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Ultimate Strength Capacity of R.C Space Frames Strengthened by FRC

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

ذَرَفْعُ دَرَجَاتٍ مِّنْ نَّشَأُ وَفَوْقَ كُلِّ ذِي عِلْمٍ عَلِيمٌ

صَبْرًا وَتَوَكَّلْ عَلَى اللَّهِ الْعَلِيِّ الْعَظِيمِ

المقاومة القصوى للهياكل الفضائية الخرسانية المسلحة والمقوات بالـ(FRC)

الخلاصة:

إن مادة الألياف المسلحة بالبولمر (FRP) والتي تستخدم لتصليح وتقوية الأعضاء الخرسانية المختلفة، تم دراستها في هذه الرسالة باستخدام طريقة العناصر المحددة وتم التحري عنها باختبارات عملية. هذه الدراسة تحرت تفاصيل سلوك الهيكل الفضائي ذو الأعضاء الخرسانية المقيدة الموشورية وغير الموشورية وباستخدام ألياف الكربون المسلحة بالبولمر (CFRP) وبطريقة العناصر المحددة بالأعتماد على أنموذج لخطيا يدعى (Region Approach) وتحت أحمال دورية.

تضمنت الدراسة متغيرات مختلفة مثل: تأثير الترابط بين عزم الانحناء والقص والقوى المحورية و لا خطية المواد والملاخطة الهندسية.

أحمال العقد المكافئة (Fixed-End Forces) تم تبنيها لعنصر متغير العمق والعرض (Tapered Member) وتحت حمل منتشر بانتظام مع الأخذ بنظر الاعتبار إمكانية أن تكون المناطق اللدنة (Plastic Zones) في النهاية اليسرى أو اليمنى أو كلاهما للعنصر.

لغرض التطبيق الكامل لهذه التقنية الجديدة في هندسة الإنشاءات، الجانب العملي تم تعزيزه بواسطة تغليف العناصر الإنشائية بمادة الكربون المسلح بالبولمر (CFRP) مثل الأعتاب (beams) والأعمدة (columns) تمثلت بعمل نماذج هياكل فضائية في هذه الدراسة.

سلوك الانثناء (Flexural Behavior) للأعضاء الخرسانية والمغلطة بمادة الكربون المسلح بالبولمر (CFRP) تم التحري عنها بالأعتماد على الجانب العملي في مختبرات جامعتي بابل والكوفة. حيث تم تطبيق طرق تغليف مختلفة.

أظهرت النتائج العملية إن التقييد بمادة الكربون المسلح بالبولمر (CFRP) يعطي زيادة محسوسة في المقاومة القصوى للانثناء وبمقدار (75 %). على أي حال، فإن إسهام الألياف يمكن أن يتنوع وفق طرق التغليف المستخدمة.

النتائج العملية تم مقارنتها مع الجانب النظري المتمثل بطريقة العناصر المحددة وباستخدام برنامج بلغة الفورتران 90 (RCSF).

خمس هياكل خرسانية فضائية مع طرق تغليف مختلفة تم اختبارها وتحليلها. ويمكن الاستنتاج بان التغليف بالألياف الطولية فقط يمكن ان يعطي زيادة جوهرية في مقاومة الانثناء للهياكل بمقدار (25 %), ولكن أيضا يقلل من التشوهات في الهيكل بمقدار (33 %).

الهياكل المغلفة بألياف طولية وعرضية امتلكت أفضل تمثيل للانثناء بشكل عام بسبب المقاومة العالية وفشل المطيلية الجيد. وبمساعدة الألياف العرضية يمكن أن تصل الزيادة في المقاومة القصوى إلى (75 %) مقارنة مع الهيكل المرجعي غير المعزز بالألياف الكربونية.

Abstract

The Carbon Fiber Reinforced Polymer (CFRP) materials, which are used to repair and strengthen various concrete members, have been studied in this dissertation by using the finite element method and by experimental tests. This study investigates the detailed space frame behavior of confined concrete prismatic and non prismatic members using CFRP fabric jackets by the non linear finite element approach (Region Approach) under cyclic loading. Different parameters have included in the present study which are: effects of coupling between bending moment, shear and axial forces, material non-linearity, and geometric non-linearity. Equivalent nodal loads (fixed – end forces) are adopted for tapered element under uniformly distributed load considering the possibility of existence of plastic zone at left, right or both ends of the member.

In order to apply this new technique to structural engineering, experiments have been conducted by wrapping CFRP fabrics to the structural members such as the beams and columns of space frames specimens considered in this study. The flexural behavior of the concrete members wrapped by CFRP fabrics is investigated based on experiments tested at the Structural Laboratories at both Babylon and Kufa Universities. Different wrapping methods are applied. Test results show that the CFRP confinement can significantly increase the ultimate flexure strength of space frame up to (75%). However, the contribution of fabrics may vary due to the different wrapping methods applied. Experimental results are compared with the finite element modeling using **RCSF** (Repair of Reinforced Concrete Space Frame by FRP Fabrics) computer program. Five reinforced concrete space frames with different wrapping methods have tested and analyzed. It can be concluded that pure longitudinal fabric wrapping has significantly increased the flexural strength of the test frames by (25 %), but it has also decreased the deformation of the frame by about

(33%). The frames wrapped with both longitudinal and transverse fabrics have the best overall flexural performance due to its high strength, high deformability and good ductile failure. With the help of transverse fabrics, the ultimate strength has increased up to (75%) over the control frame.

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NOTATIONS

Symbol	Definition
A	Cross – sectional area on a section.
a, b	Constant used in calculation of concrete cylinder strength
A_1, A_2	Parameters used in steel cyclic behavior model.
a_1 to a_4	Polynomial coefficients.
A_c	Constant for concrete stress – strain envelope equation.
$\{a_f\}$	Flow vector.
a_{f1} to a_{f4}	Coefficient in flow vector.
$A_{force}, B_{force}, C_{force}, D_{force}$	Coefficients in calculation of fixed end forces.
A_s	Reinforcing bar area.
$A_{s(ij)}$	Area of steel region (ij).
b'	Slope of lines $A - B$ and $A' - B'$ as shown in Fig. (4.7b).
B_1, b_2	Over all width of the element at node 1 and 2 respectively.
$B_A(x, y, z)$	Axial strain – displacement matrix.
B_c	Constant for concrete stress – strain envelope equation.
$[B_{CA}]$	Concrete axial strain – displacement matrix.
$[B_{cxy}]$	Concrete shear strain – displacement matrix in xy – plane.
$[B_{cxz}]$	Concrete shear strain – displacement matrix in xz – plane.
$[B_{cyz}]$	Concrete shear strain – displacement matrix in yz – plane.
b_{ij}	Width of region (ij).
$[B_{fA}]$	CFRP axial strain – displacement matrix.
$[B_{SA}]$	Steel axial strain – displacement matrix.
$B_{x,y}$	Shear strain – displacement matrix in xy – plane.
$B_{x,z}$	Shear strain – displacement matrix in xz – plane.
$B_{y,z}$	Shear strain – displacement matrix in yz – plane.
C_c	Constant for concrete stress – strain envelope equation.
D_c	Constant for concrete stress – strain envelope equation.
$[D_{con}]$	Concrete stress – strain relationship matrix.
$[D_{con}]_{ep}$	Elasto – plastic concrete stress – strain relationship matrix.
$[D_f]$	CFRP stress – strain relationship matrix.
$[D_s]$	Steel stress – strain relationship matrix.
E_1	First slope in the stress-strain curve of confined concrete.
E_2	Second slope in the stress-strain curve of confined concrete.
E_f	Effective modulus of elasticity of CFRP.
$d\bar{\lambda}$	Plastic multiplier.
E_{co}	Initial tangent flexural modulus of elasticity of concrete.
$E_{c(ij)}$	Flexural modulus of elasticity of concrete region (ij).
E_{ct}	Tangent flexural modulus of elasticity of concrete.
$[f]$	Vector of nodal forces.
f_c'	Cylinder strength of concrete.

$f'_c(28)$	Cylinder strength of concrete at an age (28) day.
$f'_c(t)$	Cylinder strength of concrete at an age (t).
f_t'	Ultimate concrete tensile strength.
f_f	Tensile strength of the CFRP fabrics.
f_r	Confining pressure of the CFRP fabric.
G	Shear modulus of elasticity of concrete.
$\overline{G}$	Shear modulus of elasticity of cracked concrete.
G_{xy}, G_{xz}, G_{yz}	Concrete shear modulus of elasticity in xy, xz and yz – plane respectively.
$G_{xy(ij)}, G_{xz(ij)}, G_{yz(ij)}$	Shear modulus of elasticity in xy, xz and yz – plane of concrete region (ij).
h_1, h_2	Over all depth of the element at node 1 and 2 respectively.
H'	Hardening / softening parameter.
I_1	First stress invariant.
I_1'	First strain invariant.
I_y	Second moment of inertia in y – direction.
J_2	Second deviatoric stress invariant.
J_2'	Second deviatoric strain invariant.
K	Shear correction factor.
K_1 to K_{12}	Coefficient in strain – displacement matrices.
$[K_{ca}]$	Concrete axial stiffness matrix.
$[K_{cxy}]$	Concrete shear stiffness matrix in xy – plane.
$[K_{cxz}]$	Concrete shear stiffness matrix in xz – plane.
$[K_{cyz}]$	Concrete shear stiffness matrix in yz – plane.
$[K_{fa}]$	CFRP axial stiffness matrix.
$[K_{sa}]$	Steel axial stiffness matrix.
L	Total length of element.
M_{iy}, M_{iz}	Bending moment at node (i) in an element in y and z – direction.
N_1 to N_4 , $\overline{N}_1$ to $\overline{N}_4$, $\underline{\underline{N}}_1$ to $\underline{\underline{N}}_4$, $\overline{\underline{\underline{N}}}_1$ to $\overline{\underline{\underline{N}}}_4$,	Shape functions.
N_1 to N_4 , $\underline{\underline{N}}_1$ to $\underline{\underline{N}}_4$, $\overline{\underline{\underline{N}}}_1$ to $\overline{\underline{\underline{N}}}_4$,	
N_{1c}, N_{2c}	Number of concrete regions in depth and width direction respectively.
N_{1f}, N_{2f}	Number of CFRP regions in depth and width direction respectively.
N_{1s}, N_{2s}	Number of steel regions in depth and width direction respectively.
p_{ix}	Axial force at node (i) in an element.
p_{iy}, p_{iz}	Shear force at node (i) in an element in y and z – direction

	respectively.
Q_1 to Q_6	Fixed end forces.
R_o	Parameter used in steel cyclic behavior model.
T_i	Torsional moment at node (i) in an element.
t_{ij}	depth of region (ij).
u_i	Axial displacement at node (i) in an element.
$u_{(x)}$	Axial displacement due to axial effect.
$u_{(x,y,z)}$	Axial displacement at point (x,y,z) in an element.
v_i, w_i	Lateral displacement at node (i) in an element in y and z – direction.
$v_{(x)}, w_{(x)}$	Lateral displacement at center line of element in y and z – direction at distance (x) from the first end of element.
$v_{(x,y,z)}, w_{(x,y,z)}$	Lateral displacement at any point (x,y,z) in an element in y and z – direction respectively.
w_{con}	Concrete unite weight.
w_{load}	Uniformly distributed load.
y_{ij}	Distance in y – direction from the center line of (ij) region to the element center line.
z_{ij}	Distance in z – direction from the center line of (ij) region to the element center line.
α	Correction factor to allow for cross – sectional warping.
α_{cr}, β_{cr}	Material parameters used in yield criterion.
$\alpha'_{cr}, \beta'_{cr}$	Material parameters used in crushing criterion.
$\gamma_{cxy}, \gamma_{cxz}, \gamma_{cyz}$	Concrete shear strain in xy, xz and yz – plane.
$\gamma_{cxy(ij)}, \gamma_{cxz(ij)}, \gamma_{cyz(ij)}$	Concrete shear strain in xy, xz and yz – plane respectively at the center of region (ij).
$\gamma_{fxy}, \gamma_{fxz}, \gamma_{fyz}$	CFRP shear strain in xy, xz and yz – plane.
$\gamma_{sxy}, \gamma_{sxz}, \gamma_{syx}$	Steel shear strain in xy, xz and yz – plane.
$\zeta \Psi$	Vector of virtual nodal displacement.
ϵ_c	Concrete strain.
ϵ_{co}	Strain at ultimate concrete strength.
$\epsilon_{cx}, \epsilon_{cy}, \epsilon_{cz}$	Concrete strrain in local direction x , y and z respectively.
$\epsilon_{\bar{cx}}$	Concrete tension unloading strain in local direction (x).
$\epsilon_{cx(ij)}, \sigma_{cx(ij)}$	Concrete stress and strain in local direction (x) at the center of region (ij).
ϵ_{cu}	Concrete ultimate total strain.
ϵ_e	Concrete elastic strain.
$\epsilon_{in}, \sigma_{in}$	Start point of the nth inversion for steel stress – strain cuve.
$\epsilon_{kn}, \sigma_{kn}$	Intersecting point of the nth inversion for steel stress – strain cuve.
ϵ_m, α_m	Tension stiffening parameters.
ϵ_p	Concrete plastic strain.

ϵ_f	CFRP strain.
ϵ_s	Steel strain.
ϵ_{s0}	Strain at yield strength of steel.
ϵ_{sx}	Steel strain in local direction (x).
$\epsilon_{sx(ij)}, \sigma_{sx(ij)}$	Steel stress and strain in local direction (x) at the center of region (ij).
ϵ_t	Concrete strain value at the ultimate concrete tensile strength (f_t').
ϵ_x	Strain in local direction (x).
$\theta_{ix}, \theta_{iy}, \theta_{iz}$	Rotation at node (i) in an element in x, y and z – direction respectively.
$\theta_{x(x)}, \theta_{y(x)}, \theta_{z(x)}$	Rotations in x, y and z – direction respectively at the element centroid axis at distance (x) from the first end of the element.
$\theta_{x(x,y,z)}, \theta_{y(x,y,z)}, \theta_{z(x,y,z)}$	Rotations at any point (x,y,z) in an element in x, y and z – direction respectively.
ν	Concrete Poisson's ratio.
ξ_n	Steel plastic deformation undergone in each half cycle (n).
σ_0	Equivalent effective stress.
$\sigma_{cx}, \sigma_{cy}, \sigma_{cz}$	Concrete stress in local direction x, y and z respectively.
$\sigma_x, \sigma_y, \sigma_z$	Stress in local direction x, y and z respectively.
σ_f	CFRP stress.
σ_s	Steel stress.
σ_{s0}	Yield strength of steel.
σ_{fx}	CFRP stress in local direction (x).
σ_{sx}	Steel stress in local direction (x).
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	Shear stress in plane xy, xz and yz respectively.
$\tau_{cxy}, \tau_{cxz}, \tau_{cyz}$	Concrete shear stress in xy, xz and yz – plane.
$\tau_{cxy(ij)}, \tau_{cxz(ij)}, \tau_{cyz(ij)}$	Concrete shear stress in xy, xz and yz – plane respectively at the center of region (ij).
$\tau_{sxy}, \tau_{sxz}, \tau_{syx}$	Steel shear stress in xy, xz and yz – plane.
ϕ	Reinforcing bar circumference.
ϕ	Bar diameter
ψ	Vector of nodal displacement.

Note: Any other notation may be explained where it appears in the thesis.

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Appendix A

The properties of the materials used in concrete mix are as follow:

1. Cement

Saudi Arabian sulfate resistance Portland cement was used in this research, which satisfies the Iraqi Standard. The cement put in bags of mass 50 kg each. Table (A.1) shows the cement quality.

Type of Cement: S.R.P.C		Weight of Sample:20 kg
Factory: Saudi Arabian		Sampling Date: 22/4/2007
Chemical Analysis (According to IQS No.5/1984)		
Oxide	Test Result	Limits According to IQS
CaO %	63.33	-
SiO ₂ %	20.66	-
Al ₂ O ₃ %	4.10	-
Fe ₂ O ₃ %	5.48	-
MgO %	1.54	≤ 5 %
SO ₃ %	2.21	≤ 2.5 % if C ₃ A < 5% ≤ 2.8 % if C ₃ A > 5%
Free Limit %	0.78	-
Loss on Ignition %	2.10	≤ 4 %
Insoluble Residue %	0.77	≤ 1.5 %
L.S.F	0.92	0.66-1.02
M.S	2.15	-
M.A	0.75	-
Total	99.43	
C ₃ S %	55.94	-
C ₂ S %	17.03	-
C ₃ A %	1.59	≤ 3.5 %
C ₄ AF %	16.67	-

Physical Test (According to IQS No.5/1984)		
Setting Time(min), Intial	130	≥ 45 min
, Final	205	≤ 600 min
Fineness (Blaine), (m^2/kg)	300	≥ 250
Compressive Strength,		
(MPa), 3 days	22	≥ 15
7 days	30	≥ 23

2. Fine Aggregate

Karbala sand (Al-Nebaei sand) of maximum size of 4.75 mm. The sand is used dried before use it. The gradation of fine aggregate lies within the upper and lower limits of B.S. specifications as shown in Figure (A.1).

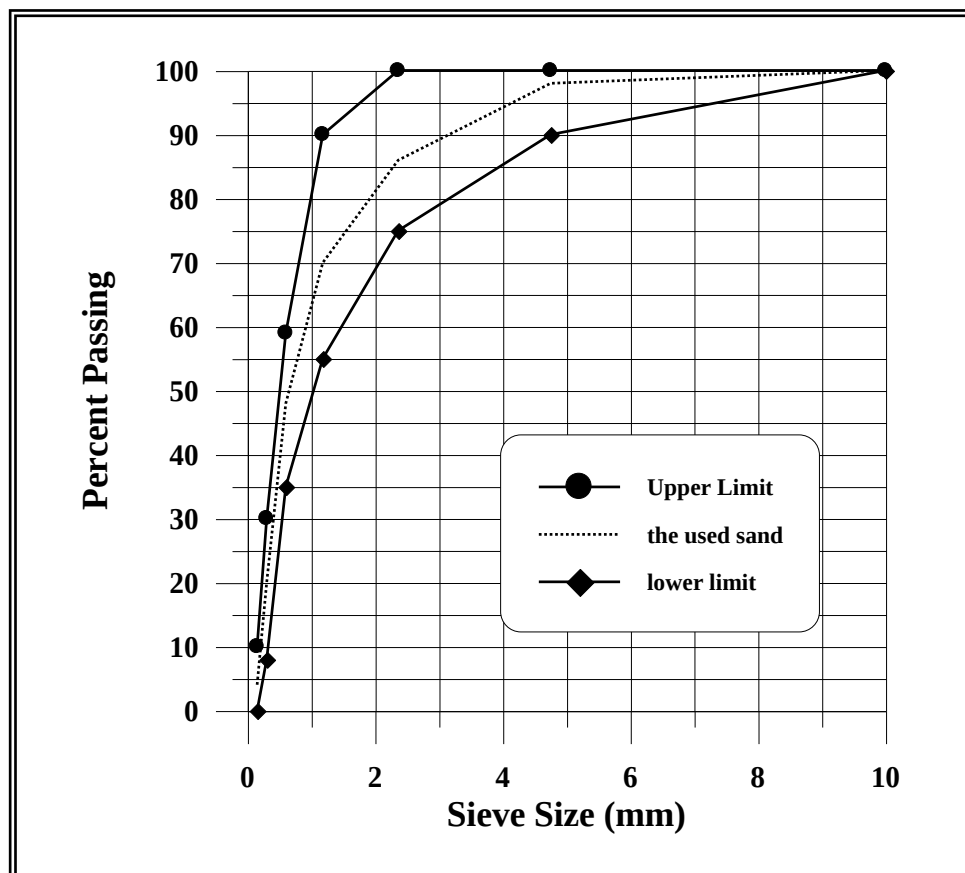


Figure (A.1) Curve for Fine aggregate (Sand)

3. Coarse Aggregate

The coarse aggregate is natural river gravel brought from Al-Nebaei region. They were mostly rounded in shape and specified the B.S. specifications. Gradation lies within the upper and lower limits, as shown in Figure (A.2). A maximum size of 19.0 mm is used in the current experimental work which satisfies ACI Code 1999 requirements for maximum aggregate size used in the reinforced concrete beam design and constructions.

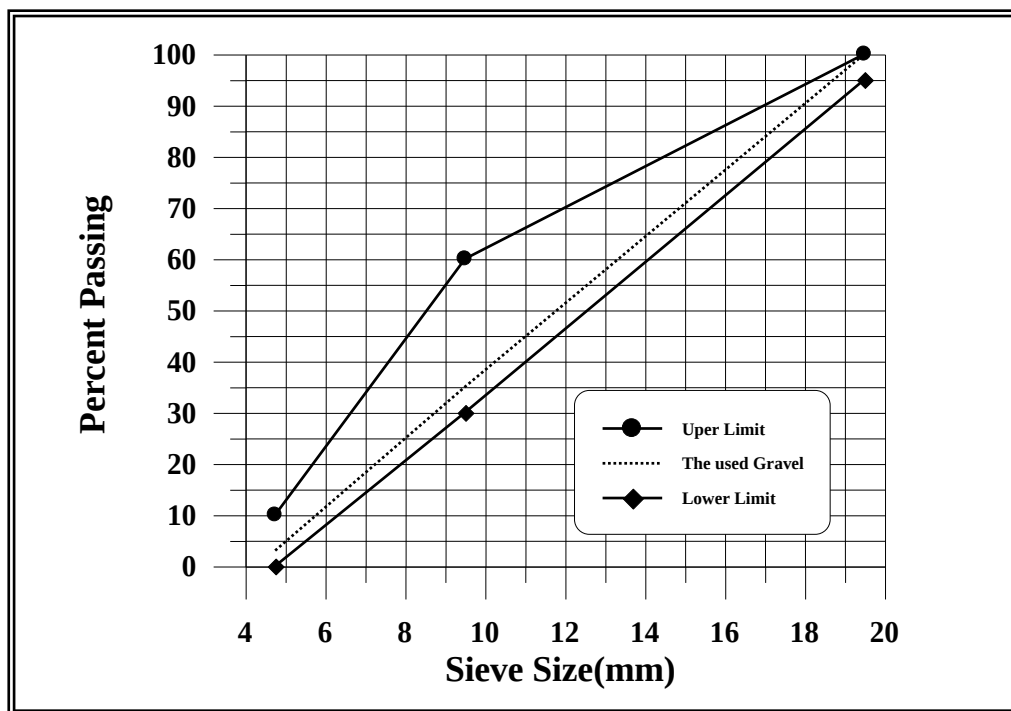


Figure (A.2) Curve for Coarse Aggregate (Gravel)

Appendix B

Design of Experimental Frames

The frame is limited in dimensions for experimental reasons (prototype model) to 1.0 m height, 0.65 m width, and 1.9 m length. The members cross section is 0.1m x 0.1m, the material properties are $f_y=400$ MPa, $f'_c=30$ MPa for steel and concrete respectively.

Design of Beams

To ensure the behavior of reinforcement concrete, ACI-Code 10.3.5 establishes a minimum net tensile strain ϵ_t , at the nominal strength of 0.004 for members subjected to axial load less than the following equation:

$$0.1f'_cA_g \quad (B.1)$$

where

f'_c = compressive strength, and

A_g = gross area of the cross section.

Using $\epsilon_t = 0.004$ as shown in Figure (B.1) provides the maximum ratio allowed by the ACI-Code for beams.

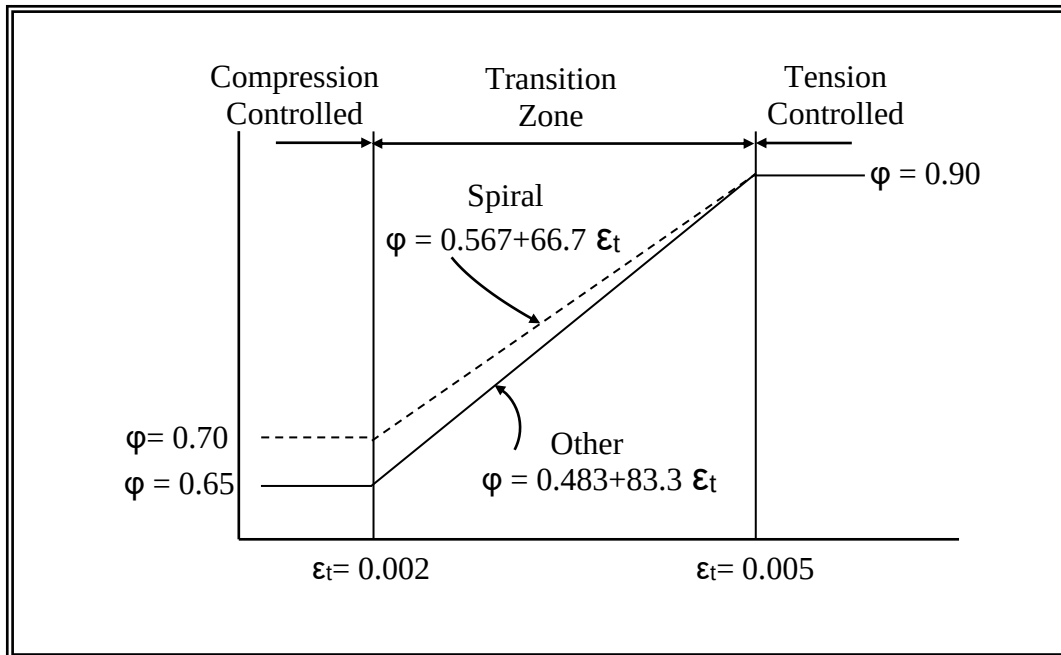


Figure (B.1) Variation of strength reduction factor with net tensile strain

$$\rho_{max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} \quad (B.2)$$

where

ρ_{max} = maximum reinforcement ratio,

β_1 = ratio between compressive result to neutral axis, and

ϵ_u = maximum strain in concrete.

$$\therefore \rho_{max} = 0.85 * 0.85 * \frac{30}{400} * \frac{0.003}{0.003 + 0.004} = 0.0232$$

$$\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{400} = 0.0035$$

$$A_s = \rho b d = 0.0232 * 100 * 80 = 185.6 \text{ mm}^2$$

Use 2Φ 8 mm, $A_s = 100.53 \text{ mm}^2$, then

$$\rho = \frac{A_s}{b d} = \frac{100.53}{100 * 80} = 0.0125 \quad \rho_{min} < \rho < \rho_{max} \text{ ok}$$

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right), \text{ and}$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{100.53 * 400}{0.85 * 30 * 100} = 15.77 \text{ mm}$$

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right) = 0.9 * 100.53 * 400 * \left(80 - \frac{15.77}{2} \right) = 2.61 \text{ kN.m}$$

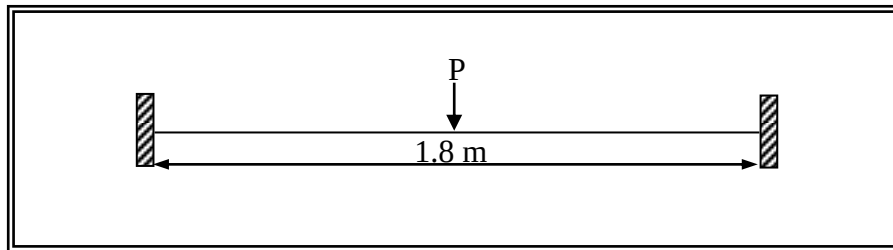


Figure (B.2) Member with Fixed Ends

For member with fixed ends, the moment as shown in Figure (B.2) is:

$$M = \frac{Pl}{8} \rightarrow P = \frac{8M}{l} = \frac{8 * 2.61}{1.8} = 11.6 \text{ kN}$$

$$V_{ud} = 11.6 / 2 = 5.8 \text{ kN}$$

$$\phi V_c = 0.75 * \left(\frac{1}{6}\right) \sqrt{f'_c} b d = 0.75 * \left(\frac{1}{6}\right) \sqrt{30} * 100 * 80 * 10^{-3} = 5.5 \text{ kN}$$

$\phi V_c < V_{ud}$, then shear reinforcement is required

$$\phi V_s = V_{ud} - \phi V_c = 5.8 - 5.5 = 0.3 \text{ kN}$$

$4 * \phi V_c = 4 * 5.5 = 22 \text{ kN} > \phi V_s$ the section dimension is ok

$\phi V_s = 0.3 < 2 * \phi V_c = 11$, then if use Φ 5 mm stirrup, the distance between stirrups is:

$$S_{max} = \begin{cases} \frac{d}{2} = 40 \text{ mm} \\ 600 \text{ mm} \\ \frac{3A_v f_y}{b_w} = \frac{3 * 2 * 19.63 * 400}{100} = 471.2 \text{ mm} \\ \frac{16A_v f_y}{b_w \sqrt{f'_c}} = \frac{16 * 2 * 19.63 * 400}{100 \sqrt{25}} = 502.6 \text{ mm} \end{cases}$$

And distance at critical section is

$$S_o = \frac{A_v f_y d}{V_s} = \frac{2 * 19.63 * 400 * 80}{670 / 0.75} = 1406.328 \text{ mm}$$

Use Φ 5 mm @ 40 mm for beams

Design of Columns

A column 0.1x 0.1 m is specified, and reinforcement distributed around the column perimeter will be used. Bar cover is estimated to be 0.02m from the column face to the steel centerline for each bar. The column parameters (assuming bending about the strong axis) are:

$$K_n = \frac{P_u}{\phi f'_c A_g} \quad (\text{B.3})$$

$$R_n = \frac{M_u}{\phi f'_c A_g h} \quad (\text{B.4})$$

where

P_u = factored load, according to the ACI safety provisions, the column must be designed for factored load $P_u = 1.2 * 11.6 = 13.92 \text{ kN}$,

Appendix B

M_u = factored moment ($M_u=2.61$ kN),

ϕ = reduction factors:

$\phi = 0.65$ for tied columns

$\phi = 0.70$ for spirally reinforced columns,

A_g = gross area of the cross section, and

h = depth of column.

With cover 20 mm the parameter γ is:

$$\gamma = \frac{h - 2d'}{h} = \frac{100 - 2 \cdot 20}{100} = 0.6$$

For this column geometry and material strength, Graph A.9 (*Nilson, and et.al* 2003) for that figure, with the calculated value of k_n and R_n as follow:

$$K_n = \frac{P_u}{\phi f'_c A_s} = \frac{13.92 \cdot 1000}{0.65 \cdot 30 \cdot 100 \cdot 100} = 0.07$$

$$R_n = \frac{M_u}{\phi f'_c A_g h} = \frac{2.61 \cdot 10^6}{0.65 \cdot 30 \cdot 100 \cdot 100 \cdot 100} = 0.14$$

The reinforcement ratio $\rho_g = 0.024$

Thus the requirement reinforcement is $A_s = 0.024 \cdot 100 \cdot 80 = 192 \text{ mm}^2$

Use $4\Phi 8$ mm $\rightarrow A_s = 201.06 \text{ mm}^2$, then if use $\Phi 5$ mm stirrup, the distance between stirrups is:

$$S \leq \begin{cases} 48 \cdot d_s = 48 \cdot 5 = 240 \text{ mm} \\ 16 \cdot d_b = 16 \cdot 8 = 128 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

Use stirrups $\Phi 5$ mm @ 128 mm.

Certification

We certify that the preparation of this thesis was made under our supervision at the University of Babylon in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Civil Engineering.

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Certificate

We certify as an examining committee that we have read this thesis entitled “***Ultimate Strength Capacity of R.C. Space Frames Strengthened by FRC***”, and examined the student **Jawad Talib Aboudi** in its content and what related to it, and found it meets the standard of thesis for the degree of Doctor of Philosophy in Civil Engineering.

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Chapter One Introduction

1.1 General

Historically, concrete members have been repaired by post tensioning or jacketing with new concrete in conjunction with a surface adhesive. This method is very time-consuming and cannot effectively enhance the properties of concrete structures. Since the mid 1960's, steel jacketing has proved to be an effective measure for retrofitting and has been widely used in practice. However, steel plates have a durability problem unique to this application, because corrosion may occur along the adhesive interface. This type of corrosion adversely affects the bond at the steel plate/concrete interface and is difficult to monitor during routine inspections. Additionally, special equipment is necessary to install the heavy plates. Engineers have to search for alternative materials.

Recently, the repair of under strengthened or damaged reinforced concrete members by the external bonding of fiber reinforced polymer materials has received considerable attention. Fiber reinforced polymers (FRP), or simply composites, wrapped or epoxy-bonded to the web or tension side of concrete, in the form of thin laminates or fabric, have been used successfully in the aerospace and automotive industry for the last two decades. They are generally constructed of high performance fibers such as carbon, aramid, or, glass which are placed in a resin matrix. By selecting among the many available fibers, geometry and polymers, the mechanical and durability properties can be tailored for particular application. This synthetic quality makes FRP a good choice for civil engineering application as well. The FRP offers the engineer an outstanding combination of properties, such as low weight (making them much easier to handle on site), immunity to corrosion, excellent mechanical strength and stiffness, and the ability of formation in very long length, thus

eliminating the need for lapping at joints. Although FRP is a relatively expensive material compare to steel, the total rehabilitation project costs is about 20% lower by using FRP than steel due to the savings in construction expenses **Chen** (2005). The FRP-strengthening technique has found wide attractiveness and acceptance among researchers and engineers today in many parts of the world.

1.2 Fiber Reinforced-Composite

A composite is a combination of two or more materials (reinforcing laments, fillers, and matrix binder) with different form or composition which, when combined into a material system, exhibit properties which are a combination of its individual components. The system constituents retain their distinct identities, which do not dissolve or merge completely into each other, but act in concert to provide an overall function. The matrix can be a ceramic, metal, or polymer. Fillers may be mineral or metallic powders. Reinforcing can be particles, fibers, rods, or bars. For example, reinforced concrete is a composite consisting of steel reinforcement, sand and gravel fillers, and a Portland cement matrix.

Fiber-reinforced composites or fiber-reinforced polymers (FRP) consist primarily of a typical reinforcement of glass, carbon or aramid fibers, and a polymer matrix. Fillers to modify the physical, mechanical, thermal, electrical, and other properties or to lower the cost or density may or may not be included. The polymer matrix may be a thermoplastic, a thermoset, or an elastomer. A thermoplastic polymer, polyethylene, polyvinyl chloride, or polystyrene for example, is one which becomes pliable or plastic when heated and then becomes hard again when cooled. A thermoset polymer changes into a cross linked, substantially infusible material when cured by heat or chemical reaction. Epoxy, polyester, and polyurethane are examples of thermosets. An elastomer is a rubber-like

polymer which recovers its original shape and size after removal of a deforming force.

The key component of an FRP is the fibrous reinforcement; it is the primary load bearing component. The matrix serves as the mechanism by which loads are transferred within the member from one fiber to another. Each type of fiber has certain advantages and disadvantages; reinforcement is selected on the basis of its physical, mechanical, and thermal properties.

Modern glass fibers were first developed in the 1930s for military purposes. Soon after, its primary commercial use was for the reinforcement of plastics. E-glass is the standard because of its electrical and mechanical properties. This fiber has a tensile strength nearly double that of steel **Brady, and Marshall** (1998) and has modified versions that resist strong acids. An interesting characteristic of glass fibers is that they are elastic — elongating until failure without yielding. After the load is released the fiber returns to its original length.

Carbon fibers are the most widely used variety of reinforcement having a very wide range of physical properties. Their strength can vary from that of steel to about four times that **Brady, and Marshall** (1998). What separates carbon fiber reinforced polymer (CFRP) from the rest is its low weight. Its performance based on stiffness to density is very high. It also has very good fatigue and damping characteristics. Manufactured carbon fibers can vary from the weakest of all fibers to among the strongest. Likewise, their price also varies from inexpensive for the weaker fibers to expensive for the strongest fibers. The most commonly produced versions of CFRP are the intermediate strength fibers. They have tensile strengths stronger than glass and somewhat weaker than aramids.

Like carbon, aramid fibers which are lightweight, have high tensile strengths and good damping and wear resistance. They also have excellent fiber toughness. A popular version of the aramid fiber is marketed under

the trademark Kevlar. However, its drawbacks are low resistance to acid attack and high cost.

As the manufacture of FRP composites improves and their mechanical properties are better understood, they are being used in a wider variety of applications. Because of the ultra-conservative nature of the civil engineering community and the relatively short history of FRP composites use, fiber-reinforced composites are just beginning to be considered as a civil engineering material alternative to steel and reinforced concrete. Although many factors, including material form, will significantly influence any design, some general differences between metals and composites may make the latter appear to be the more attractive choice. Differences between composites and metals are as follows ***Brady, and Marshall*** (1998):-

- Unidirectional aramid and carbon fiber reinforced epoxies provide a specific tensile strength (ratio of material strength to density) that is approximately four to six times greater than that of steel or aluminum.
- Unidirectional carbon fiber reinforced epoxies provide a specific modulus (ratio of material stiffness to density) that is approximately 3½ to 5 times greater than that of steel or aluminum. Aramid falls between carbon and glass fiber reinforced epoxies.
- Comparing efficiently designed structural elements, the fatigue endurance limit for aramid and carbon fiber reinforced epoxies may approach 60% of the ultimate tensile strength. For aluminum and steel, this value is considerably lower.
- Because of the properties listed above, aramid, carbon, and hybrid fiber reinforced plastics can provide structures that are 25 to 45% lighter than aluminum structures designed to the same functional requirements. Impact energy values for aramid-epoxy composites are significantly higher than those for carbon fibers and aerospace aluminum alloys.

- Fiber-reinforced polymers can be designed with excellent structural damping features to provide lower vibration transmission than metals.
- Fibrous composites are more versatile than metals and can be tailored to meet performance needs and complex design requirements. Design requirements sometimes cannot be satisfied by metal alloys within the critical weight limitations.
- The properties mentioned above can be balanced with cost by hybridization (mixing different fibers in a given composite to attain an optimum combination of properties).
- Corrosion and other attributes of fibrous composites will contribute to reduced lifecycle cost.
- Composite parts can eliminate joints/fasteners, providing part simplification and integrated design.

FRP composites consist primarily of fiber reinforcement and a polymer. Fibers that are typically used for civil and structural engineering applications are Eglass, carbon, and aramid; polymers are either polyester, vinyl ester, or epoxy. A major reason of using these polymers is because they cure by chemical reaction at ambient temperature. FRP composites may take several forms. The fibers can be in a woven or stitched fabric, or unidirectional sheet, tow or yarn. The composite may be a prepreg (fabric with uncured polymer infusion at the factory), perform (extruded, cast, or shaped at the factory), laminate plate, rod/cable, or a hybrid of these. Various methods exist for applying composites to a structural member. They include hand lay-up, filament winding, vacuum resin transfer molding, and any compaction process. When performs or laminate plates are used for repair or upgrade, matrix binders or adhesives made of polyester, vinyl ester, epoxy, or polyurethane are used to bond them to the structural members. Depending upon the composite specifications,

additives, fillers, or coatings may also be incorporated in the composite to provide and/or fire resistance and special moisture or chemical resistance.

The high material cost of FRP might be a deterrent to its use, but upon a closer look, FRP can be quite competitive. In addition to their resistance to corrosion, FRPs have high ratios of strength and stiffness to density. The light weight of FRP provides considerable cost savings in terms of labor. FRP plates and fabric come in great lengths, which can be cut to size in the field, as compared with welding of steel plates. Fiber-reinforced polymer plates or fabric are thin, light, and flexible enough to be inserted behind pipes, electrical cables, etc., further facilitating installation. With heat curing, epoxy can reach its design strength in a matter of hours, resulting in rapid bonding of FRP to the structure and consequently, minimum disruption to its use. The tensile strength of FRP can exceed 3,000 MPa (compared to 400 MPa for reinforcing steel), and their stiffness ranges from slightly greater than that of steel for high-modulus carbon to about 1/3 that of steel for S glass. Fiber-reinforced polymers do not exhibit plastic yielding as steel does, however, and behave elastically up to an ultimate strain in the range from 0.15 to 0.5% ***Brady, and Marshall*** (1998).

1.3 Ductility of FRP Fabrics

Ductility is a desirable structural property because it allows stress redistribution and provides warning of impending failure. Steel-reinforced concrete beams are underreinforced by design, so that failure is initiated by yielding of the steel reinforcement, followed after considerable deformation at no substantial loss of load carrying capacity, by concrete crushing and ultimate failure. This mode of failure is ductile and is guaranteed by designing the tensile reinforcement ratio to be substantially below (ACI 318-02 requires at least 25% below) the balanced ratio ***Duthinh, and Starnes*** (2004), which is the ratio at which steel yielding and concrete crushing occur simultaneously. The reinforcement ratio thus provides a

metric for ductility, and the ductility corresponding to the maximum allowable steel reinforcement ratio provides a measure of the minimum acceptable ductility.

In a beam reinforced internally with steel and externally with FRP, there is usually substantial reserve capacity at steel yielding. After the steel reinforcement the beam can still carry increasing loads, albeit at a lower rate (with respect to deflections) than prior to steel yielding, and the FRP maintains elastic behavior until failure occurs suddenly. Failure is precipitated by debonding, or rupturing of the FRP plate, or crushing of concrete. All of these modes of failure are brittle, i.e., load capacity is reached with little inelastic deformation.

1.4 Properties of FRP

1.4.1 Non-Corrosive

When dealing with infrastructure, corrosion is a major concern. The high degree of deterioration can be largely attributed to the corrosive property of metals as shown in Figure (1.1). Composites alleviate the problem of corrosion because they do not rust.



Figure (1.1): Corrosion of Reinforced Concrete Member

1.4.2 Fatigue Life

Because of the nature of the loading on bridge structures, the fatigue life is of great concern. Cyclic loading from traffic can lead to the distress of a structure. Most FRPs have an enhanced resistant to fatigue in a certain loading range as compared to conventional materials as shown in Figure (1.2).



Figure (1.2) FRP Application in a Bridge

1.4.3 Non-Magnetic Properties

Steel reinforcement and members often inhibit the design of a structure when the interference of electromagnetic waves is of concern. Using FRP in place of the magnetic elements may be a solution to this problem.

1.4.4 High Specific Strength and Stiffness

Two of the most important advantages to using FRP in civil infrastructure applications are the high specific strength to weight ratio and high stiffness to weight ratio. With the improvement of strength to weight

ratio over other conventional materials the designer has the ability to use lower weights and thicknesses. With this in mind, designers are no longer limited by the ratios of conventional materials thus allowing new design concepts to be explored. By reducing the weight installation time is reduced which results in a large reduction in cost. Once the size of the superstructure is reduced this will directly affect the required dimensions and capacity of the foundation and substructure.

1.4.5 Brittle Material Response

FRPs do exhibit definite ductile behavior like metals as shown in Figure (1.3). The ductile behavior is desirable in a structure to provide ample warning to occupants prior to failure. However, FRP components can be designed to exhibit a sequence of damage mechanisms to ensure a relatively slow failure with extensive deformations.

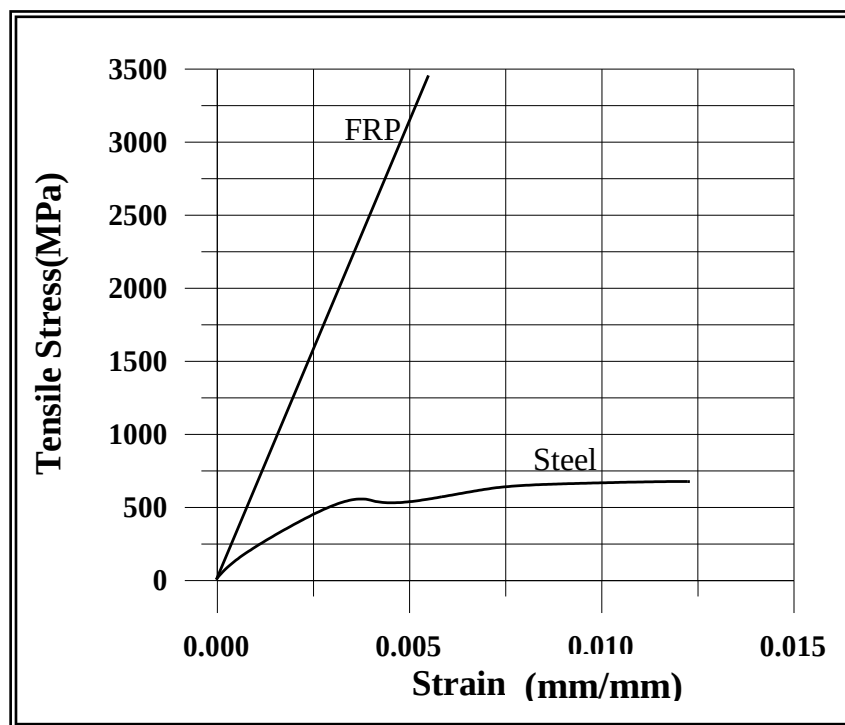


Figure (1.3) Stress-Strain Comparison between FRP and Steel

1.5 Objectives of the Present Study

The new technique of using FRP materials to repair and strengthen various concrete members is becoming more and more popular in structural retrofitting field. Among all the confining materials, FRP fabric jackets are identified as an effective way to enhance the strength and ductility of concrete members due to its superior mechanical properties. In order to take full advantage of this application, detailed structural behavior of confined concrete using FRP jackets need to be investigated. Therefore, both analytical and experimental studies become an urgent task in this field.

To further understand the structural behavior of various structural members confined with CFRP fabric jackets, the following objectives of this research have been established:

1. To investigate the flexural behavior of RC space frame strengthening/repair with CFRP fabric jackets on the tension face.
2. To investigate the behavior of RC members confined with CFRP fabrics.
3. To investigate the behavior of RC members strengthening/repair with both longitudinal and transverse CFRP fabrics.
4. To propose an analytical method to estimate the flexural strength of the RC members confined with CFRP jackets.
5. To investigate the suitability of the finite element method to analyze RC space frames strengthening/repair with FRP under cyclic loading.

1.6 The Scope of the Thesis

This study is presented in eight chapters:

- ❖ **Chapter One:** presents the properties of FRP fabrics and their advantages and disadvantages.
- ❖ **Chapter Two:** presents a review of the related studies in literature. Theoretical and experimental, monotonic and cyclic behavior of reinforced concrete flexural beams strengthened with FRP laminates and reinforced concrete frames without strengthening are reviewed.
- ❖ **Chapter Three:** presents the experimental work with description of material properties used. Also, the chapter describes cast, strengthening, and test procedures for five reinforced concrete frames used in the present work.
- ❖ **Chapter Four:** presents the stress-strain behavior of normal concrete and confined with CFRP fabrics under monotonic and cyclic loading.
- ❖ **Chapter Five:** illustrates the finite element formulation by developed analytical region formulation technique and the numerical integration rule used to evaluate the element stiffness matrix.
- ❖ **Chapter Six:** deals with the presentation, discussion and evaluation of experimental results.
- ❖ **Chapter Seven:** presents comparison between experimental results and finite element formulation (region formulation) results.
- ❖ **Chapter Eight:** gives conclusions drawn from this study, and recommendations and suggestions for further studies.

Chapter Two Literature Review

2.1 General

The repair of reinforced concrete (RC) members by the external bonding of fiber reinforced polymers (FRP) is becoming increasingly popular in the construction industry. One of the FRP materials called FRP fabric or composite jacket receives more and more attention in recent years. Although not strong enough as compared with FRP laminates and plates, FRP fabrics are extremely easy to handle and can be applied to almost every type of structure members.

2.2 Stress-Strain Behavior of Concrete with FRP Fabrics.

Fiber reinforced polymer (FRP) composite materials have recently been applied to retrofit of structural members. Several composite Jacketing systems have been developed and validated in laboratory or field conditions.

Cusson, and Paultre (1995) proposed a stress-strain model for confined high-strength concrete and calibrated against the test results from 50 large-scale high-strength concrete tied columns tested under concentric loading. The effects of the concrete compressive strength, tie yield stress, tie configuration, transverse reinforcement ratio, tie spacing, and longitudinal reinforcement ratio were accounted for in the proposed stress-strain model. The determination of the strength and ductility of confined concrete is based on the computation of the effective confinement pressure, which depends on the effectively confined concrete area. A method was proposed to compute the stress in the transverse reinforcement at maximum strength of confined concrete. Their proposed equation of stress-strain model is:

$$f_c = f_{cc} \left[\frac{k \left(\varepsilon_c / \varepsilon_{cc} \right)}{k - 1 + \left(\varepsilon_c / \varepsilon_{cc} \right)^k} \right]; \varepsilon_c \leq \varepsilon_{cc}; k = \frac{E_c}{E_c - \left(f_{cc} / \varepsilon_{cc} \right)} \quad (2.1)$$

$$f_c = f_{cc} \cdot \exp \left[k_1 (\varepsilon_c - \varepsilon_{cc})^{k_2} \right]; \varepsilon_c \geq \varepsilon_{cc} \quad (2.2)$$

$$k_1 = \frac{\ln 0.5}{(\varepsilon_{c50c} - \varepsilon_{cc})^{k_2}}; k_2 = 0.58 + 16 \left(f_{le} / f_{co} \right)^{1.4} \quad (2.3)$$

where k = controls the initial slope and the curvature of the ascending branch ; E_c = tangent modulus of concrete ; k_1 = controls the general slope of the descending branch, was adjusted to force the stress-strain curve pass through the point $(\varepsilon_{c50c} , 0.5f_{cc})$, and k_2 = controls the curvature of the descending branch.

Hoshikuma and Kawashima (1997) proposed a stress-strain model of concrete that takes confinement effects into account, based on the results of a series of compression loading tests of reinforced concrete column specimens. Their specimens have circular, square, and wall-type cross sections, with various arrangements of hoop reinforcement so as to cover practical bridge column sections designed in Japan. Test results showed that three parameters-peak stress; strain at peak stress, and deteriorating rate-are significant factors for the stress-strain curve of confined concrete. These parameters were analyzed statistically based on test data and equations for estimating them were proposed. An equivalent confined section was also proposed to evaluate effect of cross ties in wall-type cross section. Test results were compared with both the proposed model and existing models. The comparison showed that the predicted stress-strain relation by the proposed model provides better agreement with the experimental results than existing models. Their proposed equation of stress-strain model is:

$$f_c = E_c \varepsilon_c \left[1 - \frac{1}{n} \left(\frac{\varepsilon_c}{\varepsilon_{cc}} \right)^{n-1} \right] \text{ for } \varepsilon_c \leq \varepsilon_o \quad (2.4)$$

$$f_c = f_{cc} - E_{des} (\varepsilon_c - \varepsilon_{cc}) \text{ for } \varepsilon_c > \varepsilon_o \quad (2.5)$$

where n = coefficient, and is given as:

$$n = \frac{E_c \varepsilon_{cc}}{E_c \varepsilon_{cc} - f_{cc}} \quad (2.6)$$

E_c = modulus of elasticity of unconfined concrete; f_{cc} and ε_{cc} = peak stress of confined concrete and axial strain corresponding to peak stress.

Based on thirty 152.4mm by 304.8mm cylindrical tests, **Samamn, et al** (1998) pointed out that performance of a concrete column under axial loading is significantly enhanced by lateral confinement. It has been suggested that the concrete be referred as a restraint-sensitive material while the FRP fabric as a restraint-passive material. Before specified level, the load is mainly carried by concrete alone. After that stress level, the ultimate behavior of the system is solely dependent on the confining properties of FRP fabric. Therefore the stress-strain behavior shows a complete bilinear behavior. In their research, a simple model was presented to predict the complete stress-strain behavior in both axial and lateral directions. This model is based on correction between the dilation rate of concrete and hoop stiffness of the restraining member. The parameters of the model are directly related to the material properties of the FRP jackets and concrete core. The proposed equation is derived from the four parameter relationship of **Richard, and Abbott** (1975) (cited by **Samamn, et al** (1998)):

$$f_c = \frac{(E_1 - E_2) \varepsilon_c}{\left[1 + \left(\frac{(E_1 - E_2) \varepsilon_c}{f_o} \right)^n \right]^{1/n}} + E_2 \varepsilon_c \quad (2.7)$$

where ϵ_c and f_c = axial strain and stress of concrete ; E_1 and E_2 = first and second slopes in the stress-strain curve; f_o = reference plastic stress at the intercept of the second slope with the stress axis; and n = a curve-shaped parameter that controls the curvature of transition zone. This equation also is applied to stress-lateral strain relationship.

