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Behavior and Optimal Design of Space Steel Frames Subjected to Earthquake Forces Based on Non-linear Analysis

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الخلاصة

تتناول هذه الدراسة التحليل النظري للحصول على التصرف و التصميم الأمثل للهياكل الحديدية المجسمة المعرضة إلى الهزات الأرضية و السلوك اللاخطي و تحتوي على مفصل حديدية ذات سلوكية لا خطية. إن مادة هذه الهياكل قد فرضت بأنها تتبع التصرف المرن-اللدن و في حالة التحليل اللدن فأن مخطط تداخل و صيغة فشل بين عزوم اللي و الانحناء و قوى القص و القوى المحورية قد افترضت لكي يسيطر على معرفة تسلسل المفصل اللدنة. كذلك اخذ بنظر الاعتبار إن لا محددات على قيم الإزاحات (Large Displacement Analysis).

تبنت هذه الدراسة طريقة العمود-العتبة (Beam-Column Approach) وهذه الدراسة تعتمد على نوعين من العناصر هي عنصر العمود-العتبة و عنصر المفصل. تم اعتبار العلاقة بين العزم والدوران في المفصل الحديدي علاقة لاخطية (Non-Linear Moment-Rotation Behavior). وقد تم الأخذ بنظر الاعتبار التحميل والتفريغ في المفصل الحديدي.

قدمت هذه الدراسة اشتقاق مصفوفة الصلابة المماسية للعنصر المجسم الحديدي ذي المقطع الثابت في نظام الإحداثيات الموقعية (Local Coordinates) والعامة (Global Coordinates) مع الأخذ بنظر الاعتبار تغيير الشكل اللاخطي (Geometrical Non-Linearity). تم اشتقاق مصفوفة صلابة مماسية مطورة جديدة (New Modified Tangent Stiffness Matrix) لتأخذ بنظر الاعتبار الأنواع الثلاثة من اللاخطية وهي (الشكل و المادة و المفصل) (Geometry, Material and Connection). بالإضافة إلى ذلك فقد تم إيضاح طريقة خاصة في حساب القوة المحورية.

تم تتبع منحنى الحمل-التشوه في التحليل الساكن باستخدام تكرارات نيوتن-رافسون التزايدية. أما في التحليل الديناميكي فقد استخدمت طريقة نيومارك- β لغرض تتبع منحنى التشوه مقابل الزمن. مثلت كتلة العنصر باستخدام طريقتين هما (النقل المتجانس) و (النقل المركز). أما ظاهرة التخميد التي تحصل في المنشآت فقد تم التعبير عنها باستخدام طريقة (Rayleigh). لقد تم استخدام صيغة خاصة للتعبير عن الهزات الأرضية المؤثرة على المنشآت. و كذلك استخدام طريقة المحدد لغرض حساب الحمل الحرج للمنشأ.

تم استخدام طريقة عامة للتصميم الأمثل للهياكل الحديدية الثلاثية الأبعاد و بالاعتماد على التحليل اللاخطي المقترح في هذا البحث. إن طريقة التصميم الأمثل المستخدمة تستند على البحث المباشر (direct search method) للحصول على أفضل تصميم (أقل وزن كلي).

ضمن متطلبات هذه الدراسة تمت كتابة البرنامج (OSDEPNONSF) الذي هو عبارة عن التصميم الأمثل تحت تأثير الأحمال الساكنة و الديناميكية و التصرف المرن-اللدن و التحليل اللاخطي للمنشآت المجسمة ذات المفاصل المرنة بلغة ميكروسوفت فيجول بيسك 6.0 (2003). لغرض تقييم كفاءة البرنامج (OSDEPNONSF) في الحصول على السلوك الكامل للهياكل المتكونة من عناصر ذات مقاطع ثابتة ومفاصل حديدية ذات سلوكية صلابة مرنة و أحمال ساكنة و ديناميكية و تحليل مرن و تحليل مرن-لدن فقد تمت مقارنة النتائج المستحصلة مع نتائج أمثلة في دراسات منشورة سابقا. كذلك تم تحميل مجموعة من الحالات الدراسية والتي تبين السلوك اللاخطي الكامل للمنشآت الحديدية. و قد تمت دراسة عدد من

العوامل المؤثرة على النتائج النظرية لتشوه المنشآت الحديدية الهيكلية. و كذلك أمثله لغرض تقييم أداء البرنامج للقيام بعملية التصميم الأمثل للمنشآت.

كنتيجة لهذه الدراسة فقد تم التوصل إلى عدة استنتاجات مهمة و من هذه الاستنتاجات هي إن تأثير الهزات الارضية على الأبنية المتعددة الطوابق يزداد بزيادة عدد الطوابق للمنشأ. أن أكبر إزاحة تزداد بنسبة (25%) لكل ثلاث طوابق. و كذلك فأن الصيغة المتعددة الحدود للتصرف اللاخطي بين العزم و التشوه للمفصل الحديدي تعطي نتائج جيدة بالمقارنة مع التصرف الدقيق بفارق لا يزيد عن (1%). و كذلك التصميم الأمثل للمنشأ الحديدي ذي التحليل المرن-اللدن هو اقتصادي أكثر من التصميم الأمثل للمنشأ الحديدي ذي التحليل المرن فقط بفارق لا يزيد عن (45%) للحالات المدروسة في هذه الدراسة. وكذلك تقديم عدة توصيات لغرض تطوير العمل المقدم.

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

وَيَسْأَلُونَكَ عَنِ الرِّيحِ قُلِ الرِّيحُ مِنْ أَمْرِ
رَبِّي وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

صدق الله العظيم

سورة الإسراء / الآية 85

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ABSTRACT

In this investigation, a theoretical analysis is presented for determining the behavior and optimal design of space steel frames subjected to earthquake excitation, based on non-linear analysis with flexible connections. The material of the frames is assumed to follow elastic-plastic behavior and for plastic analysis, an interaction diagram and yield criterion including bending, torsion, shear and axial forces is suggested to know the sequence of plastic hinge formulation in the space steel frames. Also, it is assumed that there is no-restrictions on the displacement values (large displacement analysis).

The analysis adopts the beam-column approach and it is based on using two types of elements, namely, the beam-column (frame) element and the connection element. The moment-rotation behavior of steel connection is considered to be linear or non-linear. The loading and unloading responses of the connection are also considered.

The formulation of a tangent stiffness matrix for a prismatic space steel member in local and global coordinates which takes into account the geometrical non-linearity has been presented. A new modified tangent stiffness matrix is formulated which takes into account the three types of non-linearities at the same time (geometric, material and connection). In addition, a special procedure which is used in the calculations of axial force has been presented.

In the static analysis, the load-deflection curves of the structures are obtained by using **N-R** method as a incremental load control technique. In the dynamic analysis, **Newmark- β** method is used as a time integration technique to plot the time-deformation curves. Two methods are used for modeling the mass of the element (Lumped and Consistent mass transfer method). Damping effect is expressed by using **Rayleigh** damping matrix. A special form is used to formulate the earthquake forces acting on the

frames. Also, the determinant technique is used for detecting the intersection of load limit points. A general optimal design procedure for space steel frames is presented based on the proposed non-linear analysis. The optimal design formulation is based on a direct search method to get the best design (minimum total weight).

In the present study, a multi-purpose computer program (**OSDEPNONSF**) which Optimal design under Static and Dynamic loads on Elastic-Plastic NON-linear analysis of Space steel structures with Flexible connections has been coded in Microsoft Visual Basic (6.0) (2003) is presented. In order to verify the effectiveness of the program (**OSDEPNONSF**) in predicting the behavior of frames with prismatic members, flexible and rigid connections, static and dynamic loads and elastic and elastic-plastic analysis, some examples reported by previous researchers are utilized. The investigation is extended to predict a complete load-displacement curve for many case studies that show the non-linear behaviors. A parametric study is made to show the effect of several selected parameters on the static and dynamic response of the structures. Also, optimal design examples are presented to verify the efficiency of the program for the optimal design problem.

As a result of this investigation, several important conclusions are obtained. Among those conclusion, the effect of the earthquake excitations on the frames becomes greatly considerable with increasing the number of stories. The maximum deflection increased by (25%) when the stories increase from one to three. Also, the polynomial modeling of the non-linear moment-rotation curve of steel connection gives excellent results with the exact non-linear behavior with difference not more than (1%) and the optimal design of steel frames based on inelastic analysis is economical than that based on elastic analysis with difference reaches (45%) for the considered cases. Also, many recommendations are suggest for further work.

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ARABIC ABSTRACT	

NOTATION

SYMBOL	DESCRIPTION
A	: Cross-sectional area.
b_{1n} and b_{2n}	: Bowing functions with respect to the (n)-axis.
[B]	: Matrix defined in equation (3.23).
$\bar{[B]}$	: Instantaneous static matrix.
C_{1n}	: Flexural stiffness factor with respect to the (n)-axis.
C_{2n}	: Moment carry over factor with respect to the (n)-axis.
C_t	: Torsional constant.
C_{bn}	: Length correction factor due to bowing action to the (n)-axis.
[C]	: Structural damping matrix.
C_w	: Warping constant.
E	: Modulus of elasticity.
{F}	: External forces.
$\{\bar{F}\}$	: Internal forces in local coordinates .
{f}	: Internal forces of the nodal points in global coordinates.
F_y	: Yield stress.
f_{bx}, f_{by}	: Computed bending tensile stresses about the major and minor axes, respectively.
F_{bx}, F_{by}	: Allowable bending tensile stresses about the major and minor axes, respectively.
F_t	: Allowable tensile stress.
GJ	: The torsional rigidity.
G_{1n}, G_{2n}	: Quantities defined in equations (3.70) and (3.71).
{g^(k)}	: Geometric matrices defined in equation (3.76).
H	: Quantity defined in equation (3.72).
I_n	: Moment of inertia with respect to the (n)-axis.

[K]	: Structural stiffness matrix.
L	: Member initial (undeformed) length.
M_{1n}, M_{2n}	: End moments with respect to the (n)-axis.
M_{pi}	: Full plastic moment of the cross-section at end i.
M_{pci}	: Reduced plastic moment capacity at end i.
[M]	: Structural mass matrix.
n	: Either y or z,
Q	: Axial force.
Q_E	: Euler load = $\pi^2 EI/L^2$.
q	: Axial force parameter = Q/Q_E .
Q_y	: Axial yield strength = $F_y \cdot A$.
r	: Radius of gyration.
R_{ki}	: Initial tangent stiffness of connection.
R_{kt}	: Tangent stiffness of connection.
[R]	: Transformation matrix.
{S}	: Member end forces in local coordinates.
[T]	: Tangent stiffness matrix in global coordinates.
[T]_{mod.}	: New modified TSM in global coordinates.
[t]	: Tangent stiffness matrix in local coordinates.
u	: Axial relative displacement.
{u}	: Member end displacements in local coordinates.
V₁, V₇	: Global displacements in X-direction of end (1) and end (2) respectively.
V₂, V₈	: Global displacements in Y-direction of end (1) and end (2) respectively.
V₃, V₉	: Global displacements in Z-direction of end (1) and end (2) respectively.
{v}	: Member end displacements in global coordinates.
V_{max.}	: Maximum shear force in the member.
X₁, X₂, Y₁, Y₂, Z₁, Z₂	: The initial global coordinates of end (1) and end (2) respectively.

$\{X^{00}\}$	: Nodal acceleration vector.
$\{X^0\}$	: Nodal velocity vector.
$\{X\}$	: Nodal displacement vector.
Z_i	: Weight of member (i).
λ_{cr}	: Elastic critical load factor.
ϕ	: Rotational deformation (angle) within connection.
ϕ_{1n}, ϕ_{2n}	: End connection rotations with respect to the (n)-axis.
Δ	: Increment.
α, α''	: The angle between the local and global element coordinates in the deformed and undeformed configuration (rad.).
θ_{1n}, θ_{2n}	: Relative end rotations (rad.) with respect to the (n)-axis.
π	: 3.1416.....
$\{\varpi_i\}$	: Mode shape vector for the ith mode of vibration.
Δt	: Time interval.

Note: Any other notation may be explained where it appears in the thesis.

**STABILITY AND BOWING FUNCTIONS WITH THEIR
DERIVATIVES FOR PRISMATIC MEMBERS**
A.1 Stability functions [2]

1. Compressive axial force, ($q > 0$):-

$$C_1 = \frac{\phi(\sin \phi - \phi \cos \phi)}{2(1 - \cos \phi) - \phi \sin \phi} \dots\dots\dots (A.1)$$

$$C_2 = \frac{\phi(\phi - \sin \phi)}{2(1 - \cos \phi) - \phi \sin \phi} \dots\dots\dots (A.2)$$

where

$$\phi^2 = \frac{QL^2}{EI} = \pi^2 q \dots\dots\dots (A.3)$$

2. Zero axial force, ($q = 0$):-

$$C_1 = 4 \dots\dots\dots (A.4)$$

$$C_2 = 2 \dots\dots\dots (A.5)$$

3. Tensile axial force, ($q < 0$):-

$$C_1 = \frac{\psi(\psi \cosh \psi - \sinh \psi)}{2(1 - \cosh \psi) + \psi \sinh \psi} \dots\dots\dots (A.6)$$

$$C_2 = \frac{\psi(\sinh \psi - \psi)}{2(1 - \cosh \psi) + \psi \sinh \psi} \dots\dots\dots (A.7)$$

where

$$\psi^2 = -\frac{QL^2}{EI} = -\pi^2 q \dots\dots\dots (A.8)$$

A.2 Bowing functions [3]:-

$$b_1 = \frac{(C_1 + C_2)(C_2 - 2)}{8\pi^2 q} \dots\dots\dots (A.9)$$

$$b_2 = \frac{C_2}{8(C_1 + C_2)} \dots\dots\dots (A.10)$$

A.3 Derivatives with respect to q [11]:-

$$C'_1 = -2\pi^2 (b_1 + b_2) \dots\dots\dots(A.11)$$

$$C'_2 = -2\pi^2 (b_1 - b_2) \dots\dots\dots(A.12)$$

$$b'_1 = -\frac{(b_1 - b_2)(C_1 + C_2) + 2C_2b_1}{4q} \dots\dots\dots(A.13)$$

$$b'_2 = \frac{\pi^2 (16b_1b_2 - b_1 + b_2)}{4(C_1 + C_2)} \dots\dots\dots(A.14)$$

A.4 Series Expressions [11]

The closed form expressions of stability and bowing functions and their derivatives become singular at $q=0$. As a result, computational difficulties are encountered in the evaluation of functions for small values of q . these difficulties can be avoided by using the series expansion of these functions as given by **Kassimali**. The series expressions which are valid for the small values of compressive and tensile axial force, were used within the range of $(-0.1 \leq q \leq 0.1)$ in the numerical solution presented in this study. The series expressions are:

$$C_1 \approx 4 - \frac{2}{15} \pi^2 q - \frac{11}{6300} \pi^4 q^2 - \frac{1}{27000} \pi^6 q^3 \dots\dots\dots(A.15)$$

$$C_2 \approx 2 + \frac{1}{30} \pi^2 q + \frac{13}{12600} \pi^4 q^2 + \frac{11}{378000} \pi^6 q^3 \dots\dots\dots(A.16)$$

$$b_1 \approx \frac{1}{40} + \frac{1}{2800} \pi^2 q + \frac{1}{168000} \pi^4 q^2 + \frac{37}{388080000} \pi^6 q^3 \dots\dots\dots(A.17)$$

$$b_2 \approx \frac{1}{24} + \frac{1}{720} \pi^2 q + \frac{1}{20160} \pi^4 q^2 + \frac{37}{604800} \pi^6 q^3 \dots\dots\dots(A.18)$$

$$C'_1 \approx -\frac{2}{15} \pi^2 - \frac{11}{3150} \pi^4 q - \frac{1}{9000} \pi^6 q^2 \dots\dots\dots(A.19)$$

$$C'_2 \approx \frac{1}{30} \pi^2 + \frac{13}{6300} \pi^4 q + \frac{11}{126000} \pi^6 q^2 \dots\dots\dots(A.20)$$

$$b'_1 \approx \frac{1}{2800} \pi^2 + \frac{1}{84000} \pi^4 q + \frac{27}{129360000} \pi^6 q^2 \dots\dots\dots(A.21)$$

$$b'_2 \approx \frac{1}{720} \pi^2 + \frac{1}{10080} \pi^4 q + \frac{1}{201600} \pi^6 q^2 \dots\dots\dots(A.22)$$

Finally, the stability and bowing functions are shown graphically in Figures (A.1) and (A.2).

