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Estimation of Reliability Function of Transmuted Survival Weibull Model

A Thesis
Submitted to the Council of the College of Education for Pure Sciences in University of Babylon in as a Partial Fulfillment of the Requirements for the Degree of Master in Education /Mathematics

By

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Contents

List of Symbols

T	**Value of random variable**
T	**Random variable**
W	**Weibull distribution of two parameters**
WDs	**Weibull distribution of three parameters**
$\boldsymbol{WTran}$	**Weibull transmuted distribution**
F(.)	**Baseline cumulative function**
$\boldsymbol{F_T(t)}$	**Formal for transmuted distribution**
Γ	Gamma function
W	**Weibull distribution**
$\boldsymbol{K}$	**Transmuted parameter**
CS	**Coefficient of Skewedness**
CK	**Coefficient of Kurtosis**
S(t)	**Survival function**
$\boldsymbol{S_{CTSW}(.)}$	**Survival function OF Cubic Transmuted Survival Distribution**
h(.)	**Hazard function**
CTSW	**Cubic Transmuted Survival Weibull Distribution**
$\boldsymbol{F_{CTSW}(t)}$	**Cumulative of Cubic Transmuted Survival Weibull Distribution**
AIC	**The Akaiki information criterion**
AICC	**Corrected Akaiki information criterion**
BIC	**Baysian information criterion**
K-S	**Kolmogorov-Smirnov Test**
MSE	**Mean Square Error**
E(.)	**MEAN OF Cubic Transmuted Survival Weibull Distribution**
Var (.)	**Variance Cubic Transmuted Survival Weibull Distribution**
mgf	**The Moment Generated Function for CTSW**
mle	**Maximum Likelihood Estimates of CTSW Distribution**
ols	**Least Square Estimation**

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Dedications

This research is for my parents ...

For their endless love, support and encouragement and for everyone who gave me advice or taught me a lesson in my life.........

And to everyone who helped me get this work done

.... Ban Jabbar ....

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Praise be to God, who has enabled us to complete this work, and who has bestowed upon us health, wellness and determination

I would like

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…. Ban Jabbar ….

Abstract

There is a great need for new distribution since there are many problems where the truth data do not follow any classical or standard statistical models. The main idea of this research is to find a new distribution using the transformed cubic formula based on reliability function transformed cubic formula based on reliability and then estimating the reliability function for this distribution.The proposed new distribution is the cubic transformed survival Weibull distribution (CTSW) with a discussion of some statistical properties.The two methods were used to estimate the parameters (Maximum Likelihood Estimates (MLE), Least Square Estimation (OLS)) .Therefor the simulation method was used assuming three different sample sizes (30, 50 and 100) and estimating the reliability function by the two above methods and find out the best estimation method by measuring the mean squared error. Finally, two sets of real data are modeled and using the statistical criteria AIC, CAIC, BIC, and K-S test.

Matlab program was used to draw functions and statistical modeling.

Introduction

The quality of the procedures that used in statistical analysis depends largely on the assumption of a probabilistic model or assumption because of this great effort has been spent in developing large classes of standard probability distributions Along with straightforward statistical methodologies. However, there are still many important problems where the truth does not follow any of the classical or standard statistical models. In this research, we used the transmuted distribution formula that was proposed by (Shaw & Bkely 2007) [31] which is the functional composition of the cumulative distribution of one distribution with the but using the survival. There use estimated the survival function of the Cubic transmute Survival Weibull distribution using the cubic formula. Consists of four chapter as:

First Chapter: This distribution contains some important definitions and concepts used such as reliability functions, coefficients of skewness, kurtosis, and quintile, as well as useful functions and statistical properties.

Second Chapter: We introduce and new distribution known as the cubic transmuted survival Weibull distribution, as well as some of its key features. Functions such as survival, hazard function, as well as various statistical and mathematical properties such

rth moment about the origin, thought about the mean, Coefficients skewness coefficient kurtosis respectively, and variation order statistics and the moment generating function

Third Chapter: The third chapter deals with estimation methods so that we use two methods Classical Estimation Methods: Maximum Likelihood Estimator, Least Square.

We have used Numerical methods such as Newton-Raphson methods.

Chapter Forth: The simulation method was used to find out which method is best for estimating the parameters of the cubic transmuted survival Weibull) (CTSW) distribution, and then the best method was chosen and applied to real data.

For that, we used the Matlab with the analysis and design of electronic systems, and used it for design in the design and simulation of systems.

1. Publication

Ban Jabbar Jawd kadim and Kareema Abed Al- kadim Makrib Al-Khafaji " Estimation of Reliability Function of Transmuted Survival Weibull Model " Journal of Positive School Psychology 2022 , Vol. 6,No.4,4630-4640 .

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Chapter one

Definitions Used and Important Concepts

1. Introduction:

The quality of procedure used in statistical analysis depends heavily on assumed distribution because of this. It has been developed large classes of standard probability distributions, however there are still many problems where the real the data are not fitted any of classical or standard probability models.

The Weibull distribution is a very popular lifetime probability distribution. It has been extensively used for modeling in the reliability, quality control, physics, medicine and others. Despite of the novelty of the idea of the transmuted distribution. There are many studies about dealt with this topic and have been applied to different distribution, so we will present some of them about those related to the Weibull distribution.

W.Shaw and I.Buckley2007, who mane the first who suggested a formula transmuted distribution to find new more flexible formulas. In contrast to the Gram –Charlier approach, the research included example of parametric distributions **[31]**

Aryal and C.Tsokos 2011,studied generalize the two parameters Weibull distribution using the quadratic rank transmuted .They described the mathematical properties and what is the intended benefit of transmuted Weibull distribution by using it to find reliability for real data**[6]**

I.Elbatal and G.Aryal 2013, derived "transmuted additive Weibull distribution " and discussed some of characteristics properties. For there they introduced an example where the (TAW) is better A.W. according to their belief **[11]**

Merovic.F 2013, developed a transformed Lindley distribution by the quadratic rank transformation Lindly distribution and provided comprehensive description of the mathematical properties of the subject distribution. Besides behavior reliability and utility Lindley transmuted distribution for modeling reliability data was studied using real data **[17]**.

Ebeaheim A. 2014, presented the generalization for a two –parameter distribution by named **''exponential transmuted Weibull ''**(ETW) distribution. The characteristics of new model estimation its parameters **[10]**

Pal M. and Tiensuwan M. 2014, gave studied distribution called "beta a transmuted Weibull " distribution and applied this model to real –life to be shown a considerable good fit **[22]**.

Ahsan UL Haq and elat 2016, presented three parameters of the transformation power function distribution which is generalization power distribution. They used two sets of real data to show flexibility of the new distribution **[7]**

AL-Kadim K.A. 2018. derived "the general formula for the transmuted distribution" which helps us is building models for a new distribution **[4]**.

Rahaman M. and Elat 2018, Suggests a general transmuted distribution where he focused in his article on " A General Transformation Family of Distribution (CT) " to provide more flexibility in bi-Midian data modeling where he discussed **'' The transformation cubic exponential distribution ''** thought its details and its various statistical characteristics, and used a real data set to verify its applicability. **[26]**

Rahaman M. Rahaman and elat 2020, made an article as a review about families of transmuted distributions. **[24].**

Mohamaad S.F. and K.A.AL-Kadim 2021. Developed a new survival model. Three different distributions were used to test this formula. The new formula could be flexible enough to be used to various distributions for modeling survival data in many fields like biostatistics, metrology, and engineering. **[19]**

1.1 The Aim of Research

Constructing a new distribution: cubic transmuted survival Weibull distribution discussing, and studying some of statistical and mathematical properties of the given distributions .Further using the proposed distribution

1.2. Some Basic Concepts and Useful Definition

1.2.1 Statistical Model [14]

A statistical model is a type of mathematical model that encapsulates a set of statistical hypotheses about the development of a statistical sample as well as analogous data from a broader statistical population. Mathematical equations that relate one or more random variables, some of which are expected to approach the distribution, are frequently used to specify a statistical model. It was used to extract a specific sample of data We may say that the model is successful. It's a statistical description of a platform that attempts to be as accurate as possible.

1.2.2 Continue Random Variable [20]

A random variable T is called continuous if there is exists function f(.) such that

$F(t) = \int_{-\infty}^{t} f(u)du$ ...... 1.1 for every real number t ∈ ℝ.

Where $f(\cdot)$ is called probability density function of T.

that is define $f: R \to [0, \infty)$ iff

1.$f(t) \geq 0 \quad \forall t$ 1.2

2.$\int_{-\infty}^{\infty} f(t)\, dt = 1$ 1.3

1.2.3 Mode [15]

The mode is one of the measures of central tendency. The mode is defined as value of the random variables that makes maximum $f(\cdot)$ in the case of continuous variables. The aim of studying the mod is to get an idea of the maximum value of function in addition of being an alternative measure of average in the case when the latter is not found it can be obtained by finding for the value or values of T that make f(t) at it maximum values and that satisfied the equation

$$\frac{\partial f(t)}{\partial t} = 0 \quad \ldots\ldots\ldots 1.4$$

where $f(.)$ the probability density function.

1.2.4 Median [15]

The median is a measure of central tendency (Location Measure) with the value specified on the x-axis. The median is defined as value of random variable T the divides an area under the curve of the function f(t) in two equal parts.

The median of continues distribution function is defined as

$$\int_{-\infty}^{x} f(t)dt = P(T \leq t)$$

That is

$$F(t) = \frac{1}{2} \ldots\ldots\ldots\ldots 1.5$$

Where $F(t)$ is cdf of continues distribution function .

1.2.5 Quantile Function. [12][28]

A random variables T, the quantile function is defined as

$t = Q(q)$

$= F^{-1}(q) \qquad where\ 0 < q < 1$

1.2.6 Useful Functions

1.Gamma Function [30]

It is an extension of the factorial function of the ordered and real numbers. It is defined as

$$\Gamma(n) = \int_0^{\infty} t^{n-1}\, e^{-t} dt \qquad \text{..... 1.6}$$

$\Gamma(n+1) = n! \ \forall n \in N$

2.The Binomial Formula [27]

$$(\boldsymbol{x}+\boldsymbol{y})^{\boldsymbol{n}} = \sum_{\boldsymbol{k}=\boldsymbol{0}}^{\boldsymbol{n}} \binom{\boldsymbol{n}}{\boldsymbol{k}}\ \boldsymbol{x}^{\boldsymbol{n}-\boldsymbol{k}}\ \boldsymbol{y}^{\boldsymbol{k}} \qquad \text{...... 1.7}$$

Where $n \in$ N

1.2.7 Coefficient of Skewedness [20]

The coefficient of skewedness is a value that gives an idea of localization of the value of variable if its values are centered in direction of the small values more than if it is centered in the direction of large value. Then the distribution of this variable is skewed towards the right and called positive skewedness.

As for if on the contrary, the skewness of this variable is negative or skewed to the left, it is extracted in the following form:

$$\boldsymbol{cs} = \frac{\boldsymbol{E}(\boldsymbol{t}-\boldsymbol{\mu})^3}{\boldsymbol{\sigma}^3} \qquad \text{........ 1.8}$$

1.2.8 The Coefficient of Kurtosis [20]

Kurtosis is defined as the flatness or flatness of the probability distribution curve of a random variable, and it .is the higher the dispersion of values. The degree of flatness of the distribution curve can be measured according to following formula proposed by scientist Pearson:

$$ck = \frac{E(T-\mu)^4}{(\sigma^2)^2} \quad \ldots\ldots\ 1.9$$

1.2.9 Weibull Distribution (w) [21]

It is a continuous probability distribution and is attributed to the Swedish scientist Waloddi Weibull. It uses the Weibull distribution in various areas of life.

A Weibull distribution random variable T if its pdf is if it has a pdf as :

$$f(t) = \frac{a}{b}\left(\frac{t}{b}\right)^{a-1} e^{-(\frac{t}{b})^a} . I_{(0,\infty)}(t) \ldots\ldots..\ 1.10$$

Where $a > 0$ is the dimensionless shape parameter, $b > 0$ is the scale parameter

The formal for the cumulative distribution is

$$F(t) = 1 - e^{-(\frac{t}{b})^a} \ldots\ldots\ldots\ 1.11$$

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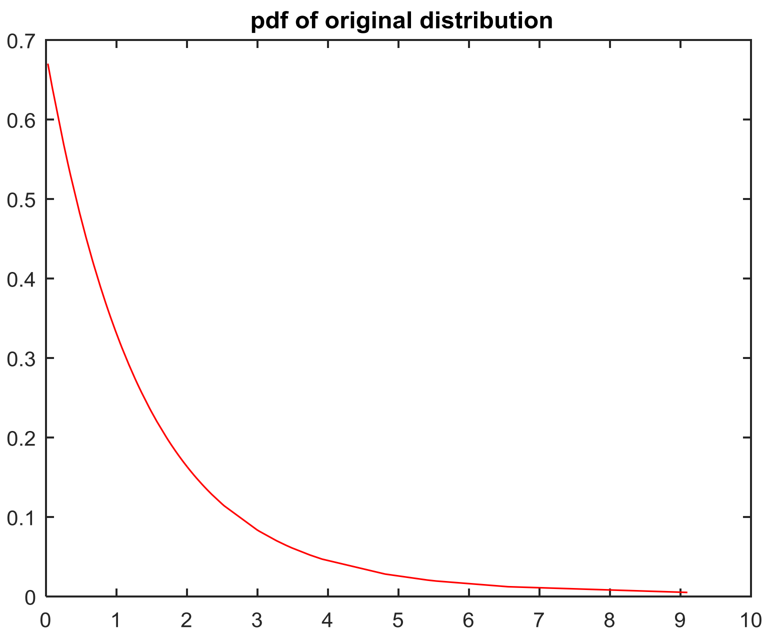

1.3 Some Function to Survival Analysis: [2][8]

1.3.1 The Survival Function (s(t))

The survival function defines the probability that the element will not fail (t,0), In other word, the probability that the organism will survives until that time determinant (t).we denoted $S(t)$

The term survival function is usually used in medical and life studies, and its mathematical formula is:

$S(t) = P(T > t)$,t> 0

$= 1 - p(T \leq t)$

$= 1 - F(t)$……. 1.12

1.3.2. The Hazard Function:

The hazard function is conditional failure rate. That is conditional a person has actually survival until time t.

The hazard function formula is:

$h(t) = \frac{f(t)}{s(t)}$……… 1.13

Where $f(t)$ ،$s(t)$ are probability density function and the survival function of distribution respectively.

1.4 Transmuted Distribution [5][9][31]

Shaw and Buckleley [31] have proposed quadric transmuted family of distribution with cdf

$F_T(t) = (1 + k)F(t) - kF^2(t)$ ….. 1.14

Where $\boldsymbol{k} \in [-1,1]$ is called parameter transmutation and $F(t)$is a cdf of base distribution.

The transmuted family of distribution of (*) has been recently generalized to cubic transmuted family by [5]. The cdf of cubic transmuted family of distribution has the form:

$F_T(t) = (1 + k)F(t) - 2kF^2$(t)+ $kF^3(t)$ $|\lambda k| \leq 1$ …….. 1.15

$f_T(t) = (1 + k)f(t) - 4kF(t)f(t) + 3kF^3(t)f(t)$………1.16

Where F(t) is the cdf of the base distribution and t $\in \mathbb{R}$, k is transmutation parameter. The cubic transmuted family is flexible enough to capture the complexity of real life data sets.

1.5 The New Formula of Distribution. [19]

In this section we refer to new distribution that is define using survival function ($S(t)$) which is defined by :

$S(t) \quad = 1 - F(t)$……. 1.17

Now, on computing a new S(t) form the 1.15 .

The transmuted survival formula:

$S(t) = (1 + k)S_1^3(t) + kS_1^2(t) - 2kS_1(t)$ …… 1.18

Where $S_1(t)$ is the survival function of base distribution.

By differentiation law

$ds(t) = -dF(t) = -f(t)$

We have :

$-f(t) = -3(1 + k)s_1^2(t)f(t) - 2k\, s_1(t)f(t) + 2kf(t)$

$f(t) = 3(1 + k)s_1^2(t)f_1(t) + 2k\, s_1(t)f_1(t) - 2kf_1(t)$ …… 1.19

1.6 Information Criterion and Curving fitting [1][16][31]

Statistical modeling is used to understand a particular phenomenon as well as to make reliable predictions in different fields of science, The objective of statistical analysis is to express information in an understandable form using a statistical model. Therefore, researchers sought to use a different statistical standard to generalize statistical models, among the models that will be used.

In the work, we used 3- different types of information criterion:

1.The Akaiki information criterion (AIC)[22]

The Akaki criterion is an important measure of the relative quality of statistical models in general applied to a set of data. It is also applied to test the appropriate statistical distribution of the data.

- The Akaike's information criterion (AIC)

$AIC = -2Ln(\widehat{\vartheta}) + 2m$ ……….. 1.20

Where $\widehat{\boldsymbol{\vartheta}}$ is number of parameters to be estimated, m is number of observed data

$L(\widehat{\vartheta}) = L(t_1, t_2, \cdots\cdots, : \hat{\vartheta})$ for the predicted model, is the maximum magnitude of the likelihood function $\hat{\vartheta}$ is MLE estimator predict of the factor ϑ ,

2.Corrected Akaiki Information Criterion (AICC)[3]

The updated Akaike's information criterion (AICC)

$AICC = AIC + \frac{2m(m+1)}{n-m+1}$ ……….. 1.21

3.Baysian Information Criterion (BIC) [3]

It is a criterion for model selection among finite set of model .it is based, in part on likelihood function and it is closely related to AIC.

$BIC = -2Ln(\widehat{\vartheta}) + m\ Ln(n)$ ………..1.22

1.6.1 Curving Fitting [23]

1. The Root Mean Square Error (RMSE)

determined utilizing:

$$RMSE = \left[\frac{1}{n}\sum_{i=1}^{n}\left(Y - \hat{Y})\right)^2\right]^{1/2}$$ ………..1.23

N: number of the data point.
Y: observed value.
$\hat{Y}$: estimator value

2. Kolmogorov-Smirnov Test (K-S) [13]

It is a test of good fit (quality of fit), a non-parametric statistical test that compares the distribution of the statistical population through independent random samples taken from the community when we have a sample that follows the assumed distribution

Consider a random sample, denoted T = (T_i,..., T ,..., T_i), coming from a continuous variable T. Let the hypothesized CDF be $F(t|\vartheta)$, where ϑ is the vector of parameters of F. We formulate the hypothesis H0 : T ∼ F(·|ϑ) r.v H1 : T ≁ F(·|ϑ), against the alternative that the random variable does not follow the claimed distribution. The Kolmogorov-Smirnov K-S test consists of rejecting H0 when the statistic

$\boldsymbol{K - S\ =\ sup\ |F(t) - Fn(t)|}$, $t\epsilon R$………..1.24

Chapter Two

Cubic Transmuted Survival Weibull Model & Statistical Properties

Cubic Transmuted Survival Weibull Model and Statistical Properties:

In this chapter , we find the cubic transmuted survival Weibull distribution, and introduce some of statistical properties of the distribution .

