

Republic of Iraq
Ministry of Higher Education and scientific Research
University of Babylon
College of Engineering
Civil Engineering Department



STUDING OF THE RUTTING IN THE FLEXIBLE PAVEMENTS USING VISCOELASTIC FINITE ELEMENT METHOD

A Thesis

*SUBMITTED TO THE COLLEGE OF ENGINEERING OF BABYLON
UNIVERSITY IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR DEGREE O MASTER SCIENCE IN CIVIL ENGINEERING
(TRANSPORTATION ENGINEERING)*

By

Ghasak Ebrahim Almothfer

(B.Sc. in Civil Engineering, 2004)

Supervised by

Assist Prof.Dr. Nahla Yassoub

Assist Prof.Dr. Haithem H. Al Daa'mi

(September, 2007)

(Shaaban, 1428)



**دراسة الروطان في التبليط المرن
بإستعمال طريقة العناصر المحددة
للمرونة-اللزجة**

الى كلية الهندسة / جامعة بابل وهي جزء من
متطلبات نيل درجة الماجستير علوم في قسم الهندسة المدني / هندسة المواصلات
الطالب

إشراف

أ.م.الدكتورة: نهلة يعسوب

أ.م. الدكتور. هيثم حسن الدعي

أيلول، 2007

1428, شعبان

Supervisor Certificate

We certify that this thesis entitled **“Formulation of The Rutting in The Flexible Pavements Using Viscoelastic Approach”** was prepared by **“Ghasak E. AL Mothfer ”**, under our supervision at Babylon University as partial fulfillment of the requirements for the master degree of science in civil engineering/ transportation engineering

Signature:

Name: Assist Professor Dr. Nahla Yassoub

Date: / / 2007

Signature:

Name: Assist Professor Dr. Haithem H. AL. Daa'mi

Date: / / 2007

***DEDICATING TO MY FAMILY, SPECIALLY TO MY
MOTHER AND FATHER WITH ALL LOVE AND
RESPECT***

ACKNOWLEDGEMENT

I express my sincere appreciation to everyone who helped me during the preparation of this thesis and offer my sincere thanks to them.

First of all, I wish to express my sincere gratitude and appreciation to the supervisors, Assist Prof. Dr. Nahla Yassoub and Assist Prof. Dr. Haithm H.AL Daa'mi for their supervisions, precious advices, technical guidance, continuous encouragements and remarkable patience in reviewing my thesis, I'm really indebted to them.

Thanks are presented to the staff of the Transportation Engineering of Babylon University for their help in using the various facilities. Special thanks to my guide M.Sc. Shaker and M.Sc. Hareth for their support.

Special thanks are also devoted to my closer friends Eng. Mohamed Al Ameedi for his Precious helping not only in Computer software but also provide me all support I need to finish this study.

I would like to thanks the staff of Mechanical Engineering Department especially to my friend M.Sc Salwan; also I would like to thank M.Sc Ali Abbas in robotics Engineering Department of Baghdad University for their notes in ANSYS commands and using it options.

Thanks to my friend Eng. Ahmed Abbas for his efforts to make all impossible things easy, providing me all books and papers up-to-date and encouragement. Also thank to my partner Eng. Anmar and my partner Eng. Ausama Ali for his deeply supports.

Finally, I would like to thank my family for their great patient and support during the period of this study.

الخلاصة

ان الهدف الرئيسي من هذه الدراسة هو تشكيل موديل تنبؤي لمقدار التحدد الاولي في طبقة التبليط الاسفلتي تحت الظروف المحلية. تم اعتماد بيانات فحص خضوع الزحف / الاسترداد الثابت لمادة التبليط الاسفلتي لتقييم قابليتها للتحدد, تحت انواع مختلفة من درجات الحرارة, كذلك تم استعمال تقنية العناصر المحدودة, دقة وكفاءة تلك التقنية جعلتها احد افضل الطرق التقريبية لحل المعادلة التفاضلية التي تصف طبقات التبليط الاسفلتي, هذا الحل يمثل الاستجابة الابتدائية والتي تؤثر مباشرة على حساسية طرق التحليل للتنبؤ بالروطان لكل من مقترب المرونة ومقترب المرونة-اللزوجة. تم استعمال طريقة الارتباط اللاخطي لتمثيل بيانات الفحص وتم استعمال هذا التمثيل في ايجاد ثوابت التشوه الدائم أ و ج , هذه الثوابت تم استعمالها في ايجاد الروطان للخرسانة الاسفلتية.

استجابة الطبقة تحت مختلف الاحمال تم حسابها بواسطة برنامج **ANSYS version 9.0**, تم استعمال برنامج **KENLAYER** لاجراء المقارنة مع تقنية العناصر المحدودة, الفرق بين طريقة التحليل بمقترب المرونة عن مقترب المرونة -لزوجة حوالي (12%). اخيرا تم حساب الروطان ومقارنته مع قيمة الروطان المحسوبة من صيغتان معتبرتان, ان نتائج المقارنة بينت وجود فرق بين نتائج طرق التحليل يتراوح بين (12%-27%).

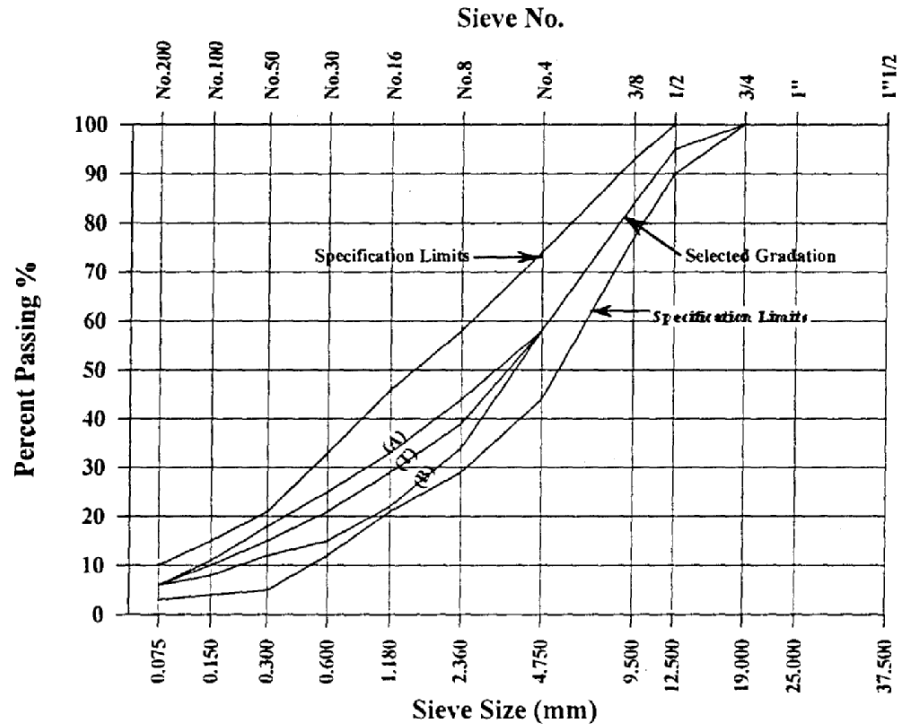
ABSTRACT

The main objective of this study is to formulate a prediction model of rutting under surrounding conditions depending on the viscoelastic approach behavior of asphalt concrete materials subjected to repeated axle loading. The static creep / recovery test previous data have been employed for characterizing deformation-time relationship at various temperatures. To analyze the system of pavement The "Finite Element Method" has been used, its accuracy and efficiency for analyze a flexible pavement made it one of the best approximate solution of the partial differential equation of multilayer pavement's system, This solution represent the primary response which is the main factor that directly affects upon analysis susceptibility of the rutting predication for both elastic and viscoelastic approach. Using a non-linear regression for simulating creep /recovery compliance data, this simulating was used to calculate the permanent deformation parameters, which parameters have been used to predict rut depth.

The pavement response under various loading has been evaluated using [ANSYS](#) (version 9.0) software. [KENLAYER](#) software has been used to calibrate the constructed model. For primary response the difference between the two approaches (Elastic; Viscoleastic) is about (12%). Finally the rut depth was calculated and compared with two considerable models, the results of comparison show that there are differences in results of model about (12%-27%).

Appendix A

MODELING OF PERMENTANT STRAIN OF ASPHALT PAVEMENT FOR OTHER MIXTURES



Sieve Size	Percent Passing %		
	A	T	B
3/4	100	100	100
1/2	95	95	95
3/8	84	84	84
No. 4	58	58	58
No. 8	44	39	34
No. 16	33	29	22
No. 30	25	21	15
No. 50	18	15	12
No. 100	11	10	8
No. 200	6	6	6

Figure (A-1): The Selected Gradations Chart (Yassoub, 2004).

A-1 Parameters α , μ for Each Mixture

- SSE=Sum of squares due to error.
- Rsquare=Coefficient of determination.
- Dfe=Degrees of freedom.
- Adjrsquare=Degree-of-freedom adjusted coefficient of determination.
- RMSE=Root mean squared error (standard error).
- MaxFunEvals -- Maximum number of function (model) evaluations allowed. The default value is 600.
- MaxIter -- Maximum number of fit iterations allowed. The default value is 400.
- TolFun -- Termination tolerance used on stopping conditions involving the function (model) value. The default value is 10-6.
- TolX -- Termination tolerance used on stopping conditions involving the coefficients. The default value is 10-6.

Table (A-1), Limitations of Nonlinear Regression Method for Each Mixture.

Type of procedure :	least square method
Algorithm :	Trust – Region method
Different min change :	1.0E-12
Different max change :	1.0E-14
Max function evaluating :	1.0E09
Max iteration :	1.0E09
Tolerance of function :	1.0E-20
Tolerance of x value :	1.0E-20

Table (A-2), Creep and Recovery Compliance in (1/psi) for Mix A1 at Temperature =25 C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.000040375	0.000031125
0.25	0.000053125	0.000033625
0.5	0.000071125	0.000035375
1	0.0000775	0.000038375
2	0.00008875	0.00003975
4	0.000100625	0.000043625
8	0.00011125	0.000047375
15	0.000144	0.000048375
30	0.000167	0.00004875
45	0.00017975	0.00004875
60	0.000183625	0.00004875

Table (A-3), Estimating Parameters of Hyperbolic Model for Mix A1 at Temperature =25 C⁰.

A	0.1452	B	0.884
Do	1.753e-005	Ro	2.664e-005
Dm	0.000467	Rm	5.162e-005
m	0.35	mp	0.63
SSE:	2.5742e-010	SSE:	6.6399e-012
R-square:	0.9901	R-square:	0.9855
Adjusted R-square:	0.9876	Adjusted R-square:	0.9819
RMSE:	5.6725e-006	RMSE:	9.1104e-007
Dfe:	8	Dfe:	8

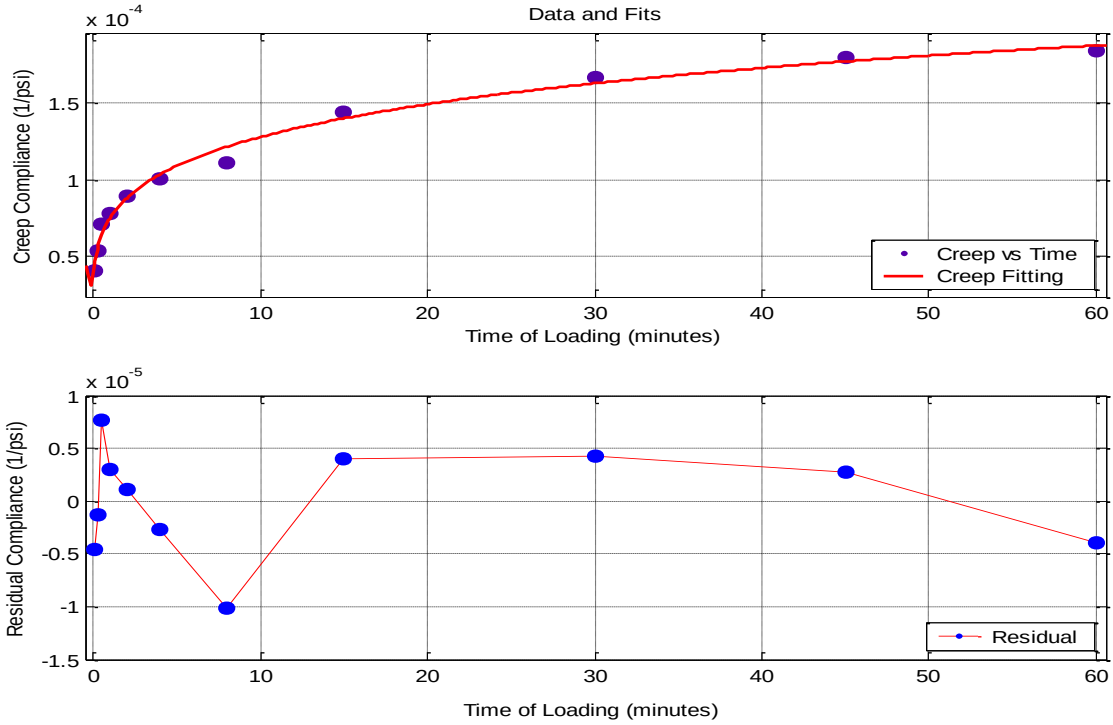


Figure (A-2), Creep Compliance Fitting and Residual of Compliance for Mix A1 at Temperature =25 C⁰.

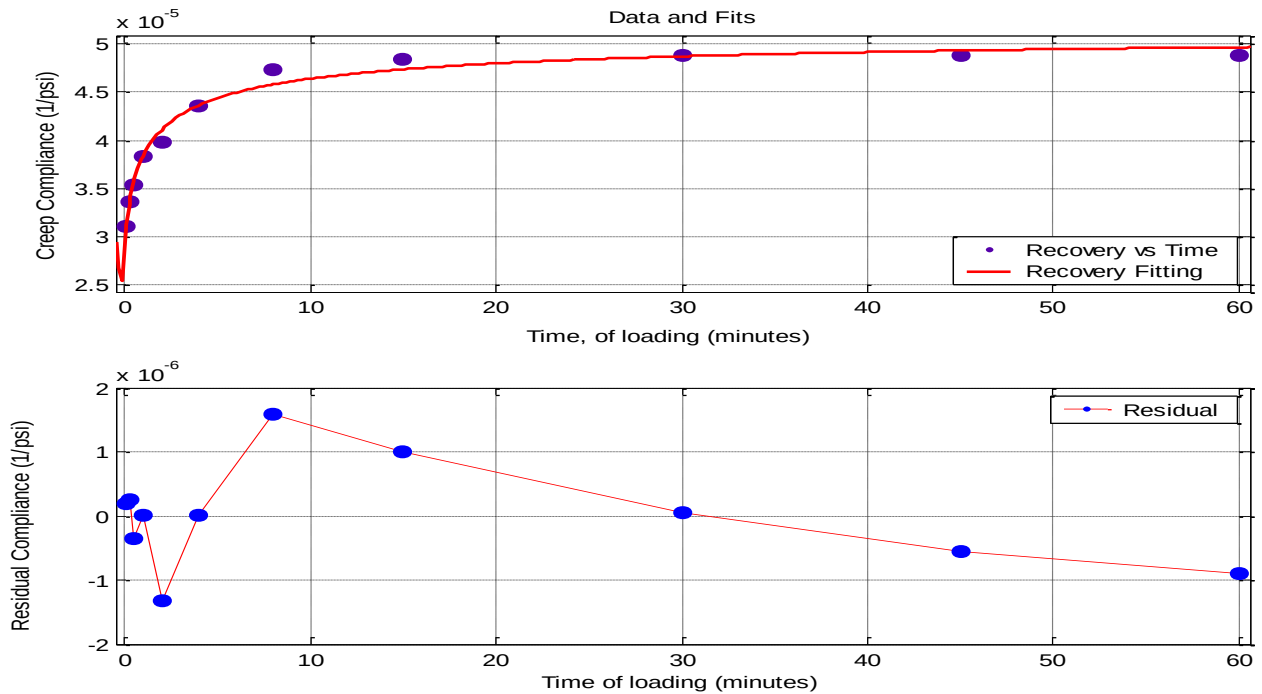


Figure (A-3), Recovery Compliance Fitting and Residual of Compliance for Mix A1 at Temperature =25 C⁰.

Table (A-4), Creep and Recovery Compliance in (1/psi) for Mix T1 at Temperature =25 C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.0000525	0.000035375
0.25	0.000069125	0.000038625
0.5	0.000091625	0.000040625
1	0.00010075	0.00004475
2	0.0001075	0.000046625
4	0.000115375	0.000051625
8	0.000130875	0.0000565
15	0.000144625	0.00005775
30	0.00018725	0.00005825
45	0.000217125	0.00005825
60	0.000233625	0.00005825

Table (A-5), Estimating Parameters of Hyperbolic Model for Mix T1 at Temperature =25 C⁰.

A	00.1214	B	2.124
Do	8.702E-006	Ro	9.057e-006
Dm	0.0004469	Rm	3.978e-005
m	0.31	mp	0.434
SSE:	1.417e-010	SSE:	1.105e-011
R-square:	0.9955	R-square:	0.959
Adjusted R-square:	0.9909	Adjusted R-square:	0.9487
RMSE:	4.208e-006	RMSE:	1.175e-006
Dfe:	8	Dfe:	8

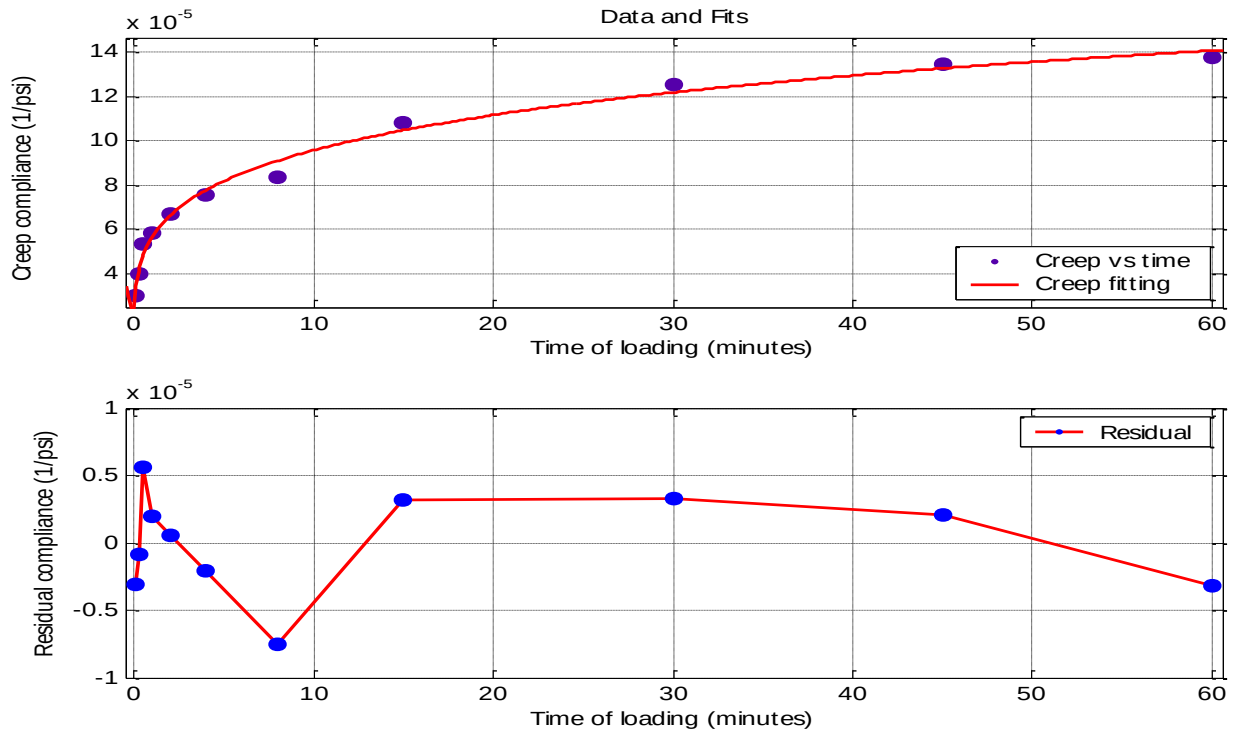


Figure (A-4), Creep Compliance Fitting and Residual of Compliance for Mix T1 at Temperature =25 C°.

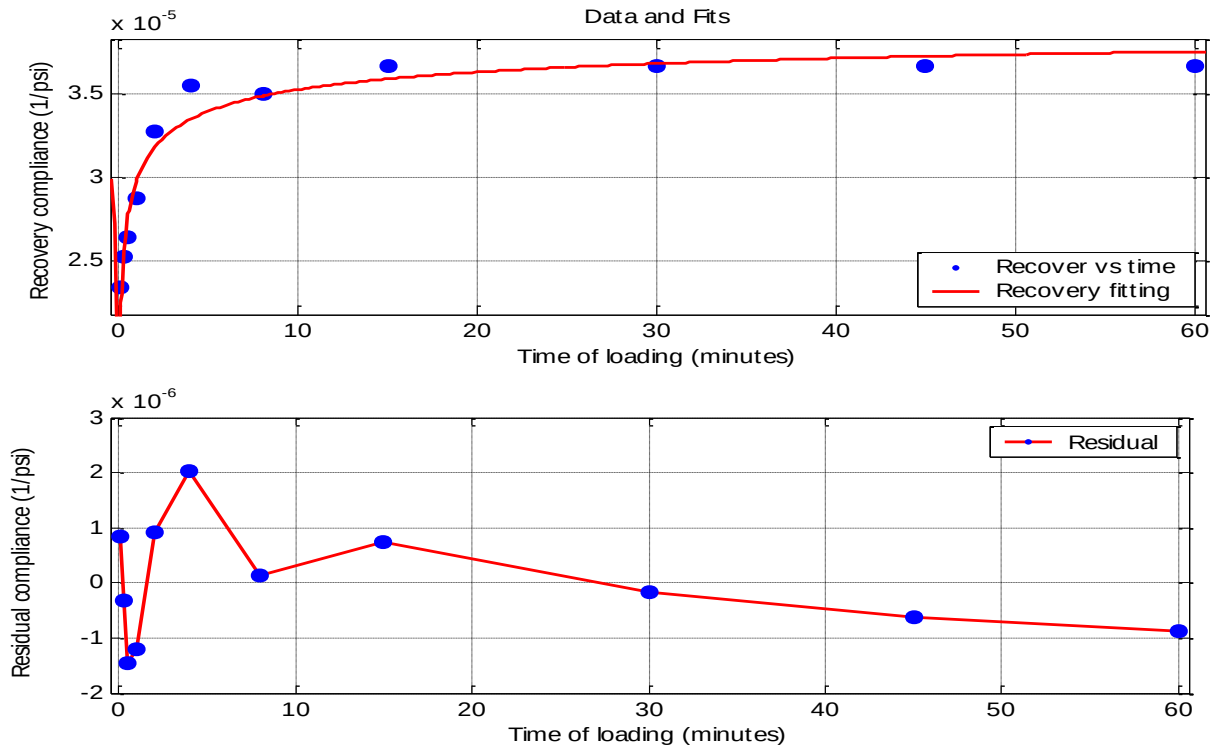


Figure (A-5), Recovery Compliance Fitting and Residual of Compliance for mix T1 at Temperature =25 C°.

Table (A-6), Creep and Recovery Compliance in (1/psi) for Mix A2 at Temperature =25 C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.000073875	0.000043
0.25	0.00007325	0.000046375
0.5	0.00009725	0.0000485
1	0.000106875	0.000053
2	0.000122375	0.000054875
4	0.00013875	0.00006025
8	0.0001535	0.000065375
15	0.000198625	0.00006675
30	0.000230375	0.00006725
45	0.000248	0.00006725
60	0.000253375	0.00006725

Table (A-7), Estimating Parameters of Hyperbolic Model for Mix A2 at Temperature =25 C⁰.

A	0.1066	B	0.8822
Do	4.434e-005	Ro	3.672e-005
Dm	0.000672	Rm	7.125e-005
m	0.389	mp	0.63
SSE:	4.1787e-010	SSE:	1.3053e-011
R-square:	0.9909	R-square:	0.9851
Adjusted R-square:	0.9887	Adjusted R-square:	0.9814
RMSE:	7.2273e-006	RMSE:	1.2774e-006
Dfe:	8	Dfe:	8

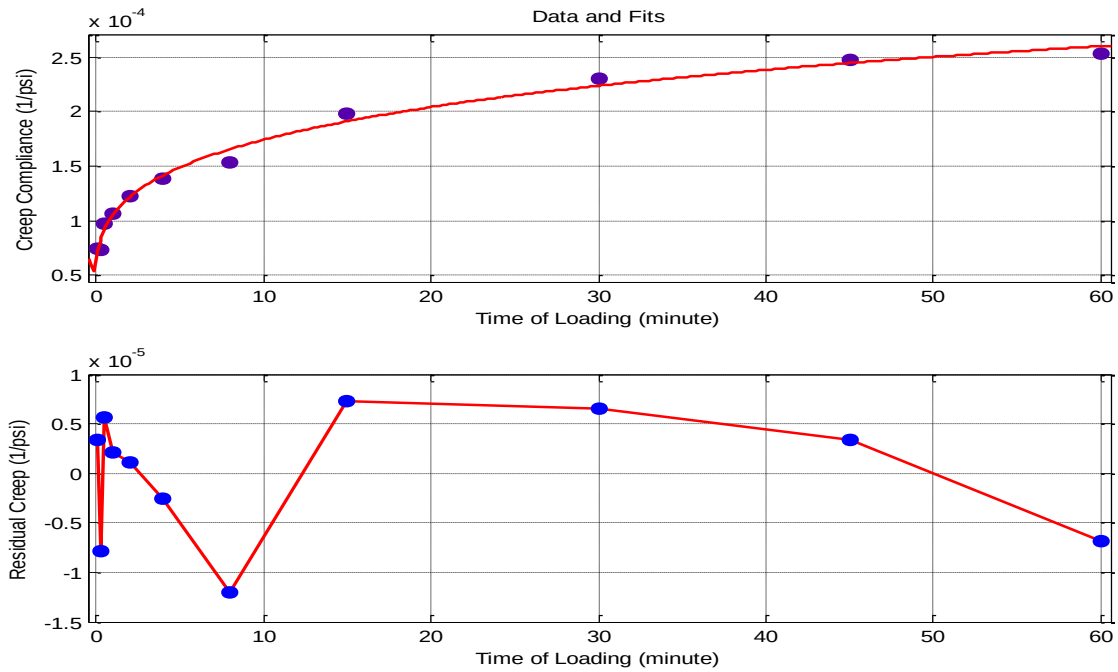


Figure (A-6) Creep Compliance Fitting and Residual of Compliance for Mix A2 at Temperature =25 C⁰.

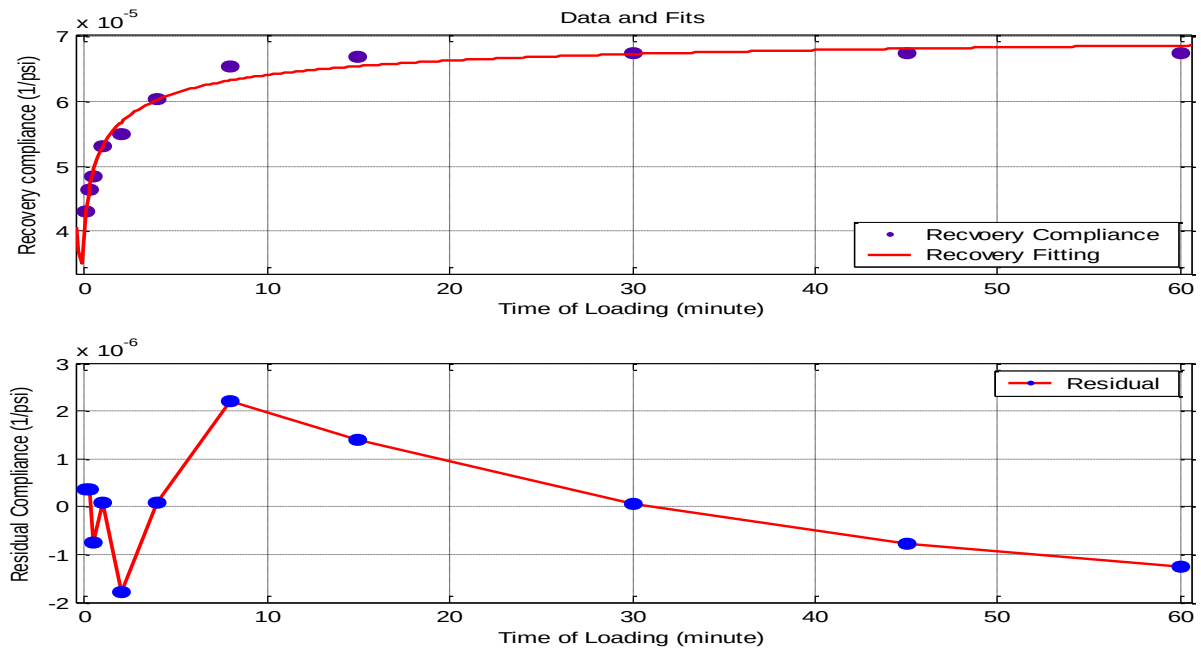


Figure (A-7), Recovery Compliance Fitting and Residual of Compliance for Mix A2 at Temperature =25 C⁰.

Table (A-8), Creep and Recovery Compliance in (1/psi) for Mix T2 at Temperature =25 C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.0000545	0.000042
0.25	0.00007175	0.000045375
0.5	0.000095125	0.000047375
1	0.000105625	0.00005175
2	0.000119875	0.000053625
4	0.000135875	0.000058875
8	0.00015025	0.000063875
15	0.000194375	0.00006525
30	0.0002255	0.00006575
45	0.000242625	0.00006575
60	0.000247875	0.00006575

Table (A-9), Estimating Parameters of Hyperbolic Model for Mix T2 at Temperature =25 C⁰.

A	0.1654	B	0.8764
Do	3.125e-005	Ro	3.592e-005
Dm	0.0005162	Rm	6.967e-005
m	0.397	mp	0.63
SSE:	4.7394e-010	SSE:	1.2403e-011
R-square:	0.9899	R-square:	0.9852
Adjusted R-square:	0.9874	Adjusted R-square:	0.9815
RMSE:	7.6969e-006	RMSE:	1.2452e-006
Dfe:	8	Dfe:	8

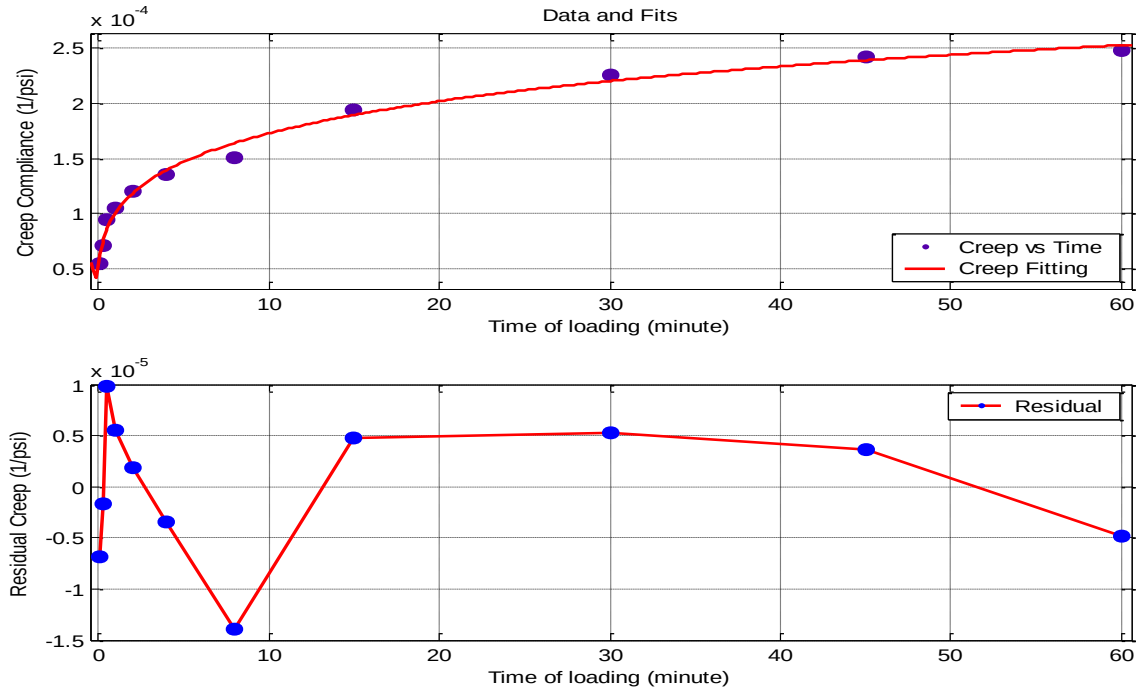


Figure (A-8) Creep Compliance Fitting and Residual of Compliance for T2 at Temperature =25 C⁰.

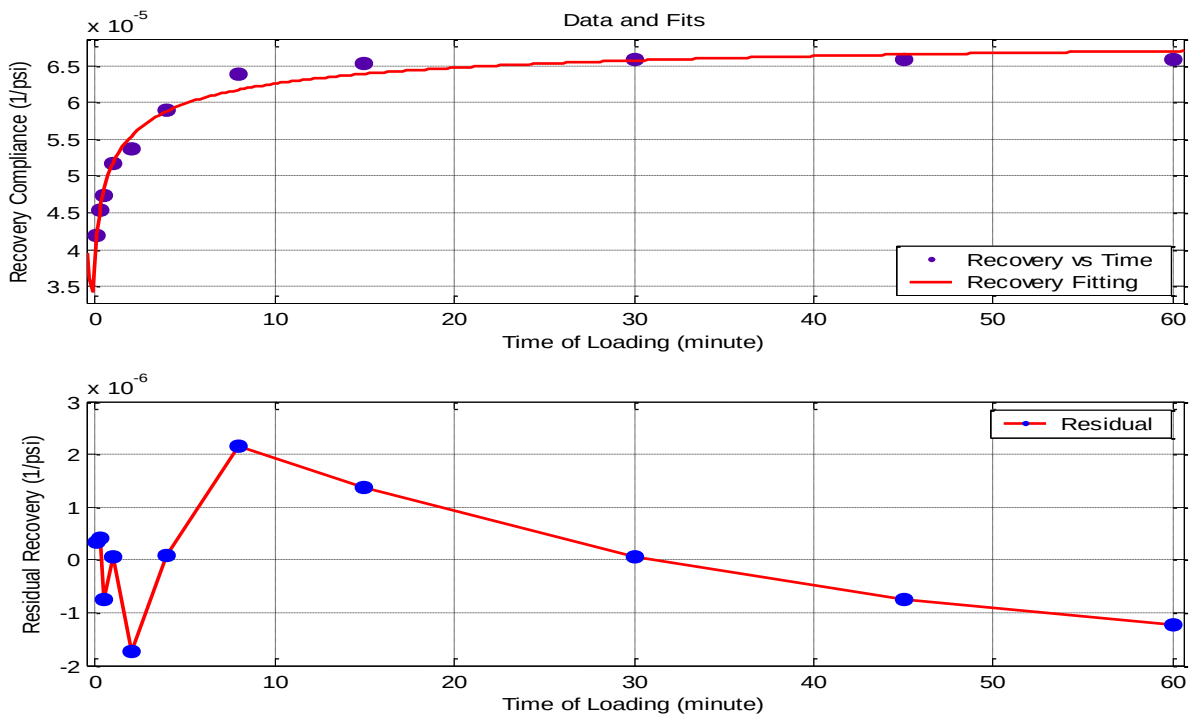


Figure (A-9) Recovery Compliance Fitting and Residual of Compliance for T2 at Temperature =25 C⁰.

Table (A-10), Creep and Recovery Compliance in (1/psi) for Mix B2 at Temperature =25 C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.000051625	0.000040125
0.25	0.0000685	0.000043375
0.5	0.000091	0.000045625
1	0.0001	0.0000495
2	0.0001145	0.00005125
4	0.00012975	0.00005625
8	0.0001435	0.000061125
15	0.00018575	0.000062375
30	0.000215375	0.000062875
45	0.000231875	0.000062875
60	0.000236875	0.000062875

Table (A-11), Estimating Parameters of Hyperbolic Model for Mix B2 at Temperature =25 C⁰.

A	0.1654	B	0.8845
Do	2.932e-005	Ro	3.435e-005
Dm	0.0004951	Rm	6.658e-005
m	0.396	mp	0.63
SSE:	4.3286e-010	SSE:	1.1154e-011
R-square:	0.9900	R-square:	0.9854
Adjusted R-square:	0.9875	Adjusted R-square:	0.9817
RMSE:	7.3558e-006	RMSE:	1.1808e-006
Dfe:	8	Dfe:	8

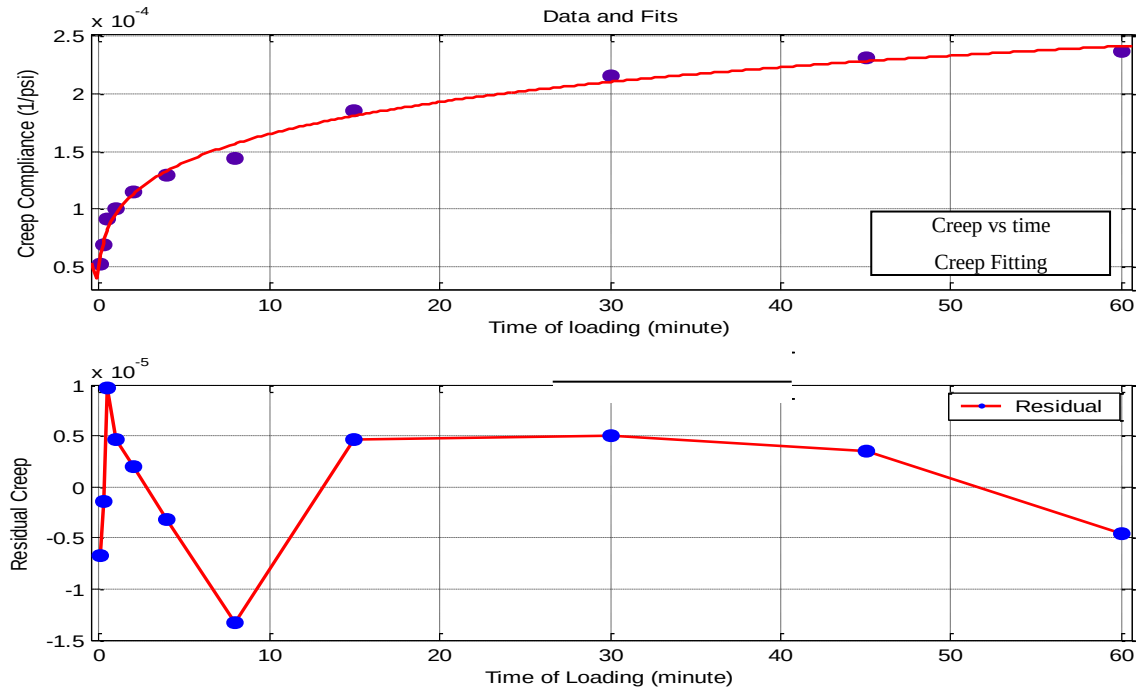


Figure (A-10), Creep Compliance Fitting and Residual of Compliance for Mix B2 at Temperature =25 C⁰.

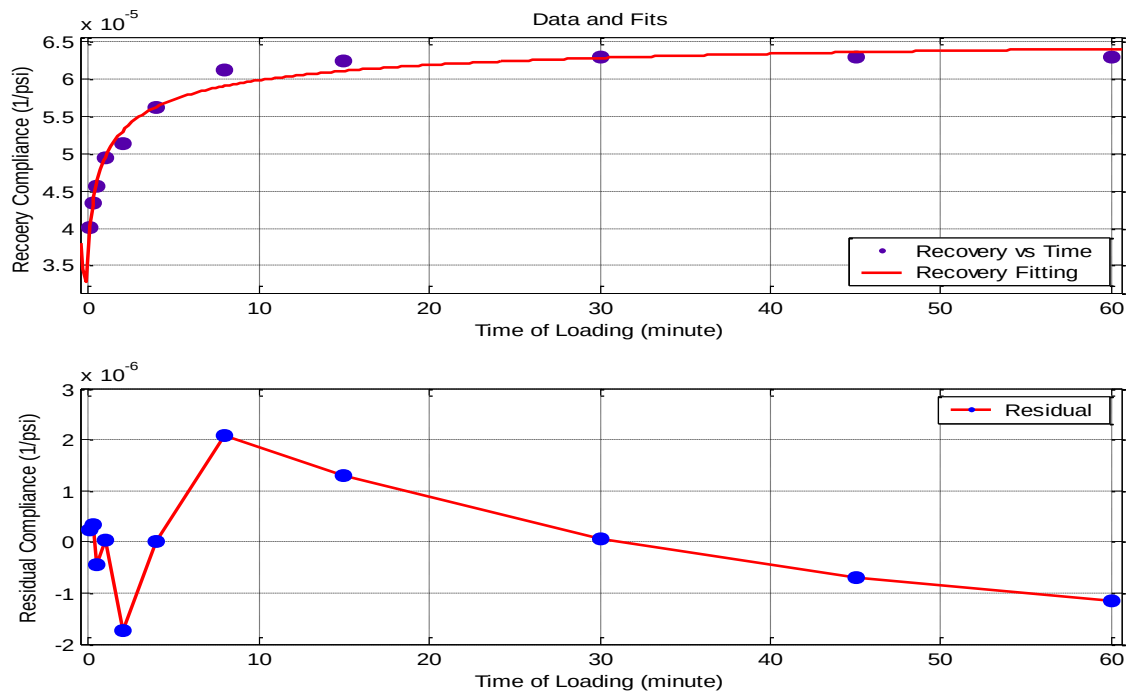


Figure (A-11), Recovery Compliance Fitting and Residual of Compliance for Mix B2 at Temperature =25 C⁰.

Table (A-12), Creep and Recovery Compliance in (1/psi) for Mix B1 at Temperature=40C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.000036875	0.00002075
0.25	0.000067125	0.00002275
0.5	0.000078	0.000022875
1	0.000106625	0.000022875
2	0.000125375	0.0000245
4	0.00013625	0.000024875
8	0.000151125	0.000024875
15	0.00017775	0.0000255
30	0.000188625	0.0000255
45	0.0002015	0.0000255
60	0.00022025	0.0000255

Table (A-13), Estimating Parameters of Hyperbolic Model for Mix B1 at Temperature=40C⁰.

A	0.6023	B	8.941
Do	8.001e-006	Ro	2.555e-007
Dm	0.0002473	Rm	2.628e-005
m	0.523	mp	0.3665
SSE:	5.1433e-010	SSE:	1.2133e-012
R-square:	0.9853	R-square:	0.9519
Adjusted R-square:	0.9816	Adjusted R-square:	0.9313
RMSE:	8.0182e-006	RMSE:	4.1633e-007
Dfe:	8	Dfe:	8

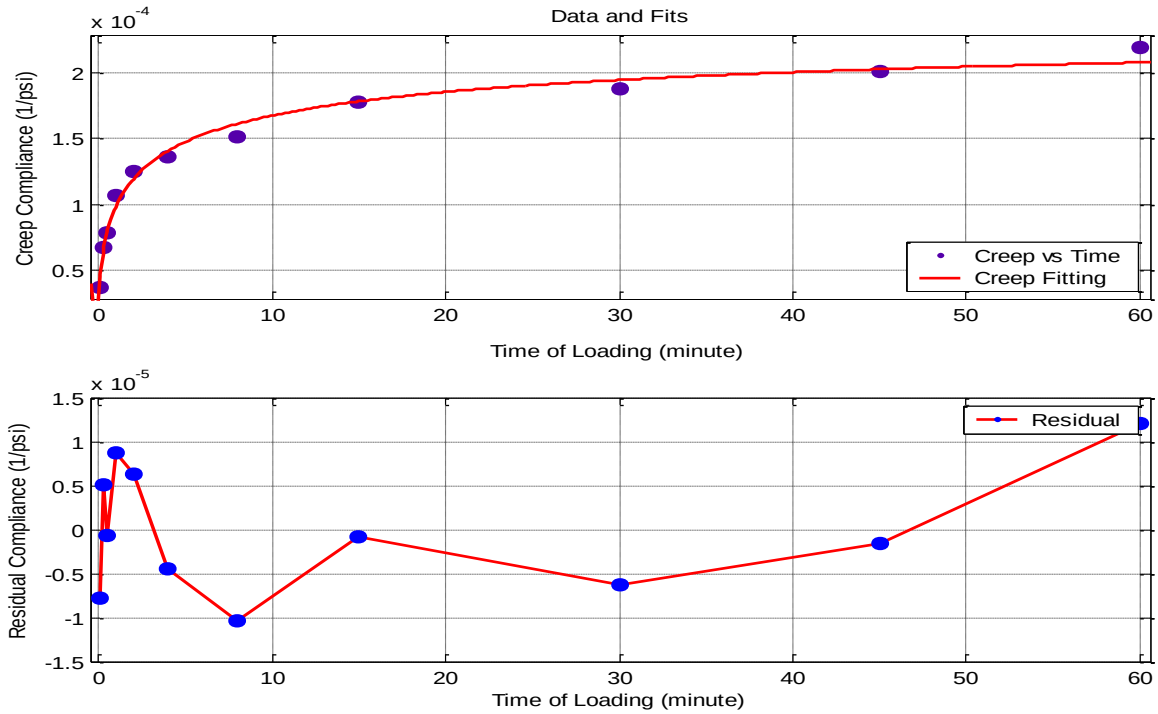


Figure (A-12), Creep Compliance Fitting and Residual of Compliance for Mix B1 at Temperature=40C⁰.

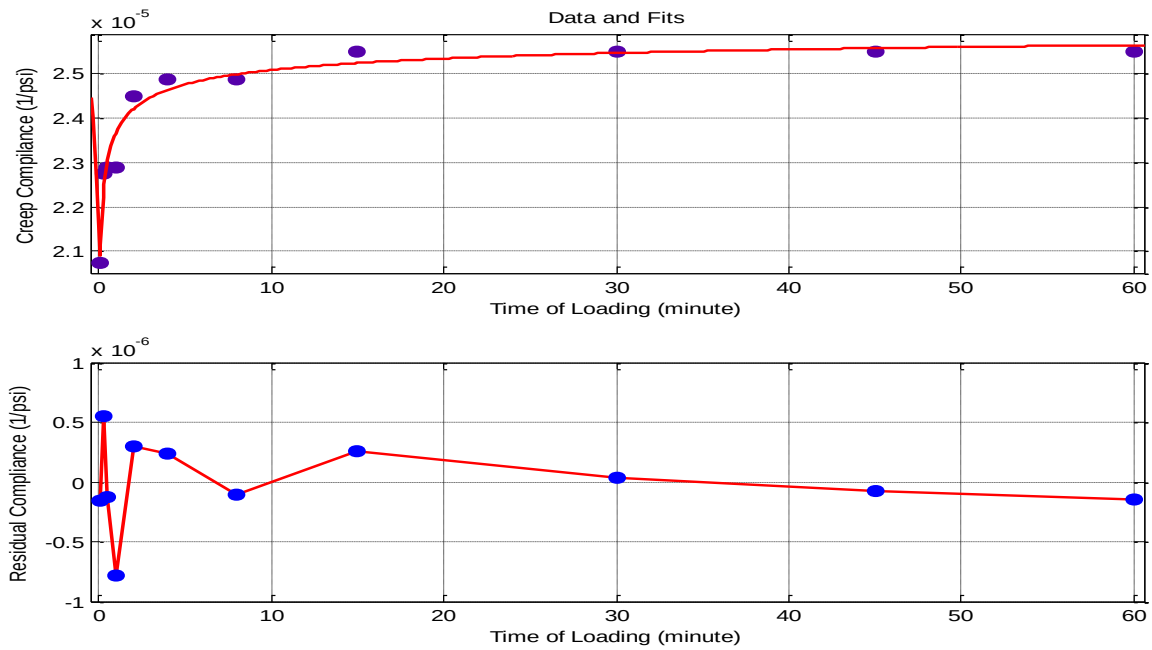


Figure (A-13), Recovery Compliance Fitting and Residual of Compliance for Mix B1 at Temperature=40C⁰.

Table (A-14), Creep and Recovery Compliance in (1/psi) for Mix B1 at Temperature=50C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.000069875	0.000084125
0.25	0.000076625	0.000086375
0.5	0.000077	0.000087375
1	0.000080375	0.000087875
2	0.000089625	0.00008925
4	0.000101375	0.00008975
8	0.00011575	0.000091
15	0.00014625	0.000091375
30	0.0001725	0.000091375
45	0.000193375	0.000091375
60	0.000215875	0.000091375

Table (A-15), Estimating Parameters of Hyperbolic Model for Mix B1 at Temperature=50C⁰.

A	0.03039	B	3.002
Do	6.707e-005	Ro	7.664e-005
Dm	0.000579	Rm	9.225e-005
m	0.632	mp	0.4961
SSE:	6.4817e-011	SSE:	8.2023e-013
R-square:	0.9976	R-square:	0.9866
Adjusted R-square:	0.9966	Adjusted R-square:	0.9809
RMSE:	3.0429e-006	RMSE:	3.4231e-007
Dfe:	8	Dfe:	8

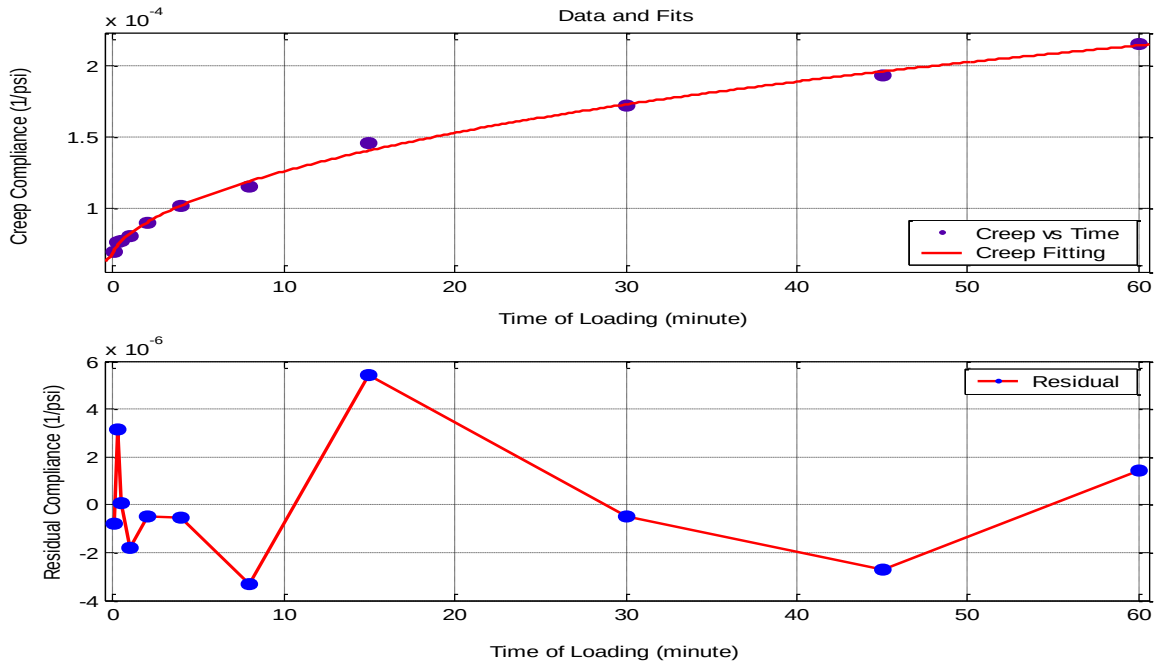


Figure (A-14), Creep Compliance Fitting and Residual of Compliance for Mix B1 at Temperature=50C⁰.

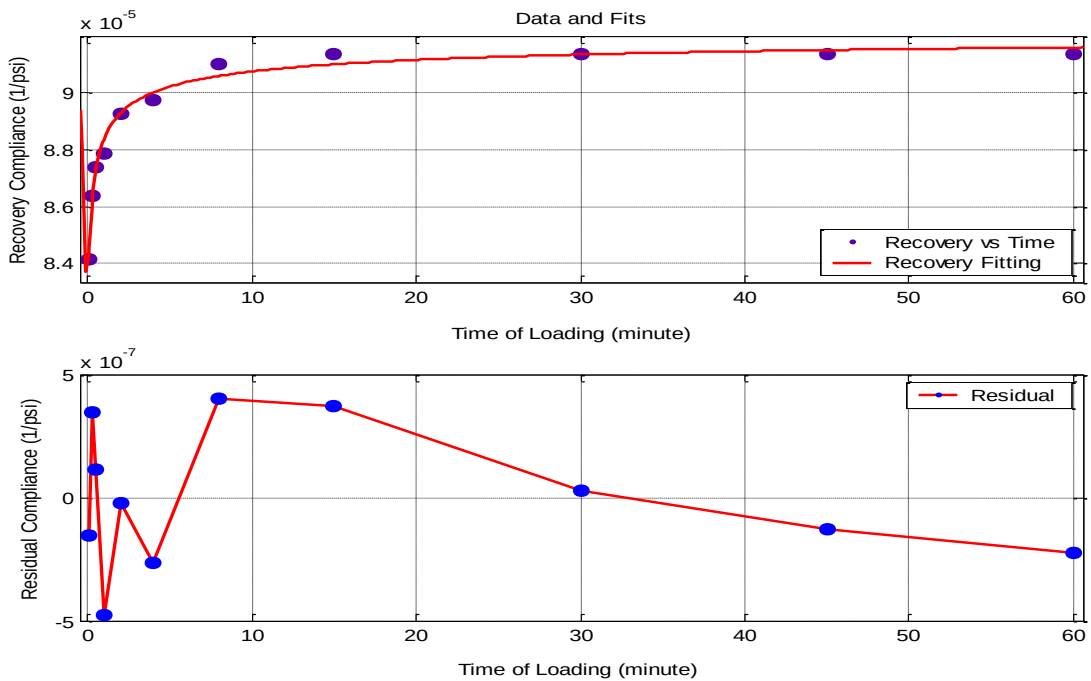


Figure (A-15), Recovery Compliance Fitting and Residual of Compliance for Mix B1 at Temperature=50C⁰.

Appendix B

SOLUTION OF KENLAYER FOR MULTILAYER FLEXIBLE PAVEMEN

B.1 KENLAYER SOLUTION

INPUT FILE NAME -C:\KENPAVE\viscoelastic\Elastic_Solution_Check.DAT

NUMBER OF PROBLEMS TO BE SOLVED = 1

TITLE -Elastic Solution

MATL = 1 FOR LINEAR ELASTIC LAYERED SYSTEM

NDAMA = 0, SO DAMAGE ANALYSIS WILL NOT BE PERFORMED

NUMBER OF PERIODS PER YEAR (NPY) = 1

NUMBER OF LOAD GROUPS (NLG) = 1

TOLERANCE FOR INTEGRATION (DEL) -- = 0.001

NUMBER OF LAYERS (NL)----- = 3

NUMBER OF Z COORDINATES (NZ)----- = 13

LIMIT OF INTEGRATION CYCLES (ICL)- = 80

COMPUTING CODE (NSTD)----- = 9

SYSTEM OF UNITS (NUNIT)----- = 1

Length and displacement in cm, stress and modulus in kPa
unit weight in kN/m³, and temperature in C

THICKNESSES OF LAYERS (TH) ARE : 10 20

POISSON'S RATIOS OF LAYERS (PR) ARE : 0.35 0.3 0.4

VERTICAL COORDINATES OF POINTS (ZC) ARE: 0 10 20 30 40 50 60
90 120

150 180 200 230

ALL INTERFACES ARE FULLY BONDED

FOR PERIOD NO. 1 LAYER NO. AND MODULUS ARE : 1 3.500E+06 2
3.500E+05

3 1.000E+05

LOAD GROUP NO. 1 HAS 1 CONTACT AREA

CONTACT RADIUS (CR)----- = 15

CONTACT PRESSURE (CP)----- = 550

RADIAL COORDINATES OF 12 POINT(S) (RC) ARE : 0 5 15 30 45 60 75
90

120 150 180 210

PERIOD NO. 1 LOAD GROUP NO. 1

5.00000	30.00000	0.03471	62.056	-96.290	-98.410	5.926
(STRAIN)		3.442E-04	-2.440E-04	-2.518E-04	.440E-04	
5.00000	40.00000	0.02947	44.182	0.001	-0.275	3.489
(STRAIN)		4.429E-04	-1.756E-04	-1.795E-04	.977E-04	
5.00000	50.00000	0.02563	33.172	-0.132	-0.282	2.228
(STRAIN)		3.334E-04	-1.329E-04	-1.350E-04	.624E-04	
5.00000	60.00000	0.02268	25.891	-0.195	-0.284	1.516
(STRAIN)		2.608E-04	-1.044E-04	-1.056E-04	.424E-04	
5.00000	90.00000	0.01685	14.310	-0.243	-0.269	0.612
(STRAIN)		1.451E-04	-5.860E-05	-5.895E-05	.171E-04	
5.00000	120.00000	0.01337	9.078	-0.222	-0.232	0.309
(STRAIN)		9.259E-05	-3.760E-05	-3.774E-05	.866E-05	
5.00000	150.00000	0.01106	6.256	-0.187	-0.192	0.178
(STRAIN)		6.407E-05	-2.613E-05	-2.619E-05	.499E-05	
5.00000	180.00000	0.00941	4.560	-0.154	-0.156	0.112
(STRAIN)		4.684E-05	-1.915E-05	-1.919E-05	.313E-05	
5.00000	200.00000	0.00856	3.782	-0.134	-0.136	0.085
(STRAIN)		3.890E-05	-1.592E-05	-1.595E-05	.238E-05	
5.00000	230.00000	0.00753	2.936	-0.109	-0.110	0.058
(STRAIN)		3.023E-05	-1.239E-05	-1.241E-05	.164E-05	
15.00000	0.00000	0.03873	550.000	813.018	955.264	0.000
(STRAIN)		-1.036E-04	1.111E-04	1.660E-04	.184E-10	
15.00000	10.00000	0.03859	141.234	-336.807	-624.695	89.378
(STRAIN)		1.365E-04	-4.788E-05	-1.589E-04	.689E-04	
15.00000	20.00000	0.03524	83.972	-21.295	-36.628	50.857
(STRAIN)		2.896E-04	-1.014E-04	-1.584E-04	.378E-03	
15.00000	30.00000	0.03252	51.541	-66.976	-82.736	14.646
(STRAIN)		2.756E-04	-1.646E-04	-2.232E-04	.109E-03	
15.00000	40.00000	0.02815	38.865	1.990	-0.166	9.129
(STRAIN)		3.814E-04	-1.349E-04	-1.651E-04	.256E-03	
15.00000	50.00000	0.02477	30.216	0.993	-0.219	6.052
(STRAIN)		2.991E-04	-1.101E-04	-1.270E-04	.169E-03	
15.00000	60.00000	0.02209	24.116	0.486	-0.247	4.219
(STRAIN)		2.402E-04	-9.062E-05	-1.009E-04	.118E-03	
15.00000	90.00000	0.01661	13.782	-0.038	-0.258	1.767
(STRAIN)		1.390E-04	-5.448E-05	-5.756E-05	.495E-04	
15.00000	120.00000	0.01325	8.866	-0.139	-0.228	0.907
(STRAIN)		9.013E-05	-3.595E-05	-3.719E-05	.254E-04	
15.00000	150.00000	0.01099	6.155	-0.147	-0.190	0.526

(STRAIN)		6.289E-05	-2.533E-05	-2.593E-05	.147E-04	
15.00000	180.00000	0.00937	4.506	-0.132	-0.155	0.331
(STRAIN)		4.621E-05	-1.872E-05	-1.905E-05	.928E-05	
15.00000	200.00000	0.00853	3.744	-0.119	-0.135	0.252
(STRAIN)		3.846E-05	-1.563E-05	-1.585E-05	.706E-05	
15.00000	230.00000	0.00751	2.913	-0.100	-0.110	0.174
(STRAIN)		2.997E-05	-1.221E-05	-1.235E-05	.487E-05	
30.00000	0.00000	0.03062	0.000	294.236	434.266	0.000
(STRAIN)		-4.965E-05	3.252E-05	8.653E-05	-.604E-12	
30.00000	10.00000	0.02942	30.936	156.171	-157.081	53.741
(STRAIN)		8.930E-06	5.723E-05	-6.359E-05	.415E-04	
30.00000	20.00000	0.02850	33.216	3.054	-24.273	43.646
(STRAIN)		1.131E-04	1.060E-06	-1.004E-04	.324E-03	
30.00000	30.00000	0.02720	30.682	-15.638	-50.593	16.631
(STRAIN)		1.444E-04	-2.761E-05	-1.574E-04	.124E-03	
30.00000	40.00000	0.02464	26.671	5.704	0.098	12.144
(STRAIN)		2.435E-04	-5.004E-05	-1.285E-04	.340E-03	
30.00000	50.00000	0.02235	22.739	3.448	-0.062	8.925
(STRAIN)		2.138E-04	-5.623E-05	-1.054E-04	.250E-03	
30.00000	60.00000	0.02036	19.318	2.134	-0.149	6.671
(STRAIN)		1.852E-04	-5.533E-05	-8.730E-05	.187E-03	
30.00000	90.00000	0.01585	12.197	0.543	-0.228	3.118
(STRAIN)		1.207E-04	-4.245E-05	-5.324E-05	.873E-04	
30.00000	120.00000	0.01285	8.202	0.111	-0.216	1.677
(STRAIN)		8.244E-05	-3.083E-05	-3.541E-05	.470E-04	
30.00000	150.00000	0.01076	5.830	-0.023	-0.184	0.997
(STRAIN)		5.913E-05	-2.281E-05	-2.507E-05	.279E-04	
30.00000	180.00000	0.00922	4.330	-0.065	-0.153	0.637
(STRAIN)		4.417E-05	-1.736E-05	-1.859E-05	.178E-04	
30.00000	200.00000	0.00842	3.622	-0.073	-0.135	0.487
(STRAIN)		3.705E-05	-1.468E-05	-1.554E-05	.136E-04	
30.00000	230.00000	0.00743	2.839	-0.073	-0.111	0.339
(STRAIN)		2.913E-05	-1.165E-05	-1.218E-05	.948E-05	
45.00000	0.00000	0.02471	0.000	139.562	224.569	0.000
(STRAIN)		-2.578E-05	1.370E-05	4.648E-05	.382E-11	
45.00000	10.00000	0.02284	8.089	126.552	-43.679	27.369
(STRAIN)		-5.976E-06	3.972E-05	-2.594E-05	.211E-04	
45.00000	20.00000	0.02257	13.814	10.360	-13.959	24.804
(STRAIN)		4.255E-05	2.972E-05	-6.060E-05	.184E-03	

45.00000	30.00000	0.02202	16.615	9.256	-27.796	12.478
(STRAIN)		6.336E-05	3.603E-05	-1.016E-04	.927E-04	
45.00000	40.00000	0.02074	16.339	7.379	0.313	10.640
(STRAIN)		1.326E-04	7.185E-06	-9.175E-05	.298E-03	
45.00000	50.00000	0.01941	15.332	5.104	0.087	8.766
(STRAIN)		1.326E-04	-1.064E-05	-8.087E-05	.245E-03	
45.00000	60.00000	0.01811	14.009	3.534	-0.045	7.125
(STRAIN)		1.261E-04	-2.052E-05	-7.062E-05	.200E-03	
45.00000	90.00000	0.01476	10.089	1.230	-0.187	3.854
(STRAIN)		9.671E-05	-2.730E-05	-4.715E-05	.108E-03	
45.00000	120.00000	0.01226	7.241	0.451	-0.196	2.221
(STRAIN)		7.139E-05	-2.367E-05	-3.273E-05	.622E-04	
45.00000	150.00000	0.01040	5.339	0.158	-0.173	1.370
(STRAIN)		5.345E-05	-1.908E-05	-2.372E-05	.384E-04	
45.00000	180.00000	0.00899	4.056	0.039	-0.146	0.896
(STRAIN)		4.099E-05	-1.524E-05	-1.785E-05	.251E-04	
45.00000	200.00000	0.00824	3.429	0.001	-0.130	0.693
(STRAIN)		3.480E-05	-1.318E-05	-1.502E-05	.194E-04	
45.00000	230.00000	0.00731	2.719	-0.027	-0.108	0.487
(STRAIN)		2.773E-05	-1.071E-05	-1.185E-05	.136E-04	
60.00000	0.00000	0.02047	0.000	77.422	131.676	0.000
(STRAIN)		-1.486E-05	6.837E-06	2.776E-05	.174E-12	
60.00000	10.00000	0.01820	3.035	81.809	-11.001	13.894
(STRAIN)		-6.214E-06	2.417E-05	-1.163E-05	.107E-04	
60.00000	20.00000	0.01811	6.819	11.326	-7.630	13.226
(STRAIN)		1.631E-05	3.306E-05	-3.735E-05	.983E-04	
60.00000	30.00000	0.01790	9.085	15.881	-15.175	8.418
(STRAIN)		2.535E-05	5.059E-05	-6.476E-05	.625E-04	
60.00000	40.00000	0.01730	9.702	7.167	0.401	8.034
(STRAIN)		6.676E-05	3.126E-05	-6.347E-05	.225E-03	
60.00000	50.00000	0.01658	9.827	5.529	0.174	7.277
(STRAIN)		7.546E-05	1.529E-05	-5.969E-05	.204E-03	
60.00000	60.00000	0.01581	9.589	4.197	0.031	6.389
(STRAIN)		7.898E-05	3.492E-06	-5.484E-05	.179E-03	
60.00000	90.00000	0.01349	7.921	1.805	-0.145	4.022
(STRAIN)		7.256E-05	-1.305E-05	-4.036E-05	.113E-03	
60.00000	120.00000	0.01152	6.142	0.800	-0.172	2.514
(STRAIN)		5.891E-05	-1.588E-05	-2.949E-05	.704E-04	
60.00000	150.00000	0.00994	4.743	0.364	-0.159	1.626

(STRAIN)		4.661E-05	-1.469E-05	-2.202E-05	.455E-04	
60.00000	180.00000	0.00869	3.710	0.164	-0.138	1.094
(STRAIN)		3.700E-05	-1.264E-05	-1.688E-05	.306E-04	
60.00000	200.00000	0.00801	3.181	0.093	-0.124	0.858
(STRAIN)		3.193E-05	-1.130E-05	-1.433E-05	.240E-04	
60.00000	230.00000	0.00714	2.562	0.033	-0.104	0.612
(STRAIN)		2.591E-05	-9.504E-06	-1.142E-05	.171E-04	
75.00000	0.00000	0.01735	0.000	47.799	84.049	0.000
(STRAIN)		-9.296E-06	3.891E-06	1.787E-05	-.404E-11	
75.00000	10.00000	0.01485	1.463	53.262	0.039	7.051
(STRAIN)		-4.912E-06	1.507E-05	-5.461E-06	.544E-05	
75.00000	20.00000	0.01483	3.641	10.166	-4.110	7.121
(STRAIN)		5.212E-06	2.945E-05	-2.358E-05	.529E-04	
75.00000	30.00000	0.01476	5.070	15.655	-8.427	5.579
(STRAIN)		8.291E-06	4.761E-05	-4.184E-05	.414E-04	
75.00000	40.00000	0.01450	5.746	6.163	0.389	5.760
(STRAIN)		3.126E-05	3.709E-05	-4.375E-05	.161E-03	
75.00000	50.00000	0.01414	6.176	5.171	0.198	5.606
(STRAIN)		4.028E-05	2.622E-05	-4.341E-05	.157E-03	
75.00000	60.00000	0.01370	6.369	4.231	0.069	5.246
(STRAIN)		4.649E-05	1.656E-05	-4.171E-05	.147E-03	
75.00000	90.00000	0.01219	5.989	2.168	-0.109	3.799
(STRAIN)		5.165E-05	-1.843E-06	-3.371E-05	.106E-03	
75.00000	120.00000	0.01070	5.042	1.094	-0.148	2.585
(STRAIN)		4.664E-05	-8.633E-06	-2.603E-05	.724E-04	
75.00000	150.00000	0.00941	4.103	0.564	-0.144	1.762
(STRAIN)		3.935E-05	-1.020E-05	-2.010E-05	.493E-04	
75.00000	180.00000	0.00834	3.323	0.296	-0.128	1.227
(STRAIN)		3.255E-05	-9.824E-06	-1.575E-05	.344E-04	
75.00000	200.00000	0.00773	2.897	0.193	-0.116	0.978
(STRAIN)		2.866E-05	-9.196E-06	-1.352E-05	.274E-04	
75.00000	230.00000	0.00694	2.378	0.100	-0.099	0.712
(STRAIN)		2.378E-05	-8.118E-06	-1.090E-05	.199E-04	
90.00000	0.00000	0.01242	0.000	-42.001	34.343	0.000
(STRAIN)		1.077E-06	-1.554E-05	1.390E-05	-.285E-11	
90.00000	10.00000	0.01237	0.738	35.693	3.919	3.544
(STRAIN)		-3.750E-06	9.732E-06	-2.524E-06	.273E-05	
90.00000	20.00000	0.01238	1.968	8.404	-2.198	3.897
(STRAIN)		3.015E-07	2.421E-05	-1.517E-05	.290E-04	

90.00000	30.00000	0.01237	2.849	13.425	-4.759	3.706
(STRAIN)		7.130E-07	3.999E-05	-2.755E-05	.275E-04	
90.00000	40.00000	0.01229	3.401	5.012	0.328	4.058
(STRAIN)		1.264E-05	3.521E-05	-3.037E-05	.114E-03	
90.00000	50.00000	0.01212	3.847	4.490	0.184	4.176
(STRAIN)		1.977E-05	2.878E-05	-3.151E-05	.117E-03	
90.00000	60.00000	0.01189	4.164	3.900	0.079	4.112
(STRAIN)		2.572E-05	2.202E-05	-3.146E-05	.115E-03	
90.00000	90.00000	0.01095	4.412	2.316	-0.081	3.370
(STRAIN)		3.518E-05	5.833E-06	-2.772E-05	.944E-04	
90.00000	120.00000	0.00987	4.034	1.304	-0.125	2.492
(STRAIN)		3.563E-05	-2.596E-06	-2.261E-05	.698E-04	
90.00000	150.00000	0.00885	3.470	0.734	-0.128	1.795
(STRAIN)		3.228E-05	-6.025E-06	-1.810E-05	.503E-04	
90.00000	180.00000	0.00795	2.920	0.419	-0.117	1.298
(STRAIN)		2.799E-05	-7.017E-06	-1.453E-05	.363E-04	
90.00000	200.00000	0.00741	2.594	0.291	-0.107	1.054
(STRAIN)		2.521E-05	-7.037E-06	-1.262E-05	.295E-04	
90.00000	230.00000	0.00672	2.177	0.169	-0.093	0.783
(STRAIN)		2.147E-05	-6.642E-06	-1.032E-05	.219E-04	
120.00000	0.00000	0.00892	0.000	-59.660	-0.347	0.000
(STRAIN)		3.452E-06	-1.612E-05	6.759E-06	.176E-11	
120.00000	10.00000	0.00909	0.171	16.761	5.066	0.822
(STRAIN)		-2.134E-06	4.265E-06	-2.459E-07	.634E-06	
120.00000	20.00000	0.00911	0.542	5.169	-0.583	1.197
(STRAIN)		-2.382E-06	1.480E-05	-6.562E-06	.889E-05	
120.00000	30.00000	0.00914	0.876	8.576	-1.575	1.681
(STRAIN)		-3.499E-06	2.510E-05	-1.260E-05	.125E-04	
120.00000	40.00000	0.00917	1.172	3.094	0.192	2.003
(STRAIN)		-1.422E-06	2.548E-05	-1.515E-05	.561E-04	
120.00000	50.00000	0.00916	1.466	3.052	0.120	2.236
(STRAIN)		1.975E-06	2.418E-05	-1.688E-05	.626E-04	
120.00000	60.00000	0.00913	1.735	2.900	0.062	2.378
(STRAIN)		5.503E-06	2.181E-05	-1.792E-05	.666E-04	
120.00000	90.00000	0.00882	2.286	2.170	-0.045	2.381
(STRAIN)		1.436E-05	1.273E-05	-1.827E-05	.667E-04	
120.00000	120.00000	0.00831	2.448	1.463	-0.087	2.047
(STRAIN)		1.898E-05	5.183E-06	-1.652E-05	.573E-04	
120.00000	150.00000	0.00771	2.359	0.949	-0.098	1.645

(STRAIN)		2.018E-05	4.499E-07	-1.421E-05	.460E-04	
120.00000	180.00000	0.00712	2.154	0.612	-0.095	1.287
(STRAIN)		1.947E-05	-2.121E-06	-1.201E-05	.360E-04	
120.00000	200.00000	0.00674	1.996	0.457	-0.090	1.088
(STRAIN)		1.849E-05	-3.052E-06	-1.071E-05	.305E-04	
120.00000	230.00000	0.00621	1.759	0.298	-0.080	0.848
(STRAIN)		1.672E-05	-3.738E-06	-9.030E-06	.237E-04	
150.00000	0.00000	0.00707	0.000	-18.495	5.630	0.000
(STRAIN)		1.221E-06	-5.824E-06	3.481E-06	-.317E-11	
150.00000	10.00000	0.00711	0.019	8.317	4.293	0.135
(STRAIN)		-1.256E-06	1.945E-06	3.931E-07	.104E-06	
150.00000	20.00000	0.00712	0.120	3.032	-0.090	0.385
(STRAIN)		-2.178E-06	8.635E-06	-2.957E-06	.286E-05	
150.00000	30.00000	0.00715	0.243	5.152	-0.536	0.807
(STRAIN)		-3.261E-06	1.497E-05	-6.155E-06	.599E-05	
150.00000	40.00000	0.00720	0.388	1.860	0.104	1.015
(STRAIN)		-3.975E-06	1.663E-05	-7.950E-06	.284E-04	
150.00000	50.00000	0.00723	0.548	1.968	0.069	1.198
(STRAIN)		-2.672E-06	1.721E-05	-9.370E-06	.335E-04	
150.00000	60.00000	0.00725	0.712	1.996	0.039	1.346
(STRAIN)		-1.018E-06	1.695E-05	-1.044E-05	.377E-04	
150.00000	90.00000	0.00719	1.152	1.772	-0.026	1.569
(STRAIN)		4.542E-06	1.321E-05	-1.196E-05	.439E-04	
150.00000	120.00000	0.00699	1.428	1.375	-0.059	1.529
(STRAIN)		9.012E-06	8.279E-06	-1.180E-05	.428E-04	
150.00000	150.00000	0.00667	1.533	1.002	-0.072	1.357
(STRAIN)		1.161E-05	4.177E-06	-1.086E-05	.380E-04	
150.00000	180.00000	0.00631	1.521	0.710	-0.074	1.147
(STRAIN)		1.266E-05	1.314E-06	-9.664E-06	.321E-04	
150.00000	200.00000	0.00605	1.472	0.561	-0.072	1.011
(STRAIN)		1.276E-05	1.160E-08	-8.853E-06	.283E-04	
150.00000	230.00000	0.00567	1.367	0.393	-0.067	0.828
(STRAIN)		1.237E-05	-1.267E-06	-7.715E-06	.232E-04	
180.00000	0.00000	0.00585	0.000	-9.136	6.378	0.000
(STRAIN)		3.190E-07	-3.263E-06	2.721E-06	.108E-11	
180.00000	10.00000	0.00583	-0.014	4.395	3.474	-0.001
(STRAIN)		-7.907E-07	9.097E-07	5.544E-07	-.794E-09	
180.00000	20.00000	0.00584	0.008	1.784	0.064	0.147
(STRAIN)		-1.560E-06	5.034E-06	-1.354E-06	.109E-05	

180.00000	30.00000	0.00586	0.053	3.084	-0.174	0.422
(STRAIN)		-2.342E-06	8.914E-06	-3.185E-06	.314E-05	
180.00000	40.00000	0.00590	0.122	1.132	0.062	0.545
(STRAIN)		-3.558E-06	1.058E-05	-4.394E-06	.153E-04	
180.00000	50.00000	0.00593	0.204	1.263	0.044	0.665
(STRAIN)		-3.189E-06	1.164E-05	-5.430E-06	.186E-04	
180.00000	60.00000	0.00596	0.295	1.344	0.027	0.776
(STRAIN)		-2.533E-06	1.215E-05	-6.290E-06	.217E-04	
180.00000	90.00000	0.00599	0.583	1.358	-0.014	1.012
(STRAIN)		4.523E-07	1.131E-05	-7.907E-06	.283E-04	
180.00000	120.00000	0.00593	0.823	1.180	-0.039	1.092
(STRAIN)		3.664E-06	8.662E-06	-8.400E-06	.306E-04	
180.00000	150.00000	0.00578	0.976	0.945	-0.052	1.056
(STRAIN)		6.182E-06	5.758E-06	-8.202E-06	.296E-04	
180.00000	180.00000	0.00557	1.045	0.726	-0.056	0.958
(STRAIN)		7.774E-06	3.306E-06	-7.650E-06	.268E-04	
180.00000	200.00000	0.00541	1.056	0.601	-0.057	0.879
(STRAIN)		8.380E-06	2.015E-06	-7.195E-06	.246E-04	
180.00000	230.00000	0.00515	1.034	0.449	-0.055	0.757
(STRAIN)		8.766E-06	5.680E-07	-6.478E-06	.212E-04	
210.00000	0.00000	0.00497	0.000	-5.601	4.919	0.000
(STRAIN)		9.260E-08	-2.101E-06	1.957E-06	.144E-11	
210.00000	10.00000	0.00495	-0.018	2.527	2.859	-0.005
(STRAIN)		-5.437E-07	4.378E-07	5.661E-07	-.423E-08	
210.00000	20.00000	0.00496	-0.013	1.080	0.110	0.080
(STRAIN)		-1.058E-06	3.004E-06	-5.999E-07	.597E-06	
210.00000	30.00000	0.00497	0.004	1.893	-0.036	0.247
(STRAIN)		-1.582E-06	5.437E-06	-1.728E-06	.183E-05	
210.00000	40.00000	0.00500	0.036	0.711	0.044	0.317
(STRAIN)		-2.659E-06	6.789E-06	-2.548E-06	.888E-05	
210.00000	50.00000	0.00503	0.078	0.825	0.033	0.392
(STRAIN)		-2.652E-06	7.808E-06	-3.284E-06	.110E-04	
210.00000	60.00000	0.00505	0.127	0.911	0.022	0.467
(STRAIN)		-2.456E-06	8.507E-06	-3.930E-06	.131E-04	
210.00000	90.00000	0.00511	0.303	1.016	-0.005	0.657
(STRAIN)		-1.015E-06	8.971E-06	-5.329E-06	.184E-04	
210.00000	120.00000	0.00511	0.478	0.964	-0.025	0.767
(STRAIN)		1.024E-06	7.825E-06	-6.014E-06	.215E-04	
210.00000	150.00000	0.00505	0.619	0.836	-0.036	0.797

(STRAIN)		2.987E-06	6.026E-06	-6.177E-06	.223E-04	
210.00000	180.00000	0.00493	0.710	0.687	-0.042	0.770
(STRAIN)		4.521E-06	4.201E-06	-6.011E-06	.216E-04	
210.00000	200.00000	0.00483	0.746	0.593	-0.044	0.732
(STRAIN)		5.264E-06	3.118E-06	-5.791E-06	.205E-04	
210.00000	230.00000	0.00466	0.768	0.467	-0.044	0.661
(STRAIN)		5.992E-06	1.770E-06	-5.379E-06	.185E-04	

Figure (B-1), Solution of KENLAYER

Certificate of the Examining Committee

We certify that we have read the entitled “**STUDYING OF THE RUTTING IN FLEXIBLE PAVEMENT USING VISCOELASTIC FINITE ELEMENT METHOD**” and as an examining committee. Examined the student (**Ghasak Ebrahim Al- Mothfer**) in its content and in what is connected with it, and that in our opinion it is adequate as a thesis for degree of **MASTER OF SCIENCE IN CIVIL ENGINEERING (TRANSPORTATION ENGINEERING)**.

Signature:

Name: Assist Prof. Ali A. Hussain Abdul-Amir.
(Member)

Date: / /2008.

Signature:

Name: Assist Prof. Dr. Nahla Y. Ahmed
(Member and supervisor)

Date: / /2008.

Signature:

Name: Prof. Dr. Nameer A. A. Alwash.
(Chairman)

Date: / /2008.

Signature:

Name: Assist Prof. Dr. Haithm H. Al-Daa'mi.
(Member and supervisor)

Date: / /2008.

Signature:

Name: Assist Prof. Ali A. A. Alwash.
(Member)

Date: / /2008.

Signature:

Name: Assist Prof. Dr. Ammar Y. Ali
(Head of the Civil Engineering Department)

Date: / /2008.

Signature:

Name: Prof. Dr. Salah Tawfeek Ali Al-Bazzaz.
(Dean of the College of Engineering)

Date: / /2008.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5. Conclusions:

Within the limitations of materials and testing data used in this work, the following conclusions are drawn:

1. The proposed model from this work to predict the permanent deformation parameters Alpha and Mu is.

$$\alpha = 1 - m$$

$$\mu = M_r * \left[\frac{D_m \cdot r \cdot m}{(1 + r \cdot N^m)} - \frac{(D_o + D_m \cdot r \cdot N^m) \cdot r \cdot m}{(1 + r \cdot N^m)^2} - \frac{R_m \cdot r_p \cdot N^{m(p-1)} m_p}{N^{(m)} (1 + r_p \cdot N^{m_p})} \right. \\ \left. + \frac{(R_o + R_m \cdot r_p \cdot N^{m_p}) r_p \cdot N^{m(p-1)} m_p}{(1 + r_p \cdot N^{m_p})^2} \right]$$

$$\epsilon_p = \int_0^N \epsilon_p(N) \cdot dN = \epsilon_r \cdot u \cdot \frac{N^{(1-\alpha)}}{(1-\alpha)} \quad \text{and} \quad Rd = \epsilon_p \times h$$

(All variables are previously defined) The differences in results of calculations of rut depth between the proposal and (Yassoub; Albayati) models are about (12%-27%)

2. Finite element approach could be used to estimate the primary response components elastic or viscoelastic such as (stress; strain; deflection etc) at critical points.
3. To analyze the system of pavement's layers by finite element approach a ANSYS 9.0 software program was used in this work, a general notes in ANSYS 9.0 environment should be maintain to analysis any pavement system:

- Load distribution applied on surface of element under type (8-nodes, Plane-182) is better than applied load on nodes.
 - The materials that used in analysis should be work as one group to transfer the response from layer to the other, to achieve that an option named (Glue) should be use between two layers meeting.
 - The viscoelastic solution should maintain a sufficient number of kernel function parameters until reach to a low value of standard mean of error and good coefficient of determination, to give a best fitting of decay moduli behavior (bulk and shear moduli).
4. Based on the creep / recovery compliance test data the hyperbolic model provides a near perfect prediction of actual creep and recovery performance of asphalt mixtures, (P) value which mean the ratio of the recovery hyperbolic modifier slope to the creep hyperbolic slope is range from (0.7008) to (1.8). For different in mixture type (P) value is could be a good indicator and its value varies extrusive with rutting depth under the same stress level and temperature.
5. Analyze the [Effective] asphalt layer as viscoelastic approach instead of elastic approach only, due to first one reflecting the actual behavior of asphalt concrete and deal with variable (Time) with its equations. Elastic finite element model give rut depth 10.78 mm while viscoelastic finite element model give rut depth 12.54 mm, with 14% percent difference.

