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Comparison Study of Some Methods to Solve Transportation Problems

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Master in Education / Mathematics

By

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

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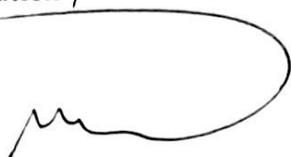
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Dedication

To my Family.

Zena Saleh Mahdi

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Abstract

In this thesis, we propose two modern techniques to find the initial solution for Transportation Problems obtained by modifying Vogel's Approximation Method, one of the best-known classic methods for finding the initial solution to Transportation Problems. Also, a new technique was introduced to find the initial solution for fuzzy transportation problems obtained by modifying the least-cost method. The working efficiency of all these new techniques proposed in this study has been tested by using them to solve many different examples, and the desired results have been obtained, where these techniques are characterized by having easier and fewer solving steps, thus reducing the cost, time and effort greatly to obtain the optimal solution to the problem.

List of Symbols

No.	Symbols	Description
1.	m	The number of sources.
2.	n	The number of destinations.
3.	a_i	Available quantities of each m sources.
4.	b_j	Available quantities for each n destinations.
5.	c_{ij}	The cost of transportation one unit of goods from origin i to destination j at every path in TP.
6.	i	It is index for origins(factory); $i = 1, 2, \ldots, m$
7.	j	It is index for destinations(warehouse); $j = 1, 2, \ldots, n$
8.	x_{ij}	The number of units shipped in every path from origin i to destination j in TP(The decision variables).
9.	D_j	The destinations.
10.	S_i	The sources.
11.	CL	The logic that deals with numerical data whose solution requires only two options: yes or no.
12.	CS	Collection of distinct well-defined objects
13.	$\widetilde{a}_i$	Available fuzzy quantities of each m sources.
14.	$\widetilde{b}_i$	Available fuzzy quantities for each n destinations.

No.	Symbols	Description
15.	$\widetilde{c}_{ij}$	The cost of transportation one unit of goods from origin i to destination j at every path in FTP.
16.	$\widetilde{x}_{ij}$	The number of fuzzy units to be transported from source i to destination j in FTP
17.	$\widetilde{A}$	Fuzzy set
18.	$\mu_{\widetilde{A}}(x)$	The membership value of $x \in X$ in the fuzzy set $\widetilde{A}$.
19.	$\widetilde{A}_{\alpha}$	α -cut of $\widetilde{A}$.

List of Acronyms

No.	Acronyms	Full Name
1.	OR	Operation Research
2.	TP	Transportation Problem
3.	BTP	Balanced Transportation Problem
4.	UBTP	Un Balanced Transportation Problem
5.	LPP	Linear Programming Problem
6.	FS	Feasible Solution
7.	BFS	Basic Feasible Solution
8.	OS	Optimal Solution
9.	ZT1	First New Technique
10.	ZT2	Second New Technique
11.	ZT3	Third New Technique
12.	NWCM	North-West Corner Method
13.	LCM	Least Cost Method
14.	VAM	Vogel's Approximation Method
15.	SSM	Stepping Stone Method
16.	MODI	Modified Distribution Method
17.	FTP	Fuzzy Transportation Problem

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CHAPTER 1

INTRODUCTION AND CONCEPTS

1.1 A Brief Review of the Transportation Problems

The globe has experienced a significant increase in the size and complexity of organizations since the beginning of the industrial revolution. Small stores run by craftspeople in the past have grown into multibillion-dollar companies today. Massive growth in the division of labor and segmentation of managerial duties in these businesses has been an intrinsic aspect of this revolutionary development. The end outcome has been incredible. However, along with its benefits, increased specialization has resulted in new problems in many businesses. One problem is that many components of any company tend to grow into empires that are completely independent with their own laws and regulations, forgetting how their goals and actions fit into the whole of the company. It is worth noting that what is excellent for one component is usually harmful to another, which leads to a conflict in the work of the components, with the growth of the company's specialization, it becomes more difficult to allocate the available resources to the various tasks in the most efficient way for the organization as a whole. These problems led to discovering a better approach to address them, which was the reason for the development of OR. The origins of OR

go back many decades when the beginning of OR activity was attributed to World War II's military services. Therefore, there was a need to allocate the sources effectively to the many military operations and activities within each operation.

As a result, the British and later American military administrations asked the help of a team of scientists to adopt a scientific approach to dealing with tactical and strategic issues. Where these scientists formed the first operating room team. These teams played a prominent role in the Air Battle of Britain. Through their research on managing convoy and anti-submarine operations, they were also instrumental in winning the Battle of the North Atlantic [17]. OR is a sub-field of mathematics, particularly applied mathematics; it is used to provide management with a scientific base so that decision-makers can make effective and rapid decisions to solve problems involving the interaction of all organization components, a solution that serves the organization's interest as a whole; this solution is called the optimal decision. New applications for OR appeared, including industry, commerce, agriculture, transportation, marketing and sales, banks, hospitals, libraries, and hotels [44].

The term "transportation" clearly defines mobility and movement from one location to another, which has made people's lives easier and has seen significant improvement in recent years. For example, in most civilizations and their differences, families can no longer produce food, clothing, and other necessities on their own. Many marketplaces provide their clients with a wide variety of innovations and requirements, which diminishes the need for self-production. Goods are produced on a large scale in factories

and farms managed by competent authorities and reach consumers at a reasonable cost. Transportation costs are spent when items are shipped from a supplier to a customer. The entire structure of modern society implies trade, economies, group activities, and the transportation of materials from one place to another. The problem that occurs in this kind of society is TP.

TP is one of the most important and earliest LPP applications. The transportation problems or models focus on the most efficient way to move a product from several factories (named supply sources) to various consumers or warehouses (named demand destinations). The goal of the classic TP is to find the best shipment schedule that minimizes overall shipping costs while meeting supply and demand constraints. When items are physically moved from the manufacturing site to the ultimate customer through various distribution channels (wholesalers, retailers, distributors, and so on), it is necessary to reduce transportation costs to improve sales profit. The application of this model is not only limited to transportation and distribution; it can also be expanded to include other problems such as plant site assignment, worker distribution to machines, assigning vehicles to routes [3, 46].

TP can be divided into linear and nonlinear categories according to the type of the cost function. The cost of a commodity per unit delivered from a source to a destination is constant in a linear transportation problem (common transportation problem), regardless of the number of goods transported. Furthermore, it is assumed that the mileage (distance) from each source to each destination is constant. However, in practice, we can

find at least two instances when TP is not linear.

1) The cost of a commodity per unit delivered may not be constant.

2) The distance traveled from every source to every destination is not specific; the reason is due to some emergency conditions, such as transporting military supplies during wars due to the destruction of the transport network or transporting emergency materials when natural disasters occur. The solution of TP in the preceding situations is not as simple as the linear transportation problem [29].

TP is one of the optimization problems. Therefore, the solution of TP is in two-phase; the first phase is finding the initial basic feasible solution (IBFS) to this problem by using the famous classical methods like "the North-West Corner Method"(NWCM), "the Least Cost Method"(LCM), and "the Vogel's Approximation Method"(VAM). The second phase is improving IBFS to obtain OS for TP by using one of the two known methods, the Modified Distribution Method (MODI) and the Stepping Stone Method (SSM) [40].

The focus and attention of the researcher in this study are to find a new and practical approach to get the initial solution to the linear transportation problems and fuzzy transportation problems. Two new modifications of the classical method VAM (the first new technique (ZT1) and the second new technique (ZT2)) have been proposed to find the initial solution to TP.

The working efficiency of these two new techniques has been tested by solving a lot of numerical examples. A few of these examples were mentioned in this thesis for clarification and made a comparison of the solution

results in the new techniques with the solution results in the traditional methods of finding the initial solution to TP.

Moreover, the researcher presented a modification to the classic method LCM (the third new technique (ZT3)) to find IBFS for FTP by converting it to the crisp form equivalent to it using the (the robust ranking technique). We applied this technique to triangular fuzzy number, trapezoidal fuzzy number, hexagonal fuzzy number, octagonal fuzzy number, and nonagonal fuzzy numbers and has been tested the efficiency of the performance of the new technique by solving a set of numerical examples were mentioned in this thesis and made a comparison of the solution obtained by ZT3 with the solution obtained by LCM that it is less or equal to the results of LCM, knowing that the results obtained using the new technique were close to OS, and sometimes equal to it. These new techniques are easy to understand and apply, have fewer steps, and take less time to solve the problems.

1.2   Historical Overview

Many companies and production industries face many difficulties of economic improvements in our daily lives, such as reducing costs to increase profits. Transportation models are an example of these rooted financial optimizations. The formulation and solution of TP as LPP was one of the first and most important uses of linear programming.

In (1781), the French mathematician Gaspard Monge [44]in cooperation with the army of Napoleon Bonaparte, published a mathematical model dealing with the transport of soil at the lowest possible cost between different construction sites to build forts military roads . Although Monge laid a theoretical basis for solving TP, an algorithm was not developed until (1941) when American mathematician Frank Hitchcock [18] published his solution to the problem of Monge. This presentation is the first significant contribution to solving TP. Hitchcock gave a method to find the initial solution to TP, now known as the (the north-west corner method).

In (1947), Koopmans [23] proposed his article to solve TP. The two studies mentioned above represent the most important advances in establishing diverse ways of solving TP. In (1951), Dantzig [14] proposed solving TP using the simplex method by describing it as an LP model. Still,TP has a lot of variables and limitations, and solving it using the simplex technique requires a lot of time and effort. In (1954), Charnes and Cooper [10] was designed the "stepping stone method" to find OS for TP. In (1957), Munkres [30] also presented techniques for solving transportation problems, and assignments. In (1959), Bell [6] introduced a new

algorithm to solve TP when the shipping cost on each route is convex. In (1984), Goyal [16] proposed a study to solve the unbalanced transportation problem by modifying VAM. In (2002), Ping and Chu [33] presented their study "The dual-matrix approach to the transportation problem". In (2006), Maurya and Garg [28] introduced an alternative approach to finding the best assignment table in administrative systems. In (2010), Pandian and Natarajan [32] suggested a new method for finding an OS for TP. In (2011), Korukoğlu and Ballı [24] both provided an improvement to the work of the VAM using the Total Opportunity Cost (TOC) matrix. In (2012), Sudhakar, et al. [43] introduced a new approach to finding OS for TP. In (2014), Babu, et al. [4] discussed a new method for finding IBFS to TP is called the "Implied Cost Method (ICM)". In (2015), Soomro, et al. [41] modified the VAM and obtained a new method to find IBFS to TP. In (2016), Ahmed, et al. [1] introduced a new approach to solve TP. In (2017), Ahmed, et al. [2] offered an innovative way to obtain IBFS for TP. In (2018), Kumar, et al. [25] proposed a new approach to getting IBFS for TP. And in (2018), Kaur, et al. [22] also described an enhancement in the Maximum Difference Method (MDM) to identify IBFS for TP. In (2019), Sasikala, Akiri and Subbara [37] introduced two methods to find IBFS for TP. In (2020), Hussein and Shiker [19] offered a new VAM modification to solve TP. In (2021), Saito, et al. [36] offered a new VAM modification to find OS by a statistical technique that provides a better feasible solution than existing methods such as NWCM and LCM. And in (2021), Basriati, et al. [5] proposed a technique for optimizing drinking water distribution costs using VAM and three modified VAM methods.

1.3   A Brief Review of the Fuzzy Transportation Problems

Classical logic (the Crisp Logic(CL)) deals with numerical data. The problem with classical logic takes only two options: black or white, on or off, long or short, cold or hot. So, it is called (binary logic or two-valued Boden), which is represented by only two values, 1 and 0. For example, the element a belongs to B, so its degree of membership will be 1, but if it does not belong to B, its degree of membership will be 0.

Likewise, we say, "John is tall because his taller is 182 cm". When we draw a straight at 180 cm, we can see Mike, whose length is 178 cm, is short. Really Mike short? According to classical logic, the answer will be yes or no, after assuming that everyone whose height is less than 180 cm is short, and those who are taller than that are tall. Therefore, we need a logic that is closer to human thinking and formulates our sense with words, a logic that leads to new systems, more intelligence, and humanity.