A uniaxial model for concrete confined with fiber-reinforced polymers (FRP), but also with steel jackets or conventional transverse reinforcement, was presented by **Spoelstra, and Monti** (1999). The model, which is suitable to be inserted into fiber-type beam-column models, explicitly accounts for the continuous interaction with the confining device due to the lateral strain of concrete, through an incremental-iterative approach. The relation between the axial and lateral strains is implicitly derived through equilibrium between the (dilating) confining concrete and the confining device. This relation allows one to trace the state of strain in the jacket and to detect its failure. The model is compared with a set of experimental tests and shows very good agreement in both the stress-strain and stress-lateral strain response. Evidence of the main aspects of the FRP-confined concrete is given those points out differences in the response when using fiberglass or carbon fiber. Predictive equations to determine the ultimate strength and strain of concrete confined with FRP were derived and tested on a number of experimental data. Their proposed equation of stress-strain model is:

$$f'_c = \frac{f'_{cc} \cdot x \cdot r}{r - 1 + x^r} \quad (2.8)$$

where:

$$x = \frac{\epsilon_c}{\epsilon_{cc}} ; \epsilon_{cc} = \epsilon_{co} \left[1 + 5 \left(\frac{f'_{cc}}{f'_{co}} - 1 \right) \right] \quad (2.9)$$

$$r = \frac{E_c}{E_c - E_{sec}}; E_{sec} = \frac{f'_{cc}}{\epsilon_{cc}} \quad (2.10)$$

where ϵ_{cc} = compressive strain at confined peak strength f'_{cc} .

Xiao and wu (2000) tested 36 concrete cylinders confined by CFRP fabric jackets with different unconfined concrete strength and thickness of CFRP jackets. It was found that the concrete strength and confinement modulus, defined as the ratio of transverse confinement stress to transverse strain, are most influential factors affecting the stress-strain behavior of confined concrete. The test results indicated that the failure of the confined concrete was mainly due to the rupture of the FRP jackets. In order to describe the main mechanical features of the confined concrete, they suggested a new bilinear stress-strain equation based on the theory of elasticity.

For the initial performance:

$$f_c = E_c \epsilon_c + 14 \left(\frac{f'_c \cdot d}{2t \cdot E_j} \right)^{0.8} \cdot f_r \quad (2.11)$$

For the stabilized performance after achieving the unconfined strength of concrete.

$$f_c = \alpha \cdot f'_c + k \cdot f_r \quad (2.12)$$

where E_c and E_j = modulus of concrete and FRP jackets respectively. f_r = confinement stress of FRP jackets. α and k = experimental coefficient based on the tests ($k=4.1$).

A confinement model for spirally confined concrete has been developed by **Assa, et al** (2001). The stress-strain curve of confined concrete was determined based on the compatibility in the lateral expansion between concrete and the rectangular ties. The results of predictions from the proposed confinement model showed good agreement with the experimental results of reinforced concrete column specimens' tests by

several researchers. The comparisons indicated that the proposed method has the potential to be used for predicting the stress-strain curve of core concrete confined by rectangular ties, with sufficient accuracy.

Moran and Pantelides (2002) presented a strain-based model for predicting the compressive behavior of normal strength concrete confined with FRP composite jackets. The model is applicable to both bonded and non-bonded FRP confined concrete and can be separated into two components: a strain-softening component, which accounts for unrestrained internal crack propagation in the concrete core, and a strain-hardening, which accounts for strength increase due to confinement provided by the FRP composite jacket. A variable strain ductility ratio described in a companion is used to develop the proposed stress-strain model. Equilibrium and strain compatibility were used to obtain the ultimate compressive strength and strain of FRP confined concrete as a function of the confining stiffness and ultimate strain of the FRP jacket.

An analytical model to predict the behavior of concrete confined with fiber reinforced plastic (FRP) composites subjected to axial compressive loads was developed by **Fuijkake, et al** (2004). First, a constitutive model for plain concrete was formulated from past experimental results obtained from triaxial compression tests of concrete, in which concrete specimens were maintained under constant confining stresses. This was an orthotropic constitutive model based on the concept of equivalent uniaxial strain. Subsequently, in the analytical model for FRP confined concrete, the proposed constitutive model for concrete materials was incorporated. The FRP was assumed to be linear elastic material. Force equilibrium and strain compatibility between the concrete and the FRP as well were satisfied. When the proposed model was applied to FRP confined concrete, the model overestimated the axial stress. To rectify this, a subsequent maximum strength criterion was introduced to control the

maximum strength in the postpeak region when confining stress was continuously increased. The proposed analytical model with the addition of the subsequent maximum strength criterion is in good agreement with the experimental results.

A series of CFRP wrapped concrete cylinder tests were conducted to study the compressive stress-strain behavior for CFRP confined concrete members were developed by **Chen** (2005). Based on the test results, the CFRP system can significantly increase the ductility and ultimate compressive strength of normal concrete specimens. Also, a complete stress-strain equation was derived to model the fabric-confined concrete and verified by various experimental tests results. The flexural behavior of the large concrete beams wrapped by CFRP fabrics was investigated. Different wrapping methods were applied. Test results show that the CFRP confinement can significantly increase the ultimate flexural strength of reinforced concrete beams. Experimental results were compared with the finite element modeling using the ANSYS computer program. A particular design method similar to (ACI-318-05) strength method is also proposed to predict the ultimate strength of the beam specimens. Good agreements are achieved for both methods.

2.3 Flexural Capacity of RC Beams Strengthened by Externally Bonded FRP Fabric

Strengthening with externally bonded FRP fabrics has been used to repair and retrofit many kinds of structures. Currently, this new technique has been applied to strengthen such structures as columns, beams, walls, slabs, etc. the use of external FRP fabrics may be classified as flexural strengthening, improving the ductility, and shear strengthening.

A lot of research work has been conducted to study the repair of RC beams using steel plates and FRP stripes. However, researches on flexural

strengthening using FRP fabric sheets or jackets only started recently. In the late 1990's, several experimental work were conducted to strengthen both virgin and precracked RC beams using FRP composite sheets. Due to its super light weight and superior mechanical properties, FRP fabric sheets and jackets quickly received much attention among various FRP materials.

Fiber reinforced polymer (FRP) materials are continuing to show great promise of use in strengthening reinforced concrete structures. These materials are excellent options for use as external reinforcing because of their light weight, resistance to corrosion, and high strength. Externally bonded FRP sheets have been used by *Khalifa, et al* (1998) to increase moment capacity of flexural members and to improve confinement in compression members. Investigations into the use of externally bonded FRP sheets for shear strengthening have also been conducted and have been shown to be viable. Methods for computing the shear capacity based on the stress level that causes tensile fracture of the FRP sheet (at times this stress may be less than ultimate due to stress concentrations) and based on delaminating of the sheet from the concrete surface were presented. Areas of research that have the potential for further development are also discussed.

24 reinforced concrete beams strengthened with carbon fiber wraps were experimentally evaluated by *Gangarao, et al* (1998) to determine improvements in flexural strength. In addition, bending test results from the wrapped beams were compared with those of other identical concrete beams strengthened with bonded steel plates. Four different carbon wrap configurations were used for strengthening beams and for evaluation and comparison of experimental data. Static responses of all the concrete beams were evaluated in terms of strength, stiffness, compositeness between wrap and concrete, and associated failure modes. Factors governing the percentage increase in the ultimate strength were discussed. Wrapped

beams were analyzed considering conventional force equilibrium equations and compared with the experimental results.

El-Mihilmy, and Tedesco (2000) investigated the flexural behavior of reinforced concrete beams strengthened with externally bonded FRP laminates. A simple and direct analytical procedure for evaluating the ultimate flexural capacity of FRP strengthened reinforced concrete flexural members was presented. The procedure was derived from equilibrium equations and compatibility of strains and is applicable to both single and doubly reinforced concrete rectangular sections, as flanged sections. The procedure is validated by comparisons with results of experimental data available in the literature. Upper and lower limits of FRP that may be boded facilitate implementation of the procedure were presented.

Park, and Oehlers (2000) strengthened reinforced concrete beams by gluing steel plates to their tension surfaces. Such plates beams were designed for flexure using conventional predication equations and the assumption that full bond will be developed between the concrete and the plates. Very advanced design rules have already been developed at the University of Adelaide for adhesive bonding steel plates to reinforced concrete beams in order to prevent premature debonding. Used continuous reinforced concrete beams to compare experimentally steel plating with FRP plating and in particular their ductility.

Yost, et al (2001) evaluated the flexural performance of simply supported concrete beams subjected to four-point monotonic loading and reinforced with a 2D fiber-reinforced plastic (FRP) grid. The main parameter of the study is the amount of longitudinal FRP reinforcement. With respect to a balanced strain condition, three underreinforced and two overreinforced FRP designs were tested with three identical beams per design. Laboratory recorded load-deflection, failure mode, cracking behavior, and reinforcement strain data were compared with theoretical

predictions calculated according to traditional steel-reinforced concrete procedures. Deflection compatibility between test results and ACI predictions employing the Branson effective moment of inertia was dependent on the percentage of longitudinal reinforcement. In general, observed flexural stiffness was less than that predicted by Branson's equation. A moment-curvature deflection procedure employing a bilinear concrete model compared very well with measured deflections. Finally, the grid configuration provides an effective force transfer mechanism. Cracking occurred at transverse bar locations only, and FRP tensile rupture was achieved with no observed deterioration in force transfer mechanics.

Finite element method (FEM) models were developed by **Kachlakev, and Miller** (2001) to simulate the behavior of four full-size beams from linear through nonlinear response and up to failure, using the ANSYS program. Comparisons were made for load-strain plots at selected locations on the beams; load-deflection plots at midspan; first cracking loads; loads at failure; and crack patterns at failure. The models were subsequently expanded to encompass the linear behavior of the Horsetail Creek Bridge. Modeling simplifications and assumptions developed during this research were presented. They compared strains from the FEM analysis with measured strains from load tests.

Lamanna, et al (2001) presented a feasibility study of an alternative method of attaching FRP strips to reinforced concrete beams. The method utilized off-the-shelf powder-actuated fasteners to attach pultruded FRP strips to the concrete. Small-sized beams strengthened with this method were tested, and the results were compared with the results from beams strengthened with conventional bonding methods and to the results from control beams. Small-size beams strengthened using powder-actuated fasteners attained (65 - 70) % of the increase in capacity of the beams strengthened using the conventional bonded method; however, the

fastening method is extremely rapid and the failure modes of the beams strengthened by the fastening method were more ductile than those strengthened by the bonded method.

Seven concrete beams reinforced internally with varying amounts of steel and externally with carbon fiber-reinforced polymer (FRP) laminates applied after the concrete had cracked under service loads were tested under four-point bending by **Duthinh, and Starnes** (2004). Strains measured along the beam depth allowed computation of the beam curvature in the constant moment region. Results show that FRP is very effective for flexural strengthening. As the amount of steel increases, the additional strength provided by the FRP laminates decreases. Compared to beam reinforced heavily with steel only, beams reinforced with both steel and carbon have adequate deformation capacity, in spite of their brittle mode of failure. Clamping or wrapping of the ends of the laminate enhances the capacity of adhesively bonded FRP anchorage. Design equations for anchorage, allowable stress, ductility, and amount of reinforcement were discussed.

Harajli (2005) presented an analytical evaluation of the effect of external confinement using fiber reinforced polymers (FRP) sheets on the response of concrete rectangular columns designed for gravity load only and having spliced longitudinal reinforcement at the column base. A general analytical scheme for evaluating the strength capacity and ductility of the columns under combined flexural-axial loads was developed. The analysis takes into account the bond strength degradation of the spliced reinforcement with increase in lateral load by incorporating a generalized bond stress-slip law, and considers the effect of FRP confinement on the stress-strain response of concrete material. Particular emphasis is placed in the analysis on the slip response of the spliced bars and the consequent fixed end rotation that develops at the column base. Results predicated by

the analysis showed very good agreement with the limits of experimental data. A parametric evaluation was carried out to evaluate the effect of different design and strength parameters on the column response under lateral load. Without confinement, the columns suffered premature bond failure and, consequently, low flexural strength capacity. Confining the concrete in the columns end zone at the splice location with FRP sheets enhanced the bond strength capacity of the spliced reinforcement, increased the steel stress that can be mobilized before bond failure occurs, and consequently improved the flexural strength capacity and ductility of the columns. A general design equation, expressed as a function of the main parameters that influence the bond strength capacity between spliced steel bars and FRP confined concrete, was proposed to calculate the area of FRP sheets needed for strengthening of the subject columns.

2.4 Shear Capacity of RC Beams Strengthened by Externally Bonded FRP Fabric

The RC beams may be shear strengthened in various ways: complete FRP wraps covering the whole cross section (i.e., complete wrapping), FRP U jackets covering the two sides and the tension face (i.e., U jacketing), and FRP strips bonded to the sides only (i.e., side bonding). Previous studies have established clearly that such strengthened beams fail in shear.

Four prestressed high-strength concrete T-beams (joists) with integral web openings were tested the effectiveness of FRP-based repair techniques by **Brady, and Marshall** (1998) on full-scale prestressed high-strength concrete joists fabricated with insufficient shear reinforcement. Calculations were based on controlling shear crack widths to maintain aggregate interlock and proper shear transfer through the concrete. The results of tests indicate that significant increase in shear strength can be achieved by the application of FRP to concrete beams deficient in shear capacity. When an FRP jacket is properly applied over the shear span of the

member the failure mode of a member may be altered from that of a brittle shear failure to a ductile flexural failure mode. However, the repaired joist was not able to achieve the strength and stiffness levels of properly reinforced specimen.

Triantafillou, and Antonopoulos (2000) presented a simple design model for the calculation of the fiber-reinforced polymer (FRP) contribution to the shear capacity of strengthened RC elements according to the design formats of the Euro code, ACI-318-2000, and Japanese concrete Institute. The key element in the model is the calculation of an effective FRP strain, which is calculated when the element reaches its shear capacity due to concrete diagonal tension. Diagonal tension failure may be combined with FRP debonding or tensile fracture, and the latter also may occur at a stage beyond the ultimate shear capacity. An upper limit (maximum) to the FRP effective strain also is defined and aimed at controlling crack opening. The effective strain, obtained through calibration with more than 75 experimental data, was shown to decrease with the FRP axial rigidity divided by the concrete shear strength. It is also demonstrated that the contribution of FRP to shear capacity is typically controlled by either the maximum effective strain or by debonding and, for a given concrete strength, it increases linearly with the FRP axial rigidity until the latter reaches a limiting value beyond which debonding controls and the gain in shear capacity is relatively small. However, proper anchoring (e.g., full wrapping) suppresses the debonding mechanism and results in considerable increase in shear capacity with FRP axial rigidity. Finally it is demonstrated that, when compared with others, the proposed model gives better agreement with most of the test results available.

Five RC beams without shear reinforcement were cast at the concrete laboratory of New Jersey Institute of Technology. After the beams were kept in the curing room for 28 days, carbon fiber strips and fabrics made by

SIKA Corp were applied on both sides of the beams at various orientations with respect to the axis of the beam. All beams were tested on 220-kip MTS testing machine by **Zhang, and Hsu** (2002). Results of test demonstrate the feasibility of using externally applied, epoxy-bonded CFRP system to restore or increase the shear capacity of RC beams. The CFRP system can significantly increase the serviceability, ductility, and ultimate shear strength of a concrete beam, thus restoring beam shear strength using CFRP is a highly effective technique. An analysis and design method for shear strengthening of externally bonded CFRP has been proposed as well.

Deniaud, and Cheng (2003) studied the interaction of concrete, steel stirrups, and fiber reinforced polymer (FRP) sheets in carrying shear loads in reinforced concrete beams. A total of eight tests were conducted on four laboratory-controlled concrete T-beams. The beams were subjected to a four-point loading. Each end of each beam was tested separately. Three types of FRP, uniaxial glass fiber, uniaxial carbon fiber, and triaxial glass fiber, were applied externally to strengthen the web of the T-beams, while some ends were left without FRP. The test results show that FRP reinforcement increases the maximum shear strengths between (15.4-42.2)% over beams with no FRP. The magnitude of increased shear capacity is dependent not only on the type of FRP but also on the amount of internal shear reinforcement. The triaxial glass fiber reinforced beam exhibited more ductile failure than the other FRP reinforced beams.

The numerical models developed by **Khomwan, and Foster** (2004) deal with the global and local behavior of reinforced concrete plane stress members (beams and walls) strengthened with FRP. One and two dimensional interface elements were used to connect the repair material to the parent concrete allowing for debonding failure to be captured. The models were tested for flexurally strengthened and shear strengthened

members with good correlations observed for the numerical results compared with the test data for the failure load and failure mechanism.

Shanthakumar, et al (2004) presented a numerical study to simulate the behavior of retrofitted reinforced concrete shear beams. The study was carried out on the unretrofitted RC beam designated as control beam and RC beams retrofitted using carbon fiber reinforced plastic (CFRP) composites with $\pm 45^\circ$ and 90° fiber orientations. The effect of retrofitting on uncracked and precracked beams was studied too. The finite elements adopted by ANSYS were used. A quarter of the full beam was used for modeling by taking advantage of the symmetry of the beam and loading. The load deflection plots obtained from numerical study show good agreement with the experimental results. There was a difference in behavior between the uncracked and precracked retrofitted beams though not significant. The crack patterns in the beams were also presented.

Cao, et al (2005) presented an experimental study on debonding failure state of FRP from the sides in which a total of 18 beams were tested. They focused on the distribution of strains in the FRP strips intersected by the critical shear crack, and the shear capacity at debonding. A simple model was presented to predict the contribution of FRP to the shear capacity of the beam at the complete debonding of the critical FRP strip.

2.5 Behavior of RC Columns Confined with FRP

Fabric

Reinforced concrete columns often have little confinement to the concrete core or lateral support to the longitudinal reinforcing bars. Poorly confined columns are recognized to behave in a brittle manner, exhibiting little deformation capacity. Increased loads, column deterioration, or seismic retrofit may require that additional confinement be provided to ensure adequate force and deformation capacity.

A total of 55 (15.25cm*30.50cm) concrete cylinders including 45 carbon-wrapped and 10 control (unconfined) specimens were tested in uniaxial compression by **Shahawy, et al** (1998). Two parameters were considered in the experimental program; concrete strength (20.64 and 41.28 MPa), and number of layers of carbon fabric (1-Slayers). Typical failure of carbon wrapped specimens was marked by fiber fracture at or near the mid-height of the specimens. Since the fabric was unidirectional (at 0°), a band or ring was typically formed as a result of the shearing off and separation of the fabric in the hoop direction. Carbon-wrapping of concrete column adds to its strength and ductility. The behavior of carbon-wrapped concrete is in general very similar to that of glass-wrapped concrete. The bilinear confinement that was developed for glass-wrapped concrete can be applied to carbon-wrapped concrete. However, a better fit can be obtained by recalibrating the model for an entire database that consists of both carbon-wrapped and glass-wrapped concrete.

Chaallal, et al (2000) presented results of a comprehensive experimental investigation on the behavior of axially loaded short rectangular columns strengthened with CFRP wrap. An extended literature review was provided discussing the existing confinement models developed for FRP confined concrete columns. Six series, a total of 90 specimens, of uniaxial compression tests were conducted on rectangular short columns. The behavior of the specimens was investigated in the axial and transverse directions. The confinement provided by the CFRP improves both the load-carrying capacity and the ductility of the column. This method of structural rehabilitation was shown to be applicable to prismatic sections. The gain in compressive strength increased with the number of layers. The maximum gain was achieved by columns with aspect ratio $a/b=0.65$ for $f'_c=20.64$ MPa and $a/b=0.5$ for 41.28 MPa. The maximum gain achieved was respectively 86% and 31% with respect to

control specimen. As f_c increased the axial strain substantially decreased and the transverse strain slightly decreased.

Yang, et al (2001) presented an experimental study focusing on the effects of corner radius on FRP mechanical properties. A unique re-usable test device was designed and used for this purpose such that plies of FRP could be applied over interchangeable corner inserts. The radius of the inserts ranged from a minimum of 0 to a maximum of 50.8 mm, and one or two plies of carbon FRP were used. The monitored parameters were strain distribution in the FRP laminate and load. It was found that only a portion of the CFRP laminate capacity was developed when failure occurred at the corner. Increasing the number of plies from one to two slightly improved the efficiency of the laminate.

The work of **Lorenzis, and Tepfers** (2001) was a systematic assessment of the performance of the existing models on confinement of concrete columns with FRP materials. Only the case of circular columns of limited size has been considered, due to the limited set of experimental data available for the case of non-circular and full-size columns. The assessment has focused on prediction of the ultimate axial stress and strain of the confined concrete, which are the most significant parameters from a design standpoint and have a major influence on the proximity of the predicted stress-strain curve to the experimental one. the experimental data on confinement of concrete cylinders with FRP available in the technical literature were classified according to the value of the significant variables; the existing empirical and analytical models were reviewed, pointing out their distinct features; the whole set of available experimental results was compared with the whole set of analytical models; strengths and weaknesses of the various models were analyzed. Finally, suggestions were indicated for possible improvements.

Fukuyama, et al (2002) described and commented on some of the design approaches provided by JBDPA guidelines for the strengthening of reinforced concrete columns. This was one of the main targets of Japanese experience on infrastructure strengthening, which become an imperative task after the post-earthquake observations of the damage caused by the Kobe earthquake in 1995. Finally, comparisons with the ACI-318-02 guidelines for the strengthening of RC members with FRP system were also formulated.

Lam, and Teng (2004) reported and compared the ultimate tensile strains of two types of FRP (carbon FRP and glass FRP) obtained from three types of tests—flat coupon tensile tests, ring splitting tests, and FRP-confined concrete cylinders were tests. Based on comparisons of these test results, it can be concluded that the FRP hoop rupture strains in FRP-confined concrete cylinders are reduced below the ultimate tensile strains from flat coupon tests by at least three factors—(1) the curvature of the FRP jacket; (2) the deformation localization of the cracked concrete; and (3) the existence of an overlapping zone. While the first factor that reduces the in situ strain capacity of FRP on confined concrete is material dependent, the last two factors that result in a nonuniform strain distribution in the jacket are independent of the FRP material properties. The third effect reduces the average hoop rupture but does not affect the distribution of the confining pressure, as the FRP jacket is thicker in the overlapping zone.

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Results from an experimental program and numerical analysis were presented by **Hosseini, and Fadaee** (2004). Four square columns with 260mm width and 1650mm length were tested under constant axial load and reversed cyclic lateral load that simulated forces from an earthquake. The test specimens were divided into two groups. Each group consisted of

two columns that were designed and detailed according to ACI 318-02 requirements for intermediate moment frames, and built from high strength concrete (i.e., $f'_c = 54$ MPa), One as control specimen and the other wrapped with carbon fiber reinforced polymers (CFRP). First and second group were reinforced longitudinally with four 18mm diameter ($\rho_{sl} = 1.5\%$) and eight 18mm diameter ($\rho_{sl} = 3\%$) respectively. Results of experimental and theoretical moment-curvature relationship were compared. This theoretical relationship was derived from numerical analysis of existing stress-strain models that were presented for FRP-confinement. Comparison between the experimental and analytical results indicates good agreement, except that ultimate curvature in wrapped specimens was underestimated. From the results of tests, it can be concluded that ductility parameters of wrapped column such as ultimate displacement and curvature ductility were enhanced 53%, 78% for S4 and 25%, 28% for S8 column respectively.

Cheng, et al (2004) presented the theoretical results of sensitivity studies performed on a simply supported FRP wrapped RC beam-column subjected to an axial load and to equal end moments. These sensitivity studies were performed to investigate the effects of the various parameters on the strength of the FRP wrapped RC beam-columns. The parameters used in the investigation include the unconfined concrete strength, the steel ratio, the thickness of FRP wraps, and the section diameter. A total of 23 cases were studied. Interaction equations were also developed in this work to provide a simplified and practical tool for engineers to evaluate the ultimate strength of the FRP wrapped columns. It is shown that the developed interaction equations for predicting the ultimate strength are reliable.

Belarbi, et al (2004) evaluated the effects of various environmental conditions on the long-term behavior of RC columns wrapped with FRP

sheets. The experimental program comprised two parts; ambient environmental effect tests and corrosion tests. The environmental conditions considered in the ambient environmental tests included (1) freeze-thaw cycles, (2) high-temperature cycles, (3) high-humidity cycles, (4) ultraviolet (UV) radiation, and (5) saline solution; while the corrosion tests dealt with the corrosion of steel reinforcement embedded in RC columns wrapped with FRP sheets. RC columns were wrapped with carbon fiber reinforced polymer (CFRP) sheets and glass fiber reinforced polymer (GFRP) sheets and conditioned under the environmental conditions listed. After the environmental conditioning, uni-axial compression tests were conducted in order to evaluate the effects on the mechanical properties, such as column capacity, stiffness, and ductility. The test results showed that the mechanical properties were affected due to the environmental conditioning and the corrosion of steel reinforcement, and the effects varied according to the types of environmental conditions. Based on these results, improved design guidelines for the RC columns wrapped with FRP sheet were proposed. The proposed design guidelines included an analytical model and reduction factors to account for the effects of various environmental conditions.

Zhu, et al (2005) undertook a work to advance the current state-of-the-art on concrete-filled FRP tube (CFFT) by comparing its seismic performance to that of the conventional RC bridge pier column at the sectional, member, and bridge system levels. A non-linear finite element model was developed for a bridge case study. Since FRP has a lower rupture strain than steel spiral, and because of its linear elastic response, CFFT section shows less ductility than RC section. However, at the member level, CFFT column distinctly outperforms its RC counterpart with almost twice the base shear capacity and over three times the lateral drift capacity. This phenomenon was attributed to the effective role of FRP tube

in extending the plastic hinge zone of the column well beyond its typical range in conventional RC columns. The implication of this behavior was shown through a seismic simulation of the entire bridge under a major historical shake with varying levels of magnified ground acceleration. The simulation showed the CFFT substructure to suffer moderate damage while maintaining structural integrity, as compared to the RC substructure with severe and irreparable damages.

The stress–strain characteristics of FRP confined concrete have been extensively studied by *Matthys, et al* (2006), testing small-scale cylindrical specimens. The stress–strain response of axially loaded large-scale columns confined with FRP wrapping reinforcement was investigated. The effective circumferential FRP failure strain and the effect of increasing confining action on the stress–strain behavior of RC columns confined with FRP were examined. Different models for the prediction of the stress–strain behavior of FRP-confined concrete have been reviewed. The existing stress–strain models were applied to predict the stress–strain response of the experimental results presented. A comparison between the experimental results and those predicted by the existing models were presented. Few of the available models that were derived based on small-size cylinders were found to accurately predict the stress–strain response of large-scale columns.

2.6 Behavior of Reinforced Concrete Frames

Hsu, et al (1981) described a computer program for the elastic perfectly plastic analysis of reinforced concrete planar frames. This computer program requires less computer time and memory space and it is intended as practical for analysis and design uses. The computer program is capable of a complete analysis of reinforced concrete frames from zero load until failure under any system of static gravity and lateral loads. The analysis of reinforced concrete frames uses a computer program as a

subroutine to calculate the moment – curvature characteristics under a constant load applied at the section centroid. The program efficiency was checked through comparing its results with experimental results for reinforced concrete plane frames that were tested by others. The program ignored each of; (1) geometric non – linearity (2) shear deformations and (3) local unloading. However, the program considered the possibility of each of collapse mechanism failure and local crushing failure following the plastic hinge model.

Al – Sarraf (1986) derived modified stability functions for two – dimensional prismatic beam – columns having any solid cross – section shapes, laced or battened built – up structural members, in terms of the shear flexibility and axial load parameters. An approximate formula for such functions were proposed which make possible the rapid prediction of the elastic critical load of structures taking into consideration the effect of shear force in the members, using a hand – computing method. The modified stability functions were related to previously exist tabulated stability functions of prismatic strut. The approximate formula predicts accurate elastic load to within 1%.

Al – Rifaie and Trikha (1988) recognized the importance of the finite size of joints and shear deformations in the analysis of plane concrete structures, especially frame – shear wall type structures. They proposed rigid – ended element for idealization to carry out a step – by – step linearised limit state analysis. Simplified axial force – moment diagrams for reinforced concrete sections had been derived for detection of a plastic hinge formation. However, the analysis ignored the effect of each of geometric non –linearity, possibility of local unloading, and possibility of crushing failure at any critical section. In addition, the analysis considered that each element has only two critical sections at its ends and could not treat the case of existence of a third critical section at the zero shear point

when the element is under distributed load. A computer program had been used to analyze an irregular multi – storied frame, an industrial gable frame and a frame – shear wall structure and the results were compared to establish the validity of the procedure.

Wong and Loi (1990) developed a method for the elastic non – linear analysis of framed structures depending on the finite element method. They described the main forms of Lagrangian coordinates systems and their relative merits were also discussed. The solution strategy rested on a combination of the Newton – Raphson iterative technique and modified “arc length method” within a finite element based partially on updated Lagrangian description. The proposed modified technique of the arc length enables a solution at the limit point to be calculated. They solved three examples and got very accurate results compared with experimental results obtained by others.

A reinforced concrete structures subjected to in plane shear and normal stress were analyzed by **Hu and Schnobrich** (1990). A non-linear material model including the smeared crack representation and rotation crack approach was adopted. This model was taken into account tension stiffening, stress degrading effect for concrete parallel to the crack direction and shear retention of concrete on the crack surface. The results obtained by this model appear good agreement with experimental results.

Alwash (1995) proposed a general non – linear method of analysis of reinforced concrete plane frames which was utilized for the analysis of reinforced concrete Vierendeel trusses. It follows a step – by – step with iterations approach. Different parameters were included in the analysis, they are: combined effect of geometric non – linearity and shear deformations, material non-linearity, moment-axial force interaction, possibility of local stress unloading and the effect of the elements ends rigidity. Fixed end forces were derived for an element under uniformly

distributed load and the possibility of existence of an additional critical section at zero shear point for such element was also treated. Three possible failure criteria were predicted (collapse mechanism, local crushing failure and stability failure). The theoretical results were compared with those obtained experimentally and rather good agreement was obtained.

Hashim (1999) presented an analytical method that describes the ultimate strength and behavior of reinforced concrete frames having prismatic and / or non-prismatic (tapered) members. Plastic zone model was suggested with an incremental approach to study the inelastic behavior of reinforced concrete frames. Different parameters were included in this method which are: effect of coupling between bending moment, shear and axial forces, material non-linearity, geometric non-linearity, moment-axial force interaction at both yield and ultimate states, and possibility of local stress unloading. A suitable bilinear moment-rotation law was suggested. Equivalent nodal loads (fixed-end forces) were presented for tapered element under uniformly distributed load considering the possibility of existence of plastic zone at left, right, or at both ends of the member. Failure was predicted by the concrete crushing at a certain section.

Kuw and Ju (2002) developed a three – dimensional (3D) non – linear beam element into a static and dynamic non –linear finite element program called (**AN** program). This 3D beam is an inelastic element for modeling beam and beam – columns to model the steel or reinforced concrete structures for the static and dynamic loading, especially for the ground motion problem. The **AN** program provides efficient solution scheme for this beam element. For example, a non – linear finite element analysis with hundred thousand nodes could be performed effectively using a personal computer. The absorbed boundary condition of the **AN** program also provides the 3D beam to perform the soil – structure interactive analysis in

the time – domain. The AN program had the ability to perform a very complex non – linear structural analysis since it incorporated this 3D beam element.

2.7 Behavior of Reinforced Concrete Members under Cyclic Loads

Accurate analysis of concrete frame structures under static and dynamic overloads is an important and difficult task in structural engineering. Eleven reinforced concrete cantilever beam specimens were constructed and tested by **Hwang and Scribner** (1983) under various conditions of inelastic reversal loading to determine the effect of variations in displacement history sequence and their magnitude on cyclic response of reinforced flexural member. Test results indicated that the member energy dissipation capacity was a function of applied shear stress intensity and the magnitude of the maximum displacements applied to the members.

Al-Sulayfani (1989) investigated the non-linear analysis of prismatic reinforced concrete frames subjected to cyclically varying loading. The stiffness method with effective stiffness was used to study the behavior of such structures considering the material and geometrical non–linearity. An analytical solution was suggested using secant stiffness with the iterative technique. The layered approach was adopted for non-linear section analysis, considering the axial force and bending moment only. The non–linear cyclic behavior model for concrete was developed in a uniaxial state of stress, and non-linear cyclic behavior for reinforcing steel was used with the assumption of perfectly fixed bonds between steel and concrete.

Test results of 10 hybrid precast concrete beam-to-column connections were presented by **Stone, et al** (1995). These tests constitute Phase IV of an experimental program on one-third-scale model precast moment-resisting connections conducted at the National Institute of

Standards and Technology (*NIST*). The objective of the test program was to develop guidelines for the design of moment-resisting precast connections in regions of high seismicity. The hybrid connections consist of mild steel used to dissipate energy and post-tensioning (*PT*) steel used to provide the required shear resistance. Variables examined were the amount and type of mild steel (*ASTM A 615*). The amount of post-tensioning steel was varied to control the relative moment capacity contributed by the *PT* and mild steel. The specimens were subjected to reverse cyclic loading in accordance with a prescribed displacement history. Connection performances were compared to previous *NIST* tests based on energy dissipation capacity, connection strength, and drift capacity. Hybrid precast connection can be designed to match or exceed the performance of a monolithic connection in terms of energy dissipation, strength and drift capacity.

A stress resultant plasticity-based formulation with shear failure criteria and post-shear failure effects was presented by **Ricles, et al** (1998) for modeling the response of nonductile reinforced concrete bridge columns subjected to biaxial seismic loading. The formulation consists of a single component model, in which plastic hinges exist at each end of an elastic beam-column. The formulation is shown to predict reasonably well the cyclic response of nonductile beam-column experimental test specimens that develop either ductile flexural yielding, a brittle shear failure, or a ductile shear failure in the flexural plastic hinge zone when subjected to uniaxial or biaxial nonproportional cyclic lateral loading. Inelastic seismic time-history analysis was also presented to illustrate the significance of including nonductile effects in a model of a bridge structure susceptible to column shear failure during an earthquake.

Four models were selected by **Pandolfi, and Taliervo** (1998), which were based on the concept of “bounding surface.” Several numerical simulations of fatigue tests were performed to assess the potentiality of

these models in the domain of fatigue. Whereas the response of two of these models was consistent with the loading conditions in most simulations, the other ones were unable to take the effect of confining pressure into account. All of the models, however, require modifications to give realistic predictions of fatigue life and cyclic creep. The parameters that mostly seem to affect the model response under fatigue were devised and slightly modified; comparisons with available experimental data support the validity of this proposal. This preliminary research was intended to be the first step toward the incorporation of rate effects into the models considered, or other theoretical models.

Perez, and Pantazopoulou (1998) attempted to establish the parametric relation-ship between shear strength and deformation demand through a nonlinear analytical model of cyclic plane-stress states in reinforced concrete. The formulation establishes tensorial, constitutive, and equilibrium relations, all expressed in terms of average stresses and strains. The model was used to conduct a parametric investigation of several design variables. The rate of strength degradation and ductility capacity were used to gauge the parametric sensitivity of the problem. It was found that the calculated concrete shear contribution degraded with increasing deformation demand and was enhanced by an increase in the level of axial load and the amount of transverse reinforcement. A simplified design equation was developed from the analytical results to express the dependence of the concrete contribution term on the intensity of imposed deformation demand.

A plastic-damage model for concrete subjected to cyclic loading was developed by **Lee, and Fenves** (1998) using the concepts of fracture-energy-based damage and stiffness degradation in continuum damage mechanics. Two damage variables, one for tensile damage and the other for compressive damage, and a yield function with multiple-hardening

variables were introduced to account for different damage states. The uniaxial strength functions are factored into two parts, corresponding to the effective stress and the degradation of elastic stiffness. The constitutive relations for elastoplastic responses were decoupled from the degradation damage response, which provides advantages in the numerical implementation. In there model, the strength function for the effective stress was used to control the evolution of the yield surface, so that calibration with experimental results was convenient. A simple and thermodynamically consistent scalar degradation model was introduced to simulate the effect of damage on elastic stiffness and its recovery during crack opening and closing. The performance of the plastic-damage model was demonstrated with several numerical examples of simulating monotonically and cyclically loaded concrete specimens.

The fiber beam element with shear modeling developed by **Petrangeli** (1999) was calibrated and verified by comparison with test data. The verification was carried out for the material constitutive behavior and for single beam and column elements using available test results from literature. A structural analysis of a shear sensitive viaduct pier subjected to ground input motion was presented. Details of the algebraic expressions used for the concrete and steel constitutive behaviors were provided.

Al-D'Ambrisi, and Filippou (1999) presented improved analytical methods for simulating the nonlinear static and dynamic response of reinforced concrete frames. A new approach in describing the nonlinear hysteretic behavior of reinforced concrete elements was proposed. This approach consists of isolating the basic mechanisms that control the hysteretic behavior of RC members into individual subelements that were connected in series to form the RC member element. A shear subelement was presented. It describes the shear sliding in the critical regions and the shear distortion along the member. In cases where a substantial axial force

was present, the proposed shear subelement was capable of describing the interaction of axial force with the opening and closing of shear cracks. To establish the validity of the proposed model, correlation studies of analytical results with experimental evidence of the load-displacement response of shear critical RC members and subassemblies under static load reversals were conducted. The analytical results generally show very good agreement with experimental results.

Barnes, and Mays (1999) examined the fatigue performance of CFRP-strengthened concrete beams as part of a project investigating the use of CFRP as an alternative to steel. Five reinforced concrete beams were tested in fatigue; two control beams and three strengthened with externally bonded CFRP plates. Three loading options were used: (1) apply the same loads to both plated and unplated beams, (2) apply loads to give the same stress range in the rebar in both beams, and (3) apply the same percentage of the ultimate load capacity to each beam. Fatigue fracture of the internal reinforcement steel would appear to be the dominant factor governing failure, and it would appear reasonable to expect the same fatigue life for plated and unplated beams with comparable values of stress range in the steel bar.

The bond mechanism of fiber-reinforced polymer (FRP) rebars to concrete was studied by **Katz** (2000) to evaluate the effect of cyclic loading. Five different types of 12.0- and 12.7-mm rebars were tested. The FRP rods were embedded in concrete blocks and were subjected to up to 450,000 cycles at service stress level. The loading was accompanied by immersion in water at 60 and 207C to accelerate deterioration effects. Pullout behavior of the rods was determined at the end of the loading period. The results indicated a reduction in the bond strength after loading. Three mechanisms of failure were identified: (1) Abrasion of the surface of the rod, which, in the case of uniform resin throughout the rod, may lead to a reduction of 20–

30% in the bond strength; (2) delamination of the outer layer of the resin at the surface of the rod, which may lead to a reduction of up to 60% in the bond strength; and (3) abrasion of cement particles entrapped between the rod and the concrete, which serves as the main source of “bond” for smooth rods leading to approximately 70% of reduction in the bond strength. There was no significant change in the effect of curing temperature on the loss of bond.

A full-scale model of a bridge deck slab isotropically reinforced with 0.3% fiber-reinforced plastic reinforcement (FRP) was tested by **Rahman, et al** (2000). The 6 m long slab of 185 mm thickness had 2 m spans and 1 m cantilevers on both sides. The slab was first loaded monotonically to crack the concrete. Then it was loaded cyclically in three stages of 4 million cycles each at a frequency of 5 Hz, with the load varying between 0 and 100 kN in the first two stages and between 0 and 125 kN in the last. Finally, it was loaded monotonically up to failure. Constructability of the slab with the FRP reinforcement was found satisfactory. The cyclic load test shows that deflections and stresses were small, and the degradation indicated by increases in deflections and stresses after 4 million cycles of overloading by 25% was very small. The ultimate load capacity of the slab was at least five times the maximum wheel load of 100 kN, and the failure mode was punching shear. Long-term performance of FRP reinforcement under the combined effect of exposure to the chemical environment and loading needs to be studied to evaluate durability, and hence cost-effectiveness, of FRP reinforcement in bridge decks.

Karabinis, and Kiouisis (2001) presented a theoretical and computational framework for the analysis of structures that were subjected to inelastic static or dynamic overloads. Current practice in structural engineering assumes elastic material response for the analysis but applies the resulting solutions to design methods that were based on elastoplastic or

perfectly plastic material response. This approach generates inaccuracies which, depending on the amount of overload, can be excessive because force distributions within statically indeterminate structures depend on the relative stiffnesses of the individual structural elements (i.e., beams and columns). The relative element stiffnesses within a structure change continuously under inelastic loading and can be significantly different from their initial elastic values. For this purpose, a new plasticity model that combines the nonlinear material response and the geometric characteristics of a structural element was developed. The model provides the computational efficiency and simplicity of matrix structural analysis and avoids modeling at the material level. Alternative analysis would require very expensive 2D or 3D finite-element computations using solid elements.

A new extension for the applied-element method for structural analysis was introduced by **Meguro, and Din** (2001). In this method, the structure was modeled as an assembly of elements made by dividing the structure virtually. They were introduced the element formulation of the applied-element method. Next, the effects of the element size and arrangement were discussed, and finally, the accuracy of the proposed method in a nonlinear material case was verified by studying the behavior of RC structures under cyclic loading. For effects caused by the size and arrangement of the elements, it was shown that accurate results of stresses and strains can be obtained if elements of small size are used. As for failure behavior simulation of RC structures, the complicated behavior of cracks in structures subjected to cyclic loading, such as crack initiation and propagation, and opening and closure of cracks during the unloading process can be simulated automatically and without any use of complicated techniques. No special knowledge about the crack location or direction of propagation was needed before the analysis. The calculated load-displacement relation and the failure load show reliable accuracy.

The selection and evaluation of finite-element modeling approaches for predicting cyclic behavior of structural concrete was presented by **Kwan, and Billington** (2001). Nonlinear material models for structural concrete were evaluated through comparison of simulations with one set of experiments. A set of models and parameters was selected and modifications were implemented by the writers. This set of models was further evaluated through comparison with experiments of cyclically loaded structural concrete components with varying geometry, loading conditions, and hysteretic response. It was found that representation of shear transfer across crack faces strongly influences hysteretic behavior. Modeling compression strain-softening and the Bauschinger effect in reinforcing steel improves predicted cyclic response relative to experiments. The selected set of material models were evaluated and found to capture flexure-dominated hysteretic behavior, indicate potential shear-dominated failure, predict local influences of bonded prestressing through joints, and capture the influences of unbonded vertical prestressing in bridge columns. Modeling bond-slip, concrete shear distortion, and steel buckling were necessary for full failure mode prediction. This research uses models available in commercial finite-element codes and was intended for researchers and practitioners interested in using nonlinear finite-element analysis to predict cyclic response of new and existing structural concrete designs.

El-Tawil, et al (2001) described an analytical model for simulating the static response and accelerated fatigue behavior of reinforced concrete beams strengthened with CFRP laminates. Static and fatigue calculations were carried out using a fiber section model that accounts for the nonlinear time-dependent behavior of concrete, steel yielding, and rupture of CFRP laminates. Analysis results were compared with experimental data from two sets of accelerated fatigue tests on CFRP strengthened beams and show

good agreement. Cyclic fatigue causes a time-dependent redistribution of stresses, which leads to a mild increase in steel and CFRP laminate stresses as fatigue life was exhausted. Based on the findings, design considerations were suggested for the repair and/or strengthening of reinforced concrete beams using CFRP laminates.

Analysis and design procedures were presented by **Pantelides, and Gergely** (2002) for a carbon-fiber-reinforced polymer (CFRP) composite seismic retrofit of a reinforced concrete (RC) three-column bridge bent. The CFRP jacket was designed using performance-based criteria to provide target displacement ductility based on seismic retrofit measures for the columns, bent cap, and bent cap-column joints. In situ quasi-static cyclic tests of a bent in the as-built condition and a bent retrofitted with the CFRP jacket were carried out in 1998. The seismic retrofit was successful, and the bridge bent retrofitted with CFRP composites reached a displacement ductility level in excess of the target ductility and double the hysteretic energy dissipation of the as-built bent. A description of the CFRP composite layout and validation of the design assumptions from the experimental results was presented. Recommendations for improvement of the original CFRP composite seismic retrofit design were offered based on the lessons learned from the in situ tests.

Kadhim (2007) presented a plastic zone model with an incremental and iterative technique to study the inelastic behavior of reinforced concrete space frames having prismatic and / or non-prismatic (tapered) members and subjected to cyclically varying loading. The effect of bond-slip, creep, and shrinkage were taking into account. The non-linear cyclic behavior model for concrete and steel were used to represent the response of these two materials under cyclic loading.

2.8 Originality of the Present Study

Considering that the rehabilitation of the infrastructure is an important matter in Iraq. The proposed research is expected to generate both the scientific knowledge and engineering method for the application of CFRP fabric in the infrastructure field.

The proposed experimental test help understanding the behavior of strengthened concrete beams and columns; it will also be used to verify the validity of previous proposed analytical models. The proposed research will greatly increase the applicability of the strengthening methods with externally epoxy-bonded CFRP fabric. It will also broaden the knowledge related to the reinforced concrete structures as well.

The originality of the proposed research can be found in the following aspects:

1. It is clear from the preceding review a detailed study was not found that deals with the inelastic non-linear behavior of reinforced concrete framed structures having prismatic and non-prismatic members under cyclic loading and strengthening/repair with CFRP.
2. The Region technique is used for the treatment of the variation in material properties through the depth and width of section.
3. The hardening rule, flow rule, tension stiffening and crushing state are taken into account.
4. Material non-linearity, geometric non-linearity and coupling between bending moment, shear and axial forces are taken into account.
5. Possibility of existence of plastic zone at left, right, or at both ends of the member.
6. Wrapping with CFRP fabrics in longitudinal and transverse directions.
7. Possibility of existence of distributed load on any member in addition to nodal forces is considered in treating this problem in the present study.

Chapter Three Experimental Work

3.1 Introduction

The aim of this chapter is to cast experimental specimens that cover some applications of this research by getting real results about these applications and to be the reference for comparison and checking the ability of the analytical program.

This experimental investigation is carried out in the structural laboratory of the Civil Engineering Department at University of Babylon. The experimental work consisted of monotonic and quasi-static (repeated) tests on five samples of reinforced concrete frames. These frames have a length of 1900 mm, width of 650 mm, and height of 1000 mm. The cross-sectional dimensions of beams and columns are 100 x 100 mm. The details configuration of frames can be seen from Figure (3.1).

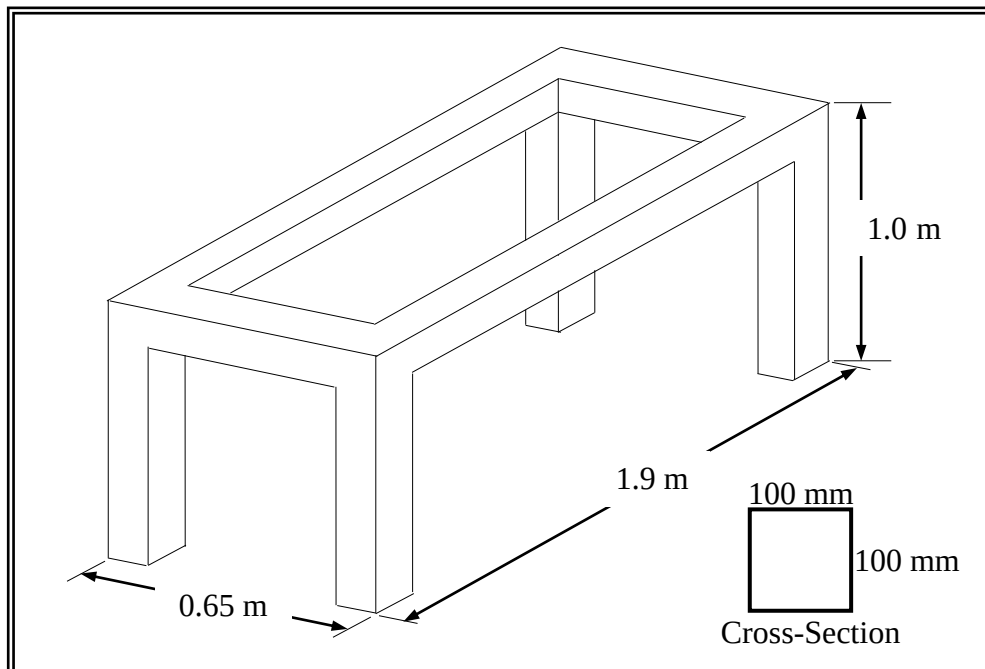


Figure (3.1) Geometry of Reinforced Concrete Tested Frames

3.2 Materials

3.2.1 Carbon Fiber Reinforced Polymer and Epoxy

Sika Corporation, Syria branch, provided the CFRP wrap used in this research. **SikaWrap Hex 230C** as shown in Figure (3.2), which is a kind of woven composite fabric, was employed in this study. CFRP fabric is unidirectional and has a longitudinal strength of (3500 MPa). The lateral strength of the fabrics is negligible. The fabric strips are elastic until break. The represented properties are given in Table (3.1).

The advantages of the CFRP are: very high strength, outstanding fatigue resistance, non-corrosion, excellent alkali-resistance, low weight, low thickness, available in any length, economical to apply. The disadvantage is its relatively high material cost.

The epoxy resin (**Sikadur-330**) as shown in Figure (3.2) used in this research was also provided by Sika, Syria branch. The epoxy consists of two components; part **A** is in white, part **B** is in dark gray. The mix ratio of part **A** and part **B** is four to one by weight. The properties are also given in Table (3.1).

The advantages of this epoxy can be listed in the following: high mechanical strength, high creep resistance under permanent load, high temperature resistance, long open life, easy to mix and apply, solvent free. This epoxy can be applied to slightly damp concrete surfaces. Also the epoxy exhibits non-sag in vertical and overhead applications, sets without tackiness, rapid curing even at low temperatures, and shrinkage-free curing.

Table (3.1) Properties of Sika's CFRP System

	Sika Wrap Hex-230C	Sikadur-330
Tensile Strength(MPa)	3450	30
Flexural E-Modulus(GPa)	230	3.8
Specific Gravity	1.5-1.9	1.25-1.45
Density	1.8(g/cc)	1.31(kg/l)
Elongation at break (%)	1.5	1.5
Thickness (mm)	0.13	-



Figure (3.2) SikaWrap Hex 230C and Sikadur-330

3.2.2 Concrete

The concrete used in the frames consists of Type I cement satisfies the Iraqi Standard, sand from local source, rounded stone with maximum aggregate size of (19.0 mm) more details in Appendix A. All frames in this research have been prepared using the same mix design. The mix properties used in this study are listed in Table (3.2).

Table (3.2) Mix Properties of Concrete Frames

Water/Cement Ratio	Cement (kg/m ³)	Sand (kg/m ³)	Gravel (kg/m ³)	Water (kg/m ³)
0.55	484.25	726.75	1453.5	266.5

3.2.3 Reinforcement

Two bars of Φ 8 mm in diameter are used in the tension face of the beams of the frames. The yield stress for steel is (543 MPa). Stirrups of Φ 5 mm in diameter spaced @ 40 mm have been used. The yield stress of steel used as stirrups is (650 MPa) which is accordance to (A706-ASTM specifications). For columns the longitudinal bars are 4 Φ 8 mm and stirrups of Φ 5 mm spaced @ 125 mm as shown in Figure (3.5) (more design details are given in Appendix B). The bars are tested in the Material Laboratory of the Civil Engineering Department at Baghdad University.

The mean data obtained from test for three specimens of each deformed bars are shown in Table (3.3).

Table (3.3) Test results for the used reinforcement bars

Φ (mm)	Φ equivalent	Pattern	Weight (kg/m)	A_s (mm ²)	f_y (MPa)	f_u (MPa)	Elongation (%)
5	4.984	C*	0.153166	19.511	650	744	5
8	7.771	C*	0.37233	47.43	543	670	10

* Deformation pattern C consists of diagonal ribs inclined at an angle of 60 degrees with respect to the axis of the bar.

3.3 Casting and Curing of Concrete Frames

Five RC frames having the same dimensions as shown in Figure (3.4) are cast at the structural laboratory of Babylon University. The framework of a steel mold as shown in Figure (3.3) is prepared and lubricated with oil before the concrete is poured. Then the steel bars are tied and put into the lubricated form. During casting, an electrical vibrator is used to compact the specimens. All specimens are cured in standard curing room for 28 days.

In order to simulate the real casting condition the researcher used two batches of concrete to fabricate each frame; the first one is casting the columns then they cured in standard curing room for 7 days. The second batch is used to cast the beams, after these 7 days the frame is cured for 28 days as shown in Figure (3.4). Three concrete cubes of 150 x 150 x 150 mm were casted for each batch of casting and then tested to determine the value of concrete compressive strength at 7 and 28 days of age, as shown in Table (3.4).

