Republic of Iraq
Ministry of Higher Education
And Scientific Research
University of Babylon
College of Education for Pure Sciences
Department of Mathematics

Using numerical approximations to solve differential and integral equations

Research

Submitted to the Council of the Council of the Education ,for Pure Science in the University of Babylon in Fulfillment of the Requirement for the Degree of Higher Diploma Education / Mathematics

By

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Supervised by

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2022 A.D. 1444 A.H.

بسم الله الرحمن الرحيم

(يَرْفَعِ اللَّهُ الَّذِينَ آمَنُوا مِنكُمْ وَالَّذِينَ أُوتُوا الْعِلْمَ دَرَجَات وَاللَّهُ بِمَا تَعْمَلُون خَبِير)

صدق الله العظيم

سوره ألمجادله أيه (١١

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وزارة التعليم العالي والبحث العلمي
جامعة بابل
كلية التربية للعلوم الصرفة
قسم الرياضيات

استخدام التقريبات العددية لحل المعادلات التفاضلية والتكاملية

بحث مقدم
إلى مجلس كلية التربية للعلوم الصرفة جامعة بابل كجزء
من متطلبات نيل درجة الدبلوم العالي تربية/ الرياضيات

مقدم من قبل

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أ.م.د. احمد صباح الجيلاوي

2022 م   1444 هـ

إهــــــــداء

إلى من غمرتنا بحبها طول حياتها....

إلى من أعطتنا بلا حدود .....

إلى الوالدة العزيزة.... أطال الله عمرها في طاعته...

شكر وتقدير

ليس بعد تمام العمل من شئ أجمل ولا أحلى من الحمد لله والشكر له كما ينبغي لجلال وجهه وعظيم سلطانه وكما ينبغي لجزيل فضله وعظيم إحسانه على ماانعم علي من إتمام هذا البحث المتواضع.

ثم أتوجه بجزيل الشكر وعظيم الأمتنان الى كل من :

- الدكتور الفاضل احمد صباح الجيلاوي لتفضله بالأشراف على البحث وتكرمه بنصحي وتوجيهي حتى اتم هذا البحث.
- أعضاء لجنه المناقشة الكرام
- زوجتي الغالية على جهدها المبذول من طباعة وتنسيق للبحث.
- كل من ساندني ودعمني

بِسْمِ اللَّهِ الرَّحْمَٰنِ الرَّحِيمِ

(يَرْفَعِ اللَّهُ الَّذِينَ آمَنُوا مِنكُمْ وَالَّذِينَ أُوتُوا الْعِلْمَ
دَرَجَات وَاللَّهُ بِمَا تَعْمَلُون خَبِير)

صدق الله العظيم

سوره المجادلة آية (١١)

الخلاصة

يعد التحليل العددي وأجهزة الكمبيوتر من الأدوات المهمة في الرياضيات التطبيقية ، والمعادلات التفاضلية هي معادلات تتضمن دالة ومشتقات غير معروفة. ستكون هناك أوقات قد يكون فيها حل المعادلة غير متوفر أو قد تكون وسائل حلها غير متوفرة. في هذه الأوقات ومعظم الوقت ، سيتم استخدام طرق صريحة وضمنية بدلاً من الحلول الدقيقة. في الحالات الأبسط ، تعتبر المعادلات التفاضلية العادية أو المعادلات التفاضلية العادية وطريقة أويلر وطريقة أويلر العكسية طرقًا فعالة للحصول على تقديرات تقريبية دقيقة إلى حد ما للحلول الفعلية. من خلال التلاعب بهذه الأساليب ، يمكن للمرء أن يجد طرقًا لتقديم تقديرات تقريبية جيدة مقارنة بالحل الدقيق ، كما قدمنا تقريبًا جيدًا لطريقة نيوتن.

Dedication

To the one who showered us with her love all her life…

To those who gave us without limits…..

To dear mother… May God prolong her life in obedience to him…

ACKNOWLEDGMENT

After the completion of the work, there is nothing more beautiful or sweeter than praise and thanks be to God, as it should be considered as the majesty of his countenance, the greatness of his authority, and the best for his great bounty and great benevolence for what he has bestowed upon me to complete this humble research.

Then I would like to express my sincere thanks and gratitude to:

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• Honorable members of the discussion committee

• My dear wife, for her efforts in printing and coordinating the research.

• Everyone who supported and supported me

Abbreviations

Symbol	Meaning
(ODE)	Ordinary Differential Equation
(PDE)	Partial Differential Equation
IVP.	Initial Value Problem
NR	Newton Raphson
(WDN)	Water Distribution Network
i.e.	That is
w.r.t.	We write

ABSTRACT

Numerical analysis and computers are important tools in Applied mathematics, the differential equations are equations that involve an unknown function and derivatives. There will be times when solving the exact solution for the equation may be unavailable or the means to solve it will be unavailable. At these times and most of the time, explicit and implicit methods will be used in place of the exact solutions. In the simpler cases, ordinary differential equations(ODEs). Euler's method, and backward Euler's method are efficient methods to yield fairly accurate approximations of the actual solutions. By manipulating such methods, one can find ways to provide good approximations compared to the exact solution, Also we provided a good approximation with Newton's method.

CONTENTS

Mathematical Background (Basic Concept)

Numerical analysis deals with the process of getting the numerical solution to complex problems. The majority of mathematical problems in the science and engineering are difficult to answer precisely, and in some cases it is impossible. To make a tough mathematical problem easier to solve, an approximation is essential. Numerical approximation has become more popular as a result of tremendous advances in computational technology. As a result, a great deal of scientific software is being developed to solve more complex challenges quickly and easily. Let us go through the definition of numerical analysis as well as the various concepts included, such as errors and interpolation .

Chapter One

Mathematical Foundations

1.1 Introduction to Numerical Analysis

Numerical analysis is a discipline of mathematics concerned with the development of efficient methods for getting numerical solutions to complex mathematical problems. There are three sections to the numerical analysis. The first section of the subject deals with the creation of a problem-solving approach. The analysis of methods, which includes efficiency analysis, is covered in the second section. The efficiency analysis shows us how fast we can compute the result, while the error analysis informs us how correct the result will be if we utilize the approach. The construction of an efficient algorithm to implement the approach as a computer code is the subject's third part. All three elements must be familiar to have a thorough understanding of the numerical analysis.

Meanwhile, there are at least three reasons to learn the theoretical foundations of numerical methods:

1. Learning various numerical methods and analyzing them will familiarize a person with the process of inventing new numerical methods. When the existing approaches are insufficient or inefficient to handle a certain problem, this is critical.
2. In many cases, there are multiple solutions to a problem. As a result, using the right procedure is critical for getting a precise answer in less time.
3. With a solid foundation, one can effectively apply methods (especially when a technique has its owns-tractions and/or drawbacks in the certain instances) and, more significantly.

1.2 Differential Equations [2]

A linear differential equations is a differential equation that is defined by a linear polynomial in the unknown function and its derivatives, which is an equation of the form .

$$a_0(x)y + a_1(x)y' + a_2(x)y'' + \cdots + a_n(x)y^{(n)} + b(x) = 0$$

where $a_{0(x)}$, ..., $a_n(x)$and b(x)are arbitrary that differentiable functions do not need to be linear, and $y', \ldots, y^n$ are the successive derivatives of the unknown function y of the variable x.

Among ordinary differential equations, linear differential equations play a prominent role for several reasons. Most elementary and special functions that are encountered in physics and applied mathematics are solutions of linear differential equations. When physical phenomena are modeled with non-linear equations, they are generally approximated by linear differential equations for an easier solution. The few non-linear ODEs that can be solved explicitly are generally solved by transforming the equation into an equivalent linear ODE .

Some ODEs can be solved explicitly in terms of known functions and integrals. When that is not possible, the equation for computing the Taylor series of the solutions may be useful. For applied problems, numerical methods for ordinary differential equations can supply an approximation of the solution.

1.3 Linear Differential Equation [3]

The linear differential equations with one independent variable. For similar equations with two or more independent variables, see partial differential equation & Linear equations of second order .

Such an equation is an ordinary differential equation (ODE). A linear differential equation may also be a linear partial differential equation (PDE), if the unknown function depends on several variables, and the derivatives that appear in the equation are partial derivatives .A linear differential equation or a system of linear equations such that the associated homogeneous equations have constant coefficients may be solved by quadrature , which means that the solutions may be expressed in terms of integrals. This is also true for a linear equation of order one, with non-constant coefficients. An equation of order two or higher with non-constant coefficients cannot, in general, be solved by quadrature. The solutions of linear differential equations with polynomial coefficients are called homonymic functions. This class of functions is stable under sums, products, differentiation, integration, and contains many usual functions and special functions such as exponential functions, logarithm, sine, cosine, inverse trigonometric functions, error function, Bessel functions and hyper geometric functions .

Their representation by the defining differential equation and initial conditions allows making algorithmic most operations of calculus, such as computation of ant derivatives, limits, asymptotic expansion, and numerical evaluation to any precision , with a certified error bound.

Examples : 1) $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$

2) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = x^2 + x + 1$

3) $\boldsymbol{\frac{dy}{dx} + y\, secx = tanx}$

4) $\boldsymbol{x^2 dy + (x+y)dx = 0}$

5) $(e^y + 1)\, cosx\, dx + e^y \sin x\, dy = 0$

1.4 Nonlinear Differential Equations [4]

A system of differential equations is said to be nonlinear if it is not a system of linear equations. Problems involving nonlinear differential equations are extremely diverse, and methods of solution or analysis are problem dependent. Examples of nonlinear differential equations are the Navier–Stokes equations in fluid dynamics and the Lotka–Volterra equations in biology.

One of the greatest difficulties of nonlinear problems is that it is not generally possible to combine known solutions into new solutions. In linear problems, for example, a family of linearly independent solutions can be used to construct general solutions through the superposition principle. A good example of this is one-dimensional heat transport with Dirichlet boundary conditions, the solution of which can be written as a time-dependent linear combination of sinusoids of differing frequencies; this makes solutions very flexible. It is often possible to find several very specific solutions to nonlinear equations, however the lack of a superposition principle prevents the construction of new solutions.

