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& Scientific Research
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College of Engineering*



INVESTIGATION OF THE OPTIMUM DESIGN OF GTX-AIRCRAFT WING STRUCTURE UNDER ACTION OF INERTIA LOADS

A Thesis

***Submitted to the College of Engineering of the University
of Babylon in Partial Fulfillment of Requirements
for the Degree of Master of Science in
Mechanical Engineering
(Applied Mechanics)***

By

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بسم الله الرحمن الرحيم

وَيَسْأَلُونَكَ عَنِ الرُّوحِ قُلِ الرُّوحُ مِنْ أَمْرِ رَبِّي وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا {85} وَلَئِنْ شِئْنَا لَنَذْهَبَنَّ بِالَّذِي أَوْحَيْنَا إِلَيْكَ ثُمَّ لَا تَجِدُ لَكَ بِهِ عَلَيْنَا وَكِيلًا {86} إِلَّا رَحْمَةً مِنْ رَبِّكَ إِنَّ فَضْلَهُ كَانَ عَلَيْكَ كَبِيرًا {87} قُلِ لَّيْنِ اجْتَمَعَتِ الْإِنْسُ وَالْجِنُّ عَلَى أَنْ يَأْتُوا بِمِثْلِ هَذَا الْقُرْآنِ لَا يَأْتُونَ بِمِثْلِهِ وَلَوْ كَانَ بَعْضُهُمْ لِبَعْضٍ ظَهِيرًا {88}

صدق الله العلي العظيم

سورة الإسراء

الآيات (85 - 88)

CERTIFICATION

We certify as an examining committee that we have read this thesis, entitled "INVESTIGATION OF THE OPTIMUM DESIGN OF GTX-AIRCRAFT WING STRUCTURE UNDER ACTION OF INERTIA LOADS ", and examining the student (NAWFEL NAZAR MOHAMMAD), in its contents and what is concerned with it, and that in our opinion it meets the standard of a thesis for obtaining the degree of Master of science in Mechanical Engineering (Applied Mechanics).

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Abstract

In this thesis, the optimum design of GTX-Aircraft wing structure which is designed by space NASA team under action of the inertia loads that resulting from its general 3-D motion was presented. The design study of the considered wing is viewed from two main aspects: the first is to study the Dynamics problem; the second is to implement the adopted structural optimization technique.

The study of Dynamics problem is subdivided into two parts: the first is to study the Kinematics problem (the General 3-D motion of GTX-Aircraft structure translationally & rotationally), whereas the second is to study the Kinetics problem to estimate the inertia loads induced in the wing structure as result of its general 3-D motion (the Impulse & Momentum Principles is used to solve kinetics problem).

The structural optimization problem is viewed topologically for the wing structure for the two types of variables: geometrical & material variables. The investigation of the geometrical variables topologically includes the variation of stiffeners location and orientation (six models were considered in the present work to represent this type of geometrical optimization, so the difference between any model and others is attributed to the appreciable variation in location and orientation of stiffeners and their arrangement). The investigation of the material variables topologically includes the structural materials changing for the wing structure using the multi-material modeling with isotropic elastic materials with assuming that each stiffener is to be consisting of single material, so two types of materials (alloys) is considered practically successful in the construction of GTX-Aircraft wing structure (29 Ti-alloys & 48 Al-alloys).

The structural weight is considered as objective function and the strength failure criterion is used with constraints on structural frequency and displacement. The design variables within the single model are limited to the set of material properties of constituent materials.

A computer program has been built using FORTRAN 77 language to investigate the dynamics problem, while FINITE ELEMENT METHOD (FEM) has been adopted using ANSYS V5.4 package to treat the structural optimization problem.

The parametric study reveals that the elastic modulus has no effect on stress variation for single-material modeling, while structural optimization results state that the elastic modulus plays the important role in stress variation among the considered materials for the multi-material modeling. The structural optimization results also reveal that both structural natural frequency and displacement values for Al-alloys structure is higher than Ti-alloys structure for the same model.

The structural optimization results also reveal that the transverse stiffeners and rectangular honeycomb sandwich core models are the most two successful designs because of they have the highest values of structural strength and stiffness among the considered models, where the transverse stiffeners model achieves weight ratio of (0.580758) to obtain minimum weight of (29.1058 kg), while the rectangular honeycomb core model achieves weight ratio of (0.58795) to obtain minimum weight of (29.3869 kg).

الخلاصة

في هذه الرسالة، تم تقديم التصميم الأمثل لمنشأ جناح طائرة (GTX) والمصممة من قبل وكالة ناسا الفضائية تحت تأثير أحمال القصور الذاتي الناتجة من حركتها العامة ثلاثية الأبعاد. تم النظر إلى الدراسة التصميمية للجناح قيد الدراسة من جانبين رئيسيين: الأول هو دراسة المسألة الديناميكية، الثاني هو تنفيذ تقنية الأمثلية الإنشائية المعتمدة.

إن دراسة المسألة الديناميكية قُسمت بدورها إلى جزأين: الأول هو دراسة مسألة الحركة (الحركة العامة ثلاثية الأبعاد لمنشأ طائرة (GTX) إنتقالياً ودورانياً)، بينما الثاني هو دراسة مسألة التحريك لتقييم أحمال القصور الذاتي المتولدة في الجناح كنتيجة لحركته العامة ثلاثية الأبعاد (وقد استخدمت مبادئ الدفع والزخم في حل مسألة التحريك).

تم النظر إلى مسألة الأمثلية الإنشائية لاكمياً لمنشأ الجناح لكلا نوعي المتغيرات: الشكلية والمادية. إن إستبيان المتغيرات الشكلية لاكمياً يتضمن تغيير موقع وتوجه المجسات (ستة نماذج تم أخذها بنظر الإعتبار في العمل الحالي لتجسيد هذا النوع من الأمثلية الشكلية، ولذلك فإن الفرق بين أي نموذج وآخر يكمن في التغير الملحوظ في موقع وتوجه المجسات وترتيبها). إن إستبيان المتغيرات المادية لاكمياً يتضمن تغيير المواد الإنشائية لمنشأ الجناح باستخدام النمذجة متعددة المواد بمواد مرنة أيزوتروبية مع فرض أن كل مجسأ يكون مكوناً من مادة واحدة، ولذلك فإن نوعين من المواد (الساباك) إعتبرت ناجحة عملياً في تركيب منشأ جناح طائرة (GTX) (29 سبيكة تيتانيوم و 48 سبيكة ألومنيوم).

الوزن الإنشائي تم اعتباره كدالة هدف للتصميم وقد تم استخدام معيارية فشل المقاومة مع وجود قيود على التردد الإنشائي والإزاحة الإنشائية. متغيرات التصميم ضمن النموذج الواحد محددة بمجموعة خواص المادة لكل مادة مكونة للمنشأ. تم هيكلة برنامج الحاسبة باستخدام لغة فورتران 77 لإستبيان المسألة الديناميكية، بينما تم إعتداد طريقة العنصر المحدد باستخدام برنامج (ANSYS V5.4) للتعامل مع مسألة الأمثلية الإنشائية.

إن دراسة التأثيرات تبين أن معامل المرونة ليس له أي تأثير على تغيير الإجهاد للنمذجة وحيدة المادة، بينما نتائج الأمثلية الإنشائية تكشف أن معامل المرونة يلعب الدور المهم في تغيير الإجهاد في المواد المأخوذة بنظر الإعتبار للنمذجة متعددة المواد. وكذلك فإن نتائج الأمثلية الإنشائية تكشف أن قيم كلاً من التردد الطبيعي والإزاحة الإنشائية لمنشأ سبائك الألومنيوم هي أعلى منها لمنشأ سبائك التيتانيوم لنفس النموذج.

كذلك فإن نتائج الأمثلية الإنشائية تكشف أن النموذج ذو المجسات العرضية والنموذج ذو الحشوات الخلوية المستطيلة الشكل هما التصميمين الأكثر نجاحاً لأنهما يمتلكان القيم العليا للمقاومة والصلابة الإنشائية بين النماذج قيد الدراسة، حيث إن النموذج ذو المجسات العرضية يحقق نسبة وزن مقدارها (0.580758) للحصول على وزن أدنى مقداره (29.1058 كيلو غرام) والنموذج ذو الحشوات الخلوية المستطيلة الشكل يحقق نسبة وزن مقدارها (0.58975) للحصول على وزن أدنى مقداره (29.3869 كيلو غرام).

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CHAPTER ONE

INTRODUCTION

1.1 General:

Due to the high bending stiffness to weight ratio of stiffened plates, they are possible to take part in feasible life so they can be applied as roof structures for supermarkets and petrol stations (**Figure (1.1a)**), orthotropic bridge decks (**Figure (1.1b)**), airplane wing structures (present work) (**Figure (1.1c)**), ship wall and deck structures (**Figure (1.1d)**), roof structure of tanks (**Figure (1.1e)**) and aero-shell corrugated panels found in spacecraft structures (**Figure (1.1f)**) (**Karoly, 2000**)⁽²⁷⁾.

In general, the wing structure is constructing of two types of plate members: the first is the two thin face sheet members (skins), the second is the set of stiffening members (stiffeners) which may be allowed to shape various honeycomb patterns. The main advantage of presence of stiffeners in such structures is to provide the supporting frame that bears the lateral action (increase the bending stiffness of the structure) of the applied loads. The task of stiffeners and honeycomb core is to resist the bending and torsional loads together (increase the bending and torsional stiffness) depending on the honeycomb arrangement, while the main task of skins is to resist the tensional and compressive loads (**Zoltan, 2004**)⁽⁷⁰⁾.

The prediction of static and dynamic behaviour of wing structure represents a problem of extreme complexity in the design of aerospace structure especially when it is subjected to the inertia loads which are of transient, sinusoidal, pulse periodic (rectangular or triangular) nature resulting from its general 3-D motion, so the analysis of such structure requires an efficient numerical tool to predict the dynamic characteristics for such structures which are represented by the detailed simulation of displacements, stresses and strains with time. The dynamic characteristics of any structure are governed by its stiffness and

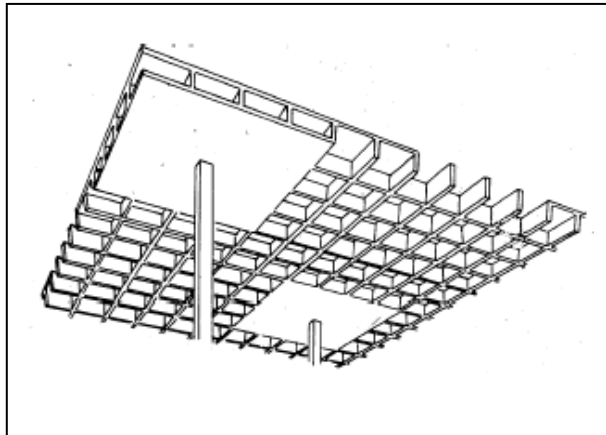
material properties. The most important dynamic characteristics of the structure are the natural frequency and response which are a function of material constants or properties, geometry and dimensions of structure (**Olson & Lindberg, 1970⁽⁴⁶⁾, Egal & Sewall, 2002⁽¹³⁾**).

The aero-shell skin panels are comprised of a sandwich section that has an Aluminum alloys core and Titanium alloys face sheets. The Titanium and Aluminum alloys were selected for their oxidation resistance under the harsh environment during flight (**William & Leslie, 2001**) ⁽⁶⁵⁾. Sandwich structures (plates, beams, panels, etc.) consist of a combination of different materials that are placed together so that the material properties of each one can be utilized for the structural advantage of the whole assembly. Sandwich panels generally consist of three significant components, two thin, stiff face sheets and a thick, light and weaker core (**Rigo, 2001**)⁽⁵⁵⁾.

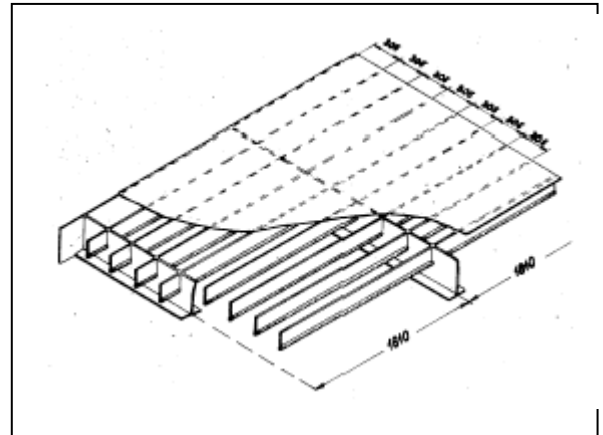
Due to high bending stiffness of sandwich construction, it results in lower lateral deformations, higher buckling resistance and higher natural frequencies than do other constructions (foam, truss ...etc.) (**Mario, K. F., 1997**) ⁽³⁹⁾.

The **NASA** approach to advance access to space is a program consisting of three phases. The third phase, known as Generation 3, includes development of revolutionary propulsion technology. **NASA** Glenn Research Center (GRC) has been developing Rocket-Based Combined-Cycle (RBCC) propulsion as a means to achieve this goal. GRC believes that for Single-Stage-To-Orbit (SSTO) to become feasible, the propulsion system must be fully integrated to a reference vehicle concept and concurrently developed. This approach lends itself to optimization at the highest possible level of the system including trajectory considerations. Consequently, the **GTX** concept was selected as a candidate integral propulsion/vehicle system of high optimization potential. The **GTX** vehicle concept would launch from sea level static conditions and accelerate to hypersonic speed while in the earth's atmosphere. The aerodynamic environment, both pressure and heating, could be very severe in the air-breathing

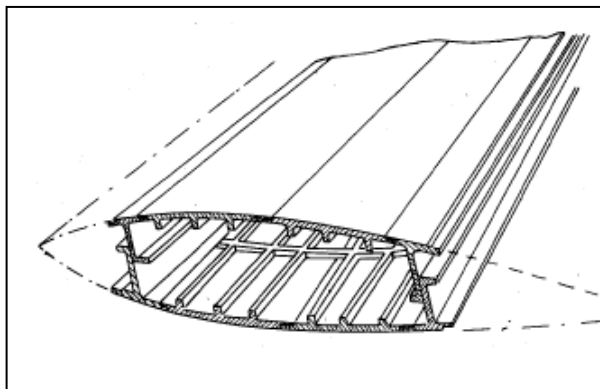
segment of the trajectory. To reduce the impact of this environment, a high thrust-to-weight vertical take-off mission profile was selected. This effectively shortens the mission, minimizes aero-heating and keeps the vehicle weight to a minimum (*Hunter, et al, 2002*)⁽²³⁾.



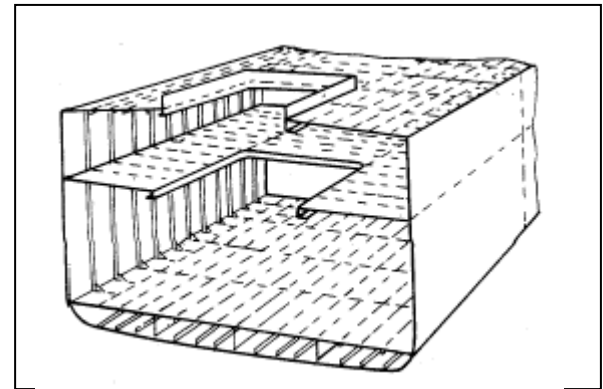
(a) Roof structure



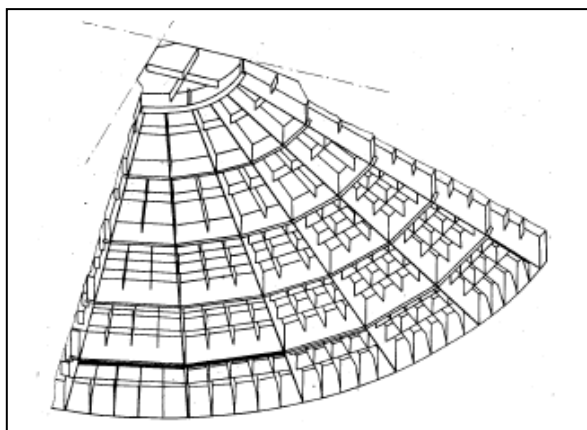
(b) Bridge deck structure



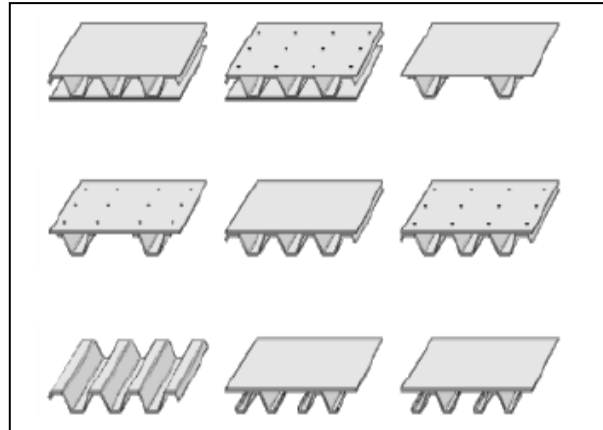
(c) Wing structure



(d) Ship deck structure



(e) Tank roof structure



(f) Aero-shell corrugated panels

Figure (1.1) Stiffened plates applications [Ref. 27]

1.2 GTX-Aircraft Design Requirements:

All design requirements for **GTX** as an integrated single-stage-to-orbit (SSTO) vehicle are derived to maximize the use of air breathing performance while minimizing structural weight. **GTX** is designed as a vertical take-off and horizontal landing concept in order to minimize the weight of wings and landing gear structure. As a design requirement, this should reduce the reliance on aerodynamics at low speed.² Sharp leading edges on the nose-tip, wings, tail, and cowl are required in order to reduce vehicle drag, especially at hypersonic speeds. The propellant requirements are liquid oxygen (LOX) and liquid hydrogen (LH2) selected for energy per unit mass advantage as well as for cooling properties. While the **GTX** is to be reusable, the number of missions the vehicle is to be capable of flying has not been specified at this point in the study. A 800-kg payload has been specified for initial structural assessments with an emphasis on bringing the integrated vehicle system to closure. A mission profile (trajectory) is required to establish critical flight environments and may have constrictions applied to offset excessive loading of the system. For example, the angle of attack at the end of the scramjet phase was limited to six degrees so that aerodynamic loads do not increase the system weight (*Hunter, et al, 2002*)⁽²³⁾. Requirements allow for scaling of the integrated vehicle to achieve greater payload capability and/or reach the ultimate goal of vehicle closure. These basic requirements and the need for structural efficiency led to the design of a reference vehicle concept.

1.3 Critical Load Determination:

The selection of the critical loads for the integrated vehicle structure assessment was made after a careful review of the ascent trajectory. **GTX-Aircraft** was imposed to more severe conditions for ascending trajectory events than those of descending trajectory events, so at the present work, the study is restricted to take the effect of inertia loads due to payloads (800 kg) borne by the

horizontal wings structures for the ascending trajectory only due to its severe in estimation of inertia loads for which the gravity acceleration (g) will be added to the global Z-Acceleration for translational motion of the vehicle.

A total of five critical events were selected for investigation of the ascent trajectory ranging from Liftoff to mach 11. The intent was to bound the worse case environment for each system by the effect of inertial loads, aerodynamic pressures and temperatures, and internal combustion effects at each critical load event. The first critical event was that of liftoff since the propellant tanks are at full capacity and will experience high inertia loads along with the tank interface structure at the fuselage. The next critical event occurs at mach 2.5 when the rocket engine is shut down just prior to the ramjet phase at which the maximum inertia loads will be attained due to the superior accelerating thrust provided by the propulsive engine. The third critical event occurs at the end of the ramjet phase near mach 5. At this event, the internal combustion pressures have reached the peak levels during ascent. The fourth critical load event selected was at mach 10 where aero heating has become critical to the outer shell of the integrated vehicle. Finally, mach 11 was selected as critical since aero heating is at a peak and the trajectory angle of attack is at six degrees as the vehicle turns up for the coast to orbit.

These five cases were thus selected as critical points in the trajectory where various motion loads reach their maximum values in various structural locations at these events. Other cases were excluded such as the induced impact loads at landing process and the relatively light inertia loads induced during the horizontal descending of **GTX-Aircraft**. The present investigation is restricted to view the critical events of inertia action for ascending trajectory at which the induced inertia loads reach their maximum values near Mach No. 2.5 as shown in **Table (1.1)**. **Table (1.1)** reports the values of translational acceleration vector as function of Mach No. for **GTX-Aircraft** at the above mentioned critical events (*Hunter, et al, 2002*)⁽²³⁾.

Table (1.1) Critical load events of GTX-Aircraft as inertia action for ascending trajectory ⁽²³⁾

Mach No.	Total Acceleration	Axial Acceleration (Tangential Acceleration)	Normal Acceleration
0	1.88 g's	1.88 g's	0.00
2.5	5.04 g's	3.78 g's	3.33 g's
5	4.14 g's	3.12 g's	2.72 g's
10	3.93 g's	2.97 g's	2.58 g's
11	3.45 g's	2.73 g's	2.11 g's

1.4 Structural Optimization:

Structural optimization is a part of design in engineering fields especially in mechanical and civil engineering. The earliest idea of optimum design found in the works of **Galileo** (stemmed from consideration of bending strength of beams), **Bernoulli**, **Lagrange**, **Navier**, and other great scientists sought the "best" shapes for structural elements to satisfy the strength requirements (**Andrzej, et al, 1986**)⁽⁴⁾.

The importance of minimum weight design of structures was first recognized by the aerospace industry where aircraft structural design is often controlled more by weight than by cost considerations (**Agarwal & Davis, 1974**)⁽²⁾ & (**Govil, et al, 1979**)⁽¹⁷⁾. In other industries dealing with civil, mechanical and automotive engineering systems, cost may be a primary consideration although the weight of the system does affect its cost and performance (**Kazuhiro, et al, 2005**)⁽²⁹⁾.

The numerical techniques were frequently used in the structural optimization to obtain more realistic and practical design where the variables in the optimization are of discrete nature. The structural optimization methods can be broadly arranged into three main areas: Topology optimization, Shape optimization and Size optimization (**Bendsoe & Rodrigues, 2002**)⁽⁷⁾. In most

engineering problems, the objective and constraint functions can be placed interchangeably, and each of them can take one of the following cases (*Kaarel, 2002*)⁽²⁶⁾:

- 1- Minimum weight (volume & weight density) of material at a constant load carrying capacity.
- 2- Maximum load carrying capacity at a constant weight of material.
- 3- Minimum weight of material at a constant load carrying capacity, subjected to stability constraints.
- 4- Minimum deformability at a constant weight of material and constant load carrying capacity, subjected to stability constraints.

Also, the design variables can take one of the following parameters:

- 1- Geometrical dimensions & shape (plate thickness, beam width, rod cross-sectional area, location & orientation of stiffeners, beams & rods for stiffened structures, frames, trusses, ...etc.); they are also called side constraints.
- 2- Material constants (density, elasticity, rigidity ...etc.).
- 3- Nature of forces (concentrated, uniformly distributed, of harmonic nature, of periodic nature, inertia loads, thermal loads, etc.) (*Rapheal & Zafer, 1993*)⁽⁵³⁾.

1.5 Objectives of the Present Work:

One: Study the Kinematics of general motion of *GTX-Aircraft* using advanced three dimensional relations considering translation, rotation, and general motion.

Two: Study the kinetics of the *GTX-Aircraft* wing structure using advanced *Euler's Eq.* which is based on impulse and momentum principles in order to estimate the inertia loads induced in the wing structure due to the general 3-D motion of *GTX-Aircraft*.

Three: Study the effect of the material properties (within the feasible domain of *GTX-Aircraft* wing construction, i.e. within Ti- & Al-alloys properties) on the structural behavior of the wing structure such as: strength, strain, frequency & displacement.

Four: Suggest an optimization criterion to perform the design optimization for ***GTX-Aircraft*** wing structure using special ***Evolution Strategy (ES)*** to obtain the minimum structural weight for the considered wing structure within the topological optimization principles (***Lagaros & papadrak, 2004***)⁽³³⁾.

Five: Perform the topological structural optimization for the considered wing structure with its geometrical concept which is represented by considering six models for the wing structure which represent the most conventional constructions as stiffeners and honeycomb sandwiches for the wing in the practical life (***Moshe & Nathan, 2005***)⁽⁴³⁾.

Six: Perform the topological structural optimization for the considered wing structure with its material concept which is enhanced by the multi-material modeling for which each of structural materials take the optimized set of material properties (two types of isotropic elastic materials were considered: Ti- & Al-alloys, so they arranged in descending order according to their specific strengths as fixed in Appendix-D-) (***Shackelford & William, 2001***)⁽⁶⁰⁾.

CHAPTER TWO

LITERATURE REVIEW

2.1 General:

The stiffened plates have the wide use in aerospace applications owing to their high specific strength. In the design of sandwich constructions panels found in aircraft structures, it is important to study the dynamic properties of such structures and then perform the optimization process, since the whole structure (aircraft) is subjected to dynamic loads caused by the general motion loads (inertia, aero-dynamic & aero-heating loads) with forcing loads (due to the propulsive system, gust loads & random excitations) also, other loads of dynamic nature (impulsive, harmonic, transient, spectrum, etc.).

The present review covers two aspects: the first includes the dynamic behaviour investigation of the stiffened plates structure, the second aspect is concerned with variety methods used as tools to obtain the optimum structural design under action of general load conditions (static and dynamic loads).

2.2 Dynamic Analysis:

Olson, M. D., & Lindberg, G. M., (1970) [46]: used shell theory to treat fluid-loaded stiffened plates of constant thickness subjected to an arbitrary dynamic loading. The infinite region outside the shell was transformed into a finite region before applying a finite difference method to obtain approximate solution. Numerical results were presented for the acoustic pressure in response to the application of a spatially uniform ramp-shaped transient forcing function. In all the works cited, except Berger's, the effects of bending resistance were ignored. In Berger's work the effects of transverse shear and rotary inertia are not included.

Rao, S. S., (1986) [52]: presented optimization of aircraft wing structures by considering the dynamic stresses and strains developed during landing impact and gust load conditions. The random nature of sinking speed and the forward

velocity at the instant of contact are considered in the calculation of landing stresses. The vertical velocity due to gust conditions is treated as a stochastic process for the computation of gust induced stresses. The optimum design of symmetric double wedge airfoil, based on beam type of analysis and supersonic airplane wing, based on finite element analysis, are considered to illustrate the procedure.

Abhijit, M., & Madhujit, M., (1988) [1]: introduced an isoparametric stiffened plate element for the free vibration analysis of eccentrically stiffened plates. The element has the ability to accommodate irregular boundaries. Moreover the formulation considered the shear deformation, hence, the formulation is applicable to both thick and thin plates. At that work, the stiffeners can be replaced anywhere within the plate element and they need not necessarily follow the nodal lines. In addition, the effects of consistent and lumped mass matrices on natural frequencies of stiffened plates are investigated. The effects of several parameters of the stiffener-eccentricity, shape, torsional stiffness, etc. on the natural frequencies of the stiffened plates are studied.

Patrick, B., (2001) [48]: gave the approximate path followed by air fighting aircraft and investigated the dynamic behaviour of the whole structure under action of motion and thermal loads during the critical instant attained by aircraft. The analytical and experimental methods for calculating the moment of inertia of the whole system (air combat aircraft) and how to find the center of gravity to calculate the trim moment of the aircraft in the longitudinal direction and aerodynamic twist when it subjected different load conditions (motion). Also the detailed structural analysis for that structure was introduced.

William, L. K., & Leslie, G., (2001) [65]: performed the thermal stresses and structural analyses on the unconventional wing structures of a Hyper-X hypersonic flight research vehicle (fighting aircraft designated as X-43) subjected to nominal Mach (8-11) aerodynamic effects and (4-5) g's axial acceleration for critical launching conditions. A wing mid-span cross section

was selected for the structural and thermal stress analyses. Structural analysis was performed on three regions of the wing skin (lower or upper); 1) a fore wing panel, 2) an aft wing panel, and 3) a unit panel at the middle of the aft wing panel. A fourth structural analysis was performed on a mid-span wing segment. The unit panel region was identified as the potential thermal buckling initiation zone. Therefore, thermal buckling analysis of the Hyper-X wing panels could be reduced to the thermal buckling analysis of that unit panel. “Buckling temperature magnification factors” were established. Structural stress-time histories were presented for the critical motion of aircraft ((4-5) g's total acceleration). The results show that the concerns of shear failure at wing and spar welded sites, and of thermal buckling of Hyper-X wing panels, may not arise under Mach 7 conditions.

Hunter, J. E., et al (2002) [23]: designed and analyzed stiffened panel concepts in air combat vehicle (**GTX-Aircraft**). The dynamic stresses and strains are investigated as function of ascending and descending trajectories and as result as function of time. A total of five critical events were selected for investigation of the ascent trajectory ranging from liftoff to mach 11. The intent was to bound the worse case environment for each system by the combining of inertial loads, aerodynamic pressures and temperatures, and internal combustion effects at each critical load event as mentioned in Chapter 1 (section 1.3). The geometry and dimensions of the integrated vehicle is shown in **Figure (2.1)**.

Salwan, O. W., (2005) [58]: dealt with two aspects: the first general numerical technique based on finite elements and calculus of variation was suggested to investigate the optimum design fiber-reinforced composite stiffened shell under certain design load, the second concerned with the dynamic analysis for some of wing structures that were created previously in order to study their dynamic responses. Newmark integration method was used to solve the dynamic equilibrium equation. Damping property was considered using Rayleigh type which is linearly related to the mass and stiffness matrices.

Systems natural frequencies were obtained by solving the eigenvalue problem by inverse iteration method. The numerical program which included all the above features was written in APDL (Ansys Parametric Design Language).

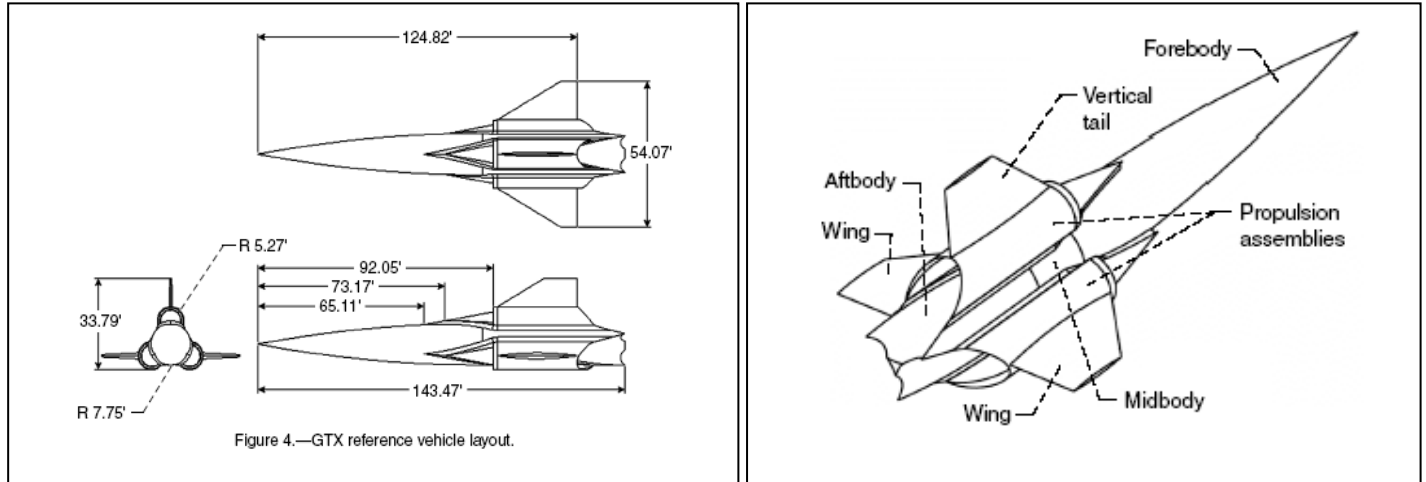


Figure (2.1) GTX-Aircraft configuration as described by Hunter, J. E., et al. [Ref. 23]

2.3 Structural Optimization:

Patnaik, S. N., & Maiti, M., (1976) [47]: presented the optimum design of stiffened cylindrical panels' weight as the objective function and constraints on static instabilities, the natural frequencies in the presence of initial stresses by using unconstrained minimization techniques of mathematical programming problem. The interaction between the buckling constraints (stability constraints) and the natural frequency constraints in the presence of initial stresses is considered. Loss of load carrying capacity due to imperfection and due to suddenly applied load is included in the buckling analysis.

Chan, A. S. L., & Turlia, E., (1978) [8]: solved the nonlinear structural optimization problem by successive approximations in which the objective and constraint functions are approximately treated as single term polynomials about a given design point at each step, and transformed into a linear programming

problem in the logarithmic space. Each feasible basic solution was constructed with the aid of pseudo constraint limits, which are obtained from the moving of specified design point in the feasible space at the previous approximated solution. The agreement of the obtained results with **Stafford & Kelly** (who used an approximate dual formulation of structural optimization) is exemplary.

Govil, A. K., et al (1979) [17]: presented the optimum design method with sub- structuring to complex wing- type structures to demonstrate its generality. The optimal design problem for idealized wing structures is defined as follows: find the design variables associated with each element of a wing structures. Such that the total weight is minimized, the equations of equilibrium hold and constraints on stress, displacement and member size are satisfied.

Rao, V. R., & Iyengar, N. G. R., (1979) [51]: attempted the investigation of minimum weight design of wing structures with restriction on strength, stability and frequency characteristics. They presented a detailed study of the theory of free vibration of thin elastic stiffened plates and demonstrated by experiment that the normal modes of vibration predicted do exist. The multi-web delta wing structure was idealized with three different kinds of finite elements. The constant stress triangular plate elements, the rectangular shear panels and pin jointed bar elements are used to present, respectively, the cover skins, web and stringers of wing structures. A parametric study is made to reduce the number of design variables. Off-design charts were obtained by performing sensitivity analysis about the final optimum design point and those data became the most suitable (convergent) in practical field.

Ding, Y., (1987) [11]: attempted the investigation of minimum weight design of sandwich construction with restriction on strength, stability and characteristics. They treated Finite Element analysis and the optimization problem of sandwich constructions. The thickness of the faceplates and the core are used as design variables. The hybrid approximation technique in combination with the dual method from mathematical programming is used.

Three examples are solved using six-nodal triangular and eight-nodal quadrilateral shell elements.

Yunliang, D., (1989) [67]: dealt with the optimum design of honeycomb sandwich constructions with restrictions on buckling characteristics. Four modes of instability for honeycomb sandwich structures were considered in buckling constraints, including the overall buckling, core shear instability, face wrinkling and mono-cell buckling. The face thickness, core depth, cell wall thickness and the diameter of an inscribed circle were taken as design variables. Eight nodal quadrilateral honeycomb sandwich isoparametric shell elements and hybrid approximation techniques in combination with the dual solution were used.

Ravi, S. B. R., (1999) [54]: presented a design methodology for the optimization of stiffened plates with frequency and buckling constraints. The basic idea of the methodology is to consider a plate with a fairly dense distribution of stiffeners. Thickness of the plate and stiffeners, and the stiffener width are the design variables. Design variable linking is accomplished by the use of rational spline surfaces. The finite element method is used for the analysis. The plate is modeled using linear Mindlin plate elements and the stiffeners by linear Timoshenko beam elements. Both the plate and the beam elements are shear- locking free by formulation, without requiring any special techniques such as reduced integration.

Karoly, J., (2000) [27]: aimed to show how to make the real structural optimization on a strong theoretical background through the making use of stiffened plates to get more stiff structure. Several calculations have been developed using stiffened plates all of them are approximations: the Massonnet & Gienke techniques. Cost calculations are also important due to the expensive welding technologies. Two applications are shown: shipdeck panel and compressed stiffened plate.

Jozsef, F., (2000) [25]: presented a new strategy to separate the material and fabrication costs to estimate the more realistic design (minimum cost design

of welded structures) and to show the achievable cost savings. The numerical examples of I-beam, stiffened box beams, stiffened plates loaded by hydraulic pressure, Vierendeel trusses, soils and bridge decks illustrated that significant cost savings can be achieved by minimum cost design.

Shenghua, Z. et al (2001) [61]: added V70.7 release of MSC.Nastran and the upcoming 2001 release to the Design Sensitivity and Optimization capability that is contained in SOL 200 to enhance its capability on analyzing the more complex problems in structural topological optimization to include more features in these techniques. A brief overview is given of all the major new optimization features. It is shown how the capability to designate material properties as design variables enables some innovative topology optimization techniques.

Rigo, P., (2001) [55]: presented the optimal design of marine structures (ships and naval structures). To perform a rational analysis, the structures modeled using stiffened plate and stiffened cylindrical shell elements in the linear elastic analysis of structures using the so-called “stiffened plate” method to provide a fast and reliable assessment of the stress pattern existing in the stiffened structure. The nonlinear constrained optimization problem was solved by iterative approach. The objective of the design was the minimum cost including raw material, labor and overhead costs. The design variables are the plate thickness and dimensions of the longitudinal stiffeners. Three types of constraints are distinguished: structural, technological and geometrical.

Kaarel, P., (2002) [26]: proved that the evolutionary structural optimization method is the most efficient technique used in the numerical analysis and design of engineering problems. Many of work examples investigated the robustness of this method to obtain the most realistic design especially in topology structural optimization under different boundary and load conditions. The multi-criteria evolutionary structural optimization was used to investigate the optimum design of thin and thick plates and many of mechanical

parts (beams, rectangular plates, circular plates, elbow frames, etc.) for which multi-criteria structural optimization is used (moment of inertia, strength, frequency, stiffness, etc.). The weight of the structural system was considered as objective function and the frequency and strength and stiffness were considered as driving functions (constraints) and the material properties and geometrical dimensions were considered as design variables.

Lagaros, N. D., Papadrak, M., (2002) [32]: presented the optimum design of stiffened shell structure. Combinatorial optimization methods and more specifically algorithms based on evolution strategies were implemented for the solution of the optimization problem. The efficiency of evolution strategies for solving optimization problems of real-scale stiffened shell structures under design codes is examined. Three optimization types have been considered: sizing, sizing combined with shape and sizing combined with shape and topology. The problem of shape- topology- sizing optimization of stiffened shell structures with the objective to minimize the weight of the structure under certain behavioral constraints on stresses and displacements is considered.

Bendsoe, M. P., & Rodrigues, H. C., (2002) [7]: presented a process for integrating 2-D structural topology optimization and shape optimization using the density contour approach. The critical issue is how to interpret topology optimization result into a representation that can be used in shape optimization. In the interpretation process presented in this research, a continuity analysis is first used to filter out the so-called “porous topology” formed by the elements with intermediate densities in the topology optimization result. Next, trivial solids and trivial voids are also filtered out. Finally, the black-and-white finite element topology optimization result is transferred into a smooth density contour represented by B-spline curves, which will be used as the initial design of shape optimization.

Eerme, M., et al (2004) [12]: suggested method for facilitating the difficulty of sandwich construction manufacturing by using the topological

optimization method, besides, they examined the strength properties of sandwich construction depending on rib configuration and the length of welds. Some of technological adaptations for construction of sandwich frames were described. They considered the chain-cutting machine body as a practical example in their works.

Yong, K., & Olivier, D. W., (2004) [66]: introduced the concept of variable chromosome lengths in the context of an adaptive genetic algorithm (GA). The (GA) concept is applied to structural topology optimization with large numbers of design variables. In traditional genetic algorithms, the chromosome length is determined when the phenotype is encoded into a genotype. Subsequently, the chromosome length does not change. They proved that this approach does not effectively solve problems with large numbers of design variables and complex design spaces, e.g. structural topology optimization, because the design spaces are extremely large, and it is very difficult to explore the design spaces in their entirety with reasonable population sizes.

Zoltan, V., (2004) [70]: investigated the optimum design of loaded stiffened plates with various plate types, loadings, and stiffener shapes. **Mikami** and **API** methods are used for the optimum design and comparison of the two methods and uni-axially compressed plates stiffened by ribs of various shapes. Both methods considered the effect of initial imperfection and residual welding stresses, but their empirical formulae are different. The elastic secondary deflection due to compression and lateral pressure is calculated using the Paik's solution of the differential equation for orthotropic plates, and the self-weight is also taken into account. Besides that deflection some more deformations were caused by lateral pressure and the shrinkage of longitudinal welds. The unknowns (design variables) were the thickness of the base plate as well as the dimensions and number of stiffeners. The cost function to be minimized included two kinds of material and three kinds of welding costs.

Chang, S.Y., (2004) [9]: presented a new developed material cloud method (MCM) for topology optimization to overcome some difficulties in traditional density distribution method and improve a numerical efficiency in topology optimization procedure. In MCM, an optimal structure can be found out through modifying sizes and positions of material clouds, which are lumps of material having specified properties. A design concept of MCM is presented and the results obtained by MCM are compared with those of density distribution method.

Klanac, A., & Kujala, P., (2004) [30]: presented the optimal design of steels sandwich panel applications in ships, which is a complex task, comprising of many subtasks such as load modeling, response calculations and optimization. The first half of the paper concentrated on presenting general aspects describing these. The second half describes the case study-a redesign of hostable car deck with the need to decrease the weight and cost of production. The design variables were the plate thickness and dimensions of the stiffeners.

Lagaros, N. D., & Papadrak, M., (2004) [33]: investigated the optimum design of stiffened shell structures by using a robust and efficient optimization algorithm where the total weight of the structure is to be minimized subject to behavioral constraints imposed by structural design codes. Evolutionary algorithms and more specifically the evolution strategies (**ES**) method specially tailored for this type of problems is implemented for the solution of the structural optimization problem. The discretization of the stiffened shell is performed by means of cost-effective and reliable shell and beam elements that incorporate the natural mode concept.

Moshe, B. F., & Nathan, P., (2005) [43]: applied optimum structural topological design methodologies to the stiffening beams of plates in bending. The purpose of the design is to use a given amount of material as orthogonal stiffeners to minimize the compliance of plates in bending. **Cheng & Olhoff** had shown that if left unchecked, that formulation included dense sets of infinitely

thin stiffeners – a solution though optimal, obviously impractical. In order to circumvent the obstacle, the formulation assumed that the plate was meshed by an orthogonal lattice with rectangular elements allowing the stiffeners be positioned only along the lines of the mesh. Assuming that the mesh has (l) longitudinal and (t) transversal lines, the basic problem is to be added in an optimal fashion, uniform beams along these lines, subjected to a constraint on the amount of material. The stiffening beams are assumed of rectangular cross-sections of constant height, with their widths or densities taken as the design variables x_i , $i=1, \dots, l+t$ of the problem. The standard topological design problem that calls for the minimization of the compliance, subjected to a constraint on the amount of material, which can be solved by classical optimization techniques has been stated.

Haider, M. A. A., (2005) [20]: dealt with the optimal elastic-plastic design of plate and shell structures based on elastic-plastic geometrically nonlinear incremental iterative finite element analysis. The modified Hooke and Jeeves are used as structural optimization method which considered the volume of the structure as objective function and dimensions as design variables with geometrical constraints.

Li, Q., & Steven, G. P., (2006) [34]: showed the equivalence of the von Mises stress criterion and the stiffness sensitivity criterion. Consequently, the criterion of Von-Mises stress in the classical ESO method is equivalent to the criterion of the compliance minimization or stiffness maximization, which has been exhaustively studied and well understood in structural optimization. It can be concluded that a stiffness optimization can at the same time yield a strength efficient design from the established stress criterion. From another point of view, the stiffness optimization problem can be solved by directly using the Von-Mises criterion, and vice versa.

2.4 Concluding Remarks:

Many remarks are extracted from reviewing the literature, which represent the basic principles in the present study, these are:

- More practical design can be reached through executing the structural optimization process with the discrete nature of its design variables and give the best design for global optimum if the probabilistic family of optimization methods is used instead of deterministic ones as done by present work.
- Topology structural optimization is very sensitive to the loading case and the governing criterion for which only numerical techniques are capable to predict the optimum distribution of structural material within feasible domain.
- The behavior constraints e.g., stress, deflections, natural frequencies & buckling etc. are functionally related through laws of structural Mechanics to the design variables.
- The evolution strategies and in particular their discrete version can be considered as robust and reliable tools for practical design optimization for stiffened plate and shell structures. With relatively few FE analyses, the evolution strategies implemented can reach the optimum design irrespective of the type of optimization, whether it is sizing or combined sizing-shape-topology optimization problem. Furthermore, the presented results indicate the beneficial effect of the optimum layout of the transverse stiffeners to the performance of these types of structures (under lateral loads) resulting in a substantial reduction of material weight compared to other types of stiffeners.
- It is evident that if the fundamental vibration frequency (the first mode only) of the loaded structure satisfies the minimum frequency restrictions instead of other modes of vibration for which the suitable stiffness within safe region can be realized, then the estimation of stiffness safety based on other modes of vibration with greater reason will satisfy the stiffness requirements.

CHAPTER FOUR

STRUCTURAL ANALYSIS & OPTIMIZATION

4.1 General:

The term "**Structural Optimization**" means the desire and drive towards the improvement in the performance of any given structure, set in a structural environment of loads, constraints and restraints. This improvement may be in anything related to the structure ranging from need to reduce the structural weight without compromising the structural integrity to the need to reduce the combined raw material demand and manufacturing cost of the structure including the operational cost through its expected lifetime (*Jozsef, 2000*)⁽²⁵⁾. The structural optimization can be implemented by finding the optimum value (maximum or minimum) of a function called "**objective function**" $F(x_1, x_2, x_3, \dots, x_n)$ of n real variables called "**design variables**" without sacrificing the functional and performance requirements which are called "**constraints functions**" (*Haider, 2005*)⁽²⁰⁾.

In structural optimization process, the multi-material model (with isotropic elastic materials) for various types of wing structure for a suitable optimality criterion was suggested. The stress failure criterion with constraints on frequency, and displacement were the drive toward the optimum design of both stiffened and sandwich construction models. In this process the structural weight will be treated as "**objective function**" and the special ability will be added to **FE program** to insert the material properties as design variables (through the multi-material modeling) instead of geometrical dimensions for enhancing the work which was implemented by [*Refs. 1, 2, 11, 51, 65, and 67*]. Two types of isotropic materials or alloys (29 Titanium alloys & 48 Aluminum alloys) were considered as successful and safe materials in the **GTX-wing** construction and each of physical, elastic, mechanical properties and thermal treatment conditions are listed in **Appendix –D-** (*Shackelford, et al, 2001*)⁽⁶⁰⁾.

4.2 Structural Optimization:

The optimization problem is classified on the basis of nature of equations with respect to design variables. In general, the design variables are real but sometimes they could be integers; for example, number of layers, orientation angle ...etc. The behavior constraints could be equality constraints or inequality constraints depending on the nature of the problem. The objective function depends on a number of parameters called **design variables**, such as dimensional parameters and material properties; each design variable is defined within a particular range.

The structural optimization problem can be expressed as (Lyengar, 1999) ⁽³⁸⁾:

Minimize or Maximize

$$F^{\circ} = F^{\circ}(x_1, x_2, x_3, \dots, x_n) \quad \dots\dots\dots$$

(4.1)

Subject to:

$$C^{\circ}_1 = C^{\circ}_1(x_1, x_2, x_3, \dots, x_n)$$

$$C^{\circ}_2 = C^{\circ}_2(x_1, x_2, x_3, \dots, x_n)$$

.

.

$$C^{\circ}_n = C^{\circ}_n(x_1, x_2, x_3, \dots, x_n) \quad \dots\dots\dots$$

(4.2a)

And

$$\phi^{\circ}_1 = \phi^{\circ}_1(x_1, x_2, x_3, \dots, x_n) \geq 0$$

$$\phi^{\circ}_2 = \phi^{\circ}_2(x_1, x_2, x_3, \dots, x_n) \geq 0$$

.

.

$$\phi^{\circ}_n = \phi^{\circ}_n(x_1, x_2, x_3, \dots, x_n) \geq 0 \quad \dots\dots\dots$$

(4.2b)

In which $x_1, x_2, x_3, \dots, x_n$ are the design variables (unknowns), $C^{\circ}_1, C^{\circ}_2, \dots, C^{\circ}_n$ are the equality constraints and $\phi^{\circ}_1, \phi^{\circ}_2, \dots, \phi^{\circ}_n$ are the inequality constraints. The nature of the mathematical programming problem

depends on the functional form of F_0 , C_0 , and ϕ_0 , if these are linear function of design variables, then the mathematical programming problem is treated as linear programming problem. On the other hand if any one of them is a nonlinear function of the design variables, then it is classified as nonlinear programming problem (Zoltan, 2004⁽⁷⁰⁾, Karoly, 2000⁽²⁷⁾, Ravi, 1999⁽⁵⁴⁾, Rapheal & Zafer, 1993⁽⁵³⁾, Ding, 1987⁽¹¹⁾).

4.2.1 Objective Function (Design Objective):

Type of optimization problem depends on the number of objective functions and the way they are treated. The notion of optimization also implies that there is some merit function $F_0(x)$ or functions ($F_01(x)$, $F_02(x)$... $F_0n(x)$) that can be improved and can be used as a measure of effectiveness of the design. Optimization with more than one objective or constraint function is generally referred to as "**Multi-criteria Optimization**". For structural optimization problems, weight, displacement, stress, strain, vibration frequency, buckling loads, cost and any combination of these can be used as objective function or constraints (Kaarel, 2002)⁽²⁶⁾.

4.2.2 Constraints Functions (Design Constraints):

The definition of load, response, strength, strain, stiffness and frequency criteria is needed to define the design constraints, all of which are evaluated numerically by using closed form solution. The formulation of the sandwich construction strains, stresses and frequencies is subjected to upper and lower limits on design variables. Because of their simplicity, these upper and lower limits on the values of the design variables are often treated in a special way by solution procedures, and are referred to as side constraints (Klanac, & Kujala, 2004)⁽³⁰⁾. Constraints which impose upper or lower limits on their quantities are by their nature inequality constraints. Some of engineering problems need to equality constraints, for example the equation provided by the equilibrium

process that structure must satisfy in terms of its design variables. Some strategies of solution of nonlinear optimization problems are unable to handle equality constraints, but are limited to inequality constraints only. In such instances it is possible to replace the equality constraints with two inequality constraints that form upper and lower bound constraints with a same limiting value (*Agarwal & Davis, 1974*)⁽²⁾. The constraints are linear or non-linear functions, either explicit or implicit of design variables ($\{x\}$). The constraints are limitations to what the user wants to impose on design variables, so they can be distinguished as:

- Technological constraints: those provide the upper and lower bounds of design variables for example $[x_i^{\min} \leq x_i \leq x_i^{\max}]$. It is also called side or practical constraints.
- Geometrical constraints: those impose relationships among design variables in order to guarantee a functional, feasible, reliable structure. They are generally based on good practice in rules that avoid local strength failures (suitable dimensions for structure) or to guarantee the welding quality and access to the welds.
- Structural constraints: those represent the limit states in order to avoid yielding, buckling, cracks etc. and to limit deflection, stress, frequency (or any other structural behavior constraints). Those constraints are based on solid Mechanics phenomena and modeled with rational equations. By rational constraints, it is meant a coherent and homogeneous group of analysis methods based on Physics, solid Mechanics, strength and stability treatises, etc.

4.2.3 Unknowns (Design Variables):

The notion of optimizing the structure implicitly presupposes some freedom to change the structural performance or behavior. The potential for change is typically expressed in terms of range of permissible changes of a

group of parameters. Such parameters are usually called "**design variables**" in structural optimization terminology and denoted by a vector $X = X(x_1, x_2, x_3, \dots, x_n)$. Design variables can be cross-sectional dimensions or member sizes or material properties, they can be parameters controlling the geometry, its material properties, etc. Design variables may take continuous or discrete values. Continuous design variables have a range of variation and can take any value in that range. Discrete design variables can take only specified values which always represent the practical field (**Andrzej, et al, 1986**)⁽⁴⁾.

4.3 Types of Structural Optimization:

Many structural optimization methods have emerged in recent decades with the development of computer technology. A large proportion of these methods use discrete Finite Elements. There are three main classes of structural optimization problems depending on the type of design variables used: sizing, shape, and topology. In sizing optimization problems, the aim is usually to minimize the structural weight under certain behavioral constraints on stresses, frequencies, and displacements. The design variables are most frequently chosen to be dimensions of the cross sectional areas of the members of the structure. In shape optimization problems, the aim is to improve the performance of the structure by modifying its shape, so the design variables are either some of the coordinates of keypoints in the boundary of the structure or some other parameters that influence the shape of the structure (i.e the position of element relative to another element in the structure). Topology optimization assists the designer to define the shape of the structure that is best suited to satisfy the operating conditions of the problem at hand (**Lagaros & Papadrak, 2004**)⁽³³⁾.

4.3.1 Sizing Optimization:

Sizing optimization defines the approach to change the sizes and dimensions of a structure to achieve the optimum design. This is obtained by finding the best possible combination of these sizes and dimensions.

This form of optimization involves the modification of parameters that generally define the size of complex structures (plate and truss structures). There are usually many parameters for each constituent making up the structure (*Lagaros & Papadrak, 2002*) ⁽³²⁾.

4.3.2 Shape Optimization:

Shape Optimization is a restricted form of topology optimization (*Bendsoe & Rodrigues, 2002*) ⁽⁷⁾. It determines the optimal boundaries of a structure for the given fixed topology. In this form of optimization, the object is to find the best shape that will have the best objective outcome as defined by the designer. Shape optimization may be added to the *Evolutionary Structural Optimization (ESO)* algorithm by adding a constraint to the method, which allows elements that exist only at the surface to be removed & (*Bendsoe & Rodrigues, 2002*) ⁽⁷⁾.

4.3.3 Topological Optimization:

Topology optimization is usually used to find the optimal distribution of material in the given design region that meets a predefined criterion. With topology optimization, regions of the structure that have the least contribution of the overall stiffness or natural frequency can be identified, so it should take into account the minimization of the structural mass with least impact on the performance of a structure. Topological optimization does not require the explicit definition of optimization parameters (independent variables to be optimized). In topology optimization, the material distribution function over a body serves as an optimization parameter. The theory of topological optimization seeks to minimize or maximize the objective function ($F(x_j)$) subjected to constraints ($C_i(x_j)$). The design variables (x_j) are internal, pseudo

densities that are assigned to each finite element (j) in the topological problem. The pseudo density of each element varies from 0 to 1, where ($x_j=0$) represents the material removed, and ($x_j=1$) represents material that should be kept (*Eerme, et al, 2004*)⁽¹²⁾, (*Gil & Yoon, 2004*)⁽¹⁵⁾, (*Yong & Olivier, 2004*)⁽⁶⁶⁾, & (*Bendsoe & Rodrigues, 2002*)⁽⁷⁾.

