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Study of Some Problems in Reliability Models

A Thesis

Submitted to the Council of College of Education for Pure Sciences, University of Babylon in Partial Fulfillment of the Requirements for the Degree of Master in Education /Mathematics.

By

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Supervised by

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بسم الله الرحمن الرحيم

هُوَ الَّذِي بَعَثَ فِي الْأُمِّيِّينَ رَسُولًا مِّنْهُمْ يَتْلُو عَلَيْهِمْ آيَاتِهِ وَيُزَكِّيهِمْ وَيُعَلِّمُهُمُ الْكِتَابَ وَالْحِكْمَةَ

وَإِن كَانُوا مِن قَبْلُ لَفِي ضَلَالٍ مُّبِينٍ

صدق الله العظيم

سورة الجمعة: الآية ٢

Dedication

To my mother.

Who was the reason for what he is today,

Thank you for your constant care of me.

Thanks to all brothers, sisters and friends.

Abd AL- Hasan

Acknowledgements

Thanks are due to Allah for giving me the ability to complete this study. Many people have provided me, in a way or another, with assistance throughout the process of writing this thesis. To each of them, I owe sincere thanks and gratitude. I am specifically and sincerely grateful to my supervisor.

Prof. Dr. Zahir Abdul Haddi Hassan for his continual support and patience. His suggestions and comments greatly helped me to improve my work. My heartfelt gratitude goes to him. My thanks also extend to all members of the Mathematics Department (University of Babylon, College of Education for Pure Sciences) for their help and encouragement. I can never express appropriately and adequately my appreciation towards them. Finally, I would like to express my deepest appreciation to my family and friends for supporting and encouraging me throughout the years of my study and writing this thesis. This work would not have been possible without them.

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Table 1: List of abbreviations

Symbol	Description
$R(t)$	Reliability
P(t)	Probability
C	Combinations
MTTF	Mean time to failure
$h(t)$	Hazard rete
$f(x)$	Probability density function(p. d .f)
$F(t)$	Cumulative distribution function (c.d.f)
R_S	Systems reliability
R_{S1}	Reliability of first component
R_i^*	Subsystem reliability requirement
λ_s^*	System failure rate
λ_J	Predicted failure rate for J^{th} subsystem
λ_J^*	Allocated failure rate for J^{th} subsystem
$I_i(R)$	Importance of reliability
K_j	Is the J^{th} subsystem difficult level
C_j	Is the J^{th} subsystem cost

Supervisor's Certification

I certify that this thesis entitled " **Study of Some Problems in Reliability Models** " by **Abd AL-Hasan Hameed Saleh** has been prepared under my supervision in Babylon University / College of Education for Pure Sciences, in a partial requirements for the degree of Master in Education / Mathematics.

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Abstract

In this thesis, we calculated the reliability of mixed system by using some suitable methods, including reduction to the series element method convert it a series system only, path tracing method, minimum cut method and sum of disjoint products method.Also, included calculating the importance of all system's components to find the effect of all these components on the functioning of system as a whole. The topic of increasing the reliability of a system has been studied based on the importance of each component of the mixed system, through the most important components in the system obtaining the best allocation from the increase in reliability over others that are followed by importance. Three methods were used to assign the reliability of the mixed system, including the basic assignment method, minimum effort method, and the ARINC approach method, and then we compared them to find out the best method.Finally, we improved system reliability through redundancy, then redundancy after reduction turned out to be better than redundancy before reduction.

Introduction

The major focus is on multistate systems with elements that are detrimental since, of the usefulness of this technique in safe assessment, analysis, and predicting, as well as the research of the effectiveness of operating processes in real technological systems. Because most information systems have a large number of components and complex operating methods have, they fall under the category of systems. As a result, establishing a system's reliability and safety may be difficult. They are typically series systems with a large number of elements. Series processes are often designated for components or subsystems, resulting in paralleled-series or series-paralleled reliabilities designs [40]. While transporting water, gas, oil, and a variety of chemical substances via the pipe systems, we come across large series systems. These systems are often utilized to distribute electrical energy on a large scale. If all of its routes could carry people, a municipal bus transportation system with numerous communication lines, each operated by a single vehicle, might be regarded a modeled series systems. We may call it a paralleled series systems or m out of n systems if the communications lines has several buses at their disposal. An electrical cable thought up of several wires is one of the simpler parallel components, or m out of n systems. Electrical pulses may be sent using an m out of n -series system instead of a paralleled series systems. Telephony, rope-based transportation, and the use of belt conveyors and elevators all make use of large-scale systems of this kind [53]. Typical cases of series-paralleled and paralleled-series systems include ship-rope elevator and port elevator, deployed at shipyards throughout undocking and docking systems. This reliability analysis should go beyond the two-state technique to include a multistate approach, since these systems safety and operational process efficiency are important considerations. Reliability in people has traditionally been held in high regard as a characteristic. For engineering system, the reliability principle have been there for over than sixty year, but it has yet to be put into practice, the phrase took on a more technical sense and was employed to evaluate the safe operation of single-, twin-and four-engine aircraft. The amount of crashes per flying hour was used to gauge a plane's reliability. Quality control for industrial items was given a theoretical basis in the early [9, 55]. Nevertheless, these ideas were not brought into practice to any considerable degree until World War started. Products created with a great number of parts usually did not operate, even though it was formed up of individually high-qualities elements. The initial missiles all detonated on the landed or launch pad too soon despite efforts to use high-quality components and pay close attention to detail (English Channels), was enlisted to serve as a consultant. He was tasked with dissecting the missile system and came up with the law of product probability of series components in record time, he performed well. This theorem applies to systems that only operate if all of their elements are operational and valid under certain conditions. It states that the reliability of such a system is comparable to incorporating a large numbers of elements. As a result, even if the individual pieces are very reliable, the system's overall dependability may be low. In the United State, efforts were undertaken to overcome for low system dependability by boosting the individual component quality. Fundamental resources required to be enhanced, as well as product designs [14, 28] . It was able to improve the system's dependability. However, a systematic analysis of the subject was definitely not conducted at that time. As more and more sophisticated products with an ever-increasing number of parts were developed after, advancements continued to be made (electronic, televisions, computer). As automating progressed, so did the need for increasingly sophisticated systems of control and safety. A great deal of attention was paid to the development of intercontinental ballistic missiles and space exploration in the United States across the late 1950s and early 1960s. Sometime after, an association for engineers working with reliability issues was formed. Transactions on reliability, the earliest publication on the area, was published in 1963, and other publications on the subject were published in the 1960s. Despite its shortcomings, this document gives the first rigorous safety review of a complex system such as a nuclear power plant. Similar investigation has previously been conducted out in Asia and Europe [13]. In today's world, risks and dependability are analyzed in practically every business. In Norwegian, the comparable is true, notably in the offshore oil business. The North Sea's offshore oil and gas development is expanding into deeper and more inhospitable seas, and several remotely controlled subsea productions lines are placed into operations. Subsea system reliability is comparable to spaceflight reliability in its importance. Significant servicing will not be able to make up for a low level of reliability. Knight (1991) and Villemeur (1988) provide a more in-depth look at the history of reliability technology and how the system as a whole is comprised of the reliabilities of its many components [57]. An item's reliability measures how likely it is, given certain environmental factors and use patterns, that it will continue to perform as intended for an extended length of time. In the years after committees and some specific projects, several research facilities, universities, and mathematicians started investigating the issues of life testing and durability. The study of numerous daily reliability mechanisms, including series, parallel, series-paralleled, paralleled-series, and hybrid systems, is popular among scholars. Lists focusing on maintenance and failures of parts and equipment researchers have sought to design unique techniques to tackle mixed systems because of the numerous diverse systems to consider. As a starting concept, let us define dependability before moving on to some crucial probability and reliability concepts. During this era, Walludi Weiball was researching statistics theories for weariness. Reliability engineering progressed in lockstep with qualities engineering (Jurun and Gryana, 1988) [16, 54].

The military's principal use of reliability engineering was for vacuum tubes employed in radars devices and related devices, where reliability had shown to be difficult and expensive, established the reliability societies. Testing for component and system reliability. A military standard 781 was developed during this time period. Radio Corporation of America (RCA) published the widely utilized (and well discussed) around this time period and employed it to anticipate component failures ratings. There has been a gradual shift away from emphasizing component reliability and empirical study on their own. Techniques borrowed from the consumer goods industry, such as the more pragmatic. Published an important work entitled cumulative damage in fatigue in the ASME [28].

Construction and operation sparked an upsurge in global concern about risk and safety, both in the United State and elsewhere in the United States, a massive study committee headed by Normen Rasmassen was put up to investigate the situation. The multimillions-dollars effort culminated in the so-calling Rasmussen report, WASH-1400 (NUREG-75/014) [23]. Introduced the quadratic case of reliability to use the convex/concave mutual dependence of slice components along the curves of constant-slice eliability to maintain or increase the circuit reliability. Clear engineering techniques to evaluate the reliability of electrical systems used in aircraft show that elements of reliability must be linearly based on time [27].

The researchers have introduced accurate ways to calculate reliability of complex systems to calculate the reliability of the electromagnetic system inside aircraft and some engineering features. Also, the researcher has studied some ways to allocate reliability to the subsystem. As well as studying the reliability allocation and optimization of the reliability of the electromagnetic system within an aircraft by the genetic algorithm and the particle swarm optimization. Presented reliability of oxygen supply systems for spacecraft and some engineering characteristics. They have presented geometries of reliabilities models of the electricity systems utilized in spacecraft [2, 58].

In this thesis, we searched for appropriate methods for calculating the reliability of mixed systems and the study of reliability allocation and improvement. This thesis aims to calculate the reliability of the mixed system as well as allocate the reliability to find out which is the best method to allocate and improve the mixed systems.

This thesis consists of five chapters, while the suggested order is the one. Chapter one, contains some definitions, general concepts, and useful life of components.

Chapter two, contains reliability systems and how to calculate the reliability of each system and optimal arrangement for series and parallel systems.

Chapter three, contains the mixed systems and some methods to evaluate reliability of mixed systems.

Chapter four, the researcher tries to discuss the methods to allocate reliability the mixed system and improve the reliability of the mixed system before and after reduction.

Chapter five consists of conclusions and future works.

Chapter 1

Probability and Reliability

1.1 Introduction

In this chapter, we will begin our discussion of some general and important definitions such as probability, reliability and other definitions by first defining the concept of reliability. As well as studying the properties of some definitions and making a study on the reliability of the life of some components, to find out what is the reliability value of components and systems before and after operation and what are the reasons that affect the loss of reliability .While there were major challenges with the construction and operation of the technologies, the repercussions of failures were not as apparent or as tragic. Reliability issues have to be taken seriously from the beginning of the industrial period [26].

1.2 Some Definitions and General Concepts

Definition 1.1 ***Probability p(t)*** *[10] It is a numerical measure of the probability of an event occurring in relation to a set of alternative event.*

Probability Properties

$P(A)$ is the probability of an occurrence A in a sample space S, and it has the following properties:

(1) $0 \leq P(A) \leq 1$.

(2) $P(A) = 1 – P(A^c)$

(3) $P(\phi) = 0$

(4) $P(S) = 1$

If A and B are any two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{1.1}$$

If A, B and C are three events, then.

$$P(A \cup B \cup C) = P(A)+P(B)+P(C)-P(A \cap B)-P(A \cap C)-P(B \cap C)+P(A \cap B \cap C) \tag{1.2}$$

Definition 1.2 ***Probability density function (p. d .f)*** *[25] A continuous random variable X take all values in a interval of number [a,b] the probability.*

$$P(a \leq x < b) \; = \int_a^b f(x)\, dx \text{ and } f(x) \geq 0, for\ all\ x \tag{1.3}$$

Definition 1.3 ***Cumulative distribution function (c.d.f)*** *[24] The cumulative distribution function (c.d.f),is a function, F(y) of a random varible Y, and is defined for a number y by:*

$$F(y) = P(Y \leq y) = \int_{-\infty}^{y} f(s)\, ds \tag{1.4}$$

The mathematical relationship between the p.d.f and c.d.f is illustrated by:
$F(y) = \int_{-\infty}^{y} f(s)\, ds$, where s is a dummy integration variable. Conversely:

$$f(y) = \frac{d(F(y))}{d(y)} \tag{1.5}$$

Definition 1.4 ***System*** *[7] It is a group of compounds arranged in a specific order of communication with each other and with external components for a specific purpose.*

Definition 1.5 ***Reliability*** *[19] Is the probability that a device will function normally for a certain amount of time under specified operating circumstances.*

The probability of survival or reliability $R(t)$ at time t, has the following properties:

(1) $0 \leq R(t) \leq 1$.

(2) R(0) = 1; the device is assumed to be working properly at time t =0, and $R(\infty) = 0$; no device can work forever without failure.

(3) $R(t)$ in general is decreasing function of time t.

$$R(t) = P\{T \geq t\}. \tag{1.6}$$

$$R(t) = 1 - \int_0^t f(t)\, dt. \tag{1.7}$$

Definition 1.6 ***Mean time to failure (MTTF)*** *[34] The average time to failure reflects the most probable timeframe for a non-repairable systems to fail. MTTF is an evaluation of the mean or average time it takes for an element to fail or cause a disruption in the operation of a process, product, design, or procedure.*

Properties of the Failure Rate

MTTF considers that the product cannot be fixed and so cannot resume regular operations. When evaluating maintenance costs, it is typically used to quantify system or equipment reliability and is relevant even if there is no steady failure rate. Only components that can be fixed and restored to service should be assigned an MTTF.It is more significant to know the average time to failure of a product than to know all of the failure specifics. The MTTF is believed to be the same for all items that are equal in design and operate under same circumstances [20, 34]. If we have data from life tests on a populations of N objects with failing times $t_1, t_2, ..., t_n$ the MTTF may be stated mathematically as:

$$MTTF = \frac{1}{N}\sum_{i=1}^{n} ti \tag{1.8}$$

The MTTF is a function of the random variables T, which represents the component's period to failing, whether it is defined by the hazard model and its reliability functions.

The MTTF (mean time between failures) is described as:

$$\begin{aligned} MTTF =& E(T) \\ =& \int_0^\infty tf(t)\ dt \end{aligned} \tag{1.9}$$

$$\begin{aligned} f(t) &= \frac{dF(t)}{dt} \\ &= -\frac{dR(t)}{dt}. \end{aligned} \tag{1.10}$$

Hence

$$\begin{aligned} \text{MTTF} =& -\int_0^\infty tdT(t) \\ =& -tR(t)|_0^\infty + \int_0^\infty R(t)\ dt \\ =& \int_0^\infty R(t)\ dt \end{aligned} \tag{1.11}$$

The MTTF may alternatively be calculated as the R(t) Laplace transform, i.e.

$$\begin{aligned} \text{MTTF} &= \int_0^\infty t(R)\ dt \\ &= \lim_{t\to 0} \int_0^t R(s)\ ds \end{aligned} \tag{1.12}$$

However

$$\begin{aligned} &\lim_{t\to\infty} \int_0^t R(s)\ ds \\ &= \lim_{s\to 0} R(s) \end{aligned} \tag{1.13}$$

In which $R(s)$ is the Laplace transform of $R(t)$

$$MTTF = \lim_{s\to 0} R(s) \tag{1.14}$$

Definition 1.7 ***Constant hazard model h(t)*** *[34] The hazard rate is assumed to be constant and does not rise considerably with component age in this model.The constant-hazard approach is ideal for when a product is working throughout its effective lifetime.*

1.3 Relations Between h(t),R(t),F(t) and f(t)

Constant hazard models reliability is:

$$R(t) = 1 - F(t) = e^{-\lambda t}. \tag{1.15}$$

Consider a constant failure rate and a probability distribution with an exponential shape. Failure rate,

$$f(t) = \lambda e^{-\lambda t}. \tag{1.16}$$

Reliability function,

$$R(t) = e^{-\lambda t}. \tag{1.17}$$

Then hazard function,

$$h(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda. \tag{1.18}$$

The constant hazard model has the form $Z(t) = \lambda$. When the hazard rate is steady and equivalent to the failure rate.

Where λ is a constant. Many components, especially electrical ones, have this characteristic. This model has been utilized in reliability studies for several decades [35, 51]. The reliability and unreliability functions for an object with a steady hazard rate are as follows.

$f(t) = \lambda e^{-\lambda t}$,

$R(t) = e^{-\lambda t}$,

$$F(t) = 1 - e^{-\lambda t}. \tag{1.19}$$

The average times of failures of the item are:

$$\begin{aligned} \text{MTTF} &= \int_0^\infty e^{-\lambda t}\, dt \\ &= \frac{1}{\lambda}. \end{aligned} \tag{1.20}$$

Definition 1.8 ***Mutually exclusive events*** *[52] If A and B are mutually exclusive, then the probability of their union satisfies.*

$$P(A \cup B) = P(A) + P(B), \quad i.e \quad P(A \cap B) = 0 \tag{1.21}$$

Definition 1.9 ***Conditional probability*** *[56] The probability that A occurs given that B occurs is called the conditional probability of A given B and is written $P(A/B)$ as.*

$$P(A/B) = \left(\frac{P(A \cap B)}{P(B)}\right) \text{ for P(B)} > 0 \tag{1.22}$$

Definition 1.10 ***Component*** *[39] The component manufacturer usually specifies the component to meet functional requirements in most applications.*

Definition 1.11 ***Reliability allocation*** *[1] It is the technique of allocating individual unit reliability criteria in order to achieve the required system reliability.*

1.4 Reliability Allocation Factors

(1) Any system's complexity, as well as the complexity of individual subsystems, varies greatly. Because the amount of components that make up a module determines its reliability, reliability allocation should be based in complexities. The failures rates of a module is equivalent to the total of the failures rates of its individual elements, as is widely acknowledged. To begin with, all module's failures rate should be proportionate to the numbers of elements that compose it up. Therefore $Z_j \propto K_j$. In which K_j is the j^{th} subsystem's difficulty level. The number of active element groups is a common metric for these difficulty aspects.

(2) Because the cost increase for reliability enhancement for a very expensive subsystem is sometimes extremely substantial, cost considerations in the reliabilities allocations

programs are critical. The assigned reliabilities must also be proved, and showing a high-reliabilities levels for a pricey system might be prohibitively expensive. From this perspective, a lower reliability number should be assigned to a more expensive subsystem. A greater failure rate is desirable for a more expensive system $Z_j \propto C_j$. In which C_j is the j^{th} subsystem's cost [32, 68].