5.1 Recommendations for Future Research:

On the bases of this study, the following recommendations are made:

1. The finite element approach is a general solution also it is able to solve challenges problems such as considering the moving load; heat transfer; viscoplastic approach; creep simulating with (time or strain) hardening that related with system of pavement's layers is still with no simulating in Iraq, Also Finite element approach could be used not only as full scale, it could be used as a prototype, many asphalt material test apparatuses molds lack to simulating and there is no idea about the response except a simple attempts represented by a dial gage.
2. Full scale field tests should be conducted on local pavements composed of different structure and subjected to different patterns and characteristics of traffic in order to obtain a comprehensive rutting model under the ambient conditions.

CHAPTER ONE

STATEMENT OF THE PROBLEM

1. Introduction

The main types of structural distress of asphalt concrete pavements are cracking and permanent deformation, also known as rutting. Rutting is the most common distress mode still persisting in today asphalt pavement and is characterized by depressions that form in the wheel paths. Rutting may occur due to repeated application of high stresses on the subgrade or by consolidation and instability ([Drakos, et al., 2001](#)). Cracking is the result of excessive consolidation of the pavement along the wheel path due to either reduction of the air voids in the asphalt concrete layer, or the permanent deformation of the base or subgrade. Rutting type, failure is attributed strictly to the asphalt mixture properties and usually occurs within the top two inches of the asphalt concrete layer. The mechanism of this mode of failure has not been clearly defined and may be expressed as the resistance of a mix to pavement rutting under varying conditions of climate and traffic loading ([Hills, 1993](#)). In Iraq, within about one year after construction, rutting distress was observed at several locations in the highway especially due to higher loadings and degree of temperature ([Yassoub, 2004](#)).

1.1. Factors Affecting Rutting

Permanent deformation is physically visible as ruts on the pavement surface. Surface rutting in excess of about 0.25 inch (6 mm) is typically considered to be a hydroplaning safety hazard. As a result, excessive rutting is often responsible for premature maintenance and rehabilitation activities, reducing the service life of the pavement. Rutting is a result of consolidation of the mix after construction

and plastic flow of the asphalt mix in the wheel path over time, as reported by (Roberts, et al., 1996).

Rutting is strongly influenced by traffic loading, but climate can also have a large influence especially when the pavement subgrade undergoes seasonal variations in bearing capacity, or when bituminous course are subjected to high temperatures (Organization for Economic Cooperation and Development (OECD, 1988). Ruts develop within pavement layers when traffic loading causes layer densification and/or when stresses induced in the pavement materials are sufficient to cause shear displacements within the materials.

Many researches performed over several decades have shown that the susceptibility to rutting can be linked to the following material attributes: excessive asphalt content, excessive fine grained aggregate, high percentages of natural sand, rounded aggregate particles, excessive permissible moisture in the mix or in granular materials and soils, temperature susceptible asphalt cement, and cold weather pavement leading to low density (Button, et al., 1990). Other factors affecting rutting are temperature; precipitation; time; type and extent of loading. The above factors when combined also determine measures such as Hveem and Marshal Stability, complex modulus, resilient modulus, and deflection that are normally used for pavement distress modeling.

1.2. Objectives of the Study:

The objective of the research work could be presented as follows:

1. Evaluating and calculating the primary response (stress; strain; .. etc) at critical location in the system of pavement layers. The effect of method of analysis for this response upon rutting depth is also investigated.

2. Evaluating the effects of the main variables related to asphalt concrete mixtures, stress levels and temperature effects on rutting potential using static creep/recovery test data.
3. Constituting the best hyperbolic model that simulating the actual behavior of creep and recovery compliance that varies with time for different types of mixtures and surrounding circumstances.
4. Developing a permanent deformation model that reflects the asphalt concrete properties depending on [VESYS](#) model over a range of loadings and local temperature.
5. Presenting a procedure for the prediction of rutting potential in asphalt mixtures taking into consideration the contributions of the layers under different variables.

1.3. Scope of the Study:

The work of this thesis consisted of the following:

1. [Chapter Two](#) is devoted to the review of literature concerning the factors that affect rutting and the criteria to assess rutting are summarized.
2. [Chapter Three](#) provides the Evaluation and calculating of the primary response (stress; strain; .. etc) at critical location in system of pavement's layers and a comparison between the exact solution represented by the [Multilayer Elastic Theory \(MLET\)](#) and approximate solution represented by the [Finite Element Method \(FEM\)](#) for determine the primary response, also a [viscoelastic](#) solution depending on the [Small Strain Theory](#) is presented.
3. The details of modeling of the permanent strain in asphalt mixture over a range of degree of temperature and differences in mixture types are

presented in [Chapter Four](#). Also the simulating of creep and recovery compliance data based on nonlinear regression analysis also permanent strain material properties represented by [Alpha](#) (α) and [Mu](#) (μ) are presented.

4. Finally, the main conclusions obtained from the results of this work and recommendations for future work are presented in [Chapter Five](#).

CHAPTER TWO

LITERATURE REVIEW

2. Introduction

Asphalt concrete (sometimes referred to as “Hot Mix Asphalt” or simply “HMA”) is a paving material that consists of asphalt binder and mineral aggregate. The asphalt binder, which can be asphalt cement or modified asphalt cement, acts as a binding agent to glue aggregate particles into cohesive mass. Because it is impervious to water, the asphalt binder also functions to waterproof the mixture. When bound by the asphalt binder, mineral aggregate acts as a stone framework to impart strength and toughness to the system. Because HMA contains both asphalt binder and mineral aggregate, the behavior of the mixture is affected by the properties of the individual components and how they react with each other in the system ([Finn, 1967](#); [Choa, 1977](#)).

In the design of asphalt concrete mixture it is generally agreed that a balance be obtained between a numbers of desirable mix properties that, inevitably, necessitates a compromise in the selection of the design asphalt cement content. These pertinent mix properties include the following.

1. Stability- resistance to deformation under load.
2. Durability- resistance to weathering.
3. Flexibility- ability to conform to long-term variations in base and subgrade elevations.
4. Fatigue resistance- ability to minimize cracking due to load repetitions.
5. Skid resistance- sufficient friction to enable a vehicle to brake to stop within a reasonable distance under a variety of environmental conditions.

6. Fracture strength- maximum strength a mix exhibits when subjected to tensile forces.

The particular interest this research is the stability which is the resistance to permanent deformation, i.e., the rutting associated with depressions in the pavement surface under the wheel paths. As shown in Figure (2-1), stability is a function of the internal mix frictional resistance, cohesion and inertia. As noted by (Finn, 1967), frictional resistance is the “major contributor to resistance to deformation”. Moreover, at high temperatures and slowly applied loads, the contribution of the antiparticle friction to stability is predominant as aggregate surface texture exerts a major influence (Finn, 1967). Improper compaction, high asphalt content, (and /or) a high percentage of fine grained material (natural sand) tend to reduce this friction permitting plastic deformations to develop more rapidly (SHRP, 1994).

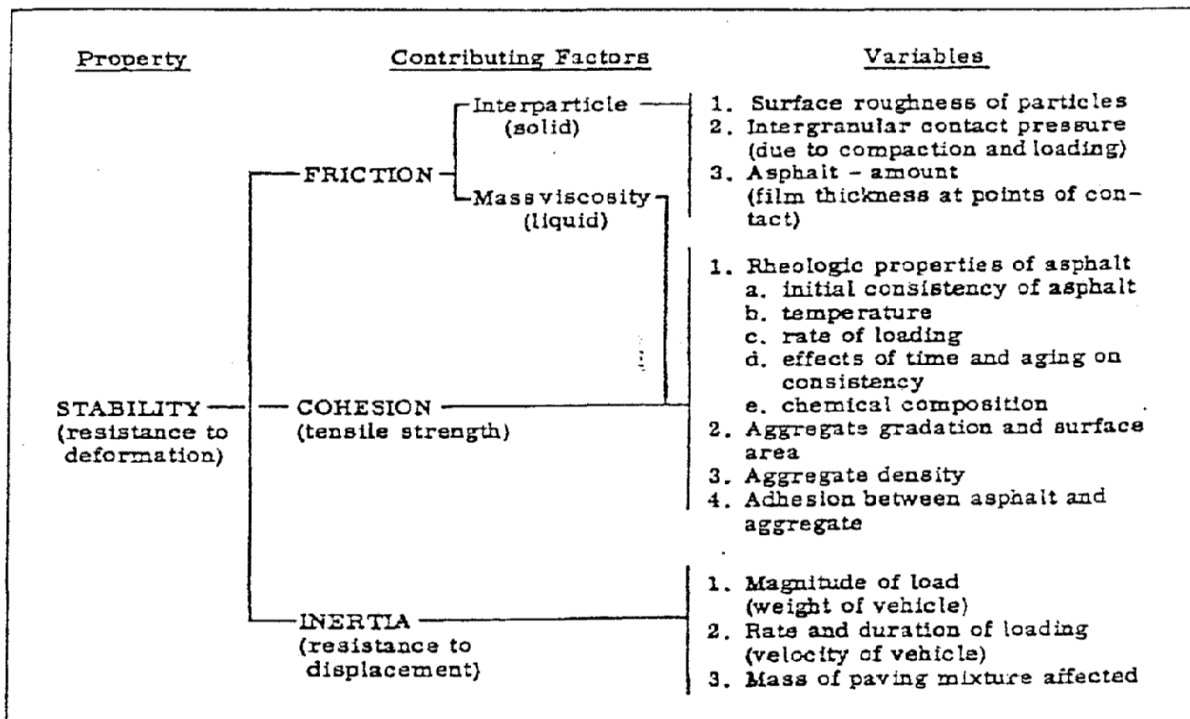


Figure (2-1), Analysis of Stability of Asphalt Paving Mixtures (Finn, 1967)

2.1 Rutting: Types, Mechanism and Distribution with Depth

Historically, the term “permanent deformation” was used to describe any distortion of the pavement surface, including shoving and pushing due to mix instability ([Goetz et. al., 1957](#)). Today, however, this form of distress refers to longitudinal depression or “ruts” that form in the wheel paths due to “consolidation and/or lateral movement in one or more of the component pavement layers due to repeated, transient load applications”. Because rutting appears only as a change in transverse surface profile, it was often erroneously blamed on surface instability. Investigations at the [AASHTO](#) road test in [1959](#), however, revealed that permanent deformation occurred in all layers of the pavement system, as illustrated in [Figure \(2-2\)](#). From 51 sections trenched in 1960, the total permanent deformation was distributed among the component layers in the following average proportions ([Morris, 1973](#)):

<u>Component layer</u>	<u>Percent permanent deformation</u>
AC surface	32
Base	14
Subbase	45
Subgrade	9

Other studies have indicated that as much as 19 percent of the permanent deformation may be attributed to the subgrade, while others have shown that for 6-inch thick bituminous pavements, rutting was entirely due to deformation within the asphalt concrete ([Majidzadeh et. al., 1978](#)).

LOOP 6 6-3-12 DESIGN
30 KIP SINGLE AXLE LOAD

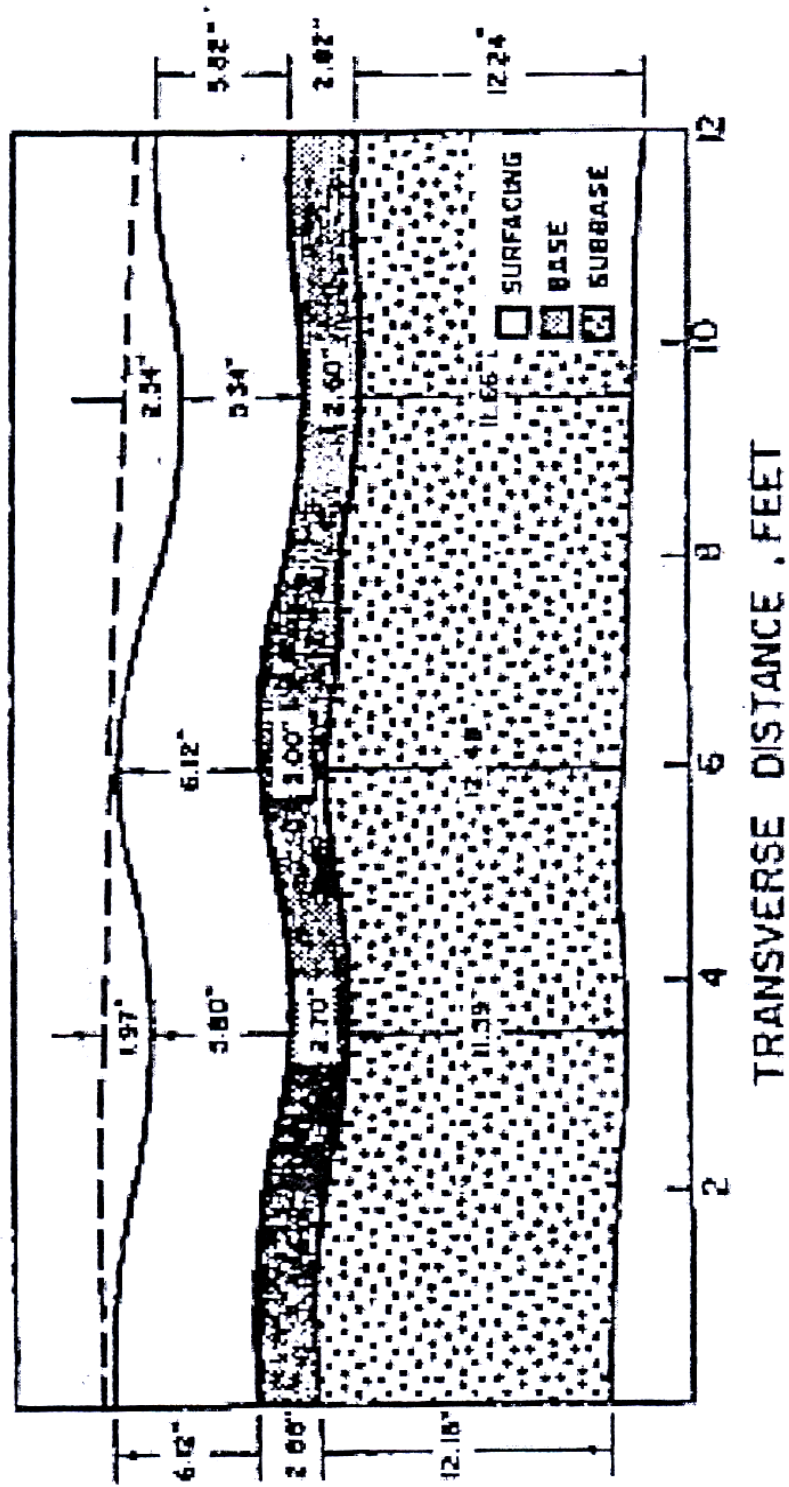


Figure (2-2), Typical Transverse Profile, 1959 AASHTO Trench Study (Morris, 1973)

(Daweley et. al. 1990) states that the majority of HMA rutting is primarily due to instability rutting. This research deals with this type of rutting essentially a reflection of the rutting.

In 2002, National Cooperative Highway Research Program (NCHRP, 2002) classified rutting into the following categories shown in Figure (2-3):

1. Wear rutting, which is due to the progressive loss of coated aggregate Particles from the pavement surface caused by combined environmental and traffic influences.
2. Structural rutting, which is due to the permanent vertical deformation of the pavement structure under repeated traffic permanent deformation loads and it is within the subgrade.
3. Instability rutting, which is due to the densification (decrease in volume and, hence, increase in density) and lateral displacement (shear deformation) of material within the pavement asphalt concrete layers.

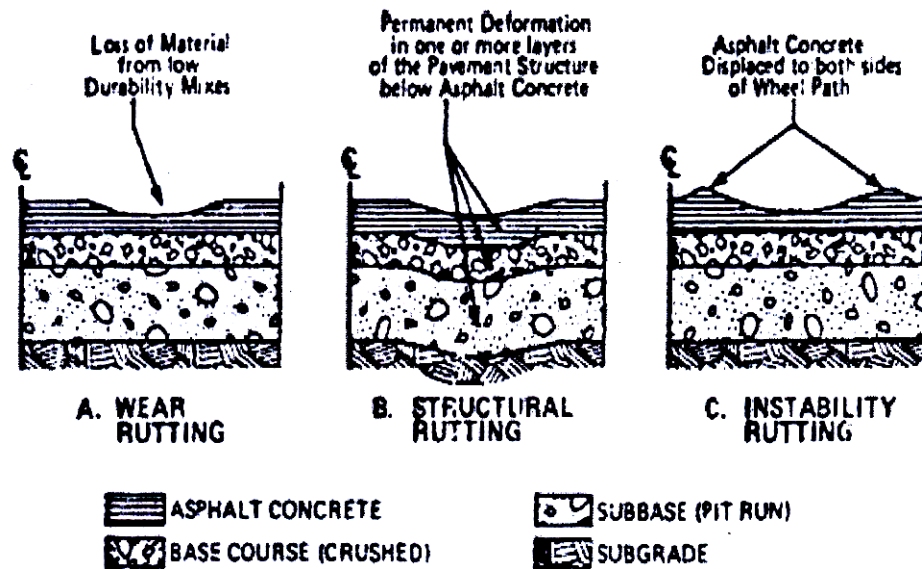


Figure (2-3) Description of Rutting Categories (NCHRP, 2002)

Undoubtedly, the mechanical responses of the HMA layer to various loading conditions are the result of a series of complex internal activates. For years, researchers tried to relate these microscopic functions to the macroscopic rutting phenomenon. Some researchers have hypothesized that the relative movements of aggregate particles result in different arrangements of the particles in order to satisfy equilibrium conditions. It is further hypothesized that this process is not a reversible one, and upon the removal of the loads, equilibrium is achieved through a recovery process which involves some permanent deformations ([Hills, 1971](#)). Others have taken the above theory one step further at the microscopic level and offered explanations of the mechanisms involved in particle reorientation and the collapse of the void network. They have suggested that the rheology (The word rheology is. derived from the Greek word rheo, which translates literally as “to flow.” and used to illustrate the dependency of the stress-strain relationship of asphaltic material on time) of asphalt concrete and its “flow like” behavior stem from viscous flow of the asphalt binder itself ([Hofstra and Klomp, 1972](#)). The flow of asphalt cement into the voids and reduction in the thickness of aggregate coating results in reduction in the relative distances between aggregate particles. Research at the University of Waterloo, as illustrated by [Figure \(2-5\)](#), suggests that nearly all of the permanent deformation in the bituminous layer occurs in the lower half of the layer as a result of the lateral tensile stresses, this conclusion is in contrast with the observations of ([Hofstra and Klomp, 1972](#)) and the predictions of Repeated load tests by ([Morris, 1973](#)) indicated that the greatest amount of permanent deformation occur at depth of about 2 inches (5.08 cm) below the surface, as illustrated by the data presented in [Figure\(2-4\)](#) ,i.e., at the point of occurrence of the largest value of stress difference ,([McClean, 1974](#)). Hence, a particle reorientation is caused by the flow of the asphalt cement into the voids ([Van de Loo, 1978](#)). As a result of

these reorientation of aggregate particles which is enhanced by the temperature effect which liquefy asphalt cement and make the sliding of aggregate particles among each other easy. Under these circumstances, rutting in asphalt concrete is appearing due to the repetition of traffic axle loads (van de Loo, 1978).

There are contradictions between the researchers with respect to distribution of rutting with depth, (Van de Loo, 1978) found that the deformation throughout the asphalt layer was greatest near the load, and gradually decreased with depth. This distribution of permanent strain with depth seems reasonable as there should be more resistance to plastic flow with increasing depth below the wheel loading, also due to the high temperature and vertical stress near the surface.

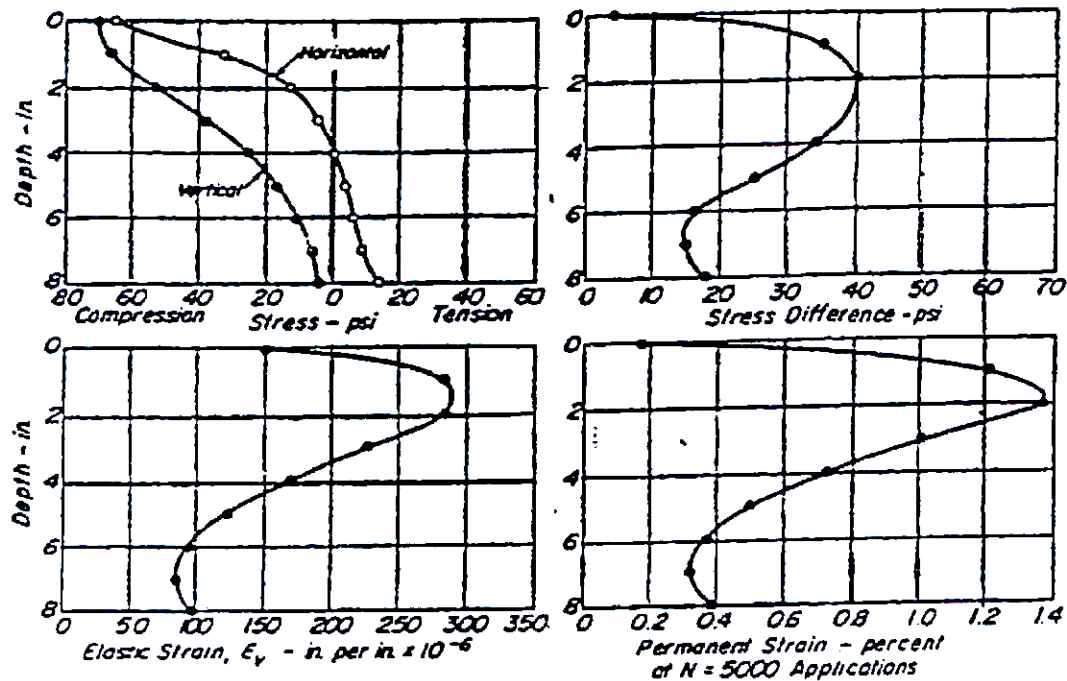


Figure (2-4), Stress and Strain Distributions in 8 inch Thick Asphalt concrete Pavement Subjected to 1500 lb Wheel Load with (70 psi Contact Pressure) Mclean (1974)

2.2 Rutting Measurement and Criteria

Rut depth is defined as the vertical distance between the valley and the crest of the ruts. Rut depth is measured by placing 1.2 m (or 3.6 m) metal straightedge across the wheel path to establish a horizontal reference line and measuring the vertical distance between the straightedge and pavement surface by 0.3 m ruler (as shown in [Figure \(2-5\)](#)). This measurement is repeated for each 6 m interval (in traffic direction) in both wheel paths and the results is averaged to give the mean rut depth ([Verstraeten et al., 1971](#)).

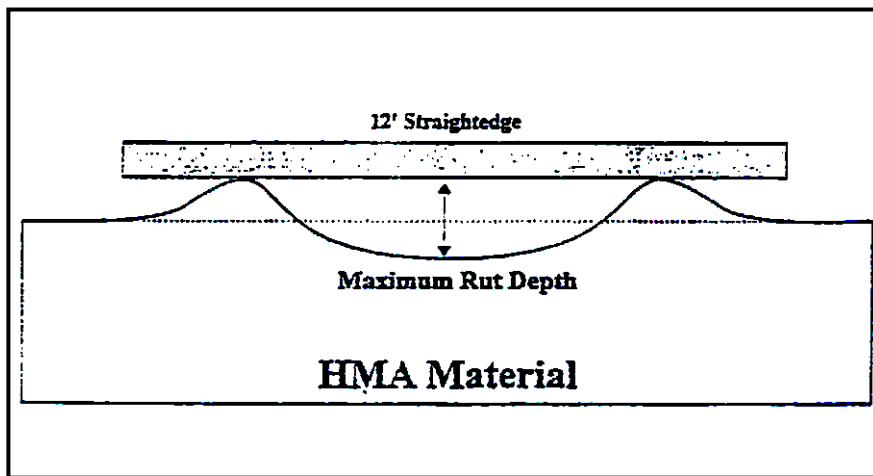


Figure (2-5) Schematic of Rut Depth Measurement with Straightedge
([Verstraeten et al., 1971](#))

The ([Federal Highway Administration FHA, 1975](#)) has classified rutting into four levels of severity:

1. Hydroplaning (0.2 to 0.25 inch)
2. Low (0.25 to 0.5 inch)
3. Medium (0.5 to 1.0 inch)
4. High (1.0 inch)

Allowable rut depth may be limited by both safety and structural as a rut depth of 0.75 to 0.8 inch (19-20mm) measured by a 6ft (1.8m) straightedge ([Lister and Addis, 1976](#)). It is suggested that rut depths up to approximately 0.4 inch (10mm) do not cause significant loss of structural strength. For a cross slope of 2.5 percent (generally used in the United Kingdom), ([Lister and Addis, 1976](#)) have found that ruts deeper than approximately 0.5 inch (13 mm) result in ponding of water which could cause hydroplaning or loss of skid resistance.

This corroborates Barksdale's finding as noted above. ([Verstraeten et al., 1977](#)) have determined that the rut slope (ratio of rut depth to one-half its width) should not exceed 2 percent for good riding quality. The occurrence of permanent deformation or rutting is one of the major problems affecting the performance of pavement structure ([Bissada, 1983](#)). It is considered as a serious safety problem encounter the traffic for two reasons:

1. Loss of vehicle control during lane changing, ruts tend to pull a vehicle towards the rut path as it is steered across the rut.
2. If the rut depth is greater than 12.5 mm for pavement with crown slopes on the order of 2 percent, this rut depth is sufficient for the accumulation of water (ponding) and possibly causing vehicles traveling at speed 80 km/hr or more to hydroplane.

Based on the ([AASHTO guide, 1993](#)), rutting is classified according to severity as exhibited in [Table \(2-1\)](#) below.

Table (2-1) Rut Depth Criteria According to (AASHTO, 1993)

Mean Rut Depth, mm	Severity Level
6-13	Low
13-25	Medium
> 25	High

2.3 Factors Affecting Rutting

Rutting in paving materials develops gradually with increasing numbers of load applications, usually appearing as longitudinal depressions in the wheel paths accompanied by small upheavals to the sides. It is caused by a combination of densification ([decrease in volume and, hence, increase in density](#)) and shear deformation and can occur in any one or more of the pavement layers as well as in the subgrade. In general the characteristics of concrete pavements summarizes in [Table \(2-2\)](#).

([Western Association of Highway and Transportation Officials](#)), [WASHTO](#), ([Betenson, 1984](#)) has made recommendations on the selection of materials to minimize the HMA rutting. The AASHTO Joint Task Force on rutting ([AASHTO, 1988](#)) expressed the opinion that there are many factors which can influence HMA rutting. Traffic and environmental (temperature) factors were identified as the major causes of HMA rutting. Unfortunately, the highway agencies have little control of these two factors. However, there are also important factors that have contributed to the HMA rutting problem that are within the control of highway agencies ([AASHTO, 1988](#)). Highway engineers have focused attention on these factors rather than the traffic and environment. With proper control of asphalt cement, aggregate quality and gradation, asphalt

content and construction The HMA rutting problem can be greatly minimized the subcommittee on materials of the

Table (2-2), Factors Effects upon Asphalt-Concrete Mixtures (Yassoub, 2004).

	Factor	Change in Factor	Effect of Change in Factor on Rutting Resistance
Aggregate	Surface texture	Smooth to rough	Increase
	Gradation	Gap to continuous	Increase
	Shape	Rounded to angular	Increase
	Size	Increase in maximum size	Increase
Binder	Stiffness ^a	Increase	Increase
Mixture	Air void content ^b	Increase	Decrease
	VMA	Increase	Decrease ^c
	Method of compaction	d	d
Test field conditions	State of stress/ strain	Increase in tire contact pressure	Decrease
	Water	Dry to wet	Decrease if mix is water sensitive

^a Refers to stiffness at temperature at which rutting propensity is being determined. Modifiers may be utilized to increase stiffness at critical temperatures, thereby reducing rutting potential.

^b When air void contents are less than about 3 percent, the rutting potential of mixes increases.

^c It is argued that very low VMA's (e.g., less than 10 percent) should be avoided.

^d The method of compaction, either laboratory or field, may influence the structure of the system and therefore the propensity for rutting.

2.3.1 Traffic Characteristics

Traffic characteristics are one of the most important factors affecting rutting. They include tire contact pressure, vehicle speed, and the number of load repetitions (traffic volume). Higher truck tire pressures and increased trucks weights since the AASHTO road tests have lead to increased HMA rutting (Betenson, 1984). Recent studies (Middleton et. al., 1986; Thompson, 1987; Kim and Bell, 1988) have shown that truck tire pressure inflation pressure have increased and are now averaging around 100 psi. Some researchers (Middleton et. al., 1986; Hudson and Seed, 1988) reported truck tire pressures to be as high as 140 psi. Responses to the 1988 AASHTO survey on the HMA rutting problem showed that 93 % of those responding expressed that heavy truck tire pressure were major cause of HMA rutting (AASHTO, 1988). In Dubai (Vallerga et al., 1989), after extensive laboratory and field studies, it was determined that the mixture and pavement had originally been well designed according to existing methodologies. The shape of contact area between a tire and the road surface is approximately circular when the load applied is small relative to the recommended maximum for the tire, but it becomes increasingly elongated as the wheel load is increased at constant inflation pressure, this is shown in

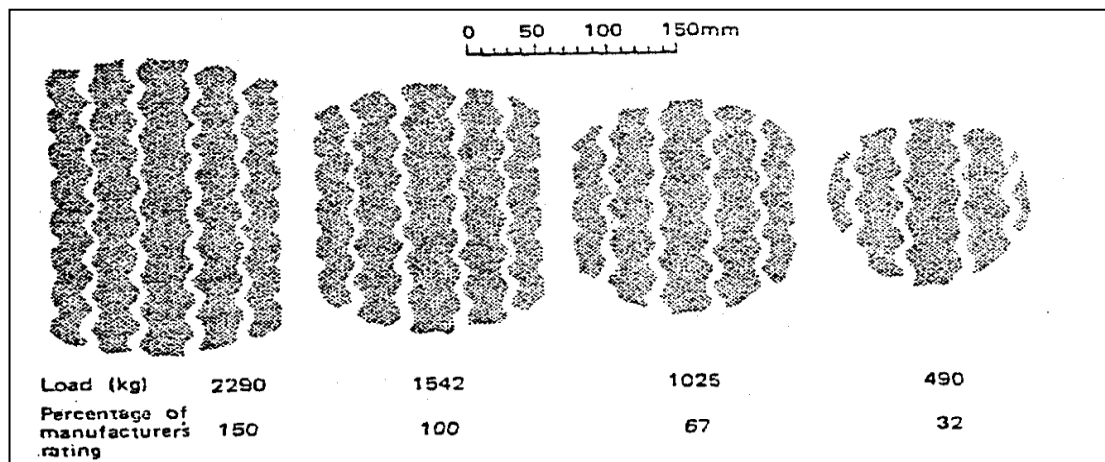


Figure (2-6) Tire prints for Different Loading
(Croney, 1991)

Figure (2-6) where the highest load considered is 50 percent above the tire manufacturer's rating (Croney, 1991). In calculating pavement stresses resulting from the passage of the traffic, it is usual to assume that the load carried by the wheel is uniformly distributed over a circular area and the radius of loading is calculated from the wheel load and the tire pressure (Yang H. Huang, 1993), this is because it is assumed that the tire pressure is equal to the contact pressure (no effect for tire wall). While the current 1993 AASHTO guide for design of pavement structure (AASHTO, 1993) is based on 18 kips loads and tire contact pressure of 75 to 80 psi. (Pereira et. al., 1999) investigated the effect of truck speed on the accumulation of the permanent deformation; measurements of the development of actual rut depth were recorded as a function of truck speed (which decreased as the trucks climbed a steep hill) for the three sites in Portugal. (Shakir, 1999) in his study for the rutting in the oil tanks road located western of Baghdad indicates that the increase in the tandem axle load by 61 percent leads to increase in rut depth by 82 percent. (Chen et. al., 2000) stated in his field study using Texas Load Simulator that such increase in tire inflation pressure from 80 to 100 psi leads to increased in rut depth about 43 percent after 1600000 axle load repetitions. But the accelerated rutting (after 10 months of trafficking) was appeared due to heavier than expected loads (up to 80,000-pound axle loads as compared to the 18,000-pound axle loads used for design) and higher tire pressures.

Another factor related to traffic that significantly affect rutting of asphalt concrete layers is the speed of traveling vehicles. When the viscoelastic approach is used to calculate the pavement response to the applied wheel load, speed is directly related to the duration of loading. If the elastic approach is used in the pavement response calculation, the elastic modulus of asphaltic concrete layers should be determined in commensurate with the vehicle speed. The investigated

roads pass through hilly terrain where long stretches of highways are on inclines and trucks frequently slow down to about 20 km/hr. Schematic diagram for the location of highway and rutting severity is shown in [Figure \(2-7\)](#). Pereira stated that the rut depth in the end of the upward direction lane with 5 percent grade and 25 km/hr average truck speed is 20 mm while in the beginning of the grade, rut depth is 10 mm. Researchers ([Uzen, 1983](#); [Al-Juraiban, 1983](#)) showed that in most cases it follows a linear trend when plotted in log scale. ([Hussein, 1985](#)) in his research on some of the important rural highways connecting Baghdad with other governorates agreed with the above observation. He also found that all the observed highways suffer from rutting particularly on the slow moving vehicle lane (right lane), which carries the bulk volume of the commercial vehicle. ([Eisenmann and Hilmer, 1987](#)) on the basis of wheel tracking test results suggest the following relationship between the rut depth (Rd) and the number of load repetitions (N):

$$Rd = a + b \times N^{0.5} \quad \dots\dots (2-1)$$

Where (a) and (b) are laboratory determined parameters.

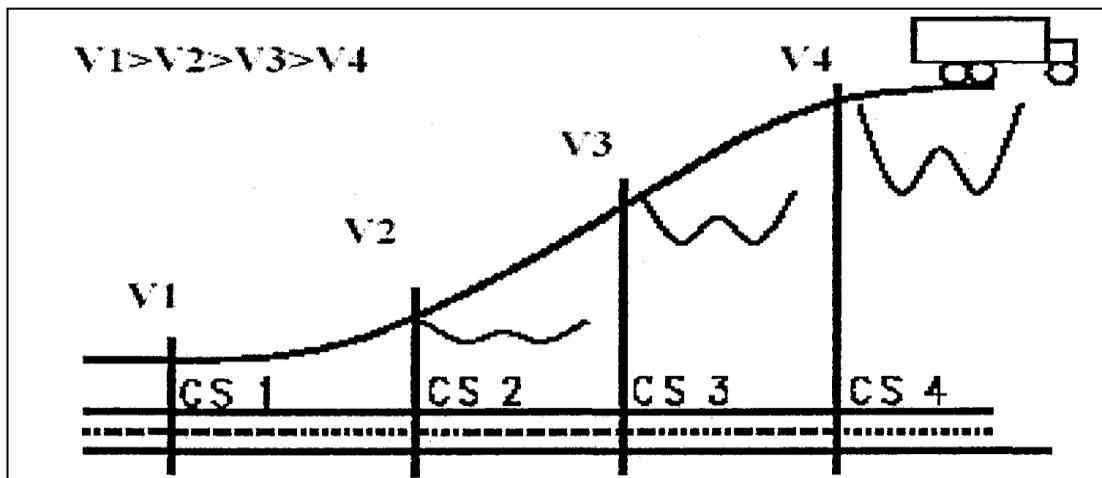


Figure (2-7), Schematic Diagram Showing the Effect of Vehicle Speed along the Grade on the Rut Depth ([Pereira et. al., 1999](#))

Also (Tien et. al, 1992) stated in his rutting survey in Singapore roads that the rutting is commonly founded in the junction. He stated also that this is an indication for the association of this problem to the slow moving vehicles.

The third component of the traffic that effect rutting is the load repetition. In general, as the load repetition increases the rutting is increase as shown in Figure (2-8).specifically, the trend of this relationship is a function several factors, such as temperature, loading conditions and mixture parameters. For the preceding discussion, it is apparent that the increased tire inflation pressure have placed the HMA mixture near the surface under high stresses, though increase from the probability of rutting. Therefore more attention should be given to the selection of high quality materials in designing the asphalt mixture and in quality control during construction of the surface course of flexible pavement in order to minimize rutting.

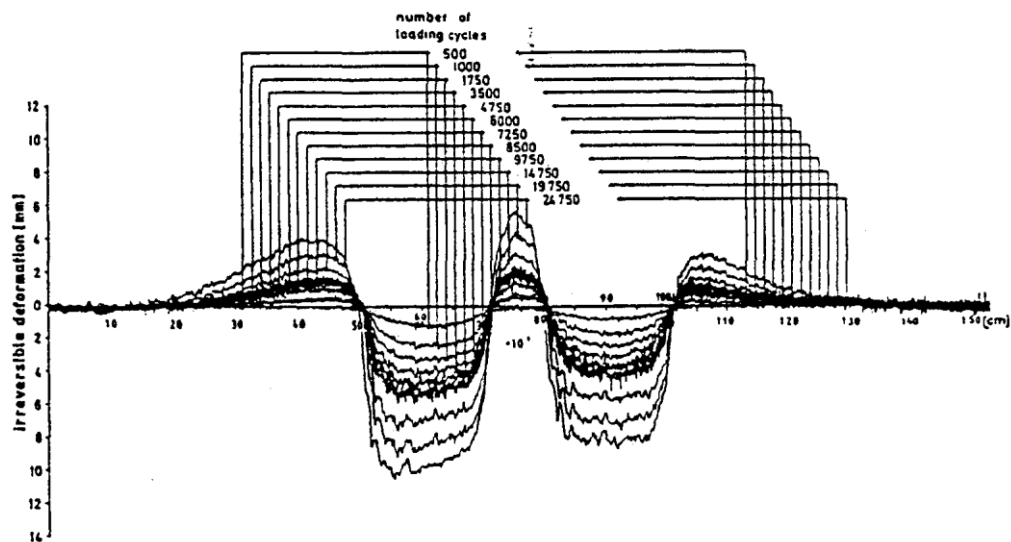


Figure (2-8) Effect of Number of Load Repetition on Rut Depth
(Eisenmann and Hilmer, 1987)

2-3-2 Temperature

Rutting is more prevalent in hot climate areas because the viscosity of the asphalt binder which is inversely related to rutting is significantly reduced with the increase in temperature resulting in amore rut susceptible HMA mix. Figure (2-

10) shows the effect of temperature on log asphalt viscosity for a wide range of asphalt cement grades (Puzinauskas, 1979). From this Figure, it is obvious that the viscosity of asphalt varies from (less than one poise) to (more than one trillion poises). Within such extreme viscosity range, asphalts are transformed from low viscosity Newtonian liquids to materials exhibiting shear-dependent viscoelastic behavior, where with decreasing temperature the elastic component tends to predominant. Thus, the gradually changing curvature of plots in Figure (2-9) indicate that the viscosity of asphalt tends to change more rapidly at low temperatures, and such change becomes far less pronounced at higher temperatures when the viscous behavior is predominant.

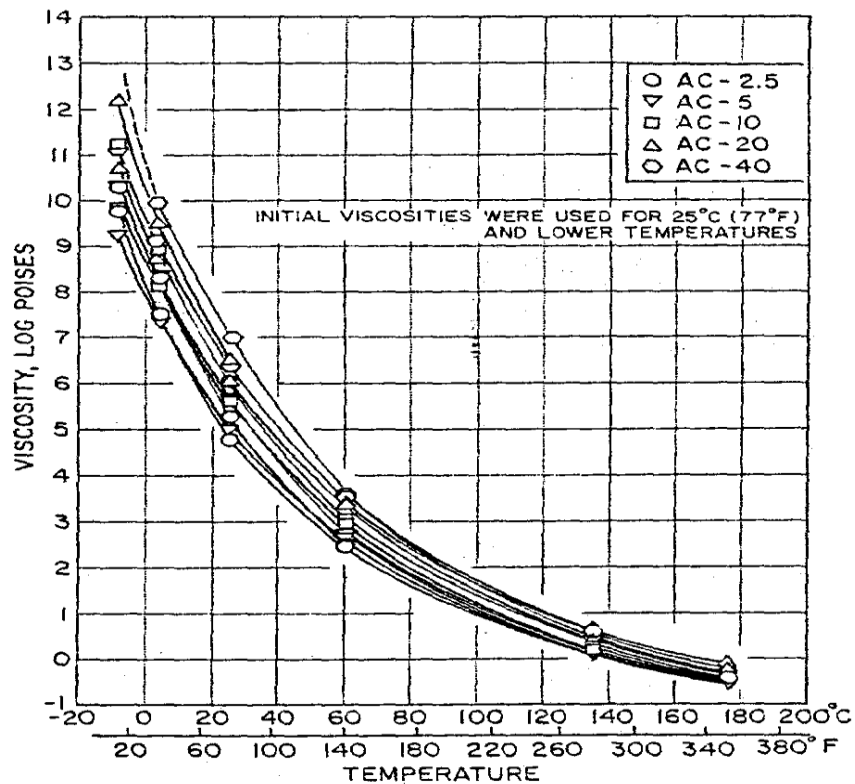


Figure 2-9 Relationship between Viscosity and Temperature for Asphalt Cements (Puzinauskas, 1979)

2-3-3 Asphalt Concrete Mixture Components and Properties

([Speer, 1962](#)) investigated the effect of different asphalt penetration grades on the rutting potential of asphalt concrete. His investigation was based on the laboratory test track and traffic simulator machine which have the capability for applying different tire pressure (30, 50, 70 and 90 psi). Based on his experiments, he stated that asphalt cement with softer grade (90) resulted in rut depth more than that of (40) penetration grade by a factor of 1.5 at the end of one million repetition of 90 psi tire pressure. The comprehensive results of Speer are shown in [Figure \(2-10\)](#) at conditions 5.0% asphalt content and 115 F temperatures. However, Speer cautioned from the construction problems associated with the use of hard grade asphalt due to the difficulty of mixing operation and pavement lay down process. There is much in literature on the relationship between the asphalt cement content and asphalt concrete rutting susceptibility. ([Mahboub and Little, 1988](#)) indicated that higher asphalt cement contents which produce lower air voids increased rutting potential [Figure \(2-11\)](#). ([Anani et. al., 1990](#)) compared the results of cores from both good performing and rutted areas of seven pavements in Saudi Arabia to identify the mix and material properties that caused rutting susceptibility. The authors reported that the rutted areas had higher asphalt cement contents than the good performing areas. Higher asphalt content is defined by the author as that produced air voids in mix less than 3 percent. ([Brown et. al., 2004](#)) stated on the basis of laboratory test track that the addition of 0.5 percent asphalt beyond the optimum content increase the permanent deformation by approximately 50 percent. They suggested that the reduction in air voids as a consequence of increased asphalt content is attributed to the fill of void space with asphalt. As a result, the increase in asphalt content is equivalent to the introduction of lubricants between aggregate particles otherwise separated by a very tight network of air voids. This

phenomenon causes the mixture containing higher asphalt content to be more susceptible to permanent deformation induced by viscous flow of asphalt between aggregate particles.

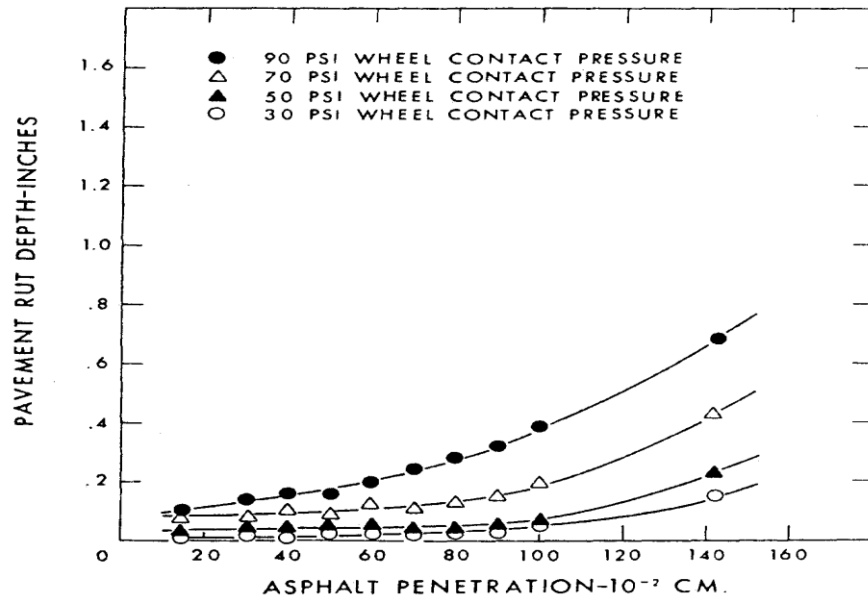


Figure (2-10) Effect of Asphalt Cement Penetration Grade on Rutting
(Speer, 1962)

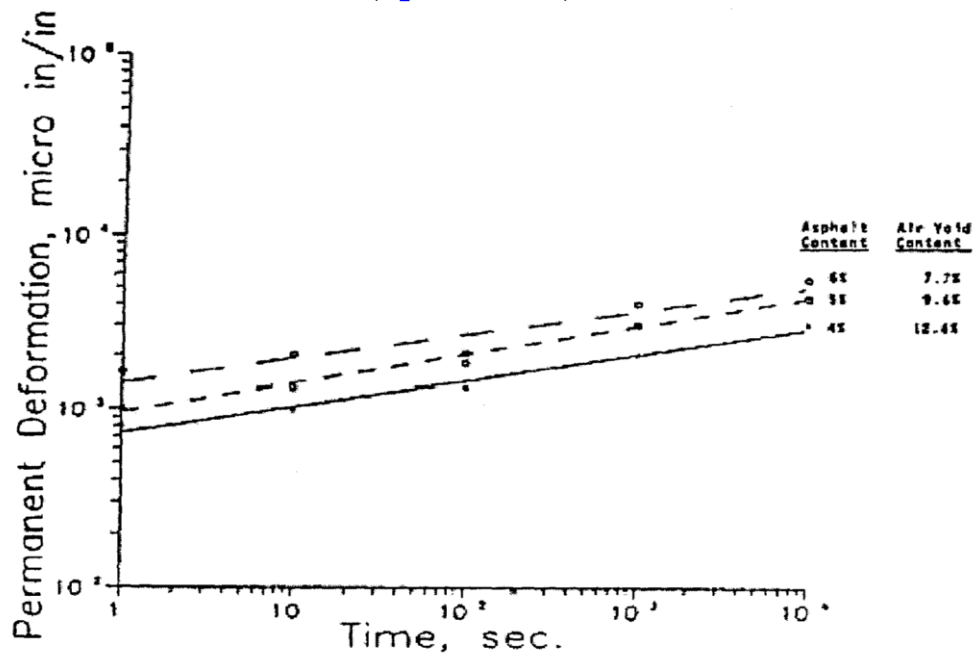


Figure (2-11) Permanent Deformation Trends for AC-5 Mixtures
(Mahboub and Little, 1988)

2.4 Permanent Deformation Predictive Models

Numerous studies have been made to characterize the permanent deformation response of asphalt concrete materials due to repeated load applications. The studies used the repeated compressive load test to investigate the variables that influence the relationship between the number of load applications, (N), and the permanent strain, (ϵ_p). Rather than reviewing each in detail, the more recent research efforts are presented in tabular form to facilitate ease of revisions.

Table (2-3) Summarized Overview of the Permanent Deformation Models (Albayati, 2006)

Author	Permanent Deformation Equation	Variables	Laboratory Test
Kirwan Snaith Glynn (1977)	$\epsilon_n = A \Delta^b$	ϵ_n = induced axial permanent strain after an elapsed time N A = function of elapsed time and material b = constant for the material Δ = applied axial compression stress	Uniaxial compression dynamic loading creep test
Monismith Inkabi Freeme McLean (1977)	$\epsilon_z^p = [\delta(T) N^\alpha (\bar{\sigma})^{\alpha-1} t] \cdot [\sigma_z - \frac{1}{2}(\sigma_x + \sigma_y)]$	ϵ_z^p = vertical permanent deformation $\delta(T)$ = function of temperature α = coefficient determined experimentally N = number of stress repetitions $\bar{\sigma}$ = equivalent stress define as a function of σ_1 , σ_2 , and σ_3 t = loading time	Repeated load triaxial compression test
Brown Bell (1979)	$\epsilon_p = \left(\frac{q}{a}\right)^b (N)$	ϵ_p = permanent shear strain q = deviator stress a, b = constant N = number of load application	Axial repeated load tests

Meyer and Haas (1979)	$\varepsilon_p = F(\sigma_1, \sigma_3, T, AV, N) \pm E$	ε_p = axial permanent deformation σ_1 = vertical stress σ_3 = lateral stress T = temperature AV = air voids N = number of load applications E = error of estimate	Repeated load triaxial test
Kenis (1977)	$\varepsilon_p(N) = e\mu N^{-\alpha}$	$\varepsilon_p(N)$ = permanent strain per pulse $\alpha = 1 - S$ S = slope of the line on a log-log plot of permanent strain versus N e = peak haversine load strain for a load pulse of duration (d) $\mu = IS/e$ I = intercept	Uniaxial repeated load tests
Thrower (1977)	$\dot{\varepsilon}_{ij} = \frac{\sigma_{ij}}{2\eta} i + j$ $\dot{\varepsilon}_{ij} = \frac{\sigma_m}{3\chi} + \frac{(9\sigma_{ij} - \sigma_m)}{18\eta}$	$\dot{\varepsilon}_{ij}$ = rate of deformation σ_{ij} = state of stress σ_m = isotropic mean stress χ = coefficient of "volume viscosity" η = coefficient of shear velocity	

Huscheck (1977)	$\varepsilon_{irr} = c \sigma t^A$ $\varepsilon_{irr}(T, \Delta t_1, t) = \frac{\sigma \Delta t_1}{[\eta(T, t)]}$ $\eta(T, t) = \frac{t^{1-A}}{(c \cdot A)}$	ε_{irr} = permanent deformation c = constant A = consolidation characteristic σ = stress level η = viscosity T = temperature t = total time of loading Δt_1 = time of loading	Uniaxial creep tests Cyclic load creep test
Khedr (1986)	$\frac{\varepsilon_p}{N} = A_a N^{-m}$	ε_p = permanent strain N = number of load cycles A_a = material properties functions of resilient modulus and applied stress m = material parameter	Multi-step dynamic tests
Battiato et al (1977)	$J(t) = J_1 t^\alpha$ $u_{ik}^{perm} = \frac{1}{\eta_s} g_{ik}(y, z)$	$J(t)$ = creep compliance function t = time J_1 = shear creep parameters α = slope of line on a log-log between $J(t)$ and time u_{ik}^{perm} = permanent deformation η_s = shear viscosity of the Maxwell element in series $g_{ik}(y, z)$ = tensor function	Uniaxial creep tests
Van de Loo (1976)	$\varepsilon_p = c \sigma N^a$	ε_p = axial permanent deformation c = constant σ = axial stress level (15 PSI) N = number of load applications a = constant	Axial creep test
Uzan (1983)	$\varepsilon_p(N) = e_r \mu N^{-\alpha}$	$\varepsilon_p(N)$ = permanent strain for N th repetition e_r = resilient strain N = number of repetitions α, μ = characteristics of materials based on intercept and slope coefficients	Repeated load testing
Leahy (1989)	Statistically derived predictive models for permanent strain, ε_p $\varepsilon_p = f(T, \sigma_d, V_{air}, N, \eta_{asp}, P'_{Wasp})$	ε_p = plastic strain T = temperature σ_d = deviator stress V_{air} = volume of air η_{asp} = asphalt viscosity P'_{Wasp} = effective asphalt content	Repeated load and creep, axial testing

2.5 Rut Depth Prediction Methods

With increases in axle loads, load repetitions, tire pressures, and asphalt-concrete thicknesses, a need has developed for methodology to predict rut depths in advance of construction to mitigate potential safety problems; For example, (Saraf et. al., 1976) presented such an equation that incorporated surface deflection as computed from elastic layered analysis. The second approach makes use of closed form viscoelastic analyses to represent pavement structure with permanent deformation characterization by means of creep test (Monismith, 1976). Examples include the Shell procedure (Claessen et al., 1977); Some have recommended limitations on vertical subgrade stress (rather than strain), e.g., (Barksdale and Miller, 1977). The Shell method (Claessen et al., 1977) exemplifies a procedure which attempts to improve the above process by including additional analysis to estimate the amount of rutting occurring in the asphalt-bound layer. and the State of Kentucky methodology (Southgate et al., 1977) the Asphalt Institute procedure (Shook et al., 1982); hydroplaning (Sousa et. al., 1991). Concomitant with the development of analysis procedures which permit estimates of stresses, strains, and deformations resulting from traffic loads, pavement design systems have evolved which include provisions for rutting considerations. These have been referred to as analytically based, mechanistic, or mechanistic-empirical procedures (Sousa et. al., 1991). A number of these procedures include criteria for limiting values of subgrade strain to levels that preclude rutting at the pavement surface. Others have utilized statistically based rut depth prediction equations. Design limitations on strain or stress are based on the assumption that, if the maximum vertical compressive strain or stress at the surface; of the subgrade is less than a critical value, then rutting will be limited to a tolerable level for a specified number of load

applications. Unfortunately, such methodology does not necessarily preclude rutting which might occur in the asphalt-bound layer.

This predictive methodology is an example of one of two analytical procedures which have evolved to estimate the amount of rutting. The procedure makes use of layered system elastic analysis and represents one of a number of such procedures termed layer-strain, in this approach repeated load test or creep test is used to characterize the permanent deformation behavior of asphalt concrete.

2.5.1 Layer-Strain Theory

The layer-strain method consists of predicting rut depths using permanent-deformation characteristics determined from laboratory tests together with an analysis procedure for the pavement structure using either linear or nonlinear elastic theory. The general principle of this method was first proposed by (Barksdale, 1972 and Romain, 1972). While nonlinear elastic theory should provide more accurate results (Brown and Bell, 1979), it has not been used very extensively because of its added complexity. To predict the amount of permanent deformation that would occur after a given number of wheel load applications, each layer of the pavement structure is divided into several sublayers, and the stress state is calculated at the center of each sublayer directly beneath the wheel load Figure (2-12) using elastic analysis. With the average stress state at the center of each sublayer, the corresponding axial plastic strain can be ascertained from the results of laboratory tests. The total rut depth for a given number of load repetitions is obtained by summing the products of the average plastic strain occurring at the center of each sublayer and the corresponding sub-layer thickness, i.e.:

$$\Delta_p = \sum_{i=1}^N [\epsilon_{pi} \times \Delta_{zi}] \quad \dots\dots (2-2)$$

where

Δ_p is the total rut depth, ϵ_{pi} is the average plastic strain in the center of the i -th sublayer, Δ_{zi} is the thickness of the i -th sublayer, and n is the total number of sublayers. This approach has been adopted in various forms by many researchers and, as noted above, the Shell design methodology represents a practical example of the use of this procedure also The **VESYS** elastic program represents another example for this method. The layer-strain method is considered a simplified engineering approach for predicting rut depth which permits the flexibility of using either linear or nonlinear elastic analysis. The following sections provide description for the **VESYS** program and the **KENLAYER** program.

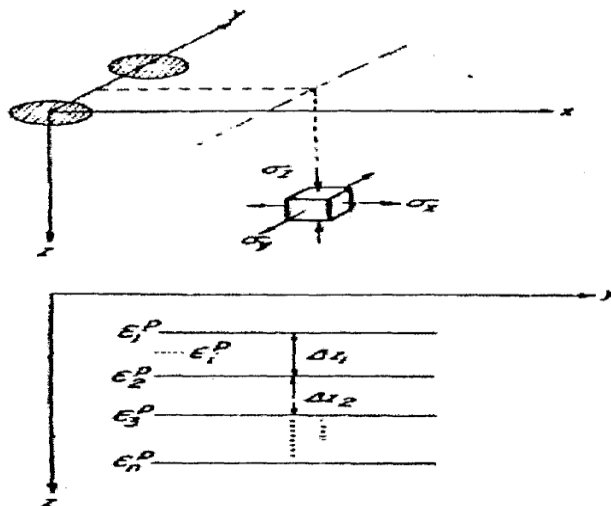


Figure (2-12) Diagram of Pavement System Used to Estimate Permanent Deformation (Sousa, 1991)

2.5.1.1. VESYS Program

VESYS is a probalistic and mechanistic flexible pavement analysis system, which was originally developed at the Massachusetts Institute of Technology (MIT) and subsequently improved further by MIT, the **Federal Highway Administration (FHWA)** and Austin Research Engineers (ARE). (Yang H. Huang, 1993). It includes four major interactive models, termed primary

response, general response, damage and performance model. The primary response model calculate the layered system response (stress, strain and deflection) using either elastic method (material characterized by resilient modulus and Poisson ratio) or viscoelastic method (material characterized by creep compliance) under static loading. The general response model incorporate the effect of moving load using [Boltzmann's principles](#), the damage model includes three sub-models for the prediction of cracking, rut depth and roughness. The calculated values for these distresses are used as an input for the performance model to establish the variation of Present Serviceability Index over the analysis period ([Yang H. Huang, 1993](#)). The [VESYS](#) system is presented as a computer program, the final version for this program which used in this research is [VESYS 5](#). The rut depth predictive sub-model which is included in this version has two options, layer rutting and system rutting. The layer rutting option calculates the rut depth for each layer in the pavement structure, while the system rutting calculates the total rut depth for the whole system. The latter option take into account the subgrade rutting and use the total deflection in the pavement surface in the rut depth calculation ([Kenis and Wang, 1997](#)). Since this research deals with the rutting in asphalt concrete layers, the discussion presented below is limited to the layer rutting. Based on the relationship of ϵ_p -N of the asphalt concrete shown in [Equation \(2-3\)](#), the permanent strain for the N^{th} repetition can be calculated as follows ([Leahy, 1998](#))

$$\epsilon_p = \epsilon_t(N) - \epsilon_r(N) = \epsilon_r \cdot \mu \cdot N^{-\alpha} \quad \text{..... (2-3)}$$

where:

- $\epsilon_p(N)$ = permanent strain for N^{th} repetition .
- $\epsilon_t(N)$ = total strain for Nth repetition
- $\epsilon_r(N)$ = resilient strain for N^{th} repetition (assumed to be independent of N)

- α is a permanent deformation parameter indicating the rate of decrease in permanent deformation as the number of load application increases.
- μ is a permanent deformation parameter representing the constant of proportionality between permanent (plastic) and resilience (elastic) strains.

$$\varepsilon_{tz} = \frac{1}{E_{lo}(N)} [\sigma_z - \gamma(\sigma_x + \sigma_y)] = \frac{\sigma_z'}{E_{lo}(N)} \quad \dots\dots (2-4)$$

$$\varepsilon_{rz} = \frac{1}{E_{un}(N)} [\sigma_z - \gamma(\sigma_x + \sigma_y)] = \frac{\sigma_z'}{E_{un}(N)} \quad \dots\dots\dots (2-5)$$

where:

- ε_{tz} = total axial strain for N^{th} repetition
- ε_{rz} = resilient axial strain for N^{th} repetition
- σ_z = axial stress
- σ_x, σ_y = radial stresses
- $E_{lo}(N)$ = modulus during loading cycle for N^{th} repetition
- $E_{un}(N)$ = modulus during unloading cycle
- γ = Poisson's ratio
- σ_z' = resultant axial stress.

By substitution of equation (2-4) and (2-5) into equation (2-3) then:

$$\sigma_z' \left(\frac{1}{E_{lo}(N)} - \frac{1}{E_{un}(N)} \right) = \frac{\sigma_z'}{E_{un}(N)} \cdot \mu \cdot N^{-\alpha} \quad \dots\dots\dots (2-6)$$

By solving equation (2-6) for $E_{lo}(N)$ yields the following:

$$E_{lo}(N) = \frac{E_{un}}{1 + \mu \cdot N^{-\alpha}} \quad \dots\dots\dots (2-7)$$

The main assumptions included in the program are:

- 1- Poisson's ratio is constant for both loading and unloading conditions.

2- ϵ_r , and consequently E_{un} , is independent of the number of load repetitions.

The model of equation (2-6) is used to evaluate rutting in the asphalt concrete layer as follows:

- Determine the value of resilient modulus, E_{un}
- For a specific value of N , determine the value of $E_{10}(N)$.
- Using $E_{10}(N)$ in multilayer elastic solution, compute the deflections due to loading ($\zeta_{10}(N)$) and unloading ($\zeta_{un}(N)$).
- Compute the rut depth for the N^{th} load repetition, $R_d(N) = \zeta_{10}(N) - \zeta_{un}(N)$.
- Compute the total rut depth for N repetitions by:

$$R_d = \int_{i=1}^N R_d(N).dN \quad \dots\dots (2-8)$$

2.5.1.2. KENLAYER

The **KENLAYER** computer program developed by (Yang H. Huang, 1993) can be applied only flexible pavements with no joints or rigid layers, the backbone of **KENLAYER** is the solution for an elastic multilayer system under a circular loaded area. The solutions are superimposed for multiple wheels, applied iteratively for nonlinear layers, and collocated at various times for viscoelastic layers. As results, **KENLAYER** can be applied to layered systems under single, dual, dual-tandem, or dual-tridem wheels with each layer behaving differently. Either linear elastic; Nonlinear elastic or viscoelastic. Damage analysis can be made by dividing each year into a maximum of 24 periods, each with a group, either single or multiple.

Over all load groups is summed up to evaluate the design life. Damage analysis is performed for permanent deformation as following:

The failure criterion for rutting is expressed as

$$N_d = f_1(\epsilon_c)^{-f_2} \quad \dots\dots (2-9)$$

In which N_d is the allowable number of load repetitions to limit permanent deformation, ϵ_c is the compressive strain on the top of subgrade, f_1 , and f_2 are constants determined from road tests or field performance. Value of f_1 and f_2 are suggested as, 6.15×10^{-7} and 4.0 by [shell \(Claussen et al., 1977\)](#) and 1.13×10^{-6} and 3.571 by the [University of Nottingham \(Brown et al., 1977\)](#) 1.365×10^{-9} and 4.477 by [Asphalt Institute \(AI, 1982\)](#).

2.6 Finite Element Programs

Several specialized finite element computer programs for flexible pavements have been developed over the years. The best known of these include [ILLI-PAVE \(Raad and Figueroa, 1980\)](#) and [MICH-PAVE \(Harichandran, Yeh, and Baladi, 1990\)](#). In addition, general-purpose finite element codes like [ABAQUS \(Hibbitt, Karlsson, and Sorensen, 1998\)](#) and [DYNA \(Livermore Software Technology Corporation, 1999\)](#) have also been used for pavement analysis.

The project team of ([AASHTO Design Guide, 2002](#)) conducted a careful and thorough evaluation of several leading finite element programs for potential implementation in the guide ([Desai and Schwartz, 2000](#)). Three candidate finite element computer programs were initially identified for in-depth evaluation; the [NCHRP](#) Project 1-37A panel subsequently requested that a fourth program ([UMPAD](#)) also be evaluated:

- [ILLI-PAVE \(University of Illinois; M.R. Thompson\)](#)
- [DSC2D \(University of Arizona; C.S. Desai\)](#)
- [DYNA3D \(Lawrence Livermore National Laboratories; J. Hallquist\)](#)
- [UMPAD \(Battelle Memorial Institute/FHWA; J. Kennedy\)](#)

The [ABAQUS](#) general purpose commercial finite element code was also an initial candidate based on its technical capabilities and its extensive past usage in

research oriented pavement analysis. However, the project team eliminated [ABAQUS](#) early on because of its high licensing costs and restrictive licensing terms.

Two flexible pavement analysis methods have been implemented in the [AASHTO 2002 Design Guide](#). For cases in which all materials in the pavement structure can realistically be treated as linearly elastic, the [JULEA](#) multilayer elastic theory program is used to determine the pavement response. [JULEA](#) provides an excellent combination of analysis features, theoretical rigor, and computational speed for linear pavement analyses. In cases where the unbound material nonlinearity is also considered, the [DSC2D](#) nonlinear finite element code is used instead for determining the pavement stresses, strains, and displacements. A major advantage of [MLET](#) solutions is very quick computation times. This method also have the advantage of requiring only minimal input data from the user; the principal inputs consist of the thicknesses and elastic properties of the layers and the tire contact radius and pressure. Solutions for multiple wheel loads can be constructed from the fundamental axisymmetric single wheel solutions via superposition, often automatically by the computer program. ([AASHTO Design Guide Report RR, 2004](#)).

The principal disadvantage of [MLET](#) solutions is the restriction to linearly elastic material behavior. Real pavement materials and the unbound materials in particular, often exhibit stress-dependent stiffness. The materials may even reach a failure condition in some locations, such as in tension at the bottom of the unbound base layer in some pavement structures. These nonlinearities vary both through the thickness of the layer and horizontally within the layer. Some attempts have been made to incorporate these material nonlinearity effects into [MLET](#) solutions in an approximate way ([Asphalt Institute, 1991](#); [Yang H. Huang, 1993](#)), but the fundamental axisymmetric [MLET](#) formulation makes it

impossible to include the spatial variation of stiffness in a realistic manner. The axisymmetric [MLET](#) formulation also makes it impossible to include any effects of vertical discontinuities in the analysis or to include nonuniform tire contact pressures in a realistic manner. The limitations of [MLET](#) solutions are the strengths of [FE](#) analysis. Finite elements are not constrained to two-dimensional axisymmetric conditions (although this is all that is required for the [AASHTO 2002 Design Guide](#) and for this study purpose), but can be easily used for two-dimensional plane stress/strain and/or generalized to a fully three-dimensional formulation in the future. This enables modeling of discontinuities and realistic nonuniform tire contact pressure distributions.

In addition, the finite element method can simulate a wide variety of nonlinear material behavior; the underlying finite element formulation is not constrained to linear elasticity, as is the case with [MLET](#). Stress-dependent stiffness, tension and shear failures, plastic flow, and material damage can all be treated within the finite element framework. Implementation of some of these nonlinear material modeling capabilities for pavement analyses in the [2002 Design Guide](#) was used. Conversely, the strengths of [MLET](#) solutions are the weaknesses of [FE](#) analysis. The input required to define the finite element mesh is substantially greater—and more time consuming to construct than the data required for a [MLET](#) analysis. This can be mitigated somewhat by use of interactive graphics pre- and post-processors for creating the mesh and viewing the results. Finite element analyses also require more computation time than do equivalent [MLET](#) solutions. Finite element solution times even with ([Infinite Boundary Elements](#)) are still 2 to 5 times longer than corresponding [MLET](#) analyses. Based on all of these considerations, it is the overwhelming judgment of the flexible pavement analysis group was that three dimensional finite element analyses are *not* appropriate for inclusion in the [2002 Design Guide](#).

CHAPTER THREE

FINITE ELEMENT METHOD FOR FLEXIBLE PAVEMENT ANALYSIS

3. Introduction

This chapter illustrates the usefulness of the finite element method in the analysis of three layer pavement systems subjected to loading. The method is capable of simulating the observed responses of pavement subjected to axle load. Finite element modeling of pavements, if validated, can be extremely useful, because it can be used directly to estimate primary response parameters without resorting to potentially costly field experiments. If accurate correlations between the calculated and the measured primary response parameters can be obtained, then the analytical model can be used to calculate primary response load equivalency factors, utilizing deflection based or strain-based equivalency factor methods. One analytical approach has been adopted for the flexible pavement response models in this chapter. For the most general case of linear material behavior the two-dimensional analysis of finite element analysis is used to predict the stresses, strains, and displacements in the pavement system under traffic loading for given environmental conditions. The purpose of this chapter is to describe the rationale behind adoption of finite element techniques for the pavement response model and the criteria governing the selection of the [ANSYS, version 9.0](#) finite element computer program. This chapter also provides comparison between finite element techniques and multilayer elastic theory ([MLET](#)).

3.1 Objectives of Pavement Response Models

The purpose of the flexible pavement response model is to determine the structural response of the pavement system due to traffic loads and environmental influences.

Environmental influences may be direct (e.g., strains due to thermal expansion and/or contraction) or indirect via effects on material properties (e.g., changes in stiffness due to temperature and/or moisture effects) ([AASHTO Design Guide Report RR, 2004](#)).

Inputs to the flexible pavement response model include:

1. Pavement geometry
 - a. Layer thicknesses
 - b. Discontinuities (e.g., cracks, layer separations)
2. Environment
 - a. Temperature vs. depth for each season
 - b. Moisture vs. depth for each season
3. Material properties (adjusted for environmental and other effects, as necessary)
 - a. Elastic properties
 - b. Nonlinear properties (where appropriate)
4. Traffic
 - a. Load spectrum—i.e., frequencies of vehicle types and weights within each vehicle type
 - b. Tire contact pressure distributions and areas

The outputs from the pavement response model are the stresses, strains, and displacements within the pavement layers. The particular interests are the critical

responses variables required as inputs to the pavement distress models in the mechanistic-empirical design procedure. Examples of critical pavement response variables include:

- Tensile horizontal strain at the bottom of the AC layer (for AC fatigue cracking)
- Compressive vertical stresses/strains within the AC layer (for AC rutting) (the problem of this study).
- Compressive vertical stresses/strains within the base/subbase layers (for rutting of unbound layers)
- Compressive vertical stresses/strains at the top of the subgrade (for subgrade rutting)

Each pavement response variable must be evaluated at the critical location within the pavement layer where the parameter is at its most extreme value. For a single wheel loading, the critical location can usually be determined by inspection. For example, the critical location for the tensile horizontal strain at the bottom of the AC layer under a single wheel load is directly beneath the center of the wheel. For multiple wheels and/or axles, the critical location will be a function of the wheel load configuration and the pavement structure. Mixed traffic conditions further complicates the problem, as the critical location within the pavement structure will not generally be the same over all vehicle types. The pavement response model must search for the critical location for each response parameter and vehicle type in these cases.

Many techniques are available for determining the stresses, strains, and deformations in flexible pavement systems. These can be categorized as follows:

- Analytical (e.g., Burmister solution)
- Multilayer Elastic Theory (MLET)

- Rate-independent [BISAR, CHEVRON]
- Viscoelastic [VESYS]
- Finite Difference Methods [FLAC]
- Finite Element Methods (FEM)
- General purpose [ABAQUS, ANSYS]
- Pavement-specific [ILLI-SLAB, ILLI-PAVE, MICH-PAVE]
- Boundary Element Methods [BEASY]
- Hybrid Methods

For the design approach proposed for any high level computer program such as ([AASHTO 2002 design guide](#)), the core analysis capabilities required in the flexible pavement response model are as follows:

- Linear material model for AC, other bound, and unbound layers (lowest hierarchical level for unbound material characterization)
- Stress-dependent material model (Nonlinear resilient modulus with tension cutoff) for unbound materials (highest hierarchical level for unbound material characterization).
- Quasi-static monotonically increasing loading from single or multiple wheel configurations
- Fully bonded, full slip, and intermediate interface conditions between layers

3.2 Selection of Analysis Method

The choice of an appropriate analysis method for the flexible pavement response model is governed by many considerations, both theoretical and practical. Key

issues include material behavior, problem dimensionality, loading conditions, computational practicality and implementation considerations.

3.2.1 Material Behavior

As a first approximation, the materials used in flexible pavement systems are often treated as linearly elastic. As shown conceptually in Figure (3-1) a, the stresses in a linearly elastic material are proportional to strain, with the proportionality constant equal to Young's modulus, E . However, unbound pavement materials (and asphalt concrete at very high temperatures) are more accurately characterized using a stress-dependent modulus Figure (3-1) b, perhaps in conjunction with a tension cut-off for the unbound materials.

Explicit consideration of the rate dependence of asphalt concrete requires viscoelastic analysis capabilities. This is needed for this study, where rate effects are incorporated by adjusting the AC stiffness using the creep and recovery compliance varies with time.

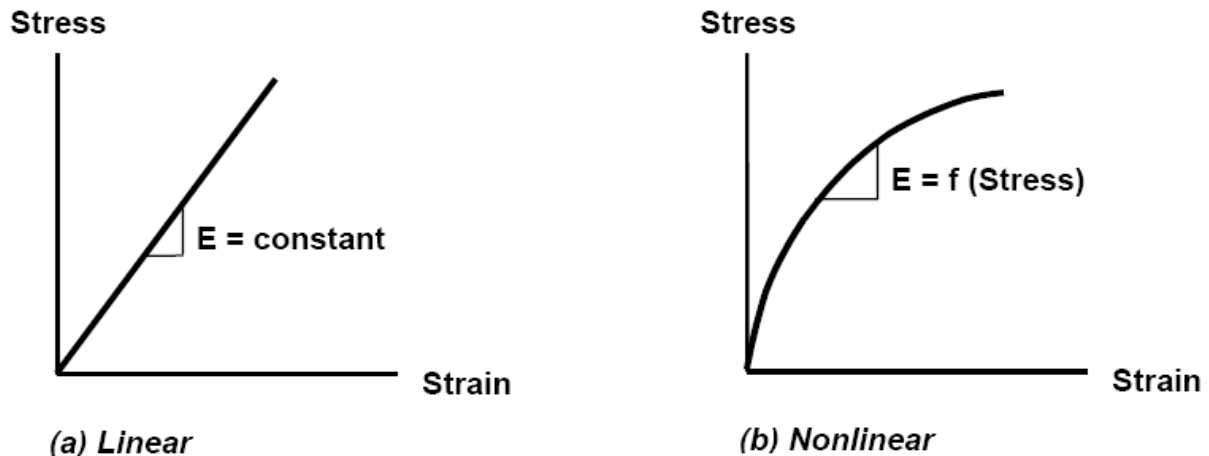


Figure (3-1) Linear and Nonlinear Material Behavior

3.2.2 Loading Condition

Because heavier axle loads have higher tire pressures and more destructive effects on pavements, the use of tire pressure as the contact pressure is therefore on the safe side. (Yang H. Huang, 1993)

These contact areas are not axisymmetric and cannot be used with the layered theory or axisymmetric 2D finite elements for one contact load. When the finite element (axisymmetric algorithm) is used for flexible pavement design, it is assumed that each tire has a circular contact area. This assumption is not correct, but the error incurred is believed to be small, to simplify the analysis of flexible pavements.

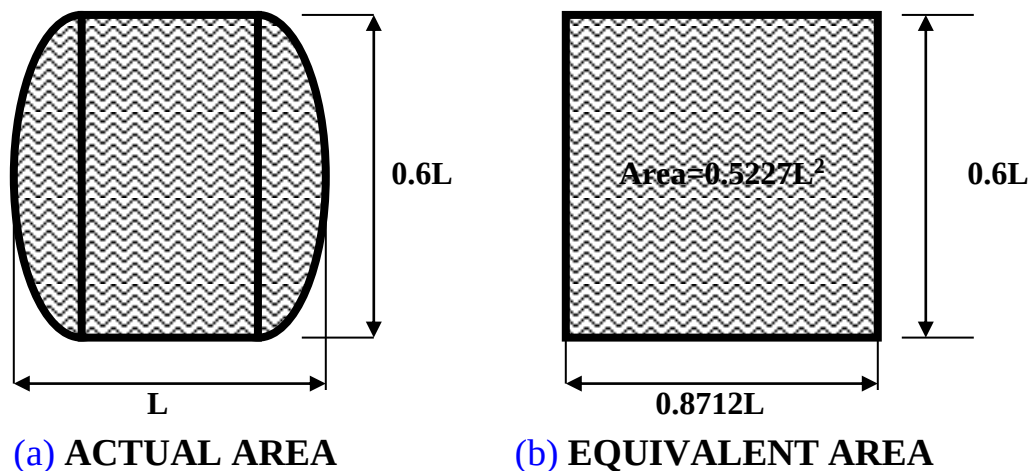
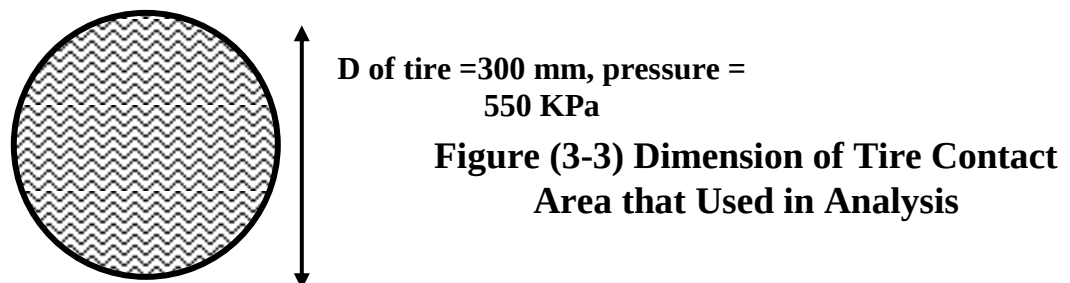


Figure (3-2) Dimension of Tire Contact Area.

Consequently it has been used a circle loading area with a radius (r) Figure (3-4)



3.2.3 Problem Dimensionality

The required dimensionality (1D vs. 2D vs. 3D) of an analysis is a function of geometry and loading conditions. It should be investigate flexible pavement analysis scenarios in which three-dimensional geometric effects may be important. The issue of two- vs. three-dimensional finite element analysis for flexible pavements has become quite controversial in recent years (GAO, 1997). The issue is not whether it can perform three-dimensional finite element analyses for pavements, including nonlinear material behavior and dynamic response, if necessary; clearly, these types of analyses are well within the capabilities of any number of available finite element programs. The real issue is whether it should implement these capabilities for design, and in particular for the flexible pavement design formulation proposed for the design pavement program.

In most flexible pavement problems, there are no vertical discontinuities (e.g., slab joints) and the only source of three-dimensionality is multiple wheel loads. If the behavior of all materials in the pavement structure can be treated as linear, then a 3D solution can be constructed from simpler 2D axisymmetric analysis can results via superposition. Although nonlinear material behavior invalidates the principle of superposition from a rigorously theoretical standpoint, superposition may still provide acceptably accurate calculations of critical pavement response parameters in most pavement structures (Schwartz, 2000). Discrete vertical discontinuities are important three-dimensional geometrical features in some flexible and composite pavement rehabilitation scenarios, in particular with regard to reflection cracking. These vertical discontinuities invalidate the two-dimensional assumption of axial symmetry. However, reflection cracking has been excluded from the scope of modern pavement programs such as the 2002 Design Guide and, as a consequence, modeling of

vertical boundaries/discontinuities is not required and a 2D axisymmetric analysis will be sufficient from a geometrical viewpoint.

3.3 Selection of Finite Element Program

Several specialized finite element computer programs for flexible pavements have been developed over the years. The best known of these include [ILLI-PAVE](#) ([Raad and Figueroa, 1980](#)) and [MICH-PAVE](#) ([Harichandran, Yeh, and Baladi, 1990](#)). In addition, general-purpose finite element codes like [ABAQUS](#) ([Hibbitt, Karlsson, and Sorensen, 1998](#)) and [DYNA](#) ([Livermore Software Technology Corporation, 1999](#)) have also been used for pavement analysis. The candidate finite element computer program is [ANSYS](#) due to the powerful, sufficient for purpose of this study, and supporting the [GUI](#) ([Graphical User Interface](#)) leading to become easy handling.

3.4 Finite Element Solutions

The computation time required for a finite element solution involves a tradeoff between solution accuracy and speed. A finer mesh (more and smaller elements) will provide a more accurate solution at the expense of increasing computation time. The question of required computation time is thus most appropriately framed as: “What computation time is required to achieve a solution of acceptable accuracy?” In order to investigate this question, a series of linearly elastic finite element analyses were performed for a typical pavement structure consisting of 100 mm of asphalt concrete ($E = 3500$ MPa, $\nu = 0.35$) over 200 mm of crushed stone base ($E = 350$ MPa, $\nu = 0.3$) over a soft subgrade ($E = 100$ MPa, $\nu = 0.4$). A single Wheel load was modeled as a uniform pressure of 550 kPa over a circular area of 150 mm radius. This single wheel load is not entirely consistent with the dual tandem loading used for the MLET part of this timing

study, but the additional computation times required to superimpose the FE solutions is expected to be minimal. All analyses were performed under linearly elastic axisymmetric conditions using [ANSYS 9.0](#) software and check against the ([MLET](#)) solution done by ([KENLAYER¹](#)). Three separate but similar finite element meshes were developed to investigate the tradeoff between accuracy and speed:

- A coarse refinement mesh consisting of 396 elements, with a smallest element size of 50 mm by 50 mm Figure (3-4). Two elements spanned the thickness of the asphalt concrete surface layer and four elements spanned the thickness of the base layer.
- A medium refinement mesh consisting of 1584 elements, with a smallest element size of 25 mm by 25 mm Figure (3-5). Four elements spanned the thickness of the asphalt concrete surface layer and eight elements spanned the thickness of the base layer.
- A fine refinement mesh consisting of 3564 elements, with a smallest element size of 16.7 mm by 16.7 mm Figure (3-6). Six elements spanned the thickness of the asphalt concrete surface layer and twelve elements spanned the thickness of the base layer.

The finite element meshes extended 2100 mm (14 tire radii) in the radial direction and 2300 mm (approximately 15 tire radii) in the vertical direction. Note that the bottom mesh boundaries are closer than conventional meshing guidelines would suggest by ([Duncan, Monismith, and Wilson, 1968](#)) recommend that the horizontal lower boundary of the finite element mesh be located no closer than 18 tire radii for a homogeneous elastic system and no closer than 50 tire radii for a layered system. The recommend that the vertical

side boundary of the finite element mesh be located at least 12 tire radii from the center of the tire.

However, this is acceptable for the purposes of the present study. Extending the meshes to greater depths would require comparatively few additional elements, as the elements are quite large in this region of the mesh. A preferable solution is to line the bottom (and vertical sides) of the meshes with infinite elements, as discussed in the following subsection.

Figure (3-7) through Figure (3-9) illustrate the trade-offs between mesh refinement and solution accuracy for the computed surface displacements and the vertical and horizontal stresses along the load centerline, respectively. The MLET solution results obtained using [KENLAYER](#) are also shown on these figures. The FE results in Figure (3-7) through Figure (3-9) were computed using [ANSYS](#). The surface displacements Figure (3-7) for the medium and fine refinement meshes are virtually identical, although they are less than the [KENLAYER](#) results because of the rigid lower boundary in the finite element meshes. The vertical and horizontal stresses in Figure (3-8); Figure (3-9) are nearly identical for the medium and fine refinement meshes, and these also agree very closely with the [KENLAYER](#) quantities. Surprisingly, even the coarse refinement mesh gave reasonable results for the surface displacements and load line stresses. Qualitatively, however, the results in Figure (3-7) through Figure (3-9) suggest that, of the three meshes studied, the medium refinement (1584 elements, with 4 element layers in the AC and 3 element layers in the base) mesh is the minimum refinement necessary to achieve acceptable solution accuracy. It should be note that all material properties, load configurations and dimensions of geometric case are reported in ([AASHTO Design Guide Report RR, 2004](#)) the units that used was SI.U ([System of International Units](#)) and the same units is

applying in [ANSYS 9.0](#) for comparison purposes, solution of [KENLAYER](#) presents in [Appendix B](#).