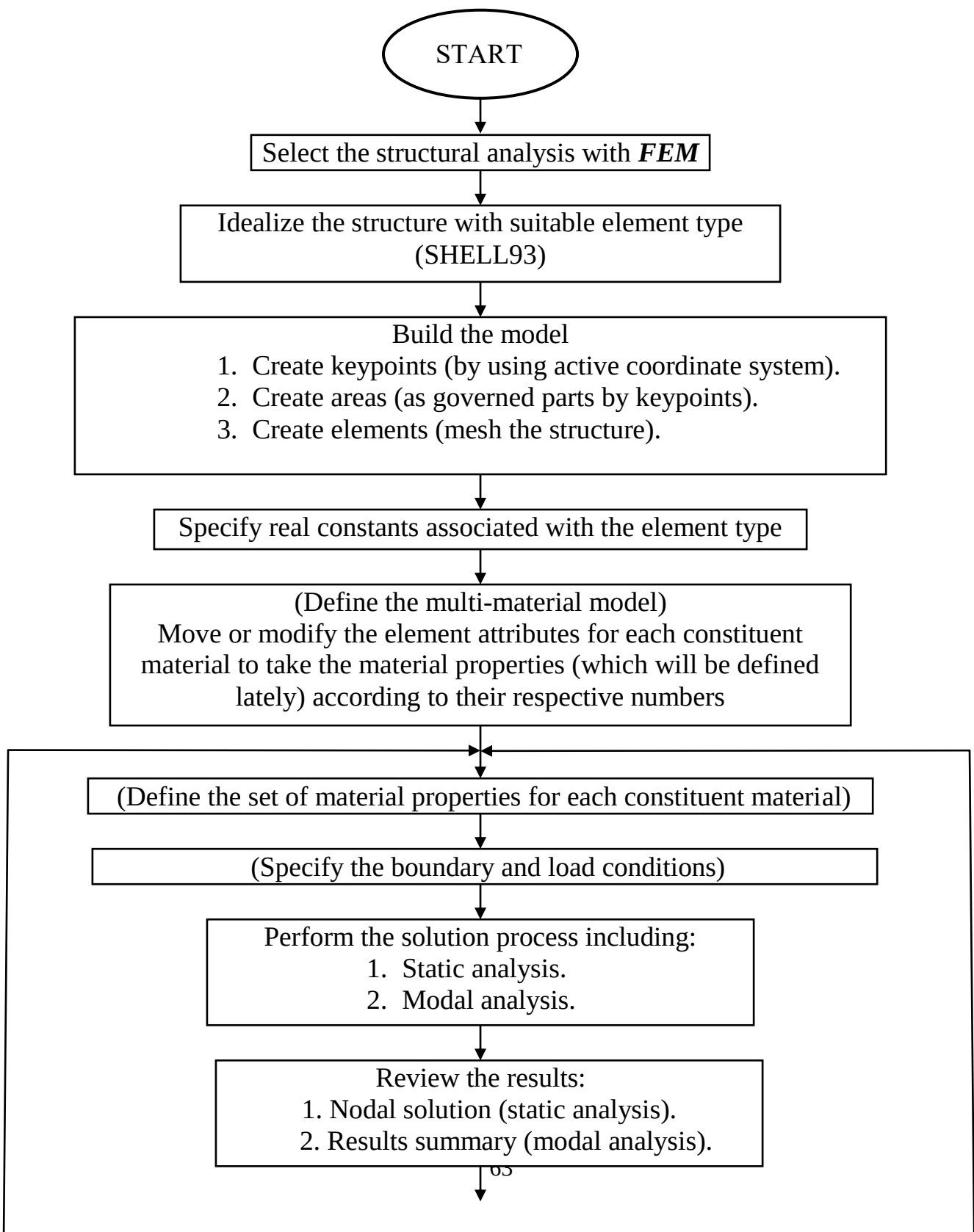
4.4 Failure Modes in Stiffened Plates & Sandwiches:

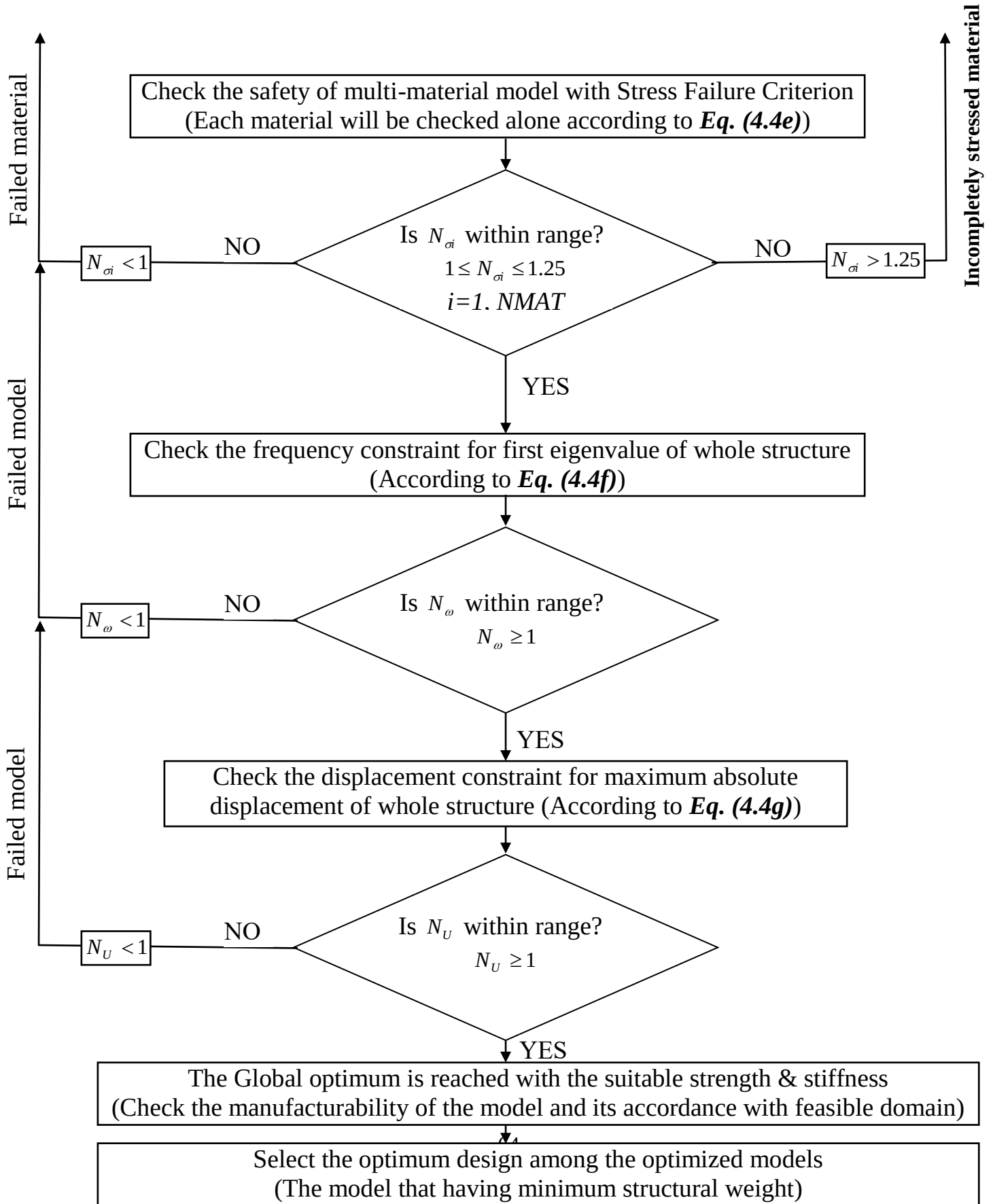
There are six basic modes of failure of stiffened plates and sandwiches; these are face tension failure, core shear failure, general buckling, face wrinkling, shear crimping, and face dimpling (*Yunliang, 1989*)⁽⁶⁷⁾. The material failure analysis (first and second modes) of the panel is done by the systematic application of maximum strain criterion to the skin and the stiffeners for any panel configuration. The remaining modes are happening due to instability of the panel under external loads. Once the critical loading is reached the effect is buckling failure of the panel. A stiffened plates structure is composed of two thin face sheets that support the membrane and bending loads, bonded to a thick core which resists the shear loads. A sandwich structure (stiffened in more than one direction) should be designed to meet strength and budding requirements (*Mario, 1997*)⁽³⁹⁾, such that: the strength of the face sheets should be enough to withstand the tensile, compressive, and in-plane shear stresses generated by the design load conditions; the thickness and shear modulus of the core should be sufficient to support the transverse shear stresses due to the design loads.

4.5 Evolution Strategy with Material Variables:

For the present work, the multi-material modeling is presented, so the **Finite Element (FE)** program is prepared to model the structure with several materials and each element that belongs to the structure can pose material properties different from the neighboring element, which in turn affects the mechanical and elastic properties of the whole structure. The variety of materials within a single structure in **FE** modeling can be implemented by modifying

material number and its properties (mechanical, elastic, physical, etc.). The proposed methodology that is used to perform the structural optimization for multi-material modeling by *FE* program is shown by Flowchart in **Fig. (4.1)**.





4.5.1 Representation of Objective Function (Structural Mass):

To represent the structural mass completely, it is necessary to describe the mass of each component as simple shapes (rectangle, triangle, circle ...etc.) separately.

Thus, the structural mass represents the summation of all structural masses of the above simple shapes, so it can be formulated in the following form:

$$m_{\text{structural}} = \sum_{i=1}^n m_i = \sum_{i=1}^n \rho_i * \forall_i \quad \dots\dots\dots (4.3)$$

n is the number of constituent materials (simple geometric shapes).

m_i is the structural mass of the i^{th} material.

ρ_i is the mass density of the i^{th} material.

$\forall_i$ is the volume of the i^{th} material (**Ji-Ho, 2004**)⁽²⁴⁾.

4.5.2 Representation of Design Constraints:

Material failure in the skins and stiffeners is estimated using a Von-Mises criterion. The design variables that can be used for optimization include all the material properties for each material in the structure that need to govern the structural strength, frequency, and displacement of the whole wing structure. Applied loads are inertia loads due to payload (excess loads borne by the wing structure) and structural weight (self-weight).

As mentioned above the strength criterion was tested for each material alone. The strength criterion will be the drive toward the optimum design (the structural mass was obtained as result of applying strength failure criterion).

The strength failure criterion given by *Eq. (B.1)* is modified, so it can be given in the following form (*Ji-Ho, 2004*)⁽²⁴⁾, (*Rao & Leyngar 1979*)⁽⁵¹⁾, (*Yunliang, 1989*)⁽⁶⁷⁾, (*Lagaros & Papadrak, 2002*)⁽³²⁾, (*Govil, et al, 1979*)⁽¹⁷⁾ & (*Sudharsan, 2003*)⁽⁶²⁾:

$$\frac{\sigma_{ypi}}{\sigma_{VMi}^{\max}} \geq 1 \quad \dots\dots\dots (4.4a)$$

Where:

$\sigma_{VMi}^{\max}$ = Maximum Von-Misses stress for the i^{th} material as defined by *Eq. (B.7)*.

σ_{ypi} = is the yield stress for the i^{th} material.

Specific strength (strength to weight ratio) of any structure can be defined as the strength of that structure divided by its mass or weight. The special feature behind the stiffened structures industry is their high specific strength fulfilled by material reduction relative to other structures where the small additive material to the stiffeners gives the philosophy behind the huge increase in their specific strengths.

There are also constraints on natural frequency (eigenvalue of first mode only) of the structure such that, (*Olson & Lindberg 1970*)⁽⁴⁶⁾, (*Patnaik & Maiti, 1976*)⁽⁴⁷⁾, (*Sudharsan, 2003*)⁽⁶²⁾, & (*Rao & Iyengar, 1979*)⁽⁵¹⁾:

$$\omega_{n1} - \omega^{\text{lower}} \geq 0 \quad \dots\dots\dots (4.4b)$$

Where ω_{n1} is the first mode natural frequency of the system (wing structure), ω^{lower} is the lower bound of natural frequency which is limited to **(40 Hz)**.

The displacement constraint may be given in the following form (*Ding, 1987*)⁽¹¹⁾, (*Agarwal & Davis, 1974*)⁽²⁾, (*Patnaik & Maiti, 1976*)⁽⁴⁷⁾ & (*Luciano, et al, 2005*)⁽³⁷⁾:

$$U^{\text{upper}} - U^{\max} \geq 0 \quad \dots\dots\dots (4.4c)$$

Where $U^{\max}$ is the maximum absolute displacement of the whole wing structure (usually equal to absolute tip point displacement) which is obtained from *FE* solution (deformational motion) by using *FE* program, U^{upper} is the

upper bound of absolute displacement which is limited to **(100 mm)** (*Chan & Turlia, 1978*)⁽⁸⁾, (*Grall & Gurdal 1992*)⁽¹⁸⁾ & (*Luciano, et al, 2000*)⁽³⁶⁾.

The manufacturing constraints are limited to the number of materials and their arrangement in away to guarantee the manufacturing possibility and easiness as illustrated in **section (4.8)**. The number of materials (which fall in technological or practical constraints class) should obey the following formula:

$$N_{MAT}^{upper} - N_{MAT} \geq 0 \quad \dots\dots\dots (4.4d)$$

Where N_{MAT} is the number of structural materials, N_{MAT}^{upper} is the upper limit of number of materials which is limited to number (50).

Eqs. (4.4a), (4.4b) & (4.4c) which categorized within behavior or structural constraints can be rewritten in non-dimensional form as in the following formulae:

$$1 - \frac{\sigma_{VMi}^{max}}{\sigma_{ypi}} \geq 0 \quad \text{or} \quad 1 - \frac{1}{N_{\sigma i}} \geq 0 \quad \dots\dots\dots (4.4e)$$

$$1 - \frac{\omega_{n1}^{lower}}{\omega_{n1}} \geq 0 \quad \text{or} \quad 1 - \frac{1}{N_{\omega}} \geq 0 \quad \dots\dots\dots (4.4f)$$

$$1 - \frac{U^{max}}{U^{upper}} \geq 0 \quad \text{or} \quad 1 - \frac{1}{N_U} \geq 0 \quad \dots\dots\dots (4.4g)$$

In which $N_{\sigma i} = \frac{\sigma_{ypi}}{\sigma_{VMi}^{max}}$, $N_{\omega} = \frac{\omega_{n1}}{\omega_{lower}}$, $N_U = \frac{U^{upper}}{U^{max}}$ are the safety factors.

Where $N_{\sigma}, N_{\omega}, N_U$ are the safety factors for strength, natural frequency and displacement respectively. i denotes to the i^{th} material in the wing structure.

4.5.3 Representation of Design Variables:

The material properties for the present work take discrete nature of design variables (taken from **Ref. [60]**). The design variables are subjected to technological constraints within feasible range which limit the design variables with upper and lower bounds ($X_{lower} \leq X_i \leq X_{upper}$) in which the design variables take discretized irregular steps between the upper and lower bounds of these variables. As strength failure criterion, the high importance parameters that

control this criterion adopted here are the yield strength, elastic modulus and mass density, so the above three parameters were combined in two terms and specified as two design variables for each material called specific strength & specific stiffness. On this basis, the specific strength represents the most important design variable among other material properties for each material, so **Table (D)** is prepared for this purpose and the material properties are arranged in a manner such that the upper bound of design variables represent the highest specific strength among adopted materials $[(\frac{\sigma_{yp}}{\rho}) \text{ for Ti1 alloy}]$ which have the value of **(0.286100 MJ/kg)** and the lower bound of design variables represent the lowest specific strength among adopted materials $[(\frac{\sigma_{yp}}{\rho}) \text{ for Al48 alloy}]$ which have the value of **(0.0104089 MJ/kg)** as in Appendix –D-. the evolution toward the optimum design requires series of changes of specific strength between the above mentioned values with discrete irregular steps according to the properties of adopted materials (other design variables, i.e. mass density, elastic modulus, poisson's ratio, etc. for each iteration will be changed accordingly). These changes can be given in the following form:

$$\left(\frac{\sigma_{yp}}{\rho}\right)^{k+1} = \left(\frac{\sigma_{yp}}{\rho}\right)^k + step^j \quad \dots\dots\dots (4.5a)$$

$step^j$ is the rate of change of specific strength, which depends on the convergence of safety factor from its limits for the material at hand.

At iteration k , i^{th} material has a discrete value X_i^k . If **Eq. (4.4e)** is not satisfied as shown by **Fig. (4.1)**, then iteration $k+1$ takes place which represents the next trial, so the discrete value of i^{th} material becomes X_i^{k+1} . This can be explained in the following fashion:

$$X_i^k > X_i^{k+1} \quad \dots\dots\dots (4.5b)$$

i : is the i^{th} material in the wing structure.

X : is the discrete value of design variable (at present, it represents the specific strength value as arranged in **Appendix –D-**).

k : is the k^{th} iteration in the evolution cycle.

X_i^k : is the discrete value for i^{th} material within feasible region (specific strength $\left(\frac{\sigma_{yp}}{\rho}\right)$) for material i in iteration k .

4.6 Finite Element Program:

The computer program (**ANSYS** program) is prepared for obtaining the optimum design through using the strength failure criterion which is detailed previously. The **FE** program is designed upon three stages. The first is to perform the transient dynamic analysis prior to optimization process which specifies the time at which the worst conditions (maximum Von-Mises stress) occurs for the whole wing structure. The second is to perform the structural optimization procedure (topological optimization with both geometrical & material variables) that fulfills **Eq. (4.4)** for strength failure criterion with frequency and displacement constraints to promote the works done by (**Zoltan, 2004⁽⁷⁰⁾**, **Karoly, 2000⁽²⁷⁾**, **Lagaros & Papadrak, 2004⁽³³⁾**, **Rao & Lyengar, 1979⁽⁵¹⁾**, **Ding, 1987⁽¹¹⁾**, **Yunliang, 1989⁽⁶⁷⁾**), in addition to the calculations of the safety factors for each of constituent materials by defining the mechanical and physical properties of all related materials. The third is to perform the transient dynamic analysis post to optimization process which investigates the dynamic behavior of wing structure and the comparison between the models to illustrate the optimum design among them.

The **Finite Element (FE)** method is considered as the most powerful numerical technique to analyze solid structures. In this technique, a complex region defining a continuum is discretized into simple geometric shapes called “**Finite Elements**”, so the **FE** technique is used as mathematical tool to perform

the structural analysis of the wing structure. The *FE* program has many *FEA* capabilities ranging from a simple, linear, static analysis to complex, nonlinear, transient dynamic analysis. Each element type has (5×8) degrees of freedom, which constitutes primary unknowns to be determined by the analysis. The data required for calculations of the element matrix which can not be determined from the node location or material properties are called real constants. At present investigation, the element *SHELL93* (isoparametric 8-node structural shell) is used in idealization of wing structure.

4.7**Assumptions:**

Numbers of restrictions were made to the present study to put the most suitable governing equations to solve the problem; these are:

1. The study is restricted to take the effect of pure inertia loads on the structural behavior of wing structure, so any effects associated with 3-D motion such as aero-dynamic pressure, induced shock wave, drag, and aero-heating loads (thermal loads) are neglected.
2. The gravity center point (**G**) of the GTX-Aircraft represents the separating point between the translational motion & the rotational motion where the **GTX-Aircraft** is translating as particle (**G**) about its path curvature center (**C**) and moving rotationally as rigid body rotating about the point (**G**).
3. The impulse and momentum method was used for Kinetics analysis.
4. The objective function is restricted to structural mass only.
5. The stress failure criterion (Von-Mises stress criterion) was used as design criterion and constraints functions are limited to frequency, and displacement constraints, so the stiffness, buckling, etc. constraints will be neglected.
6. The design variables within the single model are limited to material properties.
7. Special Evolution Strategy for structural optimization is used as a tool to obtain the optimum design for each model of wing structure.

8. Plane stress elements are considered in modeling process (the stress along the normal to the plate surface has zero value) (*Mukhopadhyay, 1981*)⁽⁴⁴⁾.
9. The variety of materials is restricted to stiffeners and sandwich honeycomb cores only; on this basis, the base plate is assumed to be consisting of a single material, for manufacturing purposes, the upper skin is assumed to be multi-material member (consisting of 8-materials (8-slices) only).
10. The wing materials are restricted to isotropic elastic materials (special Ti & Al alloys) as in *Appendix –D- (Shackelford & William, 2001)*⁽⁶⁰⁾.
11. The inertia loads were interpreted as point loads for each model.
12. Transient dynamic analysis (available in *FE* program) was adopted to investigate the dynamic behavior of wing structure (prior and post to optimization process).
13. The displacement constraints (the boundary conditions for which the displacement of all **DOF** equal to zero) were made at the region where the wing joined the fuselage structure as shown by *Fig. (4.6)*.

4.8

Finite Element Modeling:

The developments of suitable methods, more accurately, for analysis various engineering structures are needed in order to investigate their behaviour under different loading conditions. Whole dimensions of *GTX-Aircraft* adopted for present work is shown in *Figure (4.2) (Hunter, et al, 2002)*⁽²³⁾.

Six *Finite Element* models are created by using *Finite Element* program for the present study to represent the geometrical topology of the wing structure: the first model is prepared for the longitudinal stiffeners wing model. The second represents transverse stiffeners wing model. The third represents diagonal stiffeners wing model. The fourth represents rectangular–honeycomb sandwich construction wing model. The fifth represents rhombic–honeycomb sandwich construction wing model. The sixth represents triangular–honeycomb sandwich construction wing model.

4.8.1 Skins Modeling (Common Members Modeling):

The external skins are assumed to be consisting of lower plate and upper plate only as shown in **Figure (4.3)**. The skins are created as governed surfaces with [161 keypoints] as shown in **Figure (4.5)**. The first step represents the keypoints creation. The second step represents the areas creation by keypoints that facilitates the element creation step where the elements are created as governed parts by the areas in the second step. The lower plate is consisting of 60 rectangular 8-noded plate elements and 8 triangular 6-noded plate elements. The lace is consisting of 12 rectangular 8-noded plate elements and 7 quadrilateral 8-noded plate elements and 1 triangular 6-noded plate element. The lower plate and its lace is assumed to be consisting of single material (single plate) while the upper plate is consisting of 8 materials (slices) to facilitate the manufacturing process while assembling the stiffeners as shown in **Figure (4.4)**. The external skins (lower and upper plates) represent the common parts for all models and used in the modeling of all models (where each model differs from other by the stiffeners patterns) to state the effect of stiffeners and honeycomb constructions cores. The above modeling was enhanced by the geometrical verification on mesh size done by the convergence test in **section (5.4)**.

The wing structure is clamped along the line joining the wing with the Aircraft fuselage as shown in **Figure (4.6)**. The aerodynamic shape and dimensions of the wing structure is illustrated in **Figure (4.7)**.

4.8.2 Stiffeners and Honeycomb Sandwich Cores Modeling:

It is not necessary to describe the specifications of the lower, upper skins or plates (since they represent the common parts for all models) where they are described in **section (4.8.1)**, so it is sufficient to describe the stiffeners modeling only for each model alone.

The longitudinal stiffeners modeling are shown in **Figures (4.8) & (4.9)** according to their elements and materials numbers respectively.

The transversal stiffeners modeling are shown in **Figures (4.10) & (4.11)** according to their elements and materials numbers respectively.

The diagonal stiffeners modeling are shown in **Figures (4.12) & (4.13)** according to their elements and materials numbers respectively.

The rectangular honeycomb sandwich modeling is shown in **Figures (4.14) & (4.15)** according to stiffeners elements and materials numbers respectively.

The rhombic honeycomb sandwich modeling is shown in **Figures (4.16) & (4.17)** according to stiffeners elements and materials numbers respectively.

The triangular honeycomb sandwich modeling is shown in **Figures (4.18) & (4.19)** according to stiffeners elements and materials numbers respectively.

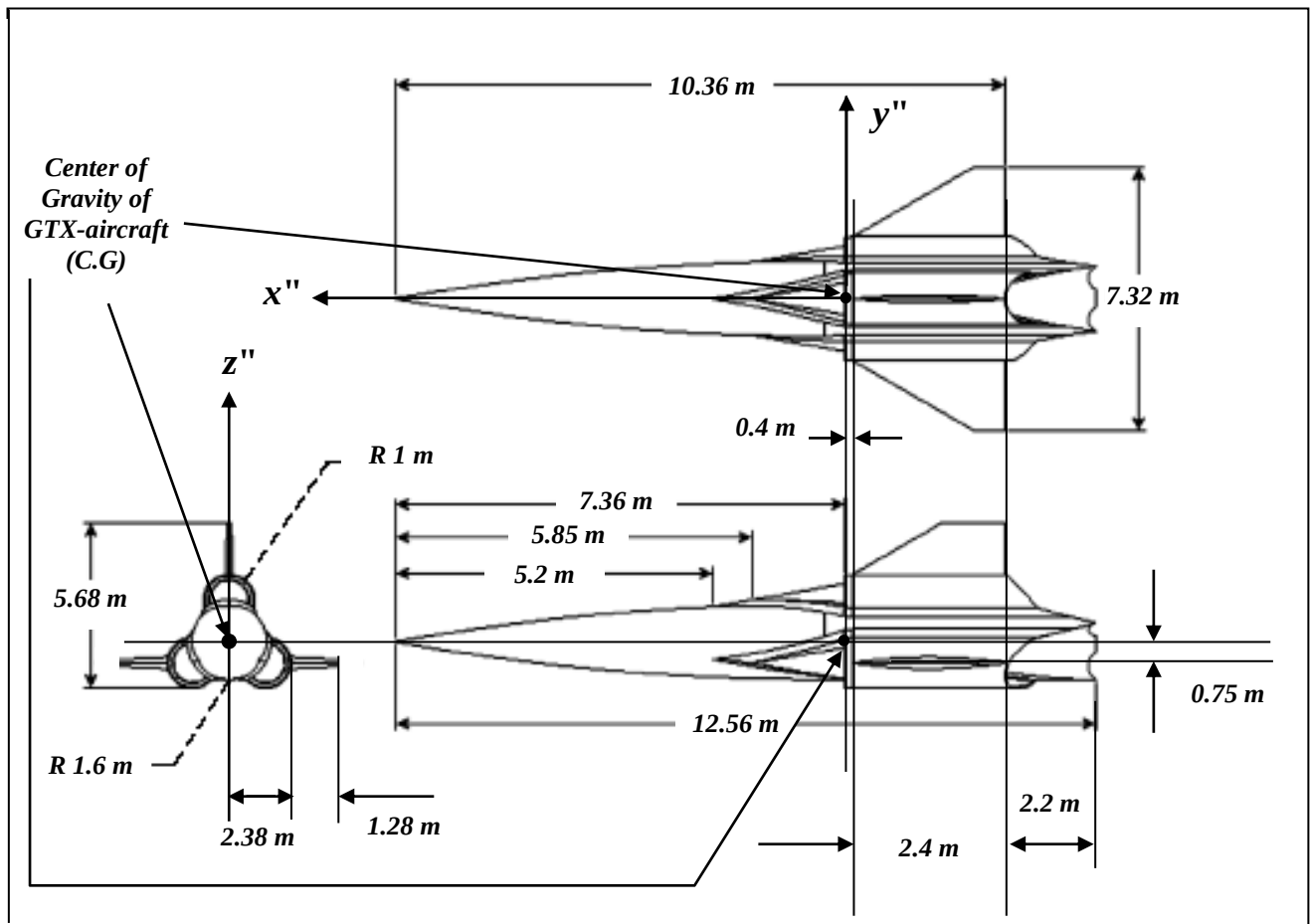
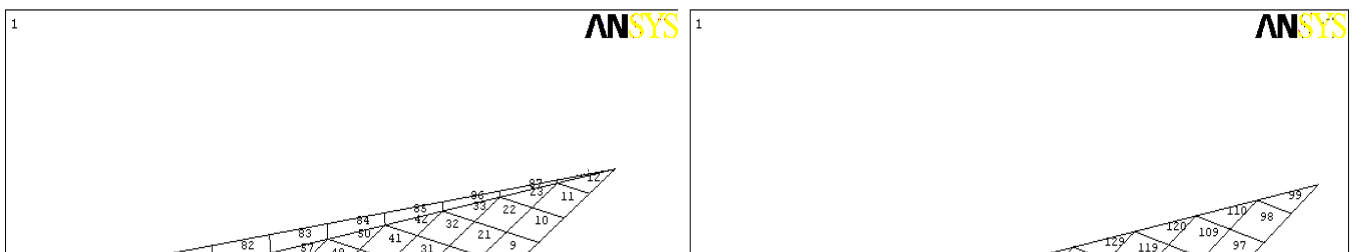
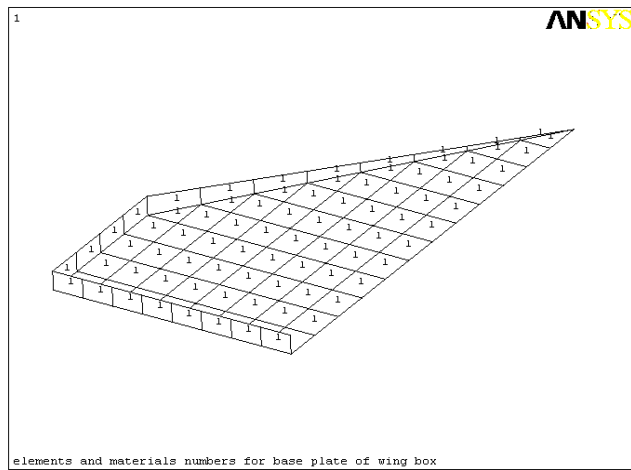
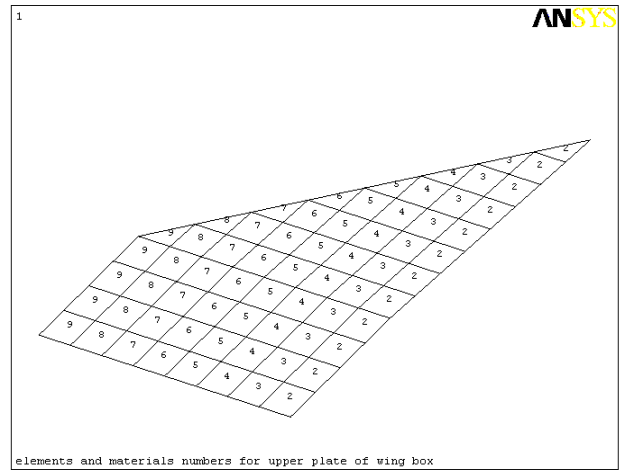


Figure (4.2) Geometry & dimensions of GTX-Aircraft used by Hunter (Ref. 23)

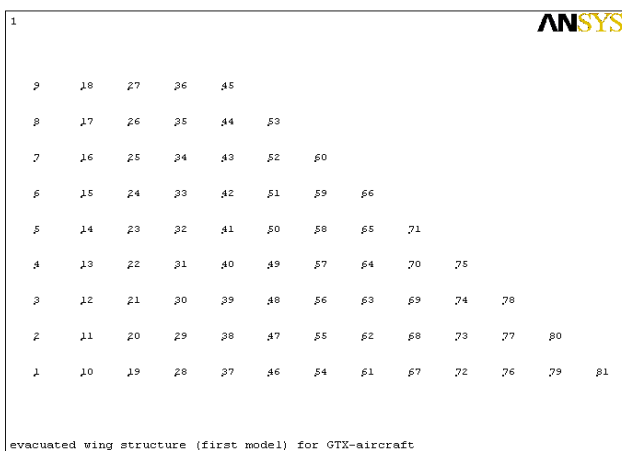




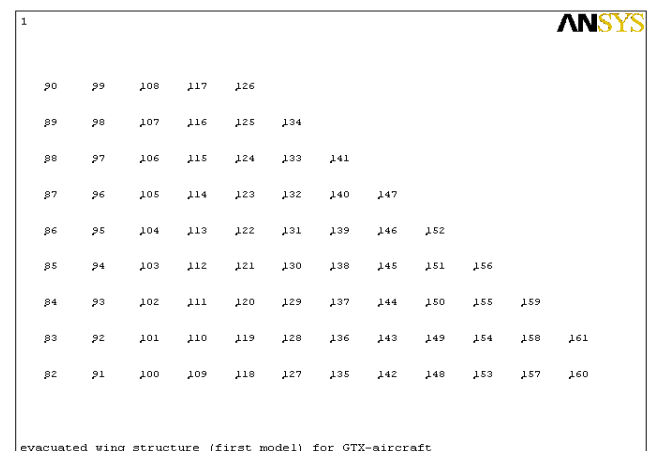
(a) Materials numbers for base plate (lower skin)



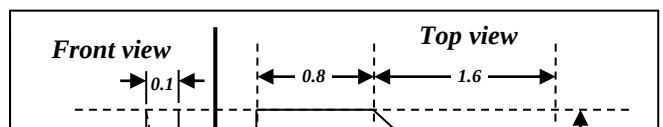
(b) Materials numbers for upper plate (upper skin)

Figure (4.4) Materials numbers for Skins modeling (lower & upper plates)

(a) Location and number of generated keypoints for lower skin



(b) Location and number of generated keypoints for upper skin

Figure (4.5) Keypoints numbers for skins

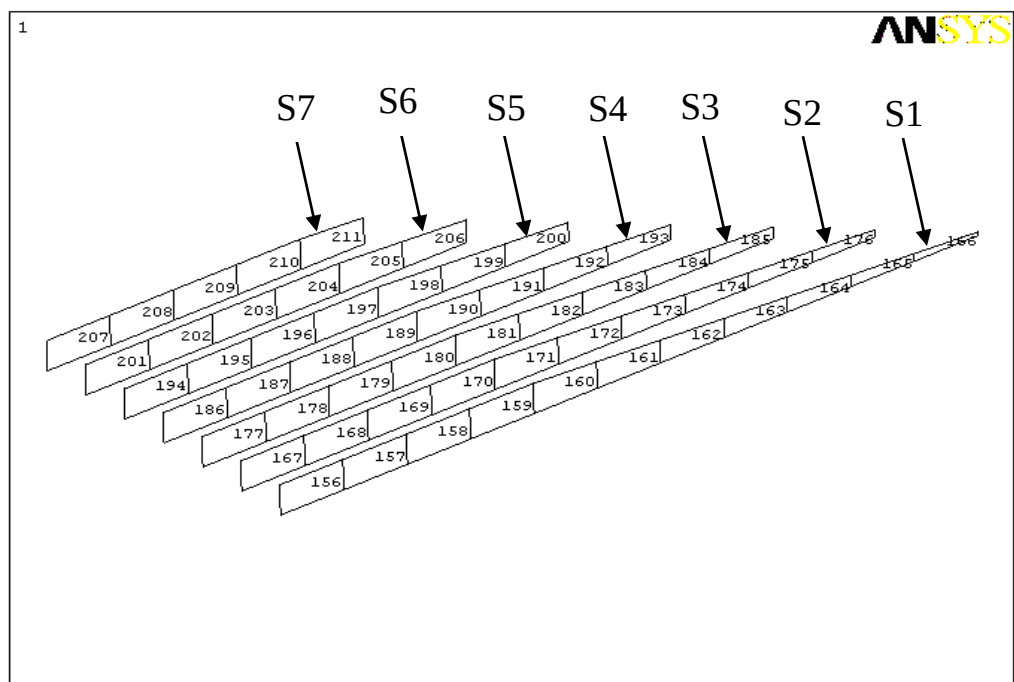
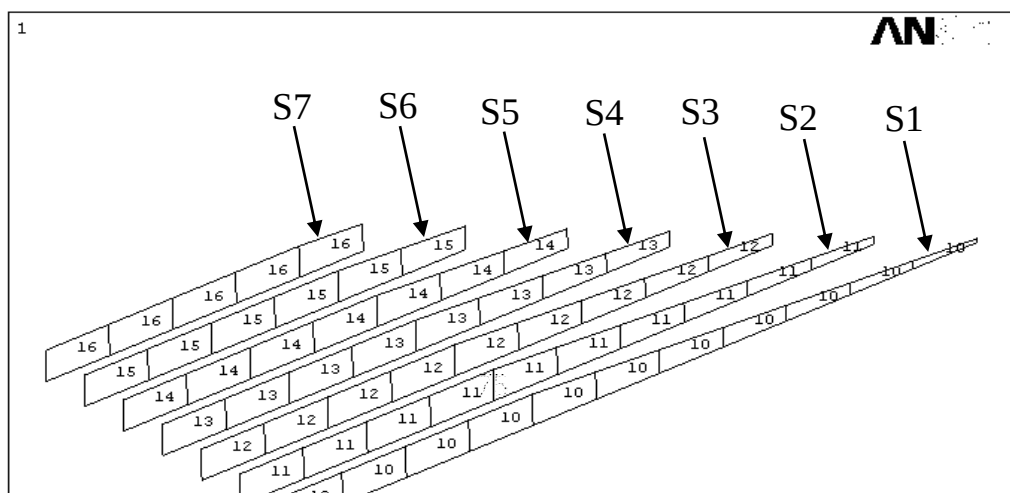


Figure (4.8) Elements numbers for longitudinal stiffeners of the first wing model



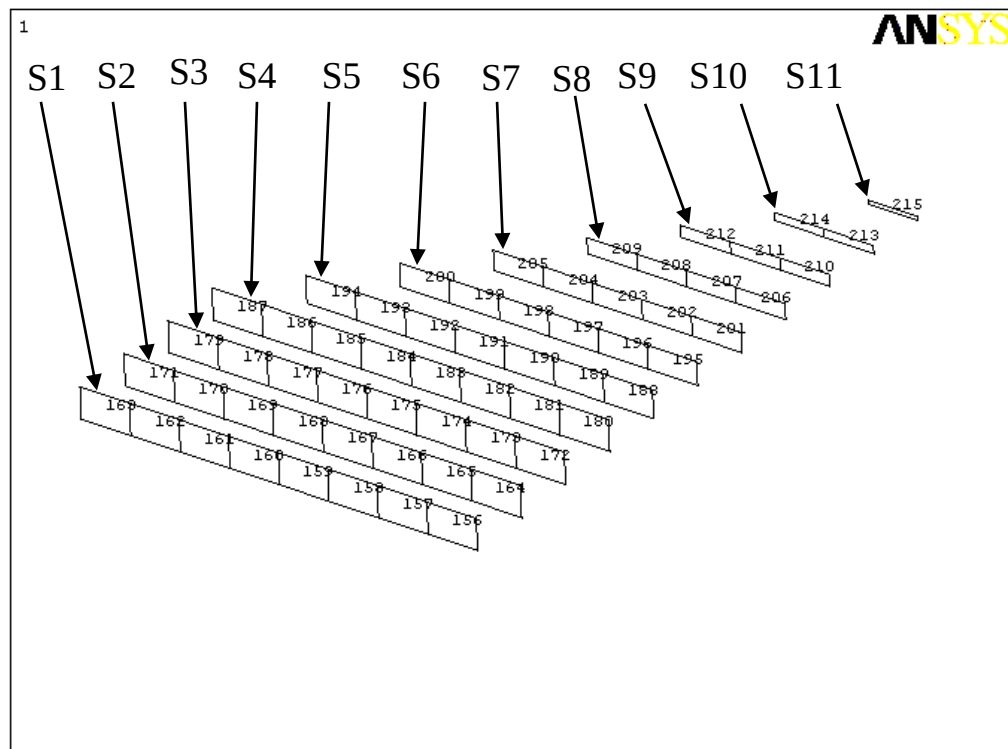
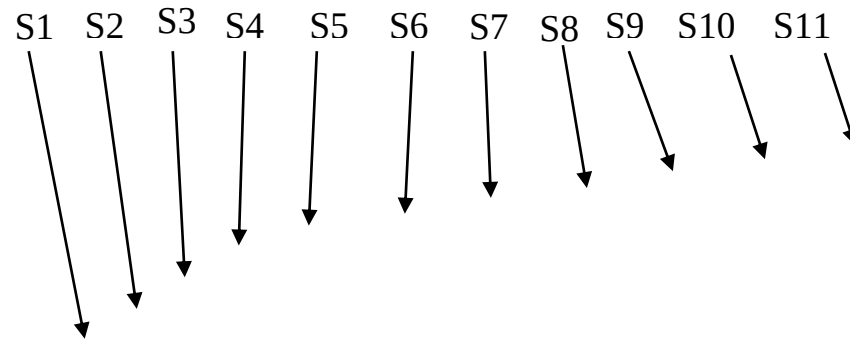
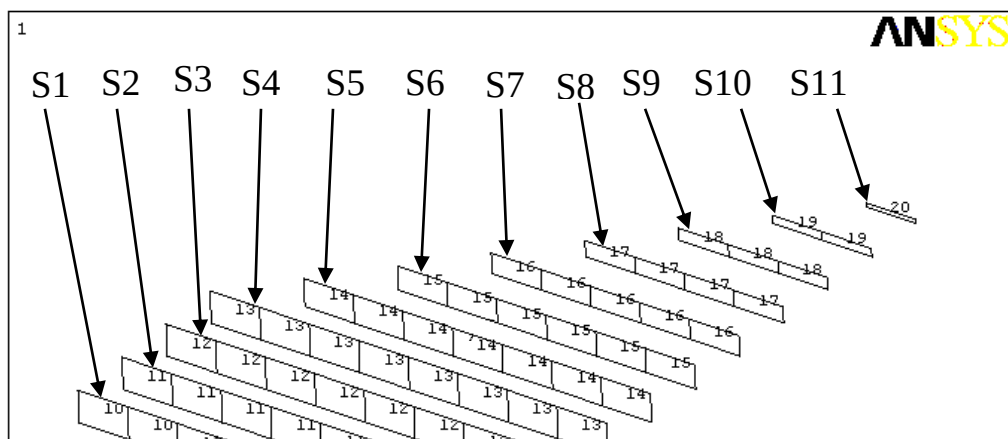


Figure (4.10) Elements numbers for transverse stiffeners of the second wing model



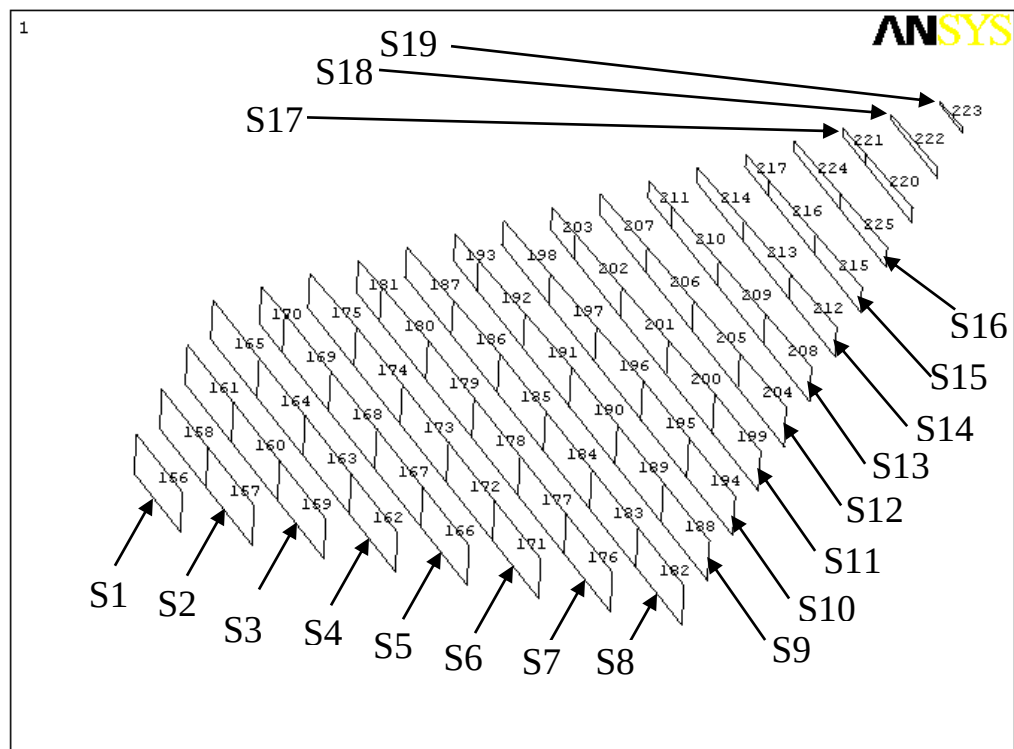
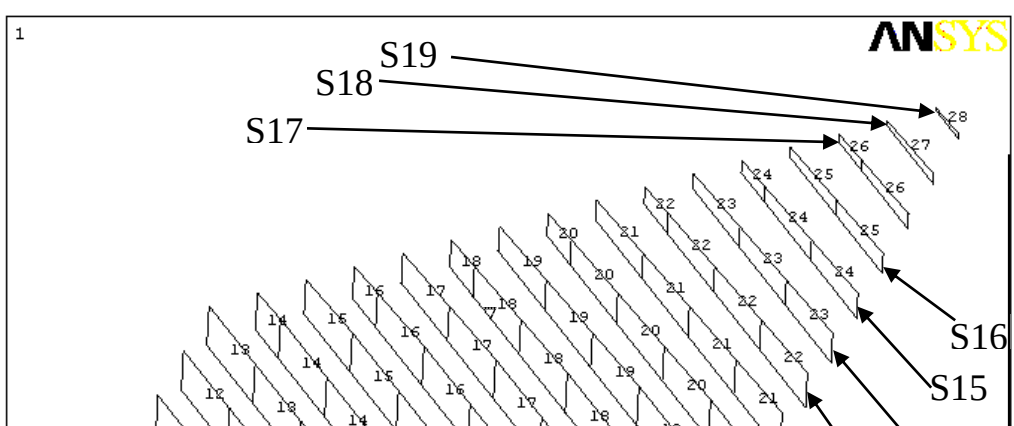


Figure (4.12) Elements numbers for diagonal stiffeners of the third wing model



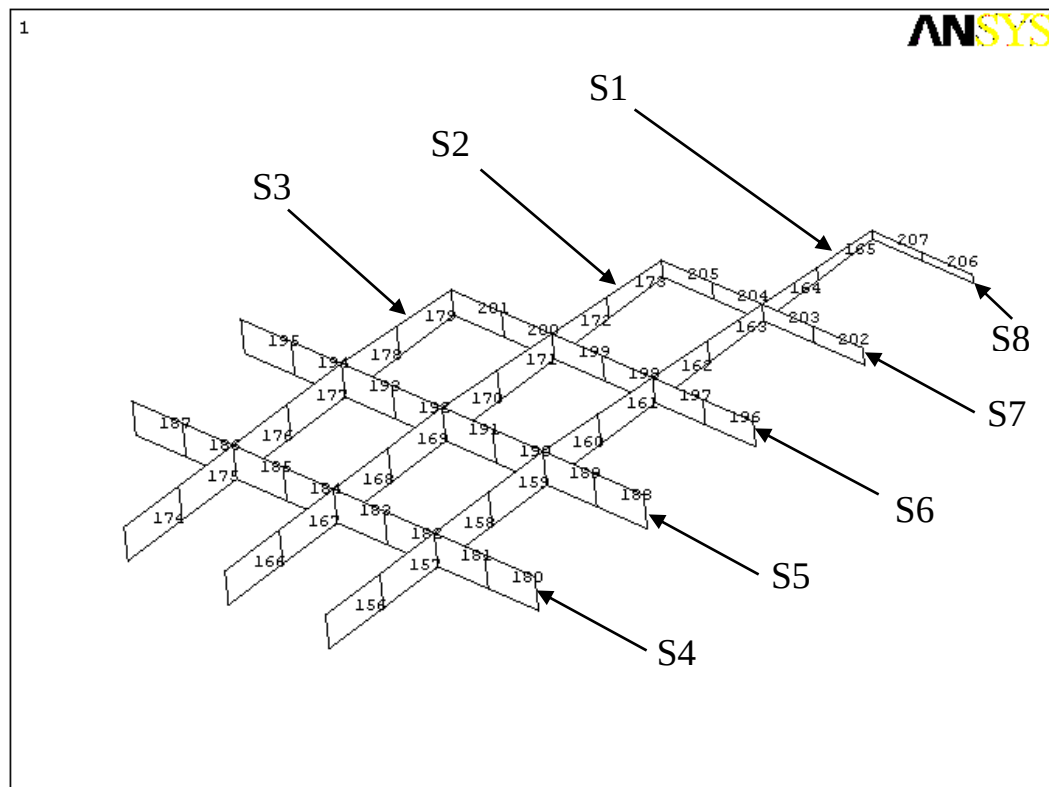
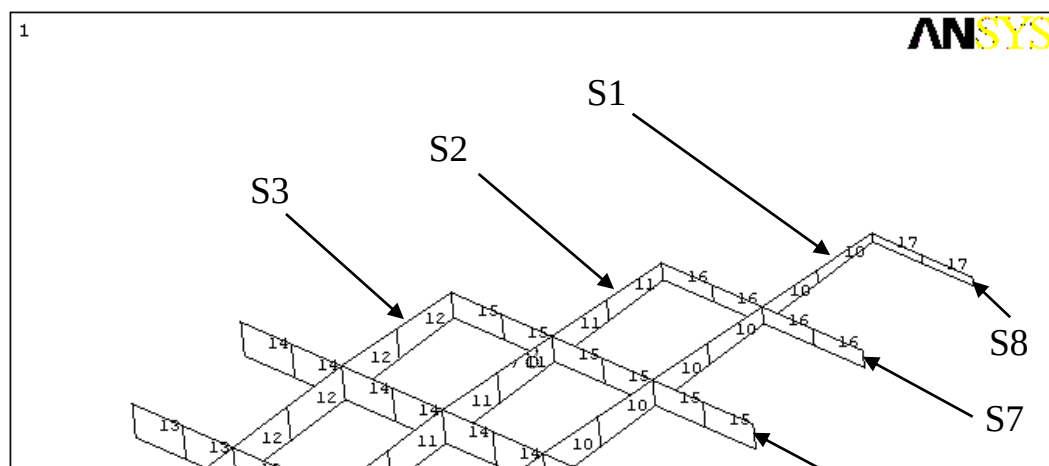


Figure (4.14) Elements numbers for rectangular honeycomb core of the fourth wing model



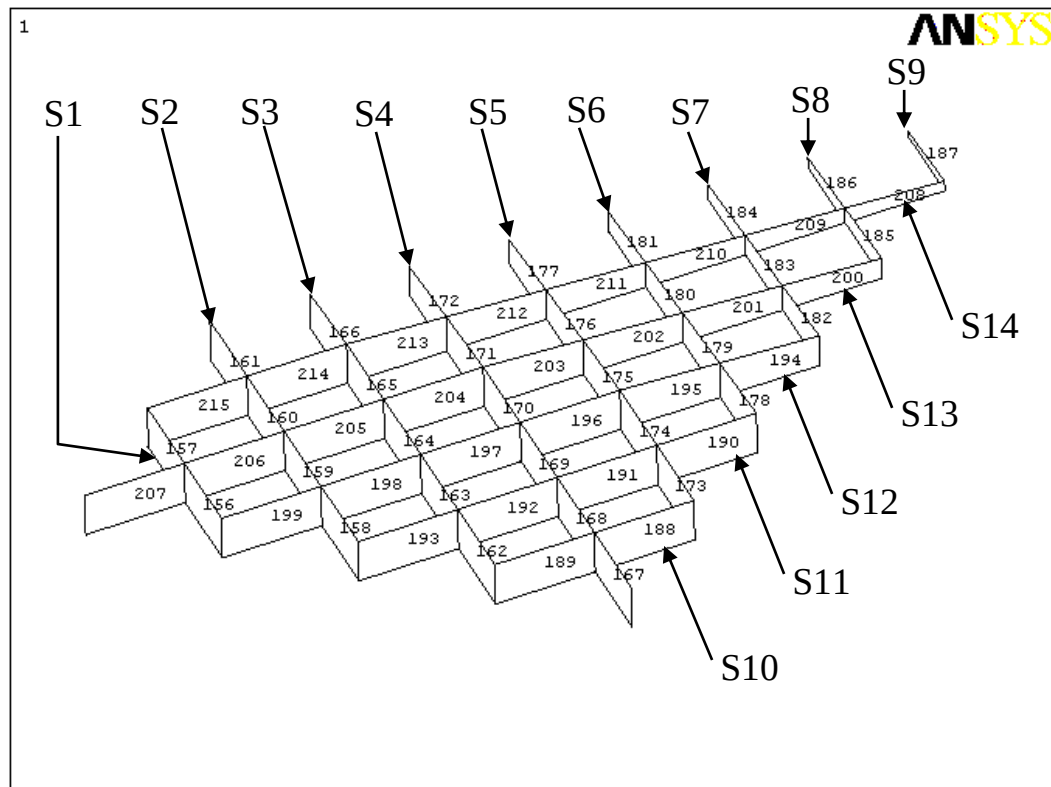
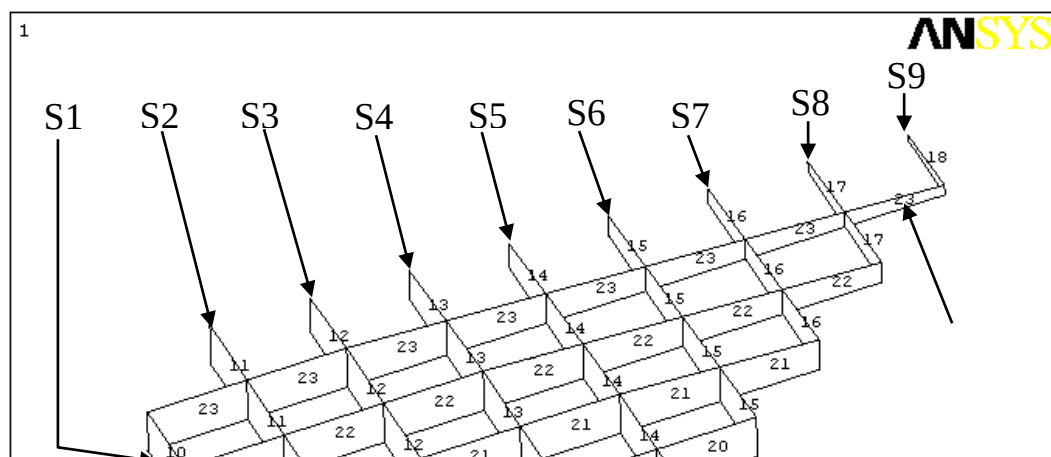


Figure (4.16) Elements numbers for rhombic honeycomb core of the fifth wing model



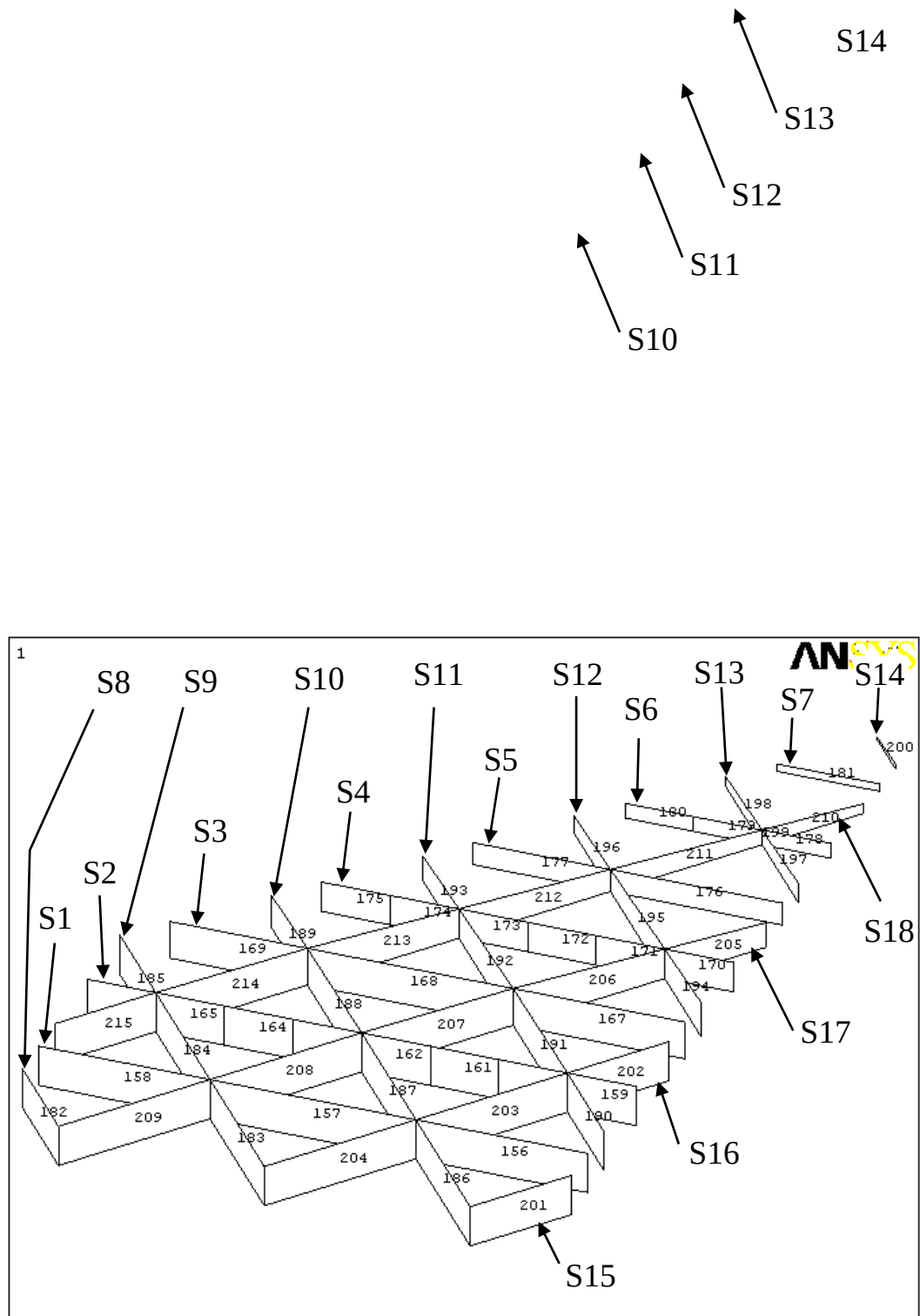
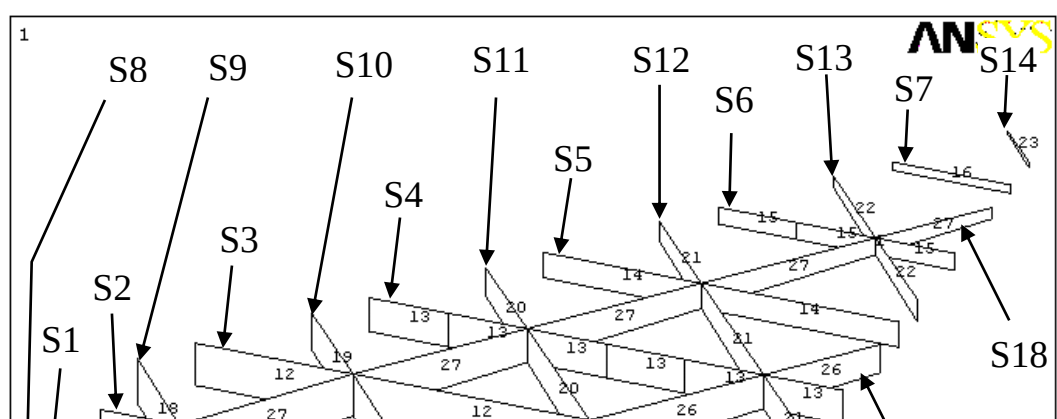


Figure (4.18) Elements numbers for triangular honeycomb core of the sixth wing model



CHAPTER FIVE

RESULTS & DISCUSSION

5.1 Case Study (*GTX-Aircraft Dynamics*):

As a case study, the *GTX-Aircraft* Dynamics was studied in details in this chapter for the considered critical period of time (from 2.895 Mach to 3.366 Mach) at which the maximum inertia loads will be attained. To describe the *GTX-Aircraft* Dynamics completely, the *GTX-Aircraft* Dynamics (*Hunter, 2002*)⁽³⁾ is viewed from two aspects taking into account sub-structuring analysis, so the wing structure was studied in details alone regardless of the other parts of aircraft (fuselage, fairing edge, engine liners, heat exchangers, etc.). The first aspect studies wing Kinematics, while the second aspect studies the wing Kinetics.

