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Dynamic Analysis of Dual-spin Spacecraft Containing a Nutation Damper

A Thesis

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ABSTRACT

Dual spin spacecraft is a type of satellites consisting of a rotor and platform, where the platform is the despun and carrying the payload and the rotor is spun to provide angular momentum (gyroscopic stiffness) which keeps the spacecraft in the specific orientation.

The dynamic of these spacecrafts containing the proposed nutation damper consists of a ring totally filled with a viscous liquid with offset center, to improving the damping characteristics of the damping system, is investigated.

The equations of motion has been developed using Newton-Lagrange approach resulting equations in terms of spacecraft's and damper's parameters. These parameters are intended to be in dimensionless in there form.

The fluid motion inside the ring is in two distinct modes; the first mode is named nutation-synchronous mode and the second is the spin-synchronous mode. The expression of the nutation angle and time constant in both modes are developed using zero-order approximation technique. The equilibria states, stability conditions, and the analytical expression for residual nutation angle has been derived. The analytical results has been compared with those found numerically using computer simulation program named MATLAB, ver. 7.

The effect of various spacecraft's and damper's parameters on the dynamic and damping characteristics are also discussed. The three dimensional graphical representation of the first and the second relative equilibria states are introduced. The numerical results are compared with the analytical for both modes of motion, where the percentage error of the time constant for nutation mode is less than ($\approx 3.6\%$), and for spin mode is less than ($\approx 8\%$).



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Appendix A

Equations of Motion

It advantageous in solving the problem to introduce the dimensionless form for the variables and to express the parameters of the system in terms of suitable dimensionless parameters.

Let the dimensionless time variable τ be defined as:

$$\tau = \Omega t \quad \text{A.1}$$

So that, the time derivative with respect to τ is given by:

$$\frac{d}{dt}(\) = \Omega \frac{d(\)}{d\tau} = \Omega(\)' \quad \text{A.2}$$

where Ω is the initial spin rate of the rotor and $(\)'$ denotes differentiation with respect to the dimensionless time variable τ . Also, let the dimensionless angular velocity components be p , q , and r defined as:

$$p = \frac{\omega_u}{\Omega}, \quad q = \frac{\omega_v}{\Omega}, \quad r = \frac{\omega_z}{\Omega} \quad \text{A.3}$$

and the terms of moment of inertia of the fluid are:

$$I_u = m \left(\frac{1}{2} R^2 + h^2 + d^2 \right)$$
$$I_v = m \left(\frac{1}{2} R^2 + h^2 \right) \quad \text{A.4}$$

$$I_z = m(R^2 + d^2)$$

$$I_{uz} = mhd$$

let the following dimensionless parameters be defined as:

$\sigma = \frac{C}{A}$: The ratio of the rotor spin axis principle moment of inertia to the total transverse principal moment of inertia of the spacecraft.

$b = \frac{h}{R}$: The ratio of the height of the ring above the spacecraft center of mass to the ring mean radius.

$\eta = \frac{C_f}{m\Omega}$: The dimensionless damping constant.

$\varepsilon = \frac{mR^2}{A}$: The ratio of the inertia of the fluid to the total transverse principal moment of inertia of the spacecraft.

$$G = \frac{d}{R}.$$

$$\varsigma = 1 + \frac{d^2}{R^2}.$$

The equations of motion 4.13, 4.14, 4.15 and 4.19, as derived in chapter four, section 4.3, are

$$(A + I_u)\dot{\omega}_u - I_{uz}\left(\dot{\omega}_z + \ddot{\alpha}\right) - (A + I_v)\left(\omega_z + \dot{\alpha}\right)\omega_v + \left[(C + I_z)\omega_z + I_z\dot{\alpha} - I_{uz}\omega_u + C_p\Omega_p\right]\omega_v = 0 \quad \text{A.5}$$

$$(A + I_v)\dot{\omega}_v + \left[(A + I_u)\omega_u - I_{uz}\left(\omega_z + \dot{\alpha}\right)\right]\left(\omega_z + \dot{\alpha}\right) - \left[(C + I_z)\omega_z + I_z\dot{\alpha} - I_{uz}\omega_u + C_p\Omega_p\right]\omega_u = 0 \quad \text{A.6}$$

$$(C + I_z)\dot{\omega}_z + I_z\ddot{\alpha} - I_{uz}\dot{\omega}_u + [(A + I_v)\omega_v]\omega_u - \left[(A + I_u)\omega_u - I_{uz}\left(\omega_z + \dot{\alpha}\right)\right]\omega_v = 0 \quad \text{A.7}$$

$$I_z\left(\dot{\omega}_z + \ddot{\alpha}\right) - I_{uz}\dot{\omega}_u - \left[(A + I_u)\omega_u - I_{uz}\left(\omega_z + \dot{\alpha}\right)\right]\omega_v + [(A + I_v)\omega_v]\omega_u = -C_f R^2 \dot{\alpha} \quad \text{A.8}$$

subtracting Eq. A.8 from Eq. A.7, results in:

$$C\dot{\omega}_z = C_f R^2 \dot{\alpha} \quad \text{A.9}$$

and one can get:

$$\dot{\omega}_z = \frac{C_f R^2 \dot{\alpha}}{C} \quad \text{A.10}$$

substituting $\dot{\omega}_z$ from Eq. A.10 into Eq. A.8, and rearranging

$$I_z \left(\frac{C_f R^2}{C} \ddot{\alpha} + \dot{\alpha} \right) - I_{uz} \dot{\omega}_u - \left[(A + I_u) \omega_u - I_{uz} \left(\omega_z + \dot{\alpha} \right) \right] \omega_v + [(A + I_v) \omega_v] \omega_u = -C_f R^2 \dot{\alpha} \quad \text{A.11}$$

rearrange:

$$\ddot{\alpha} + \frac{C_f R^2}{C} \dot{\alpha} - \frac{I_{uz}}{I_z} \dot{\omega}_u + \frac{I_v - I_u}{I_z} \omega_v \omega_u + \frac{I_{uz}}{I_z} \left(\omega_z + \dot{\alpha} \right) \omega_v + \frac{C_f R^2}{I_z} \alpha = 0 \quad \text{A.12}$$

substituting of Eqs. A.2, A.3, and A.4 and using the definition of the dimensionless parameters σ, b, ε and η , then Eq. A.12 becomes:

$$\alpha'' + C_1 \alpha' + C_2 p q + C_3 [(r + \alpha') q - p'] = 0 \quad \text{A.13}$$

where the constants are given by:

$$C_1 = \frac{\eta \lambda_n}{\zeta \sigma^2} \left(1 + \frac{\varepsilon \zeta}{\sigma} \right), \quad C_2 = -\frac{G^2}{\zeta}, \quad C_3 = \frac{bG}{\zeta}$$

substituting for α'' from Eq. A.13, then Eq. 5 becomes:

$$\left(1 + \frac{I_u}{A} - \frac{I_{uz} C_3}{A} \right) p' + \left[\left(\frac{C}{A} - 1 \right) r - \alpha' + \frac{C_p \Omega_p}{A \Omega} \right] q + \left[\left(\frac{I_{uz} C_3}{A} - \frac{I_v}{A} + \frac{I_z}{A} \right) r + \left(\frac{I_{uz} C_3}{A} - \frac{I_v}{A} + \frac{I_z}{A} \right) \alpha' + \left(\frac{I_{uz} C_2}{A} - \frac{I_{uz}}{A} \right) p \right] q - \frac{I_{uz}}{A} r' + \frac{I_{uz} C_1}{A} \alpha' = 0 \quad \text{A.14}$$

dividing Eq. A.14 by the factor of p' and using the definition of the dimensionless parameters σ, b, ε and η , then Eq. A.14 becomes:

$$p' + \frac{(\lambda r - \alpha' + \lambda_s)}{D_1} q + \frac{(A_1 r + A_2 \alpha' + A_3 p)}{D_1} q - \frac{A_4}{D_1} r' + \frac{A_5}{D_1} \alpha' = 0 \quad \text{A.15}$$

where:

$$D_1 = 1 + \varepsilon \left(\frac{1}{2} + \frac{h^2}{R^2} + \frac{d^2}{R^2} - \frac{b^2 G^2}{\zeta} \right)$$

$$A_1 = \varepsilon \left(\frac{1}{2} + b^2 G^2 + G^2 - b^2 \right)$$

$$A_2 = \varepsilon \left(\frac{1}{2} + b^2 G^2 + G^2 - b^2 \right) = A_1$$

$$A_3 = -\varepsilon \left(bG \left(1 + \frac{G^2}{\zeta} \right) \right)$$

$$A_4 = \varepsilon bG$$

$$A_5 = \varepsilon C_1 bG$$

note that, $\lambda = \sigma - 1$, $\lambda_s = \frac{C_p \Omega_p}{A \Omega}$, $\lambda_n = \lambda + \lambda_s$, $\Omega_n = \lambda_n \Omega$

rearranging Eq. A.6 and substituting Eqs. A.2 and A.3, then Eq. A.6 becomes:

$$\begin{aligned} \left(1 + \frac{I_v}{A} \right) q' - \left[\left(\frac{C}{A} - 1 \right) r - \alpha' + \frac{C_p \Omega_p}{A \Omega} \right] p \\ + \left[\left(\frac{I_u - I_z}{A} \right) r + \left(\frac{I_u - I_z}{A} \right) \alpha' + \frac{I_{uz}}{A} p \right] p - \frac{I_{uz}}{A} (r + \alpha')^2 = 0 \end{aligned} \quad A.16$$

dividing Eq. A.16 by the factor of q' and using Eq. A.4 and using the definition of dimensionless parameters σ, b, ε and η , then Eq. A.16 becomes:

$$q' - \frac{(\lambda r - \alpha' + \lambda_s)}{D_2} p + \frac{(B_1 r + B_2 \alpha' + B_3 p)}{D_2} p - \frac{B_4}{D_2} (r + \alpha')^2 = 0 \quad A.17$$

where:

$$D_2 = 1 + \varepsilon \left(\frac{1}{2} + \frac{h^2}{R^2} \right)$$

$$B_1 = \varepsilon \left(\frac{h^2}{R^2} - \frac{1}{2} \right)$$

$$B_2 = \varepsilon \left(\frac{h^2}{R^2} - \frac{1}{2} \right) = B_1$$

$$B_3 = \varepsilon bG$$

$$B_4 = \varepsilon bG = B_3$$

by the same way Eqs. A.10 will be:

$$r' - C_4 \alpha' = 0 \quad A.18$$

where

$$C_4 = \frac{\eta}{\sigma} \varepsilon$$

thus equations of motion of the spacecraft provided with nutation damper are given by:

$$\alpha'' + C_1\alpha' + C_2pq + C_3[(r + \alpha')q - p'] = 0 \quad \text{A.19}$$

$$p' + \frac{(\lambda r - \alpha' + \lambda_s)}{D_1}q + \frac{(A_1r + A_2\alpha' + A_3p)}{D_1}q - \frac{A_4}{D_1}r' + \frac{A_5}{D_1}\alpha' = 0 \quad \text{A.20}$$

$$q' - \frac{(\lambda r - \alpha' + \lambda_s)}{D_2}p + \frac{(B_1r + B_2\alpha' + B_3p)}{D_2}p - \frac{B_4}{D_2}(r + \alpha')^2 = 0 \quad \text{A.21}$$

$$r' - C_4\alpha' = 0 \quad \text{A.22}$$

Appendix B

Solution of the Equation of the Nutation Angle Rate

The differentiation of the nutation angle rate θ' may be given by:

$$\theta' = \frac{ph_v - qh_u}{h_t} \quad \text{B.1}$$

where

$$h_u = (A + I_u)\omega_u - I_{uz}\left(\omega_z + \dot{\alpha}\right)$$

$$h_v = (A + I_v)\omega_v$$

$$h_t = \sqrt{h_u^2 + h_v^2}$$

substituting for h_u , h_v , and h_t in Eq. (B.1) then

$$\theta' = \frac{p(A + I_v)\omega_v - q\left[(A + I_u)\omega_u - I_{uz}\left(\omega_z + \dot{\alpha}\right)\right]}{\sqrt{(A + I_v)^2\omega_v^2 + \left[(A + I_u)\omega_u - I_{uz}\left(\omega_z + \dot{\alpha}\right)\right]^2}} \quad \text{B.2}$$

or in term of dimensionless variables:

$$\theta' = \frac{pq(I_v - I_u) + I_{uz}(r + \alpha')q}{A\sqrt{\left(1 + \frac{I_v}{A}\right)^2 q^2 + \left(1 + \frac{I_u}{A}\right)^2 p^2 - 2\left(1 + \frac{I_u}{A}\right)\frac{I_{uz}}{A}(r + \alpha')p + \left(\frac{I_{uz}}{A}\right)^2 (r + \alpha')^2}} \quad \text{B.3}$$

and in term of dimensionless parameters will be:

$$\theta' = \frac{pq\frac{(I_v - I_u)}{A} + \frac{I_{uz}}{A}(r + \alpha')q}{\left(1 + 2f(\varepsilon) + f(\varepsilon)^2\right)q^2 + \left(1 + 2f(\varepsilon) + f(\varepsilon)^2\right)p^2 - 2f(\varepsilon)(1 + f(\varepsilon))(r + \alpha')p + (f(\varepsilon))^2(r + \alpha')^2} \quad \text{B.4}$$

where $f(\varepsilon)$ represent the terms containing the parameter ε .

Substituting for I_u , I_v , and I_{uz} and taken into account $\varepsilon \ll 1$, so that,

Eq. (B.4) will be:

$$\theta' = \frac{(-pG^2 + bG(r + \alpha'))q\varepsilon}{\sqrt{p^2 + q^2}} \quad \text{B.5}$$

now $\tan \theta$ in terms of dimensionless parameters is developed.

from Ch.4 it is Known:

$$\tan \vartheta = \frac{h_t}{h_z} \quad \text{B.6}$$

from above the expression of the transverse angular momentum vector, h_t , in terms of dimensionless parameters and variables is given by:

$$h_t = A\sqrt{p^2 + q^2} + f(\varepsilon, \varepsilon^2) \quad \text{B.7}$$

the spin axis angular momentum component in terms of dimensionless parameters and variables is given by:

$$h_z = (C + I_z)r + I_z\alpha' - I_{uz}p + \frac{C_p\Omega_p}{\Omega} \quad \text{B.8}$$

sub. (B.7) and (B.8) in (B.6) to get:

$$\tan \theta = \frac{A\sqrt{p^2 + q^2}}{(C + I_z)r + I_z\alpha' - I_{uz}p + \frac{C_p\Omega_p}{\Omega}} \quad \text{B.9}$$

Or

$$\tan \theta = \frac{\sqrt{p^2 + q^2}}{\frac{(C + I_z)}{A}r + \frac{I_z}{A}\alpha' - \frac{I_{uz}}{A}p + \frac{C_p\Omega_p}{A\Omega}} \quad \text{B.10}$$

and Eq. (B.10) in terms of dimensionless parameters and variables is given by:

$$\tan \theta = \frac{\sqrt{p^2 + q^2}}{r(\sigma + f(\varepsilon)) + f(\varepsilon)\alpha' - f(\varepsilon)p + \lambda_s} \quad \text{B.11}$$

neglect ε terms:

$$\tan \theta = \frac{\omega_t}{r\sigma + \lambda_s} \quad \text{B.12}$$

where

ω_t is the transverse angular velocity component, and it is given by:

$$\omega_t = \sqrt{p^2 + q^2} \quad \text{B.13}$$

Appendix C

C.1- Solution of the Equation of Motion in the Spin-Synchronous Mode:

In this mode the spacecraft is being close to the nominal state and the fluid goes to the stationary state and oscillating. Since the change in the speed of the fluid is small compared to the change in the relative spin of the spinning rotor $\lambda_n \tau$. So the fluid motion can be expressed as:

$$\alpha = \alpha_0 + \tilde{\alpha} \quad \text{C.1}$$

such that $\alpha_0 \gg \tilde{\alpha}$.

where α_0 is the initial value of α and $\tilde{\alpha}$ represents small changes occurs in α , such that:

$$\begin{aligned} \sin(\alpha - \alpha_0) &= \sin \tilde{\alpha} = \tilde{\alpha} \\ \cos(\alpha - \alpha_0) &= \cos \tilde{\alpha} = 1 \end{aligned} \quad \text{C.2}$$

the following relations also can be established:

$$\begin{aligned} \sin(\alpha - \lambda_n \tau) &\approx \sin(\alpha_0 - \lambda_n \tau) \\ \cos(\alpha - \lambda_n \tau) &\approx \cos(\alpha_0 - \lambda_n \tau) \end{aligned} \quad \text{C.3}$$

the basis of the above assumptions is that the change in α is small compared with $\lambda_n \tau$.

using the above assumption, then Eq. (4.71) becomes:

$$\alpha'' + \frac{\eta \lambda_n}{\zeta \sigma^2} \alpha' - \left[\frac{bG}{\zeta} (1 + \lambda_n) \right] \omega_t \sin(\alpha_0 - \lambda_n \tau) = 0 \quad \text{C.4}$$

assume the forced oscillating solution of Eq. (C.4) is given by

$$\tilde{\alpha} = \alpha - \alpha_0 = A \sin(\alpha_0 - \lambda_n \tau) + B \cos(\alpha_0 - \lambda_n \tau) \quad \text{C.5}$$

to find the constants A and B, substituting Eq. (C.5) in (C.4), then

$$\begin{aligned} -\lambda_n^2 A \sin(\alpha_0 - \lambda_n \tau) - \lambda_n^2 B \cos(\alpha_0 - \lambda_n \tau) - \frac{\eta \lambda_n^2}{\zeta \sigma^2} A \cos(\alpha_0 - \lambda_n \tau) + \\ \frac{\eta \lambda_n^2}{\zeta \sigma^2} B \sin(\alpha_0 - \lambda_n \tau) - \left[\frac{bG}{\zeta} (1 + \lambda_n) \right] \omega_t \sin(\alpha_0 - \lambda_n \tau) = 0 \end{aligned} \quad \text{C.6}$$

then,

$$A = - \left[\frac{bG\sigma^2(1+\lambda_n)}{\zeta^2 \lambda_n^2 \left(\sigma^2 + \frac{\eta^2}{\zeta^2 \sigma^2} \right)} \right] \omega_t, \quad B = \left[\frac{bG\eta(1+\lambda_n)}{\zeta^2 \lambda_n^2 \left(\sigma^2 + \frac{\eta^2}{\zeta^2 \sigma^2} \right)} \right] \omega_t$$

substitute A and B in Eq. (C.5) then the expression of $\tilde{\alpha}$ becomes:

$$\tilde{\alpha} = - \left[\frac{bG\sigma^2(1+\lambda_n)}{\zeta^2 \lambda_n^2 \left(\sigma^2 + \frac{\eta^2}{\zeta^2 \sigma^2} \right)} \right] \omega_t \sin(\alpha_0 - \lambda_n \tau) + \left[\frac{bG\eta(1+\lambda_n)}{\zeta^2 \lambda_n^2 \left(\sigma^2 + \frac{\eta^2}{\zeta^2 \sigma^2} \right)} \right] \omega_t \cos(\alpha_0 - \lambda_n \tau) \quad C.7$$

rearrange Eq. (C.7) and substitute for ω_t , then becomes:

$$\tilde{\alpha} = F \tan \theta_s \left[-\zeta \sin(\alpha_0 - \lambda_n \tau) + \frac{\eta}{\sigma^2} \cos(\alpha_0 - \lambda_n \tau) \right] \quad C.8$$

where θ_s referred to the nutation angle in the spin-synchronous mode and the constant F is given by:

$$F = \frac{\sigma_n}{\lambda_n^2} \left[\frac{bG\sigma^2(1+\lambda_n)}{\zeta^2 \left(\sigma^2 + \frac{\eta^2}{\zeta^2 \sigma^2} \right)} \right]$$

C.2- Solution of the Nutation Angle θ_s in the Spin-Synchronous Mode

The differential equation of the nutation angle rate Eq. (4.50) is given by:

$$\theta'_s = - \left[-G^2 \sigma_n \tan \theta_s \cos(\alpha - \lambda_n \tau) + bG(1+\alpha') \right] \sin(\alpha - \lambda_n \tau) \quad C.9$$

it was mentioned that $\tilde{\alpha}$ represents small variation in α such that

$\alpha = \alpha_0 + \tilde{\alpha}$, where $\alpha_0 \gg \tilde{\alpha}$. using the approximation of small angle, then

$$\sin(\alpha - \alpha_0) = \sin \tilde{\alpha} \approx \tilde{\alpha}$$

$$\cos(\alpha - \alpha_0) = \cos \tilde{\alpha} \approx 1$$

$$\sin(\alpha + \alpha_0) \approx \sin \alpha_0 \quad C.10$$

$$\sin(\alpha + \alpha_0) = \sin(2\alpha_0 + \tilde{\alpha}) \approx \sin 2\alpha_0$$

$$\cos(\alpha + \alpha_0) = \cos(2\alpha_0 + \tilde{\alpha}) \approx \cos 2\alpha_0$$

using the above mentioned relations, then the expression of $\sin(\alpha - \lambda_n \tau)$ may be given by:

$$\sin(\alpha - \lambda_n \tau) = \sin(\alpha_0 + \tilde{\alpha} - \lambda_n \tau) = (\sin \alpha_0 + \tilde{\alpha} \cos \alpha_0) \cos \lambda_n \tau - (\cos \alpha_0 - \tilde{\alpha} \sin \alpha_0) \sin \lambda_n \tau$$

C.11

substituting for $\tilde{\alpha}$ from Eq. (C.8), then Eq. (C.11) becomes:

$$\sin(\alpha - \lambda_n \tau) = \left[\sin \alpha_0 + F \tan \theta_s \cos \alpha_0 \left(-\varsigma \sin(\alpha_0 - \lambda_n \tau) + \frac{\eta}{\sigma^2} \cos(\alpha_0 - \lambda_n \tau) \right) \right] \cos \lambda_n \tau$$

$$- \left[\cos \alpha_0 - F \tan \theta_s \sin \alpha_0 \left(-\varsigma \sin(\alpha_0 - \lambda_n \tau) + \frac{\eta}{\sigma^2} \cos(\alpha_0 - \lambda_n \tau) \right) \right] \sin \lambda_n \tau$$

C.12

since that in the spin-synchronous mode, the nutation angle θ_s is small enough such that $\tan \theta_s = \theta_s$.

using the above mentioned assumption and Eq. (C.12), and using the differentiation of $\tilde{\alpha}'$ term from Eq. (C.8), then Eq. (C.9) becomes:

$$\theta'_s = -\varepsilon \left[-\sigma_n \theta_s G^2 \cos(\alpha - \lambda_n \tau) + bG \left[1 + F \theta_s \left(\varphi \lambda_n \cos(\alpha_0 - \lambda_n \tau) + \frac{\eta \lambda_n}{\sigma^2} \sin(\alpha_0 - \lambda_n \tau) \right) \right] \right]$$

$$\times \left[\begin{aligned} & \left[\sin \alpha_0 + F \theta_s \cos \alpha_0 \left(-\varsigma \sin(\alpha_0 - \lambda_n \tau) + \frac{\eta}{\sigma^2} \cos(\alpha_0 - \lambda_n \tau) \right) \right] \cos \lambda_n \tau \\ & - \left[\cos \alpha_0 - F \theta_s \sin \alpha_0 \left(-\varsigma \sin(\alpha_0 - \lambda_n \tau) + \frac{\eta}{\sigma^2} \cos(\alpha_0 - \lambda_n \tau) \right) \right] \sin \lambda_n \tau \end{aligned} \right]$$

C.13

expanding the trigonometric terms, then Eq. (C.13) becomes:

$$\theta'_s = -\varepsilon \left\{ -\sigma_n \theta_s G^2 \left[\sin \alpha_0 \cos \alpha \cos^2 \lambda_n \tau + \sin \lambda_n \tau \cos \lambda_n \tau (\sin \alpha_0 \sin \alpha - \cos \alpha_0 \cos \alpha) - \cos \alpha_0 \sin \alpha \sin^2 \lambda_n \tau \right] \right.$$

$$+ bG \left[\sin(\alpha_0 - \lambda_n \tau) + F \theta_s \left[\begin{aligned} & -\varsigma \left(\sin \alpha_0 \cos \alpha_0 \cos^2 \lambda_n \tau - \cos \alpha_0^2 \sin \lambda_n \tau \cos \lambda_n \tau \right) \\ & + \sin \alpha_0^2 \sin \lambda_n \tau \cos \lambda_n \tau - \sin \alpha_0 \cos \alpha_0 \sin^2 \lambda_n \tau \right) \\ & + \frac{\eta}{\sigma^2} \left(\cos \alpha_0^2 \cos^2 \lambda_n \tau + 2 \sin \alpha_0 \cos \alpha_0 \sin \lambda_n \tau \cos \lambda_n \tau \right) \\ & + \sin \alpha_0^2 \sin^2 \lambda_n \tau \right) \\ & + \varphi \lambda_n \left(\sin \alpha_0 \cos \alpha_0 \cos^2 \lambda_n \tau - \cos \alpha_0^2 \sin \lambda_n \tau \cos \lambda_n \tau \right) \\ & + \sin \alpha_0^2 \sin \lambda_n \tau \cos \lambda_n \tau - \sin \alpha_0 \cos \alpha_0 \sin^2 \lambda_n \tau \right) \\ & + \frac{\eta \lambda_n}{\sigma^2} \left(\sin \alpha_0^2 \cos^2 \lambda_n \tau - 2 \sin \alpha_0 \cos \alpha_0 \sin \lambda_n \tau \cos \lambda_n \tau \right) \\ & + \cos \alpha_0^2 \sin^2 \lambda_n \tau \right) \end{aligned} \right] \right] \right\}$$

C.14

since that the nutation angle is small then the terms of θ_s^2 can be neglected and using Eq. C.11 and the following relationships:

$$\sin^2 x + \cos^2 x = 1, \quad \sin 2x = 2 \sin x \cos x, \quad \sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

then Eq. C.14 is written as:

$$\theta_s' = -\theta_s \left[E_1 \cos 2\lambda_n \tau + E_2 \sin 2\lambda_n \tau + \frac{FbG\eta(1+\lambda_n)}{2\sigma^2} \varepsilon \right] + bG\varepsilon \sin(\alpha_0 - \lambda_n \tau) \quad \text{C.15}$$

where:

$$E_1 = \left[-\frac{\zeta}{2} \sin 2\alpha_0 + \frac{\eta}{2\sigma^2} \cos 2\alpha_0 + \frac{\zeta\lambda_n}{2} \sin 2\alpha_0 - \frac{\eta\lambda_n}{2\sigma^2} \cos 2\alpha_0 \right] FbG\varepsilon + \frac{G^2\sigma_n}{2} \sin 2\alpha_0$$

$$E_2 = \left[\frac{\zeta}{2} \cos 2\alpha_0 + \frac{\eta}{2\sigma^2} \sin 2\alpha_0 - \frac{\zeta\lambda_n}{2} \cos 2\alpha_0 - \frac{\eta\lambda_n}{2\sigma^2} \sin 2\alpha_0 \right] FbG\varepsilon - \frac{G^2\sigma_n}{2} \cos 2\alpha_0$$

Eq. C.15 can be written in terms of spin-synchronous time constant as:

$$\theta_s' = -\theta_s \left[E_1 \cos 2\lambda_n \tau + E_2 \sin 2\lambda_n \tau + \frac{1}{\tau_{cs}} \right] + bG\varepsilon \sin(\alpha_0 - \lambda_n \tau) \quad \text{C.16}$$

where:

$$\tau_{cs} = \frac{2\sigma^2}{FbG\eta(1+\lambda_n)\varepsilon} = \frac{2\lambda_n^2\zeta^2 \left(\sigma^2 + \frac{\eta^2}{\zeta^2\sigma^2} \right)}{\sigma_n b^2 G^2 \eta (1+\lambda_n)^2 \varepsilon} \quad \text{C.17}$$

one can see that the nutation angle time history consists of dominant exponential decay super imposed on it an oscillation of small amplitude represents the effect of the trigonometric terms. So that, the general solution of Eq. C.16 is given by:

$$\theta_s = \theta_{sc} + \theta_{sp.I.} \quad \text{C.18}$$

where:

θ_{sc} : Is the complementary part of the solution.

$\theta_{sp.I.}$: is the particular part of the solution.

and these solution may be given by:

$$\theta_{sc} = c e^{-\frac{\tau}{\tau_{cs}}} \quad \text{C.19a}$$

$$\theta_{sp.I.} = A' \sin(\alpha_0 - \lambda_n \tau) + B' \cos(\alpha_0 - \lambda_n \tau) \quad \text{C.19b}$$

substitute Eq. C.19b in Eq. C.16 to get:

$$A' = \frac{bG\varepsilon}{\tau_{cs} \left(\lambda_n^2 + \frac{1}{\tau_{cs}^2} \right)}, \quad B' = \frac{\lambda_n bG\varepsilon}{\left(\lambda_n^2 + \frac{1}{\tau_{cs}^2} \right)}$$

substituting the solution of Eq. C.19b in Eq. C.18 and using the initial condition ($\theta_s = \theta_{s0}$ at time $\tau = \tau_0$) then the constant c is given by:

$$c = \theta_{s_0} - \frac{\left(\left(\frac{\sin(\alpha_0 - \lambda_n \tau_0)}{\tau_{cs}} \right) + \lambda_n \cos(\alpha_0 - \lambda_n \tau_0) \right)}{\left(\lambda_n^2 + \frac{1}{\tau_{cs}^2} \right)} bG\varepsilon$$

and from Eq. C.18 the complete solution of the nutation angle θ_s is given by:

$$\theta_s = \left[\theta_{s_0} - \frac{\left(\left(\frac{\sin(\alpha_0 - \lambda_n \tau_0)}{\tau_{cs}} \right) + \lambda_n \cos(\alpha_0 - \lambda_n \tau_0) \right)}{\left(\lambda_n^2 + \frac{1}{\tau_{cs}^2} \right)} bG\varepsilon \right] e^{\frac{-(\tau - \tau_0)}{\tau_{cs}}} + \frac{\left(\left(\frac{\sin(\alpha_0 - \lambda_n \tau_0)}{\tau_{cs}} \right) + \lambda_n \cos(\alpha_0 - \lambda_n \tau_0) \right)}{\left(\lambda_n^2 + \frac{1}{\tau_{cs}^2} \right)} bG\varepsilon \quad \text{C.20}$$

Appendix D

Simulation Program figures

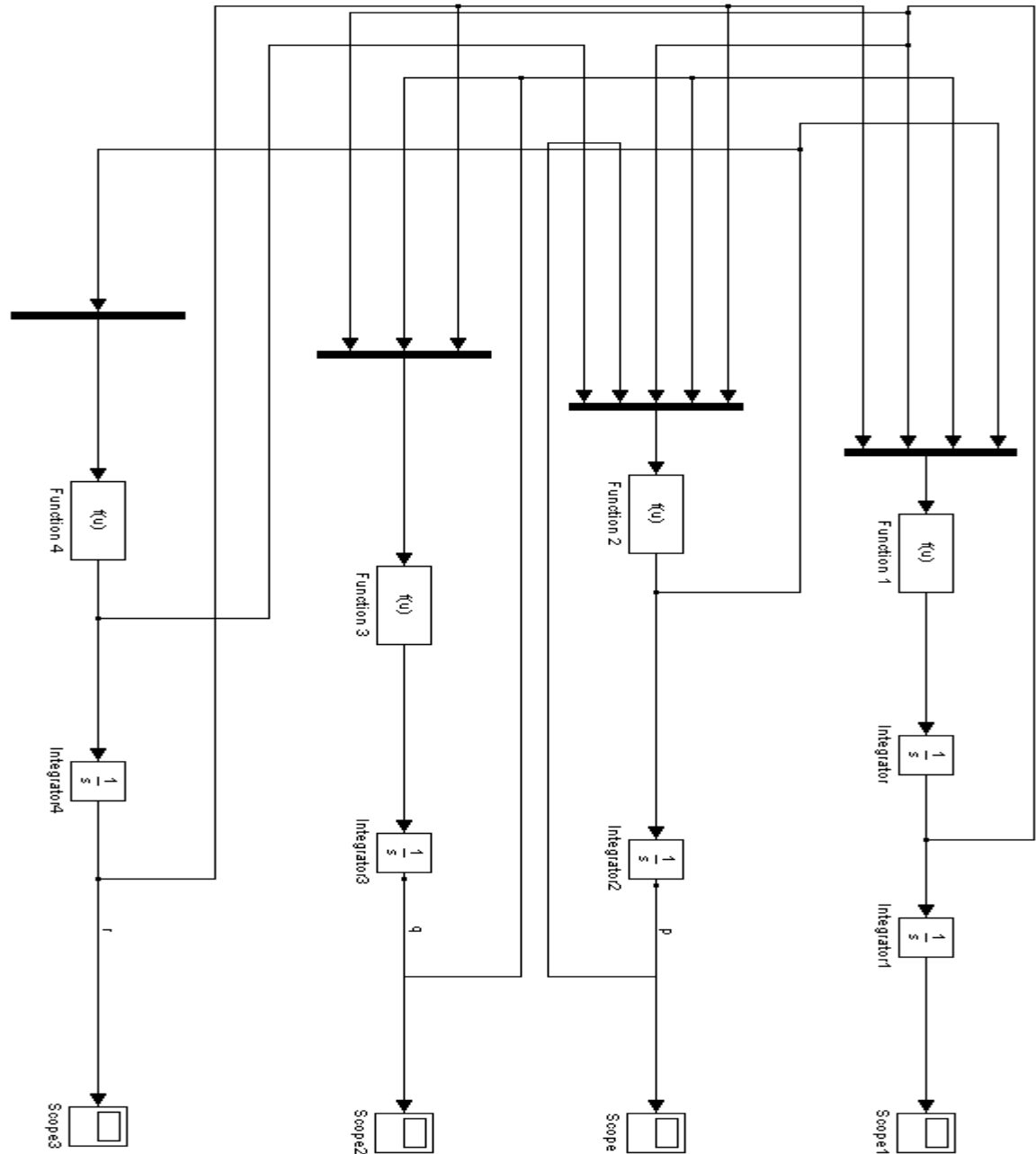


Fig. (D.1) Simulink program for calculation of the α , p , q , and r components.

