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Nonlinear Analysis of Reinforced Concrete Space Frames under Cyclic Loading Including Time Dependent Effects

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جمهورية العراق
وزارة التعليم العالي و البحث العلمي
جامعة بابل
كلية الهندسة
قسم الهندسة المدنية

التحليل اللاخطي للهياكل الخرسانية المسلحة الفضائية تحت الأثقال الدورية متضمنا التأثيرات المعتمدة على الزمن.

رسالة مقدمة لكلية الهندسة - جامعة بابل كجزء من متطلبات الحصول على درجة الدكتوراه
في الفلسفة في الهندسة المدنية (الانشاءات)

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ABSTRACT

An investigation of the nonlinear analysis of reinforced concrete space frames having prismatic and / or non-prismatic (tapered) members and subjected to cyclic varying loading is presented in this study. Plastic zone model is utilized in this study with an incremental and iterative technique to study the inelastic behavior of reinforced concrete space frames.

A new proposed approach namely (**regions approach**) is used to take into account the variation of material properties through the depth and width of the section. Many parameters are included in the present approach, they are: effects of coupling between bending moments, shear and axial forces, material non-linearity, geometrical non-linearity, and time dependent effects (creep and shrinkage).

A bond stress – slip model and a new technique are proposed to solve the bond – slip problem. This technique describes the hysteretic behavior of a single reinforcing bar in a reinforcement concrete beam-column element. This technique focuses attention on the stress transfer between reinforcing bars and surrounding concrete through bond.

Equivalent nodal loads (fixed – end forces) are presented for tapered element under uniformly distributed load (in xy – plane or xz – plane) considering the possibility of existence of plastic zone anywhere in the element.

Kupfer et.al. yield criterion is used as a limitation for the concrete behavior. The hardening / softening rule, flow rule, and tension stiffening rule for concrete are taken into account. Failure can be predicted by the concrete crushing at a certain region. The non-linear cyclic behavior model for concrete and the non-linear cyclic behavior model for steel are used to represent the response of these two materials under cyclic loading.

To verify the validity of the proposed algorithm, solutions of several examples of frames are presented and the method presented in this study shows good accuracy in comparison with the previous experimental and theoretical studies with difference not more than (16 %) for the problems considered.

Among the conclusions drawn, for the same amount of steel and the same volume of concrete, tapering the members of a structure having prismatic members increases overall stiffness, strength and integrity of that structure and as a result increases the number of load cycles up to failure. Bond between concrete and steel has a significant influence on the deformation characteristics of reinforced concrete frame elements under flexural loading. The analysis which takes slip into account gives deflection greater than the deflection obtained by the analysis without slip by about (6 %).

التحليل اللاخطي للهياكل الخرسانية المسلحة الفضائية تحت الأثقال الدورية متضمنا التأثيرات المعتمدة على الزمن.

الخلاصة:

الغرض من هذا البحث هو التحليل اللاخطي للهياكل الخرسانية المسلحة الثلاثية الأبعاد و التي تتكون من أعضاء موشورية و \ أو غير موشورية (tapered) و المتعرضة إلى الأحمال المتغيرة دوريا. تم اعتماد انموذج المناطق اللدنة (plastic zone model) في دراسة السلوك غير المرن للهياكل الخرسانية ثلاثية الأبعاد مع إتباع أسلوب الزيادة الجزئية خطوة – خطوة (step by step incremental approach) مع إجراء عمليات المحاولة و الخطأ. تم اقتراح أسلوب جديد سمي أسلوب المناطق (regions approach) للأخذ بنظر الاعتبار التغير في خواص المواد خلال عمق و عرض المقطع. عدد من العوامل تم أخذها بنظر الاعتبار في هذه الطريقة و هي: تداخل التأثير بين العزوم و قوى القص و القوى المحورية، لاختية التصرف للمادة، اللاخطية الهندسية للمنشأ و التأثيرات المعتمدة على الزمن (time dependent effects) (الزحف و الانكماش) (creep and shrinkage). تم اقتراح انموذج جديد لتمثيل العلاقة بين إجهاد الربط (bond stress) والانزلاق (slip) كما تم اقتراح أسلوب جديد لحل مسألة انزلاق الربط (bond – slip problem) . هذا النموذج يصف ال Heysteretic behavior لقضيب تسليح مفرد موضوع في reinforced concrete beam column element، هذه التقنية تركز على الاجهادات المنتقلة بين حديد التسليح و الخرسانة المحيطة به من خلال الربط. تم اشتقاق صيغ جديدة للقوى في المساند الثابتة (fixed end forces) للأعضاء المتغيرة العمق أو العرض أو كلاهما خطيا (tapered members) و المعرضة لأحمال موزعة بشكل منتظم (في المستوي xy أو المستوي xz أو كلاهما) مع الأخذ بنظر الاعتبار احتمالية وجود المنطقة اللدنة (plastic zone) في أي مكان من ذلك العضو. تم استخدام آلية الخضوع التي اقترحها **Kupfer ، Hilsdrof و Rusch** كمحدد لتصرف الخرسانة. كذلك تم الأخذ بنظر الاعتبار قاعدة الصلابة النعومة Hardening / Softening Rule، قاعدة الجريان Flow Rule و تقوية الشد Tension stiffening للخرسانة. آلية الانهيار في هذا البحث تتم بالسحق الموضعي (Crushing Failure) للخرسانة خلال منطقة معينة.

استخدم انموذج التصرف اللاخطي للخرسانة و انموذج التصرف اللاخطي للحديد لتمثيل استجابة تلك المادتين تحت تأثير الأحمال الدورية.

لغرض إثبات صحة و جدوى الأسلوب المقترح، تم تحليل عدة هياكل خرسانية وظهرت النتائج دقة جيدة بالمقارنة مع دراسات سابقة حيث لم يتجاوز الفارق نسبة (16.23%) . من الاستنتاجات التي تم الحصول عليها لنفس كمية حديد التسليح و لنفس الحجم من الخرسانة، استخدام الأعضاء المتغيرة العمق أو العرض أو كلاهما خطيا (غير موشورية) للمنشآت الخرسانية سوف يزيد من الجساء الكلية للمنشأ و كذلك مقاومة وحدة المنشأ بالمقارنة مع مثيلاتها المكونة من أعضاء منتظمة المقطع (الموشورية) و بالنتيجة يزيد من عدد الدورات اللازمة للتحميل وصولا لحالة الفشل.

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Appendix A: Numerical Integration

It may be noted that there are many formulas to carry out the numerical integration, but one of the most formulas which used for this purpose is **Gussion – Quadrature** formula. To estimate the values of the integration $\int_a^b f(x)dx$ according to **Gussion – Quadrature** formula, the function $f(x)$ is approximated by an interpolating polynomial $P_{2n-1}(x)$, and integrated as follows:

$$\int_a^b f(x)dx = \int_a^b P_{2n-1}(x)dx + Err. \quad (A - 1)$$

where, Err. is the error term for the interpolating and n is the number of Gauss points.

$$f(x) = \sum_{i=1}^n L_i(x) f(x_i) + Err. \quad (A - 2)$$

The interval of integration will be change from [a,b] to [-1,1] by a suitable transformation of variable. Let the new variable z, where $-1 \leq z \leq 1$, be defined by

$$z = \frac{2x - (a+b)}{b-a} \quad (A - 3)$$

Also defined a new function $F(z)$ so that

$$F(z) = f(x) = f\left(\frac{(b-a)z + (a+b)}{2}\right) \quad (A - 4)$$

Then, the Eq. (A – 2) becomes

$$F(z) = \sum_{i=1}^n L_i(z) F(z_i) + Err. \quad (A - 5)$$

New integration of both sides of Eq. (A – 5) between the integration limits -1 and 1 gives

$$\int_{-1}^1 F(z) dz = \int_{-1}^1 L_i(z) F(z_i) dz + Err. \quad (A - 6)$$

Since the $F(z_i)$ are fixed values, the summation operator can be taken outside the integral sign. Eq. (A – 6)

$$\int_{-1}^1 F(z) dz = \sum_{i=1}^n w_i F(z_i) + Err. \quad (A - 7)$$

where

$$w_i = \int_{-1}^1 L_i(z) dz$$

Assume the function $F(z)$ to be polynomial of $2n-1^{\text{th}}$ degree as follows

$$F(z) = c_0 + c_1 z + c_2 z^2 + \dots + c_{2n-1} z^{2n-1} \quad (A - 8)$$

Then, from Eq. (A – 7), we have

$$\sum_{i=1}^n w_i F(z_i) = w_1 F(z_1) + w_2 F(z_2) + \dots + w_n F(z_n) \quad (A - 9)$$

To illustrate the method of finding the base points and the corresponding weight factor, take the following example for two base points.

From Eq. (A – 8), one get

$$n=2, 2n-1=3$$

$$F(z) = c_0 + c_1 z + c_2 z^2 + c_3 z^3$$

and from Eq. (A – 9), one get

$$\sum_{i=1}^n w_i F(z_i) = w_1 F(z_1) + w_2 F(z_2)$$

Then, assume $z_1 = a$, $z_2 = -a$ and the error term (Err.) is found from Eq. (A – 7)

$$\begin{aligned} Err. &= \int_{-1}^1 (c_0 + c_1 z + c_2 z^2 + c_3 z^3) dz - (w_1 F(a) + w_2 F(-a)) \\ &= 2c_0(1-w) + c_2(2/3 - 2a^2 w) \end{aligned}$$

After that, minimize the resulting error with respect to c_0 and c_2 and equating the resulting equation to zero, one get

$$\frac{\partial Err.}{\partial c_0} = 0 = 2(1-w) \Rightarrow w = 1$$

$$\frac{\partial Err.}{\partial c_2} = 0 = \frac{2}{3} - 2a^2 w \Rightarrow a = \pm \frac{1}{\sqrt{3}}$$

The values of the appropriate base points and weight factors for $n = 1, 2, 3, \dots, \dots, \dots$, and 15 points formulas are shown in table (1).

Table (1): Roots of the Legendre Polynomial $P_{n+1}(z)$ and the Weight Factor for the Gauss – Legendre Quadrature

$\int_{-1}^1 F(z) dz = \sum_{i=1}^n w_i F(z_i)$		
Roots (z_i)	Weight Factors (w_i)	
$\pm 0.57735\ 02691\ 89626$	Two – Point Formula	1.00000 000000 000000
0.00000 00000 00000 $\pm 0.77459\ 66692\ 41483$	Three – Points Formula	0.88888 88888 88889 0.55555 55555 55556
$\pm 0.33998\ 10435\ 84856$ $\pm 0.86113\ 63115\ 94053$	Four – Points Formula	0.65214 51548 62546 0.34785 48451 37454
0.00000 00000 00000 $\pm 0.53846\ 93101\ 05683$ $\pm 0.90617\ 98459\ 38664$	Five – Points Formula	0.56888 88888 88889 0.47862 86704 99366 0.23692 68850 56189
$\pm 0.23861\ 91860\ 83197$ $\pm 0.66120\ 93864\ 66256$ $\pm 0.93246\ 95142\ 03152$	Six – Points Formula	0.46791 39345 72691 0.36076 15730 48139 0.17132 44923 79170
$\pm 0.14887\ 43389\ 81631$ $\pm 0.43339\ 53941\ 29247$ $\pm 0.67940\ 95682\ 99024$ $\pm 0.86506\ 33666\ 88935$ $\pm 0.97390\ 65285\ 17172$	Ten – Points Formula	0.29552 42247 14753 0.26926 67193 09996 0.21908 63625 15982 0.14945 13491 50581 0.06667 13443 08688
0.00000 00000 00000 $\pm 0.20119\ 40939\ 97435$ $\pm 0.39415\ 13470\ 77563$ $\pm 0.57097\ 21726\ 08539$ $\pm 0.72441\ 77313\ 60170$ $\pm 0.84820\ 65834\ 10427$ $\pm 0.93727\ 33924\ 00706$ $\pm 0.98799\ 25180\ 20485$	Fifteen – Points Formula	0.20257 82419 25561 0.19843 14853 27111 0.18616 10001 15562 0.16626 92058 16994 0.13957 06779 26154 0.10715 92204 67172 0.07036 60474 88108 0.03075 32419 96117



هو الذي بعث في الاميين رسولا منهم

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CERTIFICATE

We certify that the preparation of this thesis entitled “**Nonlinear Analysis of Reinforced Concrete Space Frames under Cyclic Loading Including Time Dependent Effects**” was made under our supervision at the University of Babylon in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Civil Engineering.

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NOTATIONS

A	Cross – sectional area on an section.
a, b	Constant used in calculation of concrete cylinder strength at an age (t).
A_1, A_2	Parameters used in steel cyclic behavior model.
a_1 to a_4	Polynomial coefficients.
A_c	Constant for concrete stress – strain envelope equation.
$\{a_f\}$	Flow vector.
a_{f1} to a_{f4}	Coefficient in flow vector.
$A_{force}, B_{force}, C_{force}, D_{force}$	Coefficients in calculation of fixed end forces.
A_{ir}	Air content in percent.
A_s	Reinforcing bar area.
$A_{s(ij)}$	Area of steel region (ij).
B	Number of (50) kg sacks of cement per cubic meter of concrete.
b'	Slope of lines $A - B$ and $A' - B'$ as shown in Fig. (3.4b).
B_1, b_2	Over all width of the element at node 1 and 2 respectively.
$B_A(x, y, z)$	Axial strain – displacement matrix.
B_c	Constant for concrete stress – strain envelope equation.
$[B_{CA}]$	Concrete axial strain – displacement matrix.
$[B_{cxy}]$	Concrete shear strain – displacement matrix in xy – plane.
$[B_{cxz}]$	Concrete shear strain – displacement matrix in xz – plane.
$[B_{cyz}]$	Concrete shear strain – displacement matrix in yz – plane.
b_{ij}	Width of region (ij).
$[B_{SA}]$	Steel axial strain – displacement matrix.
$B_{x,y}$	Shear strain – displacement matrix in xy – plane.
$B_{x,z}$	Shear strain – displacement matrix in xz – plane.
$B_{y,z}$	Shear strain – displacement matrix in yz – plane.
C_c	Constant for concrete stress – strain envelope equation.
C_t	Creep coefficient.
C_u	Ultimate creep coefficient.
CF^s	Shrinkage correction factor.
D_c	Constant for concrete stress – strain envelope equation.
$[D_{con}]$	Concrete stress – strain relationship matrix.
$[D_{con}]_{ep}$	Elasto – plastic concrete stress – strain relationship matrix.
$[D_s]$	Steel stress – strain relationship matrix.
$d\bar{\lambda}$	Plastic multiplier.
E_{co}	Initial tangent flexural modulus of elasticity of concrete.
$E_{c(ij)}$	Flexural modulus of elasticity of concrete region (ij).
$E_{co}(t)$	Initial tangent flexural modulus of elasticity of concrete at an age (t).

E_{ct}	Tangent flexural modulus of elasticity of concrete.
f	Constant in calculating the concrete shrinkage strain.
$[f]$	Vector of nodal forces.
f_c'	Cylinder strength of concrete.
$f_c'(28)$	Cylinder strength of concrete at an age (28)day.
$f_c'(t)$	Cylinder strength of concrete at an age (t).
f_t'	Ultimate concrete tensile strength.
$f_t'(t)$	Ultimate concrete tensile strength at an age (t).
F_A^c	Correction factor for air content in calculation ultimate creep coefficient.
F_A^s	Shrinkage correction factor for air content in calculation the shrinkage correction factor.
F_B^s	Shrinkage correction factor for cement content in calculation the shrinkage correction factor.
F_f	Percent of fines by weight.
F_F^c	Correction factor for percent of fines in calculation ultimate creep coefficient.
F_F^s	Shrinkage correction factor for percent of fines in calculation the shrinkage correction factor.
F_H^c	Correction factor for humidity in calculation ultimate creep coefficient.
F_H^s	Shrinkage correction factor for humidity in calculation the shrinkage correction factor.
F_S^c	Correction factor for slump in calculation ultimate creep coefficient.
F_S^s	Shrinkage correction factor for slump in calculation the shrinkage correction factor.
F_T^c	Correction factor for loading age in calculation ultimate creep coefficient.
F_{TH}^c	Correction factor for minimum member thickness in calculation ultimate creep coefficient.
F_{TH}^s	Shrinkage correction factor for minimum member thickness in calculation the shrinkage correction factor.
G	Shear modulus of elasticity of concrete.
$\bar{G}$	Shear modulus of elasticity of cracked concrete.
G_{xy}, G_{xz}, G_{yz}	Concrete shear modulus of elasticity in xy, xz and yz – plane respectively.
$G_{xy(ij)}, G_{xz(ij)}, G_{yz(ij)}$	Shear modulus of elasticity in xy, xz and yz – plane of concrete region (ij).
H	Ambient relative humidity in percent.
h_1, h_2	Overall depth of the element at node 1 and 2 respectively.
H'	Hardening / softening parameter.
I_1	First stress invariant.
I_1'	First strain invariant.

I_y	Second moment of inertia in y – direction.
J_2	Second deviatoric stress invariant.
J_2'	Second deviatoric strain invariant.
K	Shear correction factor.
K_1 to K_{12}	Coefficient in strain – displacement matrices.
$[K_{ca}]$	Concrete axial stiffness matrix.
$[K_{cxy}]$	Concrete shear stiffness matrix in xy – plane.
$[K_{cxz}]$	Concrete shear stiffness matrix in xz – plane.
$[K_{cyz}]$	Concrete shear stiffness matrix in yz – plane.
$[K_{sa}]$	Steel axial stiffness matrix.
L	Total length of element.
M_{iy}, M_{iz}	Bending moment at node (i) in an element in y and z – direction.
N_1 to N_4 , $\overline{N}_1$ to $\overline{N}_4$, $\underline{N}_1$ to $\underline{N}_4$, $\equiv N_1$ to $\equiv N_4$, $\equiv N_1$ to $\equiv N_4$, $\equiv N_1$ to $\equiv N_4$, $\equiv N_1$ to $\equiv N_4$, $\equiv N_1$ to $\equiv N_4$,	Shape functions.
N_{1c}, N_{2c}	Number of concrete regions in depth and width direction respectively.
N_{1s}, N_{2s}	Number of steel regions in depth and width direction respectively.
p_{ix}	Axial force at node (i) in an element.
p_{iy}, p_{iz}	Shear force at node (i) in an element in y and z – direction respectively.
Q_1 to Q_6	Fixed end forces.
R_o	Parameter used in steel cyclic behavior model.
$R_{(\xi n)}$	Parameter represent the bauschinger effect.
S	Slump in mm.
T	The age in days of concrete at load application.
t	Time in days after casting of concrete.
t_0	Concrete curing time.
T_i	Torsional moment at node (i) in an element.
t_{ij}	depth of region (ij).
TH	Minimum member thickness in mm.
u	Relative slip of reinforcing bar with respect to surrounding concrete.
u_i	Axial displacement at node (i) in an element.
$u(x)$	Axial displacement at center line of element in x direction at distance (x) from node (i).
$u(x,y,z)$	Axial displacement at point (x,y,z) in an element.
v_i, w_i	Lateral displacement at node (i) in an element in y and z –

	directions.
$V(x), W(x)$	Lateral displacement at center line of element in y and z – directions at distance (x) from the first end of element.
$V(x,y,z), W(x,y,z)$	Lateral displacement at any point (x,y,z) in an element in y and z – direction respectively.
W_{con}	Concrete unite weight.
W_{load}	Uniformly distributed load.
y_{ij}	Distance in y – direction from the center line of (ij) region to the element center line.
z_{ij}	Distance in z – direction from the center line of (ij) region to the element center line.
α	Correction factor to allow for cross – sectional warping.
α_{cr}, β_{cr}	Material parameters used in yield criterion.
$\alpha'_{cr}, \beta'_{cr}$	Material parameters used in crushing criterion.
$\gamma_{cxy}, \gamma_{cxz}, \gamma_{cyz}$	Concrete shear strain in xy, xz and yz – plane.
$\gamma_{cxy}(ij), \gamma_{cxz}(ij), \gamma_{cyz}(ij)$	Concrete shear strain in xy, xz and yz – plane respectively at the center of region (ij).
$\gamma_{sxy}, \gamma_{sxz}, \gamma_{syx}$	Steel shear strain in xy, xz and yz – plane.
$\zeta \Psi$	Vector of virtual nodal displacement.
ϵ_c	Concrete strain.
ϵ_{co}	Strain at ultimate concrete strength.
$\epsilon_{co}(t)$	Concrete uniaxial compressive strain at ultimate concrete strength at time (t).
$\epsilon_{cx}, \epsilon_{cy}, \epsilon_{cz}$	Concrete strrain in local direction x , y and z respectively.
$\epsilon_{\bar{cx}}$	Concrete tension unloading strain in local direction (x).
$\epsilon_{cx}(ij), \sigma_{cx}(ij)$	Concrete stress and strain in local direction (x) at the center of region (ij).
$\epsilon_{cx}(t)$	Total concrete strain in local direction (x) at any time (t).
$\epsilon_{cx}^c(t)$	Concrete creep strain in local direction (x) at any time (t).
$\epsilon_{cx}^{nm}(t)$	Concrete no – mechanical strain in local direction (x) at any time (t).
$\epsilon_{cx}^m(t)$	Concrete mechanical strain in local direction (x) at any time (t) caused by a short time loading.
$\epsilon_{cx}^m(T)$	Concrete mechanical strain in local direction (x) at load application time (T).
$\epsilon_{cx}^s(t)$	Concrete shrinkage strain in local direction (x) at any time (t).
$\epsilon_{cx}^s(u)$	Ultimate concrete shrinkage strain.
ϵ_{cu}	Concrete ultimate total strain.
$\epsilon_{cu}(t)$	Crushing concrete uniaxial compressive strain at time (t).
ϵ_e	Concrete elastic strain.

$\epsilon_{in} , \sigma_{in}$	Start point of the nth inversion for steel stress – strain cuve.
$\epsilon_{kn} , \sigma_{kn}$	Intersecting point of the nth inversion for steel stress – strain cuve.
ϵ_m , α_m	Tension stiffening parameters.
ϵ_p	Concrete plastic strain.
ϵ_s	Steel strain.
ϵ_{so}	Strain at yield strength of steel.
ϵ_{sx}	Steel strain in local direction (x).
ϵ_{sxa}	Strain in steel bar after slip occurrence.
ϵ_{sxb}	Strain in steel bar before slip occurrence.
$\epsilon_{sx(ij)} , \sigma_{sx(ij)}$	Steel strain and stress in local direction (x) at the center of region (ij).
ϵ_t	Concrete strain value at the ultimate concrete tensile strength (f_t').
ϵ_x	Strain in local direction (x).
$\theta_{ix} , \theta_{iy} , \theta_{iz}$	Rotation at node (i) in an element in x, y and z – directions respectively.
$\theta_{x(x)} , \theta_{y(x)} , \theta_{z(x)}$	Rotations in x, y and z – directions respectively at the element centroid axis at distance (x) from the first end of the element.
$\theta_{x(x,y,z)} , \theta_{y(x,y,z)} , \theta_{z(x,y,z)}$	Rotations at any point (x,y,z) in an element in x, y and z – directions respectively.
ν	Poisson's ratio.
ξ_n	Steel plastic deformation undergone in each half cycle (n).
σ_0	Equivalent effective stress.
$\sigma_{cx} , \sigma_{cy} , \sigma_{cz}$	Concrete stress in local direction x , y and z respectively.
$\sigma_x , \sigma_y , \sigma_z$	Stress in local direction x, y and z respectively.
$\sigma_{\overline{CX}}$	Concrete tension unloading stress in local direction (x).
σ_s	Steel stress.
σ_{so}	Yield strength of steel.
σ_{sx}	Steel stress in local direction (x).
τ_b	Bond stress.
τ_f	Residual bond strength.
$\tau_{xy} , \tau_{xz} , \tau_{yz}$	Shear stress in plane xy, xz and yz respectively.
$\tau_{cxy} , \tau_{cxz} , \tau_{cyz}$	Concrete shear stress in xy, xz and yz – plane.
$\tau_{cxy(ij)} , \tau_{cxz(ij)} , \tau_{cyz(ij)}$	Concrete shear stress in xy, xz and yz – plane respectively at the center of region (ij).
$\tau_{sxy} , \tau_{sxz} , \tau_{syx}$	Steel shear stress in xy, xz and yz – plane.

φ	Reinforcing bar circumference.
ϕ	Bar diameter
ψ	Vector of nodal displacement.

Note: Any other notation may be explained where it appears in the thesis.

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CHAPTER ONE

(INTRODUCTION)

1.1 General:

Most of the reinforced concrete structures are three dimensional in behavior. Such structures may be modeled as several two dimensional smaller problems to discretize their spatial configuration and simplify their analyses. Generally, the analysis of such structures as a fully three dimensional problems has been made possible largely by the development of the modern digital computer. Due to the complexity of these structures and the nature of the materials involved, there is a need to employ highly sophisticated numerical analysis procedures, such as the nonlinear finite element formulation.

Reinforced concrete structures may be under the action of cyclic loads. The cyclic loads can be defined as the loads which act on the structures in the form of successive cycles having constant or variable intensity and these cycles may be in the same direction or in reversed direction.

The effects of cyclic loads on structures can be divided into fatigue effect which resulted from a large number of cycles of relatively low stress levels and, incremental deformations which occur under a relatively small number of load cycles of rather high stress, each of which causes additional deformations on the structure.

Examples of cyclic loads are those due to wind, pumping of water, , earthquakes, and effects of sea water on offshore structures, etc. Although that these loads have dynamic effects, the study of the behavior of structures under static cyclic loads is critical to the success of the dynamic analysis. On the other hand, the analysis of reinforced concrete space structures under equivalent static (Quasi static) cyclic load is recommended in many cases.

Since inelastic deformations are typically concentrated at certain critical regions within the structure, the accurate prediction of the mechanical behavior of the structure during the load cycles depends on

the development of reliable analytical models. Ideally these models should be based on an accurate representation of material behavior taking into account the controlling states of stress or strain and identifying the main parameters which influence the hysteretic behavior of each critical region in order to predict the behavior up to failure of any structural component during the load cycle.

Due to the practical and economical considerations, structural inelastic deformations must be anticipated during the design of the reinforced concrete structure especially when the structure is under cyclic loading.

The linear elastic methods do not account for changes in structural properties or the extent of residual strains. The nonlinear behavior is due to either material nonlinearity, i.e., nonlinear stress – strain relationship or geometrical nonlinearity, i.e., the deformed geometry differs significantly from the undeformed state, or the combination of the two sources. In addition, nonlinear behavior of flexible connection may be considered as another source of nonlinearity.

In modeling the mechanical behavior of the reinforced concrete members, the composite action of steel and concrete has to be account for. Because of the weakness of the concrete in tension, reinforced concrete members crack early in their response history and a breakdown in a bond between steel and concrete leads to a relative slippage of reinforcing bars. Moreover cyclic load reversals result in accelerated bond deterioration, which can lead to significant reinforcing bar slippage. So the bond – slip between steel and concrete is taken into account in the present study.

All the problems of reinforced concrete structures can be considered as time dependent problems, since several concrete properties are time dependent such as strength, modulus of elasticity, ultimate strains...etc, and also due to the external load and environmental changes

which are varying through the service life of the structure. A numerical procedure based on the finite element method for the material and geometrical nonlinear analysis of three dimensional reinforced concrete structures including time dependent effects due to load history, creep and shrinkage is developed in the present study.

1.2 Objective and Scope:

The objective of this investigation is to develop a procedure for the nonlinear analysis of reinforced concrete space frames with prismatic and/or nonprismatic members under cyclic loading. This procedure must be capable to predict the displacements (horizontal, vertical, lateral and rotational), internal forces, and stresses for each concrete and steel region in each section in the structure during the whole range of cyclic loading up to failure.

The present work investigates the suitability of the finite element method with tangential stiffness approach to analyze reinforced concrete space frames under cyclic loading. Material and geometric nonlinearity are taken into account by suggesting region approach and suitable stress – strain relationship for concrete and steel. Joint coordinates are updated at the beginning of each load stage.

In chapter two, the previous works dealing with the elastic – plastic analysis of the reinforced concrete frames under cyclic loading and other related problems are reviewed.

Suitable stress – strain relationships for concrete and steel which account for the crushing of concrete and yielding of steel are presented in chapter three. The 12 – degrees of freedom stiffness matrix and fixed end forces for tapered element, are presented in chapter four. A suitable yield criterion which takes into account the interaction between axial, shear, and flexural stress, hardening / softening rule for concrete, flow rule for

concrete, tension stiffening, creep and shrinkage, and crushing condition for concrete are detailed in chapter five. Also a suitable bond – slip model is presented in the same chapter. The applications and the presentation of results with discussion are given in chapter six. Parametric studies are given in chapter seven. Chapter eight introduces the conclusions and recommendations for future work.

CHAPTER TWO

(REVIEW OF LITERATURE)

2.1 General:

The objective of this chapter is to give a brief review of the previous studies dealing with nonlinear analysis of the reinforced concrete space frames, the behavior of the reinforced concrete members under cyclic loading, the bond – slip between steel and concrete, and the time dependent effects on the behavior of the reinforced concrete structures.

2.2 Nonlinear Behavior of Plane and Space frames:

Denato and Maier (1972)^[1] formulated a mathematical programming method “Imposed rotation method” for an inelastic analysis of the reinforced concrete frames. The problem of determining the moments and rotations at critical sections of the reinforced concrete frame subjected to a given load in term of linear complementarily problem was formulated.

A significant computational development in the “Imposed rotation method” was made by **Kaneko (1977)^[2]** to analyze the reinforced concrete frames. The proposed method was essentially a reformulation of these developments by **Denato and Maier^[1]**. However, the formulation results in a new mathematical programming model for which an efficient algorithm is developed. The advantage of the proposed method is mainly computational since it requires roughly one half computer time and a storage space as compared with those of **Denato and Maier^[1]** which were employed to solve the same problem.

Chugh (1977)^[3] presented a simple and direct procedure for the formulation of an element stiffness matrix on element coordinates for a two dimensional beam and beam – column member including shear deformations. The resulting stiffness matrices are compared with those obtained by using alternative formulation in terms of member

flexibilities. The relative effects of axial and shear forces on stiffness coefficients were presented. The critical buckling loads, considering the effects of shear force, were computed and compared with other works. Only prismatic member was considered.

Gunnin et al. (1977)^[4] described a computational technique for the general nonlinear analysis of large planar frames under static loading. The material nonlinearity was included considering moment – axial force interaction. The effect of joint displacements during loading was accommodated by analysis through updating the nodal coordinates after each iteration in each load increment. The members were prismatic and the loads were applied at joints within the plane of the frames and all displacements of the frame were within that plane.

Krishnamorthy and Panneerselvam (1978)^[5] presented a computer program for the nonlinear finite element analysis of the reinforced concrete framed structures. A brief description of the material properties used and the nonlinear finite element formulation to account for the material nonlinearity, cracking and yielding was given. The computer program was explained with the aid of a flow chart and the computational steps involved in the program were described in details. Incremental iterative technique based on “Initial stress method” was adopted. Shear deformations and geometric nonlinearity were not included. The application of the program was illustrated by selected examples.

Cohn and Franchi (1979)^[6] presented the main concepts; theoretical bases, means of implementation and some illustrations of the potential use of a computer system for structural plasticity. The program is applicable to analyze two dimensional steel or reinforced concrete structures. The analysis neglects both geometric and the effect of shear

deformations. Several numerical examples were investigated to test the accuracy and efficiency of the computer program.

Khalifa (1979)^[7] considered the problem of the premature instability of the optimized steel space frames. As loads increased, plastic hinges start forming in succession, resulting in the possible instability of the deteriorating frame before a plastic mechanism is reached. The problem was formulated, and a method of analysis, which provides a lower limit on local failure load, was presented. The analysis establishes the order in which the plastic hinges form, but avoids the repeated inversion of the frame stiffness matrix. The method developed was applied on two simple space frames.

Rad and Furlong (1980)^[8] investigated theoretically and experimentally the behavior of one story, one bay portions of multistory reinforced concrete plane frames under gravity plus lateral load. In the analytical part, the combined effect of geometric and material nonlinearities were considered. The material was assumed to be elastic – perfectly plastic. However, the effect of shear deformation and local unloading were not considered in the analytical study.

Owen and Hinton (1980)^[9] adopted the non-linear finite element procedure for the inelastic stress analysis of reinforced concrete plates and reinforced concrete beams. The layering technique was used for the treatment of the variation in material properties through the thickness. **Kupfer et.al.** yield criterion was used as a limitation for the concrete behavior. The hardening rule, flow rule, tension stiffening and crushing state were taken into account.

Hsu et al. (1981)^[10] described a computer program for the elastic perfectly plastic analysis of reinforced concrete planar frames. This computer program needed less computation time and memory space and

it is intended as practical for analysis and design uses. The program efficiency was checked through comparing its results with the experimental results for the reinforced concrete plane frames that were tested by others. The program ignored each of: (1) geometric nonlinearity (2) shear deformations and (3) local unloading.

Ford et al. (1981)^[11] presented a physical and analytical modeling of a series of unbraced multi – panel concrete frames. Nine frames were tested under vertical and lateral loads. The objectives of the test were to study the behavior and redistribution capacities of highly indeterminate concrete plane frames and, as a result, to develop correct analytical modeling technique for nonlinear analysis of reinforced concrete frames. Material and geometric nonlinearities were considered. The effect of shear deformation was ignored.

Meyer and Bathe (1982)^[12] addressed their attention to the structural non-linear analysis problems for the structures under static and dynamic loading, using the finite elements method as a tool in the analysis procedure. Some of models, which described the materials behavior and failure criteria were available. Guidelines were given to select the appropriate material model.

Some aspects of nonlinear inelastic analysis of skeletal reinforced concrete structures were discussed by **Cohn et al. (1983)**^[13]. The capabilities of computer program developed by the authors were illustrated. This program can solve a variety of structural plasticity problems. This program has options related to:

1. Loading: proportional or non-proportional loading may be considered.
2. Material model: elastic-perfectly plastic or strain hardening / softening material were considered.

3. Material behavior: material can be assumed to exhibit a non-linear reversible behavior or a non-linear irreversible behavior.
4. Yield criteria: single or multi stress yield condition can be handled.

A nonlinear design method for the reinforced concrete framed structures was adopted by **Ali and Grierson (1986)^[14]** with basic strength and deformability criteria and taken into account strain-hardening effect of high grade steel reinforcement and its effect in reducing the material cost.

Al – Sarraf (1986)^[15] derived modified stability functions for two dimensional prismatic beam – columns having solid cross section shapes, laced or battened built up structural members, in terms of the shear flexibility and axial load parameters. An approximate formula for such functions was proposed which make possible the rapid prediction of the elastic critical load of structures taking into consideration the effect of shear force in the members, using hand computing method.

Y.L.Mo (1986)^[16] carried out three types of tests involving different load paths on nine model portal reinforced concrete plane frames. The vertical load was monolithically increased until failure in the first group. In the second group, the lateral load was increased to failure while the vertical load was maintained at 53% of the ultimate load. In the third group, the vertical load was increased to failure while a lateral working load was maintained at 53% of the ultimate load according to the plastic theory. The measured results were compared with previously tested corresponding prototype frames in a previous research by others. Also, the results were compared with those obtained theoretically.

Lai (1987)^[17] studied the hysteretic behavior of reinforced concrete column through by using finite filament approach with three models: triaxial spring model, trilinear degrading model and biaxial bilinear

model. Adequacy of the models was discussed with respect to six laboratory specimens.

Chajes and Churchill (1987)^[18] presented a review of the basic concepts involved when constructing nonlinear load–deflection curves of framed structures by the finite elements method. This work was limited to the geometrically nonlinear behavior of elastic structures. Three different procedures were considered: the linear incremented method, the nonlinear incremented method and the direct method. For each method, the governing equations were derived, and the procedure used to plot the load-deflection curve was outlined.

Al – Rifaie and Trikha (1988)^[19] recognized the importance of the finite size of joints and shear deformations in the analysis of plane concrete structures by using beam-column approach, especially frame – shear wall type structures. They proposed rigid – ended element for idealization to carry out a step by step linearized limit state analysis. Simplified axial force – moment diagrams for reinforced concrete sections had been derived for detection of plastic hinge formulation. However, the analysis ignored the effect of each of geometric nonlinearity, possibility of local unloading, and possibility of crushing failure at any critical section. In addition, the analysis considered that each element has only two critical sections at its ends and could not treat the case of existence of a third critical section at the zero shear point when the element is under distributed load. A computer program had been used to analyze an irregular multi – storied frame, an industrial gable frame and frame – shear wall structure. The results were compared to establish the validity of the procedure.

Qaqish (1988)^[20] compared the linear and nonlinear structural analyses for the same structural system. Rotation at different points of a cantilever beam with large deflections were calculated and compared

with results for corresponding cantilever beams with small deflections. The difference between the rotation obtained by nonlinear analysis and that obtained by linear analysis is about 58% at ($P=0.45$).

Barzegar (1988)^[21] used the layering technique with the finite elements method to analyze the reinforced concrete panels, reinforced concrete slab and reinforced concrete beam. The results indicated the efficiency of layering technique in simulating material behavior under membrane as well as flexural loading cases.

A simplified macro element model was developed by **Breysse and Mazars (1988)**^[22]. The damage mechanics was used to model the behavior of concrete macro elements. The solution algorithm was developed to allow the nonlinear problem solution. The results given by this model were compared with laboratory beam tests and traditional finite element approach. This comparison showed that this model gives quick results with sufficient accuracy.

Funk and Wang (1988)^[23] proposed a program for rigid frames consisting of non-prismatic members. In this method the member should be divided into "n" equal spaces. For simplicity, four or five sections along the member are chosen and the cross sectional area and moment of inertia at each section are calculated. Then, the program would generate the cross sectional area and moment of inertia at all sections. This analytical method can be used in any rigid frame program .

An experimental study was conducted by **Elbehairy et al. (1989)**^[24] to study the general deformational behavior, inelastic rotation and plastic hinge length at the critical sections of a reinforced concrete rectangular frames under concentrated loads. A comparison between the experimental and theoretical results shows good agreement.

Wong and Tin – Loi (1990)^[25] developed a method for the elastic nonlinear analysis of framed structures depending on the finite element method. The main forms of Lagrangian coordinates systems and their relative merits were also discussed. The solution strategy rested on a

combination of the Newton – Raphson iterative technique and modified “arc length method” within a finite element based partially on updated Lagrangian description.

Kwak and Filippou(1990)^[26] deals with the finite element analysis of the monotonic behavior of reinforced concrete beams, slabs and beam-column joint sub assemblages. The concrete stress-strain relation is derived from equivalent uniaxial relations in the axes of orthotropy. The behavior of cracked concrete is described by a system of orthogonal cracks, which follow the principal strain directions and are thus rotating during the load history. Crushing or cracking of concrete takes place when the strains lie outside the ultimate surface in the biaxial strain space. The bond – slip and the tension – stiffening are taken into account. Many examples were analyzed, among of these examples is a simply supported beam under concentrated load at midspan and give good agreement with the experimental results (difference is about 13%) (see Fig.(2.1)).

Reinforced concrete structures subjected to in plane shear and normal stress were analyzed by **Hu and Schnobrich (1990)**^[27]. A non-linear material model including the smeared crack representation and rotation crack approach was adopted. This model took into account tension stiffening, stress degrading effect for concrete parallel to the crack direction and shear retention of concrete on the crack surface. The results obtained by this model show good agreement with experimental results.

El-Mezaini et al. (1991)^[28] investigated the linear elastic behavior of frames with non-prismatic members by using isoparametric plane stress finite elements. Comparison of the fixed end moments, stiffness and carry-over factors of non–prismatic members available in the literature with those computed by the finite element analysis reveals large discrepancies. Based on an extensive study, two sources of errors were

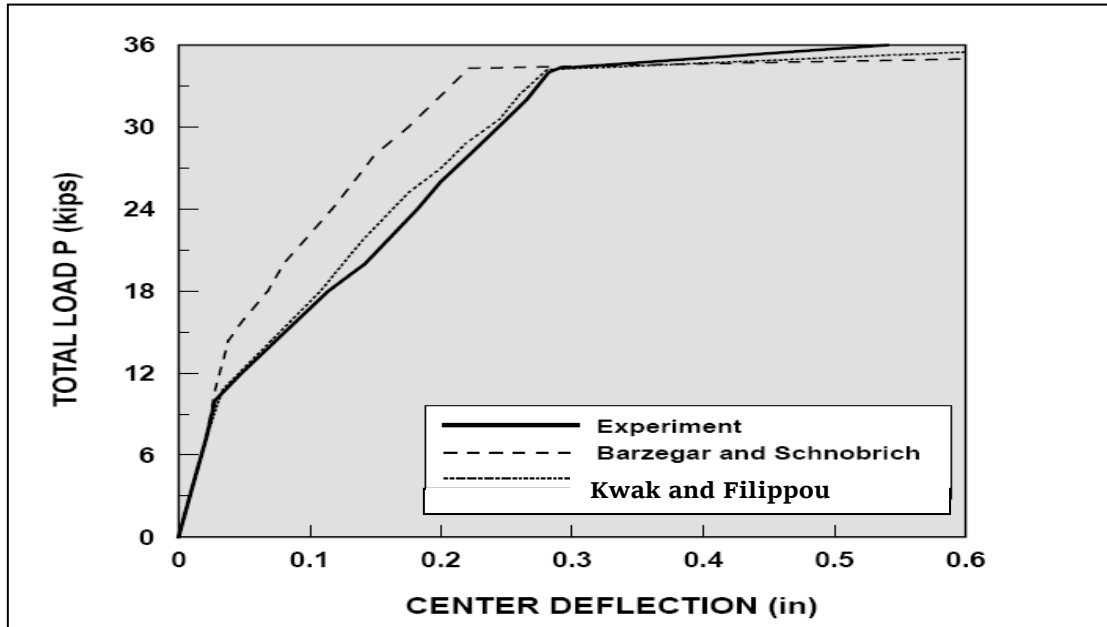


Fig.(2.1): Load – Deflection Relationship of Beam Analyzed by Kwak and Filippou^[26].

presented: (1) neglecting the discontinuity of the centroidal axis results in neglecting the coupling between axial and other end forces, which becomes a major sources of error. (2) neglecting the existence of kinks and null areas, especially in stepped members, results in overestimating the member stiffness.

Dumir et al. 1993^[29] investigated the geometric nonlinearity static analysis of steel space frames subjected to discrete and distributed transverse loads using the beam – column method. An exact expression of the beam – column equivalent load vector had been derived and it had been proved that the first term in its Taylor’s series expansion is identical to the static consistent equivalent load vector of the finite element method.

Alwash (1995)^[30] proposed a general nonlinear method to analyse reinforced concrete plane frames which was utilized for the analysis of reinforced concrete Vierendeel truss. It follows a step by step with iterations approach. Different parameters were included in the analysis, they are: combined effect of geometric nonlinearity and shear

deformations, material nonlinearity, moment axial force interaction, possibility of local stress unloading and the effect of the elements end rigidity. Fixed end forces were derived for an element under uniformly distributed load and the possibility of existence of an additional critical section at zero shear point for such element was also treated. Three possible failure criteria were predicated (collapse mechanism, local crushing failure and stability failure). The theoretical results were compared with those obtained experimentally and rather good agreement was obtained.

Hashim (1999)^[31] presented an analytical method that describes the ultimate strength and behavior of reinforced concrete frames having prismatic and / or non-prismatic (tapered) members. Plastic zone model was suggested with an incremental approach to study the inelastic behavior of reinforced concrete frames. Different parameters were included in this method which are: effect of coupling between bending moment, shear and axial forces, material non-linearity, geometric non-linearity, moment-axial force interaction at both yield and ultimate states, and possibility of local stress unloading. A suitable bilinear moment-rotation law was suggested. Equivalent nodal loads (fixed-end forces) were presented for tapered element under uniformly distributed load considering the possibility of existence of plastic zone at left, right, or at both ends of the member. Failure was predicted by the concrete crushing at a certain section.

Al – Asady (2001)^[32] developed a procedure for the elastic – plastic analysis of steel space frames. It follows step by step linearized elastic – plastic analysis which predicts the sequence and location of plastic hinges and the corresponding load factor at each stage of analysis. The procedure was based on the matrix displacement method. The

material was assumed to be elastic perfectly plastic, and yielding was considered to be concentrated at the members ends in the form of plastic hinges. The members were assumed to remain elastic between plastic hinges.

Theoretical and experimental investigations were carried out by **Terezia et al. (2001)**^[33] for the determination of the moment vs. curvature, and shear force vs. shear deformation relationships based on the stress – strain curves given in (European pre – standard design of concrete structures) (1991), with material characteristics obtained from the test of concrete and steel specimens were presented. A comparison of the theoretical values with the results of tests for seven reinforced concrete beams subjected to a concentrated force in the middle of the beam spans shows good agreement for both curvature and shear deformations.

Kuw and Ju (2002)^[34] developed a three dimensional (3D) nonlinear beam element into a static and dynamic nonlinear finite element program called (AN program). This 3D beam is an inelastic element for modeling beam and beam – columns to model the steel or reinforced concrete structures for the static and dynamic loading. The AN program had the ability to perform a very complex nonlinear structural analysis since it incorporated this 3D beam element.

Al – Janabi (2003)^[35] studied the inelastic behavior of reinforced concrete (plane or space) frames. Step by step incremental with iteration approach was used. Material nonlinearity, geometric nonlinearity in addition to shear deformation and moment axial force interaction were taken into account. Fixed end forces were derived for element under uniformly distributed load. Crushing failure, plastic collapse mechanism

and stability failure were taken into account. One model of reinforced concrete space frames was casted and tested experimentally up to failure.

2.3 Behavior of Reinforced Concrete Members under Cyclic Loading:

Sinha et al. (1964)^[36] carried out an experimental investigation that was performed on the behavior of plain concrete under cyclic loading. Experimental stress – strain curves were drawn. Mathematical expressions were proposed for usual stress-strain, envelope, unloading and reloading curves. A comparison with the experimental results shows a good performance of the proposed expressions.

The effect of cyclic loading on the behavior of reinforced concrete members was studied by **Sinha et al. (1964)**^[37] where the tests were performed to investigate the response of simply supported beams to arbitrary cyclic load histories. The equations, which represented the usual stress-strain, envelope, unloading and reloading curves were compared with test results. This comparison gives rough indication about the behavior of reinforced concrete beams under cyclic loading.

Bertero and McClure (1964)^[38] developed a bending theory of reinforced concrete beams and studied the possibility of disregarding the problem of alternating plasticity for the case of reinforced concrete frames. Five model frames were tested. Two of the model frames were tested under proportional loading to apparent collapse to determine the instantaneous collapse load. Each of the other three frames were subjected to large number of cycles of ultra severe alternating over loads and then loaded to apparent collapse.

Shah and Chandra (1970)^[39] studied the mechanism of failure of concrete subjected to slowly applied cyclic or sustained stresses. Specimens of concrete were tested under the maximum cyclic or

sustained compressive stresses of 60 to 90 percent of the ultimate stress. Volumetric strains, ultrasonic velocity and attenuation, and internal micro crack propagation were examined.

Takeda et al.(1970)^[40] proposed a realistic conceptual model which recognized the continually varying stiffness and energy absorbing characteristic of the structure .The applicability of this model to reinforced concrete was tested experimentally with the use of specimens subjected to dynamic base motions.

Anderson and Townsend (1977)^[41] introduced an improved hysteretic model as far as the ends of reinforced concrete members were concern.The hysteretic behavior was represented by four branches as shown in Fig. (2.2). The first branch of the hysteretic model was drawn from zero moment to apparent yield moment with slope equal to initial secant stiffness, k_o . The second branch is the strain-hardening branch. A straight line with slope equal to $0.085k_o$ represented this branch. When the loading is reduced, the unloading stiffness is nearly linear and equal to the initial stiffness, k_o . The fourth branch was constructed by drawing a straight line from the end of the third branch to the point where the moment – rotation curve intersects the moment axis and extended it until the yield moment is reached .The slope of the fourth branch is a function of ductility requirement, μ , and as shown in the equation (2 – 1):

$$k = k_o \left(\frac{1}{\mu} \right)^{1.5} \quad (2 - 1)$$

The effect of structural modeling on comparing the dynamic response of a structure using different structural discretization was investigated.

An analytical method was presented by **Thompson and Park (1980)**^[42] for the derivation of the moment-curvature relations for pre stressed, partially prestressed and reinforced concrete sections subjected

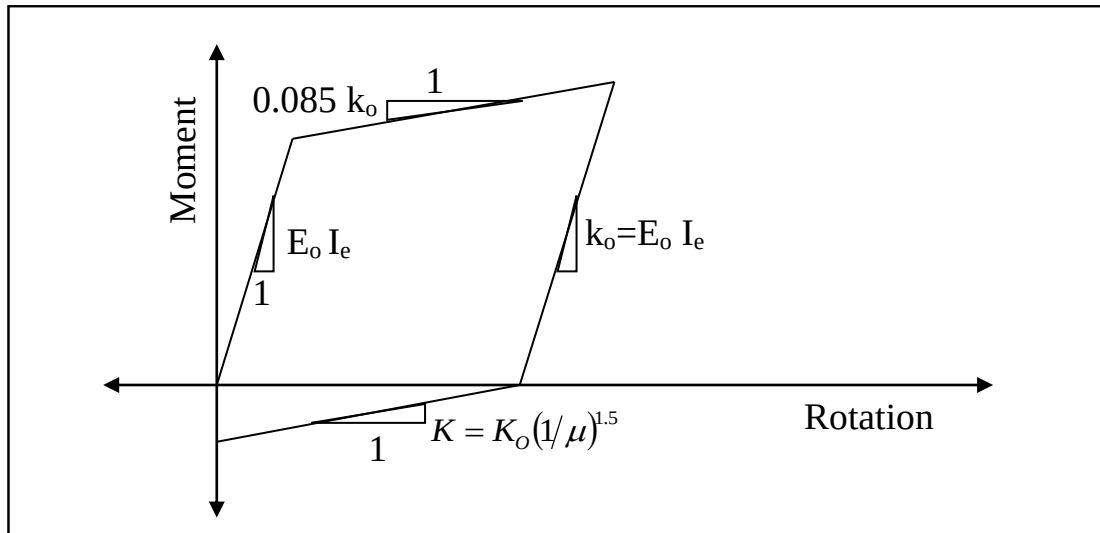


Fig. (2.2) : Moment – Rotation Relationship for Reinforced Concrete Members by Anderson and Townsend ^[41].

to cyclic loading in the inelastic range. The method utilizes stress-strain models for concrete, reinforcing steel and prestressing steel subjected to cyclic stresses. The concrete stress – strain model chosen for this study was that proposed by **Blakeley and Park**. The stress–strain model used to describe the stress – strain behavior of steel reinforcing bars under cyclic loading employs the monotonic stress – strain curve given by the modified form of the **Ramberg – Osgood** function as the envelope curve.

Jimenez et al. (1982)^[43] investigated the transfer of reversed cyclic shear forces across cracks in reinforced concrete by interface shear transfer and by dowel action. Experimental cyclic shear load-shear displacement results were given for the interface shear transfer mechanism alone, the dowel action mechanism alone, and combined mechanism. Empirical prediction expressions were presented for the two independent mechanisms, these mechanisms were combined to provide a model for predicting the load-displacement relationship for the combined interface shear transfer and dowel action including the effects of load cycling. Model predictions were compared reasonably with experimental results.

Shah et al. (1983)^[44] examined the validity of the envelope curve for spirally confined concrete by testing spirally confined normal weight and light weight concrete specimens under cyclic loading. Analytical expressions were developed to predict the envelope curve as well as the peak strength and the corresponding strain.

Eleven reinforced concrete cantilever beam specimens were constructed and tested by **Hwang and Scribner (1984)^[45]** under various conditions of inelastic reversal loading to determine the effect of variations in displacement history sequence and their magnitude on cyclic response of reinforced flexural member. Test results indicated that the member energy dissipation capacity was a function of applied shear stress intensity and the magnitude of the maximum displacements applied to the members.

A mathematical model has been presented by **Roufaiel and Meyer (1987)^[46]** that was capable of simulating the response of reinforced concrete frame members to cyclic loads. The finite size of plastic regions was accounted for, instead of assuming fictitious plastic hinges of zero dimension. The model could be accurately reproduced the complete behavior of general reinforced concrete frame members under cyclic loads up to advanced state of deterioration and up to the point of failure.

A one-dimensional model for random cyclic compression in concrete was developed by **Yankelevsky and Reinhardt (1987)^[47]**. The model determines several geometrical loci in the uniaxial stress – strain plane that govern the unloading – reloading curves. The model laws for reproducing the unloading and reloading curves were presented. By this laws the curves may be plotted graphically with no accompanying calculations. Comparisons of the model predictions with many test results

showed good agreement. The same investigators ^[48], in 1989, developed a one-dimensional model for random cyclic tension in concrete

Filippou and Issa (1988)^[49] developed improved analytical methods for predicting the nonlinear static and cyclic response of multistory reinforced concrete frames. The analytical predictions generally show excellent agreement with the experimental results.

Desai et al. (1989)^[50] studied the hysteretic behavior of axially loaded reinforced concrete bracing members. Eight specimens were subjected to tension-compression cyclic loading to study the reduction in stiffness in relation to the number of cycles, the energy dissipation capacity and the effects of longitudinal steel, slenderness ratio and confinement of concrete.

Otter and Naaman (1989)^[51] developed a model to predict the stress-strain behavior of concrete under random cyclic compressive loading. The model uses the envelope unloading strain as an index of load history. The plastic strain and reloading strain were predicted as functions of the envelope unloading strain for both full and partial unloading and reloading. Expressions for the unloading and reloading curves were also given.

Mahmood (1989)^[52] investigated the nonlinear analysis of prismatic reinforced concrete frames subjected to cyclically varying loading. The stiffness method with effective stiffness was used to study the behavior of such structures considering the material and geometric nonlinearities. An analytical solution was suggested using secant stiffness with the iterative technique. The layered approach was adopted for nonlinear section analysis, considering the axial force and bending moment only. The nonlinear cyclic behavior model for concrete was developed in a uniaxial state of stress, and nonlinear cyclic behavior for reinforcing

steel was used with the assumption of perfectly bond between steel and concrete.

Ricles et al. (1998)^[53] presented a stress resultant plasticity-based formulation with shear failure criteria and post-shear failure effects for modeling the response of nonductile reinforced concrete bridge columns subjected to biaxial seismic loading. The formulation was shown to predict reasonably well the cyclic response of nonductile beam-column.

Hoehler and Ožbolt (2001)^[54] presented the state of the micro plane based on finite element analysis program MASA for three-dimensional, nonlinear, reversed-cyclic analysis of reinforced concrete members. The program was shown to accurately represent global hysteretic behavior up to failure for cases with a limited number of cycles, while simultaneously providing information about the local behavior such as concrete cracking and steel to concrete bond interaction.

The bond of the high strength reinforced concrete under cyclic loading was investigated by **Alavi-Fard and Marzouk (2002)^[55]** under different parameters, including load history, confining reinforcement, bar diameter, concrete strength, and the rate of pull out. The bond strength, cracking, and deformation are highly dependent on the bond slip behavior between the bar and the concrete under cyclic loading. The results of cyclic testing indicate that an increase in cyclic displacement will lead to more severe bond damage.

Iemura et al. (2004)^[56] evaluated the seismic performance of unbonded bar reinforced Concrete (UBRC) structures, cyclic loading tests and pseudo dynamic tests were carried out. The results lead the conclusion that UBRC structures can improve the seismic performance of concrete structure easily and economically.

The reinforced concrete plane framed structure, having linearly tapered and/or prismatic elements was analyzed by using an inelastic

analytical procedure which was developed by **Kadhim (2001)**^[76]. This method follows an incremental procedure with simple iterative technique based on finite element method with layered approach which was developed by **Owen and Hinton**^[9]. The method ignored the effect of finite size of joints at member ends as well as the relative movement (slip) between steel and concrete. On the other hand, the previous method took into account some effects as follows:

1. Derivation of a numerical form stiffness matrix for tapered plane frame element takes into account ; (a) the interaction between bending moment , shear and axial forces ; (b) the possibility of existence of plastic zone at left , right or both sides of the element. In addition , for the case of element under uniformly distributed load , fixed end forces were derived considering the possibility of existence of plastic zone wherever it is formed .
2. Layered approach was used to consider the material non –linearity through the depth of the element.
3. Geometric non – linearity was included through adopting the incremental procedure in which the load was applied as a series of small increments and for each increment the changes in the nodal displacements were determined and the nodal coordinates were updated at the end of each load increment.
4. At the end of each load increment and before updating the nodal coordinates, the constitutive matrix for each steel and concrete layer would be calculated. By using these values, the structure would be reanalyzed and a new values of the constitutive matrix for each steel and concrete layer would be calculated. If the difference between these new and old values is less than 1 MPa, then the next load increment would be applied. Otherwise the iteration would be continued until the constitutive matrix values converge.

- 5.A suitable yield criterion was adopted which takes into account the interaction between axial , shear and flexural stress , hardening / softening rule for concrete ,flow rule for concrete , tension stiffening and crushing condition for concrete.
- 6.This method treats the possibility of existence of critical section at a point of zero shear when the element is under uniformly distributed loads, in addition to those existing at its two extremities.

2.4 Bond – Slip Between Steel and Concrete:

Ngo and Scrodelies (1967)^[57] carried out a linear elastic analysis of a beam with predefined crack patterns. The purpose of the study was to determine bond stresses and stresses in the concrete and steel for a particular crack configuration.

Nonlinear material properties and bond – slip relationship were first introduced by **Nilson (1968)** (as found in **Ref.[58]**) who treated the concrete material as an orthotropic material. The concrete cracking using the discrete cracking approach was studied.

Popov (1984)^[59] emphasized the importance of reinforcing bond and anchorage problems in inelastic seismic analysis of reinforced concrete structures. The basic nature of the bond and anchorage problems was pointed out and relevant experiments were carried out.

Ali (1990)^[60] evaluated the behavior of reinforcing steel lap splices in reinforced concrete beams subjected to reversed cyclic loads within the inelastic range of steel stress. The main variables were studied: concrete cover, reinforcing bar diameter and the intensity of load reversals.

Yannopoulos and Tassios (1991)^[61] used an analytical computerized method to determine steel, concrete and bond stress, as well as bond slip along the steel bar of reinforced concrete axial elements under monotonic and cyclic loading. A comparison of the predictions

with those found by various experimental investigations seems to be satisfactory.

Yankelevsky et al. (1992)^[62] proposed a mathematical model to illustrate the bond-slip behavior of reinforcing steel bar embedded in a well confined concrete and subjected to monotonic and cyclic loading. This model was developed according to experimental bond slip tests for well confined concrete reported in the literature and based on their observation. Results predicted by this model were compared with experimental results and showed good correspondence.

A new theoretical model which is especially suited for detailed three-dimensional analyses was developed by **Lundgren (1999)^[63]**. In this model, the splitting stresses of the bond action are included; furthermore, the bond stress depends not only on the slip, but also on the radial deformation between the reinforcement bar and the concrete. In addition, this model includes the simulation of cyclic loading.

Ayoub and Filippou (1999)^[64] presented a critical evaluation of existing anchored reinforcing bar models and proposed a new model based on a two-field mixed formulation in which independent interpolation functions are used for the relative slip and the reinforcing steel forces.

Lundgren et al. (2002)^[65] developed a model for the bond between reinforcement and concrete and used in detailed three-dimensional analyses, where the concrete is modeled with non-linear fracture mechanics. The loss of bond due to splitting cracks in the concrete or yielding of the reinforcement can be simulated, as well as the Poisson's effect at prestress release.

Limkatanyu and Samakrattakit (2003)^[66] presented the formulation and application of the flexibility-based, reinforced concrete frame model with bond-slip. This numerical model can be applied to both

monotonic and cyclic loadings. The reinforced concrete column experimentally tested under cyclic loads was used to illustrate the model accuracy and to show the importance of bond-slip inclusion.

2.5 Time Dependent Effects on The Behavior of Reinforced Concrete Structures:

Concrete is considered as a time dependent material, because its several properties change with time. The time dependent behavior of concrete can be of two different origins, producing stress originated and non stress originated strains. Shrinkage is an example of non stress originated time dependent effect, while creep strains is an example of stress originated time dependent effect, i.e., creep can be as time dependent strain due to stress, shrinkage is time depended volume or length change in a stress free specimen. While the physical mechanism of creep and shrinkage and their inter dependence are as yet incompletely understood, it is convenient from a mathematical point of view to consider them as independent phenomena ^[67].

A finite element model which was capable of simulating the behavior of reinforced concrete slabs at working loads including the effects of non uniform reinforcing, tensile cracking, shrinkage and creep, was developed by **Scanlon** (as found in **Ref.[68]**), in 1974. The inelastic strains resulting from creep and shrinkage behavior were treated by the initial strain method. The CEB expression based on available experimental data was used for evaluation of creep and shrinkage strains.

Aldstedt and Bergan (1978)^[69] presented a finite element model that was capable of predicting the behavior of plane slender reinforced concrete frames and arches subjected to loads up to their ultimate carrying capacities. The material and geometric non-linearity, creep, cracking and bond-slip mechanism between concrete and reinforcement were taken into account. The results obtained by this model agreed well

with analytical and experimental data available in the literature. Either material and / or geometrical non-linearity of reinforced concrete frames was taken into account by using different theoretical approaches.

Material and geometric nonlinear analysis of planar reinforced and prestressed concrete frames including the time dependent effects due to load history, creep, shrinkage, aging of concrete and relaxation of prestress was developed by **Kang (1980)**^[70].

Chow (1984)^[71] developed the finite element analysis including the effects of creep and shrinkage in reinforced concrete beams. The ACI method was used to model the time dependent effects in the concrete.

Al – Naimi (1996)^[68] concerned with stress analysis of three dimensional reinforced concrete structures subjected to static or dynamic loadings. Nonlinear material and time dependent effects were included in the analysis. Material nonlinearities as a result of tension cracking, strain softening after cracking, the nonlinear response of concrete in compression, crushing of concrete and the yielding of the reinforcement were considered. A comparison between the analytical and theoretical results was made and showed good agreement.

Al – Saadi (2000)^[72] studied the crack caused by restrained shrinkage in reinforced concrete walls form the base. A finite elements method was used to idealize the time dependent base restrained shrinkage cracking in plain and reinforced concrete walls. A computer program was written in (Q – basic language) and was constructed to idealize the problem of shrinkage cracking in base restrained concrete walls. Material nonlinearity, tension stiffening, stress degradation of concrete parallel to the crack direction and shear retention of concrete on the crack surface were included in the analysis.

It is clear from the preceding review that there is limited detailed study as reviewed that deals with the inelastic nonlinear behavior of reinforced concrete space framed structures having non-prismatic

members under cyclic loading. This problem is treated well in this study. The (regions technique) is suggested in the present study for the treatment of the variation in material properties through the thickness and width. Kupfer, Hilsdrof and Rusch yield criteria are used as a limitation for the concrete behavior. The hardening rule, flow rule, tension stiffening, creep and shrinkage effects on concrete, bond slip between concrete and steel and crushing state are taken into account. Material nonlinearity, geometric nonlinearity and coupling between bending moment, shear and axial forces are taken into account. Possibility of existence of plastic zone any where in the structure, and possibility of existence of distributed load on any member in addition to nodal forces are considered in treating this problem in the present study.

CHAPTER THREE (CYCLIC BEHAVIOR OF MATERIALS)

3.1 Introduction:

Reinforced concrete represents a challenging matter for the structural analyst because of its inherent inhomogeneity and non – linearity ^[73]. Sophisticated material model are required for representing the observed response of the reinforced concrete structure under cyclic loading. The development in computers and powerful numerical methods like the finite element method for analysis have facilitated incorporation of such models in addition to the complicated geometry, loading and boundary conditions. As a result, many sophisticated material models have been developed by many investigators to predict the response of reinforced concrete.

The model adopted by **Mahmood** ^[52] (**Al-Sulayfani model**) is used in the present study to represent the nonlinear behavior of concrete under cyclic loading. **Menegotto and Pinto model** as reported by **Mahmood** ^[52] is used to represent the nonlinear behavior of reinforcing steel under cyclic loading. In this chapter, these two models are illustrated and simulated by computer programs.

