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New Techniques to Find the Feasible and Optimal Solution for the Transportation Problems

A Dissertation

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

﴿وَعَلَّمَكَ مَا لَمْ تَكُن تَعْلَمُ وَكَانَ فَضْلُ اللَّهِ عَلَيْكَ عَظِيمًا﴾

صَدَقَ اللهُ الْعَلِيُّ الْعَظِيْم

من سورة النساء– الآية ﴿١١٣﴾

الإهداء

الى أمل الإنسانية ومنقذ البشرية ونور الله الذي لا يطفى الأمام الحجة بن الحسن (عجل الله فرجة)

الى كل من ساعدني في انجاز اطروحتي . . .

الى أساتذتي . . .

الى عائلتي . . .

الى زملائي . . .

اهدي جهدي المتواضع هذا ...

يعقوب

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Table 1: The Mathematical Symbols

Symbol	Description
a_i	Available quantities of each m capacity
b_j	Available quantities for each n requirements
C_{ij}	The cost of transport one quantity of goods from origin i to destination j at every path in TP.
i	The index for origins (factory); i= 1,2,...,m
j	The index for destinations (warehouse); j= 1,2,...,n
m	The number of resources (the number of rows)
n	The number of destinations (the number of columns)
x_{ij}	The number of quantities shipped in every path from origin i to destination j in TP.
A^T	Transpose of the matrix A
$\blacksquare$	End the proof.

Table 2: The abbreviations

TP	Transportation Problem.
UBTP	Unbalanced Transportation Problem.
FS	Feasible Solution.
BFS	Basic Feasible Solution.
IBFS	Initial Basic Feasible Solution.
OS	Optimal Solution
NWCM	North-West Corner Method.
LCM	Least Cost Method.
VAM	Vogel's Approximation Method.
MODI	Modified Distribution Method.
SS	Stepping Stone Method.
MST	Minimum Spanning Tree.
LDM	Largest Deference Method.
YM	Yaqoob - Mushtak Method.
DAM	Deference Absolute Method.

Abstract

In this work, three new methods are proposed to find and test the optimal solution (OS) to transportation problems (matrix method based on matrix multiplication with its transpose, YM method and absolute difference method based on graph theory). The distinguishing feature of these techniques is that the calculations are easy and more logical so that the person can understand and benefit from them, especially for those who make a decision. This work also presented a new method for finding the basic solution, the method of the largest difference, which can be developed into a direct method for finding the optimal solution, representing a new approach in solving transportation problems. We also proved that there are special cases that do not need methods to solve them, but the solution is always optimal in any way, even if the filling is random, and we explained it with theories and examples that show that special case.

Chapter 1

General Introduction

1.1 Introduction

The transportation problem (TP) is a type of linear programming problem that involves transporting a single homogeneous product from several sources (such as manufacturing or supply hubs) to multiple sinks (destinations or warehouses). When dealing with TP, the practitioner generally has a certain amount of capacity at each supply point and a certain amount of demand at each demand point [1]. Transportation problems are of great importance in public life and in particular for decision makers and logistics management to achieve appropriate and required solutions. It requires institutions and factories, whether public or private, to satisfy customers as much as possible in terms of product quality and the lowest cost of the product. Transportation plays a very important role in reducing the cost to customers until institutions and factories achieve the revenues they seek. Transportation is an essential necessity for assignment, which allows the production and consumption of products in different locations. Whereas, according to TP model, many of the problems that face us in the process of transporting goods can be solved in terms of increasing revenue and profits, reducing the total cost of transportation, or reducing the total transportation time and so on. The model of TP has a great importance not only in the problem of transportation, but also is important in several areas, including the assignment of a number of operators to a number of jobs, the appropriate distribution of teachers to courses according to specialization, in the assignment of clerks for different counters, and so on. The importance of the issue of TP, as some of them were mentioned in this study, made researchers focus of their attention and conducting extensive studies on this topic.

1.2 Historical Introduction

In (1941), Hitchcock devised a transportation mathematical model problem[2]. In (1955), Charnes and Cooper created the stepping stone approach[3]. In (1957), Munkres, introduced algorithms to solve the public assignment problem (AP) and TP[4]. In (1963), Dantzig employed the simplex approach to solve transportation difficulties [5]. In(1967), Klein devised a primitive method for calculating minimum cash flows with applicability to problems of assignment and transportation [6]. In (1972), Hadley also mentioned the issue of transportation [7]. Linear Programming is the title of his book. Goal programming was researched by Lee in (1972) [8] and Ignizio in (1976) to solve TP [9]. In (1979), Kwak used a goal programming paradigm to the topic of improved transportation [10]. In (1986), Ahuja introduced a minimax algorithm [11]. Also, in (1986), Currin investigated TP with unacceptable pathways [12]. In (1988), Sultan and Goyal investigated the initial basic feasible solution(IBFS) and degeneracy resolution in TP [13]. In (1989), Arsham and Kahn, proposed a new algorithm to solve the TP. The proposed method used only one process, the Gauss Jordan axis, which is used in the Simplex method. The final table can be used for post-optimality analysis of a TP. This algorithm looks faster than simplex, more general than the stepping stone, and simpler than both to solve the public TP[14]. In (1990), Kirca and Statir are a couple offered a method for finding a first solution to TP [15]. In (1995), Krzysztof and Goczyla presented a public transportation network [16]. In (1999), Adlakha and Kowalski proposed a different approach based on the notion of absolute points, a solution method for a specific TP has been developed [17]. In(2000), Sharma proposed the heuristic 2 for tackling the problem of uncapacitated transportation [18]. In(2002), Sun exclusionary side restrictions and the branch and bound technique were used to solve the transportation problem [19]. In (2002), Schrijver studied transportation history and maximum flows [20]. In (2004), Okunbor worked on the project and the management decision making for TP through goal programmin [21]. In(2008), Żółkiewski presented a numerical application for analysis and modelling dynamical flexible systems in transportation [22]. In (2010), Klibi et al. discussed the stochastic multi period location transportation problem [23]. In (2011), Korukogluand and Balli proposed an improved Vogel's approximation method (VAM) for TP [24]. Also, in (2011), Sharma et al. solved the transportation problem using the phase-II approach of the simplex [25]. In (2012), Sharma et al. used the integer programming problem to aid them [26]. Also, in (2012), Sudhakar et al. made a suggestion a novel method for determining the optimal solution to a transportation problem [27]. In (2013), Rekha and Joshi explored the topic of optimization [28]. In (2014), Rekha et al. recommended a maximum minimum value [29]. Also in (2014), Utpal Kanti Das, et al. identified in their study an arithmetic error in VAM. The basic concept of VAM is to select the penalty cost for rows and columns that get from finding the difference between the two smallest costs in each row and each column and then allocating the maximum in the least expensive cell to that row or column that has the most penalty. Difficulty arises when the two smallest costs are equal. In this case, they put a logical concept to solve this problem and developed a new algorithm “Advanced Vogel’s Approximation Method (AVAM)” to find an initial solution to the transportation problems [30]. In (2015), Sarbjit and Singh, offered a degeneracy resolution in the transportation problem [31]. In (2017), Ibrahim and Abaas, applied study to find the best solution to the problem of transportation in midland refineries by using the zero-poin [32]. In (2020), Hussein and Shiker, introduced new modification to VAM to solve TP[33]. In (2020), Karagul and Sahin, proposed a new method to find an initial basic feasible solution(IBFS) to TP called the Karagul Sahin approximation method (KSAM). KSAM is a 5-step iterative method [34].

1.3 Mathematical Formulation of TP

TP can be viewed in a special table using the mathematical model for linear programming. TP model can be represented in a tabular format with all relevant parameters. In the transportation schedule, the availability of supply (a_i) is displayed in each source in the rightmost column and the destination requirement (b_j) appears in the under most row. Every cell appears one path. The oneness transfer cost (C_{ij}) is displayed in the top- right angle of the cell, and the quantity of shipped matter (x_{ij}) appears in the center of the cell. If we have m origins and n destinations, the plants P_i ($i = 1, 2, \ldots, m$) transport the products a_i to the storages W_j ($j = 1, 2, \ldots, n$) which requires b_j units. Let C_{ij} be the transporting cost of one unit of product from i^{th} origin to j^{th} destination and let x_{ij} be the amount transported from i^{th} origin to j^{th} destination. The following equation (1) represents the transportation cost which its objective is to determine the number of units to be transported from i^{th} to j^{th}:

$$Minimize \;\; Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

Such that

$$\sum_{j=1}^{n} x_{ij} = a_i \quad , \quad i = 1, 2, \ldots m \text{ (supply constraints)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad , \quad j = 1, 2, \ldots n \text{ (demand constraints)}$$

where $x_{ij} \geq 0 \;\; \forall \;\; i$ and j [14]. The following table represents the mathematical formulation to TP.

Table 1.1: The Mathematical Formulation to TP

Origins (i)	**Destinations (j)**							**Supply (a_i)**
	D_1		D_2		$\cdots$	D_n		
S_1	x_{11}	c_{11}	x_{12}	c_{12}	$\cdots$	x_{1n}	c_{1n}	a_1
S_2	x_{21}	c_{21}	x_{22}	c_{22}	$\cdots$	x_{2n}	c_{2n}	a_2
$\vdots$	$\vdots$		$\vdots$		x_{ij} c_{ij}	$\vdots$		$\vdots$
S_m	x_{m1}	c_{m1}	x_{m2}	c_{m2}	$\cdots$	x_{mn}	c_{mn}	a_m
Demand(b_j)	b_1		b_2		$\cdots$	b_n		$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

1.4 Network diagram to TP

TP can be represented by the network diagram as in figure (1.4). The goal of the network diagram and formulation table is to determine the value of variable x_{ij} that will minimize the total cost of TP with the fulfilment of the supply and demand conditions. The arrows that link the origin and the destination are the path through which the goods are transported [30].

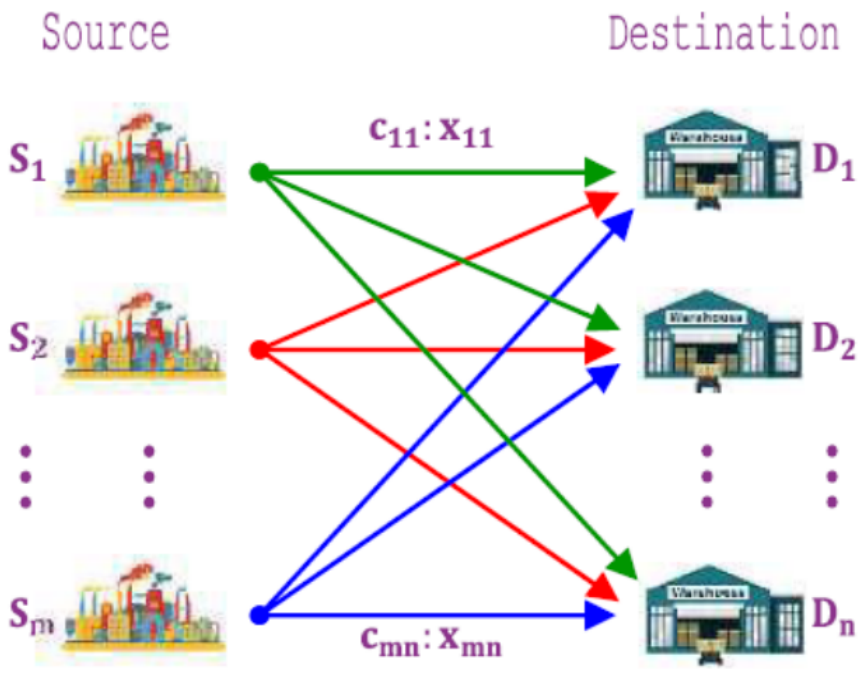


Figure 1.1: The Digram Representation of TP

1.5 Matrix Terminology

The matrix used in transportation models is made up of squares called (cells) that are stacked vertically from (columns) and horizontally from (rows). The row and column headings identify the cell at the intersection of two rows and columns. As a result, cell $(A, 4)$ is placed at the intersection of row A and column 4. Each cell contains the unit transportation expensev. Each source's availability is listed in the last column, while the criteria are listed in the last row [1].

1.6 Structure of the Study

This dissertation includes seven chapters; chapter one contains introduction and review, where the reset of the dissertation as follows:

- Chapter two introduces basic concept, important definitions and presents the basics to TP and explains them in detail in regard to the three classic methods of finding the initial solution to this problem and the two known classic methods to find OS to TP. Also introduces basic concepts to graph theory.

- Chapter three introduces a new technique named largest deference method (LDM) to find IBFS to TP.

- Chapter four introduces some important theorems about a special case of TP.

- Chapter five introduces a new two methods (matrix method and YM method) to test and find OS.

- Chapter six introduces a novel technique based on graph theory, it is called the difference absolute method (DAM) to testing and finding OS to TP.

- Chapter seven contains conclusions and future works, summarizes the conclusion that were obtained from this research study and also mentions the suggested future works.

Chapter 2

Important Definitions and Basic Concepts

2.1 Introduction

In this chapter, some basic concepts and important definitions, are introduced, we'll used them in the next chapters.

2.2 Some Important Definitions of TP

In this section we introduced basic concept and important definitions of TP [1, 11, 17].