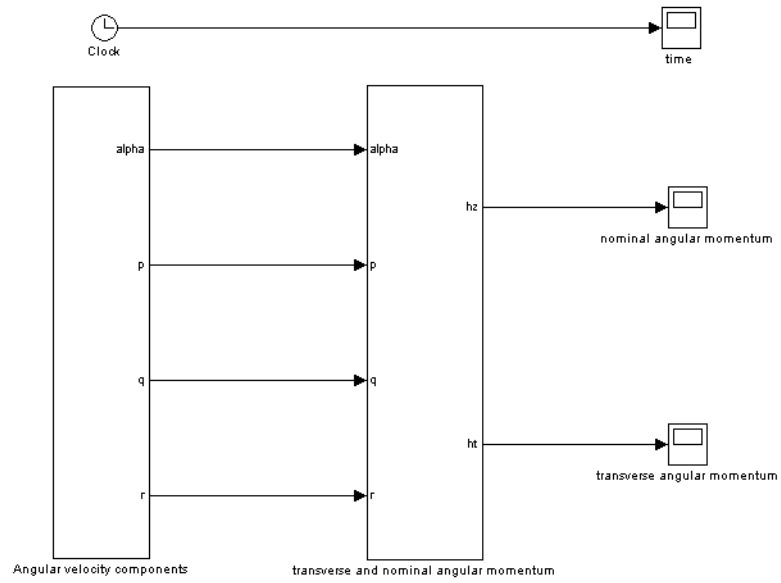


Fig. (D.2) Simulink program for calculation transverse and nominal angular momentum.

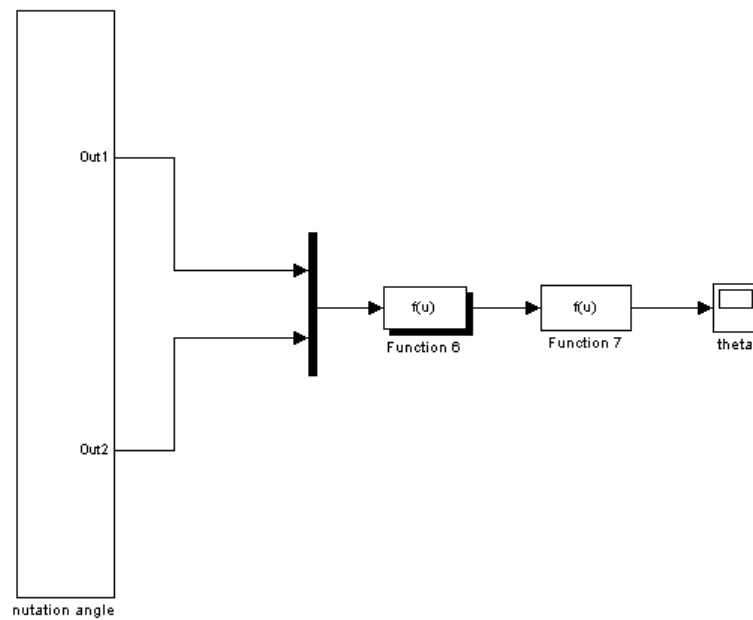


Fig. (D.3) Simulink program for calculation nutation angle time history.

CERTIFICATION

We certify that this thesis entitled "**Dynamic Analysis of Dual-Spin Spacecraft Containing a Nutation Damper**" and prepared by "**Ali Mohammed Hussein Yusif**" has been carried out completely under our supervision at the University of Babylon, Department of Mechanical Engineering, as in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering.

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Asst. Prof. Dr. Ahmed A. Al-Ragihy
(Supervisor)

Date / /2007

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Date / /2007

EXAMINING COMMITTEES CERTIFICATE

We certify that this thesis entitled "Dynamic Analysis of Dual-Spin Spacecraft Containing a Nutation Damper" and as examining committee, examined the student " Ali Mohammed Hussein Yusif" in its contents and that in our opinion it meets the standard of thesis for the degree of Master of Science in Mechanical Engineering.

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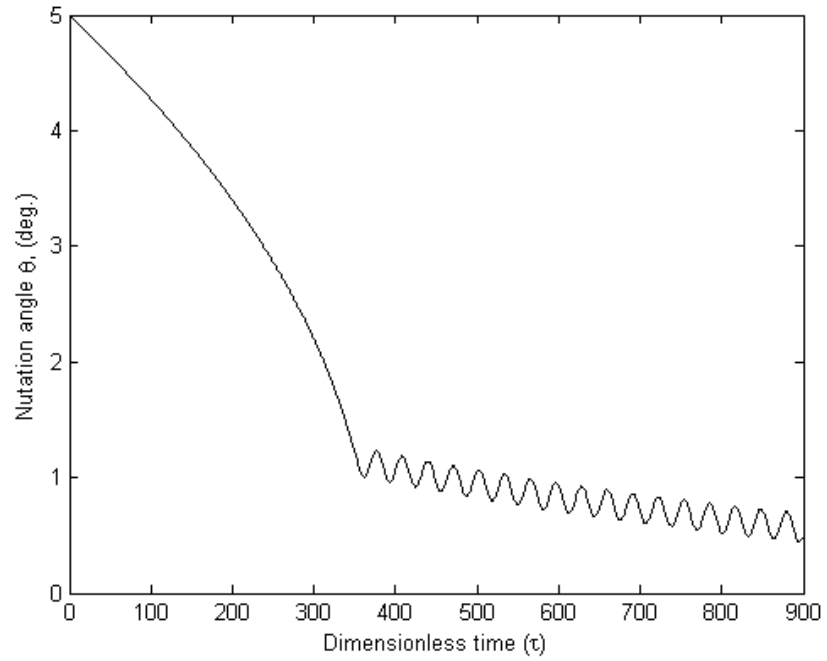


Fig. (5.1). Nutation angle time history of present work (analytical).

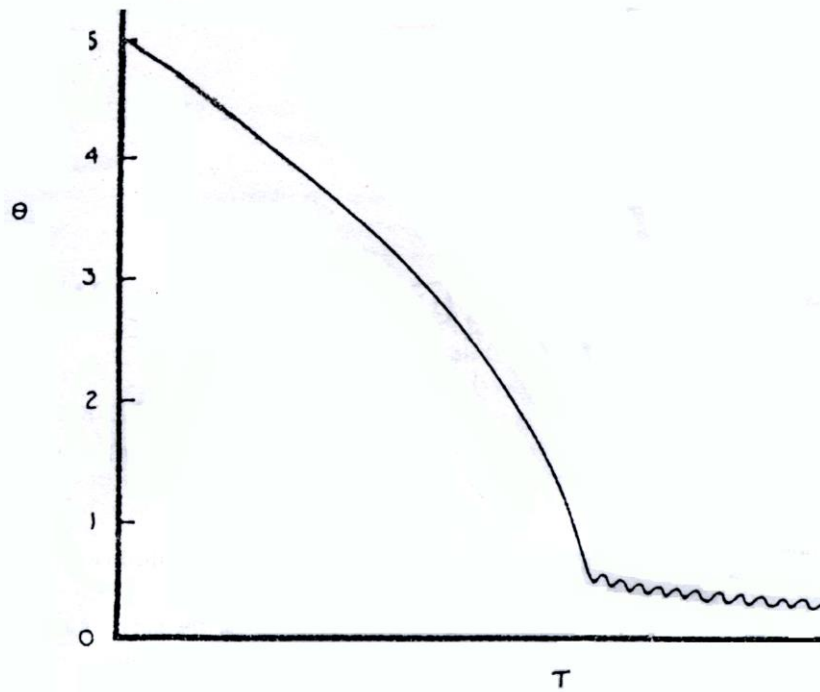


Fig. (5.2). Typical nutation angle time history of the ref.(2).

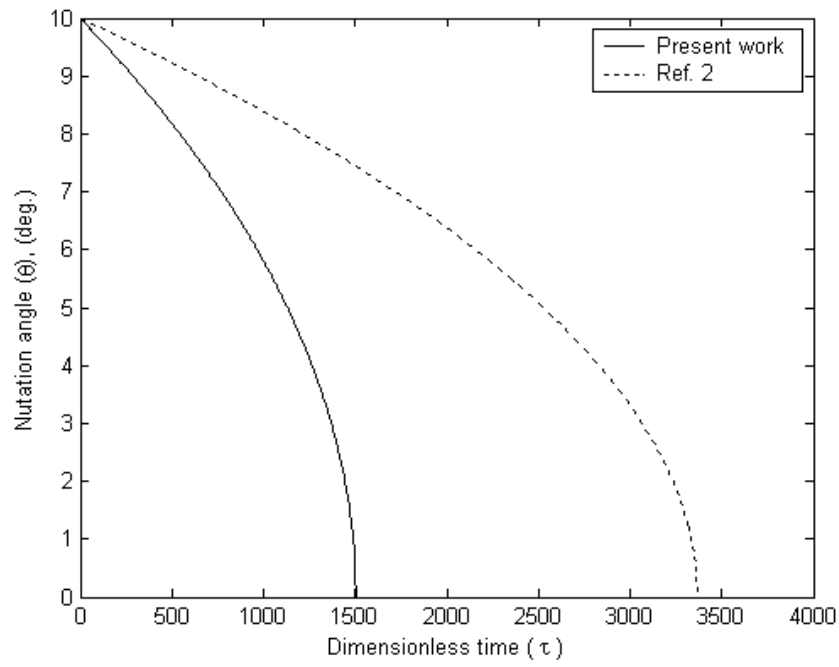


Fig. (5.3). Comparison of the nutation angle time history of present work with ref.(2) for the nutation-synchronous mode.

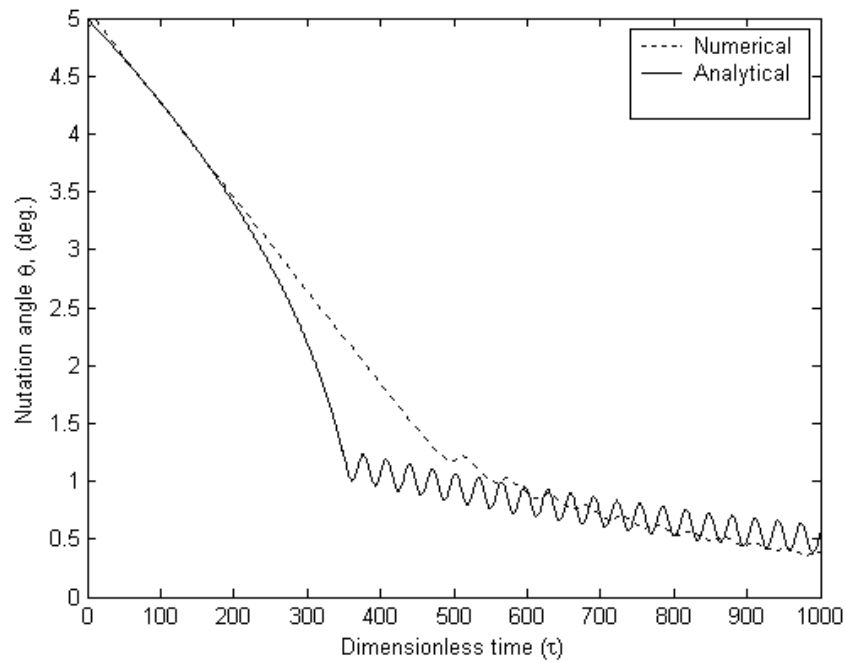


Fig. (5.4). Comparison between numerical and analytical solution of the nutation angle time history.

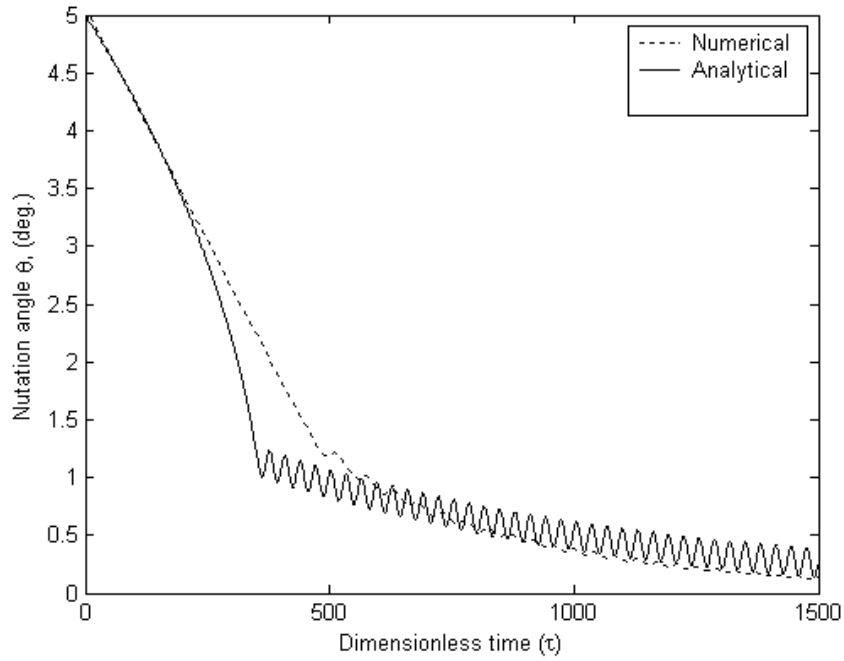


Fig. (5.5). Comparison between numerical and analytical solution of the nutation angle time history.

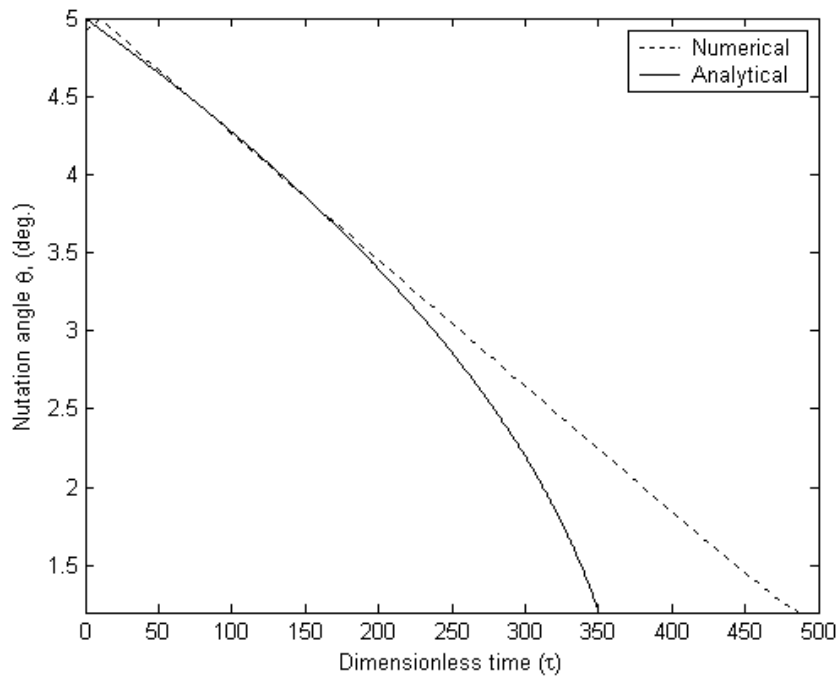


Fig. (5.6). Comparison of the nutation angle time histories for nutation-synchronous mode.

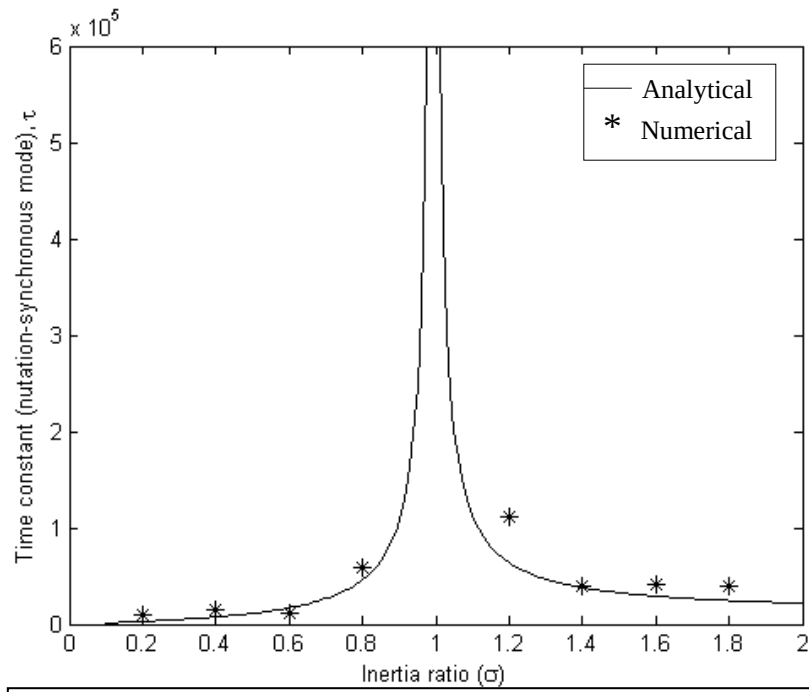


Fig. (5.7). Influence of the inertia ratio (σ) on the time constant for the nutation-synchronous mode.

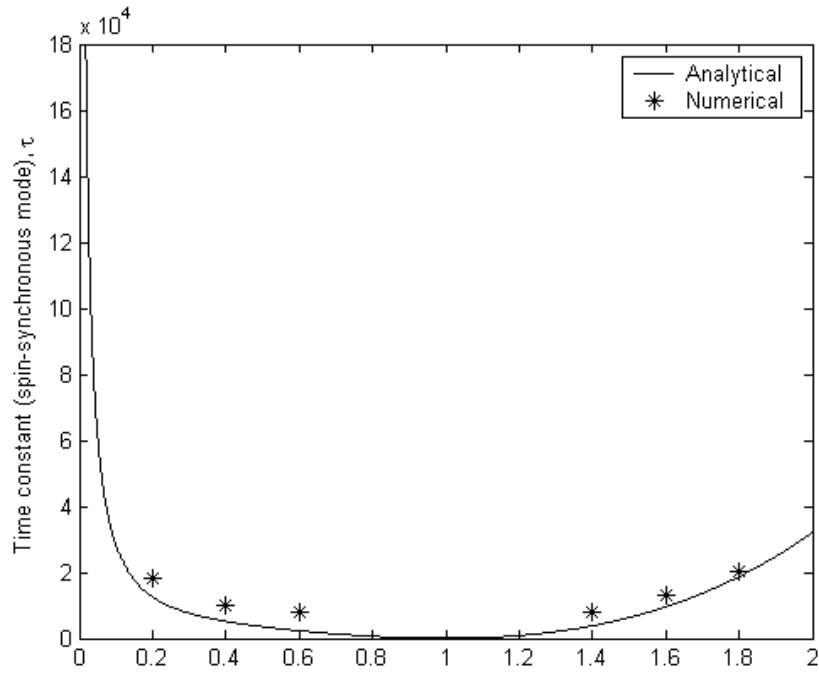


Fig. (5.8). Influence of the inertia ratio (σ) on the time constant for the spin-synchronous mode.

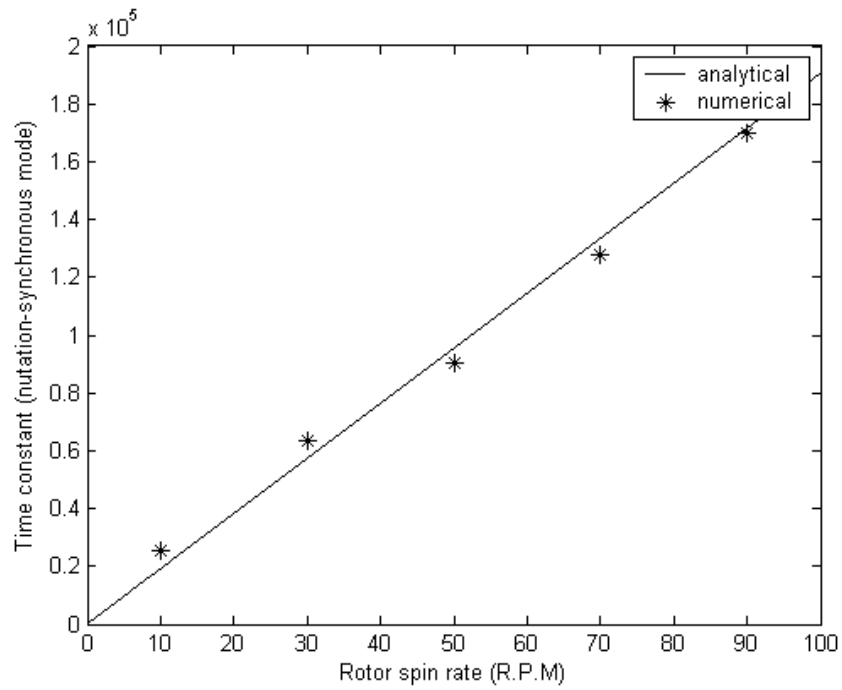


Fig. (5.9) Influence of the rotor spin rate (Ω) on the time constant for nutation-synchronous mode.

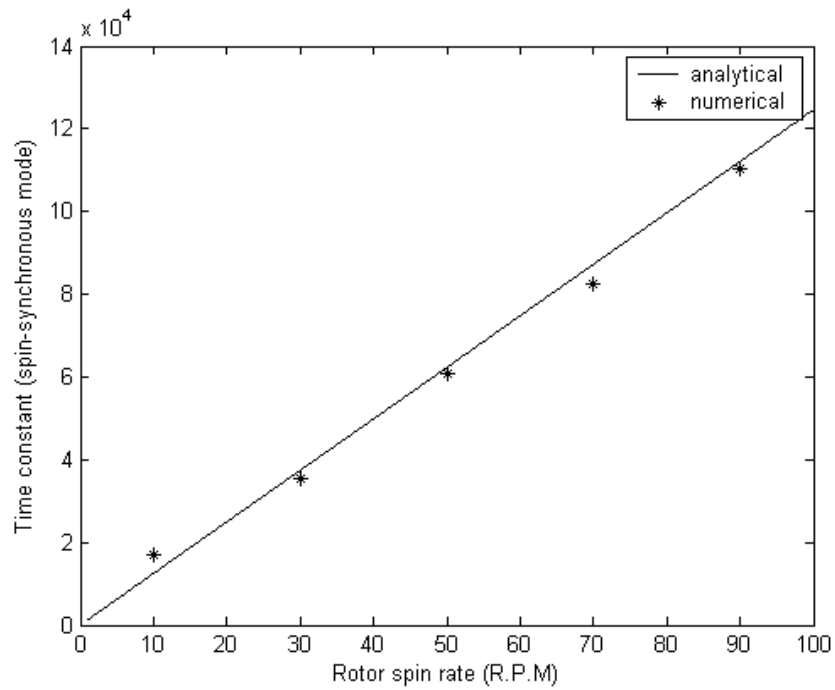


Fig. (5.10) Influence of the rotor spin rate (Ω) on the time constant for spin-synchronous mode.

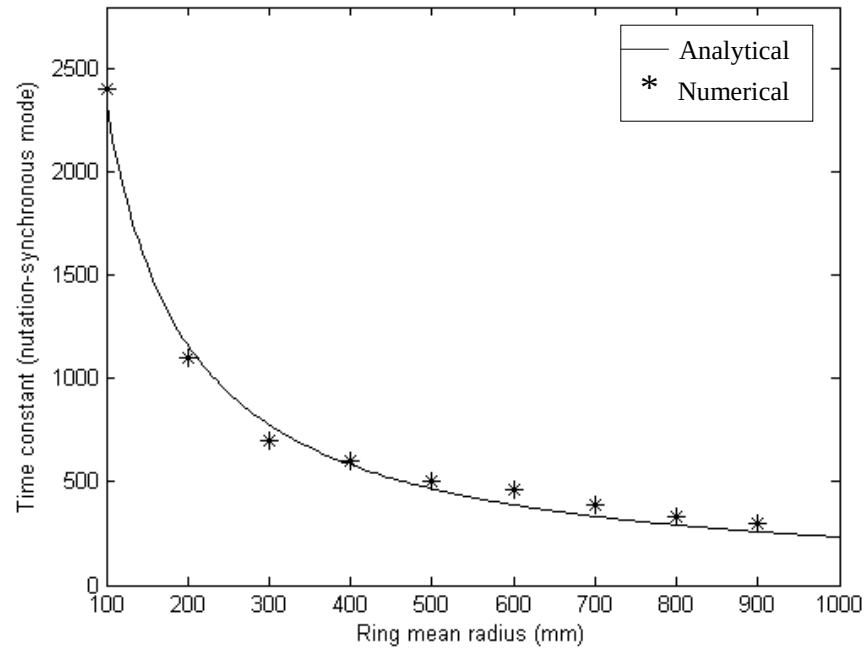


Fig. (5.11) Influence of the ring mean radius on the time constant for nutation-synchronous mode.

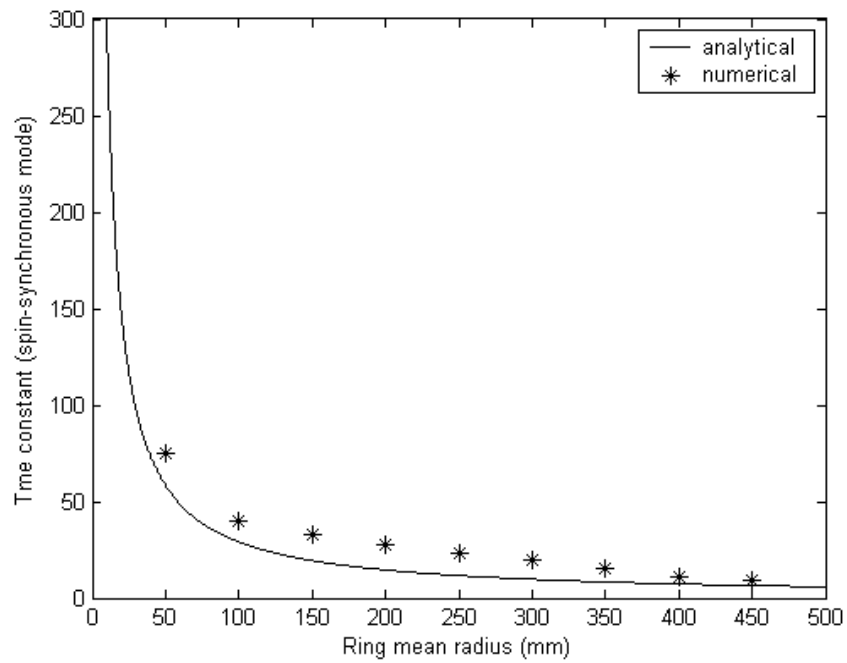


Fig. (5.12) Influence of the ring mean radius on the time constant for spin-synchronous mode.

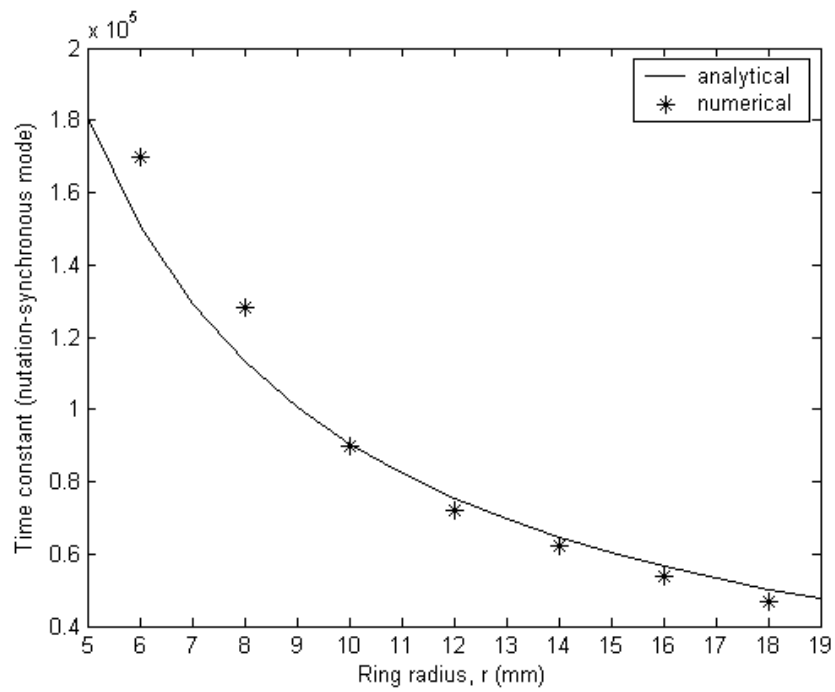


Fig. (5.13) Influence of the ring radius on the time constant for nutation-synchronous mode.

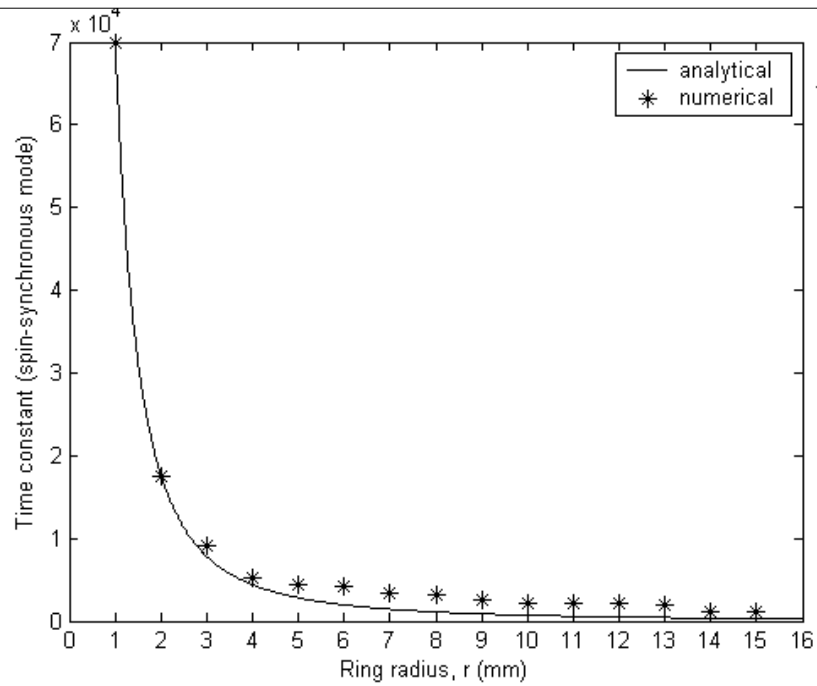


Fig. (5.14) Influence of the ring radius on the time constant for spin-synchronous mode.

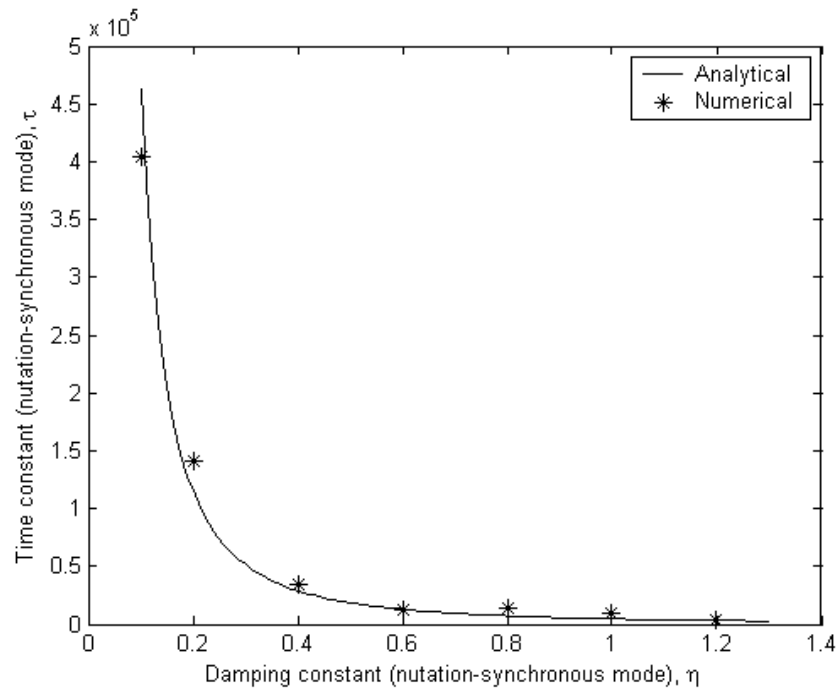


Fig. (5.15) Influence of the damping constant on the time constant for nutation-synchronous mode.

