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Ministry of Higher Education
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University of Babylon
College of Education for Pure Sciences
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Study of some reliability models

*Research
Submitted to College of Education for Pure
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Requirement for the Degree of
Higher Diploma Education / Mathematics*

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Dedication

Give this humble effort

To My Wife

To my brothers and sisters

To everyone who supported me and

Gave me support

Acknowledgments

I would like to express my thanks to “**Allah**” the Most Gracious and Most Merciful. I am deeply indebted to, “My Supervisor” **Prof. Dr. Zahir Abdul Haddi Hassan**, who has faithfully worked hard to bring this work into big success. This research wouldn't have been possible without his sincere help and assistance.

Abstract

In this project, we study some mathematical models in reliability systems such as; series , parallel , series – parallel , parallel – series and mixed systems by using some methods like: path tracing method , reduction to series element method , minimal cut method, event space method and decomposition method . Reliability allocation had been studied. The aim of it is establish a goal or objective for the reliability of each component. Two methods had been used to find the reliability allocation for series and mixed systems.

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1.1 Introduction

The Reliability is defined as the probability that a component, device, equipment, or system will perform its intended function for specified period of time under a given set of conditions. Committees and laboratories studied different problems, such as maintenance problems, failure of equipment's and components. During the decades following of the war many research laboratories and universities initiated and mathematicians interested in the study of life testing and reliability problems. There are many researchers focused on the study of different reliability systems. like the series, the parallel, the series – parallel ,the parallel – series systems. and the complex systems .that enters in different life fields. Since there are different available systems and because of their importance, the researchers attempted to find more than one method to solve these complex systems, and depend the best methods through making a comparison between them, and there are researchers focused on the study of Markov model to help them in the calculation of different reliability systems. And Markov model assumes that the future is independent of the past given the present. When using Markov the random variable is indexed in time, which can be either discrete or continuos, these researchers as *Sandler G.H., (1963), [20]* he studied the calculation of reliability for the series system and for the parallel system without maintenance. Also the calculation of mean time to failure for these selected items by determining the transition matrix $P=[P_{ij}]$ for each model, P holds the relations $P_{ij} \geq 1, i,j =1,2,3,\dots$ and solving the obtained equations by using Laplace transformations and its inverse. *Bhat N.U., (1972), [4]* He studied stochastic process and classified it to two types depending on two important things, which are parameter space, and state space and these are Markov chain and Markov process.

Govil, (1983), [8] Studied the series and parallel systems, which consist of n components .

Danielsson H. and Olsson K., (1985), [6] studied a K - out of n system, and this system requires at least K number elements being active for the system to be active.

Ansell J.I., and Phillips M.J., [3] he studied the mixed and complex systems, and he used three methods to solve these systems and these methods are path tracing method, reduction to series element and composite method and they use variable examples to state that. *Al-Ali A.A., (1998), [1]* they used a path set method to calculate the reliability of a power plant which is considered as a complete. *Hoang P., (2003), [10]* which is studied the complex systems and used other different methods to find the reliability of these systems. Like event space method, decomposition method and minimal cut method. *Al-Ali A.A. and Hassan Z.A.A., (2005), [2]* They studied type of complex systems by using some of the common (useful) methods to solve complex systems, and doing some of the comparison between these methods to reach the best reliability for complex systems.

Srinath, [11] studied the problem of reliability allocation of the series system and reliability allocation, which means the allocation of the required reliability for individual components to attain the specified system reliability. The reliability allocation is the converse of calculating the system reliability. And they use two methods to calculate the reliability allocation. While *U.Subrie, (1996), [12]* take care of studying the reliability allocation used one single method to calculate the reliability allocation of the complex system while this project includes two methods to solve the complex systems and the series systems. But *Hoyland, A. and Rausand M., (1994), [11]* they studied the reliability allocation in the series- parallel system.

Mushtak A.K., (1980), [16] evaluate the reliability of three different models of computer programs. Such reliability evaluations are quit useful in

testing algorithms for these programs are derived and discussed depending on the nature of logic flow of the program.

A. A. Al-Ali, (1988), [1] studied two unit repairable system subject to random shocks, the operating units are put under two types of repair due to their failure, Failure time of the operating unit is exponential distributed and then the researcher obtain the mean time to system failure and draw its graph against shock rates.

Subrie U.A., (1985), [21] studied two systems each consisting of one unit, the unit subject to random shocks which occurs at random times.

System 1: with out maintenance.

System 2: with maintenance.

So he find mean time to system failure for these systems.

Malte L. and Johan R., (1998), [17] this study shows that is possible specify the requirement for at least one quality aspect, the reliability, then we have full control over these requirement through the whole development process and even during the operational phase.

Suri P.K. and Aggarwal K.K., (1980), [22] he studied providing guidance for assessing the reliability of mechanical and electrical systems of navigation locks and dams.

Dileep M.J., et. al., (2015), [5] introduced a general algorithm to determine the optimum reliability system K out of n.

1.2 Some definitions and general terminologies

The concepts to follow is very important in creating and analyzing reliability block diagrams.

Definition (1.1)[1] Probability (P (t))

Probability is a numerical measure of the likelihood of an event relative to a set of alternative events. [4]

Probability Properties[2]

The probability of an event A is expressed as P (A), and has the following properties:

- 1- $0 \leq P (A) \leq 1$,
- 2- $P (A)=1- P (A^c)$,
- 3- $P (\Phi)=0$,
- 4- $P (S) =1$.

In other words, when an event is certain to occur it has a probability equal to 1 and when it is impossible to occur, it has a probability equal to 0. It can be shown that the probability of the union of two events A and B is:

$$P (A \cup B) =P (A) + P (B) - P (A \cap B) \quad \dots(1.1)$$

Similarly, the probability of the union of three events, A, B and C, is given by:

$$P (A \cup B \cup C) = P (A) + P (B) + P(C) - P (A \cap B) - P (A \cap C) - P (B \cap C) + P (A \cap B \cap C) \quad \dots(1.2)$$

Definition (1.2) Mutually Exclusive Events[6]

Two events A and B are defined as being mutually exclusive if it is impossible for them to occur simultaneously ($A \cap B = \Phi$). In such cases, the expression for the union of these two events reduces to:

$$P(A \cup B) = P(A) + P(B) \quad \dots(1.3)$$

Since the probability of the intersection of these events is defined as zero.

Definition (1.3) Conditional Probability[4]

Conditional probability of two events, A and B, is defined as the probability of one of these events occurring knowing that the other event has already occurred. The expression below denotes the probability of an occurring given that B has already occurred.

$$P(A | B) = \frac{P(A \cap B)}{P(B)} \quad \dots(1.4)$$

Note that knowing that event B has occurred reduces the sample space.

Definition (1.4) Independent Events[5]

If knowing B gives no information about A, then the events are said to be independent and the conditional probability expression reduces to:

$$P(A / B) = P(A) \quad \dots(1.5)$$

From the definition of conditional probability, Eqn. (1.4) can be written as:

$$P(A \cap B) = P(A / B) P(B) \quad \dots(1.6)$$

Since events A and B are independent, the expression reduces to:

$$P(A \cap B) = P(A) P(B) \quad \dots(1.7)$$

Definition (1.5) Probability density function p.d.f.[1]

If X is a continuous random variable, then the *probability density function*, *p.d.f.*, of X is a function, $f(x)$, such that for two numbers, a and b with $a \leq b$:

$$P(a \leq X \leq b) = \int_a^b f(x)dx \quad \text{and } f(x) \geq 0, \text{ for all } x \quad \dots(1.8)$$

Definition (1.6) Cumulative distribution function c.d.f.[1]

The cumulative distribution function, *c.d.f.*, is a function, $F(x)$, of a random variable X , and is defined for a number x by:

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(s)ds \quad \dots(1.9)$$

The *c.d.f.* is used to measure the probability that the item in question will fail before the associated time value, t , and is also called ***unreliability***.

The mathematical relationship between the *P.d.f.* and *c.d.f.* is given by:

$$F(x) = \int_{-\infty}^x f(s)ds$$

Where s is a dummy integration variable.

Conversely:

$$f(x) = \frac{d(F(x))}{dx} \quad \dots(1.10)$$

The value of the *c.d.f.* at x is the area under the probability density function up to x , if so chosen. It should also be pointed out that the total area under the *p.d.f.* is always equal to 1, or mathematically,

$$\int_{-\infty}^{\infty} f(x)dx = 1 \quad \dots(1.11)$$

Definition (1.7) System[7]

An orderly arrangement of components that interact among themselves and with external components, other systems, and human operators to perform some intended function. (unit, component or subsystem are mean the same).

Definition (1.8) Reliability $R(t)$ [8]

Is the probability that the system survives from some specified period of time. This may be expressed in terms of random variable T the time to system failure:

$$R(t) = P\{T > t\} \quad \dots(1.12)$$

\equiv Probability that the system operates without failure for length of time.

The probability of survival or reliability $R(t)$ at time t , has the following properties:

- i) $0 \leq R(t) \leq 1$
- ii) $R(0) = 1$; the device is assumed to be working properly at time $t = 0$, and $R(\infty) = 0$; no device can work for ever without failure.
- iii) $R(t)$ in general is a decreasing function of time t .

$$\text{Let } F(t) = P\{T \leq t\} \quad \dots(1.13)$$

\equiv Probability that the failure takes places at time less than or equal to t .

Since a system that does not fail for $T \leq t$ must fail at same $T > t$, we have

$$R(t) = 1 - F(t) \quad \dots(1.14)$$

Or equivalent

$$R(t) = 1 - \int_0^t f(t) dt \quad \dots(1.15)$$

$$\text{Or } R(t) = \int_0^{\infty} f(t) dt \quad \dots(1.16)$$

Definition (1.9) Failure[5]

A failure is any inability of a system (component) to carry out its specified function.

Definition (1.10) Failure Rate [9]

The failure rate is the probability that a failure per unit time occurs in the interval, say $[t, t + \Delta t]$, given that a failure has not occurred before t . That is, the failure rate is the rate at which failures occur in $[t, t + \Delta t]$,

$$\begin{aligned} \text{Failure rate} &\equiv \frac{P[t \leq T < t + \Delta t | T > t]}{\Delta t} \quad \dots(1.17) \\ &= \frac{P[(t \leq T < t + \Delta t) \cap (T > t)]}{\Delta t P[T > t]} \\ &= \frac{F(t + \Delta t) - F(t)}{\Delta t R(t)} \end{aligned}$$

Definition (1.11) Hazard Rate $h(t)$ [9]

The hazard rate is defined as the limit of the failure rate as the interval approaches zero, that is, $\Delta t \rightarrow 0$. Thus, we obtain **the hazard rate** at time t as

$$h(t) = \lim_{\Delta t \rightarrow 0} \frac{F(t + \Delta t) - F(t)}{\Delta t R(t)} = \frac{f(t)}{R(t)} \quad \dots(1.18)$$

The hazard rate is an instantaneous rate of failure at time t , given that the system survives up to t . In particular, the quantity $h(t) dt$ represents the probability that a system of age t will fail in the small interval t to $t + \Delta t$, we can explain the hazard rate in the following figure.

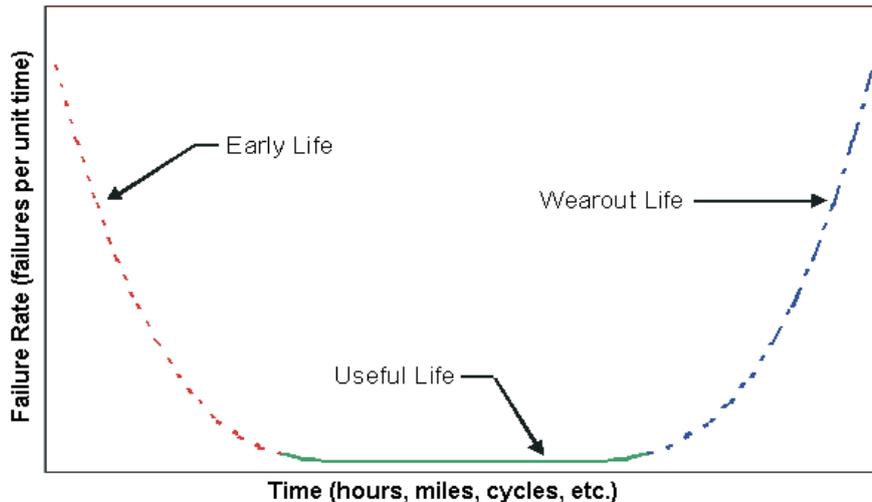


Figure (1.1): Bathtub Curve

The hazard rate will change over the lifetime of a system. The curve above (**Bathtub Curve**) is typical for many systems and components. It may divide into three distinct regions:

1. **Early life** or **Burn-in** or debugging or infant mortality phase – early failures due to material or manufacturing defect or improper design.
2. **Useful life** or **normal operating** or chance failures phase - failures caused by sudden stress of extreme conditions.
3. **Wear-out life** – **fatigue failures** caused due to wearing out of components these failures occur if the system is not maintained properly or not maintained at all and frequency of such failures increases rapidly with time.

