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Use innovative methods to increase the reliability of complex and mixed networks

A Thesis
Submitted to the Council of College of Education for Pure Sciences, Babylon University in Partial Fulfillment of the Requirements for the Degree of Master in Education /Mathematics.

By

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بسم الله الرحمن الرحيم

اللَّهُ نُورُ السَّمَاوَاتِ وَالْأَرْضِ ۚ مَثَلُ نُورِهِ كَمِشْكَاةٍ فِيهَا مِصْبَاحٌ ۖ الْمِصْبَاحُ فِي زُجَاجَةٍ ۖ الزُّجَاجَةُ كَأَنَّهَا كَوْكَبٌ دُرِّيٌّ يُوقَدُ مِنْ شَجَرَةٍ مُبَارَكَةٍ زَيْتُونَةٍ لَا شَرْقِيَّةٍ وَلَا غَرْبِيَّةٍ يَكَادُ زَيْتُهَا يُضِيءُ وَلَوْ لَمْ تَمْسَسْهُ نَارٌ ۚ نُورٌ على نُورٍ ۗ يَهْدِي اللَّهُ لِنُورِهِ مَنْ يَشَاءُ ۚ وَيَضْرِبُ اللَّهُ الْأَمْثَالَ لِلنَّاسِ ۗ وَاللَّهُ بِكُلِّ شَيْءٍ عَلِيمٌ

صدق الله العلي العظيم

الآية ٣٥ من سورة النور

Dedication

To my parents.

Who were the reason of what I become today,

thanks for your great support and continuous care.

To my brothers, sisters,wife and my children

And to everyone who supported me to reach this success.
I am very grateful for your love and care.

Haider Saleh

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Finally, I would like to express my deepest appreciation to my family and friends for supporting and encouraging me throughout the years of my study and writing this thesis. This work would not have been possible without them.

The Supervisor Certificate

I certify that this thesis entitled " Use Some Mathematical Models In Reliability Of Systems " was prepared by the student " Haider Saleh Howeidi Jawad " under my supervision at the University of Babylon, College of Education for Pure Sciences as a partial fulfillment of the requirement the Degree of Master in Education /Mathematics.

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Table 1: List of abbreviations and symbols

Abb. and symbol	Description
G	A graph defined by an ordered pair (V,E)
V	vertex set
E	edge set
$P_r(x)$	Probability of element x
$E[x]$	Expected value of element x
R_s	Reliability of the system
R_i	Reliability of the component i
$MTTF$	Mean time to failure
EMS	Electromagnetic System
CM	Connection matrix of the system
a_{ij}^k	New element k of new matrix k
IM	Incidence matrix of all minimal paths
R_U	Upper bound on reliability
R_L	Lower bound on reliability
R_{sub}	Reliability of subsystem
m_i	The number of modules in subsystem i.
w_i	The importance of subsystem i.
t_i	The operating time of subsystem i.
m	The total number of modules in the system= $\sum_{i=1}^n = m_i$.
λ_S	System failure rate.
C_k	Complexity of subsystem (k)
$Rs(0_i, R_i)$	Reliability of the system when R$_i = 0$.
$Rs(1_i, R_i)$	Reliability of the system when R$_i = 1$.
NA	Unknown quantity.
$R_{i,min}$	Minimum reliability of component i
$R_{i,max}$	Maximum reliability of component i
$C_i(R_i)$	Cost of component i
$C(R_1, ..., R_n)$	Total system cost
R_j	Reliability of component j
EA	Evolutionary algorithm
Rs°	Objective function for system reliability.
R_i°	unit target reliability.
$R(M_{P_i^\circ})$	Target reliability for the path.
$R(M_{C_i^\circ})$	Target reliability for the cut.
$R_{Us}^\star$	Unit redundancy.
$R_{Es}^\star$	Element redundancy.
$R_i^\star$	Redundancy of unit i.

Abstract

In this thesis, we will study the reliability of some known systems such as mixed and complex systems, which often have known minimal paths and cut-sets. In addition we study some techniques to evaluate the reliability importance for each unit in the system, we will address the following measurement in reliability importance (Birnbaum's measure, the improvement potential(1) measure, the improvement potential(2) measure, risk achievement worth measure, risk reduction worth measure, Fussell-Vesely's measure and two innovative methods have been introduced to compute the reliability importance of components in reliability system with independent identical units). We will address the defects of these measures and its problems in the systems, and then develop these measures to work with all systems. We will also apply this development to all systems and compare the results of the development with the Birnbaum's measure. A comparison among these techniques has been done in order to know which methods are effective in calculating the importance of each unit of the system. As we will show reliability redundancy (element redundancy and unit redundancy). Furthermore redundancy technique have been highlighted. Two types of redundancy technique has been studied, these are considered effective ways to increase the reliability of complex systems. In order to know the need for each unit of the system, the permissible increase in reliability, the subject of reliability allocation has been studied, and then we tried to understand the relationship among the reliability importance, redundancy and allocation to know the mechanism by which the reliability of the systems is increased or improved. We will discover the relationship between reliability importance and cost. In addition, five Theorems were presented and proved to study the problems of increasing and decreasing the reliability of systems. We will use properties of Howeidi's Theorem to find new techniques to address reliability allocation, reliability redundancy and reliability importance. The issues of domination, factoring algorithm and isomorphism in graph and in reliability network was discussed. We will explore that there is a difference between domination in reliability network and domination in graphs, and mathematically mimicking the topic of isomorphism in graph theory to the reliability of networks.

Introduction

The poet Samuel Taylor Coleridge is first credited with coining the term "reliability" in 1816. Prior to second war time [8], the phrase was largely associated with repeatability. If the same results could be produced repeatedly, a test was called "reliable" in any field of science. Dr. Walter A. Shewhart of Bell Labs pushed product enhancement using statistical process control in the 1920s [52]. Waloddi Weibull was working on statistical models for fatigue at the time [45]. During World War II, reliability theory took shape [49]. The theory of complex systems is essential in a variety of fields, including physics,engineering and computer science, as well as nuclear, and population research.These disciplines' technical advancements which caused finding a new way to calculate the efficiency of mechanisms and equipment [32]. Many of these machines and equipment are expensive, and their reliability is becoming increasingly important. There is an increasing demand for very dependable electronics, particularly in places where repair prices are expensive (such as space and subsea) [2].

The ability of a system to fulfill the task for which it is accountable at a certain moment is described by reliability theory [51]. It is one of the engineering cornerstones. This helps to increase the performance of systems and lessen the likelihood of failure, such as aircraft, linear accelerators, and other products. The rise of reliability has taken on recent years, a new dimension has emerged, owing to the consequences of failures in today's complex systems [52, 53], which can result in daily annoyance to operational efficiency and costly repair, even to the point of putting life on our planet at jeopardy, where compromising on quality and reliability could be disastrous. Complex networks have grown more important in a wide range of fields of study, including biology, engineering, and the social sciences.[57, 79]

In network science, graph theory theorems are utilized to model systems of interest. Network research is primarily concerned with the deployment of dynamic processes across networks. For global epidemics, reported operations include energy, information, or commodity exchange of chemical atoms. To investigate these diffusive processes on networks and the various research difficulties that have arisen around this topic, we employ network reliability [77, 75].

In 1950 investigation of the reliability of relay circuits, Shannon and Moore [6] introduced network reliability for the first time. They defined network reliability as the likelihood that if the individual contacts that make up the circuit are all closed with a specific probability, the circuit will remain closed [8]. Since their work was first published more than half a century ago, a great deal of research has been done on the subject [37,70]. Per formability, survivability, and performance are all phrases that are used to describe the same thing in different disciplines. As a result, network reliability is a probabilistic metric that determines whether a network can continue to function even if one or more of its components fails at random [53,68]. Because the definition of functionality varies depending on the situation at hand, network reliability offers a lot of potential as a unifying framework for studying a wide range of issues that arise in complex network contexts [12]. Analysis and design of network reliability, we may roughly distinguish two primary areas of research [17]. The goal of analysis is to determine how reliable a network is, The purpose of design, on the other hand, is to teach design engineers how to increase their ability to design increased networks. In an ideal world, one would wish to create a network. In recent years, there has been a significant growth in interest in network reliability, particularly telecommunications network reliability [21, 24]. Graph theory has grabbed the curiosity of scientists and engineers in recent decades. The demonstrated ability of graph theory to address issues from a wide range of domains is the fundamental reason for its increased popularity [55,66]. Because of their simple diagrammatic structure, systems originating in physical science, engineering, social sciences, and economic challenges, such as reliability engineering, are easy to understand. A diverse range of disciplines [47, 75].

In the present study, the researcher has introduced some accurate ways to calculate reliability of complex systems. In this thesis we will introduce a new method to compute the importance of components for different reliability models which has independent identical units relative to minimal cut and minimal path sets. We will compare the results of this method with the method known as Birnbaum's measure, which is used in calculating the importance of units for different systems, as a result, it order to prove the correctness of the new proposed method's results. In this thesis, we will study two main topics that are closely related to the process of increasing the reliability of complex systems, and these two topics are importance and redundancy, and studying the relationship between them will help in understanding the mechanism of increase that can be obtained by a complex system. And relationship among the reliability importance and redundancy, allocation of improving the reliability of systems. And relationship between reliability importance and cost. Created and proved five theories related to the study of the behaviour of different systems through an increase or decrease in their reliability. Used properties of Howeidi's theory to find new techniques to address the reliability allocation, reliability redundancy and reliability importance. And explore that there is a difference between domination in reliability network and domination in graphs, and mathematically mimicking topic of isomorphism in graph to the reliability of networks.

Subject of this thesis

The goal is to find mathematical models and new methods that contribute to solving the problem of increasing or decreasing the reliability of simple and complex systems.

Contributions

1. Developed some methods of measuring reliability importance for series, parallel and complex systems. Through this development, we were able to address some of the disadvantages in the techniques of calculating the reliability importance of units.

2. We compared the developed techniques with the a known Birnbaum's measure and found that our development of these techniques achieved results that are completely identical to Birnbaum's measure.

3. Also we created new methods to compute reliability importance like:

   - The improvement potential (2) measure.
   - A new method to compute the importance of components in reliability system with independent identical units.
   - We finding new mathematical relationships through what we studied from the methods of calculating the importance and applying that to the series, parallel and complex systems.

4. We explored the relationship between the reliability importance and redundancy in order to increases system reliability.

5. We discovered the relationship between reliability importance and cost.

6. We created and proved five theorems related to the study of the behavior of different systems through an increase or decrease in their reliability, (Theorem (4.1), Theorem (4.2), Theorem (4.3), Theorem (4.4)), Theorem (4.5)).

7. We used properties of Howeidi's Theorem to find a new techniques to address reliability allocation, reliability redundancy and reliability importance.

8. We explored that there is a difference between domination in reliability network and domination in graphs.

9. We mathematically mimicked topic of isomorphism in graphs to the reliability of networks.

Thesis Outlines

This Thesis consists of six chapters.

1. **Chapter one** contains some definitions and concepts such as fundamental graph theory.

2. **Chapter two** includes two techniques for calculating the reliability networks.

3. **chapter three** contains some methods of measuring importance and finding its defects with the systems and then developing these measures to work with all systems and comparing the results of development with the importance for Birnbaum's measure and adding new methods and relationships in measuring the importance of the reliability of the units.

4. **Chapter four** discusses relationship between the reliability importance and redundancy, relationship among the reliability importance, redundancy and allocation of improving the reliability of systems. Discovered the relationship between reliability importance and cost. Create and prove five theories related to the study of the behaviour of different systems through an increase or decrease in their reliability. Using properties of Howeidi's theory to find a new techniques to address the reliability allocation, reliability redundancy and reliability importance.

5. **Chapter five** consists a difference between domination in reliability networks and domination in graphs. And mathematically mimicking topic of isomorphism in graphs to the reliability of networks.

6. **Chapter six** consists of conclusions and future works.

Related Works

In 1970 [48] because of their intuitive diagrammatic depiction. The literature has provided a variety of methodologies, strategies, and approaches to consider. The use of graph theory in network reliability evaluation has become inexorably connected. Big and complicated network in its entirety while undertaking reliability analysis. The sensible approach in the design and operation of today's massive, sophisticated, and expensive networks, reliability has emerged as a critical component [50]. It is no longer economically feasible to over design network facilities and to introduce excessive redundancy. network reliability due to their intuitive diagrammatic description. Certain characteristics (cost, reliability, and weight) can be classified as a problem of combinatorial optimization, where either dependability or costs can be reduced. In either case, there are constraints on price, weight, or network reliability targets due to their simple diagrammatic portrayal. The problem with reliability redundancy distribution is that it is difficult to choose the best combination of parts and redundancy levels in order to optimize network reliability while minimizing network expenses due to many constraints. There are three types of reliability optimization problems: reliability allocation (network component reliability), redundancy allocation (redundant component number), and reliability redundancy allocation (network component reliability and redundant component number) [54]. K. K. Aggarwal and Shashwati Guha published Aggarwal, K. K., and Shashwati Guha in 1993. The researchers provide a broad strategy for increasing reliability, with the caveat that it can only be applied in non-series-parallel systems with distinct effort functions assumed for separate subsystems [57]. In 1997, Ellis Horowitz, Sartaj Sahni, and Sanguthevar Rajasekaran [73] demonstrated how to solve a problem with a multiplicative optimization function using dynamic programming. The task is to construct a system that consists of numerous devices connected in series, with device D's reliability (that is, n being the likelihood that device I will work properly) being determined by the reliability of device [1, 8]. Hoang Pham, [10] in 2006, The researcher goes into great depth about how to improve the reliability of systems that are subject to two different types of failure. The system constituent countries are considered to be statistically independent and identical, and there are no limitations on the number of components that can be used [2]. Improvement of series, parallels, parallel–series, and series–parallel reliability [6, 14]. In 2007 Liu, Chuanquan, and Yan Zhang conducted research on the electrical distribution network's dependability while taking distribution generation into account [61]. In 2009, Boesch and Charles published a number of findings about the analysis and synthesis of reliable or non-vulnerable networks, as well as research into some reliability analysis applications and associated ideas in the creation of the most reliable network [63]. Hassan created a set of equivalent methodologies in 2015 [65] for evaluating the reliability of electric aircraft systems (EAS). In 2016, Mutar and Hassan published the engineering characteristics of the oxygen supply system for spacecraft as well as the geometry of the electrical system's reliability models [40,53]. Kumar looked at the feasibility of using multifunctional particle cluster optimization involving congestion distance (MOPSO-CD) to address a challenging reliability improvement problem with the diametrically opposed objectives of lowering system cost and raising system dependability in 2017. [74]. In 2018, Feng, Zhao, and Li investigated the reliability and failure probability of electric vehicles fitted with wind turbines, identified the primary reasons for failures, and increased reliability using the Particle Swarming (PSO) method [70]. In order to find the best system design under a number of limitations and maximize system reliability, Ouyang and Zhiyuan submitted a work in 2019 examining the topic of intensive reliability and redundancy assignment [2]. In 2020 Abdullah, Ghazi, and Hassan published research on how to handle the problem of allocation and improve system reliability by utilizing the Genetic algorithm to determine the reliability of each component of a complex system [18]. In 2021, the two researchers discussed reliability of the same system and calculated reliability allocation and optimization for a complex network using GA algorithm, PSO algorithm, Ant Colony Algorithm (ACA), and Bee's Colony Optimization, with a comparison between these algorithms to choose the best algorithm that gives the highest reliability and lowest cost [13,27].

Chapter 1

Mathematical concepts and basic definitions

1.1 Introduction

This chapter introduces key definitions, basic concepts, and background information for this thesis. This chapter is divided into two portions. The first provides some of the fundamental definitions and graph theory that we will require in this thesis. The second section covers the fundamental definition of reliability, as well as specific types, characteristics, and reliability systems.

1.2 Graphs for network reliability modeling

Network-like structures can be found in technical systems such communication networks, electromagnetic systems, power grids, traffic systems, and command and control systems [6,11]. Consider a common computer network scenario. Computer network topology is defined by two major components [3] computers (servers, routes, and terminals) and inter computer transmission lines (copper cable, fiber optic lines, and wireless channels). A list of vertices and edges, as well as an incidence function that identifies the end vertices of each edge, is one way to express a network topology or graph. We describe edges as lines that connect those two circles that correspond to their end vertices by small circles and edges [12,18], as can be seen in Figure (1.1). Given the foregoing, it is necessary to understand the fundamental ideas of the graph on which this thesis is based.

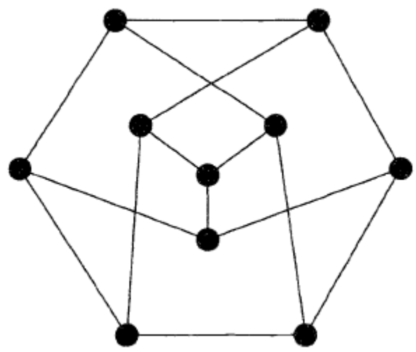


Figure 1.1: Graph.

Definition 1.1 [17] A *graph* is a pair (V, E) , with V being a set of objects known as vertices and E being a two-element subset of V known as edges.

Definition 1.2 [68] *Vertices* are commonly represented by single letters such as u or v. Edges will be denoted by pairs of letters, such as uv for the *edge* between vertices u and v. Graphs will be denoted by capital letters, such as G or H.

Definition 1.3 [71] *Adjacent* vertices are those that are joined by an edge.

Definition 1.4 [73] The number of edges that a vertex is an endpoint of determines its *degree* . The degree of vertex v is denoted by the notation $deg(v)$.

Definition 1.5 [75] The vertex set, or collection of vertices, of a graph G is denoted by $V(G)$. Similarly, the graph's edge set is indicated by $E(G)$.

Example 1.1 In the Figure (1.2), the bottommost edge is between vertices d and e . We denote it as edge de. That edge is incident on d and e. Vertex d is adjacent to vertex e, as well as to vertices b and c. The neighbors of d are b,c and e. And d has degree 3.

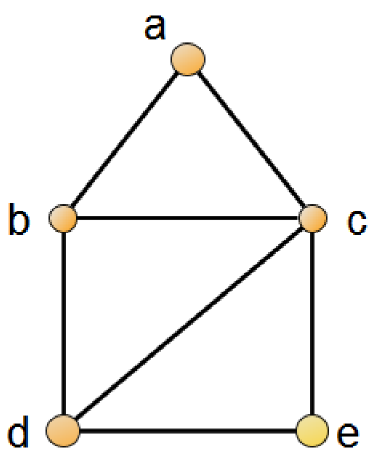


Figure 1.2: Graph G.

Definition 1.6 [77] A *loop* It is possible to have an edge from a vertex to itself. As an example, see Figure (1.3)

Definition 1.7 [61] If two or more edges in a graph G have the same endpoints, they are called *parallel or multiple* (end vertices). As an example, see Figure (1.3), there is a loop at vertex a and multiple edges (three of them) between c and d .

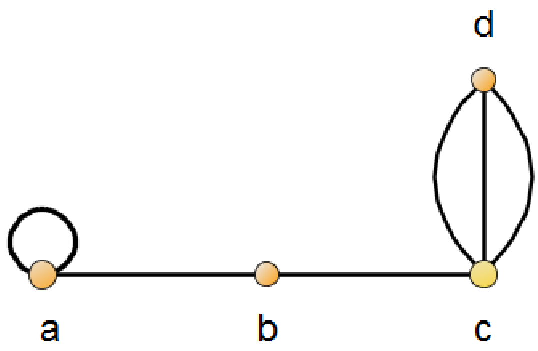


Figure 1.3: Graph G with loop and multiple edges.

Definition 1.8 [47] A graph which has no loops and multiple edges is called a *simple graph*. As an example, see Figure (1.4).

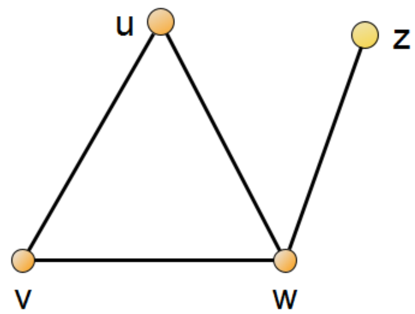


Figure 1.4: Simple graph.

Definition 1.9 [19] A graph which may have loops or multiple edges is called a *multigraph*.

Definition 1.10 [10] A simple graph in which every vertex is adjacent to every other vertex is called a *complete graph*. As an example, see Fig.(1.5)

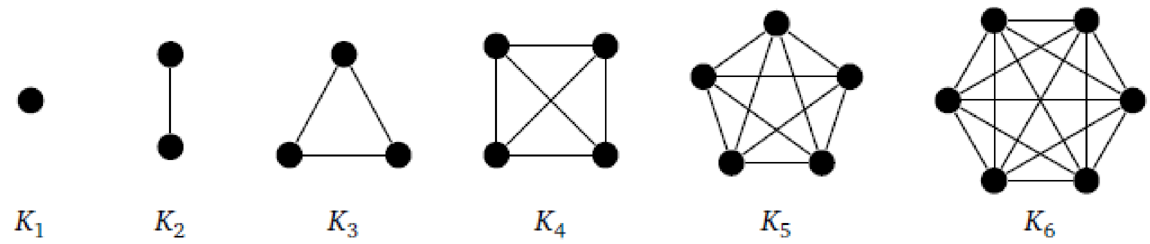


Figure 1.5: Complete graphs.

Definition 1.11 [30] If the vertices of H are a subset of the vertices of G and the edges of H are a subset of the edges of G, then H is a*subgraph* of G.

Definition 1.12 [37] A graph G is connected if for every pair u,v of vertices there exist a $u - v$ path, otherwise G is disconnected.

Definition 1.13 [41] A graph's *cut vertex* is a vertex that, if removed, divides the graph into more components than it had before.

Definition 1.14 [54] A*cut edge* is defined. It's an edge that, if removed, divides the graph into more components than it had before.

Example 1.2 In the graph (Figure 1.6) the vertices a, b and c are cut vertices because removing any of them would cause the graph to be disconnected. Similarly, because deleting edge bc causes the graph to be disconnected, bc is a cut edge. In the graph, there are no other cut vertices or cut edges.

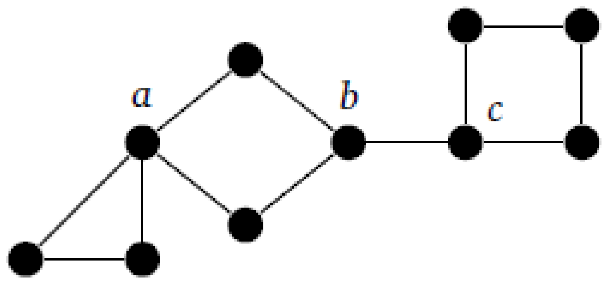


Figure 1.6: Cut vertices and cut edges.

Definition 1.15 [77] The length of the shortest path between two vertices in the graph, denoted $d(u, v)$, represents the *distance* between them. It is essentially how many "steps" the vertices are away from each other.

Example 1.3 In (Figure 1.7) b, c and d are at a distance of 1 from a, vertex **e** is at distance of 2 from a, and vertex f is at a distance of 3 from a.

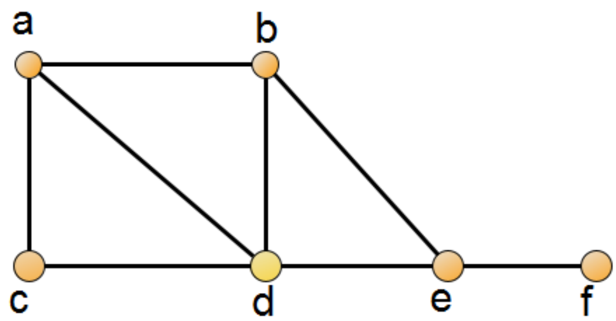


Figure 1.7: Distance between two vertices.

Definition 1.16 [79] Graphs G and H are *isomorphic* provided there exists a one-to-one and onto function $f : V(G) \to V(H)$ such that v is adjacent to w in G if and only if $f(v)$ is adjacent to $f(w)$ in H.

Example 1.4 In Figure (1.8), to accomplish this, we align the vertices as follows:
$a \leftrightarrow v, b \leftrightarrow w, c \leftrightarrow x, d \leftrightarrow z, e \leftrightarrow y.$
Written in function notation, this matching is:
$f(a) = v$, $f(b) = w$, $f(c) = x$, $f(d) = z$, and $f(e) = y$. It is one-to-one and onto.
We also need to check that all the edges work out. For instance, ab is an edge in the left graph. Since a and b are matched with v and w, we need vw to be an edge in the right graph, and it is. Similarly, there is no edge from a to e in the left graph. Since a and e are matched to v and y, we cannot have an edge in the right graph from v to y, and there is none. Though it's a little tedious, it's possible to check all the edges and nonedges in this way.

Figure 1.8: Isomorphic graphs.

Definition 1.17 [4] A *directed graph* is one in which all of the edges of the graph G are directed edges. See Fig.(1.9) for an example.

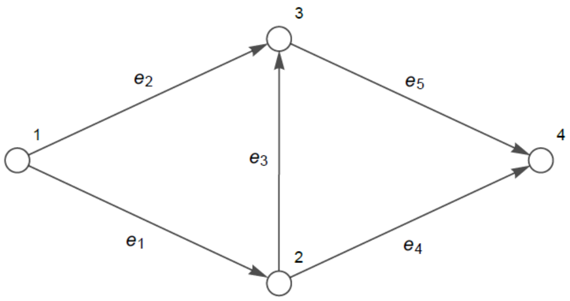


Figure 1.9: Directed graph.

Definition 1.18 [7] The graph G is called an *undirected graph* if all of its edges are undirected, as seen in Fig.(1.10)

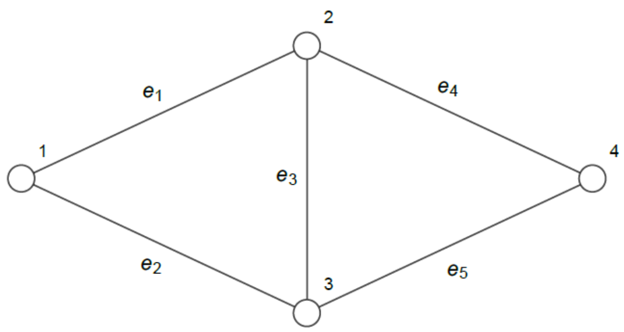


Figure 1.10: Undirected graph.

Definition 1.19 [12] A *mixed graph* is a graph G with both directed and undirected edges, as seen in Fig. (1.11).

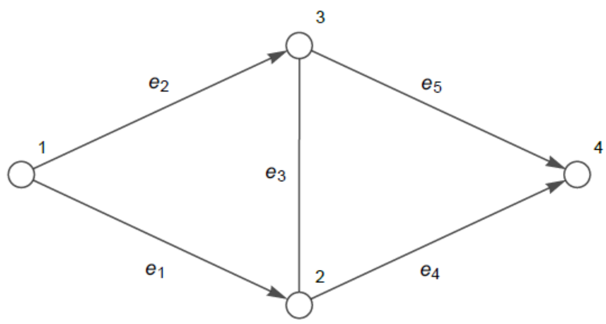


Figure 1.11: Mixed graph.

Definition 1.20 [15] In a graph, *two vertices are said to be adjacent*, if there is an edge between the two vertices. Here, the adjacency of vertices is maintained by the single edge that is connecting those two vertices.

Definition 1.21 [19] In a graph, *two edges are said to be adjacent*, if there is a common vertex between the two edges. Here, the adjacency of edges is maintained by the single vertex that is connecting two edges. Example of this is given in Fig.(1.12).

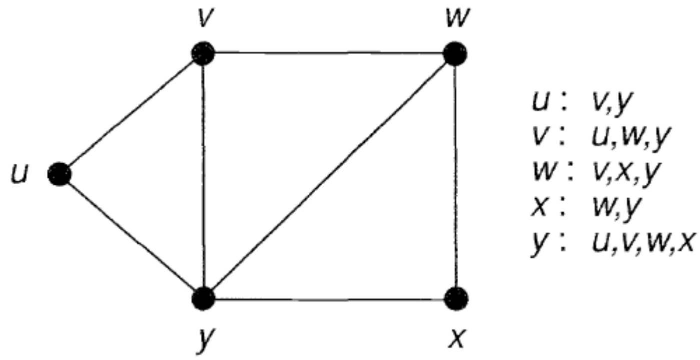
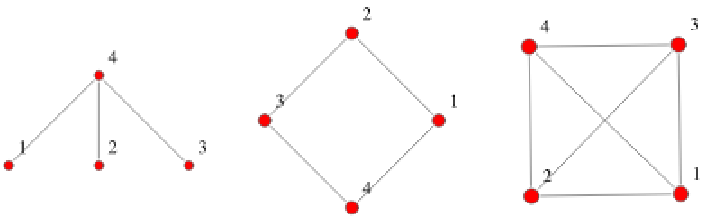


Figure 1.12: Vertices adjacent.