The motion of *GTX-Aircraft* was described by using its rectangular coordinates each alone according to the following:

1. Each rectangular component of linear acceleration vector is varying independently and assumed to be linear function of time $[f(t)]$. Thus each velocity and position vectors can be found in terms of their rectangular components by successive integration of the corresponding components of acceleration vector as done by *Eqs. (3.2e) & (3.2f)* respectively.
2. The spinning acceleration of *GTX-Aircraft* (rotational acceleration about x-axis) is assumed to be constant with time (constant acceleration), so the spinning velocity and position can be found as in item (1).
3. The precession rotational motion of *GTX-Aircraft* was studied that makes a condition “ $x' - y'$ plane of aircraft is coincident with $\bar{v}_t - \bar{a}_n$ plane” to maintain an ideal angle of attack (in which airflow meets the leading edge smoothly, resulting in a zero pressure differential across the leading edge) with radius

of precession (curvature) no less than 10 km to ensure small amounts of roll, pitch and yaw moments.

5.2 Wing Kinematics:

As mentioned in Ch. 3, wing Kinematics includes the study of translational, rotational & deformational motions.

5.2.1 Translational Motion:

The translational motion of **GTX-Aircraft** was described using the three conventional types of coordinates as follows:

- A- Rectangular coordinates: the rectangular coordinates of the translational motion for **GTX-Aircraft** at its critical period of time (during 4 seconds with time step of 0.25 second) was described in **Figures (5.1a), (5.1b) & (5.1c)** for acceleration, velocity and position vectors respectively according to **Eq. (3.4)**. Noting that x -component of acceleration is the most effective component and the greatest effect of this component can be shown evidently in velocity vector. Also it is shown that y -component of acceleration has negative value while the same component for velocity vector has positive value, this will results in turning of GTX-Aircraft with respect to curvature center (C) approximately about vertical axis (Z-axis) as shown in **Fig (3.15a)**, since the z -component has relatively low effects on both acceleration and velocity.
- B- Polar coordinates: This in turn is subdivided to cylindrical and spherical coordinates for general 3-D translational motion.
 1. Cylindrical coordinates: the cylindrical coordinates for **GTX-Aircraft** were illustrated in **Figures (5.2a), (5.2b), (5.2c), (5.2d), (5.2e) & (5.2f)** for $[r, \dot{r}, \ddot{r}, \theta, \dot{\theta}, \ddot{\theta}]$ respectively according to **Eq. (3.5)**. In this form of coordinates and other coordinates, the translational motion of GTX-Aircraft is described dependently which represents the restricted form of rectangular

- coordinates (i.e. depending on the nature of rectangular coordinates). **Fig. (5.2c)** shows that the acceleration in radial direction ($\ddot{\bar{r}}$) has approximately constant value for the period (2.5 $\rightarrow$ 4) sec.; this occurring when GTX-Aircraft motion (especially its plane motion vectors, i.e. its motion vectors lying in x-y plane) is taking less curvature approaching from the radial position vector ($\bar{r}$). The above interpretation was enhanced by **Figs. (5.2e), (5.2f)** for which the angular acceleration value $\ddot{\bar{\theta}}$ passing through the zero value at special instant (t=3.125 sec.), so the minimum angular velocity $\dot{\bar{\theta}}$ is obtained at this instant which is belonging to the period (2.5 $\rightarrow$ 4) sec.
2. Spherical coordinates: the spherical coordinates for **GTX-Aircraft** were illustrated in **Figures (5.3a), (5.3b), (5.3c), (5.3d), (5.3e) & (5.3f)** for $[R, \dot{R}, \ddot{R}, \phi, \dot{\phi}, \ddot{\phi}]$ respectively according to **Eq. (3.6)**. **Fig. (5.3c)** shows that less consistency for the radial acceleration ($\ddot{\bar{R}}$) at the above period (mentioned in item 1), this may be attributed to the additive effects of *z-component* for acceleration and velocity vectors for spherical coordinates. According to **Fig. (5.3d, e, & f)** there is appreciable reduction in angle (ϕ) associated with the negative values of angular velocity ($\dot{\bar{\phi}}$) which resulting from negative values of the difference angle between the velocity and position vectors in $\bar{R}-\bar{r}$ plane (i.e. $\phi_v - \phi_s < 0$) as illustrated by **Eq. (3.6d)**. The positive values of ($\ddot{\bar{\phi}}$) may be attributed to verification of the condition ($\phi_a - \phi_s > 0$) which resulting in decrease of the magnitude of ($\dot{\bar{\phi}}$) until obtaining zero value at special instant of motion.
- C- Normal-tangent coordinates: the normal-tangent coordinates for **GTX-Aircraft** were illustrated in **Figures (5.4a), (5.4b), (5.4c) & (5.4d)** for tangential acceleration, normal acceleration, radius of curvature and position of curvature center vectors respectively according to **Eq. (3.7)**. According to **Fig. (5.4a)**, it is observed that *x-component* of $\bar{a}_t$ approximately form the total value of tangential acceleration vector, while in **Fig. (5.4b)**, it is noted

that $\bar{a}_n$ is greatly affected by its y – component (i.e. has the greatest effect on its magnitude), so it is concluding that **GTX-Aircraft** is precessing approximately about the vertical axis (Z-axis) with respect to the global frame (CXYZ) shown by **Fig. (3.15a)**.

The acceleration vector (in terms of its rectangular coordinates) with respect to frame (Gx"y"z") (as illustrated by **Figure (3.15)**) is shown in **Figure (5.5)**. The acceleration vector mentioned above puts the idea behind the maximization of maximum Von-Mises stress induced in the wing structure at critical instant (t=2.258895 sec.), where the lateral acceleration (a_z) is behind this variation (variation of σ_{VM}^{max} with time), where the time at which the maximum lateral acceleration (a_z) occurs (i.e. when $\bar{a}_n$ become normal to the wing structure at the spinning frame (Gx"y"z")) represents the time at which the maximum Von-Mises stress achieves its maximum value due to neglecting the variations resulting from small rotational effects of **GTX-Aircraft** in comparison with the huge effects of the normal acceleration ($\bar{a}_n$) on the lateral direction of the wing structure due to its spinning. Up on this fact, the task of inspecting the perpendicularity of the normal acceleration vector $\bar{a}_n$ to the wing structure as result of its spinning motion about the gravity center of **GTX-Aircraft** is executed using Kinematics program. The perpendicularity of $\bar{a}_n$ to the wing structure in the transverse direction (i.e. $\bar{a}_z \cong \bar{a}_n$) is occurring at the instant (t=2.258895 sec.) according to **Fig. (5.5)**.

5.2.2 Rotational Motion:

The rotational motion of the wing about point (G) was described in terms of rectangular coordinates as shown in **Figures (5.6a), (5.6b), (5.6c), (5.6d) & (5.6e)** for precession & spinning motions as given by **Eqs. (3.8) & (3.10)**. The curvature of spinning position shown by **Fig. (5.6e)** is not appreciable in the mentioned **Figure** due to the relatively short period of time considered for this

description, where the curvature of the curve become more appreciable for longer periods ($t > 20$ sec.).

As excessive information about the rotational motion of *GTX-Aircraft*, the translational motion for the wing tip point (keypoint 126 as shown by **Figure (4.5b)**) about gravity center of *GTX-Aircraft* (*G*) was described in terms of its rectangular coordinates (w.r.t Frame (*Gxyz*) shown by **Fig. (3.15a)**) is shown in **Figs. (5.7a), (5.7b) & (5.7c)** as given by **Eq. (3.10a)**. According to **Fig. (5.7a)**, there is slight change in the absolute acceleration as compared to the huge changes in direction of its components that resulting from the rotational motion of *GTX-Aircraft* (especially its spinning) and the same observation can be seen for absolute velocity, while the absolute position should be constant during the rotational motion of *GTX-Aircraft*. Through tracing the path followed by wing tip, it is observed to be completely satisfying Newtons law's of motion though the complex relation used to find this behavior (coriolis and centrifugal components of acceleration vector as gyroscopic effects). According to **Fig.(5.7a)**, there are three inflection points in acceleration curve for the components (x, y, z) occurring at instants (4, 3.17, & 0.6 sec.) for x-, y-, & z-components respectively, these points represent the zero slope of the corresponding components of velocity vector shown by **Fig. (5.7b)**. The same observation can be investigated to state the relation between the components of velocity and position vectors, so the inflection points of velocity vector components are occurring at the instants (0.76, 0.54, & 3.19 sec.) for x-, y-, & z-component respectively which are reflecting the motion nature of the corresponding components of position vector illustrated by **Fig (5.7c)**, i.e. the points of zero slope for position vector is occurring at the same instants (0.76, 0.54, & 3.19 sec.) for x-, y-, & z-component respectively.

5.2.3 Deformational Motion:

The elastic deformations (translational degrees of freedom only) for keypoint 126 in terms of rectangular coordinates were described in **Figure (5.8)**. These deformations were investigated after performing the solution process in **FE** program on the optimum structural design for each of wing Models. It is noticed that Models 2 & 4 always have the minimum displacement.

5.3

Wing Kinetics:

In Kinetics investigation, it is needed to take the first two aspects of Kinetics problem, upon this any inertia effects due to elastic deformations will be neglected in comparison with those of particle and rigid body considerations.

5.3.1 Translational Motion:

In translational motion, it is required to study the linear inertia forces (produced by linear acceleration vector for translational motion which is given by **Eq. (3.17b)**) as shown in **Fig. (5.9)**. it should be remarked that there is approximately constant value for inertia forces induced in all Models due to the geometrical selection (within topological concept) of the Models for which there is small difference in their structural masses. **Fig. (5.9)** also shows the minimum value of y-Force occurs at $t=2.258895$ sec. while the maximum value of z-Force occurs at the same instant due the same reason mentioned in **item (C)** at **section (5.2.1)**. The symmetry of z-Force found in **Fig. (5.9c)** is attributed to the spinning motion of **GTX-aircraft** where the normal acceleration (regardless of rotationally induced accelerations) is analyzed into two components ($\bar{a}_{y'}$ & $\bar{a}_{z'}$) as given by rotation matrix [B] for which $\bar{a}_{z'}$ is directed upward for the first quarter of the spinning cycle, so the inertia force associated with this acceleration (i.e. translational z-Force as interpreted to **ANSYS** Package) is directed downward at the considered period of time (within first quarter).

5.3.2 Rotational Motion:

In rotational motion, it is required to present inertia forces resulting from inertia moments of rotational motion according to *Eq. (3.17c)* with time as shown in *Fig. (5.10)*.

5.3.3 Combined Motion:

The combined inertia forces (translational and rotational inertia forces) for the whole wing were shown in *Figure (5.11)*, which represent the superposition of translational and rotational inertia forces as given by *Eq. (3.17d)*. The transient dynamic analysis available in **ANSYS** Package was used to find the stress-response prior to optimization process to specify the time at which maximum Von-Mises stress occurs as shown in *Fig. (5.12)*. The worst load conditions for all Models were indicated in *Appendix–C-* (design loads are given as keypoint forces according to the respective numbers of keypoints illustrated in *Fig. (4.5)*). The effect of rotational inertia action is so small as compared to translation inertia action, so the dramatic changes in combined inertia forces may be attributed to the translational effect only as mentioned in *section (5.3.1)*.

According to *Fig. (5.12)*, the overall shape (chord wise corrugated path) of the curves is similar to the curve nature followed by each of transverse local acceleration $\bar{a}_z$, shown by *Fig. (5.5)*, the translational inertia z-Force shown By *Fig. (5.9c)*, and the combined inertia z-Force shown by *Fig. (5.11c)* at which the critical time occurs at the instant ($t=2.258895$ sec.) such that $\bar{a}_n$ become normal to the wing structure transversely (translational inertia forces appear their maximum effects) at the spinning frame ($Gx''y''z''$) as mentioned in *item (C)* at *section (5.2.1)*.

Also it is observed that Models 2 & 4 posses the minimum values of max Von-Mises stress which reflect the superior strength and stiffness of these

Models. The design loads (shown in **Appendix-C-**) is detached from curves of **Fig. (5.12)** at the critical instant ($t=2.258895$ sec.).

5.4 Convergence Test for Mesh Size:

The convergence test is performed to state the practicality of the present meshing size to give acceptable results within limited tolerance (with percentage error no more than 1 %). Model 2 is used for this purpose and many Finite Element analyses were executed under different meshing sizes (using mesh refinement found in **ANSYS** Package) to state the effect of mesh size and the percentage error in results produced for these meshes. The present mesh (with Number of Elements=215) give an acceptable error for all parameters of the structural behavior ($\sigma_{VM}^{\max}, \omega_{n1}, U_{abs}^{\max}$) as measured w.r.t the eventual asymptotes which are reported in the **Table** of **Fig. (5.13)**.

5.5 Parametric Study:

The parametric study was implemented here to facilitate the structural optimization process to take the shortest way conducted by design variables to achieve optimum design. This study was performed on the second Model (considering single material modeling), which is modeled as shown by **Figs. (4.3), (4.4), (4.10) & (4.11)**. This study is performed by taking the worse conditions (that achieve the maximum Von-Mises stress induced in that Model as a result of inertia loads). The parametric study was taken from three aspects which reveal the variation nature of each of Von-Mises stress, frequency, and displacement. The first aspect studies the variation of the structural strength with design variables as shown by **Figures (5.14a), (5.14b) & (5.14c)** for E, ρ & ν variations respectively. The second aspect studies the variation of the structural natural frequency with design variables as shown by **Figures (5.15a), (5.15b) & (5.15c)** for E, ρ & ν variations respectively. The third aspect studies the variation of the structural displacement (maximum displacement) with design variables as shown by **Figures (5.16a), (5.16b) & (5.16c)** for E, ρ & ν

variations respectively. The parametric study reveals that the structural strength did not affect E variation as single material, but the elastic modulus has considerable effects for multi-material modeling as relative strength. The Von-Mises stress is increasing with the increase of ρ & ν values. The structural natural frequency increases with E & ν increase, while decreases with ρ increase. The structural displacement is increases with ρ increase, while decreases with E & ν increase. In general, the increase in stress and displacement as ρ increases is attributed to the increase in structural inertia loads, while the decrease in ω_{n1} with ρ increase is attributed to the noticeable decrease in specific stiffness of the structure. The increase in E will result in structural stiffness increase which in turn increases the natural frequency while decreases the structural displacement. The effects of ν on the structural behavior are attributed to the complex nature of the minor effects in other directions.

5.6 Investigation of the Optimum Design:

As topological optimization, the investigation of optimum design needs to seek for the minimum induced stresses (high structural strength) for stress failure criterion, the highest natural frequency, and the minimum displacement (high structural stiffness), so the comparison among the considered Models will be performed according to structural strength and stiffness with achieving minimum structural weight.

5.6.1 Strength Investigation:

The common parts (lower and upper skins) were investigated (for strength) as a comparative study for each material to illustrate the effect of stiffeners alignment and honeycomb core patterns as geometrical variables and selection of materials as material variables. The minimum value of Von-Mises stress, the maximum structural strength is obtained. On this basis, the structural strength was investigated to state the highest strength Model among the adopted models.

5.6.1.1 Skins Investigation (Comparative Study):

As topological study, the skins represent the common parts that are found in all Models (as modeled by **Figs. (4.3) & (4.4)**), so the magnitude of the maximum Von-Mises stress (stress level) for each material in these parts will reflect the effect of stiffeners for that Model to increase its structural strength, so the stress level for each material may attributed to geometrical and material factors (within topological optimization interpretation).

Figures (5.17a) & (5.17b) present the stress, safety factor-Iteration history for material 1. Material 1 (lower plate) in Model 1 appears the maximum value of Von-Mises stress as final design ($\sigma_{VM}^{max} = 1930.611 \text{ Mpa}$) which exceeds the maximum value of yield stress ($\sigma_{yp} = 1379 \text{ Mpa}$) among the available range of materials listed in **Appendix –D-**, so Model 1 is considered as failing Model (failed with strength failure criterion). While material 1 in Model 2 has the value ($\sigma_{VM}^{max} = 490.7606 \text{ Mpa}$) which is suitable to take high strength Aluminum alloy (Al3) which has a yield point of ($\sigma_{yp} = 505 \text{ Mpa}$) as final design (ss=17), so the reduction factor achieved by max Von-Mises of lower skin in Model 2 is (0.93736) in comparison with rectangular honeycomb sandwich construction Model and (0.246185) in comparison with longitudinal stiffeners Model.

The structural materials is changing discretely, so there is a relative small changes in stress level as compared to the huge decrease in the limiting values (yield stresses) associated with the change in material properties for each iteration during the evolution cycle adopted to obtain the optimum design (optimized set of materials).

Figures (5.18a) & (5.18b) present the stress, safety factor-Iteration history for material 2. Material 2 (first slice in upper skin) in Model 1 appears the maximum value of Von-Mises stress ($\sigma_{VM}^{max} = 1145.338 \text{ Mpa}$), while for Model 3, the maximum value of Von-Mises stress is ($\sigma_{VM}^{max} = 820.7976 \text{ Mpa}$). While material 2 in Model 2 has the minimum value of max Von-Mises stress

($\sigma_{VM}^{max} = 279.0849 \text{ Mpa}$) among the Models which is suitable to take the sheet annealed Al-alloy (Al15) which has yield point of ($\sigma_{yp} = 295 \text{ Mpa}$), so the safety factor begins with value of ($N_{\sigma} = 4.650927$) as initial design (ss=0) and terminate with value of ($N_{\sigma} = 1.057026$) as final design (ss=17) which exactly verify **Eq. (4.4e)** for strength.

Figures (5.19) & (5.20) appear that materials 3 and 4 have the maximum values of max Von-Mises stress in Models 4 & 5, while they have the minimum values for Model 2.

In general, the minimum values of max Von-Mises stress for other materials (mats. 5 to 9) of the upper skin can be found in Model 2 (see **Figs. (5.21) to (5.25)**) where the values of max Von-Mises stresses are ($\sigma_{VM}^{max} = 127.2766 \text{ Mpa}$, $\sigma_{VM}^{max} = 92.35158 \text{ Mpa}$, $\sigma_{VM}^{max} = 63.17184 \text{ Mpa}$, $\sigma_{VM}^{max} = 40.16115 \text{ Mpa}$, $\sigma_{VM}^{max} = 23.76099 \text{ Mpa}$) for materials 5, 6, 7, 8 & 9 respectively, which achieve the stress reduction factors of (0.3302, 0.26122, 0.19205, 0.1335, & 0.084799 respectively) in comparison with rectangular honeycomb sandwich construction modeling (Model 4), where Models 2 & 4 achieved their objectives in structural weight minimization without sacrificing the stress failure criterion. The above values of max Von-Mises stress attained by Model 2 reflect the superior success in achieving the highest specific strength and stiffness as compared to other Models.

5.6.1.2: Stiffeners and Honeycomb Cores Investigation:

The strength investigation for the stiffeners of each Model represents the strength investigation of that Model, since the comparative study was performed on the skins materials which represent the common parts for all Models.

5.6.1.2.1: Model 1 (Longitudinal Stiffeners):

It is shown from **Fig. (5.40)** that the maximum Von-Mises stresses occurs at the root of the wing and decreases toward the tip of the wing, so that the

strongest material should be selected for root parts and the graduation of strength, so the contours shown by **Figs. (5.35) to (5.40)** illustrate the remarkable distribution of stress and displacement for all Models. Also it is seen from **Fig. (5.40)** that the absolute displacement has the zero value at the wing root (constraint value) and it increases toward the tip of the wing where it becomes maximum value at the wing tip. According to parametric study, the deflection is increasing with the decreasing of the stiffness properties. **Fig. (5.17) & Table (5.1)** show that the base plate of this Model fails according to the strength failure criterion with the limited range of design variables, while other materials appear their worthy to be successful members (**Figs. (5.18) to (5.25)**), so the recommendations about the strength investigation of its stiffeners are not necessary at this section, however, the stress evolution for the stiffeners of Model 1 is shown by **Fig. (5.26)**. **Table (5.1)** shows the best set of materials for Model 1.

Table (5.1) Optimum set for materials of Model 1

Material No.	Optimum set	Safety factor (N_{σ})	$[\sigma_{yp}]$ (Mpa)	$[\sigma_{VM}^{\max}]_{safe\ region}$ (Mpa)
1	Ti1	0.691762	1379	1993.461
2	Ti4	1.047726	1200	1145.338
3	Ti18	1.011194	896	886.0812
4	Ti25	1.004567	586	583.336
5	Al5	1.043308	455	436.1129
6	Al10	1.070305	370	345.696
7	Al15	1.051069	295	280.667
8	Al19	1.076811	230	213.5937
9	Al23	1.052848	150	142.4707
10	Ti8	1.067433	1103	1033.32
11	Ti23	1.051184	745	708.7249
12	Al5	1.036464	455	438.9926
13	Al18	1.048399	240	228.9205

14	Al15	1.066981	295	276.4809
15	Al21	1.080034	195	180.5494
16	Al26	1.045986	140	133.845
Whole wing	---	1.02788	---	---

5.6.1.2.2: Model 2 (Transverse Stiffeners):

The multi-material modeling requires the study of each stiffener separately. As initial design the material properties of superior mechanical properties Titanium alloy (Ti1) (as in **Appendix-D-**) are selected as initial design variables for all materials of this Model and other Models. In general, the change in structural behavior for each constituent material in multi-material modeling is governed by mass density $[\rho]$ as inertia loads, specific stiffness $[E/\rho]$, stiffness $[E]$ & poisson's ratio $[\nu]$ as structural strength. As evolution proceeds, the gradual change of material properties (especially specific strength $[\sigma_{yp}/\rho]$, since the satisfaction of strength failure criterion is the most important requirement for the structural design) of constituent materials takes place according to their strengths so to satisfy **Eq. (4.4e)** as a single material and (**Eqs. (4.4f), (4.4g)**) as a whole wing. According to **Fig. (5.27)**, the Maximum Von-Mises stress for stiffener No.1 (S1) as optimum design (ss=17) has the value of ($\sigma_{VM}^{\max} = 312.5383 \text{ Mpa}$), so the solution heat treated, artificially aged & cold worked Al-alloy (Al12) is the best material suited for this stiffener. For verification of strength failure criterion (minimum value of strength safety factor without violating failure limits $[N_{\sigma} \geq 1]$ according to **Eq. (4.4e)**), the stiffeners (S2, S3 & S5) as optimum design have the values of ($\sigma_{VM}^{\max} = 297.9258 \text{ Mpa}$, $\sigma_{VM}^{\max} = 294.0765 \text{ Mpa}$ & $\sigma_{VM}^{\max} = 292.6037 \text{ Mpa}$ respectively), so the same Al-alloy of S1 (Al12) is the best material suited for these stiffeners (S2, S3 & S5) (the strength safety factors for (S2, S3 & S5) have the values $N_{\sigma} = 1.090875$, $N_{\sigma} = 1.105155$, & $N_{\sigma} = 1.110718$ respectively), the optimum material for S4 that

exactly verify **Eq. (4.4e)** is the thermal treated Al-alloy (Al11), S6 & S7 select sheet annealed Al-alloy (Al15) as optimum design and so on for other stiffeners as in **Table (5.2)**.

In general, the sudden increase in safety factor between (ss=6 & ss=7) for all optimization plots is attributed to the noticeable increase in yield strength through the transition from the last Ti-alloy (Ti29) properties with ($\sigma_{yp} = 241 \text{ Mpa}$) to the first Al-alloy (Al1) properties with ($\sigma_{yp} = 525 \text{ Mpa}$). For most of optimization plots, the high slope of $N_\sigma - \text{Iteration No.}$ diagram between (ss=0 & ss=6) is mostly associated with the yield point reduction within Ti-alloys field. While the slope of less ramping of $N_\sigma - \text{Iteration No.}$ diagram between (ss=7 $\rightarrow$ optimum) is mostly associated with the yield point reduction within Al-alloys field. This gradual reduction in safety factor is attributed to the ordering for each type of alloys that appears the descending nature for the distribution of specific strengths of these alloys for each type as in **Appendix-D**.

Table (5.2) Optimum set for materials of Model 2

Material No.	Optimum set	Safety factor (N_σ)	$[\sigma_{yp}]$ (Mpa)	$[\sigma_{VM}^{\max}]_{\text{safe region}}$ (Mpa)
1	Al3	1.029015	505	490.7606
2	Al15	1.0570.26	295	279.0849
3	Al19	1.033979	230	222.4416
4	Al21	1.152277	195	169.2301
5	Al25	1.099966	140	127.2766
6	Al34	1.050334	97	92.35158
7	Al39	1.092259	69	63.17184
8	Al45	1.045787	42	40.16115
9	Al48	1.178402	28	23.76099
10	Al12	1.039873	325	312.5383
11	Al12	1.090875	325	297.9258
12	Al12	1.105155	325	294.0765

13	Al11	1.012263	345	340.8206
14	Al12	1.110718	325	292.6037
15	Al15	1.069725	295	275.7718
16	Al15	1.071688	295	275.2666
17	Al16	1.066312	255	239.1419
18	Al20	1.036183	215	207.4924
19	Al21	1.135777	195	171.6886
20	Al26	1.169793	140	119.6793
Whole wing	---	1.082370	---	---

5.6.1.2.3: Model 3 (Diagonal Stiffeners):

As shown in **Fig. (5.18)**, the increase in Max Von-Mises stress for material No.2 as iteration No. progresses may be attributed to two reasons; the first is relatively the high density within Ti-alloys field (which results in small changes in inertia forces), the second is due to selecting Al-alloy for the stiffeners under this material i.e the inertia loads are relatively borne by material No.2 more than by other materials due to the relatively high stiffness for it as material effects.

According to **Fig. (5.28)**, the steep reduction in stress for stiffener No.11 or material No.20 (from $\sigma_{VM}^{\max} = 531.41 \text{ Mpa}$ as initial design (ss=0) to $\sigma_{VM}^{\max} = 419.4386 \text{ Mpa}$ as optimum design (ss=17)) may be attributed to two reasons, the first is the replacement of the same material (S11) during optimization process with Al-alloy which has more specific stiffness than initially selected Ti-alloy (see the specific stiffnesses of respective alloys listed in **Appendix -D-** where the increase in structural strength (decrease in max Von-Mises stress) is proportional to the increase in the specific stiffness for the constituent material as multi-material modeling), the second is the replacement of Ti-alloy by Al-alloy for this stiffener (S11) with maintaining the neighboring stiffeners (S10 & S12) with the same field of Ti-alloys without crossing to Al-alloys field, so the lateral loads may be greater borne by those neighboring stiffeners than by S11 (similar to statically indeterminate problems). The same

reason is behind the steep reduction in Max Von-Mises stress of material No.16 during iteration process. The increase in $\sigma_{VM}^{\max}$ of material 27 (S18) may be attributed to replacement of S19 by Al-alloy (which is of less stiffness but more specific stiffness than of S18). In general, the rapid decrease in slope of stress curve for materials (16, 20, 22 & 28) between ss=6 and ss=7 may be attributed to the selection of Al12, Al5, Al7 & Al48 for these materials (stiffeners) respectively and maintaining other stiffeners or materials within the same field of Ti-alloys where the relatively high stiffness for materials (17, 18, 19, 21, 23, ... & 27) makes the most of inertia loads to be borne by the more stiffness members (Ti-alloys stiffeners). **Table (5.3)** shows the best set of materials for Model 3.

Table (5.3) Optimum set for materials of Model 3

Material No.	Optimum set	Safety factor (N_{σ})	$[\sigma_{yp}]$ (Mpa)	$[\sigma_{VM}^{\max}]_{safe\ region}$ (Mpa)
1	Ti6	1.022269	1172	1146.469
2	Ti21	1.016085	834	820.7976
3	Al5	1.064039	455	427.616
4	Al14	1.008328	270	267.77
5	Al20	1.027147	215	209.3177
6	Al20	1.091114	215	197.0463
7	Al20	1.021049	215	210.5678
8	Al22	1.057641	160	151.2801
9	Al22	1.052978	160	151.95
10	Al39	1.165415	69	59.20638
11	Al34	1.043957	97	92.91573
12	Al26	1.166068	140	120.0617
13	Al26	1.044249	140	134.0677
14	Al24	1.055189	145	137.4162
15	Al21	1.224637	195	159.2308
16	Al12	1.050351	325	309.4203
17	Ti20	1.043351	807	773.4693

18	Ti23	1.221064	745	610.1234
19	Ti23	1.119675	745	665.3744
20	Al5	1.084783	455	419.4386
21	Ti20	1.074267	807	751.21
22	Al7	1.028841	395	383.9272
23	Ti15	1.023647	993	970.0613
24	Ti25	1.073556	586	545.8496
25	Ti4	1.022663	1200	1173.407
26	Ti18	1.0229.24	896	875.9203
27	Ti5	1.023589	1214	1186.023
28	Al48	1.111998	28	25.1799
Whole wing	---	1.092813	---	---

5.6.1.2.4: Model 4 (Rectangular Honeycomb Core Stiffeners):

More homogenization in Von-Mises stress and strain distribution can be found in this Model due to the high degree of binding between the skins and stiffeners as shown by **Fig. (5.36)**. According to **Fig. (5.29)**, the steep reduction in Max Von-Mises stress of material 13 (from $\sigma_{VM}^{\max} = 505.3 \text{ Mpa}$ as initial design (ss=0) to $\sigma_{VM}^{\max} = 460.1413 \text{ Mpa}$ as optimum design (ss=14)) and the increase in the Max Von-Mises stress of material 14 (from $\sigma_{VM}^{\max} = 543.5 \text{ Mpa}$ as initial design (ss=0) to $\sigma_{VM}^{\max} = 557.22 \text{ Mpa}$ as optimum design (ss=14)) may be attributed to the same reason in **section (5.6.1.2.3)**, where the maintaining of stiffener on the Ti-alloys field and replacement of neighboring stiffener or element with Al-alloys always results in increasing the max Von-Mises stress (structural strength reduction) of the stiffener and decrease in max Von-Mises stress of the neighboring stiffener due to the same above reason. **Table (5.4)** shows the best set of materials for Model 4. Local displacements may occur in this Model (the skin is stressed approximately with the same degree of stiffeners and due to the relatively high vacuum in this Model small displacement occurs at the upper and lower skins).

The optimization process is finishing with reaching the optimized value of safety factor (**Fig. (4.1)**). Up on this, the required number of iterations (or ss number) is completely governed by achieving the structural homogenization of strength safety factor N_σ for whole structure within the range that mentioned by **Fig. (4.1)**.

Table (5.4) Optimum set for materials of Model 4

Material No.	Optimum set	Safety factor (N_σ)	$[\sigma_{yp}]$ (Mpa)	$[\sigma_{VM}^{\max}]_{safe\ region}$ (Mpa)
1	Al1	1.002762	525	523.5541
2	Al3	1.087408	505	464.4072
3	Al3	1.102767	505	457.9389
4	Al6	1.037113	415	400.1505
5	Al7	1.024703	395	385.4776
6	Al10	1.046569	370	353.5361
7	Al11	1.048849	345	328.9319
8	Al12	1.080336	325	300.8322
9	Al15	1.05281	295	280.2025
10	Al34	1.053701	97	92.05647
11	Al34	1.019179	97	95.17467
12	Al34	1.051537	97	92.24591
13	Al3	1.097489	505	460.1413
14	Ti25	1.051651	586	557.221
15	Al5	1.099409	455	413.8587
16	Al11	1.025385	345	336.459
17	Al16	1.018050	255	250.4788
Whole wing	---	1.054351	---	---

5.6.1.2.5: Model 5 (Rhombic Honeycomb Core Stiffeners):

According to **Fig. (5.30)**, the maximum element Von-Misses stress is plotted versus the steady state number (ss) for the set 0 to 14. It is shown that there is noticeable decrease in max Von-Mises stress as the steady state number increase between ss=6 and ss=7 due to the replacement of Ti-alloys by Al-alloys

for most of constituent materials (the more light structure, the less inertia loads) besides the relative effects among stiffeners due to changes in other properties E & ν . The sudden variation of structural behavior within same field of alloys (Ti-alloys or Al-alloys fields) may be attributed to the accumulative effects of material properties on the structural behavior of structural materials (the parametric study is done as shown in **Figs. (5.14) to (5.16)**). The small values of $\sigma_{VM}^{\max}$ for materials 10, 11, & 12 are due to their connection with wing structure away from wing root (where the stresses will concentrate at wing root) so the stiffeners that connected to wing root is greatly affected with inertia loads. The relatively large difference between initial and final values of $\sigma_{VM}^{\max}$ for the stiffener (S10) may be attributed to selecting fore-stiffeners (S5 & S6) and aft-stiffeners (S11 & S12) to be made of Ti-alloys, while the mentioned stiffener (S10) is selected to be made of Al-alloy with the same reason mentioned in **section (5.6.1.2.3)**. **Table (5.5)** shows the best set of materials for Model 5.

Table (5.5) Optimum set for materials of Model 5

Material No.	Optimum set	Safety factor (N_{σ})	$[\sigma_{yp}]$ (Mpa)	$[\sigma_{VM}^{\max}]_{safe\ region}$ (Mpa)
1	Al1	1.037781	525	505.887
2	Al3	1.095551	505	460.9557
3	Al10	1.030801	370	358.944
4	Al1	1.032786	525	508.334
5	Al3	1.101747	505	458.3628
6	Al5	1.077093	455	422.4335
7	Al5	1.088824	455	417.8822
8	Al6	1.049779	415	395.3214
9	Al15	1.055838	295	279.3988
10	Al33	1.101883	90	81.67883
11	Al26	1.144688	140	122.3041
12	Al21	1.157039	195	168.5336
13	Al6	1.048917	415	395.6464

14	Ti23	1.006009	745	740.55
15	Ti19	1.007916	862	855.23
16	Ti15	1.033775	993	960.5569
17	Ti10	1.004488	1034	1029.38
18	Ti13	1.016874	965	948.9864
19	Al3	1.025631	505	492.38
20	Ti17	1.025313	827	806.5828
21	Ti20	1.009742	807	799.2139
22	Ti20	1.049405	807	769.0075
23	Al3	1.116414	505	452.3411
Whole wing	---	1.057317	---	---

5.6.1.2.6: Model 6 (Triangular Honeycomb Core Stiffeners):

The greater variation in Von-Mises stresses, the greater steady state number requires reaching the optimum design (because the structural materials will cover approximately the whole range of materials), so the relatively more homogeneous Models (Models 4 & 5) requires relative small steady state No. to reach the optimum design. This Model is consisting of combination transverse and diagonal stiffeners in two directions. As shown in **Fig. (5.31)**, the increase in Von-Mises stress for S1, S2 & S3 is due to the replacement of S8, S9, S10 & S11 by Al-alloys where the load is approximately borne by the stiffeners (S1, S2 & S3); this may be attributed to the stiffeners S1, S2 & S3 (which maintain Ti-alloy during iteration process) have more stiffness than that of stiffeners S8, S9, S10, S11 & S13 within the same structure as multi-material modeling, so the lateral displacement for S1, S2, S3 is expected to be less than of stiffeners S8, S9, S10, & S12 which makes the load to be relatively carried more by transverse stiffeners S1, S2 & S3 (making it more stressed parts) than by diagonal stiffeners S8, S9, S10, & S12 to realize the homogeneity in lateral displacement distribution as shown in **Fig. (5.39)**. **Table (5.6)** shows the best set of materials for Model 6.

Table (5.6) Optimum set for materials of Model 6

Material No.	Optimum set	Safety factor (N_{σ})	$[\sigma_{yp}]$ (Mpa)	$[\sigma_{VM}^{\max}]_{safe\ region}$ (Mpa)
1	Ti16	1.063875	896	842.2039
2	Ti23	1.185792	745	628.2722
3	Al5	1.074945	455	423.2775
4	Al9	1.020404	350	343.0015
5	Al12	1.060068	325	306.5842
6	Al8	1.072803	380	274.9807
7	Al13	1.035513	275	265.5693
8	Al13	1.033043	275	266.2039
9	Al17	1.046768	250	238.8303
10	Ti20	1.037217	807	778.0433
11	Ti23	1.197529	745	622.1145
12	Ti23	1.166306	745	638.7688
13	Al15	1.155309	295	255.343
14	Al22	1.136262	160	137.5445
15	Al44	1.124962	48	42.6881
16	Al47	1.073327	34	31.6772
17	Al26	1.122205	140	124.7544
18	Al12	1.143903	325	284.1151
19	Al26	1.218522	140	114.8933
20	Al10	1.141424	370	324.1565
21	Al17	1.068093	250	234.0621
22	Al28	1.067923	115	107.6857
23	Al48	1.238461	28	22.6087
24	Al9	1.058907	350	330.5295
25	Al12	1.139946	325	285.1012
26	Al20	1.080961	215	198.897
27	Al3	1.078964	505	468.0415
Whole wing	---	1.106312	---	---

5.6.2: Frequency Investigation (Comparative Study):

Fig. (5.32) shows the frequency evolution during the optimization study. The study reveals that the frequency is related to elastic property (specific stiffness) of the structural materials. Frequency characteristics of any structure are related to geometrical and material properties of the structure. For the present work, both geometrical and material variables play an important role in variation of stiffness properties among the adopted Models which in turn affect the frequency characteristics. According to **Fig. (5.32)**, as optimum design, Model 2 & 4 have the highest values of natural frequencies among the Models which reflect the highest stiffness of the considered structures; this may be attributed to two factors; the first is the geometrical factor (in which the stiffeners are situated at their optimum positions and orientations), the second belongs to the material properties possessed by the structure (Model 2 is optimized to be pure Al-alloys structure, while Model 4 has only one material made of Ti-alloy (S5 is optimized to be made of Ti25 alloy) and other materials are optimized with Al-alloys as optimum designs and it is observed that pure Al-alloys structure has stiffness properties higher than pure Ti-alloys structure), so the natural frequency is increasing with evolution toward the optimum. The investigation also reveals that Models 1 & 6 are failing with frequency requirement as shown in **Fig. (5.32b)** (according to **Eq. (4.4f)**) as result of geometrical and material factors as shown in **Table (5.7)**.

Table (5.7) Frequency investigation of the models (first mode frequency)

		Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Initial design	$\omega_{n1} (Hz)$	36.508	72.418	49.911	64.369	55.917	7.8153
	N_{ω}	0.9127	1.81045	1.247775	1.609225	1.397925	0.195383
Final design	$\omega_{n1} (Hz)$	36.921	79.046	55.086	70.78	61.922	8.1196
	N_{ω}	0.923025	1.97615	1.37715	1.7695	1.54805	0.20299

5.6.3: Displacement Investigation (Comparative Study):

The displacement distribution as structural behavior is related directly to stiffness property, the stiffness of constituent materials and geometrical arrangement of stiffeners are the most two parameters that governing the displacement behavior of the structure. The Max absolute displacement-iteration history for all Models is comparatively shown in **Fig. (5.33)**. The displacement study reveals that both of Models 1 & 6 are failing with displacement constraints as shown in **Fig. (5.33b)** i.e. violate the displacement constraint with safety factors of 0.725253 & 0.449461 respectively (according to **Eq. (4.4g)**), so Models 1 & 6 will be considered as failing Models due to violating displacement & frequency constraints according to the specified limit of design variables. The smallest values of max total displacement can be found in Models 2 & 4 (the most two successful designs). **Fig. (5.33)** also reveals that the max total displacement is increasing with replacement of Ti-alloy by Al-alloy for the constituent materials; this may be attributed to the noticeable reduction in elastic modulus for the transition from Ti-alloy to Al-alloy i.e. the elastic modulus is more governing factor than the mass density & poisson's ratio in the effect on structural displacement (see **Fig. (5.16)**). Where the mass density reduction will slightly tend to reduce the displacement by decreasing inertia loads induced in the structure and the decrease in elastic modulus tends to increase the structural displacement with greater effect. **Table (5.8)** presents the values of max absolute displacement and the corresponding safety factors for initial and final designs.

Table (5.8) Displacement investigation of the adopted models

		Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Initial design	$U_{total}^{max} (mm)$	130.2714	39.213	70.4178	54.0504	63.2236	200.155
	N_U	0.767628	2.550175	1.4201	1.85	1.5817	0.4996
Final design	$U_{total}^{max} (mm)$	137.883	54.1998	74.1318	72.813	72.2986	222.488
	N_U	0.725253	1.845	1.3489	1.3734	1.383	0.44946

5.6.4: Structural Mass Investigation (Comparative Study):

Without violating the prescribed constraints, the strength failure criterion is the driving function toward the optimum design i.e. the value of Max Von-Mises stress is the most governing parameter for structural mass, where in highly stressed material, it is impossible to select any of Al-alloys to obtain suitable safety factor within safe region, so selecting Ti-alloy with more suitable yield stress than initially selected Ti-alloy (Ti1) will result in satisfaction of strength failure criterion (which makes this material a passive element to share in structural mass reduction for the whole structure). While in relatively light stressed materials, it is possible to select the suitable Al-alloy that is more suitable to verify the strength failure criterion without violating the constraints (which makes this material an active element to share in structural mass reduction for the whole structure). According to **Fig. (5.34)**, as optimum design, Models 2 & 4 have the lowest values of structural masses among the successful Models, while Model 5 has the moderate reduction of structural mass. **Table (5.9)** shows the values of structural mass for the successful designs (that not violate the requirements of the design).

Table (5.9) Structural mass investigation for the successful designs

Successful designs		Model 2	Model 3	Model 4	Model 5
Structural mass (kg)	Initial design	50.1169	55.6605	49.982	55.1359
	Final design	29.1058	44.065	29.3869	34.5928

Figs. (5.35) to (5.40) show the illustrative contours for Von-Mises stress and displacement distribution for initial and optimum designs for all Models.

CHAPTER SIX

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORKS

6.1 Conclusions:

With regard to the results of the presented optimum design of **GTX-Aircraft** wing structure subjected to pure inertia loads which are described in the previous chapter, the following conclusions are drawn:

1. the precession motion $\vec{\beta}, \ddot{\beta}$ can not be found through the direct application of Newton's laws of motion, so the iterative solution is needed to obtain the values of precession parameters $\vec{\beta}, \ddot{\beta}, \vec{s}_{G/C}, \ddot{s}_{G/C}, \ddot{s}_{G/C}$ at each stage of solution (at least 8 cycles are needed for exact solution) as illustrated by iterative sequence of *Eq. (3.8)*.
2. Impulse and Momentum method (especially advanced Euler's Equation) represents the suitable way to attack the Kinetics problem and estimation of inertia loads for rotational motion in general 3-D motion.
3. The most dangerous failure modes in the stiffened structure may be attributed to the failure of the honeycomb core by high shear stresses induced in it.
4. During the iteration process, the replacement of structural materials properties from Ti-alloys to Al-alloys properties results in increase in natural frequency, displacement (elastic modulus-dependent parameter) and noticeable increase in structural strength (reduction in Von-Mises induced stresses).
5. The natural frequency, Von-Mises stresses and strains slightly increase with poisson's ratio increase, but structural displacement decreases with poisson's ratio increase.

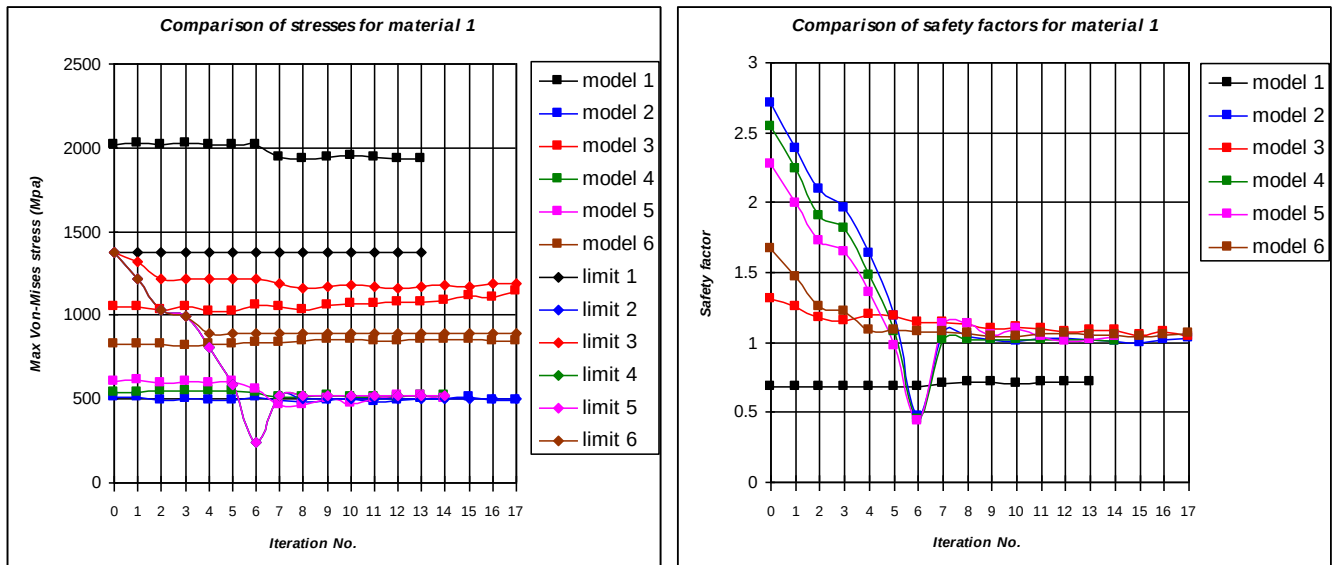
6. The results of design optimization reveal that the transverse stiffeners Model (Model 2) and rectangular honeycomb core Model (Model 4) are the most two successful designs, so they are achieving mass reductions of (0.580758) and (0.58795) respectively, in which the optimum structural masses are (29.1058 kg) and (29.3869 kg) respectively.
7. More uniform distribution of displacement (max displacement accruing at tip point) can be found in models of unidirectional stiffeners than in models of honeycomb sandwich cores (bi-, tri-directional stiffeners), while more stress homogenization can be fulfilled with honeycomb sandwich core models.
8. As inertia action, the mass density of modeling materials has the direct effect on the structural behavior of the whole structure. As structural behavior, for multi-material modeling only, the elastic modulus has the greatest effect on stress level of each material as compared to other properties of the structural materials. As strength failure criterion, the yield stress of modeling materials has the great effect on strength criterion, so the two dual terms called specific strength (σ_{yp}/ρ) and specific stiffness (E/ρ) as material properties represent the most important properties among the set of properties listed in Appendix –D–.

6.2 *Suggestions for Future Works:*

Many aspects may be concerned with the objectives of the current work, in addition to other features that are not considered in it. Thus it is possible to suggest the following items as further objectives in the future:

1. The dynamic behavior and structural optimization were performed on aircraft structures. The suggestion is to study the dynamic behavior for other structures (spacecrafts, marines, satellites, etc.).

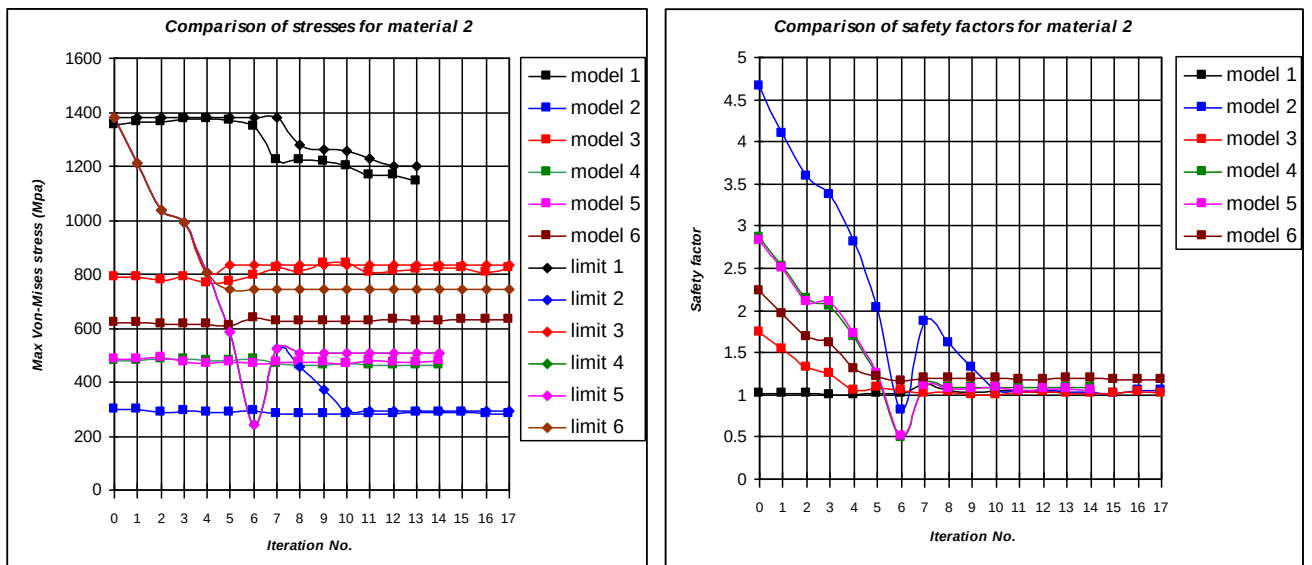
2. There are many effects associated with general motion (regardless of inertia loads) such as aero-heating, drag, aerodynamic pressure loads. The suggestion is to include these effects in the investigation of optimum design with the same optimality criterion and design variables to achieve the minimum structural weight.
3. Using structural optimization for other types of honeycomb cores such as Hexagonal-honeycomb cores, cylindrical-honeycomb cores and corrugated panels, and performing the comparison between these types and the present work types.
4. The objective function is restricted to structural weight only for aerospace industry. In marine industry the achievement of cost includes a raw material, manufacturing, and labor work (with material variables) represents the important objective in practical field.
5. The strength failure criterion does not govern the instability of the model, so including the buckling constraints (with multi-material modeling) will overcome such problems.
6. Implementing different evolution algorithm such as genetic algorithm will give the best results for global optimum.
7. Include large period of time (for steady state response) instead of the relatively short period of time considered for the present work to apply fatigue failure theories for strength failure criterion for the same objective function and design variables.