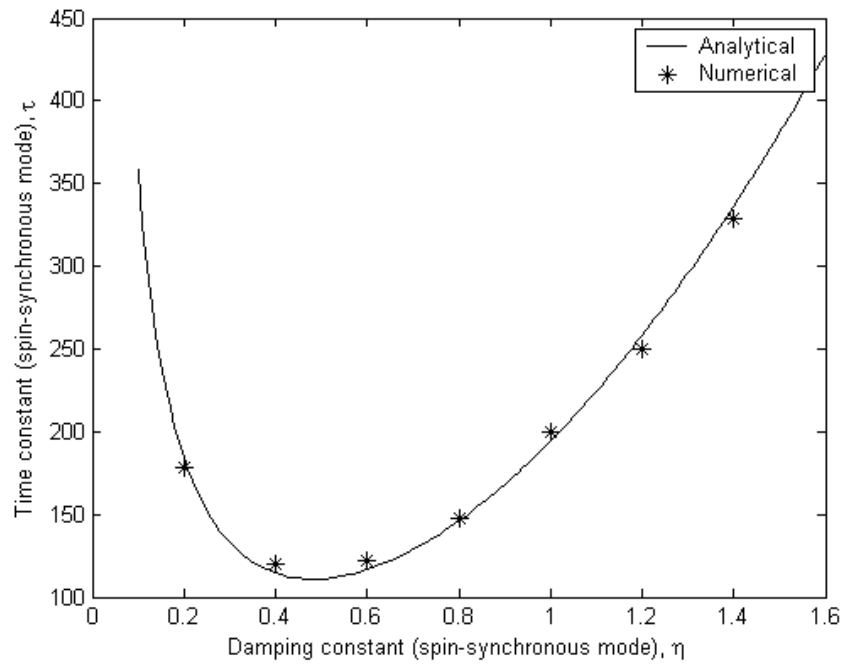


Fig. (5.16) Influence of the damping constant on the time constant for spin-synchronous mode.

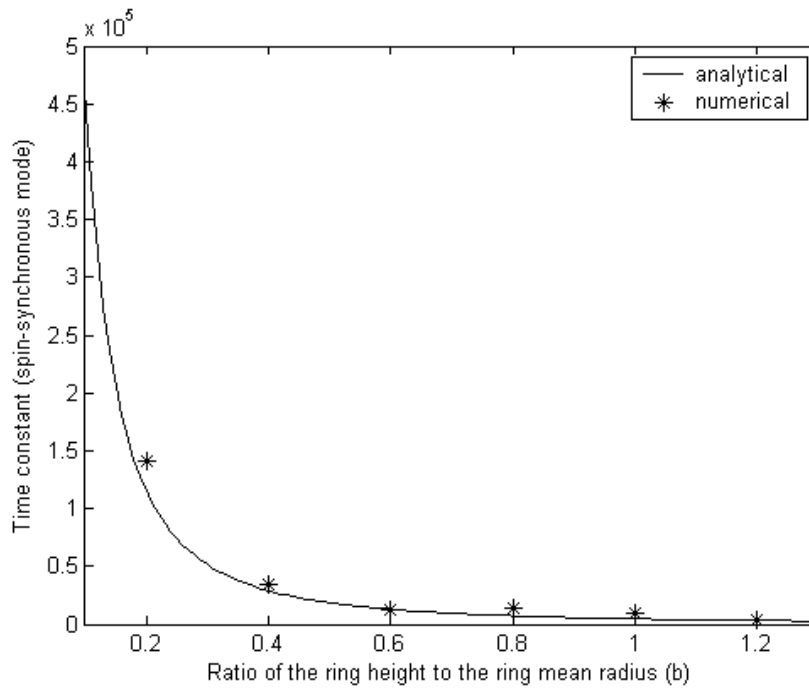


Fig. (5.17) Influence of the ratio of the ring height to the ring mean radius, b on the time constant for spin-synchronous.

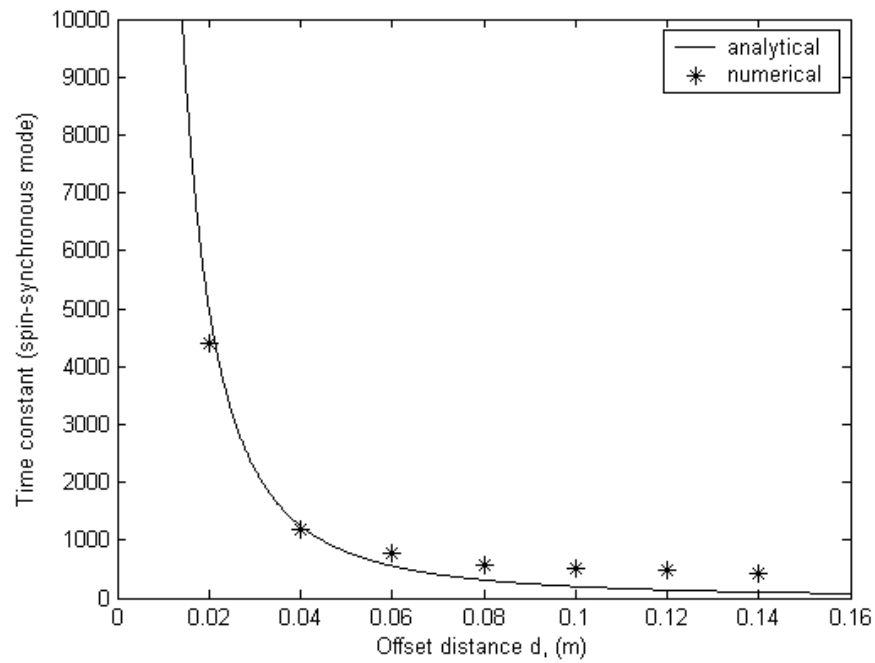


Fig. (5.18) Influence of the offset distance, d on the time constant for spin-synchronous mode.

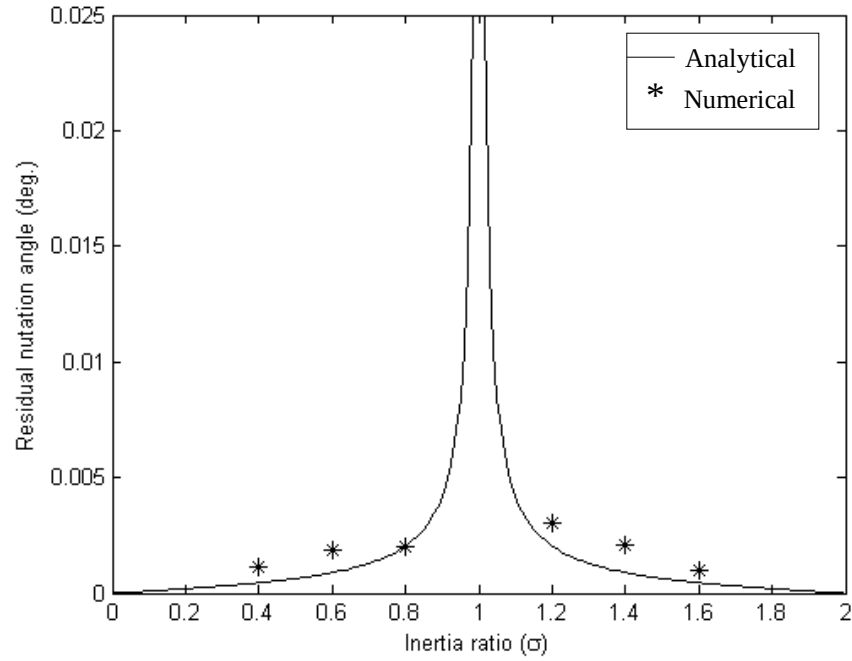


Fig. (5.19) Variation of the residual nutation angle with inertia ratio (σ).

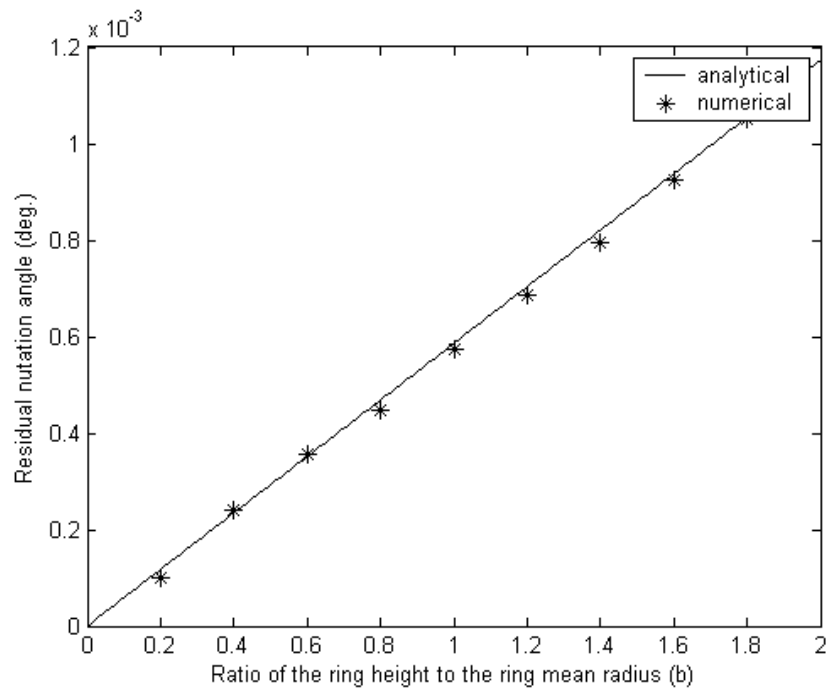


Fig. (5.20) Variation of the residual nutation angle with the ratio of the ring height to the ring mean radius (b).

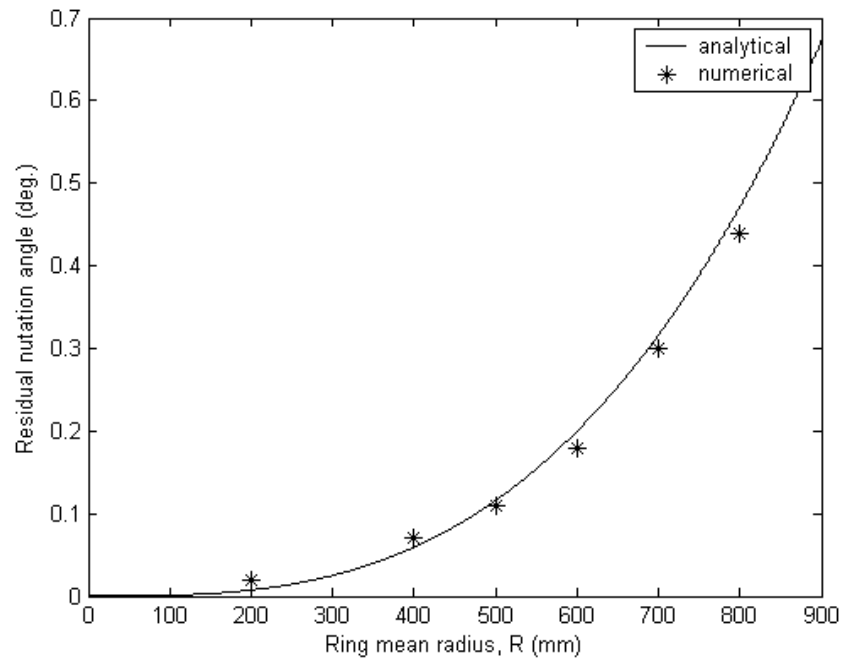


Fig. (5.21) Variation of the residual nutation angle with the ring mean radius (R).

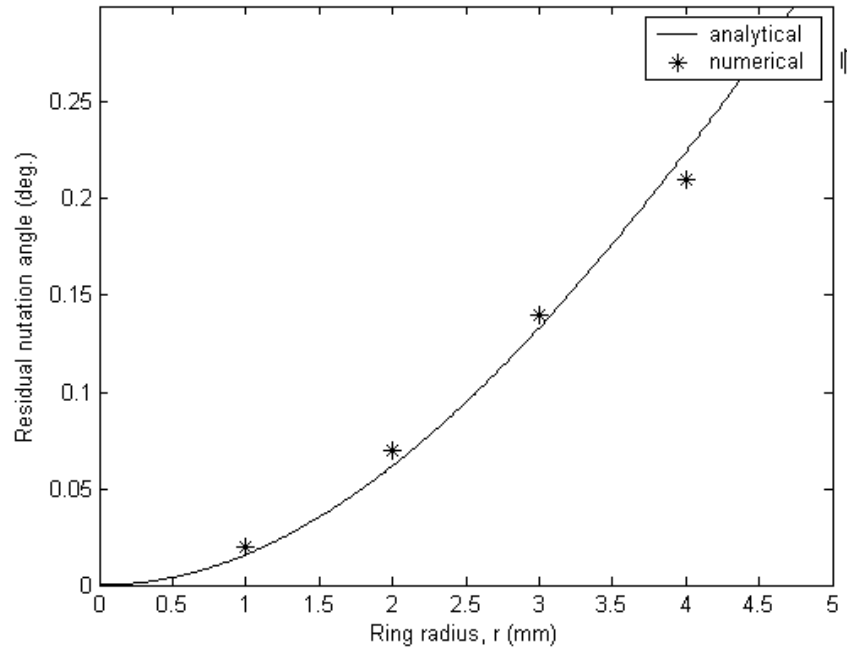


Fig. (5.22) Variation of the residual nutation angle with the ring radius (r).

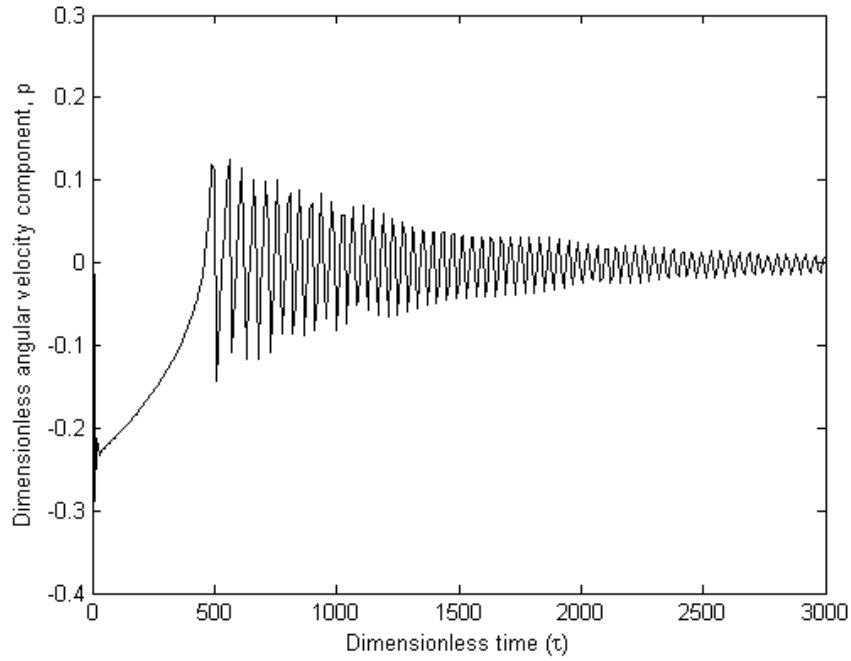


Fig. (5.23) Time history of dimensionless angular velocity component (p) for condition $(p_0, q_0, r_0)^T = (0.15, 0.3, 0.9)^T$.
First relative equilibrium state.

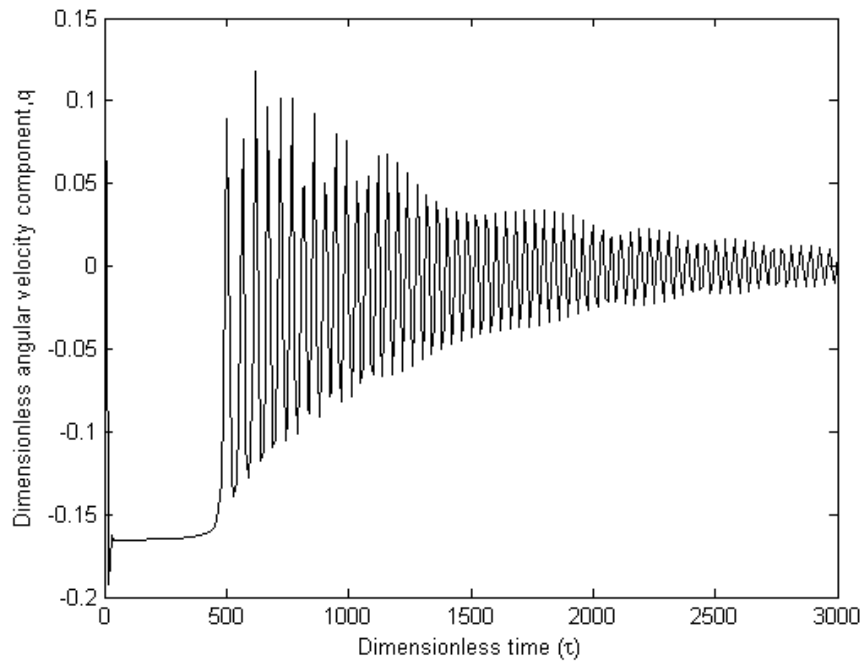


Fig. (5.24) Time history of dimensionless angular velocity component (q) for condition $(p_0, q_0, r_0)^T = (0.15, 0.3, 0.9)^T$.
First relative equilibrium state.

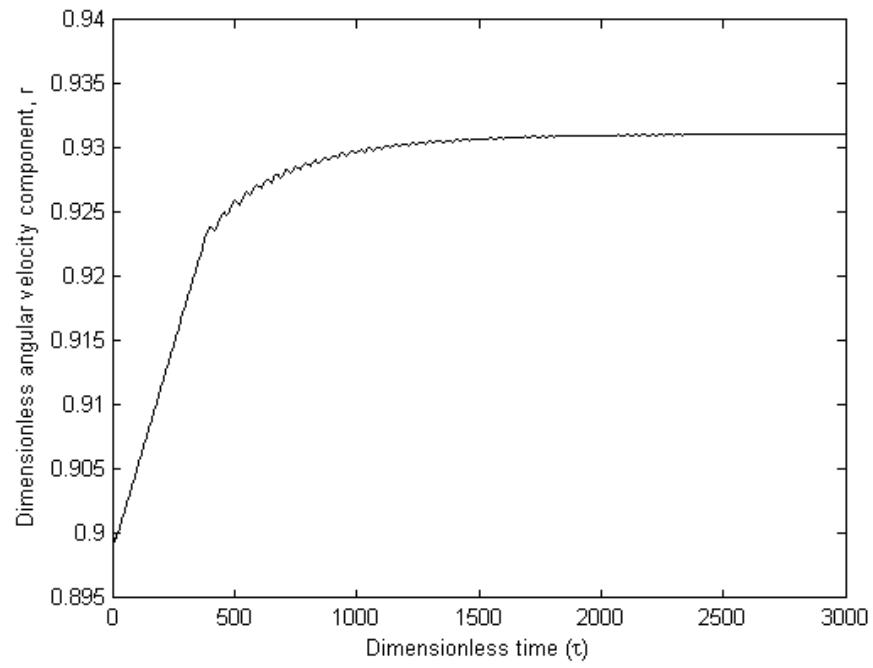


Fig. (5.25) Time history of dimensionless angular velocity component (r) for condition $(p_0, q_0, r_0)^T = (0.15, 0.3, 0.9)^T$.
For the first relative equilibrium state.

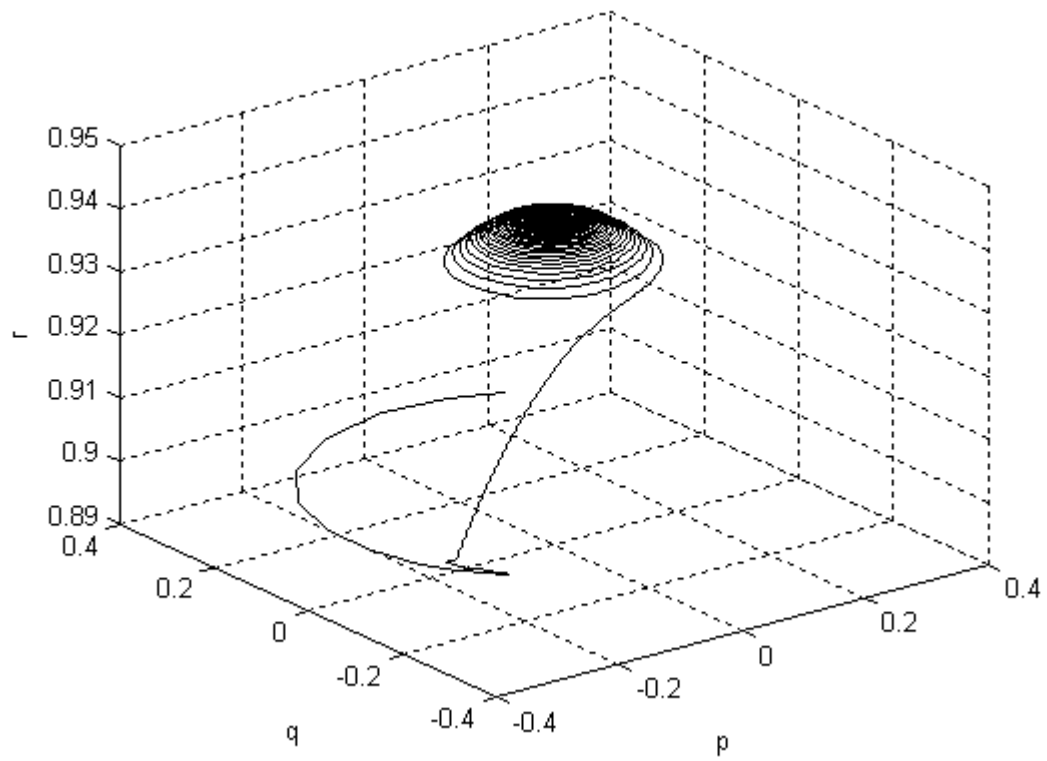


Fig. (5.26) Three dimensional visualization shows that the spacecraft diverge from second relative stability and reaches to the first relative stability, $\sigma=1.2$, $(p_o, q_o, r_o)^T=(0.15, 0.3, 0.9)^T$.

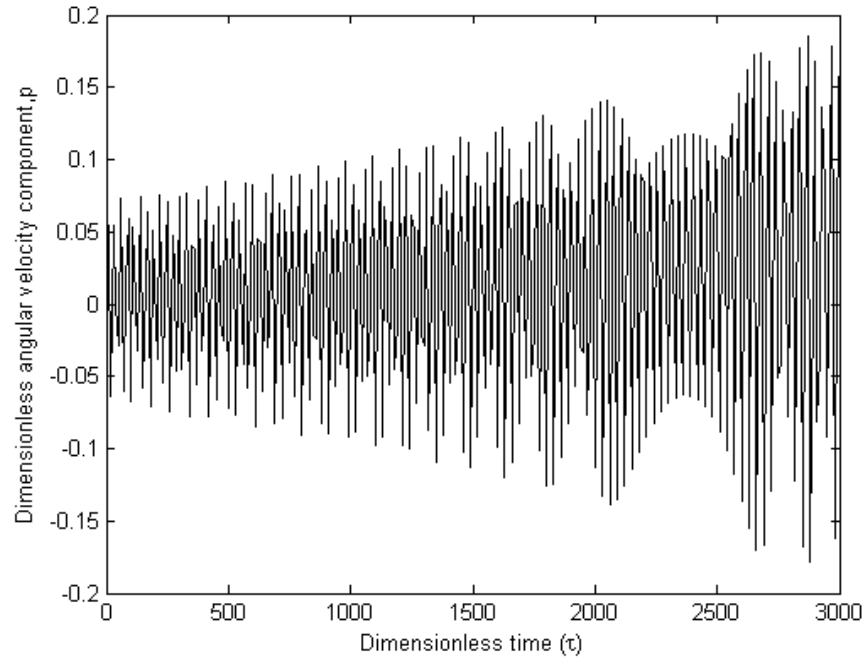


Fig. (5.27) Time history of dimensionless angular velocity component (p) for condition $(p_0, q_0, r_0)^T = (0.01, 0.07, 0.95)^T$.
Second relative equilibrium state.

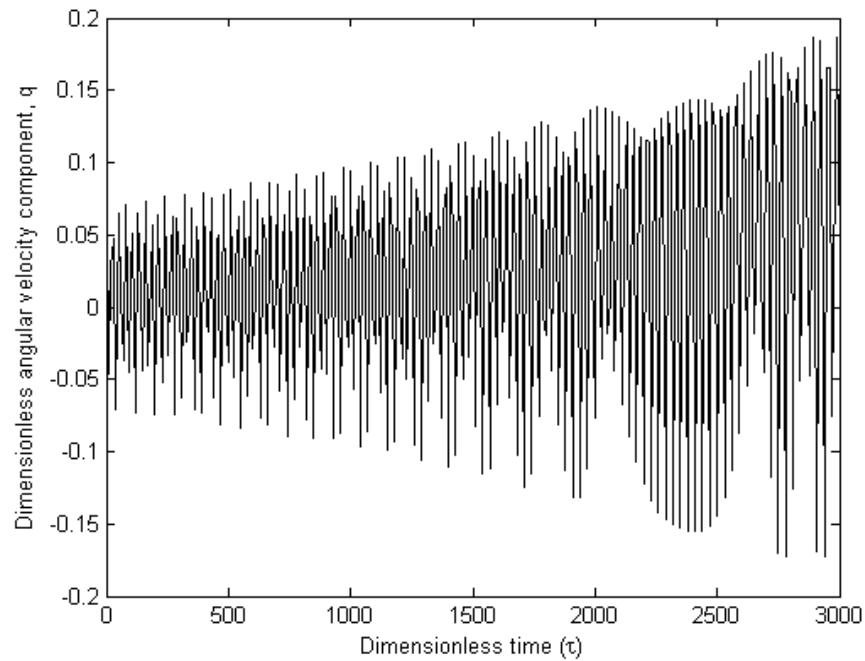


Fig. (5.28) Time history of dimensionless angular velocity component (q) for condition $(p_0, q_0, r_0)^T = (0.01, 0.07, 0.95)^T$.
Second relative equilibrium state.

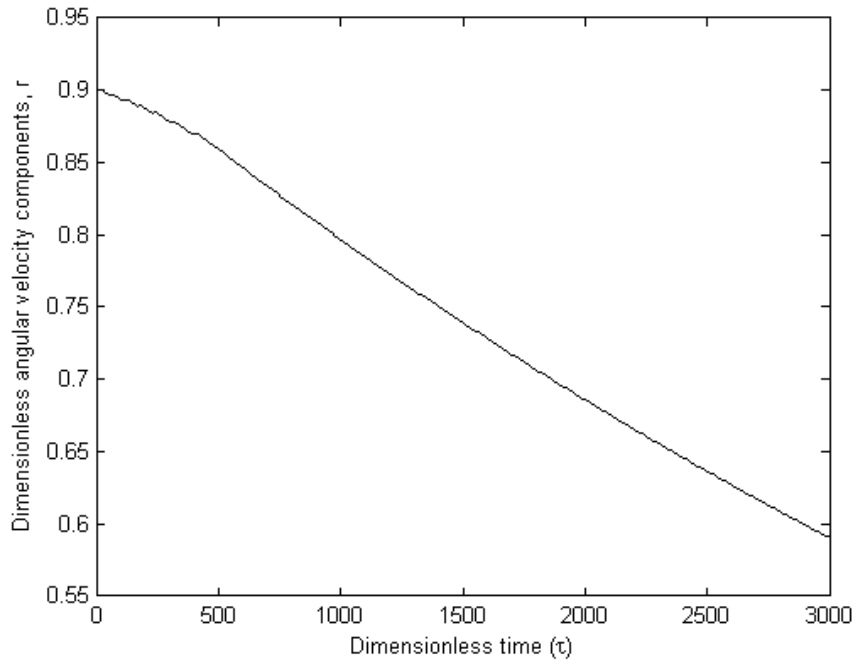


Fig. (5.29) Time history of dimensionless angular velocity component (r) for condition $(p_0, q_0, r_0)^T = (0.01, 0.07, 0.95)^T$ For the second relative equilibrium state.

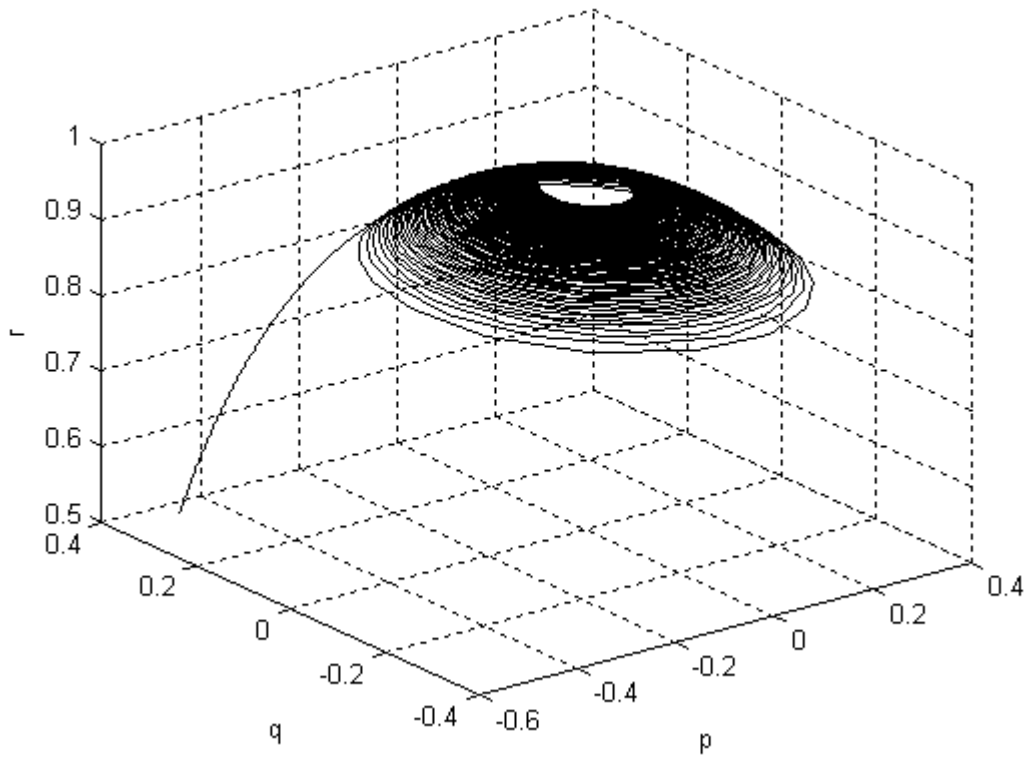


Fig. (5.30) Three dimensional visualization shows that the spacecraft diverge from first relative stability and reaches to the second relative stability, $\sigma = 0.75$, $(p_o, q_o, r_o)^T = (0.01, 0.07, 0.095)^T$.

