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Nonlinear Analysis of Reinforced Concrete Folded Plates by Finite Strip Method

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(STRUCTURAL ENGINEERING)**

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

وَالْبَاقِيَاتُ الصَّالِحَاتُ خَيْرٌ

عِنْدَ رَبِّكَ ثَوَابًا وَخَيْرٌ أَمَلًا

صدق الله العلي العظيم

سورة الكهف الآية (46)

الخلاصة

في الدراسة الحالية، استخدمت الحلول العددية المعتمدة على طريقة الشرائح المحددة ثلاثية الأبعاد لتحري التصرف الإنشائي و سعة تحمل الألواح المطوية الخرسانية المسلحة المنتظمة المقطع ذات المساند البسيطة.

في هذا البحث، استخدمت طريقة الشرائح المحددة غير الخطية ثلاثية الأبعاد (المواشير المحددة). لتمثيل الكونكرت تم استخدام ثلاثة أنواع من العناصر الموشورية هي المواشير ذات الأربع والثمانية والاثنا عشر عقدة. بينما حديد التسليح تم تمثيله كعناصر محورية مضمورة داخل الموشور مع فرض وجود ترابط تام بين الخرسانة وحديد التسليح. أعتبر تصرف الخرسانة تحت تأثير اجهادات الانضغاط تصرفاً مرناً-لدناً يتبعه تصرف لدن تام ينتهي عند تهشم الخرسانة. أما بالنسبة للمنطقة الواقعة تحت تأثير اجهادات الشد فقد تم استخدام أنموذج الشق المنتشر (Smeared Crack Model) لتمثيل تصرف الخرسانة مع اعتماد أنموذج تصلب الشد (Tension-Stiffening Model) لتمثيل الاجهادات المتبقية بعد مرحلة التشقق. إن تدهور مقاومة القص للخرسانة نتيجة التشقق تم أخذه بعين الاعتبار من خلال تبني نموذج استبقاء القص (Shear-Retention Model).

تم حل معادلات التوازن اللاخطية باستخدام طريقة تزايدية-تكرارية (Incremental-Iterative Technique) اعتماداً على طريقة نيوتن-رافسون. ثلاثة أنواع من التكاملات العددية (20 , 15a , 8) تم استخدامها لحل التكامل العددي لمصفوفة الجساسة .

اللاخطية الهندسية (Geometrical Nonlinearity) تم حلها بالاعتماد على أسلوب لاجرانج (Lagrange) الكلي مع الأخذ بعين الاعتبار فرضيات (Von Karman). تم تمثيل الأحمال المطبقة بمجموعة من الأحمال المكافئة من خلال اشتقاق مصفوفات الأحمال. تم تطوير برنامج حاسبة للطريقة السابقة.

تم أخذ مسائل توضيحية لمنشآت مختلفة تملك أشكالاً مختلفة ومواد مختلفة وتعرض لحالات تحميل مختلفة. بصورة عامة توافق جيد بين طريقة الشرائح المحددة والنتائج العملية. كما تضمنت الدراسة تأثير بعض من العوامل المهمة في تصرف منشآت الألواح المطوية الخرسانية. هذه العوامل تتضمن: تغير نسبة الفضاء إلى العرض، تغير نسبة الفضاء إلى السمك وتأثير أنواع قواعد التكامل. وجد إن الحمل الأقصى يزداد إلى (36.2%) من قيمته عندما تزداد زاوية الميلان للسقوف الجانبية من (30°) إلى (37.96°).

Abstract

In the present study, a numerical analysis technique based on the nonlinear three-dimensional finite strip method has been used to predicted the overall structural behavior and load carrying capacity of prismatic reinforced concrete folded plate structures.

The three dimensional nonlinear finite strip (or finite prism) model have been used in the present research. Three types of prism element (4-nodal line linear, 8-nodal line quadrilateral, and 12-nodal line cubic prisms) are used to simulate the concrete while the reinforcing bars are model as axial members embedded within the prism. Perfect bond between concrete and reinforcing bars was assumed to occur. The elasto-plastic strain work hardening model followed by perfectly plastic response is used to model the compressive behavior of concrete. This response is terminated at the beginning of crushing. In tension, linear elastic behavior prior to cracks is assumed. Smeared crack models with fixed orthogonal cracks are used to monitor the cracking, with a tension-stiffening model to represent the retained post-cracking tensile stresses. The degradation of the shear strength of concrete due to cracking is accounted by employing a shear-retention model.

The nonlinear equations of equilibrium have been solved using an incremental-iterative technique operating under load control. Modified Newton-Raphson method has been employed. Various integration rules (8, 15 and 20 points) have been used to carry out the numerical integration of the stiffness matrix.

A nonlinear geometrical model based on the total Lagrangian approach and taking into account von Karman assumptions is considered. The applied loads are simulated as equivalent nodal line forces by

deriving the force matrices. A computer program has been developed for this foregoing method.

Several problems are tested using the present analysis method. In general, good agreement is obtained between the finite strip method and the experimental results by others with difference not more than 15.7%. Among the conclusions drawn, the ultimate load of folded plates is increased by about 36.2% when the inclination angle of side plates increased from 30° to 37.96° .

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Words of love are due to my parents, brothers and sisters for their care, patience and encouragement.

Furthermore, I would like to thank all my friends and colleagues.

AKRM 2008

Certificate of the Examining Committee

We certify that we have read this thesis titled "**Nonlinear Analysis of Reinforced Concrete Folded Plates by Finite Strip Method**" and as an examining committee, examined the student (**Akram Rashed Hassan**) in its contents and in what is connected with it, and that in our opinion it is adequate as a thesis for degree of **Master of Science in Civil Engineering (Structural Engineering)**.

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List of Symbols

a	Length of prism
$\{a\}^p$	Nodal displacements
$[B]$	Strain – displacement matrix
u, v, w	Displacement components in x, y and z – direction respectively
N_i	Shape function at the ith nodal line
u_{im}, v_{im}, w_{im}	Nodal line displacement parameters at ith nodal line for mth series terms
x, y, z	Global or cartesian coordinates
x_i, y_i, z_i	Global coordinates of ith nodal line
$[J]$	Jacobian matrix
n_e	Number of nodal line for prism element
f_t	Maximum tensile strength of concrete
IPLQ	Isoparametric polynomial linear quadrilateral
IPQQ	Isoparametric polynomial quadrilateral quadrilateral
IPCQ	Isoparametric polynomial cubic quadrilateral
$[D]$	Constitutive matrix
W_{int}	Internal work
W_{ext}	External work
I_1	First stress invariant
J_2	Second deviatoric stress invariant
C_p	Plasticity coefficient
f'_c	Ultimate compressive strength of concrete
E_1	Reduced modulus of elasticity
G	Shear modulus of elasticity
l, m, n	Directional cosines in the x, y, z directions respectively
T	Surface traction
V	Volume
S	Part of the surface of the body where external traction are prescribed
$\{u\}^p$	Displacement vector at any point within the prism
$[A]$	Matrix which contains the differential operators
$[N]$	Matrix containing interpolation functions
$[k]$	Stiffness matrix
$\{f\}$	prism assemblage external nodal force vector
F_u	Uniformly distributed load
F_p	Point load

$[S]$	Distortion matrix in the global coordinates
$[R]$	Rotation matrix
$[\bar{S}]$	Distortion matrix in the local coordinates
$[k']$	Stiffness matrix of the bar element
W_i	Weight of the sampling point
ε	Strain
$\{\sigma\}$	Stress vector
σ	Stress
α, β	Material parameters
σ_o	Equivalent effective stress at the onset of plastic deformation
$\{\varepsilon_p\}$	Effective accumulated plastic strain vector
$\bar{\sigma}$	Equivalent uniaxial stress
ε'_o	Total strain corresponding to the parabolic part of the curve
ε_c	Total effective strain of concrete
ε_e	Elastic component of total effective strain of concrete
ε_p	Plastic component of total effective strain of concrete
H'	Hardening parameter
$\bar{I}_1$	First strain invariant
$\bar{J}_1$	Second deviatoric strain
ε_{cu}	Ultimate concrete compression strain
σ_{cr}	Cracking stress of concrete
β_1	Reduction factor
D_{cr}	Cracked material stiffness in the local axis
ε_{cr}	Cracking strain
σ_n	Stress normal to the cracked plane
ε_n	Strain normal to the cracked plane
α_1, α_2	Tension stiffening parameters
$\gamma_1, \gamma_2, \gamma_3$	Shear Retention Parameters

Appendix A

In this Appendix, the modified and additional subroutines that are used to modify (Al-shaarbaf 1990) computer program in order to construct NFSRCP computer program for nonlinear finite strip analysis of reinforced concrete plates are presented.

1) Subroutine STIFFK to calculate the stiffness matrix of concrete and reinforcement, also takes into account the geometric nonlinearity.

```
SUBROUTINE STIFFK(DOFNOD,NGAUS,TOTELS,
* DIMEN,IINCS,NUMSS,ITER,KITER,
* NCRIT,NTEMP,WGHT,NG,MTAP1,MTAP2,MTAP3,MTAP4,
* HBAND,TOTDOF,NQP,NBAR,WGHTR,NQPR)
IMPLICIT REAL*8 (A-H,O-Z)
INTEGER DIM1,DIM2,DIM3,DIM4,DIM5,DOFNOD,DOFEL,TOTELS,
* DIMEN,HBAND,ELTOP,TOTDOF,STEER,ELTOPB,DIM6,DIM7,DIM8,
* DIM9
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C CCCCC PSMLTH====PRISM LENGTH CCCCC
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
PARAMETER (PSMLTH=864.0)
PARAMETER (DIM1=504,DIM2=60,DIM3=27,DIM4=188,DIM5=1000,
* DIM6=600, DIM7=20,DIM8=224,DIM9=500)
REAL *8 JAC,JACIN,INLOAD,LOADS
DIMENSION ABSS(3,DIM3),B(6,DIM2),BT(DIM2,6),ABSSR(5),
* DB(6,DIM2),DER(3,20),WGHTR(5),GL(9,DIM2),EGK(DIM2,DIM2),
* ELK(DIM2,DIM2),FUN(20),PD(6,6),BR(DIM2), GDM(9,DIM2),
* STEER(DIM2),DC(6,6),T(6,6),COSST(6,6),COSS(6,6),
* GEOM(20,3),JAC(3,3), JACIN(3,3),BTDB(DIM2,DIM2),
* STRS(6),WGHT(DIM3),DEVIA(6),AVECT(6),DVECT(6),
* DERIV(3,20),BL(6,DIM2),AL(6,9),SAMM(9,9)
COMMON SYSK(DIM1,DIM9),TDISP(DIM1), INLOAD(DIM1),
* COORD(DIM4,3),D(6,6),CURY(DIM6),
* LOADS(DIM1),FTFT(6),FCFC(6),
* DDC(DIM6,6,6),DCOSN(DIM6,3,3),DAMM(DIM6,2),
* RLNTH(DIM8,3),STRNG(6,DIM5),HH(6),DDN(DIM6,3,3),
* FYFY(6),EE(6),AA(6), POS(6),H1H1(DIM8,3),
* STRSG(6,DIM5),PLOAD(DIM1),EFFST(DIM5),ECRK(3,DIM6),
* DDET(DIM7,DIM3),
* EPSTN(DIM5),
* DCOSS(DIM6,6,6),
```

Appendix A

```
* TRSTRN(6,DIM6),TPSTRN(3,DIM6),
* ABSLNC(3,DIM6),SCR(3,DIM6),
* TRSTRS(6,DIM6),TPSTRS(3,DIM6),
* INDCT(3,DIM5),ELTOP(DIM7,22),
* MPROP(DIM7+DIM8),ELTOPB(DIM8,2),NOUTP(2),NF(DIM4,3)
DATA IB /6/, IBT /DIM2/, IBTDB /DIM2/,ISTEER/DIM2/,
* ID /6/, IDB /6/, IDER /3/, IDERIV /3/,IABSS/3/,
* IELK /DIM2/, IELTOP /DIM7/, IFUN/20/, IGEOM/20/,
* IJAC /3/, IJACIN /3/,INF/DIM4/,JNF/3/,IWGHT/27/,
* JB/DIM2/,JBT /6/,JDER/20/,ISYSK/DIM1/,JSYSK/DIM9/,
* JBTDB /DIM2/, JCOORD /3/, JD /6/, JDB /DIM2/,
* JDERIV/20/, JELK /DIM2/, JELTOP /22/, JGEOM /3/,
* JJAC /3/, JJACIN /3/,ICOORD/DIM4/,JABSS/27/,
* IGL/9/,JGL/DIM2/,ILOADS/DIM1/,IBL/6/,JBL/DIM2/,
* ISTRSG/6/,JSTRSG/DIM5/IEGK/DIM2/,JEGK/DIM2/,
* IGDM/9/,JGDM/DIM2/
ITEST=0
REWIND(NTEMP)
REWIND (MTAP1)
REWIND (MTAP2)
REWIND (MTAP3)
REWIND (MTAP4)
CALL MATNUL(SYSK, ISYSK, JSYSK, TOTDOF, HBAND, ITEST)
NODEL=20
IF(NGAUS.EQ.1)THEN
CALL QUD27(WGHT, ABSS, NQP)
ELSEIF (NGAUS.GT.1.AND.NGAUS.LT.5)THEN
CALL QUD15(WGHT, ABSS, NQP,NGAUS)
ELSEIF(NGAUS.EQ.5)THEN
CALL QBRK8(WGHT,ABSS,NQP)
ENDIF
CALL QLIN2(WGHTR,5,ABSSR,5,NQPR)
C
C      LOOP OVER EACH ELEMENT
C
KGAUS=0
KGAUSC=0
DO 1140 NELE=1,TOTELS
IMAT=MPROP(NELE)
FY=FYFY(IMAT)
H=HH(IMAT)
NU=POS(IMAT)
E=EE(IMAT)
FT=FTFT(IMAT)
FC=FCFC(IMAT)
G=E/(2.0+2.0*NU)
```

Appendix A

```
ST=FT/E
DOFEL = NODEL * DOFNOD
CALL ELGEOM(NELE, ELTOP, IELTOP, JELTOP, COORD, ICOORD,
*   JCOORD, GEOM, IGEOM, JGEOM, DIMEN, ITEST)
CALL MATNUL(ELK, IELK, JELK, DOFEL, DOFEL, ITEST)
C
C   CONCRETE ELEMENT STIFFNESS MATRIX
C
DO 1150 IQUAD=1,NQP
KGAUS=KGAUS+1
KGAUSC=KGAUSC+1
IF(KITER.EQ.2)THEN
XI = ABSS(1,IQUAD)
ETA = ABSS(2,IQUAD)
ZETA= ABSS(3,IQUAD)
IF(ELTOP(NELE,1).EQ.5)THEN
CALL PRSM8 (FUN, IFUN, DER, IDER, JDER, XI, ETA)
ELSE IF(ELTOP(NELE,1).EQ.6)THEN
CALL PRSM12 (FUN, IFUN, DER, IDER, JDER, XI, ETA)
ELSE IF(ELTOP(NELE,1).EQ.7)THEN
CALL PRSM4 (FUN, IFUN, DER, IDER, JDER, XI, ETA)
ENDIF
CALL MATMUL(DER, IDER, JDER, GEOM, IGEOM, JGEOM, JAC, IJAC,
*   JJAC, DIMEN, NODEL, DIMEN, ITEST)
CALL AKRM1(JAC, IJAC, JJAC, JACIN, IJACIN, JJACIN, DIMEN,
*   DET)
DDET(NELE, IQUAD) = DET*0.5*PSMLTH
CALL MATMUL(JACIN, IJACIN, JJACIN, DER, IDER, JDER, DERIV,
*   IDERIV, JDERIV, DIMEN, DIMEN, NODEL, ITEST)
WRITE (MTAP1) ((DERIV(I,J),I=1,3),J=1,20)

C           FORMATION OF STRAIN-DISPLACEMENT
C           MATRIX B AND OUTPUT TO WORK FILE FOR
C           LATER RECOVERY PROCESS

IKK=IKK+1
CALL BGMAT(B,IB,JB,GL,IGL,JGL,DERIV,IDERIV,JDERIV,
*ZETA,FUN,IFUN, NODEL, PSMLTH,IKK)
WRITE (MTAP3) ((B(I,J),I=1,NUMSS),J=1,DOFEL)
WRITE (MTAP2) ((GL(I,J),I=1,9),J=1,DOFEL)
ELSE
READ (MTAP3) ((B(I,J),I=1,NUMSS),J=1,DOFEL)
READ (MTAP2) ((GL(I,J),I=1,9),J=1,DOFEL)
READ (MTAP1) ((DERIV(I,J),I=1,3),J=1,20)
ENDIF
IF (NG.EQ.1.AND.KITER.GT.2)THEN
```

Appendix A

```
IHH=IHH+1
CALL BGN(B,IB,JB,BL,IBL,JBL,GL,IGL,JGL,DERIV,IDERIV,
*JDERIV,TDISP,ILOADS,NODEL,NELE,DOFNOD,DOFEL,ELTOP,
*ZETA,FUN,IFUN,
*IELTOP,JELTOP,STEER,ISTEER,NF,INF,JNF,PSMLTH)
ENDIF
IF(IINCS.NE.1)THEN
IF(INDCT(1,KGAUS).GT.1.OR.INDCT(2,KGAUS).GT.1.OR.
* INDCT(3,KGAUS).GT.1)THEN
DO 2020 I=1,NUMSS
DO 2020 J=1,NUMSS
COSS(I,J)=DCOSS(KGAUSC,I,J)
2020 DC(I,J)=DDC(KGAUSC,I,J)
CALL MATMUL(COSS,ID,JD,DC,ID,JD,T,ID,JD,NUMSS,NUMSS,NUMSS,
* ITEST)
CALL MATRAN(COSS,ID,JD,COSST,ID,JD,NUMSS,NUMSS, ITEST)
CALL MATMUL(T,ID,JD,COSST,ID,JD,DC,ID,JD,NUMSS,NUMSS,NUMSS,
* ITEST)
CALL MATMUL(DC, ID, JD, B, IB, JB, DB, IDB, JDB, NUMSS, NUMSS,
* DOFEL, ITEST)
ELSEIF(INDCT(1,KGAUS).EQ.-4)THEN
DO 1170 I=1,NUMSS
1170 STRS(I)=STRSG(I,KGAUS)
CALL INVAR(NCRIT,AVECT,STRS,
* YIELD,NUMSS)
CALL FLOWPL(AVECT,DVECT,D,NUMSS,ID,JD,ITEST,
* ABETA,EPSTN,KGAUS,E,FC)
DO 1180 I=1,ID
DO 1180 J=1,JD
1180 PD(I,J)=D(I,J)-ABETA*DVECT(I)*DVECT(J)
CALL MATMUL(PD, ID, JD, B, IB, JB, DB, IDB, JDB, NUMSS, NUMSS,
* DOFEL, ITEST)
ELSE
CALL MATMUL(D, ID, JD, B, IB, JB, DB, IDB, JDB, NUMSS, NUMSS,
* DOFEL, ITEST)
ENDIF
ELSE
CALL MATMUL(D, ID, JD, B, IB, JB, DB, IDB, JDB, NUMSS, NUMSS,
* DOFEL, ITEST)
ENDIF
CALL MATRAN(B, IB, JB, BT, IBT, JBT, NUMSS, DOFEL, ITEST)
CALL MATMUL(BT, IBT, JBT, DB, IDB, JDB, BTDB, IBTDB, JBTDB,
* DOFEL, NUMSS, DOFEL, ITEST)
QUOT = ABS(DDET(NELE,IQUAD))*WGHT(IQUAD)
DO 1190 I=1,DOFEL
DO 1190 J=1,DOFEL
```

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```
1190 BTDB(I,J) = BTDB(I,J)*QUOT
      CALL MATADD(ELK,IELK,JELK,BTDB,IBTDB,JBTDDB,DOFEL,DOFEL,ITEST)
      IF (NG.EQ.0.OR.KITER.EQ.2) GO TO 1150
      CALL KGEOM (ELK,IELK,JELK,GL,IGL,JGL,STRSG,ISTRSG,JSTRSG,
        *DOFEL,QUOT,KGAUS,EGK,IEGK,JEGK,GDM,IGDM,JGDM)
1150 CONTINUE
C
C      REINFORCEMENT ELEMENT STIFFNESS MATRIX
C
      DO 1910 IBAR=1,NBAR
      IF(ELTOPB(IBAR,1).EQ.NELE)THEN
      DO 1920 IQUAD=1,NQPR
      KGAUS=KGAUS+1
      IF(IINCS.EQ.1.AND.IITER.EQ.1)THEN
      IF(ELTOPB(IBAR,2).EQ.1)THEN
      XI=ABSSR(IQUAD)
      ETA=RLNTH(IBAR,2)
      ZETA=RLNTH(IBAR,3)
      ELSEIF(ELTOPB(IBAR,2).EQ.2)THEN
      XI=RLNTH(IBAR,1)
      ETA=ABSSR(IQUAD)
      ZETA=RLNTH(IBAR,3)
      ELSE
      XI=RLNTH(IBAR,1)
      ETA=RLNTH(IBAR,2)
      ZETA=ABSSR(IQUAD)
      ENDIF
      IF(ELTOP(NELE,1).EQ.5)THEN
      CALL PRSM8 (FUN, IFUN, DER, IDER, JDER, XI, ETA)
      ELSE IF(ELTOP(NELE,1).EQ.6)THEN
      CALL PRSM12 (FUN, IFUN, DER, IDER, JDER, XI, ETA)
      ELSE IF(ELTOP(NELE,1).EQ.7)THEN
      CALL PRSM4 (FUN, IFUN, DER, IDER, JDER, XI, ETA)
      ENDIF
      CALL MATMUL(DER, IDER, JDER, GEOM, IGEOM, JGEOM, JAC, IJAC,
        * JJAC, DIMEN, NODEL, DIMEN, ITEST)
      CALL AKRM1(JAC, IJAC, JJAC, JACIN, IJACIN, JJACIN, DIMEN,
        * DET)
      CALL MATMUL(JACIN, IJACIN, JJACIN, DER, IDER, JDER, DERIV,
        * IDERIV, JDERIV, DIMEN, DIMEN, NODEL, ITEST)
      IF (ELTOPB(IBAR,2).EQ.3)THEN
      H1=0.5*PSMLTH
      ELSEIF (ELTOPB(IBAR,2).EQ.1)THEN
      C4=JAC(1,1)*JAC(1,1)
      C5=JAC(1,1)*JAC(1,2)
      C6=JAC(1,2)*JAC(1,2)
```

Appendix A

```
H1=SQRT(C4+C6)
ENDIF
IF(H1.EQ.0.0)H1=0.000001
H1H1(IBAR,IQUAD)=H1
A1=1+ZETA
UM=(3.1416/PSMLTH)
AAA=SIN(0.5*3.1416*A1)
DO 1940 J=1,NODEL
L=3*(J-1)+1
M=L+1
N=L+2
IF (ELTOPB(IBAR,2).EQ.3)THEN
BR(L)=0.0
BR(M)=0.0
BR(N)=-FUN(J)*UM*AAA
ELSEIF (ELTOPB(IBAR,2).EQ.1)THEN
BR(L)=((DERIV(1,J)*AAA*C4)+(DERIV(2,J)*AAA*C5))/(H1*H1)
BR(M)=((DERIV(1,J)*AAA*C5)+(DERIV(2,J)*AAA*C6))/(H1*H1)
BR(N)=0.0
ENDIF
1940 CONTINUE
WRITE(MTAP4)(BR(J),J=1,DOFEL)
ELSE
READ(MTAP4)(BR(J),J=1,DOFEL)
ENDIF
DO 1990 I=1,DOFEL
IF(BR(I).NE.0.0)THEN
IF(ABS(BR(I)).LT.0.1D-38)THEN
IF(BR(I).LT. 0.0 )BR(I)=-0.1D-38
IF(BR(I).GT. 0.0 )BR(I)= 0.1D-38
ENDIF
ENDIF
DO 1990 J=1,DOFEL
IF(BR(J).NE.0.0)THEN
IF(ABS(BR(J)).LT.0.1D-38)THEN
IF(BR(J).LT. 0.0 )BR(J)=-0.1D-35
IF(BR(J).GT. 0.0 )BR(J)= 0.1D-35
ENDIF
ENDIF
BTDB(I,J)=BR(I)*BR(J)
1990 CONTINUE
IMAT=MPROP(IBAR+TOTELS)
E=EE(IMAT)
H=HH(IMAT)
A=AA(IMAT)
FMULT=1.0-E/(E+H)
```

Appendix A

```
IF(IINCS.EQ.1)FMULT=1.0
IF(INDCT(1,KGAUS).EQ.0.OR.INDCT(1,KGAUS).EQ.5)FMULT=1.0
E=E*FMULT
QUOT=A*E*WGHTR(IQUAD)*H1
DO 1950 I=1,DOFEL
DO 1950 J=1,DOFEL
1950 BTDB(I,J)=BTDB(I,J)*QUOT
CALL MATADD(ELK,IELK,JELK,BTDB,IBTDB,JBTDDB,DOFEL,DOFEL,ITEST)
1920 CONTINUE
ENDIF
1910 CONTINUE
C          ASSEMBLY OF SYSTEM STIFFNESS MATRIX
C          SYSK
C
CALL DIRECT(NELE, ELTOP, IELTOP, JELTOP, NF, INF, JNF,
* DOFNOD, STEER, ISTEER, ITEST)
CALL ASSYM(SYSK, ISYSK, JSYSK, ELK, IELK, JELK, STEER,
* ISTEER, HBAND, DOFEL, ITEST)
1140 CONTINUE
RETURN
END
```

2) Subroutine PRSM4 to calculate the shape function and their derivative with respect to natural coordinates for the 4-nodal line linear prism.

```
SUBROUTINE PRSM4(FUN,IFUN,DER,IDER,JDER,X,Y)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION FUN(IFUN),DER(IDER,JDER)
C
FUN(1)=0.25*(1-X)*(1-Y)
DER(1,1)=-0.25*(1-Y)
DER(2,1)=-0.25*(1-X)
DER(3,1)=0.0
C
FUN(2)=0.25*(1-X)*(1+Y)
DER(1,2)=-0.25*(1+Y)
DER(2,2)=0.25*(1-X)
DER(3,2)=0.0
C
FUN(3)=0.25*(1+X)*(1+Y)
DER(1,3)=0.25*(1+Y)
DER(2,3)=0.25*(1+X)
DER(3,3)=0.0
C
FUN(4)=0.25*(1+X)*(1-Y)
```

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```
DER(1,4)=0.25*(1-Y)
DER(2,4)=-0.25*(1+X)
DER(3,4)=0.0
RETURN
END
```

3) Subroutine PRSM8 to calculate the shape function and their derivative with respect to natural coordinates for the 8-nodal line quadrilateral prism.

```
SUBROUTINE PRSM8(FUN,IFUN,DER,IDER,JDER,X,Y)
  IMPLICIT REAL*8 (A-H,O-Z)
  DIMENSION FUN(IFUN),DER(IDER,JDER)
C
  FUN(1)=-0.25*(1-X)*(1-Y)*(X+Y+1)
  DER(1,1)=-0.25*(1-Y)*((1-X)-(X+Y+1))
  DER(2,1)=-0.25*(1-X)*((1-Y)-(X+Y+1))
  DER(3,1)=0.0
C
  FUN(2)=0.5*(1-X)*(1-(Y*Y))
  DER(1,2)=-0.5*(1-(Y*Y))
  DER(2,2)=-(1-X)*Y
  DER(3,2)=0.0
C
  FUN(3)=-0.25*(1-X)*(1+Y)*(X-Y+1)
  DER(1,3)=-0.25*(1+Y)*((1-X)-(X-Y+1))
  DER(2,3)=-0.25*(1-X)*((-1-Y)+(X-Y+1))
  DER(3,3)=0.0
C
  FUN(4)=0.5*(1-(X*X))*(1+Y)
  DER(1,4)=-X*(1+Y)
  DER(2,4)=0.5*(1-(X*X))
  DER(3,4)=0.0
C
  FUN(5)=0.25*(1+X)*(1+Y)*(X+Y-1)
  DER(1,5)=0.25*(1+Y)*((1+X)+(X+Y-1))
  DER(2,5)=0.25*(1+X)*((1+Y)+(X+Y-1))
  DER(3,5)=0.0
C
  FUN(6)=0.5*(1+X)*(1-(Y*Y))
  DER(1,6)=0.5*(1-(Y*Y))
  DER(2,6)=-(1+X)*Y
  DER(3,6)=0.0
C
  FUN(7)=0.25*(1+X)*(1-Y)*(X-Y-1)
  DER(1,7)=0.25*(1-Y)*((1+X)+(X-Y-1))
```

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```
DER(2,7)=0.25*(1+X)*((Y-1)-(X-Y-1))
DER(3,7)=0.0
```

C

```
FUN(8)=0.5*(1-(X*X))*(1-Y)
DER(1,8)=-X*(1-Y)
DER(2,8)=-0.5*(1-(X*X))
DER(3,8)=0.0
RETURN
END
```

4) Subroutine PRSM12 to calculate the shape function and their derivative with respect to natural coordinates for the 12-nodal line cubic prism.

```
SUBROUTINE PRSM12(FUN,IFUN,DER,IDER,JDER,X,Y)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION FUN(IFUN),DER(IDER,JDER)
```

C

```
A=1-X
B=1-Y
C=(-10+(9*((X*X)+(Y*Y))))
D=1-(Y*Y)
E=1-(3*Y)
EE=1+(3*Y)
BB=1+Y
F=1-(X*X)
S=1-(3*X)
SS=1+(3*X)
AA=1+X
```

C

```
FUN(1)=0.03125*A*B*C
DER(1,1)=0.03125*B*((18*X*A)-C)
DER(2,1)=0.03125*A*((18*Y*B)-C)
DER(3,1)=0.0
```

C

```
FUN(2)=0.28125*A*D*E
DER(1,2)=-0.28125*D*E
DER(2,2)=0.28125*A*((-3*D)-(2*Y*E))
DER(3,2)=0.0
```

C

```
FUN(3)=0.28125*A*D*EE
DER(1,3)=-0.28125*D*EE
DER(2,3)=0.28125*A*((-2*Y*EE)+(3*D))
DER(3,3)=0.0
```

C

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FUN(4)=0.03125*A*BB*C
DER(1,4)=0.03125*BB*((18*X*A)-C)
DER(2,4)=0.03125*A*((18*Y*BB)+C)
DER(3,4)=0.0

C

FUN(5)=0.28125*F*BB*S
DER(1,5)=0.28125*BB*((-3*F)-(2*X*S))
DER(2,5)=0.28125*F*S
DER(3,5)=0.0

C

FUN(6)=0.28125*F*BB*SS
DER(1,6)=0.28125*BB*((3*F)-(2*X*SS))
DER(2,6)=0.28125*F*SS
DER(3,6)=0.0

C

FUN(7)=0.03125*AA*BB*C
DER(1,7)=0.03125*BB*((18*X*AA)+C)
DER(2,7)=0.03125*AA*((18*Y*BB)+C)
DER(3,7)=0.0

C

FUN(8)=0.28125*AA*D*EE
DER(1,8)=0.28125*D*EE
DER(2,8)=0.28125*AA*((3*D)-(2*Y*EE))
DER(3,8)=0.0

C

FUN(9)=0.28125*AA*D*E
DER(1,9)=0.28125*D*E
DER(2,9)=0.28125*AA*((-3*D)-(2*Y*E))
DER(3,9)=0.0

C

FUN(10)=0.03125*AA*B*C
DER(1,10)=0.03125*B*((18*X*AA)+C)
DER(2,10)=0.03125*AA*((18*Y*B)-C)
DER(3,10)=0.0

C

FUN(11)=0.28125*F*B*SS
DER(1,11)=0.28125*B*((3*F)-(2*X*SS))
DER(2,11)=-0.28125*F*SS
DER(3,11)=0.0

C

FUN(12)=0.28125*F*B*S
DER(1,12)=0.28125*B*((-3*F)-(2*X*S))
DER(2,12)=-0.28125*F*S
DER(3,12)=0.0
RETURN
END

Appendix A

5) Subroutine AKRM1 to calculate the two dimensional Jacobian matrix.

```
SUBROUTINE AKRM1(JAC,IJAC,JJAC,JACIN,IJACIN,JJACIN,DIMEN,DET)
  IMPLICIT REAL *8 (A-H,O-Z)
  INTEGER DIMEN
  REAL *8 JAC,JACIN
  DIMENSION JAC(IJAC,JJAC),JACIN(IJACIN,JJACIN),COF(2,2)
  DET=((JAC(1,1)*JAC(2,2))-(JAC(1,2)*JAC(2,1)))
  COF(1,1)=JAC(2,2)
  COF(1,2)=-JAC(2,1)
  COF(2,1)=-JAC(1,2)
  COF(2,2)=JAC(1,1)
  IF (DET.EQ.0.0)THEN
    WRITE(*,*)'DET IS ZERO'
    STOP
  END IF
  DO 10 I=1,2
    DO 10 J=1,2
10  JACIN(I,J)=COF(J,I)/DET
  RETURN
  END
```

6) Subroutine BGMAT to calculate the linear part of the strain-displacement matrix $[B_o]$ for concrete and the matrix $[G]$ which is used for geometrical nonlinear analysis.