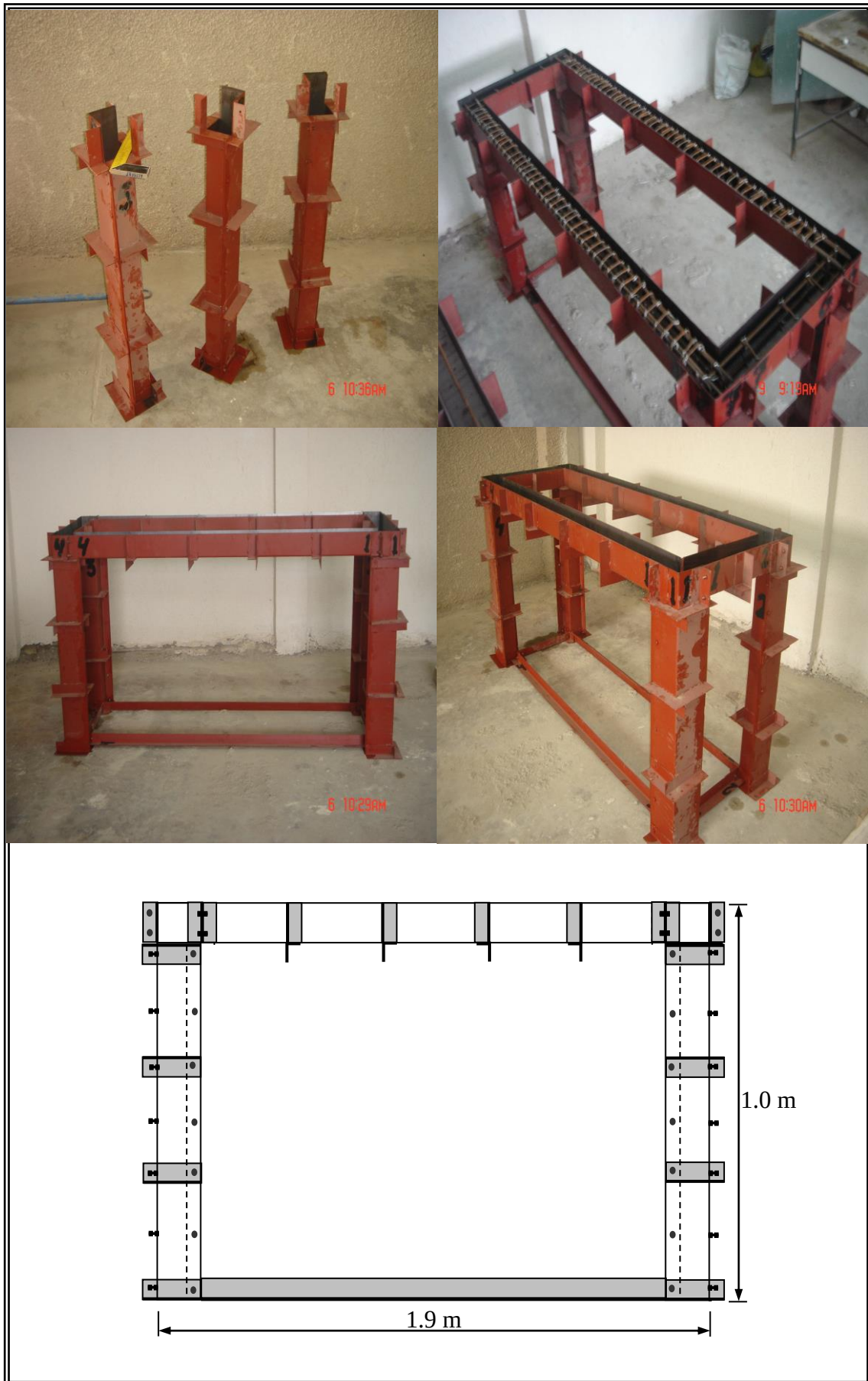


Figure (3.3) Details of Steel Mold



Figure (3.4) Reinforced Concrete Frames

Table (3.4) Concrete Properties

Concrete Batch	Average Compressive Strength f'_c (MPa)		Density (kN/m ³)
	7 days	28 days	
CF1	17	29	23.71
BF1	19	32	24.04
CF2	17	30	24.28
BF2	18	31	24.27
CF3	19	32.5	24.26
BF3	19.25	33	24.64
CF4	18.5	32	24.17
BF4	16.5	28	24.59
CF5	18.5	31	24.95
BF5	19	33	24.80
Average	18	31.15	24.37

Assuming cylinder strength $= (0.8-0.85) \times$ cube strength (*J. Newman, and B. S. Choo 2003*), then:

Average Cylinder Strength at 7 days= $0.8 \times 18 = 14.4$ (MPa), and

Average Cylinder Strength at 28 days= $0.8 \times 31.15 = 24.92$ (MPa)

In table (3.4) (CFi) refers to columns of frame (i), and (BFi) refers to beams of frame (i).

3.4 Installation of CFRP Strengthening System

3.4.1 Member Surface Treatment

The concrete surface of the members to be strengthening has to be sound and clean. Sandpaper, sand blasting, and metal brush are utilized to treat the concrete surface in this research see Figure (3.5). All the dust particles are cleaned using an airbrush. Some hard part is removed by the sand blasting. All the specimens were washed and treated carefully before the epoxy was applied.



Figure (3.5) Member Surface Treatment

3.4.2 Bonding of Composite Materials to Members

The epoxy used is mixed by hand according to ratio of 4:1 by weight. The epoxy is applied uniformly to both carbon fiber and concrete substrate. The fabric wraps are compressed to the concrete substrate with an ordinary paint roller. The extra epoxy is squeezed out and removed. The specimens are cured at room temperatures for at least 24 hours before testing. The whole procedure can be seen from Figures (3.6) and (3.7). In order to investigate the effect of the wrapping configuration on the final load-deflection behavior of RC frames, two different wrapping methods are used in this research; the first one is wrapping longitudinal at the bottom of the long beams, and the other is the transverse wrapping. In addition to that, two control frames have no wrapping at all in monotonic static and repeated load. The fifth specimen is loaded to 30% of ultimate load and then repair with both longitudinal and transverse wrapping. The details of the wrapping configuration are shown in Figures (3.8) - (3.10).



Figure (3.6) Materials and Mixing Tools



Figure (3.7) Installation of CFRP Strengthening System

3.5 Frames Configuration

All frames configuration are listed below. As mentioned earlier, there are two different wrapping configurations. All frames have the same dimensions in length, width, height, cross sectional area, and the same compressive strength for concrete and reinforcement details. The control frames FRAME1 and FRAME2 (Figure 3.8), with no fabric around the beams or columns are tested for monotonic static and repeated load respectively. FRAME3 (Figure 3.9), has one layer of CFRP fabric, which is wrapped into two directions at joints, which means the fiber direction of fabrics composites is longitudinally and perpendicular to the beam span. This frame is tested under monotonic static loads. FRAME4 is similar to FRAME3 as shows in Figure (3.10), but the longitudinal CFRP fabrics is tied at ends and middle by ties of CFRP fabrics while FRAME5, which is made for repairing sample that is loaded by (30%) of the ultimate strength. The longitudinal CFRP fabrics in FRAME5 have more ties at (30 cm) on centers as shows in Figure (3.11).

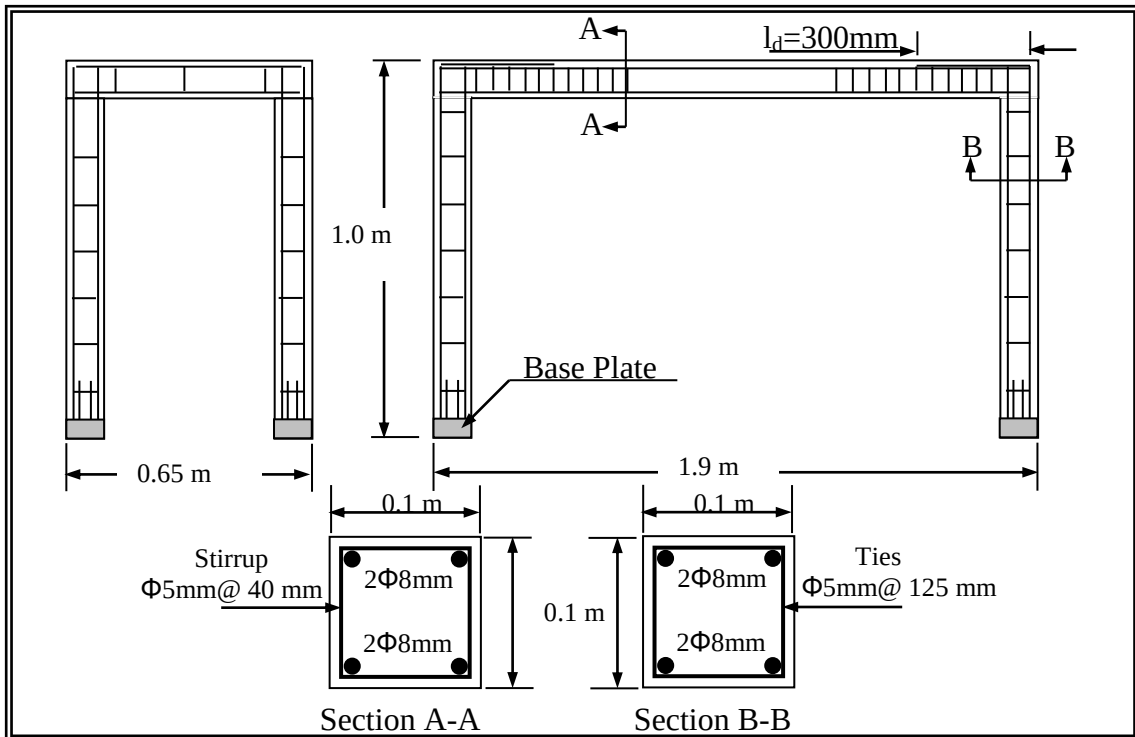


Figure (3.8) Reinforcement Details for all Frames

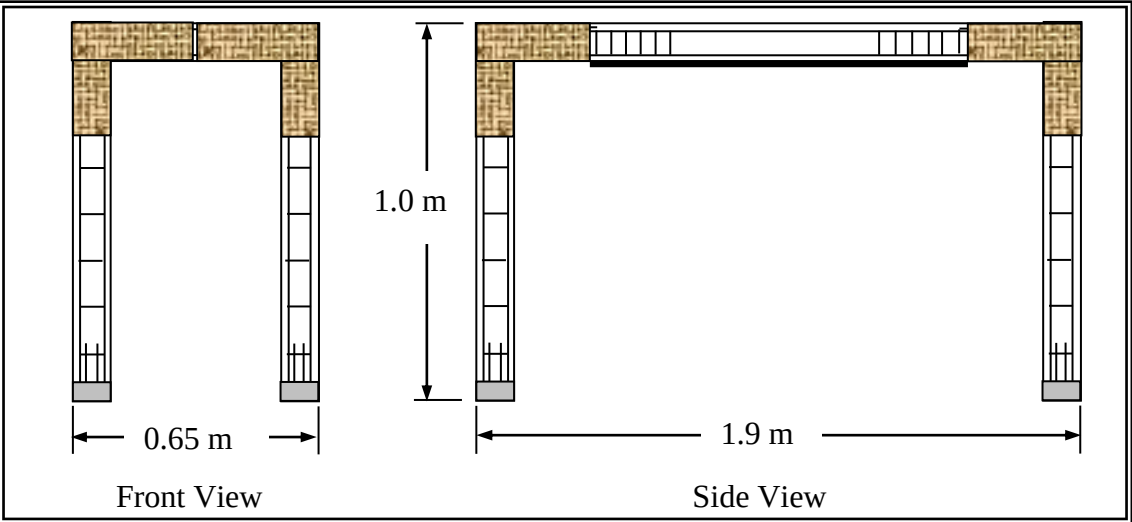


Figure (3.9) Specimen Details of FRAME3

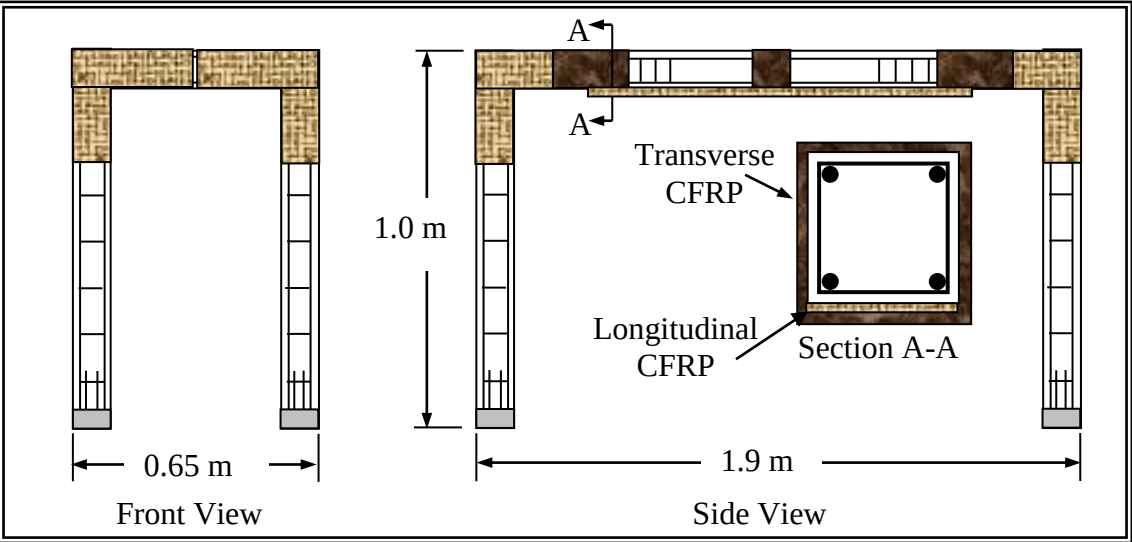


Figure (3.10) Specimen Details of FRAME4

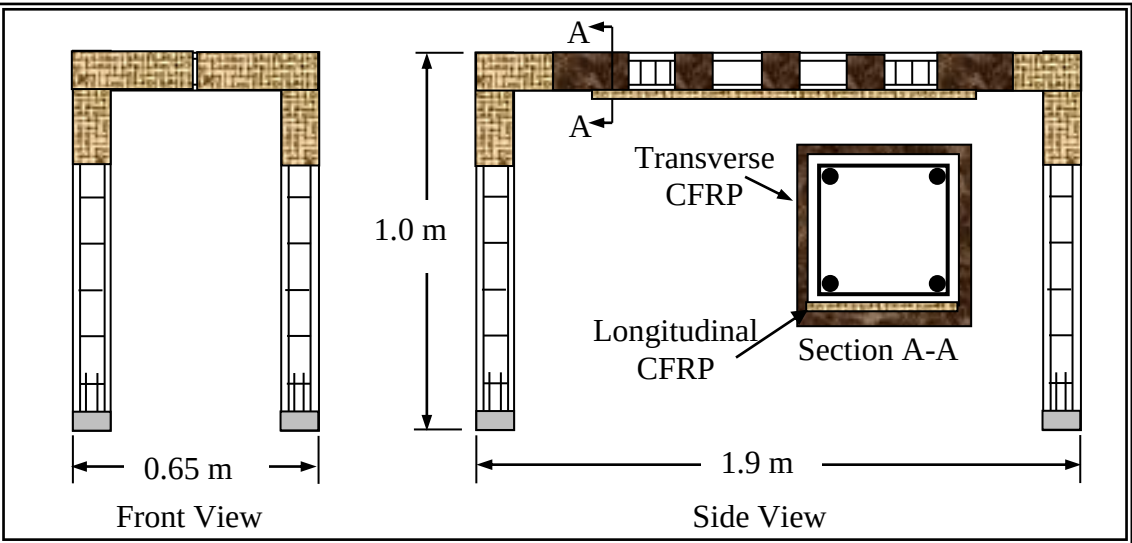


Figure (3.11) Specimen Details of FRAME5

3.6 Experimental setup and test procedures

All frames are tested by compressive testing machine (at strength materials laboratory in Kufa University) as shown in Figure (3.12). Some modifications have been introduced to the testing machine to be useful to test the frames such as using steel frame as support as shown in Figure (3.13). Partially fixed end is used at the bottom of each column. Two points load are applied right at the mid-span of each beam as shown in Figure (3.14). Six pairs of mechanical strain gages (or called demec gages) are installed at mid-span of one side of the longitudinal beam of frame before the test got started. They are located (10cm) at each side from the mid-span of the beam as shown in Figure (3.15). Dial gages are used at five points in the direction of the longitudinal beams of the frame. The strains are measured at different loading steps during the entire test. A recorded data is used to monitor the loading as well as the mid-span deflection. The load is applied by a hydraulic jack and measured by a load cell.



Figure (3.12) Compressive Testing Machine

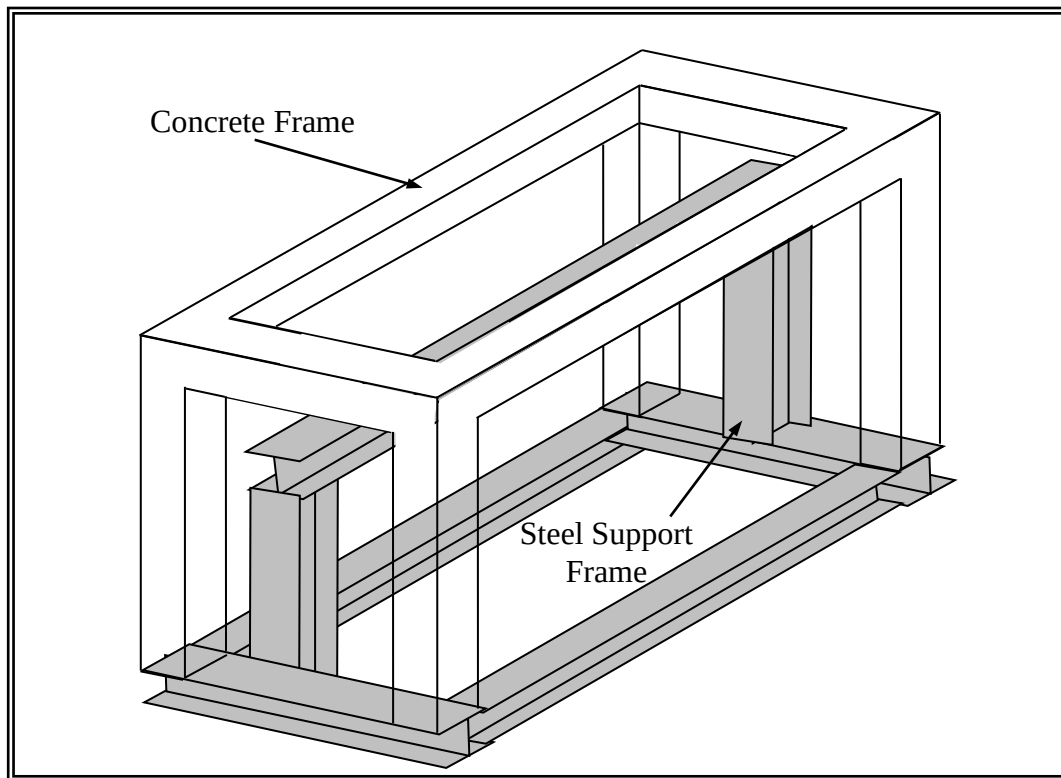


Figure (3.13) Steel Frame Support to Hold the Concrete Frame

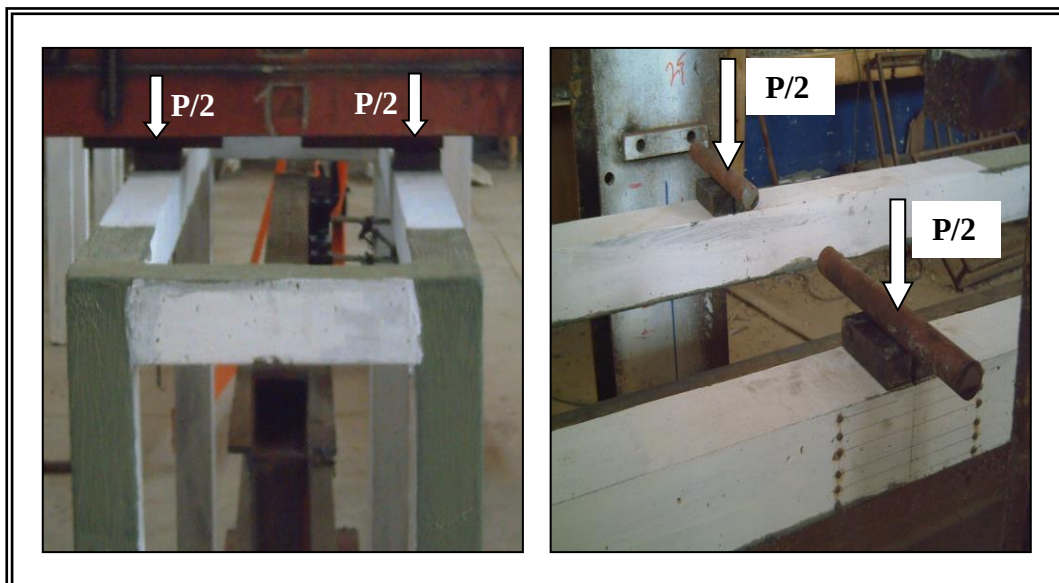


Figure (3.14) Loading Tools on Frame

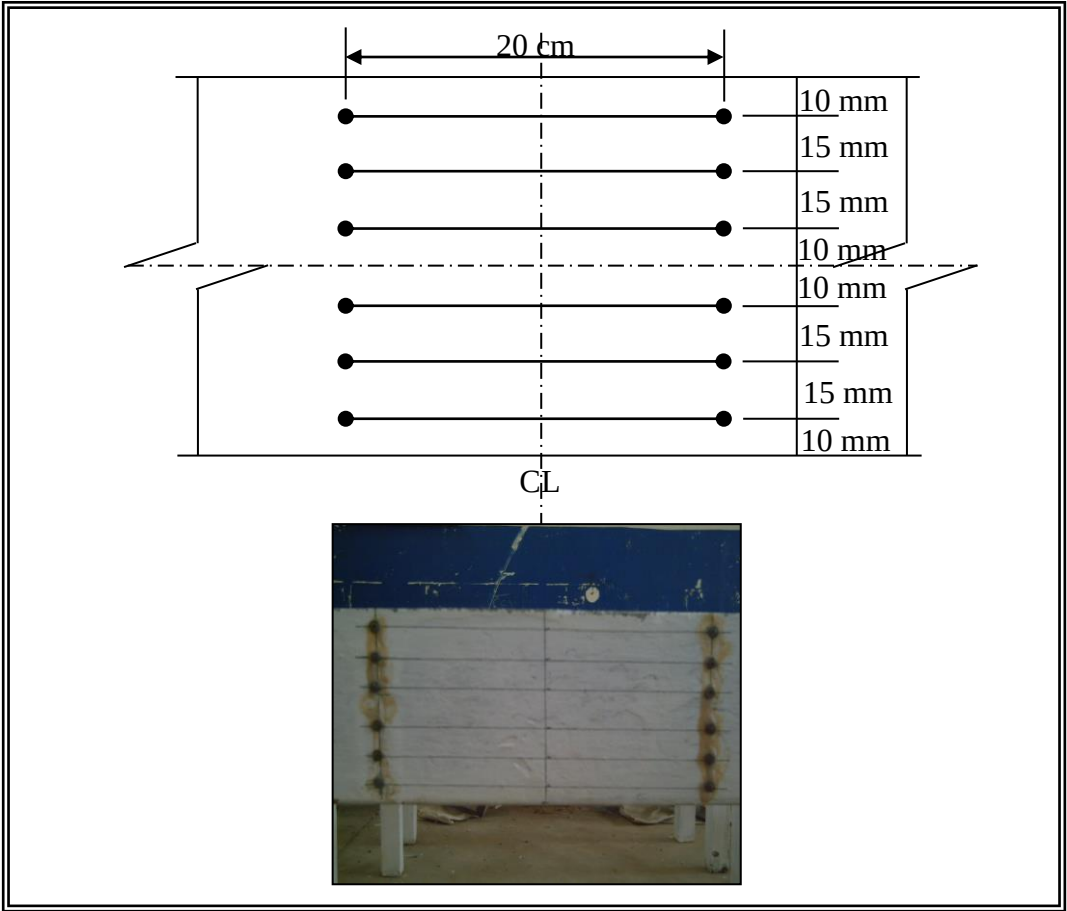


Figure (3.15) Mechanical Strain Gages (Demec Gages)

Chapter Four

Constitutive Laws for Normal and Confined Concrete

4.1 Introduction

In recent years, normal concrete strengthened or repaired with Carbon Fiber Reinforced Polymer (CFRP) fabrics has gained acceptance by engineers and researchers. Many old or damaged building and bridge members have been repaired using CFRP wrapping as a new and efficient retrofit measure. Since concrete is strengthened with CFRP fabric, a new stress-strain relationship for the confined concrete is needed.

4.2 Monotonic Stress-Strain Curve for Normal Concrete in Compression

The monotonic stress – strain curve with the general shape as shown in Figure (4.1) represents the envelope curve for normal concrete. For cyclic behavior curves, the following equations were used for the envelope curve cited by **Kadhim** (2007):

$$\bar{\sigma} = \frac{A_c * \bar{\varepsilon}}{\bar{\varepsilon}^3 + B_c * \bar{\varepsilon}^2 + C_c * \bar{\varepsilon} + D_c} \quad (4.1)$$

where:

$$\bar{\sigma} = \frac{\sigma_c}{f'_c} \quad ; \quad \bar{\varepsilon} = \frac{\varepsilon_c}{\varepsilon_{co}} \quad (4.2)$$

The coefficients (A_c, B_c, C_c, D_c) depend on the concrete maximum compressive strength f'_c ; these can be calculated from the curves shown in Figure (4.2). In the present study proposed polynomial equations to represent these coefficients according to the curves as follow:

$$A_c = 3.79598 + 0.00541903 * (f'_c) + 0.00360188 * (f'_c)^2 - 0.000647446 * (f'_c)^3 + 0.0000216942 * (f'_c)^4 - 0.000000224396 * (f'_c)^5 \quad (4.3a)$$

$$B_c = -1.11865 + 0.0324684 * (f'_c) + 0.00133292 * (f'_c)^2 - 0.000144436 * (f'_c)^3 + 0.00000204615 * (f'_c)^4 \quad (4.3b)$$

$$C_c = 2.96063 - 0.0122869 * (f'_c) - 0.00863826 * (f'_c)^2 + 0.000347041 * (f'_c)^3 - 0.00000377512 * (f'_c)^4 \quad (4.3c)$$

$$D_c = 0.872131 + 0.0509283 * (f'_c) - 0.000464236 * (f'_c)^2 - 0.0000793013 * (f'_c)^3 + 0.00000131489 * (f'_c)^4 \quad (4.3d)$$

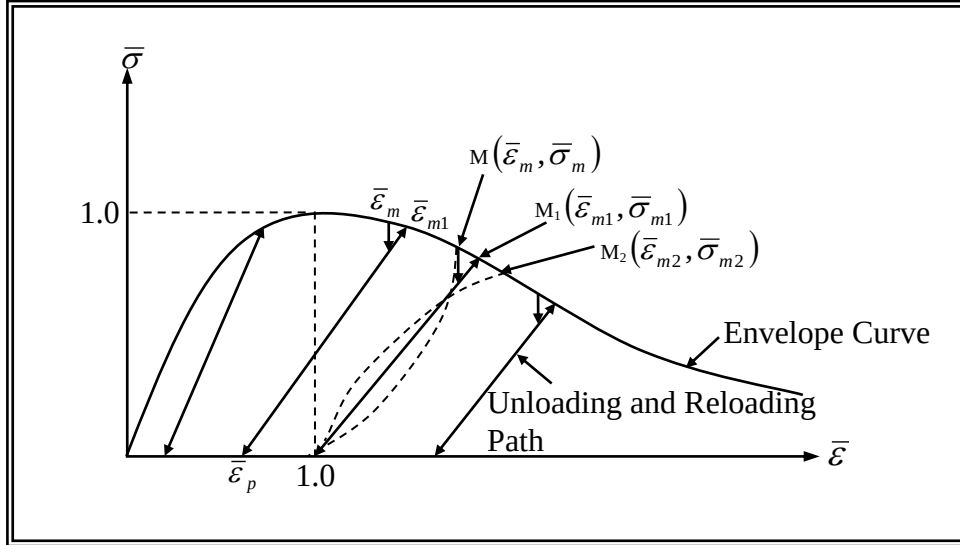


Figure (4.1): Adopted Normalized Stress-Strain Relation for Normal Concrete under Cyclic Loading Al-Sulayfani (1989)

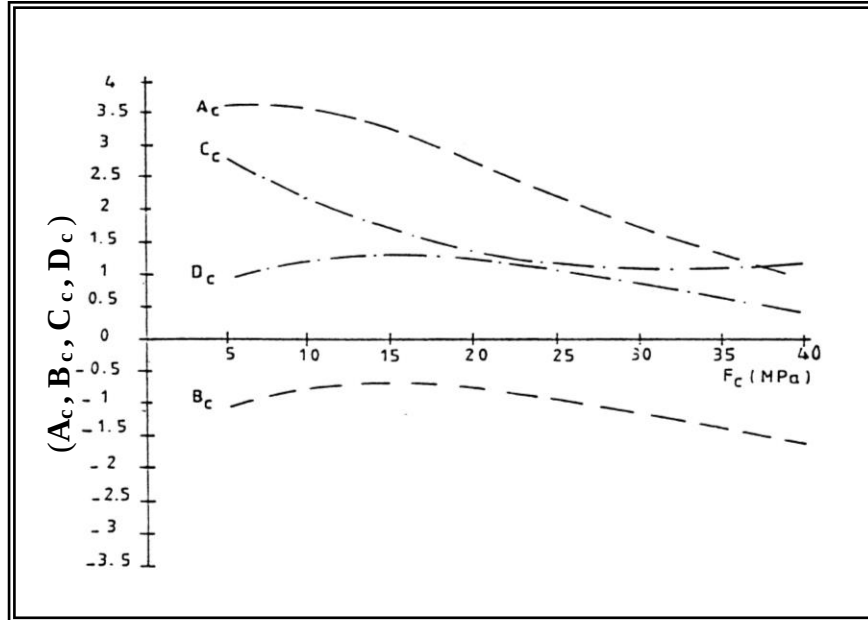


Figure (4.2): Evaluation of the Parameters for Concrete Envelope Equation (4.1) Al-Sulayfani (1989)

4.3 Monotonic Stress-Strain Curve for Fabric Confined Concrete

In order to fully understand the structural behavior of various structural members confined by CFRP fabric, a complete stress-strain equation of the confined concrete is needed. In this research a simple analytical equation is adopted to describe the whole stress-strain curve of confined concrete based on experimental results of **Chen** (2005). The following assumptions are used to develop such the adopted empirical equation.

1. Fabric wraps are considered as the confinement of the concrete core. Concrete and fabric are working together as a whole system. The adopted equation is good for the confined concrete only.
2. The Epoxy between concrete and fabric will not fail.
3. The stress-strain behavior of confined concrete can be approximated as a bilinear behavior. The second part is totally dependent on the Fabric properties.
4. The adopted equation will not describe the descending part of the stress-strain behavior due to its very brittle failure characteristic of the confined concrete.

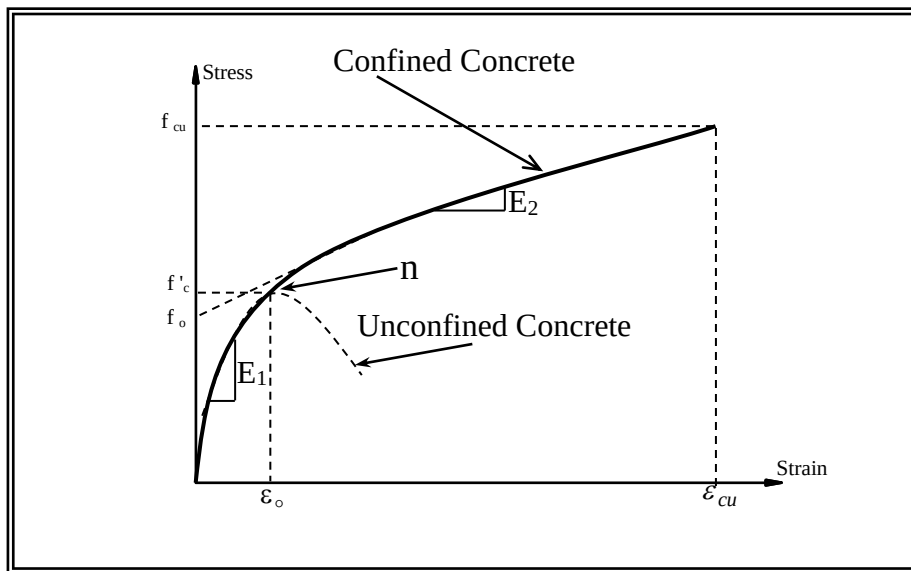


Figure (4.3): Adopted Bilinear Stress – Strain Curve of Fabric-Confined Concrete *Chen* (2005)

Figure (4.3) shows a typical stress-strain curve for fabric confined concrete. The curve has an approximate bilinear shape and has no obvious descendent part. According to experimental results of **Chen** (2005), the behavior of the concrete-fabric system can be defined by two phases. In the first phase, the axial loading is mainly carried by the concrete core. So the stress-strain curve almost follows the shape of unconfined concrete. After the concrete reaches its ultimate value of unconfined compressive strength, the dilation rate of the concrete starts to increase due to the microcracks developed within the concrete core those results in a significant confinement pressure force from the fabric jackets. Since the concrete core has already cracked, the behavior of the confined system is mainly dependent on the fabric jackets in the second phase. Therefore, the stress-strain curve follows a different slope that is mainly determined by material properties of the fabric. Also, because the final failure of the fabric is explosive and brittle, discussion of the descendent part of the stress-strain curve become meaningless.

To represent the bilinear response of fabric confined concrete, the relationship of **Samaan, and et al** (1998), which is used to define the fabric confined concrete as follows:

$$f_c = \frac{(E_1 - E_2)\epsilon_c}{\left[1 + \left(\frac{(E_1 - E_2)\epsilon_c}{f_o}\right)^n\right]} + E_2\epsilon_c \quad (4.4)$$

where f_c and ϵ_c are axial stress and axial strain of the laminate confined concrete.

E_1 is the first slope in the stress-strain curve, since the initial performance of the confined concrete is very similar to the unconfined concrete. The ACI-318M-05 equation for secant modulus of normal concrete can be used here.

$$E_1 = 4730 \sqrt{f'_c} \quad (\text{MPa}) \quad (4.5)$$

where f'_c is the compressive strength of unconfined concrete.

E_2 is the second slope in the stress-strain curve of the confined concrete. According to test results **Chen** (2005), as the concrete core approaches its unconfined strength, microcracks grow to the extent that the Poisson's ratio can no longer describe the relationship between the lateral and axial strains. And the laminate becomes the sole remaining device against catastrophic failure. Therefore, E_2 is a function of the stiffness of the laminate, and to less extent, the unconfined strength of the concrete, as follows:

$$E_2 = 189.2 f'_c{}^{0.2} + 1.345 \frac{E_f t_f}{D} \quad (\text{MPa}) \quad (4.6)$$

where: E_f , t_f are the effective modulus of elasticity and thickness of laminates, and D is the diameter of the concrete core. In the present study use equivalent diameter because use rectangular cross section Figure (4.4), then:

$$D = 1.1284 \sqrt{(h - 2d_1)(b - 2b_1)} \quad (4.7)$$

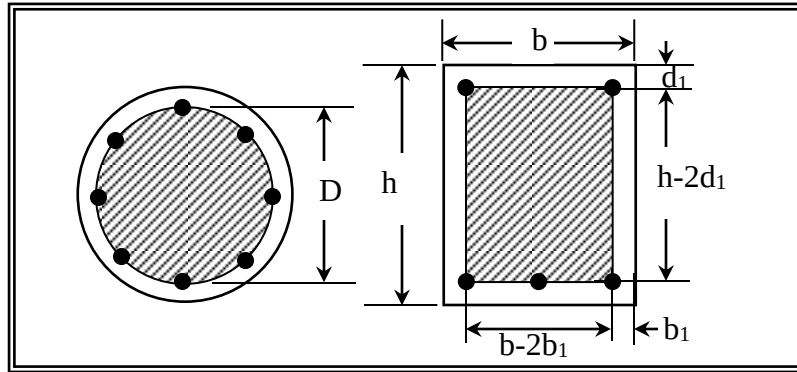


Figure (4.4): Equivalent Concrete Core

f_o is the referent stress at the interception of the second slope with the stress axis. According to the curve fitting of the experimental results **Chen** (2005), the equation of (f_o) can be estimated as follows:

$$f_o = 0.85 f'_c + 1.9 f_r + 6.89 \quad (\text{MPa}) \quad (4.8)$$

f_r is the confining pressure of the fabric. According to Figure (4.5), a simple equation for f_r can be derived from the equilibrium condition and

deformation compatibility condition. f_f is the tensile strength of the fabrics. Obviously, f_r will increase as the number of layers of the laminates increases.

$$f_r = \frac{2f_f t_f}{D} \quad (4.9)$$

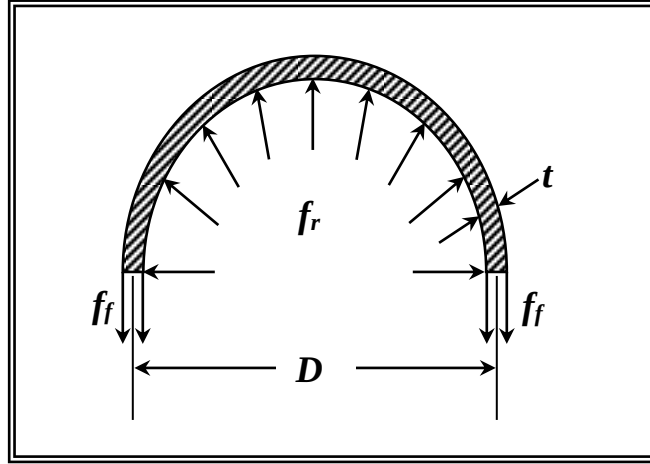


Figure (4.5): Definition of the confining pressure

n is a shape parameter that mainly controls the curvature of the transition zone of the stress-strain curve. It can be shown that the shape curve is not very sensitive to this parameter. A constant value of 0.95 is used based on curve fitting of the test results **Chen** (2005).

4.3.1 Prediction of Ultimate Stress and Strain

Although the adopted stress-strain equation can fully describe the stress-strain behavior of the fabric-confined concrete, a simple empirical equation is needed to predict the ultimate stress and ultimate strain of the confined concrete. From various literatures, it is generally assumed that the ultimate confined concrete strength (f_{cu}) is related to the confining pressure f_r . Therefore, a simple equation can be used to predict the ultimate stress of the confined concrete.

$$f_{cu} = f'_c + k_1 f_r \quad (4.10)$$

In equation (4.10), k_1 is coefficient of effectiveness that represents how much the confined strength (f_{cu}) of concrete is increased over its unconfined

value (f'_c). Based on **Chen** (2005) assumption, the following equation for k_1 is adopted by regression analysis.

$$k_1 = 6.14 f_r^{-0.25} \text{ (MPa)} \quad (4.11)$$

By substituting equation (4.11) in equation (4.10), then the ultimate confined strength (f_{cu}) can be related to the confining pressure by:

$$f_{cu} = f'_c + 6.14 f_r^{0.75} \text{ (MPa)} \quad (4.12)$$

Based on the ultimate stress of the confined concrete, the ultimate strain (ϵ_{cu}), i.e., the point at which the stress-strain response terminated is determined from the geometry of the bilinear curve as shown in Figure (4.3).

$$\epsilon_{cu} = \frac{f_{cu} - f_o}{E_2} \quad (4.13)$$

4.4 Stress – Strain Behavior under Tension

A linear path will be adopted for the stress – strain behavior of concrete under tension with the modulus of elasticity equal to the nominal modulus of elasticity in compression in both normal and confined concrete. This will be valid up to the cracking strength f'_t which has the following expression **Chen** (2005):

for normal concrete $f'_t = 0.633 \rightarrow 0.669 \sqrt{f'_c} \text{ (MPa)} \quad (4.14a)$

for confined concrete $f'_{tu} = 1.05 \rightarrow 1.4 \sqrt{f'_c} \text{ (MPa)} \quad (4.14b)$

This range of ratio to determine the ultimate tensile strength of confined concrete is depended on the thickness of the fabric jackets and the unconfined compressive strength f'_c .

Beyond f'_t for normal concrete or f'_{tu} for confined concrete, the concrete is considered to be incapable of transmitting tensile stresses.

4.5 Normal Concrete Cyclic Behavior

Figure (4.1) showed the stress-strain hysteresis loops for cyclic loading in compression range for normal concrete. This behavior is illustrated through the following points *Al-Sulayfani* (1989) (cited by *Kadhim* (2007)):

1. Assign the point where unloading occurs $M (\bar{\epsilon}_m, \bar{\sigma}_m)$.
2. Determine the value of the residual strain (plastic strain) $\bar{\epsilon}_p$ for this unloading case depending on the unloading strain value $\bar{\epsilon}_m$ which has the following equation:

$$\bar{\epsilon}_p = 0.055 * \bar{\epsilon}_m + 0.127 * (\bar{\epsilon}_m)^{3.1} ; \bar{\epsilon}_m \leq 2.2 \quad (4.15a)$$

$$\bar{\epsilon}_p = 1.584 + (\bar{\epsilon}_m - 2.2) * 0.2675 ; \bar{\epsilon}_m > 2.2 \quad (4.15b)$$

3. Where the hysteretic cycle completes after reloading, the strain path connects the envelope curve at point $M_2 (\bar{\epsilon}_{m2}, \bar{\sigma}_{m2})$. The value of $\bar{\epsilon}_{m2}$ could be determined from the relation:

$$\bar{\epsilon}_{m2} = 1.22 * \bar{\epsilon}_m \quad (4.16)$$

the main value for the strain between $\bar{\epsilon}_m$ and $\bar{\epsilon}_{m2}$ is used:

$$\bar{\epsilon}_{m1} = 1.11 * \bar{\epsilon}_m \quad (4.17)$$

then point $M_1 (\bar{\epsilon}_{m1}, \bar{\sigma}_{m1})$ was determined.

4. The unloading and reloading paths are represented by the line connecting the two points $(\bar{\epsilon}_p, 0)$, $(\bar{\epsilon}_{m1}, \bar{\sigma}_{m1})$.

5- If unloading continues for $\bar{\epsilon} < \bar{\epsilon}_p$ in compression, then the model gives tensile stress.

6- For tension strain, the same linear path is followed for unloading and reloading as for monotonic tensile envelope until the concrete tensile strength is reached where the concrete begins to crack and the stress drops to zero;

cracked layers will remain cracked and no tension stress will be allowed, while compression stress could be carried after residual strain is overcome.

7- For reloading from tension zone the same unloading path is followed until reaching the previous level of maximum compressive strain, point M_1 . Henceforth, the concrete stress – strain relation is governed by the monotonic stress – strain curve.

4.6 Confined Concrete Cyclic Behavior

To represent the behavior of confined concrete under cyclic loading, the present study modified the hysteretic loops for cyclic loading in compression range by *Al-Sulayfani* (1989) as shown in figure (4.6). This behavior is illustrated through the same above points with some modification as following:

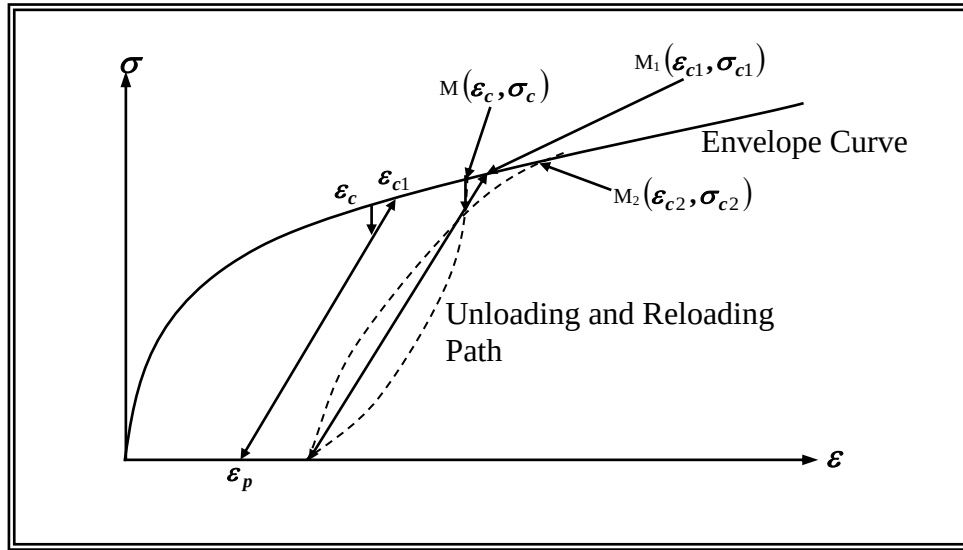


Figure (4.6): Adopted Normalized Stress-Strain Relation for Confined Concrete under Cyclic Loading

1. Assign the point where unloading occurs $M(\epsilon_c, \sigma_c)$.
2. Determine the value of the residual strain (plastic strain)(ϵ_p) for this unloading case depending on the unloading strain value (ϵ_c) which has the following equation:

$$\varepsilon_p = \left(0.055 * \left(\varepsilon_c / \varepsilon_{co} \right) + 0.127 * \left(\varepsilon_c / \varepsilon_{co} \right)^{3.1} \right) * \varepsilon_{co} \quad ; \quad \varepsilon_c \leq 2.2 * \varepsilon_{co} \quad (4.18a)$$

$$\varepsilon_p = \left(1.584 + \left(\left(\varepsilon_c / \varepsilon_{co} \right) - 2.2 \right) * 0.2675 \right) * \varepsilon_{co} \quad ; \quad \varepsilon_c > 2.2 * \varepsilon_{co} \quad (4.18b)$$

3. Where the hysteretic cycle completes after reloading, the strain path connects the envelope curve at point $M_2 (\varepsilon_{c2}, \sigma_{c2})$. The value of (ε_{c2}) could be determined from the relation:

$$\varepsilon_{c2} = 1.22 * \varepsilon_c \quad (4.19)$$

the mean value for the strain between (ε_c) and (ε_{c2}) is used:

$$\varepsilon_{c1} = 1.11 * \varepsilon_c \quad (4.20)$$

then point $M_1 (\varepsilon_{c1}, \sigma_{c1})$ was determined.

4. The unloading and reloading path is represented by the line connecting the two points $(\varepsilon_p, 0)$, $(\varepsilon_{c1}, \sigma_{c1})$.

5- If unloading continues for $\varepsilon < \varepsilon_p$ in compression, then the model gives tensile stress.

6- For tension strains the same linear path is followed for unloading and reloading as for monotonic tensile envelope until the concrete tensile strength is reached where the concrete begins to crack and the stress drops to zero; cracked layers will remain cracked and no tension stress will be allowed, while compression stress could be carried after residual strain is overcome.

7- For reloading from tension zone the same unloading path is followed until reaching the previous level of maximum compressive strain, point M_1 . Henceforth, the concrete stress – strain relation is governed by the monotonic stress – strain curve.

4.7 Stress-strain Behavior of Reinforcing Steel under Cyclic Loading

The hysteretic behavior model which was previously adopted by **Menegotto and Pinto** (cited by **Kadhim** (2007)), is used in this study. This model is shown in Figure (4.7). The stress-strain curves of all cycles lie within the two parallel lines A-B and A'-B' which are defined by the monotonic curve and passing through the yield points $(\epsilon_{so}, \sigma_{so})$ and $(-\epsilon_{so}, -\sigma_{so})$ respectively. All the curves, which represent the hysteretic behavior of steel, have the same initial slope equal to the slope (E_{so}) of the monotonic curve. The following equation was proposed by **Menegotto and Pinto** (cited by **Kadhim** (2007)) and adopted in the present study for each half stress-strain curve in normalized form:

$$\bar{\sigma} = (1 - b') * \frac{\bar{\epsilon}}{(1 + \bar{\epsilon}^R)^{1/R}} + b' * \bar{\epsilon} \quad (4.21)$$

For first loading, $(\bar{\epsilon})$ and $(\bar{\sigma})$ is calculated as shown below:

$$\bar{\sigma} = \left| \frac{\sigma_s}{\sigma_{so}} \right| \quad (4.22a)$$

$$\bar{\epsilon} = \left| \frac{\epsilon_s}{\epsilon_{so}} \right| \quad (4.22b)$$

For curves after the first load inversion:

$$\bar{\sigma} = \left| \frac{\sigma_s - \sigma_{in}}{\sigma_{kn} - \sigma_{in}} \right| \quad (4.23a)$$



(a) Hysteretic Behavior Model of Steel Reported by *Al-Sulayfani* (1989)
(b) Normalized Behavior Model of Steel Reported by *Al-Sulayfani* (1989)

(4.23b)

where the point $(\epsilon_{in}, \sigma_{in})$ is the start of the n th inversion. (ϵ_{kn}) and (σ_{kn}) are the coordinates of the intersection points which are shown in Figure (4.7). The coordinates of the intersection points depend on $\epsilon_{in}, \sigma_{in}, E_{so}$ values and yield point of reinforcing steel.

Equation (4.21) is asymptotic to one of the hardening lines A-B or A'-B' (with slope b'). The exponent R varies after every inversion, which affects the curvature of each half cycle to represent the Bauschinger effect. It's value is given by the following equation:

$$R(\xi_n) = R_o - \frac{A_1 * \xi_n}{A_2 + \xi_n} \quad (4.24)$$

$$\xi_n = \left| \bar{\epsilon}_i(n) - \bar{\epsilon}_k(n-1) \right| \quad (4.25)$$

The values of $(\bar{\epsilon}_i(n), \bar{\epsilon}_k(n-1))$ is determined from equation (4.22b) or equation (4.23b) by using the value of $\epsilon_{i(n)}$ and $\epsilon_{k(n-1)}$ respectively.

4.8 Stress-strain Behavior of FRP Fabric under Cyclic Loading

Since the CFRP fabrics are perfectly elastic in tension before they reach their ultimate strain, a simple linear relationship is used to model the longitudinal fabrics in Figure (4.8) (f_{yf}) and (ϵ_{uf}) represent the ultimate tensile stress and strain of the fabric, respectively. The compression part of the stress-strain curve is neglected since the fabrics can not take any compression forces.

To represent the hysteretic behavior of FRP fabric, all relations have the same slope (E_f) of the monotonic curve. The following equation is proposed to represent the stress-strain relationship under cyclic loading:

$$\sigma_f = E_f * \varepsilon_f \quad (4.26)$$

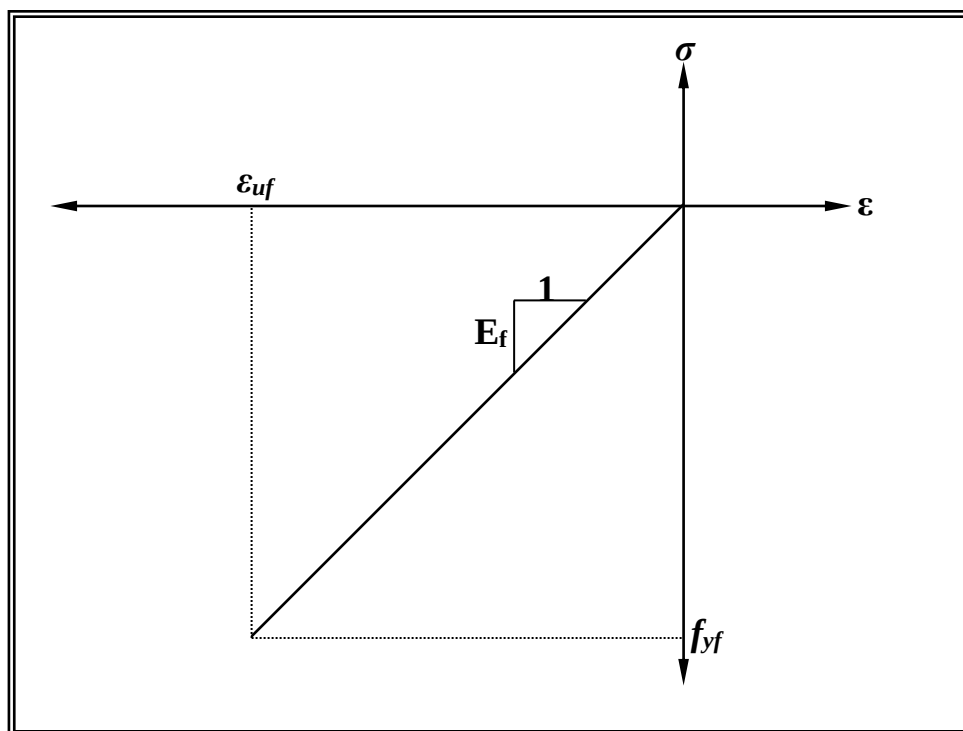


Figure (4.8): Stress-Strain Curve for Longitudinal Fabric Warping

Chapter Five

Finite Element Formulation

5.1 Formulation of Stiffness Matrix and Fixed End Forces

5.1.1 General

In this chapter, the stiffness matrix and fixed end forces are formulated based on finite element method with a developed form approach (namely region approach) *Kadhim* (2007). The derivation of a numerical form of stiffness matrix for prismatic or non – prismatic element takes into account; (a) the interaction between bending moments, shear and axial forces; (b) the possibility of existence of plastic zone any where in the element; (c) effect of FRP fabrics in longitudinal and transverse direction. In addition, for the case of element under uniformly distributed load, fixed end forces are derived considering the possibility of existence of plastic zone wherever it is formed.