CONTENTS

<i>Certificate</i>	I
<i>Examination</i>	II
<i>Acknowledgment</i>	III
<i>Abstract</i>	IV
<i>List of contents</i>	V
<i>Nomenclature</i>	IX
<i>List of figures</i>	XVII
<i>List of tables</i>	XX
 <i>Chapter One – Introduction</i>	 1
1.1 Introduction	2
1.2 Background.....	7
1.3 Objective and Approach.....	9
1.4 Overview.....	10
 <i>Chapter Two – Literature Survey</i>	 11
2.1 Introduction.....	12
2.2 Spin Stabilization	12
2.3 Attitude Dynamic of Dual-Spin Spacecraft	14
2.4 Specific Damping Devices.....	19
 <i>Chapter Three – Theoretical Analysis</i>	 23
3.1 Introduction	24
3.2 Rigid Body Dynamics	25
3.2.1 Coordinate Systems.....	25
3.2.1.1 Inertial Reference Frame (IRF).....	25
3.2.1.2 Spherical Coordinate System.....	26
3.2.1.3 Perifocal frame.....	27
3.2.1.4 Orbital coordinate frame	28
3.2.1.5 Body Fixed Coordinate System.....	29

3.2.2	Angular Momentum of a Rigid Body.....	30
3.2.3	Rotational Kinetic Energy of a Rigid Body.....	35
3.3	Attitude Dynamic of Spinning Spacecraft	37
3.3.1	Equations of Motion	37
3.3.2	Mathematical Modeling of Dual-Spinning Spacecraft.....	39
3.3.2.1	Description of the Present Model.....	39
3.3.2.2	Equations of Motion of Dual-Spin Spacecraft.....	41
3.3.2.3	Torque-free motion of axisymmetric spacecraft.....	43
3.4	Environmental Disturbing Torques	45
3.4.1	Calculation of the Environmental Disturbing Torques.....	46
3.4.1.1	Gravity Gradient Torques.....	47
3.4.1.2	Aerodynamic Drag Torque.....	48
3.4.1.3	Solar Radiation Torque.....	49
3.4.1.4	Magnetic Field Torques.....	50
3.5	Satellite Kinematics	52
3.5.1	Quaternion.....	53
3.6	Attitude Stability of Spinning Spacecraft.....	54
3.6.1	Stability of Rotation about Principal Axes.....	54
3.6.2	Dual-Spin Stabilization of an Energy-Dissipation Spacecraft.....	57

Chapter Four – Dynamic of Dual-Spin Spacecraft

	With Nutation Damper.....	63
4.1	Introduction.....	64
4.2	Description of the Proposed Nutation Damper.....	65
4.3	Equations of Motion.....	66
4.4	Approximate Analysis (Zero-Order Approximation).....	71

4.5	Approximate Solution of the Problem.....	72
4.6	Damper Motion.....	75
4.6.1	Nutation-Synchronous Mode.....	76
4.6.2	Spin-Synchronous Mode.....	78
4.7	Relative Equilibria State of Dual-Spin Spacecraft.....	81
4.7.1	introduction.....	81
4.7.2	Relative Equilibria State	81
Chapter Five – Results and discussions.....		85
5.1	Introduction	86
5.2	Numerical Determination of the Time Constant.....	87
5.3	Calculation of the damping constant (η).....	89
5.4	Comparison Between Analytical Solution and Numerical Solution Obtained From the Simulation Program.....	90
5.5	Parametric Study.....	92
5.5.1	Influence of Spacecraft and Nutation Damper Parameters on the Dynamic Characteristics	92
5.5.1.1	Influence of Inertia Ratio (σ)	93
5.5.1.2	Influence of the Rotor Spin Rate (Ω).....	93
5.5.1.3	Influence of Ring Mean Radius (R).....	94
5.5.1.4	Influence of Ring Radius (r).....	94
5.5.1.5	Influence of the Damping Constant (η).....	94
5.5.1.6	Influence of the Ratio of the Ring Height to the Ring Mean Radius (b).....	95
5.5.1.7	Influence of Distance of Offset Center (d).....	95
5.5.2	Influence of the Damper and Spacecraft Parameters on the residual nutation angle.....	95
5.5.2.1	Influence of Inertia Ratio (σ).....	96

5.5.2.2	Influence of the Ratio of the Ring Height to the Ring Mean Radius (b).....	96
5.5.2.3	Influence of Ring Mean Radius (R) and Ring Radius (r).....	96
5.6	Relative Equilibria States.....	97
 Chapter Six – Conclusions and Recommendations for Future Work.....115		
6.1	Conclusions	116
6.2	Recommendations For Future Work.....	118
 References		119
 Appendix A Equations of motion		128
Appendix B Solution of the equation of the nutation angle rate		133
Appendix C Solution of the equation of motion in the spin-synchronous		135
Appendix D Simulation programs figures.....		140
		VIII

List of figures

1.1	Intelsat IV.....	5
1.2	Intelsat V.....	5
1.3	Explorer I.....	7
1.4	TACSAT I.....	8
3.1	Inertial reference frame.....	26
3.2	Spherical coordinate system.....	27
3.3	Perifocal frame.....	28
3.4	Orbital coordinate frame.....	29
3.5	Body fixed coordinate system.....	30
3.6	Rigid body angular momentum.....	31
3.7	Spacecraft model.....	40
3.8	Environmental disturbance torques.....	46
3.9	Disk shaped spacecraft.....	57
3.10	Rod shaped spacecraft.....	58
4.1	Spacecraft model with viscous ring damper.....	65
4.2	Full filled viscous ring damper.....	84
5.1	Nutation angle time history of present work.....	98
5.2	Typical nutation angle time history of Ref.(2).....	98
5.3	Comparison of the nutation angle time history of this research with ref.(2) for the nutation-synchronous mode.....	99
5.4	Comparison between numerical and analytical solution of the nutation angle time history.....	99
5.5	Comparison between numerical and analytical solution of the nutation angle time history.....	100
5.6	Comparison of the nutation angle time histories for nutation-synchronous mode.....	100
5.7	Influence of the inertia ratio (σ) on the time constant for the nutation-synchronous mode.....	101
5.8	Influence of the inertia ratio (σ) on the time constant for the spin-synchronous mode.....	101

5.9 Influence of the rotor spin rate (Ω) on the time constant for nutation-synchronous mode.....	102
5.10 Influence of the rotor spin rate (Ω) on the time constant for spin-synchronous mode.....	102
5.11 Influence of the ring mean radius on the time constant for nutation-synchronous mode.....	103
5.12 Influence of the ring mean radius on the time constant for spin-synchronous mode.....	103
5.13 Influence of the ring radius on the time constant for nutation-synchronous mode.....	104
5.14 Influence of the ring radius on the time constant for spin-synchronous mode.....	104
5.15 Influence of the damping constant on the time constant for nutation-synchronous mode.....	105
5.16 Influence of the damping constant on the time constant for spin-synchronous mode.....	105
5.17 Influence of the ratio of the ring height to the ring mean radius, b on the time constant for spin-synchronous.....	106
5.18 Influence of the offset distance, d on the time constant for spin-synchronous mode.....	106
5.19 Variation of the residual nutation angle with inertia ratio (σ)..	107
5.20 Variation of the residual nutation angle with the ratio of the ring height to the ring mean radius (b).....	107
5.21 Variation of the residual nutation angle with the ring mean radius (R).....	108
5.22 Variation of the residual nutation angle with the ring radius (r).....	108
5.23 Time history of dimensionless angular velocity component (p) for condition $(p_0, q_0, r_0)^T = (0.15, 0.3, 0.9)^T$, First relative equilibrium state.....	109
5.24 Time history of dimensionless angular velocity component (q) for condition $(p_0, q_0, r_0)^T = (0.15, 0.3, 0.9)^T$, First relative equilibrium state.....	109

5.25	Time history of dimensionless angular velocity component (r) for condition $(p_0, q_0, r_0)^T = (0.15, 0.3, 0.9)^T$, For the first relative equilibrium state.....	110
5.26	Three dimensional visualization shows that the spacecraft diverge from second relative stability and reaches to the first relative stability, $\sigma = 1.2$, $(p_0, q_0, r_0)^T = (0.15, 0.3, 0.9)^T$	111
5.27	Time history of dimensionless angular velocity component (p) for condition $(p_0, q_0, r_0)^T = (0.01, 0.07, 0.95)^T$, Second relative equilibrium state.....	112
5.28	Time history of dimensionless angular velocity component (q) for condition $(p_0, q_0, r_0)^T = (0.01, 0.07, 0.95)^T$, Second relative equilibrium state.....	112
5.29	Time history of dimensionless angular velocity component (r) for condition $(p_0, q_0, r_0)^T = (0.01, 0.07, 0.95)^T$, For the second relative equilibrium state.....	113
5.30	Three dimensional visualization shows that the spacecraft diverge from first relative stability and reaches to the second relative stability, $\sigma = 0.75$, $(p_0, q_0, r_0)^T = (0.01, 0.07, 0.095)^T$	114
D.1	Simulink program for calculation of the α , p, q, and r component.....	140
D.2	Simulink program for calculation transverse and nominal angular momentum.....	141
D.3	Simulink program for calculation nutation angle time history..	141

List of tables

Table (1.1). Attitude control systems.....	6
Table (5.1). Nutation damper characteristics	91
Table (5.2). Inertia parameter for symmetrical dual-spin spacecraft.....	91

NOMENCLATURE

The following symbols are generally used throughout the text:-

Symbol	Description	Units
A	Principal transverse moment of inertia of the spacecraft.	kg m^2
A_a	Is the decay constant.	-
A_d	Frontal area (surface area) of the spacecraft.	m^2
A_i	Surface area of each piece of spacecraft.	m^2
A_x, A_y	Moment of inertia of the spacecraft measured along x and y axes respectively.	kg m^2
B	A rigid body.	-
b	Ratio of the ring height to the ring mean radius (h/R)	-
$C=C_R$	Moment of inertia of the rotor along z-axis.	kg m^2
C_{dr}	Diffuse reflection of the panel.	-
C_f	Coefficient of viscous friction between the liquid and the ring damper.	N s/m
C_p	Moment of inertia of the platform along z-axis.	kg m^2
C_s	Specular reflection of the panel.	-
d	Distance of the offset.	m
F_d	Drag force.	N
G	Ratio of the offset distance to the ring mean radius.	-
I_1, I_2, I_3	Principal moments of inertia.	kg m^2
I_R	Centroidal inertia tensor of the rotor section.	kg m^2
$[I_{s/c}]$	Spacecraft inertia matrix.	kg m^2
I_u, I_v, I_z	Moments of inertia of the damping viscous liquid measured along u, v, and z axes.	kg m^2
I_{uz}	Product moment of inertia of the fluid.	kg m^2

I_x, I_y, I_z	Moments of inertia of the body B about xyz coordinates respectively.	kg m^2
I_{xy}, I_{yz}, I_{zx}	Products moments of inertia of the body B about xyz coordinates respectively.	kg m^2
L	Distance between center of each surface and the center of mass of the spacecraft.	m
L_i	Moment arm between the center of pressure of surface to the spacecraft center of mass.	m
m_p	Mass of the platform section.	kg
m_R	Mass of the rotor section.	kg
T_x, T_y, T_z	Components of the applied torque.	N m
m	Mass of the fluid.	kg
n	Total number of the surfaces exposed to solar radiation pressure.	-
O_p	Mass center of the platform.	-
O_R	Mass center of the rotor.	-
O_s	Mass center of the spacecraft.	-
P, Q, W	Perifocal frame.	-
$\bar{P}$	Platform of the dual-spin spacecraft.	-
p, q, r	Dimensionless angular velocity components of the spacecraft.	-
p_a	Amplitude.	-
p_r	Radiation pressure.	N/m^2

Q	Column matrix of the moment component of the non conservative forces.	N m
Q_α	Generalized moment associated with the generalized coordinate α .	N m
q_1, q_2, q_3	Quaternions components.	-
q_4	Scalar component of the quaternion vector.	-
q_i	Reflectance factor.	-
q_o	Response of the system.	-
R	Ring mean radius of the nutation damper.	m
R_o	Orbit radius.	m
$\bar{R}$	Rotor of the dual spin spacecraft.	-
r_c	Distance between the center of masses of the platform and the rotor.	m
T	Total kinetic energy of the spacecraft equipped with nutation damper.	N m
T_a	Aerodynamic drag torque.	N m
T_b	Total torque about bearing axis.	N m
T_f	Torque produced by bearing friction.	N m
T_m	Torque produced by DC motor.	N m
T_n, T_t	Normal and tangential torque components.	N m
T_{rot}	Rotational kinetic energy of body B.	N m
T_s	Total torque.	N m
t	Time.	sec
X, Y, Z	Orbital coordinate frame.	-
X_o, Y_o, Z_o	Inertial reference frame at the earth center of mass.	-

x, y, z	Body fixed frame.	-
x_p, y_p, z_p	Coordinate system located at the platform center of mass.	-
x_R, y_R, z_R	Coordinate system located at the rotor center of mass.	-

Greek Symbols

Symbol	Description	Units
α	Relative angular displacement between the fluid and the spacecraft.	rad
α_0	Initial value of α in the spin-synchronous mode.	rad
$\tilde{\alpha}$	Small variation in α in the spin-synchronous mode.	rad
$\dot{\alpha}$	Relative angular velocity of the fluid.	rad/s
γ, ψ, L_s	Spherical coordinate system.	-
γ_r	Angle between the radiation direction and the normal to the area.	rad
ε	Inertia ratio of the fluid to the transverse inertia of the spacecraft.	-
η	Damping constant of the damping fluid.	-
η_n	Damping constant of the nutation damping in the nutation-synchronous mode.	-
η_s	Damping constant of the nutation damping in the spin-synchronous mode.	-
θ	Nutation angle of the spacecraft	rad
θ_n	Nutation angle in nutation-synchronous mode.	rad
θ_r	Residual nutation angle of the spacecraft.	rad
θ_s	Nutation angle in spin-synchronous mode.	rad
θ_{so}	Initial value of nutation angle in the spin-synchronous mode.	rad

λ_p	Nutation frequency of the platform.	rad/s
λ_R	Nutation frequency of the rotor.	rad/s
μ_g	Local gravitational constant.	m ³ /s ²
ρ	Density of the damping fluid.	kg/m ³
σ	Inertia ratio.	-
τ	Dimensionless time.	-
τ_n	Time constant in the nutation-synchronous mode of the spacecraft equipped with the nutation damper.	-
τ_s	Time constant in the spin-synchronous mode of the spacecraft equipped with the nutation damper.	-
ω_{pz}	Angular velocity component of the platform about the z-axis.	rad/s
$\omega_x, \omega_y, \omega_z$	Angular velocity components of the spacecraft along u, v, and z axes.	rad/s
Ω	Inertial spin rate of the spacecraft.	rad/s
ϕ	Phase angle .	rad
ξ	The angle between the axis of rotation (direction of ω) and the spin axis (z-axis).	rad
ψ	Quaternions angle rotation.	rad

Vector Notations

$\vec{B}_M$	Local earth geomagnetic field vector.
$\vec{h}$	Angular momentum vector of spacecraft.
$\vec{h}_o$	Angular momentum vector of the body about point O.
$\vec{h}_{oi}$	Angular momentum vector of mass m_i about point O.
$\vec{h}_p$	Angular momentum component vector of the platform along the z-axis.
$\vec{M}$	Applied torque vector.
$\vec{M}_M$	Magnetic dipole torque vector produced by electromagnets in the spacecraft.
$\vec{P}_i$	Linear momentum vector of particle m_i .
$\vec{R}_i$	Position vector of element mass m_i with respect to O' .
$\vec{R}_o$	Position vector of center of mass of body B with respect to O' .
$\vec{r}_i$	Position vector of element mass m_i with respect to O.
$\vec{T}_c$	Desired control torque vector.
$\vec{T}_M$	Maximum torque vector.
$\dot{T}_p$	Energy dissipation rate on the platform.
$\dot{T}_R$	Energy dissipation rate on the rotor.
$\vec{V}_i$	Velocity vector of particle measured relative to XYZ.
$\vec{V}_o$	Velocity vector of the origin O measured relative to XYZ.

$\vec{\omega}$	Angular velocity vector of the viscous liquid.
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CHAPTER ONE

INTRODUCTION

Chapter One

Introduction

1.1 Introduction

The attitude control system of the spacecraft is to control the attitude and position of the spacecraft as it performs its mission. The attitude of a spacecraft is its orientation in space with respect to some reference system; it describes how the spacecraft body axes are oriented relative to an inertial or rotating coordinated system. The position and velocity of a spacecraft are similarly referenced to some coordinate frame; they give the location, speed and direction of motion of the spacecraft. The motion of a spacecraft is described by four parameters: position, velocity, attitude, and attitude motion. Position and velocity are the subjects of orbital mechanics. Attitude and attitude motion, however, are the subjects of

attitude dynamics, i.e. the motion of the spacecraft about its center of mass. Since environmental torques (gravity gradient, solar wind, aerodynamic drag) are a function of spacecraft position and velocity, the spacecraft's orbit and attitude are coupled. All spacecrafts require some type of attitude determination and control. The techniques that provide attitude stabilization and control of spacecraft are; passive control system, semi-passive and active control system. The type of the system adopted in the present study is the passive type system. Passive systems do not require any external power source, once they are in place, they use gravity or momentum to create the necessary control forces and moments. The spacecraft attitude passive control system can be classified into four main categories^{(57), (76), (85)}:

- Gravity gradient.
- Spinner with nutation damper.
- Dual-Spinner with nutation damper.
- Three axis stabilization.

Dual-spin stabilization type is the method of attitude stabilization adopted in the present study. A spin and Dual-spin stabilized spacecraft, or spinners, utilizes its own spinning motion to keep it's self aligned in a certain inertial direction. The spinning motion creates stiff angular momentum vector, which tend to resist external disturbance torques. A spinner is stable if it is spun about the axis of largest principal moment of inertia, if it is spun about a different axis, any disturbance could cause the spin axis to shift to the major axis.

In single spin stabilization the whole body rotates about the axis of maximum principal moment of inertia. Early communication satellites, such Syncom I, ATS I, II and Inelsat I, II are single spin stabilized⁽⁷⁷⁾. Its advantages are simple, reliable, and long life time but the main limitations of these satellites are that they could not

use earth oriented antennas. These limitations are overcome in a dual spin spacecraft.

Whereas dual spin spacecraft consisting of spinning rotor producing gyroscopic stiffness and a platform rotating at a much slower rate in accordance with the desired attitude of the spacecraft. There are two types which are commonly known as the external rotor and body stabilized spacecraft, each type employs a different method of attitude stabilization. The external rotor type or “Gyrostat” uses spin stabilization where the rotor of relatively large moment of inertia rotates to provide gyroscopic stiffness, while the platform usually containing communication equipment and antennas are despun. A typical example of an external rotor dual spin spacecraft is the Intel sat IV Fig. (1.1).

This satellite is utilized for public switched telephony, private network business, broadcasting, and domestic/ regional communication and provides a variety of voice video and data services.

In the case of the body stabilized spacecraft, gyroscopic stiffness is achieved via spinning momentum wheel contained within a despun body. These are also referred to as bias momentum satellites. A typical example of this type of dual spin spacecraft is in Fig. (1.2).

The momentum wheel is typically has a much smaller moment of inertia than the rotor of the external rotor spacecraft and thus has a high spin rate to provide the same gyroscopic stiffness. It should also be noted that whereas the external rotor spacecraft usually contains much of the rotational inertia of the satellite and spin relatively slowly (~50 rpm) the momentum wheel in a bias momentum satellite

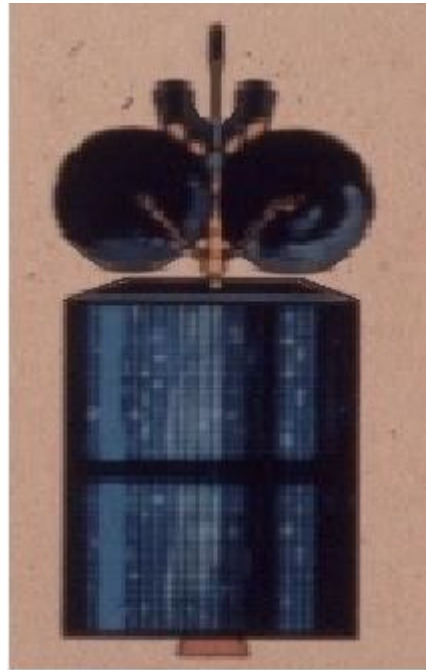


Fig. (1.1). Intelsat IV⁽⁷⁷⁾.

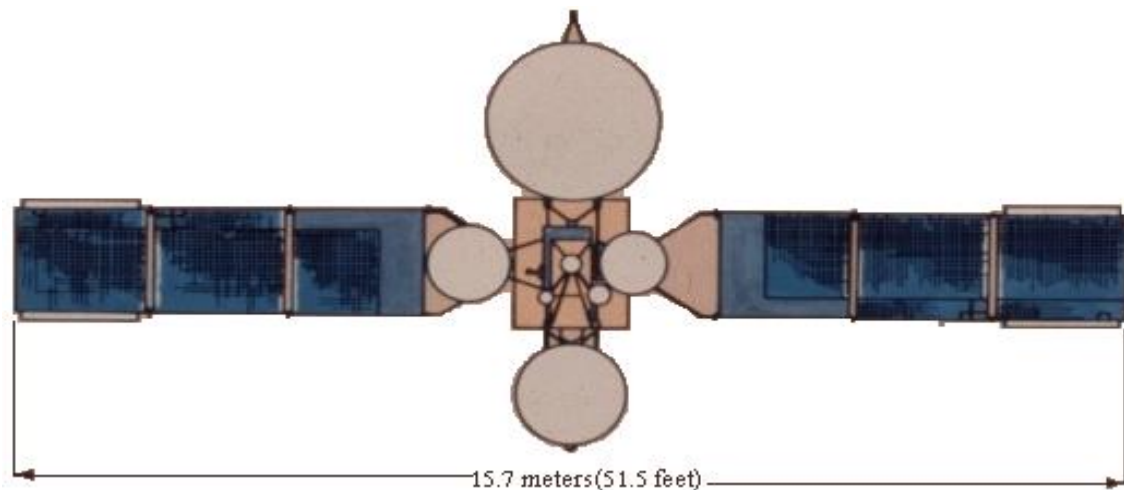


Fig. (1.2). Intelsat V⁽⁷⁷⁾.

has negligible rotational inertia but spin rapidly (~ 3000 rpm)⁽⁵⁷⁾. Table (1) shows some types of attitude control systems.

Passive techniques include nutation damping are used for stabilization of spinning satellite. Nutational motion of a spinning spacecraft can occur when nominal spinning motion is perturbed e.g. by a thruster firing or, by the effect of the environmental disturbing torque, such as aerodynamic torque and solar

radiation pressure. Nutation arises when the spin axis becomes misaligned with the principle axis (rotation spin axis). Principle axis usually stays aligned with the original mass symmetrically axis. The spin axis does not remain fixed, but precesses around the principle axis.

Table (1.1). Attitude control system types⁽⁷⁶⁾.

<i>Passive</i>	<i>Semi-passive</i>	<i>Active</i>
Gravity gradient	Momentum bias with magnets	Propellant
Spinner with nutation damper	Reaction wheels with magnetcs for momentum dumping	Reaction wheels with propellant for momentum dumping
Dual spinner with nutation damper	Control moment gyros with magnetcs for momentum dumping	Control moment gyros with propellant for momentum dumping

The nutational motion will continue indefinitely in the absence of energy dissipation, unless it is corrected. Although nutation can in principle be controlled by thruster firing, it is normally controlled passively. Nutation causes a problem with the pointing of instruments and communications antenna. It is usually detected by accelerometers and indirectly by analysis of instrument data, e.g. a periodic fluctuation, in transmitted signal strength, Hence control of nutation is a major objective. When there is flexure of antenna or structure, or fuel slosh etc., energy may dissipated, and for a spacecraft rotating about it is maximum principal moment of inertia, the precession will be damped, and the nutation angle will gradually decreases with time. The dissipation of energy may be exploited by use of the

nutration damper, a device that deliberately uses motion to dissipate energy. One of the damping mechanisms which have been proposed and analyzed, the ones containing fluids in a closed tube are especially desirable since they do not involve any moving parts, other than fluid.

1.2 Background

In retrospect, it is seen that the analytical beginning of spacecraft attitude dynamic and control in the spinning body dynamic of Euler (1758) or the lunar dynamics of Lagrange (1764). But for the new age there was another beginning, it was the launching of Sputnik by Russia in 1957 and Explorer I by USA, Fig. (1.3) In 1958.

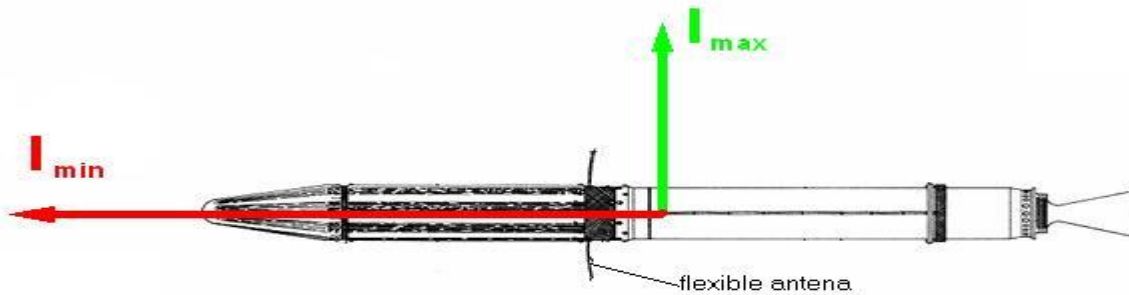


Fig. (1.3) Explorer I

By that time Sputnik had been in orbit for more than three months, but neither the general public nor the engineers preparing Explorer I at Jet Propulsion Laboratory (JPL) knew very much about the rotational dynamics of the pioneering Russian satellite. So it was from Explorer I that JPL engineers learned their first lessons in attitude stability of spinning spacecraft, and learned that the effect of energy dissipation can not be ignored in stability analysis of spinning satellite, where within ninety minutes it was essentially tumbling end over end, and the engineers at JPL were scrambling to explain why. Designers of Explorer I applied

the classic rigid body spin stability results—a spinning rigid body is stable for spin about either a major or minor axis. Explorer I was designed to spin about its minor axis. Like all real systems, Explorer I was not a rigid body, but instead had several means of dissipating energy including flexible antenna. Following the Explorer I experience, engineers analytically showed that with energy dissipation, or damping, an otherwise rigid spinning body is only stable when spinning about the major axis. The “major axis rule” for rigid bodies is rather limiting for satellite designers. A relatively flat disk-shaped satellite required for stability does not efficiently use the space in a typical launch vehicle fairing. However, engineers recognized fairly early that the addition of a spinning rotor within the satellite could contain enough angular momentum to stabilize a satellite spin axis, even for minor or intermediate axis⁽³³⁾. This led to the development of dual-spin satellites, with a relatively fast spinning rotor and a slowly spinning platform. TACSAT I (tactical satellite) 1969 was the first demonstration of this concept Fig. (1.4)

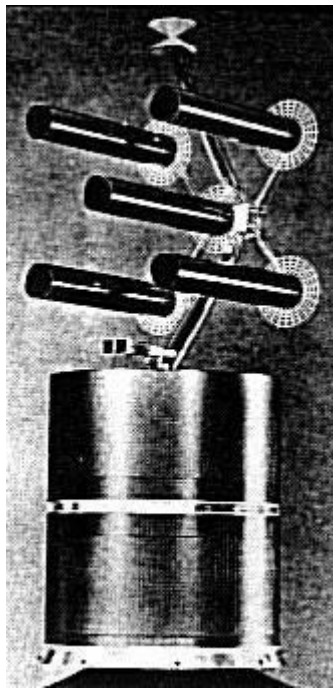


Fig. (1.4) TACSAT I.

The large rotor spun about the system's minor axis while keeping the communications payloads antenna pointed at the earth. Prior to launch, extensive analysis and simulation of a gyrostat with damping was completed to ensure the stability of TACSAT I. it is on orbit success validated the conclusion that the dual spin concept could stabilize a satellite even about a minor axis. Much of the original stability analysis for spinning bodies and gyrostats with damping was done with approximate methods, generally referred to as energy-sink methods. The stability conclusions from these methods are useful, but they often produce necessary but not sufficient stability criteria. For precise stability criteria, the actual damping mechanism must be included in the system model. Of course this higher precision comes with the price of dealing with additional degrees of freedom needed to model the damper. As mentioned above, that the damping mechanism employed here is a ring filled with a fluid and offsets from the axis of nominal spin, almost all the work done in partially filled ring damper aligned or perpendicular to the spin axis, or fully filled but aligned with the spin axis. The current work will be on the full filled ring damper perpendicular to the spin axis with offset center.

1.3 Objective and Approach

This research investigates the dynamics of a spinning, torque-free, satellites by studying a specific satellite model with energy dissipation, and developing suitable dimensionless equations of motion using Newton-Lagrange approach, and evaluates the analytical expressions of the time constant, stability condition and residual nutation angle expressions, using zero-order approximation technique, and determines the influence of different spacecraft and damper parameters on the time constant, where the analytical results are compared with those of numerical results which obtained by using computer simulation program named MATLAB ver.7.

1.4 Overview

In chapter 2, previous researches related to satellites especially spinning satellites are summarized.

In chapter 3, the attitude dynamic and stability of a specific Dual-spin satellite model are considered, and expressions describing different types of environmental disturbance torques are presented.

Chapter 4 develops equations of motion using Newton-Lagrange approach. Which are nondimensionalized in terms of suitable dimensionless variables and parameters. The nutation angle time history and the time constants are developed using zero-order approximation technique. Also the stability condition of the nominal spin state and the residual nutation angle expressions are derived.

Chapter 5 discusses the results and discussions obtained from the analytical and numerical solution and shows a comparison between them.

Chapter 6 includes the conclusions of this study and presents some suggestions for further work.

CHAPTER TWO

LITERATURE

REVIEW

Chapter 2

Literature Review

2.1 Introduction

In this chapter, the significant works related to satellite spin dynamics and stability are summarized. Firstly the results for spinning bodies are presented. Then the development of dual spin satellite with damping is discussed. Finally, the works that consider various damping devices are reviewed.

2.2 Spin Stabilization

The instability of Explorer I made engineers (at JPL) immediately aware of the effect of energy dissipation on satellite spin stability. The Explorer I instability was actually anticipated by Bracewell and Garriott⁽⁴¹⁾, who recognized that with energy dissipation, an otherwise rigid body in torque free motion is destined to spin about an axis of maximum inertia. Ronald Bracewell was a radio

astronomer at Stanford and had determined, using Sputnik's radio signals that the Soviet satellite was spinning about an axis of symmetry. Using ideas from his understanding of rotational dynamics of galaxies, he concluded that the Sputnik must be spinning about its major axis.

The key aspect of this condition is that it minimizes kinetic energy for a particular momentum value. Bracewell attempted to contact the Explorer I engineers to make sure they were aware of this effect, but the security surrounding the project prevented any contact. Bracewell published his ideas with Owin Garriott in September 1958, too late to help Explorer I.

An engineer named Vernon Landon preceded Bracewell by deducing the same major axis spin criteria in early 1957⁽⁴¹⁾. Landon's laboratory notes showed that he recognized that energy dissipation violates the assumptions behind the classic rigid body results of Euler and Poinset, resulting in an unstable minor axis spin. Unfortunately, Landon did not publish until later.

Then much of research has studied the spinning body damping and stability.

Bhuta and Koval [1966]⁽⁷⁾, studied the utilizing viscous nutation damper for removing the wobble motion of a spin-stabilized satellite, their damper was aligned with the spin axis, they used energy sink approach, and they founds the decay rates and the time is constant.

At that time the topic of spacecraft attitude dynamic and control was at beginning, and the best, reliable and cheapest attitude control techniques was not configured yet, so more ideas were followed later.

Sen [1973]⁽⁶³⁾, studied the use of a spinning dissipater for attitude stability of earth orbiting satellites. His study concentrated on the attitude stabilization system which consisting of four identical wheels mounted coaxially. He utilized rigid body rotation dynamics and deduced that a damping time constant in the

neighborhood of only one second can be easily achieved with a reasonable design of the stabilizer.

Then Mingori and Harisson [1974]⁽⁴⁸⁾, investigated the circularly concentrated particle motion in spinning and coning bodies. Their equation of motion was expressed as a nonlinear, nonautonomus, ordinary differential equation. They used phase plane techniques and they deduced that the nature of the motion for large times depends on initials conditions.

Later Kane and Levinson [1980]⁽³⁸⁾, utilized energy sink analysis to investigate system containing driven rotors. They concluded that, one is well advised to undertake numerical solutions of differential governing equations, rather than to use analytical analysis, and showed the effect of constants of the driven rotor on spacecraft behavior.

Then in 1999, Fowell, Millford and Yocum⁽¹⁹⁾, studied the spacecraft spin-stabilization using a transverse wheel. They used rigid body rotational dynamics and discussed the history of active control and some global principles for spin stabilization using a single wheel and they compared their solution with ground simulations.

Lately, Sandfry [2001]⁽⁵⁵⁾, studied the equilibrium of a Gyrostat with a discrete damper. He utilized Newton-Euler approach to investigate a gyrostat with a spring-mass-dashpot damper. He gave new insights into dynamics of spin-stabilization satellites, and studied the stability bifurcations very deeply.

2.3 Attitude Dynamic of Dual-Spin Spacecraft

A number of investigations of spinning spacecraft stability have been performed in the recent past years. After abnormalities were observed in dual-spin spacecraft in practice, many of these instabilities have been found to arise from

excitations from structural imperfection in combination with inherent nonlinearity within the dynamic of the system. For instance, a vast amount of research has been performed, investigating the attitude stability and behavior of a dual spin spacecraft due to parametric fluctuation resulting from structural imperfection.

Mingori [1969]⁽⁴⁹⁾, studied the effect of energy dissipation on the attitude stability of dual spin satellites including the effect of internal moving parts. It was concluded that a dual-spin spacecraft with a prolate rotor might only be stable provided energy dissipation on the platform is dominant over that in the rotor.

Scher and Farrenkopf [1974]⁽⁵⁷⁾, identified two trap states for spacecraft having asymmetric or unbalanced rotors. The first is a minimum energy state to which the spacecraft settles when the spin motor is deactivated. The second, encountered while despinning the platform; is characterized by a stalling of despin process and violent increase in nutation. Their analysis demonstrated that, in both trap states, the spacecraft possesses two modes of attitude motion, each with distinct frequency.

They have also shown that the excitation by the motor at an appropriate natural frequency can lead to escape from either trap, even when the motor torque is insufficient to permit direct despin.

Then Mingori with cooperation of Gebman [1976]⁽²¹⁾, expanded his work by using perturbation solution for flat spin maneuver of spacecraft with asymmetric rotor and asymmetric platform. Their analysis based upon asymptotic parameter expansions instead of the linearization technique.

They found that, the recovery time and the residual nutation angle are algebraically related to the initial flat spin rate, the spacecraft inertia properties, and the bearing axis torque. Also their results have shown that for simple case in which the bearing axis coincides with the centroidal principal axis of minimum

moment of inertia represent a significant first step in adopting asymptotic perturbation method in dynamical behavior.

Cochran [1977]⁽¹³⁾, adopted the spacecraft model studied by Scher and Farrenkopf in an attempt to focus his study on the nonlinear resonance trap state, because the trap state was explained in a heuristic manner only. Attitude equations were derived in an unconventional form that allows easy incorporation of the integral of angular momentum. He obtained an approximate solution for the attitude motion near resonance for the case of zero Despin motor torque by using the generalized method of averaging that the presence of a nonlinear resonance is shown to depend on the existence of the rotor asymmetric and platform dynamic unbalance.

Then he followed by Adams [1980]⁽¹⁾, which studied the attitude dynamics of a dual spin spacecraft of dynamically imbalance platform instead of asymmetric platform chosen by Mingori and Gebman⁽²¹⁾, and a symmetric rotor, during platform spin up which occurs if despin control is temporarily absent.

He illustrated that the dynamic interactions arise from platform dynamic imbalance, transverse inertia asymmetry, and from rotor transverse inertia asymmetry. Also he concluded that when inertia ratio (ratio of bearing axis moment of inertia to the transverse moment of inertia), is greater than one, it is generally desirable to minimize platform dynamic imbalance and rotor transverse asymmetry, and when inertia ratio is less than one, an asymmetric and/ or unbalanced platform and a symmetry rotor are preferred.

Then Tsuchya [1989]⁽⁷²⁾, treated a trap phenomenon different from that treated by Scher and Farrenkopf in that it requires an asymmetry in one body and a dynamic imbalance on the other and occurs when the nutation rate of the symmetric body equals the relative spin rate. Nonstationary attitude motions in passing through the unstable region have been examined by an analytical method

that utilizes the KB (Krylove-Bobgulove) method. Tsuchya demonstrated that under certain conditions, a trap phenomenon might occur, and to avoid this trap phenomenon the following practice are recommended:

1-If transverse moment of inertia of a rotor differ significantly, a powerful motor should be used.

2-a nutational body motion should be completely damped at the beginning of the despin maneuver.

One of the trap states occurs where the spin bearing assembly on a dual spin spacecraft can encounter undesirable torque which exceed the available spin bearing motor torque, resulting in a failure to despin the stator (hence a “Trap state”). This trap state torque is caused by static imbalance, dynamic imbalance, and asymmetry on the rotor and platform, and is also function of the rotational kinetic energy.