Definition (1.10)[8] Mean Time To Failure (MTTF)

In addition to the functions $R(t)$ and $h(t)$, a third useful reliability measure for non-repairable components is the mean time to failure (MTTF), or expected life

$$MTTF = E(T) = \int_0^t t f(t) dt \quad \dots(1.19)$$

Or in terms of the reliability function,

$$MTTF = \int_0^{\infty} R(t) dt \quad \dots(1.20)$$

In most applications, the reliability of a non-repairable component can be described by a single parameter, such as the reliability R or the mean time to failure MTTF.

1.3 Relationships among $f(t)$, $F(t)$, $R(t)$ and $h(t)$ [6],[9]

The functions $f(t)$, $F(t)$, $R(t)$ and $h(t)$ can be transformed with one another

For example:

$$h(t) = \frac{dF(t)}{dt} \frac{1}{R(t)} \quad \text{Since} \quad f(t) = \frac{dF(t)}{dt}$$

From (1.16) we know that $\frac{dR(t)}{R(t)} = -h(t)dt$ which yields

$$dF(t) = -\frac{dR(t)}{dt} \quad \dots(1.21)$$

Then by integration,

$$\ln R(T) = -\int_0^t h(x) dx + c, R(0) = 1 (c=0) \quad \dots(1.22)$$

and taking the exponential of both sides of the equation, one obtains the fundamental equation relating reliability a failure rate:

$$R(t) = \exp\left[-\int_0^t h(x) dx\right] \quad \dots(1.23)$$

Finally, by differentiation, we can write $f(t)$ in terms of $h(t)$,

$$f(t) = h(t) \exp\left[-\int_0^t h(x) dx\right] \quad \dots(1.24)$$

Triangle Relationship

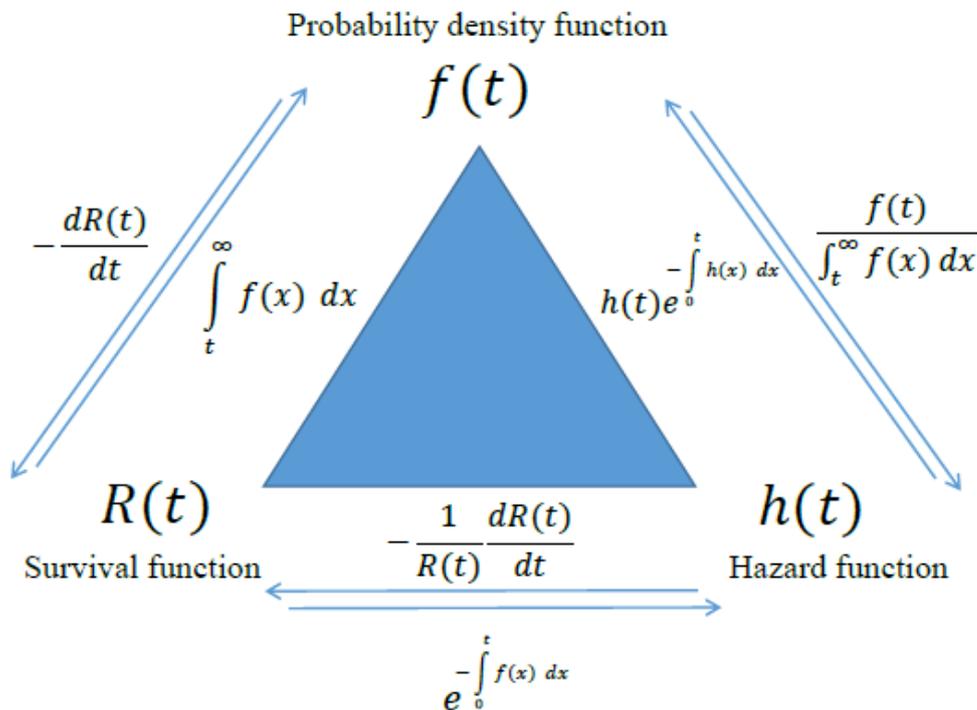


Figure (1.2): Triangle relationship

Example (1.2)[5]: The exponential distribution is a useful model of the time (or length) between failures in situations where the failures are happening at random and at a known rate, λ . The p.d.f. is then given by

$f(t) = \lambda e^{-\lambda t}$ for $t > 0$, We can find associated functions of exponential distribution

(i) Then the c.d.f is

$$\begin{aligned}
 F(t) &= P(T \leq t) \\
 &= \int_0^t f(t) dt \\
 &= \int_0^t \lambda \exp(-\lambda t) dt \\
 &= \lambda \left[\frac{\exp(-\lambda t)}{-\lambda} \right]_0^t \\
 &= \frac{\lambda}{-\lambda} [\exp[-\lambda t] - \exp[0]] \\
 &= -1 [\exp[-\lambda t] - 1]
 \end{aligned}$$

$$\text{i.e. } F(t) = 1 - e^{-\lambda t} \quad \dots(1.25)$$

(ii) Reliability Function

Then since

$$\begin{aligned}
 R(t) &= 1 - F(t) \\
 &= 1 - [1 - e^{-\lambda t}]
 \end{aligned}$$

$$\text{i.e. } R(t) = e^{-\lambda t} \quad \dots(1.26)$$

If the item's lifetime follows an exponential distribution.

(iii) Also, the hazard function is given by

$$h(t) = f(t)/R(t) \quad \dots(1.27)$$

So in this case

$$h(t) = \lambda e^{-\lambda t} / e^{-\lambda t} \quad \dots(1.28)$$

$$h(t) = \lambda \quad (\text{i.e. a constant})$$

(iv) *The mean time to failures (MTTF) is*

$$\begin{aligned} MTTF &= \int_0^{\infty} R(t) dt \\ &= \int_0^{\infty} \exp[-\lambda t] dt \\ &= \left[\frac{\exp[-\lambda t]}{-\lambda} \right]_0^{\infty} \\ &= \frac{\exp[-\infty] - \exp[-0]}{-\lambda} \\ &= \frac{1}{\lambda} \quad \dots(1.29) \end{aligned}$$

2.1 Simple systems

Suppose that we have to calculate the Reliability of a system made up of several components. The total reliability can be calculated by calculating the reliability of each individual component, and combining these individual reliabilities. The way in which they are combined depends on the way in which the components are connected. That is, whether they are connected [1-8]:

- 1– series
- 2– parallel
- 3– series – parallel.
- 4– parallel – series.
- 5– Combination of series and parallel.

2.2 Series Systems [7]

Consider a system of n components connected in series so that the system will only work (i.e. a signal will pass from I to O) if all of the components work.

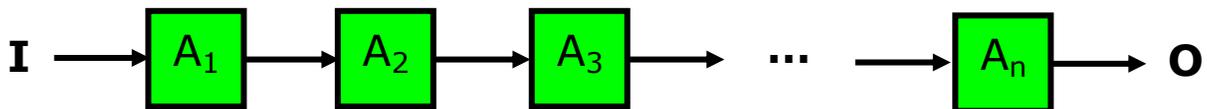


Figure (2.1)

If the components fail independent of each other it is easy to show that if R_1, R_2, \dots, R_n are the reliabilities of the individual components, then the reliability of the system is given by:

$$R_{\text{SYSTEM}} = R_1 \times R_2 \times \dots \times R_n \quad \dots(2.1)$$

Example (2.1)[8]

Consider a system of 3 components connected in series, each component having a constant failure rate. (In other words, the components have exponential lifetimes). These rates for components A, B and C are 0.3, 0.4 and 0.6 per 10,000 hours respectively. Thus we have



For constant failure rate λ , reliability

$$R(t) = e^{-\lambda t} \quad \dots(2.2)$$

Thus for component A, $R_A = e^{-0.3t}$

Similarly, $R_B = e^{-0.4t}$ and $R_C = e^{-0.6t}$.

Hence the reliability of the system is:

$$\begin{aligned} R(t) &= e^{-0.3t} \times e^{-0.4t} \times e^{-0.6t} \quad \dots(2.3) \\ &= e^{-0.3t - 0.4t - 0.6t} \\ &= e^{-1.3t} \end{aligned}$$

Then, for example, the probability that the system is still working after 30,000 hours = $R(3) = e^{-3.9} = 0.02024191145$.

2.3 Parallel Systems [5]

If n components are connected in parallel so that the system works (signal from I to O) as long as at least one of the components works.

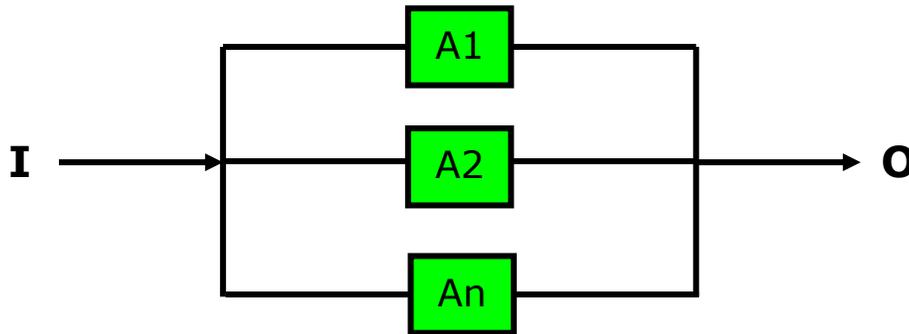


Figure (2.2): Parallel system

The reliability of the system (again assuming independent failures) is then:

$$\begin{aligned}
 R_{SYSTEM} &= 1 - P(\text{all fail}) \\
 &= 1 - [P(A1 \text{ fails}) \times P(A2 \text{ fails}) \times \dots \times P(A_n \text{ fails})] \\
 &= 1 - (1 - R_1)(1 - R_2) \dots (1 - R_n) \\
 R_{SYSTEM} &= 1 - \prod_{i=1}^n (1 - R_i) \quad \dots(2.4)
 \end{aligned}$$

Example (2.2) [4]

Component A with a constant failure rate of **1.5** per 1000 hrs and component B with a constant failure rate of **2** per 1000 hrs are connected in parallel. Find the overall reliability of this system. (Note that components A and B have exponential lifetimes).

Solution:

For the individual components, assuming a constant failure rate

$$R_A(t) = e^{-1.5t}$$

$$R_B(t) = e^{-2t}$$

Then if these components are connected in parallel,

$$\begin{aligned} R_{\text{system}}(t) &= 1 - [(1 - R_A(t)) \times (1 - R_B(t))] \\ &= 1 - [(1 - e^{-1.5t})(1 - e^{-2t})] \\ &= 1 - [1 - e^{-1.5t} - e^{-2t} + e^{-1.5t} e^{-2t}] \\ &= e^{-1.5t} + e^{-2t} - e^{-1.5t-2t} \\ &= e^{-1.5t} + e^{-2t} - e^{-3.5t} \quad \dots(2.5) \end{aligned}$$

Then, for example, the probability that the system is still working after 1000 hours is

$$\begin{aligned} R(1) &= e^{-1.5(1)} + e^{-2(1)} - e^{-3.5(1)} \\ &= 0.3283 \end{aligned}$$

i.e. there is a 33% chance that such a system will still be working after 1000 hours.