1.5 Importance of Reliability

Birnbaum's measurement of element I reliability relevance at period f equals the likelihood that the systems is in a really states at period t where component I is essential for the systems.Since the complexities of more extensive system and the consequences of its failures, the issue of reliability has taken on new dimensions in recent years. Today's modern-day unreliability may risk human life and generate operational inefficiency and uneconomical maintenance. Although the transition to thinking about reliability is challenging since it requires a conceptual shift, difficult does not imply impossible. The term reliability has an essential significance in its broadest sense: From developers, producers, investigators, and suppliers to users and everybody else involved in keeping a system dependable, reliability simply means that it is a liability [30]. More than ever before, engineers are paying close attention to the qualities and reliabilities of their system.

1.6 Useful Life of Components

Take a vast number of elements and put them through their paces, replacing them when they fail. In sufficiently lengthy intervals of similar length, about the same amount of failures will occur. A rapid buildup of stresses operating on and in the component is the physical mechanism of such failures. The unpredictability of the occurrence of accidental failures is clear since these quick stress accumulations happen at random. Let's say we plot the fail rates vs. T of a large sample of homogenous element populations. We put a large number of additional components of one type into operation at $T = O$. If this population contains any weak or inferior specimens, it would have a high failing rate at first. The failing rate drops dramatically throughout the burn-in or debugging phase, when these weak elements fail one at a period. Once the weaker elements have faded away, it stabilizes to a virtually constant value at time T. After being burned in or debugged, the component population reache its minimum fail rate, that is virtually constant. The exponential rules is a good approximation throughout this time, which is termed as the useable life period [39, 67]. Whenever the components near the end of their useful lives T_w they begin to show signs of wear. The failure rate begins to rise quickly beyond this point. If just a small fraction of the component population has failed up to the time T_w nearly half of the remaining components will fail between T_w and M. The population's mean wear out life is measured in time M. We call it basically mean life, as opposed to $m = \frac{1}{\lambda}$ .Which is the mean duration between failures. During the relevant life span. If the failing rates is reasonably low throughout the useful life's range, the mean time between failings may approach hundreds of thousands or possibly millions of hour. A component can't be used in operations for 100,000 hours only though its mean time among failings is confirmed to be 100,000 hours (or because its failures rate is 0.00001). The average duration among failures indicates the component's reliability. These information is critical throughout the useful life of a product. For every 10-hour operating period, a component with an average duration among failings of 100,000 hours would yield a reliabilities of 0.9999 or 99.99 percentage. Furthermore, we might anticipate just one component of this quality to fail if we ran 100,000 of them for an hour. Could we anticipate just one failure if we ran 10,000 components for 10 hours, 1000 elements for 100hrs, or 100 elements for 1000hrs under the same conditions. Although of the continual rating of failure of components over its effective life, any replacement scheme will not be able to avoid accidental failures. We could make no progress if we attempted to replace excellent, non-failed components within their valuable lives. We would be more likely to do injury since some of the new components might not have been fully burnt in, and their existence would just raise the rate of failure. As a result, the optimum approach is to replace components only when they fail throughout their useful life. Therefore, we must emphasize that no component should be permitted to stay in operation after its worn-out replacement time T_w has passed. Instead, the likelihood of component failure skyrockets, and the likelihood of system failure skyrockets even further. As a result, the main criterion of reliabilities are to replace elements as they fail during their effective lives and substitute each element, particularly if it had not failed, no late than whenever it approaches the end of its effective life. The burn-in technique is very necessary for missiles and space technologies. After the vehicle lifts flight, no component replacements are available, and any single component failure might result in the system's failure. Another golden rule of reliability is to burnin components before assembly, accompanied by a system debugging method [8, 70] .

Chapter 2

Reliability Systems

2.1 Introduction

In this chapter, we looked at some concepts of constant quality, such as the hazard- rate mean time to failure, and so on, as well as a representative portion of the work that has been done . This section examines the dependability of the framework. This necessitates a thorough understanding of the physical structure of the mechanism, which may range from small to large and include various segments. It also necessitates an appropriate colleague with he framework's concept in the event of subsystem failure. This chapter is divided into two parts. The second section includes a basic and includes series , parallel , series – parallel parallel - series, and a combination of series and parallel and k - out of – n framework. In the last section of this chapter, we will look at the mixed system, which is useful for difficult situations or systems where it is difficult to identify the components whether series or parallel, and we will show some ways to solve the mixed system. We also touch on optimal arrangement for series and parallel systems as well as maximizing the reliability of the parallel-serial system shown by the theory [15, 29].

2.2 System Reliability

Certain types of systems arise frequently in practice and serve to illustrate the concept . If a problem cannot be solved using the simple structure of this section, it may be possible to solve it by viewing it as a combination of simple structures [3, 47]. The components of systems can be combined in the following ways:

1. Series system.

2. Parallel system.

3. Parallel – series system.

4. Series – parallel system.

5. Mixed system.

6. K – out – of – n system.

2.2.1 Series System

Even if a system fails as a whole due to the loss of one or more elements, it is still regarded a series system. To put in another way, for a system to function, all of its components must be operational. A generic series system's reliability would be determined as following. Assuming a series system has n elements. R_i stands for component i is reliability, whereas R_s stands for system reliability. By meaning, a system's effective operation necessitates the functioning of all of its components [3, 31]. From theoretical probability.

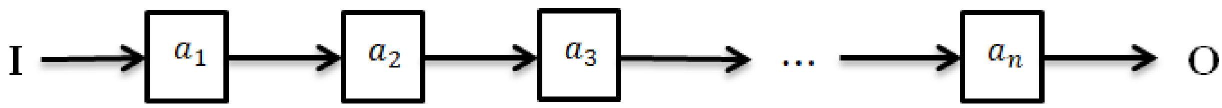

Figure 2.1: Series system.

The reliability of the system is given by:

$$R_S = R_{a_1} \times R_{a_2} \times \ldots \times R_{a_n} \tag{2.1}$$

Example 2.1 *Suppose a three-components system is connected in series, each with a fixed rate of failure. Components A, B, and C, respectively, have rates of* 0.4, 0.3, *and* 0.5 *per* 10, 000 *hours. As a result.*

Figure 2.2: Exampel of series system.

For constant failure rate λ, reliability.

$R(t) = e^{-\lambda t}$

Solution

$R_A = e^{-0.4t}$

$R_B = e^{-0.3t}$

$R_C = e^{-0.5t}$

Hence, the system's reliability is :

$$\begin{aligned} R(t) &= e^{-0.4t} \times e^{-0.3t} \times e^{-0.5t} \\ &= e^{-0.4t-0.3t-0.5t} \\ &= e^{-1.2t} \end{aligned}$$

The system's probability will continue to function beyond 30,000 hours is then calculated.

$$\begin{aligned} R(3) &= e^{-1.2\times 3} \\ &= e^{-3.6} \\ &= 0.0273 \end{aligned}$$

2.2.2 Parallel System

A system is considered a paralleled systems if and exclusively if the failures of all of its elements causes the whole systems to fail. In certain words, if one or more components are functioning, a parallel system succeeds. A lighting system with three lights in rooms, for instance, is a paralleled systems since room darkness occurs solely whenever all three lamps fail [22]. The following formula is used to determine the reliabilities of a generic paralleled systems. Assume a paralleled system consisting of mutually independent n components.

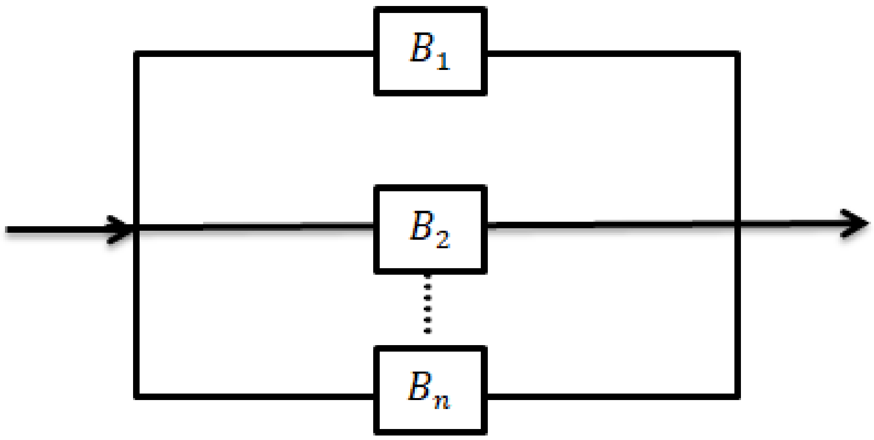

Figure 2.3: Parallel system.

The system reliability is then:

$$\begin{aligned} R_S &= 1 - P(\text{ all fail }) \\ &= 1 - [P\,(B_1 \text{ fails }) \times P\,(B_2 \text{ fails }) \times \ldots \times \mathrm{P}\,(B_n \text{ fails })] \\ &= 1 - (1 - R_1)\,(1 - R_2)\ldots(1 - R_n) \\ R_S &= 1 - \prod_{i=1}^{n}(1 - R_i) \end{aligned} \tag{2.2}$$

Example 2.2 *Parallel connections are made between component A, which has a constant rate of failure of* 0.3 *per* 1000 *hours, component B, which has a constant rate of failure of* 0.2 *per* 1000 *hours, and component C, which has a constant rate of failure* 0.6 *per* 1000 *hours. Determine the system's reliability.*

Figure 2.4: Example of parallel system.

Solution

$R\left(t\right) = e^{-\lambda t}$

$R_A\left(t\right) = e^{-0.3t}$

$R_B\left(t\right) = e^{-0.2t}$

$R_C\left(t\right) = e^{-0.6t}$

$$\begin{aligned} R_S(t) &= 1 - \left[\left(1 - R_A\right)\left(1 - R_B\right)\left(1 - R_C\right)\right] \\ &= 1 - \left[\left(1 - e^{-0.2t} - e^{-0.3t} + e^{-0.5t}\right)\left(1 - e^{-0.6}\right)\right] \\ &= e^{-0.2t} + e^{-0.3t} - e^{-0.5t} + e^{-0.6t} - e^{-0.8t} - e^{-0.9t} + e^{-1.1t} \end{aligned}$$

The system's probability will continue to function after 1000 hours, for example.

$$\begin{aligned} R_S(1) &= e^{-0.2(1)} + e^{-0.3(1)} - e^{-0.5(1)} + e^{-0.6(1)} - e^{-0.8(1)} - e^{-0.9(1)} + e^{-1.1(1)} \\ &= 0.97 \end{aligned}$$

After 1000 hours, a system like this had a 97% probability of remaining operating.

2.2.3 Parallel–series System

A generic paralleled–series system is made up of m subsystems that are connected in series and n components that are contained inside the parallel subsystem. We firstly convert every series subsystem to an equivalent reliability block to compute system reliability [3]. Then the system's reliability is given by.

The parallel–series system

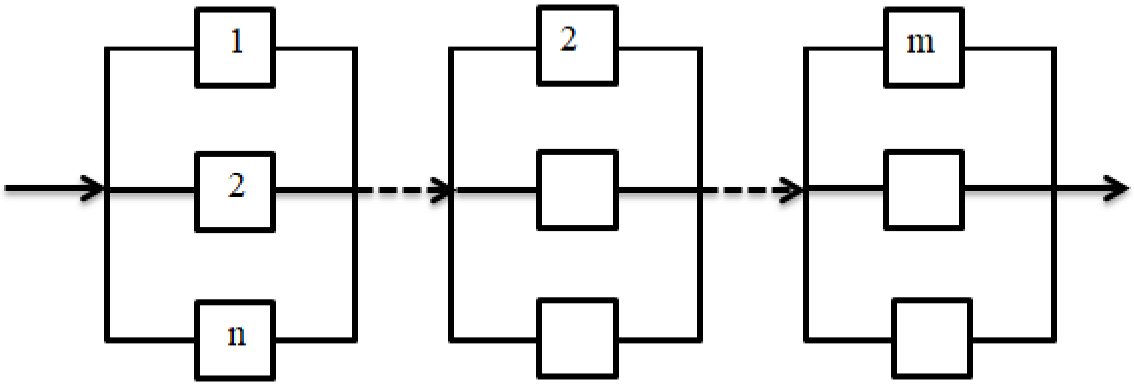

Figure 2.5: Parallel - series system.

$$R_S = [(1 - R_1)^n(1 - R_2)^n....(1 - R_n)^n]^m \tag{2.3}$$

$$R_S = [1 - (1 - R)^n]^m \tag{2.4}$$

Example 2.3 *Compute the reliability of the system in fig (2.6)*

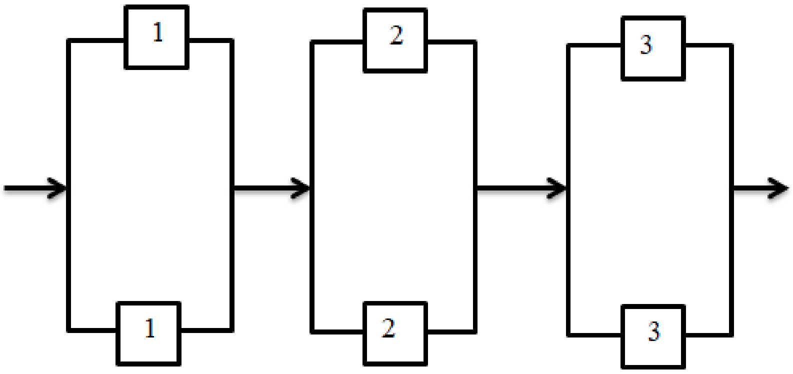

Figure 2.6: Example of parallel- series.

Solution

Assum that the reliability of each component is 0.80 here we are given $n = 2, m = 3$, using eq(2.4), we get.

$R_S = [1 - (1 - R)^n]^m$

$= \left[1 - (1 - 0.80)^2\right]^3$

$= (0.96)^3$

$= 0.88\%$

2.2.4 Series- parallel System

A system of series-parallel is made up of n subsystems running with m components in paralleled in series in all subsystems [31]. We may calculate the system's reliability as follows:

$$R_s = 1 - (1 - R)^n \tag{2.5}$$

Where $R = \prod_i^m r_i$, and r_i is the reliability of the i the component in series. Thus

$$R_S = 1 - \left[1 - \prod_{i=1}^{m} r_i\right]^n \tag{2.6}$$

The series-parallel system.

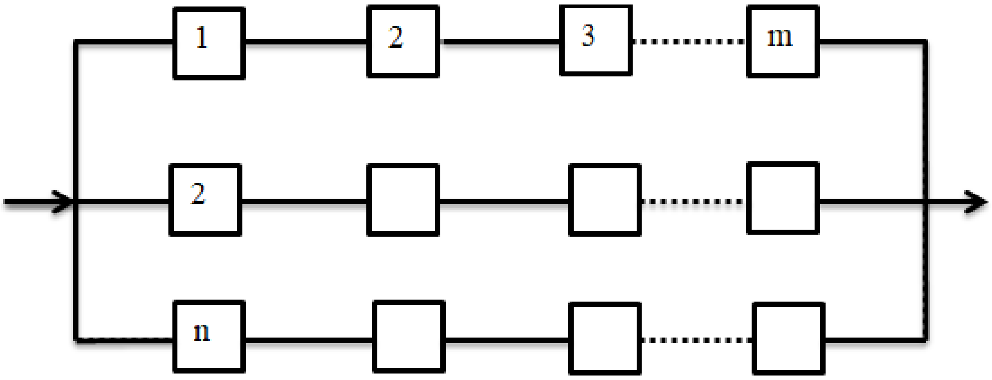

Figure 2.7: Series – parallel system .

Example 2.4 *Compute the reliability of the system in fig (2.8)*

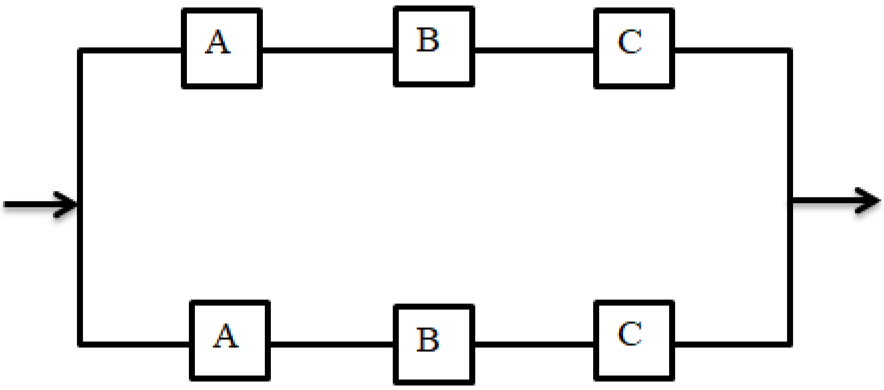

Figure 2.8: Example of series - parallel.

The reliability of $A = 0.87, B = 0.75, C = 0.95$, using the eq (2.6), we get the system reliability for $m = 3$ and $n = 2$.

$$R_S = 1 - \left[1 - \prod_{i=1}^{3} r_i\right]^2$$
$$= 1 - [1 - (0.87 \times 0.75 \times 0.95)]^2$$
$$= 1 - (1 - 0.619)^2$$
$$= 1 - 0.145$$
$$= 0.85$$

2.2.5 Mixed System

In order to meet functional or reliability requirements, serial and parallel arrangements are often blended into a system design. Series-parallel and parallel–series arrangements result from the combination [65].

Example 2.5 *Evaluate the system's dependability. Assum that the reliability of* $A = 0.2, B = 0.6, C = 0.8, D = 0.7$

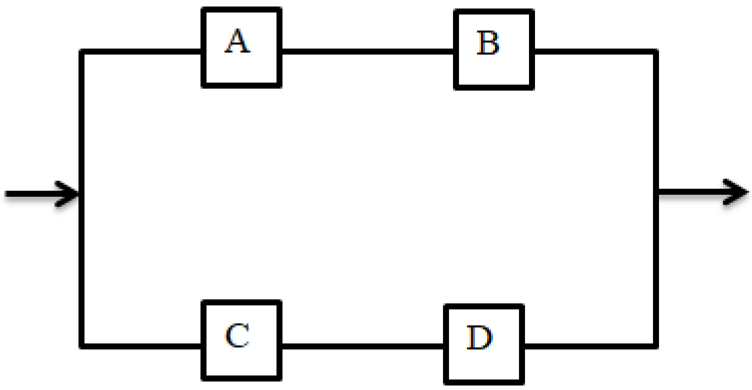

Figure 2.9: Mixed system.