Then the trap states and other topics studied deeply, where Rand and Hall⁽²⁹⁾, studied the spinup dynamics of axial dual-spin spacecraft composed of an asymmetric platform and an axisymmetric rotor parallel to a principal axis of the platform. They used conservation of angular momentum and the method of the averaging to reduce the problem to a single first order differential equation, which is studied numerically. They concluded that the oplate spinup can be described accurately by the averaged equation.

Then Or [1998]⁽⁵¹⁾, made more expansion than Adams⁽¹⁾, where he studied the nonlinear despin dynamics of a gyrostator with a symmetric balanced platform and an asymmetric rotor (can be balanced or unbalanced). A numerical scheme was developed for computing the multiple equilibrium as well as their bifurcation properties.

The major purpose was to determine the boundary curve between normal spin and flat spin transition in terms of motor torque magnitude and initial conditions, rotor asymmetry, and imbalance, when the torque is not considered small.

The numerical results indicated that besides the upper normal spin and the lower flat spin regions, there exist two more regions in between: a narrow strip region in which a normal spin transition can occur and a middle region above it corresponding to a flat spin transition. Thus calculated that, if despin is designed to minimize fuel consumption, then the strip region can serve as an optimal window for spacecraft maneuver.

Nonlinear phenomena known as “trap states” have been identified for dual spin spacecraft with asymmetrical components. These have been described as regions in state space in which the attitude variables never leave. It is of interest that such a description may be used to describe quasi periodic or chaotic motions.

These trap states are highly undesirable, as the attitude variables never converges to the desired operational values corresponding to a directionally stabilized equilibrium. In practice “trap states” have been shown to occur when despin control is deactivated or during despinning of the platform. The second occurrence is characterized by a stalling of the despin process and a violent increase in nutation.

After that Hall [1998]⁽²³⁾, presented two procedures to escape form an undesirable trap state of gyrostat containing two asymmetric rotors where one of the rotors is viscously damped and the other is subjected to a controllable torque. He has shown that the first method, uses a periodic torque that is similar to that used by Scher and Frankopf, is relatively ineffective in the presence of significant damping; while the second method which is based on the noncanonical Hamiltonian description of gyrostat dynamics, is highly effective and results in a

satisfactory escape regardless of the strength of the damping or the magnitude of the motor torque.

Then Meehan and Asokanthan [2002]⁽⁴⁴⁾, studied the control of chaotic instability resulting from sinusoidally varying torque applied to the spacecraft. They utilized energy sink method to handle a dual-spin spacecraft with an asymmetric platform and axisymmetric rotor. They found the instabilities of the system and designed a control system and showed the effectiveness of the control method in eliminating chaotic instabilities.

2.4 Specific Damping Devices

Researchers have studied the performance of different damper devices and configurations with rigid bodies. Two major types of damping mechanisms that were modeled and analyzed include spring-mass-dashpot dampers and viscous-ring nutation dampers.

The first analysis of the viscous ring damper on spinning satellite was performed by Carrier and Miles [1960]⁽⁴⁶⁾, where they employed a partially filled viscous ring damper. They utilized Navier-stoke equation in their analysis, assuming the annular cavity is rectangular, and they found that for small nutation angles the fluids spreads out around the outer part of the ring, and they find the decay time.

Then Alfrend⁽²⁾ cited in 1961; Cartwright, Missingill and Trueblood modeled the fluid as a particles of equal masses moving in a tube resisted by a viscous damping force. They identified two distinct motions which they named the nutation-synchronous and spin-synchronous modes. For larger nutation angles the fluid moves within the ring roughly synchronous with satellite precession. As damping continues, nutation angle decreases until the spin-synchronous mode

begins. This mode is roughly synchronous with the satellite spin rate and continuous damping the nutation angle, although in an oscillatory manner.

After that, Miles [1963]⁽⁴⁷⁾, repeat his proceeded study by developing laminar boundary layer of the liquid oscillating over a rigid wall. His study is deeper than the first research with Carrier. He concluded that the decay time is found to be greater than that predicted in the previous analysis.

Then after that E. Yu [1963]⁽⁷⁵⁾, made a small change to this damper, where he studied the dynamic of spinning satellite equipped with a ball in curved tube precession damper filled with neon gas. The analysis was based on the energy sink approach and the study includes an experimental measurement to determine the rolling resistance moment and the coefficient of viscous friction between the ball and damping fluid.

He was estimated that the energy dissipation due to rolling resistance moment could be neglected compared to that due to the viscous friction force and the analysis restricted for small nutation angle approximation only.

After Yu⁽⁷⁵⁾; Sen [1973]⁽⁶³⁾, suggest a Four mass nutation damper. he studied a nutation damper that consists of a four mass connected to the center of mass of the spacecraft and shows the stability criteria developed for this damper do not involve the kind of design which restricted the performance of the other available nutation dampers.

After that where different damper's configurations are developed and studied, a contentment is grow that the cheapest and reliable damper is that the viscous ring and the spring mass dashpot dampers, especially the viscous ring damper. where its needed just fluid inside a tube. So more deep studies and more configurations for these two dampers are followed.

Alfriend [1974]⁽²⁾, re-examined the problem that studied by Cartwright, Missingill and Trueblood for partial filled ring damper, He employed the zero-

order approximation technique for solving the differential equations. And generated an approximate solution for the nutation angle time history for both modes.

Then Cochran and Thompson [1980]⁽¹⁴⁾, re-examined the same problem adopted by Schneider and Likins using a different method for approximating the functions in the energy sink method, they validated the superiority of precession dampers for small coning angles. Also they achieved better agreement with numerical simulation than Schneider and Likins for larger coning angles, where nutation dampers had larger average dissipation rates.

Then Alfriend, Chang, and Liu [1996]⁽⁹⁾, studied the dynamic and stability of an inertially symmetric, spinning, rigid body with a partial filled viscous-ring damper mounted normal to the spin axis. They used the nonlinear equations directly by using center manifold theory to isolate the stable subspace and determine stability in the center manifold, and they generated the stability criteria and the decay time constant.

Sandfry [2001]⁽⁵⁵⁾ mentioned that Schneider and Likins studied the effectiveness of spring-mass-dashpot dampers in passively damping the coning motion of spinning rigid bodies. They studied dampers mounted parallel and normal to the desired satellite spin axis. The damper mounted parallel to the spin axis, sometimes called an axial damper in the literature, is excited by the precession, or coning motion of the symmetry axis about the inertially fixed angular momentum vector, and was therefore referred to as a precession damper.

The damper aligned normal to the spin axis, often referred to as a circumferential damper, is excited by nutation, or the nodding oscillation of the spin axis about its conical path, and was called a nutation damper.

Despite the more precise definitions, the term "nutation damper" has been used throughout the literature for any device which damps the coning motion. In the

same reference mentioned above⁽⁵⁵⁾, Schneider and Likins used an energy sink method and numerical simulation to support the conclusion that for any physically feasible spacecraft the precession damper dissipates more energy per unit mass than the nutation damper for small coning angles. For larger coning angles and certain rigid body asymmetries, nutation dampers have superior average energy dissipation rates.

Then Hamed [2003]⁽³¹⁾, study the ball in ring nutation damper deeply, and his study is more expanded than that Yu⁽⁷⁴⁾, where he utilized neon gas and many percentage of glycerin water mixture as damping fluid in his study.

It is seems from the above review there is no one (to our knowledge) handle the full filled ring damper mounted normally to the spin axis with offset center. So the present work represents an attempt to study the full filled viscous ring damper mounted normally to the spin axis with offset center (d) from the spin axis.

CHAPTER THREE

ATTITUDE

DYNAMICS AND

STABILITY OF

DUAL-SPIN

SPACECRAFT

Chapter 3

Attitude Dynamics and Stability of Dual-Spin Spacecraft

3.1 Introduction

The problem of the dynamic and stability of dual spin satellite is studied in this chapter. Development of mathematical model of spacecraft is important to understand the dynamic behavior of the system as well as the design and performance evaluation of spacecraft control systems. Most spacecraft systems are dynamically complex, usually having multi-degrees of freedom, which in most cases are strongly coupled and nonlinear through rotation kinematics. Consequently, a linearization technique is adopted to obtain the solution for attitude equations of motion.

3.2 Rigid Body Dynamics

The attitude dynamics of a spacecraft is based primarily on rigid body dynamical equations. These equations, however, are modified for flexibility in the cases of flexible spacecraft requiring high-accuracies⁽⁵⁷⁾. The attitude motions are normally represented in the spacecraft body fixed coordinate systems, which are rotating and accelerating. Hence the subject of coordinates transformation and relative motion plays an important role in rotating coordinate system, and derives the equations of motion of a rigid body.

3.2.1 Coordinate Systems

Several coordinate frames are used to define the position of a satellite in space. Each of these coordinate systems has particular properties, which makes them appropriate to a limited number of applications. The fundamental coordinate systems, to which all motions must be referred, are:

- The inertial reference frame.
- Spherical coordinate frame.
- Perifocal frame.
- Orbital coordinate frame.
- Body fixed coordinate frame.

3.2.1.1 Inertial Reference Frame (IRF)

Or earth-centered inertial (ECI), also called celestial coordinates⁽²⁷⁾. To define the position of a satellite in space, it is necessary to define the orbit of the satellite relative to an inertial frame and the location of the satellite in the orbit. Fig. (3.1) shows an inertial system, which is located at the center of mass of the earth.

The direction of the X_o -axis is along the line of the vernal equinox, which is the intersection of the earth equatorial plane and the earth orbital plane about the sun axis, where the later is known as the ecliptic plane. The Z_o -axis of the inertial frame is in the direction along the earth pole⁽²⁷⁾. The last axis, which is the Y_o -axis, lies in the equatorial plane and it completes the right- handed orthogonal triad.

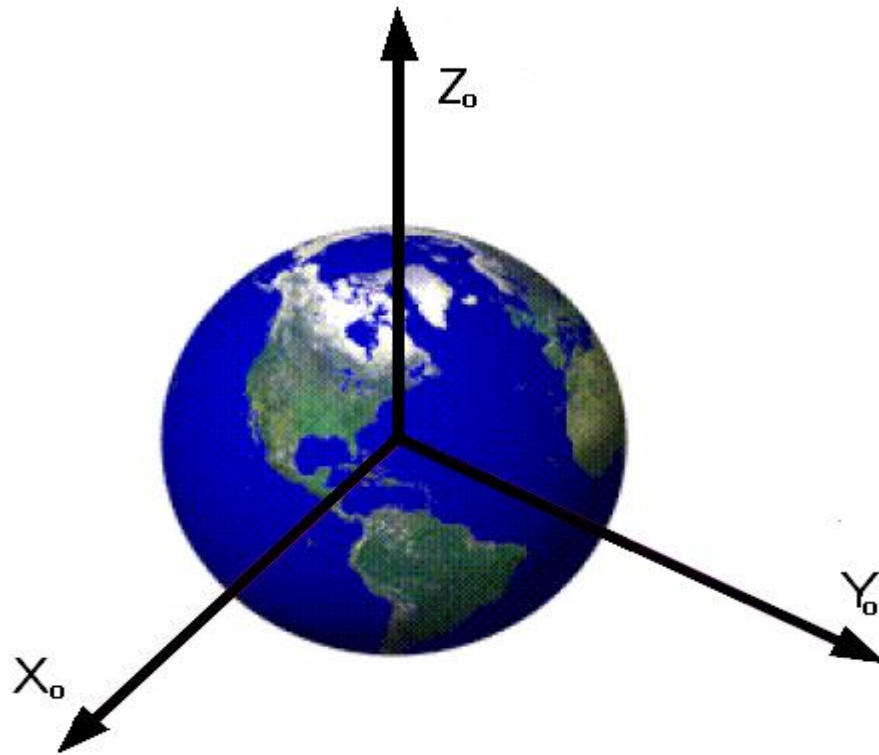


Fig. (3.1) Inertial reference frame.

3.2.1.2 Spherical Coordinate System

Spherical coordinates, as shown in Fig. (3.2), are located at the center of mass of the earth and they are ordinarily used to define the position of a satellite such that: L_s represent the radial distance of the satellite measured from the earth center of mass (origin o), γ is the right ascension and it is the angle in the equatorial plane from the vernal equinox eastward to the projection of the radial

vector L_s on the equatorial plane, and the declination angle ψ is the angle from the equatorial plane of the radial vector L_s ⁽²⁷⁾.

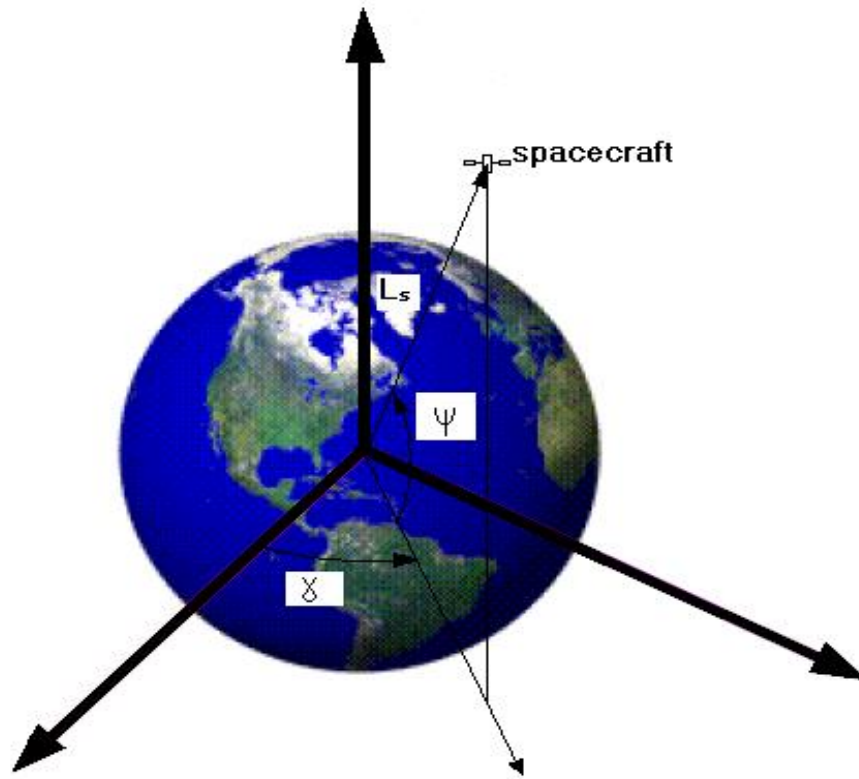


Fig. (3.2) Spherical coordinate system.

3.2.1.3 Perifocal frame

Figure (3.3) shows this coordinate system, where P-axis lies in the orbital plane in periapsis direction. W, is perpendicular to orbital plane in direction of orbit angular momentum vector, and Q, is in the orbital plane and finishes the triad of the unit vector.

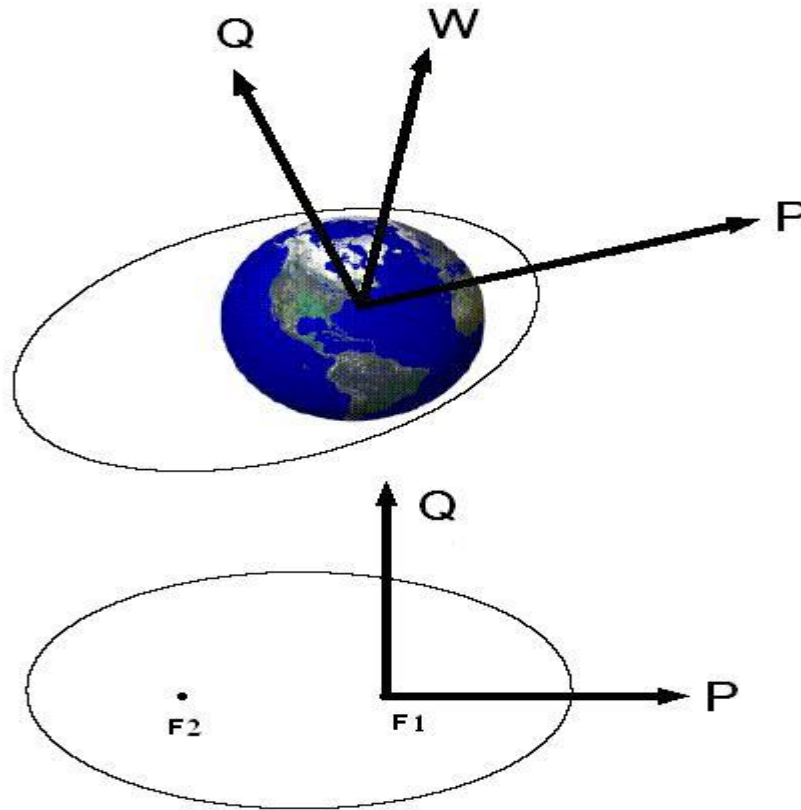


Fig. (3.3) Perifocal frame.

3.2.1.4 Orbital coordinate frame (aircraft plane)

This coordinate system is shown in Fig. (3.4), whereas the inertial reference frame X_o, Y_o, Z_o , previously defined is used to determine the orbital position of the satellite while the attitude motion of a spacecraft is most commonly described in terms of an “airplane” three axis coordinate system, namely, roll, yaw, and pitch. These axes are specified such that, the normal roll axis, X , is along the tangent to the orbit velocity vector, the nominal yaw axis, Z , is along the vector which is joining the center of mass of spacecraft to the center of mass of the earth. The last axis is the nominal pitch axis, Y , is normal to the orbit plane in such away that the X, Y, Z , coordinate system is a right-handed orthogonal triad⁽²⁷⁾. This coordinate

system is also called the orbit coordinate system. The origin of the coordinate system is at the center of mass of the spacecraft and it is rotating with respect to the inertially fixed coordinate system X_o, Y_o, Z_o , at the orbital rate.

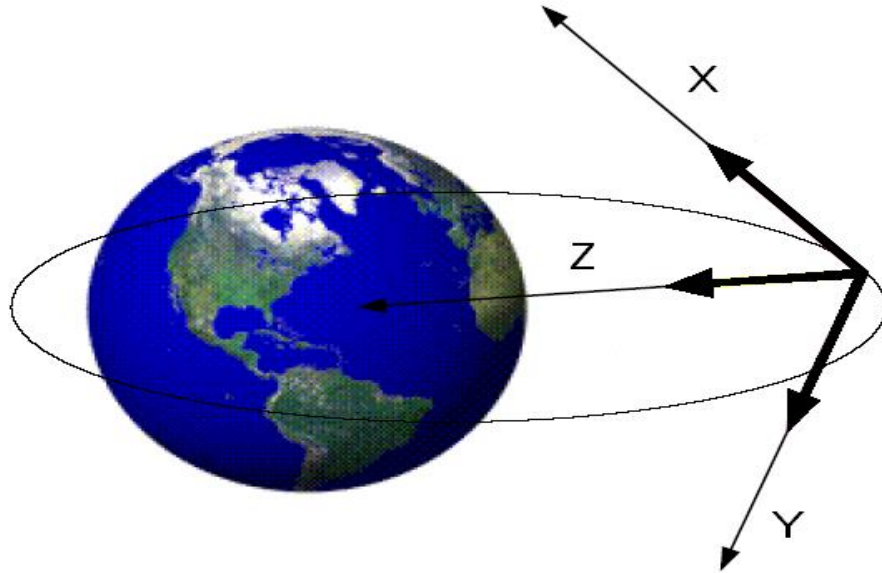


Fig. (3.4) Orbital coordinate frame.

3.2.1.5 Body Fixed Coordinate System

This coordinate system is referred to as the satellite coordinate system called x, y, z , axes as shown in Fig. (3.5). This coordinate x, y, z , is used to describe the perturbed attitude motion of the spacecraft with respect to the orbit coordinate system⁽⁵⁷⁾. Referring to Fig. (3.5), it can be seen that the satellite coordinate x, y, z , is not aligned with X, Y, Z , coordinate system, hence the attitude control goal is to matching these axes coordinates to achieve stabilization⁽⁵⁷⁾. In this research two body fixed coordinate frames are used; the rotor fixed frame x_r, y_r, z_r , and the platform fixed frame x_p, y_p, z_p , as shown in Fig. (3.7).

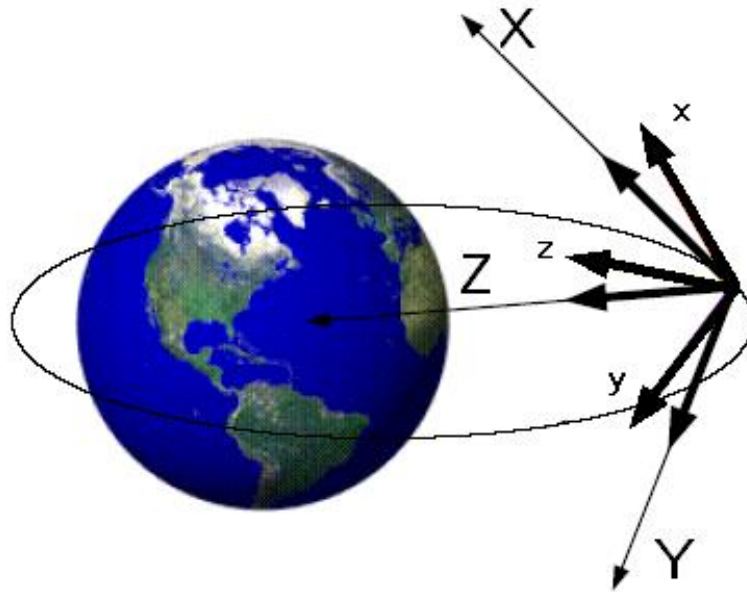


Fig. (3.5) Body fixed coordinate system.

3.2.2 Angular Momentum of a Rigid Body

A basic treatment of particle motion in space should be balanced with an introduction and the fundamental equations of rigid body motion. This is begun by developing an expression for the moment of momentum (angular momentum) of a general rigid body. Consider the situation shown in Fig. (3.6), in which body B contains fixed coordinate system xyz attached at its center of mass, O . In the more general case, the coordinate system xyz is capable not only of rotation relative to XYZ but also of translation.

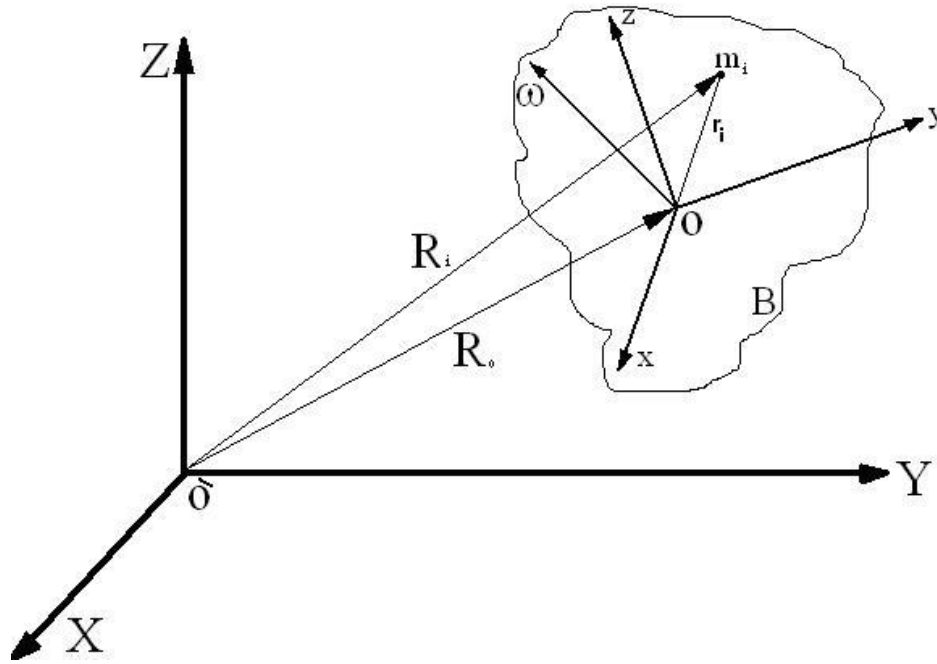


Fig. (3.6) Rigid body angular momentum.

Denoting the origin of xyz by O , the position vector of a particle m_i relative to frame XYZ can be written in the form⁽⁸⁷⁾;

$$\vec{R}_i = \vec{R}_0 + \vec{r}_i \quad 3.1$$

Where R_0 is the radius vector from O' to O and r_i is the position vector of m_i relative to frame xyz .

The absolute velocity of the particle m_i can be obtained by differentiating Eq. 3.1, with the time as^{(85), (87)}:

$$\vec{V}_i = \vec{V}_0 + \vec{\omega} \times \vec{r}_i + [\vec{V}_i]_B \quad 3.2$$

Where:

$\vec{V}_i = \dot{\vec{R}}_i$: velocity of the particle measured relative to frame XYZ .

$\vec{V}_0 = \dot{\vec{R}}_0$: velocity of the origin O measured relative to frame XYZ .

$[V_i]_B$: velocity of particle mass m_i measured relative to frame xyz , and it is equal to zero because B is a rigid body.

ω : angular velocity vector and it is equal to: $\omega i + \omega j + \omega k$

The linear momentum of a particle is defined as the product of its mass and its velocity, then⁽⁸⁷⁾:

$$\vec{P}_i = m_i \vec{V}_i \quad 3.3$$

Where $\vec{P}_i$ is the linear momentum.

The angular momentum of a particle about point O is defined as the moment of the linear momentum about O , ^{(85), (87)}.

$$\vec{h}_{oi} = \vec{r}_i \times \vec{P}_i = \vec{r}_i \times m_i \vec{V}_i \quad 3.4$$

Where $\vec{h}_{oi}$ is the angular momentum of mass m_i about point O .

Substituting for V_i from Eq. 3.2 then Eq. 3.4 becomes:

$$\vec{h}_{oi} = \vec{r}_i \times \vec{P}_i = \vec{r}_i \times m_i (\vec{V}_o + \vec{\omega} \times \vec{r}_i) \quad 3.5$$

if the body is considered as a large number of small masses, the total momentum about O of B is :

$$\vec{h}_o = \sum_i \vec{h}_{oi} = \sum_i \vec{r}_i \times m_i (\vec{V}_o + \vec{\omega} \times \vec{r}_i) \quad 3.6$$

Or, it is more conveniently can be written as;

$$\vec{h}_o = \sum_i \vec{r}_i \times m_i (\omega \times r_i) - \vec{V}_o \times \sum_i m_i \vec{r}_i \quad 3.7$$

However the center of mass is defined as the point at which⁽⁸⁷⁾:

$$\sum_i m_i \vec{r}_i = 0 \quad 3.8$$

Using this definition, then Eq. 3.7 becomes:

$$\vec{h}_o = \sum_i \vec{r}_i \times (\omega \times r_i) m_i \quad 3.9$$

Which it also true, if $\vec{V}_O = 0$, i.e. point O is fixed in space.

Now, if the mass m_i is allowed to decrease and the number of the mass element increases, expression of the angular momentum given in Eq. 3.9 becomes:

$$\vec{h}_O = \int_B \vec{r} \times (\vec{\omega} \times \vec{r}) dm \quad 3.10$$

This expression can be evaluated by the vector products in x, y, z components form.

First, consider the vector:-

$$\vec{\omega} \times \vec{r} = (\omega_y z - \omega_z y)\mathbf{i} + (\omega_z x - \omega_x z)\mathbf{j} + (\omega_x y - \omega_y x)\mathbf{k}$$

And note that the three coefficients are the x, y, and z, components of linear momentum of a unit mass with respect to O , respectively. The complete integral in component form becomes Eq. 3.10:

$$\begin{aligned} \vec{r} (\vec{\omega} \times \vec{r}) = & [\omega_x(y^2 + z^2) - \omega_y(xy) - \omega_z(xz)]\mathbf{i} \\ & + [-\omega_x(xy) + \omega_y(x^2 + z^2) - \omega_z(yz)]\mathbf{j} \\ & + [-\omega_x(xz) - \omega_y(yz) + \omega_z(y^2 + x^2)]\mathbf{k} \end{aligned} \quad 3.11$$

Each coefficient here is the x, y, or z component of angular momentum of a unit mass about point O , respectively.

Integration of from 3.11 over the body dimensions is strictly a function of mass distribution with recognition of the fact that the components of the angular velocity $\vec{\omega}$ are invariant with respect to the integrals over the body because they are independent of the body shape and integral location. Hence, the angular velocity components are become constant multipliers of the integrals. Thus any given rigid body can be characterized by a set of constants for the purpose of studying angular momentum and ultimately attitude motion.

These constants are defined⁽⁸⁷⁾.

$$I_x = \int_B (y^2 + z^2) dm, \quad I_y = \int_B (x^2 + z^2) dm, \quad I_z = \int_B (x^2 + y^2) dm. \quad 3.12$$

$$I_{xy} = \int_B (xy) dm, \quad I_{xz} = \int_B (xz) dm, \quad I_{yz} = \int_B (yz) dm. \quad 3.13$$

Where I_x , I_y , and I_z , are the moments of inertia of the body about x, y, and z axes, respectively, and I_{xy} , I_{xz} , I_{yz} are the products of moment of inertia of body B.

Note that, $I_{xy} = I_{yx}$, $I_{xz} = I_{zx}$, and $I_{yz} = I_{zy}$. And these products of inertia may have positive or negative values while the moments of inertia can never be negative, and it should be also noted that, for a coordinate system fixed in the body, the moments and products of inertia are constant quantities.

Finally the angular momentum of body B, Eq.3.10 will be written as:

$$h_o = h_x i + h_y j + h_z k \quad 3.14$$

Where the components of this vector have been defined as

$$h_x = I_x \omega_x - I_{xy} \omega_y - I_{xz} \omega_z$$

$$h_y = -I_{xy} \omega_x + I_y \omega_y - I_{yz} \omega_z \quad 3.15a,b,c$$

$$h_z = -I_{xz} \omega_x - I_{yz} \omega_y + I_z \omega_z$$

the above expression of the angular momentum component appears to be a set of three simultaneous linear equations in terms of angular velocity components, which may be written in matrix form as:-

$$h_o = \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad 3.16$$

The matrix containing the moments and products of inertia is known as the inertia tensor. This equation can be written in a vector form as:

$$\vec{h}_o = I \vec{\omega} \quad 3.17$$

3.2.3 Rotational Kinetic Energy of a Rigid Body

The kinetic energy of a rigid body about its center of mass is referred to as rotational kinetic energy. This can be shown to be related to angular momentum h (where the subscript has been dropped and the angular momentum is assumed to be taken about the center of mass in the following section, unless otherwise stated) by considering the definition of kinetic energy of an element of mass $dm^{(87)}$,

$$dT = \frac{1}{2} V^2 dm \quad 3.18$$

Where V is the magnitude of absolute velocity of an element of mass dm . The velocity of an element mass dm located at vector position $\vec{r}$ (see Fig. (3.6)), is given by⁽⁸⁷⁾:

$$\vec{V} = \vec{V}_o + \vec{\omega} \times \vec{r} \quad 3.19$$

Taking the dot product of the velocity vector $\vec{V}$ as:

$$\vec{V} \cdot \vec{V} = V^2 = V_o^2 + (\vec{\omega} \times \vec{r}) \cdot (\vec{\omega} \times \vec{r}) + 2\vec{V}_o \cdot (\vec{\omega} \times \vec{r}) \quad 3.20$$

Substituting for V , in the definition of the kinetic energy, gives:

$$T = \frac{1}{2} \int_B V^2 dm = \frac{1}{2} m_B V_o^2 + \frac{1}{2} \int_B (\vec{\omega} \times \vec{r}) \cdot (\vec{\omega} \times \vec{r}) dm + V_o \cdot \vec{\omega} \times \int_B \vec{r} dm \quad 3.21$$

The last integral term in the above expression of the kinetic energy must be zero because $\vec{r}$ is referred to the center of mass. The first term of the remaining two terms represents the translational kinetic energy associated with a particle of mass m_B concentrated at the body center of mass and the second term represents the rotational kinetic energy about the center of mass.

Thus, the kinetic energy of rotation, T_{rot} , is defined by:

$$T_{rot} = \frac{1}{2} \int_B (\vec{\omega} \times \vec{r}) \cdot (\vec{\omega} \times \vec{r}) dm \quad 3.22$$

Or,

$$T_{\text{rot}} = \frac{1}{2} \int_B [(\omega_y z - \omega_z y)^2 + (\omega_z x - \omega_x z)^2 + (\omega_x y - \omega_y x)^2] dm \quad 3.23$$

Expanding the above expression, this produces

$$T_{\text{rot}} = \frac{1}{2} \int_B [\omega_x^2 (y^2 + z^2) + \omega_y^2 (x^2 + z^2) + \omega_z^2 (x^2 + y^2) - 2\omega_x \omega_y (xy) - 2\omega_y \omega_z (yz) - 2\omega_x \omega_z (xz)] dm \quad 3.24$$

Using the definition of the moments and products of inertia given in Eqs. 3.12 and 3.13 gives:

$$T_{\text{rot}} = \frac{1}{2} [\omega_x^2 I_x + \omega_y^2 I_y + \omega_z^2 I_z - 2\omega_x \omega_y I_{xy} - 2\omega_y \omega_z I_{yz} - 2\omega_x \omega_z I_{xz}] \quad 3.25$$

Or, T_{rot} , can be expressed in the matrix form as:

$$T_{\text{rot}} = \frac{1}{2} \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix} \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad 3.26$$

Or, in vector notation

$$T_{\text{rot}} = \frac{1}{2} \vec{\omega}^T \mathbf{I} \vec{\omega} \quad 3.27$$

In terms of the angular momentum components, the rotational kinetic energy can be given by:

$$T_{\text{rot}} = \frac{1}{2} \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix} \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} \quad 3.28$$

The important use of the expression of the rotational kinetic energy ultimately appears in the next chapter, when the spacecraft is equipped with a viscous nutation damper. However the derivation of the equations, which describes the relationship between the angular momentum component and the corresponding viscous damping torque due to the presence of the nutation damper, is achieved by using Lagrange's equation for quasi coordinates.