Examples : 1) $y''' + 3yy'' + y = e^x$

2) $\frac{d^2y}{dx^2} + 2(\frac{dy}{dx})^2 + y^2 = e^x$

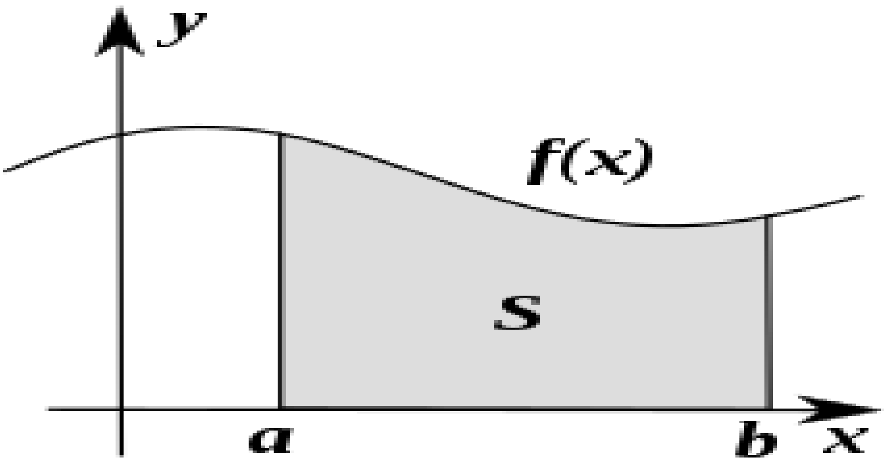
3) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y^2 = \sin x$

4) $(\frac{dx}{dt})^2 - 2t = 0$

5) $\sqrt{\frac{dy}{dx}} + 3y = sinx$

1.5 Numerical Integration [5]

In analysis, numerical integration comprises a broad family of algorithms for calculating the numerical value of a definite integral, and by extension, the term is also sometimes used to describe the numerical solution of differential equations.



Numerical integration is used to calculate a numerical approximation for the value S,

the area under the curve defined by $f(x)$.

The term numerical quadrature (often abbreviated to quadrature) is more or less a synonym for numerical integration, especially as applied to one-dimensional integrals. Some authors refer to numerical integration over more than one dimension as cubature; others take quadrature to include higher-dimensional integration.

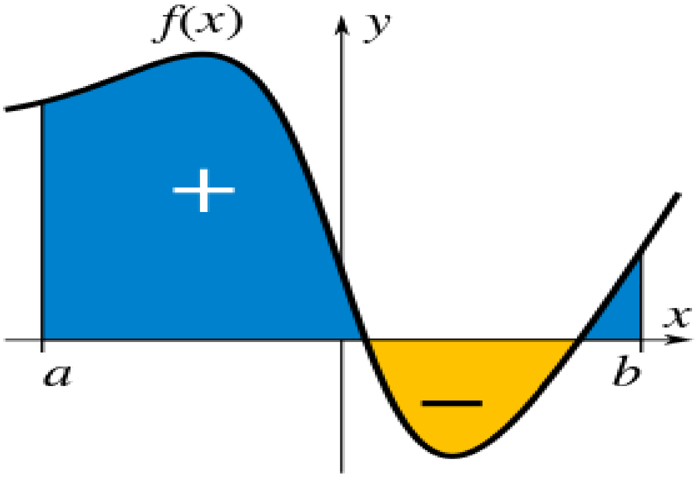
The basic problem in numerical integration is to compute an approximate solution to a definite integral

$$\int_a^b f(x)dx$$

to a given degree of accuracy. If $f(x)$ is a smooth function integrated over a small number of dimensions, and the domain of integration is bounded, there are many methods for approximating the integral to the desired precision.

1.5.1 Integral [5] [6]

In mathematics, an integral assigns numbers to functions in a way that describes displacement, area, volume, and other concepts that arise by combining infinitesimal data. The process of finding integrals is called integration. Along with differentiation, integration is a fundamental ,essential operation of calculus ,and serves as a tool to solve problems in mathematics and physics involving the area of an arbitrary shape, the length of a curve, and the volume of a solid .



A definite integral of a function can be represented as the signed area

of the region bounded by its graph.

The integrals enumerated here are those termed definite integrals, which can be interpreted as the signed area of the region in the plane that is bounded by the graph of a given function between two points in the real line. Conventionally, areas above the horizontal axis of the plane are positive while areas below are negative. Integrals also refer to the concept of an ant derivative, a function whose derivative is the given function. In this case, they are called indefinite integrals. The fundamental theorem of calculus relates definite integrals with differentiation and provides a method to compute the definite integral of a function when its ant derivative is known.[5]

Although methods of calculating areas and volumes dated from ancient Greek mathematics, the principles of integration were formulated independently by Isaac Newton and Gottfried Wilhelm Leibniz in the late 17th century, who thought of the area under a curve as an infinite sum of rectangles of infinitesimal width. Bernhard Riemann later gave a rigorous definition of integrals, which is based on a limiting procedure that approximates the area of a curvilinear region by breaking the region into thin vertical slabs.[6]

Integrals may be generalized depending on the type of the function as well as the domain over which the integration is performed. For example, a line integral is defined for functions of two or more variables, and the interval of integration is replaced by a curve connecting the two endpoints of the interval. In a surface integral, the curve is replaced by a piece of a surface in three-dimensional space.[5]

1.5.1.1 Trapezoid Method

In mathematics , the trapezoid rule is a numerical integration method , that is ,a method to calculate approximately the value of the definite integral.

$$I = \int_a^b f(x)\, dx\,.$$

$$T_n \approx \int_a^b y\, dx = \frac{\Delta x}{2}[\,(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \cdots + y_{n-1})]$$

$$= \frac{\Delta x}{2}\,[(\text{ Sum of the first and last ordinates })$$

$$+\; 2(\text{Sum of the remaining ordinates})]$$

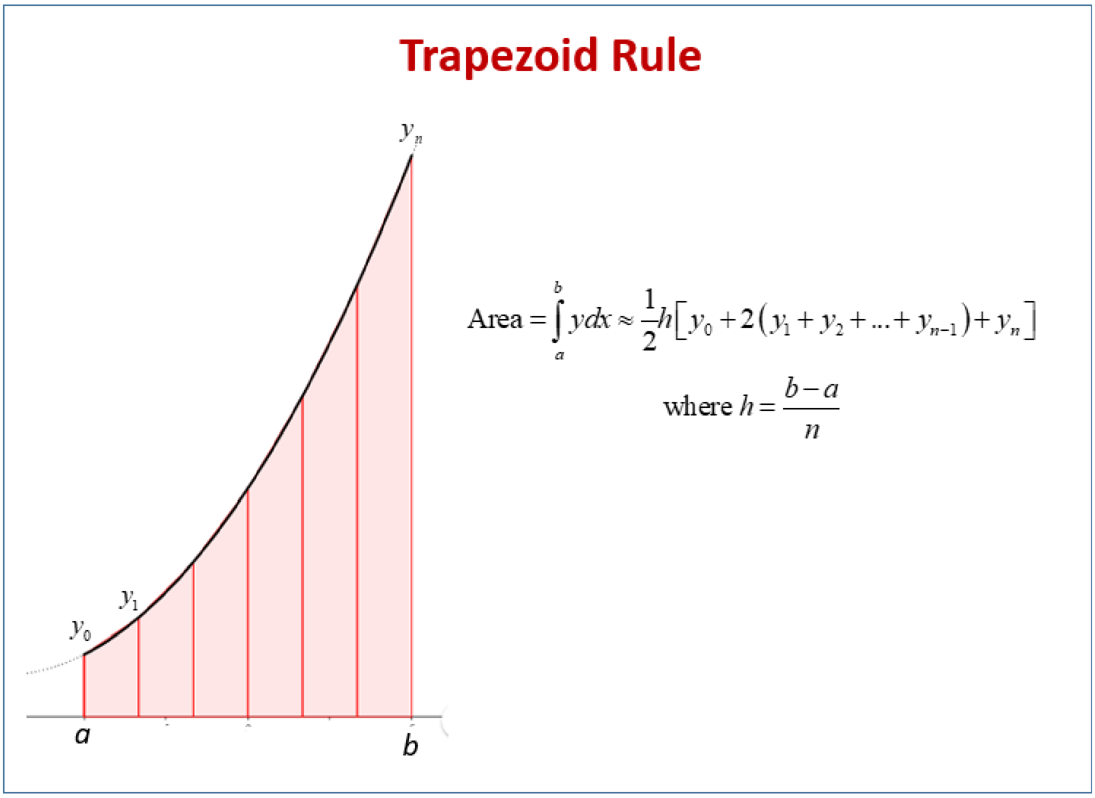
This is known as Trapezoid Rule . We have , $\Delta x = \frac{b-a}{n}$, and $x_i = a + i\Delta x$

$$I = I_0 + \cdots + I_{n-1}$$

where

$$I_K = \int_{x_k}^{x_{k+1}} f(x)dx$$

Where $x_i = a + i\Delta x$. We can add up approximations to the I_k



Example Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h = 0.2$. Hence obtain an approximate value of π . Can you use other formulae in this case .

Solution Let $y(x) = \frac{1}{1+x^2}$

Interval is $(1-0) = 1$ ∴ The value of y are calculated as points taking $h = 0.2$

x	0	0.2	0.4	0.6	0.8	1.0
$y = \frac{1}{1+x^2}$	1	0.96154	0.86207	0.73529	0.60976	0.50000

By Trapezoidal rule

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\,[(y_0 + y_n) + 2(\,y_1 + y_2 + \cdots + y_{n-1})]$$

$$= \frac{0.2}{2}\,[\,(1 + 0.5) + 2(0.96154 + 0.86207 + 0.73529 + 0.60976)]$$

$$= (0.1)[\,1.5 + 6.33732\,]$$

$$= 0.783732$$

By actual integration,

$$\int_0^1 \frac{dx}{1+x^2} = (\,\tan^{-1}x)_0^1 \;= \frac{\pi}{4}$$

$\therefore \quad \frac{\pi}{4} \approx 0.783732$

$\therefore \quad \pi \;\approx 3.13493$ (approximately) .

In this case, we cannot use Simpson′s rule

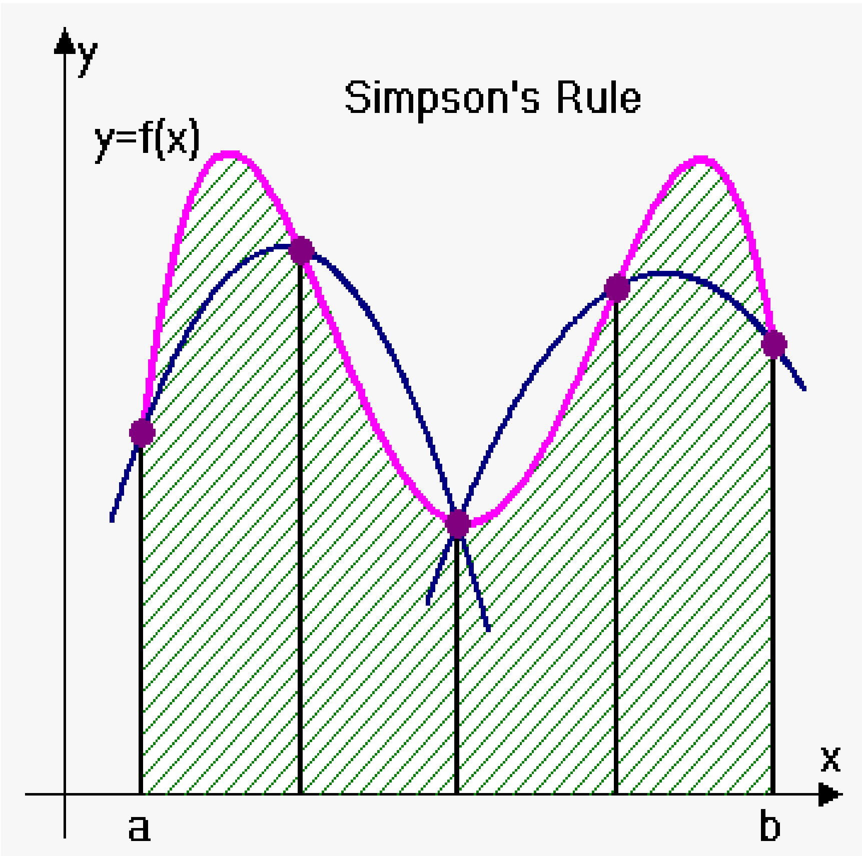
(since number of intervals is 5) .