Definition 2.2.1 (Origin (Source)): It is a location from which shipment are dispatched.

Definition 2.2.2 (Destinations (Warehouses)): It is the location to which shipment are transported.

Definition 2.2.3 (Capacities (Supplies)): The product available in an origin to satisfy the requirement of the request center's is said to be the offer limit of that origin.

Definition 2.2.4 (Demands (Requirements)): The quantity required for a product to meet the demand is called the demand requirement.

Definition 2.2.5 (Unit Transportation Cost): It is the cost of transporting one unit of the consignment from a source to a destination.

Definition 2.2.6 (Feasible Solution (FS)): A feasible solution to transportation problem is a collection of non - negative assigned quantities $x_{ij} \geq 0$ that meet the constraints (supply constraints and demand constraints in TP).

Definition 2.2.7 (Basic Feasible Solution(BFS)): We say that a feasible solution for TP is a basic feasible solution (BFS) when no more than $(m+n-1)$ includes the positive assigned quantities, such that m represents the number of rows while n represents the number of columns in TP matrix.

Definition 2.2.8 (Optimal Solution(OS)): A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

Definition 2.2.9 (Non-Degenerate Basic Feasible Solution): A basic feasible solution to a $(m \times n)$ TP that contains $(m+n-1)$ allocations in independent position

Definition 2.2.10 (Degenerate basic feasible solution) : A basic feasible solution that contains less than $(m+n-1)$ non-negative allocations.

Definition 2.2.11 (Algorithm): A step-by-step problem-solving procedure, especially an established, recursive computational course of action for solving a problem in a finite number of steps is called an Algorithm.

Definition 2.2.12 (Balanced (TP)): The (TP) is said to be balanced transportation problem if total number of supply is same as total number of demand. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

Definition 2.2.13 (Unbalanced (TP)): The (TP) is said to be unbalanced transportation problem if total number of supply is same as total number of demand. $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$,

Definition 2.2.14 (Loop or Closed Path): An ordered set of at least four cells in transportation table is said to be a loop or closed path provided, it includes two cases:

(a) Any two adjacent cells of the ordered set lie either in the same row or in the same column, see figure(2.1).

(b) No three or more adjacent cells in the ordered set lie in the same row or column, see figure(2.2).

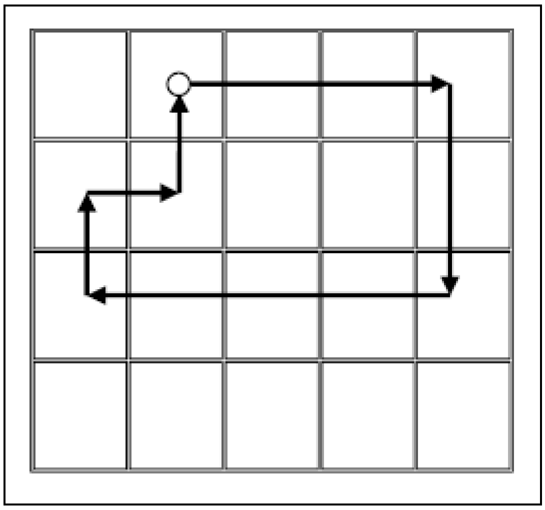


Figure 2.1: A loop is Formed

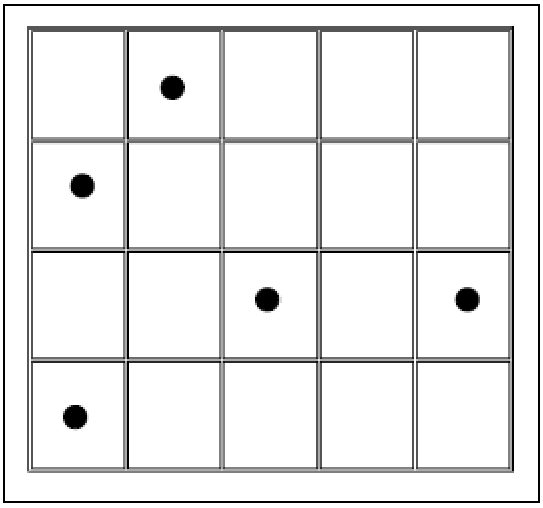


Figure 2.2: Do not Form A loop

2.3 The Classical Approach to Solve TP

The classic approach includes several steps, the following are the basic operations to find the optimal solution to the TP:

- **Step 1**: Formalize the TP mathematically(matrix of TP).

- **Step 2**: Find initial basic feasible solution (IBFS) by the classical methods.
- **Step 3**: Determine and test if the solution is optimal or not by MODI or SS method. If not, it should be improved. Until you reach the goal(OS).

2.4 The Classical Methods to Find IBFS

This section introduces the algorithms to NWCM, LCM and VAM [24].

2.4.1 North West Corner Method

Algorithm 1 North West Corner Method Algorithm (NWCM)

Step 1 : Select the north west (upper left hand) corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand,

Step 2 : Adjust the supply and demand numbers in the respective rows and columns,

Step 3 : If the demand for the first cell is satisfied, then move horizontally to the next cell in the second row,

Step 4 : If the supply for the first row is exhausted, then move down to the first cell in the second row,

Step 5 : If for any cell, supply equals demand, then the next allocation can be made in either in the next row or column,

Step 6 : Continue the process until all supply and demand values are exhausted.

Although NWCM gives a speedy answer due to the low computing time, it produces a poor result since it is far from optimal.

Example 2.4.1 Take the following TP matrix

	1	2	3	4	5	Supply
1	9	3	6	4	1	50
2	6	8	14	10	5	30
3	2	17	13	12	7	40
Demand	27	25	35	18	15	

The rim values for 1=50 and 1=27 are compared. The smaller of the two that is, min(50,27) = **27** is assigned to(1,1). This meets the complete demand of 1 and leaves 50 - 27=23 units with 1

	1	2	3	4	5	Supply
1	9**(27)**	3	6	4	1	23
2	6	8	14	10	5	30
3	2	17	13	12	7	40
Demand	0	25	35	18	15	

The rim values for 1=23 and 2=25 are compared. The smaller of the two that is, min(23,25) = **23** is assigned to(1,2). This exhausts the capacity of 1 and leaves 25 - 23=2 units with 2

	1	2	3	4	5	Supply
1	9**(27)**	3**(23)**	6	4	1	0
2	6	8	14	10	5	30
3	2	17	13	12	7	40
Demand	0	2	35	18	15	

So

	1	2	3	4	5	Supply
1	9 **(27)**	3 **(23)**	6	4	1	50
2	6	8 **(2)**	14 **(28)**	10	5	30
3	2	17	13 **(7)**	12 **(18)**	7 **(15)**	40
Demand	27	25	35	18	15	

So IBFS=$9\times27+3\times23+8\times2+14\times28+13\times7+12\times18+7\times15 = 1132$

2.4.2 Least Cost Method

Algorithm 2 Least Cost Method Algorithm (LCM)

Step 1 : Identify the box having minimum unit transportation cost in the cost matrix of the transportation table. If a tie occurs, choose any one of them randomly,

Step 2 : Allocate as many units as possible equal to the minimum between available supply and demand,

Step 3 : Adjust the supply and demand numbers in the respective rows and columns,

Step 4 : No further consideration is required for the row or column which is satisfied. If both the row and column are satisfied at a time, delete only one of the two, and the remaining row or column is assigned zero supply (or demand),

Step 5 : Continue the process for the resulting reduced transportation table until all the rim conditions are satisfied.

Example 2.4.2 Take the following TP matrix

	1	2	3	4	5	Supply
1	8	3	8	4	1	150
2	6	1	14	1	3	30
3	22	7	13	11	7	240
Demand	77	125	85	68	65	

The smallest transportation cost is 1 in cell(1,5).

The allocation to this cell is min(150,65) = **65**.

This satisfies the entire demand of 5 and leaves 150 - 65=85 units with 1

	1	2	3	4	5	Supply
1	8	3	8	4	1(**65**)	85
2	6	1	14	1	3	30
3	22	7	13	11	7	240
Demand	77	125	85	68	0	

So IBFS is

	1	2	3	4	5	Supply
1	8	3(**85**)	8	4	1(**65**)	150
2	6	1(**30**)	14	1	3	30
3	22(**77**)	7(**10**)	13(**85**)	11(**68**)	7	240
Demand	77	125	85	68	65	

Then, IBFS$= 3 \times 85 + 1 \times 65 + 1 \times 30 + 22 \times 77 + 7 \times 10 + 13 \times 85 + 11 \times 68 = 3967$

2.4.3 Vogel's Approximation Method

Algorithm 3 Vogel's Method Algorithm (VAM)

Step 1 : If either (total supply $>$ total demand) or (total supply $<$ demand), balance the transportation problem,

Step 2 : Determine the penalty cost for each row by taking difference of lowest cell cost in the row and next to lowest cell cost in the same row and put in front of the row on the right. In a similar fashion, calculate the penalty cost for each column and write them in the bottom of the cost matrix below corresponding columns,

Step 3 : Choose the highest penalty costs and observe the row or column to which this corresponds. If a tie occurs, choose any one of them randomly,

Step 4 : Make allocation min (Si,dj) to the cell having lowest unit transportation cost in the selected row or column,

Step 5 : No further consideration is required for the row or column which is satisfied. If both the row and column are satisfied at a time, delete only one of the two, and the remaining row or column is assigned zero supply (or demand),

Step 6 : Calculate fresh penalty costs for the remaining sub-matrix as in Step 2 and allocate following the procedure of Steps 3, 4 and 5. Continue the process until all rows and columns are satisfied,

Step 7 : Compute total transportation cost for the feasible allocations using the original balanced transportation cost matrix.

Example 2.4.3 Take the following TP matrix

	1	2	3	4	Supply
1	3	22	7	14	60
2	6	9	3	8	27
3	2	4	13	1	40
4	20	1	6	15	13
Demand	29	25	75	11	

	1	2	3	4	Supply	Row Penalty
1	3	22	7	14	60	7-3=4
2	6	9	3	8	27	6-3=3
3	2	4	13	1	40	2-1=1
4	20	1	6	15	13	6-1=5
Demand	29	25	75	11		
Column Penalty	3-2=1	4-1=3	6-3=3	8-1=7		

The maximum penalty,7, occurs in column 4. The minimum C_{ij} in this column is $C_{34} = 1$. The maximum allocation in this cell is min(40,11) = **11**. It satisfy demand of 4 and adjust the supply of 3 from 40 to 29 (40-11=29).

	1	2	3	4	Supply	Row Penalty
1	3	22	7	14	60	7-3=4
2	6	9	3	8	27	6-3=3
3	2	4	13	1(**11**)	29	4-2=2
4	20	1	6	15	13	6-1=5
Demand	29	25	75	0		
Column Penalty	3-2=1	4-1=3	6-3=3	--		

Then IBFS is

	1	2	3	4	Supply	Row Penalty
1	3(**12**)	22	7(**48**)	14	60	4 | 4| 4 | 4 | 4 | 4 | 3 |
2	6	9	3(**27**)	8	27	3 | 3 | 3 | 3 | 3 | -- | --|
3	2(**17**)	4(**12**)	13	1(**11**)	40	1| 2 | 2 | 11 | -- | -- | -- |
4	20	1(**13**)	6	15	13	5 | 5 |-- | -- | -- | -- | --|
Demand	29	25	75	11		
Column Penalty	1 1 1 1 3 3 3	3 3 5 -- -- -- --	3 3 4 4 4 7 --	7 -- -- -- -- -- --		

IBFS$=3\times12+7\times48+3\times27+2\times17+4\times12+1\times11+1\times13=559$

2.5 Optimality Test

The optimality test process is examining all unoccupied cell to see if putting an allocation in it decreases the overall transit cost. The stepping-stone method(SS) and the modified distribution method(MODI) are the two methods utilized for this goal [24].

2.5.1 Stepping Stone Method

Algorithm 4 Stepping Stone Method Algorithm

Step 1 : Determine an initial basic feasible solution

Step 2 : Make sure that the number of occupied cells is exactly equal to m+n-1, where m is the number of rows and n is the number of columns.

Step 3 : Select an unoccupied cell. Beginning at this cell, trace a closed path, starting from the selected unoccupied cell until finally returning to that same unoccupied cell,

Step 4 : Assign plus (+) and minus (-) signs alternatively on each corner cell of the closed path just traced, beginning with the plus sign at unoccupied cell to be evaluated,

Step 5 : Add the unit transportation costs associated with each of the cell traced in the closed path. This will give net change in terms of cost,

Step 6 : Repeat steps 3 to 5 until all unoccupied cells are evaluated,

Step 7 : Check the sign of each of the net change in the unit transportation costs. If all the net changes computed are greater than or equal to zero, stop; an optimal solution has been reached. Otherwise, go to Step 8,

Step 8 : Select the unoccupied cell having the most negative net cost change and determine the maximum number of units that can be assigned to this cell. The smallest value with a negative position on the closed path indicates the number of unit that can be shipped to the entering cell. Add this number to the unoccupied cell and to all other cells on the path marked with a plus sign. Subtract this number from cells on the closed path marked with a minus sign,

Step 9 : Go to Step 3.