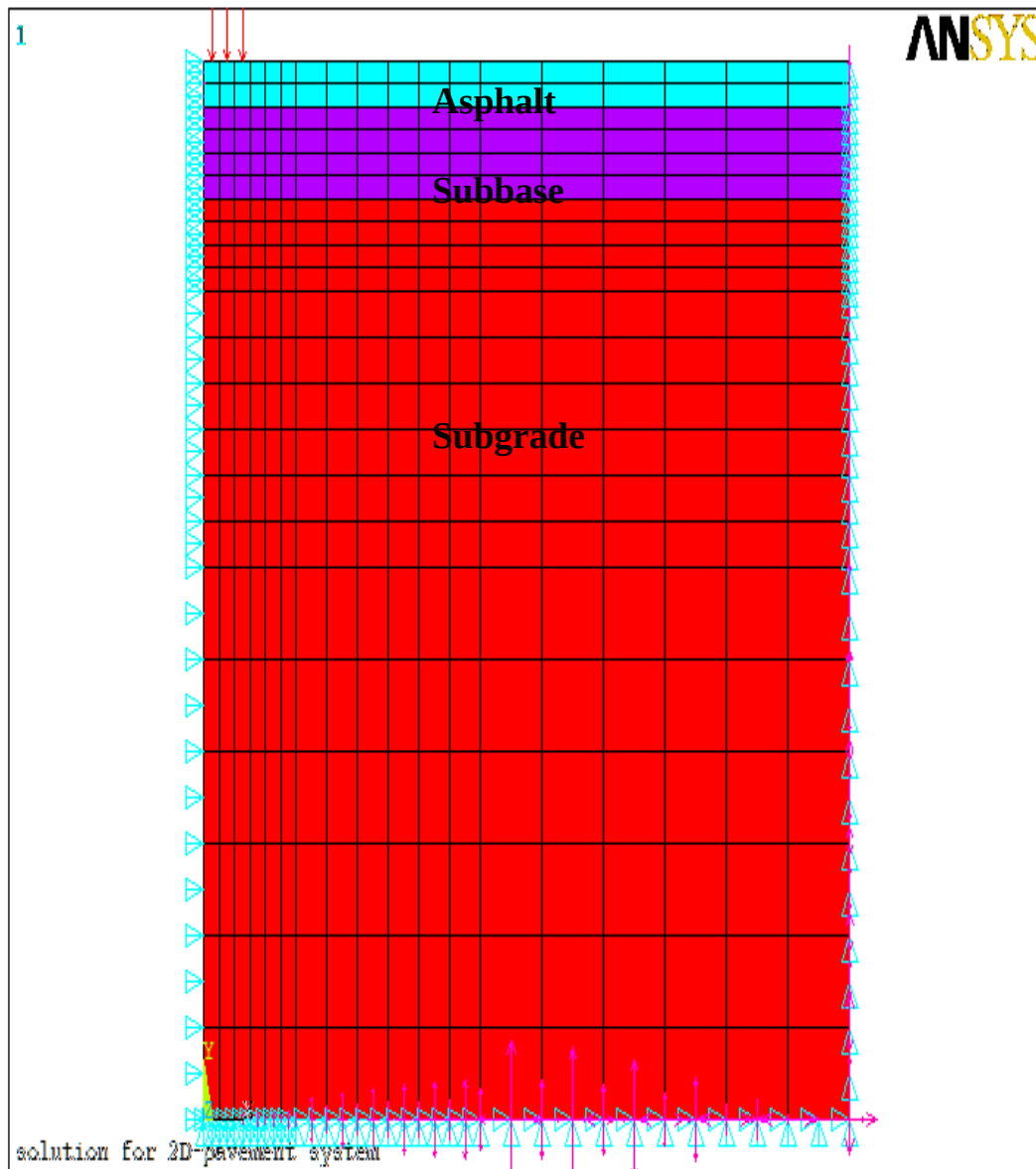


Figure (3-4). Coarse Refinement Mesh (396 elements)

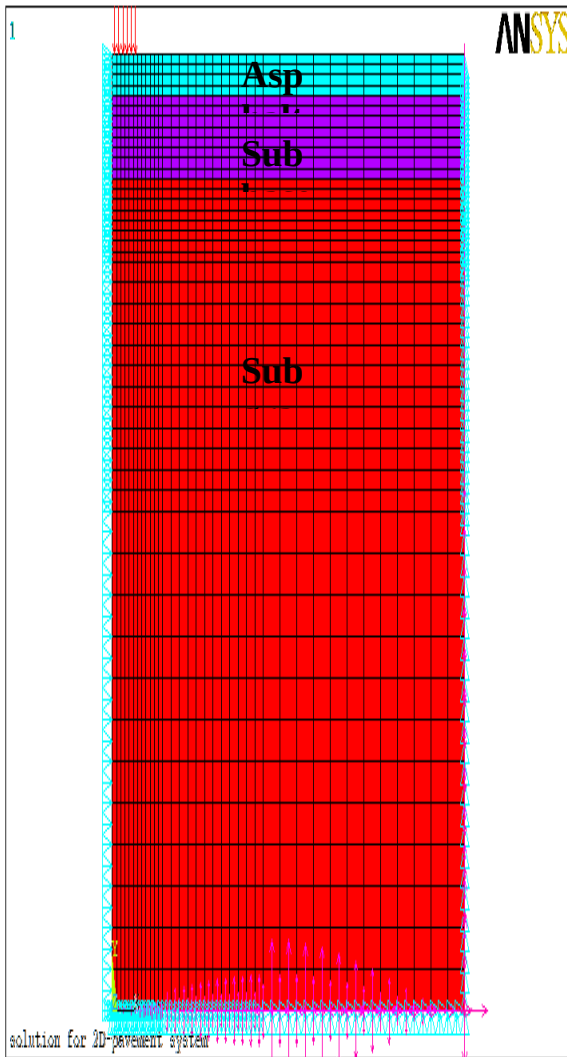


Figure (3-6). Medium Refinement Mesh (1584 elements)

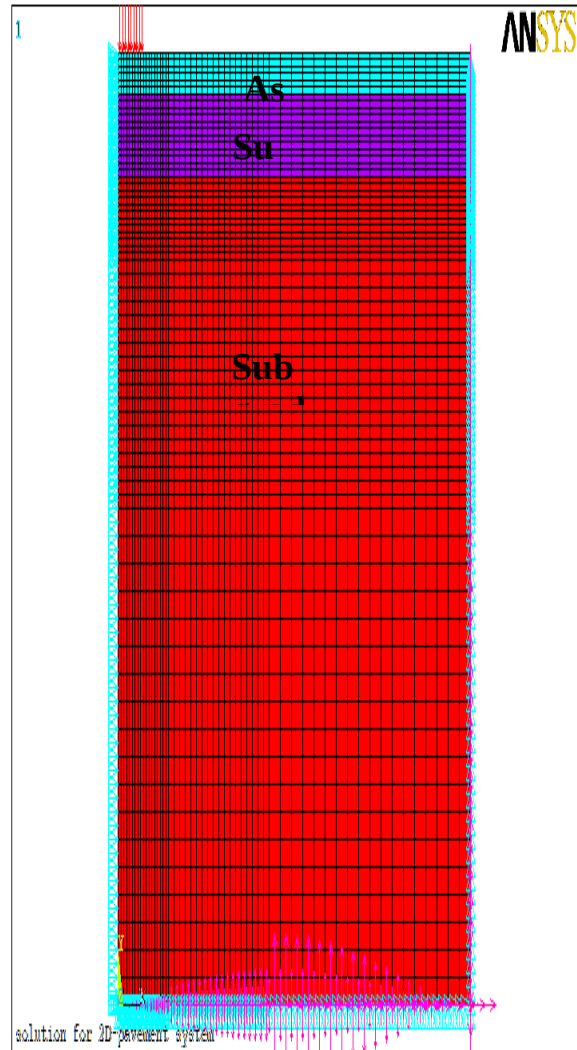


Figure (3-5). Fine Refinement Mesh (3564 elements)

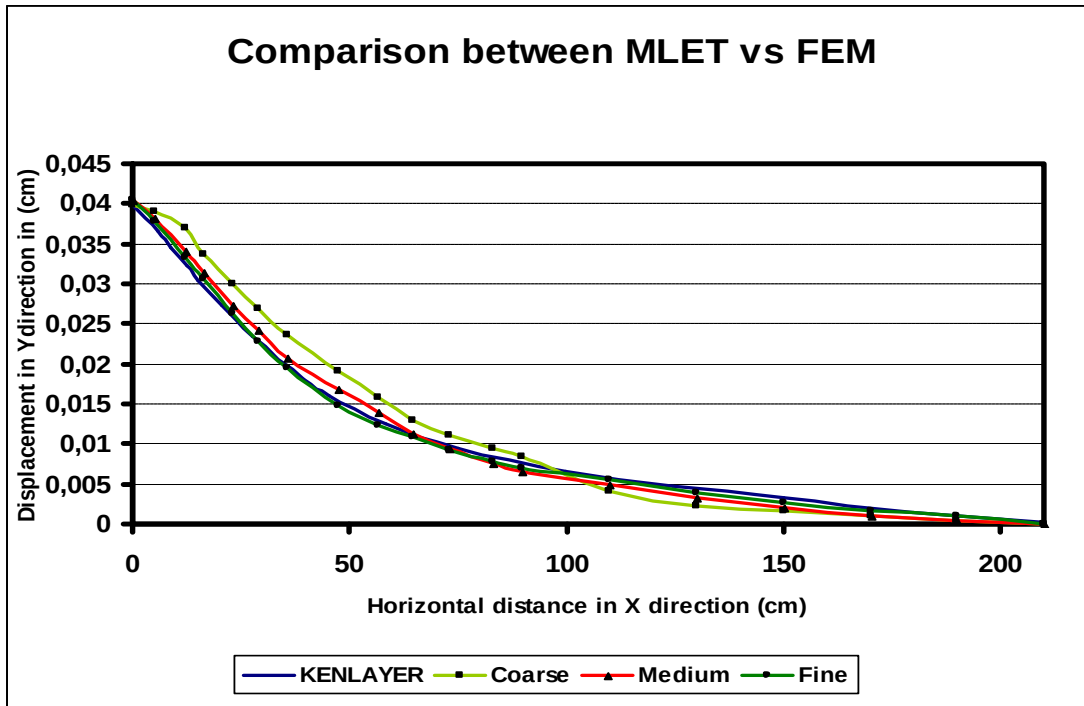


Figure (3-7). Surface Displacements Computed at Different Mesh Refinements.

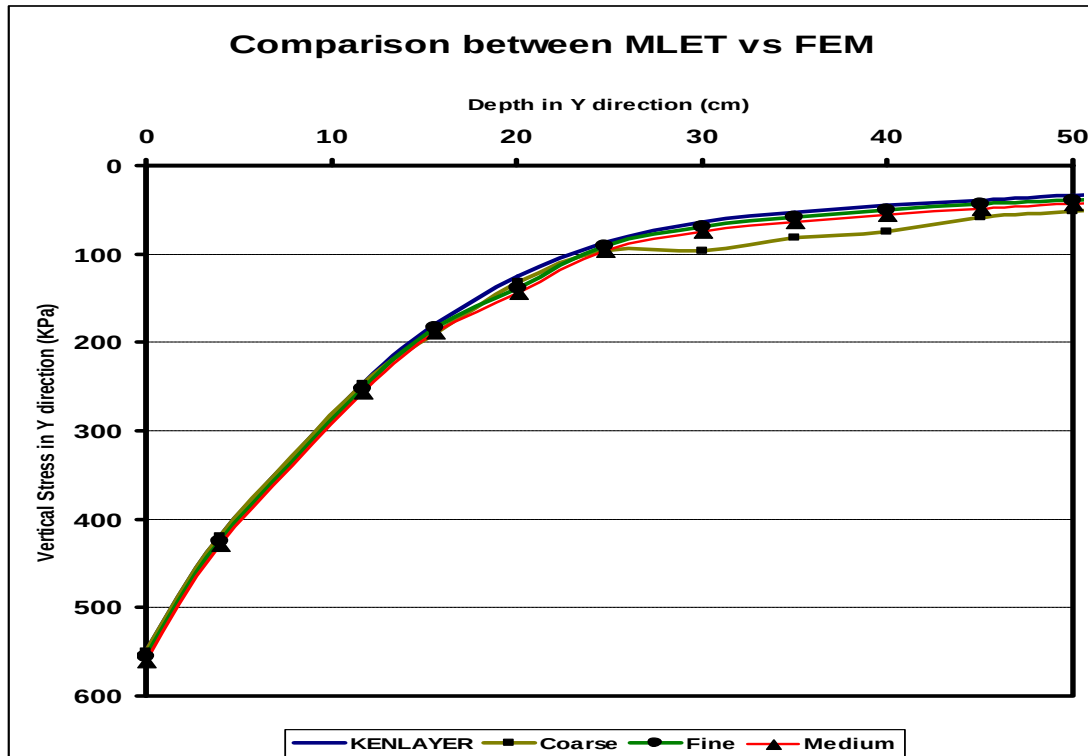


Figure (3-8). Vertical Stresses along Centerline at Different Mesh Refinements.

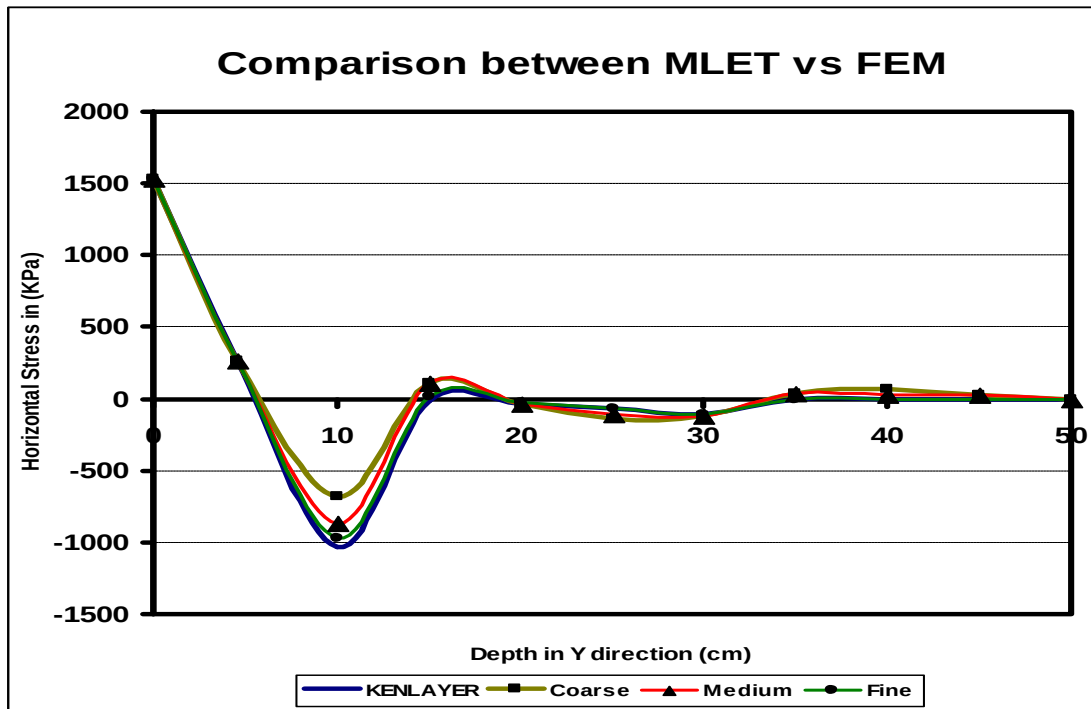


Figure (3-9).Horizontal Stresses along Centerline at Different Mesh Refinements.

3.5 Primary Response

3.5.1 Elastic Solution

A two-dimensional finite element model was developed, using [ANSYS version 9.0](#), to simulate a typical pavement structure. The simulation was developed to initially compare pavement responses to those obtained from the classical theory of linear elasticity as shown in previous section. The analysis simulated a single tire has nominal tire inflation pressure of 83 psi (572 KPa). The following sections describe some of the features of the developed FE model. The dimensions of the modeled portion were 6.889 ft (2.1m) in horizontal direction and 7.54 ft (2.3 m). These dimensions were selected based on limitations of ([Duncan, Monismith and Wilson, 1968](#)), while keeping the elements' sizes within acceptable limits. The generated mesh was designed to give an optimal

accuracy medium mesh around the load and coarse mesh far from it. To improve the rate of convergence, 8-node linear rectangular was used.

The materials that use in this model are shown in [Table \(3-1\)](#).The Elastic solution shown in [Figure \(3-12\)](#) through [Figure \(3-15\)](#) and [Table \(3-3\)](#) through [\(3-4\)](#).

Table (3-1), Material Properties for Each Layer (Yassoub,2004).

Layer	Resilience modulus, psi	Possion's ratio
Wearing course	187,580	0.35
Binder course	161000	0.35
Base course	127500	0.35
Subbase course	15100	0.4
Subgrade course	5800	0.45

3.5.2 Section Transformation (Equivalent Thickness).

The " method of equivalent thicknesses" used for layered structures was first suggested by Odemark in 1949 ([Ullidtz et. al, 1979](#)).Subsequently, their use in the analysis and performance prediction of the pavement structure becomes widespread ([Glynn, et. al, 1977](#); [Nemesdy et. al, 1977](#); [Ullidtz et. al, 1979](#)).The basic principle of the method is that the layered system with N layers can be transformed into equivalent layer by fixing the stiffness constant for the case under consideration. The stiffness is defined by ([Odemark](#)) as the product of moment of inertia and the modulus of elasticity for two layers system, the derivation of the transforming relationship becomes as following:

Stiffness of layer (1) +stiffness of layer (2) =stiffness (equivalent layer)

Where E is the modulus of elasticity for asphalt which using Mr (modulus of resilience or E* is the complex modulus of asphalt

$$EI_1 + EI_2 = EI_{eq}.$$

$$E_1\left(\frac{b_1 h_1^3}{12}\right) + E_2\left(\frac{b_2 h_2^3}{12}\right) = E_{eq}\left(\frac{b_{eq} h_{eq}^3}{12}\right)$$

For $b_1=b_2=b_{eq}$.

The relation becomes: $E_1 h_1^3 + E_2 h_2^3 = E_{eq} \cdot h_{eq}^3$

Divide by E_{eq} : $\frac{E_1 h_1^3}{E_{eq}} + \frac{E_2 h_2^3}{E_{eq}} = h_{eq}^3$ This equation can be simplified by

rearranging and raising both side of equation to power 1/3, this led to

$$h_{eq} = h_1 \cdot \left(\frac{E_1}{E_{eq}}\right)^{1/3} + h_2 \cdot \left(\frac{E_2}{E_{eq}}\right)^{1/3} \dots\dots (3-2)$$

This relationship can be generalized for N layers:

$$h_{eq} = h_1 \cdot \left(\frac{E_1}{E_{eq}}\right)^{1/3} + h_2 \cdot \left(\frac{E_2}{E_{eq}}\right)^{1/3} + h_3 \cdot \left(\frac{E_3}{E_{eq}}\right)^{1/3} + \dots\dots + h_n \cdot \left(\frac{E_n}{E_{eq}}\right)^{1/3} \dots (3-3)$$

where:

- h=layer thickness.
- b=the width of pavement section.
- I=moment of inertia (for rectangular section =bh³/12).
- E=modulus of elasticity (characterized as resilient modulus).

The Equation (3-2) was used to convert the thicknesses of binder and base courses into an equivalent layer thickness for wearing course. This transformation will save the effort for testing the asphalt concrete mixtures for both the binder and base courses for permanent deformation characterization, and will simplify the analysis of pavement structure and rut depth prediction.

For the practical solution of finite element model (Primary response model), according to the resilient modulus of the layers to be transformed (binder and base).

That for wearing layer, the thicknesses and the transformation factor $f=(E_i/E_{eq})^{1/3}$ are shown in Table (3-3) below. The resulting geometry of the transformed pavement section is shown in Figure (3-14).

Table (3-2), Equivalent Thicknesses

Layer	Initial thickness cm (inch)	Transformation factor, f	Equivalence thickness cm (inch)
Wearing	5 (2)	1	5 (2)
Binder	7 (2.8)	0.9503	6.65 (2.66)
Base	10 (4)	0.88	8.79 (3.52)
Total Equivalent Thickness			= 20.44 (8.2)

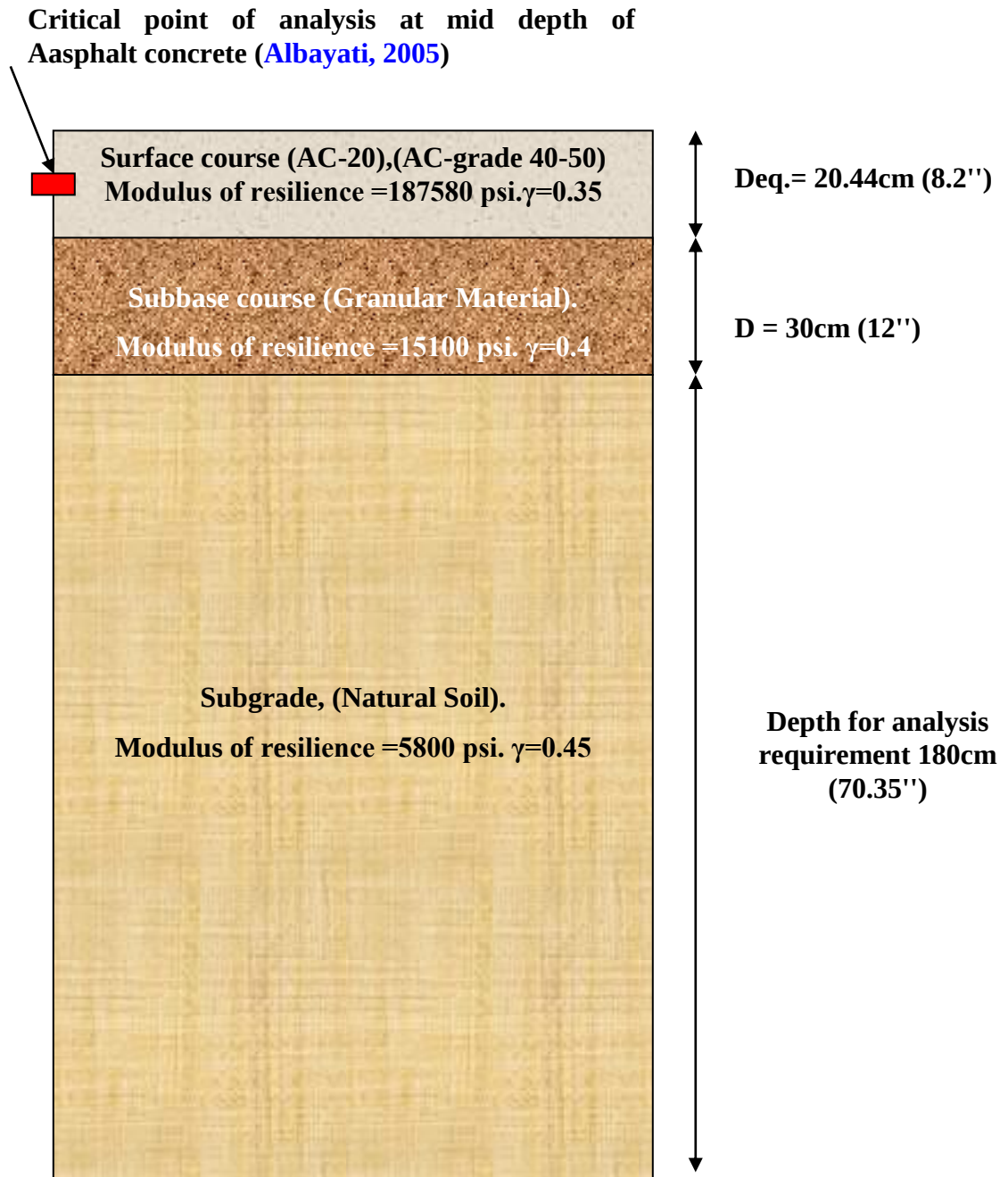


Figure (3-10), Geometry of Transformed Pavement Section.

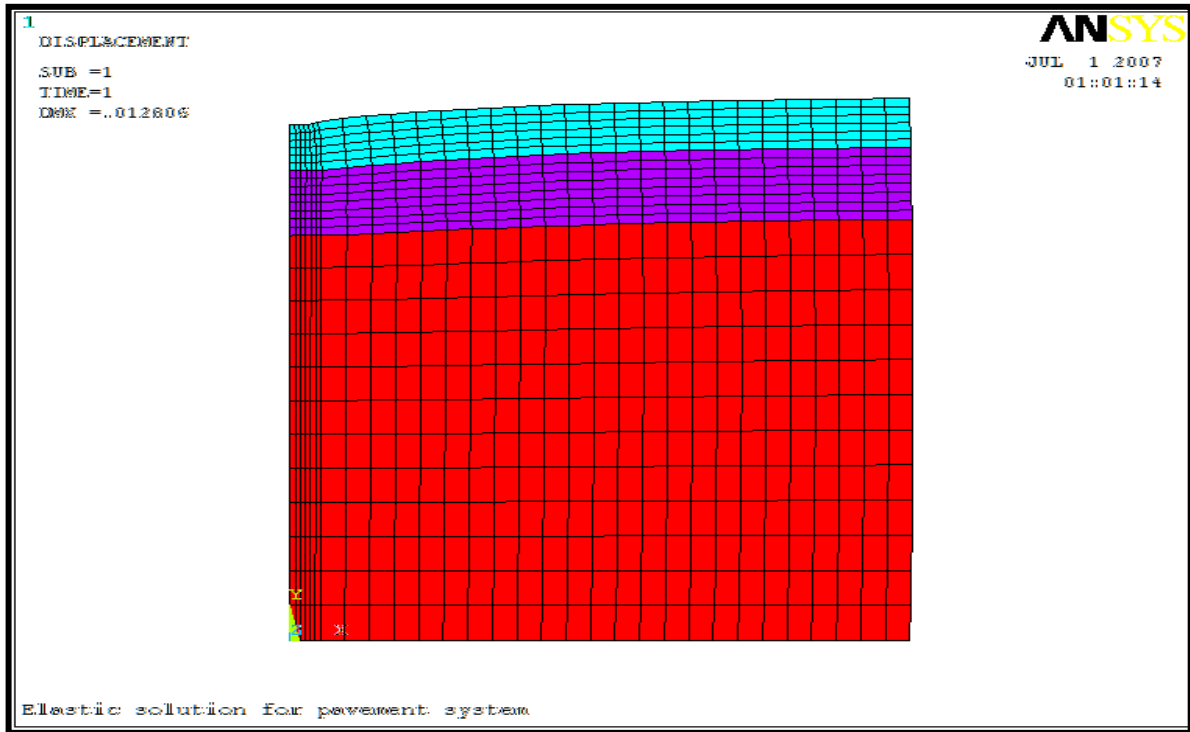


Figure (3-11), Deformation Shape and Mesh Generation for Elastic Finite Element Model.

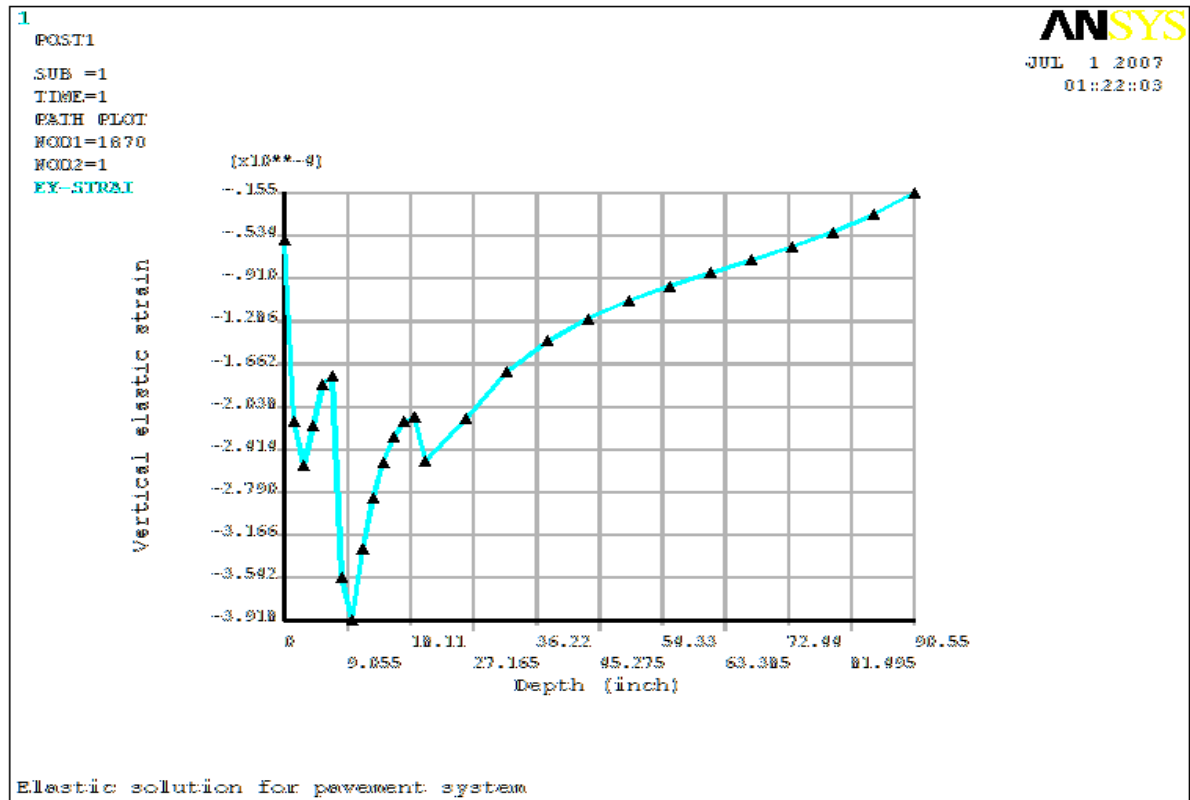


Figure (3-12), Solution of Vertical Elastic Strain Varies with Depth at Center Line of Tire.

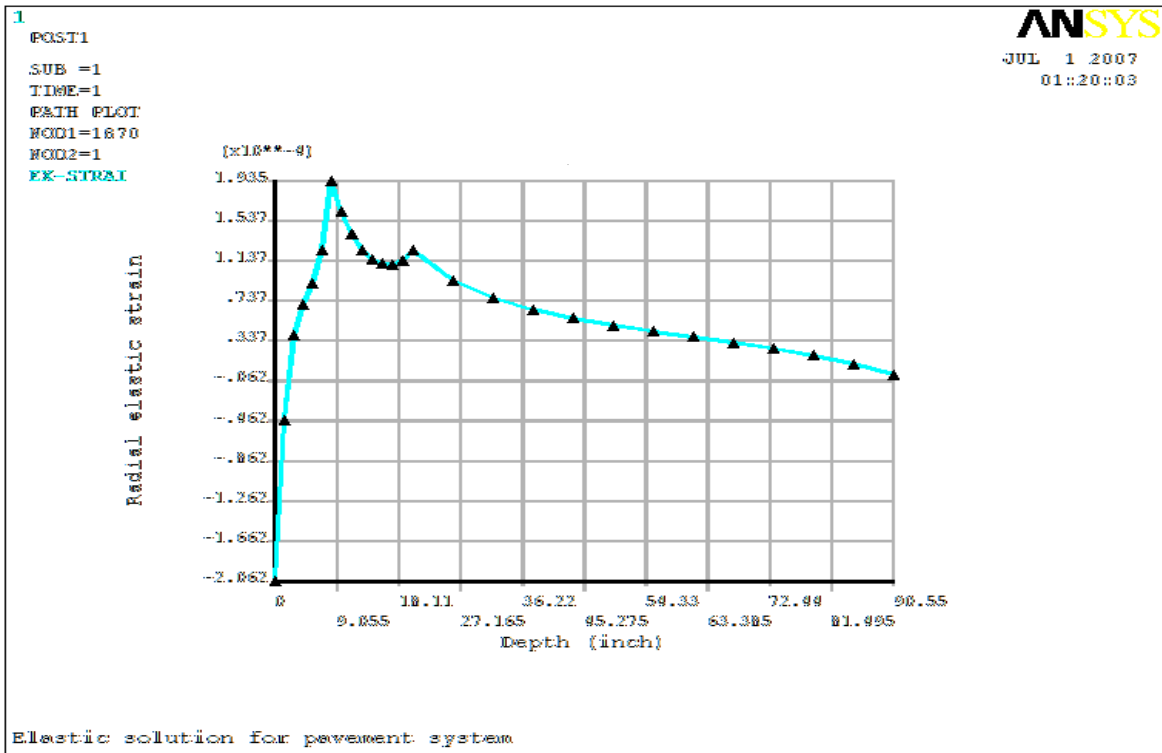


Figure (3-13), Solution of Radial Elastic Strain Varies with Depth at Center Line of Tire.

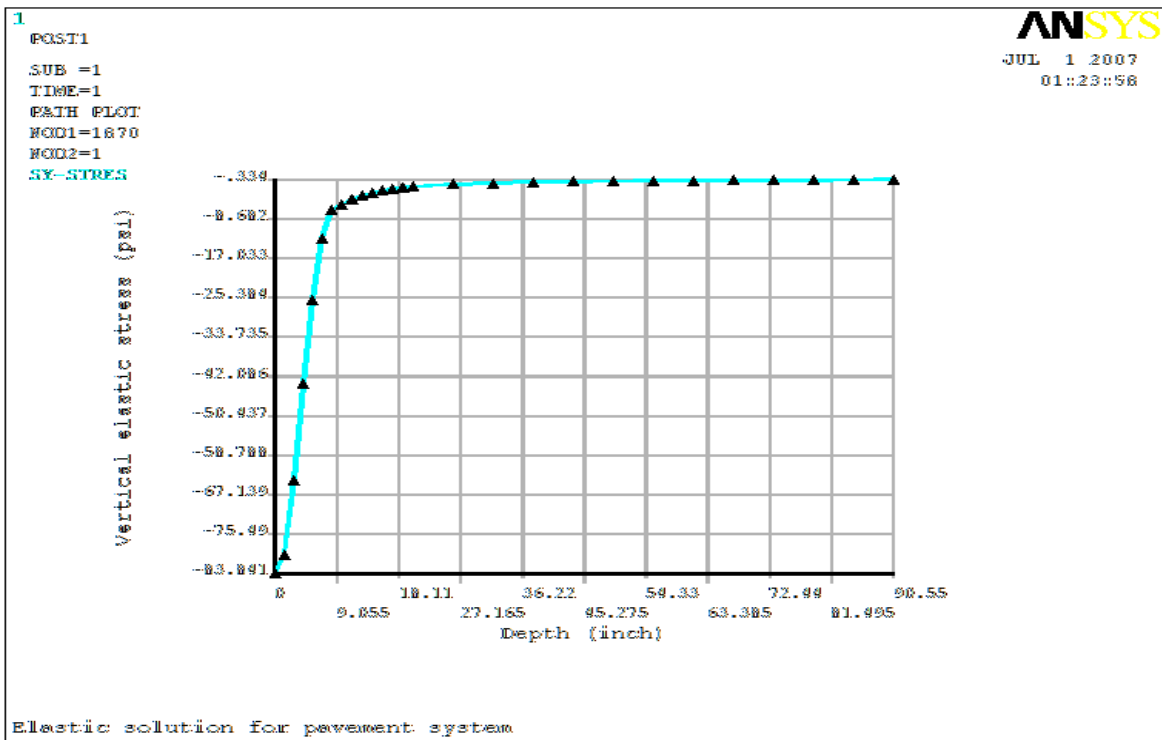


Figure (3-14), Solution of Vertical Elastic Stress Varies with Depth at Center Line of Tire.

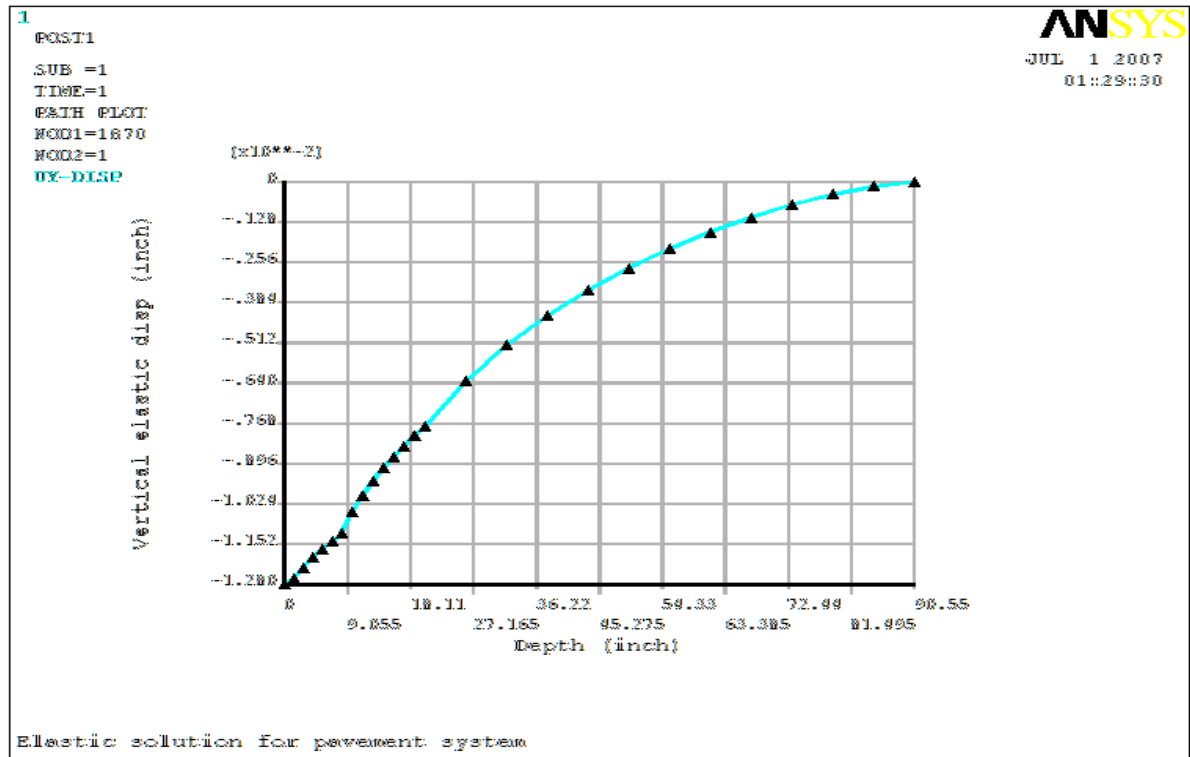


Figure (3-15), Solution of Vertical Elastic Displacement Varies with Depth at Center Line of Tire.

Table (3-3), Solution of Elastic Responses Vary with Depth under Center Line of Tire, Bold Value Refer to Critical Point.

Coordinates system (inch)	YG-depth (inch)	ϵ_y -strain	ϵ_x -strain	σ_y -stress psi	U_y -displacement inch
90.55	0	-5.64E-05	-2.06E-04	-83.841	-1.28E-02
89.183	1.3667	-2.17E-04	-4.51E-05	-79.817	-1.26E-02
87.817	2.7333	-2.55E-04	3.94E-05	-64.119	-1.23E-02
86.45	4.1	-2.21E-04	7.06E-05	-43.67	-1.20E-02
85.083	5.4667	-1.84E-04	9.14E-05	-25.84	-1.17E-02
85.083	5.4667	-1.84E-04	9.14E-05	-25.84	-1.17E-02
85.083	5.4667	-1.84E-04	9.14E-05	-25.84	-1.17E-02
83.717	6.8333	-1.76E-04	1.25E-04	-12.713	-1.14E-02
82.35	8.2	-3.54E-04	1.94E-04	-6.7565	-1.12E-02
80.85	9.7	-3.92E-04	1.62E-04	-5.6784	-1.05E-02
79.35	11.2	-3.30E-04	1.40E-04	-4.6289	-9.98E-03
77.85	12.7	-2.85E-04	1.25E-04	-3.8233	-9.52E-03
76.35	14.2	-2.52E-04	1.15E-04	-3.1898	-9.12E-03
74.85	15.7	-2.30E-04	1.10E-04	-2.6854	-8.76E-03
73.35	17.2	-2.17E-04	1.10E-04	-2.2861	-8.42E-03

71.85	18.7	-2.13E-04	1.14E-04	-1.9818	-8.10E-03
70.35	20.2	-2.51E-04	1.24E-04	-1.7722	-7.77E-03
64.487	26.062	-2.14E-04	9.39E-05	-1.3249	-6.32E-03
58.625	31.925	-1.74E-04	7.66E-05	-1.0597	-5.18E-03
52.762	37.788	-1.46E-04	6.49E-05	-0.88188	-4.25E-03
46.9	43.65	-1.26E-04	5.62E-05	-0.75573	-3.45E-03
41.038	49.512	-1.11E-04	4.92E-05	-0.6628	-2.75E-03
35.175	55.375	-9.77E-05	4.32E-05	-0.59195	-2.14E-03
29.313	61.237	-8.60E-05	3.77E-05	-0.53617	-1.60E-03
23.45	67.1	-7.48E-05	3.21E-05	-0.49054	-1.13E-03
17.587	72.962	-6.31E-05	2.60E-05	-0.4512	-7.28E-04
11.725	78.825	-5.01E-05	1.91E-05	-0.41467	-3.96E-04
5.8625	84.687	-3.47E-05	1.08E-05	-0.37727	-1.48E-04
0	90.55	-1.55E-05	2.19E-07	-0.33421	0

Table (3-4), General Elastic Solution Components at Mid Depth of Asphalt Layer (Critical Point Lays at, [4.1 inch] under Tire Contact).

Type of solution	Stress (psi)	Vertical elastic strain	Displacement (inch)	Radial elastic strain
Finite element model	43.67	-2.21E-04 ¹	-0.012 ¹	7.06E-05
Multilayer elastic model(KENLAYR)	43.892	2.191E-04	0.01450	6.804E-05

The solution of multilayer elastic model (MLET) was done by (KENLAYER) software as shown in Figure (3-16):

INPUT FILE NAME -C:\KENPAVE\KenElasticywao.DAT

NUMBER OF PROBLEMS TO BE SOLVED = 1

TITLE -elastic solution

MATL = 1 FOR LINEAR ELASTIC LAYERED SYSTEM

NDAMA = 0, SO DAMAGE ANALYSIS WILL NOT BE PERFORMED

NUMBER OF PERIODS PER YEAR (NPY) = 1

NUMBER OF LOAD GROUPS (NLG) = 1

TOLERANCE FOR INTEGRATION (DEL) -- = 0.001

NUMBER OF LAYERS (NL)----- = 3

NUMBER OF Z COORDINATES (NZ)----- = 1

LIMIT OF INTEGRATION CYCLES (ICL)- = 80

COMPUTING CODE (NSTD)----- = 9

SYSTEM OF UNITS (NUNIT)-----= 0

Length and displacement in in., stress and modulus in psi
unit weight in pcf, and temperature in F

THICKNESSES OF LAYERS (TH) ARE : 8.2 12

POISSON'S RATIOS OF LAYERS (PR) ARE : 0.35 0.4 0.45

VERTICAL COORDINATES OF POINTS (ZC) ARE: 4.1

ALL INTERFACES ARE FULLY BONDED

FOR PERIOD NO. 1 LAYER NO. AND MODULUS ARE : 1 1.876E+05 2 1.510E+04
 3 5.800E+03

LOAD GROUP NO. 1 HAS 1 CONTACT AREA

CONTACT RADIUS (CR)----- = 4.05

CONTACT PRESSURE (CP)----- = 83

RADIAL COORDINATES OF 1 POINT(S) (RC) ARE : 0

PERIOD NO. 1 LOAD GROUP NO. 1

RADIAL COORDINATE	VERTICAL COORDINATE	VERTICAL DISPLACEMENT	VERTICAL STRESS (STRAIN)	RADIAL STRESS (STRAIN)	TANGENTIAL STRESS (STRAIN)	SHEAR STRESS (STRAIN)
0.00000	4.10000	0.01450	43.892	3.998	3.998	0.000
(STRAIN)			2.191E-04	-6.804E-05	-6.804E-05	.000E+00

Figure (3-16), Solution of Elastic Response by Multilayer Theory

(KENLAYER).

3.5.3 Viscoelastic Solution.

Hot-mix asphalt is a viscoelastic material that is characterized by a certain level of rigidity of an elastic solid, but at the same time, flows and dissipates energy by frictional losses as a viscous fluid. At high temperatures or under slow moving loads, HMA may exhibit a purely viscous flow, which results in load-associated distresses such as pavement rutting. However, at low service temperatures or under rapidly applied loading, HMA becomes progressively harder and eventually brittle, which makes it vulnerable to non-load-associated distresses such as low-temperature cracking. When a vehicle travels on top of a HMA pavement layer, three different types of strain develop in the material: elastic recoverable strain, and viscous recoverable and irrecoverable strain. Elastic strain is associated with the ability of HMA to respond to the applied load and to recover instantaneously the energy released during deformation. On the other hand, viscous strain involves the time-dependent deformation of HMA. Only part of the viscous strain is recoverable and time-dependent. In general, the total strain can be expressed based on the following relation ([ERES Consultants, 2002](#)):

$$\epsilon_T = \epsilon_e + \epsilon_{ve} + \epsilon_{vp} \quad \dots (3-4)$$

where: ϵ_T =total strain;

ϵ_e =elastic strain;

ϵ_{ve} =viscoelastic strain;

ϵ_{vp} =viscoplastic strain.

Mathematical modeling of HMA response to loading demands the prediction of strain accumulation under variable loading time and temperature. Viscous deformation depends on three parameters, which are the stress level, time, and temperature. To describe the isotropic viscoelastic behavior of HMA, a

generalized Kelvin model, which consists of a spring and (K) Voigt elements connected in series, was selected in this study. In the case of a viscoelastic solid, the creep compliance at time t of a generalized Kelvin model is given as follows (Park and Schapery, 1999):

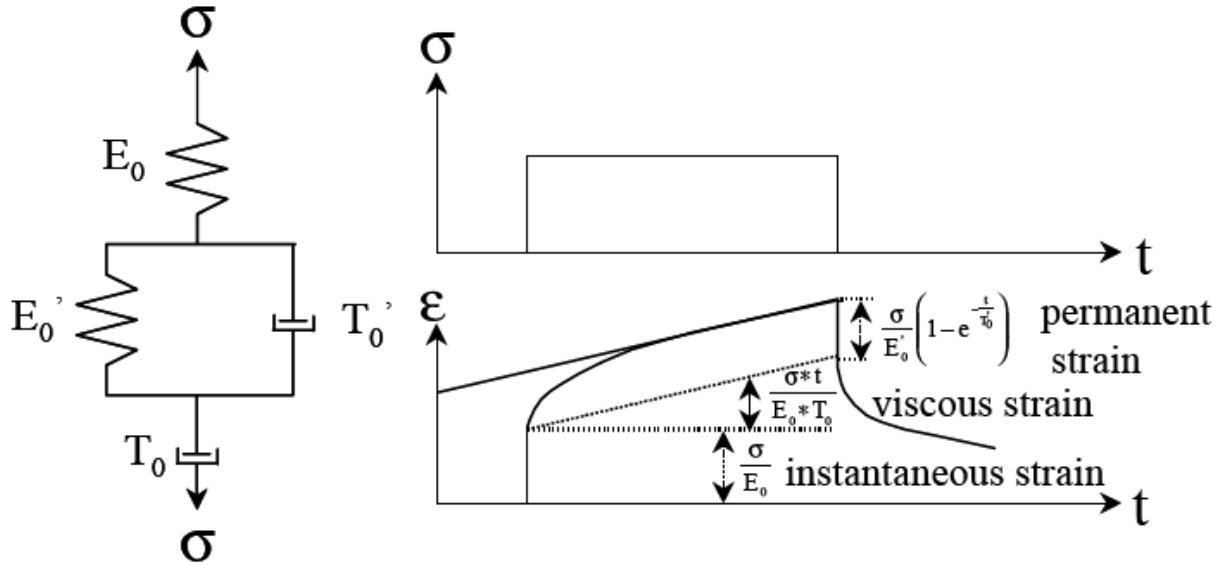


Figure (3-17), Mechanical Model, and Response to Stress of Viscoelastic Material.

here

- σ : stress level (constant).
- E_0 : elastic modulus for Maxwell's model.
- E_0' : elastic modulus for Kelvin's model.
- T_0 : relaxation time for Maxwell's model.
- T_0' : retardation time for Kelvin's model.

$$D(t) = D_0 + \sum_{i=1}^k D_i (1 - e^{-t/\tau_i}) \quad \dots\dots\dots (3-5)$$

Where $D(t)$ =creep compliance at time t ; D_i =material constants; and τ_i =retardation times. Eq. (3-4) is also known as a Prony or a Dirichlet series expansion. Since it was simpler in this study to measure the creep compliance of HMA, it was used as the base of interconversion to other viscoelastic functions that were used to model the material time-dependent properties. For instance, the fitted creep compliance was used to predict the relaxation modulus, $E(t)$.The

interconversion process was based on the approximation developed by (Kim et al., 2002). Assuming that both the creep compliance and the relaxation modulus may be described by a power law model, the relationship between the two material properties was defined as follows:

$$E(t).D(t) = \frac{\sin(n.\pi)}{n.\pi} \quad \dots\dots\dots(3-6)$$

Where n =positive constant, obtained by fitting a localized power law model $D(t)=[D_1.t^n]$ to the different regions of behavior. Similarly, a Prony series expansion was fitted to the relaxation modulus variation with time. Then, the bulk $K(t)$ and shear $G(t)$ moduli variation with time were estimated using the following relations assuming that Poisson's ratio does not change with time, assuming that the material is isotropic and homogeneous and that resulting strain is small (Elseifi A. Mostafa 1; Al-Qadi L. Imad, F.ASCE2; and Yoo Jun Pyeong 3, 2006):

$$K(t) = \frac{E(t)}{3(1-2\nu)} \quad \dots\dots\dots(3-7); \quad G(t) = \frac{E(t)}{2(1+\nu)} \quad \dots\dots\dots(3-8)$$

The simulating has done by using ANSYS version 9.0; a material is viscoelastic if its stress response consists of an elastic part and viscous part. Upon application of a load, the elastic response is instantaneous while the viscous part occurs over time. Generally, the stress function of a viscoelastic material is given in an integral form within the context of small strain theory, the constitutive equation for an isotropic viscoelastic material can be written as:

$$\sigma = \int_0^t 2G(t-\tau) \cdot \frac{de}{dt} \cdot dt + I \int_0^t K(t-\tau) \cdot \frac{d\Delta}{dt} \cdot dt \quad \dots\dots\dots(3-9).$$

where:

- σ = Cauchy stress
- e = deviatoric part of the strain
- Δ = volumetric part of the strain
- $G(t)$ = shear relaxation kernel function
- $K(t)$ = bulk relaxation kernel function
- t = current time
- τ = past time
- I = unit tensor.

For the element type (plan-183) that use in this study the kernel functions are represented in terms of Prony series, which assumes that:

$$G = G_{\infty} + \sum_{i=1}^{n^G} G_i \cdot \exp\left(-\frac{t}{\tau_i}\right) \quad \dots\dots\dots (3-10)$$

$$K = K_{\infty} + \sum_{i=1}^{n^K} K_i \cdot \exp\left(-\frac{t}{\tau_i}\right) \quad \dots\dots\dots (3-11)$$

In the above, G_{∞} ; G_i are shear elastic moduli, K_{∞} ; K_i are bulk elastic moduli τ_i^G ; τ_i^K are the relaxation times for each Prony component. Introducing the relative moduli as following:

$$\alpha_i^G = \frac{G_i}{G_o} \quad \dots\dots\dots (3-12); \quad \alpha_i^K = \frac{K_i}{K_o} \quad \dots\dots\dots (3-13)$$

where:

$$G_o = G_{\infty} + \sum_{i=1}^{n^G} G_i, \quad K_o = K_{\infty} + \sum_{i=1}^{n^K} K_i$$

The kernel functions can be equivalently expressed as:

$$G = G_o \left[\alpha_{\infty}^G + \sum_{i=1}^{n^G} \alpha_i \cdot \exp\left(-\frac{t}{\tau_i}\right) \right], \quad \dots\dots\dots (3-14)$$

$$K = K_o[\alpha_{\infty}^K + \sum_{i=1}^{n^K} \alpha_i \cdot \exp(-\frac{t}{\tau_i})] \quad \dots\dots (3-15).$$

Data input in [ANSYS version 9.0](#) platform of nonlinear material are exhibit in [Table \(3-6\)](#), while the same geometric model, load configuration, boundary condition present in elastic solution is use except asphalt material will be treat as a viscoelastic material and the analysis is change to nonlinear solution.

Table (3-5), Viscoelastic Material Properties for Asphalt Layer.

Time of loading	Creep compliance (1/psi)	Relaxation modulus E(t), (psi)	shear modulus* G(t),(psi)	bulk modulus* K(t),(psi)
0.1	0.00003025	30250.83586	11204.01328	33612.03985
0.25	0.000039875	22948.90996	8499.596283	25498.78885
0.5	0.000053375	17144.50182	6349.81549	19049.44647
1	0.000058125	15743.44576	5830.905837	17492.71751
2	0.000066625	13734.90109	5087.000402	15261.00121
4	0.0000755	12120.36801	4489.025189	13467.07557
8	0.0000835	10959.13515	4058.938944	12176.81683
15	0.000108	8473.035045	3138.161128	9414.483383
30	0.00012525	7306.090098	2705.959296	8117.877887
45	0.000134875	6784.710175	2512.85562	7538.566861
60	0.00013775	6643.105516	2460.40945	7381.228351

An efficient method of fitting Prony-series models to viscoelastic experimental data with power law pre-smoothing is presented. A direct fitting of a Prony-

37_____

* Obtains this column based on equation (3-7).

♦ Obtains this column based on equation (3-6).

series function to experimental data without appropriate pre-smoothing is difficult when the data have significant variance. A power-law series comprising multiple power law terms is found capable of portraying a globally smooth, broadband viscoelastic behavior with minimal impact from local variance in the data (Park, S. W., and Kim, Y. R., 2001). However, from a computational point of view, a Prony series representation is preferred to a power-law series representation because of the computational efficiency associated with the exponential basis functions of a Prony series, a procedure involving pre-smoothing of experimental data via power-law-series representation followed by fitting of a Prony-series model to the pre-smoothed data is illustrated. Illustration focuses on how experimental data with significant variance can be effectively pre-smoothed through power-law series representation and how pre-smoothing improves the quality of the subsequent Prony series fit.

The feasibility and effectiveness of the approach are demonstrated through numerical examples in which Prony-series representations of the shear modulus and bulk modulus of asphalt concrete are determined from available experimental data. It should be note that power law for creep compliance varies with time is $D(t) = [6E-05(\text{time})^{0.2302}]^{\wedge}$, as shown in Figure (3-20) , then $n=0.2302$ from the power law ,by applying the Equation (3-5) this led to :

$$E(t).D(t) = \frac{\sin(0.2302.\pi)}{0.2302.\pi} = 0.915087785,$$

This relationship between creep compliance and relaxation modulus is used to obtain column (3) in Table (3-6). The fitting of shear modulus, bulk modulus vary with time were done by ANSYS version 9.0, given a good parameters convergences with a low mean residual of error equal to (0.000107), Prony series parameters fitting for both shear; bulk modulus are shown in Table (3-7),

ANSYS platforms result in Figure (3-20), the decay of shear, bulk with time are present in Figure (3-20).

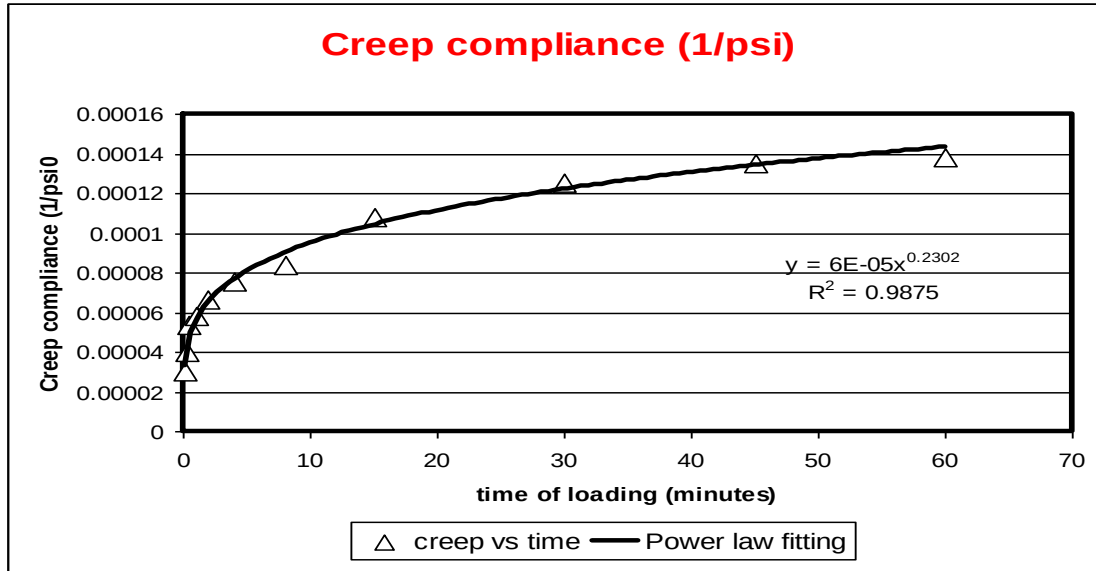


Figure (3-18), Creep Compliance Varies with Time and Power Law Fitting.

Table (3-6), Prony Series Parameters for both Shear; Bulk Modulus Decay with Time

Prony parameters for shear decays with time	Value	Prony parameters for bulk decays with time	Value
$\alpha 1$	0.097952	$\alpha 1$	0.11251
$\tau 1$	9.2719	$\tau 1$	5.9762
$\alpha 2$	0.10067	$\alpha 2$	0.1835
$\tau 2$	12.933	$\tau 2$	0.15972
$\alpha 3$	0.077162	$\alpha 3$	0.21599
$\tau 3$	1.0341	$\tau 3$	0.15962
$\alpha 4$	0.29116	$\alpha 4$	0.20815
$\tau 4$	0.17229	$\tau 4$	0.31042
$\alpha 5$	0.27135	$\alpha 5$	0.11667
$\tau 5$	0.17229	$\tau 5$	15.136

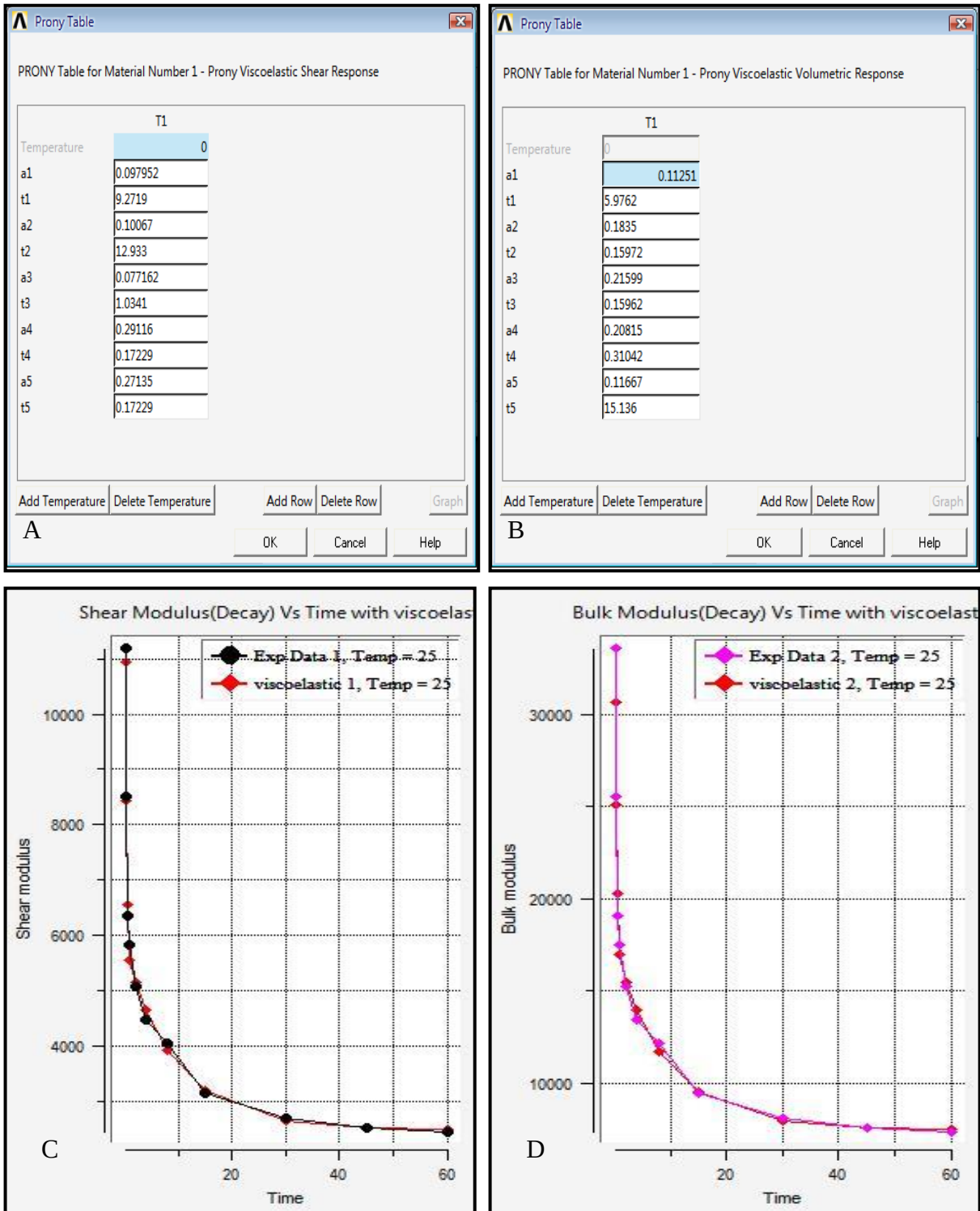


Figure* (3-20), [A, B] are Platforms of Viscoelastic Parameters Output, [C; D] are Data & Prony Series Function of Shear; Bulk Modulus Decay with Time.

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* Platforms were obtain from ANSYS 9.0 by capturing pictures

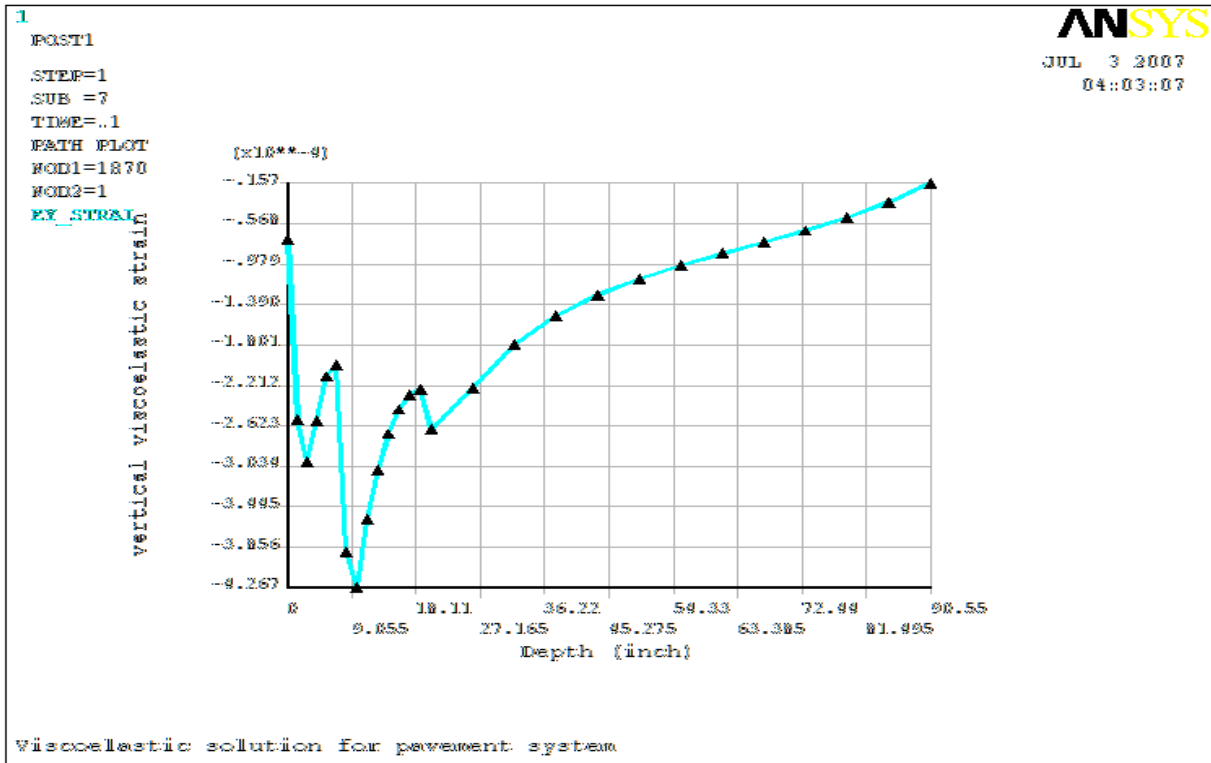


Figure (3-21), Solution of vertical viscoelastic strain varies with depth at center line of tire at time (0.1 sec).

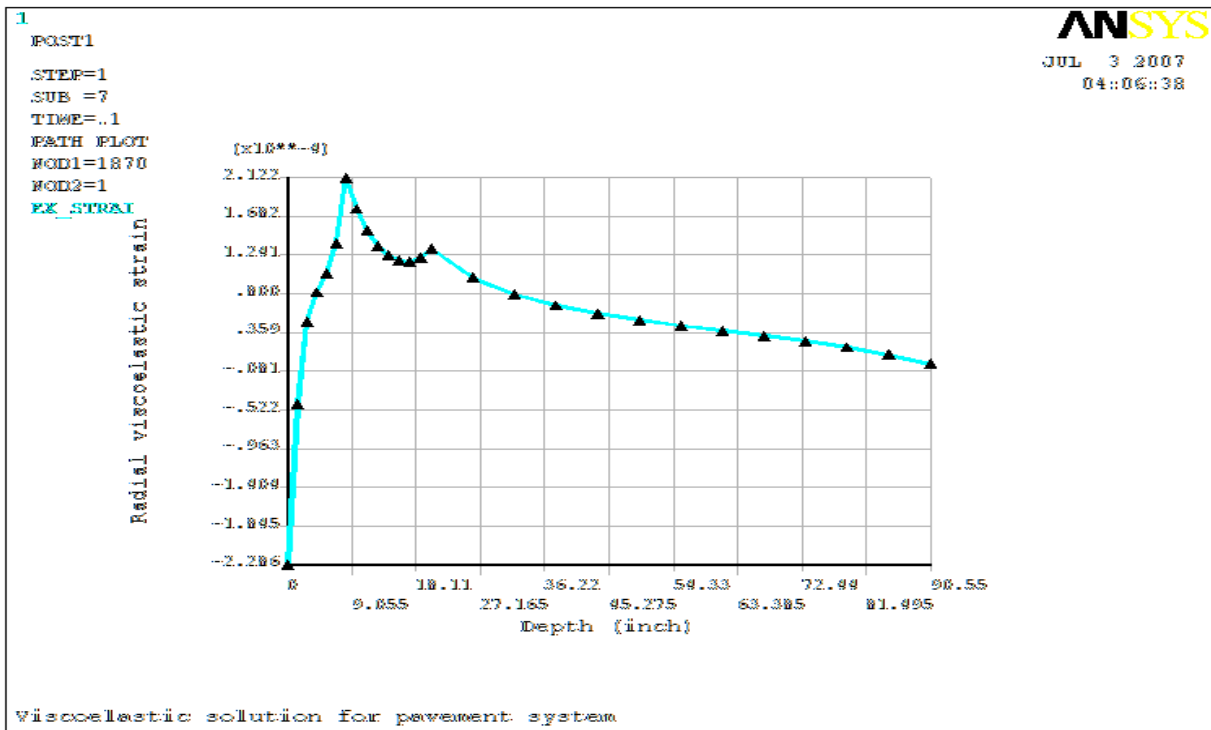


Figure (3-22), Solution of Radial Viscoelastic Strain Varies with Depth at Center Line of Tire at Time (0.1 sec).

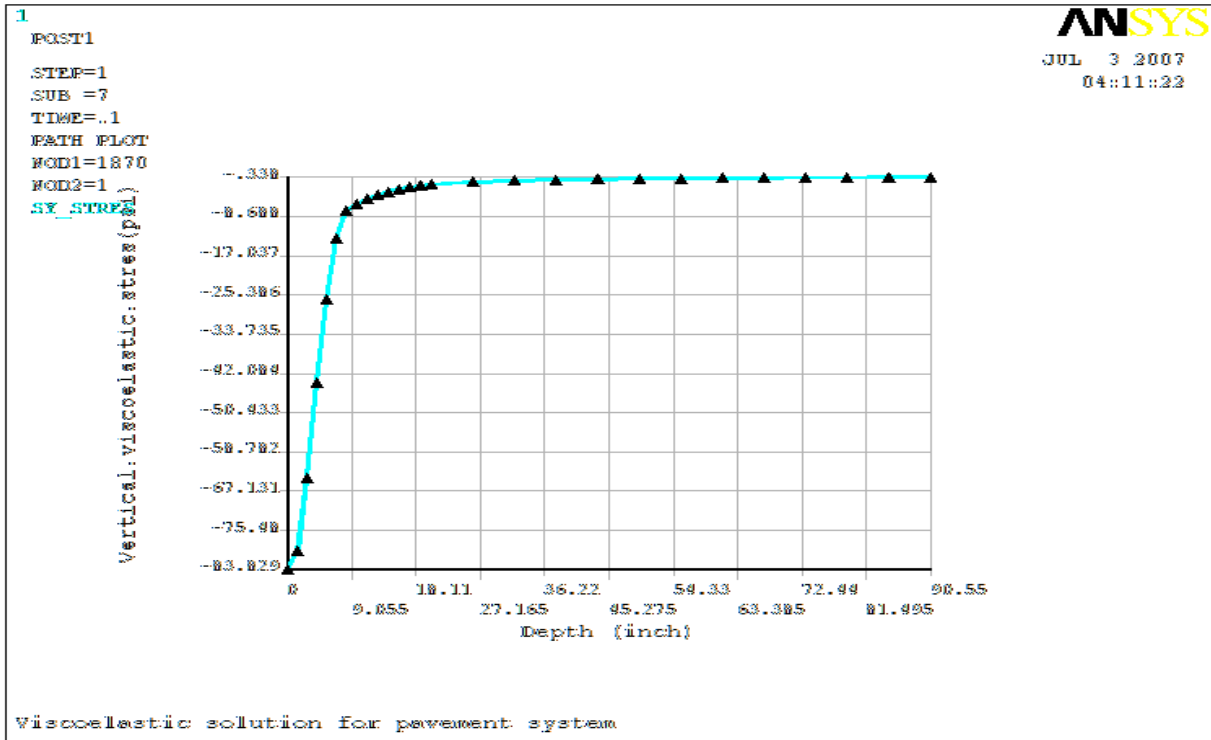


Figure (3-23), Solution of Vertical Viscoelastic Stress Varies with Depth at Center Line of Tire at Time (0.1 sec).

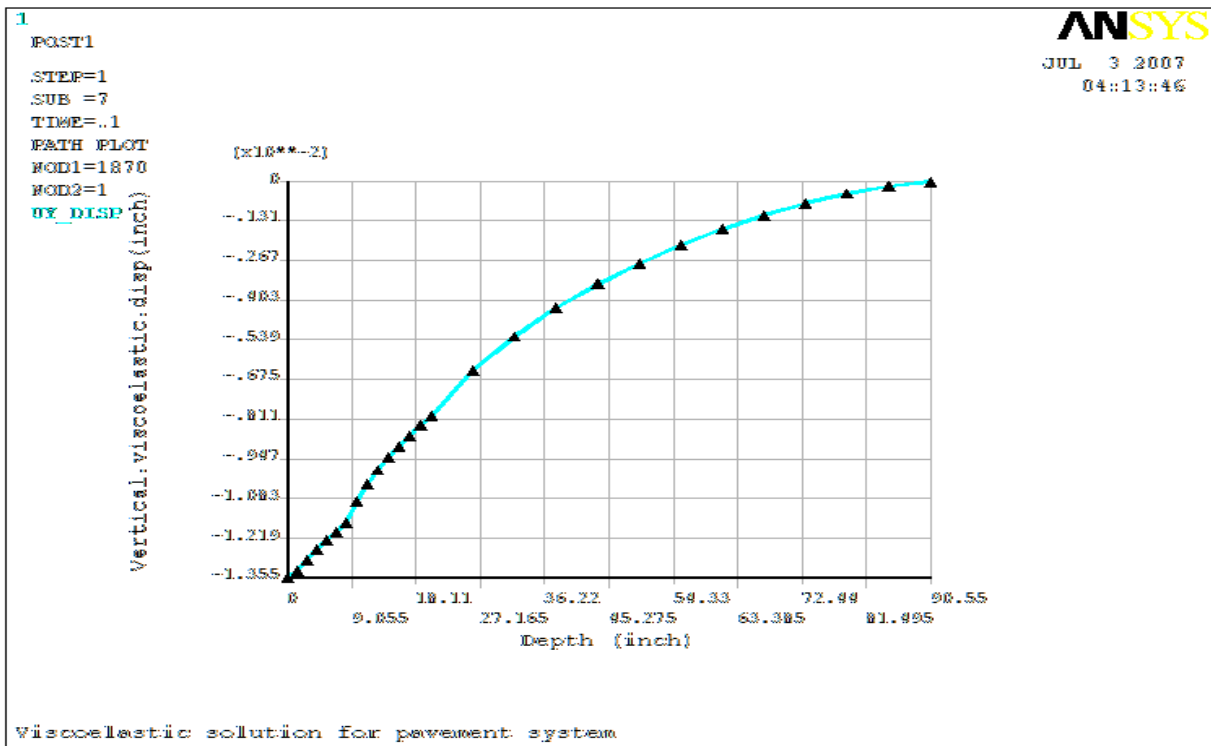


Figure (3-24), Solution of Vertical Viscoelastic Displacement Varies with Depth at Center Line of Tire at Time (0.1 sec).

Table (3-7), Solution of Viscoelastic Responses Vary with Depth under Center Line of Tire, Bold Value Refer to Critical Point.

Coordinates system (inche)	YG-depth (inche)	ϵ_y -strain	ϵ_x -strain	σ_y -stress	Uy-displecment
90.55	0	-7.34E-05	-2.29E-04	-83.829	-1.36E-02
89.183	1.3667	-2.56E-04	-4.59E-05	-79.87	-1.33E-02
87.817	2.7333	-2.99E-04	4.86E-05	-64.324	-1.30E-02
86.45	4.1	-2.57E-04	8.18E-05	-44.065	-1.26E-02
85.083	5.4667	-2.12E-04	1.03E-04	-26.424	-1.23E-02
83.717	6.8333	-2.00E-04	1.38E-04	-13.429	-1.20E-02
82.35	8.2	-3.90E-04	2.12E-04	-7.4265	-1.17E-02
80.85	9.7	-4.27E-04	1.77E-04	-6.1734	-1.10E-02
79.35	11.2	-3.57E-04	1.52E-04	-5.0077	-1.04E-02
77.85	12.7	-3.07E-04	1.35E-04	-4.1159	-9.87E-03
76.35	14.2	-2.70E-04	1.24E-04	-3.4167	-9.43E-03
74.85	15.7	-2.45E-04	1.18E-04	-2.8613	-9.05E-03
73.35	17.2	-2.31E-04	1.17E-04	-2.4228	-8.69E-03
71.85	18.7	-2.26E-04	1.21E-04	-2.0893	-8.35E-03
70.35	20.2	-2.66E-04	1.31E-04	-1.8604	-8.00E-03
64.487	26.062	-2.24E-04	9.84E-05	-1.3768	-6.48E-03
58.625	31.925	-1.80E-04	7.95E-05	-1.0934	-5.29E-03
52.762	37.788	-1.51E-04	6.68E-05	-0.90491	-4.33E-03
46.9	43.65	-1.29E-04	5.76E-05	-0.77218	-3.51E-03
41.038	49.512	-1.13E-04	5.03E-05	-0.67503	-2.80E-03
35.175	55.375	-9.94E-05	4.40E-05	-0.60139	-2.17E-03
29.313	61.237	-8.74E-05	3.83E-05	-0.54374	-1.63E-03
23.45	67.1	-7.58E-05	3.25E-05	-0.49682	-1.15E-03
17.587	72.962	-6.39E-05	2.64E-05	-0.45659	-7.37E-04
11.725	78.825	-5.07E-05	1.94E-05	-0.4194	-4.01E-04
5.8625	84.687	-3.52E-05	1.09E-05	-0.38143	-1.49E-04
0	90.55	-1.57E-05	2.22E-07	-0.33778	0

Figure (3-25) show the deformation shape while Table (3-8), Figures (3-26) through (3-29) show the different in response due to analysis type.

Table (3-8), General Response Solution Components at Mid Depth of Asphalt Layer (Critical Point Lays at, [4.1 inch] under Tire Contact).

Type of solution	Vertical stress (psi)	Vertical strain	Displacement (inch)	Radial strain
Elastic finite element model	43.67	2.21E-04	0.012	7.06E-05
Multilayer elastic model(KENLAYR)	43.892	2.191E-04	0.01450	6.804E-05
Viscoelastic finite element model at time (0.1 sec)	44.065	2.57E-04	0.0126	8.18E-05

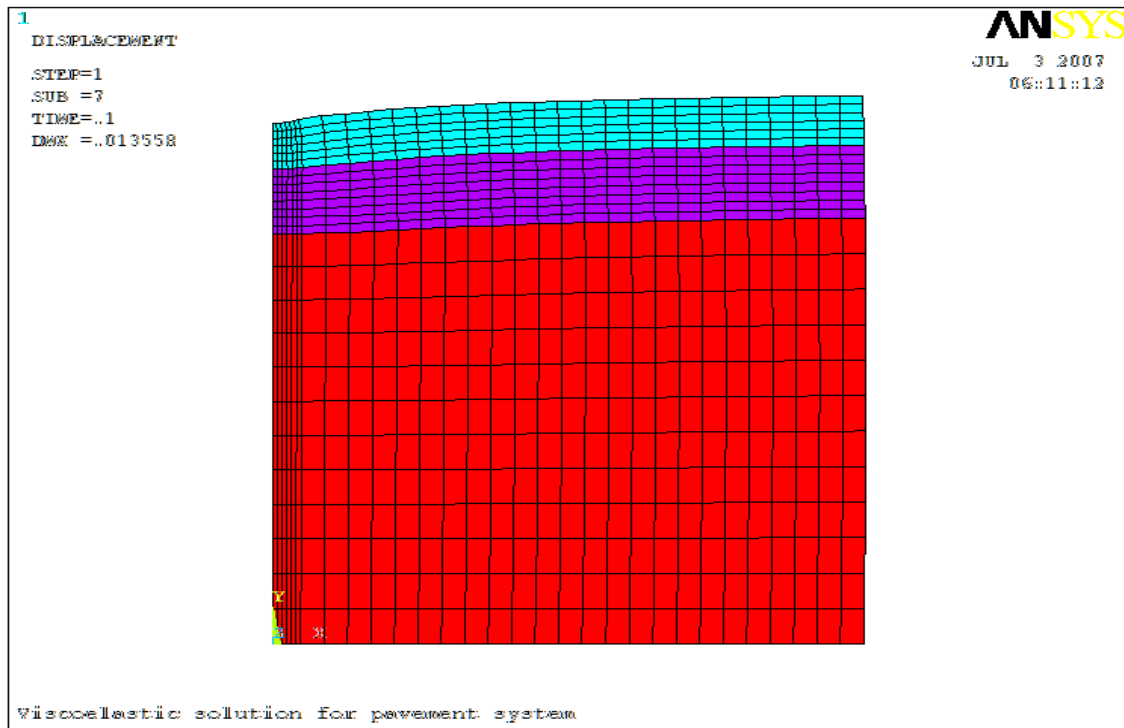


Figure (3-25), Deformation Shape and Mesh Generation for Viscoelastic Finite Element Model.

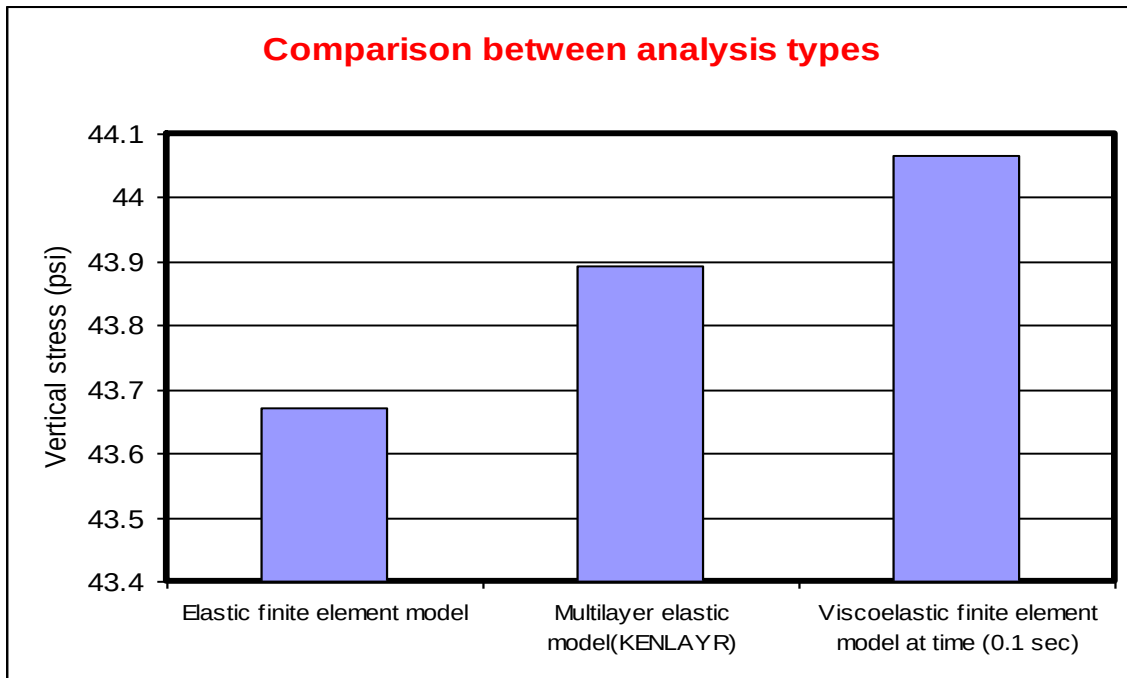


Figure (3-26), Comparison of Vertical Stress at Critical Point Varies with Analysis Types.