1.5.1.2 Simpson's Rule

We can also approximate a definte integral

$$I = \int_a^b f(x)\,dx\,.$$

$$S_n \approx \int_a^b y\,dx = \frac{\Delta x}{3}[(y_0 + y_n) + 2\sum y_{even} + 4\sum y_{odd}]\,.$$



Example Evaluate $\int_0^1 e^{-x^2}\, dx$ using Simpson's one third rule.

Solution

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
e^{-x^2}	1	0.9900	0.9608	0.9139	0.8521	0.7788	0.6977	0.6126	0.5273	0.4449	0.3679
y	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

By Simpson's one third rule ,we have

$$\int_0^1 y dx = \frac{h}{3}[(y_0 + y_{10}) + 2(y_2 + y_4 + y_6 + y_8) + 4(y_1 + y_3 + y_5 + y_7 + y_9)]$$

$$= \frac{0.1}{3}[(1 + 0.3679) + 2(0.9608 + 0.8521 + 0.6977 + 0.5273)$$
$$+ 4(0.9900) + 0.9139 + 0.7788 + 0.6126 + 0.4449)]$$

$$= \frac{0.1}{3} \; [1.3679 + 2(3.0379) + 4(3.7420)]$$

$$= \frac{0.1}{3} \; [\, 1.3679 + 6.0758 + 14.9608\,]$$

$$= \frac{0.1}{3} \; \times 22.4045 \; = 0.7468$$

Which is the required value of $\int_0^1 \mathrm{e}^{-\mathrm{x}^2}\, dx$.

1.5.1.3 Midpoint Rule

If $\boldsymbol{f}$ is integrable on $[\boldsymbol{a}, \boldsymbol{b}]$, then

$$I = \int_a^b f(x)\, dx\,.$$

$$M_n \approx \int_a^b f(x)dx = \Delta x\, [(f(x_1^*) + f(x_2^*)) + f(x_3^*) + f(x_n^*)]$$

We have $\Delta x = \frac{b-a}{n}$ and $x_i^* = \frac{x_{i+1}+x_i}{2}$, the midpoint of $[x_{i+1}, x_i]$

Example Using the Midpoint Rule to evaluate $\int_0^1 x^2\, dx \quad n = 4$

Solution

We have $\Delta x = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

The intervals

$$\left[0, \frac{1}{4}\right] \; \left[\frac{1}{4}, \frac{1}{2}\right] \; \left[\frac{1}{2}, \frac{3}{4}\right] \; \left[\frac{3}{4}, 1\right]$$

The midpoints

$x_1^* = \frac{0+\frac{1}{4}}{2} = \frac{1}{8}$, the midpoint of $[\,0, \frac{1}{4}\,]$

Then the midpoint of $\boldsymbol{x_2^*} = \frac{\mathbf{3}}{\mathbf{8}}$ ...

$$\left\{\frac{\mathbf{1}}{\mathbf{8}}, \frac{\mathbf{3}}{\mathbf{8}}, \frac{\mathbf{5}}{\mathbf{8}}, \frac{\mathbf{7}}{\mathbf{8}}\right\}$$

$$I = \int_0^1 x^2\, dx$$

$$\approx \frac{1}{4}[f\left(\frac{1}{8}\right)+f\left(\frac{3}{8}\right)+f\left(\frac{5}{8}\right)+f\left(\frac{7}{8}\right)]$$

$$= \frac{1}{4}[\frac{1}{64}+\frac{9}{64}+\frac{25}{64}+\frac{49}{64}] = \frac{1}{4}\left(\frac{84}{64}\right) = \frac{21}{64}$$

Example By dividing the range into ten equal parts, evaluate $\int_0^{\pi} \boldsymbol{sinx\, dx}$ by Trapezoidal and Simpson's rule. Verify your answer with integration .

Solution Range $= \pi - 0 = \pi$,

Hence $h = \frac{\pi}{10}$

x	0	$\frac{\pi}{10}$	$\frac{2\pi}{10}$	$\frac{3\pi}{10}$	$\frac{4\pi}{10}$	$\frac{5\pi}{10}$	$\frac{6\pi}{10}$	$\frac{7\pi}{10}$	$\frac{8\pi}{10}$	$\frac{9\pi}{10}$	π
$\sin x$	0	0.3090	0.5878	0.8090	0.9511	1.0	0.9511	0.8090	0.5878	0.3090	0
y	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

$\left(\textit{Note that the values are symmetrical about } x = \frac{\pi}{2}\right)$

(i) Trapezoidal rule

$$I = \frac{h}{2}\,[(y_0 + y_{10}) + 2(y_1 + y_2 + \cdots + y_9)]$$

$$= \frac{\pi}{20}\,[\,(0+0) + 2(0.3090 + 0.5878 + 0.8090 + 0.9511 + 0.1 + 0.9511 + 0.8090 + 0.5878 + 0.3090)]$$

$= 1.9843$ nearly .

(ii) By Simpson$'s$ $\frac{1}{3}$ rule

$$I = \frac{h}{3}[(y_0 + y_{10}) + 2(y_2 + y_4 + y_6 + y_8) + 4(y_1 + y_3 + y_5 + y_7 + y_9)]$$

$$= \frac{\pi}{30}[(0 + 0) + 2(0.5878 + 0.9511 + 0.9511 + 0.5878)$$

$$+4(0.3090 + 0.8090 + 1.0 + 0.8090 + 0.3090)]$$

$$= 2.00011$$

(iii) By actual integration, $I = (-\boldsymbol{cosx})_{\mathbf{0}}^{\boldsymbol{\pi}} = \mathbf{2}$

Hence, Simpson′s rule is more accurate that the Trapezoidal rule .

1.6 Application of Numerical Differential Equations and Integral Equations [1] [5] [6]

1.6.1 Physics

Physics is the natural science that studies matter its fundamental constituents , its motion and behavior through space and time, and the related entities of energy and force. Physics is one of the most fundamental scientific disciplines, disciplines, with its main goal being to understand how the universe behaves.

FIGURE 1.1 Various examples of physical phenomena

Physics is one of the oldest academic disciplines and, through its inclusion of astronomy, perhaps the oldest. Over much of the past two millennia, physics, chemistry, biology, and certain branches of mathematics were a part of natural philosophy, but during the Scientific Revolution in the 17th century these natural sciences emerged as unique research endeavors in their own right. Physics intersects with many interdisciplinary areas of research, such as biophysics and quantum chemistry, and the boundaries of physics are not rigidly defined. New ideas in physics often explain the fundamental mechanisms studied by other sciences and suggest new avenues of research in these and other academic disciplines such as mathematics and philosophy.

Advances in physics often enable advances in new technologies. For example, advances in the understanding of electromagnetism, solid-state physics, and nuclear physics led directly to the development of new products that have dramatically transformed modern-day society, such as television, computers, domestic appliances, and nuclear weapons advances in thermodynamics led to the development of industrialization; and advances in mechanics inspired the development of calculus.

1.6.2 Chemistry

Chemistry is the scientific study of the properties and behavior of matter. It is a natural science that covers the elements that make up matter to the compounds composed of atoms, molecules and ions: their composition, structure, properties, behavior and the changes they undergo during a reaction with other substances.

FIGURE 1.2 An oil painting of a chemist

(Ana Kansky, painted by Henrika Šantel in 1932)

In the scope of its subject, chemistry occupies an intermediate position between physics and biology. It is sometimes called the central science because it provides a foundation for understanding both basic and applied scientific disciplines at a fundamental level. For example, chemistry explains aspects of plant growth (botany), the formation of igneous rocks (geology), how atmospheric ozone is formed and how environmental pollutants are degraded (ecology), the properties of the soil on the moon (cosmochemistry), how medications work (pharmacology), and how to collect DNA evidence at a crime scene (forensics).

Chemistry addresses topics such as how atoms and molecules interact via chemical bonds to form new chemical compounds. There are two types of chemical bonds : 1. primary chemical bonds-e.g., covalent bonds, in which atoms share one or more electron(s); ionic bonds, in which an atom donates one or more electrons to another atom to produce ions (cations and anions); metallic bonds and 2. secondary chemical bonds-e.g., hydrogen bonds; Van der Waals force bonds; ion-ion interaction; ion-dipole interaction.

1.6.3 Biology

Biology is the scientific study of life. It is a natural science with a broad scope but has several unifying themes that tie it together as a single, coherent field . For instance, all organisms are made up of cells that process hereditary information encoded in genes, which can be transmitted to future generations. Another major theme is evolution, which explains the unity and diversity of life. Energy processing is also important to life as it allows organisms to move, grow, and reproduce. Finally, all organisms are able to regulate their own internal environments.

FIGURE 1.3 Biology deals with the study of life.

top: *E. coli* bacteria and gazelle

bottom: Goliath beetle and tree fern

Biologists are able to study life at multiple levels of organization, from the molecular biology of a cell to the anatomy and physiology of plants and animals, and evolution of populations. Hence, there are multiple sub disciplines within biology, each defined by the nature of their research questions and the tools that they use. Like other scientists, biologists use the scientific method to make observations, pose questions, generate hypotheses, perform experiments, and form conclusions about the world around them.

Life on Earth, which emerged more than 3.7 billion years ago, is immensely diverse. Biologists have sought to study and classify the various forms of life, from prokaryotic organisms such as archaea and bacteria to eukaryotic organisms such as protists, fungi, plants, and animals. These various organisms contribute to the biodiversity of an ecosystem, where they play specialized roles in the cycling of nutrients and energy through their biophysical environment.

Chapter Two

A New techniques for solving Ordinary Differential Equations Approxmality

Numerical Solutions of Ordinary Differential Equations [1] [7]

Introduction

Differential equations represent a number of problems in the field of engineering and science .The analytical methods of solving differential equations are applicable to a limited type of differential equations .The numerical solutions of the differential equations have become easy for manipulation .These methods are of even greater importance, as the computing machines are now readily available.