The fuzzy logic (or multi-valued logic)(FL) came to describe the uncertainty and lack of clarity in everything around us, such as temperatures, lengths, speeds, distances, beauty, and other things that CL cannot describe. FL provides us with accuracy and time to solve problems that cannot be solved, and there are no mathematical techniques to solve them and smoothly handle information mysterious and uncertain and creates systems that reflect human thinking.

FL was first introduced in 1930 by the Polish scientist Jan Lukasiewicz,

who studied methods of mathematical description of fuzzy phrases, such as (long, old, hot) where classical logic was limited to describing these phenomena with yes or no. Jan expanded this description to include all values between 0 and 1, using a specific number from the previous period to describe the probability of a particular phrase being false or true as shown in Fig 1.1.
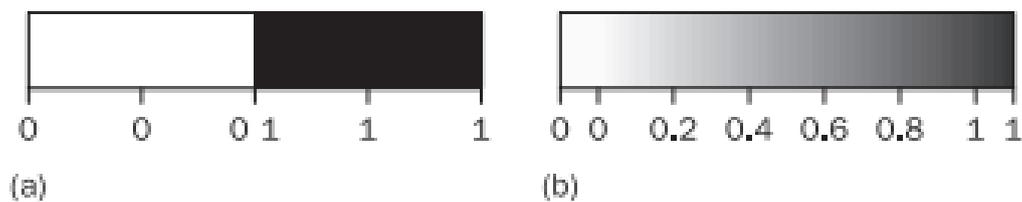
Figure 1.1: Range of Values in (a) Crisp Logic, (b) Fuzzy Logic

For instance, when the probability that a person is 182 cm tall is set to a value of 0.86, this indicates that this person is rather tall. From this expansion emerged (the possibility theory).

In 1937, the British-American scientist Max Black introduced the first simple definition of the fuzzy set and described the basic concepts of operations on fuzzy set [31, 35]. Later, Lotfi Zadeh introduced his famous research paper "Fuzzy sets" in 1965 [47]. He expanded the work on (probability theory) in a more formal mathematical form. He introduced the possibility of using natural language in logic. Therefore, Zadeh was considered the leading professor of fuzzy set.

The essential principle behind the fuzzy set theory is that an element with a specific degree of membership belongs to fuzzy set. As a result, a proposition can be partially true (or partly false) to any degree rather than being either true or false. These degrees are generally expressed as real

numbers in the specified period between 0 and 1. We discuss the classic example (tall french men) in fuzzy set theory. Fuzzy set has elements that represent 'tall french men'; all of them are men, but their degrees of membership depend on their tall, as shown in Table 1.1 all men of intermediate tall have intermediate degrees of membership. Fuzzy set asks, "How tall is this man?" the answer would be the partial membership in Fuzzy set, such as John's height is 0.82. We also note that Mike, whose height is 178 cm, which is 4 cm less than John, is suddenly no longer a very tall (or short) man as in crisp set(CS).

Degrees of Membership			
Names	**Tall, cm**	**Crisp Set**	**Fuzzy Set**
David	207	1	1.00
Bob	204	1	1.00
Steven	197	1	0.98
John	182	1	0.82
Mike	178	0	0.78
Chris	171	0	0.24
Mark	166	0	0.15
Tom	157	0	0.06
Peter	150	0	0.01
Bill	51	0	0.00

Table 1.1: Degrees of Membership for (Tall French Men)

It is worth mentioning fuzzy set can provide a graceful cross-border transition, as shown in Figure1.2. The figure's horizontal axis displays the range of all potential values for a given variable. The variable in our situation is men's height. The membership value of fuzzy set is represented by the vertical axis in the figure. Fuzzy set of "tall men" in our situation

gives height values to the corresponding membership values. We can think of new options such as (shorter men), (very shorter men), (partly tall), and (very taller men) [9, 31].
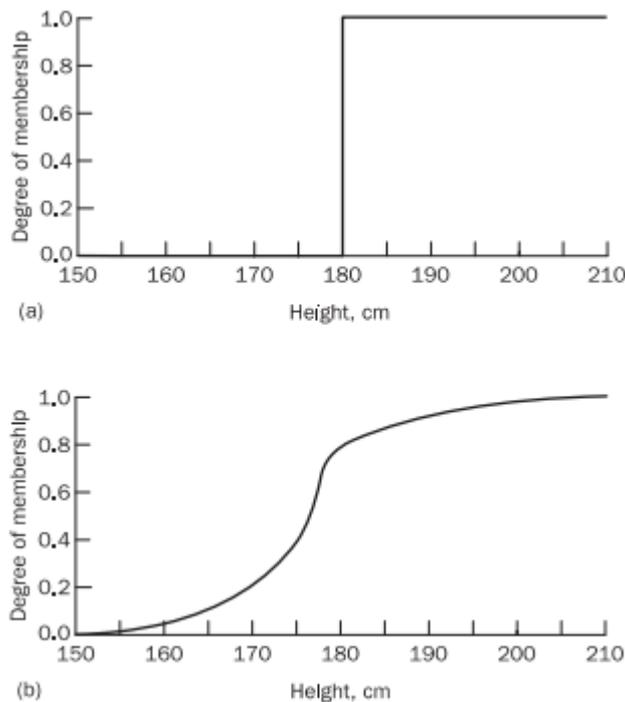
Figure 1.2: (a) Crisp Set (b)Fuzzy Set of (Tall French Men)

We can note the applications of fuzzy set theory in a wide range of areas of daily life, such as expert systems, control engineering, artificial intelligence, robotics, computer science, medicine, meteorology, industrial applications, operations research, decision theory, and management science, and mathematical modeling, etc. In addition to embracing classical mathematical fields such as algebra, graph theory, and topology [48].

In today's highly competitive market, firms are under increasing pressure to develop better ways to distribute items to customers cost-effectively. Models of transportation give a robust framework for addressing this challenge. The decision-maker should be confident of the exact values of transportation cost, product availability, and demand in classic transportation

problems. In real life, the parameters of TP may not be precisely known because of a combination of uncontrollable factors. Probability techniques and concepts can be used to deal with inaccurate information. However, probability distributions require predictable regularity in advance. Alternatively, and more particularly, can be used the membership functions of fuzzy set to express uncertain values.

Methodologies based on fuzzy theory have the advantage that they do not require predictable regularity. They can handle inaccurate information and data that contain specific emotions based on the subjective judgment of decision-makers. Therefore, many research scientists have represented all or part of the parameters of TP with fuzzy numbers and suggested new and different ways to solve these problems.

So, the problem of fuzzy transportation is TP in which demand, supply, and transportation costs are unclear. FTP aims to define a shipping table that decreases the overall cost of FTP and meets fuzzy supply and demand constraints. The balanced condition is sufficient and necessary to obtain a practical solution to TP.

FTP solution consists of two phases, the first phase is to obtain IBFS for this problem, and the second phase is to improve IBFS to get OS after using (the robust ranking technique) [11, 21, 26].

During the first half of the 1990s, fuzzy set theory was developed for fuzzy technology very rapidly. Approximately 15,000 publications have appeared since 1960. Most of them have developed the fuzzy set theory in many fields. These publications reflect applications of fuzzy set theory to real problems.

The concept of fuzzy set theory was first introduced by Zadeh, is used for solving different types of linear programming problems.

In (1973), Tanaka, et al.[45] introduced fuzzy mathematical programming for the first time. In (2013), Jayaraman and Jahirhussian [20] improved zero suffix method via robust rank techniques to solve FTP. In (2014), Ebrahimnejad [15] introduced a simplified new approach for solving FTP with generalized trapezoidal fuzzy numbers. In (2017), Satheeshkumar, et al. [38] introduced a comparative study on ZSM and LCM in FTP. In (2017), Christi [12] introduced new solutions to fuzzy transportation problems using the different ranking techniques. In (2020), Bisht and Srivastava [7] proposed a trisectional fuzzy trapezoidal approach to optimize interval data-based transportation problems. In (2021), Maheswari, et al. [27] proposed a modified method for finding a feasible initial solution for fuzzy transportation problems involving trapezoidal fuzzy numbers.

1.4 The Objectives of This Thesis

This thesis discusses the problem of transporting homogeneous goods with the aim of obtaining a new method to find an initial solution to this problem that gives the best results. Transportation costs have a major impact on production costs. One of the main goals is to find an appropriate solution method for the TP that reduces the total cost of transportation while meeting offer and demand restrictions so it shall be possible to provide a commodity at a relatively least cost. As a result, the price of the goods shall decrease, and the quality of the commodity will be preserved.

The researcher has studied and analyzed traditional algorithms in addition to recently developed algorithms, and the researcher has developed two effective techniques to solve TP in addition to developing new technique to solve FTP.

The main objectives of this research are to:

1. Study TP in detail.

2. Design and develop new techniques that solve TP to obtain an effective OS or near to it.

3. Find a new technique to get IBFS to solve FTP.

4. Test the effectiveness of new techniques by solving some examples.

5. Comparison of the results solution of new techniques with the results solution of the classical methods.

1.5   Structure of Thesis

This thesis describes TP, FTP, and the various concepts related to these problems. New techniques have been proposed to find IBFS for TP, and a new technique has been proposed to find IBFS for FTP.

This thesis is organized into five chapters, each chapter introducing successive ideas from start to finish.

The reset of this thesis is organized as follows:-

Chapter 1, Introduction and Concepts, this chapter provides a brief review of TP, FTP, and the history of these problems with a literature review. It also reviews the most important basic definitions and concepts related to TP and FTP.

Chapter 2, Transportation Problems, this chapter provides the basics of TP and explains them in detail regarding the general mathematical model of TP, special problems in transportation models, the three classic methods of finding the initial solution to this problem, and the two classic methods to find OS to TP.

Chapter 3, Solving Transportation Problems by Using Modification to Vogel's Approximation Method, discusses two new techniques to find an initial solution to TP and compare the solution with the classical methods.

Chapter 4, A New LCM Modification for Finding an IBFS for Fuzzy Transportation Problems, discusses a new technique to find an initial solution to FTP and compare the solution with the classical method LCM.

Chapter 5, Conclusions and Future Works , summarizes the conclusion obtained from this research study and mentions suggestions for future research and study.

1.6 Basic Definitions and Concepts

In this section, we will present some concepts and terminology that we think are important for understanding the content of the thesis.

1.6.1 Source

It is the location from which shipments or goods are sent out, also called (origin) [46].

1.6.2 Destination

It is the location where shipments or goods are delivered, also called (warehouses) [46].

1.6.3 Supplies

It is available quantities of goods and shipments at each distribution center (sources), also called (capacities) [42].

1.6.4 Demands

It is the quantities of goods and shipments necessary to fulfill the demand center's needs, also called (requirements) [42].

1.6.5 Feasible Solution (FS)

In TP, the set of non-negative assignments x_{ij} is called a feasible solution if it fulfills (supply and demand constraints). TP always has a feasible solution [34].

1.6.6 Basic Feasible Solution(BFS)

An FS of TP is defined as a basic feasible solution if it contains positive assignments equal to $(m + n - 1)$, where m and n are the numbers of rows and columns of TP, respectively [34].

1.6.7 Optimal Solution (OS)

An FS (not necessarily basic) that outputs the objective function in its best (minimizing or maximizing) is said to be an optimal solution [37].

1.6.8 Fuzzy Transportation Problem (FTP)

FTP deals with m sources and n destinations where $S_1, S_2, ..., S_m$ the sources with supply $\widetilde{a}_i$ $(i = 1, 2, ..., m)$ and $D_1, D_2, ...D_n$ are destinations with demand $\widetilde{b}_j$ $(j = 1, 2, .., n)$. Let $\widetilde{c}_{ij}$ represents the unit fuzzy transportation cost from source i to destination j and $\widetilde{x}_{ij}$ be the number of fuzzy units to be transported from source i to destination j, then the mathematical formulation of the problem is stated as [27]:

$$MinimizeF = \sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c}_{ij} \widetilde{x}_{ij} \qquad (1.1)$$

Subject to the constraints,

$$\sum_{j=1}^{n} \widetilde{x}_{ij} \approx \widetilde{a}_i, i = 1, 2, ...m \tag{1.2}$$

$$\sum_{i=1}^{m} \widetilde{x}_{ij} \approx \widetilde{b}_j, j = 1, 2, ...n \tag{1.3}$$

Where

$$\widetilde{x}_{ij} \geq \widetilde{0}, \quad \forall \quad i \quad and \quad j \tag{1.4}$$

1.6.9 Fuzzy Set (FS)

A Fuzzy set is characterized by a membership function mapping element of a domain, space of the universe of discourse X to the unit interval $[0,1]$, i.e., $\widetilde{A} = \{(\mu_{\widetilde{A}}(x)), x \in X\}$. Here $\mu_{\widetilde{A}}(x) : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set $\widetilde{A}$ and $\mu_{\widetilde{A}}(x)$ is called the membership value of $x \in X$ in the fuzzy set $\widetilde{A}$. These membership grades are often represented by real numbers ranging from $[0,1]$ [38].