Example 2.5.1 Take the TP matrix in (Example 2.4.3), so IBFS by VAM,

	1	2	3	4	Supply
1	3 **(12)**	22	7 **(48)**	14	60
2	6	9	3 **(27)**	8	27
3	2 **(17)**	4 **(12)**	13	1 **(11)**	40
4	20	1 **(13)**	6	15	13
Demand	29	25	75	11	

Iteration-1 of optimality test

1. Create closed path for unoccupied cells, get

Unoccupied cell	Closed path	Net cost change
12	12→11→31→32	22 - 3 + 2 - 4=17
14	14→11→31→34	14 - 3 + 2 - 1=12
21	21→23→13→11	6 - 3 + 7 - 3=7
22	22→23→13→11→31→32	9 - 3 + 7 - 3 + 2 - 4=8
24	24→23→13→11→31→34	8 - 3 + 7 - 3 + 2 - 1=10
33	33→31→11→13	13 - 2 + 3 - 7=7
41	41→42→32→31	20 - 1 + 4 - 2=21
43	43→42→32→31→11→13	6 - 1 + 4 - 2 + 3 - 7=3
44	44→42→32→34	15 - 1 + 4 - 1=17

Since all net cost change 0. So final optimal solution is arrived.

	1	2	3	4	Supply
1	3 **(12)**	22	7 **(48)**	14	60
2	6	9	3 **(27)**	8	27
3	2 **(17)**	4 **(12)**	13	1 **(11)**	40
4	20	1 **(13)**	6	15	13
Demand	29	25	75	11	

OS=$3\times12+7\times48+3\times27+2\times17+4\times12+1\times11+1\times13=559$

2.5.2 Modified Distribution Method

Algorithm 5 The Modified Distribution Method Algorithm

Step 1 : Determine an initial basic feasible solution

Step 2 : Make sure that the number of occupied cells is exactly equal to m+n-1, where m is the number of rows and n is the number of columns.

Step 3 : Compute the row indices, u_i and column indices v_j,

Step 4 : Determine the opportunity cost for non basic variable using . $d_{ij} = C_{ij} - u_i - v_j$,

Step 5 : If $d_{ij} \geq 0$ for all unoccupied cells, Stop; the solution is optimal. Otherwise, go to Step 6,

Step 6 : Identify the unoccupied cell with the negative opportunity cost and select it as the incoming cell. If more than one unoccupied cell contains negative opportunity cost, select the cell with most negative opportunity cost,

Step 7 : Draw a closed loop or stepping stone path associated with the incoming cell,

Step 8 : Assign an alternate plus and minus signs on the corner points of the closed path with a plus sign at the cell selected as the incoming cell,

Step 9 : The smallest value among the cell with a minus sign on the closed path indicates the number of units that can be shipped to the incoming cell. Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs, and subtract it from those cells marked with minus signs. If there is a tie, select any one of the tied cell. The tied cells that are not selected will be artificially occupied with a zero allocation,

Step 10 : Go to Step 3.

Example 2.5.2 Take the TP matrix in (Example 2.4.3), so IBFS by VAM method,

	1	2	3	4	Supply
1	3 **(12)**	22	7 **(48)**	14	60
2	6	9	3 **(27)**	8	27
3	2 **(17)**	4 **(12)**	13	1 **(11)**	40
4	20	1 **(13)**	6	15	13
Demand	29	25	75	11	

Iteration-1 of optimality test

1. Find u_i and v_j for all occupied cells(i,j),where $C_{ij} = u_i + v_j$

Substituting,$u_3 = 0$, we get

- $C_{31} = u_3 + v_1 \Rightarrow \text{v}_1 = C_{31} - u_3 \Rightarrow \text{v}_1 = 2 - 0 \Rightarrow \text{v}_1 = 2$
- $C_{11} = u_1 + v_1 \Rightarrow \text{u}_1 = C_{11} - v_1 \Rightarrow \text{u}_1 = 3 - 2 \Rightarrow \text{u}_1 = 1$
- $C_{13} = u_1 + v_3 \Rightarrow \text{v}_3 = C_{13} - u_1 \Rightarrow \text{v}_3 = 7 - 1 \Rightarrow \text{v}_3 = 6$
- $C_{23} = u_2 + v_3 \Rightarrow \text{u}_2 = C_{23} - v_3 \Rightarrow \text{u}_2 = 3 - 6 \Rightarrow \text{u}_2 = -3$
- $C_{32} = u_3 + v_2 \Rightarrow \text{v}_2 = C_{32} - u_3 \Rightarrow \text{v}_2 = 4 - 0 \Rightarrow \text{v}_2 = 4$
- $C_{42} = u_4 + v_2 \Rightarrow \text{u}_4 = C_{42} - v_2 \Rightarrow \text{u}_4 = 1 - 4 \Rightarrow \text{u}_4 = -3$
- $C_{34} = u_3 + v_4 \Rightarrow \text{v}_4 = C_{34} - u_3 \Rightarrow \text{v}_4 = 1 - 0 \Rightarrow \text{v}_4 = 1$

	1	2	3	4	Supply	u_i
1	3 **(12)**	22	7 **(48)**	14	60	$u_1 = 1$
2	6	9	3 **(27)**	8	27	$u_2 = -3$
3	2 **(17)**	4 **(12)**	13	1 **(11)**	40	$u_3 = 0$
4	20	1 **(13)**	6	15	13	$u_4 = -3$
Demand	29	25	75	11		
v_j	$v_1 = 2$	$v_2 = 4$	$v_3 = 6$	$v_4 = 1$		

2. Find d_{ij} for all unoccupied cells(i,j), where $d_{ij} = C_{ij} - (u_i + v_j)$

- $d_{12} = C_{12} - (u_1 + v_2) = 22 - (1 + 4) = 17$
- $d_{14} = C_{14} - (u_1 + v_4) = 14 - (1 + 1) = 12$
- $d_{21} = C_{21} - (u_2 + v_1) = 6 - (-3 + 2) = 7$

- $d_{22} = C_{22} - (u_2 + v_2) = 9 - (-3 + 4) = 8$
- $d_{24} = C_{24} - (u_2 + v_4) = 8 - (-3 + 1) = 10$
- $d_{33} = C_{33} - (u_3 + v_3) = 13 - (0 + 6) = 7$
- $d_{41} = C_{41} - (u_4 + v_1) = 20 - (-3 + 2) = 21$
- $d_{43} = C_{43} - (u_4 + v_3) = 6 - (-3 + 6) = 3$
- $d_{44} = C_{44} - (u_4 + v_4) = 15 - (-3 + 1) = 17$

	1	2	3	4	Supply	u_i
1	3 **(12)**	22[17]	7**(48)**	14[12]	60	$u_1 = 1$
2	6[7]	9[8]	3**(27)**	8[10]	27	$u_2 = -3$
3	2**(17)**	4**(12)**	13[7]	1**(11)**	40	$u_3 = 0$
4	20[21]	1**(13)**	6[3]	15[17]	13	$u_4 = -3$
Demand	29	25	75	11		
v_j	$v_1 = 2$	$v_2 = 4$	$v_3 = 6$	$v_4 = 1$		

Since all $d_{ij} \geq 0$. So final optimal solution is arrived.

	1	2	3	4	Supply
1	3 **(12)**	22	7 **(48)**	14	60
2	6	9	3 **(27)**	8	27
3	2 **(17)**	4 **(12)**	13	1 **(11)**	40
4	20	1 **(13)**	6	15	13
Demand	29	25	75	11	

OS=$3 \times 12 + 7 \times 48 + 3 \times 27 + 2 \times 17 + 4 \times 12 + 1 \times 11 + 1 \times 13 = 559$

2.6 Some Basic Concepts of Graph Theory

In this section, some basic concepts of graph theory that have been used in this work are discussed as follows.

Definition 2.6.1 (Graph) [35]: A graph $G = (V, E)$ consists of two finite sets. The vertex set V of the graph, which is a non-empty set of elements that are called vertices, and the edge set E of the graph, which is a possibly empty set of elements that are called edges, such that each edge e in E is assigned as an unordered pair of vertices (u, v), called the end vertices of e.

Example 2.6.1 : Let $V = \{v_1, v_2, v_3, v_4\}$ be vertex set and the edge set is $E = \{e_1, e_2, e_3, e_4, e_5\}$, where $e_1 = v_1v_2$, $e_2 = v_2v_3$, $e_3 = v_3v_4$, $e_4 = v_1v_4$ and $e_5 = v_1v_3$ are formed the graph G. The graph $G(V, E)$ is shown in the following figure.

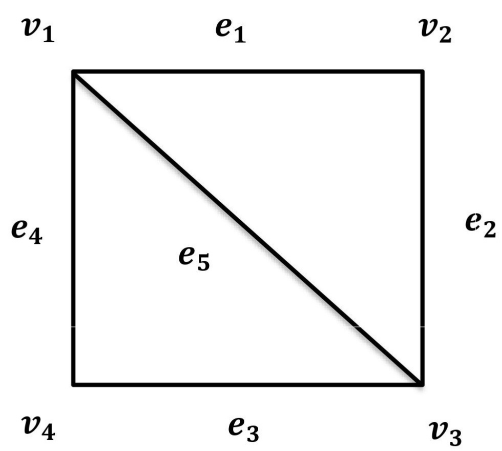


Figure 2.3: Example of Graph.

Definition 2.6.2 (Subgraph) [36]: Let H be a graph with vertex set $V(H)$ and edge set $E(H)$, and similarly let G be a graph with vertex set $V(G)$ and edge set $E(G)$. Then, we say that H is a subgraph of G

if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

Definition 2.6.3 (Spanning Subgraph) [36]: A spanning subgraph of G is a subgraph H with $V(H) = V(G)$,that is H and G have exactly the same vertex set.

Definition 2.6.4 (Order and Size of a Graph) [36]: Let $G = (V, E)$ be a graph. The order of G is defined by $\mid V \mid= n$ and $\mid E \mid= m$ is defined to be the size of G.
In figure 2.3, $\mid V \mid= 4$ and $\mid E \mid= 5$.

Definition 2.6.5 (Weighted Graph) [35]: A weighted graph is a graph in which each edge has a numerical weight. So, a weighted graph consider as a special type of a labeled graph in which the labels are numbers.

Example 2.6.2 The weighted graph is given in Figure 2.4.

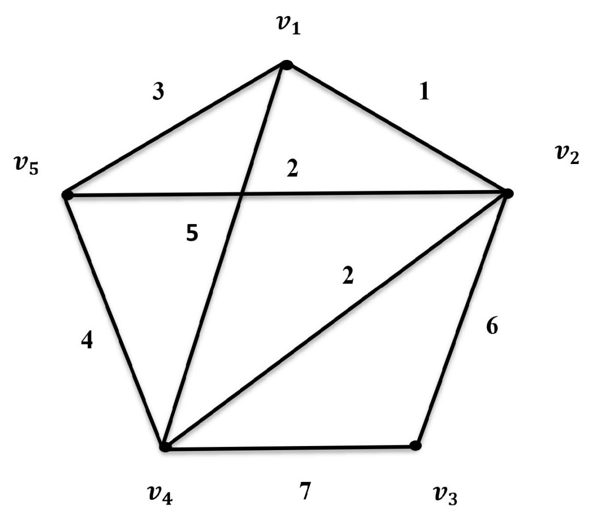


Figure 2.4: A weighted graph G.

Definition 2.6.6 (Tree) [36]: A connected graph with no cycle is called a tree.

Definition 2.6.7 (Spanning Tree) [36]: A tree T is called a spanning tree of a connected graph G if T is a subgraph of G and if T contains all the vertices of G. In other words, a spanning tree of a graph G is a spanning subgraph of G that is a tree.

Definition 2.6.8 (Minimum Spanning Tree)[36]: Let G be a weighted graph in which each edge e has been assigned a real number $w(e)$, called the weight of the edge e. If H be a subgraph of a weighted graph, the weight $w(H)$ of H, is the sum of the weights $w(e_1) + w(e_2) + ... + w(e_k)$, where $e_1, e_2, ..., e_k$ is the set of edges of H. A spanning tree T of a weighted graph G is called a minimal spanning tree if its weight is minimum. In other words, $w(T)$ is minimum, where $w(T) = w(e_1) + w(e_2) + ... + w(e_k)$ and $e_1, e_2, ..., e_k$ is the set of edges of T. Algorithm 6 is used for finding the minimum spanning tree.

Algorithm 6 Kruskal's Algorithm

Let $G = (V, E)$ be a weighted connected graph.

- **Step 1**: Select one edge e_i of G such that its weight $w(e_i)$ is minimum.

- **Step 2**:

  1: If edges $e_1, e_2, ..., e_k$ have been chosen, then select an edge e_{k+1} such that $e_{k+1} \neq e_i$ for $i = 1, 2, ..., k$.

2: The edges $e_1, e_2, ..., e_k, e_{k+1}$ does not form a circuit.

3: The weight of $w(e_{k+1})$ is as small as possible subject to the condition number 2 of step-2 above.

- **Step 3**:Stop, when all the vertices of G are in T which is the required spanning tree of G with $n-1$ edges.

Example 2.6.3 Figure 2.5 shows how determining the minimum spanning tree of the weighted graph in Figure 2.4 by using Kruskal's algorithm.

