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College of Education for Pure Sciences
Department of Mathematics

Using Fuzzy Fault Tree Analysis in Reliability and Safety Applications

A Thesis

Submitted to the Council of the College of Education for Pure Sciences in University of Babylon as a Partial Fulfillment of the Requirements for the Degree of Master in Education / Mathematics

By

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Supervised by

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بسم الله الرحمن الرحيم

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Dedications

To my family that was supportive for me at this stage and to everyone who supported me. Thank you from my heart.

Harith safaa

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Table of Contents

List of Figures

List of Symbols and Abbreviations

Symbols	Description
$\mathbf{R}_S$	Reliability system
SF	Structure Function
$\mathbf{R}_i$	Reliability of component i
$\mathbf{P}_j$	Path set j
$\mathbf{C}_i$	Minimal cut set
$\boldsymbol{I}_{C_i}$	Importance of component
$\tilde{\mathbf{A}}$	Fuzzy set
$\mu_{\tilde{A}}\,(\boldsymbol{x})$	Membership function of fuzzy set A
FT	Fault tree
SFT	Static Fault Tree
DFT	Dynamic Fault Tree
FTA	Fault Tree Analysis
FFT	Fuzzy Fault Tree
T	Top Event
RBD	Reliability Block Diagram (RBD)
ST	Success Tree
SEQ	Sequent Enforcing Gate
FDEP	Functional Dependency Gate
PAND	Priority Gate
DRBD	Dynamic Reliability Block Diagram
$\tilde{\mathbf{R}}_S$	Fuzzy reliability system

List of Tables

Abstract

The current thesis include via the four chapters that it contained, presenting basic concepts and definitions related to its title, presenting different types of reliability models and proving two theorems about the models of series and parallel based on the idea of complement, with various solving methods as reduction method, composite method to find models reliability. Present fault tree and its gates with construction, and two theorems related to the two gates (AND,OR), minimal path sets and minimal cut sets are used in (dual of FT), i.e. success tree ST and fault tree FT respectively to determine the reliability of models illustrative examples are appended.

Presenting a new simplified method for calculating the component importance (I_C), discussing sub fault tree (sub-FT) concept, domain and determine whether sub-FT is independent or not. Analyzing multistate dynamic fault tree (DFT) with the behavior of DFT gates. Converting FT to RBD, moreover the study extend to the DFT and its relationship to the DRBD with transformation among them, as well as converting the DFT into an SFT.

Present the concept of the fuzzy set and its operations with various applications taken from the reality of daily life related to safety as protection of devices, and reliability. Different applications as the protection the inverter, the project of laying the pure water pipeline to a region in Hilla city, the techniques of civil defense to active treatment fires, as well as calculating the reliability of the electrical feeding system of the central library at the University of Babylon, further the concept of the vague sets and its operations was used in applications above since it is considered the best among the concepts in dealing with engineering reliability problems. By an analytical study that showing the relationship between safety, reliability and human factors we conclude the current study.

Introduction

Reliability is The probability that a component or device will perform its intended function for a given period of time under a given set of conditions [54].Considerations for reliability become more essential in engineering techniques. Though the specifics of enforcement varies depending on whether chemical, mechanical, or electrical systems are examined, and the reliability connotations cut across engineering disciplines.

H. A. Watson of Bell telephone laboratories invented the fault tree (FT) in 1961 in connection with a U.S. air force contract to research the minuteman missile launch control system [66].

Sandler G. 1963 researched system reliability engineering and the estimation of reliability for series and parallel systems without maintenance [49].

Fault Tree Analysis (FTA) is one of the methods or activities that is used to determine reliability. It is also a valuable tool for doing system safety analysis [9]. (FTA) is a graphical design tool for assessing the top event that leads to system failure.

This approach has evolved over time into the more flexible Dynamic Fault Tree (DFT) Analysis. The main concepts for using the DFT include sequence-dependent events , spares and dynamic redundancy management ,and failure event priority [40]. Since a result, the approach is generally used to analyze the reliability of complex engineering systems from both qualitative and quantitative viewpoints, as it can quickly offer a simple depiction of failure behavior .

Although a human does not have the ability to deal with large amounts of numerical information and correct facts, is very clever in making difficult decisions. He is totally dependent on the computer, which can do the most complex mathematical, calculations in fractions of a second. However, if it is not given in a numerical format, the computer is completely worthless in comparison to simple human operations. This evident of human advantages, along with the inability of number systems, drove D.lutfy Zadah to look for and develop a fuzzy logic theory in 1965. After that, the Japanese employed this reasoning to build their manufacturing and industries, and it grew to cover the most technical aspects.

Fuzzy sets are used by experts to informally define the uncertainties of each given failure occurrence, and then mathematical operations are performed to determine system reliability . These fuzzy sets are used to assess the likelihood of failure occurrences occurring. As a result, the task is to compute the potential of failure of the top event as a fuzzy set given the failure possibilities of the fundamental events. Because fuzzy fault tree (FFT) analysis is a critical tool for estimating the reliability of complicated systems, the current study is focused on the (FFT) method and its link to other approaches.

The likelihood that does not severe incidents will occur during system operation during a certain period of time is referred to as safety. In general, safety can be treated as a part of reliability.

There have been several previous studies on the subject, and the following is a summary of the most of them.

In (1970) [61] V. Postelnicu et al. introduced a Non-dichotomous multi component structure.

In (1976) [52] Smith .O. defined the reliability of an object as the likelihood that it will fulfill its intended function for a set interval under specified conditions.

In (1978) [70] L.A. Zadeh studied a fuzzy set as the basis for a theory of possibility.

In (1983) [5] K.T. Atanassov introduced intuitionistic fuzzy sets.

In (1985) [54] Srinath L.S. defined concepts in reliability engineering.

In (1990) [21] Singer D. showed how to apply a fuzzy set to fault tree and reliability analysis.

In (1993) [29] Gau et al. presented the concepts of vague sets.

In (1996) [57] Subrie U.A. employed some reliability system subject to corrective maintenance.

In (2008) [67] Jing-Shing Yao, et al used the fuzzy system reliability analysis using triangular fuzzy numbers based on statistical data.

In (2010) [39] Wang Limin used fault tree analysis in studying the reasons of an oil tank fire and explosion.

In (2011) [33] Huang Y. L. et al. suggested modeling dynamic gates and calculating dynamic fault tree structure functions (DFT).

In (2018) [35] Jiang, Ge, et al. presented a unique technique to fuzzy dynamic fault tree analysis based on the weakest n-dimensional t-norm arithmetic.

In (2019) [31] Ghadhab, et al. performed a safety study for dynamic fault trees in vehicle guiding systems, engineering of reliability and system safety.

In (2021) [6] Sejin Baek and Gyunyoung Heo suggested the use of dynamic fault tree analysis to prioritize electric power systems in nuclear power plants energy.

The current thesis contains four chapters, the first chapter includes introduction and recall basic concepts and definitions related to the current work, the second chapter involves models of systems reliability, some methods to evaluate systems reliability as reduction method with illustrative examples, and introducing a method to find component importance I_C. Chapter Three deals with FTA and the construction of it, success tree as a dual of fault tree is presented with examples. More over dynamic fault tree DFT with its gates are presented with the method of converting a DFT to SFT as a simpler approach for users. Chapter Four is devoted to fuzzy fault tree analysis with different application taken from real life for safety and reliability.

Objectives of Thesis

The current thesis aims at:

1. Introducing the importance of component for system via a new simple method to determine the danger component which cause the system failure and to rank of component and for the classification of the component.

2. Utilizing static fault tree, dynamic fault tree and fuzzy fault tree as in active tools in quantitative and qualitative analysis for system reliability and safety via in real life applications.

3. Converting dynamic fault tree to static fault tree via relation between their gates which is easier for the users.

4. Studying the behavior of the dynamic fault tree gates for the sake of choosing the proper gate according to a kind of application.

Publication

Audi Sabri Abd ALRazaq and Harith Safaa Faisal **“ Using Fault Tree for inverter Safety System Analysis** “ International Journal of Mechanical Engineering For pure sciences, (2022), 7.2: 0974-5823.

Chapter One

Basic Concepts and Definitions

1.1 Introduction

This chapter involved one section that describes the fundamentals of the current work in terms of ideas and terminology linked to probability, reliability, fault trees, fuzzy sets, and what is a fuzzy number, as well as examples and a comparison between safety and reliability.

1.2 Some Basic Definitions and Concepts

In this section, some concepts and definitions that are related to the current work are reviewed.

Definition 1.2.1 [56]:

A **Components** is a piece of equipment or a part of a system that is evaluated as a separate unit, i.e. its reliability is unaffected by the reliability of other components.

Definition 1.2.2 [19,41]:

System is a collection of components in a defined sequence that connect in order to perform with each other and with external components or other structures for a given purpose.

Definition 1.2.3 [56]**:**

A **Failure** is the termination of the a device to perform its required function.

Definition 1.2.4 [64]:

The **Sample Space**, which is denoted as Ω is the set of all possible outcomes of a random phenomenon.

Definition 1.2.5 [64]:

A **Random Variable** (r.v) say X is continuous if the sample space Ω is uncountable set. There exist another type called discrete random variable.

Definition 1.2.6 [54]:

The **Probability** of an event A is defined as the number of positive outcomes divided by the total number of equally likely outcomes in the experiment's sample space Ω.

1.2.7 Probability Properties [38]:

P(C) is the probability of an occurrence D, and it has the following properties:

a) $0 \leq P(C) \leq 1$

b) $P(C) = 1 - P(\bar{C})$

c) $P(\emptyset) = 0$

d) $P(S) = 1$

In other words, when an event is allowed to occur, it has a probability equal to (1), and when it is impossible to occur, it has a probability equal to (0). It can be proved that the probability of the union of two occurrences C and D equals

$$P(C \cup D) = P(C) + P(D) - P(C \cap D) \tag{1.1}$$

Similarly, the likelihood of the union of three occurrences C, D and E is given by:

$$P(C \cup D \cup E) = P(C) + P(D) + P(E) - P(C \cap D) - P(C \cap E) - P(D \cap E) + P(C \cap D \cap E) \tag{1.2}$$

Definition 1.2.8 [55]:

Probability Density Function (pdf) is a symbol of continuous functions only and is symbolized by the symbol (pdf) If x is a random variable the probability function is f(x) and $a \leq x \leq b$ so it is a function,

$$p(a \leq x \leq b) = \int_a^b f(x)dx \quad \text{and } f(x) \geq 0, \quad \text{for all } x \tag{1.3}$$

Definition 1.2.9 [53]:

The **Cumulative Distribution Function (cdf)** is a random variable x function F(x) for a number x defined as:

$$F(x) = (X \leq x) = \int_{-\infty}^{x} f(s)ds \qquad (1.4)$$

The (cdf) formula calculates the probability that an item will fail before the time value t (unreliability). The mathematical connection between (pdf) and (cdf) is represented by:

$$F(x) = \int_{-\infty}^{x} f(s)ds \qquad (1.5)$$

Conversely :

$$f(x) = d(F(x))/dx \qquad (1.6)$$

It should also be noted that the entire area under the (pdf) is always equal to or mathematically equivalent to

$$\int_{-\infty}^{\infty} f(x)dx = 1 \qquad (1.7)$$

Definition 1.2.10 [59]:

If two events C and D cannot occur at the same time, they are said to be **Mutually Exclusive** ($C \cap D = \emptyset$). In such cases, the combination of these two occurrences is stated as a

$$P(C \cup D) = P(C) + P(D) \qquad (1.8)$$

Since the probability of the intersection of these events is defined as zero.

Definition 1.2.11 [51]:

The probability of one of two events C and D happening knowing that the other event has already occurred is known as **Conditional**

Probability. It is provided that D has already occurred, the expression below denotes the likelihood of an occurring

$$\mathrm{P}(C/D) = \frac{P(C \cap D)}{P(D)} \qquad (1.9)$$

Definition 1.2.12 [60]:

Independent Events when understanding D provides no knowledge about C, the events are said to be separate, and the conditional probability expression falls to:

$$\mathrm{P}\ (\mathrm{C}/\ \mathrm{D}) = \mathrm{P}(\mathrm{C}) \qquad (1.10)$$

From the definition of conditional probability.

$$\mathrm{P}(C/D) = \frac{P(C \cap D)}{P(D)}$$

$$\mathrm{P}(C/D) = \frac{P(C).P(D)}{P(D)} \qquad (1.11)$$

It can be written as follows:

$$P(C \cap D) = \mathrm{P}(C/D)\,.\,P(D) \qquad (1.12)$$

Definition 1.2.13 [36,57]:

The **Reliability of a System** is the probability that the system will adequately perform its intended function under stated environmental for specified interval of time.

Definition 1.2.14 [7]:

An **Availability** is probability that an equipment will be available for operation within a given period of time.

Definition 1.2.15 [49]:

A **Graph** is a collection of nodes connected by branches, with each branch terminating in a node at either end.

Definition 1.2.16 [49]:

A graph G is considered to be linked if every two of its vertices are **Connected**. A vertex u is said to be an u - v path in G, i.e., a graph G is connected if it cannot be written as the union of two graphs else it is **disconnected**.

Definition 1.2.17 [36]:

The **Network Reliability** is the probability that a network element thereof will do satisfactory for a given period of time.

Definition 1.2.18:

A **Static Fault Tree (SFT)** or abbreviated to **(FT)** is a logic diagrammatic description of all potential events that cause system failure using logical gates

Definition 1.2.19 [56]:

A **Path Set** is a group of fault tree initiators which if none of them occurs will guarantee that the top event cannot occur path set.

Definition 1.2.20 [54]:

A **Minimal Cut Set** is the smallest collection of fault tree initiators that, if all occur, will cause the top event to occur.

1.2.21 Structure Function [65]:

Any unit in crisp reliability has two states: working and failing. The Boolean variables represent the state of element i.

$$x_i = \begin{cases} 1 & if \text{ unit } i \text{ working} \\ 0 & if \text{ unit } i \text{ failing} \end{cases} \qquad (1.13)$$

The state of the system is also denoted by the Boolean variable
i =1,2,...,n.

$$X = \begin{cases} 1 & if\ system\ i\ works \\ 0 & if\ system\ i\ fails \end{cases} \qquad (1.14)$$

Definition 1.2.22 [37,56]:

If X is a collection of objects, denoted generally by x, then a **Fuzzy Set** $\widetilde{A}$ in X is a set of ordered pairs $\widetilde{A} = \{ (x , \mu_{\widetilde{A}} (x)) \mid x \in X \}$

Where $\mu_{\widetilde{A}}(x)$ is called the membership function of x in $\widetilde{A}$ which maps X to the membership space [0,1].

1.2.23 Fuzzy Number [16]:

It is difficult to determine the exact failure probability or relationship between events. According to the fuzzy set theory proposed by Zadeh [32], fuzzy numbers are used to handle the imprecise information and represent the possibilities of events and relationship of events. Fuzzy sets theory provides a useful tool for directly working with linguistic expression in reliability analyses. Its basic idea is to view the absolute membership of the elements as a set based on classic set theory. The membership of the element x to set A is not defined as 0 or 1, but is valued between 0 and 1, which reflects the membership of the element x to set A. Thus, it is well suited for handling ambiguous and imprecise information obtained in system safety engineering.

Example 1.1

Let us assume that :

$\tilde{A}$= " x considerably >10 "

$\tilde{B}$= " x approximately 11 "

Characterized by

$\widetilde{A}=\{(x, \mu_A(x)) \mid x \in X\}$

Where $\mu_A(x) = \begin{cases} 0 & x \le 10 \\ (1+(x-10)^{-2})^{-1} & x > 10 \end{cases}$

And $\tilde{B}=\{(x,\mu_B(x))|x \in X\}$

Where $\mu_B(x) = (1+(x-11)^4)^{-1}$

$$\mu_A(x) \cap \mu_B(x) = \begin{cases} \min\{(1+(x-10)^{-2})^{-1}, (1+(x-11)^4)^{-1}\} & X > 10 \\ 0 & x \le 10 \end{cases}$$

And

$\mu_A(x) \cup \mu_B(x) = \max\{(1+(x-10)^{-2})^{-1}, (1+(x-11)^4)^{-1}\ x \in X\}$

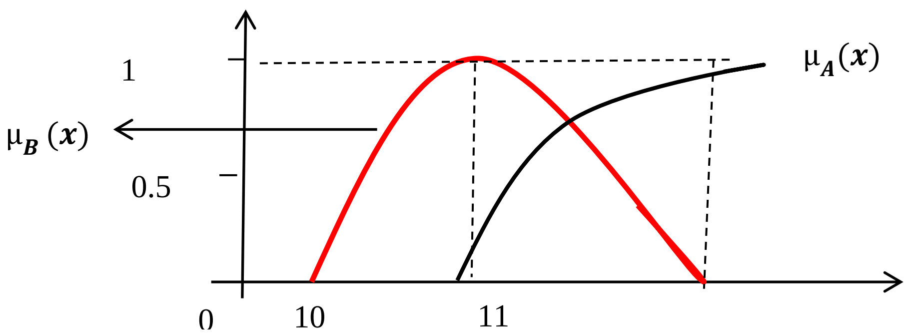


Figure (1.1) The Graph of Example 1.1

1.2.24 Arithmetic Operations on Fuzzy Number Triangular [67]:

The four basic arithmetic operations that are addition, subtraction, multiplication, and division on two triangular vague sets are defined as follows.

$\widetilde{A} =< (a_1, b_1, c_1); \mu_1, \nu_1 >$ and $\widetilde{B} =< (a_2, b_2, c_2); \mu_2, \nu_2 >$ with

$\mu = \min(\mu_1, \mu_2)$ and $\nu = \min(\nu_1, \nu_2)$ are given below :

a) Addition : $\widetilde{A} \oplus \widetilde{B} = [a_1 + a_2, b_1 + b_2, c_1 + c_2]$

b) Subtraction : $\widetilde{A} \ominus \widetilde{B} = [a_1 - c_2, b_1 - b_2, c_1 - a_2]$

c) Multiplication : $\widetilde{A} \otimes \widetilde{B} = [a_1 a_2, b_1 b_2, c_1 c_2]$

d) $\tilde{A} \oslash \tilde{B} = [\ a_1/c_2\ , b_1/b_2\ , c_1/a_2]$

Example 1.2

Let $\tilde{A} = (3, 4, 7)$ and $\tilde{B} = (2, 4, 6)$ be two fuzzy numbers then

i) $\tilde{A} \oplus \tilde{B} = (5, 8, 13)$

ii) $\tilde{A} \ominus \tilde{B} = (-3,0,5)$

iii) $\tilde{A} \otimes \tilde{B} = (6,16,42)$

iv) $\tilde{A} \oslash \tilde{B} = (\frac{3}{6}, 1, \frac{7}{2})$

Definition 1.2.25 [44]:

Safety is the probability, that no catastrophic accidents will occur during system operation, over a specified period of time.

1.2.26 The relationship between reliability and safety [15]:

Reliability is concerned with whether a system can operate properly without failure. In the probability context, it can be defined as a quantitative measure. That is, reliability of a system is the probability that the system operates properly without failure within a predetermined time interval under a specified environment. It is noted that time can be continuous or discrete, whereas safety is concerned with a special kind of failure, i.e., safety critical failure which may cause disastrous consequences. Obviously, in a broad sense, safety can be treated as a part of reliability. However stringent safety critical requirements usually advocate serious challenges to 'how-to-achieve' and ‘how-to-validate' problems. So people may prefer to treat safety a separate part of system failure engineering. The above explain the answer of the question about the relation between reliability and safety.

Chapter Two

Models of Systems and the Methods of Solving the Reliability Systems

2.1 Introduction:

This chapter has been divided into three sections. The first section is a basic forms that contains, series, parallel, series-parallel, parallel-series, k-out-of-n and mixed and complex models.

The second section, deals with the methods of solving these systems mentioned above, and they were as follows: the path-tracing method, the composite method, the reduction to the elements of the series, and the minimal cuts method, with an example solution for each method. The last section, concentrates on the importance of the component.

2.2 Models of Reliability System (Configurations):

Many models of systems are simple, mixed and complexes as in the following :

2.2.1 Series Model [68]:

The series model can only work if all of its components are functioning. This model is depending on both of them.

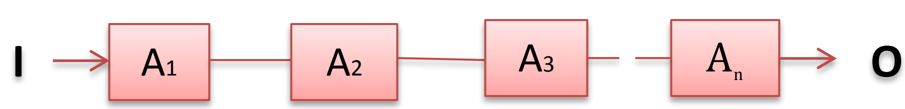


Figure (2.1) A Series Model

The system's reliability is provided by

$$R_s = R_{A_1} \times R_{A_2} \times \dots \times R_{A_n} \quad (2.1)$$

$$R_s = \prod_{i=1}^{n} R_{A_i} \quad (2.2)$$

2.2.2 Observation:

A series configuration system works only if all the components are working . It fails if one or more components fail .

Example 2.1

Consider a model with four series-connected components, each with a set failure rate.

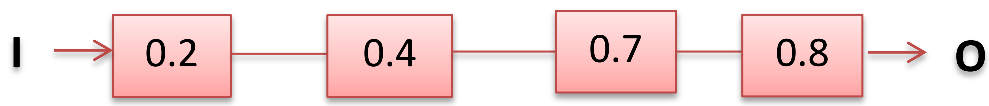


Figure (2.2) A Model Consist of 4 Components for Example 2.1

Then

$R_S = R_1 \times R_2 \times R_3 \times R_4$

$= 0.2 \times 0.4 \times 0.7 \times 0.8$

$= 0.0448$

2.2.3 Parallel Model [26]:

If n components are linked in parallel so that the model works as long as at least one of them is in good working order, the model will fail only if all of the system's components fail at the same time.

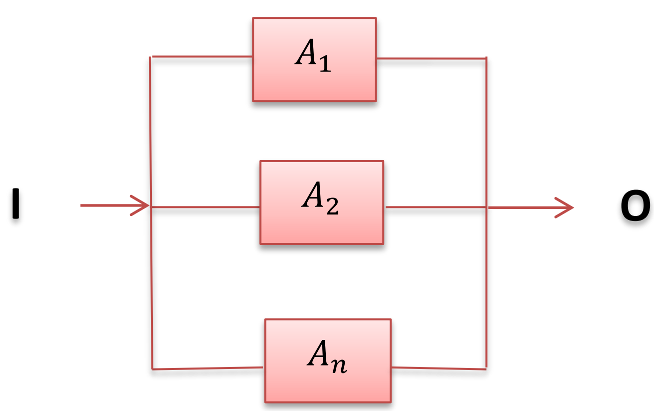


Figure (2.3) A Parallel Model

The system's reliability (again assuming independent failures) is the

$$R_s = 1 - P(\,all\ \ fail\,)$$

$$= 1 - [\,P(A_1\ fails) \times P(A_2\ fails) \times \ldots \times P(A_n\ fails)]$$

$$= 1 - (1 - R_1)(1 - R_2)\ldots(1 - R_n)$$

$$R_s = 1 - \prod_{i=1}^{n}(1 - R_i) \qquad (2.3)$$

2.2.4 Observation:

A parallel configuration system works if at least one of the components is working. It fails only if all of its components fail.

2.2.5 Proposition:

Let C_i refer to the component i is operate and $\bar{C}_i$ refer to the component i is fail assuming that the component independent.

i) To prove the reliability of series model denoted by R_s is :

$$R_s = \prod_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{P_r}(\mathrm{C_i}) \qquad (2.4)$$

proof :

R_s can be evaluated using failure component C_i. In this case

$$R_s = 1 - (\mathrm{P_r}\ \text{of the model failure}\ )$$

From observation (2.2.2) the series model fails if one or more components of its fail.

So $R_s = 1 - (\mathrm{P_r}(\bar{C}_1 \cup \bar{C}_2 \cup \ldots \ldots .\cup \bar{C}_n)$ (2.5)

From the relation $P_r(\bar{C}_i) = 1 - P_r(C_i)$

Hence

$$R_s = 1 - \prod_{i=1}^{n}(1 - P_r(C_i)) \qquad (2.6)$$

Equations (2.4) and (2.6) are identical so results obtained

ii) For a parallel model from observation (2.2.4) the model fails if all its components fail. Let R_p denote the parallel model reliability. To have

$$R_p = 1 - \prod_{i=1}^{n}\big(1 - P_r(C_i)\big) \qquad (2.7)$$

Proof :

Components of R_s are taken to obtaining R_p (i. e $R_p = 1 - \prod_{i=1}^{n} P_r(\bar{C}_i)$)

$$= 1 - P_r(\bar{C}_1 \cap \bar{C}_2 \cap \dots \cap \bar{C}_n)$$

$$= 1 - P_r(\bar{C}_1).P_r(\bar{C}_2) \dots \dots . P_r(\bar{C}_n)$$

$$= 1 - \prod_{i=1}^{n} q_i(\bar{C}_i)$$

Where $q_i(\bar{C}_i) = P_r(\bar{C}_i)$

$q_i(\bar{C}_i) = 1 - P_r(C_i)$

Hence

$$R_p = 1 - \prod_{i=1}^{n}\big(1 - P_r(C_i)\big)$$

So the result are obtained.

Example 2.2

Consider a model with three components connected in a parallel structure, each having a constant failure rate.

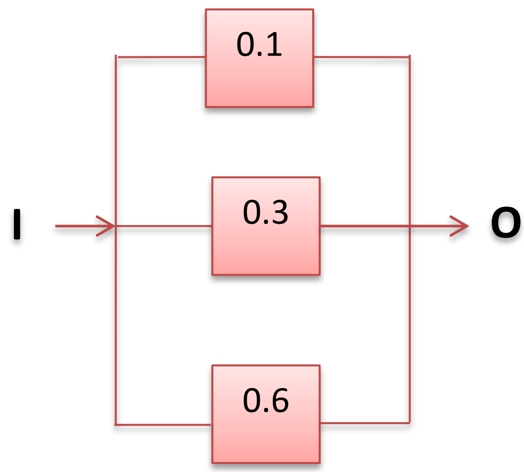


Figure (2.4) A Model Consist of 3 Components for Example 2.2

Then :

$R_s = 1 - \prod_{i=1}^{3}(1 - R_i)$

$= 1 - (1 - R_1)(1 - R_2)(1 - R_3)$

$= 1 - (1 - 0.1)|(1 - 0.3)(1 - 0.6)$

$= 1 - (0.9)(0.7)(0.4) = 0.748$

2.2.6 Parallel-Series Model [22]:

$R_s = 1 - (1 - \text{R})^n$ provides the reliability of n components that are linked in parallel. Where R signifies the reliability of a particular component. If m such sets are linked in series, with each set consisting of parallel components, the system's reliability is defined by:

$$R_S = [1 - (1 - \text{R})^n]^m \quad (2.8)$$

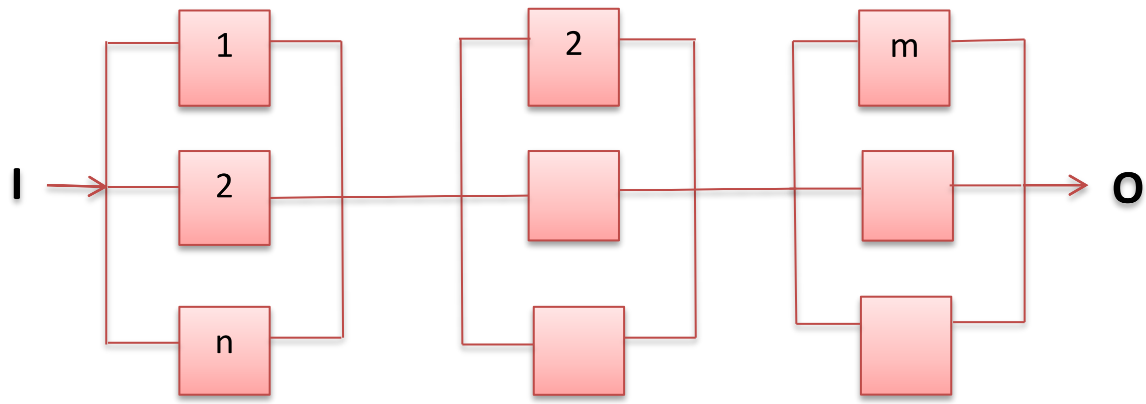


Figure (2.5) Parallel-Series Model

Example 2.3

To determine the model's reliability for the links depicted in Figure.

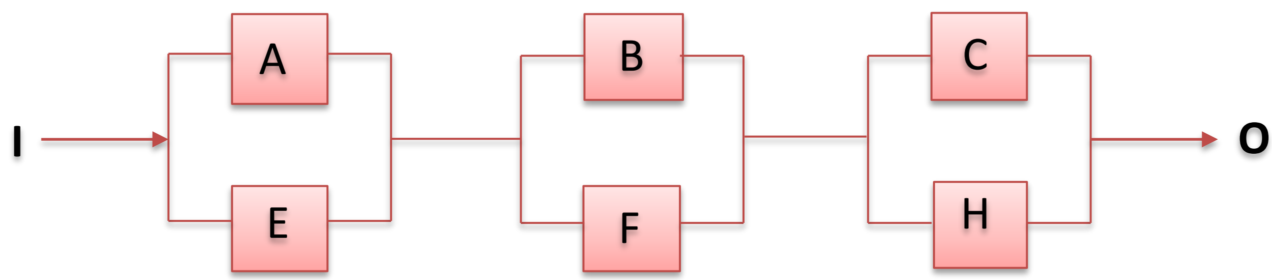


Figure (2.6) A Model of 6 Components for Example 2.3

Assuming each component's reliability is 0.8. So we have

n = 2 and m = 3 in this case.

$$R_S = [1-(1-\text{R})^{\text{n}}]^{m}$$

$$R_S = [1 - (1 - 0.8)^2]^3$$

$$= [1 - 0.04]^3 = 0.88$$

2.2.7 Series–Parallel Model [45]:

$R_s = 1 - (1 - \mathrm{R})^n$ is the reliability of a model built of n components joined in parallel redundancy. Where R denotes the reliability of a single component. If the sets are placed in parallel, each with m components in series.

Thus:

$$R_s = 1 - \left(1 - \prod_{i=1}^{m} R_i\right)^n \qquad (2.9)$$

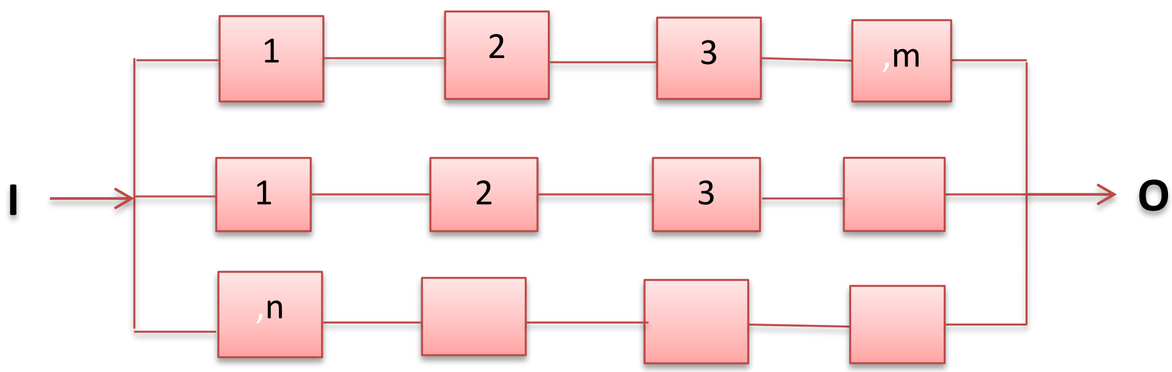


Figure (2.7) Series-Parallel Model

Example 2.4

Calculate the system's reliability for the connection shown in Figure below.

