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An Approximation Solution of Transportation Problems

A Thesis

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By

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Dedication

To the master of creation.... and the lover of truth... The Prophet Muhammad (Allah bless him and his family and grant them peace) To the one who planted in me the love of knowledge and made efforts over the years to put me on the path of prosperous life .I dedicate it cientifically and practically to dear dad To the one who surrounded me with the wall of her tenderness and humiliated me with her prayers I give you my whole life without regretmy dear mother To the one who carried the burdens with me and kept helping me my wife I dedicate this precious effort.

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Table 1: LIST OF SYMBOLS

NO.	THE SYMBOLS	DESCRIPTION
1	a_i	Available quantities of each m capacity
2	b_j	Available quantities for each n requirements
3	c_{ij}	The cost of transshipme one quantity of goods than origin i to destination j at every path in TP. The cost to assignment of i^{th} resource (worker) to j^{th} task (job) in AP.
4	i	It is index for origins (factory); $i = 1, 2, \dots, m$
5	j	It is index for destinations (warehouse); $j = 1, 2, \dots, n$
6	m	The number of resources (the number of rows)
7	n	The number of destinations (the number of columns)
8	x_{ij}	The decision variables. The number of quantities shipped in every path than origin i to destination j in TP. The assignment of i^{th} worker to j^{th} job in AP.
9	D_j	The destinations
10	S_i	The resources

Table 2: LIST OF ACRONYMS

NO.	THE SYMBOLS	DESCRIPTION
1	OR	Operations Research
2	TP	Transportation Problem
3	GTT	General Transportation Table
4	BFS	Basic Feasible Solution
5	FS	Feasible Solution
6	IBFS	Initial Basic Feasible Solution
7	LPP	Linear Programming Problem
8	MCM	Minimum Cost Method
9	NWCM	North-West Comer Method
9	VAM	Vogel's Approximation Method
11	OS	Optimal Solution
12	SSM	Stepping Stone Method
13	MODI	Modified Distribution Method
14	GRM	Golden Ratio Method
15	MODM	Modulo Method
16	AMM	Arithmetic mean Method
17	MM	Median Method

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Abstract

In this thesis, the researcher proposed four new solution methods where we modified and developed new techniques for solving TP, two new methods for finding the initial solution to various transport problems obtained by employing the Golden Ratio and employing modulo (mod). Also, two new technologies were introduced to find the initial solution of various transport problems using some laws of Statistics . The working efficiency of all these new technologies proposed in this study was tested by using them to solve many different examples and the desired results were obtained. The results of these examples obtained with new technologies were also compared with the results obtained by classical methods of the transport problem it turned out that the solutions are better or equal to the solutions of classical methods (MCM , NWCM and VAM) . The results of the proposed methods were better than the results of classical methods (MCM , NWCM and VAM), and some of them had equal results. One of the advantages of the proposed new technologies found in this research study is that it included fewer steps that can be understood and used very easily and reduced a lot of time and effort to get the optimal solution to the problem.

CHAPTER 1

INTRODUCTION AND CONCEPTS

1.1 Introduction

The decision-making process and the management of logistics in the social and commercial environment have become a complex task for a variety of reasons. These reasons include the cost of raw materials, technology, employee benefits, and a variety of other factors that greatly increase the difficulty of administrative decision-making. It is not sufficient for people to make judgments just by using their own experiences or by using a process of trial and error or perception in order to establish the suitable answer to this problem and to concentrate more on the nature of the issue. This is because of the repercussions of making poor judgments, which may have detrimental and expensive implications, and are thus undesirable. [46]

A subfield of applied mathematics known as operations research (OR) sometimes goes by the name decision science. It is known as mathematical programming, and its primary focus is on enhancing certain procedures and techniques with the goal of finding an optimum solution (OS) to a given issue. The fields of engineering, economics, management, and marketing are just few of the fields that may benefit from (OR). [29]

In operational research (OR), techniques including mathematical modeling and statistical analysis are used in order to get at the best answer and arrive at the right judgments. (OR) overlaps with a number of other disciplines, including industrial engineering, operations management, and transportation management, as a result of the breadth and depth of the applications to which it may be used. In mathematics, optimization refers to the process of selecting the optimal solution (OS) to a problem from among a collection of potential solutions that are candidates for selection. The challenge may be stated as

either optimizing the goal function to produce the greatest amount of profit or optimizing the objective function to produce the lowest amount of expense [43]. This item has made life faster and has observed vast and notable improvement over the last several years. The word "Transportation" has a definite definition that is centered on movement and movement from one location to another. Technology that facilitates global connection and information exchange has unquestionably had a positive impact on the lives of all people. For instance, it is not feasible for families in today's society to create all of their own food, clothes, and other necessities by depending only on themselves because of the myriad of variances that exist amongst cultures. The numerous marketplaces provide a diverse and ever-expanding variety of innovations and needs to their customers, which lessens the significance of the necessity for self-production. Products are manufactured on a huge scale in factories and farms that are regulated by competent authorities. [18]

These products are then distributed to customers at any time and at a suitable cost as much as is practicable. Costs associated with transportation are incurred whenever products are transported from their point of origin to the end user. Trade between different economies, participation in group activities, and the movement of people and resources from one location to another are all essential components of the foundation of modern civilization. It has been suggested that the issue at hand in a culture like this is one involving transportation issues (TP). [32]

The TP was given its moniker due to the fact that a significant portion of its applications include determining the most effective means by which to convey things. Even though difficulties related to contemporary management are always evolving, the majority of businesses and organizations strive to either minimize

expenses or increase revenues while working with constrained resources. Therefore, many issues can be summed up in the problems of linear programming. The most significant of them are transportation problems, which have a model that can be optimized and acquire an OS via it. This allows linear programming to be used to solve many difficulties. [33]

The TP is a broad research problem in (OR), and its goal is to determine the schedule for transporting homogeneous goods from the source (production centers or factories) to the destination (warehouses or markets) in a method that reduces the cost of shipping or the amount of time it takes to ship the goods, and that also satisfies supply and demand restrictions. Even if this problem (TP) may be addressed as a linear programming problem (LPP) due to the fact that it falls into one of the LPP categories, it is still possible to get the requisite answer by the use of other efficient approaches. [41]

The simplex approach, which is used to solve linear programming models, is intrinsically difficult, and when applied to the solution of a TP problem, it requires a significant amount of time. One of the models that was designed to ease the calculation and solution methods was the transportation model. This model was one of many that were developed. The use of this model is not restricted to only transportation and distribution; rather, it may be broadened to incorporate other challenges as well, such as the assignment of plant sites, the distribution of workers to machines, the assignment of vehicles to routes, product mixes, and so on. The TP includes the use of a huge number of different shipping techniques, which are used to transport goods from a variety of supply sources to a variety of destinations. The transportation model accounts for a predetermined maximum quantity of commodities that may be stored at each point of supply and a predetermined minimum quantity of

commodities that must be stored at each site of demand. This reflects supply limits as well as demand restrictions for the transportation concept. The purpose of developing the transportation model is to ascertain the number of possible units of the commodity that need to be transported from a particular source to a particular destination with the smallest possible transportation cost or the shortest possible time for transportation in order to meet the required quantity of goods or services in each destination center [3,13].

This can be accomplished by comparing the amount of time or money spent on transportation to the number of possible units of the commodity.

The linear transportation issue, also known as the normal transportation problem, requires that the cost of carrying each unit of a product delivered from a certain source to a particular destination be constant regardless of the amount of the product that is shipped. In addition, it is generally assumed that the amount of distance that must be traveled in order to deliver a product from every exporter to every destination is the same. In practice, there is a possibility of coming across at least two situations in which linear solutions to transportation issues are not appropriate. [15]

To begin, there is a possibility that the cost of moving each unit of merchandise may fluctuate owing to the availability of huge discounts that are periodically made available for large shipments. Because of this, the cost function will take on the shape of either a linear intermittent or an independent concave function. For the sake of this discussion, the issue may be phrased as a discontinuous or concave LPP with linear limitations. When there are extraordinary circumstances, such as the transportation of emergency materials when a natural disaster occurs or the transportation of military supplies during times of war when the carrier network may be destroyed, the distance from some origins

to some destinations is not specified. Examples of these kinds of circumstances include. Consequently, using a variety of distance metrics will result in a non-linear objective function (concave, convex, quadratic, ...) [1].

The TP is one of the challenges that need to be optimized. As a result, the solution to the TP consists of two stages: the first stage involves locating the initial basic feasible solution (IBFS) to this issue, and the second stage involves enhancing the original solution in order to reach the optimal answer to the TP. Before obtaining the operating system, it is necessary to locate the IBFS. In point of fact, one may claim that locating IBFS and the kind of solution to acquire will be extremely vital in order to get the operating system. As a result, IBFS has an impact on the TP's operating system. The north-west corner technique (NWCM), the minimal cost method (MCM), and the vogel's approximation approach are the well-known classical methods that may be used to achieve the first solution to the TP (VAM). One of the two known approaches, the stepping stone method (SSM) or the modified distribution method (MODI method), may be used to enhance the original solution and acquire the OS to the TP [30].

These methods are respectively known as the SSM and the MODI method.

Finding a fresh and efficient strategy for obtaining the first solution to the linear transportation issue is the primary objective and primary emphasis of the researcher who is doing this study. To identify the original answer to the various transportation difficulties, three new strategies were initially provided (first new technique (Golden ratio technique), second new technique (arithmetic mean technique), and third new technique (median technique) for solving the TP). This thesis mentions a few of these examples for clarity and makes a comparison of the results of the solution in the new techniques with the results of the

solution in the traditional methods of finding the initial solution to the TP.

As a result, the results of the solution in the new techniques were superior to the results of the solution in the traditional methods. The working efficiency of these three new techniques has been tested by solving a large number of numerical examples.

This thesis presents three innovative approaches to resolving issues related to transportation. These approaches are scientific, methodical, and adhere to established protocols. They are simple to comprehend and put into practice, and they may be used to resolve a wide variety of transportation issues.

1.2 Historical Review

Formulating and solving transportation problems as a LPP is one of the oldest and more common applications of linear programming techniques.

In (1781), the French mathematician Gaspard Monge (1746–1818), in cooperation with the army of Napoleon Bonaparte, published a mathematical model dealing with the transport of soil at the lowest possible cost between different construction sites for the purpose of building forts and military roads. Although Monge laid a theoretical basis for solving the TP, yet an algorithm was not developed until (1941) when American mathematician Frank L. Hitchcock (1875–1957) published his solution to the problem of Monge [19, 43].