In mathematics, an ordinary differential equation (ODE) is a differential equation containing one or more functions of one independent variable and the derivatives of those functions .The term ordinary is used in contrast with the term partial differential equation which may be with respect to more than one independent variable.

2.1 Method of Ordinary Differential Equations [7]

We will discuss the following methods to solve ordinary differential equations

1- Euler's method .

2- Euler's modified formula .

3- Backward Euler method

2.1.1 Euler's Method

This is purely numerical method for solving the first order differential equations .This is an elementary method . this method should not be used for practical solution. Consider the differential equation

$$\frac{dy}{dx} = f(x,y) \qquad \dots (1)$$

$$\text{Let } y = \emptyset(x) \text{ be the soution } of\ (1). \qquad \dots (2)$$

Let $(x_0, y_0), (x_1, y_1), \dots . (x_n, y_n), (x_{n+1}, y_{n+1})$ be the points on the curve of (2)

$x_0 . x_1 \dots \dots x_n . x_{n+1} \dots \dots$ are equispaced at equal interval h

$$y_{n+1} = \emptyset(x_{n+1}) \qquad [(x_{n+1}, y_{n+1}) \text{ lies on } (2)]$$

$$= \emptyset(x_n + h) \qquad (x_{n+1} = x_n + h)$$

$$= \emptyset(x_n) + h\ \emptyset'(x_n) + \frac{1}{2}h^2\emptyset''(x_n) + \cdots \qquad [\text{Taylor's series}] \dots . (3)$$

$$= \emptyset(x_n) + h\ \emptyset'(x_n) \qquad (h\ is\ very\ small)$$

$$= \emptyset(x_n) + h\, f(x_n, y_n) \qquad [\,since\ \frac{dy}{dx} = f(x,y)]$$

$$y_{n+1} = y_n + hf(x_n, y_n) \qquad [\,since\ y_n = \emptyset(x_n)\ from\ (2)] \dots \dots (4)$$

This formula (4)can be used to find y_{n+1},where y_n is known.

On substituting the value of $y_0, (n = 0) in\ (4)$,we get y_1.

similarly putting the value of $y_1, (n = 1) in (4)$,we obtain y_2 and so on.

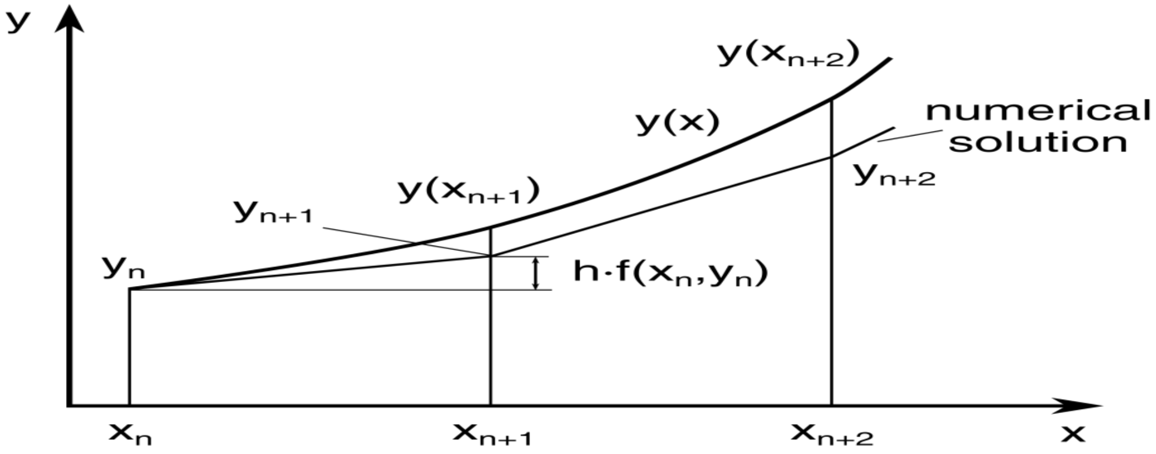


FIGURE 2.1 In Euler's method, you can approximate the curve of the solution by the tangent in each interval (that is, by a sequence of short line segments), at steps of h. ***In general***, if you use small step size, the accuracy of approximation increases.

Example 1 : Use Euler's method to solve $\frac{dy}{dx} = -1.2y + 7e^{-0.3x}$ from $x = 0$ to $x = 2$

with the initial condition $y = 3$ at $x = 0$ .Take $h = 0.5$

Solution :

n	x_n	y_n
0	0	3
1	0.5	4.7
2	1	4.8925
3	1.5	4.5499
4	2	4.0517

$y_{n+1} = y_n + h\,f(x_n, y_n)$

$n = 0 \rightarrow y_1 = y_0 + hf(x_0, y_0)$

$= 3 + 0.5[-1.2(3) + 7e^{-0.3(0)}]$

$= 4.7 = y(0.5)$

$n = 1 \rightarrow y_2 = y_1 + hf(x_1, y_1)$

$= 4.7 + 0.5[-1.2(4.7) + 7e^{-0.3(0.5)}] = 4.8925 = y(1)$

$n = 2 \rightarrow y_3 = y_2 + hf(x_2, y_2)$

$= 4.8925 + 0.5\ [-1.2(4.8925) + 7e^{-0.3(1)}] = 4.5499 = y(1.5)$

$n = 3 \rightarrow y_4 = y_3 + hf(x_3, y_3)$

$= 4.5499 + 0.5\ [-1.2(4.5499) + 7e^{-0.3(1.5)}] = 4.0517 = y(2).$

Example 2 : Use Euler's method to find $y(0.4)$ from the differential equation

$$\frac{dy}{dx} = xy \ , y(0) = 1, h = 0.1$$

Solution :

n	x_n	y_n
0	0	1
1	0.1	1
2	0.2	1.01
3	0.3	1.0302
4	0.4	1.061106

$y_{n+1} = y_n + h\, f(x_n, y_n)$

$n = 0 \rightarrow y_1 = y_0 + hf(x_0, y_0)$

$= 1 + 0.1\big((0)(1)\big) = 1 = y(0.1)$

$n = 1 \rightarrow y_2 = y_1 + hf(x_1, y_1)$

$= 1 + 0.1\big((0.1)(1)\big) = 1.01 = y(0.2)$

$n = 2 \rightarrow y_3 = y_2 + hf(x_2, y_2)$

$= 1.01 + 0.1\big((0.2)(1.01)\big) = 1.0302 = y(0.3)$

$n = 3 \rightarrow y_4 = y_3 + hf(x_3, y_3)$

$= 1.0302 + 0.1\big((0.3)(1.0302)\big) = 1.061106 = y(0.4)$

Example 3 : Use Euler's method to find $y(0.1)$ from the differential equation

$$\frac{dy}{dx} = \frac{y-x}{y+x} \ , \ y(0) = 1, h = 0.02$$

Solution :

n	x_n	y_n
0	0	1
1	0.02	1.02
2	0.04	1.0392
3	0.06	1.0577
4	0.08	1.0756
5	0.1	1.0928

$y_{n+1} = y_n + h\, f(x_n, y_n)$

$n = 0 \rightarrow y_1 = y_0 + hf(x_0, y_0)$

$$= 1 + 0.02\left(\frac{1-0}{1+0}\right) = 1.02 = y(0.02)$$

$n = 1 \rightarrow y_2 = y_1 + hf(x_1, y_1)$

$$= 1.02 + 0.02\left(\frac{1.02-0.02}{1.02+0.02}\right) = 1.0392 = y(0.04)$$

$n = 2 \rightarrow y_3 = y_2 + hf(x_2, y_2)$

$$= 1.0392 + 0.02\left(\frac{1.0392-0.04}{1.0392+0.04}\right) = 1.0577 = y(0.06)$$

$n = 3 \rightarrow y_4 = y_3 + hf(x_3, y_3)$

$$= 1.0577 + 0.02\left(\frac{1.0577-0.06}{1.0577+0.06}\right) = 1.0756 = y(0.08)$$

$n = 4 \rightarrow y_5 = y_4 + hf(x_4, y_4)$

$$= 1.0756 + 0.02\left(\frac{1.0756-0.08}{1.0756+0.08}\right) = 1.0928 = y(0.1)$$

2.1.1.1 Initial-Value and Boundary-Value Problems [8]

Boundary value problems and initial value problems are very similar to each other but there is a small difference between them. The condition in the boundary value problem is specified on the extremes of the independent variable. It means that the conditions are given by two points of the boundary curve whereas in the initial value problem there is the conditions which are specified for the same value of independent variables means only for one value the condition is there.

A boundary value is basically a solution to the differential equation where it corresponds to a parameters of the system which are studied. If there exist time t as independent variable for the domain[0,1], then the boundary value problem will consider both the initial and final domain condition such as there is specified $y(0)$ and $y(1)$. But for the initial value problem, the independent variable only has the condition for the same initial value that is 0. In this case, for the initial value problem, the initial condition is y(0) and $y'(0)$.

A differential equation with a set of constraints is called a boundary value problem and these constraints are called the boundary conditions. So, the solution of the boundary value problem is the solution of the differential equation which also satisfy the boundary conditions. The initial value problem is a differential equation with set of conditions and these conditions have to satisfy the initial value of independent variable and in case of partial differential equation the condition satisfied by the independent variables.

Initial Value Problem	Boundary Value Problem
$y(t=0)=y_0$ $y'(t=0)=y'_0$ **There is only one unique solutions.** **Note : Both y and y' are evaluated at the same time /point .**	$y(t=0)=y_0$ $y(t=c)=y_1$ **or** $y'(t=0)=y'_0$ $y'(t=c)=y'_1$ **There are many possible solutions.** **Note : There are evaluated at Different point of time .**

FIGURE 2.2 difference between an Initial- Value and Boundary-Value Problems

2.1.1.2 Applications of Initial- Value and Boundary-Value Problems [8]

Boundary value problem and initial value problem has several applications in various fields of science and engineering. It is the governing concept of the calculus. Differential equations are used in the many physical problems where the boundary value conditions and initial value conditions are used.

The wave theory has a major application of boundary condition. The normal modes which occur in the wave equations are often solved by the boundary conditions. Electric potential for a region is find out by the boundary conditions and sometimes by initial conditions in electrostatics. There is a region in which the observer wants to find the potential and if there is no charge present in the region, then the potential is the solution of Laplace's equation. If there is no charge in the region of the boundary conditions, then the interface conditions for electromagnetic fields is used to find the potential.

2.1.1.3 Advantages/Disadvantages of Euler's Methods [1] [7]

Advantages:

1 -Euler's method is simple and direct.

2-It can be used for nonlinear IVPs.