1.6.10 α - Cuts

α - cuts are slices through a fuzzy set producing regular (non-fuzzy) sets. If $\widetilde{A}$ is a fuzzy subset of universal set X, then the α -cut of $\widetilde{A}$, written $\widetilde{A}_{\alpha}$, is defined as $\widetilde{A}_{\alpha} = \{x \in X | \mu_{\widetilde{A}}(x) \geq \alpha\}$, for all $0 < \alpha \leq 1$ [8, 9].

1.6.11 Normal Fuzzy Set

A Fuzzy set $\widetilde{A}$ of the universe of discourse X is called a normal fuzzy set if there exist at least one $x \in X$ such that $\mu_{\widetilde{A}}(x) = 1$ [20].

1.6.12 Fuzzy Number

A Fuzzy subset $\widetilde{A}$ is a fuzzy number on the real line R If [39]:

1. $\widetilde{A}$ is normal.

2. $\widetilde{A}$ is strictly increasing on $[a, b]$ and strictly decreasing on $[b, c]$.

3. $\widetilde{A}$ is an upper continuous .

1.6.13 Triangular Fuzzy Number

A Fuzzy number $\widetilde{A}(x) = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is

$$\mu_{\widetilde{A}}(x) = \begin{cases} x - a/b - a & a \leq x < b \\ 1 & x = b \\ c - x/c - b & b < x \leq c \\ 0 & otherwise \end{cases} \tag{1.5}$$

Where a, b, c are real numbers such that $a < b < c$, the triangle's base is the interval $[a, c]$. A Triangular fuzzy number is represented in Fig.1.3
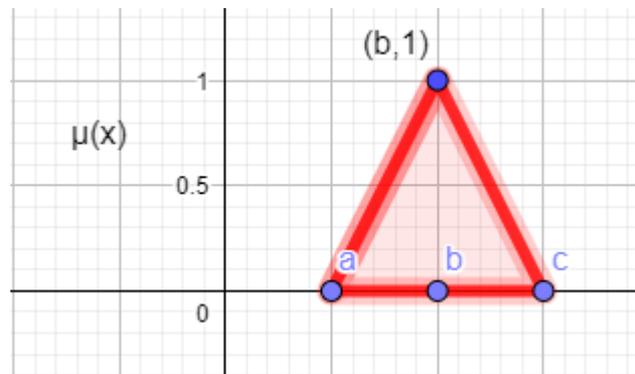
Figure 1.3: Triangular Fuzzy Number $\widetilde{A}(x)$

α - cuts of a triangular fuzzy number $\widetilde{A}(x) = (a, b, c)$ can be defined as the equation 1.6 [38]:

$$\widetilde{A}_{\alpha} = \{a + (b - a)\alpha, c + (b - c)\alpha\} \tag{1.6}$$

1.6.14 Trapezoidal Fuzzy Number

A Fuzzy number $\widetilde{A}(x) = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is

$$\mu_{\widetilde{A}}(x) = \begin{cases} x - a/b - a & a \leq x < b \\ \\ 1 & b \leq x \leq c \\ \\ d - x/d - c & c < x \leq d \\ \\ 0 & otherwise \end{cases} \tag{1.7}$$

Where a, b, c, d are real numbers such that $a < b < c < d$, the trapezoid's base is the interval $[a, d]$. A Trapezoidal fuzzy number is represented in Fig 1.4 [7].
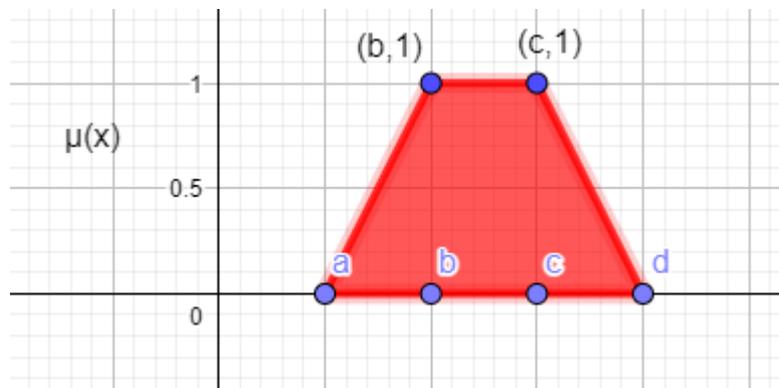
Figure 1.4: Trapezoidal Fuzzy Number $\widetilde{A}(x)$

α - cuts of a trapezoidal fuzzy number $\widetilde{A}(x) = (a, b, c, d)$ can be defined as the equation 1.8:

$$\widetilde{A}_{\alpha} = \{a + (b - a)\alpha, d + (c - d)\alpha\} \tag{1.8}$$

1.6.15   Hexagon Fuzzy Number

A Fuzzy number $\widetilde{A}(x) = (a, b, c, d, e, f)$ is said to be a hexagonal fuzzy number if its membership function is

$$\mu_{\widetilde{A}}(x) = \begin{cases} 1/2(x - a/b - a) & a \leq x \leq b \\[2mm] 1/2 + 1/2(x - b/c - b) & b \leq x < c \\[2mm] 1 & c \leq x \leq d \\[2mm] 1 - 1/2(x - d/e - d) & d < x \leq e \\[2mm] 1/2(f - x/f - e) & e \leq x \leq f \\[2mm] 0 & otherwise \end{cases} \tag{1.9}$$

Where a, b, c, d, e, f are real numbers such that $a < b < c < d < e < f$, the hexagonal's base is the interval $[a, f]$. A Hexagonal fuzzy number is represented in Fig 1.5 [13].
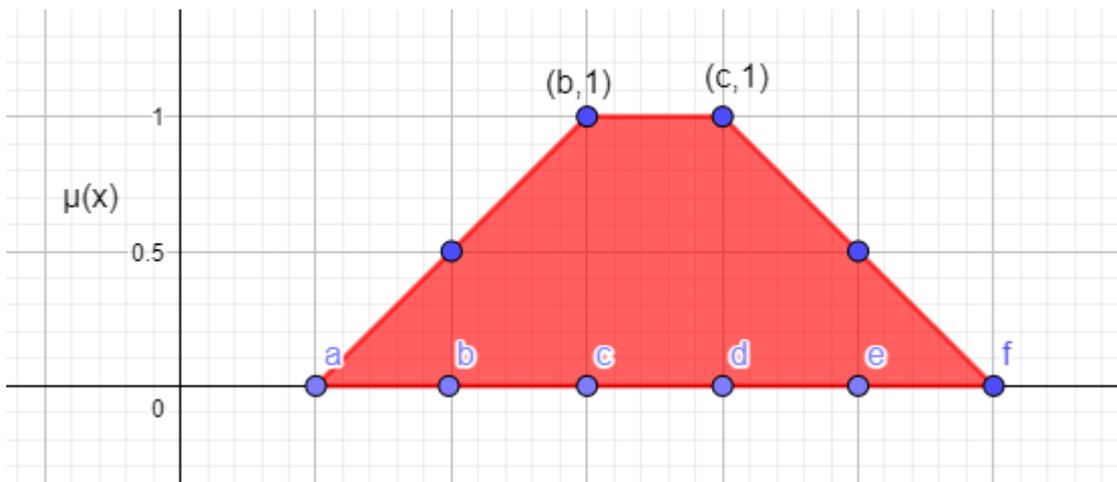
Figure 1.5: Hexagonal Fuzzy Number $\widetilde{A}(x)$

α - cuts of a hexagonal fuzzy number $\widetilde{A}(x) = (a, b, c, d, e, f)$ can be defined as the equation 1.10:

$$\widetilde{A}_\alpha = \{(b - a)\alpha + a, d - (d - c)\alpha\} + \{(d - c)\alpha + c, f - (f - e)\alpha\} \quad (1.10)$$

1.6.16   Robust's Ranking Method

Robust's ranking method satisfies compensation, linearity, and additivity properties and provides results consisting of human intuition. If $\widetilde{A}$ is a fuzzy number, then the robust ranking is defined by:

$$R(\widetilde{A}) = \int_0^1 0.5(A_\alpha^L, A_\alpha^U) d\alpha \quad (1.11)$$

Where $(A_\alpha^L, A_\alpha^U) = \widetilde{A}_\alpha$ is the α - level cut of the fuzzy number $\widetilde{A}$. The roubast ranking index $R(\widetilde{A})$ gives the representative value of the fuzzy number $\widetilde{A}$ [12].

CHAPTER 2

TRANSPORTATION PROBLEMS

2.1 General Mathematical Model of Transportation Problem

The transportation model is a particular type of LPP. The aim of the transportation model is to convey a homogenous item at a total minimum cost or in the shortest time feasible from numerous origins or factories to various destinations or marketplaces. To have a better understanding of the problem, assume that there are m sources $(S_1, S_2, ..., S_m)$ having units of supply $(a_i, i = 1, 2, ..., m)$ to be carried to n destinations $(D_1, D_2, ..., D_n)$ having units of demand $(b_j, j = 1, 2, ..., n)$. Assume that c_{ij} is the cost of delivering a single product unit from source i to destination j. The quantity of units delivered from (source i) to (destination j) denotes x_{ij} (x_{ij} are called decision variables), then the cost is $c_{ij} \ x_{ij}$. The important thing is obtaining the value of $x_{ij} \geq 0$ (x_{ij} are $m \times n$ decision variables in table of TP) which satisfies the constraints $m + n$. TP can be expressed as LPP, as shown below.

The objective function is,

$$Minimize \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} \tag{2.1}$$

Subject to the constraints,

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, ...m \tag{2.2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, ...n \tag{2.3}$$

Where

$$x_{ij} \geq 0, \quad \forall \quad i \quad and \quad j \tag{2.4}$$

For the accuracy of the solution and ease of presentation of the problem, the transportation problem data with all relevant parameters are generally represented, as shown in Table 2.1 [22, 25].

	D_1	D_2	$\cdots$	$\cdots$	D_n	Supply
S_1	c_{11} $\quad$ x_{11}	c_{12} $\quad$ x_{12}	$\cdots$ $\cdots$	$\cdots$ $\cdots$	c_{1n} $\quad$ x_{1n}	a_1
S_2	c_{21} $\quad$ x_{21}	c_{22} $\quad$ x_{22}	$\cdots$ $\cdots$	$\cdots$ $\cdots$	c_{2n} $\quad$ x_{2n}	a_2
S_3	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	a_3
$\cdot$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdot$
$\cdot$	c_{m1}	c_{m2}	$\cdots$	$\cdots$	c_{mn}	$\cdot$
S_m	x_{m1}	x_{m2}	$\cdots$	$\cdots$	x_{mn}	a_m
Demand	b_1	b_2	$\cdots$	$\cdots$	b_n	

Table 2.1: Schedule of TP

Remark

1. In TP, The total supply for all sources must be equal to the total demand for all destinations, i.e., $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. In this case, TP is called a balanced transportation problem (BTP)

2. The supply and demand must be equal in the last cell to be allocated to ensure that the solution is correct.

3. The cells with a positive allocation (i.e., $x_{ij} \geq 0$) in the transportation table are called occupied cells, otherwise are called unoccupied (or empty cells). The number of occupied squares in the transportation table is $m + n - 1$ and these are named (basic variables), the unoccupied squares are named (non-basic variables)[40, 46].