2.1 Cubic Transmuted Survival Weibull Distribution (CTSW)

The survival of Weibull distribution

$S_W(t) = 1- [1- e^{-(\frac{t}{b})^a}]$ ……….. 2.1

Now we substitute it into the equation 1.18 we get:

$$S_{CTSW}(t) = (1+k)\, exp^{-3(\frac{t}{b})^a} + k\, exp^{-2(\frac{t}{b})^a} - 2k\, exp^{-(\frac{t}{b})^a} \text{ ….. } 2.2$$

The cdf of cubic transmuted survival Weibull distribution (CTSW) is;

$$F_{CTSW}(t) = 1 - S_{CTSW}(t) \text{ ………. } 2.3$$

$$F_{CTSW}(t) = 1 - (1+k)\, exp^{-3(\frac{t}{b})^a} - k\, exp^{-2(\frac{t}{b})^a} + 2k\, exp^{-(\frac{t}{b})^a} \text{ ... } 2.4$$

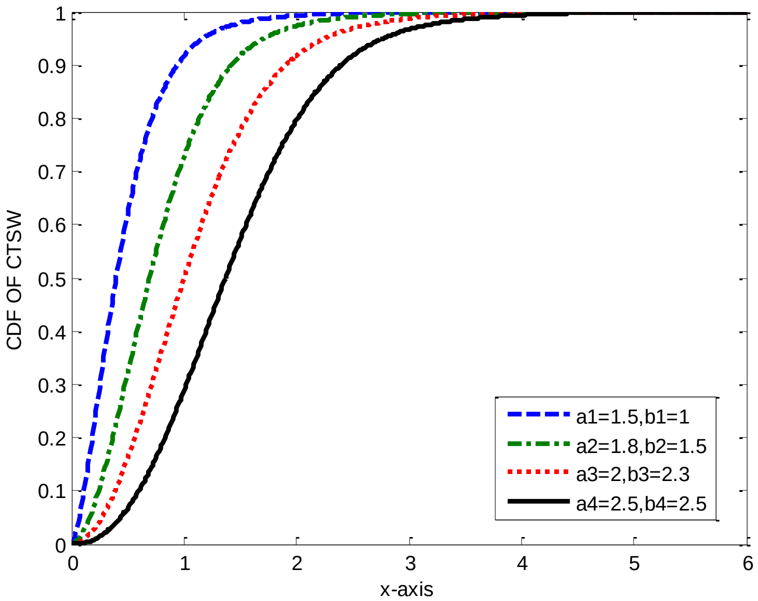

Figure 2.1 The cdf of the cubic transmuted survival weibull distribution with different values of a , b and (k=-0.06)

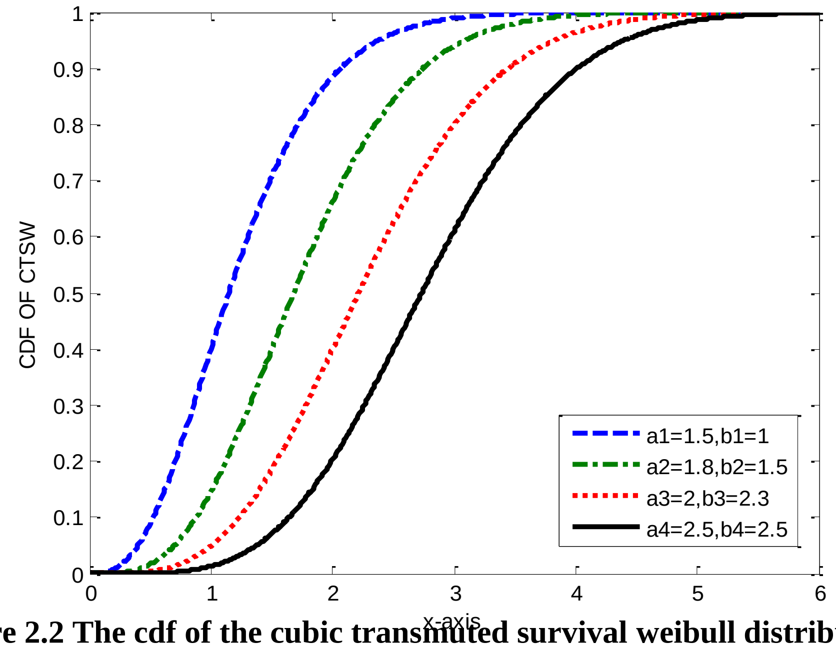

Figure 2.2 The cdf of the cubic transmuted survival weibull distribution with different values of a , b and (k=-1)

The pdf of (CTSW) distribution:

$$f_{ctsw}(t) = 3(k+1)\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-3\left(\frac{t}{b}\right)^{a}} + 2k\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-2\left(\frac{t}{b}\right)^{a}} - 2k\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-\left(\frac{t}{b}\right)^{a}}$$

$$= \left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-\left(\frac{t}{b}\right)^{a}} \left[3(k+1)e^{-2\left(\frac{t}{b}\right)^{a}} + 2ke^{-\left(\frac{t}{b}\right)^{a}} - 2k\right] \quad \text{….2.5}$$

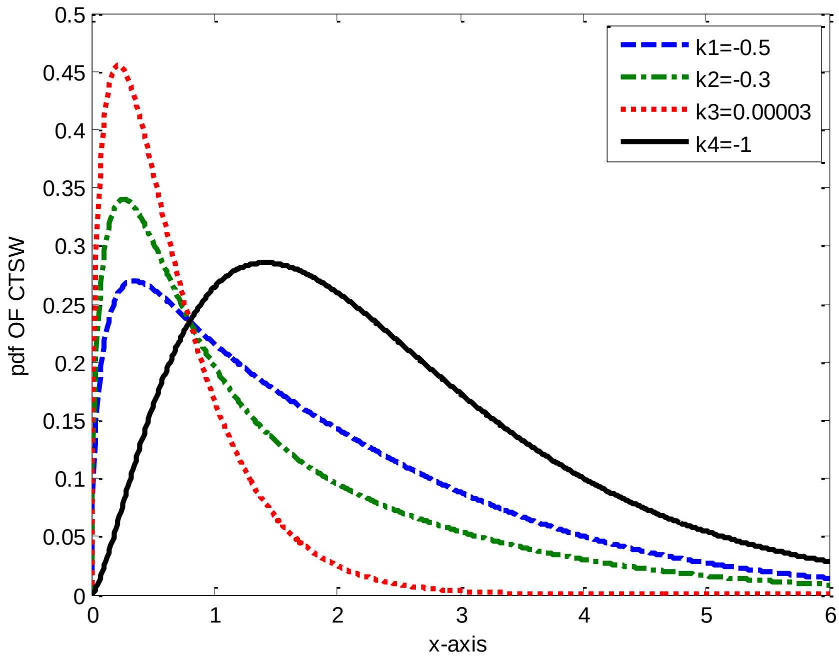

Figure 2.3 The pdf of the CTSW with different values of k , and fixed a =1.5,b=2

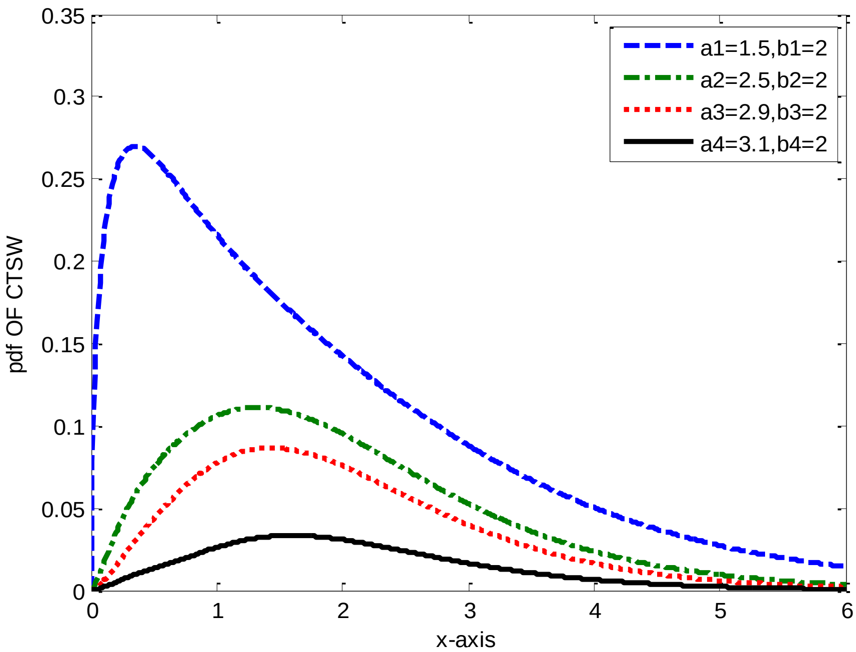

Figure 2.4 The pdf of the CTSW with different values of a, and fixed b=2 , k=-0.5

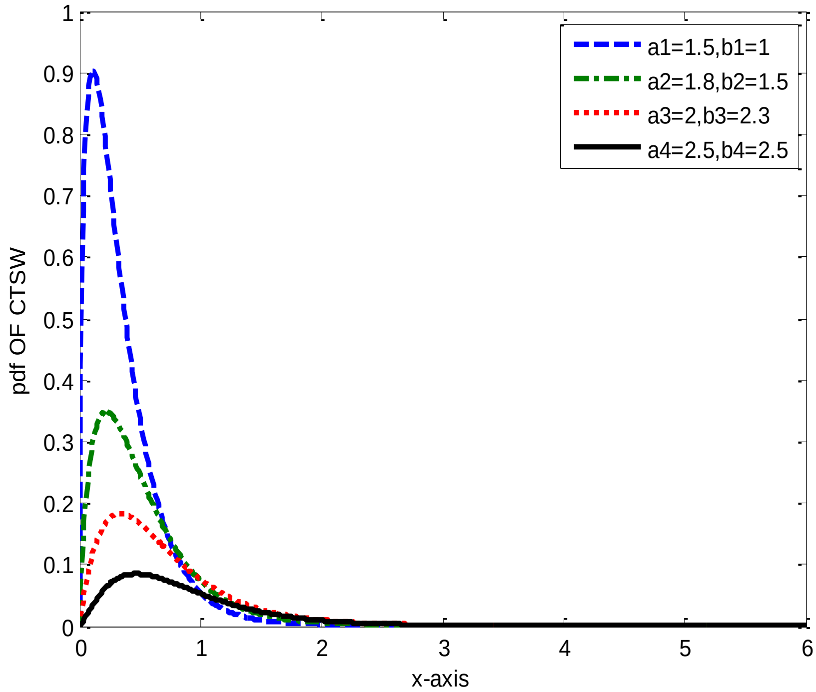

Figure 2.5 The pdf of the cubic transmuted survival Weibull distribution with different values of a, b and (k=-0.01)

Figure (2.5) demonstrates that the beaus (mode) are the same when the values of k are fixed, and this is corroborated by Figure (2.3), which shows that the limits differ when the parameter k is given different values and the two parameters a,b are fixed . The shape of the function curve differs from that in figure (2.4), in which the two parameters a, k are fixed but the parameter b is given different values, resulting in a in descending curve.

That is the curve of the pdf of CTSW Distribution has the same mod at fixed values of k and different value of the parameters, So we can say in general that increasing and stabilizing decreasing.

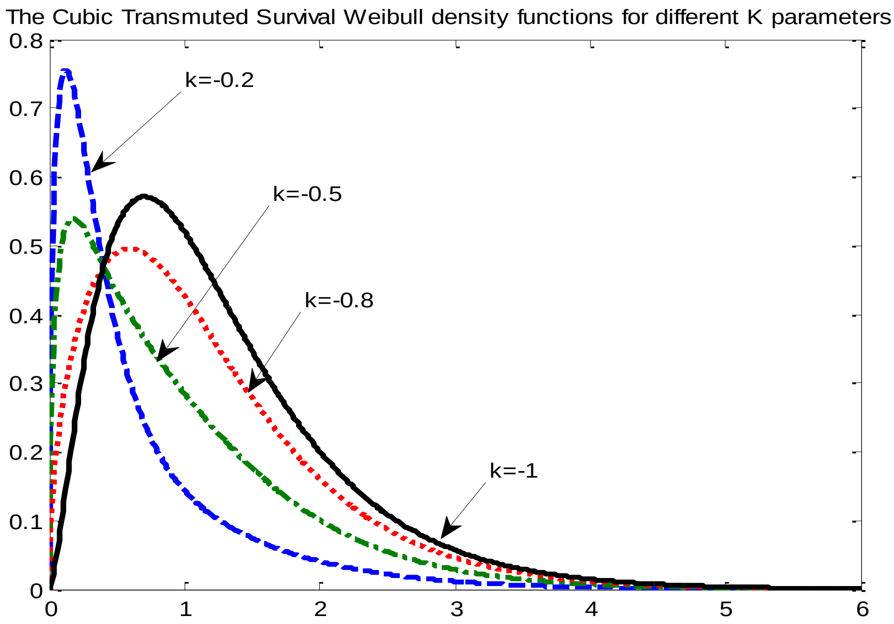

Figure 2.6 The pdf of the cubic transmuted survival Weibull distribution with different values of k

Now we want to prove that the function in 2.5 is pdf of CTSW distribution that satisfies two conditions of property density function:

1. $f(t) \geq 0$.

2. $\int_{-\infty}^{\infty} \boldsymbol{f(t) = 1}$.

Proof 1: it is clear that

$$\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} \exp^{-\left(\frac{t}{b}\right)^{a}}\left[3(k+1)\exp^{-2\left(\frac{t}{b}\right)^{a}} + 2k\exp^{-\left(\frac{t}{b}\right)^{a}} - 2k\right] \geq 0$$

$$3(k+1)\exp^{-2\left(\frac{t}{b}\right)^{a}} + 2k\exp^{-\left(\frac{t}{b}\right)^{a}} - 2k \geq 0$$

$$\frac{3(k+1)}{2k}\exp^{-2\left(\frac{t}{b}\right)^{a}} \geq 1 - \exp^{-\left(\frac{t}{b}\right)^{a}}$$

$$\frac{3(k+1)}{2k} \geq \frac{1-\exp^{-\left(\frac{t}{b}\right)^{a}}}{\exp^{-2\left(\frac{t}{b}\right)^{a}}}$$

$$\frac{2k}{3(k+1)} \leq \frac{\exp^{-2\left(\frac{t}{b}\right)^{a}}}{1-\exp^{-\left(\frac{t}{b}\right)^{a}}}$$

$\forall t \geq 0$ the inequality is true, then the function is positive.

Proof 2:

$$\int_0^\infty \boldsymbol{f_{CTSW}(t)} = \int_0^\infty \Big[\mathbf{3(k+1)} \left(\frac{\mathbf{a}}{\mathbf{b}}\right)\left(\frac{\mathbf{t}}{\mathbf{b}}\right)^{\mathbf{a-1}} \mathbf{e}^{-3\left(\frac{t}{b}\right)^{a}} +$$

$$\mathbf{2k}\left(\frac{\mathbf{a}}{\mathbf{b}}\right)\left(\frac{\mathbf{t}}{\mathbf{b}}\right)^{\mathbf{a-1}} \mathbf{e}^{-2\left(\frac{t}{b}\right)^{a}} - \boldsymbol{2k}\left(\frac{\mathbf{a}}{\mathbf{b}}\right)\left(\frac{\mathbf{t}}{\mathbf{b}}\right)^{\mathbf{a-1}} \mathbf{e}^{-\left(\frac{t}{b}\right)^{a}} \Big] \mathbf{dt}\,.$$

$$= [-(\mathbf{k+1}) \int_0^\infty -\mathbf{3}\left(\frac{\mathbf{a}}{\mathbf{b}}\right)\left(\frac{\mathbf{t}}{\mathbf{b}}\right)^{\mathbf{a-1}} \mathbf{e}^{-3\left(\frac{t}{b}\right)^{a}} -$$

$$\boldsymbol{k}\int_0^\infty -\mathbf{2}\left(\frac{\mathbf{a}}{\mathbf{b}}\right)\left(\frac{\mathbf{t}}{\mathbf{b}}\right)^{\mathbf{a-1}} \mathbf{e}^{-2\left(\frac{t}{b}\right)^{a}} + \boldsymbol{2k}\int_0^\infty \left(\frac{\mathbf{a}}{\mathbf{b}}\right)\left(\frac{\mathbf{t}}{\mathbf{b}}\right)^{\mathbf{a-1}} \mathbf{e}^{-\left(\frac{t}{b}\right)^{a}}]\, \boldsymbol{dt}$$

$$= [-(\mathbf{k+1})[\, \mathbf{e}^{-3\left(\frac{t}{b}\right)^{a}}\,]_0^\infty - \boldsymbol{k}[\mathbf{e}^{-2\left(\frac{t}{b}\right)^{a}}\,]_0^\infty + \boldsymbol{2k}[\mathbf{e}^{-\left(\frac{t}{b}\right)^{a}}\,]_0^\infty]$$

$$= [-(\mathbf{k+1})[\, \boldsymbol{e}^{-\infty} - \boldsymbol{e}^{\mathbf{0}}] - \boldsymbol{k}[\, \boldsymbol{e}^{-\infty} - \boldsymbol{e}^{\mathbf{0}}] + \boldsymbol{2k}[\, \boldsymbol{e}^{-\infty} - \boldsymbol{e}^{\mathbf{0}}]]$$

$$= (\mathbf{1} + \boldsymbol{k}) + \boldsymbol{k} - \boldsymbol{2k}$$

= 1.