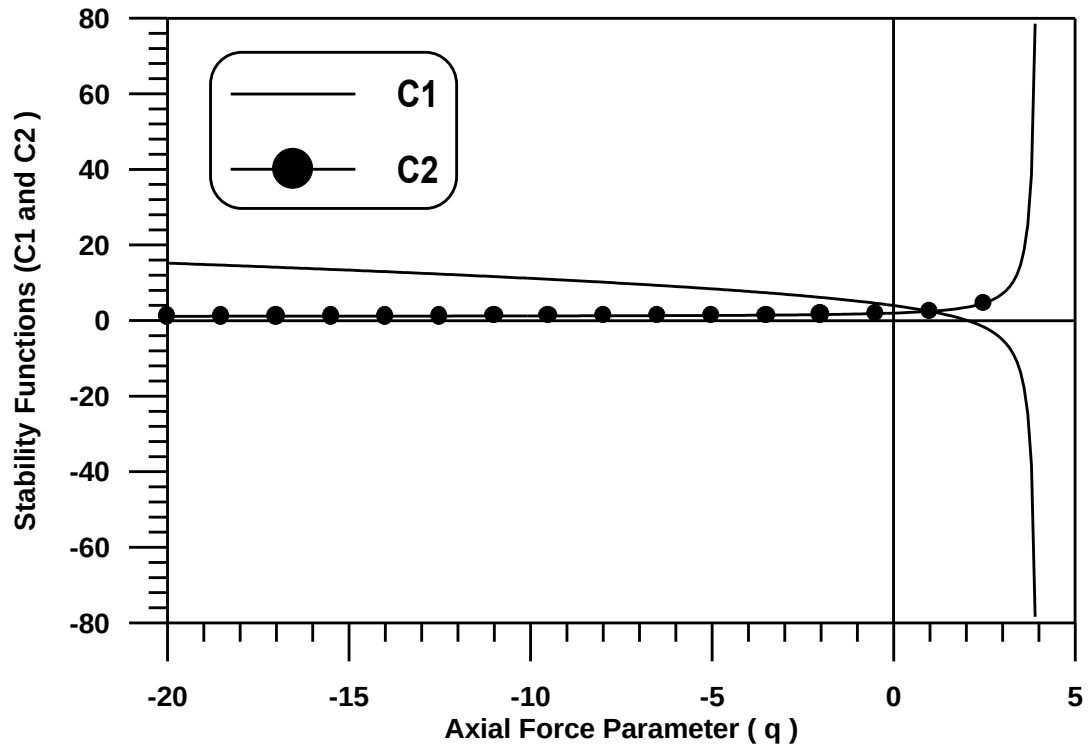


Fig.(A-1): Stability Functions-Axial Force Parameter Curves .

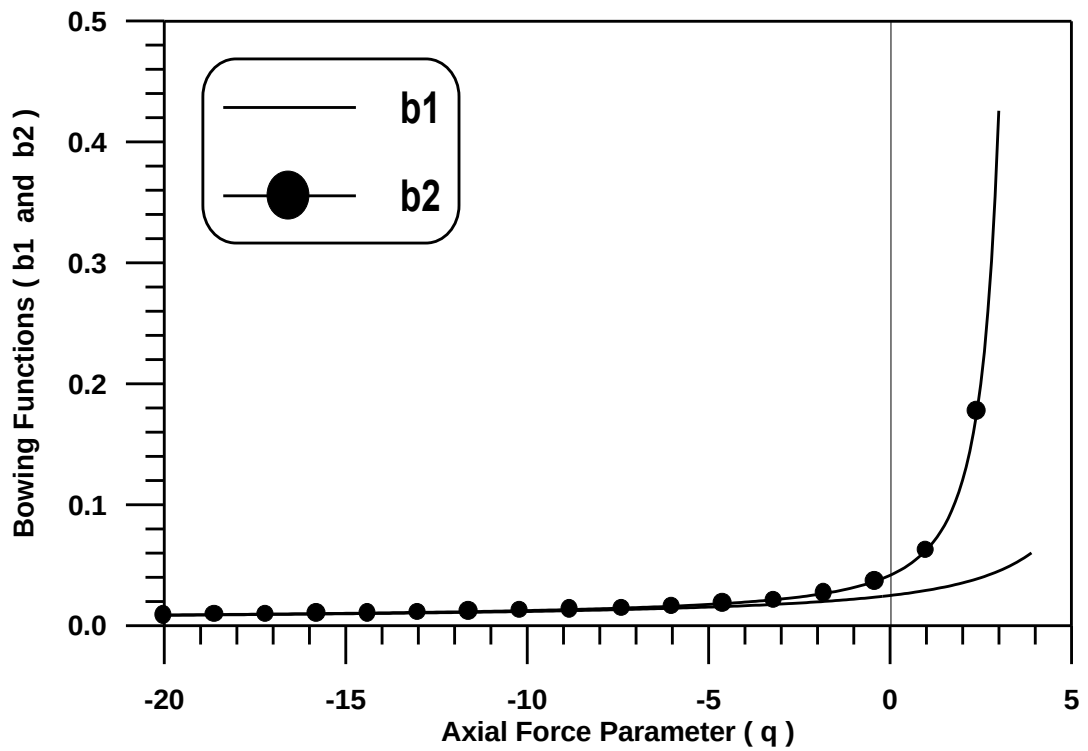


Fig.(A-2): Bowing Functions-Axial Force Parameter Curves .

Certification

We certify that the preparation of this thesis titled "Behavior and Optimal Design of Space Steel Frames Subjected to Earthquake Forces Based on Non-linear Analysis" was accomplished by Hayder Mohammed Jawad Ali Al-Khafaji under our supervision at the University of Babylon in fulfillment of partial requirements for the degree of Doctor of Philosophy in Civil Engineering.

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Certification

We certify as an examining committee that we have read this thesis entitled "Behavior and Optimal Design of Space Steel Frames Subjected to Earthquake Forces Based on Non-linear Analysis", and examined the student "Hayder Mohammed Jawad Ali Al-Khafaji" in its content and what related to it, and found it meets the standard of thesis for the degree of Doctor of Philosophy in Civil Engineering.

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Chapter

One

Introduction

1.1 General

Although the initial behavior of structural systems can very well be described by a linear computational model, all real structures exhibit a pronounced non-linear behavior when the loads and deformations become sufficiently large, and while the connections constitute a small percentage of the weight of a structure, they have relatively high labor content and hence represent a substantial percentage of the total framing cost. In addition, connection deformation is sometimes responsible for a substantial proportion of the overall deflection of a structure and often it has a significant bearing on the internal force distribution.

Also, in recent years, the analysis and design of earthquake resistant structures have taken more attention by the structural engineers and researchers. Therefore, the optimal design of the non-linear behavior of the structure under the effect of earthquake forces becomes the subject in considerable research.

1.2 Non-Linear Behavior of Structures

In recent years, the non-linear behavior of structures has attracted a great deal of attention from designers and researchers. In flexible jointed structures there are three types of non-linearity, geometrical, material and connectional. The present study considers all the above non-linearities in space steel structures with prismatic members under effect of earthquake forces.

Material non-linearity occurs due to non-linear moment-curvature thrust relationships of a given beam-column section. One of the most important structural elements in a steel framework is the beam-column. A beam-column is the type of a structural member which is subjected to the combined action of bending moments and axial force.

The effect of geometrical non-linearity may be separated into three categories:

1.2.1 Stability and Bowing Effects

These effects are caused by the interaction between bending, twisting and axial forces in the members. The lateral deflection and twisting of the member cause an additional moment when subjected to simultaneously applied axial forces. This alters the flexural and lateral stiffness of the member. In a like manner, the presence of bending and torsional moments affect the axial stiffness of the member due to an apparent shortening of the member which is called bowing caused by the bending and twisting deformation. The problem then may be solved by introducing stability functions and bowing functions which are dependent on the axial force parameter ($q=Q/Q_E$)[1], [2], [3].

1.2.2 Large Displacement Effect

The third effect of geometrical non-linearity is the large displacement problem in which the non-linearity occurs when the deformation becomes large enough to cause significant changes in the geometry of the structure, so that the equilibrium equation must be formulated for the updated configuration.

The difference between the large displacement analysis and small deformation analysis comes from including higher order terms in the large displacement analysis. **Fig(1.1)**, shows the deflected shape of a beam-column element and it may be seen that the curvature can be expressed as:

$$\phi = \frac{1}{R} = \frac{-\frac{d^2w}{dx^2}}{\left[1 + \left(\frac{dw}{dx}\right)^2\right]^{1.5}} \dots\dots\dots(1.1)$$

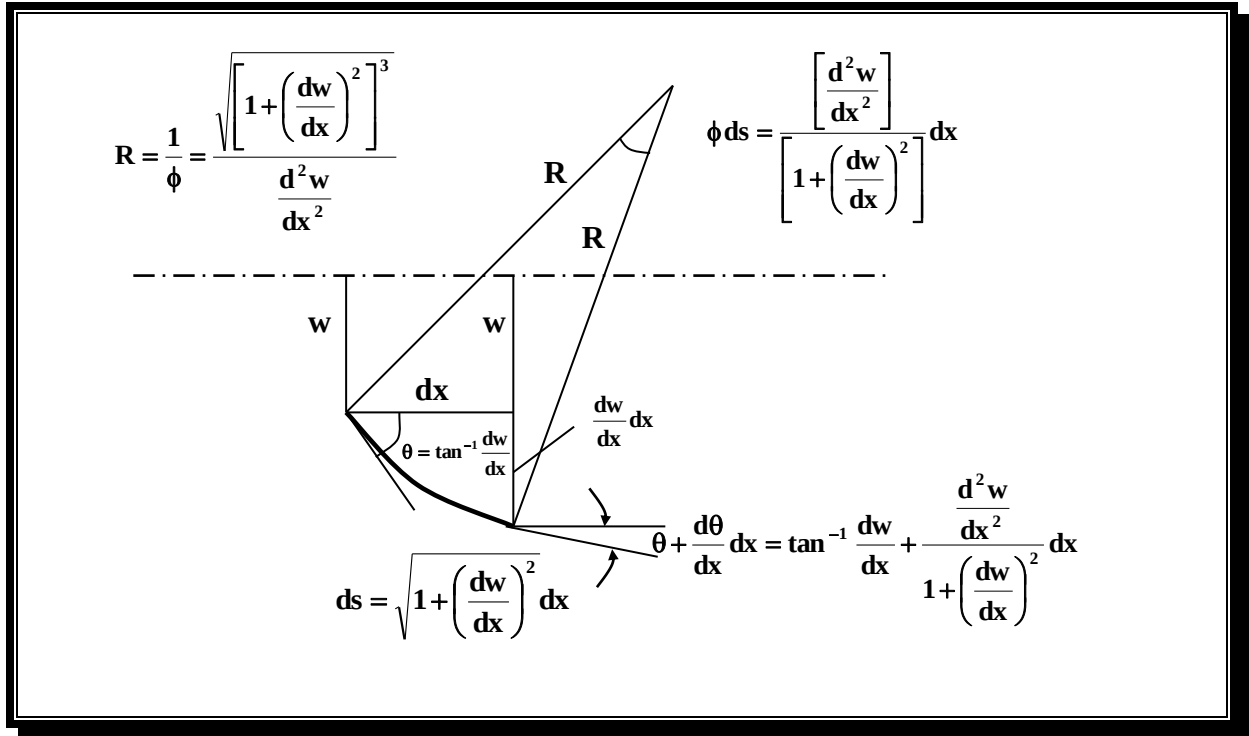


Fig.(1.1): Curvature and Deflection of Beam-Column Element.

The third type of non-linearity is the connection non-linearity, which is resulted due to the use of semi-rigid beam-to-column joints in unbraced steel structures. Since the relationship between the moment in the connection and the rotation (slip angle) is non-linear, it follows that the stiffness matrix will contain the rotations and stiffnesses of the connections which are unknown in advance, and therefore it requires an iteration technique to be solved.

The assumption of rigid joint implies full displacement and slope continuities between adjoining members and full of gravity moment being transferred from beam to column. On the other hand the assumption of ideally pinned connection implies that the beams will behave as if they were simply supported and that the columns will carry no gravity moments from the beams. Although the assumptions of fully rigid or ideally pinned connection behavior drastically simplify the analysis and design procedure, the validity of these assumptions may be questionable in some cases in which connection rigidities are intermediate between the fully rigid and pinned cases[4].

Since the connection is a highly indeterminate element, rigorous analysis study of its behavior is quite formidable task. However, because of the absence of reliable analytical methods for predicting accurately the connection behavior and since there is still a lack of semi-rigid frame analysis approach suitable for office practice, the flexible connection is rarely considered in analysis.

The problem will be more complicated if the applied load on the structure is a function of time, then the analysis is said to be “dynamic”. In the structural dynamic analysis, the natural frequency of the structure looks to be the most important parameter in the analysis. For linear problem, the natural frequencies may be found simply by solving the equation of motion for free vibration, but because the stiffness of the structure is not constant in the non-linear analysis, the eigenvalues obtained from the free vibration in non-linear analysis will be changed depending on load condition and structural

configuration. Therefore, the non-linear problems in dynamic analysis are usually solved by using numerical integration method.

1.3 Member Coordinates System

Different member coordinates may be chosen to derive the member stiffness matrix. They will be discussed by using a plane element[5].

1.3.1 Eulerian Coordinate System

In **Fig.(1.2)**, the configurations of the element before and after deformations are shown. (X_0-Y_0) are initial coordinate axes and $(\bar{X}-\bar{Y})$ denote the deformed configuration axes. For Eulerian coordinates, the generalized displacements are the end rotations θ_1, θ_2 and the relative axial displacement u , these displacements are called "relative deformations" and the corresponding end forces are the end bending moments $M1, M2$ and member axial force Q .

1.3.2 Moving Local Lagrange Coordinate System

The stiffness matrix in the coordinate $\{\bar{V}\}$ may be derived directly in terms of these coordinates rather than the Eulerian coordinates. The stiffness matrix which is derived in local Eulerian coordinates can be transformed to the local Lagrange "intermediate" coordinates using local static instantaneous transformation matrix. Then, it can be transformed to the global coordinates $\{V\}$ by using the current transformation matrix $[R]$.

1.3.3 Fixed Local Lagrange Coordinate System

In this case, the generalized member displacements are measured with respect to the undeformed configuration $\{\bar{V}_0\}$. The stiffness matrix in this case can be transformed to the global coordinates by using the initial transformation matrix $[R_0]$.