2.4 Series - Parallel systems [12]

The reliability of a system comprising n component connected in parallel redundancy, is

$$R_p = 1 - (1 - R)^n \quad \dots(2.6)$$

Where R is the reliability of an individual component.

If we place n sets in parallel, where each set has m components connected in series, we get the reliability of such a system as

$$R_s = 1 - (1 - R)^n$$

Where $R = \prod_{i=1}^m r_i$, and r_i is the reliability of the i th component in series. Thus,

$$R_s = 1 - \left(1 - \prod_{i=1}^m r_i\right)^n \quad \dots(2.7)$$

The series – parallel configuration is shown in fig (2.3)

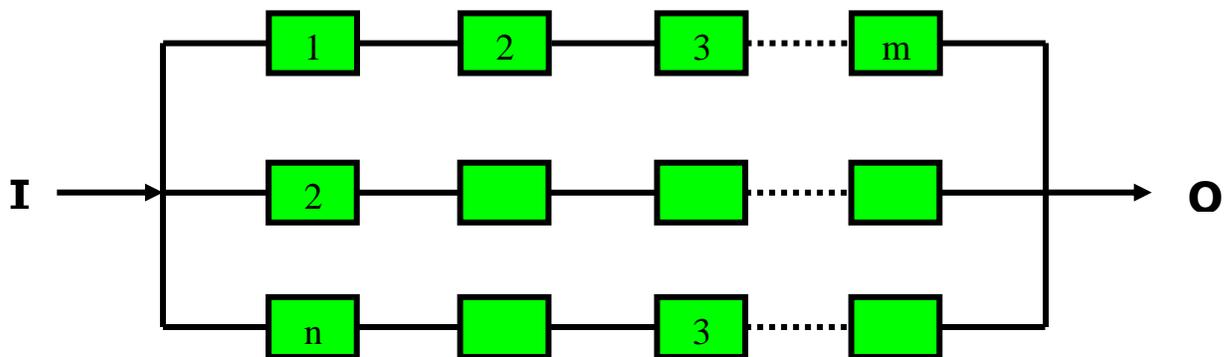


Fig (2.3) series – parallel system

Example (3)[9]

Compute the reliability of the system for the connection given in fig (2.4)

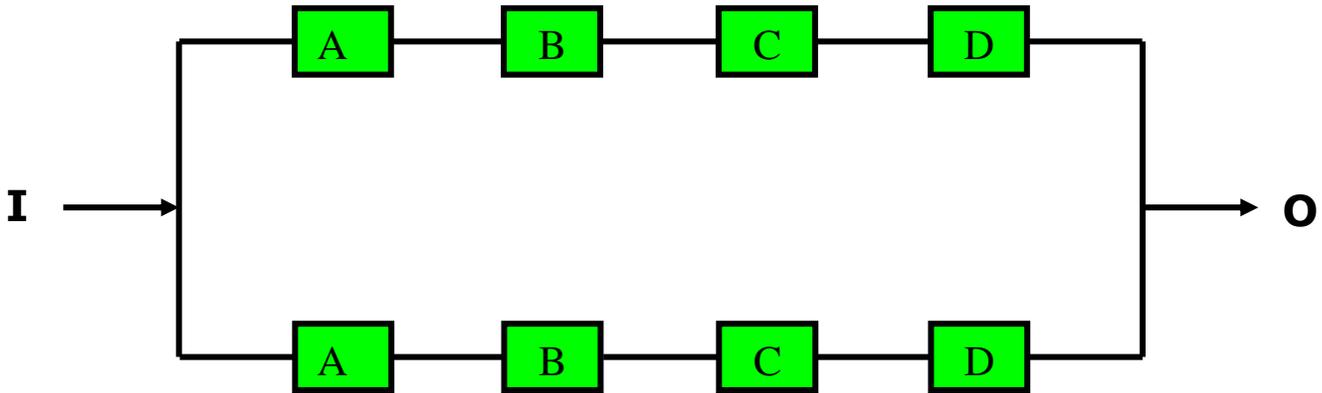


Figure (2.4) series – parallel system

The reliability of A, B, C and D are 0.95, 0.99, 0.90 and 0.96 respectively.

Making use of equation (2.7), we get the system reliability for $m = 4$ and $n = 2$.

$$\begin{aligned}
 R_S &= 1 - \left(1 - \prod_{i=1}^4 r_i\right)^2 \\
 &= 1 - [1 - (0.95 \times 0.99 \times 0.90 \times 0.96)]^2 \\
 &= 1 - (1 - 0.813)^2 \\
 &= 0.9650 \text{ or } 96.50\%
 \end{aligned}$$

2.5 Parallel – Series systems [13]

The reliability of n components connected in parallel is given by R_p

$$R_p = 1 - (1 - R)^n \quad \dots(2.8)$$

Where R is the reliability of an individual component.

If m such sets are connected in series, where each set consists of n component in parallel, then the reliability of the system is given by

$$R_s = [1 - (1 - R)^n]^m \quad (2.9)$$

The parallel – series configuration is depicted in fig (2.5)

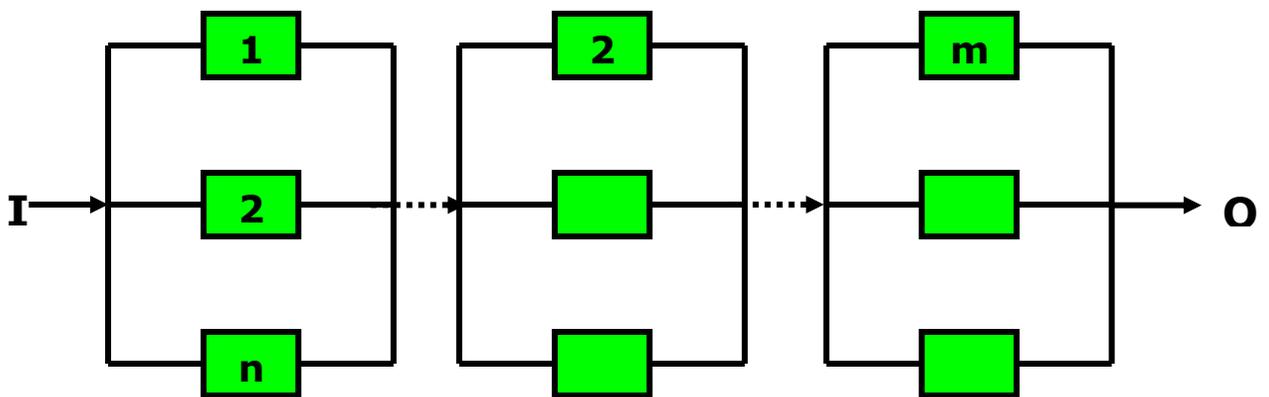


Fig. (2.5) parallel – series system

Example (4)[16]

Compute the reliability of the system for the connections given in fig (2.6):

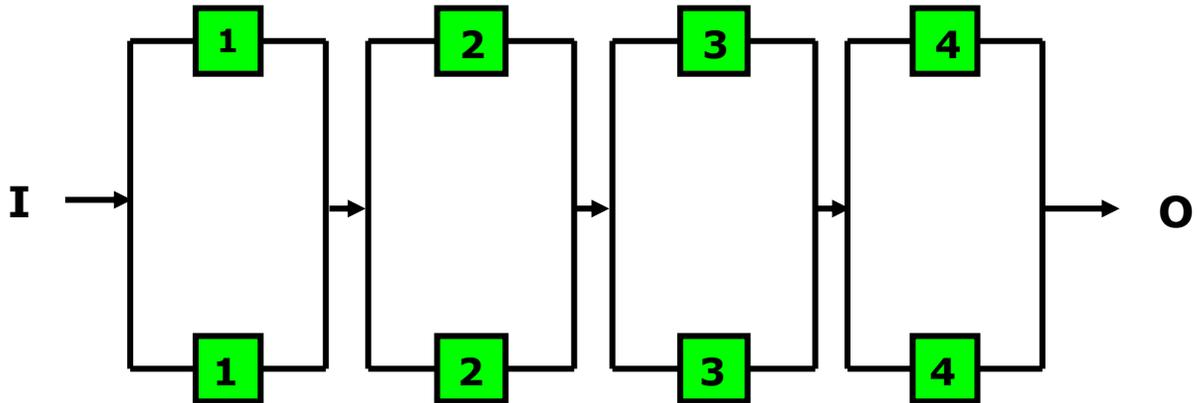


Fig (2.6) parallel – series system

Assuming the reliability of each component is 0.95

Here, we are given $n=2$, $m= 4$, using eq. (2.9), We get

$$\begin{aligned}
 R_S &= [1 - (1 - R)^n]^m \\
 &= [1 - (1 - 0.95)^2]^4 \\
 &= (0.9975)^4 = 0.9900 = 99\%
 \end{aligned}$$

3.1 Mixed Systems

If the system structure is not one of the simple forms, it becomes difficult to compute the exact reliability. To deal with the more general situation, we introduce a graphical network model in which it is possible to determine whether a system is working correctly by determining whether a successful path exists through the system. The system fails when no such path exists. We present some methods for computing the reliability [15-20].

- 1- The decomposition method.
- 2- Reduction to series elements.
- 3- The event space method.
- 4- The path-tracing method.
- 5- Minimal cut method.

3.2 Decomposition method [16]

One way to determine the reliability of complex system is decomposition method [dm] According to dm , a component is chosen close to the left or to the right end of the system block diagram this component is called "keystone" .then ,the conditional reliability of the system given that the "keystone" survives and the conditional system reliability given that The "key stone fails is computed the reliability of the whole system is then determined as a weighted average of these two conditional reliabilities, where the weights are the reliability of the "keystone " R and $1-R$, respectively. Let consider the key stone by C_x and the corresponding reliability by R_x ,then

$$R_{sys} = R_x \cdot R_{sys}/C_x + (1-R_x) R_{sys}/\bar{C}_x, \quad \dots(3.1)$$

Where R_{sys}/C_x ,

Is conditional reliability when the key stone survives

And R_{sys}/\bar{C}_x is conditional reliability when the keystone fail [13].

Example (3.1)[20]

Consider the following system:

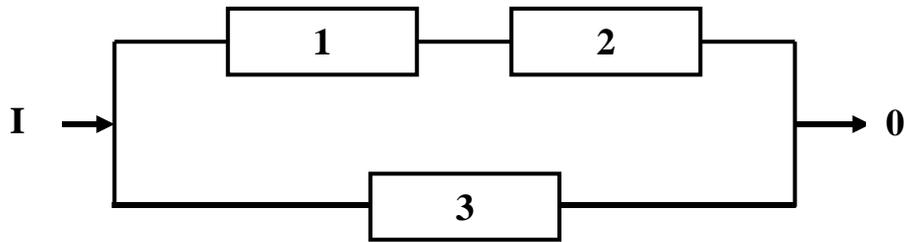


Figure (3.1) Complex Systems.

A is the event of Unit 1 success.

B is the event of Unit 2 success.

C is the event of Unit 3 success.

S is the event of system success

Selecting Unit 3 as the "key" component, the system reliability is:

$$R_s = P(S/C)P(C) + P(S/\bar{C})P(\bar{C}) \quad \dots (3.2)$$

If Unit 3 survives, then $P(S|C) = 1$

That is, since Unit 3 represents half of the parallel section of the system, then as long as it is operating, the entire system operates.

If Unit 3 fails, then the system is reduced to:



Figure(3.2)

$$P(s | \bar{C}) = R_1R_2$$

The reliability of the system is given by:

$$R_S = P(C) + R_1 R_2 P(\bar{C}) = R_3 + R_1 R_2 (1 - R_3)$$

Or:

$$R_S = R_3 + R_1 R_2 - R_1 R_2 R_3 \quad \dots (3.3)$$

3.3 Reduction to series element [18]

In this method we systematically replace each parallel path by an equivalent single path and ultimately reduce the given system to one system consisting of series element we can explain the solving of this method by the following steps[12].

