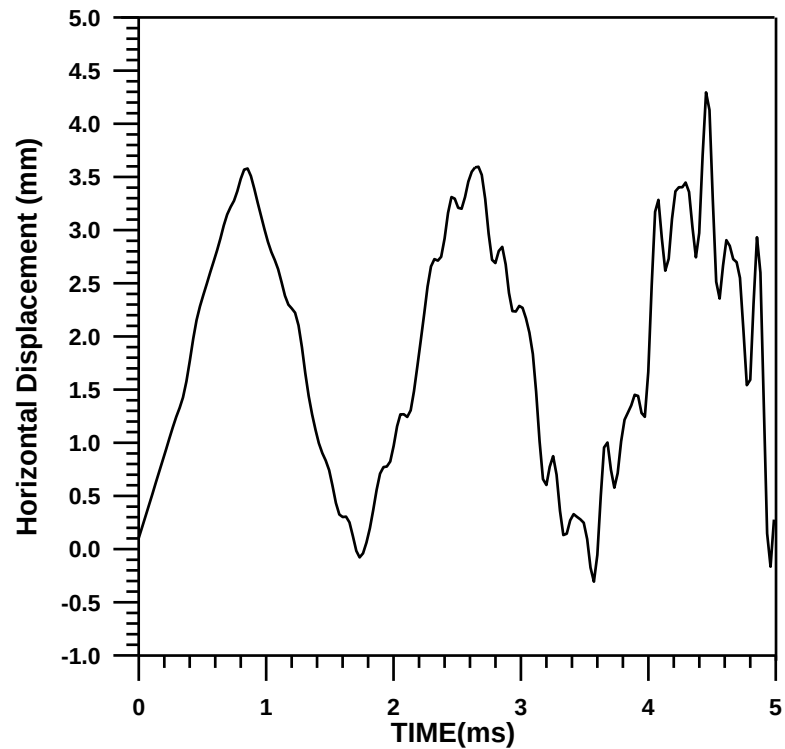
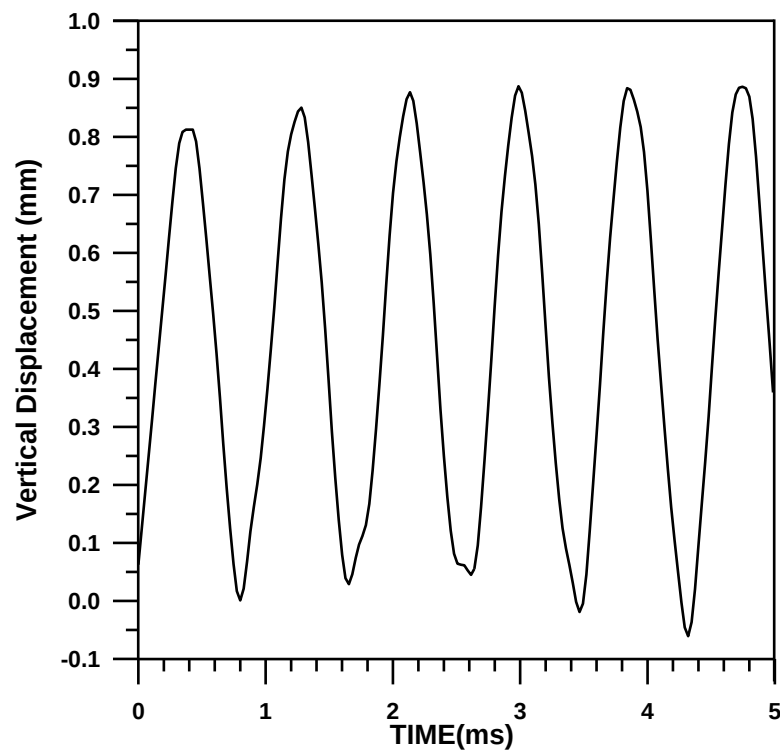


Figure (5-17): Details of Example No.7



(5-18A): Response of Horizontal Displacement at Joint (4)



(5-18B): Response of Vertical Displacement at Joint (4)

Figure (5-18): Dynamic Response for Truss structure of Example No.7

EXAMPLE NO.8 :

A tower truss structure of seven members with the geometry and dynamic characteristics are given in Figure (5-19), is studied.

The dynamic behavior of this truss structure including displacement histories in both directions are simulated as shown in Figure (5-20). The dynamic behavior of this truss structure is going to the instability after (4ms). This instability may be attributed to errors. These types of errors may be resulted from one of the following:

1. There is some numerical errors resulted from the interpolation of the Method of Boundary Characteristics (time domain), these errors has a recognized effect.
2. Due to the neglecting of damping effect, the reflected waves at boundaries or the truss joints has a considerable effect which is called noise waves.

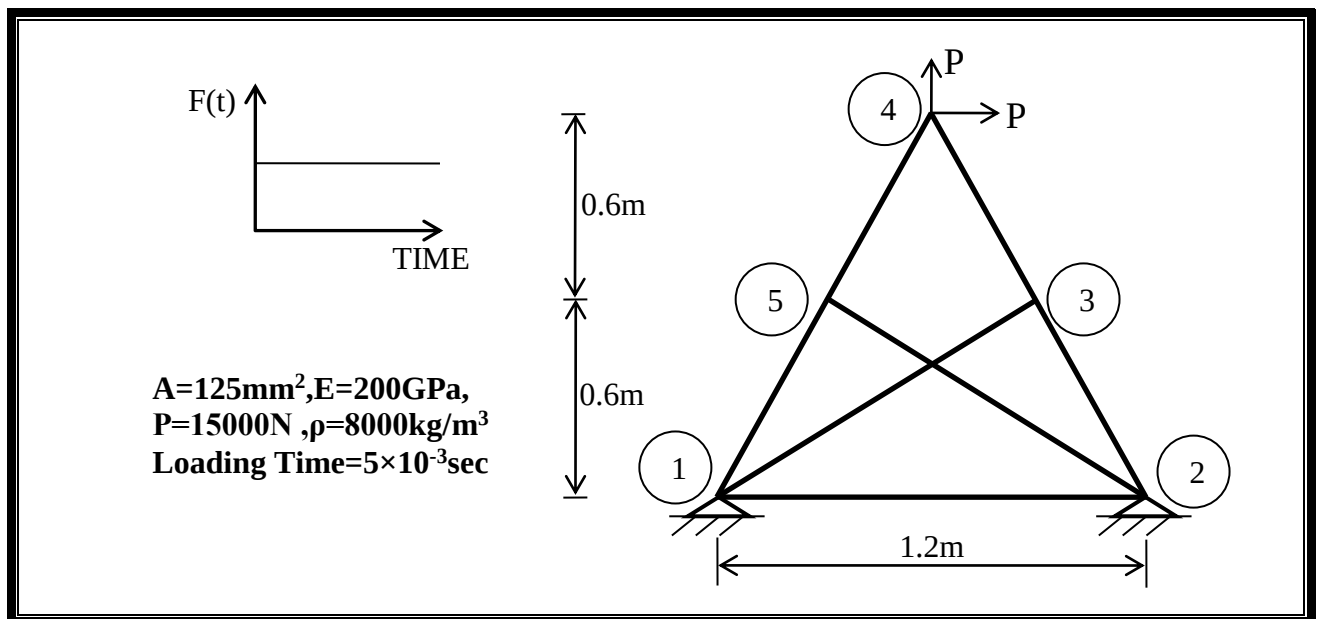
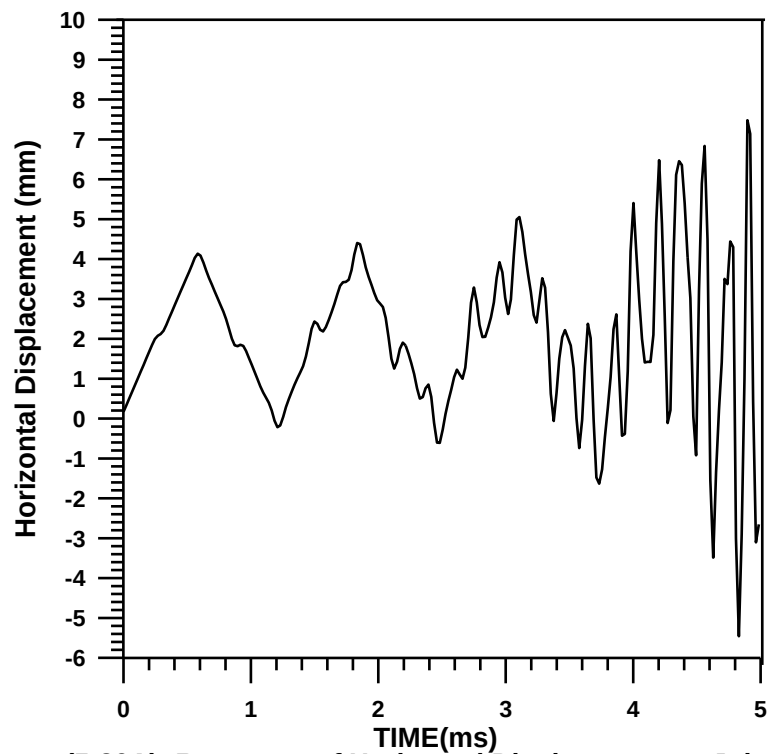
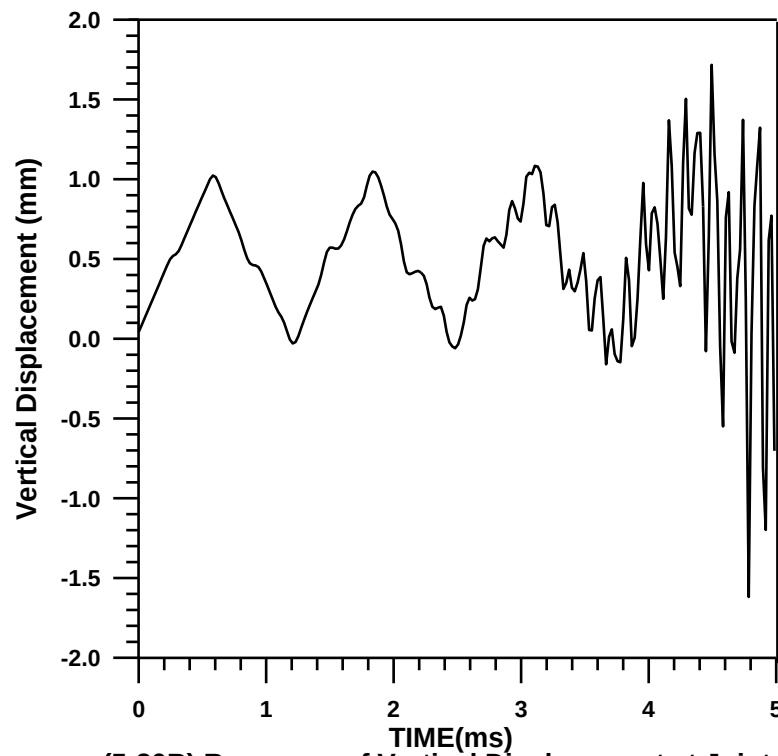


Figure (5-19): Details of Example No.8



(5-20A): Response of Horizontal Displacement at Joint (4)



(5-20B) Response of Vertical Displacement at Joint (4)

Figure (5-20): Dynamic Response for Truss structure of Example No.8

EXAMPLE NO.9 :

Simple truss structure of two members is considered. The geometry and dynamic characteristics are detailed in Figure (5-21). This truss is subjected to horizontal impulse forcing function. The geometrically non-linear (large deformation) behavior is predicted. Both displacement histories in (X-direction and Y-direction) are shown in Figure (5-22). The general behavior of the truss has changed after (1.25ms) (i.e. after the removal of the subjected load), where the vibration before (1.25ms) is above the zero (forced vibration), while after (1.25ms) the vibration becomes about the zero (free vibration).

