

Republic of Iraqi Government
Ministry of Higher Education
And Scientific Research

Babylon University
Engineering Collage
Civil Department

***Two -Dimensional Numerical Model for
Thermal Pollution of Single -Point Sources in
Rivers at Different Discharge Depths***

A Thesis
Submitted to the College of Engineering of
The University of Babylon in Partial
Fulfillment of the Requirements
For the Degree of Master
Of Science in Civil
Engineering

By

Nawras Nomass Shati Obaid

(B.Sc CIVIL ENG.)

Supervised by:

Asst. Prof. Dr. Mohammad Abid. M. Al -Tufaily
Asst. Prof. Dr. Jabbar H. Al-Baidhani

August 2007

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جمهورية العراق
وزارة التعليم العالي
والبحث العلمي
جامعة بابل
كلية الهندسة
القسم المدني

**نموذج عددي ثنائي البعد للتلوث الحراري من مصادر
نقطية أحادية في الأنهار عند أعماق طرح مختلفة**

رسالة

**مقدمة الى كلية الهندسة في جامعة بابل
كجزء من متطلبات نيل درجة الماجستير في علوم
الهندسة المدنية**

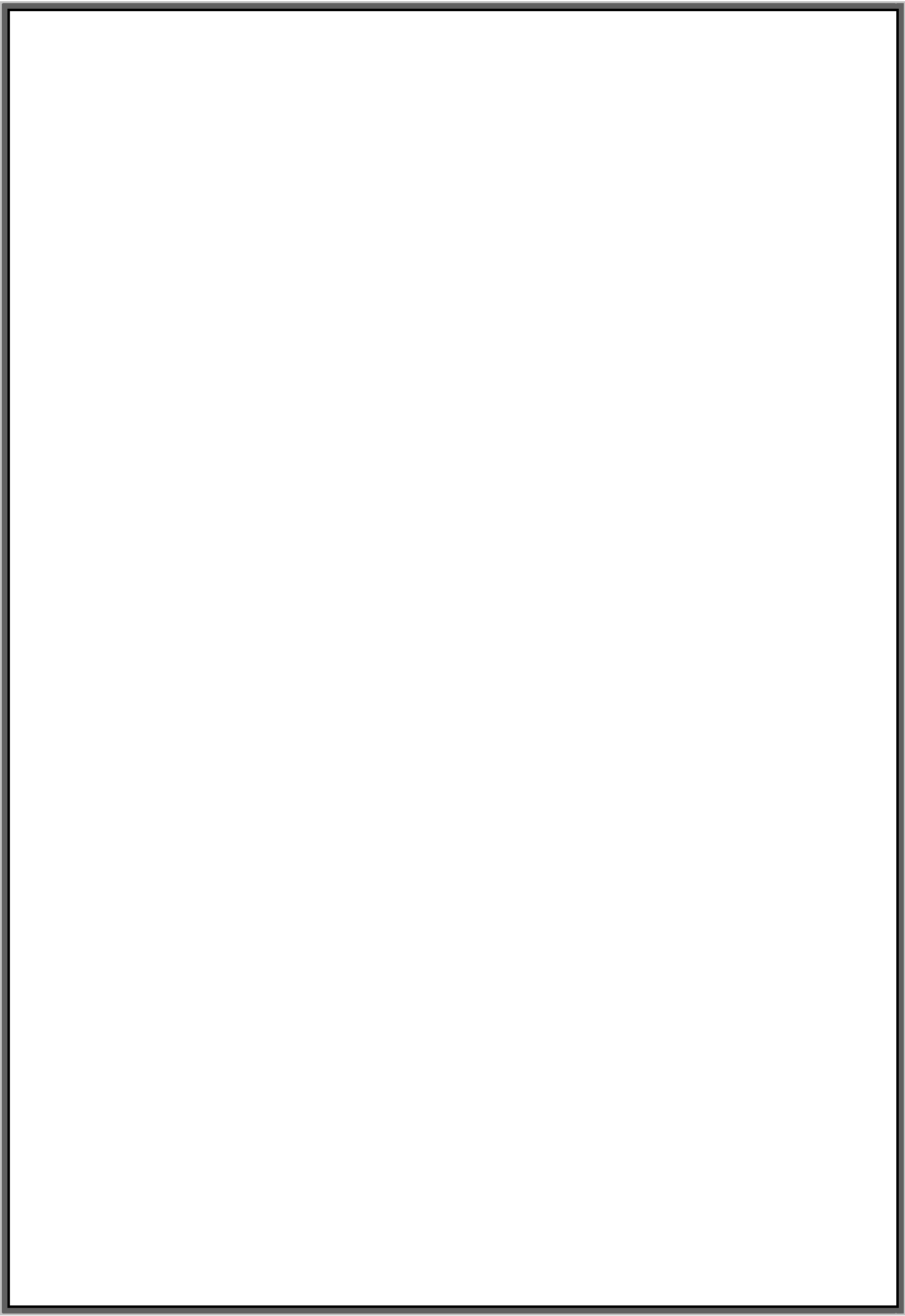
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نورس نوماش شاطي عبيد

باشراف

**أ.م.د. محمد عبد مسلم الطفيلي
أ.م.د. جبار حمود البيضاني**

أب 2007



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

﴿قَالُوا تَبِعَانَاهُ لَا عِلْمَ لَنَا
بِإِلَٰهٍ مَا جَعَلْنَا إِيَّاهُ زُنًى وَنَحْنُ
وَالْعَالِمِينَ﴾

صِرَافُ اللَّهِ الْعَظِيمِ

سُورَةُ الْبَقَرَةِ

وَاللَّهُ (32) .

ABSTRACT

A two-dimensional numerical model has been developed depending on finite-difference techniques to simulate the temperature and flow patterns resulting from unsteady continuous thermal discharge applied to river stream as a single-point source at different thermal discharge locations.

In this study three cases of thermal discharge to the river reach stream are adopted, first case is at the water surface of the river, second case is as submerged outfall lying at 0.42 from the river depth below the water surface, while the third case of discharge is a submerged outfall at 0.83 from the river depth. The latest discharge case is considered the basic contribution in this study by discharging the thermal pollutant at deep levels approach from the river bed. At each case the temperature and velocity patterns are obtained by solving the numerical model equations adopted by this study. The three cases are used to investigate which is the best for river stream from environmental point of view by achieving the fast recovery for the river stream .A comparisons between the three cases are established .The results showed that the submerged thermal discharge as single point source at 0.83 from the river depth causes a greater increase in the river temperature at deeper levels, and an earlier formation of uniform vertical distribution of temperature much faster than for the surface discharge and for discharge lies at 0.42 from the river depth.

To obtain a range for the river recovery from the thermal discharge a mathematical equation was derived as follows:

$$d_f = \frac{(td - tr) - (t_{i,j} - tr)}{(td - tr)} = \frac{(td - t_{i,j})}{(td - tr)}$$

The above equation obtains an indication of the dilution process for the river after applying the thermal discharge expressed as dilution factor (d_f).The dilution factor is a dimensionless scale range from (0.0 to 1.0), where the 0.0 value indicates that the dilution is zero at the discharge region, and the 1.0 value indicates that the river water recovers its normal temperature. The (td, tr) refers to the thermal discharge and river water temperature respectively.

Comparisons between the dilution factor variations with the horizontal distance from the discharge outfall for the three cases are obtained. These comparisons indicate that the dilution factor varies greatly with the three cases for the surface case for example; the dilution factor at distance 400 m far from the outfall is equal to 0.42, while it is increased to be 0.48 and

0.78 respectively for the first and second case of submerged discharge. Also the results showed that the submerged outfall lying at deeper depth (0.83 from the river depth) enhance the ability of the river reach to recover fast from the thermal shock.

In the present model, a comparisons between the dilution factor with two time steps (500,750 sec) for the three discharge cases was obtained to determine the best thermal discharge scheme for the river stream with minimum impact on the living organisms especially the bacteria .

Momentum conservation and thermal-energy equations were used to describe the distribution and diffusion of velocity and temperature along the flow field. Furthermore, the distribution of turbulent dynamic viscosity was calculated by the modified (k- ϵ) model which incorporates buoyancy term which tends to increase the turbulent kinetic energy and its rate of dissipation and in return increases the turbulent dynamic viscosity.

The pressure distribution was determined using hydrostatic pressure equation.

The partial differential equations were formalized and simplified in order to be solved using Alternative Direction Implicit-Explicit method (ADI) with up-winding technique. The resulting system of linear simultaneous equations was then solved using Gauss-Siedel point-by-point method. The results of the system were verified by using a numerical model data adopted by Al-Chalabi, M. A (1993). A sensitivity analysis was carried out to study the effect of variations of different parameters on the pollutant transport resulting from the application of the model. The model was found to be sensitive to the variations in turbulent dynamic viscosity, the variations in the river slope, initial river velocity and time of disposal in case of submerged discharge disposals in general. While in case of surface discharge it was sensitive to the variations in turbulent dynamic viscosity, initial river velocity and time of disposal, and insensitive to the variations in the river slope.

الخلاصة

نموذج عددي ثنائي البعد طور اعتمادا على طريقة الفروقات المحددة لاستنباط مخططات الحرارة والجريان الناتجة من طرح حراري مستمر غير مستقر على النهر من مصدر نقطي من مواقع طرح مختلفة. ثلاث حالات لطرح الحرارة لنهر افتراضي بعمق كلي مقداره 3م، الأولى (طرح الملوثات على سطح النهر) والثانية (طرح الملوثات تحت سطح الماء من مصدر نقطي عند 0.42 من عمق النهر) والثالثة (طرح الملوثات تحت سطح الماء من مصدر نقطي عند 0.83 من عمق النهر)، حالة الطرح الثالثة تعتبر الاسهام الرئيسي لهذه الدراسة من حيث طرح الملوثات الحرارية من مستويات عميقة تقترب من قاع النهر. في كل حالة يتم الحصول على مخطط لتوزيع الحرارة و الجريان من النموذج العددي. من خلال حالات الطرح الحرارية الثلاثة (سطحي وحالتي طرح غاطس) يتم التحقق أي من الحالات الثلاثة هي الأفضل للنهر من الناحية البيئة ومن ناحية سرعة استعادة النهر لحالته الطبيعية قبل الطرح الحراري. تم اجراء مقارنة بين حالات الطرح الثلاثة ونتائج هذه المقارنة اثبتت ان التلوث الحراري الغاطس عند 0.83 من عمق النهر يسبب زيادة اكبر في حرارة النهر عند المستويات العميقة، و التكوين المبكر للتوزيع العمودي المنتظم لدرجة الحرارة بصورة اسرع من حالتي الطرح السطحي وحالة الطرح الحراري الغاطس عند عمق 0.42 من عمق النهر. للحصول على مدى للتخفيف الحاصل للنهر بعد الطرح الحراري تم اشتقاق معادلة رياضية للتعبير عن هذا التخفيف يعبر عنه ب (معامل التخفيف) كما يلي:

$$d_f = \frac{(td - tr) - (t_{i,j} - tr)}{(td - tr)}$$

ان معامل التخفيف اعلاه يعطي مؤشر لا بعدي يتراوح من (0.0 الى 1.0) حيث يدل الصفر على ان التخفيف هو في منطقة الطرح الحراري وهو معدوم في البداية وبتصاعد القيمة الى (1.0) يدل على ان النهر استعاد حالته الطبيعية قبل الطرح وان حرارة النهر وصلت الى مستواها الطبيعي.

تم اجراء مقارنة لتغاير معامل التخفيف مع المسافة الافقية من نقطة الطرح لحالات الطرح الثلاثة للموديل حيث اشارت الى ان معامل التخفيف على سبيل المثال للطرح الحراري السطحي وعلى مسافة 400 م من منطقة الطرح يساوي 0.42 ولحالتي الطرح الغاطس الاولى والثانية على التوالي، معامل التخفيف ازداد ليساوي في الحالتين (0.48 و 0.78). ان نتائج المقارنة تظهر ان المصدر النقطي الغاطس عند المستويات العميقة (0.83 من عمق النهر) يحسن كثيرا من قابلية النهر على استعادة حرارته الطبيعية قبل الطرح الحراري.

تم الحصول على توزيع معامل التخفيف مع الزمن لفترتين زمنيتين (500 و 750 ثانية) لحالات الطرح الثلاثة (السطحي وحالتي الغاطس) و اجراء مقارنة في التخفيف الحاصل لحالات الطرح الثلاثة للحصول على افضل نمط لطرح الحرارة في النهر باقل اثر بيئي للكائنات الحية وخاصة البكتيريا.

من خلال تطبيق حالات الطرح الثلاثة للحرارة في النهر للدراسة الحالية تبين ان طرح الحرارة من خلال مصدر نقطي غاطس قريب من قاع النهر (0.83 من عمق النهر) يؤدي الى سرعة وصول النهر الى درجة حرارته الطبيعية لان التخفيف الناتج في العمق اكثر، والى تجنب الصدمة الحرارية الناتجة من الطرح الحراري السطحي.

تم استخدام معادلات حفظ الزخم ومعادلة حفظ الطاقة لمعرفة توزيع وانتشار درجات الحرارة خلال منطقة الدراسة وكذلك تضمن النموذج العددي معادلات الاضطراب ($K \cdot \varepsilon$) المطور الذي تضمن حد الطفو والذي يؤدي الى زيادة الجريان الاضطرابي بزيادة k و ε وبالتالي

زيادة معاملات اللزوجة. كما تم اعتماد معادلة الضغط الهيدروستاتيكي لحساب الضغط في كل نقطة من الدراسة .

تم تبسيط المعادلات التفاضلية المختلفة الاشكال بطريقة الاتجاه المتناوب (الضمني- الصريح) للفروق المحددة (ADI) مع تطبيق منظومة Up Winding للحصول على معادلات جبرية خطية تم حلها بطريقة كاوس-سيدل (point-by-point Gauss – Siedel) .

تمت مطابقة النتائج بمقارنتها مع نتائج دراسة سابقة (الباحثة مي الجلبي) (1993). واستخدام فحص الحساسية لمعرفة مدى تاثر النموذج بتغير عدة عوامل مؤثرة على الجريان بصورة عامة حيث وجد ان النموذج حساس الى سرعة النهر والى تغير معاملات اللزوجة و تغير معامل الزمن للملوث (الحرارة) وغير حساس لتغير عامل الميل السطحي بالنسبة للطرح السطحي للحرارة. و ان النموذج (لحالتي الطرح الغاطس) حساس الى تغير سرعة النهر والى تغير معاملات اللزوجة وتغير معامل الزمن للملوث (الحرارة) و تغير عامل الميل السطحي .



SYMBOLS AND ABBREVIATIONS

Table of Symbols

<u>Symbols</u>	<u>Description</u>	<u>Unit</u>
A	Cross sectional area of river	m^2
a	Coefficient in the general discretization equation	
A_s	Water surface area	m^2
b	Coefficient in the general discretization equation	
c	Coefficient in the general discretization equation	
C_p	Specific heat	W.s/ kg.K
C_μ, C_1, C_2	Constants in K- ϵ Turbulence model	
D	Total depth of the river	m
D_T	Turbulent diffusion coefficient	N.s/m ²
d_f	Dilution factor	
E	Longitudinal dispersion coefficient	m ² /s
G	Acceleration due to gravity	m/s ²
h	Water depth	m
K	Turbulence kinetic energy	m ² /s ²
k_d	Turbulence kinetic energy at the discharge region	m ² /s ²
k_v	Vertical eddy viscosity	Ns/m ²
L	Total length of river reach	m
P	Pressure	N/m ²
Q	Total Water flow	m ³ /s
R	Hydraulic radius	m
S	Water surface slope	(m/m)
S_T	Volumetric source term according to heat	
T	Water temperature	°C
tr	River water temperature	°C
td	Heated water temperature of sources	°C
t	Time	sec
U_1	Surface wind velocity at 10m above the water surface	m/s
U_2	Wind velocity at 1 m above the water surface	m/s
u	Velocity in X-direction	m/s
U_f	Friction velocity	m/s
V	Velocity in Y-direction	m/s
V	Water volume	m ³
w	Velocity in Z-direction	m/s
X	Horizontal coordinate	m
Y	Vertical coordinate	m
G_k	Rate of production of turbulence kinetic energy.	N.m ⁴ /s. kg ²
G_b	Rate of production of turbulence kinetic energy due to buoyancy.	N.m ⁴ /s. kg ²

Greek symbols

Φ	Rate of surface heat exchange between water and atmosphere	W.s/m ²
μ	Dynamic Viscosity	N.s/m ²
ρ	Density	kg/m ³
ν	Kinematics viscosity	m ² /s
ε	Dissipation rate of turbulent kinetic energy	m ² /s ³
ε_d	Dissipation rate of turbulent kinetic energy at the discharge region	m ² /s ³
σ	Prandtle number	
σ_k	Turbulent prandtle number for the turbulent kinetic energy	
σ_T	Constant in thermal –energy equation	
σ_ε	Turbulent prandtle number for the dissipation of turbulent kinetic energy	
σ_b	Turbulent prandtle number for the turbulent kinetic energy due to buoyancy	
α	Heat transfer coefficient	W/m ² .k
Δt	Time increment	sec
Δx	Horizontal increment	m
Δz	Vertical increment	m

A

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NAWRAS NOMASS SHATI OBIAD

CERTIFICATION

We certify that the thesis, entitled "**Two-Dimensional Numerical Model for Thermal Pollution of Single-Point Sources in Rivers at Different Discharge Depths**" was prepared by "**Nawras Nomass Shati Obaid**" under our supervision at Babylon University, College of Engineering in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

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Chapter One

Introduction

1.1 General

The thermal pollution can be defined as the excessive change in the natural or ambient water temperature by the addition or removal of heat through mans' activities.

The major waste-heat producing industries are: steam-electric generation plants, petroleum refineries, steel mill, chemical plants (Mathur, 1976).

Heating up a river to a relatively high temperature, can have considerable consequences on the river water system as follows:

1. Higher temperature reduces the solubility of river oxygen and increased the rivers' chemical reactions and the river water becomes anaerobic with disastrous effects on its odor and appearance.
2. The changing in river water temperature due to the hot wastewater can harm fish and the river water organisms which adapted to a practical water temperature regime.
3. Thermal pollution affecting ecosystem composition: the metabolic rate of aquatic animals, as enzyme activity, is increased by heat meaning that the river organisms will consume more food in a shorter time than if their environment was not changed.

The discharge of heated water directly to the river can be more dangerous to the health of the receiving water than organic pollution. (Rute and Siliva, 1997).

Environmental issues related to the release of cooling water are becoming increasingly important. The consequences of climate change and an increased consumption of electric energy will result in substantial releases of heated water into rivers that may have extremely low discharges. (Baptist, 2005)

Heating up a river to relatively high temperatures can have considerable consequences for the ecological system. The release of thermal effluents can cause damages to the environment. The aquatic fauna usually do not support temperature variations in its habitat. Water temperature variations above 3 °C can affect the metabolisms of certain species, even causing the death of these species. When warm water need to be released in any river reach, we should have effective mechanisms that can predict the field of temperatures in the water environment. The prognostic of the temperature field can be made through physical models simulation, that are more expensive, or through the use of numeric modeling.

Positively buoyant surface jets and plumes are formed when a lighter water mass is being discharged continuously through a well-defined source, often an open channel, near or parallel to the free surface of a receiving water body. Such discharges are common both in natural geophysical situations, but also in hydraulic, environmental and industrial applications, including various types of wastewater discharges into water courses or flows in treatment facilities. The buoyancy of these discharges is usually due to differences in temperature, salinity. Depending on the geometric and dynamic characteristics of the receiving water that may be stagnant or flowing a number of complex fluid mechanical processes will occur affecting the shape and extent of the resulting discharge plume and its degree of mixing. (Gilbert, et al.; (2005)).

The major environmental engineering application concerns pollutant discharges from municipal or industrial sources, or from mining activities into lakes, rivers, estuaries or coastal waters. This is oldest and, in form of a simple surface discharge at the river water surface, the least-costly form of wastewater discharges. Recent water quality and mixing zone regulations in most industrialized countries prohibit or discourage the surface discharge mode, since it often impacts the ecologically sensitive river (pelagic) zone and usually has low degrees of dilution. Much higher dilution rates can be achieved with offshore submerged discharges, as required by modern water quality regulations, often in combination with advanced pre-discharge treatment processes. Nevertheless, the surface discharge mode continues to be an option and reliable predictive planning methods are required, in particular for the re-licensing of already existing discharges, for small sources, or for heavily polluted water courses.(Gilbert , et al.; (2005)).

In order to evaluate the effect of various discharge schemes on the environment, it is necessary to be able to predict the resulting temperature distribution.

1.2 Major Pollutant Transport Mixing Process:

Most important pollutant transport processes in water bodies (Fisher et al.; 1981, Baptist et al; 2005) are:

1. Advection: Transport by an imposed current.
2. Diffusion (molecular): The scattering of particles by random molecular motions.
3. Diffusion (turbulent): The scattering of particles by turbulent motion, considered roughly analogous to molecular diffusion, but with “eddy” diffusion coefficients, which are much larger than molecular diffusion coefficients and not of equal size in all directions.

4. Shear: The advection of a fluid at different velocities at different positions; this may be simply the normal velocity profile for a turbulent flow where the water flows faster with increasing elevation above the bed; or shear may be the changes in both magnitude and direction of the velocity vector with depth in complex flows such as in estuaries or coastal waters.
 5. Dispersion: The scattering of particles or a cloud of contaminants by the combined effects of shear and transverse diffusion.
 6. Mixing: Diffusion or dispersion as described above; turbulent diffusion in buoyant jets and plumes; any process which causes one parcel of water to be mingled with or diluted by another.
 7. Evaporation: The transport of water vapor from a water or soil surface to the atmosphere.
 8. Radiation.: The flux of radiant energy, such as at the water surface.
- The mathematical models of pollution mixing and spreading in open channels can be effectively used for determination of transport, mixing and self purifying characteristics of channels, optimum location of outlet structure in streams, delineation of mixing zones, predication of spreading of accidental contaminant waves, etc. the ability to solve these problems may have a big influence upon the improvement of water quality, and thus upon ecology (Veliskova, 2002)

1.3 Present Contributions and Objectives:

The present study is a two dimensional numerical model designed to solve a complete set of unsteady momentum conservation, thermal energy, and the modified $k-\varepsilon$ models equations to find the temperature distribution expressed as dilution factor resulting from the temperature distribution through the river reach. This study contributes to the water pollution modeling field in the following areas:

1. The buoyancy effects are considered in the present study by using the modified $k-\varepsilon$ (Ahmadi, 1985), which differs from the standard $(k-\varepsilon)$

model by the buoyancy term $(-g \frac{\mu}{\sigma_b \rho} \frac{\partial \rho}{\partial Z})$, the buoyancy term increase the

level of k and its rate of dissipation (ε) , which intern increase the turbulent dynamic viscosity, and hence the ability of the jet to entrain ambient fluid.

2. Three limiting schemes are used for the discharge of heated water to a theoretical river with a proposed depth equal to a 3m. First scheme is the discharge of heated water at the river surface resulting in minimizing of initial dilution, while, maximizing the rate of heat loss to the atmosphere, second is a single point source submerged at depth equal to 0.42 of the river water depth which equal to 3 m to promote much initial dilution of the temperature. The third scheme is a single point source submerged at 0.83 from the river water depth to promote more initial dilution for the temperature. The third scheme is considered the basic contribution for this study.

3. Derivation of a dilution factor formula from the temperature distribution resulting from the thermal discharge applied to the river stream .This dilution factor is a dimensionless scale ranges from 0.0 to 1.0 which describes the dilution level of the river reach subjected to any thermal discharge.

The main objectives of the present work are:

1. Developing and verifying a two-dimensional numerical model to simulate the spreading and mixing of heated water disposed at and beneath the water surface of the river from a single point source.
2. Determining which method of the three-thermal discharges cases is the best for the river stream from environmental point of view.

3. Investigate the effect of various parameters on heated water distribution process depending on some of the hydraulic conditions of the river.

1.4 Thesis Presentation:

The research procedure which undertakes its presentation in this thesis is as follows:

- Chapter one presents the main objects concerns with the thermal pollution and its main effects on the river water system.
- Chapter Two presents the main differential equation which is used in the present numerical model (momentum conservation, thermal energy, and $k-\varepsilon$ turbulence model equations) to simulate the temperature and flow patterns resulting from unsteady continuous thermal discharge applied to river stream as a single-point source. And a review of both early and recent studies on thermal discharge applied to any river reach.
- Chapter Three, presents the discretization process for the differential equations mentioned in chapter two by using the finite-difference technique (Alternating Direction Implicit – Explicit Finite – Difference Technique) (ADI) to obtain a set of linear algebraic equations solved by guess –siedel point-by-point method.
- Chapter Four, presents the results of the numerical model and discussion for each method of thermal disposal , also this chapter presents the model verification which include the sensitivity analysis and comparing the present numerical data by previous data conducted by a another researcher.
- Chapter Five contains the conclusions obtained from this study and the recommendations for future works.

Chapter Two

Theoretical Background and Literature Review

2.1 Prediction of Water Velocity and Temperature Field:

A mathematical model was established to predict the temperature distribution due to a single- point source thermal discharge of heated water applied into a river, and to determine the transport, mixing and dilution of heated water in water bodies. The governing differential equation should be known .These equations should be set with the proper boundary and initial conditions if unsteady state is assumed.

2.2 Governing Equations:

The main equations which established the numerical model are:

1. Momentum Conservation Equations (Navier-Stokes Equation).
2. Thermal-Energy Equation.
3. Modified K- ϵ turbulence model.

2.2.1 Momentum Conservation Equations:

Daily and Harleman (1966) present the equation governing the conservation of momentum (Navier-stokes equation) for incompressible fluid with constant viscosity and density as follows:

$$\rho G - \nabla p + \mu \nabla^2 q + \frac{\mu}{3} \nabla(\nabla \cdot q) = \rho \frac{\partial q}{\partial t} + \rho(q \nabla)q \quad \dots 2.1$$

Where

ρ : Density (kg/m³)

p : Pressure (N/m²)

μ : Viscosity (N.s/m²)

t : Time (s)

G : Acceleration due to gravity (m/s²) with component given by: $g = i g_x + j g_y + k g_z$

Where i, j, k are vectors of magnitude equal to unity and pointing the direction of the X, Y, Z –Axes respectively.

∇ : Vector operation (differential operation) $= i \left(\frac{\partial}{\partial x} \right) + j \left(\frac{\partial}{\partial y} \right)$

$+ k \left(\frac{\partial}{\partial z} \right)$

q : vector of velocity field $= iU + jV + kW$

For an incompressible fluid in which the time rate of volume expansion of fluid element is zero, ($\nabla \cdot q = 0$) equation (2.1) becomes:

$$\mu^2 q - \nabla p + \rho G = \rho \frac{\partial q}{\partial t} + \rho(q \nabla)q \quad \dots 2.2$$

The cartesian components of this equation were given by Bird et al. (1960) as follows:

X- Component.

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial X} + v_y \frac{\partial v_x}{\partial Y} + v_z \frac{\partial v_x}{\partial Z} \right) = - \frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 v_x}{\partial X^2} + \frac{\partial^2 v_x}{\partial Y^2} + \frac{\partial^2 v_x}{\partial Z^2} \right) + \rho g_x$$

... (2.3)

Y-component:

$$\rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial X} + v_y \frac{\partial v_y}{\partial Y} + v_z \frac{\partial v_y}{\partial Z} \right) = - \frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v_y}{\partial X^2} + \frac{\partial^2 v_y}{\partial Z^2} + \frac{\partial^2 v_y}{\partial Y^2} \right) + \rho g_y$$

... (2.4)

Z- Component.

$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial X} + v_y \frac{\partial v_z}{\partial Y} + v_z \frac{\partial v_z}{\partial Z} \right) = - \frac{\partial P}{\partial Z} + \mu \left(\frac{\partial^2 v_z}{\partial X^2} + \frac{\partial^2 v_z}{\partial Y^2} + \frac{\partial^2 v_z}{\partial Z^2} \right) + \rho g_z$$

... (2.5)

If h is taken as the vertical direction, the component of the acceleration due to gravity is (Patanker and Spalding, 1982)

$$g_x = g \frac{\partial h}{\partial x}, \quad g_y = g \frac{\partial h}{\partial y}, \quad g_z = g \frac{\partial h}{\partial z} \quad \dots (2.6)$$

$$\frac{\partial p}{\partial x} = \rho g \frac{\partial h}{\partial x}, \quad \frac{\partial p}{\partial y} = \rho g \frac{\partial h}{\partial y}, \quad \frac{\partial p}{\partial z} = \rho g \frac{\partial h}{\partial z} \quad \dots (2.7)$$

If the cartesian axes are oriented so that h and z coincide (such as in rivers and canals where the slope is very small and the water surface is planer)

Then assuming constant water depth results in:

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial y} = 0 \text{ thus } g_x = g_y = 0 \text{ while } g_z = g \text{ and } \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0 \dots (2.8)$$

(Daily & harleman, 1966).

2.2.2 Thermal -Energy Equation:

Harleman (1982) express the one - dimensional conservation of thermal energy for the river as:

$$\frac{1}{A} \frac{\partial}{\partial t} (AT) + \frac{1}{A} \frac{\partial}{\partial x} (QT) = \frac{1}{A} \frac{\partial}{\partial x} \left[AE \frac{\partial T}{\partial X} \right] + \frac{A_s}{V} \frac{\Phi(T)}{\rho C_p} \quad \dots (2.9)$$

Where

A: Cross section area of the river (m²)

A_s: Water surface area (m²)

Q: Total water flow (m³/s)

T: Water temperature (°C)

V : Water volume (m³)

E: Longitudinal dispersion coefficient (m²/s)

Φ: Rate of surface heat between water and atmosphere. (W.sec/m²)

C_p: Specific heat (W.sec/kg. k)

Patanker (1980) presented three dimensional unsteady conservation of heat equation as follows:

$$\begin{aligned} \frac{\partial}{\partial t} (\rho T) + \frac{\partial}{\partial X_i} (\rho U T) + \frac{\partial}{\partial Z_j} (\rho W T) &= \frac{\partial}{\partial X_i} \left(D_T \frac{\partial T}{\partial X_i} \right) \\ + \frac{\partial}{\partial Z_j} \left(D_T \frac{\partial T}{\partial Z_j} \right) + S_T &\quad \dots 2.10 \end{aligned}$$

Where:

D_T: Turbulent diffusion coefficient (N.s/m²)

S_T: Volumetric source term accounting for heat like the heat gain or loss from the water surface (kg.k/m².sec) and expressed :

$$S_T = \frac{\alpha(t_{i,j} - t_r)}{C_p \cdot h} \quad \dots 2.11$$

Where:

α : Heat transfer coefficient (W/m².k)

h : Water depth (m)

The turbulent diffusion coefficient (D_T) is directly related with the turbulent dynamic viscosity (μ) by the following relation (Babartutsi and Chu, 1998)

$$D_T = \frac{\mu}{\sigma_T} \quad \dots 2.12$$

Where σ_T is the prandtle -Schmidt number, for which a value of (0.5) was suggested by (Rodi, 1981).

Reddy, G. S. (1998) presented a three-dimensional unsteady conservation of pollutant transport equations as:

$$\begin{aligned} \frac{\partial T}{\partial t} + \frac{\partial uT}{\partial X_i} + \frac{\partial vT}{\partial Y_k} + \frac{\partial wT}{\partial Z_j} = \frac{\partial}{\partial Z_j} \left(k_v \frac{\partial T}{\partial Z_j} \right) + \frac{\partial}{\partial X_i} \left(k_v \frac{\partial T}{\partial X_i} \right) + \\ \frac{\partial}{\partial Y_k} \left(k_v \frac{\partial T}{\partial Y_k} \right) \end{aligned}$$

..2.12'

Where:

k_v : vertical eddy viscosity(N.s / m²)

i, j ,k: subscripts represent the x, y ,z axes.

2.2.3 Turbulence Models

The turbulent viscosity/diffusivity is not a fluid property but depends strongly on the state of turbulence and may vary considerably over the flow field. Therefore, the distribution of turbulent viscosity/diffusivity over the whole flow field must be determined (ASCE Task Committee; 1988) a wide variety of turbulence models exist for this purpose. Among them are:

1. Prandtl's Mixing –Length Hypothesis:

This hypothesis was proposed by Prandtl in (1925). Who assumes that the eddy viscosity (ν_t) is proportional to a mean representation of the fluctuating velocity ($\hat{v}$) and a mixing length (l_m) (ASCE Task Committee on turbulence in models hydraulic computations ; 1988) ,ozoe et.;1986).

$$\nu_t \propto \hat{v} l_m \quad \dots (2.13'a)$$

Where:

$$\hat{v} = l_m \frac{\partial U}{\partial Z_j} \quad \dots (2.13'b)$$

Then

$$\nu_t = l_m^2 \left| \frac{\partial U}{\partial Z_j} \right| \quad \dots (2.13'c)$$

The mixing –length model has been applied mainly to two-dimensional shear-layer flows with only one velocity gradient. In more general flows with several significant mean-velocity gradients, the turbulent viscosity should be related to all specify the mixing-length distribution. To overcome this, turbulence models were developed which account for transport of turbulence quantities by solving differential transport equations for them.

2. One-Equation Model:

This model was proposed by Kolmogorov (1942) and Prandtl (1945). It employs the square root of the turbulence kinetic energy as the viscosity determining velocity scale, $\sqrt{k}$, e.

$$\nu_t = cu' \sqrt{k} L \quad \dots (2.13'd)$$

Where: cu' is an empirical constant and (L) is a length scale characterizing the turbulent motion (ASCE Task Committee.; 1988). This formula is known as Kolmogorov-Prandtl formula. The unknown (k) is determined from differential equation of transport; this is the reason why it is called one equation model.