5.1.2 Derivation of Stiffness Matrix for Tapered Element

Consider the non – prismatic element of length (L) shown in Figure (5.1). The element is rectangular in cross section. Six degrees of freedom are assumed at each node. The bending about the y and z axes is considered. All displacements, stresses and forces are positive if they are in the directions shown in Figure (5.2).

5.1.2.1 Basic Assumptions

Before the analytical model is presented, the following assumption should be considered:

1. The element is divided into the number of concrete, steel, and FRP regions along the depth and width as shown in Figure (5.1).

2. Reinforcing steel and longitudinal FRP are treated as equivalent uniaxial region materials placed at depth of the center line of the bar for steel and center line of FRP region of FRP fabrics.
3. If the concrete region in the section under consideration does not pass into inelastic range, the flexural and shear modulus of elasticity will be equal to the initial tangential modulus of elasticity for flexure and shear respectively. Otherwise flexure and shear modulus of elasticity will be calculated according to the flow rule which will be illustrated later in this chapter.
4. The flexural modulus of elasticity for steel or longitudinal FRP regions will equal the tangential modulus of elasticity at any range of loading for steel or FRP respectively (see items 4.7 and 4.8 in chapter four).
5. If transverse FRP fabrics are used then the concrete will be confined and the stress-strain curve for concrete will change to the model of confined concrete (for more detail see chapter four).
6. Plane sections before bending remain plane after bending.
7. Cracking of concrete is assumed to occur mainly in plane perpendicular to the axis of bending.
8. The stress in each concrete or steel or FRP fabric region is assumed to be uniform throughout the area of the region, and is found from the proposed behavior model (stress – strain curve).
9. Perfect bond between reinforcing steel and concrete and between FRP-fabric and concrete is assumed to occur.

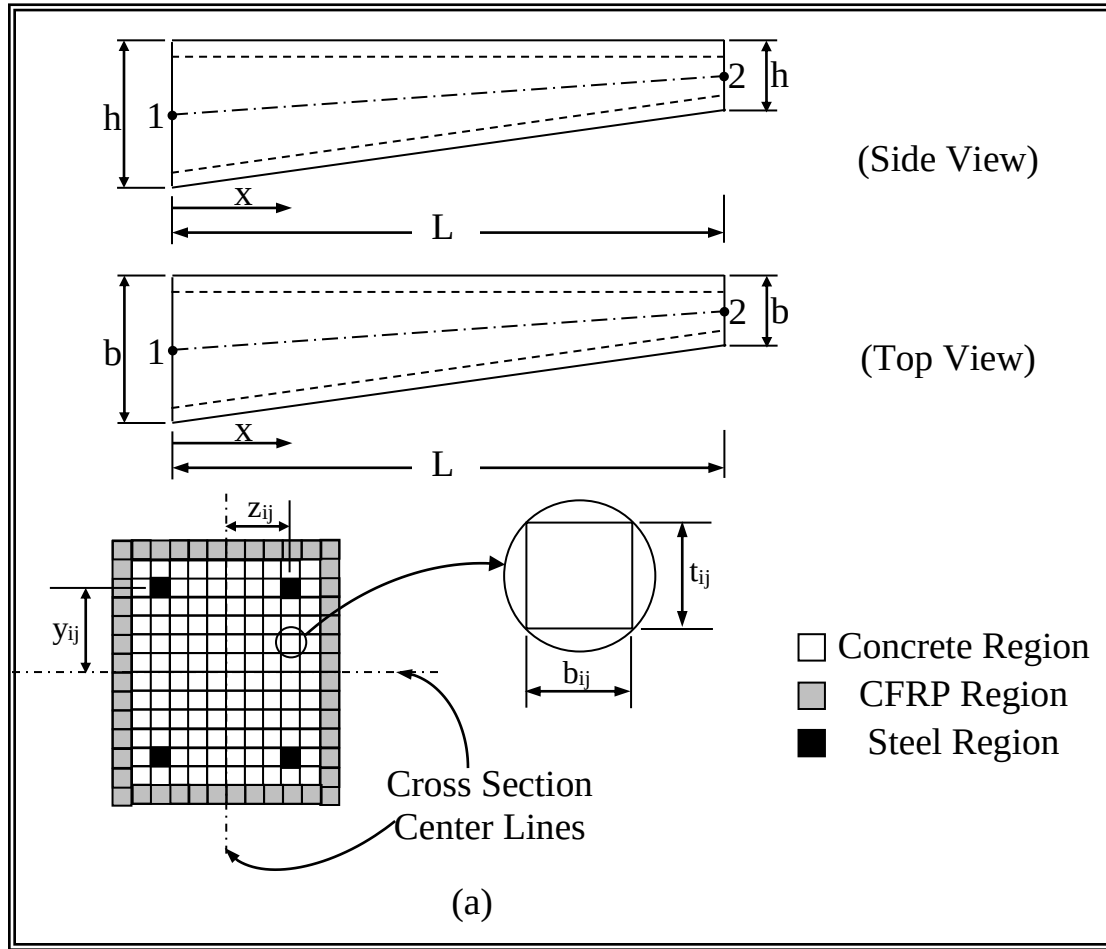


Figure (5.1): Non – Prismatic Space Frame Element
(a)Section Discrimination into Concrete, Steel, and FRP Regions

5.1.2.2 Assumed Displacement Field

The axial displacement (u) at any point (x, y, z) may be expressed directly in terms of $\theta_{y(x)}$ and $\theta_{z(x)}$ which are the rotations of the normal to the centroidal x – axis due to flexural effect, and $u_{(x)}$ which is the axial displacement due to axial effect.

$$u_{(x,y,z)} = u_{(x)} + z\theta_{y(x)} - y\theta_{z(x)} \quad (5.1)$$

where (y) is the distance from the centroidal axis of the cross section to the point (x,y,z) in y – direction, (z) is the distance from the centroidal axis of the cross section to the point (x,y,z) in z – direction. The normal rotation $\theta_{y(x)}$

is equal to the rotation β_{xz} which is due to the transverse shear deformation minus the slope of the centroidal axis after deformation. The normal rotation $\theta_{z(x)}$ is equal to the slope of the centroidal axis after deformation minus the rotation β_{xy} which is due to the transverse shear deformation.

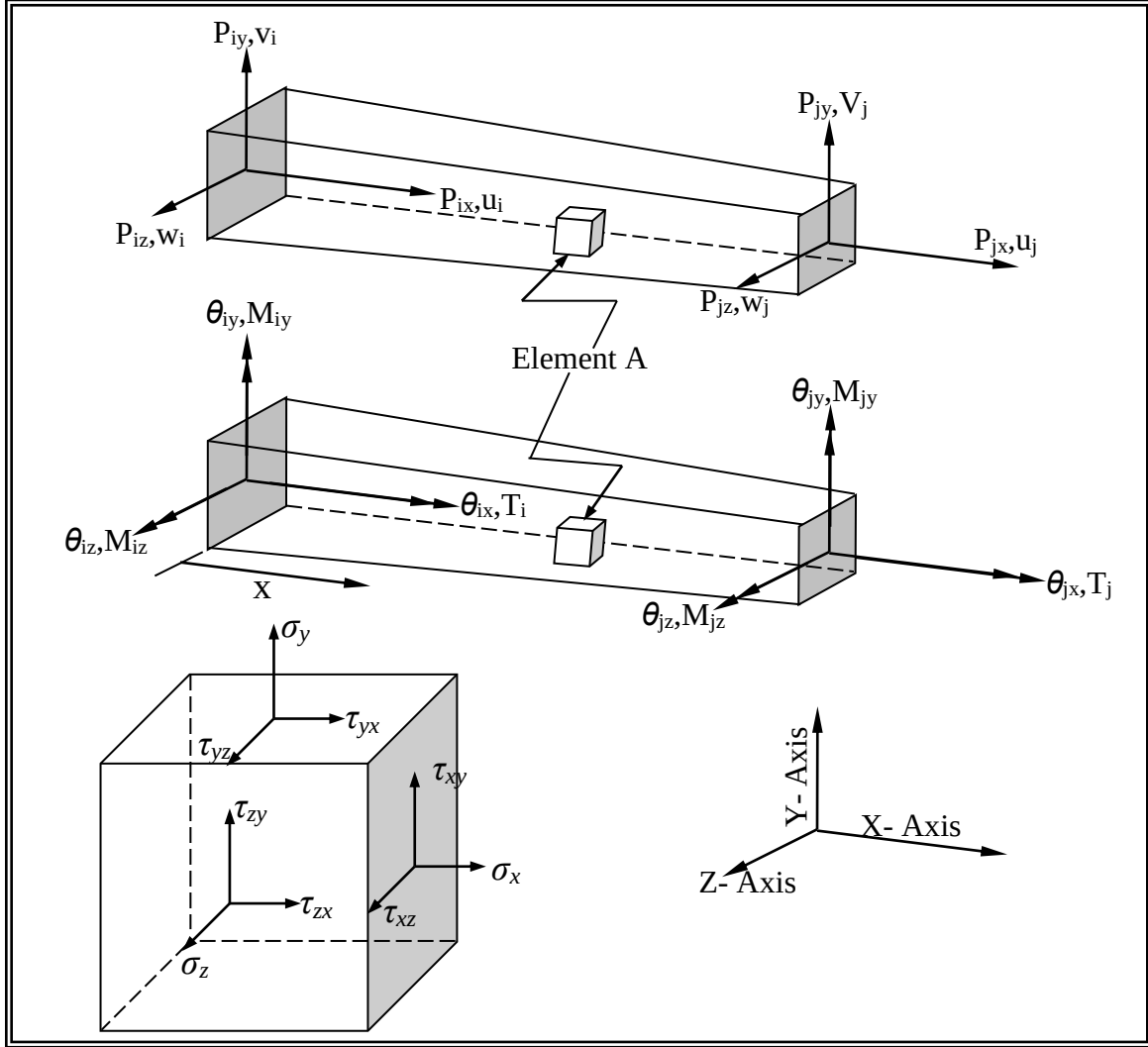


Figure (5.2): Degrees of freedom and sign convention

$$\theta_{y(x,y,z)} = -\frac{dw_{(x)}}{dx} + \beta_{xz} = \theta_{y(x)} \quad (5.2)$$

$$\theta_{z(x,y,z)} = \frac{dv_{(x)}}{dx} - \beta_{xy} = \theta_{z(x)} \quad (5.3)$$

$$\theta_{x(x,y,z)} = \theta_{x(x)} \quad (5.4)$$

$$w_{(x,y,z)} = w_{(x)} \quad (5.5)$$

$$v_{(x,y,z)} = v_{(x)} \quad (5.6)$$

where (β) is the rotation due to transverse shear deformation, $\theta_{x(x)}$ is the rotation about x – axis at any point (x,y,z), (w) is the lateral displacement in z - direction at any point (x,y,z) and equal to the lateral displacement at the central axis, (v) is the lateral displacement in y - direction at any point (x,y,z) and is equal to the lateral displacement at the central axis.

5.1.2.3 Concrete Stress – Strain Relationship

For an isotropic elastic material the relevant elastic stress – strain relationships are:

$$\tau_{cxy} = KG \cdot \gamma_{cxy} \quad (5.7)$$

$$\tau_{cxz} = KG \cdot \gamma_{cxz} \quad (5.8)$$

$$\tau_{cyz} = KG \cdot \gamma_{cyz} \quad (5.9)$$

$$\sigma_{cx} = \lambda \cdot e + 2G \epsilon_{cx} \quad (5.10)$$

$$\sigma_{cy} = \lambda \cdot e + 2G \epsilon_{cy} \quad (5.11)$$

$$\sigma_{cz} = \lambda \cdot e + 2G \epsilon_{cz} \quad (5.12)$$

where:

$$e = \epsilon_x + \epsilon_y + \epsilon_z \quad (5.13)$$

$$\lambda = \frac{\nu E_{co}}{(1 + \nu)(1 - 2\nu)} \quad (5.14)$$

ν : Concrete Poisson's ratio (0.11 – 0.27).

G is the shear modulus of elasticity and for an isotropic material within elastic range is equal to:

$$G = \frac{E_{co}}{2(1+\nu)} \quad (5.15)$$

The correction factor K which is used here because of the assumption that the distribution of transverse shear strain through the element thickness is uniform. This correction factor equal to 5/6 **Owen & Hinton** (1980).

In a matrix form:

$$\begin{bmatrix} \sigma_{cx} \\ \sigma_{cy} \\ \sigma_{cz} \\ \tau_{cxy} \\ \tau_{cxz} \\ \tau_{cyz} \end{bmatrix} = \begin{bmatrix} \lambda + 2G & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2G & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2G & 0 & 0 & 0 \\ 0 & 0 & 0 & KG & 0 & 0 \\ 0 & 0 & 0 & 0 & KG & 0 \\ 0 & 0 & 0 & 0 & 0 & KG \end{bmatrix} \begin{bmatrix} \varepsilon_{cx} \\ \varepsilon_{cy} \\ \varepsilon_{cz} \\ \gamma_{cxy} \\ \gamma_{cxz} \\ \gamma_{cyz} \end{bmatrix} \quad (5.16)$$

If σ_{cy} and σ_{cz} are assumed to be equal to zero then,

$$\varepsilon_{cy} = -\nu \cdot \varepsilon_{cx} \quad (5.17)$$

$$\varepsilon_{cz} = -\nu \cdot \varepsilon_{cx} \quad (5.18)$$

And by eliminating (ε_{cy}) and (ε_{cz}) from equation (5.16), it is possible to write the following elastic stress – strain relationship:

$$\begin{bmatrix} \sigma_{cx} \\ \tau_{cxy} \\ \tau_{cxz} \\ \tau_{cyz} \end{bmatrix} = [D_{con}] \begin{bmatrix} \varepsilon_{cx} \\ \gamma_{cxy} \\ \gamma_{cxz} \\ \gamma_{cyz} \end{bmatrix} \quad (5.19)$$

where $[D_{con}]$ is the concrete stress – strain relationship matrix which takes the following form within elastic range:

$$[D_{con}] = \begin{bmatrix} E_{co} & 0 & 0 & 0 \\ 0 & KG_{xy} & 0 & 0 \\ 0 & 0 & KG_{xz} & 0 \\ 0 & 0 & 0 & KG_{yz} \end{bmatrix} \quad (5.20)$$

For the plastic range, $[D_{con}]$ will be calculated according to the flow rule which will be illustrated later in this chapter.

5.1.2.4 Steel Stress – Strain relationships

The steel region deforms only in the direction of the bar and having strength and stiffness characteristics in the bar direction only. The dowel action of the reinforcing steel is neglected. The elastic stress – strain relationship can be written as:

$$\begin{bmatrix} \sigma_{sx} \\ \tau_{sxy} \\ \tau_{sxz} \\ \tau_{syx} \end{bmatrix} = [D_s] \begin{bmatrix} \epsilon_{sx} \\ \gamma_{sxy} \\ \gamma_{sxz} \\ \gamma_{syx} \end{bmatrix} \quad (5.21)$$

where $[D_s]$ is the steel stress – strain relationship matrix which takes the following form:

$$[D_s] = \begin{bmatrix} E_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (5.22)$$

E_s equal to the tangential modulus of elasticity which is calculated from stress – strain model for steel.

5.1.2.5 FRP Stress – Strain relationships

As explained in chapter three, the effect of FRP is included in two ways; longitudinal FRP used for flexural members, and transverse FRP for confined concrete. Herein the FRP is longitudinal and region deforms only in the direction of the member and having strength and stiffness characteristics in the member direction only because of the unidirectional characteristics of FRP. The elastic stress – strain relationship can be written as:

$$\begin{bmatrix} \sigma_{fx} \\ \tau_{fxy} \\ \tau_{fxz} \\ \tau_{fyz} \end{bmatrix} = [D_f] \begin{bmatrix} \varepsilon_{fx} \\ \gamma_{fxy} \\ \gamma_{fxz} \\ \gamma_{fyz} \end{bmatrix} \quad (5.23)$$

where $[D_f]$ is the FRP stress – strain relationship matrix which takes the following form:

$$[D_f] = \begin{bmatrix} E_f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (5.24)$$

E_f equal to the tangential modulus of elasticity which is calculated from stress – strain model for FRP.

The effect of transverse FRP is included by changing the normal stress-strain relationship of concrete to confined stress-strain relationship for concrete (see chapter three).

5.1.2.6 Strain – Displacement Relationships

According to equation (5.1) the axial strain in the x – direction is given as:

$$\varepsilon_x = \frac{\partial u_{(x,y,z)}}{\partial x} = \frac{du_{(x)}}{dx} - y \frac{d\theta_{z(x)}}{dx} + z \frac{d\theta_{y(x)}}{dx} \quad (5.25)$$

Similarly the shear strain (γ_{xy}) is given as:

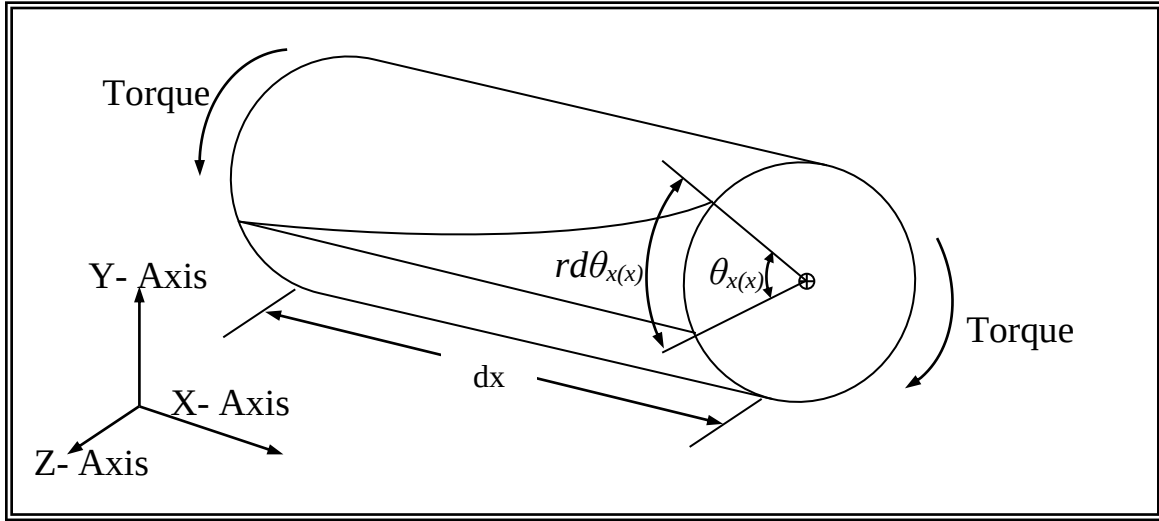


Figure (5.3): Deformation Shape of Circular Shaft by Torsion

$$\gamma_{xy} = \frac{\partial u_{(x,y,z)}}{\partial y} + \frac{\partial v_{(x,y,z)}}{\partial x} \quad (5.26)$$

and according to equation (5.3)

$$\gamma_{xy} = -\theta_{z(x)} + \frac{dv_{(x,y,z)}}{dx} = \beta_{xy} \quad (5.27)$$

The shear strain γ_{xz} is given as:

$$\gamma_{xz} = \frac{\partial u_{(x,y,z)}}{\partial z} + \frac{\partial w_{(x,y,z)}}{\partial x} \quad (5.28)$$

in the same manner, according to equation (5.2)

$$\gamma_{xz} = \theta_{y(x)} + \frac{dw_{(x,y,z)}}{dx} = \beta_{xz} \quad (5.29)$$

Also, the shear strain γ_{yz} is given as (see Figure (5.3)):

$$\gamma_{yz} = r \frac{d\theta_{x(x)}}{dx} = \sqrt{y^2 + z^2} \frac{d\theta_{x(x)}}{dx} \quad (5.30)$$

5.1.2.7 Finite Element Representation of Displacements and Strains

In the frame element representation, the lateral displacement $w_{(x)}$ and the rotation $\theta_{y(x)}$ are represented by **Muhaisin** (2003) as:

$$w_{(x)} = \bar{N}_1 w_1 + \bar{N}_2 \theta_{1y} + \bar{N}_3 w_2 + \bar{N}_4 \theta_{2y} \quad (5.31)$$

$$\theta_{y(x)} = \bar{N}_1 w_1 + \bar{N}_2 \theta_{1y} + \bar{N}_3 w_2 + \bar{N}_4 \theta_{2y} \quad (5.32)$$

where w_1 , θ_{1y} and w_2 , θ_{2y} are the nodal displacements at local nodes 1 and 2 of element respectively.

$\bar{N}_1$ to $\bar{N}_4$ and $\bar{N}_1$ to $\bar{N}_4$ are shape functions.

Also the lateral displacement $w_{(x)}$ in the element can be represented by cubic polynomial as:

$$w_{(x)} = a_1 + a_2 x + a_3 x^2 + a_4 x^3 \quad (5.33)$$

From equation (5.29), the relation between the transverse shear strain γ_{xz} ,

$\frac{dw_{(x)}}{dx}$ and $\theta_{y(x)}$ is :

$$\frac{dw_{(x)}}{dx} = \gamma_{xz} - \theta_{y(x)} \quad (5.34)$$

The transverse shear strain γ_{xz} is consistent with the cubic polynomial of $w_{(x)}$, and is assumed to be independent of x , i.e.

$$\gamma_{xz} = b_0 \quad (5.35)$$

Then, moment equilibrium of infinitesimal beam section yields:

$$\frac{dM_y}{dx} - p_z = 0 \quad (5.36)$$

The introduction of equilibrium equations as equation (5.36) in the formulation is formally equivalent to constrain to facilitate inclusion of transverse shear deformation in displacement formulations without involving additional grid point degrees of freedom.

The bending moment – curvature relation is:

$$M_y = -EI_y \frac{d\theta_{y(x)}}{dx} \quad (5.37)$$

The shearing force (p_z) is related to transverse shear strain γ_{xz} by

$$p_z = KG_{xz} A \gamma_{xz} \quad (5.38)$$

where A is the cross sectional area, K is the shear correction factor. Solving of equations (5.33) to (5.38) gives

$$b_o = 6.a_4.g \quad (5.39)$$

where

$$g = \frac{E I_y}{K G_{xz} A} \quad (5.40)$$

Using of equations (5.33) to (5.35) and (5.39) allows $\theta_{y(x)}$ to be expressed as a polynomial in (x) as follows:

$$\theta_{y(x)} = -a_2 - 2a_3x + (6g - 3x^2)a_4 \quad (5.41)$$

Using equations (5.33) to (5.41), it is now possible to express the coefficients a_1 through a_4 in terms of the displacement $w_{(x)}$ and rotation $\theta_{y(x)}$ of the element at the ends $x = 0$ (w_1, θ_{1y}) and $x = L$ (w_2, θ_{2y}).

The results are

$$a_1 = w_1 \quad (5.42)$$

$$a_2 = F \left[-12gw_1 + 12gw_2 - (6gL - L^3)\theta_{1y} + 6gL\theta_{2y} \right] \quad (5.43)$$

$$a_3 = F \left[3w_1L - 3w_2L + (6g - 2L^2)\theta_{1y} - (6g + 2L^2)\theta_{2y} \right] \quad (5.44)$$

$$a_4 = F \left[-2w_1 + 2w_2 + \theta_{1y}L + \theta_{2y}L \right] \quad (5.45)$$

where

$$F = \frac{1}{L(12g - L^2)} \quad (5.46)$$

Substitution of these coefficients into equation (5.33) yields

$$\begin{aligned} w_{(x)} = & \left[1 - 12gF.x + 3LF.x^2 - 2F.x^3 \right] w_1 + \left[\left(L^3 - 6gL \right) F.x + \left(6g - 2L^2 \right) F.x^2 + LF.x^3 \right] \theta_{1y} \\ & + \left[12gF.x - 3LF.x^2 + 2F.x^3 \right] w_2 - \left[6gLF.x - \left(6g + 2L^2 \right) F.x^2 + LF.x^3 \right] \theta_{2y} \end{aligned} \quad (5.47)$$

by comparison between equation (5.47) and (5.31) one can find that

$$\bar{N}_1 = 1 - F[12gx - 3Lx^2 + 2x^3] \quad (5.48)$$

$$\bar{N}_2 = F[-(6gL - L^3)x + (6g - 2L^2)x^2 + Lx^3] \quad (5.49)$$

$$\bar{N}_3 = F[12gx - 3Lx^2 + 2x^3] \quad (5.50)$$

$$\bar{N}_4 = F[6gLx - (6g + L^2)x^2 + Lx^3] \quad (5.51)$$

and similarly by substituting the coefficients a_2 to a_5 into equation (5.41) and comparing with equation (5.32) yields:

$$\bar{\bar{N}}_1 = F[6x^2 - 6xL] \quad (5.52)$$

$$\bar{\bar{N}}_2 = F[12gL - L^3 - (12g - 4L^2)x - 3x^2L] \quad (5.53)$$

$$\bar{\bar{N}}_3 = F[6xL - 6x^2] \quad (5.54)$$

$$\bar{\bar{N}}_4 = F[-3x^2L + (12g + 2L^2)x] \quad (5.55)$$

In the same way:

$$\mathbf{v}_{(x)} = \bar{\bar{N}}_1 \mathbf{v}_1 + \bar{\bar{N}}_2 \theta_{1z} + \bar{\bar{N}}_3 \mathbf{v}_2 + \bar{\bar{N}}_4 \theta_{2z} \quad (5.56)$$

$$\theta_{z(x)} = \bar{\bar{N}}_1 \mathbf{v}_1 + \bar{\bar{N}}_2 \theta_{1z} + \bar{\bar{N}}_3 \mathbf{v}_2 + \bar{\bar{N}}_4 \theta_{2z} \quad (5.57)$$

where:

$$\bar{\bar{N}}_1 = 1 - \bar{F}[12\bar{g}x + 3Lx^2 - 2x^3] \quad (5.58)$$

$$\bar{\bar{N}}_2 = \bar{F}[(6\bar{g}L + L^3)x - (6\bar{g} + 2L^2)x^2 + Lx^3] \quad (5.59)$$

$$\bar{\bar{N}}_3 = \bar{F}[12\bar{g}x + 3Lx^2 - 2x^3] \quad (5.60)$$

$$\bar{\bar{N}}_4 = \bar{F}[-6\bar{g}Lx + (6\bar{g} - L^2)x^2 + Lx^3] \quad (5.61)$$

$$\bar{\bar{N}}_1 = \bar{F}[6x^2 - 6xL] \quad (5.62)$$

$$\bar{\bar{N}}_2 = \bar{F}[L^3 + 12\bar{g}L - (12\bar{g} + 4L^2)x + 3x^2L] \quad (5.63)$$

$$\bar{\bar{N}}_3 = \bar{F}[6xL - 6x^2] \quad (5.64)$$

$$\bar{\bar{\bar{N}}}_4 = \bar{F} [3x^2 L - (2L^2 - 12\bar{g})x] \quad (5.65)$$

and
$$\bar{F} = \frac{1}{L(12\bar{g} + L^2)}, \quad (5.66)$$

$$\bar{g} = \frac{E I_z}{K G_{xy} A} \quad (5.67)$$

The other displacements are represented as:

$$u_{(x)} = N_1 u_1 + N_2 u_2 \quad (5.68)$$

$$\theta_{x(x)} = N_1 \theta_{1x} + N_2 \theta_{2x} \quad (5.69)$$

where:

$$N_1 = \frac{L-x}{L} = 1-m, \quad \frac{dN_1}{dx} = -\frac{1}{L} \quad (5.70)$$

$$N_2 = \frac{x}{L} = m, \quad \frac{dN_2}{dx} = \frac{1}{L} \quad (5.71)$$

The axial strain – displacement relationship can be expressed as:

$$\varepsilon_x = \frac{du_{(x)}}{dx} - y \frac{d\theta_{z(x)}}{dx} + z \frac{d\theta_{y(x)}}{dx} \quad (5.72a)$$

$$\begin{aligned} \varepsilon_x = & \left[u_1 \frac{dN_1}{dx} + u_2 \frac{dN_2}{dx} \right] - y \left[v_1 \frac{d\bar{\bar{\bar{N}}}_1}{dx} + \theta_{1z} \frac{d\bar{\bar{\bar{N}}}_2}{dx} + v_2 \frac{d\bar{\bar{\bar{N}}}_3}{dx} + \theta_{2z} \frac{d\bar{\bar{\bar{N}}}_4}{dx} \right] \\ & + z \left[w_1 \frac{d\bar{\bar{\bar{N}}}_1}{dx} + \theta_{1y} \frac{d\bar{\bar{\bar{N}}}_2}{dx} + w_2 \frac{d\bar{\bar{\bar{N}}}_3}{dx} + \theta_{2y} \frac{d\bar{\bar{\bar{N}}}_4}{dx} \right] \end{aligned} \quad (5.72b)$$

$$\varepsilon_x = \begin{bmatrix} -1/L \\ -K_1 \\ K_2 \\ 0 \\ K_3 \\ -K_4 \\ 1/L \\ -K_5 \\ K_6 \\ 0 \\ K_7 \\ -K_8 \end{bmatrix}^T \cdot \begin{bmatrix} u_1 \\ v_1 \\ w_1 \\ \theta_{1x} \\ \theta_{1y} \\ \theta_{1z} \\ u_2 \\ v_2 \\ w_2 \\ \theta_{2x} \\ \theta_{2y} \\ \theta_{2z} \end{bmatrix} \quad (5.72c)$$

$$\varepsilon_x = B_A(x, y, z) \psi \quad (5.72d)$$

where:

$$K_1 = y\bar{F}(12x - 6L) \quad (5.73)$$

$$K_2 = zF(12x - 6L) \quad (5.74)$$

$$K_3 = zF(-12g + 4L^2 - 6xL) \quad (5.75)$$

$$K_4 = y\bar{F}(-12\bar{g} - 4L^2 + 6xL) \quad (5.76)$$

$$K_5 = y\bar{F}(6L - 12x) = -K_1 \quad (5.77)$$

$$K_6 = zF(6L - 12x) = -K_2 \quad (5.78)$$

$$K_7 = zF(-6xL + 12g + 2L^2) \quad (5.79)$$

$$K_8 = y\bar{F}(6xL - 2L^2 - 12\bar{g}) \quad (5.80)$$

$B_A(x, y, z)$ is the axial strain – displacement matrix.

The shear strain – displacement relationship can be expressed as:

$$\gamma_{xy} = \frac{dv_{(x)}}{dx} - \theta_{z(x)} \quad (5.81a)$$

$$\gamma_{xy} = \left[v_1 \frac{d\bar{\bar{N}}_1}{dx} + \theta_{1z} \frac{d\bar{\bar{N}}_2}{dx} + v_2 \frac{d\bar{\bar{N}}_3}{dx} + \theta_{2z} \frac{d\bar{\bar{N}}_4}{dx} \right] - \left[v_1 \bar{\bar{N}}_1 + \theta_{1z} \bar{\bar{N}}_2 + v_2 \bar{\bar{N}}_3 + \theta_{2z} \bar{\bar{N}}_4 \right] \quad (5.81b)$$

$$\gamma_{xy} = \begin{bmatrix} 0 \\ K_9 \\ 0 \\ 0 \\ 0 \\ K_{10} \\ 0 \\ -K_9 \\ 0 \\ 0 \\ 0 \\ K_{10} \end{bmatrix}^T \cdot \begin{bmatrix} u_1 \\ v_1 \\ w_1 \\ \theta_{1x} \\ \theta_{1y} \\ \theta_{1z} \\ u_2 \\ v_2 \\ w_2 \\ \theta_{2x} \\ \theta_{2y} \\ \theta_{2z} \end{bmatrix} \quad (5.81c)$$

$$\gamma_{xy} = \mathbf{B}_{x,y} \boldsymbol{\psi} \quad (5.81d)$$

where:

$$K_9 = -12\bar{F}\bar{g} \quad (5.82)$$

$$K_{10} = -6\bar{F}\bar{g}L \quad (5.83)$$

$\mathbf{B}_{x,y}$ is the shear strain (γ_{xy}) – displacement matrix.

Also,

$$\gamma_{xz} = \frac{dw_{(x)}}{dx} - \theta_{y(x)} \quad (5.84a)$$

$$\gamma_{xz} = \left[w_1 \frac{d\bar{N}_1}{dx} + \theta_{1y} \frac{d\bar{N}_2}{dx} + w_2 \frac{d\bar{N}_3}{dx} + \theta_{2y} \frac{d\bar{N}_4}{dx} \right] - \left[w_1 \bar{N}_1 + \theta_{1y} \bar{N}_2 + w_2 \bar{N}_3 + \theta_{2y} \bar{N}_4 \right] \quad (5.84b)$$

$$\gamma_{xz} = \begin{bmatrix} 0 \\ 0 \\ K_{11} \\ 0 \\ K_{12} \\ 0 \\ 0 \\ 0 \\ -K_{11} \\ 0 \\ K_{12} \\ 0 \end{bmatrix}^T \cdot \begin{bmatrix} u_1 \\ v_1 \\ w_1 \\ \theta_{1x} \\ \theta_{1y} \\ \theta_{1z} \\ u_2 \\ v_2 \\ w_2 \\ \theta_{2x} \\ \theta_{2y} \\ \theta_{2z} \end{bmatrix} \quad (5.84c)$$

$$\gamma_{xz} = \mathbf{B}_{x,z} \boldsymbol{\psi} \quad (5.84d)$$

where:

$$K_{11} = -12Fg \quad (5.85)$$

$$K_{12} = 6FgL \quad (5.86)$$

$\mathbf{B}_{x,z}$ is the shear strain (γ_{xz})– displacement matrix.

$$\begin{aligned} \gamma_{yz} &= r \frac{d\theta_{x(x)}}{dx} \\ &= \sqrt{y^2 + z^2} \frac{d\theta_{x(x)}}{dx} \\ &= \left(\sqrt{y^2 + z^2} \theta_{x1} \frac{dN_1}{dx} \right) + \left(\sqrt{y^2 + z^2} \theta_{x2} \frac{dN_2}{dx} \right) \\ &= \left(\frac{-\sqrt{y^2 + z^2}}{L} * \theta_{x1} \right) + \left(\frac{\sqrt{y^2 + z^2}}{L} * \theta_{x2} \right) \end{aligned} \quad (5.87a)$$

$$\gamma_{yz} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\sqrt{y^2 + z^2} / L \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\sqrt{y^2 + z^2} / L \\ 0 \\ 0 \end{bmatrix}^T \cdot \begin{bmatrix} u_1 \\ v_1 \\ w_1 \\ \theta_{1x} \\ \theta_{1y} \\ \theta_{1z} \\ u_2 \\ v_2 \\ w_2 \\ \theta_{2x} \\ \theta_{2y} \\ \theta_{2z} \end{bmatrix} \quad (5.87b)$$

$$\gamma_{yz} = \mathbf{B}_{y,z} \boldsymbol{\psi} \quad (5.87c)$$

where $\mathbf{B}_{y,z}$ is the shear strain (γ_{yz})– displacement matrix.

5.1.2.8 Stiffness Matrix Evaluation

According to first assumption of section (5.1.2.1), each element will be divided into two zones in x – direction and into a chosen number of steel, concrete, and FRP regions in y and z – directions as shown in Figure (5.1). The element is subjected to external applied forces $[\mathbf{f}]$. If the element undergoes a set of virtual displacement $[\boldsymbol{\varsigma} \boldsymbol{\psi}]$ which has the form:

$$[\boldsymbol{\varsigma} \boldsymbol{\psi}] = [\boldsymbol{\varsigma} u_1 \quad \boldsymbol{\varsigma} v_1 \quad \boldsymbol{\varsigma} w_1 \quad \boldsymbol{\varsigma} \theta_{1x} \quad \boldsymbol{\varsigma} \theta_{1y} \quad \boldsymbol{\varsigma} \theta_{1z} \quad \boldsymbol{\varsigma} u_2 \quad \boldsymbol{\varsigma} v_2 \quad \boldsymbol{\varsigma} w_2 \quad \boldsymbol{\varsigma} \theta_{2x} \quad \boldsymbol{\varsigma} \theta_{2y} \quad \boldsymbol{\varsigma} \theta_{2z}] \quad (5.88)$$

then the virtual work equation can be written as:

$$\begin{aligned}
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left(\mathbf{b}_{ij} \mathbf{t}_{ij} \varepsilon_{cx(ij)} \sigma_{cx(ij)} \right) d\mathbf{x} + \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left(\mathbf{b}_{ij} \mathbf{t}_{ij} \gamma_{cxy(ij)} \tau_{cxy(ij)} \right) d\mathbf{x} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left(\mathbf{b}_{ij} \mathbf{t}_{ij} \gamma_{cxz(ij)} \tau_{cxz(ij)} \right) d\mathbf{x} + \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left(\mathbf{b}_{ij} \mathbf{t}_{ij} \gamma_{cyz(ij)} \tau_{cyz(ij)} \right) d\mathbf{x} + \\
 & \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} \left(\varepsilon_{sx(ij)} \sigma_{sx(ij)} \mathbf{A}_{s(ij)} \right) d\mathbf{x} + \int_0^L \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} \left(\mathbf{b}_{fij} \mathbf{t}_{fij} \varepsilon_{fx(ij)} \sigma_{fx(ij)} \right) d\mathbf{x} = [\mathbf{f}]^T [\varsigma \psi]
 \end{aligned} \tag{5.89}$$

where N_{1c} and N_{2c} are equal to the total number of concrete regions in width and depth respectively, N_{1s} and N_{2s} are equal to the total number of steel regions in width and depth respectively, N_{1f} and N_{2f} are equal to the total number of FRP regions in width and depth respectively. According to equation (5.72) and (5.87), equation (5.89) can be rewritten as:

$$\begin{aligned}
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cA}]^T [\varsigma \psi] \sigma_{cx(ij)} \right] d\mathbf{x} + \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cxy}]^T [\varsigma \psi] \tau_{cxy(ij)} \right] d\mathbf{x} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cxz}]^T [\varsigma \psi] \tau_{cxz(ij)} \right] d\mathbf{x} + \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cyz}]^T [\varsigma \psi] \tau_{cyz(ij)} \right] d\mathbf{x} + \\
 & \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} \left[[\mathbf{B}_{sA}]^T [\varsigma \psi] \sigma_{sx(ij)} \mathbf{A}_{s(ij)} \right] d\mathbf{x} + \int_0^L \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} \left[\mathbf{b}_{fij} \mathbf{t}_{fij} [\mathbf{B}_{fA}]^T [\varsigma \psi] \sigma_{fx(ij)} \right] d\mathbf{x} = [\mathbf{f}]^T [\varsigma \psi]
 \end{aligned} \tag{5.90}$$

By utilizing equations (5.19), (5.21) and (5.23); equation (5.90) becomes:

$$\begin{aligned}
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cA}]^T [\varsigma \psi] \mathbf{E}_{c(ij)} \varepsilon_{cx(ij)} \right] d\mathbf{x} + \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cxy}]^T [\varsigma \psi] \mathbf{G}_{xy(ij)} \gamma_{cxy(ij)} \right] d\mathbf{x} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cxz}]^T [\varsigma \psi] \mathbf{G}_{xz(ij)} \gamma_{cxz(ij)} \right] d\mathbf{x} + \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cyz}]^T [\varsigma \psi] \mathbf{G}_{yz(ij)} \gamma_{cyz(ij)} \right] d\mathbf{x} + \\
 & \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} \left[[\mathbf{B}_{sA}]^T [\varsigma \psi] \mathbf{E}_{s(ij)} \varepsilon_{sx(ij)} \mathbf{A}_{s(ij)} \right] d\mathbf{x} + \int_0^L \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} \left[\mathbf{b}_{fij} \mathbf{t}_{fij} [\mathbf{B}_{fA}]^T [\varsigma \psi] \mathbf{E}_{f(ij)} \varepsilon_{fx(ij)} \right] d\mathbf{x} \\
 & = [\mathbf{f}]^T [\varsigma \psi]
 \end{aligned} \tag{5.91}$$

then according to equation (5.72) and (5.87) the results of equation (5.91) can be rewritten as:

$$\begin{aligned}
& \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cA}]^T [\boldsymbol{\varsigma} \boldsymbol{\psi}] E_{c(ij)} \varepsilon_{cx(ij)} [\mathbf{B}_{cA}] [\boldsymbol{\psi}]^T \right] d\mathbf{x} + \\
& \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cxy}]^T [\boldsymbol{\varsigma} \boldsymbol{\psi}] G_{xy(ij)} \gamma_{cxy(ij)} [\mathbf{B}_{xy}] [\boldsymbol{\psi}]^T \right] d\mathbf{x} + \\
& \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cxz}]^T [\boldsymbol{\varsigma} \boldsymbol{\psi}] G_{xz(ij)} \gamma_{cxz(ij)} [\mathbf{B}_{xz}] [\boldsymbol{\psi}]^T \right] d\mathbf{x} + \\
& \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cyz}]^T [\boldsymbol{\varsigma} \boldsymbol{\psi}] G_{yz(ij)} \gamma_{cyz(ij)} [\mathbf{B}_{yz}] [\boldsymbol{\psi}]^T \right] d\mathbf{x} + \\
& \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} \left[[\mathbf{B}_{sA}]^T [\boldsymbol{\varsigma} \boldsymbol{\psi}] E_{s(ij)} \varepsilon_{sx(ij)} \mathbf{A}_{s(ij)} [\mathbf{B}_{sA}] [\boldsymbol{\psi}]^T \right] d\mathbf{x} + \\
& \int_0^L \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} \left[\mathbf{b}_{fij} \mathbf{t}_{fij} [\mathbf{B}_{fA}]^T [\boldsymbol{\varsigma} \boldsymbol{\psi}] E_{f(ij)} \varepsilon_{fx(ij)} [\mathbf{B}_{fA}] [\boldsymbol{\psi}]^T \right] d\mathbf{x} = [\mathbf{f}]^T [\boldsymbol{\varsigma} \boldsymbol{\psi}] \quad (5.92)
\end{aligned}$$

By differentiating equation (5.92) with respect to $[\boldsymbol{\varsigma} \boldsymbol{\psi}]$ then:

$$\begin{aligned}
& \left[\int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[E_{c(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cA}]^T [\mathbf{B}_{cA}] \right] d\mathbf{x} + \right. \\
& \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\frac{G_{xy(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij}}{\alpha} [\mathbf{B}_{cxy}]^T [\mathbf{B}_{xy}] \right] d\mathbf{x} + \\
& \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\frac{G_{xz(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij}}{\alpha} [\mathbf{B}_{cxz}]^T [\mathbf{B}_{xz}] \right] d\mathbf{x} + \\
& \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\frac{G_{yz(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij}}{\alpha} [\mathbf{B}_{cyz}]^T [\mathbf{B}_{yz}] \right] d\mathbf{x} + \\
& \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} \left[E_{s(ij)} \mathbf{A}_{s(ij)} [\mathbf{B}_{sA}]^T [\mathbf{B}_{sA}] \right] d\mathbf{x} + \\
& \left. \int_0^L \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} \left[E_{f(ij)} \mathbf{b}_{fij} \mathbf{t}_{fij} [\mathbf{B}_{fA}]^T [\mathbf{B}_{fA}] \right] d\mathbf{x} \right] [\boldsymbol{\psi}]^T = [\mathbf{f}]^T \quad (5.93)
\end{aligned}$$

where α is a correction factor to allow for cross – sectional warping. For rectangular section α is taken as (1.5) **Owen and Hinton** (1980).

Equation (5.93) can briefly be written as:

$$[K_{ca} + K_{cxy} + K_{cxz} + K_{cyz} + K_{sa} + K_{fa}][\psi] = [f] \quad (5.94)$$

where:

$$K_{ca} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} \begin{bmatrix} -1/L \\ -K_1 \\ K_2 \\ 0 \\ K_3 \\ -K_4 \\ 1/L \\ -K_5 \\ K_6 \\ 0 \\ K_7 \\ -K_8 \end{bmatrix} \begin{bmatrix} -1/L & -K_1 & K_2 & 0 & K_3 & -K_4 & 1/L & -K_5 & K_6 & 0 & K_7 & -K_8 \end{bmatrix} dx \quad (5.95)$$

$$K_{aa} = \int_0^L \sum_{i=1}^{N_{1a}} \sum_{j=1}^{N_{2a}} E_{c(ij)} b_{ij} t_{ij} \begin{bmatrix} \frac{1}{L^2} & \frac{K_1}{L} & -\frac{K_2}{L} & 0 & -\frac{K_3}{L} & \frac{K_4}{L} & -\frac{1}{L^2} & \frac{K_5}{L} & -\frac{K_6}{L} & 0 & -\frac{K_7}{L} & \frac{K_8}{L} \\ \frac{K_1}{L} & K_1^2 & -K_1 K_2 & 0 & -K_1 K_3 & K_1 K_4 & -\frac{K_1}{L} & K_1 K_5 & -K_1 K_6 & 0 & -K_1 K_7 & K_1 K_8 \\ -\frac{K_2}{L} & -K_1 K_2 & K_2^2 & 0 & K_2 K_3 & -K_2 K_4 & \frac{K_2}{L} & -K_2 K_5 & K_2 K_6 & 0 & K_2 K_7 & -K_2 K_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_3}{L} & -K_1 K_3 & K_2 K_3 & 0 & K_3^2 & -K_3 K_4 & \frac{K_3}{L} & -K_3 K_5 & K_3 K_6 & 0 & K_3 K_7 & -K_3 K_8 \\ \frac{K_4}{L} & K_1 K_4 & -K_2 K_4 & 0 & -K_3 K_4 & K_4^2 & -\frac{K_4}{L} & K_4 K_5 & -K_4 K_6 & 0 & -K_4 K_7 & K_4 K_8 \\ -\frac{1}{L^2} & -\frac{K_1}{L} & \frac{K_2}{L} & 0 & \frac{K_3}{L} & -\frac{K_4}{L} & \frac{1}{L^2} & -\frac{K_5}{L} & \frac{K_6}{L} & 0 & \frac{K_7}{L} & -\frac{K_8}{L} \\ \frac{K_5}{L} & K_1 K_5 & -K_2 K_5 & 0 & -K_3 K_5 & K_4 K_5 & -\frac{K_5}{L} & K_5^2 & -K_5 K_6 & 0 & -K_5 K_7 & K_5 K_8 \\ -\frac{K_6}{L} & -K_1 K_6 & K_2 K_6 & 0 & K_3 K_6 & -K_4 K_6 & \frac{K_6}{L} & -K_5 K_6 & K_6^2 & 0 & K_6 K_7 & -K_6 K_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_7}{L} & -K_1 K_7 & K_2 K_7 & 0 & K_3 K_7 & -K_4 K_7 & \frac{K_7}{L} & -K_5 K_7 & K_6 K_7 & 0 & K_7^2 & -K_7 K_8 \\ \frac{K_8}{L} & K_1 K_8 & -K_2 K_8 & 0 & -K_3 K_8 & K_4 K_8 & -\frac{K_8}{L} & K_5 K_8 & -K_6 K_8 & 0 & -K_7 K_8 & K_8^2 \end{bmatrix} dx \quad (5.96)$$

$$K_{cxy} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} G_{xy(ij)} \frac{b_{ij} t_{ij}}{\alpha} \begin{bmatrix} 0 \\ K_9 \\ 0 \\ 0 \\ 0 \\ K_{10} \\ 0 \\ -K_9 \\ 0 \\ 0 \\ 0 \\ K_{10} \end{bmatrix} \begin{bmatrix} 0 & K_9 & 0 & 0 & 0 & K_{10} & 0 & -K_9 & 0 & 0 & 0 & K_{10} \end{bmatrix} dx \quad (5.97)$$

$$K_{cxy} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \frac{G_{xy(ij)} b_{ij} t_{ij}}{\alpha} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_9^2 & 0 & 0 & 0 & K_9 K_{10} & 0 & -K_9^2 & 0 & 0 & 0 & K_9 K_{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_9 K_{10} & 0 & 0 & 0 & K_{10}^2 & 0 & -K_9 K_{10} & 0 & 0 & 0 & K_{10}^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -K_9^2 & 0 & 0 & 0 & -K_9 K_{10} & 0 & K_9^2 & 0 & 0 & 0 & -K_9 K_{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_9 K_{10} & 0 & 0 & 0 & K_{10}^2 & 0 & -K_9 K_{10} & 0 & 0 & 0 & K_{10}^2 \end{bmatrix} dx \quad (5.98)$$

$$K_{\alpha z} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \frac{G_{xz(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij}}{a} \begin{bmatrix} 0 \\ 0 \\ K_{11} \\ 0 \\ K_{12} \\ 0 \\ 0 \\ -K_{11} \\ 0 \\ K_{12} \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & K_{11} & 0 & K_{12} & 0 & 0 & 0 & -K_{11} & 0 & K_{12} & 0 \end{bmatrix} dx \quad (5.99)$$

$$K_{\alpha z} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \frac{G_{xz(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij}}{a} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{11}^2 & 0 & K_{11}K_{12} & 0 & 0 & 0 & -K_{11}^2 & 0 & K_{11}K_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{11}K_{12} & 0 & K_{12}^2 & 0 & 0 & 0 & -K_{11}K_{12} & 0 & K_{12}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -K_{11}^2 & 0 & -K_{11}K_{12} & 0 & 0 & 0 & K_{11}^2 & 0 & -K_{11}K_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{11}K_{12} & 0 & K_{12}^2 & 0 & 0 & 0 & -K_{11}K_{12} & 0 & K_{12}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (m-m^2)L^2 & 0 & -mL & 0 & 0 & 0 & 0 \end{bmatrix} dx \quad (5.100)$$

$$K_{\alpha x} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \frac{G_{y(ij)} b_{ij} t_{ij}}{\alpha} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{\sqrt{y^2+z^2}}{L} \\ L \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{\sqrt{y^2+z^2}}{L} \\ L \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & -\frac{\sqrt{y^2+z^2}}{L} & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{y^2+z^2}}{L} & 0 & 0 \end{bmatrix} dx \quad (5.101)$$

$$K_{\alpha x} = \frac{1}{L^2} \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \frac{G_{y(ij)} b_{ij} t_{ij}}{\alpha} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (y^2+z^2) & 0 & 0 & 0 & 0 & 0 & -(y^2+z^2) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(y^2+z^2) & 0 & 0 & 0 & 0 & 0 & (y^2+z^2) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} dx \quad (5.102)$$

$$K_{sa} = \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} \begin{bmatrix} -\frac{1}{L} \\ \frac{1}{L} \\ -K_1 \\ K_2 \\ 0 \\ K_3 \\ -K_4 \\ \frac{1}{L} \\ -K_5 \\ K_6 \\ 0 \\ K_7 \\ -K_8 \end{bmatrix} \begin{bmatrix} -\frac{1}{L} & -K_1 & K_2 & 0 & K_3 & -K_4 & \frac{1}{L} & -K_5 & K_6 & 0 & K_7 & -K_8 \end{bmatrix} dx \quad (5.103)$$

$$K_{so} = \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} \begin{bmatrix} \frac{1}{L^2} & \frac{K_1}{L} & -\frac{K_2}{L} & 0 & -\frac{K_3}{L} & \frac{K_4}{L} & -\frac{1}{L^2} & \frac{K_5}{L} & -\frac{K_6}{L} & 0 & -\frac{K_7}{L} & \frac{K_8}{L} \\ \frac{K_1}{L} & K_1^2 & -K_1 K_2 & 0 & -K_1 K_3 & K_1 K_4 & -\frac{K_1}{L} & K_1 K_5 & -K_1 K_6 & 0 & -K_1 K_7 & K_1 K_8 \\ -\frac{K_2}{L} & -K_1 K_2 & K_2^2 & 0 & K_2 K_3 & -K_2 K_4 & \frac{K_2}{L} & -K_2 K_5 & K_2 K_6 & 0 & K_2 K_7 & -K_2 K_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_3}{L} & -K_1 K_3 & K_2 K_3 & 0 & K_3^2 & -K_3 K_4 & \frac{K_3}{L} & -K_3 K_5 & K_3 K_6 & 0 & K_3 K_7 & -K_3 K_8 \\ \frac{K_4}{L} & K_1 K_4 & -K_2 K_4 & 0 & -K_3 K_4 & K_4^2 & -\frac{K_4}{L} & K_4 K_5 & -K_4 K_6 & 0 & -K_4 K_7 & K_4 K_8 \\ -\frac{1}{L^2} & -\frac{K_1}{L} & \frac{K_2}{L} & 0 & \frac{K_3}{L} & -\frac{K_4}{L} & \frac{1}{L^2} & -\frac{K_5}{L} & \frac{K_6}{L} & 0 & \frac{K_7}{L} & -\frac{K_8}{L} \\ \frac{K_5}{L} & K_1 K_5 & -K_2 K_5 & 0 & -K_3 K_5 & K_4 K_5 & -\frac{K_5}{L} & K_5^2 & -K_5 K_6 & 0 & -K_5 K_7 & K_5 K_8 \\ -\frac{K_6}{L} & -K_1 K_6 & K_2 K_6 & 0 & K_3 K_6 & -K_4 K_6 & \frac{K_6}{L} & -K_5 K_6 & K_6^2 & 0 & K_6 K_7 & -K_6 K_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_7}{L} & -K_1 K_7 & K_2 K_7 & 0 & K_3 K_7 & -K_4 K_7 & \frac{K_7}{L} & -K_5 K_7 & K_6 K_7 & 0 & K_7^2 & -K_7 K_8 \\ \frac{K_8}{L} & K_1 K_8 & -K_2 K_8 & 0 & -K_3 K_8 & K_4 K_8 & -\frac{K_8}{L} & K_5 K_8 & -K_6 K_8 & 0 & -K_7 K_8 & K_8^2 \end{bmatrix} dx \quad (5.104)$$

$$K_{cf} = \int_0^L \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} E_{f(ij)} \mathbf{b}_{fij} \mathbf{t}_{fij} \begin{bmatrix} -1/L \\ -K_1 \\ K_2 \\ 0 \\ K_3 \\ -K_4 \\ 1/L \\ -K_5 \\ K_6 \\ 0 \\ K_7 \\ -K_8 \end{bmatrix} \begin{bmatrix} -1/L & -K_1 & K_2 & 0 & K_3 & -K_4 & 1/L & -K_5 & K_6 & 0 & K_7 & -K_8 \end{bmatrix} dx \quad (5.105)$$

$$K_{cf} = \int_0^L \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} E_{f(ij)} \mathbf{b}_{fij} \mathbf{t}_{fij} \begin{bmatrix} \frac{1}{L^2} & \frac{K_1}{L} & -\frac{K_2}{L} & 0 & -\frac{K_3}{L} & \frac{K_4}{L} & -\frac{1}{L^2} & \frac{K_5}{L} & -\frac{K_6}{L} & 0 & -\frac{K_7}{L} & \frac{K_8}{L} \\ \frac{K_1}{L} & K_1^2 & -K_1 K_2 & 0 & -K_1 K_3 & K_1 K_4 & -\frac{K_1}{L} & K_1 K_5 & -K_1 K_6 & 0 & -K_1 K_7 & K_1 K_8 \\ -\frac{K_2}{L} & -K_1 K_2 & K_2^2 & 0 & K_2 K_3 & -K_2 K_4 & \frac{K_2}{L} & -K_2 K_5 & K_2 K_6 & 0 & K_2 K_7 & -K_2 K_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_3}{L} & -K_1 K_3 & K_2 K_3 & 0 & K_3^2 & -K_3 K_4 & \frac{K_3}{L} & -K_3 K_5 & K_3 K_6 & 0 & K_3 K_7 & -K_3 K_8 \\ \frac{K_4}{L} & K_1 K_4 & -K_2 K_4 & 0 & -K_3 K_4 & K_4^2 & -\frac{K_4}{L} & K_4 K_5 & -K_4 K_6 & 0 & -K_4 K_7 & K_4 K_8 \\ -\frac{1}{L^2} & -\frac{K_1}{L} & \frac{K_2}{L} & 0 & \frac{K_3}{L} & -\frac{K_4}{L} & \frac{1}{L^2} & -\frac{K_5}{L} & \frac{K_6}{L} & 0 & \frac{K_7}{L} & -\frac{K_8}{L} \\ \frac{K_5}{L} & K_1 K_5 & -K_2 K_5 & 0 & -K_3 K_5 & K_4 K_5 & -\frac{K_5}{L} & K_5^2 & -K_5 K_6 & 0 & -K_5 K_7 & K_5 K_8 \\ -\frac{K_6}{L} & -K_1 K_6 & K_2 K_6 & 0 & K_3 K_6 & -K_4 K_6 & \frac{K_6}{L} & -K_5 K_6 & K_6^2 & 0 & K_6 K_7 & -K_6 K_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_7}{L} & -K_1 K_7 & K_2 K_7 & 0 & K_3 K_7 & -K_4 K_7 & \frac{K_7}{L} & -K_5 K_7 & K_6 K_7 & 0 & K_7^2 & -K_7 K_8 \\ \frac{K_8}{L} & K_1 K_8 & -K_2 K_8 & 0 & -K_3 K_8 & K_4 K_8 & -\frac{K_8}{L} & K_5 K_8 & -K_6 K_8 & 0 & -K_7 K_8 & K_8^2 \end{bmatrix} dx \quad (5.106)$$

Integration of equations (5.96), (5.98), (5.100), (5.102), (5.104) and (5.106) has been made using Gaussian Quadrature method. Five points at each zone are found to be suitable for obtaining acceptable results when this method is used to calculate K_{ca} , K_{sa} , and K_{fa} **Kadhim(2007)**. If K_{cxy} , K_{cxz} , K_{cyz} is

calculated exactly using a 2 – points or more than two points Gaussian Quadrature ,over stiff solutions will be obtained. This phenomenon may be cured by using a 1 – point Gaussian Quadrature method **Owen and Hinton** (1980).