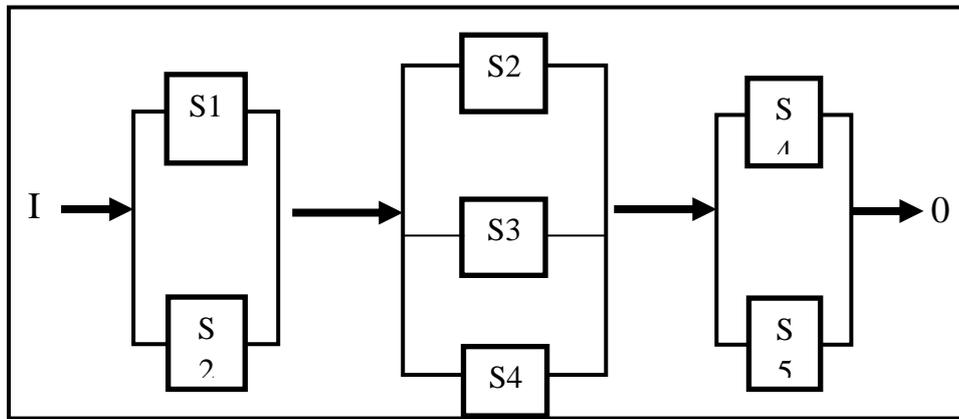


Figure (3.3) show reduction to series element.

step 1:- the **parallel path** s_1, s_2 will be first replaced by an equivalent series element say P_1

$$\begin{aligned} R_{p1}(t) &= 1 - [(1-R_1(t))(1-R_2(t))] \\ &= 1 - [1 - R_2(t) - R_1(t) + R_1(t)R_2(t)] \\ &= 1 - 1 + R_2(t) + R_1(t) - R_1(t)R_2(t) \\ &= R_2(t) + R_1(t) - R_1(t)R_2(t) \quad \dots (3.4) \end{aligned}$$

If we take $R_1 = R_2 = 0.9$, with independent identical units we get

$$R_{p1}(t) = R_2(t) + R_1(t) - R_1(t)R_2(t) = 0.9 + 0.9 - 0.9 \times 0.9 = 0.99$$

Step(2) :- parallel element S_2, S_3, S_4 can be replaced by an equivalent element whose reliability is obtained from the rule that say P_2

$$\begin{aligned}
 R_{p2}(t) &= 1 - [(1 - R_2(t))(1 - R_3(t))(1 - R_4(t))] \\
 &= 1 - [1 - R_3(t) - R_2(t) + R_2(t)R_3(t)](1 - R_4(t)) \\
 &= 1 - [1 - R_3(t) - R_2(t) + R_2(t)R_3(t) - R_4(t) + R_3(t)R_4(t) + \\
 &\quad R_2(t)R_4(t) - R_2(t)R_3(t)R_4(t)] \\
 &= R_3(t) + R_2(t) - R_2(t)R_3(t) + R_4(t) - R_3(t)R_4(t) - R_2(t)R_4(t) + \\
 &\quad R_2(t)R_3(t)R_4(t) \quad \dots(3.5)
 \end{aligned}$$

If we take $R_2=R_3=R_4 = 0.9$, with independent identical units we get

$$\begin{aligned}
 R_{p2}(t) &= R_3(t) + R_2(t) - R_2(t)R_3(t) + R_4(t) - R_3(t)R_4(t) \\
 &\quad - R_2(t)R_4(t) + R_2(t)R_3(t)R_4(t) \\
 &= 0.9 + 0.9 - 0.9 \times 0.9 + 0.9 - 0.9 \times 0.9 - 0.9 \times 0.9 \\
 &\quad + 0.9 \times 0.9 \times 0.9 = 0.999
 \end{aligned}$$

step (3):- parallel element S_4, S_5 can be replaced by an equivalent element whose reliability obtained from the rule that say P_3

$$\begin{aligned}
 R_{p3}(t) &= 1 - [(1 - R_4(t)) (1 - R_5(t))] \\
 &= 1 - [1 - R_5(t) - R_4(t) + R_4(t)R_5(t)] \\
 &= R_5(t) + R_4(t) - R_4(t)R_5(t) \quad \dots (3.6)
 \end{aligned}$$

If we take $R_3=R_4 = 0.9$, with independent identical units we get

$$R_{p3}(t) = R_5(t) + R_4(t) - R_4(t)R_5(t) = 0.9 + 0.9 - 0.9 \times 0.9 = 0.99$$

Step (4):- the system has now been reduced to a system Contains series element as :

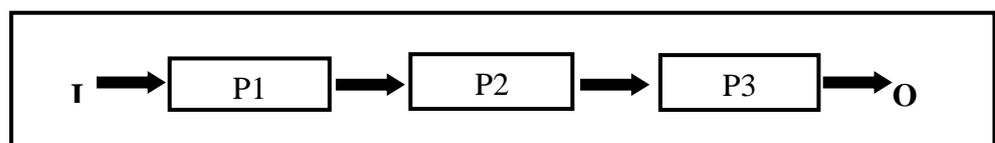


Figure (3.4) The system reliability

The system reliability of figure (3.4) is

$$R_s(t) = R_{p1}(t) \times R_{p2}(t) \times R_{p3}(t)$$

$$= 0.99 \times 0.999 \times 0.99 = 0.97$$

Example (3.2)[19]

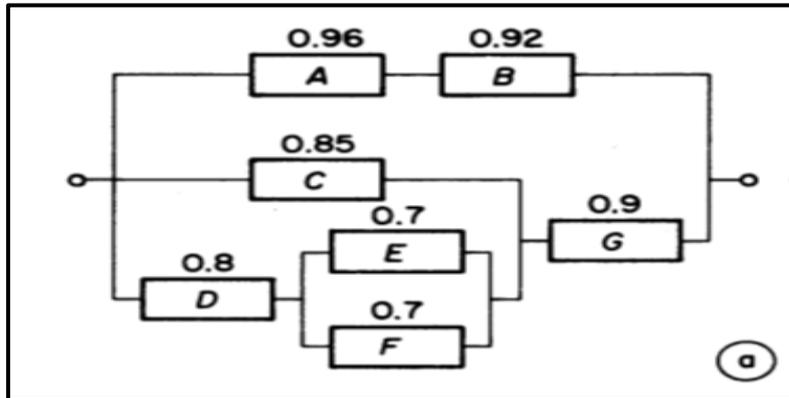


Figure (3.5)

$$R_{AB} = R_A \times R_B \quad \dots (3.7)$$

$$= 0.96 \times 0.92 = 0.88$$

$$R_{EF} = 1 - (1 - R_E)(1 - R_F)$$

$$= R_E + R_F - R_E R_F \quad \dots (3.8)$$

$$= 0.7 + 0.7 - 0.7 \times 0.7$$

$$= 1.4 - 0.49 = 0.91$$

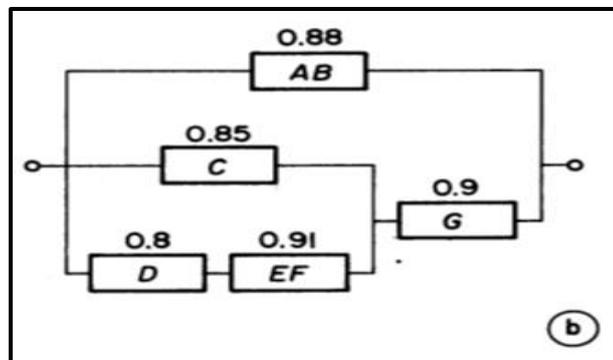


Figure (3.6)

$$R_{DEF} = R_D \times R_{EF} = 0.8 \times 0.91 = 0.73 \quad \dots (3.9)$$

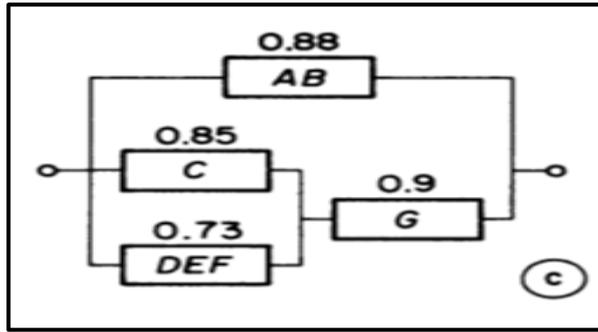


Figure (3.7)

$$\begin{aligned}
 R_{CDEF} &= 1 - (1 - R_C) (1 - R_{DEF}) \\
 &= R_C + R_{DEF} - R_C \times R_{DEF} \quad \dots (3.10) \\
 &= 0.85 + 0.73 - 0.85 \times 0.73 \\
 &= 1.58 - 0.6205 = 0.96
 \end{aligned}$$

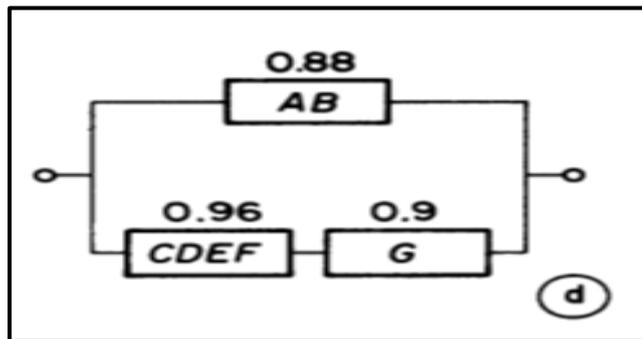


Figure (3.8)

$$\begin{aligned}
 R_{CDEFG} &= R_{CDEF} \times R_G \quad \dots (3.11) \\
 &= 0.96 \times 0.9 = 0.86
 \end{aligned}$$

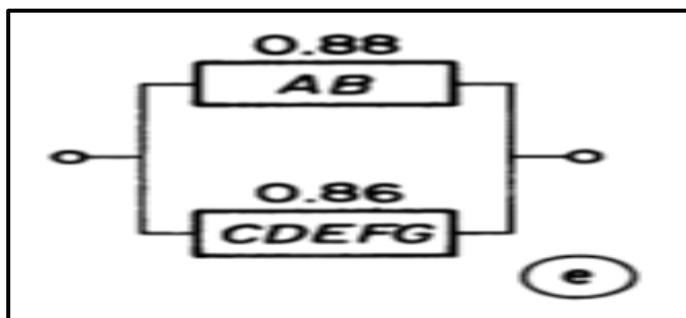


Figure (3.9)

$$R_{ABCDEF G} = 1 - (1 - R_{AB}) (1 - R_{CDEFG})$$

$$\begin{aligned}
 &= R_{AB} + R_{CDEFG} - R_{AB} \times R_{CDEFG} \quad \dots (3.12) \\
 &= 0.88 + 0.86 - 0.88 \times 0.86 \\
 &= 1.74 - 0.7568 \\
 &= 0.98
 \end{aligned}$$

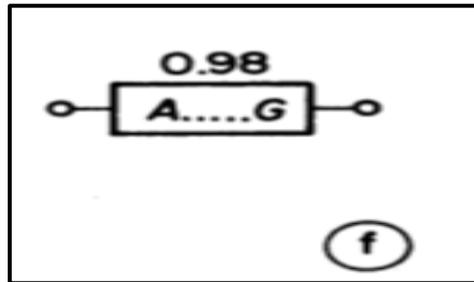


Figure (3.10)

3.4 Event space method [22]

The event space method is an application of the mutually exclusive event. The reliability of the system is simply the probability of the union of all mutually exclusive events that yield a system success.