Whereas, the derivative terms of this Lagrange's equation are expressed in terms of quasi-coordinates (i.e. in terms of the angular velocity components of the system), and the required expression for evaluating these derivative terms is completely meeting by using the corresponding expression of the rotational kinetic energy of the system in question.

3.3 Attitude Dynamic of Spinning Spacecraft

The attitude dynamic is concerned with the problem of spacecraft orientation in space. Equations of attitude motion of spacecraft are similar to the general equation of attitude motion of a rigid body, which they are known as Euler's moment equations.

Generally, earth satellites are required to point either toward the earth or toward a fixed point in an inertial space⁽⁵⁷⁾. As was mentioned previously, this can be accomplished by active control system or by passive means. It was shown that, the passive method is the one adopted, and among means of this method, spins stabilization and specifically dual-spin is chosen in the present study.

In this section, a complete description of the adopted dual spin satellite is presented and equations of attitude motion of this model are established in terms of dimensionless parameters of the system.

3.3.1 Equations of Motion (Euler's Moment Equations)

The equations of motion are based on Newton's second law, which states that the rate of change of the linear momentum of a particle is equal to the force acting on the particle. It should be noted that, in applying this law one must measure motion relative to an inertial coordinate system.

The above definition of the linear momentum can be extended to define the rotational motion of a rigid body about its center of mass (see Fig. (3.6)), which becomes, the rate of change of the angular momentum is equal to the sum of the applied torques. Or in vector form is given by⁽⁸⁷⁾:

$$\vec{M} = \left[\frac{d \vec{h}}{dt} \right]_{XYZ} \quad 3.29$$

Where $\vec{M}$ is the vector of the applied torques.

In body coordinate system, Eq. 3.29 becomes^{(84), (87)}:

$$\vec{M} = \left[\frac{d \vec{h}}{dt} \right]_{XYZ} + \vec{\omega} \times \vec{h} \quad 3.30$$

Substituting for the angular velocity vector and angular momentum vector, then the equations of motion along the axes x, y, z, can be written⁽⁸²⁾:

$$M_x = \dot{h}_x + \omega_y h_z - \omega_z h_y \quad 3.31a$$

$$M_y = \dot{h}_y + \omega_z h_x - \omega_x h_z \quad 3.31b$$

$$M_z = \dot{h}_z + \omega_x h_y - \omega_y h_x \quad 3.31a$$

These three differential equations relate applied torque components to angular momentum changes, and they are known as "Euler's equations". Thus the general attitude motion of a rigid body and/or a satellite is given by these three simply written equations. However there is no general solution because $\vec{M}$ is not specified. Fortunately, many situations of interest permit specialization of these expressions in such a way that meaning solutions are obtainable in closed form. It is interesting to note that the set of Euler's equations appear to be three first order in terms of angular momentum vector. These differential equations may be converted to first order equations in terms of angular velocity components through

application of expressions 3.15. Or these equations can be converted into second order equations when it is required to measure the orientation of a rigid body in inertial space through the Euler angles and rates.

3.3.2 Mathematical Modeling of Dual-Spinning Spacecraft

Generally, dual-spin satellite consists of two rigid bodies which are connected by means of a bearing axis. One of the bodies, the rotor, is spun at high rate to stabilize the spacecraft via gyroscopic stiffness. The second body, the platform, is nearly despun so that it is payload instrument can be inertially pointed toward the earth or other point in the space. The relative motion between two bodies is restricted to rotation about the bearing axis. The mass centers of the two bodies must lie on the axis of rotation and the axis of rotation is a common principle axis of the two bodies.

3.3.2.1 Description of the Present Model

The above description, represents a general description of dual-spin spacecraft, because each model being under investigation must have its own characteristics. These characteristics are greatly related to the dynamic imbalance problem and the problem of symmetry or asymmetry appears in the rotor or the platform or both of them.

The dual-spin spacecraft, chosen for the present study, is shown schematically in Fig. (3.7). The system is comprised of a rigid rotor $\bar{R}$ and a rigid platform $\bar{P}$ whose respective mass centers, O_R and O_P , lie on the common axis (bearing axis), and are separated by a distance r_c . Two sets of reference axes ($O_P x_P y_P z_P$) and ($O_R x_R y_R z_R$) are introduced at the mass centers C_P and C_R respectively, and the x , y , z coordinates are located at the system center of mass and are fixed relative to the

rotor. The axes z_p , z_r , and z are coincided with the axis of rotation, and the relative rotation of body $\bar{P}$ with respect to body $\bar{R}$ about the z axis is denoted by the angle α_r .

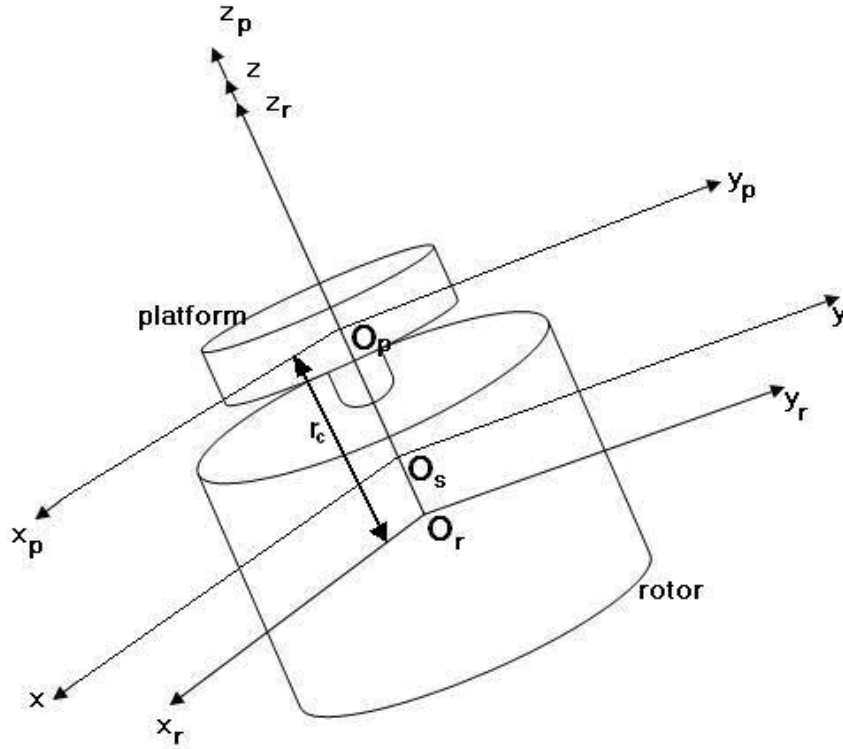


Fig. (3.7) Spacecraft model.

The platform $\bar{P}$ and the rotor $\bar{R}$ have masses m_p and m_R respectively. The individual centroidal inertia tensors, $[I_p]$ and $[I_R]$, referred to their respective axis sets $(O_p x_p y_p z_p)$ and $(O_R x_R y_R z_R)$ may take the following forms:

$$[I_p] = \begin{bmatrix} B_p & 0 & 0 \\ 0 & B_p & 0 \\ 0 & 0 & C_p \end{bmatrix}, \quad [I_R] = \begin{bmatrix} A_R & 0 & 0 \\ 0 & A_R & 0 \\ 0 & 0 & C_R \end{bmatrix} \quad 3.32$$

In the present study, both bodies are statically balanced, since their mass centers lie on the bearing axis, and are also dynamically balanced, since their

common axis (bearing axis) is on their principal axes. The bearing axis of symmetric as evidence from the double entry of B_p , and A_p in their inertia tensors.

3.3.2.2 Equations of Motion of Dual-Spin Spacecraft

The essential dynamic properties of the model, which is adopted in the present study, are described in the previous section. Therefore, the basic components of the satellite (the platform and the rotor), that are shown in Fig. (3.7) are two symmetrical rigid bodies with the system center of the mass O_s , and the relative motion between the two bodies is restricted to rotation about the common axis (bearing axis).

The angular velocity components of the platform and the rotor, referred to their reference axes are;

$$\vec{\omega}_p = \omega_x \mathbf{e}_x + \omega_y \mathbf{e}_y + \Omega_p \mathbf{e}_z \quad 3.33$$

and

$$\vec{\omega}_r = \omega_x \mathbf{e}_x + \omega_y \mathbf{e}_y + \omega_z \mathbf{e}_z \quad 3.34$$

Where Ω_p is the inertial spin rate of the platform about the spin axis.

The system angular momentum can be obtained using expression of Eq. 3.16, after omitting the inertia products due to symmetry, then one get:

$$\vec{h} = [I]_{s/c} \cdot \vec{\omega}_r + \vec{h}_p$$

3.35

Where $[I]_{s/c}$ is the inertia matrix of the entire spacecraft, and it is given by;

$$[I]_{s/c} = \begin{bmatrix} A_x & 0 & 0 \\ 0 & A_y & 0 \\ 0 & 0 & C \end{bmatrix}$$

and the entries of the inertia matrix are the principal moments of inertia referred to the system center of mass O_s , and are defined as^{(84), (87)}:

$$\begin{aligned}
A_x &= A_R + B_p + r_c^2 \cdot m_p m_R / (m_p + m_R) \\
A_y &= A_R + B_p + r_c^2 \cdot m_p m_R / (m_p + m_R)
\end{aligned}
\tag{3.36}$$

It is evidenced that, the two principal moments of inertia about the x and y axes are equal, where A_R and B_p principal moments of inertia of the rotor and platform respectively, because the platform and the rotor are assumed to be symmetrical, and $\vec{h}_p$ is the angular momentum vector of the platform about the bearing axis and it is given by the following expression, $\vec{h}_p = [0 \ 0 \ C_p \Omega_p]^T$,

Substituting the angular momentum and their time derivatives in the Euler's equations Eq. 3.31, then the attitude equations of motion of symmetric spacecraft ($A_x = A_y$) are given by:

$$A \dot{\omega}_x + (C - A) \omega_z \omega_y + C_p \Omega_p \omega_y = \Sigma M_x \tag{3.37a}$$

$$A \dot{\omega}_y - (C - A) \omega_z \omega_x - C_p \Omega_p \omega_x = \Sigma M_y \tag{3.37b}$$

$$C \dot{\omega}_z - C_p \dot{\Omega}_p = \Sigma M_z \tag{3.37c}$$

$$C_p \dot{\Omega}_p = T_b \tag{3.37d}$$

$$\dot{\alpha}_r = \Omega_p - \omega_z \tag{3.37e}$$

where,

$A = A_x = A_y$: transverse principal moments of inertia along x, and y axes respectively,

C : spin axis principal moment of inertia of the rotor,

C_p : spin axis principal moment of inertia of the platform,

$T_b = T_m + T_f$: total torque,

T_m : torque produced by DC motor on the platform,

T_f : torque produced by bearing friction,

α_r : Relative rotation rate of the platform with respect to the rotor.

3.3.2.3. Torque-Free Motion of Axisymmetric Spacecraft

In the study of spacecraft attitude dynamics, the motion of axisymmetric spacecraft is the most common situation. An axisymmetric body is a body, which contains two planes of symmetry. A general condition of axisymmetric bodies is that moments of inertia about two of the axes are equal (e.g. $A_x = A_y$).

The phrase “torque-free” simply refers to the condition that no external moments act on the body.

The equation of motion of axisymmetric bodies with moments of inertia $A_x = A_y = A$, were given in the previous section. Using a body fixed frame of reference about the principal axes, the torque components are assumed to be zero (torque-free), and by noting that the inertial spin rate of the platform is maintained constant during its normal operating mode by suitable despun control loop (i.e. $\dot{\Omega}_p = 0$), then Eqs. 3.37a, b, c can be reduced to:

$$A \dot{\omega}_x + (C-A) \omega_z \omega_y + C_p \Omega_p \omega_y = 0$$

$$A \dot{\omega}_y - (C-A) \omega_z \omega_x - C_p \Omega_p \omega_x = 0 \quad 3.38a, b, c$$

$$C \dot{\omega}_z = 0$$

Although these are still nonlinear and coupled, a solution of motion can be obtained on the basis of perturbation analysis. It is known that, the common axis (z-axis) is the spin axis of the spacecraft, and then the initial condition can be described by $\omega_x = \omega_y = 0$, and $\omega_z = \Omega$, where Ω is inertial spin rate of the spacecraft, and the perturbed motion is given by $\omega_z = \Omega + \varepsilon_d$, and $\omega_x \approx 0$, $\omega_y \approx 0$ where ω_x and ω_y are small.

Substituting, these conditions in Eqs. 3.38a, b, c, and neglecting the product of small quantities, then the resulting linearized equations are:

$$\dot{\omega}_x + \lambda_n \Omega \omega_y = 0 \quad 3.39a$$

$$\dot{\omega}_y - \lambda_n \Omega \omega_x = 0 \quad 3.39b$$

$$\dot{\varepsilon}_d = 0 \quad 3.39c$$

where:

$$\lambda_n = \lambda + \lambda_s$$

$$\lambda = \sigma - 1$$

$$\lambda_s = \frac{\Omega_p}{\Omega} \frac{C_p}{A}$$

$\sigma = \frac{C}{A}$: inertia ratio of the principal moment of inertia of the spin axis to the transverse principal moment of inertia of the spacecraft.

From Eq. 3.39c it can be concluded that ε_d remains constant. Differentiating Eq. 3.39a and substituting ω_x and ω_y from Eqs.3.39b respectively, then one get:

$$\ddot{\omega}_x + \lambda_n^2 \Omega^2 \omega_x = 0 \quad 3.40a$$

$$\ddot{\omega}_y + \lambda_n^2 \Omega^2 \omega_y = 0 \quad 3.40b$$

From Eq. 3.40a, it can be obtained that⁽⁸⁷⁾:

$$\omega_x = P_a \sin(\lambda_n \tau - \varphi) \quad 3.41$$

where:

P_a : the amplitude.

φ : the phase angle.

τ : the dimensionless time and it is given by $\tau = t \Omega$.

substituting Eq. 3.41 into Eq. 3.39a gives:

$$\omega_y = P_a \cos(\lambda_n \tau - \varphi) \quad 3.42$$

The resultant transverse angular velocity, ω_t is the sum of velocity ω_x and ω_y , and introducing complex variable notation, then ω_t is given by:

$$\omega_t = \omega_x + i\omega_y \quad 3.43$$

$$\omega_t = P_a e^{(\lambda_n \tau - \phi)}$$

The resultant transverse angular velocity is a vector in the xy plane, rotating with the an angular velocity of $(\lambda_n \Omega)$.

3.4 Environmental Disturbing Torques

Generally, two types of torques have been found to influence a spacecraft attitude, namely internal and external torques. It may be noted that, both types may affect the total rotational energy of a spacecraft; however, only an external torque will affect the total angular momentum. External torques result from the interaction of the spacecraft with its environment, the most influential components including gravity gradient, solar radiation pressure and aerodynamic torques.

Internal torques may be considered to arise due to internal motion of the spacecraft such as fuel sloshing, crew motion, flexing appendages. These torques cannot change the total angular momentum of the system but can cause significant changes in the orientation of critical parts of the spacecraft such as antennas, cameras, telescopes or attitude sensors⁽⁴²⁾, such torques are to be expected when the spacecraft is non rigid, and there is relative motion between components of the configuration. Perhaps the most general class of internally generated torques is those arising from spacecraft structural flexibility.

Environmental disturbing torques are either secular, which accumulate overtime, or cyclic, which vary sinusoidally over an orbit. Different environmental torques are also more prevalent at different attitudes, and the relationship between attitude and disturbance torque strength is shown in Fig (3.8). In low earth orbits (LEO), the

largest environmental torques are gravity gradient, magnetic, and aerodynamic and the most dominant torques at geosynchronous orbit (geostationary orbit) are solar radiation pressure and gravity gradient⁽⁵⁸⁾.

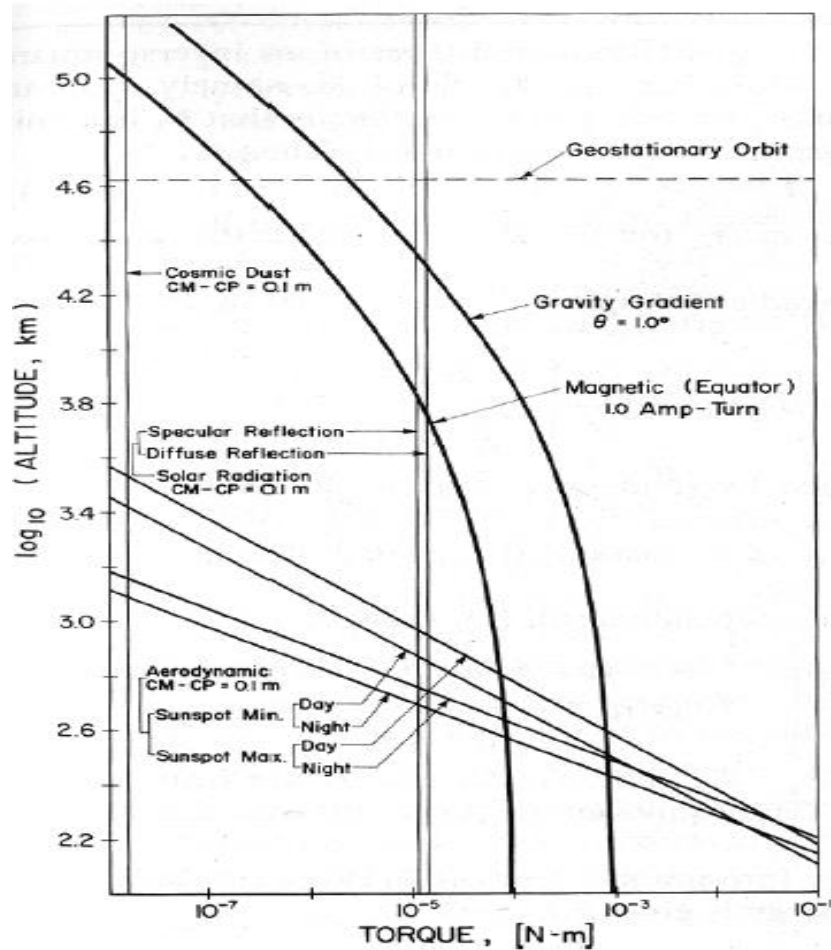


Fig. (3.8) Environmental disturbance torques⁽⁴²⁾.

3.4.1 Calculation of the Environmental Disturbing Torques

As it is mentioned above (section 3.4), the most important disturbance torques are caused by the effects of gravity gradients, solar radiation, aerodynamic, and magnetic torques. Each of the above mentioned disturbing torques are discussed in turn below with more descriptive calculation procedures.

3.4.1.1 Gravity Gradient Torques

The gravity gradient torque is caused by the slight change in the gravitational contraction between the top and bottom of the satellite, i.e. the force of gravity is weaker on those parts of the spacecraft which are furthest away from the earth and stronger on those parts which are closer. Because of this gravity gradient, the spacecraft tend to align its long axis with the local vertical. If this is the desired orientation of the spacecraft, this torque acts to stabilize the satellite; otherwise, it is a perturbing torque and must be counteracted by other means. Gravity gradient torque is usually important only for spacecraft in low orbit. For nadir pointing spacecraft, this torque is nearly constant, while for inertial pointing vehicles it is periodic⁽⁵⁸⁾.

The magnitudes of the torques themselves are given by the following equations⁽⁵⁸⁾:

$$\begin{aligned}
 T_x &= \left(\frac{3\mu}{R_0^3} \right) (I_{xy} a_x a_z - I_{xz} a_x a_y + (I_{zz} - I_{yy}) a_y a_z + I_{yz} (a_z^2 - a_y^2)) \\
 T_y &= \left(\frac{3\mu}{R_0^3} \right) (I_{yz} a_y a_z - I_{xy} a_y a_z + (I_{xx} - I_{zz}) a_x a_z + I_{xz} (a_x^2 - a_z^2)) \\
 T_z &= \left(\frac{3\mu}{R_0^3} \right) (I_{xz} a_z a_y - I_{yz} a_z a_x + (I_{yy} - I_{xx}) a_x a_y + I_{xy} (a_y^2 - a_x^2))
 \end{aligned} \tag{3.44a, b, c}$$

where

T_x, T_y, T_z : torques about the x, y, and z axes respectively,

μ : local gravitational constant,

R_0 : orbit radius,

I_{xx}, I_{yy}, I_{zz} : moments of inertia,

I_{xy}, I_{yz}, I_{xz} : products of moments of inertia,

a_x, a_y, a_z : cosines of the directions between the spacecraft's coordinate axes and the orbit radius vector.

If there is no product of inertia for the spacecraft, these equations can be reduced to:

$$T_x = \left(\frac{3\mu}{R_0^3} \right) (I_{zz} - I_{yy}) a_y a_z$$

$$T_y = \left(\frac{3\mu}{R_0^3} \right) (I_{xx} - I_{zz}) a_x a_z \quad 3.45a, b, c$$

$$T_z = \left(\frac{3\mu}{R_0^3} \right) (I_{yy} - I_{xx}) a_x a_y$$

The local gravitational constant for the earth may taken as $3.986 * 10^5 \text{ km}^3/\text{s}^2$ and should be noted that the cosines of direction a_x , a_y , and a_z are used here with their maximum deviation values from the vertical, which is determined by the pointing accuracy⁽⁵⁸⁾.

3.4.1.2 Aerodynamic Drag Torque

Although there is obviously much less drag at orbital altitudes than near the surface of the earth, aerodynamic drag can still cause large torque on a satellite, and often determines its lifetime.

It should be noted that symmetric areas of spacecraft cause no net aerodynamic torque. A spherical satellite, for example, would feel no aerodynamic at all, although the drag would eventually slow it down enough for reentry. Torque due to aerodynamic is nearly constant for nadir pointing spacecraft.

The maximum torque on a spacecraft due to aerodynamic drag can be estimated by the following equation⁽¹⁰⁾:

$$T_a = \frac{1}{2} \rho V^2 A_d C_d L \quad 3.46$$

where,

T_a : aerodynamic drag torque,

ρ : atmospheric density,

V : satellite velocity (orbital velocity),

A_d : frontal area (surface area),

C_d : drag coefficient,

L : distance between the center of the surface and the center of mass of the spacecraft.

The drag coefficient C_d depends on the configuration of the spacecraft and on its orientation relative to the velocity vector. It is often valid to use the approximation $C_d=2$,⁽⁵⁸⁾. Density for a given altitude can be obtained from tables. The orbital velocity of the satellite V can be calculated from the square root of (μ/R_0) , where μ is the local gravitational constant and R_0 is the orbit radius. Then, the torque should be calculated about all three axes, with the necessary changes in A_d and L .

For a spacecraft that can modeled with a number of simple pieces, the aerodynamic torque due to drag can be estimated by:

$$T_a = \frac{1}{2} \rho V^2 \sum_i^n A_{di} C_{di} L_i \quad 3.47$$

where, n is the total number of the surfaces experienced aerodynamic drag.

3.4.1.3 Solar Radiation Torque

Solar radiation torque is caused by solar radiation pressure, which arises from the transfer of linear momentum from solar radiation that absorbed and/or reflected by satellite surfaces. It is a function of the configuration and orientation of the spacecraft and on the surface reflectance of the vehicle. Solar radiation torque is cyclic for nadir pointing spacecraft and nearly constant for inertial pointing satellite⁽³¹⁾.

Due to solar radiation two components of forces are induced, usually they are called normal and tangential components. These torque components which produced due to the force components are represented by⁽⁵⁸⁾:

$$T_n = P_r A [(1+C_s) \cos 2\gamma_r + \frac{2}{3} C_{dr} \cos \gamma_r] \quad 3.48a, b$$

$$T_t = P_r A [(1- C_s) \sin \gamma_r \cos \gamma_r]$$

where,

P_r : radiation pressure (approximately 4.617×10^{-6} N/m² for near earth⁽⁵⁸⁾).

γ_r : angle between the radiation direction and the normal to the area.

C_s : specular reflection (approximately 2, ⁽⁵⁸⁾).

C_{dr} : diffuse reflection (approximately 5, ⁽⁵⁸⁾).

T_t, T_n : normal and tangential torque components.

3.4.1.4 Magnetic Field Torques

Magnetic coils (or magnetic torquers, or sometimes-called torque rods) interact with the earth's magnetic field in order to produce torque. These devices are extremely simple, they consist of wire-wound ferrous rods on each axis. When the current is passes through the wire coil, the device becomes an electromagnet, which exerts a torque against the local magnetic field.

Magnetic torquers have the advantage of simplicity and reliability. They also represent the only practical active method available to exert external torques on a spacecraft without expelling propellant. This makes them useful for momentum damping from wheels and control gyros.

Unfortunately, the magnetic field of the earth is not strong enough to permit these devices to exert much torque, and limit the usefulness for applications.

Also, the magnetic torque can only act about two axes, since no torque can be produced in the direction of the local earth geomagnetic field vector. The fact that

their performance directly depends on the strength of the local magnetic field is also a serious drawback. Magnetic torques are useless for geosynchronous spacecraft, since the earth's magnetic field is very weak at such high attitudes.

The relation for the magnetic torque produced by the electromagnetic dipole of a spacecraft is found as follows⁽⁵⁸⁾:

$$\vec{T}_M = \vec{M}_M \times \vec{B}_M \quad 3.49$$

where,

$\vec{M}_M$: Magnetic dipole vector produced by electromagnets in the spacecraft. The units used are A.m² (ampere meter²).

$\vec{B}_M$: Local earth geomagnetic field vector, in Gauss.

From this equation, it is seen that the vector T_m is always perpendicular to the vector B_M .

The magnitude of T_m is maximum when the vector M_M perpendicular to the vector B_M , and it is zero when the vector M_M parallel to the vector B_M . The direction of the vector B_M depends on spacecraft position and the orientation of it is plane relative to earth.

The required magnetic dipole vector M_M can be calculated from the following equation⁽⁵⁸⁾:

$$\vec{M}_M = \frac{\vec{B}_M \times \vec{T}_C}{B_M^2} \quad 3.50$$

where,

$\vec{T}_C$: Desired control torque vector.

In order to counteract an undesired torque, T_c should have the same magnitude but opposite direction.

The size of the magnetic coil can be found from the largest component of the vector M_M , which is the dipole moment needed to counteract the torque. The coil must be large enough to provide a magnetic dipole greater than the largest component of the vector M_M .

The largest magnetic torques has strength of around 4000 A.m², This yields a maximum of 0.16 N.m. Magnetic torquers typically have masses ranging from less than 1 kg to around 50 kg, and their power requirements range from 0.6 to 16W,⁽⁵⁸⁾.

3.5 Satellite Kinematics

The formulation of spacecraft attitude dynamics and control problem involves consideration of kinematics. Satellite kinematics is primarily interested in describing the orientation of the satellite that is in rotational motion. Since the relative orientation of the satellite frame and the inertial frame is time dependent, so that, the time dependent relationship between the reference frames can be described by the so called Kinematics differential equations.

There are a number of schemes available for determining the kinematic differential equations. These schemes include: direction cosine matrix, Euler angles, Euler's eigen axis, and quaternion. Of these schemes, quaternion is the most widely used, due to its easy of use since it uses only multiplication and addition operation. Hence it is very useful in digital computer and it may reduce the computation time by 40% over the equivalent direction cosine matrix solution⁽³⁷⁾. The major disadvantage is that there is no obvious physical interpretation of quaternion⁽⁴⁰⁾. In this section the kinematics' differential equations for the quaternion will be explained in detail.

3.5.1 Quaternion

The attitude of the spacecraft is represented using the quaternions. The quaternion is a four element column matrix that is defined in terms of the Euler angles and Euler axis of the rotation it represents. Quaternions are hyper complex number.

Hamilton invented the quaternions as a result of searching for hyper complex number. Euler's theorem states that any pure rotational displacement can be generalized into a unit vector, Euler axis, defining the axis of the rotation and an angle, Euler angle, defining the magnitude of the rotation. An excessive analysis of the quaternion is in reference^{(36), (37), (40), (68), (81) and (85)}.

The elements of quaternions are given by^{(40), (81)}:

$$q_1 = e_1 \sin\left(\frac{\psi}{2}\right) \quad 3.51$$

$$q_2 = e_2 \sin\left(\frac{\psi}{2}\right) \quad 3.52$$

$$q_3 = e_3 \sin\left(\frac{\psi}{2}\right) \quad 3.53$$

$$q_4 = \cos\left(\frac{\psi}{2}\right) \quad 3.54$$

$$\vec{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

$$\therefore \vec{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

The first derivative of quaternion is defined as⁽⁴⁰⁾:

$$\dot{\vec{q}} = \frac{1}{2} \vec{\omega} \times \vec{q} \quad 3.55$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} q_4 & -q_3 & q_2 & q_1 \\ q_3 & q_4 & -q_1 & q_2 \\ -q_2 & q_1 & q_4 & q_3 \\ -q_1 & -q_2 & -q_3 & q_4 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ 0 \end{bmatrix} \quad 3.56$$

The quaternion has a straight forward method to mathematically "add" successive rotations⁽⁴³⁾ and so have not inherent geometric singularity as Euler angles do. The body angular velocities ω_x , ω_y and ω_z are measured by rate gyros, and the kinematics differential equation 3.56 is then integrated numerically using onboard flight electronic card to determine the orientation of the spacecraft in terms of quaternions.

The quaternions are related to each other by the relation⁽⁸¹⁾:

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \quad 3.57$$

3.6 Attitude Stability of Spinning Spacecraft

The term stability here is used to describe the motion of a torque-free rigid body after being disturbed from steady spin. Thus motion is stable if amplitudes of disturbed quantities are bounded by initial values.

The stability of spinning spacecraft is investigated in the next section for two cases, when the spacecraft is without energy dissipation and with energy dissipation, then stability is obtained in terms of spacecraft inertia ratio.

3.6.1 Stability of Rotation about Principal Axes

Consider a rigid body that is rotating about one of its principal axes. If there are external torques acting on the body and it, for example, the rotating was initially about the z-axis (see Fig. (3.7)), then the body will continue to spin about

the z-axis; i.e. $\omega_z = \Omega = \text{constant}$. And $\omega_x = \omega_y = 0$, on the other hand, if a small impulsive disturbance torque is applied to the body, then the body will no longer rotate purely about the spin axis.

In the absence of external moments, Euler's equations of a rigid body (for general case, (i.e. $I_x \neq I_y \neq I_z$) are given by:

$$I_x \dot{\omega}_x + (I_z - I_y) \omega_y \omega_z = 0$$

$$I_y \dot{\omega}_y + (I_x - I_z) \omega_x \omega_z = 0 \quad 3.58a, b, c$$

$$I_z \dot{\omega}_z + (I_y - I_x) \omega_x \omega_y = 0$$

Where I_x , I_y , and I_z are the principal moments of inertia.

For linear stability analysis, it is assumed that the perturbation terms ω_x and ω_y , are much smaller in magnitude than ω_z . Neglecting, the products of the small perturbation terms and substituting for $\omega_z = \Omega$, then, the linearized rotational equations of motion becomes:

$$I_x \dot{\omega}_x + (I_z - I_y) \omega_y \Omega = 0$$

$$I_y \dot{\omega}_y + (I_x - I_z) \omega_x \Omega = 0 \quad 3.59a, b, c$$

$$I_z \dot{\Omega} = 0$$

Where Ω is the spin rate about the z-axis, and it is constant as shown by Eq. 3.59c.

Combining these equations, it can be obtained that:

$$\ddot{\omega}_x + \frac{(I_x - I_z)(I_y - I_z)}{I_x I_y} \Omega^2 \omega_x = 0 \quad 3.60$$

The characteristic equation is:

$$S^2 + \frac{(I_x - I_z)(I_y - I_z)}{I_x I_y} \Omega^2 = 0 \quad 3.61$$

And the characteristic roots are:

$$S = \pm i\Omega \sqrt{\frac{(I_x - I_z)(I_y - I_z)}{I_x I_y}} = 0 \quad 3.62$$

If the spin axis (the z-axis) is either the major axis ($I_y < I_z$, and $I_x < I_z$) or minor axis ($I_y > I_z$, and $I_x > I_z$), then the characteristic roots become pure imaginary numbers and the rotational motion is said to be (Lyapunov) stable though not asymptotically stable. If the spin axis is the intermediate axis then one of the characteristic roots is positive real number and motion is said to be unstable.

In the case of axisymmetric spacecraft ($I_x = I_y = I_T$), two possible cases, $I_z > I_T$ and $I_z < I_T$, are shown on Fig. (3.9) and Fig.(3.10), and the motion of the spacecraft can be represented in terms of the body cone and the space cone.

The body cone is fixed in the body and its axis coincides with the spin axis while the space cone is fixed in the space and its axis is along the direction of the angular momentum $\vec{h}$. The total angular velocity is along the line of the contact between the two cones. The plane containing angular momentum $\vec{h}$, total angular velocity $\vec{\omega}$, and spin axis rotates about $\vec{h}$ at angular velocity of $\frac{I_z}{I_T}\Omega$.

For a disk-shaped body $\frac{I_z}{I_T} > 1$, the inside surface of the body cone rolls on the outside surface of the space cone, as shown in Fig. (3.9). For a rod-shaped body $\frac{I_z}{I_T} < 1$, the outside surface of the body cone rolls on the outside surface of the space cone as shown in Fig. (3.10).

The nutation angle θ , which is the angle of inclination of the spin axis, with respect to the initial direction of the angular momentum $\vec{h}$, is given by:

$$\sin\theta = \frac{I_T \omega_T}{H} \quad 3.63$$

Where H is the magnitude of the angular momentum vector $\vec{h}$.