```
SUBROUTINE BGMAT(B,IB,JB,GL,IGL,JGL,DERIV,IDERIV,JDERIV,
  *ZETA,FUN,IFUN,NODEL,PSMLTH,IKK)
  IMPLICIT REAL *8 (A-H,O-Z)
  DIMENSION FUN(IFUN),B(IB,JB),DERIV(IDERIV,JDERIV),GL(IGL,JGL)
  WRITE (16,*)'*****SUB. BGMAT*****',IKK
  A1=1+ZETA
  UM=(3.1416/PSMLTH)
  AAA=SIN(0.5*3.1416*A1)
  BBB=COS(0.5*3.1416*A1)
  N=0
  DO 30 I=1,NODEL
    L=N+1
    M=L+1
    N=M+1
    B(1,L)=DERIV(1,I)*AAA
    B(1,M)=0.0
    B(1,N)=0.0
```

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```
B(2,L)=0.0
B(2,M)=DERIV(2,I)*AAA
B(2,N)=0.0
B(3,L)=0.0
B(3,M)=0.0
B(3,N)=FUN(I)*(-UM)*AAA
B(4,L)=DERIV(2,I)*AAA
B(4,M)=DERIV(1,I)*AAA
B(4,N)=0.0
B(5,L)=0.0
B(5,M)=FUN(I)*UM*BBB
B(5,N)=DERIV(2,I)*BBB
B(6,L)=FUN(I)*UM*BBB
B(6,M)=0.0
B(6,N)=DERIV(1,I)*BBB
GL(1,L)=0.0
GL(1,M)=0.0
GL(1,N)=0.0
GL(2,L)=0.0
GL(2,M)=0.0
GL(2,N)=0.0
GL(3,L)=0.0
GL(3,M)=0.0
GL(3,N)=0.0
GL(4,L)=0.0
GL(4,M)=DERIV(1,I)*AAA
GL(4,N)=0.0
GL(5,L)=0.0
GL(5,M)=0.0
GL(5,N)=0.0
GL(6,L)=0.0
GL(6,M)=FUN(I)*UM*BBB
GL(6,N)=0.0
GL(7,L)=0.0
GL(7,M)=0.0
GL(7,N)=0.0
GL(8,L)=0.0
GL(8,M)=0.0
GL(8,N)=0.0
GL(9,L)=0.0
GL(9,M)=0.0
GL(9,N)=0.0
30 CONTINUE
RETURN
END
```

Appendix A

7) Subroutine BGN to calculate the nonlinear strain-displacement matrix $[B_{nl}]$ for the concrete and being added to the linear part $[B_o]$.

```
SUBROUTINE BGN (B,IB,JB,BL,IBL,JBL,GL,IGL,JGL,DERIV,
*IDERIV,JDERIV,UZ,IUZ,NODEL,NELE,DOFNOD,DOFEL,ELTOP,
*ZETA,FUN,IFUN,
*IELTOP,JELTOP,STEER,ISTEER,NF,INF,JNF,PSMLTH)
IMPLICIT REAL *8 (A-H,O-Z)
INTEGER STEER,DOFEL,ELTOP,DOFNOD
DIMENSION B(IB,JB),BL(IBL,JBL),GL(IGL,JGL),UZ(IUZ),
*ELTOP(IELTOP,JELTOP),STEER(ISTEER),NF(INF,JNF),AL(6,9),
*ZZZ(60),DERIV(IDERIV,JDERIV),FUN(IFUN)
L=ELTOP(NELE,2)
JJ=0
DO 123 I=1,L
K=I+2
KK=ELTOP(NELE,K)
DO 123 J=1,DOFNOD
JJ=JJ+1
123 STEER(JJ)=NF(KK,J)
DO 11 I=1,DOFEL
JSTEER=STEER(I)
IF (JSTEER.NE.0)THEN
ZZZ(I)=UZ(JSTEER)
ELSE
ZZZ(I)=0.0
ENDIF
11 CONTINUE
DVDX=0.0
DVDZ=0.0
A1=1+ZETA
UM=(3.1416/PSMLTH)
AAA=SIN(0.5*3.1416*A1)
BBB=COS(0.5*3.1416*A1)
N=0
DO 207 I=1,NODEL
L=N+1
M=L+1
N=M+1
DVDX=DVDX+(DERIV(1,I)*AAA*ZZZ(M))
DVDZ=DVDZ+(FUN(I)*UM*BBB*ZZZ(M))
207 CONTINUE
AL(1,1)=0.0
AL(1,2)=0.0
AL(1,3)=0.0
AL(1,4)=DVDX
```

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AL(1,5)=0.0
AL(1,6)=0.0
AL(1,7)=0.0
AL(1,8)=0.0
AL(1,9)=0.0
AL(2,1)=0.0
AL(2,2)=0.0
AL(2,3)=0.0
AL(2,4)=0.0
AL(2,5)=0.0
AL(2,6)=0.0
AL(2,7)=0.0
AL(2,8)=0.0
AL(2,9)=0.0
AL(3,1)=0.0
AL(3,2)=0.0
AL(3,3)=0.0
AL(3,4)=0.0
AL(3,5)=0.0
AL(3,6)=DVDZ
AL(3,7)=0.0
AL(3,8)=0.0
AL(3,9)=0.0
AL(4,1)=0.0
AL(4,2)=0.0
AL(4,3)=0.0
AL(4,4)=0.0
AL(4,5)=DVDX
AL(4,6)=0.0
AL(4,7)=0.0
AL(4,8)=0.0
AL(4,9)=0.0
AL(5,1)=0.0
AL(5,2)=0.0
AL(5,3)=0.0
AL(5,4)=0.0
AL(5,5)=DVDZ
AL(5,6)=0.0
AL(5,7)=0.0
AL(5,8)=0.0
AL(5,9)=0.0
AL(6,1)=0.0
AL(6,2)=0.0
AL(6,3)=0.0
AL(6,4)=DVDZ
AL(6,5)=0.0

Appendix A

```
AL(6,6)=DVDX
AL(6,7)=0.0
AL(6,8)=0.0
AL(6,9)=0.0
DO 208 I=1,DOFEL
DO 208 J=1,6
BL (J,I)=0.0
DO 208 K=1,9
208 BL(J,I)=BL(J,I)+AL(J,K)*GL(K,I)
DO 209 I=1,DOFEL
DO 209 J=1,6
209 B(J,I)=B(J,I)+BL(J,I)
RETURN
END
```

8) Subroutine KGEOM to calculate the geometric stiffness matrix $[K_G]$ for the concrete and being added to the main concrete stiffness matrix $[K]$.

```
SUBROUTINE KGEOM (ELK,IELK,JELK,GL,IGL,JGL,STRSG,ISTRSG,
  *JSTRSG,DOFEL,QUOT,KGAUS,EGK,IEGK,JEGK,GDM,IGDM,JGDM)
  IMPLICIT REAL *8 (A-H,O-Z)
  INTEGER DOFEL
  DIMENSION ELK(IELK,JELK),GL(IGL,JGL),STRSG(ISTRSG,JSTRSG),
  *EGK(IEGK,JEGK),GDM(IGDM,JGDM),SAMM(9,9)
  DO 5 I=1,DOFEL
  DO 5 J=1,DOFEL
5 EGK(I,J)=0.0
  DO 6 I=1,9
  DO 6 J=1,9
6 SAMM(I,J)=0.0
  SAMM(1,1)=STRSG(1,KGAUS)
  SAMM(1,2)=STRSG(4,KGAUS)
  SAMM(1,3)=STRSG(6,KGAUS)
  SAMM(2,1)=STRSG(4,KGAUS)
  SAMM(2,2)=STRSG(2,KGAUS)
  SAMM(2,3)=STRSG(5,KGAUS)
  SAMM(3,1)=STRSG(6,KGAUS)
  SAMM(3,2)=STRSG(5,KGAUS)
  SAMM(3,3)=STRSG(3,KGAUS)
  SAMM(4,4)=STRSG(1,KGAUS)
  SAMM(4,5)=STRSG(4,KGAUS)
  SAMM(4,6)=STRSG(6,KGAUS)
  SAMM(5,4)=STRSG(4,KGAUS)
  SAMM(5,5)=STRSG(2,KGAUS)
```

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```
SAMM(5,6)=STRSG(5,KGAUS)
SAMM(6,4)=STRSG(6,KGAUS)
SAMM(6,5)=STRSG(5,KGAUS)
SAMM(6,6)=STRSG(3,KGAUS)
SAMM(7,7)=STRSG(1,KGAUS)
SAMM(7,8)=STRSG(4,KGAUS)
SAMM(7,9)=STRSG(6,KGAUS)
SAMM(8,7)=STRSG(4,KGAUS)
SAMM(8,8)=STRSG(2,KGAUS)
SAMM(8,9)=STRSG(5,KGAUS)
SAMM(9,7)=STRSG(6,KGAUS)
SAMM(9,8)=STRSG(5,KGAUS)
SAMM(9,9)=STRSG(3,KGAUS)
DO 10 I=1,9
DO 10 J=1,DOFEL
GDM(I,J)=0.0
DO 10 K=1,9
10 GDM(I,J)=GDM(I,J)+SAMM(I,K)*GL(K,J)
DO 20 I=1,DOFEL
DO 20 J=1,DOFEL
DO 20 K=1,9
20 EGK(I,J)=EGK(I,J)+GL(K,I)*GDM(K,J)*QUOT
DO 30 I=1,DOFEL
DO 30 J=I,DOFEL
30 ELK(I,J)=ELK(I,J)+EGK(I,J)
RETURN
END
```

9) Subroutine QLIN5 to calculate the numerical integration for the reinforcement stiffness matrix.