This presentation is the first important contribution to solving TP. Hitchcock gave a method to find the initial solution to the TP now known as the north west corner method. A study of the TP was conducted by A.N. Tolstói (1930). As he published, in a book on transportation planning issued by the National

Transportation Commission of the Soviet Union, an article entitled “Methods of finding the minimal total kilometrage in cargo-transportation planning in space”, where he lessons the TP and described a number of approaches to the solution [39].

In (1939), the Russian economist Leonid V. Kantorovich’s brochure “The Mathematical Method of Planning and Organizing Production” actually laid the foundation for linear programming today. However, Kantorovich was not aware of the Monge (1781) paper until (1947) when he immediately realized the similarities between his work and Monge. Meanwhile, Dutch-American Tjalling C. Koopmans (1910–1985) was studying the problem of transportation independently to support the efforts of World War II, and only in the late 1950s Kantorovich’s work in linear programming and transportation were discovered. Koopmans was helpful in reprinting the Kantorovich booklet in the United States, which resulted in the publication of Kantorovich’s works in the West. By that time, it was American mathematician George B. Dantzig who already developed his simple style in (1947) to solve any LPP, including the transportation model. In (1975), Leonid V. Kantorovich and Tjalling C. Koopmans participated Nobel Prize in Economics. [44]

In (1951), George Dantzig developed the simplex method and describes the formulation of linear programming and its connected method of solution that was developed by allocating the general simplex method to the special structure of the TP [10].

The TP is a standard LPP that can be solved by the simplex method. However, due to its very special computational structure, it was recognized early on that the simplex method applied to the TP could be made very effective in terms of how to evaluate the necessary simplex method information (variable for basis

entry, variable for basis departure and optimality conditions). However, the transportation model can be solved using highly efficient calculation methods and it is simpler than the simplex method.

Relevant literary review is an important and essential component of any research study because it enables the researcher to understand previous research interests, research patterns, and the volume of research outputs in the field in which he is researching. Regarding the field of transportation problems, the literature on it is numerous and some of it has been covered on this topic. The TP is a significant linear programming model that emerges in many cases and has deserved a lot of interest in literature.

The TP is the oldest problem discovered in linear programming problems and is of great importance. In the transportation model there is a specific capacity (supply) of products at each exporter and a certain number of requirements (demand) for the products at each destination. This model is not limited to transportation only, but this form can be used to control inventory, scheduling recruitment, plant location, assigning employees to machines, cash flow data, assigning sales staff to sales areas and many more and in all cases the goal is to reduce total time or reduce total cost or increase sales and revenue. Transportation models or problems primarily relate to the optimal method in which a commodity produced in different sources or factories (called supply assets) can be transported to a number of warehouses or clients (called demand destinations). The goal in the TP is to fully meet the destination requirements of the required goods within the constraints of the operational production capacity at the source and the restrictions of demand at the destination and with the lowest possible transportation cost or the least possible time for transportation [2,48].

In (1989), H. Arsham and A. B. Kahn [6] proposed a new algorithm to solve the TP. The proposed method uses only one process, the Gauss Jordan axis, which is used in the Simplex method. The final table can be used for post-optimality analysis of a TP. This algorithm looks faster than simplex, more general than the stepping stone, and simpler than both to solve the public TP. In (2009), Vivek A. Deshpande [12] developed a method for finding the initial solution to the TP with a few modifications to the VAM method. In (2011), Serdar Korukoğlu and Serkan Balli [24] both provided an improvement to the work of the VAM using the Total Opportunity Cost (TOC) matrix. The TOC matrix is got by creating a “row opportunity cost matrix” (for every row the lowest cost for this row is deduct from every an item in the same row) and “column opportunity cost matrix” (for every column the lowest cost for this column is deduct from every an item in the same column). Then, the penalty costs for each row and column are calculated by subtracting the lowest cost of a cell in the row or column from the lowest next cell cost in the same row or column. After that, the largest allocation as possible is allocated to the cell with the lowest unit transportation cost between the row or column containing the three highest penalty costs.

In (2013), Md. Ashraful Babu, et al [8] discussed a new method for finding an initial feasible solution to the TP is called “Lowest Allocation Method (LAM)” is dependent on supply and demand.

In (2014), Utpal Kanti Das, et al [11] identified in their study an arithmetic error in the VAM. The basic concept of VAM is to select the penalty cost for rows and columns that get from finding the difference between the two smallest costs in each row and each column and then allocating the maximum in the least expensive cell to that row or column that has the most penalty. Difficulty

arises when the two smallest costs are equal. In this case, they put a logical concept to solve this problem and developed a new algorithm “Advanced Vogel’s Approximation Method (AVAM)” to find an initial solution to the transportation problems.

In (2015), Abdul Sattar Soomro, et al [17] have modification to the VAN and obtained a new method to find an IBFS to the TP.

Multiple and different methods are included in the literature to solve transportation. These methods are usually studied and developed in order to obtain the best initial solution or an OS to this problem. In (2016), A. Amaravathy, et al [5] found a new method to solve the (TP) and called it “Maximum Divide Minimum Allotment (MDMA)” which depends on determining the largest cost value considered in the transportation schedule and after that all costs in the schedule are divided by the largest cost identified.

In (2017), Sushma Duraphe, et al [14] proposed a method based on finding the Arithmetic Mean of rows and columns in the transportation model table and then the row or column that corresponds to the largest arithmetic mean was chosen so that the cell with the lowest cost was allocated with the largest possible allocation according to supply and demand.

In (2021), SAM’AN, et al [38] introduced a new five-step solution algorithm to find the initial solution to the transportation model. Basically, the main

principle of doing this algorithm is to fill in the number zero in the cell that has the highest cost from the transportation table and assign the maximum number of units in the cell that have the lowest cost to both the row and the column that share the cell with the zero assignment (the largest cost cell).

In (2020), Kenan Karagul and Yusuf Sahin [22] proposed a new method to find an initial feasible solution to the (TP) called the “KaragulSahin Approximation Method (KSAM)”. KSAM is a 5-step iterative method. Dissolution procedures begin with a change initially applied to the transportation schedule. Then find the ratio (r_{ij} and r_{ji}) and multiply it by the cost, and then create two new matrices A (wcd) and B (wcs) to use in the assignments.

In (2020), Md. Ashraful Babu, et al [7] have developed an “Improved Vogel’s Approximation Method (IVAM)” by correcting some of the errors they identified in the VAM. In (2020), B. S. Surya Prabhavati and V. Ravindranath [36] gave a method to find a best initial solution to the problem of balanced or unbalanced transportation. In this method the non-base input variable is chosen based on both the lowest cost and the corresponding supply or demand quantity.

In (2021), Lekan, et al [26] optimized the Maximum Difference Method (MDM) to obtain the best initial feasible solution to the (TP). Where a rule was set to address the appropriate choice of the cost of the largest penalty (the biggest difference) in case it is repeated more than once in rows and columns.

In (2022), Yuniarti, et al [47] added two methods to find (IBFS) for transportation problems. The SouthEast Corner Method (SEAM) with the NorthEast Corner Method (ECM) have been adopted for the IBFS account for the (TP).

The problems of transportation in the past and present have been extensively studied by many researchers and they have found wide applications of this problem in various fields such as engineering, economics, statistics, and others. Whereas, transportation currently plays a vital and active role for the economic and financial development of countries.

1.3 Application of Transportation Model

The objective of the (TP) is to establish a plan for the shipment of products that will either cut down on the overall amount of time or money spent on shipping while still satisfying the requirements imposed by the restrictions of supply and demand. Applications of the transportation model may be found in areas such as planning and industry, as well as transportation scheduling, communication networks, and assignment, amongst other places. The (TP) presents a challenge in terms of logistics for businesses, particularly those in the industrial and transportation industries. The process of issue allocation and suitable decision-making at these institutions may both benefit from the use of this technique as an effective instrument. The transportation model may also be used to help make choices on site locations. When there are two or more potential locations for a manufacturing facility, new institution, or bureau, the

model provides assistance in making a decision between them. Utilizing the model should result in a reduction in either the gross expenses of transportation or manufacture. Utilizing the transportation model as a strategy also allows for the maximization of income and profits.

Some transportation model applications:

- I) Determining the transportation of new and fundamental components of the manufacturing process, in the form of raw materials, from various centers to various facilities that produce the component. This is especially helpful in the event that there are many factories located in various areas.
- II) Figure out how the final items will be transported from the various production sites to the various destinations and distribution locations. This is useful for companies who have many factories and sell their products in multiple markets.
- III) Solve assignment problems with its various applications.

1.4 Reasons For Studying Transportation Problems

- 1) Transportation problems are of utmost significance in public life, and in particular for the decision-makers and those in charge of logistics management who are tasked with finding solutions that are suitable and necessary.
- 2) It is necessary for organizations and factories, regardless of whether they are

public or private, to please clients to the greatest extent feasible, both in terms of the quality of the product and the cost of the product.

- 3) Transportation has a very significant part in lowering the costs incurred by clients while waiting for institutions and industries to reach the revenue goals they have set.
- 4) According to the (TP) model, the majority of the challenges that arise during the process of transporting goods can be overcome by increasing revenue and profits, decreasing the total cost of transportation, or decreasing the total amount of time spent in transportation, amongst other potential solutions. When addressing the (TP), best results may be achieved by combining the transportation model with the research studies that have been conducted on this subject and the new approaches that have been developed for solving transportation issues as a consequence of these studies.
- 5) Not only does the model of the (TP) have a significant impact on the issue of transportation, but it also has a significant impact on a number of other areas. These areas include the allocation of a certain number of operators to a certain number of jobs, as well as the appropriate distribution of teachers to courses in accordance with their areas of specialization, as well as the allocation of clerks to various counters and so on. Because of the significance of the problem of transportation issues, which were some of the issues that were discussed in this study, researchers decided to concentrate their attention and energy on this subject and perform comprehensive research on it. [45]

1.5 Objective of Thesis

The goal of our proposed methods is to figure out how much to ship from each source (factory / origin) to each destination (warehouse) so that the total transportation cost (TTC) is as low as possible, which is the goal.

1.6 The Structure of Thesis

This thesis explains the transportation problems (TP) and the various concepts related to this problem. Examples of large size have been solved and the optimal solution to the quadratic assignment problems has been organized. The thesis has been organized in five chapters . Rest of this thesis is organized of as follow :

The rest of this chapter (Chapter One) includes the most important basic and important definitions in the thesis related to transportation problems (TP).