2.2 The Limit of cdf and pdf of (CTSW) Distribution:

The limit of cdf : Taking the limit for the equation 2.4 as

$$\lim_{t\to\infty} F_{CTSW} = \lim_{t\to\infty}\left[1-(1+k)exp^{-3\left(\frac{t}{b}\right)^{a}} - k\,exp^{-2\left(\frac{t}{b}\right)^{a}} + 2k\,exp^{-\left(\frac{t}{b}\right)^{a}}\right] \quad \ldots 2.6$$

= 1

$$\lim_{t\to 0} F_{CTSW} = \lim_{t\to 0}\left[1-(1+k)e^{-3\left(\frac{t}{b}\right)^{a}} - ke^{-2\left(\frac{t}{b}\right)^{a}} + 2ke^{-\left(\frac{t}{b}\right)^{a}}\right] \quad \ldots.. 2.7$$

$$= 0$$

The limit of pdf : Now Taking the limit for the equation 2.5 as

$$\lim_{t\to\infty} f_{CTSW}(\mathrm{t}) = \lim_{t\to\infty}\left[\, 3(\mathrm{k}+1)\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} + \right.$$

$$\left. 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} - 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}\right]$$

$= 0$

$$\lim_{t\to 0} f_{CTSW}(\mathrm{t}) = \lim_{t\to 0}[3(\mathrm{k}+1)\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} +$$

$$2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} -$$

$$2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}] \ldots\ldots 2.8$$

$= 0$

This result confides by figure 2.3 , figure 2.4, figure 2.5

The survival function of cubic transmuted survival Weibull distribution by the equation 2.2 :

$$\boldsymbol{s(t)_{CTSW} = (1+k)\, e^{-3(\frac{t}{b})^{a}} + k\, e^{-2(\frac{t}{b})^{a}} - 2k\, e^{-(\frac{t}{b})^{a}}}$$

and

Then the $\lim_{t\to 0} S_{CTSW}(t) = S_{CTSW}(0) = 1.$

$$\lim_{t\to\infty} S_{CTSW}(t) = S_{CTSW}(\infty) = 0$$

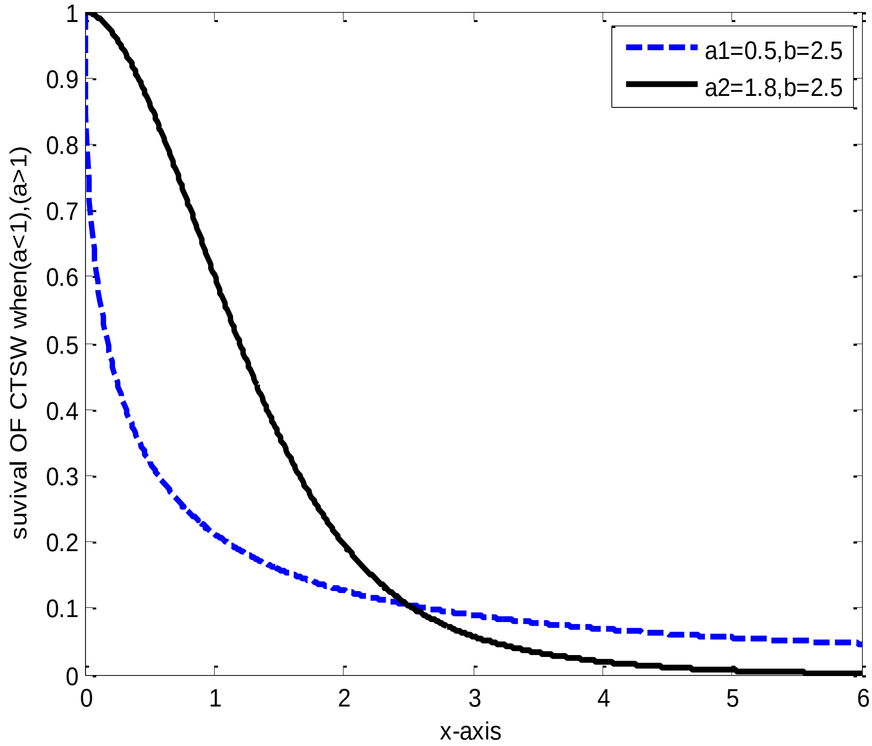

Figure 2.7 The survival of the cubic transmuted survival Weibull distribution (a <1 & a >1)

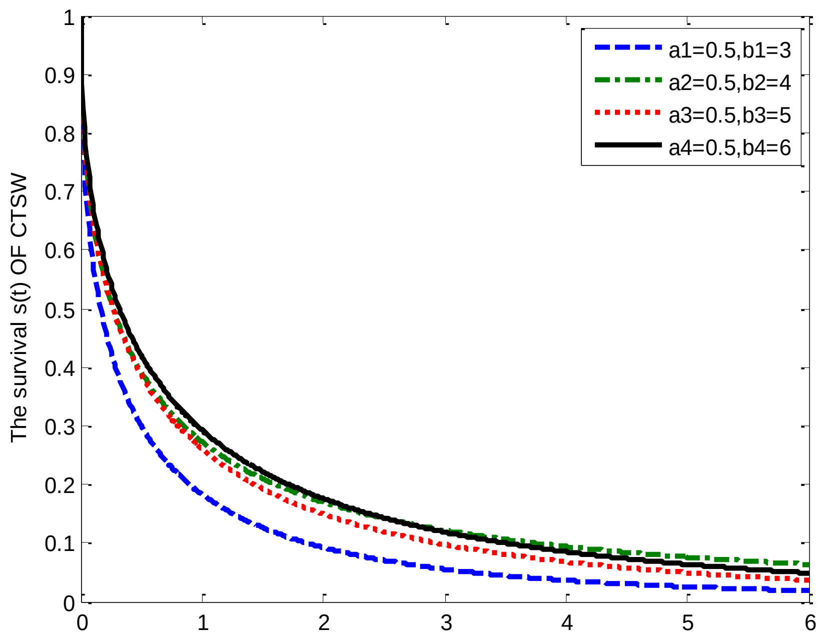

Figure 2.8 The survival function of CTSW distribution with different value of b,k and fixed a,(k1= - 0.01;k2= - 0.08;k3= 0.001;k4= 0.002)

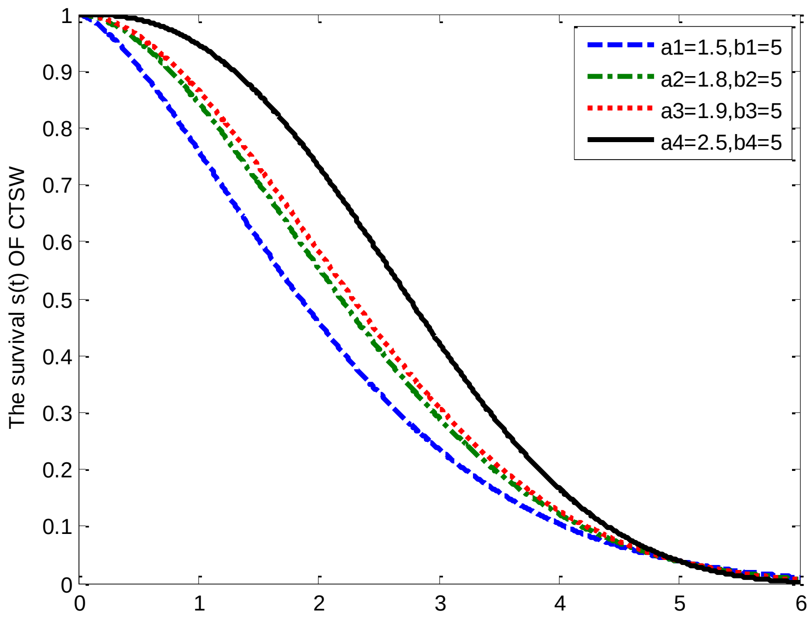

Figure 2.9 The survival function of CTSW distribution with different value of a =0.5 and fixed b,(k= 0.02)

2.3 The Hazard Function of CTSW Distribution:

Now the hazard function of CTSW distribution

$$\boldsymbol{h(t)} = \frac{\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-\left(\frac{t}{b}\right)^{a}}\left[3(k+1)e^{-2\left(\frac{t}{b}\right)^{a}} + 2ke^{-\left(\frac{t}{b}\right)^{a}} - 2k\right]}{\left(e^{-\left(\frac{t}{b}\right)^{a}}\right)\left[(1+k)\,e^{-2\left(\frac{t}{b}\right)^{a}} + K\,e^{-\left(\frac{t}{b}\right)^{a}} - 2k\right]}$$

$$= \left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} \frac{\left[3(k+1)\exp^{-2\left(\frac{t}{b}\right)^{a}} + 2k\exp^{-\left(\frac{t}{b}\right)^{a}} - 2k\right]}{\left[(1+k)\,exp^{-2\left(\frac{t}{b}\right)^{a}} + K\,exp^{-\left(\frac{t}{b}\right)^{a}} - 2k\right]} \quad \text{........ 2.9}$$

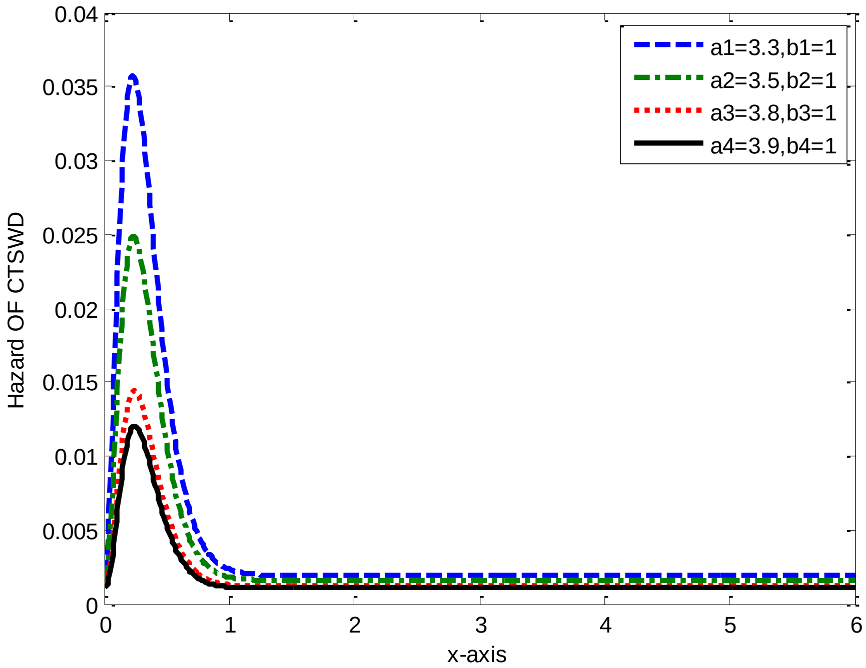

Figure 2.10 The Hazard function of CTSW distribution with different value of a and fixed b , (k=- 0.01)

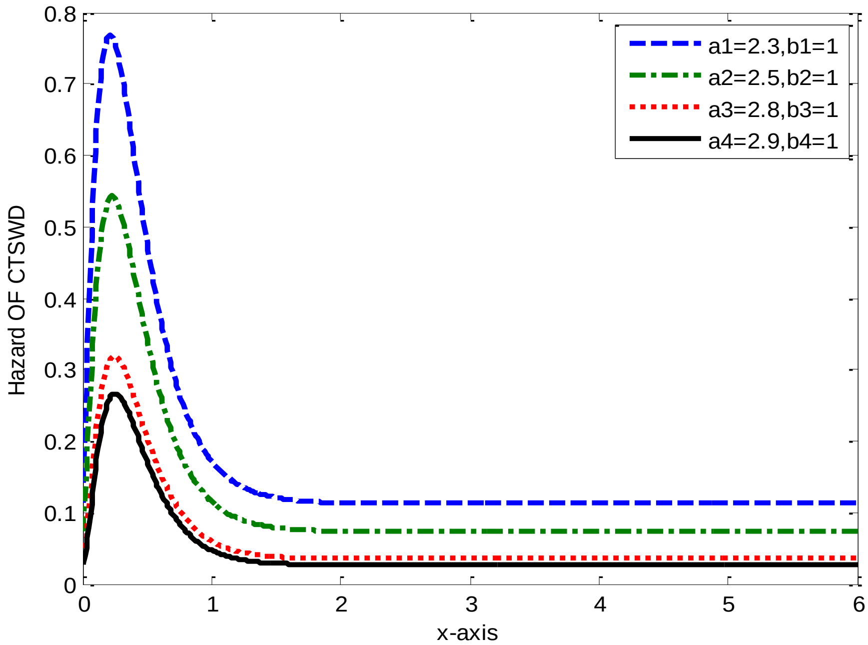

Figure 2.11 The Hazard function of CTSW distribution with different value of a and fixed b , (k=1)

We note in the figure 2.9. and figure 2.10 that the hazard function increase its maximum value (peak), and it is decreasing until it becomes convergent In other words, the function is decreasing until it becomes stable.

2.4 Shapes of (CTSW) Distribution:

Consider the equations 2.4 and 2.5

limit of (cdf) and (pdf) by section 2.2 , also we can more detail about the curve of f_{ctsw} by first and second such that :

$$\frac{\partial f_{ctsw}}{\partial t} = 3(1+k)\left(\frac{a}{b}\right)\left[e^{-3\left(\frac{t}{b}\right)^{a}} \cdot \left(\frac{a-1}{b}\right)\left(\frac{t}{b}\right)^{a-2} - \left(\frac{3a}{b}\right)e^{-3\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{2a-2}\right] + 2k\left(\frac{a}{b}\right)\left[e^{-2\left(\frac{t}{b}\right)^{a}} \cdot \left(\frac{a-1}{b}\right)\left(\frac{t}{b}\right)^{a-2} - \left(\frac{2a}{b}\right)e^{-2\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{2a-2}\right] - 2k\left(\frac{a}{b}\right)\left[e^{-\left(\frac{t}{b}\right)^{a}} \cdot \left(\frac{a-1}{b}\right)\left(\frac{t}{b}\right)^{a-2} - \left(\frac{a}{b}\right)e^{-\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{2a-2}\right] \quad \ldots\ldots.2.10$$

$$\frac{\partial^2 f_{ctsw}}{\partial t^2} = 3(1+k)\left(\frac{a}{b}\right)\left[e^{-3(\frac{t}{b})^{a}}\,(\frac{t}{b})^{a-3}\,\frac{(a-1)(a-2)}{b^2} - \left(\frac{3a}{b}\right)e^{-3(\frac{t}{b})^{a}}(\frac{t}{b})^{2a-3}\frac{(a-1)}{b} - \left(\frac{3a}{b}\right)e^{-3(\frac{t}{b})^{a}}(\frac{t}{b})^{2a-3}\frac{(2a-2)}{b} + \left(\frac{9a^2}{b^2}\right)e^{-3(\frac{t}{b})^{a}}(\frac{t}{b})^{3a-3}\right] + 2k\left(\frac{a}{b}\right)\left[e^{-2(\frac{t}{b})^{a}}\,(\frac{t}{b})^{a-3}\,\frac{(a-1)(a-2)}{b^2} - \left(\frac{a}{b}\right)e^{-2\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{2a-3}\frac{(a-1)}{b} - 2\left(\frac{a}{b}\right)e^{-2\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{2a-3}\frac{(2a-2)}{b} + 4\left(\frac{a^2}{b^2}\right)e^{-2(\frac{t}{b})^{a}}(\frac{t}{b})^{3a-3}\right] - 2k\left(\frac{a}{b}\right)\left[e^{-(\frac{t}{b})^{a}}\,(\frac{t}{b})^{a-3}\,\frac{(a-1)(a-2)}{b^2} - \left(\frac{a}{b}\right)e^{-\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{2a-3}\frac{(a-1)}{b} - \left(\frac{a}{b}\right)e^{-\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{3a-3}\frac{(2a-2)}{b} + \left(\frac{a^2}{b^2}\right)e^{-(\frac{t}{b})^{a}}(\frac{t}{b})^{3a-3}\right] \quad \ldots\ldots.\ 2.11$$

The first and second derivative can be to have clear image about the f_{CTSW}.

2.5 Some Statistical Properties of CTSW Distribution

2.5.1 Mode of CTSW Distribution

Using equation 2.10 and 1.4

$$\frac{\partial f_{ctsw\,(t)}}{\partial t} = 0$$

$$3(1+k)(\tfrac{a}{b})[e^{-3\left(\frac{t}{b}\right)^{a}}(\tfrac{a-1}{b})(\tfrac{t}{b})^{a-2} - \tfrac{3a}{b}e^{-2\left(\frac{t}{b}\right)^{a}}(\tfrac{t}{b})^{2a-2}] +$$
$$2k(\tfrac{a}{b})[e^{-2(\frac{t}{b})^{a}}(\tfrac{a-1}{b})(\tfrac{t}{b})^{a-2} - \tfrac{2a}{b}e^{-2(\frac{t}{b})^{a}}(\tfrac{t}{b})^{2a-2}] -$$
$$2k(\tfrac{a}{b})[e^{-(\frac{t}{b})^{a}}(\tfrac{a-1}{b})(\tfrac{t}{b})^{a-2} - \tfrac{a}{b}e^{-(\frac{t}{b})^{a}}(\tfrac{t}{b})^{2a-2}] = 0$$

$$3(1+k)(\tfrac{a}{b})e^{-\left(\frac{t}{b}\right)^{a}}(\tfrac{t}{b})^{a-2}[e^{-2\left(\frac{t}{b}\right)^{a}}(\tfrac{a-1}{b}) - \tfrac{3a}{b}e^{-2\left(\frac{t}{b}\right)^{a}}(\tfrac{t}{b})^{a}] +$$
$$2k(\tfrac{a}{b})e^{-\left(\frac{t}{b}\right)^{a}}(\tfrac{t}{b})^{a-2}[e^{-(\frac{t}{b})^{a}}(\tfrac{a-1}{b}) - \tfrac{2a}{b}e^{-(\frac{t}{b})^{a}}(\tfrac{t}{b})^{a}] -$$
$$2k(\tfrac{a}{b})e^{-\left(\frac{t}{b}\right)^{a}}(\tfrac{t}{b})^{a-2}[(\tfrac{a-1}{b}) - (\tfrac{a}{b})(\tfrac{t}{b})^{a}] = 0$$

$$(\tfrac{a}{b})e^{-\left(\frac{t}{b}\right)^{a}}(\tfrac{t}{b})^{a-2}\left[3(1+k)\left[e^{-2\left(\frac{t}{b}\right)^{a}}\left(\frac{a-1}{b}\right) - \frac{3a}{b}e^{-2\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{a}\right] +\right.$$
$$\left.2k\left[e^{-\left(\frac{t}{b}\right)^{a}}\left(\frac{a-1}{b}\right) - \frac{2a}{b}e^{-\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{a}\right] - 2k\left[\left(\frac{a-1}{b}\right) - \left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a}\right]\right] = 0$$

$$\left[3(1+k)\left[e^{-2\left(\frac{t}{b}\right)^{a}}\left(\frac{a-1}{b}\right) - \frac{3a}{b}e^{-2\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{a}\right] + 2k\left[e^{-\left(\frac{t}{b}\right)^{a}}\left(\frac{a-1}{b}\right) -\right.\right.$$
$$\left.\left.\frac{2a}{b}e^{-\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{a}\right] - 2k\left[\left(\frac{a-1}{b}\right) - \left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a}\right]\right] = 0 \quad \ldots\ldots\ldots\ 2.12$$

We can solve the above equation numerically.

2.5.2 The Median of CTSW Distribution

$F_{CTSW}(t) = \frac{1}{2}$

$$1-(1+k)e^{-3\left(\frac{t}{b}\right)^a} - ke^{-2\left(\frac{t}{b}\right)^a} + 2k\, e^{-\left(\frac{t}{b}\right)^a} = \frac{1}{2}$$

$$(1+k)e^{-3\left(\frac{t}{b}\right)^a} + ke^{-2\left(\frac{t}{b}\right)^a} - 2k\, e^{-\left(\frac{t}{b}\right)^a} = 1-\frac{1}{2}$$

$$(1+k)e^{-3\left(\frac{t}{b}\right)^a} + ke^{-2\left(\frac{t}{b}\right)^a} - 2k\, e^{-\left(\frac{t}{b}\right)^a} = \frac{1}{2}$$

Let $x = \left(\frac{t}{b}\right)^a$

$$(1+k)e^{-3x} + ke^{-2x} - 2k\, e^{-x} - \frac{1}{2} = 0$$

$e^{-3x} + \frac{k}{(1+k)}e^{-2x} - \frac{2k}{(1+k)}e^{-x} - \frac{1}{2(1+k)} = 0$ …….. 2.13

Solving the above equation numerically.