```
SUBROUTINE QLIN5(WGHTR,IWGHTR,ABSSR,IABSSR,NQPR)
  IMPLICIT REAL *8 (A-H,O-Z)
  DIMENSION WGHTR(IWGHTR),ABSSR(IABSSR)
  NQPR=5
  ABSSR(1)=0.000000000
  ABSSR(2)=0.5384693101
  ABSSR(3)=-0.5384693101
  ABSSR(4)=0.906179845
  ABSSR(5)=-0.906179845
  WGHTR(1)=0.568888888
  WGHTR(2)=0.47862867
  WGHTR(3)=0.47862867
  WGHTR(4)=0.236926885
  WGHTR(5)=0.236926885
```

Supervisor Certificate

We certify that this thesis titled " **Nonlinear Analysis of Reinforced Concrete Folded Plates by Finite Strip Method** " was prepared by " **Akram Rasheed Hassan** " under our supervision at Babylon University in partial fulfillment of the requirements for the degree of Master of Science in civil Engineering.

Signature:

Name: Professor Dr. Nameer A. Alwash

Date: / / 2008

Signature:

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Date: / / 2008

CHAPTER ONE

INTRODUCTION

1.1 General

Reinforced concrete folded plates, or as they are sometimes called, prismatic shells or as previously called hipped plates, provide a useful and economical method of construction for roof and floor systems in wide variety of structures.

Folded plate structures are similar to the shells in structural action, forming for folded plates is simpler than for shell structures and the cost of construction is therefore generally less (Thannon 1988). It may be made of reinforced concrete, prestressed concrete, ferrocement, metal, or wood and have been used extensively in the construction of long-span roof systems because of their economy and their interesting architectural appearance (Scordelis and Stubbs 1961).

Folded plate structure composed of a series of individual flat plates, jointed together to form composite surface. The lines of intersection between the individual plates are usually termed fold lines, ridges, or joints. The length to width ratio of each plate is usually greater than 2 and when this ratio is less than 2, the structure called short span folded plate. The end diaphragms are important parts in the overall structural action, and are considered to be perfectly stiff in its plane but flexible normal to it (Iffland 1979). When the individual plates are jointed together so that the entire cross section of the structure is constant throughout its length, the resulting structure is called a prismatic folded plate, Fig. (1.1). If the cross section is not constant, a non prismatic folded plate structure is formed as shown in Fig. (1.2), also when the folded plate supported only at the ends by diaphragms, the structure is

defined as a single span folded plate structure, Fig. (1.1). If there are more than one intermediate supports between the exterior supports, the structure is defined as a continuous folded plate structure, Fig. (1.3). If there are only two plates intersect at a junction of the structure, the structure is termed a simple folded plate structure. If more than two plates intersect, the structure is termed multiple folded plate structures, Fig. (1.4).

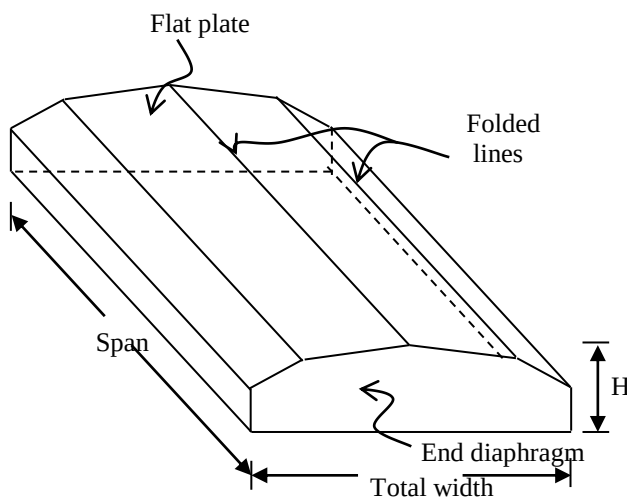


Fig. (1.1) Prismatic single span folded plate.

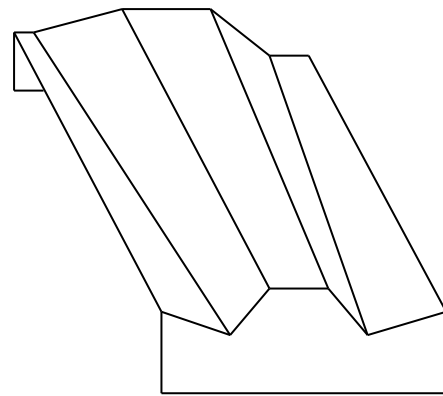


Fig. (1.2) Non prismatic folded plate.

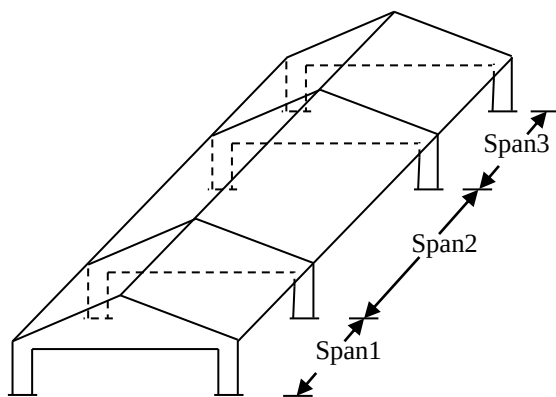


Fig. (1.3) Continuous folded plate.

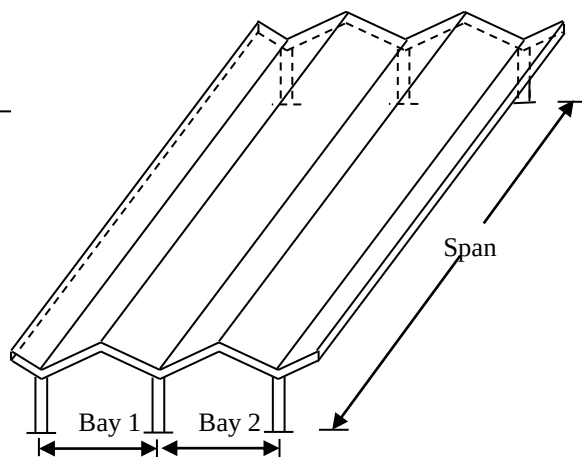


Fig. (1.4) Multiple folded plate.

1.2 Nonlinear Analysis of R/C Structures

During the last three decades, nonlinear analysis of reinforced concrete structures has become increasingly important and the interest in this field has increased steadily. This is due to the wide use of plain, reinforced and prestressed concrete as a structural material, and because the development of relatively powerful analysis techniques implemented on electronic digital computers.

Reinforced concrete exhibits a very complex behavior involving various important material nonlinearity phenomena like nonlinear stress-strain behavior, tensile cracking and compression crushing material failures, yielding of steel and interactive effect between concrete and reinforcement (Bergan and Holand 1979). Also, for some structures, such as folded plate, it is necessary to include the effect of geometric nonlinearity in addition to material nonlinearity due to the large displacement that may alter the shape of such structures and its elements.

In the present study, the finite strip method is adopted in the analysis of reinforced concrete folded plates which include the effect of both material and geometrical nonlinearities.

1.3 Objective and Scope

The aim of the present study is to investigate the overall structural behavior of reinforced concrete folded plates up to failure and taking into account both material and geometric nonlinearities due to large displacements. Cracking of concrete, plastic flow and crushing of concrete in compression and yielding of reinforcement are also included. In this study the finite strip method has been

used, formulation of stiffness matrix for three types of concrete prism elements (IPLQ, IPQQ, IPCQ) and stiffness matrix for embedded bars are adopted. A computer program designated as **NFSARCP** (Non linear finite strip analysis of reinforced concrete plates), is used in the present work to achieve this aim and this program is prepared in the present study as a modified version of (Al-Shaarbaf 1990) computer program for nonlinear analysis of reinforced concrete members by three dimensional finite element method.

1.4 Thesis Layout

This thesis contains six chapters, general introduction to the problem within hand presented in chapter one. Chapter two presents a review of the existing methods for the elastic analysis of folded plate structures and some of the theoretical studies and experimental work on the behavior of folded plates are also presented in this chapter. In chapter three, the finite strip formulation is illustrated by describing the derivation of the governing equilibrium equations and derivation of the stiffness matrix for embedded reinforcement bars. The constitutive relationships together with the material models used in the analysis are presented in chapter four. The derivation of basic matrices related to the geometric nonlinearities, numerical rules used to evaluate the prism stiffness matrix and the nonlinear solution techniques are also incorporated in this chapter. In chapter five, the results of the nonlinear analysis of six different problems are presented. The numerical analysis results carried out are compared with the available experimental and theoretical results. Parametric studies to investigate the effects of some variables on the behavior of folded plates are also presented in this chapter. The conclusions of the current research work and recommendations for further studies are given in chapter six.

CHAPTER TWO

LITERATURE REVIEW

2.1 General

In this chapter, the literature review is divided into two parts. The first is devoted for reviewing the existing methods of the elastic analysis of folded plate structures and the second is about the experimental and theoretical work on reinforced concrete folded plates.

2.2 Methods of Elastic Analysis of Folded Plate Structures

Folded plate structures have become increasingly popular in modern buildings as a result of the ease with which a series of large span rectangular plate roof can be constructed with the efficient use of the covered area. Consequently, considerable interest has been given to the analysis of folded plate structures. The existing elastic methods of analysis will be reviewed in this section.

2.2.1 Beam Method

Application of the conventional beam theory to folded plate structures has been considered by a number of researchers using diverse approaches. The beam method assumes that the variation of longitudinal stresses across the entire cross section of the structure is planner (varying linearly across the depth but remaining constant in the transverse direction of the cross section), or in the same meaning is

that the structure assumed to deflect in such a manner that all points on the same cross section deflect the same amount so that the cross section maintains its original shape through the deflection with out any transverse distortion of cross section.

Applying of the beam theory to the folded plate structures having intermediate transverse stiffing ribs yields, where ribs are spaced and designed so as to maintain the shape of the structure.

There is a difference between the results obtained by more rigorous theories and by actual experiments from the results obtained by the beam theory. So, this method fails to give good results in comparison with the other methods of analysis.

Craemer (1953) investigated a particular cross section approximating a cylindrical shell shape to assure the range of values of thickness, plate width, and span for which the results by the beam method are sufficiently accurate.

Also, Gaafar (1954) made trial to correct the results obtained by the beam method through suggesting modified beam theory.

2.2.2 Folded Plate Theory

The folded plate theory is an approximate elastic method of analysis. The simplified structural behavior is assumed in order to reduce the amount of computation required by comparing that with the computational efforts needed by using the elasticity method and resulted with a good approximation to the structural behavior. This theory considers longitudinal supporting action of each plate to be governed by the beam theory and the transverse supporting action to be treated as a continuous one-way slab. The assumptions made by the adoption of beam theory in each plate are:

1. The longitudinal normal stresses in each plate vary linearly across the width of the plate but the transverse rate of variation of stress may be different in the various plates.
2. Membrane (or in-plane) shearing stresses in each plate have negligible effect on the deflection of the structure.
3. Normal stresses in each plate in the transverse direction are included in equilibrium consideration but have negligible effect on the deflection of the structure.

The assumptions implied by the adoption of one-way slab action may be listed as follows:

1. The effect of longitudinal slab bending is negligible.
2. Transverse Shearing stresses (normal to the slab) have negligible effect on the deflection of the structure.
3. Torsional resistance of each plate is negligible. Torsional stresses due to twisting and relative deflections of the plates due to those stresses may be ignored.

Winter & Pei (1947) proposed a solution by neglecting the relative displacements of the joints and assumed that the transverse moments in the transverse strip are negligible, the solution involved two steps. The first step was analyzing the transverse slab system rigidly supported at the joints under the external loads. The second step was analyzing the longitudinal plate system under the effect of the slab reactions.

Gaafar (1954) modified the Winter & Pei method by considering the effect of relative joint displacements on the transverse moments and membrane

stresses. The method may be used for longitudinally symmetric loading on a simple span structure. Experimental verification of the theoretical analysis for a model subjected to concentrated loads was furnished with satisfactory correspondence of the results.

Simpson (1958) developed a new method, which involves moment and stress distribution. In the case of no rotation, the transverse moment can be calculated by moment distribution method and the longitudinal stresses can be obtained by stress distribution procedure. Then, an arbitrary rotation was given at each joint to allow for joint displacement correction. The stress distribution procedure at each joint was used and resulted in a set of simultaneous equations. This set of equations involves deflection due to no rotation and those due to rotation solution. By solving these equations, the final stresses and moments may then be obtained.

2.2.3 Elasticity Method

The elasticity method can be regarded as an accurate method for the analysis of folded plate structures. This method requires extensive computations and may only become practical when programmed for a digital computer.

Goldberg & Leve (1957) developed a solution for the stresses in a folded plate structure by combining the equations of the classical plate bending theory for loads normal to the plane of the plates and the elasticity equations defining the plane stresses for loads in the plane of the plates. Applied loading is approximated by a Fourier series. Equations are derived for the case of a longitudinally simple span structure and relate each joint force to a linear combination of the joint displacements.

Four displacements are considered at each ridge, two components of translation, one rotation in the plane parallel to the end diaphragm and one

displacement along the ridge. Hence, four equilibrium and four compatibility equations could be established at each ridge.

This method has its main areas of application where component plates of the structure are relatively short compared to the width, and where translation of individual joints is completely prevented.

Later, this method was developed by Leve & Lee (1967) for the solution of prismatic shells continuous over transverse diaphragms.

2.2.4 Nodal Section Method

Rockey and Evans (1972) developed the nodal-section method for the analysis of straight box girder bridges assuming that the box girder is a certain type of folded plate structures. The assessment of the accuracy of the nodal-section method for the analysis of folded plate structures was carried out by Al-Rifaie and Abdul-Majeed (1979). The method was particularly simple to apply and may be considered as a hand computation method for the analysis of folded plate structures.

The following assumptions are required for the developed nodal section method:

1. The structural material is homogeneous, isotropic and linearly elastic.
2. The actual deflections are small compared to the overall structural dimensions.
3. The principle of superposition holds.
4. The longitudinal joints between plates are fully monolithic.
5. Each end or intermediate diaphragm is infinitely stiff in its own plane and perfectly flexible normal to its plane.
6. The in-plane bending action of an individual plate is similar to that of a beam spanning between the end diaphragms.

7. The bending action of an individual plate normal to its plane is represented by considering a transverse one-way slab strips.

2.2.5 Finite Segment Method

The finite segment method firstly presented by Lo and Scordelis (1969). The approach is based on the usual ordinary theory of folded plates with the exception that the effect of shearing stresses and transverse normal stresses were included in calculating deflections.

In this method, the basic structural element is assumed to be formed by dividing each plate into a finite number of segments longitudinally. Compatibility and equilibrium conditions were then satisfied at selected points along the four edges of each segment in the structure.

Good agreement was obtained between the results evaluated from finite segment method and the results calculated by using finite element method. But the computation time required to conduct the analysis by finite segment method is less than that required by using the finite element method because of the degrees of freedom considered for the individual segment is less than that required by the finite element method.

2.2.6 Finite Element Method

The finite element method has become a powerful tool for the numerical solution of a wide range of engineering problems. With the advances in computer technology, complex problems can be modeled with relative ease.

Rockey and Evans (1967) presented a method of analysis of folded plates using the finite element method. The solution can be applied to all folded plate structures composed of rectangular plate elements and also continuously curved

shells (after approximating the curved surface by connected flat rectangular elements). The method is capable of dealing with structures supported on interior diaphragms, frames and also with plates containing openings.

In this method, the folded plate is divided into a finite number of elements. These elements may be triangular or rectangular. Two-dimensional finite element idealization has been widely used for the analysis of folded plate structures by this method. The stiffness matrix of the element is obtained by superimposing two independent parts of matrices. The first part is the stiffness matrix of a plane stress element, where the membrane forces are considered to act. While, the second part is the stiffness matrix of the plate bending element, where the shear forces and bending moments perpendicular to the plane are considered.

Numerical results are presented for a simple folded plate structure and compared with those obtained with the elasticity method, excellent agreement has been obtained.

The finite element method can be applied to deal with a wide range of loading and support conditions, and also with plates of varying thickness.

2.2.7 Finite Strip Method

The finite strip method (FSM) is a powerful and effective contemporary technique mainly for general linear elastic static, dynamic and stability analysis of structures with a constant geometry and stiffness in one direction (usually in length) and simple (one type) boundary conditions at the cross ends. First of all the long single-span or multiple-spans (continuous) thin-walled prismatic or curve structures are appropriate for this analysis. Many of thin-walled bridge structures, some roof and floor structures, some reservoirs, channels, tunnels, subways,

layered plates, shear-walls, shells and others can be studied by (FSM). This method is especially effective if two opposite ends of structure are simply supported, for example by the ideal rigid diaphragms in its plane and infinitely flexible out of it.

Cheung (1969) was the first who used this method successfully for the analysis of folded plate structures. Each strip is considered to be subjected to membrane and transverse bending actions. In the initial stage of development, cubic variation of transverse displacement combined with linear variation of membrane displacement was assumed in the solution. The method was further used by Scordelis and Mayer (1971) for the analysis of box-girders and cylindrical shells.

A higher order flat shell strip with one internal nodal line was developed by Loo and Cusens (1971) and applied to analysis of right box girder bridge decks. In this case considerable improvement in results was noticed using relatively small number of strips. Also in this paper, the direct approach takes into consideration the nature and location of the applied load.

In order to improve the efficiency of the higher order finite strip developed, Loo and Maitra (1982) showed that the static condensation technique is well suited for higher order strips with auxiliary nodal lines. This technique is employed to eliminate the internal nodal line degrees of freedom.

2.2.8 Finite Strip-Element Method

The finite strip-element method is a combination of the finite element and finite strip methods which is firstly proposed by Golley and Grice [1989] and mainly used for the analysis of prismatic folded plates structures and box girders.

Displacement functions within strip-elements are combinations of polynomials used with conventional finite elements plus displacement functions used with simply supported finite strips, which are products of truncated trigonometric series and polynomials.

2.3 Experimental and Theoretical Work on R/C Folded Plates

The available experimental and theoretical works on reinforced concrete folded plates will be reviewed in this section.

Senler (1958) experimentally tested a small reinforced concrete folded plate slab up to failure. Span of the structure was 3050 mm, thickness 25.4 mm and the slant height of the plate elements was 386.1 mm, the angle between side plates and with horizontal reference was 45°. Transverse reinforcement was 12-Gauge wire at 101.6 mm centers and longitudinal reinforcement at the bottom of each plate segment was three bars of diameter 6.35 mm. Uniformly distributed load with sand bags was applied on folded plate. Failure occurred at a load level of 2.8 times the design load. There was a little deflection but the slab had many tension cracks. Failure of one-half of the structure was so sudden that the immediate cause could not be proved.

In 1959, reinforced concrete research council at Syracuse University presented an experimental investigation of a large reinforced concrete folded plate model. Overall plane dimensions were 4.17 m by 7.93 m. The tested structure was approximately a one-half scale model and subjected to a uniformly applied load on the horizontal plane using sand bags. The structure was initially loaded until the full design live load was achieved, and then it was reloaded with

the design load on one-half of the structure. The structure collapsed under a load larger than 4 times the full design live load. Strain readings were made on the reinforcement at several places and extensive vertical and horizontal deflection measurements were taken.

An experimental work to reinforced concrete folded plate models was presented by Dykes (1960). The tested specimens consisted of eight models having inverted V cross-section with span 749.3 mm and chord 520.7 mm, concrete thickness was 12.7 mm and inclination angle of plates with horizontal reference was 33.5 degree. One of these models used to assist in developing and testing of the loading device and one collapsed prematurely. Thus, six units were available for actual test purposes. The models were made of cement mortar and equal quantity of reinforcement in two directions at right angles was provided by a steel wire mesh placed in the middle surface of the concrete and giving a steel ratio of 1.3%. All structures were subjected to a symmetrical distribution of loads normal to the surfaces. All the tested models were identical in all respects except for the manner in which the structure is supported. Ultimate moments, flexural rigidity, ultimate loads and crack patterns were reported. The main objective of the paper was to study the effect of support condition on the ultimate capacity and structural behavior of reinforced concrete folded plates. The measured ultimate loads and collapse forms show good agreement with those obtained by the theoretical analysis using the fracture line theory which was extended to include the effects of primary membrane forces.