Chapter Two, This chapter includes a the classic algorithms to solution of a transportation problem.

Chapter Three , This chapter includes a new algorithms for finding an approximate solution to transport problems .

Chapter four ,This chapter includes using statistical laws to find an approximate solution to transport problems.

Chapter five ,This chapter includes Conclusions, suggestions, and references , It includes the conclusions of the thesis, suggestion for future works, and references related to the thesis.

1.7 The General TP Is Formulated in Mathematical Formulation

Transporting homogenous resources from multiple sources to different destinations is defined as transferring them at the lowest feasible cost or in the shortest amount of time, depending on the sort of issue we are dealing with, while meeting both supply and demand limitations at the same time.

Assume that the corporation has m manufacturing units located at S_1, S_2, \dots, S_m locations, and that there is a demand for items manufactured at n various destinations located at D_1, D_2, \dots, D_m locations.

Consider the shipping cost of one unit of the commodity transported on each path from the production unit S_i (i is index for origins (factory); $i = 1, 2, \dots, m$) to the demand center D_j (j is index for destinations (warehouse); $j = 1, 2, \dots, n$).

If C_{ij} is the shipping cost of one unit of the commodity transported on each path from the production unit S_i to the demand center D_j (C_{ij} is given a $m \times n$ in cost matrix), To solve the schedule of transportation problem (TP), one must discover the value of $X_{ij} \geq 0$ (X_{ij} are $m * n$ choice variables in the issue) that is within the restrictions of $m + n$ (see Figure 1).

When the aim is to decrease the cost or time required for transportation, the TP may be expressed as a linear programming problem (LPP) as follows [6, 36]:

$$\min(Z) \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$$

Subject to

$$\sum_{j=1}^n X_{ij} = a_i \quad ; i = 1, 2, \dots, m \quad (\text{supply constraints})$$

$$\sum_{i=1}^m X_{ij} = b_j \quad ; j = 1, 2, \dots, n \quad (\text{demand constraints})$$

$$X_{ij} \geq 0 \quad \forall \quad i \quad \text{and} \quad j \quad (\text{Non-negative constraint})$$

For this reason, made the TP one of the special cases of the LPP. Note that Z is the total shipping cost and it is a linear, a_i are the quantities available per m capacity, and b_i are the quantities required in each n requirement. In the TP have to finding the value of X_{ij} so that the cost of transportation Z is minimum [3].

1.8 General Transportation Table (GTT)

Transportation tables are specially designed tables that are built and used to solve transportation problems. The transportation table is one such table.

Origin (S_i)	Destination (j)				Supply (a_i)
	D_1	D_2	...	D_n	
S_1	C_{11} X_{11}	C_{12} X_{12}	...	C_{1n} X_{1n}	a_1
S_2	C_{21} X_{21}	C_{22} X_{22}	...	C_{2n} X_{2n}	a_2
...
S_m	C_{m1} X_{m1}	C_{m2} X_{m2}	...	C_{mn} X_{mn}	a_m
Demand (b_j)	b_1	b_2	...	b_n	$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$

Table 1.1: General Transportation Tableau

m rows and n columns are contained inside the inner rectangles of the GTT shown above, whereby signifies the number of rows and n represents the number of columns. Every rectangle is referred to as a cell. In the i th row and the j^{th} column, a cell is referred to as cell (i,j) , and each unit cost component c_{ij} is put in the centre of the cell corresponding to that component. The left-top of c_{ij} is to be filled with the components of a viable solution with X_{ij} , $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$ if there is one, and the right-top with the other components. Various origins' supply capacity a_i are indicated on the rightmost column of each row of the table, while different destinations' requests (b_j) are listed on the lowermost row of each column in the table's lowermost row corresponding to each column. It is possible to solve this problem with an infinite number of variables and constraints. However, the

total number of allocation cells required in a viable solution is $m + n$, while the total number of allocation cells required in an impractical solution is $m + n - 1$.

1.9 Network Of Transportation Problem

The network design shown in Figure 1.1 may be used to describe TP. as well as the formulation table seen in Table 1.1 Using the network diagram and formulation table, the purpose is to discover the value of variable X_{ij} that will result in the lowest overall cost of the TP while meeting the supply and demand requirements. The arrows that connect the point of origin to the point of destination represent the course that the items take on their way to their destination. The public TP network might emerge in the manner shown in the following picture [2, 12, 22].

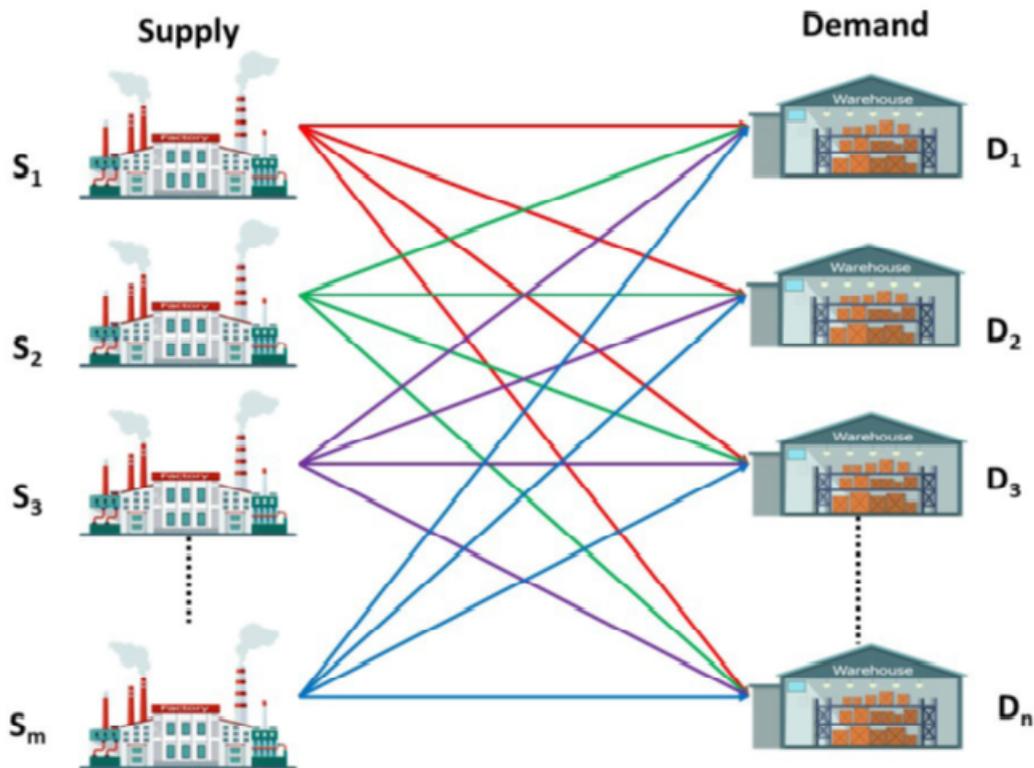


Figure 1.1: Network representation of general transportation problem

1.10 Kinds Of Transportation Problem

1. Balanced Transportation Problem (BTP) :

When the entire quantity needed at destinations is exactly the same as the total quantity available at sources, the situation is said to be a balanced transportation problem, i.e.

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

2. Unbalanced Transportation Problem (UBTP):

When the full quantity required at destinations is different from the total quantity available at sources, the situation is said to be an unbalanced transport problem, i.e.

$$\sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j$$

An UBTP may be converted into a BTP. It is necessary to include a dummy column (dummy destination) if the source of the item is bigger than the request in order to make the problem as balanced as possible, and every c_{ij} of that column should be equal to zero. If the request is more than the source, then the dummy row (dummy origin) should be added to convert the provided UBTP to BTP, and every c_{ij} in that row should be equal to zero [25].

1.11 Definitions of Some Terminologies in Transportation Problem

Definition 1.11.1. Source (Origin) : It is the location from which goods are despatched that is referred [4].

Definition 1.11.2. Destination (or warehouses): It is the place to which the

shipments are transported [31].

Definition 1.11.3. Capacities (or supplies): The offer limit of an origin refers to the product that is offered in that origin in order to meet the requirements of the request center [4].

Definition 1.11.4. Demands (or requirements): The amount of product necessary to fulfill the demand of a demand center is referred to as the demand requirement of the product [4].

Definition 1.11.5. Feasible Solution (FS): A feasible solution to the TP consists of a set of non-negative assigned numbers $X \geq 0$ that satisfy the constraints and are non-negative themselves (supply constraints and demand constraints in TP) [25].

Definition 1.11.6. Basic Feasible Solution (BFS): It is considered a basic feasible solution to the m -origin and n -destination problem if the number of positive allocations in the transportation table equals $(m + n - 1)$. In the transportation table, m represents the number of rows in the transportation table, and n represents the number of columns in the transportation table [25].

Definition 1.11.7. Degenerate BFS: If is a BFS that comprises less allocations than $m + n - 1$ The degeneracy can occur when solving a TP in :

- If the number of occupied cells is less than $(m + n - 1)$ in the initial Basic Feasible Solution (IBFS).
- In the course of moving toward (OS), there is the possibility that two or more occupied cells may become empty at the same time [34].

Definition 1.11.8. Optimal Solution (OS): A feasible solution (which may or may not be basic feasible) is considered to be optimum if it reduces the overall transportation cost to the greatest extent possible [28].

1.12 Assumptions Needed For Transport Problems

1. The capacity of each supply point (or source): the quantity of items that can be manufactured in each factory or stored in each warehouse. This is referred to as "capacity" or "supply."
2. The number of commodities necessary at each point of demand (or destination): The quantity of goods required in every client or in every shop This is referred to as a necessity or a request.
3. The cost of transferring commodities from supply sites to demand points is referred to as the third cost.
4. From point of origin to point of destination, only one kind of homogenous product is transported [34].

1.13 Solution Algorithm Using The Transportation Method

step 1 : The objective function (minimize or maximize) must be determined in accordance with the problem's constraints.

- step 2** Creating a formal problem for the issue presented in the matrix model.
- step 3** Identifying an initial solution that makes use of all available supply while also meeting all demand.
- step 4** Verifying that the first solution is optimum in terms of performance.
- step 5** Update the solution in accordance with the approach used, and continue steps 4 and 5 until the OS is reached.

1.14 Advantages to IBFS of TP

The first solution acquired via conventional solution techniques and the initial solution obtained through innovative solution methods should be characterized by the following [34]:

- i) The solution must be feasible, which means that it must satisfy both the offer and the request limits.
- ii) It must adhere to the non-passivity constraint.
- iii) The solution should be basic, that is, the number of positive allocations must equal $(m + n - 1)$, where m is the number of rows and n is the number of columns.