1.3.4 Global Coordinate System

The local matrix of each element may be transformed to the generalized global coordinates $\{V\}$ and then by assemblage, the total stiffness matrix $[T]$ is obtained. Then, the structural problem can be solved to determine the displacements, internal forces, etc.

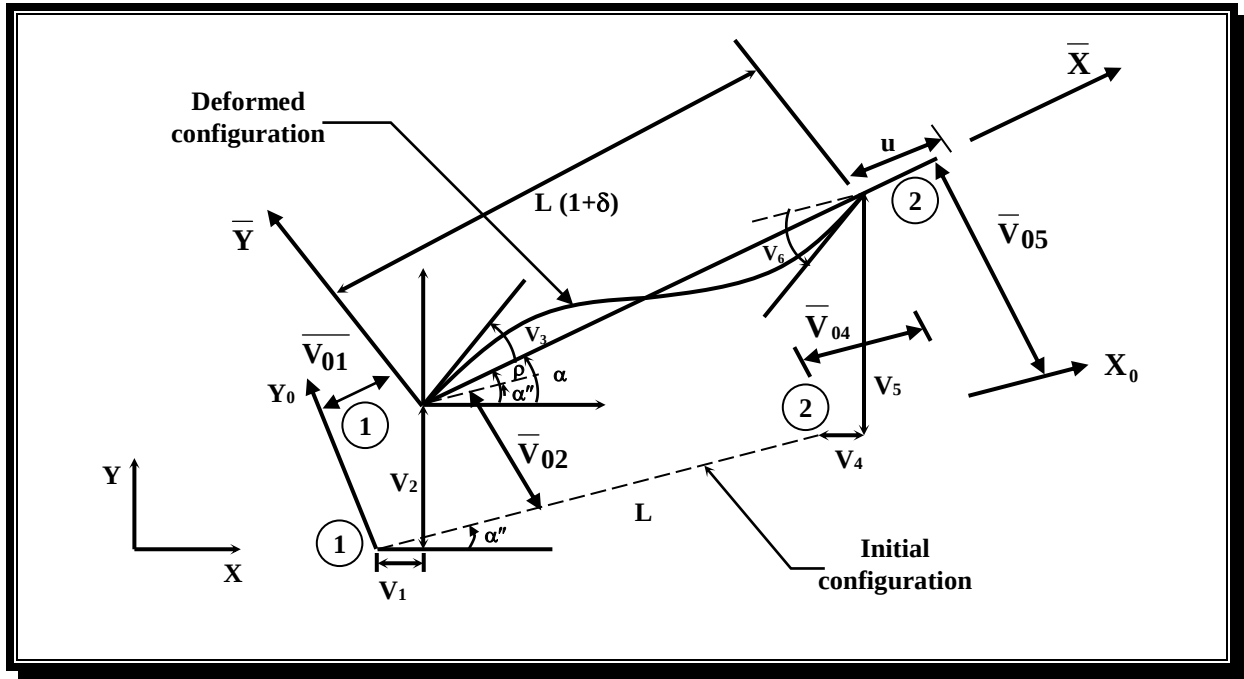


Fig.(1.2): Member Coordinate System for plane element.

1.4 Earthquake Effect

This special type of dynamic loading is taken into consideration in the analysis of structures in this thesis. In recent years, the analysis and the design of earthquake resistant structures have taken more attention from the structural engineers and researchers.

During the last three or four decades, a number of major developments were presented in the non-linear analysis and design. The tremendous growth of research interest has resulted in a considerable amount of knowledge about the analysis of earthquake resistant structures in the non-linear behavior. For

the design of earthquake resistant structures, it is being increasingly realized that large deformations can happen and should be permitted to produce economical and safe design. Also, various methods and scales were developed and modified to calculate and predict seismic forces on building members.

The force transmitted by an earthquake to a structure results from the vibration excitation of the ground. As the soil is the only medium for the earthquake waves to transform from the focus of the earthquake to the structure, it has a great influence on the earthquake severity. Another effect is the free field motion, when the wave transform from the bed rock to the surface of the earth. Although, the type of soil has a major effect on the wave propagation, but the important effect is the soil-structure interaction as the soil behavior for dynamic response is affected by the structure or vice versa. The behavior is also dependent on the earthquake parameters such as depth and distance of the focus, duration, frequency, amplitude of earthquake and amount of energy released during the earthquake[6].

1.5 Optimization

The aim of optimization of structural systems is to find out the best combination of design variables that minimize an objective function (cost, weight, etc.) without sacrificing the functional and performance constraints. The simplest approach to design such structures is to perform analyses for all possible combinations of design variables. However, it is almost impossible to perform such analyses since a huge number of combinations has to be considered in actual cases. Therefore, many kinds of structural optimization techniques have been proposed in the past years [7].

Structural optimization problems are characterized by various objective and constraint functions that are generally non-linear functions of the design variables. The mathematical formulation of structural optimization problems

with respect to the design variables, the objective and the constraint functions depend on the type of application. However, all optimization problems can be expressed in standard mathematical terms as a non-linear programming problem, which in general form can be stated as follows:

Min. $f(X_1, X_2, X_3, \dots, X_n)$

Subject to $g_i(X_1, X_2, X_3, \dots, X_n) \leq b_i \quad (i= 1, 2, \dots, m)$

where:

$(X_1, X_2, X_3, \dots, X_n)$: vector of design variables,

$f(X_1, X_2, X_3, \dots, X_n)$: objective function to be minimized,

$g_i(X_1, X_2, X_3, \dots, X_n)$: behavioral constraints,

b_i : the bounds of the constraints,

n : number of the design variables, and

m : number of the constraints.

1.6 Present Work

The objective of the present study is to perform an optimal design of space steel structures based on the non-linear analysis. Also, to derive a new modified tangent stiffness matrix in global coordinates which takes into account the three types of non-linearity at the same time under static and dynamic loads with considering the damping effects.

This study is presented in **seven chapters**. **Chapter one** introduces the problem and the aim of the study. In **chapter two**, a brief review of early studies and the more advanced studies on the subject is given with an interpretation of results as possible.

Chapter three establishes the derivation of the conventional tangent stiffness matrix for the steel space structures in local and global coordinates and the derivation of the modified tangent stiffness matrix in global coordinates which takes into account the three types of non-linearity. In

addition, the iterative technique to be used for calculating the slip angle in the presence of flexible connections or plastic hinges is presented. Also, it introduces the special technique used for accounting the axial forces when the flexural bowing in the member is included with the effect of the axial force on the plastic moment capacity. The numerical **Newton-Raphson** method which is used in solving the non-linear simultaneous equations is also presented in this chapter.

In **chapter four**, the derivation of mass and damping matrices is presented and the earthquake reasons, type of measurements and some effects on the damages due to earthquake are also introduced. Also, the analytical solution of the non-linear dynamic analysis is presented in this chapter.

The formulation of the design optimization problem involving the objective function, design variables, constraints and the optimization method used in the present study is given in **chapter five**.

In **chapter six**, three parts are obtained. In the first part, the multi-purpose computer program (**OSDEPNONSF**) which have been coded in Microsoft Visual Basic (6.0) (2003) have been presented. In addition, computer applications for testing the validity of the program (**OSDEPNONSF**), have been made and the results were compared with case studies reported by other researchers.

In the second part, non-linear analysis of space frames with rigid and flexible connections under static and dynamic loads are presented to discuss six parametric studies. In the final part of chapter six, the optimal design examples are presented and discussed to verify the validity of the formulation of the optimal design and program (**OSDEPNONSF**).

Finally, **chapter seven** gives several conclusions and recommendations for the future studies on this subject.



Chapter

Review of Literature

2.1 General

This review covers five aspects. The first one is the developments in the elastic-plastic and large displacement analysis of steel structures under static loads. The second one concerns with the developments in the elastic-plastic and large displacement analysis of steel structures under dynamic loads. In the third one the developments in the analysis of frames with flexible connections is reviewed. The fourth one is the developments in the analysis of structures under earthquake forces. Finally, the fifth one deals with the developments of the optimal design of the structures.

2.2 Non-Linear Static Analysis of Steel Structures

In 1963, **Saafan [3]** presented a non-linear formulation in plane frames. He took into account the effects of instability caused by axial force, bowing of deformed members and large deflections. He developed a computer program

for analyzing elastic plane frames subjected to plane forces. The effect of rigid gusset plates were included in the program. The conventional Newton-Raphson method with one load step has been used in his iteration procedure.

In 1973, **Oran [2]** derived a tangent stiffness matrix for a linear elastic plane prismatic member according to the conventional beam-column theory. The effects of axial force and flexural bowing on the stiffness of the member were included and made no restrictions on the displacements of the frame. Although large displacements were used, relative member deformations were assumed to be small enough to satisfy the beam-column theory and loads were assumed to be conservative and applied at joints only.

In 1973, **Oran [5]** derived a tangent stiffness matrix for a linear elastic space prismatic member using the conventional beam-column theory. The effects of axial load on the stiffness of the member were included, in addition to the effect of flexural bowing of the deformed member.

In 1974, **Noor [8]** presented a mixed method and a computational procedure for the analysis of pin-jointed plane and space trusses with geometric non-linearity. The problem was formulated in terms of both member forces and joint displacements, and the governing equations consist of the two sets of joint equilibrium equations and member constitutive relations. The resulting of non-linear algebraic equations were solved by means of an iterative technique based on the method of successive substitutions and Newton-Raphson method.

In 1976, **Oran and Kassimali [9]** suggested a method for the analysis of large deformations and stability of elastic plane framed structures using the general beam-column type method previously developed by **Oran [5]**. Two techniques in solving non-linear problems were used. In the first one, the loads were applied in small increments then the corresponding changes in the configuration of the structure from a sequence of linearized analysis were determined. The second technique is iterative, that is for any desired load level, an approximate solution is first assumed or calculated and it is then

improved step by step by Newton-Raphson iteration until the joints equilibrium is satisfied within a prescribed tolerance. Two coordinate systems were used, Lagrange for initial geometry and Eulerian for the updated configuration of the member.

In 1982, **Argyris, et al. [10]** presented a finite element analysis of large deformation in elastic-plastic frames. They used both theoretical development and numerical procedure for determining the load-displacement relationships of an arbitrary three dimensional frame. They concluded that the ultimate load capacity of the structures, when considering a large deformation in the analysis, becomes less than that when large deformations are not considered.

In 1983, **Kassimali [11]** presented a numerical procedure for a large deformation analysis of elastic-plastic frames. The procedure is based on the Eulerian formulation, which was developed initially for elastic systems and then extended to include elastic-plastic effects. Local member force-deformation relationships were based on the beam-column approach and changes in chord length due to axial strain and flexural bowing were taken into account.

In 1983, **Wen and Rahimzadeh [12]** presented a paper about the non-linear static analysis of plane and space frames using a finite element approach. The member end rotations relative to the chord of the deformed member were assumed to be small and large translations and rotations of the chord were allowed.

In 1984, **Yang and McGuire [13]** derived stiffness matrices for three dimensional beam elements that include the effect of warping restraint on elastic torsional response. The use of warping springs makes the boundary condition simple in concept and meaningful in physics. At the same time, the introduction of warping indicator facilitates determination of warping spring constants.

In 1986, **Yang and McGuire [14]** derived a new stiffness matrix for the analysis of a thin-walled beam element in space frames. Starting from the

principle of virtual displacements, an updated Lagrangian procedure was developed for non-linear analysis. A finite element model in terms of conventional stress resultants was derived based on the theory of thin-walled beams.

In 1988, **Nee and Halder [15]** presented a paper, which deals with the analysis of space structure with large deformation by using second order finite element method which involved both geometric and material non-linearity. They used stress method derive the governing equations, which satisfy joint equilibrium and displacement compatibility equations. The proposed method was verified with experimental and numerical results and it was observed that the proposed method was very reliable and accurate in studying the behavior of the finite deformed space frames.

In 1989, **Shi and Atluri [16]** presented a paper, which deals with a large deformation analysis of space frames with the effect of sime-rigid connections under static and dynamic response. It was based on a complementary energy approach and Newton-Raphson was employed to solve the incremental tangent stiffness equations under the effect of static loads and Newmark- β method was employed to solve the dynamic effect.

In 1990, **Chandra, et al [17]** proposed and used a new concept of instantaneous secant stiffness for non-linear elastic-plastic analysis of steel space structures the method replaces the Newton-Raphson technique with a new concept of instantaneous secant stiffness. It was found to be more stable and converge rapidly.

In 1997, **Al-Hachami [18]** presented a theoretical analysis for predicting the large displacement elastic stability of plane and space structures subjected to general static loading. The beam-column theory was used in this analysis taking into accounts both bowing and axial force effects. The general equations of fixed end moments of beam subjected to lateral loads were also derived. The study employed this analysis to study the behavior of beams with elastic foundations, piles driven into soil and large displacements of submarine

pipelines. Approximate approaches for determining the modified stability and bowing functions of the beam-column resting on elastic foundation and the fixed end action were derived.

In 2001, **Al-Asady [19]** presented a theoretical analysis of plane frames and the method has been developed for the elastic-plastic analysis of space frames. A step by step linearized procedure of elastic-plastic analysis has been developed to deal with the analysis of space frame, which predicts the sequence and location of plastic hinges and the corresponding load factor at each stage of analysis. The procedure is based on the matrix displacement method. In this method, the members are assumed to possess infinite rigidity for elastic-plastic behavior of the connection. The material is assumed to be elastic-perfect plastic and yielding is considered to be concentrated at the member ends in the form of plastic hinges and the members are assumed to remain elastic between the plastic hinges.

In 2002, **Sivaselvan and Reinhorn [20]** presented a theoretical analysis of plane frames with a large deformation inelastic problem. A beam-column element that includes the effects of geometric and material non-linearities is developed. The non-linear axial force and bending moment versus axial strain and curvature relationships of cross sections, including the effect of the interaction between the two stress resultants, at locations along the element axis are chosen to be presented.

In 2004, **Al-Sarraf, et al. [21]** presented a theoretical analysis for estimating the in-space large displacement elastic stability behavior of rigidly jointed steel frames consisting of members with rigid gusset plates subjected to static loads applied at the joints of these structures. The analysis adopts the beam-column approach and models the structural members as beam-column elements. The formulation of the beam-column element was based on Eulerian approach, allowing for the influence of the axial force on bending stiffness and the changes in member chord length due to axial deformation and flexural bowing. The formulation of a modified tangent stiffness matrix in local and

global coordinates, which takes into account the effect of geometrical non-linearity and the effect of the presence of the gusset plates. Incremental-iterative method was used for tracing the non-linear behavior of structures was the conventional Newton-Raphson method of iteration performed at each load step to satisfy equilibrium conditions.

2.3 Non-Linear Dynamic Analysis of Steel Structures

In 1959, **Newmark [22]** generalized certain direct numerical integration methods that have been in use up to that time. Equations for approximating the velocity and displacement of a single degree of freedom system have been presented. These equations involve two constants (γ) and (β). The parameter (β) controls the variation of the acceleration. Therefore, the technique is referred to as “Newmark’s Generalized Acceleration Method” or “Newmark- β Method”.

In 1976, **Oran and Kassimali [9]** presented a non-linear dynamic analysis using Newmark average acceleration method. It was assumed that the masses are lumped at joints, the effects of rotary inertia on mass matrix and the effect of damping of the system were neglected. In this work, the incremental procedure in the formulation of the equation of motion at time ($t + \Delta t$) was used. A fixed ended beam with an iteration process was solved and the results were compared with others, and it was found that this technique gives an accurate solution compared with others.

In 1979, **Remseth [23]** presented a study concerned with the method of non-linear static and dynamic analysis of structures, practically for application to geometrical non-linear space frames. The displacement formulation of the finite element method was adopted. Large displacements were accounted by using a Lagrangian description of motion from a fixed reference frame and assuming small strains and moderately large rotations.