Disadvantages:

We notice in Euler that we make two kinds of mistakes

1- The error resulting from deleting terms containing powers of h of the second degree and above from the Tyler series.

2- The error caused by replacing $f(x_n, y_n)$ with $f(x_n, y(x_n))$

-It is less accurate and numerically unstable.

-Approximation error is proportional to the step size h.

Hence good approximation is obtained with a very small h. This requires a large number of time discretization leading to a large computation time.

2.1.2 Euler's Modified Formula

In equation (3) of Art 2.1.1 the expansion of $\boldsymbol{y_{n+1}}$ is

$$y_{n+1} = y_n + hf(x_n, y_n) + \frac{1}{2!}h^2\emptyset''(x_n, y_n) + \frac{1}{3!}h^3\emptyset'''(x_n, y_n) + \ldots\ldots.(1)$$

In Euler's formula we omit $\frac{1}{2}h^2\emptyset''(x_n, y_n)$ and higher powers of h .

The error due to this omission is called **Truncation error** .

Now a formula is derived with small error .

Differentiating (1) $w.r.t.$ $x,$ we get

$$(\frac{dy}{dx})_{n+1} = (\frac{dy}{dx})_n + hf'(x_n, y_n) + \frac{1}{2!}h^2\emptyset'''(x_n, y_n) + \frac{1}{3!}h^3\emptyset^{iv}(x_n, y_n) + \cdots$$

$$\therefore \quad f(x_{n+1}, y_{n+1}) = f(x_n, y_n) + hf'(x_n, y_n) + \frac{1}{2}h^2\emptyset'''(x_n, y_n) + \cdots \qquad \ldots(2)$$

$$= f(x_n, y_n) + h^2\emptyset''(x_n, y_n) + \frac{1}{2}h^2\emptyset'''(x_n, y_n) + \cdots$$

Multiplying (2) by $\frac{h}{2}$ and subtracting from (1),we get

$$y_{n+1} - \frac{1}{2}hf(x_{n+1}, y_{n+1}) = y_n + \frac{h}{2}f(x_n, y_n) - \frac{h^3}{12}\emptyset'''(x_n, y_n)$$

Neglecting terms containing h^3 and higher powers, we obtain

$$y_{n+1} = y_n + h\,[\frac{f(x_n, y_n) + f(x_{n+1}, y_{n+1})}{2}] \quad \ldots\ldots\ldots\ldots(3)$$

Equation (3) is the Euler's modified formula.

But $f(x_{n+1}, y_{n+1})$ which occurs on the right hand side of equation (3), cannot be calculated since $y_{n+1}$is unknown. So first we calculate y_{n+1} from Euler's first formula.

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Thus for each stage we use the following tow formulae.

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_{n+1})]$$

Example1: Use Euler's modified method to compute y for $\boldsymbol{x = 0.05}$ Given that

$\frac{dy}{dx} = \boldsymbol{x + y}$ with initial condition $\boldsymbol{x_0 = 0, y_0 = 1}$ result correct up to three decimal places.

Solution $\quad \frac{dy}{dx} = \boldsymbol{x + y} \quad \Longrightarrow \quad \boldsymbol{f(x, y) = x + y}$

$$x_0 = 0 \qquad y_0 = 1, \qquad h = 0.01$$

$$y_{n+1} = y_n + hf(x_n, y_n) \qquad \text{(Euler's formula)}$$

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_{n+1})] \qquad \text{(Euler's modified formula)}$$

Value of $\boldsymbol{y_{(0.01)}}$

$$y_{(0.01)} = y_0 + 0.01[x_0 + y_0]$$

$$= 1 + 0.01(0 + 1) = 1.01 \qquad \text{(Euler's formula)}$$

$$y_{(0.01)} = y_0 + \frac{0.01}{2}[(0 + 1) + (0.01 + 1.01)] \qquad \text{(Euler's modified formula)}$$

$$= 1+0.005(1+1.02) =1.0101$$

Value of $y_{(0.02)}$

$$y_{(0.02)} = y_{(0.01)} + 0.01[\ 0.01 + 1.0101]$$

$$=1.0101 + 0.01(1.0201) = 1.0203 \qquad \text{(Euler's formula)}$$

$$y_{(0.02)} = y_{(0.01)} + \frac{0.01}{2}[(0.01 + 1.0101) + (0.02 + 1.0203)\ ]$$

(Euler's modified formula)

$$= 1.0101+0.005(1.0201+1.0403) =1.0204$$

Value of $y_{(0.03)}$

$$y_{(0.03)} = y_{(0.02)} + 0.01[\ 0.02 + 1.0204]$$

$$=1.0204 + 0.01(1.0404) = 1.0308 \qquad \text{(Euler's formula)}$$

$$y_{(0.03)} = y_{0.02} + \frac{0.01}{2}[(0.02 + 1.0204) + (0.03 + 1.0308)\ ]$$

(Euler's modified formula)

$$= 1.0204 + 0.005(1.0404 + 1.0608) =1.0309$$

Value of $y_{(0.04)}$

$$y_{(0.04)} = y_{(0.03)} + 0.01[\ 0.03 + 1.0309]$$

$$=1.0309 + 0.01(1.0609) = 1.0415 \qquad \text{(Euler's formula)}$$

$$y_{(0.04)} = y_{(0.03)} + \frac{0.01}{2}[(0.03 + 1.0309) + (0.04 + 1.0415)]$$

(**Euler's modified formula**)

$= 1.0309 + 0.005(1.0609 + 1.0815) = 1.0416$

Value of $y_{(0.05)}$

$y_{(0.05)} = y_{(0.04)} + 0.01[0.04 + 1.0416]$ (**Euler's formula**)

$= 1.0416 + 0.01(1.0816) = 1.0524$

$$y_{(0.05)} = y_{(0.04)} + \frac{0.01}{2}[(0.04 + 1.0416) + (0.05 + 1.0524)]$$

(Euler's modified formula)

$= 1.0416 + 0.005(1.0816 + 1.1024)$

$= 1.0416 + 0.005(2.1840) = 1.0525$

Hence $y_{(0.05)} = 1.0525$

n	x_n	y_n
0	0	1
1	0.01	1.0101
2	0.02	1.0204
3	0.03	1.0309
4	0.04	1.0416
5	0.05	1.0525

Example 2: Solve the following by Euler's modified method the equation

$$\frac{dy}{dx} = log_{10}(x+y), y(0) = 2 \ at \ x = 1.2 \ and \ 1.4 \ with \ h = 0.2 \ .$$

Solution : $\frac{dy}{dx} = log_{10}(x+y) \quad \Longrightarrow \quad f(x,y) = log_{10}(x+y)$

with $x_0 = 0, \quad y_0 = 2, \quad h = 0.2$

Value of $y_{(0.2)}$

$$y_{n+1} = y_n + hf(x_n + y_n) \qquad \text{(Euler's fo rmula)}$$

$$\mathrm{y}_{(0.2)} = \mathrm{y}_{(0)} + 0.2 \ log_{10}(x_0 + y_0)$$

$$= 2 + 0.2 \ log_{10}2 = 2 + 0.2(0.3010)$$

$$= 2.0602$$

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n + y_n) + f(x_{n+1}, y_{n+1}) \ ]\text{(Euler's modified formula)}$$

$$= 2 + 0.1 \ [\log(x_n + y_n) + \log(x_{n+1} + y_{n+1})]$$

$$\mathrm{y}_{(0.02)} = 2 + 0.1 \ [\log(0+2) + \log(0.2 + 2.0602)]$$

$$= 2 + 0.1 \ [\log 2 + \log 2.2602]$$

$$= 2 + 0.1[0.3010 + 0.3541]$$

$$= 2 + 0.1(0.6551) = 2.0655$$

Value of $y_{(0.4)}$

$$y_{(0.4)} = y_{(0.2)} + 0.2\, log(0.2 + 2.0655)$$ (Euler's formula)

$$= 2.0655 + 0.2\,(0.3552) = 2.1365$$

$$y_{(0.4)} = y_{(0.2)} + 0.1\ \ [\log(0.2 + 2.0655) + \log(0.4 + 2.1365)]$$

(Euler's modified formula)

$$= 2.0655 + 0.1[0.3552 + 0.4042]$$

$$= 2.0655 + 0.0759\ = 2.1414$$

Value of $y_{(0.6)}$

$$y_{(0.6)} = y_{(0.4)} + 0.2\, log(0.4 + 2.1414)$$ (Euler's formula)

$$= 2.1414 + 0.2\,(0.4051) = 2.2224$$

$$y_{(0.6)} = y_{(0.4)} + 0.1\ \ [\log(0.4 + 2.1414) + \log(0.6 + 2.2224)]$$

(Euler's modified formula)

$$= 2.1414 + 0.1(0.4051 + 0.4506) = 2.2270$$

Value of $y_{(0.8)}$

$$y_{(0.8)} = y_{(0.6)} + 0.2\, log(0.6 + 2.2270)$$ (Euler's formula)

$$= 2.2270 + 0.2\,(0.4513) = 2.3173$$

$$y_{(0.8)} = y_{0.6} + 0.1\ \ [\log(0.6 + 2.2270) + \log(0.8 + 2.3173)]$$

(Euler's modified formula)

$$= 2.2270 + 0.1(0.4513 + 0.4938) = 2.3215$$

Value of $y_{(1)}$

$$y_{(1)} = y_{(0.8)} + 0.2\ log(0.8 + 2.3215)$$ (Euler's formula)

$$= 2.3215 + 0.2\ (0.4944) = 2.4204$$

$$y_{(1)} = y_{(0.8)} + 0.1\ [\log(0.8 + 2.3215) + \log(1 + 2.4204)]$$

(Euler's modified formula)

$$= 2.3215 + 0.1(0.4944 + 0.5341) = 2.4244$$

Value of $y_{(1.2)}$

$$y_{(1.2)} = y_{(1)} + 0.2\ log(1 + 2.4244)$$ (Euler's formula)

$$= 2.4244 + 0.2\ (0.5346) = 2.5313$$

$$y_{(1.2)} = y_1 + 0.1\ [\log(1 + 2.4244) + \log(1.2 + 2.5313)]$$

(Euler's modified formula)

$$= 2.4244 + 0.1(0.5346 + 0.5719) = 2.5351$$

Value of $y_{(1.4)}$

$$y_{(1.4)} = y_{(1.2)} + 0.2\ log(1.2 + 2.5351)$$ (Euler's formula)

$$= 2.5351 + 0.2\ (0.5723) = 2.6496$$

$$y_{(1.4)} = y_{(1.2)} + 0.1\ [\log(1.2 + 2.5351) + \log(1.4 + 2.6496)]$$

(Euler's modified formula)

$$= 2.5351 + 0.1(0.5723 + 0.6074) = 2.6531$$

Hence

$y_{(1.2)} = 2.5351$ and $y_{(1.4)} = 2.6531$

n	x_n	y_n
0	0	2
1	0.2	2.0655
2	0.4	2.1414
3	0.6	2.2270
4	0.8	2.3215
5	1	2.4244
6	1.2	2.5351
7	1.4	2.6531