Solution

First, assume the (A B) and (C D) branches. These are subsystem series. Reliability of top branch

$$R_{\text{top}}\,(t) = R_A(t) \times R_B(t)$$
$$= 0.2 \times 0.6$$
$$= 0.12$$

Reliability of bottom branch

$$R_{\text{bot}}\,(t) = R_C(t) \times R_D(t)$$
$$= 0.8 \times 0.7$$
$$= 0.56$$

Therefore, the two subsystems (A B) and (C D) may be recognized as part of a bigger system.

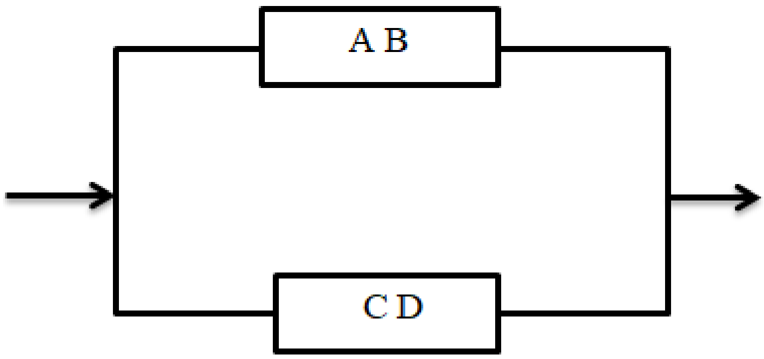

Figure 2.10: Example of mixed system.

And for this parallel system.

$$R_S(t) = 1 - [(1 - R_{AB}(t)) \times (1 - R_{CD}(t))]$$
$$= 1 - [(1 - 0.12) \times (1 - 0.56)]$$
$$= 1 - 0.3872$$
$$= 0.61$$

2.2.6 k-out-of-n systems

If at least one component of a parallel system works, it is operationally successful. In actuality, some systems need the success of several components in order for the overall system to function. These types of systems are common. An electricity-generating systems with four engines in derating mode might require at least two engines to operate at maximum capacity at the same time to create adequate power. Web hosts may have up to five servers installed, but at least three of them must be operational to ensure that the web service is not disrupted. To determine the location of an item, a positioning systems with five sensors needs have at least three sensors working. The $k-out-of-n$ system is a common name for this kind of system: G system, in which n is the total number of elements in the systems, k denotes the least number of n elements that might work for the system to operates appropriately, and G denotes good, which denotes success [41, 50] . A paralleled system is one that is one out of n:G, while a series systems are one that is n out of n:G. Occasionally, we could be captivated with describing a systems from the standpoint of failures. The system is referred to as $ak-out-of-n : F$ system if and solely if the failures of at least k elements provokes the n-component systems to failling, where F stands for failure. According to this concept, a paralleled system is an $n-out-of-n : F$ systems, while a series systems is $a_1 - out - of - n : F$ systems. An $k - out - of - n : G$ systems seems to be identical to a $(nk + 1) - out - of - n : F$ system. Consider just the $k - out - of - n : G$ system due to the equivalence connection. Assume that the failure times of n elements in an $k - out - of - n : G$ system are distributed independent and equally. Let x represent the system's numbers of operating components. x is thus a random variables with a binomially distribution. It is unlikely that all k components will be operating at the same time.

$$P_r\,(x = k) = C_n^k\; R_0^k(1 - R_0\;)^{n-k},\; k = 0, 1, \ldots, n \tag{2.7}$$

In which, R_0 is the component's reliability. Therefore, the system reliability R is a $k - out - of - n : G$ systems that needs at least k elements to operate.

$$R = P_r\,(x \geq k) = \sum_{i=k}^{n} C_n^i \; R_0^i \; (1 - R_0)^{n-i} \tag{2.8}$$

If $k = 1$, which is, so if n elements are in paralleled, the result is ,eq (2.8).

$$R = 1 - (1 - R_0)^n \tag{2.9}$$

This is comparable to, eq (2.9) might be expressed as $R = R_0^n$ since $k = n$ and the n elements are in series. When the duration to fail is exponentially, the system's reliability is exponential as well.

$$R\,(t) = \sum_{i=k}^{n} C_n^i \; e^{-\lambda i t}(1 - e^{-\lambda t}\;)^{n-i} \tag{2.10}$$

When the duration to failings is exponential, so is the system's dependability.

$$MTTF = \int_0^{\infty} R\,(t)\,dt = \frac{1}{\lambda}\sum_{i=k}^{n}\frac{1}{i} \tag{2.11}$$

Note that eq (2.11) is the identical when $k = 1$.

Example 2.6 *Five separate and similar servers are linked in parallel via a web host. For the online service to remain available, at least three should be operational. The exponential is distributed with $\lambda = 2.7 \times 10^{-5}$ failures per hour are used to simulate the server life. Over one year of continuous operation, compute the average time among failures (MTTF) and the web host's reliability [33].*

Solution

The hosting provider is a_3–out–of–$5 : G$ system. The MTTF and may be determined from the MTTF of a failed server that is promptly restored to a good-as-new state eq (2.11).

$MTTF = \frac{1}{2.7\times10^{-5}} \;\; \sum_{i=3}^{5} \frac{1}{i} = 2.9 \times 10^{-4}$ hours.

When the supplied data is substituted into eq (2.10) the web host's reliability is calculated at 8760 hrs (1 yr.).

$$R(8760) = \sum_{i=3}^{5} C_5^i e^{-2.7\times10^{-5}\times8760\times i}\left(1 - e^{-2.7\times10^{-5}\times8760}\right)^{5-i}$$

$$= 0.93$$

As previously stated, $a_1 - out - of - n$: G systems is a pure paralleled systems. A paralleled system with C nk paths, every with k different components in general, could be created from $ak - out - of - n$: G system. To demonstrate this shift, use $a_2 - out - of - 3 : G$ systems C_2^3 $= 3$ parallel pathways, every with two components, make up the comparable parallel system in fig (2.11) depicts the parallel system's reliability schematic diagram. The probability of a parallel system failure may be stated as

$$F = P_r\left(\overline{E_1 \cdot E_2} \cdot \overline{E_1 \cdot E_3} \cdot \overline{E_2 \cdot E_3}\right)$$

$$= P_r\left[\left(\overline{E_1} + \overline{E_2}\right) \cdot \left(\overline{E_1} + \overline{E_3}\right) \cdot \left(\overline{E_2} + \overline{E_3}\right)\right]$$

This equation is simplified by using the boolean principles.

$$F = P_r(\bar{E}_1 .\bar{E}_2 + \bar{E}_1.\bar{E}_3 + \bar{E}_2.\bar{E}_3) \tag{2.12}$$

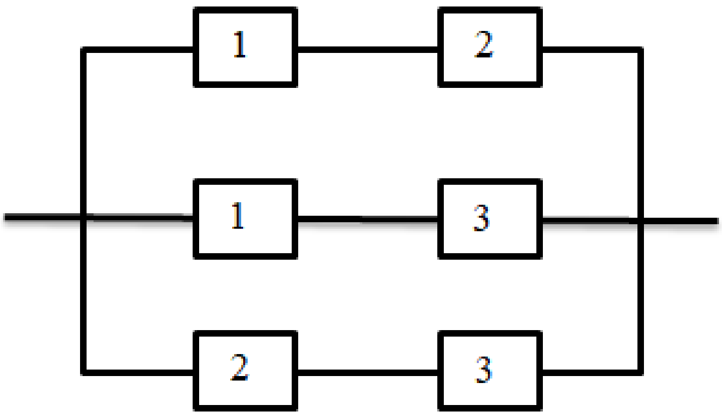

Figure 2.11: Reliability block diagram equivalent to : G system.

If any of the three $\bar{E}_1.\bar{E}_2, \bar{E}_1.\bar{E}_3, or \bar{E}_2.\bar{E}_3$ events happens, the system fails, according to the formula. A minimum cut set is $a_2 - out - of - 3$ event: the G system contains three minimal cutting sets with two components. A $k - out - of - n : G$ systems, in general, has C_n^{n-k+1} minimum cut sets, each of which has precisely k elements. Let's keep working

on the failure probability calculation, eq (2.12) for example, may be enlarged to.
$F = P_r\left(\bar{E_1}.\ \bar{E_2}\right) + P_r\left(\bar{E_1}.\bar{E_3}\right) + P_r\left(\bar{E_2}.\bar{E_3}\right) - 2P_r(\bar{E_1}.\bar{E_2}.\bar{E_3})$
Because the sines $\bar{E_1},\ \bar{E_2}\ \ and\ \ \bar{E_3}$ are mutually independent, the system's reliability may be expressed as.

$$\begin{aligned} R =& 1 - F \\ =& 1 - (1 - R_1)(1 - R_2) - (1 - R_1)(1 - R_3) - (1 - R_2)(1 - R_3) + \\ & 2(1 - R_1)(1 - R_2)(1 - R_3) \end{aligned} \tag{2.13}$$

When any components are similar and possess the same reliability R_0 , eq (2.13) produces $R = 1 - (1 + 2R_0)(1 - R_0)^2$
The reliability is similar to what it was before eq (2.8). It's worth noting that, unlike, eq (2.8), eq (2.13) does not need identical components to identify systems reliability. In circumstances whenever elements reliabilities are uneven, the translation of $ak-out-of-n : G$ systems to an similar paralleled system gives way to determining system reliability.

2.3 Optimal Arrangement for Series and Parallel Systems

Considering the challenge of maximizing the predicted number of such completed systems that would function properly by optimizing component assembly into series of systems. Every system series is made up of n elements, each being of a unique kind. There are n kinds of components accessible, everyone having k components; so, a total of nk components are accessible. The reliabilities of these k type m components are arranged in such a way that $P_1^m \leq P_2^m \leq \ldots \leq P_k^m.(1 \leq m \leq n)$. These nk components must be divided into k groups, every one containing n components of various sorts [46, 61]. A series system is created by the n components in every group. The challenge is how to build these k systems out of existing components in such a way that the anticipated number of such systems, E(N), that would function adequately is maximized. The solution is that if the most of every kind is placed in one system, the leftover best of every type is placed in the next system, and the worst of every type is placed in the ultimate system, E(N) will be maximized. To put it another way, if system j includes components with $P_j^1,\ P_j^2,\ \ldots, P_j^n$. reliabilities, E(N) is maximized. Regardless of the actual reliabilities numbers, this arrangement is best. Only the order of the reliabilities determines the best distribution. An invariant optimum arrangement is a kind of optimal allocation or arrangement (or assignment or design). Imagine another issue of optimum system assembling in which each of the k systems to be built is a parallel system. These are referred to as $S_1, S_2, \ldots, S_k$. S_j system have n_j components $(1 \leq j \leq k)$ and the values of $n_j for j = 1, 2, \ldots, k$. k does not need to be similar. The accessible components might be presumed to be of a similar kind. There is a total of $n = \sum_{j=1}^{k} n_j$ components accessible for selection and the reliabilities of these n components are ordered as.$P_1 \leq P_2 \leq ... \leq P_n$. $E(N)$ is maximized under these circumstances if a partition could be discovered that brings the reliabilities of these k systems as near together as feasible. There is no invariant optimum split or arrangement of the components if only the ordering of the relevant components is known [21]. A heuristic technique is proposed to find improved partitions via pairwise component exchange. The goal is to identify partitions that tend to equalize the parallel system's unreliabilities. Assume we begin with a partition $S_1, S_2, ..., S_k$. Use Q_j to symbolize the system's unreliability.

$S_j(1 \leq j \leq k) : Q_j = \prod_{i \in S_j} q_i$

Where $q_i = 1 - p_i for 1 \leq i \leq n$.However, we have identified the best partition since these Q_j are not all comparable to one another. We may pick the two partitions with the highest and lowest Q_j values to enhance on the existing partition, say, S_1 with $Q_1 = min_{1 \leq j \leq k} Q_j$ and S_k with $Q_k = max_{1 \leq j \leq k} Q_j$. Now pick a component from S_1 that has umeliabilily q_s $(s \in S_1)$ and a component from S_k that has unreliability $q_t(t \in S_k)$ such that $q_s < q_t \ or \ s > t$. If the inequality.

$\left| \frac{q_s Q_k}{q_t} - \frac{q_t Q_1}{q_s} \right| < |Q_k - Q_1|$

If this condition is met, the existing partition will be improved by exchanging the two components q_s of system S_1 and q_t of system S_k. The use of this heuristic does not ensure the best possible assignment. Furthermore, via pairwise component exchange, this heuristic provides a technique for refining a given design [64].

Chapter 3

Addressing the Problem of Finding Reliability for a Mixed System

3.1 Introduction

In this chapter, we will look at several reliability assignments models for mixed systems. The majority of real-world technological systems are extremely mixed, making it challenging to evaluate their reliability. The identification, assessment, prediction, and optimization of their reliability are mixed because of the huge number of components and subsystems and their mixed operating modes. In real life, the complexity of system operation procedures and their impact on changes in time, system architecture, and component reliability characteristics are often encountered. As a result, the practical significance of combining system reliability and system operation processes models into a single general model in reliability evaluations of genuine technological systems is clear [4, 38]. To find out what an assignment is and its purpose, the assignment is to use a reliability model to assign reliability to subsystems in order to achieve a system's specified reliability. We are also developing objective models to assign reliability to the mixed system. We have applied some methods and techniques to find system reliability, including reduction to series element method, path tracing method, minimal cut method, and sum of disjoint products method.

3.2 Mixed System

To figure out how reliable a system with both parallel and series parts, split the system into subsystems that only have parallel or parallel parts [65].

3.3 Some Methods to Evaluate Reliability of Mixed Systems

In this section, we will review some important ways to find the reliability of a mixed systems.

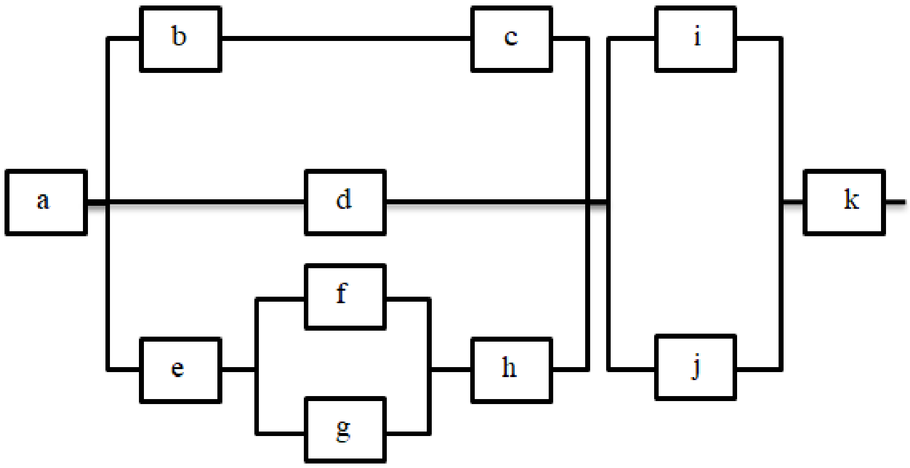

Figure 3.1: Mixed system.

3.3.1 Reduction to Series Element Method

Independent series, paralleled, series–paralleled, paralleled–series, $k - out - of - n$, and redundant subsystems are all used in certain systems. The system reduction approach involves breaking down a system into the subsystems listed above, all with a corresponding reliability series [6]. The diagram of reliability series is simplified until the complete system is described by a single dependability series.

Step 1

q = b and c are series

$R_q = R_b \, R_c$

Step 2

m = f and g are parallel

$$\begin{aligned} R_m &= 1 - [(1 - R_f)(1 - R_g)] \\ &= 1 - [1 - R_g - R_f + R_f R_g] \\ &= R_g + R_f - R_f R_g \end{aligned}$$

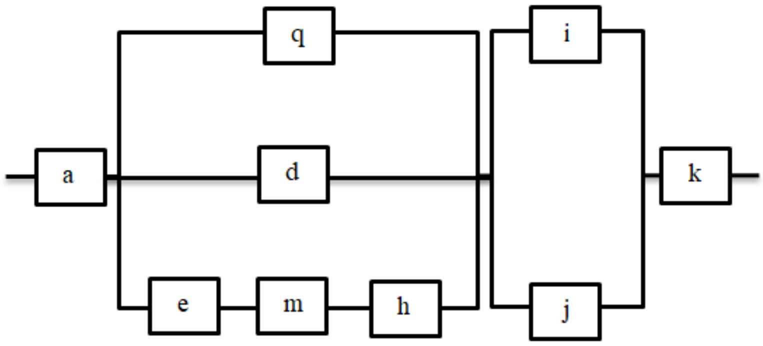

Figure 3.2: Modified (a) of a mixed system.

Step3

n = e and m and h are series

$$
\begin{aligned}
R_n &= R_e\ R_m\ R_h \\
&= R_e\,[R_g + R_f - R_f R_g]\,R_h \\
&= R_e R_g R_h + R_e R_f R_h - R_e R_f R_g R_h
\end{aligned}
$$

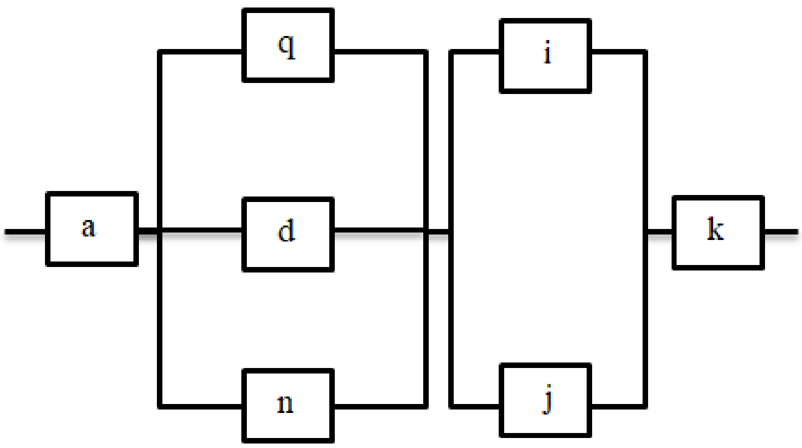

Figure 3.3: Modified (b) of a mixed system.