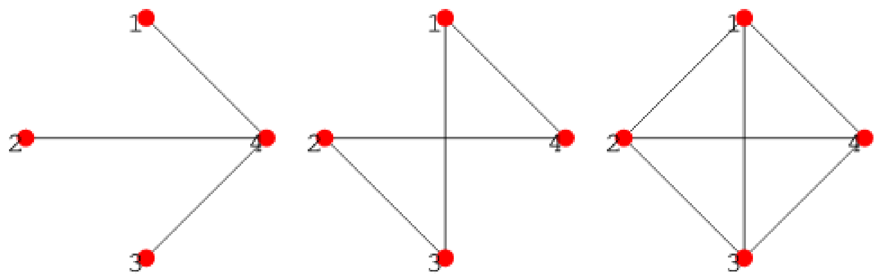
Definition 1.22 [23] A*adjacency matrix*, sometimes also called the connection matrix, of a simple graph is a matrix with rows and columns by graph vertices, with a 1 or 0 in position (u,v) according to whether u and v are adjacent or not.
Example of this is given in Fig.(1.13).



$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Figure 1.13: Adjacency matrix for n=4.

Definition 1.23 [27] A *incidence matrix* of a graph gives the (0,1) matrix which has a row for each vertex and column for each edge, and $(v, e) = 1$ iff vertex v is incident upon edge e.
Example of this is given in Fig.(1.14)



$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

Figure 1.14: Incidence matrix.

1.3 Basics of reliability polynomial

This section will provide a rudimentary grasp of reliability polynomials. It includes polynomial reliability, structural function, reliability block diagrams, and reliability system.

Definition 1.24 [30] *Reliability function* probability that the system will survive for given amount of time is indicated *R(t)*, commonly known as the survival function. This can be represented in terms of the random variable T, which represents the time it takes for a system to fail:

$$R(t) = P_r\{T > t\} \tag{1.1}$$

The probability that the system will work without failure for an extended period of time is rewarded.

Definition 1.25 [33] A reliability *block diagram* of a system is a graph whose edges are the system components, while there are a pair of nodes called terminal nodes in the backup power supply diagram. It shows the functional relationship between the components, and indicates there is a path between the terminal nodes which contains only edges with functional components, the entire system is functional. Otherwise it is not functional, see Fig.(1.15).

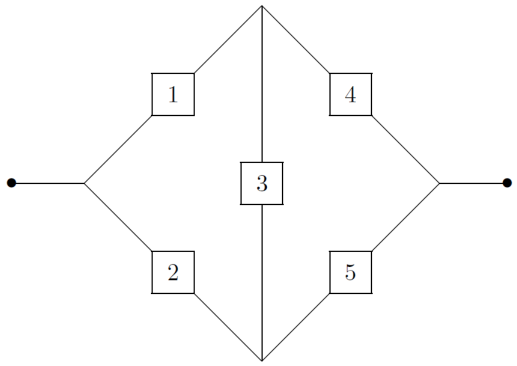


Figure 1.15: Complex or bridge network.

Definition 1.26 [44] A *complex network* is a network made up of interconnected or interwoven pieces (components) that makes it difficult to assess its reliability or solve a specific problem due to the limitations imposed by present techniques, algorithms, and software (such as programming languages and operating networks), see Fig.(1.16).

Definition 1.27 [54] The networks are considered operational if there is a path between a pair of nodes usually labeled: In put (source) and out put (sink), then the probability of a message successfully reaching the sink node from the source node is termed Two-terminal reliability, as an example, see Fig.(1.16).

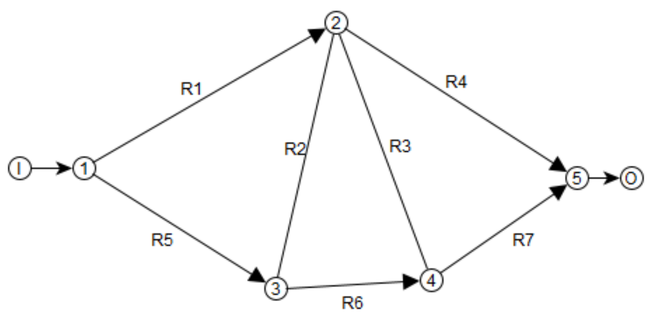


Figure 1.16: Complex network and Two-terminal reliability.

Definition 1.28 [10] A *path* is a set of components that, when working together, ensure that the system is working.

Definition 1.29 [10] A path set is called *minimal path set* if it cannot be reduced without losing its status as a path set.

Definition 1.30 [4] A *cut set* is a set of components that by failing causes the system to fail

Definition 1.31 [10] A cut set is *minimal cut set* if it cannot be reduced without losing its status as a cut set

Example 1.5 In the Figure (1.15), $\{1, 4, 5\}$ is path set, but $\{1, 4\}$ is minimal path set. And $\{1, 2, 4\}$ cut set, but $\{1, 2\}$ is minimal cut set

1.4 Structure functions

Throughout most of this thesis, we assume that the system S consists of n components $c_1, c_2, \ldots, c_n$ can only be in one of the two states, working or failing. Then, we can assign to each components or system S are Boolean (binary) indicator variables (x=0,1) for these states, respectively [59]. Let x_i indicates the state of component i for $1 \leq i \leq n$, as follows

$$x_i = \begin{cases} 1 & \text{if} \quad \text{component } c_i \text{ is working,} \\ 0 & \text{if} \quad \text{component } c_i \text{ has failed .} \end{cases} \tag{1.2}$$

And, the structure function of system as follows

$$x_s = \begin{cases} 1 & \text{if} \quad \text{system S is working,} \\ 0 & \text{if} \quad \text{system S has failed .} \end{cases} \tag{1.3}$$

We assume that the component states are the sole determinants of the system's state. As a result, the so-called structural function determines the relationship between the state of the system and its elements. $\phi(x)$ as $x_s = \phi(x)$ where $x = (x_1, x_2, \ldots, x_n)$ is called the state vector of the system [64]. Then $\phi(x)$ is a well-known fact of reliability theory that ϕ can be expressed as a multiple function of the component states (for more details, see [62,57]. Despite the fact that the variables that define the structural function of a system are Bernoulli random variables, we assume that the structural functions can be modified to produce the desired results because they only hold 1 or 0 values. $x_i^n = x_i$

1.5 Reliability system

The system can range in complexity from simple to complicated. The system can be broken down into smaller subsystems and the reliability of each subsystem estimated to determine the overall system reliability. Series, parallel, series-parallel, parallel-series, and complex reliability systems are among the several configurations (systems) accessible. Some of these systems will be mentioned because of their prominence in the research among reliability analysts [14, 33].

1.5.1 Series system

Any component failure in a series system causes the entire system to fail. In addition, a successful system is dependent on the success of all components [7,43]. As an example, see Figure (1.17). The reliability of the system is then given by:

$$\phi(x) = min\{x_1, ..., x_n\} = \prod_{i=1}^{n} x_i \tag{1.4}$$

In other word if $R_1, R_2, \ldots, R_n$ are the reliabilities of the individual components, then the reliability of the system is giving by :

$$R_{System} = R_1.R_2.....R_n = \prod_{i=1}^{n} R_i \tag{1.5}$$

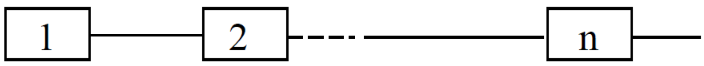


Figure 1.17: Series system.

1.5.2 Parallel system

For a parallel system to succeed, at least one of the components must succeed. For a system with n statistically independent parallel components, the probability of failure, or unreliability, is the likelihood that component 1 fails, component 2 fails, and all other components in the system fail. To put it another way, the system succeeds if component 1 or component 2 or any of the n components succeeds [6,38]. In a parallel system, the part with the highest quality has the biggest impact on the system's efficiency, because the most secure component is the one most likely to fail last. As the number of parts in a parallel arrangement grows, the system's reliability improves. As an example, see Figure (1.18). The structural function of a system is determined by:

$$\phi(x) = max\{x_1, ..., x_n\} = 1 - \prod_{i=1}^{n}(1 - x_i) \tag{1.6}$$

In other word if $R_1, R_2, .., R_n$ are the reliabilities of the individual components, then the reliability of the system is giving by :

$$R_{System} = 1 - \prod_{i=1}^{n}(1 - R_i) \tag{1.7}$$

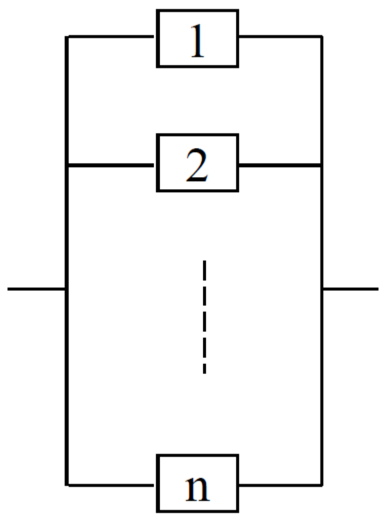


Figure 1.18: Parallel system.

1.5.3 Series-Parallel system

The reliability of this system that consist m series redundancy connected components,is

$$R_{System} = R_1.R_2.....R_n = \prod_{i=1}^{n} R_i$$

where R_i is the reliability of an individual component [16, 17]. As an example, see Figure (1.19). If we place in sets in parallel, where each has m component in series. Thus :

$$R_s = \prod_{i=1}^{m}(1 - \prod_{i=1}^{n}(1 - R_i)) \tag{1.8}$$

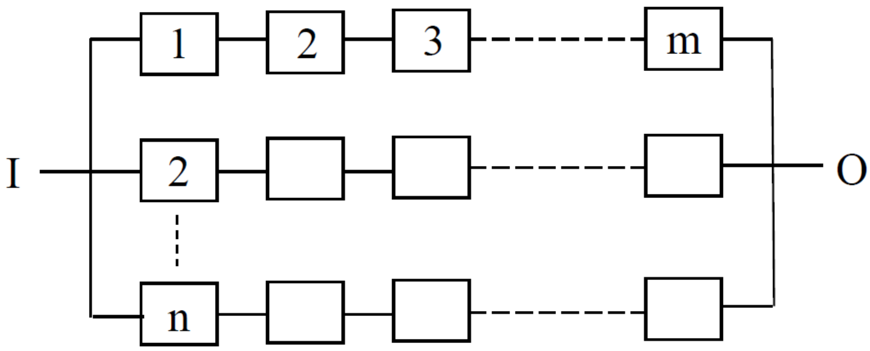


Figure 1.19: Series - parallel system.

1.5.4 Parallel-Series system

The reliability of the parallel linked n element is determined by :

$$R_s = 1 - (1 - R_i)^n$$

Where R_i is a single component's reliability [30, 51]. If m such sets are connected in series, where each set consists of n parallel components, as an example, see Figure (1.20), then the system's reliability is determined :

$$R_s = 1 - \prod_{i=1}^{m}(1 - \prod_{i=1}^{n} R_i). \tag{1.9}$$

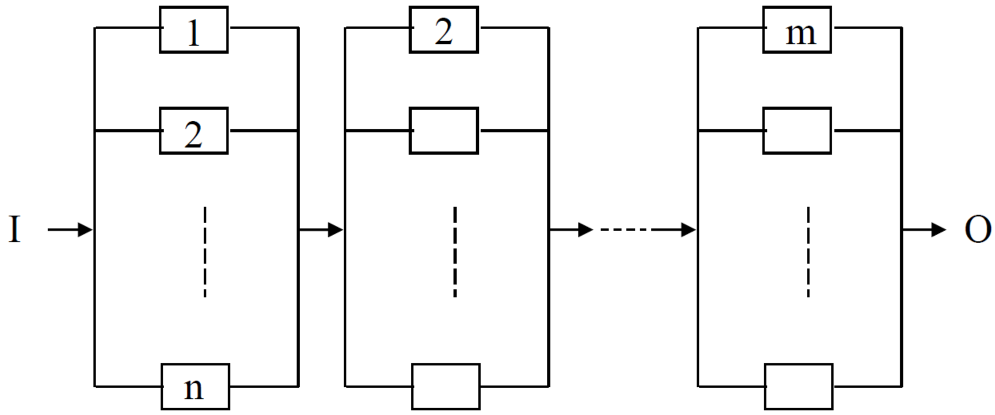


Figure 1.20: Parallel – series system.

1.5.5 Complex reliability systems

A complicated structure system is one that cannot be classified directly into the previous fundamental structure instances [9,52]. For the study of structures with intricate structures, the concepts of minimum pathways and cuts (in the case of monotonous structures) must be applied. The system in Fig.(1.15), below is a complicated structure scheme. This system is difficult to divide into traditional modules with traditional structures. If this system's input / output is situated at component 3 extremities, it will then involve a system with elementary structure. There are various approaches for determining a complicated system's reliability:

1. Path tracing method (PTM).
2. Minimal cut method (MCM).

Definition 1.32 [31] **Reliability allocation** is fundamentally a process that must be repeated. It's done early in the design process to help with concept design when the amount of information accessible is restricted. The overall reliability target should be redistributed as the design process progresses and more details of the design and materials are identified, in order to reduce the cost and risk of meeting the reliability goal. Due to technological restrictions, the allocation procedure may be triggered by the failure of one or more components to achieve the allotted reliability. Whenever a substantial design change occurs, the process is repeated.

Definition 1.33 [27] **Reliability redundancy** is commonly used in system design to enhance systems reliability, especially when it is difficult to increase component reliability itself.

Definition 1.34 [11] **Reliability importance** defines component importance as the probability of the component being critical to system failure, where being critical means that the component failure coincides with the system failure.

Chapter 2

Some methods to evaluate the reliability of networks

2.1 Introduction

In this chapter, we discuss how to extract the minimal path sets and the minimal cut sets, and it deals with the calculation of the reliability of the complex system by minimal path sets and minimal cut sets.

2.2 Generation of minimal paths and minimal cuts

To determine the reliability of complex systems, earlier methods relied on the structure of the subsystem (series, parallel, series-parallel, and parallel-series). In the path tracing approach and the minimal cut method, respectively, minimal paths and minimal cuts play an essential role. The minimal paths to be constructed for a two-terminal system's reliability are between the source node and the sink node, however the minimal cuts break all of the system's minimal paths. Then we'll go over how to calculate these concepts using mathematics.

2.2.1 Generation of minimal paths

We will use this strategy to develop minimal paths by constructing a connection matrix [2,47]. The edges of a two-terminal network diagram are prone to failure, but the nodes are flawless. We will combine the identity matrix with the $n \times n$ adjacency matrix of a simple graph as follows:

$$Cm = \begin{bmatrix} nodes & 1 & 2 & .... & n \\ \hline 1 & 0 & a_{12} & ... & a_{1n} \\ 2 & a_{21} & 0 & ... & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n & a_{n1} & a_{n2} & ... & 0 \end{bmatrix} + \begin{bmatrix} nodes & 1 & 2 & ... & n \\ \hline 1 & 1 & 0 & ... & 0 \\ 2 & 0 & 1 & ... & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n & 0 & 0 & ... & 1 \end{bmatrix}$$

Then, as seen below, we get the connection matrix.

$$Cm = \left[\begin{array}{c|cccc} nodes & 1 & 2 & ...... & n \\ \hline 1 & 1 & a_{12} & ... & a_{1n} \\ 2 & a_{21} & 1 & ... & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n & a_{n1} & a_{n2} & ... & 1 \end{array}\right] \tag{2.1}$$

Where $\{1, 2, \ldots, n\}$ represents the set of nodes and $a_{ij} = (i, j)$ represent the edge between node i and node j, if there exist a connection between node i and node j then $a_{ij} = x_{ij}$. Otherwise $a_{ij} = 0$. So Cm has a properties as the following characteristics

- Due to the fact that nodes are always perfect, the Cm's primary diagonal elements are 1.
- The connection matrix in undirected networks, such as a two-terminal system, is symmetric, i.e. if there is an underact link between node i and node j, then a_{ij} and a_{ji} represent system components (edges).
- If no link (directed edge) exists between node i and node j, $a_{ij} = a_{ji} = 0$.

As a result, nodes that are neither the source nor the sink are removed from Cm one by one until only the source and sink nodes are left in the matrix [2,47]. The elements of the connection matrix with the remaining nodes are updated when a node is removed using the following equation:

$$a^1_{i\,j} = a_{ij} + a_{il}a_{lj} \tag{2.2}$$

If node l is removed, where
$i \neq j, i \neq l, j \neq l, 1 < n, 1 < j \leq n$
for $i = 1, 2, \ldots, n$. Otherwise $a^1_{i\,j} = 1$, if and only if i = j.
Labeling the source node as the first node and the sink node as the last node is a recommended practice when using this strategy. Each intermediary node is deleted one by one until a 2×2 matrix with a_{ji} representing the sum of all minimum pathways remains. Hence removal of node i is equivalent to the removal of row i and column i of the original connection matrix where
$2 \leq i \leq n - 1$, for a network with n perfect nodes

Example 2.1 Let us consider the bridge system in Fig (2.1). The edges have been numbered from x_1 through x_8 with the nodes labeled as $1, 2, 3, 4$, and 5. Based on equation (2.1) and above descriptions, the connection matrix of bridge system is as follows:

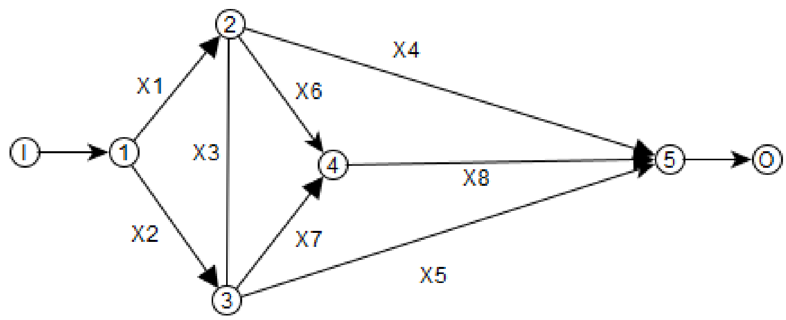


Figure 2.1: Complex systems.

$$Cm = \left[\begin{array}{c|ccccc} nodes & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & x_1 & x_2 & 0 & 0 \\ 2 & 0 & 1 & x_3 & x_6 & x_4 \\ 3 & 0 & x_3 & 1 & x_7 & x_5 \\ 4 & 0 & 0 & 0 & 1 & x_8 \\ 5 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Now, we remove node 2. The modified entries of the original matrix become the following according to equation;

$a^1_{13} = x_2 + x_1x_3$, $a^1_{14} = 0 + x_1x_6 = x_1x_6$, $a^1_{15} = 0 + x_1x_4$,

$a^1_{31} = 0$, $a^1_{34} = x_7 + x_3x_6$, $a^1_{35} = x_5 + x_3x_4$, $a^1_{41} = 0$, $a^1_{43} = 0$, $a^1_{45} = x_8$

The modified connection matrix becomes the following 4×4 matrix:

$$Cm^1 = \left[\begin{array}{c|cccc} nodes & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & x_2 + x_1x_3 & x_1x_6 & x_1x_4 \\ 2 & 0 & 1 & x_7 + x_3x_6 & x_5 + x_3x_4 \\ 3 & 0 & 0 & 1 & x_8 \\ 4 & 0 & 0 & 0 & 1 \end{array}\right]$$

Now, we remove node 2. The modified entries of the original matrix become the following according to equation:

$a_{13}^2 = x_1x_6 + (x_2 + x_1x_3)(x_7 + x_3x_6) = x_1x_6 + x_2x_7 + x_2x_3x_6 + x_1x_3x_7 + x_1x_3x_6$
$\quad = x_1x_6 + x_2x_7 + x_2x_3x_6 + x_1x_3x_7$
$a_{14}^2 = x_1x_4 + (x_2 + x_1x_3)(x_5 + x_3x_4) = x_1x_4 + x_2x_5 + x_2x_3x_4 + x_1x_3x_5 + x_1x_3x_4$
$\quad = x_1x_4 + x_2x_5 + x_2x_3x_4 + x_1x_3x_5$
$a_{34}^2 = x_8$

connection matrix becomes the following 3×3 matrix:

$$Cm^2 = \begin{bmatrix} 1 & x_1x_6 + x_2x_7 + x_2x_3x_6 + x_1x_3x_7 & x_1x_4 + x_2x_5 + x_2x_3x_4 + x_1x_3x_5 \\ 0 & 1 & x_8 \\ 0 & 0 & 1 \\ \end{bmatrix}$$

Now, we remove node 2. The modified entries of the original matrix become the following according to equation:
$a_{13}^3 = x_1x_4 + x_2x_5 + x_2x_3x_4 + x_1x_3x_5 + (x_1x_6 + x_2x_7 + x_2x_3x_6 + x_1x_3x_7)(x_8)$
$a_{13}^3 = x_1x_4 + x_2x_5 + x_2x_3x_4 + x_1x_3x_5 + x_1x_6x_8 + x_2x_7x_8 + x_2x_3x_6x_8 + x_1x_3x_7x_8$
connection matrix becomes the following 2×2 matrix:

$$Cm^3 = \begin{bmatrix} 1 & x_1x_4 + x_2x_5 + x_2x_3x_4 + x_1x_3x_5 + x_1x_6x_8 + x_2x_7x_8 + x_2x_3x_6x_8 + x_1x_3x_7x_8 \\ 0 & 1 \end{bmatrix}$$

The minimal path sets
$S = \{\{x_1, x_4\}, \{x_2, x_5\}, \{x_1, x_3, x_5\}, \{x_2, x_3, x_4\}, \{x_1, x_6, x_8\}, \{x_2, x_7, x_8\},$
$\quad \{x_1, x_3, x_7, x_8\}, \{x_2, x_3, x_6, x_8\}\}$
Are represent all minimal paths of bridge system, it shown in Figure (2.2).

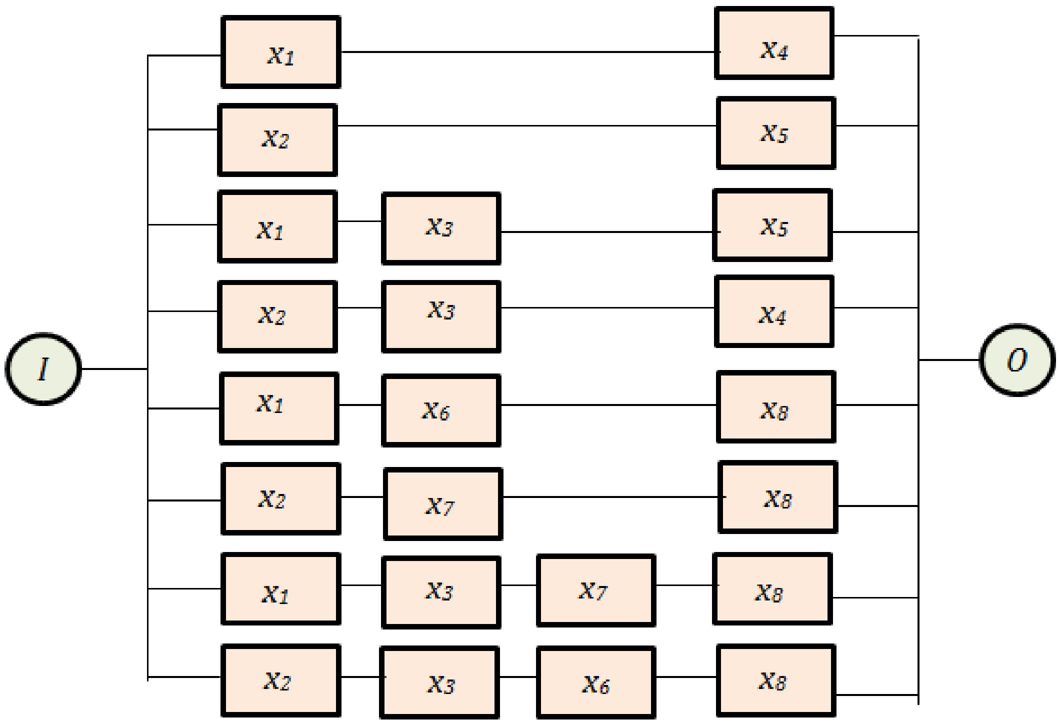


Figure 2.2: All minimal paths of complex system.

2.2.2 Generation of minimal cut sets

We will use the set of all minimal paths [20,22] to generate the incidence matrix IM of all minimal paths in this manner. Assuming there are n minimum pathways indicated by $P_1, P_2, ..., P_n$, we may derive the incidence matrix of all minimal paths IM as follows:

$$IM = \left[\begin{array}{c|cccc} nodes & x_1 & x_2 & ... & x_n \\ \hline P_1 & a_{11} & a_{12} & .... & a_{1n} \\ P_2 & a_{21} & a_{22} & .... & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_m & a_{m1} & a_{m2} & ... & a_{mn} \end{array}\right]$$

where $a_{ij} \in \{0,1\}$ with $(i = 1,2,\ldots,m, j = 1,2,\ldots,n)$. A state is an n-dimensional raw consisting of $1s$ and $0s$ such that $a_{ij} = 1$ if and only if $x_j \in P_i$, otherwise $a_{ij} = 0$. For the generation of minimal cut sets, there are three phases to follow [25,26].

- Step 1: If $\forall \mathrm{a}_{ij} \neq 0$ of any column x_j of IM, then x_j forms a firstorder cut.

- Step 2: Combine two columns of IM at a time, If $\forall i; a_{ij} + a_{ik} \neq 0$ where $k > j(k = 1, \ldots.., n)$, then $x_j x_k$ form a second order cut. To deliver the second order minimal cuts, delete any cut that contains first order cuts [4,29].

- Step 3: Repeat step (2) with three columns at a time, eliminating any cuts that contain first and second order cuts this time, and continuing until the maximum order of cut is reached.

Example 2.2 The bridge structure given by Fig.(2.1). Has the minimal path sets
$P_1 = \{x_1, x_4\}, P_2 = \{x_2, x_5\}, P_3 = \{x_1, x_3, x_5\}, P_4 = \{x_2, x_3, x_4\}$,
$P_5 = \{x_1, x_6, x_8\}, P_6 = \{x_2, x_7, x_8\}, P_7 = \{x_1, x_3, x_7, x_8\}, P_8 = \{x_2, x_3, x_6, x_8\}$

The IM of the bridge system is as follows, based on our descriptions above:

$$IM = \left[\begin{array}{c|cccccccc} paths & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ \hline P_1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ P_2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ P_3 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ P_4 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ P_5 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ P_6 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ P_7 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ P_8 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \end{array}\right]$$

Hence, no single column in IM exists in which all elements are non-zero, then there are no first order cuts. Then, applying Step (2) we found only.
$a_{i1} + a_{i2} \neq 0, \forall i$. So the second order cut is: $\{x_1, x_2\}$. Repeat with three columns at a time, we get, $\{x_4, x_5, x_8\}$. Finally, compare four columns at a time, we get,
$\{\{x_4, x_5, x_6, x_7\}, \{x_1, x_3, x_5, x_7\}, \{x_2, x_3, x_4, x_6\}, \{x_1, x_3, x_5, x_8\}, \{x_2, x_3, x_4, x_8\}\}$
Hence, The minimal cut sets

$$C = \{\{x_1, x_2\}, \{x_4, x_5, x_8\}, \{x_4, x_5, x_6, x_7\}, \{x_1, x_3, x_5, x_7\}, \{x_2, x_3, x_4, x_6\}, \{x_1, x_3, x_5, x_8\}, \{x_2, x_3, x_4, x_8\}\}$$

As shown in Figure (2.3).

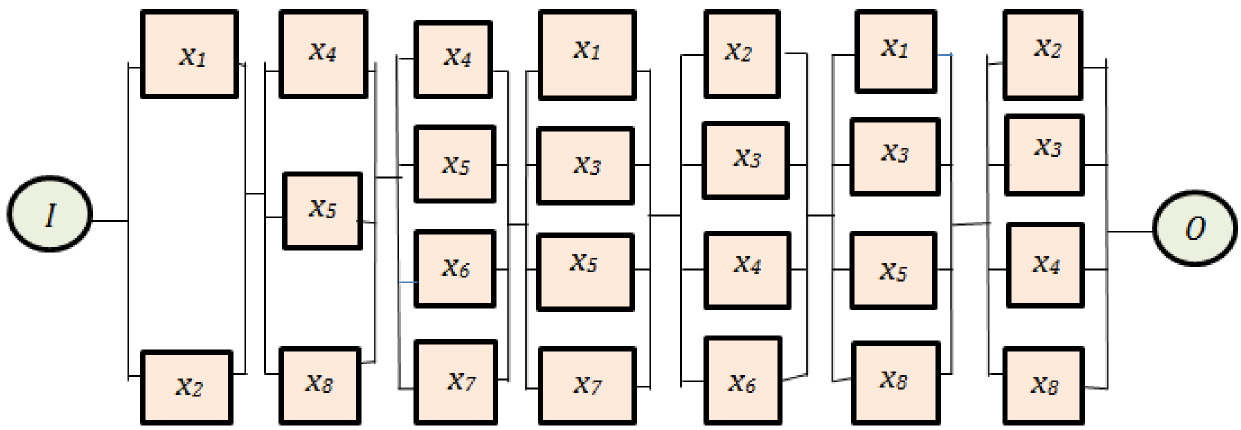


Figure 2.3: All minimal cuts of complex system.