CHAPTER 2

THE CLASSIC ALGORITHMS TO SOLUTION
OF A TRANSPORTATION PROBLEM

2.1 Introduction

As a first step toward idealization, we will discuss the first three methods that are classical algorithms for generating basic elementary solutions (IFPS) in this section. After that, we will discuss the two methods that are also special classical algorithms for finding the optimal solution. Two kinds of algorithms will be discussed in detail.

2.2 The Three Classic Algorithms to Find IBFS of TP

Obtaining the IBFS, also known as the solution that satisfies the needs of the constraints, is the first stage in the process of solving the TP in order to reach the optimal solution. This is because the IBFS is the solution that comes closest to satisfying all of the criteria (i.e. supply and demand requirements). There are a few different routes one might take to get at the initial feasible solution. The following are the most well-known approaches:

- I)** North-West Corner Method (NWCM).
- II)** Minimum Cost Method (MCM).
- III)** Vogel's Approximation Method (VAM).

There is a difference in the quality of the beginning solution that is produced by each of the three different procedures. The starting solutions that produce a

lower objective value are the ones that are considered to be the best. The first strategy is of a mechanical character in the sense that its primary objective is to provide an initial answer (basic feasible) regardless of the expense. The last two are heuristics that look for a beginning solution that is of higher quality (smaller objective value). The Vogel heuristic performs the best overall, whereas the NWC scores the lowest. The most advantageous aspect of the NWCM is that it requires the fewest number of calculations [43].

2.2.1 North west corner method (NWCM)

Hitchcock presented this method, which is the easiest way to get the IBFS. However, given that the cost component is not taken into consideration, this does not guarantee a superior choice. This method was previously described by Salvemini and Frechet in 1939 and 1951, respectively. This is the oldest method known [39]. The (NWCM) is a method for accounting an IBFS of a TP in which the fundamental variables are selected rather than the North-West Corner (where the start cell is (1,1)) in each step of the sub matrix consisting of nonzero row and nonzero column requirements. This is the reason why the NWCM is referred to as the north-west corner method. In this method the only motive is to have a balance between demand and supply.

Algorithm (NWCM) [1]:

Step 1: Putting together the cost matrix for TP (if it is not given). Examine whether or not the aggregate supply is equal to the aggregate demand; if it is not, then the TP has to be balanced.

Step 2: Locating the cell that is located in the top northwest corner of the TP table (cell (1,1)).

Step 3: Appointing the maximum number of units that may be allocated to this cell corresponding to the lowest among ready offer and request criteria, i.e. $X_{11} = \text{minimum } (S_1, D_1)$ After then, at least one of these conditions will have been satisfied.

Step 4: This would result in the offer being depleted at exporter i and/or the demand being satisfied at destination j . As a direct consequence of this, the depleted row or column can no longer accept any further units, and those units must be discarded. Next, allot as much as feasible to the cell to the right or left of the one that was deleted in the row or column. When both the row and the column have been used up, go to the next cell by moving across the diagonal.

Step 5: Continuing to carry out the operation in the same manner until all of the available supplies have been depleted and all of the criteria for the order have been satisfied.

Step 6: Using the initial balanced transportation cost matrix compute the total transportation cost for the viable allocations,

i.e. total cost $Z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$

2.2.2 Minimum cost method (MCM)

When calculating the IBFS of a TP using the minimal cost method (MCM), also known as the matrix minimum method (MMM), the fundamental variables are

selected based on the unit cost of transportation. Given that the objective is to cut overall transportation costs, it is necessary to move as much as possible via those cells with the lowest per-unit costs of transportation. Due to the fact that it takes into consideration the many cost factors that are involved in the issue, this technique often produces a better starting BFS than the north west corner method [21].

Algorithm (MCM) [20]:

- Step 1:** Putting together the cost matrix for TP (if it is not given). Examine whether or not the aggregate supply is equal to the aggregate demand; if it is not, then the TP has to be balanced.
- Step 2:** Determine which of the cells has the lowest per-unit transportation cost (C_{ij}).
- Step 3:** Appointing the maximum number of units that may be allocated to this cell corresponding to the lowest among ready offer and request criteria, i.e. $\min(C_{ij}, X_{ij}) = \min(S_i, D_j)$ After then, at least one of these conditions will have been satisfied.
- Step 4:** In the case that, there is an equal between two or more cells with the lowest possible cost, the cell with which the greatest possible quantity of allocation may be accomplished is selected and given preference over the other cells that are being allocated.
- Step 5:** There is no need for any more consideration on the column or row that has been fulfilled. In the event that supply is depleted, remove that row. If there is no longer a need for that column, it may be removed. If a row and

a column are both fulfilled at the same time, then just one of the two will be crossed out, and the zero supply will be allocated to the row (or column) that is left over (demand).

Step 6: Repeat steps 2–4 until all of the constraints have been satisfied.

Step 7: Compute the total cost that the TP will cost. It is possible to compute it by making use of the equation that follows: $Z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$

2.2.3 Vogel's approximation method (VAM)

An iterative process for calculating an IBFS of the TP, the VAM is referred to as "the VAM." The VAM is an upgraded variant of the MCM that often generates better beginning solutions, however this is not always the case (better than the NWC and the MCM). Reduced opportunity costs are the premise upon which VAM was established (or penalty). Reinfeld and Vogel (1958) came up with the idea for VAM and defined penalty as the difference between the lowest and next to lowest cost in each row and column of a transportation schedule. This led to the development of the VAM concept. There is another name for this approach, which is the Penalty technique [25].

Algorithm (VAM) [7, 12]:

Step 1: Putting together the cost matrix for TP (if it is not given). Examine whether or not the aggregate supply is equal to the aggregate demand; if it is not, then the TP has to be balanced.

Step 2: In each column of the transportation cost matrix, specify the two lower costs available. Find the difference between these two costs, which will be

referred to as the penalty cost, and place it in the column to the right of that row in the new table that was created by extending the table to the right. Find such differences between two minimum costs that are available (called penalty cost), and position it beneath that column in a novel row that was formed for this purpose by extending the table below. In the same manner, for each column of the transportation cost matrix, find such differences between two minimum costs that are available.

Step 3: Find the column or row in step 2 that has the highest possible penalty cost among the many penalty charges that were shown there (the biggest difference).

Step 4: Allocate the maximum number of units to the smaller cost cell in the selected row (or selected column), that is, for minimum (C_{ij}) , $X_{ij} = \text{minimum}(S_i, D_j)$. After then, at least one of these conditions will have been satisfied.

Step 5: If step 2's greatest penalty costs are equal to one another, choose the cell in the row or column that corresponds to the biggest equal penalty that has the lowest cost. Additionally, if there is an equality among the cells with the lowest cost, choose the cell with the lowest cost that contains the highest feasible allocation quantity. If there is an equality in the maximum potential allocation quantity, then choose the cell that corresponds to the greatest supply or demand in either the row or the column. The maximum number of units that may possibly be used is allotted to the selected cell, and the exhausted row or column that corresponds to it is eliminated (or disregarded) from any further consideration.

Step 6: There is no need for any further consideration for the column or row that has fulfilled supply or demand. If there is sufficient supply, you may eliminate that column. Delete that entry if there is no longer any supply. If both rows and columns are fulfilled at the same time, just one of the two will be crossed out, and either the supply or demand in the remaining row or column will be set to zero. If there is just one row or column with a positive supply or demand left uncrossed out, then you should use the MCM to calculate the value of the fundamental variables X_{ij} in that row or column.

Step 7: After that, you will need to compute the difference for each row and per column (penalty cost) for the shortened transportation timetable, and then you will need to continue the method until the totals for the rows and columns have been used up.

Step 8: Compute the total cost that the TP will cost. It is possible to compute it by making use of the equation that follows: $Z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$

2.3 The Classic Algorithms to Find the OS of TP

The following phase which comes after an initial solution that has been found, is to check the optimization of the outcome to see whether or not the existing solution can be made better. The OS may be achieved by making incremental enhancements to the IBFS in such a way that the cost of transportation is not reduced any more. That is, the present solution cannot be made any better than it now is. The operating system is shown here. Will take into account both of

the more conventional testing approaches in an effort to get the most effective answers. These are the methods:

I) Stepping Stone Method (SSM).

II) Modified Distribution Methods (MODI Method).

These methods reveal whether the IBFS is the OS or not. It also improves the solution until the OS is obtained [17, 47].

2.3.1 Stepping stone method (SSM)

It starts with the IBFS and then determines, for each of the variables that are not important, whether or not an optimum solution can be reached by including one of the non-essential variables in the base. In 1945, Cooper and Charles introduced the idea that would later become known as the stepping stone technique (SSM). The formation of a loop, which is a locked route followed by an ordered series of cells that begins from a non-occupied cell despite the fact that the rest of the cells are occupied, is an important part of this strategy. Cells in the ring that cascade into one another are always located in either the same row or the same column [16].

Algorithm (SSM) [16, 20]:

Step 1 : Determine the IBFS by using the most appropriate strategy.

step 2 : Check to see that the number of occupied cells is exactly equal to

$m + n - 1$ where m is the number of rows and n is the number of columns.

- Step 3:** Find a cell that is not being used, and then beginning at that cell, draw a locked route with angles that goes through at least three cells that are being used before returning to the original cell that is not being used. Take note that there are an even number of cells in each ring, and keep in mind that there shouldn't be any diagonal movement in this ring at all. It is stated that the "Stepping Stones" on the journey are the cells that are located at the rotation points.
- Step 4 :** Beginning with the plus mark in the unfilled cell to be assessed, set the plus sign (+) and the negative sign (-) alternatively on every utilized angle cell for each loop that was just recorded.
- Step 5:** Compute the optimization index by adding the unit cost numbers in each cell that is marked with a plus sign and subtracting the unit cost numbers from each cell that has a negative sign. This would result in the allocation of the net change in cost.
- Step 6:** Repeat steps 3 through 5 until the optimization index has been computed for each of the cells that are vacant.
- Step 7:** Analyze the impact that each and every net adjustment has had on the mark for the unit cost of transportation. Stop calculating if every indication you've computed is either greater than zero or equal to zero. The OS has been arrived . If this is not the case, it indicates that the BFS that has been provided is not the best one, and it also indicates that the cost of transportation may be decreased even lower before moving on to step 8.
- Step 8:** Determine which of the tracks or loops includes the more significant negative number, and use this information to make the solution better. If

you want the present solution to be improved even more, you should give the lower value in the column that has a minus sign (-). This number is added to the cells in the track that are marked with a plus sign (+), and it is subtracted from the cells in the track that are marked with a minus sign (-). Using this approach, an empty cell that must contain data is converted into a filled one while the necessary row and column spacing is preserved. This results in one of the basic cells having a value of zero, while the other cells maintain their positive sign. The basic cell in question, whose allocation was changed to zero, will be included in the subsequent tests.