2.2 Network of Transportation Problem

The network in Figure 2.1 depicts the transportation model. Each node represents one of m sources and n destinations. The arrows indicate the paths that connect the sources and destinations. $Arc(i, j)$ connects source i to destination j containing two parts of information: product shipping cost per unit c_{ij} and product shipping quantity x_{ij}. S_i is the quantity of supply at source i, D_j is the quantity of demand at destination j. The aim of the network diagram is to find x_{ij} (i.e., the amount of quantity that will be transferred from sources to destinations) that will minimize overall transportation costs while fulfilling supply and demand constraints [1, 19].
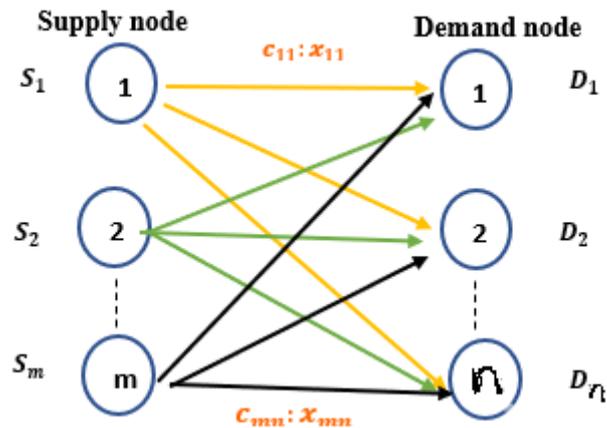
Figure 2.1: Network Diagram of TP

2.3 The Basic Assumptions of the Transportation Model

To solve any transportation model, the following assumptions must be made available in the model.

1. Each source and center of the demand has a specific and fixed energy.

2. Only one type of homogeneous goods is shipped from the origin to the destination.

3. The path between the original source and the destination must be direct. It is not permissible to transfer units from one source to another source and then re-transfer them to the requesting destination.

4. Assume that the quantities supplied in different sources are equal to the quantities required for the various demand destinations. However, there are some situations in which this assumption is not achieved. Despite that, the transportation model provides suitable solutions by

adding the quantity to the part in which the deficit (sources or demand centers).

5. Product shipping cost per unit from all origins to destinations is certain and accurate.

6. Product shipping cost per unit on a particular path is directly proportional to the number of units shipped.

7. The aim is to reduce the overall transportation cost for the entire organization and thus reduce the transportation cost for individual supply and distribution centers [34].

2.4   The Transportation Algorithm

The early transportation algorithm presented below was developed when hand computations were the norm. This interest in the development and coordination of the transportation algorithm has facilitated the modeling of several situations that do not directly deal with the transportation of goods. Today, robust computer codes can solve transportation models of any size [44].

The basic steps of the transportation algorithm are [40]:

Step 1: Create a transportation matrix. Enter the supply data a_i in the last column of the matrix, and enter the demand data b_j in the last row of the matrix; the rest cells of the matrix are for unit costs c_{ij}

Step 2: Find IBFS using one of the classic methods mentioned later, all supply and demand restrictions must be met.

Step 3: Check the optimality of the solution.

Step 4: If the solution is not optimal, repeat step 3 until OS is found.

2.5   Properties of IBFS of TP

An IBFS obtained through any of the traditional transportation problems methods must fulfill the conditions below:

1. The solution must satisfy all the (supply and demand) constraints, i.e., The solution is feasible (rim conditions).

2. The solution must satisfy the non-negative constraints.

3. The number of allocations must be exactly $(m+n-1)$, where m and n are the number of rows and columns, respectively [40].

2.6 Classic Methods for Obtaining IBFS of TP

To get OS of TP, we first start by finding IBFS, the solution that meets the supply and demand. IBFS can be obtained by using the following classic methods:

    1) North-West Corner Method (NWCM).

    2) Lest Cost Method (LCM).

    3) Vogel's Approximation Method (VAM).

The context of the work of the first method is "mechanical" in nature since its primary goal is to offer (basic feasible solutions) at any cost. The remaining two methods look for a higher quality starting solution (lower objective value). VAM outperforms NWCM, although NWCM requires the fewest calculations. Through the previous, we note that the quality of the total starting solution differs between the three ways, and the best starting solutions are those with a lower objective value [44].

2.6.1 North-West Corner Method (NWCM)

This method is the easiest of the classical methods to find an IBFS where the basic variables are chosen than the north-west corner (the start cell is (1,1)) in each step of the sub matrix consists of nonzero row and nonzero column requirements; for this reason, this method is called (the north-west corner method). The only motive for this method is to balance demand

and supply. Therefore, it is unlikely to provide a good (low-cost) initial solution because it ignores cost factors when making allocations. We list the solution steps for this method below [29, 46]:

Step 1: Construct a transportation table from the information of the problem. Ensuring that aggregate supply meets aggregate demand; if vice versa, we balance TP.

Step 2: Choose the north-west corner (top left corner) cell of the table.

Step 3: Assign the maximum possible units for this cell equivalent to the lower among ready supply and demand, i.e., x_{ij} =minimum (S_i, D_j). The transportation cost is wholly ignored during allocation (not considered).

Step 4: Cross out the column or row with no values (entirely depleted) for demand or supply.

Step 5: Now we have a new minimized transportation table, so select again the north-west corner cell and repeat step 3.

Step 6: Repeated steps 4 and 5 to depleted all demand and supply values (become zero).

Step 7: Find IBFS for TP by applying equation 2.1.

2.6.2   Least Cost Method (LCM)

The least cost method is also called (the matrix minimum method, row and column minima method). The steps of this method begin by making the allocation to the cell that has the lowest transportation cost per unit. As a result, this method provides a better beginning IBFS than NWCM because it considers the cost variables in the problem. We list the solution steps for this method below [29, 46]:

Step 1: Construct a transportation table from the information of the problem. Ensuring that aggregate supply meets aggregate demand; if vice versa, we balance TP.

Step 2: Identify the cell with the lowest transportation cost in the whole table.

Step 3: Assign the maximum possible units for this cell equivalent to the lower among ready supply and demand, i.e., $x_{ij} =$ minimum (S_i, D_j).

Step 4: Cross out the column or row without values (completely exhausted) for demand or supply. The crossed-out column (or row) should not be considered for additional assignments.

Step 5: Again, identify the least cost cell in the current table and repeat step 3. (If there are two or more minimum costs, choose the cells where the maximum allocations may be made).

Step 6: Continue the process until all supply and demand requirements are satisfied.

Step 7: Find IBFS for TP by applying equation 2.1.

2.6.3 Vogel's Approximation Method (VAM)

VAM is an iterative procedure for calculating IBFS to TP, which is better than both of the above methods. VAM was proposed by Reinfeld and Vogel (1958), they defined penalty as the difference between the smallest and next-smallest cost in each column and row of a transportation table.

VAM takes longer when compared to the other classical methods. Since this method provides a superior basic feasible solution for a startup. Generally, VAM produces an OS in 80 percent of the problems. We also call this method (Penalty method). We list the solution steps for this method below [2, 29, 44]:

Step 1: Construct a transportation table from the information of the problem. Ensuring that aggregate supply meets aggregate demand; if vice versa, we balance TP.

Step 2: Select the two lower costs available in each row of the transportation table. After that, find the difference between these two costs (named penalty cost), then insert it to the right of that row in a new column created by extending the table to the right. Likewise, select the two lower costs available in each column of the transportation table. After that, find the difference between these two costs

(called penalty cost), then insert it below that column in a new row created by extending the table to the bottom. The new row and column created by extending the table to the right and bottom are labeled (penalty row) and (penalty column), respectively.

Step 3: Amongst penalty column values and the penalty row values, we choose the largest value (largest difference). Then, assign the maximum possible units to the smallest cost cell in the selected column or row, i.e., $x_{ij} =$ minimum (S_i, D_j)

Step 4: In step 3, If equality happens in the largest penalties costs, choose the smallest cost cell in the column or row corresponding to the largest equal penalty. The selected cell is allocated the maximum possible units, and the corresponding exhausted row or column is removed (or ignored).

Step 5: Repeat steps 2 and 3 until all column and row totals are exhausted.

Step 6: Find IBFS for TP by applying equation 2.1.

2.7 The Classic Algorithms to Find OS for TP

After obtaining an initial solution, the following steps determine whether the solution is optimum which means there are no alternative transportation routes (positive allocations) in OS that can decrease the overall transportation cost. As a result, we must assess each non-base variable (unoccupied cell) in the transportation table for the possibility of lowering overall transportation costs. We choose the highest negative opportunity cost for an unoccupied cell and include it with the transport paths (positive assignments); we will call this cell the (incoming variable). At the same time, the outgoing variable represented the occupied cell (the base variable) in the closed path, whose assignment will be zero. Such an exchange (converting the non-basic variable to basic) lowers overall shipping costs. The process is repeated so that a negative opportunity cost does not remain. In other words, the existing solution cannot be improved anymore. This is the best choice; it is OS.

To check whether IBFS is OS or not we use the following classic methods:

1) Stepping Stone Method (SSM).

2) Modified Distribution Methods (MODI).

These methods also work to improve the solution till OS is achieved [46].

2.7.1 Stepping Stone Method (SSM)

In 1945, Cooper and Charnes proposed SSM. For each unoccupied cell in the SSM, a closed path is traced, then cell evaluations are found, and the cell with the largest negative evaluation becomes the basic cell. We list the solution steps for this method below [29, 5]:

Step 1: Finding IBFS using any of the previous classic methods. Then it must be ensured that the number of occupied cells fulfills $(m + n - 1)$, where m and n are the number of rows and columns in TP table, respectively.

Step 2: Choose (an unoccupied cell), and start at that cell by drawing a closed path with at least three occupied cell corners and back for the same unoccupied cell. Notice that each closed path should have an even number of cells, and keep in mind that there should be no diagonal movement in this path.

Step 3: Assign (+) and (-) marks alternately to each cell corner in the closed path you traced, beginning with the unoccupied cell you want to evaluate.

Step 4: Compute the (improvement index) by adding unit cost numbers in each cell with a plus mark (+) and deducting unit costs in every cell that includes a minus mark (-).

Step 5: Repeat steps 2, 3, and 4 for all unoccupied cells.

Step 6: When the improvement indices are negative, the provided

BFS is not OS, and transportation costs can be further reduced. Therefore, identify the closed path that contains the more negative value, choose the least number within the cells specified with a minus mark (-) , then assign this number to the selected unoccupied cell. This number is added to each cell marked with a plus mark (+) and subtracted from each cell marked with a minus mark (-) in the path.

In this method, the unoccupied cell becomes an occupied cell while maintaining the column and row requirements. That led to one basic cell becoming zero while the remaining cells stayed non-negative.

Step 7: Repeat step 6 until all improvement indicators to become positive or zero, this indicating OS has been achieved.

2.7.2 Modified Distribution Method (MODI)

MODI is a modified version of SSM (also called the u-v method or method of multipliers). We previously observed in SSM that there is a closed loop for each unoccupied cell, then we find cells evaluation, and the cell with the largest negative evaluation is the basic cell.

In MODI, we calculate the improvement index for all unoccupied cells simultaneously, after that, we find an evaluation only one for closed path, which represents the most negative cell. We list the solution steps for this method below [40, 37]:

Step 1: Find IBFS using any of the previous classic methods. Then it must be ensured that the number of occupied cells fulfills ($m + n - 1$), where m and n are the number of rows and columns in TP table,

respectively.

Step 2: Calculate the values of dual variables, V_j and U_i, corresponding to columns and rows respectively, first; assign zero to V_j or U_i where there is the largest number of assignments in the corresponding columns and rows, respectively, as this will decrease the computational work drastically. Then we calculate the remaining dual variables by using equation 2.5 for all occupied cells. Notice that if there are m sources and n destinations, there will be $m + n$ dual variables.

$$C_{ij} = U_i + V_j \tag{2.5}$$

Step 3: Calculate the opportunity cost using equation 2.6 for an unoccupied cell.

$$D_{ij} = C_{ij} + (U_i - V_j) \tag{2.6}$$

Step 4: Examine all marks of (opportunities costs) D_{ij}

(i) If $D_{ij} > 0$, then the specific BFS is OS.

(ii) If $D_{ij} = 0$, then the specific BFS is OS; there is an alternative solution.

(iii) If one or more $D_{ij} < 0$, then the specific BFS isn't OS, and there is another OS so that the transportation cost can be reduced further.