Chacos and Scalzi (1961) described an experimental test of long simply supported folded plate model consisted of three-plate configuration was selected as a typical cross-section. The thickness of the plate was established at 12.7 mm because this was the minimum which could be used to cover the reinforcement

properly. A width of 304.8 mm was selected on the bases of minimizing local buckling effects. The width to thickness ratio of 24 resulted in an elastic buckling stress much greater than the ultimate compressive strength of concrete. The span of the structure was 1524 mm making a span to width ratio of 5 which is reasonable for folded plate action, while the thickness of the end diaphragms was 76.2 mm. The tested structure was loaded with bricks on the top horizontal plate. Objective of this investigation was to determine the behavior of the folded plate structure as a simple beam with an irregular cross section and to verify the ultimate moment capacity by the rectangular stress block method. The predicted ultimate moment capacity by the rectangular stress block method was compared with the ultimate moment developed in a reinforced mortar structure loaded to failure. It was shown that the prediction of the ultimate moment based on the assumption of simple beam was valid for this particular structure and the stress in the tensile steel was agreed reasonably well with the beam and folded plate methods of analysis which state that the tension steel behave elastically until collapse occurred.

Dykes (1963) experimentally tested a number of reinforced concrete and aluminum folded plates. These folded plates are of inverted V cross-section subjected to a uniformly distributed load normal to the surface. Two cases were tested with the aim of investigating the effect of different boundary conditions along the longer edge. In the first case, a fully free edge and in the second, only rotational freedom is present with no translations. The experiment has been very well documented and detailed results are available for the second case, while only the linear response is given for the first case of boundary conditions. Freedom of rotation at the edges with no translation has been achieved by means of longitudinal hinge, Uniform reinforcement over the whole structure was provided by two meshes of 2.1 mm wires at 75 mm centers in both the

longitudinal and transverse directions. These tests were conducted on both full scale and model reinforced concrete folded plate structures. The overall dimensions of the full scale structure were 1848 mm by 3277 mm and having a total depth of 533 mm with 19 mm plate thickness. While, the model concrete structures were constructed to a one-quarter scale of the full scale structure. The experimental failure load was reported to be 5.2 kN/m² after which, the concrete structures effectively ceased to exist and further load was carried by tensile membrane action of the reinforcement mesh.

Scodelis and Gerasimenko (1966) performed an experimental investigation for two simple supported reinforced concrete folded plate models identical in all respects except for the manner and quantity of reinforcement. The purpose of this study was to compare the behavior of folded plates through the elastic, inelastic and ultimate ranges. The butterfly type cross section used is approximately one-sixth geometrical scale of a prototype used on an actual construction, the overall dimensions were 762 mm, 1778 mm. Reinforcement requirements were determined for first model by folded plate analysis based on the elasticity method, and for second model by elementary beam theory. Reinforcement diameter was 2mm, the applied loads were uniformly line loads acts at the interior joints. The ultimate predicted load was four and one-half times the full design load.

Thannon (1988) developed a finite element technique for nonlinear analysis of reinforced concrete stiffened shells and folded slabs. A layered degenerated thin shell element with five degrees of freedom was employed for the shells and, for the beams a three node curved beam element formulation was used based on the beam theory and incorporating transverse shear deformation.

For the stiffened structures, the modification of the degenerated shell element was made which was needed for the connection with curved beam element. The skeletal and wire mesh reinforcement were represented by smeared layers of equivalent thickness. Perfect bond between the mortar and reinforcement has been assumed. Both an elastic-perfectly plastic and strain hardening models have been employed to model the compressive behavior of the mortar. A smeared fixed crack approach was used to model the behavior of cracked mortar. A reduced shear modulus was used in the crack zone. The steel was considered as an elastic-perfectly plastic material with possible linear strain hardening. A nonlinear geometric model, based on total Lagrangian approach and Von-Karman sense was considered. Reinforced concrete beams, stiffened shells, and folded slabs were analyzed using the proposed models and show good agreement with several experimental results. The numerical solutions obtained for the reinforced concrete folded slabs revealed that the inclusion of geometric nonlinearity plays an important role in predicting the behavior of reinforced concrete structures which exhibit large deformations before failure.

Alrawi (2000) presented a numerical analysis of reinforced concrete folded plate structures by equivalent space grillage (or framework) method. This method based on the conversion of the folded plate structure into an assemblage of beams in space. The section properties of the beams were derived from the properties of the actual folded plate. Uncracked sections were assumed, but the orthotropic properties of the reinforced concrete plates were fully considered.

Numbers of orthotropic folded plate structures with different load and support conditions have been analyzed by the proposed method for checking the efficiency and accuracy of the suggested method. The examples were solved by

finite element method using rectangular flat shell element with orthotropic properties. Results of deflections and moments by the proposed framework method compares well to the results by finite element method.

Mohammed (2001) studied both two and three-dimensional nonlinear finite element models suitable for the analysis of reinforced concrete folded plates. The 9-node Lagrangian degenerated shell elements and the 20-node isoparametric brick elements have been used to model the concrete. While, the reinforcing bars were modeled as smeared layers of equivalent thickness throughout the shell element or as axial members embedded within the brick elements for the two and three-dimensional models respectively. Perfect bond between the concrete and the reinforcing bars was assumed to occur. The finite element results showed good agreement with the experimental results when the effect of large deformations is taken into consideration. Effect of simulating the reinforced concrete folded plates using two or three-dimensional finite elements were also studied. Several parametric studies have been carried out to investigate the effect of some numerical and material parameters on the predicted finite element results. The influence of increasing concrete compressive strength, amount of reinforcement, plate thickness, span/width ratio, and span/depth ratio, and inclination angle has been also investigated. The distribution of longitudinal stresses in the concrete and reinforcing bars was also studied.

In 2003, Altaey investigated the nonlinear behavior of reinforced concrete shells and folded plates by the finite element method. The 8, 14 and 20-node brick elements have been used to model the concrete. The reinforcing bars are modeled as axial members embedded within the brick element. Perfect bond between the concrete and the reinforcing bars was assumed to occur.

The elasto-plastic strain hardening model followed by perfectly plastic response is used to simulate the compressive behavior of concrete. This response is terminated at the beginning of crushing. In tension, linear elastic behavior prior to cracking is assumed. A smeared crack model with fixed orthogonal cracks is adopted to represent the fracture of concrete. Nonlinear geometrical model based on the total Lagrangian approach and taking into account Von-Karman assumptions is considered.

The study was applied on reinforced concrete shell structures and folded plates including edge beams to test the validity of the program in the analysis. Good agreement has been achieved between the examples analyzed in this research work and with the published experimental and theoretical results.

Also the study included effect of some parameters on the behavior of shell and folded plate structures. The parameters include; change of the thickness, edge beams height, change of the reinforcement ratio and change the inclination angle. For The folded plates, the inclination angle has significant effect on increasing or decreasing the ultimate load. The ultimate load is decreased 73%, when the inclination angle is increased from 30° to 60°. The increasing in the percent of short direction steel reinforcement to 200% gives increasing in the failure load equals to 152%.

The finite strip method has proved to be an accurate and efficient tool for the analysis of structures having regular cross-section and mechanical properties. So it has been used in the present study, with three types of prism elements to simulate the concrete, the reinforcement are represented by using embedded representation, perfect bond between concrete and reinforcing bars was assumed to occur. The effect of geometric nonlinearity in addition to material nonlinearity has been used in this study.

CHAPTER THREE
FINITE STRIP FORMULATION OF FOLDED PLATE
STRUCTURES

3.1 Introduction

The finite strip method may be regarded as a special form of the finite element method using the displacement approach and has been shown to be simple and efficient tool for the analysis of folded plate structures. In this method the structure is divided into two dimensional (strips) or three dimensional (prisms, layers). The geometry of the structure is usually constant along one or two coordinate axes so that the width of a strip or the cross section of prism or layer will not change from one end to the other.

Unlike the standard finite element method, which uses polynomial displacement functions in all directions, the finite strip method calls for use of simple polynomials in some directions and continuously differentiable smooth series in the other directions, with the stipulation that such series should satisfy a priori the boundary conditions at the ends of the strips or (prisms), and also specify the deflected shape in there directions. The general form of the displacement function is given as a product of polynomials and series. (Cheung 1976)

In the present study, the three dimensional finite strip method will be used to analyze the simply supported folded plates by dividing the structure into a finite number of prisms, each prism contain isoparametric shape function in its own section and smooth series in the direction of its length to satisfy the boundary conditions in that direction, (four nodal lines linear

prism, eight nodal lines quadratic prism, and twelve nodal line cubic prism were used in the present research work.

3.2 Three Dimensional Isoparametric Prism

In the finite element approach it is often convenient to define the coordinates of the more complex element shapes by the nodal coordinates.

$$\xi = \sum_{i=1}^s \phi_i \xi_i \quad (3.1)$$

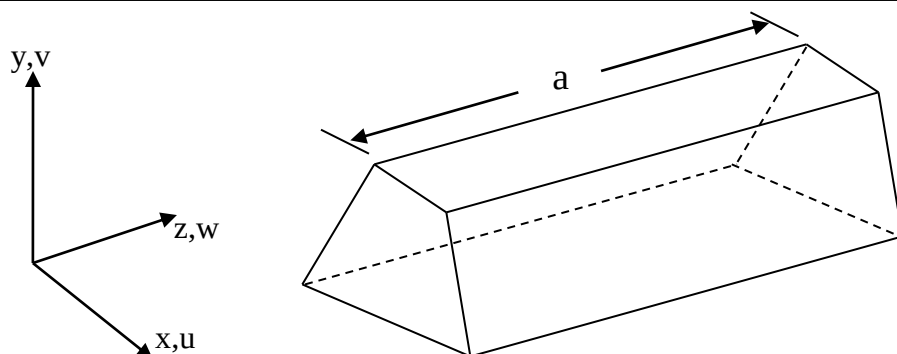
In which (s) refers to the number of nodes of the element, similarly the displacements δ of a point within the element can also be expressed in terms of the nodal displacements (δ_i)

$$\delta = \sum_{i=1}^s \psi_i \delta_i \quad (3.2)$$

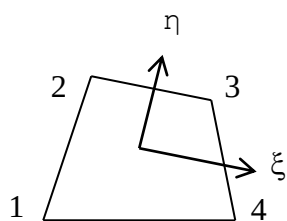
Where ϕ_i , and ψ_i are functions of a coordinate system (Cartesian, skew, curvilinear...etc). These two functions are, in general not the same. For the special case in which ϕ_i and ψ_i are identical, the element is termed isoparametric.

The concept of isoparametric elements was first introduced by taig (1961) and by irons (1963) and later extended by Ergatoudis et al (1968). In the present study the concept is further extended to formulate the stiffness matrix of straight simply supported prism.

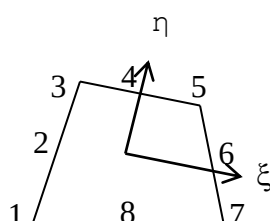
Referring to Fig. (3.1), it can be seen that in the plane of the cross section of prism the shape function given in Table (3.1) should be used, while in the axial direction, (Z_m) there are again some Fourier series which satisfy the simply support end conditions ($u = v = \frac{\partial w}{\partial z} = 0$ at $z = 0, z = a$).



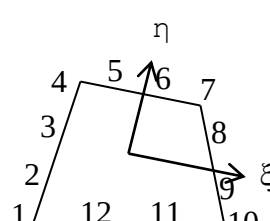
A typical 3-D finite strip.



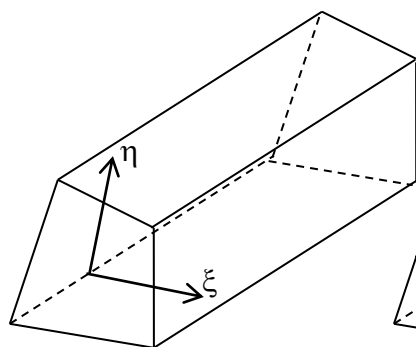
IPLQ ELEMENT



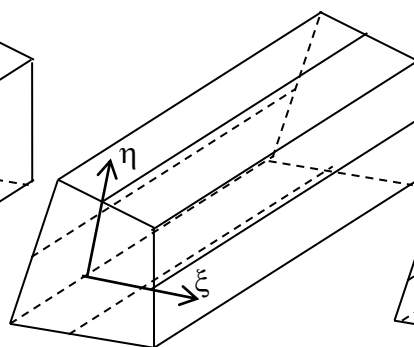
IPQQ ELEMENT



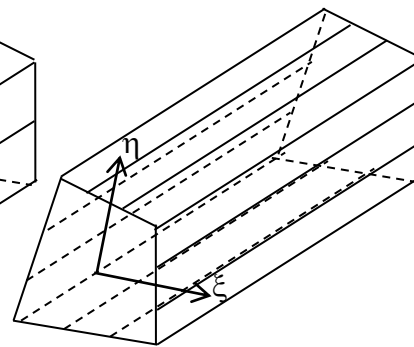
IPCQ ELEMENT



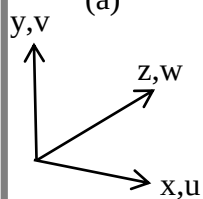
IPLQ
3-D STRIP
(a)



IPQQ
3-D STRIP
(b)



IPCQ
3-D STRIP
(c)



Isoparametric quadrilateral 3-D finite strips.

3.3 Displacement Field

Fig. (3.1) Isoparametric quadrilateral 3-D finite strips.

The natural local coordinates system may be used through the cross section of prism (x – y plane). These local coordinates (ξ, η) are scattered in the center of cross section of prism. Each prism has (3 d.o.f) per nodal line. The starting point for the stiffness matrix derivation is the prism displacement field. The assumed displacement field is:

$$\begin{aligned} u &= \sum_{m=1}^r \sum_{i=1}^{ne} N_i \cdot \sin(km \cdot z) \cdot u_{im} \\ v &= \sum_{m=1}^r \sum_{i=1}^{ne} N_i \cdot \sin(km \cdot z) \cdot v_{im} \\ w &= \sum_{m=1}^r \sum_{i=1}^{ne} N_i \cdot \cos(km \cdot z) \cdot w_{im} \end{aligned} \quad (3.3)$$

where:-

u, v, w : the displacements in x, y, z directions respectively.

u_{im}, v_{im}, w_{im} : The nodal line displacement parameters at ith nodal line for mth series terms.

N_i : shape function in terms of local curvilinear coordinate system for ith nodal line.

ne: number of nodal lines for prism element.

$\sin(km \cdot z), \cos(km \cdot z)$: series which satisfy the boundary conditions at the two ends of prism.

$km = \frac{m\pi}{a}$ Where (a= length of prism)

r: represents the number of series terms

3.3.1 Series Part of displacement field

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The series part is derived from the beam vibration equation. It represents the variation of the displacements in the longitudinal direction of the structure. This part of displacement field should satisfy the boundary conditions at the ends of the longitudinal direction of the folded plate structures (cheung 1976).

$$Z'''' = \frac{\mu^4}{a^4} Z \quad (3.4)$$

where

Z'''' : represents the fourth derivative of the series part (Z), with respect to z direction.

a: length of the prism.

μ : parameter.

the general form of the solution of eq. (3.4) is

$$Z(z) = C_1 \sin\left(\frac{\mu z}{a}\right) + C_2 \cos\left(\frac{\mu z}{a}\right) + C_3 \sinh\left(\frac{\mu z}{a}\right) + C_4 \cosh\left(\frac{\mu z}{a}\right) \quad (3.5)$$

The coefficients C_1 , C_2 , C_3 and C_4 are to be determined from the end conditions.

For simply supported ends $Z(0) = Z''(0) = Z(a) = Z''(a) = 0$, the series part become.

$$Z_m(z) = \sin \frac{\mu_m z}{a} \quad (3.6)$$

$$(\mu_m = \pi, 2\pi, 3\pi, \dots, m\pi)$$

3.3.2 Shape Function

The shape function in terms of curvilinear coordinates for the (IPLQ, IPQQ, and IPCQ elements) are shown in Table (3.1).

Table (3.1) Shape function of isoparametric quadrilateral elements.

IPLQ ELEMENT			
Nodal line number	ξ_i	η_i	$N_i(\xi, \eta)$
1,2,3,4	-1,1	-1,1	$\frac{1}{4}(1+\xi_o)(1+\eta_o)$
IPQQ ELEMENT			
Nodal line number	ξ_i	η_i	$N_i(\xi, \eta)$
1,3,5,7	-1,1	-1,1	$\frac{1}{4}(1+\xi_o)(1+\eta_o)(\xi_o+\eta_o-1)$
4,8	0	-1,1	$\frac{1}{2}(1-\xi^2)(1+\eta_o)$
2,6	-1,1	0	$\frac{1}{2}(1+\xi_o)(1-\eta^2)$
IPCQ ELEMENT			
Nodal line number	ξ_i	η_i	$N_i(\xi, \eta)$
1,4,7,10	-1,1	-1,1	$\frac{1}{32}(1+\xi_o)(1+\eta_o)\left[-10+9\left(\xi^2+\eta^2\right)\right]$

2,3,8,9	-1,1	$-\frac{1}{3}, \frac{1}{3}$	$\frac{9}{32}(1+\xi_o)(1-\eta^2)(1+9\eta_o)$
5,6,11,12	$-\frac{1}{3}, \frac{1}{3}$	-1,1	$\frac{9}{32}(1-\xi^2)(1+\eta_o)(1+9\xi_o)$

where:

$$\xi_o = \xi \xi_i \quad \eta_o = \eta \eta_i$$

ξ_i, η_i :local coordinates of nodal line i

In the isoparametric definition the shape function are also used to define the geometry of the prism and the global coordinates of any line P(ξ , η) in terms of the natural local coordinates by using the relations:

$$\begin{aligned} x &= \sum_{i=1}^{ne} N_i(\xi, \eta) x_i \\ y &= \sum_{i=1}^{ne} N_i(\xi, \eta) y_i \end{aligned} \quad (3.7)$$

where : x_i, y_i are the global coordinate of the nodal line i.

3.4 Definition of Stresses and Strains

In order to take into account the geometric nonlinearity of a structure, the general strain-displacement equations that are used for small and large deformation problems can be written as (Zienkiewicz 1977):

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \partial u / \partial x + \frac{1}{2} \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial x} \right)^2 \right] \\ \partial v / \partial y + \frac{1}{2} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \\ \partial w / \partial z + \frac{1}{2} \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} + \frac{\partial u}{\partial y} \frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} + \frac{\partial u}{\partial x} \frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial z} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial z} \end{Bmatrix} \quad (3.8)$$

where u , v , and w are the displacement components in the x , y , and z directions respectively. Equation (3.8) reveals the nonlinearity of the strain-displacement relationships. If the displacements are small, the general first order linear strain approximation is obtained by neglecting the quadratic terms. The engineering strains are considered valid for the new strains measured in the original configuration (Timoshenko and Goodier 1951).

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \sum_{m=1}^r \sum_{i=1}^{ne} \begin{bmatrix} \frac{\partial N_i}{\partial x} Sm + \frac{1}{2} \frac{\partial N_i}{\partial x} \frac{\partial N_i}{\partial x} Sm^2 u_{im} & \frac{1}{2} \frac{\partial N_i}{\partial x} \frac{\partial N_i}{\partial x} Sm^2 v_{im} & \frac{1}{2} \frac{\partial N_i}{\partial x} \frac{\partial N_i}{\partial x} Cm^2 w_{im} \\ \frac{1}{2} \frac{\partial N_i}{\partial y} \frac{\partial N_i}{\partial y} Sm^2 u_{im} & \frac{\partial N_i}{\partial y} Sm + \frac{1}{2} \frac{\partial N_i}{\partial y} \frac{\partial N_i}{\partial y} Sm^2 v_{im} & \frac{1}{2} \frac{\partial N_i}{\partial y} \frac{\partial N_i}{\partial y} Cm^2 w_{im} \\ \frac{1}{2} N_i^2 Km^2 Cm^2 u_{im} & \frac{1}{2} N_i^2 Km^2 Cm^2 v_{im} & -N_i Km Sm + \frac{1}{2} N_i^2 Km^2 Sm^2 w_{im} \\ \frac{\partial N_i}{\partial y} Sm + \frac{\partial N_i}{\partial x} \frac{\partial N_i}{\partial y} Sm^2 u_{im} & \frac{\partial N_i}{\partial x} Sm + \frac{\partial N_i}{\partial x} \frac{\partial N_i}{\partial y} Sm^2 v_{im} & \frac{\partial N_i}{\partial x} \frac{\partial N_i}{\partial y} Cm^2 w_{im} \\ N_i Km Sm Cm \frac{\partial N_i}{\partial y} u_{im} & N_i Km Cm + N_i Km Sm Cm \frac{\partial N_i}{\partial y} v_{im} & \frac{\partial N_i}{\partial y} Cm - N_i Km Sm Cm \frac{\partial N_i}{\partial y} w_{im} \\ N_i Km Cm + N_i Km Sm Cm \frac{\partial N_i}{\partial x} u_{im} & N_i Km Sm Cm \frac{\partial N_i}{\partial x} v_{im} & \frac{\partial N_i}{\partial x} Cm - N_i Km Sm Cm \frac{\partial N_i}{\partial x} w_{im} \end{bmatrix} \begin{Bmatrix} u_{im} \\ v_{im} \\ w_{im} \end{Bmatrix} \quad (3.9)$$

Where: $Sm = \sin(km.z)$, $Cm = \cos(km.z)$

$$Km = \frac{m\pi}{a}$$

a =length of prism.

ne :number of nodal line for prism element.

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Since the shape functions, N_i , are functions of local coordinates rather than Cartesian coordinates, the Jacobian matrix $[J]$, is needed to relate the derivatives in the two coordinate systems,

$$[J] = \sum_{i=1}^{ne} \begin{bmatrix} \frac{\partial N_i}{\partial \xi} x_i & \frac{\partial N_i}{\partial \xi} y_i \\ \frac{\partial N_i}{\partial \eta} x_i & \frac{\partial N_i}{\partial \eta} y_i \end{bmatrix} \quad (3.10)$$

then, the shape function derivatives with respect to x , and y can be obtained as:

$$\begin{Bmatrix} \partial N_i / \partial x \\ \partial N_i / \partial y \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \partial N_i / \partial \xi \\ \partial N_i / \partial \eta \end{Bmatrix} \quad (3.11)$$

In the z -direction the coordinate transform from Cartesian (z) coordinate to curvilinear coordinate (ζ) by using the interpolation equation.

$$z = \frac{a}{2}(1 + \zeta) \quad (3.12)$$

$$dz = \frac{a}{2} d\zeta \quad (3.13)$$

The vector of stresses is given by:

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \quad (3.14)$$

and the stress – strain relationship is represented as:

$$\{\sigma\} = [D].\{\varepsilon\} \quad (3.15)$$

where $[D]$ is the constitutive matrix.

3.5 Minimum Total Potential Energy Principle

The principle of virtual displacements of a deformable body is used to establish the governing equations of static equilibrium. Consider a general three-dimensional body acted upon by a system of external forces, which may comprise surface tractions, acting over part or the whole external surface of the body,

$$\{t\} = [t_x \quad t_y \quad t_z]^T \quad (3.16)$$

and body forces, acting through the volume of the body,

$$\{b\} = [b_x \quad b_y \quad b_z]^T \quad (3.17)$$