Step 9: Proceed to Step 2 and any other steps beyond that in order to determine whether or not an OS has been reached.

2.3.2 Modified distribution method (MODI)

The modified distribution technique, which is also known as the MODI method or the $(u - v)$ method, offers a solution for the TP that has the lowest possible cost. It is feasible to determine, via the use of MODI, whether or whether the FS that is gained lowers the overall cost of transportation. A mathematical equation is used in lieu of the stepping-stone routes in the MODI technique, which is a modified form of the stepping-stone method. For the SSM assessment, you need to plot the greatest number of closed pathways that are proportional to the number of empty cells. When using the MODI technique, on the other hand, only the closed route leading to the vacant cell that has the largest negative opportunity cost is depicted [48].

Algorithm (MODI) [47]:

Step 1 : Determine the IBFS by using the most appropriate strategy.

step 2 : Check to see that the number of occupied cells is exactly equal to $m + n - 1$ where m is the number of rows and n is the number of columns.

step 3 : Give the values for the variables u_i and v_i (dual variables), respectively, that correspond to the i^{th} row and the j^{th} column. Put a u_i in the first position of each i^{th} row, and a v_i in the last position of each j^{th} column. If there are m different sources and n different destinations, then there will be $m + n$ dual variables.

step 4 : For basic cells that already have allocations, compute $u_i + v_j = c_{ij}$. This relationship specifies the value for all u_i and v_j by using algebraic calculation. Assume that the value of the variable that occurs the most frequently, u_i or v_j , is equal to zero. This will significantly reduce the amount of work that needs to be done in terms of computation.

step 5 : Calculate the opportunity cost using $K_{ij} = C_{ij} - (u_i + v_j)$ to unoccupied cells.

step 6 : Verify the sign of each opportunity cost [35].

- i) If the opportunity costs of all of the cells that are now empty are positive, then the specified solution is the best and only solution to the problem(OS).
- ii) The particular solution is the (OS), however there is also an alternate option to consider if the opportunity costs of any non-occupied cell are zero.

iii) If one or more of the vacant cells have an opportunity cost that is negative, then the particular solution is not the (OS), and more cost reductions in transportation are possible.

Step 7 : Determine which of the unoccupied cells has the highest opportunity cost deficit and choose that one as the cell to be included in the next solution. Everyone has an equal chance of being selected if there is more than one equal cell in the greatest negative opportunity cost.

Step 8 : Within this matrix, trace a closed route or loop by drawing a sequence of lines that alternate between horizontal and vertical orientations. The journey starts and finishes in the cell that was designated as having no occupants in the stage before this one. The route may go over any number of cells, regardless of whether they are occupied or unoccupied. The path's corners are always located in the cells for which allocations have been made. Take note that the number of cells in each loop is even, and keep in mind that there should not be any movement in a diagonal direction inside this loop.

Step 9 : Place a plus sign and a negative sign in the cells that are over the angle points of the locked track, alternating between the two, while leaving the plus mark in the cell that is being assessed.

Step 10 : Determine the maximum amount of units that need to be relocated to this cell that is currently vacant. The number of possible units that may be added to the entering cell is indicated by the value that is the least and has a location that is negative on the closed route. Now, add this number to each of the cells on the corner points of the closed route that are indicated

with plus signs, and then subtract those values from each of the cells that are marked with minus signs. Using this approach, an empty cell may be converted into a filled one while still satisfying the criteria for the row and column spacing. This results in one of the basic cells having a value of zero, while the other cells maintain their positive sign. The basic cell in question, whose allocation was changed to zero, will be included in the subsequent tests.

Step 11 : Go to Step 2 and repeat the whole procedure until an OS is obtained when all $K_{ij} \geq 0$ for unoccupied cells.

2.4 Numerical Examples

Example 2.4.1. Wheat is grown in the Midwest and stored in wheat depots in three different towns: Kansas City, Omaha, and Des Moines. Kansas City is the largest of these cities. These wheat depots provide their wares to three flour mills, which are situated in the cities of Chicago, St. Louis, and Cincinnati. Railway carriages, each of which can hold one ton of wheat, are used to transport grain to mills before it is ground into flour. On a monthly basis, every wheat depot has the capacity to provide the mills with the amount of tons of wheat shown in Table 2.1.

Wheat Stores	Kansas City	Omaha	Des Moines	Total
Supply	150	175	275	600

Table 2.1: The Supply

The following quantity of tons of wheat must be delivered to each mill every month in accordance with Table 2.2.

Mill	Chicago	St. Louis	Cincinnati	Total
Demand	200	100	300	600

Table 2.2: The Demand

The transporting cost one ton of wheat varies from each depot (source) to each mill (destination) based on the distance and the rail system. These expenses are in USD and are represented in the following Table 2.3.

Wheat stores	Mill		
	D1	D2	D3
S1	6	8	10
S2	7	11	11
S3	4	5	12

Table 2.3: The Costs

In order to cut down on the overall cost of transportation, it is necessary to figure out how many tons of wheat are moved from each wheat depot to each mill on a monthly basis. The challenge comes in establishing how to do this.

Build the transportation table as in Table 2.4.

Wheat stores	Mill			Supply
	D ₁	D ₂	D ₃	
S ₁	6	8	10	150
S ₂	7	11	11	175
S ₃	4	5	12	275
Demand	200	100	300	600

Table 2.4: The TP Schedule

The mathematical model for Example 2.4.1 is: The objective function is to,

$$Z = \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} X_{ij}$$

Subject to the constraints,

Supply Constraints

$$X_{11} + X_{12} + X_{13} = 150$$

$$X_{21} + X_{22} + X_{23} = 175$$

$$X_{31} + X_{32} + X_{33} = 275$$

Demand Constraints

$$X_{11} + X_{21} + X_{31} = 200$$

$$X_{12} + X_{22} + X_{32} = 100$$

$$X_{13} + X_{23} + X_{33} = 300$$

Where $X_{ij} \geq 0$, $\forall i = 1, 2, 3$ and $j = 1, 2, 3$

The given transportation table (in Table 2.4) is balanced since total supply = total demand = 600 .

The Table 2.5. represents the solution according to algorithm of NWCM

Wheat stores	Mill			Supply
	D1	D2	D3	
S1	6 150	8	10	150
S2	7 50	11 100	11 25	175
S3	4	5	12 275	275
Demand	200	100	300	600

Table 2.5: Represents the Solution by NWCM

The total cost of the IBFS of TP is:

$$Z = \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} X_{ij}$$

$$Z = (150 * 6) + (50 * 7) + (100 * 11) + (25 * 11) + (275 * 12)$$

$$Z = 5925$$

The table 2.6. represents the solution according to algorithm of MCM

Wheat stores	Mill			Supply
	D1	D2	D3	
S1	6	8 25	10 125	150
S2	7	11	11 175	175
S3	4 200	5 75	12	275
Demand	200	100	300	600

Table 2.6: Represents the Solution by MCM

The total cost of the IBFS of TP is:

$$Z = \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} X_{ij}$$

$$Z = (25 * 8) + (125 * 10) + (175 * 11) + (200 * 4) + (75 * 5)$$

$$Z = 4550$$

The table 2.7 represents the solution according to algorithm of VAM

Wheat stores		Mill			Supply	Penalty					
		D1	D2	D3		Row Diff					
S1		6	8	10	150	150	2	2	4		
S2		7	175	11	11	175	4	-	-		
S3		4	25	5	100	12	150	275	1	1	8
Demand		200	100	300	600						
Penalty	Column Diff	2	3	1							
		2	3	2							
		2	-	2							

Table 2.7: Represents the Solution by VAM

The total cost of the IBFS of TP is:

$$Z = \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} X_{ij}$$

$$Z = (150 * 10) + (175 * 7) + (25 * 4) + (100 * 5) + (150 * 12)$$

$$Z = 5125$$

Now improving the initial solution to find the OS. Choose the initial feasible solution obtained by applying MCM (in Table 2.6)

Wheat stores	Mill			Supply
	D1	D2	D3	
S1	6	8 25	10 125	150
S2	7	11	11 175	175
S3	4 200	5 75	12	275
Demand	200	100	300	600

Table 2.8: Represents the Solution by SSM

Name of the Unused Cell	Closed Path	Indirect cost
C (1,1)	C(1,1) → C(3,1) → C(3,2) → C(1,2)	+6-4+5-8=-1
C (2,1)	C(2,1) → C(3,1) → C(3,2) → C(1,2) → C(1,3) → C(2,3)	+7-4+5-8+10-11=-1
C (2,2)	C(2,2) → C(2,3) → C(1,3) → C(1,2)	+11-11+10-8=2
C (3,3)	C(3,3) → C(1,3) → C(1,2) → C(3,2)	+12-10+8-5=5

Table 2.9: Represents the Solution by SSM

Wheat stores	Mill			Supply
	D1	D2	D3	
S1	6	8 25	10 125	150
S2	7 +	11	11 175	175
S3	4 200	5 75	12	275
Demand	200	100	300	600

Table 2.10: Represents the Solution by SSM

Wheat stores	Mill			Supply
	D1	D2	D3	
S1	6	8	10 150	150
S2	7 25	11	11 150	175
S3	4 175	5 100	12	275
Demand	200	100	300	600

Table 2.11: Represents the Solution by SSM

$$Z = \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} X_{ij}$$

$$Z = (C_{13}X_{13}) + (C_{21}X_{21}) + (C_{23}X_{23}) + (C_{31}X_{31}) + (C_{32}X_{32})$$

$$Z = (150 * 10) + (25 * 7) + (150 * 11) + (175 * 4) + (100 * 5) = 4525$$

Wheat stores	Mill			Supply
	D1	D2	D3	
S1	6	8	10	150
S2	7	11	11	175
S3	4	5	12	275
Demand	200	100	300	600

Table 2.12: Represents the Solution by SSM

Name of the Unused Cell	Closed Path	Indirect cost
C (1,1)	$C(1,1) \rightarrow C(2,1) \rightarrow C(2,3) \rightarrow C(1,3)$	$+6-7+11-10=0$
C (1,2)	$C(1,2) \rightarrow C(3,2) \rightarrow C(3,1) \rightarrow C(2,1) \rightarrow C(2,3) \rightarrow C(1,3)$	$+8-5+4-7+11-10=1$
C (2,2)	$C(2,2) \rightarrow C(2,1) \rightarrow C(3,1) \rightarrow C(3,2)$	$+11-7+4-5=3$
C (3,3)	$C(3,3) \rightarrow C(2,3) \rightarrow C(2,1) \rightarrow C(3,1)$	$+12-11+7-4=4$

Table 2.13: Represents the Solution by SSM

Wheat stores	Mill			Supply
	D1	D2	D3	
S1	6	8	10	150
S2	7	11	11	175
S3	4	5	12	275
Demand	200	100	300	600

Table 2.14: Represents the Solution by SSM

The indirect cost of unoccupied cells are positive numbers, so the occupancy of any of these cells will not reduce costs, so the solution to the last table is the optimal solution and the costs are 4525.