In 1981, **Papadrakakis [24]** presented a study concerning the application of two vector iteration methods namely “The Dynamic Relaxation” and “The Conjugate Gradient”, in the investigation of the large deflection behavior of plane and space structures.

In 1988, **Kassimali and Bedhendi [25]** studied the stability behavior and the large deformation response of space trusses subjected to dynamic loads. The method of analysis was based on an Eulerian formulation accounting for arbitrarily large displacements. Three types of dynamic forcing functions were considered these are step loads of infinite duration, triangular loads and sinusoidal loads. Load-deflection curves for these load cases were presented and the influence of the triangular impulse duration and the period of the sinusoidal load on the critical load magnitudes of the structures were examined.

In 1989, **Subbaraj and Donkainish [26]** presented a survey of the direct time integration methods in computational structural dynamics using the implicit procedures viz. Newmark- β , Wilson- θ and Houbolt- α methods. The formulation and computational step by step procedures for linear and non-linear systems were given. Stability aspects of these methods were also described.

In 1996, **Al-Rauanduzy [27]** presented a theoretical analysis for estimating the plane large displacement elastic stability behavior of steel frames subjected to either proportional or non-proportional increasing static or dynamic loads applied at joints only using beam-column approach. The formulation of the beam-column element was based on Eulerian approach allowing for the interaction between the bending moment and the axial force stiffness. Changes in member chord length due to axial strain and flexural bowing were taken into account. The load-deflection responses of frames were obtained by using an incremental load control with Newton-Raphson technique.

In 1997, **Al-Rikabi [28]** presented a theoretical analysis for the estimation of the plane large displacement elastic stability behavior of steel frames and offshore structures supported or driven into elastic Winkler type foundation subjected to either proportional or non-proportional increasing static or dynamic loads applied at joints only.

In 1998, **Al-Bidairi [29]** presented theoretical bases for the large displacement elastic stability analysis of plane steel frames and beam like lattice structures with various end conditions, subjected to static and dynamic loads. Also, the effect of shear deformation on the geometric non-linear behavior and the effect of damping on the dynamic response were considered.

In 2000, **Al-Mutairee [30]** presented a theoretical analysis for predicting the pre- and post-buckling behavior of plane steel frames with non-prismatic members subjected to static and dynamic loads. The analysis for large displacement behavior adopts the beam-column approach. Also, he made a comparison between the different iterative strategies and the different incremental strategies to decide which of each group is best, more efficient and general.

In 2002, **Al-Alwash [31]** presented a theoretical analysis of pile driving in soil using the finite element method. Two models for idealizing the pile-soil system were adopted. The first one is the lumped spring model and the other is the consistent spring model. The dynamic analysis is performed by using Newmark method with $\gamma = 0.5$ and $\beta = 0.25$, (i.e., constant average acceleration method).

2.4 Analysis of Frames with Flexible Connection

Although much research works have been conducted on the force-deformation behavior of steel framing connections, only a few attempts have

been made to incorporate the connection deformations into structural analysis of the frame.

In 1963, **Monforton and Wu [32]** developed a modification to slope-deflection equations to account for the deformation of linearly elastic connections (analogous to rotational springs) at the ends of the beam. They also presented modifications to the stiffness and carry-over factors and the fixed-end moments to permit frames with elastic connections to be analyzed by the moment distribution method.

In 1975, **Frye and Morris [33]** presented an iterative analysis procedure for planar linear, rectangular steel frames incorporating the non-linear elastic behavior of seven beam-to-column connection types. The analysis procedure involved repeated cycles of linear analysis, to determine a set of connection secant stiffness, K_s , that could be used to predict the displacement and internal forces in real structure. As illustrated in **Fig. (2.1)**, the final secant stiffness at a given connection would produce a moment, M_f , and a rotation, ϕ_f , corresponding to a point on the non-linear moment-rotation curve.

In 1979, **Uzgider [34]** developed a numerical method for the analysis of three dimensional frames loaded dynamically into the inelastic range. The elastic-plastic deformation behavior at the ends of the frame member was presented by an equation which corresponded essentially to the inverse Ramberg-Osgood function for the moment-rotation relationship.

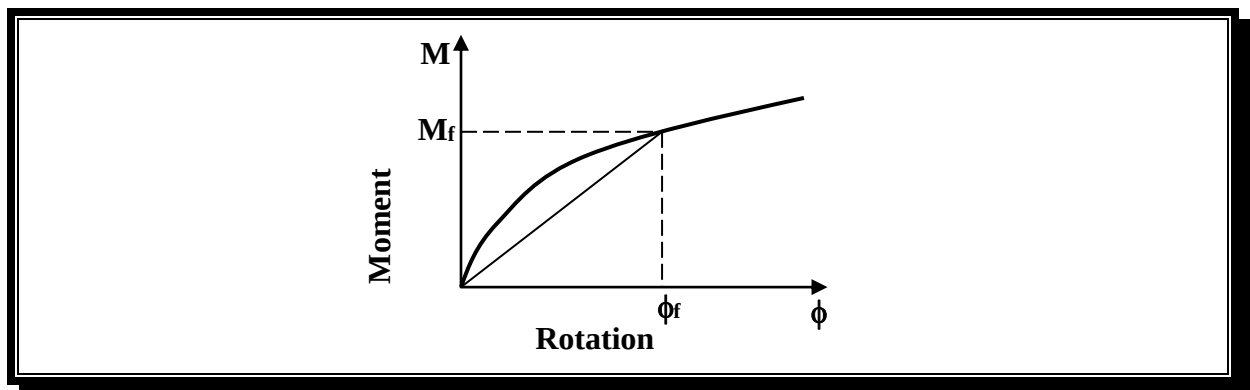


Fig.(2.1): Connection Moment-Rotation Curve and Secant Stiffness .

In 1981, **Moncarz and Gerstle [35]** presented an iterative non-linear elastic analysis procedure by modifying the conventional linear elastic stiffness matrix to take into account the presence of non-linear elastic connection. The method uses two rotational springs at the ends of a beam-column element and then determines the modified stiffness matrix after applying the static condensation technique to eliminate the additional two rotational D.O.F. and thus determining the modified fixed end force vector.

In 1984, **Ang and Morris [36]** generalized the **Frye and Morris [33]** procedure to permit the analysis of three-dimensional rectangular frames with non-linear flexible connections. They considered only one non-linear connection force-displacement relationship which they modeled by using Ramberg-Osgood function. They analyzed an 11- story unbraced frame, first modeling the non-linear behavior of the top and set an angle beam-to-column connection and then assuming the connection to be rigid.

In 1986, **Lui and Chen [37]** presented a procedure for analyzing flexibly jointed plane frames by using the finite element approach. They took into account three types of non-linearity, geometrical, material and connectional non-linearity. The beam-column element formulation was based on updated Lagrangian approach, allowing for the coupling effect of the axial force and bending moment as well as for the formation of plastic hinges in the member.

In 1987, **Blandford [38]** presented the elastic response of plane frames including the non-linear semi-rigid connection behavior, post-buckling strength of thin-walled steel members and first-order geometric non-linearity. Semi-rigid connection behavior was modeled by using an exponential approximation of the connection flexibility-moment relationship.

In 1988, **Yu and Shanmugam [4]** developed a computer program for solving the matrix method to study the stability of space frames. The factors such as geometrical effects, effects of flexural on axial stiffness and semi-rigidity of the end connections influencing the stability have been incorporated

through modified stiffness matrices. Single-story and double-story, single bay frames have been analyzed by varying the fixing factors. The results show that the elastic critical load for the frames can be increased significantly even by a marginal increase in fixity factors. Furthermore, the increase in fixity factors does not affect the elastic buckling capacity of the frames but also the loading position on the frame has considerable influence on the buckling load.

In 1989, **Shi and Alturi [16]** presented a procedure for non-linear analysis of space frames with non-linear flexible joints and subjected to static and dynamic loads. A complementary energy approach based on a weak form of the compatibility condition as a whole of a frame member, and of the joint equilibrium conditions for the frame, was found to be best suited for the analysis of flexibly jointed frames. In this study, semi-rigid connection behavior is modeled by using Ramberg-Osgood function. Several examples, with both quasi-static and dynamic loading were included to illustrate the accuracy and efficiency of the developed methodology.

In 1993, **Al-Musallam and Richard [39]** presented an elastic-plastic plane frames analysis by a finite element method which included the effect of non-linear connections behavior to predict the ultimate load carrying capacity of steel frames. The structural model comprises members which are assumed to behave elastically with a non-linear connection moment-rotation behavior.

In 1994, **Al-Sadder [40]** presented a theoretical analysis for estimating the in-plane large displacement elastic-plastic stability behavior of steel frames having non-linear steel connections using beam-column approach. Yielding is considered to be concentrated at the member ends, and changes in member chord length due to the axial strain and flexural bowing are taking into account. The moment-rotation behavior of steel connection is considered to be non-linear. The standardized Ramberg-Osgood function is used to model the non-linear behavior of connection.

In 2002, **Al-Khafaji [41]** presented a theoretical analysis for the pre- and post-buckling behavior of plane steel frames with non-prismatic members,

non-linear flexible steel connections and subjected to static loads applied at joints only. The analysis adopts the beam-column approach and it is based on using two types of elements, namely, beam-column (frame) element and connection element. Also, he made a comparison between the different iterative strategies and the different incremental strategies to decide which of each group was best, more efficient and general.

2.5 Analysis of Frames under Earthquake Effects

In 1976, **Cheng and Oster [42]** studied the effect of coupling earthquake motions on inelastic structural models by deterministic analysis in which the vertical ground motion is represented by a sinusoidal function. The responses of two structural models subjected to the coupling earthquake motions of the vertical ground motion together with the horizontal component of the (El- Centro) earthquake were studied by using a bilinear material model in the analysis.

In 1984, **Neuss and Maison [6]** presented a matrix formulation to account for $(P-\Delta)$ effects in a computer seismic analysis of multistory buildings. The method employs a linear solution approach requiring no iteration and can be used for static or dynamic elastic analyses. The method has been implemented in a computer program and sample seismic analyses of a 31-multistory building are presented. They made observations on the appropriate use and the interpretation of analysis.

In 1985, **Neuss and Maison [43]** performed an extensive computer analysis of an existing 44-story steel frame high-rise building to study the influence of various modeling aspects on the predicted dynamic properties and computed seismic response behavior. The predicted dynamic properties were compared to the buildings true properties as previously determined from experimental testing. They provided interpolations of the analysis results.

In 1996, **Zaidan [44]** presented a theoretical analysis for estimating the plane large displacement elastic stability behavior of steel frames subjected to seismic loads. The analysis adopts the beam-column approach and was based on using the beam-column frame element. The formulation of the beam-column element was based on Eulerian approach, changes in member chord length due to axial strain and flexural bowing were taken into account.

In 1997, **Liu [45]** presented a theoretical analysis to the dynamic behavior and performance of steel moment resisting frames subjected to horizontal and vertical ground motion components of selected earthquake records. Towards the goal, three buildings of 5-, 10- and 20-storey were analyzed as structural models. The results show that in the elastic range, the vertical ground motion has almost no influence on the seismic response of the structures except the axial forces and in the inelastic range, the vertical ground motion becomes a significant factor in contributing to the extent and pattern of the structural damage.

In 2003, **Al-Janaby [46]** presented a numerical solution to find the hydrodynamic pressure developed due to the earthquake in the dam-reservoir system. The solution was found for the following assumptions: (1) the base of the reservoir is considered to be horizontal and rigid, (2) the water is considered to be incompressible. Three cases were considered horizontal acceleration only, vertical acceleration only and combined acceleration. Simplified procedure has been adopted to analyze concrete gravity dam with sloping face including hydrodynamic effect; a direct integration method was used to solve this problem.

In 2004, **Selim [47]** presented a theoretical analysis of two dimensional frames subjected to seismic loads by the spectrum method. A new computer program was developed in order to define all geometric and loading data and to perform non-linear analysis of rigid and semi-rigid steel frames from which the performances will be evaluated. In this study 3-floor steel frames that have different bay numbers were investigated in various forms according to the

rigid and different semi-rigid connection types. In addition, the performance of these frames for various seismic regions and soil conditions were compared.

In 2004, **Fattah and Kadhim [48]** formulated the analysis of guyed towers subjected to normal operating loads and earthquake loads. In that paper, a non-linear dynamic finite element analysis was carried out for a typical guyed tower and the tower was subjected to (El-Centro) earthquake excitation. The computer program SAP2000 was used in the analysis and the method of non-linear time-history analysis used in SAP2000 program was an extension of the fast non-linear analysis method.

In 2006, **Jarenprasert, et al. [49]** presented a consistent approach for deriving an inelastic design spectrum and estimating the maximum displacements directly through statistical studies of inelastic response spectrum without the need to restore first to the elastic response spectrum.

2.6 Optimal Design of Structures

In 1970, **Rozvany and Cohn [50]** presented an optimal design approach for reinforced concrete frames. The only design variable was the amount of steel reinforcement, the lengths and the cross sections of all members were assumed to be fixed. The analysis approach was based on the lower bound theorem to determine the limit load and the force in the critical sections. Yield condition, serviceability and plastic compatibility were considered as constraints on the design optimization problem. There were no constraints on the deflection and cracking. The design for shear and bond was not considered.

In 1980, **Gerlein and Beaufait [51]** presented a formulation of preliminary optimal design of steel multistory-multibay frames. The design variables were the plastic moment capacity of sections. The problem was

solved by a linear programming method namely “Simplex Method” applied to story by story under the kinematic constraints and design constraints satisfying building code requirements and practical design considerations and the design was based on a rigid fully plastic material behavior using upper bound approach. The plastic hinges were assumed to be formed in beams only and shear deformations as well as axial deformations were all neglected.

In 1996, **Striz and Sobieski [52]** presented a research for a new and highly efficient methodology for the subset of structural optimization. The proposed method was detailed i.e., for the weight minimization of a given structure under size, strength, and displacement constraints. Specifically, the finite element based multilevel optimization of structures was performed. In the system level optimization, the design variables were the coefficients of assumed polynomial based global displacement functions, and the load unbalance resulting from the solution of the global stiffness equations was minimized. In the subsystems level optimizations, the weight of each element was minimized under the action of stress constraints, with the cross sectional dimensions as design variables.

In 2000, **Papadrakakis, et al. [53]** presented a paper to perform a structural optimization under seismic loading. Combinatorial optimization methods and in particular algorithms based on the evolution strategies were implemented for the solution of large-scale structural optimization problems under seismic loading. In this work the efficiency of a rigorous approach in treating dynamic loading is investigated and compared with a simplified dynamic analysis in the framework of finding the optimum design of a structure with the minimum weight. A number of accelerograms was produced from the design spectrum of the region. These accelerograms constituted the multiple loading conditions under which the structures were optimally designed.

In 2005, **Olhoff [54]** presented a devoted paper to fundamental aspects and methods for topology and shape design optimization of engineering

structures. The methods were selected from conditions of versatility and suitability for integration into an engineering design optimization system which realizes the design process as an iterative solution procedure of multicriterion optimization problem based on the concept of integration of finite element analysis, design sensitivity analysis, and optimization by mathematical programming.

In 2005, **Tsompanakis, et al. [55]** presented a robust and efficient methodology for treating large-scale reliability based structural optimization problems under seismic loads using the pushover analysis method. The optimization problem was solved with the “Evolution Strategies Method”, while the reliability analysis was carried out with the “Monte Carlo simulation Method”.