Step 5: Choose the unoccupied cell that represented the highest (negative opportunity cost) to include it in the next solution. If

there are many equal cells in the highest negative opportunity cost, any one of them can be selected.

Step 6: Start with the unoccupied cell specified in step 5, draw a closed path and differentiate this cell with a plus mark (+). Trace a path to the nearest occupied cell and differentiate this cell with a minus mark (−), and continue to another occupied cell with alternating marks. Close this path with the chosen unoccupied cell.

Step 7: Choose the least number within the cells specified with a minus mark (-), then assign this number to the selected unoccupied cell. This number is added to each cell marked with a plus mark(+) and subtracted from each cell marked with a minus mark (-) in the path. In this method, the unoccupied cell becomes an occupied cell while maintaining the column and row requirements.

Step 8: Repeat steps 2, 3, 4, 5, 6, and 7 until you've found OS (i.e., all $D_{ij} \geq 0$ for unoccupied cells).

We have previously discussed two different ways to obtain the optimal solution from a possible basic solution. And we noticed that both methods are similar in the basic idea, based on the steps we mentioned.

In SSM, we have to find the $C_{ij} - (U_i + V_j)$ values of all the non-basic variables by drawing a closed path.

In MODI, we first find the values of U_i and V_j and then we calculate $C_{ij} - (U_i + V_j)$.

Once the non-basic variables are defined, both methods will lead to the same set of operations, so both methods have the same number of

iterations. It should be noted that SSM is used for small problems (in other words, the number of non-basic variables is less), while MODI is used when the problem is large size.

The solution steps of MODI are shown in Fig.2.2 [42].
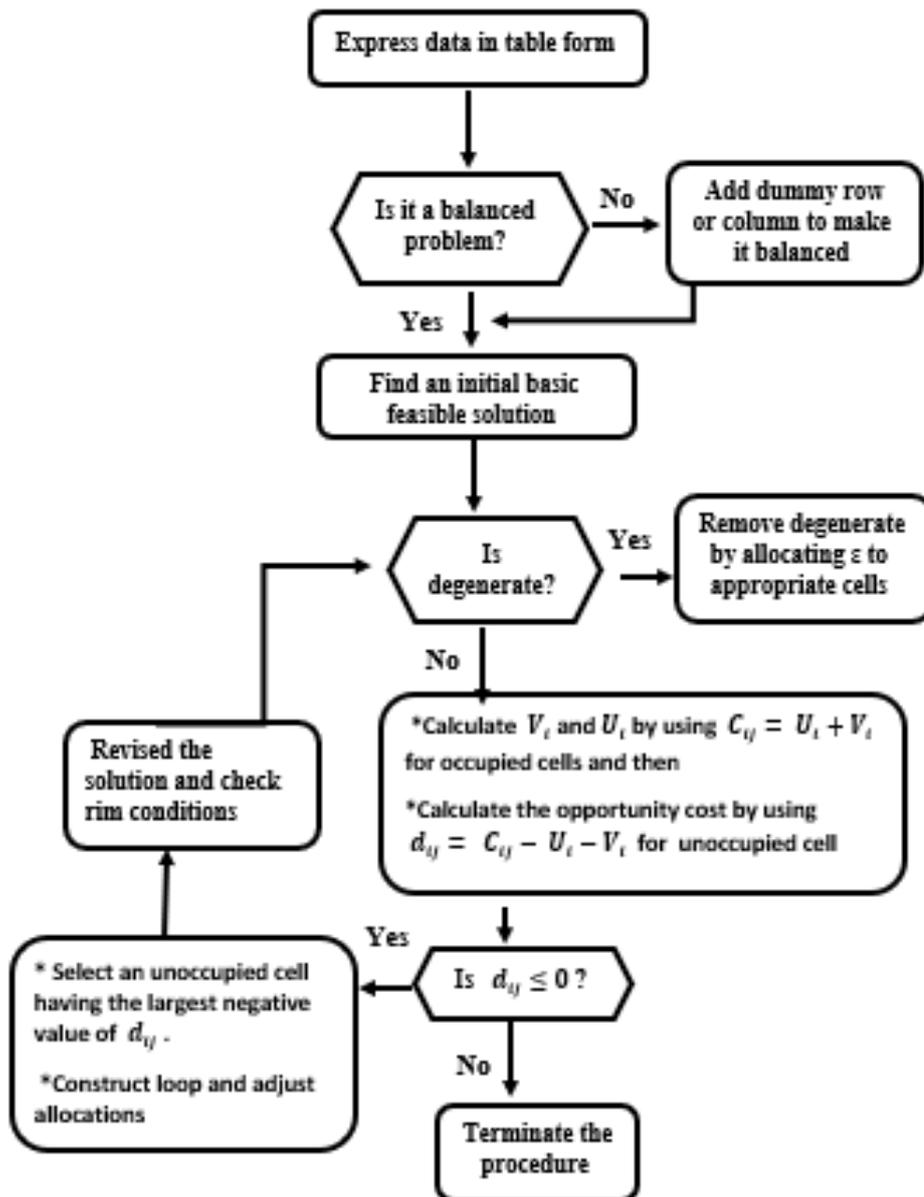
Figure 2.2: Chart of Solution Steps of MODI

2.8 Special Problems in Transportation Models

When we solve TP, we pass some special cases, which we list below with the means to address them.

2.8.1 Unbalance in Transportation Problem

This case occurs in TP when the total supply at all sources are not equal the demand at all destinations, i.e., $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$, therefore, TP is called an unbalanced transportation problem (UBTP).

Unbalanced problems can be addressed and solved easily and quickly by adding dummy destinations or dummy sources.

To clarify the idea more, if the capacity of factories (total supply) is bigger than the total needs of the sales areas or warehouses (total demand), i.e., $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$, therefore must be created a dummy warehouse with a capacity equal to the amount of increase in the factories' capacity (insert dummy destination to TP table).

For example, the total capacity of factories is 7000 units, and the needs for sales areas or warehouses are 6500 units. Then, a sales area or a fictitious store must be added with a capacity of 500 units, and the cost of transportation in the cells of that area or the fictional store is zero. Then the transportation table is made and resolved in one of the classic methods.

But if the total needs of the sales areas or warehouses (total demand) are greater than the total capacity of the factories (total supply), i.e.,

$\sum_{j=1}^{n} b_j > \sum_{i=1}^{m} a_i$, therefore, a dummy factory must be created with a capacity equal to the amount of increase in the warehouses' capacity (insert dummy source to TP table). In a real transportation process, it is a dummy factory that does not exist in reality. For example, the total capacity of factories is 8000 units, and the whole need for sales areas is 9,000 units. In this case, a dummy factory should be created with a capacity of 1000 units, and the cost of transporting the unit into the cells of that factory is zero [46].

2.8.2   The Degradation in Transportation Problem

We mentioned earlier that an IFBS to TP must contain exactly $(m+n-1)$ occupied cell (positive assignments) in the transportation table. When the number of occupied cells is fewer than $(m + n - 1)$, the solution is said to degenerate. The degeneracy problem does not give any dangerous difficulty, but it can give an arithmetic problem when specifying OS.

In SSM, it is not feasible to create loops (close paths) for each unoccupied cell, and therefore the evaluation of all unoccupied cells cannot be computed. When we use MODI, we will not be able to detect all the double variables U and V because the number of assigned cells and their associated C_{ij} values are insufficient.

As a result, it is very important to identify deteriorating TP and fix them to avoid lengthy computations[34].

Degeneracy in TP can happen in two stages [40]:

I) At IBFS, when the number of occupied cells is fewer than $(m +$

$n-1$). To fix this problem, select the unoccupied cell and assign it a very small quantity close to zero to reach $(m+n-1)$. This quantity is symbolized by the Greek letter ϵ (epsilon), which affects neither the total cost nor the supply and demand values.

Remark

- In TP of the miniaturization type, it is better to assign to the unoccupied cells with lower costs, whereas in TP of the maximizing type, it must be assigned to the cell with a large yield value.

- The specialization can be for more than unoccupied cell if necessary.

II) At any stage of finding the optimal solution by using the appropriate solution methods, two or more occupied cells simultaneously may turn into unoccupied cells.

2.9 Numerical Example

The following example is taken from [1].

The home appliance company has a sales team of 74 women working in three distribution centers. The company manager has a feeling that 10 saleswomen are needed to distribute production line A, 18 to distribute production line B, 22 to distribute production line C, and 24 to distribute product line D; the cost (in $) per month. Table 2.2 shows the appointment of saleswomen in each office.

Sources	A	B	C	D
1	50	60	100	50
2	80	40	70	50
3	90	70	30	50

Table 2.2: The Cost Per Day for Appointing Sellers from Each of the Offices

Currently, 20 saleswomen are available in position 1, 38 in position 2, and 16 in position 3. How many saleswomen must be assigned to each sales center to reduce costs?

Solution

Build the transportation table as in Table 2.3.

Sources	A	B	C	D	Supply
1	50	60	100	50	20
2	80	40	70	50	38
3	90	70	30	50	16
Demand	10	18	22	24	74

Table 2.3: TP Table

The mathematical model for the example is:

The objective function is,

$$MinZ = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij}$$

Subject to the constraints,

Supply Constraints	Demand Constraints
$x_{11} + x_{12} + x_{13} + x_{14} = 20$	$x_{11} + x_{21} + x_{31} = 10$
$x_{21} + x_{22} + x_{23} + x_{24} = 38$	$x_{12} + x_{22} + x_{32} = 18$
$x_{31} + x_{32} + x_{33} + x_{34} = 16$	$x_{13} + x_{23} + x_{33} = 22$
	$x_{14} + x_{24} + x_{34} = 24$

Where $x_{ij} \geq 0$, $\forall i = 1, 2, 3$ and $\forall j = 1, 2, 3, 4$

The given transportation table (in Table 2.3) is balanced since total supply = total demand = 74.

Table 2.4 represents the solution according to the algorithm of NWCM.

Sources	A	B	C	D	Supply
1	50 10	60 10	100	50	20
2	80	40 8	70 22	50 8	38
3	90	70	30	50 16	16
Demand	10	18	22	24	74

Table 2.4: Represents the Solution by NWCM

IBFS for TP is:

$$MinZ = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij}$$

$MinZ = (50 \times 10) + (60 \times 10) + (40 \times 8) + (70 \times 22) + (50 \times 8) + (16 \times 50) = 4160\$.$

Table 2.5 represents the solution according to the algorithm of LCM.

Sources	A	B	C	D	Supply
1	50	60	100	50 20	20
2	80 10	40 18	70 6	50 4	38
3	90	70	30 16	50	16
Demand	10	18	22	24	74

Table 2.5: Represents the Solution by LCM

IBFS for TP is:

$$MinZ = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}x_{ij}$$

$MinZ = (50 \times 10) + (60 \times 10) + (50 \times 10) + (40 \times 8) + (70 \times 22) + (8 \times 50) + (16 \times 50) = 3620\$.$

Table 2.6 represents the solution according to the algorithm of VAM.

Sources	A	B	C	D	Supply	Row Diff
1	50 10	60	100	50 10	20	0 0 10 10
2	80	40 18	70 6	50 14	38	10 10 10 10
3	90	70	30 16	50	16	20 0 0 0
Demand	10	18	22	24	74	
Column Diff	30 30 - - -	20 20 20 20 -	40 30 30	0 0 0 0 0		

Table 2.6: Represents the Solution by VAM

IBFS for TP is:

$$MinZ = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}x_{ij}$$

$MinZ = (50 \times 10) + (50 \times 10) + (40 \times 18) + (70 \times 6) + (50 \times 14) + (16 \times 30) = 3320\$.$

Now we are improving the initial solution to find OS by using MODI. Choose IBFS obtained by applying VAM (in Table 2.6) because it represents the least solution output.

IBFS obtained by VAM is (not degenerate) because it satisfies the equation ($m + n–1 = 6$).