3.2 Concrete Cyclic Behavior Model:

3.2.1 Monotonic Stress-Strain Curve in Compression:

The monotonic stress – strain curve with the general shape as shown in Fig. (3.1) represents the envelope curve. For cyclic behavior curves, the following equations were used for the envelope curve:

$$\bar{\sigma} = \frac{A_c \bar{\epsilon}}{\bar{\epsilon}^3 + B_c \bar{\epsilon}^2 + C_c \bar{\epsilon} + D_c} \quad (3-1a)$$

where:

$$\bar{\sigma} = \frac{\sigma_c}{f'_c} \quad (3-1b)$$

$$\bar{\epsilon} = \frac{\epsilon_c}{\epsilon_{co}} \quad (3-1c)$$

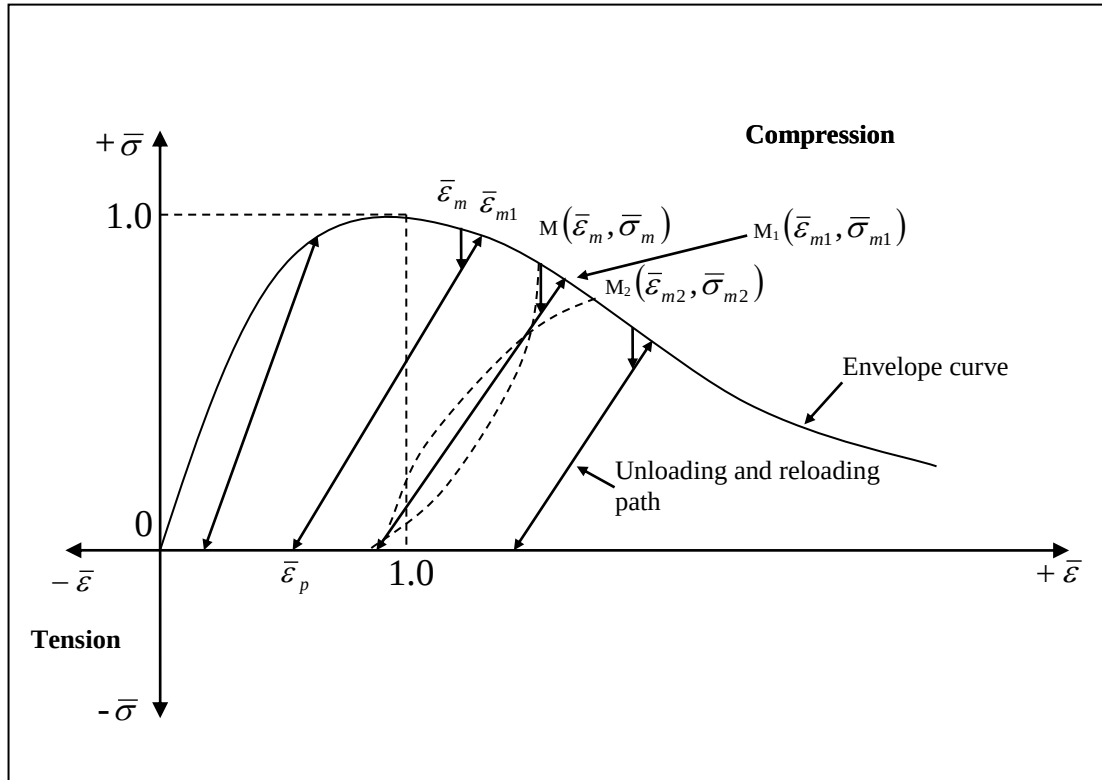


Fig. (3.1): Adopted Normalized Stress-Strain Relation for Concrete under Cyclic Loading. [52]

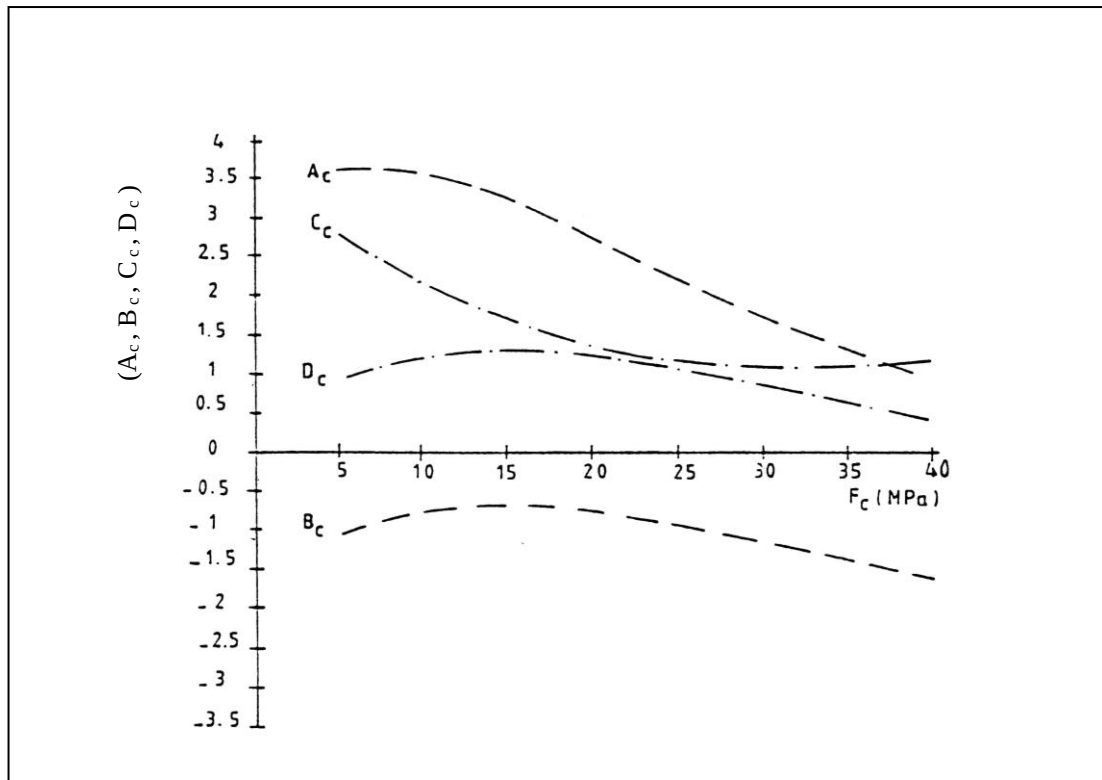


Fig. (3.2) : Evaluation of The Parameters for Concrete Envelope Equation (3 – 1a). [52]

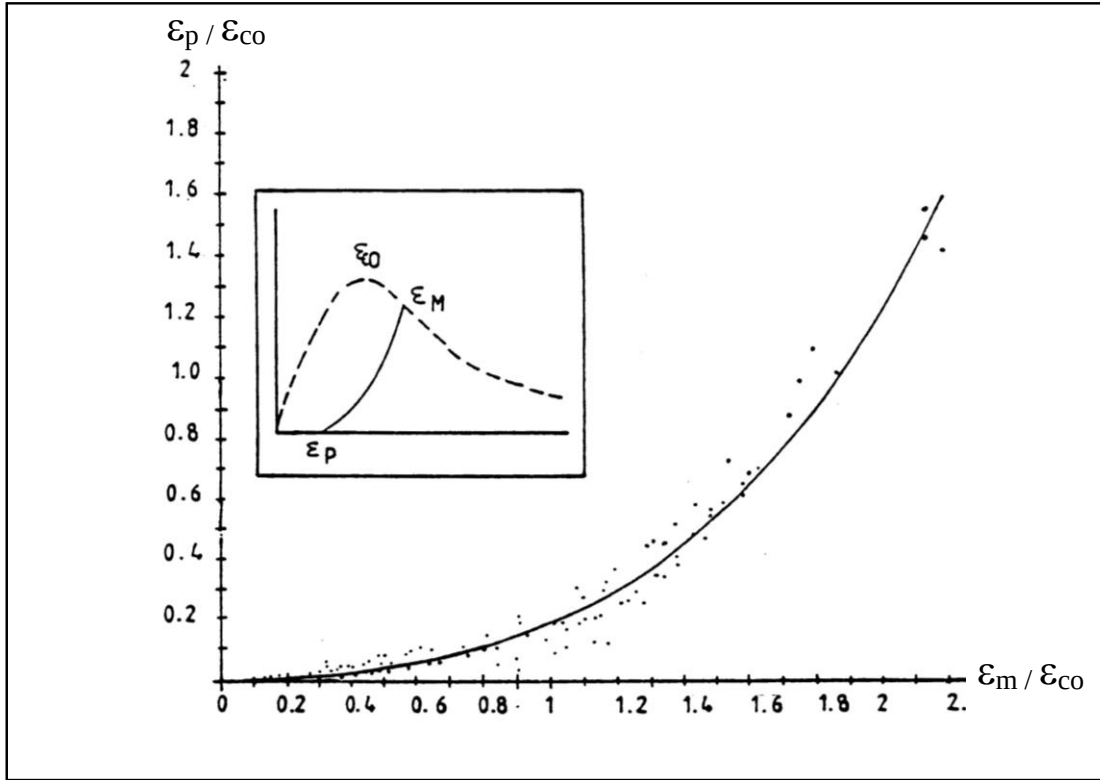


Fig. (3.3) : The Relationship Between Unloading Strain And Plastic Strain. ^[52]

The coefficients (A_c , B_c , C_c and D_c) depend on the concrete compressive strength f'_c ; these can be calculated from the curves shown in Fig. (3-2).

3.2.2 Stress – strain Behavior under Tension ^[52]:

A linear path will be adopted for the stress – strain behavior of concrete under tension with the modulus of elasticity equals to the nominal modulus of elasticity in compression. This will be valid up to the cracking strength f'_t which has the following expression:(as found in Ref.[52])

$$f'_t = 0.625 \sqrt{f'_c} \quad (3 - 2)^{[52]}$$

where f'_c is in N / mm^2 . Beyond f'_t the concrete is considered to be incapable of transmitting tensile stresses.

3.2.3 Concrete Cyclic Behavior ^[52]:

The assumed shape of the stress-strain hysteresis loops for cyclic loading in compression range is shown in Fig. (3.1). This behavior is illustrated through the following points: (as found in Ref.[52])

- 1- Assign the point where unloading occurs $M (\bar{\varepsilon}_m, \bar{\sigma}_m)$.
- 2-Determine the value of the residual strain (plastic strain) $\bar{\varepsilon}_p$ for this unloading case depending on the unloading strain value $\bar{\varepsilon}_m$ from Fig. (3.3) which was simulated by the following equations:

$$\bar{\varepsilon}_p = 0.055\bar{\varepsilon}_m + 0.127(\bar{\varepsilon}_m)^{3.1} \quad (3 - 3)^{[52]}$$

This is valid for $\bar{\varepsilon}_m \leq 2.2$.

For $\bar{\varepsilon}_m > 2.2$ a linear relationship between $\bar{\varepsilon}_m$ and $\bar{\varepsilon}_p$ was assumed and derived as follows :

For $\bar{\varepsilon}_m = 2.2$ from equation (3 - 3) $\bar{\varepsilon}_p = 1.584$, from the following equation:

$$\bar{\varepsilon}_p = 0.31(\bar{\varepsilon}_m)^{1.6} \quad (3 - 4)$$

for $\bar{\varepsilon}_m = 3, \bar{\varepsilon}_p = 1.798$.

From these two points this linear relation was derived:

$$\bar{\varepsilon}_p = 1.584 + 0.2675 (\bar{\varepsilon}_m - 2.2) \quad (3 - 5)$$

This equation was used for $\bar{\varepsilon}_m > 2.2$.

3- Where the hysteretic cycle completes after reloading, the strain path connects the envelope curve at point $M_2 (\bar{\varepsilon}_{m2}, \bar{\sigma}_{m2})$. The value of $\bar{\varepsilon}_{m2}$ could be determined from the relation:

$$\bar{\varepsilon}_{m2} = 1.22 \bar{\varepsilon}_m \quad (3 - 6)$$

the mean value for the strain between $\bar{\varepsilon}_m$ and $\bar{\varepsilon}_{m2}$ is used:

$$\bar{\varepsilon}_{m1} = 1.11 \bar{\varepsilon}_m \quad (3 - 7)$$

then point $M_1 (\bar{\varepsilon}_{m1}, \bar{\sigma}_{m1})$ was determined.

4- The unloading and reloading path are represented by the line connecting the two points $(\bar{\varepsilon}_p, 0)$, $(\bar{\varepsilon}_{m1}, \bar{\sigma}_{m1})$.

5- If the reloading in reverse direction are represented $(\bar{\varepsilon} < \bar{\varepsilon}_p)$ in compression, then the model gives tensile stress.

6- For tension strain the same linear path is followed for unloading and reloading as for monotonic tensile envelope until the concrete tensile strength is reached where the concrete begins to crack and the stress drops to zero; cracked regions will remain cracked and no tension stress will be allowed, while compression stress could be carried after residual strain is overcome.

7- For reloading from tension zone the same unloading path is followed until reaching the previous level of maximum compressive strain, point M_1 . Henceforth, the concrete stress – strain relation is governed by the monotonic stress – strain curve.

3.3 Stress-strain Behavior of Reinforcing Steel under Cyclic Loading^[52]:

The hysteretic behavior model which was previously adopted by **Menegotto and Pinto** (as found in **Ref.[52]**), is used in this study. This

model is shown in Fig. (3.4). The stress-strain curves of all cycles lie within the two parallel lines A-B and A'-B' which are defined by the monotonic curve and passing through the yield points $(\epsilon_{so}, \sigma_{so})$ and $(-\epsilon_{so}, -\sigma_{so})$ respectively. All the curves, which represent the hysteretic behavior of steel, have the same initial slope equal to the slope E_{so} of the monotonic curve. The following equation was proposed by **Menegotto and Pinto** and adopted in the present study for each half stress-strain curve in normalized form:

$$\bar{\sigma} = (1 - b') \frac{\bar{\epsilon}}{(1 + \bar{\epsilon}^R)^{1/R}} + b' \bar{\epsilon} \quad (3-8)$$

For first loading, $\bar{\epsilon}$ and $\bar{\sigma}$ is calculated as shown below:

$$\bar{\sigma} = \left| \frac{\sigma_s}{\sigma_{so}} \right| \quad (3-9a)$$

$$\bar{\epsilon} = \left| \frac{\epsilon_s}{\epsilon_{so}} \right| \quad (3-9b)$$

For curves after the first load inversion:

$$\bar{\sigma} = \left| \frac{\sigma_s - \sigma_{in}}{\sigma_{kn} - \sigma_{in}} \right| \quad (3-10a)$$

$$\bar{\epsilon} = \left| \frac{\epsilon_s - \epsilon_{in}}{\epsilon_{kn} - \epsilon_{in}} \right| \quad (3-10b)$$

Where the point $(\epsilon_{in}, \sigma_{in})$ is the start of the nth inversion. ϵ_{kn} and σ_{kn} are the coordinates of the intersection points which are shown in Fig.(3.4).

The coordinates of the intersection points depends on ϵ_{in} , σ_{in} , E_{so} values and the yield point of reinforcing steel.

Equation (3-8) is asymptotic to one of the hardening lines A-B or A'-B' (with slope b'). The exponent R varies after every inversion, which

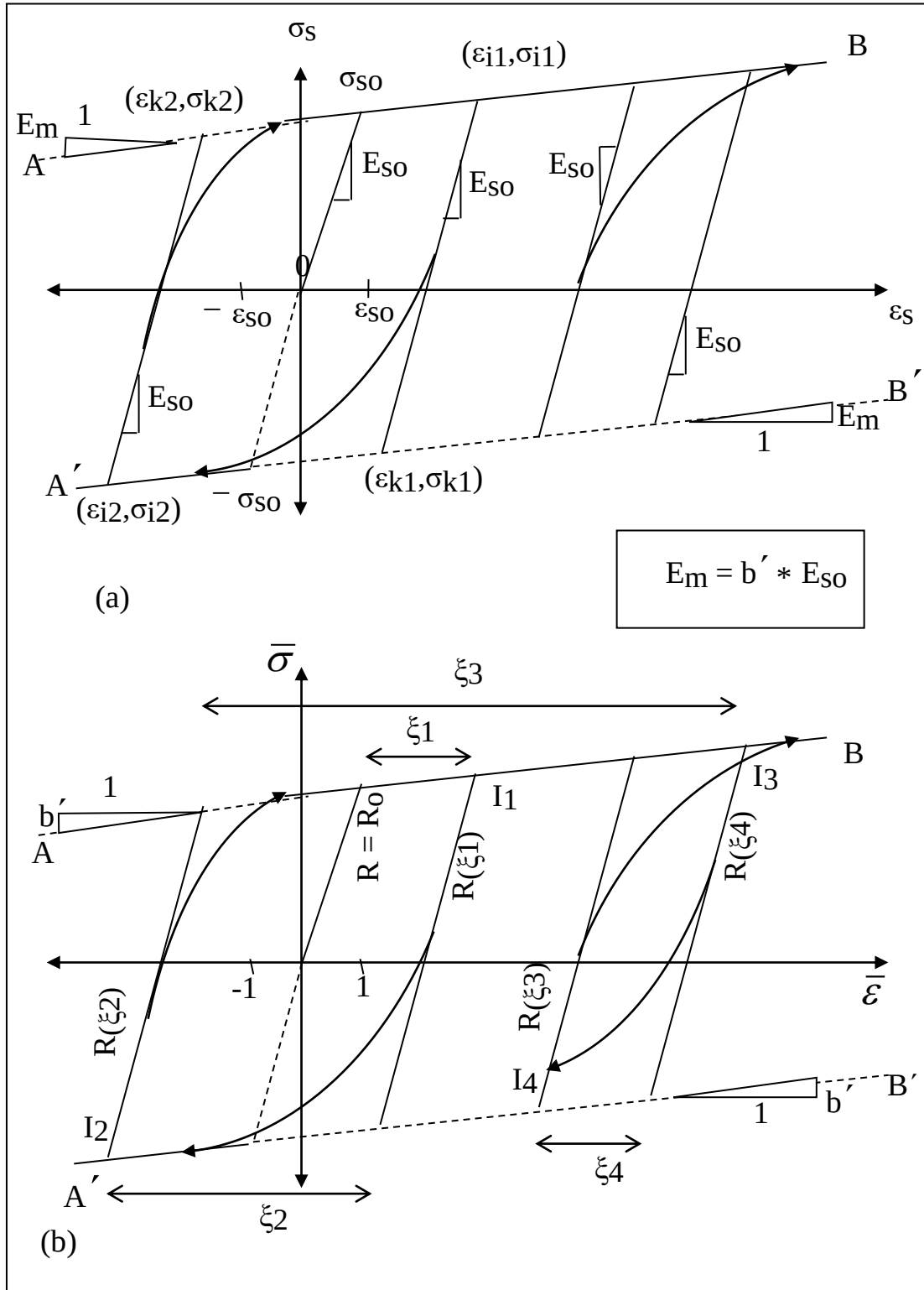


Fig. (3.4) : Cyclic Behavior Model of Steel

(a) Hysteretic Behavior Model of Steel by Menegotto and Pinto [52].

(b) Normalized Behavior Model of Steel by Menegotto and Pinto [52].

affects the curvature of each half cycle to represent the Bauschinger effect. Its value is given by the following equation:

$$R_{(\xi_n)} = R_o - \frac{A_1 \xi_n}{A_2 + \xi_n} \quad (3 - 11)$$

$$\xi_n = |\bar{\epsilon}_{i(n)} - \bar{\epsilon}_{k(n-1)}| \quad (3 - 12)$$

The values of $(\bar{\epsilon}_{i(n)}, \bar{\epsilon}_{k(n-1)})$ are determined from equation (3 -9b) or equation (3 -10b) by using the value of $\epsilon_{i(n)}$ and $\epsilon_{k(n-1)}$ respectively. The values of the parameters used for this investigation were those recommended by **Menegotto and Pinto** ^[52] which showed a satisfactory agreement between experimental and predicated hysteretic loops, ($R_o = 5.3$, $A_1=3.446$, $A_2 = 1.766$, $b' = 0.015$).

3.4 Computer Program:

Subroutines for the cyclic behavior prediction for concrete (**concy**) and steel (**stecy**) were developed on the basis of the previously mentioned algorithms and they are outlined in Fig. (3.5) and Fig. (3.6), respectively.

For concrete behavior the program should be supplied by the concrete maximum compressive strength (f_c), corresponding peak strain (ϵ_{co}), the parameters for envelope equation A_c , B_c , C_c and D_c , and tensile strength.

For steel behavior model, the program (stecy) should be supplied by the values of steel bar yield strength (f_y), modulus of elasticity (E_{so}), and parameters for the proposed model R_o , A_1 , A_2 , and b' .

These two subroutines are capable of predicting the stress in each region of concrete and steel under any strain condition of a prescribed loading history.

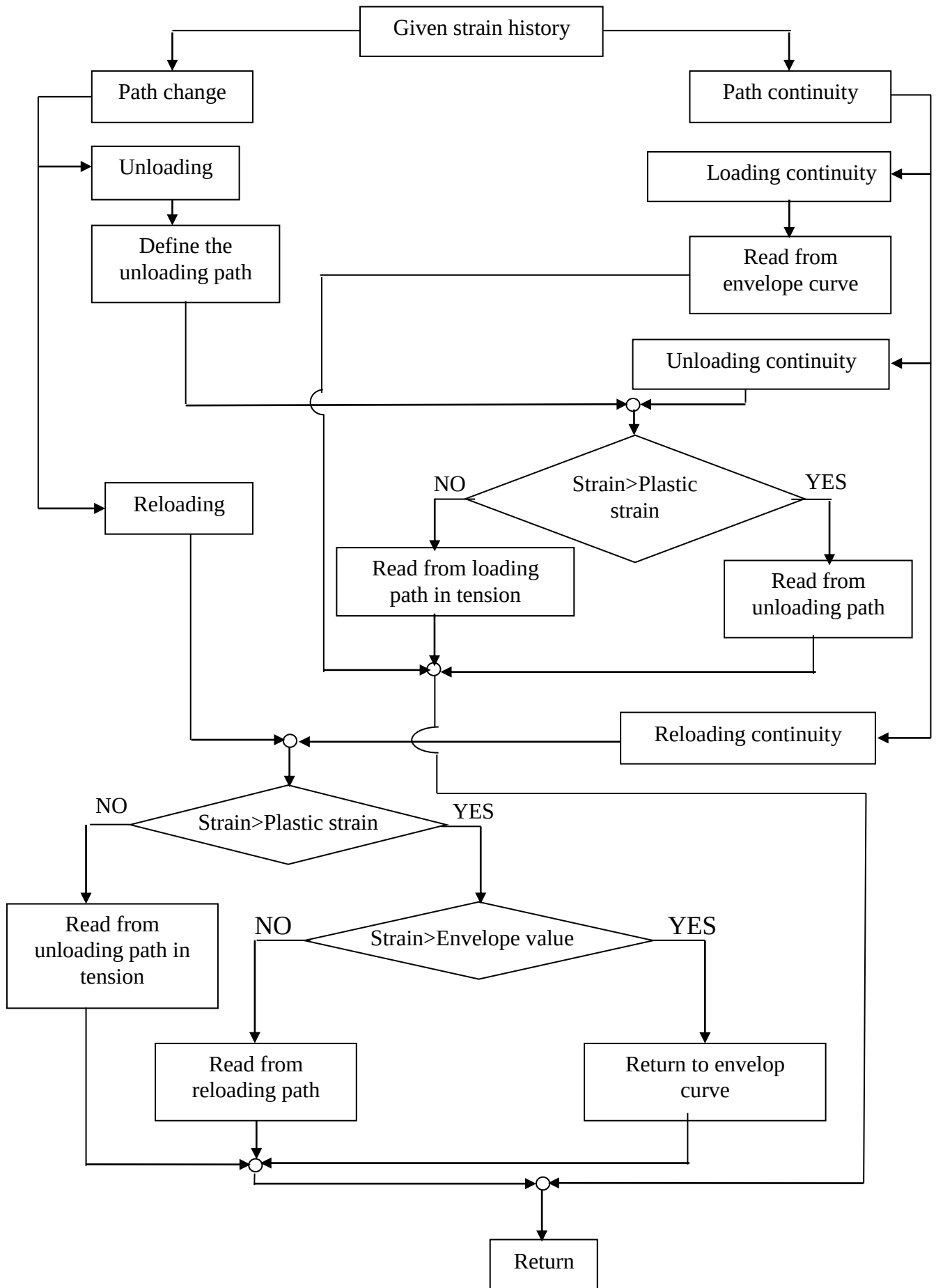


Fig. (3.5): The Flow Chart for Concrete Cyclic Behavior Subroutine (concy).

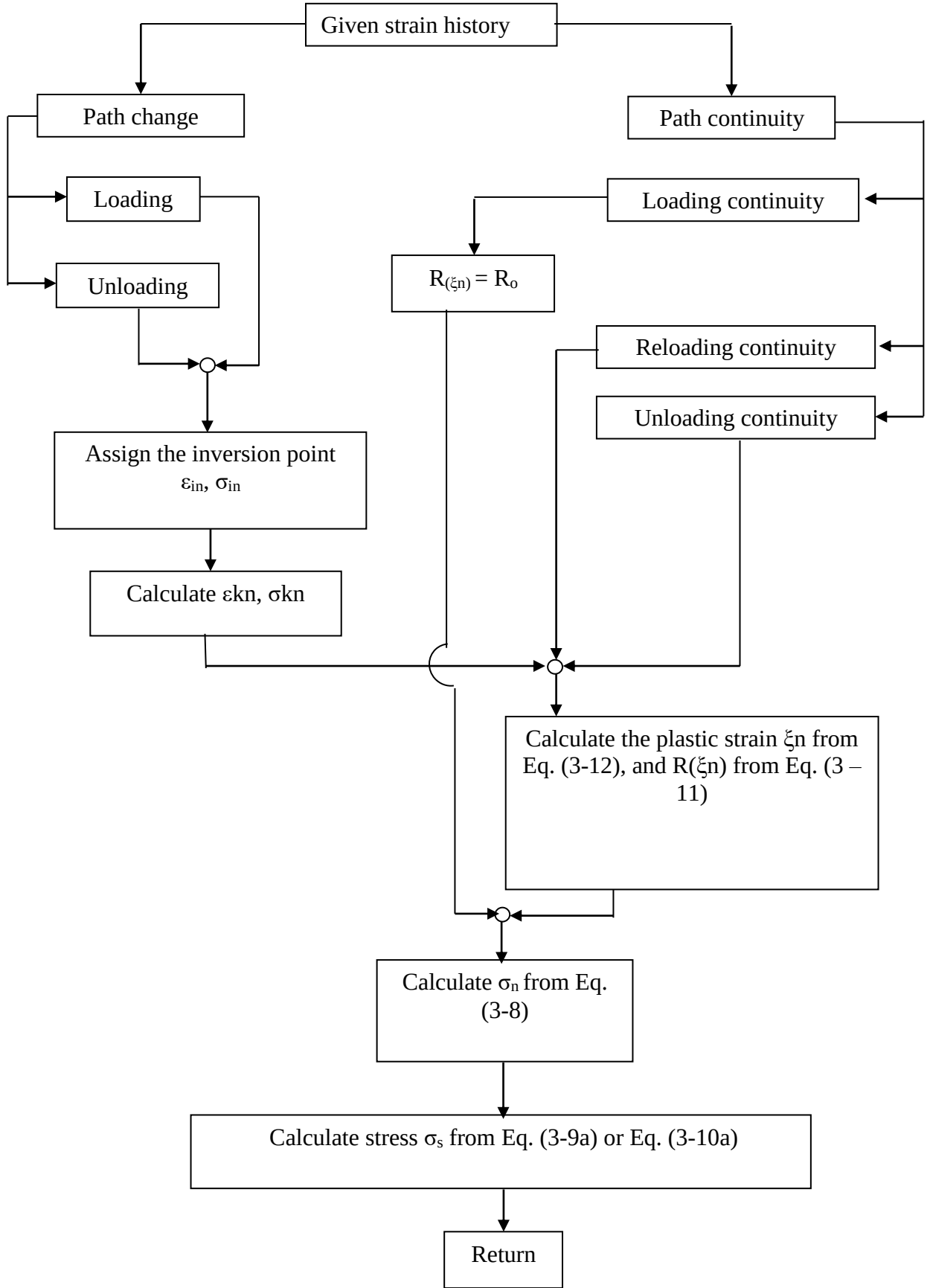


Fig. (3.6): The Flow Chart For Steel Cyclic Behavior Subroutine (stecy)

CHAPTER FOUR

(FORMULATION OF STIFFNESS MATRIX AND FIXED END FORCES)

4.1 General:

In this chapter, the stiffness matrix and fixed end forces are formulated based on finite element method with a developed form approach (namely region approach). The derivation of a numerical form of stiffness matrix for prismatic or non – prismatic element takes into account; (a) the interaction between bending moments, shear and axial forces; (b) the possibility of existence of plastic zone any where in the element. In addition, for the case of element under uniformly distributed load, fixed end forces are derived considering the possibility of existence of plastic zone wherever it is formed.

4.2 Derivation of Stiffness Matrix for Tapered Element:

Considering the non – prismatic element of length (L) shown in Fig. (4.1). The element is rectangular in cross section. Six degrees of freedom are assumed at each node. All displacements, stresses and forces are positive if they are in the directions shown in Fig. (4.2).

4.2.1 Basic Assumptions:

1. The element is divided into the number of concrete and steel regions along the depth and width as shown in Fig. (4.1). Reinforcing steel is treated as an equivalent uniaxial region material placed at depth of the center line of the bar.
2. If the concrete region in the section under consideration does not pass into inelastic range, the flexural and shear modulus of elasticity will be equal to the initial tangential modulus of elasticity for flexure and shear respectively. Otherwise flexure and shear modulus of elasticity will be calculated according to the flow rule which will be illustrated later in chapter five.
3. The flexural modulus of elasticity for steel regions will be equal to the

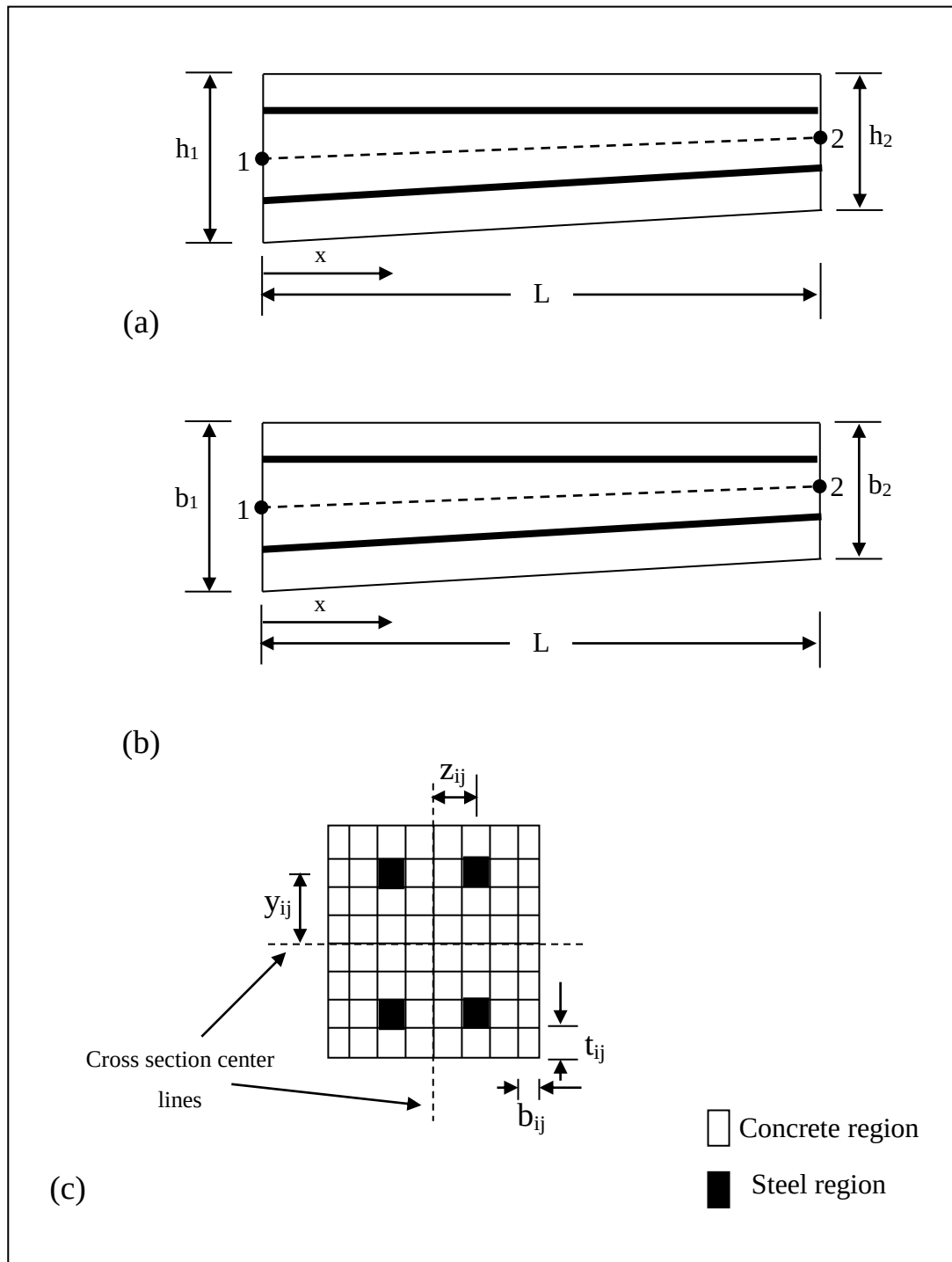


Fig. (4.1): Non – Prismatic Space Frame Element
(a) Typical Non – Prismatic Element (Side View)
(b) Typical Non – Prismatic Element (Top View)
(c) Section Discretization into Concrete and Steel Regions

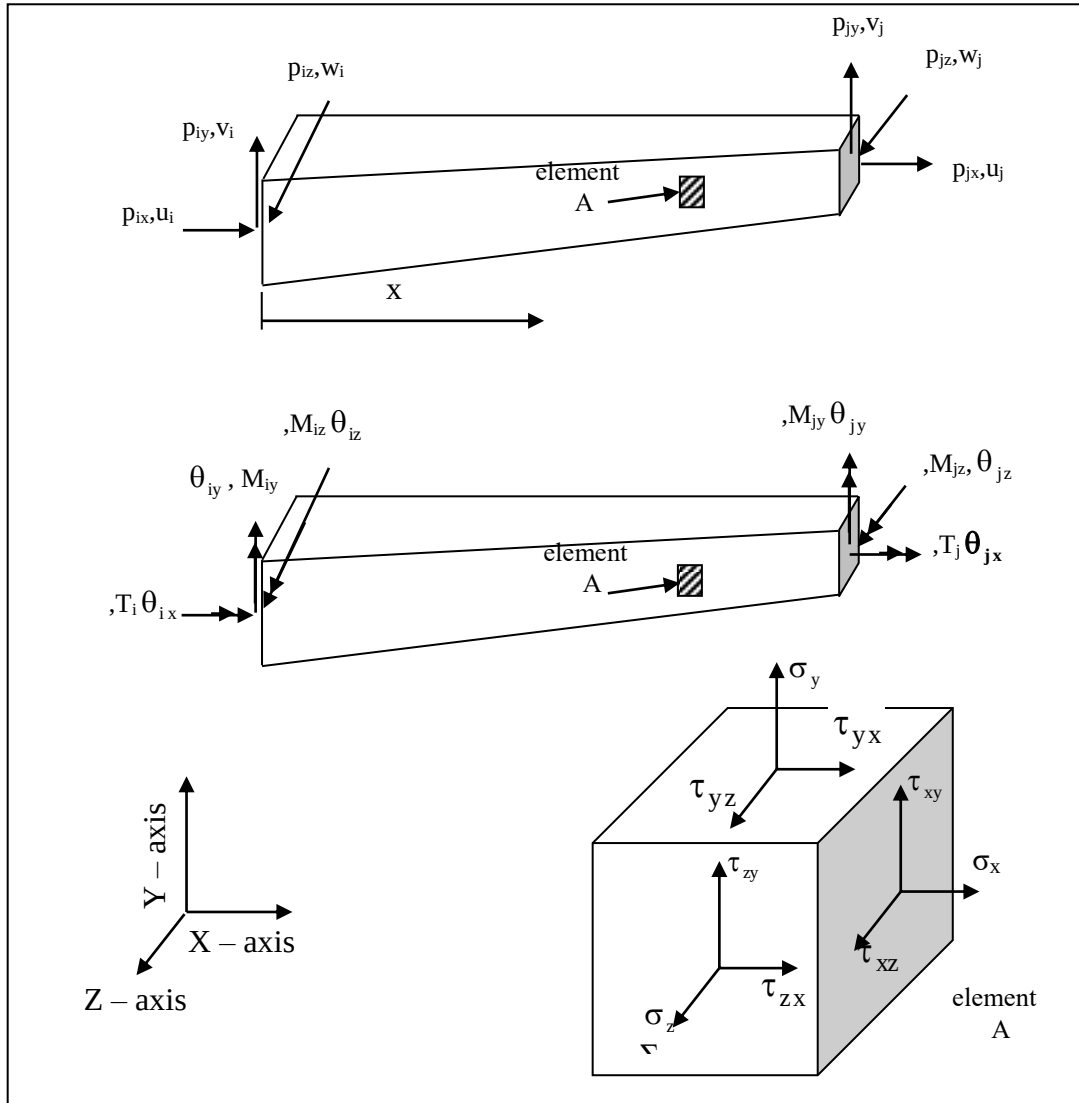


Fig. (4.2): Degrees of Freedom and Sign Convention.

tangential modulus of elasticity at any range of loading (see stress – strain model for steel in chapter three).

4. Plane sections before bending remain plane after bending.
5. Cracking of concrete is assumed to occur mainly in plane perpendicular to the axis of bending.
4. The stress in each concrete or steel region is assumed to be uniform throughout the area of the region, and is found from the proposed behavior model (stress – strain curve).

4.2.2 Assumed Displacement Field:

The axial displacement u at any point (x, y, z) may be expressed directly in terms of $\theta_{y(x)}$ and $\theta_{z(x)}$ which are the rotations of the normal to the centroidal x – axis due to flexural effect, and $u_{(x)}$ which is the axial displacement due to axial effect.

$$\mathbf{u}_{(x,y,z)} = \mathbf{u}_{(x)} + \mathbf{z}\theta_{y(x)} - \mathbf{y}\theta_{z(x)} \quad (4-1)$$

Where y is the distance from the centroidal axis of the cross section to the point (x,y,z) in y – direction, z is the distance from the centroidal axis of the cross section to the point (x,y,z) in z – direction.

The normal rotation $\theta_{y(x)}$ is equal to the rotation β_{xz} which is due to the transverse shear deformation minus a slope of the centroidal axis after deformation. The normal rotation $\theta_{z(x)}$ is equal to the slope of the centroidal axis after deformation minus a rotation β_{xy} which is due to the transverse shear deformation.

$$\theta_{y(x,y,z)} = -\frac{d\mathbf{w}_{(x)}}{dx} + \beta_{xz} = \theta_{y(x)} \quad (4-2)$$

$$\theta_{z(x,y,z)} = \frac{d\mathbf{v}_{(x)}}{dx} - \beta_{xy} = \theta_{z(x)} \quad (4-3)$$

$$\theta_{x(x,y,z)} = \theta_{x(x)} \quad (4-4)$$

$$\mathbf{w}_{(x,y,z)} = \mathbf{w}_{(x)} \quad (4-5)$$

$$\mathbf{v}_{(x,y,z)} = \mathbf{v}_{(x)} \quad (4-6)$$

where β is a rotation due to transverse shear deformation, $\theta_{x(x)}$ is a rotation about x – axis at any point (x,y,z) , w is the lateral displacement in z - direction at any point (x,y,z) and equals to the lateral displacement at the central axis, v is the lateral displacement in y - direction at any point (x,y,z) and equals to the lateral displacement at the central axis.

4.2.3 Concrete Stress – Strain Relationship:

For an isotropic elastic material the relevant elastic stress – strain relationships are:

$$\sigma_{cx} = \lambda e + 2G\epsilon_{cx} \quad (4-7)$$

$$\sigma_{cy} = \lambda \mathbf{e} + 2\mathbf{G}\epsilon_{cy} \quad (4-8)$$

$$\sigma_{cz} = \lambda \mathbf{e} + 2\mathbf{G}\epsilon_{cz} \quad (4-9)$$

$$\tau_{cxy} = \mathbf{KG} \gamma_{cxy} \quad (4-10)$$

$$\tau_{cxz} = \mathbf{KG} \gamma_{cxz} \quad (4-11)$$

$$\tau_{cyz} = \mathbf{KG} \gamma_{cyz} \quad (4-12)$$

where: $\mathbf{e} = \epsilon_x + \epsilon_y + \epsilon_z$

$$\lambda = \frac{\nu E_{co}}{(1 + \nu)(1 - 2\nu)}$$

ν : Concrete Poisson's ratio (0.11 – 0.27) [74].

G is the shear modulus of elasticity and for an isotropic material within elastic range is equal to:

$$\mathbf{G} = \frac{\mathbf{E}_{co}}{2 * (1 + \nu)}$$

The correction factor K which is used here because of the assumption that the distribution of transverse shear strain through the element thickness is uniform.

This correction factor equals to 5/6. [75]

The stress – strain relationship can be expressed in a matrix form.

$$\begin{bmatrix} \sigma_{cx} \\ \sigma_{cy} \\ \sigma_{cz} \\ \tau_{cxy} \\ \tau_{cxz} \\ \tau_{cyz} \end{bmatrix} = \begin{bmatrix} \lambda + 2\mathbf{G} & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mathbf{G} & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mathbf{G} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{KG} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{KG} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{KG} \end{bmatrix} \begin{bmatrix} \epsilon_{cx} \\ \epsilon_{cy} \\ \epsilon_{cz} \\ \gamma_{cxy} \\ \gamma_{cxz} \\ \gamma_{cyz} \end{bmatrix} \quad (4-13)$$

The normal stresses σ_{cy} and σ_{cz} can be assumed to be equal to zero then [77]

$$\epsilon_{cy} = -\nu \epsilon_{cx} \quad (4-14)$$

$$\epsilon_{cz} = -\nu \epsilon_{cx} \quad (4-15)$$

And by eliminating ϵ_{cy} and ϵ_{cz} from equation (4 – 13), it is possible to write the following elastic stress – strain relationship

$$\begin{bmatrix} \sigma_{cx} \\ \tau_{cxy} \\ \tau_{cxz} \\ \tau_{cyz} \end{bmatrix} = [\mathbf{D}_{con}] \begin{bmatrix} \epsilon_{cx} \\ \gamma_{cxy} \\ \gamma_{cxz} \\ \gamma_{cyz} \end{bmatrix} \quad (4-16)$$

Where $[D_{con}]$ is the concrete stress – strain relationship matrix which is taking the following form within elastic rang:

$$[D_{con}] = \begin{bmatrix} E_{co} & 0 & 0 & 0 \\ 0 & KG_{xy} & 0 & 0 \\ 0 & 0 & KG_{xz} & 0 \\ 0 & 0 & 0 & KG_{yz} \end{bmatrix} \quad (4-17)$$

For the plastic range, $[D_{con}]$ will be calculated according to the flow rule which will be illustrated later in chapter five.

4.2.4 Steel Stress – Strain Relationships:

The steel region deforms only in the direction of the bar and having strength and stiffness characteristics in the bar direction only. The dowel action of the reinforcing steel is neglected. The elastic stress – strain relationship can be written as:

$$\begin{bmatrix} \sigma_{sx} \\ \tau_{sxy} \\ \tau_{sxz} \\ \tau_{syx} \end{bmatrix} = [D_s] \begin{bmatrix} \epsilon_{sx} \\ \gamma_{sxy} \\ \gamma_{sxz} \\ \gamma_{syx} \end{bmatrix} \quad (4-18a)$$

Where $[D_s]$ is the steel stress – strain relationship matrix which takes the following form

$$[D_s] = \begin{bmatrix} E_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (4-18b)$$

E_s equals to the tangential modulus of elasticity which is calculated from stress – strain model for steel (see Fig.(3.4)).

4.2.5 Strain – Displacement Relationships:

According to equation (4 – 1) the axial strain in x – direction is given as:

$$\epsilon_x = \frac{\partial u_{(x,y,z)}}{\partial x} = \frac{du_{(x)}}{dx} - y \frac{d\theta_{z(x)}}{dx} + z \frac{d\theta_{y(x)}}{dx} \quad (4-19)$$

Similarly the flexural shear strain γ_{xy} is given as:

$$\gamma_{xy} = \frac{\partial u_{(x,y,z)}}{\partial y} + \frac{\partial v_{(x,y,z)}}{\partial x}$$

and according to equation (4 – 3)

$$\gamma_{xy} = -\theta_{z(x)} + \frac{dv_{(x,y,z)}}{dx} = \beta_{xy} \quad (4-20)$$

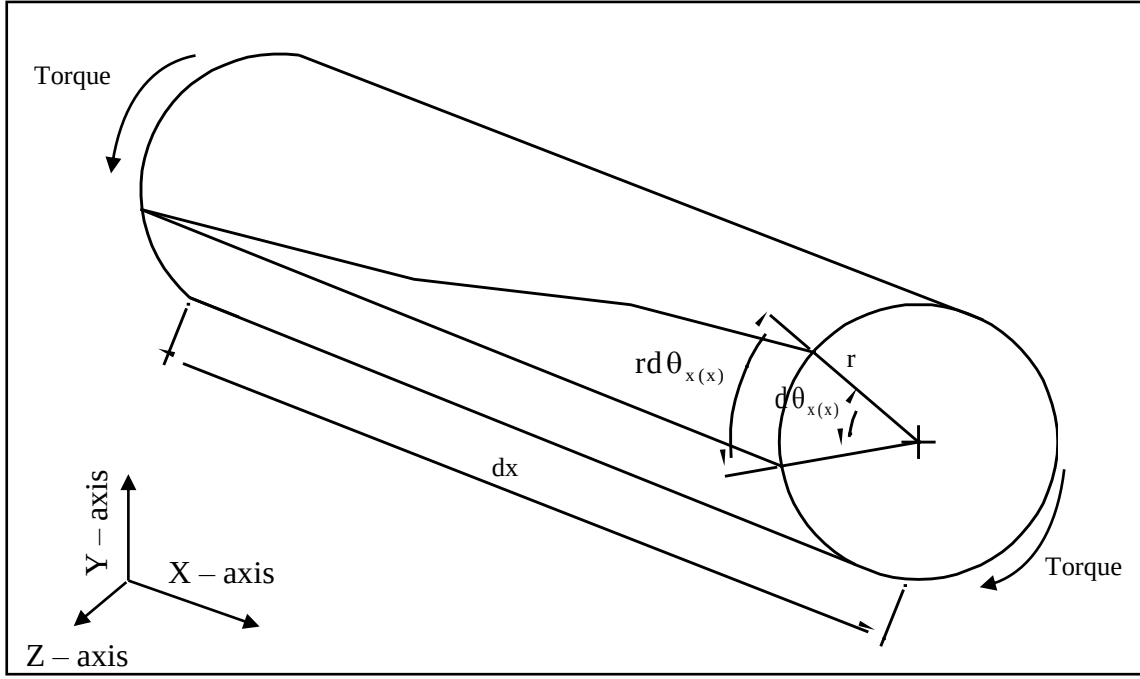


Fig. (4.3): Deformation Shape of Circular Shaft by Torsion.

The shear strain γ_{xz} is given as:

$$\gamma_{xz} = \frac{\partial u_{(x,y,z)}}{\partial z} + \frac{\partial w_{(x,y,z)}}{\partial x}$$

in the same manner, according to equation (4 – 2)

$$\gamma_{xz} = \theta_{y(x)} + \frac{dw_{(x,y,z)}}{dx} = \beta_{xz} \quad (4 - 21)$$

Also, the torsional shear strain γ_{yz} is given as (see Fig. (4.3)):

$$\gamma_{yz} = r \frac{d\theta_{x(x)}}{dx} = \sqrt{y^2 + z^2} \frac{d\theta_{x(x)}}{dx} \quad (4 - 22)$$

4.2.6 Finite Element Representation of Displacement and Strain:

In the frame element representation, the lateral displacement $w_{(x)}$ and the rotation $\theta_{y(x)}$ are represented as [73]:

$$w_{(x)} = \bar{N}_1 w_1 + \bar{N}_2 \theta_{1y} + \bar{N}_3 w_2 + \bar{N}_4 \theta_{2y} \quad (4 - 23)$$

$$\theta_{y(x)} = \bar{\bar{N}}_1 w_1 + \bar{\bar{N}}_2 \theta_{1y} + \bar{\bar{N}}_3 w_2 + \bar{\bar{N}}_4 \theta_{2y} \quad (4 - 24)$$

Where w_1, θ_{1y} and w_2, θ_{2y} are the nodal displacement at local nodes 1 and 2 of element respectively.

$\bar{N}_1$ to $\bar{N}_4$ and $\bar{\bar{N}}_1$ to $\bar{\bar{N}}_4$ are shape functions.

Also the lateral displacement $w_{(x)}$ in the element can be represented by cubic polynomial as:

$$w_{(x)} = a_1 + a_2 x + a_3 x^2 + a_4 x^3 \quad (4 - 25)$$

The relation between the transverse shear strain γ_{xz} , $\frac{dw_{(x)}}{dx}$ and $\theta_{y(x)}$ is (from eq.(4 – 21)):

$$\frac{dw_{(x)}}{dx} = \gamma_{xz} - \theta_{y(x)} \quad (4 - 26)$$

The transverse shear strain γ_{xz} , consistent with the cubic polynomial of $w_{(x)}$, is assumed to be independent of x .

$$\text{i.e. } \gamma_{xz} = b_0 \quad (4 - 27)$$

Then, moment equilibrium of infinitesimal beam section yields:

$$\frac{dM_y}{dx} - p_z = 0 \quad (4 - 28)$$

The introduction of equilibrium equations as equation (4 – 28) in the formulation is formally equivalent to constrain to facilitate inclusion of transverse shear deformation in displacement formulations without involving additional grid point degrees of freedom.

The bending moment – curvature relation is:

$$M_y = -EI_y \frac{d\theta_{y(x)}}{dx} \quad (4 - 29)$$

The shearing force p_z is related to transverse shear strain γ_{xz} by

$$p_z = KG_{xz} A \gamma_{xz} \quad (4 - 30)$$

Where A is the cross sectional area, K is the shear correction factor. Using equations (4 – 25 to 4 – 30) gives

$$b_0 = 6a_4 g \quad (4 - 31)$$

$$\text{Where } g = \frac{EI_y}{KG_{xz} A} \quad (4 - 32)$$

Using equations (4 – 25 to 4 – 27 and 4 – 31) allow $\theta_{y(x)}$ to be expressed as a polynomial in x as follows:

$$\theta_{y(x)} = -a_2 - 2a_3 x + (6g - 3x^2) a_4 \quad (4 - 33)$$

Using equations (4 – 25) to (4 – 33), it is now possible to express the coefficients a_1 through a_4 in terms of the displacement $w_{(x)}$ and rotation $\theta_{y(x)}$ of the element at the ends $x = 0$ (w_1, θ_{1y}) and $x = L$ (w_2, θ_{2y}).

The results are

$$a_1 = w_1 \quad (4 - 34)$$

$$a_2 = F \left[-12g w_1 + 12g w_2 - (6gL - L^3) \theta_{1y} + 6gL \theta_{2y} \right] \quad (4 - 35)$$

$$a_3 = F \left[3w_1 L - 3w_2 L + (6g - 2L^2) \theta_{1y} - (6g + 2L^2) \theta_{2y} \right] \quad (4 - 36)$$

$$a_4 = F \left[-2w_1 + 2w_2 + \theta_{1y} L + \theta_{2y} L \right] \quad (4 - 37)$$

$$\text{where } F = \frac{1}{L(12g - L^2)}$$

Substitute these coefficients into equation (4 – 25) yields

$$\begin{aligned} w_{(x)} = & \left[1 - 12gFx + 3LFx^2 - 2Fx^3 \right] w_1 + \left[(L^3 - 6gL)Fx + (6g - 2L^2)Fx^2 + LFx^3 \right] \theta_{1y} \\ & + \left[12gFx - 3LFx^2 + 2Fx^3 \right] w_2 + \left[6gLx - (6g + 2L^2)Fx^2 + LFx^3 \right] \theta_{2y} \end{aligned} \quad (4 - 38)$$

by comparison between equation (4 – 38) and (4 – 23) one can find that

$$\bar{N}_1 = 1 - F \left[12gx - 3Lx^2 + 2x^3 \right] \quad (4 - 39)$$

$$\bar{N}_2 = F \left[- (6gL - L^3)x + (6g - 2L^2)x^2 + Lx^3 \right] \quad (4 - 40)$$

$$\bar{N}_3 = F \left[12gx - 3Lx^2 + 2x^3 \right] \quad (4 - 41)$$

$$\bar{N}_4 = F \left[6gLx - (6g + L^2)x^2 + Lx^3 \right] \quad (4 - 42)$$

and similarly by substituting the coefficients a_2 to a_5 into equation (4 – 33) and comparing with equation (4 – 24) yields:

$$\bar{\bar{N}}_1 = F \left[6x^2 - 6xL \right] \quad (4 - 43)$$

$$\bar{\bar{N}}_2 = F \left[12gL - L^3 - (12g - 4L^2)x - 3x^2L \right] \quad (4 - 44)$$

$$\bar{\bar{N}}_3 = F \left[6xL - 6x^2 \right] \quad (4 - 45)$$

$$\bar{\bar{N}}_4 = F \left[-3x^2L + (12g + 2L^2)x \right] \quad (4 - 46)$$

In the same way:

$$\mathbf{v}_{(x)} = \bar{\bar{\bar{N}}}_1 \mathbf{v}_1 + \bar{\bar{\bar{N}}}_2 \boldsymbol{\theta}_{1z} + \bar{\bar{\bar{N}}}_3 \mathbf{v}_2 + \bar{\bar{\bar{N}}}_4 \boldsymbol{\theta}_{2z} \quad (4-47)$$

$$\boldsymbol{\theta}_{z(x)} = \bar{\bar{\bar{N}}}_1 \mathbf{v}_1 + \bar{\bar{\bar{N}}}_2 \boldsymbol{\theta}_{1z} + \bar{\bar{\bar{N}}}_3 \mathbf{v}_2 + \bar{\bar{\bar{N}}}_4 \boldsymbol{\theta}_{2z} \quad (4-48)$$

where:

$$\bar{\bar{\bar{N}}}_1 = 1 - \bar{F} [12\bar{g}x + 3Lx^2 - 2x^3] \quad (4-49)$$

$$\bar{\bar{\bar{N}}}_2 = \bar{F} [(6\bar{g}L + L^3)x - (6\bar{g} + 2L^2)x^2 + Lx^3] \quad (4-50)$$

$$\bar{\bar{\bar{N}}}_3 = \bar{F} [12\bar{g}x + 3Lx^2 - 2x^3] \quad (4-51)$$

$$\bar{\bar{\bar{N}}}_4 = \bar{F} [-6\bar{g}Lx + (6\bar{g} - L^2)x^2 + Lx^3] \quad (4-52)$$

$$\bar{\bar{\bar{N}}}_1 = \bar{F} [6x^2 - 6xL] \quad (4-53)$$

$$\bar{\bar{\bar{N}}}_2 = \bar{F} [L^3 + 12\bar{g}L - (12\bar{g} + 4L^2)x + 3x^2L] \quad (4-54)$$

$$\bar{\bar{\bar{N}}}_3 = \bar{F} [6xL - 6x^2] \quad (4-55)$$

$$\bar{\bar{\bar{N}}}_4 = \bar{F} [3x^2L - (2L^2 - 12\bar{g})x] \quad (4-56)$$

$$\bar{F} = \frac{1}{L(12\bar{g} + L^2)} \quad (4-57)$$

$$\bar{g} = \frac{EI_z}{KG_{xy}A} \quad (4-58)$$

The other displacements are represented as:

$$\mathbf{u}_{(x)} = \mathbf{N}_1 \mathbf{u}_1 + \mathbf{N}_2 \mathbf{u}_2 \quad (4-59)$$

$$\boldsymbol{\theta}_{x(x)} = \mathbf{N}_1 \boldsymbol{\theta}_{1x} + \mathbf{N}_2 \boldsymbol{\theta}_{2x} \quad (4-60)$$

where:

$$\mathbf{N}_1 = \frac{L-x}{L} = 1-m, \quad \frac{d\mathbf{N}_1}{dx} = -\frac{1}{L} \quad (4-61)$$

$$\mathbf{N}_2 = \frac{x}{L} = m, \quad \frac{d\mathbf{N}_2}{dx} = \frac{1}{L} \quad (4-62)$$

The axial strain – displacement relationship can be expressed as:

$$\varepsilon_x = \frac{d\mathbf{u}_{(x)}}{dx} - y \frac{d\boldsymbol{\theta}_{z(x)}}{dx} + z \frac{d\boldsymbol{\theta}_{y(x)}}{dx}$$

$$\varepsilon_x = \left[u_1 \frac{dN_1}{dx} + u_2 \frac{dN_2}{dx} \right] - y \left[v_1 \frac{d\bar{N}_1}{dx} + \theta_{1z} \frac{d\bar{N}_2}{dx} + v_2 \frac{d\bar{N}_3}{dx} + \theta_{2z} \frac{d\bar{N}_4}{dx} \right] + z \left[w_1 \frac{d\bar{N}_1}{dx} + \theta_{1y} \frac{d\bar{N}_2}{dx} + w_2 \frac{d\bar{N}_3}{dx} + \theta_{2y} \frac{d\bar{N}_4}{dx} \right]$$

$$\varepsilon_x = \begin{bmatrix} -\frac{1}{L} & -K_1 & K_2 & 0 & K_3 & -K_4 & \frac{1}{L} & -K_5 & K_6 & 0 & K_7 & -K_8 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ w_1 \\ \theta_{1x} \\ \theta_{1y} \\ \theta_{1z} \\ u_2 \\ v_2 \\ w_2 \\ \theta_{2x} \\ \theta_{2y} \\ \theta_{2z} \end{bmatrix}$$

$$= B_A(x, y, z) \psi \quad (4-63)$$

where:

$$K_1 = y\bar{F}(12x - 6L) \quad (4-64)$$

$$K_2 = zF(12x - 6L) \quad (4-65)$$

$$K_3 = zF(-12g + 4L^2 - 6xL) \quad (4-66)$$

$$K_4 = y\bar{F}(-12\bar{g} - 4L^2 + 6xL) \quad (4-67)$$

$$K_5 = y\bar{F}(6L - 12x) = -K_1 \quad (4-68)$$

$$K_6 = zF(6L - 12x) = -K_2 \quad (4-69)$$

$$K_7 = zF(-6xL + 12g + 2L^2) \quad (4-70)$$

$$K_8 = y\bar{F}(6xL - 2L^2 + 12\bar{g}) \quad (4-71)$$

$B_A(x, y, z)$ is the axial strain – displacement matrix.

The shear strain – displacement relationship can be expressed as:

$$\gamma_{xy} = \frac{dv_{(x)}}{dx} - \theta_{z(x)}$$

$$\gamma_{xy} = \begin{bmatrix} \mathbf{v}_1 \frac{d\bar{\mathbf{N}}_1}{dx} + \theta_{1z} \frac{d\bar{\mathbf{N}}_2}{dx} + \mathbf{v}_2 \frac{d\bar{\mathbf{N}}_3}{dx} + \theta_{2z} \frac{d\bar{\mathbf{N}}_4}{dx} \\ - \left[\mathbf{v}_1 \bar{\mathbf{N}}_1 + \theta_{1z} \bar{\mathbf{N}}_2 + \mathbf{v}_2 \bar{\mathbf{N}}_3 + \theta_{2z} \bar{\mathbf{N}}_4 \right] \end{bmatrix}$$

$$\gamma_{xy} = \begin{bmatrix} 0 & K_9 & 0 & 0 & 0 & K_{10} & 0 & -K_9 & 0 & 0 & 0 & K_{10} \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{v}_1 \\ \mathbf{w}_1 \\ \theta_{1x} \\ \theta_{1y} \\ \theta_{1z} \\ \mathbf{u}_2 \\ \mathbf{v}_2 \\ \mathbf{w}_2 \\ \theta_{2x} \\ \theta_{2y} \\ \theta_{2z} \end{bmatrix}$$

$$= \mathbf{B}_{x,y} \psi \quad (4-72)$$

where:

$$K_9 = -12\bar{\mathbf{F}}\bar{\mathbf{g}} \quad (4-73)$$

$$K_{10} = -6\bar{\mathbf{F}}\bar{\mathbf{g}}\bar{\mathbf{L}} \quad (4-74)$$

$\mathbf{B}_{x,y}$ is the shear strain (γ_{xy}) – displacement matrix.

Also,

$$\gamma_{xz} = \frac{d\mathbf{w}_{(x)}}{dx} + \theta_{y(x)}$$

$$\gamma_{xz} = \begin{bmatrix} \mathbf{w}_1 \frac{d\bar{\mathbf{N}}_1}{dx} + \theta_{1y} \frac{d\bar{\mathbf{N}}_2}{dx} + \mathbf{w}_2 \frac{d\bar{\mathbf{N}}_3}{dx} + \theta_{2y} \frac{d\bar{\mathbf{N}}_4}{dx} \\ + \left[\mathbf{w}_1 \bar{\mathbf{N}}_1 + \theta_{1y} \bar{\mathbf{N}}_2 + \mathbf{w}_2 \bar{\mathbf{N}}_3 + \theta_{2y} \bar{\mathbf{N}}_4 \right] \end{bmatrix}$$

$$\gamma_{xz} = \begin{bmatrix} 0 & 0 & K_{11} & 0 & K_{12} & 0 & 0 & 0 & -K_{11} & 0 & K_{12} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{v}_1 \\ \mathbf{w}_1 \\ \theta_{1x} \\ \theta_{1y} \\ \theta_{1z} \\ \mathbf{u}_2 \\ \mathbf{v}_2 \\ \mathbf{w}_2 \\ \theta_{2x} \\ \theta_{2y} \\ \theta_{2z} \end{bmatrix}$$

$$= \mathbf{B}_{x,z} \boldsymbol{\psi} \quad (4 - 75)$$

where:

$$\mathbf{K}_{11} = -12\mathbf{Fg} \quad (4 - 76)$$

$$\mathbf{K}_{12} = 6\mathbf{FgL} \quad (4 - 77)$$

$\mathbf{B}_{x,z}$ is the shear strain (γ_{xz})– displacement matrix.

$$\begin{aligned} \gamma_{yz} &= \mathbf{r} \frac{d\theta_{x(x)}}{dx} \\ &= \sqrt{y^2 + z^2} \frac{d\theta_{x(x)}}{dx} \\ &= \left(\sqrt{y^2 + z^2} \theta_{x1} \frac{dN_1}{dx} \right) + \left(\sqrt{y^2 + z^2} \theta_{x2} \frac{dN_2}{dx} \right) \\ &= \left(\frac{-\sqrt{y^2 + z^2}}{L} * \theta_{x1} \right) + \left(\frac{\sqrt{y^2 + z^2}}{L} * \theta_{x2} \right) \end{aligned}$$

$$\gamma_{yz} = \begin{bmatrix} 0 & 0 & 0 & \frac{-\sqrt{y^2+z^2}}{L} & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{y^2+z^2}}{L} & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ w_1 \\ \theta_{x1} \\ \theta_{y1} \\ \theta_{z1} \\ u_2 \\ v_2 \\ w_2 \\ \theta_{x2} \\ \theta_{y2} \\ \theta_{z2} \end{bmatrix}$$

$$= \mathbf{B}_{y,z} \boldsymbol{\psi} \quad (4-78)$$

Where $\mathbf{B}_{y,z}$ is the shear strain (γ_{yz})– displacement matrix.

4.2.7 Stiffness Matrix Evaluation :

According to first assumption, each element will be divided into chosen number of steel and concrete regions in y and z – directions as shown in Fig. (4.1). The element is subjected to external applied forces $[f]$. If the element undergoes a set of virtual displacement $[\zeta \boldsymbol{\psi}]$ which has the form

$$[\zeta \boldsymbol{\psi}] = [\zeta u_1 \quad \zeta v_1 \quad \zeta w_1 \quad \zeta \theta_{1x} \quad \zeta \theta_{1y} \quad \zeta \theta_{1z} \quad \zeta u_2 \quad \zeta v_2 \quad \zeta w_2 \quad \zeta \theta_{2x} \quad \zeta \theta_{2y} \quad \zeta \theta_{2z}] \quad (4-79)$$

Then the virtual work equation can be written as:^[9]

$$\begin{aligned} & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} (b_{ij} t_{ij} \epsilon_{cx(ij)} \sigma_{cx(ij)}) dx + \\ & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} (b_{ij} t_{ij} \gamma_{cxy(ij)} \tau_{cxy(ij)}) dx + \\ & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} (b_{ij} t_{ij} \gamma_{cxz(ij)} \tau_{cxz(ij)}) dx + \\ & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} (b_{ij} t_{ij} \gamma_{cyz(ij)} \tau_{cyz(ij)}) dx + \\ & \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} (\epsilon_{sx(ij)} \sigma_{sx(ij)} A_{s(ij)}) dx = [f]^T [\zeta \boldsymbol{\psi}] \end{aligned} \quad (4-80)$$

where N_{1c} and N_{2c} are equal to the total number of concrete regions in width and depth respectively, N_{1s} and N_{2s} are equal to the total number of steel regions in width and depth respectively. According to the equation (4 – 63), (4-72), (4-75) and (4 – 78) the equation (4 – 80) can be rewritten as:

$$\begin{aligned}
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} [\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cA}]^T [\boldsymbol{\varsigma}\boldsymbol{\Psi}] \boldsymbol{\sigma}_{cx(ij)}] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} [\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cxy}]^T [\boldsymbol{\varsigma}\boldsymbol{\Psi}] \boldsymbol{\tau}_{cxy(ij)}] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} [\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cxz}]^T [\boldsymbol{\varsigma}\boldsymbol{\Psi}] \boldsymbol{\tau}_{cxz(ij)}] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} [\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cyz}]^T [\boldsymbol{\varsigma}\boldsymbol{\Psi}] \boldsymbol{\tau}_{cyz(ij)}] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} [[\mathbf{B}_{sA}]^T [\boldsymbol{\varsigma}\boldsymbol{\Psi}] \boldsymbol{\sigma}_{sx(ij)} \mathbf{A}_{s(ij)}] \mathbf{dx} = [\mathbf{f}]^T [\boldsymbol{\varsigma}\boldsymbol{\Psi}]
 \end{aligned} \tag{4 – 81}$$

By utilizes equations (4 – 16) and (4 – 18); equation (4 -81) becomes:

$$\begin{aligned}
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} [\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cA}]^T [\boldsymbol{\varsigma}\boldsymbol{\Psi}] \mathbf{E}_{c(ij)} \boldsymbol{\varepsilon}_{cx(ij)}] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} [\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cxy}]^T [\boldsymbol{\varsigma}\boldsymbol{\Psi}] \mathbf{G}_{xy(ij)} \boldsymbol{\gamma}_{cxy(ij)}] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} [\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cxz}]^T [\boldsymbol{\varsigma}\boldsymbol{\Psi}] \mathbf{G}_{xz(ij)} \boldsymbol{\gamma}_{cxz(ij)}] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} [\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cyz}]^T [\boldsymbol{\varsigma}\boldsymbol{\Psi}] \mathbf{G}_{yz(ij)} \boldsymbol{\gamma}_{cyz(ij)}] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} [[\mathbf{B}_{sA}]^T [\boldsymbol{\varsigma}\boldsymbol{\Psi}] \mathbf{E}_{s(ij)} \boldsymbol{\varepsilon}_{sx(ij)} \mathbf{A}_{s(ij)}] \mathbf{dx} = [\mathbf{f}]^T [\boldsymbol{\varsigma}\boldsymbol{\Psi}]
 \end{aligned} \tag{4 – 82}$$

Then according to the equation (4 – 63) , (4-72), (4-75) and (4 – 78) the results of equation (4 – 82) can be rewritten as:

$$\begin{aligned}
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cA}]^T [\varsigma \psi] \mathbf{E}_{c(ij)} [\mathbf{B}_{cA}] [\psi]^T \right] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\frac{\mathbf{b}_{ij} \mathbf{t}_{ij}}{\alpha} [\mathbf{B}_{cxy}]^T [\varsigma \psi] \mathbf{G}_{xy(ij)} [\mathbf{B}_{xy}] [\psi]^T \right] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\frac{\mathbf{b}_{ij} \mathbf{t}_{ij}}{\alpha} [\mathbf{B}_{cxz}]^T [\varsigma \psi] \mathbf{G}_{xz(ij)} [\mathbf{B}_{xz}] [\psi]^T \right] \mathbf{dx} + \quad (4-83) \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\frac{\mathbf{b}_{ij} \mathbf{t}_{ij}}{\alpha} [\mathbf{B}_{cyz}]^T [\varsigma \psi] \mathbf{G}_{yz(ij)} [\mathbf{B}_{yz}] [\psi]^T \right] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} \left[[\mathbf{B}_{sA}]^T [\varsigma \psi] \mathbf{E}_{s(ij)} \mathbf{A}_{s(ij)} [\mathbf{B}_{sA}] [\psi]^T \right] \mathbf{dx} = [\mathbf{f}]^T [\varsigma \psi]
 \end{aligned}$$

by differentiating the equation (4 – 83) with respect to $[\varsigma \psi]$ then:

$$\left[\begin{aligned}
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\mathbf{E}_{c(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_{cA}]^T [\mathbf{B}_{cA}] \right] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\frac{\mathbf{G}_{xy(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij}}{\alpha} [\mathbf{B}_{cxy}]^T [\mathbf{B}_{xy}] \right] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\frac{\mathbf{G}_{xz(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij}}{\alpha} [\mathbf{B}_{cxz}]^T [\mathbf{B}_{xz}] \right] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \left[\frac{\mathbf{G}_{yz(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij}}{\alpha} [\mathbf{B}_{cyz}]^T [\mathbf{B}_{yz}] \right] \mathbf{dx} + \\
 & \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} \left[\mathbf{E}_{s(ij)} \mathbf{A}_{s(ij)} [\mathbf{B}_{sA}]^T [\mathbf{B}_{sA}] \right] \mathbf{dx}
 \end{aligned} \right] [\psi]^T = [\mathbf{f}]^T \quad (4-84)$$

where α is a correction factor to allow for cross – sectional warping. For rectangular section α is taken as 1.5 [9].