(a) Stress evolution

(b) Safety factor evolution

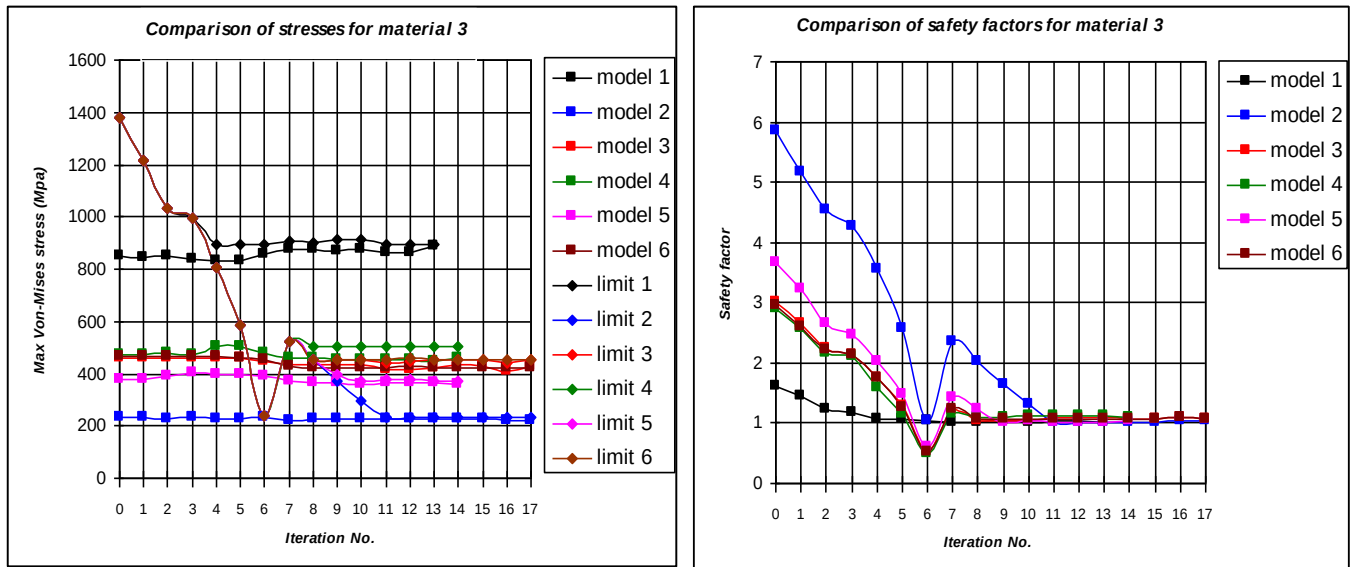
Figure (5.17) Strength investigation for material 1 (lower skin)



(a) Stress evolution

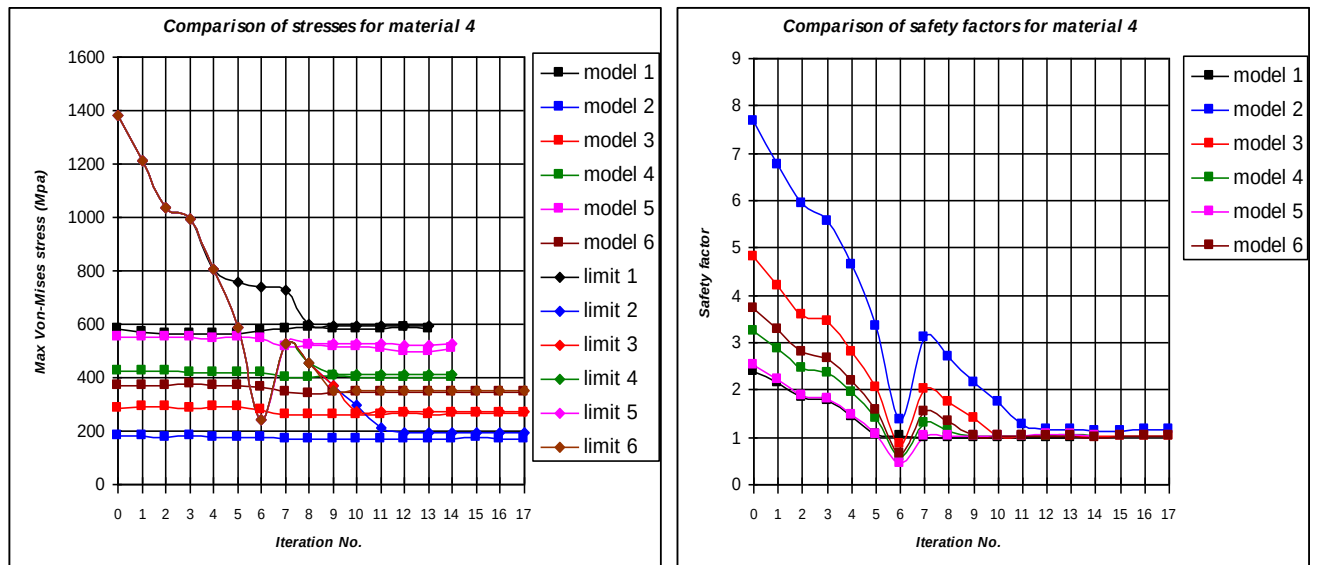
(b) Safety factor evolution

Figure (5.18) Strength investigation for material 2 (first slice of upper skin)



(a) Stress evolution

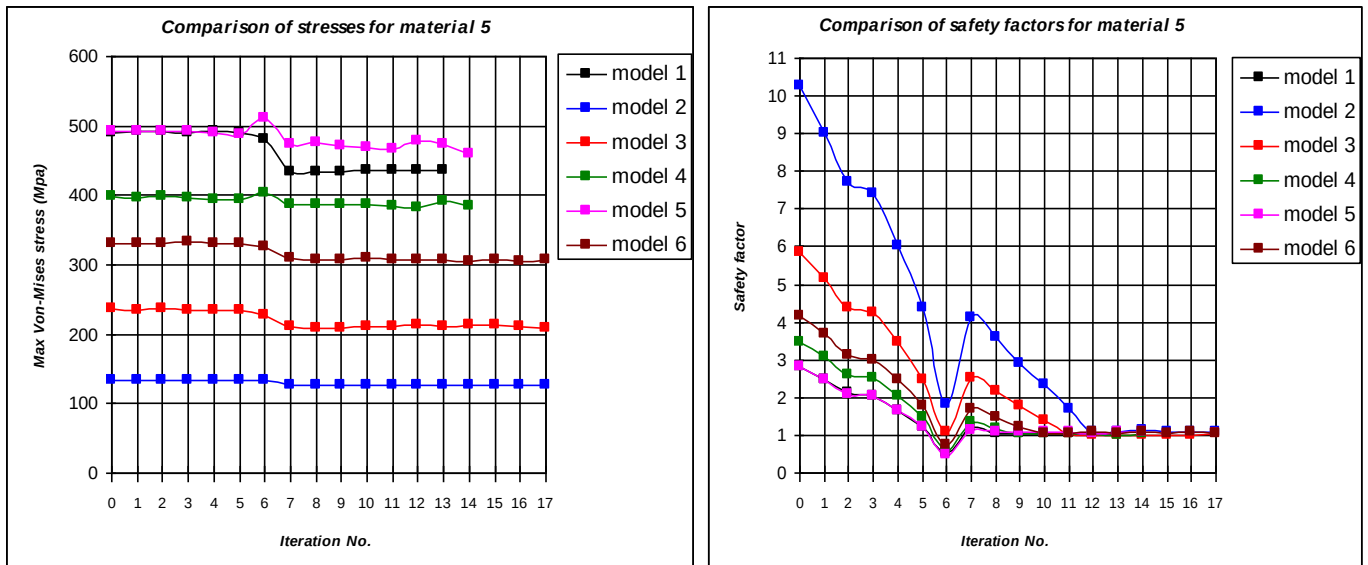
(b) Safety factor evolution

Figure (5.19) Strength investigation for material 3 (second slice of upper skin)

(a) Stress evolution

(b) Safety factor evolution

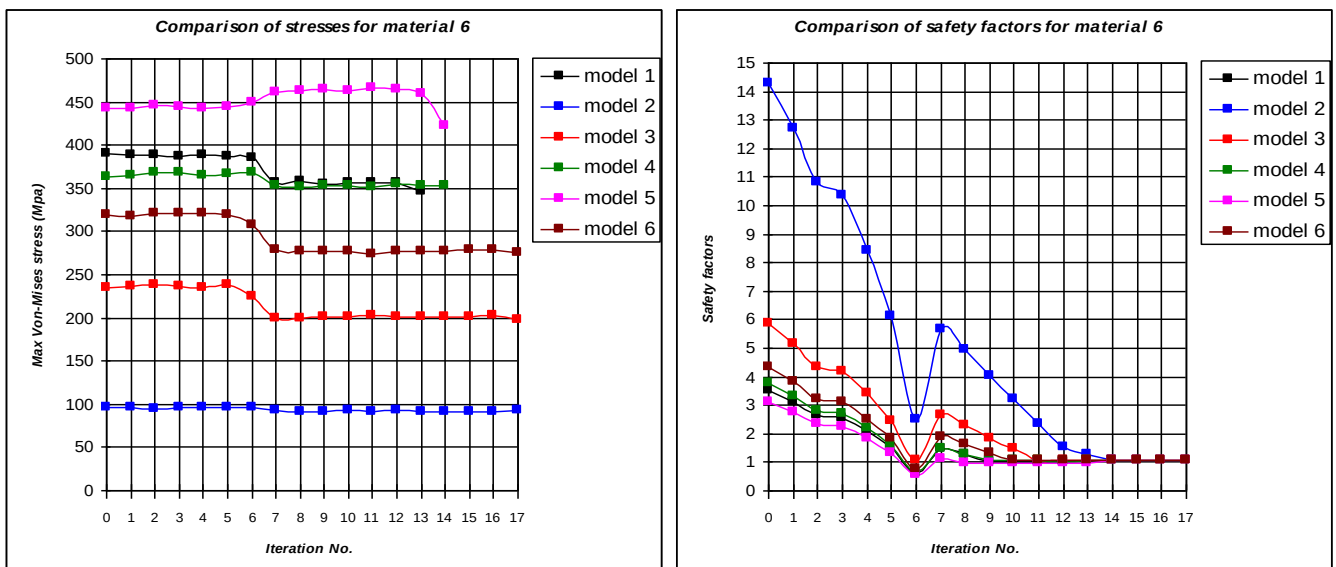
Figure (5.20) Strength investigation for material 4 (third slice of upper skin)



(a) Stress evolution

(b) Safety factor evolution

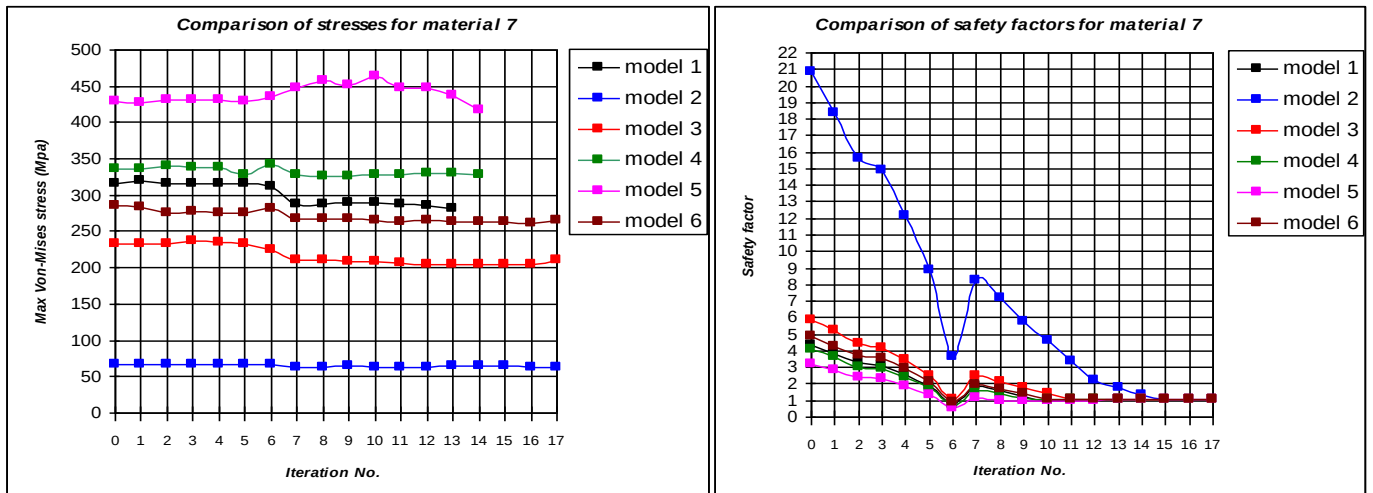
Figure (5.21) Strength investigation for material 5 (fourth slice of upper skin)



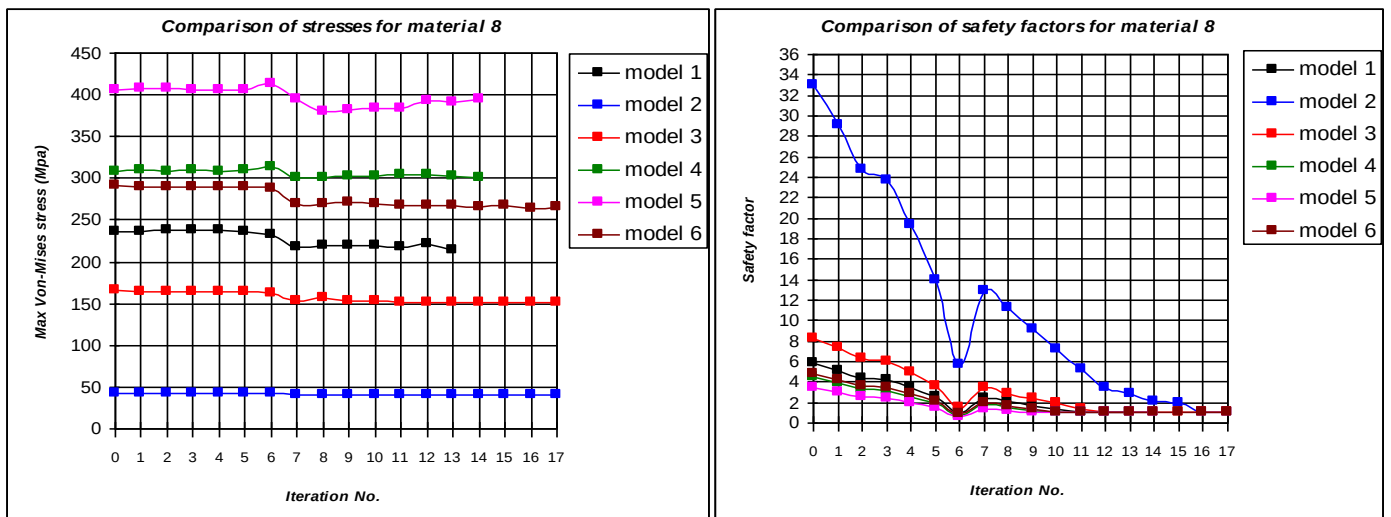
(a) Stress evolution

(b) Safety factor evolution

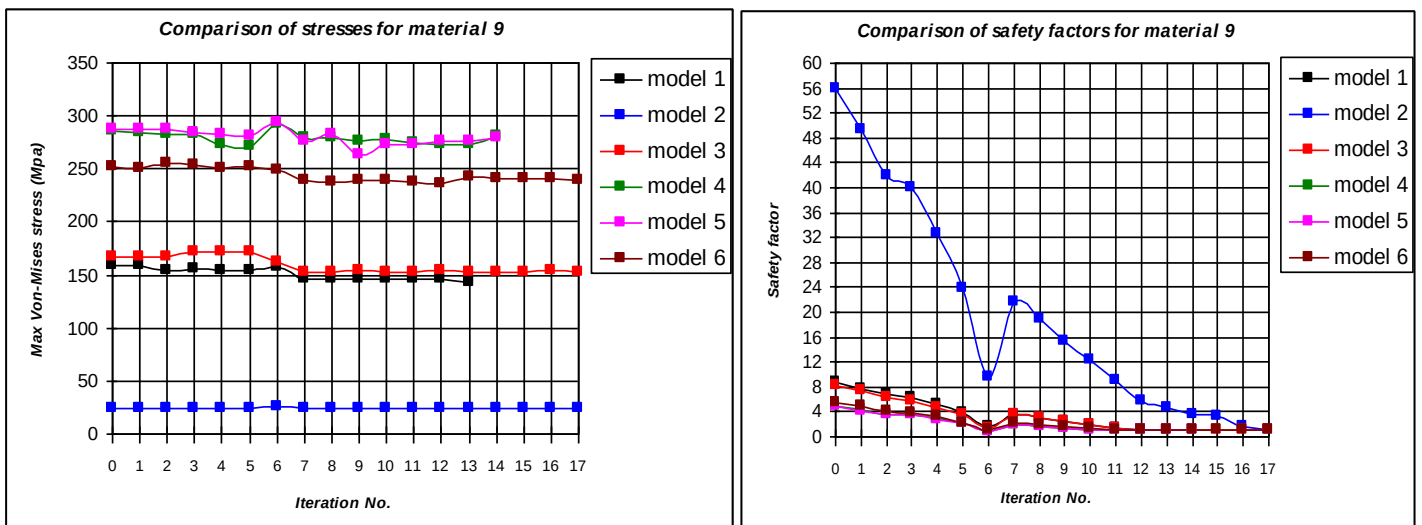
Figure (5.22) Strength investigation for material 6 (fifth slice of upper skin)



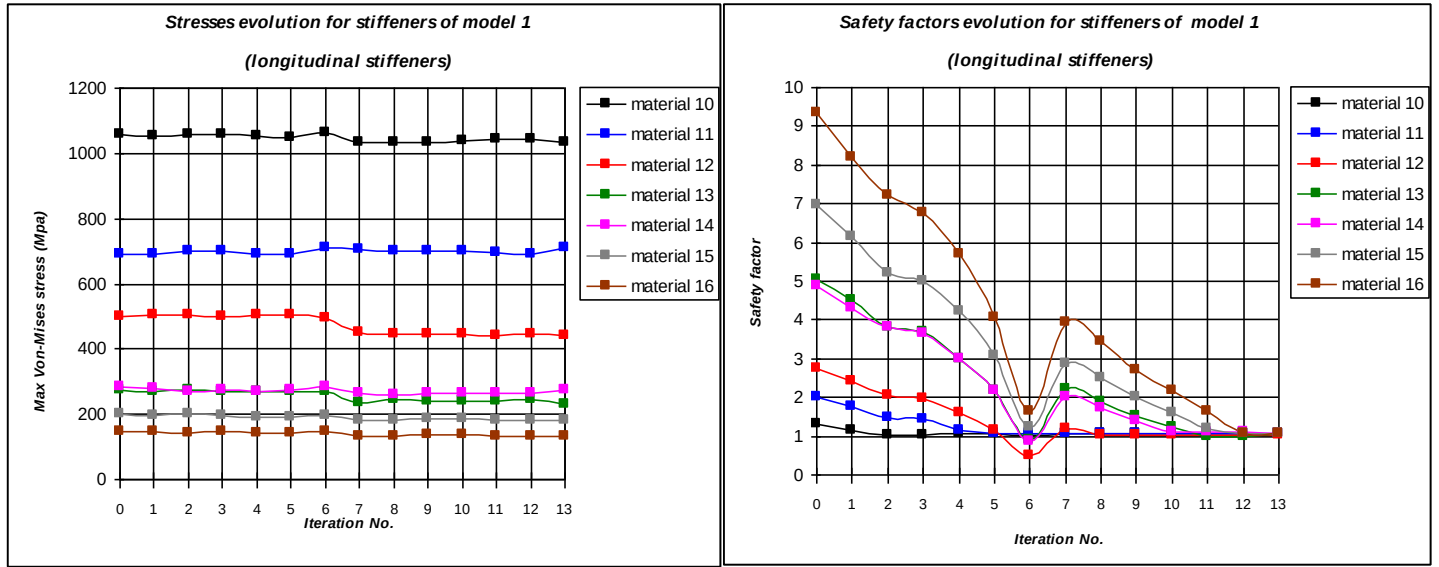
(a) Stress evolution
(b) Safety factor evolution
Figure (5.23) Strength investigation for material 7 (sixth slice of upper skin)



(a) Stress evolution
(b) Safety factor evolution
Figure (5.24) Strength investigation for material 8 (seventh slice of upper skin)



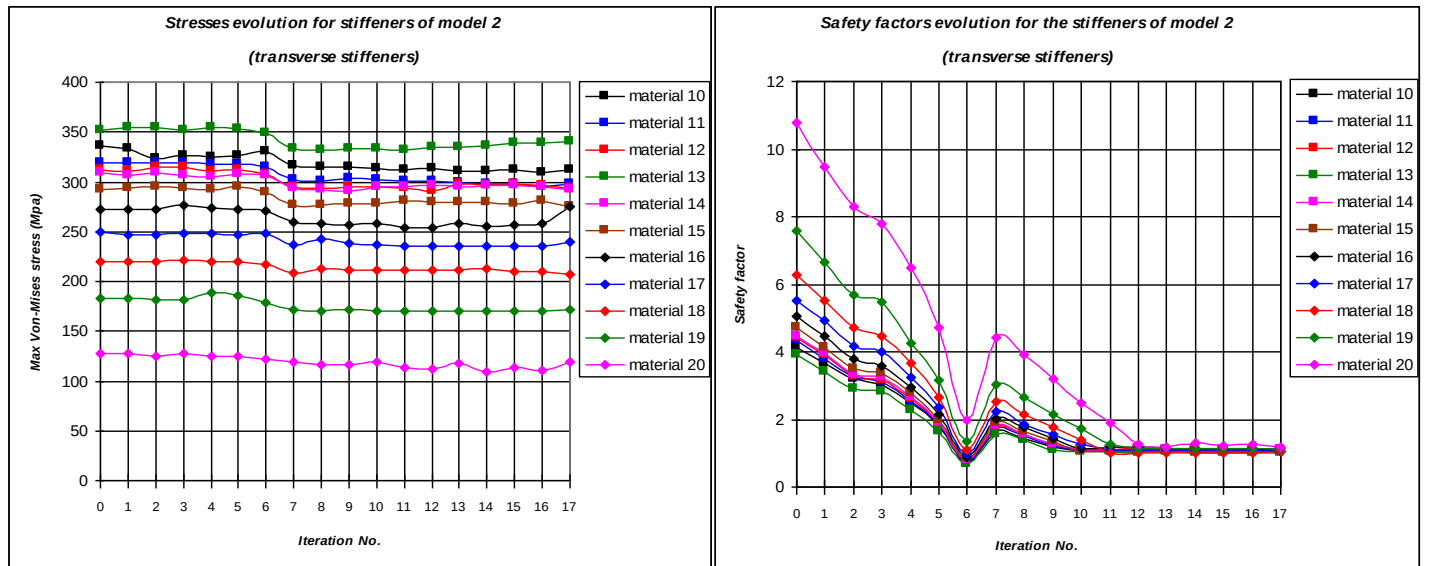
(a) Stress evolution
(b) Safety factor evolution
Figure (5.25) Strength investigation for material 9 (eighth slice of upper skin)



(a) Stress evolution for materials 10 to 16
(stiffeners S1 to S7)

(b) Safety factor evolution for materials 10 to 16
(stiffeners S1 to S7)

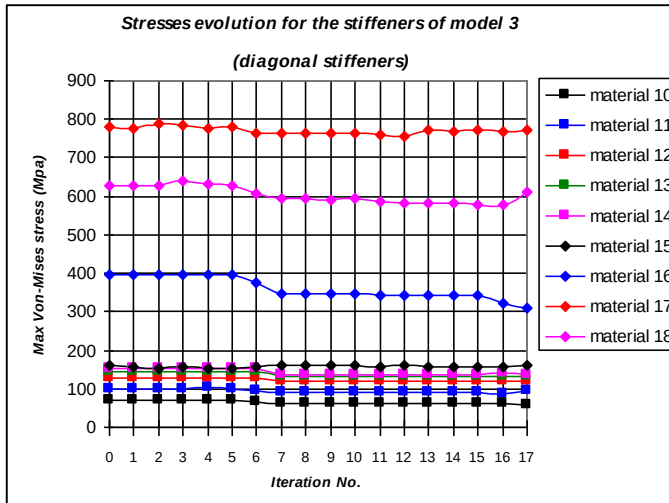
Figure (5.26) Strength investigation for stiffeners of Model 1



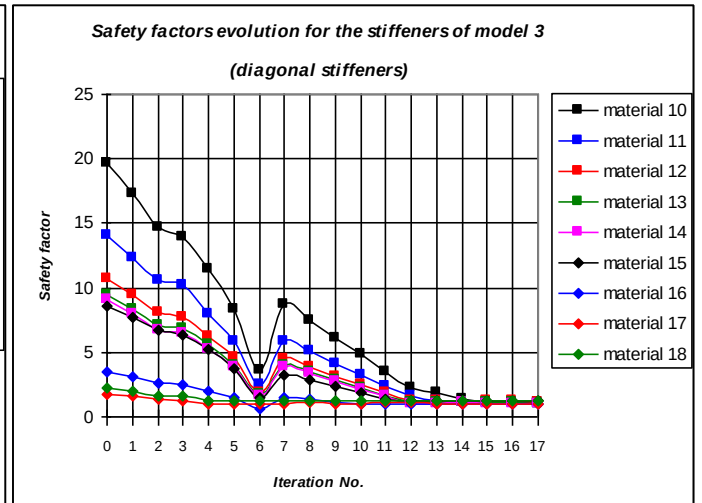
(a) Stress evolution for materials 10 to 20
(stiffeners S1 to S11)

(b) Safety factor evolution for materials 10 to 20
(stiffeners S1 to S11)

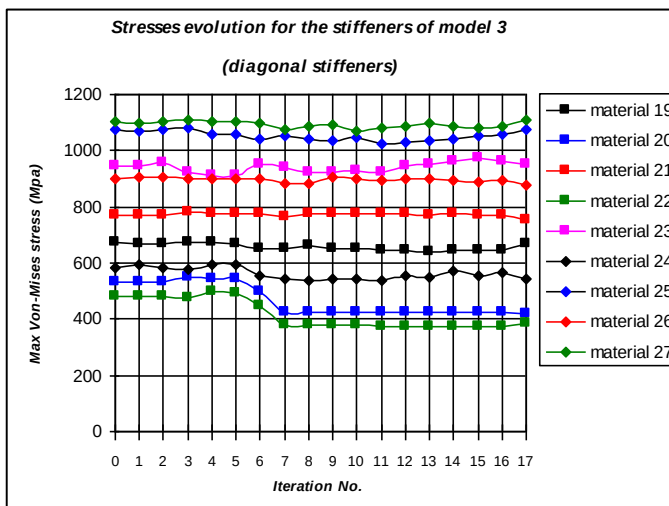
Figure (5.27) Strength investigation for stiffeners of Model 2



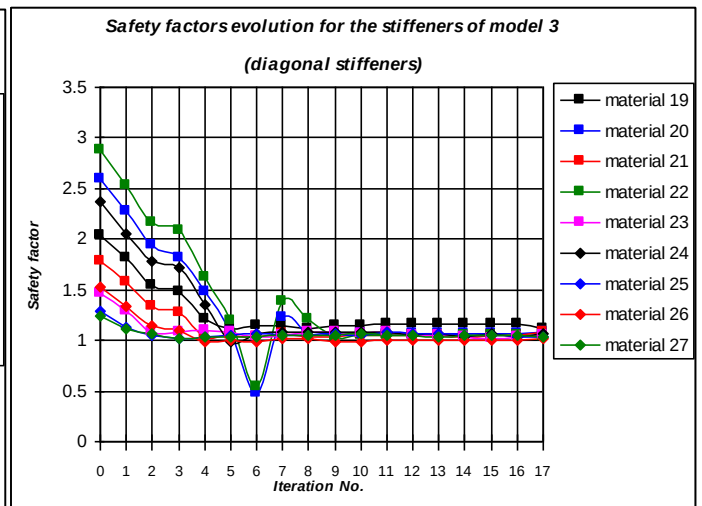
(a) Stress evolution for materials 10 to 18 (stiffeners S1 to S9)



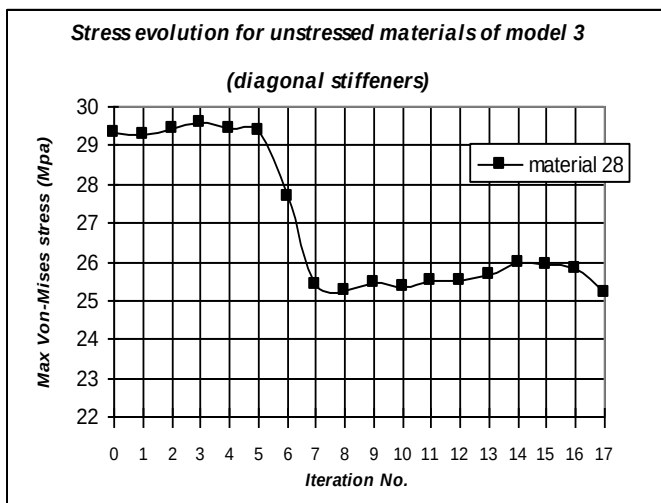
(b) Safety factor evolution for materials 10 to 18 (stiffeners S1 to S9)



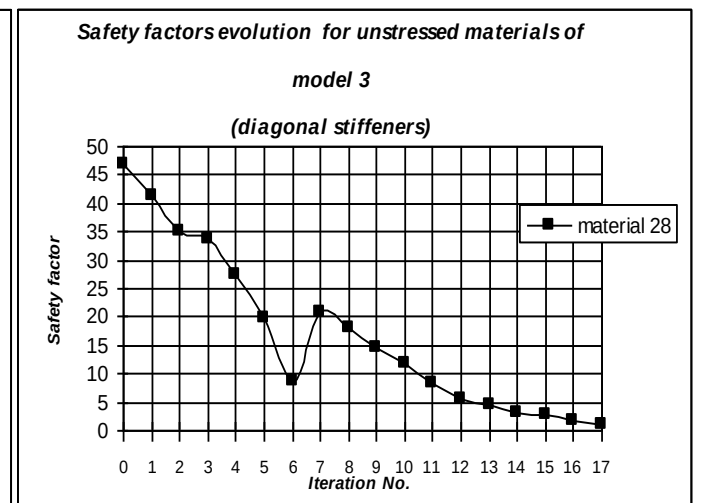
(c) Stress evolution for materials 19 to 27 (stiffeners S10 to S18)



(d) Safety factor evolution for materials 19 to 27 (stiffeners S10 to S18)



(e) Stress evolution for material 28 (S19)



(f) Safety factor evolution for material 28 (S19)

Figure (5.28) Strength investigation for stiffeners of Model 3

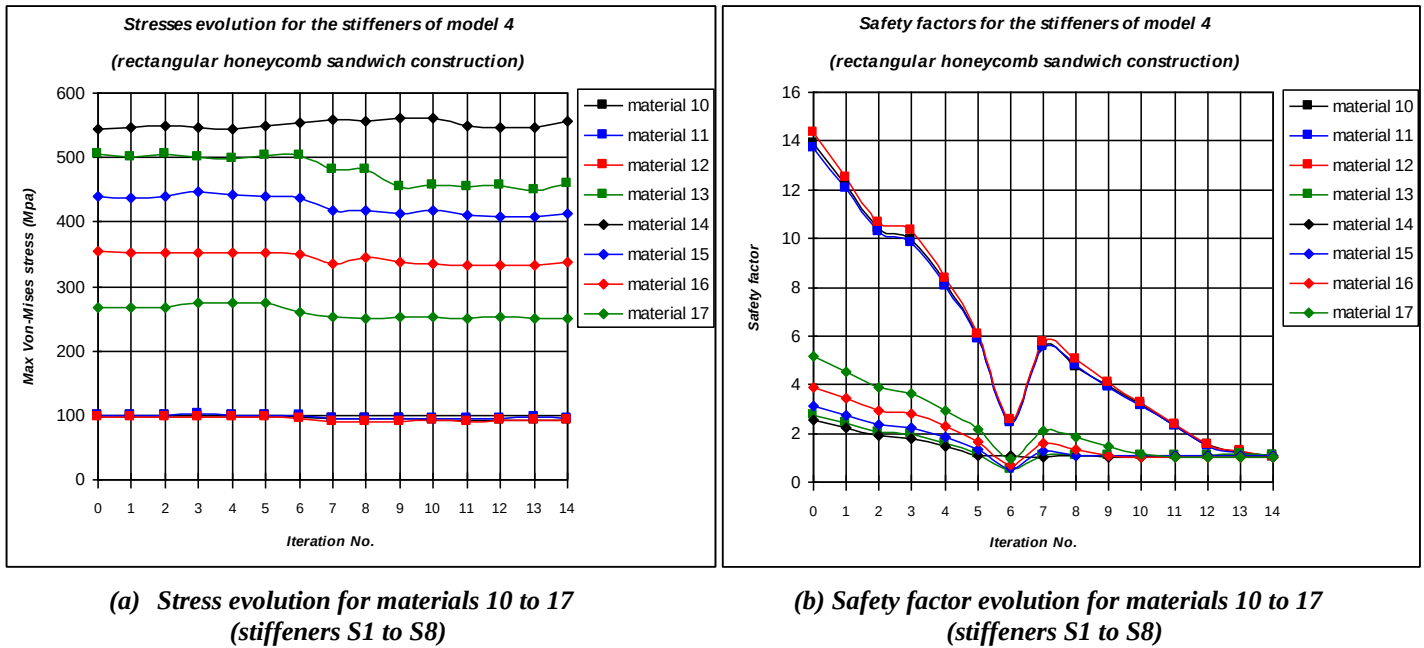


Figure (5.29) Strength investigation for honeycomb core stiffeners of Model 4

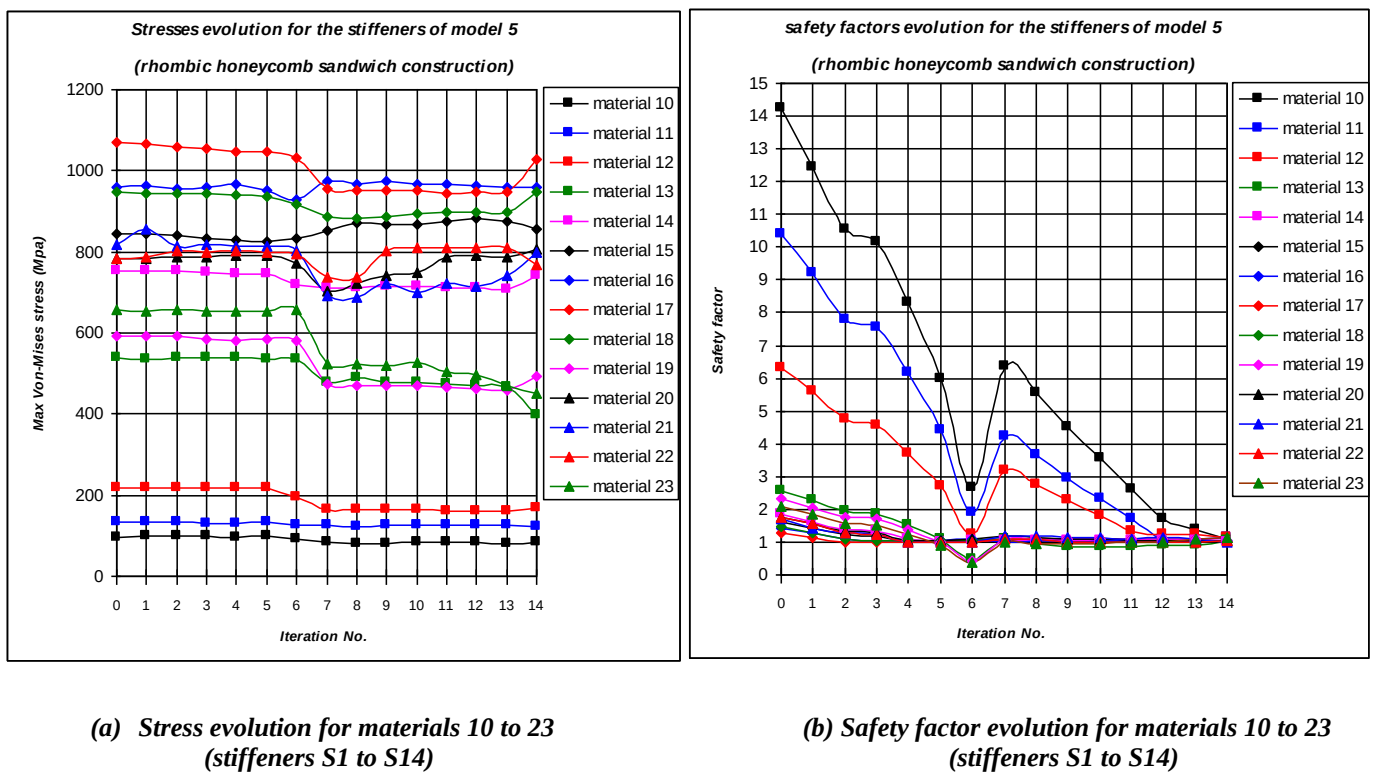
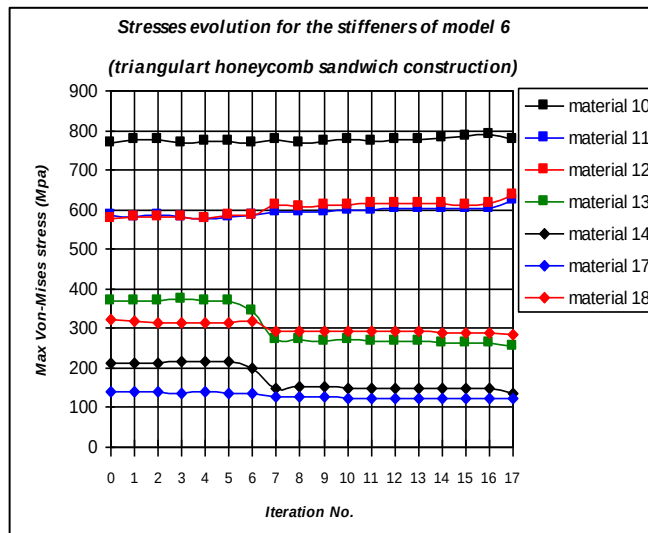
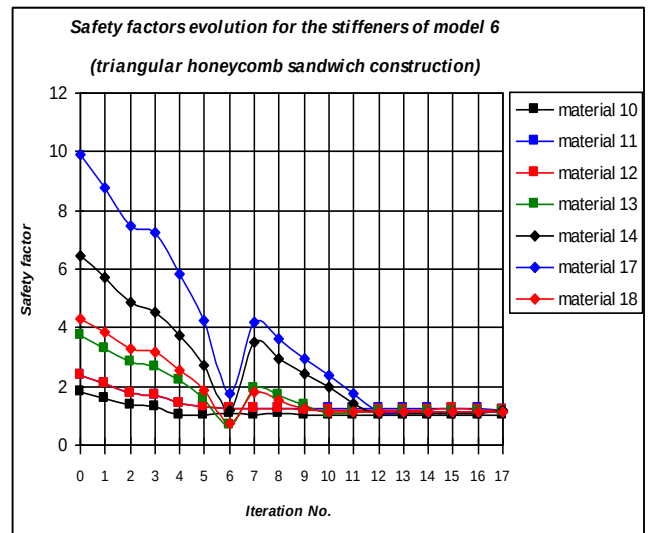


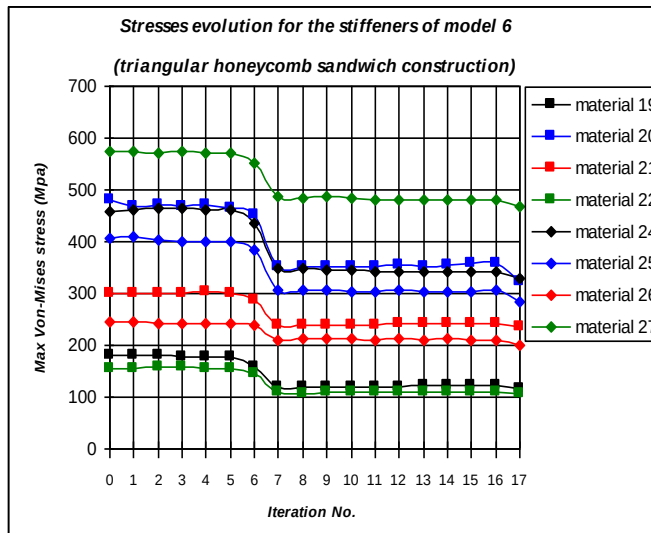
Figure (5.30) Strength investigation for honeycomb core stiffeners of Model 5



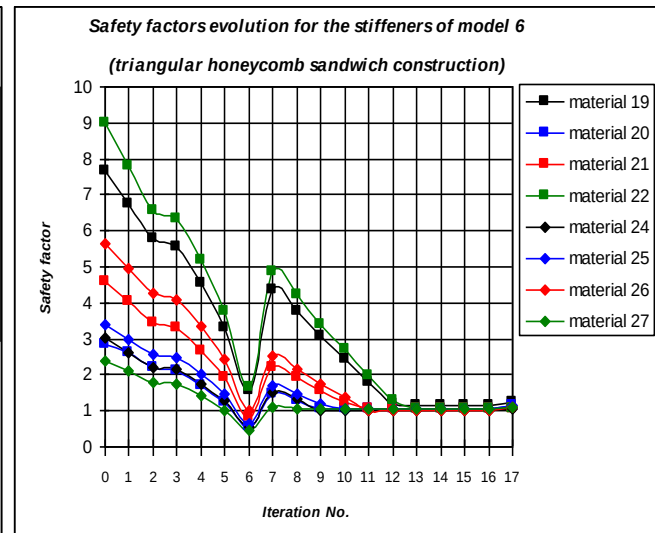
(a) Stress evolution for materials 10 to 18



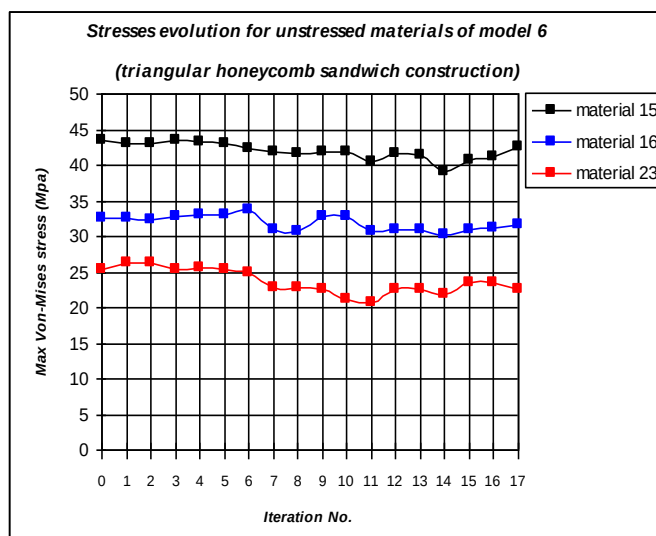
(b) Safety factor evolution for materials 10 to 18



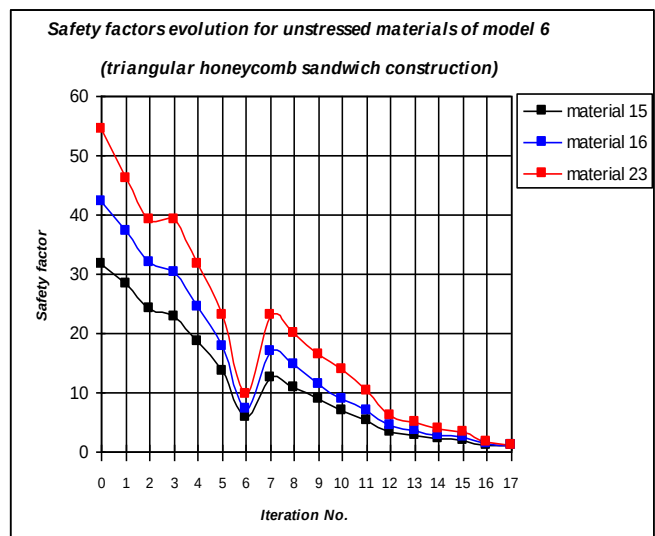
(c) Stress evolution for materials 19 to 27



(d) Safety factor evolution for materials 19 to 27

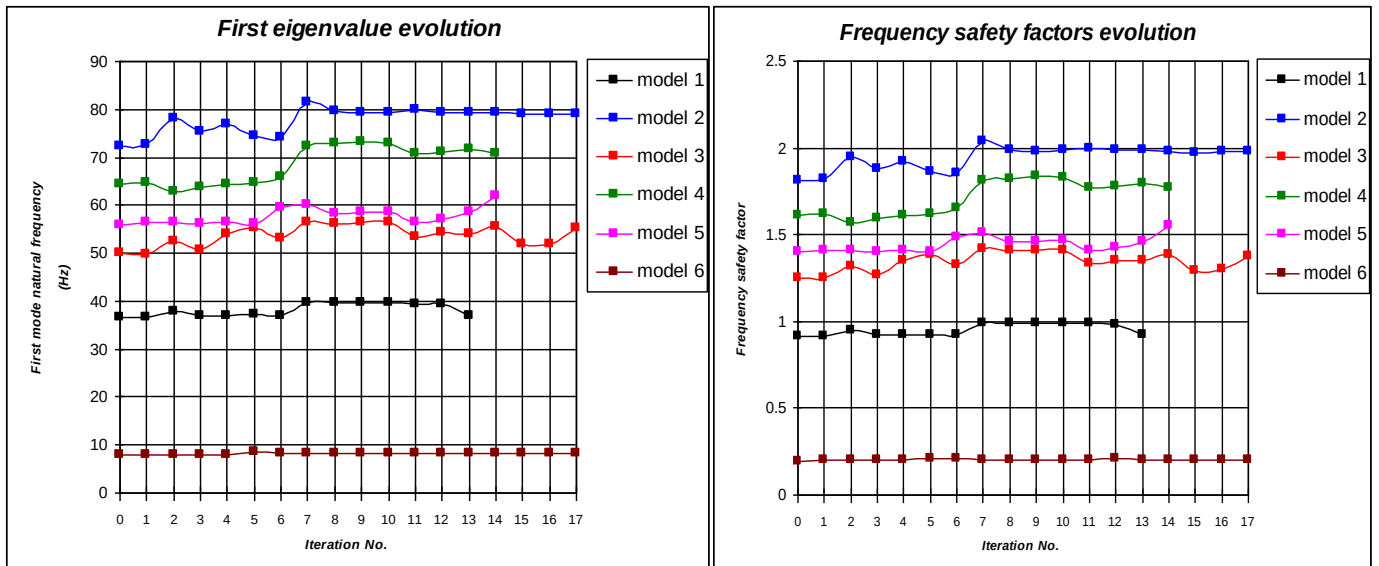


(e) Stress evolution for materials 15, 16 & 23



(f) Safety factor evolution for materials 15, 16 & 23

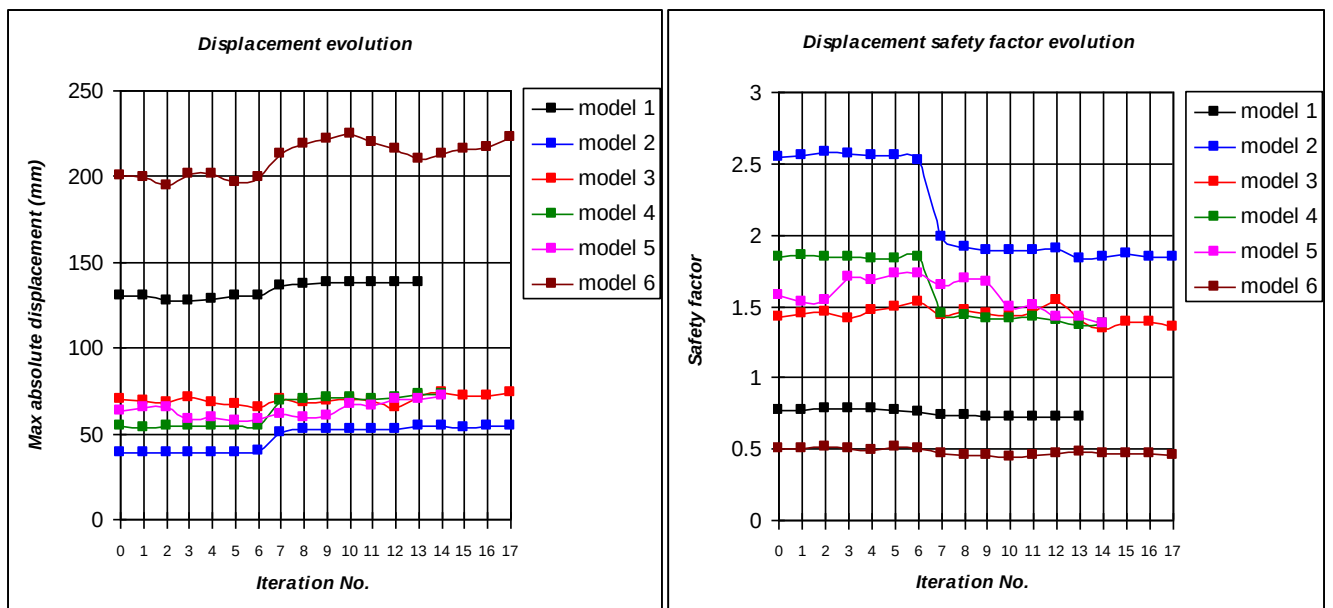
Figure (5.31) Strength investigation for honeycomb core stiffeners of Model 6



(a) Frequency (first mode (Hz))

(b) Frequency safety factor

Figure (5.32) Frequency investigation (comparative study)



(a) Maximum absolute displacement (mm)

(b) Displacement safety factor

Figure (5.33) Displacement investigation (comparative study)

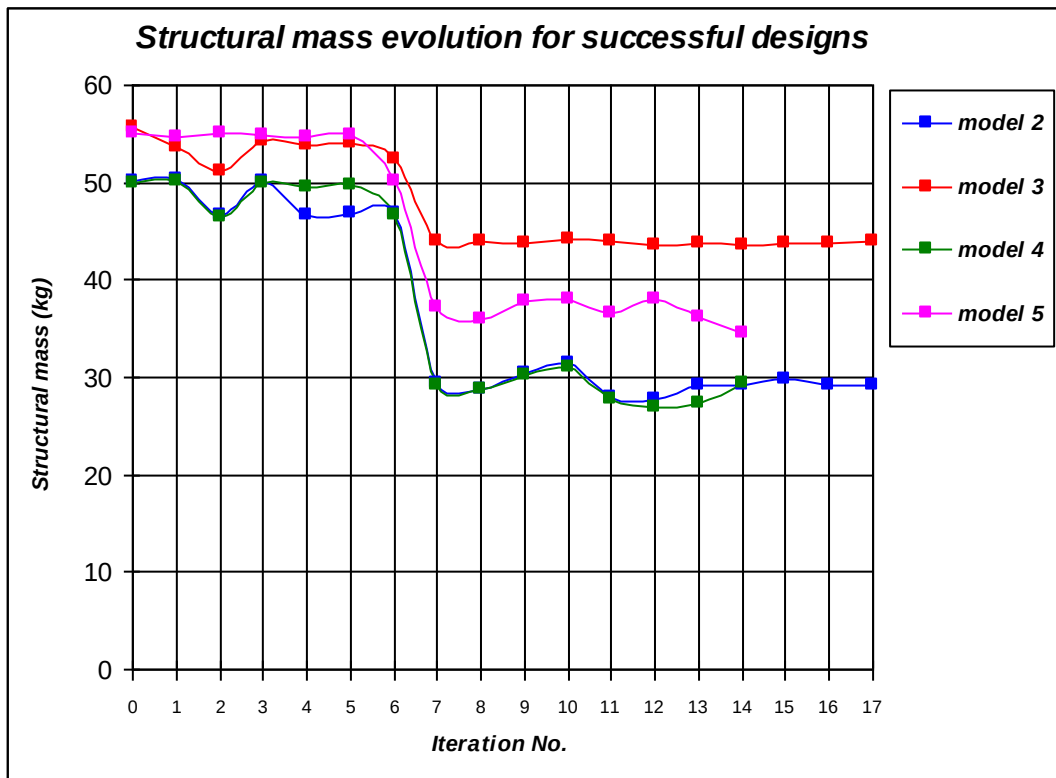
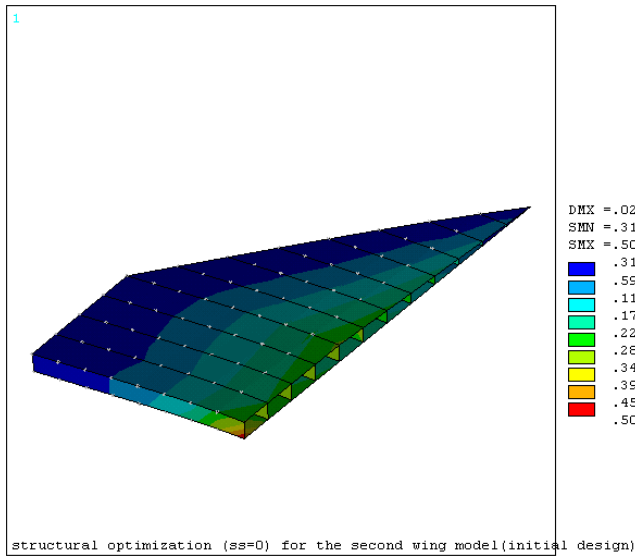
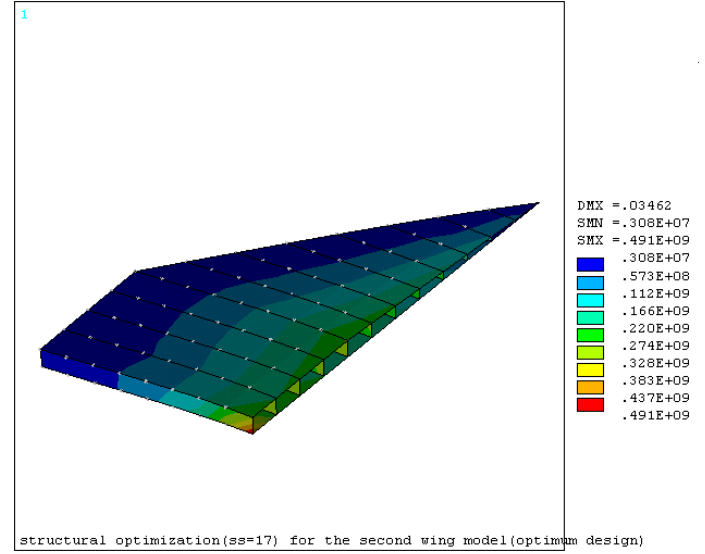


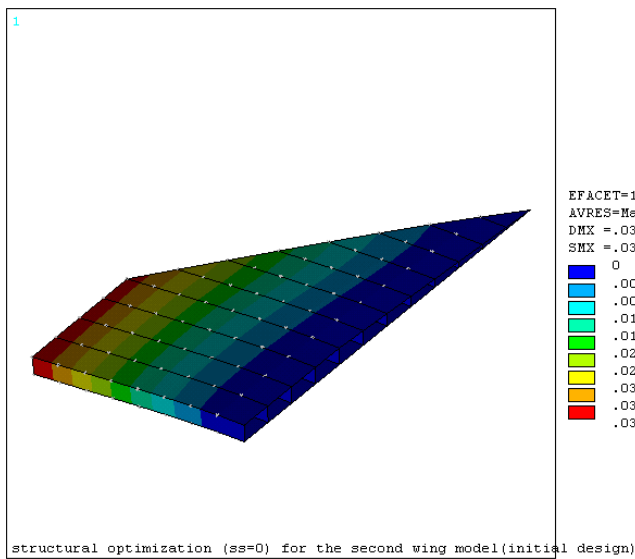
Figure (5.34) Structural mass evolution for successful designs (comparative study)