We calculate the values of dual variables, V_j and U_i, corresponding to columns and rows by using equation 2.5

$$C_{22} = U_2 + V_2 \Rightarrow V_2 = 40 - 0 \Rightarrow V_2 = 40$$

$$C_{23} = U_2 + V_3 \Rightarrow V_3 = 70 - 0 \Rightarrow V_3 = 70$$

$$C_{24} = U_2 + V_4 \Rightarrow V_4 = 50 - 0 \Rightarrow V_4 = 50$$

$$C_{14} = U_1 + V_4 \Rightarrow U_1 = 50 - 50 \Rightarrow U_1 = 0$$

$$C_{33} = U_3 + V_3 \Rightarrow U_3 = 30 - 70 \Rightarrow U_3 = -40$$

$$C_{11} = U_1 + V_1 \Rightarrow V_4 = 50 - 0 \Rightarrow V_1 = 50$$

Sources	A		B		C		D		Supply	U_i
1		50		60		100		50	20	$U_1 = 0$
	10						10			
2		80		40		70		50	38	$U_2 = 0$
			18		6		14			
3		90		70		30		50	16	$U_3 = -40$
					16					
Demand	10		18		22		24		74	
V_j	$V_1 = 50$		$V_2 = 40$		$V_3 = 70$		$V_4 = 50$			

Table 2.7: Represents the Solution by MODI

We calculate the opportunity cost using equation 2.6 for unoccupied

cells.

$$C_{12} = 60 - 0 - 40 = +$$

$$C_{13} = 100 - 0 - 70 = +$$

$$C_{32} = 70 + 40 - 40 = +$$

$$C_{21} = 80 - 0 - 50 = +$$

$$C_{31} = 70 + 40 - 40 = +$$

Since all opportunity costs for unoccupied cells are nonnegative, the solution obtained using VAM is OS.

Also, we improved the initial solution to find OS by using SSM. Choose IBFS obtained by applying VAM (in Table 2.6) because it represents the least solution output.

IBFS obtained by VAM is (not degenerate) because it satisfies the equation $(m + n - 1 = 6$).

In Table 2.8 , the improvement index is calculated for the unoccupied cells that can have a closed path from occupied cells, with an indication of the status of the specified cell's ability to improve the solution or not.

The unoccupied Cell	The improvement index	Cell Status
C_{12}	$60 - 50 + 50 - 40 = 20$	It cannot be improved
C_{13}	$100 - 50 + 50 - 70 = 30$	It cannot be improved
C_{21}	$80 - 50 + 50 - 50 = 30$	It cannot be improved
C_{31}	$90 - 50 + 50 - 50 + 70 - 30 = 80$	It cannot be improved
C_{32}	$70 - 40 + 70 - 30 = 70$	It cannot be improved
C_{33}	$50 - 50 + 70 - 30 = 40$	It cannot be improved

Table 2.8: Unoccupied Cells Test Table

Since all the improvement index values in Table 2.8 to unoccupied cells are non negative, then the solution obtained using VAM is OS.

2.10   A Comparison of the Results

We place IBFS for TP indicated above in Table 2.9 to compare the performance of classical methods NWCM, LCM, and VAM; the comparison is based on the lowest cost; the optimal algorithm provides the lowest possible cost answer.

Also we mention OS results for the previous example.

Method	NWCM	LCM	VAM	SSM	MODI
Results	4160	3620	3320	3320	3320

Table 2.9: A Comparison of the Methods

As shown in Table 2.9, VAM provides the best IBFS compared to NWCM and LCM because it has the lowest cost.

This method produces a better answer most of the time, It represents OS or close to it.

CHAPTER 3

SOLVING TRANSPORTATION PROBLEMS BY USING MODIFICATION TO VOGEL'S APPROXIMATION METHOD

In this chapter, we will modify VAM to solve TP, and these modifications have been applied to many examples.

3.1 First New Technique to Find IBFS of TP

We can find IBFS for TP of the minimization type using the first new technique (ZT1) according to the following solution steps:

Step 1: Create a transportation table based on TP information (if not provided). Ensuring that aggregate supply meets aggregate demand; if vice versa, we balance TP.

Step 2: a) Select the least cost element in the first column of TP table and subtract that cost element from each element in that column. Repeat this process for all columns in the transportation problems table. As a result, each column in the new table will contain at least one zero. This new table is referred to as the (reduced column matrix table).

b) Select of the least cost element in the first row in the (reduced column matrix table). Subtract that cost element from each element in that row. Repeat this process for all row in the (reduced column matrix table). As a result, each row in the new table will contain at least one zero. This new table is referred to as the (reduced row matrix table).

Step 3: Locate the locations of the zero cells in the original transportation problem table depending on the location of the zero cells in the (reduced row matrix table).

Step 4: In the first row of the (original transportation problem table), allocate the uttermost possible units to the max zero cells in the chosen $x_{ij} =$minimum (S_i, D_j).

- If the zero cell is satisfied, the remainder of the task is added to a nonzero cell (with the lowest cost) in the same row.

- If the specified row does not contain zero cells, the assignment starts with the nonzero cell (with the lowest cost), i.e., minimum (c_{ij}).

Step 5: Repeat step 4 for all rows in the transportation problems table.

Step 6: Find IBFS for TP by applying equation 2.1.

3.2   Second New Technique to Find IBFS of TP

We can find IBFS for TP of the minimization type using the second new technique (ZT2) according to the following solution steps:

Step 1: Create a transportation table based on TP information (if not provided). Ensuring that aggregate supply meets the aggregate demand; if vice versa, we balance TP.

Step 2: Select the least cost element in the first column of TP table and subtract that cost element from each element in that column. Repeat this process for all columns in the transportation problems table. As a result, each column in the new table will contain at least one zero. This new table is referred to as the (reduced column matrix table).

Step 3: Locate the locations of the zero cells in the original transportation problem table depending on the location of the zero cells in the (reduced column matrix table).

Step 4: In the first row of the (original transportation problem table), allocate the uttermost possible units to the max zero cells in the chosen $x_{ij} =$ minimum (S_i, D_j).

- If the zero cell is satisfied, the remainder of the task is added to a nonzero cell (with the lowest cost) in the same row.

- If the specified row does not contain zero cells, the assignment starts with the nonzero cell (with the lowest cost), i.e., minimum (c_{ij}).

Step 5: Repeat step 4 for all rows in the transportation problems table.

Step 6: Find IBFS for TP by applying equation 2.1.

3.3   Numerical Examples

The numerical experiments are used to assess the efficiency of the new techniques ZT1 and ZT2. We compare them with the three classical methods NWCM, LCM, and VAM; we also mention the results of OS to these mentioned examples of TP.

Example 3.3.1. A factory has three machines with a production capacity of 120, 70, and 50 units (in 10 s) per day of a product. These units are to be shipped to four shops with the requirement of 60, 40, 30, and 110 units (in 10 s) per day. The costs of transportation (in dollars) per unit between factory to shops are given in Table 3.1:

Sources	A	B	C	D
M_1	20	22	17	4
M_2	24	37	9	7
M_3	32	37	20	15

Table 3.1: The Costs of TP

The problem is determining the daily amounts of product carried from each factory to each shop in order to lower total transportation costs [3].

Solution:

Build the transportation table as in Table 3.2

Sources	A	B	C	D	Supply
M_1	20	22	17	4	120
M_2	24	37	9	7	70
M_3	32	37	20	15	50
Demand	60	40	30	110	240

Table 3.2: TP Table to Example 3.3.1.

The mathematical model for TP is:

The objective function is,

$$MinZ = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij}$$

Subject to the constraints,

	Supply Constraints	Demand Constraints

$$x_{11} + x_{12} + x_{13} + x_{14} = 120 \qquad x_{11} + x_{21} + x_{31} = 60$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 70 \qquad x_{12} + x_{22} + x_{32} = 40$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 50 \qquad x_{13} + x_{23} + x_{33} = 30$$

$$x_{14} + x_{24} + x_{34} = 110$$

Where $x_{ij} \geq 0$, $\forall i = 1, 2, 3$ and $\forall j = 1, 2, 3, 4$

The given TP table (in Table 3.2) is balanced since total supply $=$ total demand $= 240$.

Build the reduced column matrix table and reduced row matrix table as in Table 3.3 and Table 3.4.

Sources	A	B	C	D
M_1	0	0	8	0
M_2	4	15	0	3
M_3	12	15	11	15

Table 3.3: Reduced Column Matrix

Sources	A	B	C	D
M_1	0	0	8	0
M_2	4	15	0	3
M_3	1	4	0	4

Table 3.4: Reduced Row Matrix

Table 3.5 represents the solution according to the technique ZT1.

Sources	A		B		C		D		Supply
		20		22		17		4	
M_1	60		40		×		20		120
		24		37		9		7	
M_2	×		×		30		40		70
		32		37		20		15	
M_3	×		×		×		50		50
Demand	60		40		30		110		240

Table 3.5: The Solution Table by Using ZT1

IBFS for TP is:

$$MinZ = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij}$$

$MinZ = (20 \times 60) + (22 \times 40) + (4 \times 20) + (9 \times 30) + (7 \times 40) + (15 \times 50) =$ 3469 units.

Now we use the technique ZT2 to solve TP. Build the reduced column matrix as in Table 3.6.

Sources	A	B	C	D
M_1	0	0	8	0
M_2	4	15	0	3
M_3	12	15	11	15

Table 3.6: Reduced Column Matrix

Table 3.7 represents the solution according to the technique ZT2.

Sources	A	B	C	D	Supply
M_1	20 60	22 40	17 $\times$	4 20	120
M_2	24 $\times$	37 $\times$	9 30	7 40	70
M_3	32 $\times$	37 $\times$	20 $\times$	15 50	50
Demand	60	40	30	110	240

Table 3.7: The Solution Table by Using ZT2

IBFS for TP is:

$$MinZ = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij}$$

$MinZ = (20 \times 60) + (22 \times 40) + (4 \times 20) + (9 \times 30) + (7 \times 40) + (15 \times 50) = 3469$ units.

Example 3.3.2. SunRay transport company ships truckloads of grain from four silos with a production capacity of 9, 8, 7, and 5 units (in 1000 s) per month of a product to four mills with the requirement of 5, 10, 5, and 9 units (in 100o s) per month. The costs of transportation (in dollars) per truckload are given in Table 3.8.

Sources	A	B	C	D
S_1	20	16	14	20
S_2	9	15	16	10
S_3	8	13	5	9
S_4	9	6	5	11

Table 3.8: The Costs of TP

The problem is determining the monthly amounts of grain shiped from silos to each mill in order to lower total transportation costs [44].

Solution:

Build the transportation table as in Table 3.9.

Sources	A	B	C	D	Supply
S_1	20	16	14	20	9
S_2	9	15	16	10	8
S_3	8	13	5	9	7
S_4	9	6	5	11	5
Demand	5	10	5	9	29

Table 3.9: TP Table to Example 3.3.2.

The mathematical model for TP is:

The objective function is,

$$MinZ = \sum_{i=1}^{4} \sum_{j=1}^{4} c_{ij} x_{ij}$$

Subject to the constraints,

Supply Constraints	**Demand Constraints**
$x_{11} + x_{12} + x_{13} + x_{14} = 9$	$x_{11} + x_{21} + x_{31} = 5$
$x_{21} + x_{22} + x_{23} + x_{24} = 8$	$x_{12} + x_{22} + x_{32} = 10$
$x_{31} + x_{32} + x_{33} + x_{34} = 7$	$x_{13} + x_{23} + x_{33} = 5$
$x_{31} + x_{32} + x_{33} + x_{34} = 5$	$x_{14} + x_{24} + x_{34} = 9$

Where $x_{ij} \geq 0$, $\forall i, j = 1, 2, 3, 4$

The given TP table (in Table 3.9) is balanced since total supply $=$ total demand $= 29$.

Build the reduced column matrix table and the reduced row matrix table as in Table 3.10 and Table 3.11.

Sources	A	B	C	D
S_1	12	10	9	11
S_2	1	9	11	1
S_3	0	7	0	0
S_4	1	0	0	2

Table 3.10: Reduced Column Matrix

Sources	A	B	C	D
S_1	3	1	0	2
S_2	0	8	10	0
S_3	0	7	0	0
S_4	1	0	0	2

Table 3.11: Reduced Row Matrix

Table 3.12 represents the solution according to the technique ZT1.