The angle ζ , which is the angle between the axis of rotation (direction of ω) and the spin axis (z axis), is given by:

$$\tan \zeta = \frac{\omega_T}{\Omega} \quad 3.64$$

3.6.2 Dual-Spin Stabilization of an Energy-Dissipation Spacecraft

In a dual-spin spacecraft, which consists of a rotor and platform, the rotor is spinning about the axis of principal moment of inertia providing gyroscopic stabilization and a despun platform providing pointing toward the earth. As shown in the preceding section, for a rigid spacecraft, stable spin can take place about the

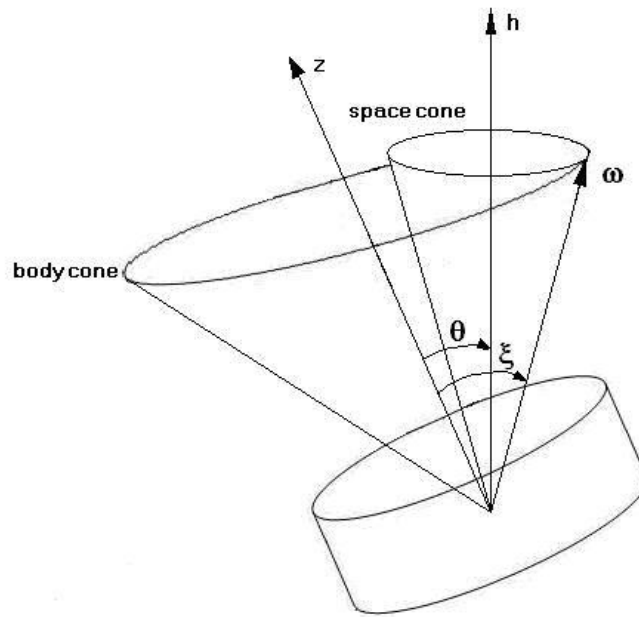


Fig. (3.9) Disk shaped spacecraft.

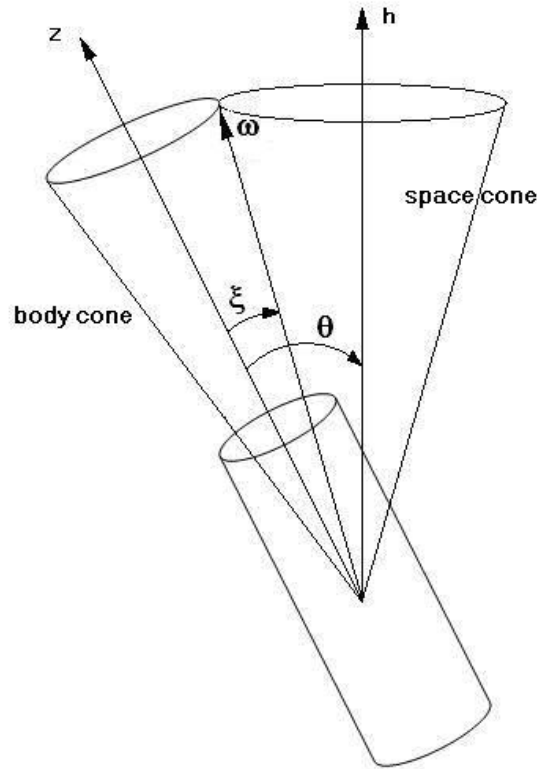


Fig. (3.10) Rod shaped spacecraft.

axis of either the maximum or the minimum principal moment of inertia. However, satellite is likely to have several components experiencing relative motion, such as elastic structural deflection and liquid slosh, so that the satellite is not rigid.

Before considering the stability criterion for dual spin satellite, stability of spinning body in terms of the rotational kinetic energy and the angular momentum is considered. In the absence of external torques, the angular momentum of the spacecraft is conserved, but the internal energy dissipation will cause the spacecraft to move toward a minimum-energy state. The kinetic energy, T_{rot} , of the spacecraft is related to the magnitude of angular momentum, $\vec{h}$ and to the moment of inertia, I , about the spin axis by⁽⁶⁸⁾:

$$T_{\text{rot}} = \frac{1}{2} \frac{H^2}{I}$$

It can be seen that, for constant angular momentum, the kinetic energy is a minimum when the moment of inertia, I , is maximum. Hence, for a non-rigid spacecraft there is only one axis of stable spin, namely, the axis of maximum moment of inertia.

In the case of dual-spin spacecraft, the stability criterion is given in terms of the moment of inertia ratio of the spin axis to the transverse axis and the amount of the energy dissipation rate in the platform and the rotor. Iorillo presented stability criterion for dual-spin spacecraft in 1965 and a similar stability criterion was discovered independently by V.D London⁽⁴¹⁾. Accordingly, in a dual-spin stabilized spacecraft, the stable spin axis can be the axis of minimum moment of inertia if the rate of energy dissipation in the platform is higher than in the rotor by a certain factor, the stability criterion of Iorilo, have been proved in a rigorous manner by many investigators. In the following section, the stability criteria for dual-spin spacecraft are obtained by using energy sink arguments.

Consider an axisymmetric dual-spin spacecraft, which is shown in Fig. (3.7) of inertia properties, defined previously in sec.3.2.2, the expression of the kinetic energy, T_{rot} , the magnitude of the angular momentum $\vec{h}$, and the precessional frequency, Ω_n , are given by:

$$T_{\text{rot}} = \frac{1}{2} (A \omega_T^2 + C\Omega^2 + C_p\Omega_p^2) \quad 3.65$$

$$H^2 = ((C\Omega + C_p\Omega_p)^2 + A^2 \omega_T^2) \quad 3.66$$

$$\Omega_n = \frac{C\Omega + C_p\Omega_p}{A} \quad 3.67$$

Assuming that there is no external torque and that the shaft is frictionless. In real systems, a servo-loop-controlled motor acts to counterbalance any shaft friction exactly.

From the conservation of angular momentum principle, $\frac{d\vec{h}}{dt} = 0$, then

$$A \omega_T \dot{\omega}_T = - \frac{C\Omega + C_p \Omega_p}{A} (C \dot{\Omega} + C_p \dot{\Omega}_p) \quad 3.68$$

Differentiating Eq. (3.65) with respect to time, then

$$\dot{T}_{rot} = \dot{T}_p + \dot{T}_R = A \omega_T \dot{\omega}_T + C \Omega \dot{\Omega} + C_p \Omega_p \dot{\Omega}_p \quad 3.69$$

Where $\dot{T}_p$ and $\dot{T}_R$ are the rate of energy dissipation of the platform and the rotor respectively.

Combining Eqs. 3.68 and 3.69, yields:

$$\dot{T}_{rot} = \dot{T}_p + \dot{T}_R = - \lambda_R C \dot{\Omega} - \lambda_p C_p \dot{\Omega}_p \quad 3.70$$

where

$$\lambda_p = \Omega_n - \Omega_p$$

$$\lambda_R = \Omega_n - \Omega_R$$

In which λ_p and λ_R are the nutation frequencies of the platform and the rotor, respectively.

In dual-spin spacecraft, it is known that the rotor and the platform are spinning about the spin axis independently, thus the reaction torques which tend to change the angular rates of the platform and the rotor can be obtained from Eq. 3.70 as:

$$\dot{\Omega}_p = - \frac{\dot{T}_p}{\lambda_p C_p} \quad 3.71a$$

$$\dot{\Omega}_R = - \frac{\dot{T}_R}{\lambda_R C} \quad 3.71b$$

Combining Eqs. 3.68 and 3.71a, b then

$$A \omega_T \dot{\omega}_T = \Omega_n \left(\frac{\dot{T}_p}{\lambda_p} + \frac{\dot{T}_R}{\lambda_R} \right) \quad 3.72$$

Due to the presence of energy dissipation, so that, the nutation angle θ is no longer constant. Differentiating Eq. 3.63 with respect to time, and using Eq. 3.72 then Eq. (3.72) becomes:

$$\dot{\theta} = \frac{2A\Omega_n}{\sin 2\theta h^2} \left(\frac{\dot{T}_p}{\lambda_p} + \frac{\dot{T}_R}{\lambda_R} \right) \quad 3.73$$

For a stable spin of the spacecraft, the nutation angle θ , must decay with the time (i.e. $\dot{\theta} < 0$), hence the stability condition is given by:

$$\left(\frac{\dot{T}_p}{\lambda_p} + \frac{\dot{T}_R}{\lambda_R} \right) < 0 \quad 3.74$$

Assuming that the rotor angular momentum is much larger than the platform angular momentum (i.e. $\Omega C \gg \Omega_p C_p$) which is a reasonable assumption in view of the fact that Ω_p is equal to the orbit rate, thus:

$$\lambda_p = \Omega_n - \Omega_p = \frac{\Omega C + \Omega_p C_p}{A} - \Omega_p = \frac{C}{A} \Omega = \sigma \Omega \quad 3.75a$$

$$\lambda_R = \Omega_n - \Omega = \frac{\Omega C + \Omega_p C_p}{A} - \Omega = \left(\frac{C}{A} - 1 \right) \Omega = (\sigma - 1) \Omega = \lambda \Omega \quad 3.75b$$

Substituting Eqs. 3.75 into inequality 3.74, the stability condition becomes:

$$\left(\frac{\dot{T}_p}{C/A} + \frac{\dot{T}_R}{C/A - 1} \right) < 0 \quad 3.76$$

There are two cases:

Case 1: $C/A > 1$. The spacecraft is stable if the energy dissipation occurs in either the platform or the rotor. This implies that a nutation damper can be placed on either the platform or the rotor.

Case 2: $C/A < 1$. For this case the first term in Eq. 3.76, $\frac{\dot{T}_p}{C/A}$ is negative and the

second term, $\frac{\dot{T}_R}{C/A - 1}$ Is positive. The stability condition becomes:

$$\left| \dot{T}_p \right| > \left| \dot{T}_R \frac{C/A}{C/A - 1} \right| \quad 3.77$$

assuming a dual-spin stabilized spacecraft with inertia ratio $C/A = 2/3$, substituting its value in Eq. 3.77, then, the magnitude of energy dissipation rate of the platform should be at least twice the dissipation rate in the rotor, hence, the damper must be placed on the platform⁽⁸²⁾.

CHAPTER FOUR
*DYNAMIC OF DUAL-
SPIN SPACECRAFT
CONTAINING A
NUTATION DAMPER*

Chapter 4

Dynamics of Dual-Spin Spacecraft With Nutation Damper

4.1 Introduction

In this chapter, the dynamic and stability of dual spinning spacecraft equipped with the proposed nutation damper are presented and investigated in terms of the damper parameters, that which may appear in the equations for attitude stability of the spinning spacecraft. Approximate equations of motion are developed in terms of suitable dimensionless variables and an approximate solution for the nutation angle time history is obtained and time constants are given as a function of suitable dimensionless parameters.

4.2 Description of the Proposed Nutation Damper

The nutation damper proposed in the present study, consists of a ring fully filled with viscous fluid to overcome the problems of spreading which occurs in the partially filled ring, and to increase damping, and it is mounted with offset from the spin axis, in order to improve its dynamic and damping performance. The damper is placed in the rotor part in a plane, which is perpendicular to the spin axis of the spacecraft above the center of mass of the spacecraft, as shown in figure (4.1).

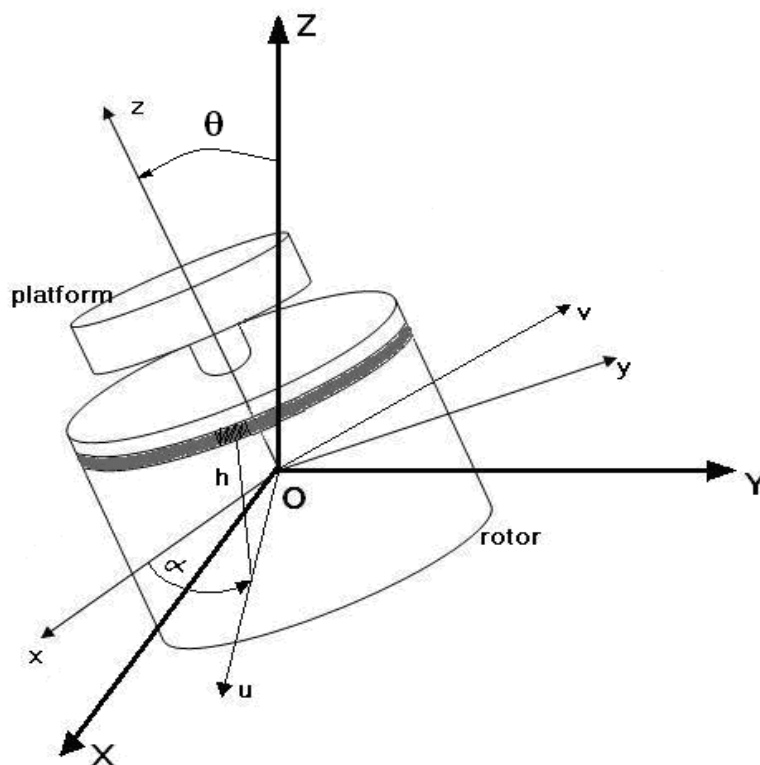


Fig. (4.1) Spacecraft model with viscous ring damper.

4.3 Equations of Motion

Consider the spacecraft model shown in figure (4.1), the coordinates system OXYZ is the inertial reference frame with the origin O fixed in space coinciding with the center of mass of inertially symmetric spacecraft (O). The local coordinate system ouvz is fixed with the geometric center of the damper ring such that the u-axis passing through the center of mass of a portion of fluid, and α is the angular position of the portion of fluid measured from the x-axis, i.e. it is the angle between the x and u axis. The angular velocity components of the spacecraft measured in the local coordinate system oxyz, take into account that the transverse angular velocity component are the same for all local coordinate system, can be given by:

$$\vec{\omega}_s = \omega_u \mathbf{e}_u + \omega_v \mathbf{e}_v + \omega_z \mathbf{e}_z \quad 4.1$$

where $\mathbf{e}_u$, $\mathbf{e}_v$, $\mathbf{e}_z$ are the unit vectors along u, v, and z axes, and ω_u , ω_v , and ω_z are the angular velocity components along the ouvz coordinate system.

The angular velocity components of the fluid portion measured in local coordinate system ouvz are given by:

$$\vec{\omega} = \omega_u \mathbf{e}_u + \omega_v \mathbf{e}_v + (\omega_z + \dot{\alpha}) \mathbf{e}_z \quad 4.2$$

where,

$\dot{\alpha}$ is the relative velocity of the fluid portion with respect to the x-axis.

The total angular momentum vector of the system in our investigation may be given in a matrix form as:

$$\vec{h} = [\mathbf{I}]_{s/c} \{\omega\}_s + [\mathbf{I}] \{\omega\} + \{h_p\} \quad 4.3$$

where:

$[\mathbf{I}]_{s/c}$: is the moment of inertia matrix of the dual-spin system not including the symmetric principle moment of inertia of the platform, which is given by:

$$[I]_{s/c} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{bmatrix} \quad 4.4$$

where A is the total transverse principal moment of inertia of the spacecraft and C is the rotor spin axis principal moment of inertia.

[I] : is the moment of inertia of fluid measured in the local coordinates $ou\bar{v}z$, and is given by (the ring is offsets from the center by “d”):

$$[I] = \begin{bmatrix} I_u & 0 & -I_{uz} \\ 0 & I_v & 0 \\ -I_{uz} & 0 & I_z \end{bmatrix} \quad 4.5$$

where:

$$\begin{aligned} I_u &= m \left(\frac{1}{2} R^2 + h^2 + d^2 \right) \\ I_v &= m \left(\frac{1}{2} R^2 + h^2 \right) \\ I_z &= m (R^2 + d^2) \\ I_{uz} &= mhd \end{aligned} \quad 4.6$$

where:

m: fluid mass,

h: height of the ring above the center of mass of the spacecraft,

d: offset center.

R: ring mean radius.

$\{h_p\} = (0, 0, C_p \omega_{pz})^T$ is the column vector.

C_p : is the symmetric principal moment of inertia of the platform about the Z-axis

ω_{pz} : is the angular velocity component along the z-axis direction.

The total angular momentum can be written in terms of angular velocity component:

$$\vec{h} = [(A+I_u)\omega_u - I_{uz}(\omega_z + \dot{\alpha})]e_u + [(A+I_v)\omega_v]e_v + [C\omega_z + I_z(\omega_z + \dot{\alpha}) - I_{uz}\omega_u + C_p\omega_{pz}]e_z \quad 4.7$$

assuming that gravity effects are neglected and the center of mass of the system remains fixed at the satellite center of mass.

Note that the inertial spin rate of the platform of the satellite adopted in the present study will be maintained at a constant rate ($\omega_{pz} = \Omega_p = \text{cons.}$) during its normal operating mode by use of a suitable despin control loop, the time derivative of the platform spin rate (ω_{pz}) will be zero.

the governing differential equations of the system is given by⁽⁸⁷⁾:

$$\vec{M} = \left[\frac{d\vec{h}}{dt} \right]_{OXYZ} \quad 4.8$$

where $\vec{M}$ is the vector sum of the external disturbance torques.

When the external torque components are zero, the system referred to as a freely precessing system, then the principle of conservation of angular momentum can be applied, such that:

$$\left(\frac{d\vec{h}}{dt} \right)_{OXYZ} = 0 \quad 4.9$$

the governing differential equation of the system and the time derivative in an inertial reference frame may be given as:

$$\vec{M} = \left(\frac{d\vec{h}}{dt} \right)_{OXYZ} = \left(\frac{d\vec{h}}{dt} \right)_{ou vz} + \vec{\omega} \times \vec{h} \quad 4.10$$

in the absence of the external disturbance torque component, then the system said to be conservative and Eq. 4.10 becomes:

$$\left(\frac{d \vec{h}}{dt} \right)_{ouVZ} + \vec{\omega} \times \vec{h} = 0 \quad 4.11$$

and can be written in terms of dot product as:

$$\left(\frac{d \vec{h}}{dt} \right)_{ouVZ} + [\omega][h] = 0 \quad 4.12$$

where,

$[\omega]$ is the skew matrix of the angular velocity components of the fluid,

$[h] = [h_u, h_v, h_z]$ is the column matrix of the angular momentum vector of the system.

rewrite Eq. 4.12 in terms of angular velocity and rearrange to get the following three equations:

$$\begin{aligned} (A + I_u) \dot{\omega}_u - I_{uz} (\dot{\omega}_z + \ddot{\alpha}) - (A + I_v) (\omega_z + \dot{\alpha}) \omega_v \\ + \left[(C + I_z) \omega_z + I_z \dot{\alpha} - I_{uz} \omega_u + C_p \Omega_p \right] \omega_v = 0 \end{aligned} \quad 4.13$$

$$\begin{aligned} (A + I_v) \dot{\omega}_v + \left[(A + I_u) \omega_u - I_{uz} (\omega_z + \dot{\alpha}) \right] (\omega_z + \dot{\alpha}) \\ - \left[(C + I_z) \omega_z + I_z \dot{\alpha} - I_{uz} \omega_u + C_p \Omega_p \right] \omega_u = 0 \end{aligned} \quad 4.14$$

$$(C + I_z) \dot{\omega}_z + I_z \ddot{\alpha} - I_{uz} \dot{\omega}_u + [(A + I_v) \omega_v] \omega_u - \left[(A + I_u) \omega_u - I_{uz} (\omega_z + \dot{\alpha}) \right] \omega_v = 0 \quad 4.15$$

the other equation of motion describes the motion of the fluid inside the ring, can be obtained by using Lagrange's equation expressed in terms of quasi-coordinate⁽²⁾,
(84).

$$\frac{d}{dt} \left[\frac{\partial T}{\partial \dot{\omega}} \right] + [\omega] \left[\frac{\partial T}{\partial \omega} \right] = [Q] \quad 4.16$$

where,

T: kinetic energy of the system.

$[\omega]$: skew symmetric matrix of angular velocity components and it is given by,

$$[\omega] = \begin{bmatrix} 0 & -\dot{\alpha} & \omega_v \\ \dot{\alpha} & 0 & -\omega_u \\ -\omega_v & \omega_u & 0 \end{bmatrix}$$

$[Q]$: column matrix of the moment components of the non-conservative forces (dissipative forces)

$$[Q] = [0 \quad 0 \quad Q_\alpha]^T$$

using Eq. 4.16, then the equation of motion corresponding to the generalized coordinate α is given by:

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\alpha}} - \omega_v \frac{\partial T}{\partial \omega_u} + \omega_u \frac{\partial T}{\partial \omega_v} = Q_\alpha \quad 4.17$$

where Q_α is the generalized moment associated with the generalized coordinate α , and it is given by:

$$Q_\alpha = C_f R^2 \dot{\alpha}$$

C_f : coefficient of viscous friction between fluid and ring wall.

The kinetic energy (T) of the system is in the form:

$$T = \frac{1}{2} \left[A(\omega_u^2 + \omega_v^2) + C\omega_z^2 + C_p \Omega_p^2 + I_u \omega_u^2 + I_v \omega_v^2 + I_z \left(\omega_z + \dot{\alpha} \right)^2 - 2I_{uz} \left(\omega_z + \dot{\alpha} \right) \omega_u \right] \quad 4.18$$

The derivative terms of the kinetic energy according to Eq. 4.17 are:

$$\frac{\partial T}{\partial \dot{\alpha}} = I_z \left(\omega_z + \dot{\alpha} \right) - I_{uz} \omega_u$$

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\alpha}} = I_z \left(\dot{\omega}_z + \ddot{\alpha} \right) - I_{uz} \dot{\omega}_u$$

$$\frac{\partial T}{\partial \omega_u} = (A + I_u) \omega_u - I_{uz} \left(\omega_z + \dot{\alpha} \right)$$

$$\frac{\partial T}{\partial \omega_v} = (A + I_v) \omega_v$$

substitute the above derivative terms into Eq. 4.17 yields the following equation:

$$I_z \left(\dot{\omega}_z + \ddot{\alpha} \right) - I_{uz} \dot{\omega}_u - \left[(A + I_u) \omega_u - I_{uz} \left(\omega_z + \dot{\alpha} \right) \right] \omega_v + [(A + I_v) \omega_v] \omega_u = -C_f R^2 \dot{\alpha} \quad 4.19$$

In order to carry out the analysis of the problem, it is interesting to introduce the dimensionless form for the equations of motion 4.13, 4.14, 4.15 and 4.19; Thus, the dimensionless form for the equations of motion are given by (details in appendix A):

$$p' + \frac{(\lambda r - \alpha' + \lambda_s)}{D_1} q + \frac{(A_1 r + A_2 \alpha' - A_3 p)}{D_1} q - \frac{A_4}{D_1} r' + \frac{A_5}{D_1} \alpha' = 0 \quad 4.20$$

$$q' - \frac{(\lambda r - \alpha' + \lambda_s)}{D_2} p + \frac{(B_1 r + B_2 \alpha' + B_3 p)}{D_2} p - \frac{B_4}{D_2} (r + \alpha')^2 = 0 \quad 4.21$$

$$\alpha'' + C_1 \alpha' + C_2 p q + C_3 [(r + \alpha') q - p'] = 0 \quad 4.22$$

$$r' - C_4 \alpha' = 0 \quad 4.23$$

4.4 Approximate Analysis (Zero-Order Approximation)

Obviously the equations of motion which are developed in the previous section are complicated in their nature, where they are strongly coupled non-linear differential equations. However expression for the parameters of the spacecraft

which describe the dynamical behavior and stability condition are required. Many researchers have adopted different methods of appropriate analysis to obtain good solution for the equations of motion. Whereas Bhuta, Koval⁽⁷⁾, and Miles⁽⁴⁷⁾ have adopted energy-sink approach in their analysis, while Annett⁽³⁾, Cochran⁽⁷³⁾, and Alfriend⁽²⁾ Adopted zero-order approximation method.

This research adopts the zero-order approximation method, in order to obtain an approximate solution for the equations of motion of the spacecraft. Zero-order approximation method is based on the fact that the mass of the fluid is much smaller than that of the spacecraft. In other words, the moment of inertia of the fluid is very small compared with that of the spacecraft. The ratio of the moment of inertia of the fluid mR^2 to the transverse moment of inertia of the spacecraft is called ε . Therefore, on the basis of zero-order approximation method any term containing the inertia ratio (ε), will be neglected.

4.5 Approximate Solution of the Problem

Before developing the solution for the attitude equations of the spacecraft, the equation which describes the nutation angle in terms of the dimensionless parameters and variables will be developed.

The cosine and sine terms of the nutation angle (θ) are given by:

$$\cos \theta = \frac{h_z}{h_{tt}} \quad 4.24$$

$$\sin \theta = \frac{h_t}{h_{tt}} \quad 4.25$$

Where h_t is the transverse angular momentum and h_{tt} is the total angular momentum of the system and it is given by:

$$h_{tt}^2 = h_t^2 + h_z^2 = \text{constant}. \quad 4.26$$

where h_t is given by:

$$h_t^2 = h_u^2 + h_v^2 \quad 4.27$$

differentiate of Eq. 4.24

$$-\sin \theta \theta' = \frac{h_z'}{h_u} \quad 4.28$$

substitute Eq. 4.25 in Eq. 4.28 gives:

$$-\frac{h_t}{h_u} \theta' = \frac{h_z'}{h_u} \quad 4.29$$

$$\therefore \theta' = -\frac{h_z'}{h_t} \quad 4.30$$

from Eq. 4.12, the time derivative of the spin axis angular momentum vector h_z , may be given by:

$$\frac{dh_z}{dt} = \omega_v h_u - h_v \omega_u \quad 4.31$$

or, in dimensionless angular velocity variables (p, q) is:

$$h_z' = q h_u - p h_v \quad 4.32$$

Substitute Eq. 4.32 into Eq. 4.30, yields the following equation:

$$\theta' = \frac{p h_v - q h_u}{h_t} \quad 4.33$$

in terms of dimensionless variables and parameters, Eq. 4.33, can be written in the following form (details given in appendix B):

$$\theta' = \frac{(-pG^2 + bG(1 + \alpha'))q\varepsilon}{\sqrt{q^2 + p^2}} \quad 4.34$$

referring to Eqs. 4.20, 4.21, 4.22, and 4.23 and apply zero-order approximation procedure, so that (appendix A shows more details);

$$p' + (\lambda r + \lambda_s - \alpha')q = 0 \quad 4.35$$

$$q' - (\lambda r + \lambda_s - \alpha')p = 0 \quad 4.36$$

$$r' = 0 \quad 4.37$$

$$\alpha'' + \frac{\eta\lambda_n}{\sigma^2\zeta} \alpha' - \frac{bG}{\zeta} p' + \left[-\frac{G^2}{\zeta} p + \frac{bG}{\zeta} (r + \alpha') \right] q = 0 \quad 4.38$$

The solution of the angular velocity component r is obtained from Eq. 4.37, with the fact that the initial value of the r -components equal to 1, then the solution of Eq. 4.37 is:

$$r=1 \quad 4.39$$

substitute the solution of the r -component Eq. 4.39 in Eqs. 4.35 and 4.36, then one get:

$$p' + (\lambda_n - \alpha')q = 0 \quad 4.40$$

$$q' - (\lambda_n - \alpha')p = 0 \quad 4.41$$

where,

$$\lambda_n = \lambda + \lambda_s$$

differentiating Eq. 4.41, substituting the result into Eq. 4.40 and using the following initial conditions,

$$q(\tau = 0) = q_0 \neq 0 \text{ and } q'(\tau = 0) = 0,$$

the solution of p and q are given by:

$$p = q_0 \cos(\lambda_n \tau - \alpha) \quad 4.42$$

$$q = q_0 \sin(\lambda_n \tau - \alpha) \quad 4.43$$

The total transverse angular velocity component (ω_t) is given by:

$$\omega_t = p + iq \quad 4.44$$

or,

$$\omega_t = q_0 [\cos(\lambda_n \tau - \alpha) + i \sin(\lambda_n \tau - \alpha)] \quad 4.45$$

this also expressible in exponential notation as:

$$\omega_t = q_0 e^{-i(\lambda_n \tau - \alpha)} \quad 4.46$$

Using the initial condition, $\alpha(\tau = 0) = 0$ then Eq. 4.46 take the following form:

$$q_0 = \omega_t \quad 4.47$$

substituting for q_0 in Eqs. 4.42 and 4.43 yields:

$$p = \omega_t \cos(\lambda_n \tau - \alpha) \quad 4.48$$

$$q = \omega_t \sin(\lambda_n \tau - \alpha) \quad 4.49$$

substitute Eqs. 4.48 and 4.49 into Eq. 4.34, then Eq. 4.34 becomes:

$$\theta' = [-G^2 \omega_t \cos(\lambda_n \tau - \alpha) + bG(1 + \alpha')] \varepsilon \sin(\lambda_n \tau - \alpha) \quad 4.50$$

Now, it is required to express for ω_t in terms of the nutation angle θ , it is given by (details given in appendix B):

$$\tan \theta = \frac{\omega_t}{\sigma r + \lambda_s} \quad 4.51$$

substitute Eq. 4.39 into Eq. 4.51 gives:

$$\therefore \tan \theta = \frac{\omega_t}{\sigma_n} \quad 4.52$$

where, $\sigma_n = \sigma + \lambda_s$

sub. p' given by Eq. 4.35 into Eq. 4.38 results:

$$\alpha'' + \frac{\eta \lambda_n}{\sigma^2 \zeta} \alpha' - \left[\frac{bG}{\zeta} (\alpha' - \lambda_n) + \frac{G^2}{\zeta} p - \frac{bG}{\zeta} (1 + \alpha') \right] q = 0 \quad 4.53$$

To find the solution of the nutation angle (θ) it is necessary to find the solution of the fluid motion in both modes of motion of the spacecraft.

4.6 Damper Motion

The symmetric rigid body, which is spinning about its axis of symmetry, has a constant nutation angle when no damping is present. The body rotates relative to ω_t at a rate of $((\sigma - 1)\Omega + \sigma_p \Omega_p)$, when no damping is present, a plane containing the

angular momentum vector $\vec{h}$, $\vec{\omega}$ and the z-axis, called the nutation plane is formed. The fluid then moves at a constant rate of $((\sigma-1)\Omega + \sigma_p \Omega_p)$ (relative spin), with respect to the body. At the same time, the fluid is subjected to centrifugal force due to the relative rotation (spin) about the z-axis. This type of motion is called "nutation-synchronous" motion. In this mode the fluid moves at a constant rate with respect to the spacecraft (damper ring). Hence, the energy dissipation rate is a constant. If $\sigma > 1$ the nutation angle decreases which cause a decrease in the centrifugal force. Eventually the component of the centrifugal force is not large enough to balance the damping and friction forces, and the fluid begins to decelerate and oscillate until becomes at rest. This type of motion is called "spin-synchronous" motion. Alfriend⁽²⁾, and Sandfry⁽⁵⁵⁾, cited that the terms nutation-synchronous and spin-synchronous firstly introduced by Cartwright, Missingll, and Trueblood.

The purpose now is to determine the fluid motion (α) and consequently θ in these two modes of motion as a function of dimensionless parameters.

4.6.1 Nutation-Synchronous Mode

Let β measure the position of the center of a portion of the fluid with respect to the nutation plane. At $\tau = 0$, $\alpha = 0$, assume:

$$\beta = \alpha - \lambda_n \tau \quad 4.54$$

substituting for β in Eq. 53 and using Eqs. 4.48 and 4.49 then Eq. 53 becomes:

$$\beta'' + \frac{\eta \lambda_n}{\sigma^2 \zeta} (\beta' + \lambda_n) - \left[\frac{bG}{\zeta} \beta' + \frac{G^2}{\zeta} p - \frac{bG}{\zeta} (1 + \beta' + \lambda_n) \right] \omega_t \sin(-\beta) = 0 \quad 4.55$$

rearrange this equation gives:

$$\beta'' + \frac{\eta \lambda_n}{\sigma^2 \zeta} \beta' + \left[\left(\frac{bG}{\zeta} - \frac{bG}{\zeta} \right) \beta' + \frac{G^2}{\zeta} p - \frac{bG}{\zeta} (1 + \lambda_n) \right] \omega_t \sin(\beta) = -\frac{\eta \lambda_n^2}{\sigma^2 \zeta} \quad 4.56$$

where, η_n refer to the damping constant in the nutation-synchronous mode.

Since that, the motion of the fluid in this mode is constant, then the solution of Eq. 4.56 is,

$\beta = \beta_s = \text{con.}$, so, $\beta'' = \beta' = 0$, substituting in Eq. 4.56 and take into account that $\lambda_n = \lambda + \lambda_s$ and $\sigma = 1 + \lambda$, then

$$\left[\frac{G^2}{\varsigma} p - \frac{bG}{\varsigma} (1 + \lambda_n) \right] \omega_t \sin \beta = - \frac{\lambda_n^2 \eta_n}{\sigma^2 \varsigma} \quad 4.57$$

rearrange this equation gives:

$$- \left[-G^2 p + bG(1 + \lambda_n) \right] \omega_t \sin \beta = - \frac{\lambda_n^2 \eta_n}{\sigma^2} \quad 4.58$$

substitute for $\beta = \beta_s$ and $\beta'_s = 0$ into Eq. 4.50, then 4.50 becomes:

$$\theta'_n = - \left[-G^2 \omega_t \cos \beta + bG(1 + \lambda_n) \right] \varepsilon \sin \beta_s \quad 4.59$$

where, θ_n refer to the nutation angle in the nutation-synchronous mode.