The application of one-equation model is restricted mainly to shear – layer situation because it is difficult to specify the length scale distribution in more complex flows. For calculating general flow, the trend has therefore been to move on to two-equation models.

3. Two-Equations Model:

The currently popular and widely used two-equation model is the $(K - \varepsilon)$ turbulence model. This model was first put forward by Harlow and Nakayama (1968), while the numerical procedure for using this model was described by Launder and Spalding (1972) as follows:

K-equation:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho U_j k) = \frac{\partial}{\partial x_j} \left(\frac{\mu}{\sigma_K} \frac{\partial K}{\partial x_j} \right) + Gk - \rho \varepsilon \quad \dots (2.13'e)$$

Where:

K : kinetic energy of turbulence flow

j : subscript takes the values 1, 2, 3 denoting the three spaces coordinates.

σ_k = empirical constant which is the Prandtl number for the turbulent kinetic energy (k) .

Gk = rate of production of turbulent kinetic energy

ε = rate of dissipation of turbulent kinetic energy.

$Gk - \rho \varepsilon$: net source term (sk).

ε - Equation:

$$\frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_j}(\rho U_j \varepsilon) = \frac{\partial}{\partial x_j} \left(\frac{\mu}{\sigma_K} \frac{\partial \varepsilon}{\partial x_j} \right) + (C1 Gk - C2 \rho \varepsilon) \left(\frac{\varepsilon}{k} \right)$$

... (2.13'f)

Where:

$C1 Gk \frac{\varepsilon}{k} - C2 \rho \frac{\varepsilon^2}{k}$: net source term ($S\varepsilon$).

$C1, C2$: empirical constant which is the Prandtl number for the rate of dissipation of turbulent kinetic energy.

4. The Modified k- ε Model:

The turbulence kinetic energy (K) and the dissipation rate (ε) can be computed from the following semi-empirical transport equations (Ahmadi, 1985)

$$\frac{\partial \rho U_i K}{\partial X_i} + \frac{\partial \rho W_j K}{\partial Z_j} = \frac{\partial}{\partial X_i} \left(\frac{\mu}{\sigma_K} \frac{\partial K}{\partial X_i} \right) + \frac{\partial}{\partial Z_j} \left(\frac{\mu}{\sigma_K} \frac{\partial K}{\partial Z_j} \right) + Gk + Gb - \rho \varepsilon$$

...2.14

$$\frac{\partial \rho U_i \varepsilon}{\partial X_i} + \frac{\partial \rho W_j \varepsilon}{\partial Z_j} = \frac{\partial}{\partial X_i} \left(\frac{\mu}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial X_i} \right) + \frac{\partial}{\partial Z_j} \left(\frac{\mu}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial Z_j} \right) + C_1 \frac{\varepsilon}{K} (Gk + Gb) - C_2 \rho \frac{\varepsilon^2}{K}$$

...2.15

Where:

σ_K : Empirical constant, which is the prandtl number for the turbulence kinetic energy.

i, j : subscripts represent the x and z axes.

σ_ε : Empirical constant, which is the prandtl number for the dissipation rate of turbulence kinetic energy.

Gk : Rate of production of turbulence kinetic energy.

Gb = Rate of production of turbulence kinetic energy due to buoyancy.

$Gk + Gb - \rho\varepsilon$: Net source term for K- equation (S_k).

$C_1(Gk + Gb) \frac{\varepsilon}{K} - C_2\rho \frac{\varepsilon}{K}$: Net source term for ε - equation (S_ε).

C_1, C_2 : Empirical constants.

The (K- ε) turbulence model involves five empirical constants. The values of these constants are as suggested by (Leschziner and Rodi, 1983)

$$C_\mu = 0.09, C_1 = 1.44, C_2 = 1.92, \sigma_\varepsilon = 1.3, \sigma_k = 1.$$

The above motioned turbulence model differs from the two-equations model which is popular in use by the buoyancy term (Gb), which lies at the right side of the equations 2.15 and 2.16. This term tends to increase k (turbulence kinetic energy) and its rate of dissipation (ε) and then increase the turbulence dynamic viscosity for the river according to equation (2.31) and increase the ability of the fluid to entrain.

Many other investigators have been used the (k- ε) turbulence model to solve many complicated flow field patterns, including those associated with thermal discharge problems (e.g. Raithby, G.D et al.; 1988)

2.3 The Expanded Form of the Governing Differential Equations:

Equations (2.1 to 2.15) can be written in an expanded form; for dense fluids like liquid which have a large heat capacity. The temperature change due to internal friction is small and then the density and viscosity

change due to internal stress is very little and both are considered constant. However because of μ and ρ are functions of temperature, they were considered as temperature dependent in this derivation.

a. Vertical-Momentum Conservation Equations:

$$\frac{\partial \rho w}{\partial t} + \frac{\partial \rho u w}{\partial X} + \frac{\partial \rho w^2}{\partial Z} = \frac{\partial}{\partial X} \left(\mu \frac{\partial w}{\partial X} \right) + \frac{\partial}{\partial Z} \left(\mu \frac{\partial w}{\partial Z} \right) - \frac{\partial P}{\partial Z} + \rho g \quad \dots (2.16)$$

b. Horizontal- Momentum Conservation Equations:

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial X} + \frac{\partial \rho w u}{\partial Z} = \frac{\partial}{\partial X} \left(\mu \frac{\partial u}{\partial X} \right) + \frac{\partial}{\partial Z} \left(\mu \frac{\partial u}{\partial Z} \right) \quad \dots (2.17)$$

c. Thermal Energy Equation:

$$\frac{\partial \rho T}{\partial t} + \frac{\partial \rho u T}{\partial X} + \frac{\partial \rho w T}{\partial Z} = \frac{\partial}{\partial X} \left(D_T \frac{\partial T}{\partial X} \right) + \frac{\partial}{\partial Z} \left(D_T \frac{\partial T}{\partial Z} \right) + S_T \quad \dots (2.18)$$

Where: $D_T = \frac{\mu}{\sigma_T}$, σ_T is the Prandtl -Schmidt number, for which a

value of (0.5) was suggested by (Rodi, et al 1981).

d. k-equation:

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \rho u K}{\partial X} + \frac{\partial \rho w K}{\partial Z} = \frac{\partial}{\partial X} \left(\frac{\mu}{\sigma_K} \frac{\partial K}{\partial X} \right) + \frac{\partial}{\partial Z} \left(\frac{\mu}{\sigma_K} \frac{\partial K}{\partial Z} \right) + Gk + Gb - \rho \epsilon \quad \dots (2.19)$$

e. ϵ -equation:

$$\frac{\partial \rho \epsilon}{\partial t} + \frac{\partial \rho u \epsilon}{\partial X} + \frac{\partial \rho w \epsilon}{\partial Z} = \frac{\partial}{\partial X} \left(\frac{\mu}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial X} \right) + \frac{\partial}{\partial Z} \left(\frac{\mu}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial Z} \right) + C1 \frac{\epsilon}{k} (Gk + Gb) - C2 \rho \frac{\epsilon^2}{k} \quad \dots (2.20)$$

Where:

$$Gk = \mu \left[2 \left(\frac{\partial u}{\partial X} \right)^2 + 2 \left(\frac{\partial w}{\partial Z} \right)^2 + \left(\frac{\partial w}{\partial X} + \frac{\partial u}{\partial Z} \right)^2 \right] \quad \dots (2.21)$$

$$Gb = -g \frac{\mu}{\rho \sigma_b} \frac{\partial \rho}{\partial Z} \quad \dots (2.22)$$

2.4 Additional Auxiliary Equations:

2.4.1. Pressure Distribution:

Many investigators have suggested methods for determining the pressure distribution as follows:

Patanker (1982) calculate the pressure field by solving the continuity and momentum equations. He found that this method is satisfactory.

Dinelli and Tozzi (1977), Cooper (1987) used the hydrostatic pressure distribution to calculate the pressure at each point in the flow domain, and used the following equation:

$$\frac{\partial P}{\partial Z_j} = \rho g \quad \dots (2.23)$$

By integrating the above equation:

$$P = g \int_0^Z \rho dz \quad \dots (2.24)$$

Where:

P: Pressure, N / m²

Z: Vertical coordinate measured positive down ward from water surface.

g: Acceleration due to gravity, m/s²

2.4.2 Density-Temperature Relationship:

The released heated water is discharged to the river from source having a lower density than that of water river. In the river, mixing will take place with a new density of mixture. Density stratification is set up in a way that may have a significant influence on the turbulence characteristic and thus on the mixing of heat water with surrounding river water. (Rodi, 1982)

The relation between the river water density and temperature is approximated by a second-degree non-linear equation as follows. (Kadhem, 2002)

$$\rho = - 0.0055T^2 + 0.0182 T + 1000.1 \quad \dots (2.25)$$

Where:

ρ : Local water density (kg/m³)

T: local water temperature (°C)

2.4.3. Heat Transfer Coefficient:

Several researches had investigated methods to determine the heat transfer coefficient:

Wust (1978) had derived from his experimental measurements and theoretical of the heat flow transferred from the water to the air, the

Following equations for the heat transfer coefficient (Glaser and Pruckner, 1982)

- For unstable air layer

$$0 < U_{0.8} < 0.9 \text{ -----} \alpha = 3.4 \text{ W/m}^2.\text{k} \quad \dots 2.26$$

$$U_{0.8} > 0.9 \text{ -----} \alpha = 2.17 + 1.42 U_{0.8} \quad \dots 2.27$$

- For stable air layer

$$\alpha = 0.09 + 1.8 U_{0.8} \quad \dots 2.28$$

Where:

$U_{0.8}$: air velocity measured at height of (0.8m) above the water surface.

Wark and Warner (1976) had calculated wind speed at height (1m) from the surface using the following equation:

$$\frac{U_1}{U_2} = \left(\frac{Z_1}{Z_2} \right)^p \quad \dots 2.29$$

Where:

U_1 : Wind speed at height ($Z_1=10\text{m}$)

U_2 : Wind speed at height ($Z_2=1\text{m}$)

p : Constant which depend on the topography of the ground and stability condition, for open area (river, lakes, oceans), he suggested A value for $p=0.16$.

Pruckner (1981) measured the heat transfer coefficient by using the energy balance in a river reach. The measured data can be represented by the following exponential law :(Glaser and Pruckner, 1982)

$$\alpha = 4.21 + 6.64 U^{0.71} \quad \dots 2.30$$

Where:

α = Heat transfer coefficient in ($\text{W}/\text{m}^2.\text{k}$)

U = Wind velocity measured at height of 1 m above the water surface.

Constant value of water specific heat (C_p) equal to (4180 W.s/kg.k) (Raznjevic, 1976) was taken according to the river water temperature.

Average wind speed in Baghdad city at height of 10m was (4.8 m/s) based on data supplied by the (Iraqi – Meteorological organization). In The present model the heat transfer coefficient was taken as a constant value equal to 0.0.

2.4.4. Turbulent Dynamic Viscosity:

The distribution of turbulent dynamic viscosity can be obtained from the following equation described by Launder and Spalding (1972).

$$\mu = C_{\mu} * \rho * \frac{K^2}{\varepsilon} \quad \dots (2.31)$$

From this equation the turbulent dynamic viscosity (μ) is a function of the turbulence kinetic energy (K) and the rate of dissipation (ε) . (Babarutsi and Chu, 1998)

Where:

C_{μ} : Empirical constant represented by the dissipation coefficient in the (K- ε) turbulence model.

K: Turbulence kinetic energy per unit mass (m^2 /s^2)

ε : Dissipation rate of turbulence kinetic energy per unit mass (m^2/s^3)

2.5 Dilution Factor Formula:

Several investigators have proposed equations that describe the dilution characteristics for the river stream suffering from the thermal pollution as following:

Adams (1972) first derived a dilution equation for unidirectional diffusers by adopting the elements of propeller theory (Prandtl, 1952) in two dimensions.

Kim and Seo (1999) define the minimum surface dilution as the discharge excess temperature divided by the highest excess temperature.

The dilution factor equation for the temperature distribution in the river was derived in the present study as:

$$d_f = \frac{(td - tr) - (t_{i,j} - tr)}{(td - tr)} \quad \dots (2.32)$$

Where

$t_{i,j}$: Temperature of the river in the selected node i , j (° C).

t_d : Thermal discharge water temperature from the electric power plants or any other thermal sources (°C).

t_r : River water temperature (°C).

The value of the dilution factor gives a dimensionless range scale from 0.0 to 1.0

Where 0.0 values denoted that there is no dilution yet till the factor value approach to reach 1.0 which means the river water reaches to the normal river water temperature.

$$t_{i,j} = t_d \Rightarrow d_f = 0 \quad \text{at the discharge region}$$

$$t_{i,j} = t_r \Rightarrow d_f = 1$$

2.6 General Differential Equation:

All previously discussed differential equations were conveniently presented in general form as:

$$\begin{aligned} \frac{\partial}{\partial t}(\rho \Phi) + \frac{\partial}{\partial X_i}(\rho u_i \Phi) + \frac{\partial}{\partial Z_j}(\rho w_j \Phi) = \frac{\partial}{\partial X_i} \left(D \Phi \frac{\partial \Phi}{\partial X_i} \right) + \frac{\partial}{\partial Z_j} \left(D \Phi \frac{\partial \Phi}{\partial Z_j} \right) \\ + S \Phi \end{aligned}$$

... (2.33)

In the above equation (Φ) identifies the dependent variables in the differential equations.

$D\Phi$: the appropriate exchange coefficient for the variable Φ .

$S\Phi$: source terms of the differential equations, and any other terms which cannot find place on the left-hand side of equation (2.33).

The detailed form of general differential equation definition is listed in Table (2-1).

Table 2-1: Summary of equations adopted by this study

<u>Equation</u>	Φ Dependent variables	D_{Φ} Exchange coefficients	S_{Φ} Source term
X-Momentum	u	μ	0
Z-Momentum	w	μ	$\rho g - \frac{\partial p}{\partial Z}$
Thermal equation	T	$\frac{\mu}{\sigma_T}$	0
Turbulence energy	k	$\frac{\mu}{\sigma_K}$	$G_K + G_b - \rho \epsilon$
Energy Dissipation	ϵ	$\frac{\mu}{\sigma_{\epsilon}}$	$C1 \frac{\epsilon}{K} (G_k + G_b) - C2 \rho \frac{\epsilon^2}{K}$

2.7 The Previous Studies Concerned With the Problem of Thermal Pollution:

Some of the most related researches of thermal pollution in water bodies are presented below:

Daws (1990) presented an analytical and numerical analysis of the cooling process of hot water discharge from a steam power plant into a

rectangular pond .Steady state two-dimensional flow in the horizontal plane was assumed. The momentum and energy equations were solved numerically to determine the temperature distribution induced in the pond as a result of the cooling process. Results are obtained for four types of configurations. The flow pattern is found to be strongly affected by the type of configuration and aspect ratio. Moreover, the dissipation of energy is found to be strongly affected by the weather conditions.

Al-Challabi (1993) developed a two- dimensional numerical model for the simulation of the spread and mixing of thermally polluted water disposed into the flow. This model considers the effect of density difference between the pollutant density and the river water density. Finite difference up winding scheme is used to solve partial differential equation, which includes the momentum conservation, energy conservation and (K- ϵ) turbulence model. The results of this model are verified by integral heat balance analysis. The model was found to be sensitive to the variation in the river velocity and density. It was also found to be insensitive to the variation in wind speed .The results show that, river with a smooth-bed-velocity-distribution causes faster spreading of the rough-bed-velocity-distribution this will cause more vertical pollutant transport toward the river bottom leading to an earlier formation of uniform vertical distribution of the pollutant.

Li-Renyu and Righetto A.M (1998) presented unsteady state two dimensional models to simulate the velocity and temperature field in the estuary of the Yangtza River in Brazil. The computation is based on the advanced turbulence depth averaged two equation (K- ϵ) model.

Variations of bottom topography and water surface elevation are included. The distribution of velocity and temperature computed by turbulence model (K- ϵ) is compared with experimental results and field data. It was found that the simulation by using (K- ϵ) model can provide more details of flow fields and temperature distribution than that once obtained by using phenomenological algebraic for models of eddy viscosity and diffusivity.

Catirolgu .C and Yurteri .C (1998) presented a mathematical model to predict the long-term effects of once-through systems on local fish population. The fish life cycle model simulates different life stages of fish by using appropriate expressions represented by growth and mortality rates. The heart of the developed modeling approach is the prediction of plant caused reduction in total fish population by estimating recruitment to adult population with and without entertainment of ichthyoplakton and impingement of small fish. The model is applied to a local fish species, githead (*Sparus aurata*), for the causes of a proposed power plant in the Aegean region of Turkey. The simulation indicates that entertainment and impingement may lead to a population reduction of about 2% to 8% in the long run.

Bormans M. And T. Webster (1998) investigated the thermal stratification dynamics of the slow flowing rivers of the Marray-Darling Basin in Australia. In particular, the development and destruction of stratification in response to changes in the river flow and meteorological condition were described. The net surface heat flux and the river discharge are the two most important parameters that determine the stratification status of these rivers with little effect from direct wind mixing. Excellent agreement between measured and

simulated temperature profiles under a wide range of river discharge and meteorological forcing is obtained.

Reddy.G.s (1998) developed three-dimensional numerical model to predict the flow and pollutant transport induced by surface discharges in rivers, estuaries and seas. The model system consists of hydrodynamic, pollutant transport models, Conservation equations have been solved by finite difference techniques. The model offers many options on selecting the open boundary conditions at the user specified locations. Based on the above model properties, a software "fluidyn- Flow coast" has been developed with an attractive graphical menu-driver interface to define the various user inputs and display the various output results graphically. This software has been run and tested against the available field data for tide forcing, density induced and wind-forced flow in an estuary. The software has also been run for various cases and the output results, water level variations, velocity currents, temperature gradients, salinity variations and pollutant concentrations levels at different locations have been discussed graphically. The efficacy of the software has been discussed in detail in the prediction of flow and pollutant transport in estuaries and coastal waters. Finally, it can be concluded that, the software can predict the spread of pollutants, thermal gradients, salinity variations, transport of miscible pollutants and flow characteristics in rivers, estuaries and seas in a realistic way by taking into account all the relevant factors.

Kim and Seo (1999) developed a three-dimensional grid –based numerical model .To simulates the mixing of heated water which is

discharged from a submerged multi-port diffuser model. The discharge momentum flux is properly incorporated into the model using the concept of a numerical slot diffuser, along with multi-jet theory. Plane or line patch was introduced to impose discharge momentum flux which plays an important role in mixing of the thermal effluent in the near field. The laboratory Experimental work has been conducted to investigate mixing characteristics of the coflowing diffuser. A comparison of model simulations with laboratory experiments shows that the proposed model properly simulates the shapes of thermal plumes and the distributions of excess temperature. The proposed model properly simulates velocity induced by a coflowing diffuser in proximity to a shoreline boundary, and this is confirmed by Lee's experiments.

Kuang and Lee (2000) studied the effect of downstream control on the mixing of a plane turbulent heated jet discharging vertically into confined depth is studied using the buoyancy extended $k-\Sigma$ model. The steady two-dimensional turbulent flow, temperature and turbulence fields are computed using the finite volume method on a high resolution grid. In the absence of a specific downstream control, the numerical predictions demonstrate three generic flow patterns for different jet discharges and environmental parameters: i) a flow with circulation cells of alternate rotation for non-buoyant discharge; ii) a stable buoyant discharge with the mixed fluid leaving the vertical jet region in a surface warm water layer; and iii) an unstable buoyant discharge with flow recirculation and re-entrainment of heated water. The computed velocity and temperature fields agree well with the laboratory flow-visualization and temperature measurements of Jirka & Harleman (1979). Numerical prediction of stability categories is in

excellent agreement with experiments. For a given discharge and depth, it is found that the jet stability can be predicted regardless of downstream control provided that the channel length exceeds about $6H$. The effect of a strong downstream control close to the discharge primarily results in a flooded internal jump and the lowering of the interface level in the stratified counter flow region. Consistent with the detailed measurements of Andreopoulos, Praturi and Rodi (1986), the predictions show a clear reduction of the bulk dilution, although the effect of downstream control on the jet discharge stability is insignificant.

Joody (2001) developed one and two dimensions numerical model for the simulation of the spread and mixing of thermally polluted water disposed at the river surface released from the AL-Daura Power Station starting from the outfall up to 1000m downstream. The momentum and thermal energy equations govern the distribution of velocity and temperature in river. The two dimensional model also incorporates the $(k-\epsilon)$ model to calculate the distribution of turbulent viscosity. The two dimensional model also discusses two cases; the first case neglects the effect of vertical velocity distribution while the second case include it. Comparison of observed data on Feb 3, 2001 and July 27, 2001 with data computed by two dimensional model shows a good agreement with percentage error of 0.57% and 1.95% respectively. The model is devoted for single point source only.

Kadhem (2002) developed a two-dimensional numerical model for simulation and prediction of the spreading and mixing of thermally polluted water disposed at the river surface released from South-Baghdad

Power station starting from the outfall up to 500m down stream of the Tigris River. Momentum conservation equation and thermal energy equation are used to describe the distribution and diffusion of temperature. Furthermore the model incorporates the (K- ϵ) model to calculate the distribution of turbulent viscosity along the study reach. Alternating Direction Implicit-Explicit finite difference technique is used to solve the partial differential equation. Many parameter were tested such as total suspended solid (T.S.S), total dissolved solid (T.D.S), electrical conductivity (EC) and dissolved oxygen (DO). Results indicate a considerable effect of temperature change on these parameters. This model was also devoted for single heat point source only.

Nasir (2005) developed a two-dimensional numerical model to simulate the mixing and the spreading of thermally polluted water released from single-point source at the river water surface starting from the outfall reign. A finite difference method was used to solve the equations governing the phenomena of Single- Point source heat disposal which were the momentum conservation equation and thermal energy equation. the model also incorporates the (k- ϵ) to calculate the distribution of turbulent viscosity along the study reach. The solution was verified by a laboratory experimental work and by field data obtained from Al-Daura power station. Then this solution is extended for solving a similar problem with Multi – Points source heat disposal using the principle of super position. The single source model was found to be sensitive to the variations in river velocity, viscosity and density difference, the multi point source model was found to be sensitive to the distance between the point sources and the heated water temperature difference between sources.

In this research a two dimensional unsteady numerical model was developed to simulate the spreading and the mixing of the heat discharge at and beneath the river water surface as a single point source for three different discharge locations(surface and two-submerged discharge cases). The equations used are the momentum conservation equation and thermal energy equation .The model also incorporates the modified (k- ϵ) to calculate the distribution of turbulent viscosity along the study reach. The finite difference method was used to solve the equations governing the phenomena of single-point source heat disposal. The present model was verified by using a previous numerical model presented by Al-Chalabi, m. A. (1993), and by using the sensitivity analysis.

Chapter Three

Numerical Modeling

3.1 Introduction:

The thermal pollution problem is represented by a mathematical model consisting of a set of partial differential equations which take into account momentum, buoyancy diffusion, density stratification. These parameters will be presented in order to obtain an algebraic equation to be solved by a numerical solution. The proper boundary and initial conditions were used to complete the solution.

3.2 Problem Description:

The problem that is discussed in the present model represents the unsteady continuous discharge process of heated water disposed into a river flow in three different discharge cases from a single-point source, first, the heated water discharge on the river surface Fig.(3.1). The second case, where the heated water discharge as submerged single source lies below the river water surface (0.42 from the river depth) Fig.(3.2). While the third case, is heated water discharge as submerged single-point source lies below the river water surface (0.83 from the river depth) Fig. (3.3).

A theoretical rectangular river reach of length (L) and depth (D) was taken starting from the outfall region, and ending at a relatively large distance (L) downstream depending on the interest of the investigation.

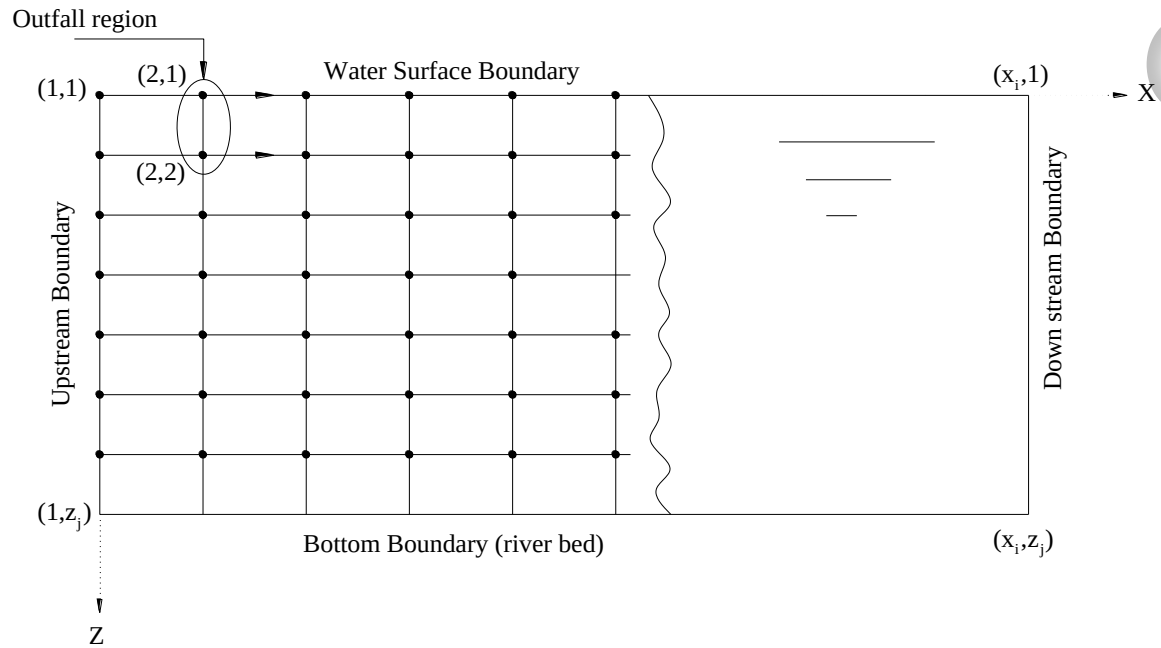


Fig. (3.1): River Reach Domain (surface discharge case)

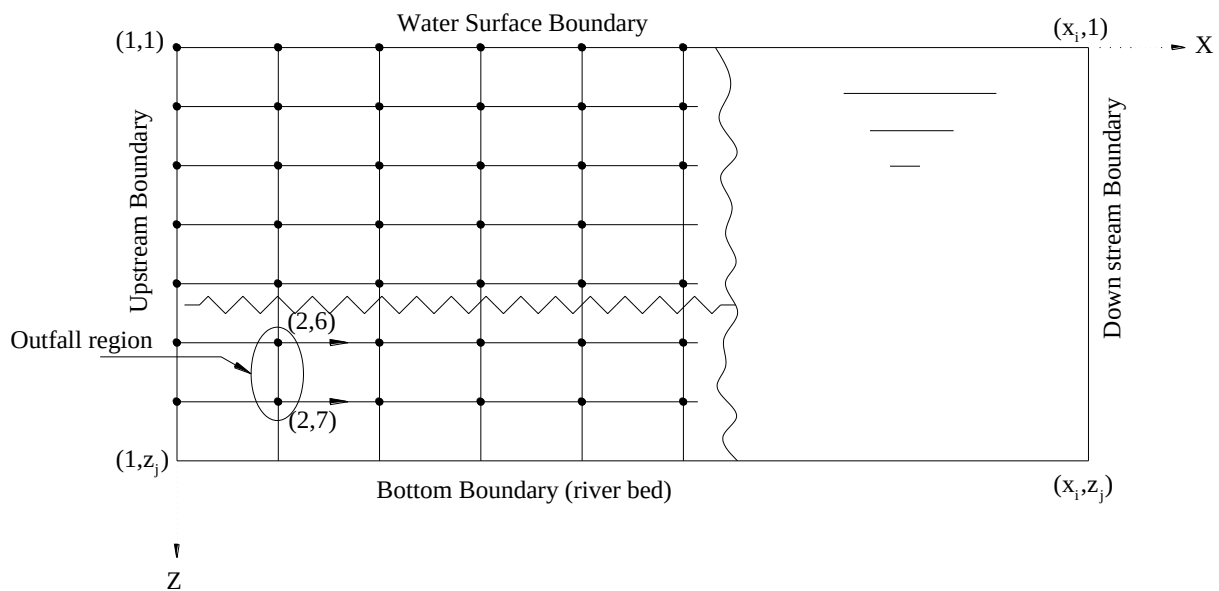


**Fig. (3.2): River Reach Domain (first case of submerged discharge)
(Outfall at 42% from the river depth)**

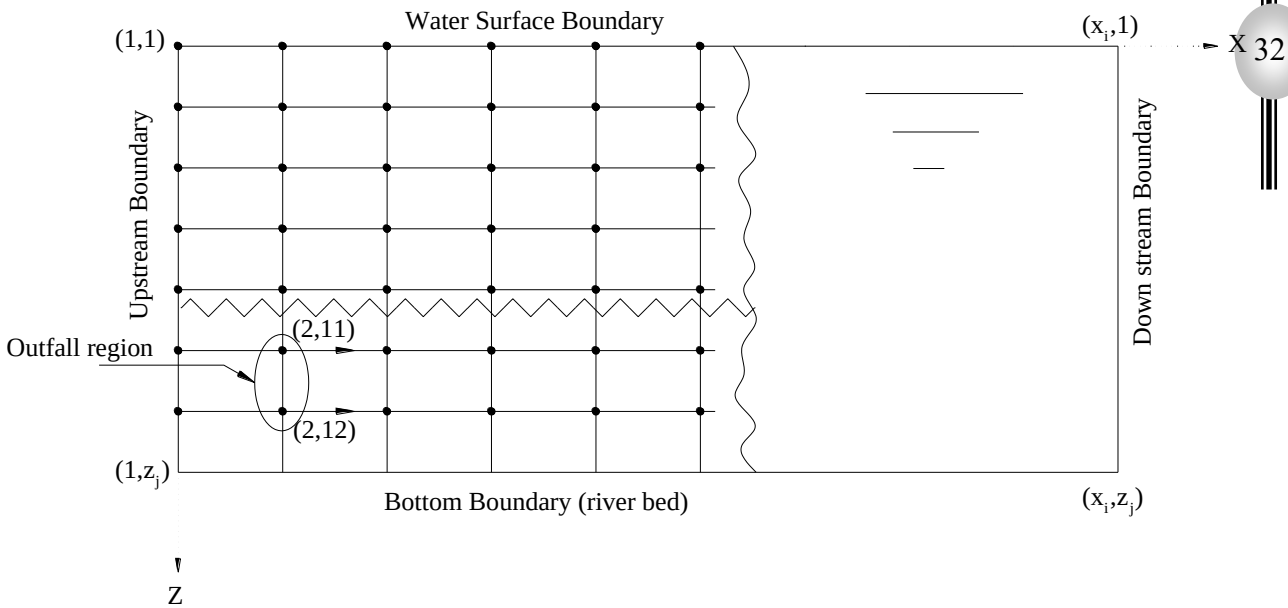


**Fig. (3.3): River Reach Domain (second case of submerged discharge)
(Outfall at 83% from the river depth)**

3.3 Initial and Boundary Conditions:

The initial and boundary conditions of the present problem are used to solve the differential equation numerically to determine the velocity and temperature distributions along the flow field.

3.3.1 Initial Conditions:-

1. In the thermal discharge, the flow domain was initialized with a temperature equal to the ambient water temperature in river except with in the outfall region where it is taken to be constant and equal to the temperature of the heated water at the outlet location.
2. The initial condition for the horizontal velocity component (u) is constant in the vertical plane, while the vertical velocity component (w) was set equal to zero at every point in the vertical plane.

3. Turbulent viscosity (μ), and (ε) dissipation rate of turbulence kinetic energy in the river are obtained from the following equations: (Babarutsi and chu, 1998):

$$\mu = 0.077 \rho U_f h \quad \dots 3.1$$

$$\varepsilon = S * g * u \quad \dots 3.2$$

Where: h = water depth (m) measured at z-axis.

S = water surface slope (m/m).

u = longitudinal velocity components (m/s).

U_f = friction velocity (m/s).

g = acceleration due to gravity (m/s²).

Using initial conditions of (μ) and (ε) as calculated from above equation is to find the initial conditions for the turbulence kinetic energy (K) in river from equation (2.31). At the outfall region, the initial condition for (K) and (ε) are obtained from the following equations (Babarutsi and chu, 1998)

$$\frac{K_d}{K_r} = \left(\frac{u_d}{u_r} \right)^2 \quad \dots 3.3$$

$$\frac{\varepsilon_d}{\varepsilon_r} = \left(\frac{u_d}{u_r} \right)^2 \quad \dots 3.4$$

The turbulent dynamic viscosity at the outfall region is obtained from equation (2.31).

3.3.2 Boundary Conditions:

The boundary conditions are shown in Figs. (3.4) (3.5) and (3.6) which can be represented as follows:-

- **At discharge region:**

$w, u, T, K,$ and ε equal to their initial condition at each discharge location.

- **At upstream boundary :**

where $X = 0$ and $0 < Z < D$ due to the assumption of no reverse flow, w, u, T, K and ε are constants and equal to their initial profiles.