Due to these forces, a particle of the body located initially at a point $P (x, y, z)$ undergoes a displacement having components u, v , and w in the x, y , and z directions respectively. Components of the displacement at point P are given by the vector:

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$$\{u\} = [u \quad v \quad w]^T \quad (3.18)$$

The principle of virtual displacements states that, “**a deformable body is in equilibrium if the total work done by all external farces plus the total work done by all internal forces during any kinematically admissible virtual displacement is zero**” (Dawe 1984). By considering the external work, W_{ext} , and the internal work, W_{int} , the principle can be written as:

$$\delta W_{int} - \delta W_{ext} = 0 \quad (3.19)$$

The external work can be expressed as the work done in moving the body forces $\{b\}$ and surface tractions $\{t\}$ through virtual displacement, $\partial\{u\}$, as:

$$\delta W_{ext} = \int_V \partial\{u\}^T \{b\} dV + \int_S \partial\{u\}^T \{t\} dS \quad (3.20)$$

where V is the volume of the body and S is that part of the surface of the body where the external tractions are prescribed. The change in strain energy, internal work, due to a set of virtual strain, $\partial\{\epsilon\}$, corresponding to the virtual displacement is:

$$\delta W_{int} = \int_V \partial\{\epsilon\}^T \{\sigma\} dV \quad (3.21)$$

Substituting of equation (3.15) into equation (3.21) yields:

$$\delta W_{int} = \int_V \partial\{\epsilon\}^T [D] \{\epsilon\} dV \quad (3.22)$$

Substitution of equation (3.20) and (3.22) into equation (3.19) yields:

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$$\int_V \partial \{\epsilon\}^T [D] \{\epsilon\} dV - \int_V \partial \{u\}^T \{b\} dV - \int_S \partial \{u\}^T \{t\} dS = \{0\} \quad (3.23)$$

This expression represents the equation of static equilibrium for a general body.

The basic concept of finite strip analysis is to discrete the continuum into arbitrary number of prisms connected together at their common nodal lines. For prism p, the displacement vector at any point is:

$$\{u\}^p = [N] \{a\}^p \quad (3.24)$$

where [N] is a matrix containing the interpolation functions which relate the displacements $\{u\}^p$, to the nodal lines displacements $\{a\}^p$. The corresponding strains $\{\epsilon\}^p$ are obtained by differentiation of the displacements such that:

$$\{\epsilon\}^p = [A] \{u\}^p \quad (3.25)$$

where [A] is a matrix which contains the differential operators. Substituting of equation (3.24) into equation (3.25) yields:

$$\{\epsilon\}^p = [B] \{a\}^p \quad (3.26)$$

where [B] is the strain-displacement matrix given by:

$$[B] = [A][N] \quad (3.27)$$

The strain-displacement matrix [B], which represents the values of the strain at any point within the prism, due to unit values of nodal line displacements. In the discrete model, the equations of equilibrium of the continuum may be written as the sum of the integration over the volume

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and surface area for all the prisms. Therefore, by making use of equation (3.24) and (3.26), equation (3.23) becomes:

$$\partial\{a\}^T \left[\sum_n \int_{V^p} [B]^T [D] [B] dV^p \{a\}^p - \sum_n \int_{V^p} [N]^T \{b\}^p dV^p - \sum_n \int_{S^p} [N]^T \{t\}^p dS^p \right] = \{0\} \quad (3.28)$$

where: n is the total number of prisms

Since the relationship must be valid for any set of virtual displacements, and since $\partial\{a\}^T$ is arbitrary, then it may be canceled from both two sides of equation (3.28) leaving:

$$\{f\} = [K] \{a\} \quad (3.29)$$

where $[K]$ is the stiffness matrix of the prism assemblage and is given by:

$$[K] = \sum_n \int_{V^p} [B]^T [D] [B] dV^p \quad (3.30)$$

$\{a\}$ is the corresponding prism assemblage nodal line displacement vector, and, $\{f\}$ is the prism assemblage of external nodal line force vector given by:

$$\{f\} = \sum_n \int_{V^p} [N]^T \{b\}^p dV^p + \sum_n \int_{S^p} [N]^T \{t\}^p dS^p \quad (3.31)$$

for a prism of volume v , the stiffness matrix is presented

implicitly such as:

$$(3.32) [K]^p = \int_{v^p} [B]^T [D] [B] dv^p$$

for three – dimensional prisms, the differential volume dv^p , may be

written as:

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$$dv^P = dx dy dz \quad (3.33)$$

Equation (3.33) can be transformed into the natural coordinates as:

$$dv^P = \frac{a}{2} |J| d\xi d\eta d\zeta \quad (3.34)$$

where $|J|$ is the determinant of the Jacobian matrix. The limits of integration in the natural coordinates become -1 and $+1$ and the prism stiffness matrix can therefore be written as:

$$[K]^P = \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [D] [B] \frac{a}{2} |J| d\xi d\eta d\zeta \quad (3.35)$$

In general it is not possible to evaluate the prism stiffness matrix explicitly. Thus, numerical integration has to be used.

3.6 Force Matrices

The vector $\{ F \}$ given in Eq. (3.31) may be written as :

$$\{ F \} = \left\{ \{ F^1 \}^T, \{ F^2 \}^T, \dots, \{ F^m \}^T, \dots, \{ F^r \}^T \right\}^T \quad (3.36)$$

where each of load vectors $\{ F^m \}$ are estimated independently for the certain harmonic term. In the present study the load vector $\{ F \}$ comprises of the uniformly distributed load $\{ F_u \}$ and point load $\{ F_p \}$ and can be expressed :

$$\{ F \} = \{ F_u \} + \{ F_p \} \quad (3.37)$$

3.6.1 Uniformly Distributed Load (U.D.L)

If an arbitrary uniform load is applied along the span with a three components of load in the x, y, and z directions named as q_x , q_y , and q_z respectively. The equivalent nodal forces $\{ F_u \}$ due to uniformly distributed load $\{ q \}$ may be expressed as. (Zienkiewics and Tylor 1989)

$$\{ F_u \} = \int [N^m]^T \{ q \} .ds \quad (3.38)$$

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S

where S is the surface area over which the load {q} is applied, then Eq. (3.38) can be expressed as :

$$\{F_u\} = \begin{Bmatrix} F_{u_x} \\ F_{u_y} \\ F_{u_z} \end{Bmatrix} = \iint \begin{Bmatrix} q_x \cdot \sin \frac{m \cdot \pi \cdot z}{a} \cdot N_i \\ q_y \cdot \sin \frac{m \cdot \pi \cdot z}{a} \cdot N_i \\ q_z \cdot \cos \frac{m \cdot \pi \cdot z}{a} \cdot N_i \end{Bmatrix} \cdot dx \cdot dz \quad (3.39)$$

Then by noting that:

$$\int_0^a \sin \frac{m \cdot \pi \cdot z}{a} \cdot dz = \frac{2 \cdot a}{\pi} \quad \text{for } m=1,3,5,\dots \quad (3.40)$$

$$\int_0^a \sin \frac{m \cdot \pi \cdot z}{a} \cdot dz = 0 \quad \text{for } m=2,4,6,\dots$$

Since the load is applied in the vertical direction, the load vector becomes {0, q, 0}, and Eq. (3.39) can be expressed as :

$$\{F_u\} = \begin{Bmatrix} F_{u_x} \\ F_{u_y} \\ F_{u_z} \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_{u_y} \\ 0 \end{Bmatrix} = \frac{2 \cdot a \cdot q}{m \cdot \pi} \int N_i \cdot dx \quad (3.41)$$

Where the shape function N_i given in the form of natural coordinates system (ξ, η) , then the integration $(\int N_i \cdot dx)$ is carried out numerically by using appropriate integration rule. For uniform load acting on the upper surface of prism element, as shown in Fig. (3.2), the integration of shape function are equal to:

b

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$$\int N_i .dx = \frac{1}{6} \quad \text{for nodes 1, 3} \quad (3.42)$$

$$\int N_i .dx = \frac{2.b}{3} \quad \text{for node No.2}$$

Then substitution of Eq. (3.42) into Eq. (3.41) yields:

$$F_{u_y} = \frac{q.b.a}{3.m.} \quad \text{for nodes 1,3} \quad (3.43)$$

$$F_{u_y} = \frac{4.q.b.a}{3.m.\pi} \quad \text{for node No.2}$$

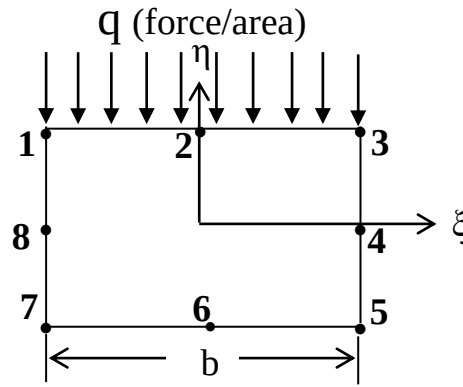


Fig. (3.2) Uniformly loaded prism element.

3.6.2 Point Load

The nodal force vector $\{F_p\}$ as a result of point load $\{f_p\}$ acting at a certain section along the span can be expressed as follows :

$$\{F_p\} = \sum_{m=1}^{\infty} \left[N^m \right]^T . \{f_p\} \quad (3.44)$$

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where $\{fp\}$ is the point load vector act at any position (ξ, η, z) which may have the load component P_x, P_y and P_z in the x, y , and z directions as:

$$\{fp\} = \begin{Bmatrix} P_x \\ P_y \\ P_z \end{Bmatrix} \quad (3.45)$$

Thus

$$\{Fp\} = \sum_{m=1}^{\infty} \sum_{i=1}^{ne} \begin{bmatrix} N_i.S_m & 0 & 0 \\ 0 & N_i.S_m & 0 \\ 0 & 0 & N_i.C_m \end{bmatrix} \begin{Bmatrix} P_x \\ P_y \\ P_z \end{Bmatrix} \quad (3.46)$$

Nevertheless, if the point load is applied at node i , thus the shape function $[N_i]$ will be equal to 1 at that point and zero elsewhere, therefore Eq. (3.46) can be rewritten as :

$$\{Fp\} = \begin{Bmatrix} P_x. \sin \frac{m. \pi z}{a} \\ P_y. \sin \frac{m. \pi .z}{a} \\ P_z. \cos \frac{m. \pi .z}{a} \end{Bmatrix} \quad (3.47)$$

3.7 Reinforcement Representation

3.7.1 Embedded Reinforcement Representation

This representation is usually used with higher isoparametric concrete elements. The reinforcing bar is considered to be an axial member built into the isoparametric concrete element as shown in Fig (3.3). Perfect bond is assumed to occur between the reinforcement and the surrounding concrete. This assumption of compatibility of displacements and strains between the steel and concrete allows the reinforcing bar to be treated as an integral part of the three dimensional element (Zienkiewicz 1977). Reinforcement bars are assumed to be capable of transmitting axial forces only and restricted to be parallel on the local coordinate lines ξ , η , and ζ of the prism.

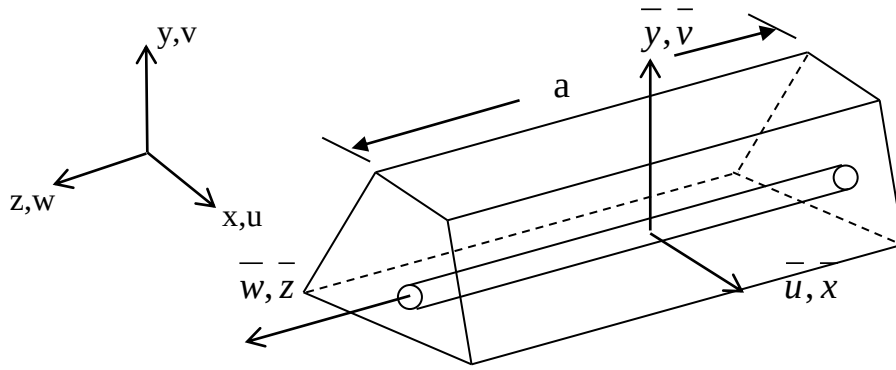


Fig. (3.3) Embedded representation of reinforcement.

3.7.2 Derivation of the stiffness matrix for embedded reinforcement bars

3.7.2.1 Reinforcement bar parallel to the local coordinate axis (ζ)

Consider a bar lying parallel to the local coordinate axis ζ , with $\xi = \xi_0$ and $\eta = \eta_0$, the coordinates of the bar are defined by using the same shape functions used for the prism.

Hence, the Cartesian coordinates of any point along this bar are obtained by submitting the constant values ξ_0 and η_0 such that:

$$\begin{aligned} x &= \sum_{i=1}^{ne} N_i(\xi_0, \eta_0) x_i \\ y &= \sum_{i=1}^{ne} N_i(\xi_0, \eta_0) y_i \end{aligned} \quad (3.48)$$

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Because full compatibility between the bar and the concrete prism is assumed, displacements of the bar are obtained from basic prism displacements field. Thus,

$$\begin{aligned} u &= \sum_{m=1}^r \sum_{i=1}^{ne} N_i(\xi_o, \eta_o) \cdot \sin km_z \cdot u_{im} \\ v &= \sum_{m=1}^r \sum_{i=1}^{ne} N_i(\xi_o, \eta_o) \cdot \sin km_z \cdot v_{im} \\ w &= \sum_{m=1}^r \sum_{i=1}^{ne} N_i(\xi_o, \eta_o) \cdot \cos km_z \cdot w_{im} \end{aligned} \quad (3.49)$$

Since the bar is capable of transmitting axial forces only, one component of displacement contributes to the strain, which defined locally as: -

$$\bar{\varepsilon} = \frac{\partial \bar{w}}{\partial \bar{z}} \quad (3.50)$$

where $\bar{x}, \bar{y}$ and $\bar{z}$ are defined as the local coordinates at point p with $\bar{x}$ and $\bar{y}$ being normal to the line of the bar, and $\bar{u}, \bar{v},$ and $\bar{w}$ are the corresponding displacements. Let a distortion matrix [S] at any point p is defined as: -

$$[S] = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{bmatrix} \quad (3.51)$$

As [S] is a second order tensor, it transforms on coordinate relation from x, y and z, to $\bar{x}, \bar{y}$ and $\bar{z}$ according to: -

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$$[\bar{S}] = \begin{bmatrix} \frac{\partial \bar{u}}{\partial \bar{x}} & \frac{\partial \bar{v}}{\partial \bar{x}} & \frac{\partial \bar{w}}{\partial \bar{x}} \\ \frac{\partial \bar{u}}{\partial \bar{y}} & \frac{\partial \bar{v}}{\partial \bar{y}} & \frac{\partial \bar{w}}{\partial \bar{y}} \\ \frac{\partial \bar{u}}{\partial \bar{z}} & \frac{\partial \bar{v}}{\partial \bar{z}} & \frac{\partial \bar{w}}{\partial \bar{z}} \end{bmatrix} = [R] [S] [R]^T \quad (3.52)$$

where $[R]$ is the rotation matrix of the direction cosines as point p defined as: -

$$[R] = \begin{bmatrix} \frac{\partial x}{\partial \bar{x}} & \frac{\partial y}{\partial \bar{x}} & \frac{\partial z}{\partial \bar{x}} \\ \frac{\partial x}{\partial \bar{y}} & \frac{\partial y}{\partial \bar{y}} & \frac{\partial z}{\partial \bar{y}} \\ \frac{\partial x}{\partial \bar{z}} & \frac{\partial y}{\partial \bar{z}} & \frac{\partial z}{\partial \bar{z}} \end{bmatrix} \quad (3.53)$$

As $\bar{z}$ and ζ coincide and different only in magnitude, hence,

$$\begin{aligned} d\bar{z} &= \left[\left(\frac{\partial x}{\partial \zeta} \right)^2 + \left(\frac{\partial y}{\partial \zeta} \right)^2 + \left(\frac{\partial z}{\partial \zeta} \right)^2 \right]^{\frac{1}{2}} . d\zeta \\ &= \left[0 + 0 + \left(\frac{a}{2} \right)^2 \right]^{\frac{1}{2}} . d\zeta \\ &= \frac{a}{2} d\zeta \end{aligned} \quad (3.54)$$

By multiplication $[R] [S] [R]^T$ the strain component equal to:

$$\begin{aligned} \frac{\partial \bar{w}}{\partial \bar{z}} &= \frac{\partial x}{\partial \bar{z}} \frac{\partial x}{\partial \bar{z}} \frac{\partial u}{\partial x} + \frac{\partial x}{\partial \bar{z}} \frac{\partial y}{\partial \bar{z}} \frac{\partial u}{\partial y} + \frac{\partial x}{\partial \bar{z}} \frac{\partial z}{\partial \bar{z}} \frac{\partial u}{\partial z} + \frac{\partial x}{\partial \bar{z}} \frac{\partial y}{\partial \bar{z}} \frac{\partial v}{\partial x} + \frac{\partial y}{\partial \bar{z}} \frac{\partial y}{\partial \bar{z}} \frac{\partial v}{\partial y} \\ &+ \frac{\partial y}{\partial \bar{z}} \frac{\partial z}{\partial \bar{z}} \frac{\partial v}{\partial z} + \frac{\partial x}{\partial \bar{z}} \frac{\partial z}{\partial \bar{z}} \frac{\partial w}{\partial x} + \frac{\partial y}{\partial \bar{z}} \frac{\partial z}{\partial \bar{z}} \frac{\partial w}{\partial y} + \frac{\partial z}{\partial \bar{z}} \frac{\partial z}{\partial \bar{z}} \frac{\partial w}{\partial z} \end{aligned} \quad (3.55)$$

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By substituting $\partial \bar{z} = \frac{a}{2} d\zeta$

$$\begin{aligned} \frac{\partial \bar{w}}{\partial \bar{z}} = \frac{1}{(a/2)^2} & \left(\frac{\partial x}{\partial \zeta} \cdot \frac{\partial x}{\partial \zeta} \cdot \frac{\partial u}{\partial x} + \frac{\partial x}{\partial \zeta} \cdot \frac{\partial y}{\partial \zeta} \cdot \frac{\partial u}{\partial y} + \frac{\partial x}{\partial \zeta} \cdot \frac{\partial z}{\partial \zeta} \cdot \frac{\partial u}{\partial z} \right. \\ & + \frac{\partial x}{\partial \zeta} \cdot \frac{\partial y}{\partial \zeta} \cdot \frac{\partial v}{\partial x} + \frac{\partial y}{\partial \zeta} \cdot \frac{\partial y}{\partial \zeta} \cdot \frac{\partial v}{\partial y} + \frac{\partial y}{\partial \zeta} \cdot \frac{\partial z}{\partial \zeta} \cdot \frac{\partial v}{\partial z} \\ & \left. + \frac{\partial x}{\partial \zeta} \cdot \frac{\partial z}{\partial \zeta} \cdot \frac{\partial w}{\partial x} + \frac{\partial y}{\partial \zeta} \cdot \frac{\partial z}{\partial \zeta} \cdot \frac{\partial w}{\partial y} + \frac{\partial z}{\partial \zeta} \cdot \frac{\partial z}{\partial \zeta} \cdot \frac{\partial w}{\partial z} \right) \quad (3.56) \end{aligned}$$

and then, substituting $\frac{\partial x}{\partial \zeta} = \frac{\partial y}{\partial \zeta} = 0$

$$\begin{aligned} \bar{\varepsilon} = \frac{\partial \bar{w}}{\partial \bar{z}} &= \frac{1}{(a/2)^2} \cdot \frac{\partial z}{\partial \zeta} \cdot \frac{\partial z}{\partial \zeta} \cdot \frac{\partial w}{\partial z} \\ &= \frac{1}{(a/2)^2} \cdot \frac{a}{2} \cdot \frac{a}{2} \cdot \frac{\partial w}{\partial z} = \frac{\partial w}{\partial z} \\ &= \sum_{m=1}^r \sum_{i=1}^{ne} -N_i(\xi_o, \eta_o) \cdot km \cdot \sin km \cdot z \cdot w_{im} \quad (3.57) \end{aligned}$$

In a matrix from

$$\begin{aligned} \bar{\varepsilon} &= [\bar{B}] \{\delta\} \\ &= \sum_{m=1}^r \sum_{i=1}^{ne} [Bi]_m \cdot \{\delta i\}_m \quad (3.58) \end{aligned}$$

where:-

$$\begin{aligned} [Bi]_m &= -N_i(\xi_o, \eta_o) \cdot km \cdot \sin km \cdot z \\ &= -N_i(\xi_o, \eta_o) \cdot \frac{m\pi}{a} \cdot \sin \frac{m\pi}{a} z \quad (3.59) \end{aligned}$$

or

$$= -N_i(\xi_o, \eta_o) \cdot \frac{m\pi}{a} \sin \frac{m\pi}{2} (1 + \zeta)$$

and

$$\{\delta i\}_m = w_{im} \quad (3.60)$$

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The stiffness matrix of an axially loaded bar may be expressed as:

$$[\bar{K}] = \int_V [\bar{B}]^T [\bar{D}] [\bar{B}] dv \quad (3.61)$$

where:

$$dv = AS. d\bar{z} = AS. \frac{a}{2}.d\zeta \quad (3.62)$$

and $[\bar{D}]$ is the constitutive matrix that represents the modulus of elasticity of steel bar and (AS) is the cross sectional area of the bar. By substituting equation (3.62) into equation (3.61), the stiffness matrix of the embedded bar can be expressed as:

$$[\bar{K}] = AS. \frac{a}{2}. \int_{-1}^{+1} [\bar{B}]^T [\bar{D}] [\bar{B}]. d\zeta \quad (3.63)$$

3.7.2.2 Reinforcement bar parallel to the local coordinate axis (η)

Consider a bar laying parallel to the local coordinate (η) with ($\xi = \xi_o, \zeta = \zeta_o$).

As a ($\bar{y}$) and η coincide and different only in magnitude hence,

$$\begin{aligned} x &= \sum_{i=1}^{ne} N_i(\xi_o, \eta). x_i \\ y &= \sum_{i=1}^{ne} N_i(\xi_o, \eta). y_i \end{aligned} \quad (3.64)$$

$$d\bar{y} = \left[\left(\frac{\partial x}{\partial \eta} \right)^2 + \left(\frac{\partial y}{\partial \eta} \right)^2 + \left(\frac{\partial z}{\partial \eta} \right)^2 \right]^{\frac{1}{2}}. d\eta \quad (3.65)$$

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By substituting $\frac{\partial z}{\partial \eta} = 0$

$$\therefore d\bar{y} = h \cdot d\eta \quad (3.66)$$

$$\text{and } h = \left[\left(\frac{\partial x}{\partial \eta} \right)^2 + \left(\frac{\partial y}{\partial \eta} \right)^2 \right]^{1/2} \quad (3.67)$$

By multiplication $[R] [S] [R]^T$ the component of strain equal to:

$$\begin{aligned} \frac{\partial \bar{v}}{\partial \bar{y}} = & \frac{\partial x}{\partial \bar{y}} \frac{\partial x}{\partial \bar{y}} \frac{\partial u}{\partial x} + \frac{\partial x}{\partial \bar{y}} \frac{\partial y}{\partial \bar{y}} \frac{\partial u}{\partial y} + \frac{\partial x}{\partial \bar{y}} \frac{\partial z}{\partial \bar{y}} \frac{\partial u}{\partial z} + \frac{\partial x}{\partial \bar{y}} \frac{\partial y}{\partial \bar{y}} \frac{\partial v}{\partial x} + \frac{\partial y}{\partial \bar{y}} \frac{\partial y}{\partial \bar{y}} \frac{\partial v}{\partial y} \\ & + \frac{\partial y}{\partial \bar{y}} \frac{\partial z}{\partial \bar{y}} \frac{\partial v}{\partial z} + \frac{\partial x}{\partial \bar{y}} \frac{\partial z}{\partial \bar{y}} \frac{\partial w}{\partial x} + \frac{\partial y}{\partial \bar{y}} \frac{\partial z}{\partial \bar{y}} \frac{\partial w}{\partial y} + \frac{\partial z}{\partial \bar{y}} \frac{\partial z}{\partial \bar{y}} \frac{\partial w}{\partial z} \end{aligned} \quad (3.68)$$

By substituting $\partial \bar{y} = h \cdot \partial \eta$

$$\begin{aligned} \frac{\partial \bar{v}}{\partial \bar{y}} = & \frac{1}{h^2} \left(\frac{\partial x}{\partial \eta} \frac{\partial x}{\partial \eta} \frac{\partial u}{\partial x} + \frac{\partial x}{\partial \eta} \frac{\partial y}{\partial \eta} \frac{\partial u}{\partial y} + \frac{\partial x}{\partial \eta} \frac{\partial z}{\partial \eta} \frac{\partial u}{\partial z} + \frac{\partial x}{\partial \eta} \frac{\partial y}{\partial \eta} \frac{\partial v}{\partial x} + \frac{\partial y}{\partial \eta} \frac{\partial y}{\partial \eta} \frac{\partial v}{\partial y} \right. \\ & \left. + \frac{\partial y}{\partial \eta} \frac{\partial z}{\partial \eta} \frac{\partial v}{\partial z} + \frac{\partial x}{\partial \eta} \frac{\partial z}{\partial \eta} \frac{\partial w}{\partial x} + \frac{\partial y}{\partial \eta} \frac{\partial z}{\partial \eta} \frac{\partial w}{\partial y} + \frac{\partial z}{\partial \eta} \frac{\partial z}{\partial \eta} \frac{\partial w}{\partial z} \right) \end{aligned} \quad (3.69)$$

By substituting $\frac{\partial z}{\partial \eta} = 0$

$$\frac{\partial \bar{v}}{\partial \bar{y}} = \frac{1}{h^2} \left(\frac{\partial x}{\partial \eta} \frac{\partial x}{\partial \eta} \frac{\partial u}{\partial x} + \frac{\partial x}{\partial \eta} \frac{\partial y}{\partial \eta} \frac{\partial u}{\partial y} + \frac{\partial x}{\partial \eta} \frac{\partial y}{\partial \eta} \frac{\partial v}{\partial x} + \frac{\partial y}{\partial \eta} \frac{\partial y}{\partial \eta} \frac{\partial v}{\partial y} \right) \quad (3.70)$$

In a matrix form

$$\bar{\varepsilon} = \frac{\partial \bar{v}}{\partial \bar{y}} = \sum_{m=1}^r \sum_{i=1}^{ne} \frac{1}{h^2} \left[\left[\frac{\partial N_i}{\partial x} \cdot S_m \quad \frac{\partial N_i}{\partial y} \cdot S_m \right] \cdot \begin{bmatrix} C_4 & C_5 \\ C_5 & C_6 \end{bmatrix} \right] \cdot \begin{Bmatrix} u_i \\ v_i \end{Bmatrix}_m \quad (3.71)$$

where :

$$C_4 = \left(\frac{\partial x}{\partial \eta} \right)^2, C_5 = \left(\frac{\partial x}{\partial \eta} \right) \left(\frac{\partial y}{\partial \eta} \right), C_6 = \left(\frac{\partial y}{\partial \eta} \right)^2$$

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Finally eq (3.71) may be written in the form

$$\bar{\varepsilon} = [\bar{B}] \{\delta\} \quad (3.72)$$

The stiffness matrix of an axially loaded bar may be expressed as:

$$[\bar{K}] = \int_v [\bar{B}]^T [\bar{D}] [B] dv \quad (3.73)$$

$$dv = AS \cdot d\bar{y} = AS \cdot h \cdot d\eta \quad (3.74)$$

$$[\bar{K}] = AS \cdot \int_{-1}^{+1} [\bar{B}]^T [\bar{D}] [\bar{B}] h \cdot d\eta \quad (3.75)$$

CHAPTER FOUR
NON LINEAR FINITE STRIP ANALYSIS

4.1 Introduction

The development of an appropriate method for the analysis of folded plate structures, considering material and geometric nonlinearities, is increasingly demanded. Finite strip method as a special form of the finite element method is an efficient method of analysis of folded plate structures with less computational effort and computer core requirements (Cheung 1976).