Sources	A		B		C		D		Supply
S_1	×	20	4	16	5	14	×	20	9
S_2	×	9	×	15	×	16	8	10	8
S_3	5	8	1	13	×	5	1	9	7
S_4	×	9	5	6	×	5	×	11	5
Demand	5		10		5		9		29

Table 3.12: The Solution Table by Using ZT1

IBFS for TP is:

$$MinZ = \sum_{i=1}^{4} \sum_{j=1}^{4} c_{ij}x_{ij}$$

$MinZ = (16{\times}4)+(14{\times}5)+(10{\times}8)+(8{\times}5)+(13{\times}1)+(9{\times}1)+(6{\times}5) =$ 306 units.

Now we use the technique ZT2 to solve TP. Build the reduced column matrix as in Table 3.13.

Sources	A	B	C	D
S_1	12	10	9	11
S_2	1	9	11	1
S_3	0	7	0	0
S_4	1	0	0	2

Table 3.13: Reduced Column Matrix

Table 3.14 represents the solution according to the technique ZT2.

Sources	A		B		C		D		Supply
S_1	× 20		4 16		5 **14**		× 20		9
S_2	× **9**		× 15		× 16		8 **10**		8
S_3	5 **8**		1 13		× **5**		1 **9**		7
S_4	× 9		5 **6**		× **5**		× 11		5
Demand	5		10		5		9		29

Table 3.14: The Solution Table by Using ZT2

IBFS for TP is:

$$MinZ = \sum_{i=1}^{4} \sum_{j=1}^{4} c_{ij}x_{ij}$$

$MinZ = (16 \times 4) + (14 \times 5) + (10 \times 8) + (8 \times 5) + (13 \times 1) + (9 \times 1) + (6 \times 5) = 306$ units.

3.4 A Comparison of the Results

We place IBFS for TP indicated above in Table 3.15 to compare the performance of the newly suggested techniques ZT1and ZT2 with the three classical methods NWCM, LCM, and VAM; the comparison is based on the lowest cost; the optimal algorithm provides the lowest possible cost answer.

Also, we mention OS results for the previous examples.

Name	NWCM	LCM	VAM	ZT1	ZT2	OS
Ex.3.3.1	3680	3670	3520	3460	3460	3460
Ex.3.3.2	392	312	308	306	306	300

Table 3.15: A Comparison of the Methods

As shown in Table 3.15, the novel approaches provides the best IBFS compared to the three classical methods because it has the lowest cost.

Our proposed techniques have been applied to many examples as shown in Fig 3.1. The results were always less than the results of VAM or equal to it. This improved strategy produces a better answer most of the time and is very close to OS (and even gives OS in many occasions).
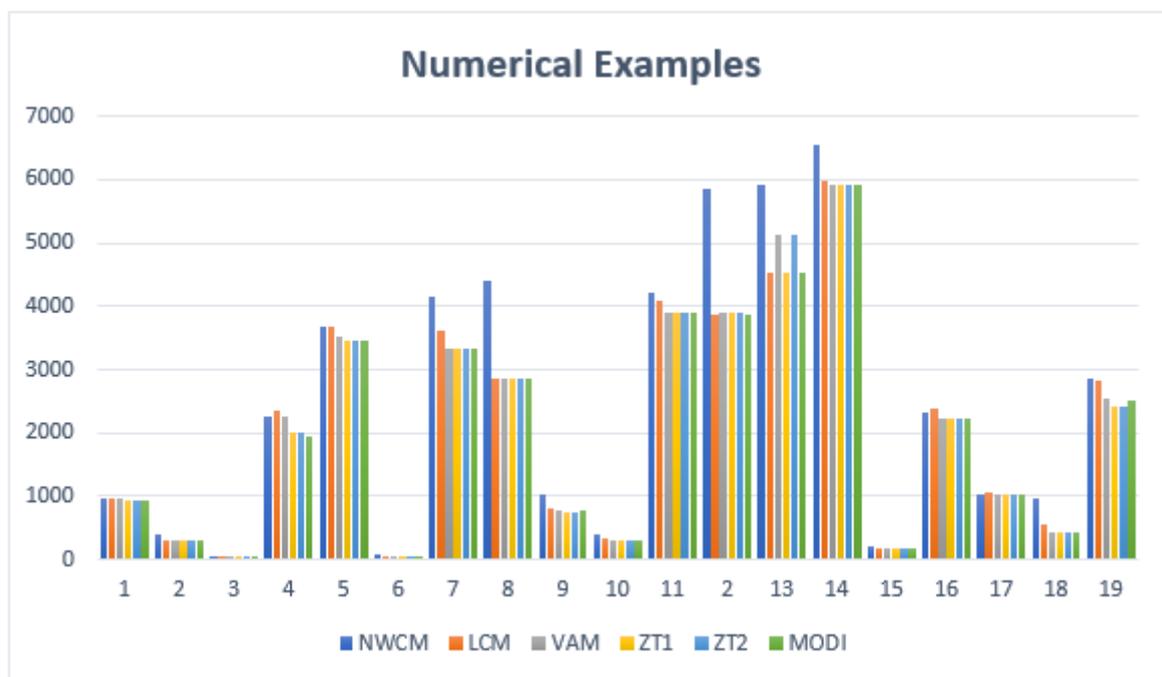
Figure 3.1: Chart Comparing Classic Methods with Suggested Techniques (ZT1, ZT2) for 19 Numerical Examples.

CHAPTER 4

A NEW LCM MODIFICATION FOR FINDING AN IBFS FOR FUZZY TRANSPORTATION PROBLEMS

In this chapter, we will modify LCM to solve FTP, and this modification has been applied to many examples.

4.1 Third New Technique to Find IBFS of FTP

We can find IBFS for FTP of the minimization type using the third new technique (ZT3) according to the following solution steps:

Step 1: Create a transportation table based on FTP information (if not provided). Ensuring that aggregate supply meets aggregate demand; If vice versa, we balance FTP.

Step 2: In the first row of the transportation table, choose the lowest cost cell minimum (c_{ij}).

Step 3: Allocate the uttermost possible units to this cell which is equivalent to the lower among ready offer and request requirements, i.e., for minimum(c_{ij}), x_{ij} =minimum (S_i, D_j). At least one of these requirements will then be met.

Step 4: Repeat steps 2,3 for all rows.

Step 5: The total cost of FTP can be calculated using equation 2.1.

4.2   Numerical Examples

Example 4.2.1. Consider FTP in Table 4.1, which represents triangular fuzzy numbers [12].

Sources	A	B	C	D	Supply
S_1	(71,75,78)	(66,70,74)	(80,85,88)	(78,80,85)	(70,80,90)
S_2	(82,86,90)	(76,82,86)	(92,96,100)	(84,90,92)	(130,150,170)
S_3	(98,102,106)	(86,90,94)	(132,136,140)	(114,120,126)	(210,230,250)
S_4	(96,100,102)	(92,98,100)	(110,115,120)	(100,112,118)	(140,170,200)
Demand	(90,100,110)	(180,200,220)	(160,180,200)	(120,150,180)	

Table 4.1: FTP Table

Solution:

The given FTP can be formulated in the following mathematical form.

$MinZ =$ R(71,75,78)x_{11} +R(66,70,74) x_{12}+ R(80,85,88)x_{13} + R(78,80,85) x_{14} +R(82,86,90)x_{21}+R(76,82,86) x_{22}+R(92,96,100) x_{23}+R(84,90,92) x_{24}+

R(98,102,106) x_{31}+R(86,90,94)x_{32}+R(132,136,140) x_{33}+R(114,120,126) x_{34}+

R(96,100,102)x_{41}+R(92,98,100)x_{42}+R(110,115,120)x_{43}+R(100,112,118)x_{44}

By using the robust ranking methodology to convert FTP to the crisp form as in Table 4.2.

$R(\widetilde{A}) = \int_0^1 0.5(A_\alpha^L, A_\alpha^U)d\alpha$, Where $(A_\alpha^L, A_\alpha^U) = \{a + (b - a)\alpha, c + (b - c)\alpha\}$

$R(71, 75, 78) = 74.5, R(66, 70, 74) = 70, R(80, 85, 88) = 84.5$

$R(78, 80, 85) = 81.5, R(82, 86, 90) = 86, R(76, 82, 86) = 81$

$R(92, 96, 100) = 96, R(84, 90, 92) = 88, R(98, 102, 106) = 102$

$R(86, 90, 94) = 90, R(132, 136, 140) = 136, R(114, 120, 126) = 120$

$R(96, 100, 102) = 99, R(92, 98, 100) = 96, R(110, 115, 120) = 115$

$R(100, 112, 118) = 111.5$

Rank of all supply

$R(70, 80, 90) = 80, R(130, 150, 170) = 150, R(210, 230, 250) = 230$

$R(140, 170, 200) = 170$

Rank of all Demands

$R(90, 100, 110) = 100, R(180, 200, 220) = 200, R(160, 180, 200) = 180$

$R(120, 150, 180) = 150$

Sources	A	B	C	D	Supply
S_1	74.5	70	84.5	81.5	80
S_2	86	81	96	88	150
S_3	102	90	136	120	230
S_4	99	96	115		170
Demand	100	200	180	150	630

Table 4.2: Crisp Form of FTP (Example 4.2.1.)

The given transportation table (in Table 4.2) is balanced since total supply = total demand = 630.

We will use ZT3 technique to obtain IBFS as in Table 4.3.

Sources	A	B	C	D	Supply
S_1	74.5 ✕	70 80	84.5 ✕	81.5 ✕	80
S_2	86 30	81 120	96 ✕	88 ✕	150
S_3	102 70	90 ✕	136 10	120 150	230
S_4	99 ✕	96 ✕	115 170	111.5 ✕	170
Demand	100	200	180	150	630

Table 4.3: Transportation Cost Using ZT3

IBFS of FTP is:

$MinZ = (80 \times 70) + (30 \times 86) + (120 \times 81) + (70 \times 102) + (10 \times 136) +$

$(150 \times 120) + (170 \times 115) = 63950$ units.

Example 4.2.2. Consider FTP in Table 4.4, which represents trapezoidal fuzzy numbers [7].

Sources	A_1	A_2	A_3	A_4	Supply
S_1	(1,2,3,4)	(1,2.67,4.33,6)	(4,6.67,9.33,12)	(5,7,9,11)	(1,4.67,8.33,12)
S_2	(0,1.33,2.64,4)	(1,2,3,4)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
S_3	(3,4.67,6.33,8)	(5,7.33,9.67,12)	(12,14.33,16.67,19)	(7,8.67,10.33,12)	(5,8.53,12.07,15.6)
Demand	(5,6.67,8.33,10)	(1,4,7,10)	(1,2.67,4.33,6)	(1,2,3,4)	

Table 4.4: FTP Table

Solution:

The given FTP can be formulated in the following mathematical form.

$MinZ = $ R(1,2,3,4)x_{11}+R(1,2.67,4.33,6) x_{12}+R(4,6.67,9.33,12) x_{13} +

R(5,7,9,11) x_{14} +R(0,1.33,2.64,4) x_{21}+R(1,2,3,4) x_{22}+R(5,6,7,8) x_{23} +

R(0,1,2,3) x_{24}+R(3,4.67,6.33,8) x_{31} +R(5,7.33,9.67,12) x_{32}+R(12,14.33, 16.67,19) x_{33} +R(7,8.67,10.33,12)x_{34}

By using the robust ranking methodology to convert FTP to the crisp form as in Table 4.5.

$R(\widetilde{A}) = \int_0^1 0.5(A_\alpha^L, A_\alpha^U)d\alpha$, Where $(A_\alpha^L, A_\alpha^U) = \{a + (b-a)\alpha, d + (c-d)\alpha\}$

$R(1,2,3,4) = 2.38, R(1,2.67,4.33,6) = 3.30, R(4,6.67,9.33,12) = 7.19,$

$R(5,7,9,11) = 7.78, R(0,1.33,2.64,4) = 2.00, R(1,2,3,4) = 2.38,$

$R(5,6,7,8) = 6.78, R(0,1,2,3) = 1.50, R(3,4.67,6.33,8) = 5.15,$

$R(5,7.33,9.67,12) = 8.4, R(12,14.33,16.67,19) = 16.37,$

$R(7,8.67,10.33,12) = 9.86$

Rank of all supply

$R(1, 4.67, 8.33, 12) = 6.25, R(0, 1, 2, 3) = 1.50, R(5, 8.53, 12.07, 15.6) = 9.10$

Rank of all Demands

$R(5, 6.67, 8.33, 10) = 7.50, R(1, 4, 7, 10) = 5.26, R(1, 2.67, 4.33, 6) = 3.30,$

$R(1, 2, 3, 4) = 2.38$

Sources	A_1	A_2	A_3	A_4	Supply
S_1	2.38	3.30	7.19	7.78	6.25
S_2	2.00	2.38	6.78	1.50	1.50
S_3	5.15	8.04	16.37	9.86	9.10
Demand	7.50	5.26	3.30	2.38	

Table 4.5: Crisp Form of FTP(Example 4.2.2.)