- **At the down stream boundary:**

Where $X = L$ and $0 < Z < D$. The gradient for w, u, T, K, ε are assumed

equal to zero
$$\frac{\partial}{\partial X_i}(w, u, T, K, \varepsilon) = 0$$

- **At the surface boundary :**

Where: $0 < X < L$ and $Z = 0$ temperature at the image point (point located outside the water surface) is equal to the surface temperature. (Oberikamp and crow, 1996) and $P = 0, w = 0$. Gradient of horizontal velocity, turbulent kinetic energy (K) and dissipation rate (ε) are assumed equal to zero.

$$\frac{\partial}{\partial Z_j}(u, K, \varepsilon) = 0$$

- **At the bottom boundary:**

Where: $0 < X < L, Z = D$ and $w = 0$, the gradient of $u, T, K,$ and ε

are equal to zero
$$\frac{\partial}{\partial Z_j}(u, T, K, \varepsilon) = 0$$

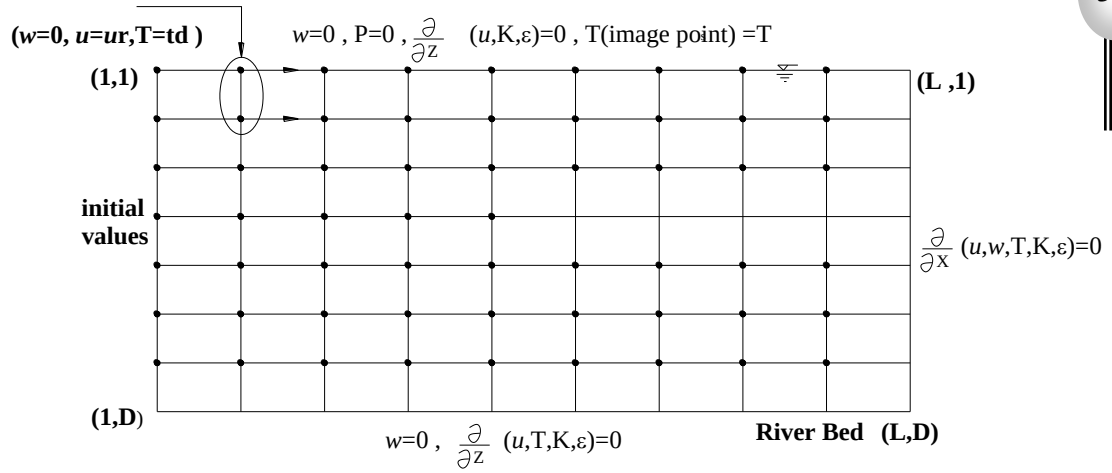


Fig. (3.4) : Boundary Conditions of The Problem (surface discharge)

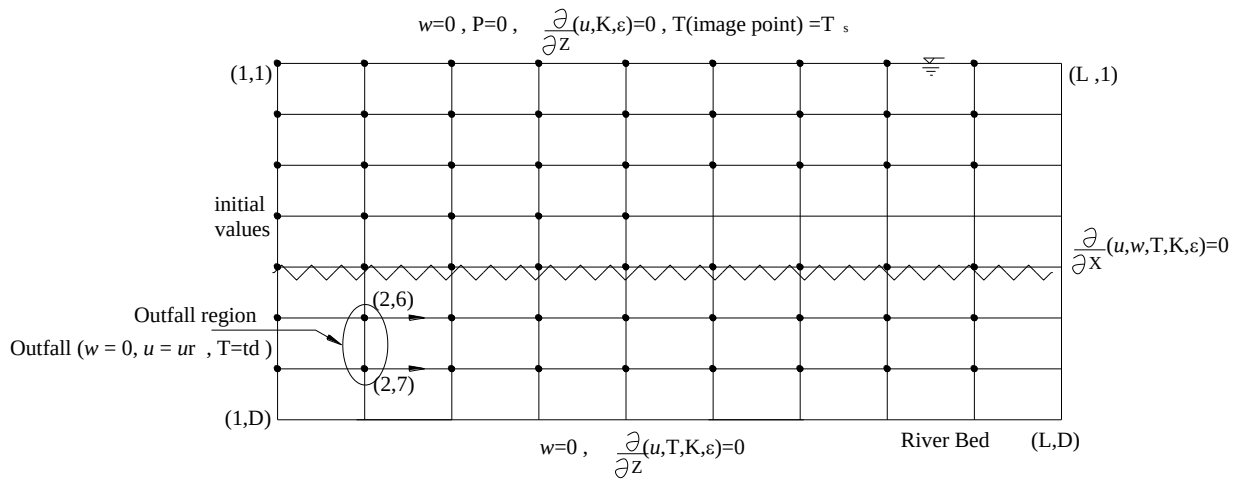


Fig. (3.5) : Boundary Conditions of the Problem (submerged discharge-first case)

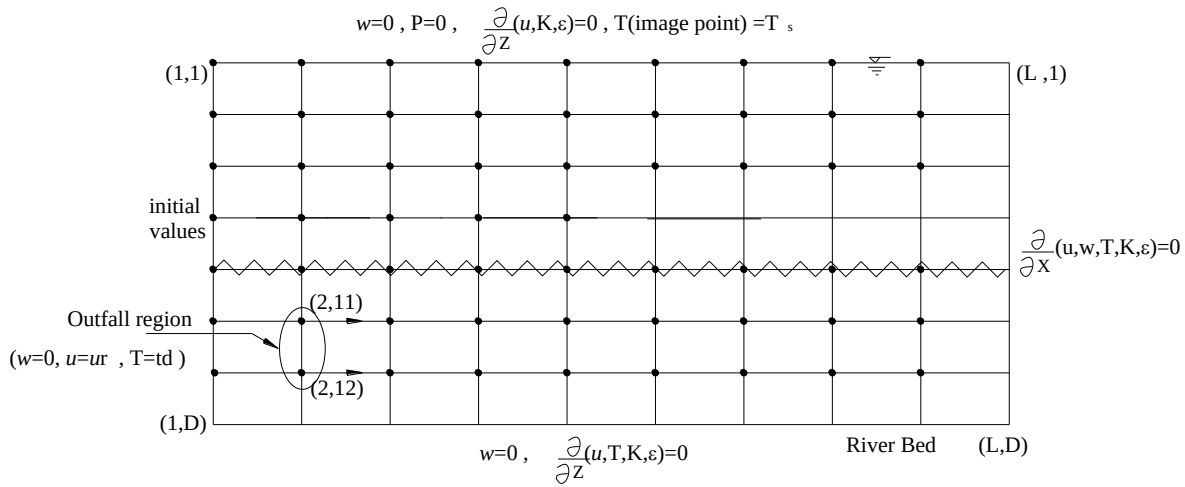


Fig. (3.6) : Boundary Conditions of the Problem (submerged discharge-second case)

3.4 Numerical Model Assumptions:

In order to simplify the analysis, the main differential equations were used with the following assumptions:

1. The river is a wide prismatic rectangular open channel with complete mixing in the transverse direction.
2. The flow is uniform ($\frac{\partial u_i}{\partial x_i} = 0$), and the heat lost or gain from the river water surface is negligible.
3. The fluid is incompressible which means $\frac{\partial \rho}{\partial X_i} = 0$.
4. The effects of wind shear on the velocity distribution are neglected, and the river surface is planer (no tides).

5. The heated water discharge is disposed with velocity equal to the river velocity at the outfall location.

6. The pressure is hydrostatic at every point in vertical direction and negligible

$\frac{\partial p}{\partial X_i}$ based on the above second assumption, $\frac{\partial p}{\partial X_i}$ can be replaced by

$(\rho g \frac{\partial h}{\partial X_i})$ (patankar and spalding ;1972,Rastogi and Rodi ;1978),where h is the

water depth, and because of constant water depth ; $\frac{\partial h}{\partial X_i}$ is equal to zero and

thus $\frac{\partial p}{\partial X_i}$ neglected.

7. The thermal discharge is constant at any time and it is disposed continuously in the direction of the river flow with only horizontal velocity component equal to the ambient river flow velocity.

8. In the equations of motion ,the horizontal and vertical velocity components (u) and (w) in the convective terms remain constants during a small time step, in order to be distinguished from the dependent (u) and (w) in the same equations, also the variables u, w, ρ , k , ε , μ and T remain constants during a small time step in the thermal energy and k- ε model equations .Other parameters such as ρ , P , μ and the empirical constants in the k- ε turbulence model $c_\mu, C_1, C_2, \sigma_k$, and σ_ε are not changed during calculations and remain constants during a small time step.

According to all assumptions mentioned previously the main simplified differential equations are as shown:

1. Vertical momentum equation:

$$\rho \frac{\partial w_j}{\partial t} + u_i \frac{\partial \rho w_j}{\partial X_i} + w_j \frac{\partial \rho w_j}{\partial Z_j} = \frac{\partial}{\partial X_i} \left(\mu \frac{\partial w_j}{\partial X_i} \right) + \frac{\partial}{\partial Z_j} \left(\mu \frac{\partial w_j}{\partial Z_j} \right) - \frac{\partial P}{\partial Z_j} + \rho g \quad \dots 3.5$$

2. Horizontal momentum equation:

$$\rho \frac{\partial u_i}{\partial t} + u_i \frac{\partial \rho u_i}{\partial X_i} + w_j \frac{\partial \rho u_i}{\partial Z_j} = \frac{\partial}{\partial X_i} \left(\mu \frac{\partial u_i}{\partial X_i} \right) + \frac{\partial}{\partial Z_j} \left(\mu \frac{\partial u_i}{\partial Z_j} \right) \quad \dots 3.6$$

3. Thermal energy equation:

$$\rho \frac{\partial T}{\partial t} + u_i \frac{\partial \rho T}{\partial X_i} + w_j \frac{\partial \rho T}{\partial Z_j} = \frac{\partial}{\partial X_i} \left[\frac{\mu}{\sigma} \frac{\partial T}{\partial X_i} \right] + \frac{\partial}{\partial Z_j} \left[\frac{\mu}{\sigma} \frac{\partial T}{\partial Z_j} \right] + ST \quad \dots 3.7$$

4. (K – ε) turbulence model equations:

$$A. \rho \frac{\partial K}{\partial t} + u_i \frac{\partial \rho K}{\partial X_i} + w_j \frac{\partial \rho K}{\partial Z_j} = \frac{\partial}{\partial X_i} \left[\frac{\mu}{\sigma_K} \frac{\partial K}{\partial X_i} \right] + \frac{\partial}{\partial Z_j} \left[\frac{\mu}{\sigma_K} \frac{\partial K}{\partial Z_j} \right] + Gk + Gb - \rho \epsilon \quad \dots$$

3.8

$$B. \rho \frac{\partial \epsilon}{\partial t} + u_i \frac{\partial \rho \epsilon}{\partial X_i} + w_j \frac{\partial \rho \epsilon}{\partial Z_j} = \frac{\partial}{\partial X_i} \left[\frac{\mu}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial X_i} \right] + \frac{\partial}{\partial Z_j} \left[\frac{\mu}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial Z_j} \right] + C_1 \frac{\epsilon}{K} (Gk + Gb) - C_2 \rho \frac{\epsilon^2}{K}$$

... 3.9

Where:

$$Gk = \mu \left[2 \left(\frac{\partial u_i}{\partial X_i} \right)^2 + 2 \left(\frac{\partial w_j}{\partial Z_j} \right)^2 + \left(\frac{\partial w_j}{\partial X_i} + \frac{\partial u_i}{\partial Z_j} \right)^2 \right] \quad \dots 3.10$$

$$Gb = -g \frac{\mu}{\rho \sigma_b} \frac{\partial \rho}{\partial Z_j} \quad \dots 3.11$$

Then the turbulent dynamic viscosity (μ) was obtained from Eq. (2.31).

3.5 Discretization Form of the Differential Equations:

Finite difference technique was used for the solution of the partial differential equations previously mentioned, which were presented in the general form (Eq. 2.33). The simultaneous equations are solved by Gauss-sidel point –by –point iteration method, the common features of this procedure are:

A. After transforming the partial differential equations from non – linear to linear equations the following terms are considered :

- In thermal energy-equation the turbulent Prandtle number (σ_r) is always constant during calculation.
- In (K - ϵ) turbulence model empirical constants c_μ , C_1 , C_2 , σ_k , σ_b and σ_ϵ are considered constant during calculations.

B. Transformation of the partial differential equation into linear algebraic differential equations by using the Alternating Direction Implicit–Explicit finite difference technique (ADI). In this formulation method the dependent variables (e.g. w , u , T , ε , K) at a grid point are connected to their values at the neighboring grid points, (the linear algebraic differential equation will be solved by Gauss – sidel point –by-point iteration method).The ADI method is based on a uniform rectangular grid as shown in Fig. (3.7).

The mesh spacing in the (X , Z) direction are ΔX and ΔZ respectively and the time is segmented into equal intervals (Δt) subscripts (i , j , and t) are associated with each mesh point.

The (ADI) method advances the solution from time level (t) to time level ($t + \Delta t$)

In two steps as shown in Fig. (3.8):

- The first step advances to the solution implicit in the Z – direction and explicit in the X -direction to intermediate level, usually termed ($t+0.5\Delta t$).
- The second step advances the solution implicit in the X – direction and explicit in the Z – direction to time level ($t+ \Delta t$).

The two steps are repeated till reaching the specified time of the study, where the temperature is printed at the final time step.

The above transformation uses "upwinding scheme" to approximate the convective term. The value of the quantity at the upwinding (upstream differencing) end of grid interval under consideration has been used as will be

described later. This scheme was found necessary to be employed in order to enhance a stability condition. Modification used by many researches to overcome the problem of unsuitability has been adopted to use the upwinding difference for the convection terms. (Shames; 1982)

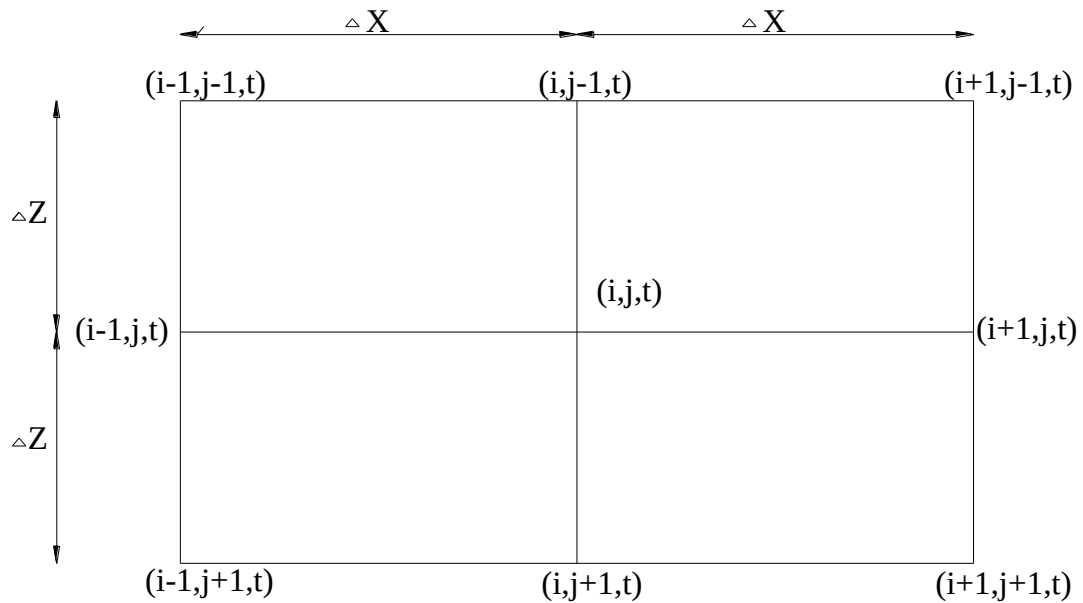


Fig. (3.7): Finite-Difference Grid

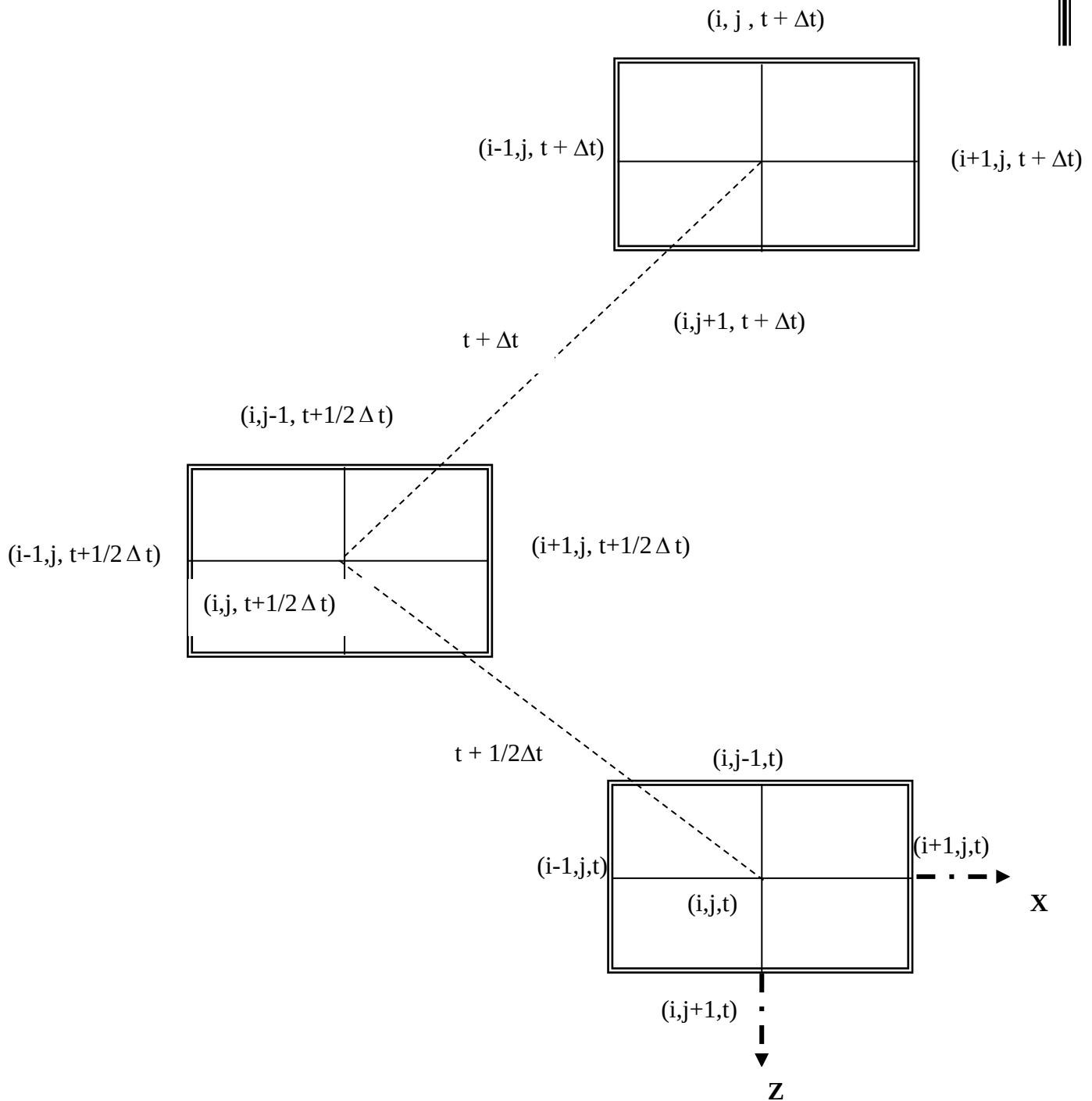


Fig. (3.8): Schematic Diagram of (ADI) Procedure

The main differential equations are discretized as following:

3.5.1 Vertical Momentum Equation

The vertical momentum conservation equ. (3.12) is converted to the finite – difference form in two steps using the (ADI) method as follows:

$$\rho \frac{\partial w}{\partial t} + u \frac{\partial \rho w}{\partial X} + w \frac{\partial \rho w}{\partial Z} = \frac{\partial}{\partial X} \left(\mu \frac{\partial w}{\partial X} \right) + \frac{\partial}{\partial Z} \left(\mu \frac{\partial w}{\partial Z} \right) - \frac{\partial P}{\partial Z} + \rho g$$

... (3.12)

In the first step, value of w at the time $(t + 0.5 \Delta t)$ are determined using the implicit scheme in the z – direction and explicit one in the x - direction. In the second step, values of w at the time $(t + \Delta t)$ are determined using the implicit scheme in the x – direction and explicit scheme in the z – direction (Smith, G.D .1978).

For the first half of the time step (i.e. $t \longrightarrow t + 0.5 \Delta t$)

$$\rho \frac{\partial w}{\partial t} = \rho_{(i,j)} \frac{w_{(i,j,t+1/2\Delta t)} - w_{(i,j,t)}}{0.5\Delta t} \quad \dots \quad (3.13a)$$

$$u \frac{\partial \rho w}{\partial X} = u_{(i,j,t)} \frac{\rho_{(i,j)} w_{(i,j,t)} - \rho_{(i-1,j)} w_{(i-1,j,t)}}{\Delta X} \quad \dots \quad (3.13b)$$

$$w \frac{\partial \rho w}{\partial Z} = w_{(i,j,t)} \frac{\rho_{(i,j)} w_{(i,j,t+1/2\Delta t)} - \rho_{(i,j-1)} w_{(i,j-1,t+1/2\Delta t)}}{\Delta Z}$$

... (3.13 c)

$$\begin{aligned}
\frac{\partial}{\partial X} \left[\mu \frac{\partial w}{\partial X} \right] &= \frac{\mu_{i+1,j} \frac{\partial w}{\partial X}(i+1, j, t) - \mu_{i,j} \frac{\partial w}{\partial X}(i, j, t)}{\Delta X} \\
&= \left(\frac{\mu_{(i+1,j)}}{\Delta X^2} \right) w_{(i+1,j,t)} - \left(\frac{\mu_{(i+1,j)} + \mu_{(i,j)}}{\Delta X^2} \right) w_{(i,j,t)} \\
&\quad + \left(\frac{\mu_{(i,j)}}{\Delta X^2} \right) w_{(i-1,j,t)}
\end{aligned}$$

... (3.13 d)

$$\begin{aligned}
\frac{\partial}{\partial Z} \left(\mu \frac{\partial w}{\partial Z} \right) &= \frac{\mu_{(i,j+1)} \frac{\partial w}{\partial Z}(i, j+1, t+1/2\Delta t) - \mu_{(i,j)} \frac{\partial w}{\partial Z}(i, j, t+1/2\Delta t)}{\Delta Z} \\
&= \left(\frac{\mu_{(i,j+1)}}{\Delta Z^2} \right) w_{(i,j+1,t+1/2\Delta t)} - \left(\frac{\mu_{(i,j+1)} + \mu_{(i,j)}}{\Delta Z^2} \right) w_{(i,j,t+1/2\Delta t)} \\
&\quad + \left(\frac{\mu_{(i,j)}}{\Delta Z^2} \right) w_{(i,j-1,t+1/2\Delta t)}
\end{aligned}$$

... (3.13 e)

$$\frac{\partial P}{\partial Z} = \frac{P_{(i,j+1)} - P_{(i,j-1)}}{2\Delta Z} \quad \dots (3.13f)$$

$$\rho_g = \rho_{i,j} * 9.806 \quad \dots (3.13g)$$

Substituting the discretization form of the terms in Eq. (3.12) and separating the unknown values at time $(t+1/2 \Delta t)$ on the left hand side from the known values at the time (t) on the right hand side yields:

$$\begin{aligned} & \left(-\frac{w_{i,j,t} \rho_{i,j-1}}{2\Delta Z} - \frac{\mu(i,j)}{2\Delta Z^2} \right) w_{i,j-1,t+1/2\Delta t} + \left(\frac{\rho_{i,j}}{\Delta t} + \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} \right. \\ & \left. + \frac{\mu(i,j+1)}{2\Delta Z^2} + \frac{\mu(i,j)}{2\Delta Z^2} \right) w_{i,j,t+1/2\Delta t} + \left(-\frac{\mu(i,j+1)}{2\Delta Z^2} \right) w_{i,j+1,t+1/2\Delta t} = \\ & \left(\frac{u_{i,j,t} \rho_{i-1,j}}{2\Delta X} + \frac{\mu(i,j)}{2\Delta X^2} \right) w_{i-1,j,t} + \left(\frac{\rho_{i,j}}{\Delta t} - \frac{u_{i,j,t} \rho_{i,j}}{2\Delta X} - \frac{\mu(i+1,j)}{2\Delta X^2} \right. \\ & \left. - \frac{\mu(i,j)}{2\Delta X^2} \right) w_{i,j,t} + \left(\frac{\mu(i+1,j)}{2\Delta X^2} \right) w_{i+1,j,t} - \left(\frac{P_{i,j+1} - P_{i,j-1}}{4\Delta Z} \right) + 4.903 \rho_{i,j} \end{aligned}$$

... (3.14)

Arranging these terms according to the general discretization equation yields:

$$w_{i,j,t+1/2\Delta t} = \frac{d_{i,j} + a_{i,j} w_{i,j-1,t+1/2\Delta t} + C_{i,j} w_{i,j+1,t+1/2\Delta t}}{b_{i,j}} \quad \dots (3.15)$$

Where:

$$a_{i,j} = \frac{w_{i,j,t} \rho_{i,j-1}}{2\Delta Z} + \frac{\mu(i,j)}{2\Delta Z^2} \quad \dots (3.15a)$$

$$b_{i,j} = \frac{\rho_{i,j}}{\Delta t} + \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} + \frac{\mu(i,j+1)}{2\Delta Z^2} + \frac{\mu(i,j)}{2\Delta Z^2} \quad \dots (3.15b)$$

$$c_{i,j} = \frac{\mu(i,j+1)}{2\Delta Z^2} \quad \dots (3.15c)$$

$$d_{i,j} = \left(\frac{u_{i,j,t} \rho_{i-1,j}}{2\Delta x} + \frac{\mu(i,j)}{2\Delta x^2} \right) w_{i-1,j,t} + \left(\frac{\rho_{i,j}}{\Delta t} - \frac{u_{i,j,t} \rho_{i,j}}{2\Delta x} \right. \\ \left. - \frac{\mu(i+1,j)}{2\Delta x^2} - \frac{\mu(i,j)}{2\Delta x^2} \right) w_{i,j,t} + \left(\frac{\mu(i+1,j)}{2\Delta x^2} \right) w_{i+1,j,t} - \left(\frac{P_{i,j+1} - P_{i,j-1}}{4\Delta Z} \right) \\ + 4.903 \rho_{i,j} \quad \dots (3.15 d)$$

For the second half of the time step (i.e. $t+0.5\Delta t \rightarrow t + \Delta t$)

$$\rho \frac{\partial w}{\partial t} = \rho(i,j) \frac{w_{(i,j,t+\Delta t)} - w_{(i,j,t+1/2\Delta t)}}{0.5\Delta t} \quad \dots (3.16a)$$

$$u \frac{\partial \rho w}{\partial X} = u_{(i,j,t)} \frac{\rho_{(i,j)} w_{(i,j,t+\Delta t)} - \rho_{(i-1,j)} w_{(i-1,j,t+\Delta t)}}{\Delta X} \quad \dots (3.16 b)$$

$$w \frac{\partial \rho w}{\partial Z} = w_{(i,j,t)} \frac{\rho_{(i,j)} w_{(i,j,t+1/2\Delta t)} - \rho_{(i,j-1)} w_{(i,j-1,t+1/2\Delta t)}}{\Delta Z}$$

.... (3.16c)

$$\frac{\partial}{\partial X} \left(\mu \frac{\partial w}{\partial X} \right) = \frac{\mu_{i+1,j} \frac{\partial w}{\partial X}(i+1, j, t + \Delta t) - \mu_{i,j} \frac{\partial w}{\partial X}(i, j, t + \Delta t)}{\Delta X}$$

$$= \left(\frac{\mu_{(i+1,j)}}{\Delta X^2} \right) w_{(i+1,j,t+\Delta t)} - \left(\frac{\mu_{(i+1,j)} + \mu_{(i,j)}}{\Delta X^2} \right) w_{(i,j,t+\Delta t)} \\ + \left(\frac{\mu_{(i,j)}}{\Delta X^2} \right) w_{(i-1,j,t+\Delta t)}$$

.... (3.16d)

$$\frac{\partial}{\partial Z} \left(\mu \frac{\partial w}{\partial Z} \right) = \left(\frac{\mu_{(i,j+1)}}{\Delta Z^2} \right) w_{(i,j+1,t+1/2\Delta t)} - \left(\frac{\mu_{(i,j+1)} + \mu_{(i,j)}}{\Delta Z^2} \right) w_{(i,j,t+1/2\Delta t)} \\ + \left(\frac{\mu_{(i,j)}}{\Delta Z^2} \right) w_{(i,j-1,t+1/2\Delta t)}$$

.... (3.16e)

$$\frac{\partial P}{\partial Z} = \frac{P_{i,j+1} - P_{i,j-1}}{4\Delta Z}$$

.... (3.16f)

$$\rho g = \rho_{i,j} * 9.806 \quad \text{.... (3.16g)}$$

Substituting in Eq. (3.12) and separating the unknown values at the time $(t+\Delta t)$ from the known values at the time $(t+1/2\Delta t)$ results:

$$\begin{aligned}
& \left(-\frac{u_{i,j,t} \rho_{i-1,j}}{2\Delta X} - \frac{\mu(i,j)}{2\Delta X^2} \right) w_{i-1,j,t+\Delta t} + \left(\frac{\rho_{i,j}}{\Delta t} + \frac{u_{i,j,t} \rho_{i,j}}{2\Delta X} + \frac{\mu(i+1,j)}{2\Delta X^2} \right. \\
& \left. + \frac{\mu(i,j)}{2\Delta X^2} \right) w_{i,j,t+\Delta t} + \left(-\frac{\mu(i+1,j)}{2\Delta X^2} \right) w_{i+1,j,t+\Delta t} = \\
& \left(\frac{w_{i,j,t} \rho_{i,j-1}}{2\Delta Z} + \frac{\mu(i,j)}{2\Delta Z^2} \right) w_{i,j-1,t+1/2\Delta t} + \left(\frac{\rho_{i,j}}{\Delta t} - \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} - \frac{\mu(i,j+1)}{2\Delta Z^2} \right. \\
& \left. - \frac{\mu(i,j)}{2\Delta Z^2} \right) w_{i,j,t+1/2\Delta t} + \left(\frac{\mu(i,j+1)}{2\Delta Z^2} \right) w_{i,j+1,t+1/2\Delta t} - \left(\frac{p_{i,j+1} - p_{i,j-1}}{4\Delta Z} \right) + 4.903 \rho_{i,j}
\end{aligned}$$

... (3.17)

Applying the finite-difference form of all terms in the equation (3.12) yields:

$$w_{i,j,t+\Delta t} = \frac{d_{i,j} + a_{i,j} w_{i-1,j,t+\Delta t} + c_{i,j} w_{i+1,j,t+\Delta t}}{b_{i,j}} \quad \dots (3.18)$$

Where:

$$a_{i,j} = \frac{u_{i,j,t} \rho_{i-1,j}}{2\Delta X} + \frac{\mu(i,j)}{2\Delta X^2} \quad \dots (3.19a)$$

$$b_{i,j} = \frac{\rho_{i,j}}{\Delta t} + \frac{u_{i,j,t} \rho_{i,j}}{2\Delta X} + \frac{\mu(i+1,j)}{2\Delta X^2} + \frac{\mu(i,j)}{2\Delta X^2} \quad \dots (3.19b)$$

$$c_{i,j} = \frac{\mu(i+1,j)}{2\Delta X^2} \quad \dots (3.19c)$$

$$d_{i,j} = R.H.S. \text{ of Eq. (3.17)} \quad \dots (3.19d)$$

The boundary conditions used in solving Eq. (3.12) are transformed to the finite-difference form as follows:

A. Discharge region boundary:

- Surface thermal boundary ($j=1$, and $j=2$):

$$w_{2,j}=0$$

- Submerged thermal boundary (first case) ($j=6$, and $j=7$):

$$w_{2,j}=0$$

- Submerged thermal boundary (second case) ($j=11$, and $j=12$):

$$w_{2,j}=0$$

B. At downstream boundary where: $X = L$ and $0 \leq Z \leq D$

$$\left. \frac{\partial w}{\partial X} \right)_{P_{X+1,j}} = 0 \text{ Is transformed to } w_{P_{X+2,j}} = w_{X,j}^P$$

C. Values of w at other boundaries are constant and equal to zero, therefore do not require a finite-difference transformation.

Modification used by many researchers to overcome the problem of instability has been adopted to use the up-winding difference for the convection terms (Shames; et .al. 1982).

In the present work the up-winding scheme (upstream differencing) was found to be necessary to employ due to non-unidirectional vertical velocity components, which leads to overflow problem in computer program due to the unstable solution for linear

simultaneous equations. Vertical velocity components are upward due to buoyancy for lower density zones and downward due to gravity for higher density zones.