5.1.3 Fixed-End Moments and Forces due to Uniformly Distribution Load

Consider the element shown in Figure (5.4) in x – y plane which is subjected to a uniformly distributed load (w_{load}) and fixed at both ends.

The unknown end moments and forces can be found by choosing Q_1 , Q_2 , and Q_3 as redundant and solving for the unknowns using the consistent deformations method with the moment area method for deflection computations. The redundant will be removed so as to form basic determinate stable structure. The basic resulting structure is loaded with external loads (w_{load}) (see Figure (5.5)).Thus,

$$A_{2,1} = \int_0^L - \frac{w_{load} x^3}{2 \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A s_{(ij)} y_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} E_{f(ij)} b_{fij} t_{fij} y_{fij}^2 \right]} dx \quad (5.107a)$$

$$A_{2,1} = - \frac{w_{load}}{2} D_{force} \quad (5.107b)$$

where

$$D_{force} = \int_0^L \frac{x^3}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A s_{(ij)} y_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} E_{f(ij)} b_{fij} t_{fij} y_{fij}^2 \right]} dx \quad (5.108)$$

$$A_{3,1} = \int_0^L - \frac{w_{load} x^2}{2 \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A s_{(ij)} y_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} E_{f(ij)} b_{fij} t_{fij} y_{fij}^2 \right]} dx \quad (5.109a)$$

$$\Delta_{3,1} = -\frac{w_{load}}{2} C_{force} \quad (5.109b)$$

where:

$$C_{force} = \int_0^L \frac{x^2}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} y_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} E_{f(ij)} b_{fij} t_{fij} y_{fij}^2 \right]} dx \quad (5.110)$$

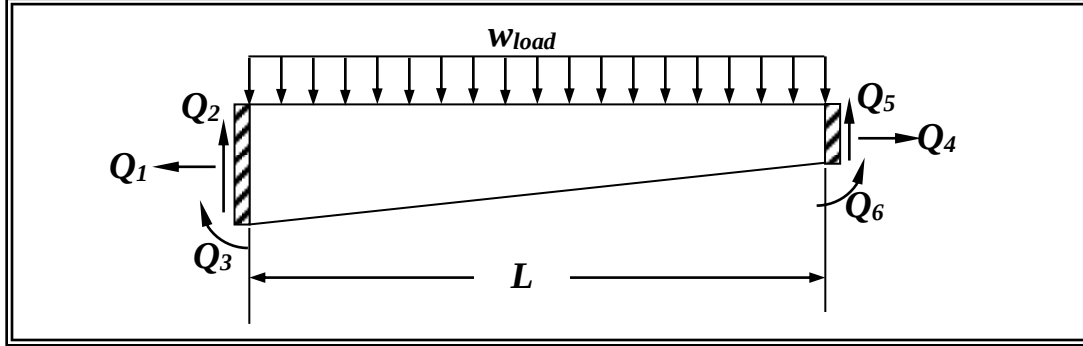


Figure (5.4): Fixed Ended Non Prismatic Frame Element under Uniformly Distributed Load

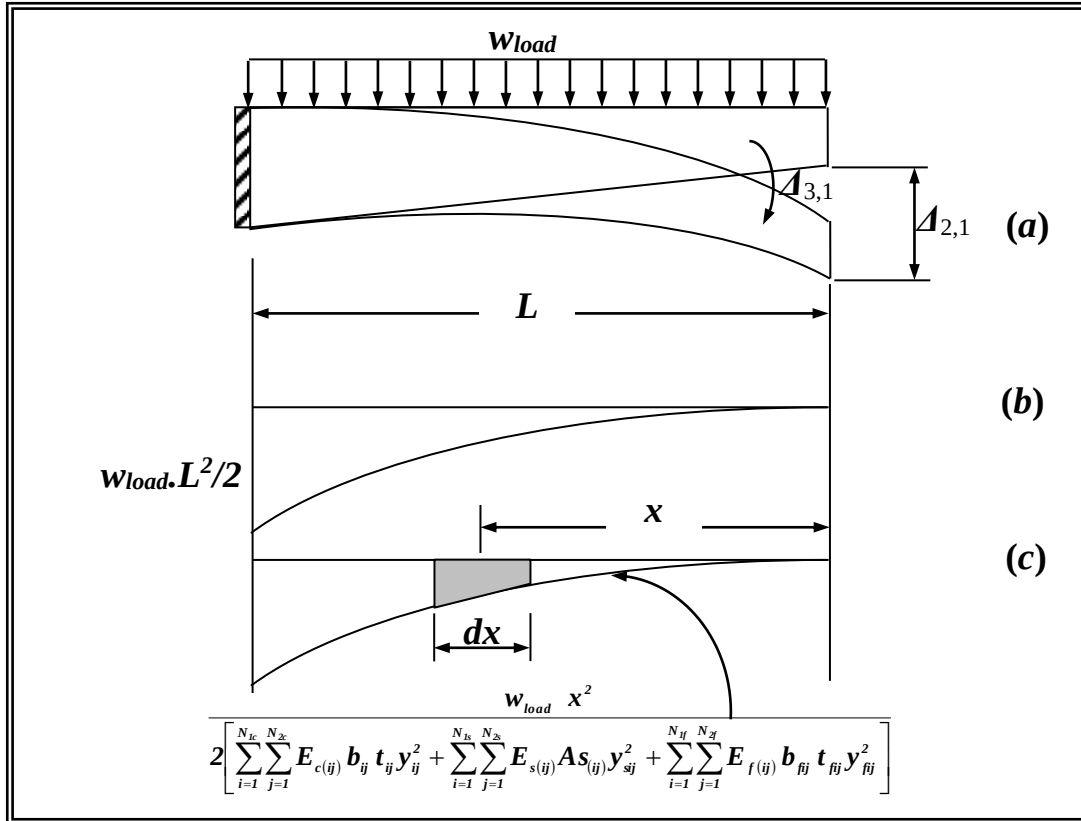


Figure (5.5): Basic Determinate Structure Under External Load (w_{load}). (a) Released Structure, (b) Bending Moment Diagram, (c) M / EI Diagram

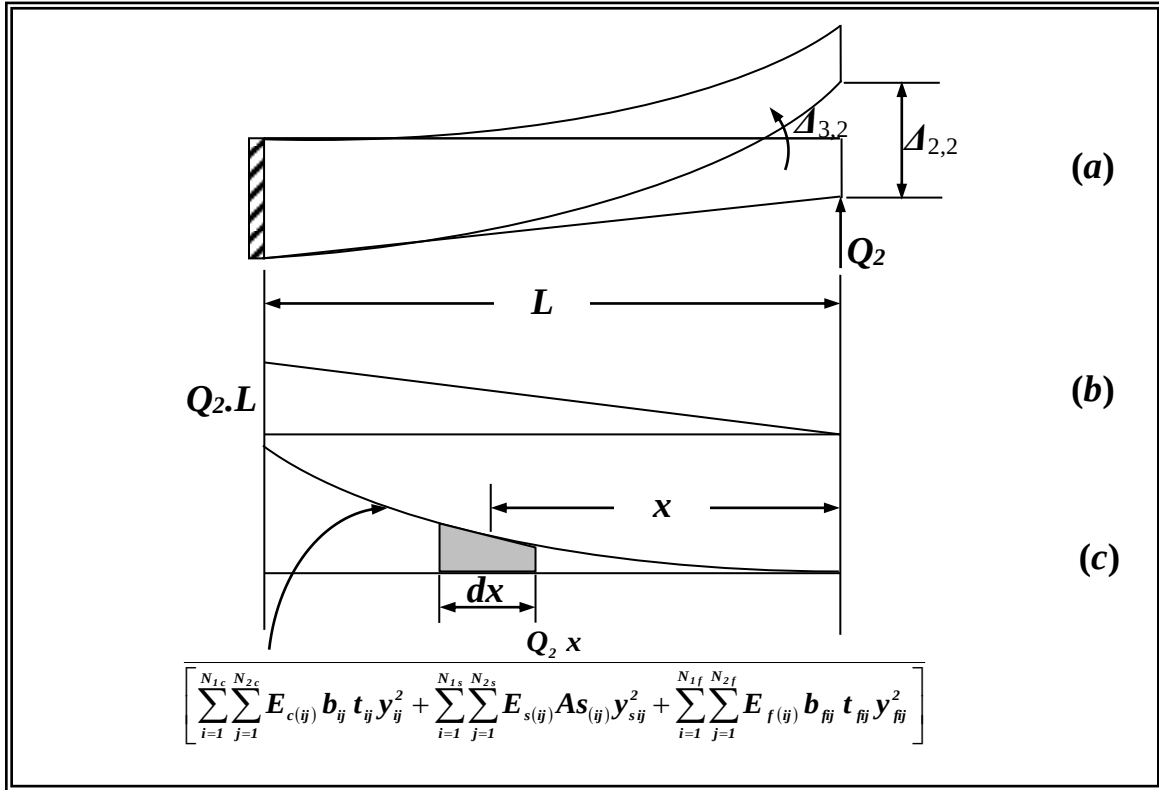


Figure (5.6): Basic Determinate Structure under Load (Q_2). (a) Release Structure, (b) Bending Moment Diagram, (c) M / EI Diagram

The redundant Q_2 is applied to the basic determinate structure as shown in Figure (5.6).

The displacement $A_{2,2}$ is computed as follows

$$A_{2,2} = \int_0^L \frac{Q_2 x^2}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A s_{(ij)} y_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} E_{f(ij)} b_{fij} t_{fij} y_{fij}^2 \right]} dx$$

or: (5.111a)

$$A_{2,2} = Q_2 C_{force} \quad (5.111b)$$

Also,

$$A_{3,2} = \int_0^L \frac{Q_2 x}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A s_{(ij)} y_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} E_{f(ij)} b_{fij} t_{fij} y_{fij}^2 \right]} dx \quad (5.112a)$$

or:

$$\mathbf{A}_{3,2} = \mathbf{Q}_2 \mathbf{B}_{force} \quad (5.112b)$$

where:

$$\mathbf{B}_{force} = \int_0^L \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \mathbf{E}_{c(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} \mathbf{E}_{s(ij)} \mathbf{A} \mathbf{s}_{(ij)} y_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} \mathbf{E}_{f(ij)} \mathbf{b}_{fij} \mathbf{t}_{fij} y_{fij}^2 \right] dx \quad (5.113)$$

The redundant $\mathbf{Q}_3$ is applied to the basic determinate structure as shown in Figure (5.7).

$$\mathbf{A}_{2,3} = \int_0^L - \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \mathbf{E}_{c(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} \mathbf{E}_{s(ij)} \mathbf{A} \mathbf{s}_{(ij)} y_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} \mathbf{E}_{f(ij)} \mathbf{b}_{fij} \mathbf{t}_{fij} y_{fij}^2 \right] dx \quad (5.114a)$$

or:

$$\mathbf{A}_{2,3} = -\mathbf{Q}_3 \mathbf{B}_{force} \quad (5.114b)$$

$$\mathbf{A}_{3,3} = \int_0^L - \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \mathbf{E}_{c(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} \mathbf{E}_{s(ij)} \mathbf{A} \mathbf{s}_{(ij)} y_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} \mathbf{E}_{f(ij)} \mathbf{b}_{fij} \mathbf{t}_{fij} y_{fij}^2 \right] dx \quad (5.115a)$$

or:

$$\mathbf{A}_{3,3} = -\mathbf{Q}_3 \mathbf{A}_{force} \quad (5.115b)$$

where:

$$\mathbf{A}_{force} = \int_0^L \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \mathbf{E}_{c(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} \mathbf{E}_{s(ij)} \mathbf{A} \mathbf{s}_{(ij)} y_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} \mathbf{E}_{f(ij)} \mathbf{b}_{fij} \mathbf{t}_{fij} y_{fij}^2 \right] dx \quad (5.116)$$

Then,

$$\mathbf{A}_{2,1} + \mathbf{A}_{2,2} + \mathbf{A}_{2,3} = 0 \quad (5.117)$$

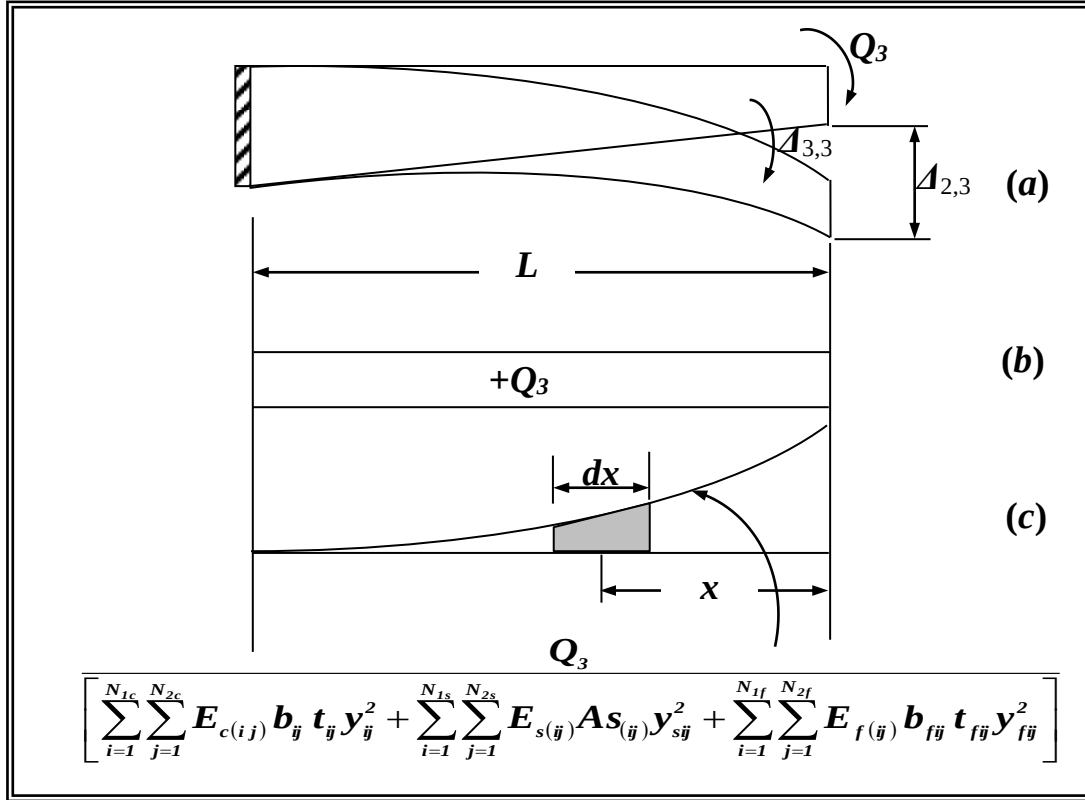


Figure (5.7): Basic Determinate Structure under Load (Q_3).

$$\left(\frac{-w_{load}}{2} D_{force} \right) + (Q_2 C_{force}) + (-Q_3 B_{force}) = 0 \quad (5.118)$$

$$Q_2 = \frac{w_{load} D_{force}}{2C_{force}} + \frac{Q_3 B_{force}}{C_{force}} \quad (5.119)$$

$$\Delta_{3,1} + \Delta_{3,2} + \Delta_{3,3} = 0 \quad (5.120)$$

$$\left(\frac{-w_{load}}{2} C_{force} \right) + (Q_2 B_{force}) + (-Q_3 A_{force}) = 0 \quad (5.121)$$

Substitute equation (5.119) in equation (5.121).

$$\left(\frac{-w_{load}}{2} C_{force} \right) + \left(\frac{w_{load} B_{force} D_{force}}{2C_{force}} \right) + \left(\frac{Q_3 B_{force}^2}{C_{force}} \right) + (-Q_3 A_{force}) = 0 \quad (5.122)$$

$$Q_3 = \frac{w_{load} (C_{force}^2 - B_{force} D_{force})}{2(B_{force}^2 - A_{force} C_{force})} \quad (5.123)$$

Substitute equation (5.123) in equation (5.119).

$$Q_2 = \frac{w_{load}}{2} \left[\frac{D_{force}}{C_{force}} + \left(\frac{B_{force}}{C_{force}} \left(\frac{C_{force}^2}{B_{force}^2} - \frac{B_{force} D_{force}}{C_{force}} \right) \right) \right] \quad (5.124)$$

From static equilibrium, the end forces at the second end of the element are:

$$Q_4 = Q_1 = 0 \quad (5.125)$$

$$Q_5 = wL - Q_2 \quad (5.126)$$

$$Q_6 = Q_2 L - Q_3 - \frac{w_{load} L^2}{2} \quad (5.127)$$

For bending in $x - z$ plane one use the same way and the same equations but:

$$A_{force} = \int_0^L \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} z_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} z_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} E_{f(ij)} b_{fij} t_{fij} z_{fij}^2 \right] dx \quad (5.128)$$

$$B_{force} = \int_0^L \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} z_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} z_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} E_{f(ij)} b_{fij} t_{fij} z_{fij}^2 \right] x dx \quad (5.129)$$

$$C_{force} = \int_0^L \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} z_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} z_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} E_{f(ij)} b_{fij} t_{fij} z_{fij}^2 \right] x^2 dx \quad (5.130)$$

$$D_{force} = \int_0^L \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} z_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} z_{sij}^2 + \sum_{i=1}^{N_{1f}} \sum_{j=1}^{N_{2f}} E_{f(ij)} b_{fij} t_{fij} z_{fij}^2 \right] x^3 dx \quad (5.131)$$

5.2 Formulation of Inelastic Analysis

5.2.1 General

An analytical method for inelastic analysis of repaired/strengthening reinforced concrete space framed structures with CFRP materials, having prismatic or non – prismatic elements has been formulated in this chapter. It follows an incremental approach with simple iterative technique based on finite element method with adopted form approach (namely region approach) **Kadhim** (2007). Many considerations have been made in this work which can briefly be given as follows:

1. Derivation of a numerical form of stiffness matrix for prismatic or non – prismatic space frame element takes into account; (a) the interaction between bending moments, shear and axial forces; (b) the possibility of existence of plastic zone any where the element. In addition, for the case of an element under uniformly distributed load, fixed end forces are derived considering the possibility of existence of plastic zone wherever it is formed (illustrated in this chapter).
2. Regions approach is adopted in the present study to take into account the material non – linearity through the depth and width of the element.
3. Geometric non – linearity is considered through adopting the incremental procedure in which the load is applied as a series of small increments and for each increment the changes in the nodal displacements are determined and the nodal coordinates are updated at the end of each load increment.
4. At the end of each load increment and before updating the nodal coordinates, the element internal forces will be calculated and compared with external applied forces. If the difference is less than 0.0001, then the next load increment will be applied. Otherwise this difference will be

applied as external forces. At the end of each load increment the constitutive matrix for steel and concrete region will be updated as shown later in this chapter.

5. A suitable yield criterion is adopted in the present study. Such a criterion includes the interaction between axial, shear and flexural stress. Hardening / softening rule for concrete, flow rule for concrete, tension stiffening and crushing condition for concrete.
6. The present method takes into consideration the possibility of existence of critical section at a point of zero shears when the element is under uniformly distributed loads, in addition to those existing at its two extremities.
7. Local stress unloading is taken into account.

In addition to these considerations, many developments have been made in this work which can briefly be given as follows:

1. Repair or strengthening of any element in the structure by CFRP-fabrics and in two ways; first longitudinal wrapping for flexure; and second transverse wrapping for confined concrete.
2. Locate the CFRP-fabrics at any region in the section such as, bottom face, top face, U-shape or any other shape.
3. At the end of each load increment, the constitutive matrix for CFRP region will be updated as shown later in this chapter.
4. The bond between the CFRP-fabrics and concrete will be considered being perfect bond.
5. The epoxy that has been used to glue the CFRP-fabrics to concrete will not fail.

5.2.2 The Concrete Yield Criterion

The yield criterion determines the stress level at which plastic deformation begins. **Owen and Hinton** (1980) proposed a triaxial yield criterion. This yield criterion takes into account the transverse shear effect and has the following form:

$$f(\sigma) = (\alpha_{cr} I_1 + 3 \beta_{cr} J_2)^{1/2} = \sigma_o \quad (5.132)$$

where α_{cr} and β_{cr} are material parameters, I_1 is the first stress invariant, J_2 is the second deviatoric stress invariant and σ_o is the equivalent effective stress taken from uniaxial test. I_1 and J_2 were written as:

$$I_1 = \sigma_{cx} + \sigma_{cy} + \sigma_{cz} \quad (5.133)$$

$$J_2 = \frac{1}{3} [(\sigma_{cx}^2 + \sigma_{cy}^2 + \sigma_{cz}^2) - (\sigma_{cx} \sigma_{cy} + \sigma_{cy} \sigma_{cz} + \sigma_{cz} \sigma_{cx})] + (\tau_{cxy}^2 + \tau_{cyz}^2 + \tau_{cxz}^2) \quad (5.134)$$

In the present study σ_{cz} and σ_{cy} are neglected. Therefore equation (5.133) and (5.134) can be rewritten as:

$$I_1 = \sigma_{cx} \quad (5.135)$$

$$J_2 = \frac{1}{3} \sigma_{cx}^2 + (\tau_{cxy}^2 + \tau_{cxz}^2 + \tau_{cyz}^2) \quad (5.136)$$

By substituting equations (5.135) and (5.136) into equation (5.132), then the equation (5.132) can be rewritten in the following form:

$$f(\sigma) = [\alpha_{cr} \sigma_{cx} + \beta_{cr} (\sigma_{cx}^2 + 3\tau_{cxy}^2 + 3\tau_{cxz}^2 + 3\tau_{cyz}^2)]^{1/2} = \sigma_o \quad (5.137)$$

If the results obtained by **Kupfer** for a failure envelope are employed for the initial yield, the value of the constant α_{cr} and β_{cr} are:

$$\alpha_{cr} = 0.355 \sigma_o \quad \text{and} \quad \beta_{cr} = 1.355 \quad (5.138)$$

In the present study it is assumed that the initial yield surface is attained when the effective stress reaches 30% of the ultimate stress f'_c for normal

concrete **Owen and Hinton** (1980) and 30% of ultimate stress f'_{cu} for confined concrete. After that, the loading (yielding) surface will expand due to hardening effect until the effective stress reaches the ultimate stress f'_c for normal concrete or f'_{cu} for confined concrete. Then, after this the loading surface will contract due to softening effect until crushing occurs.

5.2.3 The Hardening / Softening Rule for Concrete

After initial yielding, the stress level at which further plastic deformation occurs is dependent on the current degree of plastic straining. Such a phenomenon is termed as strain hardening. Thus, the yield surface will vary at each stage of the plastic deformation, with the subsequent yield surfaces being dependent on the plastic strains in some way. Some alternative models which describe strain hardening in a material are illustrated in Figure (5.8). A perfectly plastic material is shown in Figure (5.8a) where the yield stress level does not depend in any way on the degree of plasticization. If the subsequent yield surfaces are a uniform expansion of original yield curve, without translation, as shown in Figure (5.8b) the strain hardening model is said to be isotropic. On the other hand, if the subsequent yield surfaces preserve their shapes and orientation but translate in the stress space as a rigid body as shown in Figure (5.8c), kinematical hardening is said to take place.

In the present study, an isotropic hardening / softening rule is adopted. At first, when a material is stressed beyond its initial yielding surface, the yielding surface will expand until the effective stress reaches the ultimate stress f'_c for normal concrete or f'_{cu} for confined concrete. After that, the yielding surface will contract due to softening effect until the failure occurs.

To calculate the uniaxial hardening / softening parameter (H'), the tangential modulus of elasticity E_{ct} must be found at first, which will be calculated according to stress – strain model for concrete which was illustrated in chapter four, and as shown below:

1. For the compression envelope curve , E_{ct} will equal the derivative of equation (4.1)for normal concrete or equation (4.4) for confined concrete with respect to ϵ_c and has the form:

$$\text{For normal concrete: } E_{ct} = \frac{\frac{A_c f'_c}{\epsilon_{co}^4} [D_c \epsilon_{co}^3 - B_c \epsilon_c^2 \epsilon_{co} - 2 \epsilon_c^3]}{\left[\left(\frac{\epsilon_c}{\epsilon_{co}} \right)^3 + B_c \left(\frac{\epsilon_c}{\epsilon_{co}} \right)^2 + C_c \frac{\epsilon_c}{\epsilon_{co}} + D_c \right]^2} \quad (5.139a)$$

For confined concrete:

$$E_{ct} = \frac{(E_1 - E_2) \left[1 + \frac{(E_1 - E_2) \epsilon_c}{f_o} \right]^n - \frac{n}{f_o} (E_1 - E_2)^2 \left[1 + \frac{(E_1 - E_2) \epsilon_c}{f_o} \right]^{n-1} \epsilon_c}{\left[1 + \frac{(E_1 - E_2) \epsilon_c}{f_o} \right]^{2n}} \quad (5.139b)$$

2. For the unloading and reloading paths in compression , E_{ct} will equal to:

$$\text{For normal concrete: } E_{ct} = \frac{\bar{\sigma}_{m1}}{\bar{\epsilon}_{m1} - \bar{\epsilon}_p} \frac{f'_c}{\epsilon_{co}} \quad (5.140a)$$

$$\text{For confined concrete: } E_{ct} = \frac{\sigma_{m1}}{\epsilon_{m1} - \epsilon_p} f'_{cu} \quad (5.140b)$$

In the state of the unloading and reloading for the uncracked concrete in tension, the value of E_{ct} will be equal E_{co} .

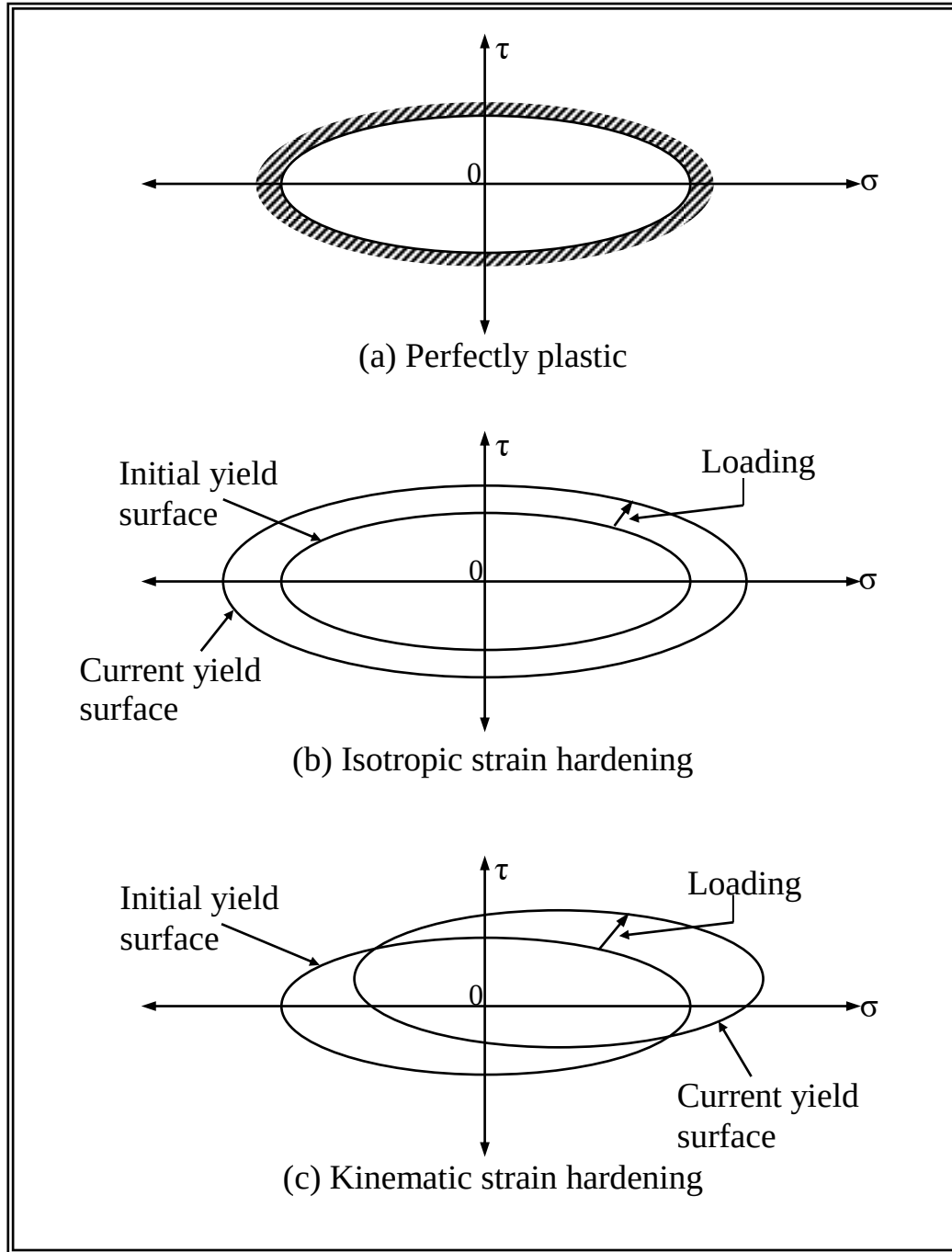


Figure (5.8): Mathematical Models for Representation of Strain Hardening Behavior *Kadhim(2007)*

3. The value of E_{α} for the cracked concrete under tension will be calculated according to tension stiffening model which will be illustrated later in this chapter.

The hardening parameter is defined as:

$$\mathbf{H}' = \frac{d\sigma}{d\epsilon_p} \quad (5.141)$$

where ϵ_p is the concrete plastic strain

$$d\epsilon_p = d\epsilon_c - d\epsilon_e \quad (5.142)$$

where ϵ_e is the concrete elastic strain. By substitution from equation (5.142) into equation (5.141):

$$\mathbf{H}' = \frac{E_{ct}}{1 - \frac{E_{ct}}{E_{co}}} \quad (5.143)$$

If $\mathbf{H}'$ equals to zero, the stressed material is perfectly plastic. When $\mathbf{H}'$ equals to infinity, then the material is still within the elastic range.

5.2.4 The Flow Rule for Concrete

The flow rule is considered to construct the stress – strain relationship in the plastic range. Assuming normality of the plastic deformation rate vector to the yield surface, the plastic strain increment can be determined for a given stress increment according to *Owen and Hinton* (1980) as:

$$d\epsilon_p = d\bar{\lambda} \frac{\partial f(\sigma)}{\partial \sigma} \quad (5.144)$$

where $d\bar{\lambda}$ is proportionality constant termed the plastic multiplier which determines the size of the plastic strain increment, and $\frac{\partial f(\sigma)}{\partial \sigma}$ is a vector

directed normal to the yield surface at the stress point under consideration.

Equation (5.132) can be rewritten as:

$$f(\sigma) = (3\beta_{cr} J_2 + 2C\sigma_o I_1)^{1/2} = \sigma_o \quad (5.145)$$

where $C = \alpha_{cr} / 2 \sigma_o$. This can be solved for σ_o , giving *Owen and Hinton* (1980):

$$f(\sigma) = C I_1 + [(C I_1)^2 + 3\beta_{cr} J_2]^{1/2} = \sigma_o \quad (5.146)$$

The yield function derivatives with respect to the stress components define the flow vector $\{a_f\}$ as:

$$[a_f] = \begin{bmatrix} \frac{\partial f(\sigma)}{\partial \sigma_{cx}} & \frac{\partial f(\sigma)}{\partial \tau_{cxy}} & \frac{\partial f(\sigma)}{\partial \tau_{cxz}} & \frac{\partial f(\sigma)}{\partial \tau_{cyz}} \end{bmatrix} \quad (5.147)$$

In which,

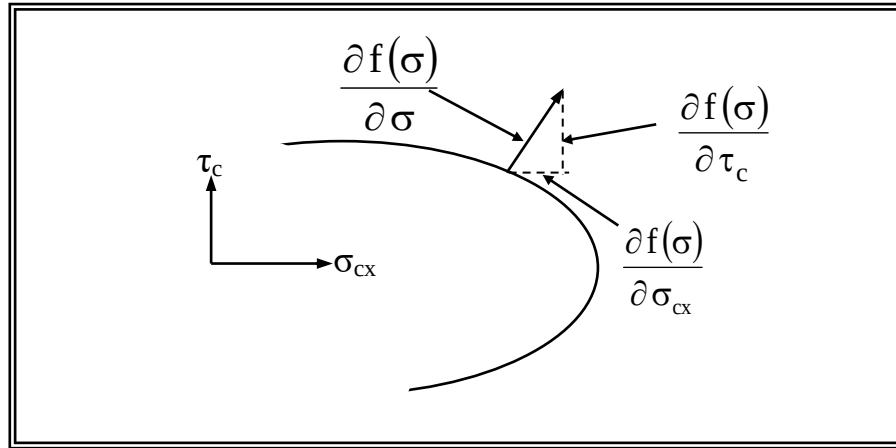


Figure (5.9): Geometrical Representation of the Normality Rule of Associated Plasticity

$$a_{f1} = \frac{\partial f(\sigma)}{\partial \sigma_{cx}} = C + \frac{2 * \sigma_{cx} * (C^2 + \beta_{cr})}{Q_{cr}} \quad (5.148)$$

Let

$$Q_{cr} = 2 \left[(C \sigma_{cx})^2 + \beta_{cr} (\sigma_{cx}^2 + 3\tau_{cxy}^2 + 3\tau_{cxz}^2 + 3\tau_{cyz}^2) \right]^{1/2} \quad (5.149)$$

$$a_{f2} = \frac{\partial f(\sigma)}{\partial \tau_{cxy}} = \frac{6\beta_{cr} \tau_{cxy}}{Q_{cr}} \quad (5.150)$$

$$\mathbf{a}_{f3} = \frac{\partial f(\boldsymbol{\sigma})}{\partial \tau_{cxz}} = \frac{6\beta_{cr} \tau_{cxz}}{Q_{cr}} \quad (5.151)$$

$$\mathbf{a}_{f4} = \frac{\partial f(\boldsymbol{\sigma})}{\partial \tau_{cyz}} = \frac{6\beta_{cr} \tau_{cyz}}{Q_{cr}} \quad (5.152)$$

The plastic multiplier $d\bar{\lambda}$ can be found by using the following formula *Owen and Hinton* (1980):

$$d\bar{\lambda} = \left[\frac{\{\mathbf{a}_f\}^T [\mathbf{D}_{con}]}{\mathbf{H}' + \{\mathbf{a}_f\}^T [\mathbf{D}_{con}] \{\mathbf{a}_f\}} \right] d\{\boldsymbol{\varepsilon}\} \quad (5.153)$$

where $[\mathbf{D}_{con}]$ is the elastic constitutive matrix, $d\boldsymbol{\varepsilon}$ is the total strain increment vector, and $\mathbf{H}'$ is the hardening parameter. The elasto – plastic increment of total strain is calculated by summing up the elastic and plastic strain components as:

$$\begin{aligned} d[\boldsymbol{\varepsilon}] &= d\{\boldsymbol{\varepsilon}_e\} + d\{\boldsymbol{\varepsilon}_p\} \\ &= [\mathbf{D}_{con}]^{-1} d\{\boldsymbol{\sigma}\} + d\bar{\lambda} \frac{\partial f(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}} \\ &= [\mathbf{D}_{con}]^{-1} d\{\boldsymbol{\sigma}\} + \frac{\{\mathbf{a}_f\} \{\mathbf{a}_f\}^T [\mathbf{D}_{con}]}{\mathbf{H}' + \{\mathbf{a}_f\}^T [\mathbf{D}_{con}] \{\mathbf{a}_f\}} d\{\boldsymbol{\varepsilon}\} \end{aligned} \quad (5.154)$$

The complete elasto – plastic incremental stress – strain relationship can be expressed as:

$$[\mathbf{D}_{con}]_{ep} = [\mathbf{D}_{con}] - \frac{[\mathbf{D}_{con}] \{\mathbf{a}_f\} \{\mathbf{a}_f\}^T [\mathbf{D}_{con}]}{\mathbf{H}' + \{\mathbf{a}_f\}^T [\mathbf{D}_{con}] \{\mathbf{a}_f\}} \quad (5.155)$$

where the second term in the bracket represents the stiffness degradation due to plastic deformations.

5.2.5 The Crushing Condition for Concrete

The crushing type of concrete fracture is a strain controlled phenomenon. A failure surface in the strain space must be defined in order to

take this type of failure into account. The failure surface is represented by the following equation **Owen and Hinton** (1980).

$$\left[\alpha'_{cr} I'_1 + \beta'_{cr} (3J'_2) \right]^{\frac{1}{2}} = \epsilon_{cu} \quad (5.156)$$

where I'_1 is the first strain invariant, J'_2 is the second deviatoric strain invariant, and ϵ_{cu} is the ultimate total strain extrapolated from the uniaxial compressive test results.

Using the material parameters α'_{cr} and β'_{cr} from Kupfer's results, the crushing condition is expressed in terms of the total strain components as:

$$0.355 \epsilon_{cu} (\epsilon_{cx} + \epsilon_{cy} + \epsilon_{cz}) + 1.355 \times \frac{4}{9} \left[(\epsilon_{cx}^2 + \epsilon_{cy}^2 + \epsilon_{cz}^2) - (\epsilon_{cx} \epsilon_{cy} + \epsilon_{cy} \epsilon_{cz} + \epsilon_{cz} \epsilon_{cx}) + 0.75 (\gamma_{cxy}^2 + \gamma_{cyz}^2 + \gamma_{cxz}^2) \right] = \epsilon_{cu}^2 \quad (5.157)$$

by using equations (5.148) and (5.149) then equation (5.157) can be rewritten as follows:

$$0.355 \epsilon_{cu} (1 - 2\nu) \epsilon_{cx} + 1.355 \times \frac{4}{9} \left[(1 + 2\nu + \nu^2) \epsilon_{cx}^2 + 0.75 (\gamma_{cxy}^2 + \gamma_{cxz}^2 + \gamma_{cyz}^2) \right] = \epsilon_{cu}^2 \quad (5.158)$$

when ϵ_{cu} reaches the ultimate value which is equal to 1.5 ϵ_{co} for normal concrete or equation (4.13) for confined concrete. The concrete is assumed to lose all its characteristics of strength and rigidity **Owen and Hinton** (1980).

5.2.6 Modeling of Concrete in Tension

A smeared representation for cracks is assumed, which implies that the cracks are distributed across a region of the finite element. A linear elastic response of concrete is assumed until the fracture surface is reached. This tensile type of cracking is governed by a maximum tensile stress criterion. The fixed orthogonal crack model is used here, in which cracks are assumed to form in a plane perpendicular to the direction of the maximum principal

stress. A sudden or gradual relaxation of the normal stress is adopted after cracking has occurred, according to the tension stiffening diagram. The modulus of elasticity and Poisson's ratio are reduced to zero in the direction perpendicular to the crack and a reduced shear modulus is employed. The stress – strain relationship in this case is: **Owen and Hinton** (1980).

$$\begin{bmatrix} \sigma_{cx} \\ \tau_{cxy} \\ \tau_{cxz} \\ \tau_{cyz} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \bar{G} & 0 & 0 \\ 0 & 0 & \bar{G} & 0 \\ 0 & 0 & 0 & KG \end{bmatrix} * \begin{bmatrix} \epsilon_{cx} \\ \gamma_{cxy} \\ \gamma_{cxz} \\ \gamma_{cyz} \end{bmatrix} \quad (5.159)$$

where $\bar{G}$ is the reduced shear modulus and the calculation of it is shown later in this chapter, K is the shear correction factor.

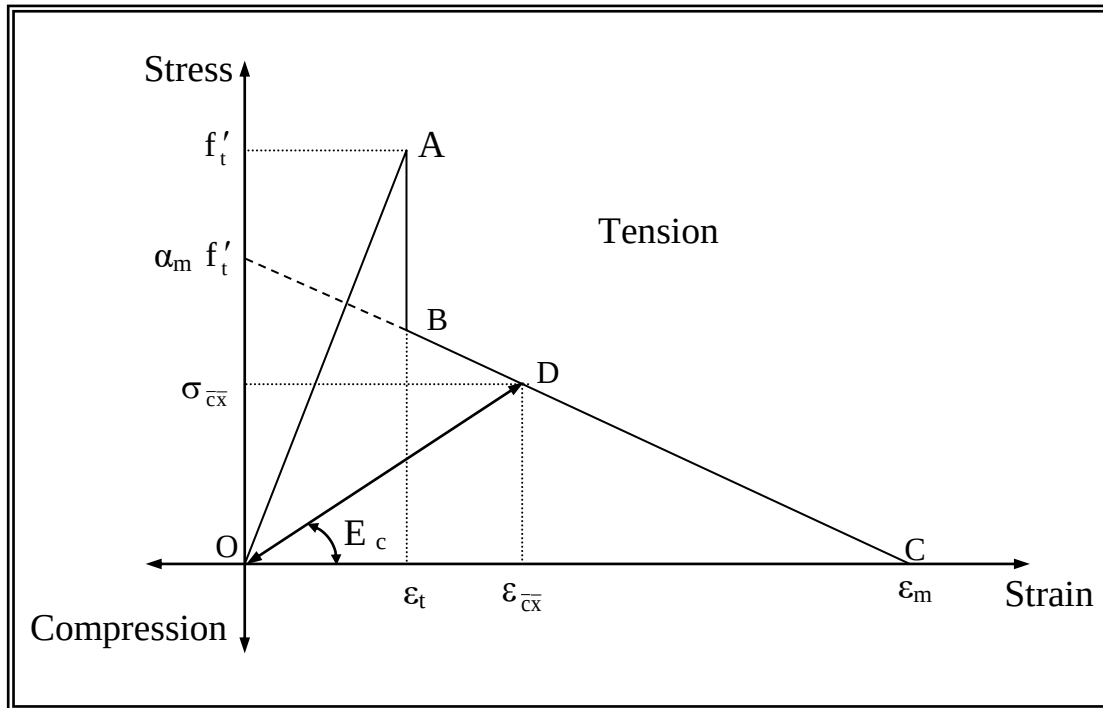


Figure (5.10): Loading and Unloading Behavior of Cracked Concrete Illustrating Tension Stiffening Behavior.

5.2.7 Tension Stiffening Rule for Concrete

Due to bond effects, cracked concrete carries between cracks a certain amount of tensile force normal to the cracked plane. The concrete adheres to the reinforcing bars and contributes to the over all stiffness of the structure: **Owen and Hinton** (1980). The assumed shape of the stress – strain hysteresis loops for cyclic loading in tension range is shown in Figure (5.10). This behavior is illustrated through the following points:

1. At first, for uncracked concrete a linear path (O – A) will be adopted for the stress – strain behavior of concrete under tension with the modulus of elasticity equal to the nominal modulus of elasticity in compression E_{co} . This will be valid up to cracking strength f_t' beyond which a gradual release of the concrete stress component normal to the cracked plane occurs along the line (B – C) as shown in Figure (5.10). The path (B – C) has the following expression.

$$\sigma_{cx} = \alpha_m f_t' \left(1 - \frac{\epsilon_{cx}}{\epsilon_m} \right) \quad (5.160)$$

where ϵ_{cx} and σ_{cx} are the current values of concrete tensile strain and stress respectively in local direction x, α_m and ϵ_m are tension stiffening parameters (α_m is taken between (0.5 – 0.7) and ϵ_m is assumed to vary from 0.0015 to 0.002) **Owen and Hinton** (1980).

2. Assign the point where unloading occurs D ($\epsilon_{cx}, \sigma_{cx}$).
3. For $\epsilon_{cx} < \epsilon_t$ the unloading and reloading path follow a linear path (O – A) with a slope equal to the nominal modulus of elasticity in compression E_{co} . Where ϵ_t is the strain value at the ultimate concrete tensile strength (f_t).

4. For $\epsilon_m \geq \epsilon_{\bar{cx}} \geq \epsilon_t$ unloading occurs along a linear path (O – D) with a slope equal to the fictitious elasticity modulus E_c given by

$$E_c = \frac{\alpha_m f'_t \left(1 - \frac{\epsilon_{\bar{cx}}}{\epsilon_m} \right)}{\epsilon_{\bar{cx}}} \quad (5.161)$$

The linear path (O – D) has the following expression:

$$\sigma_{cx} = \frac{\sigma_{\bar{cx}}}{\epsilon_{\bar{cx}}} \epsilon_{cx} \quad (5.162)$$

5. If the crack closes, i.e., if the strain component normal to the crack plane becomes negative, the concrete acquires the uncracked behavior, but the crack direction and the maximum tensile strain $\epsilon_{\bar{cx}}$ continue to be stored.
6. For $\epsilon_{cx} < \epsilon_{\bar{cx}}$ reloading occurs along the linear path (O – D) with a slope equal to the fictitious elasticity modulus E_c given by Equation (5.161). The path (O – D) has the expression shown in Equation (5.162).
7. For $\epsilon_{\bar{cx}} \leq \epsilon_{cx} \leq \epsilon_m$ reloading occurs along the linear path (D – C) which has the expression shown in Equation (5.160)

5.2.8 Shear Modulus of Cracked Concrete

The value of $\bar{G}$ is linearly decreasing with the strain normal to the cracked plane and calculated according to the following formula: **Owen & Hinton** (1980).

$$\text{for } \epsilon_{cx} < 0.004 \quad \bar{G} = 0.25 * G \left(1 - \frac{\epsilon_{cx}}{0.04} \right) \quad (5.163a)$$

$$\text{for } \epsilon_{cx} \geq 0.004 \quad \bar{G} = 0 \quad (5.163b)$$

When the crack is closed, the uncracked shear modulus is again assumed in the corresponding direction.

5.2.9 Algorithm for the Procedure of Analysis

The adopted approach of the analysis, which is called the incremental approach, treats the problem of nonlinear behavior as a sequence of linear problems. During every loading step of the sequence, the structure supports a new increment of external loads. Each step is based on material and geometry properties appropriate to that step, i.e., the stiffness of the structure is updated at the beginning of each step. The procedure of analysis can be illustrated as follows:

1. The analysis starts by considering the structure as a linear structure subjected to applying loads $\{ \mathbf{f} \}$, and solving the following equations of equilibrium

$$[\mathbf{K}_{stiff}] \{ \boldsymbol{\psi} \} = \{ \mathbf{f} \} \quad (5.164)$$

where $[\mathbf{K}_{stiff}]$ is the initial global stiffness matrix of the structure. Then these equations (5.164) is solved by **Gauss - Jordan's Elimination** method.

2. Local forces and moments at the member ends are found from the local nodal displacements that are extracted from the global nodal displacements found from equations (5.164). At every zero shear point, if it exists, for the element under uniformly distributed load (in xy – plane or xz – plane), an additional node is indicated and the element having such point is subdivided into two elements and the first step of analysis is repeated for the resolved configuration of structural elements.
3. For each load cycle, the external load will be divided into a series of increments. Then these load increments are applied in a sequence form to the structure.
4. Transform all the sections into elastic concrete sections and find the position of the central axis for each section.

5. Calculate the global stiffness matrix of the structure and solve the following equations of equilibrium.

$$[K_{stiff}] \{\Delta \psi\} = \{\Delta f\} \quad (5.165)$$

Equations (5.165) are solved by **Gauss - Jordan's Elimination** method. Local forces and moments at the member ends are found from the local displacements that are extracted from the global nodal displacements found from equations (5.165).