Step 4

p = q and d and n are parallel

$$R_p = 1 - [(1 - R_q)(1 - R_d)(1 - R_n)]$$
$$= 1 - [(1 - R_d - R_q + R_qR_d)(1 - R_n)]$$
$$= 1 - [1 - R_d - R_q + R_qR_d - R_n + R_dR_n + R_qR_n - R_qR_dR_n]$$
$$= R_d + R_q - R_qR_d + R_n - R_dR_n - R_qR_n + R_qR_dR_n$$
$$= R_d + R_bR_c - R_bR_cR_d + R_eR_gR_h + R_eR_fR_h - R_eR_fR_gR_h - R_eR_gR_hR_d$$
$$- R_eR_fR_hR_d + R_eR_fR_gR_hR_d - R_eR_gR_hR_bR_c - R_eR_fR_hR_bR_c + R_eR_f$$
$$R_gR_hR_bR_c + R_eR_gR_hR_bR_cR_d + R_eR_fR_hR_bR_cR_d - R_eR_fR_gR_hR_bR_cR_d$$

Step 5

u = i and j are parallel

$$R_u = 1 - [(1 - Ri)(1 - Rj)]$$
$$= R_j + R_i - R_iR_j$$

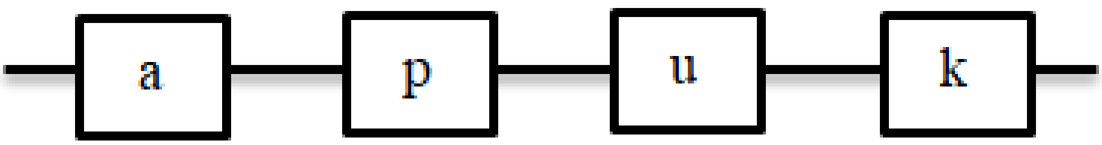

Figure 3.4: Modified (c) of a mixed system.

$$
\begin{aligned}
\mathrm{R_s} &= \mathrm{R_a\ R_p\ R_u\ R_k} \\
&= \mathrm{R_aR_dR_iR_k + R_aR_dR_jR_k + R_aR_bR_cR_iR_k + R_aR_bR_cR_jR_k - R_aR_dR_iR_jR_k} \\
&- \mathrm{R_aR_bR_cR_dR_iR_k - R_aR_bR_cR_dR_jR_k - R_aR_bR_cR_iR_jR_k + R_aR_eR_fR_hR_i} \\
&\mathrm{R_k + R_aR_eR_fR_hR_jR_k + R_aR_eR_gR_hR_iR_k + R_aR_eR_gR_hR_jR_k + R_aR_bR_c} \\
&\mathrm{R_dR_iR_jR_k - R_aR_dR_eR_fR_hR_iR_k - R_aR_dR_eR_fR_hR_jR_k - R_aR_dR_eR_gR_hR_i} \\
&\mathrm{R_k - R_aR_dR_eR_gR_hR_jR_k - R_aR_eR_fR_gR_hR_iR_k - R_aR_eR_fR_gR_hR_jR_k - R_a} \\
&\mathrm{R_eR_fR_hR_iR_jR_k - R_aR_eR_gR_hR_iR_jR_k - R_aR_bR_cR_eR_fR_hR_iR_k - R_aR_bR_c} \\
&\mathrm{R_eR_fR_hR_jR_k - R_aR_bR_cR_eR_gR_hR_iR_k - R_aR_bR_cR_eR_gR_hR_jR_k + R_aR_dR_e} \\
&\mathrm{R_fR_gR_hR_iR_k + R_aR_dR_eR_fR_gR_hR_jR_k + R_aR_dR_eR_fR_hR_iR_jR_k + R_aR_d} \\
&\mathrm{R_eR_gR_hR_iR_jR_k + R_aR_eR_fR_gR_hR_iR_jR_k + R_aR_bR_cR_dR_eR_fR_hR_iR_k +} \\
&\mathrm{R_aR_bR_cR_dR_eR_fR_hR_jR_k + R_aR_bR_cR_dR_eR_gR_hR_iR_k + R_aR_bR_cR_dR_eR_g} \\
&\mathrm{R_hR_jR_k + R_aR_bR_cR_eR_fR_gR_hR_iR_k + R_aR_bR_cR_eR_fR_gR_hR_jR_k + R_aR_b} \\
&\mathrm{R_cR_eR_fR_hR_iR_jR_k + R_aR_bR_cR_eR_gR_hR_iR_jR_k - R_aR_dR_eR_fR_gR_hR_iR_j} \\
&\mathrm{R_k - R_aR_bR_cR_dR_eR_fR_gR_hR_iR_k - R_aR_bR_cR_dR_eR_fR_gR_hR_jR_k - R_aR_bR_c} \\
&\mathrm{R_dR_eR_fR_hR_iR_jR_k - R_aR_bR_cR_dR_eR_gR_hR_iR_jR_k - R_aR_bR_cR_eR_fR_gR_h} \\
&\mathrm{R_iR_jR_k + R_aR_bR_cR_dR_eR_fR_gR_hR_iR_jR_k}
\end{aligned}
\tag{3.1}
$$

It is difficult to calculate this equation manually, so it was calculated in MATLAB program to extract system reliability R_S in eq (3.1).

3.3.2 Path Tracing Method

Every route from a beginning point to an endpoint is investigated while using the path tracing technique.Because system performance requires at least one way to be accessible from one end of the dependability block diagram to the other, the system has not fallen because at most one path from the start to the finish of the path is accessible.The chance of these pathways intersecting represents the system's dependability [6, 36].

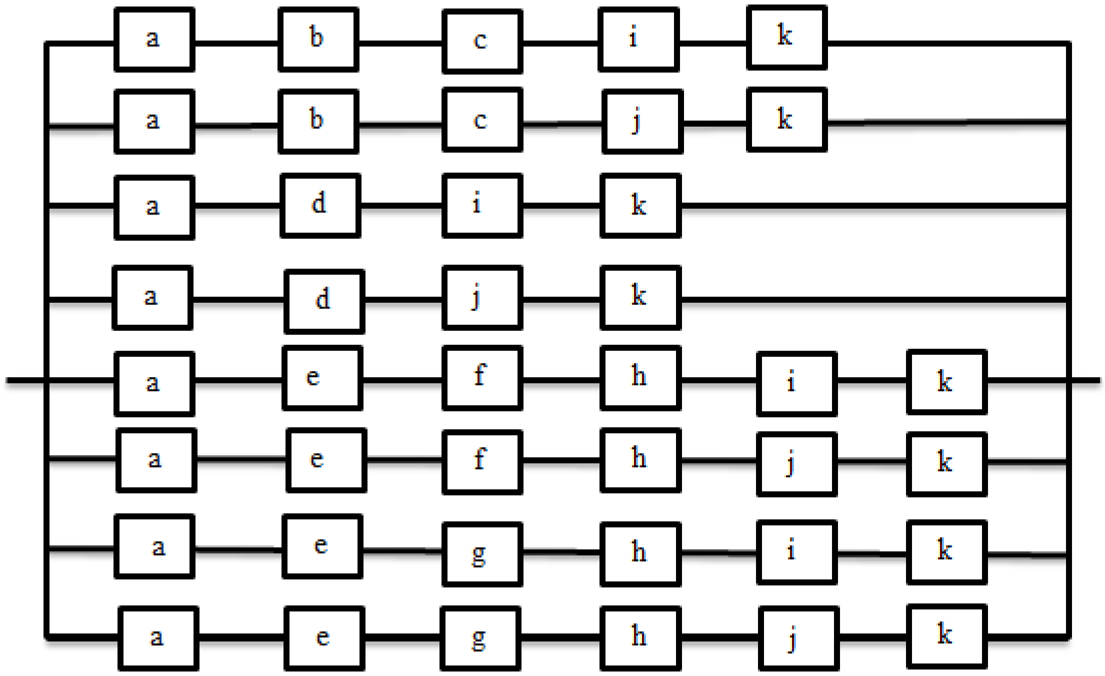

Figure 3.5: Minimal path sets as series-parallel configuration.

From fig (3.5) the reliability of minimal path sets are.

$$P_1 = \mathrm{abcik} \rightarrow R_{P_1} = R_a R_b R_c R_i R_k$$

$$P_2 = \mathrm{abcjk} \rightarrow R_{P_2} = R_a R_b R_c R_j R_k$$

$$P_3 = \mathrm{adik} \rightarrow R_{P_3} = R_a R_d R_i R_k$$

$$P_4 = \mathrm{adjk} \rightarrow R_{P_4} = R_a R_d R_j R_k$$

$$P_5 = \mathrm{aefhik} \rightarrow R_{P_5} = R_a R_e R_f R_h R_i R_k$$

$$P_6 = \mathrm{aefhjk} \rightarrow R_{P_6} = R_a R_e R_f R_h R_j R_k$$

$$P_7 = \mathrm{aeghik} \rightarrow R_{P_7} = R_a R_e R_g R_h R_i R_k$$

$$P_8 = \mathrm{aeghjk} \rightarrow R_{P_8} = R_a R_e R_g R_h R_j R_k$$

$$R_s = 1 - [(1 - R_{P1})(1 - R_{P2})(1 - R_{P3})(1 - R_{P4})(1 - R_{P5})(1 - R_{P6})(1 - R_{P7})(1 - R_{P8})]$$
$$= 1 - [(1 - R_aR_bR_cR_iR_k)(1 - R_aR_bR_cR_jR_k)(1 - R_aR_dR_iR_K)(1 - R_aR_dR_jR_k)$$
$$(1 - R_aR_eR_fR_hR_iR_k)(1 - R_aR_eR_fR_hR_jR_k)(1 - R_aR_eR_gR_hR_iR_k)(1 - R_aR_e$$
$$R_gR_hR_jR_k)]$$

The same polynomial in eq (3.1)

3.3.3 Minimal Cut Method

The minimum cut approach is a strong tool for studying systems that are linked in form of a hybrids. A cut is a group of elements so that when the elements in the cut are omitted from the system,there are no route from one endpoint to another [18, 62]. We obtain the following result when we implement this strategy to fig (3.1).

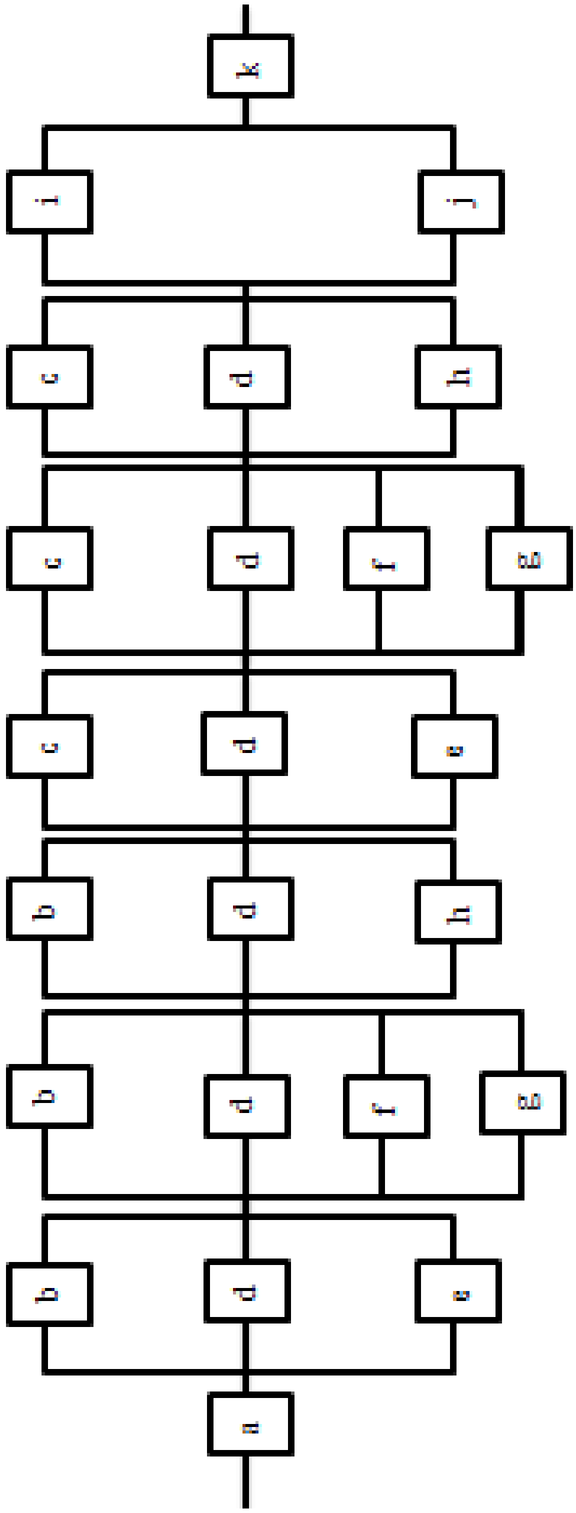

Figure 3.6: Minimal cut sets as parallel-series configuration.

$$\boldsymbol{c_1} = \text{a} \longrightarrow \boldsymbol{R_{c_1}} = \mathrm{R_a}$$

$$\boldsymbol{c_2} = \text{b and d and e are parallel} \rightarrow$$

$$\boldsymbol{R_{c_2}} = 1 - [(1-\mathrm{R_b})(1-\mathrm{R_d})(1-\mathrm{R_e})]$$

$$= 1 - [(1-\mathrm{R_d}-\mathrm{R_b}+\mathrm{R_bR_d})(1-\mathrm{R_e})]$$

$$= 1 - [1-\mathrm{R_d}-\mathrm{R_b}+\mathrm{R_bR_d}-\mathrm{R_e}+\mathrm{R_dR_e}+\mathrm{R_bR_e}-\mathrm{R_bR_dR_e}]$$

$$= \mathrm{R_d}+\mathrm{R_b}-\mathrm{R_bR_d}+\mathrm{R_e}-\mathrm{R_dR_e}-\mathrm{R_bR_e}+\mathrm{R_bR_dR_e}$$

$$\boldsymbol{c_3} = \text{b and d and f and g are parallel}$$

$$\boldsymbol{R_{c_3}} = 1 - [(1-\mathrm{R_b})(1-\mathrm{R_d})(1-\mathrm{R_f})(1-\mathrm{R_g})]$$

$$= 1 - [(1-\mathrm{R_d}-\mathrm{R_b}+\mathrm{R_bR_d})(1-\mathrm{R_g}-\mathrm{R_f}+\mathrm{R_fR_g})]$$

$$= \mathrm{R_g}+\mathrm{R_f}-\mathrm{R_fR_g}+\mathrm{R_d}-\mathrm{R_dR_g}-\mathrm{R_dR_f}+\mathrm{R_dR_fR_g}+\mathrm{R_b}-\mathrm{R_bR_g}-\mathrm{R_bR_f}+\mathrm{R_b}$$
$$\mathrm{R_fR_g}-\mathrm{R_bR_d}+\mathrm{R_bR_dR_g}+\mathrm{R_bR_dR_f}-\mathrm{R_bR_dR_fR_g}$$

$$\boldsymbol{c_4} = \text{b and d and h are parallel} \rightarrow$$

$$\boldsymbol{R_{c_4}} = 1 - [(1-\mathrm{R_b})(1-\mathrm{R_d})(1-\mathrm{R_h})]$$

$$= \mathrm{R_d}+\mathrm{R_b}-\mathrm{R_bR_d}+\mathrm{R_h}-\mathrm{R_dR_h}-\mathrm{R_bR_h}+\mathrm{R_bR_dR_h}$$

$$\boldsymbol{c_5} = \text{c and d and e are parallel} \rightarrow$$

$$\boldsymbol{R_{c_5}} = 1 - [(1-\mathrm{R_c})(1-\mathrm{R_d})(1-\mathrm{R_e})]$$

$$= \mathrm{R_d}+\mathrm{R_c}-\mathrm{R_cR_d}+\mathrm{R_e}-\mathrm{R_dR_e}-\mathrm{R_cR_e}+\mathrm{R_cR_dR_e}$$

$$\boldsymbol{c_6} = \text{c and d and f and g are parallel} \rightarrow$$

$$\boldsymbol{R_{c_6}} = 1 - [(1-\mathrm{R_c})(1-\mathrm{R_d})(1-\mathrm{R_f})(1-\mathrm{R_g})]$$

$$= \mathrm{R_g}+\mathrm{R_f}-\mathrm{R_fR_g}+\mathrm{R_d}-\mathrm{R_dR_g}-\mathrm{R_dR_f}+\mathrm{R_dR_fR_g}+\mathrm{R_c}-\mathrm{R_cR_g}$$
$$-\mathrm{R}_c\mathrm{R_f}+\mathrm{R_cR_fR_g}-\mathrm{R_cR_d}+\mathrm{R}_c\mathrm{R_dR_g}+\mathrm{R}_c\mathrm{R_dR_f}-\mathrm{R_cR_dR_fR_g}$$

$$\boldsymbol{c_7} = \text{c and d and h are parallel} \rightarrow$$

$$\boldsymbol{R}_{c7} = 1 - [(1-\mathrm{R_c})(1-\mathrm{R_d})(1-\mathrm{R_h})]$$

$$= \mathrm{R_d}+\mathrm{R_c}-\mathrm{R_cR_d}+\mathrm{R_h}-\mathrm{R_dR_h}-\mathrm{R_cR_h}+\mathrm{R_dR_h}\mathrm{R}_c$$

$$\boldsymbol{c_8} = \text{i and j are parallel} \rightarrow$$

$$\boldsymbol{R_{c_8}} = 1 - [(1 - \mathrm{R_i})(1 - \mathrm{R_j})]$$

$$= \mathrm{R_i} + \mathrm{R_j} - \mathrm{R_i R_j}$$

$$\boldsymbol{c_9} = \mathrm{k} \rightarrow \boldsymbol{R}_{c_9} = \mathrm{R_k}$$

$$\boldsymbol{R_s} = \boldsymbol{R_{c_1}}\ \boldsymbol{R_{c_2}}\ \boldsymbol{R}_{c_3}\ \boldsymbol{R}_{c_4}\ \boldsymbol{R}_{c_5}\ \boldsymbol{R}_{c_6}\ \boldsymbol{R}_{c_7}\ \boldsymbol{R_{c_8}}\ \boldsymbol{R}_{c_9}$$

The same polynomial in eq (3.1)

3.3.4 Sum of Disjoint Products Method (SDP)

The SDP technique evaluates the possibility of the combination of numerous minimal pathways using minimum paths or minimal cuts. The logic function of the system may represent the union of the minimum pathways. A union of many phrases may be used to define this logic function. All sets are covered in the SDP approach, none are double-counted, and all terms exist. The SDP equation is shorter than the equation for all but the tiniest systems. Terms that are incompatible addition to the law. The basic explanation for the SDP approach is the addition rule of probability [42, 73]. The likelihood that at least one of the minimum pathways will happen is the sum of the possibilities of the minimal single paths if two or many minimal paths had no components in similar. If two minimal paths a and b have elements in common, the union of these two minimal paths, $a \cup b$, may be expressed as the union of minimal path a with minimal paths. $\overline{a}b$, where $\overline{a}$ denotes the complement of a. Then we have the following equation for evaluation of the probability of $a \cup b$:

$P_r(a \cup b) = P_r(a) + P_r(\overline{a}b)$

Similarly with three minimal paths a, b, and c, we have.