Applying the up-winding scheme to the convective term $(w \frac{\partial \rho}{\partial Z})$ for the case where $w < 0$ (negative value of w means directed upward).

$$w \frac{\partial \rho}{\partial Z} = w_{i,j,t} \frac{\rho_{i,j+1} w_{i,j+1,t+1/2\Delta t} - \rho_{i,j} w_{i,j,t+1/2\Delta t}}{\Delta Z} \quad \dots(3.20)$$

Substituting in Eq.(3.12) instead of Eq.(3.13 c) in the case of $w < 0$, and arranging the following coefficients were obtained for the time interval $(t \rightarrow t+1/2 \Delta t)$.

$$a_{i,j} = \frac{\mu(i,j)}{2\Delta Z^2} \quad \dots (3.21a')$$

$$b_{i,j} = \frac{\rho_{i,j}}{\Delta t} - \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} + \frac{\mu(i,j+1)}{2\Delta Z^2} + \frac{\mu(i,j)}{2\Delta Z^2} \quad \dots (3.21b')$$

$$c_{i,j} = -\frac{w_{i,j,t} \rho_{i,j+1}}{2\Delta Z} + \frac{\mu(i,j+1)}{2\Delta Z^2} \quad \dots (3.21c')$$

$$\text{While } d_{i,j} \text{ will not change as equation (3.15d)} \quad \dots(3.21 d')$$

For the time interval $(t+1/2 \Delta t \rightarrow t+ \Delta t)$

By applying the up-winding scheme, all terms of equ. (3.12) is not change except the convective term $(w \frac{\partial \rho}{\partial Z})$ as shown:

$$w \frac{\partial \rho}{\partial Z} = w_{i,j,t} \frac{\rho_{i,j+1} w_{i,j+1,t+1/2\Delta t} - \rho_{i,j} w_{i,j,t+1/2\Delta t}}{\Delta Z} \quad \dots(3.22)$$

Substituting in Eq.(3.12) ,coefficients a,b, and c will not change same as equations (3.19 a , b , and c) while coefficient d becomes:

$$d_{i,j} = \left(\frac{\mu(i,j)}{2\Delta Z^2} \right) w_{i,j-1,t+1/2\Delta t} + \left(\frac{\rho_{i,j}}{\Delta t} + \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} - \frac{\mu(i,j+1)}{2\Delta Z^2} \right. \\ \left. - \frac{\mu(i,j)}{2\Delta Z^2} \right) w_{i,j,t+1/2\Delta t} + \left(-\frac{w_{i,j,t} \rho_{i,j+1}}{2\Delta Z} + \frac{\mu(i,j+1)}{2\Delta Z^2} \right) w_{i,j+1,t+1/2\Delta t} \\ - \left(\frac{p_{i,j+1} - p_{i,j-1}}{4\Delta Z} \right) + 4.903 \rho_{i,j} \quad \dots\dots(3.23d)$$

3.5.2. Finite-Difference Form of the Horizontal Momentum Equation

The horizontal momentum conservation is converted to the finite -difference form in two steps by using the same procedure applied to equ.(3.12).as follows:

$$\rho \frac{\partial u}{\partial t} + u \frac{\partial \rho u}{\partial X} + w \frac{\partial \rho u}{\partial Z} = \frac{\partial}{\partial X} \left[\mu \frac{\partial u}{\partial X} \right] + \frac{\partial}{\partial Z} \left[\mu \frac{\partial u}{\partial Z} \right] \quad \dots (3.24)$$

Arranging the terms in the general discretization equation as follows:

For the first half of the time step (i.e $t \rightarrow t + 1/2 \Delta t$):

$$u_{i,j,t+1/2\Delta t} = \frac{d_{i,j} + a_{i,j} u_{i,j-1,t+1/2\Delta t} + c_{i,j} u_{i,j+1,t+1/2\Delta t}}{b_{i,j}} \quad \dots (3.25)$$

In which $a_{i,j}$, $b_{i,j}$ and $c_{i,j}$ are the same as equs. (3.15 a, 3.15 b, and 3.15 c) respectively.

While

$$d_{i,j} = \left(\frac{u_{i,j,t} \rho_{i-1,j} + \frac{\mu(i,j)}{2\Delta x^2}}{2\Delta x} \right) u_{i-1,j,t} + \left(\frac{\rho_{i,j}}{\Delta t} - \frac{u_{i,j,t} \rho_{i,j}}{2\Delta x} \right. \\ \left. - \frac{\mu(i+1,j)}{2\Delta x^2} - \frac{\mu(i,j)}{2\Delta x^2} \right) u_{i,j,t} + \left(\frac{\mu(i+1,j)}{2\Delta x^2} \right) u_{i+1,j,t}$$

... (3.25d)

By applying the upwinding scheme, the finite-difference form of all equation terms will not change except the convective term ($w \frac{\partial \rho u}{\partial Z}$) which will be:

$$w \frac{\partial \rho u}{\partial Z} = w_{(i,j,t)} \frac{\rho_{(i,j+1)} u_{(i,j+1,t+1/2\Delta t)} - \rho_{(i,j)} u_{(i,j,t+1/2\Delta t)}}{\Delta Z}$$

... (3.26)

Substitute the above equation in equation (3.24) and arranging the equation, the coefficients $a_{i,j}$, $b_{i,j}$, $c_{i,j}$ and $d_{i,j}$ are the same as equs (3.21a', 3.21b', 3.21c' and 3.21'd) respectively.

For the second half of the time step ($t+1/2\Delta t \rightarrow t+\Delta t$):

Implicit in the x-direction for the time interval ($t+1/2\Delta t \rightarrow t+\Delta t$), and by applying the finite-difference form same as equation (3.12) to all terms in equation (3.24) and arranging these terms according to the general discretization form as :

$$u_{i,j,t+\Delta t} = \frac{d_{i,j} + a_{i,j} u_{i-1,j,t+\Delta t} + c_{i,j} u_{i+1,j,t+\Delta t}}{b_{i,j}} \quad \dots (3.27)$$

Where: $a_{i,j}$, $b_{i,j}$, and $c_{i,j}$ same as equs.(3.19a,3.19b,and 3.19 c)

While:

$$d_{i,j} = \left(\frac{w_{i,j,t} \rho_{i,j-1}}{2\Delta Z} + \frac{\mu(i,j)}{2\Delta Z^2} \right) u_{i,j-1,t+1/2\Delta t} + \left(\frac{\rho_{i,j}}{\Delta t} - \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} - \frac{\mu(i,j+1)}{2\Delta Z^2} \right. \\ \left. - \frac{\mu(i,j)}{2\Delta Z^2} \right) u_{i,j,t+1/2\Delta t} + \left(\frac{\mu(i,j+1)}{2\Delta Z^2} \right) u_{i,j+1,t+1/2\Delta t} \quad \dots(3.27d)$$

By applying the upwinding scheme, all terms in equation (3.24) will not change except the convective term $(w \frac{\partial \rho u}{\partial Z})$ will change and yields to:

$$w \frac{\partial \rho u}{\partial Z} = w_{i,j,t} \frac{\rho_{i,j+1} u_{i,j+1,t+1/2\Delta t} - \rho_{i,j} u_{i,j,t+1/2\Delta t}}{\Delta Z} \quad \dots (3.28)$$

By substituting in equation (3.24) and arranging the equation, the resulting coefficients $a_{i,j}$, $b_{i,j}$, and $c_{i,j}$ will not be changed while the coefficient $d_{i,j}$ becomes:

$$d_{i,j} = \left(\frac{\mu(i,j)}{2\Delta Z^2} \right) u_{i,j-1,t+1/2\Delta t} + \left(\frac{\rho_{i,j}}{\Delta t} + \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} - \frac{\mu(i,j+1)}{2\Delta Z^2} \right. \\ \left. - \frac{\mu(i,j)}{2\Delta Z^2} \right) u_{i,j,t+1/2\Delta t} + \left(-\frac{w_{i,j,t} \rho_{i,j+1}}{2\Delta Z} + \frac{\mu(i,j+1)}{2\Delta Z^2} \right) u_{i,j+1,t+1/2\Delta t} \quad \dots (3.28d)$$

The boundary conditions used in solving Eq. (3.24) are transformed to the finite- difference form as follows:

A. at discharge region boundary:

- Surface thermal discharge ($j=1$,and $j=2$):

$$u_{2,j} = u$$

- Submerged thermal discharge (first case) ($j=6$,and $j=7$):

$$u_{2,j} = u$$

- Submerged thermal discharge (second case) ($j=11$,and $j=12$):

$$u_{2,j} = u$$

B. At upstream boundary where: $x = 0$ and $0 \leq Z \leq D$ u is constant and equal to initial velocity distribution (uniform velocity distribution), therefore does Not require a finite –difference transformation.

C. At downstream boundary where: $x = L$ and $0 \leq Z \leq D$

$$\left. \frac{\partial u}{\partial X} \right) p_{X+1}, j = 0 \quad \text{Is transformed to } u) p_{x+2}, j = u) p_x, j$$

D. At surface boundary where: $0 \leq X \leq L, z = 0$

$$\left. \frac{\partial u}{\partial Z} \right) i, 1 = 0 \quad \text{Is transformed to } u) i, 0 = u) i, 2$$

E. At bottom boundary where: $0 \leq X \leq L$ and $Z = D$

$$\left. \frac{\partial u}{\partial Z} \right) i, pz + 1 = 0 \quad \text{Is transformed to } u) i, pz + 2 = u) i, pz$$

3.5.3. Finite –Difference Form of the Thermal-Energy Equation:

The thermal –energy equation is converted to finite difference form as follows:

$$\rho \frac{\partial T}{\partial t} + u \frac{\partial \rho T}{\partial X} + w \frac{\partial \rho T}{\partial Z} = \frac{\partial}{\partial X} \left[\frac{\mu}{\sigma_T} \frac{\partial T}{\partial X} \right] + \frac{\partial}{\partial Z} \left[\frac{\mu}{\sigma_T} \frac{\partial T}{\partial Z} \right] + ST \quad \dots (3.29)$$

The vertical and the horizontal components resulted from solving the equations of motion, are used to solve the thermal-energy equation to obtain the temperature distribution in a river due to the thermal discharges from single –point source. Using the surface boundary conditions presented in sec. (3.3.2), where the heat flux at the water surface $q = \alpha (T_s - T_e)$, the thermal –energy equation (Eq3.29) at the water surface can be represented as follows:

$$\rho \frac{\partial T}{\partial t} + U \frac{\partial \rho T}{\partial X} + W \frac{\partial \rho T}{\partial Z} = \frac{\partial}{\partial X} \left[\frac{\mu}{\sigma_T} \frac{\partial T}{\partial X} \right] + \frac{\partial}{\partial Z} \left[\frac{\mu}{\sigma_T} \frac{\partial T}{\partial Z} \right] - \frac{q}{c_w h} \quad \dots (3.30)$$

By setting the term ST equal to $(-\frac{q}{c_w h})$, where c_w is the specific heat of water and (h) represent the depth of flow. In this model the heat flux from the water surface was neglected therefore ST is equal to zero and Eq.(3.29) becomes:

$$\rho \frac{\partial T}{\partial t} + u \frac{\partial \rho T}{\partial X} + w \frac{\partial \rho T}{\partial Z} = \frac{\partial}{\partial X} \left[\frac{\mu}{\sigma_T} \frac{\partial T}{\partial X} \right] + \frac{\partial}{\partial Z} \left[\frac{\mu}{\sigma_T} \frac{\partial T}{\partial Z} \right] \quad \dots (3.31)$$

Equation (3.31) is converted to the finite-difference form using the implicit scheme in the z-direction and explicit scheme in the x-direction

For the time interval $(t \rightarrow t+1/2\Delta t)$:

Terms $\rho \frac{\partial T}{\partial t}$, $u \frac{\partial \rho T}{\partial X}$, and $w \frac{\partial \rho T}{\partial Z}$ are discretized using the same procedure which

has been applied for the terms $\rho \frac{\partial w}{\partial t}$, $u \frac{\partial \rho w}{\partial Z}$, $w \frac{\partial \rho w}{\partial Z}$ in Eq.(3.12). Other terms are

discretized as follows:

$$\begin{aligned} \frac{1}{\sigma_T} \frac{\partial}{\partial X} \left(\mu \frac{\partial T}{\partial X} \right) &= \frac{1}{\sigma_T} \frac{\left(\mu \right)_{i+1,j} \frac{\partial T}{\partial X} \Big|_{i+1,j,t} - \left(\mu \right)_{i,j} \frac{\partial T}{\partial X} \Big|_{i,j,t}}{\Delta X} \quad \dots (3.32) \\ &= \left(\frac{\mu(i+1,j)}{\sigma_T \Delta X^2} \right) T_{i+1,j,t} - \left(\frac{\mu(i+1,j) + \mu(i,j)}{\sigma_T \Delta X^2} \right) T_{i,j,t} + \left(\frac{\mu(i,j)}{\sigma_T \Delta X^2} \right) T_{i-1,j,t} \end{aligned}$$

$$\begin{aligned} \frac{1}{\sigma_T} \frac{\partial}{\partial Z} \left(\mu \frac{\partial T}{\partial Z} \right) &= \left(\frac{\mu(i,j+1)}{\sigma_T \Delta Z^2} \right) T_{i,j+1,t+1/2\Delta t} - \left(\frac{\mu(i,j+1) + \mu(i,j)}{\sigma_T \Delta Z^2} \right) T_{i,j,t+1/2\Delta t} \\ &\quad + \left(\frac{\mu(i,j)}{\sigma_T \Delta Z^2} \right) T_{i,j-1,t+1/2\Delta t} \end{aligned}$$

Substituting in Eq.(3.31), arranging the terms to fit the general

discretization equation yields:

$$T_{i,j,t+1/2\Delta t} = \frac{d_{i,j} + a_{i,j} T_{i,j-1,t+1/2\Delta t} + c_{i,j} T_{i,j+1,t+1/2\Delta t}}{b_{i,j}} \quad \dots (3.33)$$

The following coefficients are obtained:

$$a_{i,j} = \frac{w_{i,j,t} \rho_{i,j-1}}{2\Delta Z} + \frac{\mu(i,j)}{2\sigma_T \Delta Z^2} \quad \dots (3.33a)$$

$$b_{i,j} = \frac{\rho_{i,j}}{\Delta t} + \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} + \frac{\mu(i,j+1)}{2\sigma_T \Delta Z^2} + \frac{\mu(i,j)}{2\sigma_T \Delta Z^2} \quad \dots (3.33b)$$

$$c_{i,j} = \frac{\mu(i,j+1)}{2\sigma_T \Delta Z^2} \quad \dots (3.33c)$$

$$d_{i,j} = \left(\frac{u_{i,j,t} \rho_{i-1,j}}{2\Delta x} + \frac{\mu(i,j)}{2\Delta x^2} \right) T_{i-1,j,t} + \left(\frac{\rho_{i,j}}{\Delta t} - \frac{u_{i,j,t} \rho_{i,j}}{2\Delta x} \right. \\ \left. - \frac{\mu(i+1,j)}{2\Delta x^2} - \frac{\mu(i,j)}{2\Delta x^2} \right) T_{i,j,t} + \left(\frac{\mu(i+1,j)}{2\Delta x^2} \right) T_{i+1,j,t} \quad \dots (3.33 d)$$

Applying the upwinding scheme to the convective term ($w \frac{\partial \rho T}{\partial Z}$), and rearranging the terms in the equation, the coefficients a,b,c, and d become:

$$a_{i,j} = \frac{\mu(i,j)}{2\sigma_T \Delta Z^2} \quad \dots (3.33a')$$

$$b_{i,j} = \frac{\rho_{i,j}}{\Delta t} - \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} + \frac{\mu(i,j+1)}{2\sigma_T \Delta Z^2} + \frac{\mu(i,j)}{2\sigma_T \Delta Z^2} \quad \dots (3.33b')$$

$$c_{i,j} = -\frac{w_{i,j,t} \rho_{i,j+1}}{2\Delta Z} + \frac{\mu(i,j+1)}{2\sigma_T \Delta Z^2} \quad \dots (3.33c')$$

While $d_{i,j}$ will not change same as equation (3.33d)

For the time interval ($t+1/2\Delta t \rightarrow t+\Delta t$):

Where Eq. (3.31) is solved implicitly in the x-direction, the resulting equation fit the general discretization equation form type as follows:

$$T_{i,j,t+\Delta t} = \frac{d_{i,j} + a_{i,j} T_{i-1,j,t+\Delta t} + c_{i,j} T_{i+1,j+1,t+\Delta t}}{b_{i,j}} \quad \dots (3.34)$$

Where:

$$a_{i,j} = \frac{u_{i,j,t} \rho_{i-1,j}}{2\Delta X} + \frac{\mu(i,j)}{2\sigma_T \Delta X^2} \quad \dots (3.34a)$$

$$b_{i,j} = \frac{\rho_{i,j}}{\Delta t} + \frac{u_{i,j,t} \rho_{i,j}}{2\Delta X} + \frac{\mu(i+1,j)}{2\sigma_T \Delta X^2} + \frac{\mu(i,j)}{2\sigma_T \Delta X^2} \quad \dots (3.34b)$$

$$c_{i,j} = \frac{\mu(i+1,j)}{2\sigma_T \Delta X^2} \quad \dots (3.34c)$$

$$d_{i,j} = \left(\frac{w_{i,j,t} \rho_{i,j-1}}{2\Delta Z} + \frac{\mu(i,j)}{2\sigma_T \Delta Z^2} \right) T_{i,j-1,t+1/2\Delta t} + \left(\frac{\rho_{i,j}}{\Delta t} - \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} - \frac{\mu(i,j+1)}{2\sigma_T \Delta Z^2} - \frac{\mu(i,j)}{2\sigma_T \Delta Z^2} \right) T_{i,j,t+1/2\Delta t} + \left(\frac{\mu(i,j+1)}{2\sigma_T \Delta Z^2} \right) T_{i,j+1,t+1/2\Delta t}$$

... (3.34d)

Applying the up winding scheme, the finite-difference form of all terms will

not change except the convective term $(w \frac{\partial \rho T}{\partial Z})$ will change and become:

$$w \frac{\partial \rho T}{\partial Z} = w_{(i,j,t)} \frac{\rho_{(i,j+1)} T_{(i,j+1,t+1/2\Delta t)} - \rho_{(i,j)} T_{(i,j,t+1/2\Delta t)}}{\Delta Z} \quad \dots (3.35)$$

By applying in equation (3.31) and arranging the equation terms, the coefficients

$a_{i,j}$, $b_{i,j}$, $c_{i,j}$ and $d_{i,j}$ are same as equs. (3.34a, 3.34b, and 3.34c)

While coefficient $d_{i,j}$ becomes:

$$d_{i,j} = \left(\frac{\mu(i,j)}{2\sigma_T \Delta Z^2} \right) T_{i,j-1,t+1/2\Delta t} + \left(\frac{\rho_{i,j}}{\Delta t} + \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} - \frac{\mu(i,j+1)}{2\sigma_T \Delta Z^2} - \frac{\mu(i,j)}{2\sigma_T \Delta Z^2} \right) T_{i,j,t+1/2\Delta t} \\ + \left(-\frac{w_{i,j,t} \rho_{i,j+1}}{2\Delta Z} + \frac{\mu(i,j+1)}{2\sigma_T \Delta Z^2} \right) T_{i,j+1,t+1/2\Delta t} \quad \dots (3.34d')$$

The boundary condition transformation to finite-difference form for the thermal-energy equation can be set as following:

A. at discharge region boundary:

- Surface thermal discharge ($j=1$,and $j=2$)

$$T_{2,j} = td$$

- Submerged thermal discharge (first case) ($j=6$,and $j=7$)

$$T_{2,j} = td$$

- Submerged thermal discharge (second case) ($j=11$,and $j=12$)

$$T_{2,j} = td$$

B. At upstream boundary where: $x = 0$ and $0 \leq Z \leq D$ temperature is constant and equal to initial value in the undisturbed river, therefore does not require a finite –difference transformation.

C. At surface boundary where: $0 \leq X \leq L$ and $Z = 0$

$$\left. \frac{\partial T}{\partial Z} \right|_{i,1=0} \quad T)_{i,j-1} = T)_{i,j} \quad \text{is transformed to } T)_{i,0} = T)_{i,1}$$

D .At downstream boundary where: $x = L$ and $0 \leq Z \leq D$

$$\left. \frac{\partial T}{\partial X} \right|_{px+1,j=0} \quad \text{Is transformed to } T)_{px+2,j} = T)_{px,j}$$

E .At bottom boundary where: $0 \leq X \leq L$ and $Z = D$

$$\left. \frac{\partial T}{\partial Z} \right|_{i,pz+1=0} \quad \text{Is transformed to } T)_{i,pz+2} = T)_{i,pz}$$

3.5.4. Finite –Difference Form of the Modified (K – ε) Turbulence Model:

In order to determine the distribution of turbulent dynamic viscosity /turbulent diffusivity, a turbulence model is required as described previously in chapter two. The resulting vertical and horizontal velocity components from the solution of momentum conservation equations are used in the solution of the (K – ε) turbulence model.

3.5.4.1 Finite –Difference Form of the K -Equation:

$$\rho \frac{\partial K}{\partial t} + u \frac{\partial \rho K}{\partial X} + w \frac{\partial \rho K}{\partial Z} = \frac{\partial}{\partial X} \left[\frac{\mu}{\sigma_K} \frac{\partial K}{\partial X} \right] + \frac{\partial}{\partial Z} \left[\frac{\mu}{\sigma_K} \frac{\partial K}{\partial Z} \right] + Gk + Gb - \rho \epsilon \quad \dots (3.36)$$

Where:

$$GK = \mu \left[2 \left(\frac{\partial u}{\partial X} \right)^2 + 2 \left(\frac{\partial w}{\partial Z} \right)^2 + \left(\frac{\partial w}{\partial X} + \frac{\partial u}{\partial Z} \right)^2 \right] \quad \dots (3.37)$$

$$Gb = -g \frac{\mu}{\rho \sigma_b} \frac{\partial \rho}{\partial Z} \quad \dots (3.38)$$

For the time interval ($t \rightarrow t+1/2\Delta t$):

Where the equation (3.36) is solved Implicit in z-direction, and explicit in x-direction.

The finite-difference form of the turbulent kinetic energy production term (GK) is:

$$GK = \mu(i, j) \left[2 \left(\frac{u_{i,j,t} - u_{i-1,j,t}}{\Delta X} \right)^2 + 2 \left(\frac{w_{i,j,t} - w_{i,j-1,t}}{\Delta Z} \right)^2 + \left(\frac{w_{i,j,t} - w_{i-1,j,t}}{\Delta X} + \frac{u_{i,j,t} - u_{i,j-1,t}}{\Delta Z} \right)^2 \right] \quad \dots (3.39)$$

The finite-difference form of the buoyancy energy production term (Gb) is

$$Gb = g \left(\frac{\mu(i, j)}{\rho(i, j) \sigma_b} \left(\frac{\rho_{i,j} - \rho_{i,j-1}}{\Delta Z} \right) \right) \quad \dots (3.40)$$

Where σ_b is equal to (0.5)

g = acceleration due to gravity = 9.81 m/s^2

By applying the finite-difference form of in equation (3.36) and arranging the equation terms to fit the general discretization equation yields:

$$K_{i,j,t+1/2\Delta t} = \frac{d_{i,j} + a_{i,j} K_{i,j-1,t+1/2\Delta t} + c_{i,j} K_{i,j+1,t+1/2\Delta t}}{b_{i,j}}$$

... (3.41)

Where:

Coefficients $a_{i,j}$, $b_{i,j}$, and $c_{i,j}$ are not changed same as equations (3.33 a, b, c)

respectively except coefficient σ_T is replaced by σ_k . While coefficient $d_{i,j}$

becomes:

$$d_{i,j} = \left(\frac{u_{i,j,t} \rho_{i-1,j}}{2\Delta X} + \frac{\mu(i,j)}{2\sigma k \Delta X^2} \right) K_{i-1,j,t} + \left(\frac{\rho_{i,j}}{\Delta t} - \frac{u_{i,j,t} \rho_{i,j}}{2\Delta X} - \frac{\mu(i+1,j)}{2\sigma k \Delta X^2} - \frac{\mu(i,j)}{2\sigma k \Delta X^2} \right) K_{i,j,t} + \left(\frac{\mu(i+1,j)}{2\sigma k \Delta X^2} \right) K_{i+1,j,t} + \mu(i,j) \left[\left(\frac{u_{i,j,t} - u_{i-1,j,t}}{\Delta X} \right)^2 + \left(\frac{w_{i,j,t} - w_{i,j-1,t}}{\Delta Z} \right)^2 + \frac{1}{2} \left(\frac{w_{i,j,t} - w_{i-1,j,t}}{\Delta X} + \frac{u_{i,j,t} - u_{i,j-1,t}}{\Delta Z} \right)^2 \right] + (-9.806 * \frac{\mu(i,j)}{\rho(i,j)\sigma_b} \left(\frac{\rho_{i,j} - \rho_{i,j-1}}{2\Delta Z} \right) - \frac{1}{2} \rho_{i,j} \varepsilon_{i,j,t})$$

.... (3.41 d)

Applying the upwinding scheme to the convective term ($w \frac{\partial \rho K}{\partial Z}$) in the case of

$w < 0$, coefficients $a_{i,j}$, $b_{i,j}$, and $c_{i,j}$ are similar to those in Eqs.(3.33 a',b',c')

Except σ_T is replaced by σ_k . The coefficient $d_{i,j}$ same as equation (3.41d).

For the time Interval ($t+1/2\Delta t \rightarrow t+\Delta t$):

Implicit in the x-direction and explicit in the z- direction, then applying the finite-difference form in equation (3.36) and arranging the equation terms to fit the general discretization equation yields:

$$K_{i,j,t+\Delta t} = \frac{d_{i,j} + a_{i,j} K_{i-1,j,t+\Delta t} + c_{i,j} K_{i+1,j+1,t+\Delta t}}{b_{i,j}} \quad \dots (3.42)$$

Where:

Coefficients $a_{i,j}$, $b_{i,j}$, and $c_{i,j}$ are similar to those in Eqs.(3.34 a, b , and c) except

σ_r is replaced by σk and coefficient $d_{i,j}$ becomes:

$$\begin{aligned} d_{i,j} = & \left(\frac{w_{i,j,t} \rho_{i,j-1}}{2\Delta Z} + \frac{\mu(i,j)}{2\sigma k \Delta Z^2} \right) K_{i,j-1,t+1/2\Delta t} + \left(\frac{\rho_{i,j}}{\Delta t} - \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} - \frac{\mu(i,j+1)}{2\sigma k \Delta Z^2} - \right. \\ & \left. \frac{\mu(i,j)}{2\sigma k \Delta Z^2} \right) K_{i,j,t+1/2\Delta t} + \left(\frac{\mu(i,j+1)}{2\sigma k \Delta Z^2} \right) K_{i,j+1,t+1/2\Delta t} + \mu(i,j) \left[\left(\frac{u_{i,j,t} - u_{i-1,j,t}}{\Delta X} \right)^2 + \left(\frac{w_{i,j,t} - w_{i,j-1,t}}{\Delta Z} \right)^2 \right. \\ & \left. + \frac{1}{2} \left(\frac{w_{i,j,t} - w_{i-1,j,t}}{\Delta X} + \frac{u_{i,j,t} - u_{i,j-1,t}}{\Delta Z} \right)^2 \right] + (-9.806 * \frac{\mu(i,j)}{\rho(i,j)\sigma_k} \left(\frac{\rho_{i,j} - \rho_{i,j-1}}{2\Delta Z} \right) - \frac{1}{2} \rho_{i,j} \varepsilon_{i,j,t} \\ & \dots (3.42 d) \end{aligned}$$

Applying the upwinding scheme to the convective term ($w \frac{\partial \rho K}{\partial Z}$) in case of $w < 0$, as

follows:

$$w \frac{\partial \rho K}{\partial Z} = w_{(i,j,t)} \frac{\rho_{(i,j+1)} K_{(i,j+1,t+1/2\Delta t)} - \rho_{(i,j)} K_{(i,j,t+1/2\Delta t)}}{\Delta Z} \quad \dots (3.43)$$

By replacing the equation (3.43) in the (3.41) equation and arranging the terms, the coefficients $a_{i,j}$, $b_{i,j}$, and $c_{i,j}$ will be same as equations (3.34 a, b, c) respectively except σT will replace by σ_k , while coefficient $d_{i,j}$ becomes:

$$\begin{aligned}
 d_{i,j} = & \left(\frac{\mu(i,j)}{2\sigma_k \Delta Z^2} \right) K_{i,j-1,t+1/2\Delta t} + \left(\frac{\rho_{i,j}}{\Delta t} + \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} - \frac{\mu(i,j+1)}{2\sigma_k \Delta Z^2} \right. \\
 & \left. - \frac{\mu(i,j)}{2\sigma_k \Delta Z^2} \right) K_{i,j,t+1/2\Delta t} + \left(-\frac{w_{i,j,t} \rho_{i,j+1}}{2\Delta Z} + \frac{\mu(i,j+1)}{2\sigma_k \Delta Z^2} \right) K_{i,j+1,t+1/2\Delta t} \\
 & + \frac{1}{2} \mu(i,j) \left[2 \left(\frac{u_{i,j,t} - u_{i-1,j,t}}{\Delta X} \right)^2 + 2 \left(\frac{w_{i,j,t} - w_{i,j-1,t}}{\Delta Z} \right)^2 + \right. \\
 & \left. \left(\frac{w_{i,j,t} - w_{i-1,j,t}}{\Delta X} + \frac{u_{i,j,t} - u_{i,j-1,t}}{\Delta Z} \right)^2 \right] - \frac{1}{2} \varepsilon_{i,j} \rho_{i,j} \frac{(\varepsilon_{i,j,t})^2}{k_{i,j,t}} + (-9.806 * \frac{\mu(i,j)}{\rho_{i,j} \sigma_b} \left(\frac{\rho_{i,j} - \rho_{i,j-1}}{2\Delta Z} \right))
 \end{aligned}$$

.... (3.42d')

3.5.4.2 Finite –Difference Form of the ε -Equation:

$$\rho \frac{\partial \varepsilon}{\partial t} + u \frac{\partial \rho \varepsilon}{\partial X} + w \frac{\partial \rho \varepsilon}{\partial Z} = \frac{\partial}{\partial X} \left[\frac{\mu}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial X} \right] + \frac{\partial}{\partial Z} \left[\frac{\mu}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial Z} \right] + C_1 \frac{\varepsilon}{K} (Gk + Gb) - C_2 \rho \frac{\varepsilon^2}{K}$$

... (3.44)

Implicit in z-direction for the time interval ($t \rightarrow t+1/2\Delta t$) and explicit in x-direction, after substitution the finite-difference form of all terms in equation (3.44), and arranging the terms to the general discretization equation yields:

$$\varepsilon_{i,j,t+1/2\Delta t} = \frac{d_{i,j} + a_{i,j} \varepsilon_{i,j-1,t+1/2\Delta t} + C_{i,j} \varepsilon_{i,j+1,t+1/2\Delta t}}{b_{i,j}} \quad \dots (3.45)$$

Where:

Coefficients $a_{i,j}$, $b_{i,j}$, $c_{i,j}$ are similar to those in Eqs.(3.33 a ,b ,and c)

respectively except σ_T is replaced by σ_ε while coefficient $d_{i,j}$ becomes:

$$\begin{aligned}
 d_{i,j} = & \left(\frac{u_{i,j,t} \rho_{i-1,j}}{2\Delta X} + \frac{\mu(i,j)}{2\sigma_\varepsilon \Delta X^2} \right) \varepsilon_{i-1,j,t} + \left(\frac{\rho_{i,j}}{\Delta t} - \frac{u_{i,j,t} \rho_{i,j}}{2\Delta X} - \frac{\mu(i+1,j)}{2\sigma_\varepsilon \Delta X^2} - \right. \\
 & \left. \frac{\mu(i,j)}{2\sigma_\varepsilon \Delta X^2} \right) \varepsilon_{i,j,t} + \left(\frac{\mu(i+1,j)}{2\sigma_\varepsilon \Delta X^2} \right) K_{i+1,j,t} + \mu(i,j) c1 \frac{\varepsilon_{i,j,t}}{K_{i,j,t}} \left[\left(\frac{u_{i,j,t} - u_{i-1,j,t}}{\Delta X} \right)^2 + \left(\frac{w_{i,j,t} - w_{i,j-1,t}}{\Delta Z} \right)^2 \right. \\
 & \left. + \frac{1}{2} \left(\frac{w_{i,j,t} - w_{i-1,j,t}}{\Delta X} + \frac{u_{i,j,t} - u_{i,j-1,t}}{\Delta Z} \right)^2 \right] + (-9.806 * \frac{\mu(i,j)}{\rho(i,j)\sigma_b} \left(\frac{\rho_{i,j} - \rho_{i,j-1}}{2\Delta Z} \right) \\
 & - \frac{1}{2} c2 \rho_{i,j} \frac{(\varepsilon_{i,j,t})^2}{K_{i,j,t}} \\
 & \dots (3.45 d)
 \end{aligned}$$

Applying the upwinding scheme, coefficient $a_{i,j}$, $b_{i,j}$, and $c_{i,j}$ are similar to those in Eqs.(3.33 a',b',c') except σ_T is replaced by σ_ε while coefficient $d_{i,j}$ will not change.

For the time interval $(t+1/2\Delta t \rightarrow t+\Delta t)$:

Implicit in the x-direction-explicit in the z-direction for solving equation (3.44), the finite-difference form are applied in all terms of this equation and arrange it to fit the general discretization form which yields:

$$\varepsilon_{i,j,t+\Delta t} = \frac{d_{i,j} + a_{i,j} \varepsilon_{i-1,j,t+\Delta t} + c_{i,j} \varepsilon_{i+1,j+1,t+\Delta t}}{b_{i,j}} \quad \dots (3.46)$$

Again coefficients a, b, and c are similar to those in Eqs.(3.34a,b, and c) except

σ_T is replaced by σ_ε when solving Eq.(3.44) implicitly in the x-direction.