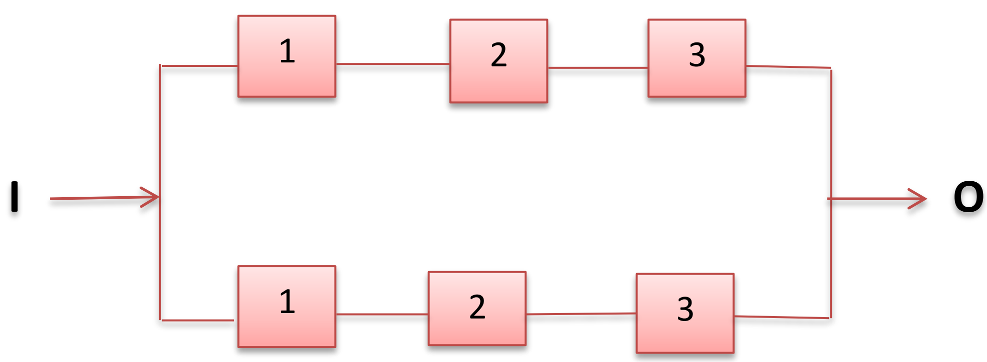


Figure (2.8) A Model Consist of 6 Components for Example 2.4

When m = 3 , n = 2

Not that the reliability of $R_1 = 0.76$, $R_2 = 0.95$, $R_3 = 0.93$ respectively.

$$R_s = 1 - \left(1 - \prod_{i=1}^{3} R_i\right)^2$$

$$= 1 - [1 - (0.76)\,(0.95)(0.93)]^2$$

$$= 1 - [1 - (0.671)]^2 = 0.89$$

Definition 2.2.8 k–out–of–n Model [2,71]:

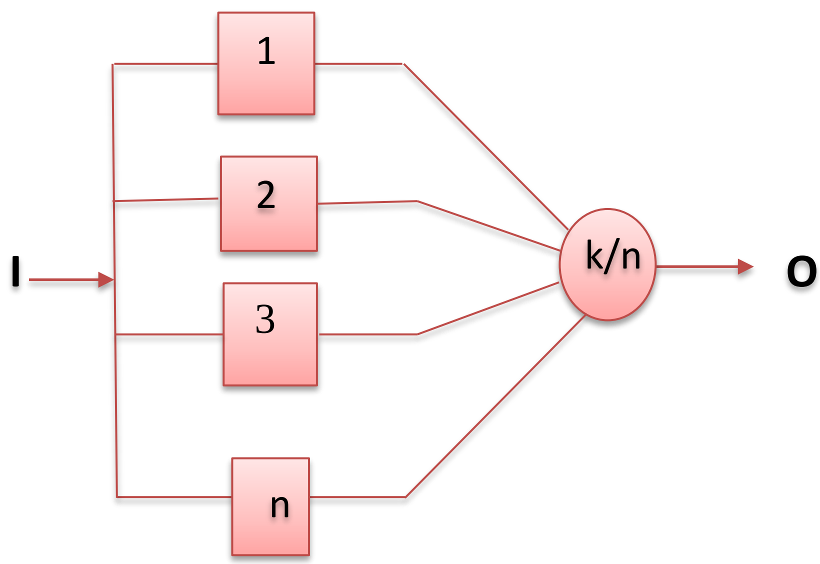


Figure (2.9) A Model k–out–of–n

This model is considered a subset of parallel redundancy in that it must function on at least the first k components of the total n components.

Although this model is a special case of parallel redundancy, it is a general configuration in some cases because the number of units required to maintain the model's success is close to the total number of units in the model and the model's behaviour is similar to a series model if the number of units requires equal number of units in the model.

Example 2.5

Consider the 2–out–of–3 model depicted in figure blow:

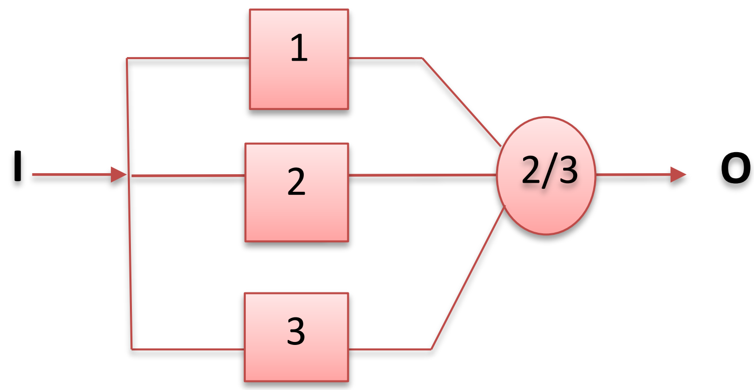


Figure (2.10) A Model 2–out– of–3 for Example 2.5

Not that the reliability of $R_1 = 0.75,\ R_2 = 0.88,\ R_3 = 0.91$ respectively

Solution:

Because it must work on at least 2- out- of -3, only one is allowed to fail. The following activities can be performed to ensure model success:

1. Everyone is working.
2. The 1 fails, but the 2 and 3 work.
3. The 2 fails, while the 1 and 3 work.
4. The 3 fails, but the 1 and 2 work.

We can calculate model reliability as follows:

$$R_s = R_1R_2R_3 + (1 - R_1)R_2R_3 + R_1(1 - R_2)R_3 + R_1R_2(1 - R_3)$$

$$= R_1R_2R_3 + R_2R_3 - R_1R_2R_3 + R_1R_3 - R_1R_2R_3 + R_1R_2 - R_1R_2R_3$$

$$= R_1R_2 + R_2R_3 + R_1R_3 - 2R_1R_2R_3 \qquad (2.10)$$

$$R_s = (0.75)(0.88) + (0.88)(0.91) + (0.75)(0.91) - 2(0.75)(0.88)(0.91)$$

$$= (0.66) + (0.8008) + (0.6825) - (1.2012) = 0.94$$

2.2.9 Mixed model [12,13,27]:

This model is a mix of a parallel and a series system, and it is merely divided into series and parallel to determine the reliability of each sub model and complete to determine the system's reliability.

Example 2.6

Consider the system represented in Figure (2.11) below, which consists of four linked components.

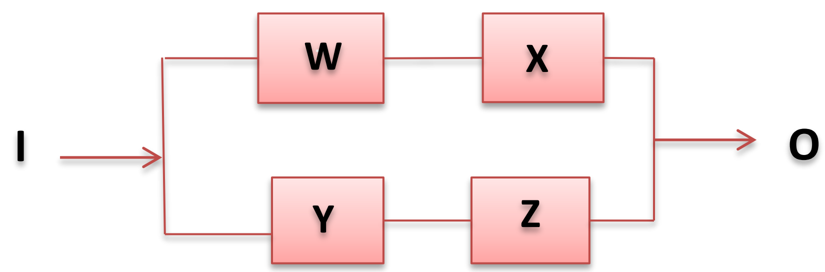


Figure (2.11) A Model Consist of 4 Components for Example 2.6

Where $R_W = 0.1$, $R_X = 0.4$, $R_Y = 0.3$, $R_Z = 0.8$

Solution :

$$R_{wx} = R_w \times R_x$$

$$= (0.1)(0.4)$$

$$= 0.04$$

And

$$R_{yz} = R_y \times R_z$$

$$= (0.3)(0.8) = 0.24$$

$$R_S = 1 - (1 - R_{wx})(1 - R_{yz})$$
$$= 1 - (1 - 0.04)(1 - 0.24)$$
$$= 1 - (0.96)(0.76) = 0.27$$

2.2.10 Complex Model [24,45]:

A model that cannot be directly classified into the preceding cases of elementary structures is called a model with complex structure. For the study of systems with complex structure, the concepts of minimal paths and cuts must be introduced. The model in Figure (2.12) is a model with complex structure. This model cannot be directly classified into modules with traditional structures . If the input-output of this model are located at the extremities of component E, it will then involve a model with elementary structure.

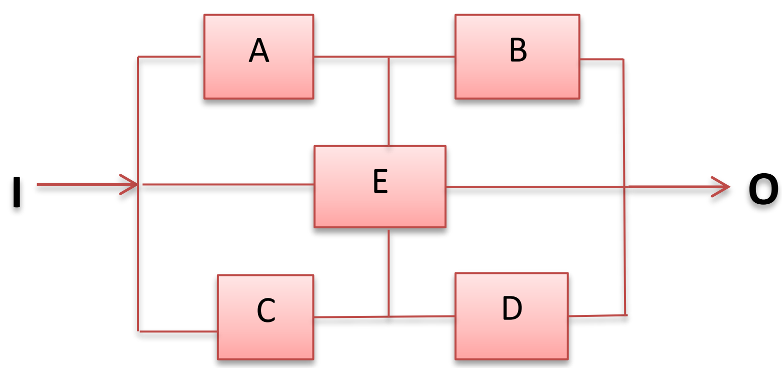


Figure (2.12) A Bridge Model

There several methods for calculating the reliability of mixed and complex systems are exist.

2.3 Some Methods Applied to Calculate the Reliability for Different Models:

2.3.1 Path-Tracing Method [4]:

This method considers any path from a source to a sink. Because of the system's performance, there must be at least one path from one end of the reliability block diagram to the other long as at least one route exists from start to finish, the device is operating, and the structure function will give by the following relationship:

$$R_s = 1 - \prod_{j=1}^{n}(1 - P_j) \qquad (2.11)$$

Where n is the number of minimal path of the model.

Example 2.7

A model is made up of 9 components that are linked together as indicated in Figure (2.13) to determine reliability.

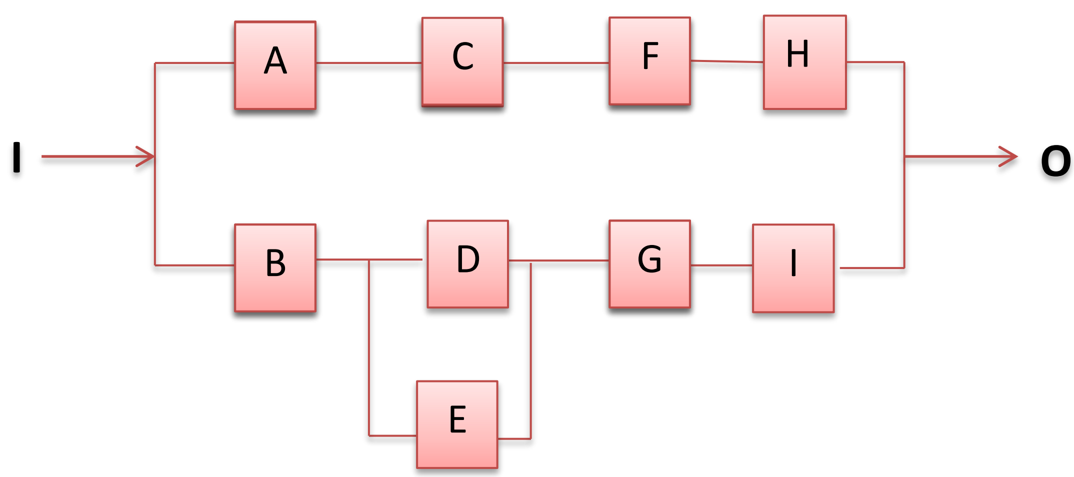


Figure (2.13) A Model Consist of 9 Components for Example 2.7

Figure (2.14) is a network representation of the complicated model described above.

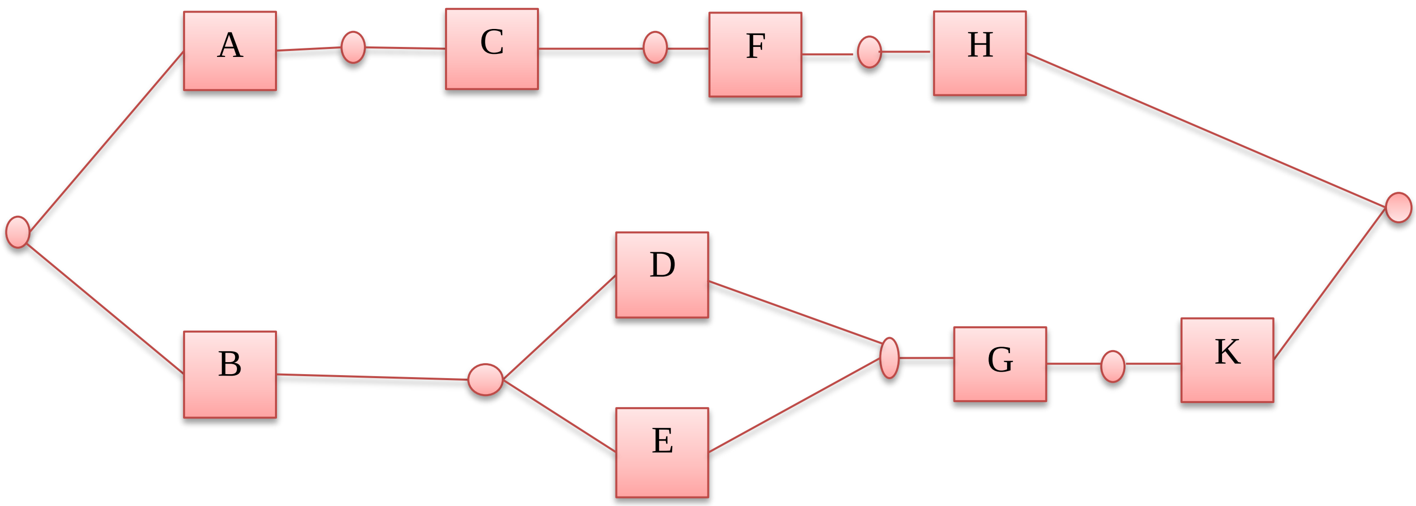


Figure (2.14) Network Representation for Figure (2.13)

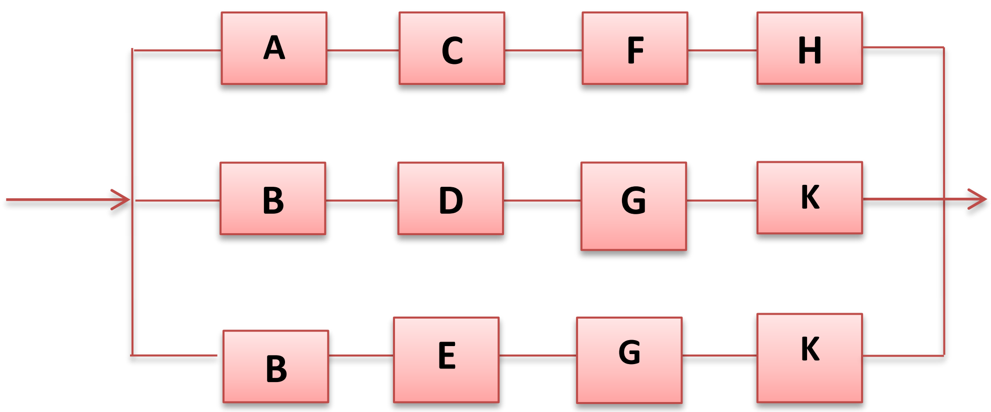


Figure (2.15) Representation for Figure (2.14)

The path sets are

$P_1 = \{ A , C , F, H \}$

$P_2 = \{ B, D , G , K \}$

$P_3 = \{ B , E , G, K \}$

Therefore the model reliability is :

$$R_s = 1 - \prod_{j=1}^{3} (1 - P_j)$$

$= 1 - (1 - P_1)(1 - P_2)(1 - P_3)$

$= P_1 + P_2 + P_3 - P_1 P_2 - P_1 P_3 - P_2 P_3 + P_1 P_2 P_3$

$= R_A R_C R_F R_H + R_B R_D R_G R_K + R_B R_E R_G R_K - R_A R_C R_F R_H R_B R_D R_G R_K -$

$R_A R_C R_F R_H R_B R_E R_G R_K - R_B R_D R_E R_G R_K + R_A R_C R_F R_H R_B R_D R_E R_G R_K$

If $R_i = R$

(That is, if the probabilities of all the components are statistically independent and identical)

Then we have

$R_s = 3R^4 - R^5 - 2R^8 + R^9$

If R = 0.8

Then the reliability of the model is

$R_s = 0.70$

2.3.2 Composite Method [7]:

This method is similar to the path tracing method but it does not list the entire parallel paths. The probability factor is derived in a comprehensive manner starting from the input end. When constructing the probability factor expressions, the series components are allocated "and " and the parallel elements are assigned "or ".

Example 2.8

Consider the following system, with reliabilities R_1, R_2 and R_3.

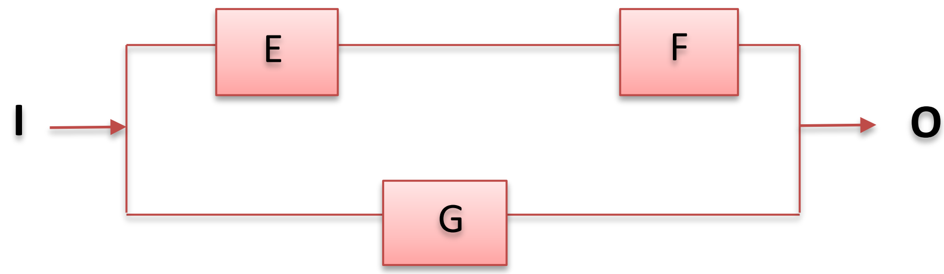


Figure (2.16) A Model Consist of 3 Components for Example 2.8

Suppose that the components are independent

Hence P(S) = P [(E and F) or G]

$$P(S) = P(E)P(F) + P(G) - P(E)P(F)P(G)$$

$$R_S(t) = R_E R_F + R_G - R_E R_F R_G$$

If $R_E = 0.93$, $R_F = 0.85$ and $R_G = 0.91$

Then

$$R_S(t) = (0.93)\,(0.85) + (0.91) - (0.93)\,(0.85)\,(0.91)$$

$$= (0.7905) + (0.91) - (0.719355) = 0.98$$

2.3.3 Reduction to Series Elements [8,14]:

In this method each parallel path is replaced with an equivalent single path , eventually reducing the supplied model to one that only consists of series elements.

Example 2.9

Consider the system, which consists of 7 linked components, as illustrated in Figure (2.17) below.

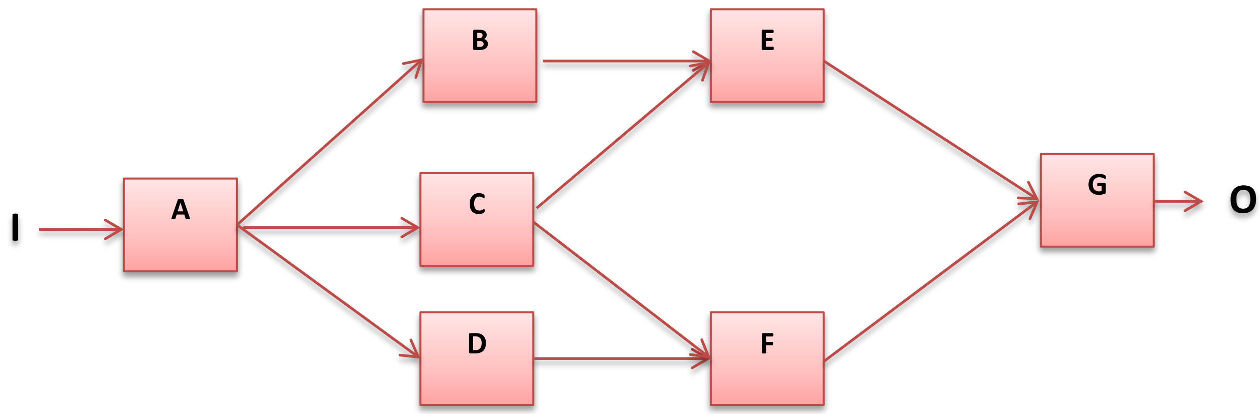


Figure (2.17) A Network for Example 2.9

Solution:

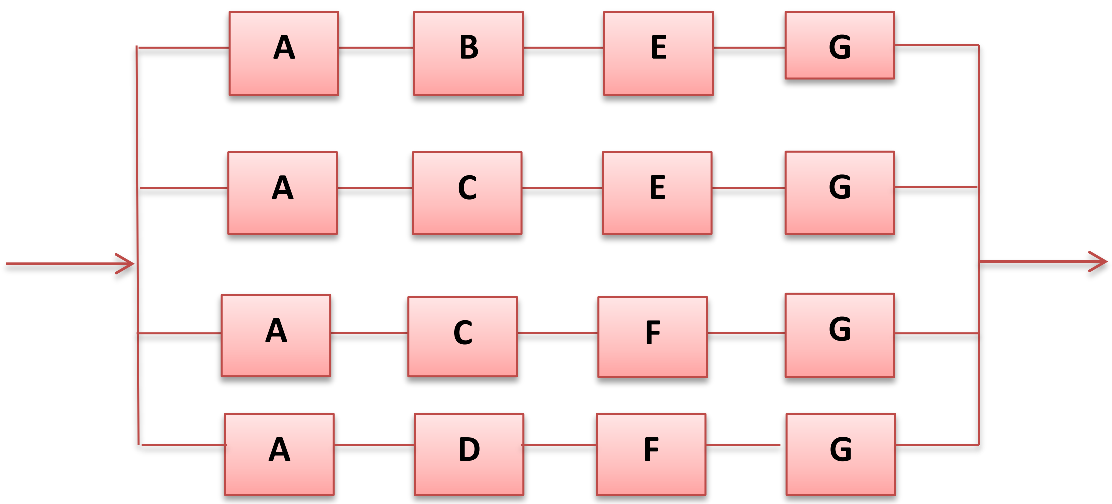


Figure (2.18) Representation for Figure (2.17)

Let O, P, Q and S paths where

O = {A , B , E , G }

P = {A , C , E , G}

Q = {A , C , F , G }

S = {A , D , F , G}

The reliability for each path

$$R_O = R_A \times R_B \times R_E \times R_G$$

$$R_P = R_A \times R_C \times R_E \times R_G$$

$$R_Q = R_A \times R_C \times R_F \times R_G$$

$$R_S = R_A \times R_D \times R_F \times R_G$$

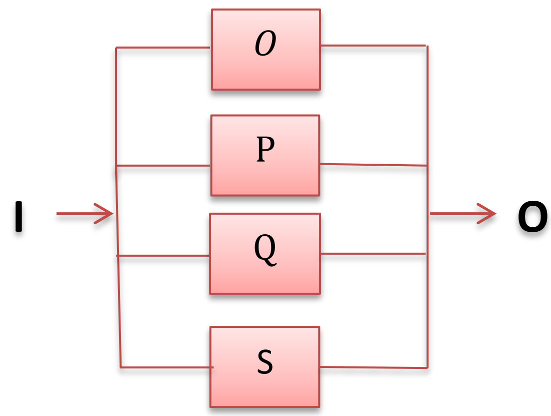


Figure (2.19) Reduction Representation for Figure (2.18)

We reduce O, P, Q and S to J

$$R_J = 1 - [(1 - R_O)(1 - R_P)(1 - R_Q)(1 - R_S)]$$

$$= R_O + R_P + R_Q + R_S - R_Q R_S - R_P R_S - R_O R_S - R_P R_Q - R_O R_Q - R_O R_P + R_P R_Q R_S + R_O R_Q R_S + R_O R_P R_S + R_O R_P R_Q - R_O R_P R_Q R_S$$

Example 2.10

Consider the system which consists of 9 linked components as illustrated in Figure (2.20) below.

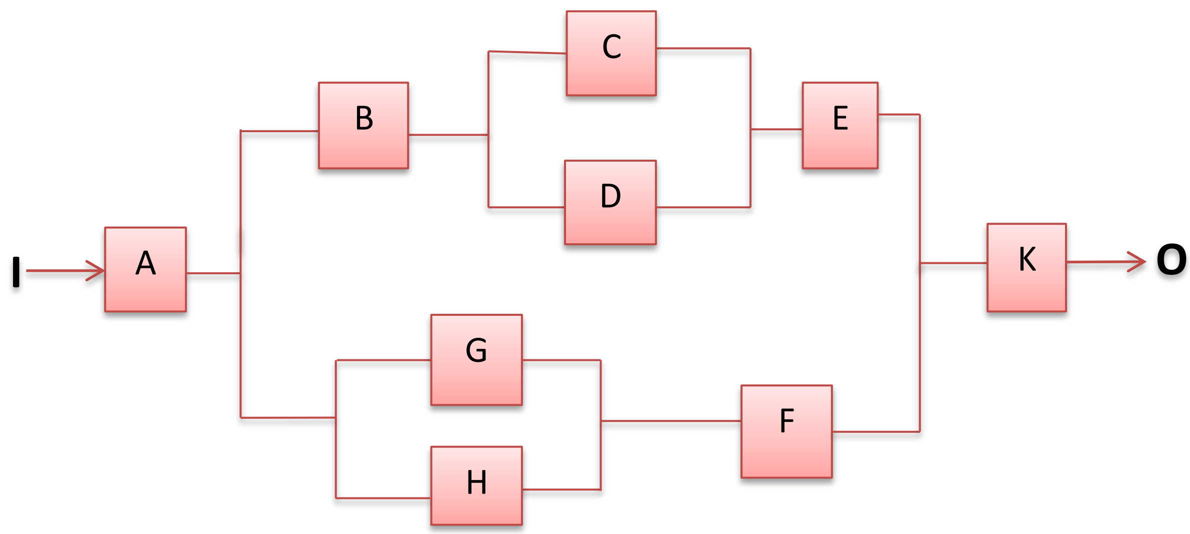


Figure (2.20) A Model Consist of 9 Components for Example 2.10

Let $R_A = 0.87$, $R_B = 0.75$, $R_C = 0.65$, $R_D = 0.80$, $R_E = 0.55$, $R_E = 0.95$, $R_G = 0.66$, $R_H = 0.3$, $R_K = 0.82$

We reduce the model in Figure (2.20) to a series model

Solution:

Let X_1 reduce to C and D , X_2 reduce to G and H

$$R_{X_1} = 1 - [(1 - R_C)(1 - R_D)]$$

$$= 1 - [(1 - 0.65)(1 - 0.80)]$$

$$= 1 - (0.35)(0.20) = 0.93$$

$$R_{X_2} = 1 - [(1 - R_G)(1 - R_H)]$$

$$= 1 - [(1 - 0.66)(1 - 0.3)]$$

$$= 1 - (0.34)(0.7) = 0.76$$

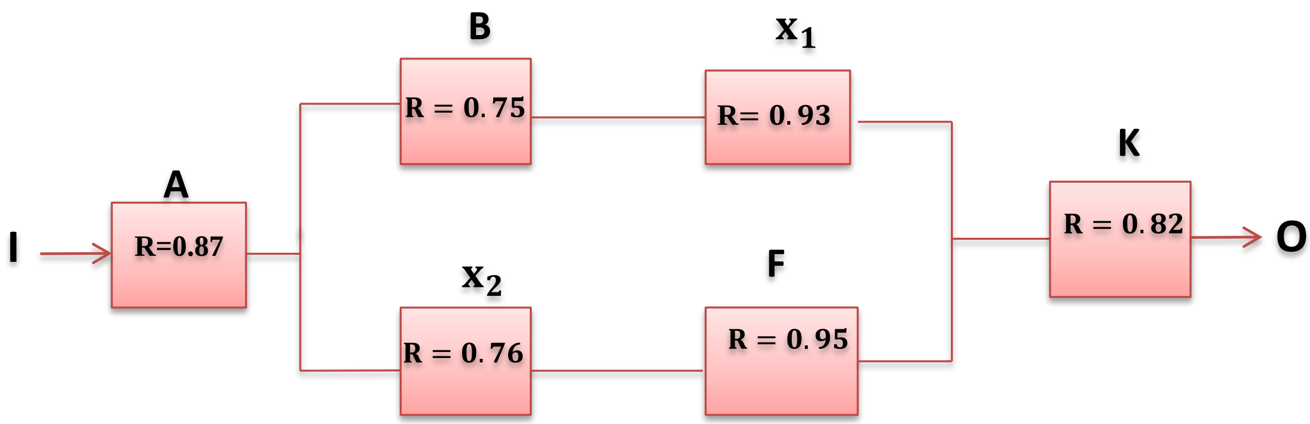


Figure (2.21) The First Step Reduction

Let X_3 reduce to B, X_1 and X_4 reduce to F and X_2

$$R_{X_3} = R_B \times R_{X_1}$$
$$= 0.75 \times 0.93 = 0.69$$
$$R_{X_4} = R_{X_2} \times R_F$$
$$= 0.76 \times 0.95 = 0.72$$

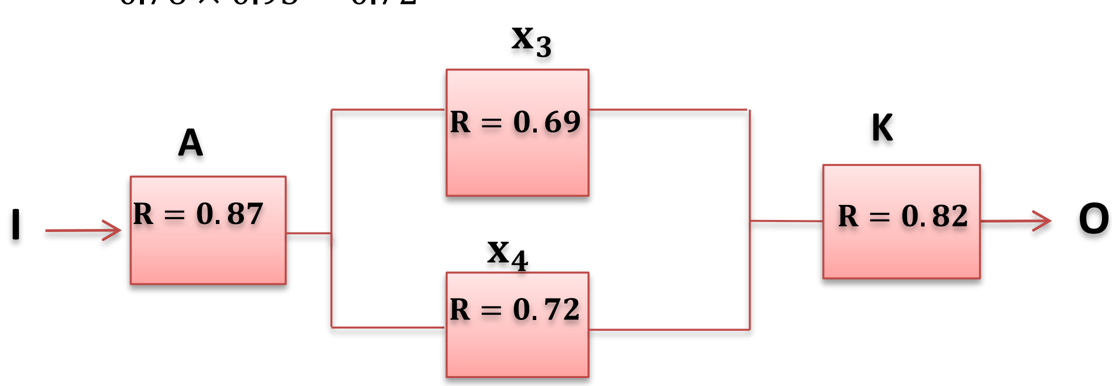


Figure (2.22) The Second Step Reduction

Let X_5 reduce to X_3 and X_4

$$R_{X_5} = 1 - [(1 - R_{X_3})(1 - R_{X_4})]$$
$$= 1 - [(1 - 0.69)(1 - 0.72)]$$
$$= 1 - (0.31)(0.28) = 0.91$$

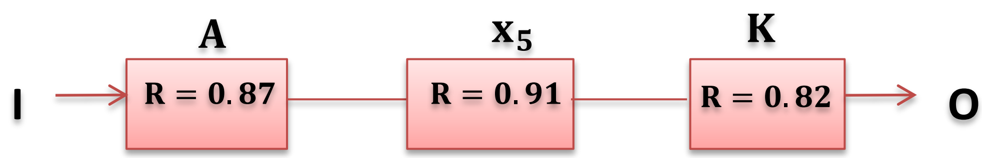


Figure (2.23) The Final Step Which Represent a Series Model

$$R_s = R_A \times R_{X_5} \times R_K$$
$$= 0.87 \times 0.91 \times 0.82 = 0.64$$

2.3.4 Minimal Cuts Method [63]:

A cut set is subset of components whose simultaneous failure causes the model to fail, regardless of the status of the other components.

A minimum cut is one in which there is no specified subset of components whose failure alone causes the system to fail.

Example 2.11

Consider the model in figure below, which has 10 components; the minimal cut of this model in Figure (2.24) is stated in the Table (2.1) below:

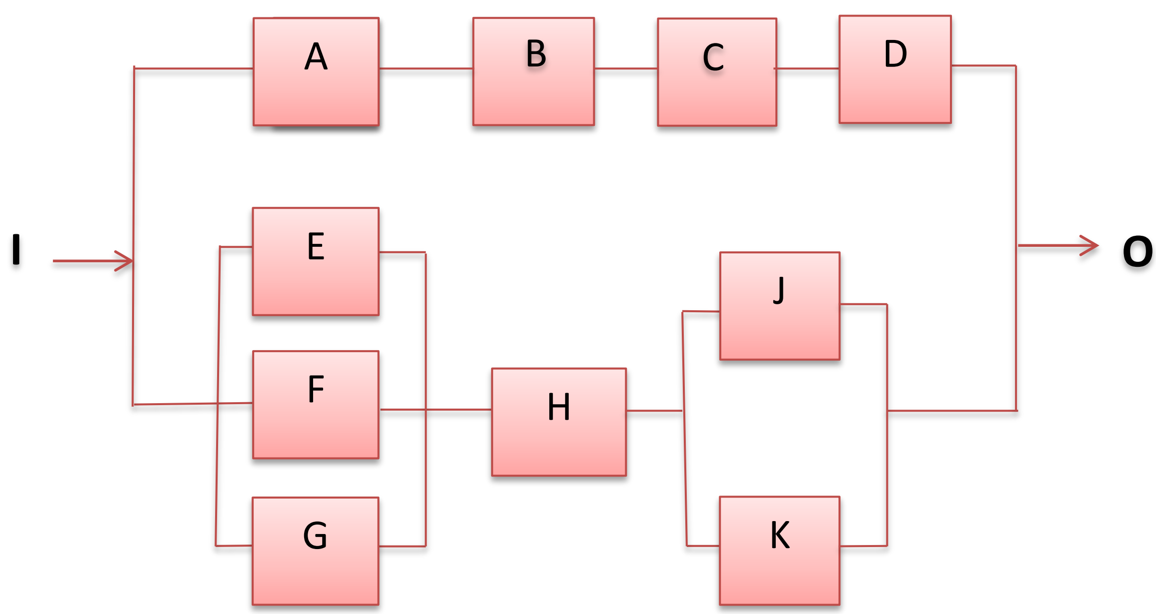


Figure (2.24) A Model of 10 Components for Example 2.11

Table (2.1) Minimal Cut Sets of the Model for Figure (2.24)

Minimal cut	Component in minimal cut
1	A , E , F ,G
2	A , H
3	A , J , K
4	B , E , F ,G
5	B , H
6	B , J , K
7	C , E , F ,G
8	C , H
9	C , J , K
10	D , E , F , G
11	D , H
12	D , J , K

The original model is identical to the system produced by its minimal cut sets in series, with each cut set represented by a parallel model with the components of the cut set as components.

2.3.5 Proposition:

Let $P_1, P_2, \ldots P_n$ be a minimal path sets and $C_1, C_2 \ldots\ldots C_m$ be the minimal cut sets of for a system (S).