2.5.3 Moments of CTSW Distribution

Theorem 1: let the r.v. T has cubic transmuted survival Weibull distribution T~ CTSW (a,b,k) distribution then given

1.The rth moment of CTSW about the origin is

$E(T^r) = b^r\Gamma\left(\frac{r}{a}+1\right)[3^{-\frac{r}{a}}(1+k) + 2^{-\frac{r}{a}}k - 2k]$… 2.14

2.the rth moment $E(T-\mu)^r$ of CTSW distribution about the mean is

$E(T-\mu)^r = \sum_{j=0}^{r}\binom{r}{j}(-1)^{r-j}\mu^{r-j}\,[\,b^r\Gamma\left(\frac{r}{a}+1\right)[3^{-\frac{r}{a}}(1+k) + 2^{-\frac{r}{a}}k - 2k]$ …… 2.15

3.The Moment Generated Function (mgf) of T for CTSW Distribution is given by:

$$M_t(z) = \sum_{j=0}^{\infty} \frac{z^j}{j!} b^j \Gamma\left(\frac{j}{a}+1\right)[3^{-\frac{j}{a}}(1+k) + 2^{-\frac{j}{a}}k - 2k] \quad \text{.....2.16}$$

Proof:

1.

$$\mathbf{E}(T^r) = \int_0^{\infty} t^r\, f(t)dt$$

$$= \int_0^{\infty} t^r \left[\, 3(\mathrm{k}+1)\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} + 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} - 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} \right] dt$$

$$= \int_0^{\infty} 3(\mathrm{k}+1)\, t^r \left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} \mathrm{dt} + \int_0^{\infty} 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} t^r \mathrm{e}^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} dt - \int_0^{\infty} 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}]dt$$

Let $u = \left(\frac{t}{b}\right)^a \rightarrow\ u^{\frac{1}{a}} = \frac{t}{b} \rightarrow b\, u^{\frac{1}{a}} = t \rightarrow \frac{b}{a}\ u^{\frac{1}{a}-1}\, du = dt$

$$= \int_0^{\infty} 3(\mathrm{k}+1)\left(b\, u^{\frac{1}{a}}\right)^r \frac{a}{b}\frac{u}{u^{\frac{1}{a}}} e^{-3u}\left(\frac{b}{a}\ u^{\frac{1}{a}-1}\right)du + \int_0^{\infty} 2\mathrm{k}\left(b\, u^{\frac{1}{a}}\right)^r \frac{a}{b}\frac{u}{u^{\frac{1}{a}}} e^{-2u}\left(\frac{b}{a}\ u^{\frac{1}{a}-1}\right)du - \int_0^{\infty} 2\mathrm{k}\left(b\, u^{\frac{1}{a}}\right)^r \frac{a}{b}\frac{u}{u^{\frac{1}{a}}} e^{-u}\left(\frac{b}{a}\ u^{\frac{1}{a}-1}\right)du$$

$$= \int_0^{\infty} 3(\mathrm{k}+1)\left(b\, u^{\frac{1}{a}}\right)^r e^{-3u}du + \int_0^{\infty} 2\mathrm{k}\left(b\, u^{\frac{1}{a}}\right)^r e^{-2u}du - \int_0^{\infty} 2\mathrm{k}\left(b\, u^{\frac{1}{a}}\right)^r e^{-u}du$$

$$= 3(k+1)b^r \int_0^{\infty} u^{\frac{r}{a}} e^{-3u}du + 2k\, b^r \int_0^{\infty} u^{\frac{r}{a}} e^{-2u}du - 2kb^r \int_0^{\infty} u^{\frac{r}{a}} e^{-u}du$$

Using the following formula of gamma function 1.6

$$\int_0^{\infty} \boldsymbol{x^{\alpha-1}\, e^{-\lambda x} dx = \frac{\Gamma(\alpha)}{\lambda^{\alpha}}\ \lambda > 0}$$

$$\int_0^{\infty} \boldsymbol{x^{\alpha-1}\, e^{-x} dx = \Gamma(\alpha)}$$

$$=3(\mathrm{k}+1)b^{r}\int_{0}^{\infty}u^{\left(\frac{r}{a}+1\right)-1}e^{-3u}+2\mathrm{k}b^{r}\int_{0}^{\infty}u^{\left(\frac{r}{a}+1\right)-1}e^{-2u}du-$$

$$2kb^{r}\int_{0}^{\infty}u^{\left(\frac{r}{a}+1\right)-1}e^{-u}du.$$

$$=3(\mathrm{k}+1)b^{r}\frac{\Gamma(\frac{r}{a}+1)}{3^{(\frac{r}{a}+1)}}+2kb^{r}\frac{\Gamma(\frac{r}{a}+1)}{2^{(\frac{r}{a}+1)}}-2kb^{r}\Gamma(\frac{r}{a}+1)$$

$$=b^{r}\Gamma\left(\frac{r}{a}+1\right)[3^{-\frac{r}{a}}(1+k)+2^{-\frac{r}{a}}k-2k].\quad r\in \mathrm{Z}^{+}$$

2.proof 2

$$E(T-\mu)^{r}=\int_{0}^{\infty}E(T-\mu)^{r}\left[\,3(\mathrm{k}+1)\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\mathrm{e}^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}+\right.$$

$$\left.2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\mathrm{e}^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}-2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\mathrm{e}^{-\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}\right]dt\ldots.\ *$$

Let $u=\left(\frac{t}{b}\right)^{a}\rightarrow\ u^{\frac{1}{a}}=\frac{t}{b}\rightarrow b\ u^{\frac{1}{a}}=t\rightarrow\frac{b}{a}\ u^{\frac{1}{a}-1}\ du=dt$

By using Binomial series

$$(T-\mu)^{r}=\sum_{j=0}^{r}\binom{r}{j}\ (-1)^{r-j}\mu^{r-j}t^{\ j}$$

$$E(T-\mu)^{r}=\sum_{j=0}^{r}\binom{r}{j}(-1)^{r-j}\mu^{r-j}\ \int_{0}^{\infty}t^{\ j}\ \mathrm{dt}$$

$$E(T-\mu)^{r}=\sum_{j=0}^{r}\binom{r}{j}(-1)^{r-j}\mu^{r-j}\ \mu_{t}$$

Where μ_t is the moment about zero (by replacing r by t in to equation 2.15

$$E(T-\mu)^{r}=\sum_{j=0}^{r}\binom{\boldsymbol{r}}{\boldsymbol{j}}(-\mathbf{1})^{\boldsymbol{r-j}}\boldsymbol{\mu}^{\boldsymbol{r-j}}\left[\,b^{r}\Gamma\left(\frac{r}{a}+1\right)[3^{-\frac{r}{a}}(1+\right.$$

$$k)+2^{-\frac{r}{a}}k-2k]$$

3.The mgf of T for CTSW Distribution

$$M_t(z) = E(e^{zt}) = \int_0^\infty e^{zt} f_{CTSW}(t)dt$$

$$= \int_0^\infty e^{zt}\Bigg[3(k+1)\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-3\left(\frac{t}{b}\right)^a} + 2k\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-2\left(\frac{t}{b}\right)^a} - 2k\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-\left(\frac{t}{b}\right)^a}\Bigg]dt$$

$$=\int_0^\infty exp^{zt}\, 3(k+1)\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-3\left(\frac{t}{b}\right)^a} dt + \int_0^\infty e^{zt}\; 2k\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-2\left(\frac{t}{b}\right)^a} dt - \int_0^\infty e^{zt}\, 2k\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-\left(\frac{t}{b}\right)^a} dt$$

Since $e^{zt} = \sum_{j=0}^{\infty} \frac{(zt)^j}{j!}$

$$=\int_0^\infty \sum_{j=0}^{\infty} \frac{(zt)^j}{j!} 3(k+1)\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-3\left(\frac{t}{b}\right)^a} dt + \int_0^\infty \sum_{j=0}^{\infty} \frac{(zt)^j}{j!} 2k\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-2\left(\frac{t}{b}\right)^a} dt - \int_0^\infty \sum_{j=0}^{\infty} \frac{(zt)^j}{j!}\; 2k\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} e^{-\left(\frac{t}{b}\right)^a} dt$$

There for

$$= 3(k+1)\sum_{j=0}^{\infty} \frac{(z)^j}{j!} b^j \frac{\Gamma(\frac{j}{a}+1)}{3^{(\frac{j}{a}+1)}} + 2k\sum_{j=0}^{\infty} \frac{(z)^j}{j!} b^j \frac{\Gamma(\frac{j}{a}+1)}{2^{(\frac{j}{a}+1)}} - 2k\sum_{j=0}^{\infty} \frac{(z)^j}{j!} b^j \Gamma(\frac{j}{a}+1)$$

$$= \sum_{j=0}^{\infty} \frac{(z)^j}{j!} b^j \Gamma\left(\frac{j}{a}+1\right)[3^{-\frac{j}{a}}(1+k) + 2^{-\frac{j}{a}}k - 2k] .$$

Corollary 1 [19]: if the random variable T is has CTSW distribution then

1 $\cdot E(T)=b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]$ ………. 2.17

2 $\cdot Var(T)=b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-$

$[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2$ …… 2.18

3.The coefficients of variation

$$CV=\frac{\sqrt{b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2}}{b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]}$$ …..

2.19

1. The Coefficients of Skewedness

$$\boldsymbol{CS}=\frac{\sum_{j=0}^{3}\binom{3}{j}(-\mu)^{3-j}[\,b^3\Gamma\left(\frac{3}{a}+1\right)[3^{-\frac{3}{a}}(1+k)+2^{-\frac{3}{a}}k-2k]}{\left(b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2\right)^{\frac{3}{2}}}$$ **….**

2.20

2. The coefficients of Skewedness

$$\boldsymbol{SK}=\frac{\sum_{j=0}^{4}\binom{4}{j}(-\mu)^{4-j}[\,b^4\Gamma\left(\frac{4}{a}+1\right)[3^{-\frac{4}{a}}(1+k)+2^{-\frac{4}{a}}k-2k]}{\left(b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2\right)^{2}}$$

…...2.21

Proof : by 2.14

1.When r= 1, $\mu=E(t)=b^r\Gamma\left(\frac{r}{a}+1\right)[3^{-\frac{r}{a}}(1+k)+2^{-\frac{r}{a}}k-2k]$.

2. when r =2

$E(T^2)=b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]$ ….. 2.22

The variance is

$$var(T) = E(T^2) - [E(T)]^2$$

Now:

$$var(T) = b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k) + 2^{-\frac{2}{a}}k - 2k] - [b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k) + 2^{-\frac{1}{a}}k - 2k]\,]^2$$

or

By equation 2.15 when r = 2

$$E(T-\mu)^2 = \sum_{j=0}^{2}\binom{2}{j}(-1)^{2-j}\mu^{2-j}\,[\,b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k) + 2^{-\frac{2}{a}}k - 2k]$$

3. The coefficient of variation (CV) is the ratio of the standard deviation ($\sqrt{var}$) to the mean:

$$CV = \frac{\sqrt{var}}{E(T)} \quad \ldots\ldots\ldots\ 2.23$$

By using 2.17 & 2.18

4.when r=3 , We substitute in an equation 2.15

$$\boldsymbol{E(T-\mu)^3 = \sum_{j=0}^{3}\binom{3}{j}(-1)^{3-j}\mu^{3-j}\,[\,b^3\Gamma\left(\frac{3}{a}+1\right)[3^{-\frac{3}{a}}(1+k) + 2^{-\frac{3}{a}}k - 2k]}\,,$$

And using 1.8 we get :

$$\boldsymbol{CS = \frac{\sum_{j=0}^{3}\binom{3}{j}(-\mu)^{3-j}[\,b^3\Gamma\left(\frac{3}{a}+1\right)[3^{-\frac{3}{a}}(1+k)+2^{-\frac{3}{a}}k-2k]}{\left(b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2\right)^{\frac{3}{2}}}}$$

$$cs=\frac{b^3\Gamma\left(\frac{3}{a}+1\right)[3^{-\frac{3}{a}}(1+k)+2^{-\frac{3}{a}}k-2k]}{[b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2]^{2/3}}-$$

$$\frac{3\left(b\,\Gamma\left(\frac{1}{a}+1\right)\left[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k\right]\right)\left(b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-\left[b\,\Gamma\left(\frac{1}{a}+1\right)\left[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k\right]\right]^2\right)}{[b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2]^{2/3}}-$$

$$\frac{\left(b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\right)^3}{[b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2]^{2/3}}$$

5.when r=4 , We substitute in an equation 2.15

$$E(T-\mu)^4=\sum_{j=0}^{4}\binom{4}{j}(-1)^{4-j}\mu^{4-j}\left[\,b^4\Gamma\left(\frac{4}{a}+1\right)[3^{-\frac{4}{a}}(1+k)+2^{-\frac{4}{a}}k-2k\right] \quad \ldots .\ 2.24$$

using by 1.9 we get :

$$CK=\frac{\sum_{j=0}^{4}\binom{4}{j}(-\mu)^{4-j}[\,b^4\Gamma\left(\frac{4}{a}+1\right)[3^{-\frac{4}{a}}(1+k)+2^{-\frac{4}{a}}k-2k]}{\left(b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2\right)^2}$$

$$CK=\frac{b^4\Gamma\left(\frac{4}{a}+1\right)[3^{-\frac{4}{a}}(1+k)+2^{-\frac{4}{a}}k-2k]}{[b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2]^2}-$$

$$\frac{4\left(b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\right)\left(b^3\Gamma\left(\frac{3}{a}+1\right)[3^{-\frac{3}{a}}(1+k)+2^{-\frac{3}{a}}k-2k]\right)}{[b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2]^2}+$$

$$\frac{6\left(b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]\right)\left(b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\right)^2}{[b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2]^2}-$$

$$\frac{3\left(b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\right)^4}{[b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2]^2}$$

2.5.4 Quintile Function of CTSW Distribution.

Consider the cdf in (2-6) and let:

$F_{CTSW}(t) \;=\; q$

$1-(1+k)\, exp^{-3(\frac{t}{b})^a} -\; k\, exp^{-2(\frac{t}{b})^a} +\; 2k\, exp^{-(\frac{t}{b})^a} = q$

$(1+k)\, exp^{-3(\frac{t}{b})^a} +\; K\, exp^{-2(\frac{t}{b})^a} -\; 2k\, exp^{-(\frac{t}{b})^a} = 1-q$

By using Taylor series representation, we get:

$$(1+k)\sum_{n=0}^{\infty}\frac{(-3\left(\frac{t}{b}\right)^a)^n}{n!}+k\sum_{n=0}^{\infty}\frac{(-2\left(\frac{t}{b}\right)^a)^n}{n!}-2k\sum_{n=0}^{\infty}\frac{(-\left(\frac{t}{b}\right)^a)^n}{n!}=1-q$$

$$(1+k)\sum_{n=0}^{\infty}\frac{(-3)^n(\frac{t}{b})^{an}}{n!}+k\sum_{n=0}^{\infty}\frac{(-2)^n(\frac{t}{b})^{an}}{n!}-2k\sum_{n=0}^{\infty}\frac{(-1)^n(\frac{t}{b})^{an}}{n!}=$$
$$1-q$$

$$(1+k)\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-3)^n}{n!}+k\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-2)^n}{n!}-$$
$$2k\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-1)^n}{n!}=1-q$$

$$\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-3)^n}{n!}+k\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-3)^n}{n!}+k\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-2)^n}{n!}-$$
$$2k\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-1)^n}{n!}=1-q$$

$$\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\;\frac{(-3)^n+k((-3)^n+(-2)^n-2(-1)^n)}{n!}=1-q \;\ldots..\; 2.25$$

Finally, the equation 2.25 can solve numerically for t.

2.5.5 Order Statistics: [23][28]

Sample values from a random sample, such as the smallest, largest, or median observation, usually provide useful information.

Let $T_1.T_2.\cdots\cdots T_n$ the random sample of size n from the distribution $f_{CTSW}(t)$ let $T_{(1)}$ minimum $\{T_1.T_2.\cdots\cdots T_n\}$

$T_{(2)}$maximum $\{T_1.T_2.\cdots\cdots T_n\}$

Similarly $T_{(r)}$ denote the rth of smallest of $\{T_1.T_2.\cdots\cdots T_n\}$ then the random variables $\{T_{(1)}.T_{(2)}.\cdots\cdots T_{(n)}\}$ are called order statistics of sample $T_1.T_2.\cdots\cdots T_n$ especially , $T_{(r)}$ is called the rth order of $T_1.T_2.\cdots\cdots T_n$

2.5.5.1 The Order Statistics of CTSW Distribution

Let $T_1.T_2.\cdots\cdots T_n$ be random sample of size n from CTSW distribution with cdf $F(t.\vartheta)$ and pdf $f(t.\vartheta)$ given by 2.4 & 2.5

Let $0 \le T_{1:n}.T_{2:n}.\cdots\cdots T_{r:n} \le \infty$ denote the order statistics $T_{r:n}$

$$f\left(t_{(r)}\right) = \frac{n!}{(r-1)!(n-r)!} f(t;\vartheta)F(t;\vartheta)^{r-1}[1 - F(t;\vartheta)]^{n-r} \ldots\ldots. 2.26$$

By the density function of $r - th$ order statistics we can determine

If $r = 1$ then minimum value is

$$f\left(t_{(1)}\right) = n \left(3(\mathrm{k}+1)\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} + 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} - 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\mathrm{e}^{-\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}\right)\left[1 - (1 - (1 + k)\, e^{-3\left(\frac{t}{b}\right)^{a}} - k\, e^{-2\left(\frac{t}{b}\right)^{a}} + 2k\, e^{-\left(\frac{t}{b}\right)^{a}})\right]^{n-k}$$

$$= n \left(3(\mathrm{k}+1)\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} + 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} - 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\mathrm{e}^{-\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}\right)\left[(1+k)e^{-3\left(\frac{t}{b}\right)^{a}} + k\, e^{-2\left(\frac{t}{b}\right)^{a}} - 2k\, e^{-\left(\frac{t}{b}\right)^{a}})\right]^{n-r}$$

…..2.27

If $r = n$ then the pdf of $f(t)$ is maximum value

$$f\left(t_{(n)}\right) = n\ \Bigg(3(\mathrm{k}+1)\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} +$$

$$2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} - 2\mathrm{k}(\frac{\mathrm{a}}{\mathrm{b}})(\frac{\mathrm{t}}{\mathrm{b}})^{\mathrm{a}-1}\mathrm{e}^{-(\frac{\mathrm{t}}{\mathrm{b}})^{\mathrm{a}}}\Bigg)\Bigg(1-(1+k)\ e^{-3(\frac{t}{b})^{a}} -$$

$$k\ e^{-2(\frac{t}{b})^{a}} +\ 2k\ e^{-(\frac{t}{b})^{a}}\Bigg)^{n-1}\Bigg[(1+k)e^{-3\left(\frac{t}{b}\right)^{a}} +\ k\ e^{-2(\frac{t}{b})^{a}} -$$

$$2k\ e^{-(\frac{t}{b})^{a}})\Bigg]^{r-r}$$

$$= n\ \Bigg(3(\mathrm{k}+1)\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} + 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1} \mathrm{e}^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} -$$

$$2\mathrm{k}(\frac{\mathrm{a}}{\mathrm{b}})(\frac{\mathrm{t}}{\mathrm{b}})^{\mathrm{a}-1}\mathrm{e}^{-(\frac{\mathrm{t}}{\mathrm{b}})^{\mathrm{a}}}\Bigg)\Bigg(1-(1+k)\ e^{-3(\frac{t}{b})^{a}} -\ k\ e^{-2(\frac{t}{b})^{a}} +$$

$$2k\ e^{-(\frac{t}{b})^{a}}\Bigg)^{n-1} \ldots\ldots.\ 2.28$$

Chapter Three

Estimation Methods

3 . Algorithms and Estimation Methods :

In this chapter, the two consider estimation methods are (Maximum Likelihood Estimation) and (Least Square Estimation) are illustrated to estimate the 3-parameter CTSW. The reason for using estimation methods for parameter is make a good prediction about a particular phenomenon. There for we present the analysis of the real data sets using the CTSW distribution and compare it with the TWD and WD in order to determine the best of them in fitting the real data.