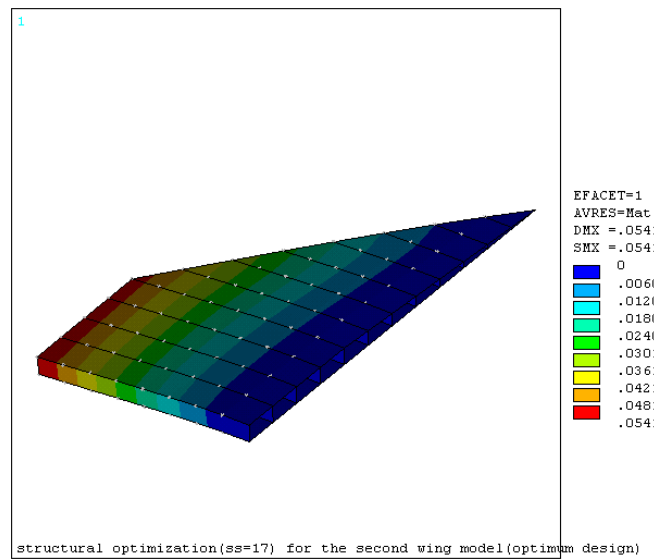
(a) Von-Mises stress distribution in Pa
(initial design)



(b) Von-Mises stress distribution in Pa
(final design)



(c) Total displacement distribution in m
(initial design)



(d) Total displacement distribution in m
(final design)

Figure (5.35) Illustrative contours for optimum design (the best design)
(Model 2)

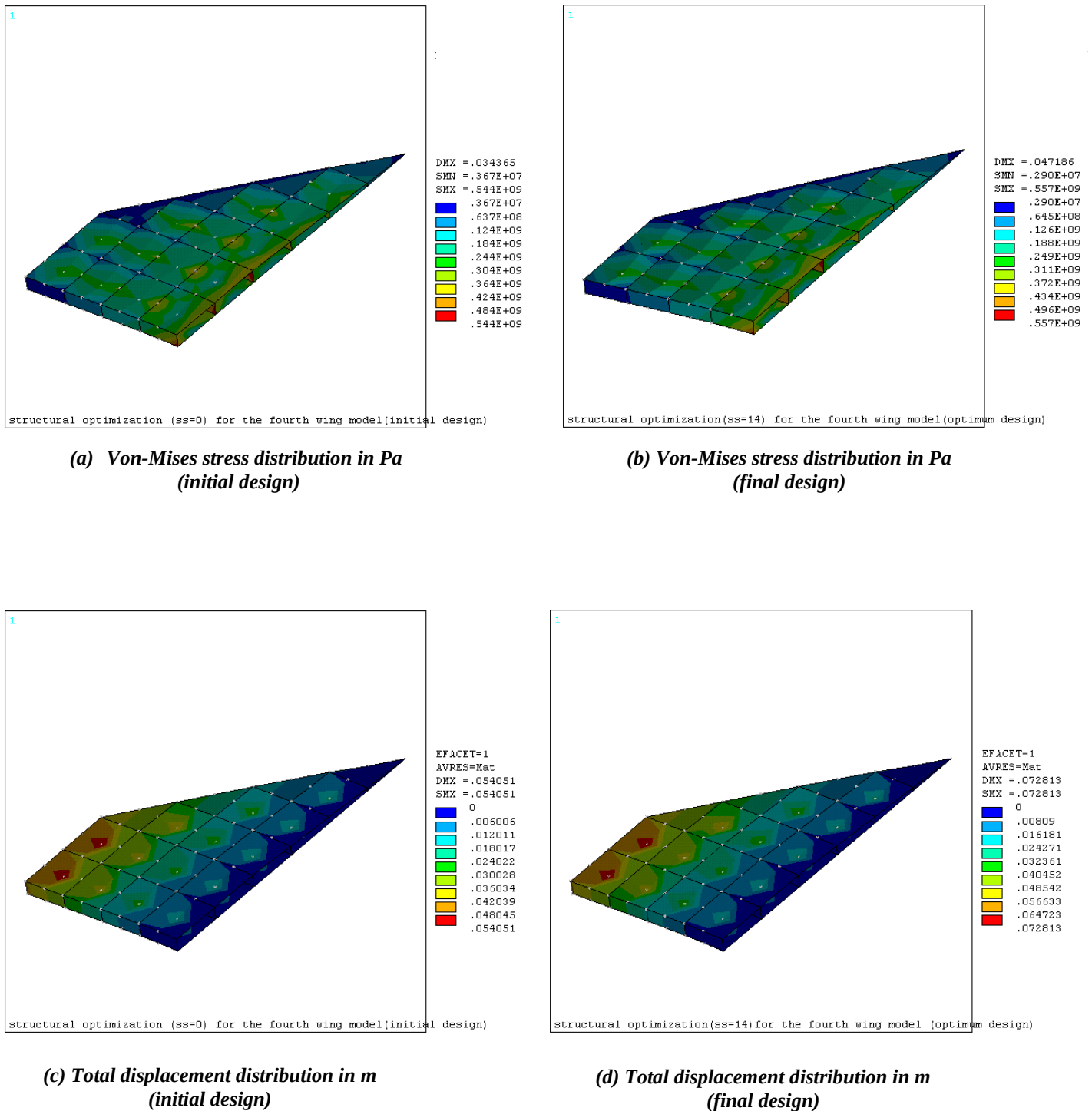
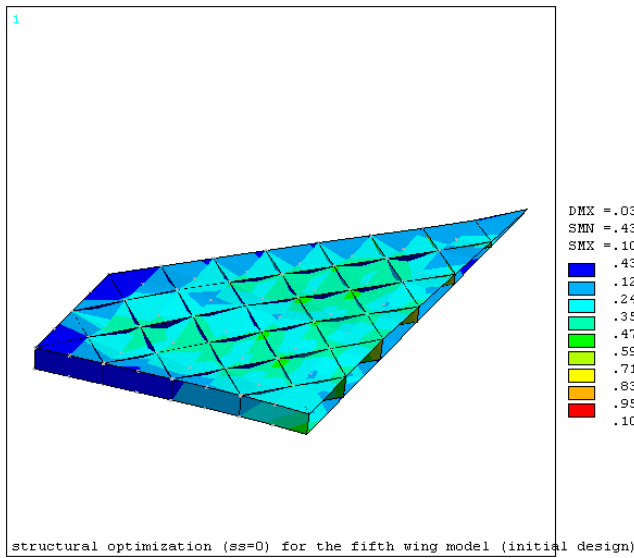
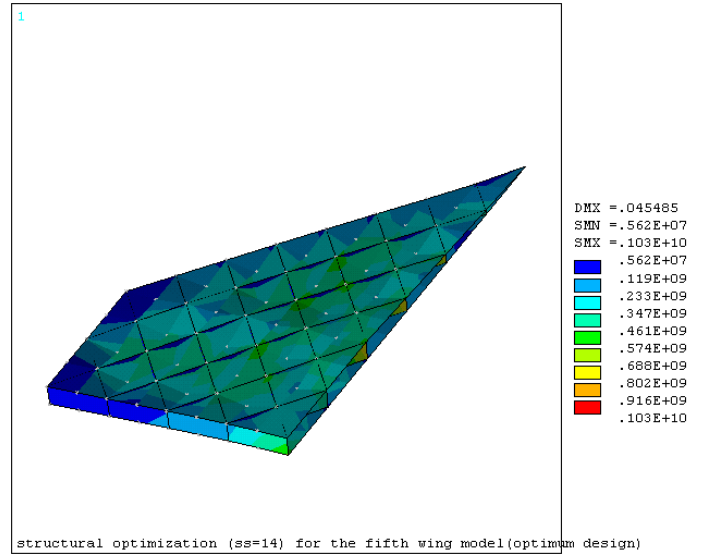


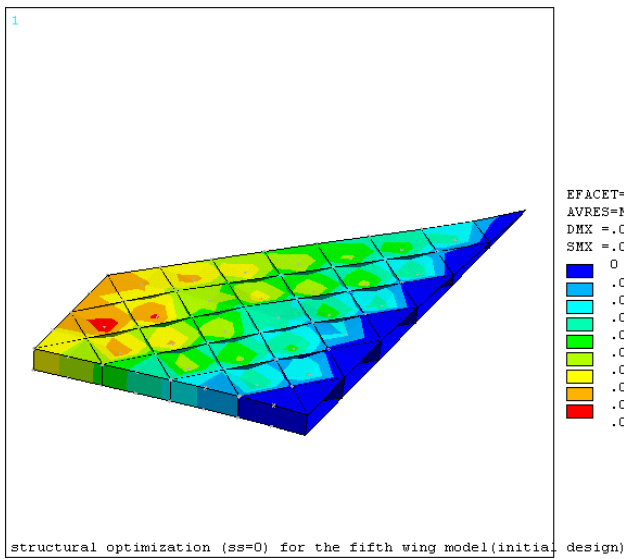
Figure (5.36) Illustrative contours for Model 4 (the second successful design)



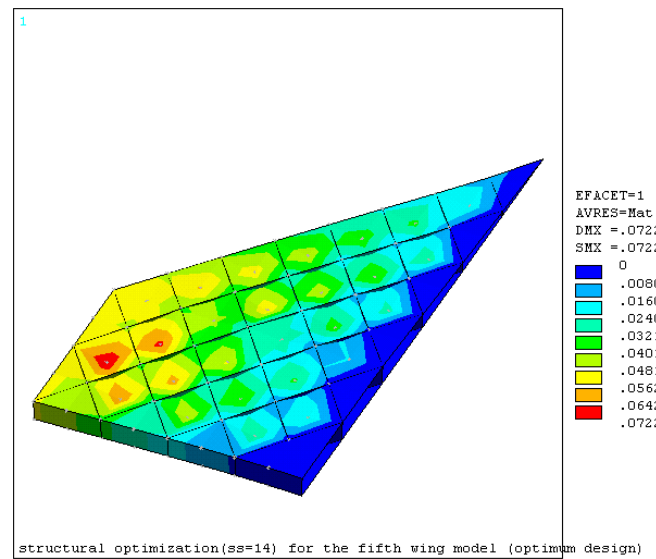
(a) Von-Mises stress distribution in Pa
(initial design)



(b) Von-Mises stress distribution in Pa
(final design)

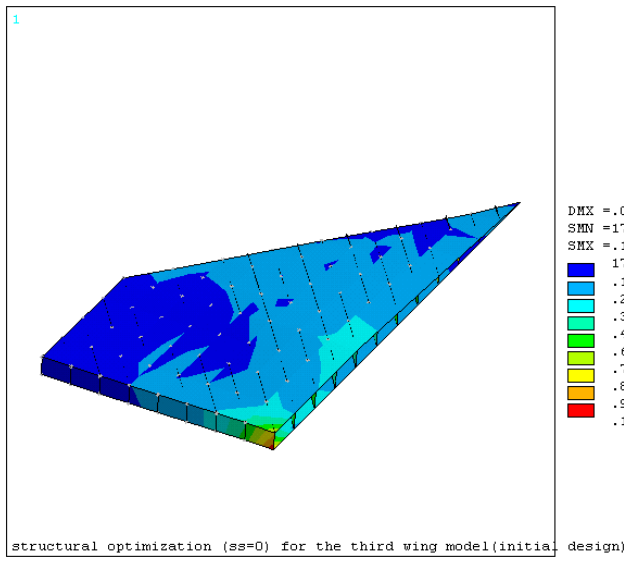


(c) Total displacement distribution in m
(initial design)

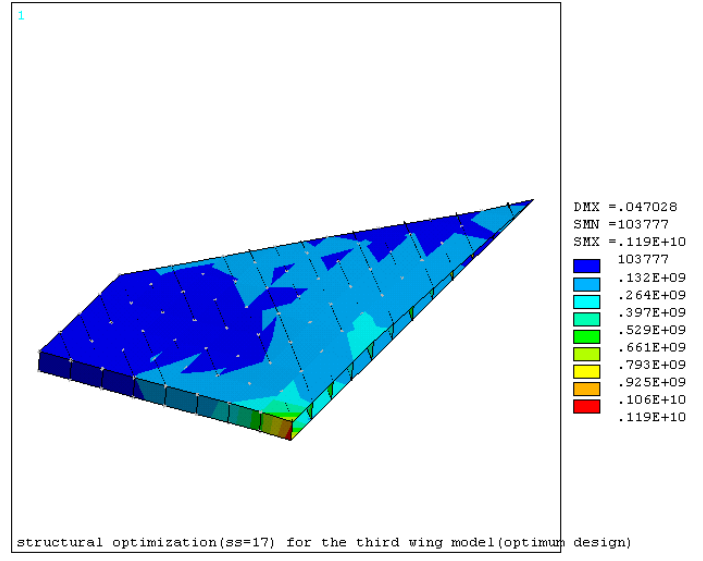


(d) Total displacement distribution in m
(final design)

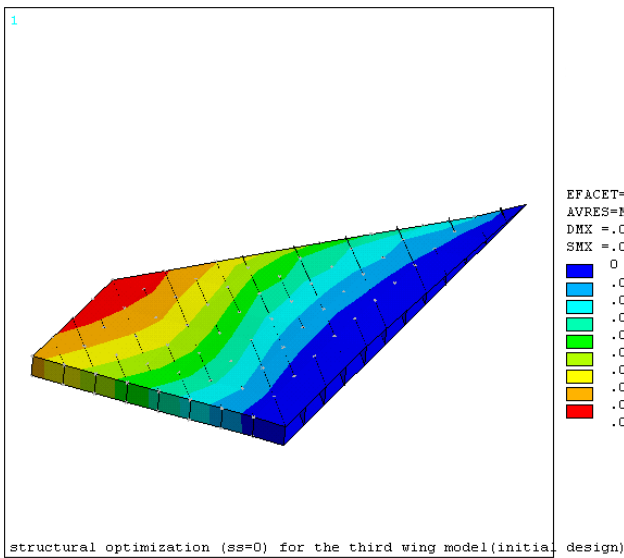
Figure (5.37) Illustrative contours for Model 5
(the third successful design)



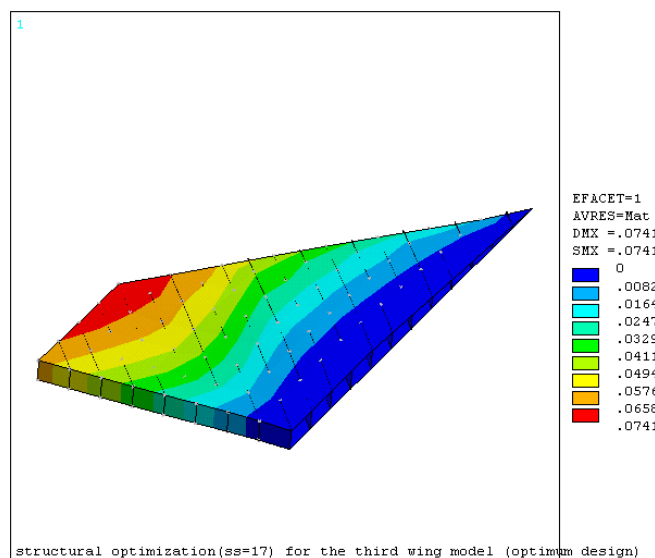
(a) Von-Mises stress distribution in Pa
(initial design)



(b) Von-Mises stress distribution in Pa
(final design)



(c) Total displacement distribution in m
(initial design)



(d) Total displacement distribution in m
(final design)

Figure (5.38) Illustrative contours for Model 3
(the fourth successful design)

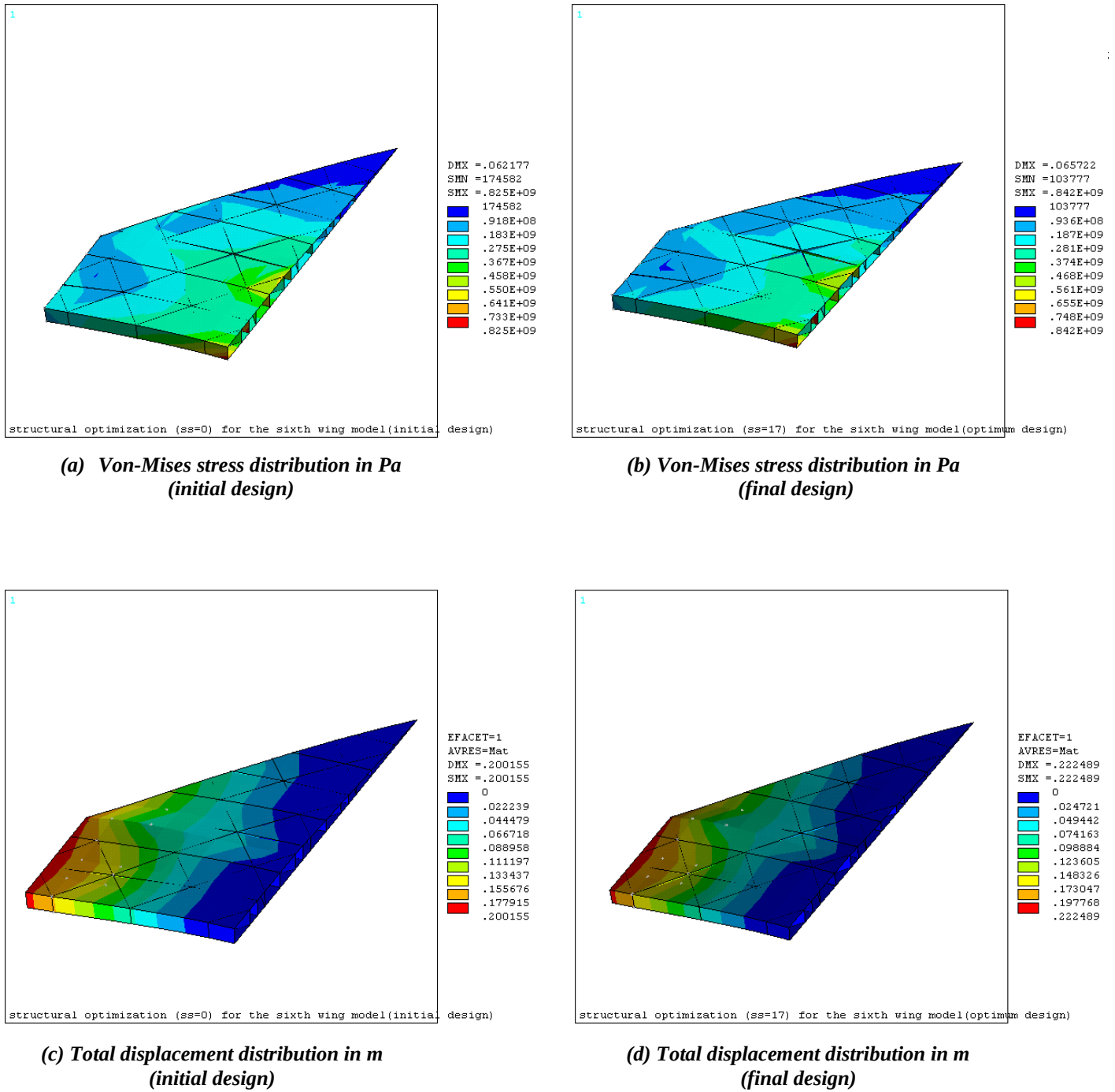
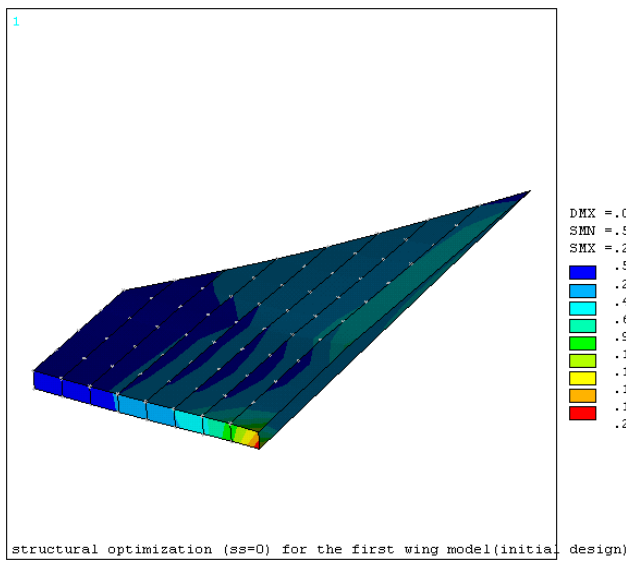
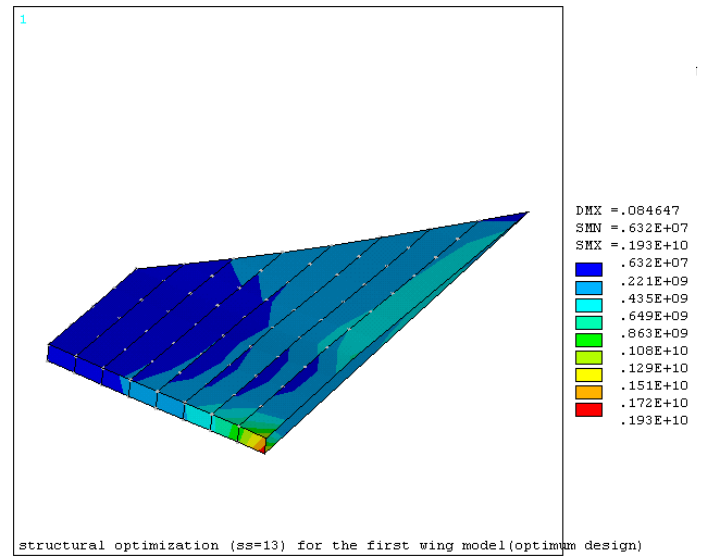


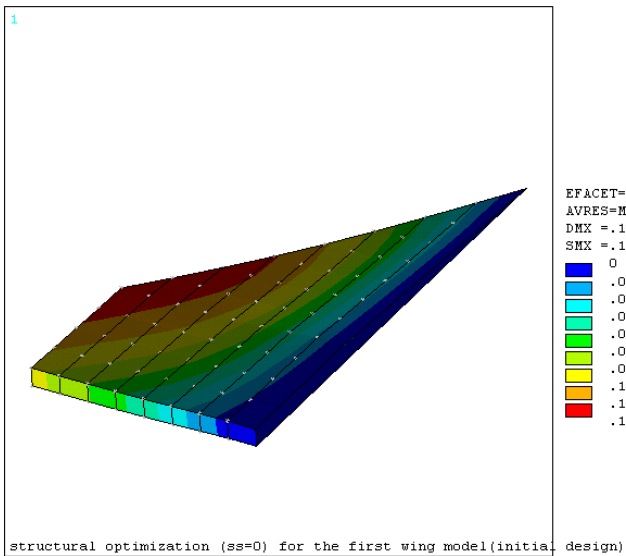
Figure (5.39) Illustrative contours for Model 6 (failed with displacement & frequency constraints)



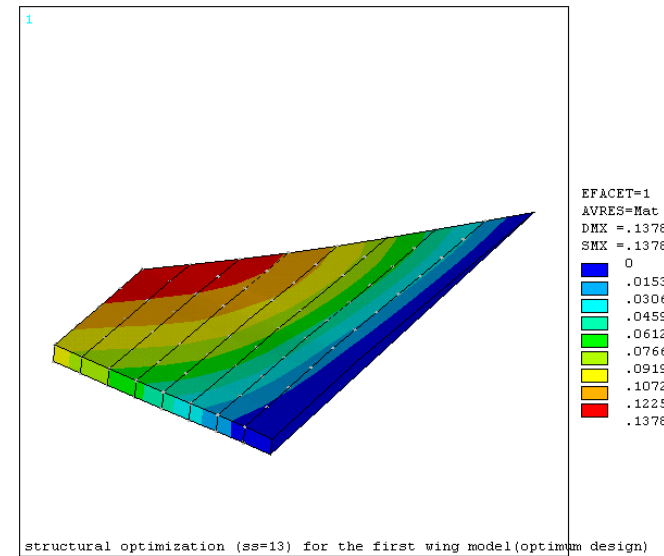
(a) Von-Mises stress distribution in Pa
(initial design)



(b) Von-Mises stress distribution in Pa
(final design)



(c) Total displacement distribution in m
(initial design)



(d) Total displacement distribution in m
(final design)

Figure (5.40) Illustrative contours for the worst design (Model 1)
(failed with stress failure criterion, displacement & frequency constraints)

5.7 Data Verification:

The data obtained in this work were verified with three aspects: the first includes the inertia loads, the second includes the modal analysis, and the third is the optimization technique. These three aspects are listed below:

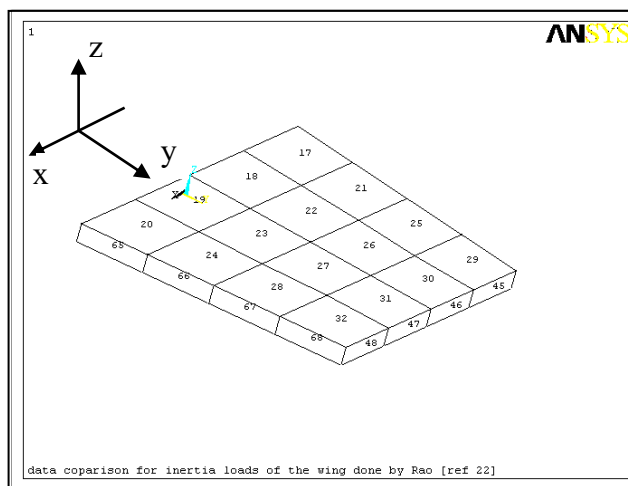
5.7.1 Inertia Loads (Structural Behaviour):

The case study discussed here includes the comparison between the work of **Rao** and the present work. The model to be studied represents the wing structure and its main dimensions were indicated in **Figure (5.41)**. The wing specification and its material properties are shown in **Table (5.10a)**.

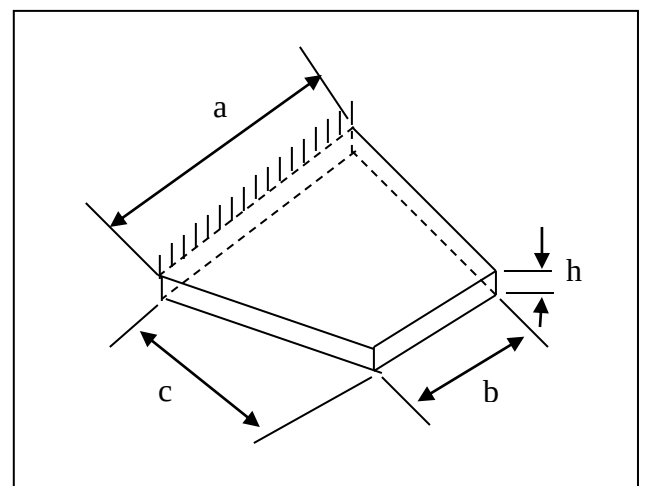
Table (5.10a) Specifications of the wing used by Rao (Ref. 51)

property	ρ	E	ν	thickness	payload	a	b	c	h	a_x	a_y	a_z
Value	4500	105	0.3	0.002	600	2.8	2.2	2.2	0.15	3.5g	0	2.5g
unit	Kg/m ³	Gpa	---	m	kg	m	m	m	m	m/s ²	m/s ²	m/s ²

The **FE** idealization of the wing box is done with 68 elements (32 quadrilateral plate elements for skin panel and 36 rectangular plate elements for stiffeners members) as shown by **Fig (5.41)**. The payload is distributed over all nodes equally. The comparison shows the best agreement with **Rao** results (**Table (5.10b)**).



(a) Finite element idealization of wing box used by Rao (Ref. 51)



(b) Main dimensions of wing box used by Rao (Ref. 51)

Figure (5.41) Wing box used by Rao (Ref. 51)

Table (5.10b) Data comparison with *Rao (Ref. 51)*

<i>Parameter</i>	<i>Rao (Ref. 51)</i>	<i>Present work</i>	<i>Error (%)</i>
$\sigma_{VM}^{max} (Mpa)$	296.54	298.82	0.7688
$u_z^{max} (m)$	$6.3523 \cdot 10^{-2}$	$6.40061 \cdot 10^{-2}$	0.7605
$\tau^{max} (Mpa)$	230.13	232.253	0.9225

5.7.2 Modal Analysis:

The same wing structure studied by *Rao (Ref. 51)* is used as verification tool for modal analysis of the present work as shown below (the first five modes of natural frequency will be taken into account) as shown in *Table (5.11)*:

Table (5.11) Comparison of modal analysis for the model used by *Rao (Ref. 51)*

<i>Mode</i>	<i>Rao (Ref. 51)</i>	<i>Present work</i>	<i>Error (%)</i>
<i>Mode (1) [Hz]</i>	25.68	26.245	1.694
<i>Mode (2) [Hz]</i>	54.88	54.22	1.202
<i>Mode (3) [Hz]</i>	114.3	115.18	0.770
<i>Mode (4) [Hz]</i>	139.66	137.34	1.660
<i>Mode (5) [Hz]</i>	154.71	153.95	0.491

5.7.3 Structural Optimization Technique:

The efficiency of the present strategy was investigated through the direct comparison with *Chan⁽⁸⁾* work. *Chan⁽⁸⁾* wing model is used for this purpose, where the wing structure is idealized by 36 isoparametric 4-noded plate elements of rectangular shape and 3 isoparametric 4-noded plate elements of quadrilateral shape as done by *Chan⁽⁸⁾*. The unknowns are 39 thicknesses of the plate elements to be designed for minimum weight as shown in *Figure (5.42a)* and defined by *Table (5.12a)*. The whole dimensions, boundary and load conditions are shown in *Fig. (5.42b)*. The largest equivalent Von-Mises stress for each element alone is subjected to a limit of ($\pm 145 Mpa$) taking the material

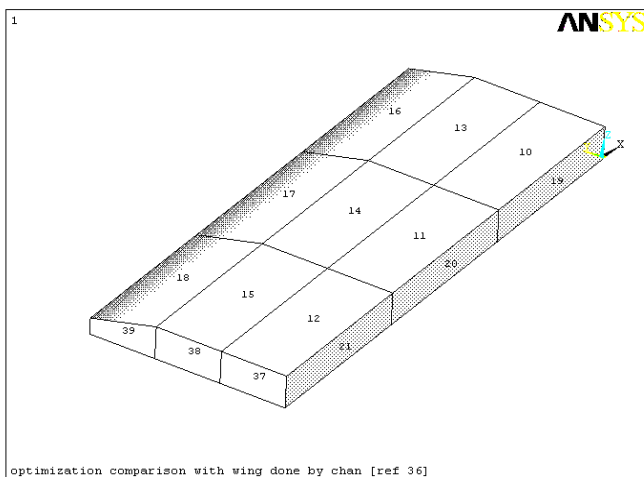
properties of Al24 listed in **Appendix-D-** and the thickness is limited to a minimum of (2mm). The comparison of final design results (Evolution Strategy) with those results obtained by **Chan**⁽⁸⁾ works (Lagrange multipliers) is shown in **Table (5.12b)**. **Figs. (5.42c) & (5.42d)** show the Von-Mises stress distribution contours for initial and optimum designs respectively. The structural mass evolution is shown in **Fig. (5.42e)** as compared with **Chan & Stafford**⁽⁸⁾.

Table (5.12a) Material and geometrical properties for the model used by Chan (Ref. 8)

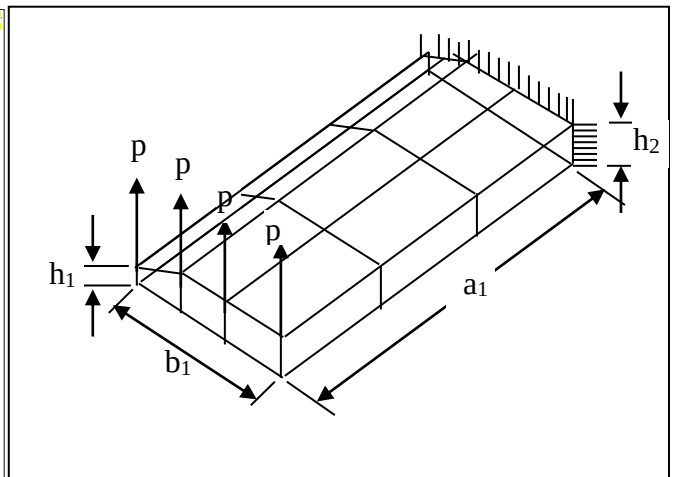
Property	No. of materials	Properties of material (Al24)				p	a_1	b_1	h_1	h_2	Initial Thickness (t)
		ρ	σ_{yp}	E	ν						
Value	1	2696	145	71.3	0.33	7500	3	1.2	0.1	0.2	0.0045
unit	---	Kg/m ³	Mpa	Gpa	---	N	m	m	m	m	m

Table (5.12b) Comparison of data with Chan (Ref. 8) for optimum design

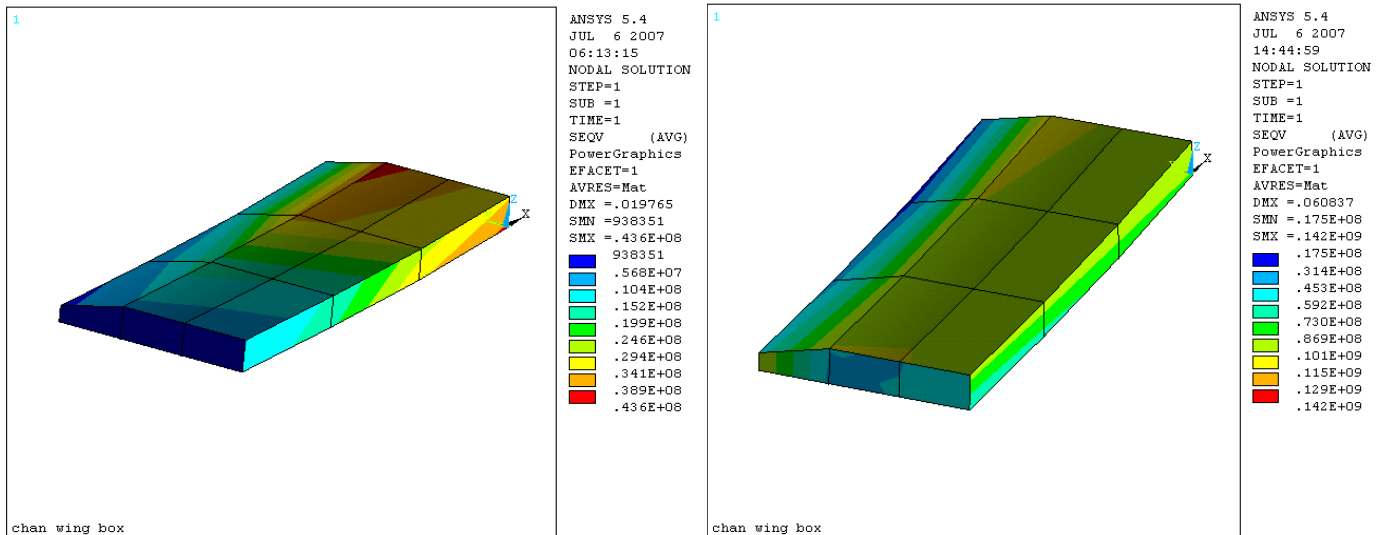
Parameter	Chan (Ref. 8)	Present work	Error (%)
σ_{VM}^{max} (Mpa)	142.684	141.544	0.7989
u_z^{tip} (m)	9.4823×10^{-2}	9.37061×10^{-2}	1.177
Mass (kg)	72.866	73.5827	0.9836



(a) Finite element idealization of Chan wing model (Ref. 8)



(b) Dimensions, boundary, and load conditions for Chan wing model (Ref. 8)



(c) Von-Mises stress distribution (initial design)

(d) Von-Mises stress distribution (final design)

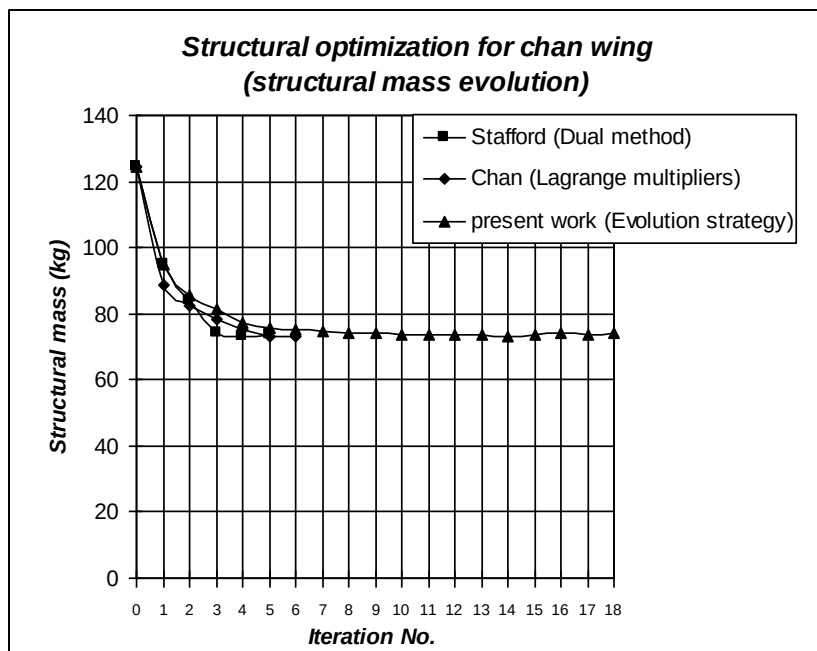
(e) Structural mass evolution for Chan ⁽⁸⁾ wing structure (comparison)

Figure (5.42) Illustrative figures for Chan wing Model (Ref. 8)

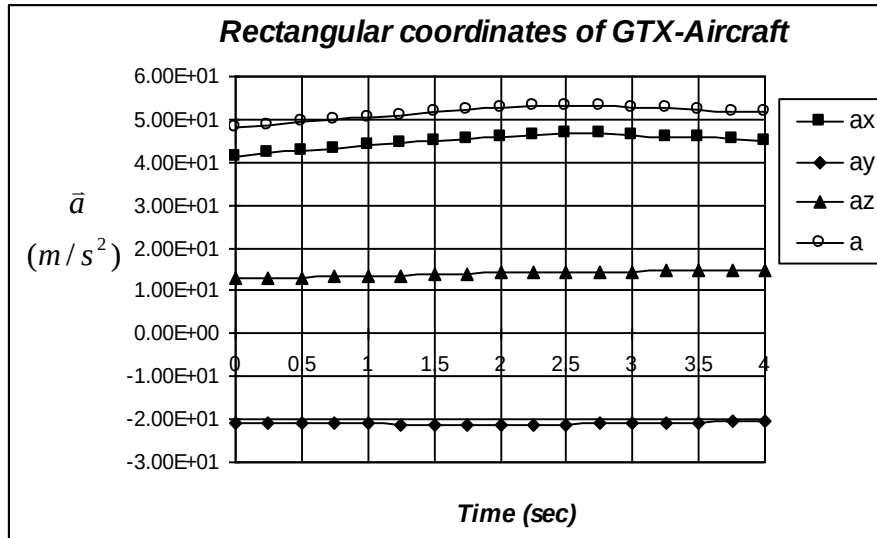


Figure (5.1a) Rectangular coordinates of acceleration vector (particle consideration)

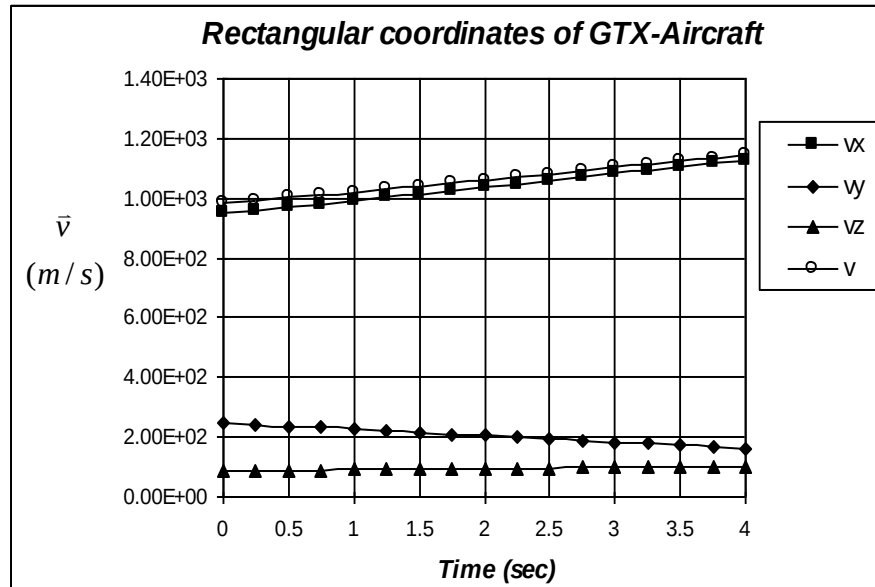


Figure (5.1b) Rectangular coordinates of velocity vector (particle consideration)

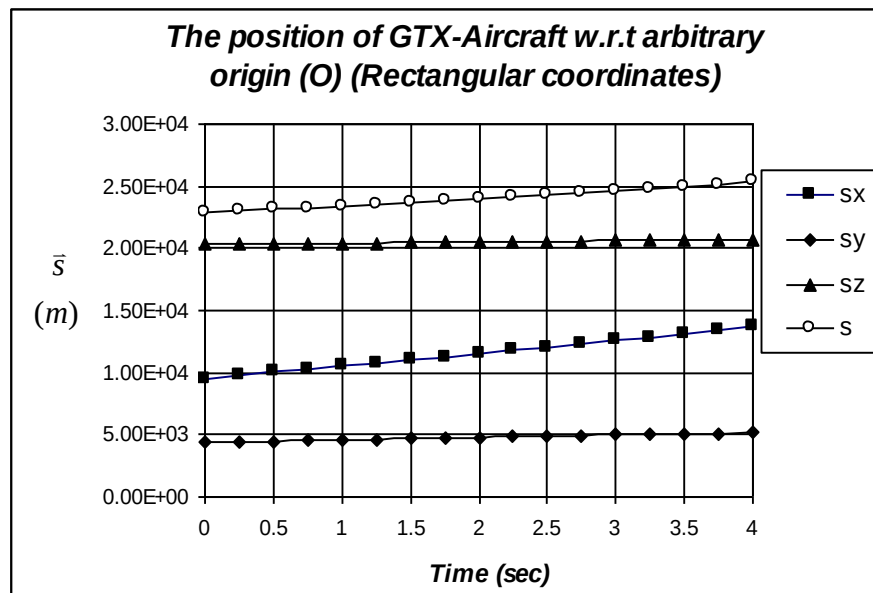


Figure (5.1c) Rectangular coordinates of position vector (particle consideration)

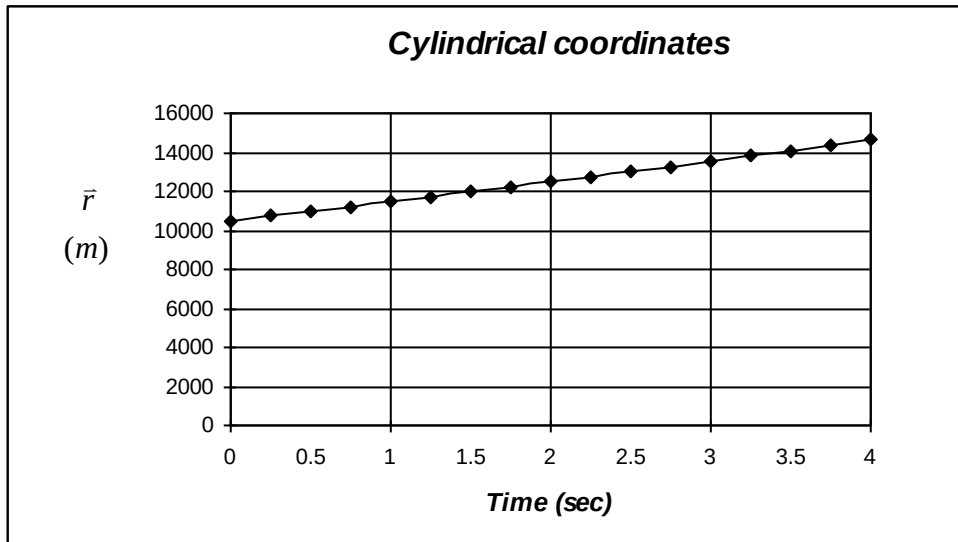


Figure (5.2a) Cylindrical coordinates (r) (particle consideration)

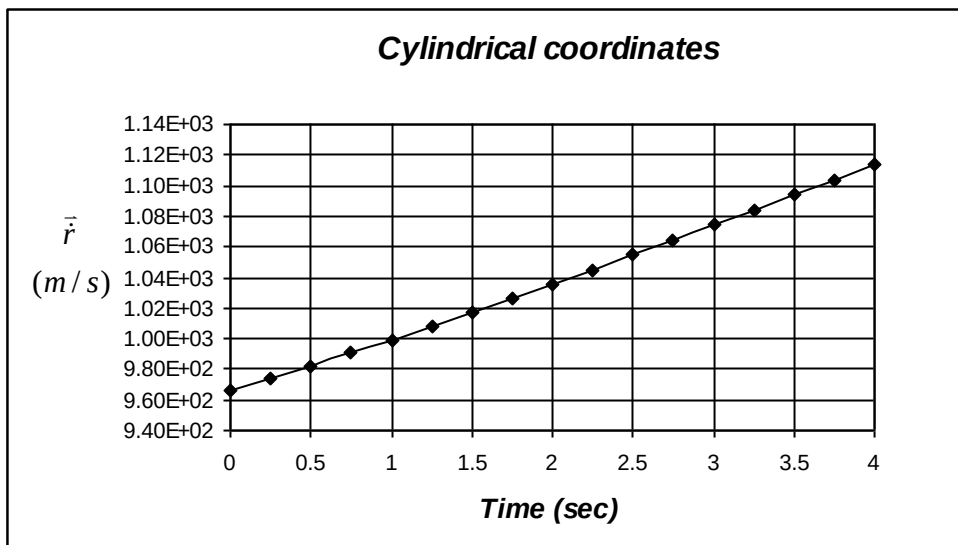


Figure (5.2b) Cylindrical coordinates ($\dot{r}$) (particle consideration)

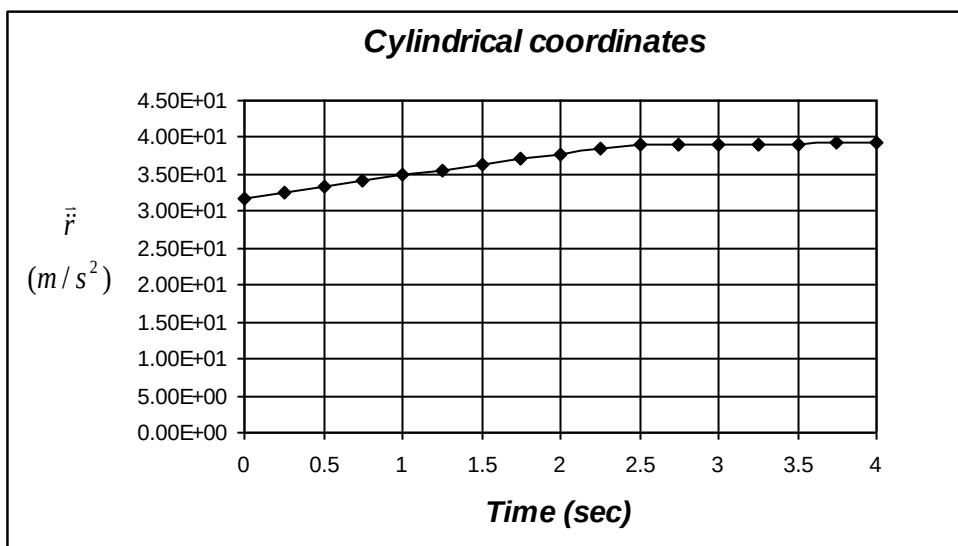


Figure (5.2c) Cylindrical coordinates ($\ddot{r}$) (particle consideration)

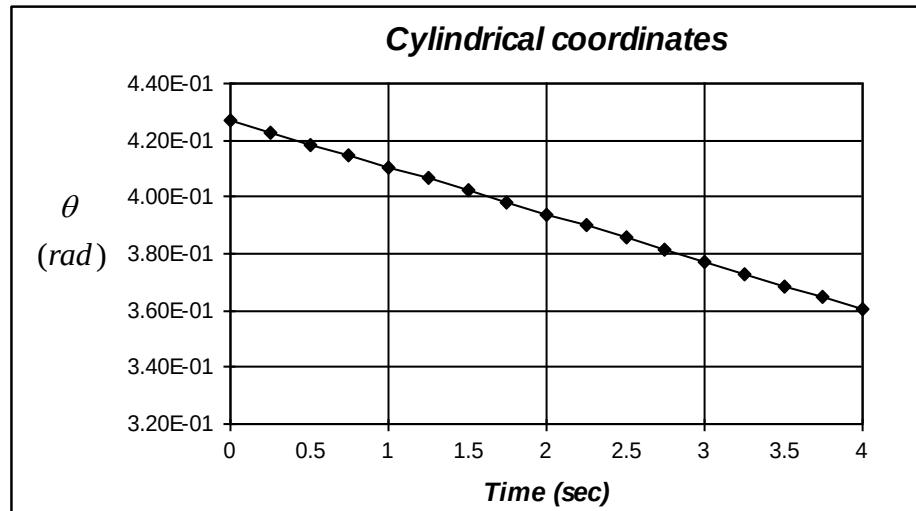


Figure (5.2d) Cylindrical coordinates (θ) (particle consideration)

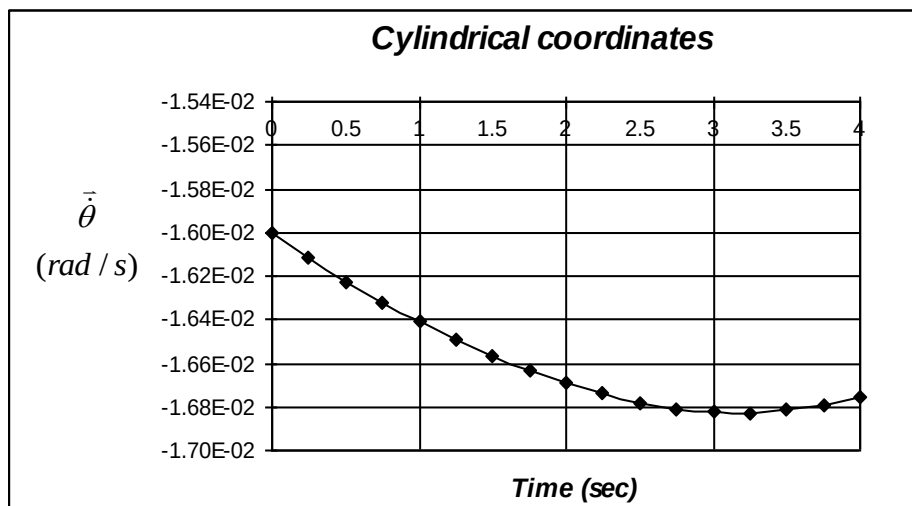


Figure (5.2e) Cylindrical coordinates ($\dot{\theta}$) (particle consideration)

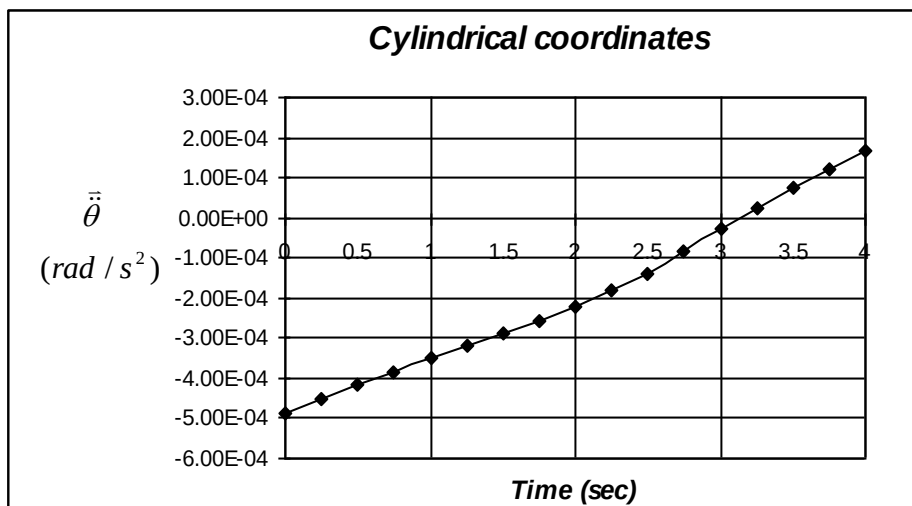


Figure (5.2f) Cylindrical coordinates ($\ddot{\theta}$) (particle consideration)

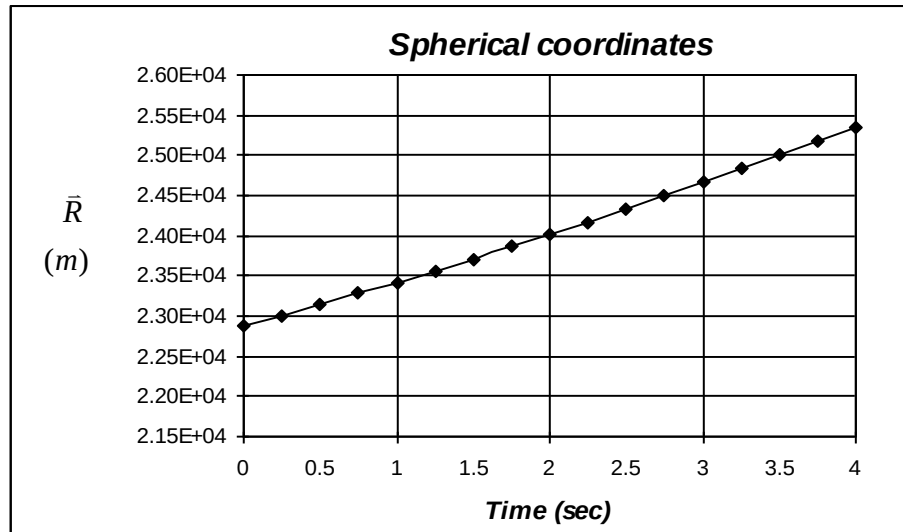


Figure (5.3a) Spherical coordinates ($\bar{R}$) (particle consideration)