Let R_i refer to the reliability of the i th of component than the model reliability R_s holds the following inequality:

$$\prod_{i=1}^{m}(C_i) \leq R_s \leq 1-\prod_{j=1}^{n}(1-P_j) \qquad (2.12)$$

R_s is bounded from below by the reliability a system of the form (parallel-series) which represent the cut sets of the system and bounded form above by the reliability of a system of the form (series-parallel) which is represent the minimal path sets of the system.

This means that R_s is bounded from above by LB (C_1, C_2 …..C_m) and bounded by UB ($P_1, P_2, \ldots P_n$), where LB refers to the lower bound, UB refers to the upper bound respectively.

Example 2.12

Consider a model consists of 8 components which are connected in Figure (2.25) calculate the reliability of the model.

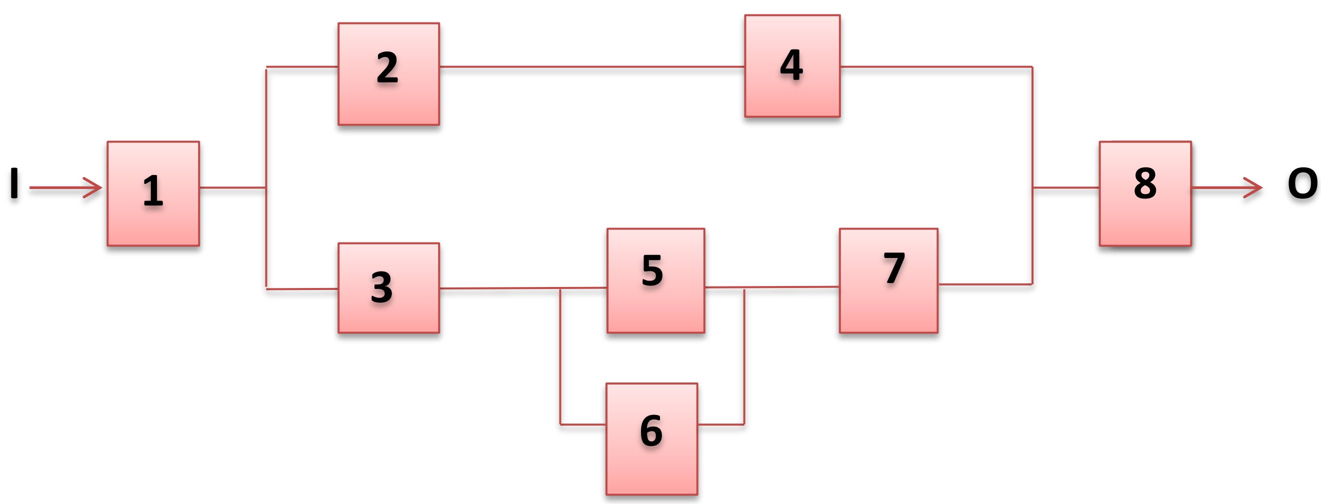


Figure (2.25) A Model Consist of 8 Components for Example 2.12

The network of figure above is :

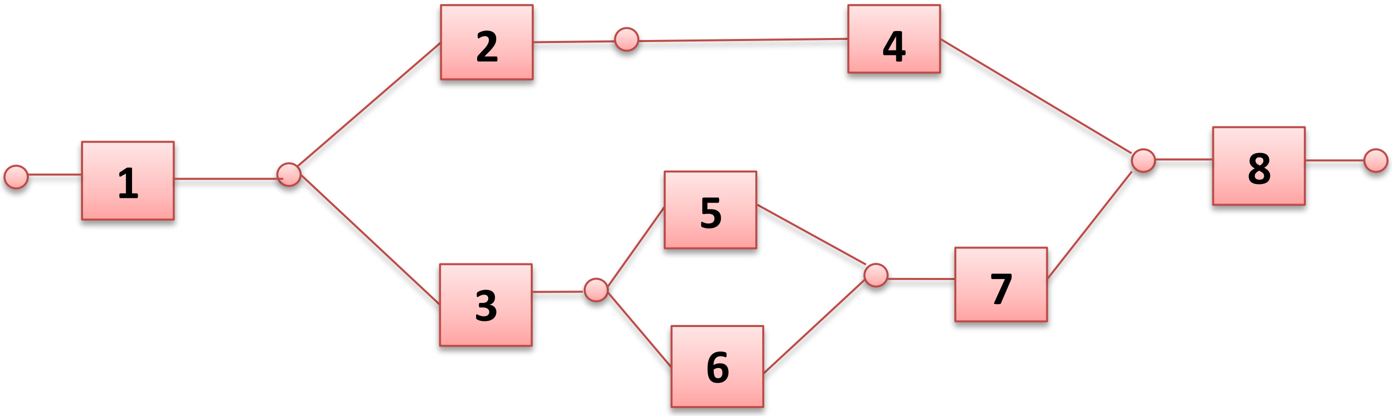


Figure (2.26) A Model Network for Figure (2.25)

Solution:

There are eight minimal cut sets :

$C_1 = \{1\}$, $C_2 = \{2,3\}$, $C_3 = \{2 ,7\}$, $C_4 = \{3,4\}$, $C_5 = \{4,7\}$,

$C_6 = \{2, 5, 6\}$, $C_7 = \{4 ,5 ,6\}$, $C_8 = \{8\}$

Then the model becomes:

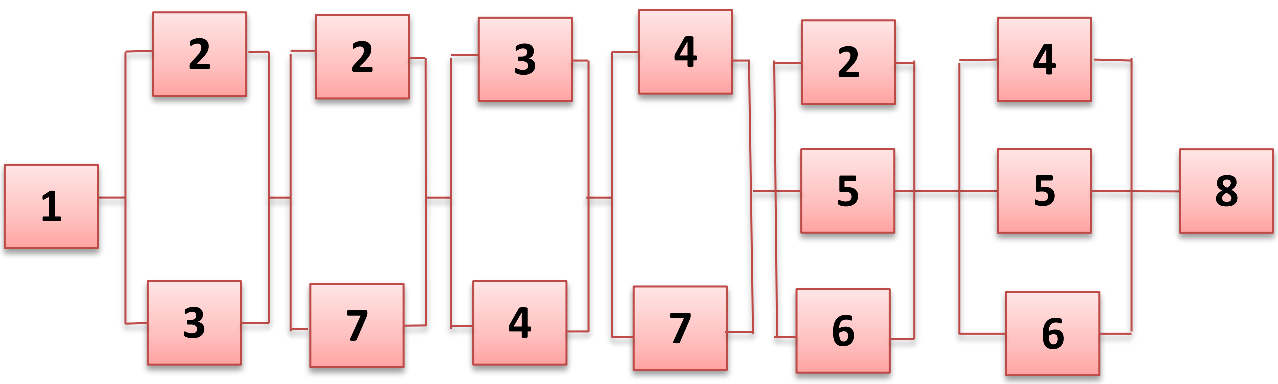


Figure (2.27) A Minimal Cut Sets for Figure (2.26)

The reliability of each cut set is C_i (where i = 1 ,… , 8) are as follows :

$R_{C1} = [\,1 - (1 - R_1)\,]$

$R_{C2} = [\,1 - (1 - R_2)\,(1 - R_3)\,]$

$R_{C3} = [\,1 - (1 - R_2)\,(1 - R_7)\,]$

$R_{C4} = [\,1 - (1 - R_3)\,(1 - R_4)]$

$R_{C5} = [\,1 - (1 - R_4)\,(1 - R_7)]$

$R_{C6} = [\,1 - (1 - R_2)\,(1 - R_5)\,(1 - R_6)\,\,]$

$R_{C7} = [\,1 - (1 - R_4)\,(1 - R_5)\,(1 - R_6)\,]$

$R_{C8} = [1 - (1 - R_8)]$

Therefore

$R_S = [\,1 - (1 - R_1)][\,1 - (1 - R_2)(1 - R_3)][\,1 - (1 - R_2)(1 - R_7)]$
$[\,1 - (1 - R_3)(1 - R_4)][\,1 - (1 - R_4)(1 - R_7)][\,1 - (1 - R_2)(1 - R_5)(1 - R_6)\,]$
$[\,1 - (1 - R_4)\,(1 - R_5)\,(1 - R_6)\,][1 - (1 - R_8)]$

Suppose the components have the same reliability (identical) R_i= R.

Then R_S is:

$R_S = [\,1 - (1 - R)]^2[\,1 - (1 - R)^2\,]^4[1 - (1 - R)^3]^2$

Example 2.13

For a system consist of nine components as shown in Figure (2.28) below, the requires:

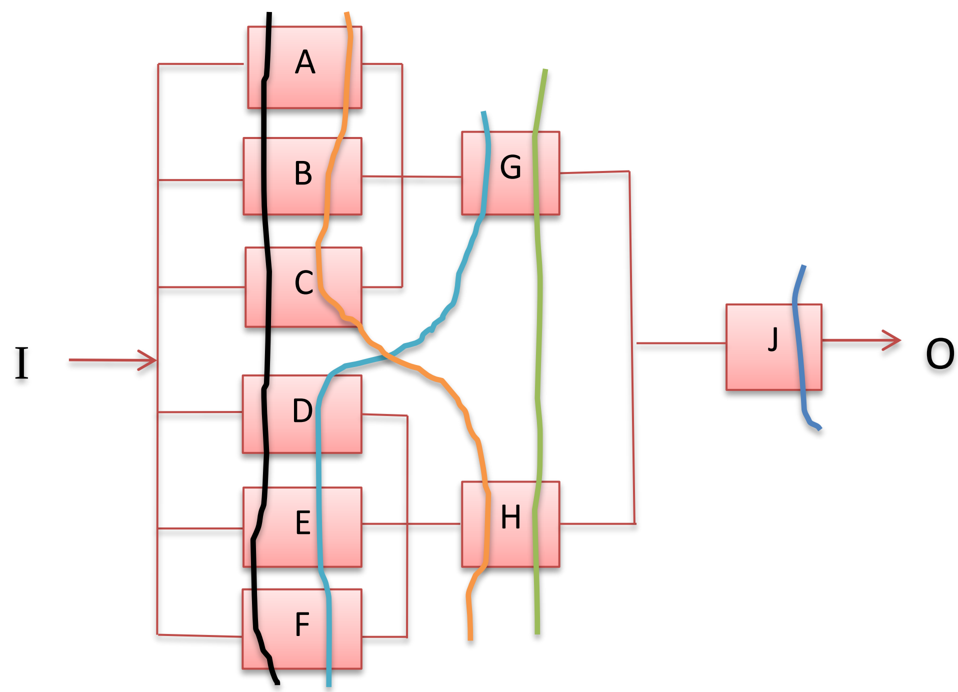


Figure (2.28) A Model Consist of 9 Components for Example 2.13

1. Determine path sets with representation.

2. Determine minimal cut sets with representation.

Solution:

1.There are six paths

$P_1 = \{A, G, J\}$, $P_2 = \{B, G, J\}$, $P_3 = \{C, G, J\}$, $P_4 = \{D, H, J\}$,

$P_5 = \{E, H, J\}$, $P_6 = \{F, H, J\}$

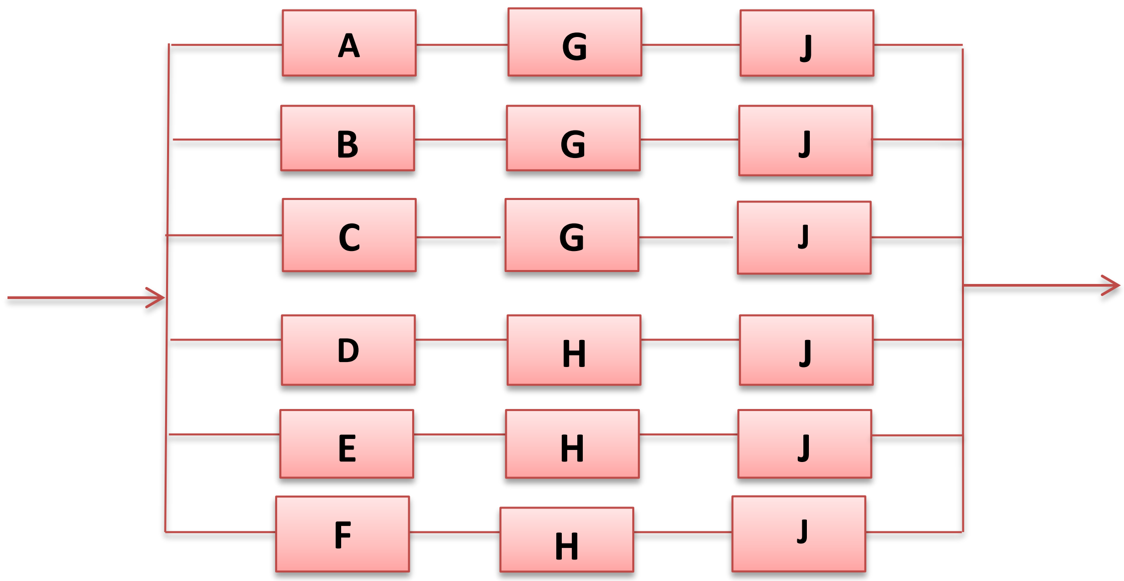


Figure (2.29) A Path Sets for Figure (2.28)

$$R_s = 1 - \prod_{j=1}^{6}(1 - P_j)$$

$= 1 - (1 - P_1)(1 - P_2)(1 - P_3)(1 - P_4)(1 - P_5)(1 - P_6)$

$= 1 - [(1 - R_A R_G R_J)(1 - R_B R_G R_J)(1 - R_C R_G R_J)(1 - R_D R_H R_J)(1 - R_E R_H R_J)$
$(1 - R_F R_H R_J)]$

Suppose the components have the same reliability (identical) $R_i = R$.We get

$R_s = 1 - [(1 - R^3)]^6$

If R = 0.75 than

$R_s = 0.96$

2. The minimal cut sets are :

$C_1 = \{A, B, C, D, E, F\}$

$C_2 = \{A, B, C, H\}, C_3 = \{G, D, E, F\}, C_4 = \{G, H\}, C_5 = \{J\}$

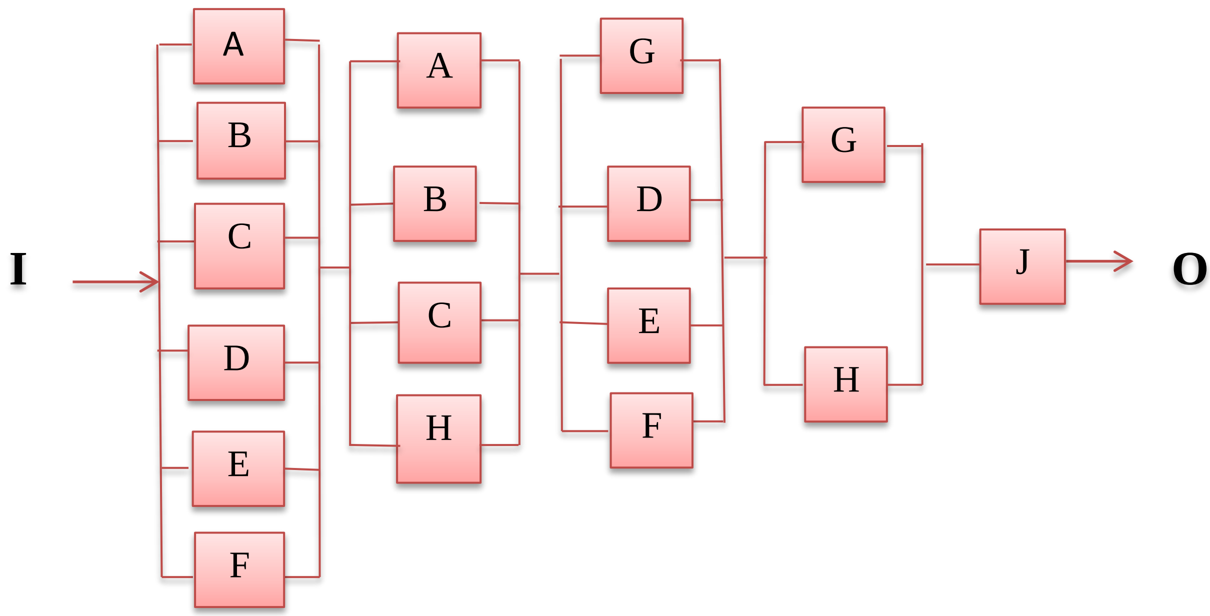


Figure (2.30) A Minimal Cut Sets of Figure (2.28)

The reliability of each cut set is C_i are as follows :

$$R_{C1} = [1-(1-R_A)(1-R_B)(1-R_C)(1-R_D)(1-R_E)(1-R_F)]$$

$$R_{C2} = [1-(1-R_A)(1-R_B)(1-R_C)(1-R_H)]$$

$$R_{C3} = [1-(1-R_G)(1-R_D)(1-R_E)(1-R_F)]$$

$$R_{C4} = [1-(1-R_G)(1-R_H)]$$

$$R_{C5} = [1-\left(1-R_J\right)]$$

Therefore

$$R_S = [1-(1-R_A)(1-R_B)(1-R_C)(1-R_D)(1-R_E)(1-R_F)][1-(1-R_A)(1-R_B)(1-R_C)(1-R_H)]\,[1-(1-R_G)(1-R_D)(1-R_E)(1-R_F)]\,[1-(1-R_G)(1-R_H)]\,[1-\left(1-R_J\right)]$$

If the components have the same probability $R_i = \mathrm{R}$ then R_S becomes :

$$R_S = [1-(1-R)^6]\,[1-(1-R)^4]^2[1-(1-R)^2]\,[1-(1-R)]$$

If R = 0.75 than

$R_S = 0.69$

$0.69 \leq R_s \leq 0.96$

Example 2.14

Return to the Figure (2.10) of 2- out- of -3 model determine the following

1. Path sets

2. Minimal cut set

Solution:

1. There are 3 path sets where i = 1, 2, 3

$P_1 = \{1,2\}, P_2 = \{2,3\}, P_3 = \{3,1\}$

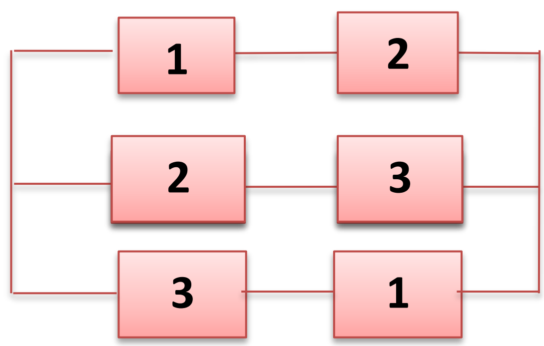


Figure (2.31) A Path Sets Representation for Figure (2.10)

So

$$R_s = 1 - \prod_{j=1}^{3} (1 - P_j)$$

$$R_s = 1 - (1 - P_1)(1 - P_2)(1 - P_3)$$

$$R_s = 1 - (1 - R_1 R_2)(1 - R_1 R_3)(1 - R_2 R_3)$$

$$R_s = R_1 R_2 + R_1 R_3 + R_2 R_3 - 2 R_1 R_2 R_3$$

Suppose the components have the same reliability (identical) $R_1 = R_2 = R_3 = R$.

We get

$R_s = 3R^2 - 2R^3$

If R = 0.7

Then $R_s = 0.78$

2. There are three minimal cut sets:

$C_1 = \{1,2\}, C_2 = \{1,3\}, C_3 = \{2,3\}$

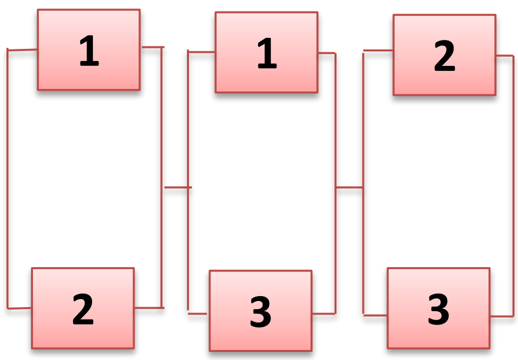


Figure (2.32) A Minimal Cut Sets of the Model for Figure (2.10)

The reliability of each cut set C_i are as follows :

$R_{C1} = [1 - (1 - R)^2]$

$R_{C2} = [1 - (1 - R)^2]$

$R_{C3} = [1 - (1 - R)^2]$

Suppose the components have the same reliability (identical) $R_i = R$.

$\therefore\ R_s = [1 - (1 - R)^2]^3$

If R = 0.7 then

$R_s = 0.75$

$0.75 \leq R_s \leq 0.78$

And this coincidence with the following relation

$$\prod_{i=1}^{3}(C_i) \leq R_s \leq 1 - \prod_{j=1}^{3}(1 - P_j)$$

2.3.6 Observations :

1.In the inequality (2.12) above at some time the lower and upper are equal.

2. Figure (2.31) depicts a series-parallel system.

3. Figure (2.32) depicts a parallel- series system.

4.Then number of path sets equal to the number of cut sets.

5. If K = 1 the model becomes a parallel model.

6. If K = n the model becomes a series model.

3.4 Reliability Importance of Component

This section deals with reliability importance of component by presenting a new method to calculate it via an illustrative .

Definition 3.4.1 [3,18]:

The **Component's Importance** is the likelihood that at least one minimal cut set including component say i failed at time t. It should be noted that a minimal cut set fails when all of its components fail.

We can calculate the importance of component i (I_{C_i}) via the following steps :

1) Let $C_1, C_2, \dots \dots . . , C_n$ be a components of a model (S).

2) $M_i(t)$ represent at least one minimal cut which contains component i is failed at time t.

3) S(t) represent the model (S) is failed at time t.

4) K(i,j) represent the component i is failed at time t amongst minimal cut j.

5) For the event $M_i(t)$ happens if at least one of events K (i,j) (t) take place i=1,2,…n , j=1,2,….m so

$M_i(t) = K(i,1)(t) \cup K(i,2)(t) \cup \dots \dots \cup K(i,m)(t)$

6) Since the component C_i are supposed independent then

$$P_r[S(t)] = Q_S(t) = 1 - R_S(t)$$

Where i = 1,2,…..,n , j = 1,2,….m by using conditional probability

$$I_{C_i}(t) = P_r[M_i(t)/S(t)]$$

$$I_{C_i}(t) = \frac{P_r[M_i(t) \cap S(t)]}{P_r(S(t))}$$, But $M_i(t)$ implies S(t)

$$I_{C_i}(t) = \frac{P_r[M_i(t)]}{P_r(S(t))} \qquad (2.13)$$

Example 2.15

As an illustration we return to Figure (2.32) in Example (2.14) 2- out -of -3 model. The model has 3 minimal cut sets: $\{C_1, C_2\}, \{C_1, C_3\}, \{ C_2, C_3\}$.

Let the reliabilities R_i of component C_i then i =1, 2, 3 are $\mathrm{R}_1 = 0.95$,

$\mathrm{R}_2 = 0.93$, $R_3 = 0.96$ .The model reliability R_s is :

$R_s = \mathrm{R}_1\mathrm{R}_2 + \mathrm{R}_1\mathrm{R}_3 + \mathrm{R}_2\mathrm{R}_3 - 2\mathrm{R}_1\mathrm{R}_2\mathrm{R}_3$

$= (0.95)(0.93) + (0.95)(0.96) + (0.93)(0.96)\_2(0.95)(0.93)(0.96)$

$= (0.8835) + (0.912) + (0.8928) - (1.69632)$

$= 0.99198$

$$Q_S = 1 - R_S$$

$= 1 - 0.99198 = 0.00802$

The minimal cut sets are : $M_1 = \{C_1, C_2\}, M_2 = \{C_1, C_3\}, M_3 = \{ C_2, C_3\}$.

Now, we find $P_r(M_1)$, $P_r(M_2)$, $P_r(M_3)$

For C_1: $\mathrm{P_r(M_1)} = \mathrm{P_r[K(1,1) \cup K(1,2)]}$

$\mathrm{P_r(M_1)} = \mathrm{P_r}K(1,1) + \mathrm{P_r}K(1,2) - \mathrm{P_r}[K(1,1) \times K(1,2)]$

We have $q_i = 1 - R_i$ so i =1,2…n

$\mathrm{P_r(M_1)} = q_1q_2 + q_1q_3 - q_1q_2q_3$

$= (0.05)(0.07) + (0.05)(0.04) - (0.05)(0.07)(0.04)$

$= 0.0035 + 0.002 - 0.00014 = 0.00536$

For C_2: $\mathrm{P_r(M_2)} = \mathrm{P_r[K(2,1) \cup K(2,3)]}$

$\mathrm{P_r(M_2)} = \mathrm{P_r}K(2,1) + \mathrm{P_r}K(2,3) - \mathrm{P_r}[K(2,1) \times K(2,3)]$

$\mathrm{P_r(M_2)} = q_1q_2 + q_2q_3 - q_1q_2q_3$

$= (0.05)(0.07) + (0.07)((0.04) - (0.05)(0.07)(0.04)$

$= (0.0035) + (0.0028) - (0.00014) = 0.00616$

For C_3: $\mathrm{P_r(M_3)} = \mathrm{P_r[K(3,2) \cup K(3,3)]}$

$\mathrm{P_r(M_3)} = \mathrm{P_rK(3,2)} + \mathrm{P_rK(3,3)} - \mathrm{P_r[K(3,2) \times K(3,3)]}$

$\mathrm{P_r(M_3)} = q_1q_3 + q_2q_3 - q_1q_2q_3$

$= (0.05)(0.04) + (0.07)(0.04) - (0.05)(0.07)(0.04)$

$= 0.002 + 0.0028 - 0.00014 = 0.00466$

And the last step is to find the reliability importance I_{C_i} by substation in (2.13) we get

$$I_{C_1}(t) = \frac{P_r[M_1(t)]}{P_r(S(t))} = \frac{0.00536}{0.00802} = 0.6683$$

$$I_{C_2}(t) = \frac{P_r[M_2(t)]}{P_r(S(t))} = \frac{0.00616}{0.00802} = 0.7680$$

$$I_{C_3}(t) = \frac{P_r[M_3(t)]}{P_r\big(S(t)\big)} = \frac{0.00466}{0.00802} = 0.5810$$

It is noted that the component C_2 with lowest reliability is the most important and noted that $I_{C_3}(t) < I_{C_1}(t) < I_{C_2}(t)$

Chapter Three
Static Fault Tree and Dynamic Fault Tree

3.1 introduction:

In this chapter, there are two types of fault trees that is dealt with. The first type is SFT divided into six sections, the first section is a SFT consisting of some concepts, definitions, logic gates, how to create a fault tree and steps to analyze it, and the second section contains sub fault tree (sub-FT) and multistate FT and some examples of dependent and independent (sub-FT) and some theories related to series and parallel. The third section is to analyze the fault tree analysis FTA in two ways, and the fourth section deal with RBD and then how to convert FT to RBD and then define the success tree ST.

The second type in this chapter, deals with the dynamic fault tree DFT. The fifth section introduces the dynamic gates, their properties, dynamic reliability block diagram, dynamic fault tree gates, how to convert the DFT to DRBD, and then DRBD versus SFT. In the sixth section, models of dynamic gates behavior. In section seventh a dynamic multistate fault tree is illustrated with an example. At the end of this type the conversion of DFT to SFT is studied.

3.2 Some Concepts and Definitions

Some concepts and definitions related to the current work are provided on static fault tree and FT construction and analysis steps.

Another definition for static fault tree (SFT) or (FT)

Definition 3.2.1 [27,35]:

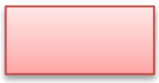
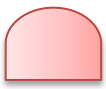
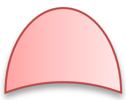
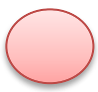
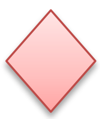
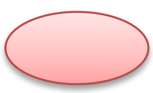
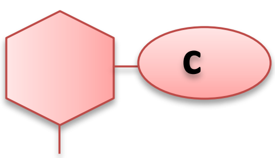
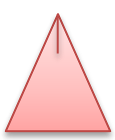
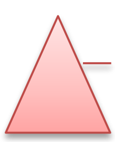
The **Static Fault Tree (SFT)** is a graphical depiction of the Boolean failure logic associated with the progression from a specific system failure (the top event) to basic failures (primary events).

Definition 3.2.2 [56]:

A **Fault Tree Diagrams** are logic block diagrams that display the state of a system (top event) in terms of the states of its components (basic event), and they are a graphical design technique.

The common logic gates used for events representation in static fault tree are listed in Table below (3.1) [54,63].

Table (3.1) Logical Gates for SFT

Graphic symbol	Meaning
	**Rectangle** Top or intermediate event.
	**AND** That the output event happens only if all of the input events happen.
	**OR** The output event occurs if one or more of the input events happens.
	**Circle** Elementary basic event .
	**Rhombus (diamond)** Non-elementary basic event (Undeveloped event).
	**Conditioning event** Any condition or constraint that applies to any logic gate is recorded using the ellipse.
	**IF** If the input exists and the state C is checked, the output is verified.
	**Transfer in** It indicates that the tree is further developed when the corresponding transfer out occurs.
	**Transfer out** It denotes that this branch of the tree must be connected at the appropriate transfer in.

3.2.3 Boolean algebra [54]:

In 1800, an Irish mathematician named James Bool invented Boolean algebra. It was proven to be very useful for building digital circuits, and it is still commonly used by electrical engineers and computer scientists to day. Boolean equations are composed of variables and operations and include algebraic equations. We used two logic gates, which are (AND, OR).

3.2.4 FT Construction [47]:

Follow these procedures to complete a full FTA.

1) Define the fault state and record the top level of failure.

2) Determine the most likely causes of the failure using technical knowledge. Note that they are level two components since they are just under the top event failure of the tree.

3) Continue to deconstruct each element by adding more gates to lower levels. Consider the relationships between the elements before deciding whether to use a "and" or "or" logic gate.

4) Finish and go through the entire diagram. The chain can only be broken by a fundamental failure: human, physical, or software.

5) If feasible, determine the likelihood of occurrence for each lowest level piece and calculate the statistical probability from the bottom to up. Assume the system shown in the Figure (3.1) below.

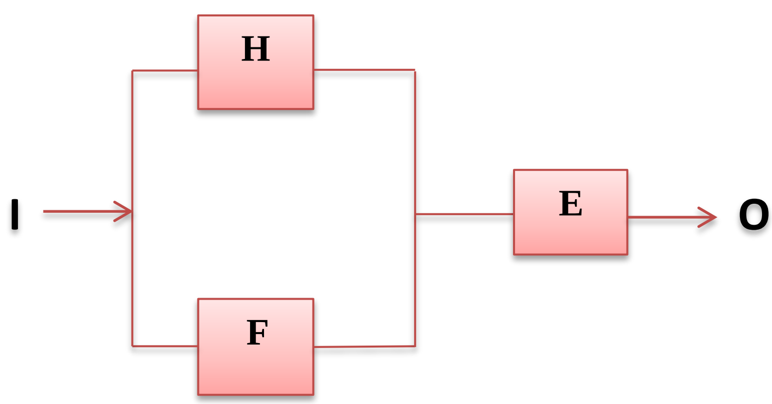


Figure (3.1) A Model of 3 Components

Let T represent the system's output T = [H ∩ F] ∪ E.

Because the relationship between components H and F is intersection [H ∩ F] and union E, the fault tree may be constructed as follows.

i) AND gate connecting occurrences H and F.

ii) OR gate connection between [H ∩ F] and E.

Because the top event represents the failure system, the model's fault tree is:

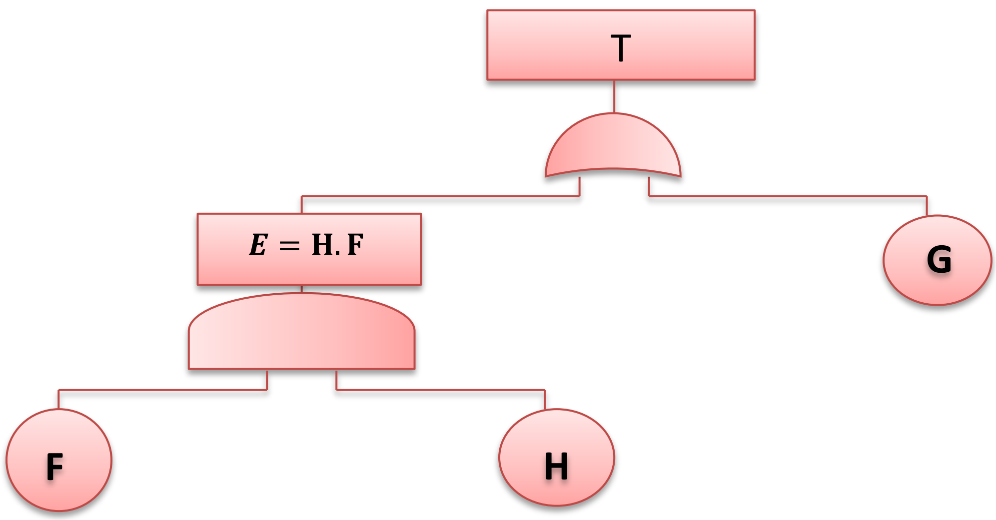


Figure (3.2) A Model FT for Figure (3.1)

Example 3.1

Return to Example (2.13), for the Figure (2.28) to construct the FT for system with analysis.