Over the last four decades the nonlinear analysis of reinforced concrete structures has become popular. This approach can be achieved by carrying out a numerical simulation of the complete structural response up to failure. In particular, it has proved that it is difficult to describe the highly complex behavior of reinforced concrete within the framework of a unified material law because of its extremely nonlinear response. This nonlinear response involving phenomena like cracking of concrete, yielding of reinforcement, and the nonlinear stress-strain relationship of concrete in compression and interactive effects between the concrete and reinforcement (Bergan and Holand 1979). These special properties of reinforced concrete must be considered for material modeling at all stages of loading. Therefore, it is necessary to model the reinforced concrete by considering the constitutive relationships of the composite material. This can be usually achieved by the superposition of constitutive models of the individual material components, concrete and reinforcement (Chen 1982).

The nonlinear behavior of structures may be due to material or geometric nonlinearities. Geometric nonlinearity is associated only with certain special structural members and systems in which the effect of displacements on initial forces must be considered in the analysis. Long columns, flexible arches, and some thin plate and shell structures are examples of such special structural members. On the other hand, material nonlinearities occur in all reinforced concrete structures and should be considered using an accurate rational analysis (Nilson 1982).

At the linear stage of behavior, the solution of the equilibrium equations can be reached in a direct way. However, at the nonlinear stage, it is not possible to solve the governing equations directly. Therefore, it is important to use more sophisticated solution strategies. A combined incremental-iterative solution technique is usually used. In the present study, the Newton-Raphson incremental-iterative solution algorithms have been used to predict the behavior of reinforced concrete folded plate structures up to failure.

4.2 Concrete Models Adopted in the Analysis

In the present study, a plasticity based model is used for simulating the nonlinear behavior of reinforced concrete members under static loads. In compression, the behavior of concrete is simulated by an elasto-plastic work hardening model followed by perfectly plastic response, which is terminated at the beginning of crushing. The model will be illustrated in terms of the following constituents:

1. Yield criterion.
2. Hardening rule.

$$f(I_1, J_2) = (\alpha I_1 + 3\beta J_2)^{1/2} = \sigma_o \quad (4.1)$$

where α and β are material parameters, I_1 is the first stress invariant given by:

$$I_1 = \sigma_x + \sigma_y + \sigma_z \quad (4.2)$$

and J_2 is the second deviatoric stress invariant given by:

$$J_2 = \frac{1}{3} \left\{ \left[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 \right] - \left[\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x \right] \right\} + \left[\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 \right] \quad (4.3)$$

and σ_o is the equivalent effective stress at the onset of plastic deformation that can be determined from the uniaxial compression test as:

$$\sigma_o = C_p \cdot f'_c \quad (4.4)$$

where, C_p , is the plasticity coefficient which is used to mark the initiation of the plastic deformation and ranged as $(0 < C_p < 1.0)$. For normal strength concrete, a value of 0.3 is usually set for C_p (Al-Shaarbaf 1990). The parameters α , and β , are determined from the uniaxial and biaxial experimental studies as:

$$\alpha = 0.35468 \cdot \sigma_o \quad \text{and} \quad \beta = 1.35468 \quad (4.5)$$

Let

$$c = \frac{\alpha}{2\sigma_o} = 0.17734 \quad (4.6)$$

Therefore, equation (4.1) can be rewritten as:

$$f(\{\sigma\}) = (2c\sigma_o I_1 + 3\beta J_2)^{1/2} = \sigma_o \quad (4.7)$$

This can be solved for σ_o ,

$$f(\{\sigma\}) = cI_1 + \left[(cI_1)^2 + 3\beta J_2 \right]^{1/2} = \sigma_o \quad (4.8)$$

4.2.1.2 The Hardening Rule

The hardening rule is required to define the growth of the subsequent loading surfaces during plastic loading. Many types of hardening rules has been proposed to describe the growth of subsequent loading surface for work-hardening materials. Some of these rules are; isotropic hardening, kinematic hardening and mixed hardening. The isotropic model applies mainly to proportional loading. For cyclic and reversed types of loading, kinematic hardening rule is more appropriate. Combinations of isotropic and kinematic hardening are called mixed hardening rules (Chen 1982).

The isotropic hardening rule has been adopted in the present study to simulate concrete behavior. This rule assumes that the yield surface expands uniformly without distortion as plastic deformation occurs. Therefore, the subsequent loading surfaces may be written as:

$$f(\{\sigma\}) = cI_1 + \left[(cI_1)^2 + 3\beta J_2 \right]^{1/2} = \bar{\sigma} \quad (4.9)$$

where the effective stress or equivalent uniaxial stress $\bar{\sigma}$ represents the stress level at which further plastic deformation will occur. A relationship between the accumulated (effective) plastic strain, (ϵ_p) and effective (equivalent) stress, $\bar{\sigma}$ is required to control the size of expansion. The concept

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of effective plastic strain and effective stress is used to allow for extrapolation from the results of a uniaxial test to a multiaxial situation. In the present model, a parabolic stress-strain curve is used for the uniaxial stress-strain relationship beyond the limit of elasticity, $C_p f'_c$. This parabolic curve represents the work-hardening stage of behavior. When the peak compressive stress is reached, a perfectly plastic response is assumed to occur. Fig (4.2) shows the equivalent uniaxial stress-strain curve at the various stages of behavior. These relationships are given by:

(a) For $\bar{\sigma} \leq C_p f'_c$

$$\bar{\sigma} = E \cdot \varepsilon_c \quad (4.10)$$

(b) For $C_p f'_c \leq \bar{\sigma} \leq f'_c$

$$\bar{\sigma} = C_p \cdot f'_c + E \left[\varepsilon_c - \frac{C_p \cdot f'_c}{E} \right] - \frac{E}{2\varepsilon'_o} \left[\varepsilon_c - \frac{C_p \cdot f'_c}{E} \right]^2 \quad (4.11)$$

(c) For $\varepsilon_c \geq (2 - C_p) \cdot \frac{f'_c}{E}$

$$\bar{\sigma} = f'_c \quad (4.12)$$

where ε'_o is the total strain corresponding to the parabolic part of the curve given by:

$$\varepsilon'_o = 2(1 - C_p) \frac{f'_c}{E} \quad (4.13)$$

For normal strength concrete, a value of 0.3 is assumed for the plastic coefficient C_p in the present study and hence plastic yielding begins at a stress level equal to $0.3 f'_c$. By substituting the value of the total strain component,

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which is the sum of the elastic and plastic strains into equation (4.11), the effective stress-plastic strain relationship can be expressed as:

$$\bar{\sigma} = C_p f'_c - E \varepsilon_p + (2E^2 \varepsilon'_o \varepsilon_p)^{1/2} \quad (4.14)$$

The slope of the effective stress-plastic strain curve, H' the hardening parameter, is used in the formulation of the elasto-plastic incremental stress-strain relationship. By differentiation of equation (4.14) with respect to the effective plastic strain, the hardening parameter can be expressed as:

$$H' = \frac{d\bar{\sigma}}{d\varepsilon_p} = E \left[\left(\frac{\varepsilon'_o}{2\varepsilon_p} \right)^{1/2} - 1.0 \right] \quad (4.15)$$

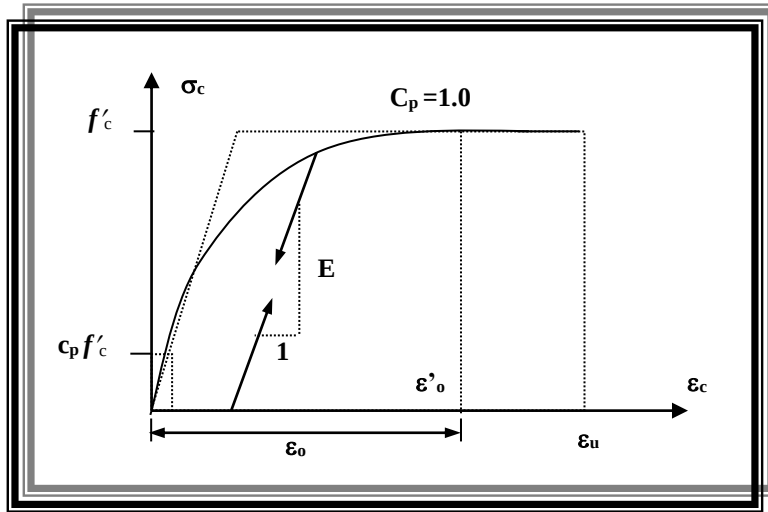


Fig. (4.2) Uniaxial stress-strain curve for concrete (Al-Shaarbaf 1990).

4.2.1.3 The Flow Rule

The flow rule is necessary to connect the loading surface with the stress-strain relationship (Chen 1982). Assuming that the plastic strain increment vector will be normal to the yield surface, the plastic strain increment can be determined for a given stress increment as:

$$d\{\epsilon_p\} = d\lambda \frac{\partial f(\{\sigma\})}{\partial \{\sigma\}} \quad (4.16)$$

where, $d\lambda$ is the hardening or the loading parameter which determines the size of the plastic strain increment and $\partial f(\{\sigma\})/\partial \{\sigma\}$ defines the direction of the plastic strain increment vector $d\{\epsilon_p\}$ as normal to the current loading surface. The yield function derivatives with respect to the stress components define the flow vector $\{a\}$ as:

$$\{a\}^T = \left[\frac{\partial f}{\partial \sigma_x}, \frac{\partial f}{\partial \sigma_y}, \frac{\partial f}{\partial \sigma_z}, \frac{\partial f}{\partial \tau_{xy}}, \frac{\partial f}{\partial \tau_{yz}}, \frac{\partial f}{\partial \tau_{zx}} \right]^T \quad (4.17)$$

where,

$$\left. \begin{aligned} a_1 &= \frac{\partial f}{\partial \sigma_x} = c + \left[2(c^2 + \beta)\sigma_x + (2c^2 - \beta)(\sigma_y - \sigma_z) \right] / Q \\ a_2 &= \frac{\partial f}{\partial \sigma_y} = c + \left[2(c^2 + \beta)\sigma_y + (2c^2 - \beta)(\sigma_x - \sigma_z) \right] / Q \\ a_3 &= \frac{\partial f}{\partial \sigma_z} = c + \left[2(c^2 + \beta)\sigma_z + (2c^2 - \beta)(\sigma_x - \sigma_y) \right] / Q \\ a_4 &= \frac{\partial f}{\partial \tau_{xy}} = 6\beta\tau_{xy} / Q \\ a_5 &= \frac{\partial f}{\partial \tau_{yz}} = 6\beta\tau_{yz} / Q \\ a_6 &= \frac{\partial f}{\partial \tau_{zx}} = 6\beta\tau_{zx} / Q \end{aligned} \right\} \quad (4.18)$$

where c and β , are the material constants, and Q is given by:

$$Q = 2 \left[\left(c^2 + \beta \right) \left(\sigma_x^2 + \sigma_y^2 + \sigma_z^2 \right) + \left(2c^2 - \beta \right) \left(\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x \right) + 3\beta \left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 \right) \right]^{1/2} \quad (4.19)$$

The plastic multiplier, $d\lambda$, can be found as:

$$d\lambda = \left[\frac{\{a\}^T [D]}{H' + \{a\}^T [D] \{a\}} \right] d\{\varepsilon\} \quad (4.20)$$

where $[D]$ is the elastic constitutive matrix, $d\{\varepsilon\}$ is the total strain increment vector and H' is the hardening parameter ($H' = \frac{d\bar{\sigma}}{d\varepsilon_p}$). The elastic strain increment is related to the stress increment by the elastic constitutive relation given by:

$$d\{\sigma\} = [D] d\{\varepsilon\}^e \quad (4.21)$$

and

$$d\{\varepsilon\} = d\{\varepsilon^e\} + d\{\varepsilon^p\} \quad (4.22)$$

Substitution of equation (4.16) and (4.21) into equation (4.22) yields:

$$d\{\varepsilon\} = [D]^{-1} d\{\sigma\} + d\lambda \frac{\partial f}{\partial \sigma} \quad (4.23)$$

$$d\{\varepsilon\} = [D]^{-1} d\{\sigma\} + \frac{\{a\}^T [D] \{a\}}{H' + \{a\}^T [D] \{a\}} d\{\varepsilon\} \quad (4.24)$$

Pre-multiplying both sides of equation (4.24) by $[D]$, the complete elastoplastic incremental stress-strain relationship can be expressed as:

$$d\{\sigma\} = [D]_{ep} d\{\varepsilon\} \quad (4.25)$$

where,

$$[D]_{ep} = \left[[D] - \frac{[D]\{a\}\{a\}^T[D]}{H' + \{a\}^T[D]\{a\}} \right] \quad (4.26)$$

In this equation, the second term in the brackets represents the degradation of stiffness matrix due to the plastic deformations.

4.2.1.4 The Crushing Condition

Crushing indicates the complete rupture and disintegration of the material under compressive stress state. After crushing, the current stresses drop rapidly to zero and the concrete is assumed to lose its resistance completely against further deformation. In the adopted model, concrete is considered to crush when the strain reaches a specified ultimate value. Hence, rewriting the yield condition from Eq (4.8) in terms of the strains instead of stress components (Chen 1982), the following crushing criterion is obtained:

$$C \bar{I}_1 + \sqrt{(C\bar{I}_1)^2 + \beta \bar{J}_2} = \varepsilon_{cu} \quad (4.27)$$

where $\bar{I}_1$: is the first strain invariant.

$\bar{J}_2$: is the second deviatoric strain.

ε_{cu} : is the ultimate concrete strain that can be extrapolated from the uniaxial compression test.

4.2.2 Modeling of Concrete in Tension

The tensile strength of concrete is much smaller than the compressive strength and its influence on the behavior of reinforced concrete structure is much sensible. In the present study smeared representation for cracks is used, a linear-elastic response of concrete is assumed until the fracture surface is reached. The fixed orthogonal crack model is used here, in which, cracks are assumed to form in a plane perpendicular to the direction of the maximum tensile principal stresses.

4.2.2.1 The Cracking Criterion

In the present study, the maximum tensile stress criterion is used to monitor cracking. Cracking occurs if the principal tensile stress exceeds the limiting tensile strength of concrete. Also a plane of fracture is assumed to develop perpendicular to the direction of principal stress. The limiting tensile stress required to define the onset of cracking can be calculated for states of triaxial tensile stress and for combinations of tension and compression principal stresses as follows (Al-Shaarbaf 1990):

- (a) For the triaxial tension zone ($\sigma_1 \geq \sigma_2 \geq \sigma_3 > 0$)

$$\sigma_i = \sigma_{cr} = f_t, \quad i = 1, 2, 3 \quad (4.28)$$

- (b) For the tension-tension-compression zone ($\sigma_1 \geq \sigma_2 > 0, \sigma_3 \leq 0$)

$$\sigma_i = \sigma_{cr} = f_t \left[1 + \frac{0.75\sigma_3}{f'_c} \right] \quad i = 1, 2 \quad (4.29)$$

- (c) For the tension-compression-compression zone ($\sigma_1 > 0, \sigma_3 \leq \sigma_2 \leq 0$)

$$\sigma_1 = \sigma_{cr} = f_t \left[1 + \frac{0.75\sigma_2}{f'_c} \right] \left[1 + \frac{0.75\sigma_3}{f'_c} \right] \quad (4.30)$$

where σ_{cr} is the cracking stress and both f_t and f'_c are given positive values. Equation (4.29) and (4.30) indicate that the presence of compressive stress in one direction favors the cracking in the other directions and reduces the tensile capacity of the material.

After cracking, the normal and shear stresses across the plane of failure and the corresponding normal and shear stiffness are reduced. However, the behavior of the material between two adjacent failure planes remains linearly elastic, i.e., concrete is assumed to be transversely isotropic with planes of isotropy being perpendicular to the major principal stress direction which violates the cracking criterion. Thus, the elastic modulus in the direction of maximum tensile stress σ_1 is reduced. Because of the lack of interaction between the orthogonal planes caused by cracking, Poisson's ratio, ν is set to zero and a reduced shear modulus $\beta_1 G$ is employed to model the shear strength deterioration. Therefore, the incremental stress-strain relationship in the local material axes may be expressed as:

$$\begin{Bmatrix} \Delta\sigma_1 \\ \Delta\sigma_2 \\ \Delta\sigma_3 \\ \Delta\tau_{12} \\ \Delta\tau_{23} \\ \Delta\tau_{31} \end{Bmatrix} = \begin{bmatrix} E_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & E/1-\nu^2 & \nu E/1-\nu^2 & 0 & 0 & 0 \\ 0 & \nu E/1-\nu^2 & E/1-\nu^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_1 G \end{bmatrix} \begin{Bmatrix} \Delta\epsilon_1 \\ \Delta\epsilon_2 \\ \Delta\epsilon_3 \\ \Delta\gamma_{12} \\ \Delta\gamma_{23} \\ \Delta\gamma_{31} \end{Bmatrix} \quad (4.31)$$

Equation (4.31) may be written as:

$$\Delta\{\sigma\} = [D_{cr}] \Delta\{\epsilon\} \quad (4.32)$$

where $[D_{cr}]$ is the material stiffness in local material axes. The elastic material between two adjacent failure planes is presented in equation (4.32) by a plane

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stress constitutive relation. The stress increments in the global axes (x, y, z) may be obtained by using the coordinate transformation matrix such that,

$$\Delta\{\sigma\} = [T]^T [D_{cr}] [T] \Delta\{\varepsilon\} \quad (4.33)$$

where [T] is the transformation matrix expressed in terms of direction cosines as follows (Al-Shaarbaf 1990):

$$[T] = \begin{bmatrix} i_1^2 & m_1^2 & n_1^2 & i_1 m_1 & m_1 n_1 & n_1 i_1 \\ i_2^2 & m_2^2 & n_2^2 & i_2 m_2 & m_2 n_2 & n_2 i_2 \\ i_3^2 & m_3^2 & n_3^2 & i_3 m_3 & m_3 n_3 & n_3 i_3 \\ 2i_1 i_2 & 2m_1 m_2 & 2n_1 n_2 & i_1 m_2 + i_2 m_1 & n_2 m_1 + n_1 m_2 & i_2 n_1 + i_1 n_2 \\ 2i_2 i_3 & 2m_2 m_3 & 2n_2 n_3 & i_2 m_3 + i_3 m_2 & n_3 m_2 + n_2 m_3 & i_3 n_2 + i_2 n_3 \\ 2i_3 i_1 & 2m_3 m_1 & 2n_3 n_1 & i_3 m_1 + i_1 m_3 & n_1 m_3 + n_3 m_1 & i_1 n_3 + i_3 n_1 \end{bmatrix}$$

where i_i , m_i and n_i represent the direction cosines of the local coordinate axes with (x, y and z) direction respectively.

For the tension-tension-compression and the triaxial tension states, the cracking criterion may be violated by the major principal stress, σ_1 and the second principal stress, σ_2 simultaneously. Thus, two sets of orthogonal failure planes develop. These planes are perpendicular to the principal axes 1 and 2 respectively. In this case, Poisson's ratio is set to zero in all directions and the constitutive matrix in the local material axes becomes a diagonal matrix,

$$[D_{cr}] = \begin{bmatrix} E_1 & 0 & 0 & 0 & 0 & 0 \\ & E_2 & 0 & 0 & 0 & 0 \\ & & E & 0 & 0 & 0 \\ & & & \beta_1 G & 0 & 0 \\ \text{Sym.} & & & & \beta_2 G & 0 \\ & & & & & \beta_1 G \end{bmatrix} \quad (4.34)$$

In the current model, a maximum of three sets of cracks are allowed to form at each sampling point.

4.2.2.2 Post Cracking Models

The tensile stresses normal to the cracked plane are gradually released, and are usually represented by an average stress-strain curve. To obtain such a relationship, the tension-stiffening concept is usually adopted (Al-Shaarbaf 1990). Since the cracked concrete can still initially carry some tensile stresses in the direction normal to the crack, the tension-stiffening effect is considered. This can be achieved by assuming a gradual release of the concrete stress component normal to the cracked plane. In the present work, the normal stress that is carried by cracked concrete can be obtained from the following expression:

a) For $\varepsilon_{cr} \leq \varepsilon_n \leq \alpha_1 \varepsilon_{cr}$

$$\sigma_n = \alpha_2 \sigma_{cr} [\alpha_1 - \varepsilon_n / \varepsilon_{cr}] / [\alpha_1 - 1.0] \quad (4.35)$$

b) For $\varepsilon_n > \alpha_1 \varepsilon_{cr}$

$$\sigma_n = 0.0 \quad (4.36)$$

where ε_{cr} is the cracking strain associated with the cracking stress σ_{cr} and α_1 and α_2 are the tension-stiffening parameters. α_1 represents the rate of stress release as the crack widens; α_2 represents the sudden loss of stress at instant of cracking. The unloading and reloading behavior can be obtained by using the secant path shown in Fig. (4.3).

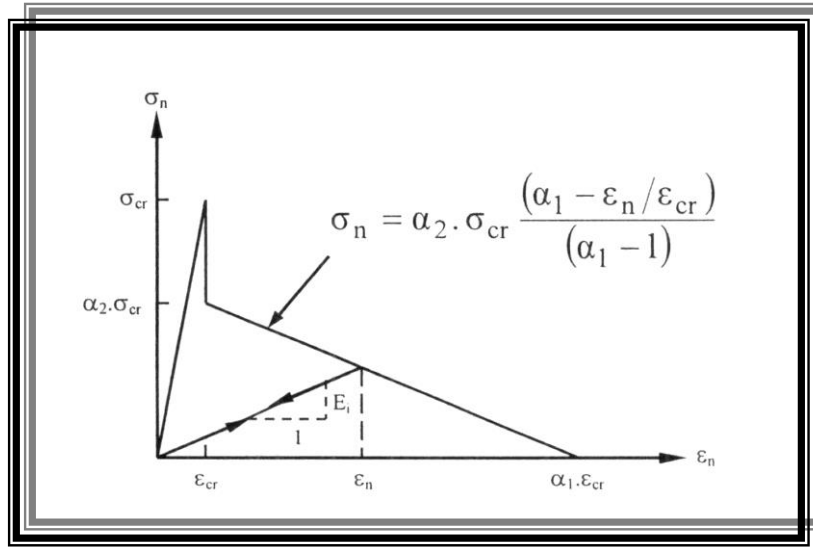


Fig. (4.3) Post cracking model for concrete (Chen 1982).

4.2.2.3 Shear Retention Model

At a cracked sampling point, the shear stiffness across the cracked plane becomes progressively smaller as the crack wideness. A reduced shear modulus βG has been used across the cracked plane. Before cracking, a value of unity is assigned to shear reduction factor β . As the crack propagates, the shear reduction factor is taken to be linearly decreased with the strain normal to the cracked plane, which represents the crack width. When the cracks have sufficiently opened, a constant value is assigned to β to account for dowel action. The shear retention model can be expressed as: (Al –Shaarbaf 1990).

(a) For $\varepsilon_n < \varepsilon_{cr}$

$$\beta = 1.0 \quad (4.37)$$

(b) For $\varepsilon_{cr} \leq \varepsilon_n \leq \gamma_1 \varepsilon_{cr}$

$$\beta = \frac{\gamma_2 - \gamma_3}{\gamma_1 - 1} \left[\gamma_1 - \varepsilon_n / \varepsilon_{cr} \right] + \gamma_3 \quad (4.38)$$

(c) For $\varepsilon_n > \gamma_1 \varepsilon_{cr}$

$$\beta = \gamma_3 \quad (4.39)$$

where, γ_1 represents the rate of decay of shear stiffness as the crack widens, γ_2 represents the sudden loss in shear stiffness at the instant of cracking, and γ_3 represents the residual shear stiffness due to dowel action.

When the crack is closed, it is assumed that the original shear modulus is retained, as shown in Fig. (4.4) and a value of unity is assigned to the shear reduction factor β .

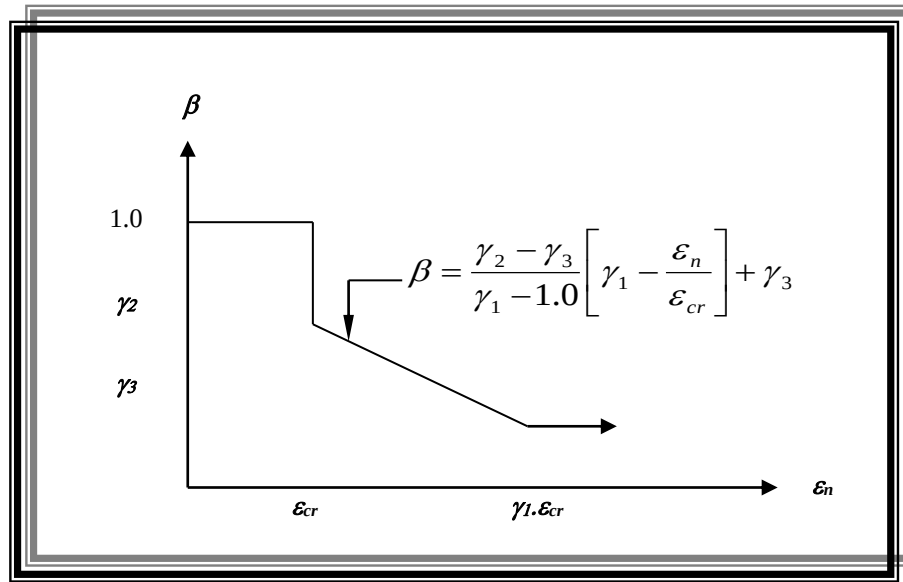


Fig (4.4) Shear-retention model for concrete (Al-Shaarbaf 1990).

4.3 Modeling of Reinforcement

The mechanical properties of steel in comparison to the properties of concrete are well-known and better understood. Steel is homogeneous material and has usually the same yield strength in tension and compression. In reinforced concrete members, reinforcing bars are usually long and relatively slender and therefore can be generally assumed to be capable of transmitting axial forces only.

In the present study, the uniaxial stress-strain behavior of steel bars has been simulated by an elastic-linear work hardening model as shown in Fig. (4.5).

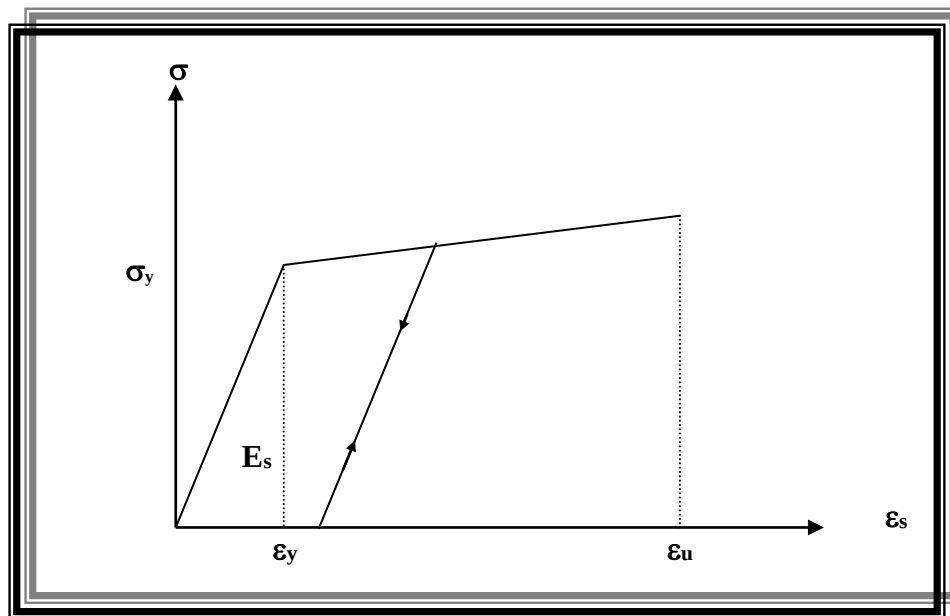


Fig (4.5) Stress-strain relationship for steel bars.

4.4 Nonlinear Geometric Analysis

Consideration of geometric nonlinearity plays an important role in the analysis of thin reinforced concrete slabs, plates, and shells (Bergan and Holand 1979). In general, it is not known in advance in what category the real structure can be classified and the geometric nonlinear analysis is needed to avoid underestimating or overestimating the failure load.

Two basically different formulations can be employed for the description of large deformation problems. The first is the total Lagrangian approach in which the current (2nd Piola-Kirchhoff) stress and (Green-Lagrange) strain fields are referred to the original geometric configuration and the displacement field gives the current configuration of the system in relation to its initial position. The second is the updated Lagrangian approach in which the current configuration of the system is used to define the current (Cauchy) stress and (Almansi) strain state. The geometry of the structure is successively updated during the incremental process and the stress and strain fields are referred to the last evaluated configuration.

In the present work, the total Lagrangian approach has been adopted for implementing the geometrical nonlinearity.

4.4.1 Geometric Nonlinear Formulations for Quadratic Prism

The strain components in equation (3.8) can be separated into a linear $\{\varepsilon_o\}$ and a nonlinear $\{\varepsilon_{nl}\}$ part, which can be written as:

$$\{\varepsilon\} = \{\varepsilon_o\} + \{\varepsilon_{nl}\} \quad (4.40)$$

where,