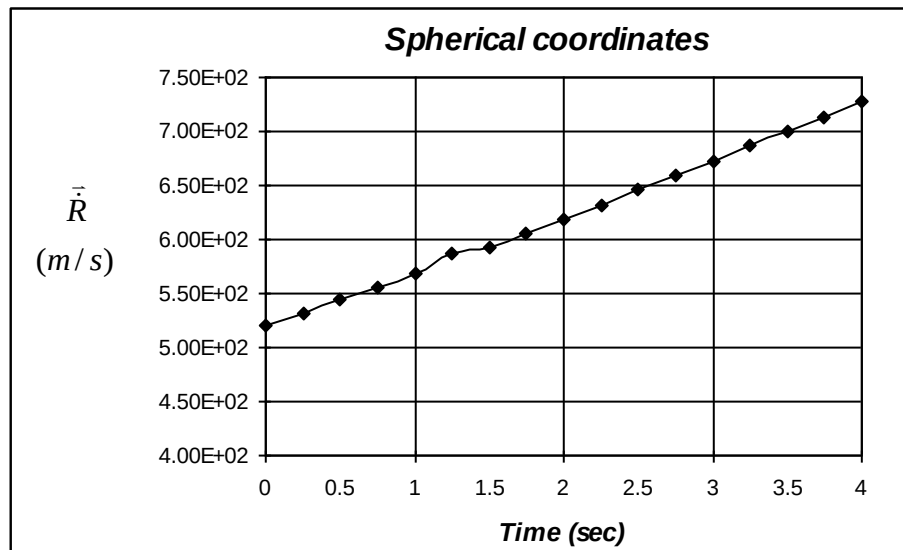


Figure (5.3b) Spherical coordinates ($\dot{\bar{R}}$) (particle consideration)

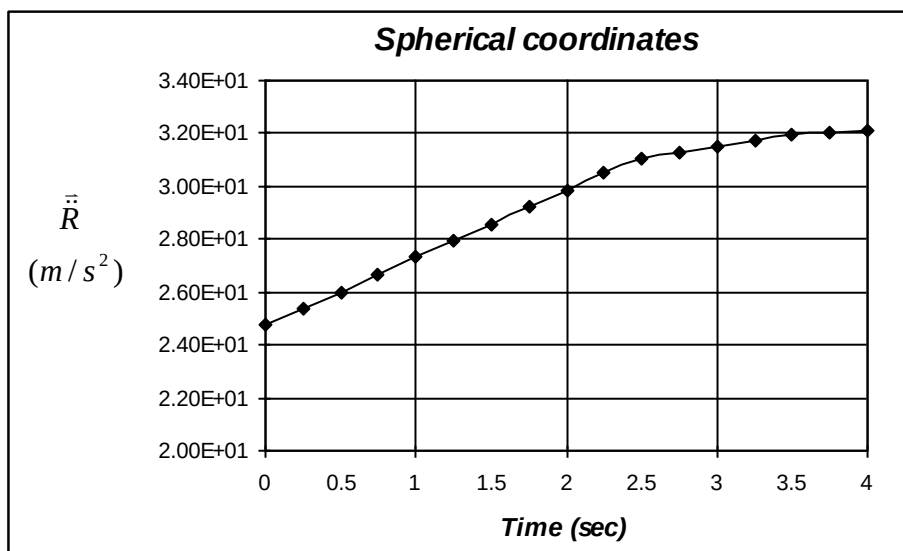


Figure (5.3c) Spherical coordinates ($\ddot{\bar{R}}$) (particle consideration)

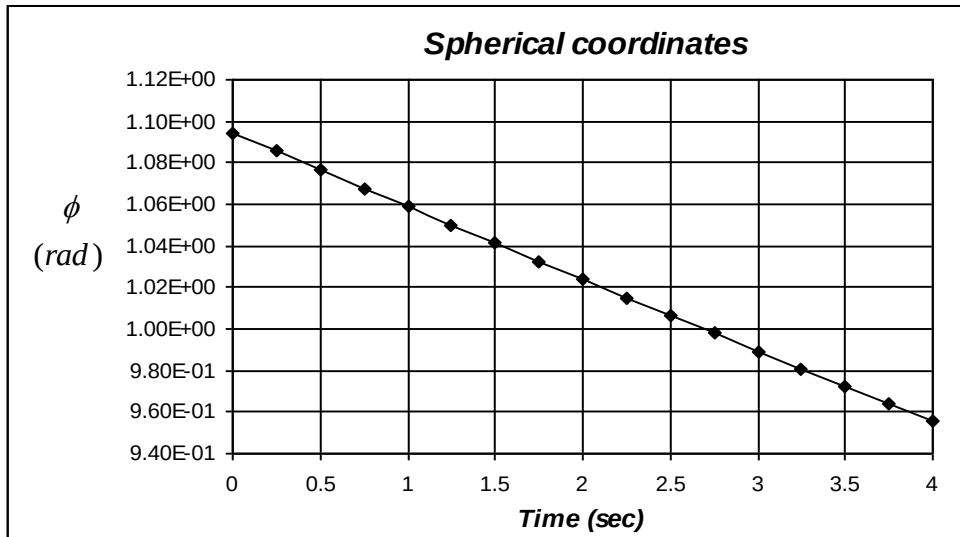


Figure (5.3d) Spherical coordinates (ϕ) (particle consideration)

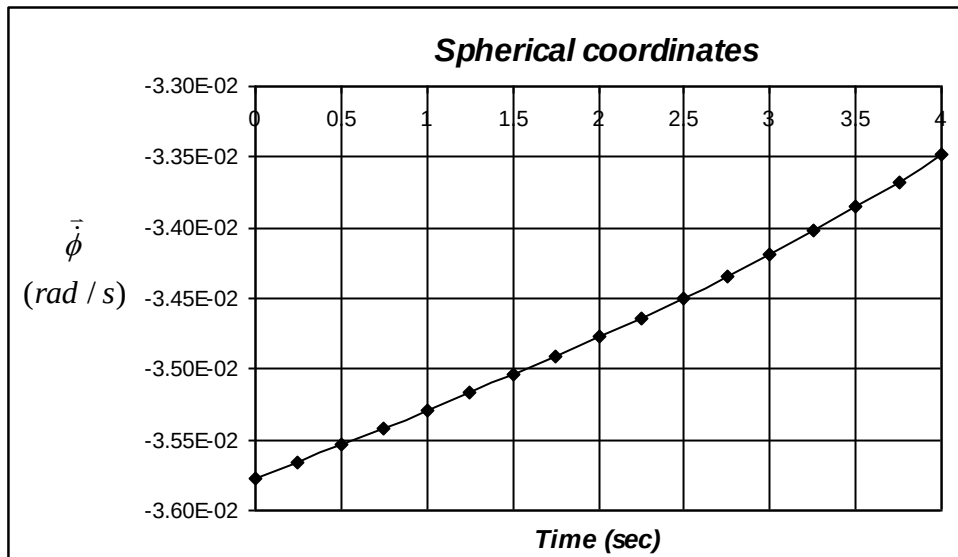


Figure (5.3e) Spherical coordinates ($\dot{\phi}$) (particle consideration)

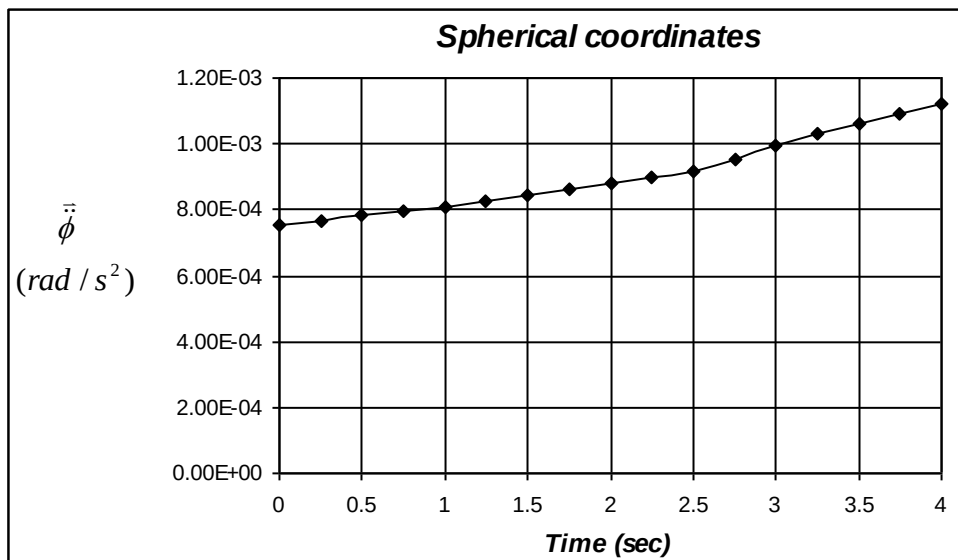


Figure (5.3f) Spherical coordinates ($\ddot{\phi}$) (particle consideration)

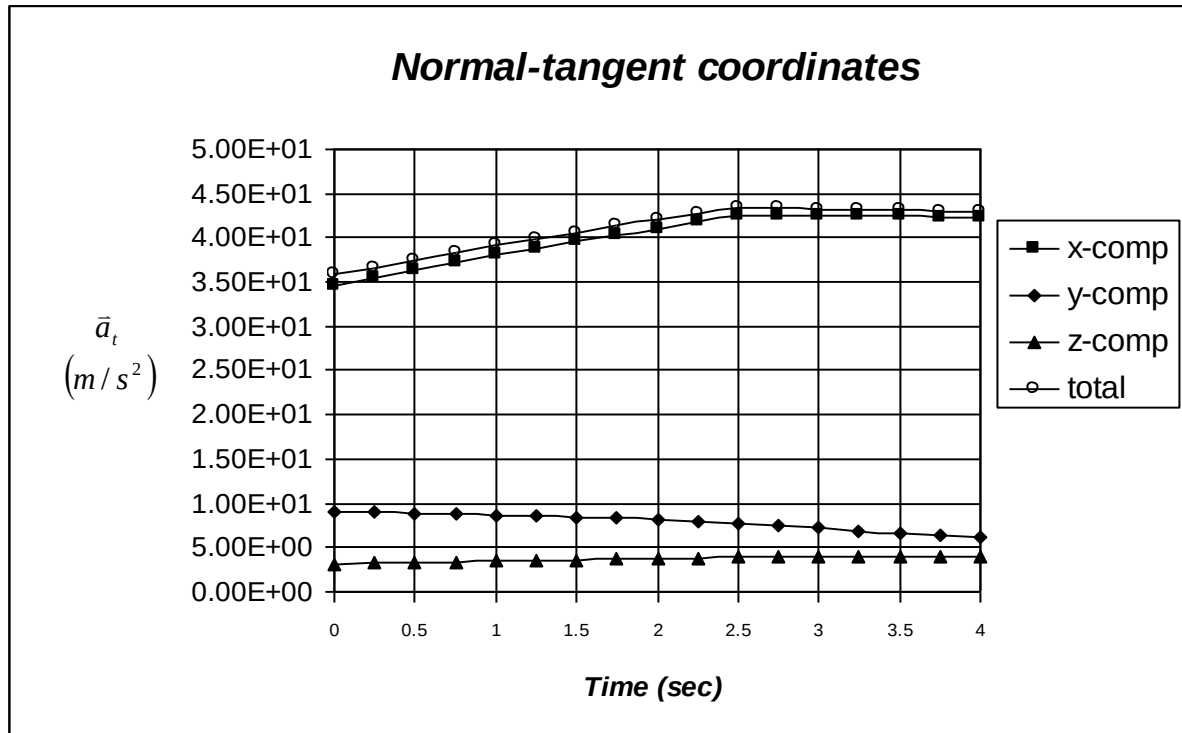


Figure (5.4a) Normal-tangent coordinates of acceleration vector ($\bar{a}_t$)
(Particle consideration)

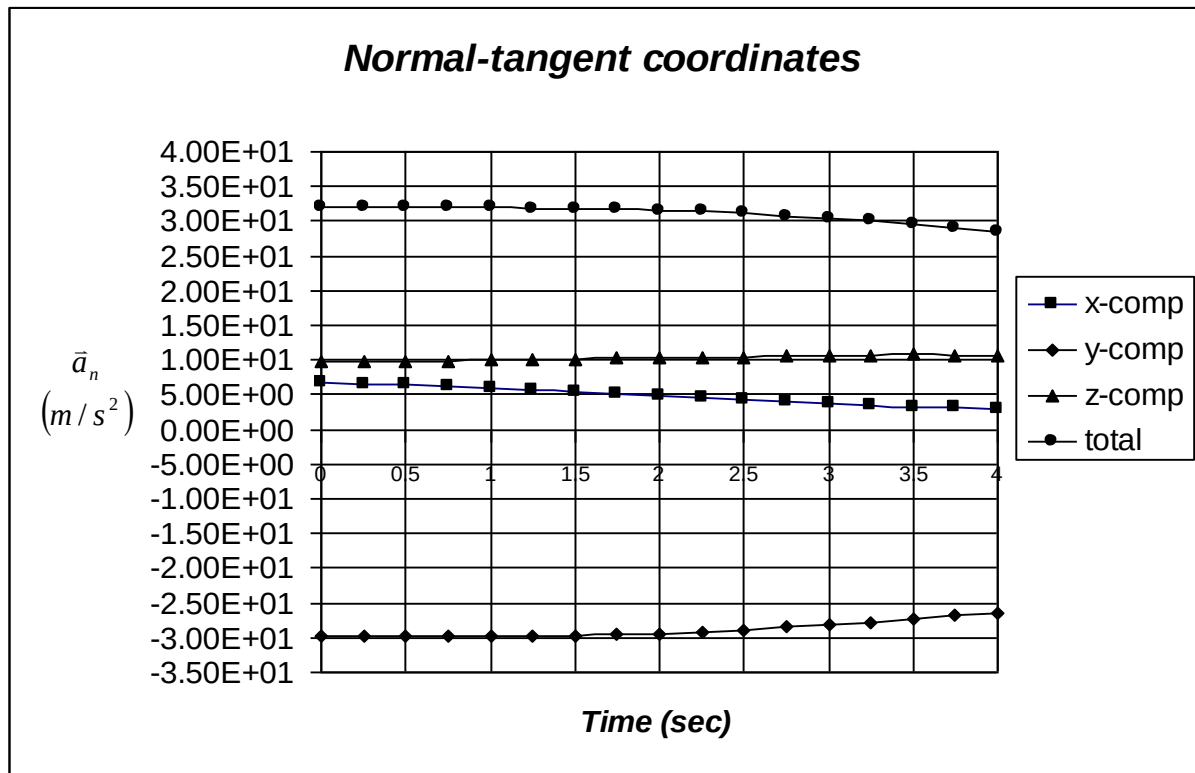


Figure (5.4b) Normal-tangent coordinates of acceleration vector ($\bar{a}_n$)
(Particle consideration)

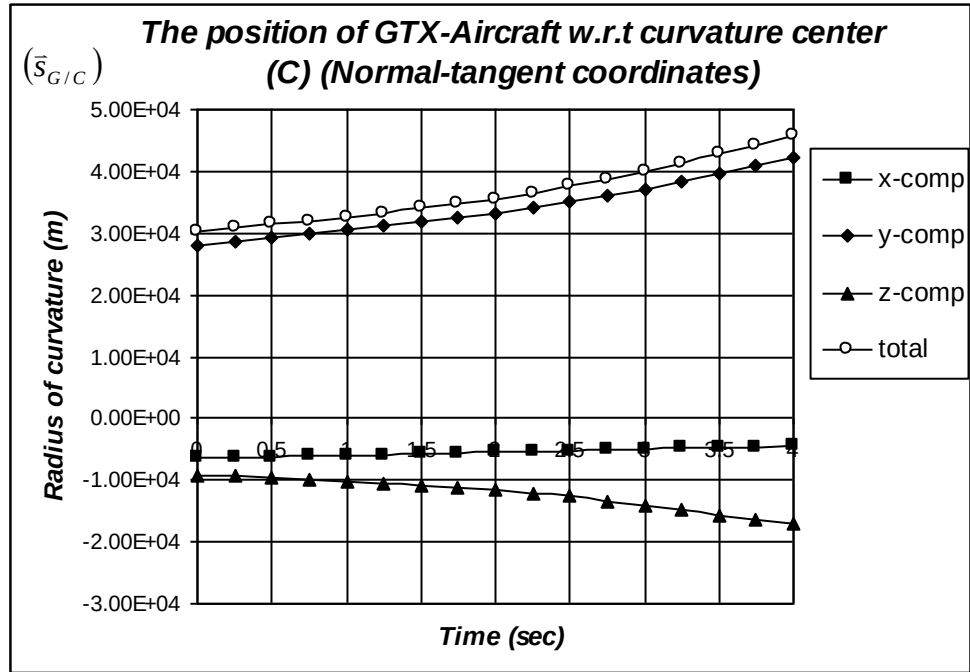


Figure (5.4c) Normal-tangent coordinates (radius of curvature vector ($\bar{s}_{G/C}$))
(Particle consideration)

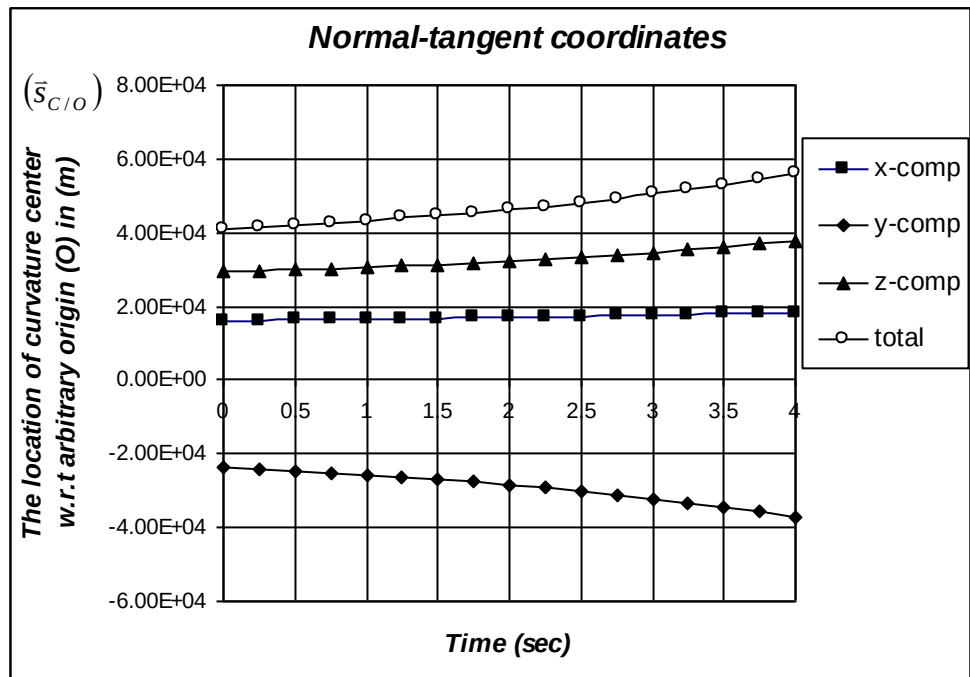


Figure (5.4d) Normal-tangent coordinates (position vector of curvature center ($\bar{s}_{C/O}$))
(Particle consideration)

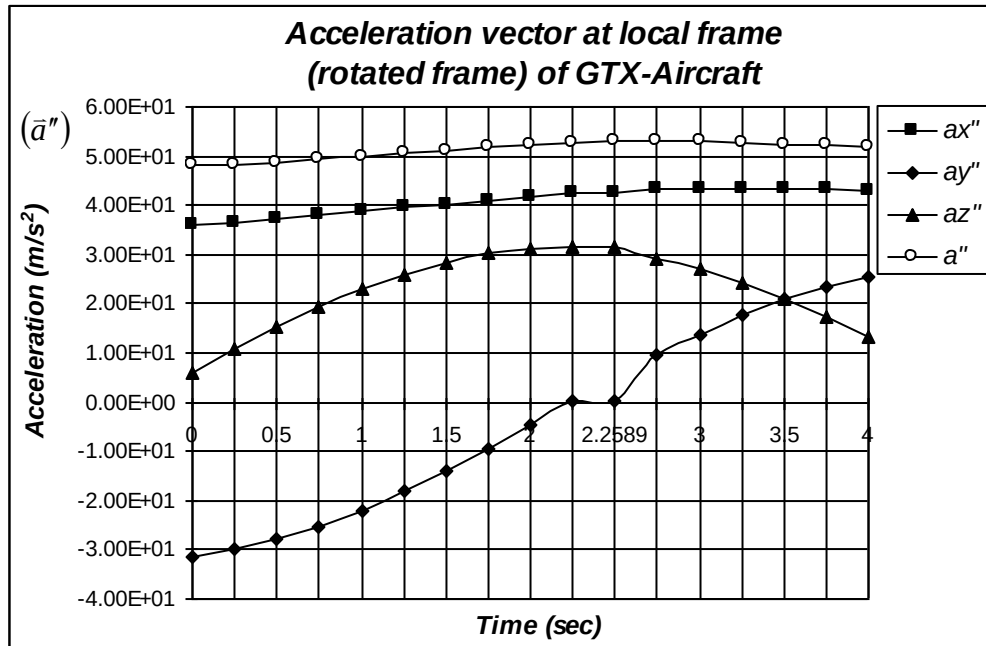
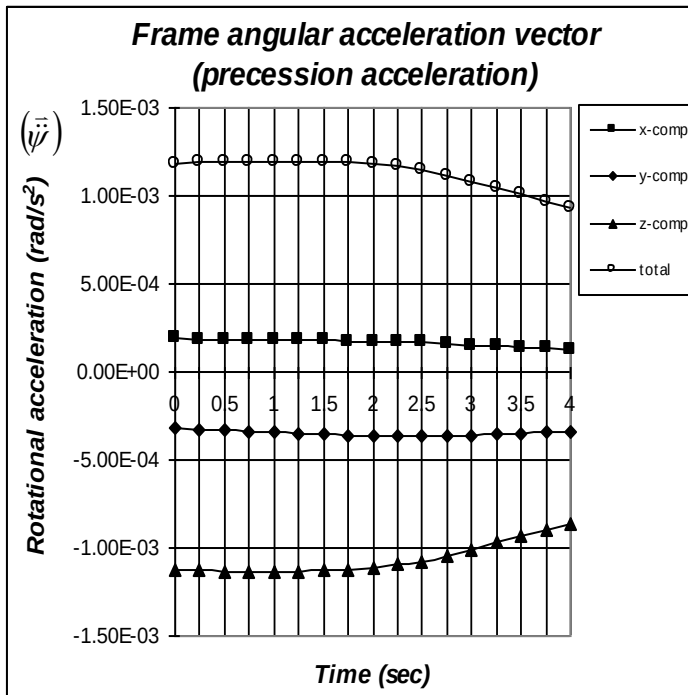
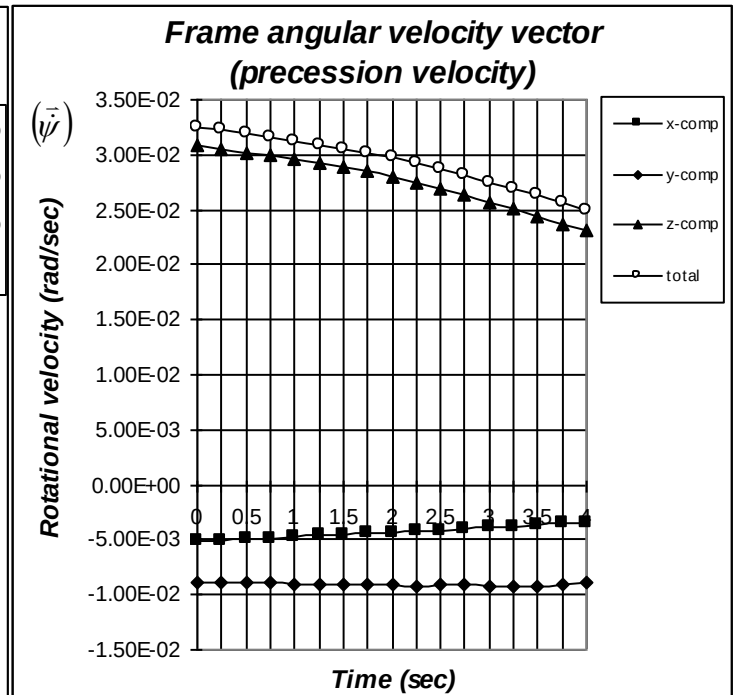


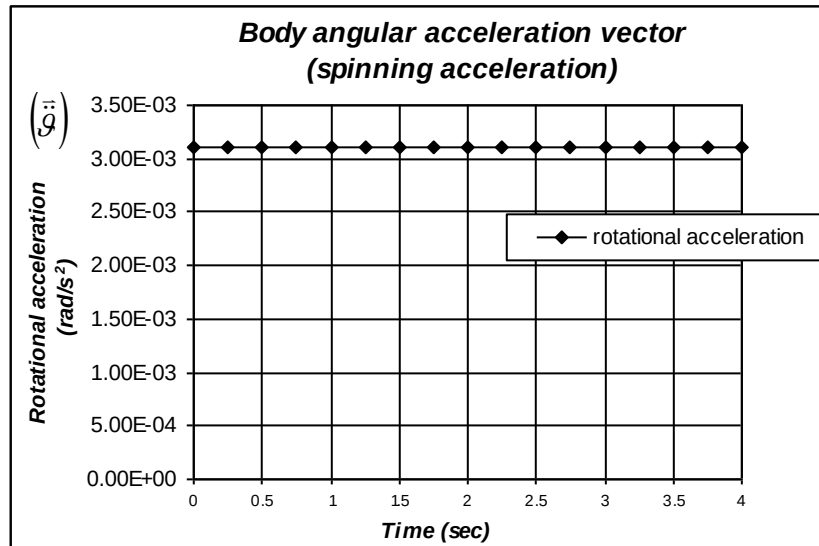
Figure (5.5) Acceleration vector with respect to the spinning *Frame* ($Gx''y''z''$)
(Used directly to investigate the maximum Von-Mises stress induced for each of wing structures)



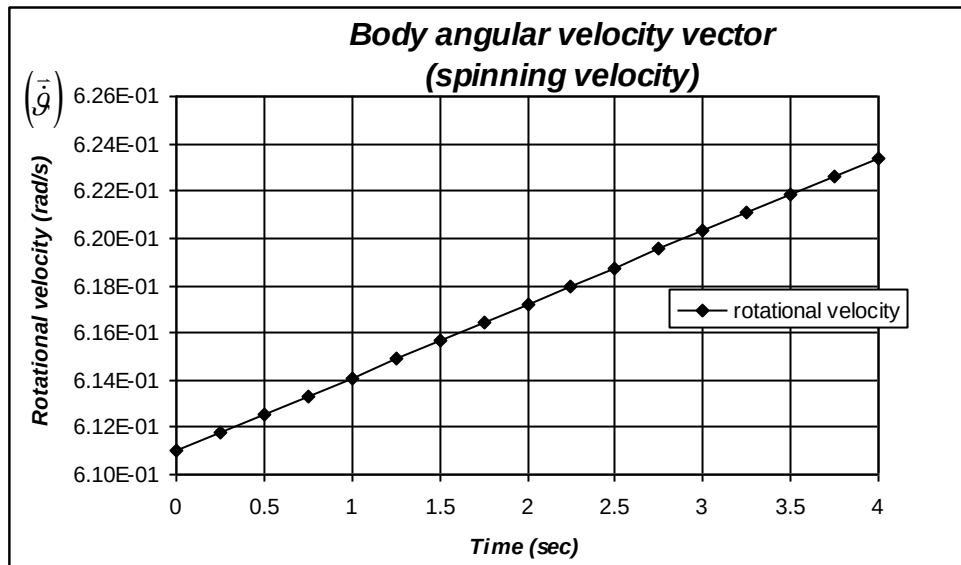
(a) Frame angular acceleration about G
(rigid body consideration)



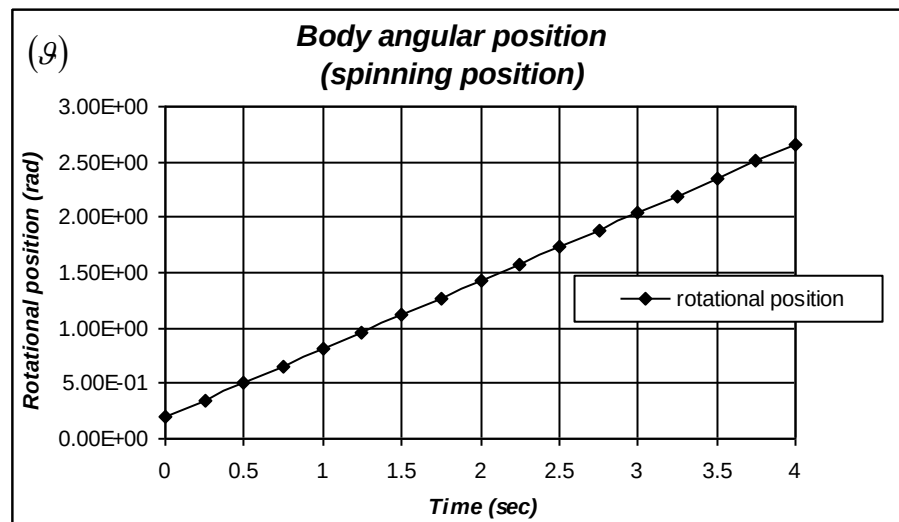
(b) Frame angular velocity about G
(rigid body consideration)



(c) Body angular acceleration about G (Rigid body consideration)



(d) Body angular velocity about G (Rigid body consideration)



(e) Body angular position about G (Rigid body consideration)

Figure (5.6) Rotational motion (Frame and body rotation) about **G** (rigid body consideration)

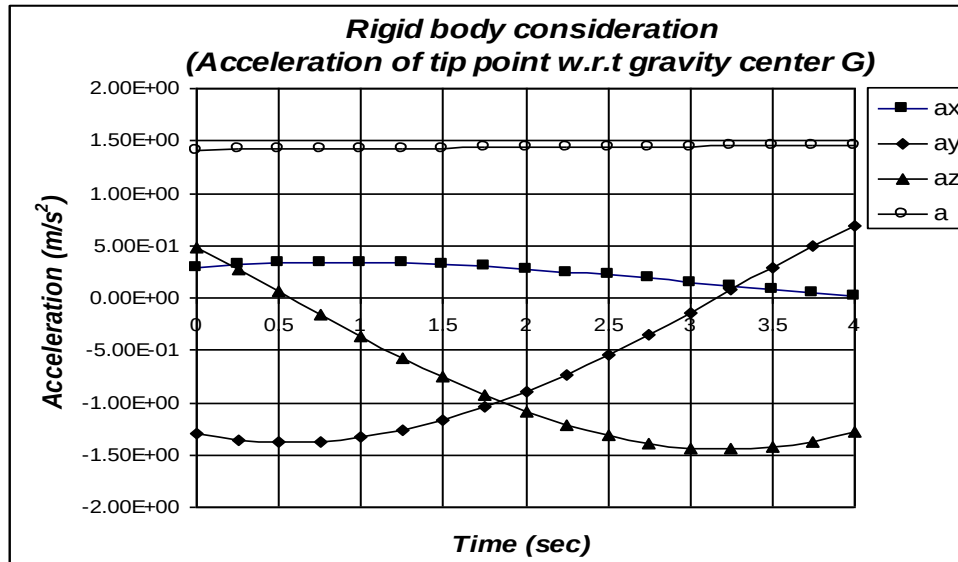


Figure (5.7a) Tip point acceleration (w.r.t Local Frame (Gxyz) shown by Fig (3.15))

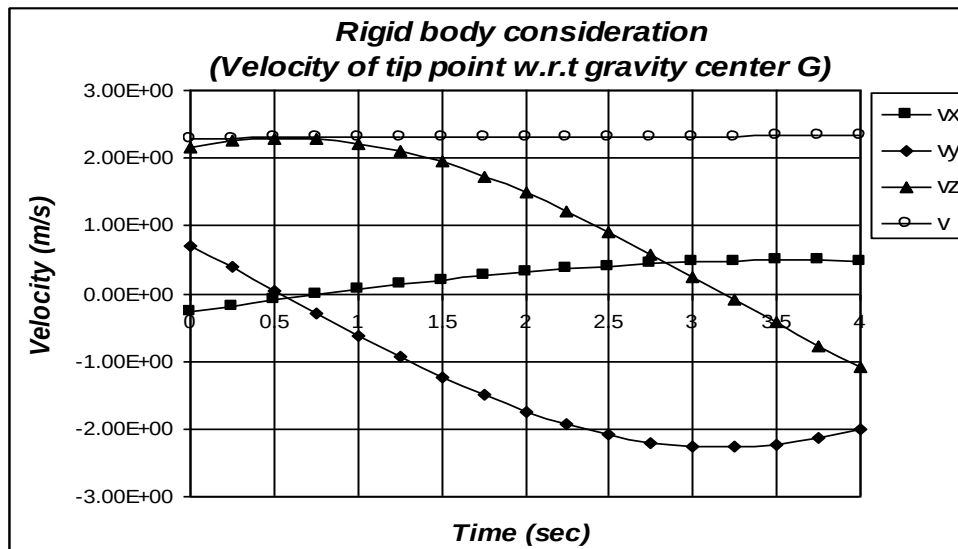


Figure (5.7b) Tip point velocity (w.r.t Local Frame (Gxyz))

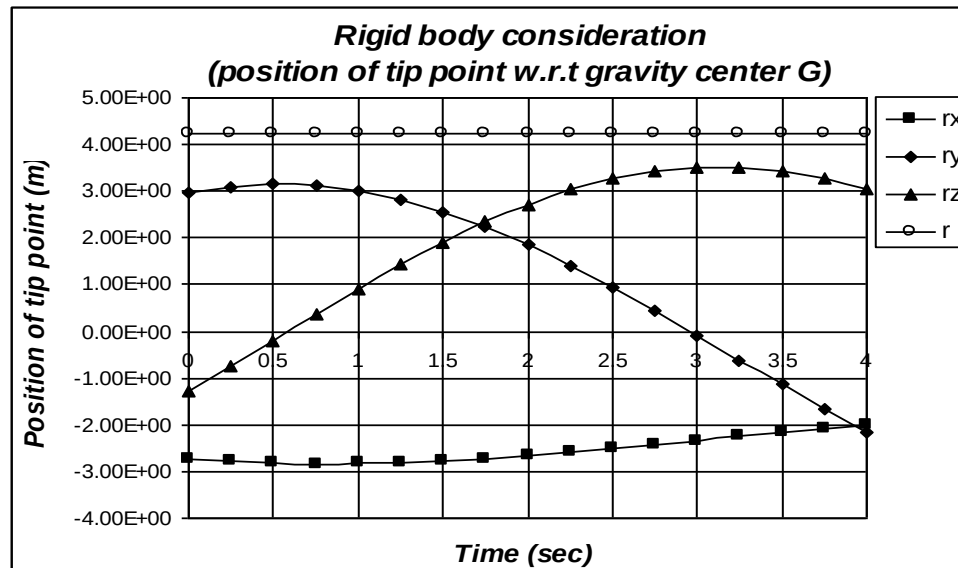


Figure (5.7c) Tip point position (w.r.t Local Frame (Gxyz))

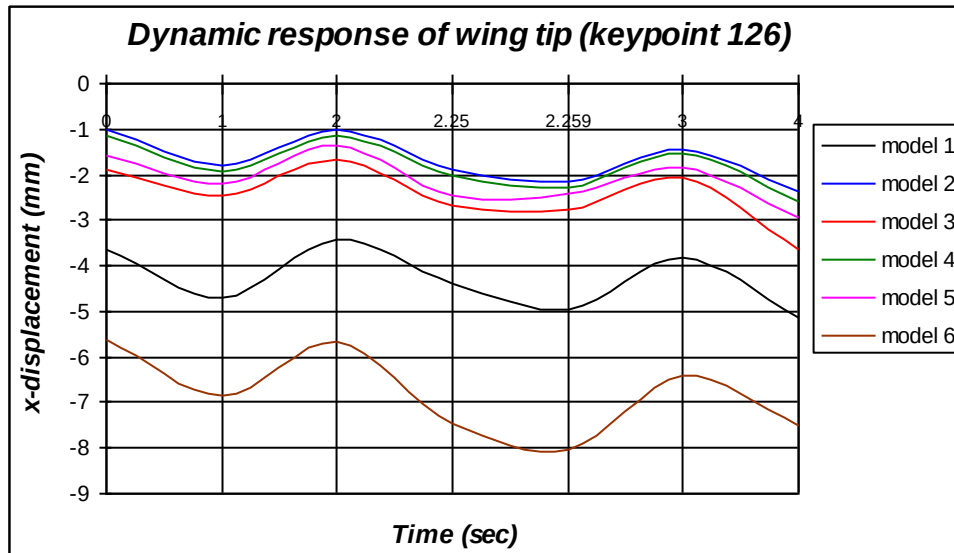


Figure (5.8a) x-displacement of tip point versus time (deformable body consideration)

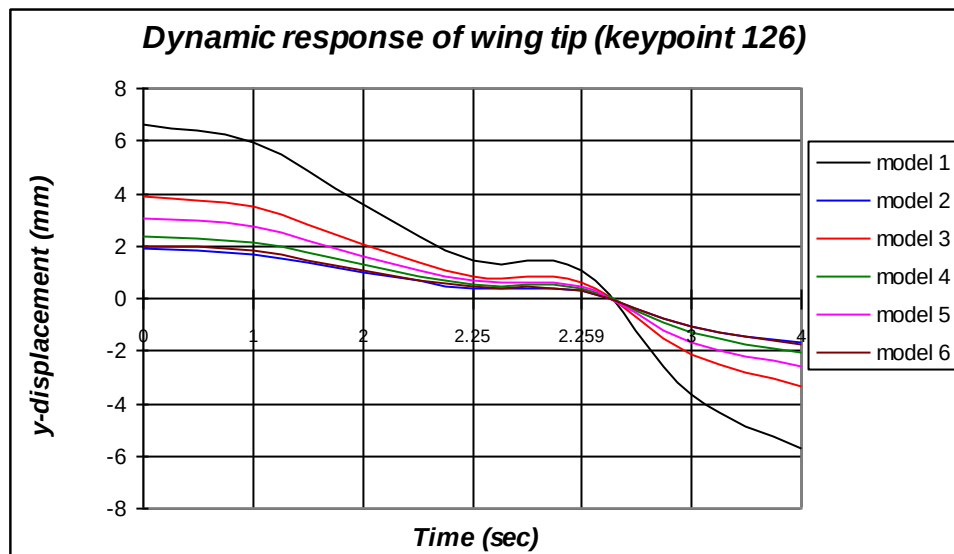


Figure (5.8b) y-displacement of tip point versus time (deformable body consideration)

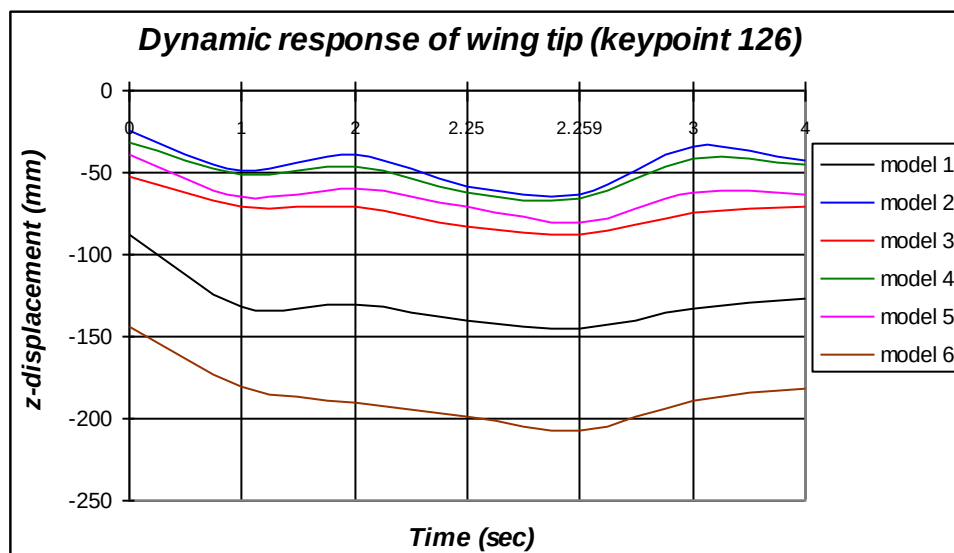


Figure (5.8c) z-displacement of tip point versus time (deformable body consideration)

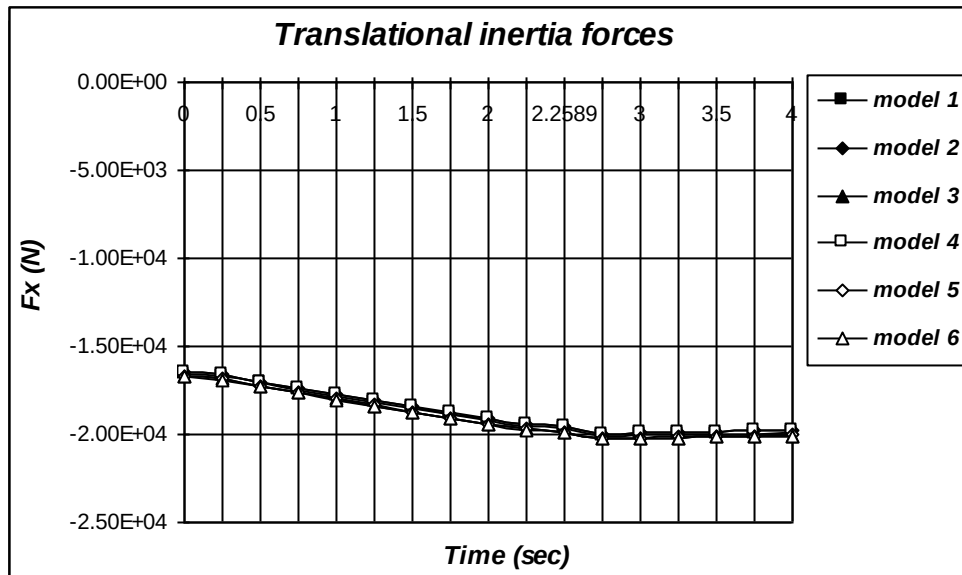


Figure (5.9a) Translational inertia forces (total x-Force)

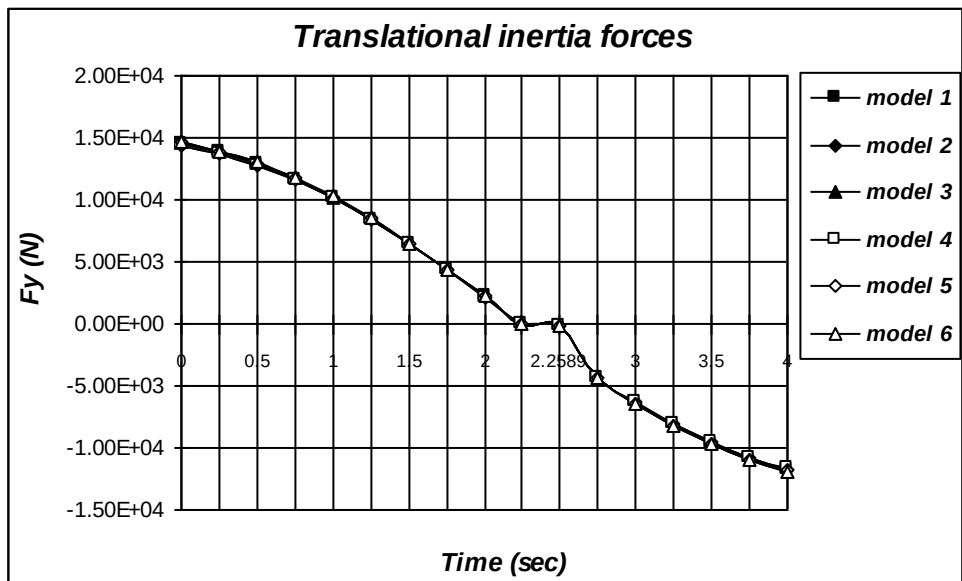


Figure (5.9b) Translational inertia forces (total y-Force)

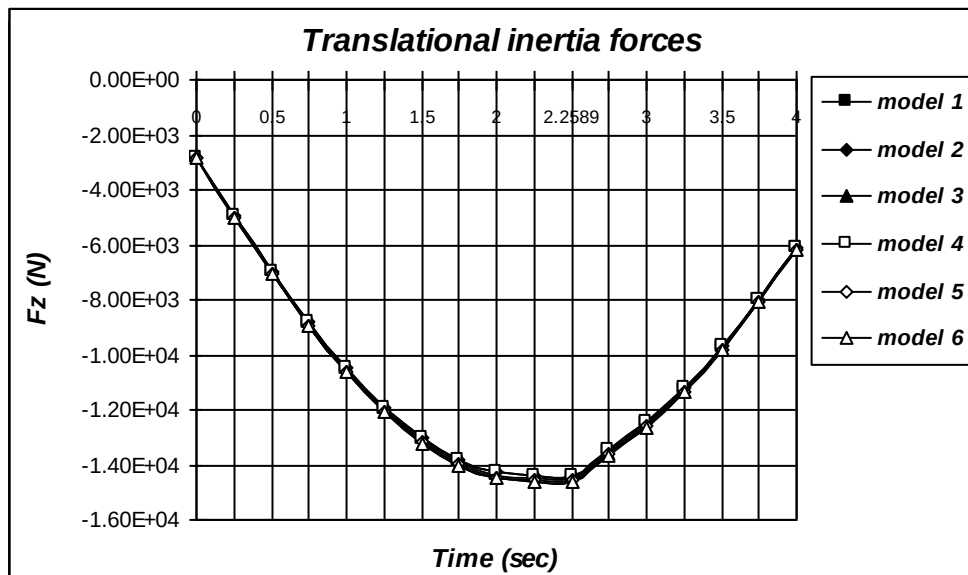


Figure (5.9c) Translational inertia forces (total z-Force)

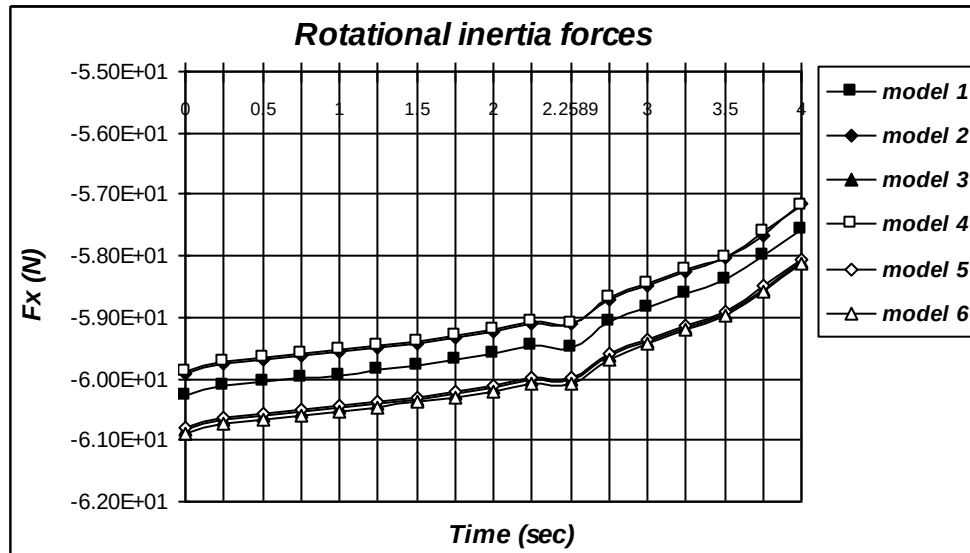


Figure (5.10a) Rotational inertia forces (total x-Force)

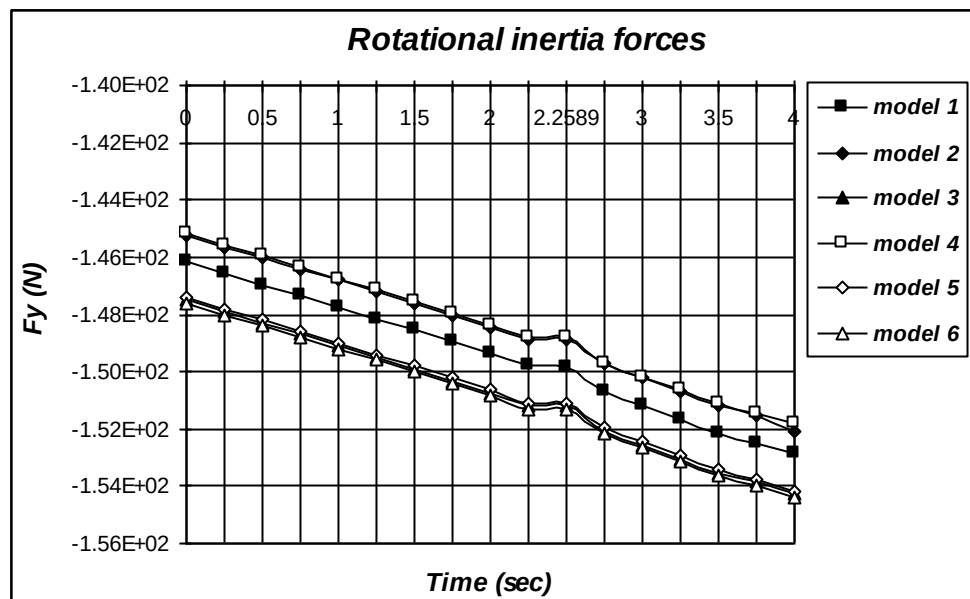


Figure (5.10b) Rotational inertia forces (total y-Force)

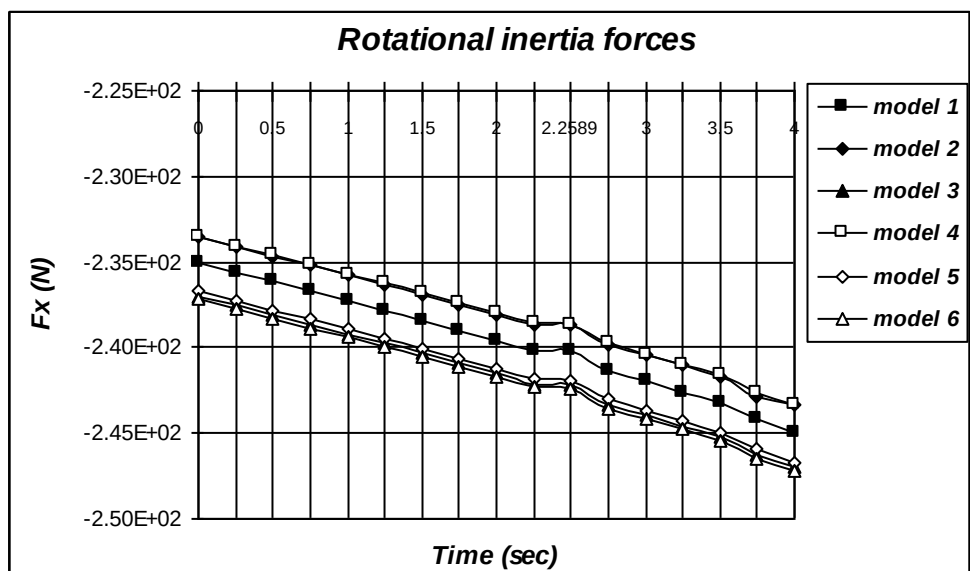


Figure (5.10c) Rotational inertia forces (total z-Force)

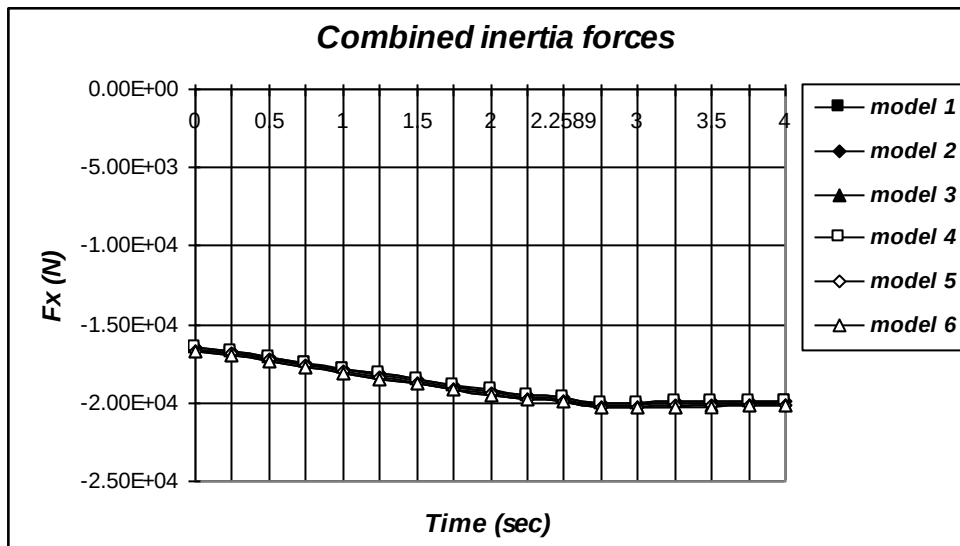


Figure (5.11a) Combined inertia forces (total x-Force)

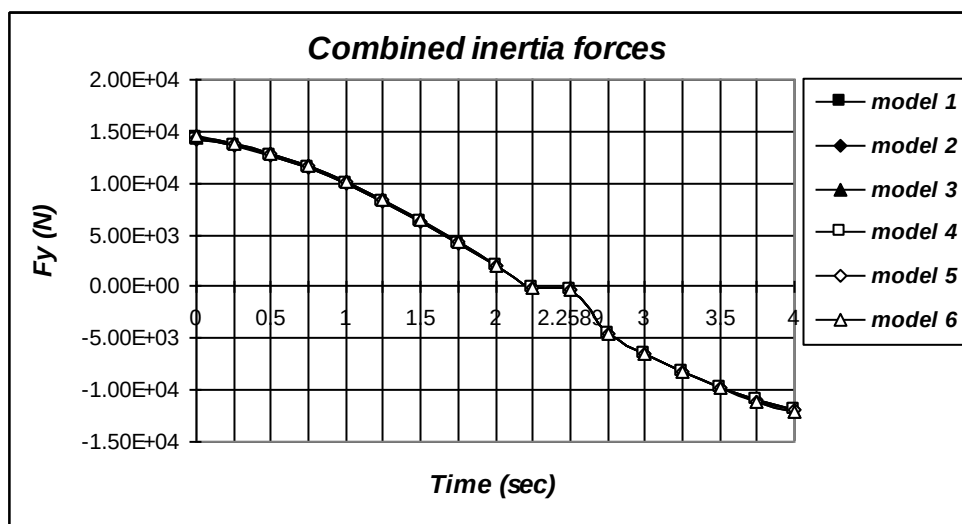


Figure (5.11b) Combined inertia forces (total y-Force)