Briefly equation (4 – 84) can be written as:

$$[\mathbf{K}_{ca} + \mathbf{K}_{cxy} + \mathbf{K}_{cxz} + \mathbf{K}_{cyz} + \mathbf{K}_{sa}] [\psi] = [\mathbf{f}] \quad (4-85)$$

where:

$$K_{ca} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} \begin{bmatrix} -\frac{1}{L} \\ \frac{1}{L} \\ -K_1 \\ K_2 \\ 0 \\ K_3 \\ -K_4 \\ \frac{1}{L} \\ -K_5 \\ K_6 \\ 0 \\ K_7 \\ -K_8 \end{bmatrix} \begin{bmatrix} -\frac{1}{L} & -K_1 & K_2 & 0 & K_3 & -K_4 & \frac{1}{L} & -K_5 & K_6 & 0 & K_7 & -K_8 \end{bmatrix} dx \quad (4-86)$$

$$K_{ca} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} \begin{bmatrix} \frac{1}{L^2} & \frac{K_1}{L} & -\frac{K_2}{L} & 0 & -\frac{K_3}{L} & \frac{K_4}{L} & -\frac{1}{L^2} & \frac{K_5}{L} & -\frac{K_6}{L} & 0 & -\frac{K_7}{L} & \frac{K_8}{L} \\ \frac{K_1}{L} & K_1^2 & -K_1 K_2 & 0 & -K_1 K_3 & K_1 K_4 & -\frac{K_1}{L} & K_1 K_5 & -K_1 K_6 & 0 & -K_1 K_7 & K_1 K_8 \\ -\frac{K_2}{L} & -K_1 K_2 & K_2^2 & 0 & K_2 K_3 & -K_2 K_4 & \frac{K_2}{L} & -K_2 K_5 & K_2 K_6 & 0 & K_2 K_7 & -K_2 K_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_3}{L} & -K_1 K_3 & K_2 K_3 & 0 & K_3^2 & -K_3 K_4 & \frac{K_3}{L} & -K_3 K_5 & K_3 K_6 & 0 & K_3 K_7 & -K_3 K_8 \\ \frac{K_4}{L} & K_1 K_4 & -K_2 K_4 & 0 & -K_3 K_4 & K_4^2 & -\frac{K_4}{L} & K_4 K_5 & -K_4 K_6 & 0 & -K_4 K_7 & K_4 K_8 \\ -\frac{1}{L^2} & -\frac{K_1}{L} & \frac{K_2}{L} & 0 & \frac{K_3}{L} & -\frac{K_4}{L} & \frac{1}{L^2} & -\frac{K_5}{L} & \frac{K_6}{L} & 0 & \frac{K_7}{L} & -\frac{K_8}{L} \\ \frac{K_5}{L} & K_1 K_5 & -K_2 K_5 & 0 & -K_3 K_5 & K_4 K_5 & -\frac{K_5}{L} & K_5^2 & -K_5 K_6 & 0 & -K_5 K_7 & K_5 K_8 \\ -\frac{K_6}{L} & -K_1 K_6 & K_2 K_6 & 0 & K_3 K_6 & -K_4 K_6 & \frac{K_6}{L} & -K_5 K_6 & K_6^2 & 0 & K_6 K_7 & -K_6 K_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_7}{L} & -K_1 K_7 & K_2 K_7 & 0 & K_3 K_7 & -K_4 K_7 & \frac{K_7}{L} & -K_5 K_7 & K_6 K_7 & 0 & K_7^2 & -K_7 K_8 \\ \frac{K_8}{L} & K_1 K_8 & -K_2 K_8 & 0 & -K_3 K_8 & K_4 K_8 & -\frac{K_8}{L} & K_5 K_8 & -K_6 K_8 & 0 & -K_7 K_8 & K_8^2 \end{bmatrix} dx \quad (4-87)$$

$$K_{cy} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} G_{xy(ij)} \frac{b_{ij} t_{ij}}{\alpha} \begin{bmatrix} 0 \\ K_9 \\ 0 \\ 0 \\ 0 \\ K_{10} \\ 0 \\ -K_9 \\ 0 \\ 0 \\ 0 \\ K_{10} \end{bmatrix} \begin{bmatrix} 0 & K_9 & 0 & 0 & 0 & K_{10} & 0 & -K_9 & 0 & 0 & 0 & K_{10} \end{bmatrix} dx \quad (4-88)$$

$$K_{cy} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \frac{G_{xy(ij)} b_{ij} t_{ij}}{\alpha} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_9^2 & 0 & 0 & 0 & K_9 K_{10} & 0 & -K_9^2 & 0 & 0 & 0 & K_9 K_{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_9 K_{10} & 0 & 0 & 0 & K_{10}^2 & 0 & -K_9 K_{10} & 0 & 0 & 0 & K_{10}^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -K_9^2 & 0 & 0 & 0 & -K_9 K_{10} & 0 & K_9^2 & 0 & 0 & 0 & -K_9 K_{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_9 K_{10} & 0 & 0 & 0 & K_{10}^2 & 0 & -K_9 K_{10} & 0 & 0 & 0 & K_{10}^2 \end{bmatrix} dx \quad (4-89)$$

$$K_{cx} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \frac{G_{xz(ij)} b_{ij} t_{ij}}{\alpha} \begin{bmatrix} 0 \\ 0 \\ K_{11} \\ 0 \\ K_{12} \\ 0 \\ 0 \\ 0 \\ -K_{11} \\ 0 \\ 0 \\ K_{12} \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & K_{11} & 0 & K_{12} & 0 & 0 & 0 & -K_{11} & 0 & K_{12} & 0 \end{bmatrix} dx \quad (4-90)$$

$$K_{\alpha\alpha} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \frac{G_{\alpha(ij)} b_{ij} t_{ij}}{\alpha} dx \quad (4-91)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{11}^2 & 0 & K_{11}K_{12} & 0 & 0 & 0 & -K_{11}^2 & 0 & K_{11}K_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{11}K_{12} & 0 & K_{12}^2 & 0 & 0 & 0 & -K_{11}K_{12} & 0 & K_{12}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -K_{11}^2 & 0 & -K_{11}K_{12} & 0 & 0 & 0 & K_{11}^2 & 0 & -K_{11}K_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{11}K_{12} & 0 & K_{12}^2 & 0 & 0 & 0 & -K_{11}K_{12} & 0 & K_{12}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (m-m^2)L^2 & 0 & -mL & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K_{\alpha\gamma} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \frac{G_{\alpha\gamma(ij)} b_{ij} t_{ij}}{\alpha} dx \quad (4-92)$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{-\sqrt{y^2+z^2}}{L} \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{\sqrt{y^2+z^2}}{L} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & \frac{-\sqrt{y^2+z^2}}{L} & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{y^2+z^2}}{L} & 0 & 0 \end{bmatrix} dx$$

$$K_{\gamma\gamma} = \frac{1}{L^2} \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \frac{G_{\gamma\gamma(ij)} b_{ij} t_{ij}}{\alpha} dx \quad (4-93)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (y^2+z^2) & 0 & 0 & 0 & 0 & 0 & -(y^2+z^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(y^2+z^2) & 0 & 0 & 0 & 0 & 0 & (y^2+z^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K_{so} = \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} \begin{bmatrix} -\frac{1}{L} \\ \frac{K_1}{L} \\ -K_1 \\ K_2 \\ 0 \\ K_3 \\ -K_4 \\ \frac{1}{L} \\ -\frac{K_5}{L} \\ K_6 \\ 0 \\ K_7 \\ -K_8 \end{bmatrix} \begin{bmatrix} -\frac{1}{L} & -K_1 & K_2 & 0 & K_3 & -K_4 & \frac{1}{L} & -K_5 & K_6 & 0 & K_7 & -K_8 \end{bmatrix} dx \quad (4-94)$$

$$K_{so} = \int_0^L \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} \begin{bmatrix} \frac{1}{L^2} & \frac{K_1}{L} & -\frac{K_2}{L} & 0 & -\frac{K_3}{L} & \frac{K_4}{L} & -\frac{1}{L^2} & \frac{K_5}{L} & -\frac{K_6}{L} & 0 & -\frac{K_7}{L} & \frac{K_8}{L} \\ \frac{K_1}{L} & K_1^2 & -K_1 K_2 & 0 & -K_1 K_3 & K_1 K_4 & -\frac{K_1}{L} & K_1 K_5 & -K_1 K_6 & 0 & -K_1 K_7 & K_1 K_8 \\ -\frac{K_2}{L} & -K_1 K_2 & K_2^2 & 0 & K_2 K_3 & -K_2 K_4 & \frac{K_2}{L} & -K_2 K_5 & K_2 K_6 & 0 & K_2 K_7 & -K_2 K_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_3}{L} & -K_1 K_3 & K_2 K_3 & 0 & K_3^2 & -K_3 K_4 & \frac{K_3}{L} & -K_3 K_5 & K_3 K_6 & 0 & K_3 K_7 & -K_3 K_8 \\ \frac{K_4}{L} & K_1 K_4 & -K_2 K_4 & 0 & -K_3 K_4 & K_4^2 & -\frac{K_4}{L} & K_4 K_5 & -K_4 K_6 & 0 & -K_4 K_7 & K_4 K_8 \\ -\frac{1}{L^2} & -\frac{K_1}{L} & \frac{K_2}{L} & 0 & \frac{K_3}{L} & -\frac{K_4}{L} & \frac{1}{L^2} & -\frac{K_5}{L} & \frac{K_6}{L} & 0 & \frac{K_7}{L} & -\frac{K_8}{L} \\ \frac{K_5}{L} & K_1 K_5 & -K_2 K_5 & 0 & -K_3 K_5 & K_4 K_5 & -\frac{K_5}{L} & K_5^2 & -K_5 K_6 & 0 & -K_5 K_7 & K_5 K_8 \\ -\frac{K_6}{L} & -K_1 K_6 & K_2 K_6 & 0 & K_3 K_6 & -K_4 K_6 & \frac{K_6}{L} & -K_5 K_6 & K_6^2 & 0 & K_6 K_7 & -K_6 K_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_7}{L} & -K_1 K_7 & K_2 K_7 & 0 & K_3 K_7 & -K_4 K_7 & \frac{K_7}{L} & -K_5 K_7 & K_6 K_7 & 0 & K_7^2 & -K_7 K_8 \\ \frac{K_8}{L} & K_1 K_8 & -K_2 K_8 & 0 & -K_3 K_8 & K_4 K_8 & -\frac{K_8}{L} & K_5 K_8 & -K_6 K_8 & 0 & -K_7 K_8 & K_8^2 \end{bmatrix} dx \quad (4-95)$$

Integration of equations ((4 – 87), (4 – 89), (4 – 91), (4 – 93) and (4 – 95)) have been made using Gaussian Quadrature method (see appendix A). Five points are found to be suitable to obtain acceptable results when this method is used to calculate K_{ca} and K_{sa} . If K_{cxy} , K_{cxz} , K_{cyz} is calculated exactly using a 2 – points or more than two points Gaussian Quadrature, over stiff solutions will be obtained. This phenomenon may be cured by using a 1 – point Gaussian Quadrature method.^[9]

4.3 Fixed – End Moments and Forces Due to Uniformly Distributed Load:

Consider the element shown in Fig. (4.4) in $x - y$ plane which is subjected to uniformly distributed load (w_{load}) and fixed at both ends.

The unknown end moments and forces can be found by choosing Q_1 , Q_2 , and Q_3 as redundant and solving for the unknowns using the consistent deformations method with the moment area method for deflection computations. The redundant will be removed so as to form basic determinate stable structure. The basic resulting structure is loaded with external loads (w_{load}) (see Fig. (4.5)). Thus,

$$\Delta_{2,1} = \int_0^L - \frac{w_{load} x^3}{2 \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(i,j)} b_{ij} t_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(i,j)} A_{s(i,j)} y_{ij}^2 \right]} dx$$

$$\Delta_{2,1} = - \frac{w_{load}}{2} D_{force} \quad (4 - 96)$$

where

$$D_{force} = \int_0^L \frac{x^3}{2 \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(i,j)} b_{ij} t_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(i,j)} A_{s(i,j)} y_{ij}^2 \right]} dx$$

$$(4 - 97)$$

$$\Delta_{3,1} = \int_0^L - \frac{w_{load} x^2}{2 \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(i,j)} b_{ij} t_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(i,j)} A_{s(i,j)} y_{ij}^2 \right]} dx$$

$$\Delta_{3,1} = - \frac{w_{load}}{2} C_{force} \quad (4 - 98)$$

where:

$$C_{force} = \int_0^L \frac{x^2}{2 \left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(i,j)} b_{ij} t_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(i,j)} A_{s(i,j)} y_{ij}^2 \right]} dx \quad (4 - 99)$$

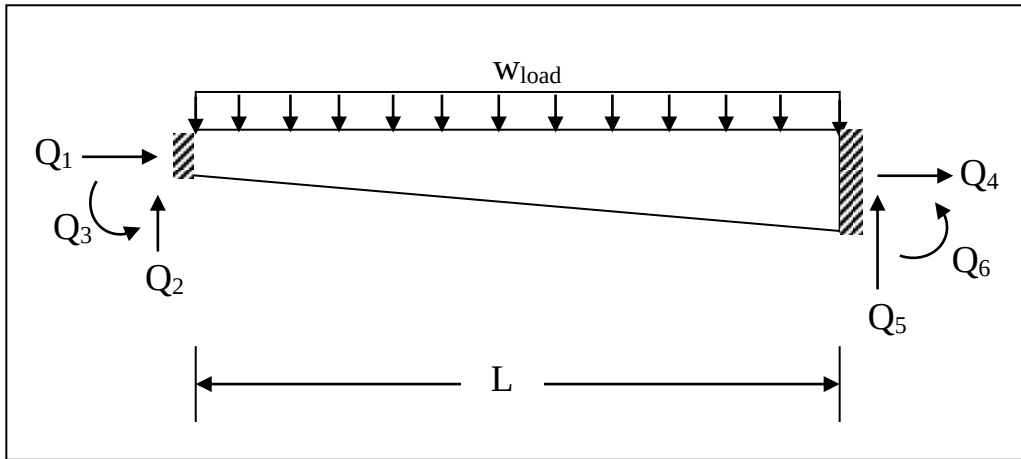


Fig. (4.4): Fixed ended non prismatic frame element under uniformly distributed load.

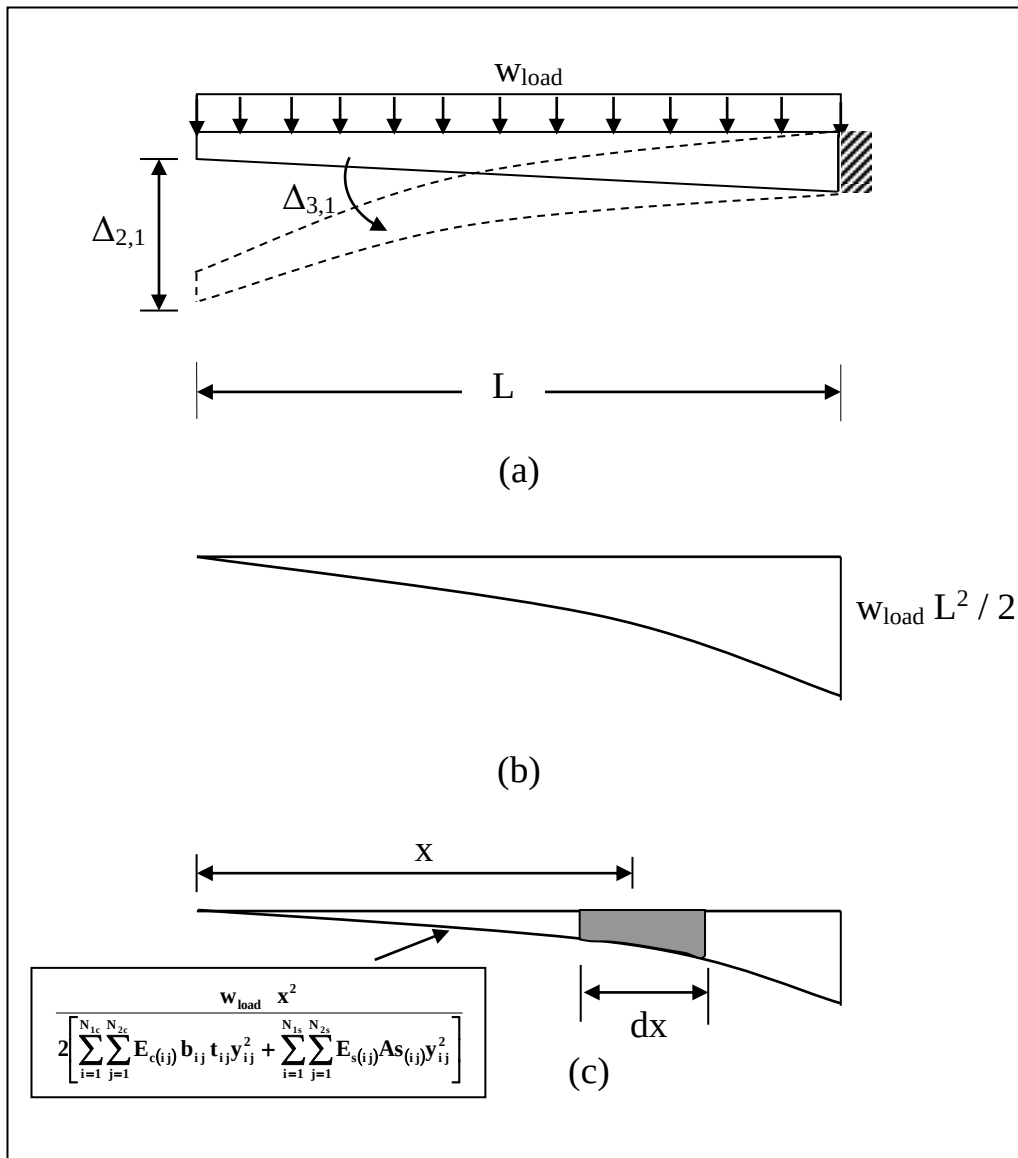


Fig. (4.5): Basic Determinate Structure under External Load (w_{load}). (a) Released Structure, (b) Bending Moment Diagram, (c) M / EI Diagram

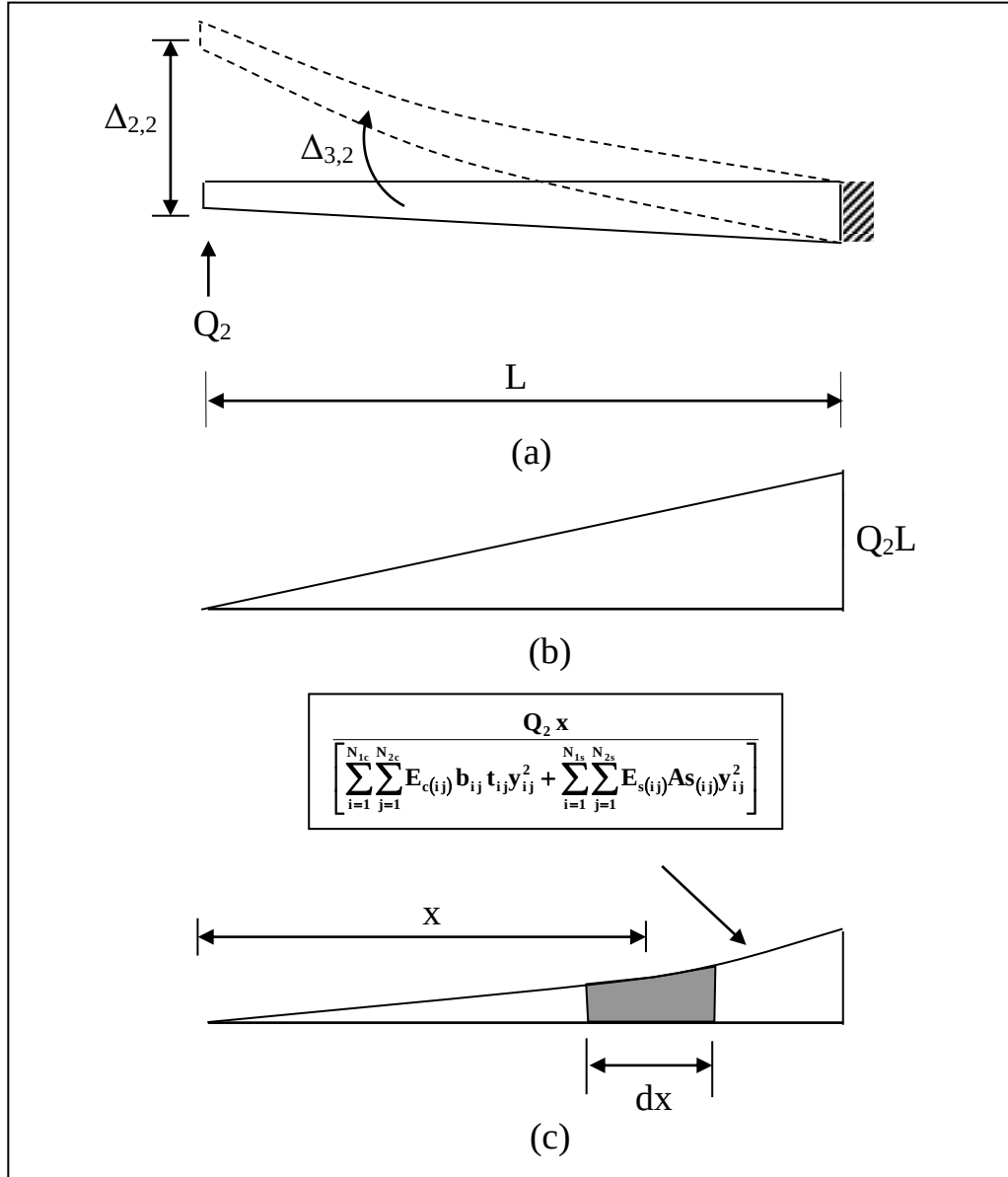


Fig. (4.6): Basic Determinate Structure under Load (Q_2). (a) Release Structure, (b) Bending Moment Diagram, (c) M / EI Diagram.

The redundant Q_2 is applied to the basic determinate structure as shown in Fig. (4.6).

The displacement $\Delta_{2,2}$ is computed as follows

$$\Delta_{2,2} = \int_0^L \frac{Q_2 x^2}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} y_{ij}^2 \right]} dx$$

or:

$$\Delta_{2,2} = Q_2 C_{\text{force}} \quad (4 - 100)$$

Also,

$$\Delta_{3,2} = \int_0^L \frac{\mathbf{Q}_2 \mathbf{x}}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} \mathbf{A}s_{(ij)} y_{ij}^2 \right]} \mathbf{dx}$$

$$\Delta_{3,2} = \mathbf{Q}_2 \mathbf{B}_{\text{force}} \quad (4-101)$$

where:

$$\mathbf{B}_{\text{force}} = \int_0^L \frac{\mathbf{x}}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} \mathbf{A}s_{(ij)} y_{ij}^2 \right]} \mathbf{dx} \quad (4-102)$$

The redundant Q_3 is applied to the basic determinate structure as shown in Fig. (4.7).

$$\Delta_{2,3} = \int_0^L - \frac{\mathbf{Q}_3 \mathbf{x}}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} \mathbf{A}s_{(ij)} y_{ij}^2 \right]} \mathbf{dx}$$

$$\Delta_{2,3} = -\mathbf{Q}_3 \mathbf{B}_{\text{force}} \quad (4-103)$$

$$\Delta_{3,3} = \int_0^L - \frac{\mathbf{Q}_3}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} \mathbf{A}s_{(ij)} y_{ij}^2 \right]} \mathbf{dx}$$

$$\Delta_{3,3} = -\mathbf{Q}_3 \mathbf{A}_{\text{force}} \quad (4-104)$$

where:

$$\mathbf{A}_{\text{force}} = \int_0^L \frac{1}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} \mathbf{b}_{ij} \mathbf{t}_{ij} y_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} \mathbf{A}s_{(ij)} y_{ij}^2 \right]} \mathbf{dx} \quad (4-105)$$

Then,

$$\Delta_{2,1} + \Delta_{2,2} + \Delta_{2,3} = 0$$

$$\left(\frac{-w_{\text{load}}}{2} \mathbf{D}_{\text{force}} \right) + (\mathbf{Q}_2 \mathbf{C}_{\text{force}}) + (-\mathbf{Q}_3 \mathbf{B}_{\text{force}}) = 0$$

$$\mathbf{Q}_2 = \frac{w_{\text{load}} \mathbf{D}_{\text{force}}}{2\mathbf{C}_{\text{force}}} + \frac{\mathbf{Q}_3 \mathbf{B}_{\text{force}}}{\mathbf{C}_{\text{force}}} \quad (4-106)$$

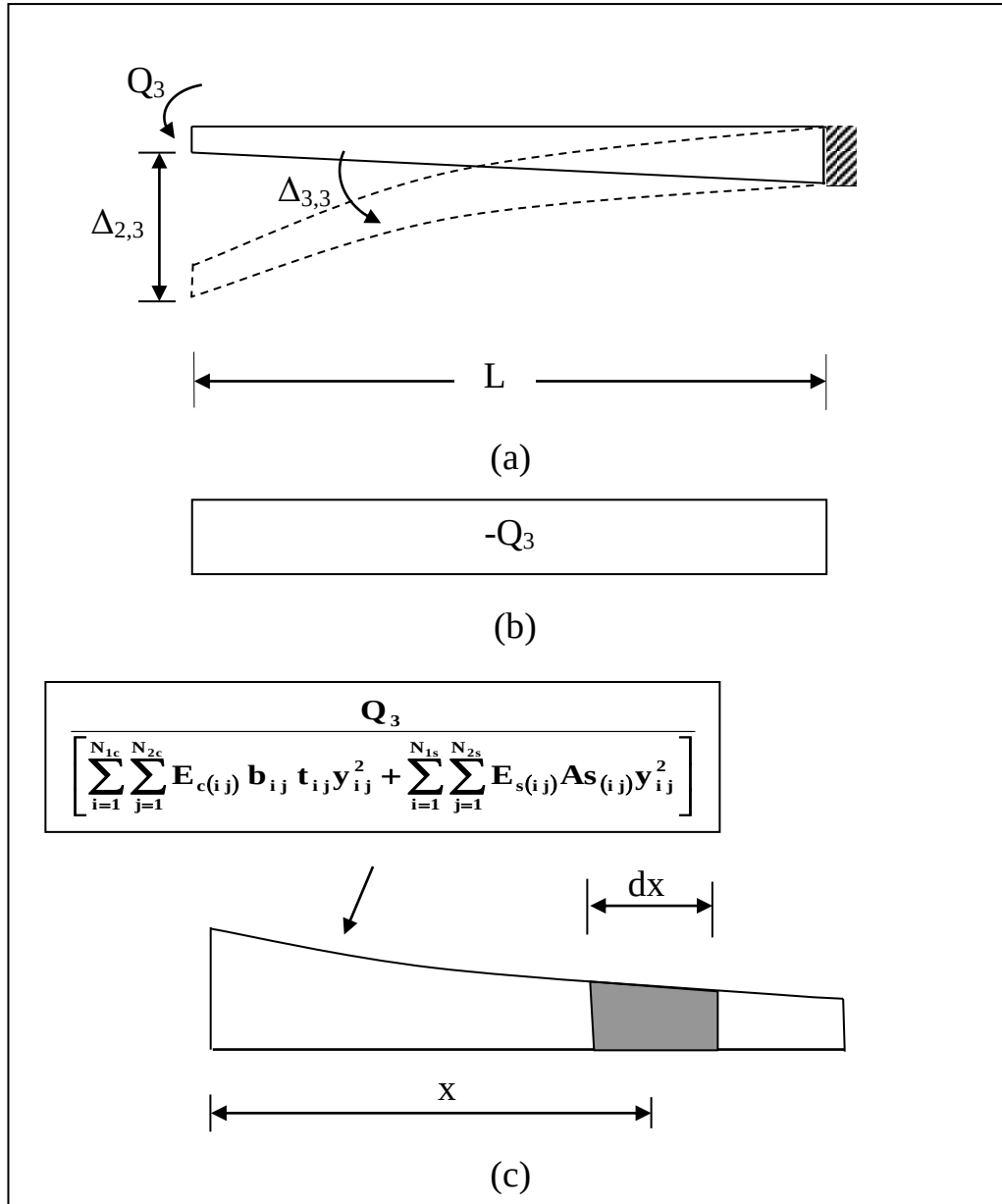


Fig. (4.7): Basic Determinate Structure under Load (Q_3). (a) Release Structure, (b) Bending Moment Diagram, (c) M / EI Diagram.

$$\Delta_{3,1} + \Delta_{3,2} + \Delta_{3,3} = 0$$

$$\left(\frac{-w_{\text{load}}}{2} C_{\text{force}} \right) + (Q_2 B_{\text{force}}) + (-Q_3 A_{\text{force}}) = 0 \quad (4-107)$$

Substitute equation (4-106) in equation (4-107).

$$\left(\frac{-w_{\text{load}}}{2} C_{\text{force}} \right) + \left(\frac{w_{\text{load}} B_{\text{force}} D_{\text{force}}}{2C_{\text{force}}} \right) + \left(\frac{Q_3 B_{\text{force}}^2}{C_{\text{force}}} \right) + (-Q_3 A_{\text{force}}) = 0$$

$$Q_3 = \frac{w_{\text{load}} (C_{\text{force}}^2 - B_{\text{force}} D_{\text{force}})}{2(B_{\text{force}}^2 - A_{\text{force}} C_{\text{force}})} \quad (4-108)$$

Substitute equation (4 – 108) in equation (4 – 106).

$$Q_2 = \frac{w_{load} D_{force}}{2C_{force}} + \left(\frac{w_{load}}{2} \frac{C_{force}^2 - B_{force} D_{force}}{B_{force}^2 - A_{force} C_{force}} \frac{B_{force}}{C_{force}} \right)$$

$$Q_2 = \frac{w_{load}}{2} \left[\frac{D_{force}}{C_{force}} + \left(\frac{B_{force}}{C_{force}} \frac{C_{force}^2 - B_{force} D_{force}}{B_{force}^2 - A_{force} C_{force}} \right) \right] \quad (4 - 109)$$

From static equilibrium, the end forces at the second end of the element are:

$$Q_4 = Q_1 = 0 \quad (4 - 110)$$

$$Q_5 = wL - Q_2 \quad (4 - 111)$$

$$Q_6 = Q_2 L - Q_3 - \frac{w_{load} L^2}{2} \quad (4 - 112)$$

For bending in $x - z$ plane one use the same way and the same equations but:

$$A_{force} = \int_0^L \frac{1}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} z_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} z_{ij}^2 \right]} dx \quad (4 - 113)$$

$$B_{force} = \int_0^L \frac{x}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} z_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} z_{ij}^2 \right]} dx \quad (4 - 114)$$

$$C_{force} = \int_0^L \frac{x^2}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} z_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} z_{ij}^2 \right]} dx \quad (4 - 115)$$

$$D_{force} = \int_0^L \frac{x^3}{\left[\sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} E_{c(ij)} b_{ij} t_{ij} z_{ij}^2 + \sum_{i=1}^{N_{1s}} \sum_{j=1}^{N_{2s}} E_{s(ij)} A_{s(ij)} z_{ij}^2 \right]} dx \quad (4 - 116)$$

CHAPTER FIVE

(FORMULATION OF INELASTIC ANALYSIS)

5.1 General:

In the present study, an analytical method for inelastic analysis of reinforced concrete space framed structures, having prismatic or non – prismatic elements has been formulated in this chapter. It follows an incremental approach with simple iterative technique based on finite element method with a developed form approach (namely region approach). In the present work, the effect of a finite size of joints at member ends is ignored. On the other hand, many developments have been made in this work which can be, briefly, given as follows:

1. Derivation of a numerical form of stiffness matrix for prismatic or non – prismatic space frame element takes into account; (a) the interaction between bending moments, shear and axial forces; (b) the possibility of existence of plastic zone anywhere in the element. In addition, for the case of element under uniformly distributed load, fixed end forces are derived considering the possibility of existence of plastic zone wherever it is formed (illustrated in chapter four).
2. Regions approach is suggested in the present study to take into account the material non – linearity through the depth and width of the element.
3. Geometric non – linearity is considered through adopting the incremental procedure in which the load is applied as a series of small increments and for each increment the changes in the nodal displacements are determined and the nodal coordinates are updated at the end of each load increment.
4. Creep and shrinkage effects on concrete are taken into account by calculating the strains caused by them.

5. Bond – slip between concrete and reinforcing steel is considered in this study.
6. At the end of each load increment and before updating the nodal coordinates, the element internal forces will be calculated and compared with external applied forces. If the difference is less than 0.0001, then the next load increment will be applied. Otherwise this difference will be applied as external forces. At the end of each load increment the constitutive matrix for each steel and concrete region will be updated as shown later in this chapter.
7. A yield criterion is adopted in the present study. Such criterion includes the interaction between axial, shear and flexural stress. Hardening / softening rule for concrete, flow rule for concrete, tension stiffening and crushing condition for concrete.
8. The present method treats the possibility of existence of critical section at a point of zero shear when the element is under uniformly distributed loads, in addition to those existing at its two extremities.
9. Local stress unloading is taken into account.

5.2 The Concrete Yield Criterion:

The yield criterion determines the stress level at which plastic deformation begins. **Kupfer et.al (as found in Ref.[75])** proposed a tri – axial yield criterion. This yield criterion takes into account the transverse shear effect and has the following form:

$$f(\sigma) = (\alpha_{cr} I_1 + 3\beta_{cr} J_2)^{1/2} = \sigma_o \quad (5 - 1)$$

where α_{cr} and β_{cr} are material parameters, I_1 is the first stress invariant, J_2 is the second deviatoric stress invariant and σ_o is the equivalent effective stress taken from uni – axial test. I_1 and J_2 were written as :

$$I_1 = \sigma_{cx} + \sigma_{cy} + \sigma_{cz} \quad (5 - 2)$$

$$J_2 = \frac{1}{3}[(\sigma_{cx}^2 + \sigma_{cy}^2 + \sigma_{cz}^2) - (\sigma_{cx}\sigma_{cy} + \sigma_{cy}\sigma_{cz} + \sigma_{cz}\sigma_{cx})] + (\tau_{cxy}^2 + \tau_{cyz}^2 + \tau_{cxz}^2) \quad (5-3)$$

In the present study σ_{cz} and σ_{cy} are neglected [77]. Therefore, equation (5-2) and (5-3) can be rewritten as:

$$I_1 = \sigma_{cx} \quad (5-4)$$

$$J_2 = \frac{1}{3}\sigma_{cx}^2 + (\tau_{cxy}^2 + \tau_{cxz}^2 + \tau_{cyz}^2) \quad (5-5)$$

By substituting equations (5-4) and (5-5) into equation (5-1), then the equation (5-1) can be rewritten in the following form:

$$f(\sigma) = [\alpha_{cr}\sigma_{cx} + \beta_{cr}(\sigma_{cx}^2 + 3\tau_{cxy}^2 + 3\tau_{cxz}^2 + 3\tau_{cyz}^2)]^{1/2} = \sigma_o \quad (5-6)$$

If the results obtained by **Kupfer** for a failure envelope are employed for the initial yield, the value of the constant α_{cr} and β_{cr} are:[75]

$$\alpha_{cr} = 0.355 \sigma_o \text{ and } \beta_{cr} = 1.355 \quad (5-7)$$

In the present study it is assumed that the initial yield surface is attained when the effective stress reaches 30 % of the ultimate stress f'_c [31][75][77]. After that, the loading (yielding) surface will expand due to hardening effect until the effective stress reaches the ultimate stress f'_c . Then, after this the loading surface will contract due to softening effect until crushing occurs.

5.3 The Hardening / Softening Rule for Concrete:

After initial yielding, the stress level at which further plastic deformation occurs is dependent on the current degree of plastic straining. Such a phenomenon is termed strain hardening. Thus, the yield surface will vary at each stage of the plastic deformation, with the subsequent yield surfaces being dependent on the plastic strains in some way. Some alternative models which describe strain hardening in a material are illustrated in Fig. (5.1). A perfectly plastic material is shown in Fig. (5.1a) where the yield stress level does not depend in any way on the degree of plastification. If the subsequent yield surfaces are a uniform expansion of

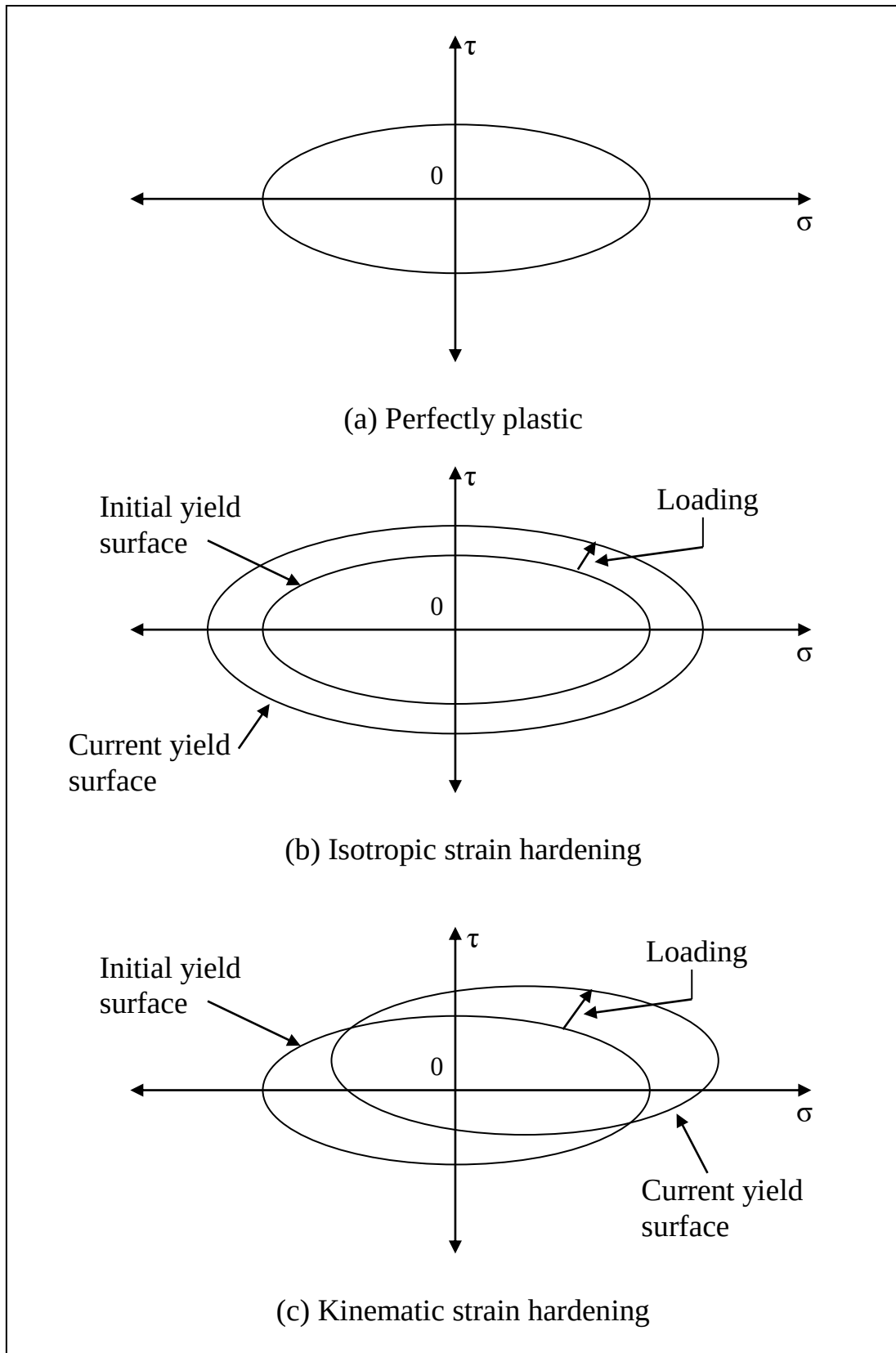


Fig. (5.1): Mathematical Models for Representation of Strain Hardening Behavior.

original yield curve , without translation , as shown in Fig. (5.1b) the strain hardening model is said to be isotropic. On the other hand, if the subsequent yield surfaces preserve their shapes and orientation but translate in the stress space as a rigid body as shown in Fig. (5.1c), kinematic hardening is said to take place.

In the present study , an isotropic hardening / softening rule is adopted. At first , when a material is stressed beyond its initial yielding surface , the yielding surface will expand until the effective stress reaches the ultimate stress f'_c . After that , the yielding surface will contract due to softening effect until the failure occur. To calculate the uni – axial hardening / softening parameter (H') , the tangential modulus of elasticity E_{ct} must be found at first , which will be calculated according to stress – strain model for concrete which was illustrated in chapter three , and as shown below:

1. For the compression envelope curve , E_{ct} will be equal to the derivative of equation (3 – 1a) with respect to ϵ_c and has the form:

$$E_{ct} = \frac{\frac{A_c f'_c}{\epsilon_{co}^4} [D_c \epsilon_{co}^3 - B_c \epsilon_c^2 \epsilon_{co} - 2\epsilon_c^3]}{\left[\left(\frac{\epsilon_c}{\epsilon_{co}} \right)^3 + B_c \left(\frac{\epsilon_c}{\epsilon_{co}} \right)^2 + C_c \frac{\epsilon_c}{\epsilon_{co}} + D_c \right]^2} \quad (5 - 8)$$

2. For the unloading and reloading paths in compression , E_{ct} will be equal to:

$$E_{ct} = \frac{\bar{\sigma}_{m1}}{\bar{\epsilon}_{m1} - \bar{\epsilon}_p} \frac{f'_c}{\epsilon_{co}} \quad (5 - 9)$$

3. In the state of the unloading and reloading for the uncracked concrete in tension , the value of E_{ct} will be equal to E_{co} .
4. The value of E_{ct} for the cracked concrete under tension will be calculated according to tension stiffening rule which will be illustrated later in this chapter.

The hardening parameter is defined as:

$$H' = \frac{d\sigma}{d\varepsilon_p} \quad (5 - 10)$$

where ε_p is the concrete plastic strain

$$d\varepsilon_p = d\varepsilon_c - d\varepsilon_e \quad (5 - 11)$$

where ε_e is the concrete elastic strain. By substitution from equation (5 – 11) into equation (5 – 10):

$$\begin{aligned} H' &= \frac{d\sigma}{d\varepsilon - d\varepsilon_e} \\ &= \frac{E_{ct} d\varepsilon}{d\varepsilon - d\varepsilon_e} \\ &= \frac{E_{ct}}{1 - \frac{d\varepsilon_e}{d\varepsilon}} \\ H' &= \frac{E_{ct}}{1 - \frac{d\sigma}{E_{co}} * \frac{E_{ct}}{d\sigma}} \\ H' &= \frac{E_{ct}}{1 - \frac{E_{ct}}{E_{co}}} \end{aligned} \quad (5 - 12)$$

If H' equals to zero, the material is stressed to be perfectly plastic. When H' equals to infinity, then the material is still within elastic range.

5.4 The Flow Rule for Concrete:

The flow rule is considered to construct the stress – strain relationship in the plastic range. Assuming normality of the plastic deformation rate vector to the yield surface, the plastic strain increment can be determined for a given stress increment [75].

$$d\varepsilon_p = d\lambda \frac{\partial f(\sigma)}{\partial \sigma} \quad (5 - 13)$$

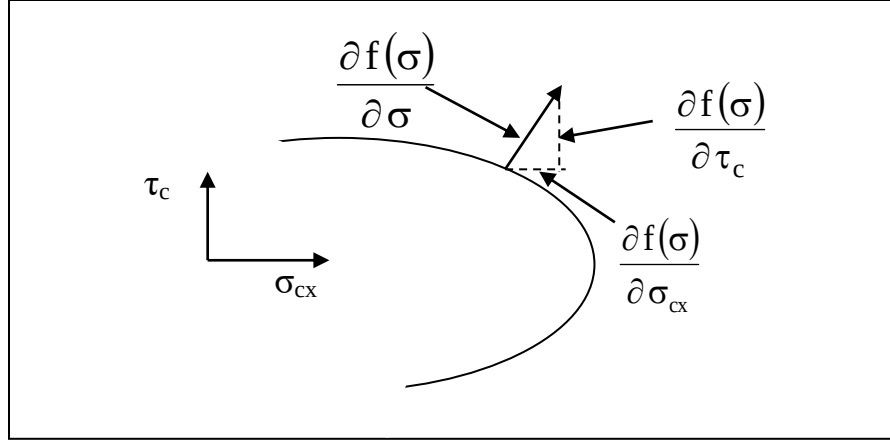


Fig. (5.2) : Geometrical Representation of The Normality Rule of Associated Plasticity.

where $d\bar{\lambda}$ is proportionality constant termed the plastic multiplier which determines the size of the plastic strain increment, and $\frac{\partial f(\sigma)}{\partial \sigma}$ is a vector directed normal to the yield surface at the stress point under consideration. Equation (5 – 1) can be rewritten as:

$$f(\sigma) = (3\beta_{cr} J_2 + 2C\sigma_o I_1)^{1/2} = \sigma_o \quad (5 - 14)$$

where $C = \alpha_{cr} / 2 \sigma_o$. This can be solved for σ_o , giving ^[75]:

$$f(\sigma) = C I_1 + [(C I_1)^2 + 3\beta_{cr} J_2]^{1/2} = \sigma_o \quad (5 - 15)$$

The yield function derivatives with respect to the stress components define the flow vector $\{a_f\}$ as:

$$[a_f] = \left[\frac{\partial f(\sigma)}{\partial \sigma_{cx}}, \frac{\partial f(\sigma)}{\partial \tau_{cxy}}, \frac{\partial f(\sigma)}{\partial \tau_{cxz}}, \frac{\partial f(\sigma)}{\partial \tau_{cyz}} \right] \quad (5 - 16)$$

In which ,

$$\begin{aligned} a_{f1} &= \frac{\partial f(\sigma)}{\partial \sigma_{cx}} \\ &= C + \frac{2 * \sigma_{cx} * (C^2 + \beta_{cr})}{Q_{cr}} \end{aligned} \quad (5 - 17)$$

$$\text{Let } Q_{cr} = 2 \left[(C\sigma_{cx})^2 + \beta_{cr} (\sigma_{cx}^2 + 3\tau_{cxy}^2 + 3\tau_{cxz}^2 + 3\tau_{cyz}^2) \right]^{1/2} \quad (5-18)$$

$$\begin{aligned} a_{f2} &= \frac{\partial f(\sigma)}{\partial \tau_{cxy}} \\ &= \frac{6\beta_{cr} \tau_{cxy}}{Q_{cr}} \end{aligned} \quad (5-19)$$

$$\begin{aligned} a_{f3} &= \frac{\partial f(\sigma)}{\partial \tau_{cxz}} \\ &= \frac{6\beta_{cr} \tau_{cxz}}{Q_{cr}} \end{aligned} \quad (5-20)$$

$$\begin{aligned} a_{f4} &= \frac{\partial f(\sigma)}{\partial \tau_{cyz}} \\ &= \frac{6\beta_{cr} \tau_{cyz}}{Q_{cr}} \end{aligned} \quad (5-21)$$

The plastic multiplier $d\bar{\lambda}$ can be found by using the following formula [9][77]:

$$d\bar{\lambda} = \left[\frac{\{a_f\}^T [D_{con}]}{H' + \{a_f\}^T [D_{con}] \{a_f\}} \right] d\{\varepsilon\} \quad (5-22)$$

where $[D_{con}]$ is the elastic constitutive matrix, $d\varepsilon$ is the total strain increment vector, and H' is the hardening parameter. The elasto – plastic increment of total strain is calculated by summing up the elastic and plastic strain components as:

$$\begin{aligned} d[\varepsilon] &= d\{\varepsilon_e\} + d\{\varepsilon_p\} \\ &= [D_{con}]^{-1} d\{\sigma\} + d\bar{\lambda} \frac{\partial f(\sigma)}{\partial \sigma} \\ &= [D_{con}]^{-1} d\{\sigma\} + \frac{\{a_f\} \{a_f\}^T [D_{con}]}{H' + \{a_f\}^T [D_{con}] \{a_f\}} d\{\varepsilon\} \end{aligned}$$

The complete elasto – plastic incremental stress – strain relationship can be expressed as:

$$[D_{con}]_{ep} = [D_{con}] - \frac{[D_{con}] \{a_f\} \{a_f\}^T [D_{con}]}{H' + \{a_f\}^T [D_{con}] \{a_f\}} \quad (5-23)$$

where the second term in the bracket represents the stiffness degradation due to plastic deformations.

5.5 The Crushing Condition for Concrete:

The crushing type of concrete fracture is a strain controlled phenomenon. A failure surface in the strain space must be defined in order to take this type of failure into account. The failure surface is represented by the following equation [75].

$$[\alpha'_{cr} I'_1 + \beta'_{cr} (3 J'_2)]^{1/2} = \epsilon_{cu}$$

where I'_1 is the first strain invariant, J'_2 is the second deviatoric strain invariant, and ϵ_{cu} is the ultimate total strain extrapolated from the uni – axial compressive test results.

Using the material parameters α'_{cr} and β'_{cr} from Kupfer's results, crushing condition is expressed in terms of the total strain components as:

$$0.355 \epsilon_{cu} (\epsilon_{cx} + \epsilon_{cy} + \epsilon_{cz}) + 1.355 \times \frac{4}{9} [(\epsilon_{cx}^2 + \epsilon_{cy}^2 + \epsilon_{cz}^2) - (\epsilon_{cx} \epsilon_{cy} + \epsilon_{cy} \epsilon_{cz} + \epsilon_{cz} \epsilon_{cx})] + 0.75 (\gamma_{cxy}^2 + \gamma_{cyz}^2 + \gamma_{cxz}^2) = \epsilon_{cu}^2 \quad (5 - 24)$$

by using equations (4 – 14) and (4 – 15) then equation (5 – 24) can be rewritten as follows:

$$0.355 \epsilon_{cu} (1 - 2\nu) \epsilon_{cx} + 1.355 \times \frac{4}{9} [(1 + 2\nu + \nu^2) \epsilon_{cx}^2 + 0.75 (\gamma_{cxy}^2 + \gamma_{cyz}^2 + \gamma_{cxz}^2)] = \epsilon_{cu}^2 \quad (5 - 25)$$

when ϵ_{cu} reaches the ultimate value which is equal to $2 \epsilon_{c0}$, the concrete is assumed to lose all its characteristics of strength and rigidity [75].

5.6 Concrete in Tension:

A smeared representation for cracks is assumed, which implies that the cracks are distributed across a region of the finite element. A linear elastic response of concrete is assumed until the fracture surface is reached. This

tensile type of cracking is governed by a maximum tensile stress criterion. The fixed orthogonal crack model is used here, in which cracks are assumed to form in a plane perpendicular to the direction of the maximum principal stress. A sudden or gradual relaxation of the normal stress is adopted after cracking has occurred, according to the tension stiffening diagram. The modulus of elasticity and Poisson's ratio are reduced to zero in the direction perpendicular to the crack and a reduced shear modulus is employed. The stress – strain relationship in this case is:^{[75],[77]}

$$\begin{bmatrix} \sigma_{cx} \\ \tau_{cxy} \\ \tau_{cxz} \\ \tau_{cyz} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \bar{G} & 0 & 0 \\ 0 & 0 & \bar{G} & 0 \\ 0 & 0 & 0 & KG \end{bmatrix} * \begin{bmatrix} \varepsilon_{cx} \\ \gamma_{cxy} \\ \gamma_{cxz} \\ \gamma_{cyz} \end{bmatrix} \quad (5 - 26)^{[75],[77]}$$

where $\bar{G}$ is the reduced shear modulus and the calculation of it is shown later in this chapter, K is the shear correction factor.

5.7 Tension Stiffening Rule for Concrete:

Due to the bond effects , cracked concrete carries between cracks a certain amount of tensile force normal to the cracked plane. The concrete adheres to the reinforcing bars and contributes to the over all stiffness of the structure ^{[75],[77]}. The assumed shape of the stress – strain hysteresis loops for cyclic loading in tension range is shown in Fig. (5.3). This behavior is illustrated through the following points:

1. At first, for uncracked concrete a linear path (O – A) will be adopted for the stress – strain behavior of concrete under tension with the modulus of elasticity equals to the nominal modulus of elasticity in compression E_{co} . This will be valid up to cracking strength f_t' beyond which a gradual release of the concrete stress component normal to the cracked plane occurs along

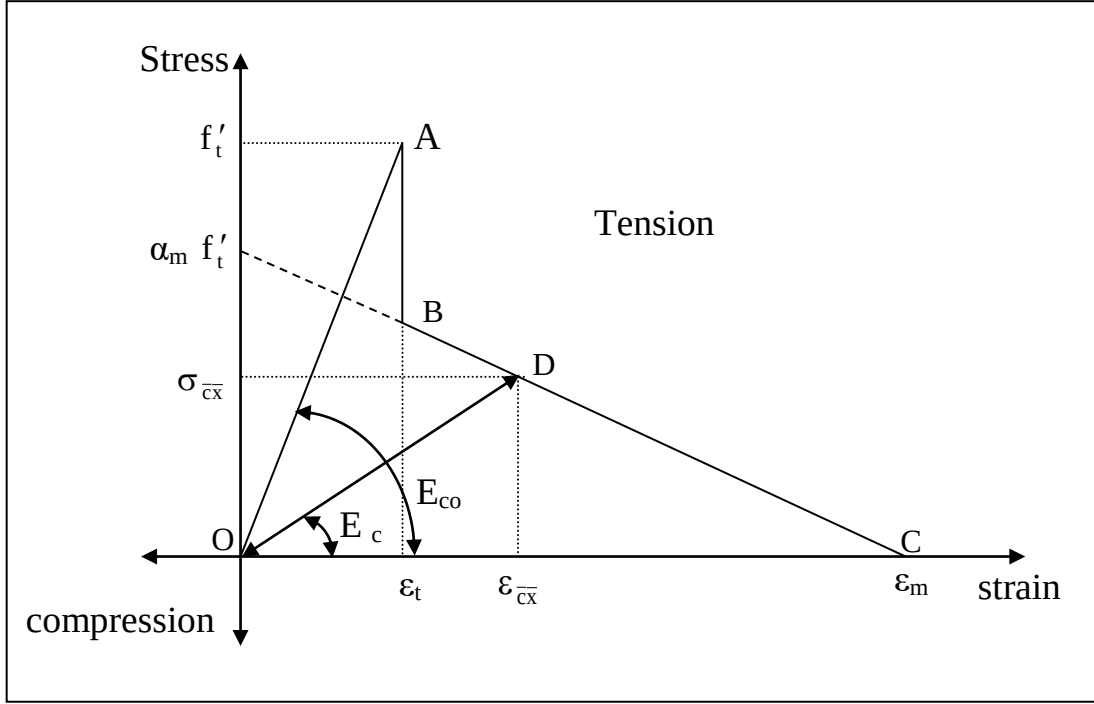


Fig. (5.3) : Loading and Unloading Behavior of Cracked Concrete Illustrating Tension Stiffening Behavior. ^{[75] [77]}

the line (B – C) as shown in Fig. (5 – 3). The path (B – C) has the following expression.

$$\sigma_{cx} = \alpha_m f'_t \left(1 - \frac{\epsilon_{cx}}{\epsilon_m} \right) \quad (5 - 27)$$

where ϵ_{cx} and σ_{cx} are the current values of concrete tensile strain and stress respectively in local direction x, α_m and ϵ_m are tension stiffening parameters (α_m is taken between 0.5 – 0.7 and ϵ_m is assumed to vary from 0.0015 to 0.002) ^{[75],[77]}.

2. Assign the point where unloading occurs D ($\epsilon_{\overline{cx}}, \sigma_{\overline{cx}}$).
3. For $\epsilon_{\overline{cx}} < \epsilon_t$ the unloading and reloading path follow a linear path (O – A) with a slope equal to the nominal modulus of elasticity in compression E_{co} . Where ϵ_t is the strain value at the ultimate concrete tensile strength (f'_t).

4. For $\varepsilon_m \geq \varepsilon_{\overline{cx}} \geq \varepsilon_t$ unloading occurs along a linear path (O – D) with a slope equal to the fictitious elasticity modulus E_c given by

$$E_c = \frac{\alpha_m f'_t \left(1 - \frac{\varepsilon_{\overline{cx}}}{\varepsilon_m} \right)}{\varepsilon_{\overline{cx}}} \quad (5 - 28)$$

The linear path (O – D) has the following expression :

$$\sigma_{cx} = \frac{\sigma_{\overline{cx}}}{\varepsilon_{\overline{cx}}} \varepsilon_{cx} \quad (5 - 29)$$

5. If the crack closes, i.e., if the strain component normal to the crack plane becomes negative, the concrete acquire the uncracked behavior, but the crack direction and the maximum tensile strain $\varepsilon_{\overline{cx}}$ continue to be stored.
6. For $\varepsilon_{cx} < \varepsilon_{\overline{cx}}$ reloading occurs along the linear path (O – D) with a slope equal to the fictitious elasticity modulus E_c given by Eq. (5 – 28). The path (O – D) has the expression shown in Eq. (5 – 29).
7. For $\varepsilon_{\overline{cx}} \leq \varepsilon_{cx} \leq \varepsilon_m$ reloading occurs along the linear path (D – C) which has the expression shown in Eq. (5 – 27)

5.8 Shear Modulus of Cracked Concrete:

The value of $\overline{G}$ is linearly decreasing with the strain normal to the crack plane and calculated according to the following formula:^[75]

for $\varepsilon_{cx} < 0.004$

$$\overline{G} = 0.25 * G \left(1 - \frac{\varepsilon_{cx}}{0.04} \right) \quad (5 - 30a)$$

for $\varepsilon_{cx} \geq 0.004$

$$\overline{G} = 0 \quad (5 - 30b)$$

When the crack is closed , the uncracked shear modulus is again assumed in the corresponding direction.

5.9 Time dependent effects:

A numerical procedure based on the finite element for the material nonlinear analysis of three dimensional reinforced concrete structures including the time dependent effects due to load history, creep and shrinkage is developed in this work. Creep and shrinkage are not independent of each other. Even under simple case of isothermal conditions, the presence of shrinkage increase the creep. This means that creep and shrinkage effects are coupled. However, from a practical point of view, it is almost always assumed that creep strains and shrinkage are independent ^[67].

Creep is defined as the increase in strain under a sustained stress, as shown in Fig. (5.4).

Shrinkage is defined as non – stress produced length or volume changes. It arises from loss of water on drying and volume changes on carbonation.

5.9.1 Time Dependent Concrete Properties:

Concrete is the time dependent material, for its properties such as the uniaxial compressive strength f'_c , uniaxial tensile strength and uniaxial modulus of elasticity.

The ACI committee 209 ^[78] proposes the following form of equation for predicating a compressive strength at any time t:

$$f'_c(t) = \frac{t}{a + bt} f'_c(28) \quad (5 - 31)$$

where; $f'_c(28)$ is 28 day strength, t is the time in days after casting of concrete, and the constant a and b depend on cement type and curing method. Following approximate average values determined by **Kang** ^[79] from some 88 test specimens are recommended:

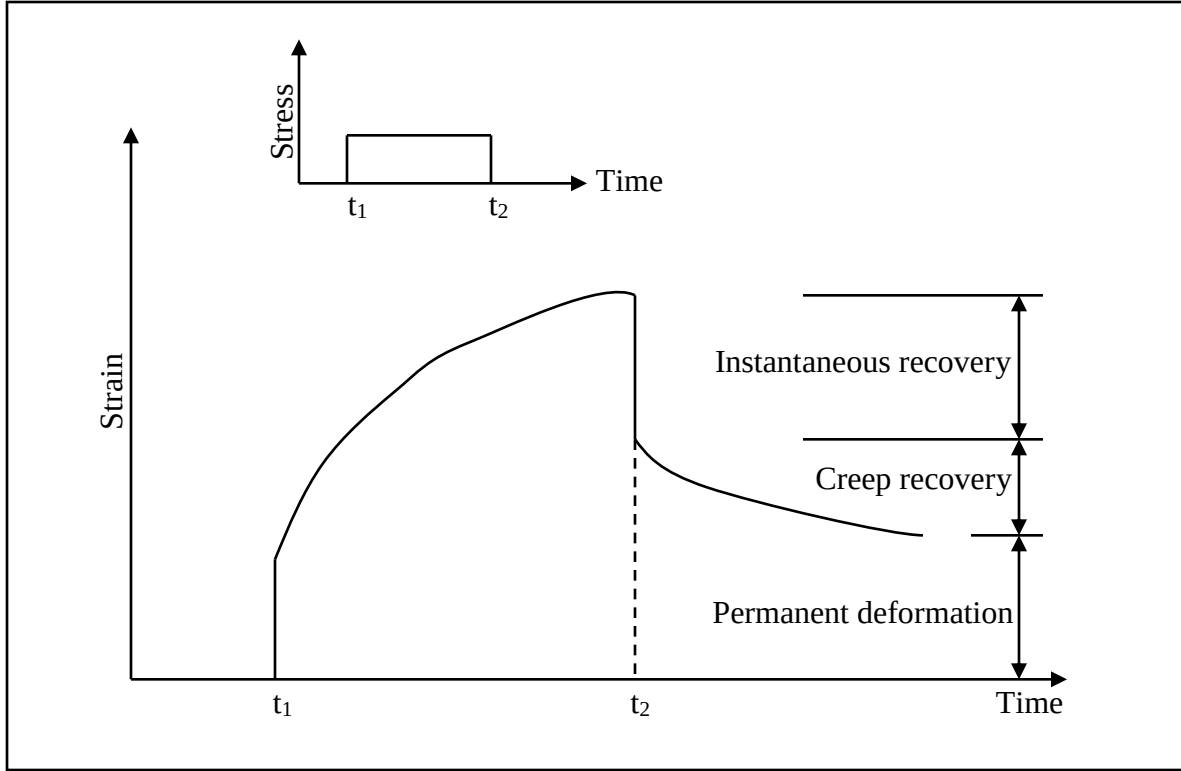


Fig. (5.4): Creep Response of Uniaxial Specimen.

Moist cured concrete, type I cement: $a = 4$, $b = 0.85$.

Moist cured concrete, type III cement: $a = 2.3$, $b = 0.92$.

Steam cured concrete, type I cement: $a = 1$, $b = 0.95$.

Steam cured concrete, type III cement: $a = 0.7$, $b = 0.98$.

The ACI committee 209 also proposes the following equations to calculate the direct tensile strength f'_t , and the modulus of elasticity E_{co} :

$$f'_t(t) = 0.007 \sqrt{W_{con} f'_c(t)} \quad (5 - 32)$$

$$E_{co}(t) = 0.043 W_{con}^{1.5} \sqrt{f'_c(t)} \quad (5 - 33)$$

$$\epsilon_{co}(t) = 2f'_c(t)/E_{co}(t) \quad (5 - 34)$$

$$\epsilon_{cu}(t) = 4f'_c(t)/E_{co}(t) \quad (5 - 35)$$

where $\epsilon_{co}(t)$ and $\epsilon_{cu}(t)$ are the maximum uniaxial compressive strain at the time t and the corresponding crushing strain at the time t , respectively. And W_{con} is the concrete unit weight in kg/m^3 , $f'_c(t)$ is the cylinder strength of

the concrete at any time t , $f'_t(t)$ is the ultimate concrete tensile strength at any time t and $E_{co}(t)$ is the initial tangent flexural modulus of elasticity of concrete at any time t . $f'_c(t)$, $f'_t(t)$ and $E_{co}(t)$ are in MPa.

5.9.2 The Strain Components of Concrete:

One of the most important assumptions in studying the deformation of concrete is that the strain of concrete may be considered as being composed of strains caused by different phenomena. For the present investigation, the total uniaxial concrete strain $\epsilon_{cx}(t)$ at any time t is assumed to be composed of the following contributions:

$$\epsilon_{cx}(t) = \epsilon_{cx}^m(t) + \epsilon_{cx}^{nm}(t) \quad (5 - 36)$$

$$\epsilon_{cx}^{nm}(t) = \epsilon_{cx}^c(t) + \epsilon_{cx}^s(t) \quad (5 - 37)$$

$\epsilon_{cx}^m(t)$ is the mechanical strain or instantaneous strain caused by a short time loading [68]. Non mechanical strain $\epsilon_{cx}^{nm}(t)$ consists of creep strain $\epsilon_{cx}^c(t)$ and shrinkage strain $\epsilon_{cx}^s(t)$. The meaning of each of these strain components is illustrated in Fig. (5.5).

5.9.3 Mathematical Formulation of Creep:

It is well known that creep has a considerable influence on the deformation of a concrete structure. There are several methods which are used to obtain creep strains. Some of them depend on experimental data which are used to find constant parameter contain in mathematical formulation. The following empirical expression for the prediction of creep deformation is recommended by **ACI – Committee 209** [78]:

$$C_t = \frac{(t-T)^{0.6}}{10 + (t-T)^{0.6}} C_u \quad (5 - 38)$$

where, C_t is the creep coefficient defined as the ratio of creep strain at (t) days after loading to the initial instantaneous strain at loading, T is the age in days of concrete at load application.

C_u is the ultimate creep coefficient which is equal to [78]

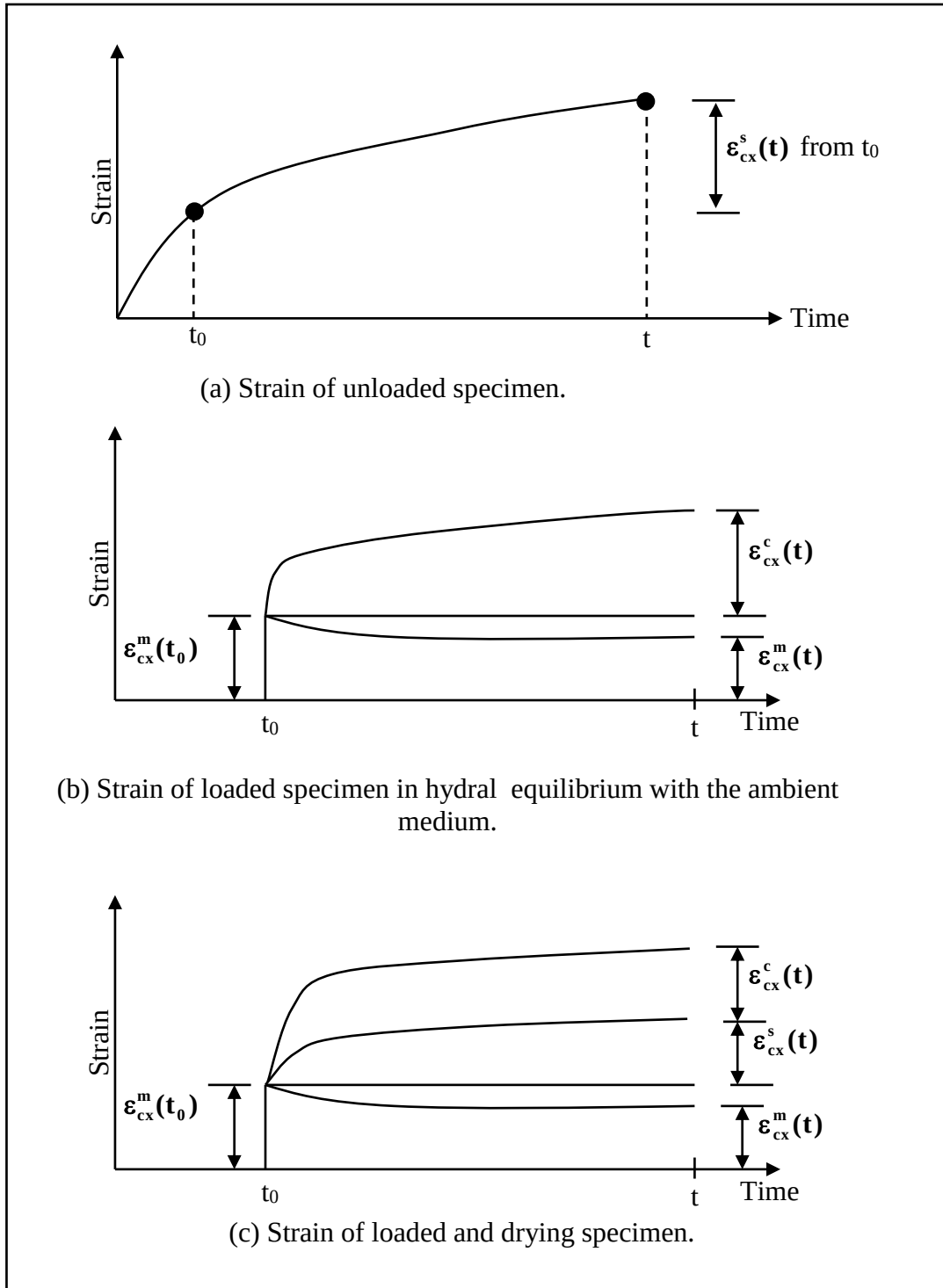


Fig. (5.5): Definition of Strain Components ^[68].

$$C_u = 2.35 \times F_T^c \times F_H^c \times F_{TH}^c \times F_S^c \times F_F^c \times F_A^c \quad (5 - 39)$$

where $F_T^c, F_H^c, F_{TH}^c, F_S^c, F_F^c, F_A^c$ are correction factor for loading age, humidity, minimum member thickness, slump, percent of fines and air

content. All the above correction factors assume the value of unity for standard conditions. The correction factors are calculated as follows:

$$\begin{aligned} F_T^c &= 1.25 T^{-0.118} && \text{for moist curing} \\ &= 1.13 T^{-0.094} && \text{for steam curing} \end{aligned} \quad (5-40a)$$

$$F_H^c = 1.27 - 0.0067H \quad H \geq 40\% \quad (5-40b)$$

$$\begin{aligned} F_{TH}^c &= 1.14 - 0.00092TH && \text{for } \leq 1 \text{ year loading} \\ &= 1.10 - 0.00067TH && \text{for ultimate loading} \end{aligned} \quad [78] \quad (5-40c)$$

$$F_S^c = 0.82 + 0.00264S \quad (5-40d)$$

$$F_F^c = 0.88 + 0.0024F_f \quad (5-40e)$$

$$\begin{aligned} F_A^c &= 1.0 && \text{for } A_{ir} \leq 6\% \\ &= 0.46 + 0.09 A_{ir} && \text{for } A_{ir} > 6\% \end{aligned} \quad (5-40f)$$

where T is the loading age in days, H is the ambient relative humidity in percent, TH is the minimum member thickness in mm, S is the slump in mm, F_f is the percent of fines by weight, and A_{ir} is the air content in percent. Creep deformation may be obtained by the super position method which was stated by **D. McHenry** as follows ^[80]:

"The strain produced in concrete in any time, t , are independent of the effects of any stress applied either earlier or later than t ".