6. Calculate the accumulating strain values for each concrete region at five sections of each element (one section at each Gaussain point) by using equations (5.72) to (5.87). The normal strain values must be multiplied by (-1) before sending it to the concrete cyclic stress – strain model to calculate the accumulating stress values for concrete regions, because in the concrete cyclic stress - strain model the compressive values are considered to be positive and tension values are considered to be negative, while in this chapter the compressive values are considered to be negative and tension values are considered to be positive. Also, the accumulating normal stress values must be multiplied by (-1) before using it in the later steps for the same reason. The accumulating shear stress values for concrete layers will be calculated according to equation (5.19).
7. Calculate the accumulating axial strain value for each steel region at five sections of each element (one section at each Gaussain point) by using equation (5.72).
8. Calculate the accumulating stress values for steel regions using the steel cyclic stress – strain model but the normal strain values must be multiplied by (-1) before sending it to the steel cyclic stress – strain

model , because in the steel cyclic stress - strain model the compressive values are considered to be positive and tension values are considered to be negative, while in this chapter the compressive values are considered to be negative and tension values are considered to be positive. Also, the accumulating normal stress values must be multiplied by (-1) before using it in the later steps for the same reason.

9. Calculate the accumulating axial strain value for each CFRP region at five sections in each element (one section at each Gaussain point) by using equation (5.72).
10. Calculate the accumulating stress values for CFRP regions using the CFRP cyclic stress – strain model but the normal strain values must be multiplied by (-1) before sending it to the CFRP cyclic stress – strain model, because in the CFRP cyclic stress - strain model the compressive values are considered to be positive and tension values are considered to be negative, while in this chapter the compressive values are considered to be negative and tension values are considered to be positive. Also, the accumulating normal CFRP values must be multiplied by (-1) before using it in the later steps for the same reason.
11. Calculate the difference [**Dif**] between the external applied forces and the internal forces calculated in step (5).

$$12. \quad \mathbf{FNORM} = \frac{\left[\left[\sum_{i=1}^{NEQ} [\mathbf{Dif}] \right]^2 \right]^{\frac{1}{2}}}{\left[\left[\sum_{i=1}^{NEQ} [\Delta \mathbf{f}_{in}] \right]^2 \right]^{\frac{1}{2}}} \quad (6.35)$$

where **NEQ** equal to the total number of external forces components applied to the structure. If **FNORM** ≤ 0.05 the next load increment will

be applied, otherwise the residual forces calculated in step (11) will be applied as external forces and the steps (5) to (12) will be repeated until convergence occurs.

13. At the end of each load increment, the nodal coordinates will be updated and the displacements and forces will be accumulated.
14. For the concrete regions, which were in compressive state, calculate the effective stress from equation (5.137). If the effective stress reaches 30% of the ultimate stress f_c' for normal concrete, then use the hardening / softening rule and flow rule to calculate the constitutive elasto – plastic matrix, otherwise the material is still within the elastic range.
15. For the concrete regions within the tension state, if the normal stress reaches f_t' , i.e. the region is cracked, and then the values of the constitutive matrix will be calculated according to tension stiffening model and the model used to calculate cracked shear modulus. Otherwise the material is still within the elastic range.
16. For the concrete regions within plastic range, calculate the effective strain by using equation (5.158). If the effective strain reaches $1.5 \epsilon_{co}$ for normal concrete or equation (4.13) for confined concrete, then the crushing failure is occurred and the solution is terminated.
17. Repeat steps (3) to (16) until the crushing failure occurs or the total cyclic load is applied.

5.2.10 Computer Program

A computer program is prepared, as apart of this work, for the analysis of a space reinforced concrete frame repair or strengthening with CFRP-fabrics of non – prismatic and / or prismatic members based on the prescribed procedure. The computer program is written in **FORTRAN90** language for the **IBM** personal computer and compatible microcomputers. The flow chart of this program is given in Figure (5.11) to outline the main steps in the computer program.

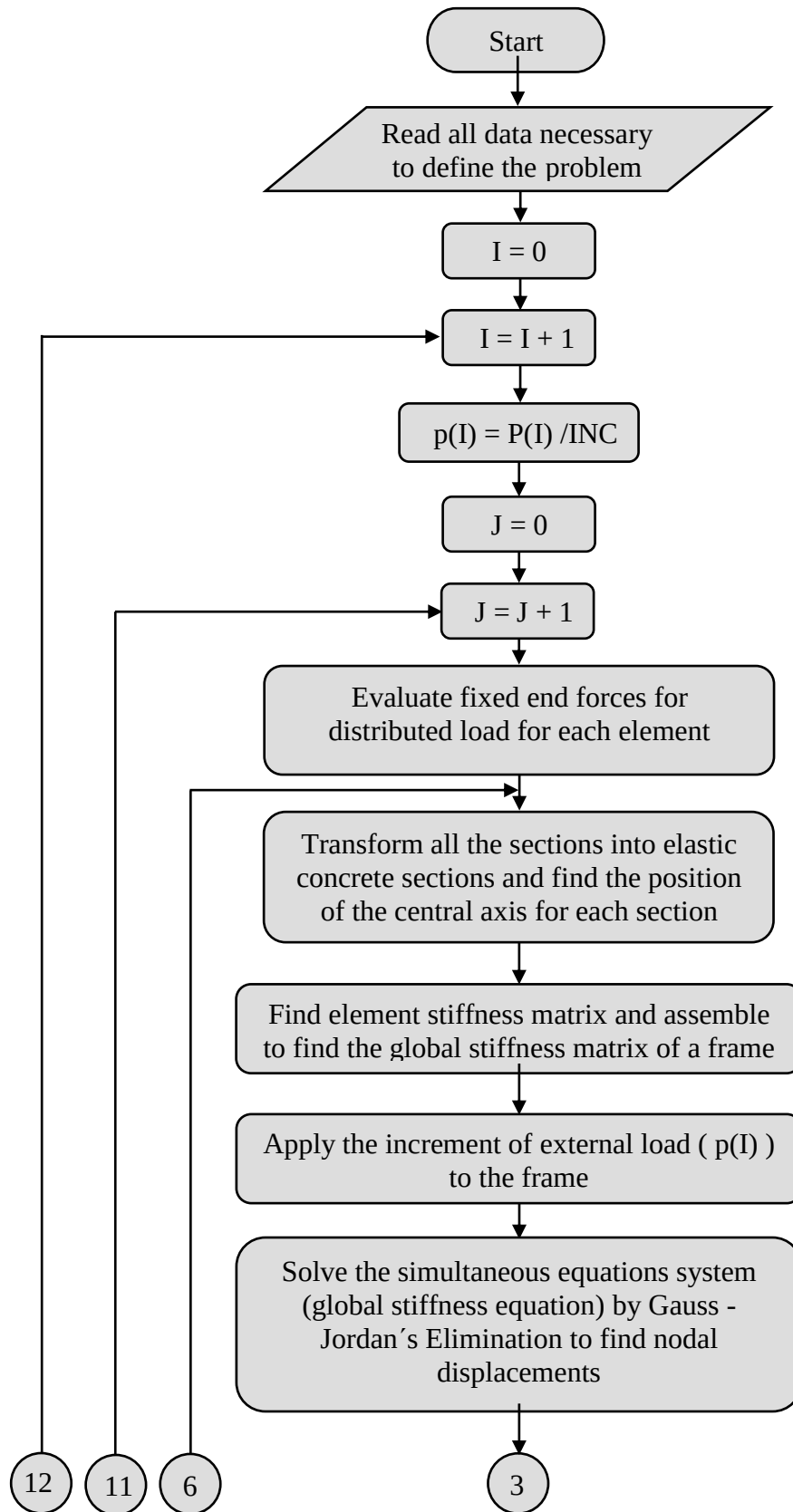


Figure (5.11): Flow Chart of the Computer Program

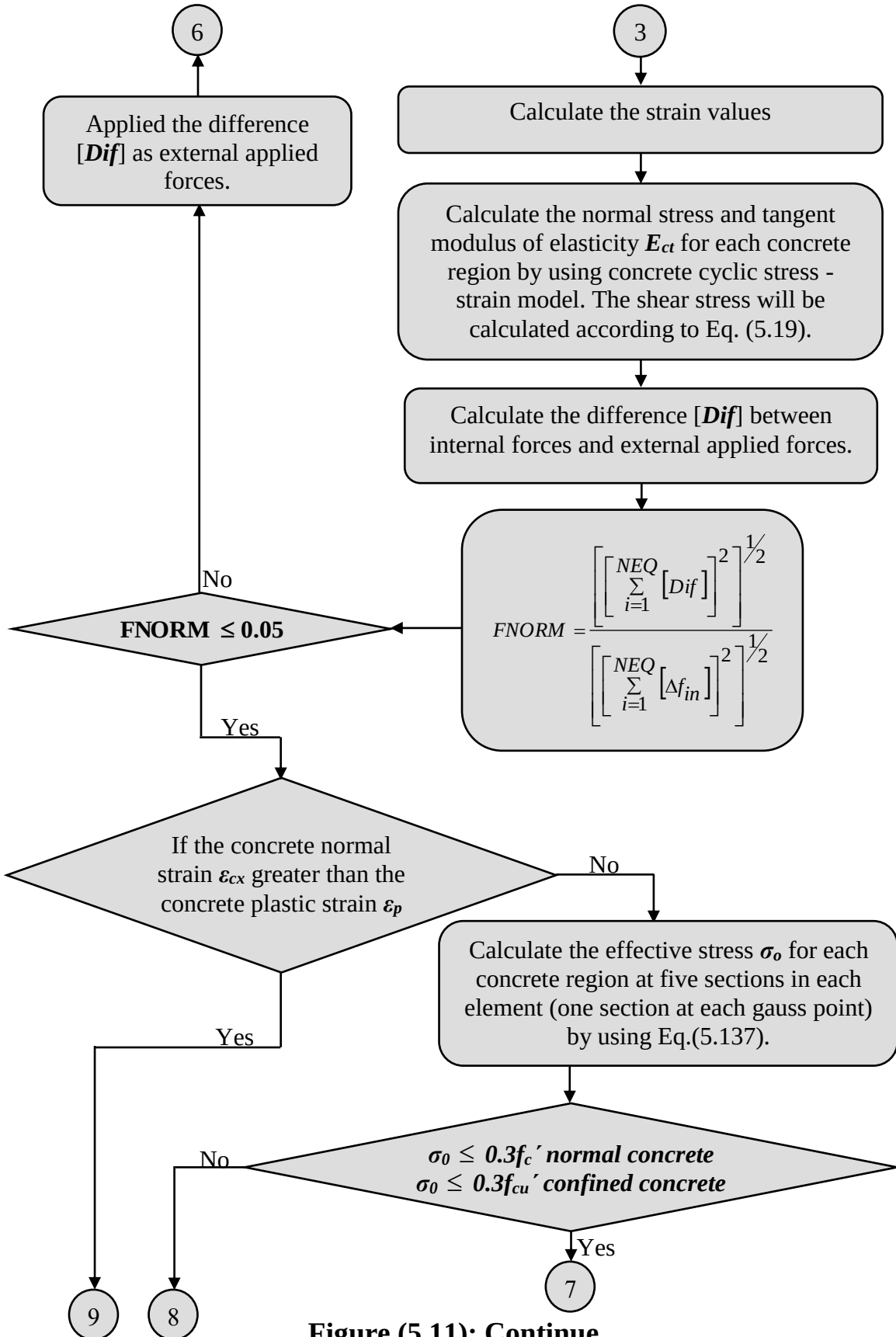


Figure (5.11): Continue

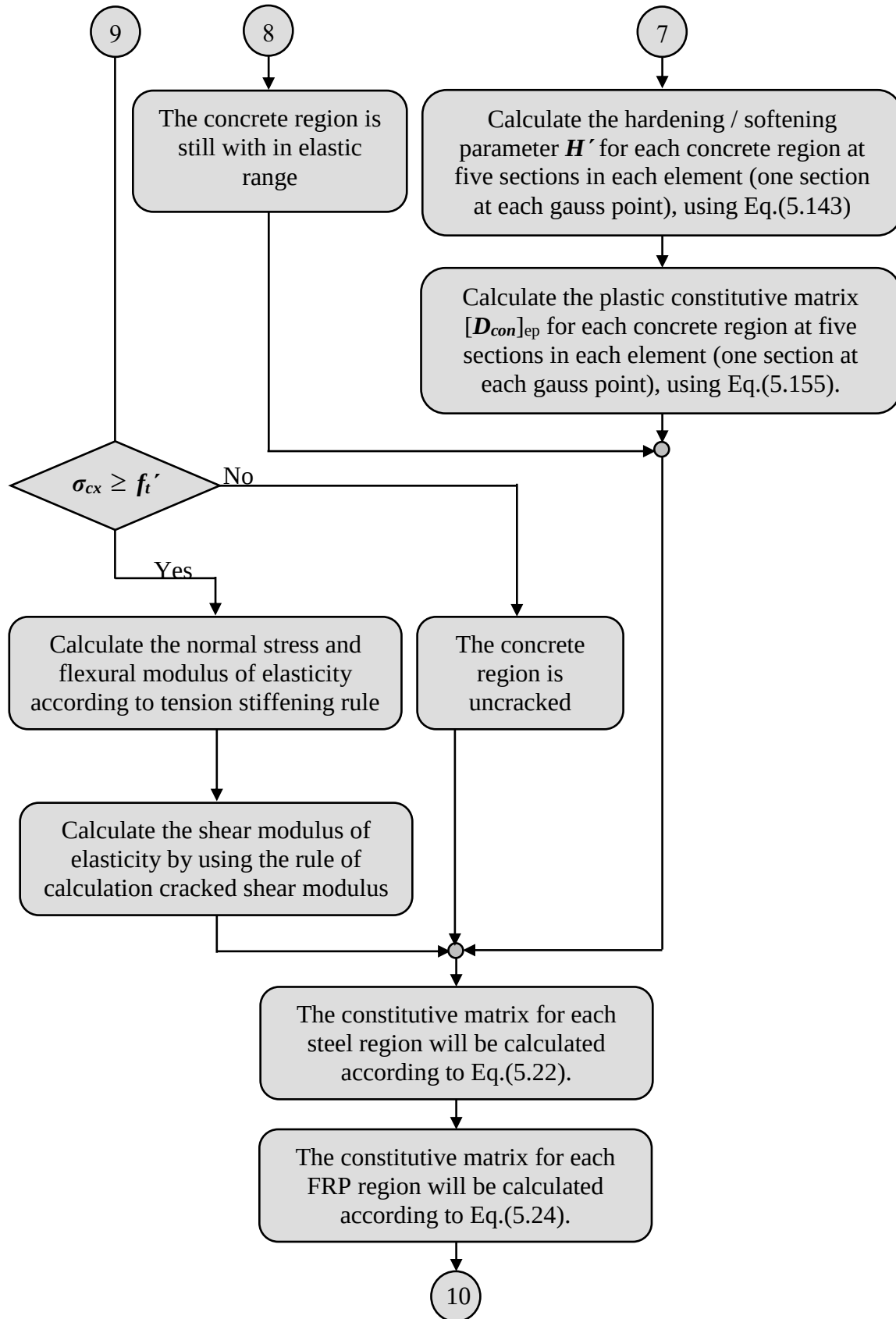
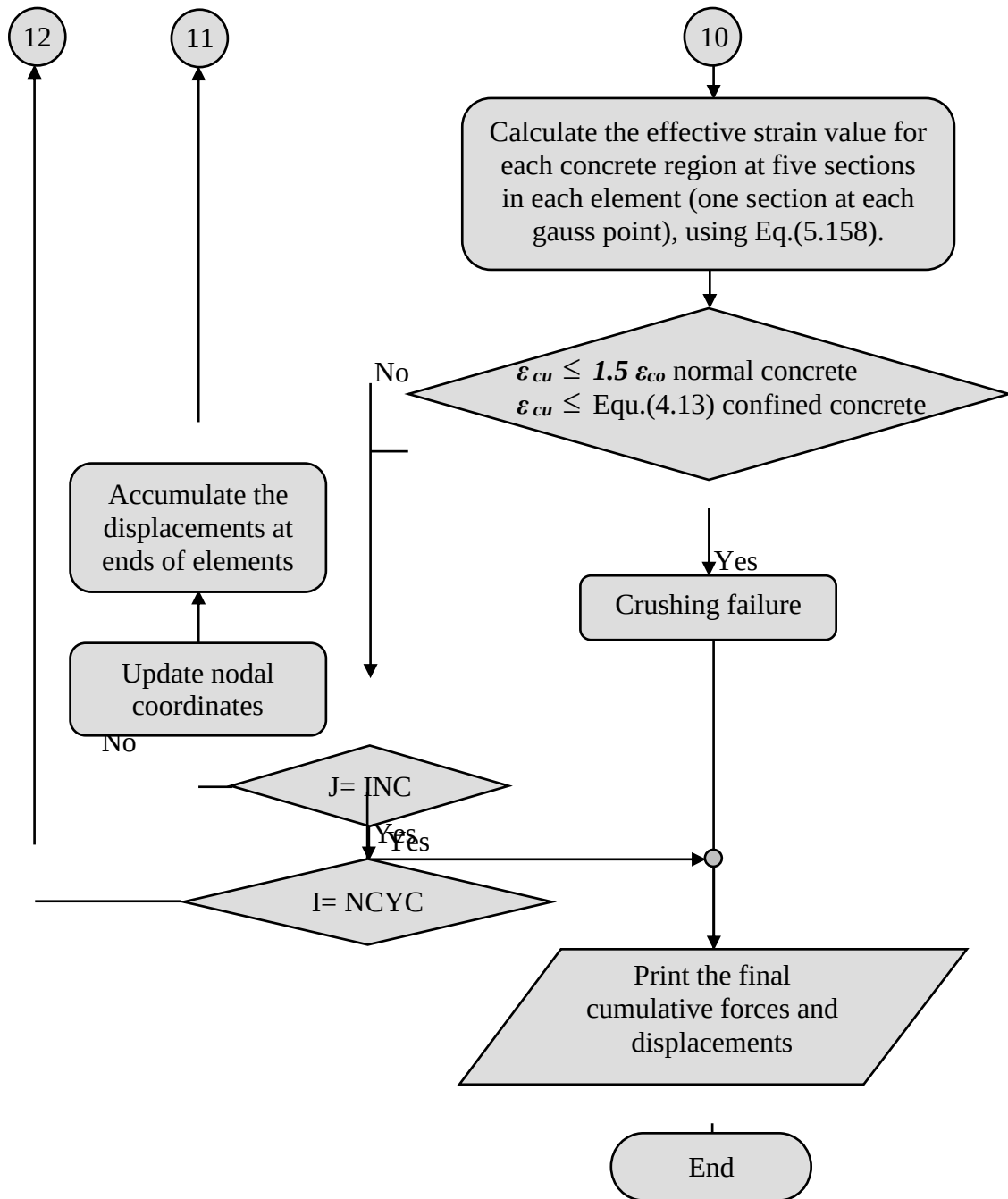


Figure (5.11): Continue



INC = total number of the load increments

NCYC = total number of the load cycles

Figure (5.11): Continue

Chapter Six

Experimental Results

6.1 General

The tested frames are designed to be controlled by bending failure. That means if the frames fail in shear first, one will be unable to determine whether the externally wrapped CFRP fabrics do help or not to carry the bending force, let alone to decide how much contribution it would make to the whole flexural resistance of the member. Thus, the shear reinforcement of the concrete should be designed to withstand large shear forces. At the same time, the span to depth ratio of the beam is small enough to ensure that the failure due to bending is more possible than that due to shear. Although the lower the flexural reinforcement ratio is, the more likely the failure will be in bending. However, ACI-318M-05 still suggests a minimum reinforcement ratio for a regular member design. Therefore, the flexural reinforcement ratio ρ should also be not less than the minimum reinforcement ratio $\rho_{\min}$ (for more details see Appendix B). Based on the these reasons, 2- Φ 8 mm bars are used as the main flexural reinforcement for beams and 4- Φ 8 mm bars are used as the main reinforcement for the columns. Φ 5 mm @ 40mm stirrups are used as shear reinforcement in the beams, and Φ 5 mm @ 125mm ties are used in the columns. 2- Φ 8 mm bars are also used in the compression area of beams to stirrup support and to cover the negative moment at joints. As shown in chapter three, FRAME1 is statically tested up to failure in a single load cycle, FRAME2 is tested under repeated load of 93.75% of it's ultimate load, FRAME3 is statically tested up to failure in a single load cycle, FRAME4 is statically tested up to failure in a single load cycle and FRAME5 is statically tested to 30% of it's ultimate load (first crack load) and then it was repaired and loaded up to failure load. The test procedures for each frame can be described as follows:

6.2) FRAME1 (FR₁)

The first tested frame was the control frame. It was failed in flexure mode. As the load increased up to (10 kN), the first tension crack started at the mid-span and the joints in longitudinal direction of the frame. Then, the cracks were extended vertically in the longitudinal beams. As the load continued increasing, more flexural cracks developed and after the load reached (40 kN), the load stopped to increased and dropped down in a constant rate until zero. This process illustrated in Figures (6.1), (6.2) and (6.3) by marking the tip ends of the cracks for each load step.

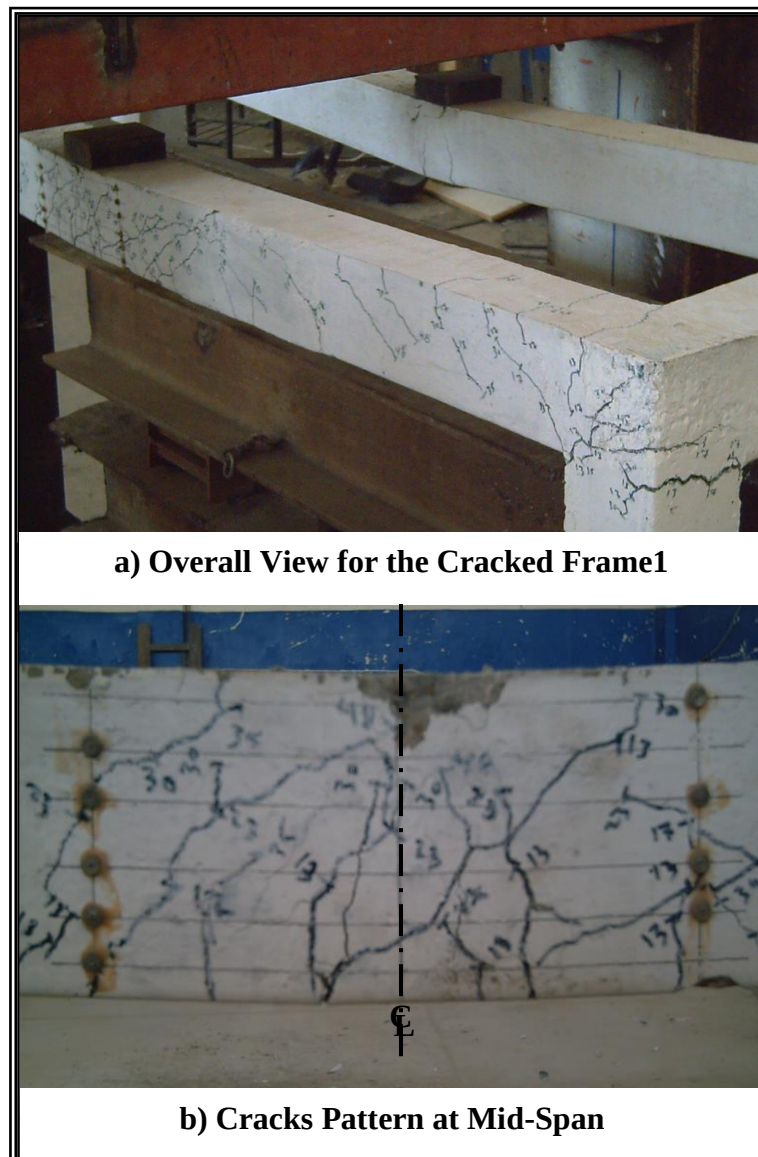


Figure (6.1) Mode of Failure for Tested FRAME1

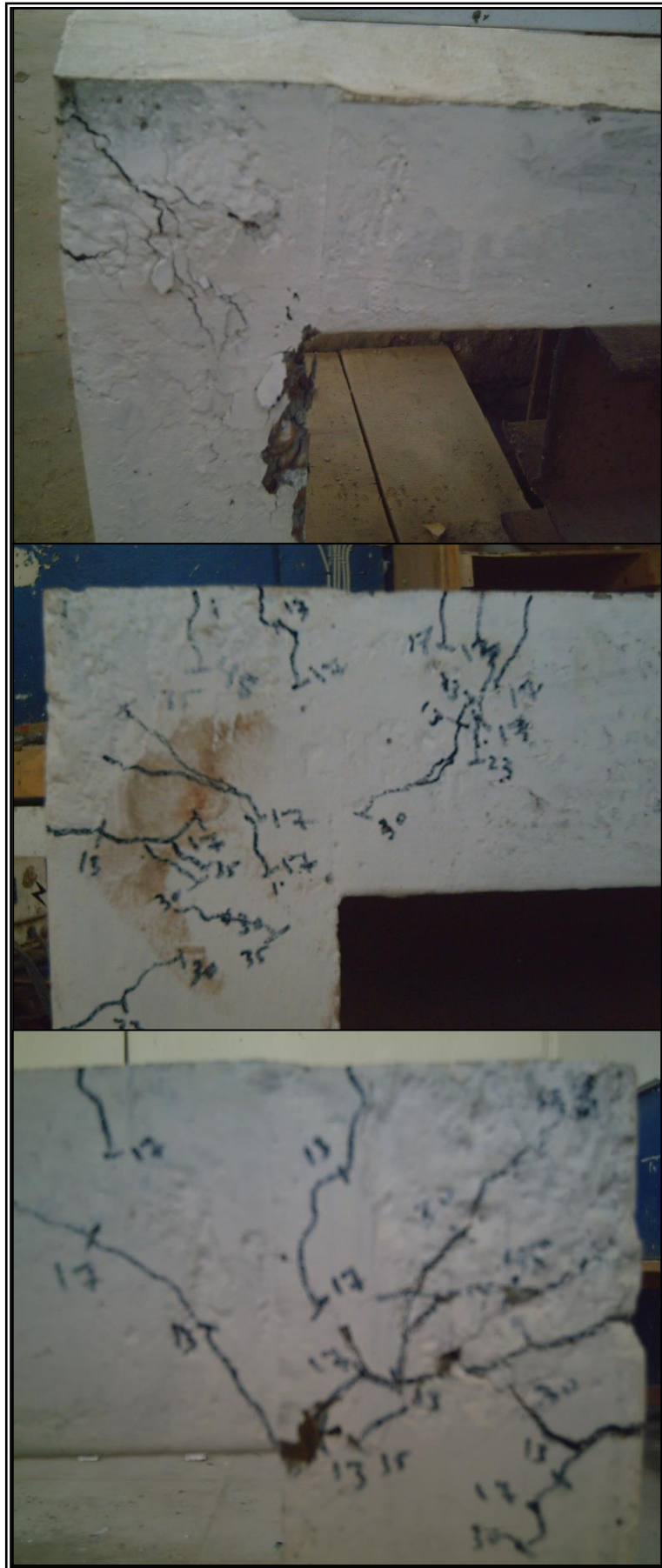


Figure (6.2) Failure of Joints for Tested FRAME1

6.2.1) Load Deflection Results

The specimen **FR₁** was loaded with a monotonic static load up to failure. The load is applied at middle of the long beams simultaneously. The loaded frame has fixed supports at the bases of all columns. The recorded data are plotted in some figures. Figure (6.3) shows the total load versus the mid-span deflection and as shown in this figure the curve can be divided into three stages, the first stage represents the elastic range (uncracked region) that approximately ended at (10 kN). The second stage represents the region beyond the initial cracked to the peak ultimate load (40 kN), and finally the region beyond the failure is represented by the decreasing curve until zero load with a residual deflection of (30 mm).

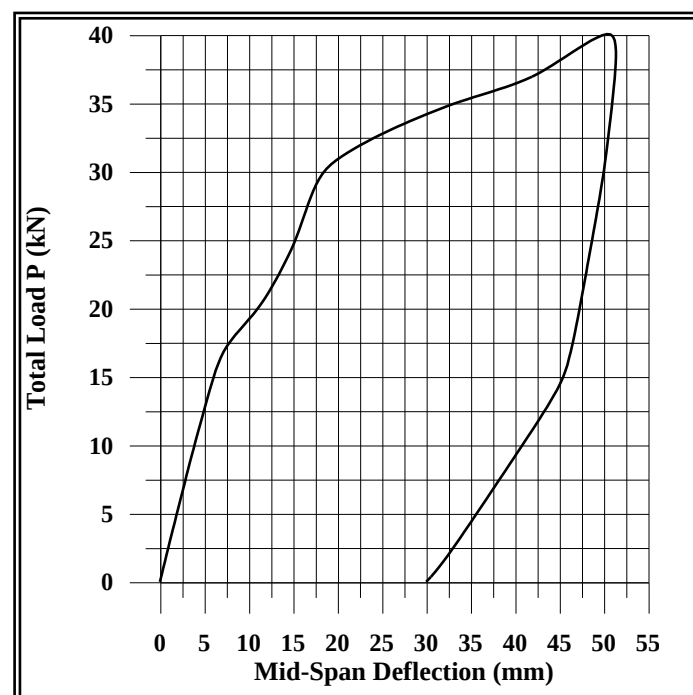


Figure (6.3) Load versus Longitudinal Mid-Span Deflection of FRAME1

Also from the recorded data, the deflected shape along the longitudinal beam of frame is plotted as shown in Figure (6.4) and it can be noticed that the deflection at mid-span increases from (3.88mm) at (10 kN) to (50.8 mm) at (40 kN) because the steel bars reach their yielded point at (30kN). Then the slope of the curve decreased rapidly after this point.

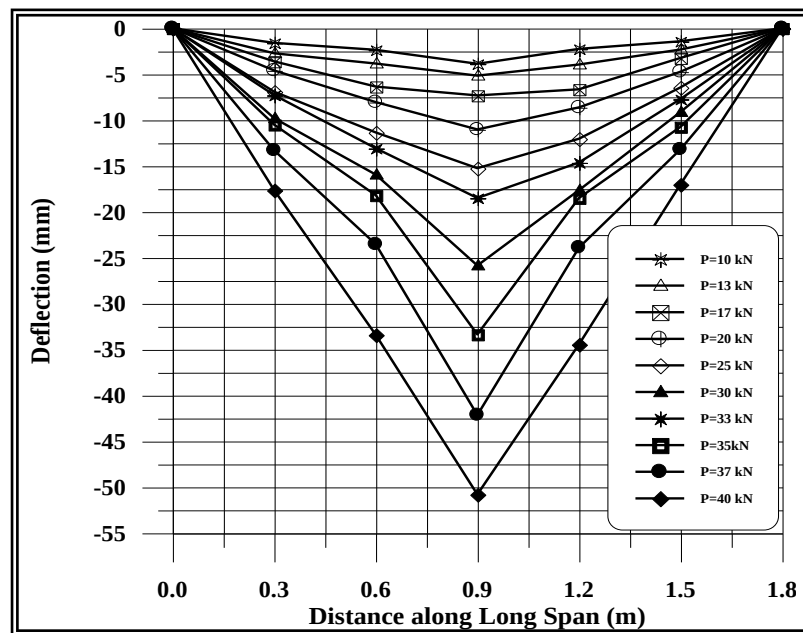


Figure (6.4) Deflection Profile along the Beam of FRAME1

6.2.2) Mid-Span Strain Results

The strain distribution along the depth of the cross section had been measured by strain gages (Demec gages) using six pairs of demec points. These gages were placed on both sides of the center line of long span beam with a distance between each pair of (20 cm). The strain variation across the depth of critical section at different loading level is illustrated in Figure (6.5). Also the variation of strain in certain depth is plotted in Figure (6.6).

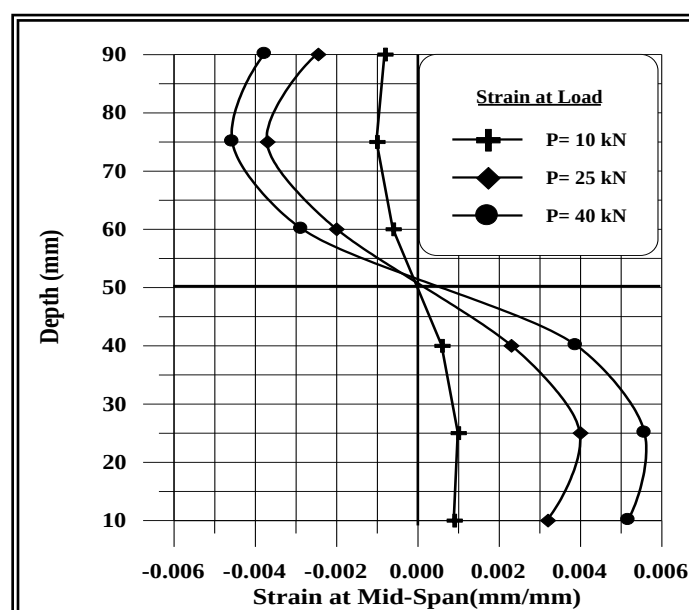


Figure (6.5) Distribution of Concrete Strain at Different Load Stages of FRAME1

Figure (6.5) shows the variation of strain at mid-span across the depth at different load stages, as shown in this figure the variation of strain across the depth is nonlinear because a crack was occur at a load level of (10 kN). At the same time the neutral axis shifted towards the compression zone. That mean the crack really occur at this load and the frame is work as it assumed in design. The maximum strain at failure load is (-0.00455) at distance of (75mm) from bottom face of the beam and (0.0056) at distance of (25 mm) from bottom face of the beam.

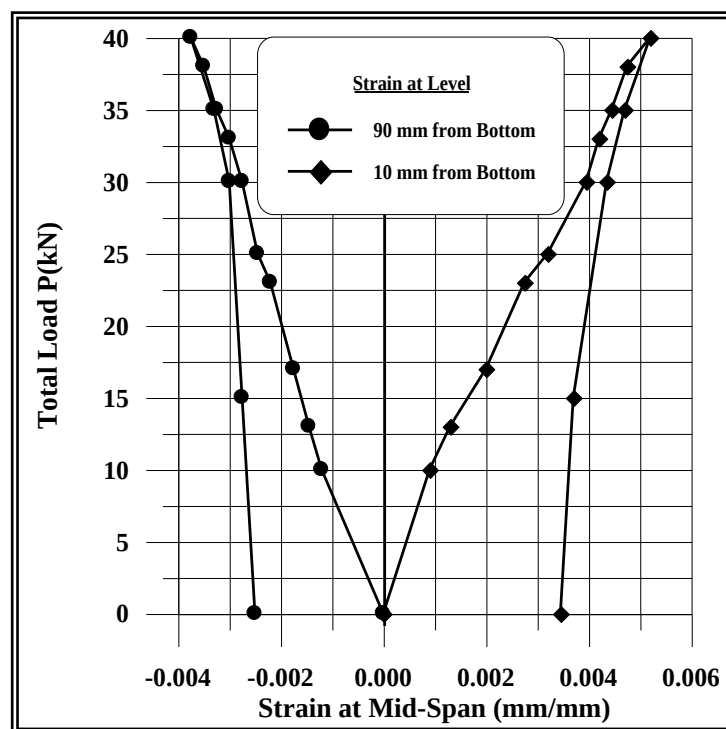


Figure (6.6) Load versus Concrete Strain for Compression and Tension Regions at Longitudinal Mid-Span of FRAME1

Figure (6.6) shows the variation of concrete strains versus load at compression and tension regions (10mm and 90mm from bottom of Longitudinal Mid-Span of FRAME1). From this figure the conclusion is that the relation of the concrete strains with load began linear until the load reach (10kN) then it will become nonlinear, for the load more than the cracking load, also for the same load the tension strain is larger than the compression strain. This mean that the crack became more and micro cracks became large cracks. After dropping in load, the residual strains

The specimen **FR₂** was loaded with a repeated load by (93.75%) of the ultimate load of frame (**FR₁**). The conditions of loading and supports are similar to those for frame (**FR₁**). Frame (**FR₂**) was failed by typical flexure failure in the third cycle at load (37.5 kN). The pattern of cracks is depicted in Figures (6.7) and (6.8). The larger percent of cracks was the flexure cracks that mean the frame was work under reinforced.

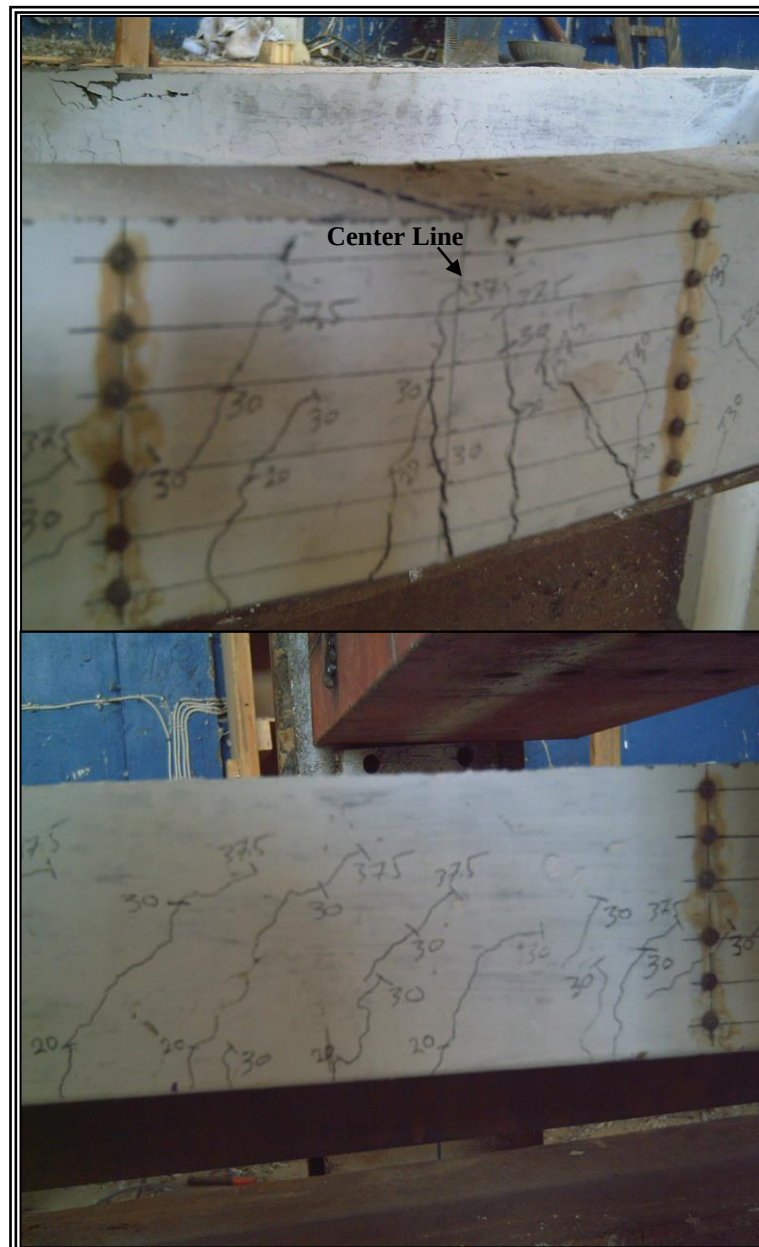


Figure (6.7) Beam Failure of FRAME2



Figure (6.8) Joint Failure of FRAME2

6.3.1) Load Deflection Results

The specimen **FR₂** was loaded with a repeated load up to failure. From the recorded data, some figures are plotted, Figure (6.9) shows the total load versus mid-span deflection which has three cycles and can be divided into three stages, the first one represent the elastic range (uncracked region) that approximately ended at (10 kN). The second stage represented by the region beyond the initial cracking that appears approximately at (10 kN), and finally the region beyond a load level of

(37.5 kN) that is represented by a decreasing curve until zero and continues for three cycles. At the first cycle the cracks are created at a certain level that made the deformation. The development of the cracks at the second cycle is in a low degree because the steel is not yielded yet. Finally, the frame failed during the third cycle and the steel reached its yield point at mid-span and at joints.

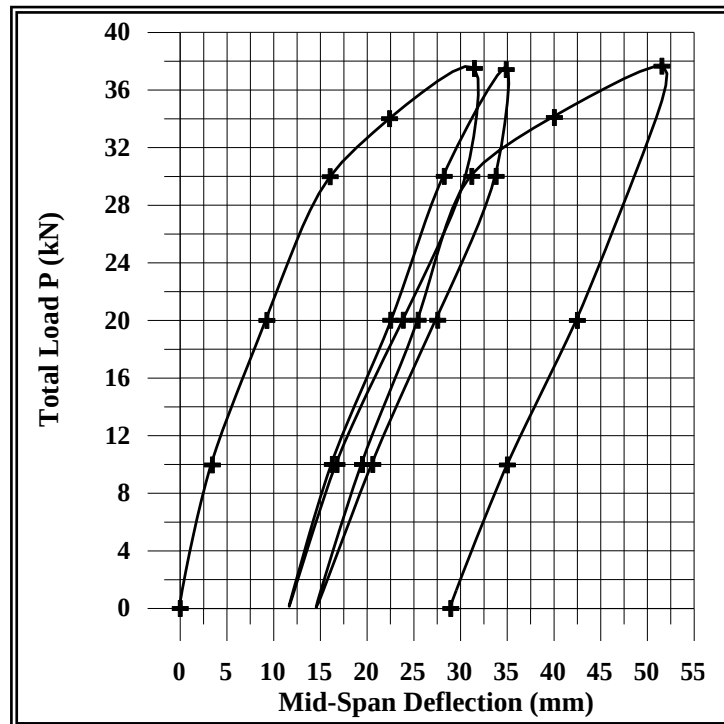


Figure (6.9) Load versus Longitudinal Mid-Span Deflection of FRAME2
6.3.2) Mid-Span Strain Results

From the recorded data, Figures (6.10) and (6.11) are obtained to trace the strain distribution across the depth of the cross section of the longitudinal beam in the frame **FR₂**.

Figure (6.10) shows the variation of strains at mid-span across the depth of the cross section at different loading stages for the first cycle. As shown in this figure the variation of strain across the depth is linear at a load level of (5 kN) because no cracks appear at this load level. Beyond this load, the cracks are initiated then the curve became nonlinear at a load level of (10 kN). At the same time the neutral axis is shifted towards the compression zone (i.e. the cracks propagate at this load level). The

maximum strain at failure load is (-0.003) at distance of (75mm) from bottom face of the beam and (0.0026) at distance of (25 mm) from bottom face of the beam.

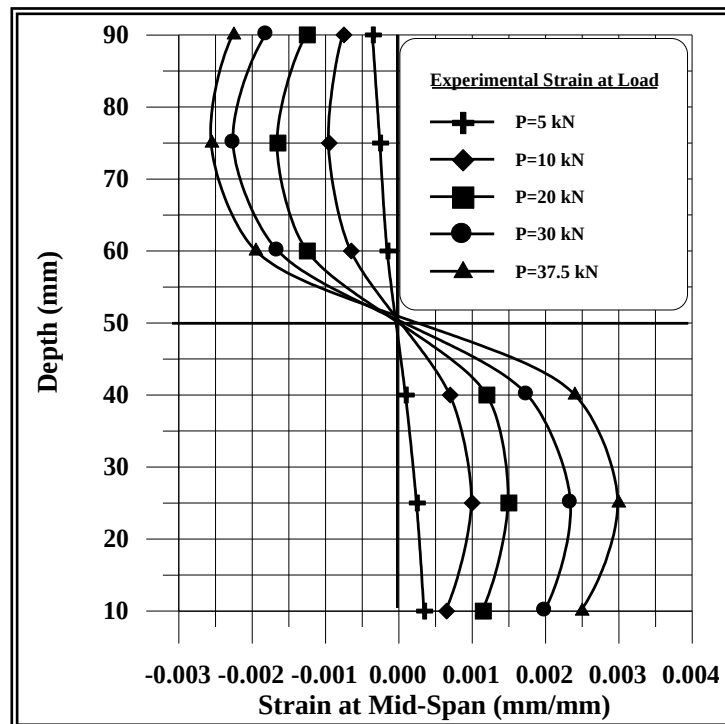


Figure (6.10) Distribution of Concrete Strains at Different Load Stages for the First Cycle of FRAME2

Figure (6.11) shows the variation of strains at different loading stages for compression and tension regions across the depth represented by a distance of (90 mm and 10 mm from the bottom of the beam) respectively. Notice from this figure that the relation of the strains with loading begin linear until (10kN). The relation become nonlinear for the load equal or larger than (10kN), also for the same load the tension strain is larger than the compression strain that mean the cracks became more and micro cracks became large cracks. Notice in this curve that the frame return to zero strain at load zero while the frame **FR₁** has residual strain at zero loads that because the frame **FR₁** is loaded to its ultimate load while the frame **FR₂** is loaded to 93.75% of its ultimate load that mean all the steel work in the frame and the number of cracks no large yet.

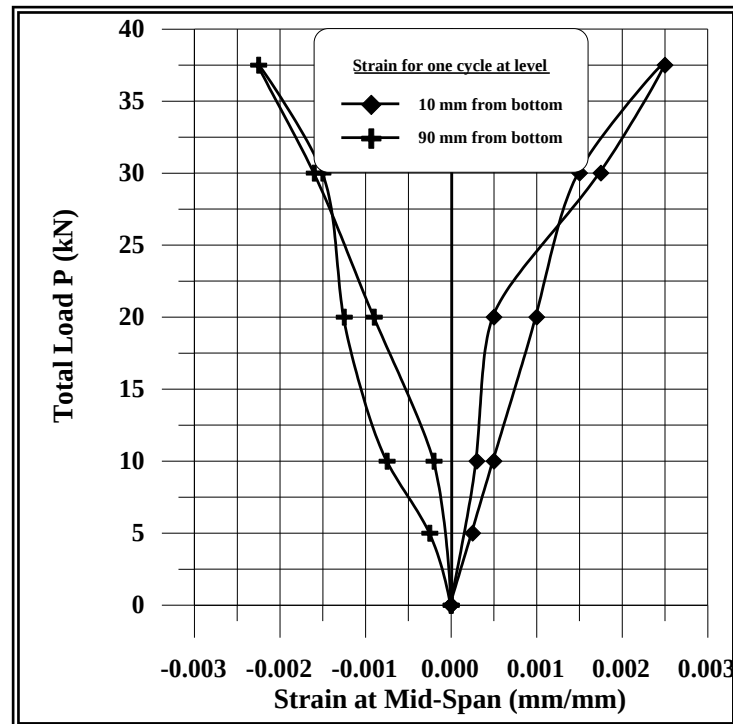


Figure (6.11) Load versus Concrete Strains for compression and Tension regions for the First Cycle of FRAME2

6.4) FRAME3 (FR_{L3})

The frame FR_{L3} is strengthening with one layer of longitudinal CFRP at the bottom of the long span beam with a length of (0.6 m) at two sides of the center line of beam and top of the same beam for a distance of (0.35 m) from its edges. Also it is strengthening with CFRP in the transverse direction (short span beam) at joints work as a confined concrete at both directions. Failure of frame FR_{L3} is a flexure debonding failure as shown in Figure (6.12). The cracks went toward the ends of strengthening and this means the strengthening work correctly to prevent created cracks. Figure (6.13) shows cracks pattern at joints. From recorded data some figures are plotted to investigate the behavior of this frame and compare it with other frames.

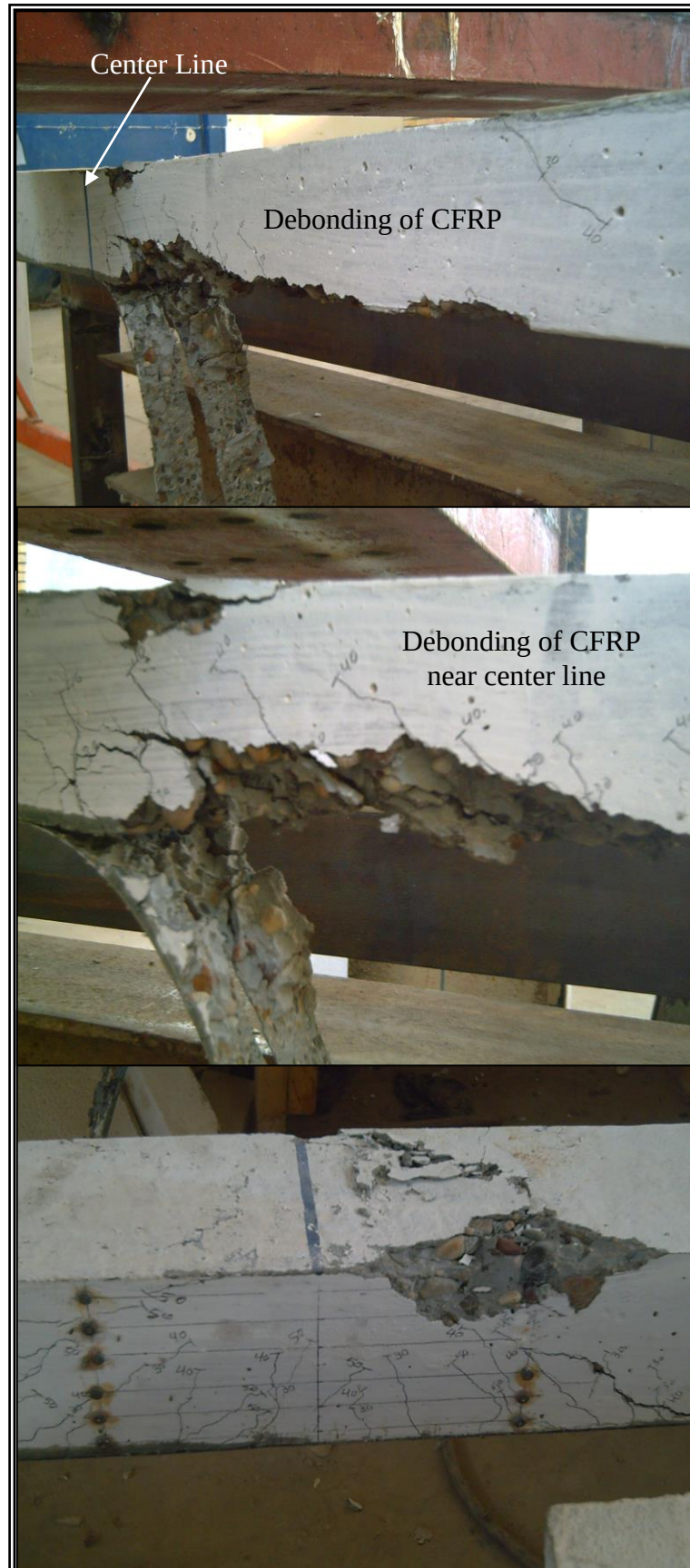


Figure (6.12) Mode of Failure for FRAME3

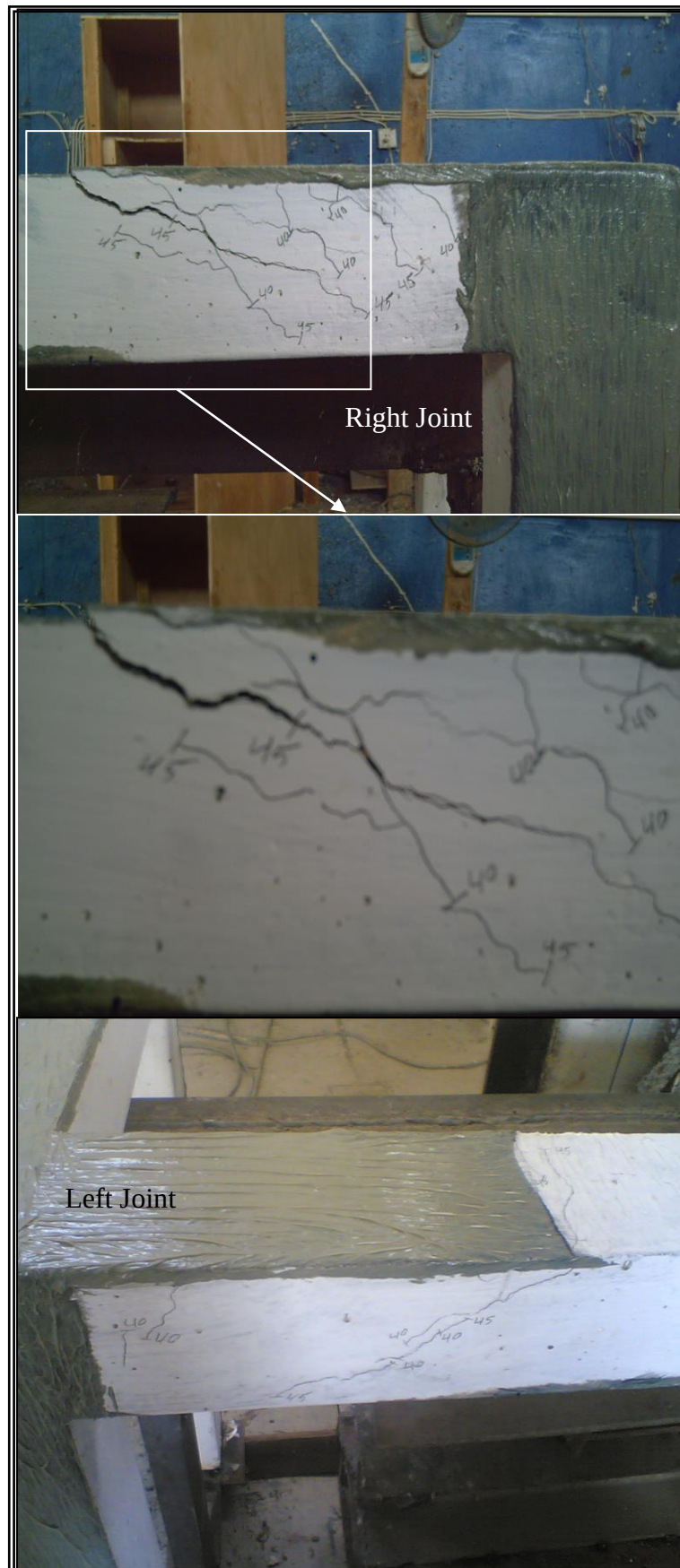


Figure (6.13) Failure of Joint for FRAME3

6.4.1) Load Deflection Results

Frame **FR_{L3}** is loaded with a monotonic static load up to failure. The load is applied at middle of the long span beam at two sides and the frame is fixed at the bottom ends of all columns. Figure (6.14) shows the relation between the applied load and deflection at mid-span of longitudinal beam of frame **FR_{L3}**. As shown in this figure the failure load was (50 kN) and the deflection at peak load was (33.95 mm). After releasing the load to zero the residual deflection was (8.54 mm). The type of strengthening in this frame gives additional strength about (25%) than control frame **FR₁** and releases in deflection at failure about (33%). The decrease in residual deflection after unloading is (71.53%) as shown in Figure (6.14).