For the time interval $(t+1/2\Delta t \rightarrow t+\Delta t)$, while coefficient $d_{i,j}$ becomes:

$$\begin{aligned}
 d_{i,j} = & \left(\frac{\mu(i,j)}{2\sigma_\varepsilon \Delta Z^2} \right) \varepsilon_{i,j-1,t+1/2\Delta t} + \left(\frac{\rho_{i,j}}{\Delta t} + \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} - \frac{\mu(i,j+1)}{2\sigma_\varepsilon \Delta Z^2} \right. \\
 & - \frac{\mu(i,j)}{2\sigma_\varepsilon \Delta Z^2} \varepsilon_{i,j,t+1/2\Delta t} + \left(-\frac{w_{i,j,t} \rho_{i,j+1}}{2\Delta Z} + \frac{\mu(i,j+1)}{2\sigma_\varepsilon \Delta Z^2} \right) \varepsilon_{i,j+1,t+1/2\Delta t} \\
 & + \frac{1}{2} c1 \frac{\varepsilon_{i,j,t}}{k_{i,j,t}} \mu(i,j) \left[2 \left(\frac{u_{i,j,t} - u_{i-1,j,t}}{\Delta X} \right)^2 + 2 \left(\frac{w_{i,j,t} - w_{i,j-1,t}}{\Delta Z} \right)^2 + \right. \\
 & \left. \left(\frac{w_{i,j,t} - w_{i-1,j,t}}{\Delta X} + \frac{u_{i,j,t} - u_{i,j-1,t}}{\Delta Z} \right)^2 \right] - \frac{1}{2} c2 \rho_{i,j} \frac{(\varepsilon_{i,j,t})^2}{k_{i,j,t}} + (-9.806 * \frac{\mu(i,j)}{\rho_{i,j} \sigma_b} \left(-\frac{\rho_{i,j} - \rho_{i,j-1}}{2\Delta Z} \right)
 \end{aligned}$$

... (3.46 d)

Applying the upwinding scheme for the convective term $(\frac{\partial \rho \varepsilon}{\partial Z})$, coefficients

$a_{i,j}$, $b_{i,j}$, and $c_{i,j}$ will not be change, while coefficient $d_{i,j}$ becomes:

$$\begin{aligned}
d_{i,j} = & \left(\frac{w_{i,j,t} \rho_{i,j-1}}{2\Delta Z} + \frac{\mu(i,j)}{2\sigma\epsilon\Delta Z^2} \right) \epsilon_{i,j-1,t+1/2\Delta t} + \left(\frac{\rho_{i,j}}{\Delta t} - \frac{w_{i,j,t} \rho_{i,j}}{2\Delta Z} - \frac{\mu(i,j+1)}{2\sigma\epsilon\Delta Z^2} - \right. \\
& \left. \frac{\mu(i,j+1)}{2\sigma\epsilon\Delta Z^2} - \frac{\mu(i,j)}{2\sigma\epsilon\Delta Z^2} \right) \epsilon_{i,j,t+1/2\Delta t} + \left(\frac{\mu(i,j+1)}{2\sigma\epsilon\Delta Z^2} \right) \epsilon_{i,j+1,t+1/2\Delta t} \\
& + \frac{1}{2} c1 \frac{\epsilon_{i,j,t}}{k_{i,j,t}} \mu(i,j) \left[2 \left(\frac{u_{i,j,t} - u_{i-1,j,t}}{\Delta X} \right)^2 + 2 \left(\frac{w_{i,j,t} - w_{i,j-1,t}}{\Delta Z} \right)^2 + \right. \\
& \left. \left(\frac{w_{i,j,t} - w_{i-1,j,t}}{\Delta X} + \frac{u_{i,j,t} - u_{i,j-1,t}}{\Delta Z} \right)^2 \right] - \frac{1}{2} c2 \rho_{i,j} \frac{(\epsilon_{i,j,t})^2}{k_{i,j,t}} + (-9.806 * \frac{\mu(i,j)}{\rho(i,j)\sigma b} \left(\frac{\rho_{i,j} - \rho_{i,j-1}}{2\Delta Z} \right)
\end{aligned}$$

... (3.46d')

The boundary condition transformation to finite-difference form for the modified k- ϵ model equations can be set as following:

A. at the discharge region:

- The surface thermal discharge (j=1 ,and j=2):

$$k_{2,j} = kd$$

$$\epsilon_{2,j} = \epsilon d$$

- The submerged thermal discharge (first case) (j=6,and j=7):

$$k_{2,j} = kd$$

$$\epsilon_{2,j} = \epsilon d$$

- The submerged thermal discharge (second case) (j=11,and j=12):

$$k_{2,j} = kd$$

$$\epsilon_{2,j} = \epsilon d$$

B. At upstream boundary where: $x=0$ and $0 \leq Z \leq D$ k, ε are constant and equal to their initial distribution in the undisturbed river, therefore does not require a finite –difference transformation.

C. At surface boundary where: $0 \leq X \leq L$ and $Z = 0$

$$\left. \frac{\partial k}{\partial Z} \right)_{i,1} = 0 \quad \text{Is transformed} \quad \text{to} \quad k_{i,0} = k_{i,2}$$

$$\left. \frac{\partial \varepsilon}{\partial Z} \right)_{i,1} = 0 \quad \text{Is transformed} \quad \text{to} \quad \varepsilon_{i,0} = \varepsilon_{i,2}$$

D. At downstream boundary where: $x=L$ and $0 \leq Z \leq D$

$$\left. \frac{\partial k}{\partial X} \right)_{px+1,j} = 0 \quad \text{Is transformed} \quad \text{to} \quad k_{px+2,j} = k_{px,j}$$

$$\left. \frac{\partial \varepsilon}{\partial X} \right)_{px+1,j} = 0 \quad \text{Is transformed} \quad \text{to} \quad \varepsilon_{px+2,j} = \varepsilon_{px,j}$$

E. At bottom boundary where: $0 \leq X \leq L$ and $Z=D$

$$\left. \frac{\partial k}{\partial Z} \right)_{i,pz+1} = 0 \quad \text{Is transformed} \quad \text{to} \quad k_{i,pz+2} = k_{i,pz}$$

$$\left. \frac{\partial \varepsilon}{\partial Z} \right)_{i,pz+1} = 0 \quad \text{Is transformed} \quad \text{to} \quad \varepsilon_{i,pz+2} = \varepsilon_{i,pz}$$

Equation (2.31) which is used to determine the distribution of the turbulent dynamic viscosity is a direct equation and does not need to be transformed to finite-difference form.

3.6 Finite – Difference Form of the Additional Auxiliary Equations:

3.6.1. Hydrostatic Pressure Equation:

The hydrostatic pressure distribution is used to calculate the pressure at each point.

$$P = g \int_0^z \rho dz \quad \dots 3.47$$

This equation is transformed to Finite – Difference form to obtain the pressure at point (i,j) as follows (Al-Chalabi,1993):

$$P_{i,j} = P_{i,j-1} + 9.81 \left(\frac{\rho_{i,j-1} + \rho_{i,j}}{2} \right) \Delta Z \quad \dots 3.48$$

The other auxiliary equations (2.25, 2.31) are applied directly and

Do not require to be transformed to finite-difference form.

3.7 Solution of the General Discretization Equations:

All the previously discussed general discretization equations can be conveniently presented in general form:

Let $\phi_{i,j}$ is a variable denoted to the unknown values of w , u , T , K , and ε .

For the first time step:

$$\phi_{i,j}^{t+\Delta t/2} = \frac{d_{i,j} \phi_{i,j-1}^{t+\Delta t/2} + f_{i,j} \phi_{i,j+1}^{t+\Delta t/2}}{b_{i,j}} \quad \dots 3.49$$

By apply equation (3.49), each nodes column lies in grid domain gives a set of Linear algebraic equations contain the unknown in that column.

For the second time step:

$$\phi_{i,j}^{t+\Delta t} = \frac{d_{i,j} + a_{i,j} \phi_{i-1,j}^{t+\Delta t} + f_{i,j} \phi_{i+1,j}^{t+\Delta t}}{b_{i,j}} \quad \dots 3.50$$

By apply equation (3.50), each nodes row lies in grid domain gives a set of linear algebraic equations contain the unknown in that row.(Smith, G.D (1978)).

There are several methods for solving a set of linear algebraic equations some more suitable than others. Direct methods for solving the linear algebraic equations are much more complicated and require rather large amount of computer storage and time .Iterative methods was therefore employed in this study. There are many iterative methods for solving the linear algebraic equations. Gauss-seidel point-by-point method was considered in the present work.

3.8 The Gauss-Seidel Point –By-Point Method:

The simplest of all iterative methods is the Gauss-seidel method in which the Values of the variable are calculated by visiting each grid point in a certain order. Only one set of $\phi_{i,j}$ is held in computer storage .In the beginning, these represent The initial guess or values from the previous iteration. As each grid point is visited, the corresponding value of $\phi_{i,j}$ in the computer storage is altered as follows:

Let $\phi_{i,j}^*$ represent the initial guess or values of $\phi_{i,j}$, then the new values

Of $\phi_{i,j}$ for the first time step are calculated from equation as follows:

$$\phi_{i,j} = \frac{d_{i,j} + a_{i,j} \phi_{i,j-1}^* + f_{i,j} \phi_{i,j+1}^*}{b_{i,j}} \quad \dots 3.51$$

At each visited grid point, the new value of $\phi_{i,j}$ becomes $\phi^*_{i,j}$ for the next node. When all grid points have been visited, first iteration of Gauss-seidel method is complete. The same procedure is repeated for the next iteration by making the new values of $\phi_{i,j}$ as initial values $\phi^*_{i,j}$. Many iterations are made until the values of $\phi_{i,j}$ are not changed between successive iterations. By using the same previous manner to calculate the new values of $\phi_{i,j}$ for the second time step from equation. (Smith, G.D (1978)).

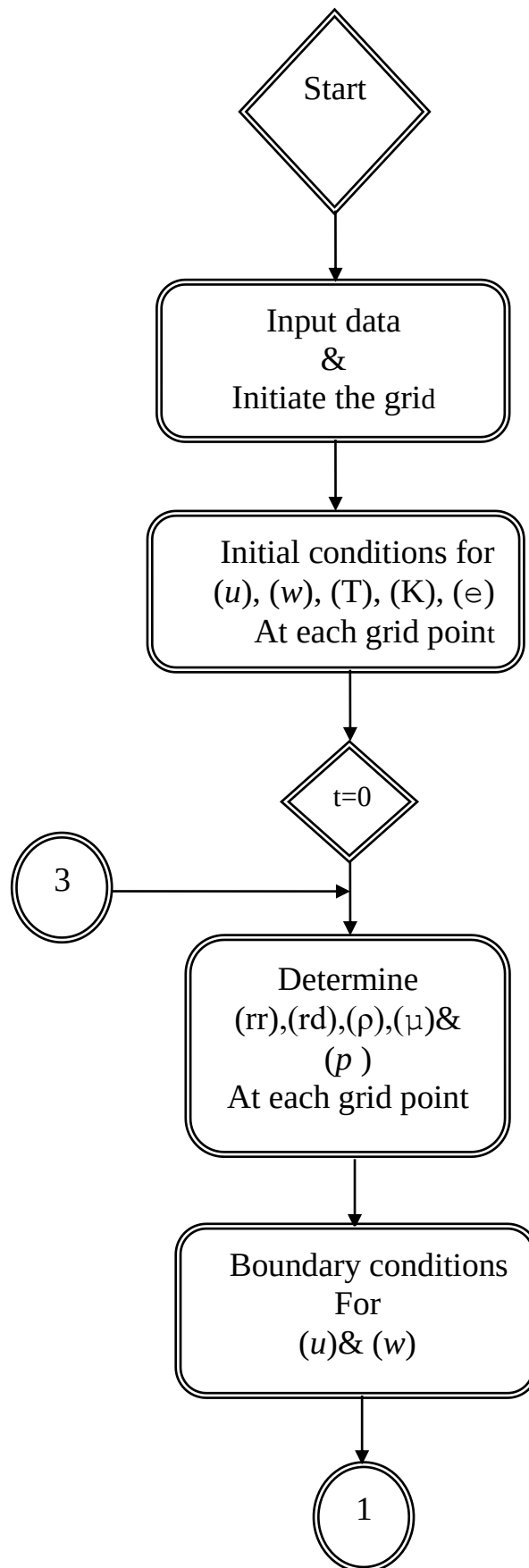
3.9 Computer Program Description:-

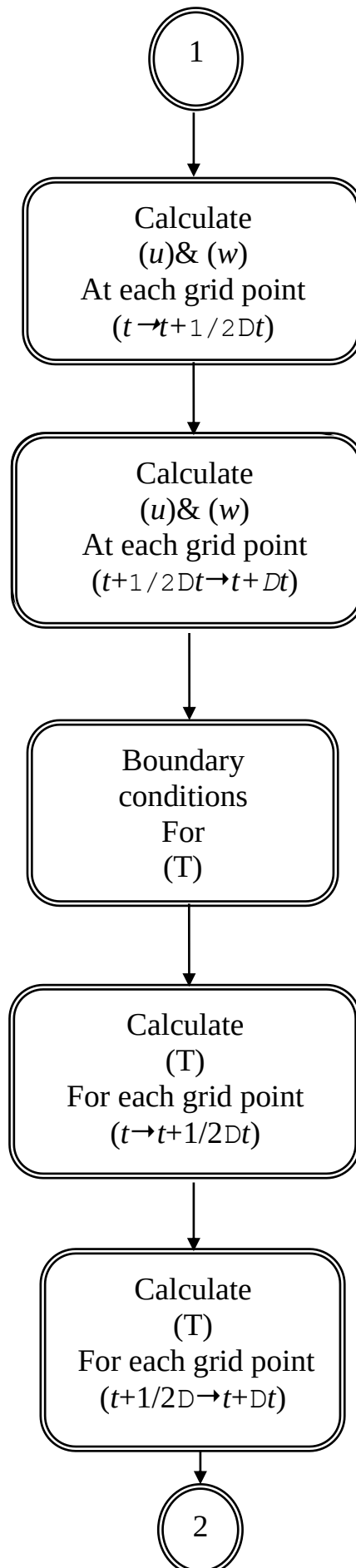
A computer program is a basic research tool used to perform computation of the simulation model. The set of the two – dimensional equations is used to describe thermal pollutant transport phenomena and finite difference procedure for solving these equations which were presented previously. A computer program was written in quick basic 4.5 languages for Ms-Dos Microsoft computers because of large memory and speed required to carryout the numerical calculations in the present work. The input data used to operate the model and run the computer programs are the length, depth, and slope of the river, time interval and final time of the study, water river characteristics (e.g. temperature , density , kinetic viscosity....etc), temperature of heated water, longitudinal and vertical increment. The flow chart of the computer program is shown in Fig. (3.9), and its steps are explained as follows:

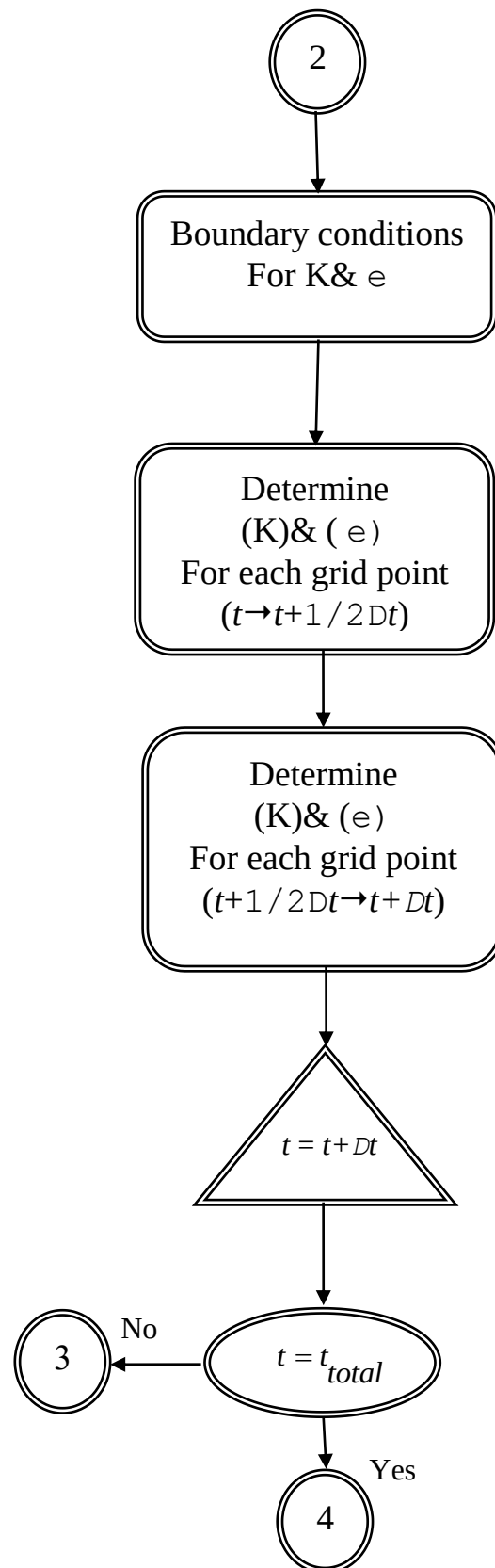
1. Reading the input data of the problem, then constructing a uniform Rectangle vertical grid, the input data required as following:
 - a. Total length of the river and its length increment.
 - b. Total depth of the river and its increment.
 - c. Total time of study and time increment.
 - d. Water surface slop of the river.
 - e. Initial horizontal velocity components of the river.

- f. Discharged water temperature at the discharge region.
 - g. River water temperature.
2. Specifying the initial condition for the problem is as shown in section (3.3.1)
 3. Calculation the density of water river and heated water discharge from equation (2.25). At each point in the rectangular grid.
 3. Calculation of viscosity of water river and heated water from equations (2.31), and depending on densities obtained at different time steps.
 4. Reading boundary conditions for the grid points on the boundaries
And for all variables is as shown in section (3.3.2).
 5. Calculation of the pressure values at each grid point is from equation (2.25).
 6. Calculation the vertical and horizontal velocity components (w, u) at each grid point at time step ($t \rightarrow t+1/2 \Delta t$) using the equations (3.15 and 3.25) respectively.
 7. Calculation the vertical and horizontal velocity components (w, u) at each grid point at time step ($t+1/2 \Delta t \rightarrow t+ \Delta t$) using the equations (3.18 and 3.27) respectively.
 8. Specification of the boundary condition for the thermal energy equation is as shown in pp. (64).
 9. The temperature of river water is determined for the time step ($t \rightarrow t+1/2 \Delta t$) at each grid point using equation (3.33).
 10. The temperature of river water is determined for the time step ($t+1/2 \Delta t \rightarrow t+ \Delta t$) at each grid point using equation (3.34).
 11. The boundary condition for the modified $K - \varepsilon$ turbulence model is specified in pp. (69.70).
 12. The kinetic energy k and its rate of dissipation ε is determined for the time step ($t \rightarrow t+1/2 \Delta t$) at each grid point using equation (3.41 and 3.42) respectively.

13. The kinetic energy k and its rate of dissipation ε is determined for the time step $(t+1/2 \Delta t \rightarrow t+ \Delta t)$ at each grid point using equation (3.45 and 3.46) respectively.
14. The time is increased by the time increment and the procedure is repeated from step (3) by using the last values of the variables till the total time of the study is reached.
15. The steps (6,7,9,10,12 and 13) incorporate the calculation of the coefficients a , b , c for the considered general discretization equation at each grid point in that time step, then the set of linear algebraic equation are solved by Gauss-seidel point-by-point method as shown in section (3.7, and 3.8) to obtain the considered variable in that step.
16. Printing the temperature field at each grid point is at the final time step.







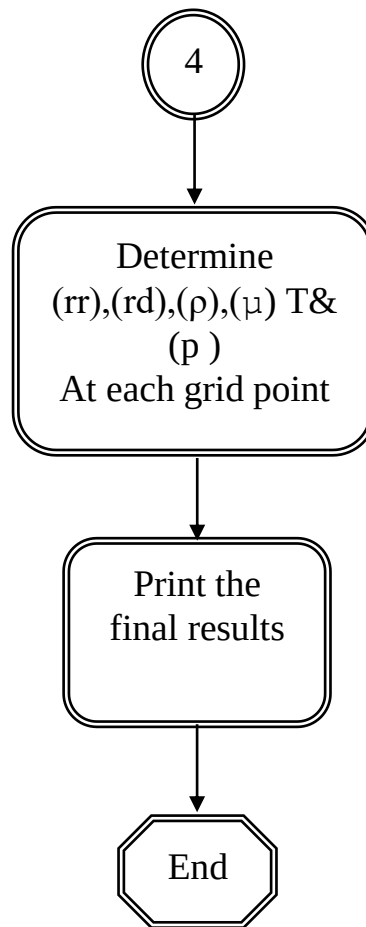


Fig. (3.9): Flow Chart of the Computer Program

Chapter Four

Results and Discussion

4.1 Introduction:

This chapter presents the results obtained from solving the algebraic equations concerned with the problem of thermal discharge which is presented as isotherms. The thermal application to the river reach is from three different discharge cases as single-point source. The temperature isotherms transformed to a dimensionless scale (dilution factor) which represent the dilution level of the river. Also, this chapter shows the effect of different parameters on the thermal pollutant released from different discharge locations.

4.2 Model Verification:

The results of any numerical model should be evaluated therefore, it is essential to test the model in order to determine its validity. The accuracy of any numerical model depends widely on available field data specially when there is no analytical solution for the equations which are involved to the relevant problem. This field data should be reliable for the model verification. In the present study, field data are not available; thus the verification was established by comparing the results of the present model results with theoretical results conducted by a previous work for Al-Chalabi, M.A (1993).

The main characteristics of Al-Chalabi M.A model can be summarized as follows:

1. A two-dimensional numerical model was developed for the simulation of the spreading and mixing of thermally polluted water

disposed into river flow with a smooth-rough-bed-velocity-distribution considering the effect of density difference between the pollutant density and the river water density.

2. The various forms of the governing differential equations in horizontal and vertical cartesian coordinates were analyzed; these are momentum conservation equations and energy conservation equation, and the $(k - \varepsilon)$ turbulence model.
3. The turbulent dynamic viscosity and diffusion coefficients were calculated using the "k- ε " turbulence model.
4. Longitudinal velocity distribution equation was used to study the vertical velocity profile in turbulent flow which indicates that, the velocity in turbulent flow is a logarithmic function of the distance Y which is the normal distance from the solid surface, and gives two forms of velocity distributions as follows:
 - a. Smooth-bed-velocity –distribution (S.B.V.D.):

$$V = 5.75 \ v_f \log \frac{9Y v_f}{\nu} \quad \dots 4.1$$

Where:

Y: normal distance from the river bottom.

V: Kinematics viscosity.

v_f : Friction velocity.

- b. Rough-bed-velocity-distribution (R.B.V.D.):

$$V = 5.75 \ v_f \log \frac{30 Y}{k} \quad \dots 4.2$$

Where:

k : Roughness height which is the effective height of irregularities forming the roughness elements.

5. The different forms of partial differential equations were discretized using the (ADI-the alternating direction implicit-explicit method) finite-difference technique with application of up-winding scheme to obtain a set of linear algebraic equations.
6. The linear equations were solved by Gauss's elimination method.

4.2.1 Numerical Model Data:

The input data used to run the computer program was to compatible with the previous work and based on some of the hydraulic conditions of the tigris river in baghdad city. The input data are as follows:

- The total length of the river reach is 2000 m and its increment is 100 m.
- The depth of the river reach is 3 m and its increment is 0.25 m.
- The slope of the river is 6 cm/km (Euphrates center for studying and design of irrigation projects, 1992).
- The hydraulic radius of the river was taken equal to the river reach depth (i.e., 3 m).
- The initial longitudinal velocity component of the river in the case of (S.B.V.D.) is (1.48) m/sec (equation (4.1)), and (0.76) m/sec in (R.B.V.D.) (equation (4.2)), (Euphrates Center for studying and design of irrigation projects, 1992).
- The temperature of disposed water is 40 °C.
- The average river water temperature is 20 °C.
- The time increment is 1 sec and the program runs for different times.

4.2.2 The Types of Discharge:

1. Surface Discharge:

In this case, the discharge region outfall is at the surface of the river which is most convenient and widely disposal method of polluted water.

Figs. (4.1, and 4.2) show a comparison in temperature isotherms of points located at the water surface with horizontal distance from the discharge region between the present model and those resulted by Al-Chalabi (1993). The heated water released mixes and spreads longitudinally by the longitudinal turbulent mixing due to the interaction of the turbulent stresses with the mean velocity gradients, the longitudinal convection due to the mean motion, and longitudinal diffusion due to turbulent fluctuations. it can be seen that the maximum temperature located at the discharge region is reduced with horizontal distance to approach the normal river temperature at distance about 900 m from the discharge region at time 400 sec, with initial river velocity equal to 0.76 m/sec, and about 1300 m when the initial river velocity increased to 1.48 m/sec (for both present and Al-Chalabi work). Also the figures shows that increasing of the river initial velocity from (0.76 m/sec) to (1.48 m/sec) causes a faster longitudinal spread of the pollutant (temperature) due to the greater longitudinal convection which leads to increase of temperature levels at the horizontal stations, (for example at distance 400 m away from the outfall location and initial river velocity equal to 0.76 m/sec the isotherm value is (27.6 °C) (present work), while this isotherm becomes (31.8 °C) (present work) with initial river velocity equal to 1.48 m/sec).

The increase in temperature levels at all longitudinal stations (except at the outfall region) of the present work, with a decrease in temperature levels for the same longitudinal stations for the previous

work, this behavior accompanied by a decrease in the vertical dispersion at the same time. For example at distance 500 m away from the outfall location with river velocity equal to 0.76 m/sec, the isotherm value is (24.6 °C) while its (21.5 °C) for the Al-Chalabi work. This can be attributed to the effect of buoyancy term on the spreading and mixing of the pollutant (temperature), since the disposal of hot water is normally accompanied by a large effect of buoyancy forces resulted from the longitudinal spreading.

When the river initial velocity increases from 0.76 to 1.48 m/sec the buoyancy forces effect on flow are reduced, and the horizontal convection will be the dominant mechanism on the temperature distribution.

Figs. (4.3 and 4.4) show a comparison in temperature variation with vertical distance for points located below the outfall region in the case of surface discharge. In the vertical direction the heated water released mixes and spreads by the vertical turbulent mixing, vertical diffusion, and by vertical convection. Temperature variation is retarded vertically (diluted slowly) in case of the Al-Chalabi work (i.e. at depth 2m from the outfall location, and river velocity =0.76 m/sec the temperature value is (26 °C) which is less than the temperature value conducted by Al-Chalabi (29.5 °C), Fig. (4.3). This is can be attributed to the buoyancy term neglected in the Al-Chalabi work which leads to increase the turbulent kinetic energy K and its rate of dissipation. But in return increasing the turbulent dynamic viscosity leads to increase the water entrainment, and reduce the temperature levels as conducted by the present work.

The increasing of the initial river velocity (1.48 m/sec) leads to a increase in the longitudinal convection, vanish the buoyancy effect, and

a increase in the longitudinal turbulent mixing which effects on reducing the vertical spreading at the same time.

The above results are in a good agreement with the statement presented by Arpaci and Larsen (1984) that "the buoyant force is the driving mechanism of natural convection. So, when the effect of temperature on fluid density is taken into account, lighter parts of heated fluid flow upward and the heavier ones downward relative to a reference state in equilibrium". The preceding analysis shows the necessity of incorporating the effect of density difference in studying the spreading and mixing of thermally polluted water having a density different from the river water density.

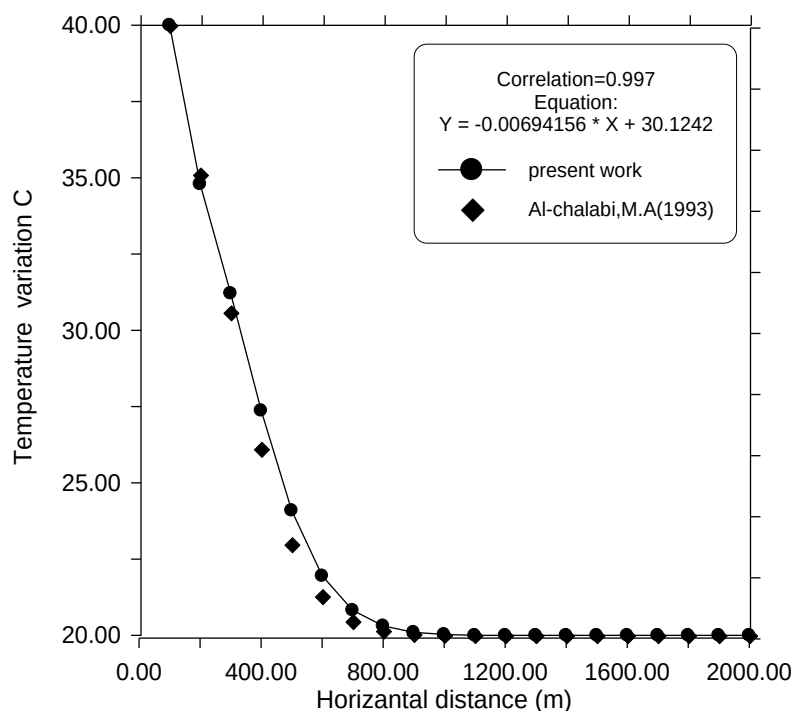


Fig. (4.1) :Comparison in Temperature Variation with Horizontal Distance (surface discharge) between the Present work and the Al-chalabi work (Tr=20 C, Td=40 C, U =0.76 m/sec , Time = 400 sec)

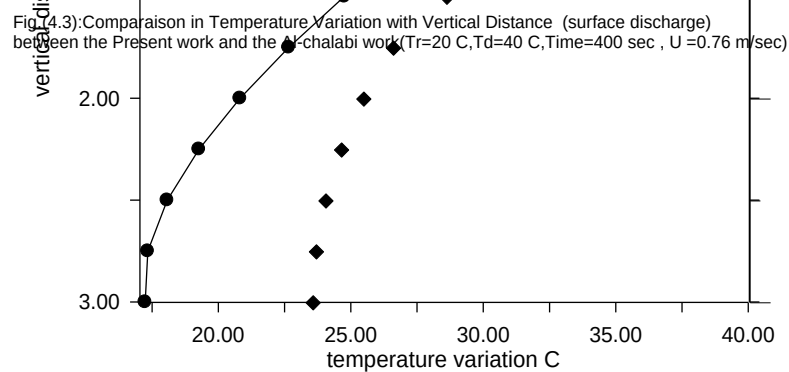
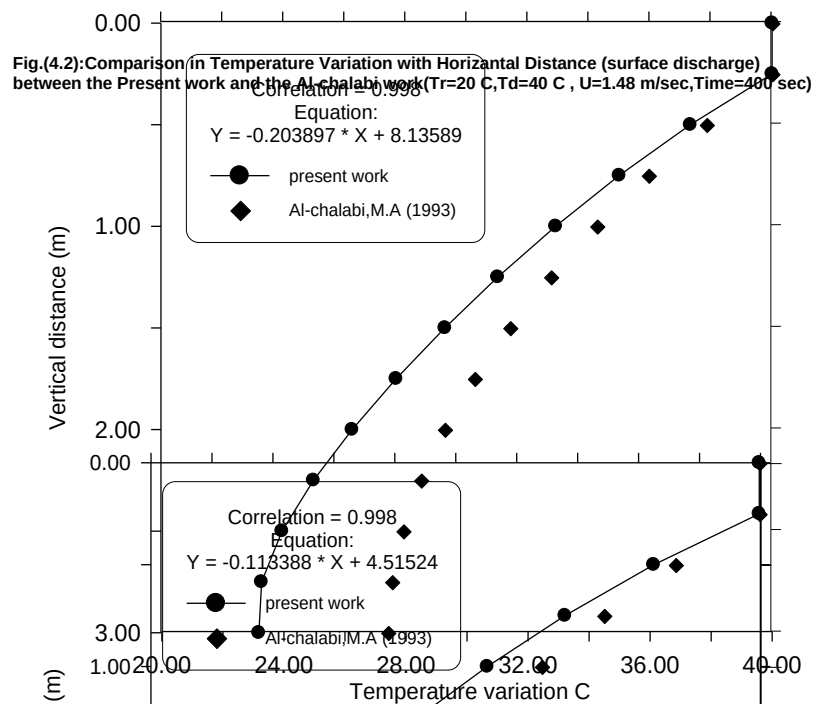
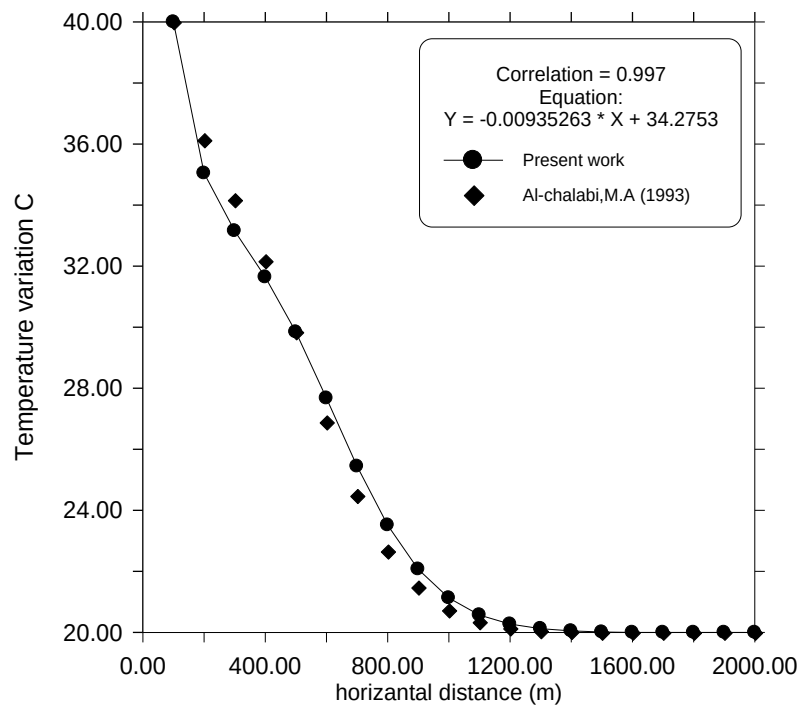


Fig.(4.4): Comparison in Temperature Variation with Vertical Distance (surface discharge) between the Present work and Al-chalabi work (Tr=20 C, Td=40 C, Time=400 sec, U=1.48 m/sec)

2. Submerged Discharge:

The submerged disposal is considered in this study with two discharge cases. The first case where the outfall location is at 0.42 of the river depth as single-point source, and the second where the outfall location is at 0.83 of the river depth as single-point source also. To verify the submerged numerical model results a comparison is made with the previous model data made by (Al-Chalabi).