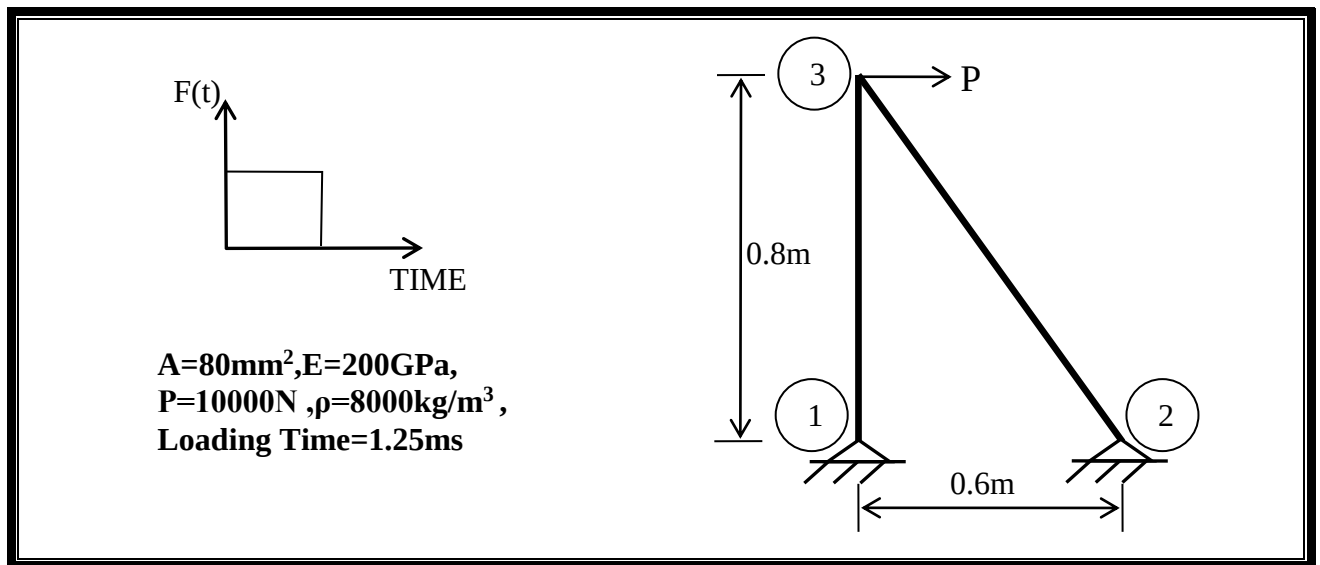
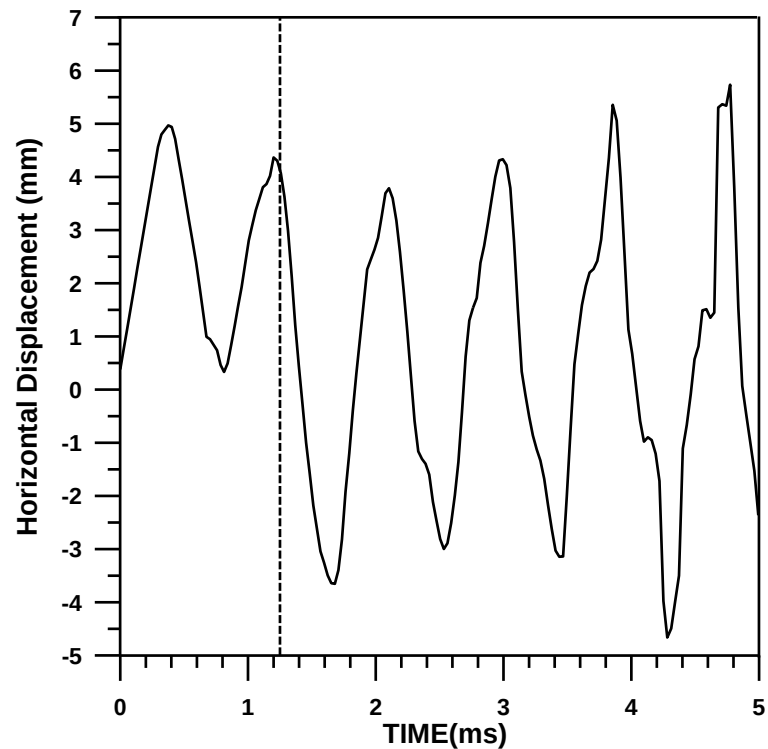
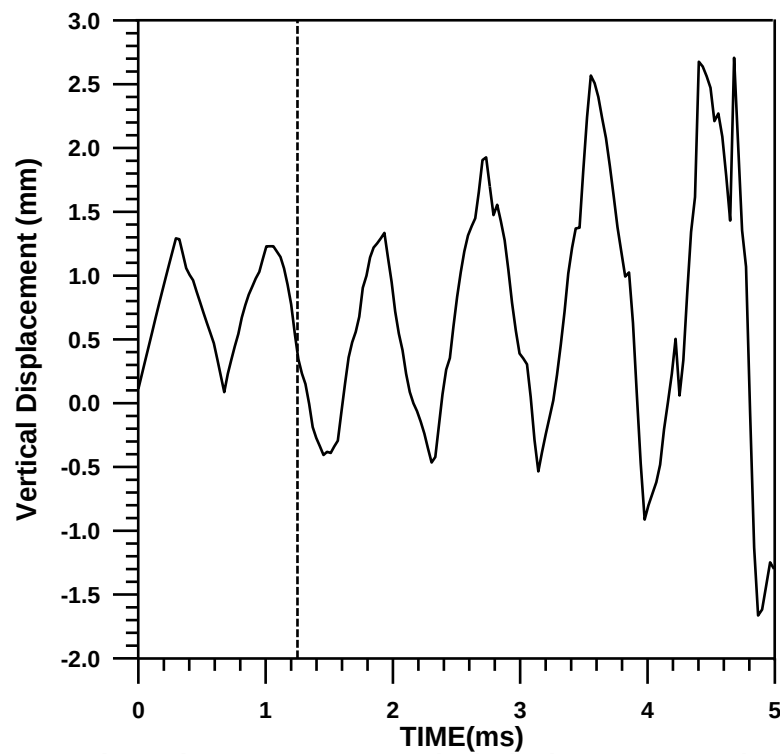


Figure (5-21): Details of Example No.9



(5-22A): Response of Horizontal Displacement at Joint (3)



(5-22B): Response of Vertical Displacement at Joint (3)

Figure (5-22): Dynamic Response for Truss structure of Example No.9

EXAMPLE NO.10 :

Toggle truss structure of two member is studied. The truss is subjected to horizontal and vertical step forcing functions of (20KN). The geometrically non-linear (large deformation) behavior including displacement histories in both directions for joint number (3) are predicted and shown in Figure (5-24).

It can be concluded that the response of this truss has slight increasing in the natural time period about (0.05ms), also has observed increasing in the amplitude about (13.5%) in the horizontal displacement and about (19.3%) in the vertical displacement, this is due to the effect of both the fundamental frequency of the truss and the geometrical non-linearity.