3.1 Maximum Likelihood Estimation: [24] [26]

It is one of important method of estimating one the basis that parameter to be measured It is one of the widely used methods because it has good and distinctive properties like as inversion, consistency. Since that method can find the estimation we make the logarithm its maximum limit.

3.1.1 Maximum Likelihood Estimates of CTSW Distribution:

Let $T_1.T_2.\cdots\cdots.T_n$be random sample of size n . ϑ is noted as the cubic transmuted survival Weibull distribution parameters . which are be estimated namely $\vartheta=(a,b,k)$ sample is given by pdf CTSW is

$$(\frac{a}{b})(\frac{t}{b})^{a-1}e^{-(\frac{t}{b})^a}[\,3(1+k)e^{-2(\frac{t}{b})^a}+2ke^{-(\frac{t}{b})^a}-2k]$$

$$L=(\frac{a}{b^a})^n\exp\left(\frac{1}{b^a}\sum_{i=1}^{\infty}t_i^a\right)\prod_{i=1}^{n}t_i^{a-1}[\,3(1+k)e^{-2(\frac{t_i}{b})^a}+$$
$$2k\,e^{-(\frac{t_i}{b})^a}-2k] \quad ..... \ 3.1$$

And the ln- likehood function $\ln L=lnL\,(t_1,t_2,\cdots\cdots,t_n:a,b,k)$ of this random sample is given by:

$$lnL = nln\frac{a}{b^a} + \frac{1}{b^a}\sum_{i=1}^{n} t_i^a + (a-1)\sum_{i=1}^{n} lnt_i + \sum_{i=1}^{n} \ln[3(1+k)e^{-2\left(\frac{t_i}{b}\right)^a} + 2ke^{-\left(\frac{t_i}{b}\right)^a} - 2k] \ \ldots\ldots\ldots.. \ 3.2$$

Differentiating above formula with respect to factor a ,b ,and k respectively and equating each derivative to zero we gain the formulas

$$n\left(\frac{1}{a}-\frac{1}{b}\right) + \sum_{i=1}^{n}[t_i^a \ln(t_i)\ b^{-a} - t_i^a\, b^{-a} lnb] + \sum_{i=1}^{n} \ln t_i + \sum_{i=1}^{n} \frac{3(1+k)(-2ln\left(\frac{t_i}{b}\right)e^{-2\left(\frac{t_i}{b}\right)^a}\left(\frac{t_i}{b}\right)^a}{3(1+k)e^{-2\left(\frac{t_i}{b}\right)^a}+2ke^{-\left(\frac{t_i}{b}\right)^a}-2k} + \frac{(2k)(-ln\left(\frac{t_i}{b}\right)e^{-\left(\frac{t_i}{b}\right)^a}\left(\frac{t_i}{b}\right)^a}{3(1+k)e^{-2\left(\frac{t_i}{b}\right)^a}+2ke^{-\left(\frac{t_i}{b}\right)^a}-2k} = 0 \ \ldots.. \ 3.3$$

$$n\left(\frac{a}{b}\right) + \frac{a\sum_{i=1}^{n} t_i^a}{b^{a+1}} + \sum_{i=1}^{n} \frac{3(k+1)(2\left(\frac{a}{b}\right)e^{-2\left(\frac{t_i}{b}\right)^a}\left(\frac{t_i}{b}\right)^a)+2k(\left(\frac{a}{b}\right)e^{-\left(\frac{t_i}{b}\right)^a}\left(\frac{t_i}{b}\right)^a)}{3(1+k)e^{-2\left(\frac{t_i}{b}\right)^a}+2ke^{-\left(\frac{t_i}{b}\right)^a}-2k} = 0 \ \ldots\ldots 3.4$$

$$\sum_{i=1}^{n} \frac{3exp^{-2\left(\frac{t_i}{b}\right)^a}+2exp^{-\left(\frac{t_i}{b}\right)^a}-2}{3(1+k)e^{-2\left(\frac{t_i}{b}\right)^a}+2ke^{-\left(\frac{t_i}{b}\right)^a}-2k} = 0\ldots\ldots\ldots \ 3.5$$

The mle estimates $\hat{a},\hat{b},\hat{k}$of the parameters a, b, k are obtained iteratively from the nonlinear system of equations (3.3 , 3.4 , 3.4).

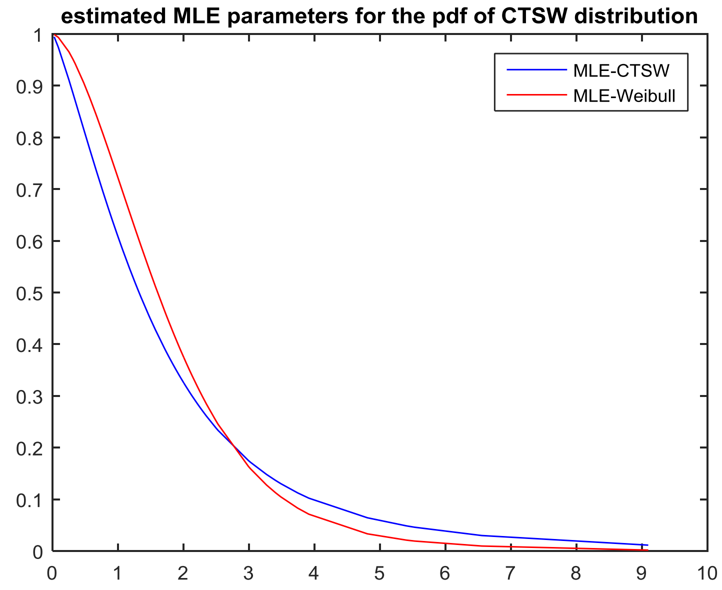

Figure 3.1 estimated MLE for original Weibull and CTSW Distributions.

3.2 Least Square Estimation(ols) [18][25]

Now we will use least squares method to estimate the parameters of the cubic transmuted survival Weibull distribution (CTSW) and to estimate the survival function of the distribution.

This method is very suitable in terms of calculating unknown parameters, and this equation makes the sum of the squares of random errors at its smallest end.

We will estimate the three parameters for CTSW distribution $(a.b.k)$, the cdf of CTSW is given by equation 2.5

$$F_{CTSW}(t) = 1-(1+k)exp^{-3\left(\frac{t}{b}\right)^{a}} - k\, exp^{-2\left(\frac{t}{b}\right)^{a}} + 2k\, exp^{-\left(\frac{t}{b}\right)^{a}}$$

And the (LS) estimators $\left(\hat{a}.\hat{b}.\hat{k}\right)$ are by minimizing:

$$\boldsymbol{LS(a.b.k) = \sum_{i=1}^{n}\left[F(a.b.k;t_i) - \frac{i}{n+1}\right]^2} \text{ ………… } \mathbf{3.6}$$

Thus, derive the equation 3.6 for the parameters $(a.b.k)$ respectively:

$$\frac{\partial\, LS(a.b.k)}{\partial a} = 0\,,\ \frac{\partial\, LS(a.b.k)}{\partial b} = 0\,,\ \frac{\partial\, LS(a.b.k)}{\partial k} = 0$$

They can also be calculated numerically by solving the following nonlinear equations.

$$\frac{\partial\, LS(a.b.k)}{\partial a} = 2\left[\sum_{i=1}^{n}\left[F(a.b.k;t_i) - \frac{i}{n+1}\right] (-(-3(k+1)\ln\left(\frac{t}{b}\right)e^{-3\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{a}) - (-2k\ln\left(\frac{t}{b}\right)e^{-2\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{a}) + (-2k\ln\left(\frac{t}{b}\right)e^{-\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{a})\right]$$

$$= \sum_{i=1}^{n}\left[F(a.b.k;t_i) - \frac{i}{n+1}\right]\left[2\left[3(k+1)\ln\left(\frac{t}{b}\right)e^{-3\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{a} + 2k\ln\left(\frac{t}{b}\right)e^{-2\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{a} - 2k\ln\left(\frac{t}{b}\right)e^{-\left(\frac{t}{b}\right)^{a}}\left(\frac{t}{b}\right)^{a}\right]\right] \text{ ………. } 3.7$$

$$\frac{\partial\, LS(a.b.k)}{\partial b} = 2\left[\sum_{i=1}^{n}\left[F(a.b.k;t_i\,) - \frac{i}{n+1}\right]\,(-3(k+1)\,\frac{a}{b^2}\;t\,e^{-3\left(\frac{t}{b}\right)^a}\left(\frac{t}{b}\right)^{a-1} - (-2k\,\frac{a}{b^2}\;t\,e^{-2\left(\frac{t}{b}\right)^a}\left(\frac{t}{b}\right)^{a-1}) + 2k\,\frac{a}{b^2}\;t\,e^{-\left(\frac{t}{b}\right)^a}\left(\frac{t}{b}\right)^{a-1})\right]$$

$$= \sum_{i=1}^{n}\left[F(a.b.k;t_i\,) - \frac{i}{n+1}\right]\left[2\left[-3(k+1)\,\frac{a}{b^2}\;t\,e^{-3\left(\frac{t}{b}\right)^a}\left(\frac{t}{b}\right)^{a-1} + 2k\,\frac{a}{b^2}\;t\,e^{-2\left(\frac{t}{b}\right)^a}\left(\frac{t}{b}\right)^{a-1} + 2k\,\frac{a}{b^2}\;t\,e^{-\left(\frac{t}{b}\right)^a}\left(\frac{t}{b}\right)^{a-1}\right]\right] \quad \ldots\ldots\ldots.\ 3.8$$

$$\frac{\partial\, LS(a.b.k)}{\partial k} = 2\left[\sum_{i=1}^{n}\left[F(a.b.k;t_i\,) - \frac{i}{n+1}\right]\right]\left(-e^{-3\left(\frac{t}{b}\right)^a} - 2e^{-2\left(\frac{t}{b}\right)^a} + 2e^{-\left(\frac{t}{b}\right)^a}\right)$$

$$= \left[\sum_{i=1}^{n}\left[F(a.b.k;t_i\,) - \frac{i}{n+1}\right]\right]\left[2\left(-e^{-3\left(\frac{t}{b}\right)^a} - 2e^{-2\left(\frac{t}{b}\right)^a} + 2e^{-\left(\frac{t}{b}\right)^a}\right)\right] \ldots\ 3.9$$

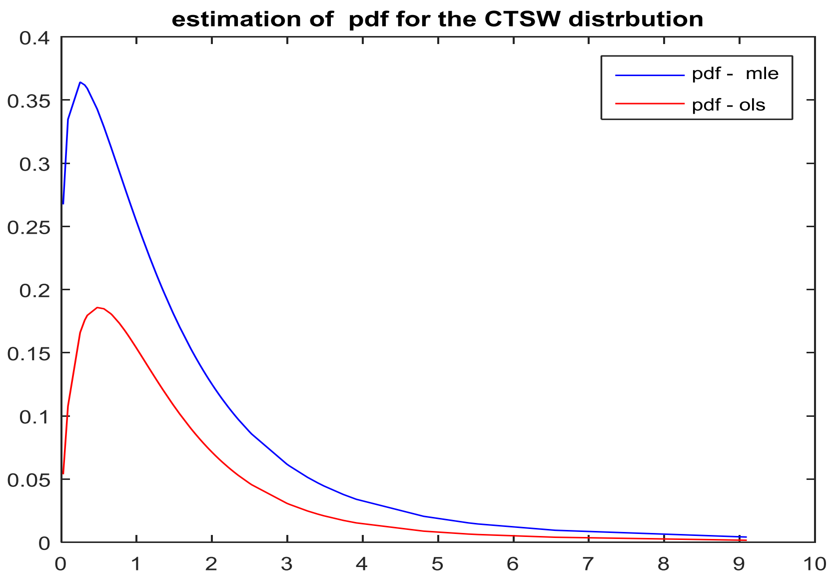

Figure 3.2 estimated MLE and OLS CTSW for pdf CTSW distribution.

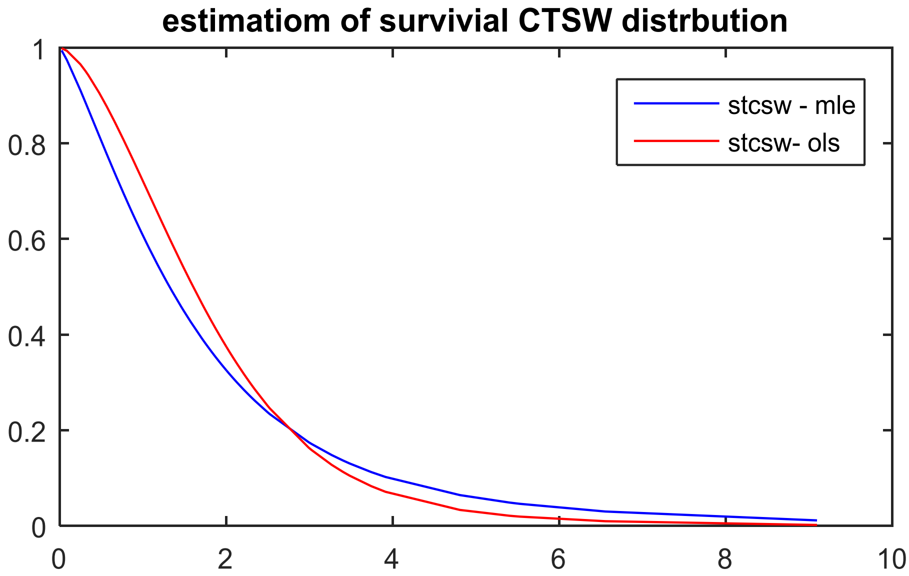

Figure 3.3 estimated MLE and OLS CTSW for Survival CTSW distribution.

Chapter Four

Empirical and Applied Part

4. Empirical and Applied Part

In this chapter, we used the two estimation methods which are discussed in chapter three to estimate the parameters (a, b and k) of the given distributions. That means, we companied between the analytic solution to the estimated methods that we derived in chapter three .The numerical technique to compute the estimators and examine these estimators with simulation.

4.1 Simulation Experiment [18][19]

It is a process of imitation of a real phenomenon in the presence of a set of mathematical equations using advanced computer programs. One of the most important features of the simulation experiment is its flexibility. It has the ability to test and repeat the experiment many times. Due to the nature of the mathematical process used, the simulation experience gives the ability to experiment and make adjustments in order to invest time and money.

This section contains the numerical results of two methods:
mle and ols are mentioned in Chapter 3 of the CTSW Distribution. Moreover, the following tables display a file the numerical results of a , b and k at n = 30, 50,100 with MSE who are they mse for a , b and k .iterations R = 500, 1000.

4.2 The Stages of as simulation Experience:

1. Choosing default values for the estimators as shown in the table below:

Table 4.1: default values for parameter for CTSW Distribution

Number of set	a	b	k
Set 1	1.5	2.5	-0.009
Set 2	1	2	0.0001
Set 3	0.5	1.5	-0.9

We choose different sample sizes small-medium-large (30-50-100) and repeat the experiment 1000 times to obtain high homogeneity.

2. We generate random data subject to a CTSW distribution by using the inverse method of the (CTSW) distribution function and generate observational values that follow a uniform on (0,1) distribution using the following formula

Assumes $\boldsymbol{F_{CTSW}(t) = u}$

$$\boldsymbol{F_{CTSW}(t) \;=\; 1-(1+k)\,exp^{-3(\frac{t}{b})^{a}} - \;k\,exp^{-2(\frac{t}{b})^{a}} + \;2k\,exp^{-(\frac{t}{b})^{a}} \sim \mathcal{U}\,(0,1)}$$

$$\boldsymbol{u = \;1-(1+k)\,exp^{-3(\frac{t}{b})^{a}} - \;k\,exp^{-2(\frac{t}{b})^{a}} + \;2k\,exp^{-(\frac{t}{b})^{a}}}$$

$$\boldsymbol{u-1 = -(1+k)\,exp^{-3(\frac{t}{b})^{a}} - \;k\,exp^{-2(\frac{t}{b})^{a}} + \;2k\,exp^{-(\frac{t}{b})^{a}}}$$

$$\boldsymbol{1-u = (1+k)exp^{-3\left(\frac{t}{b}\right)^{a}} + \;k\,exp^{-2\left(\frac{t}{b}\right)^{a}} - 2k\,exp^{-(\frac{t}{b})^{a}}}$$

The last equation is nonlinear and it can be solved numerically.

3. This stage is considered important and the parameters and survival function (reliability) of the CTSW distribution are estimated using (ols & mle), which we have already discussed in Chapter Three.

4. Using the statistical criterion mean squared error mse , a comparison was made between the two methods above, and the best method was chosen according to the following formula:

$$MSE\left(S(t)\right) = \frac{1}{R}\sum_{i=1}^{R}((\hat{S}(t_i) - \hat{S}(t_i))^2 \ldots\ldots\ 4.1$$

5. At this stage, the best estimator is selected based on (MSE) and the simulation results are analyzed and presented using the reliability(survival) function estimation of the Cubic Transmuted Survival Weibull (CTSW) Distribution.