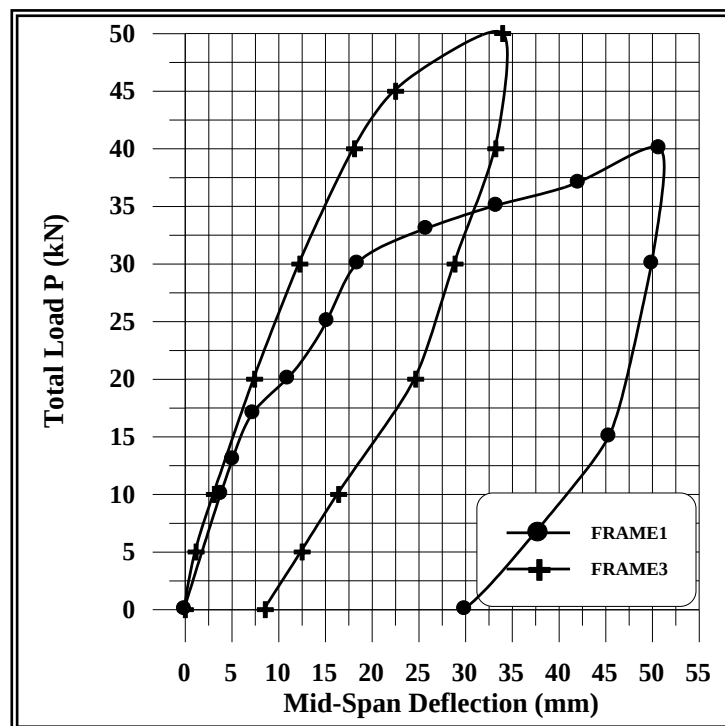


Figure (6.14) Load versus Longitudinal Mid-Span Deflection of FRAME3 and FRAME1

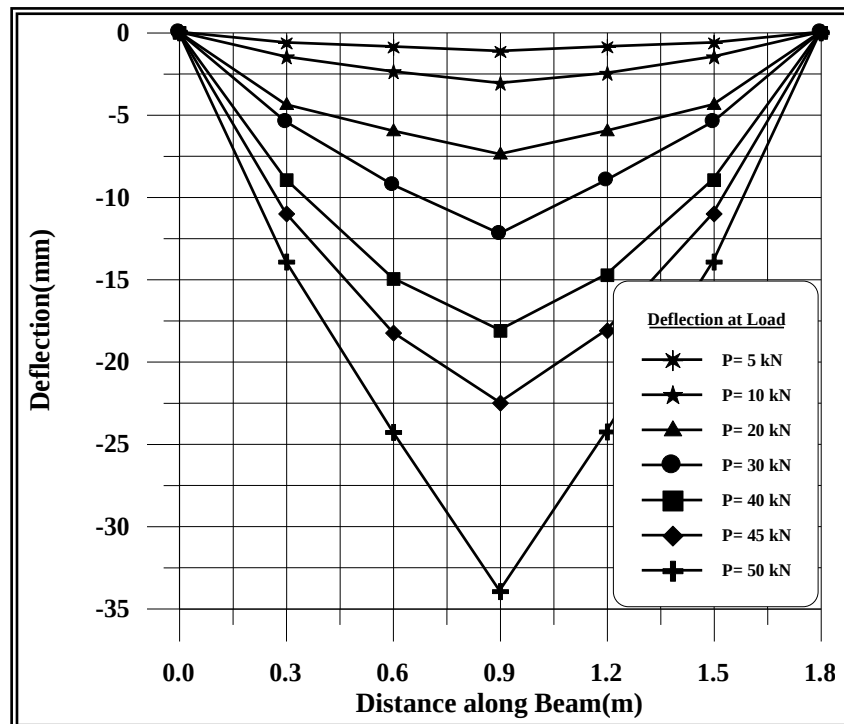


Figure (6.15) Deflection Profile along the Beam of FRAME3

Figure (6.15) shows the deflected shape along the long span beam in FR_{L3} with loading. Figure (6.16) shows a comparison between frame FR_1 and frame FR_{L3} with regard to the deflection in the direction of the long span beam.

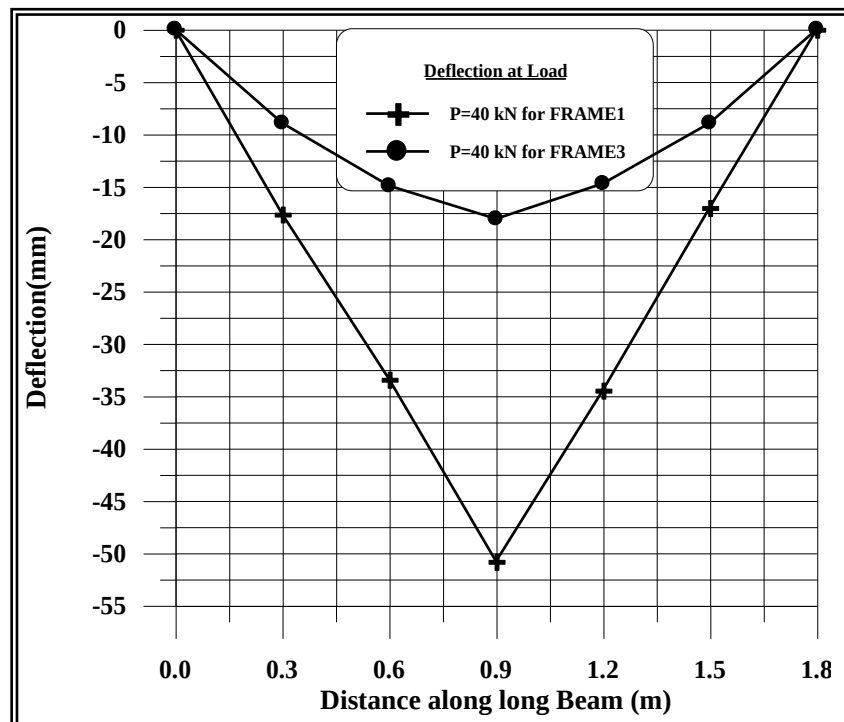


Figure (6.16) Deflection Profile Comparison at Load (40 kN) along the Beam between FRAME1 and FRAME3

6.4.2) Mid-Span Strain Results

The strain distribution at mid-span cross section of the longitudinal beam of frame **FR_{L3}** is recorded from test and presented in Figures (6.17) and (6.18) to trace the strains across the depth of long span beam in the **FR_{L3}** and Figure (6.19) to investigate the strain distribution of **FR_{L3}** with the load.

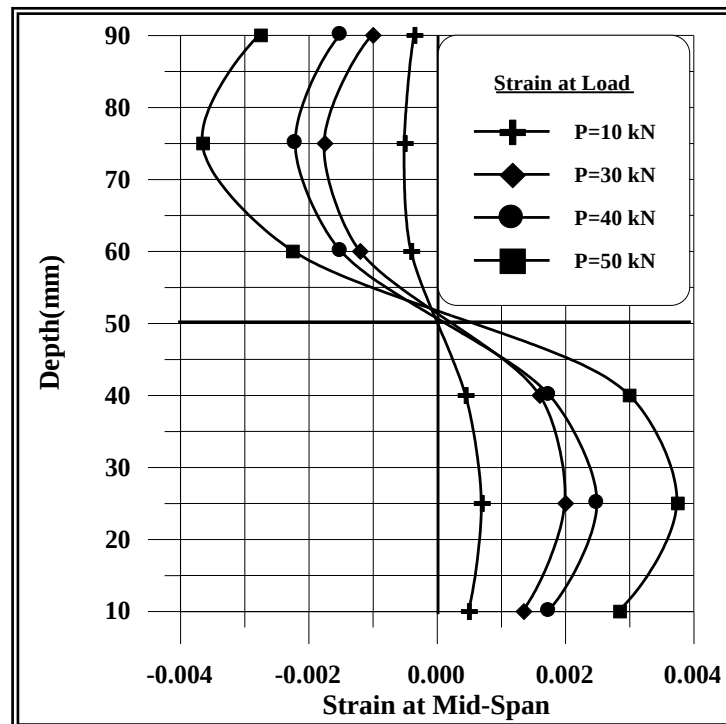


Figure (6.17) Distribution of Concrete Strains at Different Load Stages for FRAME3

Figure (6.17) shows the variation of strain at mid-span across the depth at different loading stages. As shown in this figure, the variation of strain across the depth is nonlinear at a load level (10 kN). At the same time the neutral axis is shifted toward the compression zone. The maximum strain at failure load is (-0.00365) at distance of (75mm) from the bottom face of the beam and (0.00375) at distance of (25 mm) from the bottom face of the beam. The percentage decrease in strain value at failure load between **FR_{L3}** and **FR₁** is (19.78%) in compression and (33.04%) in tension as shown in Figure (6.18). Figure (6.19) shows the development of strains with load at compression and tension regions represented by distances

(90mm and 10mm) from the bottom of the beam for frame **FR₁** and frame **FR_{L3}** respectively.

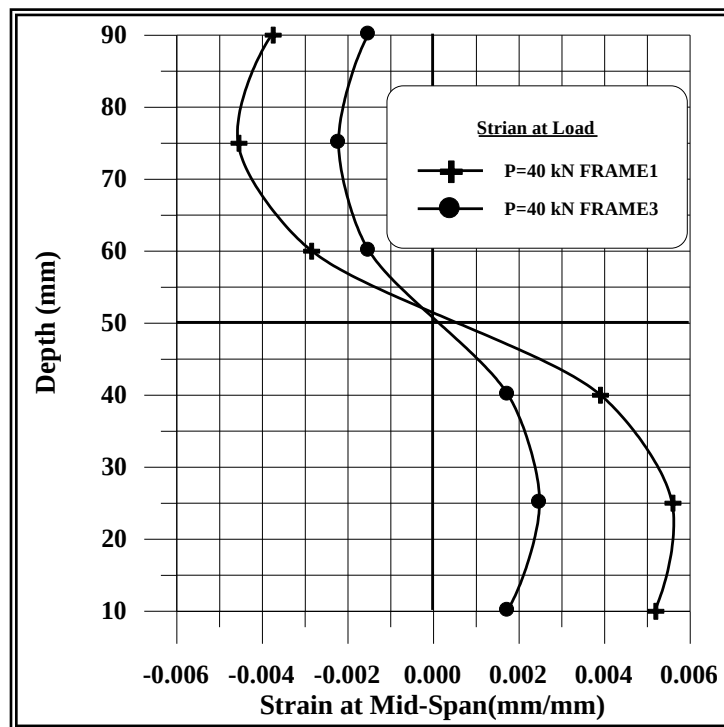


Figure (6.18) Comparison of Concrete Strain Distribution across the depth between FRAME1 and FRAME3 at Failure Load

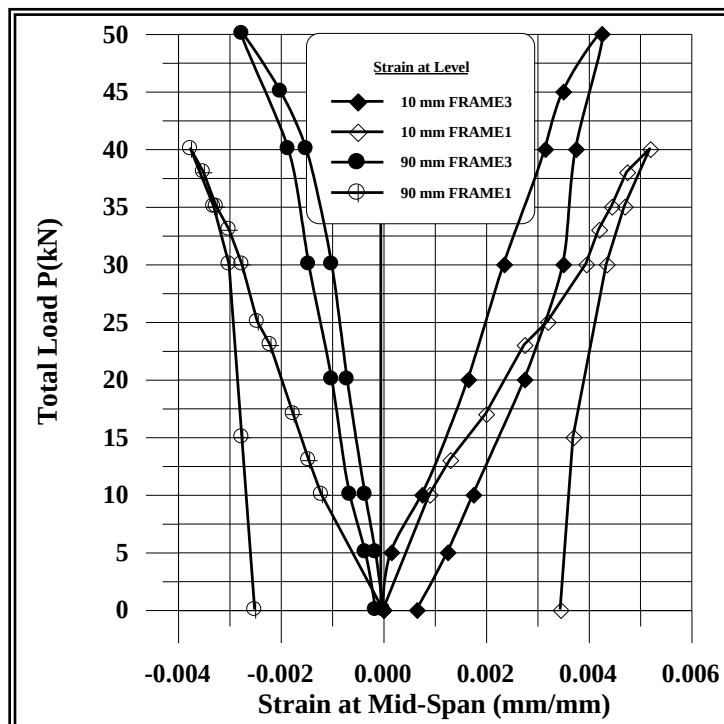


Figure (6.19) Comparison of Concrete Strain at Compression and Tension Regions of FRAME1 and FRAME3

6.5) FRAME4 (FR_{LT4})

Frame **FR_{LT4}** is strengthened with one layer of longitudinal CFRP at the bottom of the long span beam with a length of (0.6 m) at two sides of center line of beam (work to resistance the positive moment) and top of the same beam for distance (0.35 m) from its edges (work to resistance the negative moment). In addition to this, strengthening CFRP fabrics put in the transverse direction at ends and middle of longitudinal CFRP fabrics in order to prevent earlier debonding in longitudinal CFRP fabrics. Also strengthening with CFRP is in the transverse direction (short span beam) at joints work as a confined CFRP at joints in both directions. Frame **FR_{LT4}** fractured and flexure debonding failure in beam and debonding failure in column occurred as shown in Figures (6.20) and (6.21). The cracks appear at the region located between transverse CFRP because of the confinement of concrete therefore debonding is prevented. Finally, fracture occurs at the region between confined of center line and confined of end of longitudinal CFRP.



Figure (6.20) Mode of Failure for FRAME4

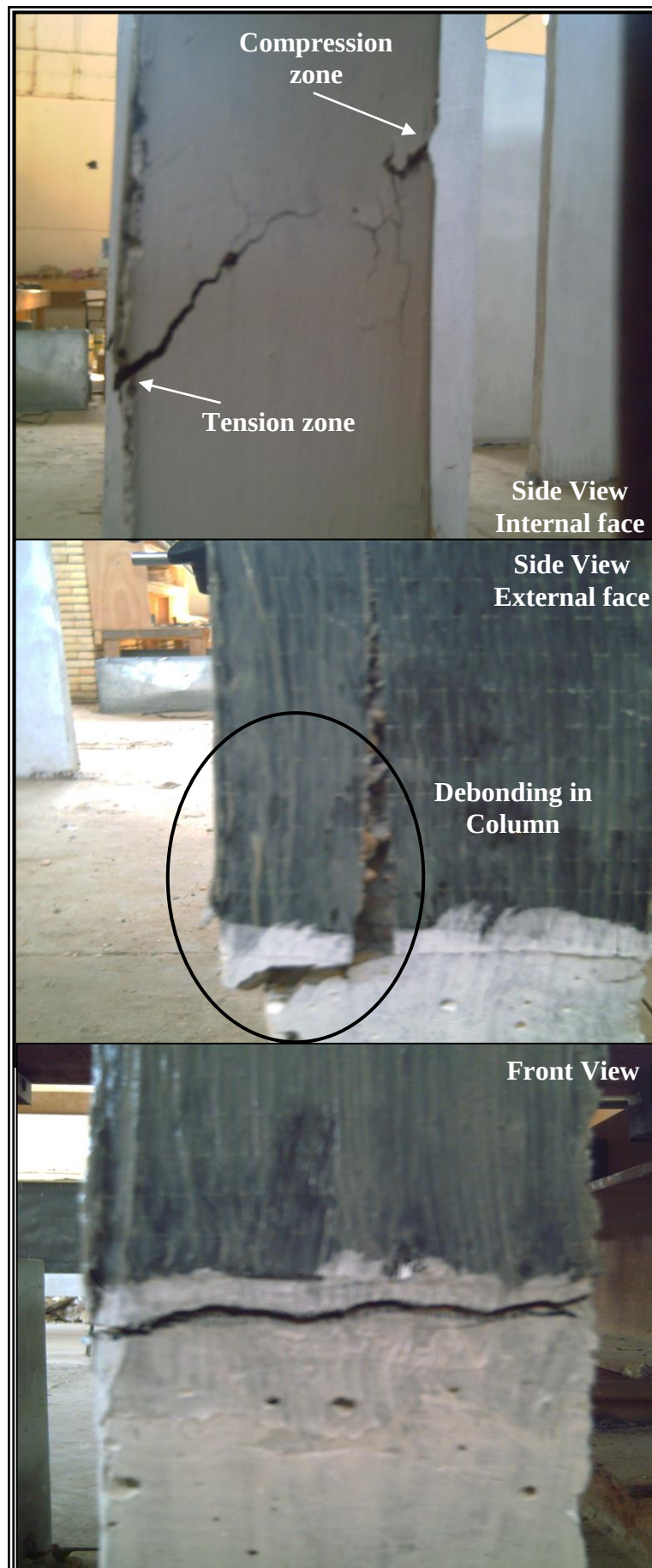


Figure (6.21) Column Failure of FRAME4

6.5.1) Load Deflection Results

Frame **FR_{LT4}** is loaded with a monotonic static load up to failure. The load is applied at middle of the long span beam at two sides and the frame is which was fixed at bottom ends of all columns. Figure (6.22) shows the relation between load and deflection at mid-span of **FR_{LT4}**. As shown in this figure, the failure load (60 kN) with a maximum deflection of (49.23 mm) and after releasing the load to zero the residual deflection was (23.13 mm). The type of strengthening of this frame gives additional strength of about (50%) larger than that of the control frame **FR₁** and the residual deflection at failure is increased by about (3.1%). The release in residual deflection after unloading is (22.9%) as shown in Figure (6.23). This behavior of frame **FR_{LT4}** is because the high resistance of the transverse CFRP fabrics to prevent the deboding occurs. By adding more transverse CFRP fabrics the resistance will increase more as shown later in frame FRAME5.

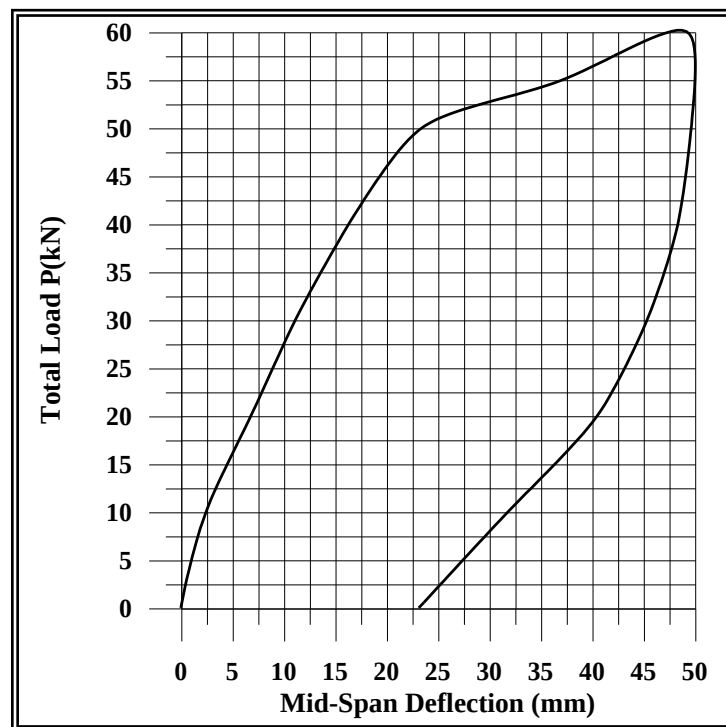


Figure (6.22) Load versus Longitudinal Beam Mid-Span Deflection of FRAME4

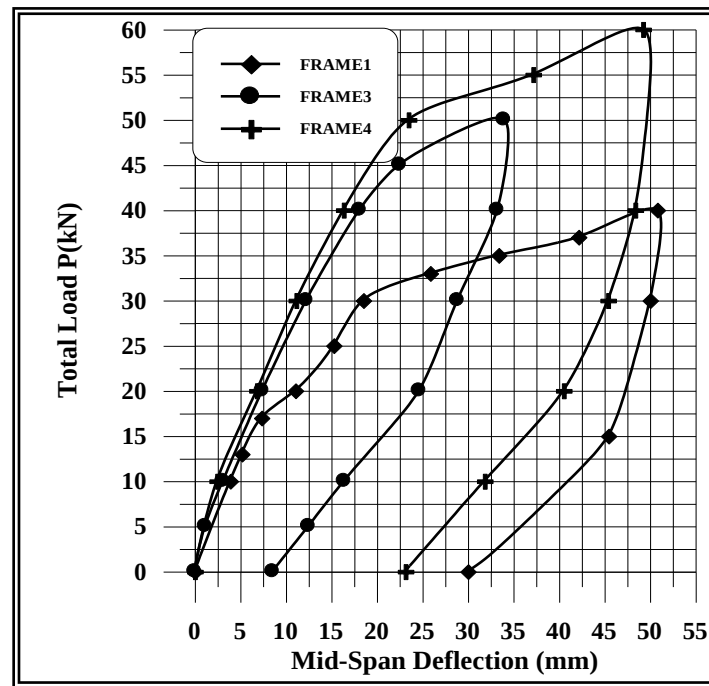


Figure (6.23) Load versus Longitudinal Beam Mid-Span Deflection of FRAME4, FRAME3 and FRAME1

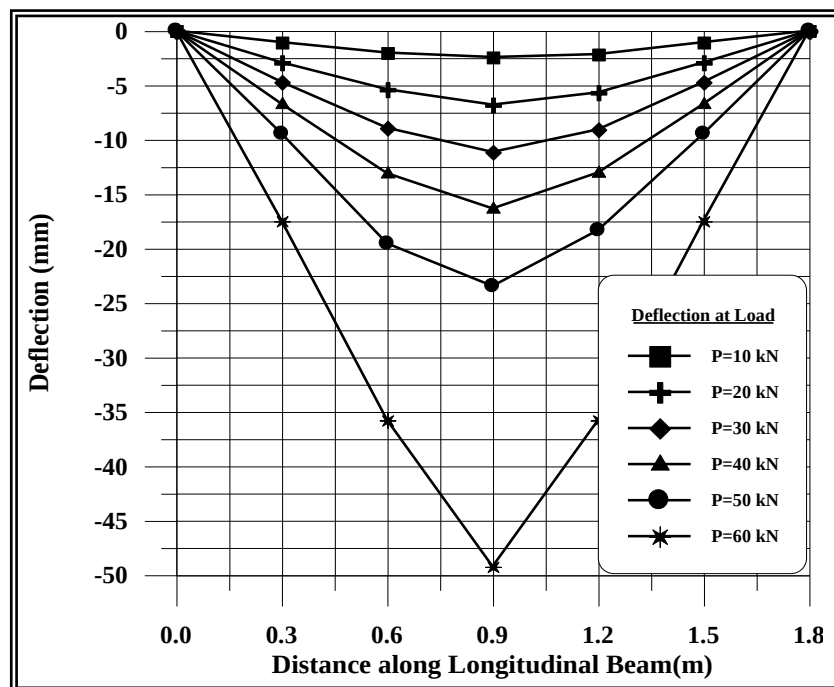


Figure (6.24) Deflection Profile along the Beam of FRAME4

Figure (6.24) shows the deflected shape along the long beam in frame FR_{LT4} with increasing loads. Figure (6.25) shows the comparison between frames FR_1 , FR_{L3} and FR_{LT4} in deflection with regard to the direction of the long span beam at a load level of (40 kN) (failure load of FR_1).

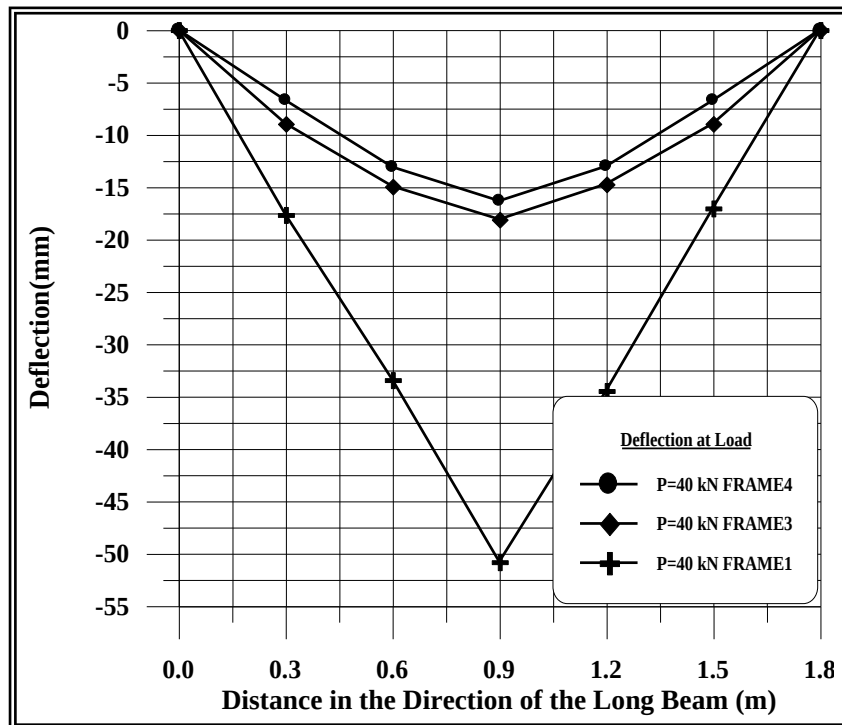


Figure (6.25) Deflection Profile Comparison at Load (40 kN) along The Beam between FRAME1, FRAME3 and FRAME4

6.5.2) Mid-Span Strain Results

The strain distribution at mid-span cross section of the longitudinal beam of frame FR_{LT4} is recorded from test and presented in Figures (6.26) and (6.27) to trace the strains across the depth of long span beam.

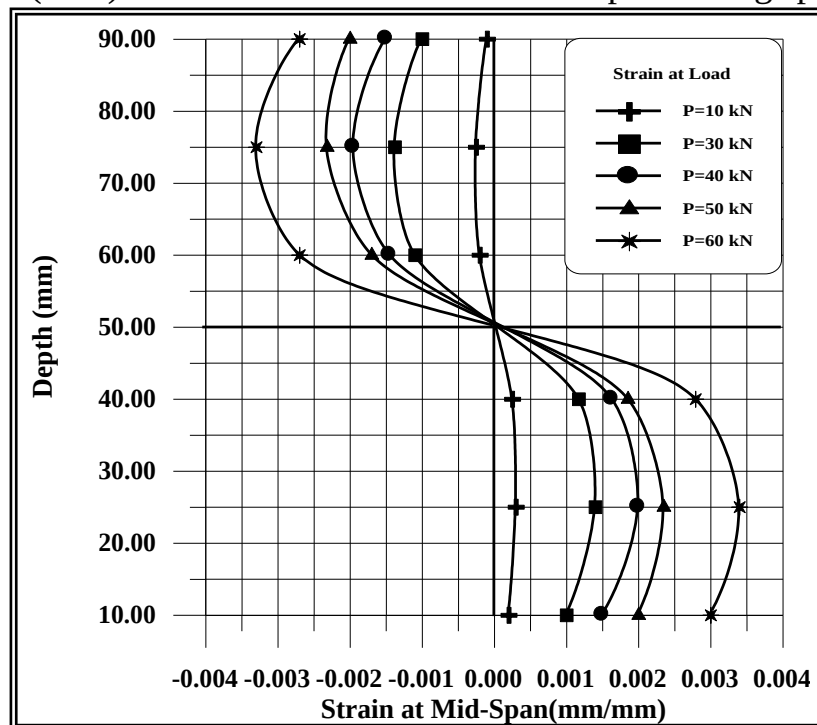


Figure (6.26) Distribution of Concrete Strains at Different Loading Stages for FRAME4

Figure (6.26) shows the variation of strain at mid-span cross section across the depth at different load stages. As shown in this figure, the variation of strain across the depth is nonlinear at a load level of (10 kN). At the same time the neutral axis is shifted toward the compression zone. The maximum strain at failure load is (0.0033 compression) at distance of (75mm) from bottom face of the beam and (0.0034 tension) at distance of (25 mm) from bottom face of the beam. The percentage decrease in strain value at failure load between **FR_{LT4}** and **FR₁** is (27.47%) in compression and (39.29%) in tension. Figure (6.27) shows the comparison among frames **FR₁**, **FR_{L3}** and **FR_{LT4}** in strain along the depth at a load level of (40 kN) (failure load of frame **FR₁**). Figure (6.28) shows the development of strain with increasing load at two levels (10mm, 90mm) from the bottom of the beam for frames **FR₁**, **FR_{L3}** and **FR_{LT4}**.

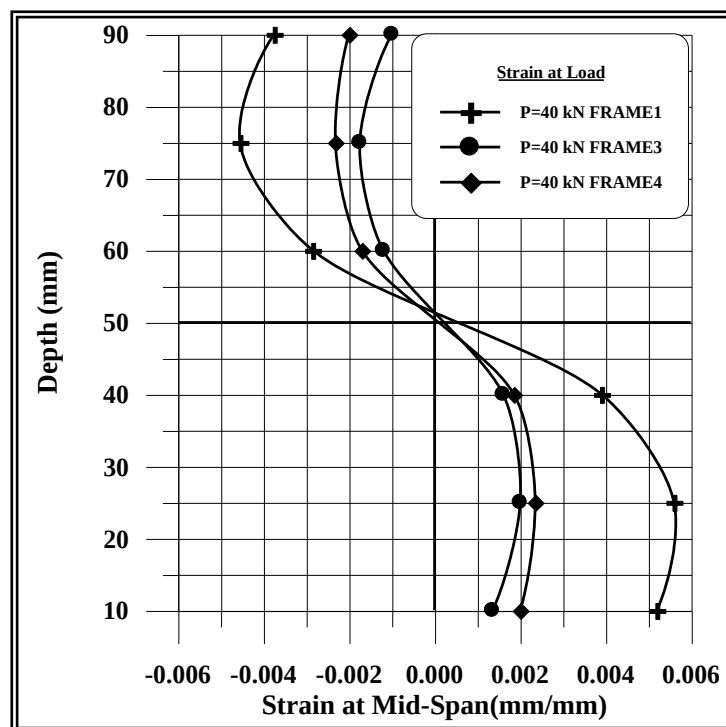


Figure (6.27) Comparison of Concrete Strain Distribution across the depth between FRAME1, FRAME3 and FRAME4

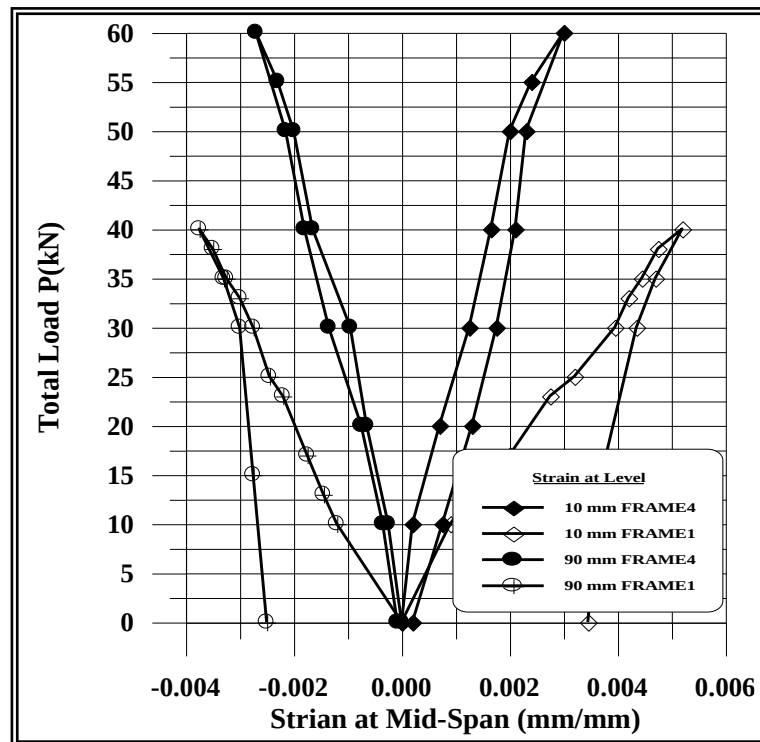


Figure (6.28) Comparison of Concrete Strain at Compression and Tension Regions of FRAME1 and FRAME4

6.6) FRAME5 (FR_{TL5})

Frame FR_{TL5} is repaired after loading up to 30% of the ultimate load of frame FR_1 . the frame is repaired with one layer of longitudinal CFRP at the bottom of the long span beam with a length (0.6 m) at two sides of the center line of beam (work to resistance positive moment) and top of the same beam for a distance (0.35 m) at its edges (work to resistance negative moment). In addition to this strengthening, CFRP fabrics are putted in the transverse direction at ends and middle of span at each (30 cm) of longitudinal CFRP fabrics in order to prevent early deboning in the longitudinal CFRP fabrics. Also repairing with CFRP in the transverse direction (short span beam) at joints works as a confined CFRP at joints in both directions. The failure of the frame FR_{TL5} is flexure debonding failure and fracture in CFRP fabrics. As shown in Figure (6.29). The cracks appear in the region between transverse CFRP because of the confinement. Figure (6.30) shows the behavior and failure of columns in frame FR_{LT5} .

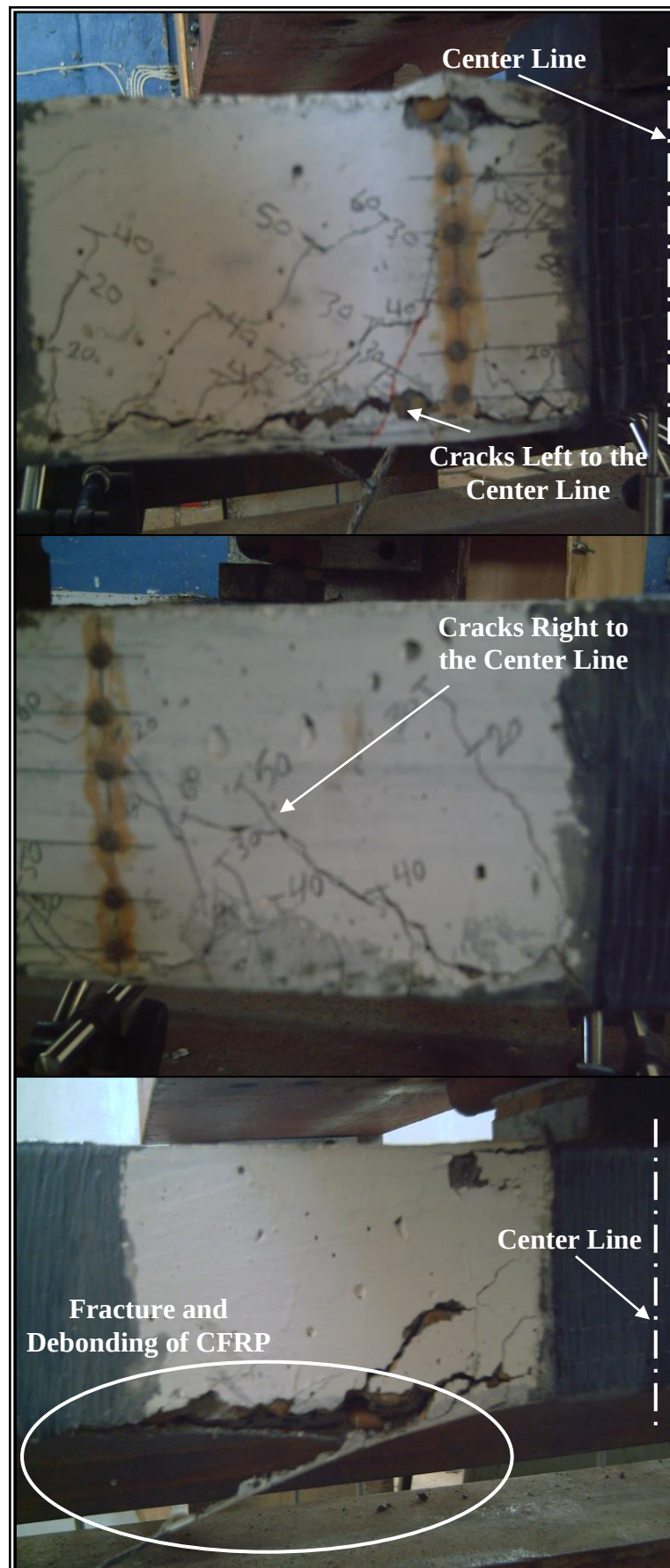


Figure (6.29) Mode of Failure for FRAME5

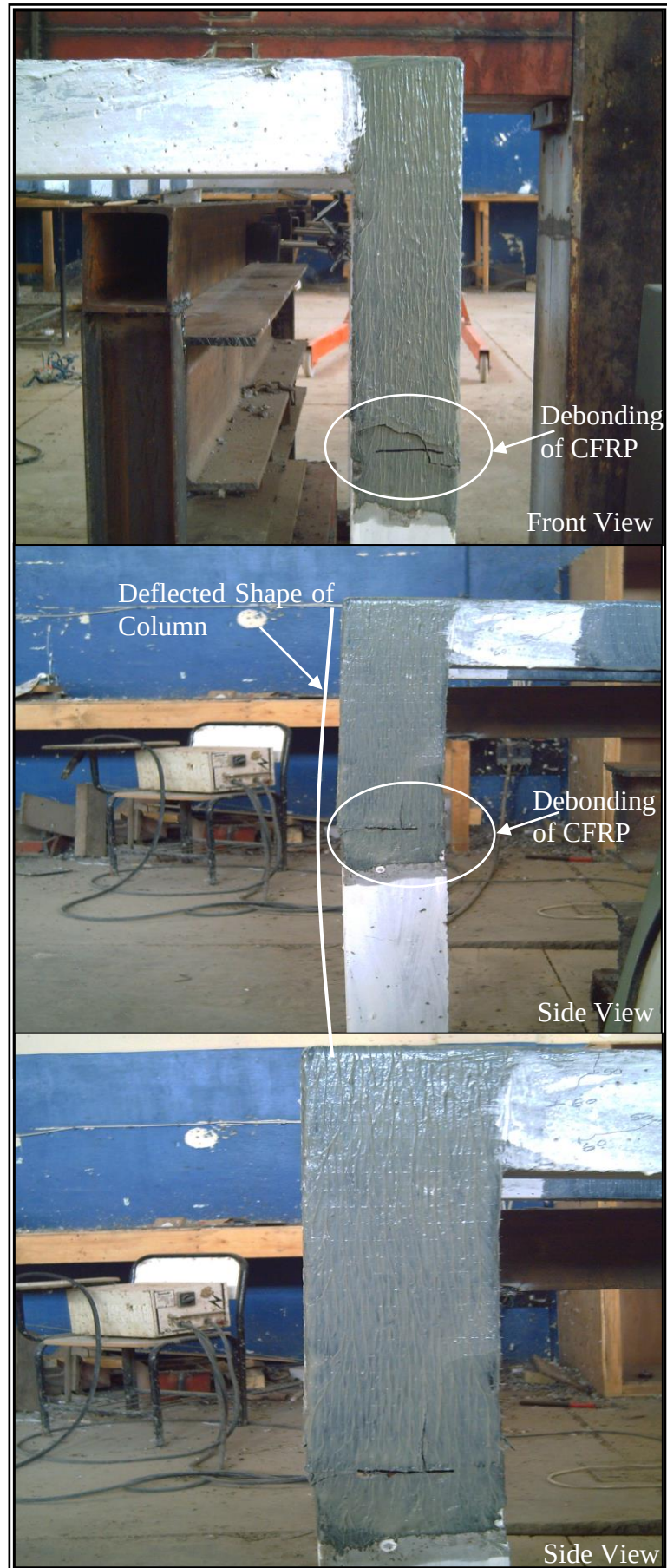


Figure (6.30) Column Failure of FRAME5

And as shows in Figure (6.30), the evident deflection that appears in column because the high bending created at columns. Also the debonding of CFRP occurs at end of CFRP fiber.

6.6.1) Load Deflection Results

Frame **FR_{LT5}** first is loaded with a monotonic static load up to 30% of the ultimate load of frame **FR₁** (first crack loading). Then the frame is repaired with CFRP fabrics as described in section (6.6) and loaded up to failure. The load is applied at middle of the long span beam at two sides and the loaded frame is fixed at the bottom of all columns. Figure (6.31) shows the relation between load and deflection at mid-span of frame **FR_{LT5}**. As shown in this figure the failure load is (70 kN) and the deflection at peak load is (52.6 mm). After release load, the residual deflection was (25.65 mm). The type of repairing with CFRP fabric in this frame results in additional strength of about (75%) as compared with the control frame **FR₁**. The deflection at failure is increased by about (3.4%). The residual deflection after unloading is increased by about (14.67%) as shown in Figure (6.31).

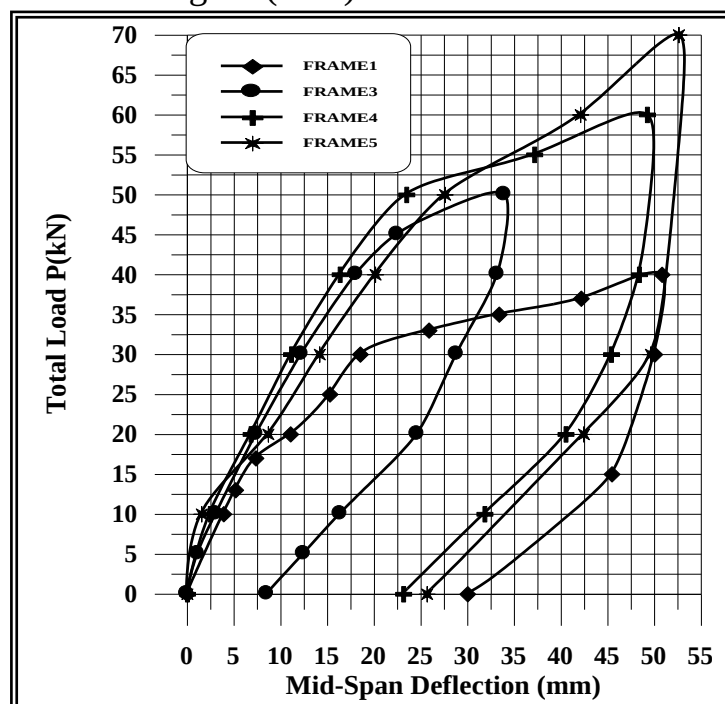


Figure (6.31) Load versus Longitudinal Beam Mid-Span Deflection of FRAME5, FRAME4, FRAME3 and FRAME1

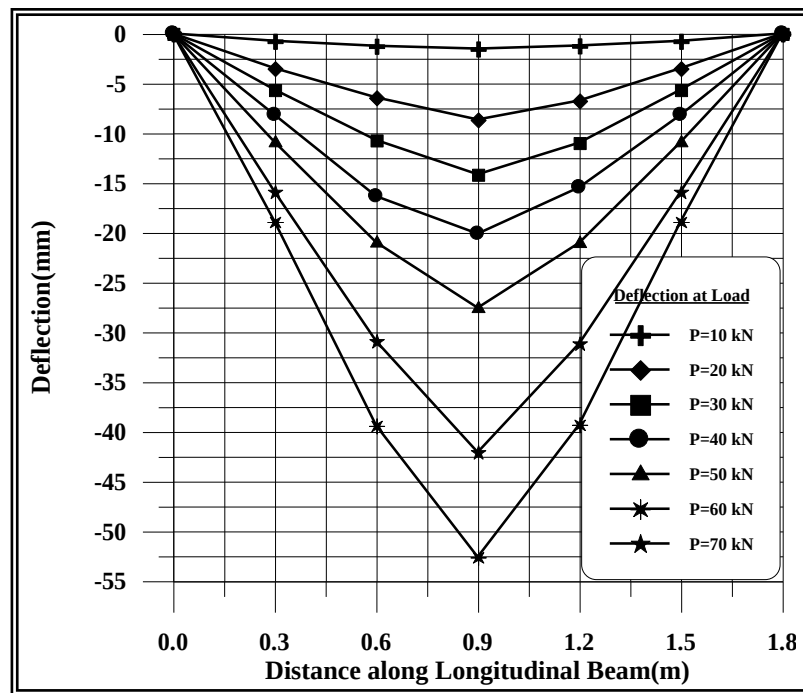


Figure (6.32) Deflection Profile along the Beam of FRAME5

Figure (6.32) shows the development of deflection in the direction of the long span beam in **FR_{LT5}** with increasing loads. Figure (6.33) shows the comparison among frames **FR₁**, **FR_{L3}**, **FR_{LT4}** and **FR_{LT5}** in deflection in the direction of the long span beam at a load level of (40 kN) (failure load of **FR₁**).

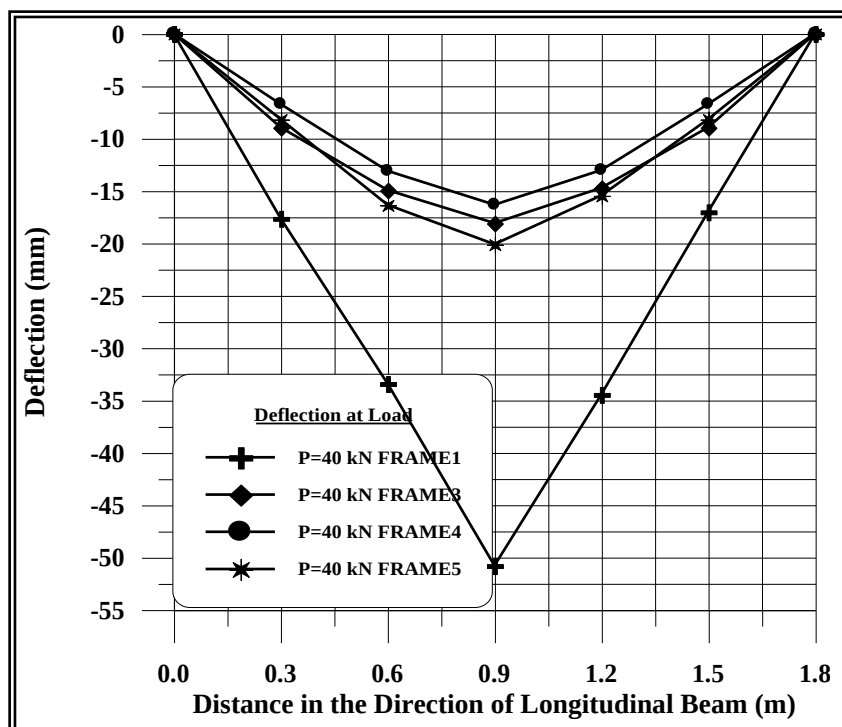


Figure (6.33) Deflection Profile Comparison at Load (40 kN) along the Beam between FRAME1, FRAME3, FRAME4 and FRAME5

6.6.2) Mid-Span Strain Results

As for the other frames, the strain distribution in FRAME5 was recorded from test and is presented in Figures (6.34) and (6.35) to trace the strain distribution across the depth of long span beam in the FRAME5.

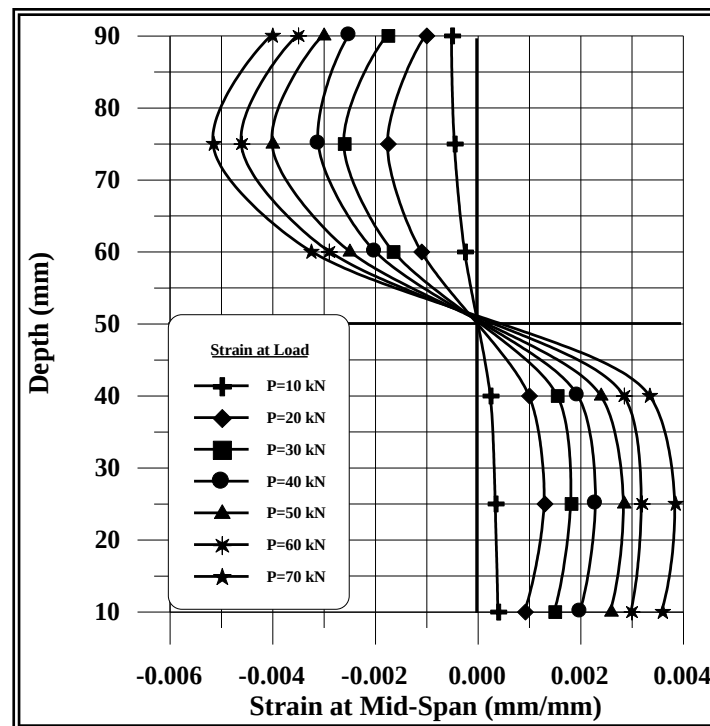


Figure (6.34) Distribution of Concrete Strain at Different Load Stages for FRAME5

Figure (6.34) shows the variation of strain at mid-span cross section across the depth at different loading stages. As shown in this figure the variation of strain cross the depth is nonlinear at a load level of (10 kN). At the same time the neutral axis is shifted toward the compression zone. The maximum strain at failure load is (0.00515 compression) at a distance of (75mm) from the bottom face of the beam and (0.00385 tension) at a distance of (25 mm) from the bottom face of the beam. The increase and decrease in the maximum strain at failure load of frame **FR_{LT5}** with respect to frame **FR₁** is (13.19%) in compression and (31.25%) in tension respectively as shown in Figure (6.35). Figure (6.36) shows the development of strain with increasing load at two levels

(10mm, 90mm) from the bottom of the beam for frames **FR₁**, **FR_{L3}**, **FR_{LT4}** and **FR_{LT5}**.

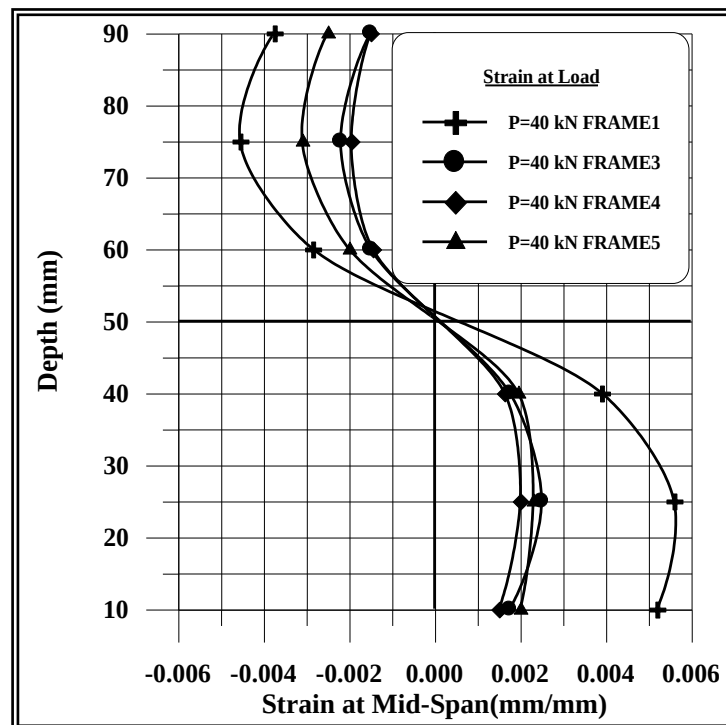


Figure (6.35) Comparison of Concrete Strain Distribution across the depth between FRAME1, FRAME3, FRAME4 and FRAME5

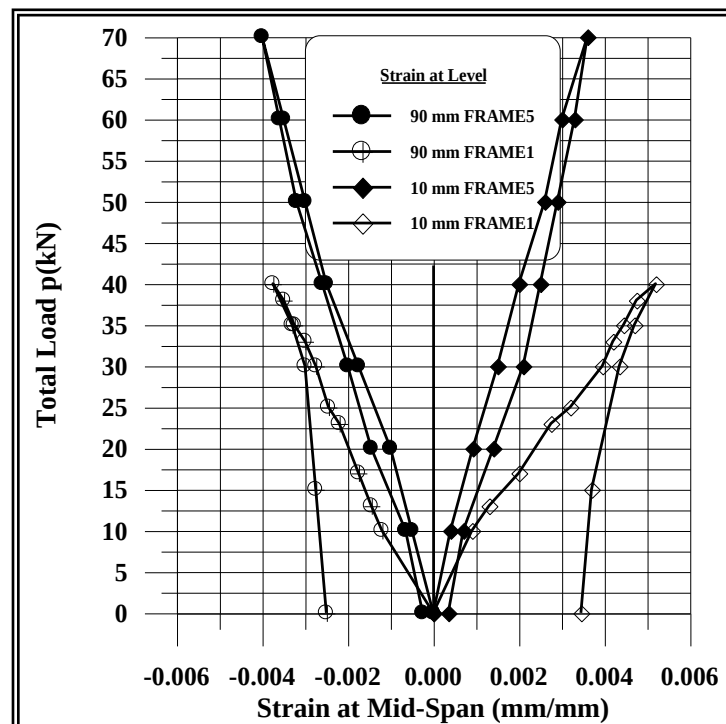


Figure (6.36) Comparison of Concrete Strain at Compression and Tension Regions of FRAME1 and FRAME5

6.7) Ductility

Energy-absorbing characteristics of a structure are denoted by ductility or deformability indices. Ductility of reinforced concrete members is defined as the ratio of the ultimate curvature, deflection, or rotation to the corresponding value at first yielding steel. However, for a concrete member enhanced with composite wrapping or steel rebars or the combination of both, a better measure of the energy absorption should be considered. The load-deflection curves are used as measurement for the ductility of reinforced concrete members and CFRP-wrapped reinforced concrete members. For different stages of loading and the corresponding ultimate deflection and deflection at first yielding for each frame, the ductility is shown in Table (6.1).