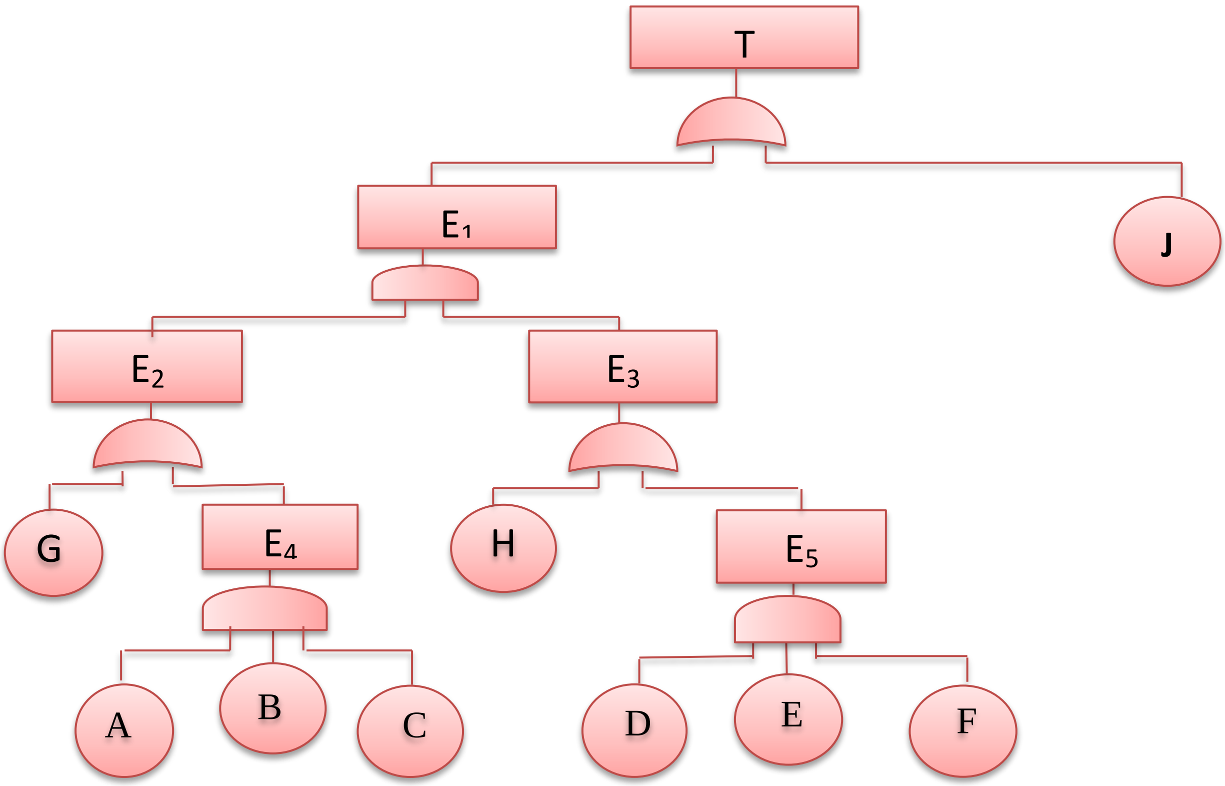


Figure (3.3) FT for a Model in Figure (2.28)

T= E_1 +J Where

$E_1 = E_2 . E_3$, $E_2 = E_4 + G$, $E_3 = E_5 + H$, $E_4 = A . B . C$,

$E_5 = D . E . F$

By substitution

T = $E_2 . E_3 + J$

$= (E_4 + G)(E_5 + H) + J$

$= (A . B . C + G)(D . E . F + H) + J$

$= (A . B . C . D . E . F) + (A . B . C . H) + (D . E . F . G) + (G . H) + (J)$ (3.1)

As analytical study we noted that.

There are five cut sets which cause failure of the system. Equation (3.1) show that the top event T which refers to the failure of the system happens where (A . B . C . D . E . F) or (A . B . C . H) or (D . E . F. G) or (G . H) or (J) happens so the probability of failure of the system F_S is:

$$F_S = P((\text{A . B . C . D . E . F}) + (\text{A . B . C . H}) + (\text{D . E . F . G}) + (\text{G. H}) + (\text{J}))$$

Assume that the components are independent

Let $P_A, P_B, P_C, P_D, P_E, P_F, P_H \text{ and } P_J$.

Represents their probabilities respectively and R_i is the reliability for the components A, B, C, D, E, F, H, J.

To get R_S we return to the relation
$R_S = 1 - F_S$

3.2.5 Fault Tree Analysis FTA [47]:

Bell Laboratories [66] created the fault tree analysis (FTA), which is currently one of the most extensively used approaches in system reliability, maintenance, and safety analysis. It is a logical method that is used to identify the possible combinations of hardware, software, and human errors that might result in unwanted events (also known as top events) at the system level.

The deductive analysis starts with a broad conclusion and then aims to identify the precise causes of the conclusion by creating a logic diagram known as a fault tree. This is sometimes referred to as a top-down strategy. The fundamental purpose of fault tree analysis is to discover potential causes of system breakdowns before they occur. Second, it may be utilized to apply analytical or statistical approaches to determine the likelihood of the top event.

System quantitative reliability and maintainability data such as failure probability, failure rate, and repair rate are included in these computations.

Following the completion of an FTA, you may focus your efforts on improving system safety and reliability.

3.2.6 Steps for Performing an FTA [46,58]:

A successful FTA involves the following steps:

1. Determine the FTA's goal.

2. Identify the FTA's top event.

3. Define the scope of the FTA.

4. Describe the FTA resolution.

5. Define the FTA's basic rules.

6. Build the FT.

7. Evaluate the FT.

8. Interpret and explain your findings.

The Diagram (3.4) indicates the interdependence of the eight phases. The Diagram below depicts the feedback.

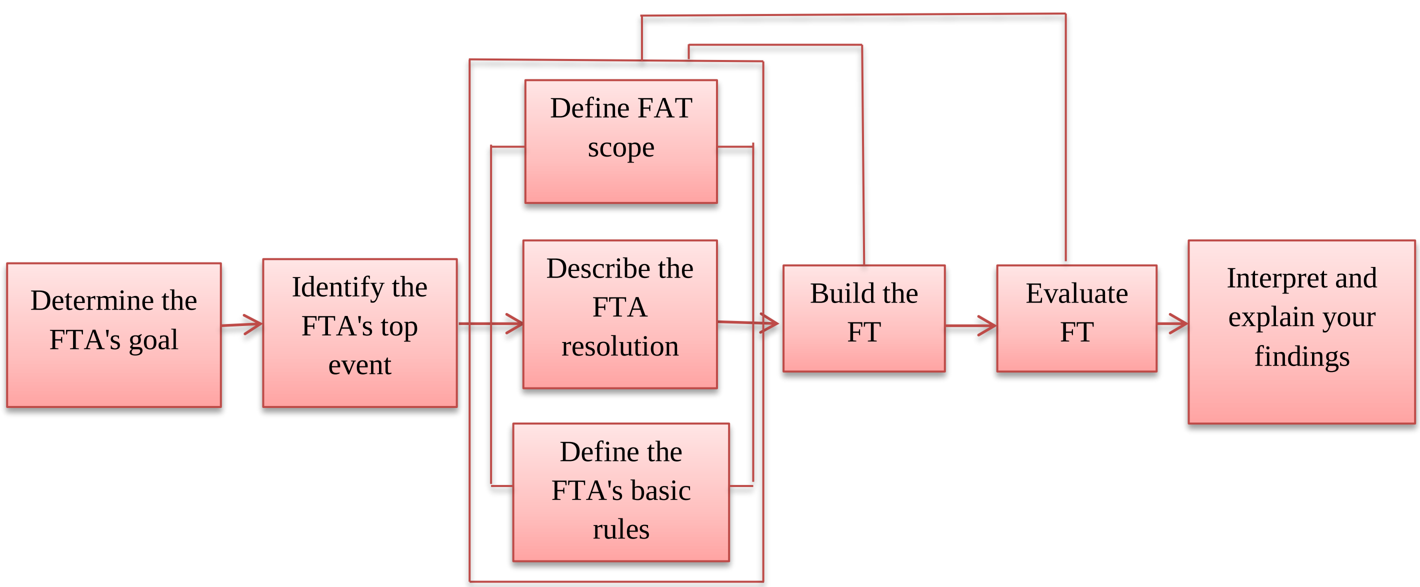


Diagram (3.4) FTA Steps

3.2.7 Observation:

1. An **AND gate** joins two or more events that must occur together. The result is a top or intermediate event, which is frequently represented as an a rectangle. The AND operation is represented by the symbol (**.**).

2 . An **OR gate** links two or more events, each of which leads to a certain intermediate or top event (a rectangle in the fault tree). The OR operation is represented by the symbol (+).

3. The goal of **qualitative analysis** for FT is to identify the (minimal cut sets) of low level failures that are generating the top level failure.

4. The objective of the failure probability of the top event and the basic events is determined by **quantitative analysis** for FT.

3.3 Sub Fault Tree (Sub-FT) and Multistate Fault Trees:

In this section some definitions related to it with illustrate example.

Definition 3.3.1 [34]:

The **Sub-FT** which correspond to the intermediate event is the biggest FT possess this intermediate event as its top event.

Example 3.2

For describing the ingredients of a fault tree, the following fault tree as an example is produced.

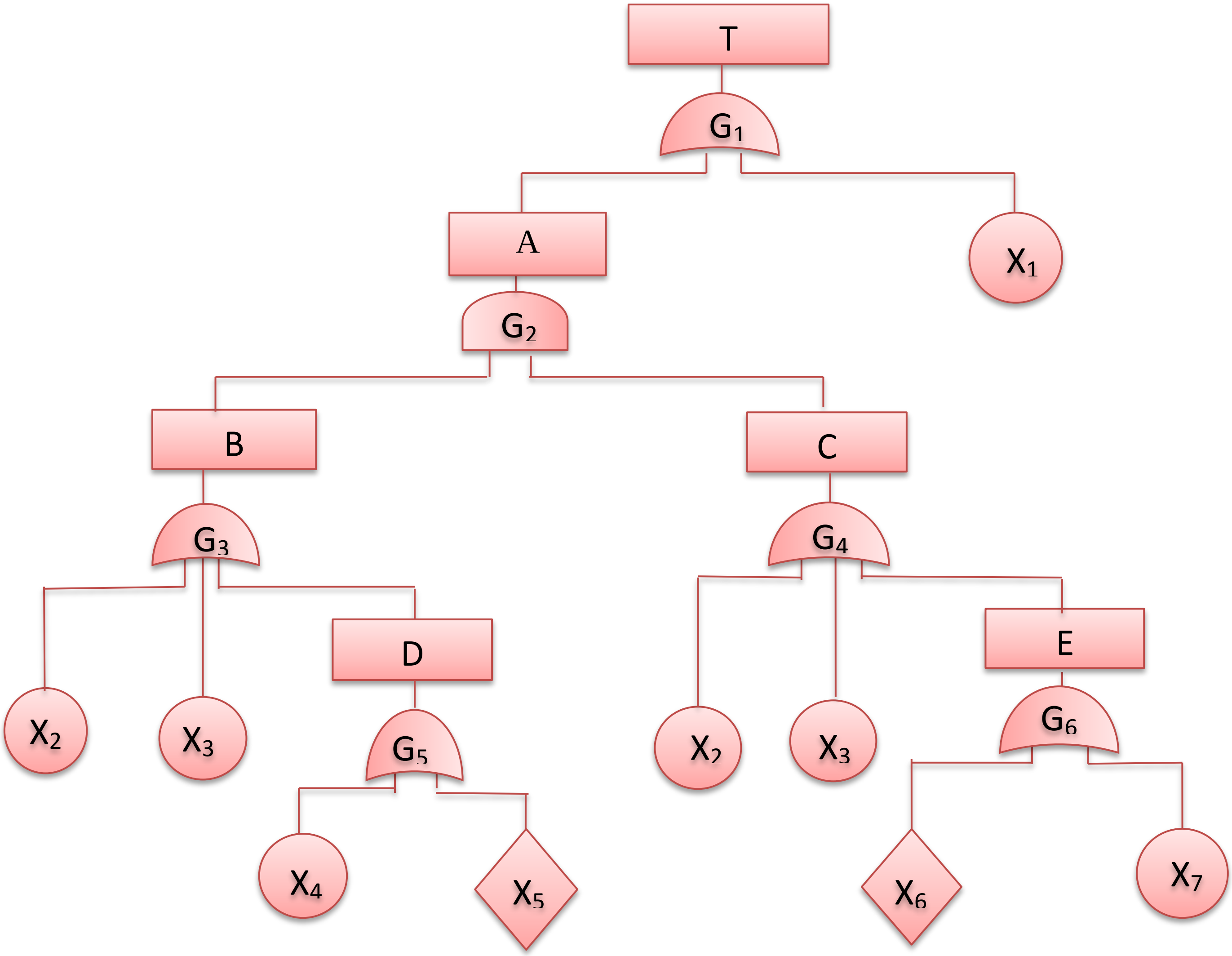


Figure (3.5) FT for Example 3.2

The ingredients of a fault tree of example above are:

1. Top event T
2. Intermediate events: A, B, C, D, E.
3. Basic events: X_1 , X_2 , X_3 , X_4 , X_5 , X_6 , X_7.
4. Events X_5 and X_6 are not elementary (undeveloped event).
5. Events X_2 and X_3 are repeated events.

6. The OR gates: G_1, G_3, G_4, G_5, G_6 and the AND gate G_2. (The gates G_5, G_6 are primary since all their inputs are basic events).

For an example to a sub-FT: the sub-FT which is correspond to the intermediate event B is the FT consist of the gates G_3 and G_5, the basic event X_2 , X_3 , X_4 , X_5 which are inputs of gates G_3 and G_5. The domain (dom) of the sub-FT B which represent the basic events of B is :

dom (B)={ X_2 , X_3 , X_4 , X_5}

3.3.2 Observation [34]:

1. A sub FT is independent if no elements of its domain appears anywhere else in it.

2. If a sub FT is independent then it's called module.

Example 3.3

Consider the FT in the figure below.

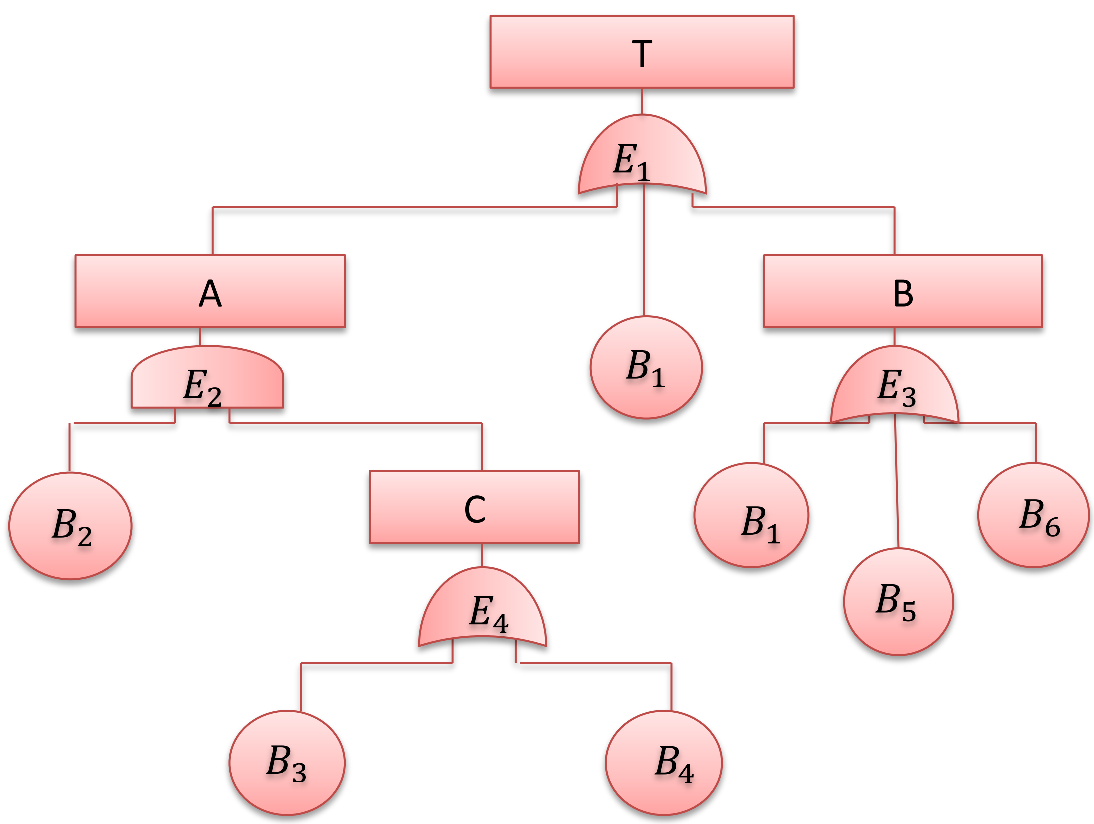


Figure (3.6) FT Model for Example 3.3

The sub-FT A is independent as no element in its domain $\{B_2, B_3\ , B_4\}$ exists anywhere in the FT. The sub-FT B on the other hand, is not independent, because the event B_1 happens elsewhere. As a result, the sub-FT A is a module of the FT (T).

Definition 3.3.3 [34]:

Multistate Fault Trees are trees in which the basic events are not events but components, the intermediate events are subsystems, and the top event is the system failure.

Definition 3.3.4 [61]:

The **Multistate Operator (MOP)** is used to apply input spaces to output spaces. The function depicted by the operator in Figure (3.7) below is:

$$f: E_{r,1} \times E_{r,2} \times \dots \dots . \times E_{r,n} \longrightarrow E_r$$

where $E_{r,i}$ (i =1,…, n) is the space of the i-th input, and E_r is the output space.

Noted that studying systems with state space $E_i = \{0,1\}$ for any component i , it's called binary system. Postelnicu (1970) as an extension introduced the state space $[0,1] \subset R$ for the components and the systems.

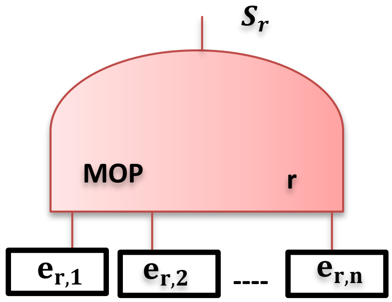


Figure (3.7) Multistate Operator with n inputs

The following theorems have been introduced concern the current work.

3.3.5 Theorem:

The reliability of FT for logic gate AND with (n) independent basic events is:

$$R_s = 1 - \prod_{i=1}^{n} P_r[B_i(t)]$$

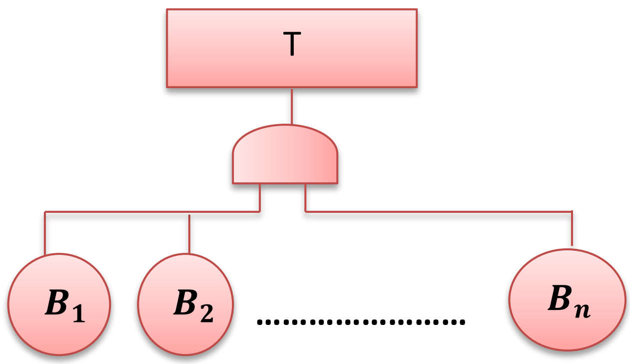


Figure (3.8) FT of AND Gate

Proof :

The top event T happens if any basic events B_i, i =1,…,n fail. Assume that B_1, B_2,..., B_n are independent. Let $B_i(t)$ repersent the basic event B_i fails at time t, i =1,2,…., n and let P_r is probability of an event. By using the Boolean algebra rules the unavialability of the top event T (i.e the failure of system say $F_S(t)$ occurs if any of the event fail).

Therefore

$$F_S(t) = P_r[B_1(t) \wedge B_2(t) \wedge \ldots.. \wedge B_n(t)]$$

$$F_S(t) = P_r[B_1(t) \times B_2(t) \times \ldots..\times B_n(t)]$$

$$F_S(t) = \prod_{i=1}^{n} P_r\big(B_i(t)\big)$$

We can obtain R_s form the relation

$$R_s(t) = 1 - F_S(t)$$

3.3.6 Theorem:

The reliability of FT for logic gate OR with (n) independent basic event is :

$R_s = 1 - (1 - \prod_{i=1}^{n}[1 - P_r(B_i(t))]$

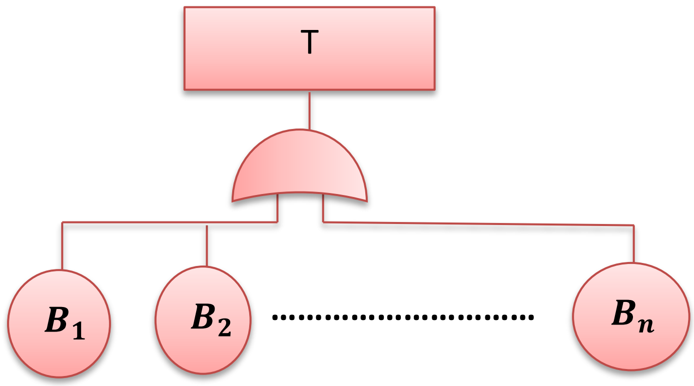


Figure (3.9) FT of OR gate

Proof :

The top event T happens if at least one of the basic events $B_1, B_2, \ldots.., B_n$ take plece. Assume that the basic events are independent. Consider $B_i(t)$ repersent the basic event happens at time t and $\bar{B}_i(t)$ repersent the basic event does not occur at t. Where $\bar{B}_i(t) = 1 - B_i(t)$. Due to the Boolean algebra rules we get the top event (the system failure) $F_S(t)$ as follow:

$F_S(t) = P_r[B_1(t) \vee B_2(t) \vee \ldots..\vee B_n(t)]$
$F_S(t) = 1 - P_r[(\bar{B}_1(t)) \wedge \bar{B}_2(t) \wedge \ldots. \wedge \bar{B}_n(t)]$

$F_S(t) = 1 - [P_r\,\bar{B}_1(t) \times P_r\,\bar{B}_2(t) \times \ldots \times P_r\,\bar{B}_n(t)]$

Hance $F_S(t) = 1 - \prod_{i=1}^{n}[P_r(\bar{B}_i(t))]$ (3.2)

Since $P_r(\bar{B}_i(t)) = 1 - P_r(B_i(t))$

$F_S(t) = 1 - \prod_{i=1}^{n}[1 - P_r(B_i(t))]$

And we get $R_s(t)$ from the relation

$R_s(t) = 1 - F_S(t)$

3.4 The FT's Evaluation:

In this section , the light will be shed at how to evaluate a fault tree for any model .

Example 3.4

Assume we have the fault tree depicted in the figure below.

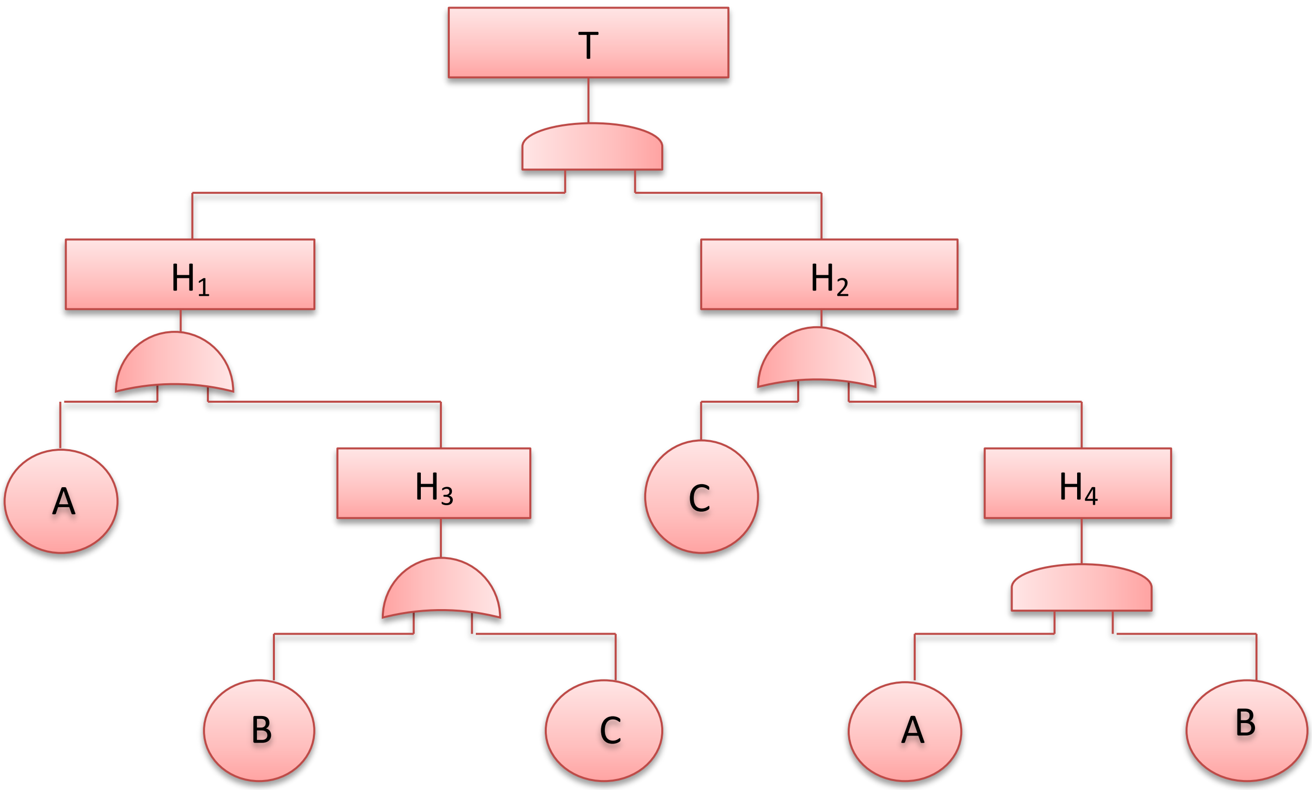


Figure (3.10) FT Model for Example 3.4

Writing equation for each gate of the tree

$\mathrm{T} = \mathrm{H}_1.\mathrm{H}_2, \mathrm{H}_1 = A + \mathrm{H}_3, \mathrm{H}_2 = C + \mathrm{H}_4, \mathrm{H}_3 = B + C, \mathrm{H}_4 = A\,.B$

There are two methods to solving the example for the model above:

3.4.1 Top to Down Method [54]:

It begins with the top event equation and then substitutes and spreads until the minimal cut set ending for the top event (T) is obtained.

$\mathrm{T} = \mathrm{H}_1.\mathrm{H}_2$

$\mathrm{T} = (A + \mathrm{H}_3).(C + \mathrm{H}_4)$

Substitute

$\mathrm{H}_3 = B + C$

$\mathrm{T} = (A + B + C).(C + \mathrm{H}_4)$

$\mathrm{T} = (A\,.\,C + B\,.\,C + C\,.\,C + \mathrm{A}\,.\,\mathrm{H}_4 + B\,.\,\mathrm{H}_4 + C.\mathrm{H}_4)$

$\mathrm{T} = (A\,.\,C + B\,.\,C + C\,.\,C + C\,.\,\mathrm{H}_4) + \mathrm{A}\,.\,\mathrm{H}_4 + B.\mathrm{H}_4)$

$\mathrm{T} = (C + \mathrm{A}\,.\,\mathrm{H}_4 + B\,.\,\mathrm{H}_4)$

$\mathrm{T} = (C + \mathrm{A}\,.\,(A\,.\,B) + B.(A\,.\,B))$

$\mathrm{T} = C + A\,.\,B + A\,.\,B$

$\mathrm{T} = C + A\,.\,B$

A . B and C are the minimal cut sets.

3.4.2 Bottom to Up Method [54]:

It begins from the bottom of the tree and works its way up using the same replacement and expansion approach. The equation only has one basic failure. When we apply this concept for example (3.4) it has been obtained that :

$\mathrm{B} + \mathrm{C} = \mathrm{H}_3\,, \mathrm{H}_3 + \mathrm{A} = \mathrm{H}_1\,, \mathrm{A} + \mathrm{B} = \mathrm{H}_4$

$\mathrm{H}_4 + \mathrm{C} = \mathrm{H}_2\,, \mathrm{A} + \mathrm{B} + \mathrm{C} = \mathrm{H}_1\,, \mathrm{C} + \mathrm{A}\,.\,\mathrm{B} = \mathrm{H}_2$

$\mathrm{H}_1\,.\,\mathrm{H}_2 = \mathrm{T}$

$(\mathrm{A} + \mathrm{B} + \mathrm{C})\,.\,(\mathrm{C} + \mathrm{A}\,.\,\mathrm{B}) = \mathrm{T}$

$\mathrm{A}\,.\,\mathrm{C} + \mathrm{A}\,(\mathrm{A}\,.\,\mathrm{B}) + \mathrm{B}\,.\,\mathrm{C} + \mathrm{B}\,(\mathrm{A}\,.\,\mathrm{B}) + \mathrm{C}\,.\,\mathrm{C} + \mathrm{C}\,(\mathrm{A}\,.\,\mathrm{B}) = \mathrm{T}$

$(\mathrm{A}\,.\,\mathrm{C} + \mathrm{B}\,.\,\mathrm{C} + \mathrm{C} + \mathrm{C}\,(\mathrm{A}\,.\,\mathrm{B})) + \mathrm{A}\,.\,\mathrm{B} + \mathrm{A}\,.\,\mathrm{B} = \mathrm{T}$

$\mathrm{T} = \mathrm{C} + \mathrm{A}\,.\,\mathrm{B}$

It has been obtained that two sets of minimal cut: A . B and C.

Example 3.5

Determine all of the minimal cut sets for an electrical motor failure in the following fault tree and compute the system failure (T) that occurs.

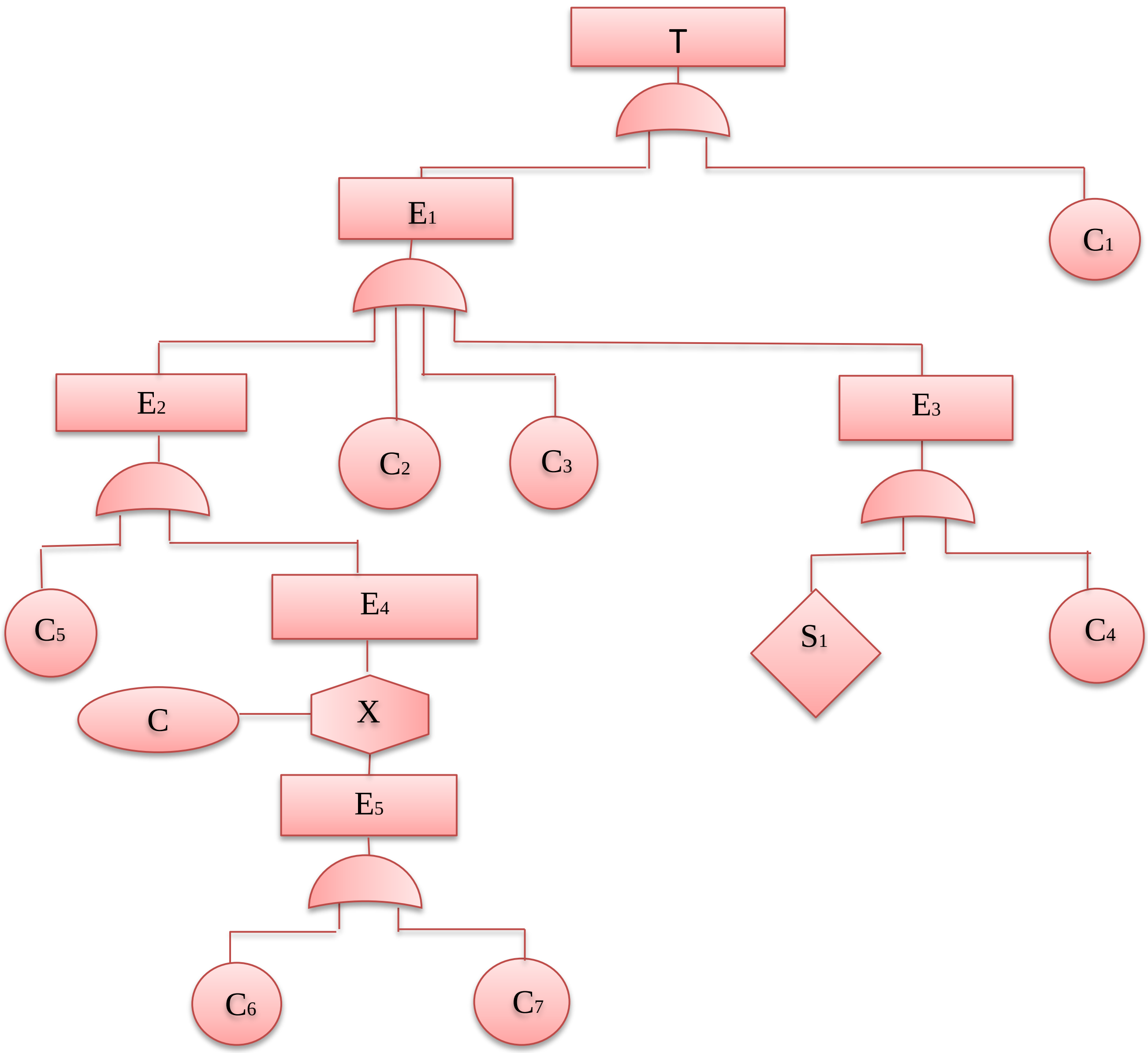


Figure (3.11) FT for Example 3.5

T Refer to the motor stops working.