Example (3.3) [11]

Consider the following system

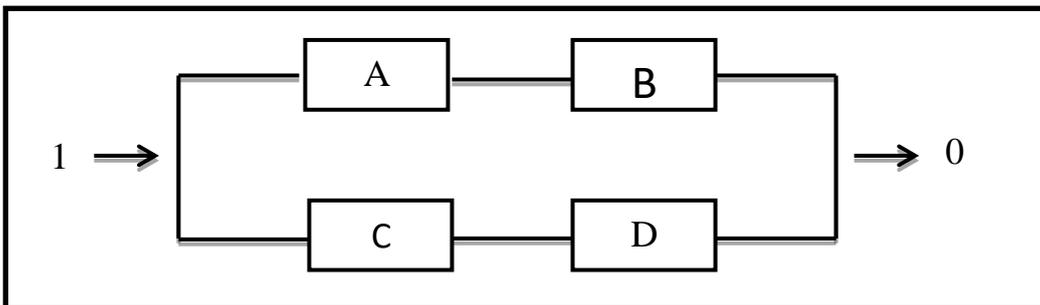


Figure (3.11)

A is event of unit 1 success

B is event of unit 2 success

C is event of unit 3 success

D is event of unit 4 success

$X_1 = ABCD$ all unit success

$X_2 = \bar{A}BCD$ unite1 fail and BCD success

$X_3 = \bar{A}\bar{B}CD$ unite2 fail and ACD success

$X_4 = \bar{A}B\bar{C}D$ unite3 fail and ABD success

$X_5 = \bar{A}BC\bar{D}$ unite4 fail and ABC success

$X_6 = \bar{A}\bar{B}\bar{C}D$ unite1,2 fail and C,D success

$X_7 = \bar{A}\bar{B}\bar{C}\bar{D}$ unite1,3 fail and B,D success

$X_8 = \bar{A}B\bar{C}\bar{D}$ unite1,4 fail and B,C success

$X_9 = \bar{A}\bar{B}\bar{C}\bar{D}$ unite2,3 fail and A,D success

$X_{10} = \bar{A}B\bar{C}\bar{D}$ unite3,4 fail and A,B success

$X_{11} = \bar{A}\bar{B}\bar{C}\bar{D}$ unite2,4 fail and A,C success

$X_{12} = \bar{A}\bar{B}\bar{C}\bar{D}$ unite1,2,3 fail and D success

$X_{13} = \bar{A}\bar{B}\bar{C}\bar{D}$ unite1,2,4 fail and C success

$X_{14} = \bar{A}\bar{B}\bar{C}\bar{D}$ unite1,3,4 fail and B success

$X_{15} = \bar{A}\bar{B}\bar{C}\bar{D}$ unite 2,3,4 fail and A success

$X_{16} = \bar{A}\bar{B}\bar{C}\bar{D}$ unite all fail

System event $X_7, X_8, X_9, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}$ result in system failure .those the probability of failure of the system

$$p_f = P(x_7 \cup x_8 \cup x_9 \cup x_{11} \cup x_{12} \cup x_{13} \cup x_{14} \cup x_{15} \cup x_{16}) \quad \dots (3.13)$$

Since event $x_7, x_8, x_9, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}$ and x_{16} are mutually exclusive then:-

$$p_f = P(x_7) + P(x_8) + P(x_9) + P(x_{11}) + P(x_{12}) + P(x_{13}) + P(x_{14}) + P(x_{15}) + P(x_{16}) \quad \dots (3.14)$$

and

$$P(x_7) = p(\overline{A}B\overline{C}D)$$

$$=(1-R_1) R_2(1-R_3) R_4$$

$$=(R_2 - R_1 R_2)(R_4 - R_3 R_4)$$

$$=R_2 R_4 - R_2 R_3 R_4 - R_1 R_2 R_4 + R_1 R_2 R_3 R_4$$

$$P(x_8) = p(\overline{A}BC\overline{D})$$

$$=(1-R_1) R_2 R_3(1-R_4)$$

$$=(R_2 - R_1 R_2)(R_3 - R_3 R_4)$$

$$=R_2 R_3 - R_2 R_3 R_4 - R_1 R_2 R_3 + R_1 R_2 R_3 R_4$$

$$P(x_9) = p(A\overline{B}C\overline{D})$$

$$=R_1 (1-R_2) (1-R_3) R_4$$

$$=(R_1 - R_1 R_2)(R_4 - R_3 R_4)$$

$$=R_1 R_4 - R_1 R_3 R_4 - R_1 R_2 R_4 + R_1 R_2 R_3 R_4$$

$$P(x_{11}) = p(A\overline{B}C\overline{D})$$

$$=R_1 (1-R_2) R_3(1-R_4)$$

$$=(R_1 - R_1 R_2)(R_3 - R_3 R_4)$$

$$=R_1 R_3 - R_1 R_3 R_4 - R_1 R_2 R_3 + R_1 R_2 R_3 R_4$$

$$P(x_{12}) = p(\overline{A}B\overline{C}D)$$

$$=(1-R_1) (1-R_2) (1-R_3) R_4$$

$$=(1-R_1) (1-R_2) (R_4 - R_3 R_4)$$

$$=(1-R_2 - R_1 + R_1 R_2) (R_4 - R_3 R_4)$$

$$=R_4 - R_2 R_4 - R_1 R_4 + R_1 R_2 R_4 - R_3 R_4 + R_2 R_3 R_4 + R_3 R_4 - R_1 R_2 R_3 R_4$$

$$P(x_{13}) = p(\overline{A}B\overline{C}\overline{D})$$

$$=(1-R_1) (1-R_2) R_3 (1-R_4)$$

$$\begin{aligned}
&= (1-R_1)(1-R_2)(R_3 - R_3R_4) \\
&= (1-R_2 - R_1 + R_1R_2)(R_3 - R_3R_4) \\
&= R_3 - R_2R_3 - R_1R_3 + R_1R_2R_3 - R_3R_4 + R_2R_3R_4 + \\
&\quad R_1R_3R_4 - R_1R_2R_3R_4
\end{aligned}$$

$$P(x_{14})=p(\overline{A}B\overline{C}D)$$

$$\begin{aligned}
&= (1-R_1)R_2(1-R_3)(1-R_4) \\
&= (1-R_1)(R_2 - R_2R_3)(1-R_4) \\
&= (R_2 - R_2R_3 - R_1R_2 + R_1R_2R_3)(1-R_4) \\
&= R_2 - R_2R_3 - R_1R_2 + R_1R_2R_3 - R_2R_4 + R_2R_3R_4 + R_1R_2R_4 + \\
&\quad R_1R_2R_4 - R_1R_2R_3R_4
\end{aligned}$$

$$P(x_{15})=p(A\overline{B}C\overline{D})$$

$$\begin{aligned}
&= R_1(1-R_2)(1-R_3)(1-R_4) \\
&= (R_1 - R_1R_2)(1-R_3)(1-R_4) \\
&= (R_1 - R_1R_3 - R_1R_2 + R_1R_2R_3)(1-R_4) \\
&= R_1 - R_1R_3 - R_1R_2 + R_1R_2R_3 - R_1R_4 + R_1R_3R_4 + R_1R_2R_4 + \\
&\quad -R_1R_2R_3R_4
\end{aligned}$$

$$P(x_{16})=p(\overline{A}B\overline{C}D)$$

$$\begin{aligned}
&= (1-R_1)(1-R_2)(1-R_3)(1-R_4) \\
&= (1-R_2 - R_1 + R_1R_2)(1-R_4 - R_3 + R_3R_4) \\
&= 1-R_4 - R_3 + R_3R_4 - R_2 + R_2R_4 + R_2R_3 - R_2R_3R_4 - R_1 + \\
&\quad R_1R_4 + R_1R_3 - R_1R_3R_4 + R_1R_2 + R_1R_2R_4 + R_1R_2R_3 - \\
&\quad R_1R_2R_3R_4
\end{aligned}$$

Combining term yields:-

$$\begin{aligned}
P_f &= R_2R_4 - R_2R_3R_4 - R_1R_2R_4 + R_1R_2R_3R_4 + \\
&R_2R_3 - R_2R_3R_4 - R_1R_2R_3 + R_1R_2R_3R_4 + R_1R_4 - R_1R_3R_4 - R_1R_2R_4 + \\
&R_1R_2R_3R_4 + R_1R_3 - R_1R_3R_4 - R_1R_2R_3 + R_1R_2R_3R_4 + R_4 - R_2R_4 -
\end{aligned}$$

$$\begin{aligned}
& R_1R_4 + R_1R_2R_4 - R_3R_4 + R_2R_3R_4 + R_1R_3R_4 - R_1R_2R_3R_4 + R_3 - \\
& R_2R_3 - R_1R_3 + R_1R_2R_3 - R_3R_4 + R_2R_3R_4 + R_1R_3R_4 - R_1R_2R_3R_4 + \\
& R_2 - R_2R_3 - R_1R_2 + R_1R_2R_3 - R_2R_4 + R_2R_3R_4 + R_1R_2R_4 - \\
& R_1R_2R_3R_4 + R_1R_3R_4 - R_1R_2 + R_1R_2R_3 - R_1R_4 + R_1R_3R_4 + \\
& R_1R_2R_4 - R_1R_2R_3R_4 + 1 - R_4 - R_3 + R_3R_4 - R_2 + R_2R_4 + R_2R_3 - \\
& R_2R_3R_4 - R_1 + R_1R_4 + R_1R_3 - R_1R_3R_4 + R_1R_2 - \\
& R_1R_2R_3R_4 \quad \dots(3.15)
\end{aligned}$$

Since

$$R_s(t) = 1 - p$$

$$\begin{aligned}
R_s(t) = & 1 - R_2R_4 + R_2R_3R_4 + R_1R_2R_4 - R_1R_2R_3R_4 - \\
& R_2R_3 + R_2R_3R_4 + R_1R_2R_3 - R_1R_2R_3R_4 - R_1R_4 + R_1R_3R_4 + R_1R_2R_4 - \\
& R_1R_2R_3R_4 - R_1R_3 + R_1R_3R_4 + R_1R_2R_3 - R_1R_2R_3R_4 - R_4 + R_2R_4 + \\
& R_1R_4 - R_1R_2R_4 + R_3R_4 - R_2R_3R_4 - R_1R_3R_4 + R_1R_2R_3R_4 - \\
& R_3 + R_2R_3 + R_1R_3 - R_1R_2R_3 + R_3R_4 - R_2R_3R_4 - R_1R_3R_4 + R_1R_2R_3R_4 \\
& - R_2 + R_2R_3 + R_1R_2 - R_1R_2R_3 + R_2R_4 - R_2R_3R_4 - R_1R_2R_4 + \\
& R_1R_2R_3R_4 - R_1 + R_1R_3 + R_1R_2 - R_1R_2R_3 + R_1R_4 - R_1R_3R_4 - \\
& R_1R_2R_4 + R_1R_2R_3R_4 - 1 + R_4 + R_3 - R_3R_4 + R_2 - R_2R_4 - R_2R_3 + \\
& R_2R_3R_4 + R_1 - R_1R_4 - R_1R_3 + R_1R_3R_4 - R_1R_2 + R_1R_2R_4 + \\
& R_1R_2R_3 + R_1R_2R_3R_4
\end{aligned}$$

Then

$$R_s(t) = R_3R_4 + R_1R_2 + R_1R_2R_3R_4$$

3.5 Path tracing method [21]

With the path –tracing method every path from a starting point to an ending point is considered. As long as at least one path from the beginning to the end of the path is available, then the system has not failed.

Example (3.4)

Consider the following system

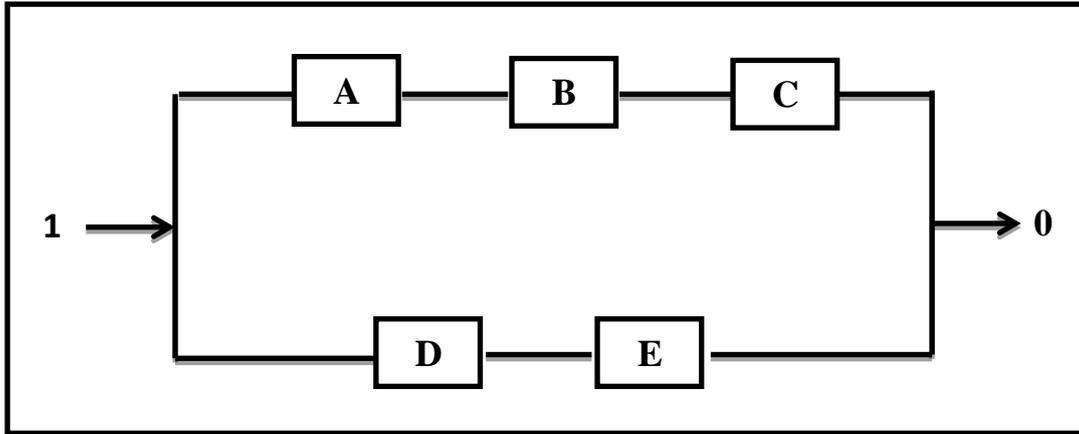


Figure (3.12)

Solution

$$X_1=A,B,C \quad \text{and} \quad X_2=D,E$$

$$\begin{aligned} P(X_1 \cup X_2) &= P(X_1) + P(X_2) - P(X_1 \cap X_2) \\ &= P(A,B,C) + P(D,E) - P(A,B,C,D,E) \end{aligned}$$

OR

$$R_S(t) = R_{A \cap B \cap C} + R_{D \cap E} - R_A R_B R_C R_D R_E$$

3.6 Minimal cut method[17]

A minimal cut set is a set of system components which, when fail, cause failure of the entire system; but when any one component in the set does not fail, the system will not fail as a whole, all components of each cut set must fail in order for the system to fail. Consequently, the components in one cut set are effectively connected in parallel. Furthermore if there are more than one cut set, then the system fails if all the components in any one of the cut set fails. Hence all the cut sets are effectively connected in series. Therefore, given a system / network, one

can a obtain a simple parallel – series model based on the cut sets for the reliability analysis.