Table 4.2 : Parameter estimation values for each method according to the sample size

N			$\hat{a}$	$\hat{b}$	$\hat{k}$
30	Set 1	mle	1.009316680569408	1.587147115768801	-0.239423924088802
		ols	0.937724040440925	2.709549115519565	-0.060672074294387
	Set 2	mle	0.844339789669403	1.574651625063901	-0.269268932956260
		ols	0.920672229433654	2.090212994273886	-0.167378540209137
	Set 3	mle	0.118114935494324	35.765276963621055	-0.829688482458374
		ols	1.168637061010145	1.398093978814544	-0.280907399313780
50	Set 1	mle	1.04451868938526	1.50417019119792	-0.185768548562497
		ols	1.11158025828219	1.79248051995724	-0.096138318042138
	Set 2	mle	0.93510859963414	1.34515276228163	-0.258281623794055
		ols	0.95434197251253	2.37143668218195	-0.028902258448883
	Set 3	mle	0.11703120490980	42.42830298260366	-0.827604764593329
		ols	1.23655448429535	1.22943300928396	-0.408677787230058
100	Set 1	mle	0.92368219886635	1.66955017433452	0.208804801128472
		ols	0.94354166939612	2.39619020415726	0.059648911393319
	Set 2	mle	0.90652727515538	1.62892109108307	-0.249334250973708
		ols	0.94314592395247	2.11588181621457	-0.161260315612490
	Set 3	mle	0.11905673007645	34.65730317256763	-0.828300998864318
		ols	1.44314268275807	1.35162257316369	-0.187534021072103

The values of the estimated parameters and the survival function values converge from the default values of the two methods and for all sample sizes, which are shown in green estimator OLS when

n= 30

(2.709549115519565, 2.090212994273886, 0.920672229433654, 1.398093978814544) and MLE (-0.829688482458374)

n= 50

estimator OLS is (1.79248051995724, 0.95434197251253, 1.22943300928396) and MLE (1.04451868938526, -0.827604764593329)

n=100 estimator OLS (2.39619020415726, 0.94314592395247, 2.11588181621457, 1.35162257316369) and MLE (-0.828300998864318) , noting that there are extreme values in red in the table

(35.765276963621055, 42.42830298260366, 34.65730317256763)

Table 4.3 : The mean squared error of the reliability function for set (1)

N	MSE($\hat{S}_{MLE(t_i)}$)	MSE($\hat{S}_{OLS(t_i)}$)	Best
30	0.001608546625452	0.001199629540190	ols
50	0.003935487067373	0.001196354025675	ols
100	0.061131616944894	0.005448092284160	ols

Table 4.3 indicates oscillating values of mean square error as sample values increasing.
We also note that the mean squared error is always smaller in the OLS method, so it is the best method for estimation

Table 4.4: The mean squared error of the reliability function for set (2)

N	MSE($\hat{S}_{MLE(t_i)}$)	MSE($\hat{S}_{OLS(t_i)}$)	Best
30	0.006509580136784	0.002281769335630	ols
50	0.006426053424803	0.001338642645791	ols
100	0.003935062874953	0.005448092284160	mle

In Table 4.4, when the mean squared error of the values (n = 30,50,100) is observed in the case of estimating the reliability function by the (MLE) method, the values are decreasing with the win(OLS) method as the best method.

Table 4.5 : The mean squared error of the reliability function for set (3)

N	MSE($\hat{S}_{MLE(t_i)}$)	MSE($\hat{S}_{OLS(t_i)}$)	Best
30	0.096387600937847	4.709986835340387e-04	ols
50	0.109557588120621	0.001045105437889	ols
100	0.084021780790087	0.001343726451918	ols

Table 4.5 indicates the fluctuating values of mean squared error with increasing sample N values It is clear weighting of the estimation method (OLS) as the best estimation method.

Now by reading the results of the above tables, we note that set 2 is the best group, as decreasing readings of the mean squared error values were recorded regularly for the (MLE) method and for three values, as well as for the ols method for two values (n =30 ,50) .and the third value n=100 kept for mean squared error less than set 1

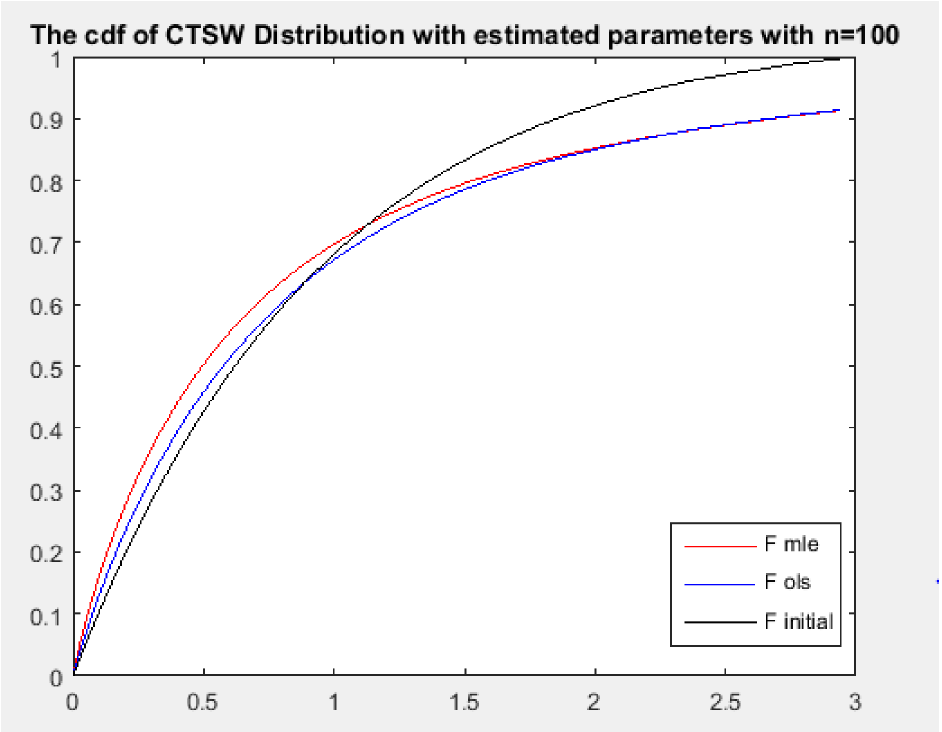

Figure 4.1 The cdf of CTSW distribution with estimated parameter by (mle & ols) when n=100

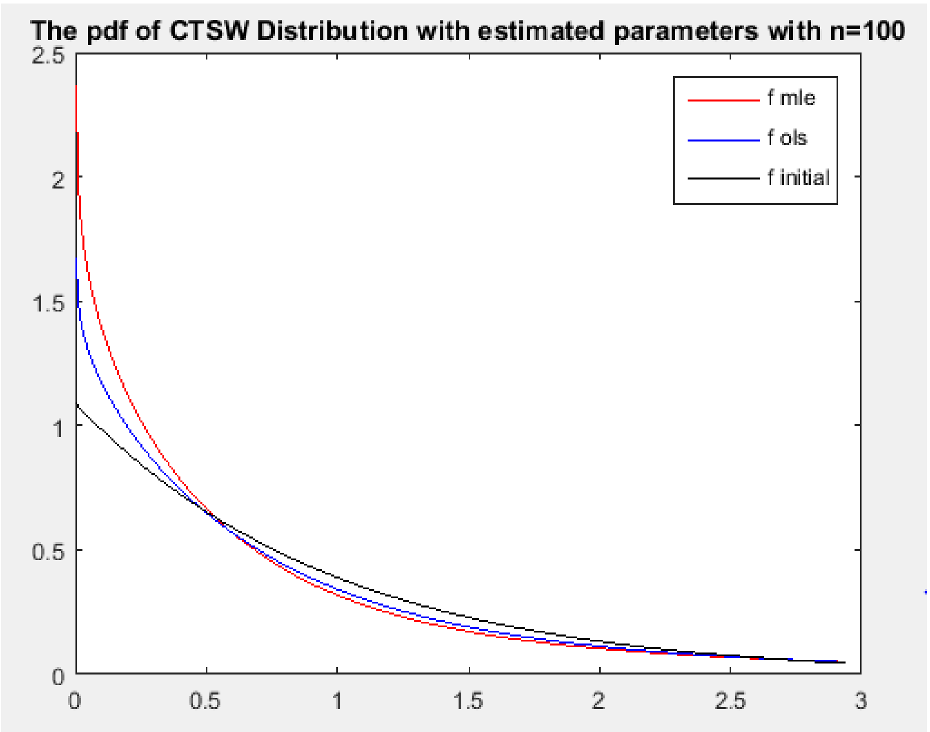

Figure 4.2 : The pdf of CTSW distribution with estimated parameter by (mle & ols) when n=100

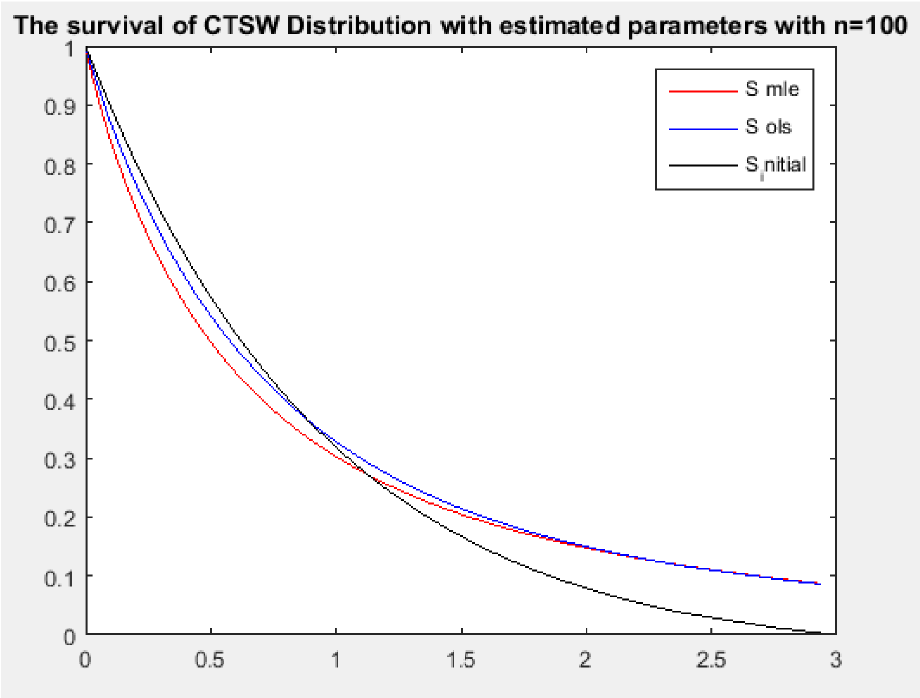

Figure 4.3 The survival of CTSW distribution with estimated parameter by (mle & ols) when n=100

4.3 Applications

In this part, we test the Reliability (survival) CTSW distribution with the following data sets and compare the result using information criterions and curve fitting tools .

Data set 1:[23] The first data set represents lifetimes of Kevlar 49/epoxy strands subjected to constant sustained pressure at 90% stress level until the strand failure

2.4951, 2.5260, 2.9911, 3.0256, 3.2678, 3.4045, 3.4846, 3.7433, 3.7455, 3.9143, 4.8073, 5.4005, 5.4435, 5.5295, 6.5541, 9.0960, 0.0251, 0.0886, 0.0891, 0.2501, 0.3113, 0.3451, 0.4763, 0.5650, 0.5671, 0.6566, 0.6748, 0.6751, 0.6753, 0.7696, 0.8375, 0.8391, 0.8425, 0.8645, 0.8851, 0.9113, 1.3503, 1.3551, 1.4595, 1.4880, 1.5728, 1.5733, 1.7083, 1.7263, 1.7460, 1.7630, 1.7746, 1.8275, 1.8375, 1.8503, 1.8808, 1.8878, 1.8881, 1.9316, 1.9558, 2.0048, 2.0408, 2.0903, 2.1093, 2.1330, 2.2100, 2.2460, 2.2878, 2.3203, 2.3470, 2.3513, 0.9120, 0.9836, 1.0483, 1.0596, 1.0773, 1.1733, 1.2570, 1.2766, 1.2985, 1.3211.

Table 4.6 criteria for data set 1

Distribution	Factor predicts	Statistical test				
		Ln L	AIC	AICC	BIC	RMSE
w (a,b)	$\hat{a}=$ 1.3256 $\hat{b}=$ 2.1328	-122.5247	249.0494	249.2094	253.7108	0.0423
WDs (a,b,c)	$\hat{a}=$ 1.3169 $\hat{b}=$ 2.1228 $\hat{c}=$ 0.0058	-122.5141	251.0282	251.3525	258.0204	0.0425
$wTran$ (a,b,λ)	$\hat{a}=$ 1.0509 $\hat{b}=$ 1.4419 $\hat{\lambda}=$ -0.7955	-121.4300	248.8600	249.1843	255.8522	0.0351
$CTSW$ (a,b,k)	$\hat{a}=$ 1.6126 $\hat{b}=$ 3.8134 $\hat{k}=$ -0.0509	-120.1507	246.3015	246.6258	253.2937	0.0228

In Table 4.6 we see that values of AIC, AICC and RMSE are smallest ones and magnitude of ln L for CTSW distribution, compared to value for 2-factor and 3- factor WDs and WTran. Hence, we can conclude that the CTSW distribution provides better fit to the data than the other three distributions.

Where the (K-S) test used for the goodness of matching with real date (k-s = 0.9967) for set 1

Table 4.7 Estimator of reliability by ols data set 1

T	R(t)- OLS	T	R(t)- OLS	T	R(t)- OLS
0.0251	0.999137	1.257	0.624775	2.1093	0.344898
0.0886	0.993423	1.2766	0.617485	2.133	0.338528
0.0891	0.993363	1.2985	0.609366	2.21	0.318425
0.2501	0.965453	1.3211	0.60102	2.246	0.309336
0.3113	0.951206	1.3503	0.590288	2.2878	0.299032
0.3451	0.94265	1.3551	0.58853	2.3203	0.291204
0.4763	0.905545	1.4595	0.55076	2.347	0.284892
0.565	0.87758	1.488	0.540623	2.3513	0.283886
0.5671	0.876894	1.5728	0.510961	2.4951	0.251824
0.6566	0.846826	1.5733	0.510788	2.526	0.245334
0.6748	0.840527	1.7083	0.465298	2.9911	0.163561
0.6751	0.840423	1.7263	0.459406	3.0256	0.1586
0.6753	0.840353	1.746	0.453008	3.2678	0.127537
0.7696	0.806903	1.763	0.447528	3.4045	0.112679
0.8375	0.782143	1.7746	0.443811	3.4846	0.104781
0.8391	0.781554	1.8275	0.427096	3.7433	0.082901
0.8425	0.780302	1.8375	0.423981	3.7455	0.082736
0.8645	0.772178	1.8503	0.420013	3.9143	0.071099
0.8851	0.764536	1.8808	0.410654	4.8073	0.033184
0.9113	0.754777	1.8878	0.408525	5.4005	0.021062
0.912	0.754516	1.8881	0.408434	5.4435	0.020412
0.9836	0.727664	1.9316	0.395364	5.5295	0.019184
1.0483	0.703264	1.9558	0.388214	6.5541	0.009582
1.0596	0.698997	2.0048	0.374001	9.096	0.001742
1.0773	0.692312	2.0408	0.363789		
1.1733	0.656113	2.0903	0.350066		

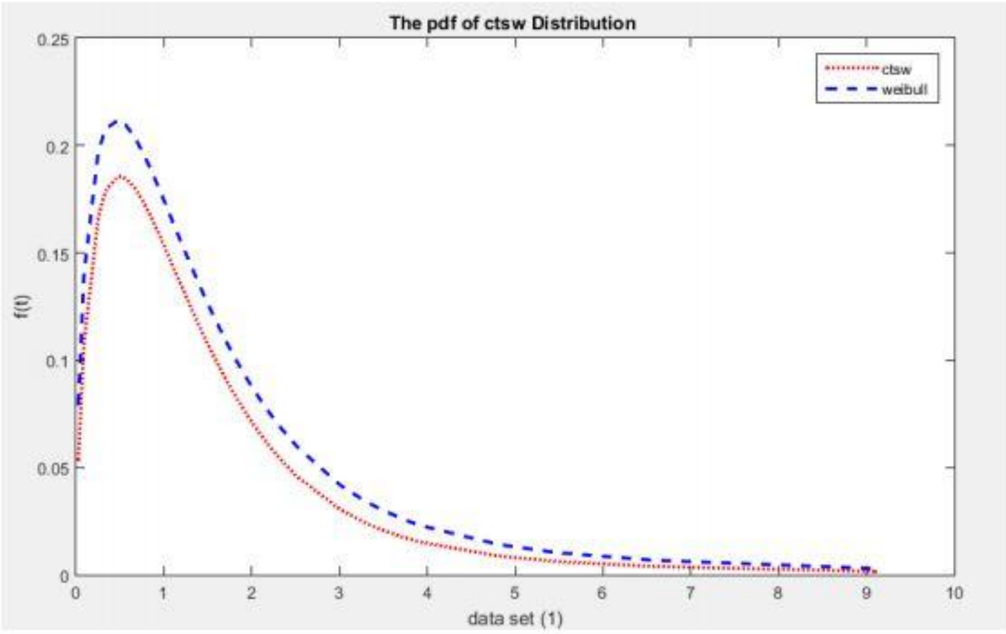

Figure 4.4 The pdf of CTSW distribution set 1

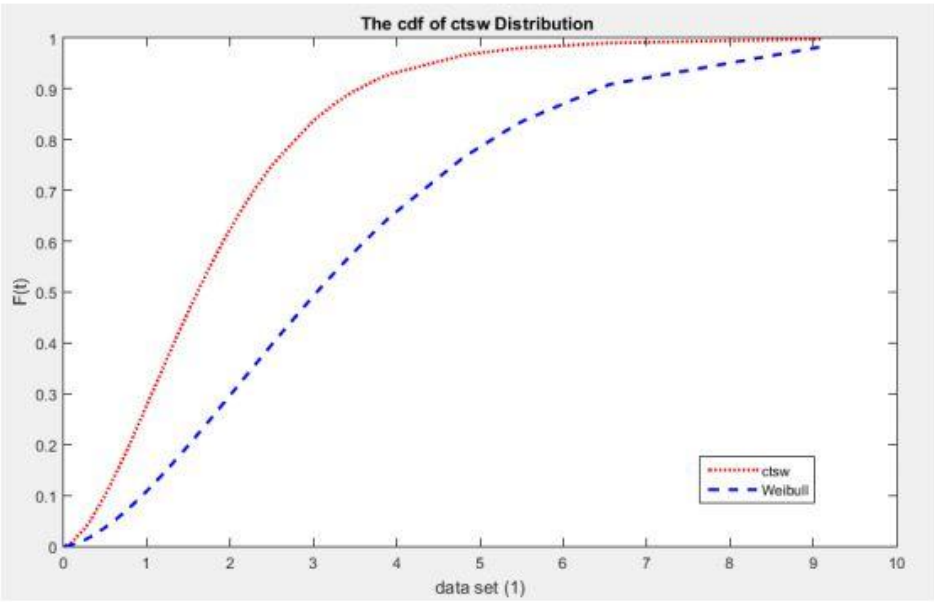

Figure 4.5 The cdf of CTSW distribution set 1

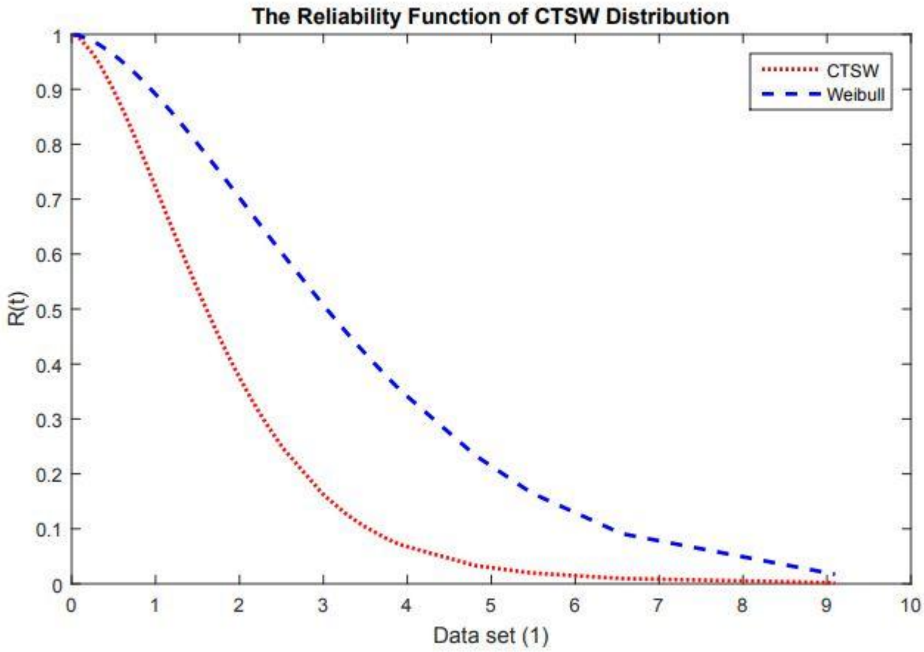

Figure 4.6 The reliability of CTSW distribution set 1

Data set 2 [23] : The data set represents age of 155 patients suffering from the breast cancer, recorded from June to November 2014, who entered the Breast Tumours Detection Unit, Benha Hospital University, Egypt.