E_1 Refer to no power to the motor.

E_2 Refer to fuse fails open.

E_3 Refer to switch open.

E_4 Refer to overload causes fuse failure.

E_5 Refer to in circuit overload .

C_1 Refer to a motor fault.

C_2 Refer to failure of a wire (open).

C_3 Refer to breakdown of the power supply.

C_4 Refer to the switch fails (open).

C_5 Refer to normal-condition fuse failure (open).

C_6 Refer to failure of a wire (shorted).

C_7 Refer to failure of power (surge).

S_1 Refer to switch accidentally flipped on.

C Refer to the fuse fails to open.

Writing equation for each gate of the tree

$T = E_1 + C_1, E_1 = E_2 + E_3 + C_2 + C_3, E_2 = C_5 + E_4$

$E_3 = S_1 + C_4, E_4 = C \,.\, E_5, E_5 = C_6 + C_7$

Using the Top To Down approach we get by substitution

$T = E_1 + C_1$

$T = C_1 + E_2 + E_3 + C_2 + C_3$

$T = C_1 + C_2 + C_3 + C_5 + E_4 + E_3$

$T = C_1 + C_2 + C_3 + C_5 + S_1 + C_4 + E_4$

$T = C_1 + C_2 + C_3 + C_4 + C_5 + S_1 + C \,.\, E_5$

$T = C_1 + C_2 + C_3 + C_4 + C_5 + S_1 + C \,.\, (C_6 + C_7)$

$T = C_1 + C_2 + C_3 + C_4 + C_5 + S_1 + (C\,.\,C_6 + C\,.\,C_7)$

Using the Bottom To Up approach we get by substitution

$C_6 + C_7 = E_5, (C\,.\,(C_6 + C_7) = E_4\,,\ S_1 + C_4 = E_3\,,$

$C_5 + C\,.\,(C_6 + C_7) = E_2$

$E_2 + E_3 + C_2 + C_3 = E_1$

$C_5 + C\,.\,(C_6 + C_7) + E_3 + C_2 + C_3 = E_1$

$C_5 + C\,.\,(C_6 + C_7) + S_1 + C_4 + C_2 + C_3 = E_1$

$T = E_1 + C_1$

Thus $T = C_5 + C\,.\,(C_6 + C_7) + S_1 + C_4 + C_2 + C_3 + C_1$
Or $T = C_1 + C_2 + C_3 + C_4 + C_5 + S_1 + (C\,.\,C_6 + C\,.\,C_7)$

3.5 The relation between FT and RBD

This section presents (RBD) and its relation with FT to answer the question is it possiable to convert the FT to RBD. Moreover we discuss the success tree ST.

3.5.1 Reliability Block Diagram (RBD) [21]:

The logic diagram in an RBD is organized to show which combinations of component failure result in system failure and which combinations of correctly operating components keep the system operational. A block in RBD symbolizes a functional physical component, and the removal of the corresponding block indicates the failure of this component. If enough blocks are removed from an RBD to break the link between the input and output locations the system fail. If there is at least one path connect the input and output then the system continues to function normally.

3.5.2 Converting FT to RBD [54]:

A FT AND gate relates to a parallel reliability block diagram. A series reliability block diagram relates to the OR gate. As seen in Figure (3.12).

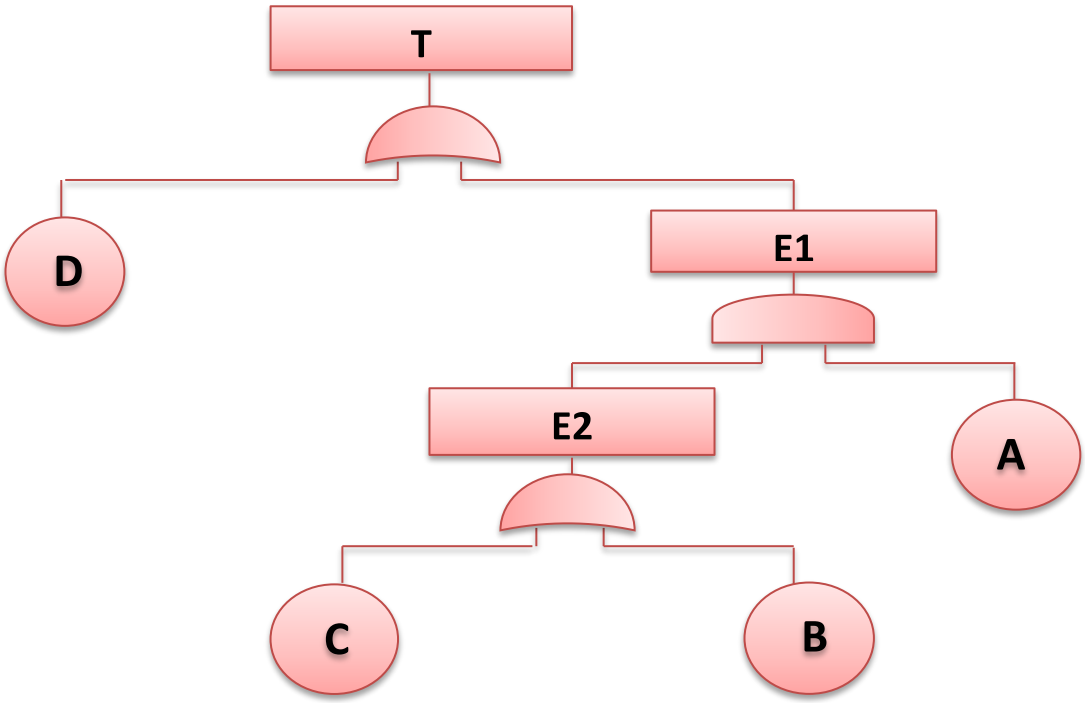


Figure (3.12) FT Consist of 4 Components

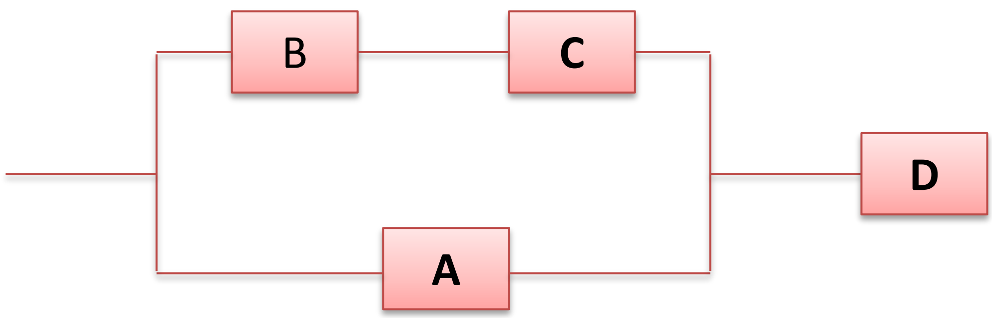


Figure (3.13) RBD of Figure (3.12)

Definition 3.5.3 [7]:

The complement of the fault tree is the **Success Tree** which is denoted as (ST). A FT may be replaced to a success tree as follows :

a) Each OR gate is being replaced with an AND gate.

b) Replace every AND gate with an OR gate.

The FT for T is shown Figure (3.14) below.

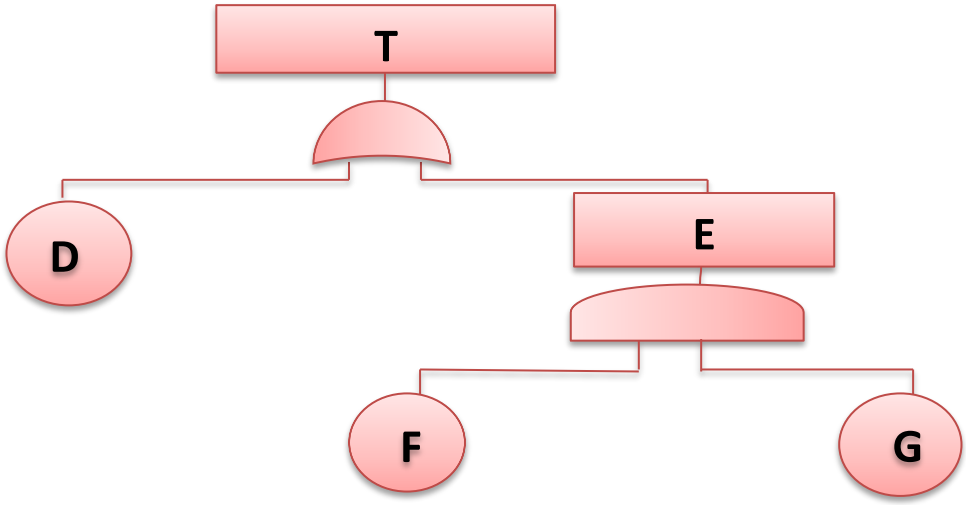


Figure (3.14) A Fault Tree

$T^C = (D + F.G)^C$

$T^C = D^C.(F.G)^C$

According to De Morgan's theorem

$T^C = D^C.(F^C + G^C)$

Than $T^C = D^C.F^C + D^C.G^C$

The equivalent (T^C) success tree (ST) Figure (3.15) is as follows:

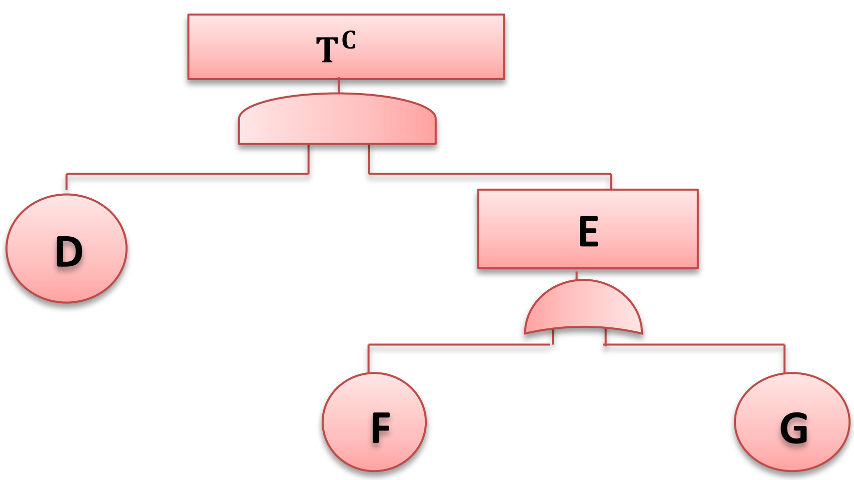


Figure (3.15) A Success Tree ST for Figure (3.14)

3.6 Some Concepts and Definitions to Dynamic Fault Tree:

In this type, some concepts and definitions related to the current work will be explained such as dynamic fault tree, DFT properties, dynamic reliability block diagram (DRBD), DFT gates, converting DFT to DRBD, DRBD versus SFT.

Definition 3.6.1 [27,35]:

This approach has evolved SFT over time into the more flexible **Dynamic fault tree DFT**. DFT is commonly utilized for the reliability study of complex systems with dynamic failure characteristics. In many cases, determining the exact value of system reliability is challenging owing to an insufficient information on failure probabilities or component failure rates or it is a graphical modeling technique that extends the (SFT) by expressing dynamic behaviors in complex systems.

3.6.2 Dynamic Fault Tree Characteristics [6,11]:

A dynamic fault tree (DFT) is a way of supplying a static fault tree (SFT) with dynamic gates that deal with sequential concepts. Modelers can describe sequence-dependent system failure behavior, spares, and dynamic redundancy management with the use of dynamic gates. Furthermore, during a failure occurrence, priorities in DFT are concise and clearly understood. They can also explore combinations that can modify a system's failure condition by implementing a component's startup, shutdown, and repair within a mission time frame.

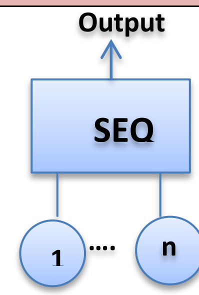
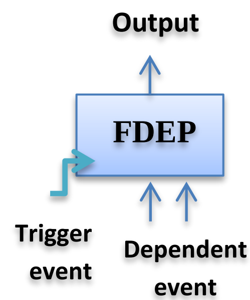
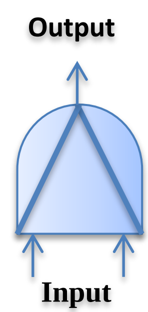
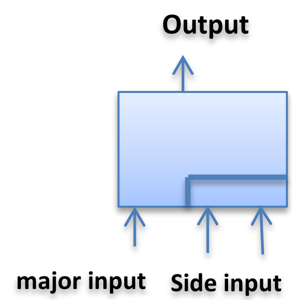
3.6.3 Dynamic Reliability Block Diagram DRBD [11,54]:

Each component's condition in the DRBD system is specified by a variable state that specifies the operational status of the component at a given time. Events that occur to a component characterize its state evolution. The states that a generic DRBD component can use are active when the component is functioning and failed when it is not.

3.6.4 Dynamic Fault Tree gates:

Dugan, et al. [23] proposed a new system to include different sort of temporal and statistical dependencies in SFT system which called dynamic fault tree (DFT). It is based on definitions of new gates as listed in Table (3.2) below.

Table (3.2) Logical Gates for DFT

The Dynamic Gate	Enter Event Details	Criteria for Failure	Figure for the Gate
Sequent enforcing gate (SEQ)	The SEQ Gate has multiple inputs.	If all the inputs occur, the output is true.	
Functional dependency gate (FDEP)	consists of trigger event and multiple dependent events.	If the trigger event occurs, all dependent events will occur, and the gate will have an output.	
Priority gate (PAND)	A gate has two inputs. A and B, which can be essential events or the outputs of other logical gates.	Gate has output if both events A and B, happened, A before B	
Spare gate	The spare gate has one major input and one or more alternative inputs.	When the major component fails, the alternate component starts operating first, and the gate has output after all inputs fail.	

3.6.5 Converting From DFT to DRBD [63]:

The mapping from the DFT domain to the DRBD domain has been defined. This mapping may be reversed if and only if the DRBD structures exactly match those listed below; otherwise, the DFT equivalent of the DRBD structure cannot exist .

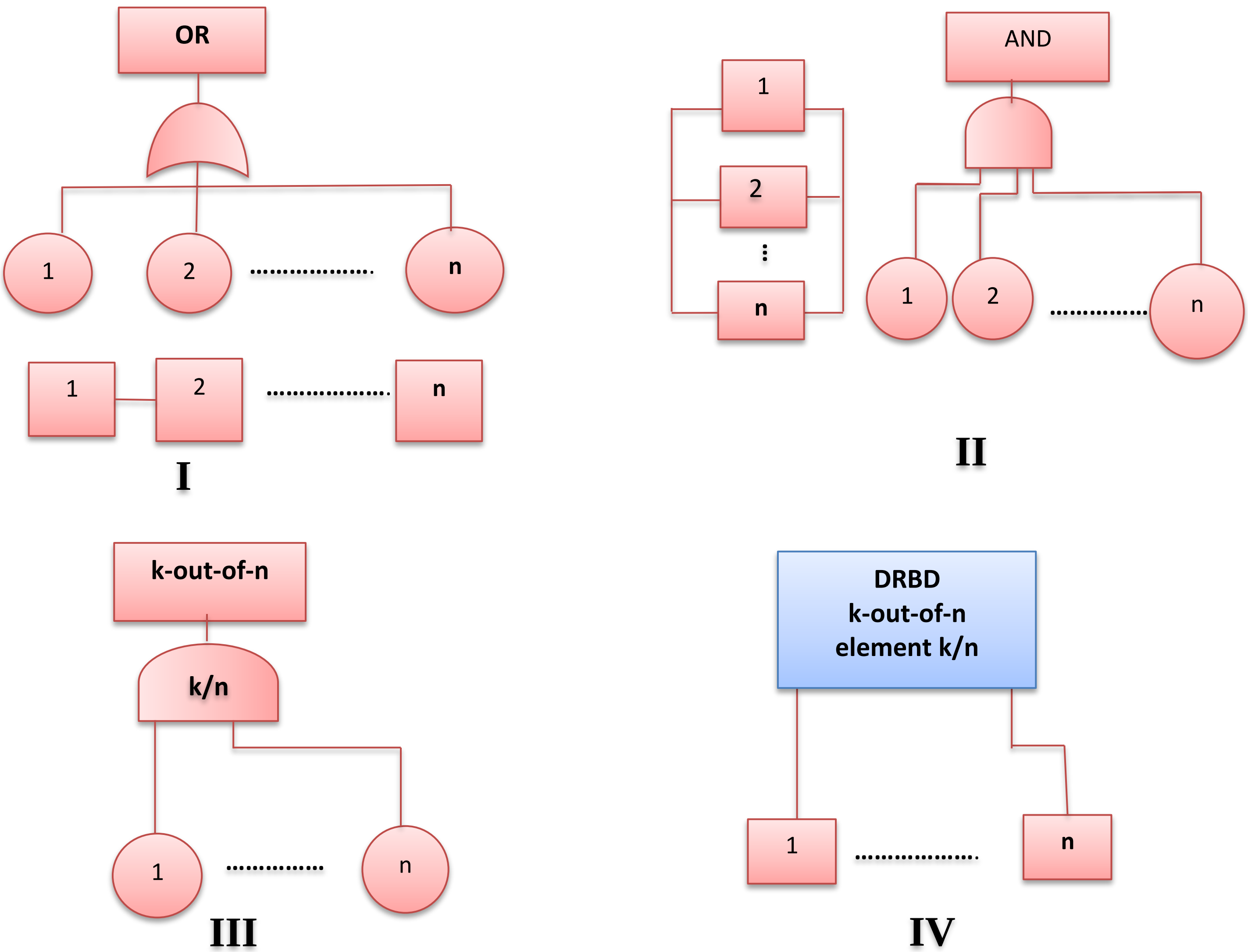


Figure (3.16) SFT vs. DRBD comparison, where I, II, and III are SFT and IV is DRBD

3.6.6 DRBD vs. SFT [21]:

RBD represents the system as a collection of components that are linked based on their function or reliability.The component failure combinations that cause the system to fail are expressed by FT.

3.6.7 Observation [21,35]:

RBD and SFT systems are equivalent, however DRBD and SFT do not hold due to the two reasons listed below.:

1. It is connected to the systeming approach: although SFT and DFT only show component failure, RBD and DRBD can also represent system representations or activations. This point is irrelevant in the static case where system dynamics are ignored and activation, thus SFT and RBD are considered identical. In fact, DRBD may perform system dynamic actions (such as repairs or activation events), but DFT cannot.

2. The technique to systemizing dynamic reliability characteristics differs between DFT and DRBD: DFT defines particular gates to illustrate failure dependency and/or common mode failure (FDFP), redundancy (spare gate), and order relationships (SEQ and PAND gates), whereas DRBD expresses and formalizes the notion as the fundamental to any dynamic component of a system. As a result, the DRBD method provides a degree of flexibility in reliability systeming, allowing for the representation of various dynamic behaviors with customized policies and schemas.

3.7 Models of Dynamic Gates Behavior:

We present an analytical extract of the time operators for the models of dynamic gates which refer to the behavior of the gates used to specify the sequence of probable element failures, for more detail see [23,34,42].

A) Priority Gates

Priority gates having two inputs, C and D. Equation (3.3) describes the behavior model of a gate shown in Figure (3.17).

$$T = (C.D).(C \leq D) = D.(C \leq D) \quad (3.3)$$

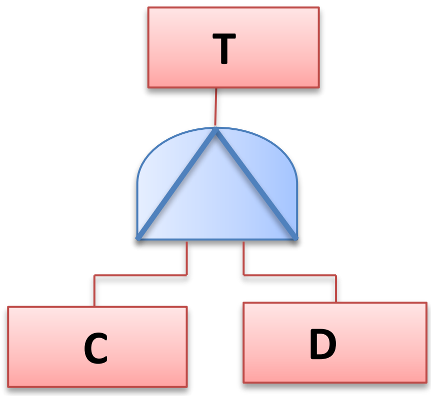


Figure (3.17) PAND Gate

B) Functional Dependency Gate

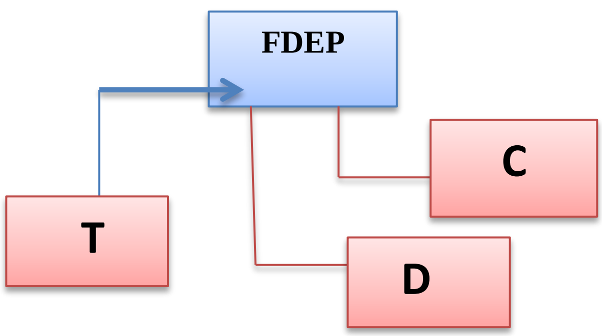


Figure (3.18) FDEP Gate

The basic dependent events C and D in Figure (3.18), might fail for their own causes or as a result of trigger event failure. Failure of events C and D because the failure of trigger events is denoted as C_T and D_T, while failure of dependent events due to their own causes is indicated as $(C \leq T)$ and $(D \leq T)$.

The FDEP gate's behavior model is:

$$C_T = T + (C \leq T) \quad (3.4)$$

$$D_T = T + (D \leq T) \quad (3.5)$$

Model simplified is:

$$C_T = T + (C \leq T) = C + T \quad (3.6)$$

$$D_T = T + (D \leq T) = D + T \quad (3.7)$$

C) Spare Gates

When evaluating the behavior of spare gates, it is assumed that there is only one type of gate, warm gates, and that cold and hot gates are special instances of warm gates. Figure (3.19) depicts a spare gate with one spare event. There are two techniques for series of failures: [C, D] and [D,C]. In a series of [C, D], D is failed in an active state (D_a), but in a series of [D,C], D is failed in a dormant state (D_d). Because the same component does not have the same operating time to failure distribution in active and inactive (dormant) modes, two marks are used.

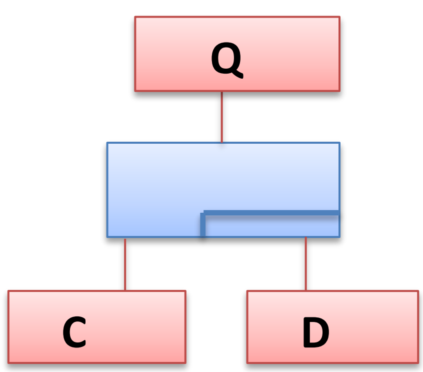


Figure (3.19) Spare Gate

The gate's behavior model with two input events may be stated as follows:

$$Q = D_a.(C \leq D_a) + C.(D_d \leq C) \qquad (3.8)$$

D) SEQ Gates

It is a like PAND gate but happens of events are can to occur in a particular manner failure of first component forces the other components to fail. No component can fail prior to the first component .

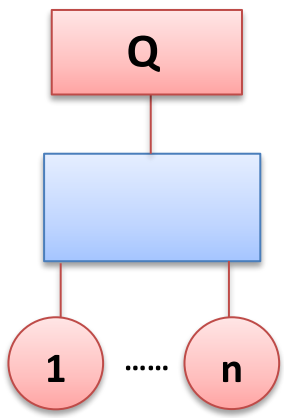


Figure (3.20) SEQ gate

3.8 Multistate Dynamic Fault Tree [34,42,61]:

This section deals with the multistate DFT with illustrative example.

Example 3.6 taken form [42] with modification

The following multistate DFT in figure below models the failure of a cardiac assist system (HCAS). Via four patterns which are as in note 4.8.1 below.

Let TS_1, TS_2 and TS_3 represent the top events of the sub-FTs 1, 2 and 3 respectively.

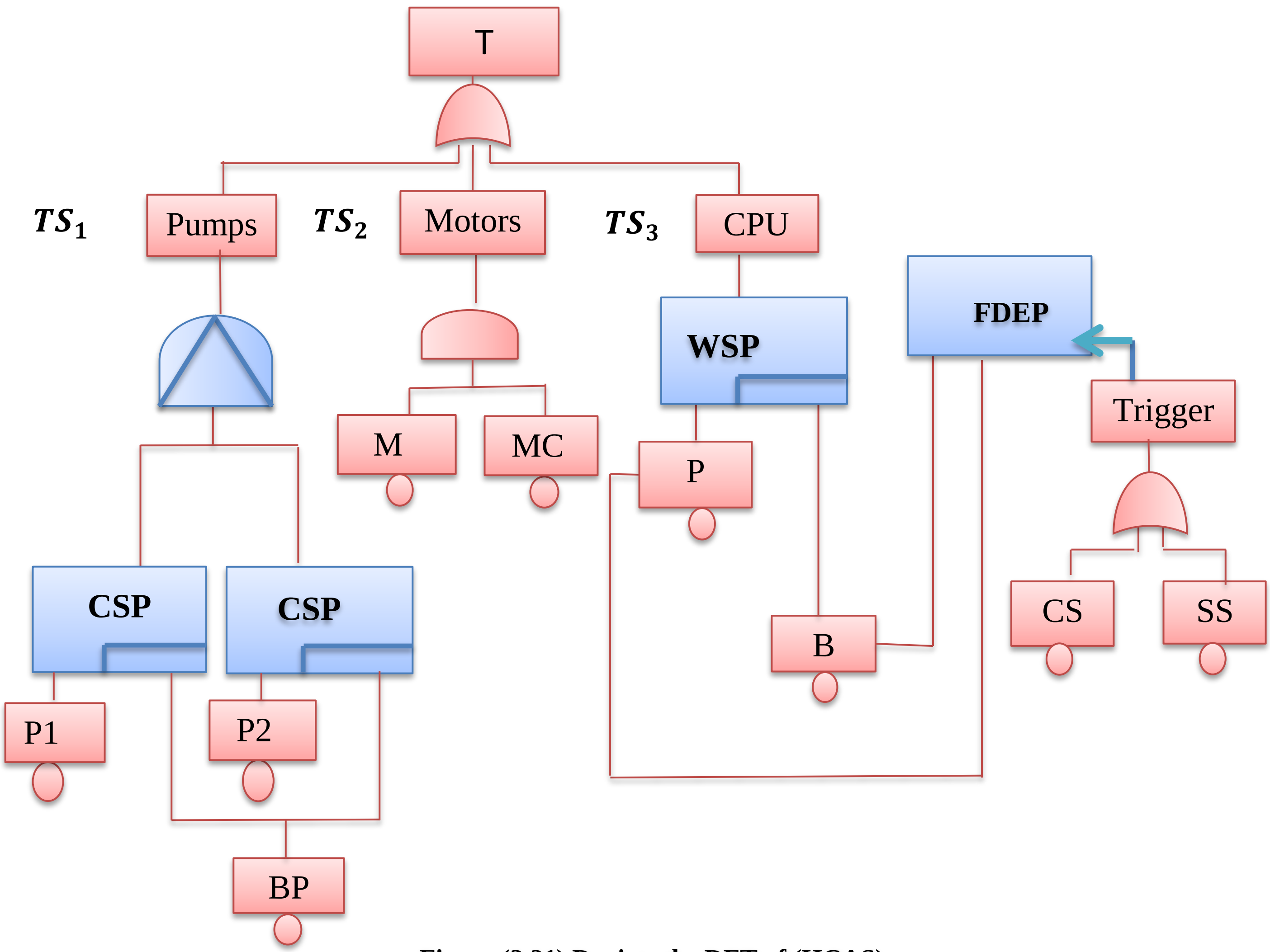


Figure (3.21) Depicts the DFT of (HCAS)

4.8.1 Note:

1. Trigger consist of a cross bar switch indicated by (CS) and a system supervisor marked by (SS). The failure of either CS or SS trigger the failure of both CPUs.

2. The CPU unit is a warm spare, with a primary unit (P) and a spare unit (B) that have a dormancy of 0.5.

3. If both motors (M) and (MC) fail, the motor section does not function.

4. The pumps unit (P) is consist of two cold spare gates, each one possesses a primary pump (P1 and P2) common in a spare pump (BP). To fail the pump unit, all three pumps must fail, and the left-hand side spare gate need

to fail before or at the same time as the right-hand side spare gate, i.e. PAND gate.

3.8.2 Boolean Operators:

The temporal definition of Boolean operators OR and AND based on the date of happens of a and b which are indicated by d(a) and d(b) respectively are:

$$1.\ d(a+b) = \begin{cases} d(a) & if\ \ d(a) < d(b) \\ d(a) & if\ \ d(a) = d(b) \\ d(b) & if\ d(a) > d(b) \end{cases} \qquad (3.9)$$

$$2.\ d\ (a\,.\,b) = \begin{cases} d(b) & if\ \ d(a) < d(b) \\ d(a) & if\ \ d(a) = d(b) \\ d(b) & if\ d(a) > d(b) \end{cases} \qquad (3.10)$$

Every element of temporal operators can be written in classical Boolean operators.

Noted that

1. $a + b$ takes place as soon as a or b happens.

2. $a.b$ happens as soon as a and b have happened.

3. OR and AND are commutative.

3.8.3 Observation:

1. In DFT the failure of top event (T) depends not on the failure of basic events but also on the order of the happening of these failures.

2. The classical Boolean function cannot represent the dynamic relation between the top event and basic events that exist in a DFT.

3. The algebraic framing permits to algebraically model dynamic gates and determine the structure function (SF) of any DFT.

4. The minimal cut sets and sequence of DFTs can be determined directly from this SF.

5. Due to the probabilistic model of dynamic gates allowed to fulfill the quantitative and qualitative analysis for DFT form its SF.

3.8.4 Remark:

Note that the three sub-FTs are statically independent. The failure probability of SFT for the DFT can be evaluated by using the inclusion-exclusion theorem. Consider each event is non-repairable, each one can be assigned only one date of appearance. The identity element of operator OR and the identity element AND in non-repairable event E_{nr} are ⊥ for OR and ⊤ for AND to which those dates can assigned .

d (⊥) = +∞ , d (⊤) = 0

where ⊥ is the never-occurring event, ⊤ the always-occurring event.

3.8.5 Temporal Operator:

To model the order of occurrence of event, an operator non-inclusion is introduced to model the sequence of occurrence of events as in below.

1. BEFORE (BE) symbolized ⊲ .

2. The operator (SIMULTANEOUS) denoted by (SM) and represented by the symbol Δ.There formal definitions based on the dates of occurrence for a and b as follows:

$$1.\ d(a \lhd b) = \begin{cases} d(a) & if\ \ d(a) < d(b) \\ +\infty & if\ \ d(a) = d(b) \\ +\infty & if\ d(a) > d(b) \end{cases} \qquad (3.11)$$

$$2.\ d(a \vartriangle b) = \begin{cases} +\infty & if\ \ d(a) < d(b) \\ d(a) & if\ \ d(a) = d(b) \\ +\infty & if\ d(a) > d(b) \end{cases} \qquad (3.12)$$

According to the two operations above. It can introduce an INCLUSIVE BEFORE (IBE) (non-strict) symbol by $a \unlhd b = a \lhd b + a \Delta b$

Where based on the occurrence of a and b

$$d(a \unlhd b) = \begin{cases} d(a) & if\ \ d(a) < d(b) \\ d(a) & if\ \ d(a) = d(b) \\ +\infty & if\ d(a) > d(b) \end{cases} \qquad (3.13)$$

3.8.6 Observation:

1. In SFTs static gates it can be modeled by means of Boolean operators and inclusion-exclusion formula which is proper to determine the failure probability of top event of FT.

2. Inclusion-exclusion formula is still used in DFTs but as modeled dynamic gates by means of temporal operators.

3.The expression obtained will have probabilities of algebraic terms including temporal operators. Hence a probabilistic model of dynamic gates needed to perform quantitative analysis. So it can propose a probabilistic model of dynamic gates based on their behavior model.

3.8.7 Some Probabilistic Formulas [42]:

It should be noted that $F(x)$ denotes the cumulative distribution function (cdf) of the event x. The relationship between (cdf) and probability density function (pdf), i.e. $f(x)$, are shown below.