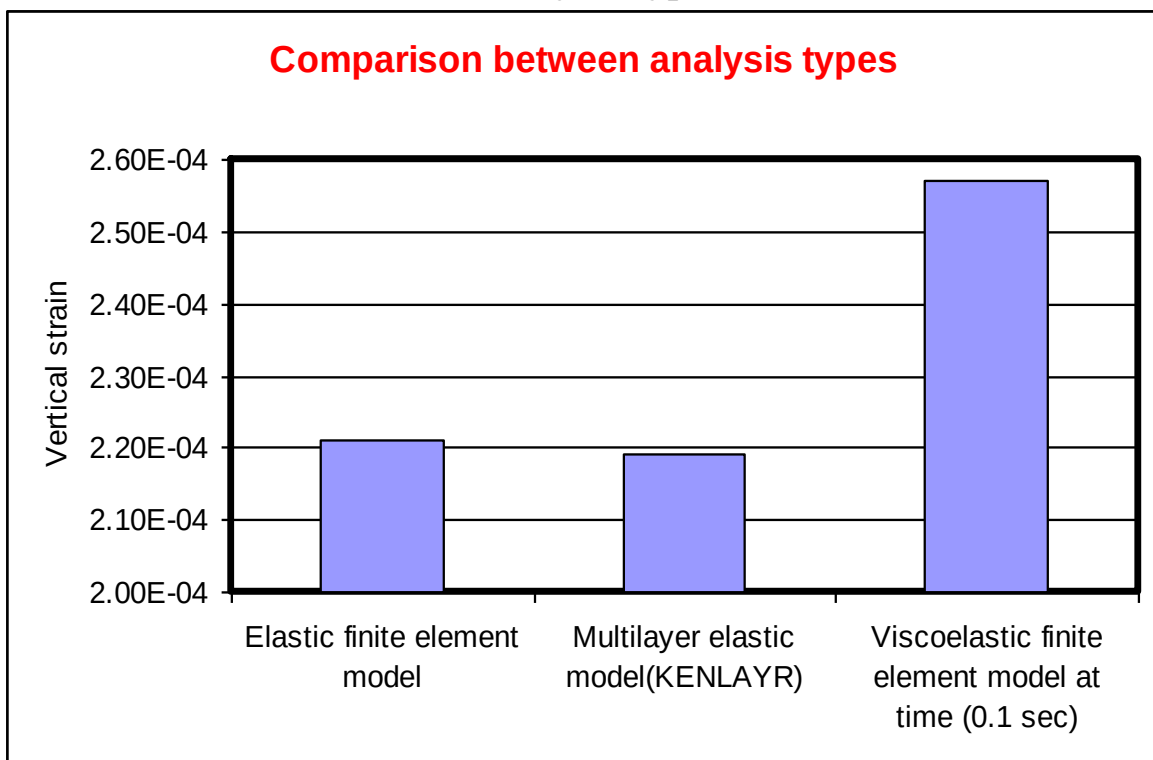


Figure (3-27), Comparison of Vertical Strain at Critical Point Varies with Analysis Types.

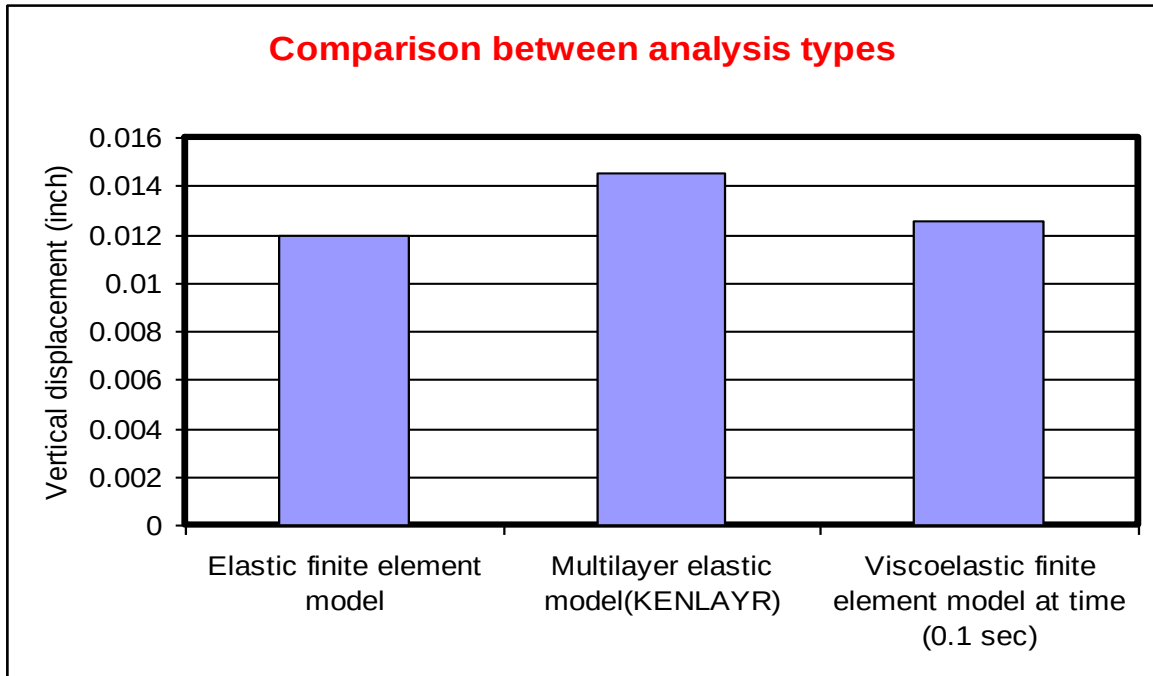


Figure (3-28), Comparison of Vertical Displacement at Critical Point Varies with Analysis Types.

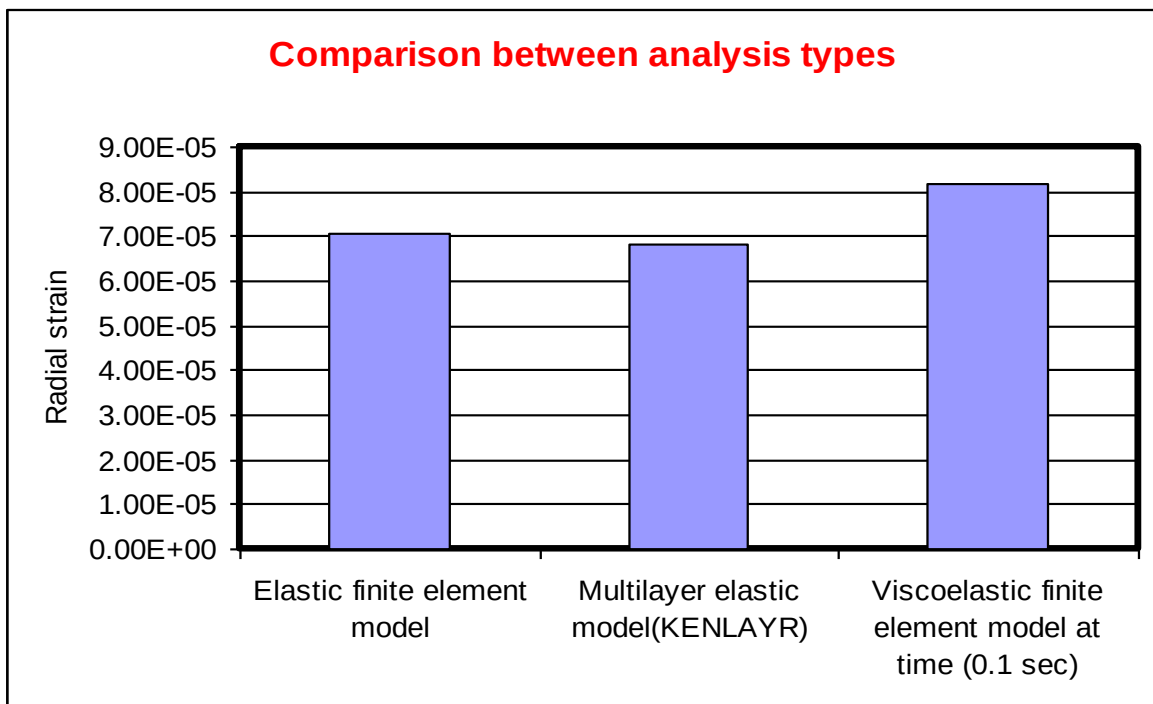


Figure (3-29), Comparison of Radial Strain at Critical Point Varies with Analysis Types.

CHAPTER FOUR

MODELING OF PERMANENT STRAIN OF ASPHALT PAVEMENT

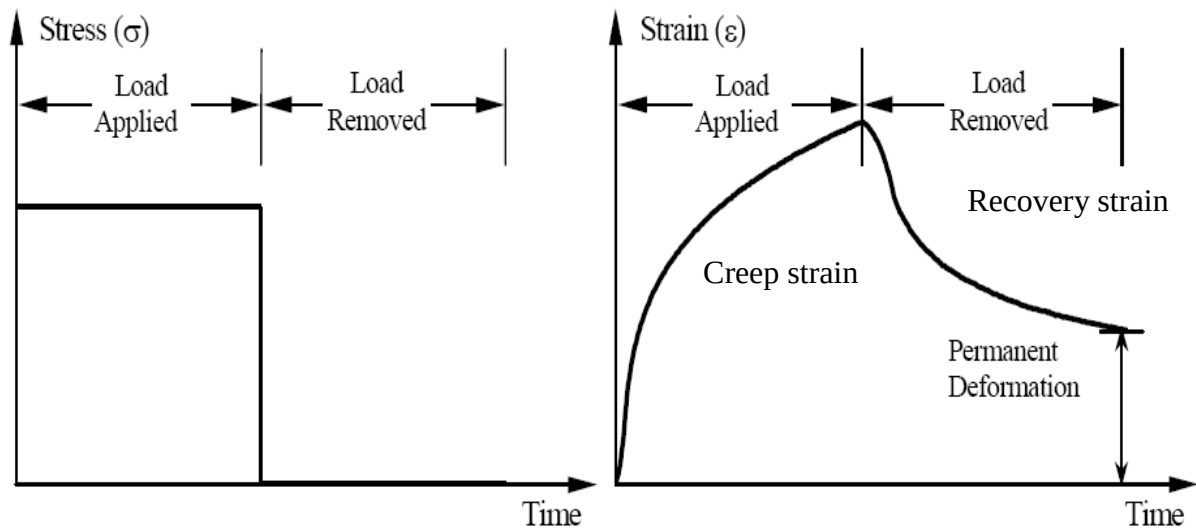
4. Introduction

The adequate and accurate characterization of asphalt mixture behavior is vital if pavement performance is to be predicted realistically. The ideal material characterization model should be able to predict the response of asphalt concrete over the full range of temperatures and stress states of interest in pavement systems. Asphalt concrete behavior may vary from elastic and linear viscoelastic at low temperatures (and/ or) fast loading rates to nonlinear visco-elastic-plastic at high temperatures and or slow loading rates.

In general, asphalt concrete deformation is composed of four strain components, elastic, plastic, viscoelastic and viscoplastic. The magnitude of the strain components depends on temperature, loading and stress state, [Figure \(4.1\)](#) presents the strain components in repeated loading after ([Uzen, 1996](#)). Work, the modeling approach adopted is to model each response component Separately, That is to develop a viscoelasto-plastic model, depending on [VESYS Model \(Kenis, W. J., 1977\)](#). The separation of the viscoelasto-plastic strain from the total strain is accomplished by conducting creep and recovery test.

4.1 Viscoelasto-plastic Model

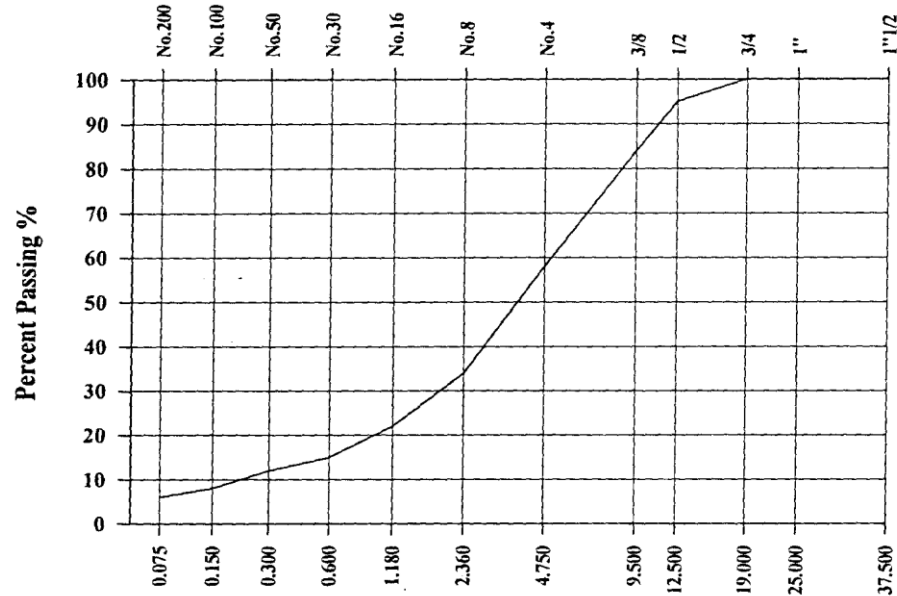
As mentioned, modeling the viscoelasto-plastic behavior of asphalt concrete mixtures calls for the separation of total strain into viscoelastic and plastic components. This can be accomplished by conducting creep and recovery compliance test. Efforts need to be made to determine proper data for simulating material by viscoelasto-plastic model. Figure (4.2) show the typical creep and recovery for creep and recovery testing. One asphalt concrete mixture is selected, in which it represent the best behavior under the conducted previous tests.



Figure(4-1): Typical Stress and Strain for Creep and Recovery test

This mixture is consisted from crushed aggregate with gradation below the restricted zone and asphalt binder from Al-Daurah refinery with (40-50) grade (AC-20). The optimum asphalt content is 4.9%. It is clear from Figure (4-3) that the Gradations of the selected asphalt mixture, approximately meet the mid point of the Specification limits (ASTM-D3515) sieve (No.8) in which it passes below

the restricted zone with fewer fines this gradation represents mix **B1[^]**. Also, represents the typical gradation for the surface layer in Iraq ([Yassoub, 2004](#)).



Sieve Size	Percent Passing Below the Restricted Zone	Specification Limit ASTMD(3515)
3/4	100	100
1/2	95	90-100
3/8	84	77-93
No.4	58	44-74
No.8	34	29-58
No.16	22	21-46
No.30	15	12-33
No.50	12	5-21
No.100	8	4-15
No.200	6	3-10

Figure (4-2): Selected Combined Gradation of Aggregate and Mineral Filler for Mixture B1 ([Yassoub, 2004](#))

[^] Appendix (A) offers all mixtures gradations that used in this study.

4.1.1 Theoretical Formulation.

Plastic strain is assumed to be linear log-log relationship with number of load repetitions as suggested by (Monismith, 1988) and (Barksdale, 1972) Equation (4-3)

$$\varepsilon_p = a.N^b \quad \dots (4-3)$$

Where

ε_p = permanent strain (plastic strain).

N = number of stress applications.

a,b= constants.

Figure (4-4) and (4-5) show this relationship on semi log and log –log plots, respectively. Using this equation as a base relationship, two coefficients may be developed to describe a material's permanent deformation potential under repeated loads (Rauhut, 1980). Differentiation of Equation (4-3) yields the following:

$$\frac{d\varepsilon_p}{dN} = \varepsilon_p(N) = abN^{b-1} \quad \dots (4-4)$$

Assuming the resilient strain, ε_r , is independent of the number of load repetitions and dividing equation (4-4) by ε_r one obtains:

$$\frac{\varepsilon_p(N)}{\varepsilon_r} = \left[\frac{abN^{b-1}}{\varepsilon_r} \right] \quad \dots (4-5)$$

For convenience, arbitrary definitions have been made for mathematical simplification:

$$\mu = \frac{ab}{\varepsilon_r} \quad \dots (4-6)$$

$$\alpha = 1-b \quad \dots (4-7)$$

By substitution, Equation (4-5) may be rewritten as:

$$\frac{\varepsilon_p(N)}{\varepsilon_r} = \mu N^{-\alpha} \quad \dots (4-8)$$

One of the main models related to this mechanistic-empirical approach is the **VESYS** rutting model that relates the plastic strain to the resilient strain or viscoelastic strain through the permanent deformation parameters (**PDPs**) μ and α (**Kenis, W. J., 1977**) as follows:

$$\varepsilon_p (N) = \frac{\partial \varepsilon_p(N)}{\partial N} = \varepsilon_r \cdot \mu \cdot N^{-\alpha} \quad \dots(4-9)$$

where:

ε_r = resilience strain.

N = number of stress applications.

α, μ = define in Equation (4-6), (4-7).

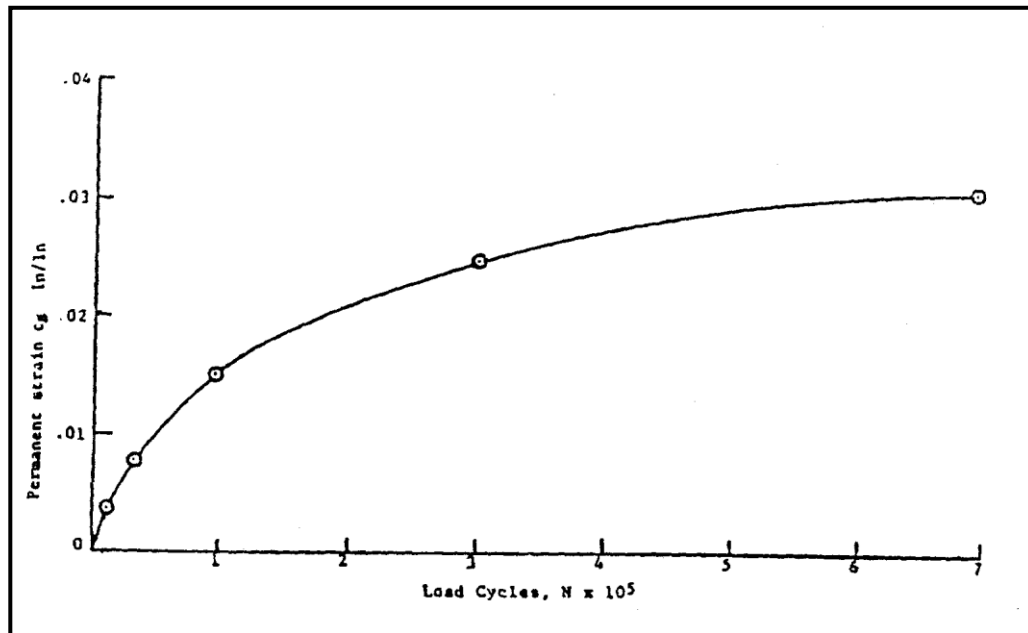


Figure (4-3) Typical Test Results for Repetitive Load Test on Asphalt Concrete (Rauhut et. al., 1976**)**

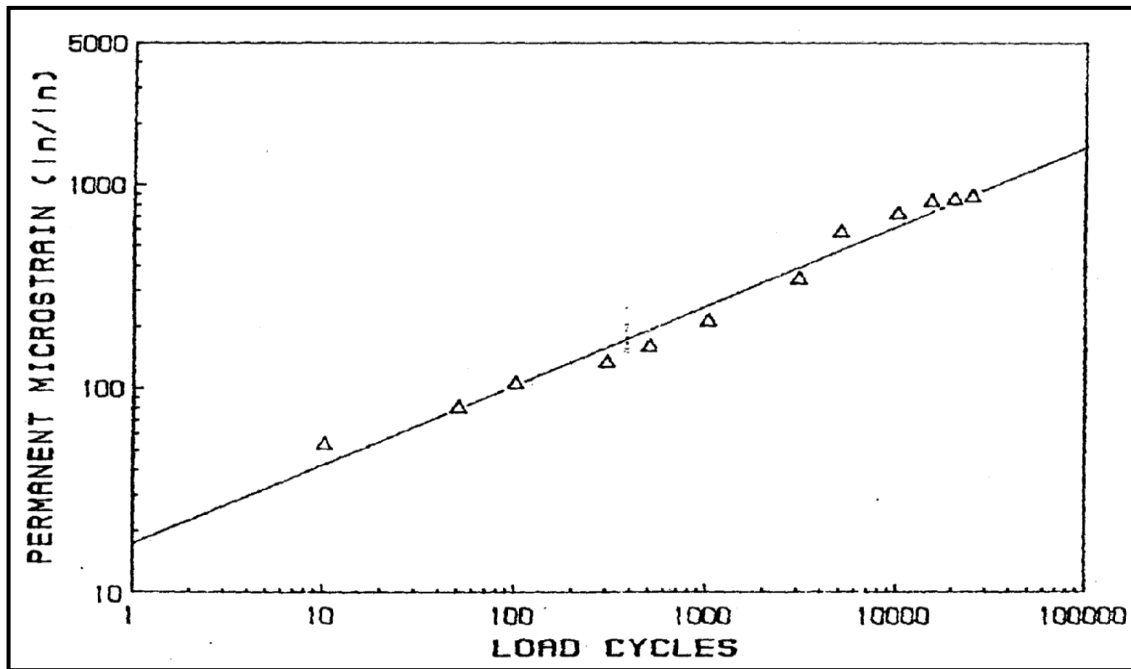


Figure (4-4) Typical Test Results for Repetitive Load Test on Asphalt Concrete (Rauhut et. al., 1976)

4.1.2 Development Of Rutting model parameters α and μ

A novel theoretical approach, founded on the creep and recovery behavior of asphalt concrete mixtures, was developed into a rutting model. The resulting hyperbolic equations accurately model the deformation performance of a paving mixture.

It has been found that both creep and recovery compliance behavior of an asphalt mixture follow curves similar to hyperbolic equation (Perdomo, Button and Lytton, 1992) for predicting the creep compliance behavior of an asphalt concrete mixture can be derived .

$$D(t) = \frac{D_o + D_m * A * t^m}{1 + A * t^m} \quad \dots\dots\dots (4-10)$$

where

D_o = initial creep compliance,

D_m = maximum creep compliance,

A = regression constant

t = time, and

m = slope factor.

The same philosophical approach can be applied to the recovery compliance, even though it shows a different slope factor and a partial strain recovery. The equation obtained for this recovery compliance is represented as follows:

$$R(t) = \frac{R_o + R_m * B * t^{mp}}{1 + B * t^{mp}} \quad \dots\dots\dots (4-11)$$

where

R_o = initial recovery compliance,

R_m = maximum recovery compliance,

B = regression constant,

t = time, and

mp = slope modifier factor.

In order to obtain all the unknown parameters in Equation (4-10) (D_o , D_m , A , m) and (4-11) (R_o , R_m , B , mp), an optimization technique known as pattern search was used. Pattern search is a technique based on an optimization method named (Trust-Region). This technique can solve difficult nonlinear problems more efficiently than the other algorithms and it represents an improvement over the popular Levenberg-Marquardt algorithm (MATLAB Version 6.5.0, 2000). This algorithm was developed by (Branch, 1999). The method consists, basically, of finding the unknown parameters that minimize the sum of the squared differences between observed data and predicted values. Six asphalt

concrete mixtures containing different aggregate gradation; asphalt type and asphalt percent were used in this study based on (Yassoub, 2004) laboratory test data, in order to obtain α and μ volumetric properties for each mixture are denoted in Table (4-1).

Table1 (4-1), Volumetric Properties and Type of Asphalt Cement for Each Mixture

Mixture Type	Ac-grade	Ac %	Av%	VMA%	VFA%	Visco	EFF %	ABS %
A1	40-50	5.2	4	14.25	76	8.014	10.26	0.062
T1	40-50	4.9	4	14.57	76	8.014	10.24	0.060
B1	40-50	4.9	4	14.56	75	8.014	10.23	0.059
A2	85-100	5.1	4	16.8	79	2.909	11.78	0.059
T2	85-100	5.2	4	16.81	80.8	2.909	11.76	0.057
B2	85-100	5.1	4	17.67	82	2.909	11.75	0.057

And the resilience modulus for each mixture have denoted in Table (4-2)

Table (4-2), Resilience Modulus for Each Mixture in (psi)

Mix. type	A1	T1	B1	A2	T2	B2
Resilience modulus in (psi)	182,890	186,080	187,580	168,860	172,270	172,540

¹ All mixtures information have been taken form the data of laboratory test from (Yassoub, 2004) all symbols of this table are define as following:

Ac-grade: Asphalt grade (40-50) or (85-100) that used.

Ac%: Selected optimum Asphalt content.

Av%: Air void in total mixture.

VMA%: Void in mineral aggregate.

VFA%: Void filled with asphalt.

Visco: Viscosity in (poise*10⁶) at 20 C⁰ for bitumen.

ABS: Percent absorbed asphalt (by weight of aggregate).

EFF: Percent effective asphalt (by volume of mixture).

A permanent deformation model properties α , μ are derived from Equation (4-10), (4-11) suppose that a load pulse of duration Δt is applied repeatedly, N times. Then, compliance equations for both creep and recovery may be used to estimate the total deformation and recovery that occurs (Perdomo, Button and Lytton, 1992):

$$D(N) = \frac{D0 + Dm.r.N^m}{1 + r.N^m} \quad \dots\dots\dots (4-12)$$

$$R(N) = \frac{R0 + Rm.rp.N^{mp}}{1 + rp.N^{mp}} \quad \dots\dots\dots (4-13)$$

where:

N = number of cycles, $D(N)$ = creep compliance (1/psi).
 $r = A^*(\Delta t)^m$, $R(N)$ = recovery compliance (1/psi).
 $rp = B^*(\Delta t)^{mp}$, $t = N(\Delta t)$.

The total accumulated strain after N repetitions for a constant stress pulse, σ_0 , is defined as:

$$\varepsilon_p(N) = \varepsilon_{\text{creep}}(N) - \varepsilon_{\text{re}}(N) = \sigma_0 [D(N) - R(N)] \quad \dots\dots(4-14)$$

where:

$\varepsilon_p(N)$ =creep strain at N of repetition.
 $\varepsilon_{\text{re}}(N)$ =recover strain at N of repetition.
 σ_0 =applied constant stress pulse.

The rate of change of the permanent strain with load repetitions is:

$$\frac{\partial \varepsilon_p(N)}{\partial N} = \sigma_0 * \left[\frac{\partial D(N)}{\partial N} - \frac{\partial R(N)}{\partial N} \right] \quad \dots\dots\dots (4-15)$$

Replacing Equation (4-12), (4-13) in Equation (4-15) and drive with respect N yield:

$$\begin{aligned} \frac{\partial \varepsilon_p(N)}{\partial N} = & \frac{Dm.r.N^m.m}{N(1+r.N^m)} - \frac{(Do + Dm.r.N^m).r.N^m.m}{(1+r.N^m)^2.N} \\ & - \frac{Rm.rp.N^{mp}mp}{N(1+rp.N^{mp})} + \frac{(R0 + Rm.rp.N^{mp})rp.N^{mp}mp}{(1+rp.N^{mp})^2.N} \end{aligned} \quad \dots (4-16)$$

By dividing both side by resilient strain (ε_r), given

$$\begin{aligned} \frac{1}{\varepsilon_r} \frac{\partial \varepsilon_p(N)}{\partial N} = & Mr * N^{(m-1)} * \left[\frac{Dm.r.m}{(1+r.N^m)} - \frac{(Do + Dm.r.N^m).r.m}{(1+r.N^m)^2} \right. \\ & \left. - \frac{Rm.rp.N^{p(m-1)}mp}{N^{(m)}(1+rp.N^{mp})} + \frac{(R0 + Rm.rp.N^{mp})rp.N^{p(m-1)}mp}{(1+rp.N^{mp})^2} \right] \end{aligned} \quad \dots (4-17)$$

where:

Mr =resilient modulus.

Now, recalling the permanent strain response model used in the [VESYS](#) approach [Equation (4-4)]

$$\frac{\partial \varepsilon_p(N)}{\partial N} = \varepsilon_r. \mu. N^{-\alpha}$$

Thus, if

$$\mu * N^{-\alpha} = \frac{1}{\varepsilon_r} \frac{\partial \varepsilon_p(N)}{\partial N}$$

Then, it is apparent from equation (4-17) that

$$\alpha = 1 - m \quad \dots\dots (4-18)$$

$$\begin{aligned} \mu = Mr * \left[\frac{Dm.r.m}{(1+r.N^m)} - \frac{(Do + Dm.r.N^m).r.m}{(1+r.N^m)^2} - \frac{Rm.rp.N^{m(p-1)}mp}{N^{(m)}(1+rp.N^{mp})} \right. \\ \left. + \frac{(R0 + Rm.rp.N^{mp})rp.N^{m(p-1)}mp}{(1+rp.N^{mp})^2} \right] \end{aligned} \quad \dots\dots (4-19)$$

where:

$$p = \frac{mp}{m} \quad \text{..... (4-20)}$$

The mixture B1 show a good resistance to rutting (Yassoub,2004), and its clearly form Table (4-2) that it has a large modulus of resilience ,which mean offering low response according to Hooke's law shown in Equation (4-1). Mixture (B1) has fitted below and used to predicting rutting in an example.

Table (4-3), Creep and Recovery Compliance in (1/psi) for Mix B1 under Temperature 25 C °.

Loading for creep unloading for recovery(sec)	Creep compliance (1/psi)	Recovery compliance(1/psi)
0.1	0.00003025	0.000023375
0.25	0.000039875	0.00002525
0.5	0.000053375	0.000026375
1	0.000058125	0.00002875
2	0.000066625	0.00003275
4	0.0000755	0.0000355
8	0.0000835	0.000035
15	0.000108	0.000036625
30	0.00012525	0.000036625
45	0.000134875	0.000036625
60	0.00013775	0.000036625

Table (4-4), Information Input Limitations of Nonlinear Regression Method For Each Mixture.

Type of procedure :	least square method
Algorithm :	Trust – Region method
Different min change :	1.0E-12
Different max change :	1.0E-14
Max function evaluating :	1.0E09
Max iteration :	1.0E09
Tolerance of function :	1.0E-20
Tolerance of x value :	1.0E-20

Table (4-5), Estimating Parameters of Hyperbolic Model for Mix B1.

A	0.148	B	2.124
Do	1.362e-005	Ro	9.057e-006
Dm	0.0003413	Rm	3.978e-005
m	0.355	mp	0.434
SSE:	1.4481e-010	SSE:	1.105e-011
R-square:	0.9901	R-square:	0.99
Adjusted R-square:	0.9876	Adjusted R-square:	0.9845
RMSE:	4.2546e-006	RMSE:	1.1753e-006
Dfe:	8	Dfe:	8

where:

- SSE=Sum of squares due to error.
- Rsquare=Coefficient of determination.
- Dfe=Degrees of freedom.
- Adjrsquare=Degree-of-freedom adjusted coefficient of determination.
- RMSE=Root mean squared error (standard error).
- MaxFunEvals -- Maximum number of function (model) evaluations allowed. The default value is 600.
- MaxIter -- Maximum number of fit iterations allowed. The default value is 400.
- TolFun -- Termination tolerance used on stopping conditions involving the function (model) value. The default value is 10-6.
- TolX -- Termination tolerance used on stopping conditions involving the coefficients. The default value is 10-6.

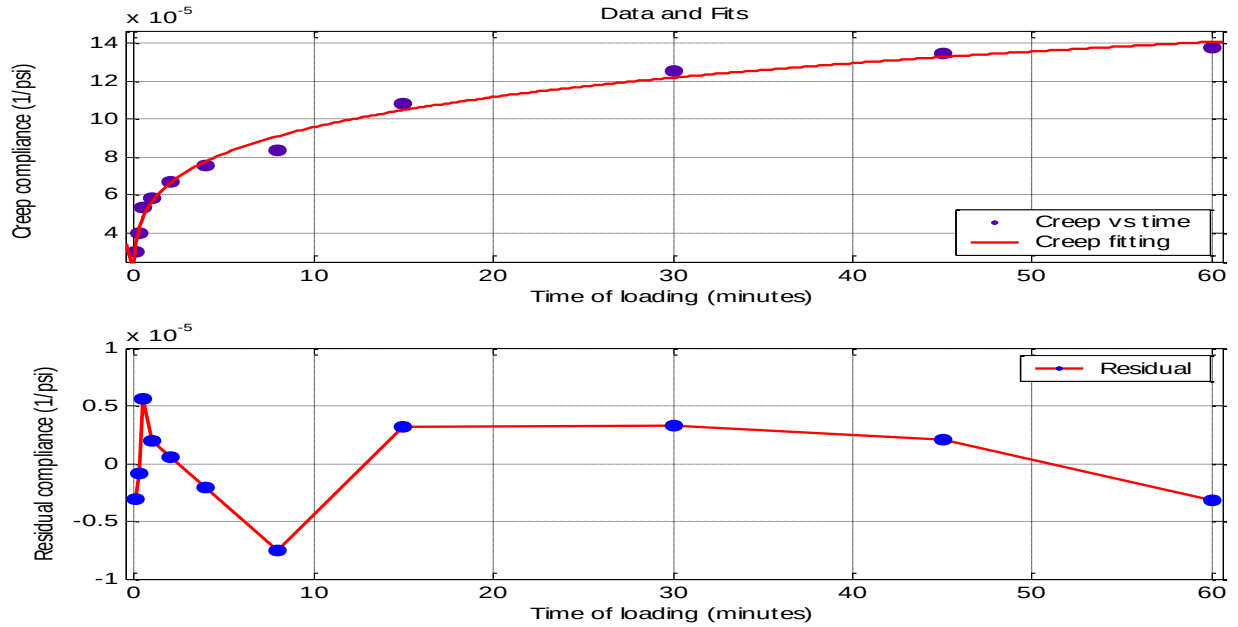


Figure (4-5) Creep Compliance Fitting and Residual of Compliance at Temperature = 25 C°.

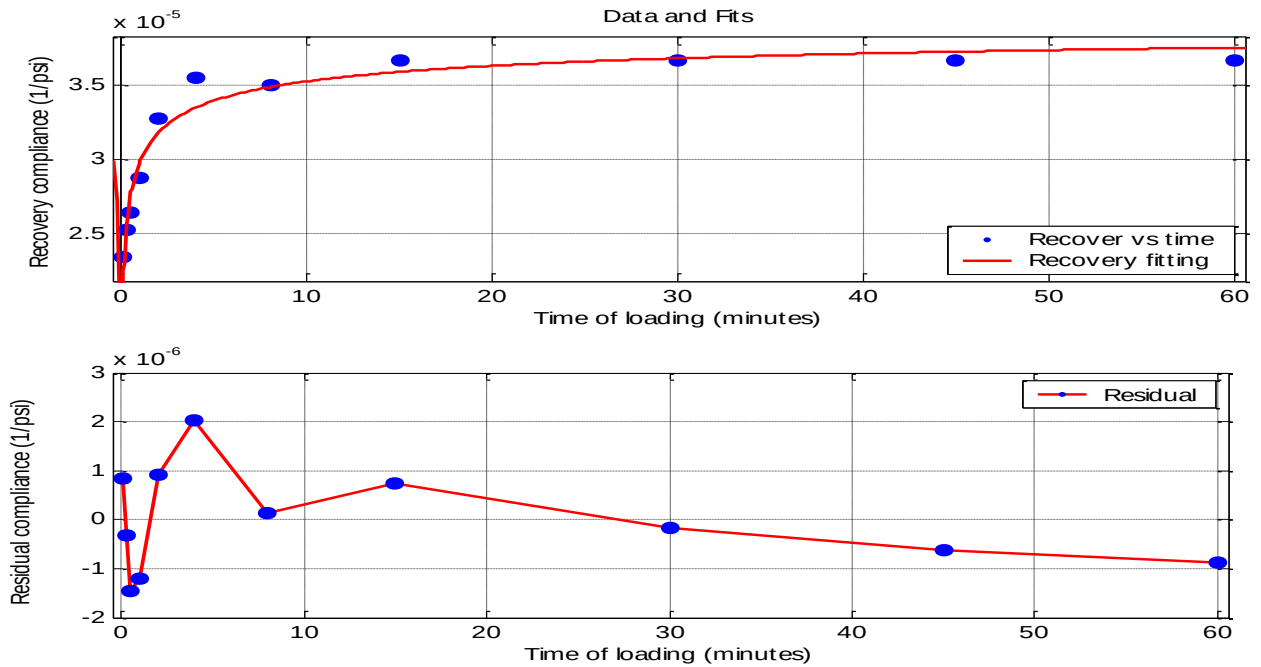


Figure (4-6) Recovery Compliance Fitting and Residual of Compliance at Temperature = 25 C°.

The results of the remaining mixtures have presented in Table (A-2) through (A-11), Figure (A-2) through (A-11) in Appendix (A).

The creep test is a simplified test and requires much less time to perform than the dynamic test. With a dynamic loading time of 0.1-s duration, a static load of t seconds is equivalent to 10t load repetitions. Therefore ,the permanent deformation after a 1000-s creep test is equivalent to that of a dynamic test after 10000, so if the number of load applications is N, then the creep time in seconds corresponding to N is (0.1*N) because the duration of each load application is $\Delta t=0.1$ -sec (Yang H. Huang ,1993). Based on this criteria it should be used ($N=(60*120)/\Delta t$), this led to use ($N=72,000$), for simulating plastic parameters (α, μ).

For verification purpose an investigation has done in order to computing α, μ for each mixture via a model developed by (Albayati,2006) based on statistical analysis for different asphalt material properties and under wide range of surrounding circumstances such as (temperature ,stress level ...etc) as following:

To compute alpha (α)

$$\alpha = 1.391 - 0.011 * TMP - 0.00541 * STR - 0.252 * Dur - 0.0212 * EFF \quad \dots (4-21)$$

To compute mu (μ).

$$\begin{aligned} \mu = & 4.1896559 + 0.02 * TMP - 0.13091 * VFA - 0.00019 * TMP^2 + 0.000312 * STR^2 - \\ & 0.04305 * AV^2 - 0.00112 * STR * VIS + 0.031432 * STR * Dur + 0.570402 * Dur * AV - \\ & 127.263 * Dur * ABS + 0.511025 * Dur * EFF - \\ & 0.08536 * AV * EFF + 9.964481 * VMA * ABS. \quad \dots (4-22) \end{aligned}$$

where:

- Av%: Air void in total mixture.
- VMA%: Void in mineral aggregate.
- VFA%: Void filled with asphalt.
- Visco: Viscosity in (poise*10⁶) at 20 C⁰ for bitumen.
- ABS: Percent absorbed asphalt (by weight of aggregate).
- EFF: Percent effective asphalt (by volume of mixture).
- Dur: duration of loading in (second).
- STR: applied stress in (psi) for specimen.
- TMP: Temperature in centigrade.

Table (4-6), Plastic Parameters for Each Mixture.

Mixture type	Alpha (α) by Albayati model*	Mu (μ) by Albayati model*	P [^] value	Alpha (α) by proposed model	Mu (μ) by proposed model
A1	0.6407	0.4814	1.800	0.6500	0.4561
T1	0.6411	0.5428	1.7549	0.6410	0.4119
B1	0.6376	0.53	1.2225	0.6450	0.3327
A2	0.6106	0.420	1.6221	0.6110	0.4392
T2	0.6028	0.4571	1.5869	0.6030	0.3535
B2	0.6034	0.452	1.5909	0.6040	0.3421

The [Table \(4-6\)](#) refers to a good agreement for values of alpha (α) but fairly for mu (μ) because of the low Coefficient of determination in ([Albayati, 2006](#)) model ($R=0.749$) make it not refer to exact solution, It should be note that the differences in the two procedure which led to different values is also affect on computing values of α, μ . It is clearly that (P) value in Table (4-6) good indicator

* The regression of Equation (4-21) and (4-22) based on Coefficient of determination is equal to $R1=0.958$, 0.749 for more information see ([Albayati, 2006](#)) at Table (6-7) page (165).

* Where (p) define by [Equation \(4-20\)](#).

to refers that mix B1 has a small plastic parameter μ (μ) also this indicator and modulus of resilience refer together to rutting depth as will see in the "rut depth prediction in asphalt concrete" Viscoelastic-plastic could be applied to creep and recovery compliance for different degree of temperature as shown in Table (4-7).

Table (4-7), Plastic Parameters for Mix B1 under Different Temperatures.

plastic parameters	Mix B1 under 25 c ⁰	Mix B1 under 40 c ⁰	Mix B1 under 50 c ⁰
Alpha (α)	0.6450	0.4770	0.3680
Mu (μ)	0.3327	0.0379	0.0184
P value	1.2225	0.7008	0.7850

Alpha (α) value is affected by creep and recovery compliance data only, while Mu (μ) value is affected by (creep, recovery compliance) and modulus of resilience, modulus of resilience is one of major factors which affecting on Mu (μ) calculation because it appears in Mu (μ) Equation (4-19) and modulus of resilience (M_r) decreases with increases the temperature. This led to lower value of Mu if temperature increasing [Figure \(4-7\)](#) shows the relationship between the modulus of resilience and temperature. The simulating data and viscoelasto-plastic parameters for [Table \(4-7\)](#) have present in Appendix (A).

Alpha (α) is a permanent (plastic) deformation parameter indicating the rate of decrease in permanent deformation as the number of load applications increases, while mu (μ) is a permanent (plastic) deformation parameter representing the constant of proportionality between permanent and elastic strains ([Yang H.Huang,1993](#)).

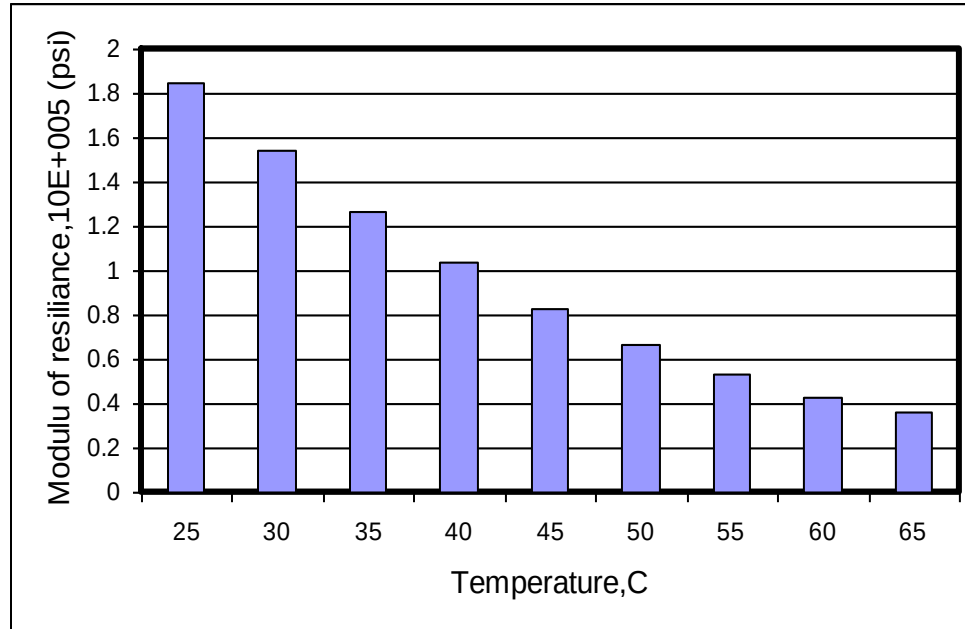


Figure (4-7), Modulus of Resilience Varies with Temperature for Mixture B1.

4.2 Rut Depth Prediction in Asphalt Concrete

4.2.1 Statement of the Problem

Asphalt concrete rutting is a longitudinal deformation that develops under the action of canalized loads. It is a bowl-shaped depression in the wheel paths resulting from pavement deformation. It occurs because of consolidation and/or plastic flow in the asphalt concrete layers within the pavement structure. Rutting often develops over a number of years due to repeat loadings ([AASHTO, 1988](#)). It has a major impact on pavement performance. According to the AASHTO design equation for flexible pavement, a (1.1") deep rut will reduce the Present Serviceability Index (PSI) of a relatively new pavement, having no other distress, from 4.2 to 2.5. The AASHTO design guide 2002 suggests that major highways be maintained at a PSI level pavement has in effect been lost.

Also, rutting affect on vehicle handling and cause a hydroplaning. Handling for smaller vehicles is impaired when they drive in a rut pattern that has been established in pavement surface by trucks. As water accumulates in the ruts, the possibility for hydroplaning Increases due to rutting ([AASHTO, 1988](#)).

Most of the existing design methods are based on the assumption that, if the maximum vertical compressive strain or stress at the surface of the subgrade is less than a critical value, then rutting will be limited to a tolerable level for a specified number of load applications. Unfortunately, such methodology does not preclude rutting which might occur in the asphalt bound layers and does not have the capability of rutting prediction.

In this section, a simplified procedure for the prediction of the rut depth in the asphalt concrete layers within the pavement structure is developed and verification for the obtained results by comparison with ([Albayati, 2006](#)) and ([Yassoub, 2004](#)) models for is presented. The proposed method is based on the permanent deformation ([Viscoelastic-Plastic](#)) model. In this method, the asphalt concrete layers that have different mechanical properties are converted into an equivalent layer using the equivalent stiffness method ([Ullidtz, 1979](#)). This has the consequence of instead of evaluating and testing the permanent deformation for each layer, only the asphalt concrete mixture for the equivalent layer will be characterized for permanent deformation prediction as present in Chapter 3.

The necessary input parameters for the proposed rut depth prediction method are:

1. Geometry of the pavement structure.
2. Materials response properties.
3. Climatic factor.
4. Traffic loading.

The proposed methodology is presented and discussed in the following sections; the steps of the methodology are further illustrated throughout a practical example for rut depth estimation.

4.2.1.1 Geometry of the pavement structure:

The geometry of the pavement structure ([Thickesses of The Layers](#)) required as input for the proposed methodology. In order to obtain the thicknesses for the Pavement layers, structural design can be performed using one of the available methods such as [AASHTO](#) or [PCASE](#) program. For the practical example, the geometry of the pavement structure is shown in [Figure \(4-8\)](#). The shown structure is a conventional flexible pavement section with five layers, these are: wearing course, binder (leveling) course, base course, subbase course and subgrade.

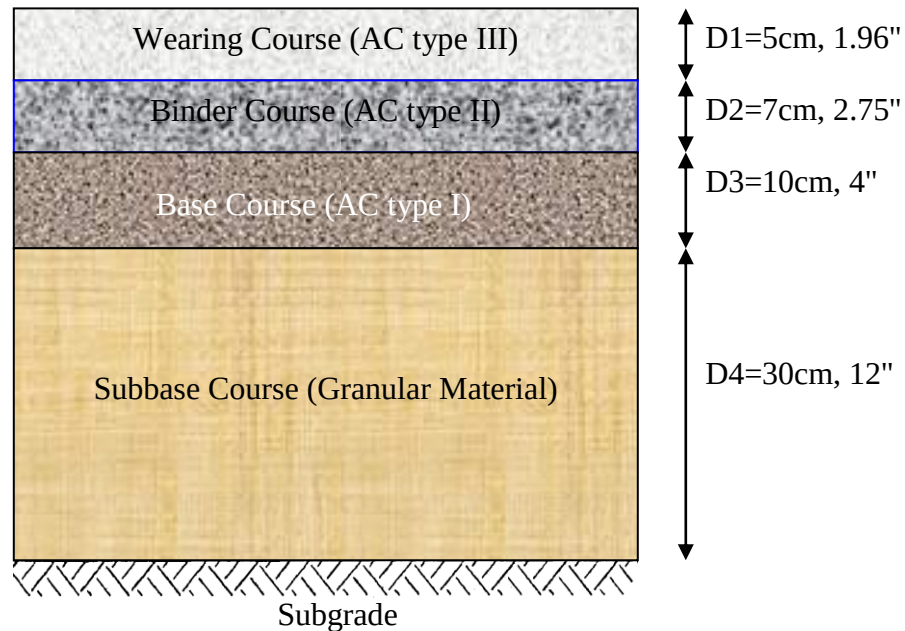


Figure (4-8), Geometry of The Pavement Structure.

4.2.1.2 Climatic Factors:

Many climatic and environmental factors influence the performance of the flexible pavement. The only environmental effect that is included in the proposed model is the temperature of asphalt layer. Creep and recovery compliance vary with time are affected by temperature. Changes in the temperature of the asphalt layer affect the elastic and viscous properties of the

bitumen. After the mean air temperature was recorded, the profile of the asphaltic material could be predicted. Predicting the temperature profile is complicated and requires details information about the thermal properties of the asphalt and the ambient conditions. Consequently, a simplified approach was used, where the maximum temperature of the asphalt layer was predicted at the (4.1"), (10.22 cm) for effect layer see ([Chapter-3-, Figure \(3-14\)](#)), at critical point below the surface and varied with the depth. This approach was reported by ([Albayati, 2006](#)) depending on the air temperature. This equation was:

$$T_{asp} = 1.217.T_{air} - 0.354.Z \quad \dots (4-23)$$

where:

T_{asp} =the effective surface layer temperature C^0

T_{air} =the measured mean air temperature C^0

Z =the depth below the surface (mm)

Mean Annual Air Temperature ([MAAT](#)) is equivalent to $23 C^0$ in Iraq was reported by ([Albayati, 2006](#)), this led to $T_{asp}=25C^0$.

4.2.1.3. Material Response Properties:

When the pavement structure is treated as multi-layer elastic system, it is necessary to quantify the material stiffness as defined by the resilient modulus of elasticity and Poisson's ration. These properties are needed for the calculation of the stresses, strains and deflections response in the pavement system under the application of traffic loading. The properties are also needed in the transformation of the pavement layers to an equivalent layer using equivalent stiffness method ([Chapter-3-, "3.5.2 Section Transformation \(Equivalent thickness\)"](#)). The finite element model both elastic or viscoelastic are need resilience modulus and Poisson's ratio as input for this purpose it should be calculating all these properties as following:

Asphalt Concrete Layers:

The resilient modulus of asphalt concrete wearing course can be predicted using the model shown below which is developed by (Albayati, 2006).

$$Mr = 46.6609 \times 10^4 - 9411.88 \times TMP + 68.77 \times TMP^2 + 724.72 \times AV^2 + 1679.443 \times TMP \times Dur - 163.46 TMP.AV + 73.46.VIS.VMA - 12053.97 Dur.AV - 69.376.Dur.EFF - 13.7435 \times 10^4 \times ABS.EFF. \quad \dots\dots\dots (4-24)$$

where:

- Mr = resilient modulus (psi).
- TMP = temperature in degree centigrade (C⁰)
- AV = percent air voids
- Dur = applied stress duration (sec)
- VIS = viscosity at 20 C⁰ (10⁶)
- VMA = percent voids in the mineral aggregate
- EFF = percent effective asphalt (by volume of mix)
- ABS = percent absorbed asphalt (by weight of aggregate).

The asphalt concrete mixture volumetric properties, asphalt cement viscosity and temperature at the mid-depth of layer (as estimated using Equation (4-23) are required to be input to the above model beside the loading duration which can be calculated as follows (Yang H.Huang,1993).

$$tw = \frac{12 \times ar}{S}. \quad \dots\dots\dots (4-25)$$

where:

- tw = loading duration (sec).
- ar = tire contact radius (cm or inch)
- S = vehicle speed (cm/sec (Km/hr x 27.8) or inch/sec (mphx17.61)

For the asphalt concrete wearing, the resilient modulus values are determined from the Equation (4-24) for type mix B1 all volumetric properties are present in Table (4-1), for the asphalt concrete binder (leveling) and base

layers, the resilient modulus are obtained for both directly from (Albayati, 2006) model determined from the repeated load test. The second response property is the Poisson's ratio (ν); it's defined as the ratio of the lateral strain to the axial strain. By measuring the axial and lateral strains during the resilient modulus test, the Poisson's ratio can be determined, because Poisson's ratio has a relatively small effect on pavement performance prediction, it is customary to assume a reasonable value for use in performance prediction, rather than to determine it from actual tests (Albayati, 2006). Table (4-8) shows the range and typical Poisson's ratio values for paving materials.

Table (4-8), Poisson Ratios for Different Paving Materials (Southgate et al., 1977)

Material	Range	Typical value
Asphalt concrete	0.30-0.4	0.35
Unstabilized granular subbase and base	0.30-0.45	0.4
Silt subgrade	0.35-0.45	0.45
Clay subgrade	0.4-0.5	0.5

Subbase layer

The method adopted for the estimation of the resilient modulus of the subbase granular material is that developed by the SHELL researchers (Claessen et.al.1977). In this method it is assumed that the subbase resilient modulus is a function of the subgrade resilient modulus according to the following relationship:

$$Mr_{subbase} = K \times Mr_{subgrade} \quad \dots\dots\dots (4-26)$$

where:

$$K = 0.2 \cdot h^{0.45} \quad \dots\dots\dots (4-27)$$

h = the thickness of subbase layer in mm

M_r = the resilient modulus in psi.

The relationship is applicable in the range $2 < k < 4$. The Poisson's ratio for the subbase layer can be obtained from table (4-8).

Subgrade layer:

Similarly, the subgrade resilient modulus was estimated using the relationship developed by the SHELL investigators (Claessen et. al., 1977). The relationship is based on the correlation between dynamic in situ tests for the subgrade resilient modulus and the corresponding laboratory measured California Bearing Ratio (CBR), CBR value for the subgrade soil can be obtained according to standard test procedure of ASTM D-1883, the relationship is given below:

$$M_{r\text{subgrade}} = 1450 * \text{CBR} \quad \dots\dots\dots (4-28)$$

Where: M_r is the subgrade resilient modulus expressed in psi, and CBR value for the subgrade soil is expressed as a percentage. The Poisson ratio for the subgrade soil can be obtained from Table (4-8).

4.3.1.4. Traffic loading:

In an effort to simplify the input traffic requirements, it is decided to convert mixed traffic to equivalent 18-Kip (80 KN) single axle loads in accordance with AASHTO load equivalency factors (AASHTO, 1993). The adopted 18-kip ESAL configuration is shown in Figure (4-10). The required traffic information for the estimation of the 18-Kip ESAL is the Annual Average Daily Traffic (AADT) and traffic composition at the start of the analysis period. These values can be obtained from the actual traffic counts on the existing pavement where the rut depth prediction is to be performed or on nearby highways with similar travel patterns for the new constructed pavement.

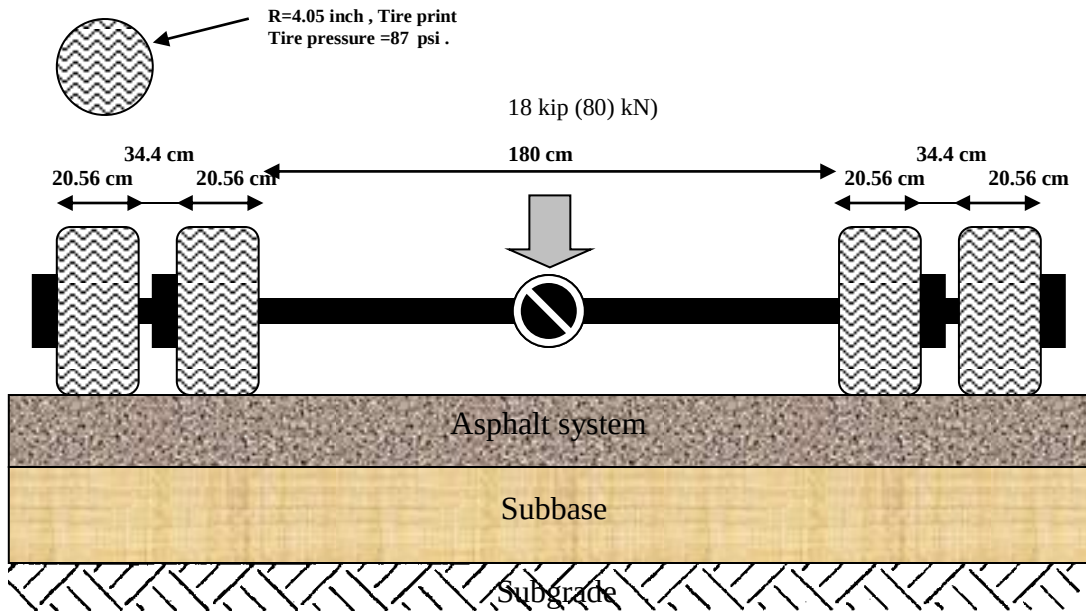


Figure (4-9), 18-Kip ESAL Configuration

For the practical example of rut depth prediction, N=6,000,000 repetitions is used based on worksheet* present by (Albayati, 2006).

For the practical example also the material response properties for each layer of the conventional pavement section are presented in the [Table \(3-2\)](#), mix B1 was used as a wearing asphaltic layer and it is represent the effective layer, all volumetric properties for mix B1 are present in [Table \(4-1\)](#).

4.3.1 Rut Depth

Rut depth of the asphalt concrete layers of the pavement structure under repeated loadings can be estimated according to the formula:

$$Rd = \varepsilon_p \times h \dots\dots\dots (4-29)$$

where:

Rd= rut depth of asphalt concrete layers (mm)

 ϵ_p = permanent strain in the asphalt concrete layers (mm/mm)

* Worksheet was presented in Albayati dissertation see " Table (8-5) ,p 227" for more details.

h= total thickness of asphalt concrete layers (mm).

ϵ_p stated as previous page is determined using the proposed model as following:

Recall Equation (4-9)

$$\epsilon_p(N) = \frac{\partial \epsilon_p(N)}{\partial N} = \epsilon_r \cdot \mu \cdot N^{-\alpha}$$

$$\epsilon_p = \int_0^N \epsilon_p(N) \cdot dN = \epsilon_r \cdot \mu \cdot \frac{N^{(1-\alpha)}}{(1-\alpha)} \quad \dots (4-30)$$

Using the value of Alpha (α), Mu (μ) that obtained in Table (4-6) for mix B1 under 25C⁰, Alpha (α)=0.6450 , Mu (μ) = 0.3327. Input ϵ_r for mix B1 that obtain form Table (3-5), $\epsilon_r^* = 2.21E-04$.

It should be noted that the rut depth predicted according to the above formula is limited for the asphalt concrete layer. But pavement structure includes also subbase and subgrade layers which may also contribute to the total pavement rutting. Therefore, the rut depth values of these layers which is out of the scope of this research should be added to that estimated according to Equation (4-29).

For the practical example, all the required variables for use of Equation (4-29) are presented in the previous sections, therefore, the total rut depth for 6,000,000 18-kip (ESAL) will be:

$$R_d = 0.05277 \cdot 204.4$$

$$= 10.78 \text{ mm}$$

For the same philosophy^{*}, it could be replaced ϵ_r in Equation (4-23) by ϵ_{ve} (viscoelastic strain that obtain from Table (3-5), $\epsilon_{ve} = 2.57E-04^{\wedge}$ at $t = 0.1 \text{ sec}$).

[♦] Obtained from elastic finite element model presents in chapter -3-.

^{*} (Yang H.Huang, 1993) offer in "Pavement Analysis and Design" at section 10.5- "Probabilistic Computer Program" VESYS method procedure and it contains this philosophy.

[^] Obtained from viscoelastic finite element model presents in chapter -3-.

$$\begin{aligned} Rd &= 0.06137 * 204.4 \\ &= 12.54 \text{ mm} \end{aligned}$$

It is clearly that general response calculations are so important and influent on rutting depth calculations. Increasing rutting depth from 10.78 mm to 12.54 mm, with 14% percent difference it should take care and treated the analysis of wearing asphalt concrete layer as viscoelastic approach instead of elastic approach only. Applying proposed model for all types of mixtures at $N=6,000,000$ repetitions, rutting depth variation with type of mixture is presented in Figure (4-10). It appropriates to use Mixture B1 because it offers a low susceptibility to rutting which is the same result of (Yassoub, 2004).

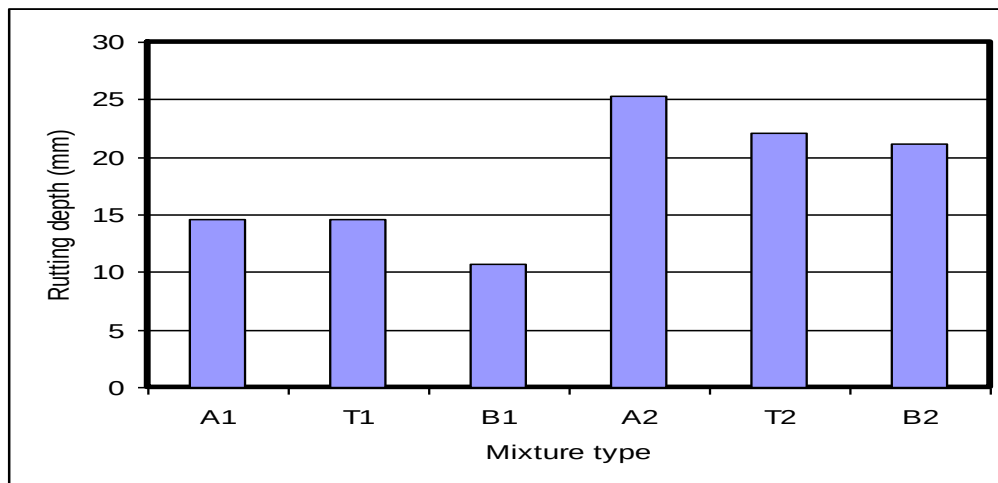


Figure (4-10), Rutting Depth Varies with Type of Mixture.

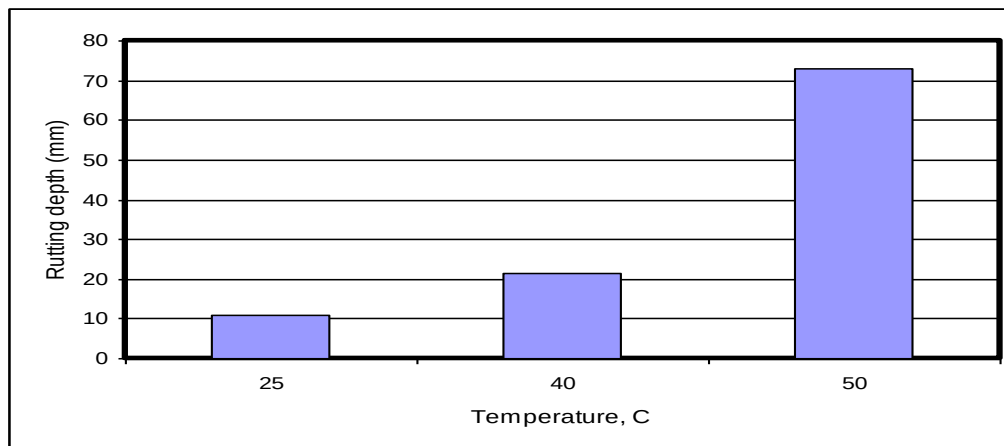


Figure (4-11), Rutting Depth Varies with Temperature for Mix B1.

4.3.2 Verification of the Results with Albayati and Yassoub Model

To ensure the reliability of the developed models and the algorithm of the rut depth predication for the asphalt concrete layers under the application of repeated loads, the obtained results for the rut depth of the practical example are verified by comparison with results of (Albayati, 2006) and (Yassoub, 2004) as following:

Albayati rutting model was developed based on the repeated loads tests, represent a statistical model applying for different test conditions and mix properties, in the following form:

$$\text{Log} \varepsilon_p = -34.463 + 0.983 \cdot \log N^{(0.00973 \text{TMP} + 0.00284 \text{STR})} + 1.961 \cdot \log \text{TMP} + 1.812 \cdot \log \text{STR} + 0.656 \cdot \log \text{Dur} - 36.277 \cdot \log \text{ABS} - 4.951 \cdot \log \text{VFA} - 1.455 \cdot \log \text{AV} - 1.93 \cdot \log \text{VMA}. \quad (4-31)$$

(All variables were previously defined).

To obtain rutting depth it should be applying Equation (4-29)

Yassoub rutting model was developed based on uniaxial repeated cyclic creep/recovery; a viscoplastic strain model was developed for local asphalt paving mixtures in the general form of:

$$\varepsilon_{vp} = 4.7195 \cdot 10^{-9} \times \sigma^{0.868} \times t^{0.658} \times T^{1.974} \quad \dots (4-32)$$

The developed model represents the variation of permanent strain with material property as asphalt mixtures, level of applied stresses (σ), time of loading (t), and degree of temperature (T). The developed form of the accumulated permanent strain as a function of the number of repetitions $\varepsilon_p(N)$ is:

$$\text{Log} \cdot \varepsilon_p(N) = \log \varepsilon_{vp} + S \cdot \log N \quad \dots (4-33)$$

For mixture B1 the slope (S) of the $\log(\varepsilon_{vp})$ vs. $\log(t)$ relationship is equal to (0.3693) and $\varepsilon_{vp} = 0.00014749$. To obtain rutting depth it should be applying Equation (4-22), the comparison is shown in Figure (4-10).

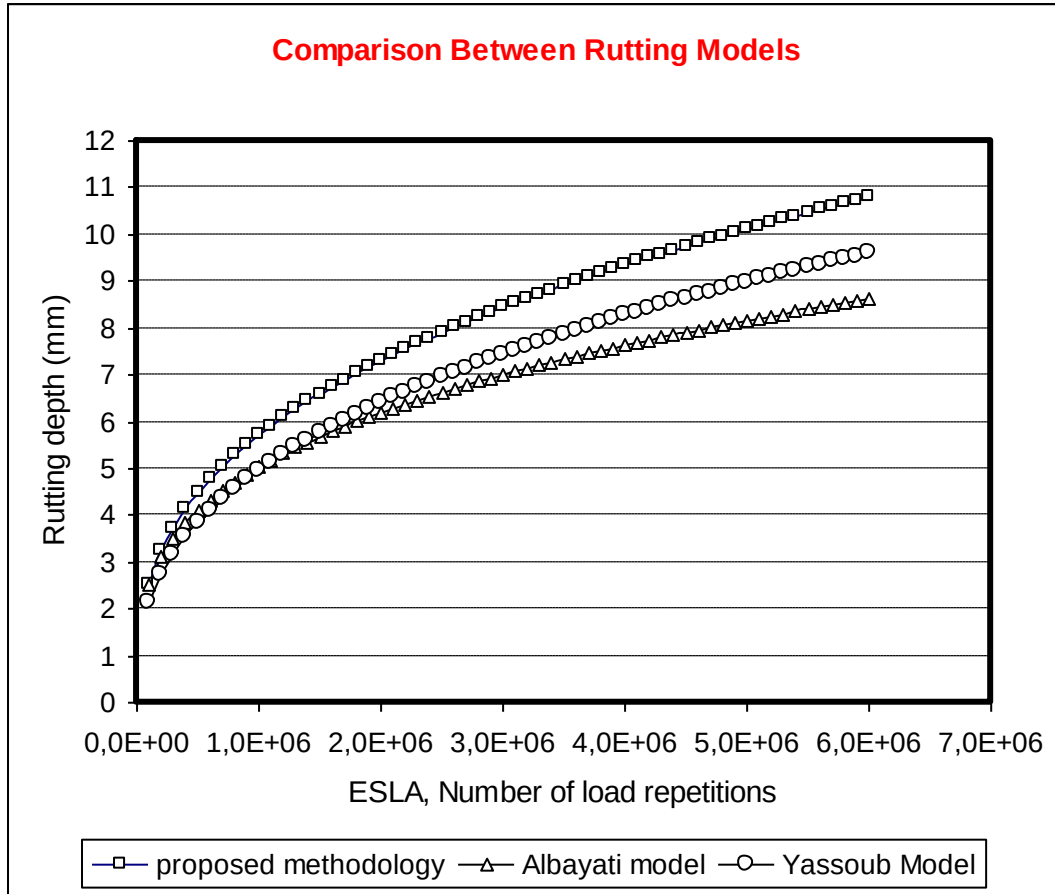


Figure (4-12), Comparison between (Albayati, Yassoub) against the Proposed Methodology, Rut Depth for Various Numbers of ESAL Applications for Mix B1.

The hyperbolic model provides a near perfect prediction of actual creep and recovery compliance performance of asphalt mixtures. While the rutting computing by (Alpha and Mu) that proposed in Equation (4-18) ;(4-19) gives a high value (because of difference in analysis method, type of data that simulated, Type and sensitivity of variables, test type and conditions e.g. differences in mold size.) if it compare with Yassoub and Albayati models, but it is still in the same range of other models, that range is denoted in FHWA specifications in the literature. Finally the procedure for prediction of rutting potential in asphalt mixtures presents in Figure (4-13).

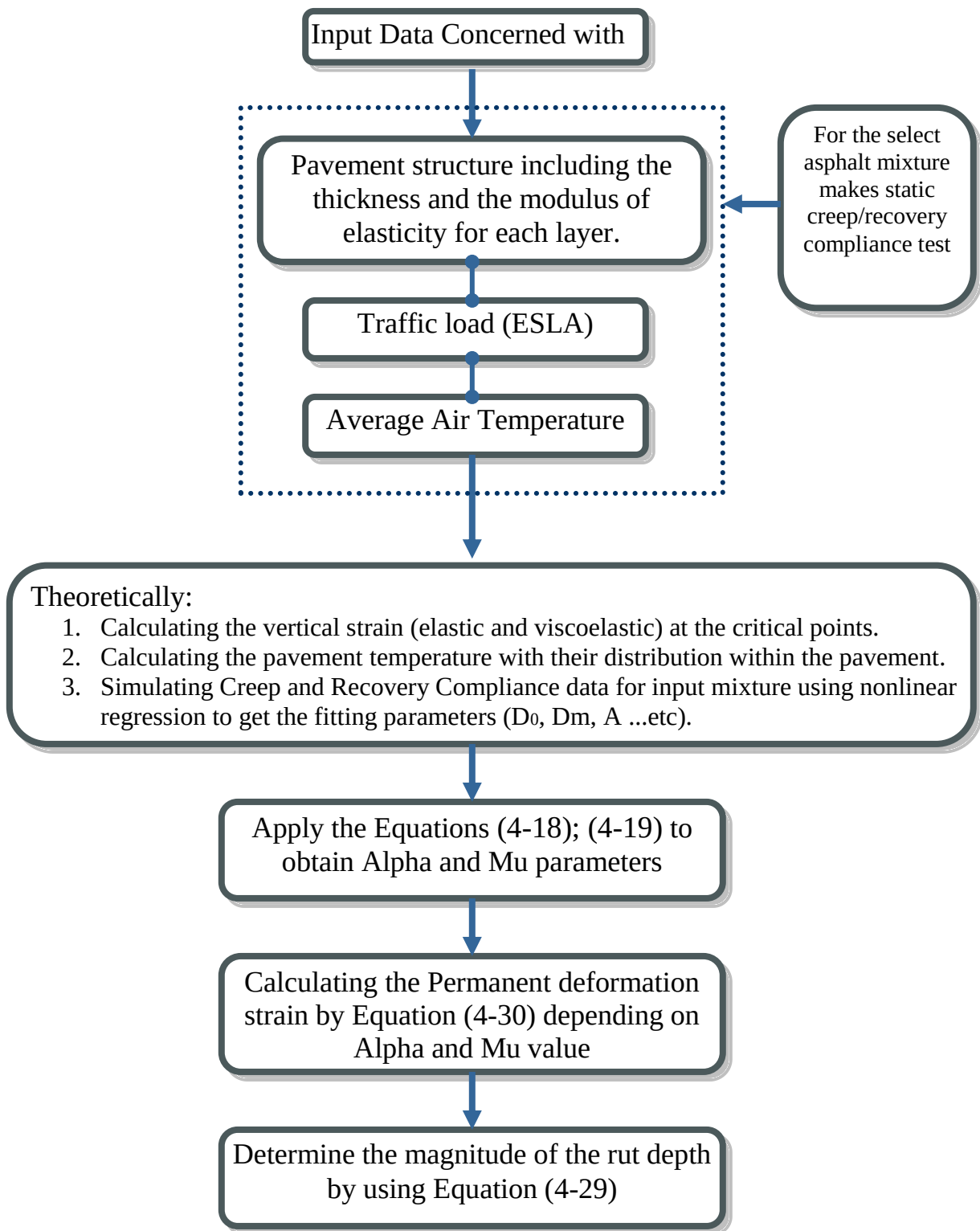


Figure (4-13), Flow Chart of the Proposed Model to Calculate the Rutting Potential.

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SYMBOLS AND ABBREVIATIONS

- **A, B**: Material Constant.
- **AADT**: Annual Average Daily Traffic.
- **AAMAS**: Asphalt Aggregate Mixture Analysis System.
- **AASHTO**: American Association of State Highway and Transportation Officials.
- **ABS**: Percent Absorbed Asphalt (By Weight of Aggregate).
- **AC**: Asphalt Content Percent %.
- **AI**: Asphalt Institute.
- **ASTM**: American society for Test of Material.
- **BTDC**: Bitumen Test Data Chart.
- **CBR**: California Bearing Ration.
- **C_m**: Correction Factors for Dynamic Effects.
- **CV**: Volume concentration of the Aggregate in the Compacted Mix.
- **D(t)**: Creep Compliance of Asphalt as function of time.
- **DGAC**: Dense Graded Asphalt Concrete.
- **E_a**: Stiffness of Asphalt Binder in kg/cm².
- **EFF**: Percent Effective Asphalt (by Volume of Mixture).
- **E_{sub}**: Elastic Modulus of the Subbase Layer.
- **FEM**: Finite Element Method.
- **FHWA**: Federal Highway Administration.
- **h_k**: Thickness of layer (K).
- **HMA**: Hot Mix Asphalt.
- **ILLI**: Illinois Finite Element Program.
- **IMO**: Iraqi Meteorological Organization.
- **I_p**: Penetration Index.
- **JMF**: Job Mix Formula.
- **m** : Slope of Creep Compliance Function.
- **MICH**: Michigan Finite Element Program.
- **MLET**: Multi layer Elastic Theory.
- **MMAT**: Mean Monthly Air Temperature.
- **mp** : Modifier Slope of Recovery Compliance Function.
- **Mr**: Resilience Modulus.
- **N**: Number of Load and Recovery Cycles in Equation (4-12, 4-13).
- **OBS**: Standard value.

- **OECD**: Organization for Economic Cooperation and Development.
- **PRED**: Predicated Values.
- **R(t)**: Recovery Compliance of Asphalt as function of time.
- **R**: Coefficient of Determination.
- **RD**: Rut Depth Accumulated up to Season j.
- **S**: Slope of ($\log \epsilon_p - \log t$) Derived from Test Results.
- **SHRP**: Strategic Highway Research Damage.
- **SORB**: State Organization of Road and Bridge.
- **Tasp**: Effective Surface Layer Temperature C^0 .
- **V**: Vehicle speed.
- **VA**: Air Void in total mix of Asphalt.
- **VESYS**: Viscoelastic System Analysis Program.
- **VFA**: Void Filled with Asphalt.
- **VMA**: Void in Mineral Aggregate.

GREEK SYMBOLS

- Δt : Loading Time of each Cycle (sec).
- α : Material Regression Constant in Equation (4-18).
- ϵ_p : Plastic Strain.
- ϵ_r : Resilience strain.
- ϵ_{ve} : Viscoelastic Strain.
- ϵ_{vp} : Viscoplastic Strain
- η : Constant Viscosity.
- κ : Bulk stress.
- σ : Applied Stress.
- τ : Shear stress.
- α, μ Plastic Strain parameters .

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1 CHAPTER ONE

STATEMENT OF THE PROBLEM

1. INTRODUCTION

The main types of structural distress of asphalt concrete pavements are cracking and permanent deformation, also known as rutting. Rutting is the most common distress mode still persisting in today asphalt pavement and is characterized by depressions that form in the wheel paths. This type of distress may occur due to repeated application of high stresses on the subgrade or by consolidation and instability ([Drakos, et al., 2001](#)). The first type is the result of excessive consolidation of the pavement along the wheel path due to either reduction of the air voids in the asphalt concrete layer, or the permanent deformation of the base or subgrade. In the second type, failure is attributed strictly to the asphalt mixture properties and usually occurs within the top two inches of the asphalt concrete layer. The mechanism of this mode of failure has not been clearly defined and may be expressed as the resistance of a mix to pavement rutting under varying conditions of climate and traffic loading ([Hills, 1993](#)). In Iraq, within about one year after construction, rutting distress was observed at several locations in the highway especially due to higher loadings and degree of temperature.

1.1. Factors Affecting Rutting

Permanent deformation physically visible as ruts on the pavement surface. Surface rutting in excess of about 0.25 inches (6 mm) is typically considered to

be a hydroplaning safety hazard. As a result, excessive rutting is often responsible for pre-mature maintenance and rehabilitation activities, reducing the service life of the pavement. Rutting is result of consolidation of the mix after construction and plastic flow of the asphalt mix in the wheel path over time, as reported by (Roberts, et al., 1996).

Rutting is strongly influenced by traffic loading, but climate can also have a large influence especially when the pavement subgrade undergoes seasonal variations in bearing capacity, or when bituminous course are subjected to high temperatures (Organization for Economic Cooperation and Development (OECD, 1988). Ruts develop within pavement layers when traffic loading causes layer densification and/or when stresses induced in the pavement materials are sufficient to cause shear displacements within the materials.

Many researcher performed over several decades have shown that the susceptibility to rutting can be linked to the following material attributes: excessive asphalt content, excessive fine grained aggregate, high percentages of natural sand, rounded aggregate particles, excessive permissible moisture in the mix or in granular materials and soils, temperature susceptible asphalt cement, and cold weather pavement leading to low density (Button, et al., 1990). Other factors affecting rutting are temperature; precipitation; time; type and extent of loading. The above factors when combined also determine measures such as Hveem and Marshal Stability, complex modulus, resilient modulus, and deflection that are normally used for pavement distress modeling.

1.2. Objective of the Study:

In this thesis, the objective of the research work could be presented as follows:

1. Evaluating and calculating the primary response (stress; strain; .. etc) at critical location in system of pavement's layers and effect of method of analysis for this response upon rutting depth.
2. Evaluating of the effects of the main variable related to asphalt concrete mixtures, stress levels and temperature effects on rutting potential using static creep/recovery test data.
3. Founding the best hyperbolic model that simulating the actual behavior of creep and recovery compliance that varies with time for different types of mixtures and surrounding circumstances.
4. Development of a permanent deformation model that reflecting the asphalt concrete properties depending on **VESYS** model over a range of loadings and local temperature.
5. Presenting a procedure for the prediction of rutting potential in asphalt mixtures taking into consideration the contributions of the layers under different variables.

1.3. Scope of the Study:

The work of this thesis consisted of the following:

1. A general idea is presented in **Chapter One** about the rutting potential problem, while **Chapter Two** is devoted to the review of literature concerning the factors that affect rutting and the criteria to assess rutting are summarized.
2. **Chapter Three** provides the Evaluation and calculating of the primary response (stress; strain; .. etc) at critical location in system of pavement's layers and a comparison between the exact solution represented by the

[Multilayer Elastic Theory \(MLET\)](#) and approximate solution represented by the [Finite Element Theory \(FET\)](#) for determine the primary response, also a [viscoelastic](#) solution depending on the [Small Strain Theory](#) is present.

3. The details of modeling of the permanent strain in asphalt mixture over a range of degree of temperature and different in mixture type are present in [Chapter Four](#) also the simulating of creep and recovery compliance data based on nonlinear regression analysis and a permanent strain material properties represented by [Alpha](#) (α) and [Mu](#) (μ) are present too.
4. Finally, the main conclusions obtained from the results of this work and recommendations for future work are presented in [Chapter Five](#).

2 CHAPTER TWO

LITERATURE REVIEW

2. ASPHALT CONCRETE

Asphalt concrete (sometimes referred to as “Hot Mix Asphalt” or simply “HMA”) is a paving material that consists of asphalt binder and mineral aggregate. The asphalt binder, which can be asphalt cement or modified asphalt cement, acts as a binding agent to glue aggregate particles into cohesive mass. Because it is impervious to water, the asphalt binder also functions to waterproof the mixture. When bound by the asphalt binder, mineral aggregate acts as a stone framework to impart strength and toughness to the system. Because HMA contains both asphalt binder and mineral aggregate, the behavior of the mixture is affected by the properties of the individual components and how they react with each other in the system. ([SHRP, 1994 spi](#)).

In the design of asphalt concrete mixture it is generally agreed that a balance be obtained between a numbers of desirable mix properties that, inevitably, necessitates a compromise in the selection of the design asphalt cement content. These pertinent mix properties include the following ([Choa, 1977; Finn, 1967](#))

1. Stability- resistance to deformation under load.
2. Durability- resistance to weathering.
3. Flexibility-ability to conform to long-term variations in base and subgrade elevations.
4. Fatigue resistance- ability to minimize cracking due to load repetitions.
5. Skid resistance- sufficient friction to enable a vehicle to brake to a stop within a reasonable distance under a variety of environmental conditions.

6. Fracture strength- maximum strength a mix exhibits when subjected to tensile forces.

The particular interest in this research is the stability which is the resistance to permanent deformation, i.e., the rutting associated with depressions in the pavement surface under the wheel paths as shown in [Figure \(2-1\)](#), stability is a function of the internal mix frictional resistance, cohesion and inertia. As noted by [Finn \(1967\)](#), frictional resistance is the “major contributor to resistance to deformation”. Moreover, at high temperatures and slowly applied loads, the contribution of the antiparticle friction to stability is predominant as aggregate surface texture exerts a major influence ([Finn, 1967](#)). Improper compaction, high asphalt content, and /or a high percentage of fine grained material (natural sand) tend to reduce this friction permitting plastic deformations to develop more rapidly.

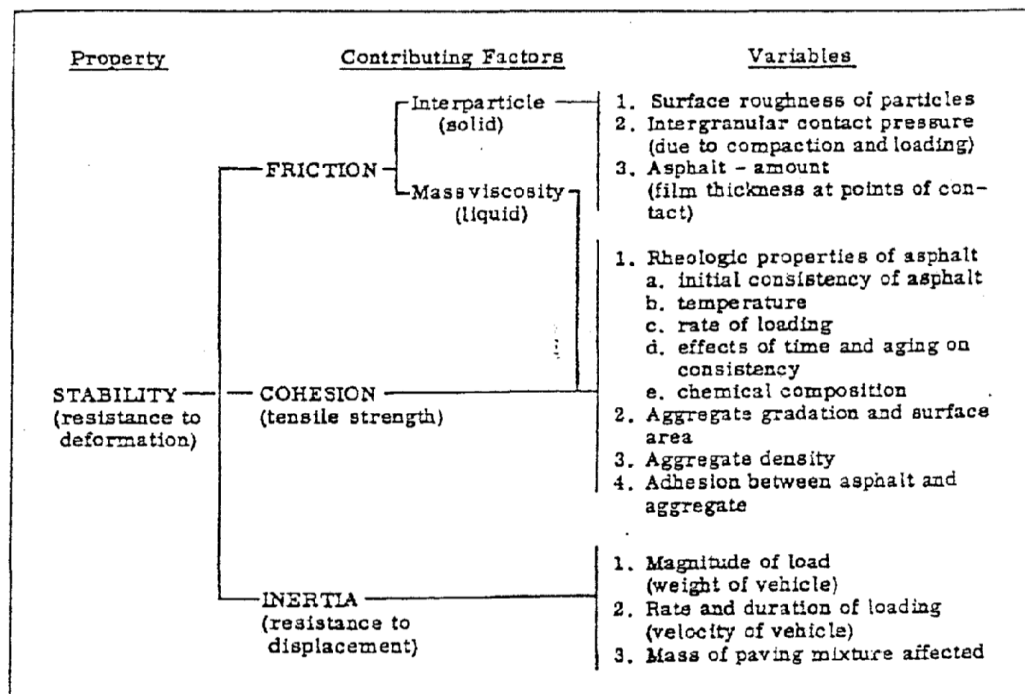


Figure 1- خطأ! لا يوجد نص من النمط المعين في المستند. **Analysis of Stability of Asphalt Paving Mixtures (Finn, 1967)**

2.1. Rutting: types, Mechanism and Distribution with Depth

Historically, the term “permanent deformation” was used to describe any distortion of the pavement surface, including shoving and pushing due to mix instability ([Goetz et. al., 1957](#)). Today, however, this form of distress refers to longitudinal depression or “ruts” that form in the wheel paths due to “consolidation and/or lateral movement in one or more of the component pavement layers due to repeated, transient load applications”. Because rutting appears only as a change in transverse surface profile, it was often erroneously blamed on surface instability. Investigations at the [AASHO](#) road test in 1959, however, revealed that permanent deformation occurred in all layers of the pavement system, as illustrated in [Figure \(2-2\)](#). From 51 sections trenched in 1960, the total permanent deformation was distributed among the component layers in the following average proportions ([Morris, 1973](#)):

<u>Component layer</u>	<u>Percent permanent deformation</u>
AC surface	32
Base	14
Subbase	45
Subgrade	9

Other studies have indicated that as much as 19 percent of the permanent deformation may be attributed to the subgrade, while others have shown that for 6-inch thick bituminous pavements, rutting was entirely due to deformation within the asphalt concrete ([Majidzadeh et. al., 1978](#); [Hofstra and Kiomps, 1972](#)).