The table 2.15 represents the solution according to algorithm of MODI

Wheat stores	Mill			Supply
	D1	D2	D3	
S1	6	8	10	150
S2	7	11	11	175
S3	4	5	12	275
Demand	200	100	300	600

Table 2.15: Represents the Solution by MODI

From the table we can see that the number of occupied cells is five cells, and thus five equations are formed as follows:

$$C12 = U1 + V2$$

$$C13 = U1 + V3$$

$$C23 = U2 + V3$$

$$C31 = U3 + V1$$

$$C32 = U3 + V2$$

Suppose that one variable is equal to zero so that we can find the values of the other variables and suppose $U1 = 0$ From the equations we will get the following results:

$$V1 = 7 \qquad U1 = 0$$

$$V2 = 8 \qquad U2 = 1$$

$$V3 = 10 \qquad U3 = -3$$

Name of the Unused Cell	Indirect cost $\overline{C_{ij}} = C_{ij} - U_i - V_j$
C (1,1)	$\overline{C_{11}} = 6 - 0 - 7 = -1$
C (2,1)	$\overline{C_{21}} = 7 - 1 - 8 = -2$
C (2,2)	$\overline{C_{22}} = 11 - 1 - 8 = 2$
C (3,3)	$\overline{C_{33}} = 12 + 3 - 10 = 5$

Table 2.16: Represents the Solution by MODI

Wheat stores	Mill			Supply
	D1	D2	D3	
S1	6	8	10	150
S2	7	11	11	175
S3	4	5	12	275
Demand	200	100	300	600

Diagram annotations in Table 2.17:
 - A horizontal arrow from D2 to D3 with a red '25' above it and a '+' sign to the right.
 - A vertical arrow from D2 to D3 with a red '175' to its right and a '-' sign below it.
 - A vertical arrow from D1 to D2 with a red '200' to its right and a '+' sign above it.
 - A horizontal arrow from D1 to D2 with a red '75' above it and a '+' sign below it.
 - A horizontal arrow from D1 to D3 with a red '200' below it and a '-' sign above it.

Table 2.17: Represents the Solution by MODI

Wheat stores	Mill			Supply
	D1	D2	D3	
S1	6	8	10	150
S2	7	11	11	175
S3	4	5	12	275
Demand	200	100	300	600

Table 2.18: Represents the Solution by MODI

$$Z = \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} X_{ij}$$

$$Z = (150 * 10) + (25 * 7) + (150 * 11) + (175 * 4) + (100 * 5) = 4525$$

Wheat stores	Mill			Supply
	D1	D2	D3	
S1	6	8	10	150
S2	7	11	11	175
S3	4	5	12	275
Demand	200	100	300	600

Table 2.19: Represents the Solution by MODI

From the table we can see that the number of occupied cells is five cells, and thus five equations are formed as follows:

$$C_{13} = U_1 + V_3$$

$$C_{21} = U_2 + V_1$$

$$C_{23} = U_2 + V_3$$

$$C_{31} = U_3 + V_1$$

$$C_{32} = U_3 + V_2$$

Suppose that one variable is equal to zero so that we can find the values of the other variables and suppose $U_1 = 0$ From the equations we will get the following results:

$$V_1 = 6 \qquad U_1 = 0$$

$$V_2 = 7 \qquad U_2 = 1$$

$$V_3 = 10 \qquad U_3 = -2$$

Name of the Unused Cell	Indirect cost $\bar{C}_{ij} = C_{ij} - U_i - V_j$
C (1,1)	$\bar{C}_{11} = 6 - 0 - 6 = 0$
C (1,2)	$\bar{C}_{12} = 8 - 0 - 7 = 1$
C (2,2)	$\bar{C}_{22} = 11 - 1 - 7 = 3$
C (3,3)	$\bar{C}_{33} = 12 + 2 - 10 = 4$

Table 2.20: Represents the Solution by MODI

The indirect cost of unoccupied cells are positive numbers, so the occupancy of any of these cells will not reduce costs, so the solution to the last table is the optimal solution and the costs are 4525.

CHAPTER 3

A NEW ALGORITHMS FOR FINDING AN
APPROXIMATE SOLUTION OF TRANSPORT
PROBLEMS

3.1 Introduction

There are now three broad solutions to the problem of transportation techniques that may be found in the literature that has previously been examined. These strategies can only provide an initial answer that is practical. On the other hand, we will talk about two novel alternative approaches that offer an initial viable solution in addition to an ideal or nearly optimal solution. In addition to the aforementioned three ways, there are two further methods known as the MODI method and SSM that provide the best result. But before we can arrive at the best answer, we need to start by locating an initial solution using one of the three ways that have been shown. However, the approaches that are covered in this chapter provide an initial solution in addition to either the ideal solution or a solution that is nearly optimal. In another sense, we may argue that using one of the two procedures results in either an initial viable solution as well as an ideal solution or a solution that is close to optimal.

3.2 Algorithm of Golden Ratio method (GRM)

The Golden Ratio or the so-called golden section, the sacred ratio, the Golden Number or the divine number, is a ratio represented by a simple number (1.61803) and this number has aroused throughout history the controversy and curiosity of mathematicians, theoretical physicists, architects, artists, natural scientists and astronomers, as this figure representing the golden ratio appears in many manifestations of nature and the universe around us, thus drawing the attention of many to it, and some have considered it a standard mathematical

law, some consider it a natural law, while others have considered it a divine holy law, others some other promised him a philosophical-aesthetic law. There is a belief that the ancient Egyptians adopted the golden ratio in the design of the pyramids, and that the Greeks designed the Parthenon using the Golden Ratio, and that the Greek sculptors designed their statues using that golden ratio, and the use of this ratio also appeared in many churches and cathedrals of the Middle Ages, and in the Renaissance the use of the Golden Ratio prevailed, where If the golden ratio has attracted human attention since ancient times and continues to date, as it is considered a measure of the aesthetic proportions of visuals, whether those visuals are made forms such as art, or forms of assets in the natural, it is an indicator of the individual's taste and response to the repetition of visuals, and determines the extent of his awareness of geometric proportions, and mathematical sequences contained in those visuals, and this golden ratio has entered the literature of some theories and trends of aesthetics and art criticism, as it was sometimes considered as a higher example this ratio has impressed mathematicians, biologists, artists and others, and the mathematician (mark Baar) this ratio is the name (PH), which is the first Greek letter with which the name Phidias begins, a Greek sculptor who is believed to have used this ratio in the construction of Parthenon and his other sculptures [9].

The Golden Ratio is a mathematical ratio that represents a sequence that exists widely in nature, and is attributed to its discoverer, the Italian mathematician (Fibonacci) and is also called the Golden Section or the golden number and is symbolized by the Greek letter (φ) and its real value is $((1 + \sqrt{5})/2)$, which is approximately equal to (1.61803). Fibonacci discovered this sequence while studying the reproduction of rabbits, he came up with a numerical visualization

of that reproduction, set it in a numerical sequence, and to find the various elements of the sequence, we combine the previous two elements [27].

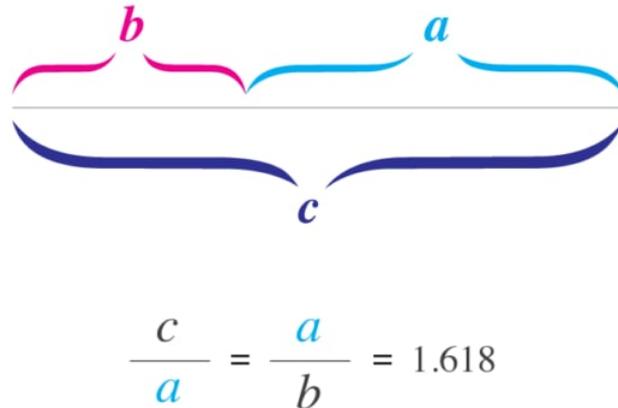


Figure 3.1: Show the Fibonacci Sequence Ratio

The numbers of this sequence are as follows : 0,1,1,2,3,5,8,13,21,... In order to understand this sequence easier, we make a straight line as in Figure No. (1) and divide that line into two different lengths so that the ratio of the whole line to the largest section is equal to the ratio of the largest section to the smallest section, which represents the ratio (1.62:1) or roughly (5:3), which is what achieves the Fibonacci sequence ratio, which is (1.61803) so that any number in this sequence divided by the next number is approximately (0.618) and any number divided the previous figure is approximately (1.61803) and one of the most famous applications of the Golden Ratio is the golden rectangle [42].