From the preceding review of literature it is clear that no study was found that considers the non-linear analysis and optimal design of space steel frames with flexible connections and elastic-plastic behavior subjected to static and dynamic loads. Such study case is presented in the present work.



Chapter

III

Static Analysis of Space Structures

3.1 General

The aim of this chapter is to provide a theoretical basis for the elastic-plastic analysis of space steel structures having non-rigid connections and subjected to static loads.

The complete derivation of the conventional tangent stiffness matrix in local, intermediate and global coordinates is presented. The new modified tangent stiffness matrix is derived and takes into account the presence of flexible linear and/or non-linear connections and the presence of plastic hinges. It is to be noted that a significant difficulty has been encountered in calculating the end bending moments and connection rotations due to presence of highly non-linear equations for each beam-column member and solving by Newton-Raphson iteration. The special procedure used in the calculations of member axial forces has been explained in details and the computational technique used in solving the non-linear simultaneous equations is also

presented. Finally, a brief description of the convergence criteria usually used in the computer programs is presented in this chapter.

3.2 Modeling of A Beam-Column Element

The modeling of the geometrically non-linear behavior of an elastic beam-column element has been the subject of researches of a number of workers. Generally, two approaches can be used:

1. Finite element approach [56, 57].
2. Beam-Column approach [5, 11].

In the first approach, the basic idea is relatively simple in theory, but a large number of elements is often needed for satisfactory accuracy. In the second approach, member force-deformation relations are determined from a more complex, but also more refined analysis, so that fewer elements are generally needed than in the case of a finite element approach.

3.2.1 Beam-Column Approach

The beam-column element developed in the present study uses the beam-column approach. The term “beam-column” generally refers to an initially straight member subjected to a prescribed combination of axial and lateral loads.

The main assumptions and consideration points used here, in the modeling of the beam-column element are as follows:

1. The member is prismatic and plane section before deformation remains plane after deformation.
2. Large rigid-body displacements are allowed and relative member deformations are assumed to be small enough to justify the use of beam-column theory.
3. Influence of axial force on member stiffness is taken into account.

4. Changes in member chord length due to axial strain and flexural bowing are taken into account.
5. Shear deformations are neglected.
6. Proportional and non-proportional increases in loading are taken into account.
7. Post-critical behavior is neglected.
8. The member has 2-axis of symmetry, so that the centroid and the shear center of the member cross-section coincide and twisting and bending of a member occur independently.
9. Plastic effects are assumed localized at plastic hinge points and elastic behavior is assumed between plastic hinges.
10. Material is assumed elastic-perfectly plastic, and any possible strain hardening is neglected as shown in **Fig.(3.1)**.
11. Reduction of the plastic moment capacity of the cross-section as a result of the presence of axial force is taken into account.
12. The steel connections considered assumed to have non-linear moment-rotation characteristics.
13. Unloading path of steel connection is taken into account.
14. Connections are of negligible size. Hence, connection deformations are assumed concentrated at the ends of the beams and columns.
15. The steel connections are assumed to behave in an elastic-perfectly plastic manner, and any possible strain hardening in connections is neglected.

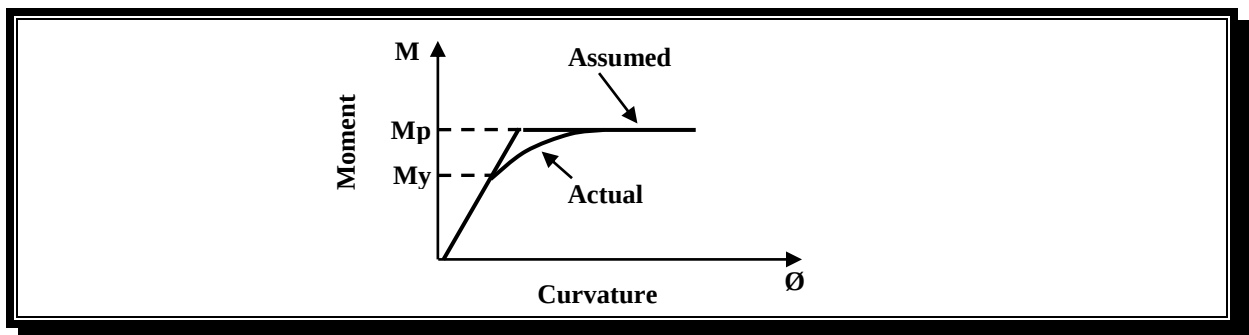


Fig.(3.1): Idealized Moment-Curvature Relationship.

3.2.2 System Equilibrium Equation

The equilibrium equations of a structure as a whole can be written symbolically [9]

$$\mathbf{f}_i(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n) - \mathbf{F}_i = 0 \quad \text{for } i=1, 2, \dots, n \quad \dots\dots\dots(3.1)$$

in which $\mathbf{x}_i$ = generalized coordinates (essentially, translations and rotations of the joints); $\mathbf{F}_i$ = load terms (prescribed external forces and moments acting at the joints); and $\mathbf{f}_i$ = non-linear functions representing resultant internal forces corresponding to $\mathbf{x}_i$. When the problem is analyzed by the beam-column approach, $\{\mathbf{f}\}$ cannot be expressed explicitly in terms of $\{\mathbf{x}\}$, because member axial force does remain in the picture as auxiliary parameters.

Equation (3.1) can be rewritten in terms of differentials as:

$$\{\Delta \mathbf{F}\} = [\mathbf{K}] \{\Delta \mathbf{x}\} \quad \dots\dots\dots(3.2)$$

where $\{\Delta \mathbf{x}\}$ and $\{\Delta \mathbf{F}\}$ represent displacement and load increments vectors respectively, and

$$[\mathbf{K}] = \left[\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_i} \right] \quad \dots\dots\dots(3.3)$$

in which, $[\mathbf{K}]$ is the tangent stiffness matrix which is a symmetric square matrix.

3.2.3 Member Force-Deformation Relations

The basic member force-deformation relations [5] for a prismatic member shown in **Fig.(3.2)** are:

$$\mathbf{M}_{1n} = \frac{EI_n}{L} (C_{1n} \theta_{1n} + C_{2n} \theta_{2n}) \quad \dots\dots\dots(3.4)$$

$$\mathbf{M}_{2n} = \frac{EI_n}{L} (C_{2n} \theta_{1n} + C_{1n} \theta_{2n}) \quad \dots\dots\dots(3.5)$$

$$\mathbf{M}_t = \frac{C_t}{L} \phi_t \quad \dots\dots\dots(3.6)$$

$$\mathbf{Q} = \frac{EA}{L} (u - C_{bz} \cdot L - C_{by} \cdot L) \quad \dots\dots\dots(3.7)$$

in which,

- n** : either y or z,
C_{1n} and **C_{2n}** : conventional elastic stability functions, where,
C_{1n} : flexural stiffness factor with respect to the (n)-axis,
C_{2n} : moment carry-over factor with respect to the (n)-axis,
C_t : torsional constant,
u : axial relative displacement.
 and,

$$C_{bn} = b_{1n}(\theta_{1n} + \theta_{2n})^2 + b_{2n}(\theta_{1n} - \theta_{2n})^2 \dots\dots\dots(3.8)$$

is the length correction factor due to bowing action,

- b_{1n}** and **b_{2n}** : bowing functions with respect to the (n)-axis,
E : modulus of elasticity,
I_n : moment of inertia with respect to the (n)-axis,
A : cross-sectional area,
L : member initial (undeformed) length.

Stability and bowing functions are dependent on axial forces parameter (**q_n**) and their explicit transcendental and series expressions are given in (**Appendix A**).

Writing $\epsilon_n = \frac{I_n}{I} \dots\dots\dots(3.9)$

where **I** : reference moment of inertia,

$$q_n = \frac{Q}{Q_E} = \frac{Q.L^2}{\pi^2 EI_n} \dots\dots\dots(3.10)$$

when if $\lambda = \frac{L}{\sqrt{I/A}}$ and $q = \epsilon_n . q_n$

Equation (3.7) can be rewritten as:

$$q = \frac{\lambda^2}{\pi^2 L} (u - C_{bz} . L - C_{by} . L) \dots\dots\dots(3.11)$$

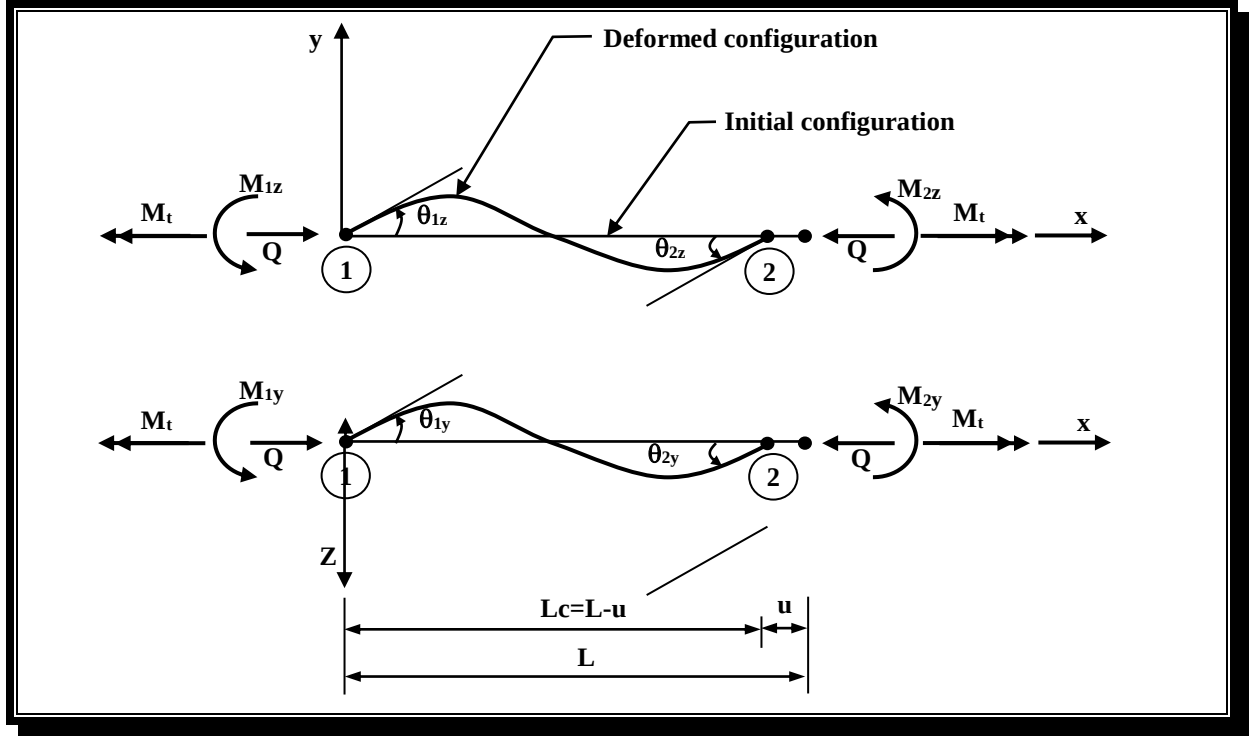


Fig.(3.2): Relative Member Deformations and Corresponding Forces.

The torsion factor “ C_t ” considers the influence of axial force on member torsional stiffness [58, 60],

for $C_w \neq 0$ and $GJ - Qr^2 > 0$

$$C_t = \frac{GJ - Qr^2}{\left(1 - \frac{2}{KL} \tanh \frac{KL}{2}\right)} \dots\dots\dots(3.12)$$

$$K = \sqrt{\frac{1}{EC_w} (GJ - Qr^2)} \dots\dots\dots(3.13)$$

for $C_w \neq 0$ and $GJ - Qr^2 < 0$

$$C_t = \frac{GJ - Qr^2}{\left(1 - \frac{2}{KL} \tan \frac{KL}{2}\right)} \dots\dots\dots(3.14)$$

$$K = \sqrt{\frac{1}{EC_w} (Qr^2 - GJ)} \dots\dots\dots(3.15)$$

for $GJ - Qr^2 = 0$

$$C_t = 0 \quad \dots\dots\dots(3.16)$$

for $C_w = 0$

$$C_t = GJ - Qr^2 \quad \dots\dots\dots(3.17)$$

in which,

GJ : is the torsional rigidity,

r : radius of gyration,

C_w : warping constant.

The warping torsional constant (C_w) measures the resistance of a structural member to non-uniform or warping torsion. It is used in calculating the buckling moment resistance of laterally unsupported beams and torsional-flexural buckling of compression members [59].

For open symmetrical sections like doubly symmetric wide flange shapes (W-shapes and I-beams) the warping torsional constant is [70]:

$$C_w = \frac{(d')^2 b^3 t}{24} \quad \dots\dots\dots(3.18)$$

$$d' = d - t \quad \dots\dots\dots(3.19)$$

in which,

d : the total depth of the section (mm),

t : the flange depth of the section (mm),

b : the flange width of the section (mm).

For closed structural sections, the warping deformations are small and the warping torsional constant is generally taken as zero.

3.2.4 Member End Effects

An arbitrary prismatic member of a space frame is considered shown in **Fig.(3.3)**, and $\{F\}$ and $\{\bar{F}\}$ are used to denote member end forces in the global and local coordinates respectively, the relationship between $\{F\}$ and $\{\bar{F}\}$ can be written as [58, 60]:

$$\{F\} = [R]\{\bar{F}\} \quad \dots\dots\dots(3.20)$$

[R]: is the (12x12) orthogonal matrix and defined as:

$$[R] = \begin{bmatrix} [r] & 0 & 0 & 0 \\ 0 & [r] & 0 & 0 \\ 0 & 0 & [r] & 0 \\ 0 & 0 & 0 & [r] \end{bmatrix} \quad \dots\dots\dots(3.21)$$

[r] : is the (3x3) member orientation matrix, which refers to the deformed configuration of the member.

Similarly the member forces $\{\bar{F}\}$ are related to the forces $\{S\}$ associated with the relative member deformations, **Fig.(3.2)** and **Fig.(3.3)**, by:

$$\{\bar{F}\} = [B]\{S\} \quad \dots\dots\dots(3.22)$$

[B] : is the instantaneous static matrix defined as:

$$[B] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ B_{y1} & B_{y2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -B_{z1} & B_{z2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ -B_{y2} & -B_{y1} & 0 & 0 & 0 & 0 \\ 0 & 0 & B_{z2} & B_{z1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \dots\dots\dots(3.23)$$

in which ,

$$B_{y1} = B_{y2} = B_{z1} = B_{z2} = \frac{1}{Lc} \quad \dots\dots\dots(3.24)$$

$$Lc = L - u = [\{(X_2 + V_7) - (X_1 + V_1)\}^2 + \{(Y_2 + V_8) - (Y_1 + V_2)\}^2 + \{(Z_2 + V_9) - (Z_1 + V_3)\}^2]^{\frac{1}{2}} \quad \dots\dots\dots(3.25)$$

Lc : is the member chord length of the deformed configuration,

X_1, X_2, Y_1, Y_2, Z_1 and Z_2 : are the initial global coordinates of end (1) and end (2) respectively,

V_1, V_7 : global displacements in X-direction of end (1) and end (2) respectively,

V_2, V_8 : global displacements in Y-direction of end (1) and end (2) respectively,

V_3, V_9 : global displacements in Z-direction of end (1) and end (2) respectively.