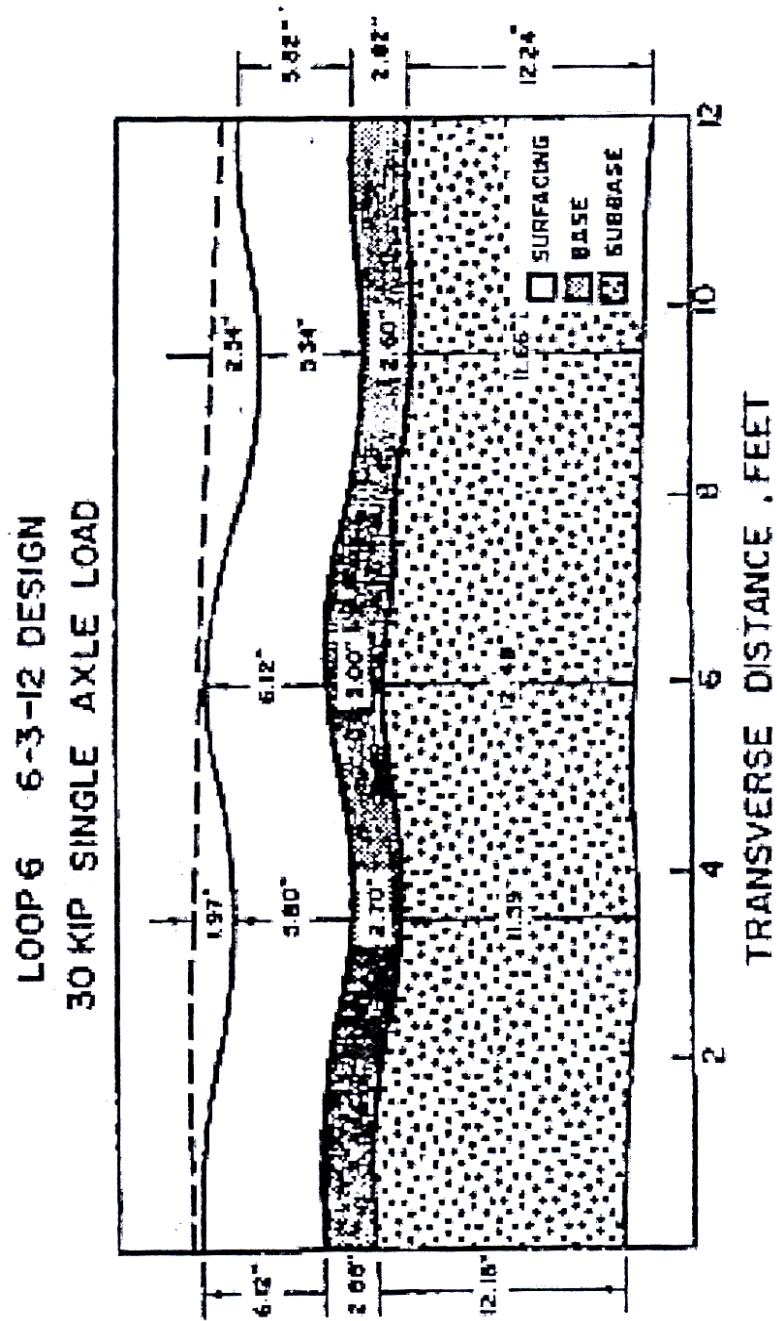


Figure 2-2, Typical Transverse Profile, 1959 AASHTO Trench Study (Morris, 1973)

In 2002, National Cooperative Highway Research Program (NCHRP, 2002) classified rutting into the following categories shown in Figure (2-3):

1. Wear rutting, which is due to the progressive loss of coated aggregate Particles from the pavement surface and caused by combined environmental and traffic influences.
2. Structural rutting, which is due to the permanent vertical deformation of the pavement structure under repeated traffic permanent deformation loads and it is within the subgrade.
3. Instability rutting, which is due to the densification (decrease in volume and, hence, increase in density) and lateral displacement (shear deformation) of material within the pavement asphalt concrete layers. (Daweley et. al. 1990) state that the majority of HMA rutting is due to primarily to instability rutting. This research is deal with this type of rutting essentially a reflection of the rutting.

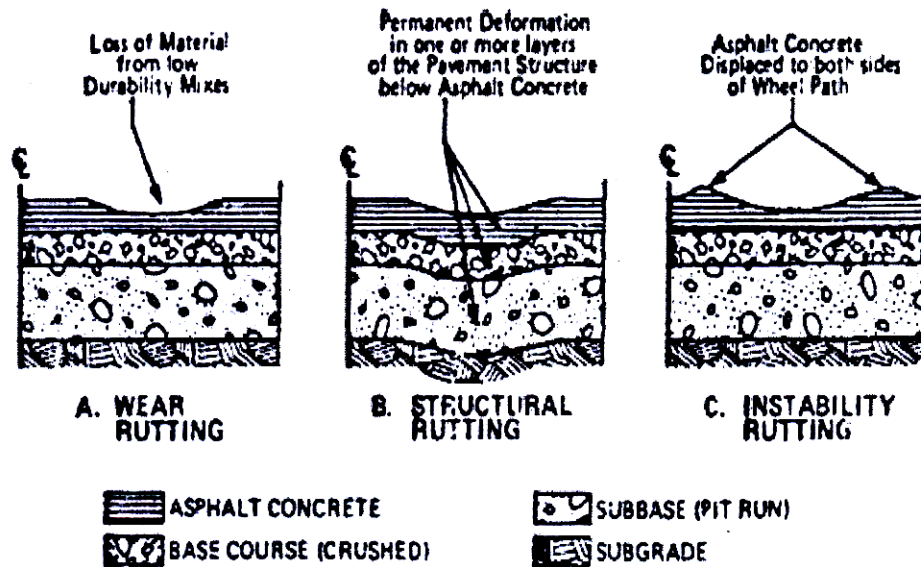


Figure 3-خطأ! لا يوجد نص من النمط المعين في المستند. 3, Description of Rutting Categories (NCHRP, 2002)

Undoubtedly, the mechanical responses of the HMA layer to various loading conditions are the result of a series of complex internal activates. For years, researchers tried to relate these microscopic functions to the macroscopic rutting phenomenon. Some researchers have hypothesized that the relative movements of aggregate particles result in different arrangements of the particles in order to satisfy equilibrium conditions. It is further hypothesized that this process is not a reversible one, and upon the removal of the loads, equilibrium is achieved through a recovery process which involves some permanent deformations ([Hills, 1973](#)). Others have taken the above theory one step further at the microscopic level and offered explanations of the mechanisms involved in particle reorientation and the collapse of the void network. They have suggested that the rheology (The word rheology is. derived from the Greek word rheo, which translates literally as “to flow.” and used to illustrate the dependency of the stress-strain relationship of asphaltic material on time) of asphalt concrete and its “flow like” behavior stem from viscous flow of the asphalt binder itself ([Van de Loo, 1978](#); [Davis, 1987](#)). The flow of asphalt cement into the voids and reduction in the thickness of aggregate coating results in reduction in the relative distances between aggregate particles. Hence, a particle reorientation is caused by the flow of the asphalt cement into the voids ([Van de Loo, 1974 and 1978](#)). As a result of these reorientation of aggregate particles which is enhanced by the temperature effect which liquefy asphalt cement and make the sliding of aggregate particles among each other easy. Under these circumstances, rutting in asphalt concrete is appearing due to the repetition of traffic axle loads ([van de Loo, 1978](#)).

There are contradictions between the researchers with respect to distribution of rutting with depth, ([Hofstra and Klomp, 1972](#)) found that the deformation throughout the asphalt layer was greatest near the load, and gradually decreased

with depth. This distribution of permanent strain with depth seems reasonable as there should be more resistance to plastic flow with increasing depth below the wheel loading, also due to the high temperature and vertical stress near the surface. repeated load tests by (Mclean, 1974) indicated that the greatest amount of permanent deformation occur at depth of about 2 inches (5.08 cm) below the surface, as illustrated by the data presented in Figure(2-4) ,i.e., at the point of occurrence of the largest value of stress difference. Research at the University of Waterloo, as illustrated by Figure (2-5), suggests that nearly all of the permanent deformation in the bituminous layer occurs in the lower half of the layer as a result of the lateral tensile stresses, this conclusion is in contrast with the observations of (Hofstra and Klomp, 1972) and the predictions of (Mclean, 1974), (Morris, 1973).

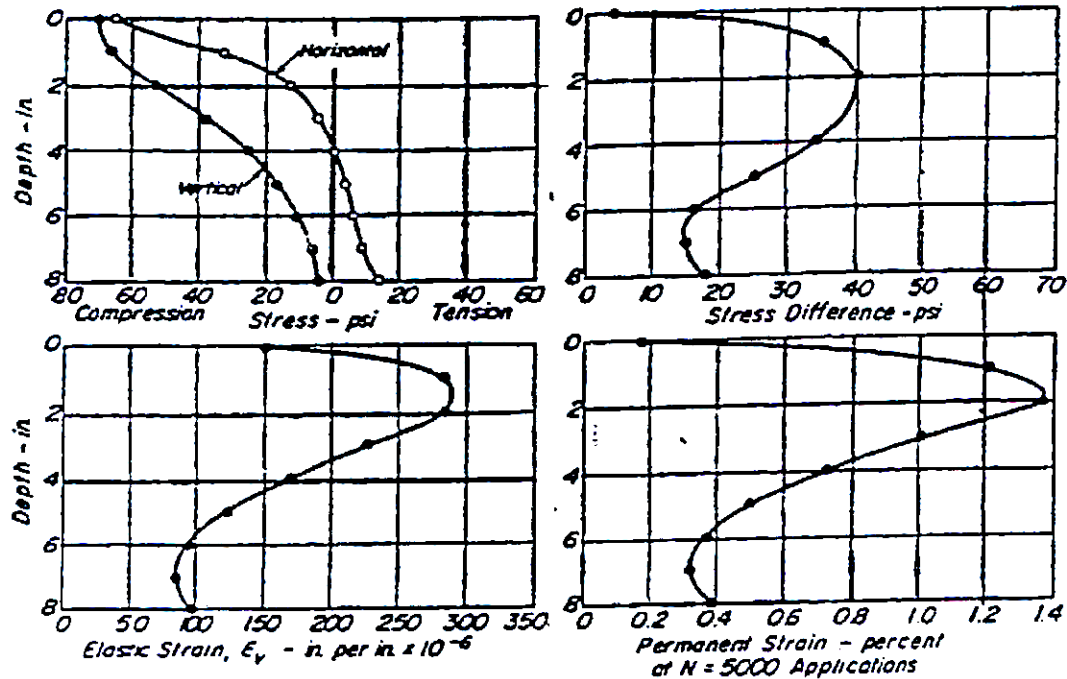


Figure 4-4, Stress and Strain Distributions in 8 inch Thick Asphalt concrete Pavement Subjected to 1500 lb Wheel Load with (70 psi Contact Pressure) Mclean (1974)

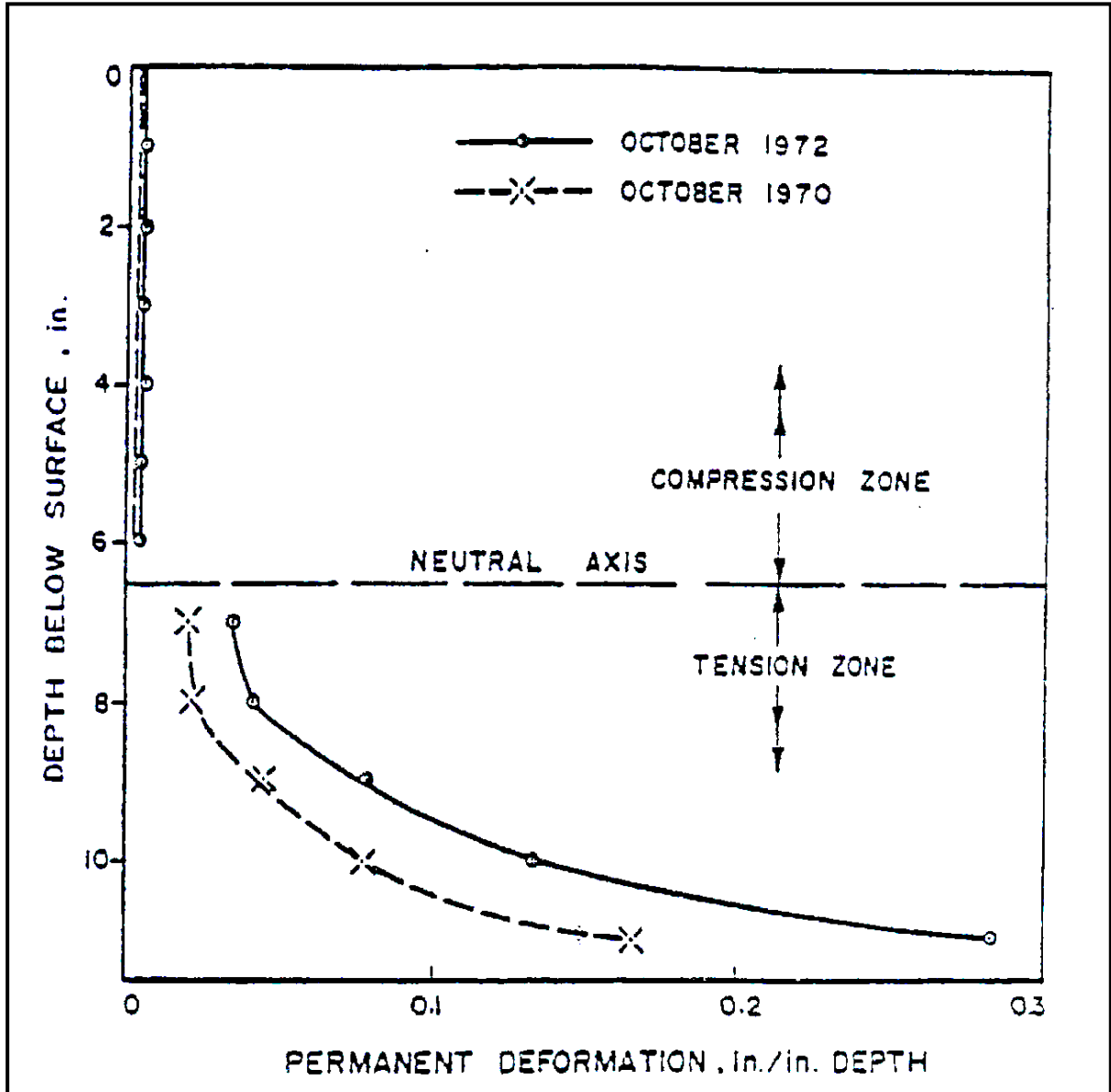


Figure 5-خطأ! لا يوجد نص من النمط المعين في المستند. Variation of Permanent Deformation with Depth (Morris, 1973)

2.2. Rutting Measurement and Criteria

Rut depth is defined as the vertical distance between the valley and the crest of the ruts. Rut depth is measured by placing 1.2 m (or 3.6 m) metal straightedge across the wheel path to establish a horizontal reference line and measuring the vertical distance between the straightedge and pavement surface by 0.3 m ruler (as shown in [Figure \(2-6\)](#)). This measurement is repeated for each 6 m interval (in traffic direction) in both wheel paths and the results is averaged to give the mean rut depth.

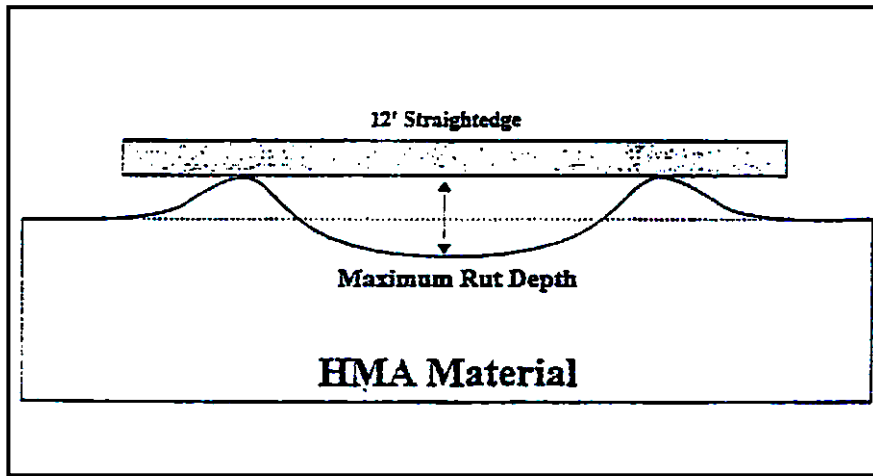


Figure 6-خطاً! لا يوجد نص من النمط المعين في المستند. Schematic of Rut Depth Measurement with Straightedge (Ahlrich, 1995)

The occurrence of permanent deformation or rutting is one of the major problems affecting the performance of pavement structure ([Bissada, 1983](#)). It is considered as a serious safety problem encounter the traffic for two reasons:

1. Loss of vehicle control during lane changing, ruts tend to pull a vehicle towards the rut path as it is steered across the rut.

2. If the rut depth is greater than 12.5 mm for pavement with crown slopes on the order of 2 percent, this rut depth is sufficient for the accumulation of water (pounding) and possibly causing vehicles traveling at speed 80 km/hr or more to hydroplane.

Based on the ([AASHTO guide, 1993](#)), rutting is classified according to severity as exhibited in [Table \(2-1\)](#) below.

Table 2-1, Rut Depth Criteria According to (AASHTO, 1993)

Mean Rut Depth, mm	Severity Level
6-13	Low
13-25	Medium
> 25	High

The ([Federal Highway Administration FHA, 1979](#)) has classified rutting into four levels of severity:

1. Hydroplaning (0.2 to 0.25 inch)
2. Low (0.25 to 0.5 inch)
3. Medium (0.5 to 1.0 inch)
4. High (1.0 inch)

Many researchers, however, consider that the only reasonable standard is that associated with hydroplaning. For pavements with crown slopes of the order of 2 percent and rut depths of about 0.5 inch, ponding is sufficient to cause vehicles traveling at speeds of 50 mph or more to hydroplane ([Barksdale, 1972](#)). Allowable rut depth may be limited by both safety and structural as a rut depth of 0.75 to 0.8 inch (19-20mm) measured by a 6ft (1.8m) straightedge ([Lister and Addis, 1976](#)). It is suggested that rut depths up to approximately 0.4 inch

(10mm) do not cause significant loss of structural strength. For a cross slope of 2.5 percent (generally used in the United Kingdom), ([Lister and Addis, 1976](#)) have found that ruts deeper than approximately 0.5 inch (13 mm) result in ponding of water which could cause hydroplaning or loss of skid resistance. This corroborates Barksdale's finding as noted above. ([Verstraeten et al., 1977](#)) have determined that the rut slope (ratio of rut depth to one-half its width) should not exceed 2 percent for good riding quality.

2.3. Factors Affecting Rutting

Rutting in paving materials develops gradually with increasing numbers of load applications, usually appearing as longitudinal depressions in the wheel paths accompanied by small upheavals to the sides. It is caused by a combination of densification (decrease in volume and, hence, increase in density) and shear deformation and can occur in any one or more of the pavement layers as well as in the subgrade. In general the characteristics of concrete pavements are summarized in [Table \(2-2\)](#).

The AASHTO Joint Task Force on rutting ([AASHTO, 1988](#)) expressed the opinion that there are many factors which can influence HMA rutting. Traffic and environmental (temperature) factors were identified as the major causes of HMA rutting. Unfortunately, the highway agencies have little control of these two factors. However, there are also important factors that have contributed to the HMA rutting problem that are within the control of highway agencies ([AASHTO, 1988](#)). Highway engineers have focused attention on these factors rather than the traffic and environment. With proper control of asphalt cement, aggregate quality and gradation, asphalt content and construction The HMA rutting problem can be greatly minimized the subcommittee on materials of the

(Western Association of Highway and Transportation Officials), WASHTO, (Betenson, 1984) has also made recommendations on the selection of materials to minimize the HMA rutting.

Table 2-2, Factors Affecting of Asphalt-Concrete Mixtures (Yassoub, 2004)

	Factor	Change in Factor	Effect of Change in Factor on Rutting Resistance
Aggregate	Surface texture	Smooth to rough	Increase
	Gradation	Gap to continuous	Increase
	Shape	Rounded to angular	Increase
	Size	Increase in maximum size	Increase
Binder	Stiffness ^a	Increase	Increase
Mixture	Air void content ^b	Increase	Decrease
	VMA	Increase	Decrease ^c
	Method of compaction	d	d
Test field conditions	State of stress/ strain	Increase in tire contact pressure	Decrease
	Water	Dry to wet	Decrease if mix is water sensitive
^a Refers to stiffness at temperature at which rutting propensity is being determined. Modifiers may be utilized to increase stiffness at critical temperatures, thereby reducing rutting potential. ^b When air void contents are less than about 3 percent, the rutting potential of mixes increases. ^c It is argued that very low VMA's (e.g., less than 10 percent) should be avoided. ^d The method of compaction, either laboratory or field, may influence the structure of the system and therefore the propensity for rutting.			

2.3.1 Traffic Characteristics

Traffic characteristics are one of the most important factors affecting rutting. They include tire contact pressure, vehicle speed, and the number of load repetitions (traffic volume). The shape of contact area between a tire and the road surface is approximately circular when the load applied is small relative to the recommended maximum for the tire, but it becomes increasingly elongated as the wheel load is increased at constant inflation pressure, this is shown in figure (2-7) where the highest load considered is 50 percent above the tire manufacturer's rating (Cronney, 1991). In calculating pavement stresses resulting from the passage of the traffic, it is usual to assume that the load carried by the wheel is uniformly distributed over a circular area and the radius of loading is calculated from the wheel load and the tire pressure (Yang H. Huang, 1993), this is because it is assumed that the tire pressure is equal to the contact pressure (no effect for tire wall).

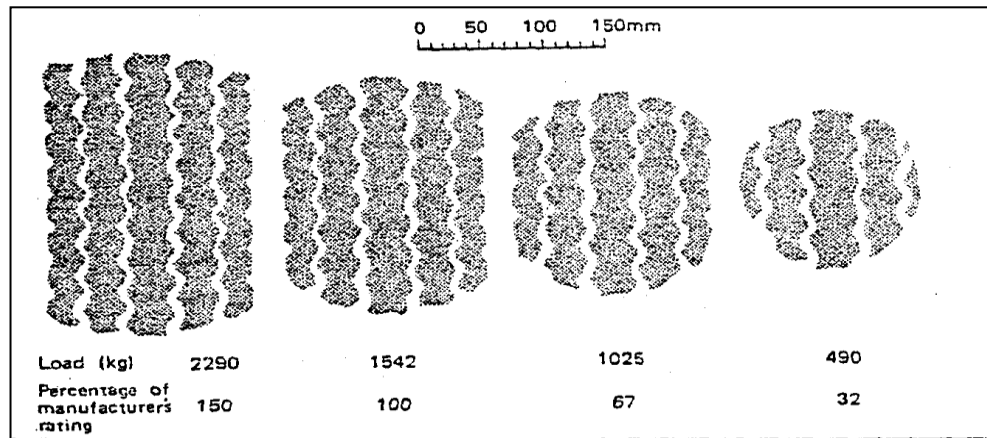


Figure 7- خطأ! لا يوجد نص من النمط المعين في المستند. Tire prints for Different Loading (Cronney, 1991)

Higher truck tire pressures and increased trucks weights since the AASHTO road tests have lead to increased HMA rutting (Betenson, 1984). Responses to the 1987 AASHTO survey on the HMA rutting problem showed that 93 % of those responding expressed that heavy truck tire pressure were major cause of

HMA rutting ([AASHTO, 1988](#)). While the current 1993 AASHTO guide for design of pavement structure ([AASHTO, 1993](#)) is based on 18 kips loads and tire contact pressure of 75 to 80 psi , recent studies ([Middleton et. al., 1986](#); [Thompson, 1987](#); [Kim and Bell, 1988](#)) have shown that truck tire pressure inflation pressure have increased and are now averaging around 100 psi. Some researchers ([Middleton et. al., 1986](#); [Hudson and Seed, 1988](#)) reported truck tire pressures to be as high as 140 psi. ([Chen et. al., 2000](#)) stated in his field study using Texas Load Simulator that such increase in tire inflation pressure from 80 to 100 psi leads to increased in rut depth about 43 percent after 1600000 axle load repetitions. In Dubai ([Vallerga et al., 1989](#)), after extensive laboratory and field studies, it was determined that the mixture and pavement had originally been well designed according to existing methodologies. But the accelerated rutting (after 10 months of trafficking) was appeared due to heavier than expected loads (up to 80,000-pound axle loads as compared to the 18,000-pound axle loads used for design) and higher tire pressures. ([Shakir, 1999](#)) in his study for the rutting in the oil tanks road located western of Baghdad indicates that the increase in the tandem axle load by 61 percent leads to increase in rut depth by 82 percent.

Another factor related to traffic that significantly affect rutting of asphalt concrete layers is the speed of traveling vehicles. When the viscoelastic approach is used to calculate the pavement response to the applied wheel load, speed is directly related to the duration of loading. If the elastic approach is used in the pavement response calculation, the elastic modulus of asphaltic concrete layers should be determined in commensurate with the vehicle speed. ([Pereira et. al., 1999](#)) investigated the effect of truck speed on the accumulation of the permanent deformation; measurements of the development of actual rut depth were recorded as a function of truck speed (which decreased as the trucks

climbed a steep hill) for the three sites in Portugal. The investigated roads pass through hilly terrain where long stretches of highways are on inclines and trucks frequently slow down to about 20 km/hr. Schematic diagram for the location of highway and rutting severity is shown in [Figure \(2-8\)](#). Pereira stated that the rut depth in the end of the upward direction lane with 5 percent grade and 25 km/hr average truck speed is 20 mm while in the beginning of the grade, rut depth is 10 mm. Also ([Tien et. al, 1992](#)) stated in his rutting survey in Singapore roads that the rutting is commonly founded in the junction. He stated also that this is an indication for the association of this problem to the slow moving vehicles. ([Hussain, 1985](#)) in his research on some of the important rural highways connecting Baghdad with other governorates agreed with the above observation. He also found that all the observed highways suffer from rutting particularly on the slow moving vehicle lane (right lane), which carries the bulk volume of the commercial vehicle.

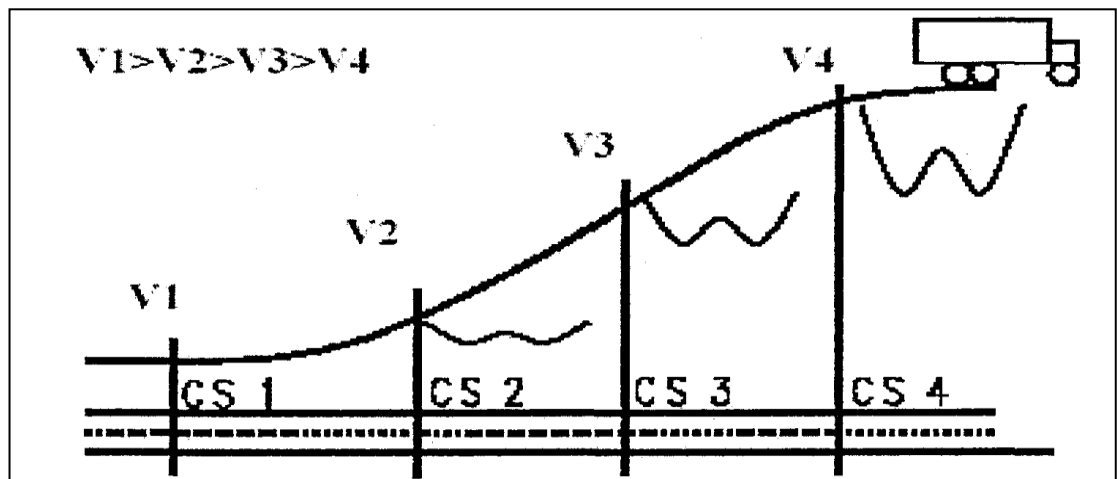


Figure 8-خطاً! لا يوجد نص من النمط المعين في المستند. Schematic Diagram Showing the Effect of Vehicle Speed along the Grade on the Rut Depth (Pereira et. al., 1999)

The third component of the traffic that effect rutting is the load repetition. In general, as the load repetition increases the rutting is increase as shown in [Figure \(2-9\)](#).specifically, the trend of this relationship is a function several factors, such as temperature, loading conditions and mixture parameters. Researchers ([Uzen,](#)

1983; Al-Juraiban, 1983) show in most cases it follows the linear trend when plotted in log scale. (Eisenmann and Hilmer, 1987) on the basis of wheel tracking test results suggest the following relationship between the rut depth (Rd) and the number of load repetitions (N):

$$Rd = a + b \times N^{0.5} \quad \dots\dots (2-1)$$

Where (a) and (b) are laboratory determined parameters. For the preceding discussion, it is apparent that the increased tire inflation pressure have placed the HMA mixture near the surface under high stresses, though increase from the probability of rutting. Therefore more attention should be given to the selection of high quality materials in designing the asphalt mixture and in quality control during construction of the surface course of flexible pavement in order to minimize rutting.

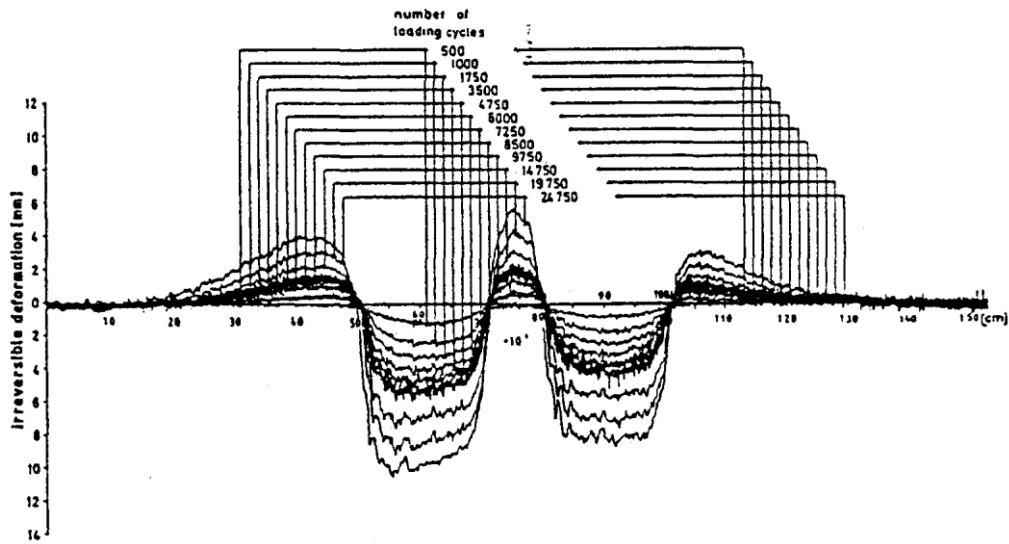


Figure 9-خطأ! لا يوجد نص من النمط المعين في المستند. Effect of Number of Load Repetition on Rut Depth (Eisenmann and Hilmer, 1987)

2-3-2 Temperature

Rutting is more prevalent in hot climate areas because the viscosity of the asphalt binder which is inversely related to rutting is significantly reduced with the increase in temperature resulting in amore rut susceptible HMA mix. [Figure \(2-10\)](#) shows the effect of temperature on log asphalt viscosity for a wide range of asphalt cement grades ([Puzinauskas, 1979](#)). From this Figure, it is obvious that the viscosity of asphalt varies from (less than one poise) to (more than one trillion poises). Within such extreme viscosity range, asphalts are transformed from low viscosity Newtonian liquids to materials exhibiting shear-dependent viscoelastic behavior, where with decreasing temperature the elastic component tends to predominant .Thus, the gradually changing curvature of plots in [Figure \(2-10\)](#) indicate that the viscosity of asphalt tends to change more rapidly at low temperatures, and such change becomes far less pronounced at higher temperatures when the viscous behavior is predominant.

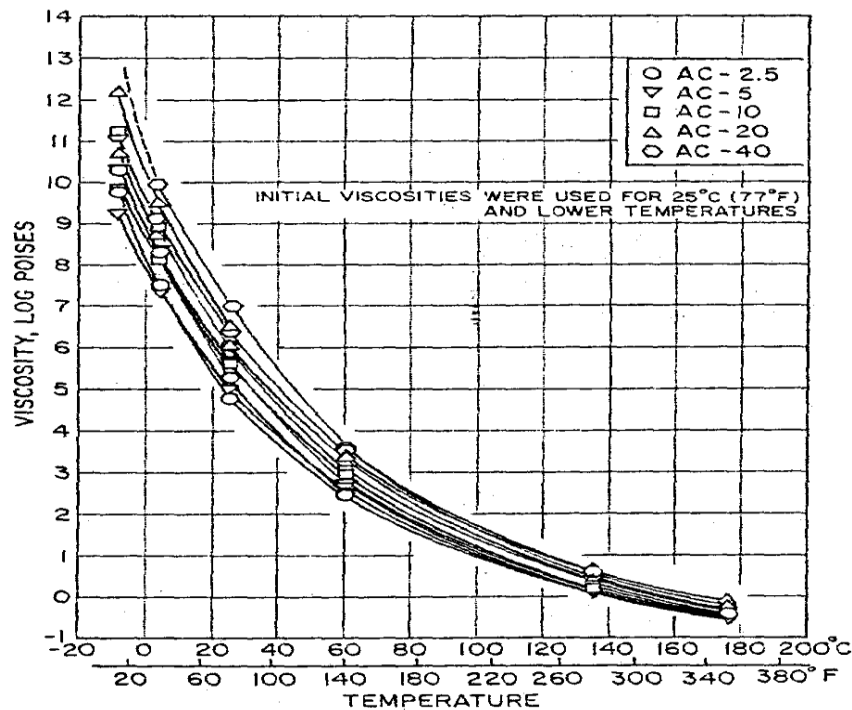


Figure 10-Relationship between Viscosity and Temperature for Asphalt Cements (Puzinauskas, 1979). لا يوجد نص من النمط المعين في المستند.

2-3-3 Asphalt Concrete Mixture Components and Properties

([Speer, 1962](#)) investigated the effect of different asphalt penetration grades on the rutting potential of asphalt concrete. His investigation was based on the laboratory test track and traffic simulator machine which have the capability for applying different tire pressure (30, 50, 70 and 90 psi). Based on his experiments, he stated that asphalt cement with softer grade (90) resulted in rut depth more than that of (40) penetration grade by a factor of 1.5 at the end of one million repetition of 90 psi tire pressure. The comprehensive results of Speer are shown in [Figure \(2-11\)](#) at conditions 5.0% asphalt content and 115 F traffic temperature. However, Speer cautioned from the construction problems associated with the use of hard grade asphalt due to the difficulty of mixing operation and pavement lay down process. There is much in literature on the relationship between the asphalt cement content and asphalt concrete rutting susceptibility. ([Anani et. al., 1990](#)) compared the results of cores from both good performing and rutted areas of seven pavements in Saudi Arabia to identify the mix and material properties that caused rutting susceptibility. The authors reported that the rutted areas had higher asphalt cement contents than the good performing areas. Higher asphalt content is defined by the author as that produced air voids in mix less than 3 percent. ([Brown et. al., 2004](#)) stated on the basis of laboratory test track that the addition of 0.5 percent asphalt beyond the optimum content increase the permanent deformation by approximately 50 percent. ([Mahboub and Little, 1988](#)) indicated that higher asphalt cement contents which produce lower air voids increased rutting potential [Figure \(2-12\)](#). They suggested that the reduction in air voids as a consequence of increased asphalt content is attributed to the fill of void space with asphalt. As a result, the increase in asphalt content is equivalent to the introduction of lubricants between aggregate particles otherwise separated by a very tight network of air voids. This

phenomenon causes the mixture containing higher asphalt content to be more susceptible to permanent deformation induced by viscous flow of asphalt between aggregate particles.

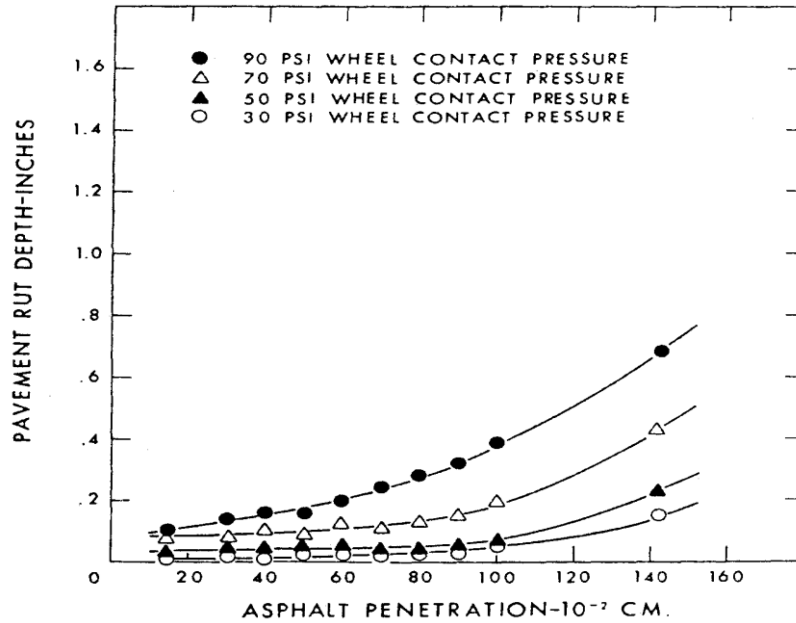


Figure 11-خطاً! لا يوجد نص من النمط المعين في المستند. Relationship between Viscosity and Temperature for Asphalt Cements (Puzinauskas, 1979)

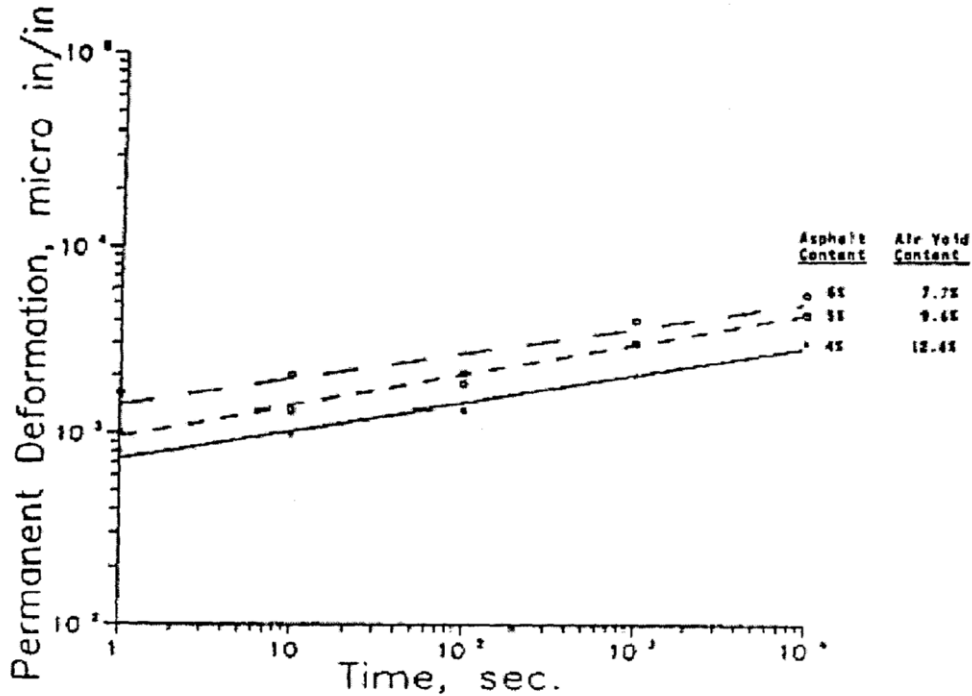


Figure 12-خطاً! لا يوجد نص من النمط المعين في المستند. Permanent Deformation Trends for AC-5 Mixtures (Mahboub and Little, 1988)

2.4. Permanent Deformation Predictive Models

Numerous studies have been made to characterize the permanent deformation response of asphalt concrete materials due to repeated load applications. The studies used the repeated compressive load test to investigate the variables that influence the relationship between the number of load applications, (N), and the permanent strain, (ϵ_p). Rather than reviewing each in detail, the more recent research efforts are presented in tabular form to facilitate ease of revisions.

Table (2-3)
Table 2-3, Summarized Overview of the Permanent Deformation Models
(Albayati, 2006)

Author	Permanent Deformation Equation	Variables	Laboratory Test
Kirwan Snaith Glynn (1977)	$\epsilon_n = A \Delta^b$	ϵ_n = induced axial permanent strain after an elapsed time N A = function of elapsed time and material b = constant for the material Δ = applied axial compression stress	Uniaxial compression dynamic loading creep test
Monismith Inkabi Freeme McLean (1977)	$\epsilon_z^p = [\delta(T) N^\alpha (\bar{\sigma})^{\alpha-1} t] \cdot [\sigma_z - \frac{1}{2}(\sigma_x + \sigma_y)]$	ϵ_z^p = vertical permanent deformation $\delta(T)$ = function of temperature α = coefficient determined experimentally N = number of stress repetitions $\bar{\sigma}$ = equivalent stress define as a function of σ_1 , σ_2 , and σ_3 t = loading time	Repeated load triaxial compression test
Brown Bell (1979)	$\epsilon_p = (\frac{q}{a})^b (N)$	ϵ_p = permanent shear strain q = deviator stress a, b = constant N = number of load application	Axial repeated load tests

**Table (2-3) Summarized Overview of the Permanent Deformation Models
(Albayati, 2006)**

Meyer and Haas (1979)	$\varepsilon_p = F(\sigma_1, \sigma_3, T, AV, N) \pm E$	ε_p = axial permanent deformation σ_1 = vertical stress σ_3 = lateral stress T = temperature AV = air voids N = number of load applications E = error of estimate	Repeated load triaxial test
Kenis (1977)	$\varepsilon_p(N) = e\mu N^{-\alpha}$	$\varepsilon_p(N)$ = permanent strain per pulse $\alpha = 1 - S$ S = slope of the line on a log-log plot of permanent strain versus N e = peak haversine load strain for a load pulse of duration (d) $\mu = IS/e$ I = intercept	Uniaxial repeated load tests
Thrower (1977)	$\dot{\varepsilon}_{ij} = \frac{\sigma_{ij}}{2\eta} i + j$ $\dot{\varepsilon}_{ij} = \frac{\sigma_m}{3\chi} + \frac{(9\sigma_{ij} - \sigma_m)}{18\eta}$	$\dot{\varepsilon}_{ij}$ = rate of deformation σ_{ij} = state of stress σ_m = isotropic mean stress χ = coefficient of "volume viscosity" η = coefficient of shear velocity	

**Table (2-3) Summarized Overview of the Permanent Deformation Models
(Albayati, 2006)**

Huscheck (1977)	$\varepsilon_{irr} = c \cdot \sigma \cdot t^A$ $\varepsilon_{irr} (T, \Delta t_1, t) = \frac{\sigma \Delta t_1}{[\eta (T, t)]}$ $\eta (T, t) = \frac{t^{1-A}}{(c \cdot A)}$	ε_{irr} = permanent deformation c = constant A = consolidation characteristic σ = stress level η = viscosity T = temperature t = total time of loading Δt_1 = time of loading	Uniaxial creep tests Cyclic load creep test
Khedr (1986)	$\frac{\varepsilon_p}{N} = A_a N^{-m}$	ε_p = permanent strain N = number of load cycles A_a = material properties functions of resilient modulus and applied stress m = material parameter	Multi-step dynamic tests
Battiato et al (1977)	$J(t) = J_1 t^\alpha$ $u_{ik}^{perm} = \frac{1}{\eta_s} g_{ik} (y, z)$	$J(t)$ = creep compliance function t = time J_1 = shear creep parameters α = slope of line on a log-log between $J(t)$ and time u_{ik}^{perm} = permanent deformation η_s = shear viscosity of the Maxwell element in series $g_{ik} (y, z)$ = tensor function	Uniaxial creep tests
Van de Loo (1976)	$\varepsilon_p = c \sigma N^a$	ε_p = axial permanent deformation c = constant σ = axial stress level (15 PSI) N = number of load applications a = constant	Axial creep test
Uzan (1983)	$\varepsilon_p(N) = e_r \mu N^{-\alpha}$	$\varepsilon_p(N)$ = permanent strain for Nth repetition e_r = resilient strain N = number of repetitions α, μ = characteristics of materials based on intercept and slope coefficients	Repeated load testing

**Table (2-3) Summarized Overview of the Permanent Deformation Models
(Albayati, 2006)**

Leahy (1989)	Statistically derived predictive models for permanent strain, ϵ_p $\epsilon_p = f(T, \sigma_d, V_{air}, N, \eta_{asp}, P_{Wasp})$	ϵ_p = plastic strain T = temperature σ_d = deviator stress V_{air} = volume of air η_{asp} = asphalt viscosity P_{Wasp} = effective asphalt content	Repeated load and creep, axial testing

2.5. Rut Depth Prediction Methods

With increases in axle loads, load repetitions, tire pressures, and asphalt-concrete thicknesses, a need has developed for methodology to predict rut depths in advance of construction to mitigate potential safety problems, e.g., hydroplaning (Sousa et al., 1991). Concomitant with the development of analysis procedures which permit estimates of stresses, strains, and deformations resulting from traffic loads, pavement design systems have evolved which include provisions for rutting considerations. These have been referred to as analytically based, mechanistic, or mechanistic-empirical procedures (Sousa et al., 1991). A number of these procedures include criteria for limiting values of subgrade strain to levels that preclude rutting at the pavement surface. Examples include the Shell procedure (Claessen et al., 1977); the Asphalt Institute procedure (Shook et al., 1982); and the State of Kentucky methodology (Southgate et al., 1977). Some have recommended limitations on vertical subgrade stress (rather than strain), e.g., (Barksdale and Miller, 1977). Others have utilized statistically based rut depth prediction equations. For example, (Saraf et al., 1976) presented such an equation that incorporated surface deflection as computed from elastic layered analysis. Design limitations on strain or stress are based on the assumption that, if the maximum vertical compressive

strain or stress at the surface; of the subgrade is less than a critical value, then rutting will be limited to a tolerable level for a specified number of load applications. Unfortunately, such methodology does not necessarily preclude rutting which might occur in the asphalt-bound layer. The Shell method (Claessen et al., 1977) exemplifies a procedure which attempts to improve the above process by including additional analysis to estimate the amount of rutting occurring in the asphalt-bound layer.

This predictive methodology is an example of one of two analytical procedures which have evolved to estimate the amount of rutting. The procedure makes use of layered system elastic analysis and represents one of a number of such procedures termed layer-strain, in this approach repeated load test or creep test are used to characterize the permanent deformation behavior of asphalt concrete. The second approach makes use of closed form viscoelastic analyses to represent pavement structure with permanent deformation characterization by means of creep test (Monismith, 1976).

2.5.1 Layer-Strain Procedure

The layer-strain method consists of predicting rut depths using permanent-deformation characteristics determined from laboratory tests together with an analysis procedure for the pavement structure using either linear or nonlinear elastic theory. The general principle of this method was first proposed by (Barksdale, 1972 and Romain, 1972). While nonlinear elastic theory should provide more accurate results (Brown and Bell, 1979), it has not been used very extensively because of its added complexity. To predict the amount of permanent deformation that would occur after a given number of wheel load applications, each layer of the pavement structure is divided into several sublayers, and the stress state is calculated at the center of each sublayer directly beneath the wheel load Figure (2-15) using elastic analysis. With the average stress state at the

center of each sublayer, the corresponding axial plastic strain can be ascertained from the results of laboratory tests. The total rut depth for a given number of load repetitions is obtained by summing the products of the average plastic strain occurring at the center of each sublayer and the corresponding sub-layer thickness, i.e.:

$$\Delta_p = \sum_{i=1}^N [\varepsilon_{pi} \times \Delta_{zi}] \quad \dots\dots (2-2)$$

where

Δ_p is the total rut depth, ε_{pi} , is the average plastic strain in the center of the i-th sublayer, Δ_{zi} is the thickness of the i-th sublayer, and n is the total number of sub layers. This approach has been adopted in various forms by many researchers and, as noted above, the Shell design methodology represents a practical example of the use of this procedure also the [VESYS](#) elastic approach represents another example for this methods. The layer-strain method is considered a simplified engineering approach for predicting rut depth which permits the flexibility of using either linear or nonlinear elastic analysis. The following sections provide description for the [VESYS](#) program and in [KENLAYER](#).

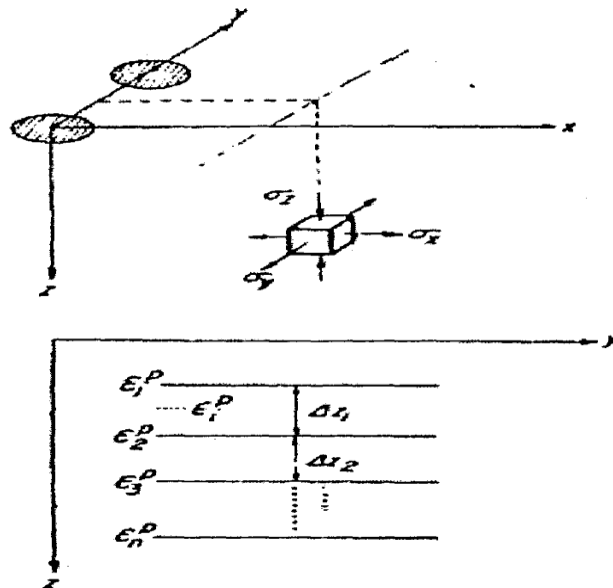


Figure 13-خطأ! لا يوجد نص من النمط المعين في المستند. Diagram of Pavement System Used to Estimate Permanent Deformation (Sousa, 1991)

1. VESYS Method

VESYS is a probabilistic and mechanistic flexible pavement analysis system, which was originally developed at the Massachusetts Institute of Technology (MIT) and subsequently improved further by MIT, the [Federal Highway Administration \(FHWA\)](#) and Austin Research Engineers (ARE). ([Yang H. Huang, 1993](#)). It includes four major interactive models, termed primary response, general response, damage and performance model. The primary response model calculate the layered system response (stress, strain and deflection) using either elastic method (material characterized by resilient modulus and Poisson ratio) or viscoelastic method (material characterized by creep compliance) under static loading, the general response model incorporate the effect of moving load using [Boltzmann's principles](#), the damage model includes three sub-models for the prediction of cracking, rut depth and roughness. The calculated values for these distresses are used as an input for the performance model to establish the variation of Present Serviceability Index over the analysis period ([Yang H. Huang, 1993](#)). The [VESYS](#) system is presented as a computer program, the final version for this program which used in this research is [VESYS 5](#). The rut depth predictive sub-model which is included in this version has two options, layer rutting and system rutting. The layer rutting option calculate the rut depth for each layer in the pavement structure, while the system rutting calculate the total rut depth for the whole system, the latter option take into account the subgrade rutting and use the total deflection in the pavement surface in the rut depth calculation ([Kenis and Wang, 1997](#)). Since this research deals with the rutting in asphalt concrete layers, the discussion presented below is limited to the layer rutting. Based on the relationship of ϵ_p -N of the asphalt concrete shown in [Equation \(2-3\)](#), the permanent strain for the N^{th} repetition can be calculated as follows ([Leahy, 1989](#))

$$\varepsilon_p = \varepsilon_t(N) - \varepsilon_r(N) = \varepsilon_r \cdot \mu \cdot N^{-\alpha} \quad \dots\dots (2-3)$$

where:

- $\varepsilon_p(N)$ = permanent strain for N^{th} repetition $\varepsilon_t(N)$ total strain for N^{th} repetition
- $\varepsilon_t(N)$ = total strain for N^{th} repetition
- $\varepsilon_r(N)$ = resilient strain for N^{th} repetition (assumed to be independent of N)
- α = is a permanent deformation parameter indicating the rate of decrease in permanent deformation as the number of load application increases.
- μ = is a permanent deformation parameter representing the constant of proportionality between permanent (plastic) and resilience (elastic) strains.

$$\varepsilon_{tz} = \frac{1}{E_{lo}(N)} [\sigma_z - \gamma(\sigma_x + \sigma_y)] = \frac{\sigma_z'}{E_{lo}(N)} \quad \dots\dots (2-4)$$

$$\varepsilon_{rz} = \frac{1}{E_{un}(N)} [\sigma_z - \gamma(\sigma_x + \sigma_y)] = \frac{\sigma_z'}{E_{un}(N)} \quad \dots\dots\dots (2-5)$$

where:

- ε_{tz} = total axial strain for N^{th} repetition
- ε_{rz} = resilient axial strain for N^{th} repetition
- σ_z = axial stress
- σ_x, σ_y = radial stresses
- $E_{lo}(N)$ =modulus during loading cycle for N^{th} repetition
- $E_{un}(N)$ =modulus during unloading cycle
- γ =Poisson's ratio
- σ_z' = resultant axial stress.

By substitution of equation (2-4) and (2-5) into equation (2-3) then:

$$\sigma_z' \left(\frac{1}{E_{lo}(N)} - \frac{1}{E_{un}(N)} \right) = \frac{\sigma_z'}{E_{un}(N)} \cdot \mu \cdot N^{-\alpha} \quad \dots\dots (2-6)$$

By solving equation (2-6) for $E_{lo}(N)$ yields the following:

$$E_{lo}(N) = \frac{E_{un}}{1 + \mu \cdot N^{-\alpha}} \quad \dots\dots (2-7)$$

The main assumptions included in the program are:

- 1- Poisson's ratio is constant for both loading and unloading conditions.
- 2- ϵ_r , and consequently E_{un} , is independent of the number of load repetitions.

The model of equation (2-6) is used to evaluate rutting in the asphalt concrete layer as follows:

- Determine the value of resilient modulus, E_{un}
- For a specific value of N , determine the value of $E_{10}(N)$.
- Using $E_{10}(N)$ in multilayer elastic solution, compute the deflections due to loading ($\zeta_{10}(N)$) and unloading ($\zeta_{un}(N)$).
- Compute the rut depth for the N^{th} load repetition, $R_d(N) = \zeta_{10}(N) - \zeta_{un}(N)$.
- Compute the total rut depth for N repetitions by:

$$R_d = \int_{i=1}^N R_d(N) \cdot dN \quad \dots\dots (2-8)$$

2.KENLAYER

The **KENLAYER** computer program developed by (Yang H. Huang, 1993) can be applied only o flexible pavements with no joints or rigid layers, the backbone of **KENLAYER** is the solution for an elastic multilayer system under a circular loaded area. The solutions are superimposed for multiple wheels, applied iteratively for nonlinear layers, and collocated at various times for viscoelastic layers. As results, **KENLAYER** can be applied to layered systems under single, dual, dual-tandem, or dual-tridem wheels with each layer behaving differently.

Either linear elastic; Nonlinear elastic or viscoelastic. Damage analysis can be made by dividing each year into a maximum of 24 periods, each with a group, either single or multiple. The damage caused by fatigue cracking and permanent deformation in each period.

Over all load groups is summed up to evaluate the design life. Damage analysis is performed for permanent deformation as following:

The failure criterion for rutting is expressed as

$$N_d = f_1(\varepsilon_c)^{-f_2} \quad \text{..... (2-9)}$$

In which N_d is the allowable number of load repetitions to limit permanent deformation, ε_c is the compressive strain on the top of subgrade, f_1 , and f_2 are constants determined from road tests or field performance. Value of f_1 and f_5 are suggested as 1.365×10^{-9} and 4.477 by [Asphalt Institute \(AI, 1982\)](#), 6.15×10^{-7} and 4.0 by [shell \(Claussen et al., 1977\)](#) and 1.13×10^{-6} and 3.571 by the [University of Nottingham \(Brown et al., 1977\)](#).

2.6. Finite Element Programs

Several specialized finite element computer programs for flexible pavements have been developed over the years. The best known of these include [ILLI-PAVE \(Raad and Figueroa, 1980\)](#) and [MICH-PAVE \(Harichandran, Yeh, and Baladi, 1990\)](#). In addition, general-purpose finite element codes like [ABAQUS \(Hibbitt, Karlsson, and Sorensen, 1998\)](#) and [DYNA \(Livermore Software Technology Corporation, 1999\)](#) have also been used for pavement analysis.

The project team of ([AASHTO Design Guide, 2002](#)) conducted a careful and thorough evaluation of several leading finite element programs for potential implementation in the guide ([Desai and Schwartz, 2000](#)). Three candidate finite element computer programs were initially identified for in-depth evaluation; the

[NCHRP](#) Project 1-37A panel subsequently requested that a fourth program ([UMPAD](#)) also be evaluated:

- ILLI-PAVE (University of Illinois; [M.R. Thompson](#))
- DSC2D (University of Arizona; [C.S. Desai](#))
- DYNA3D (Lawrence Livermore National Laboratories; [J. Hallquist](#))
- UMPAD (Battelle Memorial Institute/FHWA; [J. Kennedy](#))

The [ABAQUS](#) general purpose commercial finite element code was also an initial candidate based on its technical capabilities and its extensive past usage in research oriented pavement analysis. However, the project team eliminated [ABAQUS](#) early on because of its high licensing costs and restrictive licensing terms.

Two flexible pavement analysis methods have been implemented in the [AASHTO 2002 Design Guide](#). For cases in which all materials in the pavement structure can realistically be treated as linearly elastic, the [JULEA](#) multilayer elastic theory program is used to determine the pavement response. [JULEA](#) provides an excellent combination of analysis features, theoretical rigor, and computational speed for linear pavement analyses. In cases where the unbound material nonlinearity is also considered, the [DSC2D](#) nonlinear finite element code is used instead for determining the pavement stresses, strains, and displacements. A major advantage of [MLET](#)([Multilayer Elastic Theory](#)) solutions is very quick computation times. This method also have the advantage of requiring only minimal input data from the user; the principal inputs consist of the thicknesses and elastic properties of the layers and the tire contact radius and pressure. Solutions for multiple wheel loads can be constructed from the fundamental axisymmetric single wheel solutions via superposition, often automatically by the computer program. ([AASHTO Design Guide Report RR, 2004](#)).

The principal disadvantage of [MLET](#) solutions is the restriction to linearly elastic material behavior. Real pavement materials and the unbound materials in particular, often exhibit stress-dependent stiffness. The materials may even reach a failure condition in some locations, such as in tension at the bottom of the unbound base layer in some pavement structures. These nonlinearities vary both through the thickness of the layer and horizontally within the layer. Some attempts have been made to incorporate these material nonlinearity effects into [MLET](#) solutions in an approximate way ([Asphalt Institute, 1991](#); [Yang H. Huang, 1993](#)), but the fundamental axisymmetric [MLET](#) formulation makes it impossible to include the spatial variation of stiffness in a realistic manner. The axisymmetric [MLET](#) formulation also makes it impossible to include any effects of vertical discontinuities in the analysis or to include nonuniform tire contact pressures in a realistic manner. The limitations of [MLET](#) solutions are the strengths of [FE](#) analysis. Finite elements are not constrained to two-dimensional axisymmetric conditions (although this is all that is required for the [AASHTO 2002 Design Guide](#) and for this study purpose), but can be easily used for two-dimensional plane stress/strain and/or generalized to a fully three-dimensional formulation in the future. This enables modeling of discontinuities and realistic nonuniform tire contact pressure distributions.

In addition, the finite element method can simulate a wide variety of nonlinear material behavior; the underlying finite element formulation is not constrained to linear elasticity, as is the case with [MLET](#). Stress-dependent stiffness, tension and shear failures, plastic flow, and material damage can all be treated within the finite element framework. Implementation of some of these nonlinear material modeling capabilities for pavement analyses in the [2002 Design Guide](#) was used. Conversely, the strengths of [MLET](#) solutions are the weaknesses of [FE](#) analysis. The input required to define the finite element mesh is substantially greater—and

more time consuming to construct than the data required for a [MLET](#) analysis. This can be mitigated somewhat by use of interactive graphics pre- and post-processors for creating the mesh and viewing the results. Finite element analyses also require more computation time than do equivalent [MLET](#) solutions. Finite element solution times even with ([Infinite Boundary Elements](#)) are still 2 to 5 times longer than corresponding [MLET](#) analyses. Based on all of these considerations, it is the overwhelming judgment of the flexible pavement analysis group was that three dimensional finite element analysis is *not* appropriate for inclusion in the [2002 Design Guide](#). Some even argued that inclusion of three-dimensional finite analysis might actually *diminish* the overall quality of the guide and the likelihood of its successful adoption by state agencies ([AASHTO Design Guide Report RR, 2004](#)).

The project team of [AASHTO 2002 guide](#) is not alone in this assessment. During the [2002 Design Guide](#) workshop at the [January 2000](#) Transportation Research Board Meetings, [Dr. Samir Shoukry](#) ,([Shoukry, S., 1998a](#)), organizer of the first two International Conferences of 3D FEM for Pavement Analysis, also voiced the opinion that 3D finite element analysis was not an appropriate tool for routine design. Although the final selection of the finite element code to be implemented in the 2002 Design Guide had not yet been made at the time of preparing ([AASHTO Design Guide Report RR, 2004](#)), [LS-DYNA](#) was one of the leading contenders.

3 CHAPTER THREE

FINITE ELEMENT PROCEDURES FOR FLEXIBLE PAVEMENT ANALYSIS

3. Introduction

This chapter illustrates the usefulness of the finite element method in the analysis of three layer pavement systems subjected to loading. The method is capable of simulating the observed responses of pavements subjected to axle load. Finite element modeling of pavements, if validated, can be extremely useful, because it can be used directly to estimate primary response parameters without resorting to potentially costly field experiments. If accurate correlations between the calculated and the measured primary response parameters can be obtained, then the analytical model can be used to calculate primary response load equivalency factors, utilizing deflection based or strain-based equivalency factor methods. One analytical approach has been adopted for the flexible pavement response models in this chapter. For the most general case of linear material behavior the two-dimension analysis of finite element analysis is used to predict the stresses, strains, and displacements in the pavement system under traffic loading for given environmental conditions. The purpose of this chapter is to describe the rationale behind adoption of finite element techniques for the pavement response model and the criteria governing the selection of the [ANSYS, version 9.0](#) finite element computer program. This chapter also provides comparison between finite element techniques and multilayer elastic theory ([MLET](#)).

3.1. Objectives of Pavement Response Models

The purpose of the flexible pavement response model is to determine the structural response of the pavement system due to traffic loads and environmental influences.

Environmental influences may be direct (e.g., strains due to thermal expansion and/or contraction) or indirect via effects on material properties (e.g., changes in stiffness due to temperature and/or moisture effects).

Inputs to the flexible pavement response model include:

1. Pavement geometry

- a. Layer thicknesses
- b. Discontinuities (e.g., cracks, layer separations)

2. Environment

- a. Temperature vs. depth for each season
- b. Moisture vs. depth for each season

3. Material properties (adjusted for environmental and other effects, as necessary)

- a. Elastic properties
- b. Nonlinear properties (where appropriate)

4. Traffic

- a. Load spectrum—i.e., frequencies of vehicle types and weights within each vehicle type
- b. Tire contact pressure distributions and areas

The outputs from the pavement response model are the stresses, strains, and displacements within the pavement layers. Of particular interest are the critical

response variables required as inputs to the pavement distress models in the mechanistic-empirical design procedure. Examples of critical pavement response variables include:

- Tensile horizontal strain at the bottom of the AC layer (for AC fatigue cracking)
- Compressive vertical stresses/strains within the AC layer (for AC rutting) (the problem of this study).
- Compressive vertical stresses/strains within the base/subbase layers (for rutting of unbound layers)
- Compressive vertical stresses/strains at the top of the subgrade (for subgrade rutting)

Each pavement response variable must be evaluated at the critical location within the pavement layer where the parameter is at its most extreme value. For a single wheel loading, the critical location can usually be determined by inspection. For example, the critical location for the tensile horizontal strain at the bottom of the AC layer under a single wheel load is directly beneath the center of the wheel. For multiple wheels and/or axles, the critical location will be a function of the wheel load configuration and the pavement structure. Mixed traffic conditions further complicates the problem, as the critical location within the pavement structure will not generally be the same over all vehicle types. The pavement response model must search for the critical location for each response parameter and vehicle type in these cases.

Many techniques are available for determining the stresses, strains, and deformations in flexible pavement systems. These can be categorized as follows:

- Analytical (e.g., Burmister solution)
- Multilayer Elastic Theory (MLET)

- Rate-independent [BISAR, CHEVRON]
- Viscoelastic [VESYS]
- Finite Difference Methods [FLAC]
- Finite Element Methods (FEM)
- General purpose [ABAQUS, ANSYS]
- Pavement-specific [ILLI-SLAB, ILLI-PAVE, MICH-PAVE]
- Boundary Element Methods [BEASY]
- Hybrid Methods

The computer programs in each category are given in the square brackets; these are not intended to be all-inclusive. Details of each technique can be found in standard textbooks (e.g., [Yang H. Huang, 1993](#)) and in comparison studies in the literature ([Chen *et al.*, 1995](#)). The finite element method is by far the most versatile of these analysis techniques, providing capabilities for three-dimensional geometric modeling, nonlinear material characterization, large strains/deformations, dynamic analysis, and other sophisticated features. Note that a general pavement response model is capable of computing much more than the critical pavement response quantities. However, the primary objective of the mechanistic-empirical design methodology in any enhancement pavement design programs is to design pavements based on predicted pavement performance. The critical pavement response quantities required by the pavement distress models are, therefore, the primary outputs of the pavement response model. All other outputs are secondary and supplementary. The pavement response model must be capable of analyzing in a sufficiently realistic manner all of the flexible pavement new construction and rehabilitation scenarios considered in this study. In addition, the flexible pavement response model may also be used to analyze certain composite pavement scenarios. For the design approach proposed for any high level computer program such as

([AASHTO 2002 design guide](#)), the core analysis capabilities required in the flexible pavement response model are as follows:

- Linear material model for AC, other bound, and unbound layers (lowest hierarchical level for unbound material characterization)
- Stress-dependent material model (Nonlinear resilient modulus with tension cutoff) for unbound materials (highest hierarchical level for unbound material characterization).
- Quasi-static monotonically increasing loading from single or multiple wheel configurations
- Fully bonded, full slip, and intermediate interface conditions between layers

3.2. Accuracy of Pavement Performance Predictions

Accurate pavement performance predictions require more than just an accurate pavement response model. There are many sources of error in pavement performance predictions, and most are more difficult to control than the response model. Validation of pavement response models--i.e., comparisons of predictions against field measurements—is nonetheless an important topic, but unfortunately one that has received comparatively little attention in this work because of the high cost of equipments and difficulty of test conditions and related circumstances all that make getting field measurements are nearly impossible.

3.2.1. Sources of Error in Performance Predictions

[Ullidtz and Peattie \(1980\)](#) have succinctly and eloquently stated the realistic practical limitations of any pavement analysis:

“The ... results obtained [from the analysis] may deviate from the exact values. These deviations, however, should be considered in relation both to the simplifications made in the analysis and to the variations of materials and structures with space and time. Real pavements are not infinite in horizontal extent, and subgrade materials are not semi-infinite spaces. The materials are nonlinear, elastic, anisotropic, and inhomogeneous, and some are particulate; viscous and plastic deformations occur in addition to the elastic deformation occur In addition to the elastic deformations; loadings are not usually circular or uniformly distributed, and so on. To these differences between real and theoretical structures should be added the very large variations in layer thicknesses and elastic parameters from point to point, and during the life of a pavement structure. Moreover, it is a fact that precise information on the elastic parameters of granular materials and subgrades is in most cases very limited. For most practical purposes, therefore, the accuracy of the...methods should be quite sufficient.”

(Ullidtz and Peattie, 1980) made these observations twenty years ago in the context of their equivalent thickness approximate method and linear elasticity, but the general thrust of the comments applies equally well today to even the most sophisticated pavement analysis techniques.

The design methodology should be based on mechanistic-empirical predictions of pavement performance. There are many components and subsystems involved in making these predictions: inputs such as traffic loading, environmental

conditions, and material properties; the pavement response model; and the empirical distress prediction models. Each of these components has an inherent inaccuracy or uncertainty, In general, the level of inaccuracy or uncertainty in the material inputs and, especially, the distress prediction models will be far greater than that of the pavement response calculations. Within this context, for example, the additional modest differences between 2D vs. 3D pavement response calculation may be insignificant in practical terms. This point was already recognized during earlier attempts to develop a mechanistic-empirical pavement design procedure ([Thompson, 1990](#)).

3.3. Selection of Analysis Method

The choice of an appropriate analysis method for the flexible pavement response model is governed by many considerations, both theoretical and practical. Key issues include material behavior, problem dimensionality, loading conditions, computational practicality and implementation considerations.

3.3.1. Material Behavior Predictions

As a first approximation, the materials used in flexible pavement systems are often treated as linearly elastic. As shown conceptually in [Figure \(1-3\) a](#), the stresses in a linearly elastic material are proportional to strain, with the proportionality constant equal to Young's modulus, E . However, unbound pavement materials (and asphalt concrete at very high temperatures) are more accurately characterized using a stress-dependent modulus [Figure \(1-3\) b](#), perhaps in conjunction with a tension cut-off for the unbound materials.

Explicit consideration of the rate dependence of asphalt concrete requires viscoelastic analysis capabilities. This is needed for this study, where rate effects

are incorporated by adjusting the AC stiffness using the creep and recovery compliance vary with time.

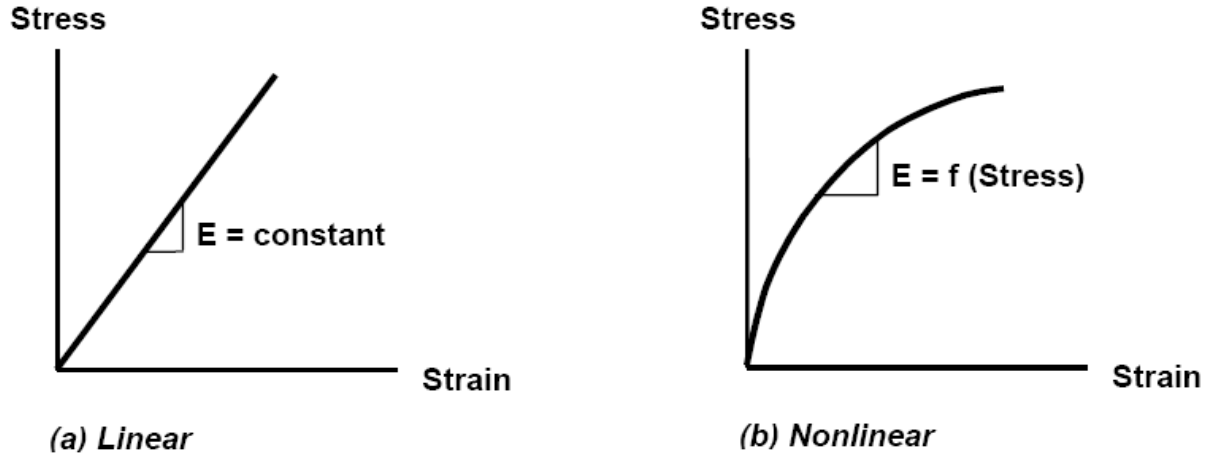


Figure 1-خطأ! لا يوجد نص من النمط المعين في المستند. Linear and Nonlinear Material Behavior

3.3.2. Loading Condition

In the mechanistic method of design, it is necessary to know the contact area between tire and pavement, so the axle load can be assumed to be uniformly distributed over the contact area. The size of contact area depends on the contact pressure. As indicated by Figure (2-3), the contact pressure is greater than the tire pressure for low-pressure tires, because the wall of tires is in compression and the sum of vertical forces due to wall and tire pressure must be equal to the force due to contact pressure; the contact pressure is smaller than the tire pressure for high- pressure tires, because the wall of tires is in tension. However, in pavement design the contact pressure is generally assumed to be equal to the tire pressure. Because heavier axle loads have higher tire pressures and more destructive effects on pavements, the use of tire pressure as the contact pressure is therefore on the safe side. (Yang H. Huang, 1993)

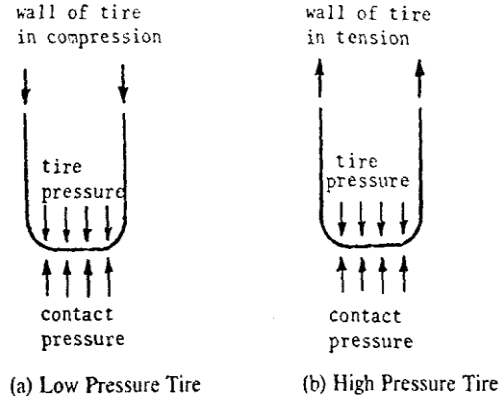


Figure 2-خطأ! لا يوجد نص من النمط المعين في المستند. Relationship between Contact Pressure and Tire Pressure (Yang H. Huang, 1993)

Heavier axle loads are always applied on dual tires. Figure (3-3) shows the approximate shape of contact area for each tire, which is composed of a rectangle and two semicircles. By assuming a length of L and a width of $0.6L$, the area of contact $A_c = \Pi * (0.3L)^2 + (0.4L)(0.6L) = 0.5227L^2$, or

$$L = \sqrt{\frac{A_c}{0.5227}} \quad \dots (3-1)$$

In which A_c = contact area, which can be obtained by dividing the load on each tire by the tire pressure. The contact area shown in Figure (3-3) was used previously by PCA (1966) for the design of rigid pavements. The current (PCA, 1984) method is based on the finite element procedure and a rectangular area is assumed with a length of $0.8712L$ and a width of $0.6L$, which has the same area of $(0.5227L^2)$, as shown in Figure (3-3) b. These contact areas are not axisymmetric and cannot be used with the layered theory or axisymmetric 2D finite elements for one contact load. When the finite element (axisymmetric algorithm) is used for flexible pavement design, it is assumed that each tire has a circular contact area. This assumption is not correct, but the error incurred is believed to be small, to simplify the analysis of flexible pavements.

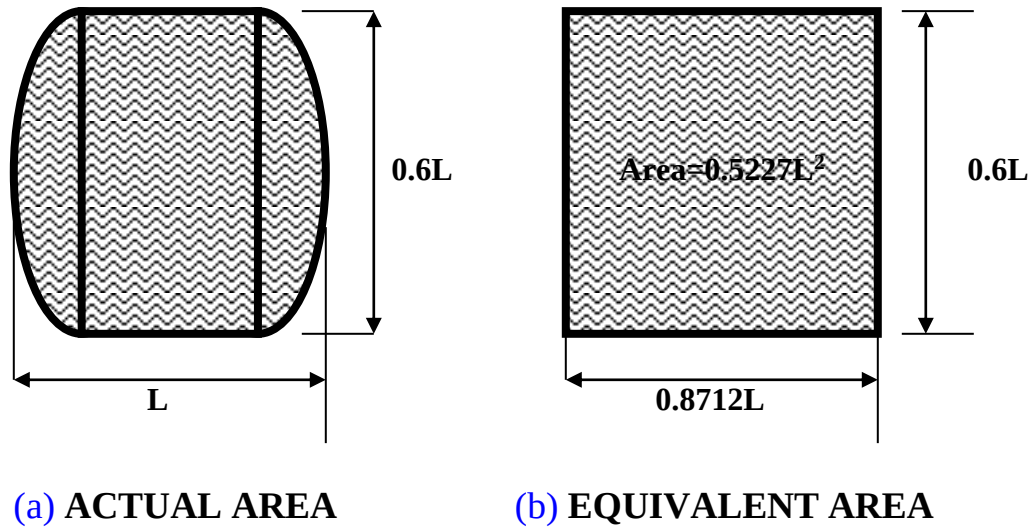


Figure 3-خطأ! لا يوجد نص من النمط المعين في المستند. 3, Dimension of Tire Contact Area

Consequently it has been used a circle loading area with a radius (r) Figure (3-4)



3.3.3. Problem Dimensionality

The required dimensionality (1D vs. 2D vs. 3D) of an analysis is a function of geometry and loading conditions. It should be investigate flexible pavement analysis scenarios in which three-dimensional geometric effects may be important. The issue of two- vs. three-dimensional finite element analysis for flexible pavements has become quite controversial in recent years (GAO, 1997). The issue is not whether it can perform three-dimensional finite element analyses for pavements, including nonlinear material behavior and dynamic response, if necessary; clearly, these types of analyses are well within the capabilities of any number of available finite element programs. The real issue is whether it should

implement these capabilities for design, and in particular for the flexible pavement design formulation proposed for the design pavement program.

In most flexible pavement problems, there are no vertical discontinuities (e.g., slab joints) and the only source of three-dimensionality is multiple wheel loads. If the behavior of all materials in the pavement structure can be treated as linear, then a 3D solution can be constructed from simpler 2D axisymmetric analysis can results via superposition. Although nonlinear material behavior invalidates the principle of superposition from a rigorously theoretical standpoint, superposition may still provide acceptably accurate calculations of critical pavement response parameters in most pavement structures ([Schwartz, 2000](#)). Discrete vertical discontinuities are important three-dimensional geometrical features in some flexible and composite pavement rehabilitation scenarios, in particular with regard to reflection cracking. These vertical discontinuities invalidate the two-dimensional assumption of axial symmetry. However, reflection cracking has been excluded from the scope of modern pavement programs such as the [2002 Design Guide](#) and, as a consequence, modeling of vertical boundaries/discontinuities is not required and a 2D axisymmetric analysis will be sufficient from a geometrical viewpoint.

3.3.4. Computational Practicality

It must be recognized that the type of flexible pavement [FEA](#) required for the purposes of the pavement response should not complex. They consist of regular meshes, simple material models with relatively gentle nonlinearities, and straightforward monotonic loadings¹. The key difficulty lies in the large number of analyses required for the incremental damage/reliability design formulations

¹ This is not to imply that all pavement finite element calculations are simple. Modeling of cyclic traffic loading with nonlinear material models, characterization of material degradation and fracturing at reflection cracks, and dynamic analysis under moving vehicle loads are challenging problems that are at or beyond the limit of current study. However, none of these advanced features is required for pavement response model.

implemented in the guide. The pavement response model must be able to perform all analysis calculations in a practically acceptable amount of time. The time required to perform a flexible pavement response analysis is a complex function of the type of analysis methodology (e.g., [MLET](#) vs. [FEM](#)), the dimensionality of the problem (e.g., [2D](#) vs. [3D](#)), the complexity of the pavement structure, the degree of material nonlinearity to be considered, and computer type and speed. Very broad estimates of required calculation times on current generation personal computers can be summarized as follows ([AASHTO 2002 design guide](#)):

- 2D linear analyses (MLET/FEM):.....Seconds
- 2D nonlinear FE analyses:.....10s of seconds
- 3D linear FE analyses:.....10s of minutes to hours
- 3D nonlinear FE analyses: Hours to 10s of hours

Conventional wisdom holds that axisymmetric multilayer elastic theory solutions (MLET) are less computation-intensive than axisymmetric two-dimensional linear finite element (FE) solutions. However, upon closer examination it is not clear how substantial this disparity will be for realistic pavement design scenarios.

The execution time for MLET solutions will increase with number of layers and with number of required stress computation points (e.g., to determine the critical locations for the critical response parameters, and for superposition of multi-wheel loading cases). In contrast, a FE solution (assuming a sufficiently fine mesh) will not require significant additional computation time as the number of layers and/or stress computation points increases. The finite element meshing already divides the pavement structure into many thin layers (theoretically, each layer of elements in the mesh could be assigned properties corresponding to different pavement layers) and the FE algorithms automatically determine the

stresses and strains at all element integration points and the 2D axisymmetric linear FE analysis is estimated to require slightly more time than a similar 2D axisymmetric MLE analysis, but still within the same order of magnitude (AASHTO 2002 design guide).see Table (3-1),for the first and second row.

It should be note that results of table (3-1) for (Ayres, 1997) were used to convert (Chen *et al.* 1995) computation times for a 25 MHz 386 PC to equivalent computation times for a 266 MHz Pentium II system; a conversion factor of 70 was used (i.e., 266 MHz Pentium II is 70 times faster than a 25 MHz 386).

Table (3-1).

Table 3-1, Computational Times for Various Analysis Methods (AASHTO Design Guide Report RR, 2004)

Analysis Method	Problem Size	Program	Computer	CPU Time	Source
2D axisymmetric MLE	2- 8 layers	JULEA	266 MHz Pentium II	0.5-2.5 sec	Ayres (1997)
2D axisymmetric linear FE	Typical	Estimate	266 MHz Pentium II	2 sec	Estimate
2D linear FE (Rigid pavement: plate elements on Winkler spring subgrade, wheel loading, and no temperature curling)	4, 000 DOF	ILLI-SLAB	200 MHz Pentium	8 sec	AASHTO desgin guide
2D axisymmetric nonlinear FE	Unknown (coarse mesh)	ILLI-PAVE	266 MHz Pentium II	5 sec	Chen <i>et al.</i> (1995)
2D axisymmetric nonlinear FE (Flexible pavement w/ material nonlinearity)	Typical	ILLI-PAVE	266 MHz Pentium II	1 min	Estimate
"2.5D" nonlinear FE (Rigid pavement: plate elements on Winkler spring subgrade, wheel loading, and temperature curling)	4, 000 DOF	ILLI-SLAB	200 MHz Pentium	12 sec	AASHTO design guide
"2.5D" nonlinear FE (Rigid pavement: plate elements on Winkler subgrade, wheel loading, and temperature curling)	8, 000 DOF	ILLI-SLAB	200 MHz Pentium	49 sec	AASHTO design guide
3D linear FE	Typical	Estimate	Workstation	15-120+ min	Hjelmstad, Kim, and Zuo (1997)
3D linear FE (flexible pavement)	26,220 DOF	ABAQUS	450 MHz Pentium II	7.5 min	Davids (1998)
3D linear FE w/ layer separation/contact (rigid pavement)	19,155 DOF	EVERFE	Pentium 166	54 min	
3D linear FE w/ layer separation/contact (rigid pavement)	~60,000 DOF	ABAQUS	Cray YMP	Several hours	Hammons (1998)
3D nonlinear FE	11,500 DOF (coarse mesh)	ABAQUS	200 MHz Pentium Pro	8.3 min	Hibbitt, Karlsson, and Sorensen
3D nonlinear FE (flexible pavement)	26,220 DOF	ABAQUS	450 MHz Pentium II	20-100 min	AASHTO design guide
3D nonlinear FE (rigid pavement plus subgrade)	86,500 DOF (fine mesh)	NIKE3D	SG Indigo workstation	11.2 hrs	Brill, Hayhoe, and Lee (1997)
3D nonlinear FE	--	Estimate	Workstation	Up to 24+ hrs	Hjelmstad, Kim, and Zuo (1997)

3.3.5. Some Factors Influencing Analysis Times for Pavement

The following points—in decreasing order of importance—will have a major impact on analysis efficiency:

1. *Two- vs. three-dimensions*: Computation time for a three-dimensional analysis is up to two to three orders of magnitude longer than for a corresponding two-dimensional calculation (see [Section 1.3.3](#)). Input (mesh) preparation and post-processing resource demands (computer and user time, disk space, etc.) are also substantially greater.
2. *Static vs. dynamic*: Computation time for a dynamic analysis is several orders of magnitude longer than for a corresponding static calculation. There is no compelling reason for performing dynamic analyses in the ([AASHTO 2002 Design Guide](#)) also in this study, especially given the level of idealization/approximation in the other areas of the design approach (e.g., material characterization).
3. *Formulation—implicit vs. explicit*. For linear static problems, implicit finite element formulations (e.g., [ILLI-PAVE](#), [DSC2D](#)) are faster than a corresponding explicit formulation of the type implemented in [DYNA3D](#).¹² This advantage decreases when nonlinear material behavior is introduced, as the computation time for the implicit formulation may increase significantly with nonlinearity (in order to achieve equilibrium convergence iteratively within each load increment) while the time for a corresponding explicit formulation will increase by a smaller extent (pure incremental formulation), fortunately the both methods are available in [ANSYS 9.0](#) program.
4. *Mesh design*: There are three major efficiency issues in the design of meshes for pavement problems:

- **Number of elements or degree of mesh refinement.** Fewer elements mean faster analyses, albeit at degraded solution accuracy. The trade-off between accuracy and solution time can only be established via parametric sensitivity studies. Note that this consideration is not specific to any particular finite element program, but applies equally to all.
- **Location of boundary conditions.** Conventional wisdom suggests that the vertical side boundaries for the mesh should be no closer than 10 to 12 tire radii; the horizontal bottom boundary at the base of the subgrade should be no closer than 50 tire radii e.g., see ([Duncan et al. 1968](#)) and ([Chen et al. 1995](#)). These minimum distances require meshes having many elements and nodes outside the primary zone of interest, i.e., the highly stressed region beneath the tire load. Special analysis features such as the infinite elements available in many codes (e.g., the dynamic compliant boundaries in [UMPAD](#)) can be used effectively to move the boundaries closer and thus reduce the number of elements and nodes.
- **Element formulation** Higher order elements (e.g., [8-node quadrilaterals](#)) can be more efficient and therefore require fewer elements than do lower order elements (e.g., [4-node quadrilaterals](#)) for comparable accuracy in linear problems. For nonlinear analyses, more but smaller low-order elements are often more effective than fewer but larger high-order elements for tracking the nonlinear response. The ideal approach is perhaps a hybrid set of elements in which high-order elements are used in the areas of primarily linear response (e.g., the AC layer and regions of the unbound layers far removed from the loading) and low-order elements are used in the

areas of nonlinear response (e.g., the regions of the unbound layers nearest the loading).