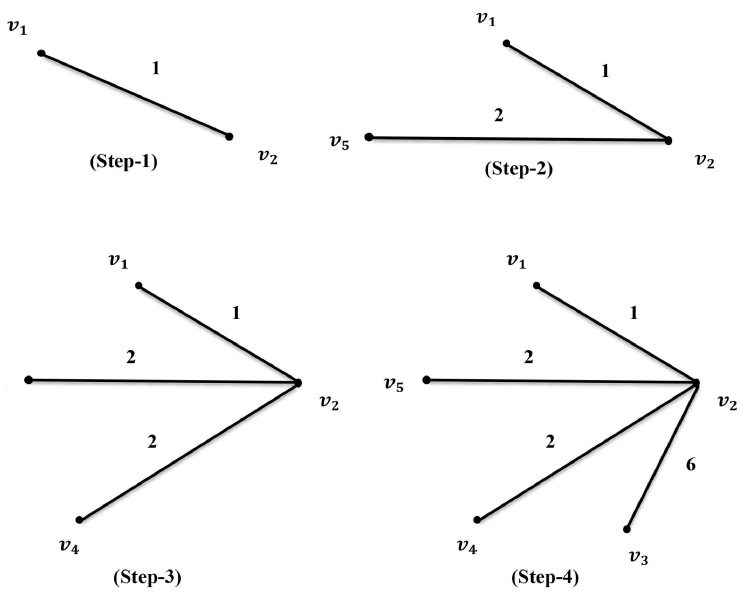


Figure 2.5: A step by step to find a minimum spanning tree

Definition 2.6.9 (Directed Graph) [35]

A directed graph (digraph), $D = (V, Ar)$ consists of a non empty finite set $V(D)$ of elements called vertices, and a finite family $Ar(D)$ of ordered pairs of elements of $V(D)$ called arcs. The digraph is given in Figure 2.6.

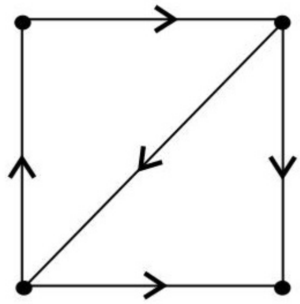


Figure 2.6: The digraph

Chapter 3

New Method to Find IBFS

3.1 Introduction

In this chapter, introduced new technique named largest deference method (LDM) to find IBFS . The largest difference method is a new method to find IBFS to TP, this new method is presented in this study. For obtaining a reasonable primary answer to a wide range of TP. Analytical examples and optimality testing of the conclusions justify these methodologies. That distinguishes these methods is that they just necessitate a few simple calculations. Also introduced some new theorems of special case TP.

3.2 Largest Deference Method Algorithm

After writing the matrix of transportation problems and making it balanced by the previously known methods, we start with the following steps one after the other.

Algorithm 7 Largest Deference Method Algorithm

Step 1 : Calculate $I_{ij} = \alpha_{ij} - C_{ij}$ where α_{ij} is largest cell's cost in row i and column j for all i, j in matrix TP except for the row or column that is added in case of unbalanced TP,

Step 2 : Designate the cell with the greatest difference than the cell that corresponds to $max_{ij}(I_{ij})$,

Step 3 : Dictate the cell with the lowest cost in the row and column of the cell corresponding to $max_{ij}(I_{ij})$

Step 4 : Recalculate I_{ij} to remaining cells and repeat the previous three steps.

3.3 Analytical Examples of Largest Deference Method

Without a new condition or constraint, this strategy has been used to many examples.

Example 3.3.1 Let's take the next transportation problem with four rows and six columns.

	1	2	3	4	5	6	Supply
1	11	13	17	14	9	4	250
2	16	18	14	10	1	22	300
3	21	24	13	10	33	12	400
4	8	6	12	8	2	11	250
Demand	200	225	275	250	100	150	

Step 1: Calculate $I_{ij} = \alpha_{ij} - C_{ij}$

Table 3.1: Calculate $I_{ij} = \alpha_{ij} - C_{ij}$

I_{ij}	The value	I_{ij}	The value	I_{ij}	The value	I_{ij}	The value
I_{11}	10	I_{21}	6	I_{31}	12	I_{41}	13
I_{12}	11	I_{22}	6	I_{32}	9	I_{42}	18
I_{13}	0	I_{23}	8	I_{33}	20	I_{43}	5
I_{14}	3	I_{24}	12	I_{34}	23	I_{44}	6
I_{15}	24	I_{25}	32	I_{35}	0	I_{45}	31
I_{16}	18	I_{26}	0	I_{36}	21	I_{46}	11

Iteration 1:$max_{ij}(I_{ij}) = I_{25} = 32$, so

	1	2	3	4	5	6	Supply
1	11	13	17	14	9	4	250
2	16	18	14	10	1(100)	22	200
3	21	24	13	10	33	12	400
4	8	6	12	8	2	11	250
Demand	200	225	275	250	0	150	

Table 3.2: Recalculate I_{ij} to Iteration 2

I_{ij}	The value	I_{ij}	The value	I_{ij}	The value	I_{ij}	The value
I_{11}	10	I_{21}	6	I_{31}	3	I_{41}	13
I_{12}	11	I_{22}	6	I_{32}	0	I_{42}	18
I_{13}	0	I_{23}	8	I_{33}	11	I_{43}	5
I_{14}	3	I_{24}	12	I_{34}	14	I_{44}	6
I_{16}	18	I_{26}	0	I_{36}	12	I_{46}	11

Iteration 2:$max_{ij}(I_{ij}) = I_{16} = 18$, so

	1	2	3	4	5	6	Supply
1	11	13	17	14	9	4(150)	100
2	16	18	14	10	1(100)	22	200
3	21	24	13	10	33	12	400
4	8	6	12	8	2	11	250
Demand	200	225	275	250	0	0	

Table 3.3: Recalculate I_{ij} to Iteration 3

I_{ij}	The value	I_{ij}	The value	I_{ij}	The value	I_{ij}	The value
I_{11}	10	I_{21}	5	I_{31}	3	I_{41}	13
I_{12}	11	I_{22}	6	I_{32}	0	I_{42}	18
I_{13}	0	I_{23}	4	I_{33}	11	I_{43}	5
I_{14}	3	I_{24}	8	I_{34}	14	I_{44}	6

Iteration 3: $max_{ij}(I_{ij}) = I_{42} = 18$, so

	1	2	3	4	5	6	Supply
1	11	13	17	14	9	4(150)	100
2	16	18	14	10	1(100)	22	200
3	21	24	13	10	33	12	400
4	8	6(225)	12	8	2	11	25
Demand	200	0	275	250	0	0	

Table 3.4: Recalculate I_{ij} to Iteration 4

I_{ij}	The value	I_{ij}	The value	I_{ij}	The value	I_{ij}	The value
I_{11}	10	I_{21}	5	I_{31}	0	I_{41}	13
I_{13}	0	I_{23}	3	I_{33}	8	I_{43}	5
I_{14}	3	I_{24}	6	I_{34}	11	I_{44}	6

Iteration 4: $max_{ij}(I_{ij}) = I_{41} = 13$, so

	1	2	3	4	5	6	Supply
1	11	13	17	14	9	4(150)	100
2	16	18	14	10	1(100)	22	200
3	21	24	13	10	33	12	400
4	8(25)	6(225)	12	8	2	11	0
Demand	175	0	275	250	0	0	

Table 3.5: Recalculate I_{ij} to Iteration 5

I_{ij}	The value	I_{ij}	The value	I_{ij}	The value
I_{11}	10	I_{21}	5	I_{31}	0
I_{13}	0	I_{23}	3	I_{33}	8
I_{14}	3	I_{24}	6	I_{34}	11

Iteration 5: $max_{ij}(I_{ij}) = I_{34} = 11$, so

	1	2	3	4	5	6	Supply
1	11	13	17	14	9	4(150)	100
2	16	18	14	10	1(100)	22	200
3	21	24	13	10(250)	33	12	150
4	8(25)	6(225)	12	8	2	11	0
Demand	175	0	275	0	0	0	

Table 3.6: Recalculate I_{ij} to Iteration 6

I_{ij}	The value	I_{ij}	The value	I_{ij}	The value
I_{11}	10	I_{21}	5	I_{31}	0
I_{13}	0	I_{23}	3	I_{33}	8

Iteration 6: $max_{ij}(I_{ij}) = I_{11} = 10$, so

	1	2	3	4	5	6	Supply
1	11(100)	13	17	14	9	4(150)	0
2	16	18	14	10	1(100)	22	200
3	21	24	13	10(250)	33	12	150
4	8(25)	6(225)	12	8	2	11	0
Demand	75	0	275	0	0	0	

Table 3.7: Recalculate I_{ij} to Iteration 7

I_{ij}	The value	I_{ij}	The value
I_{21}	5	I_{31}	0
I_{23}	2	I_{33}	8

Iteration 7: $max_{ij}(I_{ij}) = I_{33} = 8$, so

	1	2	3	4	5	6	Supply
1	11(100)	13	17	14	9	4(150)	0
2	16	18	14	10	1(100)	22	200
3	21	24	13(150)	10(250)	33	12	0
4	8(25)	6(225)	12	8	2	11	0
Demand	75	0	125	0	0	0	

Table 3.8: Recalculate I_{ij} to Iteration 8

I_{ij}	The value
I_{21}	0
I_{23}	2

Iteration 8:$max_{ij}(I_{ij}) = I_{23} = 2$, so

	1	2	3	4	5	6	Supply
1	11(100)	13	17	14	9	4(150)	0
2	16	18	14(125)	10	1(100)	22	75
3	21	24	13(150)	10(250)	33	12	0
4	8(25)	6(225)	12	8	2	11	0
Demand	75	0	0	0	0	0	

Iteration 9,

	1	2	3	4	5	6	Supply
1	11(100)	13	17	14	9	4(150)	0
2	16(75)	18	14(125)	10	1(100)	22	0
3	21	24	13(150)	10(250)	33	12	0
4	8(25)	6(225)	12	8	2	11	0
Demand	0	0	0	0	0	0	

Hence

	1	2	3	4	5	6	Supply
1	11(100)	13	17	14	9	4(150)	250
2	16(75)	18	14(125)	10	1(100)	22	300
3	21	24	13(150)	10(250)	33	12	400
4	8(25)	6(225)	12	8	2	11	250
Demand	200	225	275	250	100	150	

And IBFS$= 11 \times 100 + 4 \times 150 + 16 \times 75 + 14 \times 125 + 1 \times 100 + 13 \times 150 + 10 \times 250 + 8 \times 25 + 6 \times 225 = 10750$

Now by NWCM

	1	2	3	4	5	6	Supply
1	11(200)	13(50)	17	14	9	4(150)	250
2	16	18(175)	14(125)	10	1	22	300
3	21	24	13(150)	10(250)	33	12	400
4	8	6	12	8	2(100)	11(150)	250
Demand	200	225	275	250	100	150	

And IBFS$= 11 \times 200 + 13 \times 50 + 18 \times 175 + 14 \times 125 + 13 \times 150 + 10 \times 250 + 2 \times 100 + 11 \times 150 = 14050$

Now by LCM

	1	2	3	4	5	6	Supply
1	11(100)	13	17	14	9	4(150)	250
2	16(100)	18	14(100)	10	1(100)	22	300
3	21	24	13(175)	10(225)	33	12	400
4	8	6(225)	12	8(25)	2	11	250
Demand	200	225	275	250	100	150	

And IBFS=$11 \times 100 + 4 \times 150 + 16 \times 100 + 14 \times 100 + 1 \times 100 + 13 \times 175 + 10 \times 225 + +6 \times 225 + 8 \times 25 = 10875$

Now by Vogel's Method

	1	2	3	4	5	6	Supply
1	11(100)	13	17	14	9	4(150)	250
2	16	18	14	10(200)	1(100)	22	300
3	21(75)	24	13(275)	10(50)	33	12	400
4	8(25)	6(225)	12	8	2	11	250
Demand	200	225	275	250	100	150	

And IBFS=$11 \times 100 + 4 \times 150 + 10 \times 200 + 1 \times 100 + 21 \times 75 + 13 \times 275 + 10 \times 50 + 21 \times 75 + 6 \times 225 = 11000$

Now by MODI method found OS

	1	2	3	4	5	6	Supply
1	11(100)	13	17	14	9	4(150)	250
2	16(75)	18	14	10(125)	1(100)	22	300
3	21	24	13(275)	10(125)	33	12	400
4	8(25)	6(225)	12	8	2	11	250
Demand	200	225	275	250	100	150	

OS$=11\times 100+4\times 150+16\times 75+14\times 125+1\times 100+13\times 150+10\times 250+8\times 25+6\times 225=10625$

Example 3.3.2 Let's take the next transportation problem with three rows and four columns.