2.3 Path tracing method (PTM)

This method considers every path from a source to a sink. Because system success necessitates at least one path from one end of the reliability block diagram to the other, the system has not failed as long as at least one path exists from beginning to finish. The processes for calculating the reliability of a complicated system using this method are as follows [17,27].

- Make a list of the system's minimum pathways (tie-set).
- System success is determined by the success of all sections in minimal paths (tie-set).
- This indicates that there are links between these components in the series.
- The system's accomplishment is generated by each set of minimal ties.
- This means that the minimal tie sets are connected in a parallel manner.
- Draw an equal system and calculate the system's dependability using series and parallel reduction.

The chance of combining these shortest paths determines the system's reliability [8,15]. The following is a general equation for the union of minimum pathways P_i:

$$\begin{aligned} p_r(p_1 + p_2 + ... + p_n) =& p_r(\sum_{i=1}^{n} p_i) \\ &= \sum_{i=1}^{n} p_r(p_i) - \sum_{i<j=2}^{n} p_r(p_i.p_j) + \sum_{i<j<k=3}^{n} p_r(p_i.p_j p_k) \\ &- ... + (-1)^{n+1} p_r(p_1.p_2....p_n). \end{aligned}$$

This expression is also known as the inclusion-exclusion expansion [46]. The reliability that at least one minimal path will work for n independent minimal paths. As a result, we end up with.

$$p_r(\sum_{i=1}^{n} p_i) = 1 - [1 - p_r(p_1)] \times [1 - p_r(p_2)] \times ... \times [1 - p_r(p_n)] \tag{2.3}$$

Example 2.3 Consider Fig.(3.3), how we will compute the system reliability depending on of all minimal path sets.

$S = \{\{x_1, x_4\}, \{x_2, x_5\}, \{x_1, x_3, x_5\}, \{x_2, x_3, x_4\}, \{x_1, x_6, x_8\}, \{x_2, x_7, x_8\},$
$\{x_1, x_3, x_7, x_8\}, \{x_2, x_3, x_6, x_8\}\}$

$$\begin{aligned} R_s = 1 - &[1 - (x_1 x_4)][1 - (x_2 x_5)][1 - (x_1 x_3 x_5)][1 - (x_2 x_3 x_4)] \\ &[1 - (x_1 x_6 x_8)][1 - (x_2 x_7 x_8)]\ [1 - (x_1 x_3 x_7 x_8)][1 - (x_2 x_3 x_6 x_8)] \end{aligned}$$

$$\begin{aligned} R_S = & R_1R_4 + R_2R_5 + R_1R_3R_5 + R_2R_3R_4 + R_1R_6R_8 + R_2R_7R_8 - R_1 \\ & R_2R_3R_4 - R_1R_2R_3R_5 - R_1R_2R_4R_5 - R_1R_3R_4R_5 - R_2R_3R_4R_5 + R_1 \\ & R_3R_7R_8 - R_1R_4R_6R_8 + R_2R_3R_6R_8 - R_2R_5R_7R_8 + 2R_1R_2R_3R_4R_5 - \\ & R_1R_2R_3R_6R_8 - R_1R_2R_3R_7R_8 - R_1R_2R_4R_7R_8 - R_1R_2R_5R_6R_8 - R_1 \\ & R_3R_4R_7R_8 - R_1R_3R_5R_6R_8 - R_2R_3R_4R_6R_8 - R_1R_2R_6R_7R_8 - R_1R_3R_5 \\ & R_7R_8 - R_2R_3R_4R_7R_8 - R_2R_3R_5R_6R_8 - R_1R_3R_6R_7R_8 - R_2R_3R_6R_7 \\ & R_8 + R_1R_2R_3R_4R_6R_8 + 2R_1R_2R_3R_4R_7R_8 + 2R_1R_2R_3R_5R_6R_8 + \\ & R_1R_2R_3R_5R_7R_8 + R_1R_2R_4R_5R_6R_8 + 2R_1R_2R_3R_6R_7R_8 + R_1R_2 \\ & R_4R_5R_7R_8 + R_1R_3R_4R_5R_6R_8 + R_1R_2R_4R_6R_7R_8 + R_1R_3R_4R_5R_7 \\ & R_8 + R_2R_3R_4R_5R_6R_8 + R_1R_2R_5R_6R_7R_8 + R_1R_3R_4R_6R_7R_8 + R_2 \\ & R_3R_4R_5R_7R_8 + R_1R_3R_5R_6R_7R_8 + R_2R_3R_4R_6R_7R_8 + R_2R_3R_5R_6 \\ & R_7R_8 - 2R_1R_2R_3R_4R_5R_6R_8 - 2R_1R_2R_3R_4R_5R_7R_8 - 2R_1R_2R_3R_4 \\ & R_6R_7R_8 - 2R_1R_2R_3R_5R_6R_7R_8 - R_1R_2R_4R_5R_6R_7R_8 - R_1R_3R_4R_5 \\ & R_6R_7R_8 - R_2R_3R_4R_5R_6R_7R_8 + 2R_1R_2R_3R_4R_5R_6R_7R_8 \end{aligned} \tag{2.4}$$

2.4 Minimal cut method

Using this method, we determine polynomial reliability using the following assumptions [24,28]:

- Make a list of all the minimal cut-sets in the system.
- In a minimal cut-set, system failure is caused by the failure of all parts.
- This indicates that these components are linked in a parallel manner.
- Each set of minimal cuts causes the system to fail.
- This indicates that the minimal cut sets in series are linked.

- Calculate system and system reliability using parallel and series decreases.

$$Rs = 1 - P_r(c_1 + c_2 + \ldots + c_i) \tag{2.5}$$

where $c_i (i = 1, 2, \ldots, n)$ represents the event i that components in minimal cut set are all in a failure state and n is the total number of minimal cut sets [29,37] . Equation (2.5) can be evaluated by applying the inclusion - exclusion rule, which is:

$$\begin{aligned} p_r(c_1 + c_2 + ... + c_n) = & \sum_{i=1}^{n} p_r(c_i) - \sum_{i<j=2}^{n} p_r(c_i.c_j) + \sum_{i<j<k=3}^{n} p_r(c_i.c_j.c_k) \\ & + ... + (-1)^{n+1} p_r(c_1.c_2....c_i). \end{aligned}$$

For n independent minimal cuts, the probability that at least one minimal cut will be working [5]. As a result we get:

$$p_r(\sum_{i=1}^{n} c_i) = [1 - p_r(c_1)] \times [1 - p_r(c_2)] \times ... \times [1 - p_r(c_n)] \tag{2.6}$$

Example 2.4 Consider Fig.(2.1) compute the system reliability depending on the sets of all minimal cut .
From equation (2.6), the sets of all minimal cut:
$C = \{\{x_1, x_2\}, \{x_4, x_5, x_8\}, \{x_4, x_5, x_6, x_7\}, \{x_1, x_3, x_5, x_7\}, \{x_2, x_3, x_4, x_6\}, \{x_1, x_3, x_5, x_8\}, \{x_2, x_3, x_4, x_8\}\}$

$$\begin{aligned} R_s = & [1 - (1 - x_1)(1 - x_2)][1 - (1 - x_4)(1 - x_5)(1 - x_8)] \\ & [1 - (1 - x_4)(1 - x_5)(1 - x_6)(1 - x_7)][1 - (1 - x_1)(1 - x_3)(1 - x_5)(1 - x_7)] \\ & [1 - (1 - x_2)(1 - x_3)(1 - x_4)(1 - x_6)][1 - (1 - x_1)(1 - x_3)(1 - x_5)(1 - x_8)] \\ & [1 - (1 - x_2)(1 - x_3)(1 - x_4)(1 - x_8)] \end{aligned}$$

The result that has obtained is the same result in eq.(2.4).

Chapter 3

Some importance measurement methods for units in reliability systems

3.1 Introduction

In this chapter, we will discuss some methods of measuring the importance of reliability and the problems of these methods with different systems. We are making improvements on some systems to eliminate their shortcomings so that they become good at measuring all systems. And comparison measures that has been improved with the Birnbaum's measure. As well as adding new methods in calculating the measurement of importance.

3.2 significance of reliability importance

- Give the units a rough ranking based on their impact in terms of system reliability (in a fault tree, or upper event probability).
- Assist in concentrating on the main causes of system unreliability.
- Assist in reducing system unreliability's lowest contributions.
- Concentrate on enhancements that have the biggest impact on reliability.
- Determine the sensitivity of model parameters.
- Assist in the focus of reviews and sensitivity studies.
- In a complex system, prioritize fault-finding.

3.3 Some reliability importance measure

- Birnbaum's measure.
- The improvement potential (1) measure.
- The improvement potential (2) measure.
- Risk achievement worth.
- Risk reduction worth.

- The criticality importance measure.
- Fussell-Vesely's measure.
- Other accurate methods of calculating an important measure of reliability.
- Use a new method to compute the importance of components in reliability system with. independent identical units

The literature does not always define significance measures consistently. The importance of something is determined by the circumstances. The various metrics are based on slightly different interpretations of the significance of thought components. Two considerations should, in theory, define the relevance of a unit:

- The system's location of the unit.
- The reliability of the unit in question.

and, possibly, the erroneous unit reliability estimation.

3.4 Birnbaum's measure

At time t,[64] A Birnbaum (1969) developed the following measure of unit i's reliability importance:

$$I(i) = \frac{\partial R_S(t)}{\partial R_i(t)} \tag{3.1}$$

The partial derivative of the system reliability R_S with respect to R_i is therefore produced as Birnbaum's measure. Classic sensitivity analysis will be familiar with this method. If the large reliability of unit i results in a comparatively large change in the system reliability at time t, a small change in the value of I(i) can have a big impact. The reliabilities of the other units stay fixed when taking this derivative; just the effect of changing R_i is evaluated. The rate of change in system reliability as a result of changes in the reliability of a particular component is measured by Birnbaum's measure. Birnbaum's measure can be stated in fault tree notation as:

$$I(i) = \frac{\partial R_f(t)}{\partial R_{fi}(t)} \tag{3.2}$$

The measure is named for Zygmund William Birnbaum, a Hungarian-American scholar (1903-2000)[64]. The system reliability is denoted R_S in the definition of Birnbaum's measure, and hence the system reliability is just a function of the unit reliabilities. When the units are dependent, such as when we have common-cause failures, this definition of Birnbaum's measure is no longer applicable.

3.5 Improvement potential (1) measure

At time t [64], unit I's improvement potential (1) measure is defined as follows:

$$I^{ip}(i/t) = R_s(1_i, R_i) - R_s \tag{3.3}$$

$I^{ip}(i/t)$ As a result, the system reliability with a perfect unit I differs from the system reliability with the actual unit I. It tells us how much the current system's reliability could be improved if we could replace the current unit I with a flawless unit. $Rs(1_i, R_i)$: reliability of the system when $R_i = 1$

3.5.1 Improvement potential (1) measure for reliability of series system

Consider a series system consisting of two-independent units, 1 and 2, each with unit reliabilities of R_1 and R_2. Assume that R_1 is greater than R_2, indicating that unit 1 is the more dependable of the 2 units. As a result, the system's reliability is increased.
$Rs = R_1R_2$
$I^{ip}(1) = (1)R_2 - R_1R_2$
$I^{ip}(2) = R_1(1) - R_1R_2$

This means that $I^{ip}(2) > I^{ip}(1)$. Hence we may deduce that the unit with the lowest reliability is the most important in a series structure when using the improvement potential (1) measure. To improve the reliability of a series structure, we must first improve the "weakest" unit.

3.5.2 Improvement potential (1) measure for reliability of parallel system

Consider a parallel system consisting of two-independent units, 1 and 2, each with unit reliability coefficient of R_1 and R_2. Assume that R_1 is greater than R_2, showing that unit 1 is the more reliable of the unit 2. As a result, the system's reliability is increased.
$R_s = R_1 + R_2 - R_1R_2$
$I^{ip}(1) = 1 - [R_1 + R_2 - R_1R_2]$
$I^{ip}(2) = 1 - [R_1 + R_2 - R_1R_2]$

This means that $I^{ip}(1) = I^{ip}(2)$. We can conclude that while measuring improvement potential (1), all units in a parallel structure are equally important. One of the problems of this method is that it may not produce accurate results, especially when the system is parallel; but, as shown in the parallel system, this method can be improved. We get, the improvement potential (1) can be stated as [64]

$$I^{ip}(i/t) = I^B(i/t)(1 - R_i) \tag{3.4}$$

Consider two independent units in a parallel system, 1 and 2, with unit consistency of 0.7 and 0.9, respectively.
$R_s = R_1 + R_2 - R_1R_2 = 0.97$
By Birnbaum's measure, $I^B(1/t) = 0.1, I^B(2/t) = 0.3$
By improvement potential (1) measure, $I^{ip}(1/t) = 0.03, I^{ip}(2/t) = 0.03$
Improve this method use equation (3.4), we get
$I^B(1/t) = \frac{0.03}{0.3} = 0.1, I^B(2/t) = \frac{0.03}{0.1} = 0.3$

3.5.3 Improvement potential (1) measure for reliability of complex system

We will discuss the following complex system (α), and the possibility of accessing the unit importance scale accurately and method of eliminating defects and errors and arriving at accurate results [36]. See Figure (3.1).

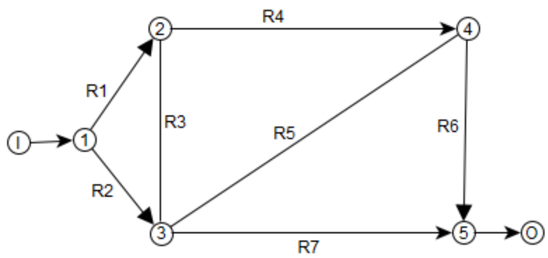


Figure 3.1: Complex system (α)[36].

The reliability of complex system (α):

$$\begin{aligned} R_S = & R_2R_7 + R_1R_3R_7 + R_1R_4R_6 + R_2R_5R_6 - R_1R_2R_3R_7 + R_1R_3R_5R_6 + R_2R_3R_4R_6 + R_1 \\ & R_4R_5R_7 - R_2R_5R_6R_7 - R_1R_2R_3R - 4R_6 - R_1R_2R_3R_5R_6 - R_1R_2R_4R_5R_6 - R_1R_2R_4R_5 \\ & R_7 - R_1R_3R_4R_5R_6 - R_1R_2R_4R_6R_7 - R_1R_3R_4R_5R_7 - R_2R_3R_4R_5R_6 - R_1R_3R_4R_6 \\ & R_7 - R_1R_3R_5R_6R_7 - R_2R_3R_4R_6R_7 - R_1R_4R_5R_6R_7 + 2R_1R_2R_3R_4R_5R_6 + R_1R_2R_3R_4 \\ & R_5R_7 + 2R_1R_2R_3R_4R_6R_7 + R_1R_2R_3R_5R_6R_7 + 2R_1R_2R_4R_5R_6R_7 + 2R_1R_3R_4R_5R_6 \\ & R_7 + R_2R_3R_4R_5R_6R_7 - 3R_1R_2R_3R_4R_5R_6R_7 \end{aligned} \quad (3.5)$$

If $R_i, i = 1, \ldots, 7$, have same reliability, put $R_i = 0.8$. By $I^B(i/t)$ method, we get, the Table (3.1) shows reliability importance and level of units in complex system (α) when have same reliability.

Table 3.1: Reliability importance and level of units in complex system (α) when have same reliability

Components	$\mathrm{I}^B(i/t)$	Level
R_2, R_7	0.2235	1
R_1, R_6	0.1864	2
R_4	0.0584	3
R_3, R_5	0.0379	4

By improvement potential (1) method, we get the Table (3.2) shows reliability importance by improvement potential (1) method in complex system (α) when have same reliability.

Table 3.2: Reliability importance by improvement potential (1) method in complex system (α) when have same reliability

Components	$Rs(1_i, R_i)$	Rs	$I^{ip}(i/t)$	$I^B(i/t)$
R_2, R_7	0.9526	0.9079	0.0447	0.2235
R_1, R_6	0.9452	0.9079	0.0373	0.1864
R_4	0.9196	0.9079	0.0117	0.0584
R_3, R_5	0.9155	0.9079	0.0076	0.0379

Note that the Table in measure improvement potential (1) keeps the levels when the reliability of the units is equal and the use equation (3.6) can be used to get matching results. These results can be converted into accurate results from the following relationship:

$$I^B(i/t) = \frac{I^{ip}(i/t)}{(1 - R_i)} \qquad R_i \neq 1 \tag{3.6}$$

$I^B(R_2/t) = I^B(R_7/t) = \frac{0.0447}{0.2} = 0.2235, I^B(R_1/t) = I^B(R_6/t) = \frac{0373}{0.2} = 0.1865$
$I^B(R_4/t) = \frac{0.0117}{0.2} = 0.0585, I^B(R_3/t) = I^B(R_5/t) = \frac{0.0076}{0.2} = 0.038.$

Consider if R_i have different reliability units in Figure (3.1). By $I^B(i/t)$ method, we get the Table (3.3). And by improvement potential (1) method, we get, the Table (3.4)
Through the above Table (3.4), it is not possible to rely on this measure when the values of the units are different, but it is possible to reach correct results and adopt it after making the following improvement:
$I^B(R_7/t) = \frac{0.081}{0.2} = 0.405, I^B(R_2/t) = \frac{0.0389}{0.2} = 0.1945, I^B(R_6/t) = \frac{0.0746}{0.4} = 0.1865$
$I^B(R_1/t) = \frac{0.0162}{0.1} = 0.162, I^B(R_3/t) = \frac{0.0208}{0.3} = 0.069, I^B(R_4/t) = \frac{0.0235}{0.4} = 0.05875$
$I^B(R_5/t) = \frac{0.0056}{0.1} = 0.056.$
After the improvement process, we obtained identical results and can be adopted in measuring the importance of systems reliability.

Table 3.3: Reliability importance and level of units in complex system (α) when have different reliability

Components	Value	$I^B(i/t)$	Level
R_7	0.8	0.4052	1
R_2	0.8	0.1946	2
R_6	0.6	0.1866	3
R_1	0.9	0.1618	4
R_3	0.7	0.0694	5
R_4	0.6	0.0588	6
R_5	0.9	0.0565	7

Table 3.4: Reliability importance by improvement potential (1) method in complex system (α) have different reliability

Components	Value	$Rs(1_i, R_i)$	Rs	$I^{ip}(i/t)$	$I^B(i/t)$
R_7	0.8	0.9571	0.8761	0.081	0.4052
R_6	0.6	0.9507	0.8761	0.0746	0.1866
R_2	0.8	0.9150	0.8761	0.0389	0.1946
R_4	0.6	0.8996	0.8761	0.0235	0.0588
R_3	0.7	0.8969	0.8761	0.0208	0.0694
R_1	0.9	0.8923	0.8761	0.0162	0.1618
R_5	0.9	0.8817	0.8761	0.0056	0.0565

3.6 Improvement potential (2) measure

Unit I's improvement potential (2) measure at time t is defined as:

$$I^{ip2}(i/t) = R_s - R_s(0_i, R_i) \tag{3.7}$$

$Rs(0_i, R_i)$: reliability of the system when $R_i = 0$ It is a method made conclusions by the researcher

3.6.1 Improvement potential (2) measure for reliability of series system

Consider a system with two-independent units, 1 and 2, each with unit consistency of R_1 and R_2. Assume that unit 1 is more reliable than unit 2 (R_1 is greater than R_2). As a result, the system's reliability is increased.
$R_s = R_1R_2$
$I^{ip2}(1) = R_1R_2 – (0)R_2 = R_1R_2$
$I^{ip2}(2) = R_1R_2 – (0)R_1 = R_1R_2$
This means that $I^{ip2}(1) = I^{ip2}(2)$. Thus we can conclude that all units in a series system are equally important when using the improvement potential (2) measure. One of the problems of this method is that it may not give accurate results, especially when the system is in series; but, as shown in the series system, this method can be improved. As a result, the improvement potential (2) measure can be written as:

$$I^B(i/t) = \frac{I^{ip2}(i/t)}{R_i}, \qquad R_i \neq 0 \tag{3.8}$$

Consider a two-component series system, 1 and 2, with unit consistency of 0.7 and 0.9, respectively. $R_s = R_1R_2 = 0.63$
By Birnbaum's measure, $I^B(1/t) = 0.9, I^B(2/t) = 0.7$
By improvement potential (2) measure, $I^{ip2}(1/t) = 0.63, I^{ip2}(2/t) = 0.63$
Improve this method use equation (3.8), we get
$I^B(1/t) = \frac{0.63}{0.7} = 0.9, I^B(2/t) = \frac{0.63}{0.9} = 0.7$

3.6.2 Improvement potential (2) measure for reliability of parallel system

Consider a parallel system consisting of two-independent units, 1 and 2, each with unit reliabilities of R_1 and R_2. Assume that R_1 is greater than R_2, showing that unit 1 is the more reliable of the two. As a result, the system's reliability is increased.
$R_s = R_1 + R_2 - R_1R_2$
$I^{ip2}(1) = (R_1 + R_2 - R_1R_2) - R_2 = R_1 - R_1R_2$
$I^{ip2}(2) = (R_1 + R_2 - R_1R_2) – R_1 = R_2 - R_1R_2$

This means that $I^{ip2}(2) > I^{ip2}(1)$. Thus we may deduce that the unit with the lowest reliability is the most important in a parallel system when using the improvement potential (2) measure. In order to improve a parallel system, we must first strengthen the "weakest" component, i.e. the unit with the lowest reliability.

3.6.3 Improvement potential (2) for reliability of complex system

We will discuss the complex system (α). If $R_i, i = 1, \ldots, 7$, have same reliability, put $R_i = 0.8$, by improvement potential (2) method, we get the Table (3.5).

Table 3.5: Reliability importance by improvement potential (2) method in complex system (α) when have same reliability

Components	Rs$(0_i, R_i)$	Rs	I$^{ip2}(i/t)$	I$^{B}(i/t)$
R_2, R_7	0.7291	0.9079	0.1788	0.2235
R_1, R_6	0.7588	0.9079	0.1491	0.1864
R_4	0.8612	0.9079	0.0467	0.0584
R_3, R_5	0.8776	0.9079	0.0303	0.0379

Note that the table in measure improvement potential (2) keeps the levels when the reliability of the units is equal and the use equation (3.8) can be used to get matching results. These results can be converted into accurate results from the following relation:
$I^B(2/t) = I^B(7/t) = \frac{0.1788}{0.8} = 0.2235, I^B(1/t) = I^B(6/t) = \frac{0.1491}{0.8} = 0.186375$
$I^B(4/t) = \frac{0.0467}{0.8} = 0.058375, I^B(3/t) = I^B(5/t) = \frac{0.0303}{0.8} = 0.037875$

Consider if R_i have different reliability by improvement potential (2) method, we get the Table (3.6).

Through the above table, it is not possible to rely on this measure when the values of the units are different, but it is possible to reach correct results and adopt it after making the following improvement:
$I^B(R_7/t) = \frac{0.3241}{0.8} = 0.405125, I^B(R_2/t) = \frac{0.1557}{0.8} = 0.194625, I^B(R_6/t) = \frac{0.112}{0.6} = 0.1866$
$I^B(R_1/t) = \frac{0.1457}{0.9} = 0.16188, I^B(R_3/t) = \frac{0.0486}{0.7} = 0.0694, I^B(R_4/t) = \frac{0.0353}{0.6} = 0.0588$
$I^B(R_5/t) = \frac{0.0509}{0.9} = 0.05655$

Table 3.6: Reliability importance by improvement potential (2) method in complex system (α) have different reliability

Components	Value	Rs($0_i, R_i$)	Rs	I$^{ip}(i/t)$	I$^{B}(i/t)$
R_7	0.8	0.5520	0.8761	0.3241	0.4052
R_2	0.8	0.7204	0.8761	0.1557	0.1946
R_1	0.9	0.7304	0.8761	0.1457	0.1618
R_6	0.6	0.7641	0.8761	0.112	0.1866
R_5	0.9	0.8252	0.8761	0.0509	0.0565
R_3	0.7	0.8275	0.8761	0.0486	0.0694
R_4	0.6	0.8408	0.8761	0.0353	0.0588

3.7 Risk achievement worth measure

The risk achievement worth (RAW) measures the proportionate increase in system unreliability when unit I is found to be failed. When it's determined that unit I isn't working,, the The nominal unreliability of a system is $1 - Rs$, the system reliability is $1 - Rs(0_i; R_i)$, As a result, the relative increase in system unreliability has occurred [64].

$$\frac{[1 - Rs(0_i; R_i)] - [1 - Rs]}{1 - Rs} = \frac{1 - Rs(0_i; R_i)}{(1 - Rs)} - 1$$

In most literature sources, however, **RAW** is defined without the minus one as:

$$I^{RAW} = \frac{1 - Rs(0_i; R_i)}{1 - Rs} \tag{3.9}$$

3.7.1 RAW measure for reliability of series system

Consider a system with two-independent units, 1 and 2, each with unit reliabilities R_1 and R_2. Assume that R_1 is greater than R_2, indicating that unit 1 is the more dependable of the 2. As a result, the system's reliability is high:

$Rs = R_1 R_2$

$I^{RAW}(1) = \frac{1}{1-R_1R_2}$

$I^{RAW}(2) = \frac{1}{1-R_1R_2}$

This means that $I^{RAW}(1) = I^{RAW}(2)$. We can therefore conclude that when using the RAW

measure, all units in a series system are equally important. One of the disadvantages of this measure is that it does not always give accurate results, and this measure can be improved to obtain accurate results in the context of this research.
These results can be converted into accurate results from the following relation:

$I^{RAW} = \frac{1-Rs(0_i;R_i)}{1-Rs}$, then $1 - Rs = \frac{1-Rs(0_i;R_i)}{I^{RAW}}$

$$Rs = 1 - \frac{1 - Rs(0_i; R_i)}{I^{RAW}} \tag{3.10}$$

But for equation (3.8), $I^B R = Rs - Rs(0_i; R_i)$
From equation (3.10) and equation (3.8) we get:

$I^B R = [1 - \frac{1-Rs(0_i;R_i)}{I^{RAW}} - Rs(0_i; R_i)] = [(1 - Rs(0_i; R_i)(1 - \frac{1}{I^{RAW}})]$

$$I^B = \frac{[(1 - Rs(0_i; R_i)(1 - \frac{1}{I^{RAW}})]}{R} \tag{3.11}$$

Through the equation (3.11), accurate results can be obtained with the $I^B(i/t)$ method, thus eliminating the defects of this measure.
Consider a two-independent units, series system, 1 and 2, with unit consistency of 0.7 and 0.9, respectively.
$R_s = R_1R_2 = 0.63$
By Birnbaum's measure, $I^B(1/t) = 0.9, I^B(2/t) = 0.7$
By risk achievement worth, $I^{RAW}(R_1) = 2.7027027027, I^{RAW}(R_2) = 2.7027027027$
Improve this method use equation (3.11), we get:

$I^B(1/t) = \frac{[(1-0)(1-\frac{1}{2.7027027027})]}{0.7} = 0.9, I^B(1/t) = \frac{[(1-0)(1-\frac{1}{2.7027027027})]}{0.9} = 0.7$

3.7.2 RAW measure for reliability of parallel system

Consider a parallel system with two-independent units, 1 and 2, each with unit consistency of R_1 and R_2. Assume that R_1 is greater than R_2, suggesting that unit 1 is more reliable than unit 2. As a result, the system's reliability has increased.
$Rs = R_1 + R_2 - R_1R_2$

$I^{RAW}(1) = \frac{1-R_2}{1-(R_1+R_2-R_1R_2)}$, $I^{RAW}(2) = \frac{1-R_1}{1-(R_1+R_2-R_1R_2)}$

This means that $I^{RAW}(1) > I^{RAW}(2)$, and we may assume that when using the RAW measure, the most important component in a parallel system is the strongest (i.e., most reliable).

3.7.3 RAW measure for reliability of complex system

We will discuss the following complex system (β) and the possibility of accessing the reliability importance measure accurately and method of eliminating defects and errors and arriving at accurate results [35]. See Figure (3.2).

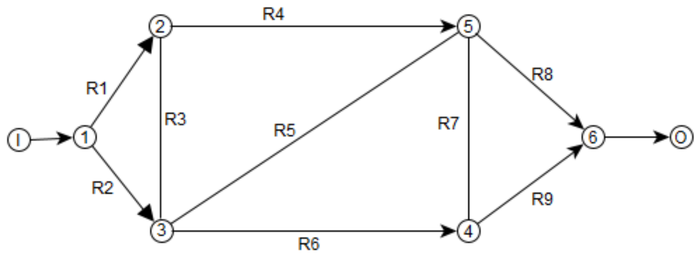


Figure 3.2: Complex system (β)[35].