[12, 17, 18, 18, 20, 24, 24, 25, 26, 28, 28, 28, 29, 30, 30, 30, 30, 31, 31, 31, 32, 32, 32, 32, 33, 34, 34, 35, 35, 35, 35, 35, 35, 35, 36, 36, 36, 36, 36, 36, 38, 38, 38, 38, 38, 38, 38, 38, 38, 38, 38, 38, 38, 39, 39, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 41, 42, 42, 42, 42, 42, 42, 42, 42, 43, 43, 44, 44, 45, 45, 45, 45, 45, 45, 45, 46, 46, 47, 48, 48, 48, 48, 48, 48, 48, 49, 49, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 52, 52, 52, 53, 54, 54, 55, 56, 56, 58, 58, 58, 59, 60, 60, 60, 60, 60, 60, 60, 63, 65, 65, 66, 69, 80, 90, 90]

Table 4.7 criteria for data set 2

Distribution	Factor predicts	Statistical test				
		Ln L	AIC	AICC	BIC	RMSE
w (a,b)	$\hat{a}$= 3.6871 $\hat{b}$= 48.1100	-610.2967	1224.5934	1224.6713	1230.6802	0.0491
WDs (a,b,c)	$\hat{a}$= 2.9515 $\hat{b}$= 38.3366 $\hat{c}$= 9.2768	-607.4815	1220.9629	1221.1189	1230.0932	0.0489
$wTran$ (a,b,λ)	$\hat{a}$= 4.0312 $\hat{b}$= 55.0570 $\hat{\lambda}$= 0.8380	-605.5600	1217.1200	1217.2768	1226.2502	0.0448
$CTSW$ (a,b,k)	$\hat{a}$=4.3181 $\hat{b}$= 47.3488 $\hat{k}$=-0.5904	-251.7991	509.5981	509.7550	518.7284	5.2520e-08

In Table 4.7 we see that value of AIC, AICC and RMSE are smallest ones and value of ln L for CTSW distribution, compared to values for 2-factor and 3- factor WDs and WTran Hence, we can conclude that the CTSW distribution provides better fit to the data than the other three distributions. Where the (K-S) test used for the goodness of matching with real date (k-s = 0.9998) for set 2

Table 4.9 Estimator of Survival by ols data set 2

T	S(t)- OLS	T	S(t)- OLS	T	S(t)- OLS	T	S(t)- OLS
12	0.996733	36	0.712718	42	0.53992	50	0.295373
17	0.985436	38	0.657975	42	0.53992	50	0.295373
18	0.981422	38	0.657975	42	0.53992	50	0.295373
18	0.981422	38	0.657975	43	0.509271	50	0.295373
20	0.970977	38	0.657975	43	0.509271	50	0.295373
24	0.938013	38	0.657975	44	0.478392	50	0.295373
24	0.938013	38	0.657975	44	0.478392	50	0.295373
25	0.629365	38	0.657975	45	0.447399	50	0.295373
26	0.914242	38	0.657975	45	0.447399	50	0.295373
28	0.884985	38	0.657975	45	0.447399	50	0.295373
28	0.884985	38	0.657975	45	0.447399	52	0.238897
28	0.884985	38	0.657975	45	0.447399	52	0.238897
29	0.868221	38	0.657975	45	0.447399	52	0.238897
30	0.629365	38	0.657975	45	0.447399	53	0.212307
30	0.629365	39	0.629365	46	0.416414	54	0.187057
30	0.629365	39	0.629365	46	0.416414	54	0.187057
30	0.629365	40	0.600078	47	0.385569	55	0.163315
31	0.830409	40	0.600078	48	0.355008	56	0.141221
31	0.830409	40	0.600078	48	0.355008	58	0.102402
31	0.830409	40	0.600078	48	0.355008	58	0.102402
32	0.809406	40	0.600078	48	0.355008	58	0.102402
32	0.809406	40	0.600078	48	0.355008	59	0.085792
32	0.809406	40	0.600078	48	0.355008	60	0.071061
32	0.809406	40	0.600078	48	0.355008	60	0.071061
33	0.787064	40	0.600078	49	0.324886	60	0.071061
34	0.763448	40	0.600078	49	0.324886	60	0.071061
34	0.763448	40	0.600078	50	0.295373	60	0.071061
35	0.738635	40	0.600078	50	0.295373	60	0.071061
35	0.738635	40	0.600078	50	0.295373	60	0.071061
35	0.738635	40	0.600078	50	0.295373	63	0.037552
35	0.738635	40	0.600078	50	0.295373	65	0.023011
35	0.738635	40	0.600078	50	0.295373	65	0.023011
35	0.738635	40	0.600078	50	0.295373	66	0.017647
35	0.738635	41	0.570226	50	0.295373	69	0.007294
36	0.712718	42	0.53992	50	0.295373	80	7.77E-05
36	0.712718	42	0.53992	50	0.295373	90	1.31E-07
36	0.712718	42	0.53992	50	0.295373	90	1.31E-07
36	0.712718	42	0.53992	50	0.295373		
36	0.712718	42	0.53992	50	0.295373		

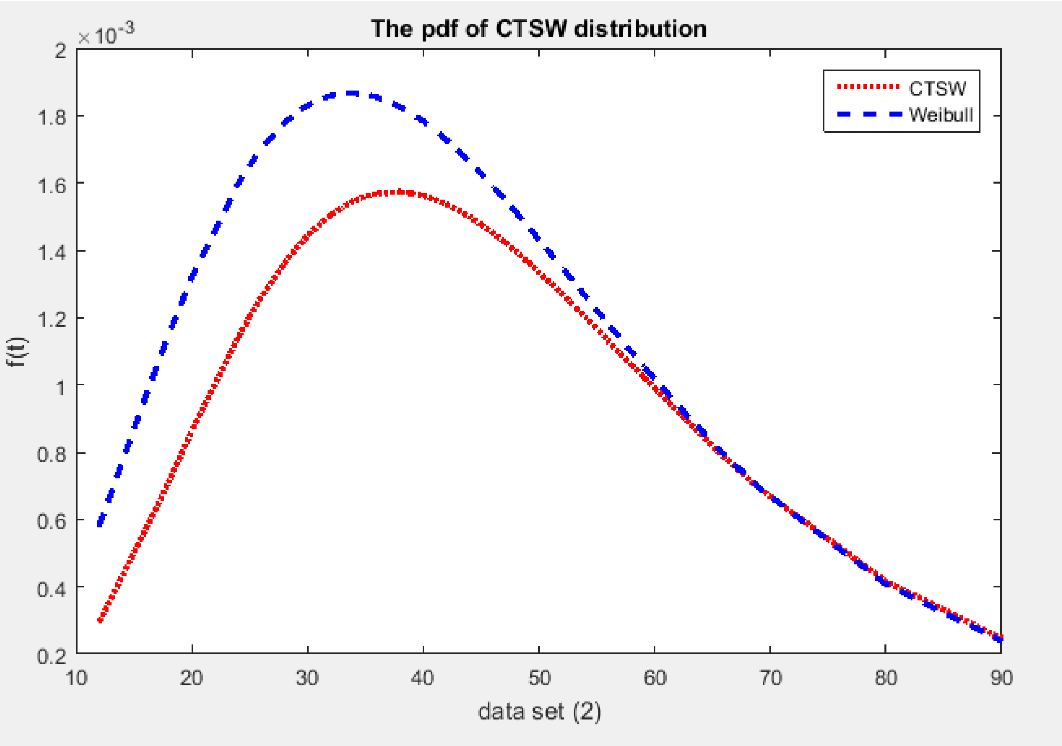

Figure 4.7 The pdf of CTSW distribution set 2

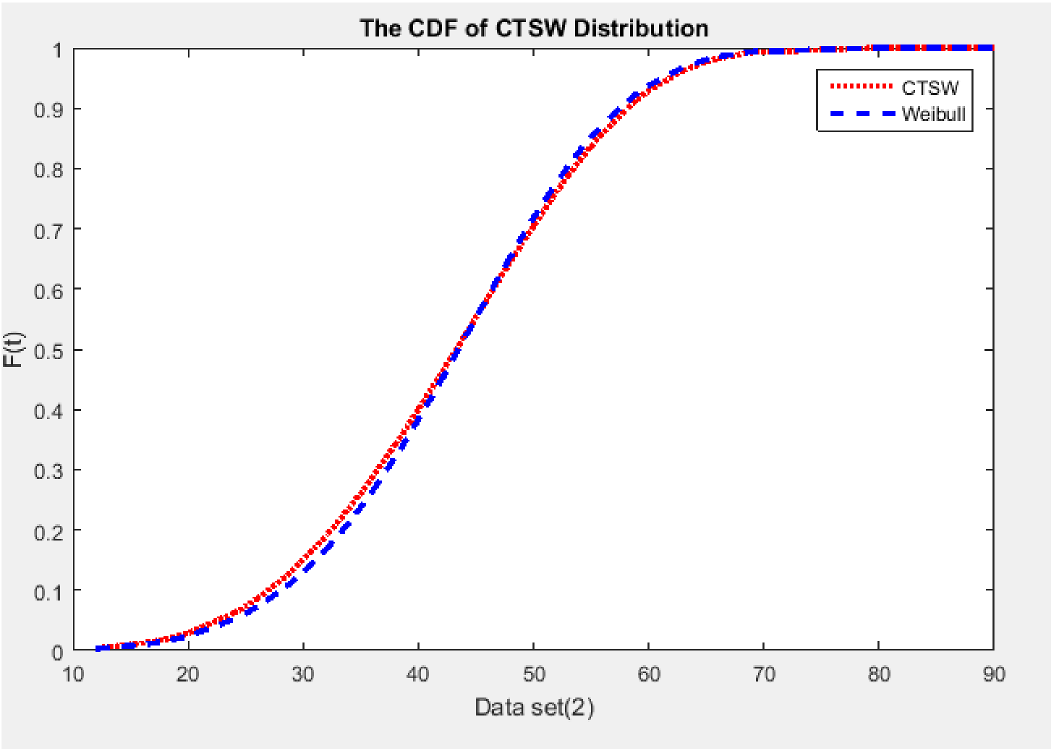

Figure 4.8 The cdf of CTSW distribution set 2

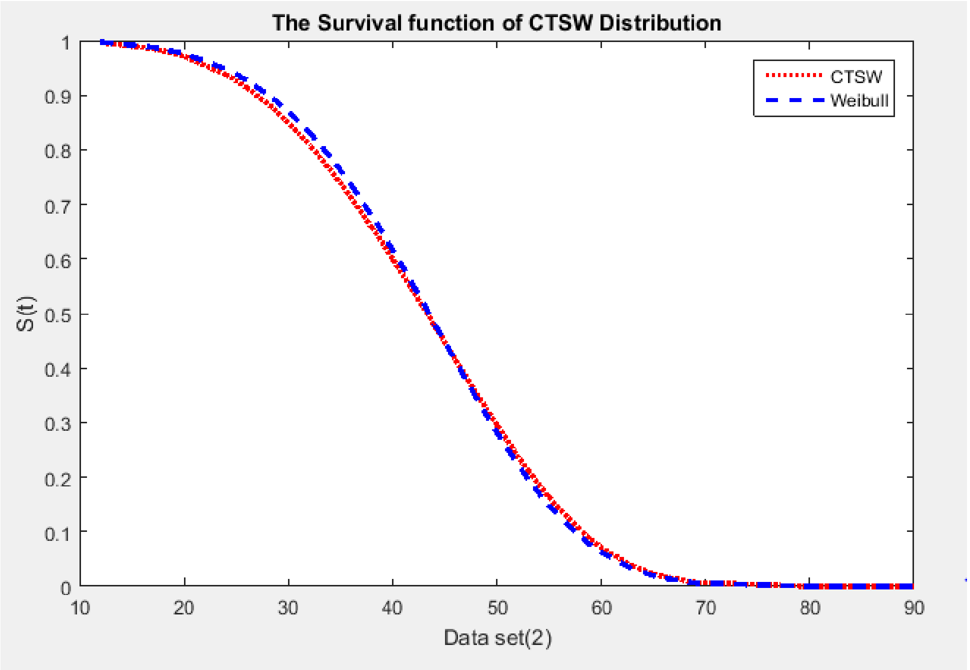

Figure 4.9 The Survival of CTSW distribution set 2

4.4 Conclusions

The following are some of the study's most important conclusions:

1. In this research, new distribution is derived for the transmuted Weibull distribution called cubic transmuted survival Weibull distribution.
2. Within the simulation study, to estimate the reliability function of the cubic Transmuted survival Weibull distribution used assuming the ols method is the best three
3. The CTSW distribution is the best model for the set 1 and set 2 of real data .

4.5 Future Works:

There are numerous key ideas and notes that can be applied in future research including;

1. Estimation of the survival function of cubic transmuted Weibull distribution for 3 – parameters
2. Using new method for estimating the cubic transmuted survival Weibull distribution and comparing it with other methods .
3. Apply the cubic transmuted survival formula to other distribution to learn more about them.

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Appendix

The plot the CDF of the cubic transmuted survival weibull distribution (k=-1)

```
syms x a1 a2 a3 a4 b1 b2 b3 b4 k;
>> x=0:.01:6;a1=1.5;a2=1.8;a3=2;a4=2.3;b1=1;b2=1.5;b3=2;b4=2.5;k=-1;
>> y1=(1-(k+1)*exp(-3*(x/b1).^a1)-k*exp(-2*(x/b1).^a1)+(2*k)*exp(-(x/b1).^a1));
>> y2=(1-(k+1)*exp(-3*(x/b2).^a2)-k*exp(-2*(x/b2).^a2)+(2*k)*exp(-(x/b2).^a2));
>> y3=(1-(k+1)*exp(-3*(x/b3).^a3)-k*exp(-2*(x/b3).^a3)+(2*k)*exp(-(x/b3).^a3));
>> y4=(1-(k+1)*exp(-3*(x/b4).^a4)-k*exp(-2*(x/b4).^a4)+(2*k)*exp(-(x/b4).^a4));
>> plot(x,y1,'--','linewidth',2.5);hold all;
>> plot(x,y2,'-.','linewidth',2.5);hold all;
>> plot(x,y3,':','linewidth',2.5);hold all;
>> plot(x,y4,'-k','linewidth',2.5);hold all;
>> legend('a1=1.5,b1=1','a2=1.8,b2=1.5','a3=2,b3=2.3','a4=2.5,b4=2.5');
>> xlabel('x-axis');
>> ylabel('CDF OF CTSW');
```