	1	2	3	4	Supply
1	12	5	7	8	50
2	16	10	2	1	90
3	9	14	5	7	150
Demand	128	72	65	25	

IBFS by NWCM is

	1	2	3	4	Supply
1	12(50)	5	7	8	50
2	16(78)	10(12)	2	1	90
3	9	14(60)	5(65)	7(25)	150
Demand	128	72	65	25	

IBFS=$12 \times 50 + 16 \times 78 + 10 \times 12 + 14 \times 60 + 5 \times 65 = 3308$

IBFS by LCM is

	1	2	3	4	Supply
1	12	5(50)	7	8	50
2	16	10	2(65)	1(25)	90
3	9(128)	14(22)	5	7	150
Demand	128	72	65	25	

IBFS=$5 \times 50 + 12 \times 65 + 1 \times 25 + 9 \times 128 + 14 \times 22 = 1865$

IBFS by VAM is

	1	2	3	4	Supply
1	12	5(50)	7	8	50
2	16	10	2(65)	1(25)	90
3	9(128)	14(22)	5	7	150
Demand	128	72	65	25	

IBFS=$5 \times 50 + 12 \times 65 + 1 \times 25 + 9 \times 128 + 14 \times 22 = 1865$

IBFS by LDM is

	1	2	3	4	Supply
1	12	5(50)	7	8	50
2	16	10	2(65)	1(25)	90
3	9(128)	14(22)	5	7	150
Demand	128	72	65	25	

IBFS=$5 \times 50 + 12 \times 65 + 1 \times 25 + 9 \times 128 + 14 \times 22 = 1865$

OS by MODI method is

	1	2	3	4	Supply
1	12	5(50)	7	8	50
2	16	10(22)	2(43)	1(25)	90
3	9(128)	14	5(22)	7	150
Demand	128	72	65	25	

IBFS=$5 \times 50 + 10 \times 22 + 2 \times 43 + 1 \times 25 + 9 \times 128 + 5 \times 25 = 1843$

Example 3.3.3 Let's take the next transportation problem with three rows and three columns.

	1	2	3	Supply
1	7	3	8	100
2	5	6	1	200
3	10	4	7	300
Demand	80	170	350	

IBFS by NWCM is

	1	2	3	Supply
1	7(80)	3(20)	8	100
2	5	6(150)	1(50)	200
3	10	4	7(300)	300
Demand	50	30	60	

IBFS=$7 \times 80 + 3 \times 20 + 6 \times 150 + 1 \times 50 + 7 \times 300 = 3670$

IBFS by LCM is

	1	2	3	Supply
1	7	3(100)	8	100
2	5	6	1(200)	200
3	10(80)	4(70)	7(150)	300
Demand	50	30	60	

IBFS=$3 \times 100 + 1 \times 200 + 10 \times 80 + 4 \times 70 + 7 \times 150 = 2630$

IBFS by VAM is

	1	2	3	Supply
1	7	3(100)	8	100
2	5	6	1(200)	200
3	10(80)	4(70)	7(150)	300
Demand	50	30	60	

IBFS=$3 \times 100 + 1 \times 200 + 10 \times 80 + 4 \times 70 + 7 \times 150 = 2630$

IBFS by LDM is

	1	2	3	Supply
1	7	3(100)	8	100
2	5	6	1(200)	200
3	10(80)	4(70)	7(150)	300
Demand	50	30	60	

IBFS=$3 \times 100 + 1 \times 200 + 10 \times 80 + 4 \times 70 + 7 \times 150 = 2630$

OS by MODI method is

	1	2	3	Supply
1	7(80)	3(20)	8	100
2	5	6	1(200)	200
3	10	4(150)	7(150)	300
Demand	50	30	60	

IBFS=$7 \times 80 + 3 \times 20 + 1 \times 200 + 4 \times 150 + 7 \times 150 = 2470$

Example 3.3.4 Let's take the next transportation problem with four rows and four columns.

	1	2	3	4	Supply
1	7	3	8	4	100
2	5	6	1	6	200
3	1	4	7	7	300
4	3	6	8	2	35
Demand	80	170	310	75	

Now, by VAM IBFS is

	1	2	3	4	Supply
1	7	3	8(60)	4(40)	100
2	5	6	1(200)	6	200
3	1(80)	4(170)	7(50)	7(40)	300
4	3	6	8	2(35)	35
Demand	80	170	310	75	

IBFS=$8\times60+4\times40+1\times200+1\times80+4\times170+7\times50+2\times35 = 2020$

IBFS by LDM is

	1	2	3	4	Supply
1	7	3(100)	8	4	100
2	5	6	1(200)	6	200
3	1(80)	4(70)	7(110)	7(40)	300
4	3	6	8	2(35)	35
Demand	80	170	310	75	

IBFS=$3\times100+1\times200+1\times80+4\times70+7\times110+7\times40+2\times35 = 1980$

OS by MODI method is

	1	2	3	4	Supply
1	7	3(60)	8	4(40)	100
2	5	6	1(200)	6	200
3	1(80)	4(110)	7(110)	7	300
4	3	6	8	2(35)	35
Demand	80	170	310	75	

IBFS=$3\times60+4\times40+1\times200+1\times80+4\times110+7\times110+2\times35 = 1900$

In terms of the IBFS, and near to OS the following table compares the LDM method with the classical methods,

Table 3.9: Comparison Between LDM and the Classical Methods

Number of Example	NWCM	LCM	VAM	LDM	OS
3.3.1	14050	10875	11000	10750	10625
3.3.2	3308	1865	1865	1865	1843
3.3.3	3670	2630	2630	2630	2470
3.3.4	3740	1980	2020	1980	1900

Through the comparison table, we notice that the solution by the proposed method is less or equal to the solution by the classical methods, so the solution by LDM method is in way that is near to OS.

Chapter 4

Analysis of Special Case to TP

4.1 Introduction

Theories of special circumstances in which IBFS is always the optimal solution and which are expected to be developed include transportation problems and construction of an ideal model are presented, so some important theories about TP are addressed in this chapter.

4.2 Some Theorems of a Special Case

In this section, we presented a special case in which the initial solution equals the optimal solution because the difference between the costs of any two identical cells in any two given rows is constant. Therefore, we presented the proofs that indicate this case and prove them in general in the following theorems,

Theorem 4.2.1 In any matrix, $C_{ij} - C_{rj} = \delta_{ir}$, where δ_{ir} is constant, for all two rows i, r and column j such that δ_{ir} is constant if and only if $C_{ij} - C_{ik} = \sigma_{jk}$, where σ_{jk} is constant, for all row i and all two columns $j, k.$ that is the deference between all two symmetric cells in all two rows is constant if and only if the deference between all two symmetric cells in all two columns is constant.

Proof:

$(\Rightarrow)$ *Let* $C_{ij} - C_{rj} = \delta_{ir}$ for any rows i, r and column j, then $C_{ij} = \delta_{ir} + C_{rk}$ so $C_{ij} - C_{ik} = \delta_{ir} + C_{rj} - (\delta_{ir} + C_{rk}) = C_{rj} - C_{rk}$, that is the deference between all two symmetric cells in all two rows is constant. Hence $C_{ij} - C_{ik} = \sigma_{jk}$

$(\Leftarrow)$ *Let* $C_{ij} - C_{ik} = \sigma_{jk}$ for any row i and columns j, k,then $C_{ij} = \sigma_{jk} + C_{ik}$ and $C_{rj} = \sigma_{jk} + C_{rk}$ so $C_{ij} - C_{rj} = \sigma_{jk} + C_{ik} - (\sigma_{jk} + C_{rk}) = C_{ik} - C_{rk}$, that is the deference between all two symmetric cells in all two columns is constant. Hence $C_{ij} - C_{rj} = \delta_{ir}$. ■

Note 4.2.1 If $C_{ij} - C_{ik} = \sigma_{jk}$ then $C_{ik} - C_{ij} = \sigma_{kj}$ so $\sigma_{kj} = -\sigma_{jk}$ and $\sigma_{jk} + \sigma_{kj} = 0$.

Note 4.2.2 If $C_{ij} - C_{rj} = \delta_{ir}$ then $C_{rj} - C_{ij} = \delta_{ri}$ so $\delta_{ir} = -\delta_{ri}$ and $\delta_{ir} + \delta_{ri} = 0$.

Note 4.2.3 In MODI method, If $C_{ij} - C_{rj} = \delta_{ir}$, for all two rows i, r and column j such that δ_{ir} is constant and since $C_{ij} = u_i + v_j$ then $\delta_{ik} + C_{jk} = u_i + v_j \Rightarrow \delta_{ik} + u_k + v_j = u_i + v_j \Rightarrow \delta_{ik} = u_i - u_k$.

Note 4.2.4 In MODI method, If $C_{ij} - C_{rj} = \delta_{ir}$, for all two rows i, r and column j such that δ_{ir} is constant and since $C_{ij} = u_i + v_j$ then $\sigma_{jk} + C_{ik} = u_i + v_j \Rightarrow \sigma_{jk} + u_i + v_k = u_i + v_j \Rightarrow \sigma_{jk} = v_j - v_k$.

Note 4.2.5 The index of net cost change =0 for any simple closed path since $C_{ij} - C_{ik} + C_{rk} - C_{rj} = \sigma_{jk} + \sigma_{kj} = 0$

Theorem 4.2.2 In balanced matrix TP, if $C_{ij} - C_{rj} = \delta_{ir}$, for all two rows i, r and column j such that δ_{ir} is constant, then IBFS=OS. that is if the deference between all two symmetric cells in all two rows is constant then IBFS by any method equal to OS.

Proof:

Since $d_{ij} = C_{ij} - u_i - v_j = \delta_{ir} + C_{rj} - u_i - v_j = \delta_{ir} + u_r + v_j - u_i - v_j = \delta_{ir} + \delta_{ri} = 0$ for all i, j, then IBFS=OS. ■

4.3 Analytical Examples of a Special Case

Example 4.3.1 Let's take the next TP with three rows and three columns.

	1	2	3	Supply
1	11	13	17	25
2	1	3	7	100
3	4	6	10	15
Demand	50	30	60	

IBFS by NWCM is

	1	2	3	Supply
1	11(25)	13	17	25
2	1(25)	3(30)	7(45)	100
3	4	6	10(15)	15
Demand	50	30	60	

IBFS=$11 \times 25 + 1 \times 25 + 3 \times 30 + 7 \times 45 + 10 \times 15 = 855$

Now by MODI method test and found OS.

Iteration 1 of optimality test

1: Find u_i and v_j for all occupied cells where $C_{ij} = u_i + v_j$

- Put,$u_2 = 0$, we get
- $C_{21} = u_2 + v_1 \Rightarrow v_1 = C_{21} - u_2 \Rightarrow v_1 = 1 - 0 = 1$
- $C_{11} = u_1 + v_1 \Rightarrow u_1 = C_{11} - v_1 = 11 - 1 = 10$
- $C_{22} = u_2 + v_2 \Rightarrow v_2 = C_{22} - u_2 = 3 - 0 = 3$
- $C_{23} = u_2 + v_3 \Rightarrow v_3 = C_{23} - u_2 = 7 - 0 = 7$
- $C_{33} = u_3 + v_3 \Rightarrow u_3 = C_{33} - v_3 = 10 - 7 = 3$

So

	1	2	3	Supply	u_i
1	11(25)	13	17	25	$u_1 = 10$
2	1(25)	3(30)	7(45)	100	$u_2 = 0$
3	4	6	10(15)	15	$u_3 = 3$
Demand	50	30	60		
v_j	$v_1 = 1$	$v_2 = 3$	$v_3 = 7$		

2: Calculate d_{ij} for all unoccupied cells (i, j) where $d_{ij} = C_{ij} - (u_i + v_j)$.

- $d_{12} = C_{12} - (u_1 + v_2) = 13 - (10 + 3) = 0$
- $d_{13} = C_{13} - (u_1 + v_3) = 17 - (10 + 7) = 0$
- $d_{31} = C_{31} - (u_3 + v_1) = 4 = (3 + 1) = 0$

- $d_{32} = C_{32} - (u_3 + v_2) = 6 - (3 + 3) = 0$

Since all $d_{ij} \geq 0$ so the solution is optimal. That is IBFS=OS=855

Now by SS method test and found OS. The following Table contains the closed paths to the unoccupied cells.

Table 4.1: Closed Paths

Unoccupied Cell	Closed Path	Index of Net Cost Change
12	12$\rightarrow$ 11 $\rightarrow$ 21 $\rightarrow$ 22	13-11+1-3=0
13	13$\rightarrow$ 11 $\rightarrow$ 21 $\rightarrow$ 23	17-11+1-7=0
31	31$\rightarrow$ 33 $\rightarrow$ 23 $\rightarrow$ 21	4-10+7-1=0
32	32 $\rightarrow$ 33 $\rightarrow$ 23 $\rightarrow$ 22	6-10+7-3=0

Since all index of Net Cost Change=0 So IBFS=OS

Example 4.3.2 Let's take the next TP with three rows and four columns.

	1	2	3	4	Supply
1	11	13	17	14	250
2	16	18	22	19	300
3	15	17	21	18	400
Demand	200	225	275	250	

IBFS by NWC method is

	1	2	3	4	Supply
1	11(200)	13(50)	17	14	250
2	16	18(175)	22(125)	19	300
3	15	17	21(150)	18(250)	400
Demand	200	225	275	250	

IBFS=$11 \times 200 + 13 \times 50 + 18 \times 175 + 22 \times 125 + 21 \times 150 + 18 \times 250$=16400 =IBFS by any method=OS.

Example 4.3.3 Let's take the next TP with three rows and four columns.