The reliability of complex system (β):

$$
\begin{aligned}
R_S = & R_1R_4R_8 + R_2R_5R_8 + R_2R_6R_9 + R_1R_3R_5R_8 + R_2R_3R_4R_8 + R_1R_3R_6R_9 + R_1R_4R_7 \\
& R_9 + R_2R_5R_7R_9 + R_2R_6R_7R_8 - R_1R_2R_3R_4R_8 - R_1R_2R_3R_5R_8 - R_1R_2R_4R_5R_8 - \\
& R_1R_2R_3R_6R_9 - R_1R_3R_4R_5R_8 - R_2R_3R_4R_5R_8 + R_1R_3R_5R_7R_9 + R_1R_3R_6R_7R_8 + \\
& R_1R_4R_5R_6R_9 + R_2R_3R_4R_7R_9 - R_2R_5R_6R_7R_8 - R_1R_4R_7R_8R_9 - R_2R_5R_6R_7R_9 - \\
& R_2R_5R_6R_8R_9 - R_2R_5R_7R_8R_9 - R_2R_6R_7R_8R_9 + 2R_1R_2R_3R_4R_5R_8 - R_1R_2R_3R_4 \\
& R_7R_9 - R_1R_2R_3R_5R_7R_9 - R_1R_2R_3R_6R_7R_8 - R_1R_2R_4R_5R_6R_9 - R_1R_2R_4R_5R_7 \\
& R_9 - R_1R_2R_4R_6R_7R_8 - R_1R_3R_4R_5R_6R_9 - R_1R_2R_4R_6R_7R_9 - R_1R_3R_4R_5R_7R_9 - \\
& R_1R_3R_4R_6R_7R_8 - R_1R_2R_4R_6R_8R_9 - R_1R_3R_4R_6R_7R_9 - R_1R_3R_5R_6R_7R_8 - R_2 \\
& R_3R_4R_5R_7R_9 - R_2R_3R_4R_6R_7R_8 - R_1R_3R_4R_6R_8R_9 - R_1R_3R_5R_6R_7R_9 - R_2R_3 \\
& R_4R_6R_7R_9 - R_1R_3R_5R_6R_8R_9 - R_1R_4R_5R_6R_7R_9 - R_2R_3R_4R_6R_8R_9 - R_1R_3R_5 \\
& R_7R_8R_9 - R_1R_4R_5R_6R_8R_9 - R_2R_3R_4R_7R_8R_9 - R_1R_3R_6R_7R_8R_9 + 2R_2R_5R_6 \\
& R_7R_8R_9 + R_1R_2R_3R_4R_5R_6R_9 + 2R_1R_2R_3R_4R_5R_7R_9 + 2R_1R_2R_3R_4R_6R_7R_8 + 2R_1 \\
& R_2R_3R_4R_6R_7R_9 + R_1R_2R_3R_5R_6R_7R_8 + 2R_1R_2R_3R_4R_6R_8R_9 + R_1R - 2R_3R_5R_6 \\
& R_7R_9 + R_1R_2R_4R_5R_6R_7R_8 + R_1R_2R_3R_4R_7R_8R_9 + R_1R_2R_3R_5R_6R_8R_9 + 2R_1R_2 \\
& R_4R_5R_6R_7R_9 + R_1R_3R_4R_5R_6R_7R_8 + R_1R_2R_3R_5R_7R_8R_9 + 2R_1R_2R_4R_5R_6R_8R_9 + \\
& 2R_1R_3R_4R_5R_6R_7R_9 + R_2R_3R_4R_5R_6R_7R_8 + R_1R_2R_3R_6R_7R_8R_9 + R1_{R2}R_4R_5 \\
& R_7R_8R_9 + 2R_1R_3R_4R_5R_6R_8R_9 + R_2R_3R_4R_5R_6R_7R_9 + 2R_1R_2R_4R_6R_7R_8R_9 + R_1R_3 \\
& R_4R_5R_7R_8R_9 + R_2R_3R_4R_5R_6R_8R_9 + 2R_1R_3R_4R_6R_7R_8R_9 + R_2R_3R_4R_5R_7R_8 \\
& R_9 + 2R_1R_3R_5R_6R_7R_8R_9 + 2R_2R_3R_4R_6R_7R_8R_9 + R_1R_4R_5R_6R_7R_8R_9 - 2R_1R_2 \\
& R_3R_4R_5R_6R_7R_8 - 3R_1R_2R_3R_4R_5R_6R_7R_9 - 3R_1R_2R_3R_4R_5R_6R_8R_9 - 2R_1R_2R_3R_4 \\
& R_5R_7R_8R_9 - 4R_1R_2R_3R_4R_6R_7R_8R_9 - 2R_1R_2R_3R_5R_6R_7R_8R_9 - 3R_1R_2R_4R_5R_6 \\
& R_7R_8R_9 - 3R_1R_3R_4R_5R_6R_7R_8R_9 - 2R_2R_3R_4R_5R_6R_7R_8R_9 + 5R_1R_2R_3R_4R_5R_6 \\
& R_7R_8R_9
\end{aligned}
\tag{3.12}
$$

If $R_i, i = 1, \ldots, 9$, have same reliability, put $R_i = 0.8$, by $I^B(i/t)$ method, we get the Table

(3.7). By risk achievement worth method, we get the Table (3.8).

Table 3.7: Reliability importance and level of units in complex system (β) when have same reliability

Components	$I^B(i/t)$	Level
R_2, R_8	0.2175	1
R_1, R_9	0.1870	2
R_4, R_6	0.0682	3
R_5	0.0453	4
R_3, R_7	0.0352	5

Table 3.8: Reliability importance by risk achievement worth method in complex system (β) when have same reliability

Components	Rs($0_i, R_i$)	1-Rs($0_i, R_i$)	1- Rs	$I^{RAW}(i/t)$	$I^B(i/t)$
R_2, R_8	0.7263	0.2737	0.0997	2.7452	0.2175
R_1, R_9	0.7507	0.2493	0.0997	2.5005	0.1870
R_4, R_6	0.8457	0.1543	0.0997	1.5476	0.0682
R_5	0.8641	0.1359	0.0997	1.3630	0.0453
R_3, R_7	0.8721	0.1279	0.0997	1.2828	0.0352

Note that the table in measure risk achievement worth method keeps the levels when the reliability of the units is equal. Through the equation (3.11), accurate results can be obtained with the $I^B(i/t)$ method, thus eliminating the defects of this measure.

$$I^B(2/t) = I^B(8/t) = \frac{[(1-0.7263)(1-\frac{1}{2.7452})]}{0.8} = 0.21749$$
$$I^B(1/t) = I^B(9/t) = \frac{[(1-0.7507)(1-\frac{1}{2.5005})]}{0.8} = 0.18699$$
$$I^B(4/t) = I^B(6/t) = \frac{[(1-0.8457)(1-\frac{1}{1.5476})]}{0.8} = 0.06824$$
$$I^B(5/t) = \frac{[(1-0.8641)(1-\frac{1}{1.3630})]}{0.8} = 0.04524$$
$$I^B(3/t) = I^B(7/t) = \frac{[(1-0.8721)(1-\frac{1}{1.2828})]}{0.8} = 0.035245$$

Consider if R_i have different reliability by $I^B(i/t)$ method, we get the Table (3.9).

Table 3.9: Reliability importance and level of units in complex system (β) when have different reliability

Components	Value	$I^B(i/t)$	Level
R_8	0.7	0.4197	1
R_9	0.6	0.2639	2
R_2	0.8	0.1742	3
R_1	0.9	0.1555	4
R_5	0.9	0.1435	5
R_4	0.6	0.0801	6
R_7	0.8	0.0730	7
R_6	0.6	0.0647	8
R_3	0.7	0.0617	9

Table 3.10: Reliability importance by risk achievement worth method in complex system (β) when different reliability

Components	Value	Rs($0_i, R_i$)	1-Rs($0_i, R_i$)	1-Rs	I^{RAW}	$I^B(i/t)$
R_8	0.7	0.5214	0.4786	0.1848	2.5898	0.4197
R_9	0.6	0.6569	0.3431	0.1848	1.8566	0.2639
R_1	0.9	0.6753	0.3247	0.1848	1.7570	0.1555
R_2	0.8	0.6759	0.3241	0.1848	1.7537	0.1742
R_5	0.9	0.6861	0.3139	0.1848	1.6985	0.1435
R_7	0.8	0.7568	0.2432	0.1848	1.3160	0.0730
R_4	0.6	0.7672	0.2328	0.1848	1.2597	0.0801
R_3	0.7	0.7721	0.2279	0.1848	1.2332	0.0617
R_6	0.6	0.7764	0.2236	0.1848	1.2099	0.0647

By risk achievement worth method, we get the Table (3.10).

Through the above Table (3.10), it is not possible to rely on this measure when the values of the units are different, but it is possible to reach correct results and adopt it after making the following improvement use equation (3.11).

$I^B(R_8) = \frac{(0.5214)(1-\frac{1}{2.5898})}{0.7} = 0.41971, I^B(R_9) = \frac{(0.3431)(1-\frac{1}{1.8566})}{0.6} = 0.2638$
$I^B(R_2) = \frac{(0.3241)(1-\frac{1}{1.7537})}{0.8} = 0.1741, I^B(R_1) = \frac{(0.3247)(1-\frac{1}{1.7570})}{0.9} = 0.1554$
$I^B(R_5) = \frac{(0.3139)(1-\frac{1}{1.6985})}{0.9} = 0.1434, I^B(R_4) = \frac{(0.2328)(1-\frac{1}{1.2597})}{0.6} = 0.07999$
$I^B(R_7) = \frac{(0.2432)(1-\frac{1}{1.3160})}{0.8} = 0.07299, I^B(R_6) = \frac{(0.2236)(1-\frac{1}{1.2099})}{0.6} = 0.0646$
$I^B(R_3) = \frac{(0.2279)(1-\frac{1}{1.2332})}{0.7} = 0.0615$
We note that we get results matching with I^B and thus get rid of the defects of this measure.

3.8 Risk reduction worth measure

When unit I is known to be functional, the risk reduction worth (RRW) measures the relative reduction in system reliability [64]. The system's unreliability has improved by a factor of two.

$$\frac{[1-Rs]-[1-Rs(1_i;R_i)]}{1-Rs(1_i;R_i)} = \frac{1-Rs}{1-Rs(1_i;R_i)} - 1$$

Most literature sources, on the other hand, define RRW without the "minus one" as:

$$I^{RRW} = \frac{1-Rs}{1-Rs(1_i;R_i)} \tag{3.13}$$

which, for all coherent systems, fulfills $I^{RRW} \geq 1$. When $I^{RRW} = 1$. It has no effect to improve component I so that it is always working.

3.8.1 RRW measure for reliability of series system

Consider a series structure with two-independent units, 1 and 2, each having R_1 and R_2 unit reliabilities. Assume that R_1 is greater than R_2, indicating that unit 1 is the more dependable of the unit 2. As a result, the system's reliability is:
$Rs = R_1R_2$
$I^{RRW}(1) = \frac{1-R_1R_2}{1-R_2}$
$I^{RRW}(2) = \frac{1-R_1R_2}{1-R_1}$
This suggests that $I^{RRW}(2) > I^{RRW}(1)$, thus we may conclude that when employing the RRW measure, the unit with the lowest reliability is the most essential in a series system.

3.8.2 RRW measure for reliability of parallel system

Consider a parallel system with two two-independent units, 1 and 2, each having R_1 and R_2 unit reliabilities. Assume R_1 is greater than R_2, indicating that unit 1 is the more reliability of the unit 2. As a result, the system's reliability is $Rs = R_1 + R_2 - R_1R_2$.
$I^{RRW}(1) = NA$
$I^{RRW}(2) = NA$
NA:unknown quantity, when we know that unit I is working in a parallel system, we know that the system is working as well, and the denominator in the $I^{RRW}(i)$ statement is always equal to zero. The parallel system's system unreliability has been reduced.

$$1 - (R_1 + R_2 - R_1R_2)$$

but because the denominator is 0, the relative reduction is not specified.
One of the disadvantages of this measure is that it does not always give accurate results, and this measure can be improved to obtain accurate results in the context of this research. These results can be converted into accurate results from the following relationship.

$I^{RRW} = \frac{1-Rs}{1-Rs(1_i;R_i)} then I^{RRW}[1 - Rs(1_i; R_i)] = 1 - Rs$

$Rs = 1 - I^{RRW}[1 - Rs(1_i; R_i)]$.
But for equation (3.4) $I^{ip} = Rs(1_i, R_i) - Rs$
$Rs = Rs(1_i, R_i) - I^{ip}$
$I^{ip} = Rs(1_i, R_i) – [1 - I^{RRW}(1 - Rs(1_i; R_i))]$
$I^{ip} = Rs(1_i, R_i) – 1 + I^{RRW}(1 - Rs(1_i; R_i)$
but, for equation (3.4) $I^{ip}(i/t) = I^B(i/t)(1 - R_i)$

$I^B(i) = \frac{Rs(1_i,R_i)-1+I^{RRW}(1-Rs(1_i;R_i)}{1-R_i}$

but, $I^{RRW} = \frac{1-Rs}{1-Rs(1_i;R_i)}$, we get:

$I^B(i) = \frac{Rs(1_i,R_i)-1+(\frac{1-Rs}{1-Rs(1_i;R_i)}(1-Rs(1_i;R_i)))}{1-R_i}$

$$I^B(i) = \frac{Rs(1_i, R_i) – Rs}{1 - R_i} \tag{3.14}$$

From the equation (3.14), this method can be improved to measure the reliability of the systems and give identical results. Thus, the defects of this method can be eliminated through the mentioned equation.

Consider parallel system two-independent units, 1,and 2, with unit reliabilities 0.7 and 0.9 respectively. $R_s = R_1 + R_2 - R_1R_2 = 0.97$
By Birnbaum's measure, $I^B(1/t) = 0.1, I^B(2/t) = 0.3$, by risk reduction worth measure, $I^{RRW}(1/t) = NA, I^{RRW}(2/t) = NA$. Improve this method use equation (3.14), from equation (3.14), this method can be improved to measure the reliability of the systems and give identical results. Thus, the defects of this method can be eliminated through the mentioned equation:
$I^B(R_1/t) = \frac{1-0.97}{0.3} = 0.1, I^B(R_2/t) = \frac{1-0.97}{0.1} = 0.3$

3.8.3 RRW measure for reliability of complex system

We will discuss the complex system (β). If R_i , i =1, . . . , 9, have same reliability, put $R_i = 0.8$. By risk achievement worth method, we get the Table (3.11).
These results can be converted into accurate results from the following relationship.

Table 3.11: Reliability importance by risk reduction worth method in complex system (β) when have same reliability

Components	Rs($1_i, R_i$)	1-Rs($1_i, R_i$)	1- Rs	I^{RRW}	I^B
R_2, R_8	0.9438	0.0562	0.0997	1.7740	0.2175
R_1, R_9	0.9377	0.0623	0.0997	1.5775	0.1870
R_4, R_6	0.9139	0.0861	0.0997	1.1579	0.0682
$R5$	0.90947	0.0906	0.0997	1.1004	0.0453
R_3, R_7	0.9073	0.0927	0.0997	1.0755	0.0352

$I^{RRW} = \frac{1-Rs}{1-Rs(1_i;R_i)} \rightarrow I^{RRW}[1 - Rs(1_i; R_i)] = 1 - Rs$

$Rs = 1 - I^{RRW}[1 - Rs(1_i; R_i)]$. But for equation (3.4), $I^{ip} = Rs(1_i, R_i) - Rs$
$Rs = Rs(1_i, R_i) - I^{ip}$

$I^{ip} = Rs(1_i, R_i)$–$[1 - I^{RRW}(1 - Rs(1_i; R_i))]$
$I^{ip} = Rs(1_i, R_i)$–$1 + I^{RRW}(1 - Rs(1_i; R_i)$
But for equation (3.4), $I^{ip}(i/t) = I^B(i/t)(1 - R_i)$

$I^B(i) = \frac{Rs(1_i,R_i)–1+I^{RRW}(1-Rs(1_i;R_i)}{1-R_i}$

$$I^B(i) = \frac{(Rs(1_i, R_i)–1)(1 - I^{RRW})}{1 - Ri} \tag{3.15}$$

From the equation (3.15), this method can be improved to measure the reliability of the systems and give identical results. Thus, the defects of this method can be eliminated through the mentioned equation.

$I^B(R_2/t) = I^B(R_8/t) = \frac{(0.9438-1)(1-1.7740)}{0.2} = 0.217494$

$I^B(R_1/t) = I^B(R_9/t) = \frac{[(1-0.9377)(1-1.5775)]}{0.2} = 0.17989$

$I^B(R_4/t) = I^B(R_6/t) = \frac{[(1-0.9139)(1-1.1579)]}{0.2} = 0.06797$

$I^B(R_5/t) = \frac{[(1-0.9094)(1-1.1004)]}{0.2} = 0.04548$

$I^B(R_3/t) = I^B(R_7/t) = \frac{[(1-0.9073)(1-1.0755)]}{0.2} = 0.03499$

Consider if R_i have different reliability by risk achievement worth method, we get the Table (3.12). Through the down Table (3.12), it is not possible to rely on this measure when the values of the units are different, but it is possible to reach correct results and adopt it after making the following improvement use equation (3.15).

$I^B(R_8/t) = \frac{(0.9412-1)(1-3.1428)}{0.3} = 0.41993$

$I^B(R_9/t) = \frac{(0.9208-1)(1-2.3333)}{0.4} = 0.26399$

$I^B(R_2/t) = \frac{(0.8501-1)(1-1.2328)}{0.2} = 0.17448$

$I^B(R_1/t) = \frac{(0.8308-1)(1-1.0921)}{0.1} = 0.15583$

$I^B(R_5/t) = \frac{(0.8296-1)(1-1.0845)}{0.1} = 0.143988$

$I^B(R_4/t) = \frac{(0.8473-1)(1-1.2102)}{0.4} = 0.08024$

$I^B(R_7/t) = \frac{(0.8298-1)(1-1.0857)}{0.2} = 0.0729$

$I^B(R_6/t) = \frac{(0.8411-1)(1-1.1629)}{0.4} = 0.0647$

$I^B(R_3/t) = \frac{(0.8337-1)(1-1.1112)}{0.3} = 0.0616$

We note that we get results matching with I^B and thus get rid of the defects of this measure.

Table 3.12: Reliability importance by risk reduction worth method in complex system (β) have different reliability

Components	Value	$\text{Rs}(1_i, R_i)$	$\text{1-Rs}(1_i, R_i)$	1-Rs	I^{RRW}	I^{B}
R_8	0.7	0.9412	0.0588	0.1848	3.1428	0.4197
R_9	0.6	0.9208	0.0792	0.1848	2.3333	0.2639
R_2	0.8	0.8501	0.1499	0.1848	1.2328	0.1742
R_4	0.6	0.8473	0.1527	0.1848	1.2102	0.0801
R_6	0.6	0.8411	0.1589	0.1848	1.1629	0.0647
R_3	0.7	0.8337	0.1663	0.1848	1.1112	0.0617
R_1	0.9	0.8308	0.1692	0.1848	1.0921	0.1555
R_7	0.8	0.8298	0.1702	0.1848	1.0857	0.0730
R_5	0.9	0.8296	0.1704	0.1848	1.0845	0.1435

3.9 Fussell-Vesely's measure

The importance measure defined by Fussell-Vesely [64], $I^{FV}(i)$ at time t, is the probability that at least one minimal cut set containing unit I fails, assuming the system fails at time t.

Fussell-Vesely's me assure can be generally estimated by

$$I^{FV}(i) \approx \frac{(1 - \prod_{j=1}^{m}(1 - \varrho_j^i(t))}{\varrho_0(t)} \approx \frac{\sum_{j=1}^{m} \varrho_j^i(t)}{\varrho_0(t)}$$

Where ϱ represents the probability that the minimal cut set j at time t, one of those having unit I fails. and $\varrho_0 \approx 1 - \prod_{j=1}^{m}(1 - \varrho_i)$.
The Fussell-Vesely measure is simple to calculate by hand and gives similar results to the Criticality Importance measure. We will discuss the Figure (3.1) complex system (α). If $R_i, i = 1, \ldots, 7$, have same reliability, put $R_i = 0.8$. The minimal cuts sets are:

$C_1 = \{R_1, R_2\} \longrightarrow \varrho_1 = q_1 q_2 = 0.04$
$C_2 = \{R_6, R_7\} \longrightarrow \varrho_2 = q_6 q_7 = 0.04$
$C_3 = \{R_2, R_3 R_4,\} \longrightarrow \varrho_3 = q_2 q_3 q_4 = 0.008$
$C_4 = \{R_4, R_5 R_7,\} \longrightarrow \varrho_4 = q_4 q_5 q_7 = 0.008$
$C_5 = \{R_1, R_3 R_5, R_7\} \longrightarrow \varrho_5 = q_1 q_3 q_5 q_7 = 0.0016$
$C_6 = \{R_2, R_3 R_5, R_6\} \longrightarrow \varrho_6 = q_2 q_3 q_5 q_6 = 0.0016$

$$\varrho_0 \approx 1 - \prod_{j=1}^{6}(1 - \varrho_i)$$

To determine Fussell-Vesely's measure of, for example, unit R_i, we have to find the minimal cut sets where unit R_i is a member.

$I^{FV}(R_2) \approx \frac{\varrho_1+\varrho_3+\varrho_6}{\varrho_0} = \frac{0.04+0.008+0.0016}{0.0921} = 0.53854$

$I^{FV}(R_7) \approx \frac{\varrho_2+\varrho_4+\varrho_5}{\varrho_0} = \frac{0.04+0.008+0.0016}{0.0921} = 0.53854$

$I^{FV}(R_1) \approx \frac{\varrho_1+\varrho_5}{\varrho_0} = \frac{0.04+0.0016}{0.0921} = 0.45168$

$I^{FV}(R_6) \approx \frac{\varrho_2+\varrho_6}{\varrho_0} = \frac{0.008+0.0016}{0.0921} = 0.451682$

$I^{FV}(R_3) \approx \frac{\varrho_3+\varrho_5+\varrho_6}{\varrho_0} = \frac{0.008+0.0016+0.0016}{0.0921} = 0.1910$

$I^{FV}(R_5) \approx \frac{\varrho_4+\varrho_5+\varrho_6}{\varrho_0} = \frac{0.0016+0.008+0.0016}{0.0921} = 0.1910$

$I^{FV}(R_4) \approx \frac{\varrho_3+\varrho_4}{\varrho_0} = \frac{0.008+0.008}{0.0921} = 0.1737$

3.10 Other accurate methods of calculating an reliability important measure

By pivotal decomposition, we have [64]:

$$Rs = R_i Rs(1_i, R_i) - (1 - R_i)Rs(0_i, R_i)$$
$$= R_i[Rs(1_i, R_i) - Rs(0_i, R_i)] + Rs(0_i, R_i)$$

This shows that Rs is a linear function of R_i (when all other reliabilities remain the same) As a result, Birnbaum's measure can be written as:

$$I^B(i/t) = \frac{\partial Rs}{\partial R_i} = Rs(1_i, R_i) - Rs(0_i, R_i)$$
$$I^B(i/t) = Rs(1_i, R_i) - Rs(0_i, R_i) \tag{3.16}$$

where $Rs(1_i, R_i)$ is the system's reliability when we know unit I is working and $Rs(0_i, R_i)$. When we know that component I isn't working, what is the system's reliability. This leads to a relatively straightforward method of calculation $I^B(i/t)$ as seen in the example on the following slide. This method is used by most computer tools for fault tree analysis to calculate Birnbaum's measure. When determining $I^B(i/t)$ for systems with common-cause failures, the same approach is sometimes utilized.
By improvement potential(1) measure of unit i at time t is defined as:

$I^{ip}(i/t) = Rs(1_i, R_i) - Rs$, but $I^B(i/t) = \frac{I^{ip}(i/t)}{1-R_i}$, we get

$$I^B(i/t) = \frac{Rs(1_i, R_i) - Rs}{1 - R_i} \tag{3.17}$$

By improvement potential(2) measure of unit i at time t is defined as:

$I^{ip2}(i/t) = Rs - Rs(0_i, R_i)$, but $I^B(i/t) = \frac{I^{ip2}(i/t)}{R_i}$, we get

$$I^B(i/t) = \frac{Rs - Rs(0_i, R_i)}{R_i} \tag{3.18}$$

Some applying formulas (3.16, 3.17 and 3.18)

3.10.1 for reliability of series system

Consider a series system of two independent units, R_1 and R_2, with unit reliabilities 0.6 and 0.8, respectively. The system reliability is therefore. $Rs = R_1R_2 \Longrightarrow Rs = 0.48$

- Use Birnbaum's measure, $I^B = \frac{\partial Rs}{\partial R_i}, I^B(R_1) = R_2 = 0.8, I^B(R_2) = R_1 = 0.6$

- Use $I^B = Rs(1_i, R_i) - Rs(0_i, R_i)$

Table 3.13: Reliability importance by $Rs(1_i, R_i) - Rs(0_i, R_i)$ method for series system

Components	Value	Rs($1_i, R_i$)	Rs($0_i, R_)$	Rs($1_i, R_i) - Rs(0_i, R_i$)
R_1	0.6	0.8	0	0.8
R_2	0.8	0.6	0	0.6

- Use equation (3.17), $I^B = \frac{Rs(1_i,R_i)-Rs}{1-R_i}$
  $I^B(R_1) = \frac{0.8-0.48}{0.4} = 0.8$
  $I^B(R_2) = \frac{0.6-0.48}{0.2} = 0.6$

- Use equation (3.18), $I^B = \frac{Rs-Rs(0_i,R_i)}{R}$
  $I^B(R_1) = \frac{0.48-0}{0.6} = 0.8$
  $I^B(R_2) = \frac{0.48-0}{0.8} = 0.6$

3.10.2 for reliability of parallel system

Consider a parallel system of two-independent units, R_1 and R_2, with unit reliabilities 0.6 and 0.8, respectively. The system reliability is therefore. $Rs = R_1+R_2-R_1R_2 \Longrightarrow Rs = 0.92$

- Use Birnbaum's measure $I^B = \frac{\partial Rs}{\partial R_i}, I^B(R_1) = 1 - R_2 = 0.2, I^B(R_2) = 1 - R_1 = 0.4$

- Use $I^B = Rs(1_i, R_i) - Rs(0_i, R_i)$

Table 3.14: Reliability importance by $Rs(1_i, R_i) - Rs(0_i, R_i)$ for parallel system

Components	Value	Rs$(1_i, R_i)$	Rs$(0_i, R_i)$	Rs$(1_i, R_i) - Rs(0_i, R_i)$
R_1	0.6	1	0.8	0.2
R_2	0.8	1	0.6	0.4

- Use equation (3.17), $I^B = \frac{Rs(1_i,R_i)-Rs}{1-R_i}$
$I^B(R_1) = \frac{1-0.92}{0.4} = 0.2$
$I^B(R_2) = \frac{1-0.92}{0.2} = 0.4$

- Use equation (3.18), $I^B = \frac{Rs-Rs(0_i,R_i)}{R}$
$I^B(R_1) = \frac{0.92-0.8}{0.6} = 0.2$
$I^B(R_2) = \frac{0.92-0.6}{0.8} = 0.4$

3.10.3 for reliability of complex system

Consider a complex system of eight independent units, $R_1, R_2, R_3, R_4, R_5, R_6, R_7$ and R_8, with unit reliabilities 0.9,0.7,0.7,0.9,0.6,0.6 and 0.8, respectively. The system reliability is therefore equation (2.4), and $Rs = 0.9536$. See Figure (3.3).

- Use Birnbaum's measure, $I^B = \frac{\partial Rs}{\partial R_i}$
- Use $I^B = Rs(1_i, R_i) - Rs(0_i, R_i)$
- Use equation (3.17), $I^B = \frac{Rs(1_i,R_i)-Rs}{1-R_i}$

$I^B(R_1) = \frac{0.9857-0.9536}{0.1} = 0.321, I^B(R_4) = \frac{0.9670-0.9536}{0.1} = 0.134$

$I^B(R_2) = \frac{0.9854-0.9536}{0.3} = 0.106, I^B(R_8) = \frac{0.9623-0.9536}{0.2} = 0.0435$

$I^B(R_5) = \frac{0.9658-0.9536}{0.4} = 0.0305, I^B(R_3) = \frac{0.9598-0.9536}{0.3} = 0.02066$

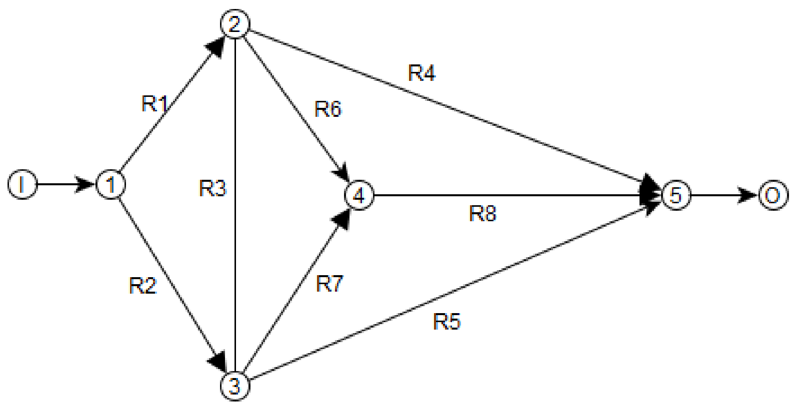


Figure 3.3: Complex systems with eight units.