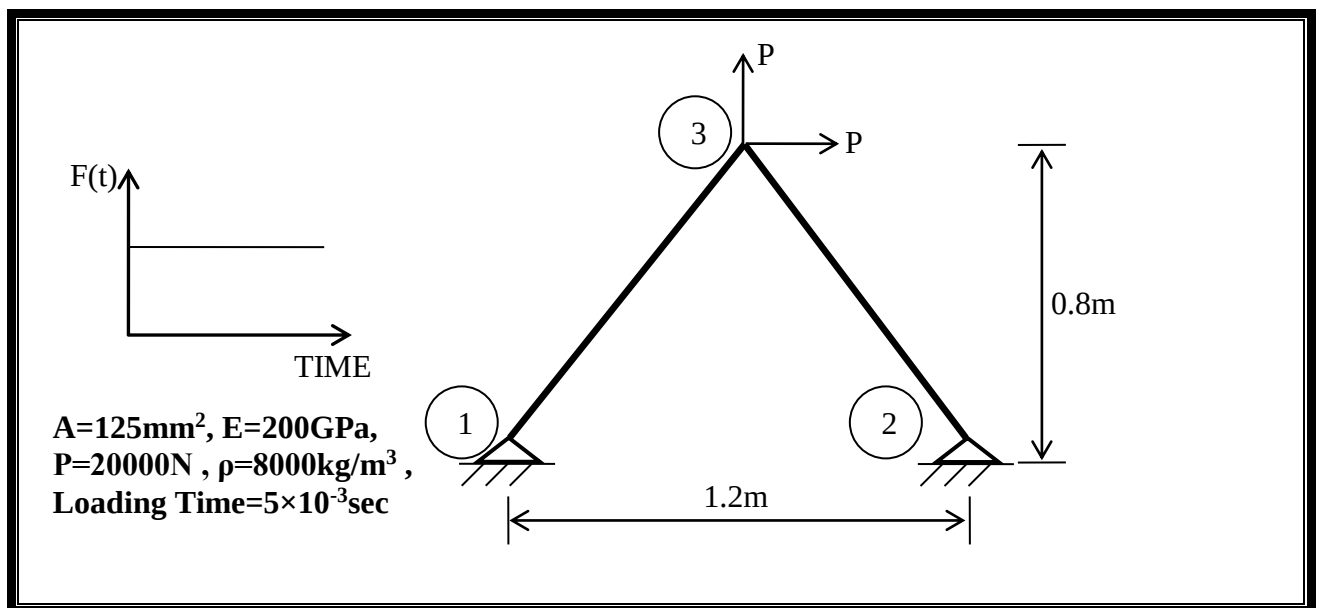
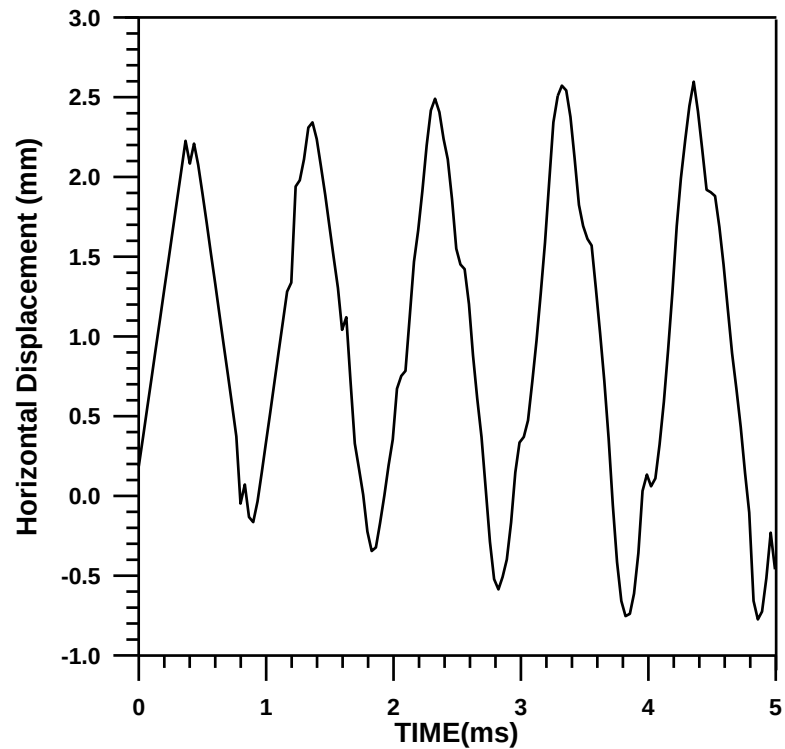
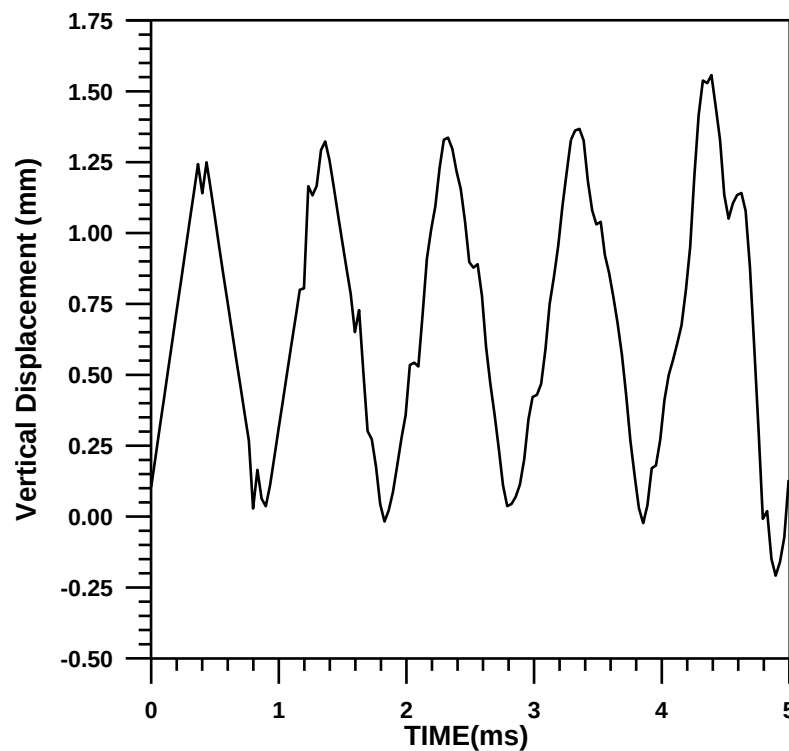


Figure (5-23): Details of Example No.10



(5-24A): Response of Horizontal Displacement at Joint (3)



(5-24B): Response of Vertical Displacement at Joint (3)

Figure (5-24): Dynamic Response for Truss structure of Example No.10

EXAMPLE NO.11 :

Three truss members jointed together at a one end is analyzed to examine the ability of the method at multi connected members. Its geometry and dynamic characteristics are given in Figure (5-25).

The geometrically non-linear (large deformation) behavior is predicted. The displacement response in X-direction and Y-direction are plotted as shown in Figure (5-26). Effect of large deformation can be noticed clearly in Figure (5-26), where as it is known for the non-linear dynamic analysis the stiffness matrix will be changed continuously with the time marching, so the response has no fixed form (i.e. variable amplitude and variable natural time period), while the linear dynamic analysis has a certain fixed shape (i.e. fixed natural frequency and fixed amplitude of vibration). Hence there is no realistic comparison can be made between the two responses.

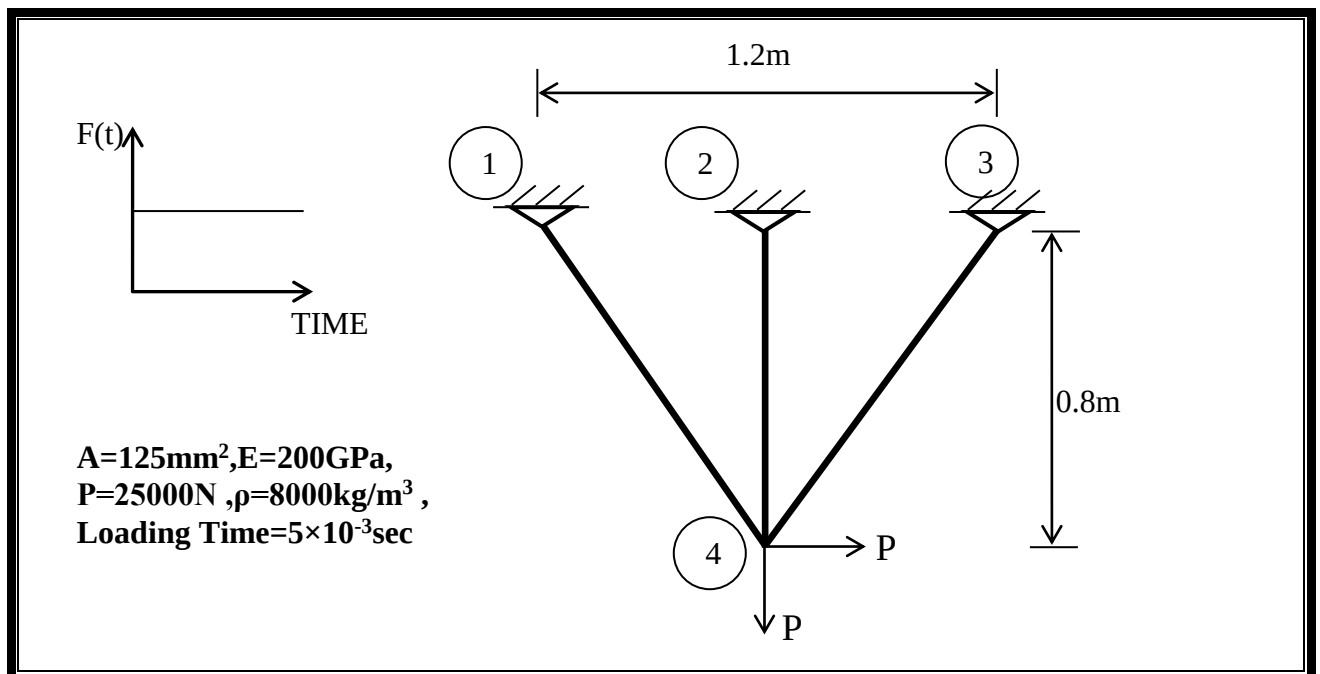
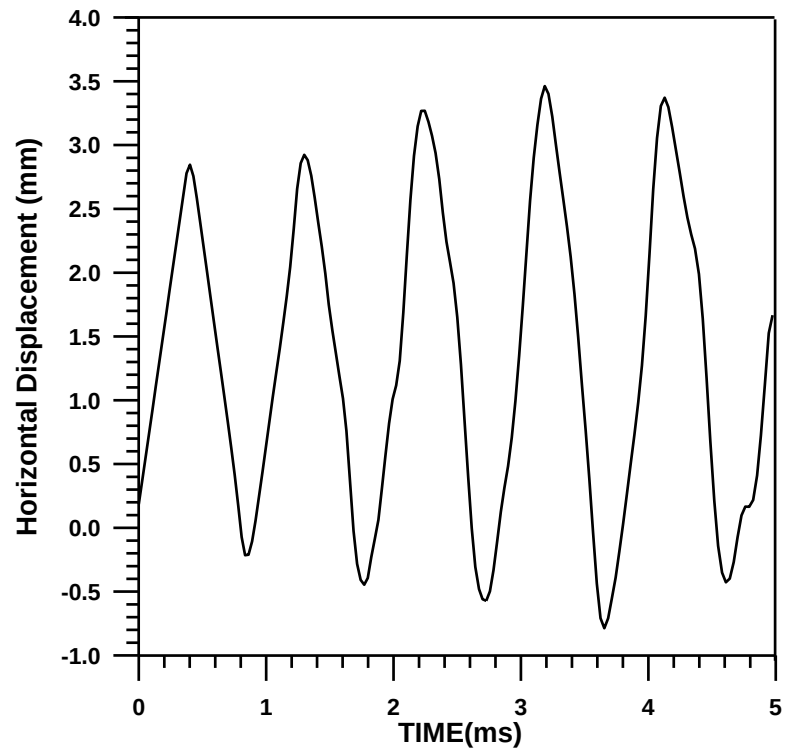
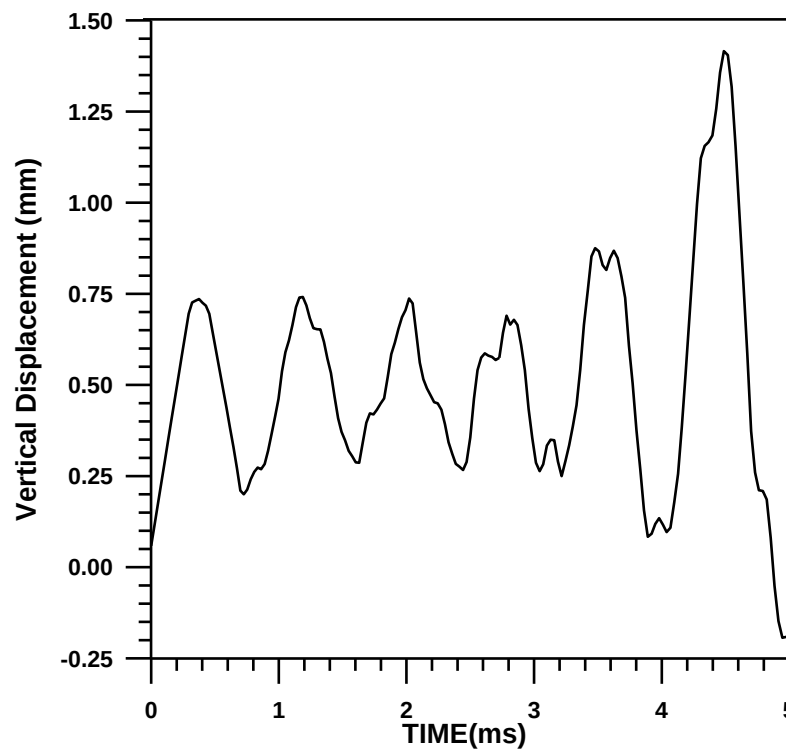