Example (3.5)

Consider the following system :-

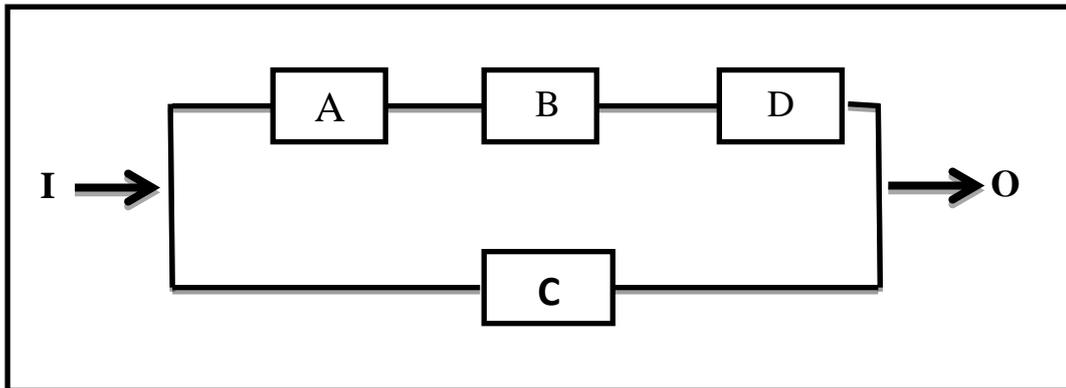


Figure (3.13) Complex systems.

Solution

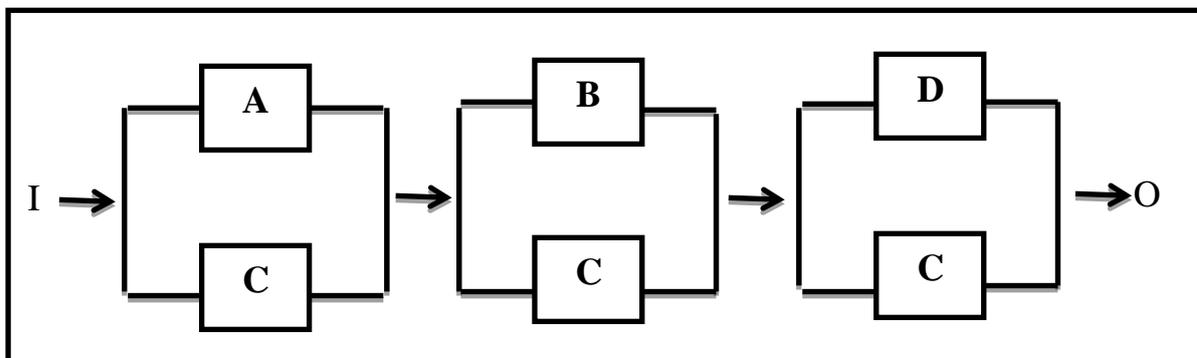


Figure (3.14)

$$R_{P1}(t) = 1 - [(1 - R_A)(1 - R_C)]$$

$$R_{P1} = R_A + R_C - R_A R_C \quad \dots (3.16)$$

$$R_{P2}(t) = 1 - [(1 - R_B)(1 - R_C)]$$

$$R_{P2} = R_B + R_C - R_B R_C \quad \dots (3.17)$$

$$R_{P3}(t) = 1 - [(1 - R_D)(1 - R_C)]$$

$$R_{P3} = R_D + R_C - R_D R_C \quad \dots (3.18)$$

$$R_s(t) = \prod_{i=1}^n R_{pi}(t) \\ = (R_A + R_C + R_A R_C)(R_B + R_C - R_B R_C)(R_D + R_C - R_D R_C) \quad \dots (3.19)$$

Now we solve the equation (3.19) we get

$$R_s(t) = \prod_{i=1}^n R_{pi}(t) \\ = [R_A R_B + R_A R_C - R_A R_B R_C + R_C R_B + R_C R_C - R_C R_B R_C - R_A R_C R_B - \\ R_A R_C R_C + R_A R_C R_B R_C](R_D + R_C - R_D R_C) \\ = R_A R_B R_D + R_A R_C R_D - R_A R_B R_C R_D + R_C R_B R_D R_C R_D + R_C R_B R_D + R_C R_C R_D - \\ R_C R_B R_C R_D - R_A R_C R_B R_D - R_A R_C R_C R_D + R_A R_C R_B R_C R_D + R_A R_B R_C + R_A R_C R_C - \\ R_A R_B R_C R_C + R_C R_C R_C - R_C R_B R_C R_C - R_A R_C R_B R_C - R_A R_C R_C R_C + R_A R_C R_B R_C R_C - \\ R_A R_B R_D R_C - R_A R_C R_D R_C + R_A R_B R_C R_D R_C - R_C R_B R_D R_C - \\ R_C R_C R_D R_C + R_C R_B R_C R_D R_C + R_A R_C R_B R_C R_D + R_A R_C R_C R_D R_C - \\ R_A R_C R_B R_C R_D R_C$$

$$R_s(t) = R^3 + R^3 - R^4 + R^3 + R^3 - R^4 - R^4 + R^5 + R^3 + R^3 - R^4 + R^3 + \\ R^3 - R^4 - R^4 - R^4 + R^5 - R^4 - R^4 + R^5 - R^4 - R^4 + R^4 + R^5 + R^5 - \\ R^5 - R^6$$

$$= 8R^3 - 12R^4 + 6R^5 - R^6$$

This same result as in equation (3.19)

3.7 A comparison between all methods

1- By using path tracing method.

Solution:- Assume starting and ending blocks that con not fail , as shown next.

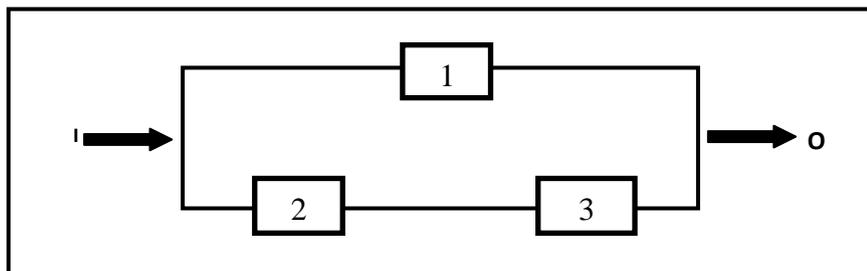


Figure (3.15) mixed system.

The path for this system are $x_1=1$ and $x_2=2,3$ the probability of success of the system is given by :-

$$P(x_1 \cup x_2) = p(x_1) + p(x_2) - p(x_1 \cap x_2) = p(1) + (p(2,3)) - p(1,2,3)$$

Or: - $R_s(t) = R_1 + R_2 R_3 - R_1 R_2 R_3$

This is of course the same result

2- By using minimal cut method.

When we apply this method to figure (3.15) , we get

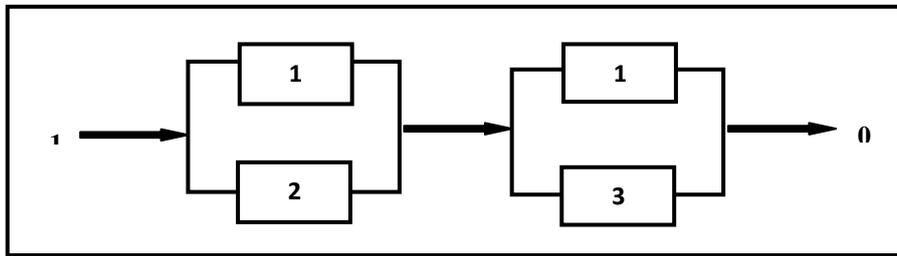


Figure (3.16)

$$R_{p1}(t) = 1 - [(1 - R_1)(1 - R_2)]$$

$$= R_1 + R_2 - R_1 \cdot R_2 \quad \dots (3.20)$$

$$R_{p2}(t) = 1 - [(1 - R_1)(1 - R_3)]$$

$$= R_1 + R_3 - R_1 \cdot R_3 \quad \dots (3.21)$$

$$R_s(t) = \prod_{i=1}^n R_{pi}(t) = (R_1 + R_2 - R_1 \cdot R_2)(R_1 + R_3 - R_1 \cdot R_3) \quad \dots(3.22)$$

Now we solve the equation (3.22) we get

$$R_s(t) = \prod_{i=1}^n R_{pi}(t) = R_1 R_1 + R_1 R_3 - R_1 R_1 R_3 + R_2 R_1 + R_2 R_3 - R_2 R_1 R_3 - R_1 R_2 R_1 - R_1 R_2 R_3 + R_1 R_2 R_3 R_4$$

This same result as in equation (3.22)

3-by using event space method.

When we apply this method to figure (3.15), we get

A is the event of Unit 1 success

B is the event of Unit 2 success

C is the event of Unit 3 success

The mutually exclusive system events are:

$X_1 = ABC$, all units succeed

$X_2 = \bar{A}BC$, only Unit 1 fails

$X_3 = A\bar{B}C$, only Unit 2 fails

$X_4 = AB\bar{C}$, only Unit 3 fails

$X_5 = \bar{A}\bar{B}C$, Units 1 and 2 fail

$X_6 = \bar{A}B\bar{C}$, Units 1 and 3 fail

$X_7 = A\bar{B}\bar{C}$, Units 2 and 3 fail

$X_8 = \bar{A}\bar{B}\bar{C}$, all units fail

System events X_5 , X_6 , and X_8 result in system failure. Thus the probability of failure of the system is:

$$P_f = p(X_5 \cup X_6 \cup X_8) \quad \dots (3.23)$$

Since events X_5 , X_6 , and X_8 are mutually exclusive, then:

$$P_f = p(X_5) + p(X_6) + p(X_8) \quad \dots (3.24)$$

And

$$P(X_5) = (\bar{A}\bar{B}C) = (1 - R_1)(1 - R_2)(R_3)$$

$$P(X_6) = (\bar{A}B\bar{C}) = (1 - R_1)(R_2)(1 - R_3)$$

$$P(X_8) = (\bar{A}\bar{B}\bar{C}) = (1 - R_1)(1 - R_2)(1 - R_3)$$

Combining terms yields:

$$P_f(t) = 1 - R_1 - R_2 R_3 + R_1 R_2 R_3 \quad \dots (3.25)$$

Since:

$$R_s(t) = 1 - p_f$$

Then:

$$R_s(t) = R_1 + R_2 R_3 - R_1 R_2 R_3$$

This is of course the same result as the one obtained previously using the decomposition method.

4-by using decomposition method.