Figs. (4.5, and 4.6) show a comparison in temperature variation of points located at 0.42 of the river depth (submerged discharge) with horizontal distance from the discharge region. The results indicate that at the beginning of the discharge the temperature isotherms values are much less than that for obtained by Al-Chalabi. The submerged discharge at low initial river velocity (0.76 m/sec) and a distance 400 m from the outfall region, the temperature isotherm is 26.1 °C, while it is (27.8 °C) for the previous work. The difference in the temperature can be attributed to the

submerged thermal discharge usually imparts a considerable horizontal momentum (strong horizontal advection) to the effluent, which possess a turbulent kinetic energy due to buoyancy induced by its elevated temperature causing vertical movement of pollutant. As the pollutant rise, it will entrain the cooler surrounding water into it. The entrainment will increase the volume of polluted area and also reduce the temperature difference between the polluted area and the surroundings. The entrainment is increased in present work due to increase in the turbulence effects and buoyancy induced by the buoyancy term.

Also from Figs. (4.5), and (4.6) show that the increasing of the river velocity from (0.76 m/sec) to (1.48 m/sec) leads to increase in the river water temperature as the distance increased from outfall location. This can be attributed to the strong longitudinal turbulent mixing and strong longitudinal convection.

Figs. (4.7 and 4.8) show a comparison in temperature isotherms with vertical distance for the first case of submerged discharge. The two figures show that the temperature isotherms are less in the vertical direction in case of present work as compared to

Fig (4.9) shows a comparison between the variation in dilution factor resulting from the temperature distribution patterns with horizontal distance for three different discharge locations and initial river velocity equal to 0.8 m/sec and with time equal to 600 sec. For the river reach subjected to thermal pollutant discharged as a single-point source with temperature equal to 40 °C. It can be clearly observed that, as the river water temperature decreases as moving downstream far from the outfall the dilution factor tends to

increase .At distance 400 m from the discharge region, the dilution factor is equal to (0.42) for the surface discharge ,and for first case of submerged disposal the dilution factor is increased to be (0.48) ,while it is(0.78) at (0.83 from the river depth) .It is also found that the maximum increase in temperature occurs at the surface and the lesser occurs at larger depths.

The maximum value of the dilution factor for the three-thermal discharge cases is obtained from the second case of submerged discharge. This means that the river recover its original condition in case of outfall at 0.83 from the river depth faster than the other two schemes of discharge.

4.3 Temperature Variation Comparison:

In order to test the most efficient method of disposal for thermal discharge, a comparison between the surface discharge method which is the most widely used and the submerged discharges is done .The outfall locations are one at 0.42, and the other at 0.83 of the river depth (total river depth =3 m). In order to make such a comparison the discharge temperature is taken to be equal to 40 °C, and the temperature of the river reach is 20 °C .The river velocity equal to 0.8 m / sec with total time of thermal disposal = 600 sec and river surface slope equal to 0.00006 m/m.

Fig. (4.10) shows the thermal discharge patterns, from this figure we conclude:

1. For submerged disposal, the increasing in temperature at deeper depth is greater than that at surface .while, for surface discharge, the increasing in temperature in deeper depth is smaller than that at deeper depth.

2. The maximum curvature of the isotherms occurs near the water surface in case of surface disposal, and almost happens at the middle of the river in case of first case of submerged disposal, while it occurs near the river bed in the second case of submerged disposal. This can be attributed to the effect of buoyant spreading and bending the isotherms, which take place inside the river and near the river bed in case of submerged disposals and near the surface in the case of surface disposal.

3. The isotherms of low temperature are longitudinally advanced in both submerged disposals cases, and this advancing is retarded in the second case of submerged disposal. For example, The isotherm equal to 21°C at distance 900 m from the outfall location in the first case of submerged disposal, while this distance becomes 780 m in the second case of submerged disposal. This can be attributed to the effect of buoyancy forces which extends further downstream in these two cases relative to that in case of surface disposal and this extending is reduced in the second case of submerged disposal due to the increase in turbulence which accompanied the submerged disposal and increase the ability of fluid to entrain. Therefore the thermal discharge diluted faster in the second case of submerged discharge.

4. The longitudinal retardation of high temperature isotherms which can be noticed by comparing the location of these isotherms at deep levels in the case of submerged disposals with their location at the surface in the case of surface disposal (the isotherm 35°C for the surface discharge case is at distance 215 m from the discharge region while this distance become 210 m and 150 m from the discharge region for the first and second submerged discharge.

5. An earlier formation of uniform vertical temperature distribution in the two cases of submerged disposal is compared to that in the case of surface disposal. The uniform vertical temperature distribution in the first case of submerged disposal for example can be noticed in isotherms of temperature less than 34°C as shown in Fig. (4.10.b).while in the second case of submerged disposal the uniform vertical temperature distribution is noticed in isotherms that have less than 25°C as shown in Fig. (4.10c) .In the surface thermal disposal, the uniform vertical temperature distribution can be noticed in isotherms of temperature less than 29°C as shown in Fig.(4.10 a).This can be attributed to the effect of upward motion of lighter parts by the effect of buoyancy forces and the downward motion of heavier parts due to the gravity effect, and an entrainment of ambient water causing a decrease in the temperature difference and an earlier formation of uniform temperature distribution in the case of submerged disposals as expected.

The Comparison Summary:

1. Surface thermal discharge condition is least-costly and oldest discharge scheme imparts less dilution rates and cause a huge

damage to the aquatic environment by reducing the oxygen solubility in the water bodies.

2. Submerged discharge schemes gives the maximum temperature value at deep levels avoiding the thermal shock to the water system, much higher rates of dilution occur due to increase in turbulent motion due to the buoyancy accompanied the submerged disposal schemes .
3. The water system recovery from the thermal discharge is much faster in the second case of submerged discharge (point of discharged at 0.83 from the river depth) than from the first case of submerged thermal discharge). This can be noticed in the earlier formation of uniform vertical temperature which occurs in second case much faster than from the other discharge schemes. (e.g. the normal river temperature is obtained at distance 910 m in the surface case and 895 m and 760 m in the, first and second case of submerged thermal discharge).

From the above summary it can be conclude that the second case of submerged disposal is the most effective and efficient method for the thermal discharge to any stream flow.

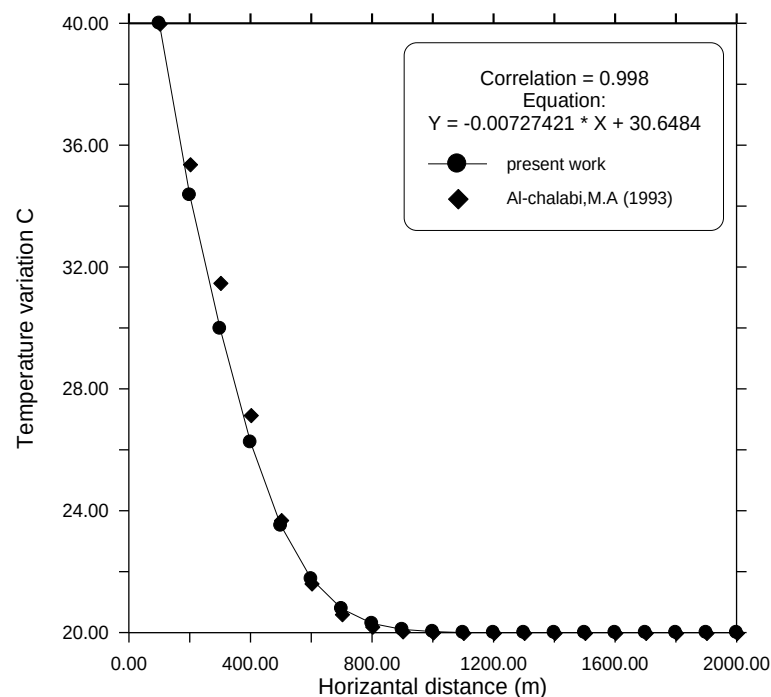


Fig.(4.5):Comparison in Temperature Variation (submerged discharge-first case) with Horizontal Distance between the Present work and the Al-chalabi work ($T_r=20$ C, $T_d=40$ C, $U=0.76$ m/sec , Time=400 sec).

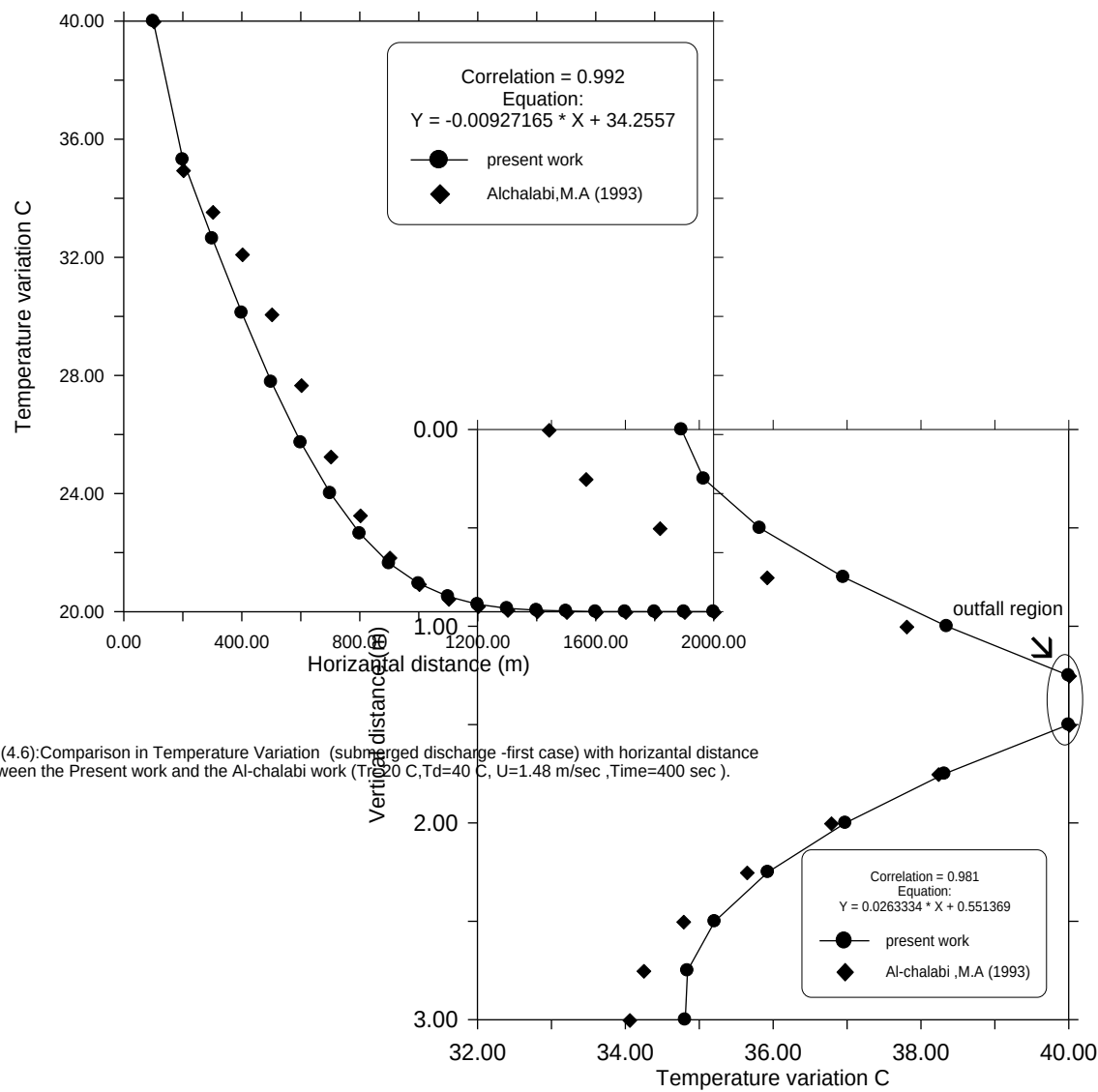


Fig.(4.6):Comparison in Temperature Variation (submerged discharge -first case) with horizontal distance between the Present work and the Al-chalabi work (Tr=20 C,Td=40 C,U=1.48 m/sec ,Time=400 sec).

Fig.(4.7):Comparison in Temperature Variation with Vertical Distance (submerged discharge -first case) between the Present work and Al-chalabi work (Tr=20 C,Td=40 C,U=0.76 m/sec,Time = 400 sec).

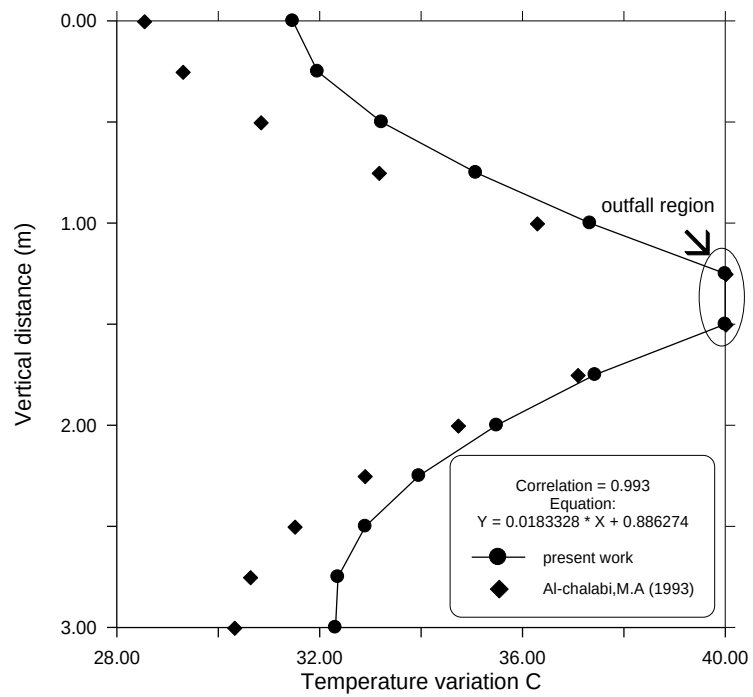


Fig.(4.8):Comparison in Temperature Variation (submerged discharge-first case) with Vertical Distance between the Present work and Al-chalabi work (Tr=20 C, Td=40 C, U=1.48 m/sec, Time=400 sec).

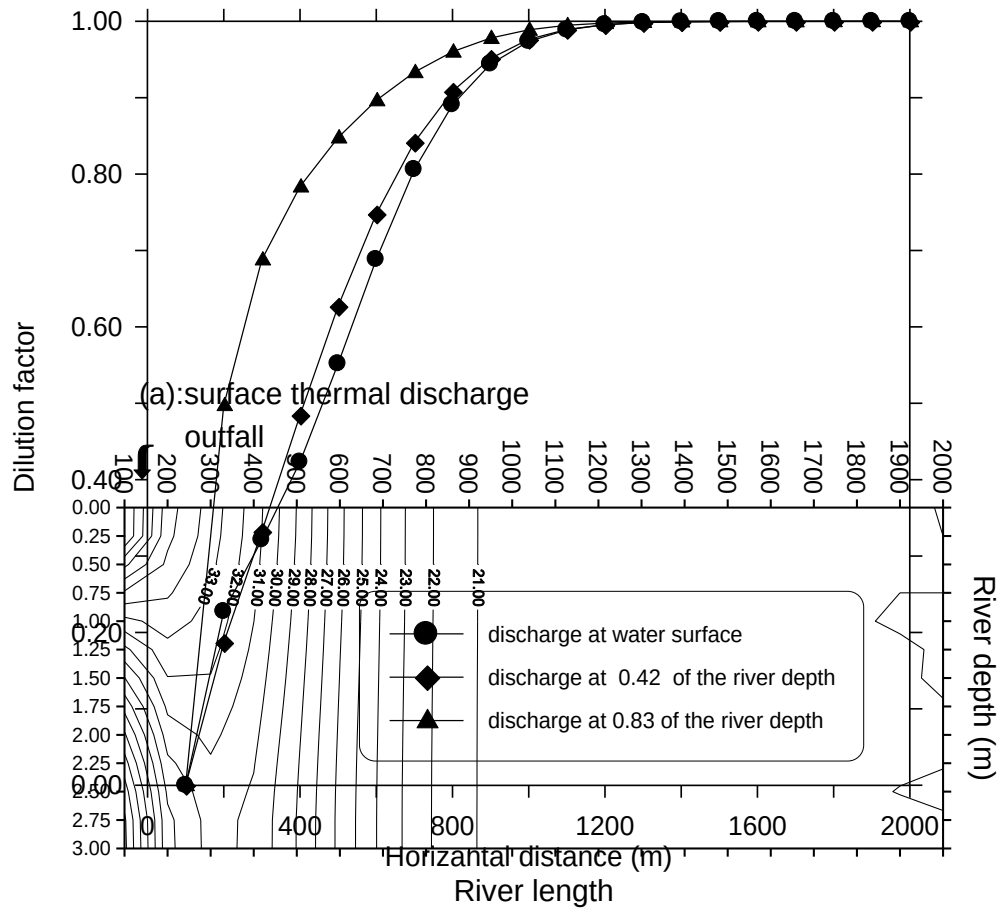


Fig.(4.9): (a) surface thermal discharge (0.42 from the river depth), and the submerged thermal discharges (0.42, 0.83 from the river depth) (Time = 600 sec , water surface slope = 0.00006 m/m , River water velocity = 0.8 m/sec)

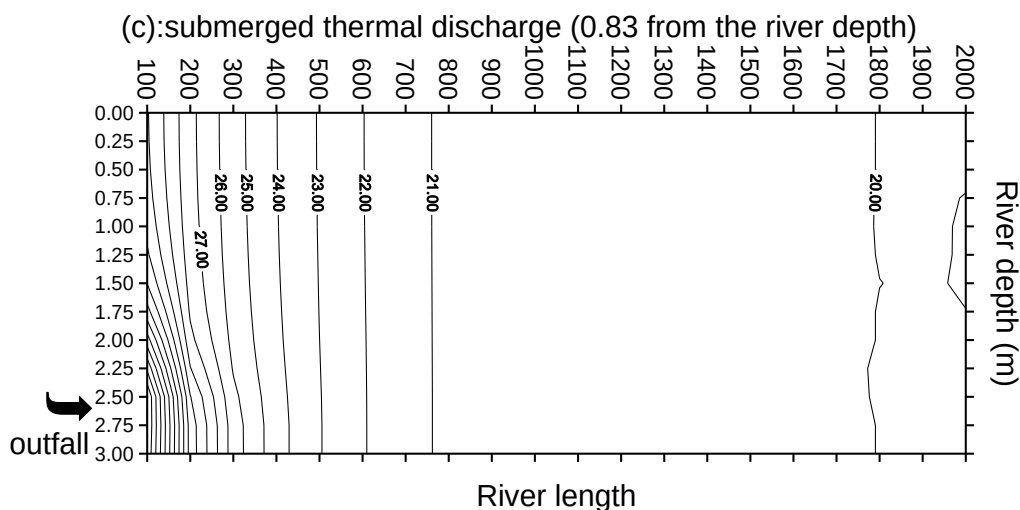
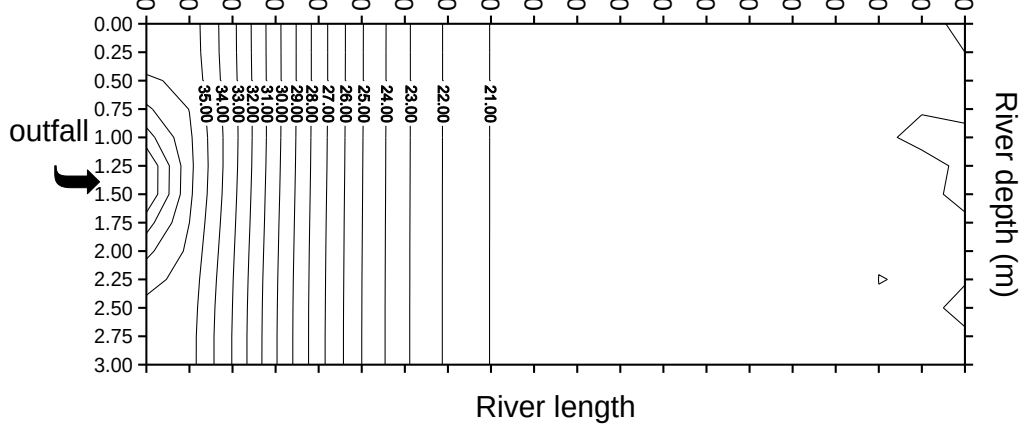


Fig.(4.10): Comparison in Temperature Variation between the Surface thermal discharge and the Submerged thermal Discharges (0.42, 0.83 from the river depth) (Tr=20 C, Td= 40 C, U=0.8 m/sec, surface water slope =0.00006 m/m , Time =600 sec)

4.4 Sensitivity Analysis:

4.4.1. Surface Discharge:

Sensitivity analysis for several of the important model parameters (such as turbulent dynamic viscosity / diffusivity, river slope, variation of temperature distribution with time ,initial river water velocity) is carried out to identify their significance on the temperature distribution resulting from the application of the present model .In order to conduct this analysis, a continuous surface disposal of a heated water with

temperature equal to 40 °C , into a river with temperature equal to 20 °C, the length of river is (2000) m and depth (3) m.

4.4.1.1: Effect of Turbulent Dynamic Viscosity and Turbulent Diffusivity:

A test was carried out assuming a constant turbulent dynamic viscosity (μ) /diffusivity (D_T) over the whole flow field during the discharge time by dropping the (k- ϵ) turbulence from the present numerical model in order to study the effect of (μ / D_T) on the spreading and mixing processes. The model was runned letting (μ / D_T) constant with time .Viscosity at ambient river water temperature is $\mu = 9.642 \text{ N.s/m}^2$.The value for D_T was found to be 19.28 N.s/m^2 .The value of turbulent dynamic viscosity is calculated by using Eq. (3.8).

The results of this run are shown in Fig. (4.11) which illustrate the temperature isotherms of time equal to 400 sec with initial river velocity equal to 0.8 m/sec ,and surface water slope equal to 0.00006 m/m as follows :

- A. Determining the distribution of variable (μ / D_T) over the whole flow field during a time period of 400 sec.
- B. Assuming a constant distribution of (μ / D_T) over the whole flow field during the same period of time 400 sec.

Fig. (4.11) shows that:

1. Isotherms in case of assuming constant (μ / D_T) are retarded longitudinally at and near the water surface , while in case of assuming variable (μ / D_T) these isotherms are advance in the

horizontal direction at the water surface more than in case of constant (μ / D_T), (e.g. the isotherm 21 °C is at distance about 680 m from the outfall region at constant (μ / D_T), while this distance become 695 m at variable (μ / D_T). Also the retardation at the river bed is obviously for isotherms of high temperature values, and becomes smaller for isotherms of low temperature. This is because of assuming a constant turbulent eddy viscosity/diffusivity over all points in the river reach means the effects of turbulence and density difference is neglected between the effluent temperature and the river reach water. This leads to reduction in buoyancy forces which cause an increasing in vertical diffusion that leads to advance the temperature contours towards the river bottom, and decrease in the horizontal convection at the same time.

2. All isotherms are advanced vertically towards the river bottom, in case of assuming constant (μ / D_T), except isotherms near the outlet location. This vertical advancing is increased in case of assuming constant (μ / D_T) as shown in Figs. (4.11 a, and b) which show that the isotherms of 29 °C is almost reached the river bottom in case of assuming (μ / D_T) constant, while it reached (2.3) m in depth in the other case (variable μ / D_T), and the isotherm (28 °C) has inclined with curvature in case of variable (μ / D_T). While it is almost vertical in the other case. This can be attributed to the effect of vertical diffusion in advancing the isotherms vertically in the case of assuming constant (μ / D_T) as compared to the strong buoyancy forces and weak vertical diffusion in the other case. The effect of vertical diffusion has reduced in the region near the outfall location due to the greater production of turbulent kinetic energy and hence increased in the ability of the pollutant to advance into depth keeping the isotherms closer to each other in both cases.

The above results of Fig. (4.11) show a good agreement with the statement presented by Harleman (1982) that "the eddy viscosity is a direct measure of the transporting capacity of the mixing process, regardless of whether the transport refers to momentum or heat" Also, the results show a good agreement with that results presented in report carried by ASCE Task Committee on turbulence models (1988). They reported that "the heat transfer cannot be separated from the diffusion effects and the assumption of constant turbulent dynamic viscosity /diffusivity is appropriate only for far –field situations where the turbulence is governed by the natural water body and not by local man –made disturbances such as thermal discharges". Therefore, the model is very sensitive to this parameter and the results are physically realistic.

4.4.1.2: Effect of Surface Water Slope:

In order to study the effect of water surface slope on the river velocity and in return to the temperature transport, the model was rerun with two different values of water surface (0.00006 m/m and 0.0001 m/m); these values are selected from the values adopted by (Euphrates Center for studying and design of irrigation projects, 1992) for Tigris river, river velocity = 0.8 m/sec, $t_r = 20^\circ \text{C}$, $t_d = 40^\circ \text{C}$. From Fig. (4.12) it can be observe:

1. Run the computer program for the present water surface slope which equal to 0.00006 m/m in the temperature distribution
2. Increasing the water surface slope from 0.00006 m/m to 0.0001 m/m and rerun the computer program for the temperature distribution.

Fig. (4.12) shows a comparison between these two cases. By examining the results of Fig. it can be one can concluded that the increasing of the water surface slope from (0.00006 to 0.0001 m/m) in such a small reach used to simulate the flow domain in the present work, showed no noticeable changes in temperature distribution.

The above analysis indicates that the model is insensitive to variations of water surface slope.

4.4.1.3: Variation of Temperature Distribution with Time:

To study the effect of time variations on the spreading of temperature in river water reach, two values of time are selected (600 sec, and 800 sec), with constant value of ($t_d = 40^\circ\text{C}$ and $t_r = 20^\circ\text{C}$) for temperature, river initial velocity = 0.8 m/sec, river slope = 0.00006 m/m. The results are shown in Fig. (4.13) which illustrate that the increasing in the time of disposal causes a significant effect on the longitudinal and vertical spreading of the released discharges. The isotherm of 32°C for example reaches a distance 370 m, 480 m at water surface at time 600 sec, and 800 sec respectively, while the vertical advancing also changes (e.g. the isotherm 31°C is at depth 2.2 m at time 600 sec while this isotherm reaches the river bed with increasing the time to a value of 800 sec). This can be attributed to the buoyancy forces and longitudinal convection which has sufficient effect when the time is increased.

The above analysis indicates that the model is sensitive to the variation in time of disposal.

4.4.1.4: Effect of Initial River Water Velocity:

Initial river water velocity is increased from 0.8 m/sec to 1 m/sec to study its influence on temperature transport and spreading. Fig. (4.14) shows the temperature isotherms at time equal to 400 sec with initial river water velocities 0.8 , and 1 m/sec respectively ,river slope=0.00006 m/m ,Time of disposal = 400 sec .From this figure it can be notice that there is a distinguish increase in temperature levels at the longitudinal stations while there is a decrease in temperature variation in the vertical direction with the river velocity increasing. For example the isotherm 31 °C is about 1.4 m away vertically from the outfall at the water surface in case of temperature having initial river water equal to 1 m/sec, while it is about 1.6 m in case of initial river water equal to 0.8 m/sec. This can be attributed to the effect of initial river water velocity on convection mechanism. The increasing of initial river water velocity will increase the horizontal convection and this will effect on the vertically spreading of the temperature at the same time.

The above analysis indicates that the model is sensitive to to initial river water velocity.

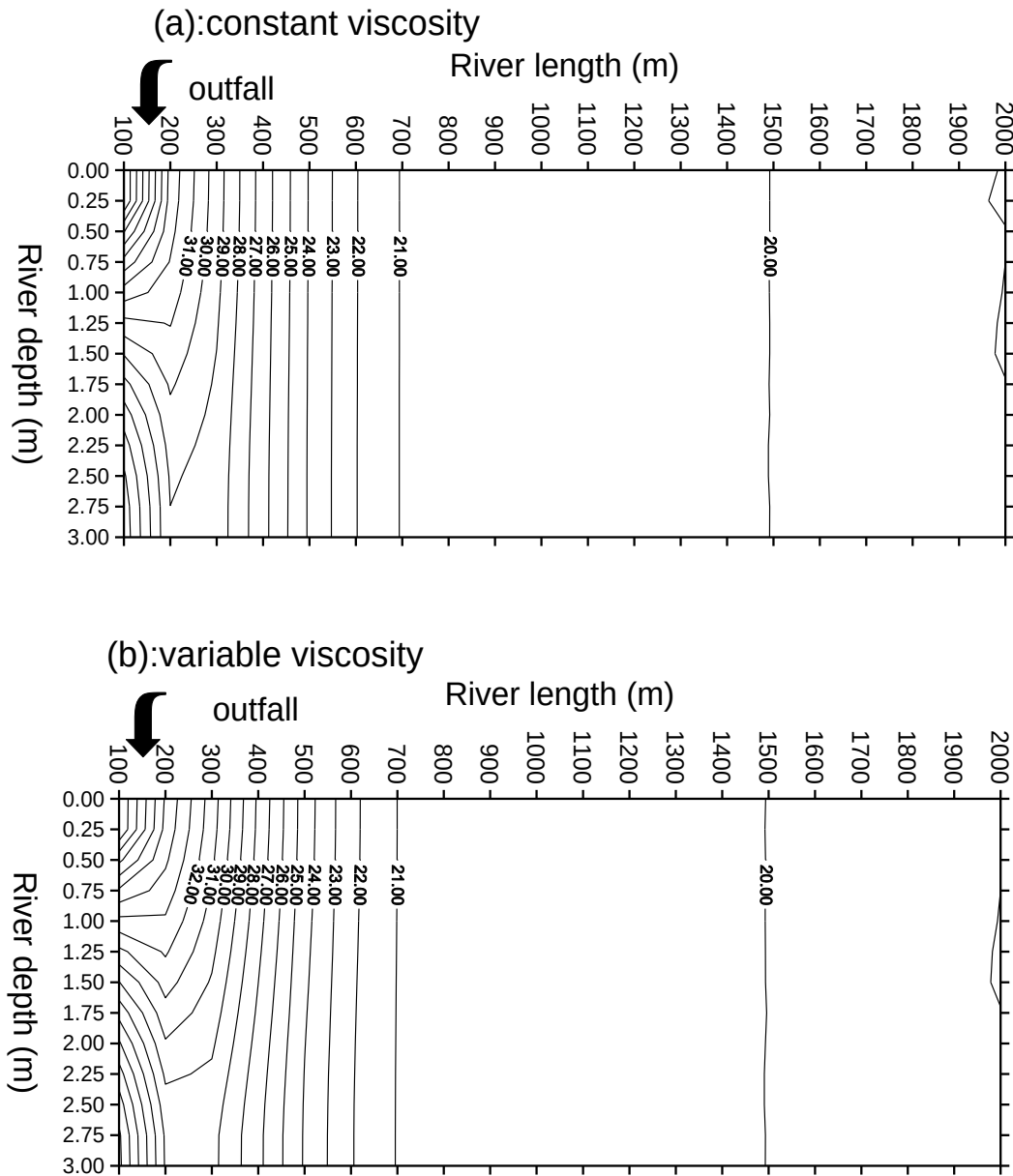


Fig .(4.11) :Effect of turbulent viscosity on temperature distriubution for surface discharge (i,0) ($T_r = 20\text{ C}$, $T_d = 40\text{ C}$, surface water slope = 0.00006 m / m , Time=400 sec, $U=0.8\text{ m/sec}$)

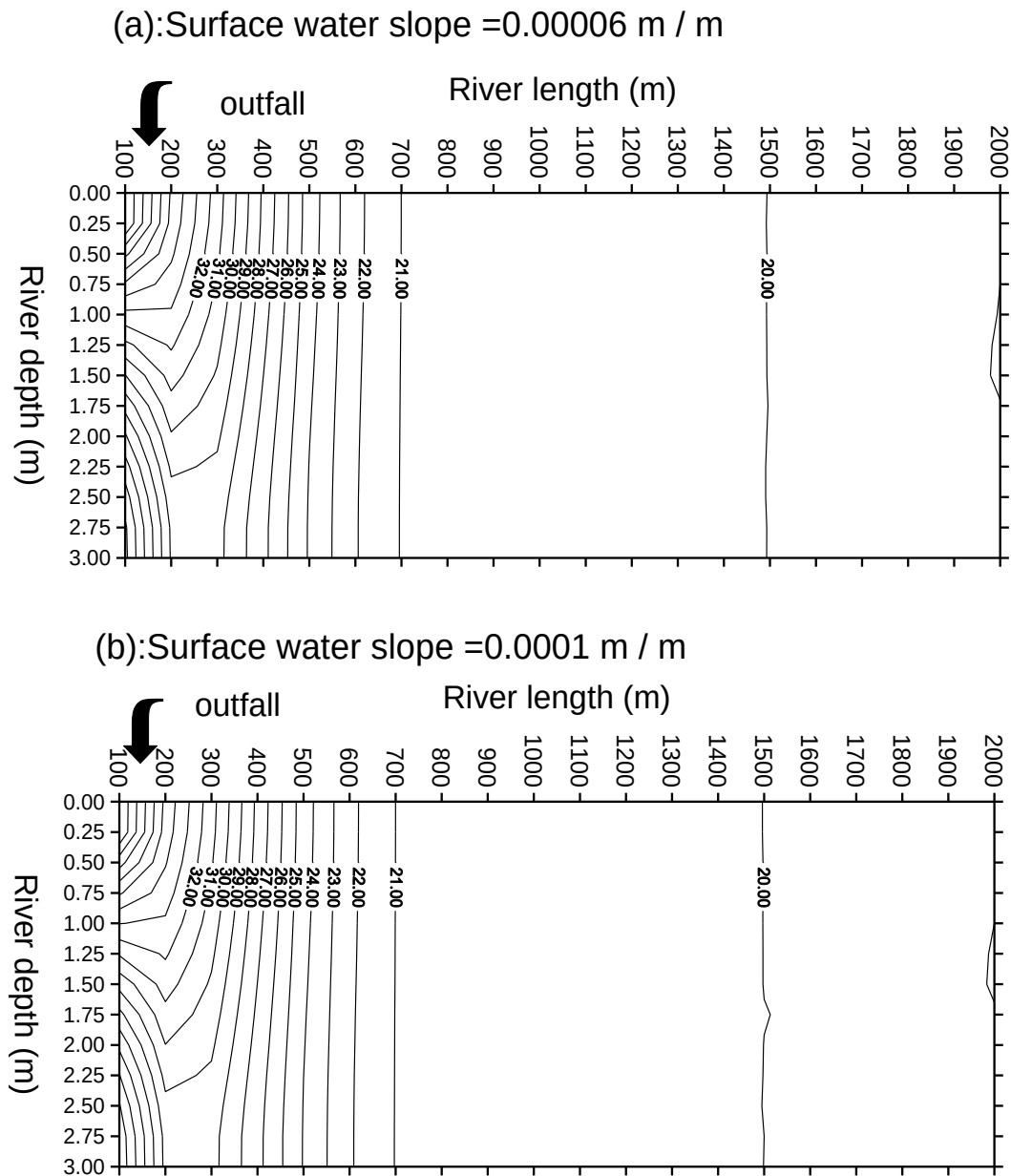


Fig .(4.12) : Effect of water surface slope on Temperature distribution surface discharge (i,0) ($T_r=20 \text{ C}$, $T_d=40 \text{ C}$, river water velocity = 0.8 m/sec , Time = 400 sec)