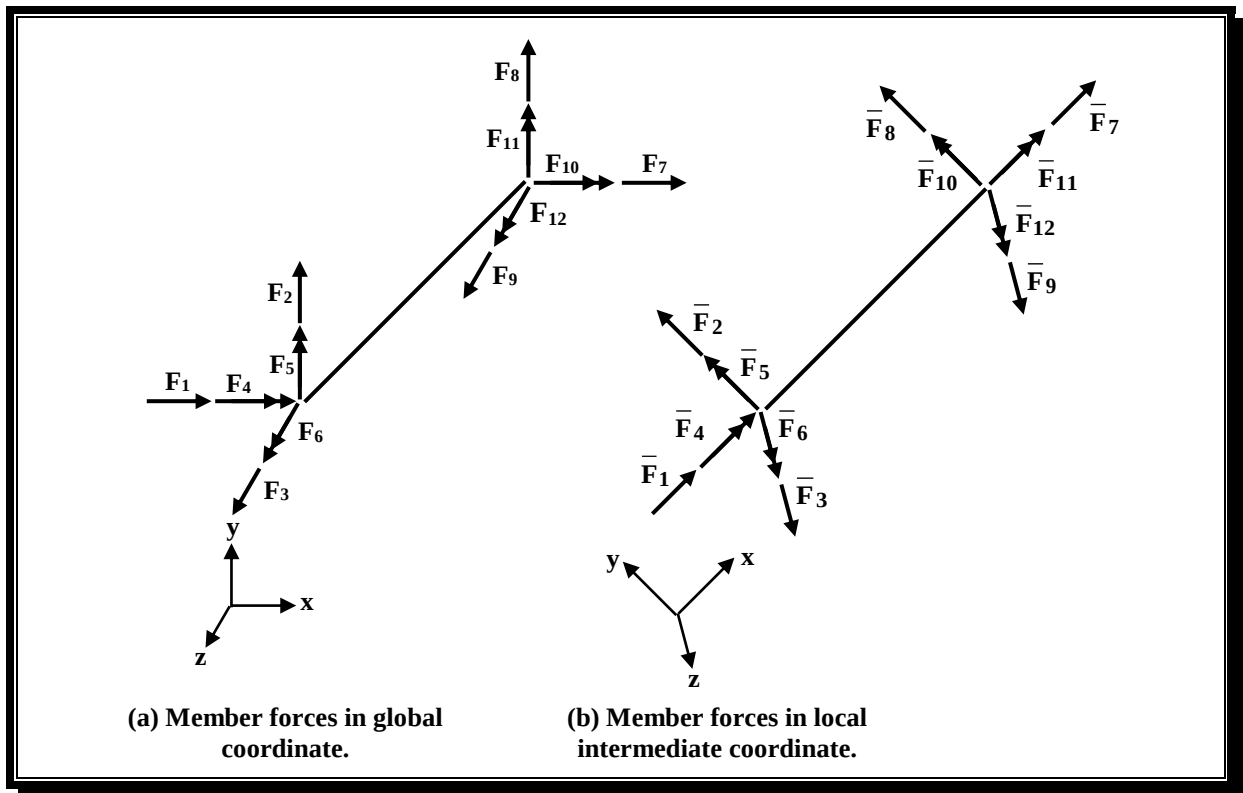


Fig.(3.3): Member End Coordinates.

3.2.5 Member Orientation Matrix

3.2.5.1 Initial Member Orientation Matrix $[r_0]$

The transformation from local member axes to the global axes may be considered to take place in three steps, **Fig.(3.4)**, [60]:

1. Rotation through the angle “ γ ” about the “**z-axis**”. This rotation places the “**x-axis**” in the position denoted “ x_β ”, which is the intersection of (**X-Z**) plane and (**X-Y**) plane, where:

$$[r_\gamma] = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \dots\dots\dots(3.26)$$

$$\cos\gamma = \sqrt{C_x^2 + C_z^2} \dots\dots\dots(3.27)$$

$$\sin\gamma = C_y \dots\dots\dots(3.28)$$

$$C_x = \frac{(X_2 - X_1)}{L} \dots\dots\dots(3.29)$$

$$C_y = \frac{(Y_2 - Y_1)}{L} \dots\dots\dots(3.30)$$

$$C_z = \frac{(Z_2 - Z_1)}{L} \dots\dots\dots(3.31)$$

2. Rotation through the angle “ β ” about the “**y-axis**”. This rotation places the “**x-axis**” in its final position, where:

$$[r_\beta] = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \dots\dots\dots(3.32)$$

$$\cos\beta = \frac{C_x}{\sqrt{C_x^2 + C_z^2}} \dots\dots\dots(3.33)$$

$$\sin\beta = \frac{C_z}{\sqrt{C_x^2 + C_z^2}} \dots\dots\dots(3.34)$$

3. Rotation through the angle “ ϕ ” about the “**X-axis**” causing the “**y** and **z**” **axes** to coincide with the principal axes of the cross-section (**X, Y, Z**), where:

$$[r_\phi] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \dots\dots\dots(3.35)$$

So, the initial orientation member matrix

$$[r_0] = [r_\gamma][r_\beta][r_\phi]$$

$$[r_0] = \begin{bmatrix} C_x \frac{-C_x C_y \cos\phi - C_z \sin\phi}{\sqrt{C_x^2 + C_z^2}} & \frac{C_x C_y \sin\phi - C_z \cos\phi}{\sqrt{C_x^2 + C_z^2}} \\ C_y \frac{\sqrt{C_x^2 + C_z^2} \cos\phi}{\sqrt{C_x^2 + C_z^2}} & -\sqrt{C_x^2 + C_z^2} \sin\phi \\ C_z \frac{-C_y C_z \cos\phi - C_x \sin\phi}{\sqrt{C_x^2 + C_z^2}} & \frac{-C_y C_z \sin\phi - C_x \cos\phi}{\sqrt{C_x^2 + C_z^2}} \end{bmatrix} \quad \text{.....(3.36)}$$

Step (3) is performed only when the member has its axes in skew directions. In many instances, a space structure member will be oriented so that the axes of the cross-section lie in horizontal and vertical planes (i.e. the angle “ θ ” is zero).

The preceding orientation matrix $[\mathbf{r}_0]$ is valid for all positions of the member. For vertical members since there is no rotation through the angle “ β ”, and with the presence of only the first rotation through the angle “ γ ” which may be either (90^0 or 270^0), the initial member orientation matrix is seen to be:

$$[r_0] = \begin{bmatrix} 0 & -C_y \cos \phi & C_y \sin \phi \\ C_y & 0 & 0 \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \dots\dots\dots (3.37)$$

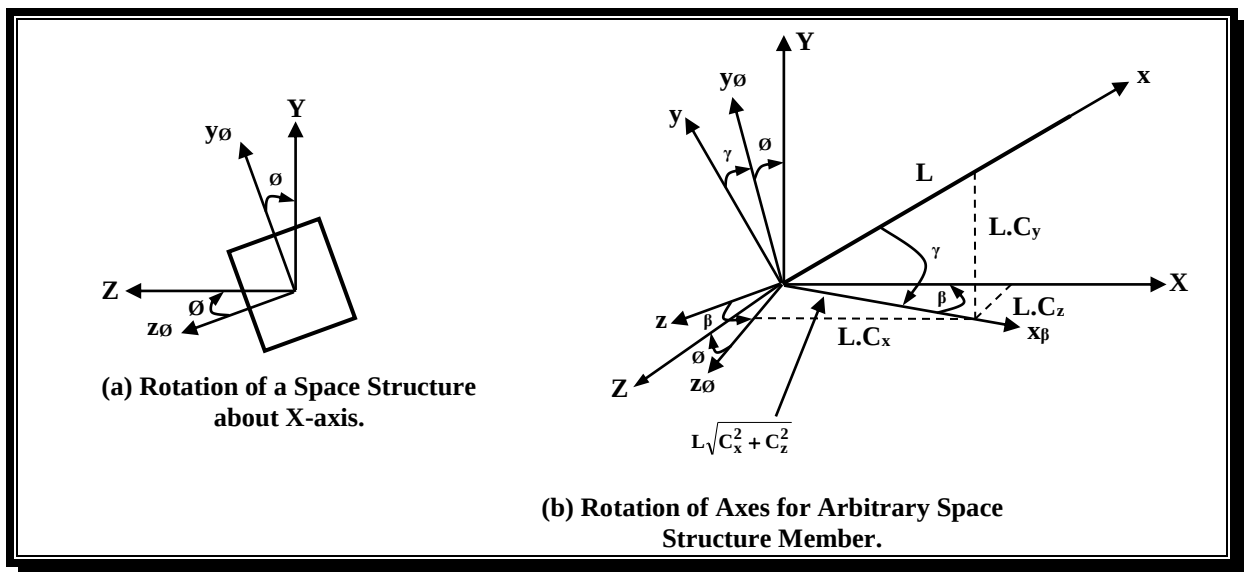


Fig.(3.4): Transformation of Axes.

3.2.5.2 Current Member Orientation Matrix

Oran had suggested a method to find the current member orientation matrix [60], this method is based on the variation of the principal directions from one section to another, the incremental joint orientation matrix is calculated in each load increment and the total joint orientation matrix is updated for the next increment. The current member orientation matrix, then is obtained by averaging the orientations of the two end sections of the member.

Let **[W]** be the joint orientation matrix:

$$[W] = \begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \end{bmatrix} \dots\dots\dots(3.38)$$

For convenience, these joint reference lines can be assumed to be parallel to the global coordinate axes in the initial undeformed configuration of the system, thus:

$$[W_0] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \dots\dots\dots(3.39)$$

The incremental rotation matrix for a joint in global coordinates is:

$$[\Delta\Omega] = \begin{bmatrix} 0 & -\Delta\Omega_3 & \Delta\Omega_2 \\ \Delta\Omega_3 & 0 & -\Delta\Omega_1 \\ \Delta\Omega_2 & \Delta\Omega_1 & 0 \end{bmatrix} \dots\dots\dots(3.40)$$

in which either,

$$\begin{aligned} \Delta\Omega_1 &= \Delta V_4, \Delta\Omega_2 = \Delta V_5, \Delta\Omega_3 = \Delta V_6 \\ \text{or} \quad \Delta\Omega_1 &= \Delta V_{10}, \Delta\Omega_2 = \Delta V_{11}, \Delta\Omega_3 = \Delta V_{12} \end{aligned} \dots\dots\dots(3.41)$$

The resulting change in the joint orientation matrix is expressed as:

$$[\Delta W]^i = [\Delta\Omega][W]^{i-1} \dots\dots\dots(3.42)$$

and the updated joint orientation matrix at “**i th**” load level or iteration, can be evaluated as:

$$[W]^i = [W]^{i-1} + [\Delta W]^i \dots\dots\dots(3.43)$$

Writing $[P^{(1)}]$ and $[P^{(2)}]$ to be end section orientation matrices defined as follows:

$$\begin{aligned} [P^{(1)}] &= [W^{(1)}] [r_0] \\ [P^{(2)}] &= [W^{(2)}] [r_0] \end{aligned} \quad \dots\dots\dots(3.44)$$

Then the first column $\{r_1\}$ of the desired current deformation member orientation matrix $[r]$, can be obtained directly from the deformed joint global coordinates, and to determine the second and third columns of $[r]$, the following is used:

$$[e^{(j)}] = \begin{bmatrix} 1 & \theta_{jz} & -\theta_{jy} \\ -\theta_{jz} & 1 & 0 \\ \theta_{jy} & 0 & 1 \end{bmatrix} \quad \text{for } j=1, 2 \dots\dots\dots(3.45)$$

Then the new end section orientation matrices will be given by the products of $[P^{(1)}][e^{(1)}]$ and $[P^{(2)}][e^{(2)}]$ respectively, thus resulting in the following expression for the member orientation matrix:

$$[r] = \frac{1}{2} \{ [D^{(1)}] + [D^{(2)}] \} \quad \dots\dots\dots(3.46)$$

$$\text{where: } [D^{(n)}] = [P^{(n)}] [e^{(n)}] \quad \dots\dots\dots(3.47)$$

3.2.6 Relative Member Deformations

The relative member deformations can be obtained directly in two stages [58, 60, 61]:

(i) In the first stage, the displacement vector for the initial (undeformed) local configuration $\{\bar{V}_0\}$, **Fig.(3.5)**, is obtained by using the initial transformation matrix $[R_0]$, thus:

$$\{\bar{V}_0\} = [R_0]^T \{V\} \quad \dots\dots\dots(3.48)$$

(ii) In the second stage, the initial displacement vector $\{\bar{V}_0\}$ is transformed to the end relative Eulerian displacements, **Fig.(3.5)**, by using the following relations:

$$u = L - Lc \quad \dots\dots\dots(3.49)$$

$$\theta_{1z} = V_6 - \rho_z \quad \dots\dots\dots(3.50)$$

$$\theta_{2z} = V_{12} - \rho_z \quad \dots\dots\dots(3.51)$$

$$\theta_{1y} = V_5 - \rho_y \quad \dots\dots\dots(3.52)$$

$$\theta_{2y} = V_{11} - \rho_y \quad \dots\dots\dots(3.53)$$

where:

$$\rho_z = \tan^{-1} \frac{\bar{V}_{0y}}{\left\{ \left(L + \bar{V}_{0x} \right)^2 + \left(\bar{V}_{0z} \right)^2 \right\}^{\frac{1}{2}}} \quad \dots\dots\dots(3.54)$$

$$\rho_y = \tan^{-1} \frac{\bar{V}_{0z}}{\left\{ \left(L + \bar{V}_{0x} \right)^2 + \left(\bar{V}_{0y} \right)^2 \right\}^{\frac{1}{2}}} \quad \dots\dots\dots(3.55)$$

and,

$$\bar{V}_{0x} = \bar{V}_{07} - \bar{V}_{01} \quad \dots\dots\dots(3.56)$$

$$\bar{V}_{0y} = \bar{V}_{08} - \bar{V}_{02} \quad \dots\dots\dots(3.57)$$

$$\bar{V}_{0z} = \bar{V}_{09} - \bar{V}_{03} \quad \dots\dots\dots(3.58)$$

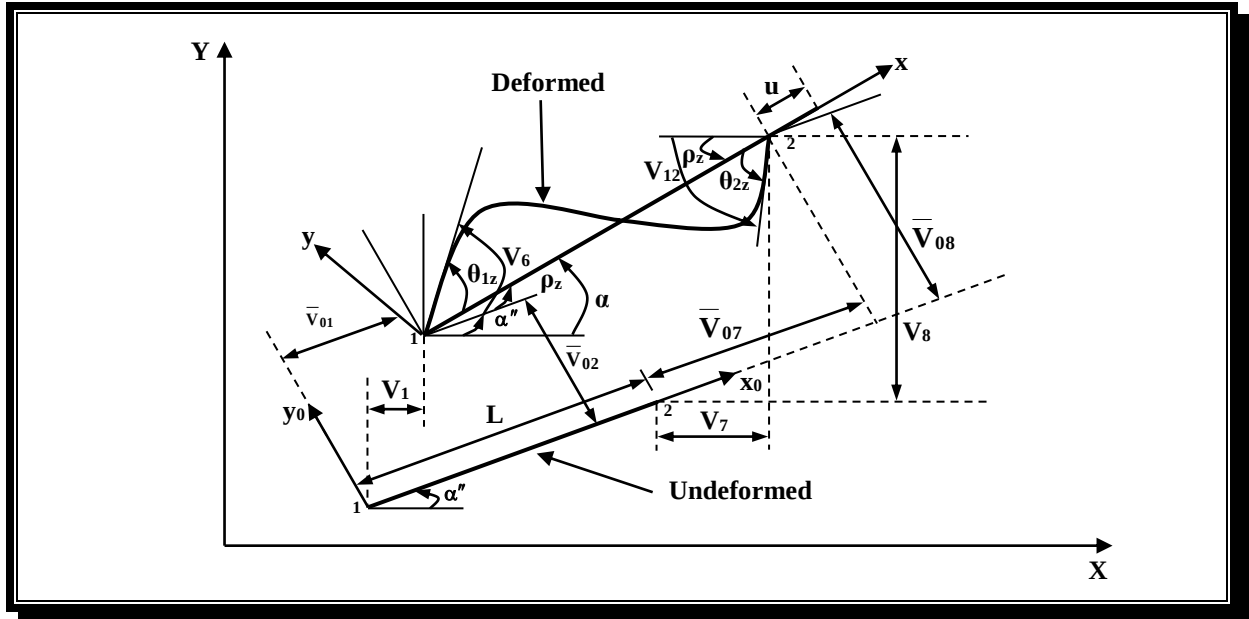


Fig.(3.5): Member End Displacements in Global and Initial Coordinates.