Figure (5-25): Details of Example No.11



(5-26A): Response of Horizontal Displacement at Joint (4)



(5-26B): Response of Vertical Displacement at Joint (4)

Figure (5-26): Dynamic Response for Truss structure of Example No.11

EXAMPLE NO.12 :

A truss structure of two members is studied. The details of the geometry, dynamic characteristics and material properties are given in Figure (5-27). The truss is subjected to horizontal and vertical step forcing functions of (30KN). The material non-linear behavior including displacement histories in both directions for joint number (3) are predicted and shown in Figure (5-28). The initial amplitudes of horizontal and vertical displacement are larger than the latest amplitudes of displacements but still larger than the static solution, this can be attributed to the varying stiffness with time of truss structure.

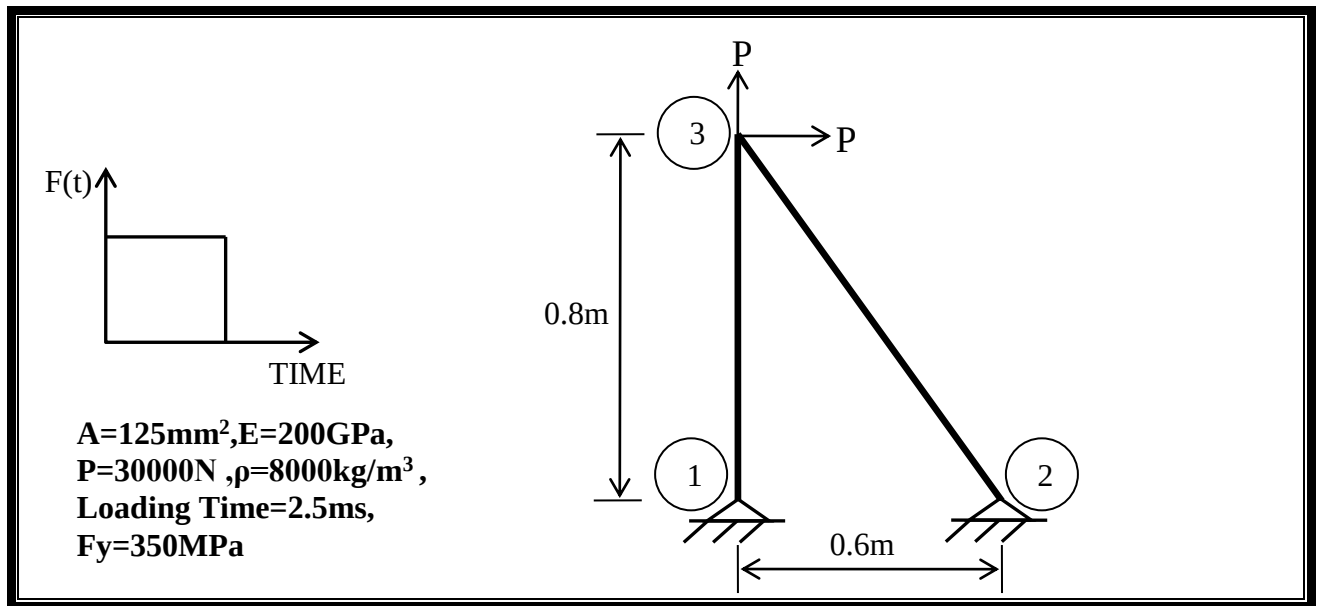
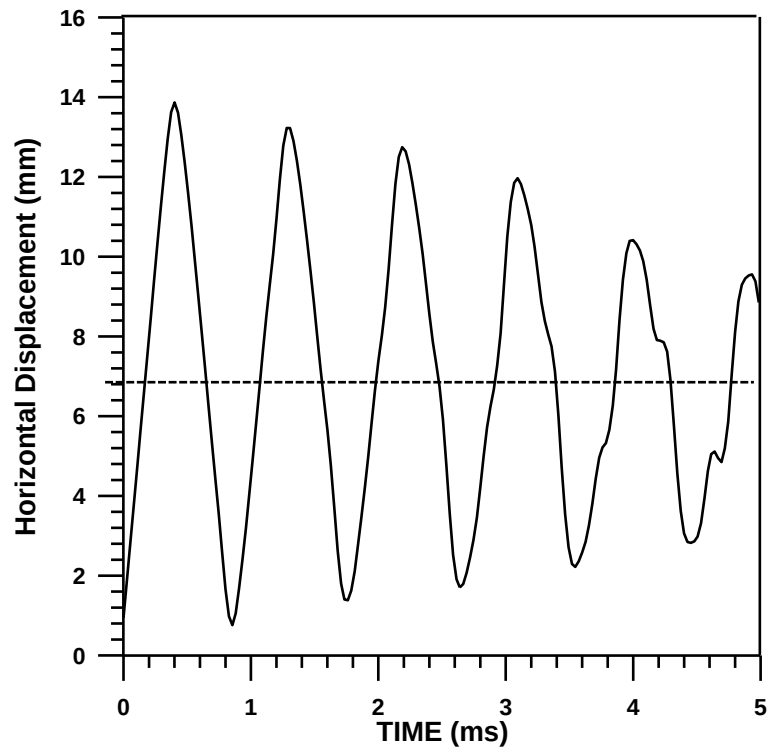
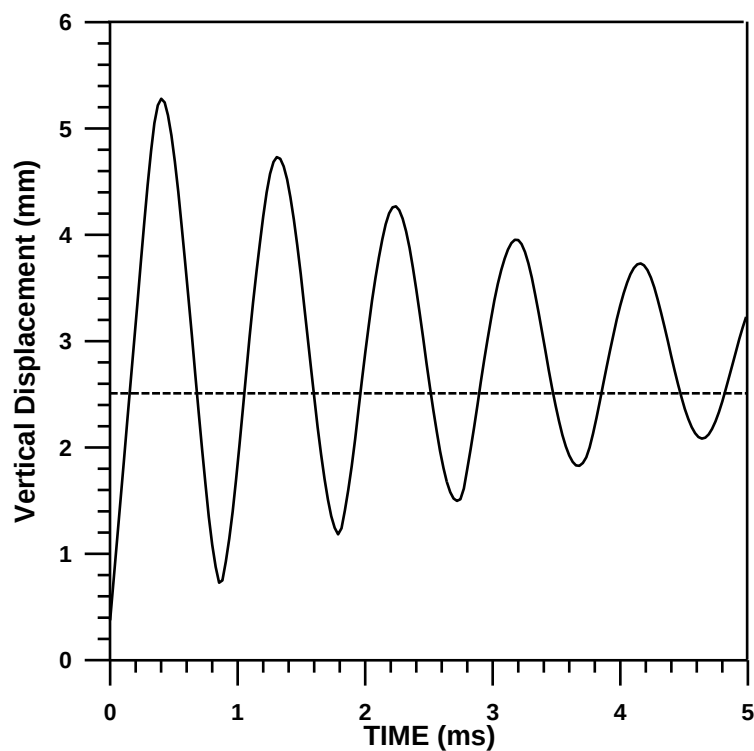


Figure (5-27): Details of the Truss Structure



(5-28A): Response of Horizontal Displacement at Joint (3)



(5-28B): Response of Vertical Displacement at Joint (3)

--- Static solution

Figure (5-28): Dynamic Response for Truss structure of Example No.12

EXAMPLE NO.13 :

A box truss structure of five members is presented in this example. The details of the geometry and dynamic characteristics of this truss are given in Figure (5-29). The step forces function shown in Figure (5-29) are subjected to the truss structure at joint (3). The dynamic behavior of this example are predicted when material non-linearity is considered. Figure (5-30) shows this behavior including the horizontal and vertical displacements response of this truss structure.

It can be noticed that the dynamic behavior of this truss structure has changed after (5ms) (i.e. after the removal of the applied loads), where the truss structure under forced vibration during the first (5ms), then the structure is being under free vibration.