Table 3.15: Reliability importance by Birnbaum's measure method

Components	Value	I^B	Level
R_1	0.9	0.3211	1
R_4	0.9	0.1338	2
R_2	0.7	0.1061	3
R_8	0.8	0.0435	4
R_5	0.6	0.0305	5
R_3	0.7	0.0205	6
R_7	0.8	0.0178	7
R_6	0.6	0.0120	8

$I^B(R_7) = \frac{0.9572-0.9536}{0.2} = 0.018, I^B(R_6) = \frac{0.9584-0.9536}{0.4} = 0.012$

- Use equation (3.18), $I^B = \frac{Rs-Rs(0_i,R_i)}{R}$

$I^B(R_1) = \frac{0.9536-0.6646}{0.9} = 0.3211, I^B(R_4) = \frac{0.9536-0.833}{0.9} = 0.13377$

$I^B(R_2) = \frac{0.9536-0.8793}{0.7} = 0.10614, I^B(R_8) = \frac{0.9536-0.9188}{0.8} = 0.0435$

Table 3.16: Reliability importance by $Rs(1_i, R_i) - Rs(0_i, R_i)$ for complex system

Components	Value	Rs$(1_i, R_i)$	Rs$(0_i, R_i)$	Rs$(1_i, R_i) - Rs(0_i, R_i)$
R_1	0.9	0.9857	0.6646	0.3211
R_4	0.9	0.9670	0.8332	0.1338
R_2	0.7	0.9854	0.8793	0.1061
R_8	0.8	0.9623	0.9188	0.0435
R_5	0.6	0.9658	0.9353	0.0305
R_3	0.7	0.9598	0.9392	0.0206
R_7	0.8	0.9572	0.9393	0.0179
R_6	0.6	0.9584	0.9464	0.012

$I^B(R_5) = \frac{0.9536-0.9353}{0.6} = 0.0305, I^B(R_3) = \frac{0.9536-0.9392}{0.7} = 0.02057$

$I^B(R_7) = \frac{0.9536-0.9393}{0.8} = 0.01787, I^B(R_6) = \frac{0.9536-0.9464}{0.6} = 0.012$

3.11 Use an innovative way to compute reliability importance for system with i.i. units

Usually, we compute the reliability importance by classical way which depend on partial derivative technique or Birnbaum's measure for all component of the systems in order to investigate which is the critical and importance components which has direct or indirect effect on the operation of the systems, while this Thesis provide a new method depend on finding minimal path sets and minimal cut sets for different reliability models which will explain now.

- **Step 1:** Finding minimal path and minimal cut sets.
- **Step 2:** Arrange minimal path sets and minimal cut sets relative to order of them, as ascending order.
- **Step 3:** Arrange minimal path sets relative to higher reliability value, as descending order.

- **Step 4:** Let's start with the minimal cut sets of lowest order (step two), if this set/sets contain one unit then we can say that it has highest importance (level one).
- **Step 5:** If minimal cut sets of lowest order (has more than one unit), we select the unit found in this sets and in the minimal path sets which has lowest order or it has higher reliability value (level one). If there is two units (or more) it has the same previous specifications, we select the unit which possesses more repeated in minimal cut sets and have lowest orderin this case, this unit is more important than the importance of other units that are less than repeatedly.
- **Step 6:** In order to find the reliability importance of other levels (two, three, ..., etc.), we repeat the steps (1-5) and move on to the minimal cut sets of higher order

3.12 Comparison between a new method and Birnbaum's measure

In this section we will consider some complex systems due to show mechanism by which the new method works and compare it with partial derivative technique (Birnbaum's measure).

Note 3.12.1 If the network has independent identical units, i.e., $R_i, i = 1, 2, \ldots, n$, have same reliability, put, $R_1 = R_2 = R_3 = ... = R_n$

Example 3.1 Electromagnetic system inside aircraft. consider a complex network in Fig.(3.4), of Electromagnetic system inside aircraft Fig.(3.4). The reliability of this system is computed as in [34]:

$$\begin{aligned} R_S =& R_1R_4 + R_1R_3R_7 + R_2R_4R_5 + R_5R_6R_7 - R_1R_2R_4R_5 - R_1R_3R_4R_7 + R_1R_2R_6R_7 + \\ & R_2R_3R_5R_7 + R_3R_4R_5R_6 + R_1R_2R - 3R_5R_7 - R_1R_2R_3R_6R_7 - R_1R_3R_4R_5R_6 - \\ & R_1R_2R_4R_6R_7 - R_2R_3R_4R_5R_6 - R_1R_2R_5R_6R_7 - R_2R_3R_4R_5R_7 - R_1R_3R_5R_6R_7 - \\ & R_1R_4R_5R_6R_7 - R_2R_3R_5R_6R_7 - R_2R_4R_5R_6R_7 - R_3R_4R_5R_6R_7 + R_1R_2R_3R_4R_5R_6 + \\ & R_1R_2R_3R_4R_5R_7 + R_1R_2R_3R_4R_6R_7 + 2R_1R_2R_3R_5R_6R_7 + 2R_1R_2R_4R_5R_6R_7 + \\ & 2R_1R_3R_4R_5R_6R_7 + 2R_2R_3R_4R_5R_6R_7 - 3R_1R_2R_3R_4R_5R_6R_7 \end{aligned} \tag{3.19}$$

Let the given complex network has independent identical units, i.e., $R_i, i = 1, \ldots, 7$, have same reliability, put $R_i = 0.8$

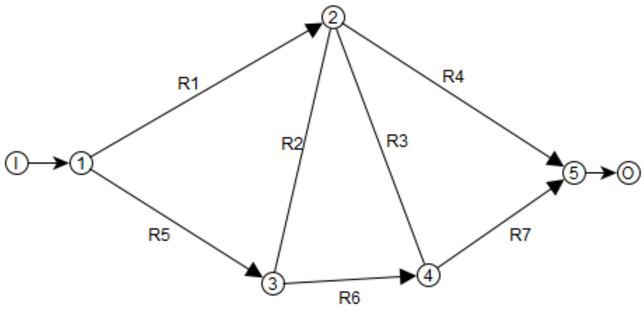


Figure 3.4: Electromagnetic system inside aircraft [34].

Now we will find the reliability importance by using Birnbaum's measure

$$\frac{\partial R_S(t)}{\partial R_1(t)} = R_4 + R_3R_7 - R_2R_4R_5 - R_3R_4R_7 + R_2R_6R_7 - R_2R_3R_5R_7 - R_2R_3R_6R_7 - R_3R_4R_5 R_6 - R_2R_4R_6R_7 - R_2R_5R_6R_7 - R_3R_5R_6R_7 - R_4R_5R_6R_7 + R_2R_3R_4R_5R_6 + R_2 R_3R_4R_5R_7 + R_2R_3R_4R_6R_7 + 2R_2R_3R_5R_6R_7 + 2R_2R_4R_5R_6R_7 + 2R_3R_4R_5R_6 R_7 - 3R_2R_3R_4R_5R_6R_7 \tag{3.20}$$

$\frac{\partial RS(t)}{\partial R_1(t)}$ = R+R$^2 - R^3 - 7R^4 + 9R^5 - 3R^6$

$$\frac{\partial R_S(t)}{\partial R_2(t)} = R_4R_5 - R_1R_4R_5 + R_1R_6R_7 + R_3R_5R_7 - R_1R_3R_5R_7 - R_1R_3R_6R_7 - R_1R_4R_6R_7 - R_3R_4R_5R_6 - R_1R_5R_6R_7 - R_3R_4R_5R_7 - R_3R_5R_6R_7 - R_4R_5R_6R_7 + R_1R_3R_4 R_5R_6 + R_1R_3R_4R_5R_7 + R_1R_3R_4R_6R_7 + 2R_1R_3R_5R_6R_7 + 2R_1R_4R_5R_6R_7 + 2R_3 R_4R_5R_6R_7 - 3R_1R_3R_4R_5R_6R_7 \tag{3.21}$$

$\frac{\partial R_S(t)}{\partial R_2(t)}$=$\mathrm{R}^2 + R^3 - 8R^4 + 9R^5 - 3R^6$

$$\frac{\partial R_S(t)}{\partial R_3(t)} = R_1R_7 – R_1R_4R_7 + R_2R_5R_7 + R_4R_5R_6 – R_1R_2R_5R_7 – R_1R_2R_6R_7 - R_1R_4R_5R_6 – R_2 R_4R_5R_6 – R_2R_4R_5R_7 – R_1R_5R_6R_7 - R_2R_5R_6R_7 – R_4R_5R_6R_7 + R_1R_2R_4R_5R_6 + R_1R_2R_4R_5R_7 + R_1R_2R_4R_6R_7 + 2R_1R_2R_5R_6R_7 + 2R_1R_4R_5R_6R_7 + 2R_2R_4R_5R_6R_7 – 3R_1R_2R_4R_5R_6R_7 \tag{3.22}$$

$\frac{\partial R_S(t)}{\partial R_3(t)}$=$\mathrm{R}^2 + R^3 - 8R^4 + 9R^5 - 3R^6$

$$\frac{\partial R_S(t)}{\partial R_4(t)} = R_1 + R_2R_5 - R_1R_2R_5 - R_1R_3R_7 + R_3R_5R_6 – R_1R_3R_5R_6 – R_1R_2R_6R_7 – R_2R_3R_5 R_6 - R_2R_3R_5R_7 – R_1R_5R_6R_7 – R_2R_5R_6R_7 - R_3R_5R_6R_7 + R_1R_2R_3R_5R_7 + R_1R_2R_3 R_5R_7 + R_1R_2R_3R_6R_7 + 2R_1R_2R_5R_6R_7 + 2R_1R_3R_5R_6R_7 + 2R_2R_3R_5R_6R_7 - 3R_1R_2R_3R_5R_6R_7 \tag{3.23}$$

$\frac{\partial R_S(t)}{\partial R_4(t)}$=$\mathrm{R}+\mathrm{R}^2 - R^3 - 7R^4 + 9R^5 - 3R^6$

$$\frac{\partial R_S(t)}{\partial R_5(t)} = R_2R_4 + R_6R_7 – R_1R_2R_4 + R_2R_3R_7 + R_3R_4R_6 – R_1R_2R_3R_7 – R_1R_3R_4R_6 – R_2R_3R_4R_6 – R_1R_2R_6R_7 – R_2R_3R_4R_7 – R_1R - 3R_6R_7 – R_1R_4R_6R_7 - R_2R_3R_6R_7 - R_2R_4R_6R_7 – R_3R_4R_6R_7 + R_1R_2R_3R_4R_6 + R_1R_2R_3R_4R_7 + 2R_1R_2R_3R_6R_7 + 2R_1R_2R_4R_6R_7 + 2R_1 R_3R_4R_6R_7 + 2R_2R_3R_4R_6R_7 - 3R_1R_2R_3R_4R_6R_7 \tag{3.24}$$

$\frac{\partial R_S(t)}{\partial R_5(t)}$=$2\mathrm{R}^2 + R^3 - 10R^4 + 10R^5 - 3R^6$

$\frac{\partial R_S(t)}{\partial R_6(t)}= R_5R_7 + R_1R_2R_7 + R_3R_4R_5 - R_1R_2R_3R_7 - R_1R_3R_4R_5 - R_1R_2R_4R_7 - R_2R_3R_4R_5 -$
$R_1R_2R_5R_7 - R_1R_3R_5R_7 - R_1R_4R_5R_7 - R_2R_3R_5R_7 - R_2R_4R_5R_7 - R_3R_4R_5R_7 + R_1$
$R_2R_3R_4R_5 + R_1R_2R_3R_4R_7 + 2R_1R_2R_3R_5R_7 + 2R_1R_2R_4R_5R_7 + 2R_1R_3R_4R_5R_7$
$+2R_2R_3R_4R_5R_7 - 3R_1R_2R_3R_4R_5R_7$

(3.25)

$\frac{\partial R_S(t)}{\partial R_6(t)}=\mathrm{R}^2 + 2R^3 - 10R^4 + 10R^5 - 3R^6$

$\frac{\partial R_S(t)}{\partial R_7(t)}= R_1R_3 + R_5R_6 - R_1R_3R_4 + R_1R_2R_6 + R_2R_3R_5 - R_1R_2R_3R_5 - R_1R_2R_3R_6 - R_1R_2R_4$
$R_6 - R_1R_2R_5R_6 - R_2R_3R_4R_5 - R_1R_3R_5R_6 - R_1R_4R_5R_6 - R_2R_3R - 5R_6 - R_2R_4R_5R_6 -$
$R_3R_4R_5R_6 + R_1R_2R_3R_4R_5 + R_1R_2R_3R_4R_6 + 2R_1R_2R_3R_5R_6 + 2R_1R_2R_4R_5R_6 + 2R_1$
$R_3R_4R_5R_6 + 2R_2R_3R_4R_5R_6 - 3R_1R_2R_3R_4R_5R_6$

(3.26)

$\frac{\partial R_S(t)}{\partial R_7(t)}=2R^2 + R^3 - 10R^4 + 10R^5 - 3R^6$

Table (3.17) shows The reliability importance and level of units in Fig.(3.4).

Table 3.17: The reliability importance and level of units in Figure (3.4)

Components	$\mathrm{I}^B(i)$	Level
R_1, R_4	0.223488	1
R_5, R_7	0.186368	2
R_6	0.058368	3
R_2, R_3	0.037888	4

Now we will use the steps of the proposed method to calculate the significance (importance) of units for a complex network in Fig.(3.4).

- **Step 1:** The sets minimal paths
  $S = \{\{R_1, R_4\}, \{R_2, R_4, R_5\}, \{R_1, R_3, R_7\}, \{R_5, R_6, R_7\}, \{R_2, R_3, R_5, R_7\},$
  $\{R_1, R_2, R_6, R_7\}, \{R_3, R_4, R_5, R_6\}, \{R_1, R_2, R_3, R_4, R_6\}\}$

  The sets minimal cuts
  C=$\{\{R_1, R_5\}, \{R_4, R_7\}, \{R_1, R_2, R_6\}, \{R_3, R_4, R_6\}\{R_1, R_2, R_3, R_7\},$
  $\{R_2, R_3, R_4, R_5\}\}$

- **Step 2 and step 3 :** Arrange minimal paths by minimal order and higher reliability
  .

  minimal path of order 2 are;$p_1 = \{R_1, R_4\}$
  minimal path of order 3 are;$p_2 = \{R_1, R_3, R_7\}, p_3 = \{R_5, R_6, R_7\},$
  $p_4 = \{R_2, R_4, R_5\}$
  minimal path of order 4 are; $p_5 = \{R_3, R_4, R_5, R_6\}, p_6 = \{R_1, R_2, R_6, R_7\},$
  $p_7 = \{R_2, R_3, R_5, R_7\}$
  minimal path of order 5 are; $p_8 = \{R_1, R_2, R_3, R_4, R_6\}$

  Arrange minimal cuts by minimal order
  minimal cuts of order 2 are : $C_1 = \{R_1, R_5\}, C_2 = \{R_4, R_7\}$
  minimal cuts of order 3 are : $C_3 = \{R_1, R_2, R_6\}, C_4 = \{R_3, R_4, R_6\}$
  minimal cuts of order 4 are : $C_5 = \{R_1, R_2, R_3, R_7\}, C_6 = \{R_2, R_3, R_4, R_5\}$

- **Step 4 and step 5 :**
  minimal cuts of order 2 are: $C_1 = \{R_1, R_5\}, C_2 = \{R_4, R_7\}$.
  Note that R_1, R_4, R_5 and R_7 belong to minimal cuts sets of order 2 (C_1 or C_2), but R_1 and R_4 found in minimal path set of order 2 (p_1). In addition to that R_1 and R_4 have same repeat in all minimal cut sets, then R_1 and R_4 have same important, therefore, we can say that R_1 and R_4 lies in the level one. Followed by R_5 and R_7 are found in minimal path sets of order 3 (p_2 or p_3 or p_4) and they have same repeat in all minimal cuts sets, and this assures us that R_5 and R_7 have same important, then R_5 and R_7 lies in the level two.

- **Step 6:**
  minimal cut sets of order 3 are: $C_3 = \{R_1, R_2, R_6\}, C_4 = \{R_3, R_4, R_6\}$ notes that R_2, R_3 and R_6 belong to minimal cuts sets of order 3 (C_3 or C_4) and all of them found in minimal path sets of order 3 (p_2 or p_3 or p_4), but R_6 belong to C_3 and C_4, thus R_6 It is on the third level of importance, followed byR_2 and R_3 lies in level four which have the same repetitions in all minimal cut sets.
  Through the foregoing, we note that the importance results in the new method are completely identical to the results presented in Table (3.17).

Example 3.2 Consider a complex network in Fig.(3.1), and Table (3.1), the reliability of this system is computed as in equation (3.5). Let the given complex network has independent identical units, i.e. $R_i, i = 1, \ldots, 7$, have same reliability, put $R_i = 0.8$.
Here we do not need to check the repetition of this unit in the remaining minimal cut sets, because repeating the unit in the same order gives more importance to this unit, regardless of the rest minimal cut sets.

Now we will use the steps of the proposed method to calculate the importance of units for a complex network.

- **Step 1:** The minimal path sets are
  $S = \{\{R_2, R_7\}, \{R_1, R_3, R_7\}, \{R_1, R_4, R_6\}, \{R_2, R_5, R_6\}, \{R_2, R_3, R_4, R_6\}, \{R_1, R_3, R_5, R_6\},$
  $\{R_1, R_4, R_5, R_7\}, \{R_2, R_3, R_4, R_5 R_7\}\}.$

  And The minimal cuts sets are
  $C = \{\{R_1, R_2\}, \{R_6, R_7\}, \{R_2, R_3, R_4, \}, \{R_4, R_5 R_7, \}, \{R_1, R_3, R_5, R_7\}, \{R_2, R_3, R_5, R_6\}\}$

- **Step 2 and step 3:** Rearrange minimal paths sets by ascending order as follows:
  minimal path of order 2 are: $p_1 = \{R_2, R_7\}$
  minimal paths of order 3 are: $p_2 = \{R_1, R_3, R_7\}, P_3 = \{R_1, R_4, R_6\},$
  $p_4 = \{R_2, R_5, R_6\}$
  minimal paths of order 4 are:$p_5 = \{R_2, R_3, R_4, R_6\}, P_6 = \{R_1, R_3, R_5, R_6\}$
  $p_7 = \{R_1, R_4, R_5, R_7\}$

minimal path of order 5 are: $p_8 = \{R_2, R_3, R_4, R_5, R_7\}$.
Rearrange minimal cuts by minimal order as follows:
minimal cuts of order 2 are:$C_1 = \{R_1, R_2\}, C_2 = \{R_6, R_7\}$
minimal cuts of order 3 are:$C_3 = \{R_2, R_3, R_4, \}$, C$_4 = \{R_4, R_5, R_7\}$
minimal cuts of order 4 are:$C_5 = \{R_1, R_3, R_5, R_7\}$, C$_6 = \{R_2, R_3, R_5, R_6\}$

- **Step 4 and step 5:** minimal cuts sets of order 2 are:$C_1 = \{R_1, R_2\}, C_2 = \{R_6, R_7\}$ notes that R_1, R_2, R_6 and R_7 belong to minimal cut sets of order 2 (C_1 or C_2), but R_2 and R_7 found in minimal path of order 2 (p_1) and both of them have the same repetition in all minimal cut sets, Therefore, we can say that both are on the same level of importance, then R_2 and R_7 lies in level one.
Followed by R_1 and R_6, found in minimal path sets of order 3 (p_2 or p_3 or p_4) and both have the same repetition in all minimal cut sets, then R_1 and R_6 have same important and lies in level two.

- **Step 6:** minimal cuts of order 3 are:$C_3 = \{R_2, R_3, R_4\}$, $C_4 = \{R_4, R_5, R_7\}$ notes that R_3, R_4 and R_5 belong to C_3 or C_4 and found in minimal path sets of order 3, but R_4 belong to C_3 and C_4, thus R_4 It is on the third level of importance, but R_3 and R_5 have the same repetition in all minimal cut sets, then R_3 and R_5 have same important and lies in level four.

Example 3.3 Consider complex network consists of 8 elements in Fig.(3.3), the reliability of this system is computed as in equation (2.4). Let the given complex network has independent identical units, i.e. R_i, i=1, ..., 8, have same reliability, put $R_i = 0.85$.

Here we do not need to check the repetition of this unit in the remaining minimal cut sets, because repeating the unit in the same order gives more importance to this unit, regardless of the rest minimal cut sets. As we did in the previous example of using the Birnbaum's measure, we will get the following Table (3.18), reliability importance and level of units in Fig.(3.3).

Table 3.18: Reliability importance and level of units in Figure (3.3)

Components	$I^B(i)$	Level
R_1, R_2	0.1534	1
R_4, R_5	0.0291	2
R_8	0.0255	3
R_3	0.00967	4
R_6, R_7	0.0051	5

Now we will use the steps of the proposed method to calculate the importance of units for a complex network in Fig.(3.3).

- Step 1: The sets minimal paths
  $S = \{\{R_1, R_4\}, \{R_2, R5\}, \{R_1, R_3, R_5\}, \{R_2, R_3, R_4\}, \{R_1, R_6, R_8\},$
  $\{R_2, R_7, R_8\}, \{R_1, R_3, R_7, R_8\}, \{R_2, R_3, R_6, R_8\}\}$
  And The minimal cuts sets are
  $C = \{\{R_1, R_2\}, \{R_4, R_5, R_8\}, \{R_4, R_5, R_6, R_7\}, \{R_1, R_3, R_5, R_7\},$
  $\{R_2, R_3, R_4, R_6\}, \{R_1, R_3, R_5, R_8\}, \{R_2, R_3, R_4, R_8\}\}$

- Step 2 and step 3: Rearrange minimal paths sets by ascending order as follows:
  minimal paths of order 2 are : $P_1 = \{R_1, R_4\}, P_2 = \{R_2, R_5\}$
  minimal paths of order 3 are :$P_3 = \{R_1, R_3, R_5\}, P_4 = \{R_2, R_3, R_4\},$
  $P_5 = \{R_1, R_6, R_8\}, P_6 = \{R_2, R_7, R_8\}$
  minimal path of order 4 are :$P_7 = \{R_1, R_3, R_7, R_8\}, P_8 = \{R_2, R_3, R_6, R_8\}$ Rearrange minimal cuts by minimal order as follows:
  minimal cuts of order 2 are :$C_1 = \{R_1, R_2\}$
  minimal cuts of order 3 are :$C_2 = \{R_4, R_5 R_8\}$
  minimal cuts of order 4 are :$C_3 = \{R_4, R_5, R_6, R_7\}, C_4 = \{R_1, R_3 R_5, R_7\},$
  $C_5 = \{R_2, R_3, R_4, R_6\}, C_6 = \{R_1, R_3, R_5, R_8\},$
  $C_7 = \{R_2, R3, R_4, R_8\}$

- Step 4 and step 5: minimal cuts of order 2 are :$C_1 = \{R_1, R_2\}$ notes that R_1 and R_2 belong to minimal cut sets of order 2 (C_1) R_1 and R_2 found in minimal path of order 2

$(p_1 or p_2)$ and both of them have the same repetition in all minimal cut sets, Therefore, we can say that both are on the same level of importance, then R_1 and R_2 lies in level one.

- Step 6: minimal cuts of order 3 are : $C_2 = \{R_4, R_5 R_8\}$ notes that R_4, R_5 and R_8 belong to minimal cut sets of order 3 (C_2) , but R_4 and R_5 and found in minimal path sets of order 3, R_4 and R_5 have same repeat in all minimal cuts than R_4 and R_5 have same important in level two. Followed by R_8 important in level three.
  Notes that R_3, R_6 and R_7 (order 4), but R_3, R_6 and R_7 found in high reliability minimal path of order 4, but R_3 repeat 4 in minimal cuts in this order, but $R_6 and R_7$ repeat 2 in minimal cuts in this order thus R_3 important in level four. Followed by R_6 and R_7 in important in level five.

Chapter 4

The relations between the measure of reliability importance and some concepts of reliability

4.1 Introduction

We discussed in the previous sections the importance of reliability and methods of measurement. In this chapter, we address the relationship is important in reliability and their relationship to the allocation and redundancy, and access to some of theorems that study the reasons for the increase and decrease in the reliability of systems, and their relationship to the concept of reliability and themes which they can improve the reliability of the system by using the new methods can reduce the cost, time, size and mass and it can be adopted in the future.

4.2 General concepts of reliability redundancy

Reliability redundancy is frequently used in system design to improve system reliability, especially when increasing component reliability is challenging. If the state of the art dictates that producing highly reliable components is either impossible or prohibitively expensive, we can improve system reliability by employing the technique of introducing redundancies, which entails the deliberate creation of new parallel paths in a system.

4.2.1 Element redundancy

A separate path is provided for each system component. In the Figure (4.1) shown, the seires components C_1 and C_2, additional components of C_1 and C_2 which are the individual components of the system are as a result, the reliability of each component and, as a result, the overall system has improved. The reliability of this system is $R_s = R_{C_1} R_{C_2}$ [13,18].

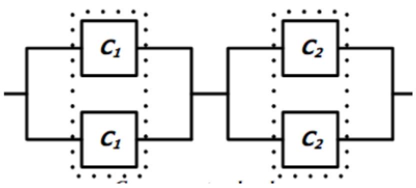


Figure 4.1: Element redundancy.

After element redundancy $R^{\star}_{C_1} = 1 - (1 - R_{C_1})^2, R^{\star}_{C_2} = 1 - (1 - R_{C_2})^2$

$$R_E s = R^{\star}_{C_1} R^{\star}_{C_2} \tag{4.1}$$

4.2.2 Unit redundancy

There is an extra path for the entire system. In the Figure (4.2) shown, C_1 and C_2 are components of a system and two such components are provided in parallel resulting in improving the reliability of the entire system [36,40].

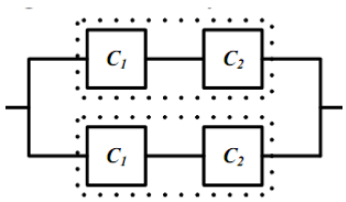


Figure 4.2: Unit redundancy.

It can be calculated unit redundancy from the following relationship:

$$R^{\star}_{Us} = 1\text{–}(1\text{–}R_s)^2 \tag{4.2}$$

When $R^{\star}_{Us}$ The reliability in unit redundancy R_s is The reliability in system

4.3 Applicability of redundancy systems

In this section, we show the relations between system reliability and the redundancy systems of units that have the highest reliability importance and compare it with methods of unit redundancy and element redundancy as in the following example.

Example 4.1 Consider mixed system with five elements A,B,C,D and F with reliabilities 0.7 , 0.7 , 0.8 ,0.6 and 0.9 respectively. See Figure (4.3)[37]. The reliability of this system:

$$R_S = R_C R_F + R_A R_D R_F + R_B R_D R_F - R_A R_B R_D R_F - R_A R_C R_D R_F - R_B R_C R_D R_F + R_A R_B R_C R_D R_F \quad (4.3)$$

The value of reliability in mixed system is $R_s = 0.8183$. Now find the reliability importance

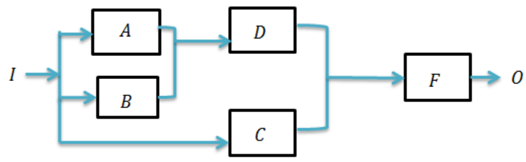


Figure 4.3: Mixed system.

of components by Birnbaum's measure:

$$\frac{\partial R_S(t)}{\partial R_A(t)} = R_D R_F + R_B R_D R_F - R_C R_D R_F + R_B R_C R_D R_F$$

$$\frac{\partial R_S(t)}{\partial R_B(t)} = R_D R_F - R_A R_D R_F - R_C R_D R_F + R_A R_C R_D R_F$$

$$\frac{\partial R_S(t)}{\partial R_C(t)} = R_F - R_A R_D R_F - R_B R_D R_F + R_A R_B R_D R_F$$

$$\frac{\partial R_S(t)}{\partial R_D(t)} = R_A R_F + R_B R_F - R_A R_B R_F - R_A R_C R_F - R_B R_C R_F + R_A R_B R_C R_F$$

$$\frac{\partial R_S(t)}{\partial R_F(t)} = R_C + R_A R_D + R_B R_D - R_A R_B R_D - R_A R_C R_D - R_B R_C R_D + R_A R_B R_C R_D$$

Table (4.1) shown reliability importance and level of units in mixed system.

Table 4.1: Reliability importance and level of units in mixed system

Components	$I^B(i/t)$	Level
R_F	0.9092	1
R_C	0.4086	2
R_D	0.1638	3
R_A, R_B	0.0324	4

4.3.1 Element and unit redundancy techniques for given mixed system

In **element redundancy** case, the shape of the system is as shown in Figure (4.4):

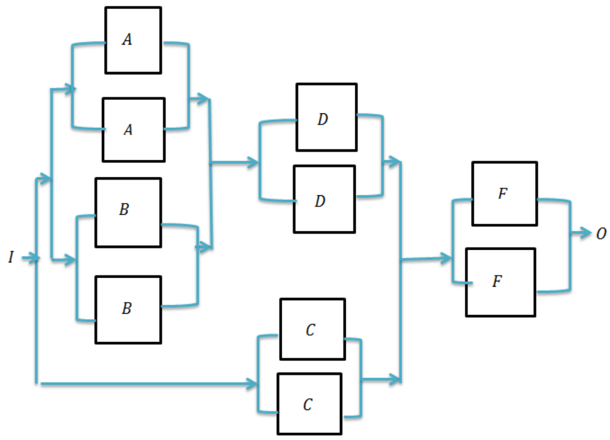


Figure 4.4: Element redundancy for mixed system.