When we apply this method to figure (3.15), we get

A is the event of Unit 1 success

B is the event of Unit 2 success

C is the event of Unit 3 success

S is the event of system success selecting unit 1 as "key" component, the system reliability is:

$$R_s = p(s/A) p(A) + p(s/\bar{A}) p(\bar{A}) \quad \dots (3.26)$$

If unit 1 success, then $p(s/A) = 1$

That is, since unit (1) represents half of the parallel section of the system then as long as it is operating, the entire system operates:-

If unit 1 fails then the system is reduced to:



Figure (3.16)

$$P(s/\bar{A}) = R_2 \cdot R_3$$

The reliability of the system is given by:

$$R_s = p(A) + R_2 R_3 p(\bar{A}) = R_1 + R_2 R_3 (1 - R_1)$$

OR:

$$R_s = R_1 + R_2 R_3 - R_1 R_2 R_3 \quad \dots (3.27)$$

5- By using Reduction to series element

In his method we systematically replace each parallel path by an equivalent single path, and ultimately reduce the given system to one consisting of only series element. we shall assume the element to be independent

step1: Accordingly, element A, B in figure are first replaced by an equivalent single element

Say s_1 .using the probability rules, we get

$$P(s_1) = p(1 \text{ and } 2) = p(1) \cdot p(2) \quad \dots(3.28)$$

Or: $R_{s1} = R_1 \cdot R_2$

Step 2: The given system has now been modified to that shown in figure

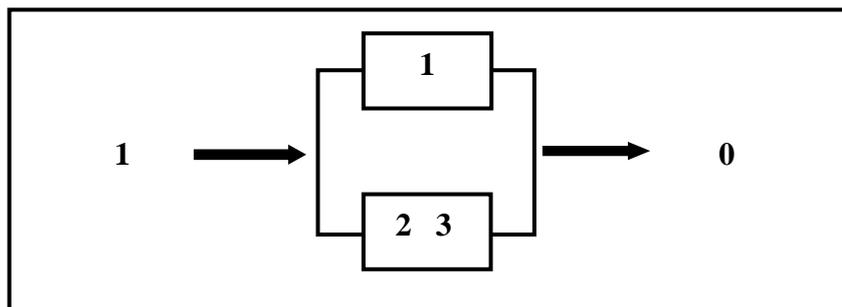


Figure (3.17)

Now $R_{s2} = 1$

Therefore, the reliability of the system is

$$R_s(t) = 1 - (1 - R_{s1}(t))$$

$$= 1 - [(1 - R_{s1}) (1 - R_{s2})] = R_{s1} + R_{s2} - R_{s1} \cdot R_{s2} \quad \dots (3.29)$$

And we substitute R_s , R_{s2} by equivalent element we obtain

$$R_s(t) = R_1 + R_2 \cdot R_3 - R_1 \cdot R_2 \cdot R_3 \quad \dots (3.30)$$

This is of course the same result as in equation (3.2).

4.1 Reliability allocation [22]

Reliability allocation is the process by which the failure allowance for a system is allocated in some logical manner to its sub-systems and elements. The purpose of reliability allocation is to establish a goal or objective for the reliability of each component so that the manufacturers can have an idea of the performance required of this product.

The reliability factor of a system is known or is specified on the basis of the overall mission required. If the system comprises many elements and units, we must have a method to determine the reliability factor for each of them.

Consider a system consisting of n components (dependent or independent) with individual reliability factors $R_1, R_2, R_3, \dots, R_n$. The system reliability $R(t)$ is a function of these components' reliability. Thus,

$$R(t) = f(R_1, R_2, R_3, \dots, R_n) \quad \dots(4.1)$$

The problem now is to determine the values of $R_1, R_2, R_3, \dots, R_n$ for a given value of $R(t)$. This problem will not have a unique answer; at the same time, the values of $R_1, R_2, R_3, \dots, R_n$ cannot be altogether arbitrary. The problem can be viewed from two aspects. First, if we allocate the reliability requirement in some logical manner among the n components, a goal will be set for the manufacturer to produce components with these reliability factors. Second, if the state of art is such that the reliability R_i of a particular component i cannot possibly be improved upon, and if the allocated reliability factor is higher.

In our project we shall discuss two methods by which the reliability can be logically apportioned among the constituent elements of a system. The problem will not be treated in its most general form as stated in eq.

(4.1), but in a slightly different manner. The reliability or the predicted failure probability of each component is obtained from its failure – data analysis. If the failure allowance specified for the system is apportioned among the components in some equitable manner [8].

4.2 Reliability allocation for a series system [8]

We discuss the reliability allocation for series system by using two methods.

1. First method

The system consisting of n components connected in series .the principle adopted in this method for subdividing the system failure allowance is that the failure allowance of each component is directly proportional to the predicted probability of failure. This rule based on the assumption that the components exhibit a constant failure rate, and if λ_i is the failure rate of component i ,

$i= 1, \dots , n$ then we can use the approximation :

$$R_s(t) = e^{-\lambda_s t} \approx 1 - \lambda_s t \quad \dots(4.2)$$

Where $\lambda_s = \sum_{i=1}^n \lambda_i$ λ_s is termed the predicted system failure rate.

λ^*_s be the specified system failure rate which is assumed to be less than λ_s .to compute the required failure rate for component I which is denoted by λ^*_i , $i = 1, 2, \dots, n$, we use the following equation :

$$\lambda^*_i t = (\lambda_i t / \lambda_s t) \cdot \lambda^*_s t \quad \dots(4.3)$$

So the reliability goal for the component i is

$$R^*_i(t) = e^{-\lambda^*_i t} \approx 1 - \lambda^*_i t \quad \dots(4.4)$$

Example (4.1)

A system is composed of 4 units connected in series the failure rate for these units are as follows:-

$$\lambda_1 = 0.05 \quad , \quad \lambda_2 = 0.2$$

$$\lambda_3 = 0.25 \quad , \quad \lambda_4 = 0.04$$

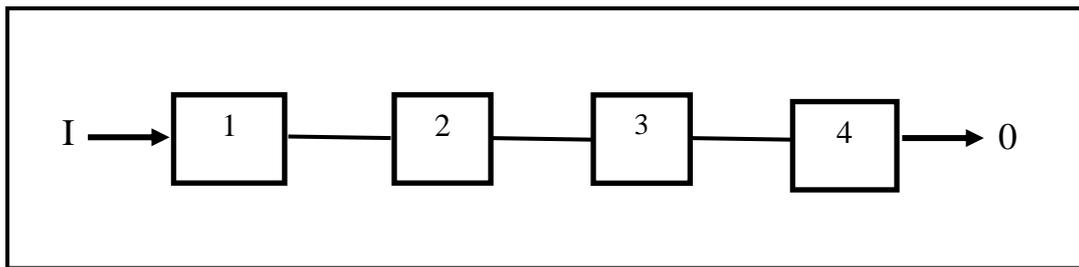


Figure (4.1)

it is desired that the maximum failure rate for the system be $\lambda^* S = 0.5$

solution :-

the sum of the unite failure rate

$$\lambda_s = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \quad \dots(4.5)$$

$$= 0.05 + 0.2 + 0.25 + 0.04$$

$$= 0.54$$

Hence , the allocated unite failure rate are

$$\lambda^* 1 = \left(\frac{0.5}{0.54}\right) \times 0.05 = 0.0462963$$

$$\lambda^* 2 = \left(\frac{0.5}{0.54}\right) \times 0.2 = 0.1851851852$$

$$\lambda^* 3 = \left(\frac{0.5}{0.54}\right) \times 0.25 = 0.2314814815$$

$$\lambda^* 4 = \left(\frac{0.5}{0.54}\right) \times 0.04 = 0.037037037$$

(ii) If $\lambda^*_1, \lambda^*_2, \lambda^*_3, \lambda^*_4$ are allocated failure rate, we get

$$R^*_s(t) = e^{-\lambda^*_1 t} e^{-\lambda^*_2 t} e^{-\lambda^*_3 t} e^{-\lambda^*_4 t} \quad \dots(4.6)$$

$$\lambda^*_1 = \left(\frac{\lambda_s}{\lambda^*_s}\right) \lambda_1 \cdot \lambda^*_2 = \left(\frac{\lambda_s}{\lambda^*_s}\right) \lambda_2$$

$$\lambda^*_3 = \left(\frac{\lambda_s}{\lambda^*_s}\right) \lambda_3 \cdot \lambda^*_4 = \left(\frac{\lambda_s}{\lambda^*_s}\right) \lambda_4$$

Hence, the allocated reliabilities are

$$R_1(t) = e^{-\left(\frac{\lambda_s}{\lambda^*_s}\right) \lambda_1 t} \quad \dots (4.7)$$

$$= e^{-\left(\frac{0.45}{0.5}\right) 0.05 t} = e^{-0.045 t} = 0.955997$$

$$R_2(t) = e^{-\left(\frac{\lambda_s}{\lambda^*_s}\right) \lambda_2 t} \quad \dots(4.8)$$

$$= e^{-\left(\frac{0.45}{0.5}\right) 0.2 t} = e^{-0.18 t} = 0.835270$$

$$R_3(t) = e^{-\left(\frac{\lambda_s}{\lambda^*_s}\right) \lambda_3 t} \quad \dots(4.9)$$

$$= e^{-\left(\frac{0.45}{0.5}\right) 0.25 t} = e^{-0.225 t} = 0.798516$$

$$R_4(t) = e^{-\left(\frac{\lambda_s}{\lambda^*_s}\right) \lambda_4 t} \quad \dots(4.10)$$

$$= e^{-\left(\frac{0.45}{0.5}\right) 0.04 t} = e^{-0.036 t} = 0.964640$$

$$R^*_S = R^*_1 \times R^*_2 \times R^*_3 \times R^*_4$$

$$= 0.95 \times 0.83 \times 0.79 \times 0.96$$

$$\cong 0.59$$

2. Second method[13],[18]

In this method, known as the minimum effort method, the reliability of only those components is increased that have a value less than a certain figure. Further, the reliability of each of those components is raised to a common value and not apportioned in the sense in which it was done in first method.

Let the system consist of n component connected in series, and let $R_1, R_2, R_3, \dots, R_n$ be their individual reliabilities .the system reliability will be

$$R_s(t) = R_1 \times R_2 \times R_3 \times \dots \times R_n$$

Let $R_s^*(t)$ be the desired reliability for the system, and assume that $R_s^*(t)$ is greater than R_s .we have now to increase the reliability of a minimum number of components to a value such that the new reliability of the system becomes $R_s^*(t)$. To achieve this we proceed as follows:

- (a) The known reliabilities of the component are arranged in an ascending order. Having done this assume that

$$R_1 \leq R_2 \leq R_3 \leq \dots \leq R_n \quad \dots(4.11)$$

- (b) The reliabilities of the first K components in eq (4.11) are increased to the same value R_0 .

The reliabilities $R_{k+1}, R_{k+2}, \dots, R_n$ at the remaining component are left unchanged. For determining k and R_0 the first is to calculate the value of

$$\left[\frac{R_s^*}{R_2 \times R_3 \times \dots \times R_n} \right]^{\frac{1}{k}} \quad \dots(4.12)$$

This is compared with R_1 .if

$$\left[\frac{R_S^*}{R_2 \times R_3 \times \dots \times R_n} \right]^{1/1} > R_1 \quad \dots(4.13)$$

Then R_1 will have to be increased to the value R_0 . The next step is to calculate the value of

$$\left[\frac{R_S^*}{R_3 \times R_4 \times \dots \times R_n} \right]^{1/2} \quad \dots(4.14)$$

This is compared with R_2 .if

$$\left[\frac{R_S^*}{R_3 \times R_4 \times \dots \times R_n} \right]^{1/2} > R_2 \quad \dots(4.15)$$

Then R_2 will have to be increased to the value R_0 . This procedure is continued till.

$$\left[\frac{R_S^*}{R_{k+1} \times R_{k+2} \times \dots \times R_n} \right]^{1/k} > R_k \quad \dots(4.16)$$

Then R_k will also have to be increased to the value R_0 . In general, k is the maximum value of j such that

$$\left[\frac{R_S^*}{\prod_{i=j+1}^{n+1} R_i} \right]^{1/j} > R_j \quad \dots(4.17)$$

Where R_{n+1} .the reason for introducing R_{n+1} is to include the possibility that in a system it may be necessary to raise the reliabilities $R_1, R_2, R_3, \dots, R_n$ of all the n components to common higher value , whose maximum value will be at most equal to $R_{n+1}=1$.

Example (4.2):-

Four units are connected in series, with reliabilities $R_1 = 0.8$, $R_2 = 0.92$, $R_3 = 0.82$ and $R_4 = 0.9$. Calculate the system reliability. If the reliability is to be increased to a value of 0.62, how should this be apportioned among the four units according to the minimum effort method?