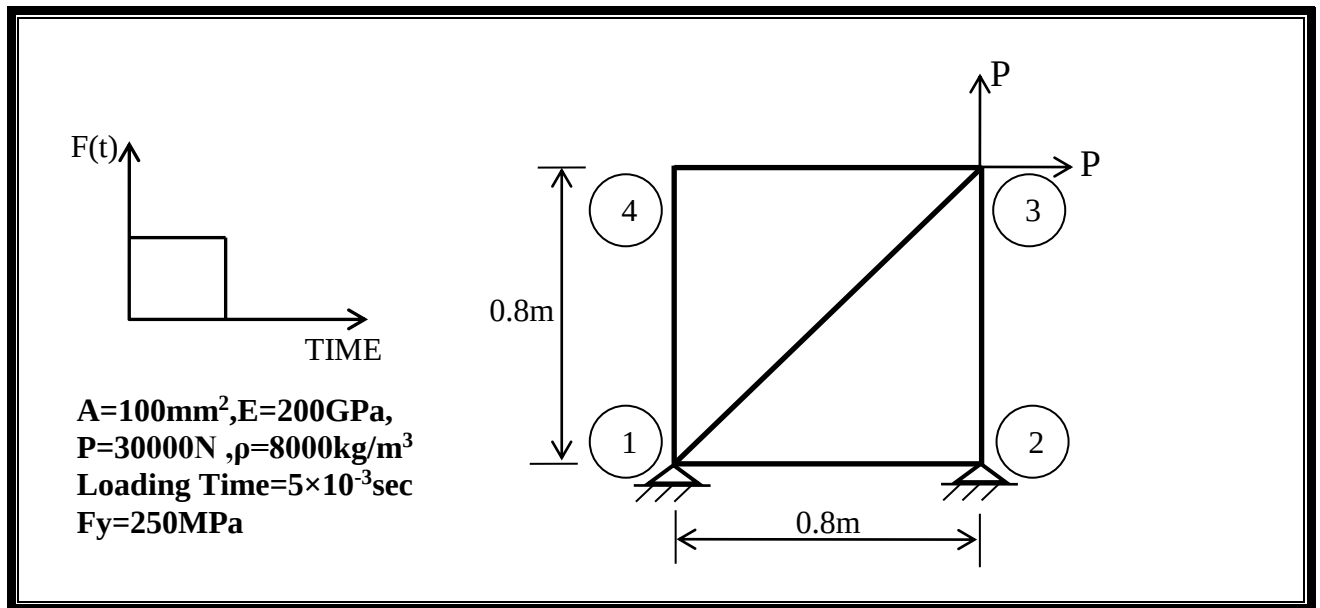
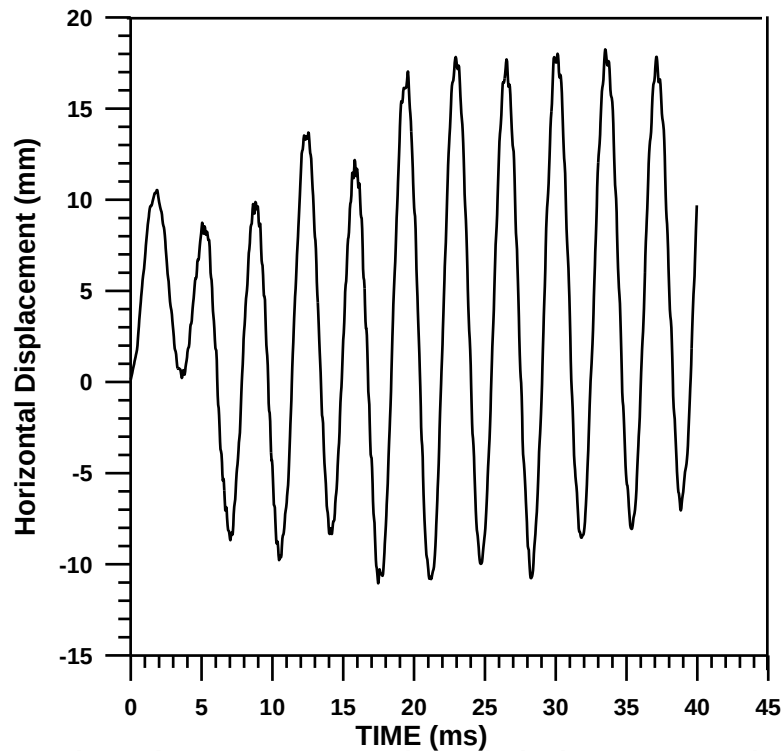
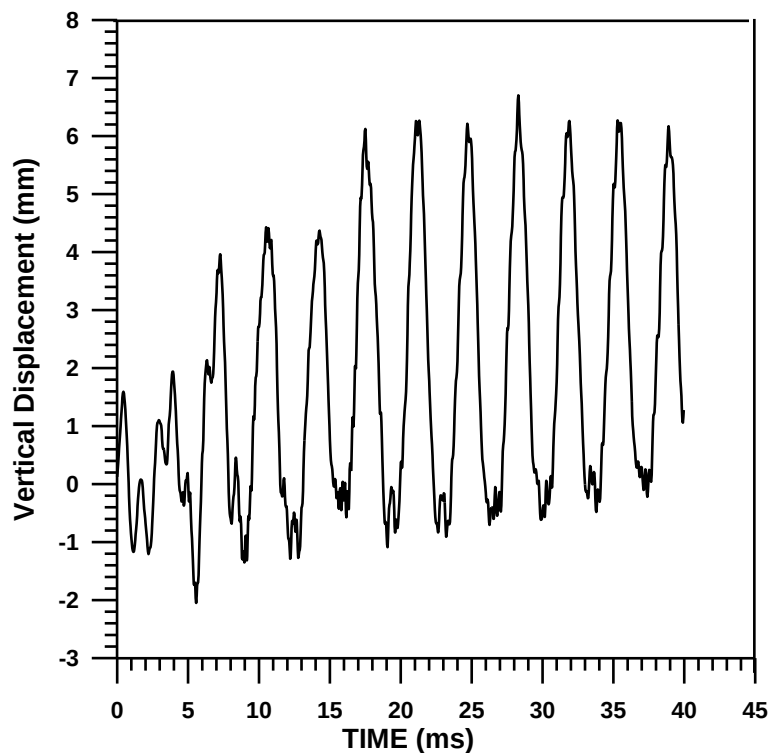


Figure (5-29): Details of the Truss Structure



(5-30A): Response of Horizontal Displacement at Joint (3)



(5-30B): Response of Vertical Displacement at Joint (3)

Figure (5-30): Dynamic Response for Truss structure of Example No.13

5.10: Effect of Large Deformation Behavior :

To assess the effect of large deformation behavior, or so-called geometrical non-linearity effect, on the elastic dynamic response of plane trusses, a simple two member truss structure is considered. All the details of this truss, including its geometry, material properties and the mechanical properties are given in Figure (5-31).

Figure (5-32) shows the response of horizontal and vertical displacement at joint (3). Figure (5-32) includes a comparison not only between small and large deformation but also with static displacement, (the ratio between displacement at dynamic loading and displacement at static loading is denoted by Dynamic Intensification Factor (D.I.F)).

At the initial time stages there is no lag time in horizontal and vertical displacement, while at the latest time stages the lag time is about (3.7%) for both horizontal and vertical displacement. There is no difference in the horizontal and vertical displacement at the initial time stages, but at the latest time stages the increase in large deformation analysis is about (34.6%) in horizontal displacement and (37%) in vertical displacement.

From these figures it is clear that the effect of large deformation can not, however, be ignored for very large deflections, due to the increasing errors in the time periods and the amplitudes of vibration. Moreover, it can be noticed that the large deformation analysis vary the amplitude of vibration and time periods from that at small deformation analysis, where the amplitude of vibration may be larger or smaller depending on the type of truss and loading direction (as will be shown later). This event is caused by the change in geometrical properties of the members, hence, the global geometry of the truss.

Figure (5-33) shows the mode shape of the same truss structure at different time periods, where after (50ms) the truss structure geometry has changed (i.e. the position of the joint (3) has changed).