$$\{\varepsilon_o\} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \end{bmatrix}, \quad \{\varepsilon_{nl}\} = \begin{bmatrix} \frac{1}{2} \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial x} \right)^2 \right] \\ \frac{1}{2} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \\ \frac{1}{2} \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] \\ \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial y} \frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial x} \frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial z} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial z} \end{bmatrix} \quad (4.41)$$

and,

$$d\{\varepsilon\} = d\{\varepsilon_o\} + d\{\varepsilon_{nl}\} \quad (4.42)$$

4.4.2 Von-Karman Assumptions

Von-Karman assumptions for large deflections of plates should take the following form when applied to plates:

- 1-Deflection magnitude (v) is of the order of the thickness.
- 2-All components of strain are small.
- 3-The thickness (h) is much smaller than the length (L), (this hypothesis is not restrictive since otherwise the displacement are not large).
- 4-The tangential displacements, u and w are small. Only nonlinear terms depend on the derivatives of v with respect to x and z ($\partial v / \partial x, \partial v / \partial z$) have to be retained in the strain-displacement relations.
- 5-Everywhere when $\partial v / \partial x \ll 1, \partial v / \partial z \ll 1$, the slope is small.

Applying these assumptions, the nonlinear part of strain can be expressed as:

$$\{\varepsilon_{nl}\} = \begin{bmatrix} \frac{1}{2} \left[\left(\frac{\partial v}{\partial x} \right)^2 \right] \\ 0 \\ \frac{1}{2} \left[\left(\frac{\partial v}{\partial z} \right)^2 \right] \\ 0 \\ 0 \\ \frac{\partial v}{\partial x} \frac{\partial v}{\partial z} \end{bmatrix} \quad (4.43)$$

1. Strain-Displacement Matrix [B]

The strain-displacement matrix [B] may be separated into two parts,

$$[B] = [B_o] + [B_{nl}] \quad (4.44)$$

where, [B_o] is the linear part, and [B_{nl}] is the nonlinear part

The strain-displacement matrix [B_o] is related to the linear components of the strain vector {ε_o}, while, the large strain-displacement matrix [B_{nl}] is related to the nonlinear components of the strain vector, {ε_{nl}}, and can be defined for use in the nonlinear process. In modeling the reinforced concrete members, the total strain has been used and the total strain-displacement relationship should be defined. For the quadratic prism, the nonlinear components of the strain vector can be expressed by:

$$\{\epsilon_{nl}\} = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 & \frac{\partial v}{\partial x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial v}{\partial z} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial v}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial v}{\partial z} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial v}{\partial z} & 0 & \frac{\partial v}{\partial x} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \partial v / \partial x \\ 0 \\ \partial v / \partial z \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (4.45)$$

The nonlinear contribution to the strain vector can be written in a compact form as:

$$\{\epsilon_{nl}\} = \frac{1}{2} [A] \{R\} \quad (4.46)$$

The derivatives of u, v and w can be calculated by summation of the contribution of each nodal line variable, and vector {R} can be written as:

$$\{R\} = \sum_{m=1}^r \sum_{i=1}^{ne} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial X} * Sm & 0 \\ 0 & 0 & 0 \\ 0 & N_i * Km * Cm & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_{im} \\ v_{im} \\ w_{im} \end{Bmatrix} \quad (4.47)$$

where: $Sm = \sin km \ z$, $Cm = \cos km \ z$

$$Km = \frac{m \pi}{a}$$

a=length of prism

Equation (4.47) can be written in the following form as:

$$\{R\} = [G] \{a\} \quad (4.48)$$

By taking the variation of equation (4.46) the following expression is obtained:

$$d\{\varepsilon_{nl}\} = \frac{1}{2}d[A]\{R\} + \frac{1}{2}[A] d\{R\} = [A] d\{R\} = [A] [G]\{a\} \quad (4.49)$$

then from the definition of the strain matrix, the following expression can be obtained:

$$[B_{nl}] = [A] [G] \quad (4.50)$$

In the present work, matrices $[B_o]$ and $[G]$ are initially calculated and stored. Using the current displacements, matrix $[A]$ is formed and then $[B_{nl}]$ is obtained from equation (4.50). The incremental strain-displacement matrix $[B]$ is evaluated by summing the matrix $[B_o]$ to the current value of matrix $[B_{nl}]$.

2. Tangential Stiffness Matrix [K]

The internal nodal line forces vector which equivalent to the internal stress level can be expressed as $\{p\} = \int_V [B]^T \{\sigma\} dV$, The tangential stiffness matrix for the current configuration $[K]$ can be derived from the variation of $\{p\}$ with respect to a displacement variation $\{a\}$.

$$d\{p\} = \int_V [B^T] d\{\sigma\} dV + \int_V d[B]^T \{\sigma\} dV = [K] d\{a\} \quad (4.51)$$

where,

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$$d\{\sigma\} = [D] \quad d\{\epsilon\} = [D] \quad [B] \quad d\{a\} \quad (4.52)$$

and,

$$d[B]^T = d[B_{nl}]^T \quad (4.53)$$

The initial stress or geometric stiffness matrix, $[K]_\sigma$, for the definition of the second term of equation (4.51) is given by:

$$[K]_\sigma d\{a\} = \int_V d[B]^T \{\sigma\} dV = \int_V d[B_{nl}]^T \{\sigma\} dV \quad (4.54)$$

then the tangential stiffness matrix can be written as:

$$[K] = [\bar{K}] + [K]_\sigma \quad (4.55)$$

where, $[\bar{K}]$ is equal to $\int_V [B]^T [D] [B] dv$. The geometric stiffness matrix $[K]_\sigma$

must be defined explicitly in order to determine the tangential stiffness matrix $[K]$. Substitution of equation (4.50) into equation (4.54) yields:

$$[K]_\sigma d\{a\} = \int_V [G]^T d[A]^T \{\sigma\} dV \quad (4.56)$$

where $d[A]^T \{\sigma\}$ can be written in the form as:

$$d[A]^T \{\sigma\} = d \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \partial v / \partial x & 0 & 0 & 0 & 0 & \partial v / \partial z \\ 0 & 0 & 0 & \partial v / \partial x & \partial v / \partial z & 0 \\ 0 & 0 & \partial v / \partial z & 0 & 0 & \partial v / \partial x \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} \quad (4.57)$$

which can be written as:

$$d[A]^T \{\sigma\} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau_{xy} & \sigma_y & \tau_{yz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau_{xz} & \tau_{yz} & \sigma_z & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_x & \tau_{xy} & \tau_{xz} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_{xy} & \sigma_y & \tau_{yz} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_{xz} & \tau_{yz} & \sigma_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_x & \tau_{xy} & \tau_{xz} \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_{xy} & \sigma_y & \tau_{yz} \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} d \left\{ \begin{array}{c} 0 \\ 0 \\ 0 \\ \partial v / \partial x \\ 0 \\ \partial v / \partial z \\ 0 \\ 0 \\ 0 \end{array} \right\} \quad (4.58)$$

The term $d[A]^T \{\sigma\}$ can be written, with the definition of $[G]$ from equation (4.48) as:

$$d[A]^T \{\sigma\} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau_{xy} & \sigma_y & \tau_{yz} & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau_{xz} & \tau_{yz} & \sigma_z & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_x & \tau_{xy} & \tau_{xz} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_{xy} & \sigma_y & \tau_{yz} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_{xz} & \tau_{yz} & \sigma_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_x & \tau_{xy} & \tau_{xz} \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_{xy} & \sigma_y & \tau_{yz} \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} [G] d\{a\} \quad (4.59)$$

By substituting equation (4.59) into equation (4.56), and denoting by $[\sigma]$ the first matrix in the right hand of (4.59), the geometric stiffness matrix can be written as:

$$[K]_{\sigma\sigma} = \int_V [G]^T [\sigma] [G] dV \quad (4.60)$$

4.5 Solution Techniques for Nonlinear Equations

The application of the finite strip method in nonlinear analysis of reinforced concrete structures needs a set of simultaneous equations can be expressed as:

$$[K]\{a\}=\{f\} \quad (4.61)$$

where,

$\{a\}$ is the unknown nodal line displacement vector.

$\{f\}$ is the nodal line applied force vector.

$[K]$ is the stiffness matrix $(\int_V [B]^T [D] [B] dV)$

The solution for these equations for linear elastic structural problems can be obtained directly. In nonlinear problems, a direct solution is no longer possible since the stiffness matrix $[K]$ depends on the displacement level ($[K]=[K](\{a\})$), and therefore, it can not be exactly calculated before the determination of the unknown nodal line displacements $\{a\}$. For the solution of a nonlinear structural problem, the state of equilibrium of the structural system corresponding to the applied load must be found. These equilibrium equations can be derived by applying the equilibrium to the structural system, and can be expressed as:

$$\{r\}=\{p\}-\{f\} \quad (4.62)$$

where $\{r\}$ is the out of balance force vector and $\{p\}$ represents the vector of the nodal line forces equivalent to the internal stress level, which is given by:

$$\{p\}=\int_V [B]^T \{\sigma\} dV \quad (4.63)$$

The equivalent internal forces also depend on the displacement level ($\{p\}=\{p\}(\{a\})$) and have to be approximated in successive steps until equation (4.62) is satisfied. The basic solution of this system is usually

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attempted by either an incremental method, an iterative method or combination of these two methods.

The incremental-iterative method is the most commonly technique used for solving nonlinear problems. The solution can be carried out effectively by dividing the total external load into small increments. Within each increment of loading, a number of iteration cycles are performed in order to satisfy the equilibrium equations according to a specified tolerance.

The incremental-iterative algorithm used in this study is the modified Newton-Raphson method, in which the stiffness matrix is updated at certain iterations of each increment of loading as shown in Fig. (4.6).

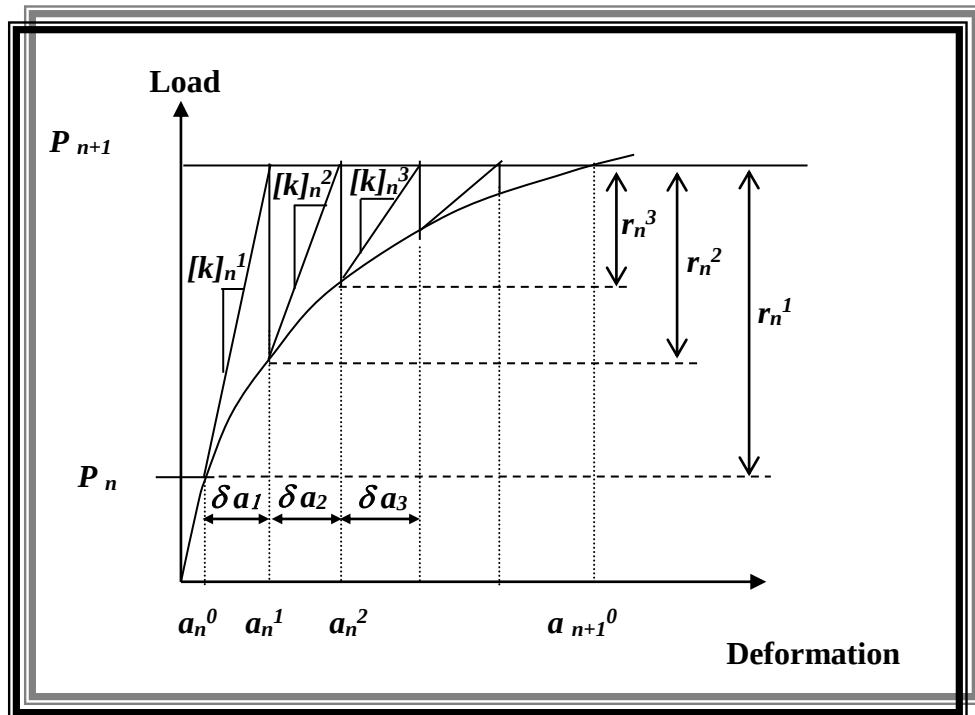


Fig. (4.6) Modified Newton–Raphson technique.

4.6 Convergence Criteria

A convergence criterion is necessary to stop the iterative process when the solution is considered to be sufficiently accurate. For nonlinear structural analysis, several convergence criteria can be used to monitor equilibrium. These criteria are usually based on displacements, out of balance forces, and internal energy.

The force based convergence criterion which has been adopted in the current study is based on the Euclidean norm and may generally written in the form:

$$\frac{\|\{r\}\|}{\|\{f\}\|} \times 100\% \leq \text{TOLER} \quad (4.64)$$

where,

$$\|\{r\}\| = (\{r\}^T \{r\})^{1/2} \quad (4.65)$$

$$\|\{f\}\| = (\{f\}^T \{f\})^{1/2} \quad (4.66)$$

and TOLER is a specified convergence tolerance.

4.7 Numerical Integration

The exact integration of Eq (3.35) is difficult to reach, so it is necessary to apply numerical integration to set up the prism stiffness matrix. The Gauss-Legendre quadratic scheme has been found to be accurate and efficient. The finite integral of the prism stiffness matrix given in Eq. (3.35), can be expressed in the form [Robinson (1973), Dawe (1984)]:

$$I = \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 F(\xi, \eta, \zeta) d\xi d\eta d\zeta \quad (4.67)$$

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which may be rewritten numerically as:

$$I = \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} \sum_{k=1}^{n_k} W_i.W_j.W_k F(\xi_i, \eta_j, \zeta_k) \quad (4.68)$$

where **n_i** , **n_j** , **n_k** are the number of Gaussian points in the ξ_i, η_j, ζ_k direction respectively. The function **$F(\xi_i, \eta_j, \zeta_k)$** represents the

matrix multiplication $([B]^T.[D].[B] \frac{a}{2} \det [J])$ at sampling points ξ_i, η_j, ζ_k .

In a similar manner, the integral of the stiffness matrix of the embedded reinforcement can be written as:

$$I = \sum_{i=1}^{n_i} W_i F(\xi_i) \quad (4.69)$$

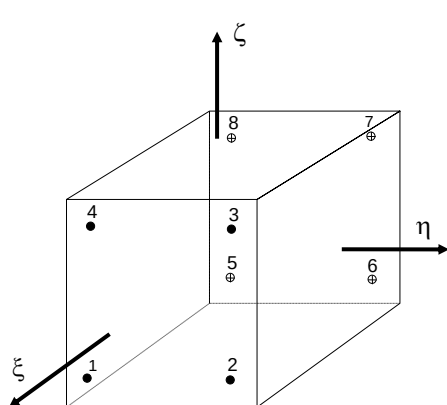
Several types of integration rules can be used such as the eight (2*2*2), and the twenty-seven (3*3*3) Gaussian point rules to integrate the stiffness matrix (Bathe 1996). Also, there is a fifteen-point Gauss type integration rule (Irons 1971).

The prescribed integration rules, which are used in the present study. The weights and abscissa of the sampling points are listed in Table (4.1), also a relative distribution of the this points over the element is given in Fig (4.7).

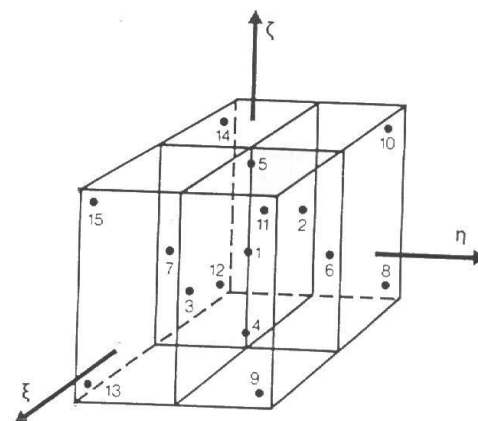
Table (4.1) Weights and abscissa for 8, 15a and 27 points integration rules.

Integration rule	Sampling point Number	ξ	η	ζ	Weight
8-Point Rule	1 - 8	+0.57734	+0.57734	+0.57734	1.0

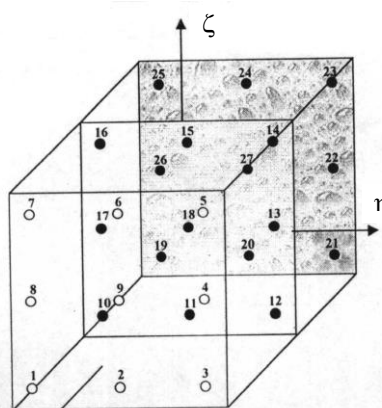
15a-point Rule	1	0.0	0.0	0.0	1.5644
	2 , 3	± 1.0	0.0	0.0	0.35556
	4 , 5	0.0	0.0	± 1.0	0.35556
	6 , 7	0.0	± 1.0	0.0	0.35556
	8 - 15	± 0.67410	± 0.67410	± 0.67410	0.53778
27-Point Rule	1, 3, 5, 7, 19, 21, 23, 25	± 0.7746	± 0.7746	± 0.7746	0.171468
	2, 4, 6, 8, 10, 12, 14, 16, 20, 22, 24, 26	± 0.7746	± 0.7746	± 0.7746	0.274348
	9, 11, 13, 15, 17, 27	± 0.7746	± 0.7746	± 0.7746	0.438957
	18	0.0	0.0	0.0	0.702332



(a) 8- point integration rule



(b) 15- point integration rule

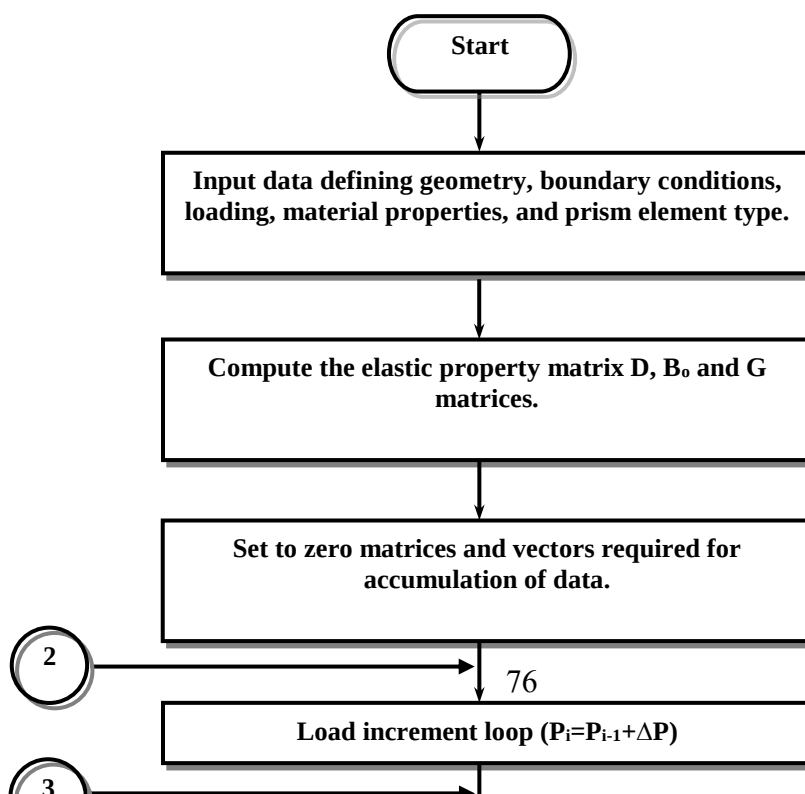


4.8 Outline of Computer Program

The computer program **NFSARCP** (nonlinear finite strip analysis of reinforced concrete plates) has been used in the present study. This program is a modified version from (Al-shaarbaf 1990) computer for nonlinear analysis of reinforced concrete members by three dimensional finite element methods. The program is coded in **FORTRAN 77** language. The main objective of the present program **NFSARCP** is to analyze reinforced concrete structures having regular geometric planes, simple boundary conditions and subjected to three-dimensional static loads up to failure.

In the present study, the **Fortran PowerStation 4.0** compiler produced by Microsoft incorporation was used to operate the program under PC Pentium IV with Intel Inc 1600 MHz processor and 256 MB RAM.

The modified and added subroutines to the original program **P3DNFEA** to construct the present program **NFSARCP** are presented in Appendix [A], and the overall structure of this program is shown in Fig. (4.8).



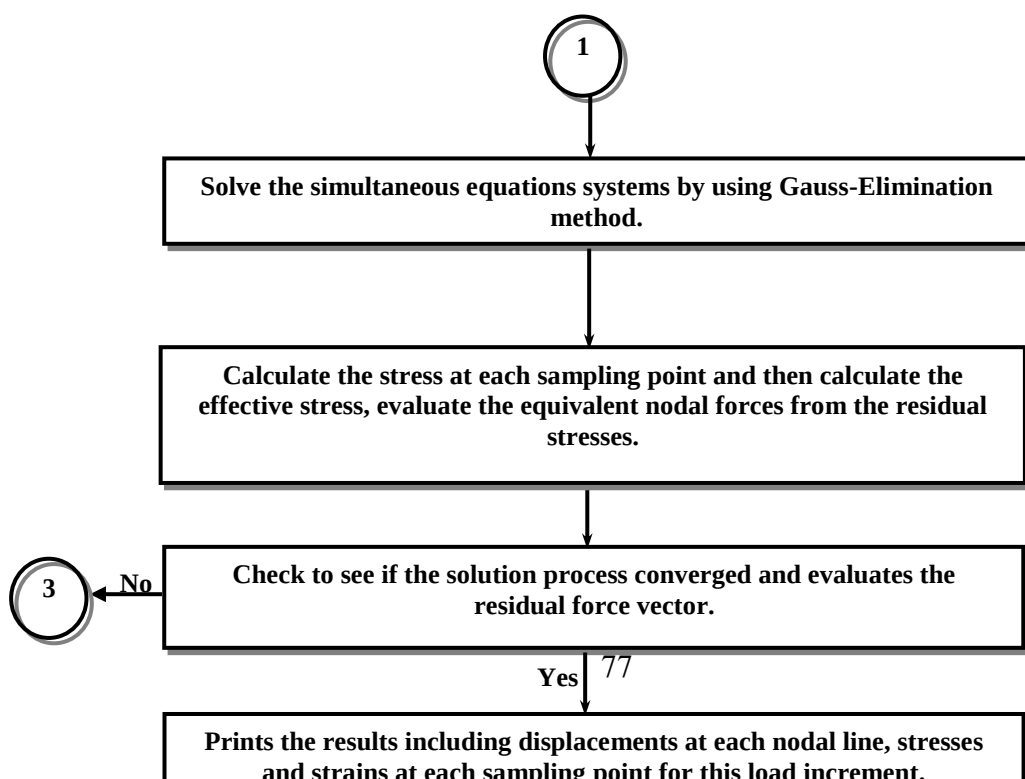


Fig. (4.8) Overall Structure of Computer Program (NFSARCP)

CHAPTER FIVE

NUMERICAL APPLICATIONS

5.1 Introduction

To study the validity, efficiency and the accuracy of the computational model and to demonstrate the versatility of the computer program, six different examples have been chosen from the available experimental published data. The examples consisted of simply supported beam, one way slab, flat slab, cylindrical shell, and folded plate. Comparison between the finite strip results and available experimental results are made to check the accuracy of the present method of analysis. The numerical analysis has been carried out using the modified Newton-Raphson method, in which the stiffness matrix is updated at the 2nd, 12th, 22nd, etc. iterations of each increment of loading. Also the tension stiffening parameters has been taken equal to ($\alpha_1 = 10$, $\alpha_2 = 0.5$) and the shear retention parameters ($\gamma_1 = 10$, $\gamma_2 = 0.5$, $\gamma_3 = 0.1$) . The 15a-point integration rule was selected to integrate the stiffness matrix of the prism element, constant value of concrete compressive strain equal to (0.003), and convergence tolerance of 5% have been used.

Parametric study to investigate the effects of some important material, geometric and solution parameters on the non-linear finite strip analysis of reinforced concrete folded plate has been carried out. These parameters are the effect of integration rule type, effect of tension stiffening parameters and effect of span/width ratio.

5.2 Bresler-Scordelis Beam

The simply supported reinforced concrete beam tested by Bresler and Scordelis (1964) was chosen to test the accuracy of the proposed method of analysis. The beam had a span of (3658 mm) with cross sectional dimensions (229*552.5 mm). It had no web reinforcement and the longitudinal reinforcement consisted of four bars with a total area of (2580 mm²). The beam was subjected to a central concentrated load as shown in Fig. (5.1), which also shows the geometry of the beam and reinforcement details.

The experimental results are considered to be very reliable, so the beam has almost become a standard benchmark for the testing of numerical methods obtained by several researchers such as [Arneson et.al. (1980), and Cervera et.al. (1987)] . Material properties of concrete and steel and additional material parameters are given in Table (5.1).

The numerical results in the present study obtained by dividing the cross section into 16 prisms of 8-nodal line quadrilateral prisms. Fig. (5.2.c) shows the details of the finite strip mesh.

The load-deflection curves at central point of the beam is shown in Fig. (5.3). Results of the finite strip analysis give good agreement with the experimental results. Also, from this figure it can be noted that including of the geometric nonlinearity in the analysis improves slightly the behavior of the beam at levels beyond cracking load up to collapse load stage. Comparison between two previous theoretical results and the present analysis is shown in Fig. (5.4). From this figure it can be seen that

the finite strip method give good simulation to the behavior of reinforced concrete beams.

Three different finite strip meshes are chosen as shown in Fig. (5.2) to study the effect of mesh size on the behavior of reinforced concrete beam, Fig. (5.5) shows the load-deflection curves for the three meshes, it is clear that increases the number of prisms gives better agreement with the experimental response.

Table (5.1) Material properties and additional parameters of Bresler-Scordelis beam.

	Material properties and material parameters	Value
Concrete	Young's modulus, E_c (N/mm ²)	22750
	Compression strength, f'_c (N/mm ²)	21.78
	Tensile strength, f_t (N/mm ²)	2.275
	Poisson's ratio, ν	0.20
Steel	Young's modulus, E_s (N/mm ²)	191665
	Yield stress, f_y (N/mm ²)	413.0
	Hardening parameter, H'	0.0

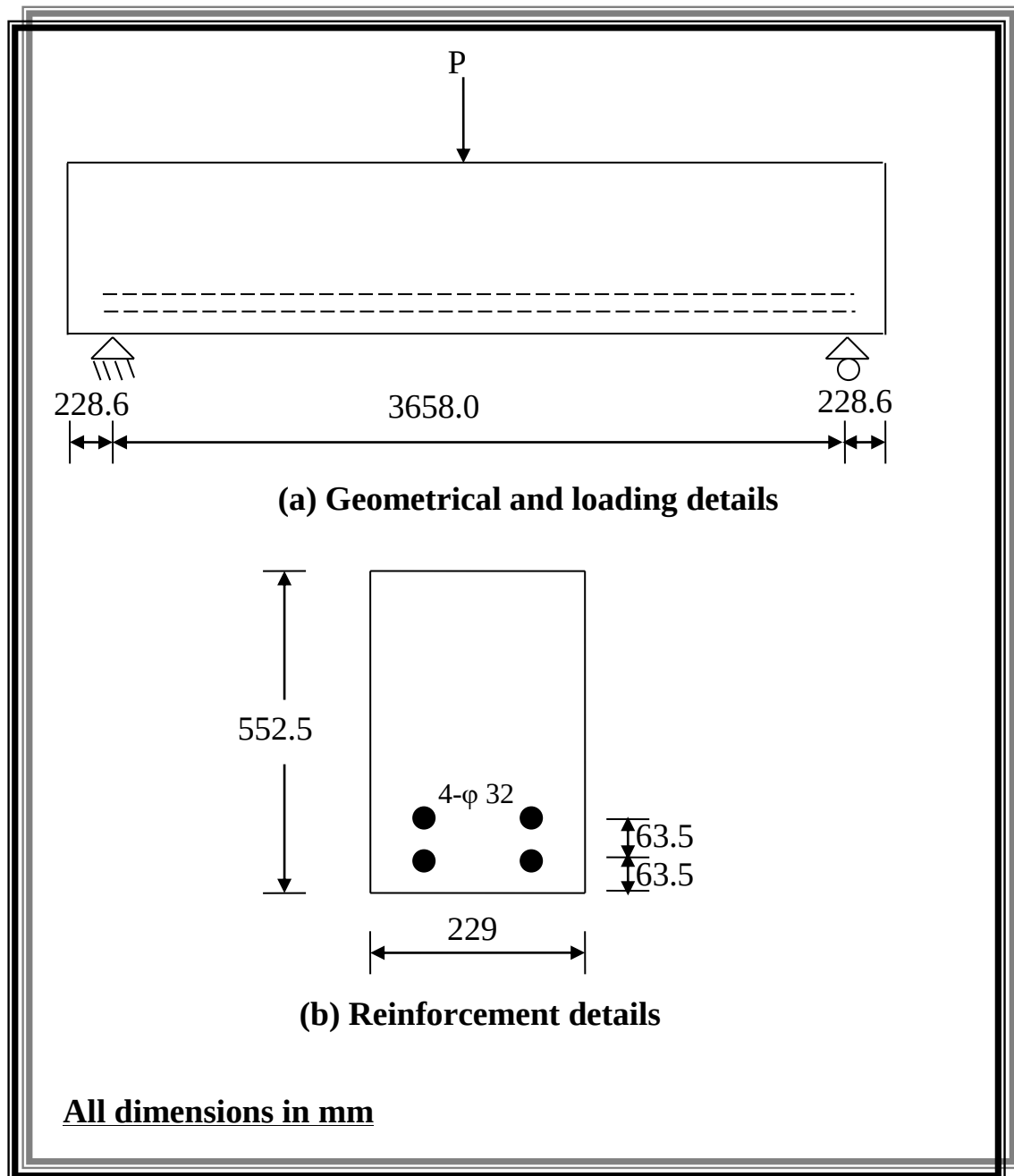
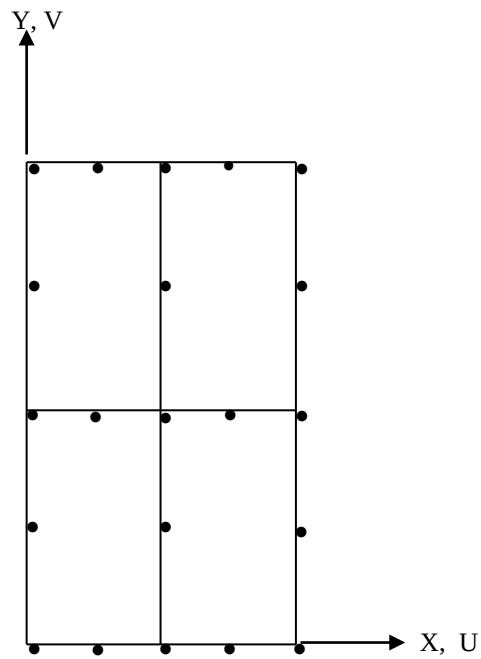
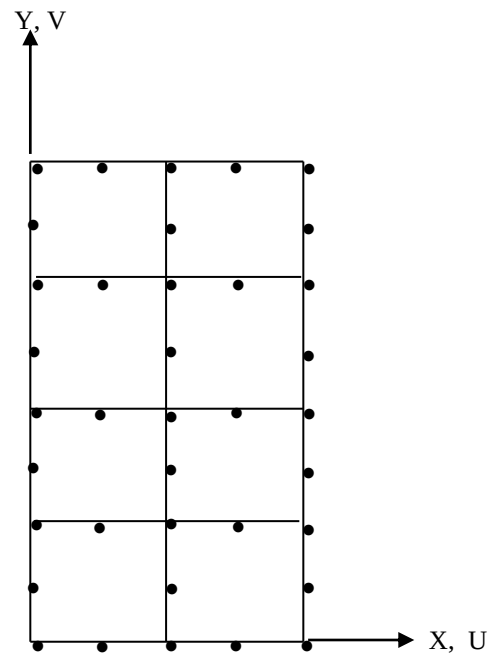


Fig. (5.1) Geometric and reinforcement details of Bresler-Scordelis beam.



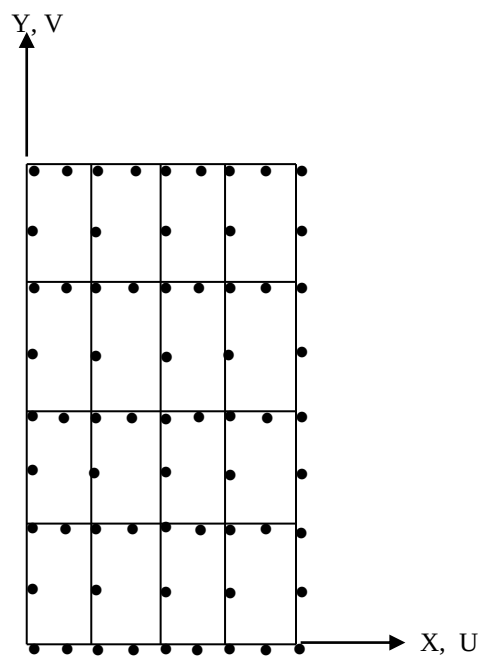
(a). Mesh No.1



(b). Mesh No.2

(4 Prism elements, 21 nodal lines)

(8 Prism elements, 37 nodal lines)



(c). Mesh No.3
(16 Prism elements, 65 nodal lines)

Fig. (5.2) Finite strip meshes of Bresler-Scordelis beam.

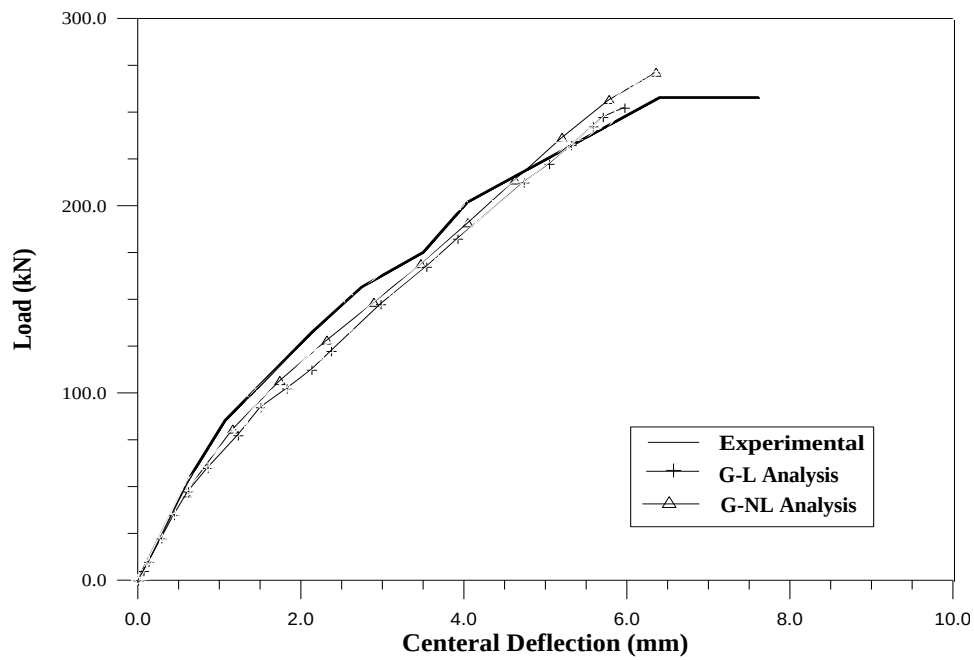


Fig. (5.3) Load-Central deflection curves of Bresler-Scordelis beam.

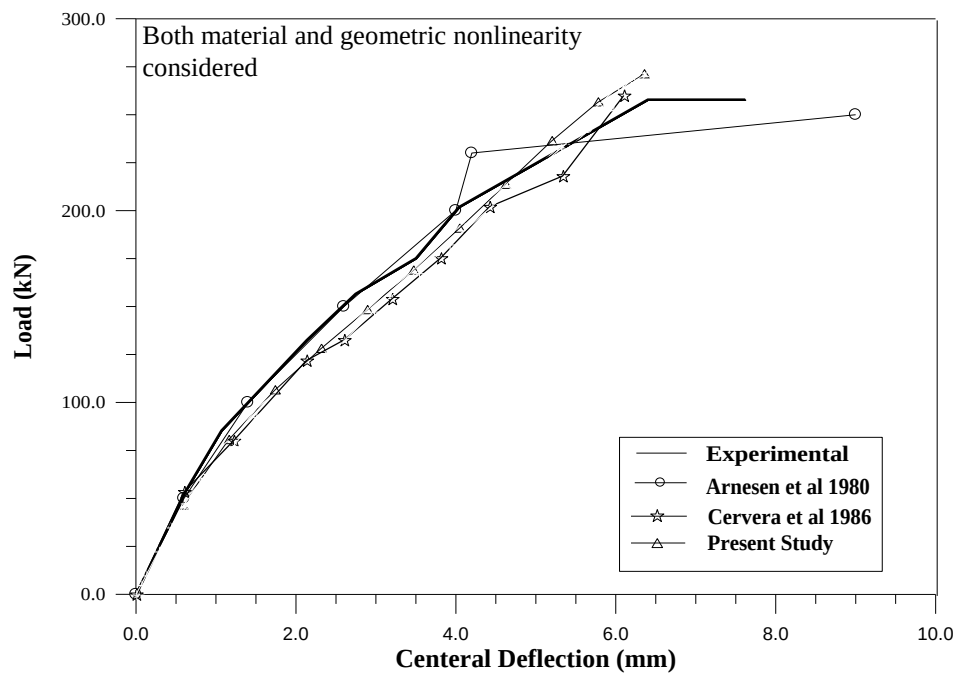


Fig. (5.4) Comparison of results for Bresler-Scordelis beam.

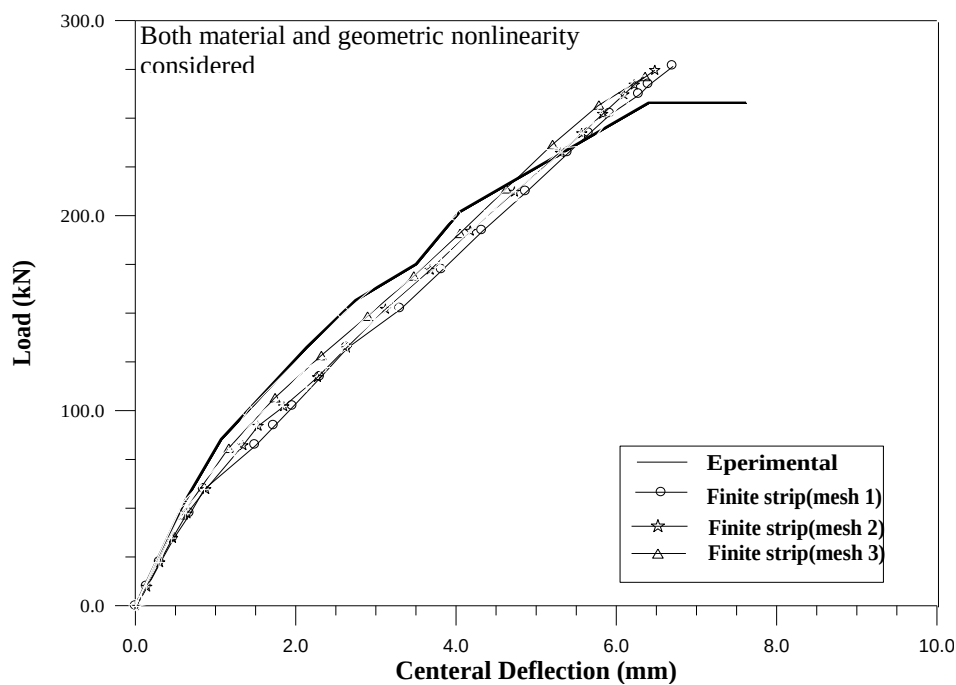


Fig. (5.5) Effect of mesh size on load-deflection response of Bresler-Scordelis beam.

5.3 One-Way Reinforced Concrete Slab

A one-way slab, simply supported at two edges was tested by Mc Niece et.al. (1971). The slab had overall dimensions (864mm*152.4mm and 44.5mm thick) and loaded by two concentrated loads at third points of span. Only steel ratio of ($\rho=0.008$) was given, so the reinforcement details are assumed as given in Fig. (5.6), which also shows the geometry of the slab and loading arrangement.

Alazzawi (2000) analyzed this slab, and thus a comparison can be made with his model. Concrete and steel material properties and the additional material parameters are given in Table (5.2).

The finite strip analysis has been carried out using twelve-prisms with 8-nodal line quadrilateral prism through cross section of slab. Fig. (5.7.b) shows the details of the finite strip mesh.

In Fig (5.8), the load-deflection curves at mid span of the slab is shown. In general very good agreement with experimental results is obtained by using the finite strip method. The effect of geometric nonlinearity was illustrated in this Figure. The inclusion of geometric nonlinearity does not change the behavior of the slab but increases the deflection at failure stage.

Two different finite strip meshes have been used as shown in Fig. (5.7) to study the effect of mesh size, the first was eight prisms and the second was twelve prisms. The load-deflection curves for these two meshes are illustrated in Fig. (5.9). Both meshes agree well with the experimental results.

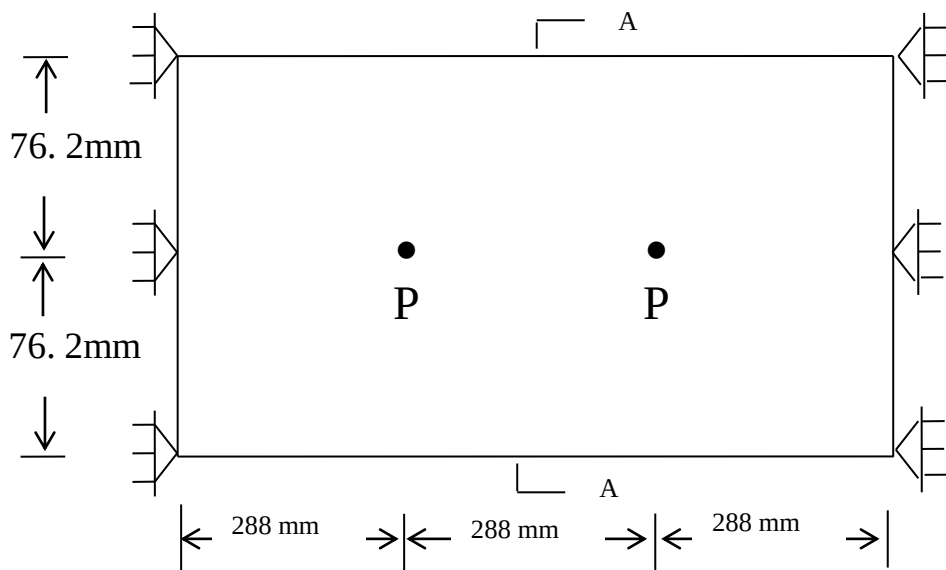
The comparison between the results of Alazzawi and by the present analysis is shown in Fig. (5.10). The Figure reveals that the present method of analysis gives better simulation to the behavior of slab than the finite element method adopted by Alazzawi.

Fig. (5.11) shows the effect of type of prism element on the load-deflection behavior of one way slab. It is clear that the eight-nodal line and twelve-nodal line prisms agree well with the experimental results in comparison with the four-nodal line prism. Because the distribution of displacements for (IPLQ prism element) is linear and higher number of prisms must be taken to improve the results while the (IPQQ, IPCQ prism elements) contains quadrilateral, cubic distribution of displacements respectively, also compatibility of displacements, strains exists in these types and less number of prisms will be needed.

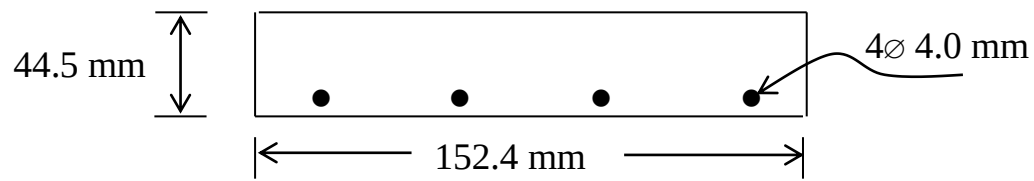
At load level of about (94%) of the experimental ultimate load, the analyses were terminated and the computer program indicates rupture of a sampling point located at the reinforcement steel.

Table (5.2) Material properties and additional parameters of Mc Niece et al one-way slab.

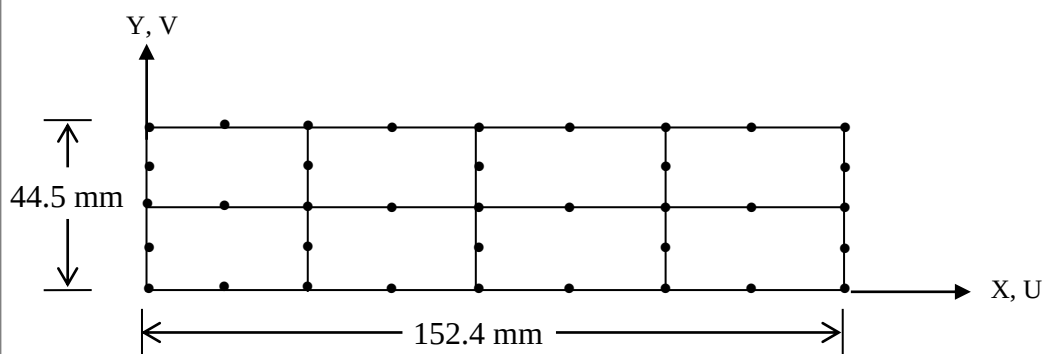
	Material properties and material parameters	Value
Concrete	Young's modulus, E_c (N/mm ²)	31000
	Compression strength, f'_c (N/mm ²)	35
	Tensile strength, f_t (N/mm ²)	3
	Poisson's ratio, ν	0.15
Steel	Young's modulus, E_s (N/mm ²)	210000
	Yield stress, f_y (N/mm ²)	345.0
	Hardening parameter, H'	0.0



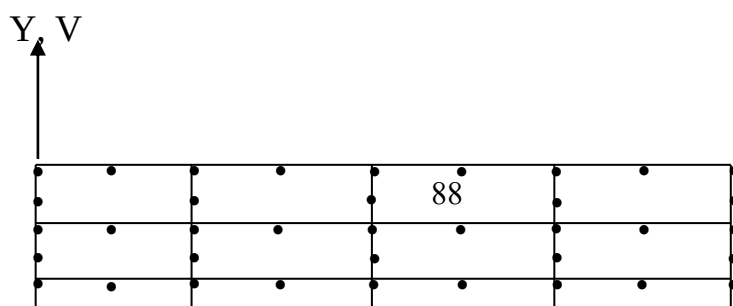
(a). Geometrical and Loading details