5. *Equation solver:* In implicit formulations (see Item 3 above), most of the computation time for larger finite element analysis is spent in solving the system of simultaneous equations. Algorithms are available that can automatically re-sequence equations to minimize bandwidth or wave front in order to optimize the computational efficiency of the equation solver. Fortunately, the regular nature of most pavement finite element meshes make optimal node numbering fairly straightforward during mesh design. The equation solver algorithms themselves must also be highly efficient, but fortunately, these routines are widely available today and incorporated in most modern finite element codes.
6. *Miscellaneous numerics:* This encompasses all of the miscellaneous details of the finite element algorithms that are related to computational efficiency (e.g., iteration algorithms for implicit formulations, numerical integration, disk I/O operations, etc). For single processor finite element codes, most of these miscellaneous items are of comparatively minor importance to the overall computational efficiency. For multiprocessor finite element codes, the degree of parallelism in the algorithms becomes a dominant consideration. However, multi-processor algorithms are not envisioned for the [AASHTO 2002 Design Guide](#).

3.4. Selection of Finite Element Program

Several specialized finite element computer programs for flexible pavements have been developed over the years. The best known of these include [ILLI-PAVE](#) (Raad and Figueroa, 1980) and [MICH-PAVE](#) (Harichandran, Yeh, and Baladi, 1990). In addition, general-purpose finite element codes like [ABAQUS](#) (Hibbitt, Karlsson, and Sorensen, 1998) and [DYNA](#) (Livermore Software Technology Corporation, 1999) have also been used for pavement analysis. The candidate finite element computer program is [ANSYS](#) due to the powerful, sufficient for purpose of this study, and supporting the [GUI](#) ([Graphical User Interface](#)) leading to become easy handling.

What is [ANSYS](#)? ANSYS is a finite element analysis ([FEA](#)) software package. It uses a preprocessor software engine to create geometry. Then it uses a solution routine to apply loads to the meshed geometry. Finally it outputs desired results in post-processing. Finite element analysis was first developed by the airplane industry to predict the behavior of metals when formed for wings. Now FEA is used throughout almost all engineering design including mechanical systems and civil engineering structures. ANSYS is used throughout industry in many engineering disciplines. This software package was even used by the engineers that investigated the World Trade Center collapse in [2001](#). More information about the ANSYS FEA package and other ANSYS products can be found at [<www.ansys.com>](http://www.ansys.com).

3.5. Finite Element Solutions

The computation time required for a finite element solution involves a tradeoff between solution accuracy and speed. A finer mesh (more and smaller elements) will provide a more accurate solution at the expense of increasing computation time. The question of required computation time is thus most appropriately framed as: “What computation time is required to achieve a solution of acceptable accuracy?” In order to investigate this question, a series of linearly

elastic finite element analyses were performed for a typical pavement structure consisting of 100 mm of asphalt concrete ($E = 3500$ MPa, $\nu = 0.35$) over 200 mm of crushed stone base ($E = 350$ MPa, $\nu = 0.3$) over a soft subgrade ($E = 100$ MPa, $\nu = 0.4$). A single Wheel load was modeled as a uniform pressure of 550 kPa over a circular area of 150 mm radius. This single wheel load is not entirely consistent with the dual tandem loading used for the MLET part of this timing study, but the additional computation times required to superimpose the FE solutions is expected to be minimal. All analyses were performed under linearly elastic axisymmetric conditions using [ANSYS 9.0](#) software and check against the ([MLET](#)) solution done by ([KENLAYER¹](#)). Three separate but similar finite element meshes were developed to investigate the tradeoff between accuracy and speed:

- A coarse refinement mesh consisting of 396 elements, with a smallest element size of 50 mm by 50 mm Figure (3-5). Two elements spanned the thickness of the asphalt concrete surface layer and four elements spanned the thickness of the base layer.
- A medium refinement mesh consisting of 1584 elements, with a smallest element size of 25 mm by 25 mm Figure (3-6). Four elements spanned the thickness of the asphalt concrete surface layer and eight elements spanned the thickness of the base layer.
- A fine refinement mesh consisting of 3564 elements, with a smallest element size of 16.7 mm by 16.7 mm Figure (3-7). Six elements spanned the thickness of the asphalt concrete surface layer and twelve elements spanned the thickness of the base layer.

The finite element meshes extended 2100 mm (14 tire radii) in the radial direction and 2300 mm (approximately 15 tire radii) in the vertical direction.

¹ See the literature review under KENLAYER title.

Note that the bottom mesh boundaries are closer than conventional meshing guidelines would suggest by (Duncan, Monismith, and Wilson, 1968) recommend that the horizontal lower boundary of the finite element mesh be located no closer than 18 tire radii for a homogeneous elastic system and no closer than 50 tire radii for a layered system. The recommend that the vertical side boundary of the finite element mesh be located at least 12 tire radii from the center of the tire.

However, this is acceptable for the purposes of the present study. Extending the meshes to greater depths would require comparatively few additional elements, as the elements are quite large in this region of the mesh. A preferable solution is to line the bottom (and vertical sides) of the meshes with infinite elements, as discussed in the following subsection.

Figure (3-11) through Figure (3-13) illustrate the trade-offs between mesh refinement and solution accuracy for the computed surface displacements and the vertical and horizontal stresses along the load centerline, respectively. The MLET solution results obtained using KENLAYER are also shown on these figures. The FE results in Figure (3-11) through Figure (3-13) were computed using ANSYS. The surface displacements Figure (3-11) for the medium and fine refinement meshes are virtually identical, although they are less than the KENLAYER results because of the rigid lower boundary in the finite element meshes. The vertical and horizontal stresses in Figure (3-12); Figure (3-13) are nearly identical for the medium and fine refinement meshes, and these also agree very closely with the KENLAYER quantities. Surprisingly, even the coarse refinement mesh gave reasonable results for the surface displacements and load line stresses. Qualitatively, however, the results in Figure (3-11) through Figure (3-13) suggest that, of the three meshes studied, the medium refinement (1584 elements, with 4 element layers in the AC and 3 element layers in the base) mesh

is the minimum refinement necessary to achieve acceptable solution accuracy. It should be note that all material properties, load configurations and dimensions of geometric case are reported in ([AASHTO Design Guide Report RR, 2004](#)) the units that used was SI.U (System of International Units) and the same units is applying in [ANSYS 9.0](#) for comparison purposes.

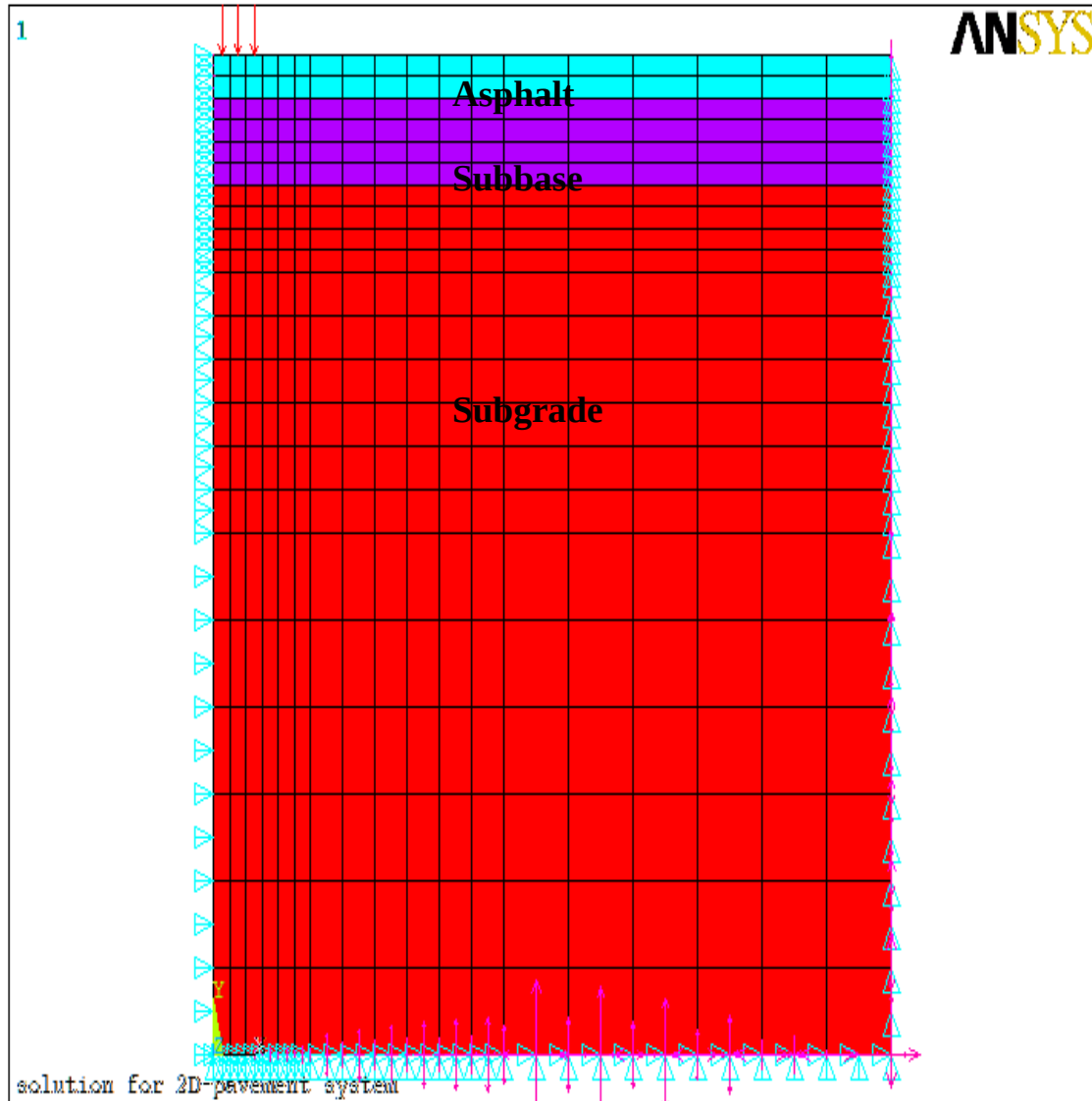


Figure 5- خطأ! لا يوجد نص من النمط المعين في المستند. Coarse Refinement Mesh (396 elements)

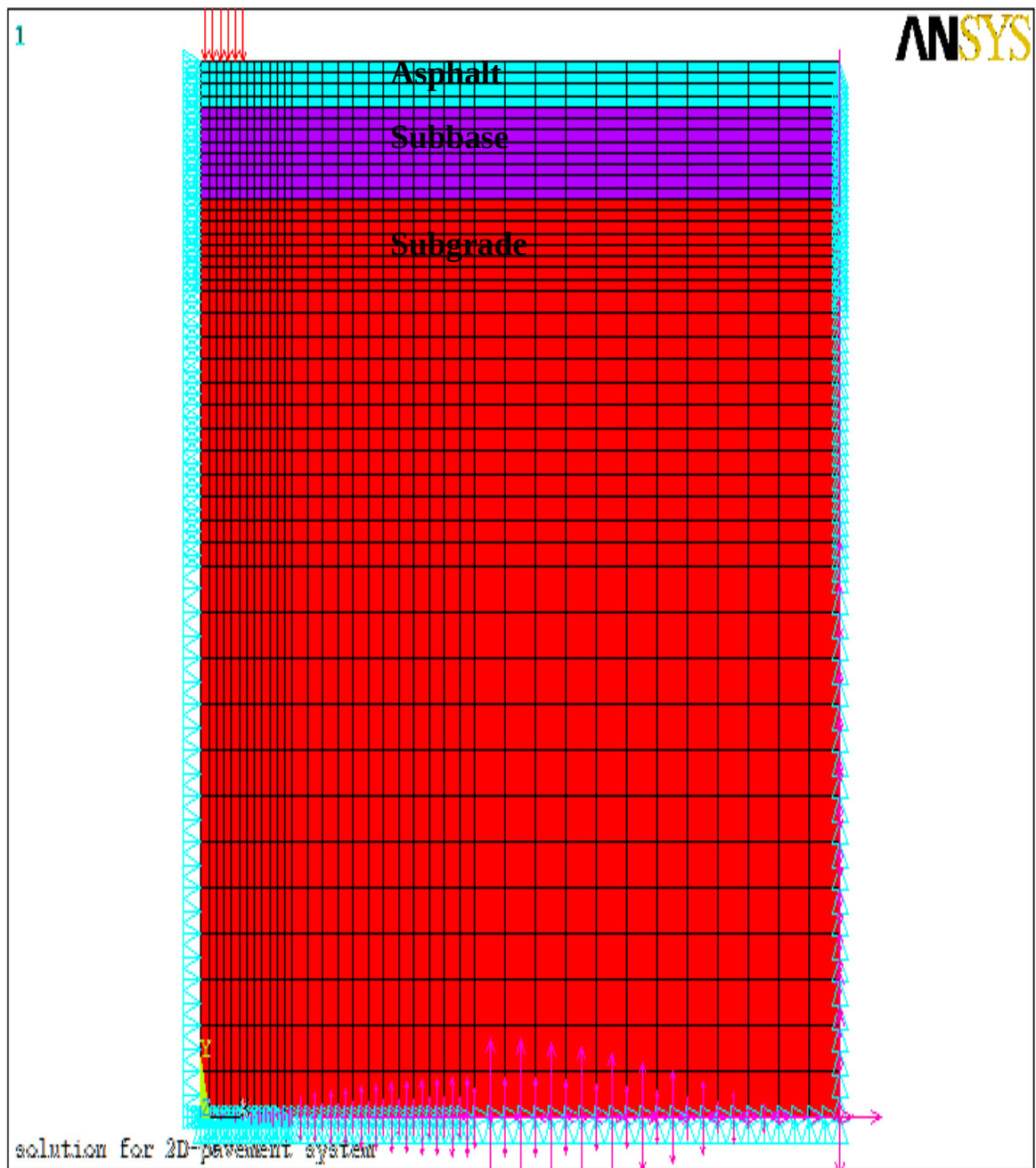


Figure 6-خطاً! لا يوجد نص من النمط المعين في المستند. Medium Refinement Mesh (1584 elements)

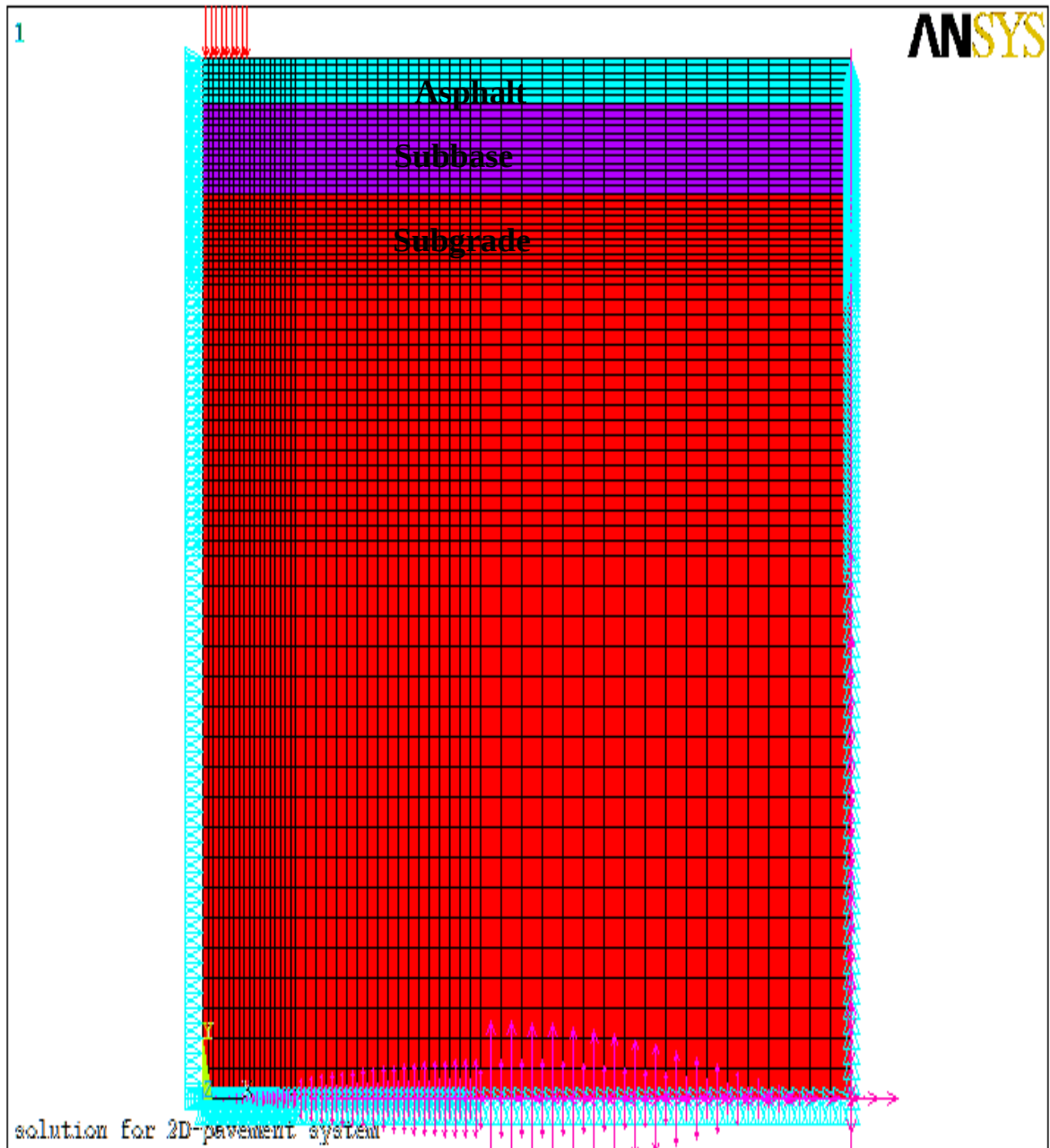


Figure 7- خطأ! لا يوجد نص من النمط المعين في المستند. Fine Refinement Mesh (3564 elements)

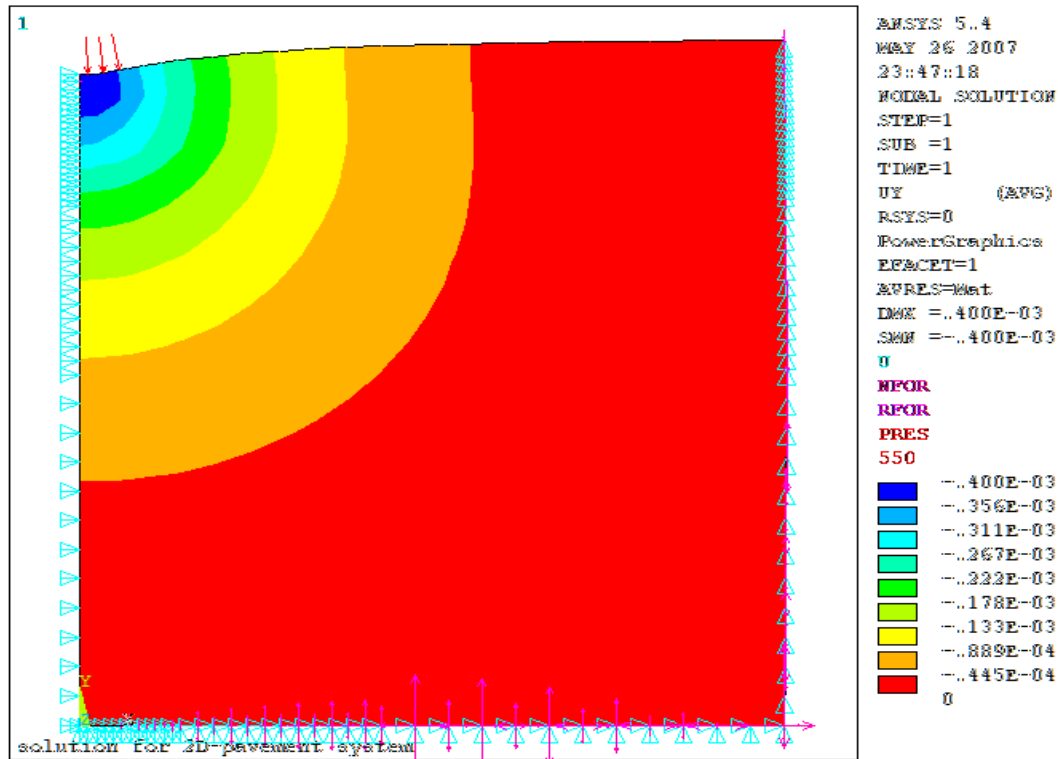


Figure 8- خطأ! لا يوجد نص من النمط المعين في المستند. Solution of Displacement in Y Direction for Coarse Mesh

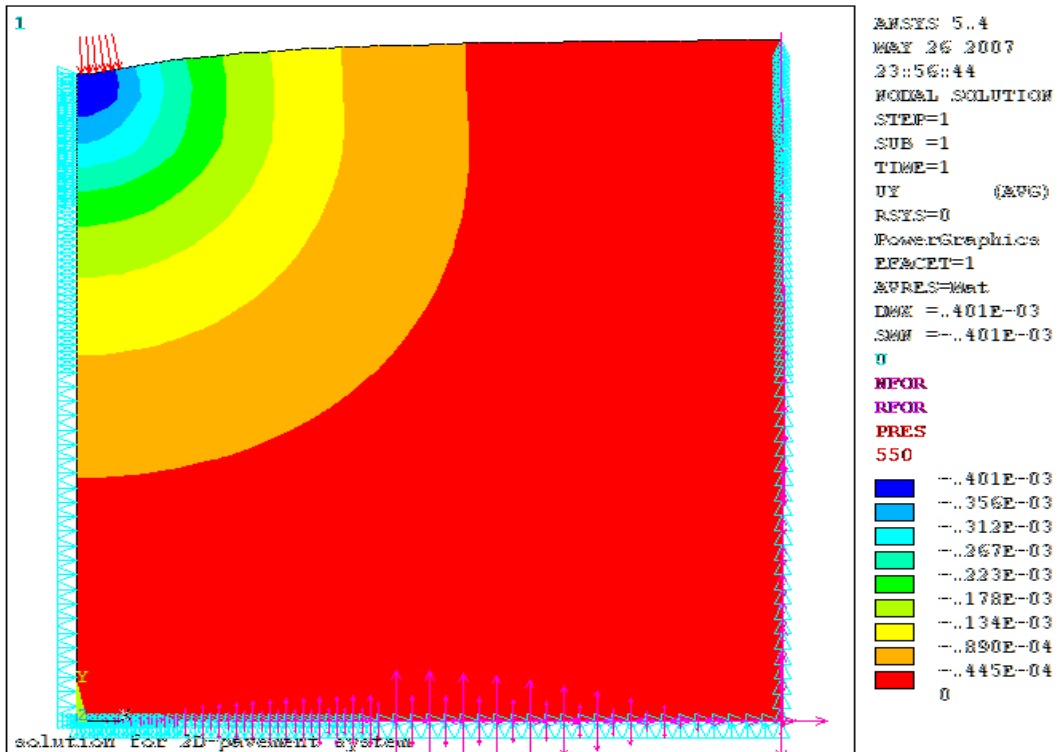


Figure 9- خطأ! لا يوجد نص من النمط المعين في المستند. Solution of Displacement in Y Direction for Medium Mesh



Figure 10, Solution of Displacement in Y Direction for Fine Mesh

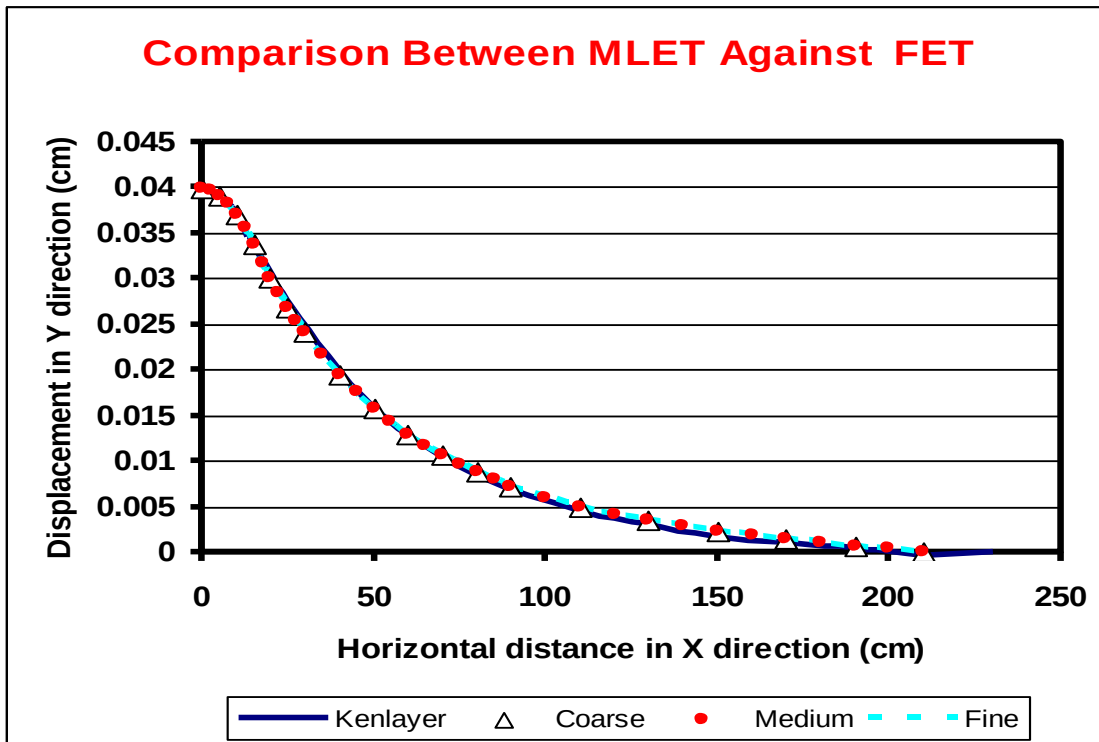


Figure 11, Surface Displacements Computed at Different Mesh Refinements.

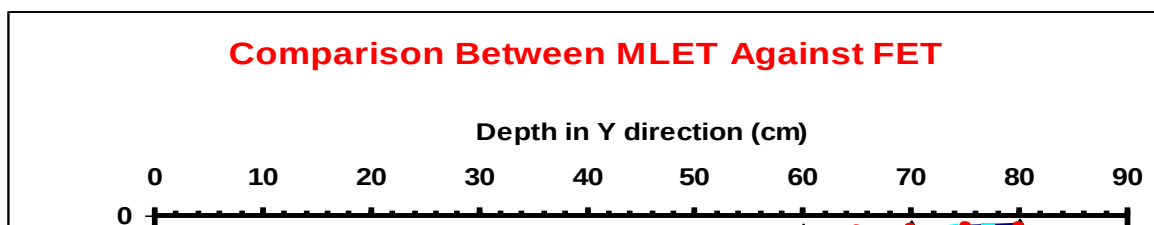


Figure 12- Vertical Stresses along Centerline at Different Mesh Refinements. خطأ! لا يوجد نص من النمط المعين في المستند.

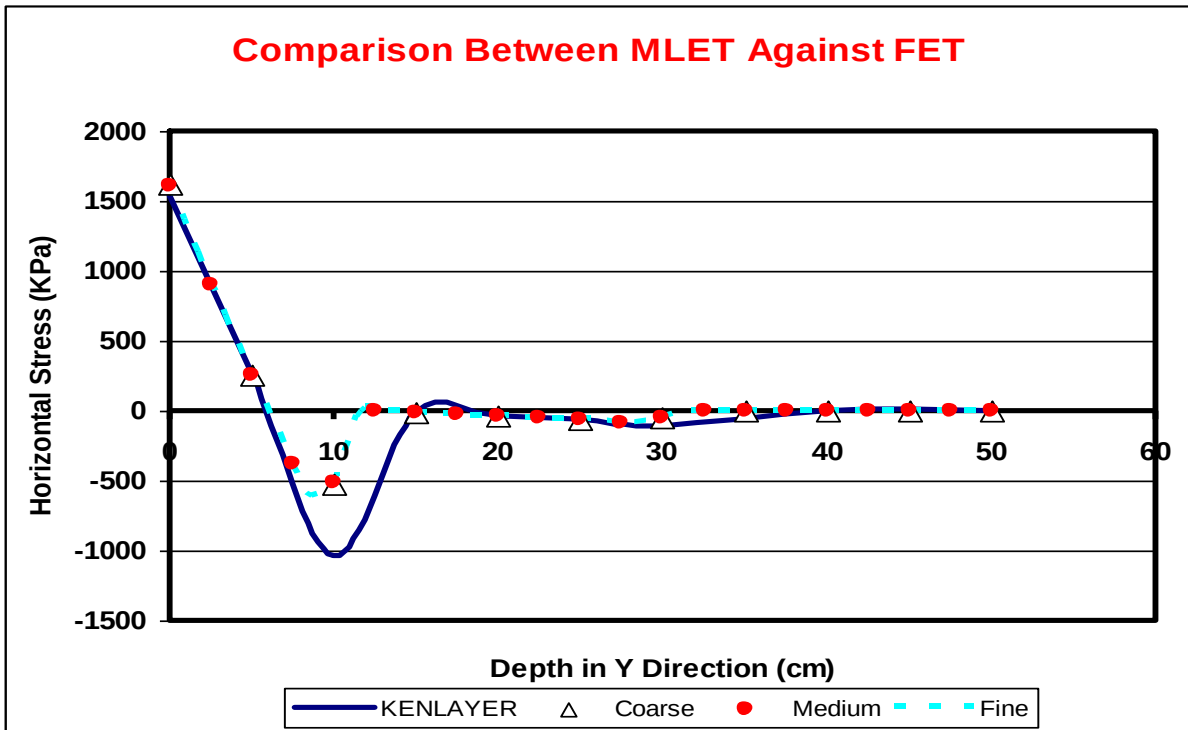


Figure 13- Horizontal Stresses along Centerline at Different Mesh Refinements. خطأ! لا يوجد نص من النمط المعين في المستند.

3.6. Primary Response

3.6.1 Elastic Solution

A two-dimensional finite element model was developed, using [ANSYS version 9.0](#), to simulate a typical pavement structure. The simulation was developed to initially compare pavement responses to those obtained from the classical theory of linear elasticity as shown in previous section. The analysis simulated a single tire has nominal tire inflation pressure of 83 psi (572 KPa). The following sections describe some of the features of the developed FE model. The dimensions of the modeled portion were 6.889 ft (2.1m) in horizontal direction and 7.54 ft (2.3 m). These dimensions were selected based on limitations of ([Duncan, Monismith and Wilson, 1968](#)), while keeping the elements' sizes within acceptable limits. The generated mesh was designed to give an optimal accuracy medium mesh around the load and coarse mesh far from it. To improve the rate of convergence, 8-node linear rectangular was used.

The materials that use in this model are shown in [Table \(3-2\)](#).

Table 3-2, Material Properties for Each Layer

Layer	Resilience modulus, psi	Possion ratio
Wearing course	187,580	0.35
Binder course	161000	0.35
Base course	127500	0.35
Subbase course	15100	0.4
Subgrade course	5800	0.45

3.6.2 Section Transformation (Equivalent thickness).

The " method of equivalent thicknesses" used for layered structures was first suggested by Odemark in 1949 (Ullidtz et. al, 1979).Subsequently, their use in the analysis and performance prediction of the pavement structure becomes widespread (Glynn, et. al, 1977; Nemesdy et. al, 1977; Ullidtz et. al, 1979).The basic principle of the method is that the layered system with N layers can be transformed into equivalent layer by fixing the stiffness constant for the case under consideration. The stiffness is defined by (Odemark) as the product of moment of inertia and the modulus of elasticity for two layers system, the derivation of the transforming relationship becomes as following:

Stiffness of layer (1) +stiffness of layer (2) =stiffness (equivalent layer)

$$EI_1 + EI_2 = EI_{eq}.$$

$$E_1\left(\frac{b_1h_1^3}{12}\right) + E_2\left(\frac{b_2h_2^3}{12}\right) = Eeq\left(\frac{beqheq^3}{12}\right)$$

For $b_1=b_2=beq$.

The relation becomes: $E_1h_1^3 + E_2h_2^3 = Eeq.h_{eq}^3$

Divide by Eeq : $\frac{E_1h_1^3}{Eeq} + \frac{E_2h_2^3}{Eeq} = h_{eq}^3$ This equation can be simplified by

rearrange and raise both side of equation to power 1/3, this led to

$$h_{eq}^3 = h_1 \cdot \left(\frac{E_1}{Eeq}\right)^{1/3} + h_2 \cdot \left(\frac{E_2}{Eeq}\right)^{1/3} \dots\dots (3-2)$$

This relationship can be generalized for N layers:

$$h_{eq}^3 = h_1 \cdot \left(\frac{E_1}{Eeq}\right)^{1/3} + h_2 \cdot \left(\frac{E_2}{Eeq}\right)^{1/3} + h_3 \cdot \left(\frac{E_3}{Eeq}\right)^{1/3} + \dots\dots + h_n \cdot \left(\frac{E_n}{Eeq}\right)^{1/3} \dots (3-3)$$

where:

- h=layer thickness.
- b=the width of pavement section.
- I=moment of inertia (for rectangular section $=bh^3/12$).
- E=modulus of elasticity (characterized as resilient modulus).

The [Equation \(3-2\)](#) was used to convert the thicknesses of binder and base courses into an equivalent layer thickness for wearing course. This transformation will save the effort for testing the asphalt concrete mixtures for both the binder and base courses for permanent deformation characterization, and will simplify the analysis of pavement structure and rut depth prediction.

For the practical solution of finite element model ([Primary response model](#)), according to the resilient modulus of the layers to be transformed (binder and base)

And that for wearing layer, the thicknesses and the transformation factor $f=(E_i/E_{eq})^{1/3}$ are shown in [Table \(3-3\)](#) below. The resulting geometry of the transformed pavement section is shown in [Figure \(3-14\)](#).

Table 3-3, Equivalent Thicknesses

Layer	Initial thickness cm (inch)	Transformation factor, f	Equivalence thickness cm (inch)
Wearing	5 (2)	1	5 (2)
Binder	7 (2.8)	0.9503	6.65 (2.66)
Base	10 (4)	0.88	8.79 (3.52)
Total Equivalent Thickness			= 20.44 (8.2)

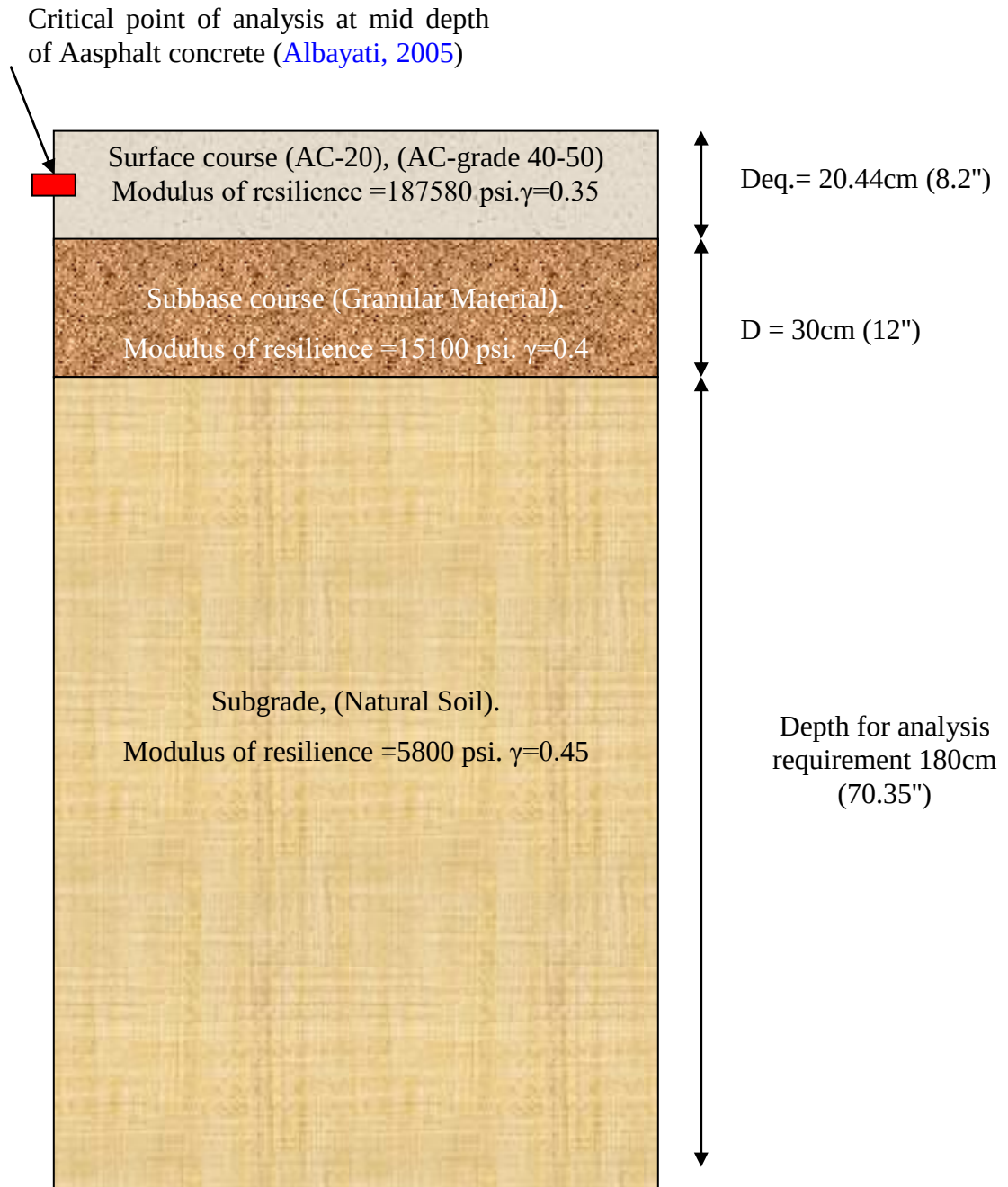


Figure 14. Geometry of Transformed Pavement Section. خطأ! لا يوجد نص من النمط المعين في المستند.

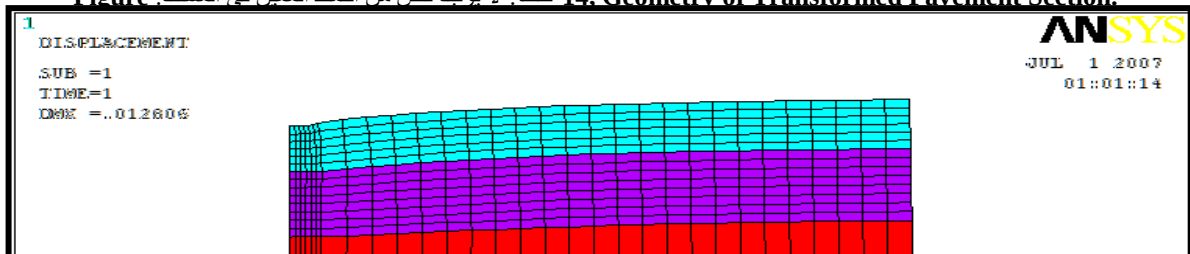


Figure 15, Deformation Shape and Mesh Generation for Elastic Finite Element Model

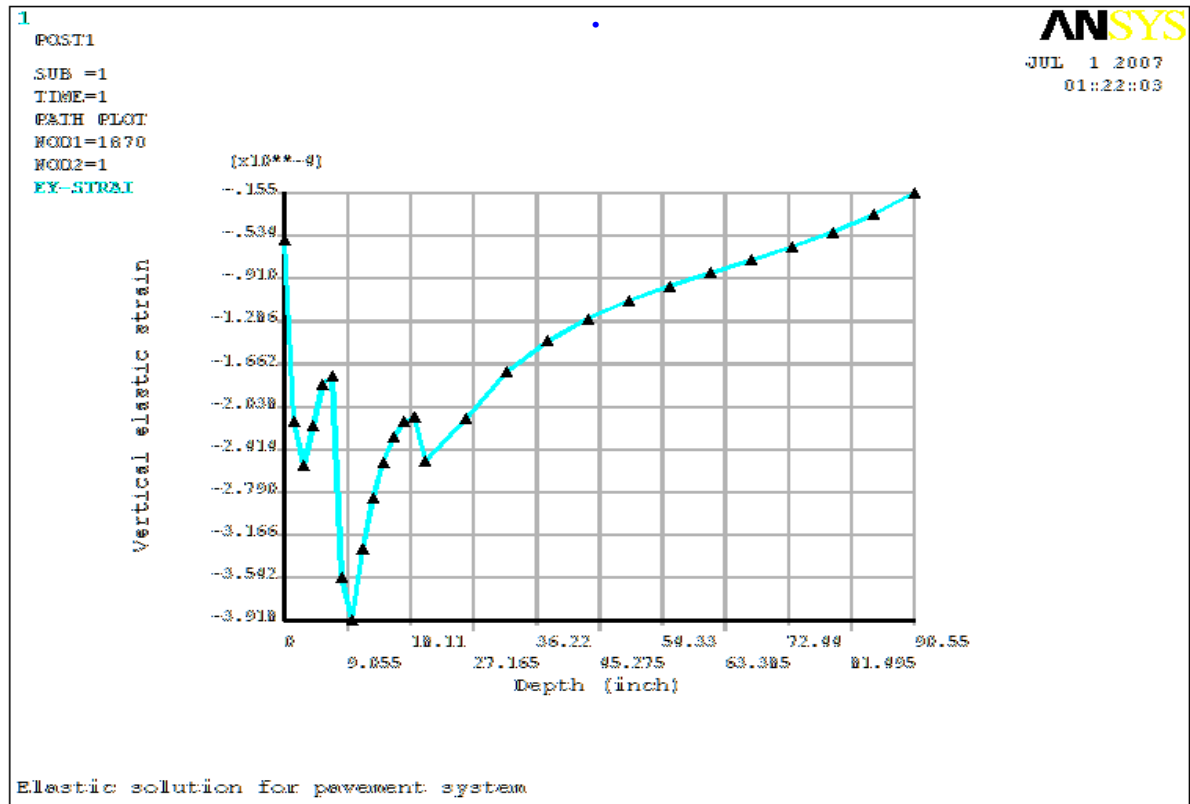


Figure 16, Solution of Vertical Elastic Strain Varies with Depth at Center Line of Tire.

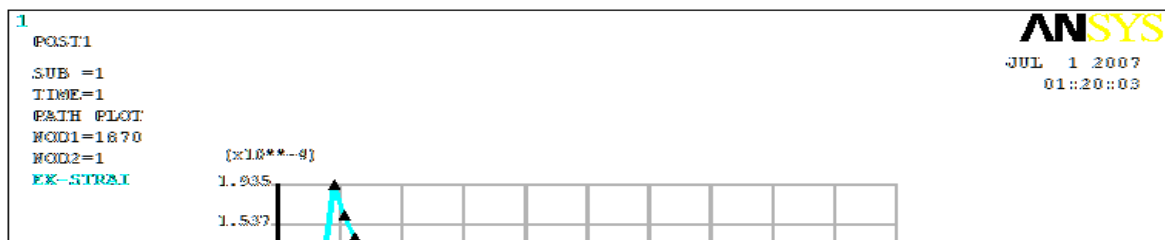


Figure 17- Solution of Radial Elastic Strain Varies with Depth at Center Line of Tire. خطأ! لا يوجد نص من النمط المعين في المستند.

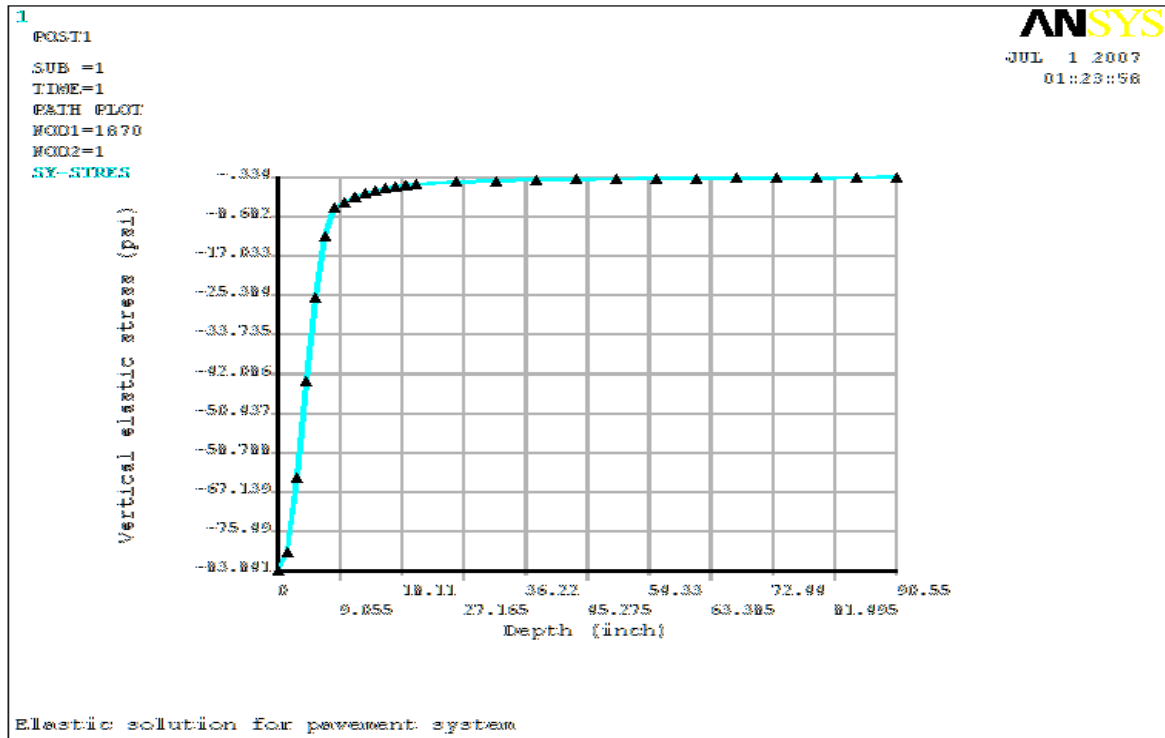


Figure 18- Solution of Vertical Elastic Stress Varies with Depth at Center Line of Tire. خطأ! لا يوجد نص من النمط المعين في المستند.

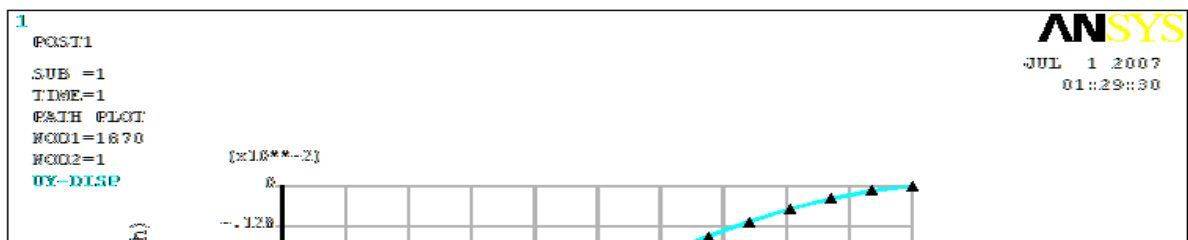


Figure 19- Solution of Vertical Elastic Displacement Varies with Depth at Center Line of Tire. خطأ! لا يوجد نص من النمط المعين في المستند.

Table 3-4, Solution of Elastic Responses Vary with Depth under Center Line of Tire, Bold Value Refer to Critical Point.

Coordinates system (inch)	YG-depth (inch)	ϵ_Y -strain	ϵ_X -strain	σ_Y -stress	U_y -displacement
90.55	0	-5.64E-05	-2.06E-04	-83.841	-1.28E-02
89.183	1.3667	-2.17E-04	-4.51E-05	-79.817	-1.26E-02
87.817	2.7333	-2.55E-04	3.94E-05	-64.119	-1.23E-02
86.45	4.1	-2.21E-04	7.06E-05	-43.67	-1.20E-02
85.083	5.4667	-1.84E-04	9.14E-05	-25.84	-1.17E-02
85.083	5.4667	-1.84E-04	9.14E-05	-25.84	-1.17E-02
85.083	5.4667	-1.84E-04	9.14E-05	-25.84	-1.17E-02
83.717	6.8333	-1.76E-04	1.25E-04	-12.713	-1.14E-02
82.35	8.2	-3.54E-04	1.94E-04	-6.7565	-1.12E-02
80.85	9.7	-3.92E-04	1.62E-04	-5.6784	-1.05E-02
79.35	11.2	-3.30E-04	1.40E-04	-4.6289	-9.98E-03
77.85	12.7	-2.85E-04	1.25E-04	-3.8233	-9.52E-03
76.35	14.2	-2.52E-04	1.15E-04	-3.1898	-9.12E-03
74.85	15.7	-2.30E-04	1.10E-04	-2.6854	-8.76E-03
73.35	17.2	-2.17E-04	1.10E-04	-2.2861	-8.42E-03
71.85	18.7	-2.13E-04	1.14E-04	-1.9818	-8.10E-03
70.35	20.2	-2.51E-04	1.24E-04	-1.7722	-7.77E-03
64.487	26.062	-2.14E-04	9.39E-05	-1.3249	-6.32E-03
58.625	31.925	-1.74E-04	7.66E-05	-1.0597	-5.18E-03

.....References Index.....

52.762	37.788	-1.46E-04	6.49E-05	-0.88188	-4.25E-03
46.9	43.65	-1.26E-04	5.62E-05	-0.75573	-3.45E-03
41.038	49.512	-1.11E-04	4.92E-05	-0.6628	-2.75E-03
35.175	55.375	-9.77E-05	4.32E-05	-0.59195	-2.14E-03
29.313	61.237	-8.60E-05	3.77E-05	-0.53617	-1.60E-03
23.45	67.1	-7.48E-05	3.21E-05	-0.49054	-1.13E-03
17.587	72.962	-6.31E-05	2.60E-05	-0.4512	-7.28E-04
11.725	78.825	-5.01E-05	1.91E-05	-0.41467	-3.96E-04
5.8625	84.687	-3.47E-05	1.08E-05	-0.37727	-1.48E-04
0	90.55	-1.55E-05	2.19E-07	-0.33421	0

Table 3-5, General Elastic Solution Components at Mid Depth of Asphalt Layer (Critical Point Lays at, [4.1 inch] under Tire Contact).

Type of solution	Stress (psi)	Vertical elastic strain	Displacement (inch)	Radial elastic strain
Finite element model	43.67	-2.21E-04 ¹	-0.012 ¹	7.06E-05
Multilayer elastic model(KENLAYR)	43.892	2.191E-04	0.01450	6.804E-05

The solution of multilayer elastic model (MLET) was done by (KENLAYER) software as following:

INPUT FILE NAME -C:\KENPAVE\KenElasticywao.DAT

NUMBER OF PROBLEMS TO BE SOLVED = 1

¹ The negative value refers to direction of response (-) down, (+) up.

TITLE -elastic solution

MATL = 1 FOR LINEAR ELASTIC LAYERED SYSTEM

NDAMA = 0, SO DAMAGE ANALYSIS WILL NOT BE PERFORMED

Figure 20- خطأ! لا يوجد نص من النمط المعين في المستند. Solution of Elastic Response by Multilayer Theory (KENLAYER).

3.6.3 Viscoelastic Solution.

Hot-mix asphalt is a viscoelastic material that is characterized by a certain level of rigidity of an elastic solid, but at the same time, flows and dissipates energy by frictional losses as a viscous fluid. At high temperatures or under slow moving loads, HMA may exhibit a purely viscous flow, which results in load-associated distresses such as pavement rutting. However, at low service temperatures or under rapidly applied loading, HMA becomes progressively harder and eventually brittle, which makes it vulnerable to non-load-associated distresses such as low-temperature cracking. When a vehicle travels on top of a HMA pavement layer, three different types of strain develop in the material: elastic recoverable strain, and viscous recoverable and irrecoverable strain. Elastic strain is associated with the ability of HMA to respond to the applied load and to recover instantaneously the energy released during deformation. On the other hand, viscous strain involves the time-dependent deformation of HMA. Only part of the viscous strain is recoverable and time-dependent. In general, the total strain can be expressed based on the following relation ([ERES Consultants, 2002](#)):

$$\epsilon_T = \epsilon_e + \epsilon_{ve} + \epsilon_{vp} \quad \dots (3-4)$$

where: ϵ_T =total strain;

ϵ_e =elastic strain;

ϵ_{ve} =viscoelastic strain;

ϵ_{vp} =viscoplastic strain.

Mathematical modeling of HMA response to loading demands the prediction of strain accumulation under variable loading time and temperature. Viscous deformation depends on three parameters, which are the stress level, time, and temperature. To describe the isotropic viscoelastic behavior of HMA, a generalized Kelvin model, which consists of a spring and (K) Voigt elements

connected in series, was selected in this study. In the case of a viscoelastic solid, the creep compliance at time t of a generalized Kelvin model is given as follows (Park and Schapery, 1999):

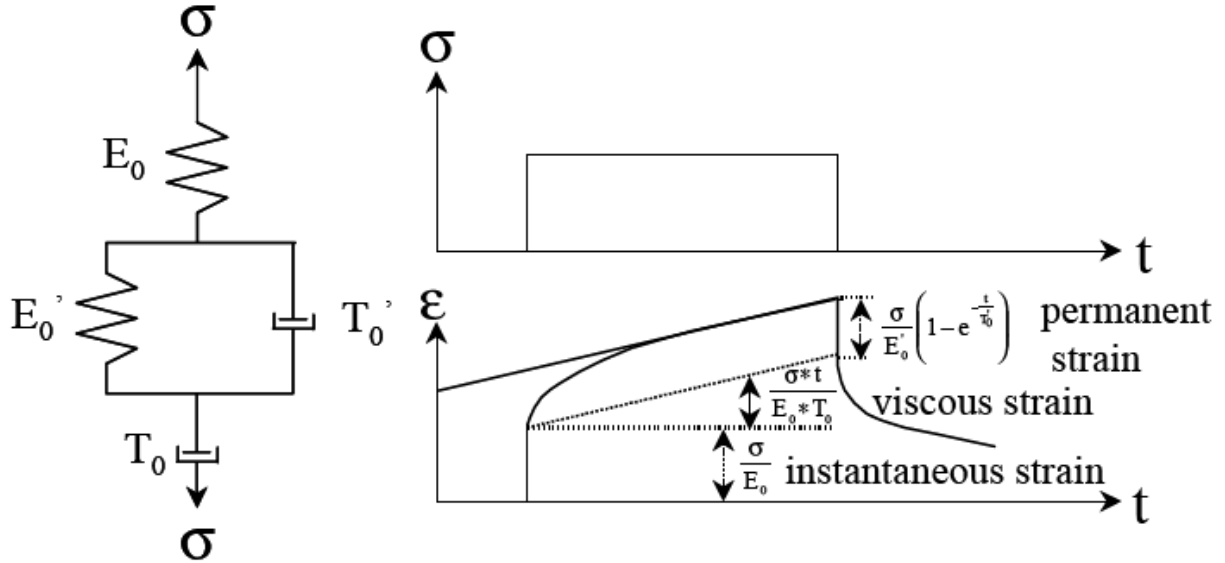


Figure 21-خطأ! لا يوجد نص من النمط المعين في المستند. **Mechanical Model, and Response to Stress of Viscoelastic Material.** □ □

here

- σ : stress level (constant).
- E_0 : elastic modulus for Maxwell's model.
- E_0' : elastic modulus for Kelvin's model.
- T_0 : relaxation time for Maxwell's model.
- T_0' : retardation time for Kelvin's model.

$$D(t) = D_0 + \sum_{i=1}^k D_i (1 - e^{-t/\tau_i}) \quad \dots\dots\dots (3-5)$$

Where $D(t)$ =creep compliance at time t ; D_i =material constants; and τ_i =retardation times. Eq. (3-4) is also known as a Prony or a Dirichlet series expansion. Since it was simpler in this study to measure the creep compliance of HMA, it was used as the base of interconversion to other viscoelastic functions that were used to model the material time-dependent properties. For instance, the fitted creep compliance was used to predict the relaxation modulus, $E(t)$. The interconversion process was based on the approximation developed by (Kim et al., 2002). Assuming that both the creep compliance and the relaxation modulus

may be described by a power law model, the relationship between the two material properties was defined as follows:

$$E(t).D(t) = \frac{\sin(n.\pi)}{n.\pi} \quad \dots\dots\dots(3-6)$$

Where n =positive constant, obtained by fitting a localized power law model $D(t)=[D1.t^n]$ to the different regions of behavior. Similarly, a Prony series expansion was fitted to the relaxation modulus variation with time. Then, the bulk $K(t)$ and shear $G(t)$ moduli variation with time were estimated using the following relations assuming that Poisson's ratio does not change with time ,assuming that the material is isotropic and homogeneous and that resulting strain is small (Elseifi A. Mostafa 1; Al-Qadi L. Imad, F.ASCE2; and Yoo Jun Pyeong 3, 2006)*:

$$K(t) = \frac{E(t)}{3(1-2\nu)} \quad \dots\dots\dots(3-7); \quad G(t) = \frac{E(t)}{2(1+\nu)} \quad \dots\dots\dots(3-8)$$

The simulating has done by using ANSYS version 9.0; a material is viscoelastic if its stress response consists of an elastic part and viscous part. Upon application of a load, the elastic response is instantaneous while the viscous part occurs over time. Generally, the stress function of a viscoelastic material is given in an integral form within the context of small strain theory[^], the constitutive equation for an isotropic viscoelastic material can be written as:

* 1 Research Scientist, Advanced Transportation Research and Engineering Laboratory, Dept. of Civil and Environmental Engineering, Univ. of Illinois at Urbana-Champaign, 1611 Titan Dr., Rantoul, IL 61866.

2 Founder Professor of Engineering and Director of Illinois Center for Transportation, Advanced Transportation Research and Engineering Laboratory, Director, Dept. of Civil and Environmental Engineering, Univ. of Illinois at Urbana-Champaign, 205 N. Mathews MC-250, Urbana, IL 61801.

3 Graduate Research Assistant, Dept. of Civil and Environmental Engineering, Univ. of Illinois at Urbana Champaign, 205 N. Mathews MC-250, Urbana, IL 61801.

[^] It's clearly that there is an agreements between assumption in above and this theory, for more information see, (ANSYS, 2004) help manual under subtitle "4.7.1. Small Strain Viscoelastic".

$$\sigma = \int_0^t 2G(t-\tau) \cdot \frac{de}{dt} \cdot dt + I \int_0^t K(t-\tau) \cdot \frac{d\Delta}{dt} \cdot dt \quad \dots\dots\dots (3-9).$$

where:

- σ = Cauchy stress
- e = deviatoric part of the strain
- Δ = volumetric part of the strain
- $G(t)$ = shear relaxation kernel function
- $K(t)$ = bulk relaxation kernel function
- t = current time
- τ = past time
- I = unit tensor.

For the element type (plan-183) that use in this study the kernel functions are represented in terms of Prony series, which assumes that:

$$G = G_{\infty} + \sum_{i=1}^{n^G} G_i \cdot \exp\left(-\frac{t}{\tau_i^G}\right) \quad \dots\dots\dots (3-10)$$

$$K = K_{\infty} + \sum_{i=1}^{n^K} K_i \cdot \exp\left(-\frac{t}{\tau_i^K}\right) \quad \dots\dots\dots (3-11)$$

In the above, G_{∞} ; G_i are shear elastic moduli, K_{∞} ; K_i are bulk elastic moduli τ_i^G ; τ_i^K are the relaxation times for each Prony component. Introducing the relative moduli as following:

$$\alpha_i^G = \frac{G_i}{G_o} \quad \dots\dots\dots (3-12); \quad \alpha_i^K = \frac{K_i}{K_o} \quad \dots\dots\dots (3-13)$$

where:

$$G_o = G_{\infty} + \sum_{i=1}^{n^G} G_i, \quad K_o = K_{\infty} + \sum_{i=1}^{n^K} K_i$$

The kernel functions can be equivalently expressed as:

$$G = G_o[\alpha_{\infty}^G + \sum_{i=1}^{n^G} \alpha_i \cdot \exp(-\frac{t}{\tau_i})], \quad \dots (3-14)$$

$$K = K_o[\alpha_{\infty}^K + \sum_{i=1}^{n^K} \alpha_i \cdot \exp(-\frac{t}{\tau_i})] \quad \dots (3-15).$$

Data input in **ANSYS version 9.0** platform of nonlinear material are exhibit in **Table (3-6)**, while the same geometric model, load configuration, boundary condition present in elastic solution is use except asphalt material will be treat as a viscoelastic material and the analysis is change to nonlinear solution.

Table 3-6, Viscoelastic Material Properties for Asphalt Layer.

Time of loading	Creep compliance (1/psi)	Relaxation modulus E(t), (psi)	shear modulus* G(t),(psi)	bulk modulus♦ K(t),(psi)
0.1	0.00003025	30250.83586	11204.01328	33612.03985
0.25	0.000039875	22948.90996	8499.596283	25498.78885
0.5	0.000053375	17144.50182	6349.81549	19049.44647
1	0.000058125	15743.44576	5830.905837	17492.71751
2	0.000066625	13734.90109	5087.000402	15261.00121
4	0.0000755	12120.36801	4489.025189	13467.07557
8	0.0000835	10959.13515	4058.938944	12176.81683
15	0.000108	8473.035045	3138.161128	9414.483383
30	0.00012525	7306.090098	2705.959296	8117.877887
45	0.000134875	6784.710175	2512.85562	7538.566861
60	0.00013775	6643.105516	2460.40945	7381.228351

* Obtain this column based on equation (3-7).

♦ Obtain this column based on equation (3-6).

An efficient method of fitting Prony-series models to viscoelastic experimental data with power law pre-smoothing is presented. A direct fitting of a Prony-series function to experimental data without appropriate pre-smoothing is difficult when the data have significant variance. A power-law series comprising multiple power law terms is found capable of portraying a globally smooth, broadband viscoelastic behavior with minimal impact from local variance in the data (Park, S. W., and Kim, Y. R., 2001). However, from a computational point of view, a Prony series representation is preferred to a power-law series representation because of the computational efficiency associated with the exponential basis functions of a Prony series, a procedure involving pre-smoothing of experimental data via power-law-series representation followed by fitting of a Prony-series model to the pre-smoothed data is illustrated. Illustration focuses on how experimental data with significant variance can be effectively pre-smoothed through power-law series representation and how pre-smoothing improves the quality of the subsequent Prony series fit.

The feasibility and effectiveness of the approach are demonstrated through numerical examples in which Prony-series representations of the shear modulus and bulk modulus of asphalt concrete are determined from available experimental data. It should be note that power law for creep compliance varies with time is $D(t) = [6E-05(\text{time})^{0.2302}]^{\Delta}$, as shown in Figure (3-20) , then $n=0.2302$ from the power law ,by applying the equation (3-5) this led to :

$$E(t).D(t) = \frac{\sin(0.2302.\pi)}{0.2302.\pi} = 0.915087785,$$

This relationship between creep compliance and relaxation modulus is used to obtain column (3) in Table (3-6). The fitting of shear modulus, bulk modulus vary with time were done by [ANSYS version 9.0](#), given a good parameters convergences with a low mean residual of error equal to (0.000107), Prony series

^Δ This power law give a Coefficient of determination $R^2 = 0.9875$.

parameters fitting for both shear; bulk modulus are shown in Table (3-7), ANSYS platforms result in Figure (3-21), the decay of shear, bulk with time are present in Figure (3-22).

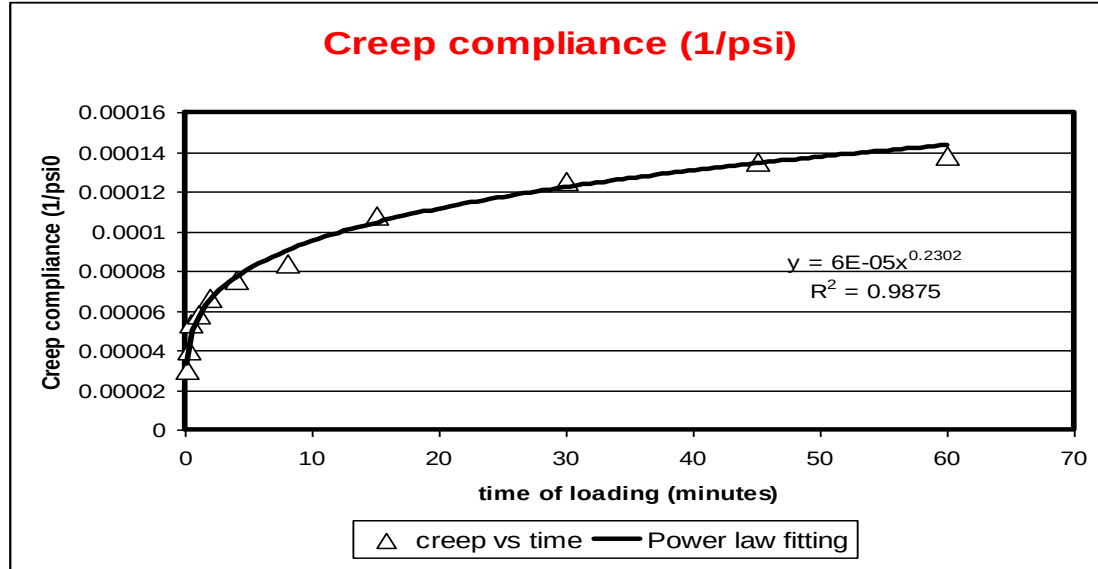


Figure 3-22, Creep Compliance Varies with Time and Power Law Fitting. لا يوجد نص من النمط المعين في المستند.

Table 3-7, Prony Series Parameters for both Shear; Bulk Modulus Decay with Time

Prony parameters for shear decays with time	Value	Prony parameters for bulk decays with time	Value
α_1	0.097952	α_1	0.11251
τ_1	9.2719	τ_1	5.9762
α_2	0.10067	α_2	0.1835
τ_2	12.933	τ_2	0.15972
α_3	0.077162	α_3	0.21599
τ_3	1.0341	τ_3	0.15962
α_4	0.29116	α_4	0.20815
τ_4	0.17229	τ_4	0.31042
α_5	0.27135	α_5	0.11667
τ_5	0.17229	τ_5	15.136

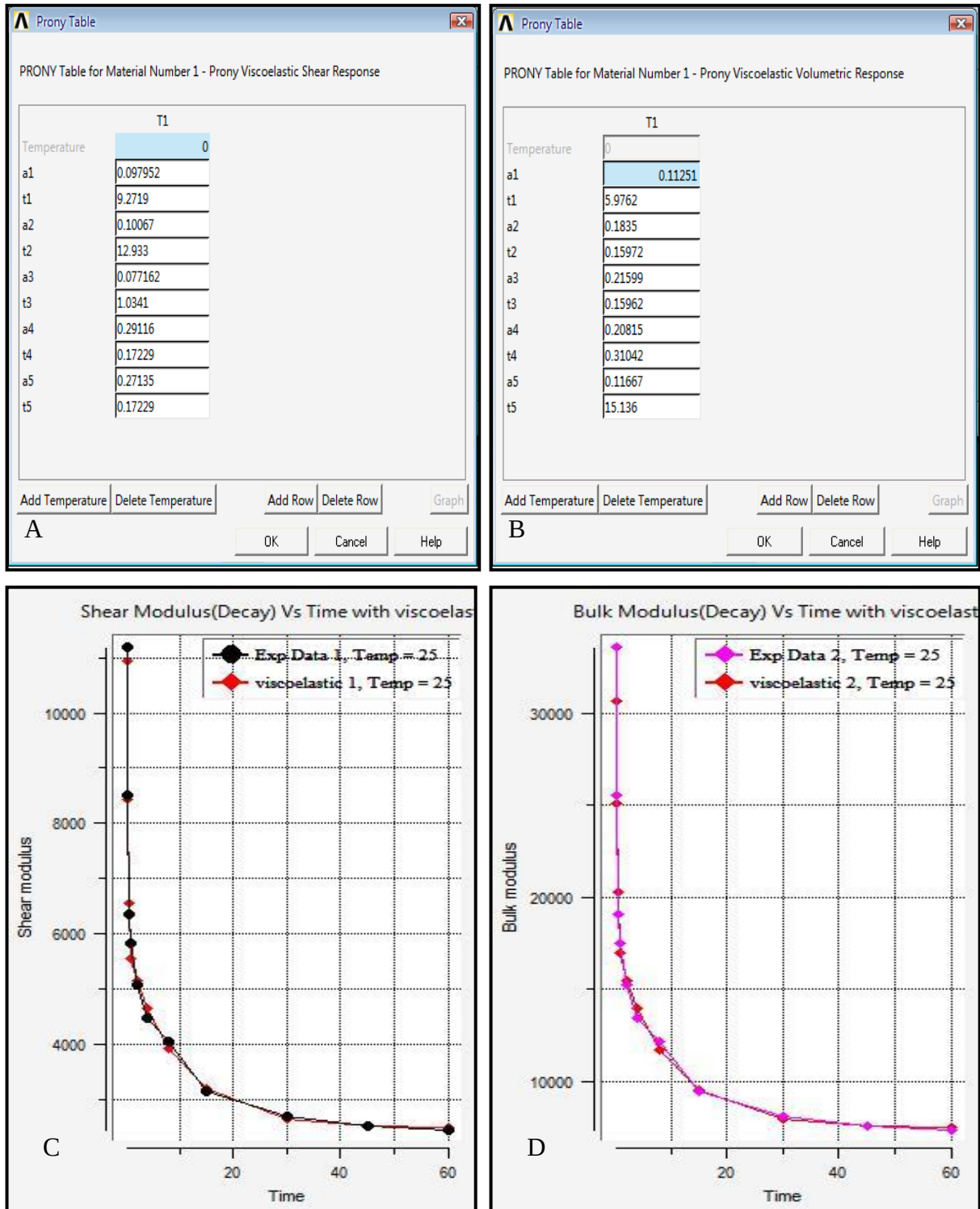


Figure (3-21),

Figure 23-خطأ! لا يوجد نص من النمط المعين في المستند. [A, B] are Platforms of Viscoelastic Parameters Output, [C; D] are Data & Prony Series Function of Shear; Bulk Modulus Decay with Time.

* Platforms were obtain from ANSYS 9.0 by capturing pictures

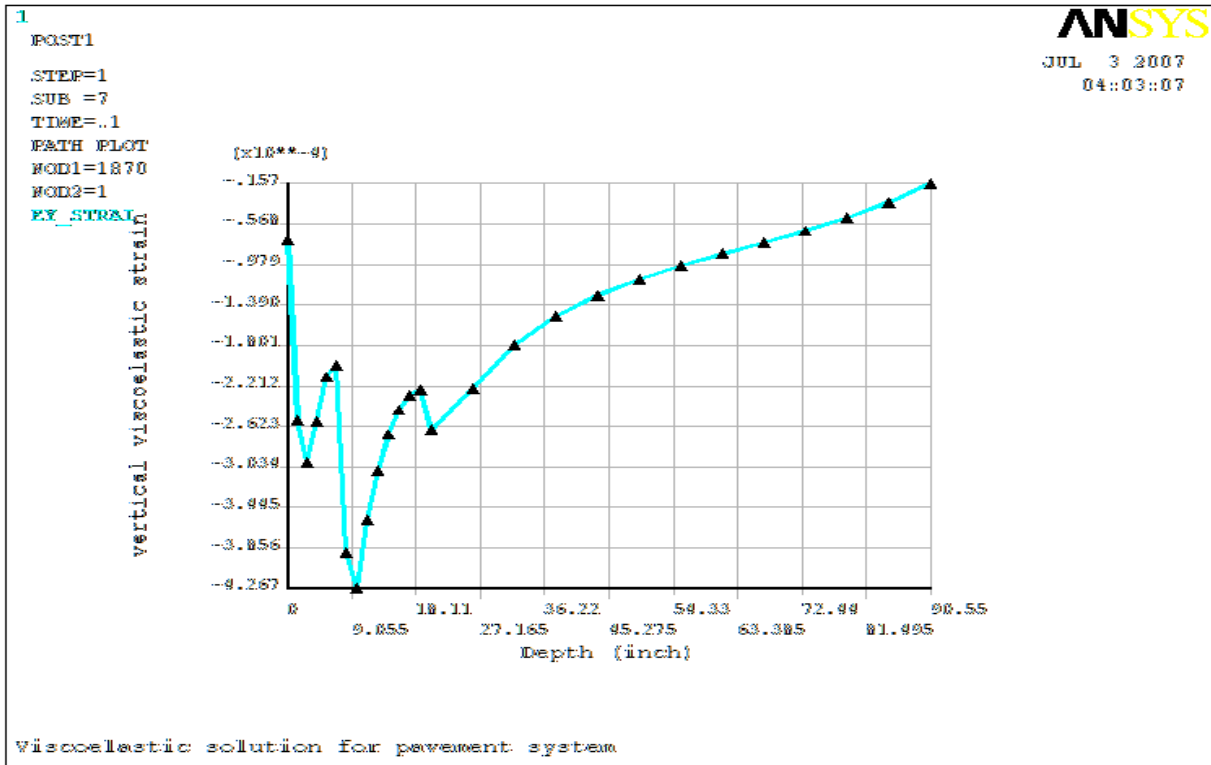


Figure 24- خطاً! لا يوجد نص من النمط المعين في المستند. Solution of vertical viscoelastic strain varies with depth at center line of tire at time (0.1 sec).

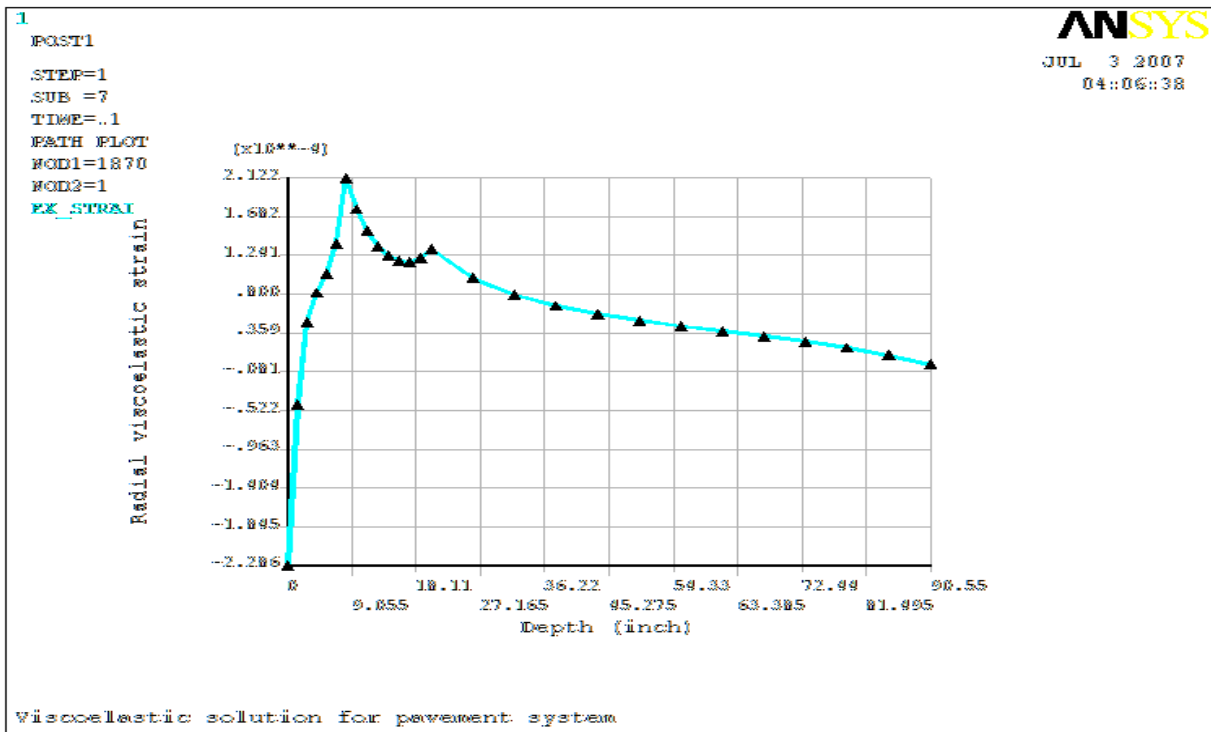


Figure 25- خطاً! لا يوجد نص من النمط المعين في المستند. Solution of Radial Viscoelastic Strain Varies with Depth at Center Line of Tire at Time (0.1 sec).

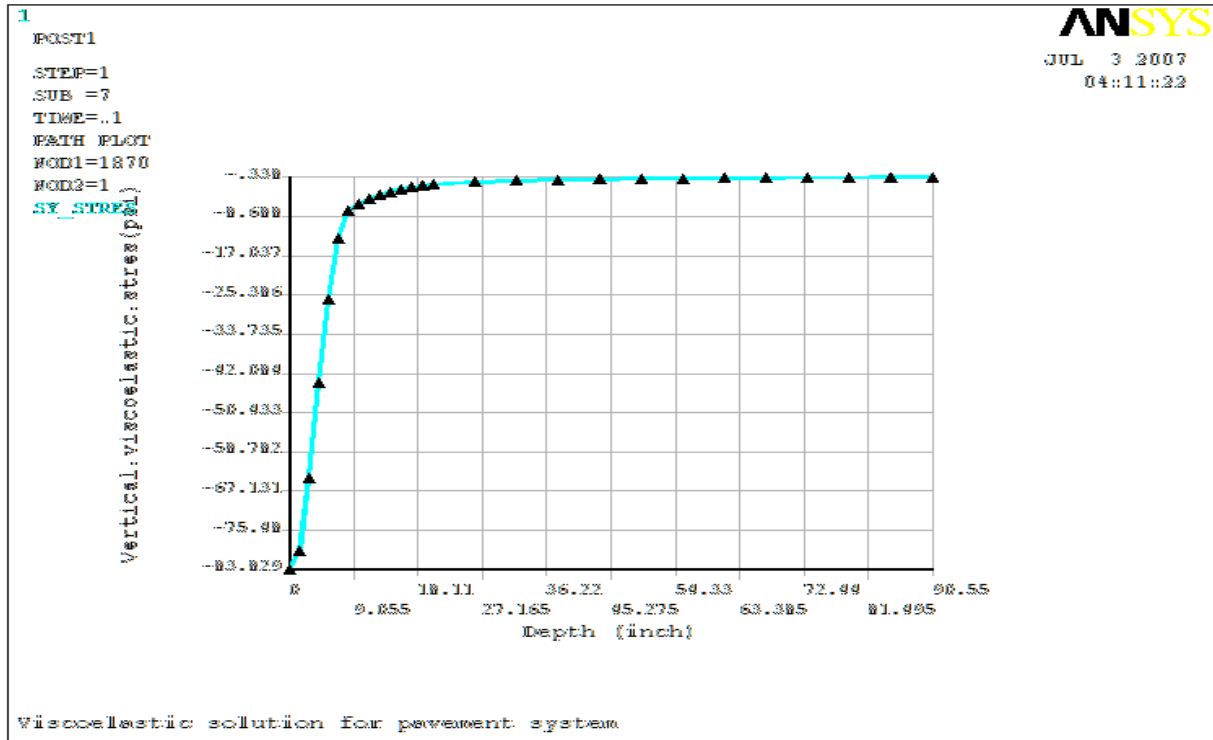


Figure 26-خطا! لا يوجد نص من النمط المعين في المستند. Solution of Vertical Viscoelastic Stress Varies with Depth at Center Line of Tire at Time (0.1 sec).

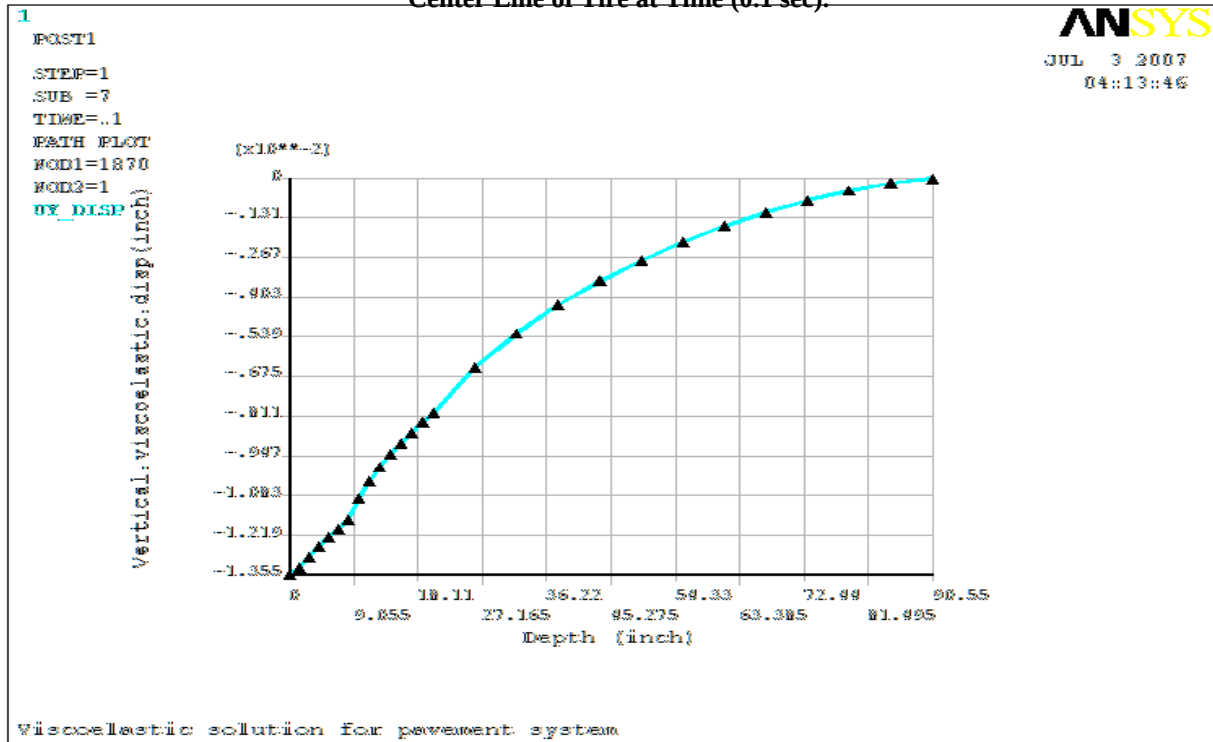


Figure 27-خطا! لا يوجد نص من النمط المعين في المستند. Solution of Vertical Viscoelastic Displacement Varies with Depth at Center Line of Tire at Time (0.1 sec).