The given transportation table (in Table 4.5)is unbalanced. Now we balanced the problem as in Table 4.6.

Sources	A_1		A_2		A_3		A_4		Supply
S_1		2.38		3.30		7.19		7.78	6.25
S_2		2.00		2.38		6.78		1.50	1.50
S_3		5.15		8.04		16.37		9.86	9.10
S_4		0		0		0		0	1.59
Demand	7.50		5.26		3.30		2.38		18.44

Table 4.6: Balanced Transportation Table

We will use ZT3 technique to obtain IBFS as in Table 4.7.

Sources	A_1		A_2		A_3		A_4		Supply
S_1	6.25	2.38	×	3.30	×	7.19	×	7.78	6.25
S_2	×	2.00	×	2.38	×	6.78	1.50	1.50	1.50
S_3	1.25	5.15	5.26	8.04	2.59	16.37	×	9.86	9.10
S_4	×	0	×	0	0.71	0	0.88	0	1.59
Demand	7.50		5.26		3.30		2.38		18.44

Table 4.7: Transportation Cost Using ZT3

IBFS of FTP is:

$MinZ = (2.38 \times 6.25) + (1.50 \times 1.50) + (5.15 \times 1.25 + (8.04 \times 5.26) + (16.37 \times 2.59) + (0 \times 0.71) + (0 \times 0.88) = 108.2512$ units.

Example 4.2.3. Consider FTP in Table 4.8, which represents hexagonal fuzzy numbers [13].

Sources	A	B	C	Supply
S_1	(3,7,11,15,19,24)	(3,5,7,9,10,12)	(11,14,17,21,25,30)	(7,9,11,13,16,20)
S_2	(3,5,7,9,10,12)	(5,7,10,13,17,21)	(7,9,11,14,18,22)	(6,8,11,14,19,25)
S_3	(7,9,11,14,18,22)	(2,3,4,6,7,9)	(5,7,8,11,14,17)	(9,11,13,15,18,20)
Demand	(6,9,12,15,20,25)	(6,7,9,11,13,16)	(10,12,14,16,20,24)	

Table 4.8: FTP Table

Solution:

The given FTP can be formulated in the following mathematical form.

$MinZ = $ R(3,7,11,15,19,24) x_{11} +R(3,5,7,9,10,12) x_{12}+ R(11,14,17,21,25,30) x_{13} + R (3,5,7,9,10,12)x_{21}+R(5,7,10,13,17,21) x_{22}+R(7,9,11,14,18,22) x_{23}+ R(7,9,11,14,18,22) x_{31}+R (2,3,4,6,7,9) x_{32}+R(5,7,8,11,14,17) x_{33}

By using the robust ranking methodology to convert FTP to the crisp form as in Table 4.2.

$R(\widetilde{A}) = \int_0^1 0.5(A_\alpha^L, A_\alpha^U)d\alpha,$

Where $(A_\alpha^L, A_\alpha^U) = \{(b-a)\alpha + a, d - (d-c)\alpha\} + \{(d-c)\alpha + c, f - (f-e)\alpha\}$

$R(3, 7, 11, 15, 19, 24) = 54.16, R(3, 5, 7, 9, 10, 12) = 31.44,$

$R(11, 14, 17, 21, 25, 30) = 78.72, R(3, 5, 7, 9, 10, 12) = 31.44,$

$R(5, 7, 10, 13, 17, 21) = 49.23, R(7, 9, 11, 14, 18, 22) = 53.91,$

$R(7, 9, 11, 14, 18, 22) = 53.91, R(2, 3, 4, 6, 7, 9) = 20.91,$

$R(5, 7, 8, 11, 14, 17) = 41.31$

Rank of all supply

$R(7, 9, 11, 13, 16, 20) = 50.40, R(6, 8, 11, 14, 19, 25) = 55.20,$

$R(9, 11, 13, 15, 18, 20) = 57.42$

Rank of all Demands

$R(6, 9, 12, 15, 20, 25) = 58.33, R(6, 7, 9, 11, 13, 16) = 41.30,$

$R(10, 12, 14, 16, 20, 24) = 63.39$

Sources	A	B	C	Supply
S_1	54.16	31.44	78.72	50.40
S_2	31.44	49.23	53.91	55.20
S_3	53.91	20.91	41.31	57.42
Demand	58.33	41.30	63.39	163.02

Table 4.9: Crisp Form of FTP (Example 4.2.3.)

The given transportation table (in Table 4.10) is balanced since total supply = total demand = 163.02.

We will use ZT3 technique to obtain IBFS as in Table 4.3.

Sources	A		B		C		Supply
S_1	9.1	54.16	41.30	31.44	$\times$	78.72	50.40
S_2	49.23	31.44	$\times$	49.23	5.97	53.91	55.20
S_3	$\times$	53.91	$\times$	20.91	57.42	41.31	57.42
Demand	58.33		41.30		63.39		163.02

Table 4.10: Transportation Cost Using ZT3

IBFS of FTP is:

$$MinZ = (54.16 \times 9.1) + (31.44 \times 41.30) + (31.44 \times 49.23) + (53.91 \times 5.97) +$$

$$(41.31 \times 57.42) = 6032.9821 \text{ units.}$$

4.3   A Comparison of the Methods

We place IBFS for FTP indicated above in Table 4.11 to compare the performance of the newly suggested technique ZT3 with the classical method LCM; the comparison is based on the lowest cost; the optimal algorithm provides the lowest possible cost answer.

Name	LCM	ZT3
Ex.4.2.1	66.460	63.950
Ex.4.2.2	116.5724	108.2512
Ex.4.2.3	7155.6034	6032.9821

Table 4.11: A Comparison of the Methods

As shown in Table 4.11, the novel approach ZT3 provides the best IBFS compared to the classical method LCM because it has the lowest cost.

Our proposed techniques have been applied to many examples as shown in Fig 4.1. The results were always less than the results of LCM or equal to it. This improved strategy produces a better answer most of the time and is very close to OS.
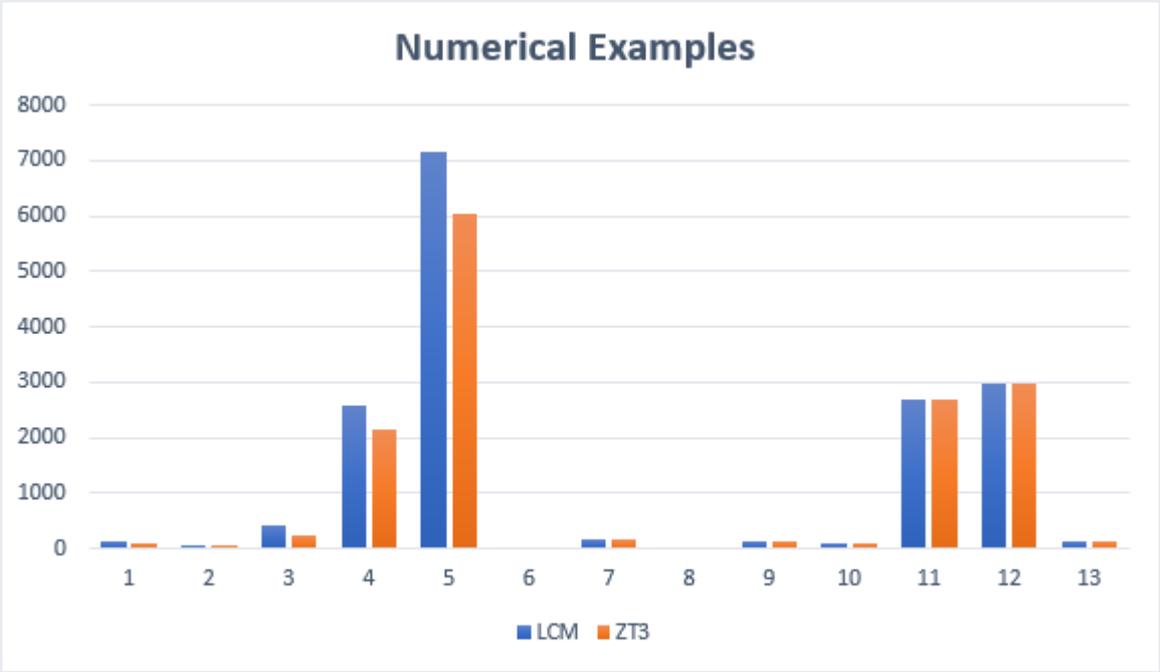
Figure 4.1: Chart Comparing LCM with the Suggested Technique (ZT3) for 13 Numerical Examples

CHAPTER 5

CONCLUSIONS AND FUTURE WORKS

5.1 Conclusions

In this thesis, we present new techniques to find the initial solution to balanced and unbalanced TP and balanced and unbalanced FTP using three new techniques that are highly efficient and gives the desired results. This research study consider both TP and FTP to be linear problem models.

We has made two modification to VAM and obtained two new techniques (the first new technique (ZT1) and the second new technique (ZT2)) to find the initial solution to different transportation problems. A comparison has made between the results of the solution to some examples using the two new techniques with the results of the solution by the standard classic methods to find the initial solution to TP. Results reached by using the new techniques (ZT1, ZT2) were:

- Better than the solution results by the known classic methods.

- Less than the results of VAM solution or equal to it.

- Represent OS or close to it.

Also, the researcher modified LCM and obtained a new technique (the third new technique (ZT3)to find the initial solution for FTP. A comparison has made between the results of the solution to some examples using the new technique with the results of the solution using LCM. Results reached by using the new techniques (ZT3) were:

- Less than the results of LCM solution or equal to it.

- Close to OS, and sometimes a few are equal to it.

- Easy to understand and apply, have fewer steps, and take less time to solve problems.

5.2 Suggestions for Future Works

Recently, many researchers have made modifications to the classic solution methods to get a better initial solution to TP and FTP.

The problem in this work depends mainly on reducing (calculating the minimum) both costs, and time (according to the problem data in the table) regarding balanced and unbalanced TP and balanced and unbalanced FTP.

There are many key points in TP and FTP that need to be investigated but cannot be within the scope of this research study because of the limited time frame. Through this thesis, some problems were identified as topics for future research and studies in this field. These include transshipment problems in TP, maximizing TP, and maximizing FTP to formulate a new method to solve these problems that can be considered in the future.

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الخلاصة

في هذه الرسالة، تم اقتراح تقنيتين حديثيتين لإيجاد الحل الأولي لمشاكل النقل الكلاسيكي وذلك بتعديل طريقة Vogel التقريبية، وهي واحدة من أشهر الطرق الكلاسيكية لإيجاد الحل الأولي لمشاكل النقل. كما تم تقديم تقنية جديدة لإيجاد الحل الأولي لمشاكل النقل الضبابي وذلك بتعديل طريقة الأقل تكلفة. تم اختبار كفاءة العمل لجميع هذه التقنيات الجديدة المقترحة في هذه الدراسة من خلال استخدامها لحل العديد من الأمثلة المختلفة، وتم الحصول على النتائج المرجوة، حيث تتميز هذه التقنيات بخطوات حل أسهل وأقل، وبالتالي تقليل التكلفة، الوقت والجهد بشكل كبير للحصول على الحل الأمثل للمشكلة.

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قسم الرياضيات

دراسة مقارنة بعض الطرق لحل مشاكل النقل

رسالة مقدمة الى

مجلس كلية التربية للعلوم الصرفة في جامعة بابل قسم الرياضيات

كجزء من متطلبات نيل درجة الماجستير في التربية / الرياضيات

من قبل

زينا صالح مهدي عبد النبي

بإشراف

أ.د. مشتاق عبدالغني شخير الجنابي

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