Solution

$$0.8 \leq 0.82 \leq 0.9 \leq 0.92$$

The system reliability R^*S for these units in series is

$$RS(t) = R_1 \cdot R_2 \cdot R_3 \cdot R_4$$

$$= 0.8 \times 0.92 \times 0.82 \times 0.9 = 0.54316$$

The value of R^*S is 0.65 following the procedure explained earlier we have.

$$\left[\frac{R^*S}{R_2 \cdot R_3 \cdot R_4 \cdot R_{n+1}} \right]^{\frac{1}{1}} = \left[\frac{0.62}{0.92 \cdot 0.82 \cdot 0.9 \cdot 1.00} \right]^{\frac{1}{1}} = \left[\frac{0.62}{0.67896} \right]^{\frac{1}{1}} = 0.9131613$$

Since $0.913 > 0.8$, the value of R_1 will have to be increased to R_0

next

$$\left[\frac{R^*S}{R_3 \cdot R_4 \cdot R_{n+1}} \right]^{\frac{1}{2}} = \left[\frac{0.62}{0.82 \cdot 0.9 \cdot 1.00} \right]^{\frac{1}{2}} = [0.84]^{\frac{1}{2}} = 0.91$$

Since $0.92 > 0.91$ the value of R_2 will have to be fixed

Next

$$\left[\frac{R^*S}{R_4 \cdot R_{n+1}} \right]^{\frac{1}{3}} = \left[\frac{0.62}{0.9 \cdot 1.00} \right]^{\frac{1}{3}} = 0.883$$

Since $0.88 > 0.82$ the value of R_3 to be increased to R_0 Next

$$\begin{aligned} \left[\frac{R^*S}{R_{n+1}} \right]^{\frac{1}{4}} &= \left[\frac{0.65}{1.00} \right]^{\frac{1}{4}} \\ &= [0.62]^{\frac{1}{4}} = 0.88 \end{aligned}$$

Since $0.9 > 0.88$ the value of R_4 will have fixed

$$\begin{aligned} R^*S &= R_1^* \times R_2^* \times R_3^* \times R_4^* \\ &= 0.91 \times 0.91 \times 0.883 \times 0.88 = 0.64 \end{aligned}$$

Example(4.3)

Four units are connected in series, with reliabilities $R_1 = 0.72$, $R_2 = 0.7$, $R_3 = 0.92$ and $R_4 = 0.82$. Calculate the system reliability. If the reliability is to be increased to a value of 0.5, how should this be apportioned among the four units according to the minimum effort method?

Solution

$$0.7 \leq 0.72 \leq 0.82 \leq 0.92$$

The system reliability R^*s for these units in series is

$$\begin{aligned} RS(t) &= R_1 \cdot R_2 \cdot R_3 \cdot R_4 \\ &= 0.72 \times 0.7 \times 0.92 \times 0.82 = 0.38 \end{aligned}$$

The value of R^*s is 0.5 following the procedure explained earlier we have.

$$\left[\frac{R^*s}{R_2 \cdot R_3 \cdot R_4 \cdot R_{n+1}} \right]^{\frac{1}{1}} = \left[\frac{0.5}{0.7 \cdot 0.92 \cdot 0.82 \cdot 1.00} \right]^{\frac{1}{1}} = \left[\frac{0.5}{0.528} \right]^{\frac{1}{1}} = 0.94$$

Since $0.94 > 0.72$, the value of R1 will have to be increased to R0 Next

$$\left[\frac{R^*s}{R_3 \cdot R_4 \cdot R_{n+1}} \right]^{\frac{1}{2}} = \left[\frac{0.5}{0.92 \cdot 0.82 \cdot 1.00} \right]^{\frac{1}{2}}$$

$$\left[\frac{0.5}{0.754} \right]^{\frac{1}{2}} = [0.66]^{\frac{1}{2}} = 0.81$$

Since $0.81 > 0.7$ the value of R2 will have to be increased to R0

Next

$$\left[\frac{R^*s}{R_4 \cdot R_{n+1}} \right]^{\frac{1}{3}} = \left[\frac{0.5}{0.82 \cdot 1.00} \right]^{\frac{1}{3}} =$$

$$\left[\frac{0.5}{0.82} \right]^{\frac{1}{3}} = [0.609]^{\frac{1}{3}} = 0.84$$

Since $0.92 > 0.84$ the value of R3 will have fixed Next

$$\left[\frac{R^*s}{R_{n+1}} \right]^{\frac{1}{4}} = \left[\frac{0.5}{1.00} \right]^{\frac{1}{4}}$$

$$= [0.5]^{\frac{1}{4}} = 0.84$$

Since $0.84 > 0.82$ the value of R4 will have to be increased to R0

$$R^*s = R^*1 \times R^*2 \times R^*3 \times R^*4$$

$$= 0.94 \times 0.81 \times 0.84 \times 0.84 = 0.53$$

4.3 Reliability allocation for a mixed system [12]

Similarly, we can find the reliability allocation for complex system by using previous methods

Example (4.4): A system consists of seven elements connected as shown in figure the predicated reliabilities of the components for a 10 hour period are also shown.

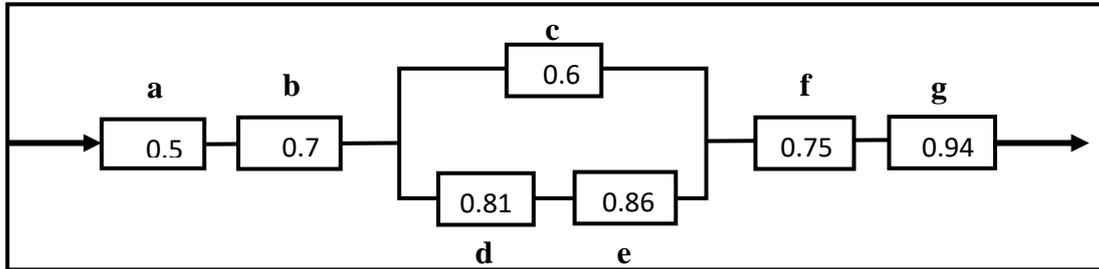


Figure (4.2)

- 1- Convert the given system into an equivalent system consisting of only series element.
- 2- Calculate the system reliability
- 3- If the system reliability is to be improved to value of 0.95 for a 10-hour period, determine the reliability goal of each component assume constant failure rate of all element without using the approximation $R(t) \approx 1 - \lambda t$

Solution :For calculating the system reliability, the given system converted into an equivalent system consisting of only series components c, d ,e mentioned in the reduction method

$$R_m(t) = R_c + R_d R_e - R_c R_d R_e = (0.6) + (0.81)(0.86) - (0.6)(0.81)(0.86)$$

(by probability rules) = 0.878

Now the given system has been reduced to a series system as shown in figure.

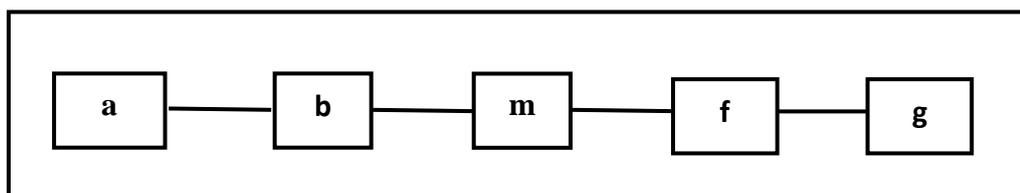


Figure (4.3)

We have, the reliability of the systems

$$R_a(t) = \exp(-\lambda_a t) = 0.5$$

$$R_b(t) = \exp(-\lambda_b t) = 0.7$$

$$R_m(t) = \exp(-\lambda_m t) = 0.87$$

$$R_f(t) = \exp(-\lambda_f t) = 0.75$$

$$R_g(t) = \exp(-\lambda_g t) = 0.94$$

The value of λt would have been

$$\lambda_a t = 0.5$$

$$\lambda_b t = 0.3$$

$$\lambda_m t = 0.12$$

$$\lambda_f t = 0.25$$

$$\text{and } \lambda_g t = 0.06$$

1- Thus the system reliability is given by

$$R_s(t) = R_a(t) \times R_b(t) \times R_m(t) \times R_f(t) \times R_g(t)$$

$$= \exp [-(\lambda_a t + \lambda_b t + \lambda_m t + \lambda_f t + \lambda_g t)]$$

$$= \exp(-\lambda_s t)$$

$$\text{Where } \lambda_s t = 0.5 + 0.3 + 0.122 + 0.25 + 0.06$$

$$= 1.232 \text{ (from eq(4.1))}$$

$$R_s(t) = \exp(-1.232) = 0.29$$

2- The desired reliability $R^*_s(t) = 0.95$

For the system ,therefore

$$R^*_{s(t)} = e^{-\lambda^* st} = 0.95 \quad \lambda^* st = 0.05$$

$$R^*_{s(t)} = e^{-\lambda^* at} e^{-\lambda^* bt} e^{-\lambda^* mt} e^{-\lambda^* ft} e^{-\lambda^* gt}$$

$$\lambda^* a = (\lambda^* s / \lambda_s) \lambda_a, \lambda^* b = (\lambda^* s / \lambda_s) \lambda_b$$

$$\lambda^* m = (\lambda^* s / \lambda_s) \lambda_m, \lambda^* f = (\lambda^* s / \lambda_s) \lambda_f$$

$$\lambda^* g = (\lambda^* s / \lambda_s) \lambda_g$$

Hence the allocated reliabilities are

$$R^*_{a(t)} = e^{-(\lambda^* s / \lambda_s) \lambda_a t}$$

$$R^*_{b(t)} = e^{-(\lambda^* s / \lambda_s) \lambda_b t}$$

$$R^*_{m(t)} = e^{-(\lambda^* s / \lambda_s) \lambda_m t}$$

$$R^*_{f(t)} = e^{-(\lambda^* s / \lambda_s) \lambda_f t}$$

$$R^*_{g(t)} = e^{-(\lambda^* s / \lambda_s) \lambda_g t}$$

$$R^*_{a(t)} = e^{\frac{-(0.051)0.5}{1.232}} = 0.96$$

$$R^*_{b(t)} = e^{\frac{-(0.051)0.7}{1.232}} = 0.95$$

$$R^*_{m(t)} = e^{\frac{-(0.051)0.878}{1.232}} = 0.94$$

$$R^*_{f(t)} = e^{\frac{-(0.051)0.99}{1.232}} = 0.95$$

$$\text{And } R^*_{g(t)} = e^{\frac{-(0.051)0.99}{1.232}} = 0.99$$

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الخلاصة

في هذا البحث تم دراسة وايجاد موثوقية بعض الأنظمة مثل متواليه، المتوازية، المتواليه المتوازية، المتوازية المتواليه والانظمة المختلطة باستخدام عدة طرق منها طريقة تحديد المسار، القطع، الاختزال الى عنصر واحد، التركيب وطريقة الحدث.

كذلك تم دراسة تخصيص الموثوقية على مركبات الأنظمة المختلطة وذلك عندما نرغب بتحديد موثوقية معينة ينبغي الوصول اليها في النظام المعني سواء كان ذلك النظام هو متوالي او نظام مختلط.

بِسْمِ اللّٰهِ الرَّحْمٰنِ الرَّحِیْمِ

﴿هُوَ الَّذِي جَعَلَ الشَّمْسَ ضِيَاءً وَالْقَمَرَ نُورًا وَقَدَّرَهُ

مَنَازِلَ لِتَعْلَمُوا عَدَدَ السِّنِينَ وَالْحِسَابَ مَا خَلَقَ

اللَّهُ ذَٰلِكَ إِلَّا بِالْحَقِّ يُفَصِّلُ الْآيَاتِ لِقَوْمٍ يَعْلَمُونَ﴾

صَدَقَ اللَّهُ الْعَلِيُّ الْعَظِيمُ

آية 5 سورة يونس



جمهورية العراق
وزارة التعليم العالي والبحث العلمي
جامعة بابل - كلية التربية للعلوم الصرفة
قسم الرياضيات

دراسة بعض نماذج الموثوقية

بحث مقدم الى

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كجزء من متطلبات نيل شهادة الدبلوم العالي تربية / الرياضيات

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