Example 3 : Solve the following by Euler's modified method the equation

$$y' = 1 - y \quad , y(0) = 0 \ at \ x = 0.1 , 0.2 \ and \ 0.3 \ with \ h = 0.1 .$$

Solution : $y' = 1 - y \quad \Longrightarrow \quad f(x, y) = 1 - y$

$x_0 = 0, \qquad y_0 = 0, \qquad h = 0.1$

Value of $y_{(0.1)}$

$$y_{(0.1)} = y_0 + 0.1(1 - y_0) \qquad \text{(Euler's formula)}$$

$$= 0 + 0.1\,(1 - 0) = 0.1$$

$$y_{(0.1)} = y_0 + \frac{0.1}{2}[(1 - 0) + (1 - 0.1)] \qquad \text{(Euler's modified formula)}$$

$$= 0 + 0.05(1 + 0.9) = 0.095$$

Value of $y_{(0.2)}$

$$y_{(0.2)} = y_{(0.1)} + 0.1(1 - 0.095) \qquad \text{(Euler's formula)}$$

$$= 0.095 + 0.1\,(0.905) = 0.1855$$

$$y_{(0.2)} = y_{(0.1)} + \frac{0.1}{2}[(1 - 0.095) + (1 - 0.1855)] \quad \text{(Euler's modified formula)}$$

$$= 0.095 + 0.05(0.905 + 0.8145) = 0.18098$$

Value of $y_{(0.3)}$

$$y_{(0.3)} = y_{(0.2)} + 0.1(1 - 0.18098) \qquad \text{(Euler's formula)}$$

$$= 0.18098 + 0.1\,(0.81902) = 0.26288$$

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n + y_n) + f(x_{n+1}, y_{n+1})]$$

$$y_{(0.3)} = y_{(0.2)} + \frac{0.1}{2}[(1 - 0.18098) + (1 - 0.26288)] \quad \text{(Euler's modified formula)}$$

$$= 0.18098 + 0.05(0.81902 + 0.73712) = 0.25879$$

n	x_n	y_n
0	0	0
1	0.1	0.095
2	0.2	0.18098
3	0.3	0.25858

2.1.2.1 Advantages/ Disadvantages of Heun's Method [1] [7]

Advantages:

-It is accurate for numerical problems.

-It requires less error then Euler's method . Hence good approximation is obtained with a hence good approximation is obtained with a very small h.

Disadvantages:

-It is not used for more than third order.

2.1.2.2 Application of Heun's Method [2]

Heun's Method considers the tangent lines to the solution curve at both ends of the interval, one which overestimates, and one which underestimates the ideal vertical coordinates. A prediction line must be constructed based on the right end point tangent's slope alone, approximated using Euler's Method.

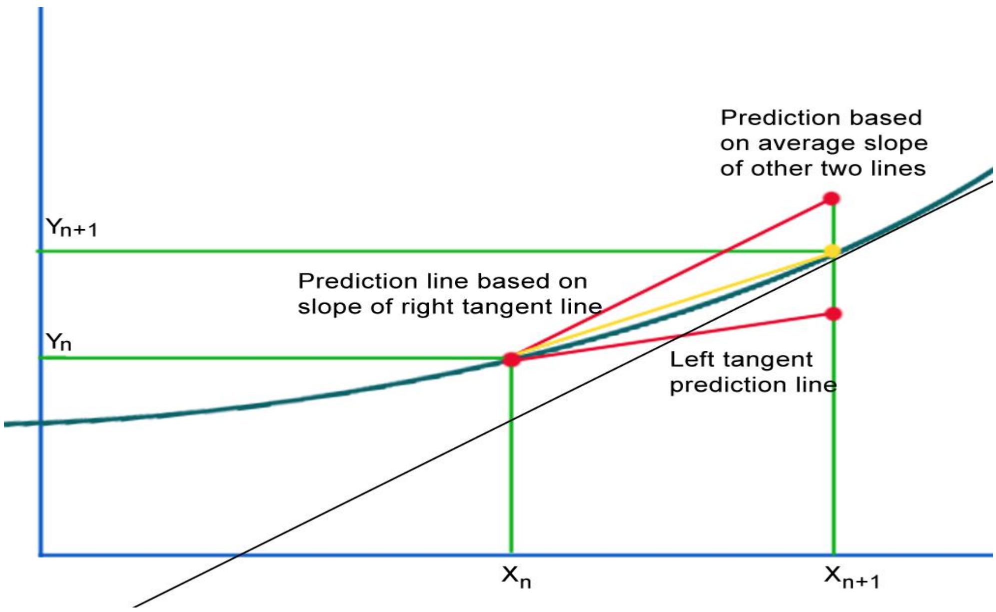


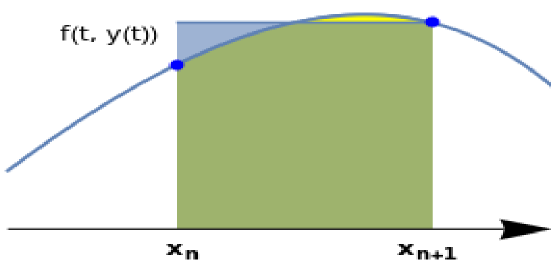
FIGURE **2.3** Heun's Method

where $y_0 = y(a)$, which is given. Note that $y(x_n)$ are unknown for n = 1, 2, … , N. s usual, we denote by y_n the approximate values of $y(x_n)$ at mesh points. Using the simple right rectangular rule for the approximation of the integral (that is, when the integrand is evaluated at the right end point) would lead to

$$\int_{x_{n-1}}^{x_n} f\left(t, y(t)\right) dt \approx f\left(x_n, y(x_n)\right)(x_n - x_{n-1}), \qquad n = 1,2,3,\ldots.$$

This yields the **backward Euler formula**

$$y_{n+1} = y_n + h\, f\,(x_{n+1}, y_{n+1}), \qquad y_0 = y(0), \qquad n = 0,1,2,\ldots\ldots$$



Example 1 : Approximate $\boldsymbol{y(0.2)}$ in the function $\boldsymbol{y' = 3x - 2y}$ when $\boldsymbol{y(0) = 1}$ and step is 0.1 using Backward Euler (or implicit method).

Solution :

$$y_{n+1} = y_n + h[\,f(x_{n+1}, y_{n+1})]$$

$$y_1 = y_0 + h[\,f(x_1, y_1)] \quad \text{where } f(x,y) = 3x - 2y$$

$$y_1 = y_0 + h[3x_1 - 2y_1] = y_0 + 3hx_1 - 2hy_1$$

$$y_1 = \frac{y_0 + 3hx_1}{1 + 2h}$$

$x_0 = 0$ $\qquad$ $y_0 = 1$

$x_1 = 0.1$ $\qquad$ $y_1 = \frac{y_0+3hx_1}{1+2h} = \frac{1+3(0.1)(0.1)}{1+2(0.1)} = 0.8583$

$x_2 = 0.2$ $\qquad$ $y_2 = \frac{y_1+3hx_2}{1+2h} = \frac{0.8583+3(0.1)(0.2)}{1+2(0.1)} = 0.76525$

n	x_n	y_n
0	0	1
1	0.1	.0.8583
2	0.2	0.76525

2.1.3 Backward Euler Method [10]

Suppose that we wish to numerically solve the initial value problem

$$y' = f(x, y), \qquad y(x_0) = y_0,$$

where $y' = dy/dx$ is the derivative of function $y(x)$ and (x_0, y_0) is a prescribed pair of real numbers. We assume that f is a smooth function so that the given initial value

problem has a unique solution. We seek a numerical solution on the interval $[a,b]$, where we usually set $x_0 = a$. Subdivide the interval $[a,b]$ with $N+1$ mesh points $x_0, x_1, \ldots, x_N$ with $x_0 = a$, $x_N = b$. Let $h = (b - a)/N$ be the step size, so that $x_n = a + hn$, $n = 1, 2, \ldots, N$. In some problems h is given, then $N = (b - a)/h$. Integrating the given differential equation, we come to the integral equation

$$y(x) = y(a) + \int_a^x f(t, y(t))\, dt$$

Now we set $x = x_n$ in the above equation and break the integral into the sum of integrals over each subinterval $[x_n, x_{n+1}]$. This leads to the sequence of integral equations for each subinterval:

$$y(x_n) = y(x_{n+1}) + \int_{x_{n-1}}^{x_n} f(t, y(t))\, dt, \qquad n = 1,2,\ldots,N,$$

Example 2 : Approximate $\boldsymbol{y(0.2)}$in the function $\boldsymbol{y' = x + y}$ when$\boldsymbol{y(0) = 0}$ and step is 0.1 using Backward Euler method.

Solution :

$y_{n+1} = y_n + h[f(x_{n+1}, y_{n+1})]$

$y_1 = y_0 + h[f(x_1, y_1)]$ where $f(x, y) = x + y$

$y_1 = y_0 + h[x_1 + y_1] = y_0 + hx_1 + hy_1$

$$y_1 = \frac{y_0 + hx_1}{1 - h}$$

n	x_n	y_n
0	0	0
1	0.1	0.01111
2	0.2	0.03457

$x_0 = 0$ $\qquad$ $y_0 = 0$

$x_1 = 0.1$ $\qquad$ $y_1 = \frac{y_0+hx_1}{1-2h} = \frac{0+(0.1)(0.1)}{1-(0.1)} = 0.01111$

$x_2 = 0.2$ $\qquad$ $y_2 = \frac{y_1+hx_2}{1-h} = \frac{0.01111+(0.1)(0.2)}{1-(0.1)} = 0.03457$

Example 3 : Compute $y(0.4)$ taking $h = 0.1$,given $\frac{dy}{dx} = -y$, $y(0) = 1$

Solution :