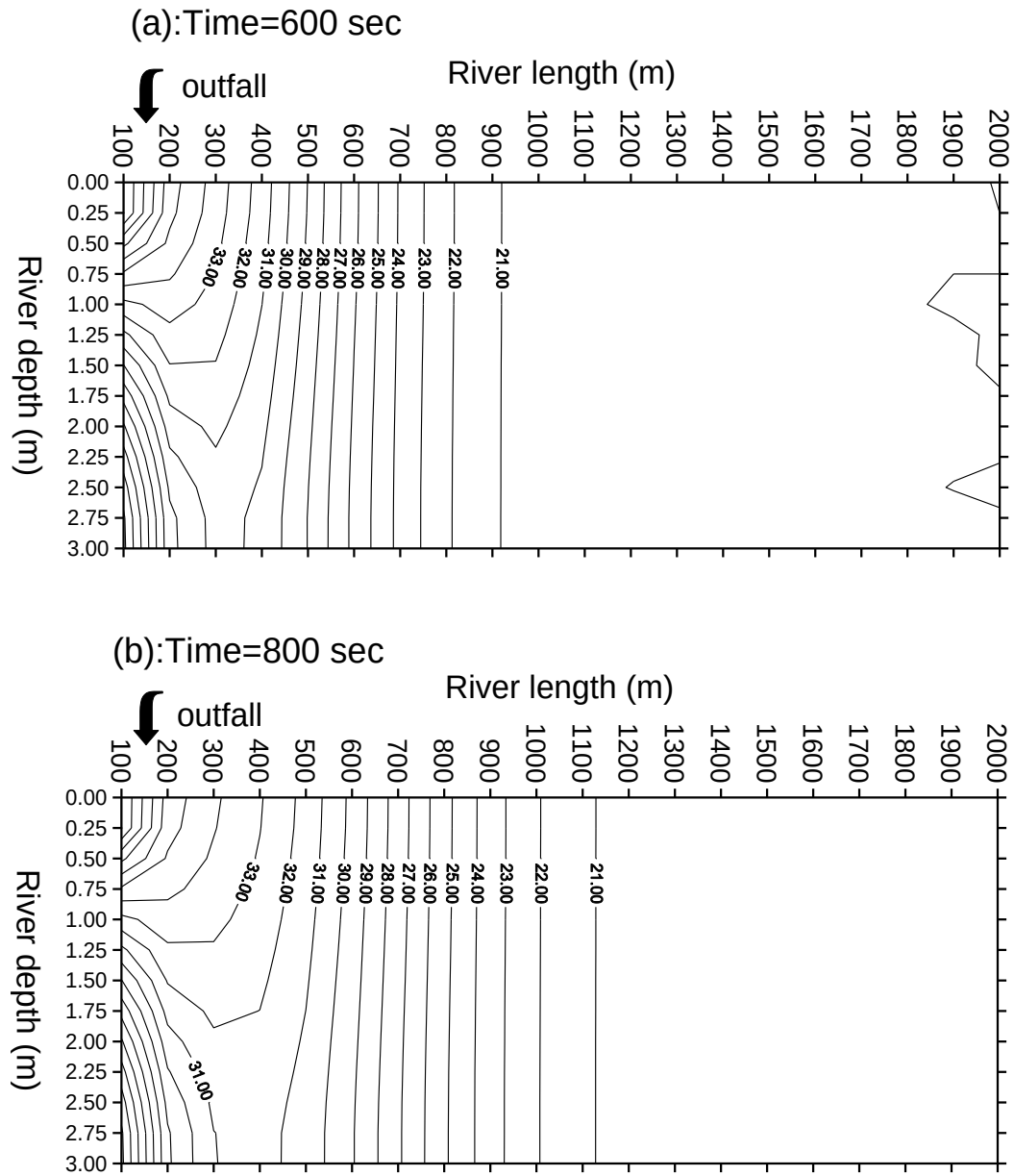
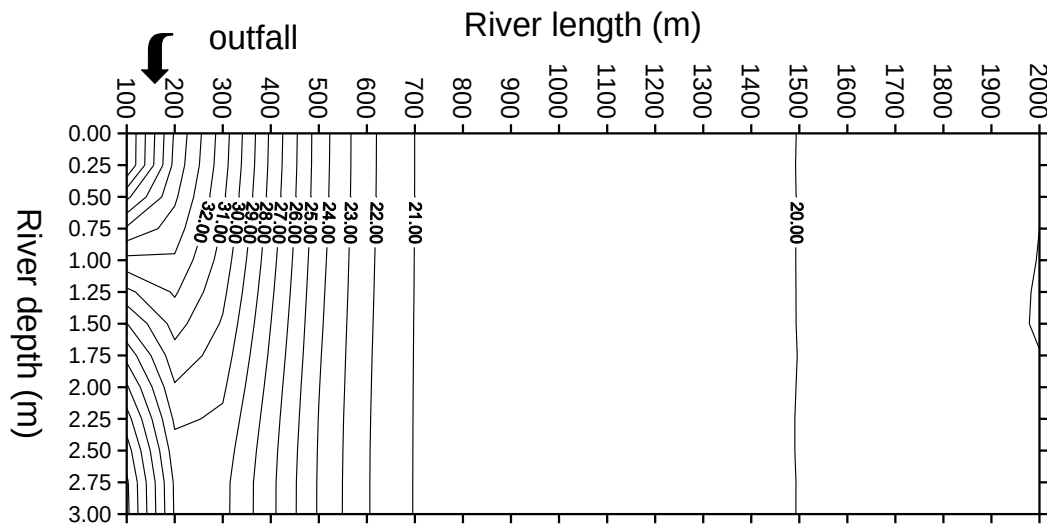


Fig .(4.13):Effect of time variation on Temperaure distribution (surface discharge (i,0) ($T_r=20$ C, $T_d=40$ C,River surface slope =0.00006 m / m, River velocity =0.8 m/sec)

(a): River water velocity = 0.8 m/sec



(b): River water velocity = 1 m/sec

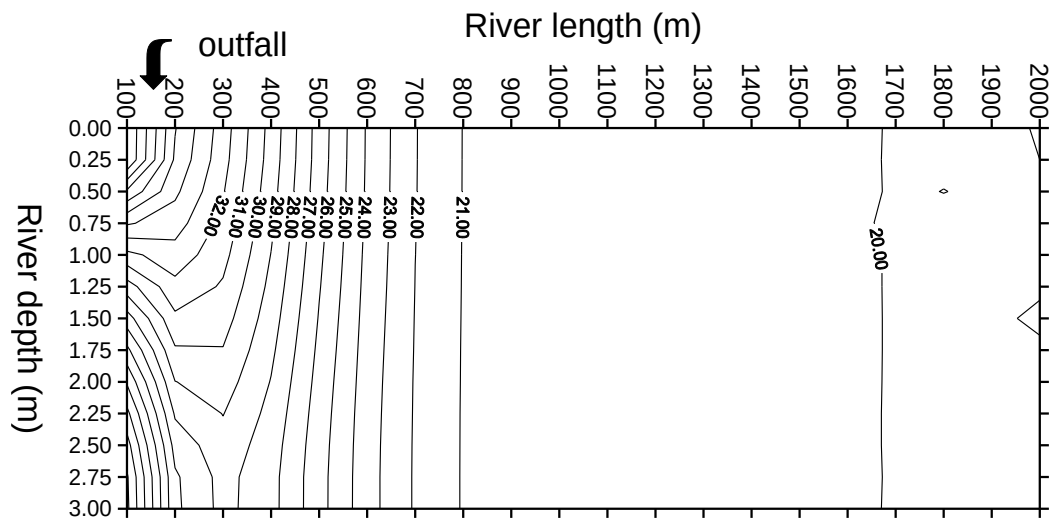


Fig .(4.14):Effect of initial River velocity on Temperaure distribution (surface discharge (i,0) ($T_r=20$ C, $T_d=40$ C,River surface slope =0.00006 m / m,Time = 400 sec)

4.4.2. Submerged Discharge:

Sensitivity analysis is applied on submerged discharge condition which consists of two different discharge cases. The first case of thermal discharge in outfall location at 0.42 from the river depth (1.26 m below water surface from theoretical river with total depth equal to 3 m), the second case of thermal discharge in outfall location at 0.83 from the river depth (2.49 m below water surface from the theoretical river with total depth equal to 3 m). The both submerged outfalls are as single-point source.

Several of the important model parameters (such as turbulent dynamic viscosity / diffusivity, river slope, variation of temperature distribution with time, effect of initial river water velocity) have been tested to investigate their effect on the temperature distribution. In order to conduct this, a continuous submerged disposal of a heated water with temperature equal to 40 °C, into a river with a temperature equal to 20 °C, the length of river is (2000) m and depth (3) m.

4.4.2.1 : (Outfall at 42% from the river depth):

4.4.2.1.1 Effect of Turbulent Dynamic Viscosity and Turbulent Diffusivity:

To examine the effect of turbulent/diffusivity on this case of submerged thermal discharge, a test was carried out assuming a constant turbulent dynamic viscosity (μ) /diffusivity (D_T) over the whole flow field by dropping the (k- ϵ) turbulence from the dispersion scheme in order to study the effect of (μ / D_T) on the spreading and mixing processes. The model was runned letting (μ / D_T) constant with time. Viscosity at ambient river water temperature is $\mu = 9.642$

N.s/m^2 . The value for DT was found to be 19.28 N.s/m^2 same as mentioned in previous case (surface discharge) as follows:

1. The temperature isotherms in case of assuming constant (μ / DT) are advanced longitudinally towards the water surface except isotherms at the submerged outfall, the advancing near the water surface is slightly more than from the second case of variable (μ / DT) , the isotherm 30°C for example is advanced from the outfall by about (320) m far away from the discharge region at the water surface in case of constant (μ / DT) . While in the second case the same isotherm advanced with (300) m. This can be attributed by assuming a constant viscosity which leads to neglect the effects of turbulence and in return this increased the horizontal convection in case of constant viscosity condition.

2. All isotherms are advanced vertically towards the river bottom, in case of assuming constant (μ / DT) , except isotherms near the outfall location as shown in Fig. (4.15) the isotherm (33°C) has inclined with curvature in case of variable (μ / DT) . While it is almost vertical in the other case. This can be attributed to the effect of strong vertical diffusion in advancing the isotherms vertically in the case of assuming constant (μ / DT) compared to the strong buoyancy forces and weak vertical diffusion in the other case. The effect of vertical diffusion has reduced in the region near the outfall location due to the greater production of turbulent kinetic energy and hence increase the ability of the temperature to advance into depth keeping the isotherms closer to each other in both cases.

The above analysis indicates that the model is sensitive to the change of viscosity /diffusivity values.

4.4.2:1:2 Effect of Water Surface Slope:

To investigate the effect of water surface slope on the temperature distribution, two different values of water surface slope were selected (0.00006 m / m and 0.0001 m /m).These values were selected from the values mention by (Euphrates Center for studying and design of irrigation projects, 1992) for Tigris river.

From Fig. (4.16) it can be found:

Increasing the water surface slope will cause an increase in the temperature levels at longitudinal stations .For example, the isotherm 31 °C is approximately at 270 , 300 m away from the submerged outfall for the slope (0.00006 m/m and 0.0001 m/m) respectively.

This is can be attributed since increasing the water surface slope causes an increase in the longitudinal velocity components , turbulent kinetic energy ,its rate of dissipation and in turbulent dynamic viscosity /diffusivity due to increase in friction velocity .Therefore ,the temperature levels increase as a result of increasing the longitudinal turbulent mixing, longitudinal convection ,and longitudinal diffusion. Therefore the buoyant spreading will increase as a result of temperature increment.

The above analysis indicates that the model is sensitive to the changes in the river velocity.

4.4.2:1:3 Variation of Temperature Distribution with Time:

The effect of time variation on the spreading of temperature in river water were studied, two values of time are selected (600 sec, and 800 sec). With constant value of ($t_d = 40^\circ\text{C}$ and $t_r = 20^\circ\text{C}$) for temperature, and river velocity equal to 0.8 m/sec. The results are shown in Fig. (4.17) which illustrates that:

Increasing the time of disposal causes a significant effect on the longitudinal spreading of the released discharges. For example, the isotherm of 35°C has reached to distance about 220 m, 300m at water surface at time 600 sec, and 800 sec respectively. This can be attributed to the buoyancy forces and longitudinal convection which has sufficient effect when time increase.

The above analysis indicates that the model is sensitive to the time variation.

4.4.2:1:4 Effect of Initial River Water Velocity:

The initial river water velocity was also increased in case of the submerged discharge disposal (first case) from 0.8 m/sec to 1 m/sec, to examine the river velocity effect on the temperature transport, the other input data for this run is the river water slope equal to 0.00006 m/m, $t_r = 20^\circ\text{C}$, $t_d = 40^\circ\text{C}$, for river reach length equal to 2000 m and river reach depth=3 m. From Fig. (4.18) it can notice that:

Increase the temperature values at the longitudinal stations from the submerged outfall below the water surface with increasing the river velocity from 0.8 m/sec to 1 m/sec, for example the isotherm 27°C is about 390 m from the discharge outfall with initial river velocity = 0.8 m/sec while this distance become 445 m with increasing the initial river velocity to 1 m/sec Fig. (4.18), and at the same time the vertical spreading is reduced with increasing the initial river velocity

for example the isotherm 34 °C is retarded near the outfall with increasing the river velocity to 1 m/sec ,while the isotherm reached the river bottom at initial river velocity equal to 0.8 m/sec.

This can be attributed to the effect of initial river velocity on increasing the horizontal convection with increasing the initial river velocity and effecting on the vertical turbulent mixing at the same time.

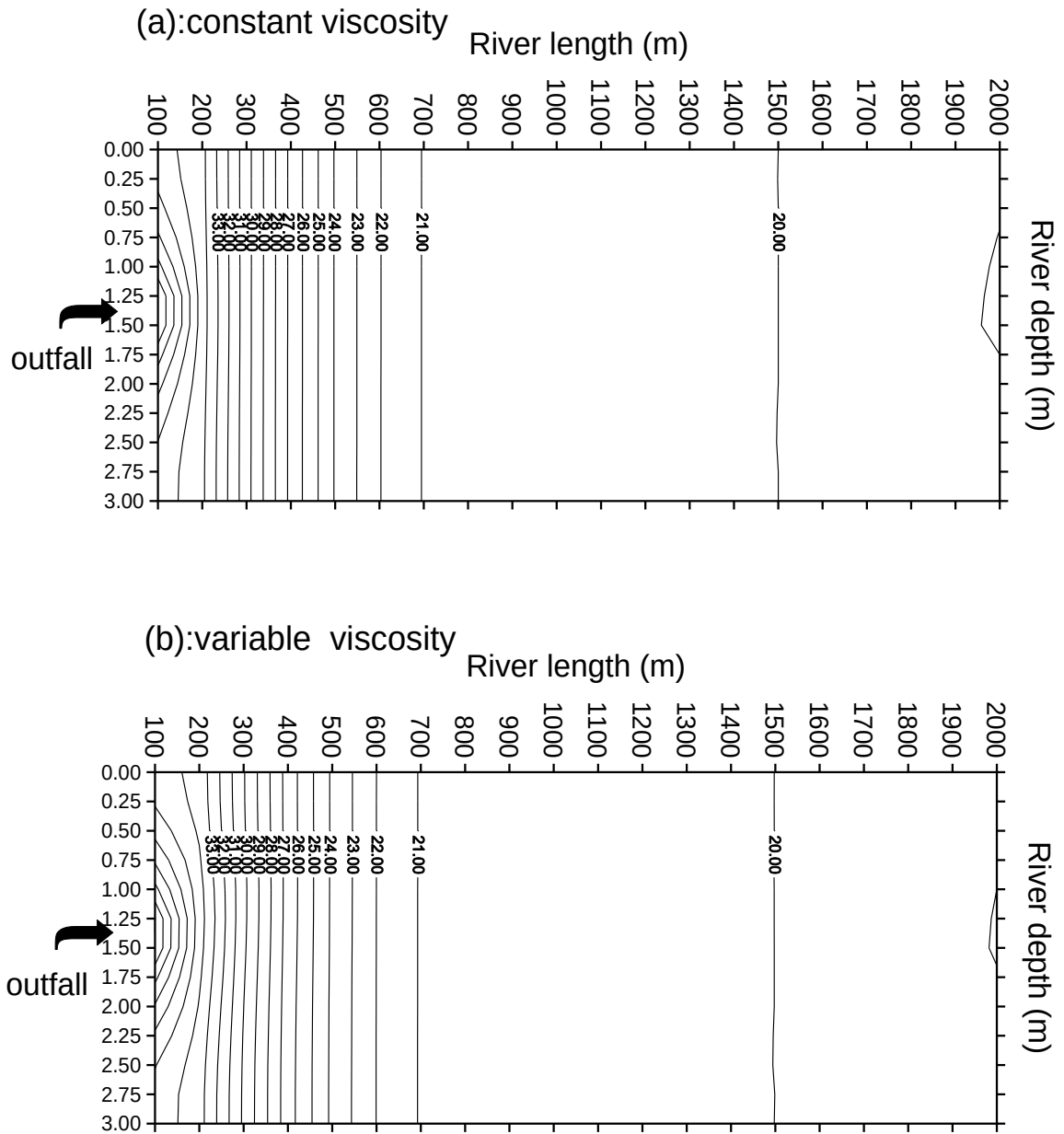


Fig. (4.15) :Effect of turbulent viscosity on temperature distribution for submerged discharge condition (first case) (outfall at 0.42 from the river depth) ($T_r = 20\text{ }^{\circ}\text{C}$, $T_d = 40\text{ }^{\circ}\text{C}$, $S = 0.00006\text{ m/m}$, $U = 0.8\text{ m/sec}$, $\text{Time} = 400\text{ sec}$)

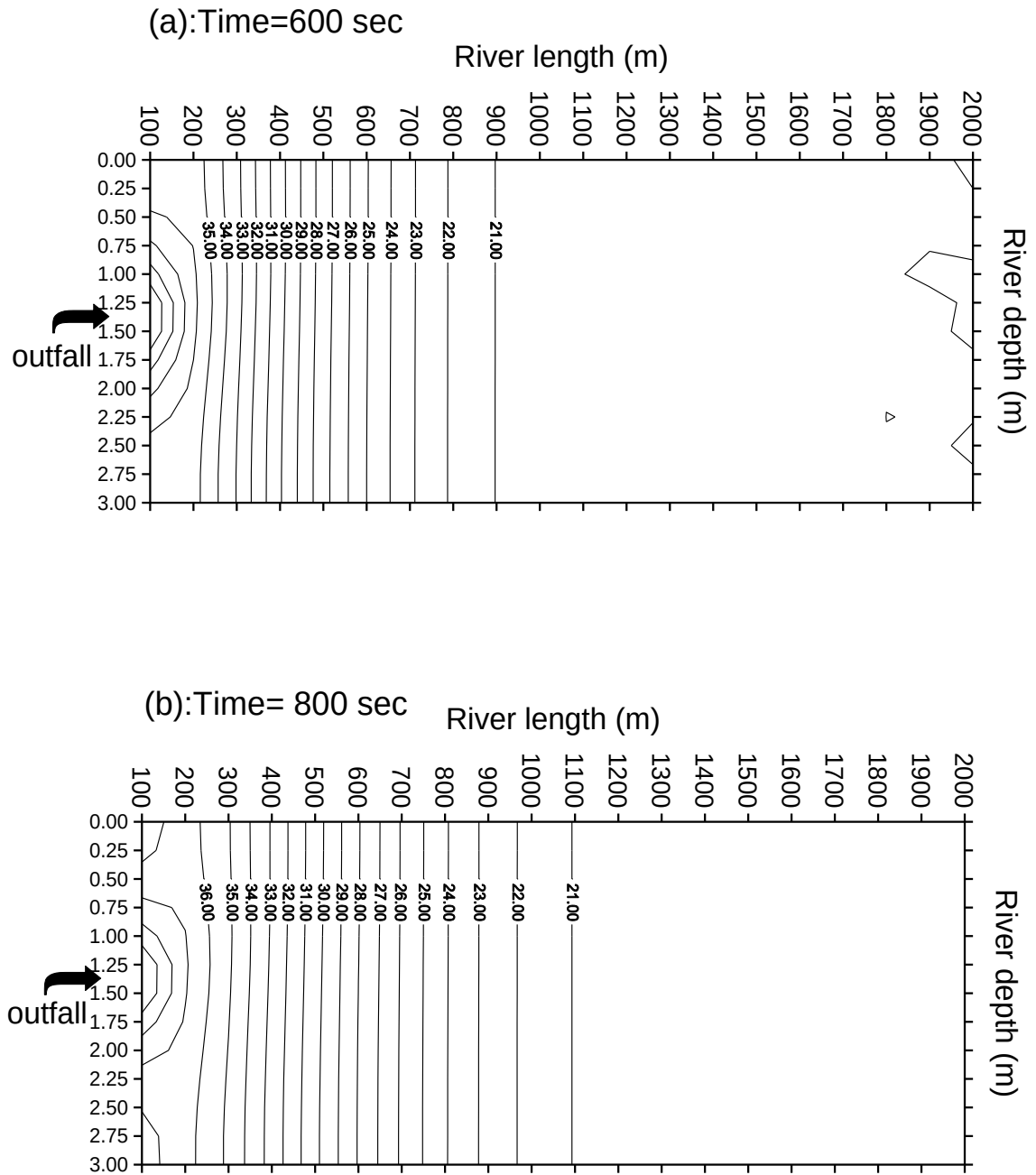
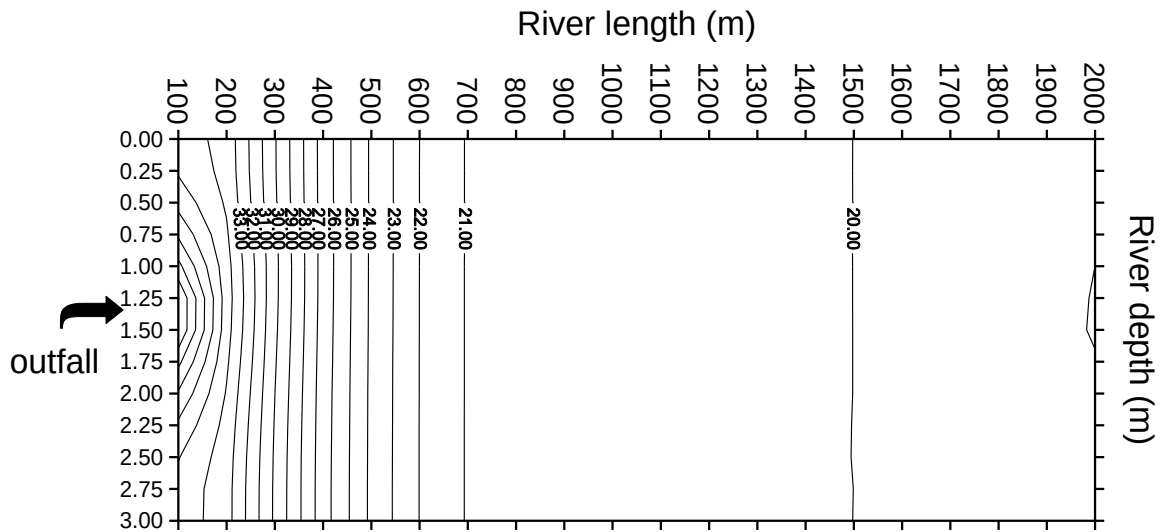


Fig.(4.17) :Effect of time variation on Temperaure distribution (submerged discharge) (first case)
(outfall at 0.42 from thr river depth)(Tr=20 C,Td=40 C,S =0.00006 m / m, U =0.8 m/sec)

(a): River water velocity = 0.8 m / sec



(b): River water velocity = 1 m/sec

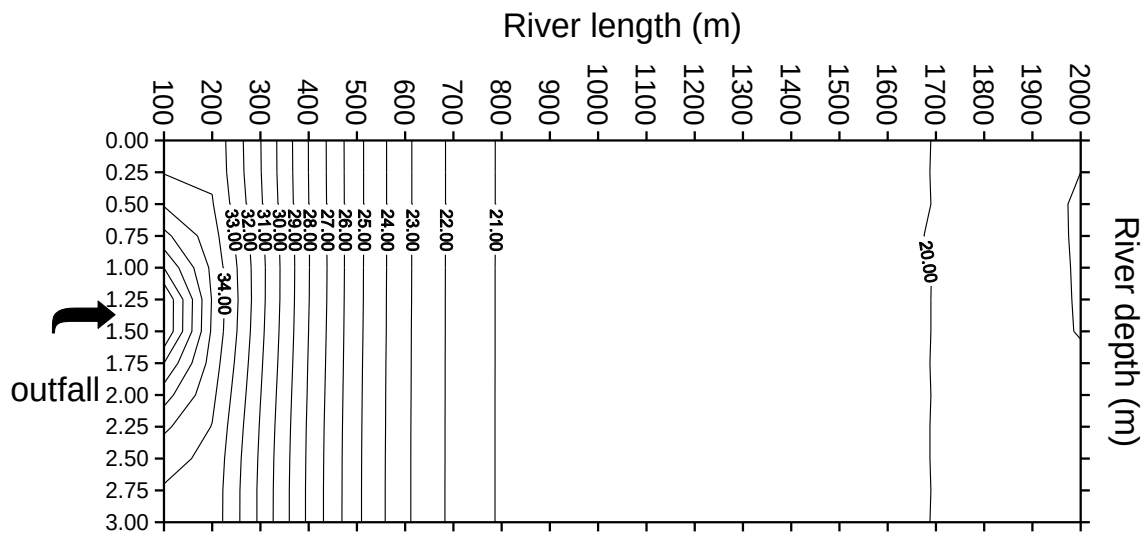


Fig.(4.18) : Effect of initial River velocity on Temperature distribution (submerged discharge (first case) (0.42 from the river depth)(Tr=20 C,Td=40 C,River surface slope =0.00006 m / m,Time = 400 sec)

4.4.2.2 (Outfall at 83% from the river depth):

4.4.2:2:1 Effect of Turbulent Dynamic Viscosity and Turbulent Diffusivity:

To examine the effect of turbulent viscosity /diffusivity on temperature distribution resulting from submerged discharge outfall as single-point source lies at 0.83 from the river depth, the $(k-\varepsilon)$ turbulence model is dropped out from numerical model results in order to study the effect of (μ / DT) on the spreading and mixing processes.

The model was runned letting (μ / DT) constant with time .Viscosity at ambient river water temperature is $\mu = 9.642 \text{ N.s/m}^2$, the value for DT was found to be 19.28 N.s/m^2 same as mentioned in previous case (surface discharge) as follows:

1. The temperature isotherms in case of assuming constant (μ / DT) are advance longitudinally near the river bed, while advancing longitudinally at the water surface. The temperature values at the horizontal stations in case of constant (μ / DT) Fig. (4.19 a) is obtained with more distance at the water surface than its values in case of variable (μ / DT) (e.g. the isotherm 25°C is about 300 m at the water surface in case of constant (μ / DT) while this isotherm is obtained at distance about 230 m in case of variable (μ / DT) . This can be attributed to effect of turbulence and buoyancy on the temperature spreading which occur fast in case of constant μ / DT as compared to the slow advancing in other case due to increase in the horizontal convection and decrease in the vertical spreading at the same time in case of constant μ / DT .

2. In the vertical direction all temperature isotherms are advanced towards the water surface (except the isotherms near the outfall location), Fig. (4.19). Also the vertical advancing in case of variable (μ / DT) is retarded extremely because of the buoyancy influence which reduce the vertical diffusion and increase the ability of fluid to entrain which reduce the temperature difference between the discharge water and its surroundings. While in case of constant (μ / DT) the turbulence and buoyancy effects are neglected and hence the temperature isotherms spreads and mixes in vertical direction by the vertical diffusion only which is less than the vertical convection resulting from turbulence and buoyancy.

The above analysis indicates that the submerged discharge model (second case) is sensitive to the change of viscosity /diffusivity values.

4.4.2:2:2 Effect of Water Surface Slope:

The effect of water surface slope on the submerged temperature distribution is studied. Two different values of water surface slope are selected (0.00006 m / m and 0.0001 m /m); these values were selected from the values adopted by (Euphrates center for studying and design of irrigation projects, 1992) for Tigris River. And the other data of this run is ($t_r = 20^\circ \text{C}$, $t_d = 40^\circ \text{C}$, river initial velocity = 0.8 m/sec, time of disposal = 400 sec).

From Fig. (4.20) it can be found:

Increasing the river water slope in case of submerged discharge condition will extend the temperature values at the horizontal stations near the submerged outfall more than at the water surface (e.g. the 25°C is at distance 310 m downstream from the discharge region near the river bed in case of surface water slope equal to

=0.00006 m/m. While this distance extend to be about 370 m in case of increasing the slope to 0.0001 m/m) .This can attributed to the effect of water discharge buoyancy and horizontal convection which increased due to the increase in water surface slope and lead to extend the temperature values at all horizontal stations.

The present model is sensitive to the change in water surface slope.

4.4.2:2:3 Variation of Temperature Distribution with Time:

The time variation effect on the spreading of temperature in river water was tested. Two values of time are selected (600 sec, and 800 sec). With constant value of ($t_d = 40^\circ\text{C}$ and $t_r = 20^\circ\text{C}$) for temperature, and river velocity equal to 0.8 m/sec. The results of this run are shown in Fig. (4.21) which illustrates that:

By increasing the time of disposal the temperature horizontal spreading is increased significantly due to the increase in horizontal convection (e.g. at time 600 sec the temperature isotherm 27°C is at distance 210 m at water surface while this distance is become about 280 for the same temperature isotherm when the time increased to be 800 sec).

The above analysis indicates that the submerged discharge model (second case) is sensitive to change the time of temperature disposal.

4.4.2:2:4 Effect of Initial River Water Velocity:

The effect of increasing the initial river water velocity is studied in case of the submerged discharge disposal (second case). Therefore the river velocity is increased from 0.8 m/sec to 1 m/sec, to examine the river velocity effect on the temperature transport. The other input data for this run is the river water slope equal to 0.00006 m/m, $t_r = 20^\circ\text{C}$, $t_d = 40^\circ\text{C}$, for river reach length equal to 2000 m and river reach depth=3 m from as shown in Fig. (4.22) it can be notice that:

Increasing in the temperature values at the longitudinal stations from the submerged outfall below the water surface with increasing the river velocity from 0.8 m/sec to 1 m/sec, for example the isotherm 26°C is about 280 m from the discharge outfall near the river bed with initial river velocity = 0.8 m/sec while this distance become 300 m with increasing the initial river velocity to 1 m/sec Fig. (4.22). And at the same time the vertical spreading is retarded with increasing the initial river velocity .This can be attributed to the effect of initial river velocity on increasing the horizontal convection with increasing the initial river velocity and effecting on the vertical turbulent mixing at the same time.

The above analysis indicates that the submerged discharge model (second case) is sensitive to change the initial river water velocity.