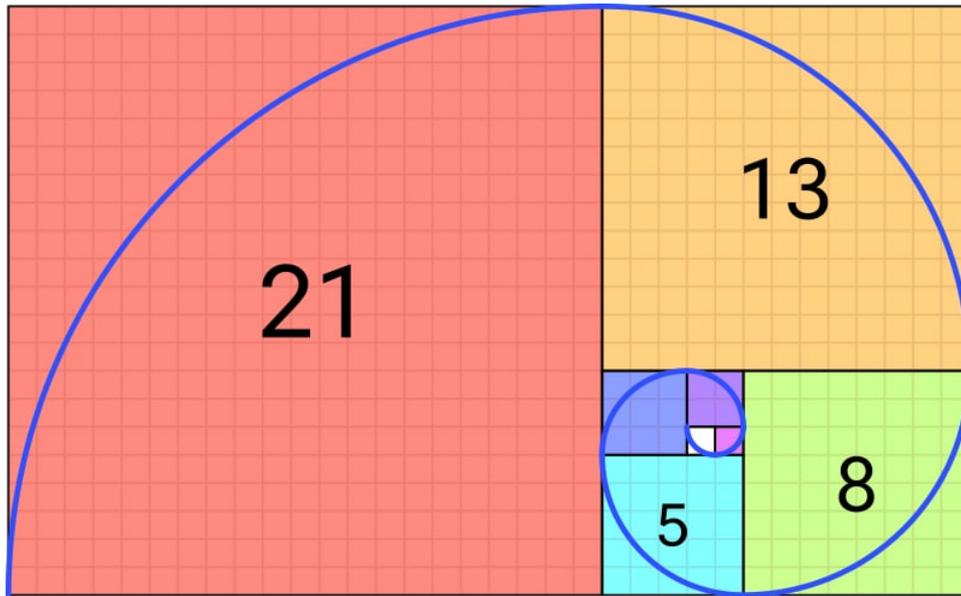


Figure 3.2: Golden Rectangle and Golden Spiral Curve

The golden rectangle is a rectangle drawn with a ratio of (5:8) to achieve a ratio of (1.61803), which is the Golden Ratio, and this rectangle if divided into a group of rectangles within the Fibonacci sequence as in Figure No. 2, this will result in the so-called Golden curve, that curve that we observe in many organic formations, in this study we used the golden ratio to solve transportation problems [23].

The steps necessary to solve the transportation problem are as follows:

step 1: The transportation table must be balanced.

step 2: Use the mathematical formula $(1.61803)^{(\text{the second lowest cost})}$ and calculate it for each of the rows and columns.

step 3: Determine the highest value that result from step (2) in all rows and columns and choose the cell with the lowest cost to give the right amount available of supply to meet the needs (demand).

step 4: If the result values are equal in more than one row or column, choose the

cell with the lowest cost to allocate the request.

step 5: The row that ran out of width or the column that was filled in the application does not enter into the following calculation.

step 6: Repeat steps (2 - 4) and calculate the total cost.

3.2.1 Numerical Examples

Below are numerical examples to make the proposed method very clear and well understood:

Example 3.2.1.

Wheat stores		Mill			supply	Penalty				
		S1	S2	S3		Row Diff				
D1	4	50	3	40	5	90	6.85	6.85	6.85	
D2	6		5	80	4	80	11.09	17.9	17.9	
D3	8	20	10		7	80	100	46.9	122.9	-
demand		70	120	80	270					
Penalty	Column Diff	17.9	11.09	11.09						
		17.9	11.09	-						
		17.9	11.09	-						

Table 3.1: Represents the Solution by GRM

$$Z = \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} X_{ij}$$

$$Z = (4 * 50) + (3 * 40) + (5 * 80) + (8 * 20) + (7 * 80)$$

$$Z = 1440$$

Example 3.2.2.

Wheat stores	Mill				supply	Penalty				
	S1	S2	S3	S4		Row Diff				
D1	3 100	4	6	0	100	4.23	---	---	---	---
D2	7	3 80	8	0	80	4.23	4.23	4.23	4.23	4.23
D3	6 10	4 30	5	0 50	90	6.85	6.85	6.85	6.85	6.85
D4	7	5	2 60	0 60	120	2.61	2.61	2.61	11.09	---
Demand	110	110	60	110	390					
Penalty	Column Diff	17.9	6.85	11.09	1					
		29.03	6.85	11.09	1					
		---	6.85	11.09	1					
		---	6.85	---	1					
		---	6.85	---	1					

Table 3.2: Represents the Solution by GRM

$$Z = \sum_{i=1}^4 \sum_{j=1}^4 C_{ij} X_{ij}$$

$$Z = (3 * 100) + (3 * 80) + (6 * 10) + (4 * 30) + (0 * 50) + (2 * 60) + (0 * 60)$$

$$Z = 840$$

3.3 Algorithm of Modulo method (MODM)

In mathematics, the remainder or remainder of a division is the quantity that "remains" after performing a calculation. In arithmetic, the remainder is known as the integer remaining after dividing an integer by another integer to result outside the division. In Algebra, the remainder is known as the polynomial remaining after dividing one polynomial by another polynomial.

If a and d are integers, and $d \neq 0$, then it can be proved that there are two unique integers q and r , where $a = qd + r$ and $0 \leq r < |d|$.

The number q is called the quotient, while r is called the remainder or remainder of the division, We adopted in this way the method proposed by the remainder of the division [40].

The steps necessary to solve the transportation problem are as follows:

step 1: The transportation table must be balanced.

step 2: Calculate the modulo $\text{mod}(\text{biggest value}, \text{smallest value})$ it for each of rows as well as for each of the columns. If in the row or column Zero is neglected (not included in the calculation)

step 3: Determine the least value resulting from Step (2) in all rows and columns and then choose the cell with the lowest cost to give the right amount available of supply to meet the needs (demand).

step 4: If the resulting values are in more than one column, we choose to allocate the cell with the lowest cost.

step 5: The row filled in the application does not enter into the following calculation.

step 6: Repeat steps (2-4) and after completing the filling of cells we calculate the total cost.

3.3.1 Numerical Examples

we are numerical examples to make the proposed method very clear and well understood:

Example 3.3.1.

Wheat stores	Mill						supply	Penalty					
	S1	S2	S3	S4	S5	S6		Row Diff					
D1	14	9	11	11 15	12 40	0 15	70	5	3	3	3	1	1
D2	11 10	7 20	6	7 20	8	0	50	5	5	4	4	1	-
D3	9	10	7 25	10	8 15	0	40	3	3	2	-	-	-
Demand	10	20	25	35	55	15	160						
Penalty	Column Diff	5	3	5	4	4	-						
		5	-	5	4	4	-						
		5	-	-	4	4	-						
		3	-	-	4	4	-						
		-	-	-	4	4	-						
		-	-	-	0	0	-						

Table 3.3: Represents the Solution by MODM

$$Z = \sum_{i=1}^3 \sum_{j=1}^4 C_{ij} X_{ij}$$

$$Z = (11 * 15) + (12 * 40) + (0 * 15) + (11 * 10) + (7 * 20) + (7 * 20) + (7 * 25) + (8 * 15)$$

$$Z = 1330$$

Example 3.3.2.

Wheat stores		Mill			supply	Penalty			
		S1	S2	S3		Row Diff			
D1	6	8	25	10	125	150	4	2	2
D2	7	11		11	175	175	4	0	-
D3	4	200	5	75	12	275	0	2	2
Demand		200	100	300	600				
Penalty	Column Diff	3	1	2					
		-	1	2					
		-	3	2					

Table 3.4: Represents the Solution by MODM

$$Z = \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} X_{ij}$$

$$Z = (8 * 25) + (10 * 125) + (11 * 175) + (4 * 200) + (5 * 75)$$

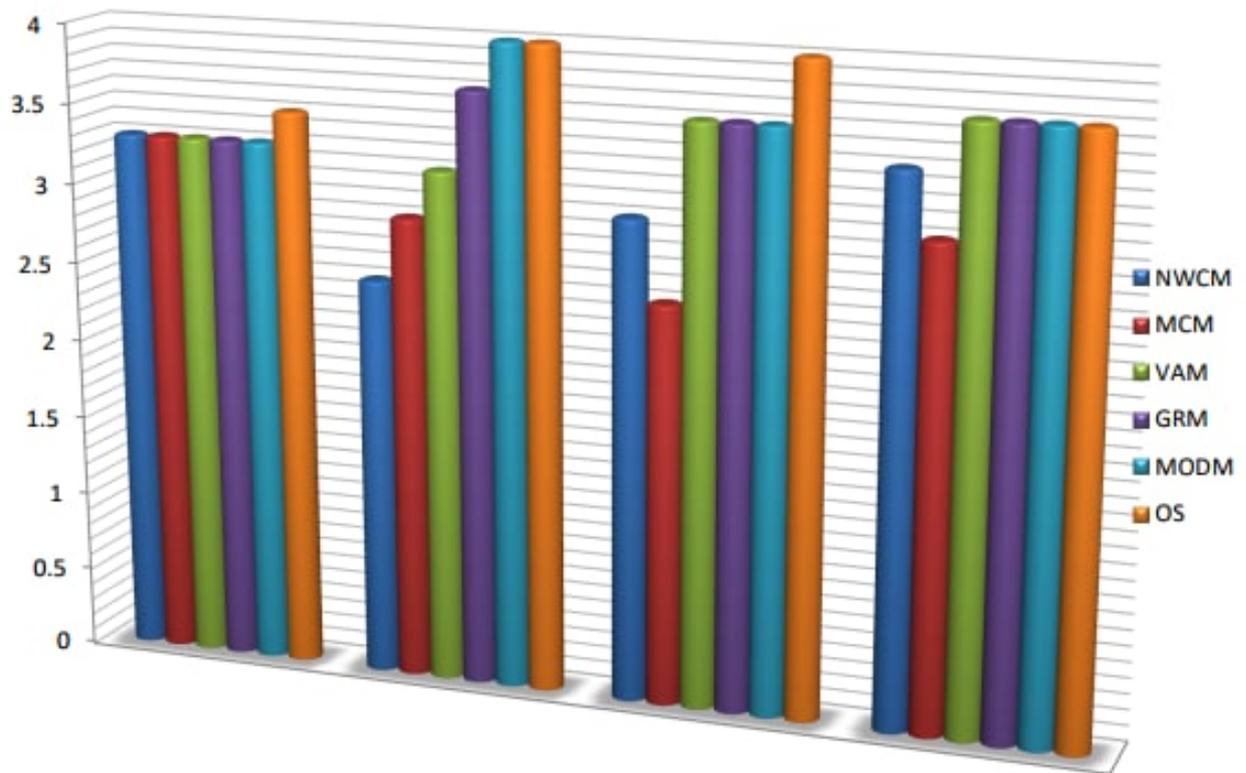
$$Z = 4550$$

3.4 Results

It is clear from Table 3.5. That the two new technique gives the best IBFS compared with the three classical methods because in general, it gives the lowest caste. A proposed new two method has been applied to many examples. The results of the solutions were better than the results of the three classic methods for solving transportation problems. In some examples, it equals the results of the VAM or OS , but in all examples, it never gives the worst results compared with any of the three classical methods. In this work, only four examples are cited in the results comparison table.