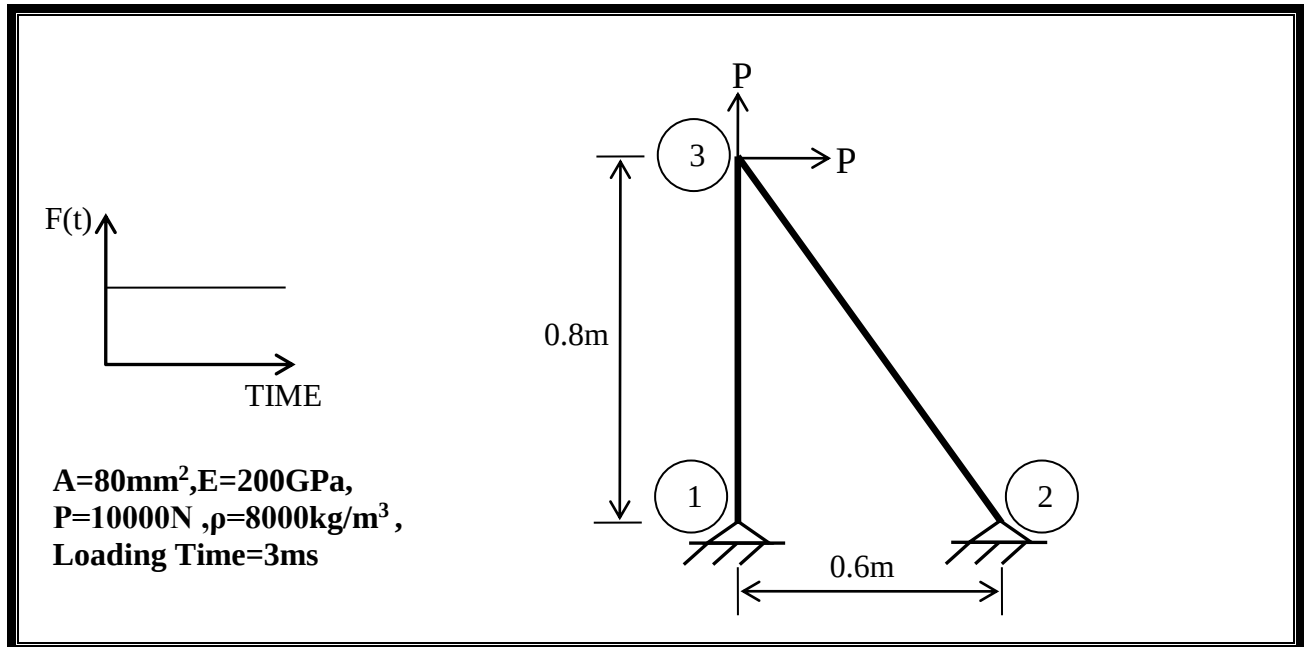
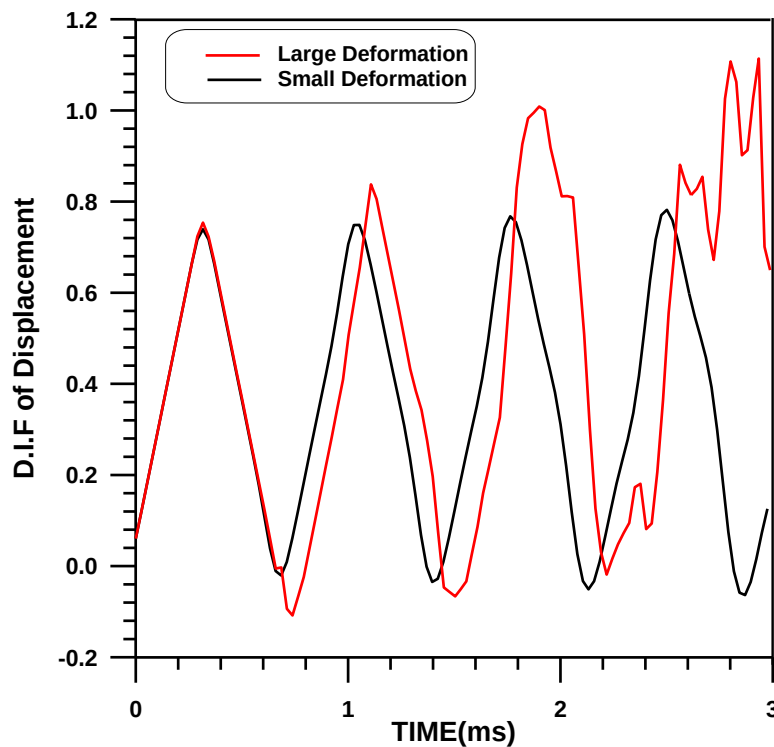
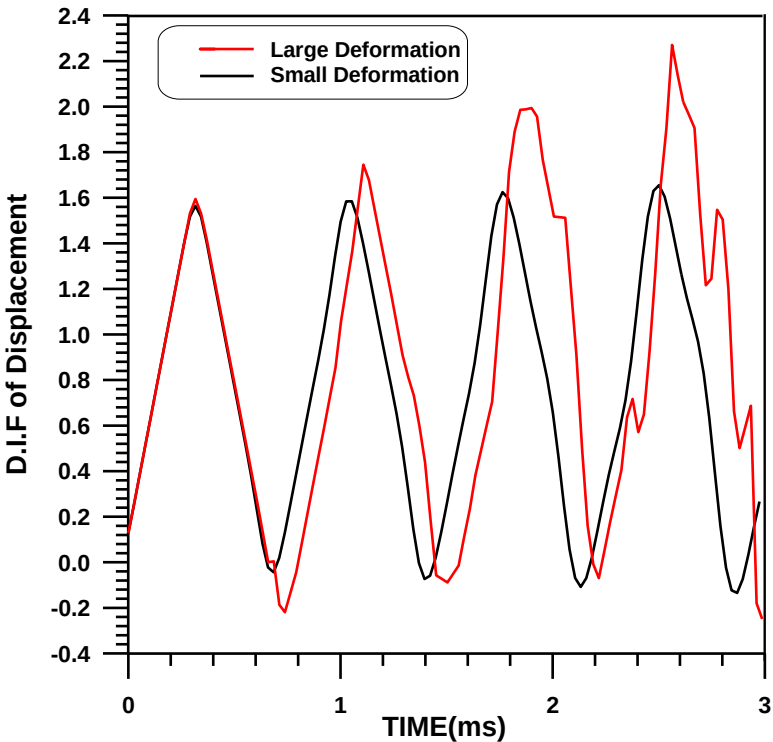


Figure (5-31): Details of the Truss Structure



(5-32A): Response of Horizontal Displacement at Joint (3)



(5-32B): Response of Vertical Displacement at Joint (3)

Figure (5-32): Effect of Large Deformation on the Truss Structure Behavior

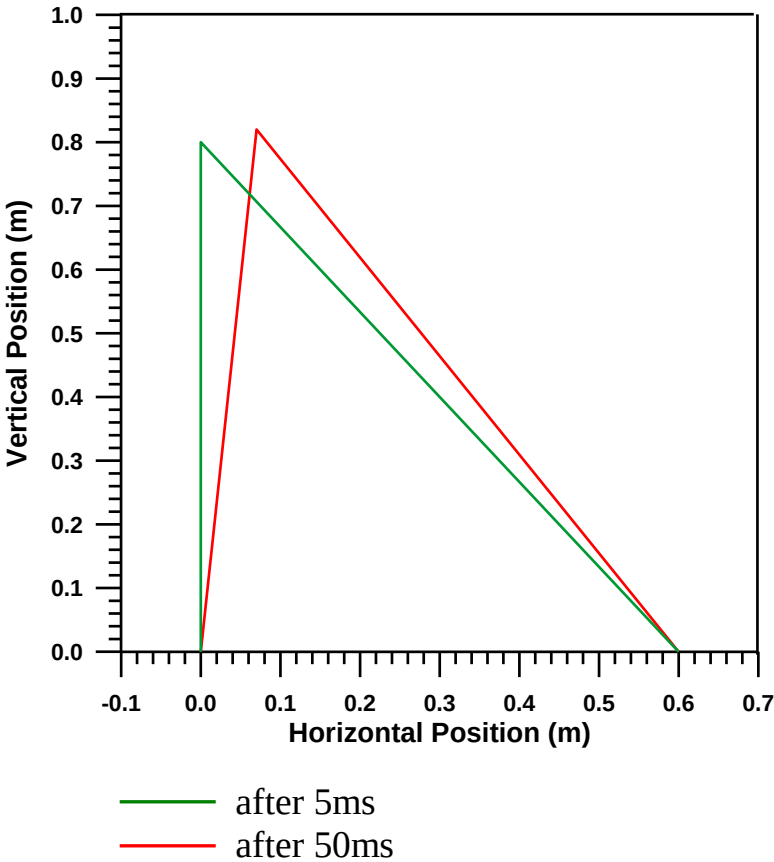


Figure (5-33): The Mode Shape of the Truss Structure

5.11: Effect of Load Intensity :

To study effect of load intensity on the elastic dynamic behavior of the plane truss structure, a truss structure of two members is subjected to the different impulse forcing functions. The details of the geometry, mechanical properties, and material properties are shown in Figure (5-34).

The three impulse forcing function are :

load (1) : $P(t)=7500 \text{ N}$, and $(\Delta T)=2.5\text{ms}$;

load (2) : $P(t)=10000 \text{ N}$, and $(\Delta T)=2.5\text{ms}$;

load (3) : $P(t)=15000 \text{ N}$, and $(\Delta T)=2.5\text{ms}$.

Figure (5-35) shows that the magnitude of the applied load can significantly affect the dynamic behavior of the truss. It can be noticed that when the structure is subjected to (15000N) the displacements history increased remarkably from the displacements history at loads (7500N and 10000N).