$R_A^\star = 1-(1-R_A)^2 = 0.91, R_B^\star = 1-(1-R_B)^2 = 0.91, R_C^\star = 1-(1-R_C)^2 = 0.96$
$R_D^\star = 1-(1-R_D)^2 = 0.84, R_F^\star = 1-(1-R_F)^2 = 0.99.$
If we put $R_A^\star, R_B^\star, R_C^\star, R_D^\star$ and $R_F^\star$ in equation (4.3), we get, $R_{Es}^\star = 0.9834$, when $R_{Es}^\star$ is the value of reliability in element redundancy.

In **unit redundancy** case, the shape of the system is as shown in Figure (4.5):

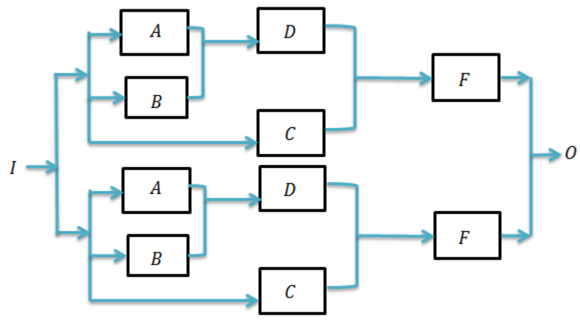


Figure 4.5: Unit redundancy for for mixed system.

The value of reliability in unit redundancy ($R^{\star}_{Us}$) for mixed system:

$$R^{\star}_{Us} = 1–(1–R_s)^2 = 1 - (1 - 0.8183)^2 = 0.9669$$

4.3.2 Redundancy for units F and C for given mixed system

In case, the shape of the system is as shown in Figure (4.6):

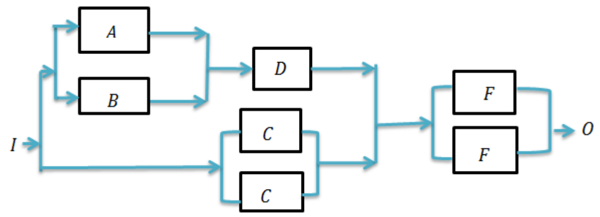


Figure 4.6: Redundancy units of F and C for mixed system.

$R^{\star}_{C} = 1 - (1 - R_C)^2 = 0.96, R^{\star}_{F} = 1 - (1 - R_F)^2 = 0.99$, if we put $R^{\star}_{C}, R^{\star}_{F}$ in equation (4.3), we get, $R^{\star}_{s} = 0.9720$. The value of reliability in redundancy of units F and C = 0.972.

Notes that the value of reliability in redundancy of units F and C in mixed system better than the value of reliability in unit redundancy for mixed system and, and it's my approximation the value of reliability in element redundancy for mixed system.

4.3.3 Redundancy for units F and D for given mixed system

In case, the shape of the system is as shown in Figure (4.7):

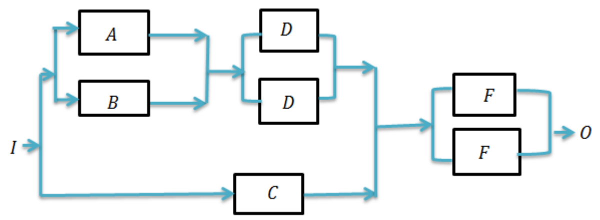


Figure 4.7: Redundancy of units F and D for mixed system.

$R_D^\star = 1-(1-R_D)^2 = 0.84, R_F^\star = 1-(1-R_F)^2 = 0.99$, if we put $R_D^\star, R_F^\star$ in equation (4.3), we get, $R_s^\star = 0.9434$. The value of reliability in redundancy of units F and D = 0.9434.

Notes that The value of reliability in redundancy of units F and C better than The value of reliability in redundancy of units F and D, because reliability importance R_C in level 2, but reliability importance R_D in level 3.

4.3.4 Redundancy for units A and B for given mixed system

In case, the shape of the system is as shown in Figure (4.8):

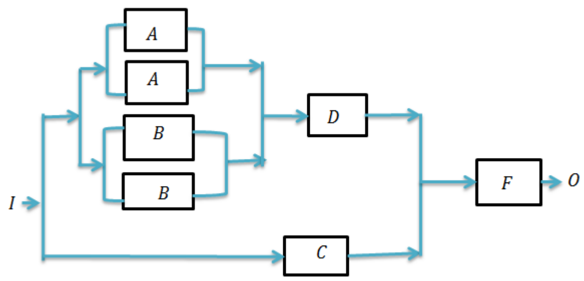


Figure 4.8: Redundancy of unit A and B for mixed system.

$R_A^\star = 1-(1-R_A)^2 = 0.91, R_B^\star = 1-(1-R_B)^2 = 0.91$, if we put $R_A^\star, R_B^\star$ in equation (4.3), we get, $R_s^\star = 0.8271$. The value of reliability in redundancy of units A and B = 0.8271.

Notes that the value of reliability in redundancy of units A and B gives a small increase in reliability because reliability importance R_A and R_B in level 4.

4.4 Application of redundancy to complex systems

Now we well show, the relations between system reliability and the redundancy systems of units that have the highest reliability importance and compare it with methods of unit redundancy and element redundancy of complex systems as in the following example.

Example 4.2 Consider complex system consists of 7 elements with reliabilities 0.6, 0.7, 0.6, 0.9, 0.8,0.9 and 0.9 respectively, the relationship reliability importance and redundancy in Figure (4.9)[79]:

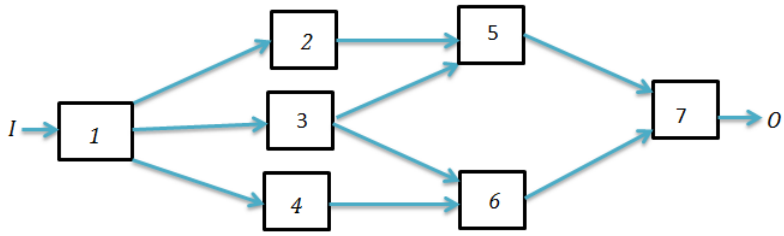


Figure 4.9: Complex system with seven units.

The reliability of this system:

$$R_S = R_1R_2R_5R_7 + R_1R_3R_5R_7 + R_1R_3R_6R_7 + R_1R_4R_6R_7 - R_1R_2R_3R_5R_7 - R_1R_3R_4R_6R_7 - R_1R_3R_5R_6R_7 - R_1R_2R_4R_5R_6R_7 + R_1R_2R_3R_4R_5R_6R_7 \tag{4.4}$$

The value of reliability in complex system $R_s = 0.5155$. Now find the reliability importance of components by Birnbaum's measure

$$\frac{\partial R_S(t)}{\partial R_1(t)} = R_2R_5R_7 + R_3R_5R_7 + R_3R_6R_7 + R_4R_6R_7 - R_2R_3R_5R_7 - R_3R_4R_6R_7 - R_3R_5R_6R_7 - R_2R_4R_5R_6R_7 + R_2R_3R_4R_5R_6R_7.$$

$$\frac{\partial R_S(t)}{\partial R_2(t)} = R_1R_5R_7 - R_1R_3R_5R_7 - R_1R_4R_5R_6R_7 + R_1R_3R_4R_5R_6R_7.$$

$$\frac{\partial R_S(t)}{\partial R_3(t)} = R_1R_5R_7 + R_1R_6R_7 - R_1R_2R_5R_7 - R_1R_4R_6R_7 - R_1R_5R_6R_7 + R_1R_2R_4R_5R_6R_7.$$

$$\frac{\partial R_S(t)}{\partial R_4(t)} = R_1R_6R_7 - R_1R_3R_6R_7 - R_1R_2R_5R_6R_7 + R_1R_2R_3R_5R_6R_7.$$

$$\frac{\partial R_S(t)}{\partial R_5(t)} = R_1R_2R_7 + R_1R_3R_7 - R_1R_2R_3R_7 - R_1R_3R_6R_7 - R_1R_2R_4R_6R_7 + R_1R_2R_3R_4R_6R_7.$$

$$\frac{\partial R_S(t)}{\partial R_6(t)} = R_1R_3R_7 + R_1R_4R_7 - R_1R_3R_4R_7 - R_1R_3R_5R_7 - R_1R_2R_4R_5R_7 + R_1R_2R_3R_4R_5R_7.$$

$$\frac{\partial R_S(t)}{\partial R_7(t)} = R_1R_2R_5 + R_1R_3R_5 + R_1R_3R_6 + R_1R_4R_6 - R_1R_2R_3R_5 - R_1R_3R_4R_6 - R_1R_3R_5R_6 - R_1R_2R_4R_5R_6 + R_1R_2R_3R_4R_5R_6.$$

Table (4.2) shown redundancy importance and level of units in complex system with seven units.

Table 4.2: Redundancy importance and level of units in complex system with seven units

Components	$I^B(i/t)$	Level
1	0.8591	Level 1
7	0.5727	Level 2
6	0.1503	Level 3
4	0.0855	Level 4
5	0.0611	Level 5
3	0.0343	Level 6
2	0.0328	Level 7

4.4.1 Applied element and unit redundancy techniques for complex system

In **element redundancy** case, the shape of the system is as shown in Figure (4.10):

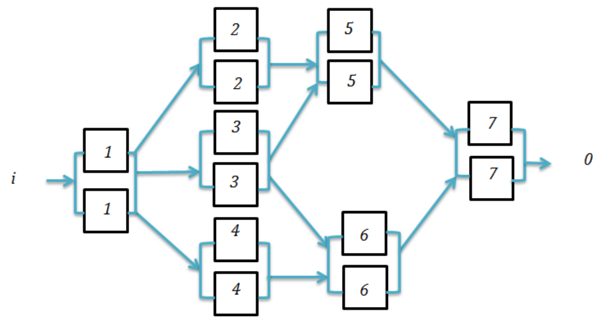


Figure 4.10: Element redundancy for complex system with seven units.

$R_1^\star = 1-(1-0.6)^2 = 0.84, R_2^\star = 1-(1-0.7)^2 = 0.91, R_3^\star = 1-(1-0.6)^2 = 0.84, R_4^\star = 1-(1-0.9)^2 = 0.99, R_5^\star = 1-(1-0.8)^2 = 0.96, R_6^\star = 1-(1-0.9)^2 = 0.99, R_7^\star = 1-(1-0.9)^2 = 0.99$, if we put $R_1^\star$, $R_2^\star$, $R_3^\star$, $R_4^\star$, $R_5^\star$, $R_6^\star$ and $R_7^\star$ in equation (4.4), we get, the value of reliability in element redundancy, $R_{Es}^\star = 0.8310$

In **unit redundancy** case, the shape of the system is as shown in Figure (4.11):

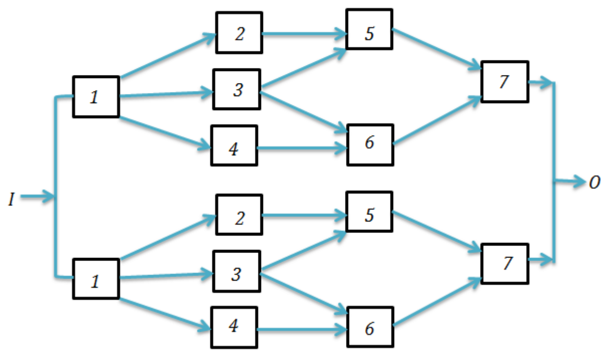


Figure 4.11: Unit redundancy for given complex system.

The value of reliability in unit redundancy, $R_{Us}^\star = 1–(1–R_s)^2 = 1-(1-0.5155)^2 = 0.7652$

4.4.2 Redundancy for units 1 and 7 for complex system

In case, the shape of the system is as shown in Figure (4.12):

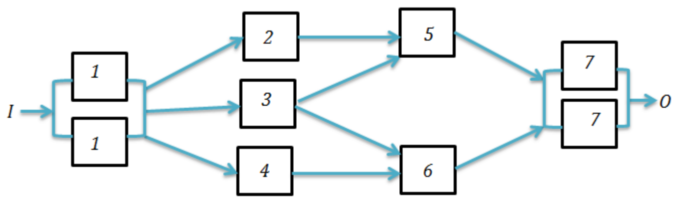


Figure 4.12: Redundancy for units 1 and 7 for given complex system

$R_1^\star = 1-(1-0.6)^2 = 0.84, R_7^\star = 1-(1-0.9)^2 = 0.99$, if we put $R_1^\star$ and $R_7^\star$ in equation (4.4), we get the value of reliability in redundancy for units 1 and 7 = 0.7938.

Notes that redundancy units 1 and 7 better than unit redundancy. And it's my approximation the value of reliability in element redundancy for complex system.

4.4.3 Redundancy for units 2 and 4 for complex system

In case, the shape of the system is as shown in Figure (4.13):

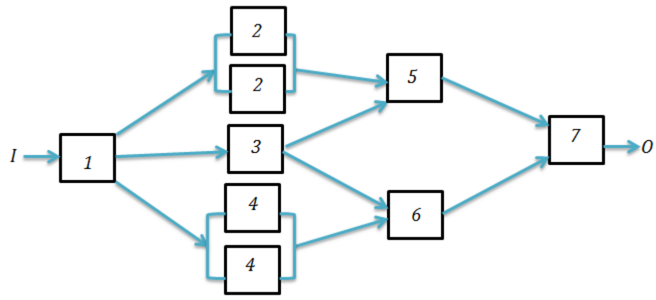


Figure 4.13: Redundancy for units 2 and 4 for given complex system.

$R_2^\star = 1-(1-0.7)^2 = 0.91, R_4^\star = 1-(1-0.9)2 = 0.99$, if we put $R_2^\star$ and $R_4^\star$ in equation (4.4), we get the value of reliability redundancy of units 2 and 4 = 0.5271.

Notes that the value of reliability redundancy of units 2 and 4 gives a small increase in reliability because reliability importance R_2 in level 7 and R_4 in level 4 and that the value of reliability redundancy of units 1 and 7 better than the value of reliability redundancy unit 2 and 4, because reliability importance R_1 in level 1 and R_7 in level 2.

4.4.4 Redundancy for units 1,6 and 7 for complex system

In case, the shape of the system is as shown in Figure(4.14):

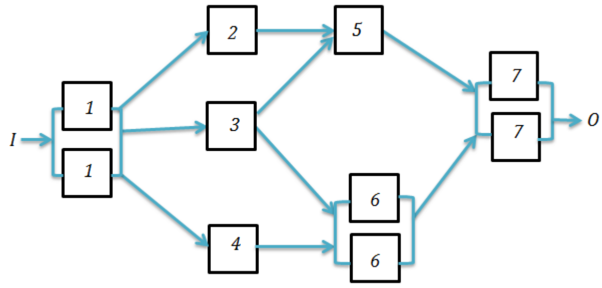


Figure 4.14: Redundancy for units 1,6 and 7 for given complex system.

$R_1^\star = 1-(1-0.6)^2 = 0.84, R_6^\star = 1-(1-0.9)^2 = 0.99, R_7^\star = 1-(1-0.9)^2 = 0.99$, if we put $R_1^\star, R_6^\star$ and $R_7^\star$ in equation (4.4), we get the value of reliability redundancy of units 1,6 and 7 = 0.8146.

Notes that the value of reliability redundancy of units 1,6 and 7, it is a small less than the value of reliability element redundancy although the number of units less than half of the units in the system this is due to the reliability importance of the units.

4.5 The relationship among the reliability importance and redundancy, allocation of improving the reliability of systems

In this section, we explain the relationship between the reliability importance, redundancy and allocation of improving the reliability of systems through the following examples.

Consider example (4.1) with equation (4.3) and importance of Table (4.1) we will stetting relationship between importance, redundancy and allocation for Figure (4.3) as following case one

- **Reliability importance:** Put $R_F = 1$ in equation (4.3) we get $Rs = 0.9092$ it is the maximum system reliability improvement that can be reached from the unit R_F (because R_F in level 1)
  In the same way we get.

If $R_C = 1 \Rightarrow Rs = 0.9$.
If $R_D = 1 \Rightarrow Rs = 0.8838$.
If $R_A = 1 \Rightarrow Rs = 0.8280$.
If $R_B = 1 \Rightarrow Rs = 0.8280$ (because R_A and R_B in level 4).

- **Reliability allocation:** If we want to increase the value from (0.8183) to (0.90) we must increase the reliability of unit F (because R_F in level 1) by in equation (4.3)
$Rs = 0.9092F$.
$Rs^\circ = 0.90$.
$R_F^\circ = \frac{0.90}{0.9092} = 0.98988$.
If we want to increase the reliability of the system to 0.90, we must increase the reliability of unit R_F to 0.98988, and the following method can be used to increase the reliability of unit R_F.

- **Using redundancy:** to increase the reliability of unit R_F to 0.98988.

$R_F^\circ = 1 - (1 - R_F)^n$.

$n = \frac{Log(1-R_F^\circ)}{Log(1-R_F)} = \frac{Log(0.01012)}{Log(0.1)} = 1.9948 \approx 2$

The number of duplicate of unit F equal to 1 because $= n–1 = 1$
accordingly, we need to duplicate unit F one time with original unit to obtain the required reliability, and connect them in parallel with the original unit in order to obtain the required reliability and approximate.

case two similarity,

- **Reliability importance:** Put $R_F = 1$ and $R_C = 1$ in equation (4.3) we get $Rs = 1$ it is the maximum system reliability improvement that can be reached from the units R_F and R_C (because R_F in level 1 and R_C in level 2).

- **Reliability allocation:** If we want to increase the reliability value from (0.8183) to

(0.99) we will increase the unit s reliability R_F and R_C by equation (4.3)

$$R^\circ s = R^\circ_C R^\circ_F + 0.42R^\circ_F + 0.42R^\circ_F - 0.294R^\circ_F - 0.42R^\circ_C R^\circ_F - 0.42R^\circ_C R^\circ_F + 0.294R^\circ_C R^\circ_F$$

$0.99 = 0.454R^\circ_C R^\circ_F + 0.546R^\circ_F$

$$R^\circ_C = \frac{0.99 - 0.546R^\circ_F}{0.454R^\circ_F} \tag{4.5}$$

but, $R^\circ_C \leq 1$ from equation (4.5), we get:

$$\frac{0.99 - 0.546R^\circ_F}{0.454R^\circ_F} \leq 1$$

$0.99 - 0.546R^\circ_F \leq 0.454R^\circ_F$

$$0.99 \leq R^\circ_F \leq 1$$

So, from equation (4.5), we get:

$$R^\circ_F = \frac{0.99}{0.454R^\circ_C + 0.546} \tag{4.6}$$

but, $R^\circ_F \leq 1$ from equation (4.6), we get:

$$\frac{0.99}{0.454R^\circ_C + 0.546} \leq 1$$

$0.99 \leq 0.454R^\circ_C + 0.546$
$0.99 - 0.546 \leq 0.454R^\circ_C$
$0.444 \leq 0.454R^\circ_C$

$$0.978 \leq R^\circ_C \leq 1$$

If we put $R^\circ_F = R^\circ_C$ in equation (4.5), we get $R^\circ_F = R^\circ_C = 0.9931$ and $R^\circ s = 0.99$

- **Using redundancy:** to increase the reliability of unit R_F to 0.999

$$n = \frac{Log(1 - R^\circ_F)}{Log(1 - R_F)} = \frac{Log(0.0069)}{Log(0.1)} = 2.1611 \approx 3$$

The number of duplicate of unit F to the system = n − 1=2
So, Using redundancy to increase the reliability of unit R_C to 0.9931.

$$n = \frac{Log(1 - R^{\circ}_C)}{Log(1 - R_C)} = \frac{Log(0.0069)}{Log(0.2)} = 3.0919 \approx 4$$

The number of duplicate of unit C to the system = n − 1=3

Consider complex system example (4.2) with equation (4.4) and reliability importance of table (4.2) we will stetting relationship between importance and redundancy for Figure(4.9) as following case one

- **Reliability importance:** Put $R_1 = 1$ in equation (4.4), we get $Rs = 0.8591$ it is the maximum system reliability improvement that can be reached from the unit R_1 (because R_1 in level 1).
  In the same way we get.
  If $R_7 = 1 \Rightarrow Rs = 0.5727$
  If $R_6 = 1 \Rightarrow Rs = 0.5305$
  If $R_4 = 1 \Rightarrow Rs = 0.5240$
  If $R_3 = 1 \Rightarrow Rs = 0.5292$
  If $R_5 = 1 \Rightarrow Rs = 0.5277$
  If $R_2 = 1 \Rightarrow Rs = 0.5253$ (because R_2 in level 7).
- **Reliability allocation:** If we want to increase the value from (0.5155) to (0.85) we must increase the reliability of unit 1 (because R_1 in level 1) by in equation (4.4):

$$Rs = 0.8591 R_1$$

$R^{\circ}s = 0.85$

$$R^{\circ}_1 = \frac{0.85}{0.8591} = 0.98940$$

- **Using redundancy:** to increase the reliability of unit 1 to 0.9894

$$n = \frac{Log(1 - R^{\circ}_1)}{Log(1 - R_1)} = \frac{Log(0.0106)}{Log(0.4)} = 4.9622 \approx 5$$

The number of duplicate of unit 1 to the system = n − 1= 4
accordingly, we need to duplicate unit 1 four time with original unit to obtain the required reliability, and connect them in parallel with the original unit in order to obtain the required reliability and approximate.

case two similarity,

- **Reliability importance:** Put $R_1 = 1$ and $R_7 = 1$ in equation (4.4), we get $Rs = 0.9546$ it is the maximum system reliability improvement that can be reached from the units R_1 and R_7 (because R_1 in level 1 and R_7 in level 2).

- **Reliability allocation:** If we want to increase the reliability value from (0.5155) to (0.95) we will increase the units reliability R_1 and R_7 in equation (4.4):

$$Rs = 0.9546 R_1 R_7$$

put $R^{\circ}s = 0.95$

$$R_1^{\circ} R_7^{\circ} = 0.995181 \tag{4.7}$$

from equation (4.7), we get

$$R_1^{\circ} = \frac{0.995181}{R_7^{\circ}} \leq 1$$

$$0.995181 \leq R_7^{\circ} \leq 1$$

similarity,

$$0.995181 \leq R_1^{\circ} \leq 1$$

put $R_1^{\circ} = R_7^{\circ}$ in equation (4.7), we get $R_1^{\circ} = R_7^{\circ} = 0.997587$ and $R^{\circ}s = 0.95$

- **Using redundancy:** to increase the reliability of unit 1 and 7 to 0.997587

$$R_1^{\circ} = 1 - (1 - R_1)^n$$

$$n = \frac{Log(1 - R_1^{\circ})}{Log(1 - R_1)} = \frac{Log(0.002413)}{Log(0.4)} = 6.577 \approx 7$$

The number of duplicate of unit 1 to the system= n − 1=6

$$n = \frac{Log(1 - R_7^\circ)}{Log(1 - R_7)} = \frac{Log(0.004)}{Log(0.1)} = 2.617 \approx 3$$

The number of duplicate of unit 7 to the system = n − 1=2

Through the previous examples, note by determining the importance of the units in the system, it is possible to know the extent of improvement that can be obtained for the system from which the reliability of this unit can be raised. It is also possible to improve more than one unit to obtain the desired reliability. This method can reduce cost, volume, mass and time when making a calculation on the reliability of a particular system.

4.6 The best reliability value at a given cost using redundancy

To improve the reliability of any within a certain cost, the cost to be improved is extracted from the following relationship:

$$A = C - \sum_{i=1}^{n} C_i \tag{4.8}$$

C represents the total available cost of the system, C_i it is the cost per unit in the system, i is the number of units in the system and

$$U_i = \lfloor \frac{A}{C_i} \rfloor + 1 \tag{4.9}$$

U_i represents the maximum number of units that can be added to the system according to the excess cost. Where similar units are connected in parallel here, switching circuit determines which devices in any given group are functioning properly. Then they make use of such devices at each stage, that result is increase in reliability at each stage. If at each stage i, contains m_i copies of devices D_i, then the probability that all m_i have a malfunction is $(1-r_i)^{m_i}$ which is very less. And the reliability of the stage i becomes $(1-(1-r_i)^{m_i})$. e.g. If $r_i = 0.99$ and $m_i = 2$, then the stage reliability $= (1-(1-r_i)^{m_i}) = 1-(1-0.99)^2 = 0.9999$.

Example 4.3 Consider complex system in in Fig.(4.9) with available cost 280.

Table 4.3: Cost of units

Units	C_i	U_i
1	40	2
2	25	3
3	30	3
4	25	3
5	35	3
6	35	3
7	40	2

Total cost of system $= \sum_{i=1}^{7} C_i = 210$, $A = C - \sum_{i=1}^{7} C_i = 70$, this is the remaining cost of improvement

$$U_1 = \lfloor \frac{70}{40} \rfloor + 1 = 2, U_2 = \lfloor \frac{70}{25} \rfloor + 1 = 3$$

$$U_3 = \lfloor \frac{70}{30} \rfloor + 1 = 3, U_4 = \lfloor \frac{70}{25} \rfloor + 1 = 3$$

$$U_5 = \lfloor \frac{70}{35} \rfloor + 1 = 3, U_6 = \lfloor \frac{70}{35} \rfloor + 1 = 3$$

$$U_7 = \lfloor \frac{70}{40} \rfloor + 1 = 2$$

The reliability of the system is 0.51546 and at a cost of 210. As for the remaining cost of improving the system, it is 70. So, how to improve the system within the remaining cost, and it will be according to the following possibilities:

Table 4.4: Possibilities to duplicate of units

duplicate of units	The reliability value of units that doubled	$\sum C_i$	Rs*
2 and 7	$R_2^\star = 0.91, R_7^\star = 0.99$	275	0.5746
3 and 7	$R_3^\star = 0.84, R_7^\star = 0.99$	270	0.5761
4 and 7	$R_4^\star = 0.99, R_7^\star = 0.99$	265	0.5755
2 two times	$R_2^\star = 0.973$	260	0.5244
2 and 3	$R_2^\star = 0.91, R_3^\star = 0.84$	260	0.5265
2 and 4	$R_2^\star = 0.91, R_4^\star = 0.99$	260	0.5271
2 and 5	$R_2^\star = 0.91, R_5^\star = 0.96$	270	0.5335
2 and 6	$R_2^\star = 0.91, R_6^\star = 0.99$	270	0.5329
3 two times	$R_3^\star = 0.936$	270	0.527
3 and 4	$R_3^\star = 0.84, R_4^\star = 0.99$	265	0.5268
3 and 5	$R_3^\star = 0.84, R_5^\star = 0.96$	275	0.5328
3 and 6	$R_3^\star = 0.84, R_6^\star = 0.99$	275	0.5349
4 two times	$R_4^\star = 0.999$	260	0.5239
4 and 5	$R_4^\star = 0.99, R_5^\star = 0.96$	270	0.5310
4 and 6	$R_4^\star = 0.99, R_6^\star = 0.99$	270	0.5310
5 two times	$R_5^\star = 0.992$	280	0.5272
5 and 6	$R_5^\star = 0.96, R_6^\star = 0.99$	270	0.5321
6 two times	$R_6^\star = 0.999$	280	0.5303
1 and 2	$R_1^\star = 0.84, R_2^\star = 0.91$	265	0.7313
1 and 4	$R_1^\star = 0.84, R_4^\star = 0.99$	265	0.7324
1 and 3	$R_1^\star = 0.84, R_3^\star = 0.84$	270	**0.7332**

We note the best connection to the system units at the given cost highest reliability that can be obtained is in the end case. i.e.; when R_1, R_3 duplicated. See Figure (4.15):

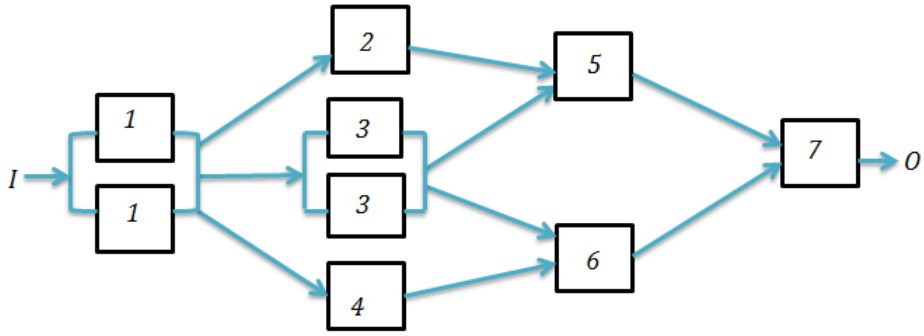


Figure 4.15: The best connection to the system units at the given cost.