Examples	NWCM	MCM	VAM	GRM	MODM	OS
Ex: 3.2.1.	1500	1450	1500	1440	1500	1390
Ex: 3.2.2.	1010	920	880	840	840	840
Ex: 3.3.1.	1340	1295	1355	1270	1330	1245
Ex: 3.3.2.	5925	4550	5125	5125	4550	4525

Table 3.5: Comparing the Results



This graph shows the efficiency of the two proposed methods , where a large set of examples have been solved, and therefore the graph shows us that in some examples their efficiency is equal to the efficiency of the solution by classical methods (NWCM, MCM and VAM) and in some examples their efficiency is better or equal to the efficiency of the solution by the method (VAM) as well as their efficiency is better than classical methods and they are equal (OS) .

CHAPTER 4

USING STATISTICAL TECHNIQUE TO FIND AN
APPROXIMATE SOLUTION TO TRANSPORT
PROBLEMS

4.1 Introduction

The data tend to be centred around a specific value called a central value. In this case, the measures of central tendency are the function used to recognise this significant value to represent the data. In some cases, the data are close to the principal value and sometimes more widespread. To measure the proximity or distance of data from that central value, measures of dispersion are used. In this chapter, some of these measures are used to find the Approximation Solution to Transportation Problems.

4.2 Algorithm of Arithmetic mean method (AMM)

The steps necessary to solve the transportation problem are as follows:

step 1: The transfer schedule must be balanced.

step 2: We apply the mathematical formula $A = \sum_{i=1}^m \frac{C_{ij}}{m}$ to each column separately .

step 3: Determine the highest value resulting from Step (2) in all columns and then choose the cell with the lowest cost to give the proper supply to meet the needs (demand).

step 4: If the resulting values are in more than one column, we choose to allocate the cell with the lowest cost.

step 5: The columns filled in the application does not enter into the following calculation.

step 6: Repeat steps (2-4) and after completing the filling of cells we calculate the total cost.

4.2.1 Numerical Examples

we are numerical examples to make the proposed method very clear and well understood:

Example 4.2.1.

Wheat Stores		Mill				Supply
		S1	S2	S3	S4	
D1		3 100	4	6	0	100
D2		7	3 80	8	0	80
D3		6 10	4 30	5	0 50	90
D4		7	5	2 60	0 60	120
Demand		110	110	60	110	390
Penalty	Column Diff	5.7	4	5.2	0	
		6.6	4	5	0	
		-	4	5	0	
		-	4	-	0	
		-	4.5	-	0	

Table 4.1: Represents the Solution by AMM

$$Z = \sum_{i=1}^4 \sum_{j=1}^4 C_{ij} X_{ij}$$

$$Z = (3 * 100) + (3 * 80) + (6 * 10) + (4 * 30) + (0 * 50) + (2 * 60) + (0 * 60) = 840$$

Example 4.2.2.

Wheat Stores		Mill					Supply
		S1	S2	S3	S4	S5	
D1		5	7	10	5 1	3 4	5
D2		8 3	6 3	9 4	12	14	10
D3		10	9	8 6	10 4	15	10
Demand		3	3	10	5	4	25
Penalty	Column Diff	7.6	7.3	9	9	10.6	
		7.6	7.3	9	9	-	
		9	7.5	8.5	11	-	
		9	7.5	8.5	-	-	
		-	7.5	8.5	-	-	

Table 4.2: Represents the Solution by AMM

$$Z = \sum_{i=1}^3 \sum_{j=1}^5 C_{ij} X_{ij}$$

$$Z = (5 * 1) + (3 * 4) + (8 * 3) + (6 * 3) + (9 * 4) + (8 * 6) + (10 * 4)$$

$$Z = 183$$

4.3 Algorithm of median method (MM)

In statistics and probability theory, the median is the value dividing the upper half from the lower half of a data sample, a population, or a probability distribution, for a data set. The median is calculated as follows: The values are arranged in increasing order of importance (or descending), if the number of costs is even, the median represents the arithmetic mean between two values of the order $\frac{m}{2}$ and $\frac{m}{2} + 1$, the median represents the value of the order $\frac{m + 1}{2}$ if the number of costs is odd. [37]

The steps necessary to solve the transportation problem are as follows:

step 1: The transfer schedule must be balanced.

step 2: We calculate the median for each column only separately .

step 3: Determine the highest value resulting from Step (2) in all columns and then choose the cell with the lowest cost to give the proper supply to meet the needs (demand).

step 4: If the resulting values are in more than one column, we choose to allocate the cell with the lowest cost.

step 5: The column filled in the application does not enter into the following calculation.

step 6: Repeat steps (2-4) and after completing the filling of cells we calculate the total cost.

4.3.1 Numerical Examples

we are numerical examples to make the proposed method very clear and well understood:

The steps necessary to solve the transportation problem are as follows:

Example 4.3.1.

Wheat Stores		Mill				Supply
		S1	S2	S3	S4	
D1		4 10	6 30	8	8	40
D2		6	8	6 10	7 50	60
D3		5 10	7	6 40	8	50
Demand		20	30	50	50	150
Penalty	Column Diff	5	7	6	8	
		5	7	6	-	
		5	-	6	-	
		4.5	-	7	-	

Table 4.3: Represents the Solution by MM

$$Z = \sum_{i=1}^3 \sum_{j=1}^4 C_{ij} X_{ij}$$

$$Z = (4 * 10) + (6 * 30) + (6 * 10) + (7 * 50) + (5 * 10) + (6 * 40) = 920$$

Example 4.3.2.

Wheat Stores		Mill				supply
		S1	S2	S3	S4	
D1		20 60	22 40	17	4 20	120
D2		24	37	9 30	7 40	70
D3		32	37	20	15 50	50
demand		60	40	30	110	240
Penalty	Column Diff	24	37	17	7	
		24	-	17	7	
		-	-	17	7	

Table 4.4: Represents the Solution by MM

$$Z = \sum_{i=1}^3 \sum_{j=1}^4 C_{ij} X_{ij}$$

$$Z = (20 * 60) + (22 * 40) + (4 * 20) + (9 * 30) + (7 * 40) + (15 * 50)$$

$$Z = 3460$$

4.4 Results

The efficiency of the work of the two new techniques and the advantage of the results obtained upon use are very clear. As noted in comparing the results of the examples mentioned in this section when solving them, the results of the solution using the two proposed new methods are much better compared to the results of the solution by classic methods of solving the TP or equal to the results of the VAM . Also, have noted that the results obtained using the two new techniques are the OS to the problem at hand or close to the outcome of the OS to the problem.

Examples	NWCM	MCM	VAM	AMM	MM	OS
Ex: 4.2.1.	1010	920	880	840	840	840
Ex: 4.2.2.	234	191	187	183	183	183
Ex: 4.3.1.	980	960	960	920	920	920
Ex: 4.3.2.	3680	3670	3520	3460	3460	3460

Table 4.5: Comparing the Results

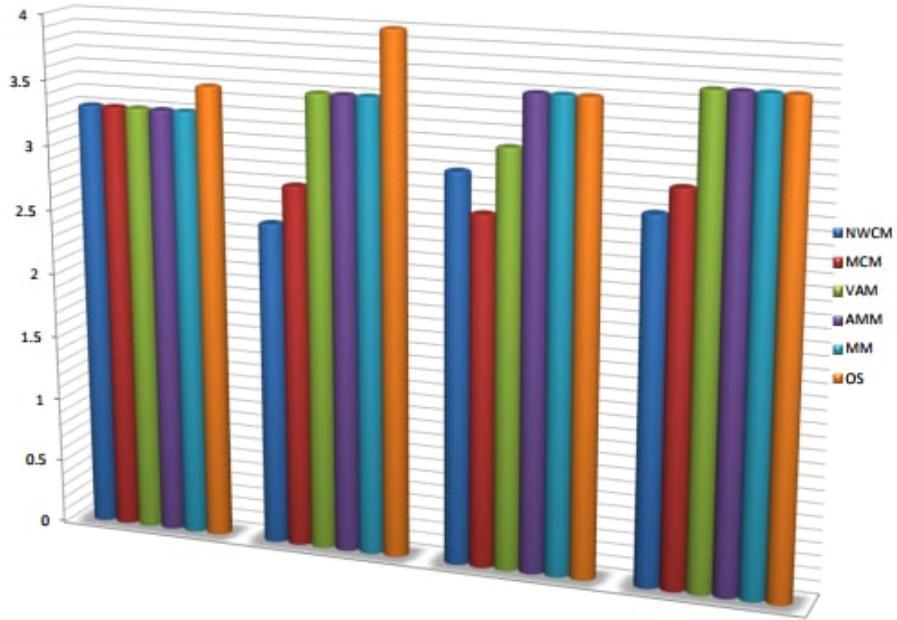


Figure 4.1: Efficiency Scheme Method (AMM and MM)

This graph shows the efficiency of the two proposed methods , where a large set of examples have been solved, and therefore the graph shows us that in some examples their efficiency is equal to the efficiency of the solution by classical methods (NWCM, MCM and VAM) and in some examples their efficiency is better or equal to the efficiency of the solution by the method (VAM) as well as their efficiency is better than classical methods and they are equal (OS) .

CHAPTER 5

CONCLUSIONS AND SUGGESTIONS

5.1 Conclusions

This thesis was supplemented by a study on an Approximation Solution to Transportation Difficulties. This solution applies to both balanced and unbalanced T.P. It uses four innovative approaches that are both very efficient and effective in achieving the desired outcomes. The researcher made a change to the Vogel's Approximation Method (VAM), and they acquired four new approaches (the first new technique (GRM), the second new technique (MODM), the third new technique (AMM), and the fourth new technique (MM)) to discover the Approximation Solution to a variety of transportation issues. To discover a solution to the transportation issue, the efficacy and efficiency of the results obtained through the solution of numerous practical examples using the new approaches was compared with the results obtained via the solution of the same problem using the traditional classic methods (TP). As a consequence of this, the results of the solution using any of the four provided ways were either superior than the results of the solution using any of classic methods or were on par with the results of the solution in the OS. It was also observed that when attempting to solve transportation problems by utilizing the four proposed methods, the majority of the results obtained were the same as the results obtained by applying the OS to this problem, and some of the results were very close to the OS. As a result, the four novel methods that may be used to obtain an approximation solution to the TP cut down significantly on the amount of time and effort required to get to the OS. In addition to this, the processes involved are straightforward and easy to comprehend. Nevertheless, they are scientific and methodical approaches that may be relied upon to determine the bare minimum for a variety of transportation issues.

5.2 Suggestions For Future Work

It is expected to obtain several works inspired by this study in the future , including:

- 1) developing the Golden Ratio method to be a direct way to obtain the (OS).
- 2) developing the Modulo method to to be a direct way to obtain the (OS).
- 3) Using other statistical laws to find (IBFS), as well as to find (OS) directly.

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