Oran, suggested the following procedure to determine the relative rotations [17, 60]:

$$\begin{Bmatrix} 1 \\ -\theta_{1z} \\ \theta_{1y} \end{Bmatrix} = [\mathbf{P}^{(1)}]^T \{\mathbf{r}_1\} \quad \dots\dots\dots(3.59)$$

$$\begin{Bmatrix} 1 \\ -\theta_{2z} \\ \theta_{2y} \end{Bmatrix} = [\mathbf{P}^{(2)}]^T \{\mathbf{r}_1\} \quad \dots\dots\dots(3.60)$$

where $[\mathbf{P}^{(1)}]$ and $[\mathbf{P}^{(2)}]$: are end section orientation matrices,
 $\{\mathbf{r}_1\}$: is the first column of the current member orientation matrix, and can be obtained directly from the deformed joints global coordinates, thus:

$$\{\mathbf{r}_1\} = \begin{Bmatrix} \bar{C}_x \\ \bar{C}_y \\ \bar{C}_z \end{Bmatrix} \quad \dots\dots\dots(3.61)$$

$$\bar{C}_x = \frac{(X_2 + V_7) - (X_1 + V_1)}{L_c} \quad \dots\dots\dots(3.62)$$

$$\bar{C}_y = \frac{(Y_2 + V_8) - (Y_1 + V_2)}{L_c} \quad \dots\dots\dots(3.63)$$

$$\bar{C}_z = \frac{(Z_2 + V_9) - (Z_1 + V_3)}{L_c} \quad \dots\dots\dots(3.64)$$

To determine the relative torsional rotation “ ϕ_t ”, the following relation is used [17, 61]:

$$\phi_t = \bar{V}_{0t} = \bar{V}_{010} - \bar{V}_{04} \quad \dots\dots\dots(3.65)$$

and **Oran** proposed the following equation:

$$\phi_t = -\{\mathbf{P}_2^{(1)}\}^T \{\mathbf{P}_3^{(2)}\} = \{\mathbf{P}_3^{(1)}\}^T \{\mathbf{P}_2^{(2)}\} \quad \dots\dots\dots(3.66)$$

3.2.7 Member Tangent Stiffness Matrix in Local Eulerian Coordinates

The relations between incremental vectors $\{\mathbf{S}\}$ and $\{\mathbf{u}\}$ can be expressed in a matrix form as [5, 17, 58]:

$$\{\Delta \mathbf{S}\} = [\mathbf{t}] \{\Delta \mathbf{U}\} \quad \dots\dots\dots(3.67)$$

in which $[\mathbf{t}]$: is the tangent stiffness matrix for relative deformations, with

$$t_{ij} = \frac{\partial S_i}{\partial U_j} + \frac{\partial S_i}{\partial q} \cdot \frac{\partial q}{\partial U_j} \quad \text{for } i, j=1, 2, \dots, 6. \quad \dots\dots\dots(3.68)$$

knowing that:

$$\frac{\partial q}{\partial \theta_{mn}} = \frac{G_{mn}}{\pi^2 H}, \frac{\partial q}{\partial \phi_t} = 0, \frac{\partial q}{\partial U} = \frac{1}{HL}, m = 1, 2, n = y, z. \quad \dots\dots\dots(3.69)$$

$$G_{1n} = C'_{1n} \theta_{1n} + C'_{2n} \theta_{2n} \quad \dots\dots\dots(3.70)$$

$$G_{2n} = C'_{2n} \theta_{1n} + C'_{1n} \theta_{2n} \quad \dots\dots\dots(3.71)$$

$$H = \frac{\pi^2}{\lambda^2} + \sum_{n=y,z} \frac{1}{\epsilon_n} \left[b'_{1n} (\theta_{1n} + \theta_{2n})^2 + b'_{2n} (\theta_{1n} - \theta_{2n})^2 \right] \quad \dots\dots\dots(3.72)$$

The element stiffness matrix in local Eulerian coordinates becomes;

$$[t] = \frac{EI}{L} \begin{bmatrix} \epsilon_z C_{1z} + \frac{G_{1z}^2}{\pi^2 H} & \epsilon_z C_{2z} + \frac{G_{1z} G_{2z}}{\pi^2 H} & \frac{G_{1z} G_{1y}}{\pi^2 H} & \frac{G_{1z} G_{2y}}{\pi^2 H} & 0 & \frac{G_{1z}}{HL} \\ \epsilon_z C_{2z} + \frac{G_{1z} G_{2z}}{\pi^2 H} & \epsilon_z C_{1z} + \frac{G_{2z}^2}{\pi^2 H} & \frac{G_{2z} G_{1y}}{\pi^2 H} & \frac{G_{2z} G_{2y}}{\pi^2 H} & 0 & \frac{G_{2z}}{HL} \\ \frac{G_{1z} G_{1y}}{\pi^2 H} & \frac{G_{2z} G_{1y}}{\pi^2 H} & \epsilon_y C_{1y} + \frac{G_{1y}^2}{\pi^2 H} & \epsilon_y C_{2y} + \frac{G_{1y} G_{2y}}{\pi^2 H} & 0 & \frac{G_{1y}}{HL} \\ \frac{G_{1z} G_{2y}}{\pi^2 H} & \frac{G_{2z} G_{2y}}{\pi^2 H} & \epsilon_y C_{2y} + \frac{G_{1y} G_{2y}}{\pi^2 H} & \epsilon_y C_{1y} + \frac{G_{2y}^2}{\pi^2 H} & 0 & \frac{G_{2y}}{HL} \\ 0 & 0 & 0 & 0 & \frac{C_t}{EI} & 0 \\ \frac{G_{1z}}{HL} & \frac{G_{2z}}{HL} & \frac{G_{1y}}{HL} & \frac{G_{2y}}{HL} & 0 & \frac{\pi^2}{L^2 H} \end{bmatrix} \quad \dots\dots\dots(3.73)$$

3.2.8 Member Tangent Stiffness Matrix in Local Member Intermediate Coordinates

Rewriting equation (3.22) in terms of differentials, thus:

$$\{\Delta \bar{F}\} = [B]\{\Delta S\} + [\Delta B]\{S\} \quad \dots\dots\dots(3.74)$$

in which $[\Delta B]$: is the change in $[B]$ resulting from $\{\Delta \bar{V}\}$.

The “kth” column of $[\Delta B]$ can be expressed as:

$$[\Delta B] = \bar{g}^{(k)} \{\Delta \bar{V}\} \quad \dots\dots\dots(3.75)$$

where $\bar{g}^{(k)}$: is the (12x12) geometry matrix [5, 17, 58], thus,

$$[\bar{g}^{(k)}] = \begin{bmatrix} [f^{(k)}] & [-f^{(k)}] \\ [-f^{(k)}] & [f^{(k)}] \end{bmatrix} \quad \dots\dots\dots(3.76)$$

in which $[f^{(k)}]$: are (6x6) matrices defined as follows:

$$\begin{array}{llll}
 \text{for } k=1, 2 & \mathbf{f}_{ij}^{(k)} = \mathbf{0} & \text{except} & \mathbf{f}_{12}^{(k)} = \mathbf{f}_{21}^{(k)} = \frac{1}{\mathbf{Lc}^2} \\
 \text{for } k=3, 4 & \mathbf{f}_{ij}^{(k)} = \mathbf{0} & \text{except} & \mathbf{f}_{13}^{(k)} = \mathbf{f}_{31}^{(k)} = \frac{1}{\mathbf{Lc}^2} \\
 \text{for } k=5 & \mathbf{f}_{ij}^{(k)} = \mathbf{0} & & \\
 \text{for } k=6 & \mathbf{f}_{ij}^{(k)} = \mathbf{0} & \text{except} & \mathbf{f}_{22}^{(k)} = \mathbf{f}_{33}^{(k)} = \frac{1}{\mathbf{L.Lc}}
 \end{array}$$

From Equation (3.67) and knowing that:

$$\{\Delta \mathbf{U}\} = [\mathbf{B}]^T \{\Delta \bar{\mathbf{V}}\} \dots\dots\dots(3.77)$$

then, Equation (3.74) can be rewritten as:

$$\{\Delta \bar{\mathbf{F}}\} = [\bar{\mathbf{T}}] \{\Delta \bar{\mathbf{V}}\} \dots\dots\dots(3.78)$$

$$\text{in which } [\bar{\mathbf{T}}] = [\mathbf{B}] [\mathbf{t}] [\mathbf{B}]^T + \sum_{k=1}^6 \left[\bar{\mathbf{g}}^{(k)} \right] \mathbf{S}_k \dots\dots\dots(3.79)$$

is the (12x12) symmetrical member tangent stiffness matrix in local member coordinates for a space beam element.

3.2.9 Member Tangent Stiffness Matrix in Global Coordinates

If $\{\Delta \mathbf{F}\}$ and $\{\Delta \mathbf{V}\}$ denote increments of member end forces and displacements respectively in global coordinates, then [5, 17, 58]:

$$\{\Delta \mathbf{F}\} = [\mathbf{R}] \{\Delta \bar{\mathbf{F}}\} \dots\dots\dots(3.80)$$

$$\{\Delta \bar{\mathbf{V}}\} = [\mathbf{R}]^T \{\Delta \mathbf{V}\} \dots\dots\dots(3.81)$$

Substituting equations (3.78), (3.79) and (3.81) into equation (3.80), the latter can be rewritten as:

$$\{\Delta \mathbf{F}\} = [\mathbf{K}] \{\Delta \mathbf{V}\} \dots\dots\dots(3.82)$$

$$\text{in which } [\mathbf{K}] = [\mathbf{R}] [\bar{\mathbf{T}}] [\mathbf{R}]^T \dots\dots\dots(3.83)$$

is the desired member tangent stiffness matrix in global coordinates.

3.3 General Information About Semi-Rigid Connections

The steel frame performs as an integrated structure deriving its strength from the individual elements, which are properly located to transmit loads throughout the structure. The connections behavior has an effect on the frame performance, because they are the basic elements and integrated part of a steel frame. The Load and Resistance Factor Design (LRFD) specification of the American Institute of Steel Construction (AISC) categorizes two basic types of steel frame construction as follows [33, 37]:

1. Fully restrained (FR) type construction, with the assumption of full continuity and adequate rigidity of beam to column connections, which can retain the initial angles between intersection members.
2. Partially restrained (PR) type construction, with the assumption that a connection possesses moment capacity in between complete fixity and the pin connection.

Conventional analysis and design of steel frameworks are performed by using the assumption that the connections are either fully rigid or ideally pinned. No relative rotation of connection occurs and the end moments of the beam are completely transferred to the columns in fully rigid connections. Contrary to this type, the pinned connection implies that no restraint for rotation of connection exists and the connection moment is always zero. The available design techniques are based on these idealized conditions. But, in reality most connections are semi-rigid and fall under PR construction type.

Most of the connections that fasten beams to columns by using angles, plates, welds, and bolts are deformable and perform in non-linear behavior between fully fixed and perfectly pinned conditions. So it is more reasonable to classify all connections under the classification of semi-rigid, whereas rigid and pinned conditions are special cases. Accurate determination of the relative

restraint of beam to column connection is important for both strength and serviceability of structural frames. Overestimating the connection restraint can result in underestimating lateral sway and underestimating the connection restraint can lead to underestimating forces developed in the beams and columns, both conditions can affect structural stability. **Fig.(3.6)** shows the comparison between connection types.

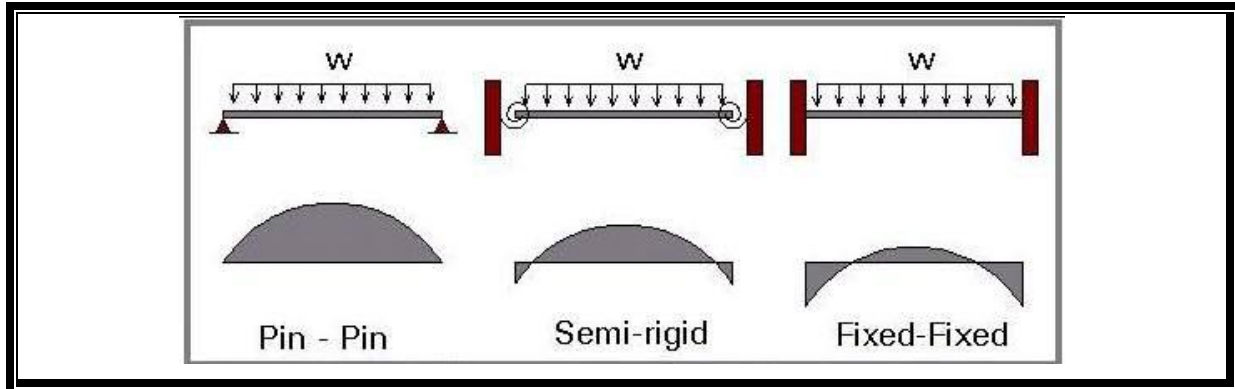


Fig.(3.6): Comparison of Semi-Rigid Connections vs. Pinned and Fixed Connections.

3.4 The Most Common Types of Semi-Rigid Connections

Fig. (3.7) shows the relative moment rotation behavior of a variety of commonly used semi-rigid connections [36, 47]. All types of connections exhibit non-linear moment-rotation behavior that falls between the two extreme cases of ideally pinned and fully rigid connections. The single web-angle connection represents a flexible joint and the T-stub connection represents a rather rigid joint.

A popular model for structural analysis is the polynomial function proposed by **Frye and Morris** [33]. The Frye-Morris model was developed based on a procedure by **Somner** [62]. They used the method of least squares to determine the constants of the polynomial. The model has the form:

$$\phi = C_1(KM) + C_2(KM)^3 + C_3(KM)^5 \dots\dots\dots(3.84)$$

where:

ϕ : is the angle of rotation of connection (rad.),

M : is the moment of the connection (Kips-in),

C_i : are curve-fitting constants,

K : is the standardization dimensionless factor.

Table (3.1) gives the size parameters (constants) of Equation (3.84), while the eight types of connections included in the table are given in **Fig.(3.8)**. All dimensions used in **Table (3.1)** are in (inches).