Based on the above principle, for initial loading applied at time t , the total deformation $\epsilon_{cx}(t)$ at time t is:

$$\epsilon_{cx}(t) = \epsilon_{cx}^m(T) + (C_t \times \epsilon_{cx}^m(T)) \quad (5-41)$$

where $\epsilon_{cx}(t)$ is the total uniaxial concrete strain at any time t , $\epsilon_{cx}^m(T)$ is the mechanical concrete strain at the load application age, C_t is the creep coefficient.

For any load increment applied at time t_i , the strain increment in the time interval t_{i-1} to t_i is:

$$\Delta \epsilon_{cx}(t_i) = \Delta \epsilon_{cx}^m(t_i) + (C_{t_i} \Delta \epsilon_{cx}^m(t_i)) \quad (5-42)$$

The total strain at time t_i is:

$$\varepsilon_{cx}(t_i) = \varepsilon_{cx}(t_{i-1}) + \sum_{j=1}^n \Delta \varepsilon_{cx}^m(t_j) + (C_{t_i} \Delta \varepsilon_{cx}^m(t_i)) \quad (5-43)$$

In simple cases, the strain history may be represented as a series of steps, knowing variation of C_t and the material properties with time of initiation and duration of loading.

5.9.4 Mathematical Formulation of Shrinkage:

The shrinkage strain can be found by using the **ACI – Committee 209 recommendations** ^[78] which are based on the **Branson et al. (1975)** ^[68].

$$\varepsilon_{cx}^s(t-t_0) = \frac{(t-t_0)}{f+(t-t_0)} \varepsilon_{cx}^s(u) \quad (5-44)$$

where $\varepsilon_{cx}^s(t-t_0)$ is the shrinkage strain for the time interval from t_0 to t , $\varepsilon_{cx}^s(u)$ is the ultimate shrinkage strain to be determined from experiment, t is the time in days since the concrete casting, t_0 is the curing time, and f is a constant which depends on the curing type.

The **ACI – Committee 209** has recommended the following values for f , t_0 and $\varepsilon_{cx}^s(u)$;

- For moist cured concrete:

$$f = 35, \quad t_0 = 7, \quad \varepsilon_{cx}^s(u) = 800 \times 10^{-6} \times CF^s \quad (5-45)$$

- For steam cured concrete:

$$f = 55, \quad t_0 = 1-3, \quad \varepsilon_{cx}^s(u) = 730 \times 10^{-6} \times CF^s \quad (5-46)$$

where CF^s is the shrinkage correction factor which is given as:

$$CF^s = F_H^s \times F_{TH}^s \times F_S^s \times F_B^s \times F_F^s \times F_A^s \quad (5-47)$$

where $F_H^s, F_{TH}^s, F_S^s, F_B^s, F_F^s, F_A^s$ are shrinkage correction factor for humidity, minimum member thickness, slump, cement content, percent of fines and air content. All these factors assume the value of unity for the standard conditions ^[78]. For conditions other than the standard ones, the correction factors are calculated as follows:

$$\begin{aligned} F_H^s &= 1.4 - 0.01H & 40\% \leq H \leq 80\% \\ &= 3.0 - 0.03H & 80\% < H \leq 100\% \end{aligned} \quad (5-48a)$$

$$\begin{aligned} F_{TH}^s &= 1.23 - 0.0015 TH && \text{for } \leq 1 \text{ year loading} \\ &= 1.17 - 0.00114 TH && \text{for ultimate loading} \end{aligned} \quad (5 - 48 b)$$

$$F_S^s = 0.89 + 0.00161 S \quad (5 - 48c)$$

$$F_B^s = 0.75 + 0.00061 B \quad (5 - 48d)$$

$$\begin{aligned} F_F^s &= 0.3 + 0.014 F_f && F_f \leq 50\% \\ &= 0.9 + 0.002 F_f && F_f > 50\% \end{aligned} \quad (5 - 48e)$$

$$F_A^s = 0.95 + 0.008 A_{ir} \quad (5 - 48f)$$

where H is the ambient relative humidity in percent, TH is the minimum member thickness in mm, S is the slump in mm, B is the number of 50 kg sacks of cement per cubic meter of concrete, F_f is the percent of fine aggregate by weight, and A_{ir} is the air content in percent. The shrinkage strains are assumed to consist of axial components only and are uniform in all directions.

5.10 Bond – Slip Effect:

The bond between concrete and steel has a significant influence on the deformation characteristics of reinforced concrete elements under external loading. A series of fundamental problems relate directly to the bond characteristics of steel concrete interface: internal distribution of stress and strains along the steel and in the surrounding concrete; variation of relative slip between steel and concrete; location, spacing, and width of cracks in concrete; tension stiffening between primary cracks; deflection of reinforced concrete members; etc. The stiffness of reinforced concrete member under cyclic loading is strongly influenced by the increase in bond slip of the reinforcing bar due to the deterioration of bond. Thus, to find a rational analytical solution of these problems, the local bond stress – slip relationship between concrete and steel should be used.

5.10.1 Proposed Analytical Model:

The model presented in this study describes the hysteretic behavior of a single reinforcing bar in a reinforcement concrete beam column element.

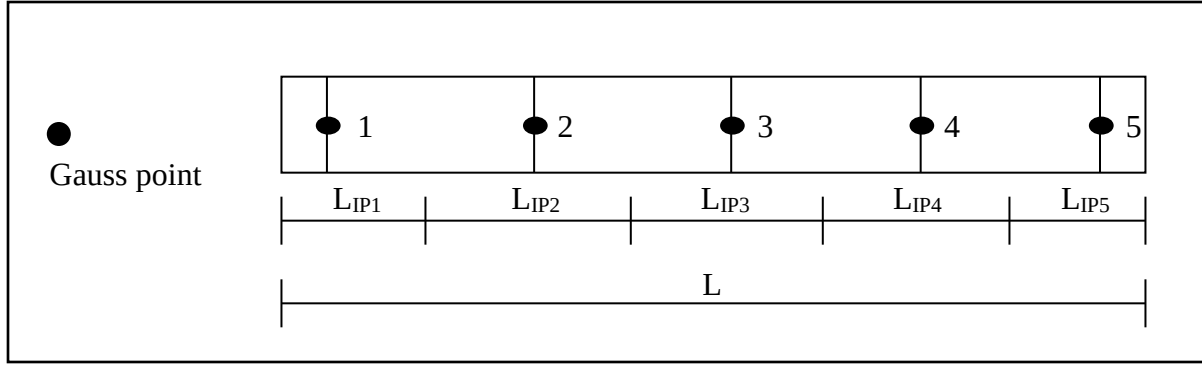


Fig. (5.6): Beam – Column with Five Gauss Point.

This model focuses attention on the stress transfer between reinforcing bars and surrounding concrete through bond ^[81]. In the present study five gauss points are used to evaluate the numerical integration to find the element stiffness so to study the bond slip effect. One section will be taken in each gauss point as shown in Fig. (5.6).

The differential equations of bond are ^[81]:

$$A_s \frac{d\sigma_{sx}}{dx} = \tau_b \phi \quad (5 - 49)$$

$$\epsilon_{sx} - \epsilon_{cx} = \frac{du}{dx} \quad (5 - 50)$$

where; A_s : reinforcing bar area; σ_{sx} : steel uniaxial stress; τ_b : bond stress as a function of relative slip; ϕ : reinforcing bar circumference; ϵ_{sx} : steel strain; ϵ_{cx} : concrete strain; u : relative slip of reinforcement bar with respect to surrounding concrete.

In the present study each section is divided into concrete and steel regions. i.e. σ_{sx} and ϵ_{sx} will be calculated for each steel region or steel bar. dx will be equal to L_{IPi} , where i vary from 1 to 5. ϵ_{sx} means the strain in steel bar after slip occurrence and can be termed as ϵ_{sxa} . ϵ_{cx} is equal to the steel region strain at the same position of concrete region before occurring the slip and can be termed as ϵ_{sxb} . Equations (5 – 49) and (5 – 50) can be rewritten as follows:

$$\text{As } \sigma_{sx} = \tau_b \phi L_{IP} \quad (5 - 51)$$

$$\epsilon_{sxa} - \epsilon_{sxb} = \frac{u}{L_{IP}} \quad (5 - 52)$$

5.10.2 Bond Stress – Slip Model:

Based on experimental results a bond stress – slip model was proposed by **Eligehausen et al** (as found in **Ref [26]**) in 1983. This model is shown in figure (5.7). In 1991, **Yannopoulos** and **Tassios** ^[61] proposed a multi – linear bond stress – slip model (see figure (5.8)).

In the present study a new bond stress – slip model is proposed. This model is a combination of monotonic bond stress – slip proposed by **Eligehausen et.al.** (as found in **[26]**) and the bond stress – slip cyclic behavior proposed by **Yannopoulos** and **Tassios** ^[61] and the details of the proposed model are given as follows:

5.10.2.1 Monotonic Bond Stress – Slip Envelope:

In fact there are two same envelopes, one is in tension and the other is in compression. The monotonic envelope consists of an initial nonlinear relation given by the following equations ^[26] (see Fig. (5.9)).

$$\tau_b = \tau_{b1} \left(\frac{u}{u_1} \right)^{\alpha_s} \text{ valid for } u \leq u_1 \quad (5 - 53)$$

where:

$$\tau_{b1} = 0.21(100 - \Phi) \text{ (units: MPa)} \quad (5 - 54)$$

$$u_1 = 0.04(1 + \Phi) \geq 0.7\text{mm} \quad (5 - 55)$$

$$\alpha_s = 0.4 \quad (5 - 56)$$

where Φ is the steel bar diameter.

$$\tau_b = \tau_{b1} \text{ for } u_1 < u \leq u_2 \quad (5 - 57)$$

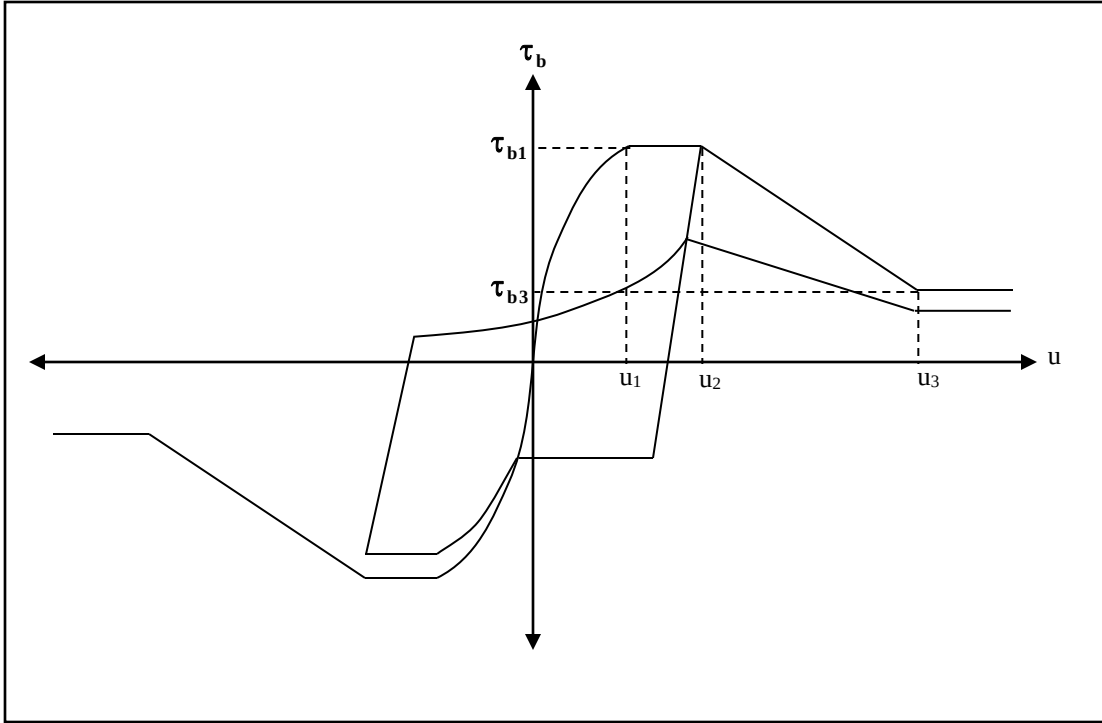


Fig. (5.7): Bond Stress – Slip Model Proposed by Eligehausen et al. [26]

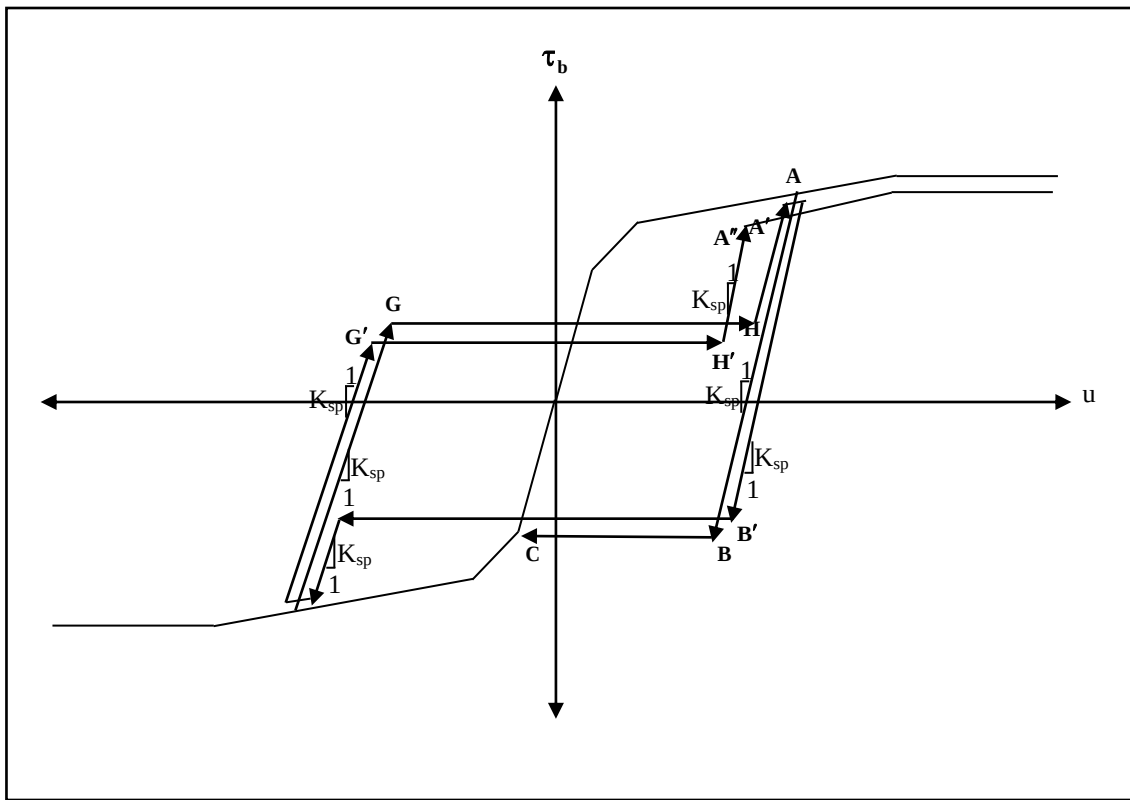


Fig. (5.8): Bond Stress – Slip Model Proposed by Yannopoulos and Tassios [61].

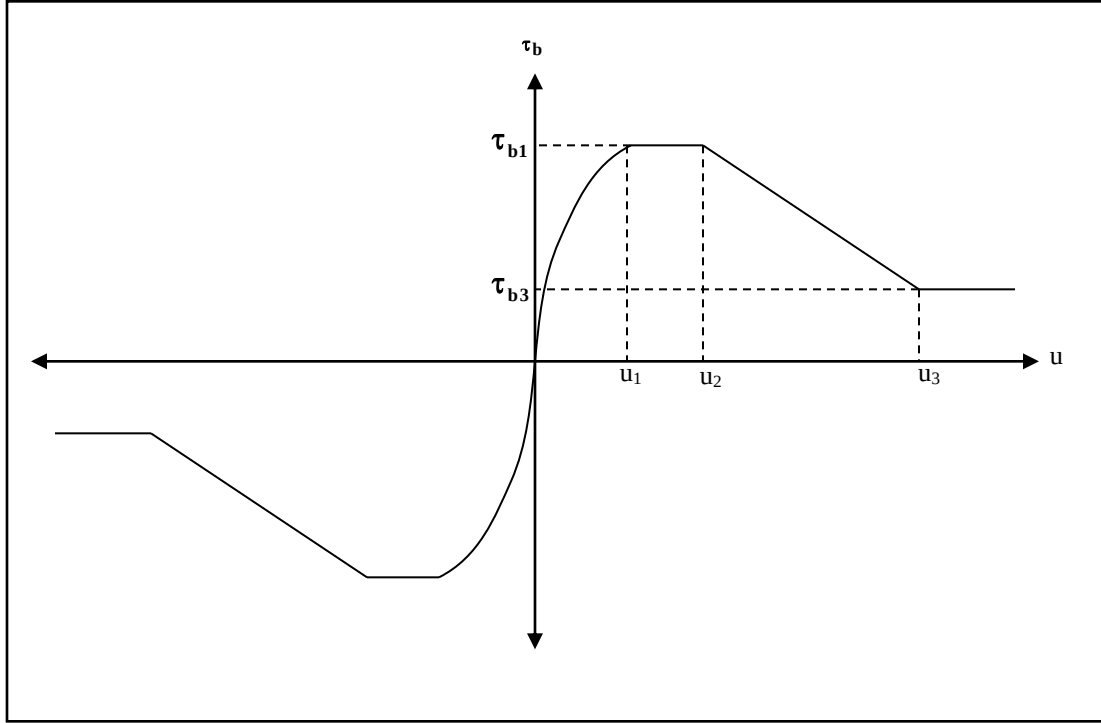


Fig. (5. 9): Monotonic Bond Stress – Slip Envelope by Eligehausen et al^[26].

$$\text{where } u_2 = 3u_1 \quad (5 - 58)$$

$$\tau_b = \frac{u_3 - u}{u_3 - u_2} (\tau_{b1} - \tau_{b3}) + \tau_{b3} \quad \text{for } u_2 < u \leq u_3 \quad (5 - 59)$$

$$\text{where } u_3 = 10 u_1 \quad (5 - 60)$$

$$\tau_{b3} = \frac{1}{3} \tau_{b1} \quad (5 - 61)$$

The prescribed bond stress – slip envelope is utilized in the following bond stress – slip cyclic behavior.

5.10.2.2 Bond Stress – Slip Cyclic Behavior:

The shape of the bond stress – slip cyclic behavior is proposed by **Yannopoulos** and **Tassios** ^[61]. The relationships which describe this behavior are illustrated as follows (see Fig. (5.10)).

1. Assign the point where unloading occurs **A**.
2. Determine the coordinate of point **B** (τ_{bB}, u_B).

$$\tau_{bB} = \tau_f \quad (5 - 62)$$

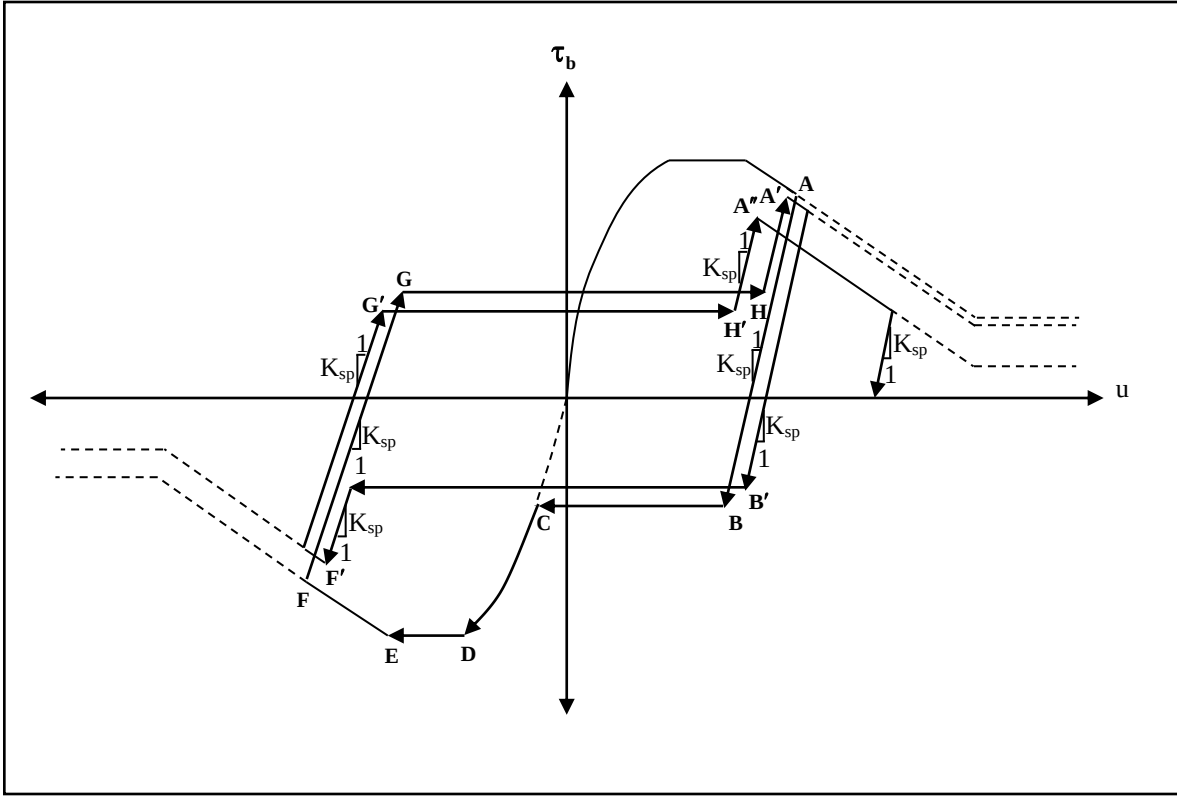


Fig. (5.10): Bond Stress – Slip Cyclic Behavior.

$$\tau_f = \frac{1}{2} \tau_{b1} \quad (5 - 63)$$

where τ_f is the residual (frictional) bond strength.

$$u_B = u_A - \frac{\tau_{bA} - \tau_{bB}}{K_{sp}} \quad (5 - 64)$$

where $K_{sp} = 180 \text{ N/mm}^3$ [82]

3. The unloading path is presented by:

$$u = u_A - \frac{\tau_{bA} - \tau_b}{K_{sp}} \quad \text{for } u_B \leq u \leq u_A \quad (5 - 65)$$

4. For reloading in reverse direction, horizontal line from B to C will be followed. After that the monotonic envelope will be followed.

5. For unloading from reverse direction, assign the point of unloading F.

6. Determine the coordinate of point G in the same manner of coordinate determination of point B. The unloading path is presented by:

$$\mathbf{u} = \mathbf{u}_F - \frac{\tau_{bF} - \tau_b}{\mathbf{K}_{sp}} \quad (5 - 66)$$

7. For reloading, a horizontal line from G to H is followed, where H is the intersection point for the straight line GH and the unloading path AB.

8. Determine the coordinate of point A'.

$$\tau_{bA'} = \rho_1 \tau_{bA} \quad (5 - 67)$$

$$\text{where: } \rho_1 = 1 - \sqrt{n/10} \quad (5 - 68)$$

n: is the number of cycles.

$$\mathbf{u}_{A'} = \mathbf{u}_A \quad (5 - 69)$$

9. Reloading will continue through the line HA'.

$$\mathbf{u} = \mathbf{u}_{A'} - \frac{\tau_{bA'} - \tau_b}{\mathbf{K}_{sp}} \quad (5 - 70)$$

10. If unloading start at the same point A', the unloading path is a straight line A'B' with slop $\mathbf{K}_{sp}$.

$$\mathbf{u}_{B'} = \mathbf{u}_B \quad (5 - 71)$$

$$\tau_{bB'} = \rho_2 \tau_{bB} \quad (5 - 72)$$

$$\text{where: } \rho_2 = 1 - \sqrt{n/40} \quad (5 - 73)$$

n: is the number of cycles.

11. If unloading does not occur, the reloading will continue through the reduced monotonic envelope (dash line in Fig. (4.15)).

5.11 Algorithm for The Proposed Procedure of Analysis:

The adopted approach of the analysis , which is called the incremental approach , treats the problem of nonlinear behavior as a sequence of linear problems. During every loading step of the sequence , the structure supports a new increment of external loads. Each step is based on material and

geometry properties appropriate to that step , i.e. , the stiffness of the structure is updated at the beginning of each step. The procedure of analysis can be illustrated as follows:

1. The analysis starts by considering the structure as a linear structure subjected to applying loads $\{ f \}$, and solving the following equation of equilibrium

$$\{ f \} = [K_{stiff}] \{ \psi \} \quad (5 - 74)$$

where $[K_{stiff}]$ is the initial global stiffness matrix of the structure. Then this equation (5 – 74) is solved by **Gauss - Jordan's Elimination** method.

2. Local forces and moments at the member ends are found from the local nodal displacements that are extracted from the global nodal displacements found from equation (5 – 74). At every zero shear point , if it exists , for the element under uniformly distributed load (in xy – plane or xz – plane), an additional node is indicated and the element having such point is subdivided into two elements and the first step of analysis is repeated for the resolved configuration of structural elements.
3. For each load cycle, the external load will be divided into a series of increments. Then these load increments are applied in a sequence form to the structure.
4. Transform all the sections into elastic concrete sections and find the position of the central axis for each section.
5. Calculate the global stiffness matrix of the structure and solve the following equation of equilibrium

$$[K_{stiff}] \{ \Delta \psi \} = \{ \Delta f \} \quad (5 - 75)$$

this equation (5 – 75) is solved by **Gauss - Jordan's Elimination** method. Local forces and moments at the member ends are found from the local

displacements that are extracted from the global nodal displacements found from equation (5 – 75).

6. Calculate the accumulating strain values for each concrete regions at five sections in each element (one section at each gauss point) by using equations (4 – 63) to (4 – 78). The normal strain values must be multiplied by (-1) before sending it to the concrete cyclic stress – strain model to calculate the accumulating stress values for concrete regions, because in the concrete cyclic stress - strain model the compressive values are considered to be positive and tension values are considered to be negative, while in this chapter the compressive values are considered to be negative and tension values are considered to be positive. Also, the accumulating normal stress values must be multiplied by (-1) before using it in the later steps for the same reason. The accumulating shear stress values for concrete regions will be calculated according to equation (4 – 16).
7. Calculate the accumulating axial strain value for each steel region at five sections in each element (one section at each gauss point) by using equation (4 – 63) .
8. Calculate the accumulating stress values for steel regions using the steel cyclic stress – strain model but the normal strain values must be multiplied by (-1) before sending it to the steel cyclic stress – strain model , because in the steel cyclic stress - strain model the compressive values are considered to be positive and tension values are considered to be negative, while in this chapter the compressive values are considered to be negative and tension values are considered to be positive. Also, the accumulating normal stress values must be multiplied by (-1) before using it in the later steps for the same reason.
9. For each steel regions at the five sections in each element, let $\epsilon_{sxa_{old}} = \epsilon_{sx}$.

10. Calculate the bond stress for each steel regions at the five sections in each element by using equation (5 – 51).
11. For each steel regions at the five sections in each element, calculate the relative slip from bond stress – slip model.
12. Calculate $\epsilon_{sxa_{new}}$ from equation (5 – 52) for each steel regions at the five sections in each element.
13. For each steel regions at the five sections in each element , calculate the error from the following equation.

$$\text{error} = \left| \frac{\epsilon_{sxa_{old}} - \epsilon_{sxa_{new}}}{\epsilon_{sxa_{old}}} \right| \quad (5 - 73)$$

14. For each steel regions at the five sections in each element if the value of error is greater or equal to 0.0001, then let $\epsilon_{sx} = \epsilon_{sxa_{new}}$ and repeat the steps 8 to 14.
15. Calculate the incremental non – mechanical strain for each concrete region at the five sections in each element by using equations (5 – 37) to (5 – 48).
16. Calculate the non – mechanical internal force increment by the following equation.

$$\{\Delta \mathbf{f}^{nm}\} = \int_0^L \sum_{i=1}^{N_{1c}} \sum_{j=1}^{N_{2c}} \mathbf{b}_{ij} \mathbf{t}_{ij} [\mathbf{B}_A]^T [\mathbf{D}_{con}] \{\Delta \epsilon_{cx}^{nm}\} d\mathbf{x} \quad (5 - 74)$$

where: $[\mathbf{B}_A]$ is the axial strain – displacement matrix, $[\mathbf{D}_{con}]$ is the concrete elasticity matrix, ϵ_{cx}^{nm} is the concrete non – mechanical strain, N_{1c} and N_{2c} are the total number of concrete region in width and depth respectively.

17. Add the non – mechanical internal forces calculated in step (16) to the mechanical internal forces calculated in step (5) to find the incremental total internal forces $\{\Delta \mathbf{f}_{in}\}$.

18. Calculate the difference [Dif] between the external applied forces and the internal forces calculated in step (17).

$$19. \quad \mathbf{FNORM} = \frac{\left[\left[\sum_{i=1}^{NEQ} [\mathbf{Dif}] \right]^2 \right]^{1/2}}{\left[\left[\sum_{i=1}^{NEQ} [\Delta \mathbf{f}_{in}] \right]^2 \right]^{1/2}} \quad (5 - 75)$$

where NEQ equal to the total number of external forces components applied to the structure. If $\mathbf{FNORM} \leq 0.05$ the next load increment will be applied, otherwise the difference calculated in step (18) will be applied as external force and the steps (5) to (19) will be repeated until convergence occur.

20. At the end of each load increment, the nodal coordinates will be updated and the displacements and forces will be accumulated.
21. For the concrete regions, which were in the compression state, calculate the effective stress from equation (5 – 6). If the effective stress reaches 30% of the ultimate stress f_c' , then use the hardening / softening rule and flow rule to calculate the constitutive elasto – plastic matrix, otherwise the material is still within elastic range.
22. For the concrete region within the tension state, if the normal stress reaches f_t' , i.e. the region is cracked , then the values of the constitutive matrix will be calculated according to tension stiffening rule and the rule of calculation of cracked shear modulus. Otherwise the material is still within elastic range.
23. For the concrete regions within plastic range, calculate the effective strain by using equation (5 – 25). If the effective strain reaches $1.5 \epsilon_{co}$, then the crushing failure is occurred and the solution is finished.
24. Repeat steps (3) to (23) until the crushing failure occurs or the total cyclic load is applied.

5.12 Computer Program:

A computer program was prepared, as a part of this work, for the analysis of a space reinforced concrete frame with non – prismatic and / or prismatic members based on prescribed procedure. The program is written in **FORTRAN77** language for the **IBM** personal computer and compatible microcomputers. The flow chart of this program is given in Fig. (5.11) to outline the main steps in the program.

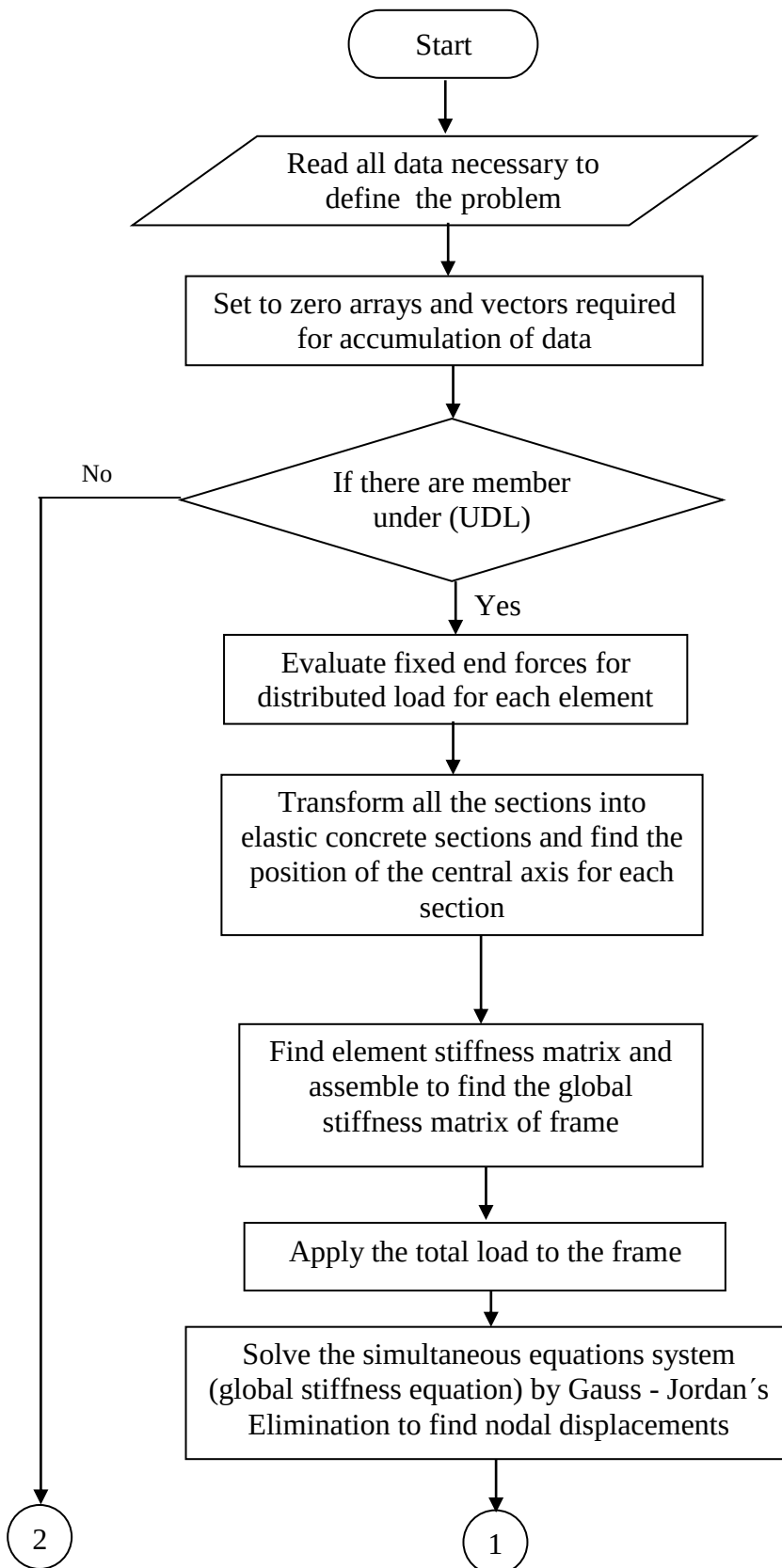


Fig. (5.11) : Flow Chart of The Program.

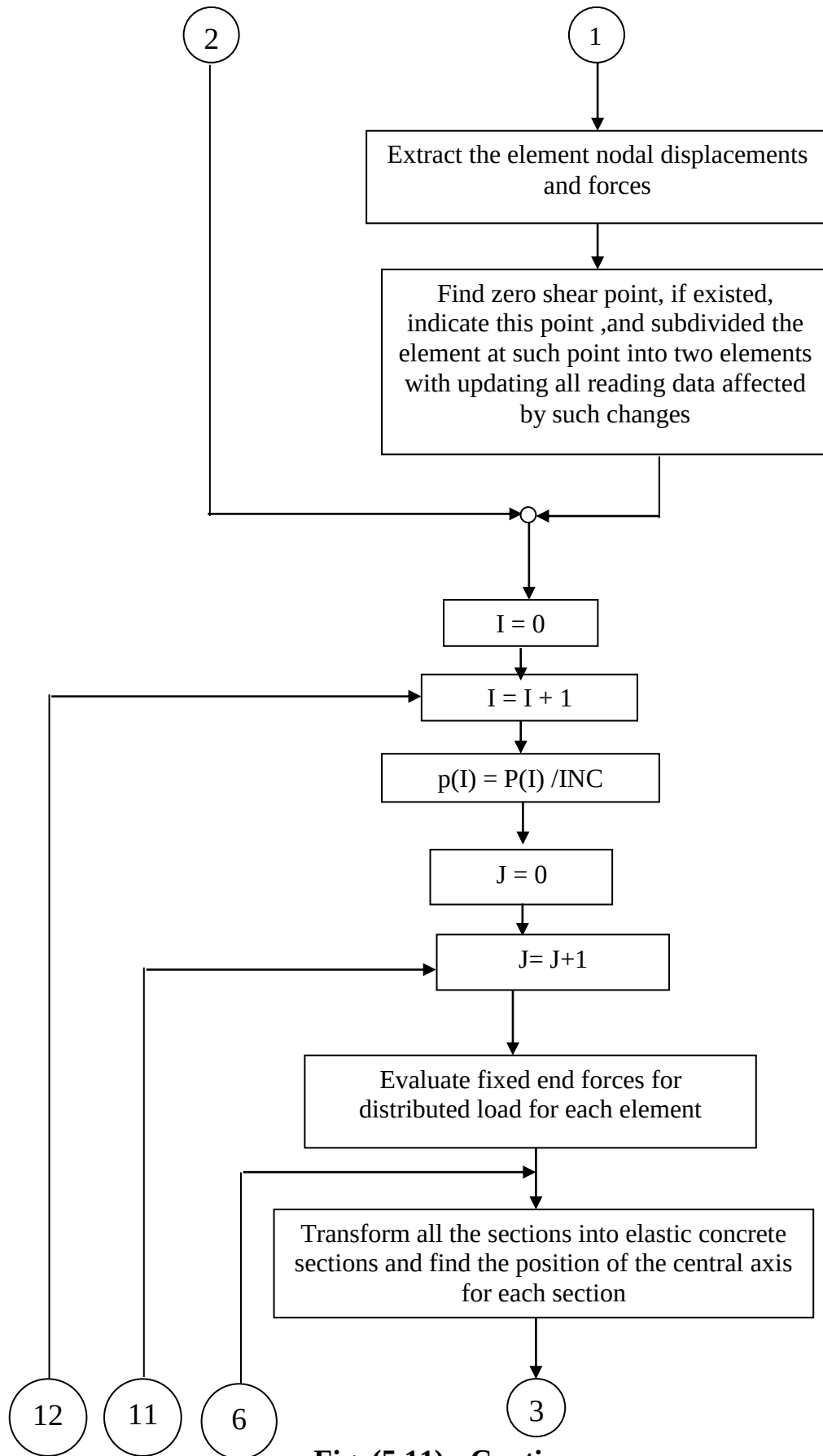


Fig. (5.11) : Continue

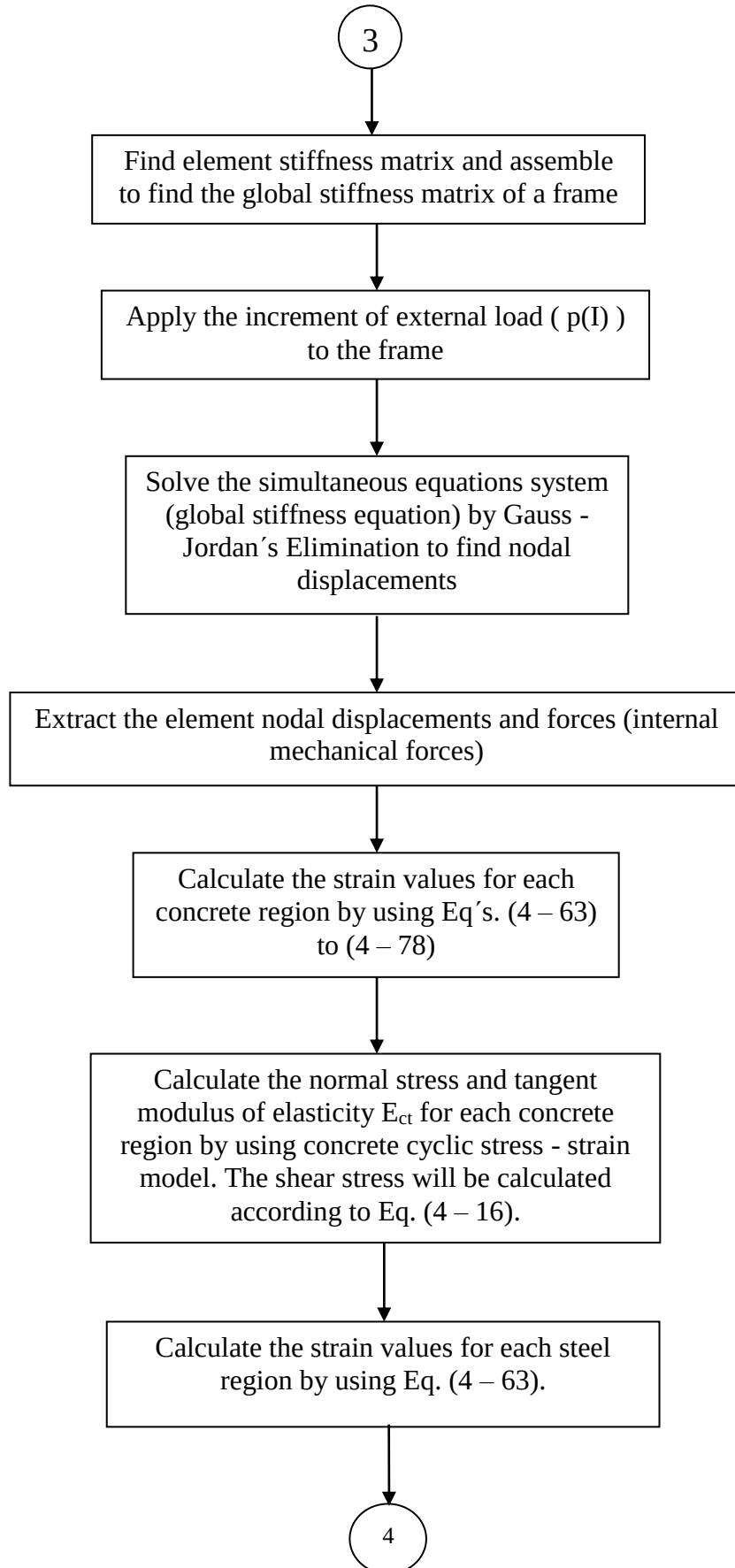


Fig. (5.11) : Continue

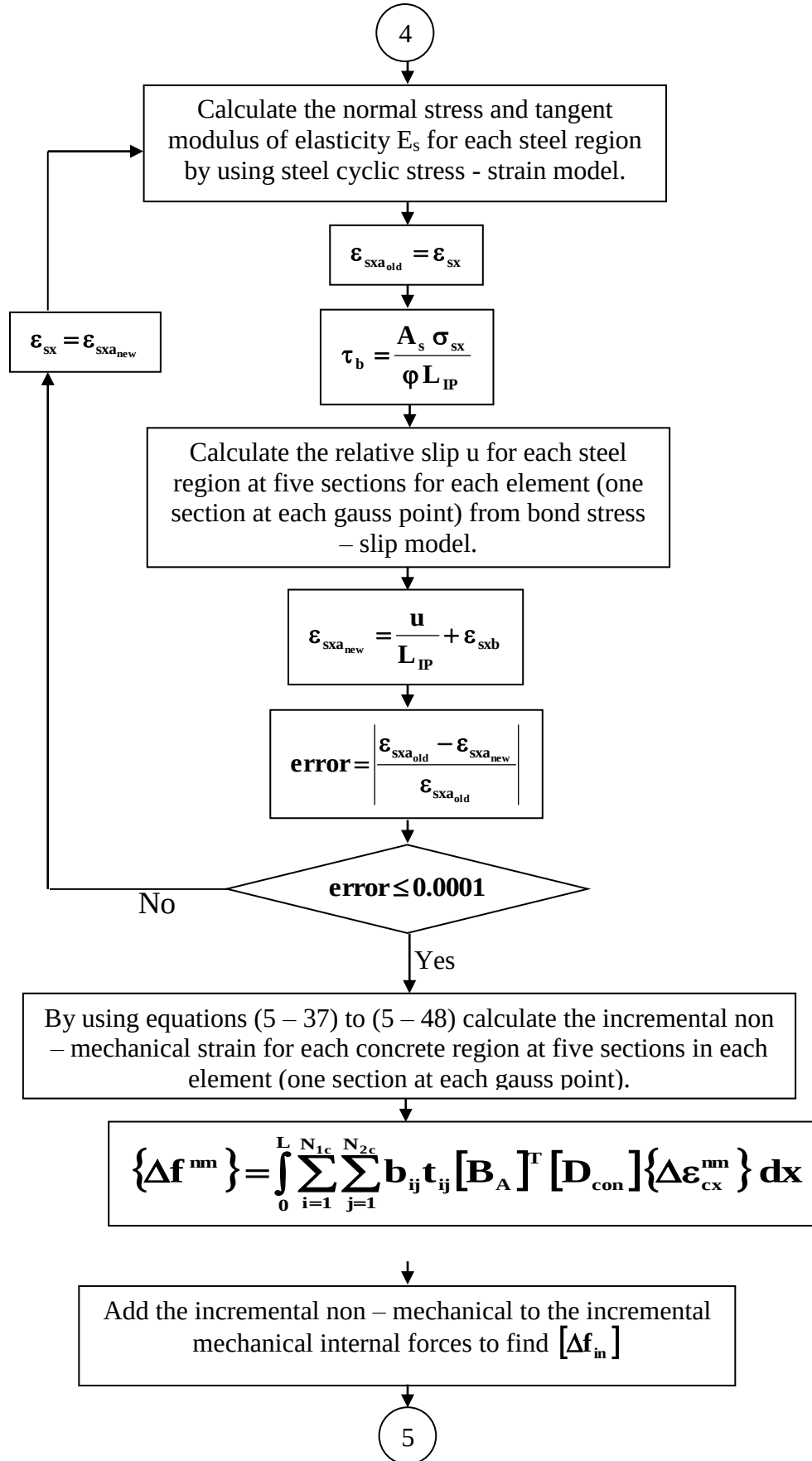


Fig. (5.11) : Continue

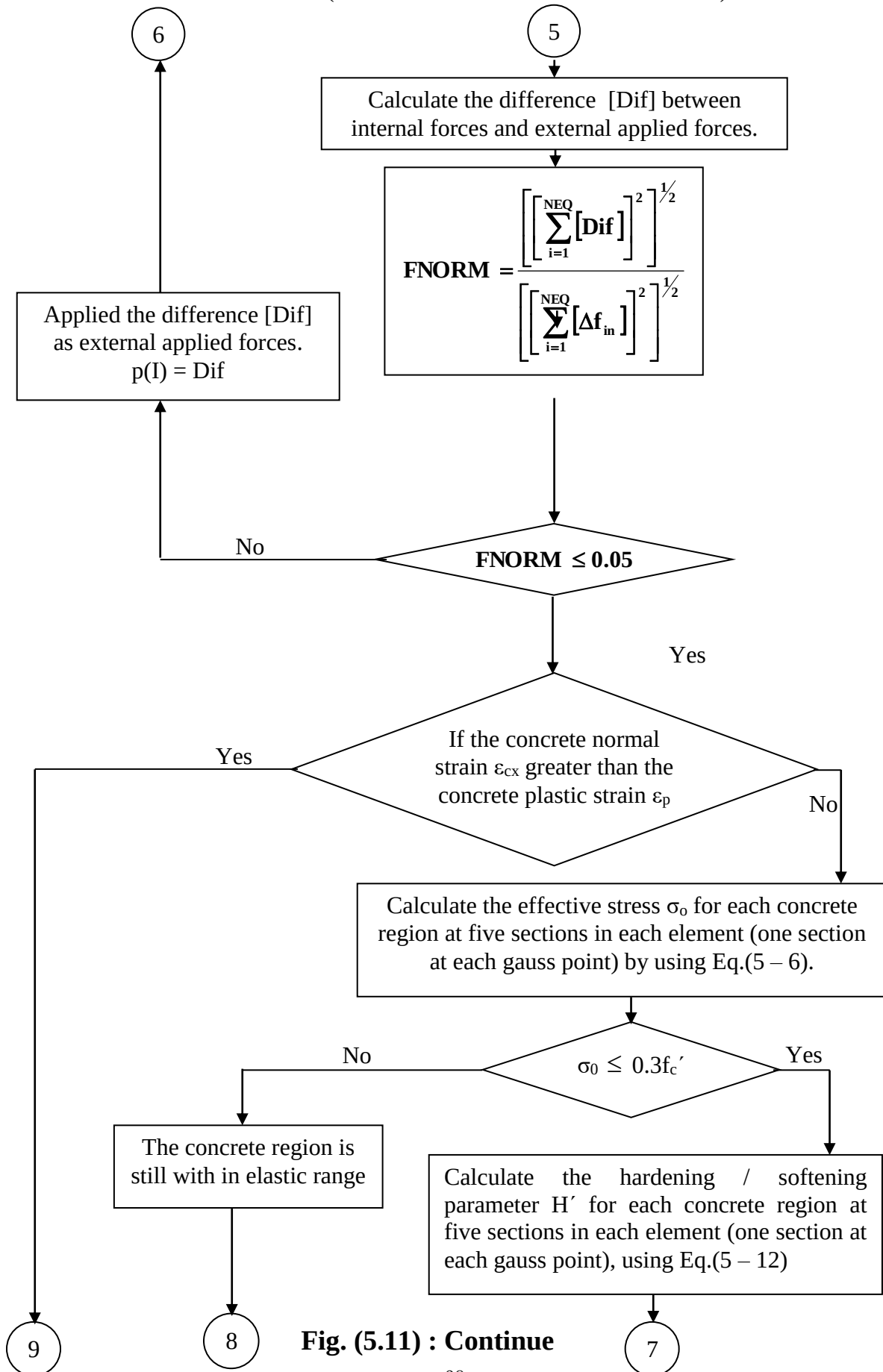


Fig. (5.11) : Continue

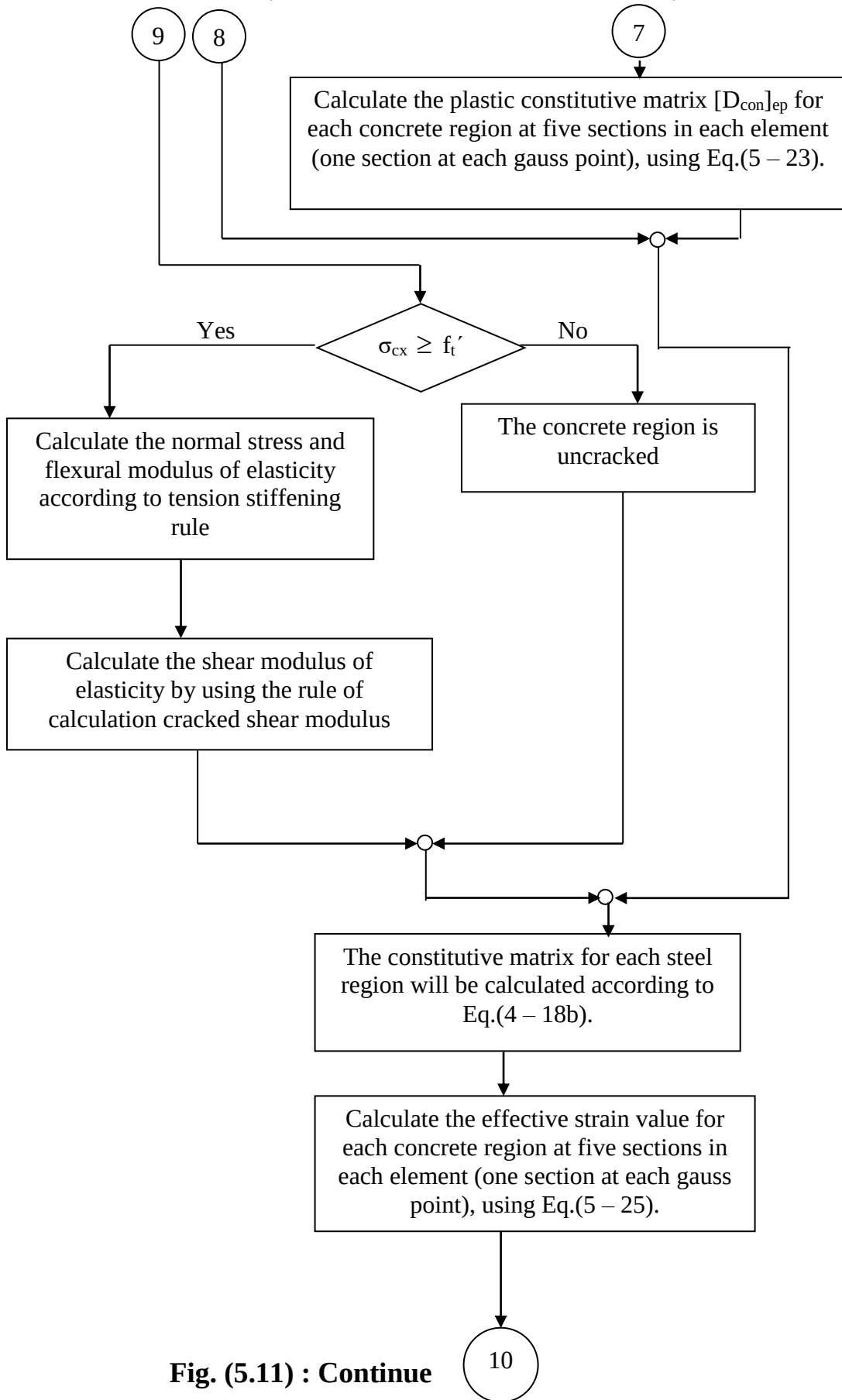
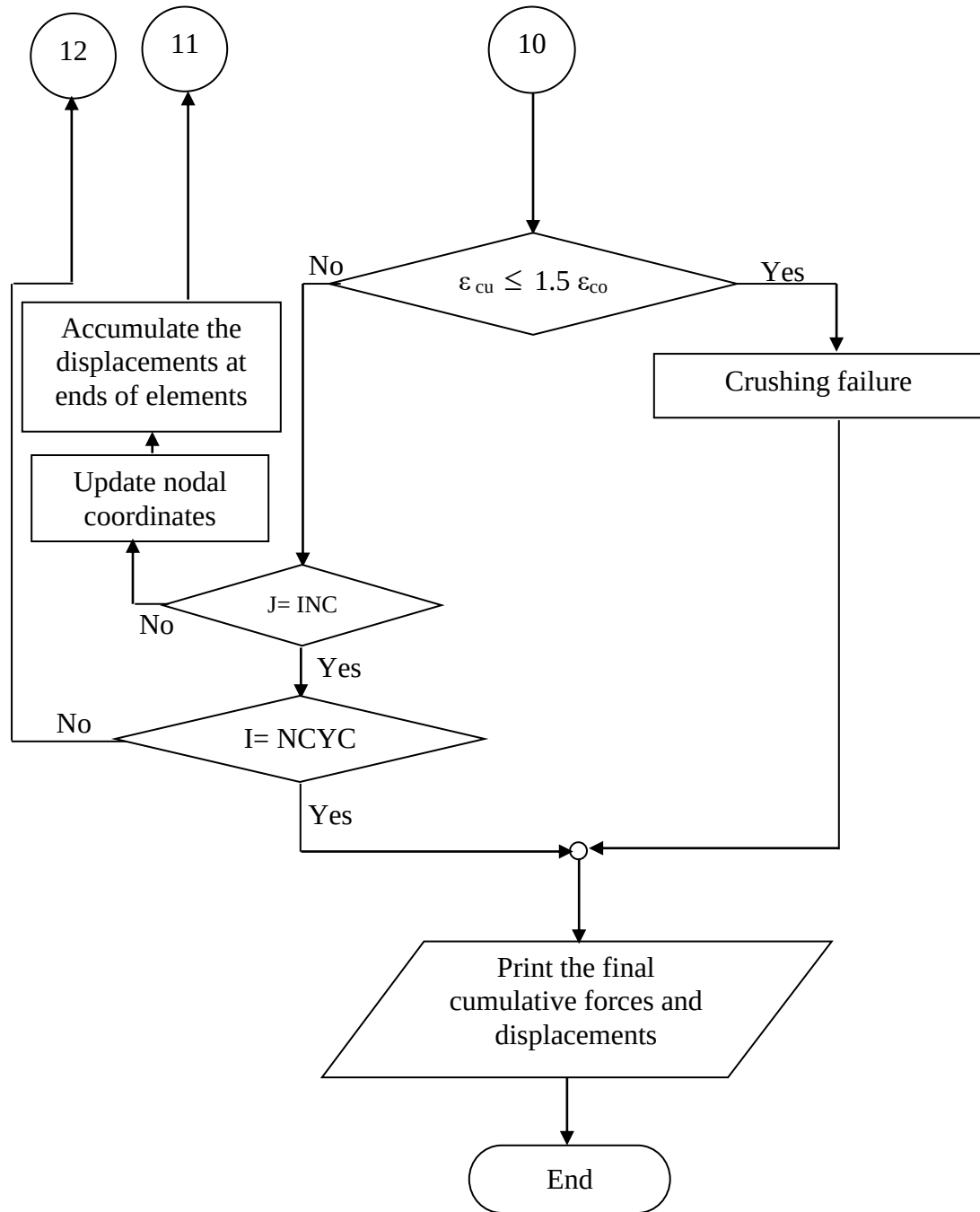


Fig. (5.11) : Continue



INC = the total number of the load increments

NCYC = the total number of the load cycles

Fig. (5.11) : Continue

CHAPTER SIX

(APPLICATIONS AND DISCUSSION)

6.1 Introduction:

Based on the proposed analysis technique that is formulated in chapter four and chapter five, the results of the analysis of several problems are presented in this chapter to check the validity of the proposed approach of analysis in solving various types of inelastic analysis of the reinforced concrete frames that were previously tested in laboratory or analyzed theoretically by others. This is done through comparing the results of the previous studies with those obtained from the new analytical approach.

It is worth to mention, that this analytical study has approximations resulting mainly from different sources, these are:

1. The material modeling of concrete and steel.
2. The modeling and idealization of bond – slip phenomenon at steel – concrete interface.
3. Algorithm used to represent the time dependent effects.
4. The finite elements discretization technique.
5. Approximation in the numerical integration technique used in this numerical analysis.
6. The algorithm used in solving the nonlinear system of equations.

6.2 Applications :

In this section, because no results are found for the reinforced concrete space frame under cyclic loading including time dependent effects, then at first a reinforced concrete plane frame under cyclic loading will be analyzed. After that, a reinforced concrete space frame under monotonic load will be

analyzed and the results will be compared with those obtained previously to check the validity of the proposed technique and computer program to solve three dimensional frame problem. Also, a reinforced concrete plane frame under monotonic loading will be analyzed taking into account the time dependent effects and the results will be compared with the results obtained from previous studies to check the capability of the proposed analytical technique and computer program to solve this type of problems.

Example No.1:

A plane one storey one bay reinforced concrete frame was analyzed theoretically by **Mahmood** ^[52], in 1989. Figure (6.1) shows the data used in the present study to analysis this frame. The loading conditions for frame (F1) are shown in figure (6.2). Frame (F1) was divided by **Mahmood** ^[52] into three elements , i.e one element for each member. According to the new proposed procedure of analysis, frame (F1) is subdivided into various number of elements as illustrated in Table (6.1), to find the suitable number of elements which give good agreement with theoretical results obtained by **Mahmood** ^[52]. Each beam or column section in the frame is subdivided into ten concrete regions in width direction by ten concrete regions in depth direction and six steel regions. The theoretical horizontal displacements obtained by **Mahmood** ^[52] and the results obtained from the proposed analytical procedure using various number of elements are presented in figures (6.3) to (6.6). Table (6.2) give the values of horizontal displacement obtained from the new proposed procedure and the percentage error with respect to theoretical results at load equal to (200 kN) and (-200 kN) for various number of elements. Figures (6.7) and (6.8) show the curve which represent the relation between the number of elements and the percentage error with respect to displacement obtained by **Mahmood** ^[52] at load equals to (200 kN) and (-200 kN) respectively. From figures (6.3) to

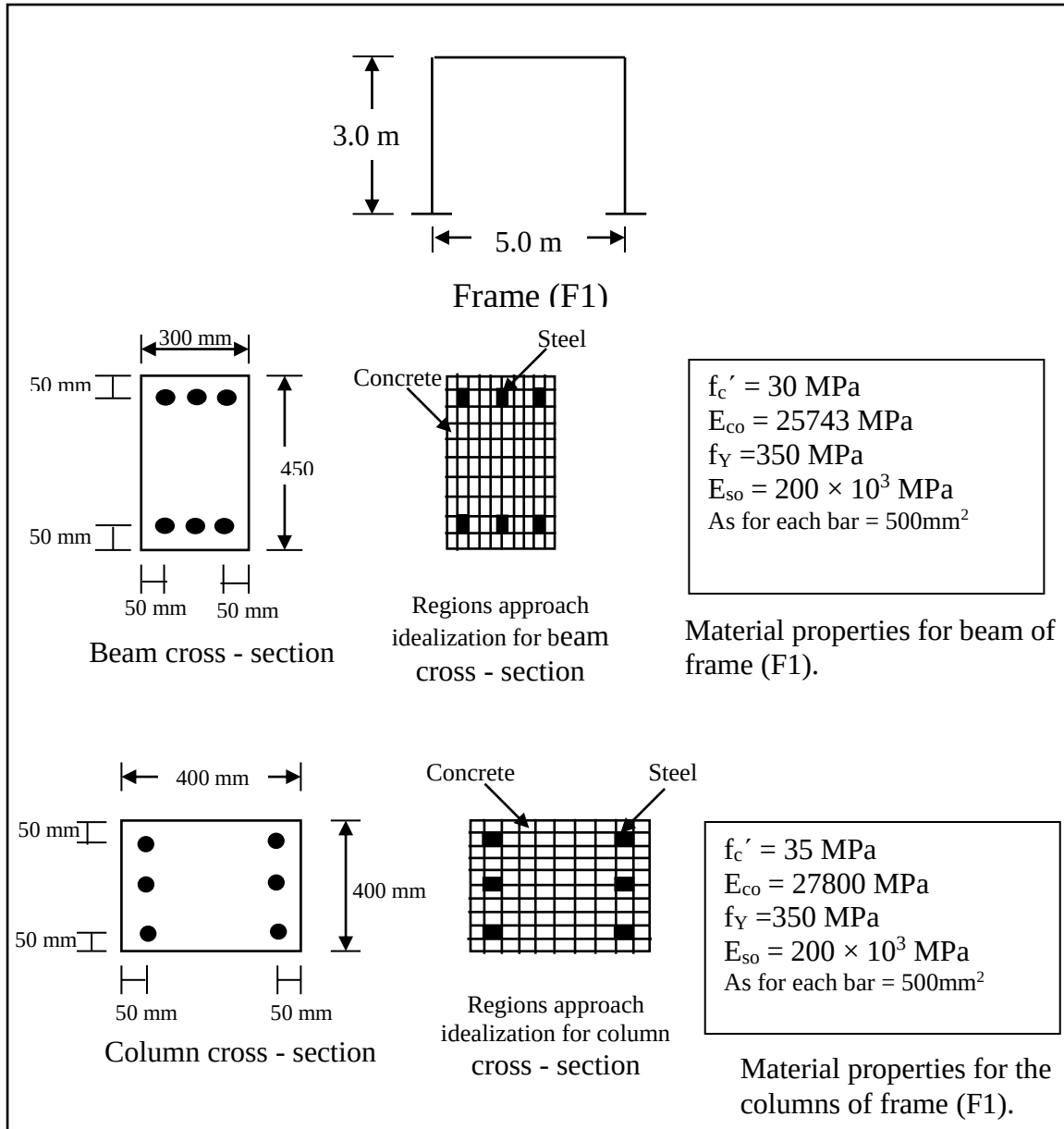


Fig. (6.1) : Details of Frame (F1)

(6.8) and table (6.2) ,the analysis using (15 elements) gives good agreement with the **Mahmood** theoretical results. For the analysis using (15 elements), the difference between the theoretical results by **Mahmood** ^[52] and the present analytical results is about (1.0%) at load equals to (200 kN) and about (18.0%) at load equals to (-200 kN).

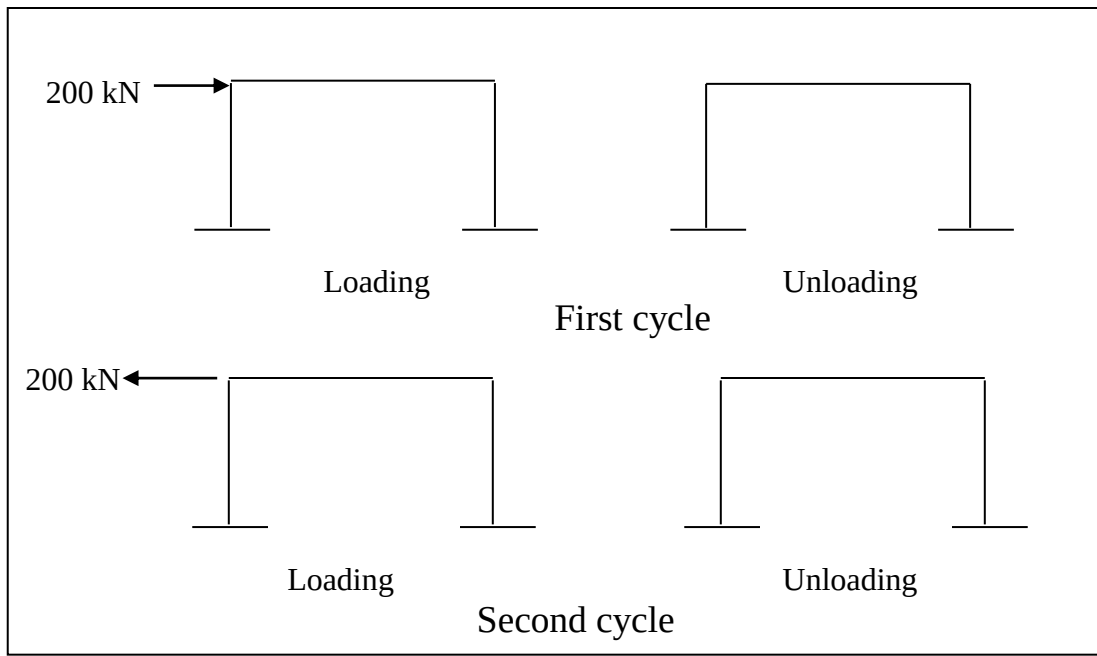


Fig. (6.2) : Test Loading Conditions for Frame (F1)

Table (6.1): Numbers of Elements in Beam and Columns of Frame (F1).