$f(x) = F\acute{(}x)$ for the event x.

i) Now consider two independent event A and B, the following hold:

1. $P_r(A.B)(t) = F_A(t) \times F_B(t)$

2. $P_r(A + B)(t) = F_A(t) + F_B(t) - F_A(t) \times F_B(t)$

3. $P_r(A \lhd B)(t) = \int_0^t f_A(u)(1 - F_B(u))du$

4. $P_r(B.(A \lhd B))(t) = \int_0^t f_B(u)\, F_A(u)du$

ii) Dynamic gate probabilistic model:

a) For FDEP which is depicted in Figure (3.18), the algebraic model to FDEP with two dependent basic events is :

$A_T = T + A$

$B_T = T + B$

We derive the probability expression of A_T , B_T which corresponding to OR expression as follows:

1. $P_r(A_T)(t) = P_r(T + A)(t) = F_T(t) + F_A(t) - F_T(t) \times F_A(t)$

2. $P_r(B_T)(t) = P_r(T + B)(t) = F_T(t) + F_B(t) - F_T(t) \times F_B(t)$

b) For PAND gate which is depicted in Figure (3.17), the algebraic model is:

$Q = B.(A \trianglelefteq B)$

C. Figure (3.19) depicts spare gates with two input events, the basic event A and one spare event B. It should be noted:

$$P_r[P.(B_d \lhd P) + B_a.(P \lhd B_a)(t)] = \int_0^t (\int_v^t fB_a(u,v)du) fP(v)dv) + \int_0^t fP(u)FB_d(u)du$$

3.8.8 The Structure Function SFT for the system (HCAS):

We may partition the DFT of (HCAS) into three sub-FTs, and the structure function can be separated into three structure functions. We make use of the behavioural models of dynamic gates to determine the structure functions of the sub-FTs.

Let the structure function of (HCAS) denoted by SFT and represent the three sub-FTs by SFT_1, SFT_2, and SFT_3respectively and be stated as follows:

i) The structure function SFT_1 of sub-FT1 can be obtained from the behavioural models of one PAND gate and two cold spare gates since it is corresponds to the failure of the pumps unit as follows:

$$SFT_1 = BP_a.(P_2 \lhd P_1).(P_1 \lhd BP_a) + P_2.(P_1 \lhd BP_a).(BP_a \lhd P_2)$$

ii) The structure function SFT_2 of sub-FT2 can be obtained directly from a single AND gate since it is corresponds to the failure of the motor section as follows:

$$SFT_2 = (M).(MC)$$

iii) The structure function SFT_3 of sub-FT3 can be obtained from the behaviorual model of a single OR gate and a single FDEP gate and warm spare gate, so it is dynamic.

$$SFT_3 = CS + SS + P.(B_d \lhd P) + B_a.(P \lhd B_a)$$

Hence

The structure function (SFT) of the DFT of (HCAS) under study is since it is corresponds to the failure of the CPU unit as follows:

$$\begin{aligned} SFT = CS + SS + (M).(MC) + P.(B_d \lhd P) + B_a.(P \lhd B_a) \\ + BP_a.(P_2 \lhd P_1).(P_1 \lhd BP_a) \\ + P_2.(P_1 \lhd BP_a).(BP_a \lhd P_2) \end{aligned} \qquad (3.14)$$

2.8.9 Observation:

The structure function of DFT possesses three terms don't contain the temporal operator (BF), i.e. they are static so it can provide three minimal cut sets of as in DFT below: CS, SS, (M).(MC).

Whereas the operator four term of SFT contains the temporal operation (BF) i.e. they are dynamic and it can provide the following so called minimal sequences of the DFT: $[B_d, P]$, $[P, B_a]$, $[P_2, P_1, BP_a]$, $[P_1, BP_a, P_2]$.

The equation (3.14) of SFT is easier for engineers and technologists to estimate the top event of DFT.

The three kinds of operators which are utilized to define the date of appear of events in dynamic gates i.e. we use the following symbols for each one as below which is more common for readers.

1. Non-inclusive BEFORE denoted is by the symbol $<$.

2. Inclusive BEFORE denoted is by the symbol $\leq$ and they are same results for the temporal operators $\lhd$ and $\unlhd$.

3. $a \leq b = a < b + a \, \Delta \, b$.

Now the final expressions of $P_r(SFT)(t)$ is:

$$P_r(SFT)(t) = P_r(SFT_1 + SFT_2 + SFT_3)(t)$$

$$\begin{aligned} &= P_r(SFT_1)(t) + P_r(SFT_2)(t) + P_r(SFT_3)(t) \\ &- P_r(SFT_1)(t) \times P_r(SFT_2)(t) - P_r(SFT_1)(t) \times P_r(SFT_3)(t) \\ &- P_r(SFT_2)(t) \times P_r(SFT_3)(t) \\ &+ P_r(SFT_1)(t) \times P_r(SFT_2)(t) \times P_r(SFT_3)(t) \end{aligned} \quad (3.15)$$

For more detail see [42].

3.9 Converting DFT to SFT :

Section 3.9 concern with converting DFT to SFT with illustrative example.

The qualitative analysis entails calculating the failure probability of the top event of the SFT. Dugan et al [23] developed a new system architecture to integrate many types of temporal and statistical relationships in the SFT system, which is known as dynamic fault tree (DFT). It is based on the obtain definitions of gets listed in Table (3.2).

Now, in the current study, a DFT from [63] is converted to an equivalent SFT by adding a series of OR gates, one for each basic event, instead of the dynamic gate FDEP, to simplify the examination of the DFT's reliability for technologists and engineers, as shown in the figures below:

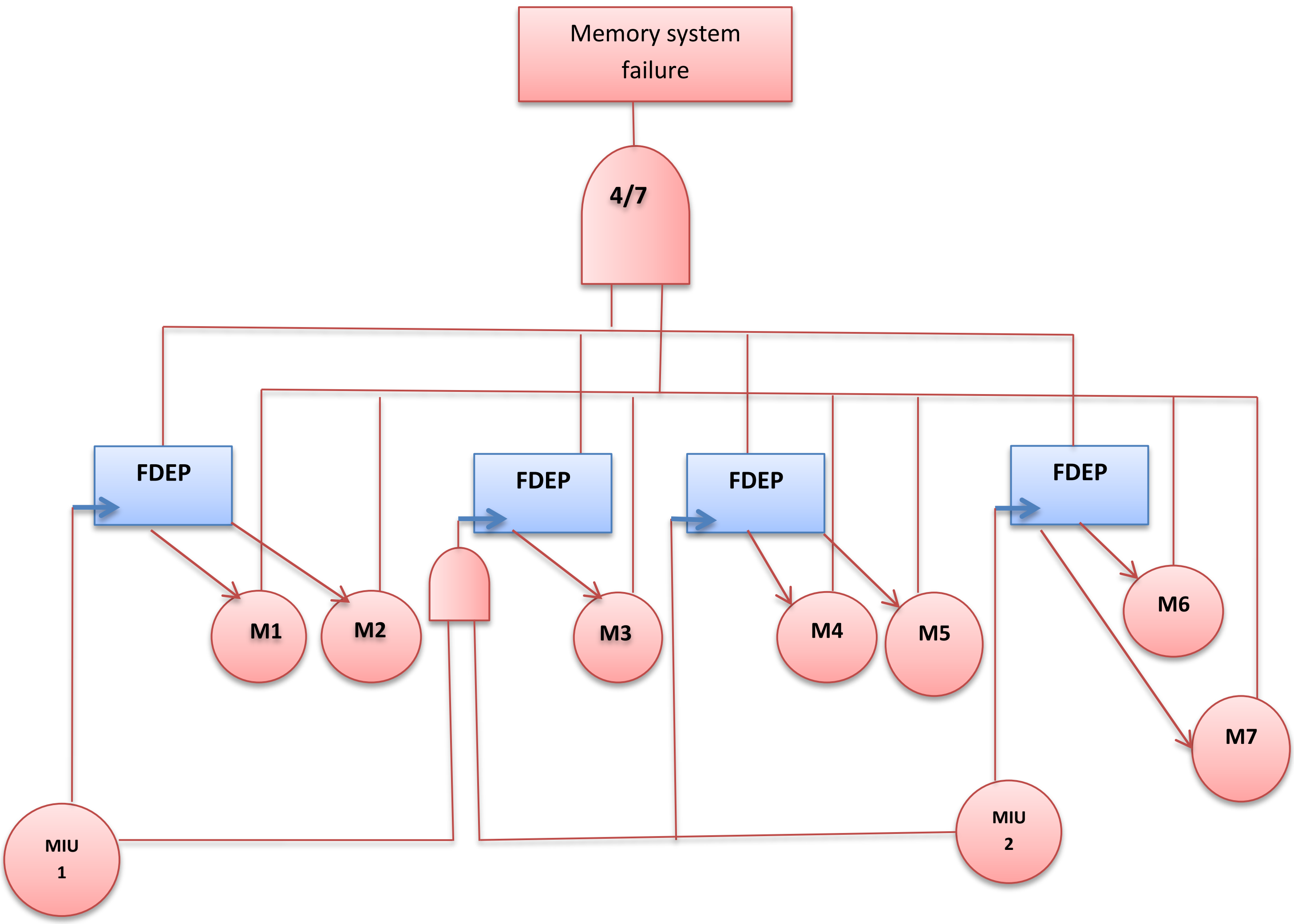


Figure (3.22) DFT

It is that the FT in Figure (3.22) consist one gate AND and four FDEP noted gates hence it can considered as DFT and equivalent the SFT in Figure (3.23) below.

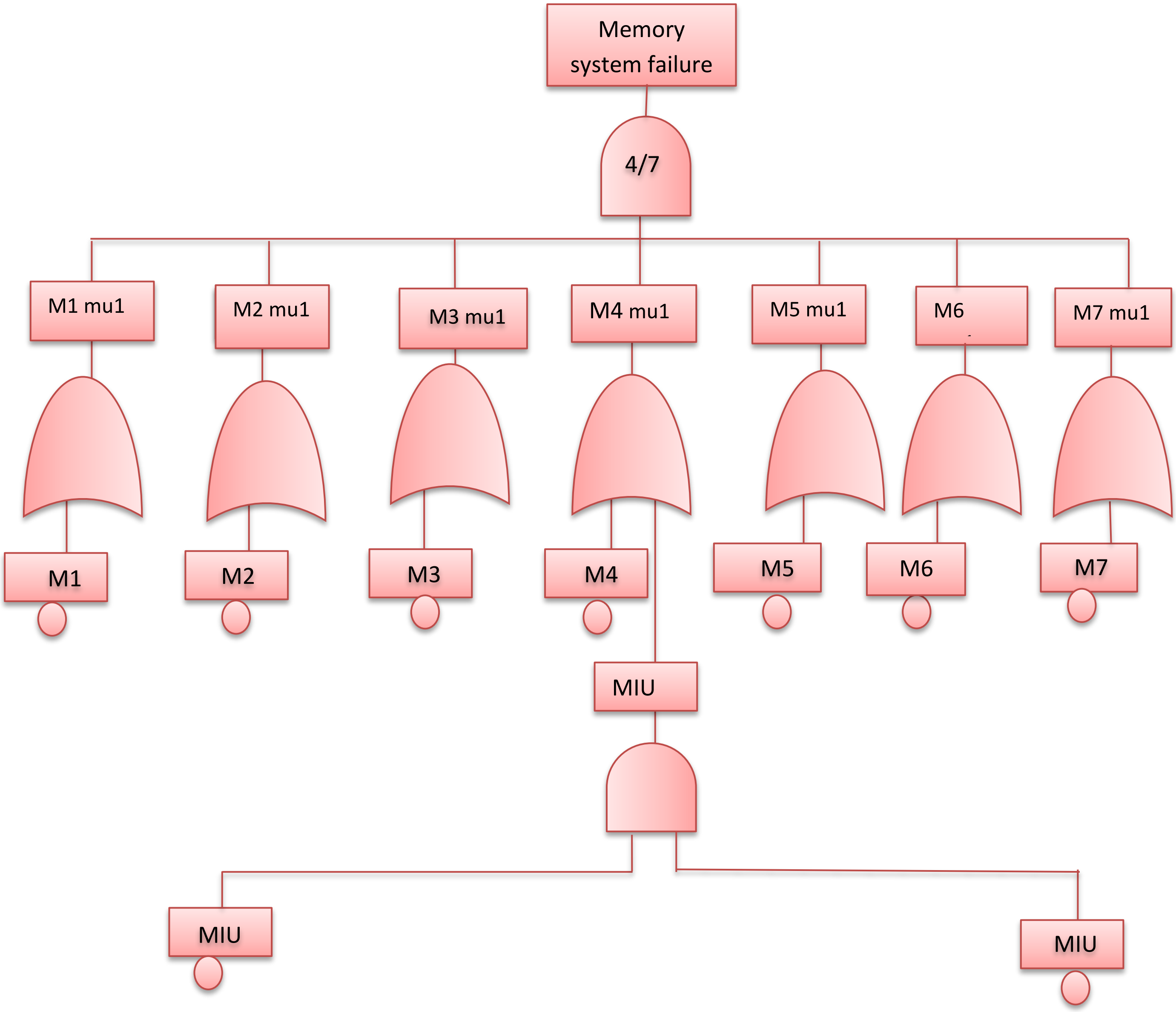


Figure (3.23) Shows an SFT to DFT Equivalency (3.22)

Chapter Four

Fuzzy Fault Tree with Applications in Safety and Reliability

4.1 Introduction

This chapter is devoted in order to display some concepts and definitions related to the current work via four sections. Section one deals with some concepts and definitions. Section two contains analysis of fuzzy system reliability using vague sets. Section three contains some illustrative examples taken from real life applications. Section four involve the relation between human factors, reliability and safety.

4.2 Some Concepts and Definitions

This section presents concepts and definitions relate to fuzzy sets.

4.2.1 Causes of Utilize Fuzzy Sets [1,32,43]:

Lotfi A. Zadeh introduced fuzzy sets in 1965, he was nearly entirely responsible for the early development of this discipline. A fuzzy set is a group of objects with graded membership $\mu_A(x)$, such that $0 \leq \mu_A(x) \leq 1$.

Fuzzy sets can be applied when:

1. There is no clear boundary between failures and success of the system.

2. The probability of system failure cannot be calculated accurately due to lack of sufficient data.

3. Subjective information, such as natural language expressions, is collected from experts and analysts.

Definition 4.2.2 [1,29]:

Let U be the universal of discourse. A **Vague Set** $\bar{\nabla}$ over U is characterized by a truth membership function $t_{\bar{\nabla}}, t_{\bar{\nabla}}: U \to [0,1]$ and a false membership function $f_{\bar{\nabla}}, f_{\bar{\nabla}}: U \to [0,1]$. If the generic element of U is denoted by x_i then the lower bound on the membership grade of x_i derived from evidence for x_i is denoted by $t_{\bar{v}}(x_i)$ and the lower bound on the negation of x_i is denoted by $f_{\bar{v}}(x_i)$. $t_{\bar{v}}(x_i)$ and $f_{\bar{v}}(x_i)$ both associate a real number in [0,1] with each point x_i in X , where $t_{\bar{v}}(x_i) + f_{\bar{v}}(x_i) \leq 1$

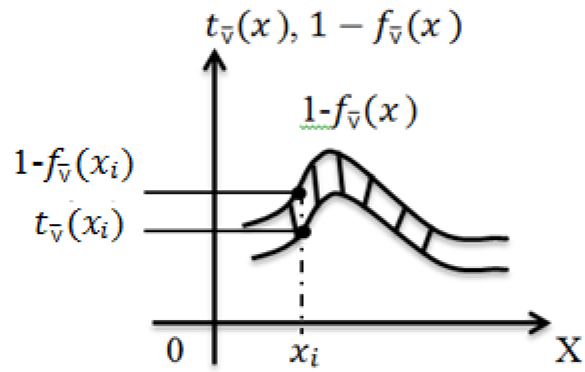


Figure (4.1) A Vague Set

Definition 4.2.3 [16,25]:

A vague set $\tilde{A}$ of the universe of discourse U is called a **Normal Vague Set** if $\exists\, u_i \in$ U, 1 - $f_{\tilde{A}}(u_i) = 1$, i.e. , $f_{\tilde{A}}(u_i) = 0$.

Definition 4.2.4 [17,68]:

A **Vague Number** is a vague subset in the universal of discourse U that is both convex and normal.

Let us consider the triangular vague set $\tilde{A}$ shown in Figure (4.2) where the triangular vague set $\tilde{A}$ can be parameterized by tuple $[(a,b,c);\mu_1]$, $[(a,b,c);\mu_2]$. For convenience, tuple $[(a,b,c);\mu_1]$, $[(a,b,c);\mu_2]$ can be also abbreviated into $[(a,b,c);\mu_1;\mu_2]$ and $0 \leq \mu_1 \leq \mu_2 \leq 1$.

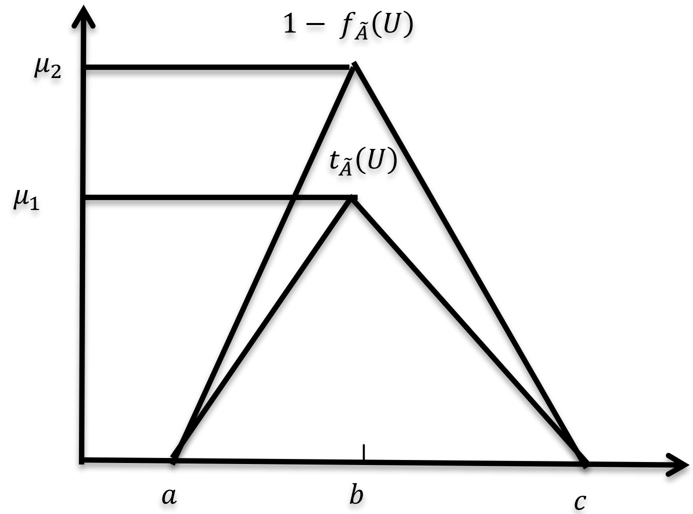


Figure (4.2) A Triangular Vague Set

Definition 4.2.5 [28,30]:

We get a crisp interval by **α-Cut** operation, interval A(**α**) shall be obtained as for each $\alpha \in[0, 1]$. Thus A(α)= $[(a_2-a_1)\alpha+a_1\,, a_3-(a_3-a_2)\alpha]$.

4.2.6 Observation α-Cut [1]:

Let A is fuzzy number such that

$$A(X)=\begin{cases}(x-a_1\,)/a_2-a_1 & for \quad a_1\le x\le a_2\\ (a_3-x\,)/a_3-a_2 & for\; a_2\le x\le a_3\\ 0 & other\ wise\end{cases}$$

Then A(α)= $[(a_2-a_1)\alpha+a_1\,, a_3-(a_3-a_2)\alpha]$

Example 4.1 (A question taken from [30] with modification)

Let $\tilde{C}$, $\tilde{D}$ be two fuzzy numbers whose membership functions which shows in Figure (4.3) are given by

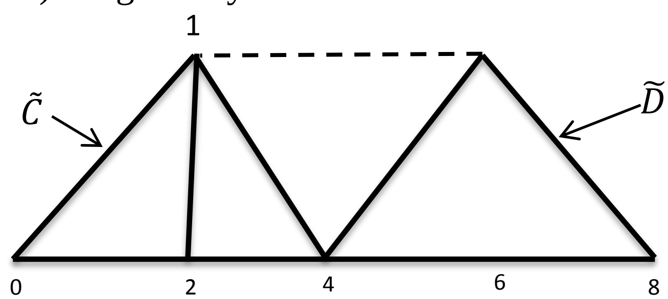


Figure (4.3) (a) Shows Membership Function for $\tilde{C}$, $\tilde{D}$

$$\tilde{C}\ (x) = \begin{cases} (x-0)/2 & for \quad 0 < x \le 2 \\ (4-x)/2 & for \quad 2 < x \le 4 \\ 0 & other\ wise \end{cases}$$

$$\widetilde{D}\ (x) = \begin{cases} (x-4)/2 & for \quad 4 < x \le 6 \\ (8-x)/2 & for \quad 6 < x \le 8 \\ 0 & other\ wise \end{cases}$$

Calculate the fuzzy numbers $\tilde{C} + \widetilde{D}$, $\tilde{C} - \widetilde{D}$

Solution

Let $x / 2 = \alpha \longrightarrow x = 2\alpha$ Such that $\alpha \in [0, 1]$

Let $(4 - x) / 2 = \alpha \longrightarrow 2\alpha = 4 - x \longrightarrow x = 4 - 2\alpha$

$C(\alpha) = [2\alpha, 4 - 2\alpha]$

Let $(x - 4)/2 = \alpha \longrightarrow 2\alpha = x - 4 \longrightarrow x = 2\alpha + 4$

Let $(8 - x) / 2 = \alpha \longrightarrow 2\alpha = 8 - x \longrightarrow x = 8 - 2\alpha$

$D(\alpha) = [2\alpha + 4, 8 - 2\alpha]$

$(\tilde{C} + \widetilde{D})(\alpha) = [4\alpha + 4, 12 - 4\alpha]$

$4\alpha + 4 = x \longrightarrow \alpha = \frac{X-4}{4}$

$\alpha = 1 \longrightarrow 4 = x - 4 \longrightarrow x = 8$

$12 - 4\alpha = x \longrightarrow 4\alpha = 12 - x \longrightarrow \alpha = \frac{12-x}{4}$

$\alpha = 0 \longrightarrow x = 12$

$\alpha = 1 \longrightarrow 12 - x = 4 \longrightarrow x = 8$

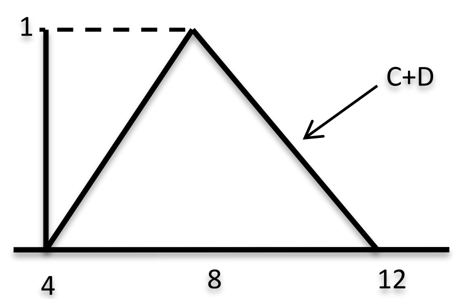


Figure (4.3) (b) Membership Function $\tilde{C}$+$\widetilde{D}$

$$(\tilde{C} + \widetilde{D})(x) = \begin{cases} 0 & if \ x \le 4, \ x \ge 12 \\ (x-4)/4 & if \quad 4 < x \le 8 \\ x - 12/-4 & if \quad 8 < x \le 12 \end{cases}$$

$(\tilde{C} - \widetilde{D})(\alpha) = [4\alpha - 8, -4\alpha]$

$4\alpha - 8 = x \longrightarrow \alpha = \frac{x+8}{4}$

$\alpha = 0 \longrightarrow x = -8$

$\alpha = 1 \longrightarrow x = -4$

$-4\alpha = x \longrightarrow \alpha = \frac{-x}{4}$

$\alpha = 0 \longrightarrow x = 0$

$\alpha = 1 \longrightarrow x = -4$

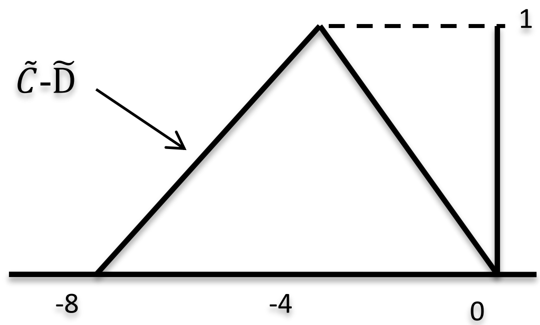


Figure (4.3) (c) Membership Function $\tilde{C}$-$\tilde{D}$

$$(\tilde{C} - \tilde{D})(x) = \begin{cases} -x/4 & if \ -4 < x \leq 0 \\ (x+8)/4 & if \ -8 \leq x \leq -4 \\ 0 \quad if & X \geq 0, X \leq -8 \end{cases}$$

4.3 Analysis of Fuzzy Model Reliability Using Vague Set [10,16]:

This section concern with a method for analyzing the reliability of a system using vague set.

4.3.1 Series Model:

Let a series model consisting of n components is considered as shown in below.

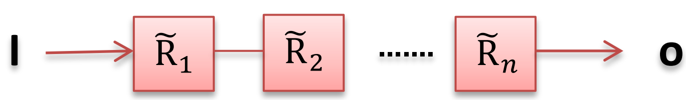


Figure (4.4) A Series Model

The fuzzy reliability $\tilde{R}_S$ of the series model is:

$$\tilde{R}_S = \tilde{R}_1 \otimes \tilde{R}_2 \otimes \ldots \ldots . . \otimes \tilde{R}_n \qquad (4.1)$$

$$= [(a_1, b_1, c_1), \ \mu_1] \otimes [(a_2, b_2, c_2), \ \mu_2] \otimes \ldots \ldots \otimes [(a_n, b_n, c_n), \ \mu_n]$$

$$= \left[\prod_{i=1}^{n} a_i \, , \ \prod_{i=1}^{n} b_i \, , \prod_{i=1}^{n} c_i \, , \ Min\,(\mu_1 , \mu_2 \ , \ldots \ \mu_n) \right] \qquad (4.2)$$

4.3.2 Parallel Model:

Let a parallel model consisting of n components is considered as shown in figure below.

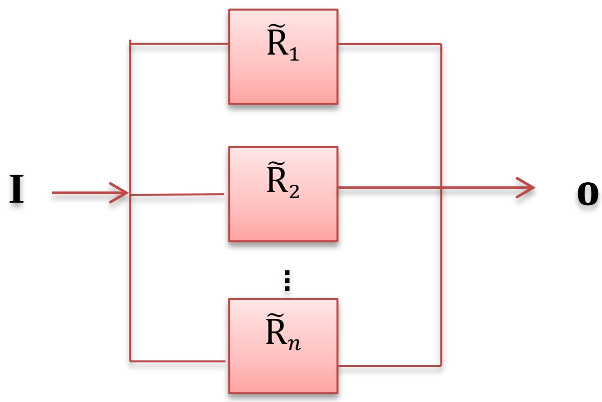


Figure (4.5) Parallel Model

The fuzzy reliability ($\tilde{R}_S$) of the parallel model is

$$\tilde{R}_S = 1 \ominus \prod_{i=1}^{n} (1 \ominus \tilde{R}_i) \quad (4.3)$$

$$= 1 \ominus [1 \ominus (a_1, b_1, c_1), \ \mu_1] \otimes [1 \ominus (a_2, b_2, c_2), \ \mu_2], \ldots \ldots$$

$$\otimes [1 \ominus (a_n, b_n, c_n), \ \mu_n]$$

$$= \left[\left(1 - \prod_{i=1}^{n}(1 - a_i), 1 - \prod_{i=1}^{n}(1 - b_i), 1 - \prod_{i=1}^{n}(1 - c_i)\right), \mathrm{Min}(\mu_1, \mu_2 \ldots, \mu_n)\right] \quad (4.4)$$

Example 4.2

Let we consider the model shown in Figure (4.6) where the reliability of the components $\tilde{R}_1$, $\tilde{R}_2$, $\tilde{R}_3$, $\tilde{R}_4$ and $\tilde{R}_5$ are as in follows.

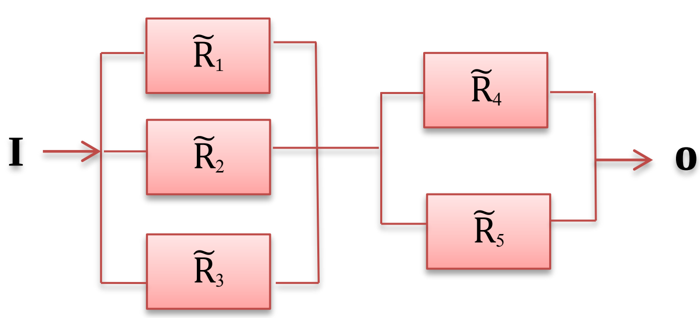


Figure (4.6) A Model Consist of 5 Components for Example 4.2

Which represent a parallel- series

$$\widetilde{R}_1 = [\,(a_1, b_1, c_1), \mu_1]$$

$$\widetilde{R}_2 = [\,(a_2, b_2, c_2), \mu_2]$$

$$\widetilde{R}_3 = [\,(a_3, b_3, c_3), \mu_3]$$

$$\widetilde{R}_4 = [\,(a_4, b_4, c_4), \mu_4]$$

$$\widetilde{R}_5 = [\,(a_5, b_5, c_5), \mu_5]$$

$$\widetilde{R}_S = [\,1 \ominus (1 \ominus \widetilde{R}_1) \otimes (1 \ominus \widetilde{R}_2) \otimes (1 \ominus \widetilde{R}_3)\,] \otimes [\,1 \ominus (1 \ominus \widetilde{R}_4) \otimes (1 \ominus \widetilde{R}_5)\,]$$

$$= [\,1 \ominus (1 \ominus [\,(a_1, b_1, c_1), \mu_1]] \otimes [(1 \ominus [\,(a_2, b_2, c_2), \mu_2]] \otimes [(1 \ominus [\,(a_3, b_3, c_3), \mu_3]] \otimes [\,1 \ominus (1 \ominus [\,(a_4, b_4, c_4), \mu_4]] \otimes [\,(1 \ominus [\,(a_5, b_5, c_5), \mu_5]]$$

$$= 1 \ominus [(1 - c_1), (1 - b_1), (1 - a_1), \mu_1] \times [(1 - c_2), (1 - b_2), (1 - a_2), \mu_2] \times [(1 - c_3), (1 - b_3), (1 - a_3), \mu_3] \otimes [\,1 \ominus [(1 - c_4), (1 - b_4), (1 - a_4), \mu_4] \times [(1 - c_5), (1 - b_5), (1 - a_5), \mu_5]$$

$$= [1 - (1 - c_1)(1 - c_2)(1 - c_3), 1 - (1 - b_1)(1 - b_2)(1 - b_3), 1 - (1 - a_1)(1 - a_2)(1 - a_3), \mathrm{Min}(\mu_1, \mu_2, \mu_3)] \otimes [1 - (1 - c_4)(1 - c_5), 1 - (1 - b_4)(1 - b_5), 1 - (1 - a_4)(1 - a_5), \mathrm{Min}(\mu_4, \mu_5)]$$

$$= [(c_1 + c_2 + c_3 - c_1c_2 - c_1c_3 - c_2c_3 + c_1c_2c_3), (b_1 + b_2 + b_3 - b_1b_2 - b_1b_3 - b_2b_3 + b_1b_2b_3), (a_1 + a_2 + a_3 - a_1a_2 - a_1a_3 - a_2a_3 + a_1a_2a_3), \text{Min}(\mu_1, \mu_2, \mu_3)] \otimes [(c_4 + c_5 - c_4c_5), (b_4 + b_5 - b_4b_5), (a_4 + a_5 - a_4a_5), \text{Min}(\mu_4, \mu_5)]$$

$$= [(c_1 + c_2 + c_3 - c_1c_2 - c_1c_3 - c_2c_3 + c_1c_2c_3)(c_4 + c_5 - c_4c_5), (b_1 + b_2 + b_3 - b_1b_2 - b_1b_3 - b_2b_3 + b_1b_2b_3)(b_4 + b_5 - b_4b_5), (a_1 + a_2 + a_3 - a_1a_2 - a_1a_3 - a_2a_3 + a_1a_2a_3)(a_4 + a_5 - a_4a_5), \text{Min}(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5)]$$

$$= [(c_1c_4 + c_2c_4 + c_3c_4 - c_1c_2c_4 - c_2c_3c_4 - c_1c_3c_4 + c_1c_2c_3c_4 + c_1c_5 + c_2c_5 + c_3c_5 - c_1c_2c_5 - c_2c_3c_5 - c_1c_3c_5 + c_1c_2c_3c_5 - c_1c_4c_5 - c_2c_4c_5 - c_3c_4c_5 + c_1c_2c_4c_5 + c_1c_3c_4c_5 + c_2c_3c_4c_5 - c_1c_2c_3c_4c_5, b_1b_4 + b_2b_4 + b_3b_4 - b_1b_2b_4 - b_2b_3b_4 - b_1b_3b_4 + b_1b_2b_3b_4 + b_1b_5 + b_2b_5 + b_3b_5 - b_1b_2b_5 - b_2b_3b_5 - b_1b_3b_5 + b_1b_2b_3b_5 - b_1b_4b_5 - b_2b_4b_5 - b_3b_4b_5 + b_1b_2b_4b_5 + b_2b_3b_4b_5 + b_1b_3b_4b_5 - b_1b_2b_3b_4b_5, a_1a_4 + a_2a_4 + a_3a_4 - a_1a_2a_4 - a_2a_3a_4 - a_1a_3a_4 + a_1a_2a_3a_4 + a_1a_5 + a_2a_5 + a_3a_5 - a_1a_2a_5 - a_2a_3a_5 - a_1a_3a_5 + a_1a_2a_3a_5 - a_1a_4a_5 - a_2a_4a_5 - a_3a_4a_5 + a_1a_2a_4a_5 + a_2a_3a_4a_5 + a_1a_3a_4a_5 - a_1a_2a_3a_4a_5), \text{Min}(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5)]$$

4.4 Some Applications in Safety and Reliability:

In this section different applications in safety and reliability form real life are presented as below.