Table 3-8, Solution of Viscoelastic Responses Vary with Depth under Center Line of Tire, Bold Value Refer to Critical Point.

Coordinates system (inche)	YG-depth (inche)	ϵ_y -strain	ϵ_x -strain	σ_y -stress	Uy-displecment
90.55	0	-7.34E-05	-2.29E-04	-83.829	-1.36E-02
89.183	1.3667	-2.56E-04	-4.59E-05	-79.87	-1.33E-02
87.817	2.7333	-2.99E-04	4.86E-05	-64.324	-1.30E-02
86.45	4.1	-2.57E-04	8.18E-05	-44.065	-1.26E-02
85.083	5.4667	-2.12E-04	1.03E-04	-26.424	-1.23E-02
83.717	6.8333	-2.00E-04	1.38E-04	-13.429	-1.20E-02
82.35	8.2	-3.90E-04	2.12E-04	-7.4265	-1.17E-02
80.85	9.7	-4.27E-04	1.77E-04	-6.1734	-1.10E-02
79.35	11.2	-3.57E-04	1.52E-04	-5.0077	-1.04E-02
77.85	12.7	-3.07E-04	1.35E-04	-4.1159	-9.87E-03
76.35	14.2	-2.70E-04	1.24E-04	-3.4167	-9.43E-03
74.85	15.7	-2.45E-04	1.18E-04	-2.8613	-9.05E-03
73.35	17.2	-2.31E-04	1.17E-04	-2.4228	-8.69E-03
71.85	18.7	-2.26E-04	1.21E-04	-2.0893	-8.35E-03
70.35	20.2	-2.66E-04	1.31E-04	-1.8604	-8.00E-03
64.487	26.062	-2.24E-04	9.84E-05	-1.3768	-6.48E-03
58.625	31.925	-1.80E-04	7.95E-05	-1.0934	-5.29E-03
52.762	37.788	-1.51E-04	6.68E-05	-0.90491	-4.33E-03
46.9	43.65	-1.29E-04	5.76E-05	-0.77218	-3.51E-03
41.038	49.512	-1.13E-04	5.03E-05	-0.67503	-2.80E-03
35.175	55.375	-9.94E-05	4.40E-05	-0.60139	-2.17E-03
29.313	61.237	-8.74E-05	3.83E-05	-0.54374	-1.63E-03
23.45	67.1	-7.58E-05	3.25E-05	-0.49682	-1.15E-03
17.587	72.962	-6.39E-05	2.64E-05	-0.45659	-7.37E-04
11.725	78.825	-5.07E-05	1.94E-05	-0.4194	-4.01E-04
5.8625	84.687	-3.52E-05	1.09E-05	-0.38143	-1.49E-04
0	90.55	-1.57E-05	2.22E-07	-0.33778	0

Figure (3-26) show the deformation shape while Table (3-5), Figures (3-26) through (3-30) show the different in response due to analysis type.

Table 3-9, General Response Solution Components at Mid Depth of Asphalt Layer (Critical Point Lays at, [4.1 inch] under Tire Contact).

Type of solution	Vertical stress (psi)	Vertical strain	Displacement (inch)	Radial strain
Elastic finite element model	-43.67 ¹	-2.21E-04 ¹	-0.012 ¹	7.06E-05
Multilayer elastic model(KENLAYR)	43.892	2.191E-04	0.01450	6.804E-05
Viscoelastic finite element model at time (0.1 sec)	-44.065 ¹	-2.57E-04	-0.0126 ¹	8.18E-05

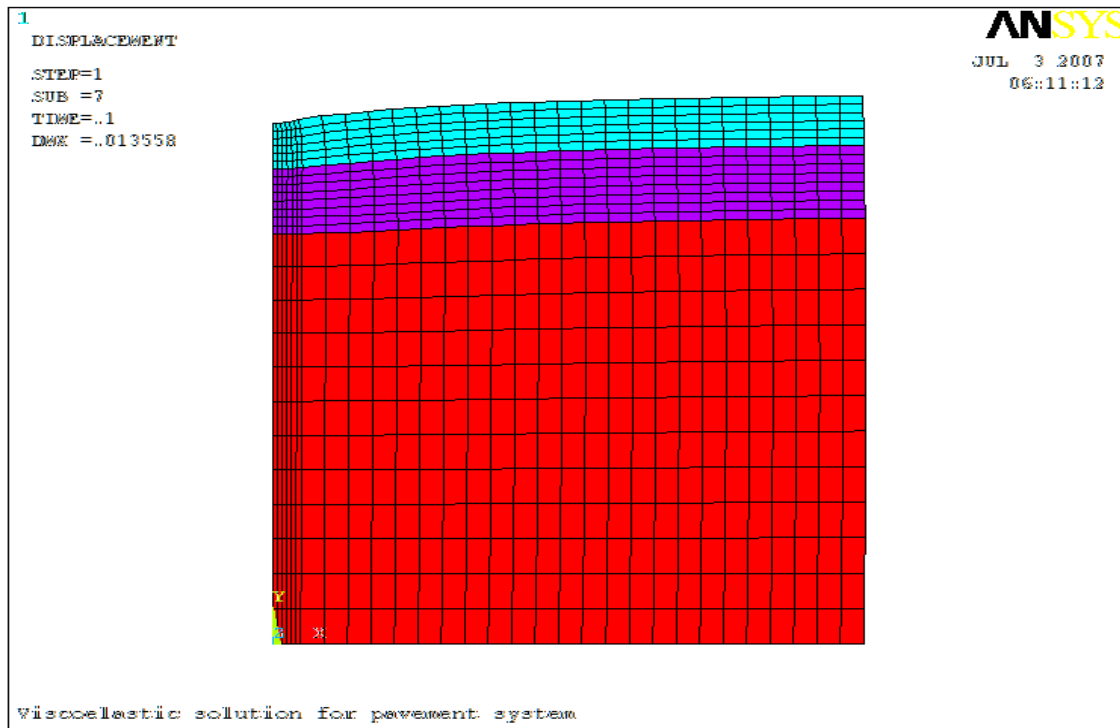


Figure 28, Deformation Shape and Mesh Generation for Viscoelastic Finite Element Model. لا يوجد نص من النمط المعين في المستند.

¹ The negative value refers to direction of response (-) down, (+) up.

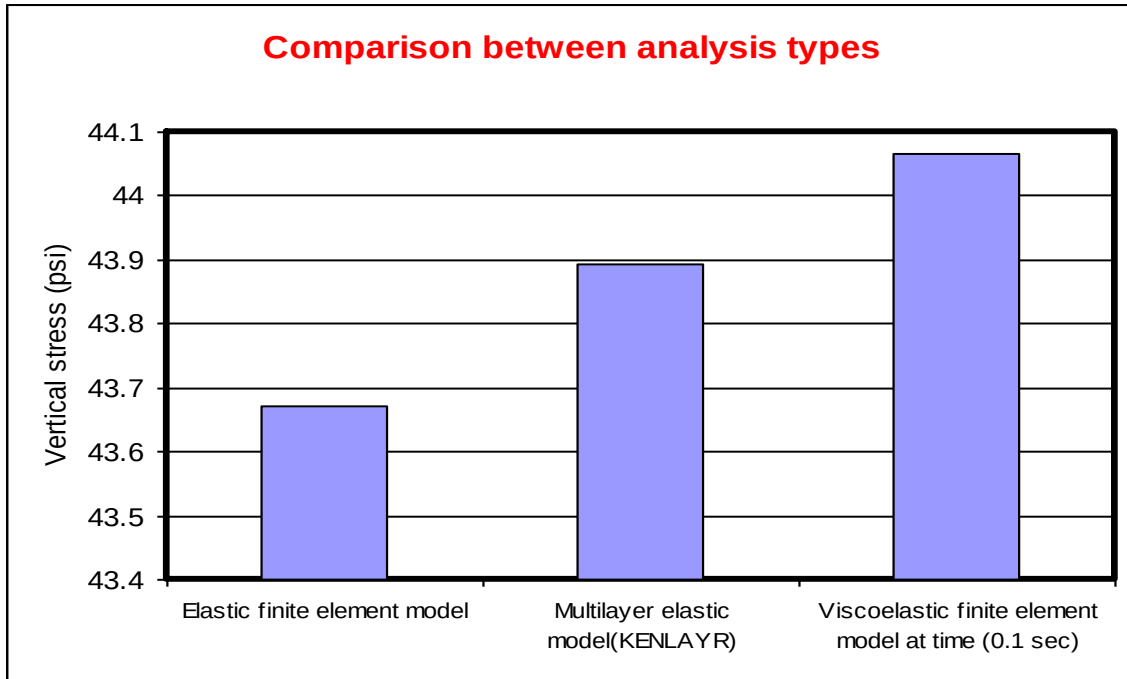


Figure 29- خطأ! لا يوجد نص من النمط المعين في المستند. Comparison of Vertical Stress at Critical Point Varies with Analysis Types.

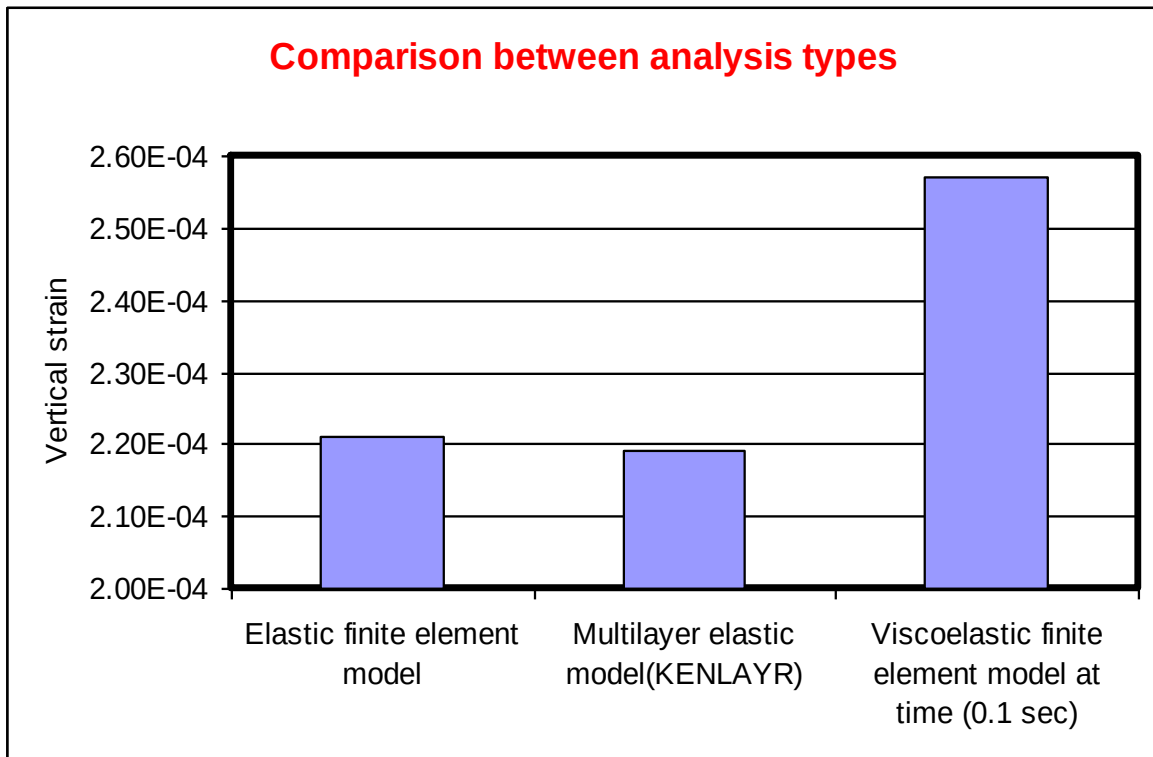


Figure 30- خطأ! لا يوجد نص من النمط المعين في المستند. Comparison of Vertical Strain at Critical Point Varies with Analysis Types.

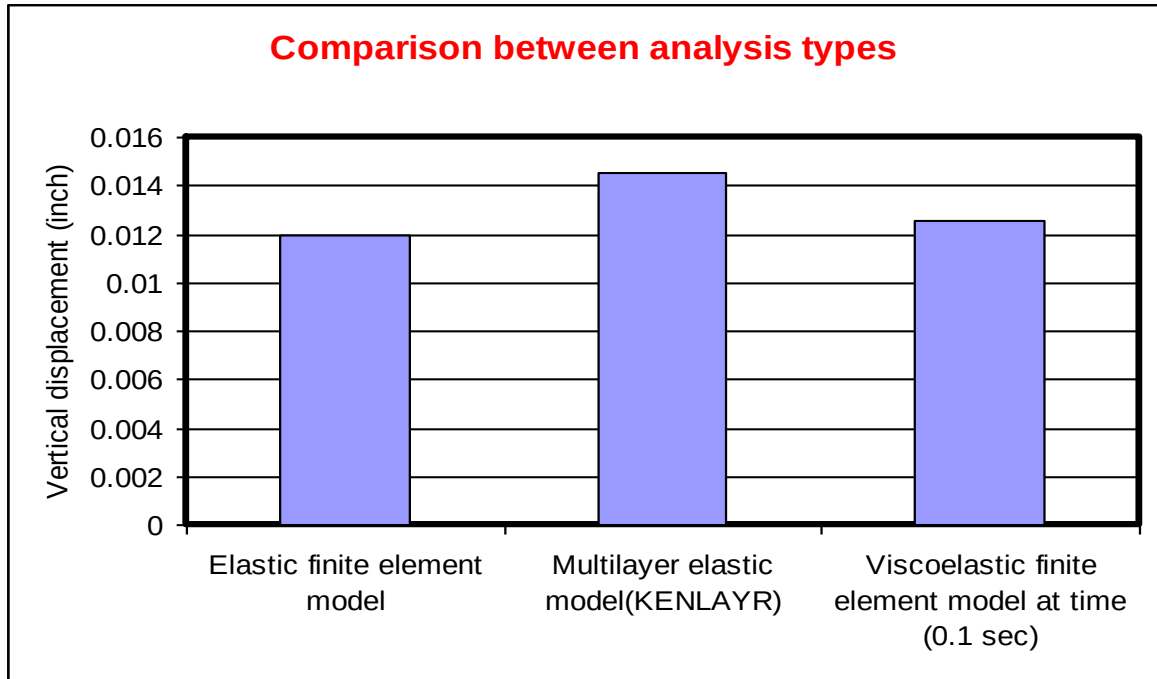


Figure 31-خطأ! لا يوجد نص من النمط المعين في المستند. Comparison of Vertical Displacement at Critical Point Varies with Analysis Types.

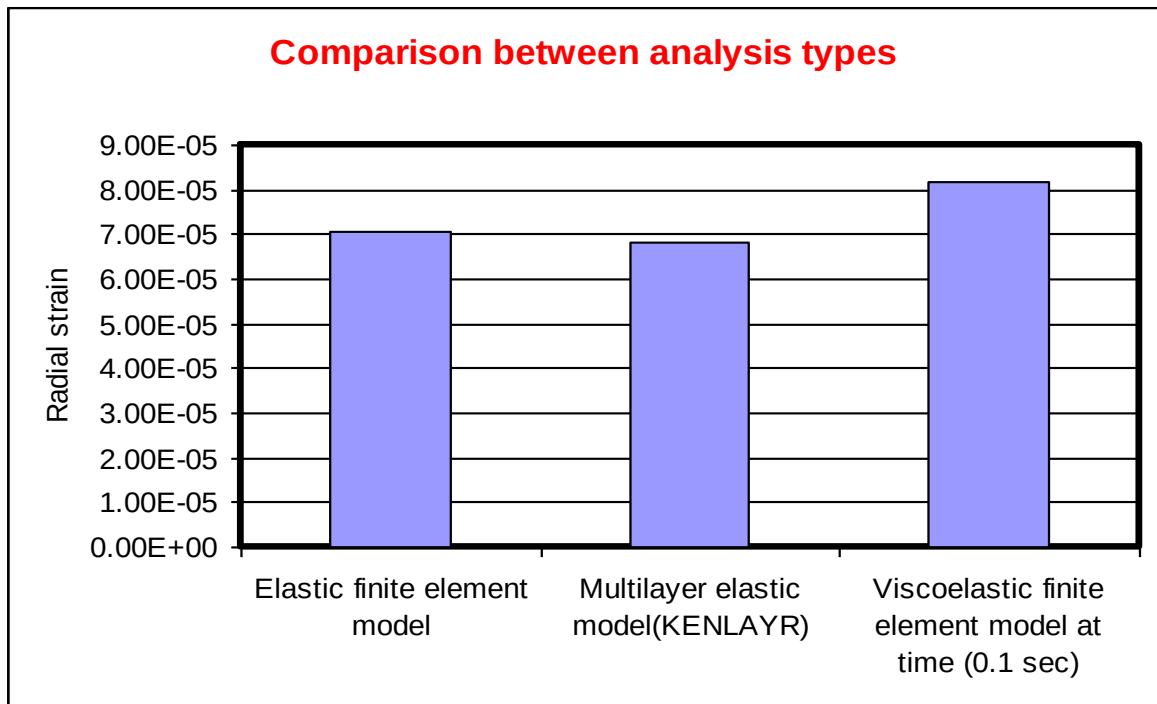


Figure 32-خطأ! لا يوجد نص من النمط المعين في المستند. Comparison of Radial Strain at Critical Point Varies with Analysis Types.

4 CHAPTER FOUR

MODELING OF PERMANENT STRAIN OF ASPHALT PAVEMENT

4. Introduction

The adequate and accurate characterization of asphalt mixture behavior is vital if pavement performance is to be predicted realistically. The ideal material characterization model should be able to predict the response of asphalt concrete over the full range of temperatures and stress states of interest in pavement systems. Asphalt concrete behavior may vary from elastic and linear viscoelastic at low temperatures and or fast loading rates to nonlinear visco-elastic-plastic at high temperatures and or slow loading rates.

In general, asphalt concrete deformation is composed of four strain components, elastic, plastic, viscoelastic and viscoplastic. The magnitude of the strain components depends on temperature, loading and stress state, [Figure \(4.1\)](#) presents the strain components in repeated loading after ([Uzen, 1996](#)).

4.1. Objectives of Pavement Response Models

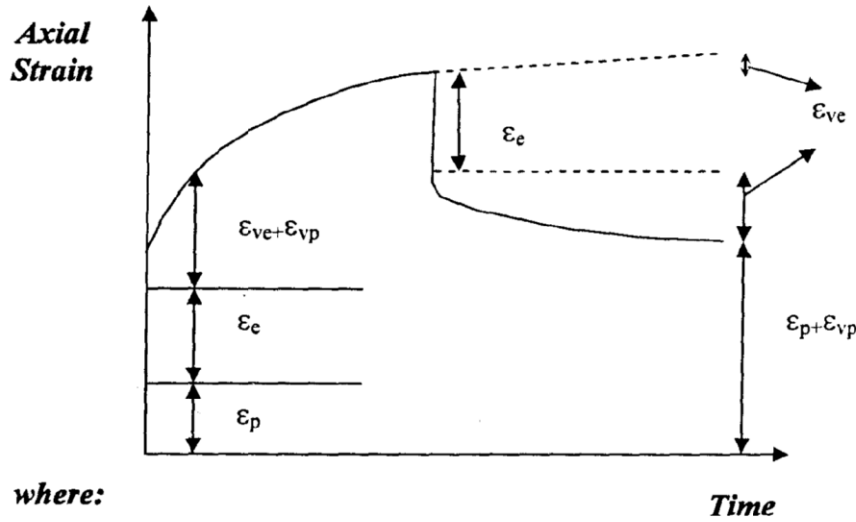
As mentioned above, the total strain is composed into four components and is explained briefly as follows:

4.1.1. Elastic strain (ϵ_e):

Elastic behavior is governed by Hooke's law which states that strain (ϵ) is proportional stress (σ);

$$\epsilon = \sigma / E \quad \text{..... (4-1)}$$

Where E is Young's modulus. The rate-independent instantaneous elastic strain is related to the underlying stiffness of the mixture structure. It is independent of the loading rate by definition and is assumed to be independent of time but may be dependent on the stress state, particularly confining pressure. All of the work required to produce elastic strain during an increment of deformation is stored and recovered during unloading; i.e. material does not dissipate energy. Young's modulus is a characteristic of material. It is not dependent on the size and shape of the object being stressed.



where:

ϵ_e : elastic strain.

ϵ_{ve} : viscoelastic strain.

ϵ_{vp} : viscoplastic strain

ϵ_p : plastic strain.

Figure 1- خطأ! لا يوجد نص من النمط المعين في المستند. Strain Components in Repetitive Loading (Yassoub, 2004)

4.1.2. Viscoelastic Strain (ϵ_{ve}):

Newton's law says that for purely viscous fluids, shear stress (τ) is directly proportional to the rate of strain(y'); That is

$$\tau = \eta \cdot \frac{\partial v}{\partial y} \quad \dots\dots\dots (4-2)$$

Where η is the constant of proportionality called viscosity. Elastic solids and viscous fluids differ widely in their deformational characteristics. Elastically deformed bodies return to their natural or undeformed state upon the removal of applied loads. However, if solid materials are heated above their melting points so that molecular bonds are broken and they become fluids, they often behave in a viscous manner. Many fluids such as water and air behave in a Newtonian manner over a broad range of loading conditions. Material behavior that incorporates both elastic and viscous behavior is called viscoelastic behavior. The elastic (Hookean) solid and viscous (Newtonian) fluid represent opposite endpoints of a wide spectrum of viscoelastic behavior.

4.1.3 Viscoplastic strain (ϵ_{vp})

Viscoplastic behavior is time-dependent, but deformation is irrecoverable. A [Bingham model](#) consisting of a dry-friction element in parallel with a linear viscous element represents linear viscoplastic behavior ([Yassoub, 2004](#)).

4.1.4. Plastic Strain (ϵ_p)

Plastic deformations result from the mechanism of slip or from dislocations at the atomic level. Such deformations occur at stress intensities above a certain value known as the elastic limit or yield stress. The phrase (plastic flow) is used to designate (irrecoverable) plastic deformation. Plastic deformation is assumed to be time-independent. Since permanent deformation typically develops at high temperatures and /or slow loading rates, it is essential that the material characterization model for permanent deformation prediction includes plastic and viscoplastic behavior. As mentioned earlier, in this work, elastic strain and viscoelastic strain are combined and referred to as viscoelastic strain, plastic

strain is likewise combined and referred to as viscoelasto-plastic strain In this work, the modeling approach adopted is to model each response component Separately, That is to develop a viscoelasto-plastic model, depending on VESYS Model (Kenis, W. J., 1977). The separation of the viscoelasto-plastic strain from the total strain is accomplished by conducting creep and recovery test.

4.2. Viscoelasto-plastic Model

As mentioned, modeling the viscoelasto-plastic behavior of asphalt concrete mixtures calls for the separation of total strain into viscoelastic and plastic components. This can be accomplished by conducting creep and recovery compliance test. Efforts need to be made to determine proper data for simulating material by viscoelasto-plastic model. Figure (4.2) show the typical creep and recovery for creep and recovery testing. One asphalt concrete mixture is selected, in which it represent the best behavior under the conducted previous tests.

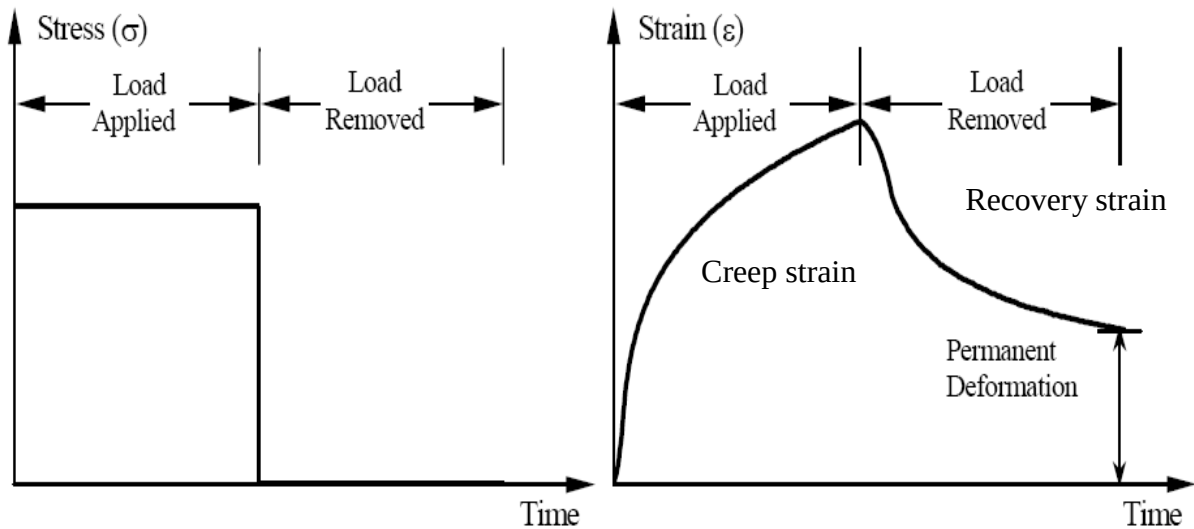
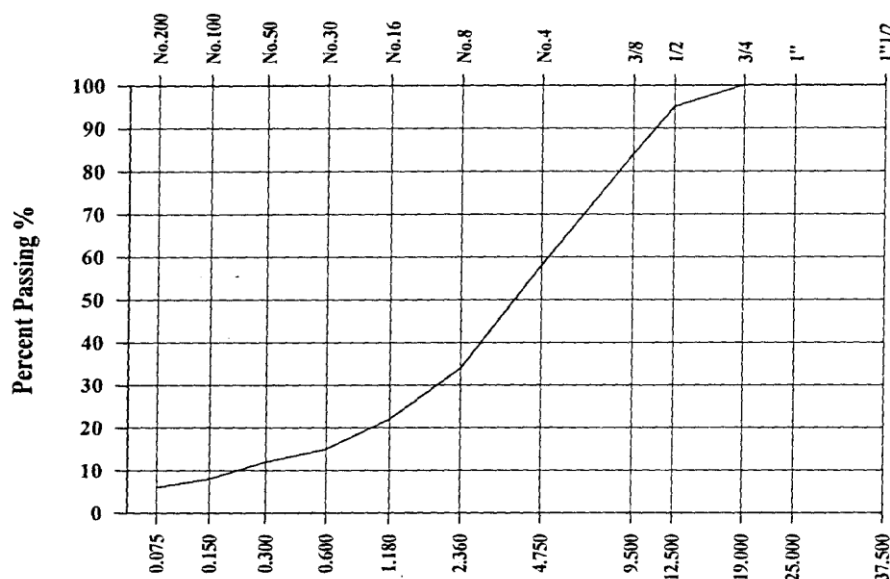


Figure 2-خطأ! لا يوجد نص من النمط المعين في المستند. Typical Stress and Strain for Creep and Recovery test
This mixture is consisted from crushed aggregate with gradation below the restricted zone and asphalt binder from Al-Daurah refinery with (40-50) grade

(AC-20). The optimum asphalt content is 4.9%. It is clear from Figure (4.3) that the Gradations of the selected asphalt mixture, approximately meet the mid point of the Specification limits (ASTM-D3515) sieve (No.8) in which it passes below the restricted zone with fewer fines this gradation represents mix B1[^]. Also, represents the typical gradation for the surface layer in Iraq (Yassoub, 2004).



Sieve Size	Percent Passing Below the Restricted Zone	Specification Limit ASTM D(3515)
3/4	100	100
1/2	95	90-100
3/8	84	77-93
No. 4	58	44-74
No. 8	34	29-58
No. 16	22	21-46
No. 30	15	12-33
No. 50	12	5-21
No. 100	8	4-15
No. 200	6	3-10

Figure 3- خطأ! لا يوجد نص من النمط المعين في المستند. Selected Combined Gradation of Aggregate and Mineral Filler for Mixture B1 (Yassoub, 2004)

[^] Appendix (A) offers all mixtures gradations that used in this study.

4.2.1. Theoretical Formulation.

Plastic strain is assumed to be linear log-log relationship with number of load repetitions as suggested by (Monismith, 1988) and (Barksdale, 1972) Equation (4-3)

$$\epsilon_p = a.N^b \quad \dots (4-3)$$

Where

ϵ_p = permanent strain (plastic strain).

N = number of stress applications.

a,b= constants.

Figure (4-1) and (4-2) show this relationship on semi log and log –log plots, respectively. Using this equation as a base relationship, two coefficients may be developed to describe a material's permanent deformation potential under repeated loads (Rauhut, 1980). Differentiation of Equation (4-3) yields the following:

$$\frac{d\epsilon_p}{dN} = \epsilon_p(N) = abN^{b-1} \quad \dots (4-4)$$

Assuming the resilient strain, ϵ_r , is independent of the number of load repetitions and dividing equation (4-4) by ϵ_r one obtains:

$$\frac{\epsilon_p(N)}{\epsilon_r} = \left[\frac{abN^{b-1}}{\epsilon_r} \right] \quad \dots (4-5)$$

For convenience, arbitrary definitions have been made for mathematical simplification:

$$\mu = \frac{ab}{\epsilon_r} \quad \dots (4-6)$$

$$\alpha = 1-b \quad \dots (4-7)$$

By substitution, Equation (2-4) may be rewritten as:

$$\frac{\varepsilon_p(N)}{\varepsilon_r} = \mu N^{-\alpha} \quad \dots (4-8)$$

One of the main models related to this mechanistic-empirical approach is the **VESYS** rutting model that relates the plastic strain to the resilient strain or viscoelastic strain through the permanent deformation parameters (**PDPs**) μ and α (Kenis, W. J., 1977) as follows:

$$\varepsilon_p (N) = \frac{\partial \varepsilon_p(N)}{\partial N} = \varepsilon_r \cdot \mu \cdot N^{-\alpha} \quad \dots(4-4)$$

where:

ε_r = resilience strain.

N = number of stress applications.

α, μ = define in Equation (4-6), (4-7).

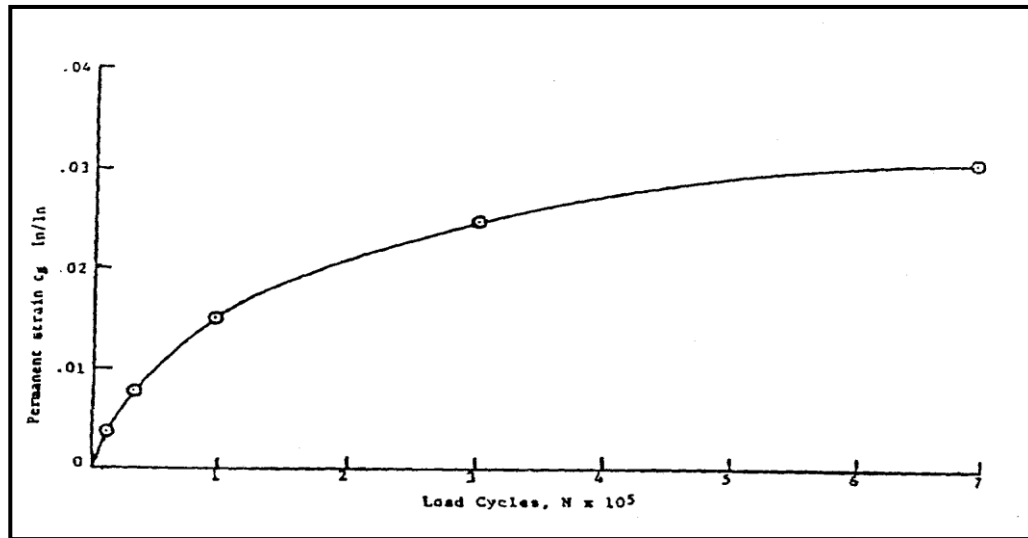


Figure 4- Typical Test Results for Repetitive Load Test on Asphalt Concrete (Rauhut et. al., 1976). لا يوجد نص من النمط المعين في المستند.

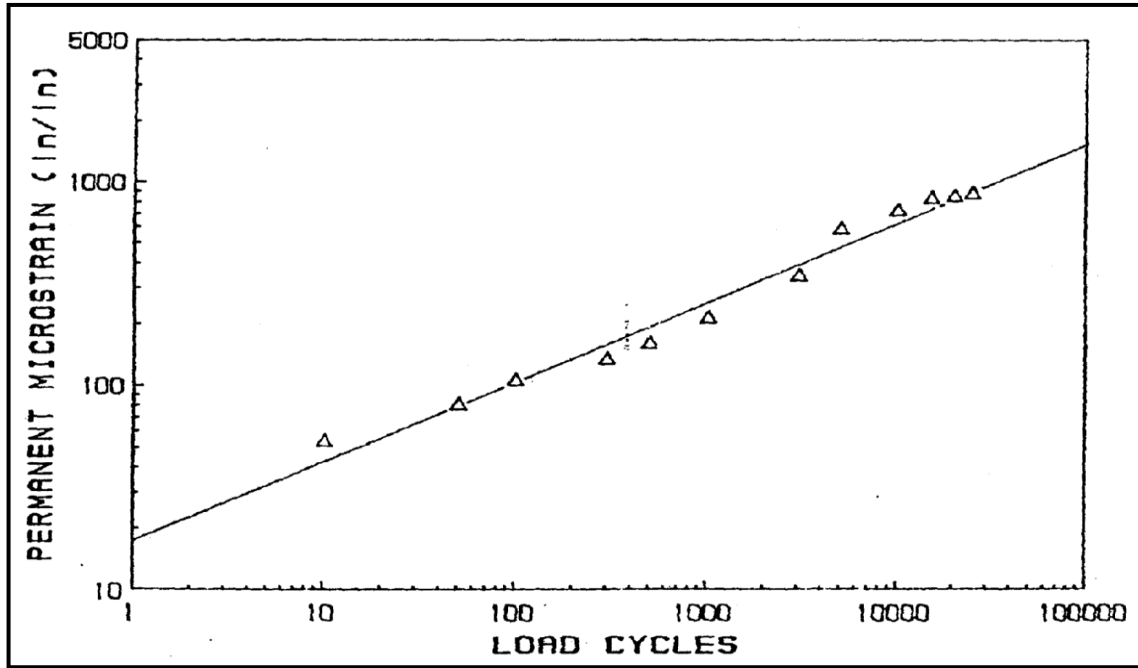


Figure 5-خطأ! لا يوجد نص من النمط المعين في المستند. Typical Test Results for Repetitive Load Test on Asphalt Concrete (Rauhut et. al., 1976)

4.2.2. Development of Rutting Model Parameters α and μ

A novel theoretical approach, founded on the creep and recovery behavior of asphalt concrete mixtures, was developed into a rutting model. The resulting hyperbolic equations accurately model the deformation performance of a paving mixture.

It has been found that both creep and recovery compliance behavior of an asphalt mixture follow curves similar to hyperbolic equation (Perdomo, Button and Lytton, 1992) for predicting the creep compliance behavior of an asphalt concrete mixture can be derived .

$$D(t) = \frac{D_o + D_m * A * t^m}{1 + A * t^m} \quad \dots\dots\dots (4-5)$$

where

Do = initial creep compliance,

Dm= maximum creep compliance,

A = regression constant

t = time, and

m = slope factor.

The same philosophical approach can be applied to the recovery compliance, even though it shows a different slope factor and a partial strain recovery. The equation obtained for this recovery compliance is represented as follows:

$$R(t) = \frac{R_o + R_m * B * t^{mp}}{1 + B * t^{mp}} \quad \dots\dots\dots (4-6)$$

where

Ro = initial recovery compliance,

Rm= maximum recovery compliance,

B= regression constant,

t = time, and

mp = slope modifier factor.

In order to obtain all the unknown parameters in [Equation \(4-5\)](#) (Do, Dm, A, m) and [\(4-6\)](#) (Ro, Rm, B, mp), an optimization technique know as pattern search was used. Pattern search is a technique based on an optimization method named ([Trust-Region](#)) this technique can solve difficult nonlinear problems more efficiently than the other algorithms and it represents an improvement over the popular Levenberg-Marquardt algorithm ([MATLAB Version 6.5.0, 2000](#)). This algorithm was developed by ([Branch, 1999](#)). The method consists, basically, of finding the unknown parameters that minimize the sum of the squared differences between Observed data and predicted values .Six asphalt concrete

mixtures containing different aggregate gradation asphalt type and asphalt percent were used in this study based on (Yassoub, 2004) laboratory testing data, in order to obtain α and μ volumetric properties for each mixture are denoted in Table (4-1).

Table1 (4-1),
Table 4-1, Volumetric Properties and Type of Asphalt Cement for Each Mixture

Mixture Type	Ac-grade	Ac %	Av%	VMA%	VFA%	Visco	EFF %	ABS %
A1	40-50	5.2	4	14.25	76	8.014	10.26	0.062
T1	40-50	4.9	4	14.57	76	8.014	10.24	0.060
B1	40-50	4.9	4	14.56	75	8.014	10.23	0.059
A2	85-100	5.1	4	16.8	79	2.909	11.78	0.059
T2	85-100	5.2	4	16.81	80.8	2.909	11.76	0.057
B2	85-100	5.1	4	17.67	82	2.909	11.75	0.057

And the resilience modulus for each mixture are denote in Table (4-2)

Table 4-2, Resilience Modulus for Each Mixture in (psi)

Mix. type	A1	T1	B1	A2	T2	B2
Resilience modulus in (psi)	182,890	186,080	187,580	168,860	172,270	172,540

¹ All mixtures information have been taken form the data of laboratory test from (Yassoub, 2004) all symbols of this table are define as following:

Ac-grade: Asphalt grade (40-50) or (85-100) that used.

Ac%: Selected optimum Asphalt content.

Av%: Air void in total mixture.

VMA%: Void in mineral aggregate.

VFA%: Void filled with asphalt.

Visco: Viscosity in (poise*10⁶) at 20 C⁰ for bitumen.

ABS: Percent absorbed asphalt (by weight of aggregate).

EFF: Percent effective asphalt (by volume of mixture).

A permanent deformation model properties α , μ are derived from Equation (4-5), (4-6) suppose that a load pulse of duration Δt is applied repeatedly, N times. Then, compliance equations for both creep and recovery may be used to estimate the total deformation and recovery that occurs (Perdomo, Button and Lytton, 1992):

$$D(N) = \frac{D0 + Dm.r.N^m}{1 + r.N^m} \quad \dots\dots\dots (4-7)$$

$$R(N) = \frac{R0 + Rm.rp.N^{mp}}{1 + rp.N^{mp}} \quad \dots\dots\dots (4-8)$$

where:

N = number of cycles, $D(N)$ = creep compliance (1/psi).
 $r = A^*(\Delta t)^m$, $R(N)$ = recovery compliance (1/psi).
 $rp = B^*(\Delta t)^{mp}$, $t = N(\Delta t)$.

The total accumulated strain after N repetitions for a constant stress pulse, σ_0 , is defined as:

$$\varepsilon_p(N) = \varepsilon_{\text{creep}}(N) - \varepsilon_{\text{re}}(N) = \sigma_0 [D(N) - R(N)] \quad \dots\dots(4-9)$$

where:

$\varepsilon_p(N)$ =creep strain at N of repetition.
 $\varepsilon_{\text{re}}(N)$ =recover strain at N of repetition.
 σ_0 =applied constant stress pulse.

The rate of change of the permanent strain with load repetitions is:

$$\frac{\partial \varepsilon_p(N)}{\partial N} = \sigma_0 * \left[\frac{\partial D(N)}{\partial N} - \frac{\partial R(N)}{\partial N} \right] \quad \dots\dots\dots (4-10)$$

Replacing equation (4-7), (4-8) in equation (4-10) and drive with respect N yield:

$$\begin{aligned} \frac{\partial \varepsilon_p(N)}{\partial N} = & \frac{Dm.r.N^m.m}{N(1+r.N^m)} - \frac{(Do + Dm.r.N^m).r.N^m.m}{(1+r.N^m)^2.N} \\ & - \frac{Rm.rp.N^{mp}mp}{N(1+rp.N^{mp})} + \frac{(R0 + Rm.rp.N^{mp})rp.N^{mp}mp}{(1+rp.N^{mp})^2.N} \end{aligned} \quad \dots (4-10)$$

By dividing both side by resilient strain (ε_r), given

$$\begin{aligned} \frac{1}{\varepsilon_r} \frac{\partial \varepsilon_p(N)}{\partial N} = & Mr * N^{(m-1)} * \left[\frac{Dm.r.m}{(1+r.N^m)} - \frac{(Do + Dm.r.N^m).r.m}{(1+r.N^m)^2} \right. \\ & \left. - \frac{Rm.rp.N^{p(m-1)}mp}{N^{(m)}(1+rp.N^{mp})} + \frac{(R0 + Rm.rp.N^{mp})rp.N^{p(m-1)}mp}{(1+rp.N^{mp})^2} \right] \end{aligned} \quad \dots (4-11)$$

where:

Mr =resilient modulus.

Now, recalling the permanent strain response model used in the [VESYS](#) approach [equation (4-4)]

$$\frac{\partial \varepsilon_p(N)}{\partial N} = \varepsilon_r. \mu. N^{-\alpha}$$

Thus, if

$$\mu * N^{-\alpha} = \frac{1}{\varepsilon_r} \frac{\partial \varepsilon_p(N)}{\partial N}$$

Then, it is apparent from equation (4-11) that

$$\alpha = 1 - m \quad \dots\dots (4-12)$$

$$\begin{aligned} \mu = Mr * \left[\frac{Dm.r.m}{(1+r.N^m)} - \frac{(Do + Dm.r.N^m).r.m}{(1+r.N^m)^2} - \frac{Rm.rp.N^{m(p-1)}mp}{N^{(m)}(1+rp.N^{mp})} \right. \\ \left. + \frac{(R0 + Rm.rp.N^{mp})rp.N^{m(p-1)}mp}{(1+rp.N^{mp})^2} \right] \end{aligned} \quad \dots\dots (4-13)$$

where:

$$p = \frac{mp}{m} \quad \dots\dots\dots (4-14)$$

The mixture B1 show a good resistance to rutting ([Yassoub,2004](#)),and its clearly form [Table \(4-2\)](#) that it has a large modulus of resilience ,which mean offering low response according to Hooke's law shown in Equation (4-1).

Mixture ([B1](#)) is fitted below and used to predicting rutting in an example.

Table 4-3, Creep and Recovery Compliance in (1/psi) for Mix B1 under Temperature 25 c °.

Loading for creep unloading for recovery(sec)	Creep compliance (1/psi)	Recovery compliance(1/psi)
0.1	0.00003025	0.000023375
0.25	0.000039875	0.00002525
0.5	0.000053375	0.000026375
1	0.000058125	0.00002875
2	0.000066625	0.00003275
4	0.0000755	0.0000355
8	0.0000835	0.000035
15	0.000108	0.000036625
30	0.00012525	0.000036625
45	0.000134875	0.000036625
60	0.00013775	0.000036625

Table 4-4, Information Input Limitations of Nonlinear Regression Method

Type of procedure :	least square method
Algorithm :	Trust – Region method
Different min change :	1.0E-12
Different max change :	1.0E-14
Max function evaluating :	1.0E09
Max iteration :	1.0E09
Tolerance of function :	1.0E-20
Tolerance of x value :	1.0E-20

Table 4-5, Estimating Parameters of Hyperbolic Model for Mix B1.

A	0.148	B	2.124
Do	1.362e-005	Ro	9.057e-006
Dm	0.0003413	Rm	3.978e-005
m	0.355	mp	0.434
SSE:	1.4481e-010	SSE:	1.105e-011
R-square:	0.9901	R-square:	0.99
Adjusted R-square:	0.9876	Adjusted R-square:	0.9845
RMSE:	4.2546e-006	RMSE:	1.1753e-006
Dfe:	8	Dfe:	8

where:

- SSE=Sum of squares due to error.
- Rsquare=Coefficient of determination.
- Dfe=Degrees of freedom.
- Adjrsquare=Degree-of-freedom adjusted coefficient of determination.
- RMSE=Root mean squared error (standard error).
- MaxFunEvals -- Maximum number of function (model) evaluations allowed. The default value is 600.
- MaxIter -- Maximum number of fit iterations allowed. The default value is 400.
- TolFun -- Termination tolerance used on stopping conditions involving the function (model) value. The default value is 10-6.
- TolX -- Termination tolerance used on stopping conditions involving the coefficients. The default value is 10-6.

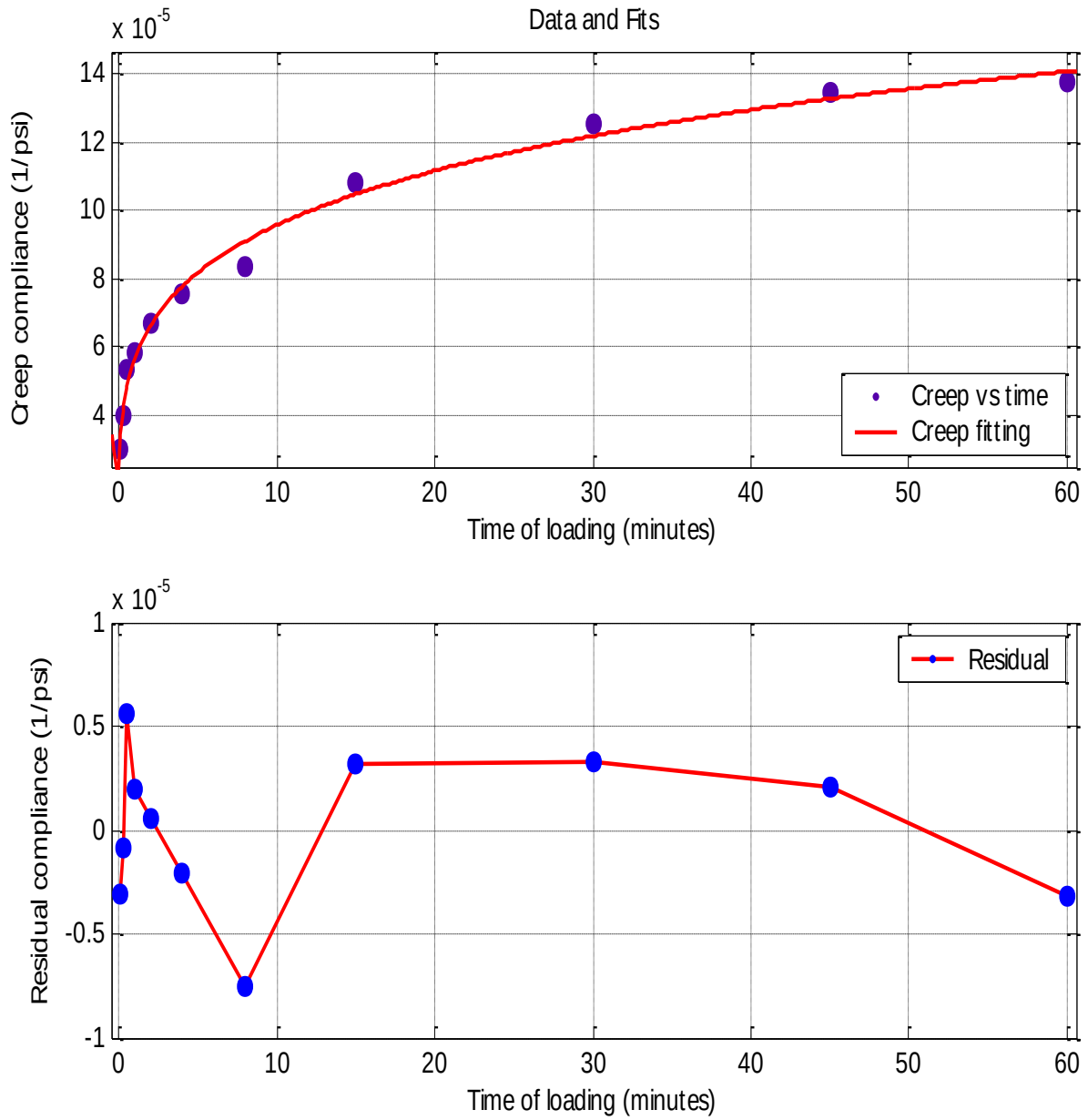


Figure 6-خطأ! لا يوجد نص من النمط المعين في المستند. Creep Compliance Fitting and Residual of Compliance.

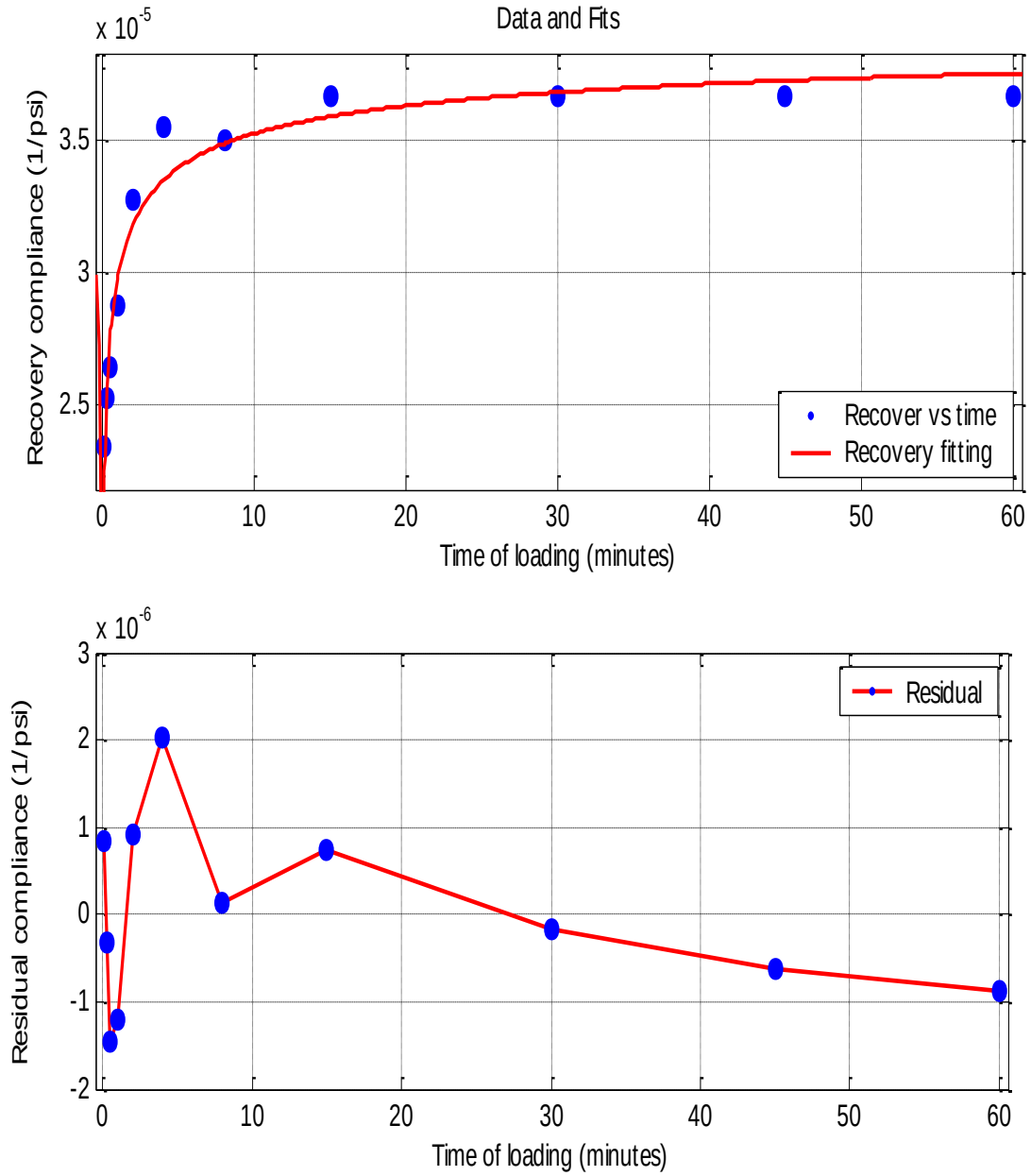


Figure 7-خطاً! لا يوجد نص من النمط المعين في المستند. Recovery Compliance Fitting and Residual of Compliance.

The results of the remaining mixtures are presented in [Table \(A-1\)](#) through ([A-](#)) in appendix ([A](#)).

The creep test is a simplified test and requires much less time to perform than the dynamic test. With a dynamic loading time of 0.1-s duration, a static load of t seconds is equivalent to 10t load repetitions. Therefore ,the permanent deformation after a 1000-s creep test is equivalent to that of a dynamic test after 10000, so if the number of load applications is N, then the creep time in seconds corresponding to N is (0.1*N) because the duration of each load application is $\Delta t=0.1\text{-sec}$ (Yang H. Huang ,1993). Based on this criteria it should be used ($N=(60*120)/\Delta t$), this led to use ($N=72,000$), for simulating plastic parameters (α,μ).

For verification purpose an investigation has done in order to computing α,μ for each mixture via a model developed by ([Albayati,2006](#)) based on statistical analysis for different asphalt material properties and under wide range of surrounding circumstances such as (temperature ,stress level ...etc) as following:

To compute alpha (α)

$$\alpha=1.391-0.011*TMP-0.00541*STR-0.252*Dur-0.0212*EFF \quad \dots (4-15)$$

To compute mu (μ).

$$\begin{aligned} \mu= & 4.1896559+0.02*TMP-0.13091*VFA-0.00019*TMP^2+0.000312*STR^2- \\ & 0.04305*AV^2-0.00112*STR*VIS+0.031432*STR*Dur+0.570402*Dur*AV- \\ & 127.263*Dur*ABS+0.511025*Dur*EFF- \\ & 0.08536*AV*EFF+9.964481*VMA*ABS. \quad \dots (4-16) \end{aligned}$$

where:

- Av%: Air void in total mixture.
- VMA%: Void in mineral aggregate.
- VFA%: Void filled with asphalt.
- Visco: Viscosity in (poise*10⁶) at 20 C⁰ for bitumen.
- ABS: Percent absorbed asphalt (by weight of aggregate).
- EFF: Percent effective asphalt (by volume of mixture).
- Dur: duration of loading in (second).
- STR: applied stress in (psi) for specimen.
- TMP: Temperature in centigrade.

Table 4-6, Plastic Parameters for Each Mixture.

Mixture type	Alpha (α) by Amjed model*	Mu (μ) by Amjed model*	p [^] value	Alpha (α) by proposed model	Mu (μ) by proposed model
A1	0.6407	0.4814	1.800	0.6500	0.4561
T1	0.6411	0.5428	1.7549	0.6410	0.4119
B1	0.6376	0.53	1.2225	0.6450	0.3327
A2	0.6106	0.420	1.6221	0.6110	0.4392
T2	0.6028	0.4571	1.5869	0.6030	0.3535
B2	0.6034	0.452	1.5909	0.6040	0.3421

The [Table \(4-6\)](#) refer to a good agreement if they be compare between alpha (α) but fairly for mu (μ) because of the low Coefficient of determination in ([Albayati, 2006](#)) model ($R=0.749$), It should be note that the different in the two procedure led to different values is also effect on compute values α, μ . It is clearly that ([P](#)) value in Table (4-6) good indicator to refers that mix B1 has a

* The regression of Equation (4-15) and (4-16) based on Coefficient of determination is equal to $R^2=0.958$, 0.749 for more information see ([Albayati, 2006](#)) at Table (6-7) page (165).

^ Where (p) define by [Equation \(4-14\)](#).

small plastic parameter μ (μ) also this indicator and modulus of resilience refer together to rutting depth as will see in the "rut depth prediction in asphalt concrete" Viscoelastic-plastic could be applied to creep and recovery compliance for different degree of temperature as shown in Table (4-7).

Table 4-7, Plastic Parameters for Mix B1 under Different Temperatures.

plastic parameters	Mix B1 under 25 c ⁰	Mix B1 under 40 c ⁰	Mix B1 under 50 c ⁰
Alpha (α)	0.6450	0.4770	0.3680
Mu (μ)	0.3327	0.0379	0.0184
P value	1.2225	0.7008	0.7850

Alpha (α) value is affected by creep and recovery compliance data only, while Mu (μ) value is affected by (creep, recovery compliance) and modulus of resilience, modulus of resilience is one of major factors which affecting on Mu (μ) calculation because it is appears in Mu (μ) equation (4-13) and modulus of resilience (M_r) is decrease with increase the temperature. This led to lower value of Mu if temperature increasing [Figure \(4-5\)](#) shows the relationship between the modulus of resilience and temperature. The simulating data and viscoelasto-plastic parameters for [Table \(4-7\)](#) are present in appendix ([A](#)).

The best define of alpha (α) is a permanent (plastic) deformation parameter indicating the rate of decrease in permanent deformation as the number of load applications increases, while mu (μ) is a permanent (plastic) deformation parameter representing the constant of proportionality between permanent and elastic strains ([Yang H.Huang,1993](#)).

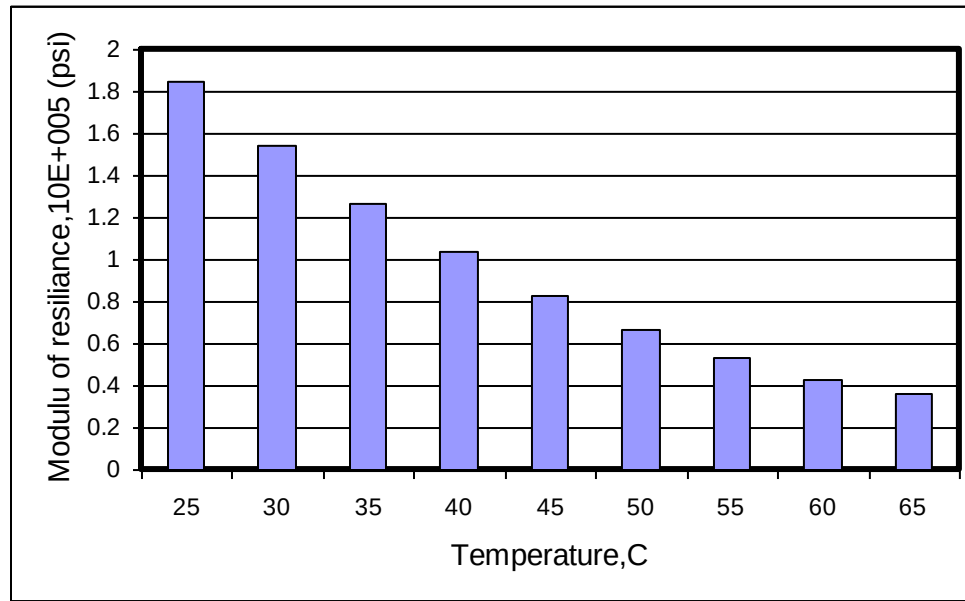


Figure 8-Modulus of Resilience Varies with Temperature for Mixture B1. لا يوجد نص من النمط المعين في المستند.

4.3 Rut Depth Prediction in Asphalt Concrete

4.3.1 Statement of the Problem

Asphalt concrete rutting is a longitudinal deformation that develops under the action of canalized loads. It is a bowl-shaped depression in the wheel paths resulting from pavement deformation. It occurs because of consolidation and/or plastic flow in the asphalt concrete layers within the pavement structure. Rutting often develops over a number of years due to repeat loadings ([AASHTO, 1988](#)). It has a major impact on pavement performance. According to the AASHTO design equation for flexible pavement, a (1.1") deep rut will reduce the Present Serviceability Index (PSI) of a relatively new pavement, having no other distress, from 4.2 to 2.5. The AASHTO design guide 2002 suggests that major highways be maintained at a PSI level pavement has in effect been lost. Also, rutting affect on vehicle handling and cause a hydroplaning. Handling for smaller vehicles is impaired when they drive in a rut pattern that has been

established in pavement surface by trucks. As water accumulates in the ruts, the possibility for hydroplaning Increases due to rutting ([AASHTO, 1988](#)).

Most of the existing design methods are based on the assumption that, if the maximum vertical compressive strain or stress at the surface of the subgrade is less than a critical value, then rutting will be limited to a tolerable level for a specified number of load applications. Unfortunately, such methodology does not preclude rutting which might occur in the asphalt bound layers and does not have the capability of rutting prediction.

In this section, a simplified procedure for the prediction of the rut depth in the asphalt concrete layers within the pavement structure is developed and verification for the obtained results by comparison with ([Albayati, 2006](#)) and ([Yassoub, 2004](#)) models for is presented. The proposed method is based on the permanent deformation ([Viscoelastic-Plastic](#)) model. In this method, the asphalt concrete layers that have different mechanical properties are converted into an equivalent layer using the equivalent stiffness method ([Ullidtz, 1979](#)). This has the consequence of instead of evaluating and testing the permanent deformation for each layer, only the asphalt concrete mixture for the equivalent layer will be characterized for permanent deformation prediction as present in Chapter 3.

The necessary input parameters for the proposed rut depth prediction method are:

1. Geometry of the pavement structure
2. Climatic factor
3. Materials response properties
4. Traffic loading

The proposed methodology is presented and discussed in the following sections; the steps of the methodology are further illustrated throughout a practical example for rut depth estimation.

1. Geometry of the pavement structure:

The geometry of the pavement structure ([Thickesses of The Layers](#)) required as input for the proposed methodology. In order to obtain the thicknesses for the Pavement layers, structural design can be performed using one of the available methods such as [AASHTO](#) or [PCASE](#) program. For the practical example, the geometry of the pavement structure is shown in [Figure \(4-6\)](#). The shown structure is a conventional flexible pavement section with five layers, these are: wearing course, binder (leveling) course, base course, subbase course and subgrade.

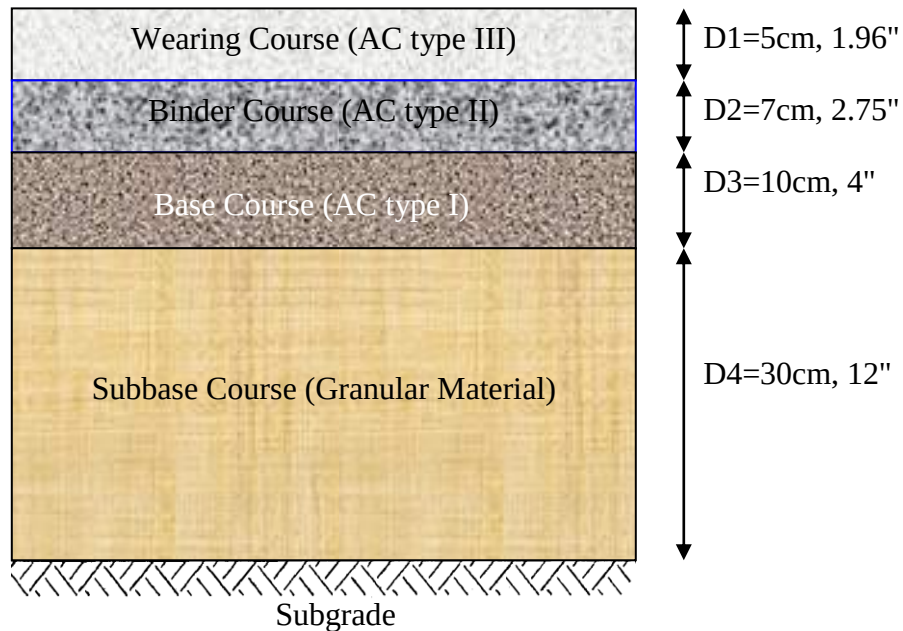


Figure 9- خطأ! لا يوجد نص من النمط المعين في المستند. Geometry of The Pavement Structure.

2. Climatic Factors:

Many climatic and environmental factors influence the performance of the flexible pavement. The only environmental effect that is included in the proposed model is the temperature of asphalt layer. Creep and recovery compliance vary with time are affected by temperature. Changes in the temperature of the asphalt layer affect the elastic and viscous properties of the bitumen. After the mean air temperature was recorded, the profile of the

asphaltic material could be predicted. Predicting the temperature profile is complicated and requires details information about the thermal properties of the asphalt and the ambient conditions. Consequently, a simplified approach was used, where the maximum temperature of the asphalt layer was predicted at the (4.1"), (10.22 cm) for effect layer see ([Chapter-3-, Figure \(3-14\)](#)), at critical point below the surface and varied with the depth. This approach was reported by ([Albayati, 2006](#)) depending on the air temperature. This equation was:

$$T_{asp} = 1.217.T_{air} - 0.354.Z \quad \dots (4-17)$$

where:

T_{asp} =the effective surface layer temperature C^0

T_{air} =the measured mean air temperature C^0

Z =the depth below the surface (mm)

Mean Annual Air Temperature ([MAAT](#)) is equivalent to $23 C^0$ was reported in a ([Albayati, 2006](#)), this led to $T_{asp}=25C^0$.

3. Material Response Properties:

When the pavement structure is treated as multi-layer elastic system, it is necessary to quantify the material stiffness as defined by the resilient modulus of elasticity and Poisson's ration. These properties are needed for the calculation of the stresses, strains and deflections response in the pavement system under the application of traffic loading. The properties are also needed in the transformation of the pavement layers to an equivalent layer using equivalent stiffness method ([Chapter-3-, "3.5.2 Section Transformation \(Equivalent thickness\)"](#)). The finite element model both elastic or viscoelastic are need resilience modulus and Poisson's ratio as input for this purpose it should be calculating all these properties as following:

Asphalt Concrete Layers:

The resilient modulus of asphalt concrete wearing course can be predicted using the model shown below which is developed by (Amjed, 2006).

$$Mr = 46.6609 \times 10^4 - 9411.88 \times TMP + 68.77 \times TMP^2 + 724.72 \times AV^2 + 1679.443 \times TMP \times Dur - 163.46 TMP.AV + 73.46.VIS.VMA - 12053.97 Dur.AV - 69.376.Dur.EFF - 13.7435 \times 10^4 \times ABS.EFF. \quad \dots\dots\dots (4-18)$$

where:

- Mr = resilient modulus (psi).
- TMP = temperature in degree centigrade (C⁰)
- AV = percent air voids
- Dur = applied stress duration (sec)
- VIS = viscosity at 20 C⁰ (10⁶)
- VMA = percent voids in the mineral aggregate
- EFF = percent effective asphalt (by volume of mix)
- ABS = percent absorbed asphalt (by weight of aggregate).

The asphalt concrete mixture volumetric properties, asphalt cement viscosity and temperature at the mid-depth of layer (as estimated using [Equation \(4-17\)](#) are required to be input to the above model beside the loading duration which can be calculated as follows ([Yang H.Huang,1993](#)).

$$tw = \frac{12 \times ar}{S}. \quad \dots\dots\dots (4-18)$$

Where:

- tw = loading duration (sec).
- ar = tire contact radius (cm or inch)
- S = vehicle speed (cm/sec (Km/hr x 27.8) or inch/sec (mphx17.61)

For the asphalt concrete wearing, the resilient modulus values are determined from the [Equation \(4-18\)](#) for type mix B1 all volumetric properties are present in [Table \(4-1\)](#), for the asphalt concrete binder (leveling) and base

layers, the resilient modulus are obtain for both directly from ([Albayati, 2006](#)) model determined from the repeated load test. The second response property is the Poisson's ratio (ν); it's defined as the ratio of the lateral strain to the axial strain. By measuring the axial and lateral strains during the resilient modulus test, the Poison's ratio can be determined, because Poison's ratio has a relatively small effect on pavement performance prediction, it is customary to assume a reasonable value for use in performance prediction, rather than to determine it from actual tests ([Albayati, 2006](#)). [Table \(4-8\)](#) shows the range and typical Poison's ratio values for paving materials.

Table 4-8, Poisson Rations for Different Paving Materials (Southgate et al., 1977)

Material	Range	Typical value
Asphalt concrete	0.30-0.4	0.35
Unstabilized granular subbase and base	0.30-0.45	0.4
Silt subgrade	0.35-0.45	0.45
Clay subgrade	0.4-0.5	0.5

Subbase layer

The method adopted for the estimation of the resilient modulus of the subbase granular material is that developed by the SHELL researchers ([Claessen et.al.1977](#)).In this method it is assumed that the subbase resilient modulus is a function of the subgrade resilient modulus according to the following relationship:

$$Mr_{subbase} = K \times Mr_{subgrade} \quad \dots\dots\dots (4-19)$$

where:

$$K = 0.2 \cdot h^{0.45} \quad \dots\dots\dots (4-20)$$

h = the thickness of subbase layer in mm

M_r = the resilient modulus in psi.

The relationship is applicable in the range $2 < k < 4$. The Poisson's ratio for the subbase layer can be obtained from table (4-8).

Subgrade layer:

Similarly, the subgrade resilient modulus was estimated using the relationship developed by the SHELL investigators (Claessen et. al., 1977). The relationship is based on the correlation between dynamic in situ tests for the subgrade resilient modulus and the corresponding laboratory measured California Bearing Ratio (CBR), CBR value for the subgrade soil can be obtained according to standard test procedure of ASTM D-1883, the relationship is given below:

$$M_{r\text{subgrade}} = 1450 * \text{CBR} \quad \dots\dots\dots (4-21)$$

Where: M_r is the subgrade resilient modulus expressed in psi, and CBR value for the subgrade soil is expressed as a percentage. The Poisson ratio for the subgrade soil can be obtained from Table (4-8).

4. Traffic loading:

In an effort to simplify the input traffic requirements, it is decided to convert mixed traffic to equivalent 18-Kip (80 KN) single axle loads in accordance with AASHTO load equivalency factors (AASHTO, 1993). The adopted 18-kip ESAL configuration is shown in Figure (4-7). The required traffic information for the estimation of the 18-Kip ESAL is the Annual Average Daily Traffic (AADT) and traffic composition at the start of the analysis period. These values can be obtained from the actual traffic counts on the existing pavement where the rut depth prediction is to be performed or on nearby highways with similar travel patterns for the new constructed pavement.

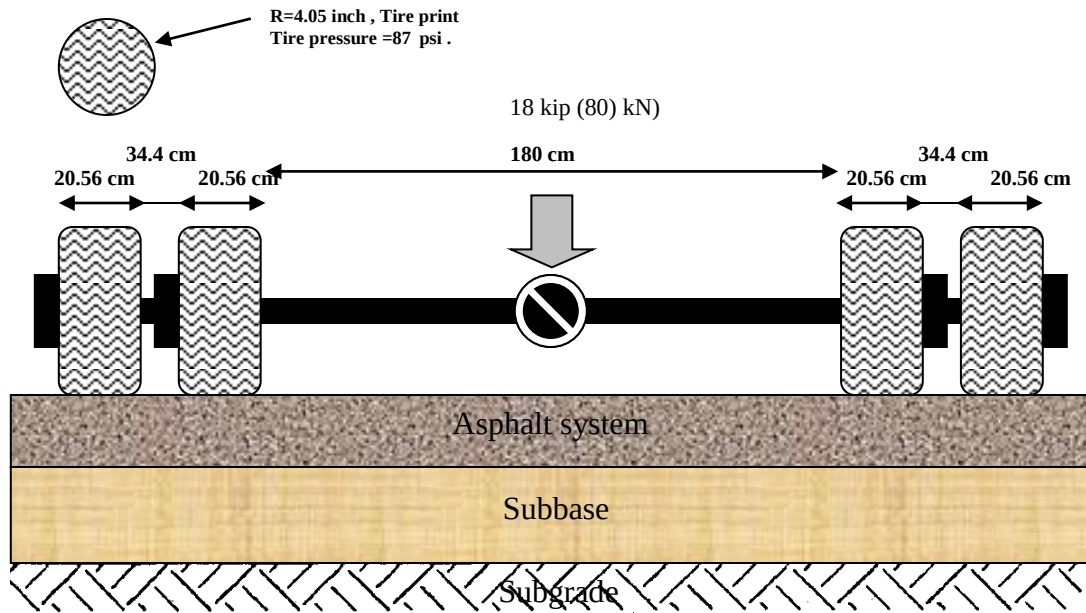


Figure 10, 18-Kip ESAL Configuration لا يوجد نص من النمط المعين في المستند.
For the practical example of rut depth prediction, N=6,000,000 repetitions is used based on worksheet* present by (Albayati, 2006).

For the practical example also the material response properties for each layer of the conventional pavement section are presented in the Table (3-2), mix B1 was used as a wearing asphaltic layer and it is represent the effective layer, all volumetric properties for mix B1 are present in Table (4-1).

4.3.2 Rut Depth

Rut depth of the asphalt concrete layers of the pavement structure under repeated loadings can be estimated according to the formula:

$$Rd = \varepsilon_p \times h \quad \dots\dots\dots (4-22)$$

where:

Rd= rut depth of asphalt concrete layers (mm)

ε_p = permanent strain in the asphalt concrete layers (mm/mm)

h= total thickness of asphalt concrete layers (mm).

* Worksheet was present in Amjed dissertation see " table (8-5) ,p 227" to Amjed dissertation for more details.

ϵ_p stated as previous page is determined using the proposed model as following:

Recall Equation (4-4):

$$\epsilon_p(N) = \frac{\partial \epsilon_p(N)}{\partial N} = \epsilon_r \cdot \mu \cdot N^{-\alpha}$$

$$\epsilon_p = \int_0^N \epsilon_p(N) \cdot dN = \epsilon_r \cdot \mu \cdot \frac{N^{(1-\alpha)}}{(1-\alpha)} \quad \dots (4-23)$$

Using the value of Alpha (α), Mu (μ) that obtained in table (4-6) for mix B1 under 25C⁰, Alpha (α)=0.6450 , Mu (μ) = 0.3327. Input ϵ_r for mix B1 that obtain from table (3-5), $\epsilon_r^* = 2.21\text{E-}04$.

It should be noted that the rut depth predicted according to the above formula is limited for the asphalt concrete layer. But pavement structure includes also subbase and subgrade layers which may also contribute to the total pavement rutting. Therefore, the rut depth values of these layers which is out of the scope of this research should be added to that estimated according to Equation (4-22).

For the practical example, all the required variables for use of Equation (4-22) are presented in the previous sections, therefore, the total rut depth for 6,000,000 18-kip (ESAL) will be:

$$\begin{aligned} \epsilon_p &= 0.05277 \cdot 204.4 \\ &= 10.78 \text{ mm} \end{aligned}$$

For the same philosophy^{*} it could replace ϵ_r in Equation (4-23) by ϵ_{ve} (viscoelastic strain that obtain from table (3-5), $\epsilon_{ve} = 2.57\text{E-}04^{\wedge}$ at $t = 0.1\text{sec}$.

$$\epsilon_p = 0.06137 \cdot 204.4$$

[♦] Obtained from elastic finite element model present in chapter -3-.

^{*} (Yang H.Huang, 1993) offer in "Pavement Analysis and Design" at section 10.5- "Probabilistic Computer Program" VESYS method procedure and it contains this philosophy.

[^] obtained from viscoelastic finite element model present in chapter -3-.

$$= 12.54 \text{ mm}$$

It is clearly that general response calculations are so important and influent on rutting depth calculations. Increasing rutting depth from 10.78 mm to 12.54 mm, with 14% percent different it should take care and treated the wearing asphalt concrete layer as viscoelastic approach instead of elastic approach only. Applying proposed model for all types of mixtures at $N=6,000,000$ repetitions, rutting depth varies with type of mixture is present in Figure (4-8). It apposite to use Mixture B1 because it offers a low susceptibility to rutting which is the same result of (Yassoub, 2004).

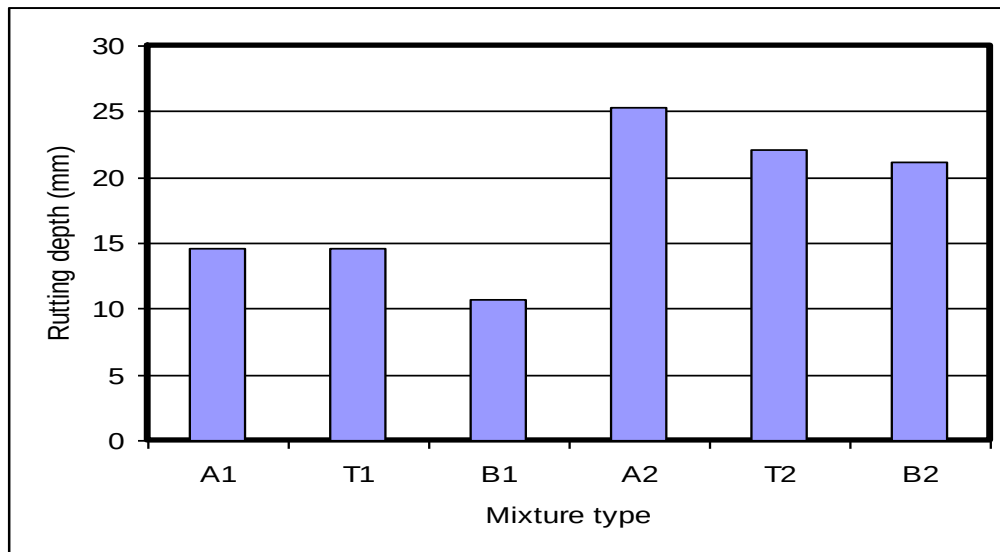


Figure 11- خطأ! لا يوجد نص من النمط المعين في المستند. Rutting Depth Varies with Type of Mixture.

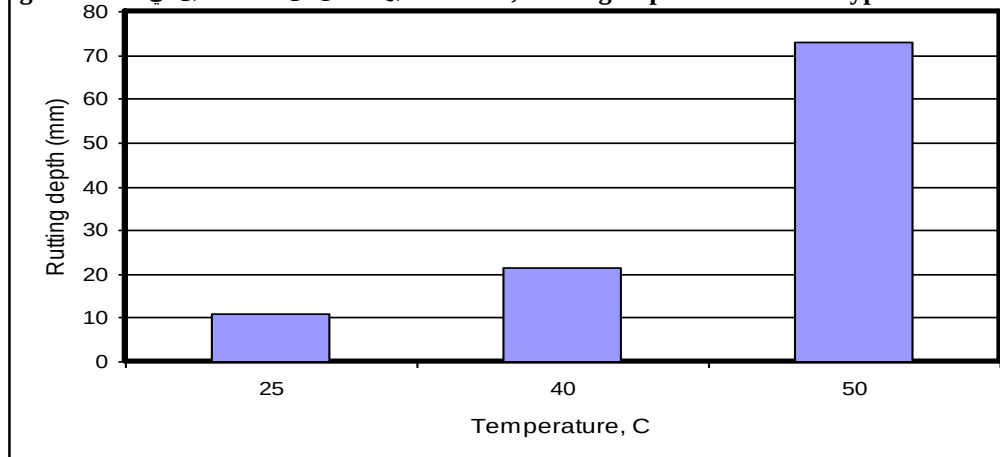


Figure 12- خطأ! لا يوجد نص من النمط المعين في المستند. Rutting Depth Varies with Temperature for Mix B1.

4.3.3 Verification of the Results with Amjed and Yassoub model

To ensure the reliability of the developed models and the algorithm of the rut depth predication for the asphalt concrete layers under the application of repeated loads, the obtained results for the rut depth of the practical example are verified by comparison with results of (Albayati, 2006) and (Yassoub, 2004) as following:

Albayati rutting model was developed based on the repeated loads tests, represent a statistical model applying for different test conditions and mix properties, in the following form:

$$\text{Log} \varepsilon_p = -34.463 + 0.983 \cdot \log N^{(0.00973 \text{TMP} + 0.00284 \text{STR})} + 1.961 \cdot \log \text{TMP} + 1.812 \cdot \log \text{STR} + 0.656 \cdot \log \text{Dur} - 36.277 \cdot \log \text{ABS} - 4.951 \cdot \log \text{VFA} - 1.455 \cdot \log \text{AV} - 1.93 \cdot \log \text{VMA} . \quad (4-24)$$

(All variables were previously defined).

To obtain rutting depth it should be applying equation (4-22)

Yassoub rutting model was developed based on uniaxial repeated cyclic creep/recovery; a viscoplastic strain model was developed for local asphalt paving mixtures in the general form of:

$$\varepsilon_{vp} = 4.7195 \cdot 10^{-9} \times \sigma^{0.868} \times t^{0.658} \times T^{1.974} \quad \dots (4-25)$$

The developed model represents the variation of permanent strain with material property as asphalt mixtures, level of applied stresses (σ), time of loading (t), and degree of temperature (T). The developed form of the accumulated permanent strain as a function of the number of repetitions $\varepsilon_p(N)$ is:

$$\text{Log} \cdot \varepsilon_p (N) = \log \varepsilon_{vp} + S \cdot \log N \quad \dots (4-26)$$

For mixture B1 the slope (S) of the $\log (\varepsilon_{vp})$ vs. $\log (t)$ relationship is equal to (0.3693) and $\varepsilon_{vp} = 0.00014749$. To obtain rutting depth it should be applying Equation (4-22), the comparison is shown in Figure (4-10).