$P_r(a \cup b \cup c) = P_r(a) + P_r(\overline{a}b) + P_r(\overline{a\overline{b}}c)$

With n minimal paths $p_1, p_2, ..., p_n$, we have.

$P_r\left(p_1 \cup \ldots \cup p_n\right) = P_r\left(p_1\right) + P_r\left(\overline{p_1} p_2\right) + P_r\left(\overline{p_1 p_2} p_3\right) + \cdots + P_r\left(\overline{p_1} \ldots \overline{p_{n-1}} p_n\right)$

From path tracing method we get.

$$\mathrm{p_1 = abcik, p_2 = abcjk, p_3 = adik, p_4 = adjk, p_5 = aefhik, p_6 = aefhjk}$$
$$\mathrm{p_7 = aeghik, p_8 = aeghjk,}$$

$$\begin{aligned} R_{(S)} = & P_r\left(p_1\right) + P_r\left(\overline{p_1} p_2\right) + P_r\left(\overline{p_1 p_2} p_3\right) + P_r\left(\overline{p_1 p_2 p_3} p_4\right) + P_r\left(\overline{p_1 p_2 p_3 p_4} p_5\right) \\ & + P_r\left(\overline{p_1 p_2 p_3 p_4 p_5} p_6\right) + P_r\left(\overline{p_1 p_2 p_3 p_4 p_5 p_6} p_7\right) \\ & + P_r\left(\overline{p_1 p_2 p_3 p_4 p_5 p_6 p_7} p_8\right) \end{aligned}$$

$$\begin{aligned} R_S = & P_r(\text{ abcik }) + P_r(\overline{\text{abcik}}\text{ abcjk}) + P_r(\overline{\text{abcik abcjk}}\text{ adik}) \\ & + P_r(\overline{\text{ abcik abcjk adik }}\text{ adjk }) + P_r(\overline{\text{ abcik abcjk adik adjk }}\text{ aefhik }) \\ & + P_r(\overline{\text{ abcik abcjk adik adjk aefhik }}\text{ aefhjk }) \\ & + P_r(\overline{\text{ abcik abcjk adik adjk aefhik aefhjk aeghik }}\text{ aeghjk}) \end{aligned}$$

$$\begin{aligned} \mathrm{R_s} = & \mathrm{R_aR_bR_cR_iR_k + (1 - R_aR_bR_cR_iR_k)R_aR_bR_cR_jR_k + (1 - R_aR_bR_cR_iR_k)(1 - R_a} \\ & \mathrm{R_bR_cR_jR_k)R_aR_dR_iR_k + (1 - R_aR_bR_cR_iR_k)(1 - R_aR_bR_cR_jR_k)(1 - R_aR_dR_i} \\ & \mathrm{R}_k\mathrm{)R_aR_dR_jR_k + (1 - R_aR_bR_cR_iR_k)(1 - R_aR_bR_cR_jR_k)(1 - R_aR_dR_iR_k)(1 - R_a} \\ & \mathrm{R_dR_jR_k)R_aR_eR_fR_hR_iR_k + (1 - R_aR_bR_cR_iR_k)(1 - RaR_bR_cR_jR_k)(1 - R_aR_d} \\ & \mathrm{R_iR_k)(1 - R_aR_dR_jR_k)(1 - R_aR_eR_fR_hR_iR_k)R_aR_eR_fR_hR_jR_k + (1 - R_aR_bR_c} \\ & \mathrm{R_iR_k)(1 - R_aR_bR_cR_jR_k)(1 - R_aR_dR_iR_k)(1 - R_aR_dR_jR_k)(1 - R_aR_eR_fR_hR_i} \\ & \mathrm{R_k)(1 - R_aR_eR_fR_hR_jR_k)R_aR_eR_gR_hR_iR_k + (1 - R_aR_bR_cR_iR_k)(1 - R_aR_bR_cR_j} \\ & \mathrm{R_k)(1 - R_aR_dR_iR_k)(1 - R_aR_dR_jR_k)(1 - R_aR_eR_fR_hR_iR_k)(1 - R_aR_eR_fR_hR_jR_k)} \\ & \mathrm{(1 - R_aR_eR_gR_hR_iR_k)R_aR_eR_gR_hR_jR_k} \end{aligned}$$

The same polynomial in eq (3.1).

Chapter 4

Some Techniques to Address Increasing Reliability of Mixed Systems

4.1 Introduction

In this chapter, we'll talk about how to enhance the mixed system. Certain system elements are clearly more significant than others in establishing whether the system is operating properly. For instance, an element that is mixed in with the entire system would be just as significant as any other element in the systems. A quantifiable estimate of the significance of the system's distinct elements might be beneficial. We must first construct such a metric. The significance of criticality is an element significance metric that is especially useful for prioritizing maintenance tasks [49]. Then we allocate the reliability of the system using some methods, including the basic allocation method, minimum effort method and ARINC approach method, and compare the methods to find out the best method. Then we work to improve the reliability of the mixed systems by deriving the reliability of the systems in relation to the main components of the system. We also do redundancy of the components to determine the best redundancy before or after reduction.

4.2 Reliability Allocation

In a mixed system, overarching system properties, such as reliability, must be translated into particular requirements for the multiple parts that making up the system. Reliability allocation is the practice of allocating required reliability to specific units in order to achieve the required systems reliability. The underlying inequality must be solved in order to allocate system reliability [69].

$$F\left(R_1^*, R_2^*, \ldots, R_n^*\right) \geq R^*. \tag{4.1}$$

Where,

R^*: required reliability system.

R_i^* : i^{th} required subsystem reliability.

The above formula is modified for a series systems as.

$$R_1^* \,.R_2^* \ldots R_n^* \geq R^*. \tag{4.2}$$

If there are no constraints on the allocation, this equation theoretically contains an endless number of solutions. The challenge is to devise a technique that produces a solution that allows for allocating consistent and acceptable reliabilities. Reliability criteria derived via an allocation mechanism might be more realistic, cost-effective, and consistent, than those derived from subjectives techniques or crash programs launched in response to bad field experiences [37].

4.3 Benefits of the Reliability Allocation Program

(1) The reliability allocations procedure makes it necessary for system designers and developers to comprehend and build links among component subsystems and system reliabilities. This contributes to a better comprehension of the design's core reliability issues.

(2) Reliability must be considered with other systems criteria like cost, weights, and performance aspects by the systems engineer.

(3) The reliability allocation system guarantees that the design, production processes, and test methods are appropriate.

The allocation procedure is approximate, and the system efficiency characteristics allotted to the subsystems, like reliability and maintainability, are utilized as guides to assess design feasibility. If a system's allotted parameters could not be satisfied with present technologies, the system should be updated and the allocation reallocated [12, 71]. At the initial levels of a system's breakdown into its key subsystems, apportionment is most useful. It's especially important at this level since each main component is usually generated by an agency or distinct division. The physically nature of the system has a big impact on how deep down into the assembly we should go to meet our system needs.

4.4 Subsystems Reliability Improvement

There might constantly be those subsystem for which reliability are recognized as in every mixed system. In an age of rapid technological innovation, on the other hand, a complex system may often include multiple components that will be utilized for the first time, and no reliability forecasts can be made for these units. There might be multiple units in the former group where reliability could be improved. At the same time, there might be certain units that we choose to use without making any attempt to increase their reliability [45]. This might be due to the fact that we have no plans to change the design of these subsystems for a variety of reasons, including the expense of redesign, the lack of alternatives, and the mission's tight timeline. It is evident that such subsystems should not be included in the reliability allocation process since we would be unable to integrate the allotted values in any way. As a result, all like subsystem are recognized, and the needed systems reliability target is split by the product of such subsystem reliabilities. As a result, a novel target is set for the leftover units to achieve. Assume the system has N subsystems, each with an R^* reliability aim. Suppose there be $m(\leq N)$ subsystems in which the evaluated or predicted reliabilities are available, and reliability enhancements are regarded achievable out of these N subsystems. Assume the remaining subsystems $(n = N - m)$ by their estimated or expected reliabilities [11]. We must assign reliabilities to these subsystems based on cost, mixed, and state of the art factors. The reliability allocations issue for the group is covered in the following part, which is outside the scope of this section. We need to divide the reliability objective into two sub-goals because we propose to break down the issue of reliability allocations into two distinct subproblems requiring m and $(n = N - m)$ unit, respectively. The objective is calculated as $(R^*)^{m/N}$ for a first m elements and $(R^*)^{n/N}$ for the last n elements in the second categories. For ease of noting

$$Assume \quad \mathrm{R}' = (\mathrm{R}^*)^{m/N} \tag{4.3}$$

$$also \quad \mathrm{R}'' = (\mathrm{R}^*)^{n/N} \tag{4.4}$$

As a result, the issue statement for these part are: Systems have m elements with projected reliabilities of $R_1, R_2, ..., R_m$. R' is the intended systems reliability [44]. Assign revised reliability values to $R_1^*, R_2^*, \ldots, R_m^*$. Three approaches to solving this challenge are discussed below.

4.4.1 Basic Allocation Method

This strategy improves the dependability of each component subsystem in order to meet the reliability target. The theory's underlying principle is to reduce the failing rates of every component by a similar factor [5].

λ_s^* failing rates of the system.

λ_J anticipated failing rates for J^{th} subsystem.

λ_J^* allocated failing rates for J^{th} subsystem.

This method's stages are as follows:

(i) If λ_s^* is the failing rates of the system, necessary allocation unit failing rates λ_J^* must be selected in order for.

$$\lambda_1^* + \lambda_2^* + \ldots + \lambda_n^* \leq \lambda_s^*. \tag{4.5}$$

(ii) The fail rates are used to calculate relative units weight.

$$W_j = \frac{\lambda_j}{\sum_{j=1}^{m} \lambda_j}. \tag{4.6}$$

(iii) Because W_J denotes the relative susceptibility of the J^{th} unit to fail and.

$$\sum W_j = 1 \tag{4.7}$$

$\lambda_J^* = w_j \lambda_s^*$.

(iv) If reliability values are to be allocated.

$$R_j^* = (R^-)_j^w. \tag{4.8}$$

Example 4.1 *I discuss the reduction system in fig(3.4) has four series with predicted failure rate of.*

$\lambda_a = 0.09, \quad \lambda_p = 0.07, \quad \lambda_u = 0.05, \quad \lambda_k = 0.1$

if system failure rate is desired to be 0.010 , allocation failure rates to four unites.

Solution

$$\sum \lambda_J = \lambda_a + \lambda_p + \lambda_u + \lambda_k$$

$$= 0.09 + 0.07 + 0.05 + 0.01$$

$$= 0.31$$

Therefore

$W_1 = \frac{0.09}{0.31} = 0.2903$

$W_2 = \frac{0.07}{0.31} = 0.2258$

$W_3 = \frac{0.05}{0.31} = 0.1613$

$W_4 = \frac{0.1}{0.31} = 0.3226$

Hence

$\lambda_a^* = 0.2903 \times 0.010 = 2.903 \times 10^{-3}$

$\lambda_p^* = 0.2258 \times 0.010 = 2.258 \times 10^{-3}$

$\lambda_u^* = 0.1613 \times 0.010 = 1.613 \times 10^{-3}$

$\lambda_k^* = 0.3226 \times 0.010 = 3.226 \times 10^{-3}$

4.4.2 Minimal Effort Method

In the prior methodology, every subsystem's reliabilities were anticipated to increase in order to meet the system reliability target. As a result, particularly high-reliability elements will need to be enhanced. It is generally recognized that increasing the dependability of such elements is prohibitively expensive [5, 43]. As a result, we will explain an approach that has been shown to take little work under the specific assumption let.

$R_1, R_2, R_3, \ldots, R_n$ be their single reliabilities, and the reliability of the system would be.

$$R_s = R_1.R_2.R_3 \ldots R_n. \tag{4.9}$$

Let R_s^* be the desired reliability for the system, and assume that R_s^* is greater than R_s. We have now to increase the reliability of a minimum number of components to a value such that the new reliability of the system becomes R_s^*. To achieve this we proceed as follows.

(1) The component's recognized reliabilities were listed in increasing order. Suppose, after doing so, that.

$$R_1 \leq R_2 \leq R_3 \leq \ldots \leq R_n \tag{4.10}$$

(2) The first K components' reliabilities in the equation $R_i^* = e^{-\lambda_i^*} \approx 1 - \lambda_i^*$ were increase to the same value R_0. The reliabilities $R_{k+1}, R_{k+2}, \ldots, R_0$ at the remaining component are left unchanged [72]. For determining K and R the first is to calculate the value of.

$$\left[\frac{R_s^*}{R_2 \times R_3 \ldots R_n}\right]^{\frac{1}{1}} \tag{4.11}$$

This is compared with R_1. If

$$\left[\frac{R_s^*}{R_2 \times R_3 \ldots R_n}\right]^{\frac{1}{1}} > R_1 \tag{4.12}$$

The amount of R_1 must then be raised to R. The following stage is to determine the quantity of.

$$\left[\frac{R_s^*}{R_3 \times R_4 \ldots R_n}\right]^{\frac{1}{2}} \tag{4.13}$$

This is compared with R_2. If

$$\left[\frac{R_s^*}{R_3 \times R_4 \ldots R_n}\right]^{\frac{1}{2}} > R_2 \tag{4.14}$$

The amount of R_2 must then be raised to R. This method is repeated until

$$\left[\frac{R_s^*}{R_{k+1} \times R_{k+2} \ldots R_n}\right]^{\frac{1}{2}} > R_k \tag{4.15}$$

Then R_k will also have to be increased to the value R. In general, K is the maximum value of j such that.

$$\left[\frac{R_s^*}{\prod_{i=j+1}^{n+1} R_i}\right]^{\frac{1}{j}} > R_j \tag{4.16}$$

Where R_{n+1} the reason for introducing R_{n+1} is to include the possibility. That in a system it may be necessary to raise the reliabilities.

$R_1, R_2, R_3, \ldots, R_n$ of all the n components to common higher value, whose maximum value will be at most equal to $R_{n+1} = 1$.

Example 4.2 *In this example we will modified mixed system in fig (3.4) to find the reliability allocation by this method and the system reliability is to be improved to value of (0.75).*

Solution

Let\ $R_1 = R_a = 0.91$

$R_2 = R_p = 0.93$

$R_3 = R_u = 0.95$

$R_4 = R_k = 0.90$

$\mathrm{R}_s = R_a \times R_p \times R_u \times R_k$

$= 0.91 \times 0.93 \times 0.95 \times 0.90$

$\cong 0.72$

$R_1 \leq R_2 \leq R_3 \leq R_4$

$0.90 \leq 0.91 \leq 0.93 \leq 0.95$

$R_1^* = \left[\frac{R_s^*}{R_2 \times R_3 \times R_4 \times R_5}\right]^{\frac{1}{1}} = \left[\frac{0.75}{0.91 \times 0.93 \times 0.95 \times 1}\right]^{\frac{1}{1}} = 0.93$

Since $0.93 > 0.90$ the value of R_1 will have to be increased to R_0. Next

$R_2^* = \left[\frac{0.75}{0.93 \times 0.95 \times 1}\right]^{\frac{1}{2}} = 0.92$

Since $0.92 > 0.91$ the value of R_2 will have to be increased to R_0. Next

$R_3^* = \left[\frac{0.75}{0.95\times1}\right]^{\frac{1}{3}} = 0.92$

Since $0.92 < 0.93$ the value of R_3 will be left unaltered. Next

$R_4^* = \left[\frac{0.75}{1}\right]^{\frac{1}{4}} = 0.93$

Since $0.93 < 0.95$ the value of R_4 will be left unaltered.

$R_s^* = R_1^* \times R_2^* \times R_3^* \times R_4^*$

$= 0.93 \times 0.92 \times 0.93 \times 0.95$

$= 0.75$

4.4.3 ARINC Approach Method

These components are, linked in series, independently of one another, exponentially dispersed, and, share a common mission duration, according to the ARINC concept developed by aeronautical research Inc. The process of selecting single element failing rate becomes reliability allocation λ_i^* [5, 66]. The following weighting variables are used to calculate λ_i^*. Which takes into consideration the chance of component failing (one of the criteria mentioned above).

$$W_i = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i} \quad , \quad i = 1, 2, \ldots, n \tag{4.17}$$

Wherein λ_i is the element i failure rate as determined by historical information or forecast. The variables represent the probability of failing. The higher the w_i number, the more likely the element will fail. As a result, the failing rates objective assigned to an element must be proportionate to the weight value: apparently.

$$\lambda_i^* = w_i.\lambda^* \quad , i = 1, 2, ..., n. \tag{4.18}$$

and

$$\lambda^* = -\frac{ln[R^*(t)]}{t}. \tag{4.19}$$

This is the element's highest permitted failing rates. The relevant dependability objective may be computed easily as follows.

$$R_i^*(t) = exp(-w_i \lambda^* t) \qquad , i = 1, 2, ..., n., \tag{4.20}$$

Example 4.3 *Suppose a communication system in fig (3.4) consists of four subsystems a, p, u and k having consecutive failure rate λ_a = 0.09 , λ_p = 0.07 , λ_u = 0.05 and λ_k = 0.1 failing month, selecting the reliability required throughout (36) moths in service to achieve total reliability it is (0.73).*

Solution

$$W_i = \frac{\lambda_i}{\sum_{i=1}^{4} \lambda_i} \qquad , \quad i = 1, 2, 3, 4$$

$$W_1 = \frac{\lambda_1}{\sum_{i=1}^{4} \lambda_i} = \frac{0.09}{0.09+0.07+0.05+0.1} = \frac{0.09}{0.31} = 0.29$$

$$W_2 = \frac{0.07}{0.31} = 0.22$$

$$W_3 = \frac{0.05}{0.31} = 0.16$$

$$W_4 = \frac{0.1}{0.31} = 0.32$$

Now, by using equation (4.19) we find λ^* where

$$\lambda^* = -\frac{\ln[R^*(t)]}{t}$$
$$= -\frac{\ln[0.73]}{36} = 8.741 \times 10^{-3}$$

By using equation (4.18) we get.

$$\lambda_a^* = 0.29 \times 8.741 \times 10^{-3} = 2.53489 \times 10^{-3}$$

$$\lambda_p^* = 0.22 \times 8.741 \times 10^{-3} = 1.92302 \times 10^{-3}$$

$\lambda_u^* = 0.16 \times 8.741 \times 10^{-3} = 1.39856{\times}10^{-3}$

$\lambda_k^* = 0.32 \times 8.741 \times 10^{-3} = 2.79712{\times}10^{-3}$

Now, we find the reliability per component by using equation (4.20) we get.