$y_{n+1} = y_n + h[\, f(x_{n+1}, y_{n+1})]$

$y_1 = y_0 + h[\, f(x_1, y_1)]$ where $f(x, y) = -y$

$y_1 = y_0 + h[-y_1] = y_0 - hy_1$

$$y_1 = \frac{y_0}{1 + h}$$

n	x_n	y_n
0	0	1
1	0.1	0.90909
2	0.2	0.82645
3	0.3	0.75132
4	0.4	0.68302

$x_0 = 0$ $\qquad$ $y_0 = 1$

$x_1 = 0.1$ $\qquad$ $y_1 = \frac{y_0}{1+h} = \frac{1}{1+0.1} = 0.90909$

$x_2 = 0.2$ $\qquad$ $y_2 = \frac{y_1}{1+h} = \frac{0.90909}{1+0.1} = 0.82645$

$x_3 = 0.3$ $\qquad$ $y_3 = \frac{y_2}{1+h} = \frac{0.82645}{1+0.1} = 0.75132$

$x_4 = 0.4$ $\qquad$ $y_4 = \frac{y_3}{1+h} = \frac{0.75132}{1+0.1} = 0.68302$

Example 4 : Compute $y(0.4)$from the differential equation $\frac{dy}{dx} = xy$

$, y(0) = 1, h = 0.1$

a) Euler's method

b) Backward Euler method

Solution a :

$$y_{n+1} = y_n + h\, f(x_n, y_n)$$

n	x_n	y_n
0	0	1
1	0.1	1
2	0.2	1.01
3	0.3	1.0302
4	0.4	1.061106

$n = 0 \rightarrow y_1 = y_0 + hf(x_0, y_0)$

$$= 1 + 0.1\big(\ (0)(1)\big) = 1 = y(0.1)$$

$n = 1 \rightarrow y_2 = y_1 + hf(x_1, y_1)$

$$= 1 + 0.1\big((0.1)(1)\big) = 1.01 = y(0.2)$$

$n = 2 \rightarrow y_3 = y_2 + hf(x_2, y_2)$

$$= 1.01 + 0.1\big((0.2)(1.01)\big) = 1.0302 = y(0.3)$$

$n = 3 \rightarrow y_4 = y_3 + hf(x_3, y_3)$

$$= 1.0302 + 0.1\big((0.3)(1.0302)\big) = 1.061106 = y(0.4)$$

Solution b :

$y_{n+1} = y_n + h[\, f(x_{n+1}, y_{n+1})\,]$

$y_1 = y_0 + h[\, f(x_1, y_1)]$ where $f(x, y) = xy$

$y_1 = y_0 + h[x_1 y_1]$

$$y_1 = \frac{y_0}{1 - hx_1}$$

n	x_n	y_n
0	0	1
1	0.1	1.0101
2	0.2	1.0307
3	0.3	1.0626
4	0.4	1.1069

$x_0 = 0$ $\quad$ $y_0 = 1$

$x_1 = 0.1$ $\quad$ $y_1 = \frac{y_0}{1-hx_1} = \frac{1}{1-0.1(0.1)} = 1.0101$

$x_2 = 0.2$ $\quad$ $y_2 = \frac{y_1}{1-hx_2} = \frac{0.0101}{1-0.1(0.2)} = 1.0307$

$x_3 = 0.3$ $\quad$ $y_3 = \frac{y_2}{1-hx_3} = \frac{1.0307}{1-0.1(0.3)} = 1.0626$

$x_4 = 0.4$ $\quad$ $y_4 = \frac{y_3}{1-hx_4} = \frac{1.0626}{1-0.1(0.4)} = 1.1069$

Example 5 : Approximate $\boldsymbol{y(0.2)}$ in the function $\boldsymbol{y'=x+y}$ when $\boldsymbol{y(0)=0}$

and step is 0.1

a) Euler's method

b) Backward Euler method

Solution a :

$y_{n+1} = y_n + h\, f(x_n, y_n)$

$n = 0 \rightarrow y_1 = y_0 + hf(x_0, y_0)$

n	x_n	y_n
0	0	0
1	0.1	0
2	0.2	0.01

$$= 0 + 0.1(\ 0 + 0) = 0 = y(0.1)$$

$$n = 1 \rightarrow y_2 = y_1 + hf(x_1, y_1)$$

$$= 0 + 0.1(0.1 + 0) = 0.01 = y(0.2)$$

Solution b :

n	x_n	y_n
0	0	1
1	0.1	0.01111
2	0.2	0.03457

$$y_{n+1} = y_n + h[\ f(x_{n+1}, y_{n+1})]$$

$y_1 = y_0 + h[\ f(x_1, y_1)]$ where $f(x, y) = x + y$

$$y_1 = y_0 + h[x_1 + y_1] = y_0 + hx_1 + hy_1$$

$$y_1 = \frac{y_0 + hx_1}{1 - h}$$

$x_0 = 0$ $\quad$ $y_0 = 0$

$x_1 = 0.1$ $\quad$ $y_1 = \frac{y_0+hx_1}{1-2h} = \frac{0+(0.1)(0.1)}{1-(0.1)} = 0.01111$

$x_2 = 0.2$ $\quad$ $y_2 = \frac{y_1+hx_2}{1-h} = \frac{0.01111+(0.1)(0.2)}{1-(0.1)} = 0.03457$

2.2 Numerical Linear and Nonlinear Algebra [3][4] [11]

This refers to problems in evolving the solution of systems of linear and nonlinear equations, possibly with a very large number of variables. Many problems in applied mathematics involve solving systems of linear equations, with the linear system occurring naturally in some cases and as a part of the solution process in other cases. Linear systems are usually written using matrix-vector notation, **Ax = b**, with A the matrix of coefficients for the system, x the column vector of the unknown variables $x_1,\ldots,x_n$, and b a given column vector. Solving linear systems with up to a n = 1000 variables is now considered rela tively straightforward in most cases. For small to moderate sized linear systems (say n ≤ 1000), the favorite numerical method is Gaussian elimination and its variants; this is simply a precisely stated algorithmic variant of the method of elimination of variables that students first encounter in elementary algebra. For larger linear systems, there are a variety of approaches depending on the structure of the coefficient matrix A. Direct methods lead to a theoretically exact solution x in a finite number of steps, with Gaussian elimination the best known example. In practice, there are errors in the computed value of x due to rounding errors in the computation, arising from the finite length of numbers in standard computer arithmetic. Iterative methods are approximate methods which create a sequence of approximating solutions of in- creasing accuracy. Linear systems are categorized according to many properties (e.g. A may be symmetric about its main diagonal), and specialized methods have been developed for problems with these special properties. Nonlinear problems are often treated numerically by reducing them to a sequence of linear problems. As a simple but important example, consider the problem of solving a nonlinear equation $f(x) = 0$. Approximate the graph of $y = f(x)$ by the tangent line at a point $x^{(0)}$ near the desired root, and use the root of the tangent line to approximate the root of the original nonlinear function $f(x)$. This leads to Newton's method for rootfinding .[4]

$$x^{(k+1)} = x^{(k)} - \frac{f(x^{(k)})}{f'(x^{(k)})} \qquad k = 0,1,2, \ldots.$$

This generalizes to handling systems of nonlinear equations. Let $f(x) = 0$ denote a system ofn nonlinear equations in n unknowns $x_1,\ldots,x_n$. Newton's method for solving this system is given by

$$x^{(k+1)} = x^{(k)} + \delta^{(k)}$$

$$f'\left(x^{(k)}\right)\delta^{(k)} = -f\left(x^{(k)}\right), \qquad k = 0,1,\ldots$$

In this, $f'(x)$ is the Jacobian matrix of $f(x)$, and the second equation is a linear system of order n. There are numerous other approaches to solving nonlinear systems, most based on using some type of approximation using linear functions. An important related class of problems occur under the heading of optimization. Given a real-valued function $f(x)$ with $\boldsymbol{x}$ a vector of unknowns, we wish to find a value of $\boldsymbol{x}$ which minimizes $f(x)$. In some cases $\boldsymbol{x}$ is allowed to vary freely, and in other cases there are constraints on the values of $\boldsymbol{x}$. Such problems occur frequently in business applications.

2.2.1 Newton Raphson Method [4] [9]

The Newton Raphson method is for solving equations of the form $f\ (x) = 0$. We make
an initial guess for the root we are trying to find, and we call this initial guess x_0.
The sequence $x_0, x_1, x_2, x_3, \ldots$ generated in the manner described below should converge to the exact root.

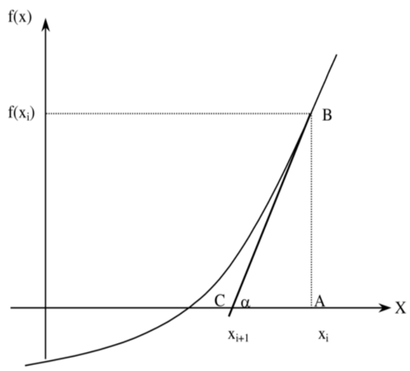
To implement it analytically we need a formula for each approximation in terms of the previous one, i.e. we need $x_n + 1$ in terms of x_n.
The equation of the tangent line to the graph $y = f(x)$ at the point $(x_0, f(x_0))$ is

The tangent line intersects the x-axis when $y = 0$ and $x = x_1$, so

Solving this for x_1 gives

and, more generally,

You should memorize the above formula. Its application to solving equations of the form $f(x) = 0$, as we now demonstrate, is called the Newton Raphson method .It is guaranteed to converge if the initial guess $x_0$is close enough, but it is hard to make a clear statement about what we mean by close enough because this is highly problem specific. A sketch of the graph of f (x) can help us decide on an appropriate initial guess $x_0$for a particular problem.



$$\tan(\alpha) = \frac{AB}{AC}$$

$$f'(x_i) = \frac{f(x_i)}{x_i - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

2.2.2 Derivation of Newton Raphson Method

The Newton-Raphson method is named after Isaac Newton; the man who discovered the method in 1736, and Joseph Raphson, the man who described the method back in 1690. Both mathematicians utilized calculus in this method in order to find the roots of an equation.

Example 1 : Derive the Newton-Raphson Formula for finding a root of a non-linear equation . Find a root of $f(x) = x^3 + 2x^2 + 10x - 20 = 0$ up to 10 iterations.

Solution.

Newton-Raphson formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$f(x) = x^3 + 2x^2 + 10x - 20 \; \dots (1) \implies f'(x) = 3x^2 + 4x + 10$$

$$f(1) = 1^3 + 2(1)^2 + 10(1) - 20 = 1 + 2 + 10 - 20 = -7$$

$$f(2) = 2^3 + 2(2)^2 + 10(2) - 20 = 8 + 8 + 20 - 20 = 16$$

Since $f(1) and\ f(2)$ are of opposite signs so the root of (1) lies between 1 and 2.

As $f(1)$ is near to zero than $f(2). So,$ 1 is better approximate root then 2.