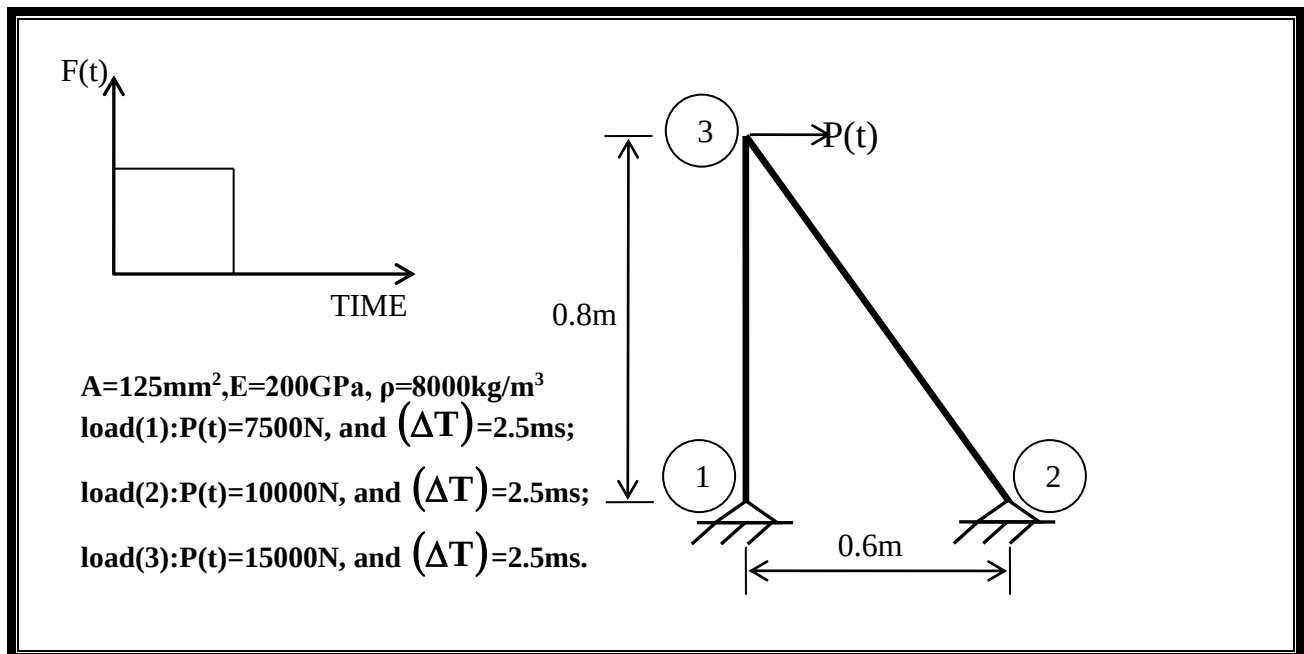
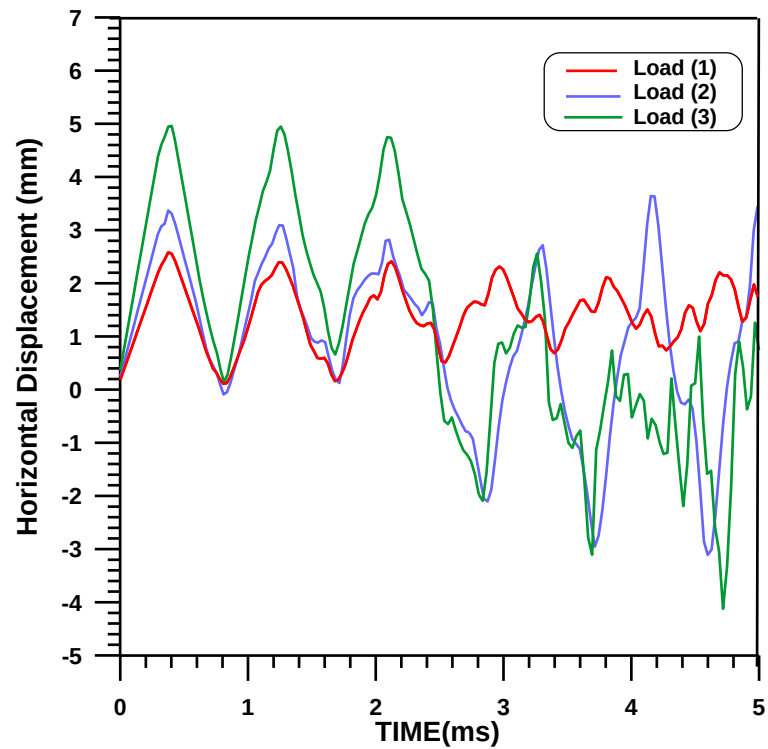
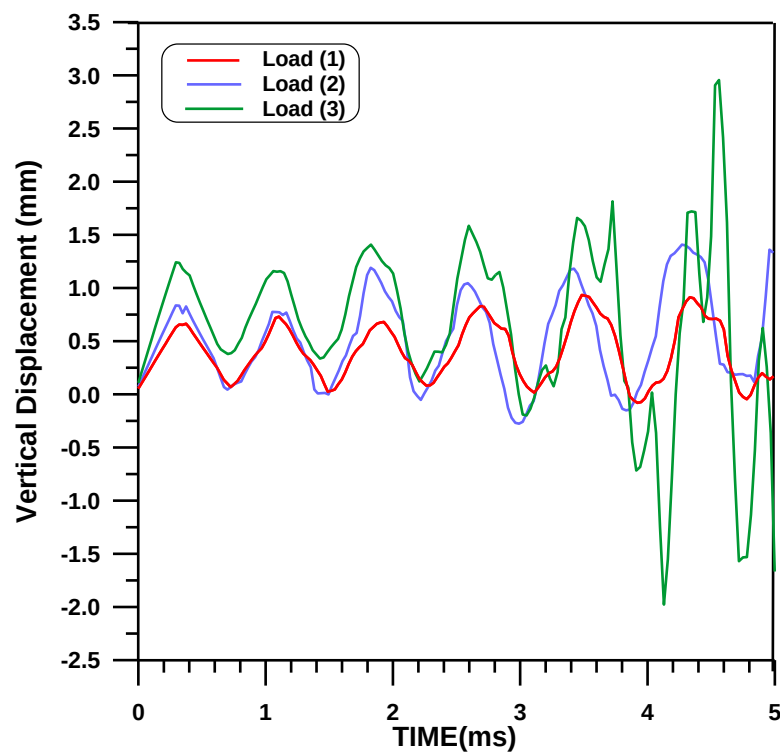


Figure (5-34): Details of the Truss Structure



(5-35A): Response of Horizontal Displacement at Joint (3)



(5-35B): Response of Vertical Displacement at Joint (3)

Figure (5-35): Effect of Load Intensity on the Truss Structure Behavior

5.12: Effect of Loading Sign (Direction) :

To show effect loading sign (direction), a truss structure of two members is considered. The details of its geometry, mechanical properties and material properties are given in Figure (5-36). This truss structure is subject once to left forcing function, and then to right forcing function. To assess the difference between effect of the two opposite directions of load, the displacement caused by left forcing function is multiplied by (-1), also the displacement curves will be in the positive region only. This comparison in dynamic behavior is predicted and shown in Figure (5-37).

From these figures it can be concluded that effect of right forcing function is larger than effect of left forcing function, where the amplitude of vibration is larger in right forcing function, that is caused by the varying stiffness at each case of loading.

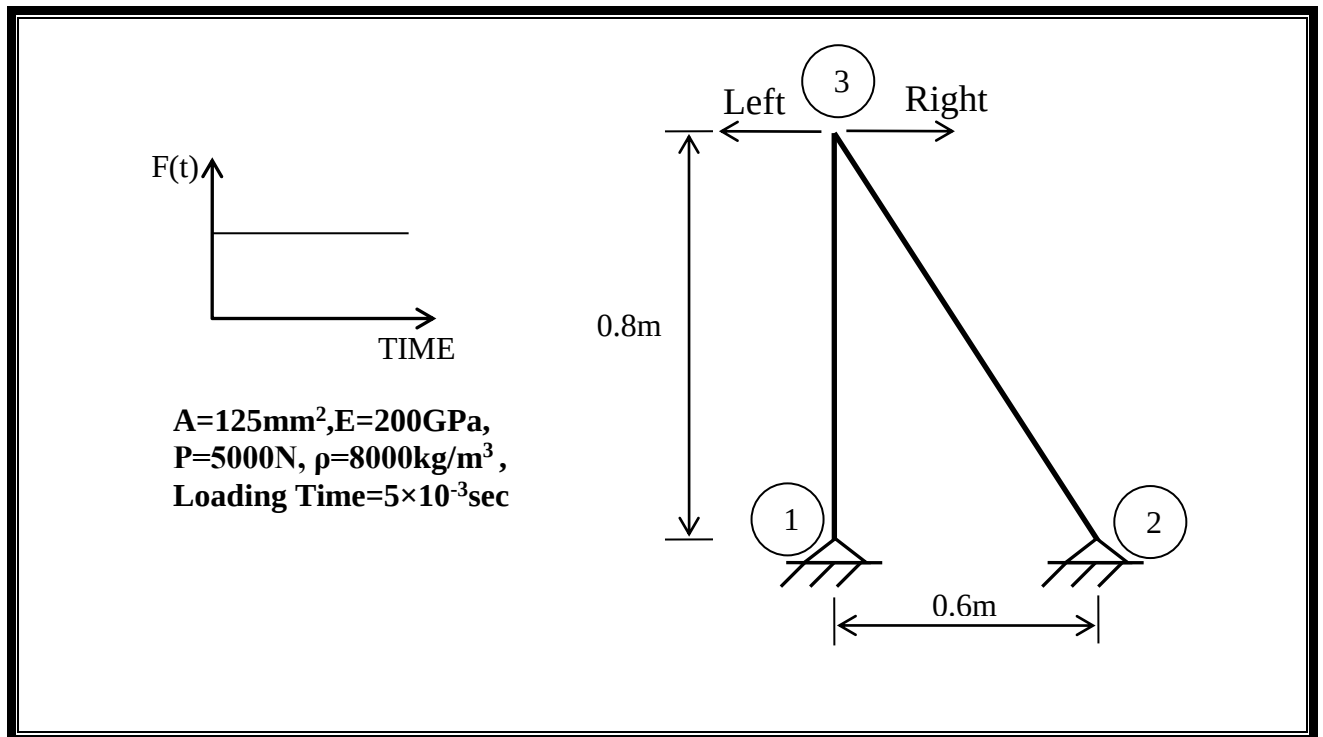
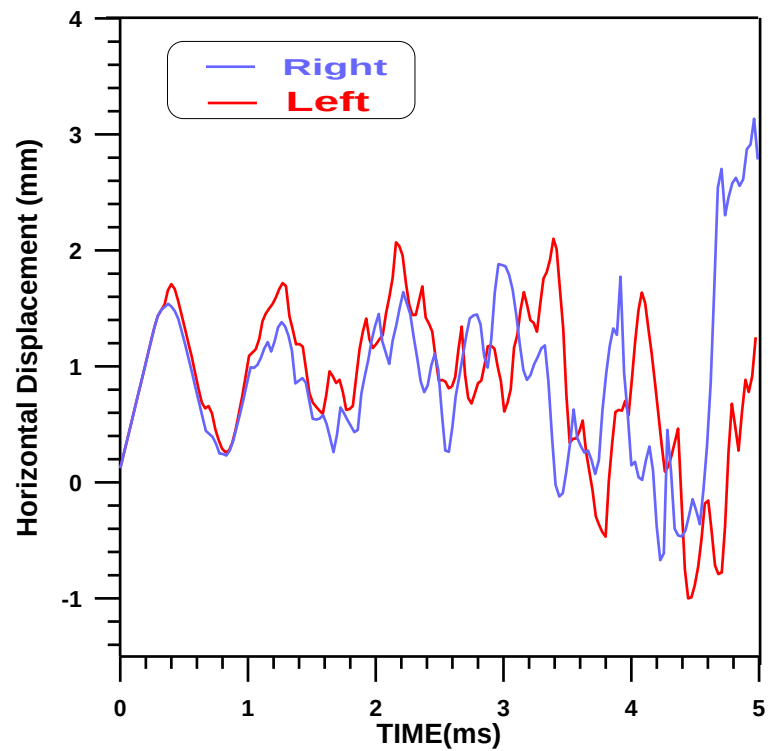
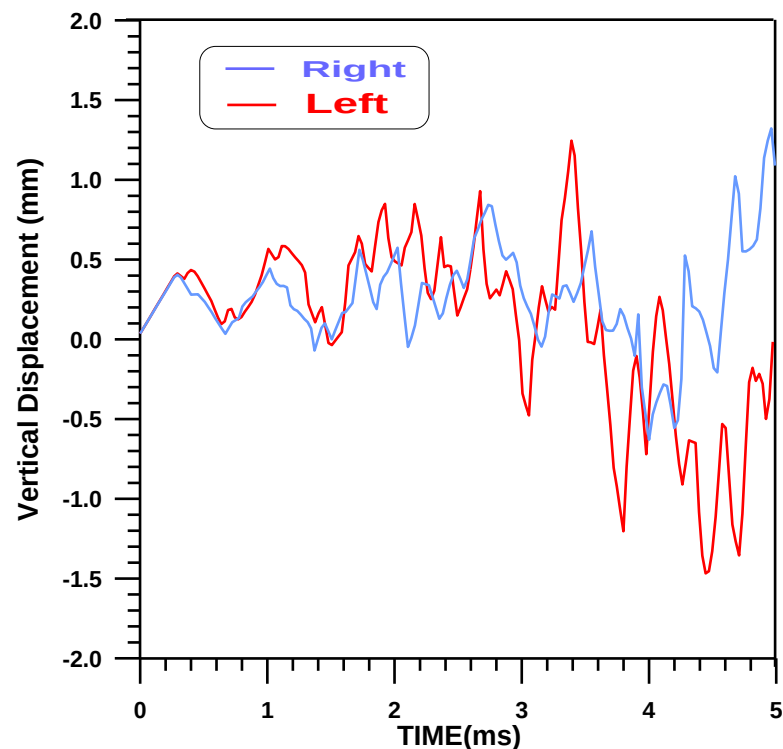


Figure (5-36): Details of the Truss Structure



(5-37A): Response of Horizontal Displacement at Joint (3)



(5-37B): Response of Vertical Displacement at Joint (3)

Figure (5-37): Effect of Loading Sign (Direction) on the Truss Behavior

5.13: Effect of Material Non-Linearity :

To study effect of material non-linearity on the elastic dynamic response of plane trusses, a truss structure of two member is considered. The geometry, material properties and the mechanical properties are shown in Figure (5-38). The response of horizontal and vertical displacement at joint (3) are predicted and shown in Figure (5-39).