$R_a^*(36) = \exp(-2.53489{\times}10^{-3} \times 36) = 0.9127$

$R_p^*(36) = \exp(-1.92303{\times}10^{-3} \times 36) = 0.9331$

$R_u^*(36) = \exp(-1.39856{\times}10^{-3} \times 36) = 0.9508$

$R_k^*(36) = \exp(-2.79712{\times}10^{-3} \times 36) = 0.9042$

Now, the reliability at 36 months is.

$$\mathrm{R}_i^*(36) = R_a^*(36) \times R_p^*(36) \times R_u^*(36) \times R_k^*(36)$$

$$= 0.9127 \times 0.9331 \times 0.9508 \times 0.9042$$

$$= 0.73$$

Reliability required.

4.5 Comparison Between ARINC and Basic Allocation Method

We will compare between these two methods to find out the failure rate through the example (4.1)

Table 4.1: Allocated method.

Basic Allocation method	ARINC Approach method
$\lambda_a^* = 2.903 \times 10^{-3}$	$\lambda_a^* = 2.53489{\times}10^{-3}$
$\lambda_p^* = 2.258 \times 10^{-3}$	$\lambda_p^* = 1.92302{\times}10^{-3}$
$\lambda_u^* = 1.613 \times 10^{-3}$	$\lambda_u^* = 1.39856{\times}10^{-3}$
$\lambda_k^* = 3.226 \times 10^{-3}$	$\lambda_k^* = 2.79712{\times}10^{-3}$

The ARINC approach method is better than the basic allocation method because allocating failures relative to the components in the ARINC approach method is less than the basic allocation method as in the above table.

4.6 Importance of Reliability

In a system with n components, the reliability significance I of component i is defined by

$$I_{(i)} = \frac{\partial R_s(t)}{\partial R_i(t)}. \tag{4.21}$$

The system reliability is $R_s(t)$, the component reliability is, $R_i(t)$, and ∂ is a partial derivative [17, 63]. The reliability significance offered by this formula is calculated by the element's reliability as well as its location in the systems.

Example 4.4 *Fig (3.1) shows a mixed system with eleven items of equipment. We attempt to determine the relevance of every system unit in order to understand how it affects the system's operation.*

$R_a = 0.91, R_b = 0.81, R_c = 0.72, R_d = 0.71, R_e = 0.7, R_f = 0.5, R_g = 0.6, R_h = 0.8, R_i = 0.81, R_j = 0.75, R_k = 0.90$

Solution

$$\begin{aligned}\frac{\partial R_s(t)}{\partial R_a} =& \mathrm{R_dR_iR_k + R_dR_jR_k + R_bR_cR_iR_k + R_bR_cR_jR_k - R_dR_iR_jR_k - R_bR_cR_dR_i} \\ & \mathrm{R_k - R_bR_c}R_d\mathrm{R_jR_k - R_bR_cR_iR_jR_k + R_eR_fR_hR_iR_k + R_eR_fR_hR_jR_k + R_e} \\ & \mathrm{R_gR_hR_iR_k + R_eR_gR_hR_jR_k + R_bR_cR_dR_iR_jR_k - R_dR_eR_fR_hR_iR_k - R_dR_e} \\ & \mathrm{R_fR_hR_jR_k - R_dR_eR_gR_hR_iR_k - R_dR_eR_gR_hR_jR_k - R_eR_fR_gR_hR_iR_k - R_e} \\ & \mathrm{R_fR_gR_hR_jR_k - R_eR_fR_hR_iR_jR_k - R_eR_gR_hR_iR_jR_k - R_bR_cR_eR_fR_hR_iR_k} \\ & \mathrm{- R_bR_cR_eR_fR_hR_jR_k - R_bR_cR_eR_gR_hR_iR_k - R_bR_cR_eR_gR_hR_jR_k + R_d} \\ & \mathrm{R_eR_fR_gR_hR_iR_k + R_dR_eR_fR_gR_hR_jR_k + R_dR_eR_fR_hR_iR_jR_k + R_dR_eR_g}\end{aligned}$$

$$\mathrm{R_hR_iR_jR_k + R_eR_fR_gR_hR_iR_jR_K + R_bR_cR_dR_eR_fR_hR_iR_k + R_bR_cR_dR_e}$$
$$\mathrm{R_fR_hR_jR_k + R_bR_cR_dR_eR_gR_hR_iR_k + R_bR_cR_dR_eR_gR_hR_jR_k + R_bR_c}$$
$$\mathrm{R_eR_fR_gR_hR_iR_k + R_bR_cR_eR_fR_gR_hR_jR_k + R_bR_cR_eR_fR_hR_iR_jR_k +}$$
$$\mathrm{R_bR_cR_eR_gR_hR_iR_jR_k - R_dR_eR_fR_gR_hR_iR_jR_k - R_bR_cR_dR_eR_fR_gR_hR_i}$$
$$\mathrm{R_d - R_bR_cR_dR_eR_fR_gR_hR_jR_k - R_bR_cR_dR_eR_fR_hR_iR_jR_k - R_bR_cR_dR_eR_g}$$
$$\mathrm{R_hR_iR_jR_k - R_bR_cR_eR_fR_gR_hR_iR_jR_k + R_bR_cR_dR_eR_fR_gR_hR_iR_jR_k}$$
$$= 0.66$$

$$\frac{\partial R_s(t)}{\partial R_b} = \mathrm{R_aR_cR_iR_k + R_aR_cR_jR_k - R_aR_cR_dR_iR_k - R_aR_cR_dR_jR_k - R_aR_cR_iR_dR_k +}$$
$$\mathrm{R_aR_cR_dR_iR_jR_k - R_aR_cR_eR_fR_hR_iR_k - R_aR_cR_eR_fR_hR_jR_k - R_aR_cR_e}$$
$$\mathrm{R_gR_hR_iR_k - R_aR_cR_eR_gR_hR_jR_k + R_aR_cR_dR_eR_fR_hR_iR_k + R_aR_cR_dR_e}$$
$$\mathrm{R_fR_hR_jR_k + R_aR_cR_dR_eR_gR_hR_iR_k + R_aR_cR_dR_eR_gR_hR_jR_k + R_aR_cR_e}$$
$$\mathrm{R_fR_gR_hR_iR_k + R_aR_cR_eR_fR_gR_hR_jR_k + R_aR_cR_eR_fR_hR_iR_jR_k + R_aR_c}$$
$$\mathrm{R_eR_fR_gR_hR_iR_jR_k - R_aR_cR_dR_eR_fR_gR_hR_iR_k - R_aR_cR_dR_eR_fR_gR_hR_j}$$
$$\mathrm{R_k - R_aR_cR_dR_eR_fR_hR_iR_jR_k - R_aR_cR_dR_eR_fR_hR_iR_jR_k - R_aR_cR_dR_eR_g}$$
$$\mathrm{R_hR_iR_jR_k - R_aR_cR_eR_fR_gR_hR_iR_jR_d + R_aR_cR_dR_eR_fR_gR_hR_iR_jR_k}$$
$$= 0.08$$

$$\frac{\partial R_s(t)}{\partial \mathrm{R_c}} = \mathrm{R_aR_bR_iR_k + R_aR_bR_jR_k - R_aR_bR_dR_iR_k - R_aR_bR_dR_jR_k - R_aR_bR_iR_jR_k + R_aR_bR_dR_iR_jR_k - R_aR_bR_eR_fR_hR_iR_k - R_aR_bR_eR_fR_hR_jR_k - R_aR_b R_eR_gR_hR_iR_k - R_aR_bR_eR_gR_hR_jR_k + R_aR_bR_dR_eR_fR_hR_iR_k + R_aR_bR_d R_eR_fR_hR_jR_k + R_aR_bR_dR_eR_gR_hR_iR_k + R_aR_bR_dR_eR_gR_hR_jR_k + R_aR_b R_eR_fR_gR_hR_iR_k + R_aR_bR_eR_fR_gR_hR_jR_k + R_aR_bR_eR_fR_hR_iR_jR_k + R_a R_bR_eR_gR_hR_iR_jR_k - R_aR_bR_dR_eR_fR_gR_hR_iR_k - R_aR_bR_dR_eR_fR_gR_hR_j R_k - R_aR_bR_dR_eR_fR_hR_iR_jR_k - R_aR_bR_dR_eR_gR_hR_iR_jR_k - R_aR_bR_eR_f R_gR_hR_iR_jR_k + R_aR_bR_dR_eR_fR_gR_hR_iR_jR_k} = 0.28$$

$$\frac{\partial R_s(t)}{\partial \mathrm{R_d}} = \mathrm{R_aR_iR_k + R_aR_jR_k - R_aR_iR_jR_k - R_aR_bR_cR_iR_k - R_aR_bR_cR_jR_k + R_aR_bR_c R_iR_jR_k - R_aR_eR_fR_hR_iR_k - R_aR_eR_fR_hR_jR_k - R_aR_eR_gR_hR_iR_k - R_aR_e R_gR_hR_jR_k + R_aR_eR_fR_gR_hR_iR_k + R_aR_eR_fR_gR_hR_jR_k + R_aR_eR_fR_hR_i R_jR_k + R_aR_eR_gR_hR_iR_jR_k + R_aR_bR_cR_eR_fR_hR_iR_k + R_aR_bR_cR_eR_fR_h R_jR_k + R_aR_bR_cR_eR_gR_hR_iR_k + R_aR_bR_cR_eR_gR_hR_jR_k - R_aR_eR_fR_gR_h R_iR_jR_k - R_aR_bR_cR_eR_fR_gR_hR_iR_k - R_aR_bR_cR_eR_fR_gR_hR_jR_k - R_aR_bR_c R_eR_fR_hR_iR_jR_k - R_aR_bR_cR_eR_gR_hR_iR_jR_k + R_aR_bR_cR_eR_fR_gR_hR_iR_j R_k} = 0.18$$

$$\frac{\partial R_s(t)}{\partial \mathrm{R_e}} = \mathrm{R_aR_fR_hR_iR_k + R_aR_fR_hR_jR_k + R_aR_gR_hR_iR_k + R_aR_gR_hR_jR_k - R_aR_dR_f R_hR_iR_k - R_aR_dR_fR_hR_jR_k - R_aR_dR_gR_hR_iR_k - R_aR_dR_gR_hR_jR_k - R_aR_f R_gR_hR_iR_k - R_aR_fR_gR_hR_jR_k - R_aR_fR_hR_iR_jR_k - R_aR_gR_hR_iR_jR_k - R_a R_bR_cR_fR_hR_iR_k - R_aR_bR_cR_fR_hR_jR_k - R_aR_bR_cR_hR_gR_iR_k - R_aR_bR_cR_g R_hR_jR_k + R_aR_dR_fR_gR_hR_iR_k + R_aR_dR_fR_gR_hR_jR_k + R_aR_dR_fR_hR_iR_j R_k + R_aR_dR_gR_hR_iR_jR_k + R_aR_fR_gR_hR_iR_jR_k + R_aR_bR_cR_dR_fR_hR_iR_k + R_aR_bR_cR_dR_fR_hR_jR_k + R_aR_bR_cR_dR_gR_hR_iR_k + R_aR_bR_cR_dR_gR_hR_j R_k + R_aR_bR_cR_fR_gR_hR_iR_k + R_aR_bR_cR_fR_gR_hR_jR_k + R_aR_bR_cR_fR_hR_i R_jR_k + R_aR_bR_cR_gR_hR_iR_jR_k - R_aR_dR_fR_gR_hR_iR_jR_k - R_aR_bR_cR_dR_fR_g R_hR_iR_k - R_aR_bR_cR_dR_fR_gR_hR_jR_k - R_aR_bR_cR_dR_fR_hR_iR_jR_k - R_aR_bR_c R_dR_gR_hR_iR_jR_k - R_aR_bR_cR_fR_gR_hR_iR_jR_k + R_aR_bR_cR_dR_fR_gR_hR_iR_j R_k} = 0.099$$

$$\frac{\partial R_s(t)}{\partial \mathrm{R_f}} = \mathrm{R_aR_eR_hR_iR_k + R_aR_eR_hR_jR_k - R_aR_dR_eR_hR_iR_k - R_aR_dR_eR_hR_jR_k - R_a R_eR_gR_hR_iR_k - R_aR_eR_gR_hR_jR_k - R_aR_eR_hR_iR_jR_k - R_aR_bR_cR_eR_hR_iR_k - R_aR_bR_cR_eR_hR_jR_k + R_aR_dR_eR_gR_hR_iR_k + R_aR_dR_eR_gR_hR_jR_k + R_a R_dR_eR_hR_iR_jR_k + R_aR_eR_gR_hR_iR_jR_k + R_aR_bR_cR_dR_eR_hR_iR_k + R_aR_b R_cR_dR_eR_hR_jR_k + R_aR_bR_cR_eR_gR_hR_iR_k + R_aR_bR_cR_eR_gR_hR_jR_k + R_a R_bR_cR_eR_hR_iR_jR_k - R_aR_dR_eR_hR_gR_iR_jR_k - R_aR_bR_cR_dR_eR_gR_hR_iR_k - R_aR_bR_cR_dR_eR_gR_hR_jR_k - R_aR_bR_cR_dR_eR_hR_iR_jR_k - R_aR_bR_cR_eR_gR_h R_iR_jR_k + R_aR_bR_cR_dR_eR_gR_hR_iR_jR_k} = 0.021$$

$$\frac{\partial R_s(t)}{\partial \mathrm{R_g}} = \mathrm{R_aR_eR_hR_iR_k + R_aR_eR_hR_jR_k - R_aR_eR_fR_hR_iR_k - R_aR_eR_fR_hR_jR_k - R_a R_eR_hR_iR_jR_k - R_aR_bR_cR_eR_hR_iR_k - R_aR_bR_cR_eR_hR_jR_k + R_aR_dR_eR_fR_h R_iR_k + R_aR_dR_eR_fR_hR_jR_k + R_aR_dR_eR_hR_iR_jR_k + R_aR_eR_fR_hR_iR_jR_k + R_aR_bR_cR_dR_eR_hR_iR_k + R_aR_bR_cR_dR_eR_hR_jR_k + R_aR_bR_cR_eR_fR_hR_iR_k + R_aR_bR_cR_eR_fR_hR_jR_k + R_aR_bR_cR_eR_hR_iR_jR_k - R_aR_dR_eR_fR_hR_iR_jR_k - R_aR_bR_cR_dR_eR_fR_hR_iR_k - R_aR_bR_cR_dR_eR_fR_hR_jR_k - R_aR_bR_cR_dR_eR_h R_iR_jR_k - R_aR_bR_cR_eR_fR_hR_iR_jR_k + R_aR_bR_cR_dR_eR_fR_hR_iR_jR_k} = 0.026$$

$$\frac{\partial R_s(t)}{\partial \mathrm{R_h}} = \mathrm{R_aR_eR_fR_iR_k + R_aR_eR_fR_jR_k + R_aR_eR_gR_iR_k + R_aR_eR_gR_jR_k - R_aR_dR_e R_gR_iR_k - R_aR_dR_eR_gR_jR_k - R_aR_eR_fR_gR_iR_k - R_aR_eR_fR_gR_jR_k - R_aR_e R_fR_iR_jR_k - R_aR_eR_gR_iR_jR_k - R_aR_bR_cR_eR_fR_iR_k - R_aR_bR_cR_eR_fR_jR_k - R_aR_bR_cR_eR_gR_iR_k - R_aR_bR_cR_eR_gR_jR_k + R_aR_dR_eR_fR_gR_iR_k + R_aR_d R_eR_fR_gR_jR_k + R_aR_dR_eR_fR_iR_jR_k + R_aR_dR_eR_gR_iR_jR_k + R_aR_eR_fR_g R_iR_jR_k + R_aR_bR_cR_dR_eR_fR_iR_k + R_aR_bR_cR_dR_eR_fR_jR_k + R_aR_bR_cR_d R_eR_gR_iR_k + R_aR_bR_cR_dR_eR_gR_jR_k + R_aR_bR_cR_eR_fR_iR_k + R_aR_bR_cR_e R_fR_gR_jR_k + R_aR_bR_cR_eR_fR_iR_jR_k + R_aR_bR_cR_eR_gR_iR_jR_k - R_aR_dR_eR_f R_gR_iR_jR_k - R_aR_bR_cR_dR_eR_fR_gR_iR_k - R_aR_bR_cR_dR_eR_fR_gR_jR_k - R_aR_b R_cR_dR_eR_fR_gR_iR_jR_k - R_aR_bR_cR_dR_eR_gR_iR_jR_k - R_aR_bR_cR_eR_fR_gR_i R_jR_k + R_aR_bR_cR_dR_eR_fR_gR_iR_jR_k} = 0.996$$