Table (6.1) Experimental Ductility Factors of the Tested Frames

Frame	Load (kN)		Deflection (mm)		Ductility Index (Δ_u/Δ_y)	Ductility Reduction (%)	Type of Failure
	P_y	P_u	Δ_y	Δ_u			
FR₁[*]	30	40	18.5	50.8	2.75	-	Flexure
FR₂	30	37.5	16.22	49.5	3.05	-	Flexure
FR_{L3}	45	50	22.5	33.95	1.51	45.1	Debonding +FRP Fracture
FR_{LT4}	50	60	23.46	49.23	2.1	23.6	Debonding +FRP Fracture
FR_{LT5}	50	70	27.57	52.6	1.91	30.5	Debonding +FRP Fracture

* Reference Frame for Monotonic Loading.

From Figure (6.31) and Table (6.1), one can see that frame **FR₁** has a very good ductility performance because of the yielding of the steel rebars. Comparing the results of frame **FR₁** to those of frame **FR_{L3}**, the deflection of frame **FR_{L3}** has greatly been decreased. The maximum deflection is decreased from (50.8 mm) for frame **FR₁** to (33.95 mm) for frame **FR_{L3}**, and the ductility is reduced by about 45.1%. However, considering that the huge load increases for frame **FR_{L3}**, the overall area

under the load-deflection curve still increase slightly. Therefore, the ductility of frame **FR_{L3}** remains intact. The transverse confinement of the fabric wrapping strengthens the concrete in three compression zones (mid-span and ends of longitudinal FRP), also strengthening with longitudinal FRP was represented by frame **FR_{LT4}**, and thus increases the ultimate strain of the confined concrete and the ductility is reduced by about 23.6%. Although the load does not increased significantly. The ultimate mid-span deflection as compared with frame **FR₁**, which shows an overall better strength and ductility than those of frame **FR₁**. Finally, the best strength and ductility can be seen in frame **FR_{LT5}**. The transverse confinement of the fabric wrapping strengthens the concrete in five compression zones (at each 30cm), and strengthening with longitudinal CFRP. The overall area of energy absorption is also highly increased as compared with that of **FR_{LT4}** and the ductility is reduced by about 30.5%. This is because the more confinement in the compression zone due to the transverse wrapping helps in preventing earlier crushing of the concrete.

6.8) Flexural Strength

From Figure (6.31), it can be concluded that CFRP fabrics do help in flexural strengthening of RC frames. However, due to different wrapping methods, the contribution of fabrics varies. The longitudinal CFRP fabric strengthening in **FR_{L3}** increases the load capacity by about (25%) as compared with the control frame. Frame **FR_{LT5}**, which is repaired with longitudinal fabrics at the bottom face of long span beam and at joints and confined at each (30cm) in the direction of long span beam, has the highest load capacity up to (70 kN) (an increase in the strength by about 75% rather than the control frame **FR₁**). Also, Figure (6.31) shows that the strength of frame **FR_{LT4}** is higher than that of frame **FR_{L3}**. Therefore, the length of confinement by CFRP can be considered

as an important factor for studying the effect of confined strengthening. Figure (6.37) shows the relation between the volumes of CFRP used in the frame versus the increasing in strength. According to the type of repair/strengthening of the frames and as shown in this figure there is a limit of amount of CFRP with no change in strength. Therefore, this figure gives an indication for the amount of CFRP that can be used in the frame with maximum strength.

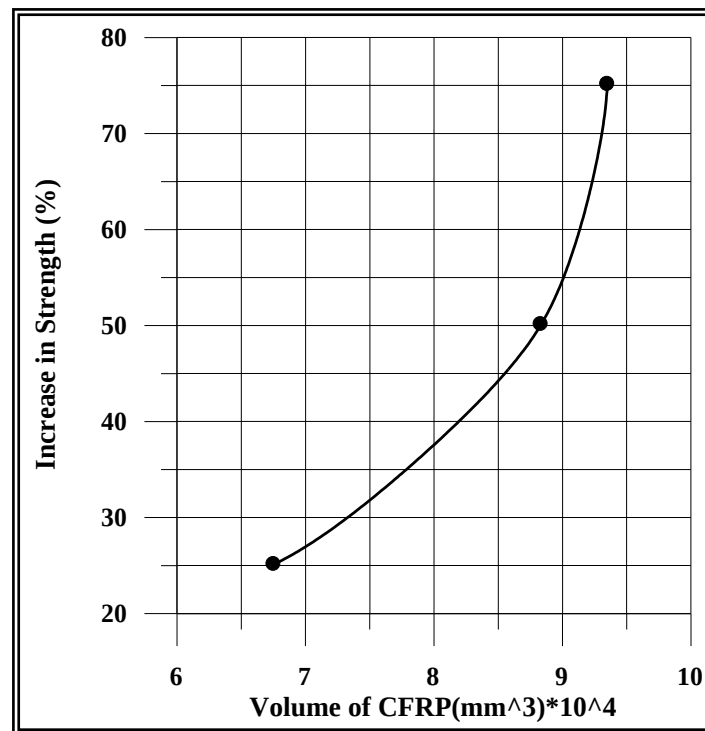


Figure (6.37) Relationship Between amounts of CFRP and Strengthening

Chapter Seven Analytical Results

7.1 General

In order to verify the computer program **RCSF** (Repair of Reinforced Concrete Space Frame by FRP Fabrics) is applicable to FRP-concrete space frames. Many parametric studies are carried out. The analytical results are compared with experimental data and ANSYS program available in literature.

7.2 BEAM1

Chen (2005) tested a number of (2.9) meter clear span control and FRP-Fabrics beams in three point bending. In order to investigate the effect of the wrapping configuration on the final load-deflection behavior of RC beams, two different wrapping methods were selected. In addition to the control beam (BEAM11) that has no wrapping at all, BEAM12 was wrapped longitudinally along its four sides and BEAM13 was wrapped vertically along its four sides as shown in Figure (7.1).

All beams have the same length, concrete compressive strength, steel reinforcement and cross sectional areas. Material properties are shown in Table (7.1).

The ANSYS computer program version 5.4 was used by **Chen** (2005) to analyze the load-deflection behavior of his test specimens. The concrete in the test beams was modeled with 3-D brick elements (Solid65) with cracking and crushing capabilities. For the beam with longitudinal fabrics, the normal concrete material properties were used. For the beam with transverse fabrics, the confined concrete material properties were used. Reinforcement was modeled with 3-D spar elements (Link8) with bilinear material properties. Longitudinal CFRP fabric was modeled with spar elements (Link8) with elastic linear properties. Transverse fabrics and steel stirrups were neglected in the modeling. At cross section, the beam

was modeled with sixteen solid65 elements with equal sizes. Meshing in the longitudinal direction was set to twenty elements along the length of the beam.

The calculated load versus deflection of the control beam and other cases, at mid-span are plotted in Figure (7.2) to Figure (7.4), together with the measured data. Good correlation is observed. As shown in Figure (7.2) the ultimate load of the experimental control beam (BEAM11) is (24.5 kN) versus (8.5 mm) deflection, and it was a flexural failure while for the analytical study the load was (27.5 kN) versus (40 mm) deflection, and failure type is crushing in concrete that means the convergence in ultimate load between the experimental and analytical was about (89.1%). Figure (7.3) shows the behavior of BEAM12 whose ultimate load for experimental was (76.5 kN) versus (36.5 mm) deflection with failure type crushing in concrete while for analytical study the load was (70.0 kN) versus (75 mm) deflection with failure type crushing in concrete. The convergence in ultimate load between the experimental and analytical was about (91.5%). Finally BEAM13 as shown in Figure (7.4) has experimental ultimate load (27.6 kN) versus (66 mm) deflection where for analytical study the load was (28.5 kN) versus (60.5 mm) deflection. The convergence in ultimate load between the experimental and analytical was about (96.8%). The different in results is natural because the analytical results based on assumptions as explained in earlier chapters and always have some error in simulation as compared with the experimental results.

Table (7.1) Material Properties used for BEAM1

Material	f'_c (MPa)	f_y (MPa)	f_{yfrp} (MPa)	E_c (MPa)	E_s (MPa)	E_{frp} (MPa)	t_{frp} (mm)
Concrete	30.96	-	-	25000	-	-	-
Steel	-	400	-	-	200000	-	-
CFRP	-	-	965	-	-	73034	0.33

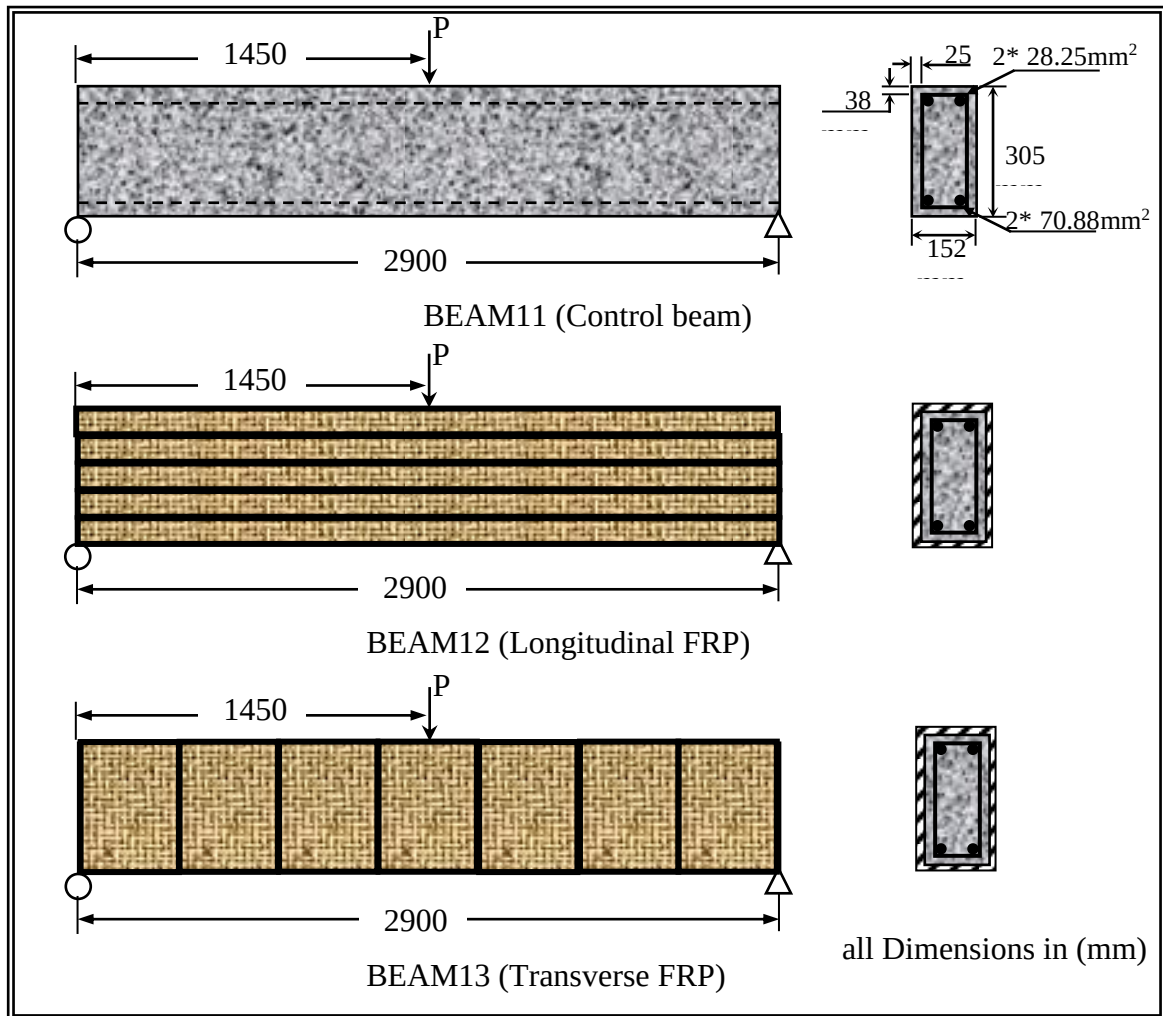


Figure (7.1) Details of BEAM1

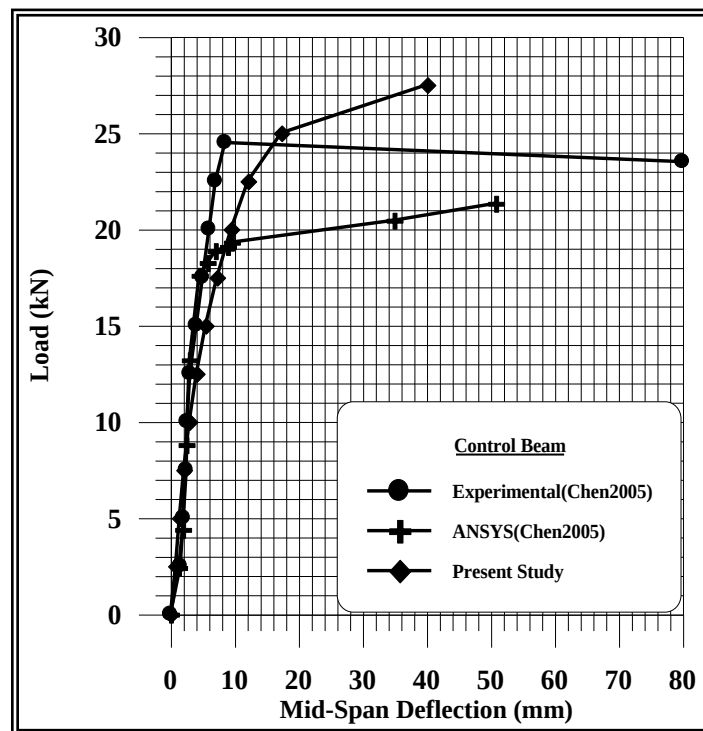


Figure (7.2) Load-Deflection Curve of BEAM11 (Control Beam)

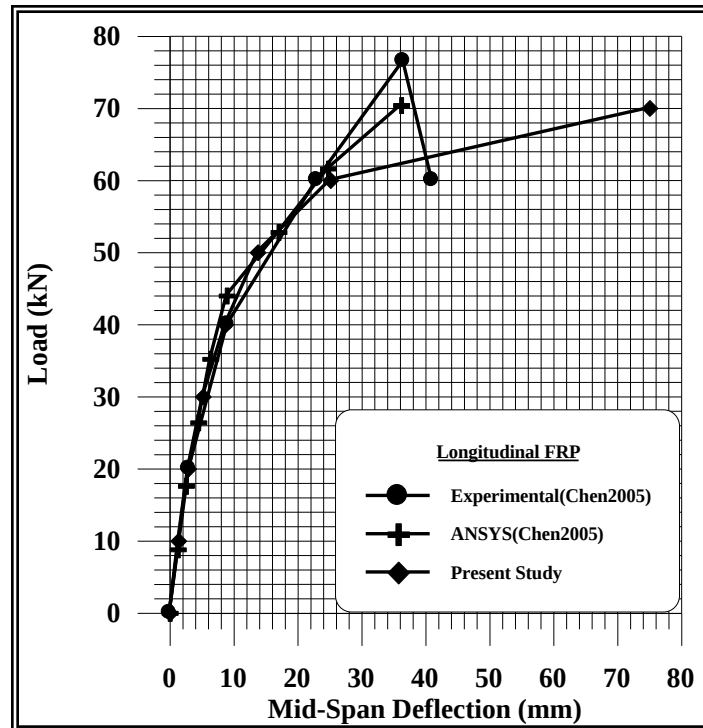


Figure (7.3) Load-Deflection Curve of BEAM12 (Longitudinal FRP)

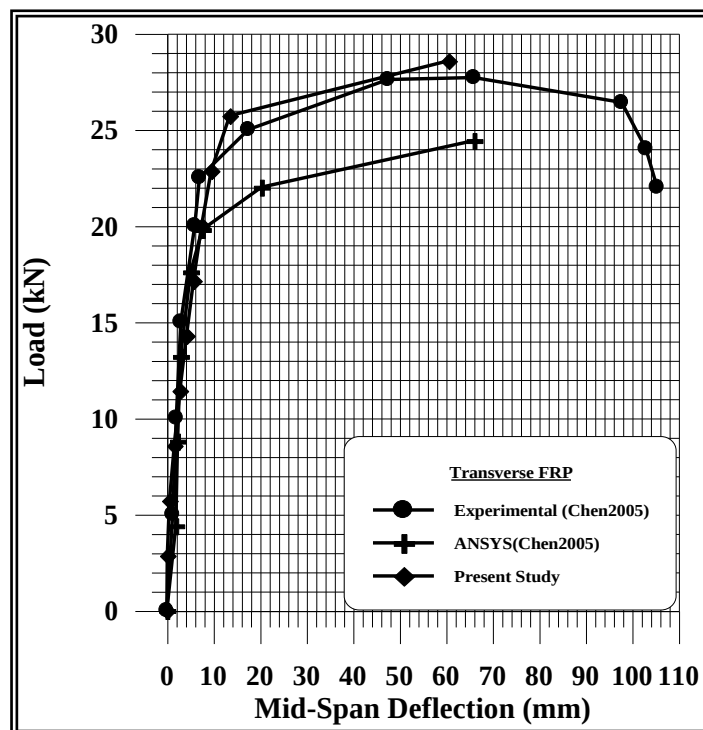


Figure (7.4) Load-Deflection Curve of BEAM13 (Transverse FRP)

7.3 BEAM2

Garden, and Hollaway (1998) tested a number of (1.0) meter long CFRP plated beams in four point bending. CFRP plates of 0.82 mm thick were glued to the underside of the beams using a 2 mm thick layer of epoxy resin. BEAM2 was selected for modeling as no anchorage to the CFRP was used other than that of the glue. Also beam2 was analyzed by **Khomwan, and Foster** (2004).

The geometry of BEAM2 is shown in Figure (7.5). The beam was analyzed assuming perfect bond. The material properties for BEAM2 are given in Table (7.2).

They used finite element models to analyze a number of strengthened reinforced concrete beams and walls. In all their analyses, 4-node concrete membrane elements were used with 2-node 1D bar elements for the reinforcing steel. The FRP elements were 2-node 1D bar elements for soffit strengthened members and 4-node membrane elements for side face strengthened members. Three mesh sizes were tested: a coarse (25 mm square), medium (12.5 mm square) and fine (6.25 mm square) mesh.

The calculated load versus deflection on the CFRP plates, at mid-span is plotted in Figure (7.6), together with the measured data. Good correlation is observed. As shown in Figure (7.6) the failure load of experimental beam is (33.5 kN) versus (8.15 mm) deflection with tensile failure through the concrete cover while for the present study is (33.33 kN) versus (7.45 mm) deflection with crushing in concrete.

Table (7.2) Material Properties used for BEAM2

Material	f'_c (MPa)	f_y (MPa)	f_{yfrp} (MPa)	E_c (MPa)	E_s (MPa)	E_{frp} (MPa)	t_{frp} (mm)
Concrete	40	-	-	25000	-	-	-
Steel	-	350	-	-	215000	-	-
FRP	-	-	1410	-	-	110000	0.82

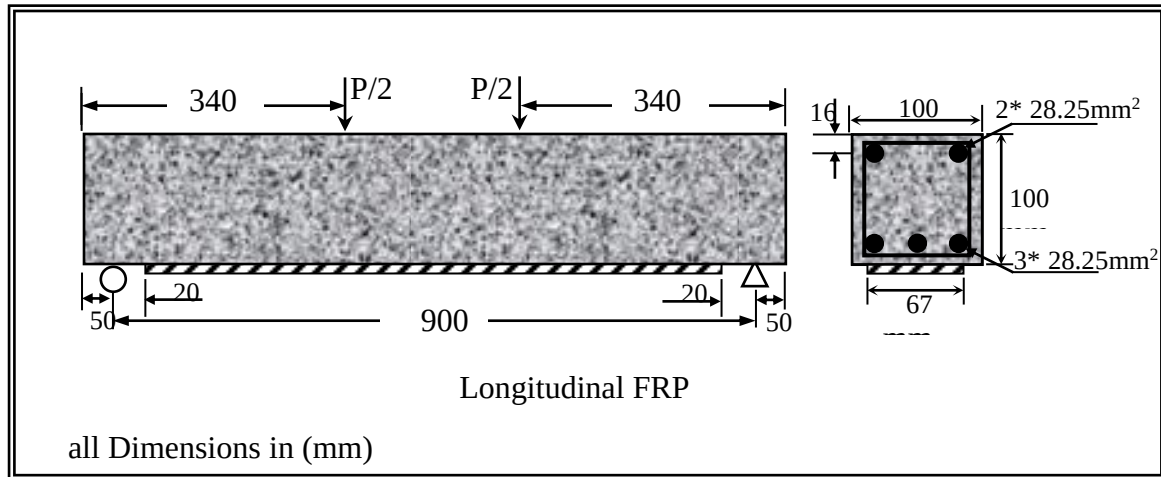


Figure (7.5) Details of BEAM2

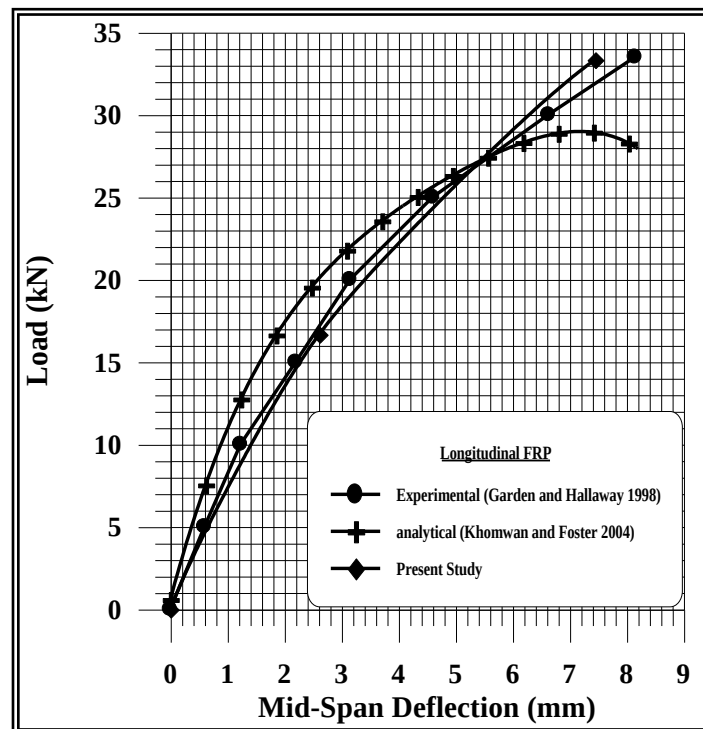


Figure (7.6) Load-Deflection Curve of BEAM2 (Longitudinal FRP)

7.4 FRAME (FR1)

The rectangular Frame (FR1) shown in Figure (7.7) was tested by *Furlong and Ferguson* (sited by *Gunnin, et al* 1977) and analyzed by *Gunnin, et al* (1977). Material properties of Frame (FR1) were showed by Table (7.3).

Figure (7.8) shows the load versus deflection of FR1, the failure load of experimental frame is (274 kN) versus (9 mm) deflection with crushing failure at columns while for the present study is (307.68 kN) versus (11.35 mm) deflection with crushing failure at columns too.

Table (7.3) Material Properties of FRAME (FR1)

Properties	Column	Beam
f'_c (MPa)	29.66	29.66
f_y (MPa)	378.62	378.62
E_c (MPa)	25760.0	25760.0
E_s (MPa)	196551.7	196551.7

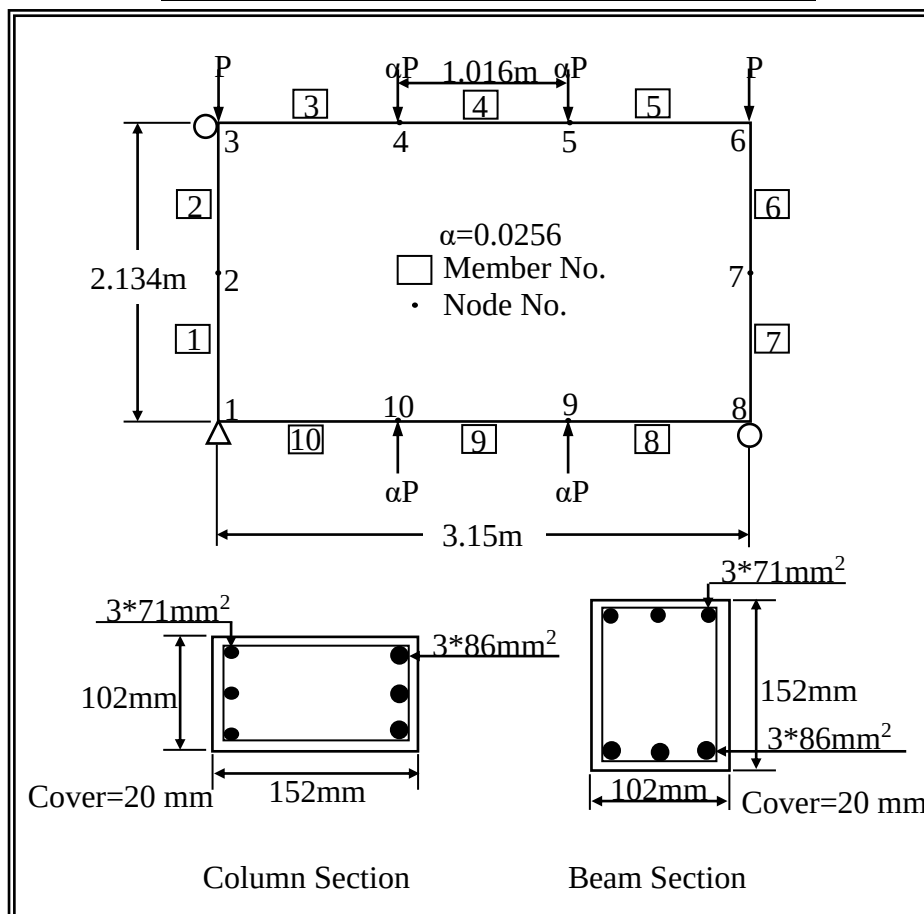


Figure (7.7) Details of FRAME FR1

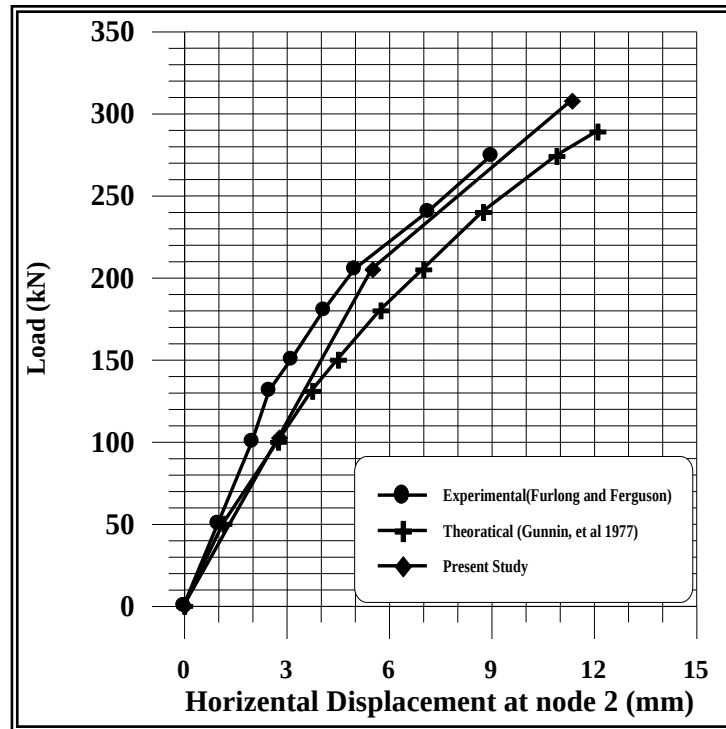


Figure (7.8) Load-Deflection Curve of FRAME FR1

7.5 FRAME (FR2)

The space frame Frame2 shown in Figure (7.9) was tested by *Al-Janabi* (2003). Properties of FR2 were showed by Table (7.4).

Figure (7.10) shows the load versus deflection of Fram2, the failure load of experimental frame is (19.5 kN) cross (6.25 mm) deflection and the crushing failure occur at nodes (12, 13, 17) while for the present study is (19.0 kN) cross (5.88 mm) deflection and the crushing failure at nodes (5, 19).

Table (7.4) Material Properties of FRAME (FR2)

Properties	Column	Beam
f'_c (MPa)	22.28	22.28
f_y (MPa)	445.63	445.63
E_c (MPa)	22184.8	22184.8
E_s (MPa)	200000.0	200000.0

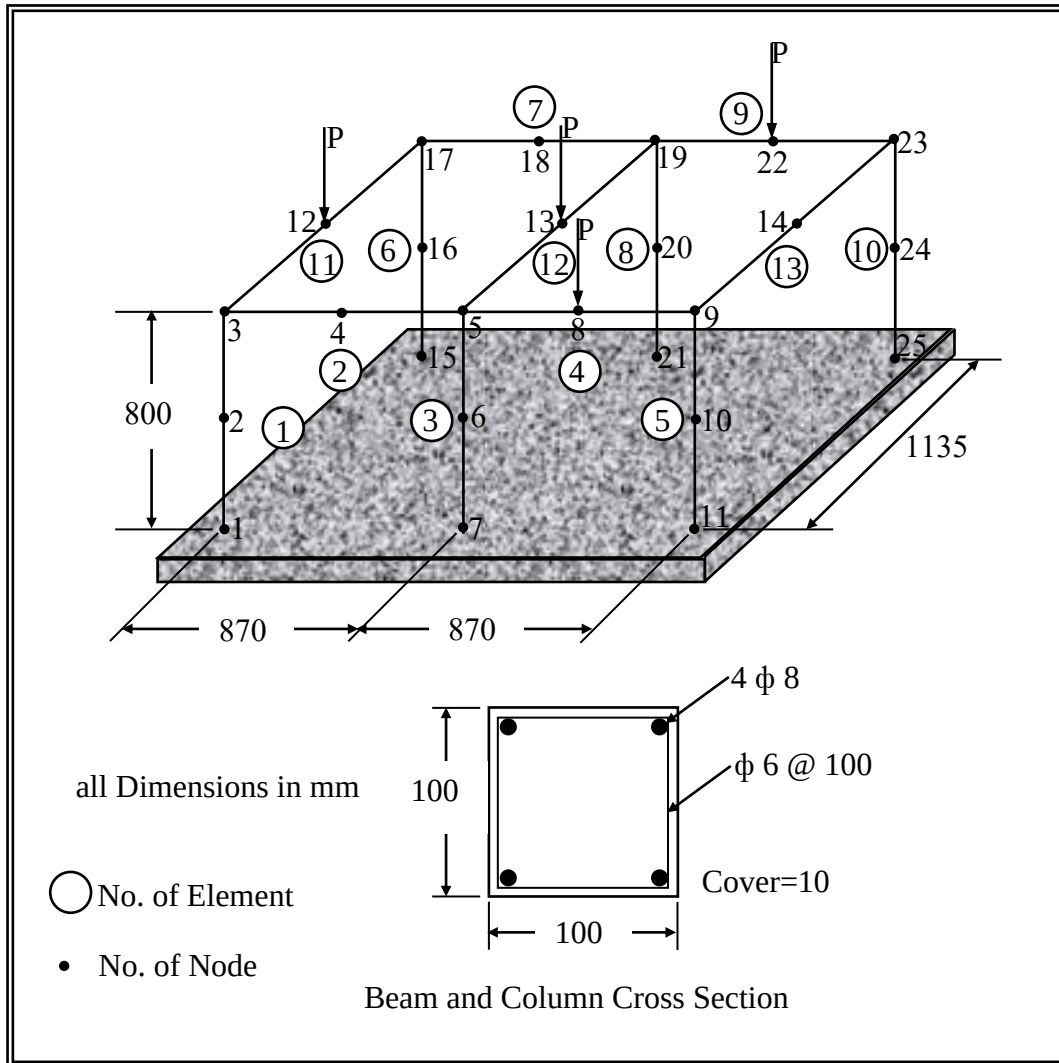


Figure (7.9) Details of FRAME FR2

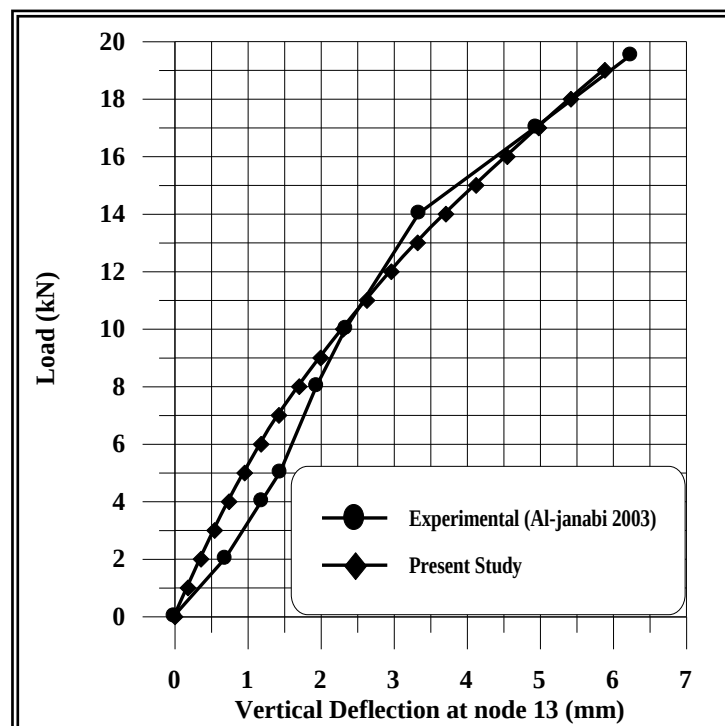


Figure (7.10) Load-Deflection Curve of FRAME FR2

7.6 Comparison of Experimental and Analytical Results

An analytical region technique formulation program is developed in the present work using FORTRAN 90 language named as **RCSF** (Repair of Reinforced Concrete Space Frame by FRP Fabrics). The computer program's theoretical background and the flow charts are mentioned in chapter five previously.

RCSF analytical computer program was used in this research to analyze the load-deflection behavior of the tested specimens. For frames with longitudinal fabrics, the normal concrete material properties are used. For frames with transverse fabrics, the confined concrete material properties are used. All the frames are modeled as fixed supported frame. Boundary conditions are applied accordingly. At a cross section, the frame is modeled with ten layers in each direction with equal sizes. The steel bars are modeled as layer in their location and the CFRP is modeled as a layer as shown in Figure (7.11).

All reinforced concrete frames tested previously in the present work, are analyzed using the region technique analysis, under monotonic static and repeated loading with two types of strengthening and different amount of CFRP fabrics to examine the effect of strengthening with these fabrics on the behavior and ultimate load capacity of these reinforced concrete frames.

The main input data for the computer program executions of the different tested frames are shown in Table (7.5).

The main experimental and analytical analysis results for the tested frames are shown in Table (7.6). Comparisons between experimental and analytical load versus mid-span deflection curves is shown in Figure (7.12), to Figure (7.16).

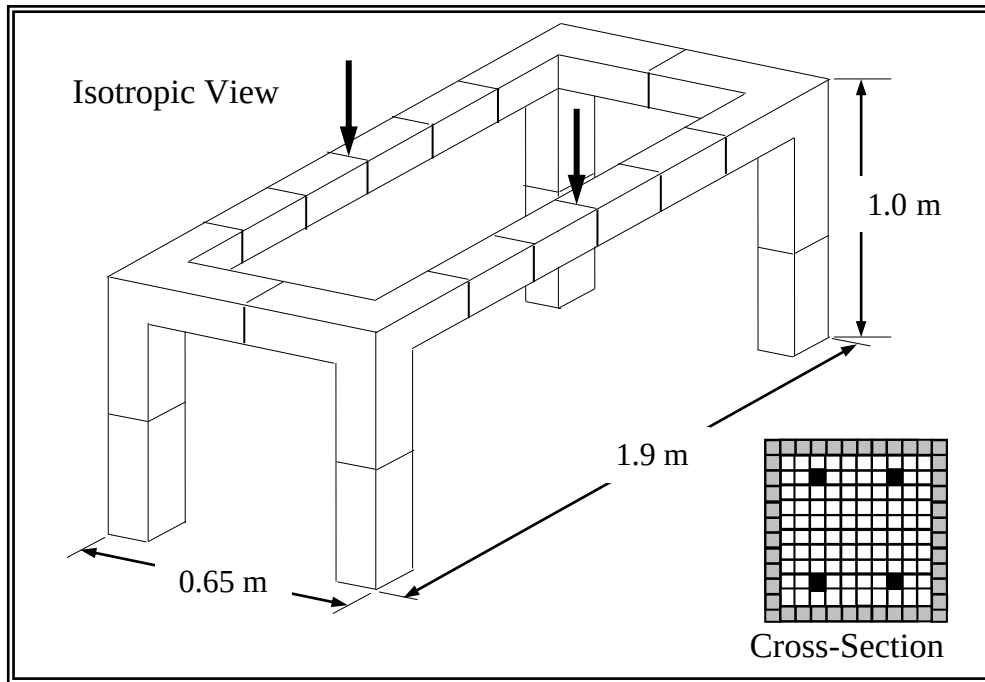


Figure (7.11) Finite Element Model for Test Frames

Table (7.5) Main input data for the Region Approach

	Young's Modulus E (GPa)	Compressive Strength f_c' (MPa)	Yield Stress (MPa)	Uniaxial Crushing Strain ϵ_{cu}	Poisson's Ratio ν
Concrete	25	25	-	0.0035 or Equ.(4.13)*	0.17
Steel	200	-	400	-	0.3
CFRP	230	-	3450	-	-

* ϵ_{cu} = 0.0035 for normal concrete, and equation(4.13) for confined concrete.

Table (7.6) Experimental and Analytical Results

Frame	Deflection (mm)				Error ¹ %	Load (kN)				Error ² %
	Experimental		Analytical			Experimental		Analytical		
	Δ_y	Δ_u	Δ_y	Δ_u		P_y	P_u	P_y	P_u	
FRAME1	18.5	50.8	20	37.92	25.4	30	40	33	40	0
FRAME2	16.22	51.95	10	57.5	10.68	30	37.5	20	37.5	0
FRAME3	22.5	33.95	25	41.13	21.15	45	50	42.5	50	0
FRAME4	23.46	49.23	24.5	47.29	3.94	50	60	46	57.14	4.77
FRAME5	27.57	52.6	20.76	55.03	4.62	50	70	44	66	5.57

$$1 \text{ Error} = \frac{(\Delta_u)_{\text{exp.}} - (\Delta_u)_{\text{ana.}}}{(\Delta_u)_{\text{exp.}}}$$

$$2 \text{ Error} = \frac{(P_u)_{\text{exp.}} - (P_u)_{\text{ana.}}}{(P_u)_{\text{exp.}}}$$

Table (7.7) Experimental and Analytical ductility

FRAME	Ductility Index		Ductility Reduction*
	Experimental	Analytical	
FRAME1	2.75	1.9	31
FRAME2	3.05	5.75	88.5
FRAME3	1.51	1.65	9.3
FRAME4	2.1	1.93	8.1
FRAME5	1.91	2.65	38.7

$$* \text{ Difference} = \frac{(\text{Ductility})_{\text{exp.}} - (\text{Ductility})_{\text{ana.}}}{(\text{Ductility})_{\text{exp.}}}$$

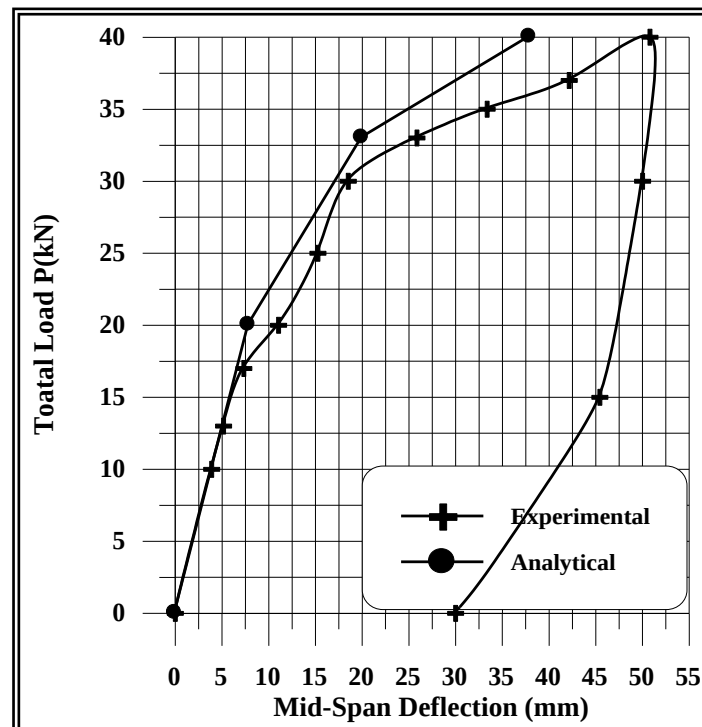


Figure (7.12) Load-Deflection Curve of FRAME1

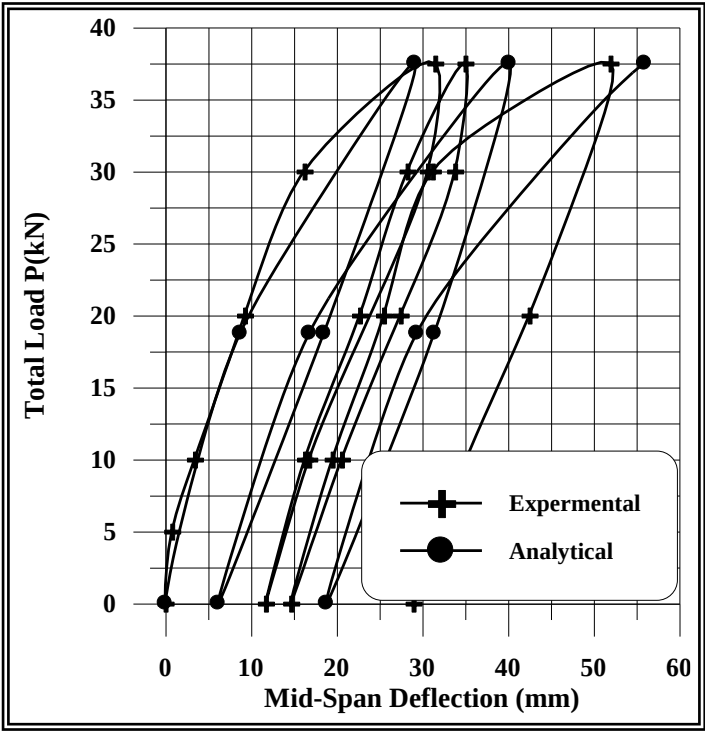


Figure (7.13) Load-Deflection Curve of FRAME2

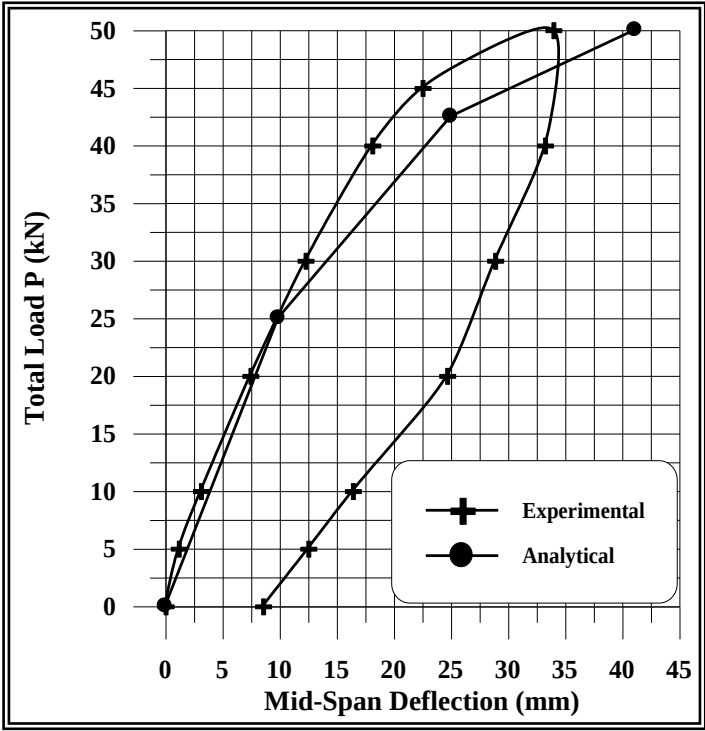


Figure (7.14) Load-Deflection Curve of FRAME3

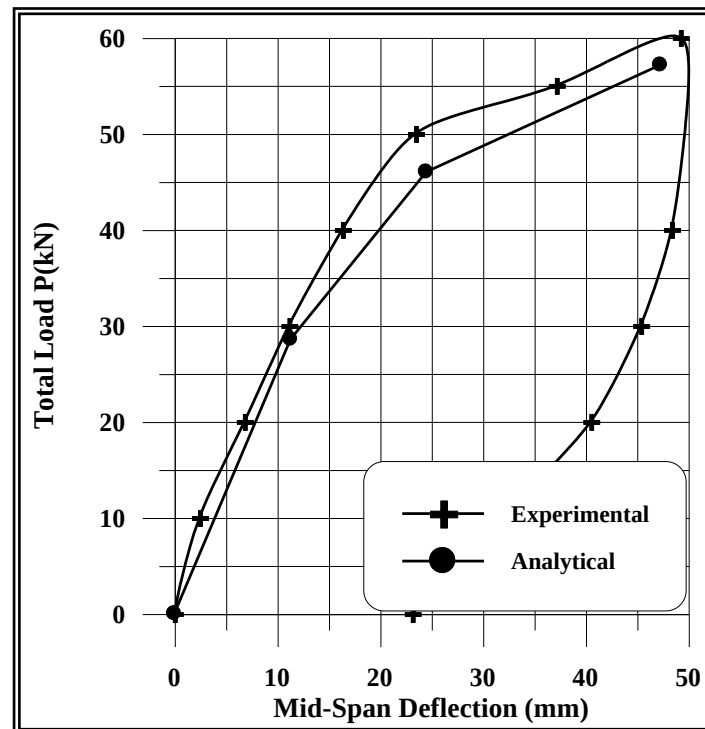


Figure (7.15) Load-Deflection Curve of FRAME4

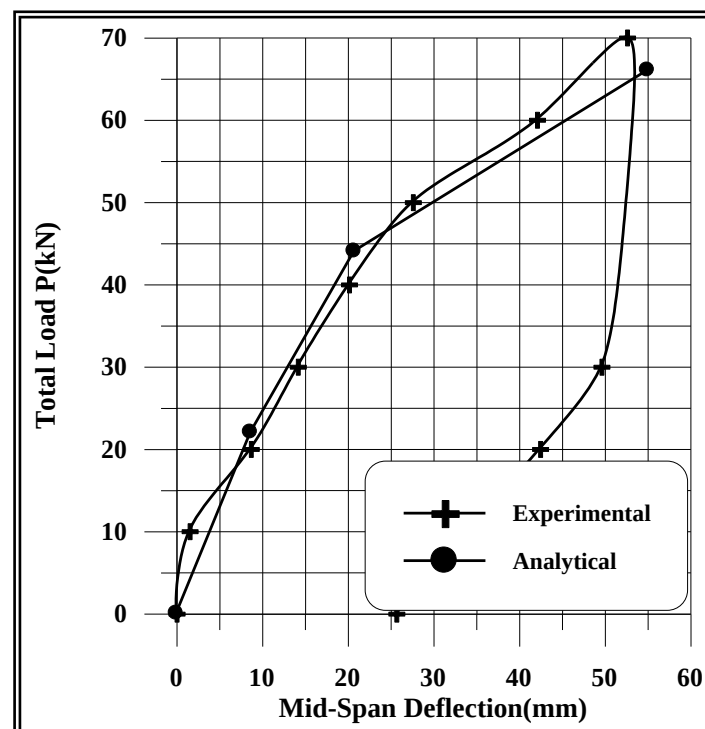


Figure (7.16) Load-Deflection Curve of FRAME5

From the figures, Table (7.6), and Table (7.7), one can clearly see that good agreements are obtained for all the frames. It is noticed that the ultimate loads obtained from analytical are usually slightly lower than the experimental results. It is noted that **RCSF** analytical computer program cannot model the descending part of concrete stress-strain behavior. But in experiments, the post-peak deformation is usually very large due to the yielding of the steel rebars. Overall, the load-deflection curves calculated from **RCSF** analytical computer program are quite satisfactory as compared with the experimental results, throughout the entire stages of loading and up to the ultimate capacity.

Chapter Eight

Conclusions and Recommendations

8.1) Conclusions

Research work performed in the present study demonstrates the feasibility of using externally applied epoxy-bonded CFRP fabrics to the concrete structures to increase the load-carrying capacity and ductility of various structural members. The following conclusions can be drawn:

1. Pure longitudinal fabric wrapping applied to testing frame (FRAME3) has significantly increased the flexural strength of the test frames by about (25 %), but it also decreases the deformation of the frame by about (33%).
2. The frames wrapped with both longitudinal and transverse fabrics as it is denoted (FRAME4 and FRAME5) have the best overall flexural performance due to its high strength, high deformability and good ductile failure. With the help of transverse fabrics, the ultimate strength has increased up to (75%).
3. The thickness of the fabrics and the unconfined concrete strength are the most important factors that affect the strength of the fabric confinement.
4. Although the longitudinal fabrics do increase the flexural strength of the fabric-confined frame, they also greatly decrease the ultimate mid-span deflection. As a result, the frame behaves in a more brittle failure conditions than the normal reinforced concrete frame. Therefore, using longitudinal fabrics only is not an efficient way to increase the ductility of the frame. The frame wrapped with both longitudinal and transverse fabrics has been found to achieve the flexural performance and ductility due to its high strength, high deformability and good ductile failure.

5. The finite element method using **RCSF** analytical program is presented to obtain the load-deflection curves, it can not predict the descending part of the load-deflection curve due to the limitation of the computer program.
6. The analytical results have good convergence with the experimental results, the error in load was (4.77%) for FRAME4 and (5.57%) for FRAME5.

8.2) Recommendations

Based on the results of the current study, the following topics need to be addressed in the future research:

1. The applications of the two-directional fabrics are recommended. However, due to the construction difficulties, the transverse wrapping is usually very hard to apply to the top of the frames in actual construction, the use of the two-directional wrapping method is still limited.
2. The ANSYS computer program can be recommended to analyze the same experimental work with CFRP fabric wrapping.
3. The experimental work carried out in this study can be recommended to use different cross-sectional area as I-section and T-section.
4. Bond slip between the CFRP fabric and concrete is recommended to be taken into consideration.
5. Tapering ratio with non-linear variation can be recommended.
6. The confinement of concrete can be recommended to introduce by steel stirrups.

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