At the initial time stages, there is no difference in the amplitude of displacements between small deformation and large deformation results, but the increase in material non-linear analysis is about (18.5%), while at the latest time stages, the average increase in the amplitude of displacement is about (18.5%, 5%) in large deformation and material non-linear analysis respectively. Also the increase in the natural time period is about (3.3%, 7.76%) for large deformation, and material non-linear analysis respectively. It can be noticed remarkably that the effect of non-linear analysis can not be ignored for high loading (i.e. when the loading is exceed the yield limit), because it will give incorrect prediction in the time periods and the amplitudes of vibration.

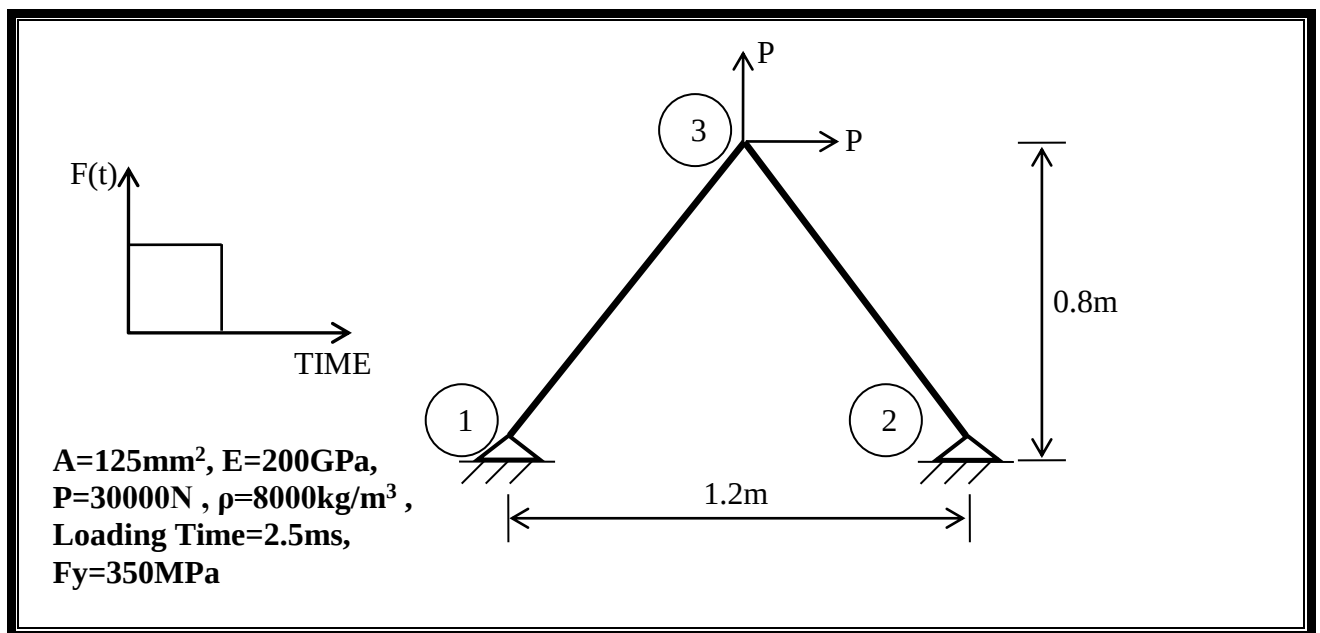
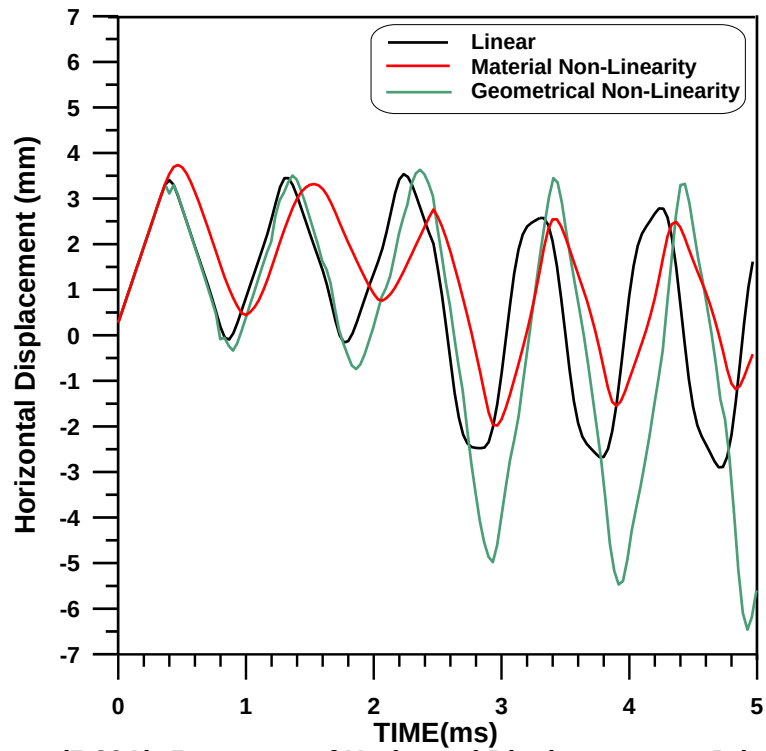
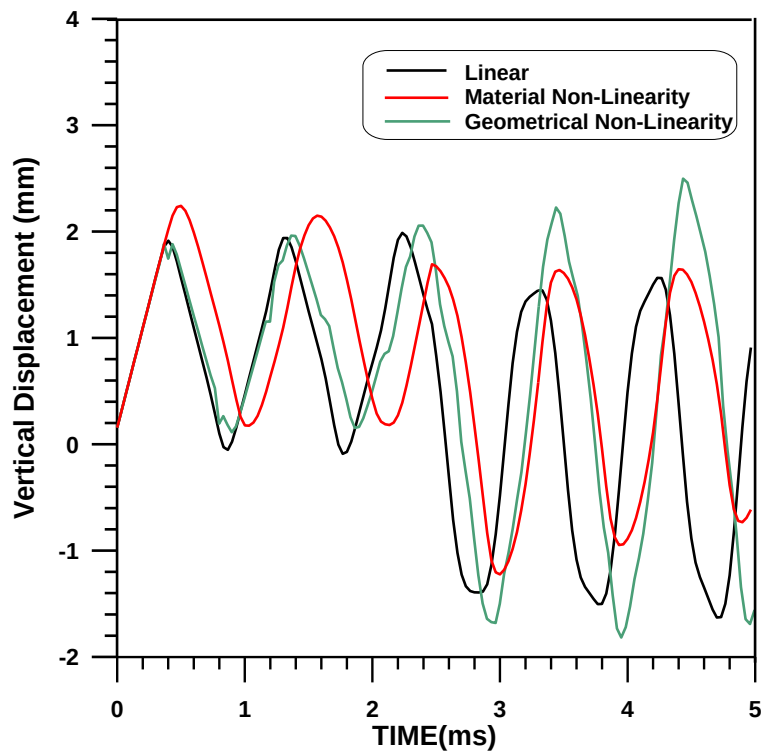


Figure (5-38): Details of the Truss Structure



(5-39A): Response of Horizontal Displacement at Joint (3)



(5-39B): Response of Vertical Displacement at Joint (3)

Figure (5-39): Comparison Between Linear and Non-Linear Analysis of the Truss Behavior

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

6.1: Conclusions:

Based on the results obtained from the present analysis procedure, the following conclusions can be drawn:

1. The truss structure in its nature is flexibly connected, this leads to a complex vibration system, the local members will vibrate individually in a certain natural frequency, while the whole truss structure vibrates in a different history.
2. The geometrical and material non-linearity vary the natural time period of the truss structure, in the result the natural frequency of the structure will vary.
3. The effect of dynamic non-linear behavior is variable, according to the geometry, characteristics of the subjected dynamic load and material properties, so the non-linear analysis must be taken into account.
4. The direction of the subjected dynamic load has a great influence on the dynamic behavior of one-dimensional members and the truss structures, when non-linear analysis is considered, where it may vary the amplitude and the natural frequency of vibration.
5. In linear analysis, the Dynamic Intensification Factor (DIF) for one-dimensional member subjected to transient loading, is equal to (2.0), while in non-linear analysis, (DIF) is variable, depending on the: characteristics of the transient subjected load which includes: (duration, magnitude, direction and type) and the material properties.
6. The method can efficiently predict forces, velocities and displacements throughout a structure under consideration at any instant.

7. The method of characteristics can be used to simulate the transient response of truss structures with prismatic straight members, subjected to any type of loading.

6.2: Recommendations:

The following topics are suggested for future studies:

1. Applying The Method of Characteristics to simulate the dynamic behavior of (3–D) Trusses.
2. Development of The Method of Characteristics to deal with the wave propagation in composite material, such as reinforced concrete.
3. Development of The Method of Characteristics to simulate the driven of piles in soil.
4. Development of The Method of Characteristics to deal with the wave propagation in structural members with thermal gradient.

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