$$
\begin{aligned}
\frac{\partial R_s(t)}{\partial R_i} =\, & R_aR_dR_k + R_aR_bR_cR_k - R_aR_dR_jR_k - R_aR_bR_cR_dR_k - R_aR_bR_cR_jR_k + R_a \\
& R_eR_fR_hR_k + R_aR_eR_gR_hR_k + R_aR_bR_cR_dR_jR_k - R_aR_dR_eR_fR_hR_k - R_aR_d \\
& R_eR_gR_hR_k - R_aR_eR_fR_gR_hR_k - R_aR_eR_fR_hR_jR_k - R_aR_eR_gR_hR_jR_k - R_a \\
& R_bR_cR_eR_fR_hR_k - R_aR_bR_cR_eR_gR_hR_k + R_aR_dR_eR_fR_gR_hR_k + R_aR_dR_e \\
& R_fR_hR_jR_k + R_aR_dR_eR_gR_hR_jR_k + R_aR_eR_fR_gR_hR_jR_k + R_aR_bR_cR_dR_e \\
& R_fR_hR_k + R_aR_bR_cR_dR_eR_gR_hR_k + R_aR_bR_cR_eR_fR_gR_hR_k + R_aR_bR_cR_e \\
& R_fR_hR_jR_k + R_aR_bR_cR_eR_gR_hR_jR_k - R_aR_dR_eR_fR_gR_hR_jR_k - R_aR_bR_c \\
& R_dR_eR_fR_hR_jR_k - R_aR_bR_cR_dR_eR_gR_hR_jR_k - R_aR_bR_cR_eR_fR_gR_hR_jR_k + \\
& R_aR_bR_cR_dR_eR_fR_gR_hR_jR_k \\
& = 0.151
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial R_s(t)}{\partial R_j} =\, & R_aR_dR_k + R_aR_bR_cR_k - R_aR_dR_iR_k - R_aR_bR_cR_dR_k - R_aR_bR_cR_iR_k + R_a \\
& R_eR_fR_hR_k + R_aR_eR_gR_hR_k + R_aR_bR_cR_dR_iR_k - R_aR_dR_eR_fR_hR_k - R_aR_d \\
& R_eR_gR_hR_k - R_aR_eR_fR_gR_hR_k - R_aR_eR_fR_hR_iR_k - R_aR_eR_gR_hR_iR_k - R_a \\
& R_bR_cR_eR_fR_hR_k - R_aR_bR_cR_eR_gR_hR_k + R_aR_dR_eR_fR_gR_hR_k + R_aR_dR_e \\
& R_fR_hR_iR_k + R_aR_dR_eR_gR_hR_iR_k + R_aR_eR_fR_gR_hR_iR_k + R_aR_bR_cR_dR_e \\
& R_fR_hR_k + R_aR_bR_cR_dR_eR_gR_hR_k + R_aR_bR_cR_eR_fR_gR_hR_k + R_aR_bR_cR_e \\
& R_fR_hR_iR_k + R_aR_bR_cR_eR_gR_hR_iR_k - R_aR_dR_eR_fR_gR_hR_iR_k - R_aR_bR_cR_d \\
& R_eR_fR_gR_hR_k - R_aR_bR_cR_dR_eR_fR_hR_iR_k - R_aR_bR_cR_dR_eR_gR_hR_iR_k - R_a \\
& R_bR_cR_eR_fR_gR_hR_iR_k + R_aR_bR_cR_dR_eR_fR_gR_hR_iR_k \\
& = 0.015
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial R_s(t)}{\partial \mathrm{R_k}} =& \mathrm{R_aR_dR_i + R_aR_dR_j + R_aR_bR_cR_i + R_aR_bR_cR_j - R_aR_dR_iR_j - R_aR_bR_cR_dR_i} \\
& \mathrm{- R_aR_bR_cR_dR_j - R_aR_bR_cR_iR_j + R_aR_eR_fR_hR_i + R_aR_eR_fR_hR_j + R_aR_e} \\
& \mathrm{R_gR_hR_i + R_aR_eR_gR_hR_j + R_aR_bR_cR_dR_iR_j - R_aR_dR_eR_fR_hR_i - R_aR_dR_e} \\
& \mathrm{R_fR_hR_j - R_aR_dR_eR_gR_hR_i - R_aR_dR_eR_gR_hR_j - R_aR_eR_fR_gR_hR_i - R_aR_e} \\
& \mathrm{R_fR_gR_hR_j - R_aR_eR_fR_hR_iR_j - R_aR_eR_gR_hR_iR_j - R_aR_bR_cR_eR_fR_hR_i -} \\
& \mathrm{R_aR_bR_cR_eR_fR_hR_j - R_aR_bR_cR_eR_gR_hR_i - R_aR_bR_cR_eR_gR_hR_j + R_aR_d} \\
& \mathrm{R_eR_fR_gR_hR_i + R_aR_dR_eR_fR_gR_hR_j + R_aR_dR_eR_fR_hR_iR_j + R_aR_dR_eR_g} \\
& \mathrm{R_hR_iR_j + R_aR_eR_fR_gR_hR_iR_j + R_aR_bR_cR_dR_eR_fR_hR_i + R_aR_bR_cR_dR_e} \\
& \mathrm{R_fR_hR_j + R_aR_bR_cR_dR_eR_gR_hR_i + R_aR_bR_cR_dR_eR_gR_hR_j + R_aR_bR_cR_e} \\
& \mathrm{R_fR_gR_hR_i + R_aR_bR_cR_eR_fR_gR_hR_j + R_aR_bR_cR_eR_fR_hR_iR_j + R_aR_bR_c} \\
& \mathrm{R_eR_gR_hR_iR_j - R_aR_dR_eR_fR_gR_hR_iR_j - R_aR_bR_cR_dR_eR_fR_gR_hR_i - R_aR_b} \\
& \mathrm{R_cR_dR_eR_fR_gR_hR_j - R_aR_bR_cR_dR_eR_fR_hR_iR_j - R_aR_bR_cR_dR_eR_gR_hR_i} \\
& \mathrm{R_j - R_aR_bR_cR_eR_fR_gR_hR_iR_j + R_aR_bR_cR_dR_eR_fR_gR_hR_iR_j} \\
& = 0.235
\end{aligned}
$$

If all units of given mixed system in fig (3.1) are independent identical, i.e. $R_a = R_b = \ldots = R_k = 0.85$, we get.

$$\frac{\partial R_s}{\partial R_a} = 2R^3 + R^4 + R^5 - 7R^6 + R^7 + 7R^8 - 5R^9 + R^{10}$$
$$= 0.820$$
$$\frac{\partial R_s}{\partial R_b} = 2R^4 - 3R^5 + R^6 - 4R^7 + 8R^8 - 5R^9 + R^{10}$$
$$= 0.026$$
$$\frac{\partial R_s}{\partial R_c} = 2R^4 - 3R^5 + R^6 - 4R^7 + 8R^8 - 5R^9 + R^{10}$$
$$= 0.026$$
$$\frac{\partial R_s}{\partial R_d} = 2R^3 - R^4 - 2R^5 - 3R^6 + 4R^7 + 3R^8 - 4R^9 + R^{10}$$
$$= 0.057$$
$$\frac{\partial R_s}{\partial R_e} = 4R^5 - 8R^6 + R^7 + 7R^8 - 5R^9 + R^{10}$$
$$= 0.024$$
$$\frac{\partial R_s}{\partial R_f} = 2R^5 - 5R^6 + 2R^7 + 4R^8 - 4R^9 + R^{10}$$
$$= 0.003$$
$$\frac{\partial R_s}{\partial R_g} = 2R^5 - 5R^6 + 2R^7 + 4R^8 - 4R^9 + R^{10}$$
$$= 0.003$$
$$\frac{\partial R_s}{\partial R_h} = 4R^5 - 6R^6 + R^7 + 7R^8 - 4R^9$$
$$= 0.813$$
$$\frac{\partial R_s}{\partial R_i} = R^3 - 4R^6 + 2R^7 + 4R^8 - 4R^9 + R^{10}$$
$$= 0.107$$
$$\frac{\partial R_s}{\partial R_j} = R^3 - 4R^6 + 2R^7 + 4R^8 - 4R^9 + R^{10}$$
$$= 0.107$$
$$\frac{\partial R_s}{\partial R_k} = 2R^3 + R^4 + R^5 - 7R^6 + R^7 + 7R^8 - 5R^9 + R^{10}$$
$$= 0.820$$

Through the above derivation, we found that the compounds a, h, k are strong and the compounds b, c, d, e, f, g, i, j are weak that need strengthening.

4.7 Redundancy

Redundancy is a method of design system that may improve reliability system. Assume that the current condition of affairs makes it impossible or prohibitively expensive to develop highly dependable components. In such instance, we may use the strategy of incorporating redundancies to increase system reliability [60]. In a system, this entails the purposeful construction of new parallel routes. When two elements with success probabilities, P(a) and P(b), are linked in parallel, the probability P(a) or P(b) equals:

$$\begin{aligned} \mathrm{P(a\ or\ b)} &= \mathrm{P(a)} + \mathrm{P(b)} - \mathrm{P(a\ and\ b)} \\ &= \mathrm{P(a)} + \mathrm{P(b)} - \mathrm{P(a)P(b)}. \end{aligned} \tag{4.22}$$

Suppose the components are selfcontained. Because P(a) and P(b) are both less than one, its product is often smaller than either P(a) or P(b) . As a result, P(a or b) is always bigger than P(a) or P(b) . When the element reliability cannot be raised, this provides a straightforward approach to enhancing the system's reliability. Despite one of the parts is necessary for the system's proper functioning, we purposely employ both elements to boost the system's chances of success, making it redundant [59]. Incorporating redundancies into a system may be done in a variety of ways. This study will take a look at a few of them.

4.7.1 Element Redundancy

Let a_1 be an element whose reliability is Ra_1. Another element a_2 , with reliability Ra_2 , is connected to it in parallel, if the system can operate with either a_1 or a_2 , then as the reliability of the system is $Ra_1 + Ra_2 - Ra_1 \times Ra_2$ which is better than the individual reliabilities of element a_1 and a_2 [48, 60].

We do redundancy before reduction in fig (3.1). Reliability of components

$R_a = 0.91, R_b = 0.81, R_c = 0.72, R_d = 0.71, R_e = 0.7, R_f = 0.5, R_g = 0.6, R_h = 0.8, R_i = 0.81, R_j = 0.75, R_k = 0.90$

So that the reliability of the system before reduction is equal to (0.73).

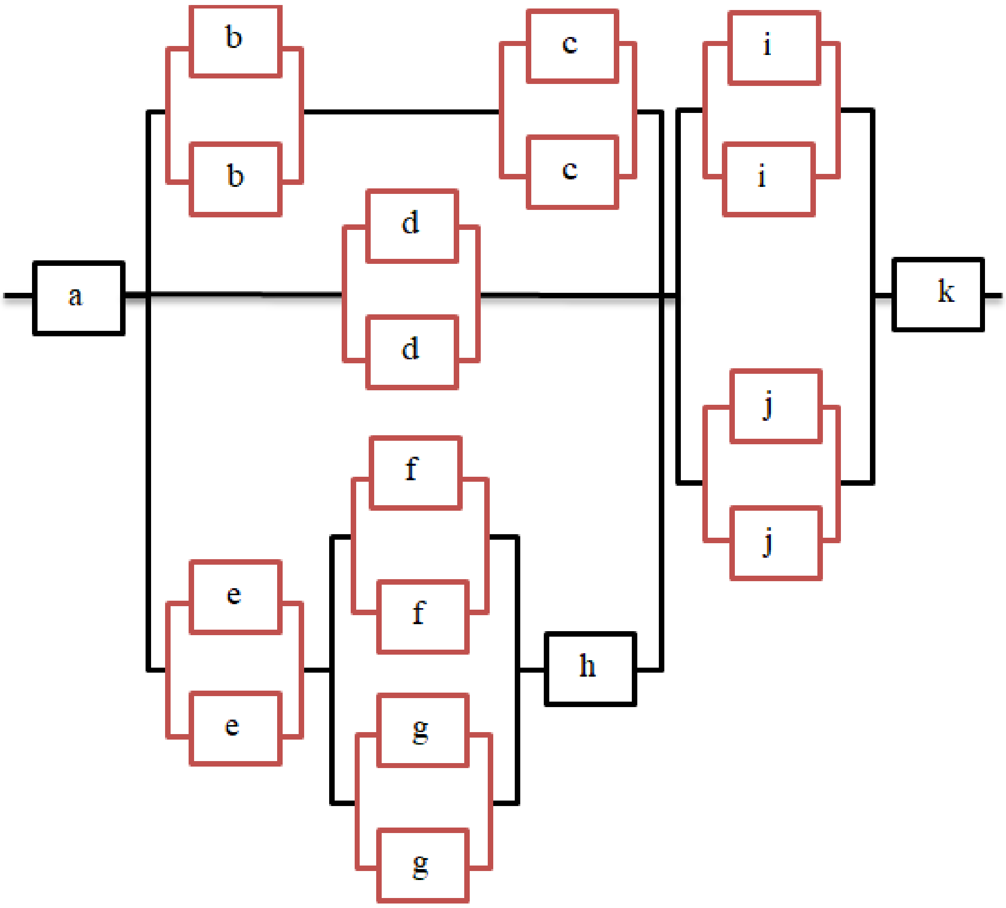

Figure 4.1: Modified (a) of a mixed system.

Step 1

$$
\begin{aligned}
\mathrm{S_1} &= 1 - [(1 - \mathrm{R_f})\,(1 - \mathrm{R_f})] \\
&= 1 - [(1 - 0.5)(1 - 0.5)] \\
&= 0.75 \\
\mathrm{S_2} &= 1 - [(1 - \mathrm{R_g})\,(1 - \mathrm{R_g})] \\
&= 1 - [(1 - 0.6)(1 - 0.6)] \\
&= 0.84 \\
\mathrm{S_3} &= 1 - [(1 - \mathrm{R_e})\,(1 - \mathrm{R_e})] \\
&= 1 - [(1 - 0.7)(1 - 0.7)] \\
&= 0.91 \\
\mathrm{S_4} &= 1 - [(1 - \mathrm{R_d})\,(1 - \mathrm{R_d})] \\
&= 1 - [(1 - 0.71)(1 - 0.71)] \\
&= 0.91 \\
\mathrm{S_5} &= 1 - [(1 - \mathrm{R_c})\,(1 - \mathrm{R_c})] \\
&= 1 - [(1 - 0.72)(1 - 0.72)] \\
&= 0.92 \\
\mathrm{S_6} &= 1 - [(1 - \mathrm{R_b})\,(1 - \mathrm{R_b})] \\
&= 1 - [(1 - 0.81)(1 - 0.81)] \\
&= 0.96 \\
\mathrm{S_7} &= 1 - [(1 - \mathrm{R_i})\,(1 - \mathrm{R_i})] \\
&= 1 - [(1 - 0.81)(1 - 0.81)] \\
&= 0.96 \\
S_8 &= 1 - [(1 - R_j)\,(1 - R_j)] \\
&= 1 - [(1 - 0.75)(1 - 0.75)] \\
&= 0.93
\end{aligned}
$$

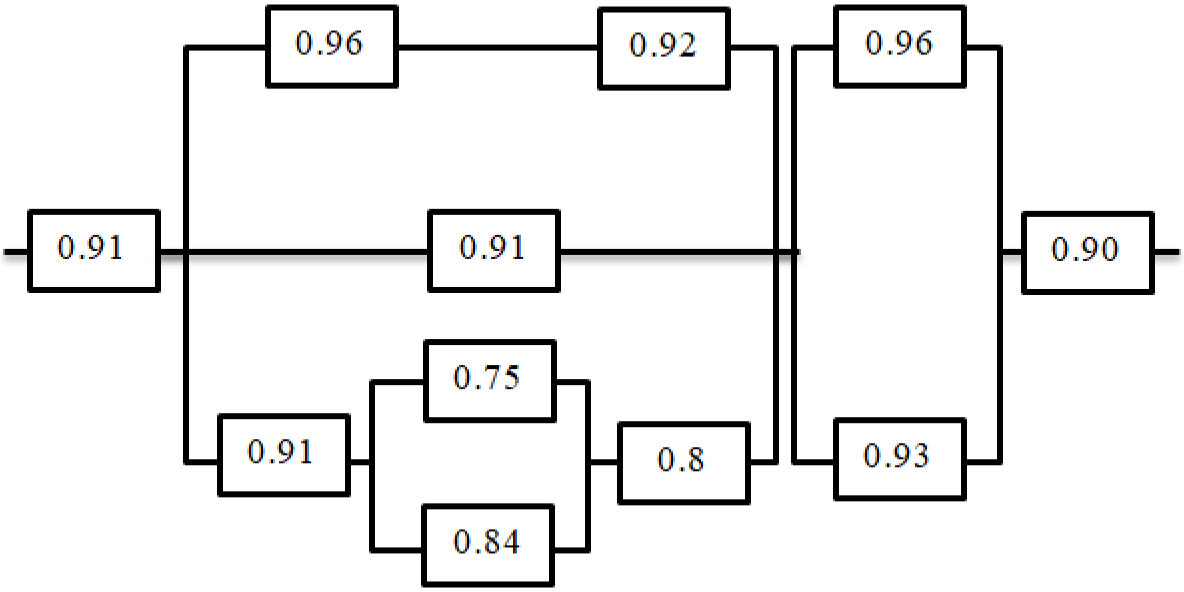

Figure 4.2: Modified (b) of a mixed system.

Step 2

$$S_9 = 0.96 \times 0.92$$
$$= 0.88$$
$$S_{10} = 1 - [(1 - 0.75)(1 - 0.84)]$$
$$= 0.96$$

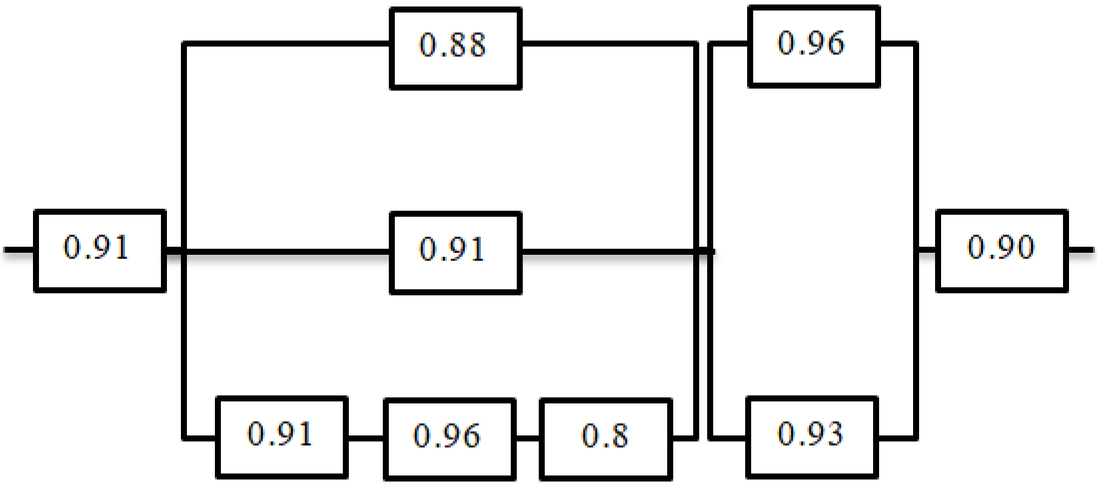

Figure 4.3: Modified (c) of a mixed system.