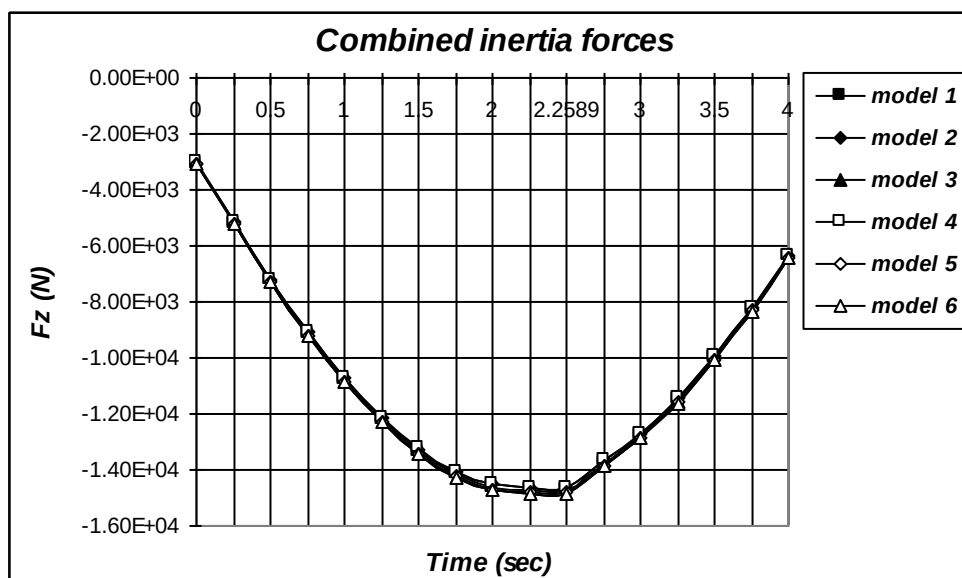


Figure (5.11c) Combined inertia forces (total z-Force)

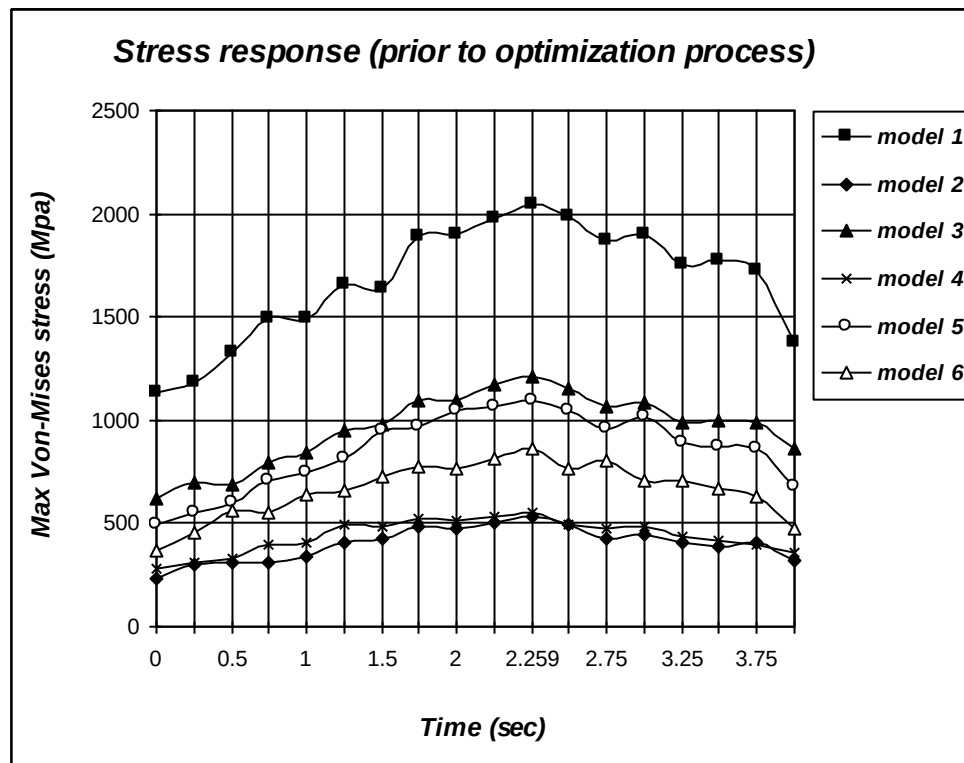


Figure (5.12) Maximum Von-Mises stress response as comparative study

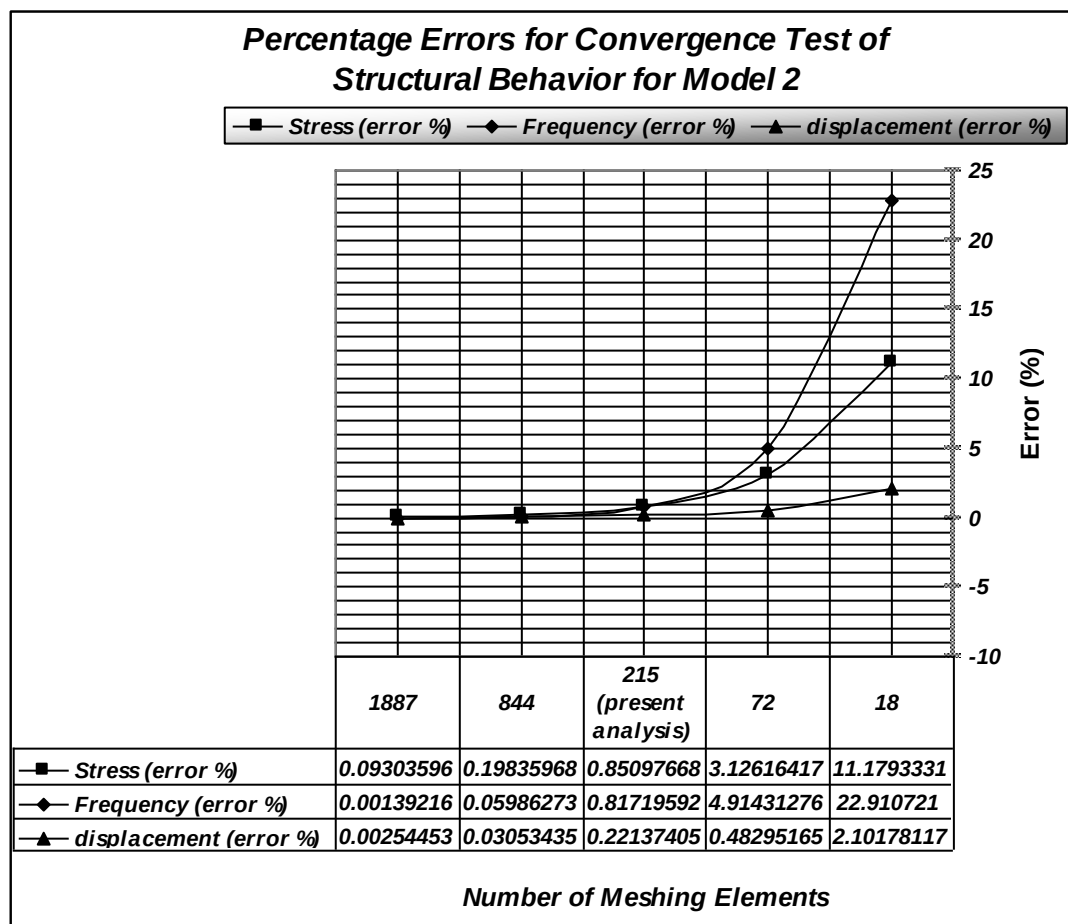


Fig. (5.13) data verification for the convergence test of the structural behavior of Model 2

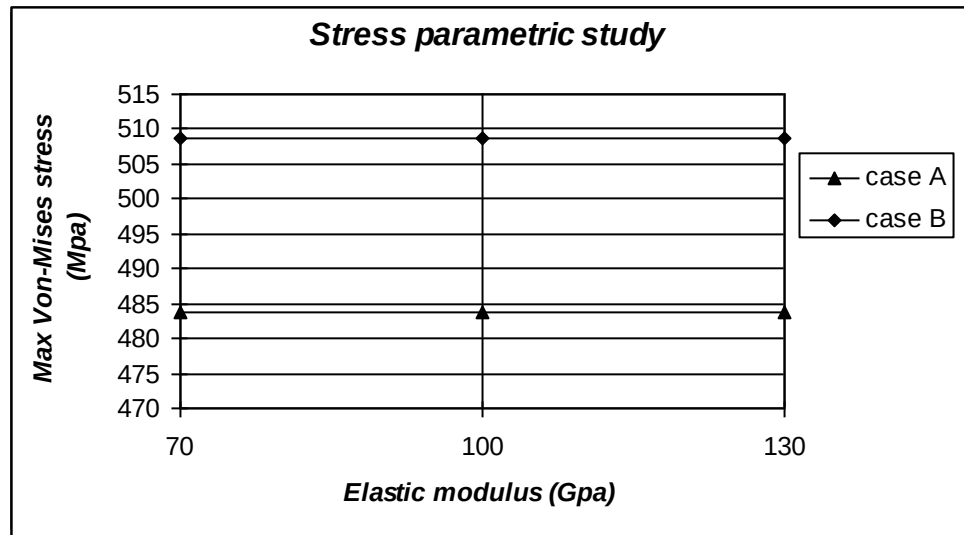


Figure (5.14a) The effect of elastic modulus on Max Von-Mises stress
Case A: $\rho = 2700 \text{ kg/m}^3, \nu = 0.33$ case B: $\rho = 4700 \text{ kg/m}^3, \nu = 0.34$

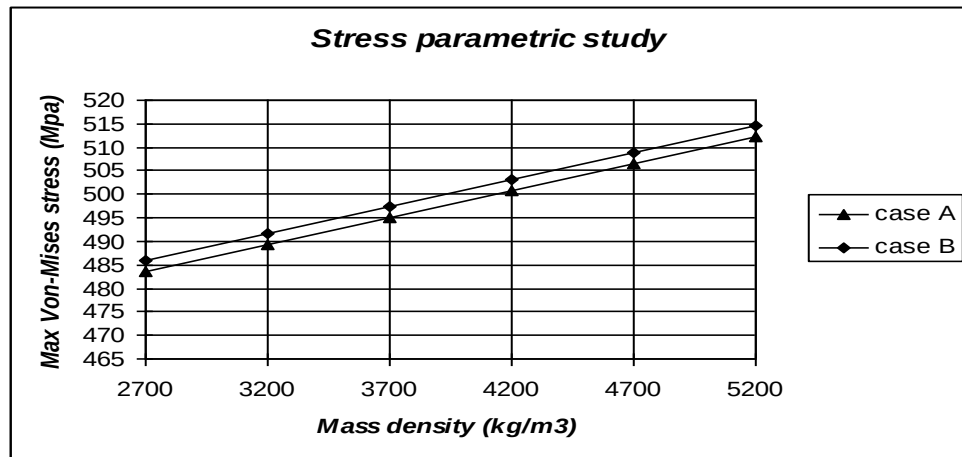


Figure (5.14b) The effect of mass density on Max Von-Mises stress
Case A: $E = 70 \text{ Gpa}, \nu = 0.33$ case B: $E = 130 \text{ Gpa}, \nu = 0.34$

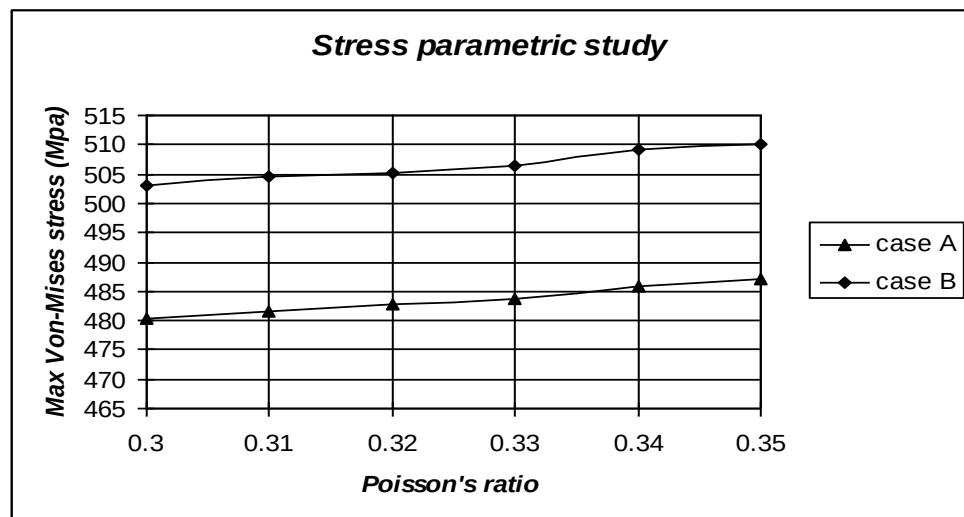


Figure (5.14c) The effect of poisson's ratio on Max Von-Mises stress
Case A: $E = 70 \text{ Gpa}, \rho = 2700 \text{ kg/m}^3$ case B: $E = 130 \text{ Gpa}, \rho = 4700 \text{ kg/m}^3$

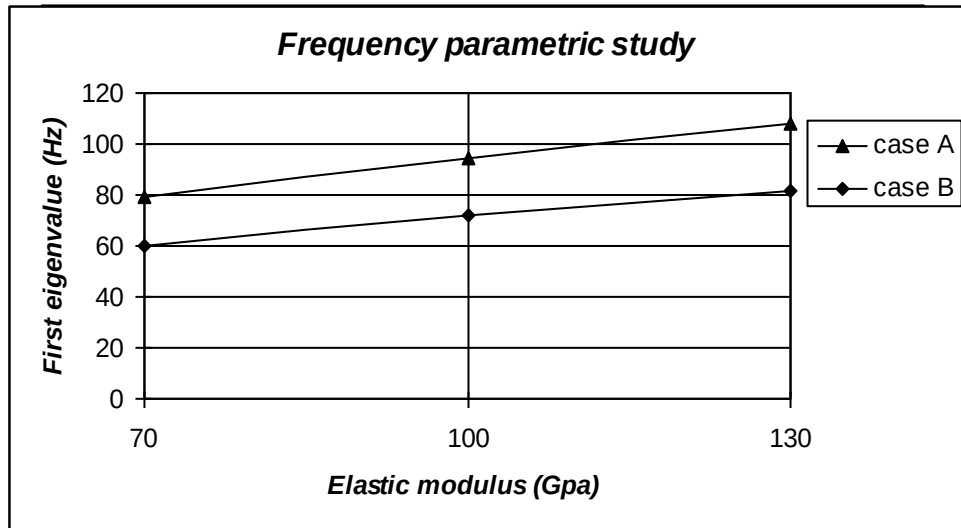


Figure (5.15a) The effect of elastic modulus on first mode natural frequency
 Case A: $\rho = 2700 \text{ kg/m}^3, \nu = 0.33$ case B: $\rho = 4700 \text{ kg/m}^3, \nu = 0.34$

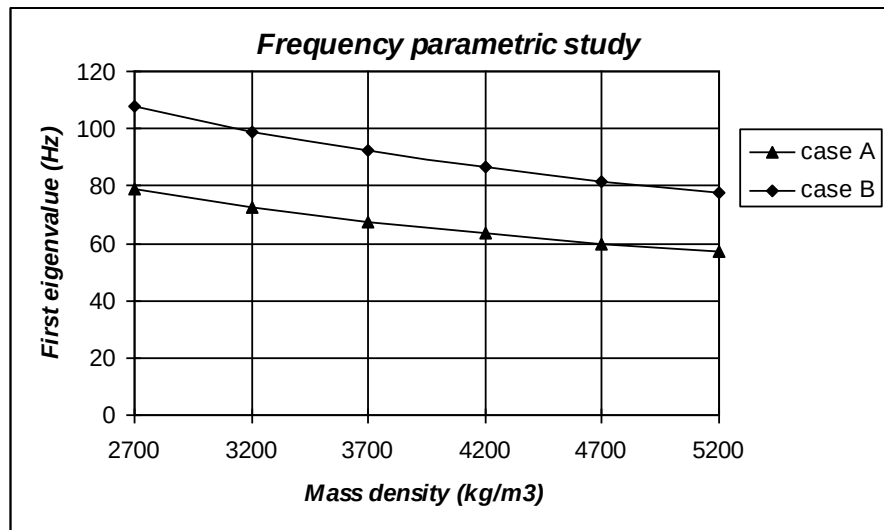


Figure (5.15b) The effect of mass density on first mode natural frequency
 Case A: $E = 70 \text{ Gpa}, \nu = 0.33$ case B: $E = 130 \text{ Gpa}, \nu = 0.34$

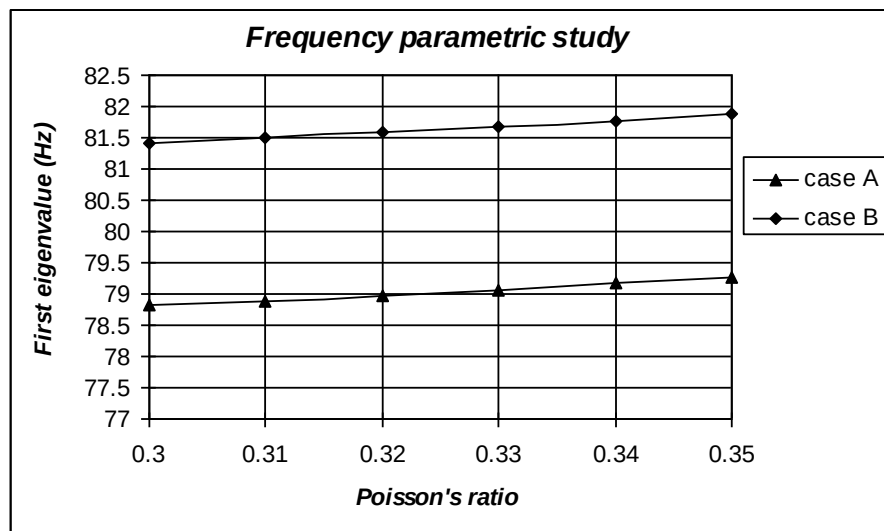


Figure (5.15c) The effect of poisson's ratio on first mode natural frequency
 Case A: $E = 70 \text{ Gpa}, \rho = 2700 \text{ kg/m}^3$ case B: $E = 130 \text{ Gpa}, \rho = 4700 \text{ kg/m}^3$

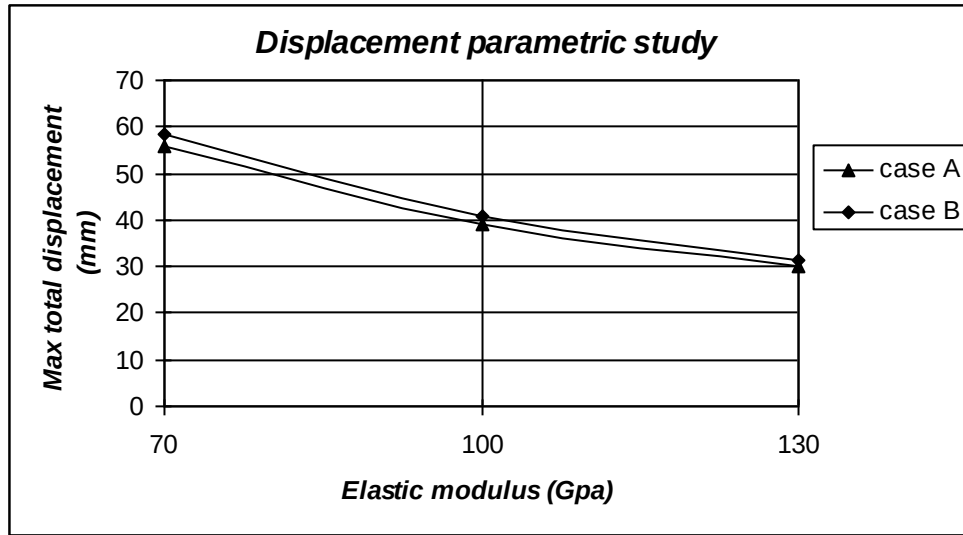


Figure (5.16a) The effect of elastic modulus on Max total displacement
 Case A: $\rho = 2700 \text{ kg/m}^3, \nu = 0.33$ case B: $\rho = 4700 \text{ kg/m}^3, \nu = 0.34$

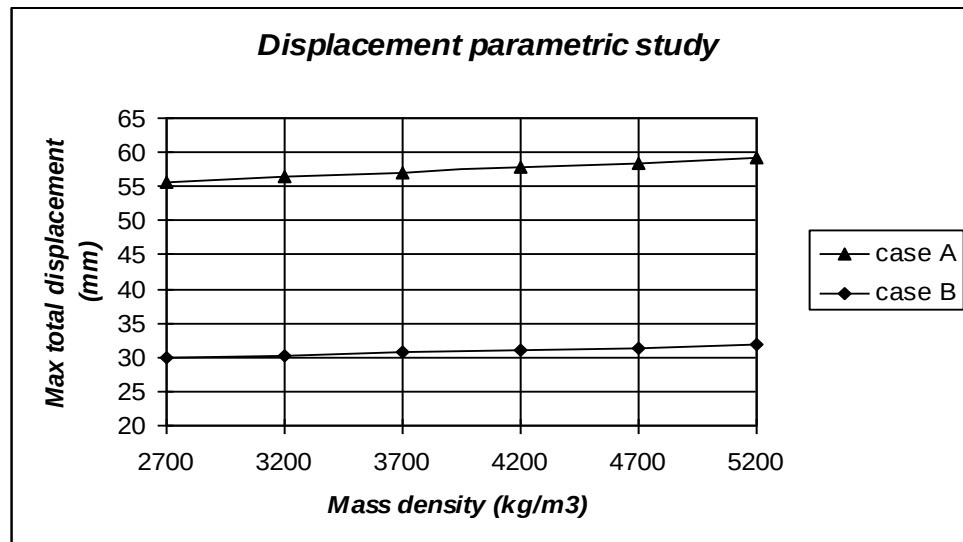


Figure (5.16b) The effect of mass density on Max total displacement
 Case A: $E = 70 \text{ Gpa}, \nu = 0.33$ case B: $E = 130 \text{ Gpa}, \nu = 0.34$

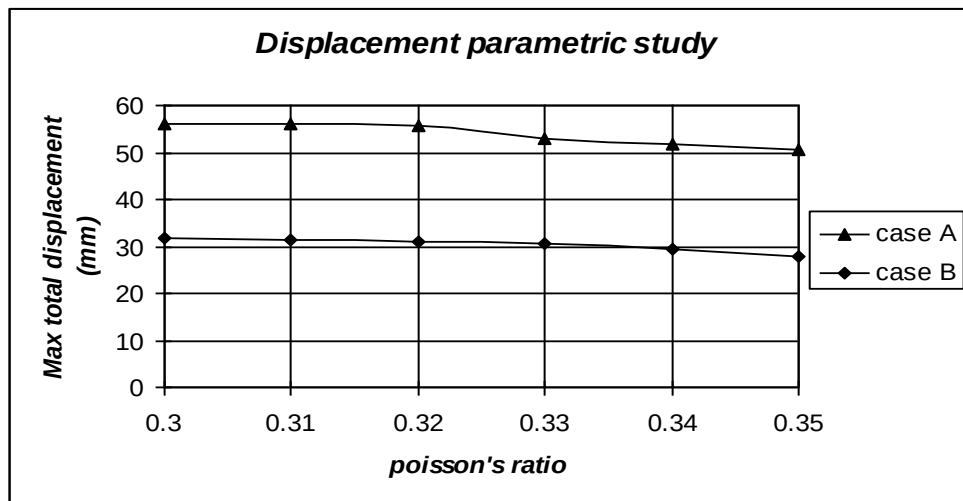


Figure (5.16c) The effect of poisson's ratio on Max total displacement
 Case A: $E = 70 \text{ Gpa}, \rho = 2700 \text{ kg/m}^3$ case B: $E = 130 \text{ Gpa}, \rho = 4700 \text{ kg/m}^3$

List of Notations

<i>English Symbols</i>	
Symbol	Meaning
Al_n	<i>Al-alloy with number n (Appendix-D-)</i>
c	<i>Plate element center</i>
C	<i>Curvature center for curved path</i>
d	<i>Differential operator</i>
e	<i>Element</i>
$\bar{e}$	<i>Unit vector for cylindrical and spherical coordinates</i>
G	<i>Gravity center of GTX-Aircraft</i>
G_{xyz}	<i>Local reference frame of GTX-Aircraft</i>
$G_{x'y'z'}$	<i>Precession reference frame of GTX-Aircraft</i>
$G_{x''y''z''}$	<i>Spinning reference frame of GTX-Aircraft</i>
$\bar{i}, \bar{j}, \bar{k}$	<i>Unit vectors for rectangular coordinates</i>
l, m, n	<i>Direction cosines w.r.t x, y, z-axes respectively</i>
ne	<i>Number of elements that bound the loading point</i>
n_{load}	<i>Number of loading points for element (e)</i>
$NMAT, NMAT^{upper}$	<i>Actual, Allowable number of constituent structural materials</i>
O	<i>Arbitrary origin for reference axes frame</i>
$OXYZ$	<i>Inertial global axes frame</i>
Sn	<i>Stiffener with number n</i>
ss	<i>Steady state number for optimization process</i>
Tin	<i>Ti-alloy with number n (Appendix-D-)</i>

English Terminologies		
Terminology	Meaning	Unit
$a, \bar{a}$	Scalar, vector representation for acceleration vector	m/s^2
b	Width of plate element	m
C°	Equality constraint function	---
E	Elastic modulus for isotropic material	N/m^2
E/ρ	Specific stiffness for isotropic material	J/kg
$F, \bar{F}$	Scalar, vector representation for force vector	N
$\bar{F}(t)$	Transient load vector (due to inertia action)	N
F°	Objective function	---
$\bar{F}', \bar{F}''$	Translational, rotational force vectors	N
g	Gravity acceleration constant	m/s^2
h	Height of the plate element	m
$\bar{H}$	Angular Momentum vector	$kg.m^2/s$
I_{ii}	Moment of inertia about i-axis (Appendix-A-)	$kg.m^2$
I_{ij}	Product of inertia w.r.t (i-j) plane (Appendix-A-)	$kg.m^2$
$L, \bar{L}$	Scalar, vector representation for linear momentum vector	$kg.m/s$
$M, \bar{M}$	Scalar, vector representation for moment vector	$N.m$
N_i	Safety factor for constraint (i)	---
$\bar{N}_i$	Direction of vector quantity (i)	---
Q_i	First moment of mass about i-axis	$kg.m$
$R, \bar{R}$	Scalar, vector representation for radial position vector (spherical coordinates)	m
$r, \bar{r}$	Scalar, vector representation for radial position vector (cylindrical coordinates)	m
$s, \bar{s}$	Scalar, vector representation for position vector (rectangular coordinates)	m
t	Time	sec
τ	Intermediate time	sec
$U^{\max}, U^{upper}$	Max, allowable value for absolute value of structural displacement	m
$u, \bar{u}$	Scalar, vector representation for translational elastic deformations vector	m
$v, \bar{v}$	Scalar, vector representation for velocity vector (rectangular coordinates)	m/s

Greek Symbols	
Symbol	Meaning
Δ	<i>Finite difference for any quantity</i>
$\dot{\vec{\delta}}$	<i>First time derivative for any vector $\vec{\delta}$</i>
$\ddot{\vec{\delta}}$	<i>Second time derivative for any vector $\vec{\delta}$</i>

Greek Terminologies		
Terminology	Meaning	Unit
$\vec{\ddot{\psi}}, \vec{\dot{\psi}}$	<i>Frame rotational acceleration, velocity (precession) of GTX-Aircraft about G</i>	<i>rad/s², rad/s</i>
$\vec{\ddot{\omega}}, \vec{\dot{\omega}}$	<i>Angular acceleration, velocity vectors for rotating body (precession & spinning) about G</i>	<i>rad/s², rad/s</i>
$\omega_n, \omega^{\text{lower}}$	<i>Nominal, allowable(lower) values of natural frequency</i>	<i>rad/s</i>
$\vec{\ddot{\beta}}, \vec{\dot{\beta}}$	<i>Orbital acceleration, velocity of GTX-Aircraft about curvature center of curved path (C)</i>	<i>rad/s², rad/s</i>
ϕ	<i>Spherical coordinates (angular position)</i>	<i>rad</i>
$\phi_{\circ}$	<i>Inequality constraint function</i>	<i>---</i>
ν	<i>Poisson's ratio of isotropic material</i>	<i>---</i>
θ	<i>Cylindrical coordinates (angular position)</i>	<i>rad</i>
$\vec{\ddot{\vartheta}}, \vec{\dot{\vartheta}}$	<i>Body rotational acceleration, velocity (spinning) of GTX-Aircraft about G</i>	<i>rad/s², rad/s</i>
ρ	<i>Mass density for isotropic material</i>	<i>kg/m³</i>
$\sigma_1, \sigma_2, \sigma_3$	<i>Principal stresses in planes 1, 2, 3 respectively</i>	<i>N/m²</i>
σ_{VM}^{max}	<i>Maximum value of Von-Mises stress</i>	<i>N/m²</i>
σ_{yp}	<i>Yield stress of isotropic material in tension</i>	<i>N/m²</i>
(σ_{yp} / ρ)	<i>Specific strength for isotropic material</i>	<i>J/kg</i>
τ_{yp}	<i>Yield stress of isotropic material in shear</i>	<i>N/m²</i>

Matrix & Vector Definitions	
Symbol	Definition
$[\]$	<i>Matrix form</i>
$[\]^T$	<i>Transpose of matrix</i>
$proj_{\vec{b}}^{\vec{a}}$	<i>Vector projection of $\vec{a}$ on $\vec{b}$</i>
$\vec{a} \cdot \vec{b}$	<i>Dot product of $\vec{a}$ with $\vec{b}$</i>
$\vec{a} \times \vec{b}$	<i>Cross product of $\vec{a}$ with $\vec{b}$</i>
$[M]$	<i>Structural mass matrix</i>
$[C]$	<i>Structural damping matrix</i>
$[K]$	<i>Structural stiffness matrix</i>
$[A], [B]$	<i>Transformation (rotation) matrices for rotational motion of GTX-Aircraft (precession & spinning respectively)</i>
$[R']$	<i>Rotation matrix for inertia of inclined stiffeners</i>

Subscripts	
Subscript	Meaning
0	<i>Initial value of variable</i>
C	<i>Center of curvature for curved path (particle motion)</i>
E	<i>Element center</i>
element inertia	<i>Structural mass (elements) inertia as combined motion</i>
G	<i>Gravity center of whole body (GTX-Aircraft)</i>
i/j	<i>Description of the vector at point (i) with respect to point (j)</i>
inertia	<i>Inertia action of moving body</i>
n, t	<i>Normal-tangent coordinates</i>
O	<i>Arbitrary origin of reference frame</i>
payload inertia	<i>Payload inertia as combined motion</i>
R, θ, ϕ	<i>Spherical coordinates</i>
r, θ, z	<i>Cylindrical coordinates</i>
x, y, z	<i>Rectangular coordinates</i>
X, Y, Z	<i>Description of vector quantity w.r.t frame Gxyz (as rigid body) which their axes are parallel to the axes of inertial reference Frame OXYZ</i>
$x'y'z'$	<i>Description of vector quantity w.r.t precession frame G x'y'z' which their axes are rotating about point G (as rigid body)</i>
$x''y''z''$	<i>Description of vector quantity w.r.t spinning frame G x''y''z'' which their axes are rotating about point G (as rigid body)</i>

Appendix-A-

Moments & Products of Inertia

A.1 Mass Center (Gravity Center): The mass center concept was completely defined in *section (3.3.2.1)*. For thin plate (assume its thickness along z-direction), the mass center of such parts represents the center point of the area formed by plate element in x-y plane. The center of gravity coordinates (location) with respect to any reference frame (**OXYZ**) is related to the first moment of mass and this relation can be defined in the following form [Refs. 21, 23, 26, 48, & 70]:

$$\begin{bmatrix} x_G \\ y_G \\ z_G \end{bmatrix} = \begin{bmatrix} \frac{\sum_{e=1}^n m^e * x^e}{\sum_{e=1}^n m^e} \\ \frac{\sum_{e=1}^n m^e * y^e}{\sum_{e=1}^n m^e} \\ \frac{\sum_{e=1}^n m^e * z^e}{\sum_{e=1}^n m^e} \end{bmatrix} = \begin{bmatrix} \frac{\sum_{e=1}^n Q_x^e}{m} \\ \frac{\sum_{e=1}^n Q_y^e}{m} \\ \frac{\sum_{e=1}^n Q_z^e}{m} \end{bmatrix} = \begin{bmatrix} \frac{Q_x}{m} \\ \frac{Q_y}{m} \\ \frac{Q_z}{m} \end{bmatrix} \dots\dots\dots (A.1)$$

Q_x, Q_y, Q_z are the first moment of mass of the body with respect to the fixed frame (**OXYZ**) about X-, Y-, Z-axes respectively and (n) is the number of elements in the structure, the superscript (e) denotes to the series of the structural elements that form the whole structure. Where x_G, y_G, z_G denote to the gravity center coordinates of the considered body, while x^e, y^e, z^e denote to the center coordinates of the constituent element (e) and m^e is the mass of element (e) .

A.2 Moments & Products of Inertia: The moments and products of inertia of plate element deal with two-dimensional (plane geometry) for small thicknesses, so in general, the moments and products of inertia of any element

(e) of mass (m^e) with respect to the reference frame (**Gxyz**) where point (**G**) is not necessarily the gravity center of whole structure is defined as in **Eq. (A.2)**. The sum of moments (or products) of inertia for all elements (whole structure) is given by **Eq. (A.3)** (**Meriam, 2002**) ⁽⁴⁰⁾, (**Beer & Johnston, 1972**) ⁽⁶⁾.

$$\begin{bmatrix} I_{xx}^e \\ I_{yy}^e \\ I_{zz}^e \\ I_{xy}^e \\ I_{yz}^e \\ I_{zx}^e \end{bmatrix} = \begin{bmatrix} \iiint_{m^e} (y^2 + z^2) * dm \\ \iiint_{m^e} (z^2 + x^2) * dm \\ \iiint_{m^e} (x^2 + y^2) * dm \\ \iiint_{m^e} (x * y) * dm \\ \iiint_{m^e} (y * z) * dm \\ \iiint_{m^e} (z * x) * dm \end{bmatrix} = \begin{bmatrix} \rho^e * \iiint_{vol^e} (y^2 + z^2) * dV \\ \rho^e * \iiint_{vol^e} (z^2 + x^2) * dV \\ \rho^e * \iiint_{vol^e} (x^2 + y^2) * dV \\ \rho^e * \iiint_{vol^e} (x * y) * dV \\ \rho^e * \iiint_{vol^e} (y * z) * dV \\ \rho^e * \iiint_{vol^e} (z * x) * dV \end{bmatrix} = \begin{bmatrix} m^e * [(y^e)^2 + (z^e)^2] \\ m^e * [(z^e)^2 + (x^e)^2] \\ m^e * [(x^e)^2 + (y^e)^2] \\ m^e * [x^e * y^e] \\ m^e * [y^e * z^e] \\ m^e * [z^e * x^e] \end{bmatrix} \dots\dots\dots (A.2)$$

$$\begin{bmatrix} I_{xx}^s \\ I_{yy}^s \\ I_{zz}^s \\ I_{xy}^s \\ I_{yz}^s \\ I_{zx}^s \end{bmatrix} = \begin{bmatrix} \sum_{e=1}^n \rho^e * \iiint_{vol^e} (y^2 + z^2) * dV \\ \sum_{e=1}^n \rho^e * \iiint_{vol^e} (z^2 + x^2) * dV \\ \sum_{e=1}^n \rho^e * \iiint_{vol^e} (x^2 + y^2) * dV \\ \sum_{e=1}^n \rho^e * \iiint_{vol^e} (x * y) * dV \\ \sum_{e=1}^n \rho^e * \iiint_{vol^e} (y * z) * dV \\ \sum_{e=1}^n \rho^e * \iiint_{vol^e} (z * x) * dV \end{bmatrix} = \begin{bmatrix} \sum_{e=1}^n m^e * [(y^e)^2 + (z^e)^2] \\ \sum_{e=1}^n m^e * [(z^e)^2 + (x^e)^2] \\ \sum_{e=1}^n m^e * [(x^e)^2 + (y^e)^2] \\ \sum_{e=1}^n m^e * [x^e * y^e] \\ \sum_{e=1}^n m^e * [y^e * z^e] \\ \sum_{e=1}^n m^e * [z^e * x^e] \end{bmatrix} \dots\dots\dots (A.3)$$

Where (n) is the number of constituent elements. The superscript (s) denotes to whole structure.

The exact relationship found between the mass moments of inertia and area moments of inertia for flat plates (small thickness). The moments (products) of inertia about x-, y-, z-axes are equal to the mass per unit area (in x-y plane) (ρt) times the area moments (products) of inertia as shown below:

$$\begin{bmatrix} I_{xx}^{mass} \\ I_{yy}^{mass} \\ I_{zz}^{mass} \\ I_{xy}^{mass} \\ I_{yz}^{mass} \\ I_{zx}^{mass} \end{bmatrix} = \begin{bmatrix} \rho t * I_{xx}^{area} \\ \rho t * I_{yy}^{area} \\ \rho t * I_{zz}^{area} \\ \rho t * I_{xy}^{area} \\ \rho t * I_{yz}^{area} \\ \rho t * I_{zx}^{area} \end{bmatrix} \dots\dots\dots (A.4)$$

It is noted that these moments and products of inertia were taken with respect to centralized reference axes (the frame **cxzy**) and they can be translated or rotated with respect to any reference frame in space by using **Eqs. (3.12) & (3.13)** for translation and rotation respectively.

In case of composite body, the mass moment of inertia of that body represents the sum of moments and products of inertia of individual parts about the same axis (moments) or in the same plane (products).

A.3 Translation of Inertia: The moments and products of inertia are often simplified by using parallel axes theorem, which states, "***the moment of inertia about any arbitrary axis parallel to other axis is equal to moment of inertia about that axis plus the mass of the body multiplied by the square of orthogonal distance between two axes***" (Meriam, 2002)⁽⁴⁰⁾.

$$\begin{bmatrix} I_{x1x1}^{mass} \\ I_{y1y1}^{mass} \\ I_{z1z1}^{mass} \\ I_{x1y1}^{mass} \\ I_{y1z1}^{mass} \\ I_{z1x1}^{mass} \end{bmatrix} = \begin{bmatrix} I_{xx}^{mass} \\ I_{yy}^{mass} \\ I_{zz}^{mass} \\ I_{xy}^{mass} \\ I_{yz}^{mass} \\ I_{zx}^{mass} \end{bmatrix} + \begin{bmatrix} \rho t b h * \left\langle (\overline{xx1})_{y-axis}^2 + (\overline{xx1})_{z-axis}^2 \right\rangle \\ \rho t b h * \left\langle (\overline{yy1})_{z-axis}^2 + (\overline{yy1})_{x-axis}^2 \right\rangle \\ \rho t b h * \left\langle (\overline{zz1})_{x-axis}^2 + (\overline{zz1})_{y-axis}^2 \right\rangle \\ \rho t b h * \left\langle (\overline{yy1})_{x-axis} * (\overline{xx1})_{y-axis} \right\rangle \\ \rho t b h * \left\langle (\overline{zz1})_{y-axis} * (\overline{yy1})_{z-axis} \right\rangle \\ \rho t b h * \left\langle (\overline{xx1})_{z-axis} * (\overline{zz1})_{x-axis} \right\rangle \end{bmatrix} \dots\dots\dots (A.5)$$

$(\overline{ii1})_{j-axis}$ Is the orthogonal distance between $i - axis$ and $i1 - axis$ along $j - axis$.

In which $(\overline{zz1})_{x-axis}, (\overline{zz1})_{y-axis}$ are not equal to zero as in area moments of inertia where plate element is allowed to move in space (not lying in x-y plane) as illustrated by **Figure (A.3)**.

A.4 Rotation of Inertia: The mass moments or products of inertia of any mass with respect to the frame $(cx1y1z1)$ which have general rotation with respect to reference frame $(cxyz)$ can be expressed in the **Eq. (A.6)** where rotation matrix $[R']$ is used as follows:

$$\begin{bmatrix} I_{x1x1}^{mass} \\ I_{y1y1}^{mass} \\ I_{z1z1}^{mass} \\ I_{x1y1}^{mass} \\ I_{y1z1}^{mass} \\ I_{z1x1}^{mass} \end{bmatrix} = [R']^* \begin{bmatrix} I_{xx}^{mass} \\ I_{yy}^{mass} \\ I_{zz}^{mass} \\ I_{xy}^{mass} \\ I_{yz}^{mass} \\ I_{zx}^{mass} \end{bmatrix} = [R']^* \rho t^* \begin{bmatrix} I_{xx}^{area} \\ I_{yy}^{area} \\ I_{zz}^{area} \\ I_{xy}^{area} \\ I_{yz}^{area} \\ I_{zx}^{area} \end{bmatrix} \dots\dots\dots (A.6)$$

Where $[R']$ is $[6 \times 6]$ matrix as it defined by **Eq. (3.13)** as shown in **Fig (A.4)**.

A.5 Inertia Properties of Some Regular Shapes:

A.5.1 Rectangular Plates: The moments or products of inertia for thin plates (regardless their shapes) can be related directly to the areas moments and products of inertia (**Ahmed, 2003**)⁽³⁾:

$$\begin{aligned} I_{xx}^{mass} &= \rho \int_{-t/2}^{t/2} \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} (y^2 + z^2) dx dy dz \\ &= \rho b t^* \left[\frac{h^3}{24} + \frac{h^3}{24} \right] = \rho t^* \left[\frac{b h^3}{12} \right] = \rho t^* I_{xx}^{area} \dots\dots\dots (A.7) \end{aligned}$$

$$\begin{aligned} I_{yy}^{mass} &= \rho \int_{-t/2}^{t/2} \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} (z^2 + x^2) dx dy dz \\ &= \rho h t^* \left[\frac{b^3}{24} + \frac{b^3}{24} \right] = \rho t^* \left[\frac{b^3 h}{12} \right] = \rho t^* I_{yy}^{area} \dots\dots\dots (A.8) \end{aligned}$$

$$\begin{aligned} I_{zz}^{mass} &= \rho \int_{-t/2}^{t/2} \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} (x^2 + y^2) dx dy dz \\ &= \rho b h t^* \left[\frac{h^2}{24} + \frac{h^2}{24} + \frac{b^2}{24} + \frac{b^2}{24} \right] = \rho t^* \left[\frac{b h^3}{12} + \frac{b^3 h}{12} \right] = \rho t^* I_{zz}^{area} \\ &\dots\dots\dots (A.9) \end{aligned}$$

$$\begin{aligned}
 I_{xy}^{mass} &= \rho \int_{-t/2}^{t/2} \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} (xy) \, dx dy dz \\
 &= \rho t * \left[\left[+\frac{b^2}{8} - \frac{b^2}{8} \right] * \left[+\frac{h^2}{8} - \frac{h^2}{8} \right] \right] = \rho t * [0] = \rho t * I_{xy}^{area} = 0 \quad \dots (A.10)
 \end{aligned}$$

$$\begin{aligned}
 I_{yz}^{mass} &= \rho \int_{-t/2}^{t/2} \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} (yz) \, dx dy dz \quad [z = 0(\text{mid-plane surface})] \\
 &= \rho t * [0] = \rho t * I_{yz}^{area} = 0 \\
 &\dots\dots\dots (A.11)
 \end{aligned}$$

$$\begin{aligned}
 I_{zx}^{mass} &= \rho \int_{-t/2}^{t/2} \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} (zx) \, dx dy dz \quad [z = 0(\text{mid-plane surface})] \\
 &= \rho t * [0] = \rho t * I_{zx}^{area} = 0 \\
 &\dots\dots\dots (A.12)
 \end{aligned}$$

As shown by **Figure (A.1)**.

A.5.2 Triangular Plates: The moments & products of inertia for thin triangular plates can be related directly to the areas moments and products of inertia (when plate element is lying in x-y plane) **Ruda, et al, 2001**)⁽⁵⁷⁾:

$$\begin{aligned}
 I_{xx}^{mass} &= \rho \int_{-t/2}^{t/2} \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} (y^2 + z^2) \, dx dy dz \\
 &= \rho b t * \left[\frac{h^3}{72} + \frac{h^3}{72} \right] = \rho t * \left[\frac{b h^3}{36} \right] = \rho t * I_{xx}^{area} \dots\dots\dots (A.13)
 \end{aligned}$$

$$\begin{aligned}
 I_{yy}^{mass} &= \rho \int_{-t/2}^{t/2} \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} (z^2 + x^2) \, dx dy dz \\
 &= \rho h t * \left[\frac{b^3}{72} + \frac{b^3}{72} \right] = \rho t * \left[\frac{b^3 h}{36} \right] = \rho t * I_{yy}^{area} \dots\dots\dots (A.14)
 \end{aligned}$$

$$\begin{aligned}
 I_{zz}^{mass} &= \rho \int_{-t/2}^{t/2} \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} (x^2 + y^2) \, dx dy dz \\
 &= \rho b h t * \left[\frac{h^2}{72} + \frac{h^2}{72} + \frac{b^2}{72} + \frac{b^2}{72} \right] \\
 &= \rho t * \left[\frac{b h^3}{36} + \frac{b^3 h}{36} \right] = \rho t * I_{zz}^{area} \dots\dots\dots (A.15)
 \end{aligned}$$

$$\begin{aligned}
 I_{xy}^{mass} &= \rho \int_{-t/2}^{t/2} \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} (xy) \, dx dy dz \\
 &= \rho t * \left[\left[+ \frac{b^2}{2} \right] * \left[+ \frac{-64h^2 + 48h^2 - 8h^2 - 3h^2}{36 * 27} \right] \right] \\
 &= \rho t * \left[\frac{b^2 h^2}{72} \right] = \rho t * I_{xy}^{area} \dots\dots\dots (A.16)
 \end{aligned}$$

$$\begin{aligned}
 I_{yz}^{mass} &= \rho \int_{-t/2}^{t/2} \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} (yz) \, dx dy dz \quad [z = 0(\text{mid-plane surface})] \\
 &= \rho t * [0] = \rho t * I_{yz}^{area} = 0 \dots\dots\dots (A.17)
 \end{aligned}$$

$$\begin{aligned}
 I_{zx}^{mass} &= \rho \int_{-t/2}^{t/2} \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} (zx) \, dx dy dz \quad [z = 0(\text{mid-plane surface})] \\
 &= \rho t * [0] = \rho t * I_{zx}^{area} = 0 \dots\dots\dots (A.18)
 \end{aligned}$$

As shown by **Figure (A.2)**.

For composite area, the mass moments and products of inertia can be found by dividing the structure to simple geometric shapes (rectangles, triangles, etc.).

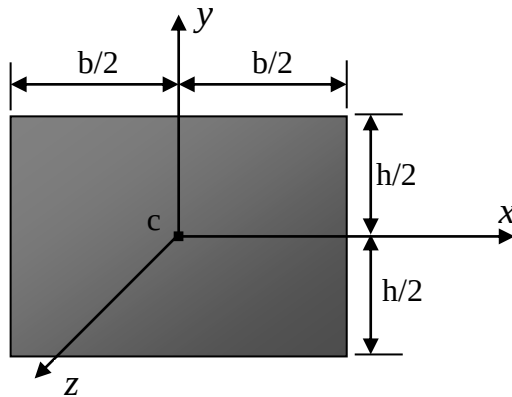


Figure (A.1) Rectangular plate

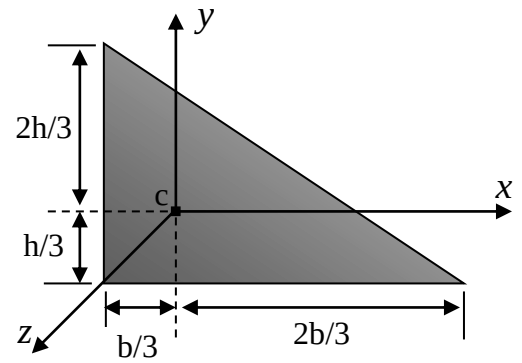


Figure (A.2) Triangular plate

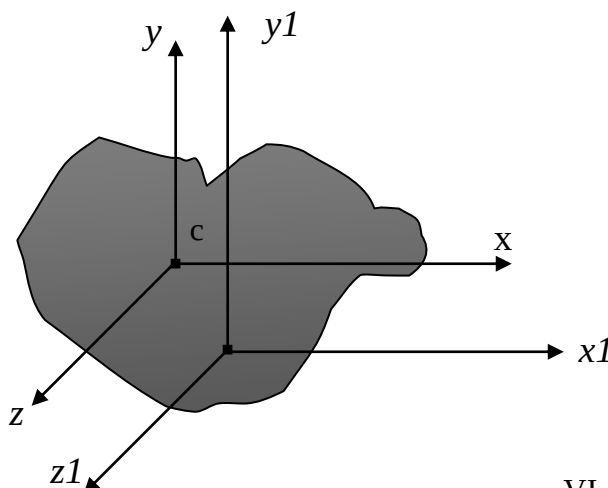


Figure (A.3) Inertia translation

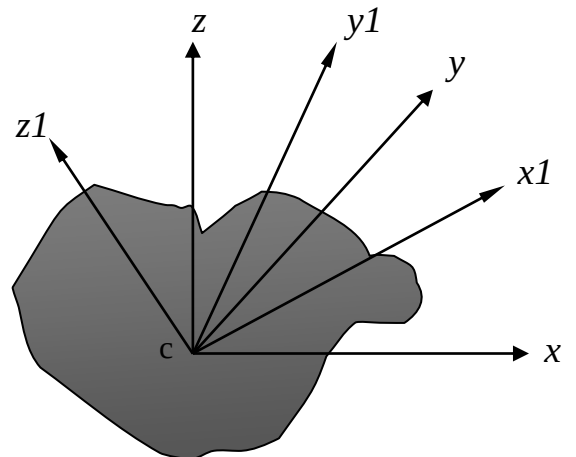


Figure (A.4) Inertia rotation

Appendix-B-

Strength Failure Criterion

B.1 Strength Failure Criterion: It is necessary that the structure in addition to being low weight must meet safety and functional requirements (Lyengar⁽³⁸⁾, 1999). The allowable stress limits for the various failure modes are available from experimental data (Ding, 1987)⁽¹¹⁾. Theoretical analyses of such problems are generally less accurate due to the complexity of the problems. The formulation of stress failure criterion for each material in simple form can be given as shown below:

$$\sigma_{ypi} - \sigma_{VMi}^{\max} \geq 0 \quad \dots\dots\dots (B.1)$$

$\sigma_{VMi}^{\max}$, σ_{ypi} are the maximum Von-Mises stress (Eq. (B.7)) and yield stress for i^{th} material respectively.

B.1.1 Failure Theories: Always the parts fail because their stresses exceed their strength and the failure kind depends on the material nature (its relative strength), the character of loading and the presence of cracks in the material. In general, ductile materials in tensile loading are limited by their shear strengths while brittle material is limited by their tensile strengths. This situation requires different failure theories for failure estimation of the two classes of materials (Norton, 1998)⁽⁷⁰⁾. For this section the failure theories for the ductile materials will be considered which represent an excellent estimation for the selected alloys.

B.1.1.1 Von-Mises Hencky Theory: Von-Mises theory assumes there is relative sliding occurring between the material atoms within their lattice structure. The sliding is caused by shear stress and is accompanied by the distortion of the shape of the part. The energy stored in the part from this distortion is an indicator of shear stress present. According to **Huber-Mises-Hencky** hypothesis, plastic yielding begins when the strain energy of distortion

per unit volume reaches a critical value characteristic for a given material. By assuming linear variation between stress and strain up to yield point, the total strain energy at any point in that range can be written as (**Lagaros & Papadrak, 2004⁽³³⁾, Haider, 2005⁽²⁰⁾, & Norton, 1998⁽⁴⁵⁾**):

$$PE = \frac{1}{2} * \sigma * \varepsilon = \frac{1}{2} * (\sigma_1 * \varepsilon_1 + \sigma_2 * \varepsilon_2 + \sigma_3 * \varepsilon_3) \quad \dots\dots\dots (B.2)$$

Where the subscript number refers to the number of plane in which zero shear stress can be obtained (often called "**principal planes**").

Von-Mises or distortion-energy theory is concerned mainly with predicting the beginning of yielding. However, in accordance with this criterion, yielding will occur when the strain energy of distortion per unit volume equals the strain energy of distortion per unit volume for specimen in uni-axial tension or compression (strained to the yield stress).

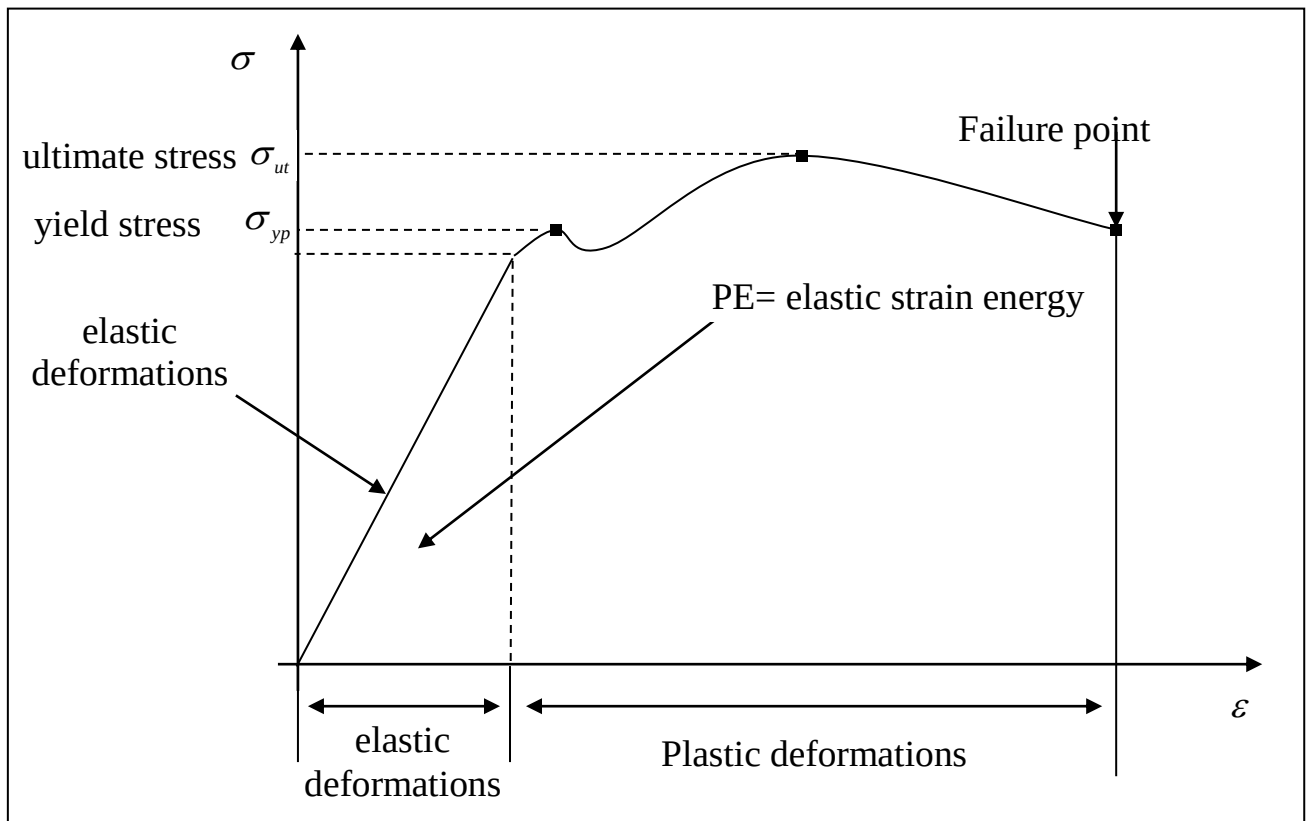


Figure (B.1) stress-strain curve

The strain energy of distortion per unit volume PE for a body subjected to a tri-axial state of stress can be expressed in terms of body principal stresses only as follows (*Norton, 1998*)⁽⁴⁵⁾:

$$PE = \frac{1+\nu}{6E} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \quad \dots\dots\dots (B.3)$$

Yielding would occur when:

$$\frac{1+\nu}{6E} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = \frac{1+\nu}{6E} * 2\sigma_{yp}^2 \quad \dots\dots\dots (B.4)$$

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_{yp}^2 \quad \dots\dots\dots (B.5)$$

For bi-axial-stress state ($\sigma_3=0$), the distortion-energy criterion for yielding can be expressed by the following formula:

$$\sigma_1^2 - \sigma_1\sigma_2 - \sigma_2^2 = \sigma_{yp}^2 \quad \dots\dots\dots (B.6)$$

This equation is that of an ellipse whose major semi-axis is $\sqrt{2} \sigma_{yp}$ and minor semi-axis is $\sqrt{\frac{2}{3}} \sigma_{yp}$ (*Zabarar, 2001*)⁽⁶⁸⁾.