	1	2	3	4	Supply
1	6	8	12	9	50
2	16	18	22	19	30
3	6	8	12	9	124
Demand	80	24	75	25	

IBFS by LCM is

	1	2	3	4	Supply
1	6	8(24)	12(26)	9	50
2	16	18	2(30)2	19	30
3	6(80)	8	12(19)	9(25)	124
Demand	80	24	75	25	

IBFS=$8 \times 24 + 12 \times 26 + 22 \times 30 + 6 \times 80 + 12 \times 19 + 9 \times 25$=2097 =IBFS by any method=OS.

Example 4.3.4 Let's take the next TP with four rows and five columns.

	1	2	3	4	5	Supply
1	1	3	17	14	5	75
2	6	8	22	19	10	15
3	0	2	16	13	4	113
4	10	12	26	23	14	87
Demand	20	123	27	50	70	

IBFS by VAM method is

	1	2	3	4	5	Supply
1	1	3(30)	17	14	5(45)	75
2	6	8	22	19	10(15)	15
3	0(20)	2(93)	16	13	4	87
4	10	12	26(27)	23(50)	14(10)	87
Demand	20	123	27	50	70	

IBFS=$3 \times 30 + 5 \times 45 + 10 \times 15 + 0 \times 20 + 2 \times 93 + 26 \times 27 + 23 \times 50 + 14 \times 10$=2643 =IBFS by any method=OS.

Example 4.3.5 Let's take the next TP with four rows and six columns.

	1	2	3	4	5	6	Supply
1	1	3	7	9	1	3	13
2	22	24	28	30	22	24	100
3	10	12	16	18	10	12	85
4	5	7	11	13	5	7	160
Demand	100	25	77	48	90	18	

IBFS by VAM method is

	1	2	3	4	5	6	Supply
1	1	3(13)	7	9	1	3	13
2	22	24(12)	28(40)	30(48)	22	24	100
3	10	12	16(37)	18	10(30)	12(18)	85
4	5(100)	7	11	13	5(60)	7	160
Demand	100	25	77	48	90	18	

IBFS=$3 \times 13 + 24 \times 12 + 28 \times 40 + 30 \times 48 + 16 \times 37 + 10 \times 30 + 12 \times 18 + 5 \times 100 + 5 \times 60$=2643 =IBFS by any method=OS.

Table 4.2: IBFS and OS

Number Example	IBFS by any method	OS by any method
4.3.1	855	855
4.3.2	16400	16400
4.3.3	2097	2097
4.3.4	2643	2643
4.3.5	4795	4795

Chapter 5

New Methods to Find OS

5.1 Introduction

In this chapter, new methods have been proposed to find OS to TP, called (Matrix method and YM method) and applied the new methods to solve many examples, in each of them and by comparing with (SS) and (MODI) methods, it was found that OS which obtained by YM method is equal to OS obtained by (SS) and (MODI) method, in addition, the index number to reach OS in the newly proposed method is less than the index number needed by the other two methods, which indicates the efficiency and quality of the newly proposed methods. The new methods are characterized by simplicity, ease, and no need for complex calculations compared to traditional (SS) and (MODI) methods used to solve TP.

5.2 Matrix Method

Algorithm 8 Matrix Method Algorithm

Step 1: Find (IBFS) by VAM method or LDM method,

Step 2: Define the solution matrix A by setting 1 for the filled cells and 0 for the other cells (empty cells),

Step 3: Find $M_1 = AA^T$ and $M_2 = A^T A$,

Step 4: Determine the zeros of the empty cells M_1 and M_2 and make a closed path to the corresponding cells in (IBFS) table, by giving $(+1)$ to the checking cell, (-1) to the next cell in the path, and so on for all cells in the path;

(a) If the improvement index for all closed paths is positive or zero, then (IBFS) is the optimal solution.

(b) If there exist a negative improvement index for any closed path, then determine the least value of products in the path cells which has (-1) and subtract this value from these cells and add it to the cells with $(+1)$.

Step5: Repeat steps $2 - 4$ until we reach an optimal solution.

5.3 Analytical Examples of Matrix Method

The new method (Matrix method) applied to solve a lot of examples, in all of them, the solutions that obtained by our new method is equal to the solutions that obtained by the classical methods, steepest stone method (SS) and modified method (MODI), moreover, the number of the index to reach the optimal solution by using the Matrix method is less than the number of the index in the other two classical method.

Example 5.3.1 Let's take the following TP matrix with five rows and five columns.

	A	B	C	D	E	Supply
1	11	13	17	14	8	250
2	16	18	14	9	9	300
3	21	24	13	10	1	400
4	8	6	12	8	3	100
5	1	3	22	7	18	40
Demand	200	225	275	250	140	1090

Firstly, to find IBFS by VAM method get:

	A	B	C	D	E	Supply
1	11(160)	13(90)	17	14	8	250
2	16	18(35)	14(15)	9(250)	9	300
3	21	24	13(260)	10	1(140)	400
4	8	6(100)	12	8	3	100
5	1(40)	3	22	7	18	40
Demand	200	225	275	250	140	1090

IBFS $=11 \times 160 + 13 \times 90 + 18 \times 35 + 14 \times 15 + 9 \times 250 + 13 \times 260 + 1 \times 140 + 6 \times 100 + 1 \times 40 = 10180$

To test the solution, whether it is optimal solution or not, we create the solution matrix as follows:

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \ A^T = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix},$$

$$M_1 = \begin{pmatrix} 2 & 1 & 0 & 1 & 1 \\ 1 & 3 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}, \ M_2 = \begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 3 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

The zeros of the empty cells in the matrices M_1 and M_2 corresponding to the cells $1C, 1D, 1E, 2E, 3A, 3D, 4A, 4C, 4E, 5B, 5C$, and $5D$, while we did not choose $3E$ and $5A$ because they correspond to filled cell. The following table contains the closed paths to the corresponding cells.

Table 5.1: Closed Paths of the Corresponding Cells

Unoccupied Cell	**Closed Path**	**Index of Net Cost Change**
1C	1C$\rightarrow 1B \rightarrow 2B \rightarrow 2C$	17 - 13 + 18 - 14=8
1D	1D$\rightarrow 1B \rightarrow 2B \rightarrow 2D$	$14 - 13 + 18 - 9 = 10$
1E	1E$\rightarrow 1B \rightarrow 2B \rightarrow 2E$	$8 - 13 + 18 - 14 + 13 - 1 = 11$
2E	2E$\longrightarrow 2C \longrightarrow 3C \longrightarrow 3E$	$9 - 14 + 13 - 1 = 7$
3A	3A$\rightarrow 3C \rightarrow 2C \rightarrow 2B \rightarrow 1B \rightarrow 1A$	$21 - 13 + 14 - 18 + 13 - 11 = 6$
3D	3D$\rightarrow 3C \rightarrow 2C \rightarrow 2D$	$10 - 13 + 14 - 9 = 2$
4A	4A$\rightarrow 4B \rightarrow 1B \rightarrow 1A$	$8 - 6 + 13 - 11 = 4$
4C	4C$\longrightarrow 4B \longrightarrow 2B \longrightarrow 2C$	$12 - 6 + 18 - 14 = 10$
4E	4E$\longrightarrow 4B \longrightarrow 2B \longrightarrow 2C \longrightarrow 3C \longrightarrow 3E$	$3 - 6 + 18 - 14 + 13 - 1 = 13$
5B	5B$\rightarrow 1B \rightarrow 1A \rightarrow 5A$	$3 - 1 + 11 - 13 = 0$
5C	5C$\rightarrow 5A \rightarrow 1A1B \rightarrow 2B \rightarrow 2C$	$22 - 1 + 11 - 13 + 18 - 14 = 23$
5D	5D$\rightarrow 5A \rightarrow 1A \rightarrow 1B \rightarrow 2B \rightarrow 2D$	$7 - 1 + 11 - 13 + 18 - 9 = 13$

Since all index of net cost change ≥ 0 then the initial solution is the optimal solution = 10180.

Example 5.3.2 Let's take the following TP with three rows and three columns.

	A	**B**	**C**	**Supply**
1	1	3	5	25
2	3	4	8	30
3	4	2	7	45
Demand	48	27	25	100

Firstly, to find IBFS by LDM we get:

	A	**B**	**C**	**Supply**
1	1(25)	3	5	25
2	3(23)	4	8(7)	30
3	4	(27)	7(18)	45
Demand	48	27	25	

IBFS$= 1 \times 25 + 3 \times 23 + 8 \times 7 + 2 \times 27 + 7 \times 18 = 330.$

To test the solution, whether it is optimal or not, and then improve it to reach the optimal solution, created the solution matrix as follows:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \; A^T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix},$$

$$M_1 = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \; M_2 = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

The zeros of the empty cells in the matrices M_1 and M_2 corresponding to the cells $1C$, $3A$ and $1B$.

The following table contains the closed paths to the corresponding cells.

Table 5.2: Closed Paths of the Corresponding Cells

Unoc.Cell	**Closed Path**	**Net Cost Change**
$1C$	$1C \rightarrow 1A \rightarrow 2A \rightarrow 2C$	$+5 - 1 + 3 - 8 = -1$
$3A$	$3A \rightarrow 3C \rightarrow 2C \rightarrow 2A$	$+4 - 7 + 8 - 3 = +1$
$1B$	$1B \rightarrow 3B \rightarrow 3C \rightarrow 2C \rightarrow 2A \rightarrow 1A$	$+3 - 2 + 7 - 8 + 3 - 1 = +2$

So the solution after improvement is

	A	B	C	Supply
1	1(18)	3	5(7)	25
2	3(30)	4	8	30
3	4	2(27)	7(18)	45
Demand	48	27	25	

IBFS $= 1 \times 18 + 3 \times 30 + 5 \times 7 + 2 \times 27 + 7 \times 18 = 323$ So

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \quad A^T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix},$$

$$M_1 = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

The following table contains the closed paths to the corresponding cells.

Table 5.3: Closed Paths Of the Corresponding Cells

Unoc. Cell	**Closed Path**	**Index of Net Cost Change**
$1B$	$1B \rightarrow 3B \rightarrow 3C \rightarrow 1C$	$+3 - 2 + 7 - 5 = +3$
$2C$	$2C$ →2A→ $1A \rightarrow 1C$	$+8 - 3 + 1 - 5 = +1$

Since all the index of net cost change ≥ 0 then we reached to the optimal solution =323.

Example 5.3.3 Take TP matrix with six rows and six columns

	1	**2**	**3**	**4**	**5**	**6**	**Supply**
1	3	3	7	1	9	8	50
2	6	8	4	7	1	10	130
3	20	2	6	9	22	1	140
4	8	2	7	1	3	8	100
5	12	8	7	1	22	6	40
6	22	11	5	4	5	9	65
Demand	20	125	80	200	55	45	525

So

	1	**2**	**3**	**4**	**5**	**6**	**Supply**
1	3(20)	3	7	1(30)	9	8	50
2	6	8	4(75)	7	1(55)	10	130
3	20	2(95)	6	9	22	1(45)	140
4	8	2(30)	7	1(70)	3	8	100
5	12	8	7	1(40)	22	6	40
6	22	11	5(5)	4(60)	5	9	65
Demand	20	125	80	200	55	45	525

OS $= 3 \times 20 + 1 \times 30 + 4 \times 75 + 1 \times 55 + 2 \times 95 + 1 \times 45 + 2 \times 30 + 1 \times 70 + 1 \times 40 + 5 \times 5 + 4 \times 60 = 1115$

Example 5.3.4 Take TP matrix with seven rows and seven columns

	1	**2**	**3**	**4**	**5**	**6**	**7**	**Supply**
1	16	9	2	3	11	8	5	70
2	14	18	9	1	6	2	1	160
3	20	11	4	6	2	33	21	90
4	1	7	8	5	32	1	8	80
5	1	12	4	1	22	5	7	40
6	30	19	11	22	4	9	1	140
7	23	7	2	1	3	9	32	24
Demand	50	120	150	100	34	70	80	

So

	1	**2**	**3**	**4**	**5**	**6**	**7**	**Supply**
1	16	9(26)	2(44)	3	11	8	5	70
2	14	18	9	1(100)	6	2(60)	1	160
3	20	11	4(90)	6	2	33	21	90
4	1(10)	7(70)	8	5	32	1	8	80
5	1(40)	12	4	1	22	5	7	40
6	30	19	11(16)	22	4(34)	9(10)	1(80)	140
7	23	7(24)	2	1	3	9	32	24
Demand	50	120	150	100	34	70	80	

OS $= 9 \times 26 + 2 \times 44 + 1 \times 100 + 2 \times 60 + 4 \times 90 + 1 \times 10 + 7 \times 70 + 1 \times 40 + 11 \times 16 + 4 \times 34 + 9 \times 10 + 1 \times 80 + 7 \times 24 = 2092$

Example 5.3.5 Take TP matrix with four rows and four columns

	1	**2**	**3**	**4**	**Supply**
1	3	3	7	1	50
2	6	8	4	7	130
3	20	2	6	9	40
4	8	2	7	1	100
Demand	20	25	75	200	320