The program plot the survival of the cubic transmuted survival weibull distribution (a <1 & a >1)

```
syms x a1 a2 b k;
>> x=0:.01:6;a1=0.5;a2=1.8;b=2.5;k=-0.1;
>> y1=((k+1)*exp(-3*(x/b).^a1)+k*exp(-2*(x/b).^a1)-(2*k)*exp(-(x/b).^a1));
>> y2=((k+1)*exp(-3*(x/b).^a2)+k*exp(-2*(x/b).^a2)-(2*k)*exp(-(x/b).^a2));
>> plot(x,y1,'--','linewidth',2.5);hold all;
>> plot(x,y2,'-k','linewidth',2.5);hold all;
>> legend('a1=0.5,b=2.5','a2=1.8,b=2.5');
>> xlabel('x-axis');
>> ylabel('suvival OF CTSW when(a<1),(a>1)');
```

```
function [F]=cdf_CTSW(t,a,b,k)
n=length(t);
for i=1:n
   F(i)=1-(k+1)*exp(-3*(t(i)/b).^a)-k*exp(-2*(t(i)/b).^a)+(2*k)*exp(-(t(i)/b).^a);
end
%%%%generate sample of CTSW distribution %%%%%
function[st]=gs_CTSW(n,a,b,k);
cctsw=@(t,a,b,k,w)(1+k)*exp(-3*(t/b)^a)+k*exp(-2*(t/b)^a)-2*k*exp(-(t/b)^a)-(1-w);
%a=1; L=-0.01;n=50
   for i=1:n
   w=rand;
   st(i)=fsolve(@(t)cctsw(t,a,b,k,w),0.1);
   end

  %gs_CTSW(20,1,0.5,0.001) for excutev

% ESTMATOR MLE
function [FF]=MLE_TCSW(t,S)
n=length(t);
a=S(1);
b=S(2);
k=S(3);
tam1=t.^(a-1);
%ta=t.^(a);
s1=0;s2=0;s3=0;s4=0;
m1=0;m2=0;m3=0;m4=0;m5=0;num1=0;num2=0;num=0;dem=0;numa=0;numb=0;numk
=0;
```

```
for i=1:n
    m1=m1+(t(i)^a*log(t(i))*b^(-a)-(t(i)^a)*b^(-a)*log(b));
    s1=s1+t(i);
end
for i=1:n
    m2=m2+log(t(i));
end
for i=1:n
    num1(i)=3*(k+1)*((-2*t(i)^a*log(t(i))*b^(-a)+2*t(i)^a*b^(-a)*log(b))*exp(-
2*t(i)^a*b^(-a)));
    num2(i)=2*k*((-t(i)^a*log(t(i))*b^(-a)+t(i)^a*b^(-a)*log(b))*exp(-t(i)^a*b^(-a)));
    numa(i)=num1(i)+num2(i);%ÈÓØ ÇáãÔÊÞÉ ÈÇáäÓÈÉ a
    dem(i)=3*(k+1)*exp(-2*(t(i)/b)^a)+2*k*exp(-1*(t(i)/b)^a)-2*k;% ÇáãÞÇã
    numb(i)=6*(k+1)*(a*t(i)^a*exp(-2*(t(i)/b)^a)*b^(-a-1))+2*k*(a*t(i)^a*exp(-
(t(i)/b)^a)*b^(-a-1));
    numk(i)=3*exp(-2*(t(i)/b)^a)+2*exp(-(t(i)/b)^a)-2;
end
for i=1:n
    m3=m3+numa(i)/dem(i);
    m4=m4+numb(i)/dem(i);
    m5=m5+numk(i)/dem(i);
end

da=(n/a)-(n*log(b))+m1+m2+m3;
db=-(n*a/b)+(a/b^(a+1))*s1+m4;
dk=m5;
FF=[da db dk];
%
%s3=(2*L+2)*s21-L*s22;
```

```
%F=[s1 s3];
% for result print ([par]=fsolve(@(S) MLE_TSW(t,S),[0.5 0.5]))
% define first vector t
% ESTMATOR  OLS
function F1=Ols_Ctsw(tn1,z)
%[par]=fsolve(@(z) Ols_Ctsw(tn1,z),z);
% ČÇä ĘćŇíÚ ćíČá ăßÚČ ÇáŐíŰÉ
a=z(1);
b=z(2);
k=z(3);
[rows, columns]=size(tn1);
m=rows;rr=columns;
mr=m*rr;
tn=tn1(:);
for i=1:mr
p(i)=(i)/(mr+1);
%a(i)=(exp((-lamda./xn(i)).^th))-p(i);
ols(i)=1-(1+k)*exp(-3*(tn(i)/b)^a)-k*exp(-2*(tn(i)/b)^a)+2*k*exp(-(tn(i)/b)^a)-p(i);
end
F1=sum(ols.^2);
close all
clear
clc
%tcarbon=[0.390000000000000,0.850000000000000,1.08000000000000,1.25000000000000,1.47000000000000,1.57000000000000,1.61000000000000,1.61000000000000,1.69000000000000,1.80000000000000,1.84000000000000,1.87000000000000,1.89000000000000,2.03000000000000,2.03000000000000,2.05000000000000,2.12000000000000,2.35000000000000,2.41000000000000,2.43000000000000,2.48000000000000,2.50000000000000,2.53000000000000,2.55000000000000,2.55000000000000,2.56000000000000,2.59000000000000,2.67000000000000,2.73000000000000,2.74000000000000,2.79000000000000,2.81000000000000,2.82000000000000,2.85000000000000,2.8700000000
```

```
0000,2.88000000000000,2.93000000000000,2.95000000000000,2.96000000000000,2.9
7000000000000,3.09000000000000,3.11000000000000,3.11000000000000,3.15000000
000000,3.15000000000000,3.19000000000000,3.22000000000000,3.22000000000000,
3.27000000000000,3.28000000000000,3.31000000000000,3.31000000000000,3.33000
000000000,3.39000000000000,3.39000000000000,3.56000000000000,3.600000000000
00,3.65000000000000,3.68000000000000,3.70000000000000,3.75000000000000,4.200
00000000000,4.38000000000000,4.42000000000000,4.70000000000000,4.9000000000
0000];
%0.001 0.1207 .0217
%t=tcarbon;n=66;
%a=input('the value of alpha= ');
%L=input('the value of lempda= ');
%b=input('input value of beta=');
%T=input('the value of Rerplication= ');
%n=input('the value of n= ');%samples sizes
a=1;n=100;b=3;
k=0.09;
T=500;
%% Section 1 Title
for i=1:n
 for tt=1:T
t=gs_CTSW(n,a,b,k);
%%%%? put t 0.0000
%t=abs((et-round(et)));
fctsw=pdf_CTSW(sort(t),a,b,k);
Fctsw=cdf_CTSW(sort(t),a,b,k);
sctsw=1-Fctsw;
 end
%% Section 2 Title
t=real(t);t=sort(t);
 [par_mle]=fsolve(@(S) MLE_TCSW(t,S),[0.5 1.5 -0.9]);
```

```
 [par_ols]=fsolve(@(S) Ols_Ctsw(t,S),[0.5 1.5 -0.9]);

a1(tt)=par_mle(1); b1(tt)=par_mle(2);k1(tt)=par_mle(3);
a2(tt)=par_ols(1) ;  b2(tt)=par_ols(2);k2(tt)=par_ols(3);

 f_mle=pdf_CTSW(sort(t),a1(tt),b1(tt),k1(tt));
 f_ols=pdf_CTSW(sort(t),a2(tt),b2(tt),k2(tt));

F_mle=cdf_CTSW(sort(t),a1(tt),b1(tt),k1(tt));
F_ols=cdf_CTSW(sort(t),a2(tt),b2(tt),k2(tt));

ks_mle(tt)=max(abs(((1:n)/n)-F_mle));
ks_ols(tt)=max(abs(((1:n)/n)-F_ols));
s_mle=1-F_mle;
 s_ols=1-F_ols;
mse_mle(tt)=immse(sctsw,(s_mle));
mse_ols(tt)=immse(sctsw,(s_ols));
% ÊÞÏíÑ ÏÇáÉ ÇáÈÞÇÁ ÇáãÍæáÉ ÇáÊßÚíÈíÉ

end
s_mle=1-F_mle;
 s_ols=1-F_ols;
 %g=par(1);L=par(2);mx=0;
% ÇáÇãßÇä ÇáÇÚÙã
%ln(2*g*(1+L)*exp(-2*g*t(i)))
%for i=1:n
   %mx=mx+log(2*g*(1+L)*exp(-2*g*t(i)));
%end
disp(par_mle)
```

```
disp(par_ols)
mse_mle(tt)
mse_ols(tt)
 ks_mle(tt)
ks_ols(tt)
 t=sort(t);
figure
plot(t,f_mle,'r',t,f_ols,'b',t,fctsw,'k')
figure
plot(t,F_mle,'r',t,F_ols,'b',t,Fctsw,'k')
figure
plot(t,s_mle,'r',t,s_ols,'b',t,sctsw,'k')
```

APLICATION FOR REAL ADTA

```
clear
close all
%a=1;b=4;k=-0.09;%good L=-120.1507
 %for set 1 good identity a=1.3 b=4 k=-0.0009 with 1.2233    3.9352   -0.0929
 %for set 1 by artical "Transmuted Weibull distribution and its
 %applications"
%a=1.5;b=4;k=-0.08; %L=-120.7454 phat= 0.6095    3.7361   -0.9691
 %for set 2
%a=3.3; b=59; k=-1.08 % good for set 3;
%a=2.1; b=150; k=-0.64 % very good for data set 2( pigs)

s1=0;s2=0;s3=0;s4=0;
%t=1:10;
```

```
%t=[0.3 0.3 4.0 5.0 5.6 6.2 6.3 6.6 6.8 7.4 7.5 8.4 8.4 10.3 11.0 11.8 12.2 12.3 13.5 14.4
14.4 14.8 15.5 15.7 16.2 16.3 16.5 16.8 17.2 17.3 17.5 17.9 19.8 20.4 20.9 21.0 21.0 21.1
23.0 23.4 23.6 24.0 24.0 27.9 28.2 29.1 30.0 31.0 31.0 32.0 35.0 35.0 37.0 37.0 37.0 38.0
38.0 38.0 39.0 39.0 40.0 40.0 40.0 41.0 41.0 41.0 42.0 43.0 43.0 43.0 44.0 45.0 45.0 46.0
46.0 47.0 48.0 49.0 51.0 51.0 51.0 52.0 54.0 55.0 56.0 57.0 58.0 59.0 60.0 60.0 60.0 61.0
62.0 65.0 65.0 67.0 67.0 68.0 69.0 78.0 80.0 83.0 88.0 89.0 90.0 93.0 96.0 103.0 105.0
109.0 109.0 111.0 115.0 117.0 125.0 126.0 127.0 129.0 129.0 139.0 154.0];

 %set 1 :.

%t=[0.0251, 0.0886, 0.0891, 0.2501, 0.3113, 0.3451, 0.4763, 0.5650, 0.5671, 0.6566,
0.6748, 0.6751, 0.6753, 0.7696, 0.8375, 0.8391, 0.8425, 0.8645, 0.8851, 0.9113, 0.9120,
0.9836, 1.0483, 1.0596, 1.0773, 1.1733, 1.2570, 1.2766, 1.2985, 1.3211, 1.3503, 1.3551,
1.4595, 1.4880, 1.5728, 1.5733, 1.7083, 1.7263, 1.7460, 1.7630, 1.7746, 1.8275, 1.8375,
1.8503, 1.8808, 1.8878, 1.8881, 1.9316, 1.9558, 2.0048, 2.0408, 2.0903, 2.1093, 2.1330,
2.2100, 2.2460, 2.2878, 2.3203, 2.3470, 2.3513, 2.4951, 2.5260, 2.9911, 3.0256, 3.2678,
3.4045, 3.4846, 3.7433, 3.7455, 3.9143, 4.8073, 5.4005, 5.4435, 5.5295, 6.5541, 9.0960]

%set 2

%t=[10, 33, 44, 56, 59, 72, 74, 77, 92, 93, 96, 100, 100, 102, 105, 107, 107, 108, 108,
108, 109, 112, 113, 115, 116, 120, 121, 122, 122, 124, 130, 134, 136, 139, 144, 146, 153,
159, 160, 163, 163, 168, 171, 172, 176, 183, 195, 196, 197, 202, 213, 215, 216, 222, 230,
231, 240, 245, 251, 253, 254, 254, 278, 293, 327, 342, 347, 361, 402, 432, 458, 555];

 %set 3

a=4;b=45;k=-0.5;

t=[12, 17, 18, 18, 20, 24, 24, 25, 26, 28, 28, 28, 29, 30, 30, 30, 30, 31, 31, 31, 32, 32, 32,
32, 33, 34, 34, 35, 35, 35, 35, 35, 35, 35, 36, 36, 36, 36, 36, 36, 38, 38, 38, 38, 38, 38, 38,
38, 38, 38, 38, 38, 38, 39, 39, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40, 40,
40, 41, 42, 42, 42, 42, 42, 42, 42, 42, 43, 43, 44, 44, 45, 45, 45, 45, 45, 45, 45, 46, 46, 47,
48, 48, 48, 48, 48, 48, 48, 49, 49, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50, 50,
50, 50, 50, 50, 50, 50, 50, 50, 52, 52, 52, 53, 54, 54, 55, 56, 56, 58, 58, 58, 59, 60, 60, 60,
60, 60, 60, 60, 63, 65, 65, 66, 69, 80, 90, 90];

 %t=0:.01:6;

n=length(t);

for i=1:n

    fw(i)=((a/b)*(t(i)/b)^(a-1)*exp(-(t(i)/b))^a);

% fw(i)=(a/b)*((t(i)/b)^a-1)*exp(-1*(t(i)/b)^a)*(3*(k+1)*exp(-2*(t(i)/b)^a)+2*k*exp(-
1*(t(i)/b)^a)-2*k);

 %stcsw=
```

```
end

figure
 plot(t,fw,'r')
for i=1:n
    s1=s1+t(i)^a;
end
for i=1:n
    s2=s2+log(t(i));
 s3=s3+log(3*(k+1)*exp(-2*(t(i)/b)^a)+2*k*exp(-1*(t(i)/b)^a)-2*k);
end
L=n*log(a/b^a)+(s1/b^a)+(a-1)*s2+s3;%log liklihood
%t^a*ln(t)*b^(-a)-t^a*b^(-a)*ln(b)
)*2-%t/b)^a*ln(t/b)*exp(-2*(t/b)^a)
%dem(i)=3*(k+1)*exp(-2*(t(i)/b)^a)+2*k*exp(-1*(t(i)/b)^a)-2*k
%num(i)=3*(k+1)((-2*t^a*ln(t)*b^(-a)+2*t^a*b^(-a)*ln(b))*exp(-2*t^a*b^(-a)))+2*k((-
t^a*ln(t)*b^(-a)+t^a*b^(-a)*ln(b))*exp(-t^a*b^(-a)))
%first dervative for a
%da=n/a-log(b)+sum(t^a*ln(t)*b^(-a)-t^a*b^(-a)*ln(b))+sum(log(t))+num/dem
] par_mle]=fsolve(@(S) MLE_TCSW(t,S),[a b k])
 a=par_mle(1);b=par_mle(2);k=par_mle(3);
 s11=0;s21=0;s31=0;
 for i=1:n
    s11=s11+t(i)^a;
end
for i=1:n
    s21=s21+log(t(i));
 s31=s31+log(3*(k+1)*exp(-2*(t(i)/b)^a)+2*k*exp(-1*(t(i)/b)^a)-2*k);
end
```

```
Lh_mle=n*log(a/b^a)+(s11/b^a)+(a-1)*s21+s31


for i=1:n
ftcsw_mle(i)=((a/b)*(t(i)/b)^(a-1)*exp(-3*(t(i)/b))^a)*3*(1+k)+((a/b)*(t(i)/b)^(a-1)*exp(-2*(t(i)/b))^a)*(2*k)-((a/b)*(t(i)/b)^(a-1)*exp(-(t(i)/b))^a)*(2*k);;
%stcsw=
end
figure
% plot (t,ftcsw_mle,'b',t,fw,'r')
%plot(t,stcsw)
%disp('L=')
%disp(L)


mse_mle=immse( ftcsw_mle,(fw));
m=3;%number of parameters
AIC=-2*L+2*m;
disp('AIC=')
disp(AIC)
AICC=AIC+(2*m*(m+1)/(n-m+1));
disp('AICC=')
disp(AICC)
BIC=-2*L+m*log(n);
disp('BIC=')
disp(BIC)
for i=1:n
    F_mle(i)=(1-(k+1)*exp(-3*(t(i)/b).^a)-k*exp(-2*(t(i)/b).^a)+(2*k)*exp(-(t(i)/b).^a));
end
%figure
%plot(t,F_mle) % plot of CDF with estimated parameters in mle method
```

```
%Stcsw_mle=1-F_mle;
%figure
%plot(t,Stcsw_mle)
]par_ols]=fsolve(@(S) Ols_Ctsw(t,S),[a b k]);% results of OLS method
 a=par_ols(1);b=par_ols(2);k=par_ols(3);
 s11=0;s21=0;s31=0;
 for i=1:n
    s11=s11+t(i)^a;
end
for i=1:n
    s21=s21+log(t(i));
 s31=s31+log(3*(k+1)*exp(-2*(t(i)/b)^a)+2*k*exp(-1*(t(i)/b)^a)-2*k);
end
 Lh_ols=n*log(a/b^a)+(s11/b^a)+(a-1)*s21+s31
% ÇáãÞÇííÓ ÈÏáÇáÉ ÇáãÑÈÚÇÊ
 Lo=Lh_ols;
 m=3;%number of parameters
 oAIC=-2*Lo+2*m;
 disp('oAIC=')
disp(oAIC)
oAICC=oAIC+(2*m*(m+1)/(n-m+1));
 disp('oAICC=')
disp(oAICC)
oBIC=-2*Lo+m*log(n);
disp('oBIC=')
disp(oBIC)
 for i=1:n
 ftcsw_ols(i)=((a/b)*(t(i)/b)^(a-1)*exp(-3*(t(i)/b))^a)*3*(1+k)+((a/b)*(t(i)/b)^(a-1)*exp(-
2*(t(i)/b))^a)*(2*k)-((a/b)*(t(i)/b)^(a-1)*exp(-(t(i)/b))^a)*(2*k);;
```

```
 %stcsw=
 end


 figure
 plot (t,ftcsw_ols,'r',t,fw,'b')
 %plot(t,stcsw)
disp('L=')
disp(L)
 mse_ols=immse( ftcsw_ols,(fw));


for i=1:n
    F_ols(i)=(1-(k+1)*exp(-3*(t(i)/b).^a)-k*exp(-2*(t(i)/b).^a)+(2*k)*exp(-(t(i)/b).^a));
    Fw(i)=1-exp(-(t(i)/b).^a);
    Sw(i)=1-Fw(i);
end
figure
plot(t,F_ols,'r',t,Fw,'b') % plot of CDF with estimated parameters in ols method
Stcsw_ols=1-F_ols;
figure
plot(t,Stcsw_ols,'r',t,Sw,'b')
%ÎÊÈÇÑ ÍÓä ÇáãØÇÈÞÉ ááÈíÇäÇÊ ÈÚÏ ÕíÇÛÉ ÇáÝ
 %ÇáÝÑÖíÉ ÇáÕÝÑíÉ :ÇáÈíÇäÇÊ ÊÊæÒÚ ÊæÒíÚ ÞíÏ ÇáÇÎÊÈÇÑ
ks=max(abs((ftcsw_ols-F_ols)));
```

المستخلص

بـالنظر لوجـود العديـد مـن المشـكلات التـي لا تتبـع بياناتهـا الحقيقـة أيـا مـن النمـاذج الإحصـائية الكلاسـيكية أو القياسـية. والفكـرة الرئيسـية لهـذا البحـث هـي إيجـاد توزيـع جديـد باسـتخدام الصـيغة التكعيبيـة المحولـة باسـتخدام دالـة الموثوقيـة ثـم تقـدير دالـة الموثوقيـة لهـذا التوزيـع .والتوزيـع الجديـد المقتـرح هـو توزيـع ويبـل المحـول (باسـتخدام دالـة البقـاء) مـن الدرجـة الثالثة (CTSW) الذي تمت مناقشة بعض خصائصه الإحصائية .

كـم تـم اسـتخدام طـريقتين لتقـدير معلمـات التوزيـع همـا طريقـة الإمكـان الأعظـم وطريقـة المربعـات الصـغرى. وتـم اسـتخدام طريقـة المحاكـاة بـافتراض ثلاثـة أحجـام مختلفـة للعينـة (30 و 50 و 100) وتقـدير دالـة الموثوقيـة بـالطريقتين المـذكورتين أعـلاه لمعرفـة أفضـل طريقـة تقـدير مـن خـلال قيـاس متوسـط الخطـأ التربيعـي. أخيـرًا ، تـم نمذجـة مجمـوعتين مـن البيانـات الحقيقية واستخدام المعايير الإحصائية اختبار AIC و CAIC و BIC و . K-S

وكان برنامج ماتلاب هو المساعد لرسم الوظائف والنمذجة الإحصائية.

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جامعة بابل

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قسم الرياضيات

تقدير دالة الموثوقية لنموذج بقاء ويبل المحول

رسالة

مقدمة إلى مجلس كلية التربية للعلوم الصرفة / جامعة بابل كجزء من متطلبات نيل درجة الماجستير في التربية / الرياضيات

من قبل

بان جبار جواد كاظم

بإشراف

أ.د كريمة عبد الكاظم مخرب الخفاجي

1444 هـ 2022 م