Through the above example, we conclude that an increase in the cost of the system does not mean an increase in the reliability of the system. Therefore, the role of determining the importance of units in the system emerges, because improving their reliability means improving the reliability of the system with better results.
If we discuss the previous example in another way that the remaining cost available to improve the system is 70, we choose the most important unit in the system, which is unit 1 as it is located in the first level of importance and its cost is 40, the remaining 30 of the cost is not enough for its value with the units 7, 6, 4, 5 (accordingly to the costs of this units), but it suffices for the cost of unit 3. It is an easier way to improve the reliability of the given system, as determining the importance of units made it easy for us a lot of time and effort.

4.7 Some basic innovative theorems that explain the increase and decrease the reliability of systems

These theorems are considered important and basic in explaining the reasons for the increase in the reliability of the systems, even though most of the reliability of these systems is not good. And also explain the reasons for the low reliability of the systems although most of the reliability of the units is good. These theorems clarified the scientific explanation for the reasons for the increase and decrease in the reliability of systems. We calculate reliability of minimal path as a series system, and calculate reliability of minimal cut as a parallel system.

Theorem 4.1 If $\forall r_i = 1$, i= 1, ..., n in a minimal path, then the reliability of this minimal path is equal to 1. or

$$\lim_{\forall r_i \to 1} R(M_{P_i}) = 1$$

Proof :

Let $M_{P_i} = \{r_1, r_2, \ldots.., r_n\}$, since the units in the minimal path are in the form of a series. Then

$$R(M_{P_i}) = r_1 r_2 ... r_n$$

but $\forall r_i \in M_{P_i} \ni r_i = 1$, then

$$R(M_{P_i}) = 1 \times 1.... \times 1 = 1$$

Theorem 4.2 If the system contains one minimal path and its reliability is equal to one, then the reliability of the system is one, i.e., If $\exists M_{P_i} \in$ S (where S is a set of all minimal path sets) $\ni R(M_{P_i}) = 1$ iff $Rs = 1$, i=1,2...,n, or

$$\lim_{R(M_{P_i}) \to 1} (Rs) = 1$$

proof:

Assume $\exists M_{P_i} \in S \ni R(M_{P_i}) = 1$, by equation (2.3):

$$Rs = 1 - [(1 - R(M_{P_1}))(1 - R(M_{P_2}))\ldots(1 - R(M_{P_i}))..(1 - R(M_{P_n}))]$$

$$Rs = 1 - (1 - R(M_{P_i}))[(1 - R(M_{P_1}))(1 - R(M_{P_2}))\ldots(1 - R(M_{P_n}))]$$

but $R(M_{P_i}) = 1$, then

$$R_s = 1 - (1 - 1)[(1 - R(M_{P_1}))(1 - R(M_{P_2}))\ldots..(1 - R(M_{P_n}))]$$

$$R_s = 1 - (0)[(1 - R(M_{P_1}))(1 - R(M_{P_2}))\ldots..(1 - R(M_{P_n}))] = 1$$

Through this theorem, it can be say that, if we want to increase the reliability of any system, we must increase the reliability of at least one minimal path set.

Example 4.4 Consider a complex network Fig.(3.3) which consists of 8 minimal path sets, and units values are as shown in the Table (4.5).

Table 4.5: Select M_{P_1}

Units	Reliability
R_1	0.99
R_2	0.1
R_3	0.1
R_4	0.99
R_5	0.1
R_6	0.1
R_7	0.1
R_8	0.1

$S = \{M_{P_1} = \{R_1, R_4\}, M_{P_2} = \{R_2, R_5\}, M_{P_3} = \{R_1, R_3, R_5\}, M_{P_4} = \{R_2, R_3, R_4\}, M_{P_5} = \{R_1, R_6, R_8\},$
$M_{P_6} = \{R_2, R_7, R_8\}, M_{P_7} = \{R_1, R_3, R_7, R_8\}, M_{P_8} = \{R_2, R_3, R_6, R_8\}\}$

By equation (2.4), we get, Rs= 0.9806, and $R(M_{P_1}) = 0.9801$.

Although most of the reliability of the units is worn out, but there is one good minimal path $M_{P_1} = \{R_1, R_4\}$ that gave good reliability to the system.

Similar, if $M_{P_3} = \{R_1, R_3, R_5\}$, we pay attention to the reliability of the units in this minimal path with the deterioration of the units in the system, the units values are as shown in the Table (4.6), also get good results for the reliability of the system, as shown in the following example.

Table 4.6: Select M_{P_3}

Units	Reliability
R_1	0.99
R_2	0.1
R_3	0.99
R_4	0.1
R_5	0.99
R_6	0.1
R_7	0.1
R_8	0.1

By equation (2.4), we get, Rs = 0.9744, and $R(P_3) = 0.970299$

Theorem 4.3 If $\forall r_i = 0$, i= 1, ..., n in a minimal cut, then the reliability of this minimal cut is equal to 0. or

$$\lim_{\forall r_i \to 0} R(M_{C_i}) = 0$$

Proof : let $M_{C_i} = \{r_1, r_2, \ldots.., r_n\}$, since the units in the minimal cut are in the form of a parallel, then

$$R(M_{C_i}) = 1 - (1 - r_1)(1 - r_2) \ldots.. (1 - r_n)$$

but $\forall r_i \in M_{C_i} \ni r_i = 0$, then

$$R(M_{C_i}) = 1 - (1 - 0)(1 - 0) \ldots \ldots (1 - 0) = 0$$

Theorem 4.4 If the system contains one minimal cut, its reliability is equal to zero, then the reliability of the system is zero, i.e., If $\exists M_{C_i} \in C$ (where C is a set of all minimal cut sets) $\ni R(C_i) = 0 \leftrightarrow Rs = 0$, i=1,2...,n, or

$$\lim_{\exists R(M_{C_i}) \to 0} (Rs) = 0$$

Proof :
Assume $\exists M_{C_i} \in C \ni R(M_{C_i}) = 0$ by equation (2.6), we get:

$$R_s = [R(M_{C_1})][R(M_{C_2})] \ldots.. [R(M_{C_i})] \ldots \ldots [R(M_{C_n})]$$

but

$$[R(M_{C_i})] = 0 \rightarrow Rs = 0$$

Then this theorem identifies the weaknesses of the system that reduce the reliability of the system, as it can be illustrated in the following example.

Example 4.5 Consider a complex network in Fig.(3.3), the units values are as shown in the Table (4.7).

$C = \{M_{C_1} = \{R_1, R_2\}, M_{C_2} = \{R_4, R_5, R_8\}, M_{C_3} = \{R_4, R_5, R_6, R_7\}, M_{C_4} = \{R_1, R_3, R_5, R_7\},$
$M_{C_5} = \{R_2, R_3, R_4, R_6\}, M_{C_6} = \{R_1, R_3, R_5, R_8\}, M_{C_7} = \{R_2, R_3, R_4, R_8\}\}$.
By equation (2.4), we get, Rs = 0.1900, and $R(M_{C_1})$ =1-(1-0.1) (1-0.1) = 0.19, $R(M_{C_1}) = Rs$

Table 4.7: Select M_{C_1}

Units	Reliability
R_1	0.1
R_2	0.1
R_3	0.99
R_4	0.99
R_5	0.99
R_6	0.99
R_7	0.99
R_8	0.99

Although most of the units' reliability is good, it does not give an acceptable reliability of the system because the C_1 minimal cut has small value of reliability, this minimal cut may cause the system to fail in general.

Similar, consider a complex network in Fig.(3.3), the units values are as shown in the Table (4.8). $R(M_{C_2}) = 1-(1-0.1)(1-0.1)(1-0.1) = 0.271$, by equation (2.4), we get, Rs = 0.2709.

Although most of the units' reliability is good, it does not give an acceptable reliability of the system because the C_2 cut has a small reliability, this minimal cut may cause the system to fail in general.

Table 4.8: Select M_{C_2}

Units	Reliability
R_1	0.99
R_2	0.99
R_3	0.99
R_4	0.1
R_5	0.1
R_6	0.99
R_7	0.99
R_8	0.1

Theorem 4.5 [Howeidi's Theorem]

$$Max\{R(M_{P_i})\} \leq Rs \leq Min\{R(M_{C_i})\}$$

Proof :

To prove $Max\{R(M_{P_i})\} \leq Rs$

If $Max\{R(M_{P_i})\}= 1$, the theory (4.2) is true, ($Max\{R(M_{P_i})\}$=Rs= 1)

If not by equation (2.3):

$$Rs = 1 - [(1 - R(M_{P_1}))(1 - R(M_{P_2}))\dots(1 - R(M_{P_i}))...(1 - R(M_{P_n}))]$$

$$1 - Rs = [(1 - R(M_{P_1}))(1 - R(M_{P_2}))\dots(1 - R(M_{P_i}))...(1 - R(M_{P_n})] \tag{4.10}$$

Select the highest reliability value of a minimal path, i. e.; $Max\{R(M_{P_i})\}$, from equation (4.10), we get:

$$\frac{1 - Rs}{1 - Max\{R(M_{P_i})\}} = [(1 - R(M_{P_1}))(1 - R(M_{P_2}))\dots..(1 - R(M_{P_n})] \tag{4.11}$$

but

$0 < [(1 - R(M_{P_1}))(1 - R(M_{P_2}))\dots..(1 - R(M_{P_n}))] < 1$

We get $Max\{R(M_{P_i})\} \leqslant Rs$......$*$

Now, to prove $Rs \leq Min\{R(M_{C_i})\}$
If $Min\{R(M_{C_i})\}$=0, the theory (4.4) is true, ($Min\{R(M_{C_i})\}$= Rs = 0)
If not by equation (2.6):
$Rs = [R(M_{C_1})][R(M_{C_2})]\ldots..[R(M_{C_i})]\ldots\ldots[R(M_{C_n})]$
Select the low reliability value of a minimal cut, i. e.; $Min\{R(M_{C_i})\}$, from equation (2.6), we get:

$$\frac{Rs}{Min\{R(M_{C_i})\}} = [R(M_{C_1})][R(M_{C_2})]\ldots\ldots[R(M_{C_n})] \tag{4.12}$$

but
$0 < [R(M_{C_1})][R(M_{C_2})]\ldots\ldots[R(M_{C_n})] < 1$
We get, $Rs \leqslant Min\{R(MC_i)\}$...... $**$
from inequality $*$ and inequality $**$, we get:

$$Max\{R(M_{P_i})\} \leq Rs \leq Min\{R(M_{C_i})\}$$

4.8 Use Howeidi's Theorem for reliability allocation

4.8.1 Applicability of Howeidi's Theorem to reliability allocation for mixed systems

Consider example (4.1) with equation (4.3)and importance of Table (4.1) use Howeidi's Theorem for reliability allocation in Figure (4.3).
S is a set of all minimal path sets.
$S = \{M_{P_1} = \{R_A, R_D, R_F\}, M_{P_2} = \{R_B, R_D, R_F\}, M_{P_3} = \{R_C, R_F\}\}$
C is a set of all minimal cut sets.
$C = \{M_{C_1} = \{R_F\}, M_{C_2} = \{R_C, R_D\}, M_{C_3} = \{R_A, R_B, R_C\}\}$

$R(M_{P_1}) = 0.378, R(M_{P_2}) = 0.378, R(M_{P_3}) = 0.72$
$R(M_{C_1}) = 0.9, R(M_{C_2}) = 0.92, R(M_{C_3}) = 0.982$

$$Max\{R(M_{P_i})\} \leqslant Rs \leqslant Mim\{R(M_{C_i})\}$$

$$R(M_{P_3}) = 0.72 \leqslant 0.8183 \leqslant R(M_{C_1}) = 0.9$$

make of

$$Max\{R(M_{P_i^\circ})\} = Rs^\circ - \in$$

$$Min\{R(M_{C_i^\circ})\} = Rs^\circ + \in$$

where Rs°,$R(M_{P_i^\circ})$, and $R(M_{C_i^\circ})$ objective reliability functions, and $0 \leq \in < 1$

If we want to increase the reliability of the system from (0.8183) to (0.95) we make $Max\{R(M_{P_i^\circ})\} = Rs^\circ - \in = 0.94999, M_{P_3} = \{R_C, R_F\}, \in = 0.00001$

$$R(M_{P_3^\circ}) = R_C^\circ R_F^\circ = 0.94999 \tag{4.13}$$

$Min\{R(M_{C_i^\circ})\} = Rs^\circ + \in = 0.95001, M_{C_1} = \{R_F\}, \in = 0.00001, R_F^\circ = 0.95001$
Put in equation (4.13), we get:

$$R_C^\circ = \frac{R(M_{P_3^\circ})}{R_F^\circ} = \frac{0.94999}{0.95001} = 0.99997894759$$

Put $R_C^\circ = 0.99997894759$ and $R_F^\circ = 0.95001$ in equation (4.3), we get, $Rs^\circ = 0.9500$.

Accurate results are obtained with Howeidi's Theorem thus, it is considered a new and accurate method of calculating reliability allocation

4.8.2 Applicability of Howeidi's Theorem to reliability allocation for complex systems

Consider complex system example (4.2) with equation (4.4) and Table (4.2) of importance use Howeidi's Theorem for reliability allocation in Figure (4.9). If we want to increase the reliability of the system from (0.5155) to more than (0.9).
S is a set of all minimal path sets.
$S = \{M_{P_1} = \{R_1, R_2, R_5, R_7\}, M_{P_2} = \{R_1, R_3, R_5, R_7\}, M_{P_3} = \{R_1, R_3, R_6, R_7\},$
$M_{P_4} = \{R_1, R_4, R_6, R_7\}\}.$
C is a set of all minimal cut sets.
$C = \{M_{C_1} = \{R_1\}, M_{C_2} = \{R_7\}, M_{C_3} = \{R_5, R_6\}, M_{C_4} = \{R_2, R_3, R_4\},$
$M_{C_5} = \{R_2, R_3, R_6\}, M_{C_6} = \{R_3, R_4, R_5\}\}.$

$R(M_{P_1}) = 0.3024, R(M_{P_2}) = 0.2592, R(M_{P_3}) = 0.2916, R(M_{P_4}) = 0.4374$
$R(M_{C_1}) = 0.6, R(M_{C_2}) = 0.9, R(M_{C_3}) = 0.98,$
$R(M_{C_4}) = 0.984, R(M_{C_5}) = 0.984, R(M_{C_6}) = 0.992$

$$MaxR(M_{P_i}) \leq Rs \leq MinR(M_{C_i})$$

$$MaxR(M_{P_i}) = 0.4374, MinR(M_{C_i}) = 0.6$$

$$R(M_{P_4}) = 0.4374 \leq Rs \leq R(M_{C_1}) = 0.6$$

$$0.4374 \leq 0.5155 \leq 0.6$$

make

$$Max\{R(M_{P_i^\circ})\} = R_1^\circ R_4^\circ R_6^\circ R_7^\circ = 0.9 \tag{4.14}$$

case: If R_1, R_4, R_6 and R_7 are independent identical units, i.e., $R_1^\circ = R_4^\circ = R_6^\circ = R_7^\circ$ in equation (4.14). Then $R(M_{P_4}) = R^4 = 0.9$, $R = (0.9)^{\frac{1}{4}} = 0.974$, and put $R_1^\circ = R_4^\circ = R_6^\circ = R_7^\circ = 0.974$ in equation (4.4), we get $Rs^\circ = 0.9372$

Similar, if we want to increase the reliability of the system from (0.5155) to more than (0.999) we make

$$Max\{R(M_{P_i^\circ})\} = R_1^\circ R_4^\circ R_6^\circ R_7^\circ = 0.999 \tag{4.15}$$

put $R_1^\circ = R_4^\circ = R_6^\circ = R_7^\circ$ in equation (4.15), we get:

$$R_1^\circ = R_4^\circ = R_6^\circ = R_7^\circ = (0.9)^{\frac{1}{4}} = 0.9997499062$$

and put $R_1^\circ = R_4^\circ = R_6^\circ = R_7^\circ = 0.9997499062$ in equation (4.4), we get, $Rs^\circ = 0.9994$

Similar, if we want to scale down the reliability of the system from (0.5155) to less than (0.3) we make

$$MinR(M_{C_1^\circ}) = 0.3$$

$M_{C_1} = \{R_1\}$ we get, $R_1^\circ = 0.3$ put $R_1^\circ = 0.3$ in equation (4.4), we get $Rs^\circ = 0.2577$.

Similar, for mixed system, if we want to increase the reliability of the complex system in Fig (4.9) from (0.5155) to (0.88)

$Max\{R(M_{P_i^\circ})\} = Rs- \in = 0.87999, M_{P_4} = \{R_1, R_4, R_6, R_7\}, \in = 0.00001$

$$Max\{R(M_{P_i^\circ})\} = R_1^\circ R_4^\circ R_6^\circ R_7^\circ = 0.87999 \tag{4.16}$$

$$MinR(M_{C_i^\circ}) = Rs^\circ + \in$$

$MinR(M_{C_i^\circ}) = 0.88001, M_{C_1} = R_1, R_1^\circ = 0.88001, \in = 0.00001$
put $R_4^\circ = R_6^\circ = R_7^\circ, and R_1^\circ = 0.88001$ in equation (4.16), we get:

$$R_4^\circ = R_6^\circ = R_7^\circ = \frac{0.87999}{0.88001} = 0.99997727299$$

If we put $R_4^\circ = R_6^\circ = R_7^\circ = 0.99997727299$ and $R_1^\circ = 0.88001$ in equation (4.4), we get, $Rs^\circ = 0.8800$.

Accurate results are obtained with Howeidi's Theorem thus, it is considered a new and accurate method of calculating reliability allocation

4.9 Use Howeidi's Theorem for reliability redundancy

This is to use duplicate for all units located in M_{P_i} who gives $Max\{R(M_{P_i})\}$ and duplicate M_{C_i} who gives $Min\{R(M_{C_i})\}$ and duplicate two times for unit located in M_{P_i} and M_{C_i}

4.9.1 Applicability of Howeidi's Theorem to reliability redundancy for mixed systems

Consider in example (4.1) with equation (4.3) and Table (4.1) of importance use Howeidi's Theorem for reliability redundancy in Figure (4.3). The reliability of the system (0.8183), the reliability in element redundancy =0.9834 and the reliability in unit redundancy= 0.9669

$$M_{P_3} = \{R_C, R_F\} \Rightarrow Max\{R(M_{P_i})\} = R(M_{P_3}) = 0.72$$

$$M_{C_1} = \{R_F\} \Rightarrow Min\{R(M_{C_i})\} = R(M_{C_1}) = 0.9$$

Now we will double (we apply redundancy technique) the unit F twice because it is located in M_{P_i} who gives $Max\{R(M_{P_i})\}$ and M_{C_i} who gives $Min\{R(M_{C_i})\}$
i.e.; $R_F^\star = 1 - (1 - R_F)^3 = 1 - (1 - 0.9)^3 = 0.999$

$R_C^\star = 1 - (1 - R_C)^2 = 1 - (1 - 0.8)^2 = 0.96$
If we put $R_F^\star = 0.999$ and $R_C^\star = 0.96$ in equation (4.3), we get $Rs^\star = 0.9809$

We note that the results of Howeidi's Theorem are very close to element redundancy, so it is another way to calculate reliability redundancy. In addition, the result we obtained was done without need to duplicate all units of the system if we used the element redundancy method.

4.9.2 Applicability of Howeidi's Theorem to reliability redundancy for complex system

Similarly, consider complex system in example (4.2) with equation (4.4) and Table (4.2) of importance use Howeidi's Theorem for reliability redundancy in Figure (4.9). The reliability of the system (0.5155), the reliability in element redundancy = 0.8310 and the reliability in unit redundancy = 0.7652.

$$M_{P_4} = \{R_1, R_4, R_6, R_7\} \Rightarrow Max\{R(M_{P_i})\} = R(M_{P_4}) = 0.4374$$

$$M_{C_1} = \{R_1\} \Rightarrow Min\{R(M_{C_i})\} = R(M_{C_1}) = 0.6$$

Now we will double (we apply redundancy technique) the unit 1 twice because it is located in M_{P_i} who gives $Max\{R(M_{P_i})\}$ and M_{C_i} who gives $Min\{R(M_{C_i})\}$
i.e.; $R_1^\star = 1 - (1 - R_1)^3 = 1 - (1 - 0.6)^3 = 0.936$
$R_4^\star = 1 - (1 - R_4)^2 = 1 - (1 - 0.9)^2 = 0.99$
$R_6^\star = 1 - (1 - R_6)^2 = 1 - (1 - 0.9)^2 = 0.99$
$R_7^\star = 1 - (1 - R_7)^2 = 1 - (1 - 0.9)^2 = 0.99$
If we put $R_1^\star = 0.936, R_4^\star = R_6^\star = R_7^\star = 0.99$ in equation (4.4), we get $R^\star s = 0.9223$

Through the above we note that note the results of Howeidi's Theorem for reliability redundancy is the first method that is better than element redundancy in calculating reliability redundancy and thus can be adopted in systems development. This method is considered the best known method and can be considered as a modern method of reliability redundancy because it gives better results in raising the reliability of the system with least number of units, which contributes to reducing cost, time, size and mass, and it can be adopted in the devel-

opment of systems reliability.Note the increase that occurred in the reliability of the system is very large if this layer theory, although the number of units added is less than the previous methods.

4.10 Applicability of Howeidi's Theorem to reliability importance

This procedure depends on the intersection of the units in minimal path, which give us the highest reliability Max $R(M_{p_i})$ and the units in minimal cut, which give us the lowest reliability Min $R(M_{C_i})$, determining the levels to which the units belong in measuring the importance of reliability. Where we will arrange the $R(M_{p_i})$ in descending order, in addition to arrange the $R(M_{C_i})$ in ascending order. If we want to determine the levels of reliability importance, we must do the following procedure:

1. Determining all minimal path and cut sets for any complex network (system)
2. To determine the level one for reliability importance, which depends on the following steps
   - Step a: Extraction all units in minimal path sets, which give us the highest reliability $R(M_{p_i})$ then move to the next step.
   - Step b: We check which units (in step a) can give us a minimal cut set which has low reliability $R(M_{C_i})$ than rest. If one unit stop. If not, then move to the next step.
   - Step c: Extract the most repeat units in minimal cuts in step(b).
3. Determining the rest of levels in the same way by exclude the units that appeared in level one.

4.10.1 Applicability of Howeidi's Theorem to reliability importance for mixed systems

Consider in example (4.1) with equation (4.3) and Table (4.1) of importance use Howeidi's Theorem for reliability importance in Figure (4.3).

1. To determine level one

    - Step a: looking for a minimal path set, which units give as Max $R(M_{p_i})$. After investigating it, we found that M_{P_3} is desired
    $M_{P_3} = \{R_C, R_F\}, R(M_{P_3}) = 0.72$
    - Step b: $M_{C_1} = \{R_F\}, R(M_{C_1}) = 0.9$
    $\{R_C, R_F\} \bigcap \{R_F\} = \{R_F\}$

    Then R_F in level one.

2. To determine level 2 (exclude unit R_F)

    - Step a: $M_{P_3} = \{R_C, R_F\}$, looking for a minimal cat set which R_c is one of its elements and gives us low reliability than the rest.
    - Step b: $M_{C_2} = \{R_C, R_D\}$
    $\{R_C, R_D\} \bigcap \{R_C\} = \{R_C\}$

    Then R_C in level 2.

3. To determine level 3 (exclude units R_F, R_C)

    - Step a: M_{P_1} and $M_{P_2} = 0.378, R(M_{P_1}) = R(M_{P_2}) = 0.378, R_A, R_B$ and R_D in minimal path sets
    - Step b: $M_{C_2} = \{R_C, R_D\}, R(M_{C_1}) = 0.92, R_D$ in minimal cut set
    $\{R_A, R_B, R_D\} \bigcap \{R_D\} = \{R_D\}$

    Then R_D in level 3.

4. Determine the level 4 (exclude units R_F, R_C and R_D)

    - Step a: M_{P_1} and $M_{P_2} = 0.378, R(M_{P_1}) = R(M_{P_2}) = 0.378, R_A$ and R_B in minimal path sets
    - Step b: $M_{C_3} = \{R_A, R_B, R_C\}, R_A$ and R_B in minimal cut set
    $\{R_A, R_B\} \bigcap \{R_A, R_B\} = \{R_A, R_B\}$
    - Step c: R_A and R_B has same repeat M_{C_3} for step b

Then R_A and R_B in level 4.

If the mixed system in Figure (4.3) has independent identical units. Consider in example (4.1) with equation (4.3) and Table (4.9) of reliability importance . With have same reliability, $R_i = 0.9$, i=1,. . . ,5 and Rs = 0.8902.

S is a set of all minimal path sets

Table 4.9: Mixed system with independent identical units

Components	$I^B(i/t)$	Level
R$_F$	0.9933	1
R$_C$	0.1651	2
R$_D$	0.0393	3
R$_A, R_B$	0.003	4

S = $\{M_{P_1} = \{R_A, R_D, R_F\}, M_{P_2} = \{R_B, R_D, R_F\}, M_{P_3} = \{R_C, R_F\}\}$
C is a set of all minimal cut sets
C = $\{M_{C_1} = \{R_F\}, M_{C_2} = \{R_C, R_D\}, M_{C_3} = \{R_A, R_B, R_C\}\}$
$R(M_{P_1}) = 0.729, R(M_{P_2}) = 0.729, R(M_{P_3}) = 0.81$
$R(M_{C_1}) = 0.9, R(M_{C_2}) = 0.99, R(M_{C_3}) = 0.999$

1. To determine level one

   - Step a: looking for a minimal path set, which units give as Max $R(M_{p_i})$. After investigating it, we found that M_{P_3} is desired
     $M_{P_3} = \{R_C, R_F\}, R(M_{P_3}) = 0.81$
   - Step b: $M_{C_1} = \{R_F\}, R(M_{C_1}) = 0.9$
     $\{R_C, R_F\} \bigcap \{R_F\} = \{R_F\}$

   Then R_F in level one.

2. To determine level 2 (exclude unit R_F)

   - Step a: $M_{P_3} = \{R_C, R_F\}, R(M_{P_3}) = 0.81$

- Step b: $M_{C_2} = \{R_C, R_D, R(M_{C_1}) = 0.99$
  $\{R_C\} \bigcap \{R_c\} = \{R_c\}$

Then R_C in level 2.

3. To determine level 3 (exclude unit R_F and R_C)

   - Step a: M_{P_1} and $M_{P_2} = 0.378, R(M_{P_1}) = R(M_{P_2}) = 0.729, R_A, R_B$ and R_D in minimal path sets
   - Step b: $M_{C_2} = \{R_C, R_D\}, R(M_{C_1}) = 0.9, R_D$ in minimal cut set
     $\{R_A, R_B, R_D\} \bigcap \{R_D\} = \{R_D\}$.

   Then R_D in level 3.

4. To determine level 4 (exclude unit R_F, R_C and R_D)

   - Step a: M_{P_1} and $M_{P_2} = 0.729, R(M_{P_1}) = R(M_{P_2}) = 0.729, R_A$ and R_B in minimal path sets
   - Step b: $M_{C_3} = \{R_A, R_B, R_C\}, = R(M_{C_3}) = 0.999, R_A$ and R_B in minimal cut set
     $\{R_A, R_B\} \bigcap \{R_A, R_B\} = \{R_A, R_B\}$
   - Step c: R_A and R_B has same repeat in M_{C_3} for step b

   Then R_A and R_B in level 4.

4.10.2 Applicability of Howeidi's Theorem to reliability importance for complex system

Consider complex system example (4.2) with equation (4.4) and Table (4.2) of importance use Howeidi's Theorem for reliability importance Figure (4.9). The reliability of system (0.5155).

1. To determine level one

   - Step a: looking for a minimal path set, which units give as Max $R(M_{p_i})$. After investigating it, we found that M_{P_4} is desired
     $M_{P_4} = \{R_1, R_4, R_6, R_7\}, R(M_{P_4}) = 0.4374$

- Step b: $M_{C_1} = \{R_1\}, R(M_{C_1}) = 0.6, R_1$ in minimal cut set
  $\{R_1, R_4, R_6, R_7\} \bigcap \{R_1\} = \{R_1\}$

Then R_1 in level one.

2. To determine level 2 (exclude unit R_1)

   - Step a: $M_{P_4} = \{R_1, R_4, R_6, R_7\}, R(M_{P_4}) = 0.4374$
   - Step b: $M_{C_2} = \{R_7\}, R(M_{C_2}) = 0.9, R_7$ in minimal cut set
     $\{R_4, R_6, R_7\} \bigcap \{R_7\} = \{R_7\}$

   Then R_7 in level 2.

3. To determine level 3 (exclude unit R_1 and R_7)

   - Step a: $M_{P_4} = \{R_1, R_4, R_6, R_7\}, R(M_{P_4}) = 0.4374$
   - Step b: $M_{C_3} = \{R_5, R_6\}, R(M_{C_3}) = 0.98, R_6$ and R_5 in minimal cut set
     $\{R_4, R_6\} \bigcap \{R_5, R_6\} = \{R_6\}$

   Then R_6 in level 3.