Step 3

$$S_{11} = 0.91 \times 0.96 \times 0.8$$
$$= 0.69$$

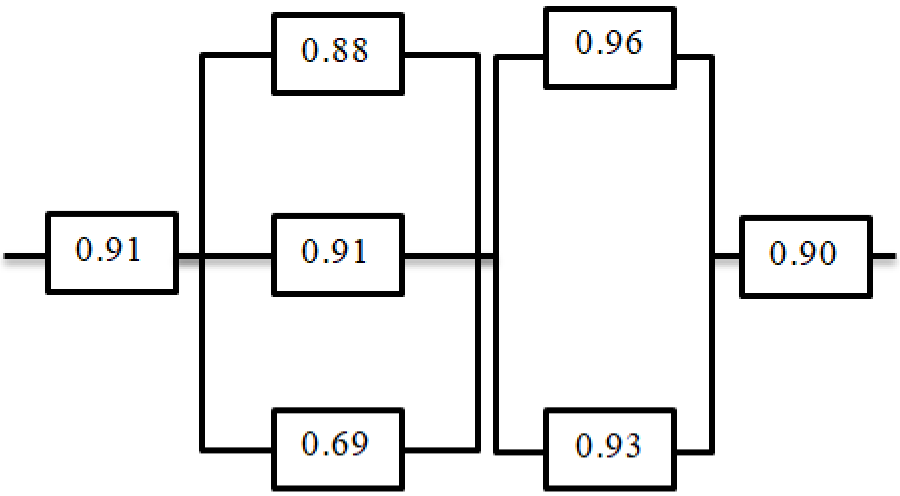

Figure 4.4: Modified (d) of a mixed system.

Step 4

$$S_{12} = 1 - [(1-0.88)(1-0.91)(1-0.69)]$$

$$= 0.99$$

$$S_{13} = 1 - [(1-0.96)(1-0.93)]$$

$$= 0.99$$

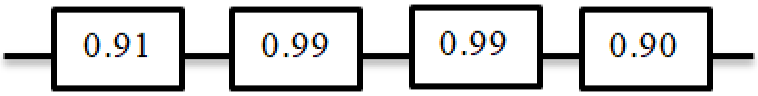

Figure 4.5: Modified (e) of a mixed system.

$$R_S = 0.91 \times 0.99 \times 0.99 \times 0.90$$

$$= 0.80$$

We do redundancy after reduction in fig (3.4). Consider fore element a, p, u and k with reliabilities 0.91 , 0.93 , 0.95 and 0.90 what the resultant system reliabilities. For series configuration with no redundancy

$$R_S = R_a R_p R_u R_k$$

$$= 0.91 \times 0.93 \times 0.95 \times 0.90$$

$$= 0.72$$

For element redundancy

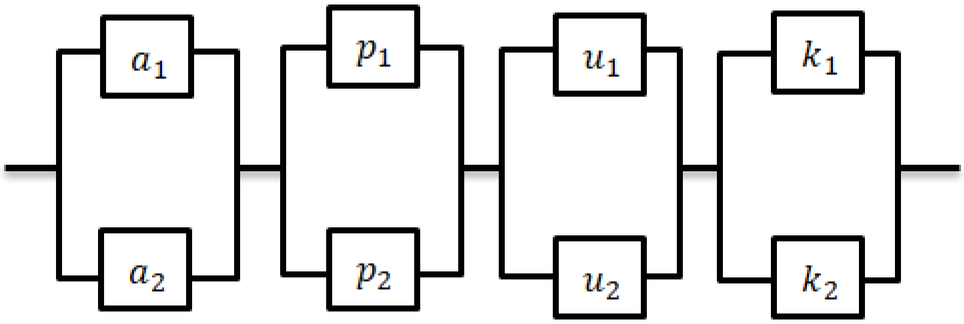

Figure 4.6: Example of element redundancy.

$$
\begin{aligned}
R_S =& R\,(a_1 + a_2 - a_1 \times a_2)\,R\,(p_1 + p_2 - p_1 \times p_2)\,R\,(u_1 + u_2 - u_1 \times u_2)\,R\,(k_1 \\
& + k_2 - k_1 \times k_2) \\
&= \left(0.91 + 0.91 - (0.91)^2\right)\left(0.93 + 0.93 - (0.93)^2\right)\left(0.95 + 0.95 - (0.95)^2\right) \\
& \left(0.90 + 0.90 - (0.90)^2\right) \\
&= 0.97
\end{aligned}
$$

Comparison between redundancy before reduction and redundancy after reduction.

It turns out that redundancy after reduction is better than before reduction.

Chapter 5

Conclusions and Future Works

5.1 Conclusions

We concluded from this thesis that, with the right methods, it is possible to convert a mixed system into a series system. Following the conversion, we worked on allocating the reliability of the mixed system to increase its reliability by using three allocation methods and selecting the best one, which is the ARINC approach method is better than the basic allocation method because allocating failures relative to the components in the ARINC approach method is less than the basic allocation method. We also worked to improve the reliability of the mixed system by employing the redundancy technique, and after learning about the weak and strong components, we concluded that redundancy after reduction is preferable to redundancy before reduction.

5.2 Future Works

According to the foregoing ,future plans can be go on to the following developments:

1 . Increasing the reliability of the systems to increase the life of the system.

2 . Reducing the failure rate of systems.

3 . Increase the reliability of the series system by placing spacers between components.

4 . Finding quick and short ways to find the reliability of the mixed system.

5 . Finding allocation techniques to increase system reliability at the lowest cost.

6 . Find allocation techniques with a lower failure rate.

References

[1] Abed, S. A. and Udriste, C., Optimization Techniques and Methods in Reliability Allocation, Ph. D. Thesis, University Politehnica of Bucharest,(2017).

[2] Abed, S. A., sulaiman, H. K., and Hassan, Z. A. H., Reliability Allocation and Optimization for ROSS of a Spacecraft by using Genetic Algorithm, in Journal of Physics: Conference Series Iop Publishing, Vol. 1294, (2019, September), No. 3, p. 032034.

[3] Abed, S. A., Udriste, C. and Tevy, I., Optimal reliability allocation for redundancy series-parallel systems, European Journal of Pure and Applied Mathematics, 10(4), (2017), pp. 877-889.

[4] Abraham, J. A., An improved algorithm for network reliability. IEEE Transactions on Reliability, April (1979), R-28: pp. 58-61.

[5] Aggarwal, K.K., Reliability engineering,Center for Excellemce in Reliability engineering, Regional engineering College ,Kurukshetra, India,(1993).

[6] Allan, R. N., Billinton, R. and De Oliveira, M. F., An efficient algorithm for deducing the minimal cuts and reliability indices of a general network configuration, IEEE Transactions on Reliability, vol. 25.4 (1976), pp. 226-233.

[7] Alghamdi, S. M. and Percy, D. F., Reliability equivalence factors for a series-parallel system of components with exponentiated Weibull lifetimes, IMA Journal of Management Mathematics, (2015), vol. 28.3, pp. 339-358.

[8] Ansell, J. I. and Phillips M. J., Practical Methods for Reliability Data Analysis, Oxford University Press, New York, (1994).

[9] Billinton, R., and R. N. Allen. 1983. Reliability Evaluation of Engineering Systems: Concepts and Techniques. Longman Scientific Technical, Essex, England.

[10] Beichelt, F. and Tittmann, P., Reliability and maintenance: networks and systems, CRC Press, (2012).

[11] Biegel, J. E., Determination of tie sets and cut sets for a system without feedback, IEEE Transactions on Reliability, vol. 26.1 (1977), pp. 39-42.

[12] Billinton, R. and Allan, R. N., Reliability evaluation of engineering systems: concepts and techniques, Springer Science and Business Media, (2013).

[13] Chelson, P. O. and Eckstein, E., Reliability computation from reliability block diagrams, IEEE Transactions on Reliability, vol. 25(1975), pp. 283-283.

[14] Crow, L. H. 1974. Reliability analysis of complex repairable systems. In Reliability and Biometry. F. Proschan,and R. J. Serfling, eds. SIAM, Philadelphia, pp. 379-410.

[15] Colbourn, C. J., The Combinatorics of Network Reliability, Oxford University Press, New York, Oxford, (1987).

[16] Dhillon, Balbir S. Maintainability, maintenance, and reliability for engineers. CRC press, (2006).

[17] Dhillon, B. S., Design Reliability Fundamentals and Applications, 1st ed., CRC Press, Taylor, Francis group, Boca Ration, Florida, (1999).

[18] Dohmen, K., Inclusion-exclusion and network reliability, The Electronic Journal of Combinatorics, Research Paper R36, 5 (1998), pp. 1 - 8.

[19] Elsayed, E. A., Reliability engineering, John Wiley and Sons, (2012).

[20] Faraci, V., Calculating failure rates of series/parallel networks, The Journal of Alion s, System Reliability Center, vol. First Quarter (2006), pp. 1-3.

[21] Feng, X., Zhu, X., Zhao, W. and Li, X., Reliability of Electric Vehicle with Wind Turbine Based on Particle Swarm Optimization, Chemical Engineering Transactions, (2018), 66, pp. 1291-1296.

[22] Gertsbakh, I. and Shpungin, Y., Network Reliability and Resilience, 1st ed., Springer-Verlag Berlin Heidelberg, (2011).

[23] Gilbert, G. T., Positive definite matrices and Sylvester's criterion, The American Mathematical Monthly, vol. 98.1 (1991), pp. 44-46.

[24] Gnedenko, B., I. Pavlov, I. Ushakov, (edited by S. Chakravarty), Statistical Reliability Engineering, John Wiley Sons, New York, (1999).

[25] Gnedenko, B. V. and Ushakov, I. A., Probabilistic Reliability Engineering, John Wiley(1995).

[26] Govil, A. K., Reliability Engineering, TaTa Mc-Graw Hill Pub. Com. Ltd., New Delhi, India, (1983).

[27] Hassan, Z. A. H. and Balan, V., Reliability extrema of a complex circuit on bi-variate slice classes, Science Direct. Karbala International Journal of Modern Science, vol. 1.1 (2015), pp. 1-8.

[28] Hamed, F. T. and Seyed, T.A, Reliability Models of Complex Systems for Robots and Automation , Ph. D. Thesis, Sharif University of Technology (2017).

[29] Hassan, Z. A. H. and Mutar, E. K., Evaluation the reliability of a high-pressure oxygen supply system for a spacecraft by using GPD method, Al-Mustansiriyah Journal of college of education, vol. special issue. 2 (2017), pp. 993 -1004.

[30] Hassan, Z. A. H. and Mutar, E. K., Geometry of reliability models of electrical system used inside spacecraft, 2017 Second Al-Sadiq International Conference on Multidisciplinary in IT and Communication Science and Applications (AIC-MITCSA). IEEE, (2017), pp. 301-306.

[31] Hassan, Z. A. H. and Udriste, C., Equivalent reliability polynomials modeling EAS and their geometries, Annals of West University of Timisoara Mathematics and Computer Science, vol. 53.1 (2015), pp. 177-195.

[32] Hassan, Z. A. H. and Udriste, C., Geometry of Reliability Models, Ph.D. Thesis, University Politehnica of Bucharest,(2016).

[33] Haynes, T. W., Hedetniemi, S. T. and Slater, P. J., Fundamentals of Domination in Graphs, Marcel Dekker, Inc., New York, (1998).

[34] Huang, Chia-Ling, and Wei-Chang Yeh. Simplified Swarm Optimization Algorithm for reliability redundancy allocation problems. International Telecommunication Networks and Applications Conference (ITNAC). IEEE, (2015).

[35] Hoyland, A. and M. Rausand, System Reliability: Models and Statistical Methods, John Wiley Sons, New York, (1994).

[36] Ireson, W. G., Coombs, C. R. and Moss, R. Y., Handbook of Reliability Engineering and Management, 2nd ed., McGraw-Hill Comp., U.S.A., (1995).

[37] Jula, Nicolae, and Cepisca Costin. Methods for analyzing the reliability of electrical systems used inside aircrafts. Recent Advances in Aircraft Technology. Intech open, (2012).

[38] Katiyar, Sapna. A Comparative Study of Genetic Algorithm and the Particle Swarm Optimization. International Journal of Technology, (2010), 2.2: pp. 21-24

[39] Kazeminia, A., Reliability Optimization of Hardware Components and System's Topology During Early Design Phase, Ph.D. Thesis, University at Duisburg Essen, (2013).

[40] Krzysztof, K., Reliability of Large and Complex Systems, Ph.D. Thesis, University Gdynia Maritime Gdynia, Poland, (2014).

[41] Kim, M. C., Reliability block diagram with general gates and its application to system reliability analysis, Annals of Nuclear Energy, vol. 38.11 (2011), pp. 2456-2461.

[42] Kim, Y. H., Case, K. E. and Ghare, P., A method for computing complex system reliability, IEEE Transactions on Reliability,vol. 21.4 (1972), pp. 215-219.

[43] Ko lowrocki, K., Reliability of large systems and Complex Systems, Wiley Online Library, (2008).

[44] Kulli, V. R. and Soner, N.D., Complementary Edge Domination in Graphs, Indian J Pure Appl.Math. 28, 1997, pp. 917-920.

[45] Kuo, S. Y., Lu, S. K. and Yeh, F. M., Determining terminal-pair reliability based on edge expansion diagrams using OBDD, IEEE Transactions on Reliability, vol. 48.3 (1999), pp. 234-246.

[46] Kuo, Way, and Ming J. Zuo. Optimal reliability modeling: principles and applications. John Wiley, Sons, 2003.

[47] Kuo,W. and Prasad, V. R., An annotated overview of system-reliability optimization, IEEE Transactions on Reliability, 49 (2000), pp. 176 - 187.

[48] Kumar, A.,. Khosla, A., Saini, J.S. and Singh, S., Meta-heuristic range based node localization algorithm for wireless sensor networks, in localization and GNSS (ICLGNSS), 2012 International Conference on IEEE, (2012) , pp. 1-7.

[49] Kumar, A., On solving complex reliability optimization problem using multi-objective particle swarm optimization. Mathematics Applied to Engineering. Academic Press, (2017), pp. 115-131.

[50] Lakey,P. B. and Neufelder, A. M. , System and Software Reliability Assurance Notebook, Rome Laboratory, Rome, (1996), pp. 6.1- 6.24.

[51] Lazzaroni, M., Cristaldi, L., Peretto, L., Rinaldi, P. and Catelani, M., Reliability Engineering Basic Concepts and Applications in ICT, Springer Verlag Berlin Heidelberg, Germany, (2011).

[52] Leemis, L.M, Reliability: Probabilistic Models and Statistical Methods, Prentice-Hall, Englewood Cliffs, NJ, (1995).

[53] Leitch, R.D., Reliability Analysis for Engineers: An Introduction, Oxford University Press, New York, (1995).

[54] Limnios, N., Failure rate of non-repairable systems, Reliability Engineering, vol. 17.2 (1987), pp. 83-88.

[55] Marin, R., System Reliability Theory, Models, Statistical Methods, and Applications, Nante Cedex France, John Wiley, Sons, (2004)

[56] Meeker, W. Q. and L. A. Escobar, Statistical Methods for Reliability Data, John Wiley, Sons, New York, (1998).

[57] Mettas, A., Reliability allocation and optimization for complex systems, Proceedings Annual Reliability and Maintainability Symposium, Los Angeles, CA, January (2000), pp. 216-221.

[58] Mutar, E. K. and Hassan, Z. A. H., On the Geometry of the Reliability Polynomials, Thesis, University of Babylon, (2017).

[59] Nedjah, N., dos Santos, L., Coelho, and de- MacedoMourelle, L., Multiobjective swarm intelligent systems: theory experiences. Springer Science Business Media, (2009), vol. 261.

[60] Ouyang and Zhiyuan, An improved particle swarm optimization algorithm for reliability redundancy allocation problem with mixed redundancy strategy and heterogeneous components. Reliability Engineering System Safety 181 (2019), pp. 62-74.

[61] Page, L. B. and Perry, J. E., Reliability of directed networks using the factoring theorem, IEEE Transactions on Reliability, vol. 38.5 (1989), pp. 556-562.

[62] Rebaiaia, M.L. and AitKadi, D., A new technique for generating minimal cut sets in nontrivial network, AASRI Procedia, vol. 5 (2013), pp. 67-76.

[63] Sandler, G. H., System Reliability Engineering, Prentice-Hall Int. Series In Space Technology, Prentice Hall Inc., Englewood Cliffs N.J., (1963).

[64] Shaghaghi, Saba, et al. "Comparative analysis of GMDH neural network based on genetic algorithm and particle swarm optimization in stable channel design." Applied Mathematics and Computation 313 (2017): pp. 271-286.

[65] Simionescu, P. A. and Beale, D., Visualization of hypersurfaces and multivariable (objective) functions by partial global optimization, The Visual Computer, vol. 20.10 (2004), pp. 665-681.

[66] Singh, S., Shivangna, S., Mittal, E., Range Based Wireless Sensor Node Localization Using PSO and BBO and Its Variants. 2013 International Conference on Communication Systems and Network Technologies,(2013), pp. 309-315.

[67] Srinath, L. S., Concepts in Reliability Engineering, East-West Press Private Ltd., (1985).

[68] Thoft-Cristensen, P. and Baker, M. J., Structural reliability theory and its applications, Springer Science and Business Media, (2012).

[69] Todinov, M. T., Risk-based reliability analysis and generic principles for risk reduction, Elsevier, (2006).

[70] Tummala, Rao R., Eugene J. Rymaszewski, and Y. C. Lee. Microelectronics packaging handbook. (1989): pp. 241-242.

[71] Venter, G. and Sobieski, J.,Particle Swarm Optimization, AIAA 2002-1235,43rd IAAA /ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Denver, CO., April (2002).

[72] Xu, L., Chen, Y., Briand, F., Zhou, F. and Givanni, M., Reliability Measurement for Multistate Manufacturing Systems with Failure Interaction, Procedia CIRP, vol. 63 (2017), pp. 242-247.

[73] Yang, Guang. Life cycle reliability engineering. John Wiley, Sons, (2007).