No. of elements in beam	No. of elements in column.	Total number of elements in the frame (F1)
1	1	3
2	2	6
3	3	9
5	5	15
6	6	18
8	8	24
10	10	30
12	12	36
14	14	42
15	15	45
16	16	48
18	18	54
19	19	57
20	20	60
21	21	63
22	22	66

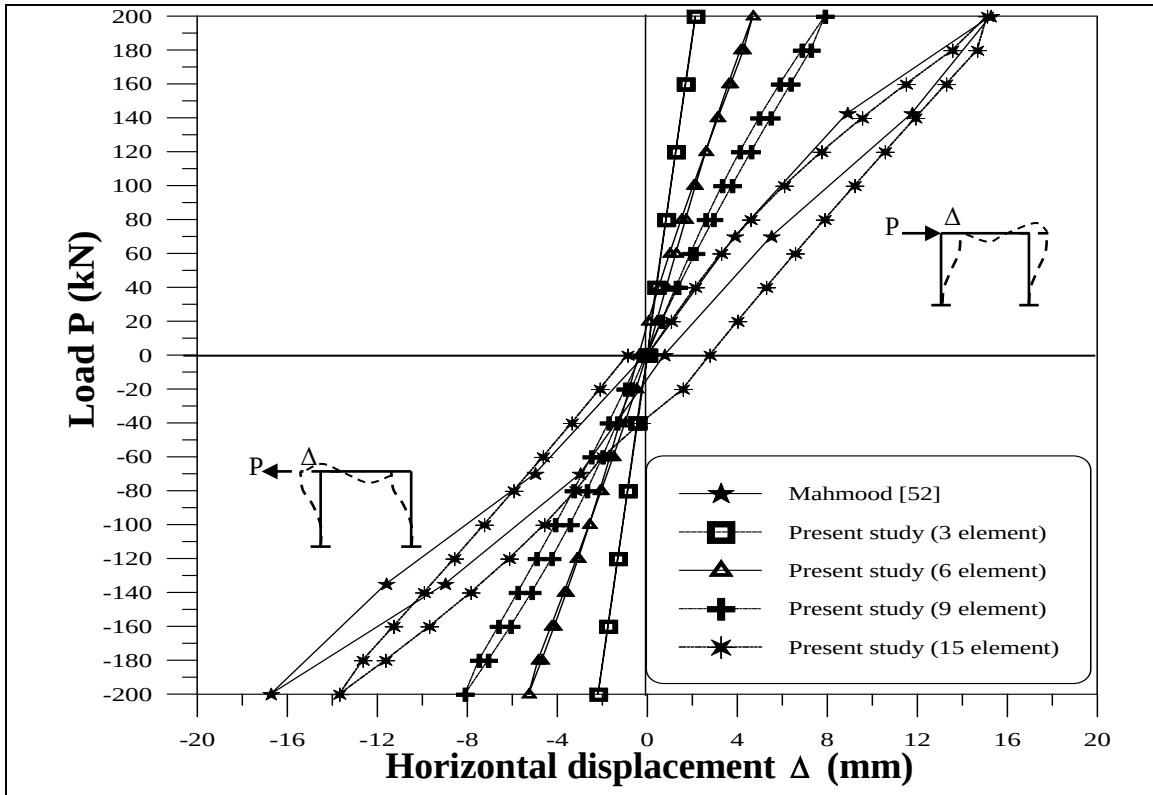


Fig. (6.3):Load – Displacement Curve for Frame (F1) Obtained by Mahmood [52] and The Present Analysis Using (3 Elements) to (15 Elements).

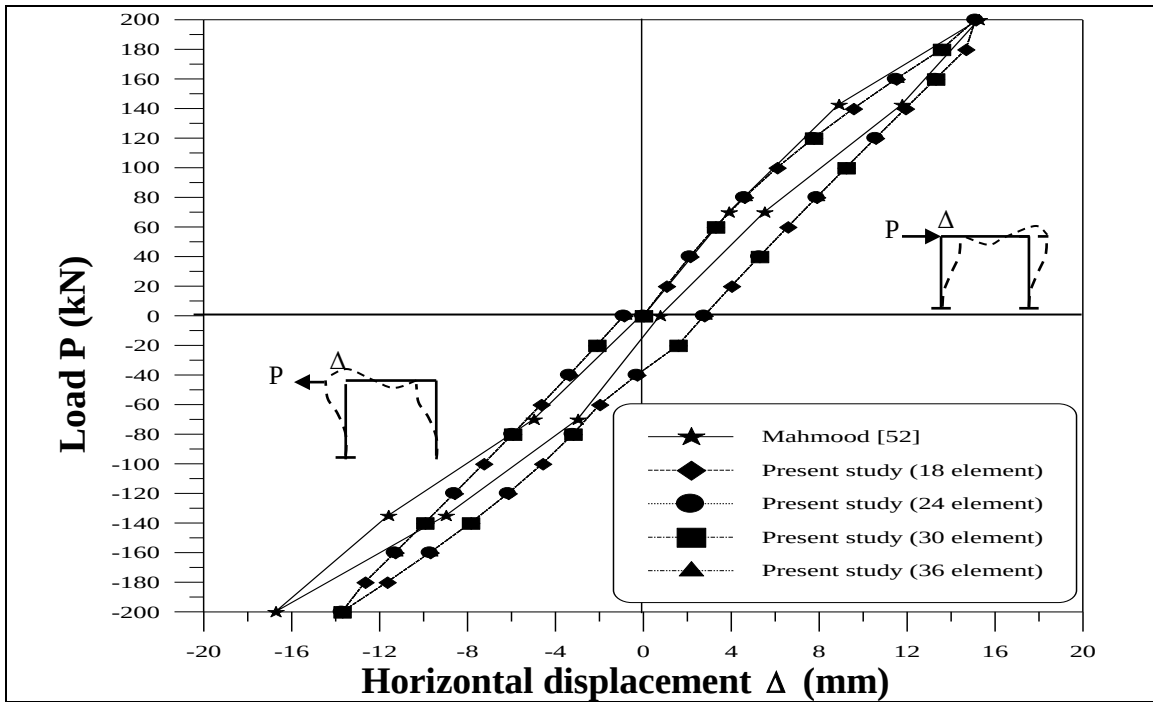


Fig. (6.4):Load – Displacement Curve for Frame (F1) Obtained by Mahmood [52] and The Present Analysis Using (18 Elements) to (36 Elements).

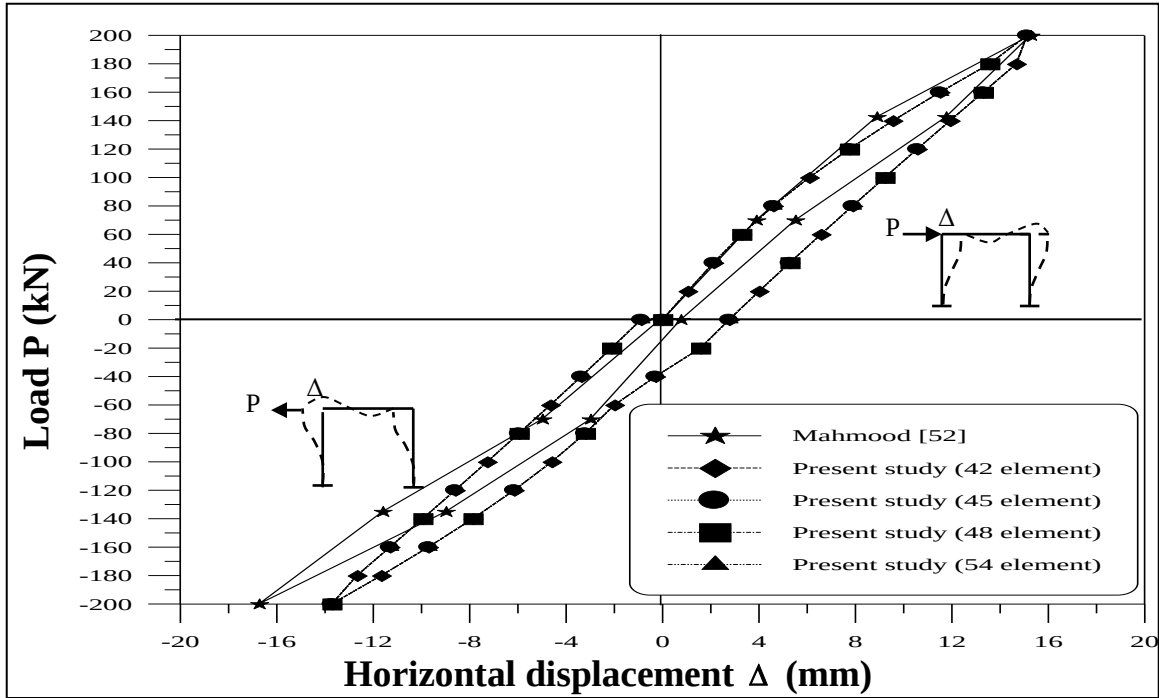


Fig. (6.5):Load – Displacement Curve for Frame (F1) Obtained by Mahmood [52] and The Present Analysis Using (42 Elements) to (54 Elements).

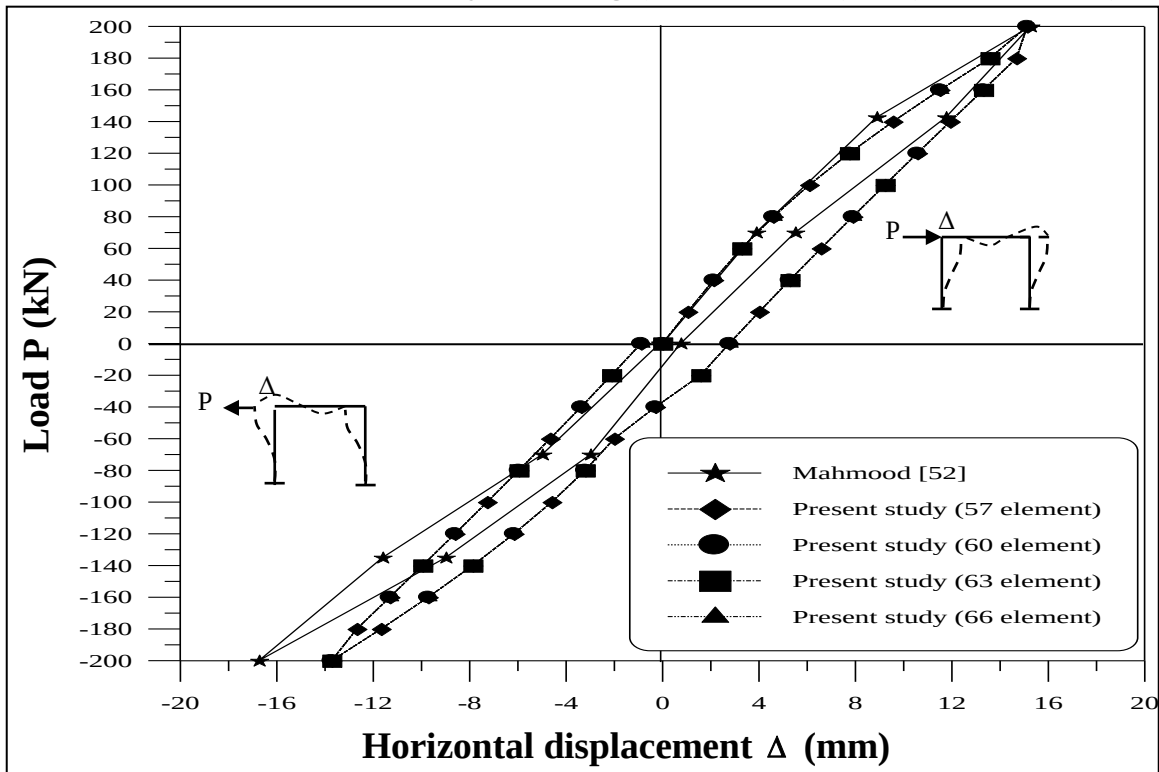


Fig. (6.6):Load – Displacement Curve for Frame (F1) Obtained by Mahmood [52] and The Present Analysis Using (57 Elements) to (66 Elements).

Table (6.2): Horizontal Displacement of Frame (F1) and Percentage Errors with Respect to Theoretical Data Obtained by Mahmood ^[52]

$$\left[Error = \frac{Mahmood\ result - Present\ result}{Mahmood\ result} \times 100 \right] \cdot$$

No. of elements	Horizontal disp. (mm) at load (200 kN)	Error(%) in disp. with respect to theo. data at load (200 kN)	Horizontal disp. (mm) at load (-200 kN)	Error (%) in disp. with respect to theo. data at load (-200 kN)
3	2.135	86	-2.188	86.937
6	4.686	69.272	-5.284	68.453
9	7.884	48.301	-8.126	51.486
15	15.093	1.029	-13.706	18.179
18	15.108	0.931	-13.719	18.095
24	15.111	0.911	-13.722	18.077
30	15.113	0.898	-13.723	18.071
36	15.114	0.891	-13.724	18.065
42	15.116	0.878	-13.726	18.053
45	15.117	0.872	-13.727	18.047
48	15.118	0.865	-13.728	18.041
54	15.12	0.852	-13.730	18.029
57	15.123	0.832	-13.733	18.011
60	15.124	0.826	-13.734	18.005
63	15.126	0.813	-13.735	18.000
66	15.126	0.813	-13.736	17.994

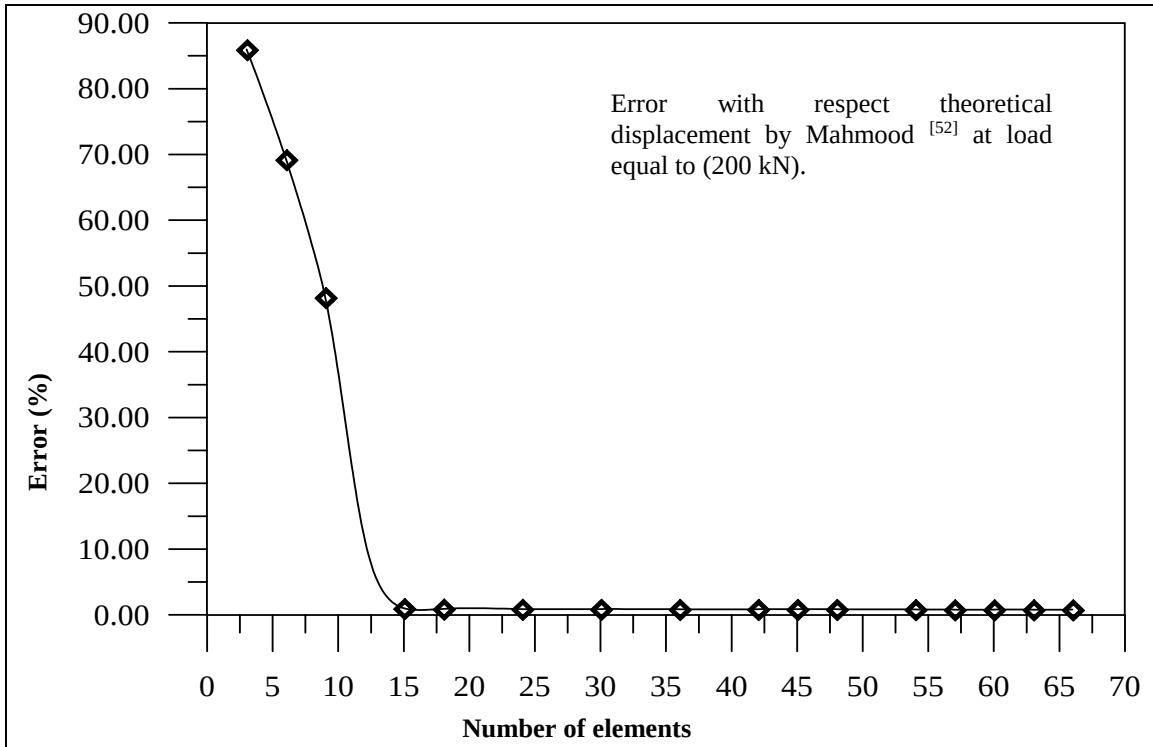


Fig. (6.7):Effect of Elements Number on The Percentage Error Value with Respect to Theoretical Results by Mahmood ^[52] at Load Equal to (200 kN).

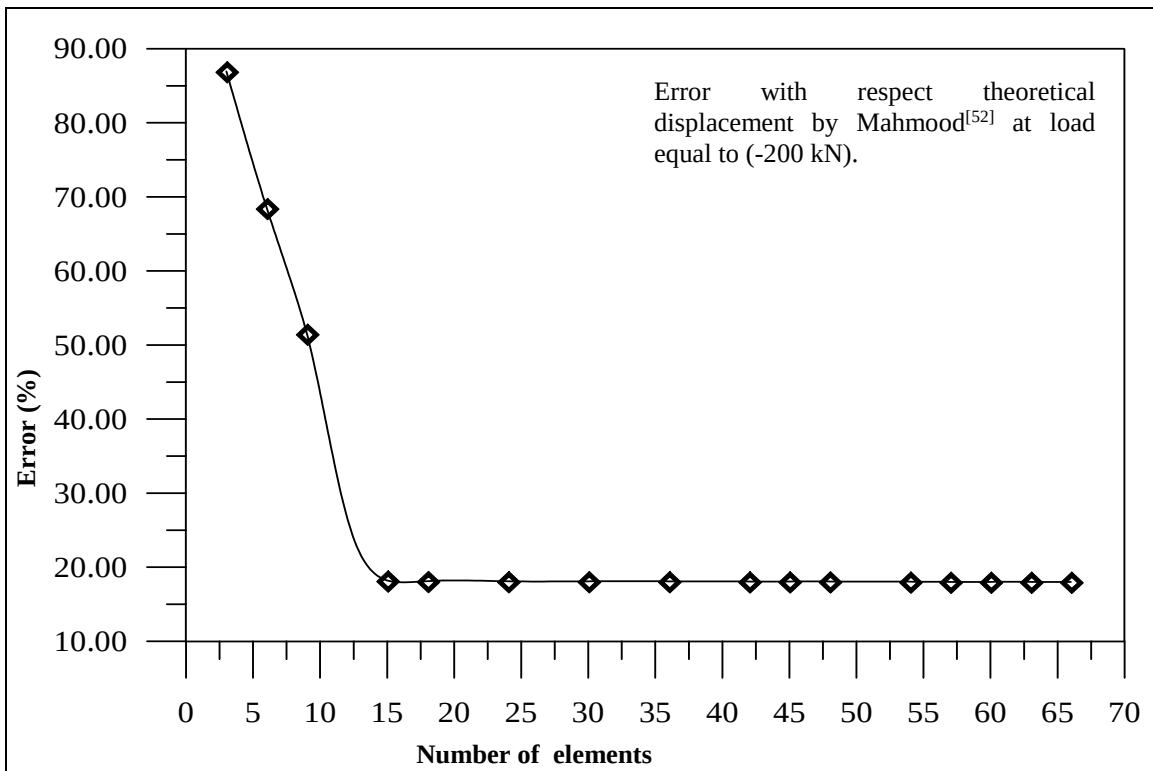


Fig. (6.8):Effect of Elements Number on The Percentage Error Value with Respect to Theoretical Results by Mahmood ^[52] at Load Equal to (-200 kN).

The (15 elements) analysis is suitable because it needs less number of elements and gives a good agreement with theoretical results by **Mahmood**^[52]. Figure (6.9) show the bending moment diagram (about z-axis) obtained from new analysis procedure using (15) elements at load equal to (200kN). Figure (6.10) show the position of concrete region (10,10) at the first gauss point in the first element in frame F1, the stress-strain curve for this region is shown in figure (6.11).

Example No.2:

In 1989, a plane one story one bay reinforced concrete frame was analyzed theoretically by **Mahmood**^[52]. Figure (6.12) shows the data used in the present study to analyze this frame. The loading conditions for frame (F2) are shown in figure (6.13). Frame (F2) was divided by **Mahmood**^[52] into nine elements , i.e three elements for each member. According to the proposed procedure of analysis, frame (F2) is subdivided into various number of elements as illustrated in table (6.3), to find the suitable number of elements which gives good agreement with theoretical data obtained by **Mahmood**^[52]. Each column section in the frame is subdivided into ten concrete regions in width direction by ten concrete regions in depth direction and six steel regions. Each beam section in the frame is subdivided into ten concrete regions in width direction by ten concrete regions in depth direction and six steel regions. The theoretical horizontal displacements obtained by **Mahmood**^[52] and the results obtained from the proposed analytical procedure using various number of elements are presented in figures (6.14)

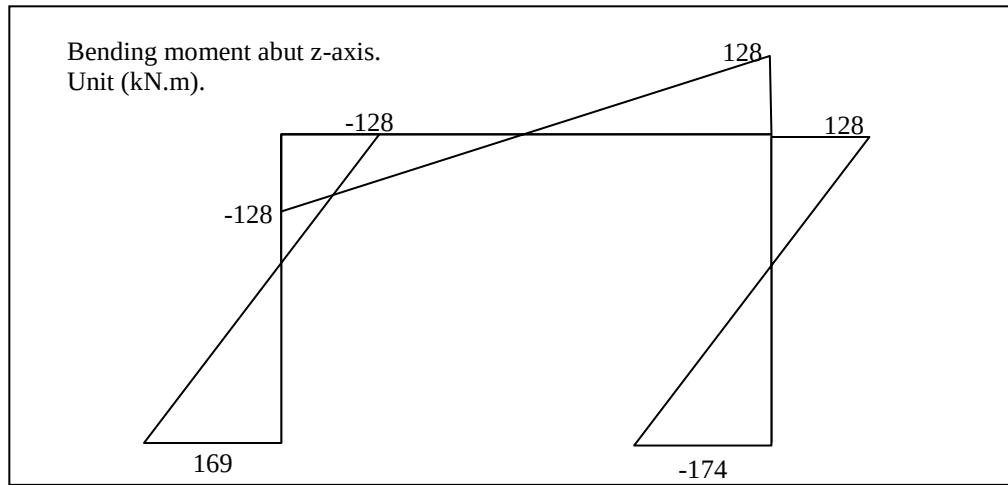


Fig.(6.9): Bending Moment Diagram (about z-axis) for Frame (F1) Using New Analysis Procedure with (15) Elements at Load (200kN).

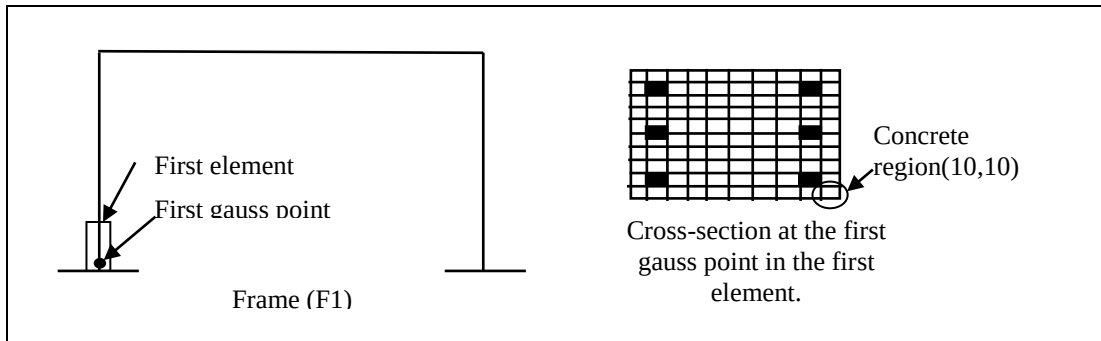


Fig.(6.10): Position of The Concrete Region (10,10) at The First Gauss Point in The first Element in Frame F1.

and (6.15). Table (6.4) gives the values of the horizontal displacement obtained from the new procedure and the percentage error with respect to theoretical data at load equals to (80 kN) and (-80 kN) for various number of elements. Figures (6.16) and (6.17) show the curve which represent the relation between the number of elements and the percentage error with respect to displacement obtained by **Mahmood** ^[52] at load equals to (80 kN) and (-80 kN) respectively. From figures (6.14) to (6.17) and table (6.4) ,the analysis using (18 elements) gives a good agreement with theoretical data. For the analysis using (18 elements), the difference between

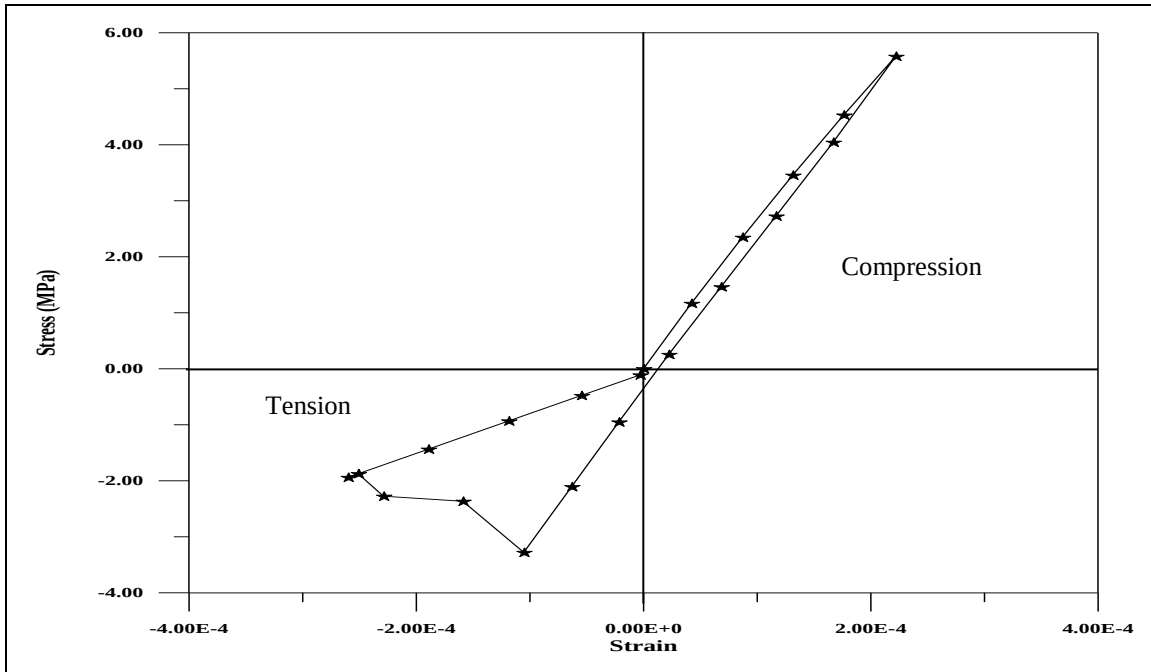


Fig.(6.11): Stress-Strain Curve for Concrete Region (10,10) at The First Gauss Point in The first Element in Frame F1 at Load (200kN).

the theoretical results by **Mahmood** ^[52] and the present analytical results is about (2 %) at load equals to (80 kN) and about (1%) at load equals to (-80 kN). Figure (6.18) show the bending moment diagram (about z-axis) obtained from new analysis procedure using six elements at load equal to (80kN). Figure (6.19) show the position of concrete region (10,10) at the first gauss point in the first element in frame F2, the stress-strain curve for this region is shown in figure (6.20).

Example No.3:

A plane one story one bay reinforced concrete frame was tested experimentally by **Bertero and McClure (1964)**^[38]. This frame was tested under the cyclic controlled horizontal and vertical displacement of its girder. Figure (6.21) shows the data used in the present study to analyze this frame.

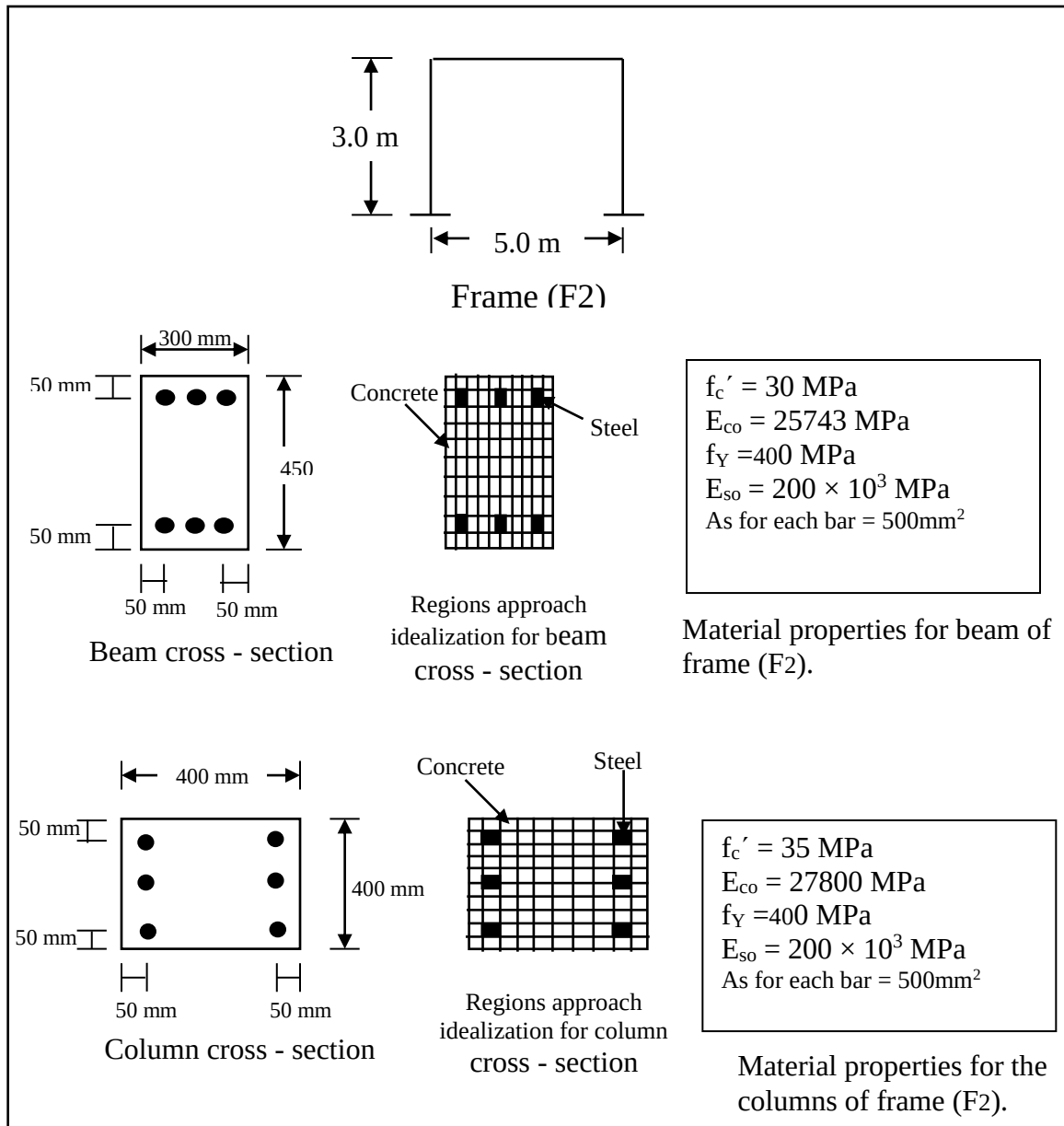


Fig. (6.12) : Details of Frame (F2).

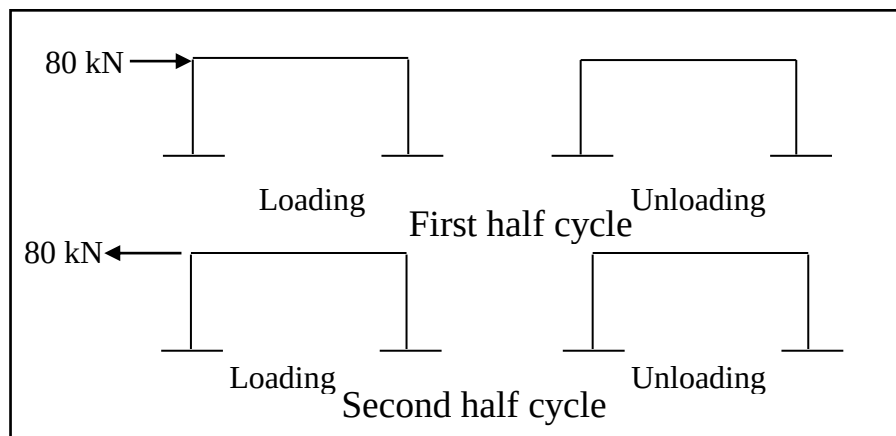
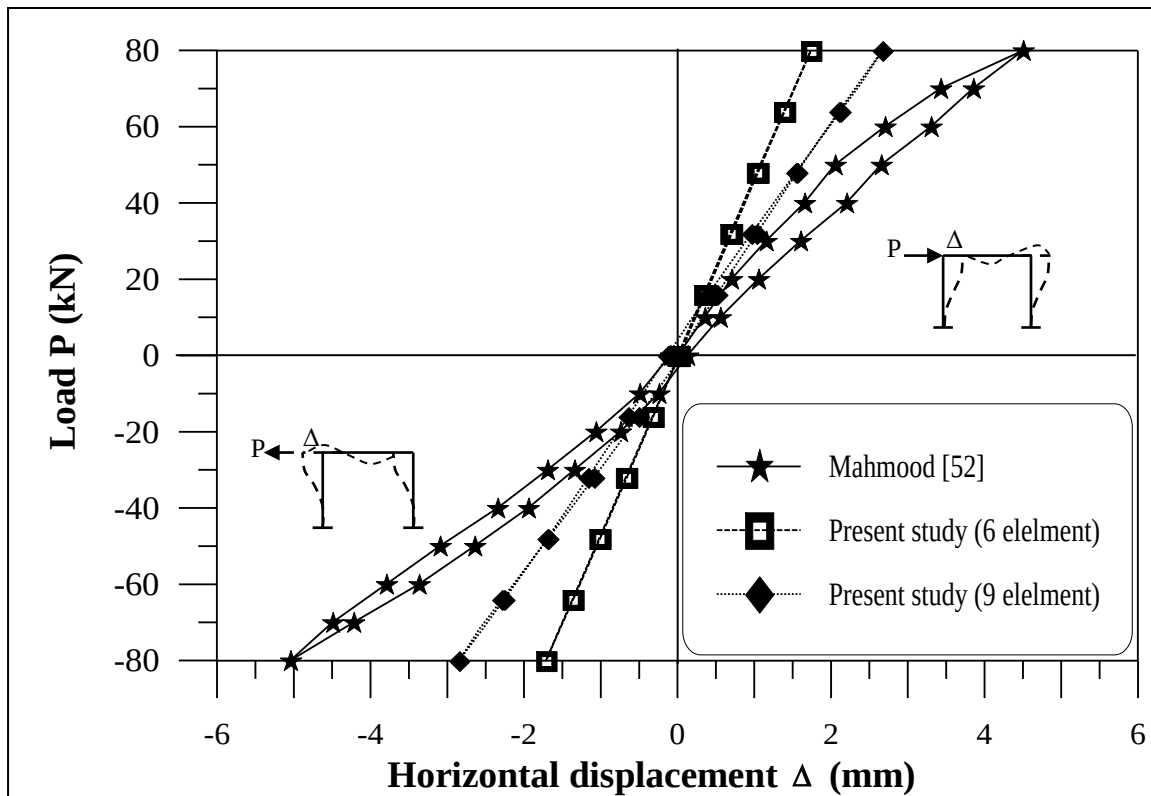


Fig. (6.13) : Test Loading Conditions for Frame (F2).

Table (6.3): Numbers of Elements in Beam and Columns of Frame (F2).

No. of elements in beam	No. of elements in column.	Total number of elements in the frame (F2)
2	2	6
3	3	9
4	4	12
5	5	15
6	6	18

**Fig. (6.14): Load – Displacement Curve for Frame (F2) Obtained by Mahmood [52] and The Present Analysis Using (6 Elements) and (9 Elements).**

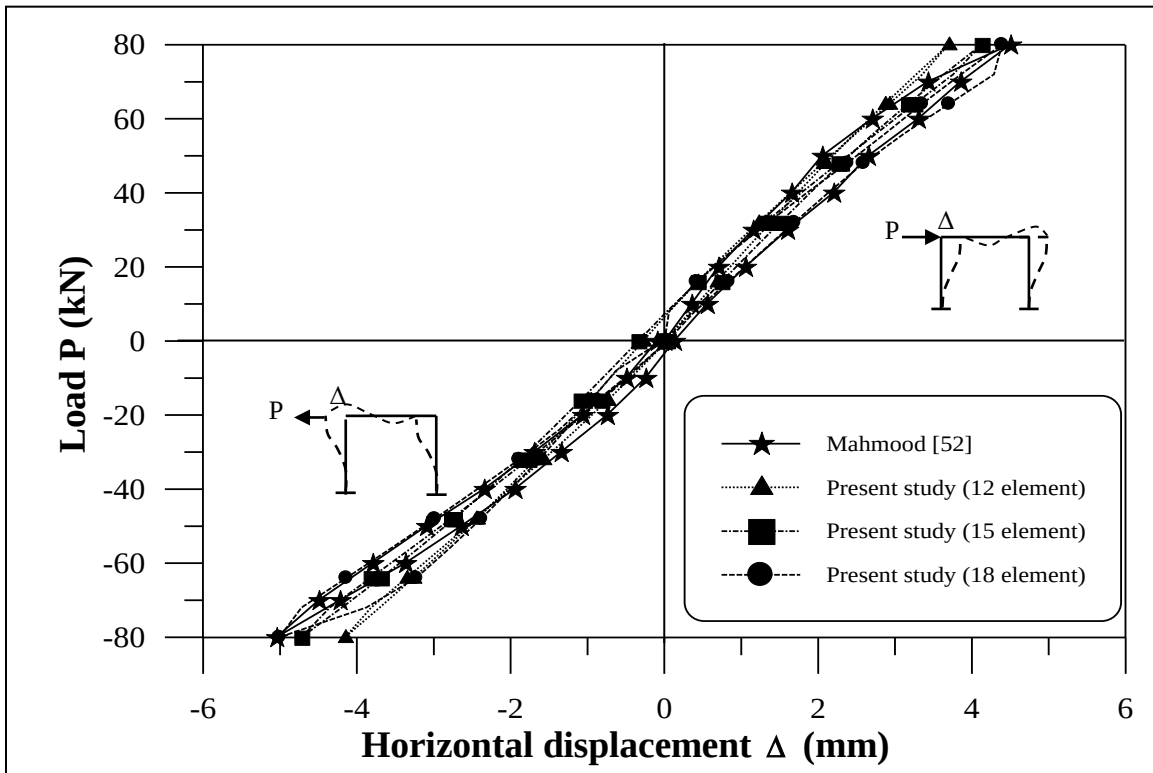


Fig. (6.15):Load – Displacement Curve for Frame (F2) Obtained by Mahmood [52] and The Present Analysis Using (12 Elements) to (18 Elements).

Table (6.4): Horizontal Displacement of Frame (F2) and Percentage Errors with Respect to Theoretical Data Obtained by Mahmood [52].

$$Error = \frac{Mahmood\ results - Present\ results}{Mahmood\ results} \times 100$$

No. of elements	Horizontal disp. (mm) at load (80 kN)	Error(%) in disp. with respect to theo. data at load (80 kN)	Horizontal disp. (mm) at load (-80 kN)	Error (%) in disp. with respect to theo. data at load (-80 kN)
6	1.739	61.355	-1.719	66.02
9	2.670	40.67	-2.849	43.58
12	3.699	17..80	-4.155	17.72
15	4.128	8.267	-4.724	6.46
18	4.392	2.40	-5.010	0.79

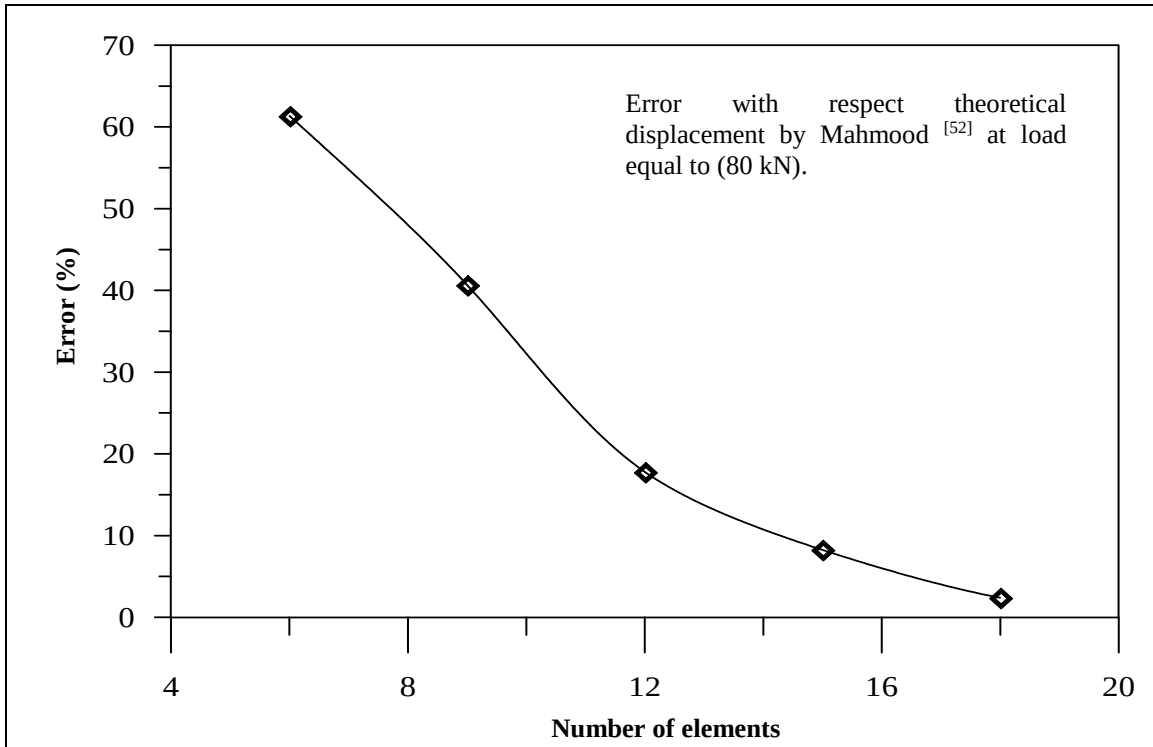


Fig. (6.16): Effect of Elements Number on The Percentage Error Value with Respect to Theoretical Results by Mahmood [52] at Load Equal to (80 kN).

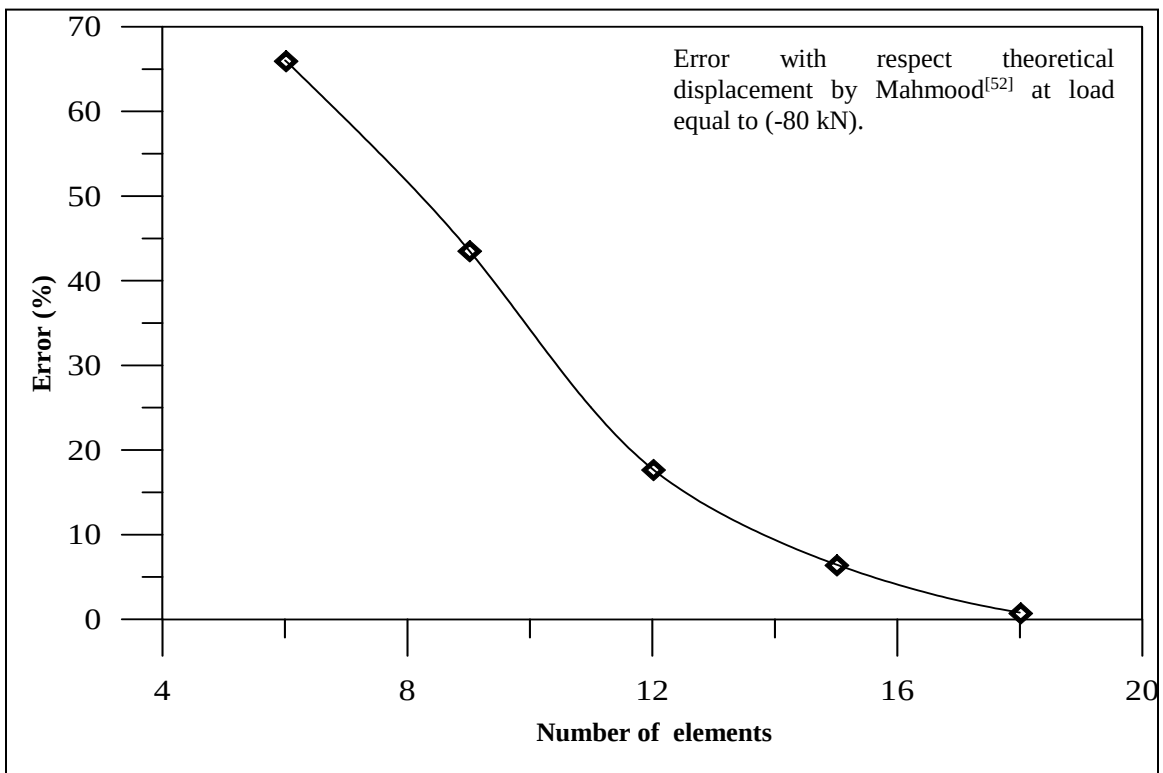


Fig. (6.17): Effect of Elements Number on The Percentage Error Value with Respect to Theoretical Results by Mahmood [52] at Load Equal to (-80 kN).

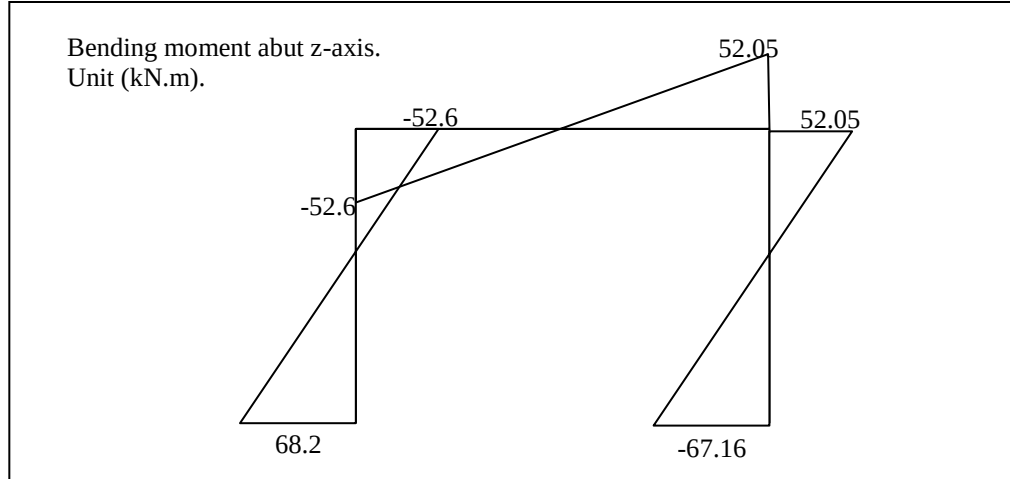


Fig.(6.18): Bending Moment Diagram (about z-axis) for Frame (F2) Using New Analysis Procedure with (18) Elements at Load (80kN).

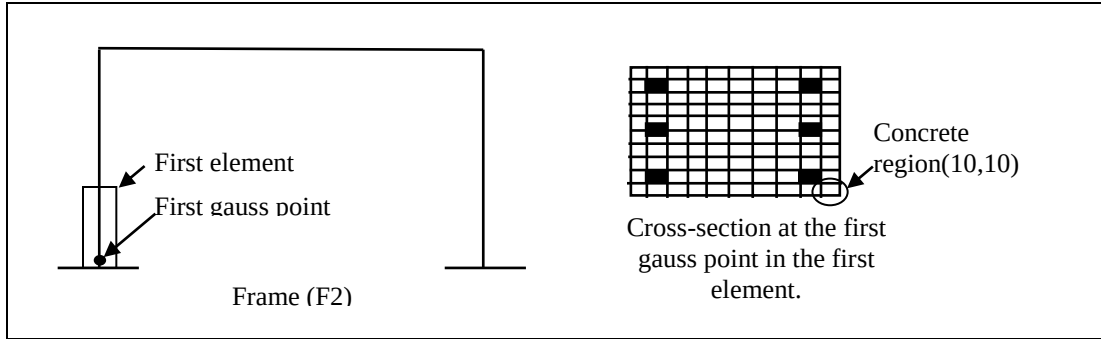


Fig.(6.19): Position of The Concrete Region (10,10) at The First Gauss Point in The first Element in Frame F2.

The test loading condition for frame (F3) is shown in figure (6.22). According to the proposed procedure of analysis, frame (F3) is subdivided into various number of elements as illustrated in table (6.5), to find the suitable number of elements which gives good agreement with experimental data obtained by **Bertero** and **McClure** ^[38]. Each section in the frame is divided into ten concrete regions in width direction by ten concrete regions in depth direction and four steel regions. The experimental horizontal displacements obtained by **Bertero** and **McClure** ^[38] and the results obtained from the proposed analytical procedure using various number of

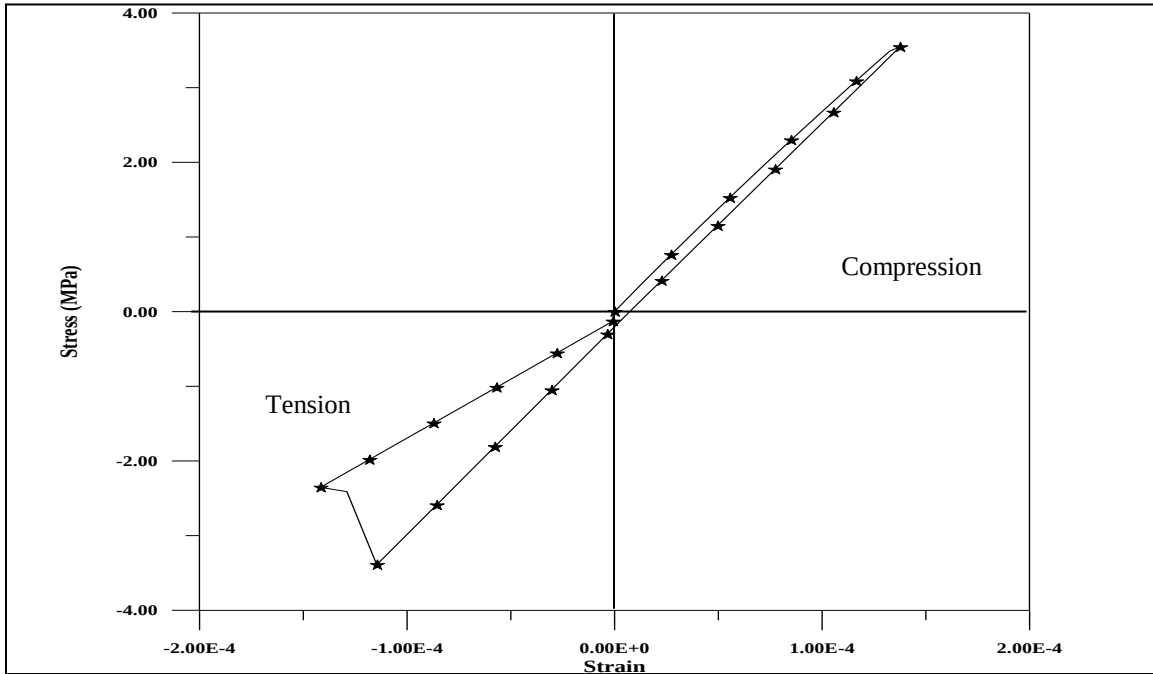


Fig.(6.20): Stress-Strain Curve for Concrete Region (10,10) at The First Gauss Point in The first Element in Frame F2 at Load (80kN).

elements are presented in figures (6.23) to (6.26). Table (6.6) gives the values of horizontal displacement obtained from the new proposed analytical procedure and the percentage error with respect to experimental data at load equals to (5.56 kN) and (-5.56 kN) for various number of elements. Figures (6.27) and (6.28) show the curve which represent the relation between the number of elements and the percentage error with respect to displacement obtained by **Bertero** and **McClure** ^[38] at load equals to (5.56 kN) and (-5.56 kN) respectively. From figures (6.23) to (6.28) and table (6.6) ,the analysis using (24 elements) gives a good agreement with experimental data. The difference between the experimental results and the present analytical results is about (3 %) at load 5.56 kN and about (7 %) at load (-5.56 kN). Figure (6.29) show the bending moment diagram (about z-axis) obtained from new analysis procedure using six elements at load equal to (5.56kN). Figure (6.30) show the position of concrete region (10,10) at the first gauss

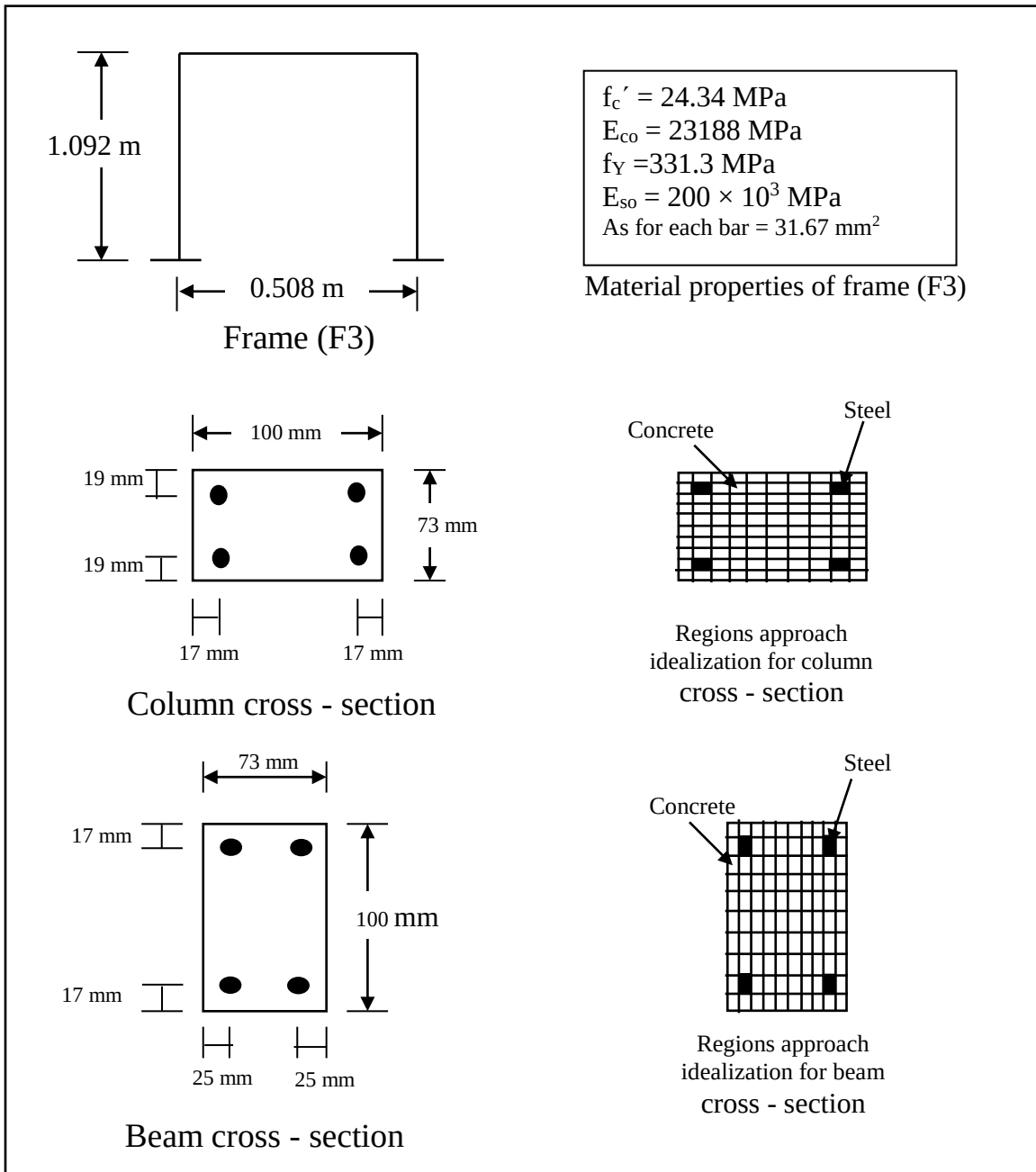


Fig. (6.21) : Details of Frame (F3).

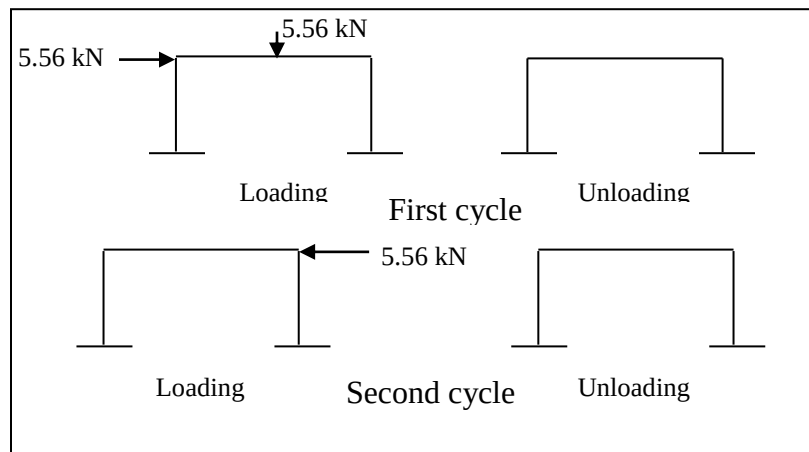
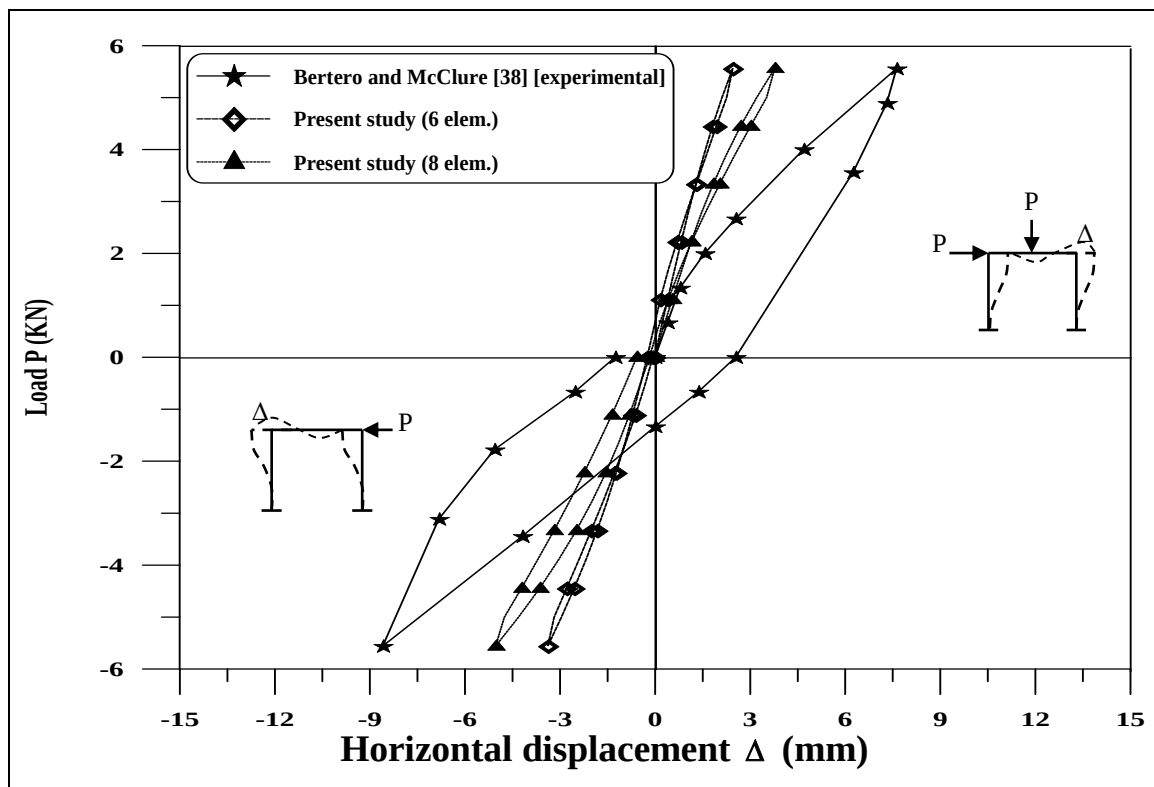


Fig. (6.22) : Test Loading Conditions for Frame (F3).

Table (6.5): Numbers of Elements in Beam and Columns of Frame (F3).

No. of elements in beam	No. of elements in column.	Total number of elements in the frame (F3)
2	2	6
2	3	8
4	3	10
4	4	12
6	5	16
6	6	18
8	8	24

**Fig. (6.23): Load – Displacement Curve for Frame (F3) Obtained by Bertero and McClure ^[38] and The present Analysis Using (6 Elements) and (8 Elements).**

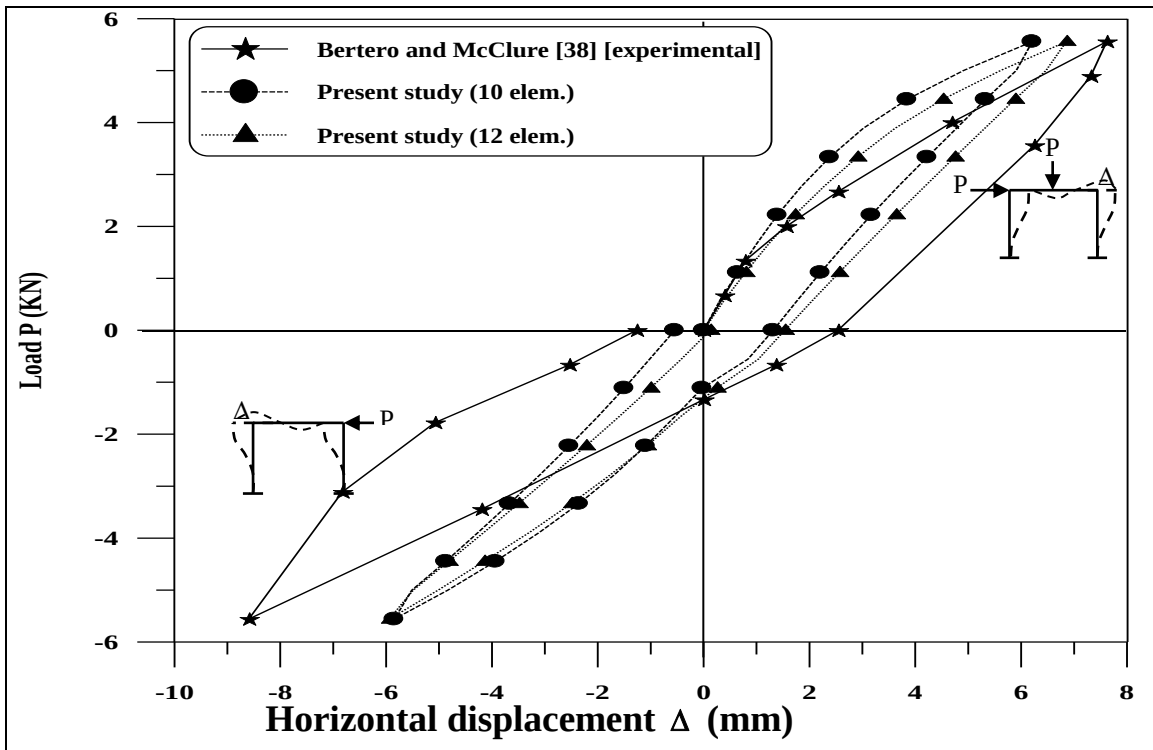


Fig. (6.24): Load – Displacement Curve for Frame (F3) Obtained by Bertero and McClure ^[38] and The Present Analysis using (10 Elements) and (12 Elements).

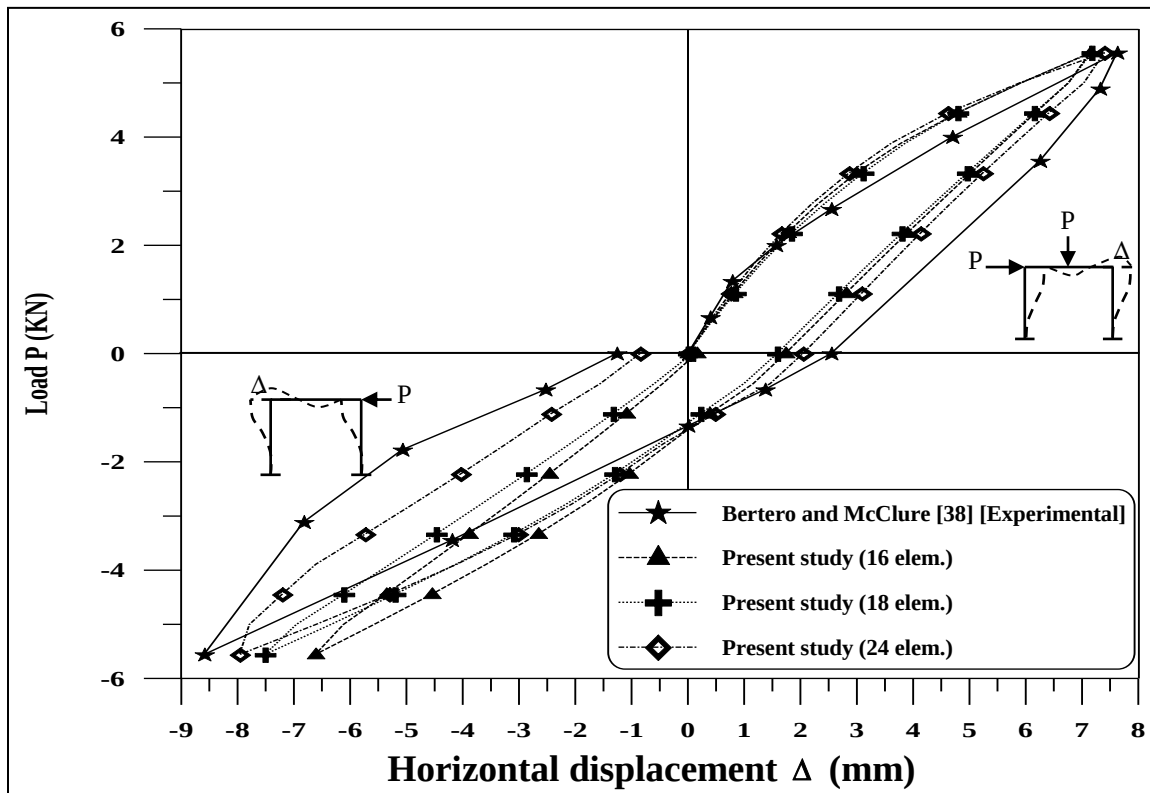


Fig. (6.25): Load – Displacement Curve for Frame (F3) Obtained by Bertero and McClure ^[38] and The present Analysis Using (16 Elements) to (24 Elements).