4. To determine level 4 (exclude unit R_1 ,R_7 and R_6)

   - Step a: $M_{P_4} = \{R_1, R_4, R_6, R_7, R(M_{P_4}) = 0.4374$
   - Step b: $M_{C_4} = \{R_2, R_3, R_4\}, M_{C_5} = \{R_2, R_3, R_6\}, = R(M_{C_4}) = R(M_{C_5}) = 0.984 R_2, R_3$ and R_4 minimal cut sets
     $\{R_2, R_3, R_4\} \bigcap \{R_4\} = \{R_4\}$

   Then R_4 in level 4 (exclude unit R_1 ,R_7 , R_6 and R_4)

5. To determine level 5

   - Step a: $M_{P_1} = \{R_1, R_2, R_5, R_7\}, R(M_{P_1}) = 0.3024$
   - Step b: $M_{C_3} = \{R_5, R_6\}, R(M_{C_3}) = 0.98, R_5$ in minimal cut set
     $\{R_2, R_5\} \bigcap \{R_5\} = \{R_5\}$

   Then R_5 in level 5.

6. To determine the level 6 (exclude unit R_1 ,R_7 , R_6 , R_4 and R_5)

    - Step a:$M_{P_1} = \{R_1, R_2, R_5, R_7\}, R(M_{P_1}) = 0.3024$
    - Step b: $M_{C_4} = \{R_2, R_3, R_4\}, M_{C_5} = \{R_2, R_3, R_6\}, R(M_{C_4}) = R(M_{C_5}) = 0.984, R_2$ and R_3 in minimal cut set
    $\{R_2, R_3\} \bigcap \{R_2\} = \{R_2\}$.

    Then R_2 in level 6. Finally,R_3 in level 7.

We note that there is a difference in the levels of importance (the sixth level and the seventh level), which is a good approximate result.

If the complex system in Figure (4.9) has independent identical units. Consider complex system example (4.2) with equation (4.4) and Table (4.10) of reliability importance. With have same reliability, R_i =0.9, i=1,. . . .,7. The reliability of system (0.7998)

Table 4.10: Complex system with independent identical units

Components	$I^B(i/t)$	Level
R$_1, R_7$	0.8887	1
R$_5, R_6$	0.0868	2
R$_3$	0.0211	3
R$_2, R_4$	0.0139	4

$R(M_{P_1}) = 0.6561, R(M_{P_2}) = 0.6561, R(M_{P_3}) = 0.6561, R(M_{P_4}) = 0.6561.$
$R(M_{C_1}) = 0.9, R(M_{C_2}) = 0.9, R(M_{C_3}) = 0.99, R(M_{C_4}) = 0.999, R(M_{C_5}) = 0.999, R(M_{C_6}) = 0.999.$

1. To determine level one

    - Step a:looking for a minimal path set, which units give as Max $R(M_{p_i})$. After investigating it, we found that M_{P_4} is desired
    $RM_{P_1} = R(M_{P_2}) = R(M_{P_3}) = R(M_{P_4}) = 0.6561$
    - Step b: $M_{C_1} = \{R_1\}, M_{C_2} = \{R_7\}, R(M_{C_1}) = R(M_{C_2}) = 0.9, R_1$ and R_7 in minimal cut set $\{R_1, R_2, R_3, R_4 R_5, R_6, R_7\} \bigcap \{R_1, R_7\} = \{R_1, R_7\}$

- Step c: R_1 and R_7 has same repeat minimal cuts M_{C_1} and M_{C_2} for step b

Then R_1 and R_7 in level 1.

2. To determine level 2 (exclude unit R_1 and R_7)

- Step a: $M_{P_1} = R(M_{P_2}) = R(M_{P_3}) = R(M_{P_4}) = 0.6561$
- Step b: $M_{C_3} = \{R_5, R_6\}, R(M_{C_3}) = 0.99, R_5$ and R_6 in minimal cut set $\{R_2, R_3, R_4 R_5, R_6\} \bigcap \{R_5, R_6\} = \{R_5, R_6\}$
- Step c: R_5 and R_6 has same repeat in M_{C_3} for step b

Then R_5 and R_6 in level 2.

3. To determine the level 3 (exclude unit R_1 ,R_7 , R_5 and R_6)

- Step a: $M_{P_1} = R(M_{P_2}) = R(M_{P_3}) = R(M_{P_4}) = 0.6561$
- Step b: $M_{C_4} = \{R_2, R_3, R_4\}, M_{C_5} = \{R_2, R_3, R_6\}, M_{C_6} = \{R_3, R_4, R_5\}, R(M_{C_4}) = R(M_{C_5}) = R(M_{C_6}) = 0.999, R_2, R_3$ and R_4 in minimal cut set $\{R_2, R_3, R_4\} \bigcap \{R_2, R_3, R_4\} = \{R_2, R_3, R_4\}$
- Step c: R_3 repeat 3 once in M_{C_4}, M_{C_5} and M_{C_6} for step b

Then R_3 in level 3.
R_2 and R_4 repeat 2 once in M_{C_4}, M_{C_5} and M_{C_6} for step b
Then R_2 and R_4 in in level 4.

From mixed and complex examples we can say that the results of reliability importance levels it can be matched with the results of Birnbaum's measure when this systems has independent identical units.
Whereas if the units for the mixed and complex systems are independent and not identical, we will get levels of reliability importance very close to the results we obtained using the Howeidi's theory method, meaning that they are acceptable results, but not with the accuracy of the first case.

Chapter 5

Some relations between the reliability and graph theory

5.1 Introduction

In this chapter, we address some relations and compare some of the concepts that link reliability theory and graph theory. These concepts are, domination in reliability theory and domination in graphs theory, relationship between the factoring algorithm and domination isomorphism in reliability theory

5.2 Domination in reliability theory

We will introduce some definitions related to domination in reliability theory as follows:

Definition 5.1 [31] Let E be the set of all edges (units) and P the set of all path sets, then we can say that (E, P) consists a system or graph, and let $e \in E$ we say that e is relevant if $e \in P$ for at least one $p \in P$. Otherwise, e is called irrelevant. i.e.; e not belong to P.

Definition 5.2 [47] Let (E, P) be a system, if all elements of E are relevant, then we can say that this system is coherent system.

Definition 5.3 [47] Spanning tree of coherent system: If a coherent system doesn't have any cycle path then there is a spanning trees compose it (i.e; the union of all spanning trees yield coherent system).

Definition 5.4 [47] A*spanning tree* can be defined as the subgraph of an undirected connected graph. It includes all the vertices along with the least possible number of edges. If any vertex is missed, it is not a spanning tree. A spanning tree is a subset of the graph that does not have cycles, and it also cannot be disconnected. For example Figure (5.1).

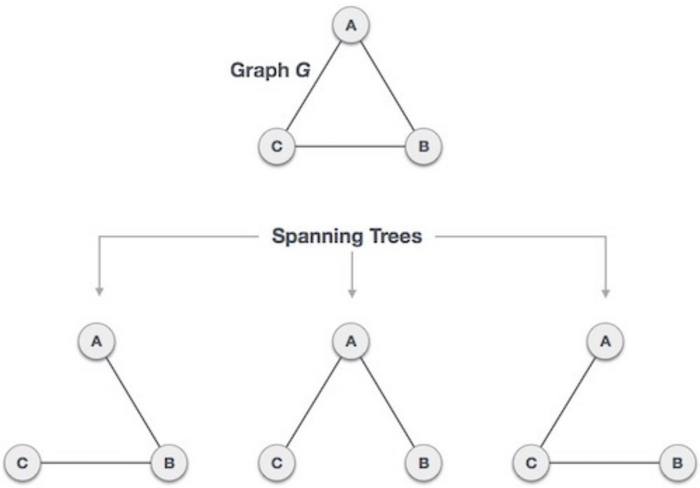


Figure 5.1: Spanning tree.

Definition 5.5 [47] A formations is nonempty subset of spanning trees of graph whose union yields the graph. The formations is termed odd, if the subset consists of an odd number of trees and even otherwise.

Definition 5.6 [2] let (E,P) be a coherent, and let d(P) be the signed domination function of the system. and let the number of odd formations denoted by N_{of}, and the number of even formations denoted by N_{ef}, then we can say that:

$$d(P) = N_{of} - N_{ef} \tag{5.1}$$

Definition 5.7 [2] A domination of the system D(P), is defined by, the absolute value of d(P) .

5.3 Domination in graphs theory

Domination in graphs theory [75] we now introduce the concept of dominating sets in graphs. A set $S \subseteq V$ of vertices in a graph $G = (V, E)$ is a dominating set if every vertex $v \in V$ is an element of S or adjacent to an element of S. Alternatively, we can say that $S \subseteq V$ is a dominating set of G if $N[S] = V(G)$. A dominating set S is a minimal dominating set if no proper subset $S' \subset S$ is a dominating set. The domination number (G) of a graph G is the minimum cardinality of a dominating set of G. We call such a set $\gamma - set$ of G.

5.4 Compare domination in reliability and domination in graphs

Example 5.1 Consider a complex system in Fig (5.2):

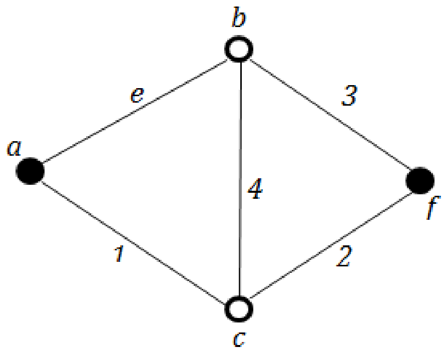


Figure 5.2: Domination in reliability and domination in graphs.

minimal dominating set of a graph
$G = \{\{b\}, \{c\}, \{a, f\}, \{a, b\}, \{a, c\}, \{b, f\}, \{b, c\}, \{f, c\}\}$
The domination number (G) of a graph $\gamma(G) = 1$
Formations=$\{1, 2\}, \{e, 3\}, \{1, 4, 2\}, \{e, 4, 3\}, \{1, 3, 4\}, \{e, 2, 4\}$
$D_k(G)$=$|(\mathrm{N}_{of} - N_{ef}\ )| = |4 - 2| = 2$

Example 5.2 Consider a complex system in Fig (5.3). Minimal dominating set of a graph $G = \{\{2, 3, 6, 8\}\}$. The domination number (G) of a graph $\gamma(G) = 4$.
The domination number (G) of a reliability system it can be calculated as follows:
Formations=$\{e, 4, 7\}, \{e, 3, 4, 7\}, \{e, 4, 6, 7\}, \{e, 3, 4, 6, 7\}, \{e, 3, 4, 5, 7\}, \{e, 4, 5, 6, 7\}$
$\{2, 5, 8\}, \{2, 3, 5, 8\}, \{2, 5, 6, 8\}, \{2, 3, 5, 6, 8\}, \{2, 3, 4, 5, 8\}, \{2, 4, 5, 6, 8\}$
$D_k(G)$= $|\mathrm{N}_{of} - N_{ef}\ |$
$\mathrm{D}_k(G) = |8 - 4| = 4$

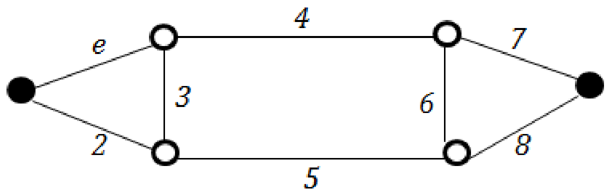


Figure 5.3: Domination in reliability and domination in graphs for example (5.2).

5.5 Relations between the factoring algorithm [73] and domination of reliability theory

Example 5.3 Us focus on the factoring theorem and see example Figure (5.4):

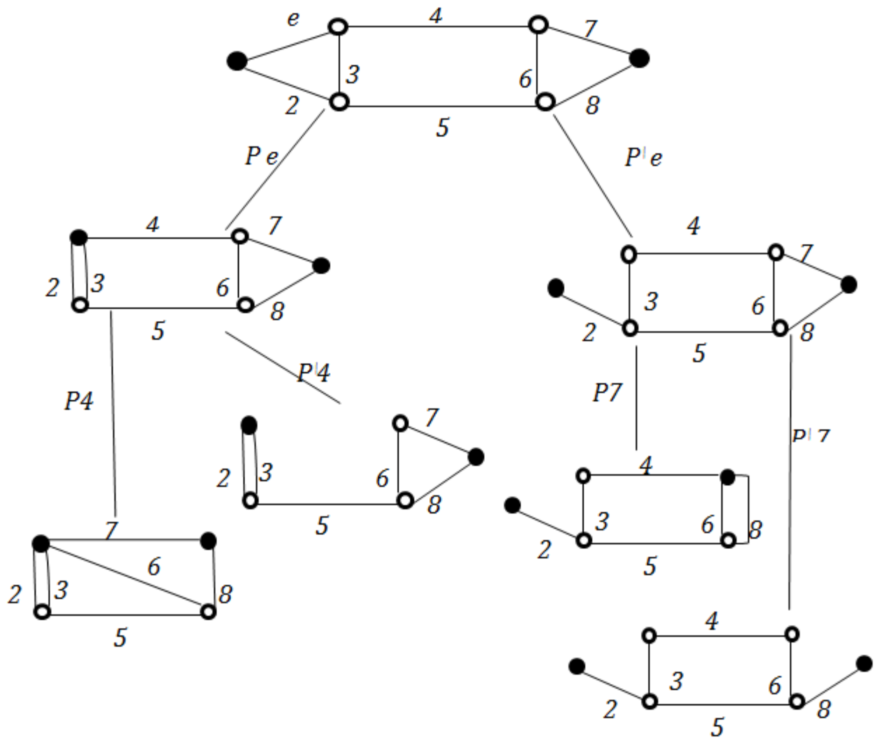


Figure 5.4: Relationship between the factoring algorithm and domination5.3.

In the initial step we pivot on edge e forming two subgraph :$G\backslash e$ corresponding to e working and G-e, corresponding to e failed. Parallel and series probability reductions are now possible. A parallel probability reduction replaces two edges say 2 and 3 in $G\backslash e$ by a single edge with associated probability $p_2+p_3–p_2p_3$. Likewise in $G\backslash e$ this new edge and edge 5 are in series. The two edges in series are then replaced by a single edge with associated reliability $(p_2 + p_3 - p_2p_3)p_5$. Pivoting now proceeds on edge 4 resulting in two additional subgraphs, each of which can be reduced to a single edge by parallel and series probability reduction.

The leaves of the binary tree are the four subgraphs at the bottom of the tree. Which is equal to the domination number (G) of a reliability system.

The domination of the top graph turns out to be $D(G) = 4$ (the number of leaves at the bottom of the tree) and the tree has $2D(G)-1 = 7$ nodes, so that the computational running time is proportional to the domination.

If R_i have same reliability and $i = 1, \ldots, 8$

$$\begin{aligned} R(G) = R^2(((((R \amalg R)R) \amalg R)R) \amalg R + R(1-R)(((R \amalg R)R)(R^2 \amalg R))+ \\ R(1-R)((R(R \amalg R)) \amalg (R^2))R + (1-R)^2((R^3 \amalg R)R^2) \\ = (R^3 + R^4 + R^5 - 5R^6 + 4R^7 - R^8) + (2R^4 - R^5 - 4R^6 + 4R^7 - R^8)+ \\ (3R^4 - 4R^5 - R^6 + 3R^7 - R^8) + (R^3 - 2R^4 + 2R^5 - 3R^6 + 3R^7 - R^8) \\ \\ = 2R^3 + 4R^4 - 2R^5 - 13R^6 + 14R^7 - 4R^8 \end{aligned}$$

Where the operator $\amalg$ corresponds to calculating the reliability of parallel edges

$$R_i \amalg R_j = R_i + R_j - R_iR_j$$

Example 5.4 Us focus on the factoring theorem and see example Figure (5.5):

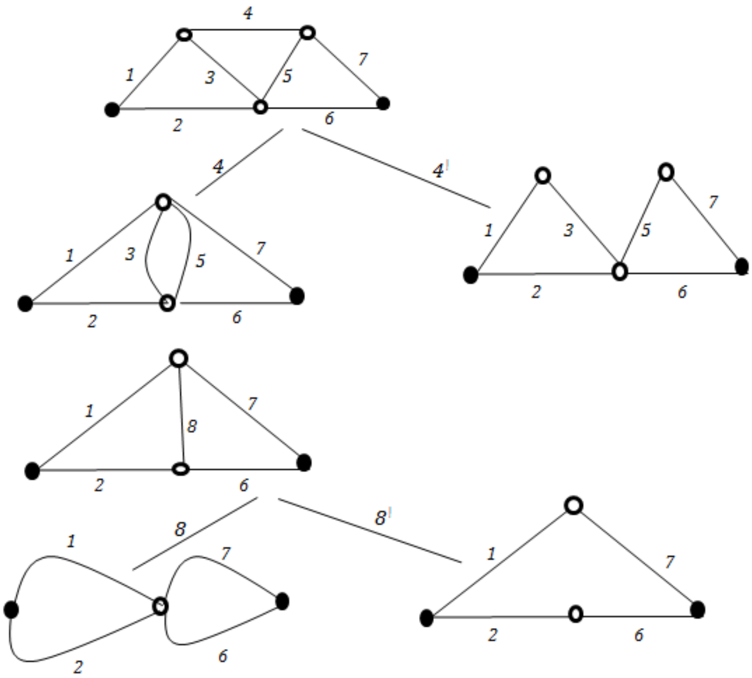


Figure 5.5: Relationship between the factoring algorithm and domination for example (5.4)

The domination of the top graph turns out to be $D(G) = 3$ (the number of leaves at the bottom of the tree) and the tree has $2D(G)-1 = 5$ nodes, so that the computational running time is proportional to the domination

$$R(G_k) = R_4(1-(1-R_3)(1-R_5))(1-(1-R_1)(1-R_2))(1-(1-R_6)(1-R_7))+ (1-R_3)(1-R_5)(R_1R_7+R_2R_6-R_1R_2R_6R_7)+R_4(R_1R_3+R_2-R_1R_2R_3)(R_5R_7+R_6-R_5R_6R_7)$$

5.6 Isomorphism of reliability

The two graphs G and H are isomorphic $G \cong H$ if there is a one to one correspondence ϕ mapping from $V(G)$ onto $V(H)$ such that ϕ preserves adjacency, i.e., for $u, v \in V(G)$, $uv \in E(G)$ if and only if $\phi(u)\phi(v) \in E(H)$. Thus the two graphs shown in Figure (5.8) are isomorphic under the correspondence:

$$u \leftrightarrow l, v \leftrightarrow m, w \leftrightarrow n, x \leftrightarrow p, y \leftrightarrow q, z \leftrightarrow r$$

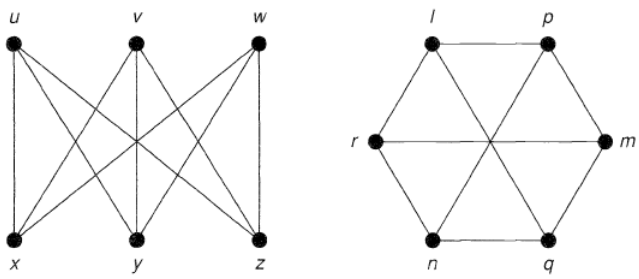


Figure 5.6: G $\cong H$.

Example 5.5 Consider the two complex systems The two systems G and H are isomorphic $G \cong H$ if there is a one to one correspondence ϕ mapping from $V(G)$ onto $V(H)$ such that ϕ preserves adjacency, i.e., for $u, v \in V(G), uv \in E(G)$ if and only if $\phi(u)\phi(v) \in E(H)$ Thus the two systems shown in Figure (5.7) are isomorphic under the correspondence:

$$R_1 \leftrightarrow R_a, R_2 \leftrightarrow R_b, R_3 \leftrightarrow R_c, R_4 \leftrightarrow R_d, R_5 \leftrightarrow R_e, R_6 \leftrightarrow R_f, R_7 \leftrightarrow R_r, R_8 \leftrightarrow R_k$$

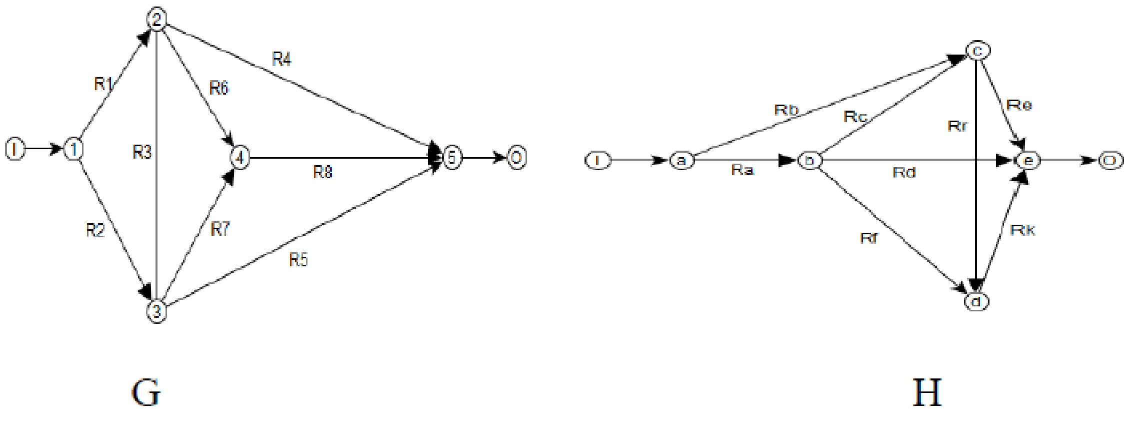


Figure 5.7: Isomorphic two complexes system.

Not's that the minimal path sets in G and H.

$S = \{\{R_1, R_4\} \cong \{R_a, R_d\}, \{R_2, R_5\} \cong \{R_b, R_e\}, \{R_1, R_3, R5\} \cong \{R_a, R_c, R_e\},$
$\{R_2, R_3, R_4\} \cong \{R_b, R_c, R_d\}, \{R_1, R_6, R_8\} \cong \{R_a, R_f, R_k\}, \{R_2, R_7, R_8\} \cong \{R_b, R_r, R_k\},$
$\{R_1, R_3, R_7, R_8\} \cong \{R_a, R_c, R_r, R_k\}, \{R_2, R_3, R_6, R_8\} \cong \{R_b, R_c, R_f, R_k\}\}$

And the minimal cuts sets in G and H

$C = \{\{R_1, R_2\}\} \cong \{R_a, R_b\}, \{R_4, R_5, R_8\} \cong \{R_d, R_e, R_k\}, \{R_4, R_5, R_6, R_7\} \cong \{R_d, R_e, R_f, R_r\},$
$\{R_1, R_3, R_5, R_7\} \cong \{R_a, R_c, R_e, R_r\}, \{R_2, R_3, R_4, R_6\} \cong \{R_b, R_c, R_d, R_f\},$
$\{R_1, R_3, R_5, R_8\} \cong \{R_a, R_c, R_e, R_k\}, \{R_2, R_3, R_4, R_8\} \cong \{R_b, R_c, R_d, R_k\}\}$

$$R_G \cong R_H$$

Example 5.6 Consider the two complex systems the two systems G and H are isomorphic $G \cong H$ if there is a one to one correspondence ϕ mapping from $V(G)$ onto $V(H)$ such that ϕ preserves adjacency, i.e., for $u, v \in V(G), uv \in E(G)$ if and only if $\phi(u)\phi(v) \in E(H)$. Thus the two systems shown in Figure (5.8) are isomorphic under the correspondence:

$$R_1 \leftrightarrow R_a, R_2 \leftrightarrow R_b, R_3 \leftrightarrow R_c, R_4 \leftrightarrow R_d, R_5 \leftrightarrow R_e, R_6 \leftrightarrow R_f, R_7 \leftrightarrow R_r$$

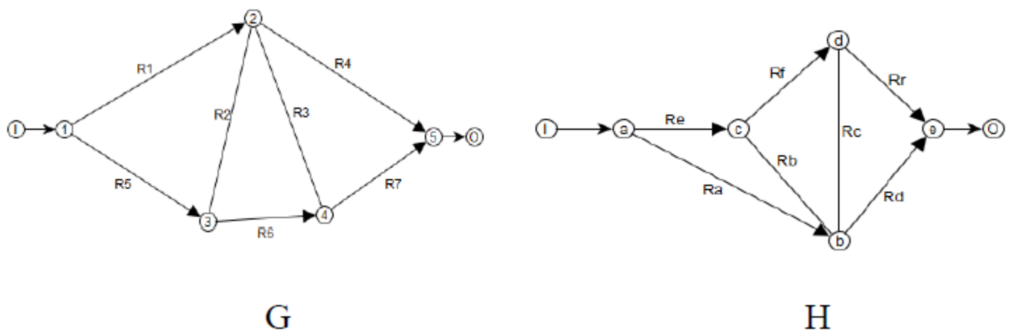


Figure 5.8: Isomorphic two complexes system for example (5.6).

Not's that the minimal path sets in G and H

$S = \{\{R_1, R_4\} \cong \{R_a, R_d\}, \{R_2, R_4, R_5\} \cong \{R_b, R_d, R_e\}, \{R_1, R_3, R_7\} \cong \{R_a, R_c, R_r\},$
$\{R_5, R_6, R_7\} \cong \{R_e, R_f, R_r\}, \{R_2, R_3, R_5, R_7\} \cong \{R_b, R_c, R_e, R_r\},$

$\{R_1, R_2, R_6, R_7\} \cong \{R_a, R_b, R_f, R_r\}, \{R_3, R_4, R_5, R_6\} \cong \{R_c, R_d, R_e, R_f\},$
$\{R_1, R_2, R_3, R_4, R_6\} \cong \{R_a, R_b, R_c, R_d, R_f\}\}$
And the minimal cuts sets in G and H
$C = \{\{R_1, R_5\} \cong \{R_a, R_e\}, \{R_4, R_7\} \cong \{R_d, R_r\}, \{R_1, R_2, R_6\} \cong \{R_a, R_b, R_f\},$
$\{R_3, R_4, R_6\} \cong \{R_c, R_d, R_f\}, \{R_1, R_2, R_3, R_7\} \cong \{R_a, R_b, R_c, R_r\},$
$\{R_2, R_3, R_4, R_5\} \cong \{R_b, R_c, R_d, R_e\}\}$

$$R_G \cong R_H$$

Chapter 6

Conclusions and Future Works

6.1 Conclusions

In the third chapter of this thesis, some methods for calculating the reliability importance of components are developed to overcome some of the defects in these methods, as well as to invent some new methods such as (Improvement potential(2) measure and new method compute the reliability importance of components in reliability system with independent identical units) and some mathematical relationships (equations) (3.17-3.18) to calculate the reliability importance of units in simple, complex and mixed systems. As for the fourth chapter, the relationship between reliability importance and redundancy has been studied, and we have concluded through this study that the technique of repetition of units that has higher levels of reliability importance has a reflection on increasing the reliability of systems and matching its value with the technique (unit redundancy) and its value is also close to the technique (element redundancy) as well as "reduction" In the size of the system, the time and cost required to increase the reliability of the systems. The relationship between the reliability importance, addition, and reliability allocation was also studied, and we concluded through this that it is possible to reach the required increase in the reliability of any system by increasing the reliability of a limited number of system units. The relationship between the importance of units and their cost has also been studied, and we have concluded that an increase in cost does not necessarily lead to an increase in the reliability of the system, unless it is relied on to increase the reliability of some units of the system that have higher levels of importance. We also discovered five theories and their proof, which can help in understanding the increase and decrease in the reliability of systems by increasing the reliability of at least one of the minimal path sets and the decreasing in reliability results from a decrease in the reliability of at least one minimal cut sets.

We used properties of Howeidi's Theorem for finding new techniques to address reliability allocation, reliability redundancy and reliability importance, accurate results are obtained with Howeidi's Theorem thus, it is considered a new and accurate method of calculating reliability allocation.

We noted that the results of Howeidi's Theorem are very close to element redundancy, so it is another way to calculate reliability redundancy. In addition, the result we obtained was done without need to duplicate all units of the system if we used the element redundancy method, we note that the results of Howeidi's Theorem for reliability redundancy is the first method that is better than element redundancy in calculating reliability redundancy and thus can be adopted in system developments. This method is considered the best known method and can be considered as a modern method of reliability redundancy because it gives better results in raising the reliability of the system with the least number of units, which contributes to reducing cost, time, size and mass, and it can be adopted in the development of system reliability. Therefore, the increase that occurred in the reliability of the system is very large If we use the properties of Howeidi's Theorem, although the number of units added is less than the previous methods.

From mixed and complex examples, we can say that the results of reliability importance levels can be matched with the results of Birnbaum's measure when these systems have independent identical units. Whereas if the units for the mixed and complex systems are independent and not identical, we get the levels of reliability importance very close to the results we obtained using Howeidi's theory method, meaning that they are acceptable results, but not with the accuracy of the first case.

In the fifth chapter, we explored that there is a difference between domination in reliability network and domination in graphs, and we mathematically mimicking topic of isomorphism in graph to the reliability of network.

6.2 Future works

1. We suggest using the failure function to find reliability importance using the previously mentioned methods.
2. We propose calculating the reliability of systems using Howeidi's theory method
3. We suggest accurately calculating the significance of the units using Howeidi's theory method when the reliability of the units is different.

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