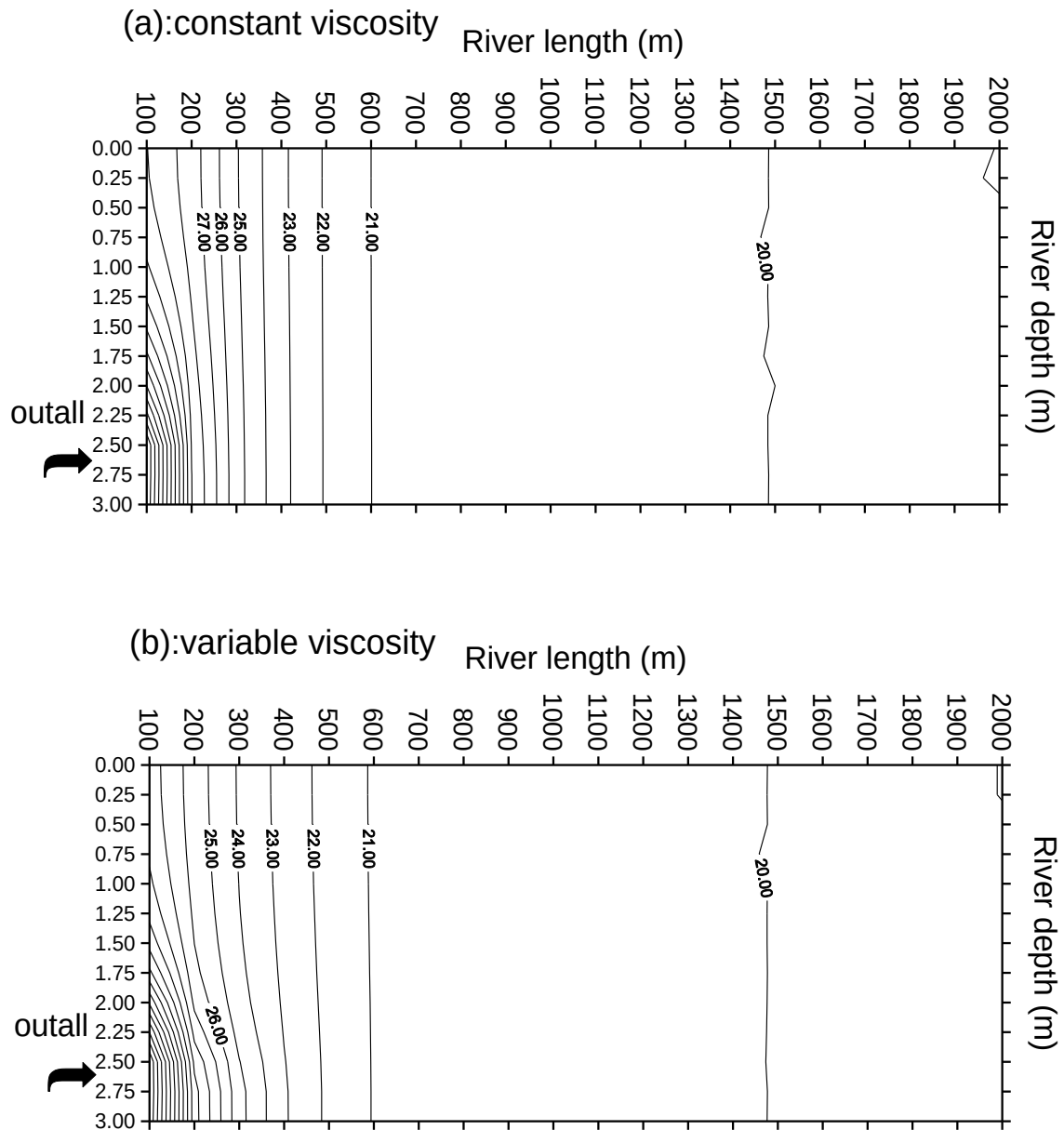
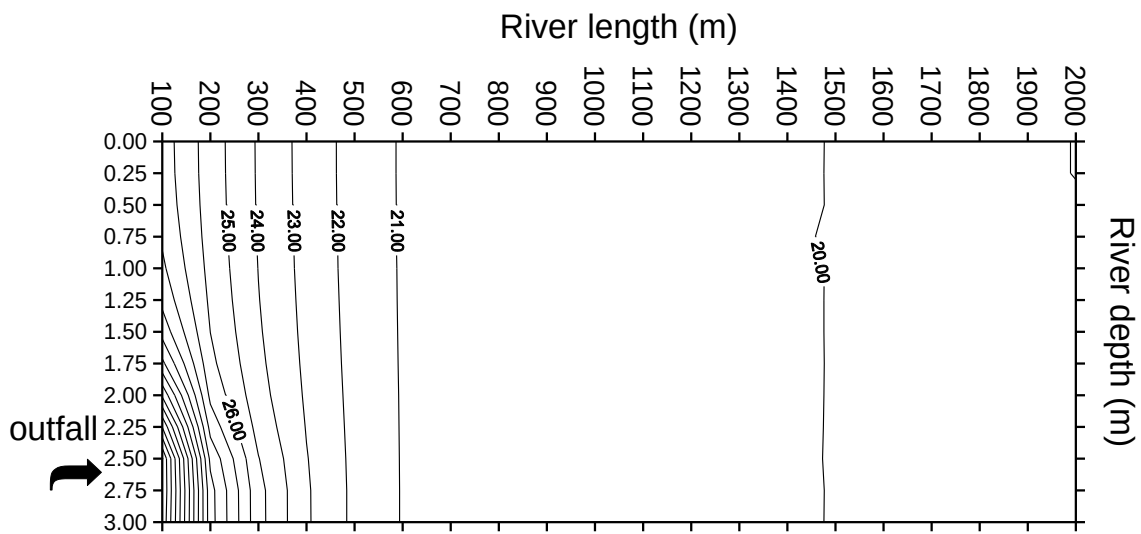


Fig. (4.19) :Effect of Turbulent viscosity on temperature distribution for submerged discharge (second case) (outfall at 0.83 from the river depth) ($T_r = 20^\circ\text{C}$, $T_d = 40^\circ\text{C}$, $S = 0.00006\text{ m/m}$, $U = 0.8\text{ m/sec}$, $\text{Time} = 400\text{ sec}$)

(a): water surface slope = 0.00006 m/m



(b): water surface slope = 0.0001 m/m

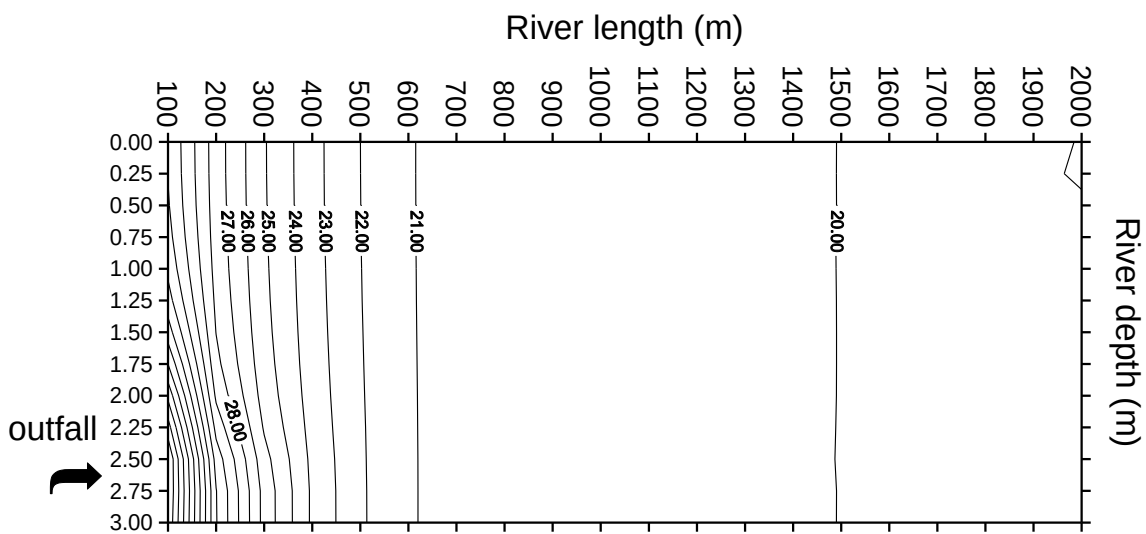


Fig.(4.20) : Effect of water surface slope on Temperature distribution submerged discharge (second case) (outfall at 0.83 from the river depth) ($T_r=20$ C, $T_d=40$ C, $U = 0.8$ m/sec , Time=400 sec)

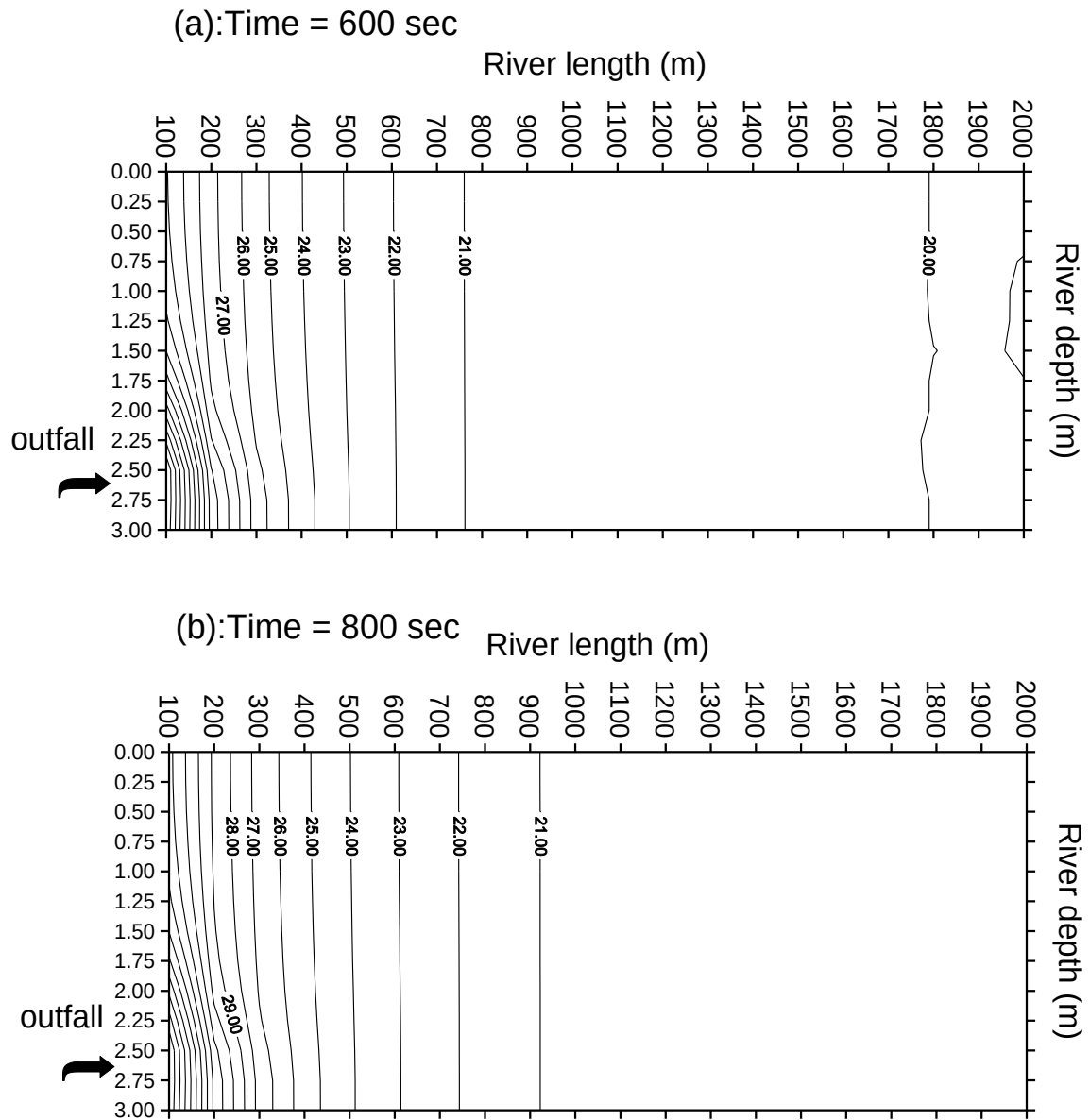
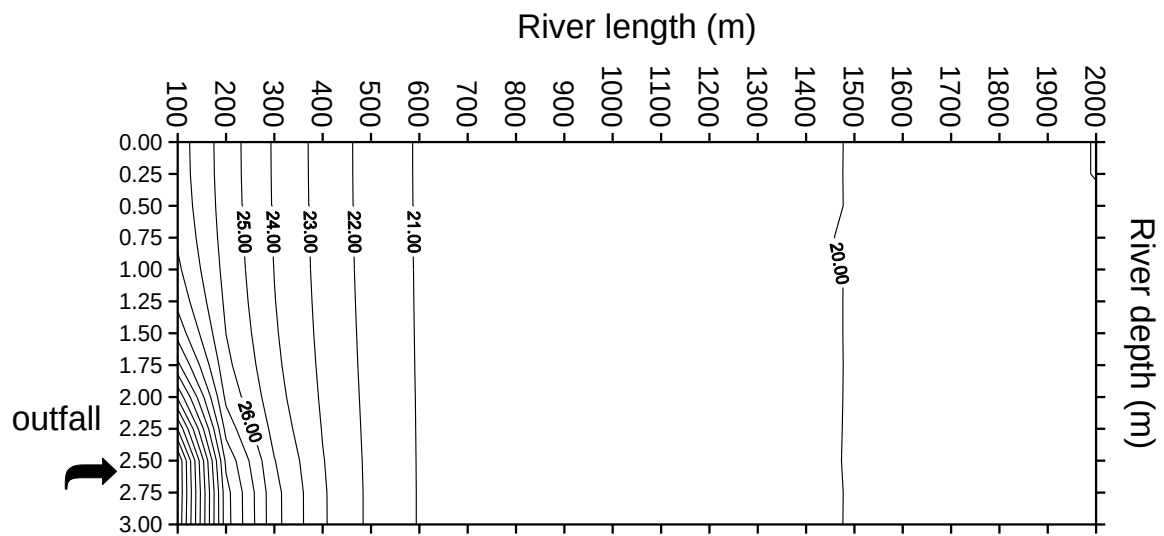


Fig .(4.21) :Effect of time variation on Temperaure distribution (submerged discharge)
(second case) (outfall at 0.83 from river depth)(Tr=20 C,Td=40 C, S =0.00006 m / m, U = 0.8 m/sec)

(a): River velocity = 0.8 m / sec



(b): River velocity = 1 m / sec

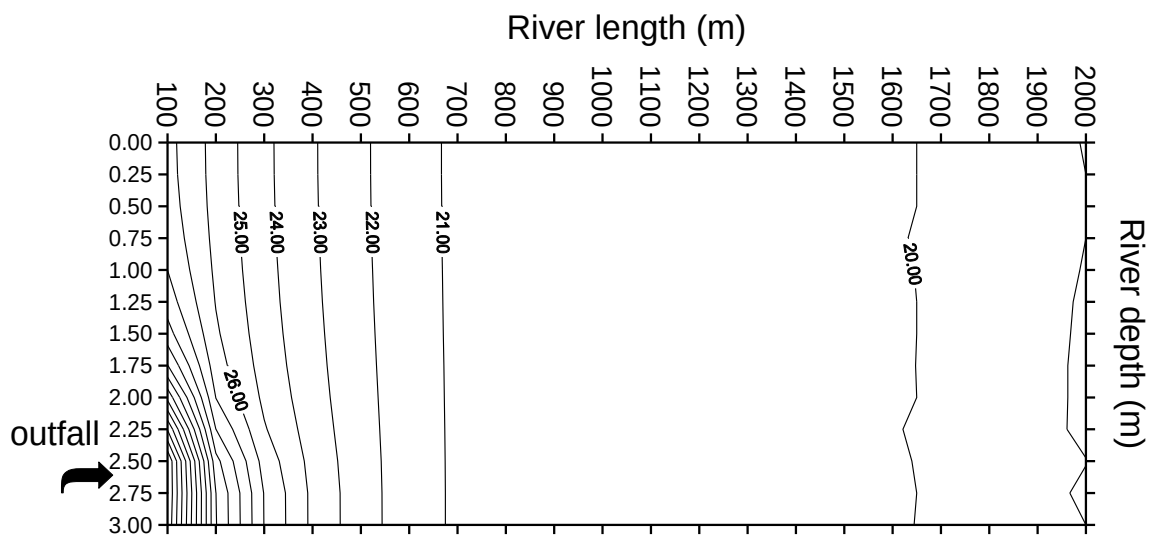


Fig. (4.22) :Effect of initial River velocity on Temperature distribution (submerged discharge)
(second case) (outfall at 0.83 from the river depth)(Tr=20 C,Td=40 C, S =0.00006 m / m,Time = 400 sec)

4.5 Rapid Environmental Assessment for the Thermal Discharge

Schemes:

The river water is the most biologically productive and economically important areas, contains a countless numbers of living organisms among them the bacteria, which involved in the productivity of the river water system through their participation in the flow of energy and cycling of materials. Bacteria can affect the fate of many contaminants in the river by contribution to their removal through uptake, degradation (for organic contaminants), transformation (metals). Thermal discharges through three different cases (surface and two submerged cases) to the river stream are applied. The effect of these cases on the bacteria microbiological activities is studied to obtain the best thermal disposal case with minimum effect on bacteria population.

For surface disposal case:

- Distinct populations of bacteria inhabit along the water column of the river, surface disposal of heat on the river water stream may have several consequences on bacteria activity range from stimulation to inhibition depending on the load of heat water.
- For submerged disposal cases:
- The densities of bacteria at river bed sediments are less from the amounts of bacteria lying at the water surface by about 100 to 10,000 times less per unit volume (N.L, Nemerow 1971).
- The decision for best thermal disposal outfall is conducted according to the following results:

Comparisons between these discharge locations according to the maximum dilution factor achieved for each outfall are

established. The maximum dilution factor used to conduct this comparison is (0.95).

The dilution factor isotherms for two time steps normally 500, and 750 sec for the three discharge cases are adopted, with initial river velocity equal to 0.8 m/sec and river surface slope equal to 0.00006 m/m.

a. Time of disposal = 500 sec

From Figs. (4.23 a), (4.24 a) and (4.25 b) it can be concluded:

1. Maximum dilution factor for the surface disposal case (0.95) is obtained at distance 805 m from the discharge region, while this distance becomes (795 m., 680 m) for the first and second case of submerged disposal.
2. The dilution level near the discharge region at distance 300 m for example is (0.35, 0.4, and 0.7) for the surface and submerged thermal discharge (first and second case) respectively.

b. Time of disposal = 750 sec

From Figs. (4.23 b), (4.24b) and (4.25 b) it can be concluded:

1. Optimum dilution factor value (0.95) is at distance 1097 m for the surface discharge case, while this distance is (1055, 890 m) for the submerged discharge (first and second case) respectively.
2. The dilution level near the discharge region at distance 400 m for example in the three thermal discharge cases is (0.35, 0.37, and 0.75) for the surface and submerged thermal discharge (first and second case). respectively.

The above results indicate that the outfall location near the river bed (0.83 from the river depth) achieves the maximum dilution factor value (0.95) with smaller distance than from outfall lies at the surface and outfall locates at 0.42 from the river depth.

The dilution level achieved near the discharge region is extremely large for outfall lies near the river bed (0.83 from the river depth) compared to that in outfalls lie at the river water surface and 0.42 from the river depth.

The outfall near the river bed gives the minimum environmental impact on the river microorganisms especially the bacterial activities which are very sensitive to the temperature variations. Achieve the maximum dilution, and the river water recovers fast from the thermal shock.

The outfall at deeper levels (0.83 from the river depth) according to the above results is the best for the river environment.

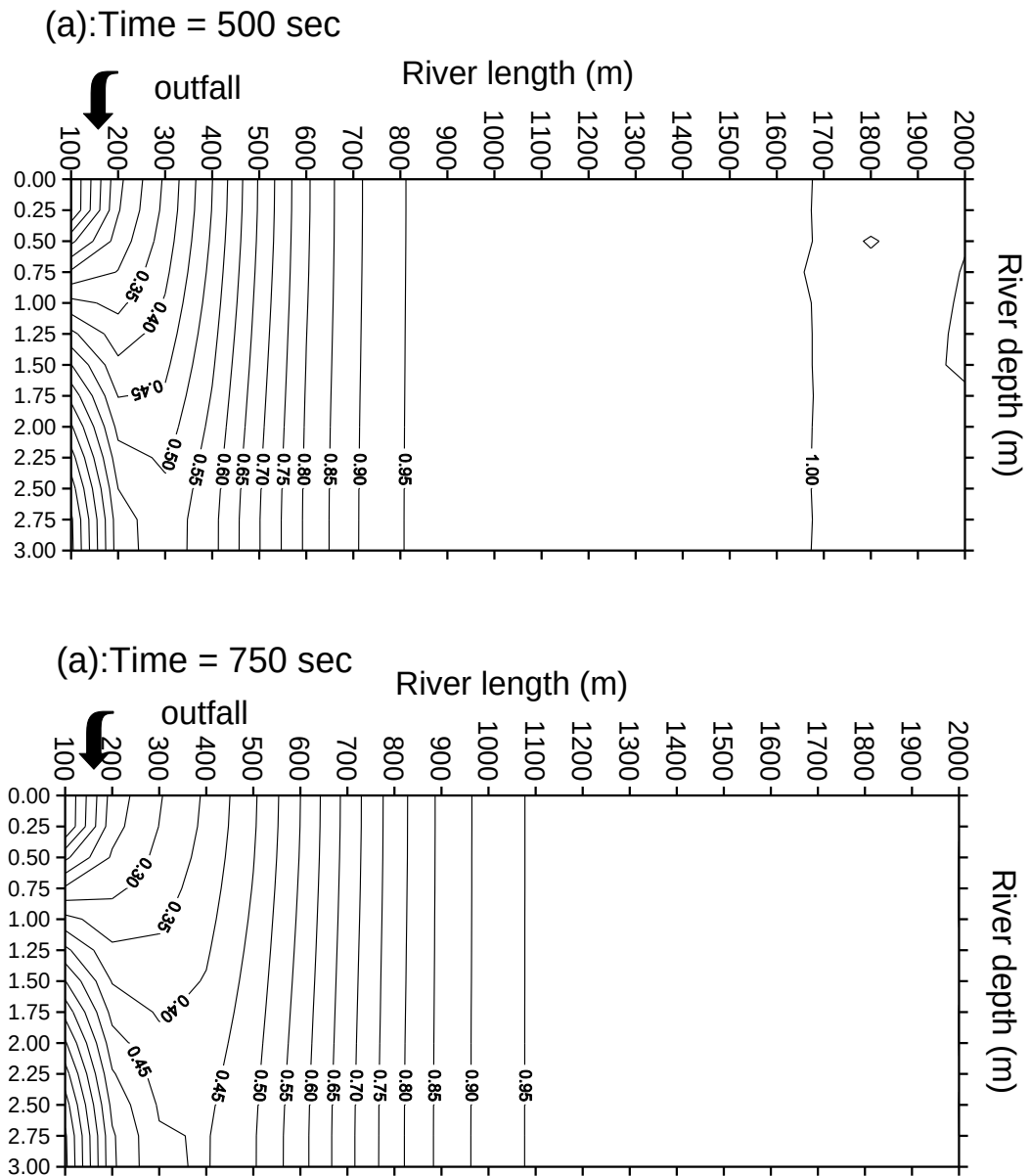


Fig (4.23) : Dilution Factor Distribution of Temperature isotherms (surface discharge)
($T_r=20^\circ\text{C}$, $T_d=40^\circ\text{C}$, Surface water slope $=0.00006\text{ m/m}$, $U=0.8\text{ m/sec}$)

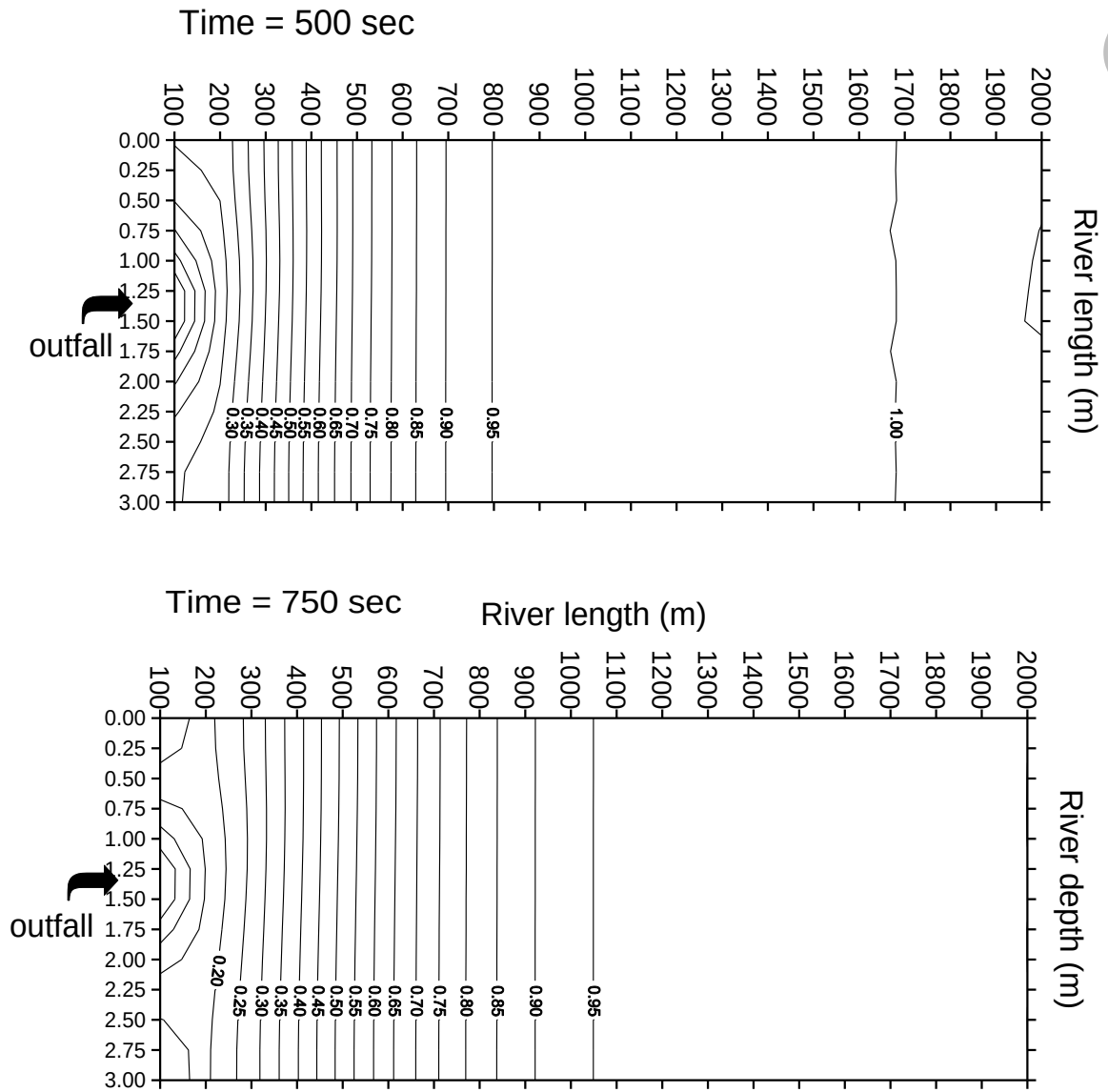


Fig.(4.24) : Dilution Factor Distribution of Temperature isotherms (first submerged discharge case)
 (outfall at 0.42 from the river depth) ($T_r=20\text{ C}$, $T_d = 40\text{ C}$, Surface water slope $=0.00006\text{ m/m}$, $U=0.8\text{ m/sec}$)

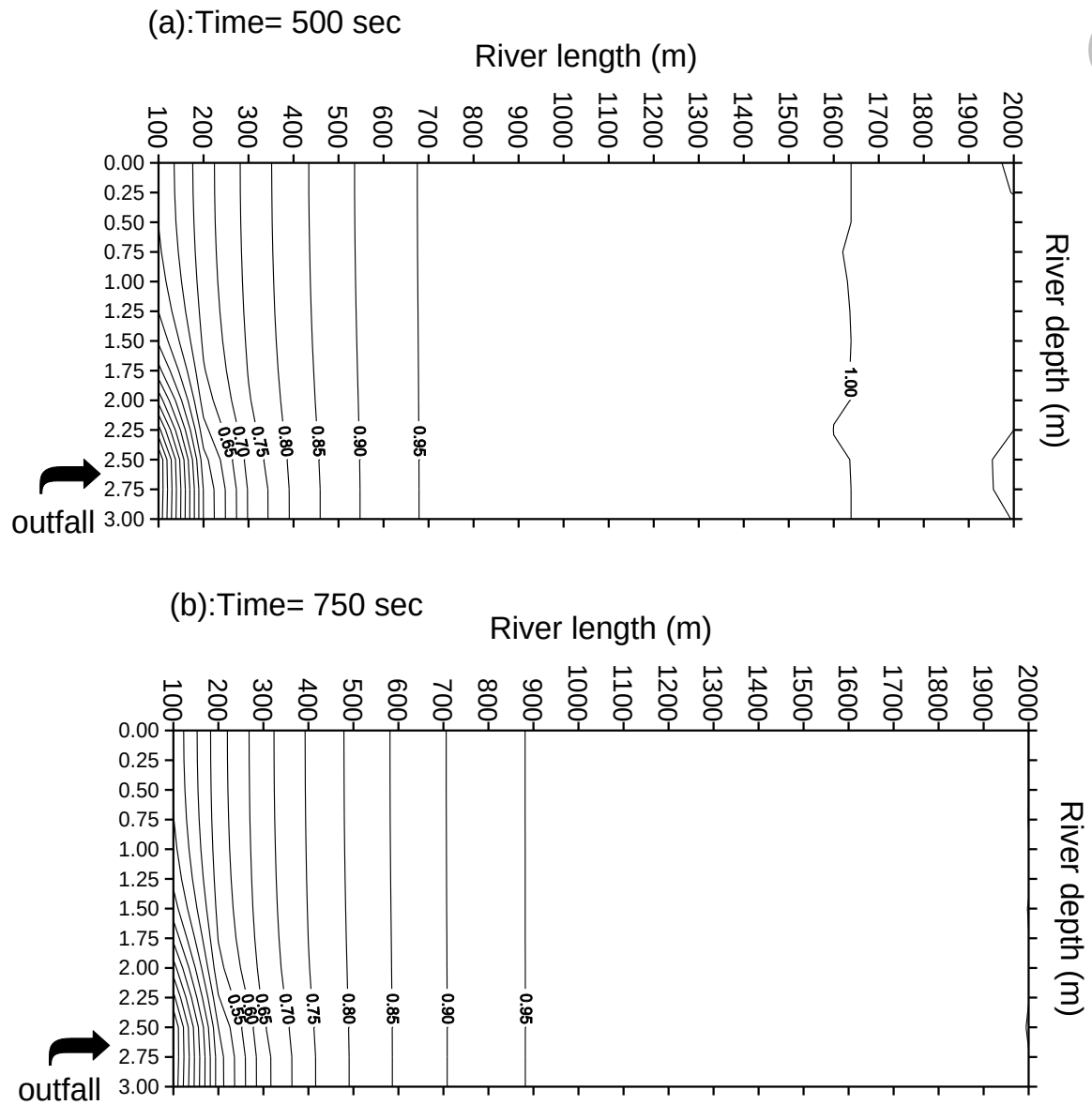


Fig . (4.25) : Dilution Factor Distribution of Temperature isotherms (second case of submerged discharge)
 (outfall at 0.83 from the river depth)(Tr=20 C , Td = 40 C ,Surface water slope =0.00006 m/m , U =0.8 m/sec)

Chapter Five

Conclusions and Recommendations

5.1 Conclusions:

The following conclusions are conducted from the present model development with all its assumptions, the conclusions are:

1. In cases where non-unidirectional vertical velocity components take place, it is necessary to apply the up-winding scheme (upstream differencing) for the solution of the convective terms in the governing differential equations in order to obtain a positive values for the coefficients a, b, and c in this equations and the solution become more stable.
2. The submerged thermal discharge for outfall lies near the river bed (0.83 from the river depth) is the best for the river environment causes an earlier formation of uniform vertical temperature faster from that in the outfall lies at 0.42 from the river depth, and the maximum curvature for the thermal plume occur at deep levels inside the river depth for both submerged discharge cases.
3. The submerged outfall at 0.42 from the river depth is not a favorable option for discharging heat because of low dilution levels achieved compared to the surface disposal scheme and navigation purposes for the rivers.

4. The dilution factor distribution gives the specific temperature value for any node inside the flow domain straightly from the contour map according to equation no. (2.32), which avoid the computer programming disturbance.
5. An assumption of constant turbulent dynamic viscosity / diffusivity in the numerical model over the whole flow field causes a longitudinal retardation of isotherm at and near the water surface, but the vertical advanced of these isotherms near the bottom of the river when the thermal discharge disposed at the river surface .While it causes a longitudinal advancing of isotherms towards the water surface, and vertically advancing towards the river bottom in case of thermal discharge dispose below the water surface (submerged discharges).
6. The model is insensitive to the variations of water surface slope in case of surface water discharge. While the model is sensitive to the variations in water surface slope, the increasing of the river surface slope in case of submerged thermal discharges at depth 0.42 and 0.83 from river depth leads to increase in temperature levels at the longitudinal stations.
7. The numerical model is sensitive to the variation in time of disposal, for surface and submerged discharges. As the time of disposal increases the longitudinal and vertical thermal spreading increases also.
8. The model is sensitive to the increase in initial river water velocity which leads to a distinguished increase in temperature levels at the

longitudinal stations and a decrease in the vertical spreading at the same time. This is because of the river velocity that affects on the convection mechanism. Hence, the increase in river velocity leads to increase in the longitudinal convection and this affects on the vertical thermal spreading.

According to the comparison results between submerged disposals with a surface disposal, a submerged disposal of thermally polluted water in the direction of the river flow causes:

- A. Greater increase in the river temperature at deeper levels.
- B. Bending of isotherms at deeper levels.
- C. Longitudinal advance of low temperature isotherms
- D. Longitudinal retardations of high temperature isotherms.
- E. Earlier formations of uniform vertical temperature distribution.

5.2 Recommendations:

The following recommendations are suggested for the future studies:

1. Experimental laboratory work /and or field work are suggested to support the theoretical investigation.
2. The model can be extended to study the effect of multi-point sources of thermal discharge on the bed material resistance and sediment transport.
3. Apply the present model by using a multi-port tee diffuser at the river bed for thermal discharge as submerged disposal.
4. Development of a three-dimensional numerical model of single-point source disposal for thermal pollution in river.

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