(b). Section A-A (Reinforcement details)



(a). Mesh No.1 (8 Prism elements, 37 nodal lines)



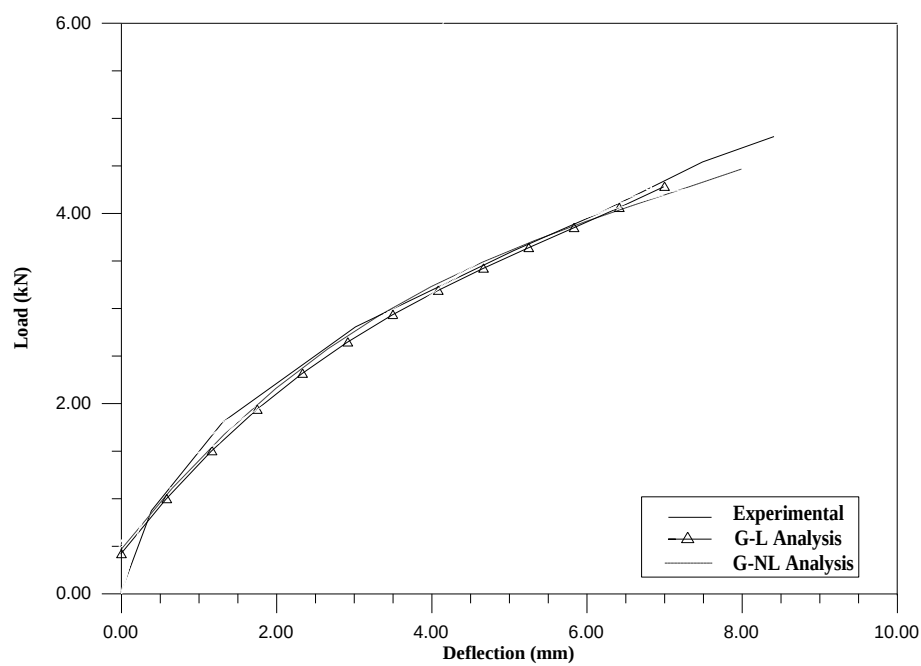


Fig. (5.8) Load-central deflection curves of Mc Niece et al one-way slab.

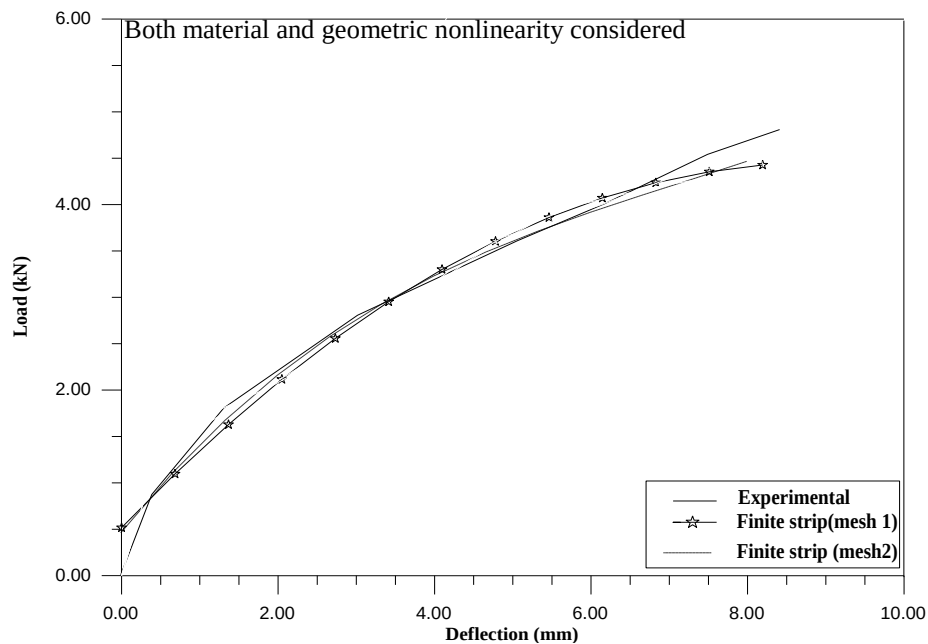


Fig. (5.9) Effect of mesh size on load-deflection response of Mc Niece et al one-way slab.

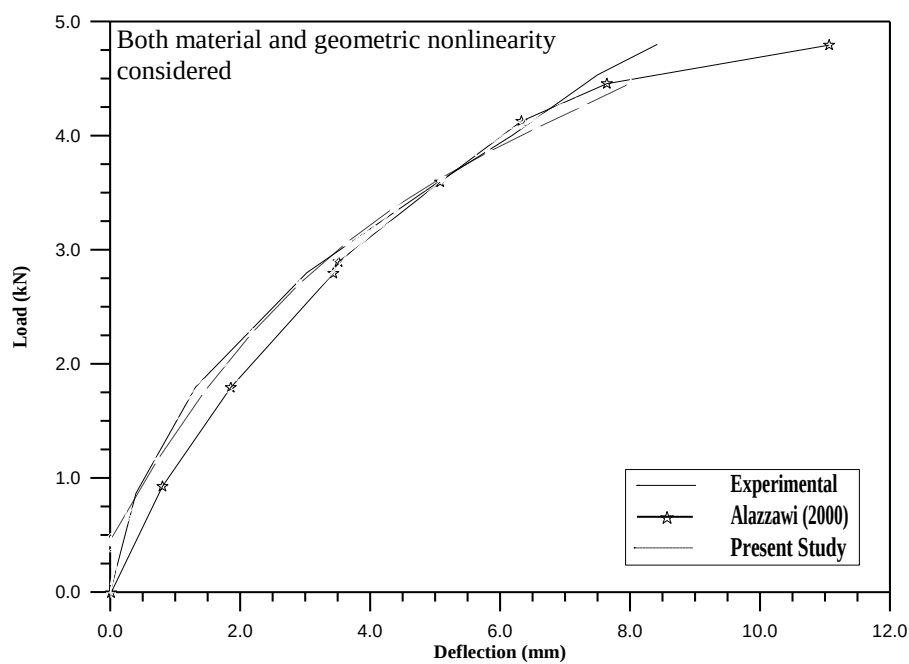


Fig. (5.10) Comparison of results for Mc Niece et al one-way slab.

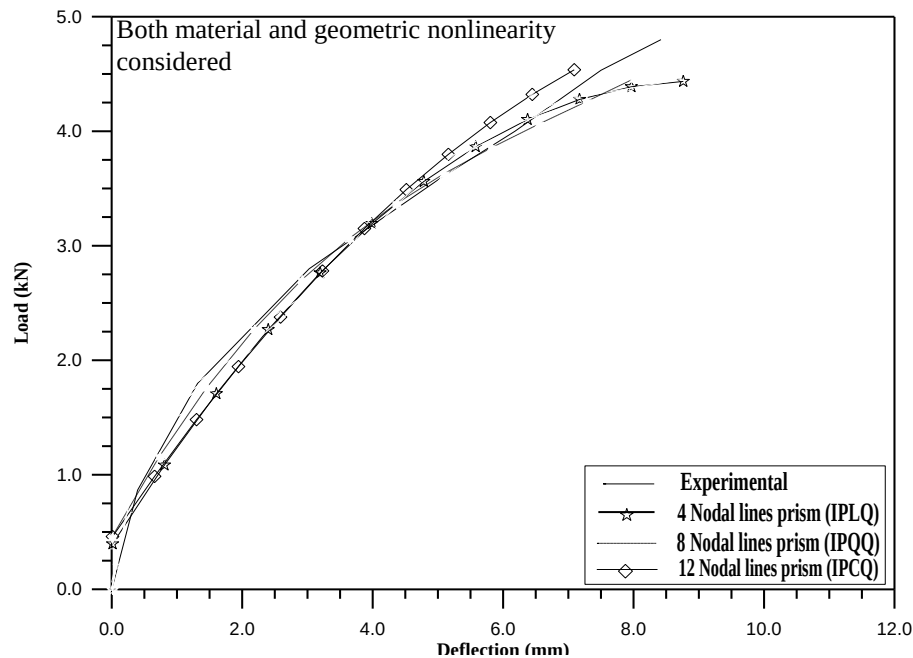


Fig. (5.11) Effect of type of prism element on load-deflection response of Mc Niece et al one-way slab.

5.4 Simply-Supported Flat Slab

A simply supported reinforced concrete flat slab was tested under uniform load by Taylor et al (1966). The overall dimensions, reinforcement details and finite strip mesh used for this slab are shown in Fig. (5.12).{Arnesen et al (1980)and Baka (2002)} have analyzed the same slab, and thus a comparison can be made with their models. Details of the material properties and additional parameters are given in Table (5.3).

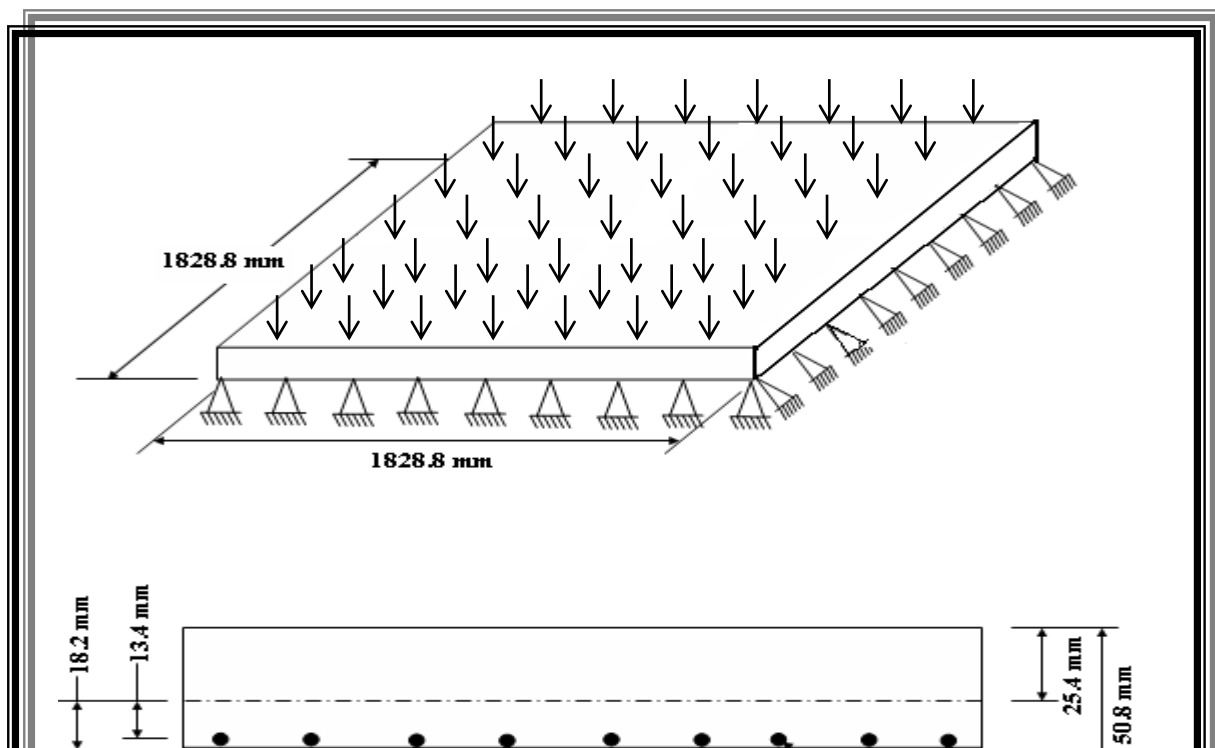
The load – deflection curve at the central point of slab are shown in Fig. (5.15). Good agreement with the experimental results of Taylor et

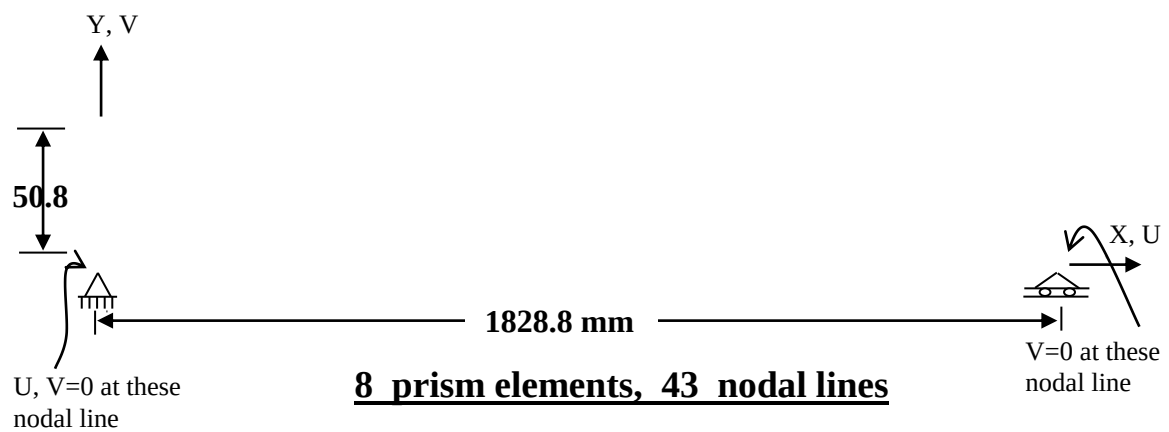
al can be seen for both geometric linear and nonlinear analyses with very small difference in the behavior for all stages up to collapse. The comparison between the two previous theoretical results and the present analysis is shown in Fig. (5.14). From this figure, it can be seen that the present model represents a good simulation to the behavior of the slab.

At a load level of about (93%) of the experimental ultimate load, the analysis was terminated and the computer program indicates crushing in sampling point located at the top surface of mid span .

Table (5.3) Material properties and additional parameters of slab.

	Material properties and material parameters	Value
Concrete	Young's modulus, E_c (N/mm ²)	30000
	Compression strength, f'_c (N/mm ²)	35
	Tensile strength, f_t (N/mm ²)	3.6
	Poisson's ratio, ν	0.15
Steel	Young's modulus, E_s (N/mm ²)	210000
	Yield stress, f_y (N/mm ²)	375.0
	Hardening parameter, H'	0.0





(b) Finite strip mesh

Fig. (5.12) Geometrical, Reinforcement details and finite strip mesh of Taylor et al square flat slab.

(b) Reinforcement details

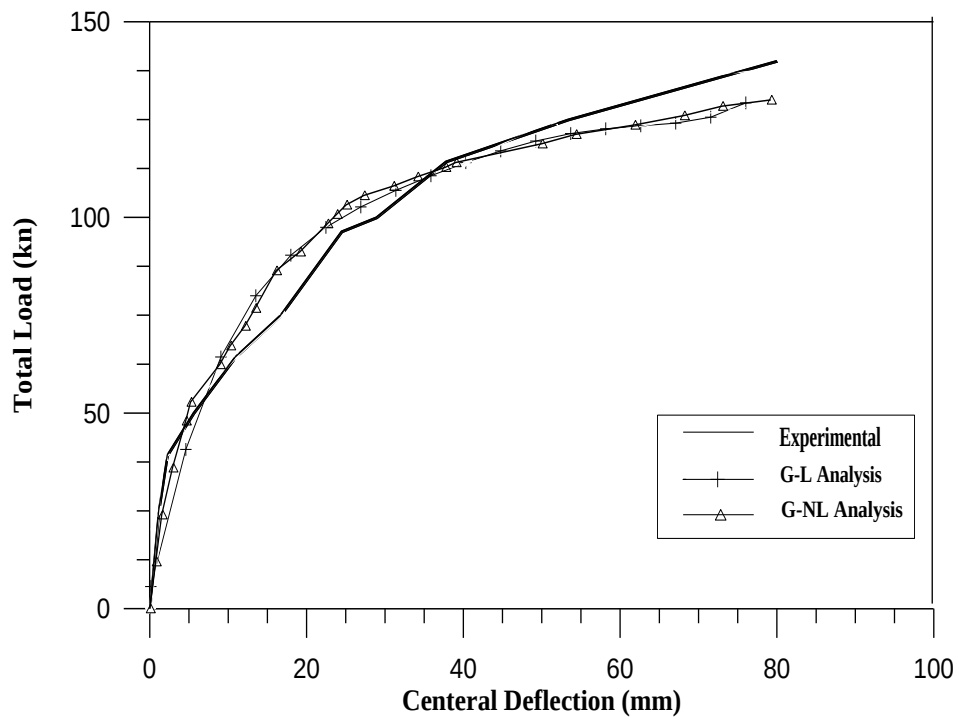


Fig. (5.13) Load-deflection curve of Taylor et al square flat slab.

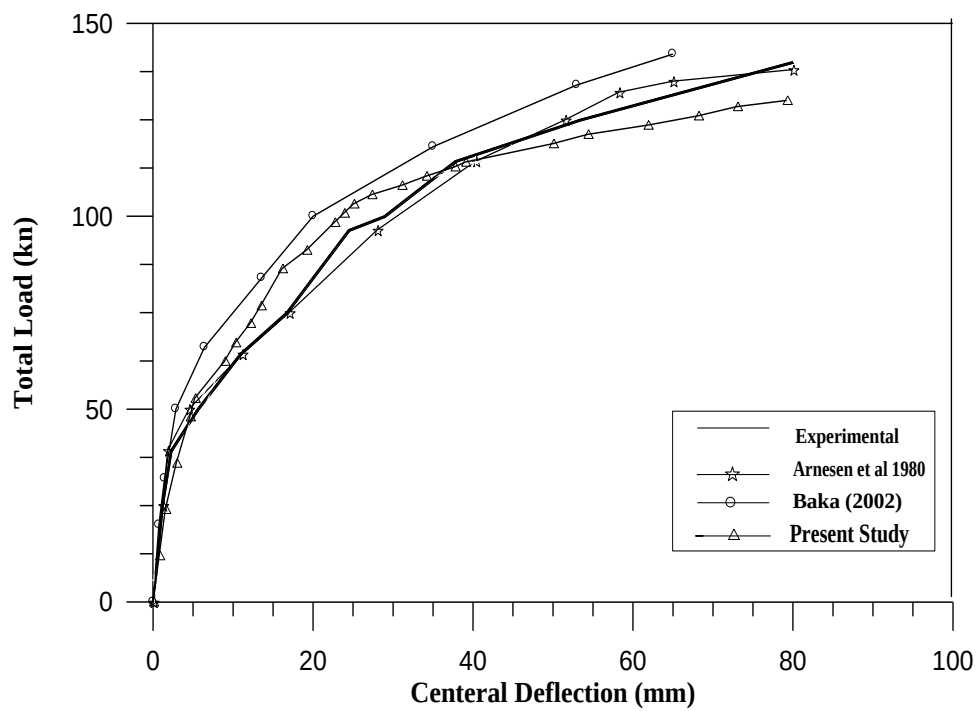


Fig. (5.14) Comparison of results for Taylor et al square flat slab.

5.5 Van Riel Cylindrical Shell

From a series of experimental studies performed on reinforced concrete cylindrical shells by Van Riel et. al in (1957), the shell no. 2 was selected for the present study because this model was studied numerically by several researchers .

The shell contained edge beams and it was assumed to be simply supported in the circumferential direction at the curved edges through end diaphragm and subjected to a uniformly distributed load per unit horizontal area in the vertical direction, geometry and reinforcement details of the shell and edge beam are shown in Fig. (5.15).

Several investigators such as [Arnesen et. al. (1980), Thannon et. al. (1988), and Altaey (2003)] have analyzed the shell, and thus a comparison can be made with their models. Material properties and the additional material parameters are given in Table (5.4).

By utilizing the symmetry due to loading and geometry properties, one half of the shell is modeled by 6 prism elements of 8-nodal line prism type. Fig. (5.16) shows the finite strip mesh and the boundary conditions used for half of the cylindrical shell.

The load–deflection curves for central point of the edge beam is shown in Figures (5.17). In general, this figure shows good agreement for the finite strip solution obtained from the present work compared with the experimental results. The comparison between the results obtained by the previous theoretical models and by the present analysis is shown in Fig. (5.18). From this figure, it can be seen that the present model represents good simulation to the behavior of the shell but the results of finite

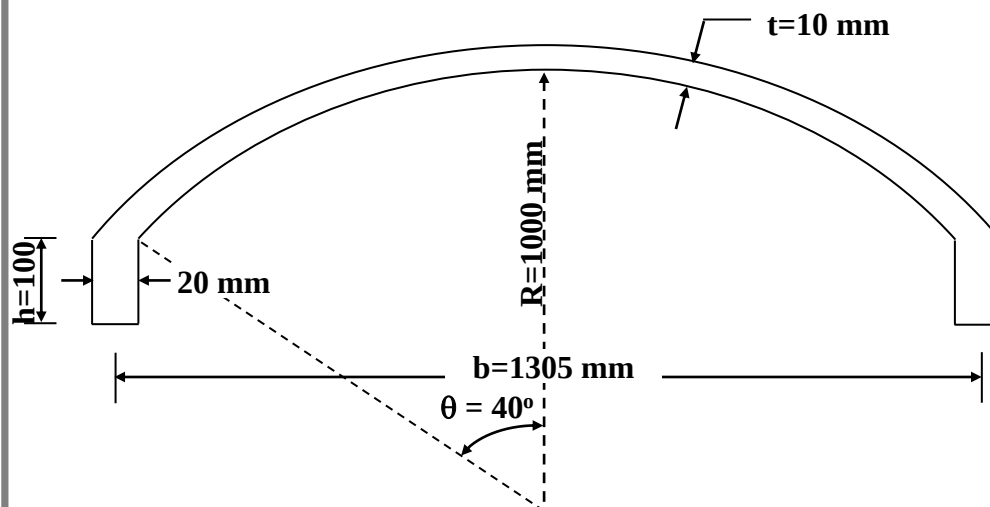
element method supported by Thannon et al (1988) and by Husain (2002) were closer to the experimental results.

Fig. (5.19) shows the deformation of mid span section for ultimate load stage. From this figure it is clear that the finite strip method gives underestimation in calculation of deflection compared with the experimental results.

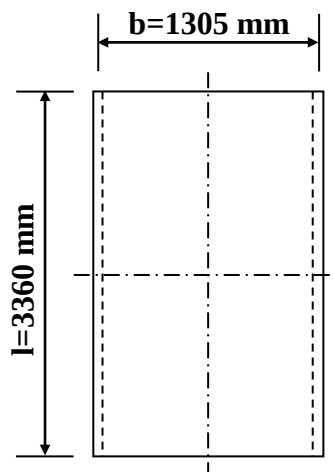
The type of failure for the shell recorded by the computer program was crushing of a sampling point located approximately in the mid span of edge beam and the ultimate load was about (90 %) of the experimental load. The crushing occurred when the compressive strain of the sampling point exceeds the ultimate strain value, then its stiffness deteriorated to zero and the analysis was terminated.

Table (5.4) Material properties and additional parameters of Van Riel et al cylindrical shell.

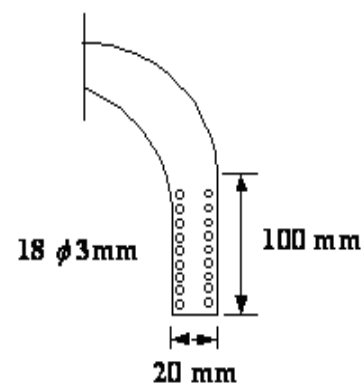
	Material properties and material parameters	Value
Concrete	Young's modulus, E_c (N/mm ²)	30000
	Compression strength, f'_c (N/mm ²)	30
	Tensile strength, f_t (N/mm ²)	1.82
	Poisson's ratio, ν	0.15
Steel	Young's modulus, E_s (N/mm ²)	210000.0
	Yield stress of the shell reinforcement, f_y (N/mm ²)	295.0
	Yield stress of the beam reinforcement, f_y (N/mm ²)	280.0
	Hardening parameter, H'	0.0



(a) Cylindrical shell cross section



**(b) Dimensions of
cylindrical shell**



**(c) Details of edge
beam reinforcement**



(d) Details of shell reinforcement

Fig. (5.15) Geometrical and Reinforcement details of Van Riel et al cylindrical shell.

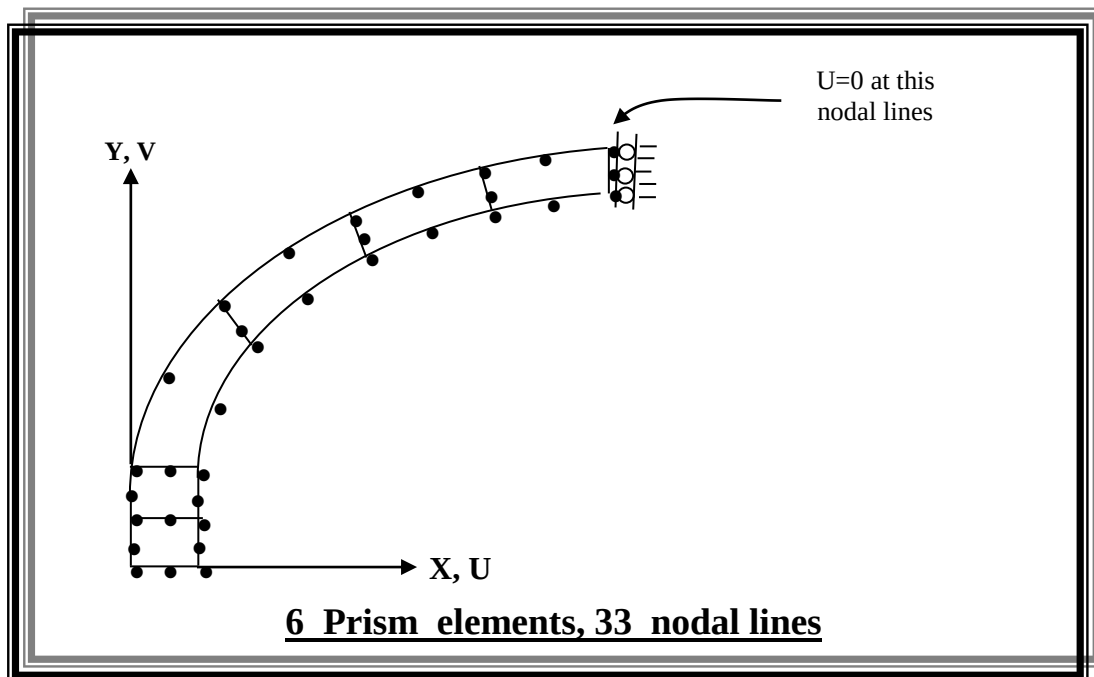


Fig. (5.16) Finite strip mesh of Van Riel et al cylindrical shell.

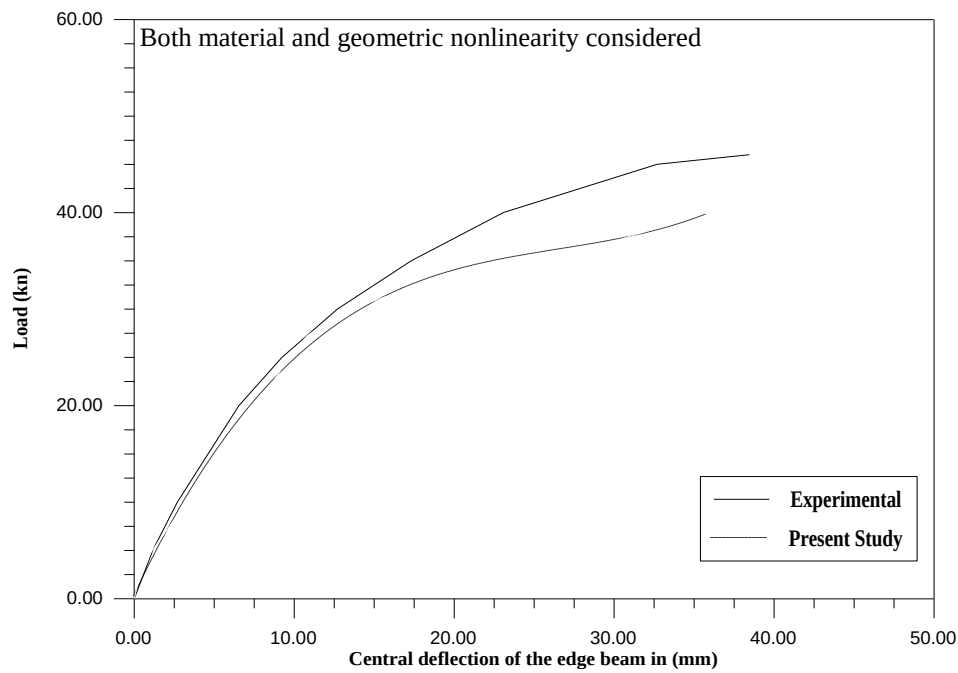


Fig. (5.17) Load-deflection curve of Van Riel et al cylindrical shell.

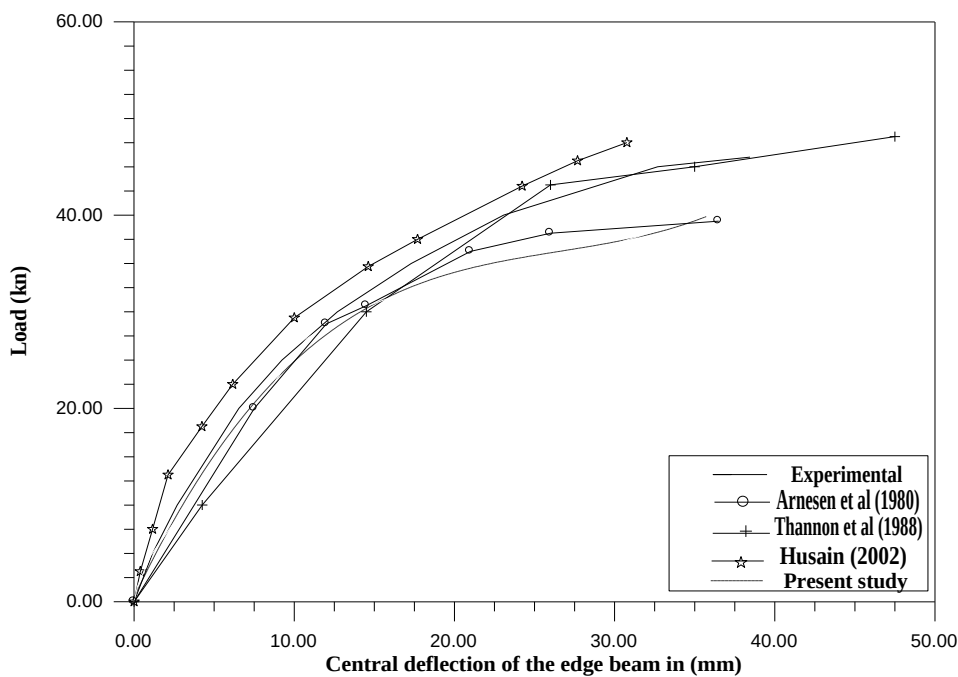


Fig. (5.18) Comparison of results for Van Riel et al cylindrical shell.

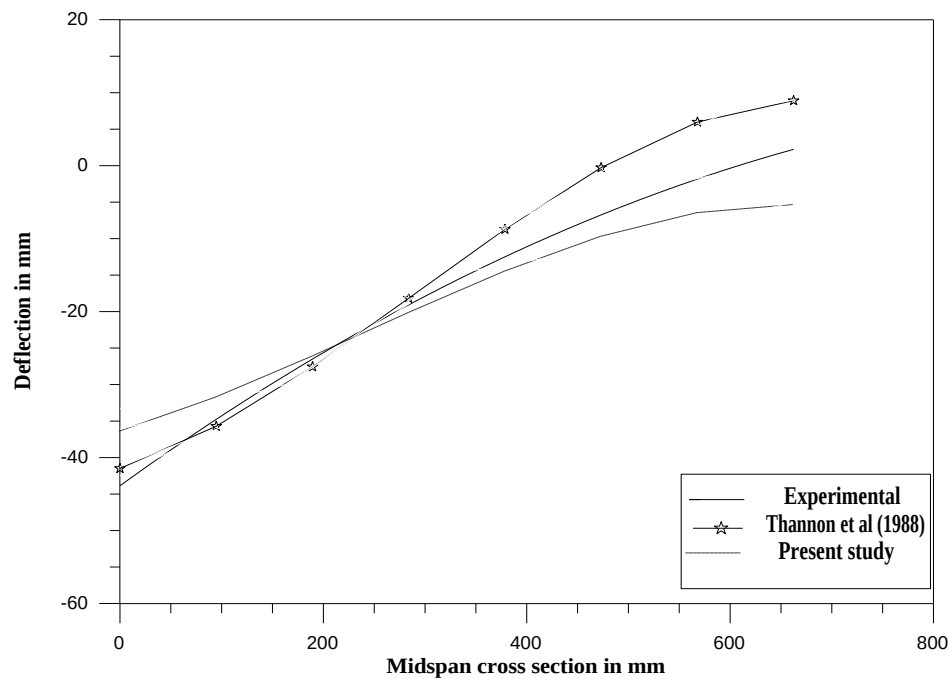


Fig. (5.19) Deformation of mid span cross section of Van Riel et al cylindrical shell.

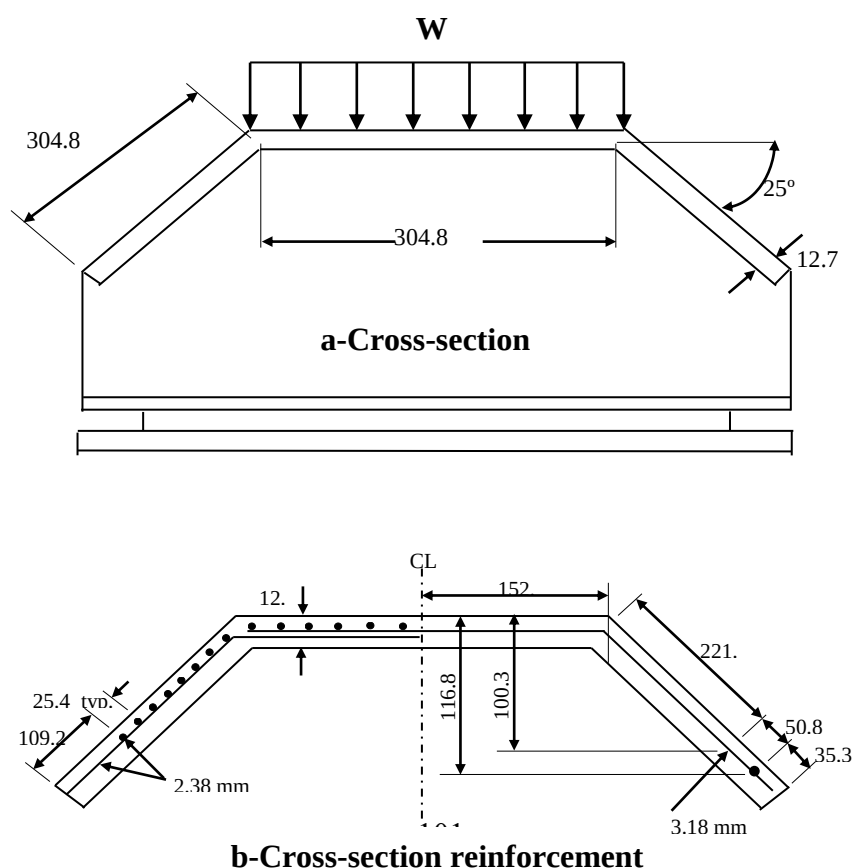
5.6 Chacos and Scalzi Folded Plate

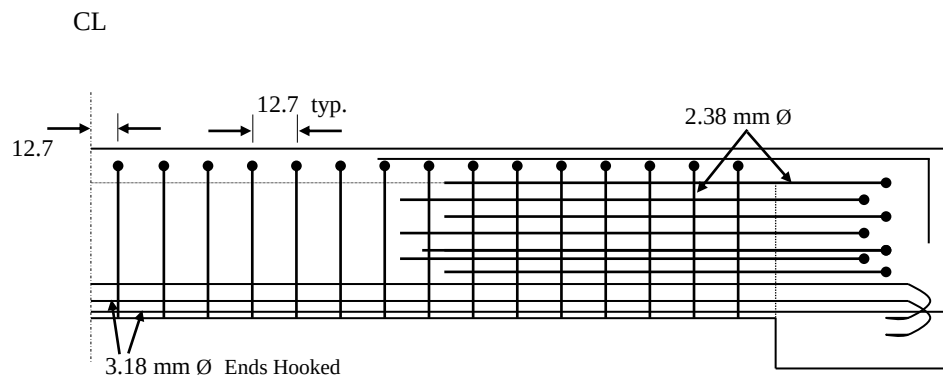
A reinforced concrete folded plate structure subjected to uniformly distributed load applied to the top plate was tested experimentally by Chacos and Scalzi (1961) as a simple beam with irregular cross section. This reinforced concrete folded plate was simply supported over 1524 mm span and had 304 mm plate width with 12.7 mm plate thickness and a 76.2 mm thick end diaphragms. The inclination angle of the side plate was 25° with the horizontal reference. The geometry, and reinforcement details of this structure are shown in Fig. (5.20). The details of material properties and additional material parameters of this structure are given in Table (5.5).

Owing to the symmetry properties due to loading and geometry, one half of the whole structure has been modeled by 8 prisms of 8-nodal

line prism elements as shown in Fig. (5.21). [Mohammed(2001)] have analyzed this folded plate and a comparison will be made with their models.

The load-vertical deflection curves for central point of the structure are shown in Fig(5.22), this figure shows good agreement for the finite strip method compared with the experimental results. The comparison between the results obtained by the previous theoretical models and by the present analysis are shown in Fig. (5.23). From this figure, it can be seen that the present model represents good simulation to the behavior of the folded plates.





c-Longitudinal reinforcement

All dimensions in (mm)

Fig. (5.20) Geometrical and Reinforcement details of Chacos and Scalzi folded plate.

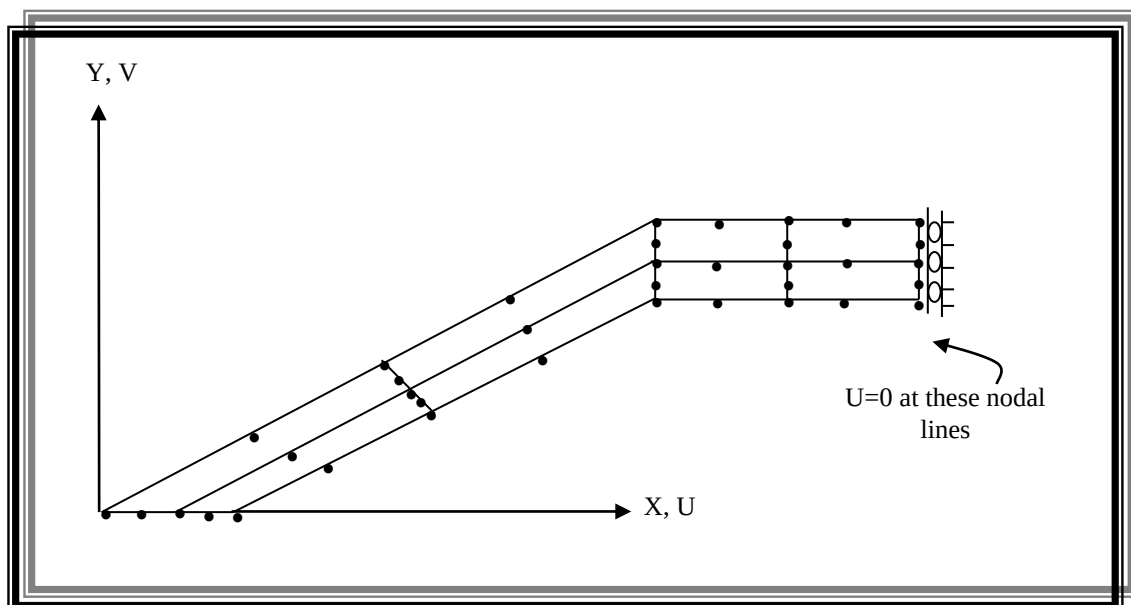


Fig. (5.21) Finite strip mesh (8 prism elements, 37 nodal lines)

Table (5.5) Material properties and additional parameters of Chacos and Scalzi folded plate.

	Material properties and material parameters	Value
Concrete	Young's modulus, E_c (N/mm ²)	17580
	Compression strength, f'_c (N/mm ²)	10.34
	Tensile strength, f_t (N/mm ²)	1.29
	Poisson's ratio, ν	0.2
Steel	Young's modulus, E_s (N/mm ²)	21582
	Yield stress, f_y (N/mm ²)	707.86
	Hardening parameter, H'	0.0

Both material and geometric nonlinearity considered

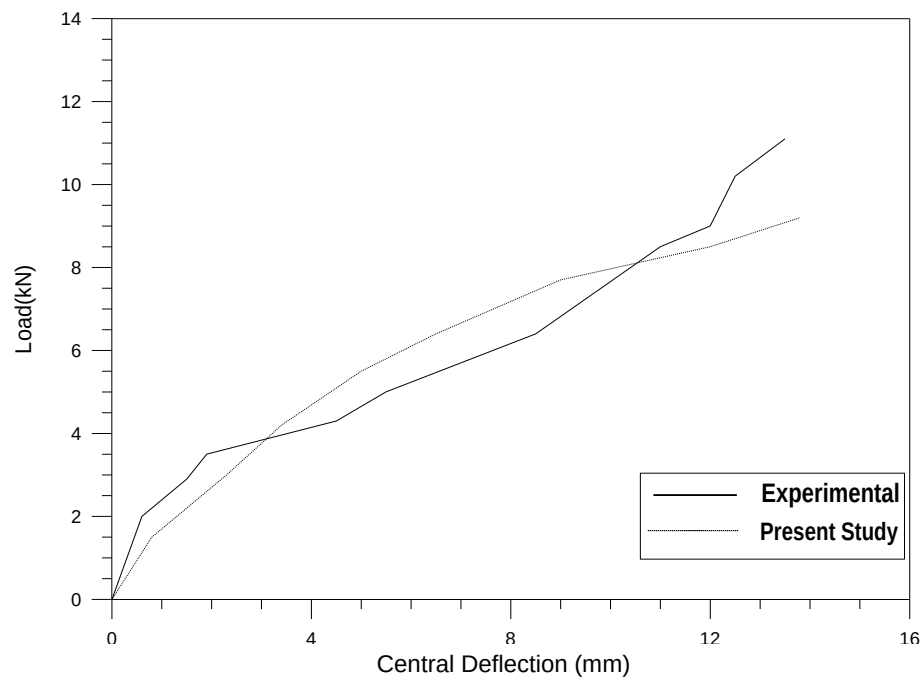


Fig. (5.22) Load-deflection curves of Chacos and Scalzi folded plate.

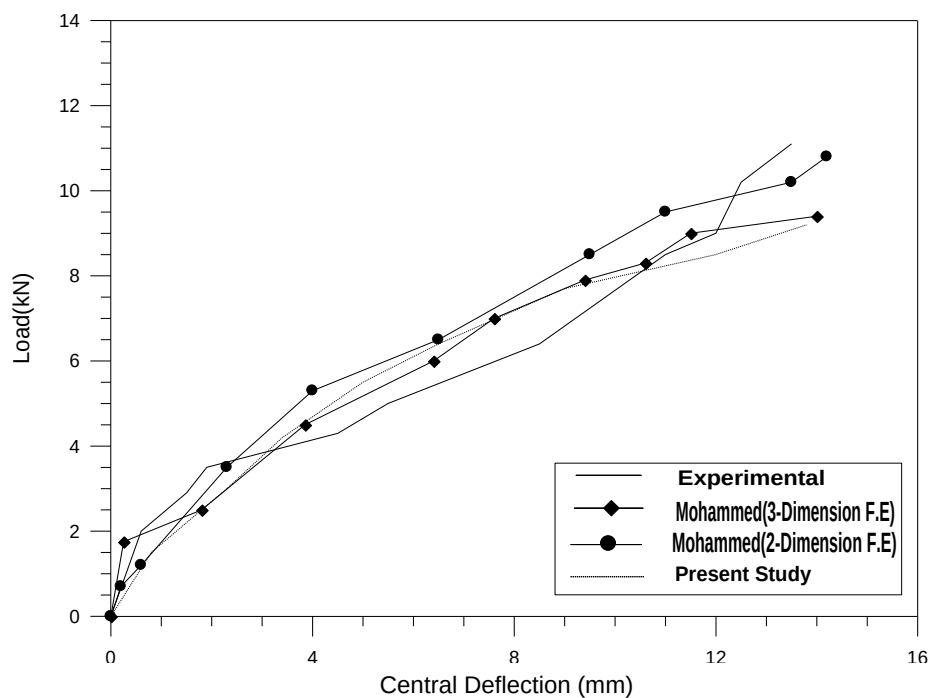


Fig. (5.23) Comparison of results for Chacos and Scalzi folded plate.

5.7 Goble and Jendikis Folded Plate

A reinforced concrete folded plate with stiffer edge beam was tested experimentally by Goble and Jendikis (1966). The plates were cast as monoliths and the end diaphragms were simply supported during the test. The applied load consisted of 4 equal concentrated loads of equal magnitude at points corresponding to third of the span as shown in Fig. (5.24), which shows also geometry, reinforcement details and finite strip mesh of the structure.

During experiment the model has been tested in three loading-unloading stages (loading from zero to 13.35 kN, followed by unloading then loading again from zero to 26.7 kN, followed by unloading and finally loading from zero up to failure 33.85 kN). Only the ultimate load has been recorded and no deflection curves or crack distributions have been reported for this test. The details of material properties and additional material parameters of this structure are given in Table (5.6).

By utilizing the symmetry properties due to loading and geometry, one half of the whole structure has been modeled by 5 prisms of 8-nodal line prism elements as shown in Fig. (5.24.c). [Arnesen et al(1980), and Altaey (2003)] have analyzed the Goble and Jendikis folded plate and a comparison will be made with their models.

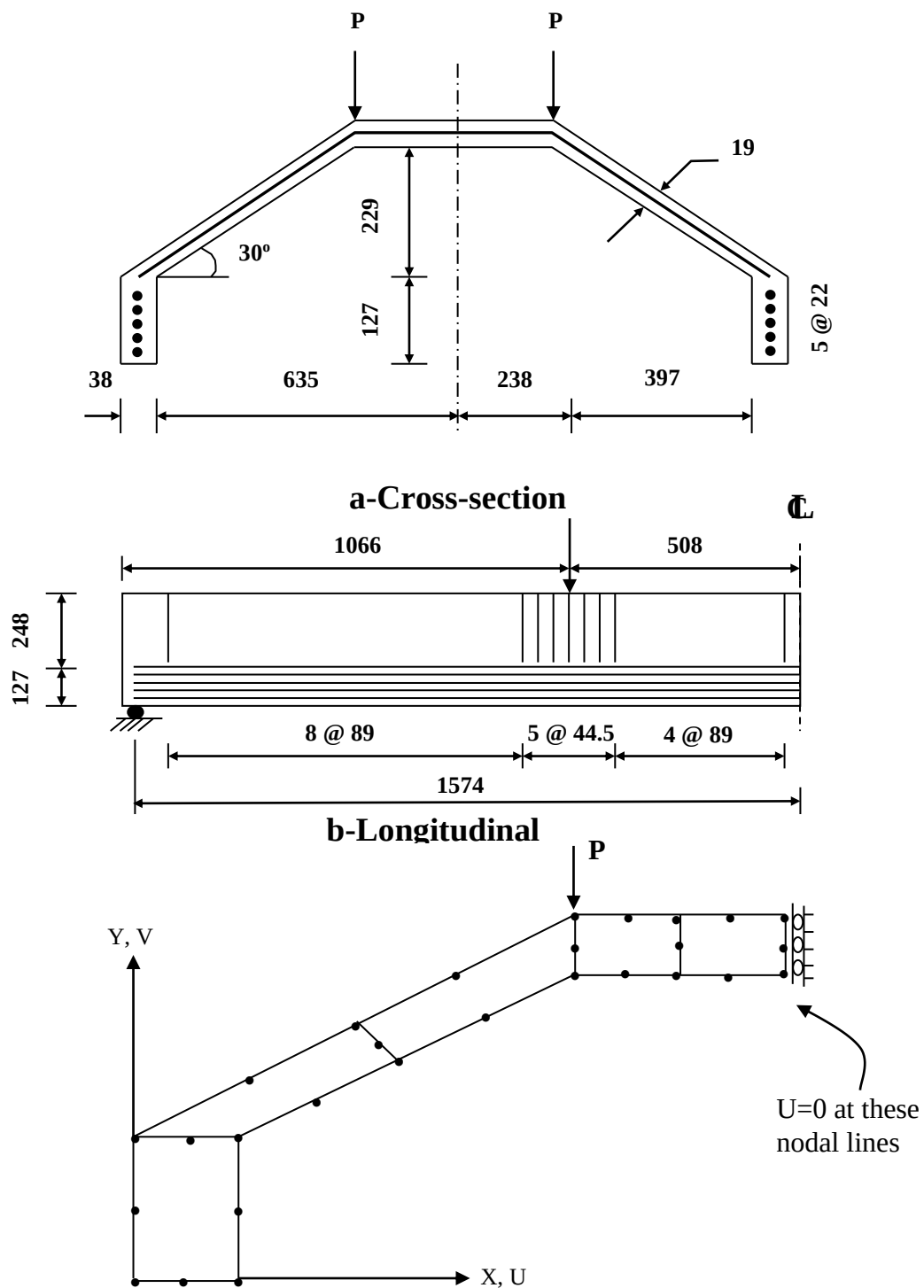
The load-vertical deflection curves for central point of the structure are shown in Fig. (5.25). It can be seen from this figure that the finite strip results consists with 2d finite element method of Thannon at initial stages up to cracking. Beyond this level the finite strip curves have softer behavior and the ultimate load agrees well the experimental ultimate load.

The distribution of the longitudinal concrete stresses (σ_z) along width of the folded plate at collapse load stage, for the mid and top surfaces is shown in Fig. (5.26a) and (5.26b) respectively.

The type of failure for the Folded plate recorded by the computer program was crushing of sampling point located in the top surface of mid span of the edge beam and the ultimate load was (37.00 kN) which represents (1.095) times the experimental load. The crushing occurred when the compressive strain of the sampling point exceeds the ultimate strain value, then its stiffness deteriorated to zero and the analysis was terminated.

Table (5.6) Material properties and additional parameters of Goble and Jendikis folded plate.

	Material properties and material parameters	Value
Concrete	Young's modulus, E_c (N/mm ²)	24290
	Compression strength, f'_c (N/mm ²)	24.2
	Tensile strength, f_t (N/mm ²)	2.07
	Poisson's ratio, ν	0.15
Steel	Young's modulus, E_s (N/mm ²)	180000
	Yield stress, f_y (N/mm ²)	331
	Hardening parameter, H'	0.0



All dimensions in mm

Fig. (5.24) Geometrical, Reinforcement details and finite strip mesh of Goble and Jendikis folded plate.

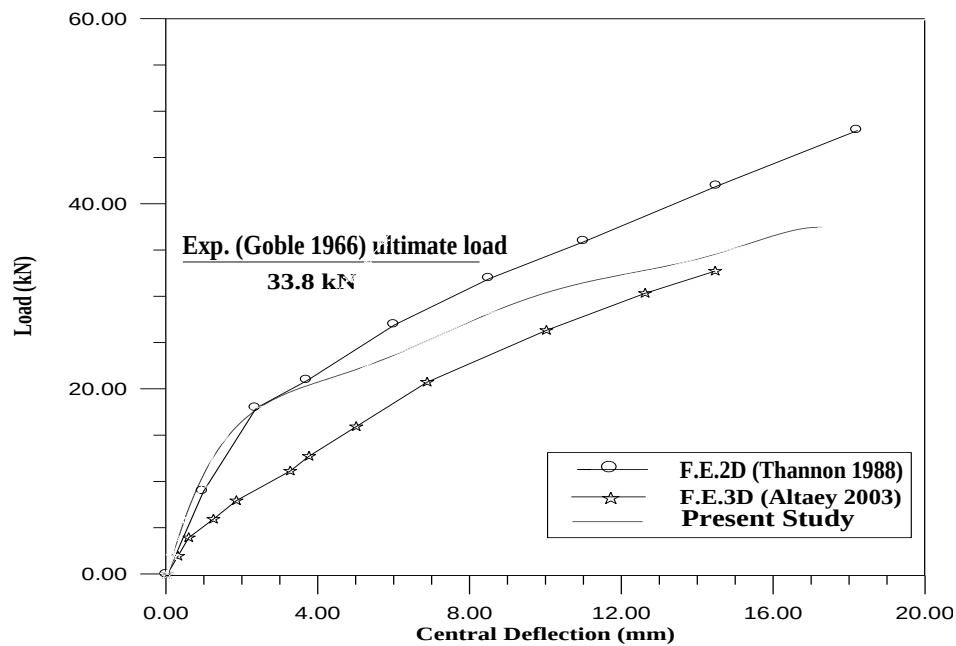


Fig. (5.25) Load-deflection curves, Comparison of results for Goble and Jendikis folded plate.

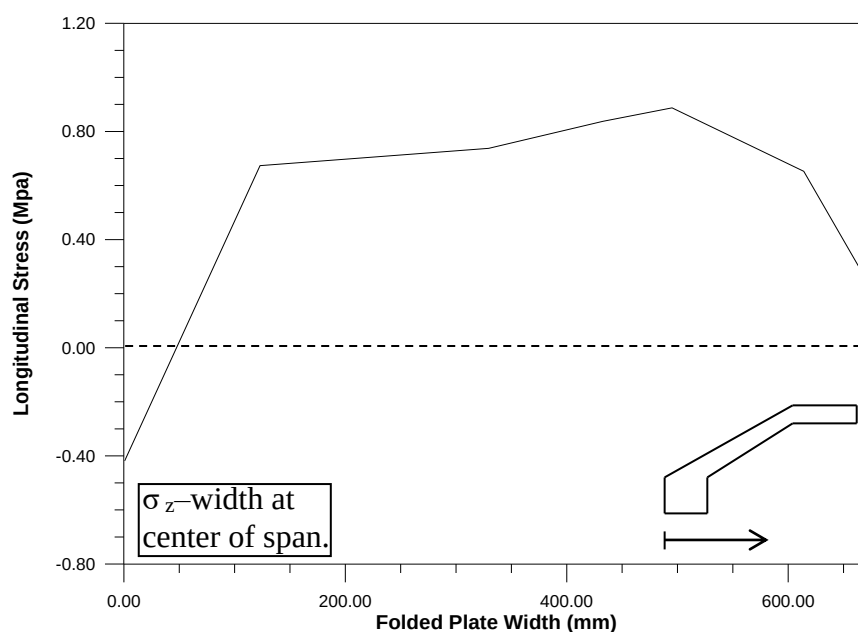


Fig. (5.26a) Distribution of longitudinal stresses along the width at middle surface for Goble and Jendikis folded plate.

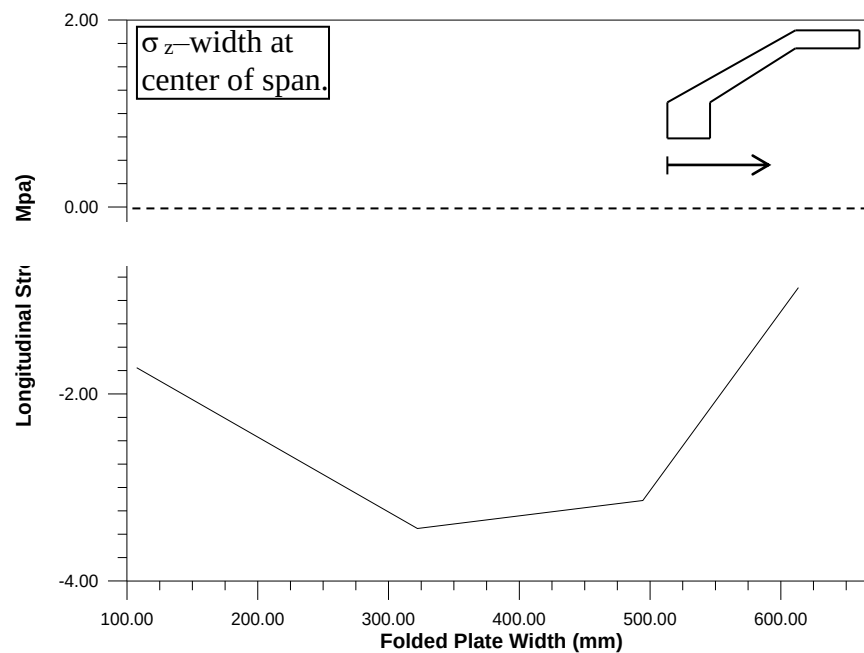


Fig. (5.26b) Distribution of longitudinal stresses along the width at top surface for Goble and Jendikis folded plate.

5.8 Parametric Study on Reinforced Concrete Folded Plate

5.8.1 Introduction

To investigate the effects of some important material, geometrical and solution parameters on the nonlinear finite strip analysis of reinforced concrete folded plates, Goble and Jendikis folded plate that was analyzed in the previous section has been chosen for this purpose.

These parameters included some variables that deals with the geometry of the folded plate like span/width ratio, and span/depth ratio, material properties like tension-stiffening parameters, some numerical and solution parameters like integration rules.

5.8.2 Effect of Span/Width Ratio

To investigate the effect of varying the span/width ratio on the behavior of folded plates, Goble and Jendikis plate has been analyzed with three different span/width ratios (2.34, 2.7, and 3.0), Fig. (5.27) illustrates the results of analysis for these ratios. The figure shows that the increase of span/width ratio results in a softer response, reduce the ultimate load and increase the ultimate deflection. Comparison between

the experimental ultimate load and the predicted finite strip load for different span/width ratios are listed in Table (5.7).

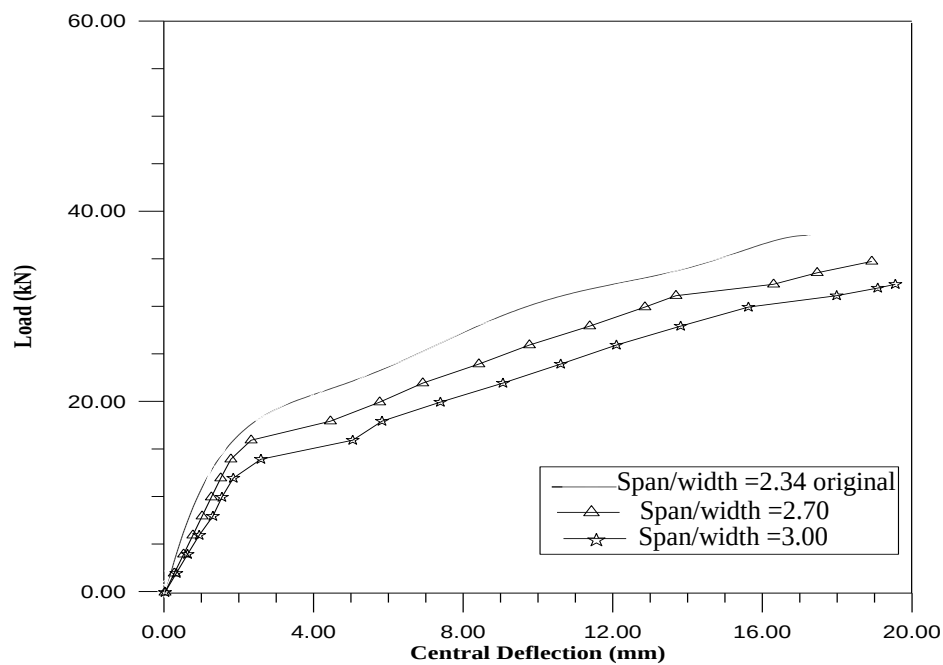


Fig. (5.27) Effect of span/width ratio on load-deflection curve of Goble and Jendikis folded plate.

Table (5.7) Comparison of experimental and predicted results for different values of span/width ratios.

Span/width ratio	Ultimate load (kN)	Load factor
Experimental (span/width=2.34)	33.80	-

Finite strip (span/width=2.34)	37.00	1.095
Finite strip (span/width=2.70)	35.00	1.036
Finite strip (span/width=3.00)	32.20	0.953

5.8.3 Effect of Span/Depth Ratio

To investigate the effect of varying the span/depth ratio, inclination angle for the same volume of folded plate Goble and Jendikis folded plate has been analyzed using different span/depth ratios. These ratios are (8.0, 8.4, and 9.0). The span are kept constant and equal to (3148mm) while the depth, and inclination angle are considered to vary for the same volume of folded plate. The effect of this factor are shown in Fig. (5.28) and Table (5.8).

A study has been made to investigate the effect of the depth, and inclination angle on the ultimate load capacity of folded plate when the span/depth ratio varying for the same volume. The values of the inclination angles were (24.55° , 30° , and 37.96°). Fig. (5.29) shows the effect of inclination angle on the load carrying capacity of folded plate, from this figure the ultimate load decrease by 6.5% when the angle increased from 24.55° to 30° . when the inclination angle increased from 30° to 37.96° the ultimate load increased by 36.2%. Fig. (5.30) shows

the effect of total depth on the load carrying capacity of folded plate, from this figure the ultimate load decrease by 26.6% when the total depth increased from 349.77 to 375mm . when the total depth increased from 375 to 393.5mm the ultimate load increased by 7%.

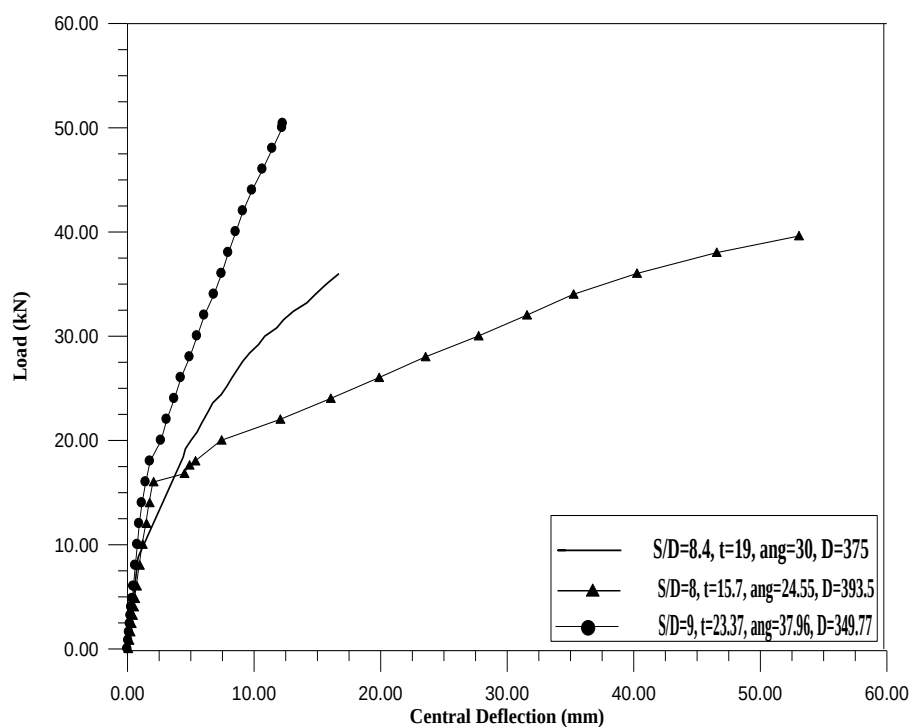


Fig. (5.28) Effect of span/depth ratio on load-deflection curve of Goble and Jendikis folded plate.

Table (5.8) Comparison of experimental and predicted results for different values of span/depth ratios.

Span/depth ratio	Ultimate load (kN)	Load factor
Experimental (span/depth=8.40)	33.80	-

Finite strip (span/ depth =8.00)	39.60	1.171
Finite strip (span/ depth =8.40)	37.00	1.095
Finite strip (span/ depth =9.00)	50.40	1.491

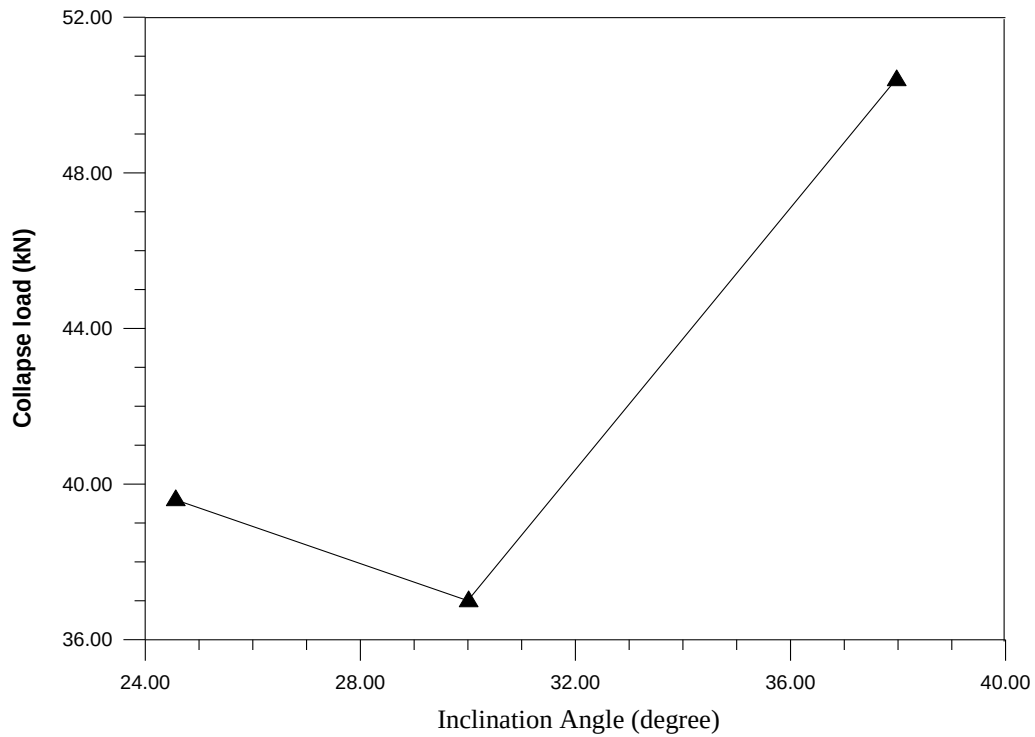


Fig. (5.29) Effect of different values of inclination angle on collapse load of Goble and Jendikis folded plate.

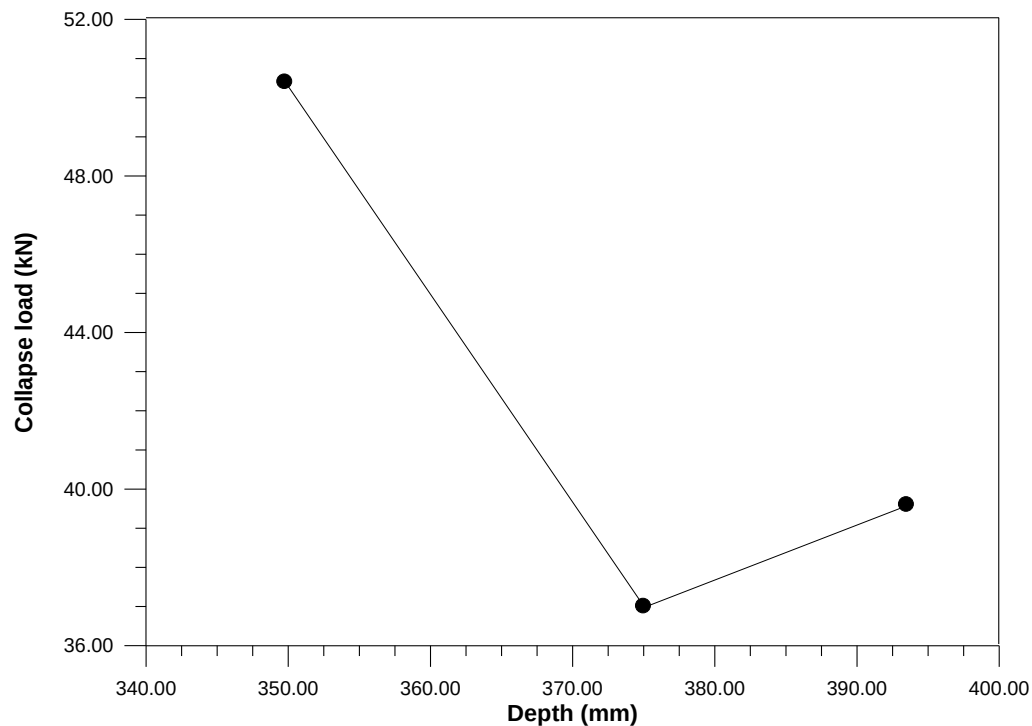


Fig. (5.30) Effect of different values of total depth on collapse load of Goble and Jendikis folded plate.

5.8.4 Effect of Tension-Stiffening Parameters

In order to investigate the effect of the tension stiffening parameters (α_1 , and α_2) on the load-deflection curves of the Goble and Jendikis folded plate. A study has been made by assigning different values of these parameters

The tension stiffening parameter α_1 represents the rate of stress release as the crack widens. The effect of this parameter was investigated by assigning three different values. These values were (10, 14, and 20) while a constant value of 0.5 was set to the parameter α_2 . The results of this study are shown in Fig. (5.31) and Table (5.9). The best fit to the experimental ultimate load was obtained for a value of α_1 equal to 10.

The tension stiffening parameter α_2 represents the sudden loss in tension stress at the instant of cracking. The effect of this parameter was investigated by assigning three different values (0.5, 0.6, and 0.7) while a constant value of 10 was set to parameter α_1 . The results of the investigation are shown in Fig. (5.32) and Table (5.10). It can be seen that the load carrying capacity of the folded plate increases as the tension stiffening parameter α_2 increases, the best fit to the experimental ultimate load has been obtained when α_2 was equal to 0.5.

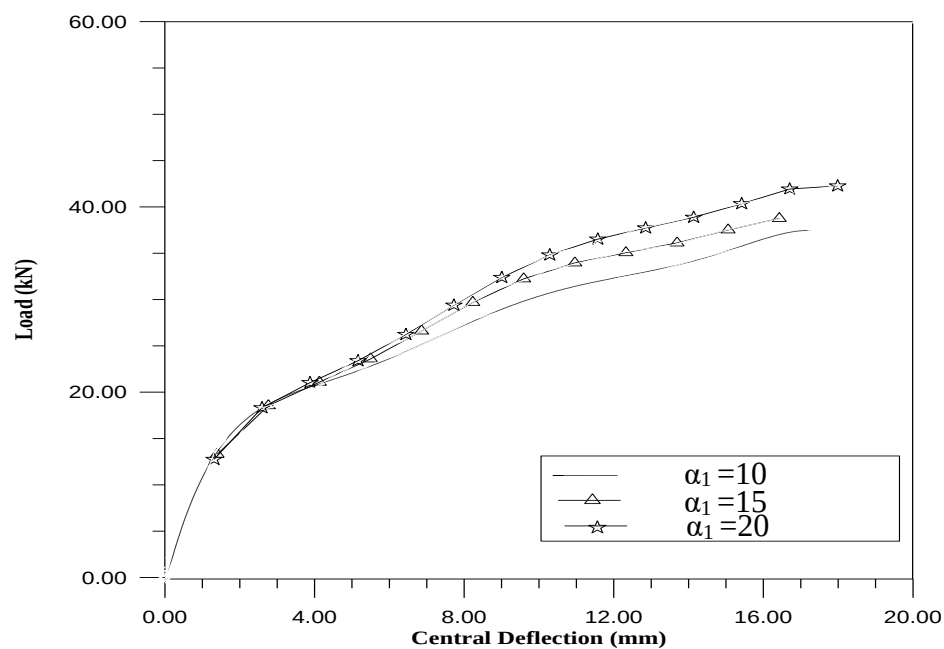


Fig. (5.31) Effect of tension stiffening parameter α_1 on load-deflection curve of Goble and Jendikis folded plate.

Table (5.9) Comparison of experimental and predicted results for different values of tension stiffening parameter α_1

Tension stiffening parameter	Ultimate load (kN)	Load factor
Experimental	33.80	-
Finite strip $\alpha_1=10.0$	37.00	1.095
Finite strip $\alpha_1=15.0$	39.10	1.15
Finite strip $\alpha_1=20.0$	43.30	1.28

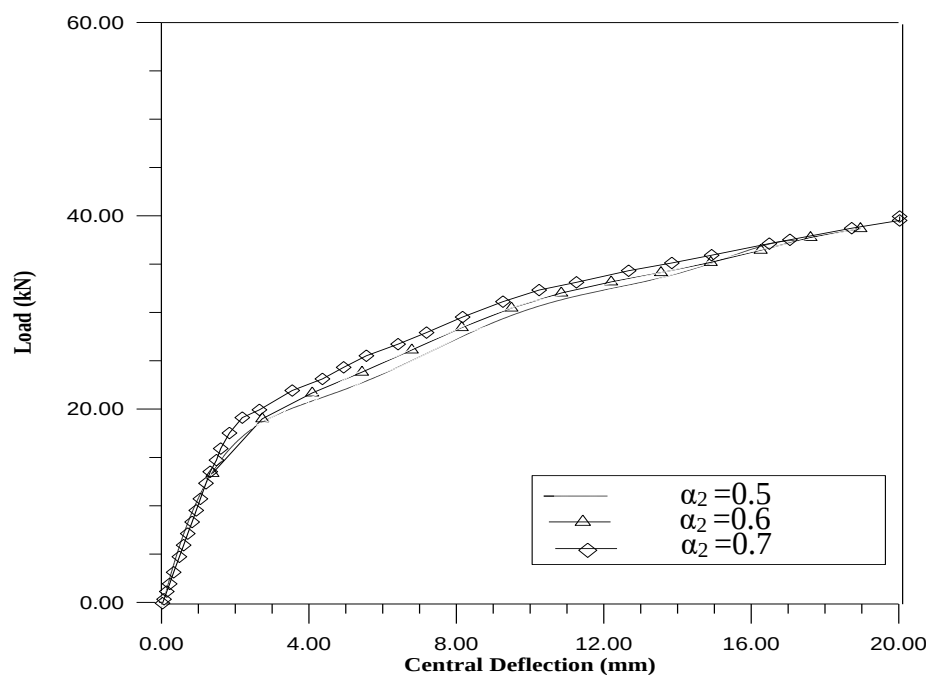


Fig. (5.32) Effect of tension stiffening parameter α_2 on load-deflection curve of Goble and Jendikis folded plate.

Table (5.10) Comparison of experimental and predicted results for different values of tension stiffening parameter α_2

Tension stiffening parameter	Ultimate load (kN)	Load factor
Experimental	33.80	-
Finite strip $\alpha_2=0.50$	37.00	1.095
Finite strip $\alpha_2=0.60$	38.50	1.14
Finite strip $\alpha_2=0.70$	40.10	1.19

5.8.5 Effect of Integration Rule

To investigate the accuracy of different integration rules that used in the computer program, Goble and Jendikis folded plate has been analyzed using 8, 15a, and 27 point integration rules. The predicted load-deflection responses and the ultimate loads are shown in Fig. (5.33) and Table (5.11). It can be can be seen from Fig. (5.33) that the results obtained by the 15a-point integration rule are closer to the experimental ultimate load, while the 27-point integration rule gives a stiffer response than 15a- rule. The 8-point integration rule gives a softer response than the other two rules.

The 15a rule has advantages of having sampling points at the centers of the element faces. These locations are convenient sampling

positions for the stress peak values (Cervera et al 1986). The stiffer response produced by full integration (27-point integration rule) is due to the shear locking phenomena which is commonly occurred in plate analysis because of the bad spreading of sampling points throughout the element.

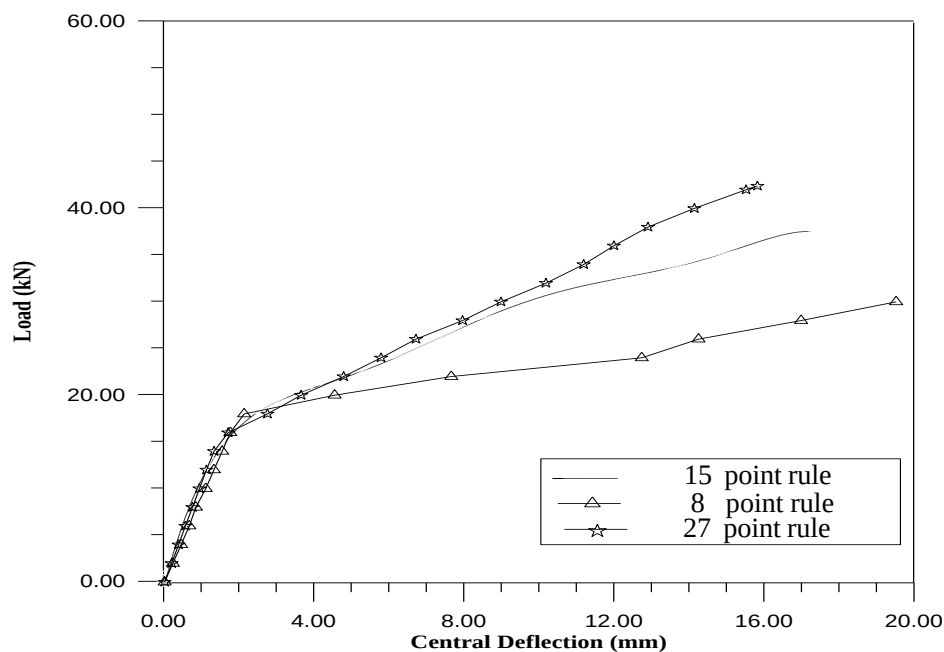


Fig. (5.33) Effect of integration rule on load-deflection curve of Goble and Jendikis folded plate.

Table (5.11) Comparison of experimental and predicted results for different integration rules.

Type of integration rule	Ultimate load (kN)	Load factor
Experimental	33.80	-
Finite strip (8-point integration rule)	30.00	0.88
Finite strip (15a-point integration rule)	37.00	1.095
Finite strip (27-point integration rule)	42.60	1.26

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

Based on the present investigation (results and discussions are given in chapter five), the following conclusions can be drawn:

- 1- In the static analysis of reinforced concrete continuum structures, especially folded plates, the numerically predicted results by the finite strip method are in good agreement with the experimental and other theoretical results.
- 2- The developed numerical model has proved to be a good tool for the analysis of reinforced concrete beam, one-way slab, flat slab, cylindrical shell, and folded plates with relatively course meshes. The maximum number of prism elements used for the considered examples are 16 prisms.
- 3- The numerical results carried out for the different types of problems showed that the inclusion of the geometric nonlinearity together with material nonlinearity in the present method of analysis gives results for the predicted load-deflection behavior closer to the experimental results. The maximum decreases of the collapse load when the geometric nonlinearity is considered is about 10%.
- 4- The numerical results revealed that as the span/width ratio of folded plates increased, the ultimate load capacity is decreased. It was found that the decrease of the collapse load is about 13.5% when the span/width ratio is increased from 2.34 to 3.0 .
- 5- The effect of varying inclination angle of side plates on the behavior of folded plates is also investigated in the current research work. the ultimate

load of folded plates is increased by about 36.2% when the inclination angle of side plates increased from 30° to 37.96° .

6- It was found that the type of integration rule has a significant effect on the behavior of the folded plate structures. The 15a-point integration rule gives a closer agreement to the experimental results.

6.2 RECOMMENDATIONS

The following recommendations are suggested for further researches:

- 1- Modify the numerical model adopted in the present work in order to analyze continuous reinforced concrete folded plates (Multiple folded plates) with interior diaphragms.
- 2- Modify the numerical model adopted in the present work to analyze the non prismatic, and curved reinforced concrete folded plates.
- 3- Modify the numerical model adopted in the present work to included long term effects (Creep and shrinkage) and the effect of cyclic loading on the behavior of reinforced concrete folded plates.
- 4- Study the nonlinear behavior of prestressed concrete folded plates.
- 5- Study the nonlinear behavior of reinforced concrete box or deck girder bridges .
- 6- Study the nonlinear behavior of reinforced concrete folded plates containing openings.

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