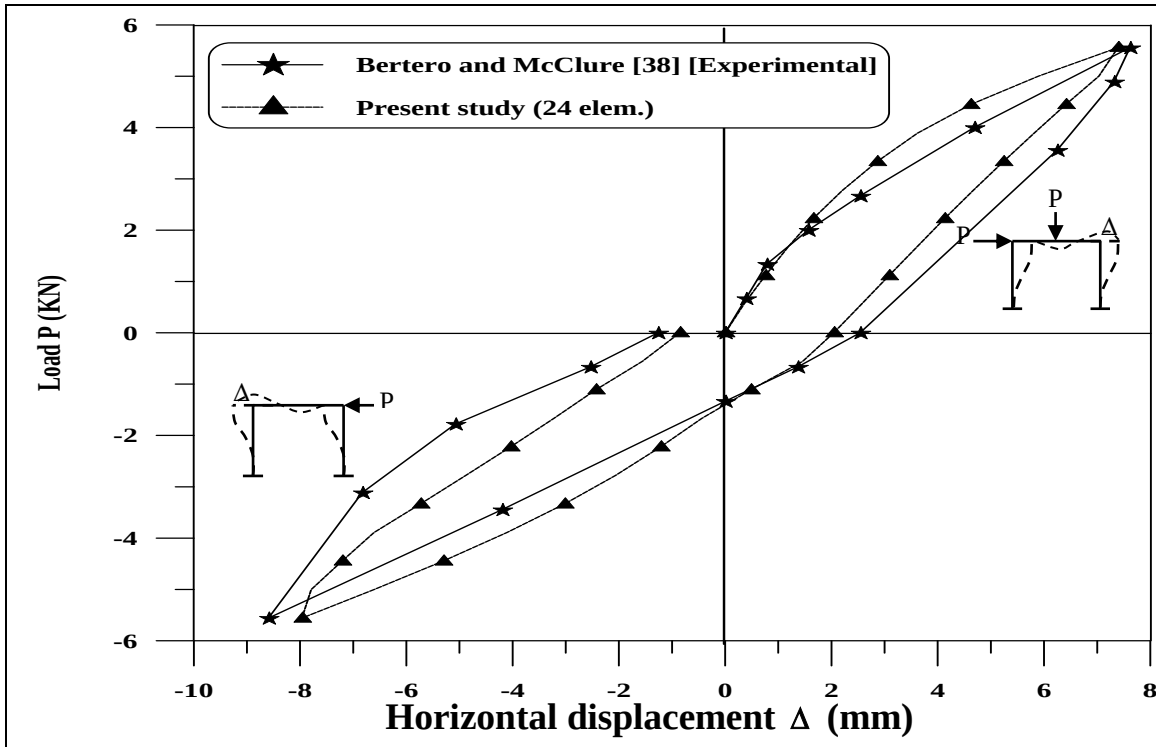


Fig. (6.26): Load – Displacement Curve for Frame (F3) Obtained by Bertero and McClure ^[38] and The Present Analysis Using (24 Elements) .

Table (6.6): Horizontal Displacement of Frame (F3) and Errors with Respect to Experimental Data Obtained by Bertero and McClure ^[38].

No. of elements	Horizontal disp. (mm) at load (5.56 kN)	Error(%) in disp. with respect to exp. data at load (5.56 kN)	Horizontal disp. (mm) at load (-5.56 kN)	Error (%) in disp. with respect to exp. data at load (-5.56 kN)
6	2.446	67.9	-3.392	60.55
8	3.776	50.45	-5.038	41.4
10	6.213	18.46	-5.846	32.0
12	6.855	10.04	-5.941	30.9
16	7.149	6.18	-6.623	22.97
18	7.167	5.94	-7.518	12.56
24	7.394	2.97	-7.968	7.33

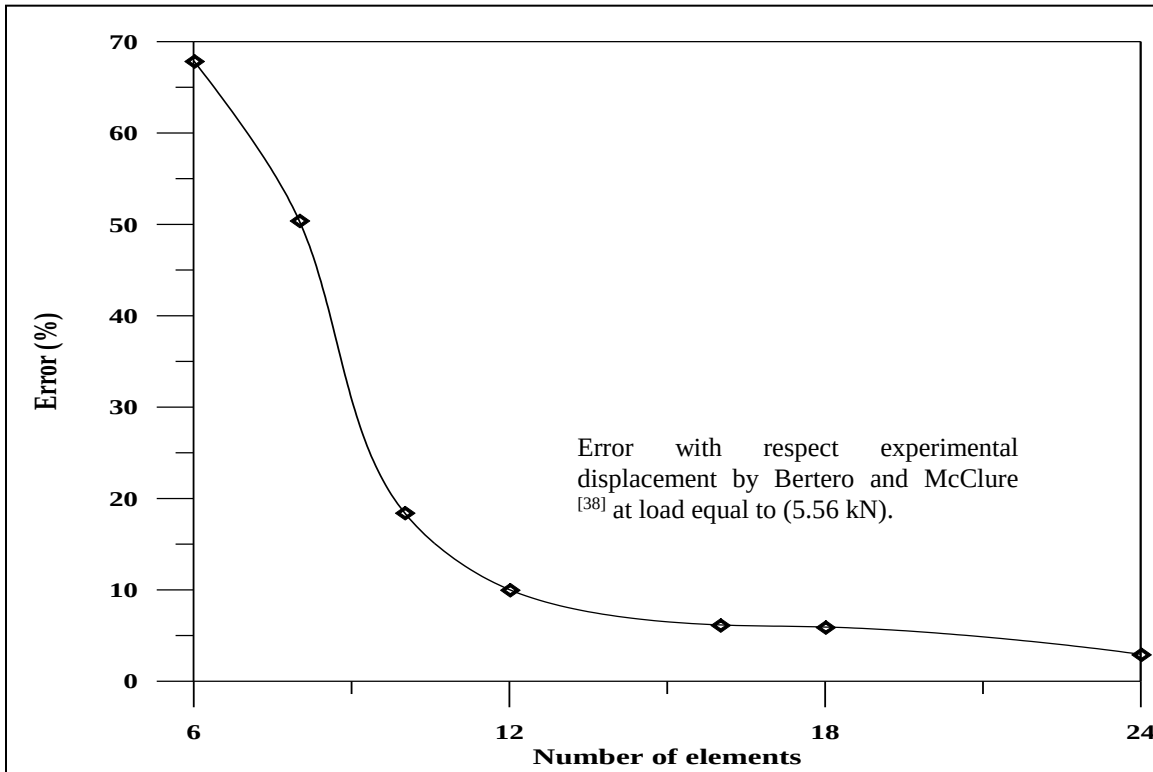


Fig. (6.27):Effect of Elements Number on The Percentage Error Value with Respect to Experimental Results by Bertero and McClure^[38] at Load Equal to (5.56 kN).

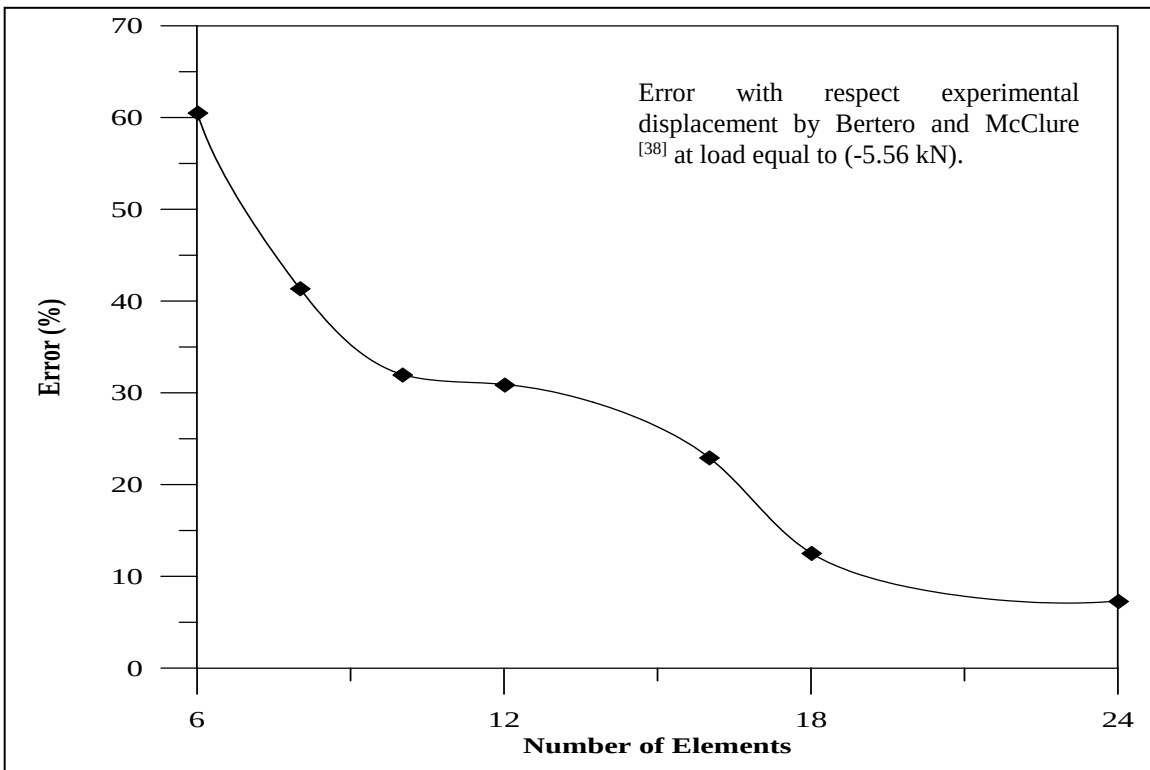


Fig. (6.28):Effect of Elements Number on The Error Value with Respect to Experimental Results by Bertero and McClure^[38] at Load Equal to (-5.56 kN).

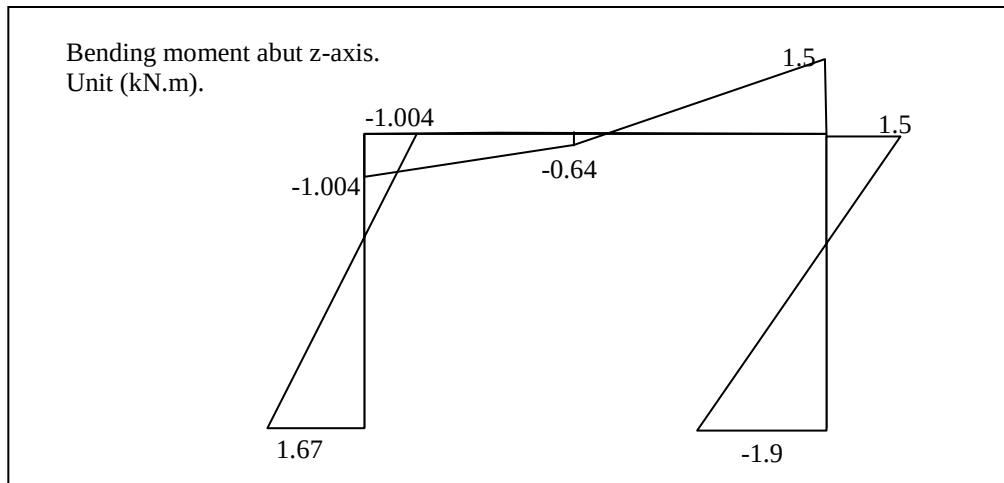


Fig.(6.29): Bending Moment Diagram (about z-axis) for Frame (F3) Using New Analysis Procedure with (24 Elements) at The First Half Cycle.

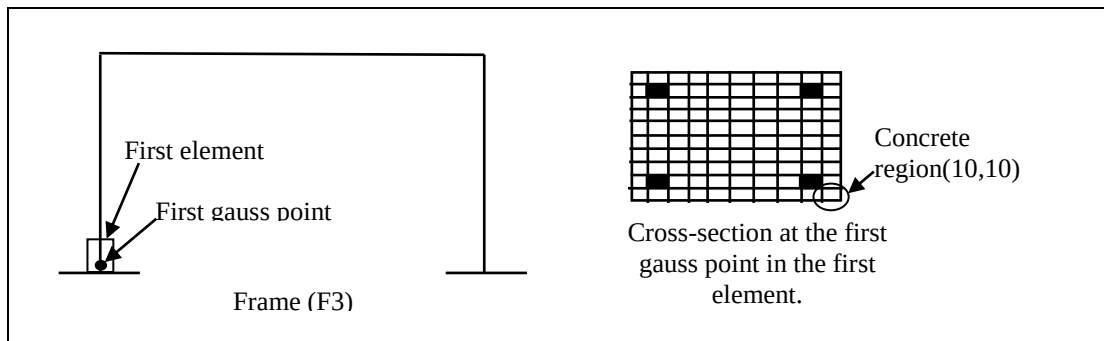


Fig.(6.30): Position of The Concrete Region (10,10) at The First Gauss Point in The first Element in Frame F3.

point in the first element in frame F3, the stress-strain curve for this region is shown in figure (6.31).

Example No.4:

In 2003, a reinforced concrete space frame was tested by **Al – Janabi** ^[35]. This frame was analyzed theoretically by the same investigator ^[35] using a step – by – step incremental with the iteration approach and taking into account the material nonlinearity, geometric nonlinearity, transverse shear deformations and moment – axial force interaction. This frame is reanalyzed using the proposed approach. The details of this frame are shown in figure (6.32). The test loading conditions for frame (F4) are shown in the same

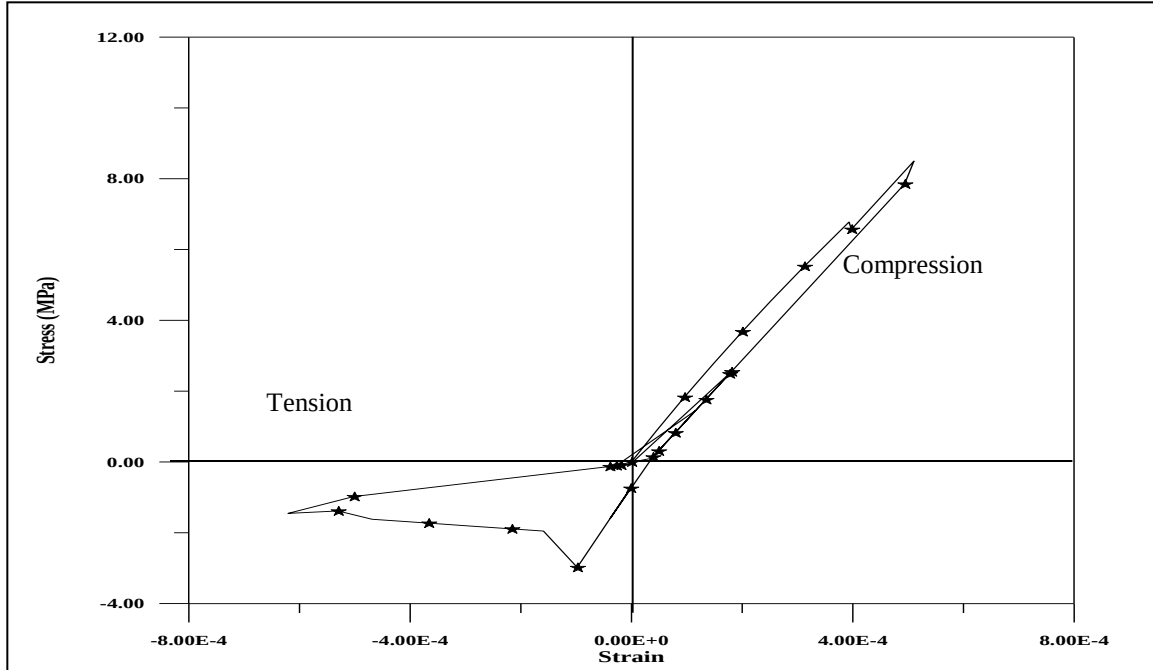


Fig.(6.31): Stress-Strain Curve for Concrete Region (10,10) at The First Gauss Point in The first Element in Frame F3 at First Half Load Cycle.

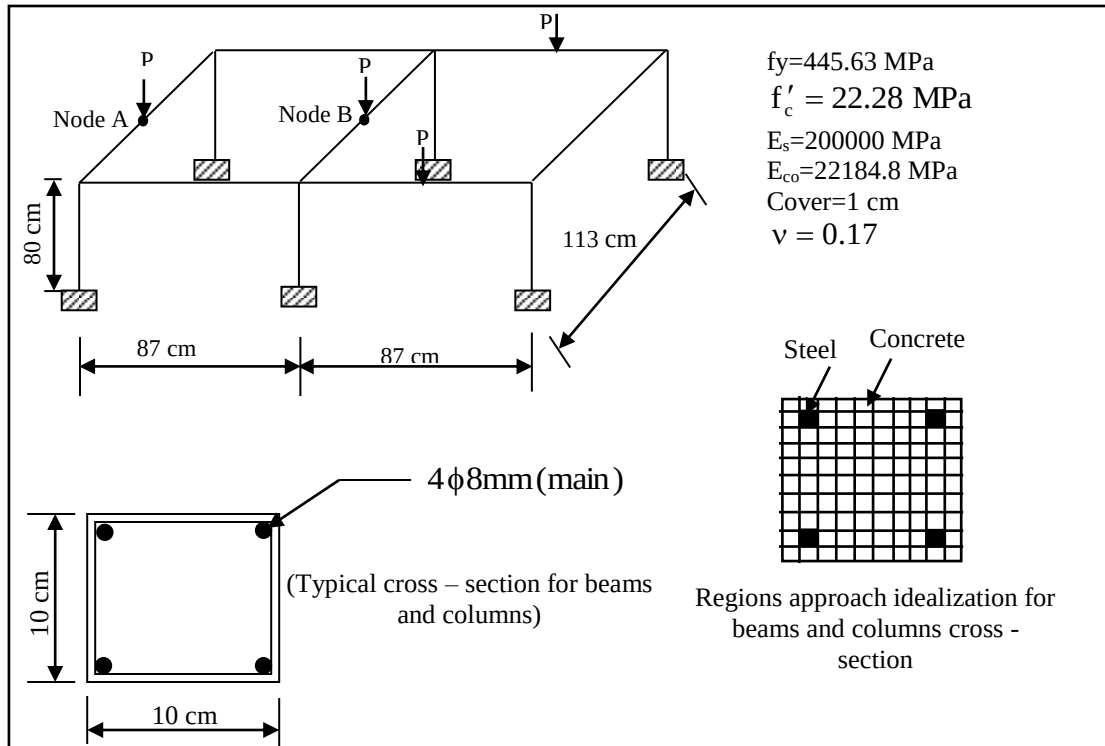


Fig. (6.32): Details of The Frame (F4).

Table (6.7): Number of Element in Each Beam and Column of Frame (F4).

No. of elements in beam	No. of elements in column.	Total number of elements in the frame (F4)
2	1	20
2	2	26
2	3	32
4	4	52
4	5	58
6	6	78

figure. This frame is subdivided into various number of elements as illustrated in table (6.7) to find the suitable number of elements which gives a good agreement with experimental data obtained by **Al – Janabi** ^[35]. Each section in this frame is subdivided into (100) concrete regions, ie. (10) regions in depth direction by (10) regions in width direction; and 4 steel regions, ie. (2) regions in depth direction by (2) regions in width directions. Experimental and theoretical load – deflection curves from **Al – Janabi** ^[35] and the theoretical load – deflection curve from the present study are shown in figure (6.33) to (6.36) at the node (A) and (B). This frame is loaded up to crushing failure. Table (6.8) gives the value of load and displacement at node A and B at crushing failure as obtained by **Al – Janabi** ^[35] and those obtained by the new proposed procedure using various number of elements. Table (6.8) also gives the value of error between the results obtained by the proposed procedure and **Al – Janabi** ^[35] experimental results. Figure (6.37) shows the curve that represent the relation between the number of elements and the percentage error with respect to experimental crushing load value.

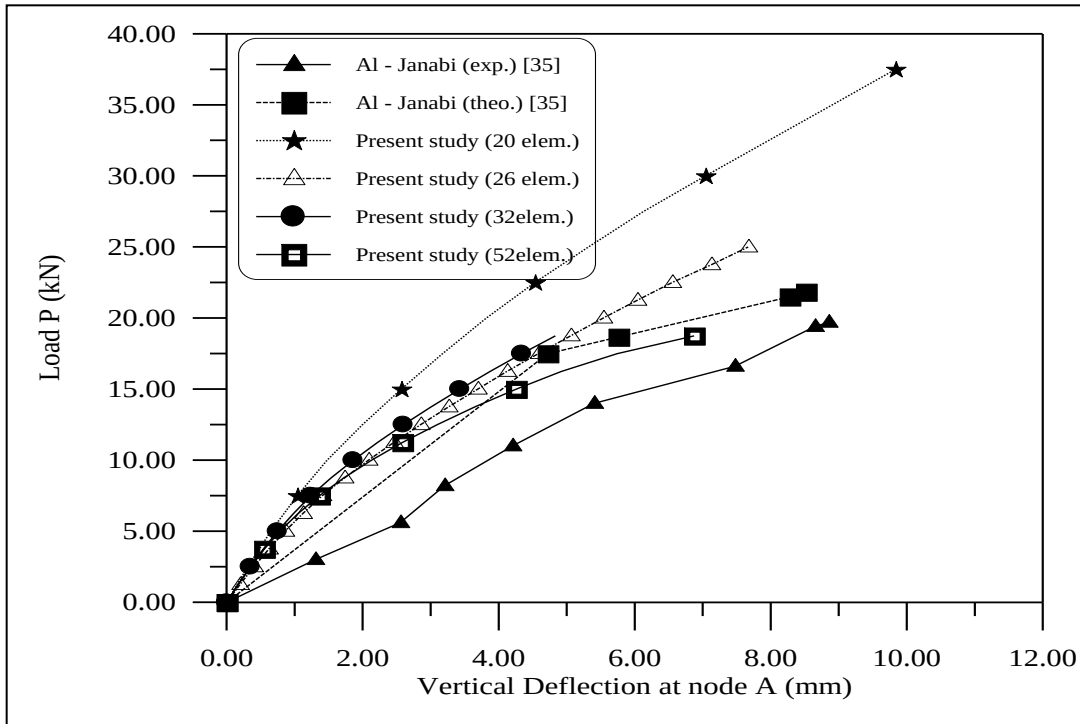


Fig. (6.33): Load – Displacement Curve for Frame (F4) at Node A as Obtained by Al – Janabi ^[35] and The Present Analysis Using (20 Elements) to (52 Elements).

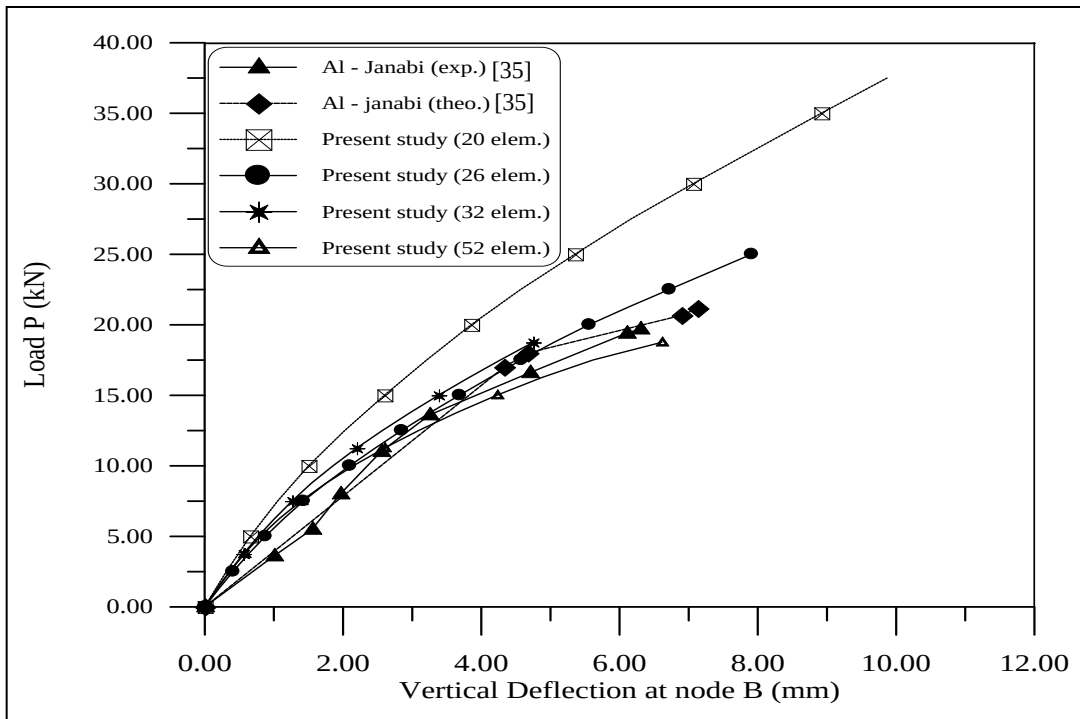


Fig. (6.34): Load – Displacement Curve for Frame (F4) at Node B as Obtained by Al – Janabi ^[35] and The Present Analysis Using (20 Elements) to (52 Elements).

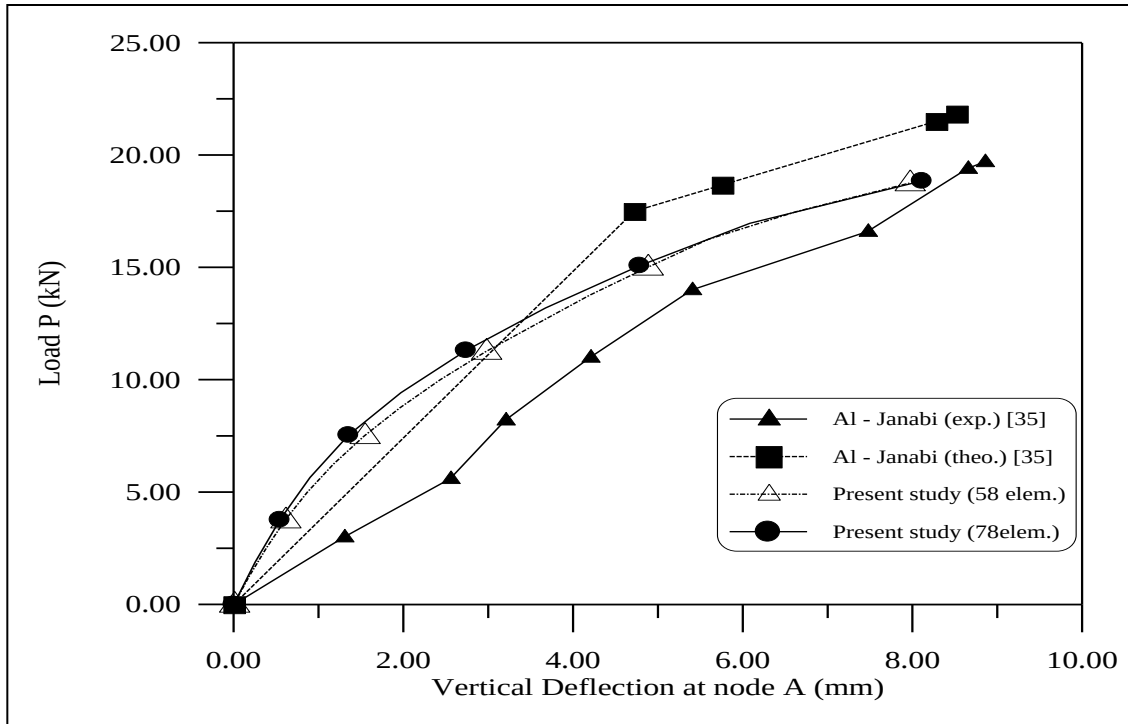


Fig. (6.35): Load – Displacement Curve for Frame (F4) at Node A Obtained by Al – Janabi ^[35] and The Present Analysis Using (58 Elements and 78 Elements) .

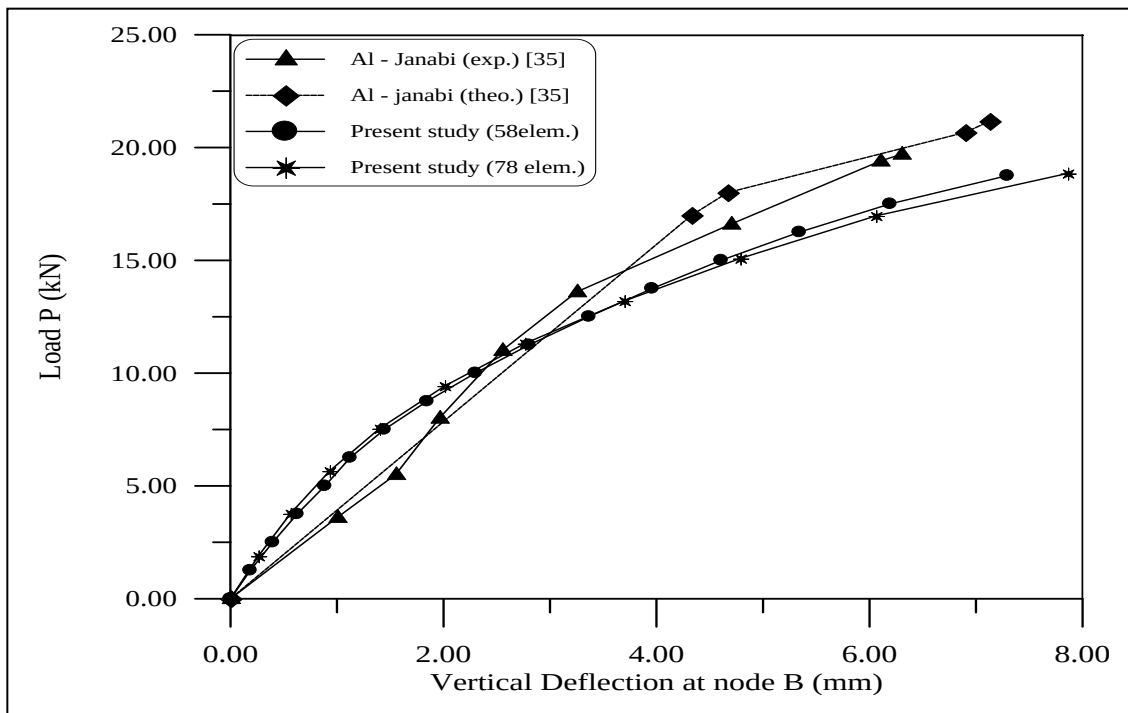
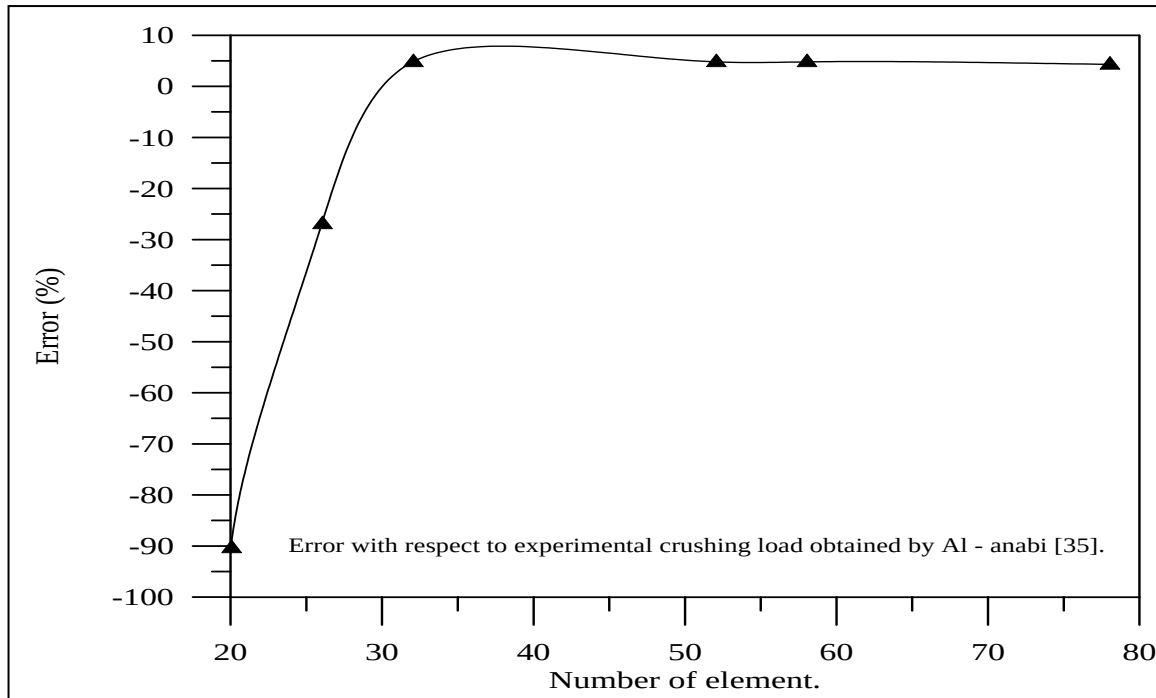


Fig. (6.36): Load – Displacement Curve for Frame (F4) at Node B as Obtained by Al – Janabi ^[35] and The Present Analysis Using(58 Elements and 78 Elements).

Table (6.8): Load, Displacement at Node A, Displacement at Node B, and The Percentage Error with Respect to Experimental Data for Frame (F4).

$$\left[\text{Error} = \frac{\text{Experimental result} - \text{Present result}}{\text{Experimental result}} \times 100 \right]$$

No. of element s.	Crushing load.	Error in load with respect to exp. crushing load.(%)	Displacement at node A. (mm)	Displacement at node B. (mm)
20	37.5	-90.35	9.35	9.87
26	25	-26.9	7.668	7.913
32	18.75	4.822	4.832	4.748
52	1875	4.822	6.874	6.61
58	18.75	4.822	7.965	7.295
78	18.85	4.314	8.113	7.862

**Fig. (6.37): Effect of Elements Number on The Percentage Error Value with Respect to Experimental Crushing Load (as Obtained by Al - Janabi ^[35]) for Frame (F4).**

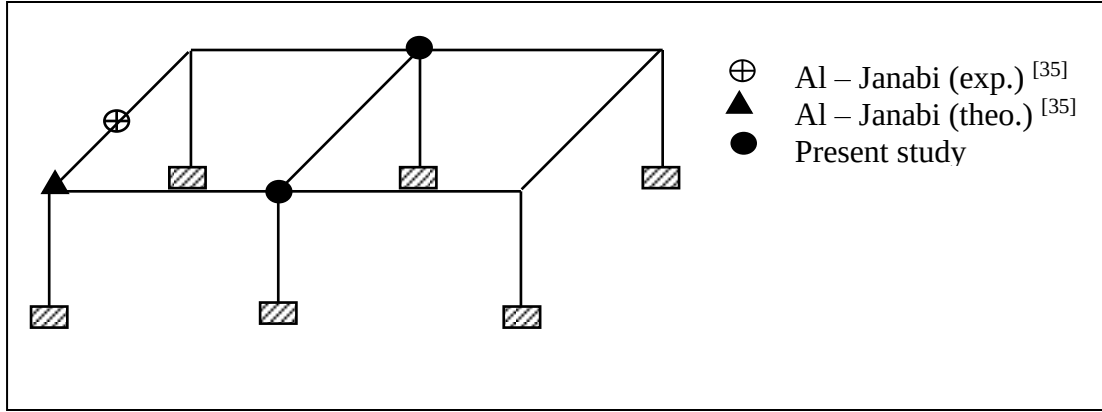


Fig. (6.38): Location of The Crushing Failure in Frame (F4).

From figures (6.33) to (6.37) and table (6.8) the analysis using (78 elements) give good agreement with experimental data. The ultimate load (P) which was obtained by **Al – Janabi** ^[35] was equal to (21.83 kN) from theoretical analysis and (19.7 kN) measured experimentally. The ultimate load obtained from the new proposed analytical procedure (using 78 elements) is (18.85 kN) (ie. The difference between the ultimate load obtained from the present study and the experiment ultimate load obtained by **Al – Janabi** ^[35] is (4 %) while the difference between the theoretical and experimental ultimate load obtained by **Al – Janabi** is (10.812 %)). The location of the crushing failure is shown in figure (6.38). Figure (6.39) shows the distribution of the elastic, plastic, and cracked regions within some sections in frame (F4) and illustrated in the same figure. Figure (6.40) show the bending moment diagram (about z-axis) obtained from new analysis procedure using (78 elements) at crushing failure load. Figure (6.41) show the position of concrete region (10,10) at the fifth gauss point in the element number 48 in frame F4, the stress-strain curve for this region is shown in figure (6.42).

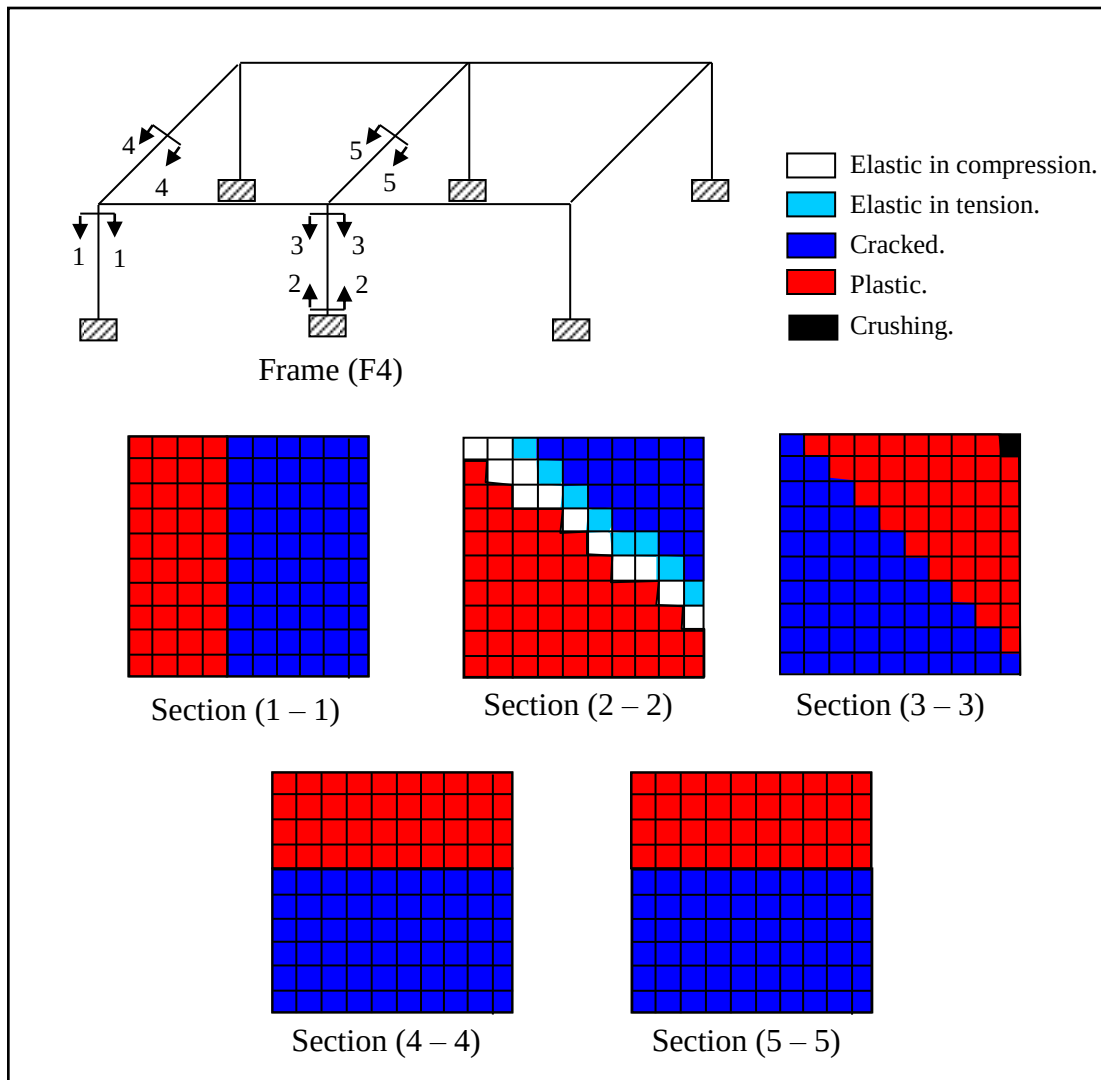


Fig. (6.39): Distribution of The Elastic, Plastic and Cracked Regions within Some Sections in The Frame (F4) at Crushing Failure State.

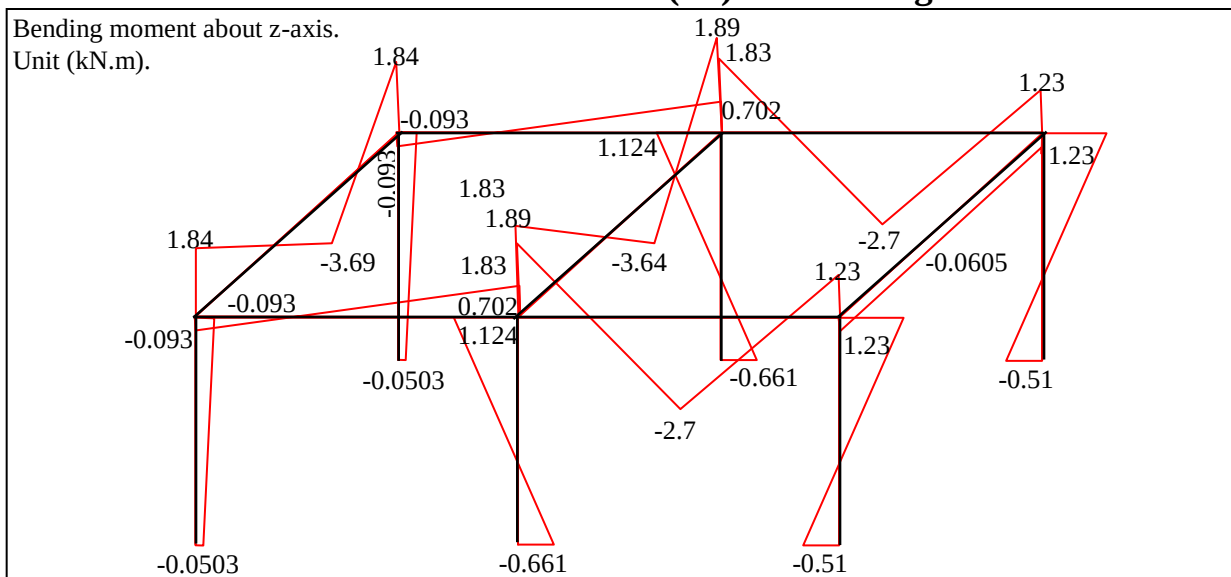


Fig.(6.40): Bending Moment Diagram (about z-axis) for Frame (F4) Using New Analysis Procedure with (78 Elements) at The Crushing Failure Load.

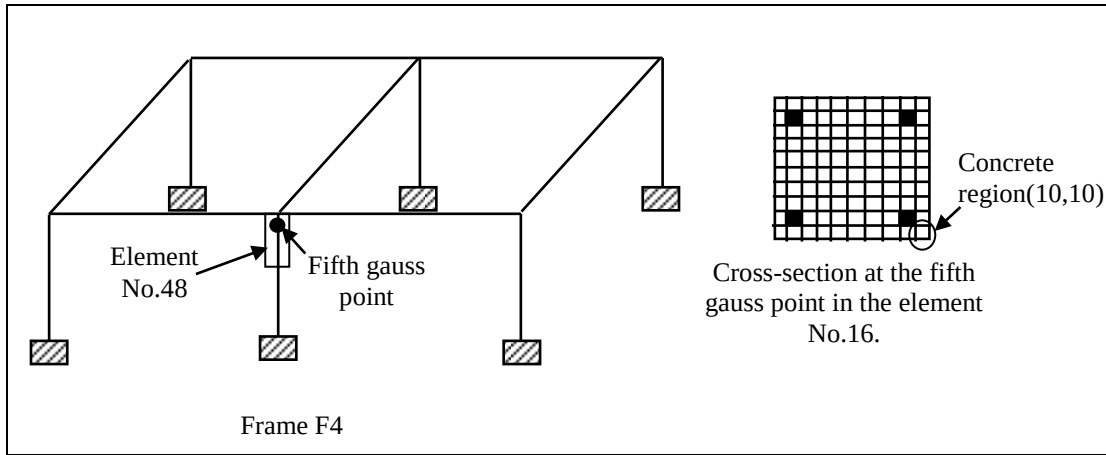


Fig. (6.41): Position of The Concrete Region (10,10) at The Fifth Gauss Point in The Element Number 48 in Frame F4.

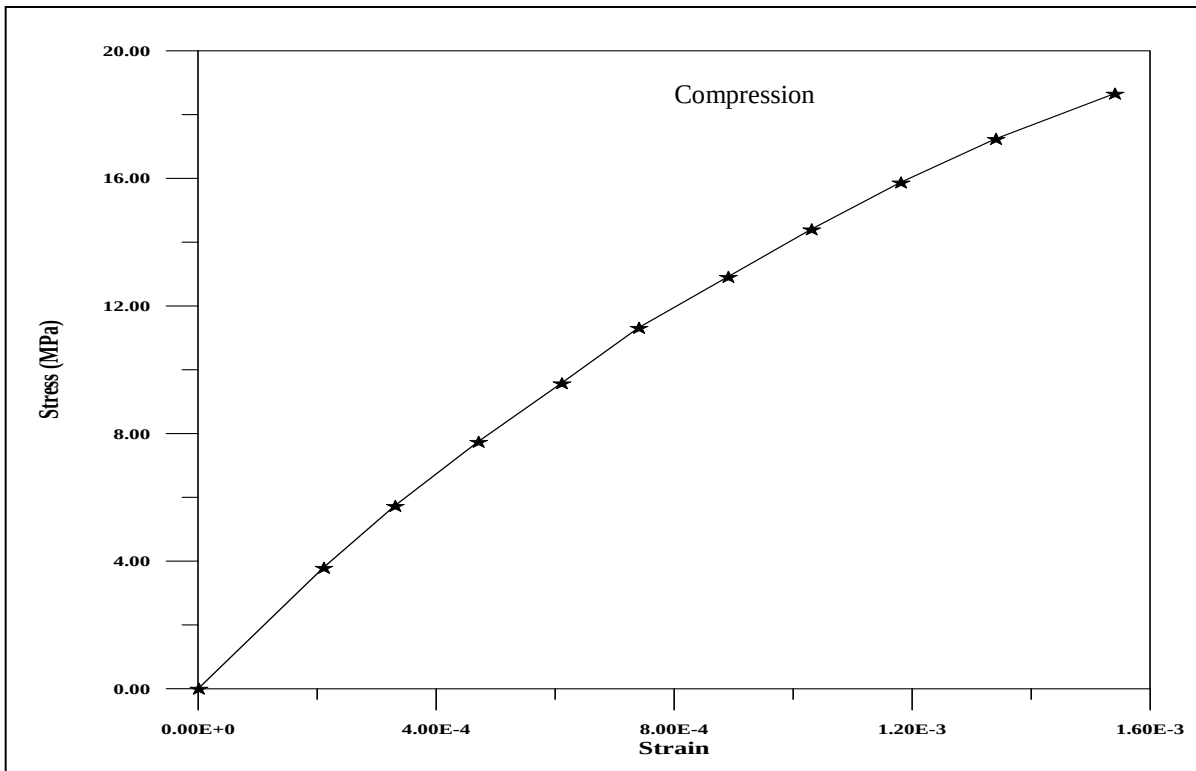


Fig.(6.42): Stress-Strain Curve for Concrete Region (10,10) at The Fifth Gauss Point in The Element Number 48 in Frame F4 at Crushing Failure Load.

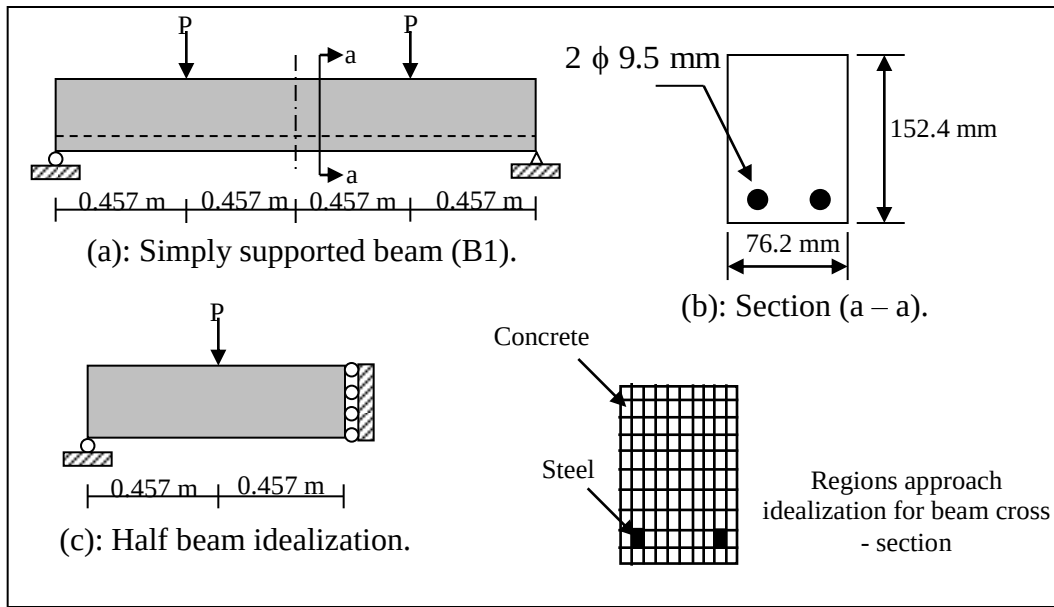


Fig. (6.43): Details of Beam (B1).

Example No.5:

The beam considered in this study was tested by **Corely** and **Sozen**, in 1966 (as found in reference [68]) and was analyzed theoretically by **Al – Naimi (1996)**^[68]. A three dimensional isoparametric element was used by **Al – Naimi**^[68] to analyze this beam. The geometric and the loading conditions of beam (B1) are shown in figure (6.43). Due to symmetry of geometry and loading, half of beam (B1) was used by **Al – Naimi**^[68] in theoretical study (see figure (6.43c)). This half beam was subdivided by **Al – Naimi**^[68] into ten elements (108 degrees of freedom). In the present study, this half beam will be analyzed by using the total number of elements vary from (2) elements to (16) elements. The section anywhere in this half beam is subdivided into (10) concrete regions in width direction by (10) concrete regions in depth direction and one steel region in depth direction by two steel regions in width direction. The beam was loaded at the quarter points of the span with two loads, the value of each is (5 kN), and kept in controlled atmosphere at (21 °C) and (50 %) relative humidity. Date of loading ranged from (7 days) to (28 days) after the concrete was placed. A

total of twenty four time steps are used to study the long – term behavior. The load is applied at five increments, which are at time (7, 10, 14, 21 and 28 days), the load factors are (10%, 10%, 20%, 30%, 30%) of the total load, respectively. The beam was designed under effects of creep and shrinkage up to (200) days . The necessary data to analysis beam (B1) as mentioned by **Al – Naimi** ^[68] is:

$E_{co}(28)=22500$ MPa, $\nu=0.15$, $f'_c(28)=21$ MPa , $f'_t(28)=3.2$ MPa , $\epsilon_{cu} = 0.0039$,
 $E_s= 200000$ MPa, $f_y = 414$ MPa, $W=23.5$ kN/m³, $t_0= 7$ days, $f = 35$, $H= 50\%$,
 $TH= 76.2$ mm, $S= 60$ mm, $B = 400$ Kg/m³, $F_f = 60\%$, $A_{ir} = 4\%$.

Where $E_{co}(28)$ is (28)day initial tangent flexural modulus of elasticity of concrete, ν is concrete Poisson's ratio, $f'_c(28)$ is (28)day concrete cylinder strength, $f'_t(28)$ is 28 day concrete direct tensile strength , ϵ_{cu} is concrete ultimate strain, E_s is tangent flexural modulus of elasticity of steel, f_y is yield strength of steel, W is the concrete unit weight , t_0 is curing time, f is a constant for the moist cured concrete, H is the relative humidity, TH is the thickness of beam, S is the slump, B is the value depending on the cement content, F_f is the percentage of the fines, and A_{ir} is the air content.

The vertical deflection in the center of the beam at the midspan is drawn in at different concrete ages in figure (6.44) for various total number of elements, also the experimental deflection – time curve by **Corely** and **Sozen** (as found in reference [68]) and the theoretical time – deflection curve by **Al – Naimi** ^[68] are shown in the same figure. Figure (6.45) shows the curve which represents the relationship between the total number of elements and the difference between the analytical results obtained from present study and the experimental results by **Corely** and **Sozen** (as found in reference [68]) at time (200) days.

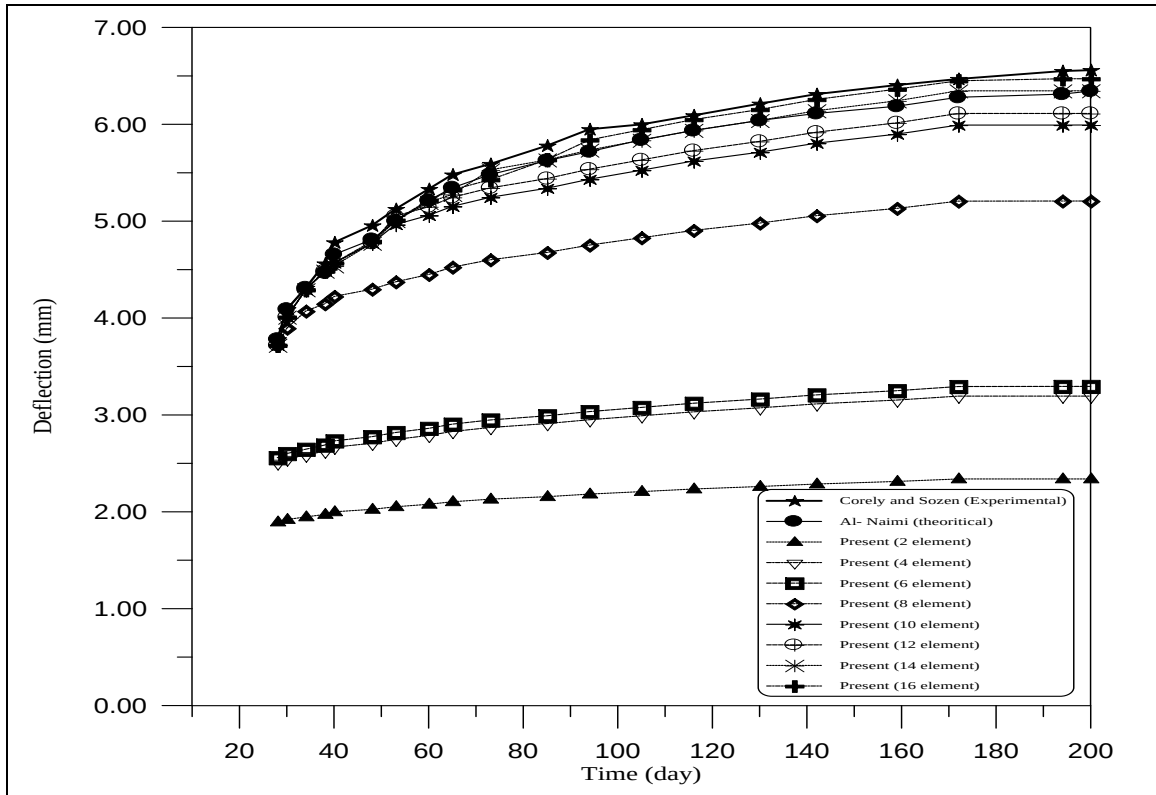


Fig. (6.44): Deflection – Time Curves for Beam (B1) as Obtained by Corely and Sozen, Al – Naimi and The present Analysis Using (2 Elements) to (16 Elements).

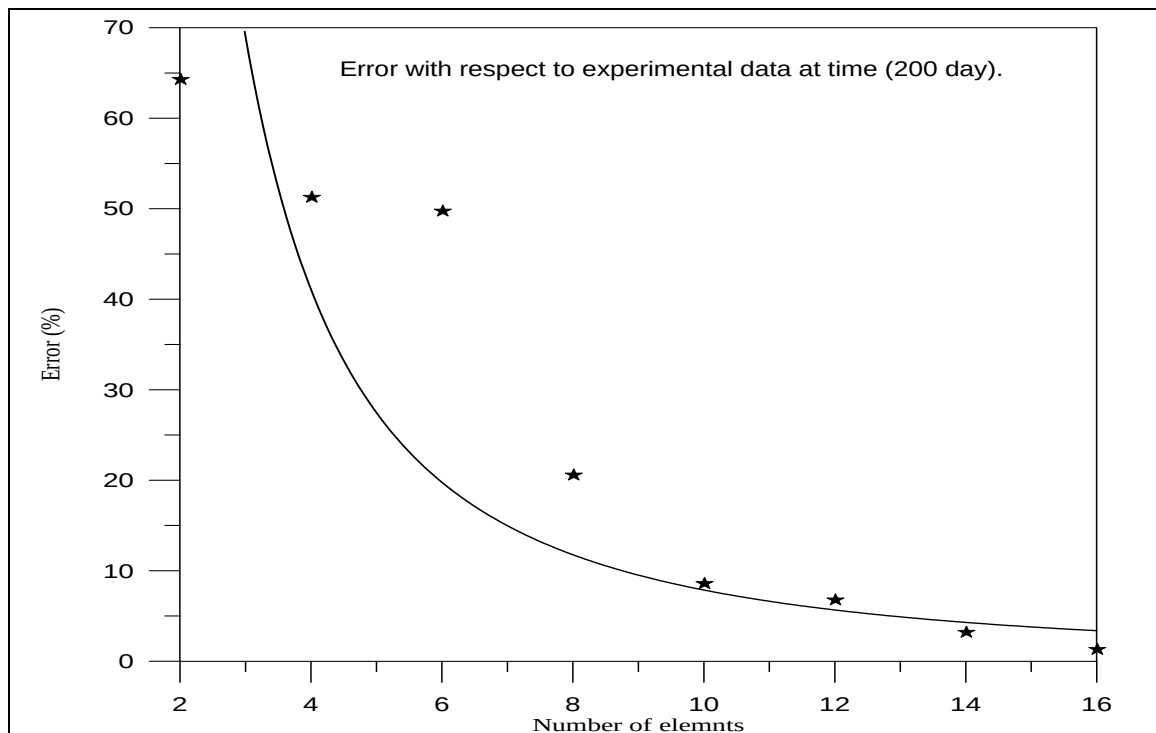


Fig. (6.45): Effect of Element Number on The Percentage Error Value with Respect to Experimental Results by Corely and Sozen (as found in reference [68]) at Time (200) Days.

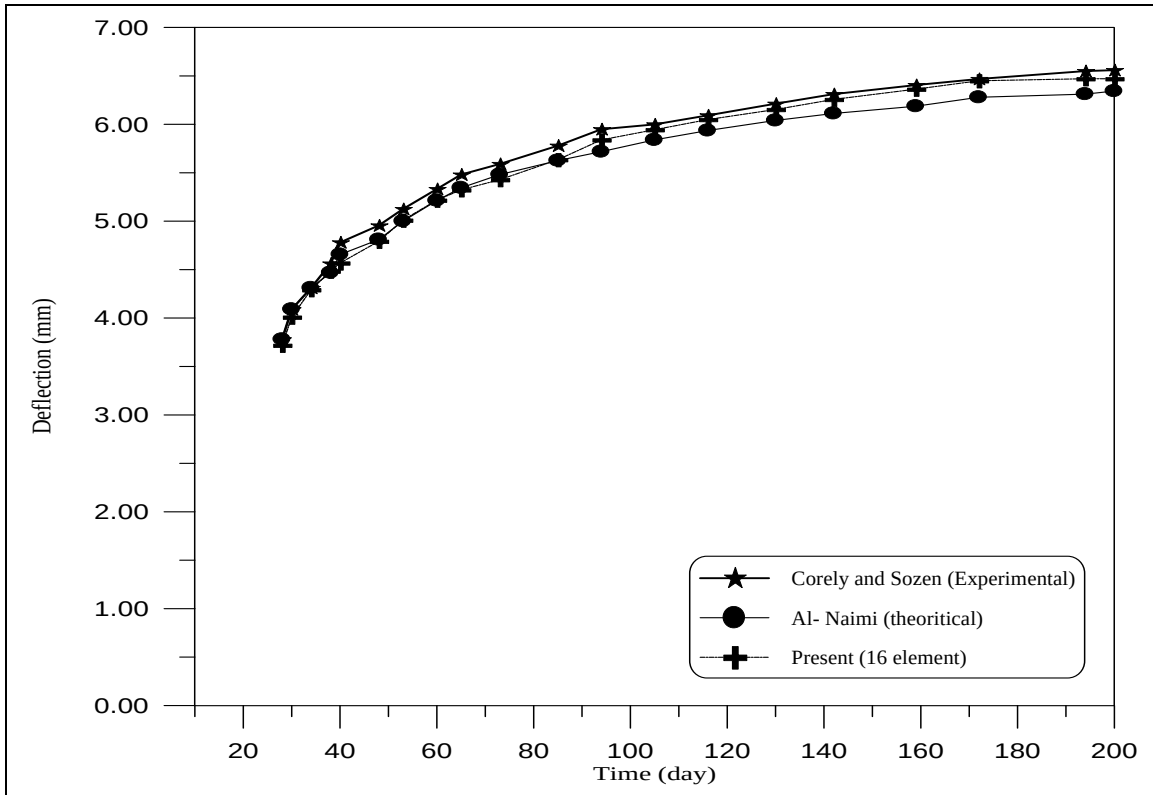


Fig. (6.46): Deflection – Time Curves for Beam (B1) as Obtained by Corely and Sozen, Al – Naimi and The Present Analysis Using (16 elements).

From figures (6.44) and (6.45), the (16) elements are suitable to analysis beam (B1). The computed curve agrees well qualitatively with the prescribed experimental and **Al – Naimi** theoretical curves. In fact, the difference between the experimental curve by **Corely** and **Sozen** and the present curve at most ages is less than (2%). Figure (6.46) shows the experimental time – deflection curve by **Corely** and **Sozen**, the theoretical time deflection curve by **Al – Naimi**, and the present time – deflection curve using 30 elements.

CHAPTER SEVEN (PARAMETRIC STUDY)

7.1 Introduction:

Based on the new analysis technique that is formulated in chapter four and chapter five and by using the developed computer program, three parametric studies are presented in this chapter and they are as follows:

1. Study (1): concerned with studying the effect of bond – slip on the nonlinear analysis of reinforced concrete space frame.
2. Study (2): concerned with finding the optimum tapering ratio for reinforced concrete space frame.
3. Study (3): concerned with studying the effect of creep and shrinkage on the nonlinear analysis of reinforced concrete space frame.

7.2 Effect of Bond – Slip:

In this parametric study the effect of bond – slip on the behavior of space frame under cyclic loading will be studied through drawing the load – deflection curve once with taking into account the effect of slip and the other without taking this effect into account for example (1) and (2) which will be shown below.

Example No.1 [Frame (F4)]:

In this example, the frame (F4) which was tested by **Al – Janabi** ^[35] under monotonic load, will be analyzed now by using the new proposed procedure under cyclic loading to study the bond – slip effect. The dimensions and all the necessary data are shown in figure (6.32). The loading conditions are shown in figure (7.1). This frame is subdivided into (78) elements , ie. Each member is divided into six elements. Also each section is subdivided into (100) concrete regions, ie. (10) regions

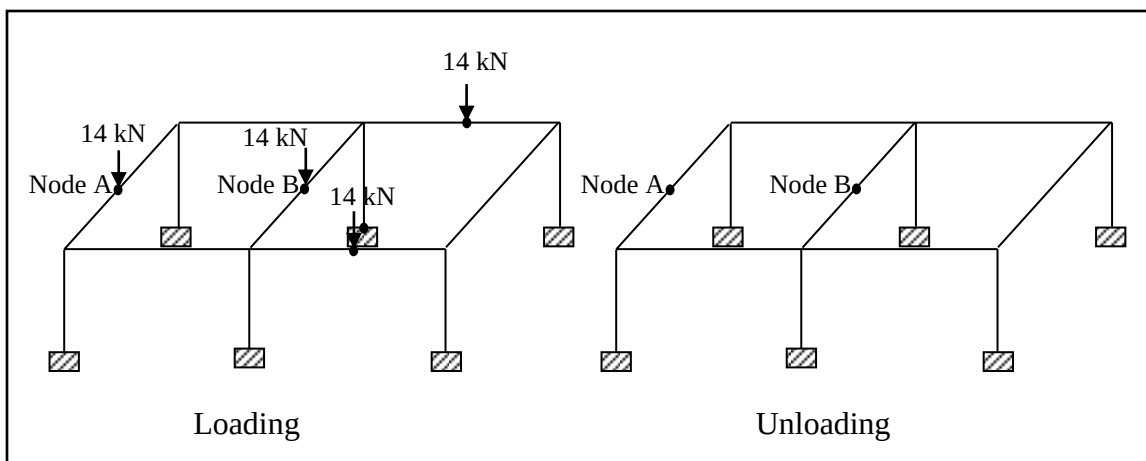


Fig. (7.1): Loading Conditions for Frame (F4).

in depth direction by (10) regions in width direction; and 4 steel regions, ie. (2) regions in depth direction by (2) regions in width directions. Figures (7.2) and (7.3) show the load – deflection curves with and without taking into account the bond – slip effect, at node A and B respectively. These two figures show that there is no difference between the load – deflection curve with bond – slip effect and the load – deflection curve without bond – slip effect up to load approximately equal to (7.84 kN) for node A and (10.08 kN) for node B, after that the difference will be occurred and increased until reaching the load (14 kN) at which the difference is about (8%). As expected, the analysis of this frame taking the bond – slip effect into account gives higher deflection rather than the results of the analysis without taking the bond – slip effect into account, since the stiffness of the frame in the first analysis is less than the frame stiffness in the second one. From the unloading part of the load – deflection curve, the residual deflection obtained from the analysis which takes the bond – slip effect into account is greater than that obtained from the analysis not taking the bond – slip into account (the difference is about 20%), because in the analysis with bond –

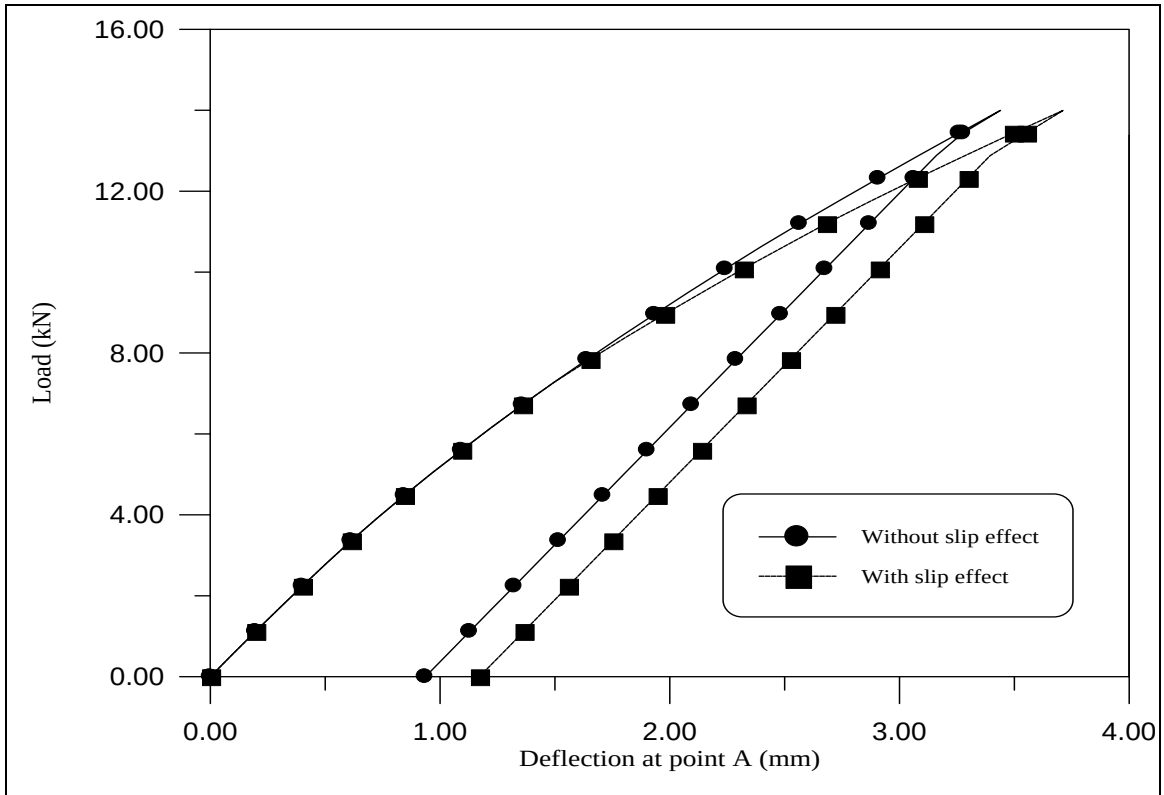


Fig. (7.2): Load – Deflection Curve for Frame (F4) at Node (A).