$f'(1) = 3(1)^2 + 4(1) + 10 = 3 + 4 + 10 = 17$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad \text{(Newton-Raphson formula)}$$

First Iteration

$$x_1 = x_0 - \frac{f(1)}{f'(1)} = 1 - \frac{-7}{17} = \frac{24}{17} \qquad f\left(\frac{24}{17}\right) = 0.9175656$$

Second Iteration

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{24}{17} - \frac{f\left(\frac{24}{17}\right)}{f'\left(\frac{24}{17}\right)} = 1.4117647 - \frac{0.9175656}{21.6262976} \qquad f'\left(\frac{24}{17}\right) = 21.6262976$$

$$= 1.4117647 - 0.04242823 = 1.36933647$$

Third Iteration

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.36933647 - \frac{f(1.36933647)}{f'(1.36933647)}$$

$$= 1.36933647 - \frac{0.01114811}{21.1025930}$$

$$= 1.36933647 - 0.00052828 = 1.36880819$$

Fourth Iteration

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.36880819 - \frac{f(1.36880819)}{f'(1.36880819)}$$

$$= 1.36880819 - \frac{0.00000173}{21.09614034} = 1.36880819 - 0.00000008$$

$$= 1.36880811$$

Fifth Iteration

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 1.36880811 - \frac{f(1.36880811)}{f'(1.36880811)}$$

$$= 1.36880811 - \frac{0.00000005}{21.09618937} = 1.36880811 - 0.0000000023 \ = 1.36880811$$

The root of the given equation is 1.36880811 ccorrect up to eight decimal place after fifth iteration. For the accuracy more than 8 decimal places, we can iterate further.

x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
1	-7	17	1.4117647
1.4117647	0.9175656	21.6262976	1.36933647
1.36933647	0.01114811	21.1025930	1.36880819
1.36880819	0.00000173	21.09614034	1.36880811
1.36880811	0.00000005	21.09618937	1.36880811

Example 2 : Find the real root of the equation $x^4 - x - 9 = 0$ by Newton-Raphson Method correct to three places of decimal.

Solution. $f(x) = x^4 - x - 9 = 0$ $\quad\quad f'(x) = 4x^3 - 1$

$$f(0) = -9$$

$$f(1) = 1^4 - 1 - 9 = 1 - 1 - 9 = -9$$

$$f(2) = 2^4 - 2 - 9 = 16 - 2 - 9 = 5$$

As $f(2)$ is nearer to zero we take 2 as an approximate root of $f(x)$.

$$f^{'}(2) = 4(2)^3 - 1 = 4(8) - 1 = 32 - 1 = 31$$

By Newton-Raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{5}{31} = 1.8387$$

$$f(1.8387) = (1.8387)^4 - 1.8387 - 9 = 11.4299 - 1.8387 - 9 = 0.5912$$

$$f'(1.8387) = 4(1.8387)^3 - 1 = 23.8652$$

$$x_2 = 1.8387 - \frac{f(1.8387)}{f'(1.8387)} = 1.8387 - \frac{0.5912}{23.8652} = 1.8387 - 0.02477 = 1.8139$$

$$f(1.8139) = (1.8139)^4 - 1.8139 - 9 = 0.01173$$

$f'(1.8139) = 4(1.8139)^3 - 1 = 22.8726$

$$x_3 = 1.8139 - \frac{f(1.8139)}{f'(1.8139)} = 1.8139 - \frac{0.01173}{22.8726}$$

$$= 1.8139 - 0.0005 = 1.8134$$

$f(1.8134) = (1.8134)^4 - 1.8134 - 9 = 10.8137 - 1.8134 - 9 = 0.0003$

$f'(1.8134) = 4(1.8134)^3 - 1 = 22.8529$

$$x_4 = 1.8134 - \frac{f(1.8134)}{f'(1.8134)} = 1.8134 - \frac{0.0003}{22.8529} = 1.8134 - 0.000013$$

$$x_4 = 1.8134$$

As x_3 and x_4 are equal,so root=1.8134

Hence, real root of the given equation is 1.8134

x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
2	5	31	1.8387
1.8387	0.5912	23.8652	1.8139
1.8139	0.01173	22.8726	1.8134
1.8134	0.0003	22.8529	1.8134

Example 3 : Find the positive root of $x = \cos x$ *using* Newton Raphson method

Solution. Let $f(x) = x - \cos x$

$$f(0) = -1\ ,\ \ f(1) = 1 - \cos 1 = 0.459697$$

∴ a root lies between 0 and 1 it is closer to 1. Therefore, take $x_0 = 0.7$.

$$f'(x) = 1 + sin\ x$$

$$x - \frac{f(x)}{f'(x)} = x - \frac{x - \cos x}{1 + \sin x} = \frac{x \sin x + \cos x}{1 + \sin x}$$

$$x_{i+1} = \frac{x_i\ sin\ x_i + cosx_i}{1 + sin\ x_i}$$

$$x_1 = \frac{0.7\ \sin\ (0.7) + \cos\ (0.7)}{1 + sin\ (0.7)} = \frac{1.21579457}{1.64421769} = 0.739436499$$

$$x_2 = \frac{0.739436499 \times \sin\ (0.739436499) + \cos\ (0.739436499)}{1 + sin\ (0.739436499)}$$

$$= \frac{1.23713372}{1.67387168} = 0.739085162$$

$$x_3 = \frac{1.23694179}{1.67361205} = 0.739085136$$

∴ Correct value of the root is 0.7390851 .

x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
0.7	- 0.06484	1.64422	0.73944
0.73944	0.00059	1.67387	0.73909
0.73909	0.000008	1.67362	0.73909

2.2.3 Advantages of Newton-Raphson Method

Here are the advantages of newton-raphson method or we can say merits of Newton's method of iteration.

1- On of the fastest methods which converges to root quickly.

2- Converges on the root quadratic ally **i.e.** rate of convergence is 2.

3- As we go near to root, number of significant digits approximately doubles with each step.

4- It makes this method useful to get precise results for a root which was previously obtained from some other convergence method.

5- Easy to convert to multiple dimension.

2.3 Application of Newton- Raphson Method In Optimal Design of Water Distribution Networks [6] [9] [11]

NR method is extensively used for analysis of flow in water distribution networks. Several efficient computer programs, using NR method, are also available for analysis of flow in large size networks. However, efficient computer software's for optimal design of multi-source looped water distribution networks are not readily available. Behave (1978) suggested a univariate method, called cost-head loss ratio criterion method, for no computer optimization of water distribution networks. Cost-head loss ratio criterion method is similar to Hardy Cross Head Correction method with the difference that assumed heads are successively corrected to satisfy cost-head loss ratio criterion instead of satisfying node-flow continuity equations. The univariate cost-head loss ratio criterion method is modified herein for rapid convergence through the application of Newton-Raphson method. Any available software for analysis of water distribution network using Newton-Raphson method can be easily upgraded for optimal design of water distribution networks.

Newton-Raphson (NR) method is an iterative scheme used to solve non-linear simultaneous equations. France (1991) described application of NR method to solve various hydraulic problems. Martin and Peters (1963) were the first to propose the application of NR method for analysis of water distribution network (WDN) having pipes and reservoirs only. McCormick and Bellamy (1968) and Zarghamee (1971) extended its use to include other network elements such as pumps and valves. Shamir and Howard (1968) applied this method to solve for all types of unknowns in a network including pipe resistances using head equations. Epp and Fowler (1970) and Gofman and Rodeh (1981) used this method for solution of loop equations. Some investigators proposed modifications to the classical NR method. Lam and Wolla (1972 a, b) modified algorithm so that it does not require the Jacobian matrix or its inverse in the iterative process and also suggested a change in the step size to minimize the error. The modification presented by Lemieux (1972) ensures the convergence of the algorithm irrespective of the starting assumption. Donachie (1974) suggested halving the step size at any node when oscillation occurs. Neilson (1989) compared NR method with linear theory method and suggested starting of NR method with single iteration by linear theory method followed by NR iterations. Andersen and Powell (1999) used a linear headloss formula in NR method for the first iteration. Several computer programs are also available for analysis of WDNs using Newton-Raphson method.

The application of NR method in optimal design of WDNs is however very limited. Young (1994) used NR method to solve nonlinear simultaneous equations which were generated through Lagrangian multiplier method for optimal design of branched WDNs. Johnson et al. (1995) discussed the limitations of Lagrangian multiplier method proposed by Young (1994). Behave (1978, 1985) developed cost head loss ratio criterion method for optimal design of WDNs. This method can be used for optimal design and expansion of single as well as multi-source branched or looped networks including pumped source nodes. Herein, cost-head loss ratio criterion method is modified for faster convergence using NR method.

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جامعة بابل
قسم الرياضيات
كلية التربية للعلوم الصرفة

استخدام التقريبات العددية لحل المعادلات التفاضلية والتكاملية

بحث مقدم

إلى مجلس كلية التربية للعلوم الصرفة جامعة بابل كجزء

من متطلبات نيل درجة الدبلوم العالي تربية/ الرياضيات

مقدم من قبل

عمار نجم عبد الله المسلماوي

إشراف

أ.م.د. احمد صباح الجيلاوي

2022 م

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إهـــــــــداء

إلى من غمرتنا بحبها طول حياتها....

إلى من أعطتنا بلا حدود .....

إلى الوالدة العزيزة.... أطال الله عمرها في طاعته...

شكر وتقدير

ليس بعد تمام العمل من شئ أجمل ولا أحلى من الحمد لله والشكر له كما ينبغي لجلال وجهه وعظيم سلطانه وكما ينبغي لجزيل فضله وعظيم إحسانه على ماانعم علي من إتمام هذا البحث المتواضع.

ثم أتوجه بجزيل الشكر وعظيم الأمتنان الى كل من :

- الدكتور الفاضل احمد صباح الجيلاوي لتفضله بالأشراف على البحث وتكرمه بنصحي وتوجيهي حتى اتم هذا البحث.
- أعضاء لجنه المناقشة الكرام
- زوجتي الغالية على جهدها المبذول من طباعة وتنسيق للبحث.
- كل من ساندني ودعمني

بِسْمِ ٱللَّهِ ٱلرَّحْمَٰنِ ٱلرَّحِيمِ

(يَرْفَعِ اللَّهُ الَّذِينَ آمَنُوا مِنكُمْ وَالَّذِينَ أُوتُوا الْعِلْمَ
دَرَجَات وَاللَّهُ بِمَا تَعْمَلُون خَبِير)

صدق الله العظيم

سوره المجادلة آية (١١)

الخلاصة

يعد التحليل العددي وأجهزة الكمبيوتر من الأدوات المهمة في الرياضيات التطبيقية ، والمعادلات التفاضلية هي معادلات تتضمن دالة ومشتقات غير معروفة. ستكون هناك أوقات قد يكون فيها حل المعادلة غير متوفر أو قد تكون وسائل حلها غير متوفرة. في هذه الأوقات ومعظم الوقت ، سيتم استخدام طرق صريحة وضمنية بدلاً من الحلول الدقيقة. في الحالات الأبسط ، تعتبر المعادلات التفاضلية العادية أو المعادلات التفاضلية العادية وطريقة أويلر وطريقة أويلر العكسية طرقًا فعالة للحصول على تقديرات تقريبية دقيقة إلى حد ما للحلول الفعلية. من خلال التلاعب بهذه الأساليب ، يمكن للمرء أن يجد طرقًا لتقديم تقديرات تقريبية جيدة مقارنة بالحل الدقيق ، كما قدمنا تقريبًا جيدًا لطريقة نيوتن.