Application 1

The data for the various components are included in the fault tree (FT) at the lowest hierarchical level and merged together using the logic of (FT) with the Boolean algebra operations to give the failure assessment of the entire system being investigated in quantitative evaluation of fault tree. The safety system for the fault tree of inverter or (ups) for generating electricity to different uses in our life as shown below . An possible analysis the failure of inverter safety system it depends on different reasons that caused by factors as defect or error in design, human errors, failure in safety circuit protection.

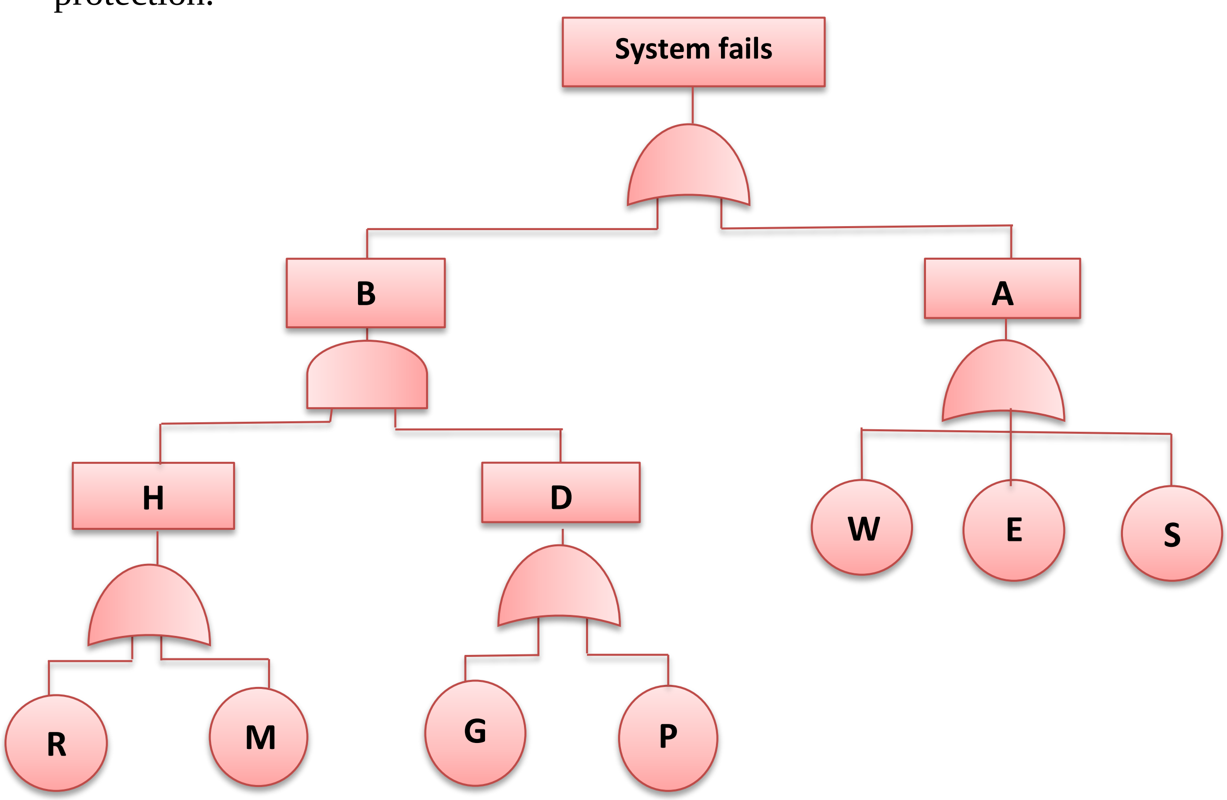


Figure (4.7) FT of (ups) Failure for Application 1

F_{IS} Refer to the inverter system's failure.

F_S Refer to a malfunction in the safety circuit protection device (anti reverse circuit).

F_W Refer to the failure caused by square wave.

F_G Refer to a design issue created by a nonstandard wire gauge (swg) that is too small, as specified by (swg table).

F_P Is the power efficiency failure caused by a short circuit.

F_R Refers to connection failure induced by human error or misuse (reverse polarity).

F_M Refer to an inverter failure caused by keeping it unmaintained after the battery has run out of power.

F_E Refer to the failure caused by environment factors.

$F_{IS} = T = A \cup B$ where $A = S \cup W \cup E$, $B = D \cap H$, $D = G \cup P$, $H = R \cup M$

By substitution $T = (S \cup W \cup E) \cup (D \cap H)$

The detecting safety device failure and inverter system failure are two key factors A and B. Each of them has sub-factors. When the failure of basic events is known, the system failure can be computed. The following is the system failure (top event) by using Boolean algebra operations.

$F_{IS} = 1 - (1 - F_A)(1 - F_B)$ and R_{IS} is :

$$R_{IS} = 1 - F_{IS}$$

$$F_A = F_S + F_W + F_E,\ F_B = F_D + F_H$$

Thus $T = [(S \cup W \cup E)] \cup [(G \cup P) \cap (R \cup M)]$

$$T = (S + W + E) + [(G + P) \times (R + M)]$$

$$T = (S + W + E) + GR + GM + PR + PM$$

The minimal cut sets are (S+W+E), GR, GM, PR and PM in that order.

Application 2 Another practical application via the following Application.

Utilizing fuzzy fault tree analysis for project to laying an underground pipelines for pure water supply project to region in Babylon city with high quality of safety. As an application of fuzzy fault tree analysis for safety of a project to supply a pure water. The study take care on the events that causes the failure in the project as human errors, some defects in engineering design, high rate of water table in some positions, depth level of digging, flood of river etc.

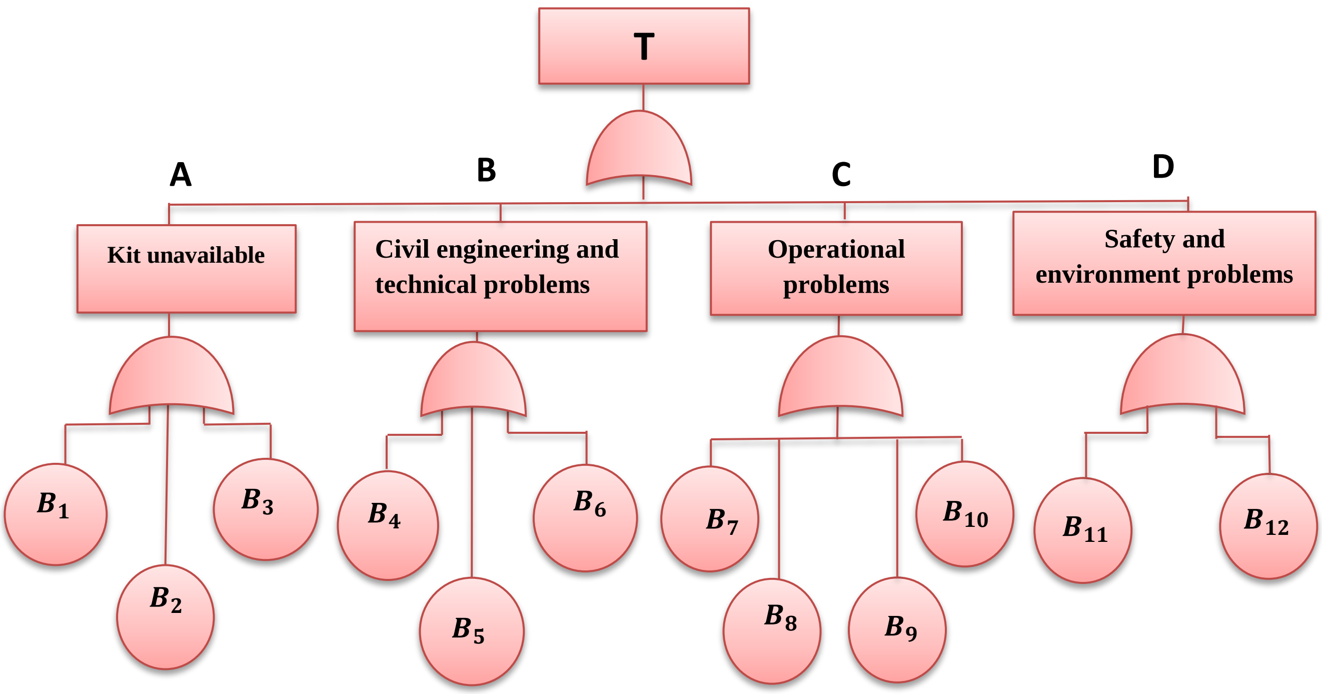


Figure (4.8) Represent the FFT for Project to Supply a Pure Water

The intermediate events A, B, C and D are:

$A = B_1 \cup B_2 \cup B_3, B = B_4 \cup B_5 \cup B_6, C = B_7 \cup B_8 \cup B_9 \cup B_{10}, D = B_{11} \cup B_{12}$

Where

$\boldsymbol{B_1}$ Indicates loss of communication with digging machine.

$\boldsymbol{B_2}$ Indicates kit breakdown.

B_3 Indicates emergency shutdown system trips.

B_4 Indicates safety incidents on location.

B_5 Indicates shortage of proper supervision.

B_6 Indicates ooze of digging fluid into waterway.

B_7 Indicates interference with water table rate.

B_8 Indicates stress of workers.

B_9 Indicates interference with bedrock and obstacle like stones.

B_{10} Indicates operator lacking required skills.

B_{11} Indicates ooze of digging fluid into soil.

B_{12} Indicates river flooding.

According the following diagram (4.9), the membership functions of fuzzy probability of occurrence with triangular fuzzy number are listed in Table (4.1).

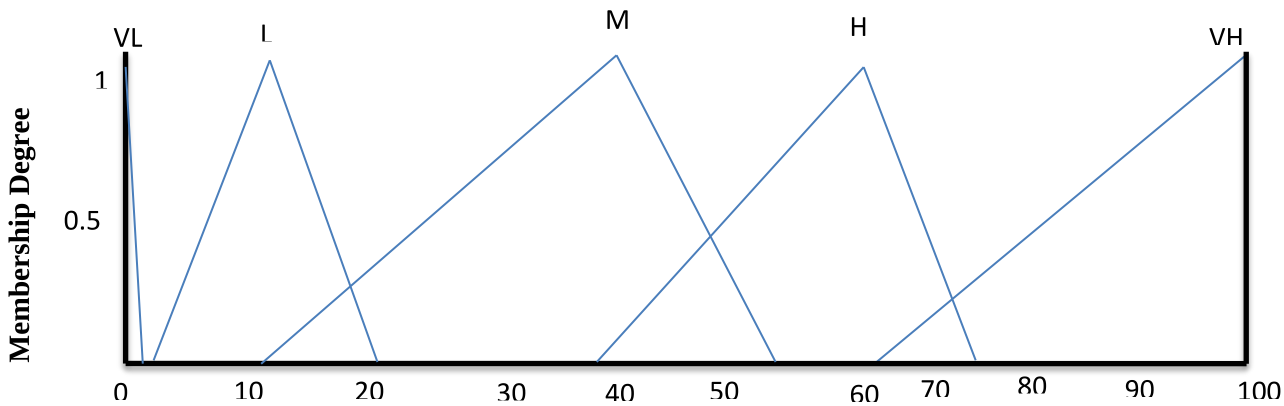


Diagram (4.9) Fuzzy Probability of Events

Table (4.1) Membership Functions for Fuzzy Probability of Occurrence of Basic Events

Linguistic Term	Triangular (a, b, c)		
	a	b	c
Very low (VL)	0	0	1
Low (L)	2	10	20
Medium (M)	10	40	55
High (H)	40	62	75
Very high (VH)	62	100	100

Table (4.2) The Basic Event and it Probability of Happens

Event	Probability of happens
B_1	Low (L)
B_2	Medium (M)
B_3	High (H)
B_4	Low (L)
B_5	Low (L)
B_6	Low (L)
B_7	Medium (M)
B_8	Very low (VL)
B_9	Medium (M)
B_{10}	Low (L)
B_{11}	Medium (M)
B_{12}	Very low (VL)

Let $\tilde{p}_i$ represent the fuzzy probability of event i and T represent top event.

$$T = \tilde{F}_S = [1 \ominus (1 \ominus \tilde{p}_\alpha(B_1)(1 \ominus \tilde{p}_\alpha(B_2)(1 \ominus \tilde{p}_\alpha(B_3)(1 \ominus \tilde{p}_\alpha(B_4)(1 \ominus \tilde{p}_\alpha(B_5)(1 \ominus \tilde{p}_\alpha(B_6)(1 \ominus \tilde{p}_\alpha(B_7)(1 \ominus \tilde{p}_\alpha(B_8)(1 \ominus \tilde{p}_\alpha(B_9)(1 \ominus \tilde{p}_\alpha(B_{10})(1 \ominus \tilde{p}_\alpha(B_{11})(1 \ominus \tilde{p}_\alpha(B_{12})] \quad (4.5)$$

Since all the gates of the tree are OR so we applying the following equation to find the top event.

$$\tilde{p}_\alpha(T) = [1 - \prod_{i=1}^{n}(1 - (a_i + (b_i - a_i)\alpha), 1 - \prod_{i=1}^{n}(1 - (c_i - (c_i - b_i)\alpha)] \quad (4.6)$$

$$\tilde{p}_\alpha(VL) = [1 - (1 - (0 + (0 - 0)\alpha), 1 - (1 - (0.01 - (0.01 - 0)\alpha)]$$

$$\tilde{p}_\alpha(VL) = [0, 0.01 + 0.01\alpha]$$

$$\tilde{p}_\alpha(L) = [1 - (1 - (0.02 + (0.10 - 0.02)\alpha, 1 - (1 - (0.20 - (0.20 - 0.10)\alpha]$$

$$\tilde{p}_\alpha(L) = [0.02 - 0.08\alpha, 0.20 + 0.10\alpha]$$

$$\tilde{p}_\alpha(M) = [1 - (1 - (0.10 + (0.40 - 0.10)\alpha), 1 - (1 - (0.55 - (0.55 - 0.40)\alpha)]$$

$$\tilde{p}_\alpha(M) = [0.10 - 0.30\alpha, 0.55 + 0.15\alpha]$$

$$\tilde{p}_\alpha(H) = [1 - (1 - (0.40 + (0.62 - 0.40)\alpha), 1 - (1 - (0.75 - (0.75 - 0.62)\alpha]$$

$$\tilde{p}_\alpha(H) = [0.40 - 0.22\alpha, 0.75 + 0.13\alpha]$$

$$\tilde{p}_\alpha(VH) = [1 - (1 - (0.62 + (1 - 0.62)\alpha), 1 - (1 - (1 - (1 - 1)\alpha)]$$

$$\tilde{p}_\alpha(VH) = [0.62 - 0.38\alpha, 1]$$

For very low basic events $\tilde{p}_\alpha(B_8), \tilde{p}_\alpha(B_{12}) = [0, 0.01 + 0.01\alpha] = \tilde{p}_\alpha(VL)$

For low basic events $\tilde{p}_\alpha(B_1), \tilde{p}_\alpha(B_4), \tilde{p}_\alpha(B_5), \tilde{p}_\alpha(B_6), \tilde{p}_\alpha(B_{10})$

$= [0.02 - 0.08\alpha , 0.20 + 0.10\alpha] = \tilde{p}_\alpha(L)$

For medium basic events $\tilde{p}_\alpha(B_2), \tilde{p}_\alpha(B_7), \tilde{p}_\alpha(B_9), \tilde{p}_\alpha(B_{11})$

$= [0.10 - 0.30\alpha , 0.55 + 0.15\alpha] = \tilde{p}_\alpha(M)$

For high basic events $\tilde{p}_\alpha(B_3) = [0.40 - 0.22\alpha , 0.75 + 0.13\alpha] = \tilde{p}_\alpha(H)$

$$T = \tilde{F}_S = [1 \ominus (1 \ominus ([0.02 - 0.08\alpha , 0.20 + 0.10\alpha]) \times (1 \ominus ([0.10 - 0.30\alpha , 0.55 + 0.15\alpha]) \times (1 \ominus ([0.40 - 0.22\alpha , 0.75 + 0.13\alpha]) \times (1 \ominus ([0.02 - 0.08\alpha , 0.20 + 0.10\alpha]) \times (1 \ominus ([0.02 - 0.08\alpha , 0.20 + 0.10\alpha]) \times (1 \ominus ([0.02 - 0.08\alpha , 0.20 + 0.10\alpha]) \times (1 \ominus ([0.10 - 0.30\alpha , 0.55 + 0.15\alpha]) \times (1 \ominus [0 , 0.01 + 0.01\alpha]) \times (1 \ominus ([0.10 - 0.30\alpha , 0.55 + 0.15\alpha]) \times (1 \ominus ([0.02 - 0.08\alpha , 0.20 + 0.10\alpha]) \times (1 \ominus ([0.10 - 0.30\alpha , 0.55 + 0.15\alpha]) \times (1 \ominus [0 , 0.01 + 0.01\alpha]]$$

And $\tilde{R}_S$ is obtained from the relation $\tilde{R}_S = 1 \ominus \tilde{F}_S$

4.4.1 Conclusions:

Many big engineering systems for different projects subject to wide range of possible future conditions which may cannot predicted with an admissible accuracy grades so it imposed a confrontation to system planning which is related with reliability of design, kinds of safety qualities and management since engineering reliability is a quantitative analysis of major sources of uncertainty as lack in knowledge, randomness, and to evaluation of safety engineering of systems. In order to perform a pure water supply project the first of all in planning to it is to valid the conditions of safety and reliability. The concept of FFT analysis is applied to deal with all the possible requirements for construction as civil engineering and others. Water supply systems include different types of interconnected components serving wide geographical regions so they are at hazard of failure according to the causes of failure whether operational errors or external reasons. The fuzzy probability of a hazard occurrence is determined to calculate the top event and the reliability of the system respectively. We suggest another project which is very important according to the need of pure water in cities, it can construct stand by one for raw water to farming watering and other aims which sharing in lowering load the use of pure water.

Application 3

We know the role of the teams in civil defense is to treatment the put out of the fires, they identity the types of factors causes the accident, since the technique for active treatment depends on the type of factor as fuel, ignition, and oxygen with their possible sub factor in order to determine a suitable kind of materials for treatment to put out fires. The main factors have three states as follow: F represent the fuel, I represent the ignition, O represent the oxygen. The following FFT depict above problem and determine the minimal cut sets which are considered the main causes of systems failure.

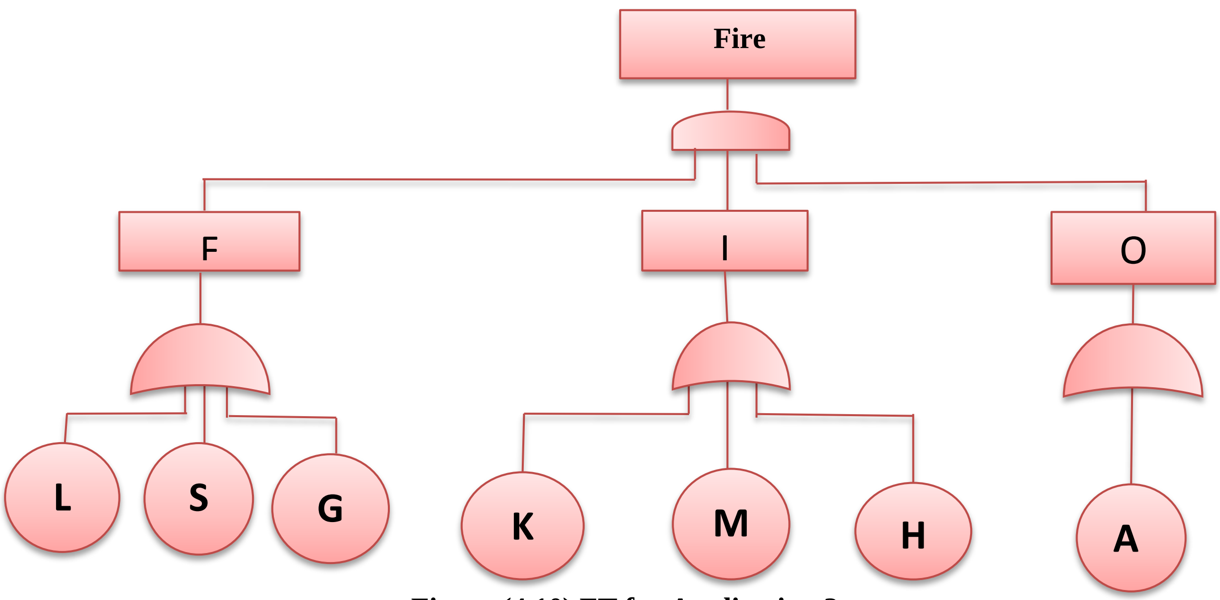


Figure (4.10) FT for Application 3

$\tilde{F}_S$ Represent the system's failure.

$\tilde{F}_{FL}$ Represent the basic event (liquid state).

$\tilde{F}_{FS}$ Represent the basic event (solid state).

$\tilde{F}_{FG}$ Represent the basic event (gases state).

$\tilde{F}_{IK}$ Represent the basic event (spark).

$\tilde{F}_{IM}$ Represent the basic event (misuse).

$\tilde{F}_{IH}$ Represent the basic event (heat or flams).

$\tilde{F}_{OA}$ Represent the basic event (atmosphere).

The system failure T is:

$T = F \cap I \cap O$

Where $F = L \cup S \cup G$, $l = K \cup M \cup H$, $O = A$

$T = (L \cup S \cup G) \cap (K \cup M \cup H) \cap A$

When the failures of the essential fault events occurring are known, the system's vague set failure may be identified. The safety system's failure is assessed as follows:

$\tilde{F}_S = \tilde{F}_F \otimes \tilde{F}_I \otimes \tilde{F}_O$,where

$\tilde{F}_F = 1 \ominus (1 \ominus \tilde{F}_{FL})(1 \ominus \tilde{F}_{FS})(1 \ominus \tilde{F}_{FG})$

$\tilde{F}_I = 1 \ominus (1 \ominus \tilde{F}_{IK})(1 \ominus \tilde{F}_{IM})(1 \ominus \tilde{F}_{IH})$

$\tilde{F}_O = \tilde{F}_A$

$$\tilde{F}_S = [[1 \ominus (1 \ominus \tilde{F}_{FL})(1 \ominus \tilde{F}_{FS})(1 \ominus \tilde{F}_{FG})] \otimes [1 \ominus (1 \ominus \tilde{F}_{IK})(1 \ominus \tilde{F}_{IM})(1 \ominus \tilde{F}_{IH})] \otimes \tilde{F}_A]$$

Since $\tilde{R}_S = 1 \ominus \tilde{F}_S$

$$\tilde{R}_S = 1 \ominus [[1 \ominus (1 \ominus \tilde{F}_{FL})(1 \ominus \tilde{F}_{FS})(1 \ominus \tilde{F}_{FG})] \otimes [1 \ominus (1 \ominus \tilde{F}_{IK})(1 \ominus \tilde{F}_{IM})(1 \ominus \tilde{F}_{IH})] \otimes \tilde{F}_A]$$

Application 4 Taken from [20] with modification

The study of a dark room electric network model. A switch and two light bulbs are present in a windowless chamber which represent as success tree we exchange the success tree for this problem into fuzzy fault tree (FFT) and by using vague set to derive $\widetilde{R}_S$.

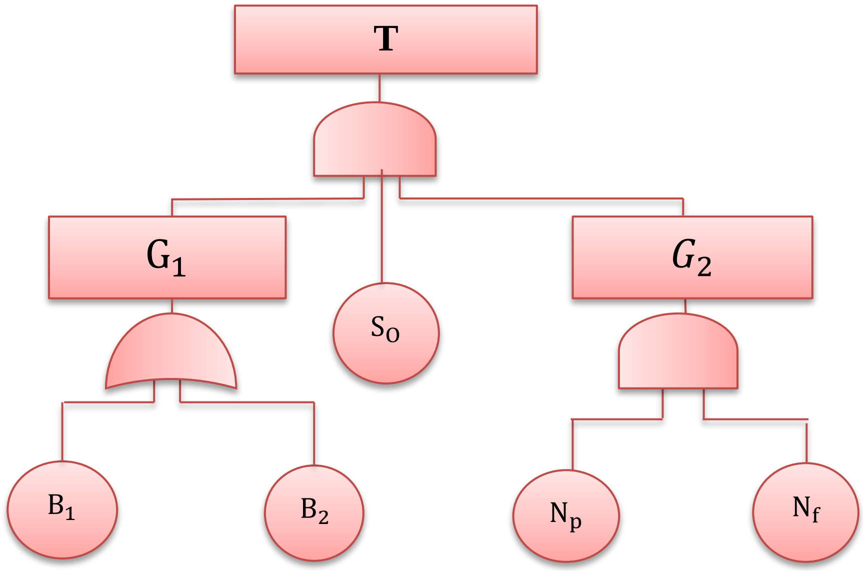


Figure (4.11) ST for Application (4)

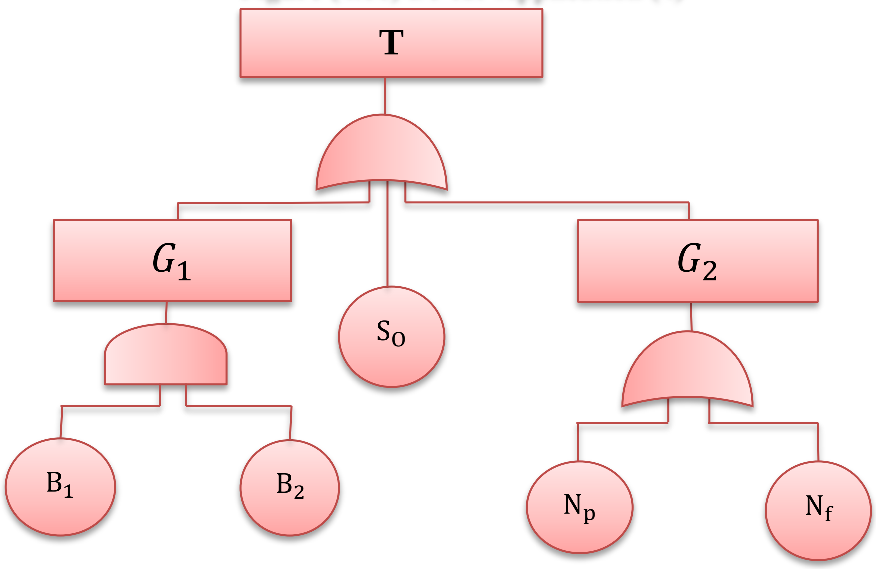


Figure (4.12) FT for Figure (4.11)

$\tilde{F}_S$ Refer to the failure illuminated dark room (T).

$\tilde{F}_{B_1}$ Refer to the failure of dark room bulb number one.

$\tilde{F}_{B_2}$ Indicates the failure of dark room bulb number two.

$\tilde{F}_{S_O}$ Refer to the failure of switch ok of dark room.

$\tilde{F}_{\mathrm{N_p}}$ Refer to power failure of electric supply of dark room.

$\tilde{F}_{\mathrm{N_f}}$ Refer to fuse failure of electric supply of dark room.

$$\mathrm{T} = \mathrm{G_1} \cup \mathrm{G_2} \cup \mathrm{S_O}$$

$$\mathrm{G_1} = \mathrm{B_1} \cap \mathrm{B_2} \,,\, \mathrm{G_2} = \mathrm{N_p} \cup \mathrm{N_f}$$

$$\mathrm{T} = (\mathrm{B_1} \cap \mathrm{B_2}) \cup (\mathrm{N_p} \cup \mathrm{N_f}) \cup \mathrm{S_O}$$

$$\mathrm{T} = (\mathrm{B_1} \times \mathrm{B_2}) + \left(\mathrm{N_p} + \mathrm{N_f}\right) + \mathrm{S_O}$$

When the failures of the essential fault events occurring are known, the vague set failure of the system may be determined. The failure of the safety system is evaluated as follows:

$$\tilde{\mathrm{F}}_\mathrm{S} = 1 \ominus (1 \ominus \tilde{\mathrm{F}}_{\mathrm{G_1}})(1 \ominus \tilde{\mathrm{F}}_{\mathrm{G_2}})(1 \ominus \tilde{\mathrm{F}}_{\mathrm{S_0}})$$

Where

$$\tilde{\mathrm{F}}_{\mathrm{G_1}} = \tilde{\mathrm{F}}_{\mathrm{B_1}} \otimes \tilde{\mathrm{F}}_{\mathrm{B_2}}$$

$$\tilde{\mathrm{F}}_{\mathrm{G_2}} = 1 \ominus (1 \ominus \tilde{\mathrm{F}}_{\mathrm{N_p}}) \otimes (1 \ominus \tilde{F}_{\mathrm{N_f}})$$

$$\tilde{\mathrm{F}}_\mathrm{S} = 1 \ominus [(1 \ominus (\tilde{\mathrm{F}}_{\mathrm{B_1}} \otimes \tilde{\mathrm{F}}_{\mathrm{B_2}})][1 \ominus \left(1 \ominus (1 \ominus \tilde{\mathrm{F}}_{\mathrm{N_p}}\right) \otimes \left(1 \ominus \tilde{F}_{\mathrm{N_f}})][(1 \ominus \tilde{\mathrm{F}}_{\mathrm{S_O}})\right]$$

$$\tilde{\mathrm{R}}_\mathrm{S} = 1 \ominus \tilde{\mathrm{F}}_\mathrm{S}$$

$$= 1 \ominus \left[1 \ominus [(1 \ominus (\tilde{\mathrm{F}}_{\mathrm{B_1}} \otimes \tilde{\mathrm{F}}_{\mathrm{B_2}})][1 \ominus \left(1 \ominus (1 \ominus \tilde{\mathrm{F}}_{\mathrm{N_p}}\right) \otimes \left(1 \ominus \tilde{F}_{\mathrm{N_f}})][(1 \ominus \tilde{\mathrm{F}}_{\mathrm{S_O}})]\right]$$

Application 5

A power supply for a central library in Babylon University is taken from the main region via a transform connected in series. For the sake to assure continual supply a subsidiary generator is used with proper switch over, as shown in Figure (4.13) below.

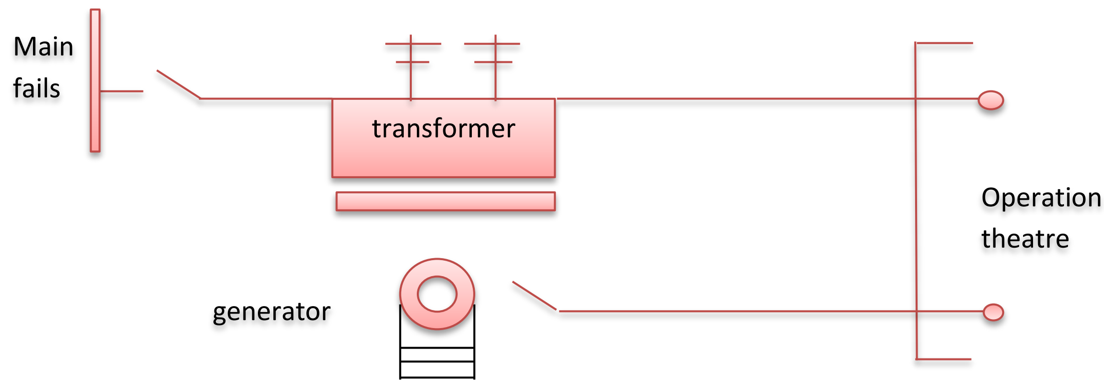


Figure (4.13) A Physical for Application 5

1. Draw

(a) FT of the system.

(b) RBD which represent FT.

2. Derive the equation of $\tilde{R}_S$ of the system.

Solution

1. (a)

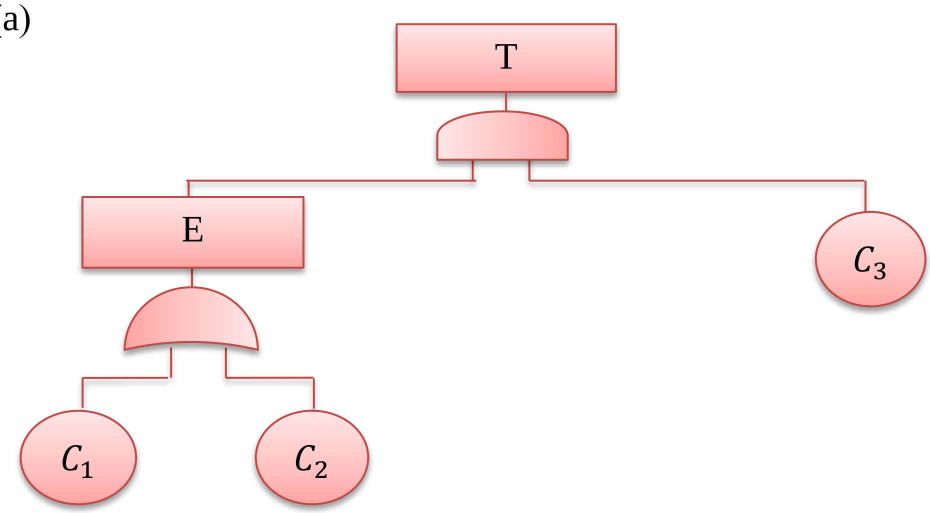


Figure (4.14) FT for Figure (4.13)

b) As illustrated in the figure below, an OR gate corresponds to series RBD and an AND gate corresponds to parallel RBD when depicting FT via RBD.