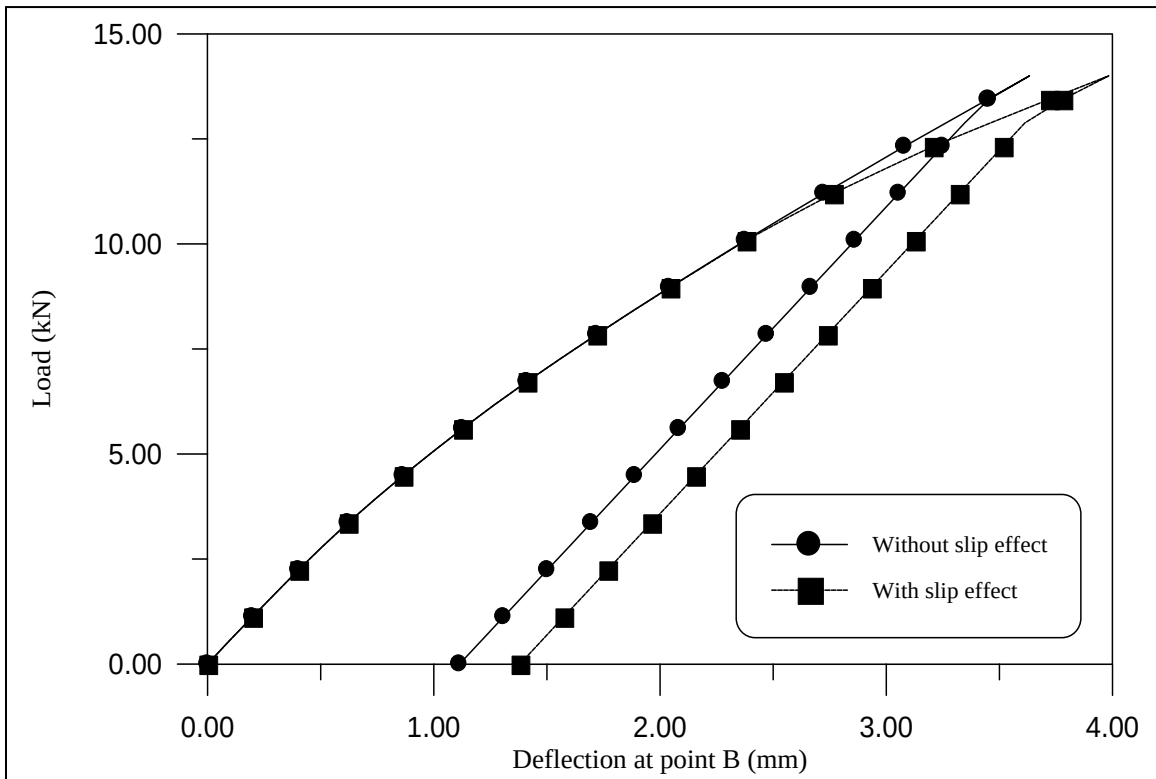


Fig. (7.3): Load – Deflection Curve for Frame (F4) at Node (B).

slip effect, the concrete will get some freedom to occupy some amount of plastic deformation rather than in the analysis with the perfect bond.

Example No.2 [Frame (F5)]:

In this example, the frame (F5) will be analyzed by using the present proposed analytical procedure to study the bond – slip effect on the behavior of reinforced concrete space frame. The details of this frame are shown in figure (7.4). The loading conditions are given in figure (7.5). This frame is subdivided into (24) elements, ie. each member is subdivided into three elements. Any section in each column is subdivided into (10) concrete regions in width direction by (10) concrete regions in depth direction, and two steel regions in width direction by three steel regions in depth direction. Any section in each beam is subdivided into (10) concrete regions in width direction by (10) concrete regions in depth direction, and two steel regions in width direction by two steel regions in depth direction. Figure (7.6) shows the load – displacement curves with and without taking the bond – slip effect into account. Again, as expected, from these curves, the displacement obtained from the analysis which takes the bond – slip effect into account is greater than the displacement obtained from the analysis not taking the bond – slip effect into account with difference reaches to amount (6%) at load equals to (250kN) and (13%) at the end of unloading path (zero load). Again this difference occurs because the stiffness of the frame in the analysis without bond – slip effect is greater than this in the analysis with bond - slip effect.

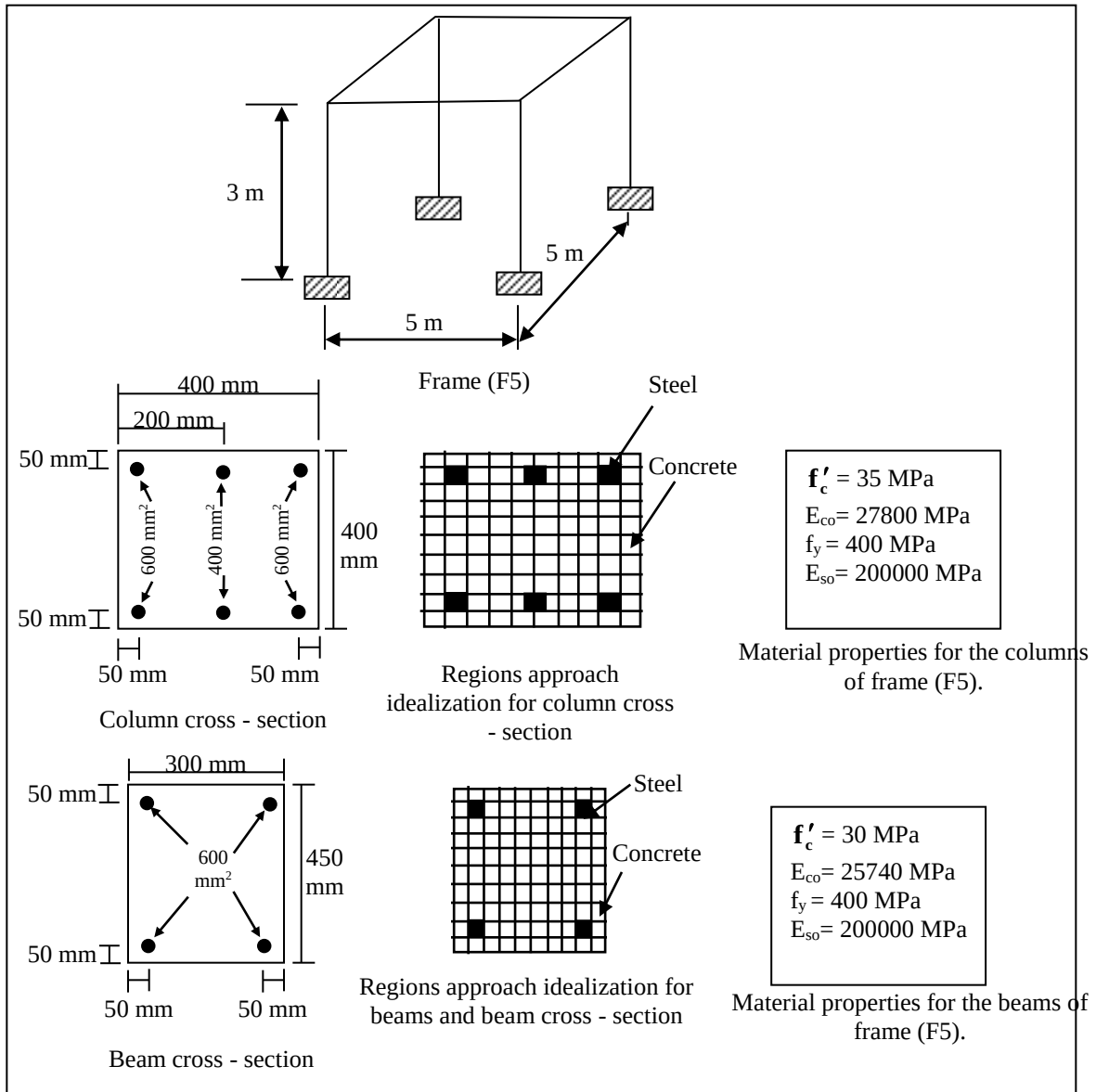


Fig. (7.4): Details of Frame (F5).

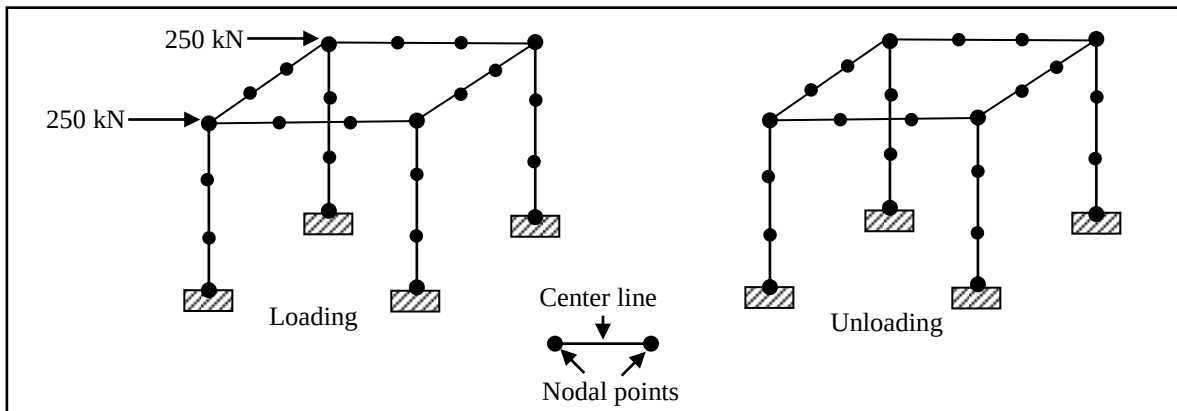


Fig. (7.5): Loading Conditions for Frame (F5).

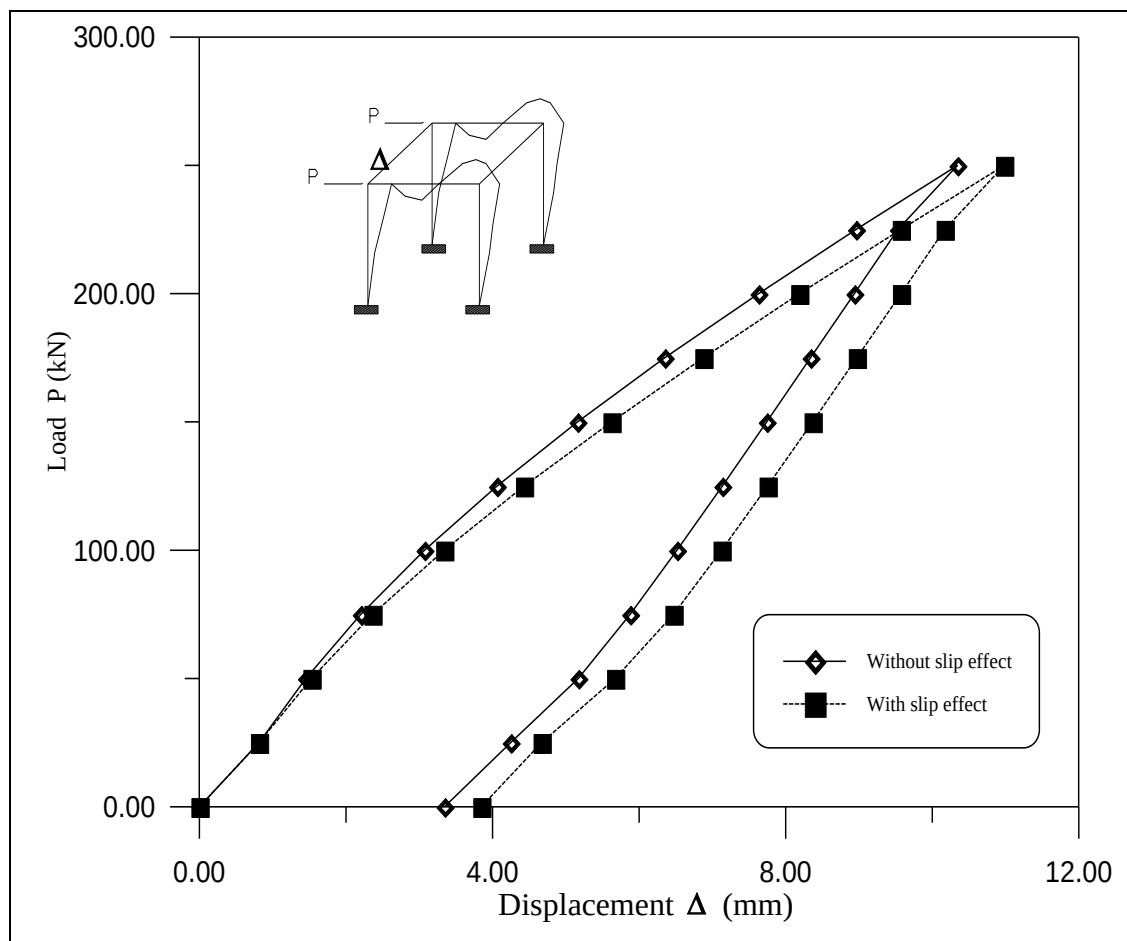


Fig. (7.6): Load – Displacement Curves for Frame (F5).

7.3 Effect of Tapering Ratio:

In order to verify the benefit of tapering in the inelastic range, the effect of tapering in depth direction only, and in width and depth directions of the space frame element on the behavior of reinforced concrete space frame under cyclic loading will be studied through drawing the load – deflection curves which are obtained from analysis of space frame having non – prismatic members for various tapering ratio.

Example No.3 [Tapering in Depth Direction Only]:

The reinforced concrete space frame (F4) will be reanalyzed now by tapering their members (tapering beams in depth direction only) with various tapering ratio for the same amount of steel and the same volume of concrete as shown in figure (7.7). The details and all the necessary data for prismatic

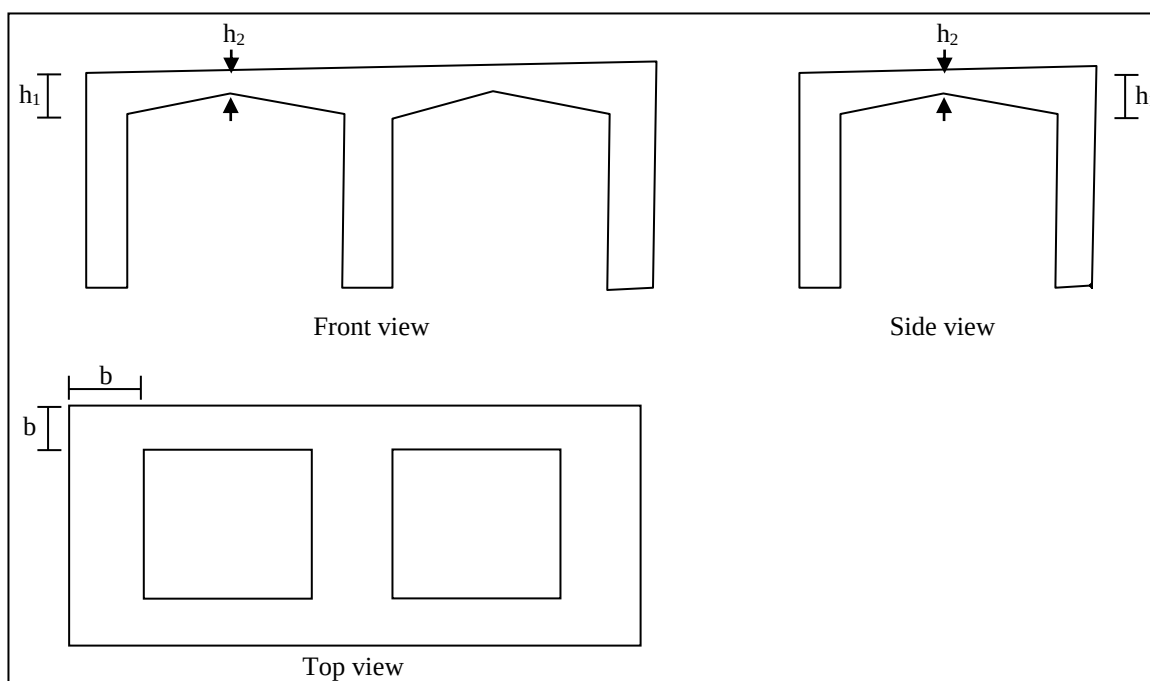


Fig. (7.7): Tapering Shape for Frame (F4) (Tapering in Depth Direction Only).

frame (F4) are shown in figure (6.32). The loading conditions for the frame (F4) with prismatic and with non – prismatic beams are shown in figure (7.1). The frame (F4) with prismatic and with non – prismatic beams is subdivided into (78) elements, ie. each member is subdivided into six elements, and each section in each element is subdivided into (10) concrete regions in width direction by (10) concrete regions in depth direction and two steel regions in width direction by two steel regions in depth direction. Figures (7.8) to (7.13) show the load – deflection curves for frame (F4) with various tapering ratio. From these figures, the prismatic frame (F4) fails in the second load cycle at load equals to (14 kN), while the non – prismatic frame (F4) (for $h_1/h_2 = 0.72$) fails in the first load cycle at load equals to (14 kN). The non – prismatic frame (F4) (for $h_1/h_2 = 1.2$) fails in the fourth load cycle at load equals to (14 kN), The other case of frame (F4) ($h_1/h_2 = 1.375$) fails in the third load cycle at load equals to (14 kN), the non – prismatic frame (F4) ($h_1/h_2 = 1.625$) fails in the third load cycle at load equals to (14 kN), and the non – prismatic frame (F4) ($h_1/h_2 = 2$) fails in the second load cycle at load equals to (14 kN). For

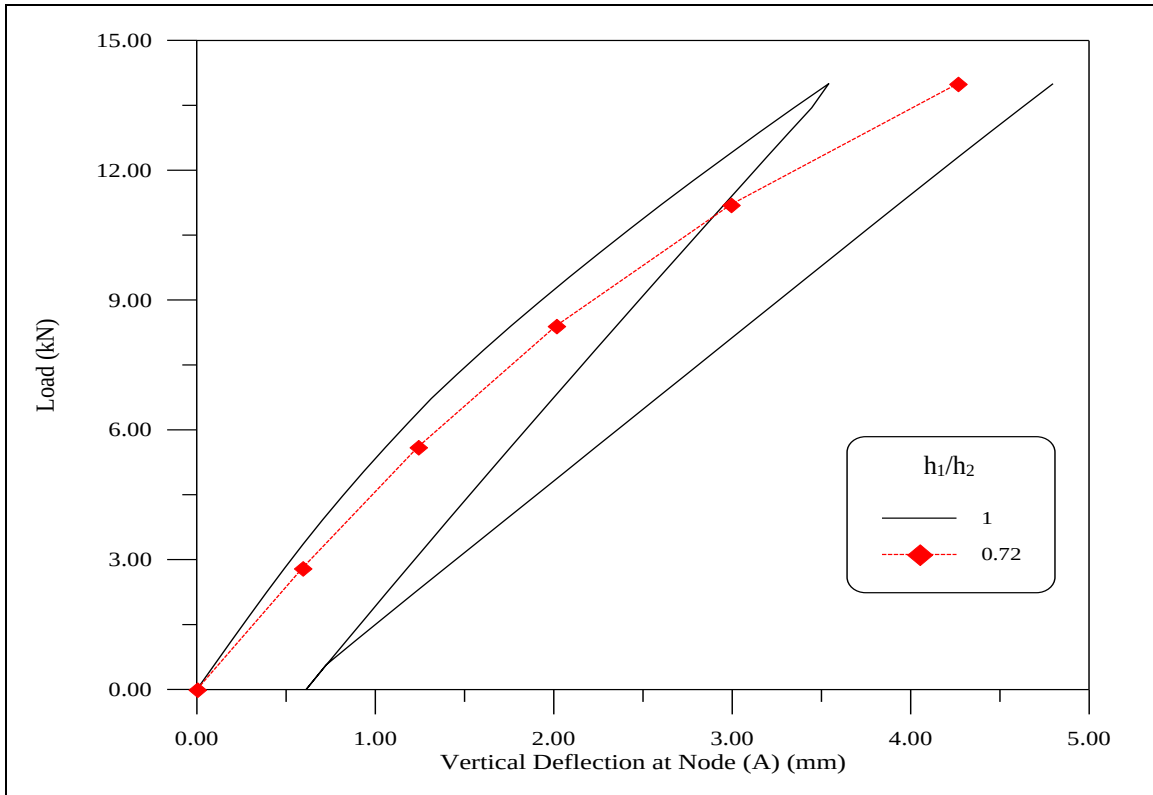


Fig. (7.8): Load – Deflection Curves for Frame (F4) with Tapered Beams ($h_1/h_2 = 0.72$ and 1.0) at Node (A).

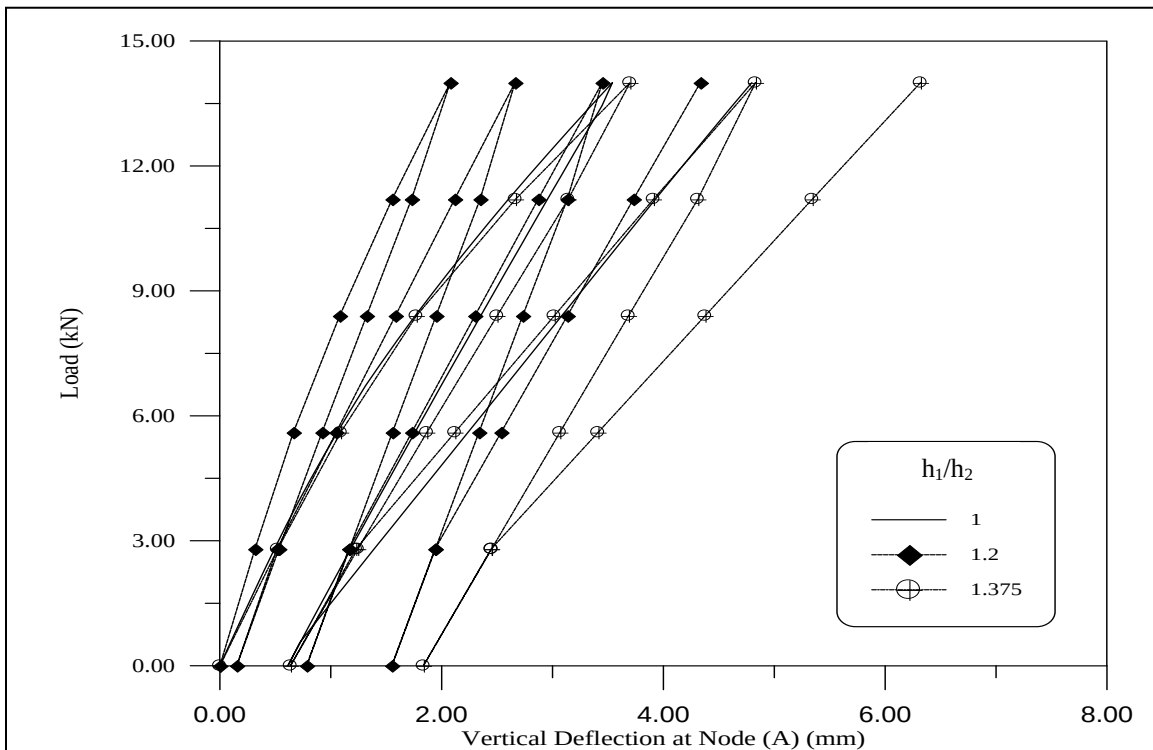


Fig. (7.9): Load – Deflection Curves for Frame (F4) with Tapered Beams ($h_1/h_2 = 1, 1.2, 1.375$) at Node (A).

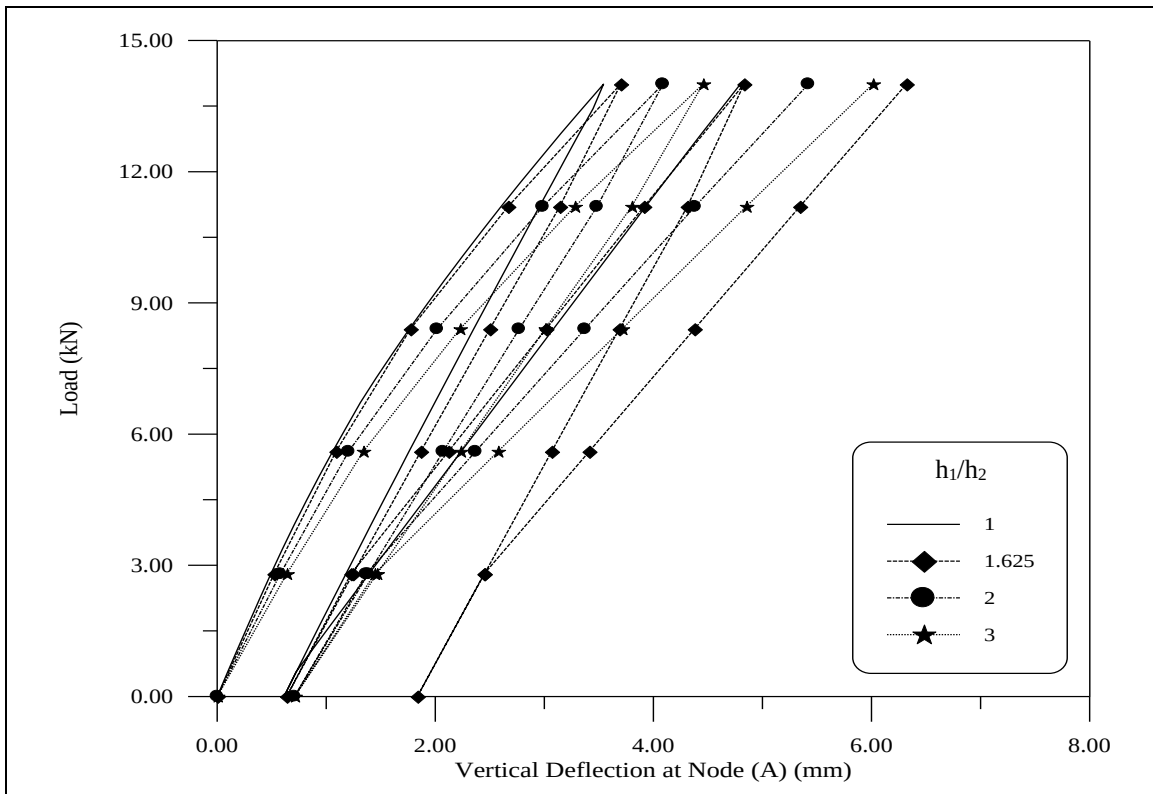


Fig. (7.10): Load – Deflection Curves for Frame (F4) with Tapered Beams ($h_1/h_2 = 1, 1.625, 2, 3$) at Node (A).

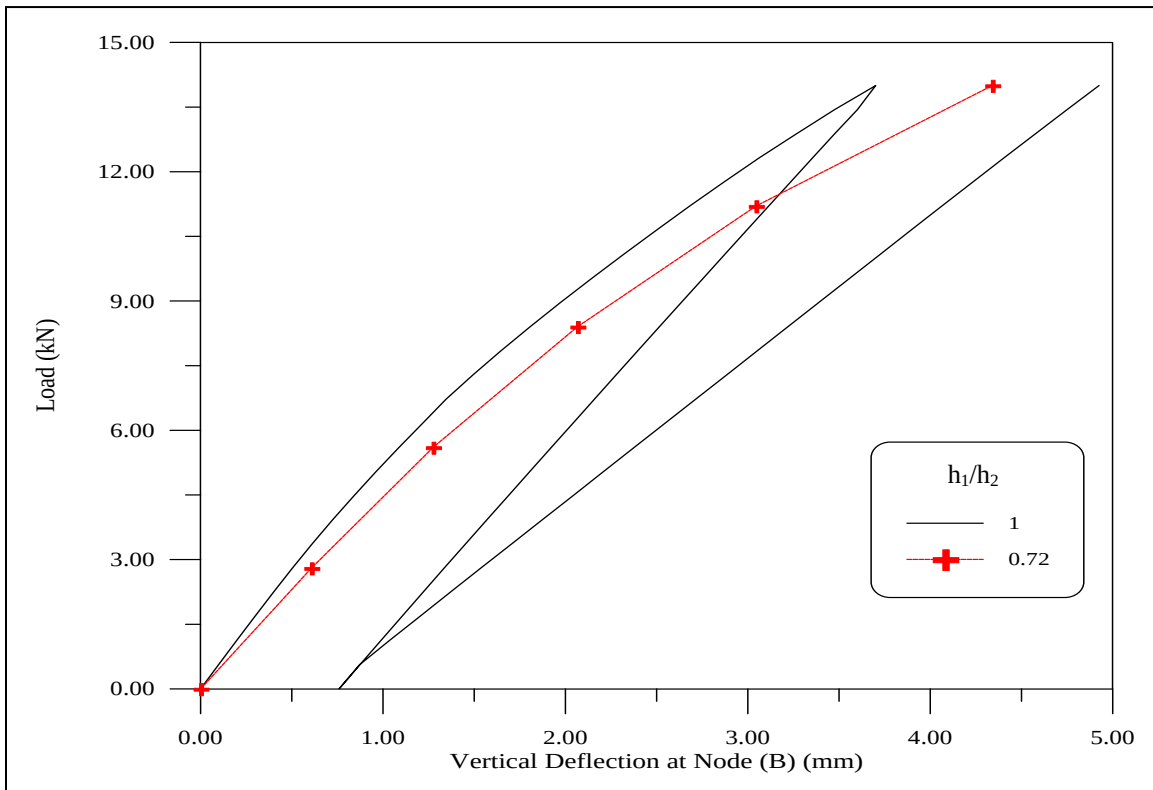


Fig. (7.11): Load – Deflection Curves for Frame (F4) with Tapered Beams ($h_1/h_2 = 0.72$ and 1.0) at Node (B).

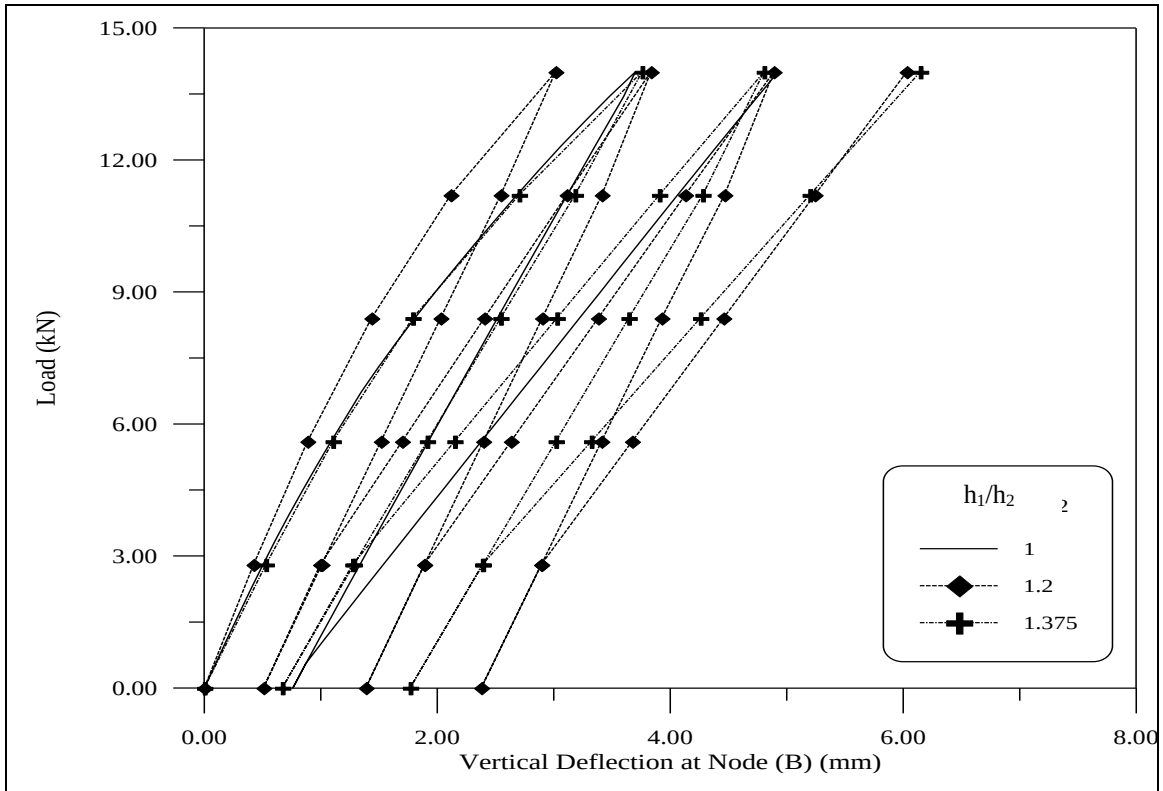


Fig. (7.12): Load – Deflection Curves for Frame (F4) with Tapered Beams ($h_1/h_2 = 1, 1.2, 1.375$) at Node (B).

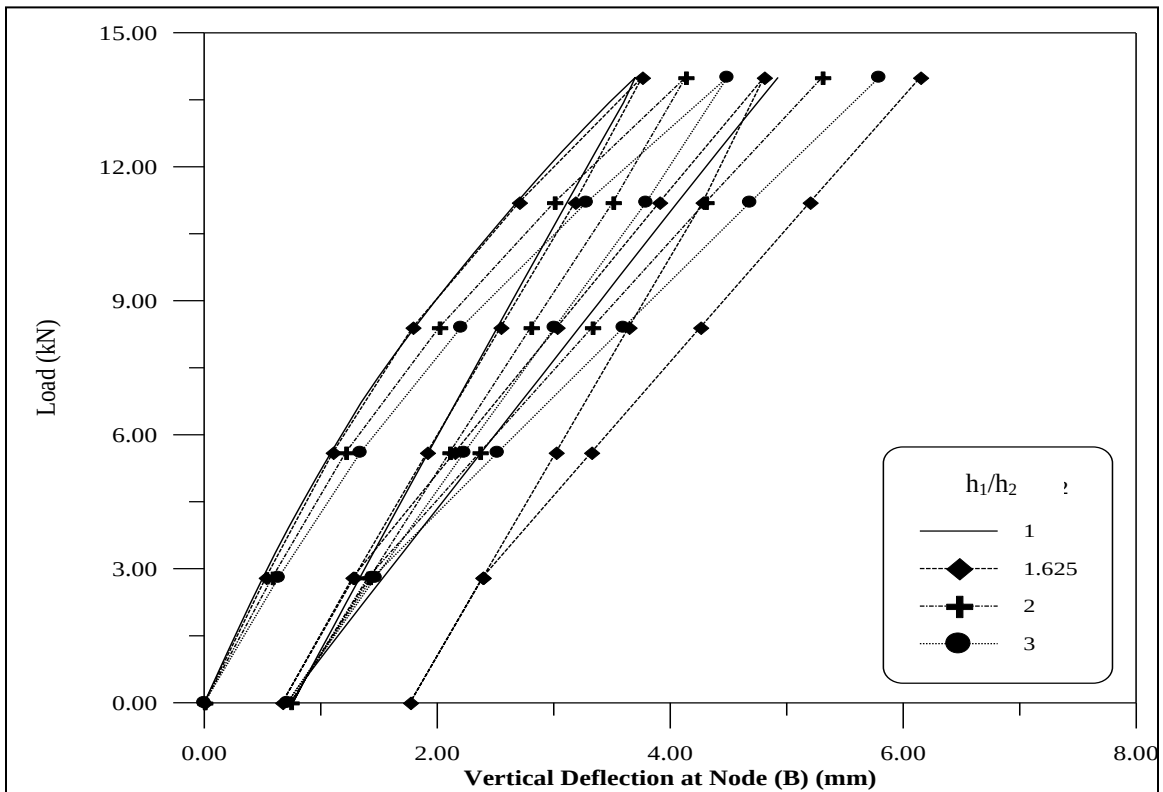


Fig. (7.13): Load – Deflection Curves for Frame (F4) with Tapered Beams ($h_1/h_2 = 1, 1.625, 2, 3$) at Node (B).

the case of non – prismatic space frame (F4) with tapering ratio (h_1/h_2) equals to (3) fails in the second load cycle at load equals to (14 kN). The position of the crushing failure for the frame (F4) with various tapering ratio is same and as shown in figure (7.14). Figure (7.15) shows the curve which represents the relation between the tapering ratio and the number of load cycle at crushing failure. From figure (7.15), the optimum tapering ratio (h_1/h_2) for beams can be assigned which lies within the limit from (1.2) to (1.625).

Example No.4 [Tapering in Two Direction]:

The reinforced concrete space frame (F4) will be reanalyzed now by tapering their members (tapering beams) with various tapering ratio for the same amount of steel and the same volume of concrete as shown in figure (7.16). The details and all the necessary data for prismatic frame (F4) are shown in figure (6.32). Figure (7.1) shows the loading conditions for the frame (F4) with prismatic and with non – prismatic beams. The frame (F4) with prismatic and with non – prismatic beams is subdivided into (78) elements, ie. each member is subdivided into six elements, and each section in each element is subdivided into (10) concrete regions in width direction by (10) concrete regions in depth direction and two steel regions in width direction by two steel regions in depth direction. Figures (7.17) to (7.22) show the load – deflection curves for frame (F4) with various tapering ratio. From these figures, the prismatic frame (F4) fails in the second load cycle at load equals to (14 kN), while the non – prismatic frame (F4) (for $h_1/h_2 = b_1/b_2 = 0.72$) fails in the first load cycle at load equals to (14 kN). The non – prismatic frame (F4) (for $h_1/h_2 = b_1/b_2 = 1.2$) fails in the third load cycle at load equals to (12.88 kN). The non – prismatic frame (F4) (for $h_1/h_2 = b_1/b_2 = 1.375$) fails in the third load cycle at load equals to (12.88 kN), The other case of frame (F4) ($h_1/h_2 = b_1/b_2 = 1.625$) fails in the second load cycle at load equals to (14 kN), the non – prismatic frame (F4)

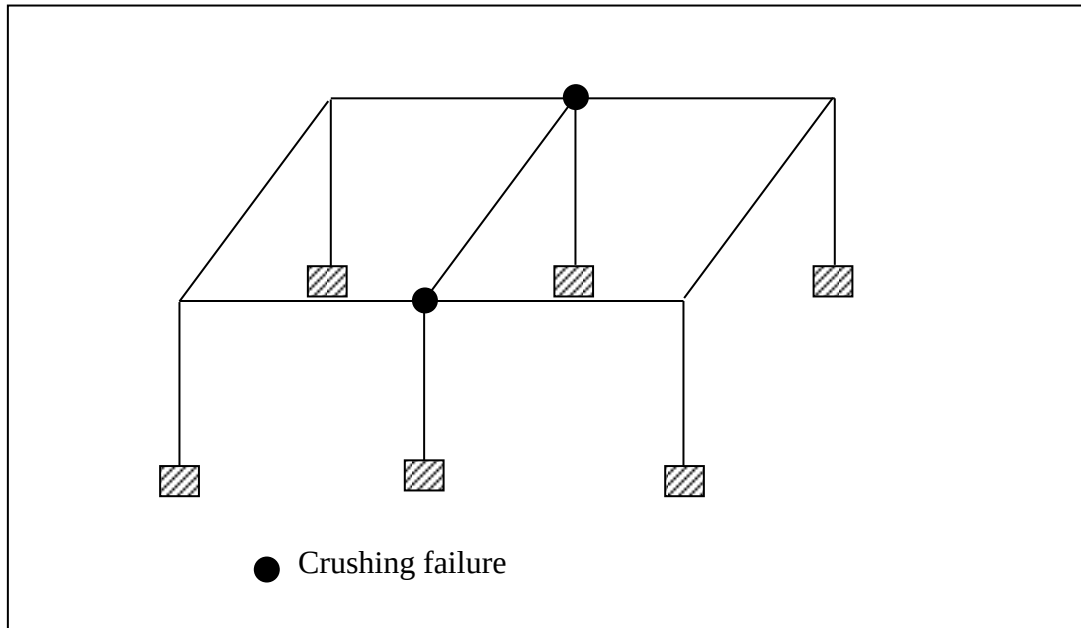


Fig. (7.14): Location of The Crushing Failure in Frame (F4) with Various Tapering Ratio (Tapering Beams).

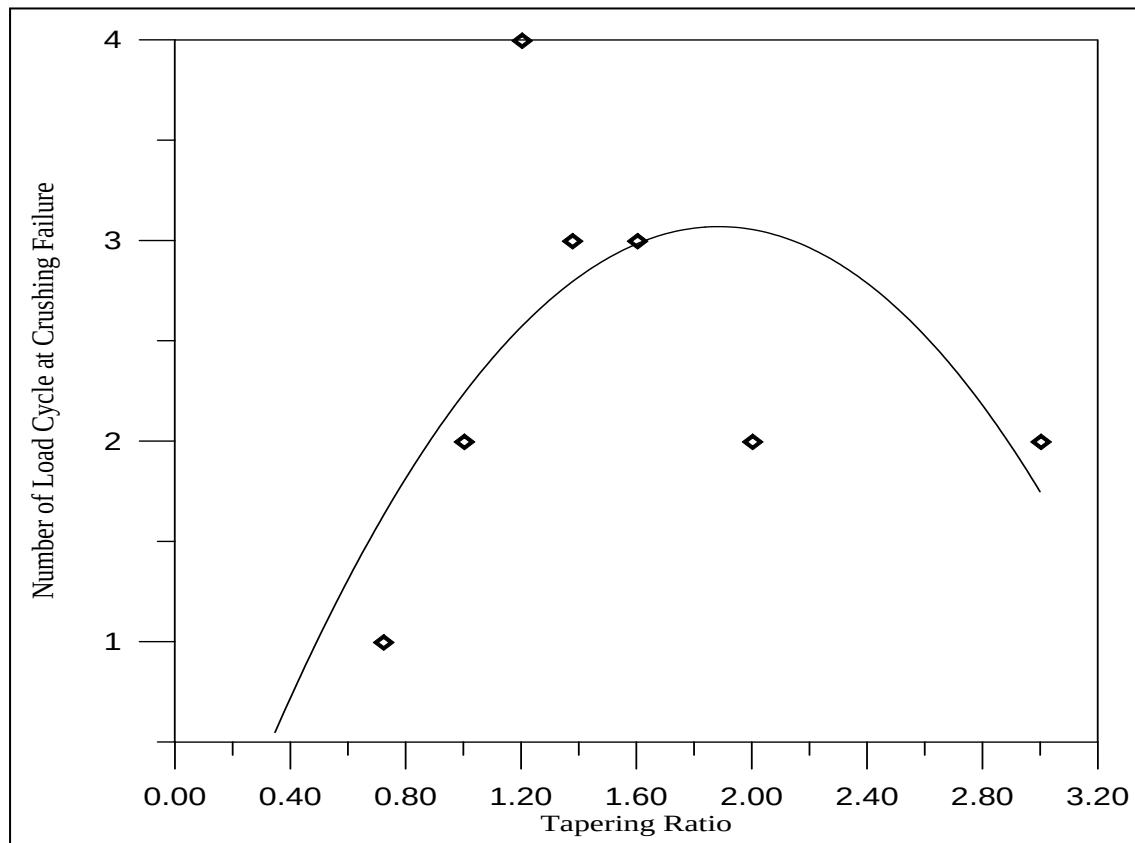


Fig. (7.15): Effect of Tapering of Frame (F4) Beams on Load Cycle Number at Crushing State (Tapering in Depth Direction Only).

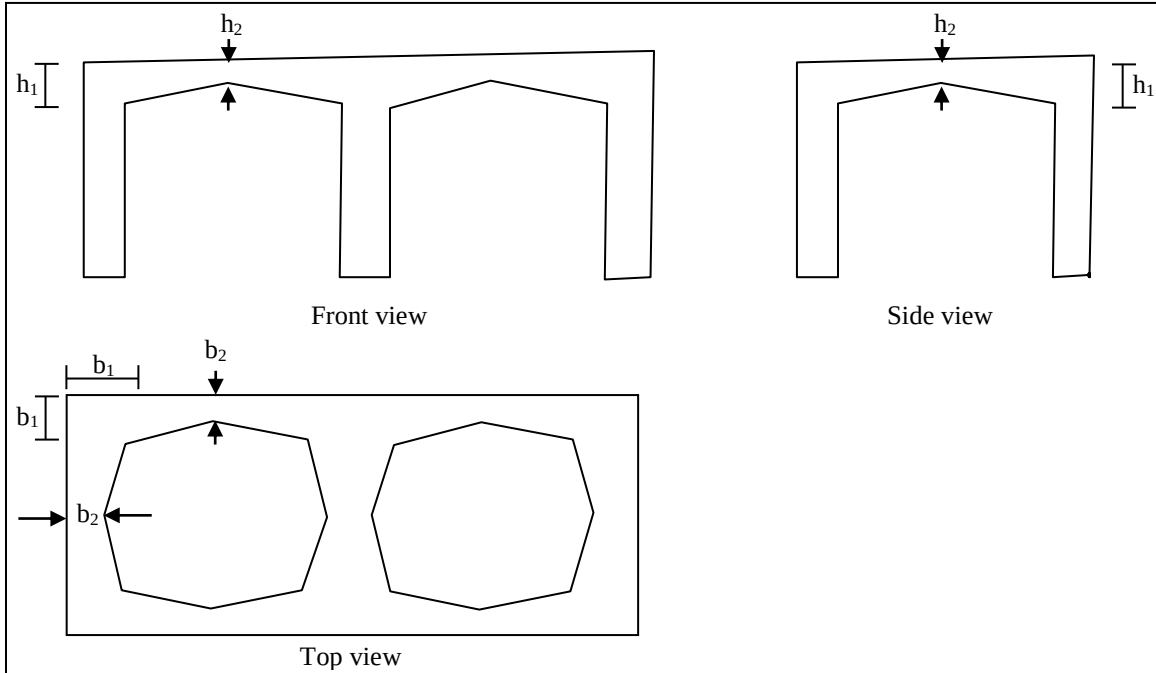


Fig. (7.16): Tapering Shape for Frame (F4) (Tapering in Width and Depth Direction).

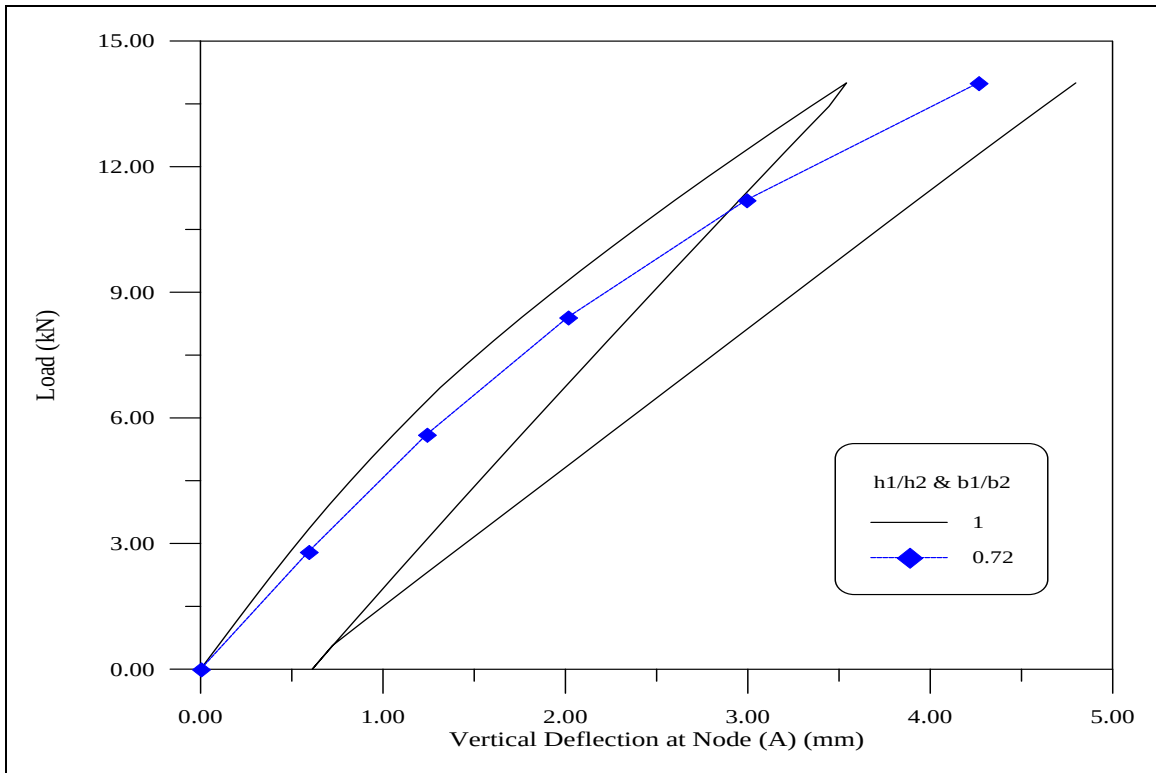


Fig. (7.17): Load – Deflection Curves for Frame (F4) with Tapered Beams ($h_1/h_2 = b_1/b_2 = 0.72$ and 1.0) at Node (A).

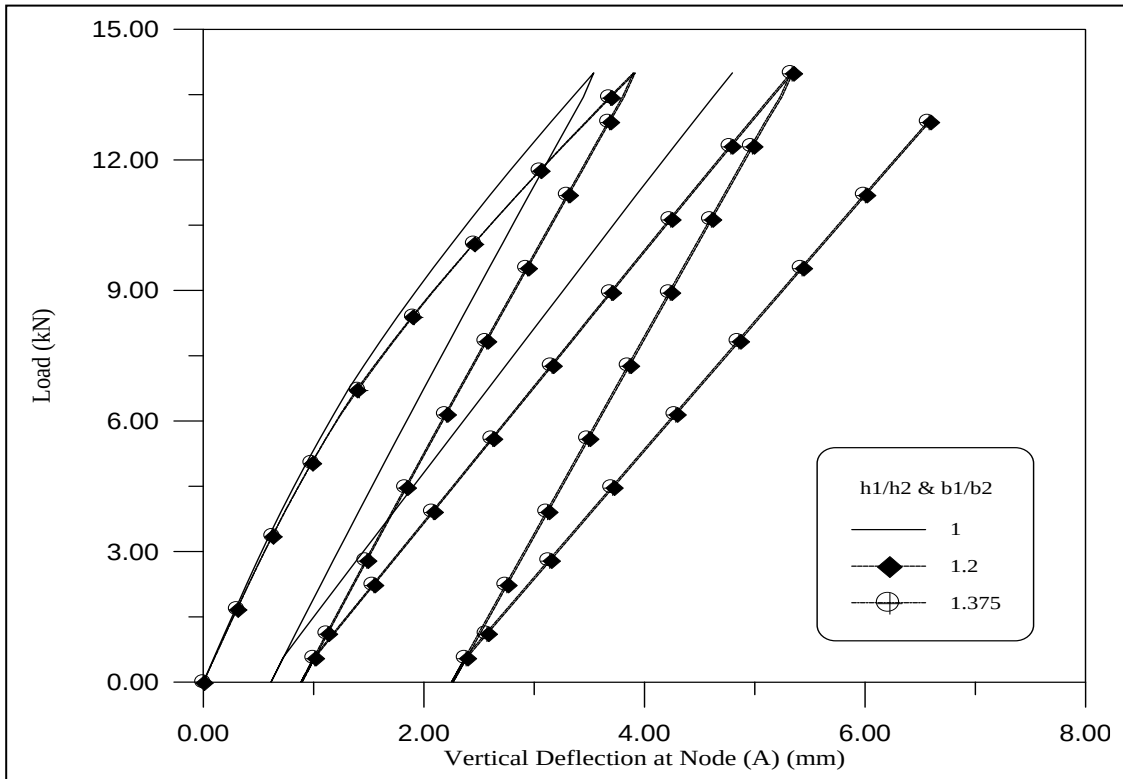


Fig. (7.18): Load – Deflection Curves for Frame (F4) with Tapered Beams ($h_1/h_2 = b_1/b_2 = 1, 1.2, 1.375$) at Node (A).

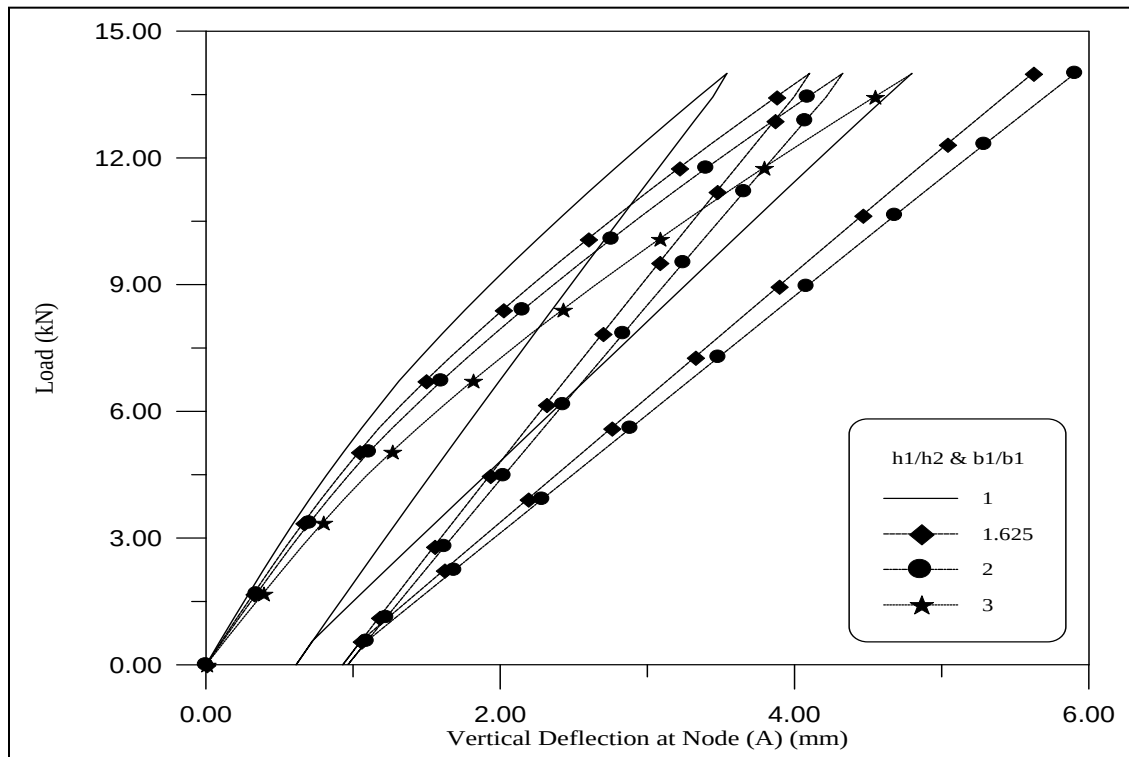


Fig. (7.19): Load – Deflection Curves for Frame (F4) with Tapered Beams ($h_1/h_2 = b_1/b_2 = 1, 1.625, 2, 3$) at Node (A).

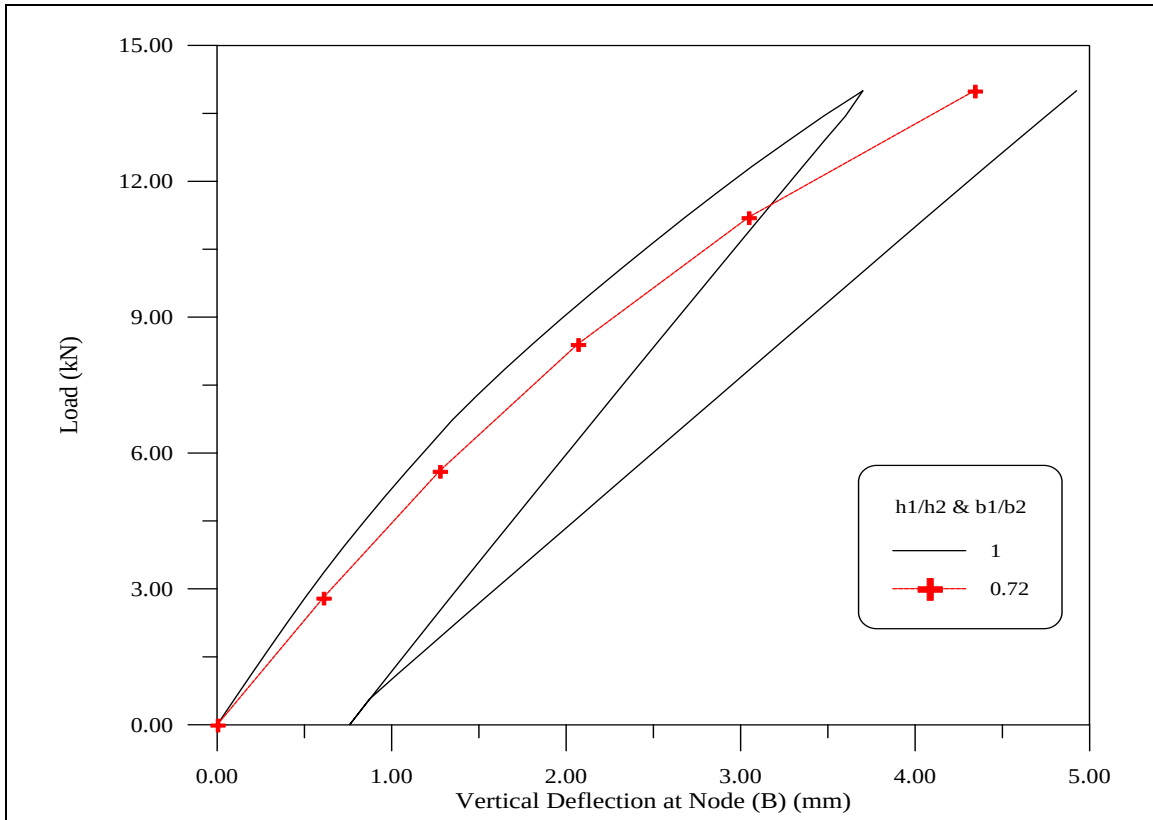


Fig. (7.20): Load – Deflection Curves for Frame (F4) with Tapered Beams ($h_1/h_2 = b_1/b_2 = 0.72$ and 1.0) at Node (B).

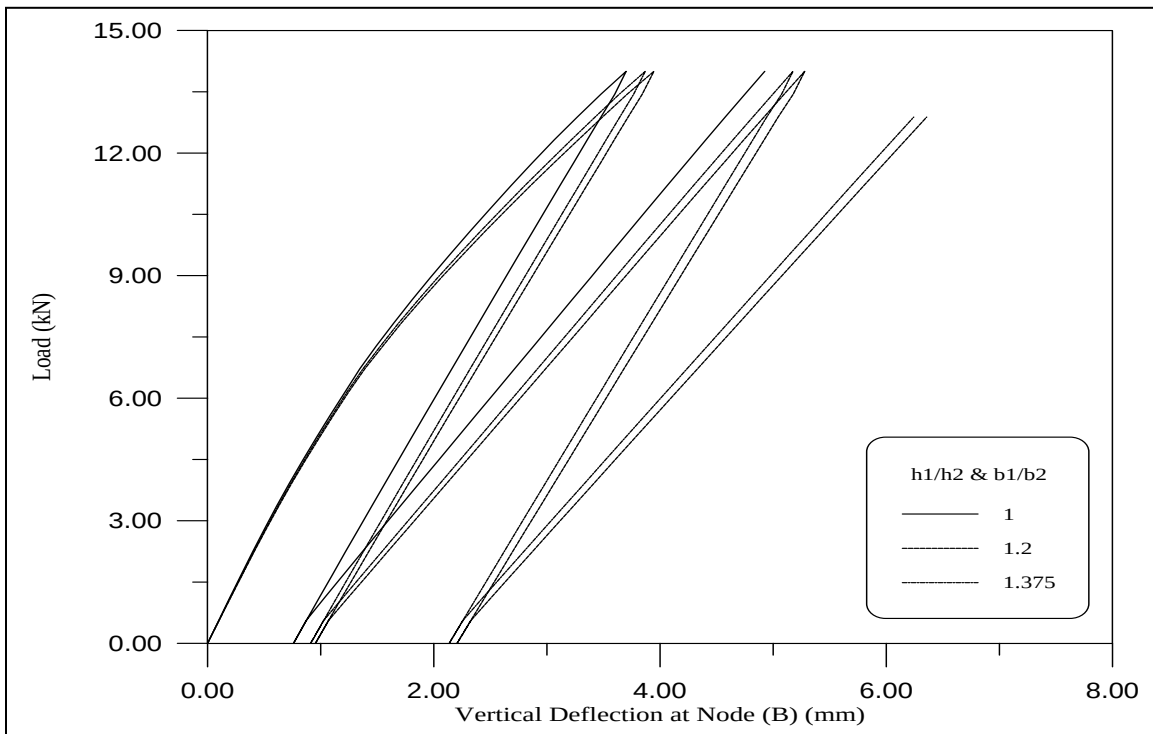


Fig. (7.21): Load – Deflection Curves for Frame (F4) with Tapered Beams ($h_1/h_2 = b_1/b_2 = 1, 1.2, 1.375$) at Node (B).

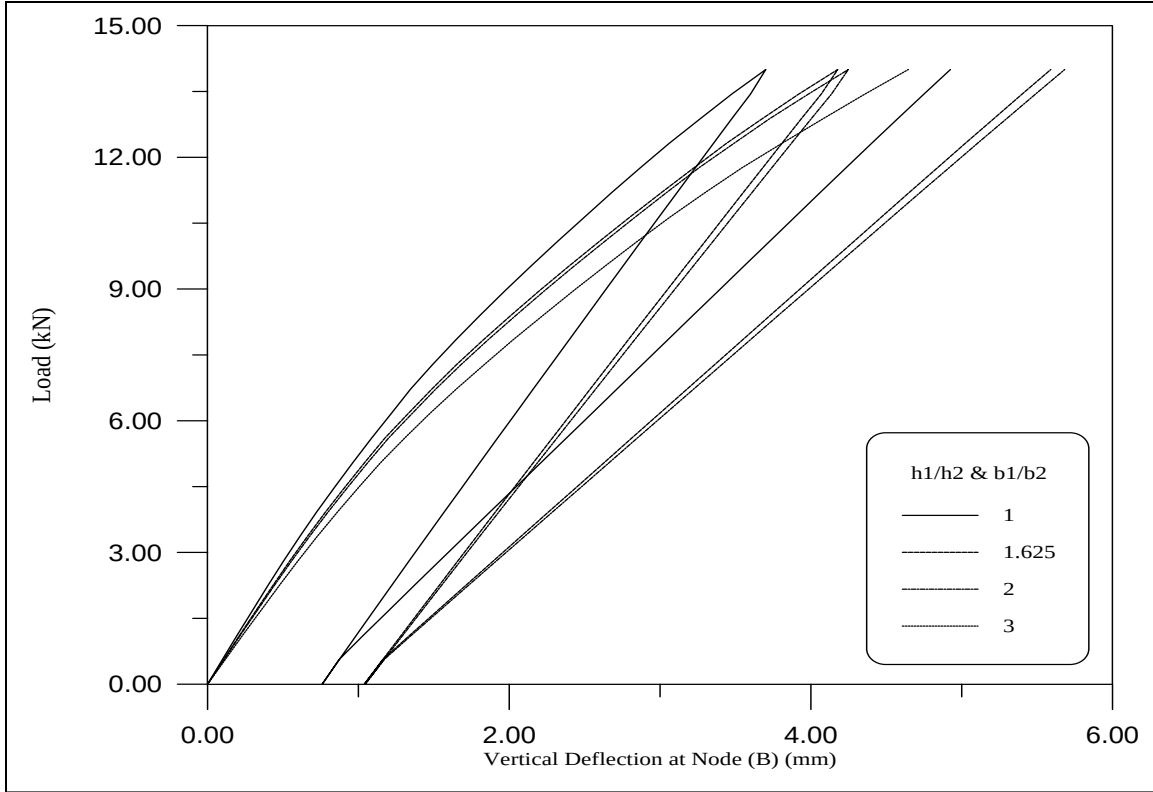


Fig. (7.22): Load – Deflection Curves for Frame (F4) with Tapered Beams ($h_1/h_2 = b_1/b_2 = 1, 1.625, 2, 3$) at Node (B).

($h_1/h_2 = b_1/b_2 = 2$) fails in the second load cycle at load equals to (14 kN), and the non – prismatic frame (F4) ($h_1/h_2 = b_1/b_2 = 3$) fails in the first load cycle at load equals to (14 kN). The position of the crushing failure for the frame (F4) with various tapering ratio (tapering in depth and width directions) is the same as for frame (F4) with various tapering ratio (tapering in depth direction only) and as shown in figure (7.14). Figure (7.23) shows the curve which represent the relation between the tapering ratio and the number of load cycle at crushing failure. From figure (7.23), the optimum tapering ratio (h_1/h_2 and b_1/b_2) for beams can be assigned which lies within the limit from (1.2) to (1.375).

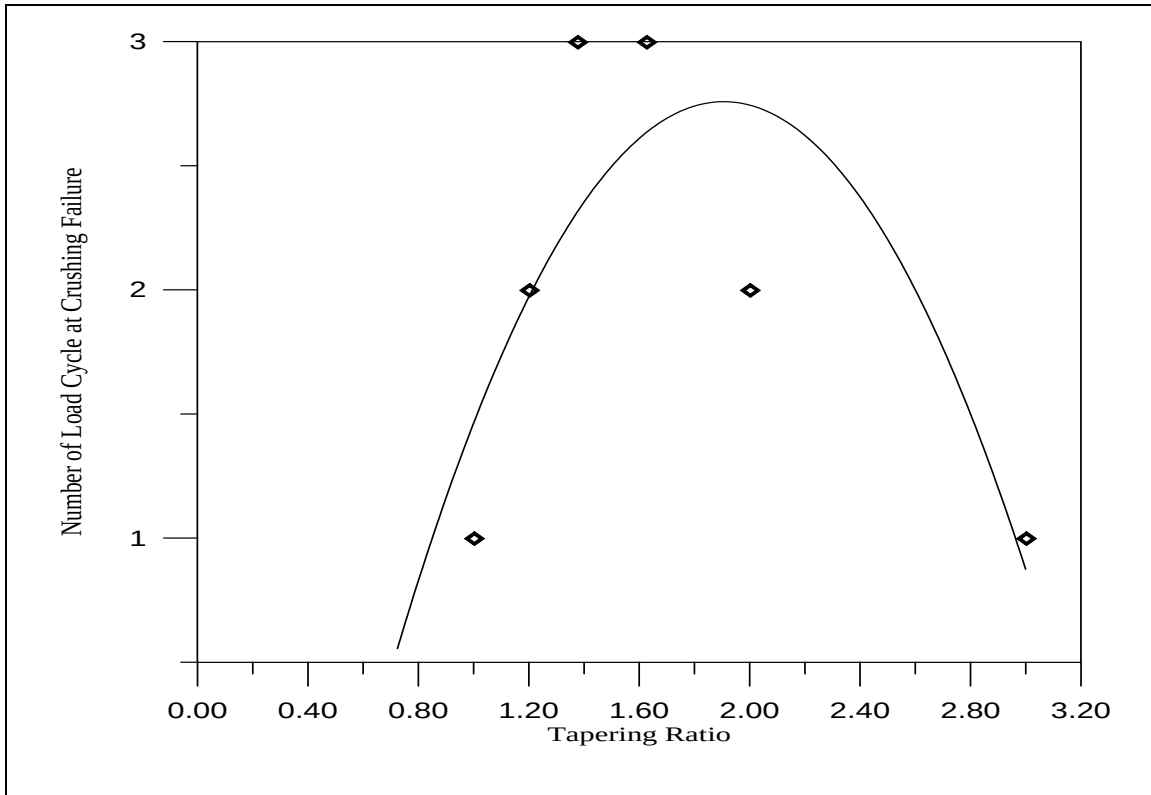


Fig. (7.23): Effect of Tapering of Frame (F4) Beams on Load Cycle Number at Crushing State (Tapering in Width and Depth directions).