So

	1	**2**	**3**	**4**	**Supply**
1	3	3	7	1(50)	50
2	6(20)	8	4(75)	7(35)	130
3	20	2(25)	6	9(15)	40
4	8	2	7	1(100)	100
Demand	20	25	75	200	320

$$\text{OS} = 1\times 50+6\times 20+4\times 75+7\times 35+2\times 25+9\times 15+1\times 100 = 1000$$

Example 5.3.6 Take TP matrix with three rows and tree columns,

	1	**2**	**3**	**Supply**
1	9	3	9	50
2	6	18	1	30
3	20	12	6	14
Demand	59	25	10	

So

	1	**2**	**3**	**Supply**
1	9(29)	3(21)	9	50
2	6(30)	18	1	30
3	20	12(4)	6(10)	14
Demand	59	25	10	

$$\text{OS} = 9 \times 29 + 3 \times 21 + 6 \times 30 + 12 \times 4 + 6 \times 10 = 612$$

Example 5.3.7 Take TP matrix with five rows and five columns,

	1	2	3	4	5	**Supply**
1	9	3	9	5	1	50
2	26	11	1	8	4	30
3	2	12	16	10	1	24
4	10	7	6	1	2	70
5	7	12	4	10	6	20
Demand	59	25	10	35	65	

So

	1	2	3	4	5	**Supply**
1	9(15)	3(25)	9	5	1(10)	50
2	26	11	1(10)	8	4(20)	30
3	2(24)	12	16	10	1	24
4	10	7	6	1(35)	2(35)	70
5	7(20)	12	4	10	6	20
Demand	59	25	10	35	65	

OS $= 9 \times 15 + 3 \times 25 + 1 \times 10 + 1 \times 10 + 4 \times 20 + 2 \times 24 + 1 \times 35 + 2 \times 35 + 7 \times 20 = 603$

Example 5.3.8 Take TP matrix with four rows and four columns,

	1	2	3	4	**Supply**
1	9	3	9	5	20
2	26	11	1	8	34
3	2	20	16	1	20
4	10	7	6	1	15
Demand	19	25	10	35	89

So

	1	2	3	4	**Supply**
1	9	3(20)	9	5	20
2	26	11(5)	1(10)	8(19)	34
3	2(19)	20	16	1(1)	20
4	10	7	6	1(15)	15
Demand	19	25	10	35	89

$OS = 3 \times 20 + 11 \times 5 + 1 \times 10 + 8 \times 19 + 2 \times 19 + 1 \times 1 + 1 \times 15 = 331$

The following table contains the comparison among Matrix method, (SS) method, and (MODI) method in terms of the optimal solutions and the number of index

Table 5.4: Comparison of the OS and the Number of Index

Example	**(SS) Method**		**(MODI) Method**		**Matrix Method**	
	OS	**NO.Index**	**OS**	**NO.Index**	**OS**	**NO.Index**
5.3.1	10180	16	10180	16	10180	12
5.3.2	323	4	323	4	323	3
5.3.3	1115	25	1115	25	1115	16
5.3.4	2092	36	2092	36	2092	30
5.3.5	1000	9	1000	9	1000	1
5.3.6	612	4	612	4	612	2
5.3.7	603	16	603	16	603	9
5.3.8	331	9	331	9	331	4

By the number of indexes we mean the number of paths that must be made or the equations required to reach the paths and therefore to reach the cells to be improved. Here we found that the proposed method reduces the number of indexes in combination with the classical methods (SS and MODI).

5.4 YM Method

Algorithm 9 YM Method Algorithm

Step 1: Find (IBFS) by by using any method,

Step 2: Select the empty cells that are less or equal cost to some of the allocation cells in their row or column ,

Step 3: Make a closed path for the cells selected from the previous step in the (IBFS) table, by giving $(+1)$ to the checking cell, (-1) to the next cell in the path, and so on for all cells in the path;

(a) If the improvement index for all closed paths is positive or zero, then (IBFS) is the optimal solution.

(b) If there exist a negative improvement index for any closed path, then determine the least value of products in the path cells which has (-1) and subtract this value from these cells and add it to the cells with $(+1)$.

Step 4: Repeat steps $2-3$ until we reach an optimal solution.

5.5 Analytical Examples of YM Method

The study applied these new method to find the optimal solution for many examples. In this section, we'll take a sample of these examples to transportation cost matrices.

Example 5.5.1 *:* Let's take the following TP matrix,

	A	**B**	**C**	**D**	**Supply**
1	11	13	17	14	250
2	16	18	14	9	300
3	21	24	13	10	400
Demand	200	225	275	250	

Firstly, to find IBFS by Vogel's method we get:

	A	**B**	**C**	**D**	**Supply**
1	11(200)	13(50)	17	14	250
2	16	18(50)	14	9(250)	300
3	21	24(125)	13(275)	10	400
Demand	200	225	275	250	

IBFS $= 11 \times 200 + 13 \times 50 + 18 \times 50 + 9 \times 250 + 24 \times 125 + 13 \times 275 =$ 12575

To test the solution, whether it is optimal solution or not, we select the empty cells that are less or equal cost to some of the allocation cells in their row or column. as follows: $2A$, $2C$, $3A$, $3D$

and make a closed path to those cells in the following table. While we did not choose $1C$ and $1D$ because its cost is higher than packed cells in its row and column.

Table 5.5: Closed Paths of the Cells

Unoccupied Cell	**Closed Path**	**Index of Net Cost Change**
2 A	2A$\rightarrow 2B \rightarrow 1B \rightarrow 1A$	16 - 18 + 13 - 11= 0
2 C	2C$\rightarrow$3 $C \rightarrow$3 B $\rightarrow$2B	14 - 13 + 24 - 18= 5
3 A	$3A \rightarrow 3B \rightarrow 1B \rightarrow 1A$	21- 24 + 13 - 11 = -1
$3D$	3D$\rightarrow$2D$\rightarrow 2B \rightarrow 3B$	10 - 9 + 18 - 24 = -5

The solution after improvement

	A	***B***	***C***	***D***	**Supply**
1	11(200)	13(50)	17	14	250
2	16	18(175)	14	9(125)	300
3	21	24	13(275)	10(125)	400
Demand	200	225	275	250	

The solution= $11 \times 200 + 13 \times 50 + 18 \times 175 + 9 \times 125 + 13 \times 275 + 10 \times 125 = 11950$. Now we repeat steps 2 to 3 until we reach the optimal solution. Blank cells whose cost is less than or equal to the cost of some assignment cells in its row or column are as follows: 2A, 2C.

Table 5.6: Closed Paths After Improvement

Unoccupied Cell	**Closed Path**	**Index of Net Cost Change**
2A	2A$\rightarrow 2B \rightarrow 1B \rightarrow 1A$	16 - 18 + 13 - 11= 0
2C	2C$\rightarrow 3C \rightarrow 3D \rightarrow 2D$	14 - 13 + 10 - 9= 2

positive indicator of net cost change, so the solution is optimal.

Example 5.5.2 Let's take TP matrix with six rows and ten columns,

	1	2	3	4	5	6	7	8	9	10	Supply
1	3	2	5	1	6	11	22	2	7	4	230
2	11	32	1	2	5	21	10	5	11	30	100
3	8	1	2	44	2	8	11	4	7	1	270
4	1	23	4	5	1	9	15	22	1	8	50
5	3	42	13	7	8	6	3	19	34	9	120
6	9	7	12	18	7	23	2	11	7	12	144
Demand	93	82	50	75	294	60	102	55	43	60	

	1	2	3	4	5	6	7	8	9	10	Supply
1	3(33)	2(82)	5	1(60)	6	11	22	2(55)	7	4	230
2	11	32	1(50)	2(15)	5(35)	21	10	5	11	30	100
3	8	1	2	44	2(210)	8	11	4	7	1(60)	270
4	1	23	4	5	1(7)	9	15	22	1(43)	8	50
5	3(60)	42	13	7	8	6(60)	3	19	34	9	120
6	9	7	12	18	7(42)	23	2(102)	11	7	12	144
Demand	93	82	50	75	294	60	102	55	43	60	

$\text{OS} = 3 \times 33 + 2 \times 82 + 1 \times 60 + 2 \times 55 + 1 \times 50 + 2 \times 15 + 5 \times 35 + 2 \times 210 + 1 \times 60 + 1 \times 7 + 1 \times 43 +$

$3 \times 60 + 6 \times 60 + 7 \times 42 + 2 \times 102 = 2256$

Example 5.5.3 Let's take TP matrix with three rows and three columns,

	1	2	3	Supply
1	3	2	5	30
2	11	8		125
3	8	1	2	70
Demand	93	82	50	

So

	1	2	3	Supply
1	3(30)	2	5	30
2	11(63)	8(12)	1(50)	125
3	8	1(70)	2	70
Demand	93	82	50	

$$\text{OS} = 3 \times 30 + 11 \times 63 + 8 \times 12 + 1 \times 50 + 1 \times 70 = 999$$

Example 5.5.4 Let's take TP matrix with six rows and six columns,

	1	2	3	4	5	6	Supply
1	11	13	17	14	8	4	250
2	16	8	3	10	1	11	300
3	1	4	13	6	3	1	400
4	8	6	12	8	2	11	250
5	1	3	2	7	18	1	40
6	20	2	1	8	5	3	60
Demand	200	225	275	250	125	225	

So

	1	2	3	4	5	6	Supply
1	11	13	17	14(25)	8	4(225)	250
2	16	8	3(275)	10	1(25)	11	300
3	1(200)	4(125)	13	6(75)	3	1	400
4	8	6	12	8(150)	2(100)	11	250
5	1	3(40)	2	7	18	1	40
6	20	2(60)	1	8	5	3	60
Demand	200	225	275	250	125	225	

OS $= 14 \times 25 + 4 \times 225 + 3 \times 275 + 1 \times 25 + 1 \times 200 + 4 \times 125 + 6 \times 75 + 8 \times 150 + 2 \times 100 + 3 \times 40 + 2 \times 60 = 4890$

The study applied the new method to solve a lot of examples, in all of them, the solutions that obtained by our new method is equal to the solutions that obtained by the classical methods, (SS) and (MODI) methods , moreover, the number of the index to reach OS by using YM method is less than the number of the index in the other two classical method. The following table contains the comparison among YM method, (SS) method, and (MODI) method in terms of OS and the number of indexes.

Table 5.7: Comparison of OS and Number of Index

No.Exam	(SS) Method		(MODI) Method		YM Method	
	OS	NO.Index	OS	NO.Index	OS	NO.Index
5.5.1	11950	6	11950	6	11950	4
5.5.2	2256	45	2256	45	2256	13
5.5.3	999	4	999	4	999	2
5.5.4	4890	25	4890	25	4890	15

Here we found that the proposed method reduces the number of indexes in combination with the classical methods (SS and MODI)

Chapter 6

New Technique Based on Graph Theory

6.1 Introduction

A novel technique called the difference absolute method (DAM) is introduced in this chapter to locate and test the optimal solution for diverse transportation challenges. The most appealing characteristic of this approach is that it just requires simple arithmetic and logical computations, making it accessible to even the most inexperienced user. This strategy will be incredibly beneficial for those who are capable decision-makers. Take care of logistics and supply chain difficulties. You may easily apply this technique to the present procedure because it is straightforward.

6.2 Deference Absolute Method

Algorithm 10 Deference Absolute Method Algorithm

Step 1: Find (IBFS) by using any method,

Step 2: Represent the solution matrix with a weighted graph by putting zero on the edges representing the allocation cell or unoccupied cells whose allocation cell in its row or column is greater than it is cost, and the weight of other cells is the difference absolute between the cost of the empty cell with the largest allocation cell in its row or column,

Step 3: Find any minimum spanning tree for this graph and take the complement of minimum spanning tree (the edges don't belong to MST and belong to weighted graph)

Step 4: Select the cell with the largest weight edge. If the weights are equal, choose the one in its row or column has the largest allocation cell and then go to the next step,

Step 5: Make a closed path for the cell with the largest weight in the minimum complement of the spanning tree, and give (+1) for the check cell, (-1) for the next cell in the path, and so on for all cells in the path. If the optimization index of the closed path is positive or zero, take the next cell with the largest weight in the minimum complement of the spanning tree and make a path for it as well,

(a) If the optimization index for a closed path is positive or zero for all paths in the previous step, then IBFS is the optimal solution,

(b) If there exist a negative improvement index for the closed path, then determine the least value of products in the path cells which has (-1) and subtract this value from these cells and add it to the cells with $(+1)$.

Step 6: Repeat steps $2 - 5$ until to reach an optimal solution.