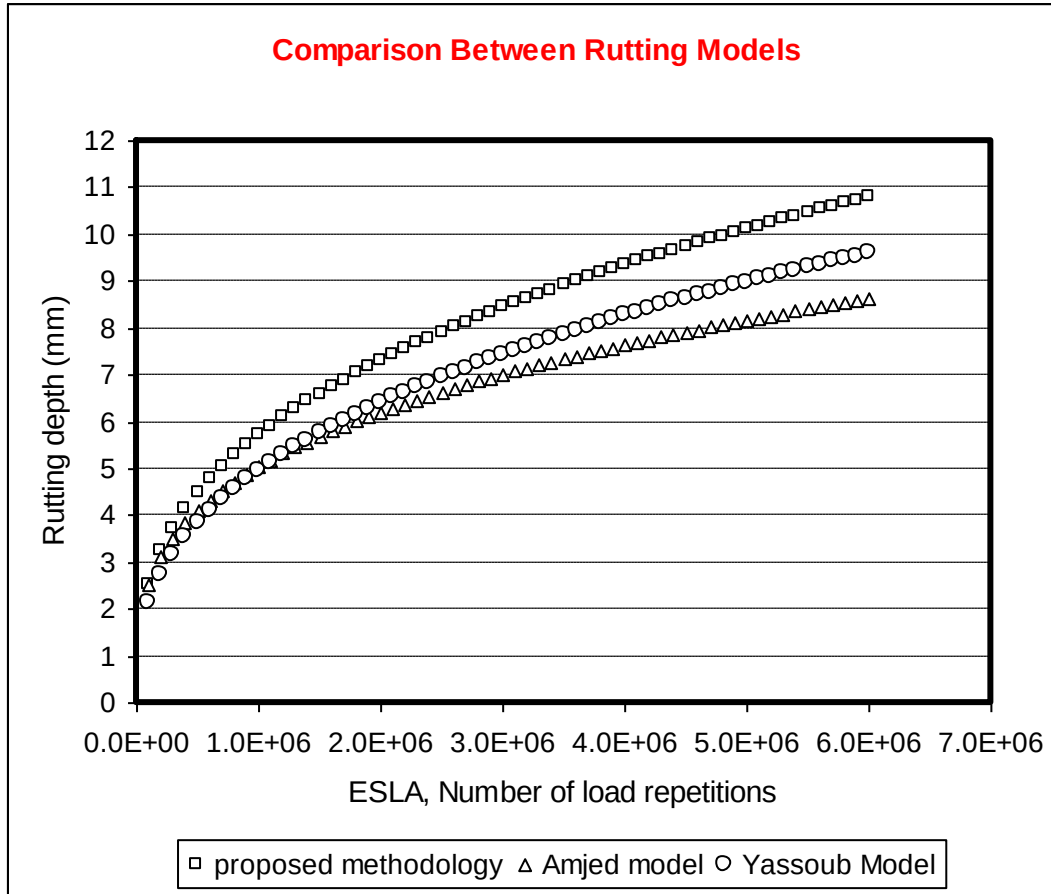


Figure 13- خطأ! لا يوجد نص من النمط المعين في المستند. Comparison between Amjed, Yassoub and proposed Methodology Rut Depth for Various Numbers of ESAL Applications for Mix B1.

The hyperbolic model provides a near perfect prediction of actual creep and recovery performance of asphalt mixtures. While the Rutting computing by (Alpha and Mu) that proposed in Equation (4-12) ;(4-13) give a high value (because of different in analysis method, type of data that simulated, Type and sensitivity of variables, test type and conditions e.g. different in mold size.) but it is still in the same range of other models, that range is denoted in FHWA specifications in the literature. Finally the procedure for prediction of rutting potential in asphalt mixtures is present in Figure (4-11).

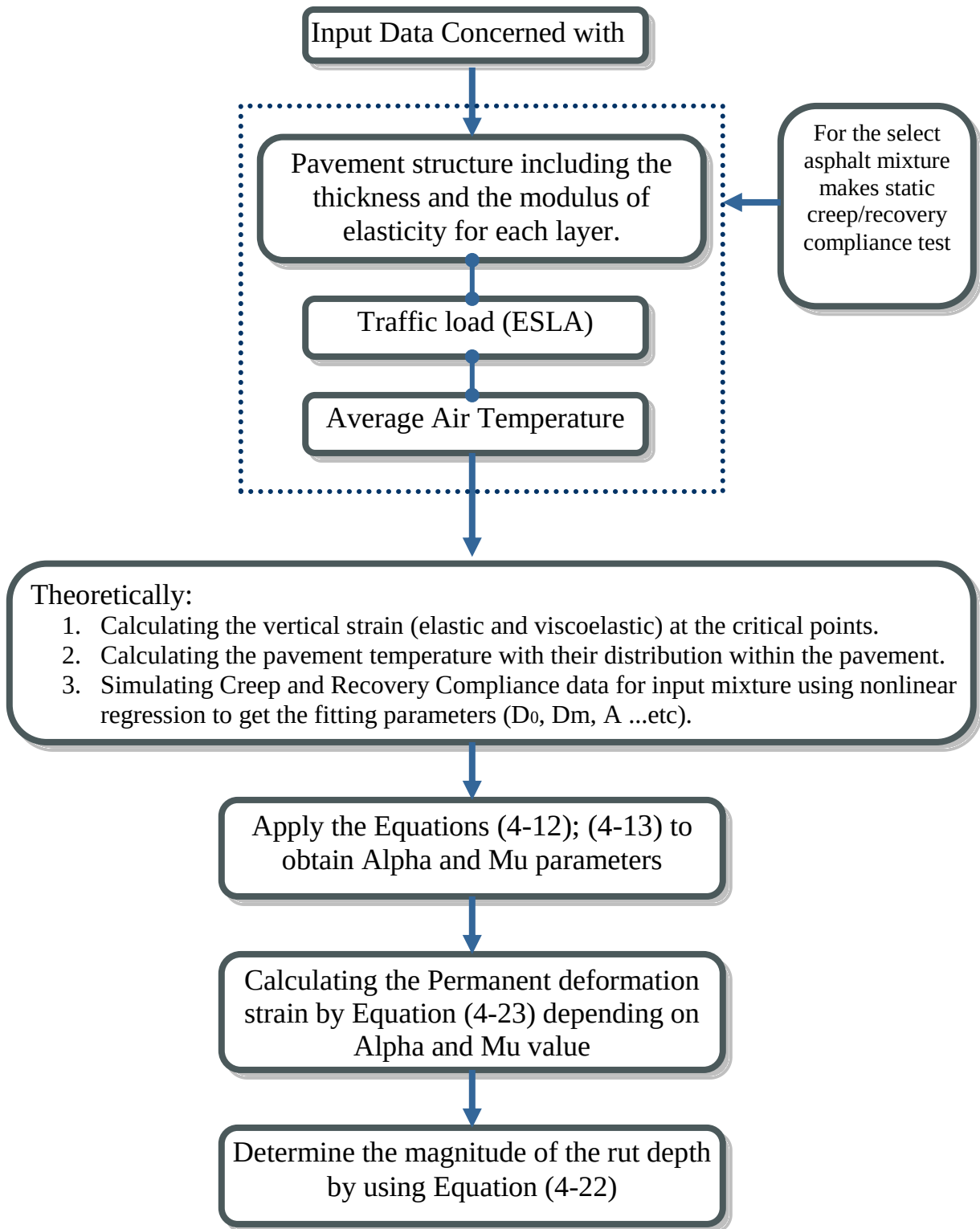


Figure 14- خطأ! لا يوجد نص من النمط المعين في المستند. Flow Chart of the Proposed Model to Calculate the Rutting Potential.

5 CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATINOS

5. Conclusions:

Within the limitations of materials and testing data used in this work, the following conclusions could be drawn:

1. Finite element approach could use to estimate the primary response components elastic or viscoelastic such as (stress; strain; deflection etc) at critical points, also finite element approach could be deal with a various types of materials and load configurations and solve the challenges problem e.g. dynamics load; heat transfer that related with system of pavement's layers.
2. Under different type of meshes, two dimensions finite element model is faster and give a reasonable accuracy. Medium mesh designs to give the primary response.
3. To analysis the system of pavement's layers by finite element approach a ANSYS 9.0 software program was used in this work, a general notes in ANSYS 9.0 environment should be maintain.
 - Load distribution applied on surface of element under type (8-nodes, Plane-182) is better than applied load on nodes.
 - The materials that used in analysis should be work as one group to transfer the response from layer to the other, to achieve that an option named (Glue) should be use between two layers meeting.
 - In viscoelastic solution after input shear and bulk moduli vary with time it should using a sufficient number of kernel function parameters until reach to a low value of standard mean of error and

good coefficient of determination, to give a best fitting of decay moduli behavior.

4. Based on the creep / recovery test data the hyperbolic model provides a near perfect prediction of actual creep and recovery performance of asphalt mixtures, (P) value as defined in equation (4-14) is range from (0.7008) to (1.8). For different in mixture type (P) value is could be a good indictor if its value small the rutting depth is small too under the same stress level and temperature.
5. Models were developed to predict the permanent deformation parameters Alpha and Mu. The developed model are:

$$\alpha = 1 - m$$

$$\mu = Mr * \left[\frac{D_m \cdot r \cdot m}{(1 + r \cdot N^m)} - \frac{(D_o + D_m \cdot r \cdot N^m) \cdot r \cdot m}{(1 + r \cdot N^m)^2} - \frac{R_m \cdot r_p \cdot N^{m(p-1)} m_p}{N^{(m)} (1 + r_p \cdot N^{m_p})} + \frac{(R_o + R_m \cdot r_p \cdot N^{m_p}) r_p \cdot N^{m(p-1)} m_p}{(1 + r_p \cdot N^{m_p})^2} \right]$$

(All variable were previously defined)

6. The proposed model for predicting rutting potential in flexible paving mixtures shall provide a suitable mean for predicting rutting distress required in the process of design and construction of asphalt pavements, in the form of:

$$Rd = \varepsilon_p \times h$$

where

Rd= rut depth of asphalt concrete layers (mm)

ε_p = permanent strain in the asphalt concrete layers (mm/mm)

h= total thickness of asphalt concrete layers (mm).

$$\text{and } \epsilon_p = \int_0^N \epsilon_p(N).dN = \epsilon_r .u. \frac{N^{(1-\alpha)}}{(1-\alpha)}$$

7. Elastic finite element model give rut depth 10.78 mm while viscoelastic finite element model give rut depth 12.54 mm, with 14% percent different it should maintain and treat the wearing asphalt concrete [Effective] layer as viscoelastic approach instead of elastic approach only.

5.1. Recommendations for Future Research:

On the bases of this study, the following recommendations are made:

1. The finite element approach is a general solution and it able to solve challenges problems such as (considering the moving load or [dynamic load]; heat transfer; viscoplastic approach; creep simulating with time or strain hardening that related with system of pavement's layers is still with no simulating in Iraq.
2. Finite element approach could be use not only as full scale, it could be use as a prototype, many asphalt material test apparatuses molds lack to simulating and there is no idea about the response except a simple attempted represented by a deal gage.
3. Full scale field tests should be conducted on local pavements composed of different structure and subjected to different patterns and characteristics of traffic in order to obtain a comprehensive rutting model under the ambient conditions.
4. Using different asphalt's mixtures contain larger aggregate, i.e., maximum size greater than 19 mm (3/4 inch); filler type; asphalt modifiers; method of specimen compaction :Kneading and gyratory.

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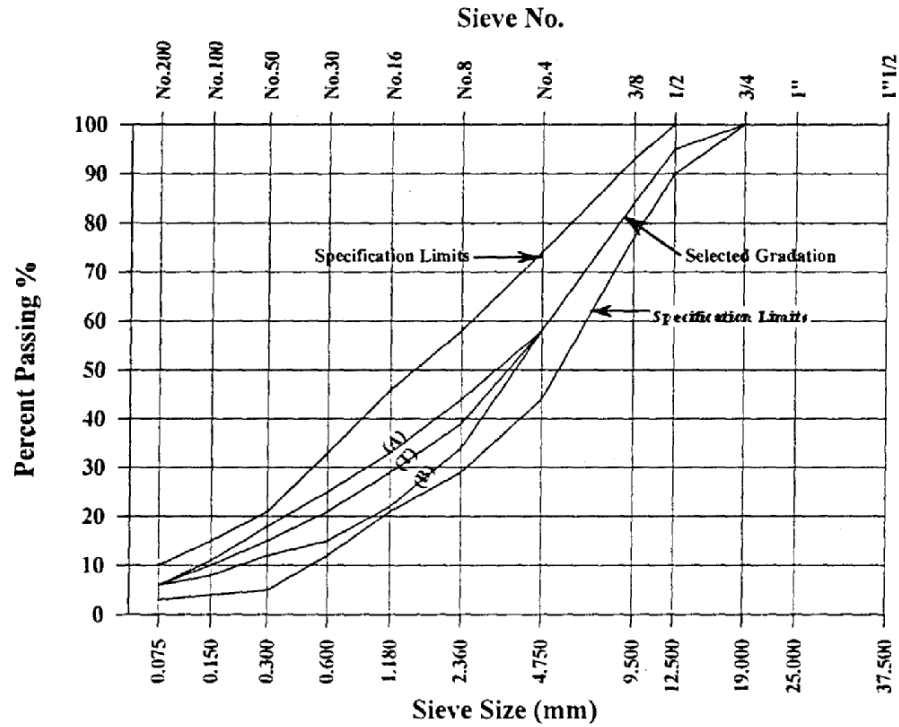
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APPENDIX A APPENDIX A **MODELING OF PERMENTANT STRAIN OF ASPHALT PAVEMENT** **FOR OTHER MIXTURES**



Sieve Size	Percent Passing %		
	A	T	B
3/4	100	100	100
1/2	95	95	95
3/8	84	84	84
No.4	58	58	58
No.8	44	39	34
No.16	33	29	22
No.30	25	21	15
No.50	18	15	12
No.100	11	10	8
No.200	6	6	6

Figure A- 1, The Selected Gradations Chart (Yassoub, 2004).

A-1.Parameters α , μ for Each Mixture

- SSE=Sum of squares due to error.
- Rsquare=Coefficient of determination.
- Dfe=Degrees of freedom.
- Adjrsquare=Degree-of-freedom adjusted coefficient of determination.
- RMSE=Root mean squared error (standard error).
- MaxFunEvals -- Maximum number of function (model) evaluations allowed. The default value is 600.
- MaxIter -- Maximum number of fit iterations allowed. The default value is 400.
- TolFun -- Termination tolerance used on stopping conditions involving the function (model) value. The default value is 10-6.
- TolX -- Termination tolerance used on stopping conditions involving the coefficients. The default value is 10-6.

Table A- 1, Limitations of Nonlinear Regression Method for Each Mixture.

Type of procedure :	least square method
Algorithm :	Trust – Region method
Different min change :	1.0E-12
Different max change :	1.0E-14
Max function evaluating :	1.0E09
Max iteration :	1.0E09
Tolerance of function :	1.0E-20
Tolerance of x value :	1.0E-20

Table A- 2, Creep and Recovery Compliance in (1/psi) for Mix A1 at Temperature 25 C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.000040375	0.000031125
0.25	0.000053125	0.000033625
0.5	0.000071125	0.000035375
1	0.0000775	0.000038375
2	0.00008875	0.00003975
4	0.000100625	0.000043625
8	0.00011125	0.000047375
15	0.000144	0.000048375
30	0.000167	0.00004875
45	0.00017975	0.00004875
60	0.000183625	0.00004875

Table A- 3, Estimating Parameters of Hyperbolic Model for Mix A1 at Temperature 25 C⁰.

A	0.1452	B	0.884
Do	1.753e-005	Ro	2.664e-005
Dm	0.000467	Rm	5.162e-005
m	0.35	mp	0.63
SSE:	2.5742e-010	SSE:	6.6399e-012
R-square:	0.9901	R-square:	0.9855
Adjusted R-square:	0.9876	Adjusted R-square:	0.9819
RMSE:	5.6725e-006	RMSE:	9.1104e-007
Dfe:	8	Dfe:	8

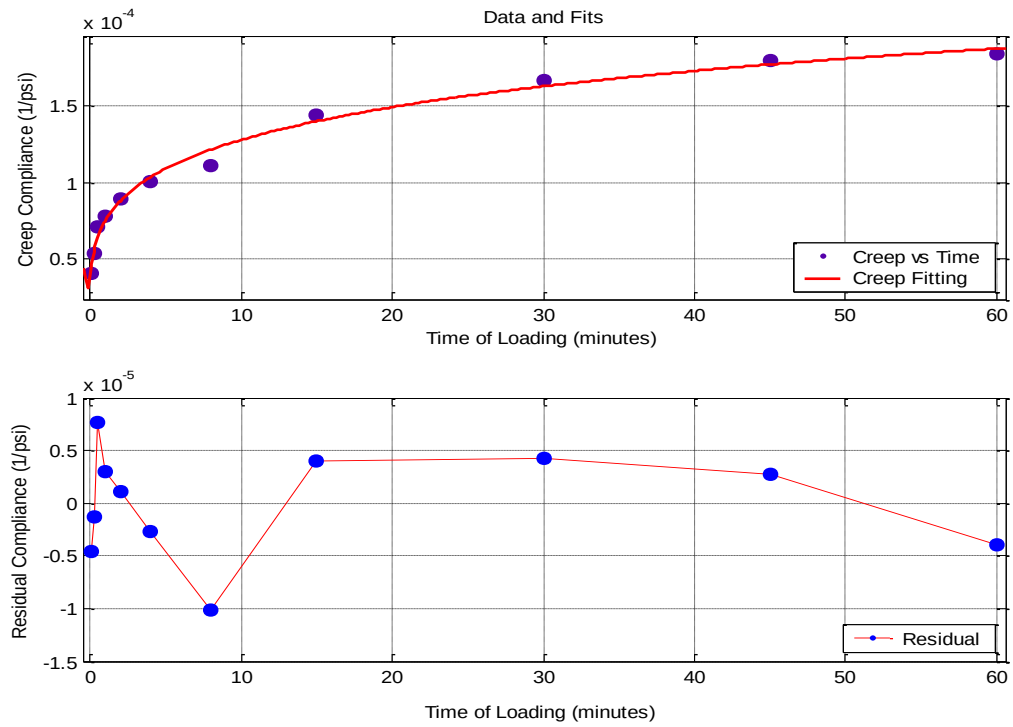


Figure A- 2, Creep Compliance Fitting and Residual of Compliance for Mix A1 at Temperature 25 C⁰.

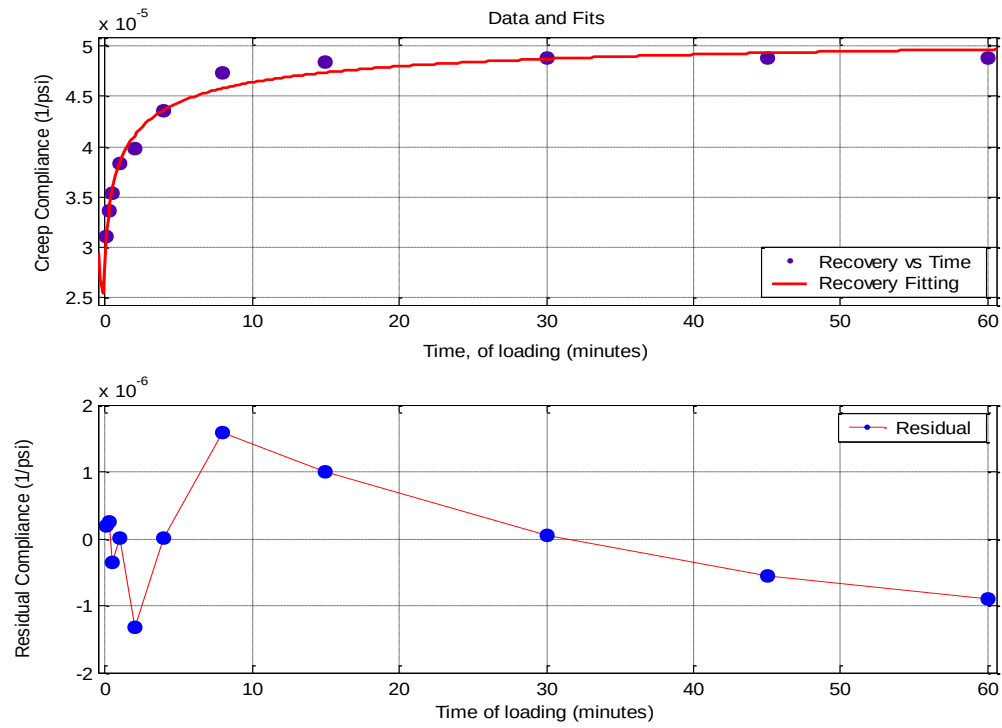


Figure A- 3, Recovery Compliance Fitting and Residual of Compliance for Mix A1 at Temperature 25 C⁰.

Table A- 4, Creep and Recovery Compliance in (1/psi) for Mix T1 at Temperature 25 C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.0000525	0.000035375
0.25	0.000069125	0.000038625
0.5	0.000091625	0.000040625
1	0.00010075	0.00004475
2	0.0001075	0.000046625
4	0.000115375	0.000051625
8	0.000130875	0.0000565
15	0.000144625	0.00005775
30	0.00018725	0.00005825
45	0.000217125	0.00005825
60	0.000233625	0.00005825

Table A- 5, Estimating Parameters of Hyperbolic Model for Mix T1 at Temperature 25 C⁰.

A	00.1214	B	2.124
Do	8.702E-006	Ro	9.057e-006
Dm	0.0004469	Rm	3.978e-005
m	0.31	mp	0.434
SSE:	1.417e-010	SSE:	1.105e-011
R-square:	0.9955	R-square:	0.959
Adjusted R-square:	0.9909	Adjusted R-square:	0.9487
RMSE:	4.208e-006	RMSE:	1.175e-006
Dfe:	8	Dfe:	8

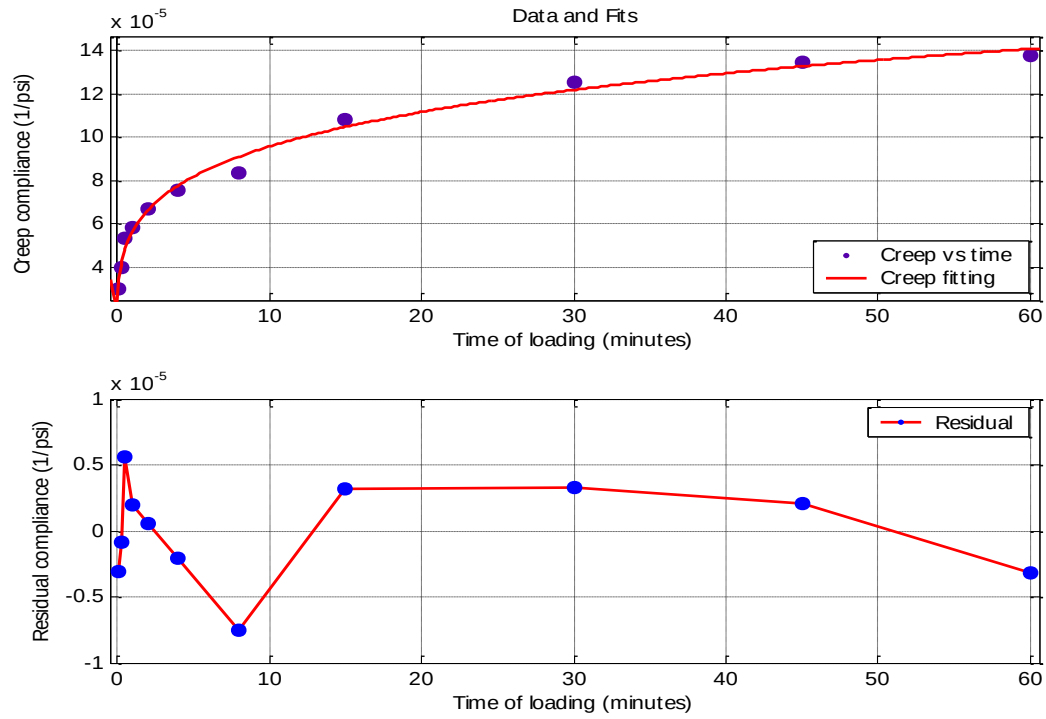


Figure A- 4, Creep Compliance Fitting and Residual of Compliance for Mix T1 at Temperature 25 C⁰.

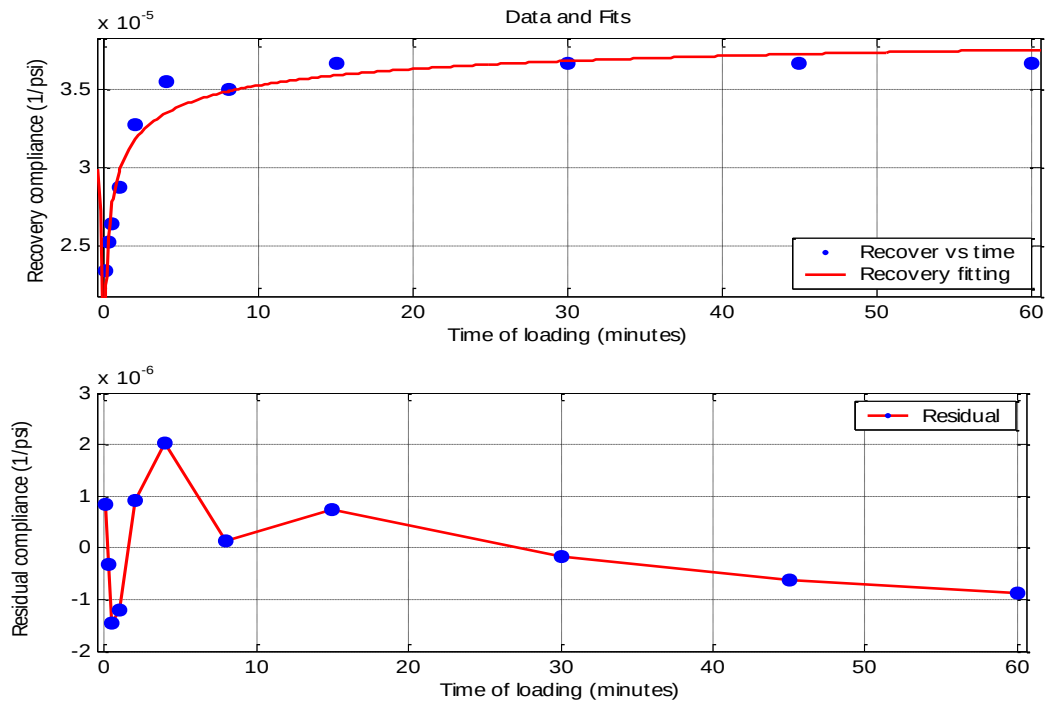


Figure A- 5, Recovery Compliance Fitting and Residual of Compliance for mix T1 at Temperature 25 C⁰.

Table A- 6, Creep and Recovery Compliance in (1/psi) for Mix A2 at Temperature 25 C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.000073875	0.000043
0.25	0.00007325	0.000046375
0.5	0.00009725	0.0000485
1	0.000106875	0.000053
2	0.000122375	0.000054875
4	0.00013875	0.00006025
8	0.0001535	0.000065375
15	0.000198625	0.00006675
30	0.000230375	0.00006725
45	0.000248	0.00006725
60	0.000253375	0.00006725

Table A- 7, Estimating Parameters of Hyperbolic Model for Mix A2 at Temperature 25 C⁰.

A	0.1066	B	0.8822
Do	4.434e-005	Ro	3.672e-005
Dm	0.000672	Rm	7.125e-005
m	0.389	mp	0.63
SSE:	4.1787e-010	SSE:	1.3053e-011
R-square:	0.9909	R-square:	0.9851
Adjusted R-square:	0.9887	Adjusted R-square:	0.9814
RMSE:	7.2273e-006	RMSE:	1.2774e-006
Dfe:	8	Dfe:	8

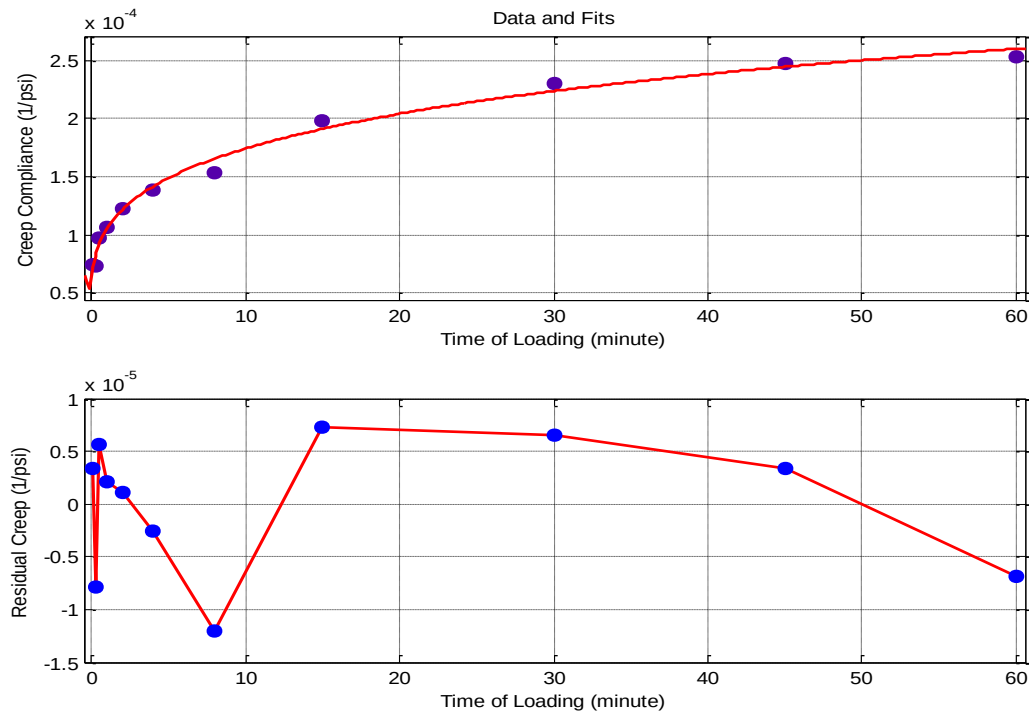


Figure A- 6, Creep Compliance Fitting and Residual of Compliance for Mix A2 at Temperature 25 C⁰.

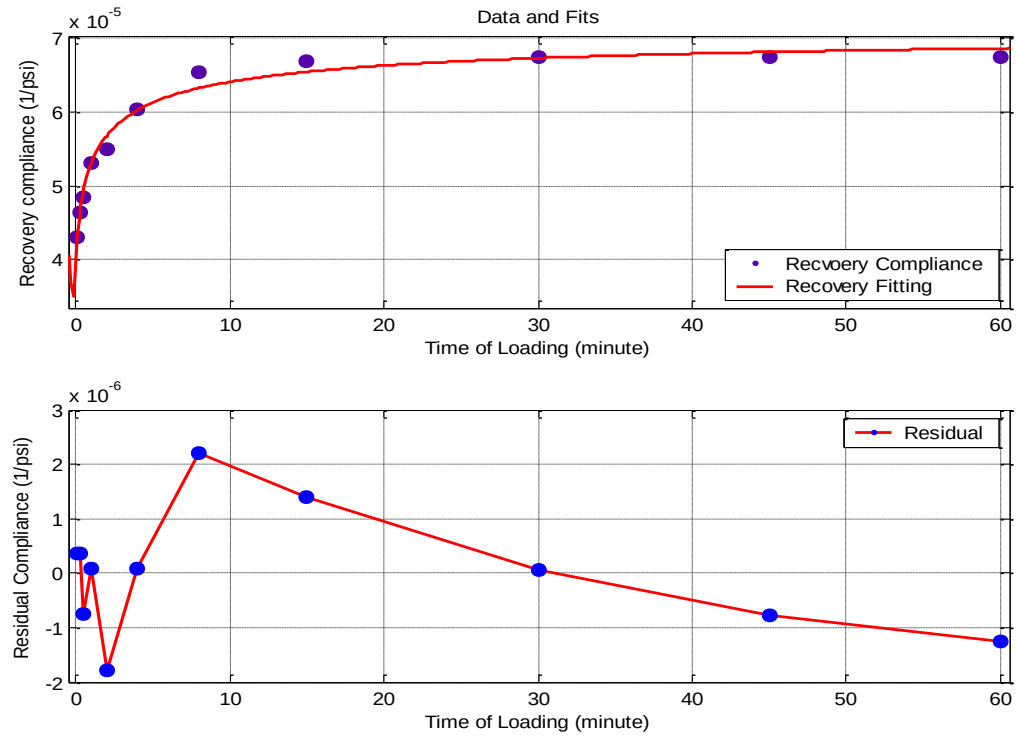


Figure A- 7, Recovery Compliance Fitting and Residual of Compliance for Mix A2 at Temperature 25 C⁰.

Table A- 8, Creep and Recovery Compliance in (1/psi) for Mix T2 at Temperature 25 C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.0000545	0.000042
0.25	0.00007175	0.000045375
0.5	0.000095125	0.000047375
1	0.000105625	0.00005175
2	0.000119875	0.000053625
4	0.000135875	0.000058875
8	0.00015025	0.000063875
15	0.000194375	0.00006525
30	0.0002255	0.00006575
45	0.000242625	0.00006575
60	0.000247875	0.00006575

Table A- 9, Estimating Parameters of Hyperbolic Model for Mix T2 at Temperature 25 C⁰.

A	0.1654	B	0.8764
Do	3.125e-005	Ro	3.592e-005
Dm	0.0005162	Rm	6.967e-005
m	0.397	mp	0.63
SSE:	4.7394e-010	SSE:	1.2403e-011
R-square:	0.9899	R-square:	0.9852
Adjusted R-square:	0.9874	Adjusted R-square:	0.9815
RMSE:	7.6969e-006	RMSE:	1.2452e-006
Dfe:	8	Dfe:	8

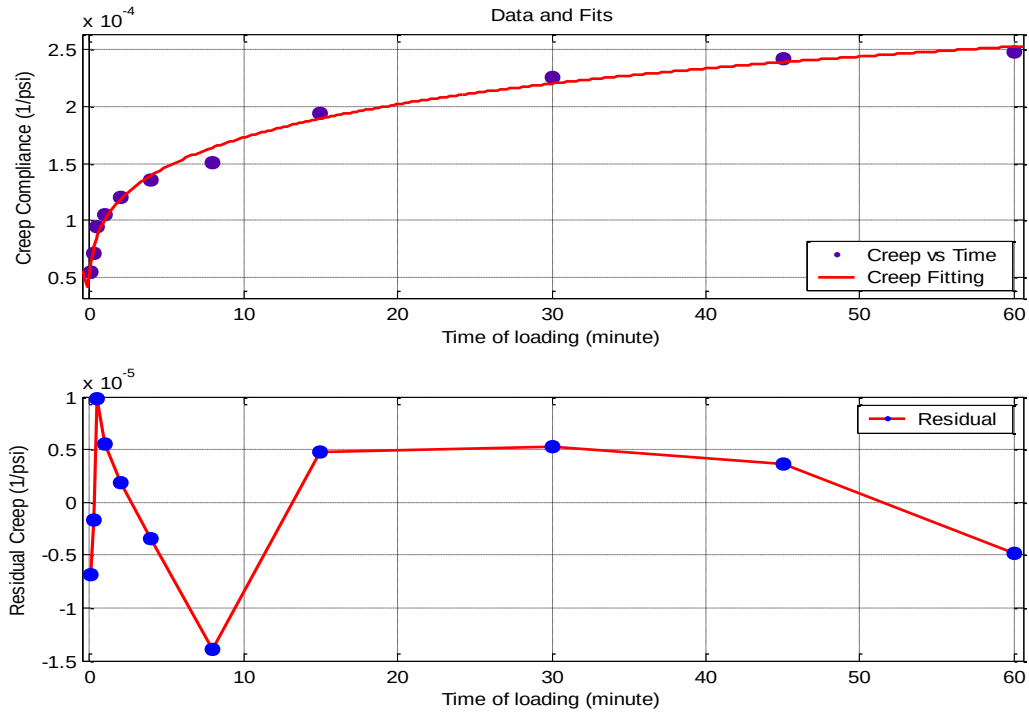


Figure A- 8, Creep Compliance Fitting and Residual of Compliance for T2 at Temperature 25 C⁰.

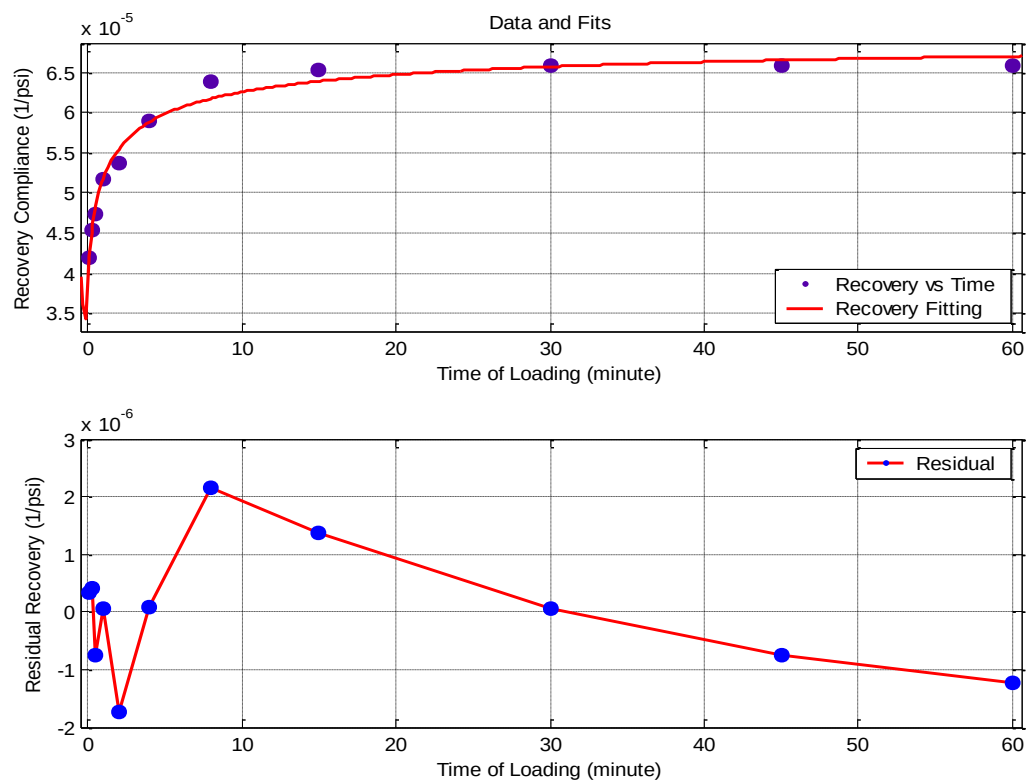


Figure A- 9, Recovery Compliance Fitting and Residual of Compliance for T2 at Temperature 25 C⁰.

Table A- 10,Creep and Recovery Compliance in (1/psi) for Mix B2 at Temperature 25 C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.000051625	0.000040125
0.25	0.0000685	0.000043375
0.5	0.000091	0.000045625
1	0.0001	0.0000495
2	0.0001145	0.00005125
4	0.00012975	0.00005625
8	0.0001435	0.000061125
15	0.00018575	0.000062375
30	0.000215375	0.000062875
45	0.000231875	0.000062875
60	0.000236875	0.000062875

Table A- 11, Estimating Parameters of Hyperbolic Model for Mix B2 at Temperature 25 C⁰.

A	0.1654	B	0.8845
Do	2.932e-005	Ro	3.435e-005
Dm	0.0004951	Rm	6.658e-005
m	0.396	mp	0.63
SSE:	4.3286e-010	SSE:	1.1154e-011
R-square:	0.9900	R-square:	0.9854
Adjusted R-square:	0.9875	Adjusted R-square:	0.9817
RMSE:	7.3558e-006	RMSE:	1.1808e-006
Dfe:	8	Dfe:	8

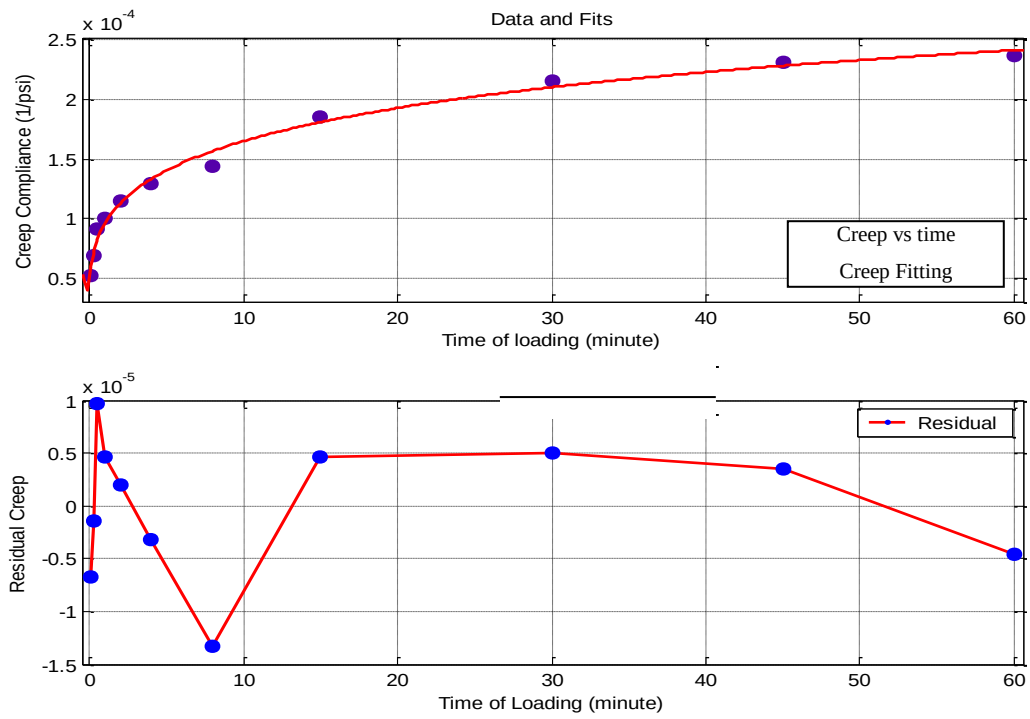


Figure A- 10, Creep Compliance Fitting and Residual of Compliance for mix B2 at Temperature 25 C⁰.

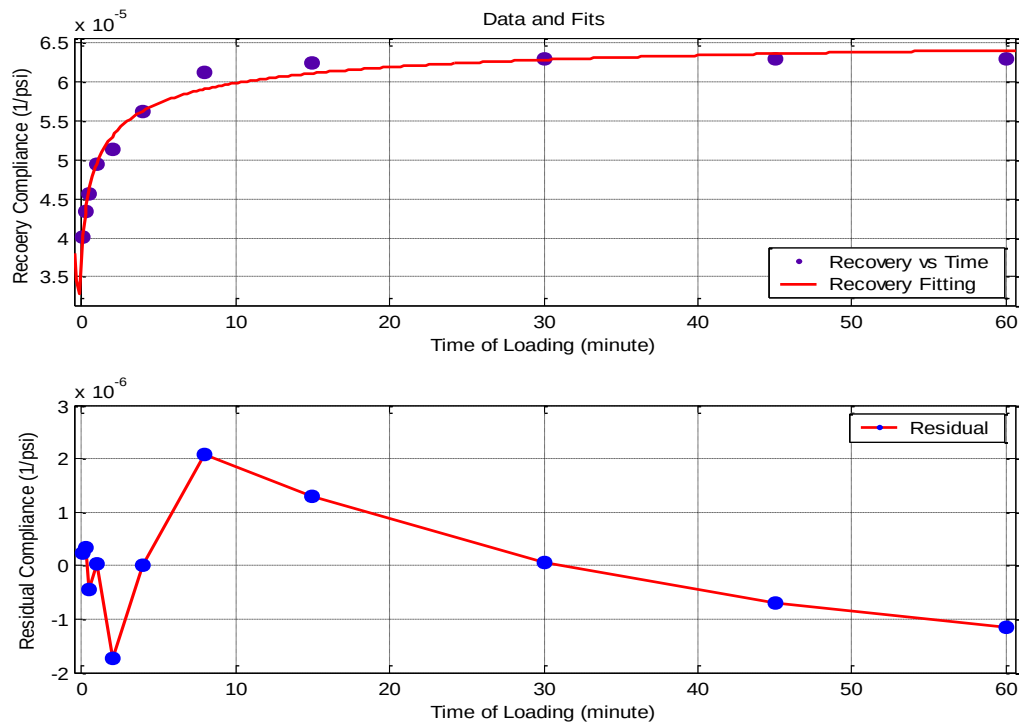


Figure A- 11, Recovery Compliance Fitting and Residual of Compliance for Mix B2 at Temperature 25 C⁰.

Table A- 12, Creep and Recovery Compliance in (1/psi) for Mix B1 at Temperature=40C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.000036875	0.00002075
0.25	0.000067125	0.00002275
0.5	0.000078	0.000022875
1	0.000106625	0.000022875
2	0.000125375	0.0000245
4	0.00013625	0.000024875
8	0.000151125	0.000024875
15	0.00017775	0.0000255
30	0.000188625	0.0000255
45	0.0002015	0.0000255
60	0.00022025	0.0000255

Table A- 13, Estimating Parameters of Hyperbolic Model for Mix B1 at Temperature=40C⁰.

A	0.6023	B	8.941
Do	8.001e-006	Ro	2.555e-007
Dm	0.0002473	Rm	2.628e-005
m	0.523	mp	0.3665
SSE:	5.1433e-010	SSE:	1.2133e-012
R-square:	0.9853	R-square:	0.9519
Adjusted R-square:	0.9816	Adjusted R-square:	0.9313
RMSE:	8.0182e-006	RMSE:	4.1633e-007
Dfe:	8	Dfe:	8

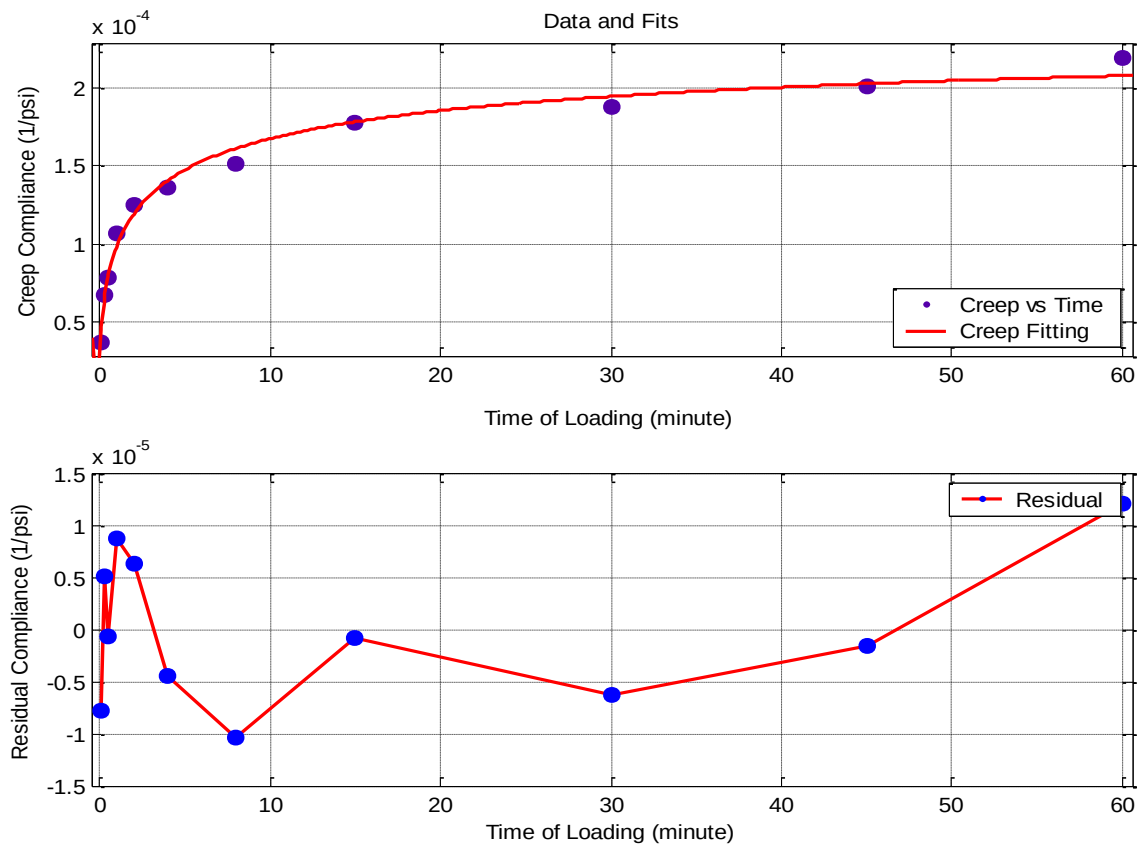


Figure A- 12,Creep Compliance Fitting and Residual of Compliance for Mix B1 at Temperature=40C⁰.

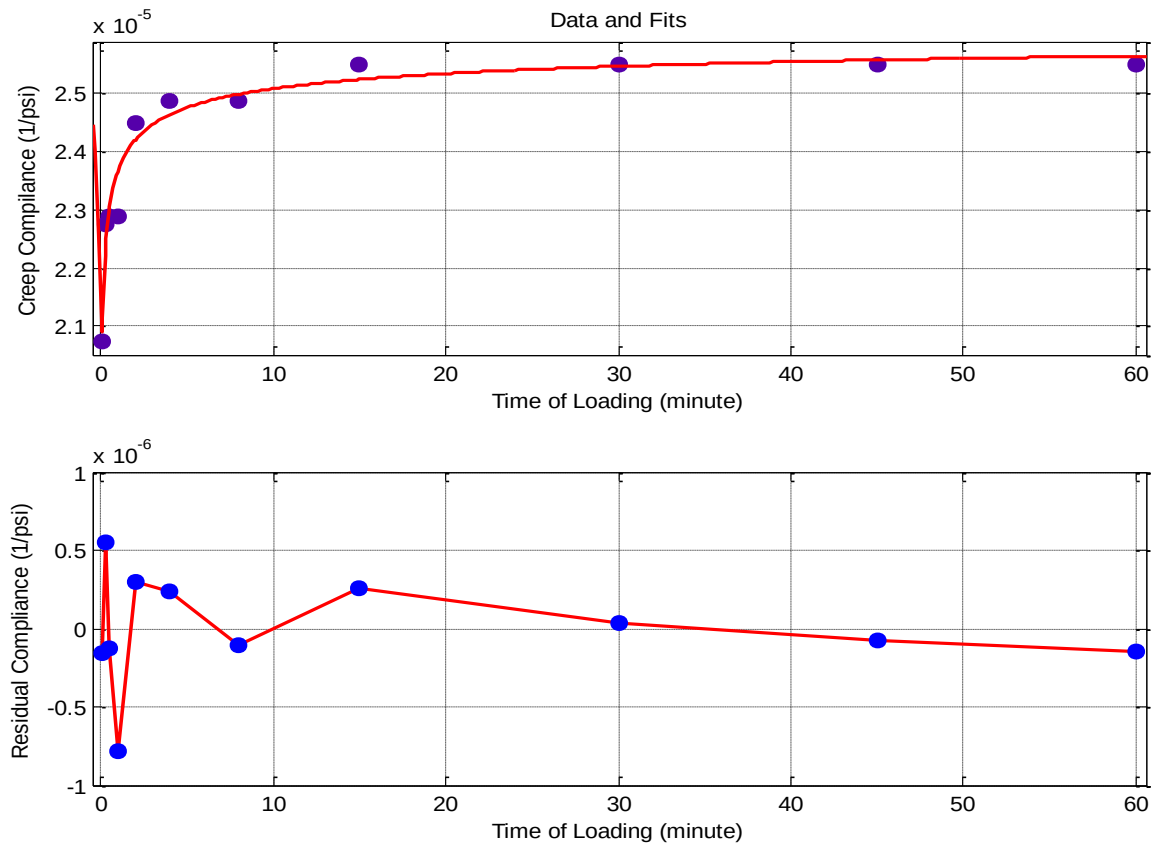


Figure A- 13, Recovery Compliance Fitting and Residual of Compliance for Mix B1 at Temperature=40C⁰.

Table A- 14, Creep and Recovery Compliance in (1/psi) for Mix B1 at Temperature=50C⁰.

Loading for creep unloading for recovery(sec)	Creep compliance(1/psi)	Recovery compliance(1/psi)
0.1	0.000069875	0.000084125
0.25	0.000076625	0.000086375
0.5	0.000077	0.000087375
1	0.000080375	0.000087875
2	0.000089625	0.00008925
4	0.000101375	0.00008975
8	0.00011575	0.000091
15	0.00014625	0.000091375
30	0.0001725	0.000091375
45	0.000193375	0.000091375
60	0.000215875	0.000091375

Table A- 15, Estimating Parameters of Hyperbolic Model for Mix B1 at Temperature=50C⁰.

A	0.03039	B	3.002
Do	6.707e-005	Ro	7.664e-005
Dm	0.000579	Rm	9.225e-005
m	0.632	mp	0.4961
SSE:	6.4817e-011	SSE:	8.2023e-013
R-square:	0.9976	R-square:	0.9866
Adjusted R-square:	0.9966	Adjusted R-square:	0.9809
RMSE:	3.0429e-006	RMSE:	3.4231e-007
Dfe:	8	Dfe:	8

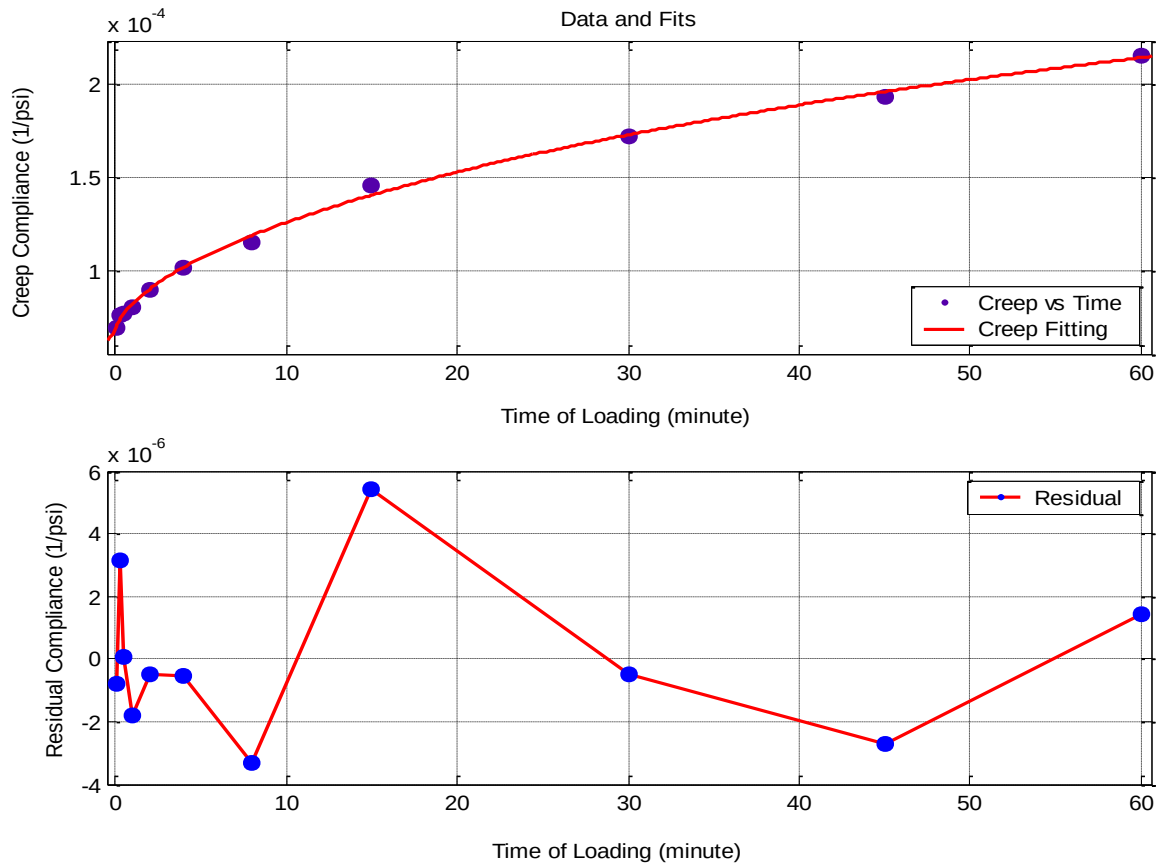


Figure A- 14, Creep Compliance Fitting and Residual of Compliance for Mix B1 at Temperature=50C⁰.

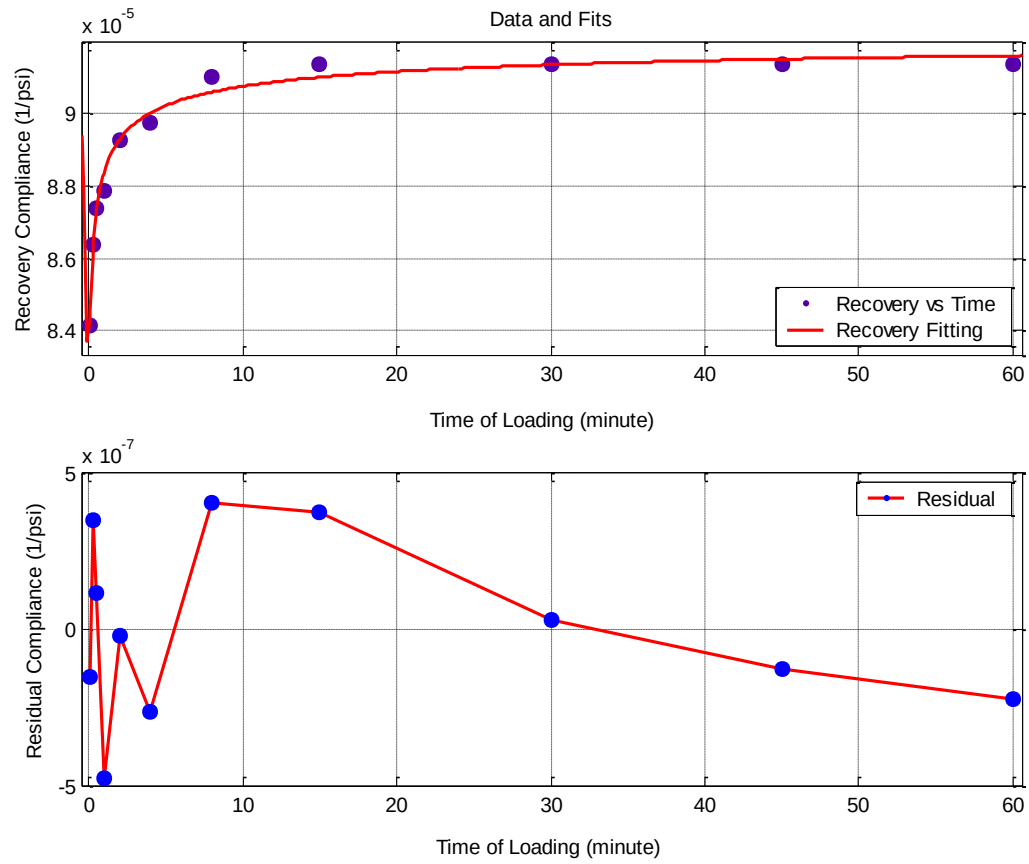


Figure A- 15, Recovery Compliance Fitting and Residual of Compliance for Mix B1 at Temperature=50C⁰.

A. APPENDIX B
SOLUTION OF KENLAYER FOR MULTILAYER FLEXIBLE
PAVEMEN

B.1 KENLAYER SOLUTION

INPUT FILE NAME -C:\KENPAVE\viscoelastic\Elastic_Solution_Check.DAT

NUMBER OF PROBLEMS TO BE SOLVED = 1

TITLE -Elastic Solution

MATL = 1 FOR LINEAR ELASTIC LAYERED SYSTEM

NDAMA = 0, SO DAMAGE ANALYSIS WILL NOT BE PERFORMED

NUMBER OF PERIODS PER YEAR (NPY) = 1

NUMBER OF LOAD GROUPS (NLG) = 1

TOLERANCE FOR INTEGRATION (DEL) -- = 0.001

NUMBER OF LAYERS (NL)----- = 3

NUMBER OF Z COORDINATES (NZ)----- = 13

LIMIT OF INTEGRATION CYCLES (ICL)- = 80

COMPUTING CODE (NSTD)----- = 9

SYSTEM OF UNITS (NUNIT)----- = 1

Length and displacement in cm, stress and modulus in kPa
unit weight in kN/m³, and temperature in C

THICKNESSES OF LAYERS (TH) ARE : 10 20

POISSON'S RATIOS OF LAYERS (PR) ARE : 0.35 0.3 0.4

VERTICAL COORDINATES OF POINTS (ZC) ARE: 0 10 20 30 40 50 60
90 120

150 180 200 230

ALL INTERFACES ARE FULLY BONDED

FOR PERIOD NO. 1 LAYER NO. AND MODULUS ARE : 1 3.500E+06 2
3.500E+05

3 1.000E+05

LOAD GROUP NO. 1 HAS 1 CONTACT AREA

CONTACT RADIUS (CR)----- = 15

CONTACT PRESSURE (CP)----- = 550

RADIAL COORDINATES OF 12 POINT(S) (RC) ARE : 0 5 15 30 45 60 75
90

120 150 180 210

PERIOD NO. 1 LOAD GROUP NO. 1

.....References Index.....

5.00000	30.00000	0.03471	62.056	-96.290	-98.410	5.926
(STRAIN)		3.442E-04	-2.440E-04	-2.518E-04	.440E-04	
5.00000	40.00000	0.02947	44.182	0.001	-0.275	3.489
(STRAIN)		4.429E-04	-1.756E-04	-1.795E-04	.977E-04	
5.00000	50.00000	0.02563	33.172	-0.132	-0.282	2.228
(STRAIN)		3.334E-04	-1.329E-04	-1.350E-04	.624E-04	
5.00000	60.00000	0.02268	25.891	-0.195	-0.284	1.516
(STRAIN)		2.608E-04	-1.044E-04	-1.056E-04	.424E-04	
5.00000	90.00000	0.01685	14.310	-0.243	-0.269	0.612
(STRAIN)		1.451E-04	-5.860E-05	-5.895E-05	.171E-04	
5.00000	120.00000	0.01337	9.078	-0.222	-0.232	0.309
(STRAIN)		9.259E-05	-3.760E-05	-3.774E-05	.866E-05	
5.00000	150.00000	0.01106	6.256	-0.187	-0.192	0.178
(STRAIN)		6.407E-05	-2.613E-05	-2.619E-05	.499E-05	
5.00000	180.00000	0.00941	4.560	-0.154	-0.156	0.112
(STRAIN)		4.684E-05	-1.915E-05	-1.919E-05	.313E-05	
5.00000	200.00000	0.00856	3.782	-0.134	-0.136	0.085
(STRAIN)		3.890E-05	-1.592E-05	-1.595E-05	.238E-05	
5.00000	230.00000	0.00753	2.936	-0.109	-0.110	0.058
(STRAIN)		3.023E-05	-1.239E-05	-1.241E-05	.164E-05	
15.00000	0.00000	0.03873	550.000	813.018	955.264	0.000
(STRAIN)		-1.036E-04	1.111E-04	1.660E-04	.184E-10	
15.00000	10.00000	0.03859	141.234	-336.807	-624.695	89.378
(STRAIN)		1.365E-04	-4.788E-05	-1.589E-04	.689E-04	
15.00000	20.00000	0.03524	83.972	-21.295	-36.628	50.857
(STRAIN)		2.896E-04	-1.014E-04	-1.584E-04	.378E-03	
15.00000	30.00000	0.03252	51.541	-66.976	-82.736	14.646
(STRAIN)		2.756E-04	-1.646E-04	-2.232E-04	.109E-03	
15.00000	40.00000	0.02815	38.865	1.990	-0.166	9.129
(STRAIN)		3.814E-04	-1.349E-04	-1.651E-04	.256E-03	
15.00000	50.00000	0.02477	30.216	0.993	-0.219	6.052
(STRAIN)		2.991E-04	-1.101E-04	-1.270E-04	.169E-03	
15.00000	60.00000	0.02209	24.116	0.486	-0.247	4.219
(STRAIN)		2.402E-04	-9.062E-05	-1.009E-04	.118E-03	
15.00000	90.00000	0.01661	13.782	-0.038	-0.258	1.767
(STRAIN)		1.390E-04	-5.448E-05	-5.756E-05	.495E-04	
15.00000	120.00000	0.01325	8.866	-0.139	-0.228	0.907
(STRAIN)		9.013E-05	-3.595E-05	-3.719E-05	.254E-04	
15.00000	150.00000	0.01099	6.155	-0.147	-0.190	0.526

.....References Index.....

(STRAIN)		6.289E-05	-2.533E-05	-2.593E-05	.147E-04	
15.00000	180.00000	0.00937	4.506	-0.132	-0.155	0.331
(STRAIN)		4.621E-05	-1.872E-05	-1.905E-05	.928E-05	
15.00000	200.00000	0.00853	3.744	-0.119	-0.135	0.252
(STRAIN)		3.846E-05	-1.563E-05	-1.585E-05	.706E-05	
15.00000	230.00000	0.00751	2.913	-0.100	-0.110	0.174
(STRAIN)		2.997E-05	-1.221E-05	-1.235E-05	.487E-05	
30.00000	0.00000	0.03062	0.000	294.236	434.266	0.000
(STRAIN)		-4.965E-05	3.252E-05	8.653E-05	-.604E-12	
30.00000	10.00000	0.02942	30.936	156.171	-157.081	53.741
(STRAIN)		8.930E-06	5.723E-05	-6.359E-05	.415E-04	
30.00000	20.00000	0.02850	33.216	3.054	-24.273	43.646
(STRAIN)		1.131E-04	1.060E-06	-1.004E-04	.324E-03	
30.00000	30.00000	0.02720	30.682	-15.638	-50.593	16.631
(STRAIN)		1.444E-04	-2.761E-05	-1.574E-04	.124E-03	
30.00000	40.00000	0.02464	26.671	5.704	0.098	12.144
(STRAIN)		2.435E-04	-5.004E-05	-1.285E-04	.340E-03	
30.00000	50.00000	0.02235	22.739	3.448	-0.062	8.925
(STRAIN)		2.138E-04	-5.623E-05	-1.054E-04	.250E-03	
30.00000	60.00000	0.02036	19.318	2.134	-0.149	6.671
(STRAIN)		1.852E-04	-5.533E-05	-8.730E-05	.187E-03	
30.00000	90.00000	0.01585	12.197	0.543	-0.228	3.118
(STRAIN)		1.207E-04	-4.245E-05	-5.324E-05	.873E-04	
30.00000	120.00000	0.01285	8.202	0.111	-0.216	1.677
(STRAIN)		8.244E-05	-3.083E-05	-3.541E-05	.470E-04	
30.00000	150.00000	0.01076	5.830	-0.023	-0.184	0.997
(STRAIN)		5.913E-05	-2.281E-05	-2.507E-05	.279E-04	
30.00000	180.00000	0.00922	4.330	-0.065	-0.153	0.637
(STRAIN)		4.417E-05	-1.736E-05	-1.859E-05	.178E-04	
30.00000	200.00000	0.00842	3.622	-0.073	-0.135	0.487
(STRAIN)		3.705E-05	-1.468E-05	-1.554E-05	.136E-04	
30.00000	230.00000	0.00743	2.839	-0.073	-0.111	0.339
(STRAIN)		2.913E-05	-1.165E-05	-1.218E-05	.948E-05	
45.00000	0.00000	0.02471	0.000	139.562	224.569	0.000
(STRAIN)		-2.578E-05	1.370E-05	4.648E-05	.382E-11	
45.00000	10.00000	0.02284	8.089	126.552	-43.679	27.369
(STRAIN)		-5.976E-06	3.972E-05	-2.594E-05	.211E-04	
45.00000	20.00000	0.02257	13.814	10.360	-13.959	24.804
(STRAIN)		4.255E-05	2.972E-05	-6.060E-05	.184E-03	

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45.00000	30.00000	0.02202	16.615	9.256	-27.796	12.478
(STRAIN)		6.336E-05	3.603E-05	-1.016E-04	.927E-04	
45.00000	40.00000	0.02074	16.339	7.379	0.313	10.640
(STRAIN)		1.326E-04	7.185E-06	-9.175E-05	.298E-03	
45.00000	50.00000	0.01941	15.332	5.104	0.087	8.766
(STRAIN)		1.326E-04	-1.064E-05	-8.087E-05	.245E-03	
45.00000	60.00000	0.01811	14.009	3.534	-0.045	7.125
(STRAIN)		1.261E-04	-2.052E-05	-7.062E-05	.200E-03	
45.00000	90.00000	0.01476	10.089	1.230	-0.187	3.854
(STRAIN)		9.671E-05	-2.730E-05	-4.715E-05	.108E-03	
45.00000	120.00000	0.01226	7.241	0.451	-0.196	2.221
(STRAIN)		7.139E-05	-2.367E-05	-3.273E-05	.622E-04	
45.00000	150.00000	0.01040	5.339	0.158	-0.173	1.370
(STRAIN)		5.345E-05	-1.908E-05	-2.372E-05	.384E-04	
45.00000	180.00000	0.00899	4.056	0.039	-0.146	0.896
(STRAIN)		4.099E-05	-1.524E-05	-1.785E-05	.251E-04	
45.00000	200.00000	0.00824	3.429	0.001	-0.130	0.693
(STRAIN)		3.480E-05	-1.318E-05	-1.502E-05	.194E-04	
45.00000	230.00000	0.00731	2.719	-0.027	-0.108	0.487
(STRAIN)		2.773E-05	-1.071E-05	-1.185E-05	.136E-04	
60.00000	0.00000	0.02047	0.000	77.422	131.676	0.000
(STRAIN)		-1.486E-05	6.837E-06	2.776E-05	.174E-12	
60.00000	10.00000	0.01820	3.035	81.809	-11.001	13.894
(STRAIN)		-6.214E-06	2.417E-05	-1.163E-05	.107E-04	
60.00000	20.00000	0.01811	6.819	11.326	-7.630	13.226
(STRAIN)		1.631E-05	3.306E-05	-3.735E-05	.983E-04	
60.00000	30.00000	0.01790	9.085	15.881	-15.175	8.418
(STRAIN)		2.535E-05	5.059E-05	-6.476E-05	.625E-04	
60.00000	40.00000	0.01730	9.702	7.167	0.401	8.034
(STRAIN)		6.676E-05	3.126E-05	-6.347E-05	.225E-03	
60.00000	50.00000	0.01658	9.827	5.529	0.174	7.277
(STRAIN)		7.546E-05	1.529E-05	-5.969E-05	.204E-03	
60.00000	60.00000	0.01581	9.589	4.197	0.031	6.389
(STRAIN)		7.898E-05	3.492E-06	-5.484E-05	.179E-03	
60.00000	90.00000	0.01349	7.921	1.805	-0.145	4.022
(STRAIN)		7.256E-05	-1.305E-05	-4.036E-05	.113E-03	
60.00000	120.00000	0.01152	6.142	0.800	-0.172	2.514
(STRAIN)		5.891E-05	-1.588E-05	-2.949E-05	.704E-04	
60.00000	150.00000	0.00994	4.743	0.364	-0.159	1.626

(STRAIN)		4.661E-05	-1.469E-05	-2.202E-05	.455E-04	
60.00000	180.00000	0.00869	3.710	0.164	-0.138	1.094
(STRAIN)		3.700E-05	-1.264E-05	-1.688E-05	.306E-04	
60.00000	200.00000	0.00801	3.181	0.093	-0.124	0.858
(STRAIN)		3.193E-05	-1.130E-05	-1.433E-05	.240E-04	
60.00000	230.00000	0.00714	2.562	0.033	-0.104	0.612
(STRAIN)		2.591E-05	-9.504E-06	-1.142E-05	.171E-04	
75.00000	0.00000	0.01735	0.000	47.799	84.049	0.000
(STRAIN)		-9.296E-06	3.891E-06	1.787E-05	-.404E-11	
75.00000	10.00000	0.01485	1.463	53.262	0.039	7.051
(STRAIN)		-4.912E-06	1.507E-05	-5.461E-06	.544E-05	
75.00000	20.00000	0.01483	3.641	10.166	-4.110	7.121
(STRAIN)		5.212E-06	2.945E-05	-2.358E-05	.529E-04	
75.00000	30.00000	0.01476	5.070	15.655	-8.427	5.579
(STRAIN)		8.291E-06	4.761E-05	-4.184E-05	.414E-04	
75.00000	40.00000	0.01450	5.746	6.163	0.389	5.760
(STRAIN)		3.126E-05	3.709E-05	-4.375E-05	.161E-03	
75.00000	50.00000	0.01414	6.176	5.171	0.198	5.606
(STRAIN)		4.028E-05	2.622E-05	-4.341E-05	.157E-03	
75.00000	60.00000	0.01370	6.369	4.231	0.069	5.246
(STRAIN)		4.649E-05	1.656E-05	-4.171E-05	.147E-03	
75.00000	90.00000	0.01219	5.989	2.168	-0.109	3.799
(STRAIN)		5.165E-05	-1.843E-06	-3.371E-05	.106E-03	
75.00000	120.00000	0.01070	5.042	1.094	-0.148	2.585
(STRAIN)		4.664E-05	-8.633E-06	-2.603E-05	.724E-04	
75.00000	150.00000	0.00941	4.103	0.564	-0.144	1.762
(STRAIN)		3.935E-05	-1.020E-05	-2.010E-05	.493E-04	
75.00000	180.00000	0.00834	3.323	0.296	-0.128	1.227
(STRAIN)		3.255E-05	-9.824E-06	-1.575E-05	.344E-04	
75.00000	200.00000	0.00773	2.897	0.193	-0.116	0.978
(STRAIN)		2.866E-05	-9.196E-06	-1.352E-05	.274E-04	
75.00000	230.00000	0.00694	2.378	0.100	-0.099	0.712
(STRAIN)		2.378E-05	-8.118E-06	-1.090E-05	.199E-04	
90.00000	0.00000	0.01242	0.000	-42.001	34.343	0.000
(STRAIN)		1.077E-06	-1.554E-05	1.390E-05	-.285E-11	
90.00000	10.00000	0.01237	0.738	35.693	3.919	3.544
(STRAIN)		-3.750E-06	9.732E-06	-2.524E-06	.273E-05	
90.00000	20.00000	0.01238	1.968	8.404	-2.198	3.897
(STRAIN)		3.015E-07	2.421E-05	-1.517E-05	.290E-04	

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90.00000	30.00000	0.01237	2.849	13.425	-4.759	3.706
(STRAIN)		7.130E-07	3.999E-05	-2.755E-05	.275E-04	
90.00000	40.00000	0.01229	3.401	5.012	0.328	4.058
(STRAIN)		1.264E-05	3.521E-05	-3.037E-05	.114E-03	
90.00000	50.00000	0.01212	3.847	4.490	0.184	4.176
(STRAIN)		1.977E-05	2.878E-05	-3.151E-05	.117E-03	
90.00000	60.00000	0.01189	4.164	3.900	0.079	4.112
(STRAIN)		2.572E-05	2.202E-05	-3.146E-05	.115E-03	
90.00000	90.00000	0.01095	4.412	2.316	-0.081	3.370
(STRAIN)		3.518E-05	5.833E-06	-2.772E-05	.944E-04	
90.00000	120.00000	0.00987	4.034	1.304	-0.125	2.492
(STRAIN)		3.563E-05	-2.596E-06	-2.261E-05	.698E-04	
90.00000	150.00000	0.00885	3.470	0.734	-0.128	1.795
(STRAIN)		3.228E-05	-6.025E-06	-1.810E-05	.503E-04	
90.00000	180.00000	0.00795	2.920	0.419	-0.117	1.298
(STRAIN)		2.799E-05	-7.017E-06	-1.453E-05	.363E-04	
90.00000	200.00000	0.00741	2.594	0.291	-0.107	1.054
(STRAIN)		2.521E-05	-7.037E-06	-1.262E-05	.295E-04	
90.00000	230.00000	0.00672	2.177	0.169	-0.093	0.783
(STRAIN)		2.147E-05	-6.642E-06	-1.032E-05	.219E-04	
120.00000	0.00000	0.00892	0.000	-59.660	-0.347	0.000
(STRAIN)		3.452E-06	-1.612E-05	6.759E-06	.176E-11	
120.00000	10.00000	0.00909	0.171	16.761	5.066	0.822
(STRAIN)		-2.134E-06	4.265E-06	-2.459E-07	.634E-06	
120.00000	20.00000	0.00911	0.542	5.169	-0.583	1.197
(STRAIN)		-2.382E-06	1.480E-05	-6.562E-06	.889E-05	
120.00000	30.00000	0.00914	0.876	8.576	-1.575	1.681
(STRAIN)		-3.499E-06	2.510E-05	-1.260E-05	.125E-04	
120.00000	40.00000	0.00917	1.172	3.094	0.192	2.003
(STRAIN)		-1.422E-06	2.548E-05	-1.515E-05	.561E-04	
120.00000	50.00000	0.00916	1.466	3.052	0.120	2.236
(STRAIN)		1.975E-06	2.418E-05	-1.688E-05	.626E-04	
120.00000	60.00000	0.00913	1.735	2.900	0.062	2.378
(STRAIN)		5.503E-06	2.181E-05	-1.792E-05	.666E-04	
120.00000	90.00000	0.00882	2.286	2.170	-0.045	2.381
(STRAIN)		1.436E-05	1.273E-05	-1.827E-05	.667E-04	
120.00000	120.00000	0.00831	2.448	1.463	-0.087	2.047
(STRAIN)		1.898E-05	5.183E-06	-1.652E-05	.573E-04	
120.00000	150.00000	0.00771	2.359	0.949	-0.098	1.645

(STRAIN)		2.018E-05	4.499E-07	-1.421E-05	.460E-04	
120.00000	180.00000	0.00712	2.154	0.612	-0.095	1.287
(STRAIN)		1.947E-05	-2.121E-06	-1.201E-05	.360E-04	
120.00000	200.00000	0.00674	1.996	0.457	-0.090	1.088
(STRAIN)		1.849E-05	-3.052E-06	-1.071E-05	.305E-04	
120.00000	230.00000	0.00621	1.759	0.298	-0.080	0.848
(STRAIN)		1.672E-05	-3.738E-06	-9.030E-06	.237E-04	
150.00000	0.00000	0.00707	0.000	-18.495	5.630	0.000
(STRAIN)		1.221E-06	-5.824E-06	3.481E-06	-.317E-11	
150.00000	10.00000	0.00711	0.019	8.317	4.293	0.135
(STRAIN)		-1.256E-06	1.945E-06	3.931E-07	.104E-06	
150.00000	20.00000	0.00712	0.120	3.032	-0.090	0.385
(STRAIN)		-2.178E-06	8.635E-06	-2.957E-06	.286E-05	
150.00000	30.00000	0.00715	0.243	5.152	-0.536	0.807
(STRAIN)		-3.261E-06	1.497E-05	-6.155E-06	.599E-05	
150.00000	40.00000	0.00720	0.388	1.860	0.104	1.015
(STRAIN)		-3.975E-06	1.663E-05	-7.950E-06	.284E-04	
150.00000	50.00000	0.00723	0.548	1.968	0.069	1.198
(STRAIN)		-2.672E-06	1.721E-05	-9.370E-06	.335E-04	
150.00000	60.00000	0.00725	0.712	1.996	0.039	1.346
(STRAIN)		-1.018E-06	1.695E-05	-1.044E-05	.377E-04	
150.00000	90.00000	0.00719	1.152	1.772	-0.026	1.569
(STRAIN)		4.542E-06	1.321E-05	-1.196E-05	.439E-04	
150.00000	120.00000	0.00699	1.428	1.375	-0.059	1.529
(STRAIN)		9.012E-06	8.279E-06	-1.180E-05	.428E-04	
150.00000	150.00000	0.00667	1.533	1.002	-0.072	1.357
(STRAIN)		1.161E-05	4.177E-06	-1.086E-05	.380E-04	
150.00000	180.00000	0.00631	1.521	0.710	-0.074	1.147
(STRAIN)		1.266E-05	1.314E-06	-9.664E-06	.321E-04	
150.00000	200.00000	0.00605	1.472	0.561	-0.072	1.011
(STRAIN)		1.276E-05	1.160E-08	-8.853E-06	.283E-04	
150.00000	230.00000	0.00567	1.367	0.393	-0.067	0.828
(STRAIN)		1.237E-05	-1.267E-06	-7.715E-06	.232E-04	
180.00000	0.00000	0.00585	0.000	-9.136	6.378	0.000
(STRAIN)		3.190E-07	-3.263E-06	2.721E-06	.108E-11	
180.00000	10.00000	0.00583	-0.014	4.395	3.474	-0.001
(STRAIN)		-7.907E-07	9.097E-07	5.544E-07	-.794E-09	
180.00000	20.00000	0.00584	0.008	1.784	0.064	0.147
(STRAIN)		-1.560E-06	5.034E-06	-1.354E-06	.109E-05	

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180.00000	30.00000	0.00586	0.053	3.084	-0.174	0.422
(STRAIN)		-2.342E-06	8.914E-06	-3.185E-06	.314E-05	
180.00000	40.00000	0.00590	0.122	1.132	0.062	0.545
(STRAIN)		-3.558E-06	1.058E-05	-4.394E-06	.153E-04	
180.00000	50.00000	0.00593	0.204	1.263	0.044	0.665
(STRAIN)		-3.189E-06	1.164E-05	-5.430E-06	.186E-04	
180.00000	60.00000	0.00596	0.295	1.344	0.027	0.776
(STRAIN)		-2.533E-06	1.215E-05	-6.290E-06	.217E-04	
180.00000	90.00000	0.00599	0.583	1.358	-0.014	1.012
(STRAIN)		4.523E-07	1.131E-05	-7.907E-06	.283E-04	
180.00000	120.00000	0.00593	0.823	1.180	-0.039	1.092
(STRAIN)		3.664E-06	8.662E-06	-8.400E-06	.306E-04	
180.00000	150.00000	0.00578	0.976	0.945	-0.052	1.056
(STRAIN)		6.182E-06	5.758E-06	-8.202E-06	.296E-04	
180.00000	180.00000	0.00557	1.045	0.726	-0.056	0.958
(STRAIN)		7.774E-06	3.306E-06	-7.650E-06	.268E-04	
180.00000	200.00000	0.00541	1.056	0.601	-0.057	0.879
(STRAIN)		8.380E-06	2.015E-06	-7.195E-06	.246E-04	
180.00000	230.00000	0.00515	1.034	0.449	-0.055	0.757
(STRAIN)		8.766E-06	5.680E-07	-6.478E-06	.212E-04	
210.00000	0.00000	0.00497	0.000	-5.601	4.919	0.000
(STRAIN)		9.260E-08	-2.101E-06	1.957E-06	.144E-11	
210.00000	10.00000	0.00495	-0.018	2.527	2.859	-0.005
(STRAIN)		-5.437E-07	4.378E-07	5.661E-07	-.423E-08	
210.00000	20.00000	0.00496	-0.013	1.080	0.110	0.080
(STRAIN)		-1.058E-06	3.004E-06	-5.999E-07	.597E-06	
210.00000	30.00000	0.00497	0.004	1.893	-0.036	0.247
(STRAIN)		-1.582E-06	5.437E-06	-1.728E-06	.183E-05	
210.00000	40.00000	0.00500	0.036	0.711	0.044	0.317
(STRAIN)		-2.659E-06	6.789E-06	-2.548E-06	.888E-05	
210.00000	50.00000	0.00503	0.078	0.825	0.033	0.392
(STRAIN)		-2.652E-06	7.808E-06	-3.284E-06	.110E-04	
210.00000	60.00000	0.00505	0.127	0.911	0.022	0.467
(STRAIN)		-2.456E-06	8.507E-06	-3.930E-06	.131E-04	
210.00000	90.00000	0.00511	0.303	1.016	-0.005	0.657
(STRAIN)		-1.015E-06	8.971E-06	-5.329E-06	.184E-04	
210.00000	120.00000	0.00511	0.478	0.964	-0.025	0.767
(STRAIN)		1.024E-06	7.825E-06	-6.014E-06	.215E-04	
210.00000	150.00000	0.00505	0.619	0.836	-0.036	0.797

(STRAIN)		2.987E-06	6.026E-06	-6.177E-06	.223E-04
210.00000	180.00000	0.00493	0.710	0.687	-0.042 0.770
(STRAIN)		4.521E-06	4.201E-06	-6.011E-06	.216E-04
210.00000	200.00000	0.00483	0.746	0.593	-0.044 0.732
(STRAIN)		5.264E-06	3.118E-06	-5.791E-06	.205E-04
210.00000	230.00000	0.00466	0.768	0.467	-0.044 0.661
(STRAIN)		5.992E-06	1.770E-06	-5.379E-06	.185E-04

Figure B- 1, Solution of KENLAYER.