7.4 Effect of Creep and Shrinkage:

In this case study the effect of creep and shrinkage on the behavior of reinforced concrete space frame will be studied through drawing the displacement – time curve with taking into account the effect of creep only, shrinkage only ; and creep and shrinkage for beam (B1) and Frame (F5) which will be shown below.

Example No.5:

In this case the beam (B1) which was tested by **Corely and Sozen** (as found in reference [68]), will be reanalyzed now with taking into account the effect of creep only, shrinkage only ; and creep and shrinkage together. The geometric and the loading conditions of beam (B1) are shown in figure (6.43) and all necessary data are mentioned in chapter six (Example No.5). Due to symmetry of geometry and loading, half of beam (B1) will be

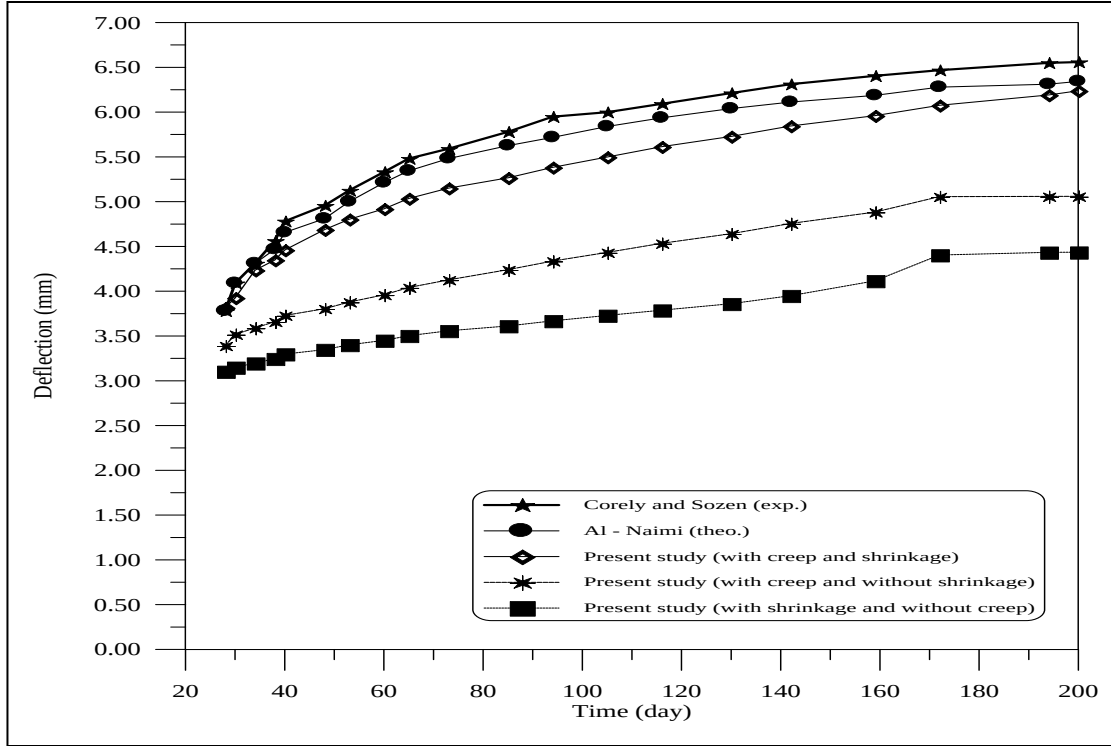


Fig. (7.24): Deflection – Time Curves for Beam (B1).

used in the analysis. This half beam is subdivided into (16) elements, ie. (8) elements from the left support to loading point and (8) elements from the loading point to the center line. The section anywhere in this half beam is subdivided into (10) concrete regions in width direction by (10) concrete regions in depth direction and one steel region in depth direction by two steel regions in width direction. Figure (7.24) shows the deflection – time curves (deflection at midspan of beam) which are obtained by **Corely and Sozen** (as found in reference [68]), **Al – Naimi** [68], and the new procedure. This figure shows that the deflection obtained by the new proposed procedure at time equals to (28 days) is equal to (3.817mm) when the analysis takes into account creep and shrinkage, (3.391mm) when the analysis takes into account creep only, and (3.103mm) when the analysis takes into account shrinkage only. The deflection obtained by the new procedure at time equals to (200 days) is equal to (6.238mm) when the

analysis takes into account creep and shrinkage, (5.059mm) when the analysis takes into account creep only, and (4.434mm) when the analysis takes into account shrinkage only. I.e., the deflection caused by creep only ,at time (28 day), is about (18.7 %) from the total deflection, and at the same time, the deflection caused by shrinkage only is about (11.16 %) from the total deflection. At a time equals to (200 days), the deflection caused by creep only is about (28.9 %) from the total deflection, and the deflection caused by shrinkage only is about (18.9 %) from the total deflection. Figure (7.24) also shows that this curve become more flat with increasing the time.

Example No.6:

In this case the frame (F5) will be reanalyzed now with taking into account the effect of creep only, shrinkage only ; and creep and shrinkage together for the cases of concentrated load and uniformly distributed load. The geometric and material properties of frame (F5) are shown in figure (7.4). This frame is subdivided into (32) elements, ie. Each member in this frame is subdivided into four elements. Any section in each column is subdivided into (10) concrete regions in width direction by (10) concrete regions in depth direction, and two steel regions in width direction by three steel regions in depth direction. Any section in each beam is subdivided into (10) concrete regions in width direction by (10) concrete regions in depth direction, and two steel regions in width direction by two steel regions in depth direction. The loading conditions for the case of concentrated load are given in figure (7.25) and the loading conditions for the case of uniformly distributed load are given in figure (7.26). Frame (F5) kept in controlled atmosphere at (21 °C) and (50 %) relative humidity. The date of loading ranged from (7 days) to (28 days) after the concrete was placed.

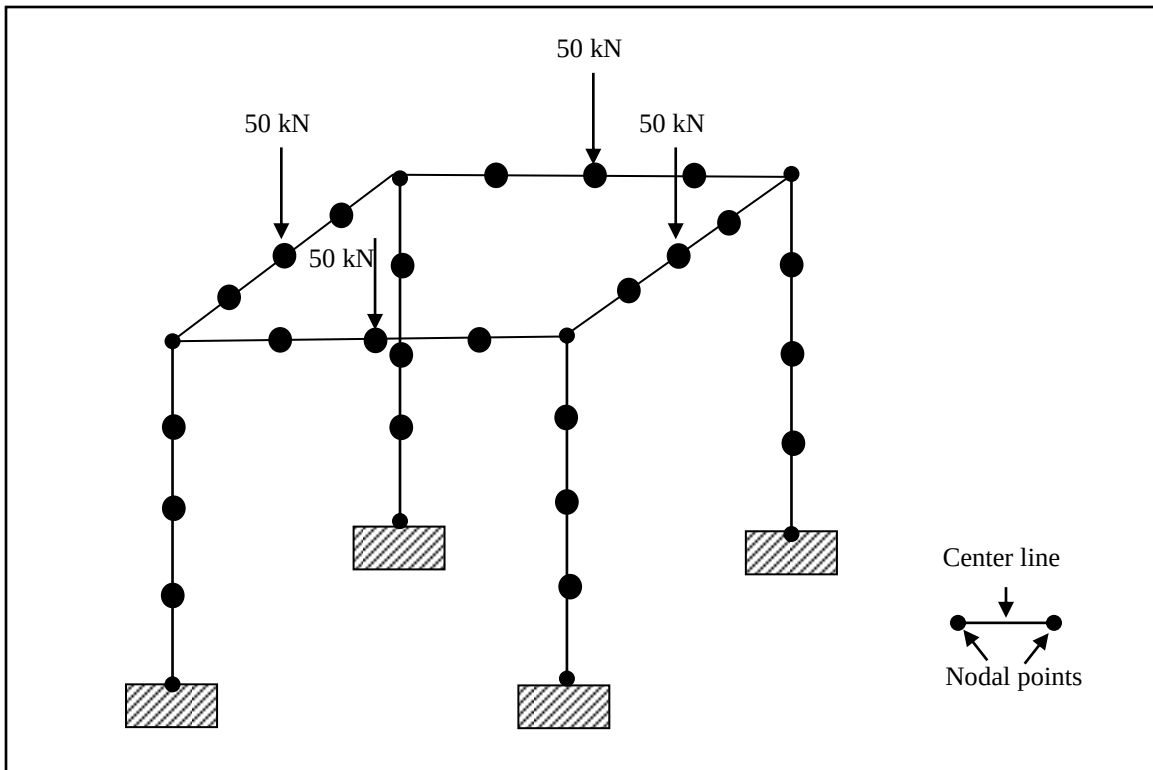


Fig.(7.25): Loading Conditions for Frame (F5) when It is under Concentrated Load.

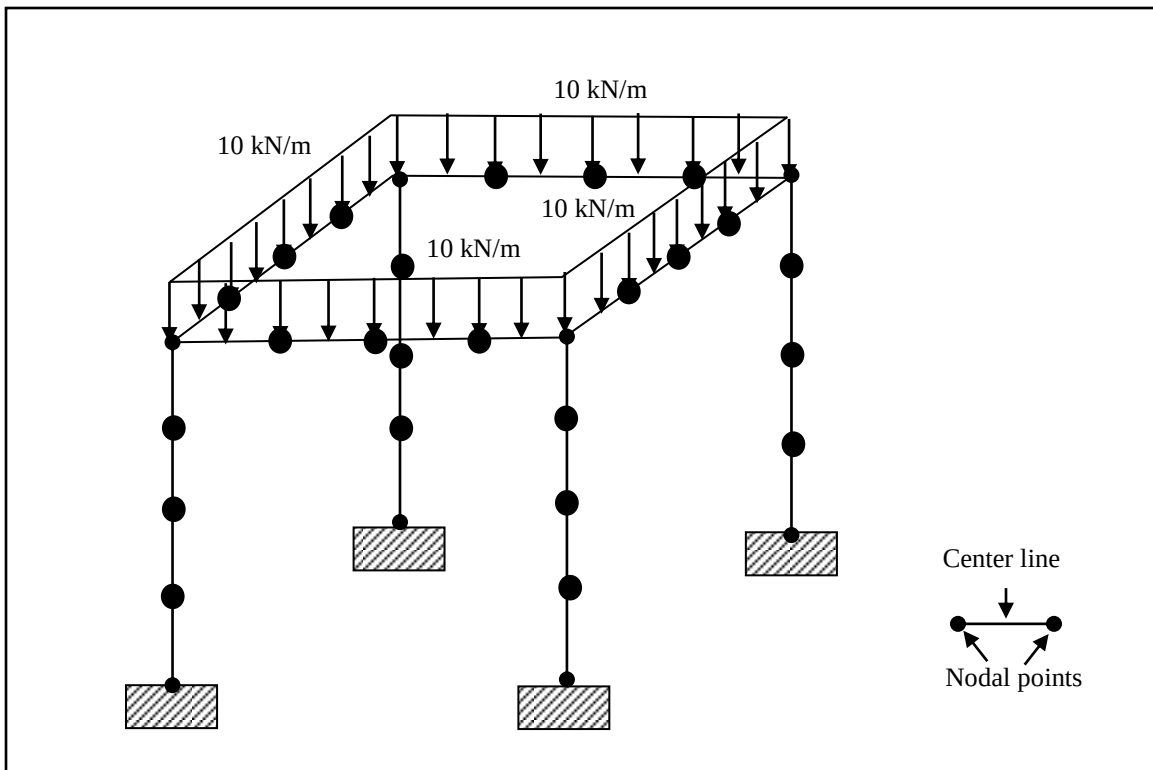


Fig.(7.26): Loading Conditions for Frame (F5) when It is under Uniformly Distributed Load.

A total of twenty four time steps are used to study the long – term behavior. The load is applied at five increments, which are at the time (7, 10, 14, 21 and 28 days), the load factors are (10%, 10%, 20%, 30%, 30%) of the total load, respectively. The frame was designed under effects of creep and shrinkage up to (200) days . The necessary data to analysis frame (F5) are:

$\nu=0.15$, $\epsilon_{cu} = 0.0039$, $W=23.5 \text{ kN/m}^3$, $t_0= 7 \text{ days}$, $f = 35$, $H= 50\%$, $TH= 76.2 \text{ mm}$, $S= 60 \text{ mm}$, $B = 400 \text{ Kg/m}^3$, $F_f = 60\%$, $A_{ir} = 4\%$.

Where ν is concrete Poisson's ratio, ϵ_{cu} is concrete ultimate strain, W is the concrete unit weight , t_0 is curing time, f is a constant for the moist cured concrete, H is the relative humidity, TH is the thickness of beam, S is the slump, B is the value depending on the cement content, F_f is the percentage of the fines, and A_{ir} is the air content.

Figure (7.27) shows the deflection – time curves (deflection at midspan of beam) which are obtained by applied the concentrated load on frame (F5). Figure (7.28) shows the deflection – time curves (deflection at midspan of beam) which are obtained by applied the uniformly distributed load on frame (F5). Figure (7.27) shows that the deflection at a time equals to (28 days) is equal to (1.082mm) when the analysis takes into account creep and shrinkage, (0.746mm) when the analysis takes into account creep only, and (0.738mm) when the analysis takes into account shrinkage only; and the deflection at a time equals to (200 days) is equal to (6.348mm) when the analysis takes into account creep and shrinkage, (5.516mm) when the analysis takes into account creep only, and (4.000mm) when the analysis takes into account shrinkage only. In other wards, the deflection caused by creep only ,at the time (28 day), is about (31.79 %) from the total deflection, and at the same time, the deflection caused by shrinkage only is about (31.05 %) from the total

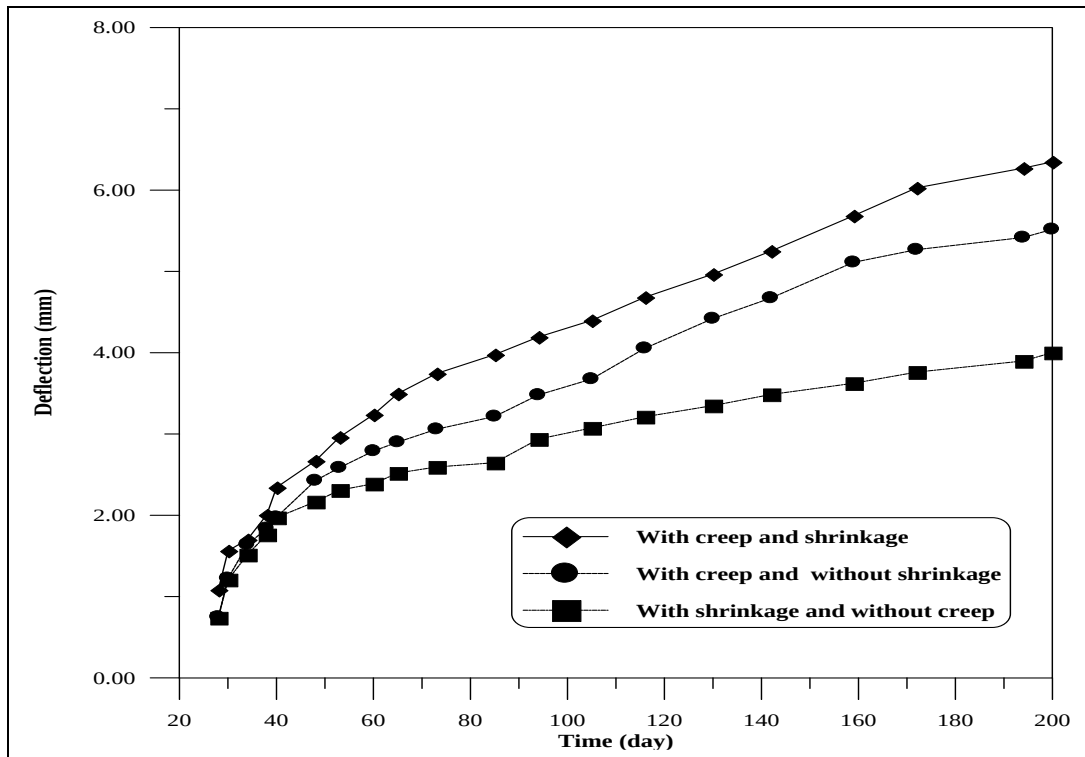


Fig. (7.27): Deflection – Time Curves for Frame (F5) under Concentrated Load.

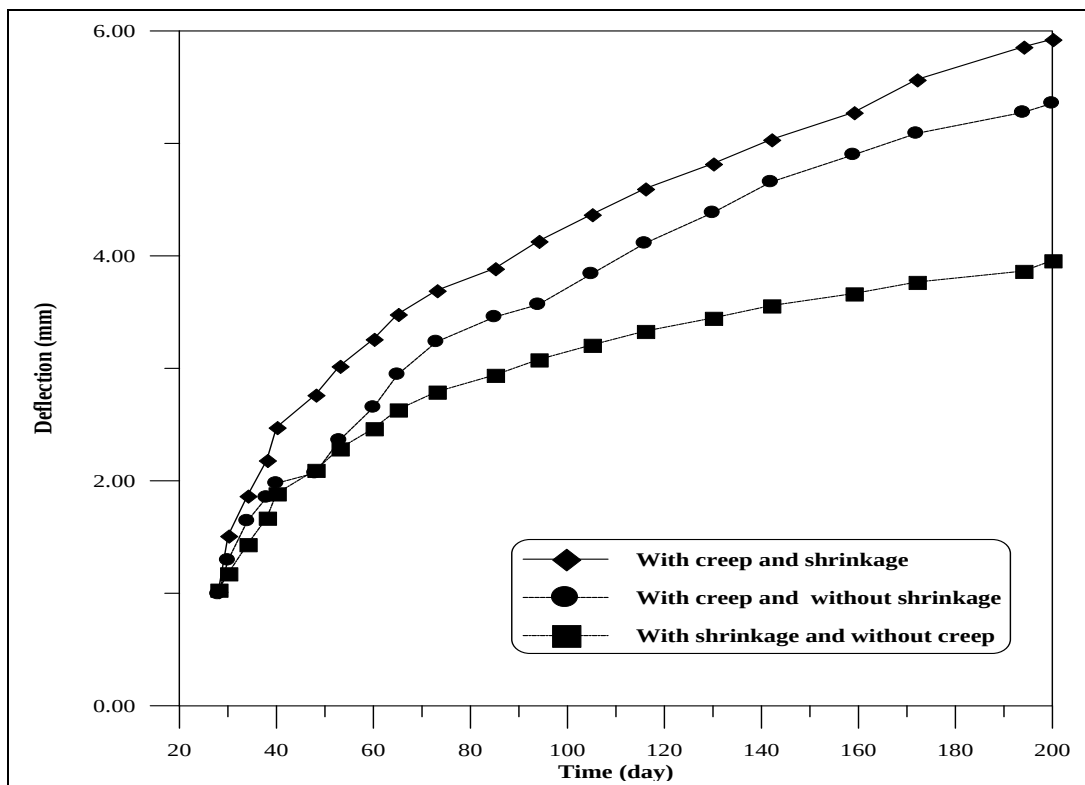


Fig. (7.28): Deflection - Time Curves for Frame (F5) under Uniformly Distributed Load.

deflection. At a time equals to (200 days), the deflection caused by creep only is about (36.99 %) from the total deflection, and the deflection caused by shrinkage only is about (13.11 %) from the total deflection. Figure (7.28) shows that the deflection at the time equal to (28 days) is equal to (1.030mm) when the analysis takes into account creep and shrinkage, (0.993mm) when the analysis takes into account creep only, and (1.017mm) when the analysis takes into account shrinkage only; and the deflection at the time equal to (200 days) is equal to (5.926mm) when the analysis takes into account creep and shrinkage, (5.358mm) when the analysis takes into account creep only, and (3.958mm) when the analysis takes into account shrinkage only. In other words, the deflection caused by creep only, at the time (28 day), is about (1.26 %) from the total deflection, and at the same time, the deflection caused by shrinkage only is about (3.59 %) from the total deflection. At the time equal to (200 days), the deflection caused by creep only is about (33.2 %) from the total deflection, and the deflection caused by shrinkage only is about (9.58 %) from the total deflection. From figures (7.27) and (7.28), at the time (28 days), the ratio of creep or shrinkage deflection to the total deflection when the frame (F5) under uniformly distributed load is lesser than the ratio of creep or shrinkage deflection to the total deflection when the frame (F5) under concentrated load, because the concentrated load gives higher strain in the section at the load point rather than uniformly distributed load. Also from figures (7.27) and (7.28), at the time (200 days), the ratio of creep or shrinkage deflection to the total deflection when the frame (F5) under uniformly distributed load is approximately equals the ratio of creep or shrinkage deflection to the total deflection when the frame (F5) under concentrated load. Figures (7.27) and (7.28) also show that these curves become more flat with increasing the time.

7.5 Effect of Tapering Ratio on The Fixed End Forces:

The effect of tapering of the space frame element on the fixed end forces of reinforced concrete space frame will be studied through studying the variation of the fixed end forces with tapering ratio and studying the variation of A_{force} , B_{force} , C_{force} and D_{force} with tapering ratio.

Example No.7:

The reinforced concrete space frame (F4) will be reanalyzed now by tapering their members (tapering beams in depth direction only) with various tapering ratio for the same amount of steel and the same volume of concrete as shown in figure (7.7). The details and all the necessary data for prismatic frame (F4) are shown in figure (6.32). The loading conditions for the frame (F4) with prismatic and with non – prismatic beams are shown in figure (7.29). The frame (F4) with prismatic and with non – prismatic beams is subdivided into (26) elements, ie. each member is subdivided into two elements, and each section in each element is subdivided into (10) concrete regions in width direction by (10) concrete regions in depth direction and two steel regions in width direction by two steel regions in depth direction. All the curves shown in figures (7.30) to (7.37) are for the element number (3) in frame (F4) which is shown in figure (7.29). Figures (7.30) to (7.33) show the relation between A_{force} , B_{force} , C_{force} and D_{force} with tapering ratio respectively. Figures (7.34) to (7.37) show the relation between fixed end forces (Q_2 , Q_3 , Q_5 and Q_6) (see Fig.(4.4)) with tapering ratio respectively. From figures (7.30) to (7.33), A_{force} , B_{force} , C_{force} and D_{force} decrease with increase the tapering ratio because of the element stiffness increase. From figures (7.34) to (7.35), the fixed end forces Q_2 and Q_3 increase with increasing the tapering ratio. From figures (7.36) to (7.37), the fixed end forces Q_5 and Q_6 decrease with increasing the tapering ratio.

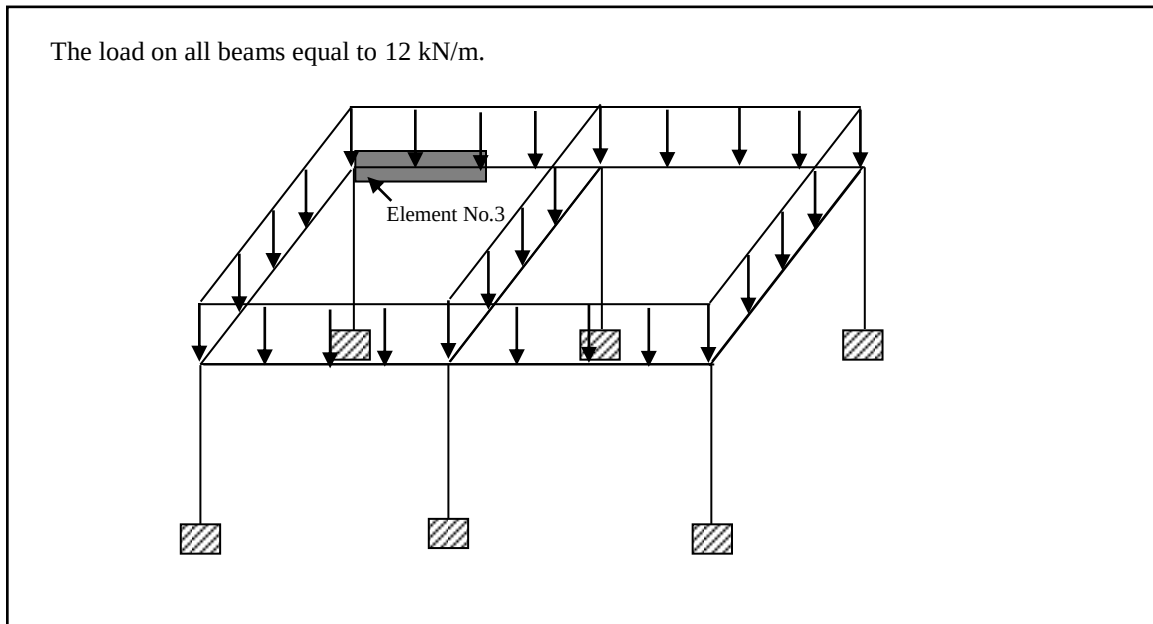


Fig. (7.29): Loading Conditions for Frame (F4) under Uniformly Distributed Load.

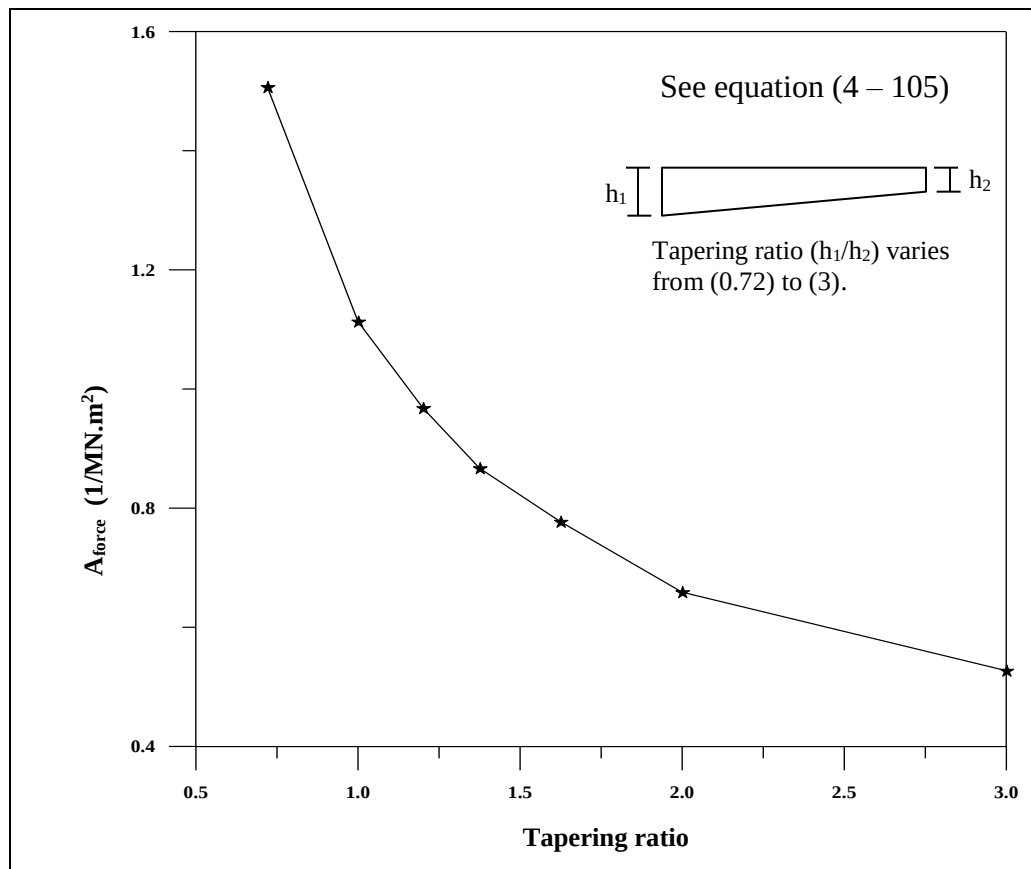


Fig.(7.30): A_{force} – Tapering Ratio Curve for Frame (F4).

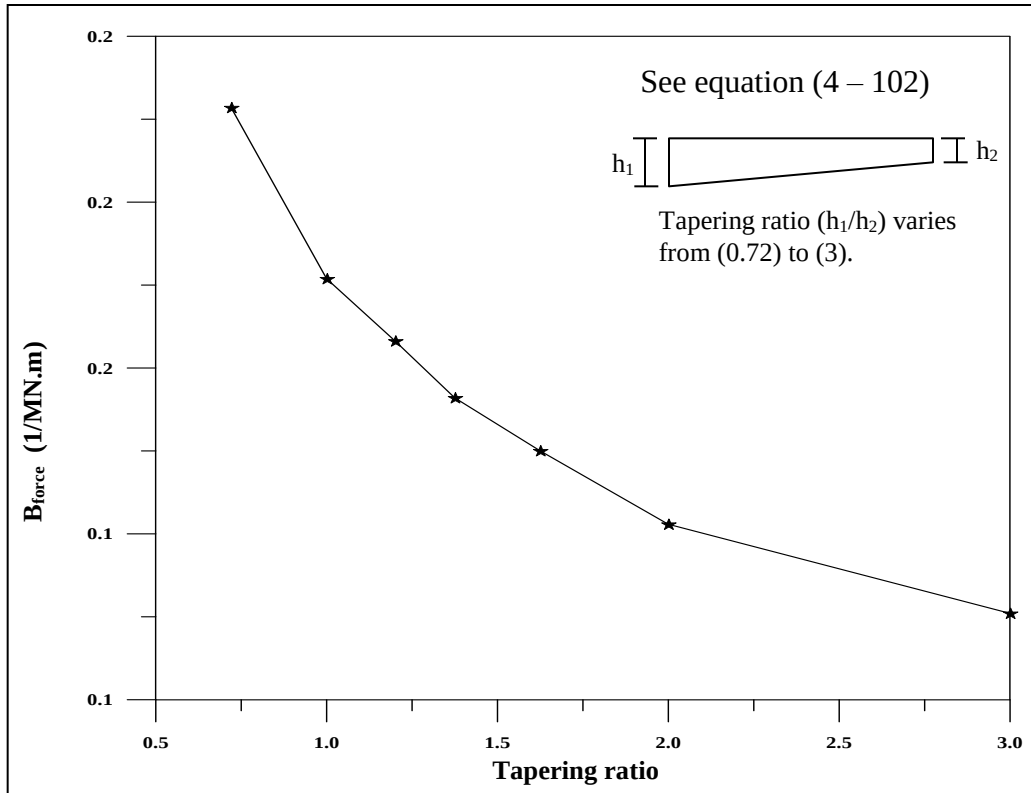


Fig.(7.31): B_{force} – Tapering Ratio Curve for Frame (F4).

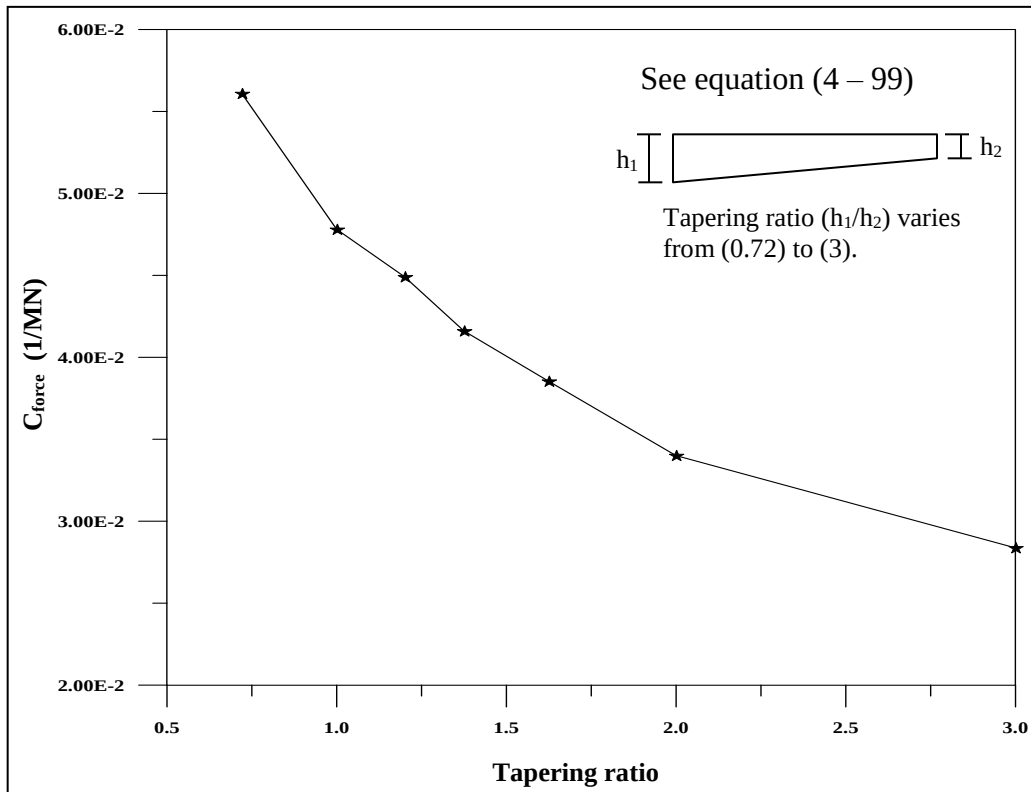


Fig.(7.32): C_{force} – Tapering Ratio Curve for Frame (F4).

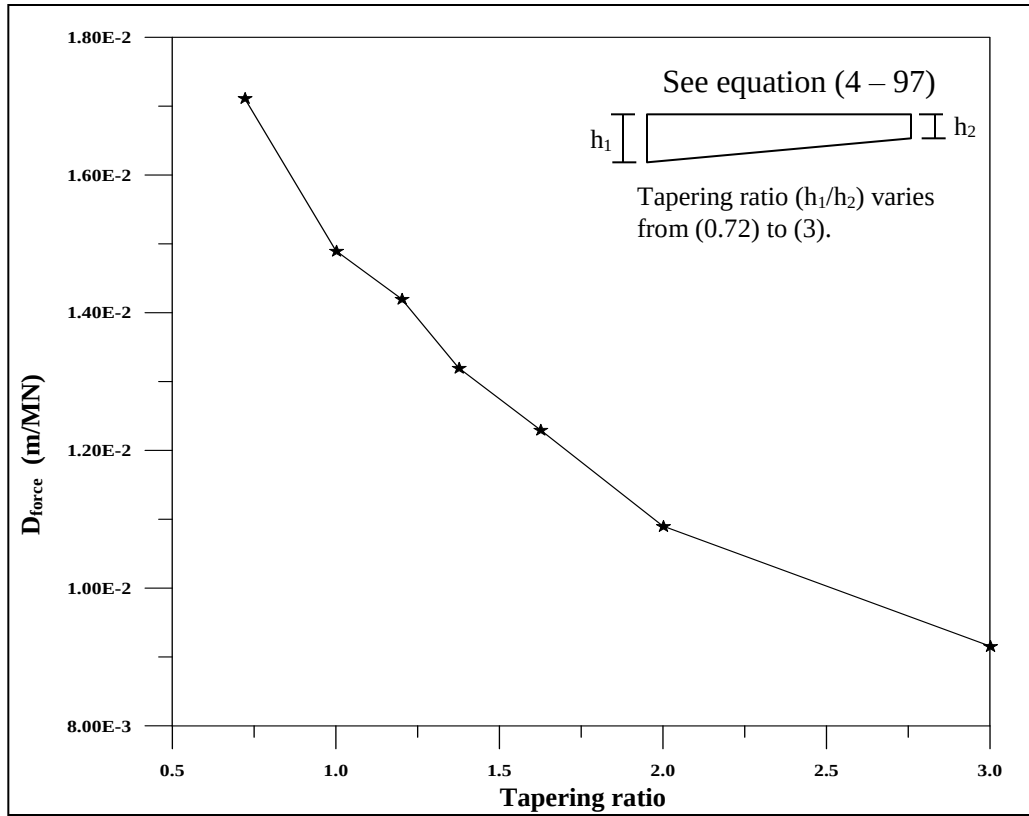


Fig.(7.33): D_{force} – Tapering Ratio Curve for Frame (F4).

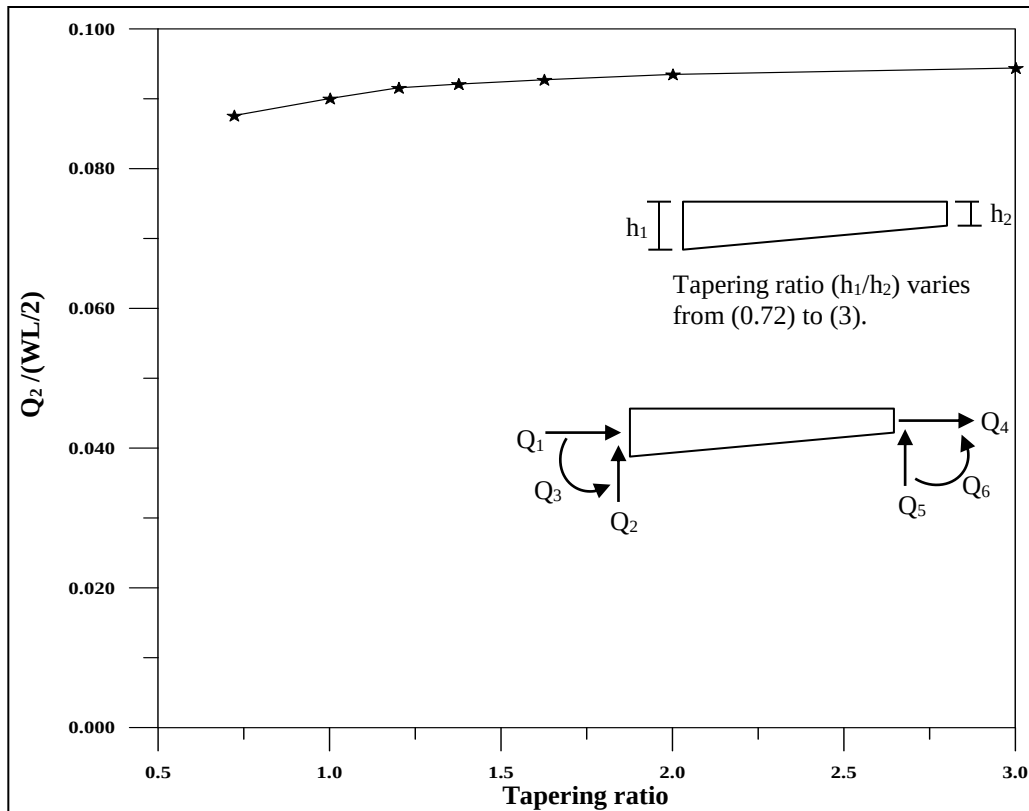


Fig.(7.34): $Q_2/(WL/2)$ – Tapering Ratio Curve for Frame (F4).

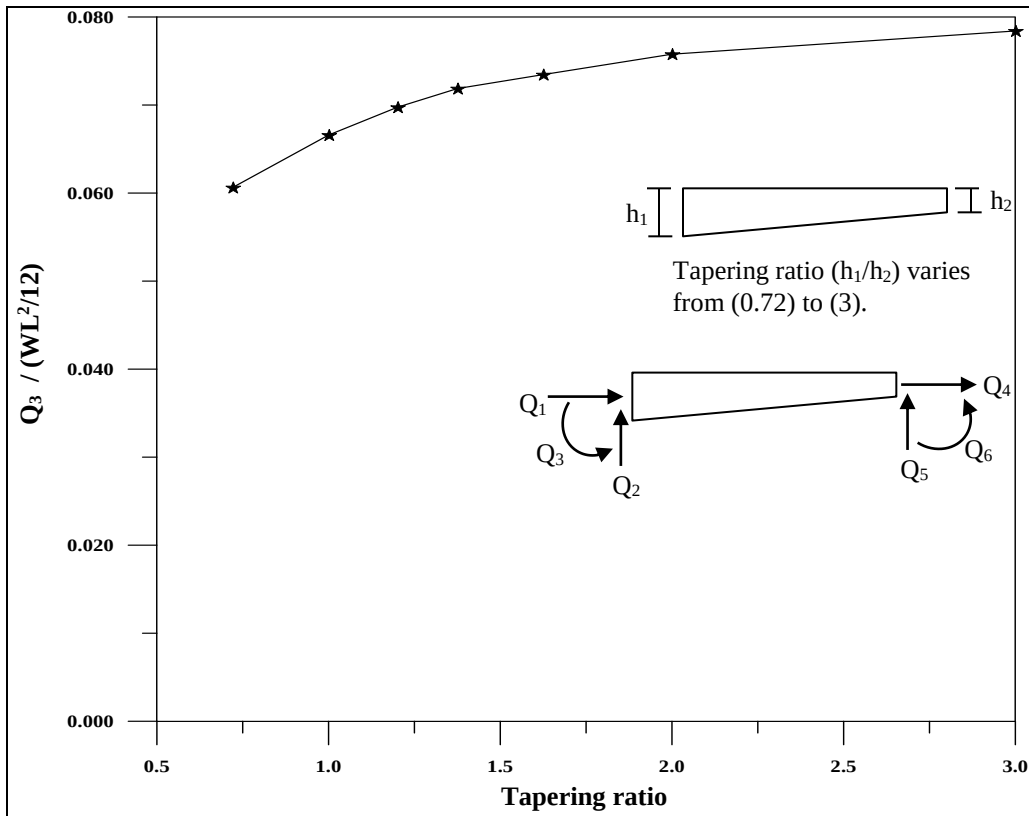


Fig.(7.35): $Q_3/(WL^2/12)$ – Tapering Ratio Curve for Frame (F4).

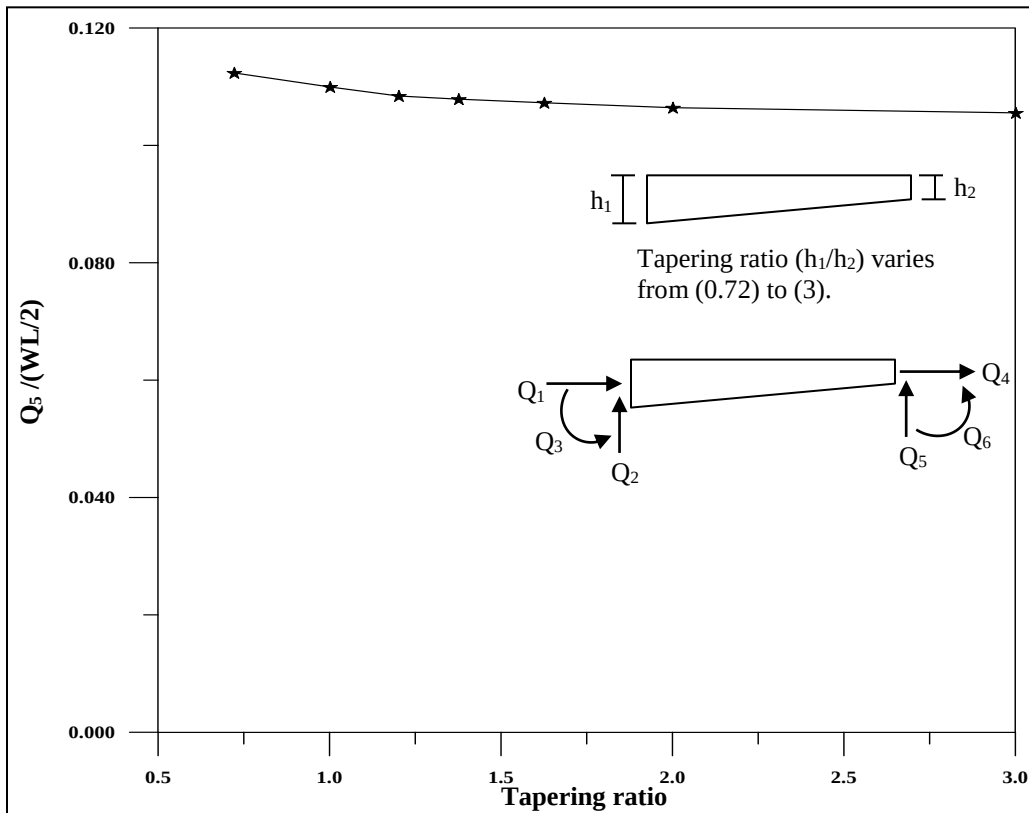


Fig.(7.36): $Q_5/(WL/2)$ – Tapering Ratio Curve for Frame (F4).

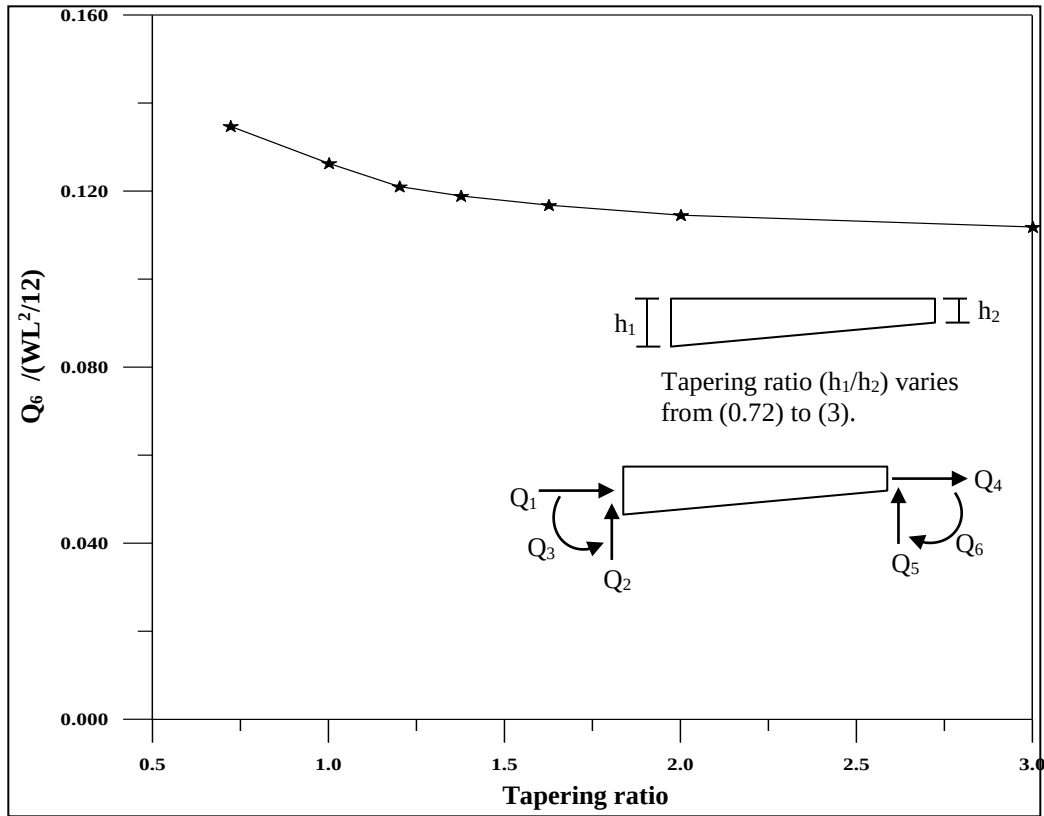


Fig.(7.37): $Q_6/(WL^2/12)$ – Tapering Ratio Curve for Frame (F4).

CHAPTER EIGHT

(CONCLUSIONS AND RECOMMENDATIONS)

8.1 Conclusions:

From the results and discussions given in the previous chapter, the following conclusions can be obtained:

1. The developed analytical technique (with proposed regions approach) and available cyclic model for behavior of concrete and steel, has been shown to be capable of predicting the nonlinear response of reinforced concrete space frame under short and long – term loading with good accuracy in comparison with available results. The algorithm of the new proposed analysis technique is capable of predicting the local crushing failure.
2. For the same amount of steel and the same volume of concrete, tapering of members may increase the overall stiffness and strength of the frame. Also, the integrity of the structure can be improved by tapering its members. The optimum tapering ratio, as found for the considered reinforced concrete frames, is approximately equals to (1.2 – 1.375) for considered problems.
3. Bond between concrete and steel has an influence on the deformation characteristics of reinforced concrete frame elements under flexural loading. The analysis which takes slip into account gives deflection greater than the deflection obtained by the analysis without slip by about (6 %). The new proposed bond stress – slip model and the proposed technique to solve the bond – slip problem are shown to be suitable.
4. The present algorithm for the time dependent analysis of three dimensional reinforced concrete structures proves to predict their behavior satisfactorily. The creep and shrinkage formulations used in the present study depending on **ACI committee 209** give good results compared with

experimental once. The creep effect is larger than shrinkage effect on the reinforced concrete space frame by about not less than 10%.

8.2 Recommendation for The Further Studies:

1. Extension of the proposed method of analysis to analyze the frames having different nonprismatic members such as a parabolic members and stepped members.
2. Extension of the proposed method of analysis to analyze the frames having prestressed concrete members.
3. Extension of the new proposed analytical procedure to analyze reinforced concrete space frame filled with brick wall.
4. Further work is recommended to verify the proposed analytical procedure for other types of concrete structures such as slabs, walls and shells.
5. Including the geometric nonlinearity effect with more exact expressions for stability functions. Also, the bowing effect could be included as another effect of the geometric nonlinearity.
6. Including the effect of confining reinforcement (spiral or stirrups) on bond – slip mechanism and global nonlinear behavior.
7. Including the effect of bond – slip in lap splices zone.
8. Study of the effect of member end rigidity.
9. Make experimental research on frames having nonprismatic members under cyclic loading which is highly desirable in order to examine the validity of analytical results that is presented in this thesis.

REFERENCES

1. Denato, O. and Maier, G. "A Mathematical Programming Method for Inelastic Analysis of R.C. Frames Allowing for Limited Rotation Capacity", *Int. J. Num. Meth. Engg.*, Vol. 4, pp. 307 – 329, (1973).
2. Kaneko, I. "A Mathematical Programming Method for The Inelastic Analysis of Reinforced Concrete Frames", *Int. J. Num. Meth. Engg.*, Vol. 11, pp. 1137 – 1154, (1977).
3. Chugh, A. K. "Stiffness Matrix for A Beam Element Including Transverse Shear and Axial Force Effects", *Int. J. Num. Meth. Engg.*, Vol. 11, pp. 1681 – 1697, (1977).
4. Gunnin, B. L., Rad, F.N. and Furlong, R. W. "A General Nonlinear Analysis of Concrete Structures and Comparison with Frame Tests", *Computers and Structures*, Vol.7, pp.257 – 265, Pergamon press, (1977).
5. Krishnamoorthy, C. S. and Panneerselvam, A. "FEPACSI – A Finite Element Program for Non – Linear Analysis of Reinforced Concrete Framed Structures", *Computers and Structures*, Vol.9, No. 5, pp.451 – 461, Pergamon press, (1978).
6. Cohn, M. Z. and Franchi, A. " Structural Plasticity Computer Sysrem: STRUPL", *J. Structural Div., ASCE*, Vol. 105, No. ST4, pp. 789 – 804, April (1979).
7. Khalifa, M. M. K. "Plastic – Instability Analysis of Space Frame", *The Structural Engineer*, Vol. 57B, No. 3, pp. 55 – 61, Sept. (1979).
8. Rad, F. N. and Furlong, R. W. "Behavior of Unbraced Reinforced Concrete Frames", *ACI Journal*, Vol. 77, No. 4, pp. 269 – 278, July – Aug.(1981).
9. Owen, D. R. J. and Hinton, E. " Finite Elements in Plasticity : Theory and Practice ", Pineridge Press Limited, Swansea, U.K., (1980).

10. Hsu, C – T. T., Mirza, M. S. and C. S. S. Sea “Nonlinear Analysis of Reinforced Concrete Frames”, Computers and Structures, Vol. 13, No. 1 – 3, pp. 223 – 227, Pergamon press, June (1981).
11. Ford, J. S., Chang, D. C. and Breen, J. E. “Experimental and Analytical Modeling of Unbraced Multipanel Concrete Frames”, ACI Journal, Vol. 78, No.1, pp 21 – 35, Jan – Feb. (1981).
12. Meyer, Christian and Bathe, Klaus – Jurgen “ Nonlinear Analysis of R/C Structures in Practice ”, J. Structural Div., ASCE, Vol.108, No.ST7, pp.1605 – 1623, Jul. (1982).
13. Cohn,M. Z.; Erbatu,Fuat and Bhat, P. D. “Computer Nonlinear Analysis of Reinforced Concrete Structures ”, ACI Journal, Vol.80, No.1, pp.28 – 32, Jan. – Feb (1983).
14. Ali, Mir M. and Grierson, Donald E. “ Nonlinear Design of Reinforced Concrete Frameworks ”, J. of Structural Engg., ASCE, Vol.112, No.10, pp.2216 – 2233, Oct. (1986).
15. Al – Sarraf, S. Z. “Shear Effect on The Elastic Stability of Frames”, The Structural Engineer, Vol. 64B, No. 2, pp. 43 – 47, June. (1986).
16. Mo, Y. L. “Moment Redistribution in Reinforced Concrete Frames”, ACI Journal, Vol. 83, No. 4, pp. 577 – 587, July – Aug.(1986).
17. S. S. Lai “ Post – Yield Hysteretic Biaxial Models for Reinforced Concrete Members”, ACI Journal, Vol.84, No.3, pp.235 – 245, May – June (1987).
18. Chajes, Alexander and Churchill, James E. “ Nonlinear Frame Analysis by Finite Element Methods ”, J. of Structural Engg., ASCE, Vol.113, No.6, pp.1221 – 1235, Jun. (1987).

19. Al – Rifaie, W. N. and Trikha, D. N. “Limit State Analysis of Reinforced Concrete Structures with Finite Size Joints” J. of Structural Engg., Vol. 14, pp. 129 – 138, Jan. (1988).
20. Qaqish, Samih “ Comparison Between Linear and Nonlinear Structural Analysis and The Use of Very Small Increments in Solving Nonlinear Structural Analysis ”, Computers and Structures, Vol.33, No.4, pp.923 – 928, Pergamon Press, (1989).
21. Barzegar, Fariborz “ Layering of RC Membrane and Plate Elements in Nonlinear Analysis ”, J. of Structural Engg., ASCE, Vol.114, No.11, pp.2474 – 2492, Nov. (1988).
22. Breysse, Denis and Mazars, Jacky “ Simplified Approach of Nonlinearity in R – C Beams ”, J. of Structural Engg., ASCE, Vol.114, No.2, pp.251 – 268, Feb. (1988).
23. Funk, Raymond R. and Wang, Kai – Tih “ Stiffnesses of Nonprismatic Member ”, J. of Structural Engg., ASCE, Vol.114, No.2, pp.289 – 294, Feb. (1988).
24. Elbehairy, S., Mohamed, I. S. and Abu Zaeid, Abdallah “Nonlinear Analysis of Reinforced Concrete Frames”, Structural Faults and Repair, pp. 65 – 74. (1989).
25. Wong, M. B. and Tin – Loi, F. “Geometrically Nonlinear Analysis of Elastic Framed Structures”, Computers and Structures, Vol.34, No. 4, pp.633 – 640, (1990).
26. H. G. Kwak and Filip C. Filippou “Finite Element Analysis of Reinforced Concrete Structures under Monotonic Loads”, Report No. UCB/SEMM-90/14, Structural Engineering, Mechanics and Materials, Department of Civil Engineering, University of California, Berkeley, November (1990).

27. Hu, Hsuan – Teh and Schnobrich, William C. “Nonlinear Analysis of Cracked Reinforced Concrete”, ACI Structural Journal, Vol.87, No.2, pp.199 – 207, Mar. – Apr (1990).
28. El – Mezaini, Nasreddin; Balkaya, Can; and Citipitioglu, Ergin “Analysis of Frames With Nonprismatic Members”, J. of Structural Engg., ASCE, Vol.117, No.6, pp.1573 – 1592, Jun. (1991).
29. Dumir, P. C., Saha, D. C. and Sengupta, S. “Beam – Column Method for Frames under Distributed Loads”, Computers and Structures, Vol.46, No. 1, pp.141 – 148, (1993).
30. Alwash, N. A. A. H. “Non – Linear Behavior and Optimal Design of Reinforced Concrete Vierendeel Trusses”, Ph.D Thesis, Department of Building and Construction, University of Technology, (1995).
31. Raad Kamil Hashim “Inelastic Analysis and Optimal Design of Reinforced Concrete Frames with Nonprismatic Members”, M. Sc. Thesis, College of Engineering, University of Babylon, Jan. (1999).
32. Al – Asady, N. H. S. “Elastic – Plastic Analysis of Steel Space Structures”, M.Sc. Thesis, Department of Building and Construction, University of Technology, (2001).
33. Terezia, N., Martin, K. and Jan, H. “Theoretical Model of The Determination of The Deformation Rates of R.C. Beams”, Construction and Building Materials, Vol. 15, pp. 169 – 176. (2001).
34. Kuw, C. T. and Ju, S. H. “Developing A Three – Dimensional Nonlinear Beam Element for Static and Dynamic Analysis”, National Science Council, Republic of China, under contract No.: NSC – 90 – 2211 – E – 006 – 068, (2002).

35. Al – Janabi, Mus’ab Aied Q. “Optimal Design of Reinforced Concrete Space Structures Based on Nonlinear Analysis”, M.Sc. Thesis, College of Engineering, University of Babylon, (2003).
36. Sinha, B. P., Gerstle, Kurt H., and Tulin, Leonard G. “ Stress – strain Relationship For Concrete Under Cyclic Loading ”, ACI Journal, Vol.2, No.2, pp.195 – 211, (1964).
37. B. P. Sinha, Kurt H. Gerstle, and Leonard G. Tulin “ Response of Single Reinforced Beams to Cyclic Loading ”, ACI Journal, Vol.61, No.8, pp.1021 – 1038, Aug. (1964).
38. Bertero, Vitelmo V. and McClure, George “Behavior of Reinforced Concrete Frames Subjected to Repeated Reversible Loads ”, ACI Journal, Vol.61, No.10, pp.1305 – 1329, Oct. (1964).
39. Shah, Surendra P. and Chandra, Sushil “ Fracture of Concrete Subjected to Cyclic and Sustained Loading ”, ACI Journal, Vol.67, No.10, pp.816 – 825, Oct. (1970).
40. Takeda, Toshikazu; Sozen, Mete A. and Nielson, N. Norby “Reinforced Concrete Response To Simulated Earthquakes ”, J. Structural Div., ASCE, Vol.96, No.ST12, pp.2557 – 2573, Dec. (1970).
41. Anderson, James C. and Townsend, William H. “ Models for RC Frames with Degrading Stiffness ”, J. Structural Div., ASCE, Vol.103, No.ST12, pp.2361 – 2376, Dec. (1977).
42. K. J. Thompson and R. Park “ Moment – Curvature Behavior of Cyclically Loaded Structural Concrete Members ”, Instn Civ. Engrs, Part 2, 1980, 69, June, pp.317 – 341.
43. Jimenez, Rafael; White, Richard N.; and Gergely, Peter “ Cyclic Shear and Dowel Action Models in R/C”, J. Structural Div., ASCE, Vol.108, No.ST5, pp.1106 – 1123, May (1982).

44. Surendra P. Shah, Apostolos Fafitis, and Richard Arnold “Cyclic Loading of Spirally Reinforced Concrete”, J. Structural Div., ASCE, Vol.109, No.7, pp.1695 – 1710, July (1983).
45. Hwang, Tze – How and Scribner, Charles F. “ R/C Member Cyclic Response During Various Loadings ”, J. Structural Div., ASCE, Vol.110, No.3, pp.477 – 489, Mar. (1983).
46. Roufaiel, Magdy S. L. and Meyer, Christian “ Analytical Modeling of Hysteretic Behavior of R/C Frames ”, J. of Structural Engg., ASCE, Vol.113, No.3, pp.429 – 444, Mar. (1987).
47. Yankelevsky, David Z. and Reinhardt, Hans W. “ Model for Cyclic Compressive Behavior of Concrete ”, J. of Structural Engg., ASCE, Vol.113, No.2, pp.228 – 240, Feb. (1987).
48. Yankelevsky, David Z. and Reinhardt, Hans W. “ Uniaxial Behavior of Concrete in Cyclic Tension ”, J. of Structural Engg., ASCE, Vol.115, No.1, pp.166 – 182, Jan. (1989).
49. Filippou, Filip C. and Issa, Ahmad “Nonlinear Analysis of Reinforced Concrete Frames under Cyclic Load Reversals”, Report on Research Conducted under Grant ECE-8657525 from the National Science Foundation, Report No. UCB/EERC–88/12 Earthquake Engineering Research Center, College of Engineering, University of California, Berkeley, September 1988.
50. Desai, J. P.; Jain, A. K.; and Arya, A. S. “Cyclic Response of Concrete Bracing Members ”, J. Structural Div., ASCE, Vol.115, No.1, pp.51 – 68, Jan. (1989).
51. Otter, Duane E. and Naaman, Antoine E. “ Model for Response of Concrete to Random Compressive Loads ”, J. of Structural Engg., ASCE, Vol.115, No.11, pp.2794 – 2809, Nov. (1989).

52. Mahmood, Abdul Kareem Darweesh “ Nonlinear Analysis of Reinforced Concrete Frames Under Cyclic Loading ”, M. Sc. Thesis, College of Engineering, University of Mosul, Sep. (1989).
53. Ricles, James M.; Yang, Yueh-Shiun; and Priestley, M. J. Nigel “Modeling Nonductile R/C Columns for Seismic Analysis of Bridges”, Journal of Structural Engineering, pp. 415 – 425, April (1998).
54. Hoehler, Matthew and Ožbolt, Joško “Three-Dimensional Reversed-Cyclic Analysis of Reinforced Concrete Members Using The Microplane Model”, Otto-Graf-Journal Vol. 12, pp.93 – 114, (2001).
55. Alavi – Fard, M. and Marzouk, H. “Bond Behavior of High Strength Concrete under Reversed Pull – Out Loading”, Can. J. Civ. Eng./Rev. Can. Ge’nie Civ. 29(2): pp. 191 – 200, (2002).
56. Iemura, Hirokazu; Takahashi, Yoshikazu; and Sogabe, Naoki “Development of Unbounded Bar Reinforced Concrete Structure”, 13th World Conference on Earthquake Engineering Vancouver, B. C., Canada, Paper No. 1537, August 1 – 6, (2004).
57. Ngo, D. and Scordelis, A. C. “Finite Element Analysis of Reinforced Concrete Beams”, ACI Journal, Title No.G4 – 14, pp. 152 – 163, March (1967).
58. Colville, J. and Abbasi, J. “Plane Stress Reinforced Concrete Finite Element”, ASCE, J. of Struct. Division, Vol.100,No. ST5, pp. 1067 – 1083, (1974).
59. Popov, Egor P. “ Bond and Anchorage of Reinforcing Bars Under Cyclic loading ”, ACI Journal, Vol.81, No.1, pp.340 – 349, Jul. – Aug. (1984).

60. Ali, Ammar Y. “ Behavior of Lap Splices in Reinforced Concrete Beams Under Inelastic Cyclic Loads ”, M. Sc. Thesis, Department of Building and Construction, University of Technology, Sep. (1990).
61. Yannopoulos, P. J. and Tassios, T. P. “Reinforced Concrete Axial Element Analyzed under Monotonic and Cyclic Actions”, ACI Structural Journal, Vol. 88, No. 1, pp. 3 – 11, January – February (1991)
62. Yankelevsky, David Z.; Adin, Moshe A.; and Farhey, Daniel N. “ Mathematical Model for Bond – Slip Behavior under Cyclic Loading ”, ACI Structural Journal, Vol.89, No.6, pp.692 – 698, Nov. – Dec. (1992).
63. Lundgren, Karin “Three – Dimensional Modeling of Bond in Reinforced Concrete”, Ph.D Thesis, Division of Concrete Structures, Department of Structural Engineering, Chalmers University of Technology, Goteborg, Sweden, (1999).
64. Ayoub, Ashraf and Filippou, Filip C. “Mixed Formulation of Bond – Slip Problems under Cyclic Loads”, Journal of structural Engineering, pp.661 – 671, June (1999).
65. Lundgren, Karin; Gustavson, Rikard; and Magnusson, Jonas “Finite Element Modeling As A Tool to Understand The Bond Mechanisms”, Bond in Concrete. From research to standards. Budapest, (2002).
66. Limkatanyu, Suchart and Samakrattakit, Agarat “Frame Element Including Effects of Reinforcement Slippage for Nonlinear Analysis of R/C Structures”, Songklanakarin J. Sci. Technol., 25(2), pp. 213 – 226, (2003).
67. Scanlon, A. and Murray, W. “Time Depended Reinforced Concrete Slab Deflection”, ASCE, J. of Struct. Division, Vol. 100, No. ST, pp. 1911 – 1924, Sep. (1974).

68. Al – Naimi, H. A. “3 – Dimensional Dynamic and Static Time Depended Finite Element Analysis of Reinforced Concrete Member”, Ph.D Thesis, University of Baghdad, College of engineering, Feb. (1996).
69. Aldstedt, E. and Bergan, P. G. “Nonlinear Time Dependent Concrete Frame Analysis”, ASCE, Struct. Division, Vol. 104, No.ST7, pp. 1077 – 1092, July (1978).
70. Kang, Y. J. and Scordelis, A. C. “Nonlinear Analysis of Prestressed Concrete Frames”, ASCE, J. of Struct. Eng., Vol. 106, No. ST2, Feb. (1980).
71. Chow, C. O. “The Finite Element Analysis of Creep and Shrinkage Effects in Reinforcing Concrete Beams”, M.Sc. Thesis, Dept. of Civil Engineering, Univer. Coll. Of Swansea, (1984).
72. Al – Saadi, A. U. K. “Shrinkage Cracks in Base Restrained Plain and Reinforced Concrete walls”, M.Sc. Thesis, Dept. of Civil Eng., College of Eng., University of Babylon, (2000).
73. Muhaisin, M. H. “Nonlinear Analysis of Reinforced Concrete Plane Frames under Moving Loads”, M.Sc. Thesis, Dept. of Civil Eng., College of Eng., University of Babylon, (2003).
74. Bahnam, M. (Lctures in Concrete Technology for Post Graduated Studies”, Civil Eng. Dept., Eng. College, Babylon University.
75. D. R. J. Owen and E. Hinton “ Finite Element Software for Plates and Shells ”. Pineridge Press, Swansea, U.K., (1984).
76. Kadhim, M. J. “Nonlinear Analysis of Reinforced Concrete Plane Frames under Cyclic Loading”, M.Sc. Thesis, Dept. of Civil Eng., College of Eng., University of Babylon, (2001).

77. Mohammad, Nesreen S. “Nonlinear Finite Element Analysis for Ferrocement Roofing Elements”, M. Sc. Thesis, Dept. of Building and Construction, University of Technology, April (1998).
78. ACI Committee 209, “Prediction of Creep, Shrinkage and Temperature Effects in Concrete Structures”, ACI Manual of Concrete Practice, Part 1, (1985).
79. Kang, Y. J. “Nonlinear Geometric Material and Time Dependent Analysis of Reinforced and Prestressed Concrete Frames”, UC SESM Report No.77 – 1, Division of Structural Engineering and Structural Mechanics, University of California, Berkeley, January (1977).
80. “Some Factors in The Investigation of Long Term Behavior of R. C. Struct.”, Report of Road Research Laboratory, Harmonds Worth, Middle Sex, February – March (1962).
81. Filippou, F. C. “A Simple Model for Reinforced Bar Anchorages under Cyclic Excitations”, Earthquake Engineering Research center, College of Engineering, University of California, Berkeley, California, Report No. UCB/EERC – 85/05, March (1985).
82. Yankelevsky, D. Z., Adin, M. A., and Farhey, D. N. “Mathematical Model for Bond – Slip Behavior under Cyclic Loading”, ACI Structural Journal, Vol. 89, No.6, pp. 692 – 698, November – December (1992).