Experiments have shown that the distortion-energy theory is in better agreement with data from the yielding of bodies under combined stress. In general, the distortion-energy theory is recommended for defining yielding of ductile materials; however, the maximum-shear theory is a good approximation. In applying the distortion-energy theory for design purpose, a criteria value is extracted from *Eq. (4.4a)* to be compared with tensile yield stress of the material. This criteria value is called Von-Misses stress σ_{VM} :

$$\sigma_{VM} = \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad \dots\dots\dots (B.7)$$

B.1.1.2 Maximum-Shear-Stress Theory (Tresca-Guest Theory): According to Tresca-Guest hypothesis, plastic yielding of a

material occurs when the largest shear stress $[\tau^{\max} = \frac{(\sigma_1 - \sigma_2)}{2}]$ reaches a critical value characteristic for that material. The maximum-shear-stress theory states that "**failure occurs when the maximum shear stress in a part exceeds the shear stress in a tensile specimen at yield (approximately one-half of the tensile yield strength)**". This theory assumes that failure (yielding or fracture) occurs for a combined stress condition when the maximum shear stress equals the value of a critical shear stress (yield shear stress or ultimate shear stress) produced in an element subjected to simple tension. For a three- dimensional stress state, the maximum shear stresses are given by one of the following expressions **(Mohammed, 2005)⁽⁴²⁾**:

$$\frac{\sigma_1 - \sigma_2}{2}, \quad \frac{\sigma_2 - \sigma_3}{2}, \quad \frac{\sigma_3 - \sigma_1}{2}, \quad \dots \dots \dots \textbf{(B.8)}$$

Where $\sigma_1, \sigma_2, \sigma_3$ are the principal stresses in 1, 2, and 3 principal directions respectively. In the case of an element subjected to simple tension at the yield condition, it can be shown that:

$$\tau_{yp} = \frac{\sigma_{yp}}{2} \quad \dots \dots \dots \textbf{(B.9)}$$

Where σ_{yp}, τ_{yp} are the tensile and shear yield stresses respectively. In accordance with the maximum-shear-stress theory, yielding will occur where the largest shear stress in the element (given by one of the terms in **Eq. (B.8)**) reaches a value which is equal to the shear yield stress τ_{yp} in simple tension. If it is assumed that the shear stress at yield for simple tension is equal to that for simple compression, then failure occurs at one of the following cases **(Norton, 1998⁽⁴⁵⁾, & Zabaras, 2001⁽⁶⁸⁾)**:

$$\left. \begin{aligned} \sigma_1 - \sigma_2 &= \pm \sigma_{yp} \\ \sigma_2 - \sigma_3 &= \pm \sigma_{yp} \\ \sigma_3 - \sigma_1 &= \pm \sigma_{yp} \end{aligned} \right\} \quad \dots \dots \dots \textbf{(B.10)}$$

Where the $\pm$ sign implies that the maximum shear stress may be positive or negative. The maximum-shear- stress theory is widely used by designers for predicting failure of ductile materials by yielding (Zoltan, 2004)⁽⁷⁰⁾.

B.1.1.3 Maximum-Normal-Stress Theory: This theory is valid for brittle materials whose ultimate tensile strength is lower than their shear and compressive strengths. The maximum normal stress theory states that "*failure will occur when the normal stress in the specimen reaches some limit on normal strength such as tensile yield strength or ultimate tensile strength*". For ductile materials, yield strength is the desired criterion. According to **Figure (B.2)**, the normal stress theory envelope is well outside both the distortion-energy ellipse and its inscribed maximum shear stress hexagon. Since experiments show that ductile material fails when its stress state is outside the ellipse region, so the normal stress theory is unsafe failure criterion in the second and fourth quadrants (Norton, 1998)⁽⁴⁵⁾.

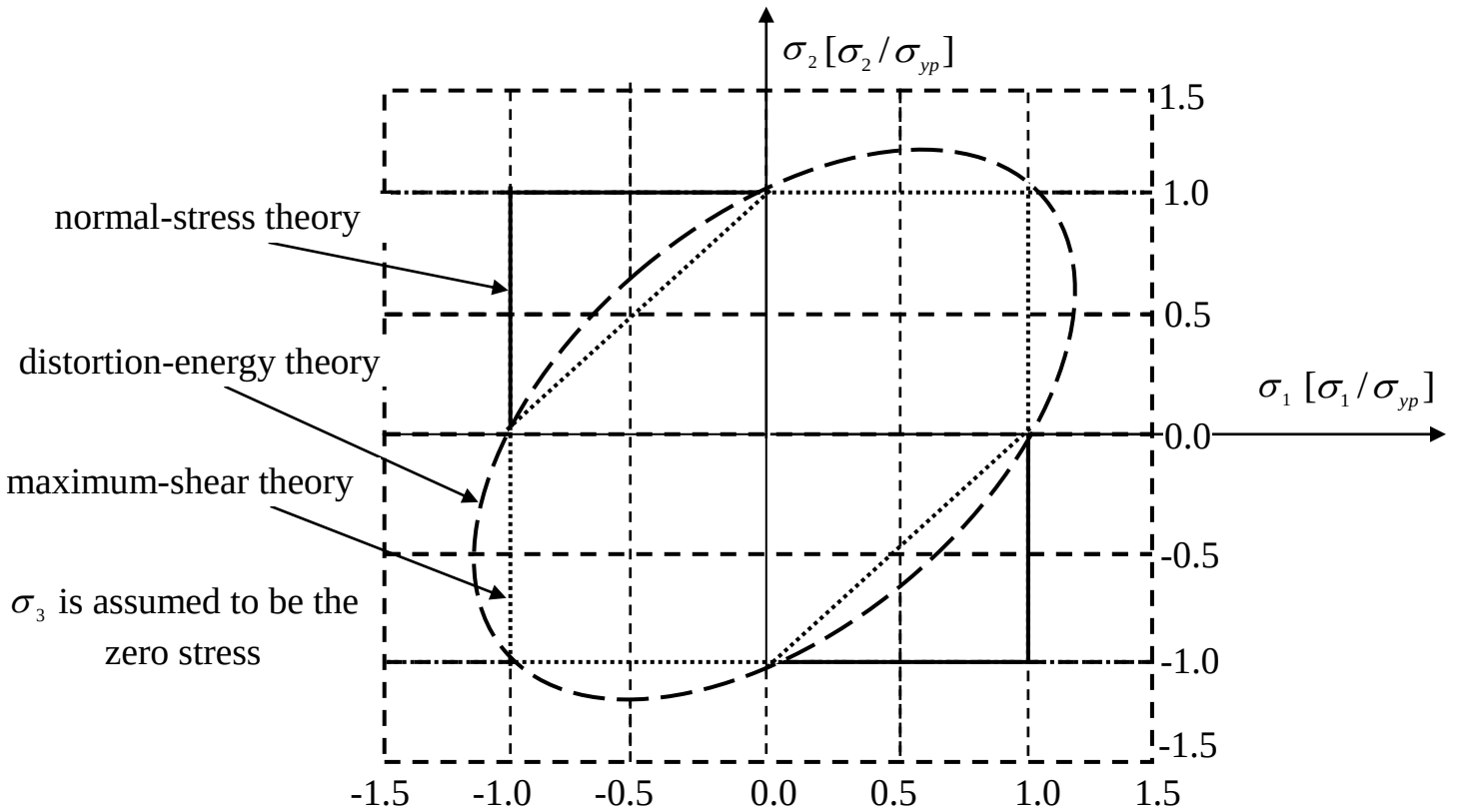


Figure (B.2) comparison of failure theories in strength failure criterion⁽⁴⁵⁾

Appendix-C-

Design Loads

In this appendix, the combined inertia forces applied to keypoints for each model in *ANSYS* program (the keypoints is illustrated in **Figure (4.5)**) were listed herein in details. These forces were estimated at critical time ($t=2.258895$ second) at which the maximum induced Von-Mises stress will be attained at the extracted period of time for the worst flight events of **GTX-Aircraft**. These combined forces for each model represent the design load conditions for that model at which the optimum design can be obtained by performing the adopted method of structural optimization.

The inertia forces applied to each keypoint in this model listed below as rectangular components (F_x , F_y , F_z) and these forces were detached from time-history of forces for each model which is the task of Kinetics program. **Table (C.1)** displays the combined inertia forces applied as keypoints loads to the *ANSYS* program for each of first, second, and third Models respectively. **Table (C.2)** displays the combined inertia forces applied as keypoints loads to the *ANSYS* program for each of fourth, fifth, and sixth Models respectively.

Table (C.1) Inertia forces for each keypoint in Models 1, 2, & 3

Keypoint No.	First Model			Second Model			Third Model		
	F_x (N)	F_y (N)	F_z (N)	F_x (N)	F_y (N)	F_z (N)	F_x (N)	F_y (N)	F_z (N)
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	-140.787	0.681	-105.322	-138.753	0.671	-103.801	-143.357	0.683	-105.749
3	-135.809	0.671	-101.646	-133.776	0.661	-100.124	-136.378	0.673	-102.072
4	-135.716	0.678	-101.622	-133.685	0.668	-100.101	-136.285	0.681	-102.047
5	-135.628	0.680	-101.600	-133.598	0.670	-100.078	-136.196	0.683	-102.025
6	-135.545	0.677	-101.578	-133.516	0.667	-100.058	-136.113	0.680	-102.003
7	-135.466	0.671	-101.557	-133.439	0.661	-100.038	-136.034	0.674	-101.983
8	-135.392	0.661	-101.539	-133.365	0.651	-100.020	-136.959	0.664	-101.964
9	-130.461	0.625	-97.875	-130.461	0.625	-97.875	-130.461	0.625	-97.875
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	-147.700	0.591	-110.444	-148.514	0.594	-111.053	-151.447	0.606	-113.246
12	-141.098	0.572	-105.555	-140.285	0.568	-104.943	-142.237	0.576	-106.403
13	-141.010	0.573	-105.527	-140.197	0.570	-104.919	-142.148	0.577	-106.378
14	-140.926	0.569	-105.504	-140.114	0.565	-104.896	-142.063	0.574	-106.354
15	-140.848	0.561	-105.482	-140.036	0.558	-104.874	-141.984	0.566	-106.333

16	-140.774	0.550	105.462	-139.962	0.547	-104.854	-141.909	0.554	-106.313
17	-140.704	0.536	-105.443	-139.894	0.533	-104.836	-141.839	0.540	-106.294
18	-134.156	0.496	-100.566	-135.777	0.502	-101.781	-136.749	0.506	-102.510
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	-147.710	0.459	-110.389	-148.523	0.461	-110.997	-151.456	0.471	-113.189
21	-141.115	0.438	-105.499	-140.302	0.436	-104.891	-142.254	0.442	-106.351
22	-141.035	0.433	-105.475	-140.222	0.431	-104.868	-142.173	0.437	-106.326
23	-140.960	0.424	-105.453	-140.148	0.422	-104.845	-142.097	0.428	-106.304
24	-140.890	0.412	-105.432	-140.048	0.410	-104.825	-142.026	0.415	-106.283
25	-140.824	0.397	-105.413	-140.013	0.395	-104.806	-141.961	0.401	-106.264
26	-140.763	0.381	-105.395	-139.952	0.378	-104.788	-141.899	0.584	-106.246
27	-134.220	0.345	-100.522	-135.842	0.350	-101.736	-136.815	0.352	-102.465
28	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
29	-147.732	0.317	-110.332	-148.546	0.319	-110.939	-151.479	0.325	-113.130
30	-141.146	0.297	-105.444	-140.333	0.296	-104.837	-142.285	0.299	-106.295
31	-141.075	0.287	-105.422	-140.262	0.285	-104.814	-142.213	0.289	-106.272
32	-141.009	0.273	-105.400	-140.197	0.272	-104.793	-142.147	0.276	-106.251
33	-140.948	0.258	-105.381	-140.136	0.256	-104.774	-142.085	0.260	-106.231
34	-140.891	0.241	-105.363	-140.079	0.239	-104.756	-142.028	0.242	-106.213
35	-140.838	0.221	-105.347	-140.027	0.221	-104.740	-141.975	0.224	-106.197
36	-134.300	0.193	-100.476	-135.922	0.195	-101.691	-136.896	0.196	-102.419
37	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
38	-147.644	0.166	-110.176	-148.585	0.168	-110.878	-151.352	0.171	-112.944
39	-141.066	0.148	-105.293	-140.380	0.147	-104.781	-142.169	0.149	-106.117
40	-141.005	0.134	-105.272	-140.319	0.133	-104.759	-142.108	0.135	-106.095
41	-140.948	0.118	-105.252	-140.263	0.117	-104.740	-142.051	0.119	-106.075
42	-140.897	0.100	-105.234	-140.211	0.099	-104.722	-141.999	0.101	-106.057
43	-140.849	0.081	-105.218	-140.164	0.081	-104.706	-141.951	0.082	-106.041
44	-139.721	0.061	-104.393	-139.036	0.061	-103.881	-140.904	0.062	-105.277
45	-137.719	0.039	-99.925	-135.342	0.040	-101.338	-136.316	0.040	-101.866
46	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
47	-147.320	0.010	-109.828	-148.033	0.001	-110.360	-150.760	0.001	-112.394
48	-140.751	-0.005	-104.951	-140.039	-0.005	-104.420	-141.748	-0.005	-105.694
49	-140.700	-0.022	-104.931	-139.988	-0.022	-104.400	-141.697	-0.022	-105.674
50	-140.653	-0.040	-104.931	-139.941	-0.040	-104.383	-141.650	-0.040	-105.657
51	-140.611	-0.058	-104.897	-139.899	-0.058	-104.367	-141.607	-0.059	-105.641
52	-139.487	-0.077	-104.074	-138.775	-0.077	-103.543	-140.564	-0.078	-104.877
53	-155.161	-0.108	-115.783	-137.535	-0.100	-102.631	-138.551	-0.096	-103.389
54	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
55	-146.892	0.151	-109.386	-147.503	-0.152	-109.841	-149.865	-0.154	-111.600
56	-140.329	-0.161	-104.513	-139.718	-0.160	-104.058	-141.184	-0.162	-105.150
57	-140.289	-0.178	-104.496	-139.678	-0.177	-104.041	-141.144	-0.179	-105.132
58	-140.252	-0.197	-104.480	-139.641	-0.196	-104.026	-141.107	-0.198	-105.117
59	-139.132	-0.214	-103.657	-138.521	-0.212	-103.203	-140.068	-0.215	-104.354
60	-137.596	-0.229	-102.524	-136.701	-0.228	-101.857	-137.596	-0.229	-102.524
61	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
62	-146.488	-0.312	-108.942	-146.997	-0.313	-109.322	-148.995	-0.317	-110.807
63	-139.931	-0.315	-104.076	-139.422	-0.314	-103.697	-140.644	-0.317	-104.606
64	-139.900	-0.332	-104.061	-139.391	-0.331	-103.682	-140.614	-0.334	-104.592

65	-138.785	-0.346	-103.240	-138.276	-0.345	-102.861	-139.579	-0.348	-103.831
66	-136.529	-0.358	-101.569	-136.147	-0.357	-101.284	-136.921	-0.359	-101.860
67	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
68	-146.111	-0.468	-108.501	-146.519	-0.470	-108.804	-148.151	-0.475	-110.016
69	-139.588	-0.463	-103.640	-139.150	-0.462	-103.337	-140.129	-0.465	-104.064
70	-138.447	-0.475	-102.820	-138.039	-0.473	-102.517	-139.099	-0.477	-103.305
71	-135.660	-0.480	-100.754	-135.328	-0.478	-100.508	-135.980	-0.481	-100.993
72	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
73	-145.762	-0.615	-108.062	-146.068	-0.616	-108.290	-147.334	-0.622	-109.228
74	-138.119	-0.596	-102.399	-138.813	-0.594	-102.172	-138.633	-0.598	-102.778
75	-135.071	-0.595	-100.143	-134.791	-0.594	-99.934	-135.322	-0.596	-100.328
76	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
77	-137.804	-0.707	-101.978	-138.010	-0.708	-102.129	-138.991	-0.713	-102.856
78	-134.221	-0.698	-99.328	-133.991	-0.697	-99.158	-134.399	-0.699	-99.460
79	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
80	-141.465	-0.833	-104.485	-141.491	-0.833	-104.504	-142.555	-0.840	-105.291
81	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
82	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
83	-140.807	0.705	-105.322	-138.773	0.671	-103.801	-143.357	0.683	-105.749
84	-135.825	0.693	-101.646	-133.793	0.661	-100.124	-136.378	0.673	-102.072
85	-135.731	0.699	-101.622	-133.699	0.668	-100.101	-136.285	0.681	-102.047
86	-135.641	0.700	-101.600	-133.611	0.670	-100.078	-136.196	0.683	-102.025
87	-135.556	0.697	-101.578	-133.527	0.667	-100.058	-136.113	0.680	-102.003
88	-135.476	0.690	-101.557	-133.448	0.661	-100.038	-136.034	0.674	-101.983
89	-135.400	0.680	-101.539	-133.376	0.651	-100.020	-136.959	0.664	-101.964
90	-130.468	0.642	-97.875	-130.467	0.625	-97.875	-130.461	0.625	-97.875
91	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
92	-147.719	0.616	-110.444	-148.514	0.594	-111.053	-151.447	0.606	-113.246
93	-141.114	0.595	-105.555	-140.285	0.568	-104.943	-142.237	0.576	-106.403
94	-141.023	0.595	-105.527	-140.197	0.570	-104.919	-142.148	0.577	-106.378
95	-140.938	0.590	-105.504	-140.114	0.565	-104.896	-142.063	0.574	-106.354
96	-140.857	0.581	-105.482	-140.036	0.558	-104.874	-141.984	0.566	-106.333
97	-140.782	0.570	-105.462	-139.962	0.547	-104.854	-141.909	0.554	-106.313
98	-140.711	0.554	-105.443	-139.894	0.533	-104.836	-141.839	0.540	-106.294
99	-134.162	0.513	-100.566	-135.777	0.502	-101.781	-136.749	0.506	-102.510
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
101	-147.726	0.484	-110.389	-148.523	0.461	-110.997	-151.456	0.471	-113.189
102	-141.129	0.461	-105.499	-140.302	0.436	-104.891	-142.254	0.442	-106.351
103	-141.047	0.455	-105.475	-140.222	0.431	-104.868	-142.173	0.437	-106.326
104	-140.970	0.445	-105.453	-140.148	0.422	-104.845	-142.097	0.428	-106.304
105	-140.898	0.432	-105.432	-140.048	0.410	-104.825	-142.026	0.415	-106.283
106	-140.831	0.416	-105.413	-140.013	0.395	-104.806	-141.961	0.401	-106.264
107	-140.769	0.398	-105.395	-139.952	0.378	-104.788	-141.899	0.584	-106.246
108	-134.224	0.362	-100.522	-135.842	0.350	-101.736	-136.815	0.352	-102.465
109	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
110	-147.746	0.342	-110.332	-148.546	0.319	-110.939	-151.479	0.325	-113.130
111	-141.157	0.319	-105.444	-140.333	0.296	-104.837	-142.285	0.299	-106.295
112	-141.085	0.308	-105.422	-140.262	0.285	-104.814	-142.213	0.289	-106.272
113	-141.017	0.293	-105.400	-140.197	0.272	-104.793	-142.147	0.276	-106.251

114	-140.954	0.277	-105.381	-140.136	0.256	-104.774	-142.085	0.260	-106.231
115	-140.896	0.258	-105.363	-140.079	0.239	-104.756	-142.028	0.242	-106.213
116	-140.843	0.238	-105.347	-140.027	0.221	-104.740	-141.975	0.224	-106.197
117	-134.303	0.207	-100.476	-135.922	0.195	-101.691	-136.896	0.196	-102.419
118	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
119	-147.674	0.190	-110.176	-148.585	0.168	-110.878	-151.352	0.171	-112.944
120	-141.087	0.170	-105.293	-140.380	0.147	-104.781	-142.169	0.149	-106.117
121	-141.025	0.154	-105.272	-140.319	0.133	-104.759	-142.108	0.135	-106.095
122	-140.967	0.136	-105.252	-140.263	0.117	-104.740	-142.051	0.119	-106.075
123	-140.914	0.117	-105.234	-140.211	0.099	-104.722	-141.999	0.101	-106.057
124	-140.865	0.097	-105.218	-140.164	0.081	-104.706	-141.951	0.082	-106.041
125	-139.717	0.076	-104.393	-139.036	0.061	-1.03.881	-140.904	0.062	-105.277
126	-133.708	0.053	-99.925	-135.342	0.040	-101.338	-136.316	0.040	-101.866
127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
128	-147.367	0.029	-109.828	-148.033	0.001	-110.360	-150.760	0.001	-112.394
129	-140.781	-0.012	-104.951	-140.039	-0.005	-104.420	-141.748	-0.005	-105.694
130	-140.729	-0.006	-104.931	-139.988	-0.022	-104.400	-141.697	-0.022	-105.674
131	-140.681	-0.025	-104.931	-139.941	-0.040	-104.383	-141.650	-0.040	-105.657
132	-140.638	-0.044	-104.897	-139.899	-0.058	-104.367	-141.607	-0.059	-105.641
133	-139.513	-0.064	-104.074	-138.775	-0.077	-103.543	-140.564	-0.078	-104.877
134	-155.176	-0.094	-115.783	-137.535	-0.100	-102.631	-138.551	-0.096	-103.389
135	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
136	-146.935	-0.135	-109.386	-147.503	-0.152	-109.841	-149.865	-0.154	-111.600
137	-140.357	-0.147	-104.513	-139.718	-0.160	-104.058	-141.184	-0.162	-105.150
138	-140.315	-0.166	-104.496	-139.678	-0.177	-104.041	-141.144	-0.179	-105.132
139	-140.278	-0.185	-104.480	-139.641	-0.196	-104.026	-141.107	-0.198	-105.117
140	-140.244	-0.204	-103.657	-138.521	-0.212	-103.203	-140.068	-0.215	-104.354
141	-137.747	-0.220	-102.524	-136.701	-0.228	-101.857	-137.596	-0.229	-102.524
142	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
143	-146.529	-0.300	-108.942	-146.997	-0.313	-109.322	-148.995	-0.317	-110.807
144	-139.957	-0.304	-104.076	-139.422	-0.314	-103.697	-140.644	-0.317	-104.606
145	-139.925	-0.323	-104.061	-139.391	-0.331	-103.682	-140.614	-0.334	-104.592
146	-138.809	-0.338	-103.240	-138.276	-0.345	-102.861	-139.579	-0.348	-103.831
147	-136.543	-0.350	-101.569	-136.147	-0.357	-101.284	-136.921	-0.359	-101.860
148	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
149	-146.150	-0.461	-108.501	-146.519	-0.470	-108.804	-148.151	-0.475	-110.016
150	-139.582	-0.456	-103.640	-139.150	-0.462	-103.337	-140.129	-0.465	-104.064
151	-138.471	-0.469	-102.820	-138.039	-0.473	-102.517	-139.099	-0.477	-103.305
152	-135.809	-0.475	-100.754	-135.328	-0.478	-100.508	-135.980	-0.481	-100.993
153	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
154	-145.800	-0.611	-108.062	-146.068	-0.616	-108.290	-147.334	-0.622	-109.228
155	-138.143	-0.592	-102.399	-138.813	-0.594	-102.172	-138.633	-0.598	-102.778
156	-135.085	-0.591	-100.143	-134.791	-0.594	-99.934	-135.322	-0.596	-100.328
157	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
158	-145.478	-0.744	-101.978	-138.010	-0.708	-102.129	-138.991	-0.713	-102.856
159	-134.371	-0.696	-99.328	-133.991	-0.697	-99.158	-134.399	-0.699	-99.460
160	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
161	-141.495	-0.833	-104.485	-141.491	-0.833	-104.504	-142.555	-0.840	-105.291

Table (C.2) Inertia forces for each keypoint in Models 4, 5, & 6

Keypoint No.	Fourth Model			Fifth Model			Sixth Model		
	F_x (N)	F_y (N)	F_z (N)	F_x (N)	F_y (N)	F_z (N)	F_x (N)	F_y (N)	F_z (N)
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	-138.753	0.671	-103.801	-138.753	0.671	-103.801	-146.564	0.708	-109.644
3	-135.809	0.671	-101.646	-138.979	0.686	-104.019	-133.776	0.661	-100.124
4	-133.685	0.668	-100.101	-133.685	0.668	-100.101	-133.685	0.668	-100.101
5	-135.628	0.680	-101.599	-138.794	0.696	-103.971	-138.795	0.695	-103.971
6	-133.516	0.667	-100.577	-133.516	0.667	-100.058	-133.516	0.667	-100.058
7	-135.466	0.671	-101.557	-138.628	0.686	-103.928	-133.439	0.661	-100.038
8	-133.365	0.651	-100.019	-133.365	0.651	-100.019	-138.552	0.676	-103.909
9	-130.461	0.625	-97.875	-133.053	0.638	-99.819	-130.461	0.625	-97.875
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	-143.633	0.575	-107.403	-159.257	0.637	-119.086	-146.076	0.584	-109.227
12	-141.089	0.572	-105.551	-137.033	0.555	-102.510	-143.863	0.583	-107.619
13	-136.947	0.556	-102.486	-147.347	0.598	-110.269	-143.773	0.584	-107.594
14	-140.926	0.569	-105.504	-136.866	0.552	-102.464	-138.490	0.559	-103.680
15	-136.790	0.544	-102.443	-147.178	0.586	-110.223	-143.607	0.572	-107.548
16	-140.770	0.550	-105.462	-136.718	0.534	-102.423	-143.532	0.561	-107.528
17	-136.651	0.521	-102.405	-147.028	0.561	-110.182	-138.272	0.527	-103.621
18	-134.156	0.546	-100.566	-134.156	0.496	-100.566	-137.560	0.508	-103.117
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	-148.523	0.461	-110.997	-143.642	0.446	-107.349	-146.082	0.454	-109.173
21	-144.368	0.448	-107.931	-147.457	0.458	-110.241	-143.880	0.447	-107.566
22	-140.222	0.431	-104.867	-136.972	0.421	-102.436	-143.798	0.442	-107.541
23	-144.209	0.434	-107.883	-147.295	0.443	-110.192	-138.524	0.417	-103.630
24	-140.078	0.410	-104.825	-136.831	0.400	-102.394	-143.650	0.420	-107.497
25	-144.070	0.406	-107.843	-147.154	0.415	-110.151	-143.583	0.405	-107.478
26	-139.952	0.378	-104.788	-136.708	0.369	-102.359	-138.330	0.374	-103.574
27	-135.842	0.349	-101.736	-139.409	0.359	-104.408	-137.625	0.354	-103.072
28	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
29	-143.663	0.308	-107.293	-159.291	0.342	-118.964	-164.169	0.353	-122.607
30	-141.146	0.267	-105.444	-137.080	0.288	-102.407	-140.333	0.295	-104.837
31	-137.011	0.278	-102.384	-147.415	0.299	-110.160	-140.262	0.285	-104.814
32	-141.010	0.274	-105.400	-136.947	0.265	-102.364	-150.597	0.292	-112.567
33	-136.887	0.251	-102.345	-147.283	0.270	-110.117	-140.136	0.256	-104.774
34	-140.891	0.241	-105.363	-136.832	0.234	-102.328	-140.079	0.239	-104.756
35	-136.781	0.215	-102.312	-136.169	0.232	-110.082	-150.414	0.236	-112.510
36	-134.300	0.193	-100.476	-134.300	0.193	-100.048	-135.922	0.195	-110.691
37	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
38	-148.585	0.168	-110.878	-143.401	0.162	-107.234	-145.990	0.165	-108.642
39	-144.320	0.152	-107.772	-147.214	0.155	-109.882	-143.776	0.151	-107.316
40	-140.319	0.133	-104.759	-137.066	0.130	-102.331	-143.714	0.137	-107.294
41	-144.200	0.121	-107.680	-147.091	0.123	-109.839	-138.535	0.116	-103.450
42	-140.211	0.099	-104.722	-136.916	0.097	-102.294	-143.604	0.102	-107.265
43	-144.100	0.083	-107.645	-146.987	0.085	-109.803	-143.555	0.083	-107.240
44	-139.036	0.061	-103.881	-135.787	0.059	-101.454	-137.310	0.060	-102.592

45	-135.342	0.041	-101.138	-136.316	0.040	-101.866	-136.316	0.040	-101.866
46	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
47	-143.758	0.009	-107.173	-157.438	0.010	-117.371	-146.048	0.010	-108.880
48	-140.751	-0.005	-104.950	-137.190	-0.005	-102.295	-143.274	-0.006	-106.832
49	-137.140	-0.021	-102.276	-146.253	-00.23	-109.072	-143.222	-0.022	-106.812
50	-140.653	-0.040	-104.913	-137.095	-0.039	-102.295	-138.620	-0.039	-103.397
51	-137.053	-0.057	-102.244	-146.160	-0.061	-109.038	-143.132	-0.060	-106.778
52	-139.487	-0.077	-104.074	-135.930	-0.075	-101.420	-142.001	-0.079	-105.594
53	-136.113	-0.094	-101.570	-138.551	-00.96	-103.389	-137.637	-0.095	-102.707
54	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
55	-147.503	-0.151	-109.841	-143.836	-0.147	-107.111	-159.072	-0.163	-118.455
56	-142.773	-0.164	-106.333	-145.094	-0.166	-108.062	-139.718	-0.160	-104.058
57	-139.678	-0.178	-104.041	-137.235	-0.175	-102.222	-139.678	-0.178	-104.041
58	-142.694	-0.200	-106.299	-145.014	-0.204	-108.028	-147.456	-0.207	-109.847
59	-138.522	-0.213	-103.202	-136.081	-0.209	-101.384	-138.522	-0.213	-103.203
60	-138.817	-0.231	-103.433	-137.596	-0.229	-102.524	-136.702	-0.228	-101.857
61	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
62	-143.940	-0.306	-107.048	-153.724	-0.327	-114.324	-145.316	-0.309	-108.071
63	-139.931	-0.315	-104.076	-137.384	-0.309	-102.181	-141.561	-0.318	-105.288
64	-137.354	-0.326	-102.167	-143.873	-0.341	-107.016	-141.530	-0.366	-105.274
65	-137.785	-0.346	-103.239	-136.239	-0.340	-101.345	-137.155	-0.342	-102.027
66	-135.128	-0.354	-100.530	-136.921	-0.359	-101.360	-136.839	-0.358	-101.799
67	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
68	-146.519	-0.470	-108.804	-144.071	-0.462	-106.986	-145.448	-0.466	-108.008
69	-141.189	-0.468	-104.852	-142.740	-0.474	-106.000	-141.047	-0.468	-104.746
70	-138.089	-0.473	-102.517	136.408	-0.468	-101.305	-137.325	-0.471	-101.987
71	-136.474	-0.483	-101.360	-135.980	-0.481	-100.993	-138.040	-0.488	-102.522
72	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
73	-144.230	-0.610	-106.927	-150.112	0-.633	-111.387	-151.950	-0.641	-112.650
74	-138.119	-0.595	-102.399	-136.588	-0.589	-101.264	-137.813	-0.594	-102.172
75	-134.178	-0.591	-99.480	-135.322	-0.596	-100.328	-134.790	-0.594	-99.934
76	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
77	-138.009	-0.708	-102.129	-136.782	-0.701	-101.222	-137.702	-0.706	-101.903
78	-134.630	-0.700	-99.631	-134.399	-0.699	-99.461	-134.318	-0.698	-99.399
79	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
80	-141.081	-0.831	-104.202	-142.064	-0.837	-104.928	-142.494	-0.840	-105.245
81	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
82	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
83	-138.753	0.671	-103.801	-138.753	0.671	-103.801	-146.564	0.708	-109.644
84	-135.809	0.671	-101.646	-138.979	0.686	-104.019	-133.776	0.661	-100.124
85	-133.685	0.668	-100.101	-133.685	0.668	-100.101	-133.685	0.668	-100.101
86	-135.628	0.680	-101.599	-138.794	0.696	-103.971	-138.795	0.695	-103.971
87	-133.516	0.667	-100.577	-133.516	0.667	-100.058	-133.516	0.667	-100.058
88	-135.466	0.671	-101.557	-138.628	0.686	-103.928	-133.439	0.661	-100.038
89	-133.365	0.651	-100.019	-133.365	0.651	-100.019	-138.552	0.676	-103.909
90	-130.461	0.625	-97.875	-133.053	0.638	-99.819	-130.461	0.625	-97.875
91	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
92	-143.633	0.575	-107.403	-159.257	0.637	-119.086	-146.076	0.584	-109.227
93	-141.089	0.572	-105.551	-137.033	0.555	-102.510	-143.863	0.583	-107.619

94	-136.947	0.556	-102.486	-147.347	0.598	-110.269	-143.773	0.584	-107.594
95	-140.926	0.569	-105.504	-136.866	0.552	-102.464	-138.490	0.559	-103.680
96	-136.790	0.544	-102.443	-147.178	0.586	-110.223	-143.607	0.572	-107.548
97	-140.770	0.550	-105.462	-136.718	0.534	-102.423	-143.532	0.561	-107.528
98	-136.651	0.521	-102.405	-147.028	0.561	-110.182	-138.272	0.527	-103.621
99	-134.156	0.546	-100.566	-134.156	0.496	-100.566	-137.560	0.508	-103.117
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
101	-148.523	0.461	-110.997	-143.642	0.446	-107.349	-146.082	0.454	-109.173
102	-144.368	0.448	-107.931	-147.457	0.458	-110.241	-143.880	0.447	-107.566
103	-140.222	0.431	-104.867	-136.972	0.421	-102.436	-143.798	0.442	-107.541
104	-144.209	0.434	-107.883	-147.295	0.443	-110.192	-138.524	0.417	-103.630
105	-140.078	0.410	-104.825	-136.831	0.400	-102.394	-143.650	0.420	-107.497
106	-144.070	0.406	-107.843	-147.154	0.415	-110.151	-143.583	0.405	-107.478
107	-139.952	0.378	-104.788	-136.708	0.369	-102.359	-138.330	0.374	-103.574
108	-135.842	0.349	-101.736	-139.409	0.359	-104.408	-137.625	0.354	-103.072
109	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
110	-143.663	0.308	-107.293	-159.291	0.342	-118.964	-164.169	0.353	-122.607
111	-141.146	0.267	-105.444	-137.080	0.288	-102.407	-140.333	0.295	-104.837
112	-137.011	0.278	-102.384	-147.415	0.299	-110.160	-140.262	0.285	-104.814
113	-141.010	0.274	-105.400	-136.947	0.265	-102.364	-150.597	0.292	-112.567
114	-136.887	0.251	-102.345	-147.283	0.270	-110.117	-140.136	0.256	-104.774
115	-140.891	0.241	-105.363	-136.832	0.234	-102.328	-140.079	0.239	-104.756
116	-136.781	0.215	-102.312	-136.169	0.232	-110.082	-150.414	0.236	-112.510
117	-134.300	0.193	-100.476	-134.300	0.193	-100.048	-135.922	0.195	-110.691
118	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
119	-148.585	0.168	-110.878	-143.401	0.162	-107.234	-145.990	0.165	-108.642
120	-144.320	0.152	-107.772	-147.214	0.155	-109.882	-143.776	0.151	-107.316
121	-140.319	0.133	-104.759	-137.066	0.130	-102.331	-143.714	0.137	-107.294
122	-144.200	0.121	-107.680	-147.091	0.123	-109.839	-138.535	0.116	-103.450
123	-140.211	0.099	-104.722	-136.916	0.097	-102.294	-143.604	0.102	-107.265
124	-144.100	0.083	-107.645	-146.987	0.085	-109.803	-143.555	0.083	-107.240
125	-139.036	0.061	-103.881	-135.787	0.059	-101.454	-137.310	0.060	-102.592
126	-135.342	0.041	-101.138	-136.316	0.040	-101.866	-136.316	0.040	-101.866
127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
128	-143.758	0.009	-107.173	-157.438	0.010	-117.371	-146.048	0.010	-108.880
129	-140.751	-0.005	-104.950	-137.190	-0.005	-102.295	-143.274	-0.006	-106.832
130	-137.140	-0.021	-102.276	-146.253	-00.23	-109.072	-143.222	-0.022	-106.812
131	-140.653	-0.040	-104.913	-137.095	-0.039	-102.295	-138.620	-0.039	-103.397
132	-137.053	-0.057	-102.244	-146.160	-0.061	-109.038	-143.132	-0.060	-106.778
133	-139.487	-0.077	-104.074	-135.930	-0.075	-101.420	-142.001	-0.079	-105.594
134	-136.113	-0.094	-101.570	-138.551	-00.96	-103.389	-137.637	-0.095	-102.707
135	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
136	-147.503	-0.151	-109.841	-143.836	-0.147	-107.111	-159.072	-0.163	-118.455
137	-142.773	-0.164	-106.333	-145.094	-0.166	-108.062	-139.718	-0.160	-104.058
138	-139.678	-0.178	-104.041	-137.235	-0.175	-102.222	-139.678	-0.178	-104.041
139	-142.694	-0.200	-106.299	-145.014	-0.204	-108.028	-147.456	-0.207	-109.847
140	-138.522	-0.213	-103.202	-136.081	-0.209	-101.384	-138.522	-0.213	-103.203
141	-138.817	-0.231	-103.433	-137.596	-0.229	-102.524	-136.702	-0.228	-101.857
142	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

143	-143.940	-0.306	-107.048	-153.724	-0.327	-114.324	-145.316	-0.309	-108.071
144	-139.931	-0.315	-104.076	-137.384	-0.309	-102.181	-141.561	-0.318	-105.288
145	-137.354	-0.326	-102.167	-143.873	-0.341	-107.016	-141.530	-0.366	-105.274
146	-137.785	-0.346	-103.239	-136.239	-0.340	-101.345	-137.155	-0.342	-102.027
147	-135.128	-0.354	-100.530	-136.921	-0.359	-101.360	-136.839	-0.358	-101.799
148	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
149	-146.519	-0.470	-108.804	-144.071	-0.462	-106.986	-145.448	-0.466	-108.008
150	-141.189	-0.468	-104.852	-142.740	-0.474	-106.000	-141.047	-0.468	-104.746
151	-138.089	-0.473	-102.517	136.408	-0.468	-101.305	-137.325	-0.471	-101.987
152	-136.474	-0.483	-101.360	-135.980	-0.481	-100.993	-138.040	-0.488	-102.522
153	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
154	-144.230	-0.610	-106.927	-150.112	-0.633	-111.387	-151.950	-0.641	-112.650
155	-138.119	-0.595	-102.399	-136.588	-0.589	-101.264	-137.813	-0.594	-102.172
156	-134.178	-0.591	-99.480	-135.322	-0.596	-100.328	-134.790	-0.594	-99.934
157	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
158	-138.009	-0.708	-102.129	-136.782	-0.701	-101.222	-137.702	-0.706	-101.903
159	-134.630	-0.700	-99.631	-134.399	-0.699	-99.461	-134.318	-0.698	-99.399
160	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
161	-141.081	-0.831	-104.202	-142.064	-0.837	-104.928	-142.494	-0.840	-105.245

Appendix –D- Material Properties

Table (D) The physical, elastic, and mechanical properties of some of selected Titanium and Aluminum alloys (isotropic elastic materials properties as mentioned by (**Ref. 60**))

(A) Some of Titanium alloys properties

No.	Alloy symbol	Alloy No.	Alloy class	condition	Chemical composition (weight %)	Mass density ρ (kg/m ³)	Elastic modulus E (Gpa)	Yield stress $\sigma_{y,p}$ (Mpa)	Specific stiffness E/ ρ (GJ/kg)	Specific strength $\sigma_{y,p}/\rho$ (MJ/kg)	Ultimate strength σ_{ut} (Mpa)	Poisson's ratio ν
1	Ti1		β	solution+age	75Ti+3Al+4Mo+8V+6Cr+4Zr	4820	105.5	1379	0.021888	0.286100	1448	0.300
2	Ti2		β	solution+age	78Ti+11.5Mo+6Zr+4.5Sn	4820	103.4	1317	0.021450	0.273230	1386	0.300
3	Ti3		$\alpha - \beta$	solution+age	86Ti+6Al+6V+2Sn	4540	110.3	1172	0.024295	0.258150	1206	0.342
4	Ti4		β	solution+age	85Ti+3Al+10V+2Fe	4650	111.7	1200	0.024020	0.258060	1276	0.340
5	Ti5		β	solution+age	79Ti+3Al+8Mo+8V+2Fe	4840	106.9	1214	0.022086	0.256400	1310	0.300
6	Ti6		$\alpha - \beta$	solution+age	82Ti+6Al+6Mo+4Zr+2Sn	4650	113.8	1172	0.024473	0.252043	1269	0.327
7	Ti7		$\alpha - \beta$	solution+age	90Ti+6Al+2Zr+2Sn	4570	122.0	1138	0.026700	0.249000	1276	0.330
8	Ti8		$\alpha - \beta$	solution+age	90Ti+6Al+4V	4430	113.8	1103	0.025688	0.249000	1172	0.342
9	Ti9		β	heat treated	73Ti+3Al+13V+11Cr	4840	101.4	1172	0.020950	0.242150	1276	0.304
10	Ti10		$\alpha - \beta$	solution+age	89Ti+7Al+4Mo	4480	113.8	1034	0.025400	0.230800	1103	0.326
11	Ti11		$\alpha - \beta$	sheet annealed	86Ti+6Al+6V+2Sn	4540	110.3	1000	0.024295	0.220264	1069	0.342
12	Ti12		near- α	duplex annealed	90Ti+8Al+1Mo+1V	4370	124.1	951	0.028400	0.217620	1000	0.320
13	Ti13		near- α	duplex annealed	85.75Ti+5Al+2Mo+2Zr+5Sn +0.25Si	4510	113.8	965	0.025230	0.213970	1048	0.326
14	Ti14		$\alpha - \beta$	duplex annealed	90Ti+6Al+4V	4430	113.8	924	0.025688	0.208580	1172	0.342
15	Ti15		near- α	duplex annealed	79.55Ti+2.25Al+2Mo+5Zr +11Sn+0.2Si	4820	113.8	993	0.023610	0.206000	1103	0.320
16	Ti16		near-	duplex	86Ti+6Al+2Mo+4Zr+2Sn	4540	113.8	896	0.025066	0.197357	979	0.326

			α	annealed								
17	Ti17		$\alpha - \beta$	solution+age (low O ₂)	90Ti+6Al+4V	4430	113.8	827	0.025688	0.186680	896	0.342
18	Ti18		β	sheet annealed	73Ti+3Al+13V+11Cr	4840	113.8	896	0.023512	0.185124	1220	0.300
19	Ti19		$\alpha - \beta$	solution+age	92Ti+8Mn	4730	113.1	862	0.023911	0.182240	945	0.342
20	Ti20		α	sheet annealed	92.5Ti+5Al+2.5Sn	4480	110.3	807	0.024620	0.180134	862	0.340
21	Ti21		β	sheet annealed	75Ti+3Al+4Mo+8V+6Cr+4Zr	4820	105.5	834	0.021888	0.172300	883	0.300
22	Ti22		near- α	plate rolled	90Ti+6Al+1Mo+2Nb+1Ta	4480	113.8	758	0.025400	0.169200	855	0.326
23	Ti23		α	sheet annealed (low O ₂)	92.5Ti+5Al+2.5Sn	4480	110.3	745	0.024620	0.166294	807	0.340
24	Ti24		$\alpha - \beta$	solution+age	94.5Ti+3Al+2.5V	4480	106.9	586	0.023861	0.130800	689	0.342
25	Ti25		pure	sheet annealed	99Ti+0.35Al +0.24Mo+0.4Ni	4510	104.1	586	0.023082	0.129933	662	0.340
26	Ti26		pure	sheet annealed	99.1Ti+0.28Mo+0.42Ni+0.2Si	4520	103.4	448	0.022876	0.099115	517	0.340
27	Ti27		pure	sheet annealed	98.9Ti+0.3Mo+0.8Ni	4540	103.0	448	0.022687	0.098678	517	0.340
28	Ti28		pure	sheet annealed	99.2Ti+0.8Al	4510	102.7	345	0.022772	0.076496	193	0.340
29	Ti29		pure	sheet annealed	99.5Ti+0.2Al+0.3V	4510	102.7	241	0.022772	0.053437	152	0.340

(B) Some of Aluminum alloys properties

No.	Alloy symbol	Alloy No.	Alloy class	condition	Chemical composition (weight %)	Mass density ρ (kg/m ³)	Elastic modulus E (Gpa)	Yield stress $\sigma_{y,p}$ (Mpa)	Specific stiffness E/ ρ (GJ/kg)	Specific strength $\sigma_{y,p}/\rho$ (MJ/kg)	Ultimate strength σ_{ut} (Mpa)	Poisson's ratio ν
30	Al1	7175		Tempered (T66)	90Al+2.4Mg+0.23Cr+6.4Zn+0.97Zr	2820	77.5	525	0.0274822	0.18617	595	0.334
31	Al2	7075		Tempered (T6,T651)	90Al+2.5Mg+0.23Cr+5.6Zn+1.6Cu +0.07Mn	2800	71.8	505	0.0256428	0.180357	525	0.334
32	Al3	7175		Tempered (T736)	90Al+2.5Mg+0.23Cr+5.6Zn+1.6Cu	2820	72.6	505	0.0257446	0.179078	570	0.334
33	Al4	7475		Tempered (T61)	90Al+2.5Mg+0.23Cr+5.6Zn+1.6Cu	2763.3	73.6	460	0.0266348	0.166467	525	0.334
34	Al5	Alclad		Tempered (T861)	93.5Al+1.5Mg+0.6Mn+4.4Cu	2769.3	72.5	455	0.02618	0.1643014	485	0.333

		2024										
35	Al6	Alclad 2024		Tempered (T851)	93.5Al+1.5Mg+0.6Mn+4.4 Cu	2769.3	72.5	415	0.02618	0.149857	450	0.330
36	Al7	Alclad 2024		Tempered (T361)	93.5Al+1.5Mg+0.6Mn+4.4 Cu	2769.3	71.8	395	0.02593	0.142635	460	0.334
37	Al8	6262		Tempered (T9)	98.4Al+0.5Mg+0.1Mn+0.1 Cr+0.1Zr +0.8Si	2701.5	72	380	0.026652	0.1406625	400	0.334
38	Al9	6070		Tempered (T6)	96.8Al+0.8Mg+0.28Cr+0.7 Mn+1.4Si +0.02Zr	2701	72.8	350	0.026953	0.129581	380	0.330
39	Al10	2618		Tempered (All)	93.7Al+2.3Cu+1.6Mg+0.07 Ti+1Ni+1.1Fe+0.18Si	2925	71.2	370	0.024341	0.126495	440	0.340
40	Al11	7005		Tempered (T53)	88.4Al+7.2Mg+0.23Cr+0.4 Si+2.7Cu +1.3Zn	2754.4	73.4	345	0.0266482	0.125254	393	0.334
41	Al12	Alclad 2024		Tempered (T351)	93.5Al+1.5Mg+0.6Mn+4.4 Cu	2769.3	71.4	325	0.0257826	0.117358	440	0.334
42	Al13	6061		Tempered (T6,T651)	97.9Al+1Mg+0.2Cr+0.6Si+ 0.02Zr +0.28Cu	2697	71.8	275	0.026622	0.101965	310	0.333
43	Al14	5154		Tempered (H38)	96.2Al+3.5Mg+0.25Cr+0.0 3.Zr+0.02Mn	2661.2	77.3	270	0.029047	0.101458	330	0.334
44	Al15	2011		Tempered (T3)	93.7Al+5.5Cu+0.4Bi+0.4Pb	3021.05	71.16	295	0.023555	0.097648	380	0.334
45	Al16	5052		Tempered (H38)	97.2Al+2.5Mg+0.25Cr+0.0 5Si	2680.1	72.8	255	0.027163	0.0951457	290	0.334
46	Al17	5154		Tempered (H36)	96.2Al+3.5Mg+0.25Cr+0.0 3.Zr+0.02Mn	2661.2	73.6	250	0.0276566	0.0939425	310	0.333
47	Al18	5052		Tempered (H36)	97.2Al+2.5Mg+0.25Cr+0.0 5Si	2680.1	72.6	240	0.0270885	0.0895488	275	0.334
48	Al19	5154		Tempered (H34)	96.2Al+3.5Mg+0.25Cr+0.0 3.Zr+0.02Mn	2661.2	72	230	0.027055	0.086427	290	0.330
49	Al20	5052		Tempered (H34)	97.2Al+2.5Mg+0.25Cr+0.0 5Si	2680.1	71.3	215	0.0266034	0.0802208	260	0.334
50	Al21	5052		Tempered (H32)	97.2Al+2.5Mg+0.25Cr+0.0 5Si	2680.1	71.3	195	0.0266034	0.0727584	230	0.334
51	Al22	5456		Tempered (O)	93.9Al+5.1Mg+0.8Mn+0.12 Cr	2692	69.1	160	0.025668	0.059430	310	0.334
52	Al23	5056		Tempered	94.76Al+5Mg+0.12Mn	2670	71	150	0.026591	0.056179	290	0.330

				(O)	+0.12Cr							
53	Al24	5083		Tempered (O)	94.7Al+4.4Mg+0.7Mn+0.15Cr	2696	71.3	145	0.026447	0.053783	290	0.330
54	Al25	5182		Tempered (O)	95.2Al+4.5Mg+0.35Mn	2675	71.8	140	0.026841	0.052336	275	0.330
55	Al26	6205		Tempered (T1)	98.4Al+0.5Mg+0.1Mn+0.1Cr+0.1Zr+0.1Si	2705.3	73.2	140	0.027058	0.051750	260	0.334
56	Al27	5154		Tempered (O)	96.2Al+3.5Mg+0.25Cr	2676	70.5	115	0.026345	0.042976	266	0.330
57	Al28	5254		Tempered (O)	96.2Al+3.5Mg+0.12Mn+0.25Cr	2685	71	115	0.026443	0.042830	360	0.334
58	Al29	5086		Tempered (O)	95.4Al+4Mg+0.4Mn+0.15Cr	2698	71	115	0.026315	0.042624	260	0.330
59	Al30	5454		Tempered (O)	96.3Al+2.7Mg+0.8Mn+0.12Cr	2715.4	70.1	115	0.025816	0.042315	250	0.334
60	Al31	6005		Tempered (T1)	98.7Al+0.5Mg+0.8Si	2692	71.8	105	0.026671	0.039000	170	0.334
61	Al32	7075		Tempered (O)	90Al+1.6Cu+2.5Mg+0.23Cr+5.6Zn	3032	72.8	105	0.024010	0.034630	270	0.340
62	Al33	5052		Tempered (O)	97.25Al+2.5Mg+0.25Cr	2685	71.1	90	0.026480	0.033519	195	0.334
63	Al34	2014		Tempered (O)	93.5Al+4.4Cu+0.5Mg+0.8Mn+0.8Si	3007	73.5	97	0.024443	0.032258	185	0.334
64	Al35	6066		Tempered (O)	95.7Al+1Cu+1.1Mg+0.8Mn+1.4Si	2784	72.8	83	0.026149	0.029813	150	0.334
65	Al36	4043		Tempered (O)	94.8Al+5.2Si	2680	71.3	69	0.026604	0.025746	145	0.334
66	Al37	2024		Tempered (O)	93.5Al+4.4Cu+1.5Mg+0.6Mn	2988	75.3	76	0.025200	0.025435	185	0.340
67	Al38	6070		Tempered (O)	96.8Al+0.28Cu+0.8Mg+0.7Mn+1.4Si	2737.4	71	69	0.025937	0.025206	145	0.334
68	Al39	3004		Tempered (O)	97.8Al+1Mg+1.2Mn	2747	70.8	69	0.025773	0.025118	180	0.334
69	Al40	2219		Tempered (O)	93Al+6.3Cu+0.06Ti+0.1V+0.3Mn+0.18Zr	3096	76.6	76	0.0247416	0.024547	170	0.334
70	Al41	5050		Tempered (O)	98.6Al+1.4Mg	2686	70	55	0.026061	0.020476	145	0.334
71	Al42	3105		Tempered	99Al+0.5Mg+0.55Mn	2722	71	55	0.026083	0.020205	115	0.334

Appendix –D-

				(O)								
72	Al43	6063		Tempered (O)	98.9Al+0.7Mg+0.4Si	2691.8	71	48	0.026376	0.017832	90	0.334
73	Al44	5457		Tempered (O)	98.7Al+1Mg+0.3Mn	2742	70	48	0.025529	0.017505	130	0.330
74	Al45	3003		Tempered (O)	98.6Al+1.2Mg+0.12Mn	2692	71.1	42	0.026412	0.015602	130	0.333
75	Al46	5005		Tempered (O)	99.2Al+0.8Mg	2692	70.2	41	0.026077	0.015230	110	0.330
76	Al47	1100		Tempered (O)	99Al+0.12Cu+0.88Mg	2688	71.3	34	0.026525	0.012649	90	0.333
77	Al48	1060		Tempered (O)	99.6Al+0.3Cu+0.1Mg	2690	71.3	28	0.0265056	0.0104089	69	0.333

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جمهورية العراق
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جامعة بابل
كلية الهندسة

إستبيان التصميم الأمثل لمنشأ جناح طائرة (GTX) تحت تأثير أحمال قصور ذاتي

رسالة
مقدمة إلى كلية الهندسة/قسم الهندسة الميكانيكية في جامعة بابل
كجزء من متطلبات نيل درجة الماجستير
في علوم الهندسة الميكانيكية
(ميكانيك تطبيقي)

أعدت من قبل الطالب
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بكالوريوس في علوم الهندسة الميكانيكية
أيلول، 2007