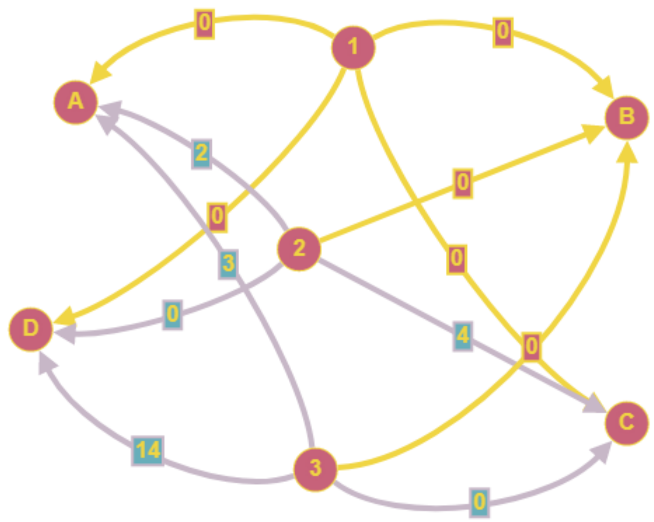
6.3 Analytical Examples of Deference Absolute Method

Example 6.3.1 *:* Let's take TP matrix in (Example 5.5.1), so IBFS is :

	A	**B**	**C**	**D**	**Supply**
1	11(200)	13(50)	17	14	250
2	16	18(50)	14	9(250)	300
3	21	24(125)	13(275)	10	400
Demand	200	225	275	250	

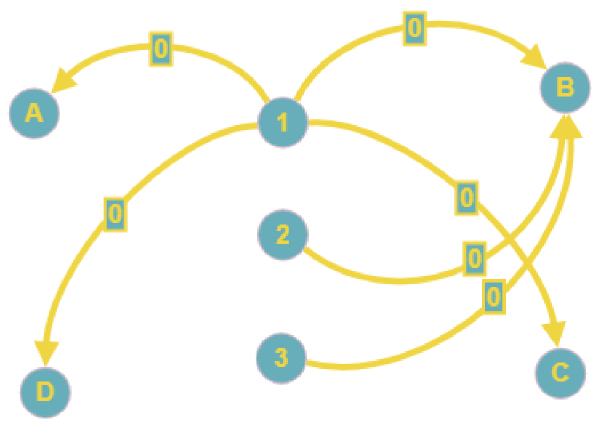
IBFS $= 11 \times 200 + 13 \times 50 + 18 \times 50 + 9 \times 250 + 24 \times 125 + 13 \times 275 = 12575$. Represent the solution matrix and locate the minimal spanning tree with yellow edges by the following graph,

Figure 6.1: Weighted Graph to Matrix Solution



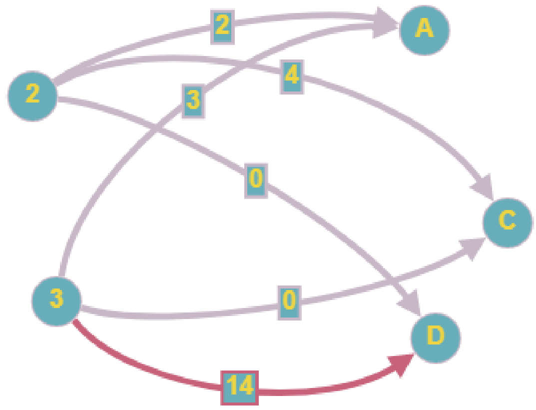
So, the minimum spanning tree represents by the following figure,

Figure 6.2: Minimum Spanning Tree



And the complement of the minimum spanning tree represents by the following figure,

Figure 6.3: Complement of Minimum Spanning Tree



The cell with the largest weight in the complement of minimum spanning is determine red edge to test the solution, whether it is optimal solution or not, make a closed path to 3D cell only in the following table,

Table 6.1: Closed Paths of the Cell 3D

Unoccupied Cell	**Closed Path**	**Index of Net Cost Change**
3D	$3D \to 2D \to 2B \to 3B$	10 - 9 + 18 - 24 = -5

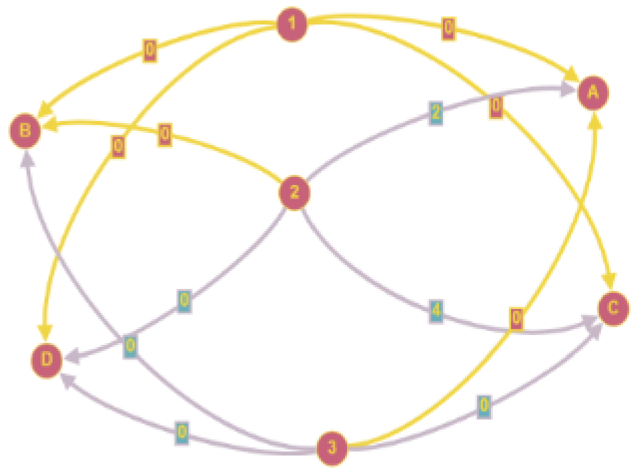
So improvement solution is:

	A	***B***	***C***	***D***	**Supply**
1	11(200)	13(50)	17	14	250
2	16	18(175)	14	9(125)	300
3	21	24	13(275)	10(125)	400
Demand	200	225	275	250	

The solution$= 11 \times 200 + 13 \times 50 + 18 \times 175 + 9 \times 125 + 13 \times 275 + 10 \times 125 = 11950$.

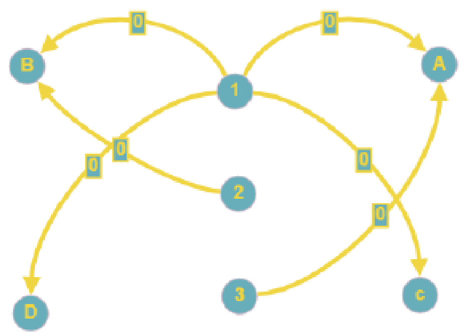
Now repeat steps 2 to 4 to the last table then, the following figure represent the solution matrix and locate the minimum spanning tree with yellow edges by the following graph,

Figure 6.4: Digraph to the Solution after Improvement



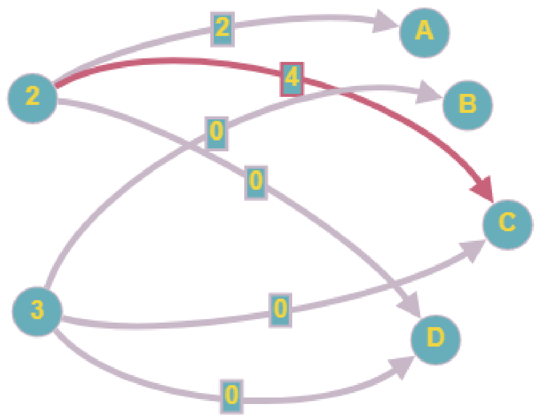
So, the minimum spanning tree represents by the following figure

Figure 6.5: Minimum Spanning Tree after Improvement



And the complement of the minimum spanning tree represents by the following figure

Figure 6.6: Complement of Minimum Spanning Tree after Improvement



The cell with the largest weight in the complement of minimum spanning is $2C$, to test the solution, whether it is optimal solution or not, make a closed path to $2C$ cell in the following table,

Table 6.2: Closed Paths of the Cell $2C$

Unoccupied Cell	**Closed Path**	**Index of Net Cost Change**
$2C$	$2C \to 2D \to 3D \to 3C$	14 - 9 + 10 - 13 = 2

Since index of net cost change $\geq 0, so \ \ IBFS \ \ is \ \ OS.$

Example 6.3.2 Let's take TP matrix with six rows and ten columns,

	1	2	3	4	5	6	7	8	9	10	Supply
1	3	2	5	1	6	11	22	2	7	4	230
2	11	32	1	2	5	21	10	5	11	30	100
3	8	1	2	44	2	8	11	4	7	1	270
4	1	23	4	5	1	9	15	22	1	8	50
5	3	42	13	7	8	6	3	19	34	9	120
6	9	7	12	18	7	23	2	11	7	12	144
Demand	93	82	50	75	294	60	102	55	43	60	

	1	2	3	4	5	6	7	8	9	10	Supply
1	3(33)	2(82)	5	1(60)	6	11	22	2(55)	7	4	230
2	11	32	1(50)	2(15)	5(35)	21	10	5	11	30	100
3	8	1	2	44	2(210)	8	11	4	7	1(60)	270
4	1	23	4	5	1(7)	9	15	22	1(43)	8	50
5	3(60)	42	13	7	8	6(60)	3	19	34	9	120
6	9	7	12	18	7(42)	23	2(102)	11	7	12	144
Demand	93	82	50	75	294	60	102	55	43	60	

$\text{OS} = 3 \times 33 + 2 \times 82 + 1 \times 60 + 2 \times 55 + 1 \times 50 + 2 \times 15 + 5 \times 35 + 2 \times 210 + 1 \times 60 + 1 \times 7 + 1 \times 43 +$

$3 \times 60 + 6 \times 60 + 7 \times 42 + 2 \times 102 = 2256$

Example 6.3.3 Let's take TP matrix with three rows and three columns,

	1	2	3	Supply
1	3	2	5	30
2	11	8	1	125
3	8	1	2	70
Demand	93	82	50	

So

	1	2	3	Supply
1	3(30)	2	5	30
2	11(63)	8(12)	1(50)	125
3	8	1(70)	2	70
Demand	93	82	50	

$$\text{OS} = 3 \times 30 + 11 \times 63 + 8 \times 12 + 1 \times 50 + 1 \times 70 = 999$$

Example 6.3.4 Let's take TP matrix with six rows and six columns,

	1	2	3	4	5	6	Supply
1	11	13	17	14	8	4	250
2	16	8	3	10	1	11	300
3	1	4	13	6	3	1	400
4	8	6	12	8	2	11	250
5	1	3	2	7	18	1	40
6	20	2	1	8	5	3	60
Demand	200	225	275	250	125	225	

So

	1	2	3	4	5	6	Supply
1	11	13	17	14(25)	8	4(225)	250
2	16	8	3(275)	10	1(25)	11	300
3	1(200)	4(125)	13	6(75)	3	1	400
4	8	6	12	8(150)	2(100)	11	250
5	1	3(40)	2	7	18	1	40
6	20	2(60)	1	8	5	3	60
Demand	200	225	275	250	125	225	

$OS = 14 \times 25 + 4 \times 225 + 3 \times 275 + 1 \times 25 + 1 \times 200 + 4 \times 125 + 6 \times 75 + 8 \times 150 + 2 \times 100 + 3 \times 40 + 2 \times 60 = 4890$

The study applied the new method to solve a lot of examples, in all of them, the solutions that obtained by our new method is equal to the solutions that obtained by the classical methods, (SS)and(MODI), moreover, the number of the index to reach OS by using DAM is less than the number of the index in the other two classical method. Using this method in most cases, the number of the index is equal to 1 in each iteration. The following table contains the comparison among DAM, (SS) method, and (MODI) method in terms of OS and the number of indexes.

Table 6.3: Comparison of OS and Number of Index

No.Exam	(SS) Method		(MODI) Method		DAM Method	
	OS	NO.Index	OS	NO.Index	OS	NO.Index
6.3.1	11950	6	11950	6	11950	1
6.3.2	2256	45	2256	45	2256	1
6.3.3	999	4	999	4	999	1
6.3.4	4890	25	4890	25	4890	1

Chapter 7

Conclusion and Future Works

7.1 Conclusion

This study proposed a new method called largest difference method (LDM) to find (IBFS) for (TP). The new method is characterized by simplicity, ease and no need for complex calculations compared to the classical method used for solving TP. Also in this work, we proposed three new methods for finding OS to, which we named (matrix method, YM method and DAM method). The new methods are characterized by their simplicity, ease of use and no need for complex calculations compared to (SS) and (MODI) methods used to solve TP. By comparing with the two methods (SS) and (MODI), we found that OS obtained by the YM method and the Matrix method is equal to OS obtained by the (SS) and (MODI) method, in addition the index of the new methods is less than the index number needed by the other two methods, which indicates the efficiency and quality of our new method. This study also showed that there are special cases in TP where IBFS is equal to OS that can be benefited from and developed to be an ideal model for decision makers to benefit from.

7.2 Future Works

It is expected to obtain several works inspired by this study in the future, including:

1) Developing the method of the largest difference to be a direct way to obtain the optimal solution.
2) Developing the matrix method to include all size to TP.
3) Forming a general model for TP in which the solution is in an optimal way to be used in practice.

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المستخلص

في هذا العمل، تم اقتراح ثلاث طرق جديدة لإيجاد واختبار الحل الأمثل لمسائل النقل (طريقة المصفوفة التي تعتمد على ضرب المصفوفة مع مدورها وطريقة YM وطريقة مطلق الفرق التي تعتمد على نظرية البيانات). السمة المميزة لهذه التقنيات هي سهولة الحسابات واكثر منطقية حتى يتمكن الشخص من فهمها والاستفادة منها وبالأخص لأصحاب اتخاذ قرار. كما قدم هذا العمل طريقة جديدة لإيجاد الحل الأساسي طريقة الفرق الاكبر التي يمكن تطويرها لتصبح طريقة مباشرة لإيجاد الحل الأمثل ليمثل نهج جديد في حل مسائل النقل. أثبتنا أيضًا أن هناك حالات خاصة لا تحتاج إلى طرق لحلها ولكن الحل هو أمثل دائماً بأي طريقة حتى لو كان المليء عشوائي وبيناه بنظريات وأمثلة تبين تلك الحالة الخاصة.

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قسم الرياضيات

تقنيات جديدة لإيجاد الحل المقبول و الأمثل لمسائل النقل

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كجزء من متطلبات نيل درجة الدكتوراه فسلفة في التربية / الرياضيات

من قبل

يعقوب علي حسين نجم

بإشراف

أ . د. مشتاق عبد الغني شخير الجنابي

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