Substituting for the left hand side of Eq. 4.58 in Eq. 4.59, then

$$\theta'_n = - \frac{\eta_n \lambda_n^2}{\sigma^2 \omega_t} \varepsilon \quad 4.60$$

substitute for ω_t from Eq. 4.52, then Eq. 4.60 can be written as:

$$\theta' \tan \theta_n = - \frac{\eta_n \lambda_n^2}{\sigma^2 \sigma_n} \varepsilon \quad 4.61$$

carrying out the integration, then the nutation angle θ_n is given by:

$$\cos \theta_n = \cos \theta_{n0} e^{-\frac{\tau}{\tau_n}} \quad 4.62$$

where, θ_{n0} is the initial value of θ_n , τ_n is the time constant of the system which it is given by:

$$\tau_n = \frac{\sigma_n \sigma^2}{\eta_n \lambda_n^2 \varepsilon} \quad 4.63$$

It is clearly that in this mode, the cosine of the nutation angle exhibits exponential behavior, no small angle approximations have been made, and Eq. 4.62 is valid for $0 < \theta < \frac{\pi}{2}$.

When small angle approximation is used, (i.e. $\tan \theta_n \approx \theta_n$), then nutation angle is given by:

$$\theta_n = \sqrt{\theta_{n0}^2 - \frac{2\eta_n \lambda_n^2 \varepsilon}{\sigma^2 \sigma_n}} \quad 4.64$$

At the end of the nutation-synchronous mode, the system goes into the spin-synchronous mode and the nutation angle θ_n gain its minimum value. Referring to Eq. 4.58, to satisfy the condition of minimum value of the nutation angle in this mode, the angle β_s should be equal to $\pm \frac{\pi}{2}$ substituting for this value and for ω_t from Eq. 4.52, then;

$$\tan \theta_n bG \left[\frac{-G}{b} \tan \theta \sigma_n \cos \beta + (1 + \lambda_n) \right] = \frac{\eta \lambda_n^2}{\sigma^2 \sigma_n} \quad 4.65$$

$$\sin \beta \tan \theta_n bG \sigma \left[\frac{-G}{b \sigma} \tan \theta \sigma_n \cos \beta + 1 + \frac{\lambda_s}{\sigma} \right] = \frac{\eta \lambda_n^2}{\sigma^2} \quad 4.66$$

$$\tan \theta = \frac{\eta \lambda_n^2}{\sigma^3 bG \sigma_n \left(1 + \frac{\lambda_s}{\sigma} \right)} \quad 4.67$$

4.6.2 Spin-Synchronous Mode

In the Spin-Synchronous mode, the spacecraft becomes more closely to the state of pure spin about the spin axis (z-axis). Accordingly, the relative speed of the fluid, about the spin axis with respect to the spinning rotor, will be decreased. Eventually the relative speed between the fluid and the spinning rotor becomes

zero, then the spacecraft spin axis is aligned with the initial direction of the total angular momentum vector. In the spin-synchronous mode, it can be shown that the fluid is moving with a small variation in its speed with respect to the damper ring and it is necessary to find the solution for the motion of the fluid (α) as a function of dimensionless time (τ)

Substituting for p and q from Eqs. 4.48, 4.49 into Eq. 4.53, yields:

$$\alpha'' + \frac{\eta\lambda_n}{\varsigma\sigma^2} \alpha' + \left[\frac{bG}{\varsigma} (\alpha' - \lambda_n) + \frac{G^2}{\varsigma} \omega_t \cos(\alpha - \lambda_n \tau) - \frac{bG}{\varsigma} (1 + \alpha') \right] \omega_t \sin(\alpha - \lambda_n \tau) = 0 \quad 4.68$$

$$\alpha'' + \frac{\eta\lambda_n}{\varsigma\sigma^2} \alpha' + \left[\left(\frac{bG}{\varsigma} - \frac{bG}{\varsigma} \right) \alpha' - \frac{bG}{\varsigma} \lambda_n - \frac{bG}{\varsigma} + \frac{G^2}{\varsigma} \omega_t \cos(\alpha - \lambda_n \tau) \right] \omega_t \sin(\alpha - \lambda_n \tau) = 0 \quad 4.69$$

$$\alpha'' + \frac{\eta\lambda_n}{\varsigma\sigma^2} \alpha' - \left[\frac{bG}{\varsigma} \lambda_n + \frac{bG}{\varsigma} \right] \omega_t \sin(\alpha - \lambda_n \tau) = 0 \quad 4.70$$

$$\alpha'' + \frac{\eta\lambda_n}{\varsigma\sigma^2} \alpha' - \left[\frac{bG}{\varsigma} (1 + \lambda_n) \right] \omega_t \sin(\alpha - \lambda_n \tau) = 0 \quad 4.71$$

as mentioned above that the fluid moving with a small variation in its speed, then the following equation can be assumed:

$$\alpha = \alpha_0 + \tilde{\alpha} \quad 4.72$$

where, α_0 is the initial value of α and $\tilde{\alpha}$ represent the small variation of the speed of the fluid such that $\alpha_0 \gg \tilde{\alpha}$. The basis of this assumption is that the change in α is small compared to $\lambda_n \tau$. An approximate steady state solution of Eq. 4.71, may be given by (details are given in appendix C):

$$\tilde{\alpha} = F \tan \theta_s [-\varsigma \sin(\alpha_0 - \lambda_n \tau) + \frac{\eta}{\sigma^2} \cos(\alpha_0 - \lambda_n \tau)] \quad 4.73$$

$$\text{where } F = \frac{\sigma_n}{\lambda_n^2} \left[\frac{\sigma^2 b G (1 + \lambda_n)}{\zeta^2 (\sigma^2 + \frac{\eta^2}{\zeta^2 \sigma^2})} \right]$$

and the suffix s refers to the spin-synchronous mode.

sub. the derivative of equation 4.73 in equation 4.50 then equation 4.50 will be (detail in appendix C):

$$\theta'_s = -\theta_s [E_1 \cos 2\lambda_n \tau + E_2 \sin 2\lambda_n \tau + \frac{1}{\tau_s}] + bG\varepsilon \sin(\alpha_0 - \lambda_n \tau) \quad 4.74$$

where E_1 and E_2 are constant and the time constant τ_s in the spin-synchronous mode is given by:

$$\tau_{cs} = \frac{2\lambda_n^2 \zeta^2 \left(\sigma^2 + \frac{\eta^2}{\zeta^2 \sigma^2} \right)}{\sigma_n b^2 G^2 \eta (1 + \lambda_n)^2 \varepsilon} \quad 4.75$$

the solution of Eq. 4.74 (detail in appendix C) is:

$$\begin{aligned} \theta_s = & \left[\theta_{s0} - \frac{\left(\left(\frac{\sin(\alpha_0 - \lambda_n \tau_0)}{\tau_{cs}} \right) + \lambda_n \cos(\alpha_0 - \lambda_n \tau_0) \right)}{\left(\lambda_n^2 + \frac{1}{\tau_{cs}^2} \right)} bG\varepsilon \right] e^{\frac{-(\tau - \tau_0)}{\tau_{cs}}} \\ & + \frac{\left(\left(\frac{\sin(\alpha_0 - \lambda_n \tau_0)}{\tau_{cs}} \right) + \lambda_n \cos(\alpha_0 - \lambda_n \tau_0) \right)}{\left(\lambda_n^2 + \frac{1}{\tau_{cs}^2} \right)} bG\varepsilon \end{aligned} \quad 4.76$$

4.7 Relative Equilibria State of Dual-Spin Spacecraft

4.7.1 Introduction

The addition of nutation damper to the satellite which is spin-stabilized may cause alteration in the stability criteria, where the parameters of the nutation damper may appear in the stability conditions. In the present section the relative equilibrium state and stability conditions are discussed.

4.7.2 Relative Equilibria State

The dynamical behavior of the spacecraft in steady state condition is a reflection of the relative state of equilibrium, where in steady state it is known that all derivatives with time will vanish.

To get equations of relative equilibria, let p_r , q_r , r_r and α_r denotes the relative equilibria state, and set, p', q', r', α' and α'' equal to zero in equations 4.20, 4.21, 4.22, and 4.23 to get:

$$\lambda r_r q_r + A_3 (r_r + p_r) q_r + \lambda_s q_r = 0 \quad 4.77$$

$$-p_r \lambda r_r - B_1 r_r p_r - B_3 p_r^2 - \lambda_s p_r + B_3 r_r^2 = 0 \quad 4.78$$

$$C_2 p_r q_r + C_3 r_r q_r = 0 \quad 4.79$$

rearrange:

$$[(\lambda + A_3)r_r + A_3 p_r + \lambda_s]q_r = 0 \quad 4.80$$

$$B_3 r_r^2 - B_3 p_r^2 - \lambda_s p_r - (\lambda + B_1)r_r p_r = 0 \quad 4.81$$

$$(C_2 p_r + C_3 r_r)q_r = 0 \quad 4.82$$

As known, the rotor angular momentum is much larger than that for platform, where $\omega_z C \gg \Omega_p C_p$, or $\lambda \gg \lambda_s$. So Equations 4.80, 4.81 and 4.82 becomes:

$$(A_3 p_r + (\lambda + A_3)r_r)q_r = 0 \quad 4.83$$

$$B_3 p_r^2 + (\lambda + B_1) r_r p_r - B_3 r_r^2 = 0 \quad 4.84$$

$$(C_2 p_r + C_3 r_r) q_r = 0 \quad 4.85$$

The solution of Eq. 4.84 is given by:

$$\text{when } q_r = 0 \Rightarrow p_r = \frac{-(\lambda + B_1) r_r + \sqrt{(\lambda + B_1)^2 r_r^2 + 4 B_3^2 r_r^2}}{2 B_3} \quad 4.86$$

$$\text{or } p_r = \frac{-(\lambda + B_1) r_r - \sqrt{(\lambda + B_1)^2 r_r^2 + 4 B_3^2 r_r^2}}{2 B_3} \quad 4.87$$

Eqs. 4.83, and 4.85 gives the following result:

$$\text{when } r_r = 0, p_r = 0, \Rightarrow q_r \neq 0 \quad 4.88$$

to find first relative equilibrium state, solution of equation 4.86 is needed, but after all the term inside the root is expanded as:

$$(x + y)^n = x^n + n x^{n-1} y + \frac{n(n-1)}{2!} x^{n-2} y^2 + \frac{n(n-1)(n-2)}{3!} x^{n-3} y^3 \quad 4.89$$

dividing equation 4.86 by $(\lambda + B_1)$ gives:

$$p_r = \frac{-1 + \sqrt{1 + Y^2}}{Y} r_r \quad 4.90$$

where,

$$Y = \frac{2 B_3}{\lambda + B_1}$$

apply relation 4.89

$$p_r = \frac{-1 + \sqrt{1^{\frac{1}{2}} + \frac{1}{2} \left(1^{\frac{1}{2}} * Y^2 \right)}}{Y} r_r = \frac{-1 + \left(1 + \frac{1}{2} Y^2 \right)}{Y} r_r \quad 4.91$$

$$\text{substitute } Y = \frac{2 B_3}{\lambda + B_1}$$

$$\therefore p_r = \frac{B_3 r_r}{\lambda + B_1} = \frac{B_3 r_r}{\lambda} = \frac{\epsilon b G}{\lambda} r_r \quad 4.92$$

this equation implies that r_r reach to the unity and p_r reach to ε , this is mean that the spacecraft spins about the z-axis, and the z-axis precesses steadily about the angular momentum vector h with a small constant residual angle which satisfies:

$$\tan \theta_r = \frac{h_t}{h_z} = \frac{\sqrt{[(A + I_u)p_r - I_{uz}r_r]^2 + (A + I_v)^2 q^2}}{(C + I_z)r_r - I_{uz}p_r + C_p \frac{\Omega_p}{\Omega}} \quad 4.93$$

it is known that $\lambda \gg \lambda_s$ and use $q_r = 0$, equation 4.93 becomes:

$$\tan \theta_r = \frac{\varepsilon b G (2 - \sigma)}{\lambda \sigma} \quad 4.94$$

this relative equilibrium state (first relative equilibrium state) is the most important state, where the spacecraft being more closely to the state of pure spin about z-axis.

For the second relative equilibrium stability, equation 4.87 can be expanded to:

$$p_r = \frac{-1 - \left(1 + \frac{1}{2} Y^2\right)}{Y} r_r \quad 4.95$$

$$\text{Sub. } Y = \frac{2B_3}{\lambda + B_1}$$

$$\therefore p_r \cong \frac{-(\lambda + B_1)}{B_3} r_r \quad 4.96$$

then:

$$r_r = \frac{-B_3}{\lambda + B_1} p_r = \frac{-\varepsilon b G}{\lambda} p_r \quad 4.97$$

Equation 4.97 shows that $r_r \approx \varepsilon$, and $p_r \approx 1$, this means that the spacecraft spins about u-axis, and u-axis precesses about (angular momentum vector $\vec{h}$) with constant residual nutation angle, which is given by:

$$\tan \theta = \frac{h_t}{h_z} = \frac{\varepsilon bG(2 - \sigma)}{\lambda(\sigma + \varepsilon\zeta)} \quad 4.98$$

For the third relative equilibrium stability, it is shown from equation 4.88 that p_r and r_r are equals to zero. This is mean that the spacecraft spins about the v-axis without precession or residual nutation angle (i.e. pure spin).

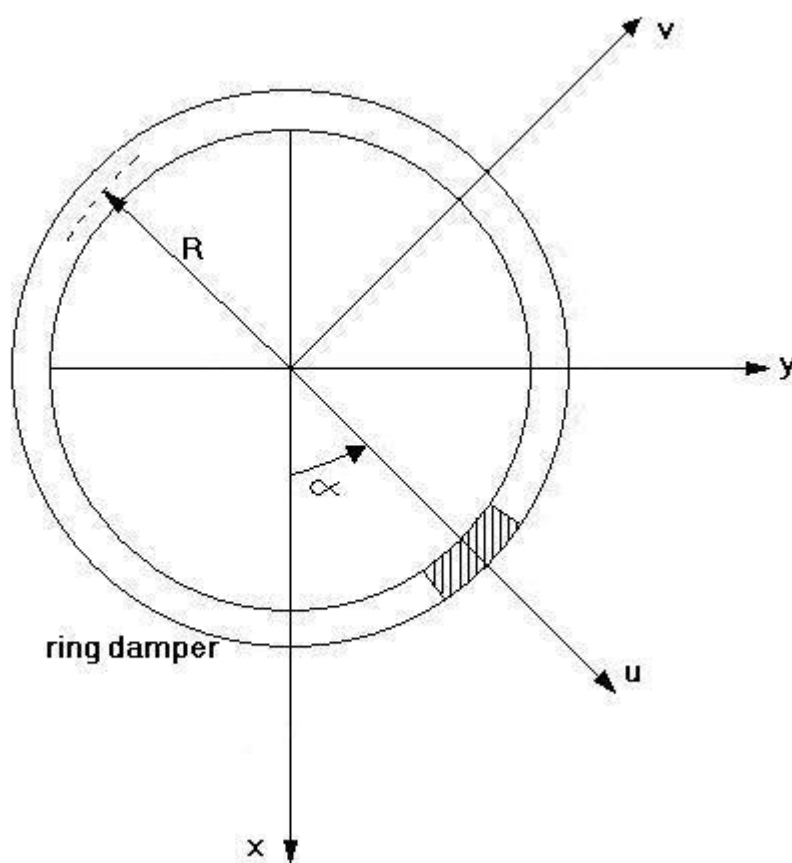


Fig. (4.2) Full filled viscous ring damper.

CHAPTER FIVE

RESULTS AND

DISCUSSIONS

Chapter Five

Results and Discussions

5.1 Introduction

The dynamics of the dual-spin spacecraft containing the proposed nutation damper has been investigated. The analysis is based on the zero-order approximation method and the solution for the angular velocities components were obtained. The differential equation for the nutation angle rate and the time constants in nutation-synchronous and spin-synchronous mode were also obtained in terms of the dimensionless parameters of the spacecraft and the nutation damper, and the stability criterion for the attitude motion of the spacecraft containing a nutation damper was obtained for the first relative equilibrium state of the spacecraft. Now at this chapter a numerical solution of the equations of motion is performed, and a comparison between numerical and the analytical solution is introduced.

5.2 Numerical Determination of the Time Constant

The time constant is the most important characteristic of any control device, and it represents the time consumed to reach the steady state. Here the time constant may be obtained as the speed of response at which the nutation damper confined the spacecraft to its initial state. It was shown that, the spacecraft has two distinct modes of motion, thus the time constant of each mode has its own characteristics. The analytical expressions of the time constants on both modes are obtained by using an approximate analytical solution (zero-order approximation). In order to compare the approximate analytical solution of the equations of motion with the numerical solution, it is required to determine the time constant numerically. Numerical determination of the time constant can be achieved by using numerical solution of the nutation angle θ obtained from the simulation program. The time constant can be determined by using the data gained from numerical solution or from an experimental test, by a variety of methods. A preferred method is replotted semilogarithmically the numerical data in order to get a better value of the time constant⁽⁸³⁾. The criteria of this method is, it is known that the exponential decaying system can be expressed as:

$$q_0 = A_a e^{-\frac{\tau}{\tau_c}} \quad 5.1$$

where:

τ : time or dimensionless time.

τ_c : dimensionless time constant of the system.

A_a : is a constant.

q_0 : the response of the system.

Equation (5.1) can be written:

$$\frac{q_0}{A_a} = e^{-\frac{\tau}{\tau_c}} \quad 5.2$$

define,

$$\Gamma = \ln\left(\frac{q_0}{A_a}\right) \quad 5.3$$

$$\therefore \Gamma = -\frac{\tau}{\tau_c} \quad 5.4$$

differentiating Γ with respect to τ , equation (5.4) becomes:

$$\frac{d\Gamma}{d\tau} = -\frac{1}{\tau_c} \quad 5.5$$

or

$$\tau_c = -\frac{1}{\frac{d\Gamma}{d\tau}} \quad 5.6$$

thus if Γ is plotted versus τ , then will get a line whose slope is equal to $\frac{1}{\tau_c}$.

This approach gives an accurate estimation of the time constant, where the slope of the best line passing through all data is used.

There are two modes of motion, nutation-synchronous mode and spin-synchronous mode, each of the modes has its own time constant and each time constant has its own characteristics.

So, in nutation-synchronous mode, the solution of the nutation angle is in term of $\cos\theta$, thus the time constant will be in an exponential behavior of $\cos\theta$ for solution obtained by numerical integration of the equations of motion.

For spin-synchronous mode, it is obvious that the solution of the nutation angle is combined from oscillation and exponential decay with the time constant. So the time constant is obtained by integration of the exact equations of motion and taking maximum value of θ in each oscillation, with assuming exponential behavior of these data.

5.3 Calculation of the damping constant (η)

The damping constant is one of the important characteristics parameters. For the spacecraft and the ring damper all of the required dimensionless parameters are well defined except for the damping constant. Since the motion of the fluid is different in two modes, so the problem must be considered for each mode.

For the nutation-synchronous mode the velocity of the fluid is constant with respect to the satellite or the ring. Assume steady laminar flow in the pipe, solution of Navier-Stoke equation with a flux of⁽²⁾:

$$Q_{flux} = \pi r^2 R \dot{\alpha} \quad 5.7$$

where, $R \dot{\alpha}$ velocity of the fluid relative to the ring, gives:

$$u_{\theta} = 2R \dot{\alpha} \left[1 - \left(\frac{a}{r} \right)^2 \right] \quad 5.8$$

where,

a: is the distance from the center of the pipe.

r: the radius of the ring.

The shear stress at any point is:

$$\tau_{shear} = \mu \frac{\partial u_{\theta}}{\partial r} \quad 5.10$$

where,

μ : is the viscosity.

Thus the total viscous force is:

$$F_{tot} = 2\pi R \gamma \tau_{shear} \Big|_{a=r} = 8\pi \mu R^2 \dot{\alpha} \gamma \quad 5.11$$

where, γ , the fullness of the ring and it's equals to (2π) , [ring is totally filled with liquid].

The force from dynamical analysis is:

$$F_{tot} = C_f V = C_f R \dot{\alpha} = \eta m \Omega R \dot{\alpha} \quad 5.12$$

gives:

$$\eta = \frac{8\nu}{r^2\Omega} \quad 5.13$$

where,

ν : kinematic viscosity. [for a mixture consists of 75% glycerin and 25% water, $\nu=5.83 \text{ mm}^2/\text{s}$, $\rho=1164.5 \text{ kg/m}^3$]⁽³¹⁾.

A mixture of glycerin with water used to get various damping by changing the properties of the mixture. Present research utilizing 75 % glycerin and 25% water, because it gives maximum energy dissipation^{(31),(7)}, other fluids considered and rejected as unsuitable, like; N_2H_4 , Lox, liquid nitrogen⁽⁶⁾.

In the spin-synchronous mode the velocity of the fluid is not constant but oscillatory with respect to the ring. So the researchers calculate the damping constant experimentally using the artificial satellite rig. Alfrend⁽²⁾ calculate the damping constant in this mode as (0.21). Hamed⁽³¹⁾ calculate it as (0.15). This research adopts Hamed's result, because the damper dimensions are the same.

5.4 Comparison Between Analytical Solution and Numerical Solution Obtained From the Simulation Program

In order to verify the results obtained by the analytical solution , a numerical solution is performed by a simulation program included in MATLAB program ver. 7, named SIMULINK. It is a very powerful package for solution of various problems in engineering and other studies, especially the topic of this research, one can find many tool box specified just for spacecraft simulation.

The differential equations 4.20, 4.21, 4.22, and 4.23 are solved in the SIMULINK employing ode 45 dormand prince method of numerical integration. The dimensions of the nutation damper used in this work is in table 5.1.

Table (5.1) nutation damper characteristics.

Description	value
Ring mean radius	0.25m
Ring radius	9.5mm
ε	0.00088

And the inertia properties of the symmetrical dual-spin spacecraft used is in table 5.2.

Table (5.2) inertia parameter for symmetrical dual-spin spacecraft

moment of inertia	Spin rate
$A = 36 \text{ kg.m}^2$	$\Omega = 1.05 \text{ rad/s}$
$C = 43.2 \text{ kg.m}^2$	$\Omega_p = 0.00058 \text{ rad/s}$
$C_p = 0.2 \text{ kg.m}^2$	

The following section shows the comparison between the numerical and analytical solution. And the effect of the spacecraft and damper parameters on these results.

Figures (5.1) and (5.2) shows the nutation angle time history of this research and Alfrend⁽²⁾ respectively. Where figure (5.2) represent the typical work of Alfrend⁽²⁾, one can see that the trend of present research is very acceptable with the trend of Alfrend⁽²⁾. Fig. (5.3) shows the comparison of the nutation angle time history of this research with that predicted by reference⁽²⁾, for nutation-synchronous mode. The figure shows that this research will decrease the time constant appreciably comparing with⁽²⁾, and the trend agrees with that provided by^{(2),(6),(18)}. In figures (5.4) and (5.5), the comparison of the nutation angle time history for both nutation-synchronous mode and spin-synchronous mode is

shown, it is seen that the analytical solution very well agrees with numerical solution. One can see that there is a little difference between the numerical and the analytical results in the transition region, i.e. the transition from the nutation-synchronous mode to the spin-synchronous mode, this is because that this region is very difficult to be predicted, and most of researchers tend to separate the plot of numerical from the analytical. Figure (5.6) shows the comparison between the numerical and the analytical solution of the nutation angle time history for the nutation-synchronous mode. The time constant for nutation-synchronous mode obtained analytically is (99981) and numerically is (96428.5) which means that the percentage error is less than ($\approx 3.6\%$), and this means that the analytical solution predicts the time constant very well, and for spin-synchronous mode, it could be seen the numerical is (866.66) and the analytical is (802.17), i.e. the percentage error is ($\approx 8\%$), again the analytical is a good solution, and these results represent a good support for the assumption adopted in the analysis that the fluid is in two modes of motion, nutation-synchronous mode and spin-synchronous mode and this conclusion is very well agrees with^{(2),(6),(7),(18)}.

5.5 Parametric Study

The aim of this parametric study is to search or investigate the influence of each parameter of the spacecraft and the proposed nutation damper on the dynamics of the spacecraft and it is damping, also on the stability of the spacecraft and the residual nutation angle.

5.5.1 Influence of Spacecraft and Nutation Damper Parameters on the Dynamic Characteristics

In this section the influence of various spacecraft and damper parameters on the dynamic characteristics are investigated.

5.5.1.1 Influence of Inertia Ratio (σ)

In figure (5.7) the variation of the time constant with the inertia ratio σ is shown. The figure shows the time constant increases as the inertia ratio tends to (1). While in figure (5.8) for spin-synchronous mode, one can see the time constant decreases as the inertia ratio approaches (1), this made a conclusion that the time constant has a singular point at $\sigma=1$. This will lead to reformulate the conclusion made from equations (4.60) and (4.75) that the time constant goes to infinity in nutation-synchronous mode and to zero in spin-synchronous mode as σ tends to (1) and it can be said these expressions are valid only for ($\sigma > 1$) according to the stability analysis.

Investigation of figures (5.7) and (5.8) for ($\sigma > 1$) shows that the time constant decreases in nutation-synchronous mode and increases in spin-synchronous mode as σ increasing, one can conclude that the requirement of nutation damper design is of sophisticated requirement for (σ). For $\sigma=1$, i.e. spacecraft of spherical, cubic shape or equal principal moments of inertia, is not included in this analysis. And for ($0 < \sigma < 1$) it is seen that the time constant in nutation-synchronous mode is rising up and for spin-synchronous mode is decreasing down, and the stability analysis still unsolved until yet, where spacecraft like that need space more than that for ($\sigma > 1$).

5.5.1.2 Influence of the Rotor Spin Rate (Ω)

Variation of the time constant with the rotor spin rate Ω is shown in figure (5.9), for nutation-synchronous mode. The increase in inertial spin rate will lead to decrease in damping constant, and this results in increasing in time constant. The physical interpretation of this, is that the higher the spin rate of spinning body, for same moments of inertia will cause higher gyroscopic stiffness of the body, thus much time is needed to bring back the body to its initial state. Figure (5.10) for spin-synchronous mode.

5.5.1.3 Influence of Ring Mean Radius (R)

Figure (5.11) shows the variation of the time constant with the ring mean radius (R), for nutation-synchronous mode, and figure (5.12) for spin-synchronous mode. For both modes it can be seen that the time constant decreases as the ring mean radius increases.

The increase in (R) will cause increase in ε and may results in changing the inertia properties of the spacecraft and may cause what is called “time varying inertia”. This is in contrast to the assumption of the analysis which is the nutation damper did not appreciably affect the precession rate ($\sigma\Omega$) of the spacecraft but serve as energy dissipation source, and this assumption is basis of zero-order approximation method. Also it is seen in these figures that the decreasing in time constant, reaches a value which is behind, no obviousable decrease in time constant as ring mean radius increasing.

5.5.1.4 Influence of Ring Radius (r)

The variation of the time constant with the ring radius is shown in figure (5.13) for nutation-synchronous mode and figure (5.14) for spin-synchronous mode, as it is seen the effect of (r) has the same effect of ring mean radius (R).

5.5.1.5 Influence of the Damping Constant(η)

The variation of the time constant with the damping constant is illustrated in figure (5.15) for nutation-synchronous mode. It is seen as increasing the damping constant, the time constant will be decreased. And in figure (5.16) for spin-synchronous mode it is seen that time constant is a concave up function with damping constant and has a minimum value , and for other values it gain the same value of time constant for two different values of damping constant, this means when two sets of parameters chosen for different damping constant will give the same time constant , so the damping fluid must be carefully chosed

to be sure of the requirement of the spacecraft and to get a very good nutation damper has a good dynamic characteristics.

5.5.1.6 Influence of the Ratio of the Ring Height to the Ring Mean Radius (b)

The variation of the time constant with the ratio of the ring height to the ring mean radius (b) for spin-synchronous mode is demonstrated in figure (5.17). one can see that the time constant decreased as (b) increased, and as seen in the analytical solution, equation (4.75), the (b) is appear in equation of time constant for spin-synchronous mode, where time constant in nutation synchronous mode is independent of (b).

5.5.1.7 Influence of Distance of Offset Center (d)

In figure (5.18), the variation of the time constant with the distance of offset center (d) is shown. The place of the damper is very important design parameter especially the amount of the offset center (d)⁽⁵⁶⁾, where it is strongly possible that the spin-synchronous mode may not be exist if the ring is not offsets from the center which placed on the spin axis. The appearance of distance (d) in time constant spin-synchronous mode term in this research is a very good result and supporting the paragraph mentioned above. One can see that the time constant decreases with increasing the offset center distance, but it should be within a reasonable value to allow much possible little change in inertia properties of the spacecraft.

5.5.2 Influence of the Damper and Spacecraft Parameters on the Residual Nutation Angle

The residual nutation angle is also a very important value for the designer

of the spacecraft, where its value must be less than the pointing accuracy of the satellite, and its for spin stabilized spacecraft is between $(0.3 - 1^\circ)^{(57)}$.

It is seen as the velocity of the fluid decreased in spin-synchronous mode, the viscous friction forces tend to vanish and the energy is no longer dissipated. So a very small angle will residue.

5.5.2.1 Influence of Inertia Ratio (σ)

In figure (5.19) the variation of the residual nutation angle with the inertia ratio (σ) is shown, whereas (σ) tends to the unity the residual nutation angle will be increased, and for ($\sigma > 1$), the residual nutation angle will decrease, this is because that for larger (σ) will get larger nutation frequency, then the larger damping force and consequently much more energy dissipation will occur, and less residual nutation angle.

5.5.2.2 Influence of the Ratio of the Ring Height to the Ring Mean Radius (b)

In figure (5.20) the variation of the residual nutation angle with (b) is shown; as increasing the ratio (b) the residual nutation angle will be increased, the physical interpretation is, as increasing (b) the inertia properties will be increased, this will demand larger damping force, where the damping force is still unchanged, then larger nutation angle introduced.

5.5.2.3 Influence of Ring mean radius (R) and ring radius (r)

The influence of the ring mean radius and ring radius on the residual nutation angle is shown in figures (5.21) and (5.22) respectively. It is seen that the residual nutation angle increased as ring mean radius increased. The effect of ring radius (r) is the same as (R), where as increasing (r), the residual nutation angle will be increased.

5.6 Relative Equilibria States

Relative Equilibria states of the spacecraft explains the final states which the spacecraft will be goes. Figure (5.25) shows the time history of the r component of the spacecraft angular velocity for the first relative equilibrium state. For inertia ratio $\sigma > 1$, which is the stability condition of the first relative equilibrium state of the spacecraft, the angular velocity component r will increase and the nutation angle will decay, while the degradation of p and q components are shown in figures (5.23) and (5.24). A three dimensional visualization of the first relative equilibrium state is shown in figure (5.26), it is shown that even the system being at a point in neighborhood of the second relative equilibrium state, it will converge to the first relative equilibrium state, this is because that the system parameters satisfy the stability condition of the first relative equilibrium state.

For the second relative equilibrium state, figure (5.29) shows the degradation of the r component, and figure (5.27) and (5.28) shows the increasing the p and q components. Figure (5.30) shows a three dimensional visualization of the second relative equilibrium state, it is shown that the system diverges from the first relative equilibrium state to the second relative equilibrium state, because the system parameters satisfy the second relative equilibrium state.

CHAPTER SIX

CONCLUSIONS, AND

SUGGESTIONS FOR

FUTURE WORK

Chapter Six

Conclusions And Suggestions

For Future Work

6.1 Conclusions

In view of the theoretical and numerical results obtained from the solutions of the governing differential equations which describe the attitude of the spacecraft, the following main conclusions can be drawn:-

1. The proposed damper overcomes the problems of the spreading and sloshing which occur in the partially filled nutation dampers.
2. Utilizing fluids with high damping coefficient will decrease the time constant in both modes of motion .
3. The inertia ratio greatly affects the time constant in both modes of motion.
4. For $\sigma > 1$, increasing inertia ratio will decrease the residual nutation angle appreciably.

-
5. Increasing the spin rate will increase the time constant and decrease the damping constant in both modes of motion.
 6. Increasing the ring mean radius will decrease the time constant in both modes of motion.
 7. The height of the nutation damper b affects the time constant just for spin-synchronous mode, and do nothing for nutation-synchronous mode.
 8. Increasing the ring radius r will decrease the time constant in both modes of motion.
 9. Increasing the ring radius r will decrease the damping constant and vice versa, for nutation-synchronous mode.
 10. The increase in the height of the nutation damper will increase the residual nutation angle.
 11. The stability analysis shows that for $\sigma > 1$, the spacecraft will go to the first relative equilibrium state, although the initial conditions lie in neighborhood of the second relative equilibrium state.
 12. The offset distance d affects the time constant in spin-synchronous mode, whereas d increases the time constant will decrease.
 13. The time constant for nutation-synchronous mode obtained analytically is (99981) and numerically is (96428.5) which means that the percentage error is less than ($\approx 3.6\%$), and this means that the analytical solution predicts the time constant very well.
 14. The time constant for spin-synchronous mode, it could be seen the numerical is (866.66) and the analytical is (802.17), i.e. the percentage error is ($\approx 8\%$), again the analytical is a good solution, and these results represent a good support for the assumption adopted in the analysis that the fluid is in two modes of motion.

6.2 Suggestions For Future Work

The category of the attitude dynamics and control of the spacecraft, precisely the topic of dual-spin spacecraft is not widespread or prevailing, especially here in Iraq. Thus, much more work is required on this category and topic to reveal all its dynamic and stability characteristics. Based on this study, the following suggestions are recommended:-

1. Perform the calculations experimentally on the artificial satellite rig.
2. Study the effect of the rotor and platform unbalance on the dynamic and stability of the spacecraft.
3. Utilizing another method for solution the differential equations, such as energy sink, and center manifold theorem.
4. Utilizing double ring or many rings instead of one ring.
5. Utilizing MR damper instead of this damper (which including dispersion a tiny metal molecules in the viscous fluid and applying electric current through a wire which is wounded around the ring, the current make the molecules aligned, then the surface area increased and the damping will be increased).

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