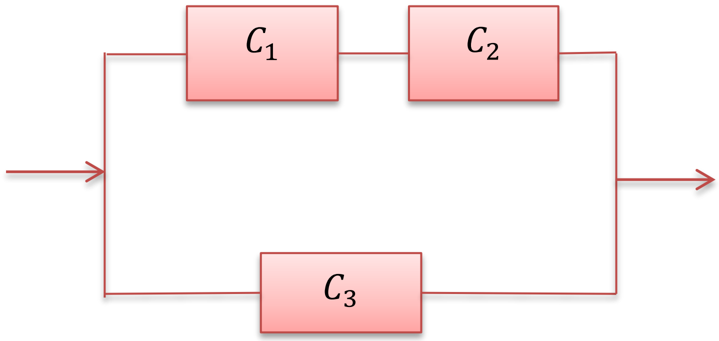


Figure (4.15) RBD for Figure (4.14)

2.

$\tilde{F}_S$ Represent the failure of system.

$\tilde{F}_{C_1}$ Represent the failure of main fails.

$\tilde{F}_{C_2}$ Represent the failure of transformer.

$\tilde{F}_{C_3}$ Represent the failure of generator.

The system failure T is:

$T = E \cap C_3$

Where $E = C_1 \cup C_2$

$T = (C_1 \cup C_2) \cap C_3$

$T = (C_1 + C_2) \times C_3$

$= C_1C_3 + C_2C_3$

When the failures of the primary fault events occurring are known, the system's vague set failure may be identified. The safety system's failure is assessed as follows:

$\tilde{F}_S = \tilde{F}_E \otimes \tilde{F}_{C_3}$

$\tilde{F}_E = 1 \ominus (1 \ominus \tilde{F}_{C_1})(1 \ominus \tilde{F}_{C_2})$

$$\tilde{F}_{IS} = [1 \ominus (1 \ominus \tilde{F}_{C_1})(1 \ominus \tilde{F}_{C_2})] \otimes \tilde{F}_{C_3}$$

$$\tilde{R}_S = 1 \ominus \tilde{F}_S$$

$$\tilde{R}_S = 1 \ominus [1 \ominus (1 \ominus \tilde{F}_{C_1})(1 \ominus \tilde{F}_{C_2})] \otimes \tilde{F}_{C_3}$$

4.5 The Relation between Human Factors, Reliability and Safety:

In this section we discussed the relation between human factors, safety and reliability.

4.5.1 Human Factors and Work Environment (Ergonomics) [62,48]:

The study of human factors is concerned with the development and use of human system interface technologies to system analysis, design, and assessment. Human-machine, human-task, human-environment, and organizational-machine interfaces are all part of this technology as illustrated in diagram below.

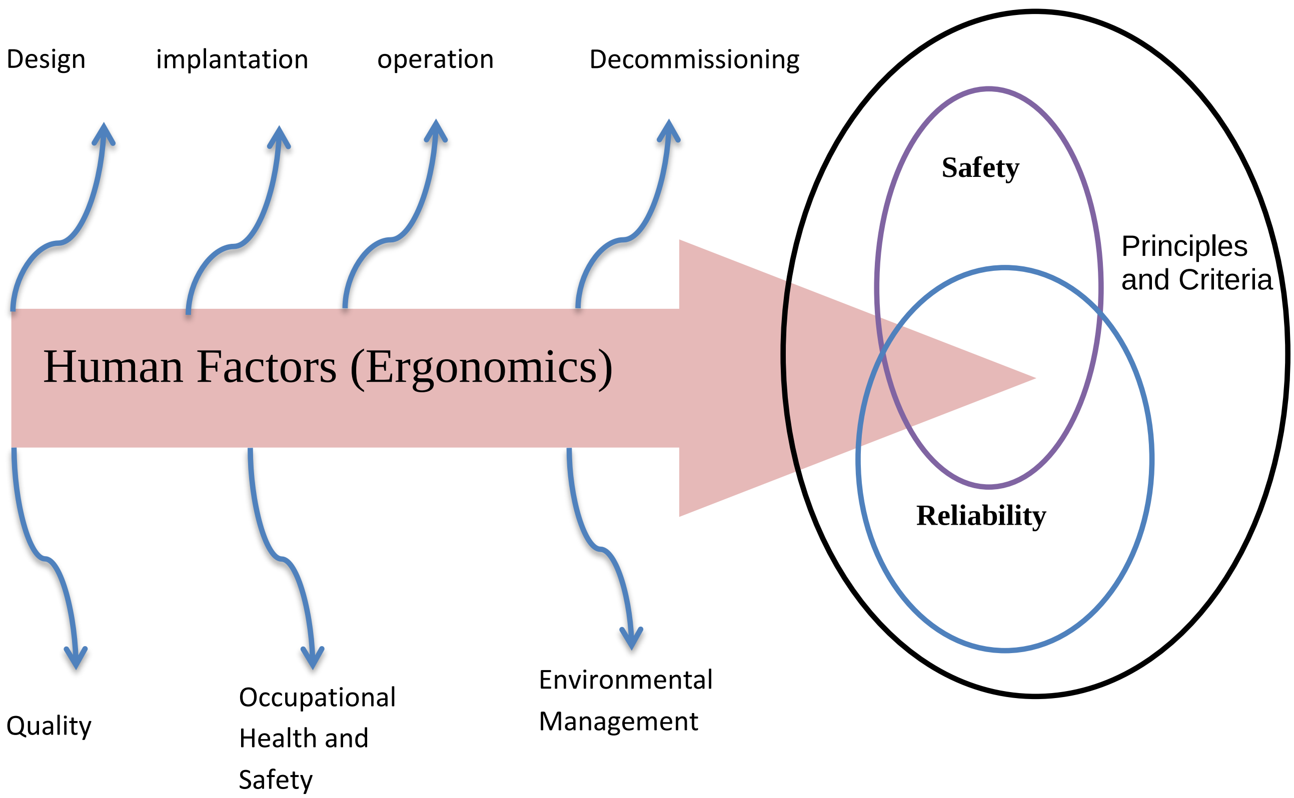


Diagram (4.16) Human Factors Integration

Note that:

i) The significance of ergonomics these days stems from the realization by businesses that creating a safe work environment may also result in increased efficiency and production. Some laws require a safe working environment. The overall architecture of the workplace has a significant effect on both safety and efficiency. The easier it is to complete a job, the more probable it is that productivity will increase owing to increased efficiency. Similarly, the safer it is to do, the more probable it is to experience productivity benefits owing to less time off for injury.

ii) Once the goal of analysis is defined, Diagram (4.16) can be used to survey an overview of the possibilities for integrating human factors (ergonomics), i.e.

1.The project's life-cycle step (design, implantation, operation, or decommissioning).

2.The target (quality, occupational health and safety, or environmental management).

3. The focus of analysis (safety, reliability). These subsequent traits will be assessed using the applicable principles and criteria.

4.5.2 Observation:

1) There is a close relationship between reliability and safety based on cases and effects as illustrate in Diagram (4.16).

2) Human factors are frequently used interchangeably with work environment, which is the practice of creating work environments to maximize safety and efficiency.

3) The systematic inclusion of human factors in the study is very clear in Diagram (4.16) by the intersection of the Human Factors (Ergonomics) arrow with the qualities under consideration (safety, reliability) or their intersections.

4) For the analysis of human factors it is grouped into six area which are:

a) Human machine interaction.

b) Organization and staffing.

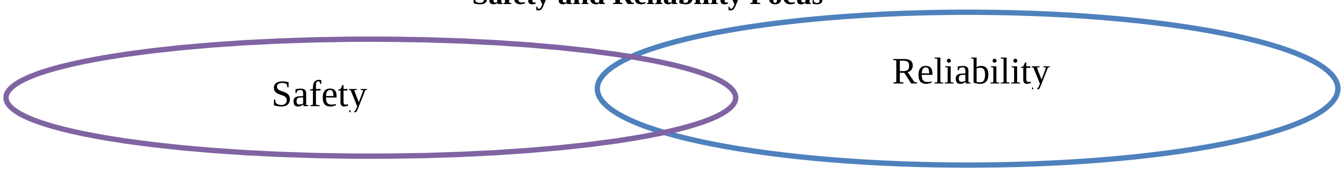
c) Procedures, roles and responsibilities.

d) Teams and communication.

e) Training and development.

f) Recovery form failures. Formore details see [48].

Table (4.3) Examples of Pertinent items and Human Factors within an Integrated Safety and Reliability Focus



Relevant Safety items	**Common relevant items and Human Factors issues**	**Relevant Reliability items**
Safety control	Safety control	Control
Safety of apparatuses	Safety of apparatuses	Apparatuses repair
Engineering safety features	Engineering safety features	Process equipment's
Critical equipment's	Repairable of safety systems	Hardware
Dangerous materials	Workload	Software
Radioactive and nuclear materials	Operation of safety systems	Power supply
	Human reliability	Reliability of repairable safety systems
Etc.	Etc.	Etc.

Table (4.4) Examples of Design and Analysis Principles and Criteria Applied to Safety, Reliability, and Human Factors

Human factors	Reliability	Safety
Environment work principles: -Work in neutral Postures -Reduce excessive force - Keep everything in easy reach - Maintain a comfortable environment - Reduce excessive motions	Standby redundancy -Variety -k-out-of-n redundancy -Fault tolerant systems -Safety factors	Fail-safe design -Double contingency -Single failure design -ALARP -Defense-in-depth -Principles of waste management

It should be noted that the ALARP concept (As Low as Reasonably Practicable) is commonly applied to hazards in fields such as radiation protection and chemical accident prevention.

Many examples in practical application in our life refer to the relation between the human factors, reliability and safety as the explosion in cheronoble and Fukushima nuclear reactors and also the accidents in civil aviation are very clear examples of the importance for the relation between human factors, reliability and safety since the human errors which causes the disaster effect on the reliability and safety condition negatively.

Now the following fault tree illustrate relation between human factors, reliability and safety.

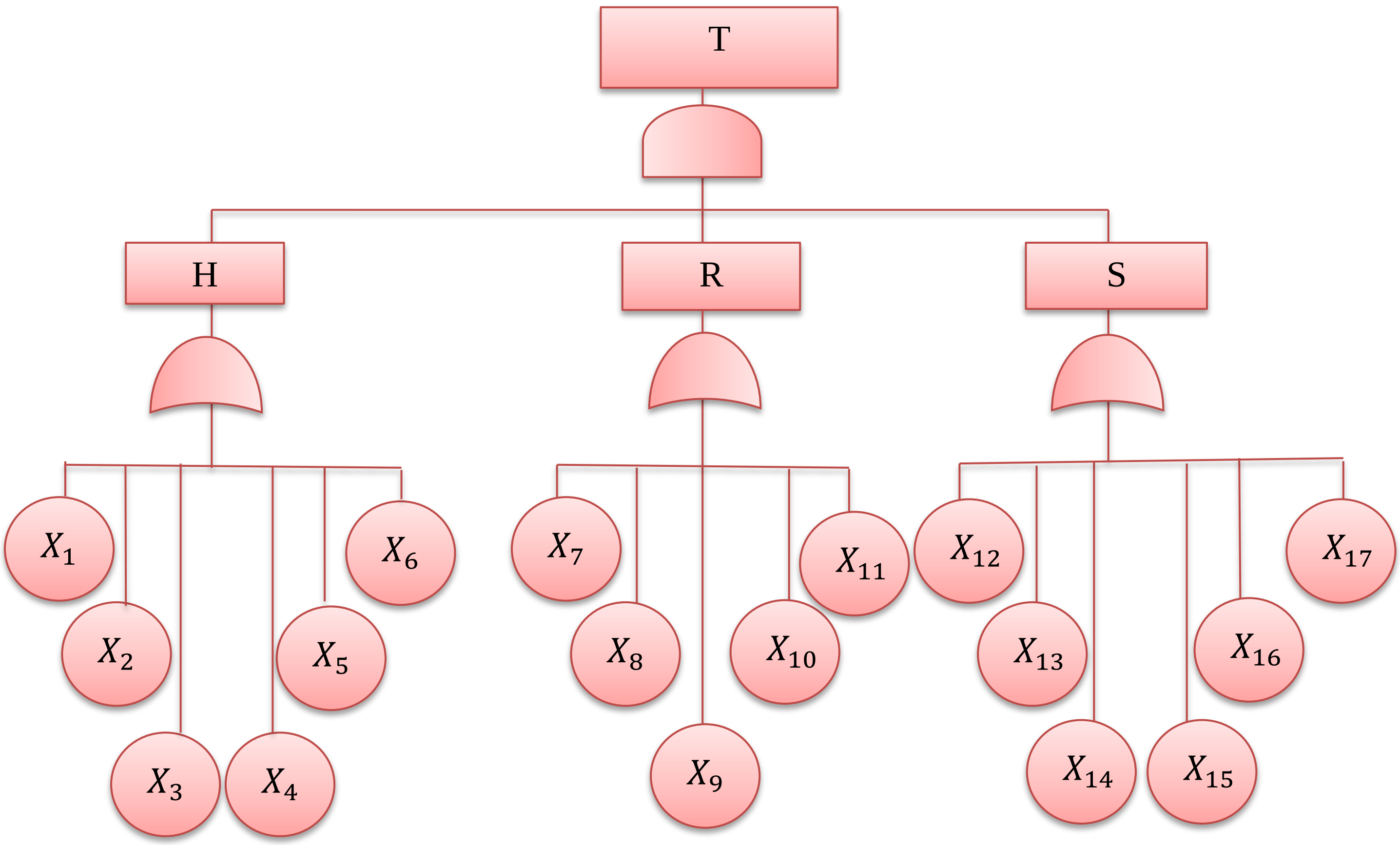


Figure (4.17) FT of Human Factors

Let H stand for human factors, R stand for reliability, S stand for safety

The top event T is :

$T = H \cap R \cap S$

Where

$\mathrm{H} = \mathrm{X}_1 \cup \mathrm{X}_2 \cup \mathrm{X}_3 \cup \mathrm{X}_4 \cup \mathrm{X}_5 \cup \mathrm{X}_6$, $\mathrm{R} = \mathrm{X}_7 \cup \mathrm{X}_8 \cup \mathrm{X}_9 \cup \mathrm{X}_{10} \cup \mathrm{X}_{11}$

$\mathrm{S} = \mathrm{X}_{12} \cup \mathrm{X}_{13} \cup \mathrm{X}_{14} \cup \mathrm{X}_{15} \cup \mathrm{X}_{16} \cup \mathrm{X}_{17}$

$$\mathrm{T} = (\mathrm{X}_1 \cup \mathrm{X}_2 \cup \mathrm{X}_3 \cup \mathrm{X}_4 \cup \mathrm{X}_5 \cup \mathrm{X}_6) \cap (\mathrm{X}_7 \cup \mathrm{X}_8 \cup \mathrm{X}_9 \cup \mathrm{X}_{10} \cup \mathrm{X}_{11}) \cap (\mathrm{X}_{12} \cup \mathrm{X}_{13} \cup \mathrm{X}_{14} \cup \mathrm{X}_{15} \cup \mathrm{X}_{16} \cup \mathrm{X}_{17})$$

$$T = (X_1 + X_2 + X_3 + X_4 + X_5 + X_6) \times (X_7 + X_8 + X_9 + X_{10} + X_{11}) \times (X_{12} + X_{13} + X_{14} + X_{15} + X_{16} + X_{17})$$

Chapter Five

Conclusions and Future Works

5.1 Conclusions

From this thesis, it has been concluded that:

1. Calculating the reliability of mixed systems by using the reduction method which depends on transform the system to series model. It permits to study the reliability allocation for the sake of increasing the reliability of a given mixed system.

2. For static fault tree analysis SFT approach it is an active tool to identification the logical occurrence of system failure starting form a specific cause to the main causes.

3. FT is allowed to identify the causes of system failure, i.e. the minimal cut sets via quantitative and qualitative analysis.

4. The determination of the component importance is very necessary to specify the danger component causes the system failure.

5. The fault tree analysis FTA helps engineers, technologist and management in decisions about failure of system, risk and safety assessment.

6. Because knowledge about the key reasons that cause system failure is sometimes incomplete, (FTA) is a useful method for doing system safety checks. In our work, the possible causes of an inverter system failure are determined through a qualitative analysis of the fault tree using minimal cut sets obtained from the analysis, which is then shared in troubleshooting for the purpose of assisting engineers and technicians in repair of various equipment.

7. Success tree represents the dual of fault tree is another tool to determine systems success via determining path sets contained in the system.

8. Moreover, the use of dynamic fault tree DFT in wide area of application more than static fault tree.

9.The partition of a FT into sub-FT is useful to check the sub-FT if it is independent or not, and to determine its domain.

10. Make use of sub-FT in multistate DFTs analysis which plays an important role in different practical applications with the use of structure function to perform the DFTs directly from its.

11. The Boolean model used to model events and gates in SFTs does not allow to take into account the order of appearance of events which is needed to model dynamic gates. To be able to take into account this temporal operators and hence model a sequence of events, we consider events as Boolean function defined on the set of positive time and which take Boolean values.

12. As well as different problems in real life possess uncertainties as lack in data or not accuracy and cloudy with hedge language. A FFT analysis is proper approach to deal with this kind of problems by utilizing the concept of fuzzy set and its operation, also fuzzy numbers in solution.

13. For different practical applications fuzzy fault tree analysis is used with vague set operations, related to safety and reliability gives rise to determine the causes of failure and danger of system. Moreover, FFT designated the common factors between human factors with both reliability and safety factors.

14. Using the concept of linguistic terms is useful in dealing with situations which are too complex or too ill defines.

5.1 Future Works

It is hoped that the suggested works of higher studies in mathematics will study the following :

1. Analytical study of fuzzy dynamic fault tree with applications.

2. Applying fuzzy soft sets in reliability and safety problems.

3. Using type-2 fuzzy set concept in safety and security applications.

4. Utilizing the uncertainty tools in GIS studies also treatment of flood and irrigation problems.

5. Making use of Markov chains to calculate the reliability of non-repairable and repairable systems.

6. Analytical study to multistate systems and multistate fault trees with applications.

[1]. A .Luay al swidi , S. A . Audi ALsaeed " Soft fuzzy and Fuzzy soft set Theory with Application " ALnoor publishment – Germany (2020).

[2]. Abdul -Haddiz Al kha, " Using Some mathematical Models in Reliability System", M.Sc. Thesis, Babylon University ,2006.

[3]. Al-Ali A.A and Subri U.A" Evaluation of reliability and main time to failure of power plant" Journal of Babylon university. Vol.7 No.3. (2002).

[4]. Al-Ali, Abdul Ameer, and K. Murari. "One unit reliability system subject to random shocks and preventive maintenance." Microelectronics Reliability 28.3 (1988): 373-377.[5]

[5]. Atanassov, K. T. "Intuitionistic fuzzy sets VII ITKR's Session." Sofia, June 1 (1983): 983.

[6]. Baek, Sejin, and Gyunyoung Heo. "Application of Dynamic Fault Tree Analysis to Prioritize Electric Power Systems in Nuclear Power Plants." Energies 14.14 (2021): 4119.

[7]. Balagurusamy, E. Reliability engineering. Tata McGraw-Hill Education, 1984.

[8]. Barlow, Richard E., and Frank Proschan. Statistical theory of reliability and life testing: probability models. Florida State Univ Tallahassee, 1975.

[9]. Bobbio, Andrea, et al. "Improving the analysis of dependable systems by mapping fault trees into Bayesian networks." Reliability Engineering & System Safety 71.3 (2001): 249-260.

[10]. Buckley, James J. Fuzzy probability and statistics. Vol. 196. Berlin: Springer, 2006.

[11]. Čepin, Marko, and Borut Mavko. "A dynamic fault tree." Reliability Engineering & System Safety 75.1 (2002): 83-91.

[12]. Chandrupatla, Tirupathi R. Quality and reliability in engineering 2. New York: Cambridge University Press, 2009.

[13]. Chang, Kuei-Hu. "A more general reliability allocation method using the hesitant fuzzy linguistic term set and minimal variance OWGA weights." Applied Soft Computing 56 (2017): 589-596.

[14]. Chatterjee, Subhashis, Jeetendra B. Singh, and Arunava Roy. "A structure-based software reliability allocation using fuzzy analytic hierarchy process." International journal of systems science 46.3 (2015): 513-525.

[15]. Chen, Qi, Yuan Wei, and Jin Qin. "Software reliability allocation with safety concerns in medical devices." 2012 Proceedings Annual Reliability and Maintainability Symposium. IEEE, 2012.

[16]. Chen, Shyi-Ming. "Fuzzy system reliability analysis based on vague set theory." 1997 IEEE international conference on systems, man, and cybernetics. Computational cybernetics and simulation. Vol. 2. IEEE, 1997.

[17]. Chen, Shyi-Ming. "Fuzzy system reliability analysis based on vague set theory." 1997 IEEE international conference on systems, man, and cybernetics. Computational cybernetics and simulation. Vol. 2. IEEE, 1997.

[18]. Chen, Wai-Kai. Graph theory and its engineering applications. Vol. 5. World Scientific, 1997.

[19]. Clark, Caitlyn E., et al. "Towards reliability-based geometry optimization of a point-absorber with PTO reliability objectives." Proceedings of the European Wave and Tidal Energy Conference, Naples, Italy. 2019.

[20]. Dhillon, Balbir S. Human reliability and error in transportation systems. Springer Science & Business Media, 2007.

[21]. Distefano, Salvatore, and Antonio Puliafito. "Dynamic reliability block diagrams vs dynamic fault trees." 2007 Annual Reliability and Maintainability Symposium. IEEE, 2007.

[22]. Dougls B. W. , “Graph Theory” , Second Edition .Prentice-Hall of India Private Limited .New Delhi (2007).

[23]. Dugan, Joanne Bechta, Kevin J. Sullivan, and David Coppit. "Developing a low-cost high-quality software tool for dynamic fault-tree analysis." IEEE Transactions on reliability 49.1 (2000): 49-59.

[24]. Ebeling, Charles E. An introduction to reliability and maintainability engineering. Tata McGraw-Hill Education, 2004.

[25]. Ebrahimnejad, Ali, and José Luis Verdegay. Fuzzy sets-based methods and techniques for modern analytics. Vol. 364. Cham: Springer, 2018.

[26]. Epstein, Benjamin, and Ishay Weissman. Mathematical models for systems reliability. Chapman and Hall/CRC, 2008.

[27]. Fahmy, Rania A. "Development of dynamic fault tree model for reactor protection system." Process Safety Progress 40.2 (2021): e12201.

[28]. Gani, A. Nagoor, and SN Mohamed Assarudeen. "A new operation on triangular fuzzy number for solving fuzzy linear programming problem." Applied Mathematical Sciences 6.11 (2012): 525-532.

[29]. Gau, W-L., and Daniel J. Buehrer. "Vague sets." IEEE transactions on systems, man, and cybernetics 23.2 (1993): 610-614.

[30]. George, J. Klir, and Yuan Bo. "Fuzzy sets and fuzzy logic: theory and applications." PHI New Delhi (1995): 443-455.

[31]. Ghadhab, Majdi, et al. "Safety analysis for vehicle guidance systems with dynamic fault trees." Reliability Engineering & System Safety 186 (2019): 37-50.

[32]. Goguen, J. A. "LA Zadeh. Fuzzy sets. Information and control, vol. 8 (1965), pp. 338–353.-LA Zadeh. Similarity relations and fuzzy orderings. Information sciences, vol. 3 (1971), pp. 177–200." The Journal of Symbolic Logic 38.4 (1973): 656-657.

[33]. Huang, Yuan-liang, Hua-ping Shao, and Qing-quan Qian. "Fault tree analysis based on logic grey gate in power system." 2011 International Conference on Network Computing and Information Security. Vol. 1. IEEE, 2011.
[34]. Janan, Xue. "On multistate system analysis." IEEE Transactions on Reliability 34.4 (1985): 329-337.
[35]. Jiang, Ge, et al. "A new approach to fuzzy dynamic fault tree analysis using the weakest n-dimensional t-norm arithmetic." Chinese Journal of Aeronautics 31.7 (2018): 1506-1514.
[36]. Keiser, Gerd. Local area networks. Information Gatekeepers Inc, 1989.
[37]. Klir, George J., Ute St. Clair, and Bo Yuan. Fuzzy set theory: foundations and applications. Prentice-Hall, Inc., 1997.
[38]. Kuo, Way, and Rui Wan. "Recent advances in optimal reliability allocation." IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans 37.2 (2007): 143-156.
[39]. Limin, Wang. "Fault Tree Analysis for oil tank fire and explosion." 2010 IEEE International Conference on Emergency Management and Management Sciences. IEEE, 2010.
[40]. Marquez, David, Martin Neil, and Norman Fenton. "Solving dynamic fault trees using a new hybrid Bayesian network inference algorithm." 2008 16th Mediterranean Conference on Control and Automation. IEEE, 2008.
[41]. Mellal, Mohamed Arezki, and Enrico Zio. "System reliability-redundancy optimization with cold-standby strategy by an enhanced nest cuckoo optimization algorithm." Reliability Engineering & System Safety 201 (2020): 106973.
[42]. Merle, Guillaume, Jean-Marc Roussel, and Jean-Jacques Lesage. "Dynamic fault tree analysis based on the structure function." 2011 Proceedings-Annual Reliability and Maintainability Symposium. IEEE, 2011.

[43]. Misra, Krishna B., and Gunter G. Weber. "A new method for fuzzy fault tree analysis." Microelectronics Reliability 29.2 (1989): 195-216.

[44]. Mulazzani, M. "Reliability versus safety." IFAC Proceedings Volumes 18.12 (1985): 141-146.

[45]. Narsingh Deo., " Graph Science Theory With Application To Engineering And Computers Science ", since Hall of India Private Limited New Delhi (2007).

[46]. Nikolaos Limnios ,"Fault Tree ISTELTD" 2007.

[47]. Pilot, Simha. "What is fault tree analysis?." Quality Progress 35.3 (2002): 120.

[48]. Sallam, Hany, Ehab Shafei, and E. A. Eisawy. "Human factors reliability analysis using fuzzy fault tree." International Journal of Engineering and Innovative Technology (IJEIT) 4.10 (2015): 145-151.

[49]. Sandler G. "System Reliability Engineering", prentice – hall Englewood Cliffs (1963).

[50]. Singer, D. "A fuzzy set approach to fault tree and reliability analysis." Fuzzy sets and systems 34.2 (1990): 145-155.

[51]. Singh, Chanan, and Roy Billinton. System reliability, modelling and evaluation. Vol. 769. London: Hutchinson, 1977.

[52]. Smith, Charles O. Introduction to reliability in design. McGraw-Hill Companies, 1976.

[53]. Souza, Ana Cláudia de, Neusa Maria Costa Alexandre, and Edinêis de Brito Guirardello. "Psychometric properties in instruments evaluation of reliability and validity." Epidemiologia e servicos de saude 26 (2017): 649-659.

[54]. Srinath L.S., "Reliability Engineering", 4tedtion Affiliated East , Wast Press Private Limited –New –Delhi, 2005.

[55]. Sriramdas, Venu, Sanjay Kumar Chaturvedi, and Heeralal Gargama. "Fuzzy arithmetic based reliability allocation approach during early design and development." Expert Systems with Applications 41.7 (2014): 3444-3449.

[56]. Stamatelatos, Michael, et al. "Fault tree handbook with aerospace applications." (2002): 1-205.

[57]. Subrie, U. A. "Some Reliability subject to Corrective Maintenance." M. Sc., university of Baghdad (1996).

[58]. Tanaka, Hideo, et al. "Fault-tree analysis by fuzzy probability." IEEE Transactions on reliability 32.5 (1983): 453-457.

[59]. Torrado, Nuria, Antonio Arriaza, and Jorge Navarro. "A study on multi-level redundancy allocation in coherent systems formed by modules." Reliability Engineering & System Safety 213 (2021): 107694.

[60]. Trivedi, Kishor S., and Andrea Bobbio. Reliability and availability engineering: modeling, analysis, and applications. Cambridge University Press, 2017.

[61]. V. Postelnicu " Non-dichotomous multi component structure " Ball Math .Soe .Sci Math .Ramanie , Vol. 14 (62) ,No.2 (1970) : 209-217.

[62]. Vasconcelos, Vanderley de, et al. "Safety, reliability, risk management and human factors: an integrated engineering approach applied to nuclear facilities." (2009).

[63]. Vesely W.E. and etal., "Fault Tree Hand book", V.S Nuclear ,Regulatory Commission Washing Ton , D.C20555 ., (2002).

[64]. VITALI, Ettore; MOTTA, Mario; GALLI, Davide Emilio. Theory and simulation of random phenomena. Springer International Publishing, (2018).

[65]. Wang, Jian, et al. "Uncertain team orienteering problem with time windows based on uncertainty theory." IEEE Access 7 (2019): 63403-63414.

[66]. Watson, Hugh A. "Launch control safety study." Bell labs (1961).

[67]. Yao, Jing-Shing, Jin-Shieh Su, and Teng-San Shih. "Fuzzy system reliability analysis using triangular fuzzy numbers based on statistical data." Journal of Information Science & Engineering 24.5 (2008).

[68]. Yoshida, Shuhei, et al. "Validity and reliability of the Patient Centred Assessment Method for patient complexity and relationship with hospital length of stay: a prospective cohort study." BMJ open 7.5 (2017): e016175.

[69]. Zadeh, Lotfi A., King-Sun Fu, and Kokichi Tanaka, eds. Fuzzy sets and their applications to cognitive and decision processes: Proceedings of the us–japan seminar on fuzzy sets and their applications, held at the university of california, berkeley, california, july 1-4, 1974. Academic press, 2014.

[70]. Zadeh, Lotfi Asker. "Fuzzy sets as a basis for a theory of possibility." Fuzzy sets and systems 1.1 (1978): 3-28.

[71]. Zohrabi, Mohammad. "Mixed method research: Instruments, validity, reliability and reporting findings." Theory and practice in language studies 3.2 (2013): 254.

المستخلص

تشمل الأطروحة الحالية من خلال الفصول الأربعة التي تضمنتها ، عرض المفاهيم الأساسية والتعريفات المتعلقة بعنوانها ، وتقديم أنواع مختلفة من نماذج الموثوقية وإثبات نظريتين حول نماذج التوالي والتوازي باعتماد على فكرة المكملة، بطرق حل مختلفة مثل طريقة الاختزال و الطريقة المركبة للحصول على موثوقية النماذج. تم استخدام شجرة الفشل وبواباتها مع كيفية بنائها واثنين من النظريات المتعلقة بالبوابتين (OR ،AND)، اقصر المسارات واصغر المجموعات القاطعة في (ثنائية من FT) ، أي شجرة النجاح ST وشجرة الفشل FT على التوالي لتحديد موثوقية النماذج مع الامثلة الموضحة.

تقديم طريقة مبسطة جديدة لحساب أهمية المكون (I_C) , ومناقشة مفهوم شجرة الأخطاء الفرعية (sub-FT) ومجالها وتحديد فيما إذا كانت sub-FT مستقلة أم لا. تحليل شجرة الخطأ الديناميكي متعدد الحالات (DFT) وسلوك بوابات DFT. تحويل FT إلى RBD ، علاوة على ذلك امتدت الدراسة إلى DFT وعلاقتها بـ DRBD مع التحويل فيما بينهما ، وكذلك تحويل DFT إلى SFT.

تم عرض مفهوم المجموعة الغامضة وعملياتها مع تطبيقات مختلفة مأخوذة من واقع الحياة اليومية المتعلقة بالسلامة كحماية الأجهزة وموثوقيتها. تطبيقات مختلفة مثل حماية العاكس ، ومشروع مد خط أنابيب المياه النقية لمنطقة في مدينة الحلة ، وتقنيات الدفاع المدني للمعالجة الفعالة للحرائق ، وكذلك حساب موثوقية نظام التغذية الكهربائية للمكتبة المركزية في جامعة بابل ، كذلك تم استخدام مفهوم Vague Sets وعملياتها في التطبيقات المذكورة أعلاه لأنها تعتبر الأفضل بين المفاهيم في التعامل مع مشاكل الموثوقية الهندسية. من خلال دراسه تحليلية تبين العلاقة بين السلامة والموثوقية والعوامل البشرية تختتم الدراسة الحالية.

جمهورية العراق
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جامعة بابل
كلية التربية للعلوم الصرفة
قسم الرياضيات

استخدام تحليل شجرة الفشل الضبابي في تطبيقات المعولية و الامان

رسالة

مقدمة الى مجلس كلية التربية للعلوم الصرفة / جامعة بابل كجزء من متطلبات نيل درجة الماجستير في التربية / الرياضيات

من قبل

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