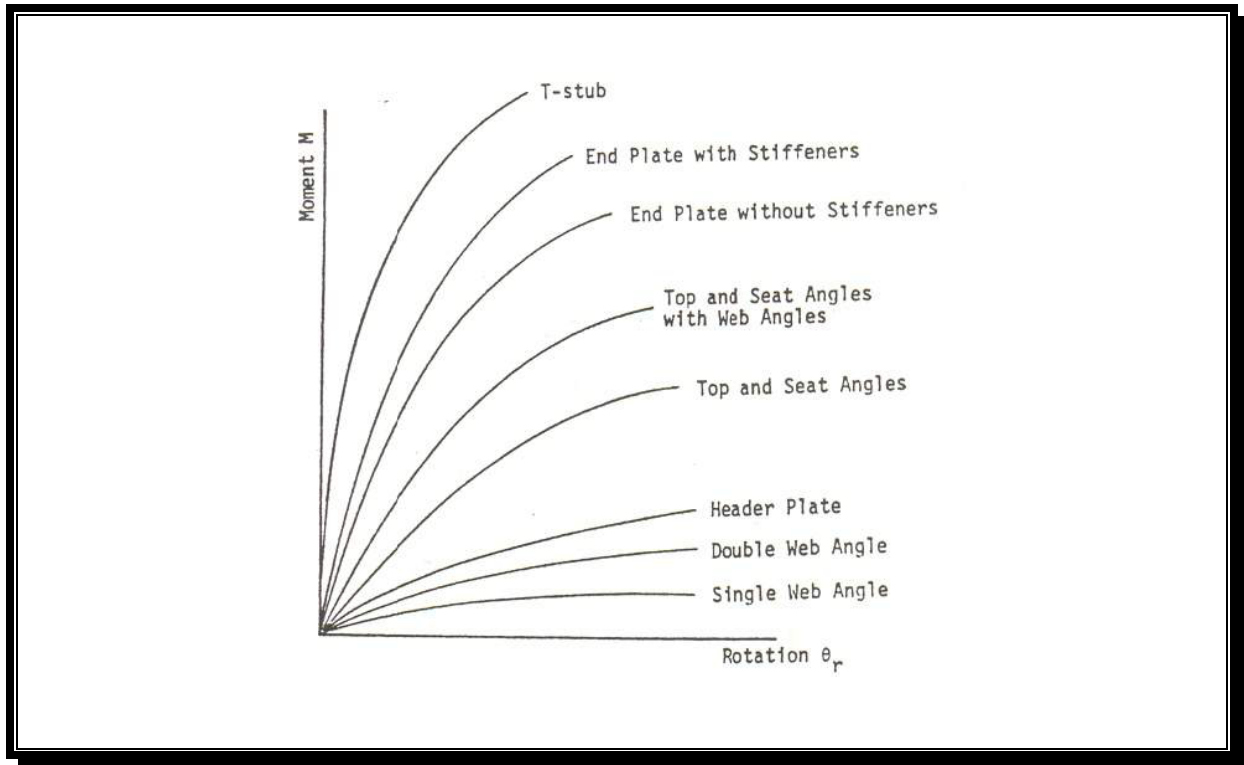


Fig.(3.7): Moment-Rotation Behavior of Commonly Used Semi-Rigid Connections [36, 47].

Table (3.1): Standardized Connections Constant [36, 47].

Connection Types	Curve-Fitting Constants	Standardization Constant
Single Web-Angle	$C_1=4.28*10^{-3}; C_2=1.45*10^{-9}$ $C_3=1.51*10^{-16}$	$K = d_a^{-2.4} t_a^{-1.81} g^{0.15}$
Double Web-Angle	$C_1=3.66*10^{-4}; C_2=1.15*10^{-6}$ $C_3=4.57*10^{-8}$	$K = d_a^{-2.4} t_a^{-1.81} g^{0.15}$
Top and Seat Angle with Double Web Angle	$C_1=2.23*10^{-5}; C_2=1.85*10^{-8}$ $C_3=3.19*10^{-12}$	$K = d^{-1.287} t^{-1.128} t_c^{-0.415}$ $L_a^{-0.694} g^{1.35}$
Top and Seat Angle without Double Web Angle	$C_1=8.46*10^{-4}; C_2=1.01*10^{-4}$ $C_3=1.24*10^{-8}$	$K = d^{-1.5} t^{-0.5} L_a^{-0.7} d_b^{-1.5}$
End Plate without Column Stiffeners	$C_1=1.83*10^{-3}; C_2=1.04*10^{-4}$ $C_3=1.24*10^{-8}$	$K = d_g^{-2.4} t_p^{-0.4} d_b^{-1.5}$
End Plate with Column Stiffeners	$C_1=1.79*10^{-3}; C_2=1.76*10^{-4}$ $C_3=2.04*10^{-4}$	$K = d_g^{-2.4} t_p^{-0.5}$
T-Stub	$C_1=2.10*10^{-4}; C_2=6.20*10^{-6}$ $C_3=-7.60*10^{-9}$	$K = d^{-1.5} t^{-0.5} L_t^{-0.7} d_b^{-1.1}$
Header Plate	$C_1=5.10*10^{-5}; C_2=6.20*10^{-10}$ $C_3=2.40*10^{-13}$	$K = d_p^{-2.3} t_p^{-1.6} t_w^{-0.5} g^{1.6}$

3.5 Modeling of Flexible Connection

The behavior of a flexible connection is usually described by the moment-rotation curves of the connection in which the slope of the curve corresponds to the rotational rigidity of the connections. The non-linear behavior of the flexible connections can be expressed approximately by bilinear or trilinear functions or by some other types of functions for example, polynomial or exponential functions [63].

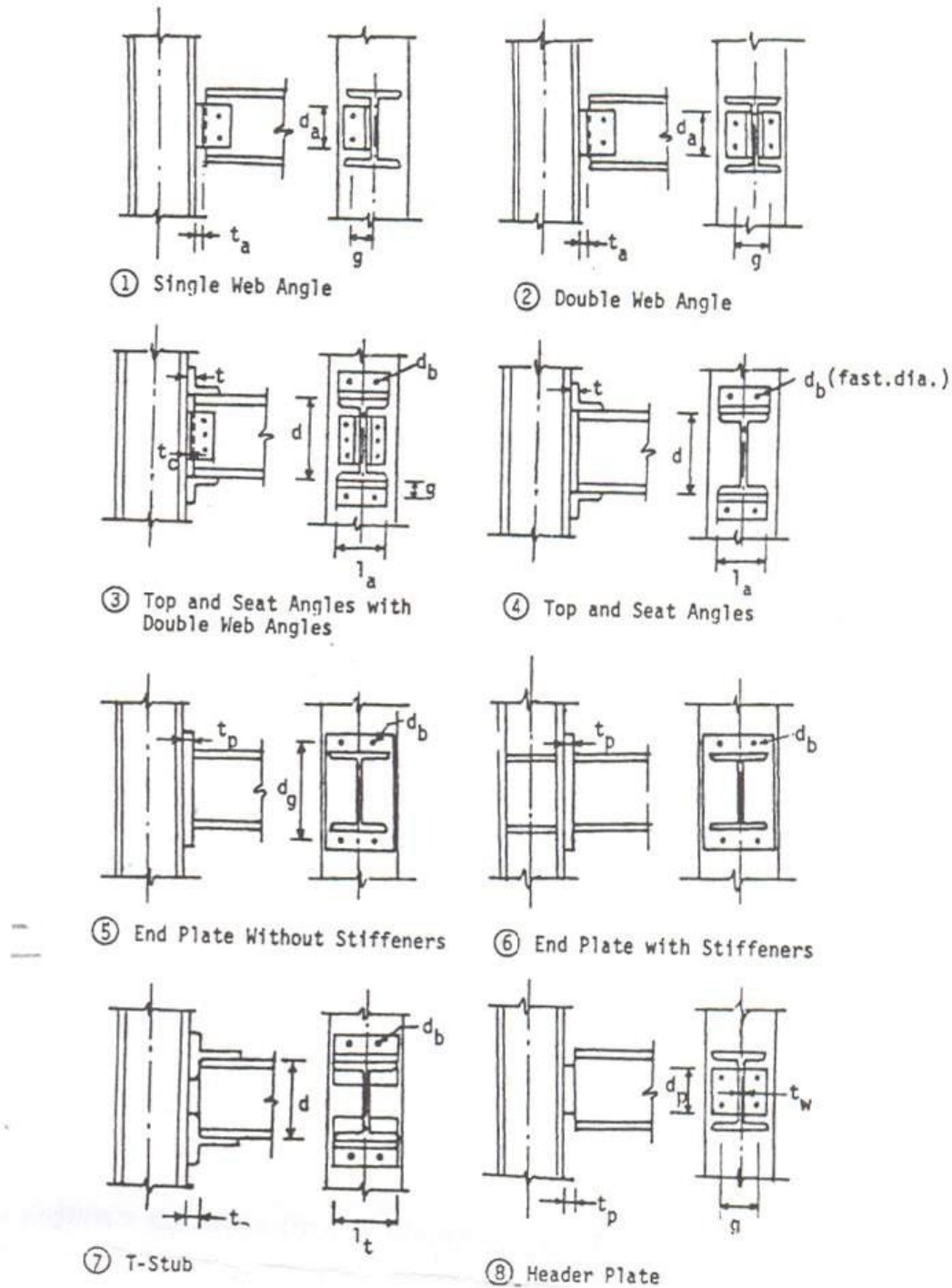


Fig.(3.8): Parameters of “8” Commonly Used Types of fastener Semi Rigid Connections [36, 47].

Because the behavior for the non-linear flexible connection is actually represented in a computational model by the instantaneous rotational stiffness of the connection (i.e., the slope of the moment-rotation curve), the property of the function used for the moment-rotation curve is very important for the numerical analysis of flexibly jointed frame structures.

By considering the properties of a moment-rotation curve, it is evident that a non-linear mathematical expression for the curve must satisfy the following requirements [64]:

1. $M=0$ at $\theta=0$ (the curve passes through the origin).
2. $dM/d\theta=R_{ki}$ at $\theta=0$ (the slope of the curve at the origin is equal to the initial elastic stiffness of the connection).
3. For any value of θ the slope of the expression corresponds to the tangent stiffness of the connection.
4. The parameters in the expression are physically meaningful.
5. The parameters are easily and accurately determinable.
6. The expression is of relatively simple form.

A polynomial expression,

$$\theta = a + bM + cM^2 + dM^3 + \dots \dots \dots (3.85)$$

can be used and has a considerable accuracy for the moment-rotation relation. Here, **a**, **b**, **c**, and **d** are obtained from a polynomial regression of experimental data. By taking a sufficient number of terms, almost any **M- θ** curve can be closely fitted, but the parameters have very little physical meaning. Also, negative values of the slope (stiffness) of the function may occur within the working range of the connection. So, it should be noted that the fitting polynomial expression must satisfy the above requirements.

The connection will have different stiffnesses during its loading or unloading conditions, **Fig.(3.9)**, [35]. If the connection is under loading condition, the instantaneous or tangent stiffness is to be used, i.e.:

$$R = R_{kt} = \left. \frac{dM}{d\phi} \right|_{\phi=\phi} \dots \dots \dots (3.86)$$

for unloading condition, the initial stiffness is to be used, i.e.:

$$R = R_{ki} = \left. \frac{dM}{d\phi} \right|_{\phi=0} \dots\dots\dots(3.87)$$

To determine whether a connection is in a loading or unloading condition, the moment at each connection of the frame is monitored at every load step. An increase in moment indicates loading and a decrease signifies unloading.

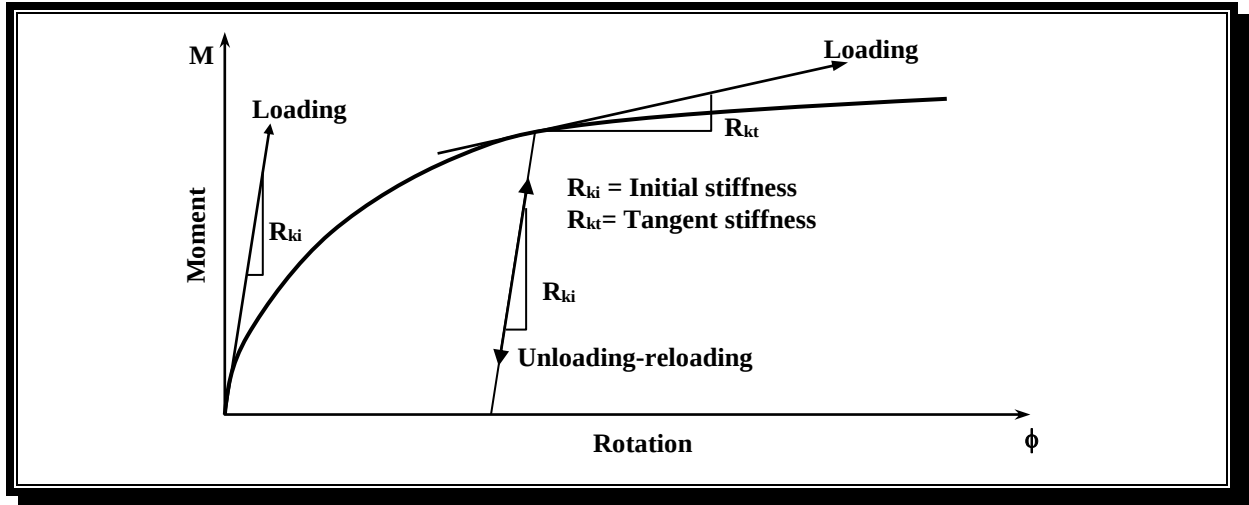


Fig.(3.9): Loading-Unloading Behavior of Steel Connections.

3.6 Modification of Incremental Beam-Column Stiffness Matrix to Account for Presence of Connections

Assuming that the axial and shearing deformation of the connection are negligible, the incremental force-displacement relationship of a connection can be written as:

$$\begin{Bmatrix} \Delta M_{1cnz} \\ \Delta M_{2cnz} \\ \Delta M_{1cny} \\ \Delta M_{2cny} \\ \Delta M_{1cnx} \\ \Delta M_{2cnx} \end{Bmatrix} = \begin{bmatrix} R_{1z} & -R_{1z} & 0 & 0 & 0 & 0 \\ -R_{1z} & R_{1z} & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{1y} & -R_{1y} & 0 & 0 \\ 0 & 0 & -R_{1y} & R_{1y} & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{1x} & -R_{1x} \\ 0 & 0 & 0 & 0 & -R_{1x} & R_{1x} \end{bmatrix} \begin{Bmatrix} \Delta V_{1cnz} \\ \Delta V_{2cnz} \\ \Delta V_{1cny} \\ \Delta V_{2cny} \\ \Delta V_{1cnx} \\ \Delta V_{2cnx} \end{Bmatrix} \dots\dots\dots(3.88)$$

where ΔV_{1cn} and ΔV_{2cn} are the incremental rotational D.O.F of the connection, **Fig.(3.10)**.

For a frame beam-column element, the incremental force-displacement relationship is:

$$\begin{Bmatrix} \Delta F_1 \\ \Delta F_2 \\ \vdots \\ \vdots \\ \Delta F_{12} \end{Bmatrix} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \cdots & \cdots & \mathbf{K}_{12} \\ \vdots & & & & \vdots \\ \vdots & & \mathbf{K}_{ij} & & \vdots \\ \vdots & & & & \vdots \\ \mathbf{K}_{121} & \cdots & \cdots & \cdots & \mathbf{K}_{1212} \end{bmatrix} \begin{Bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \vdots \\ \Delta V_{12} \end{Bmatrix} \quad \dots\dots\dots(3.89)$$

where $\mathbf{K}_{ij}$ are the entries of the (12x12) frame element tangent stiffness matrix, Equation (3.83).

The seven elements (six springs and the beam) shown in **Fig.(3.10-a)**, can be jointed together as in **Fig.(3.10-b)** by enforcing equilibrium and compatibility at their junctions. In particular, the kinematics matrix relation for the D.O.F. of the elements in **Fig.(3.10-a, 3.10-b)** can be written as:

$$[\mathbf{r}_r] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \dots\dots\dots(3.90)$$

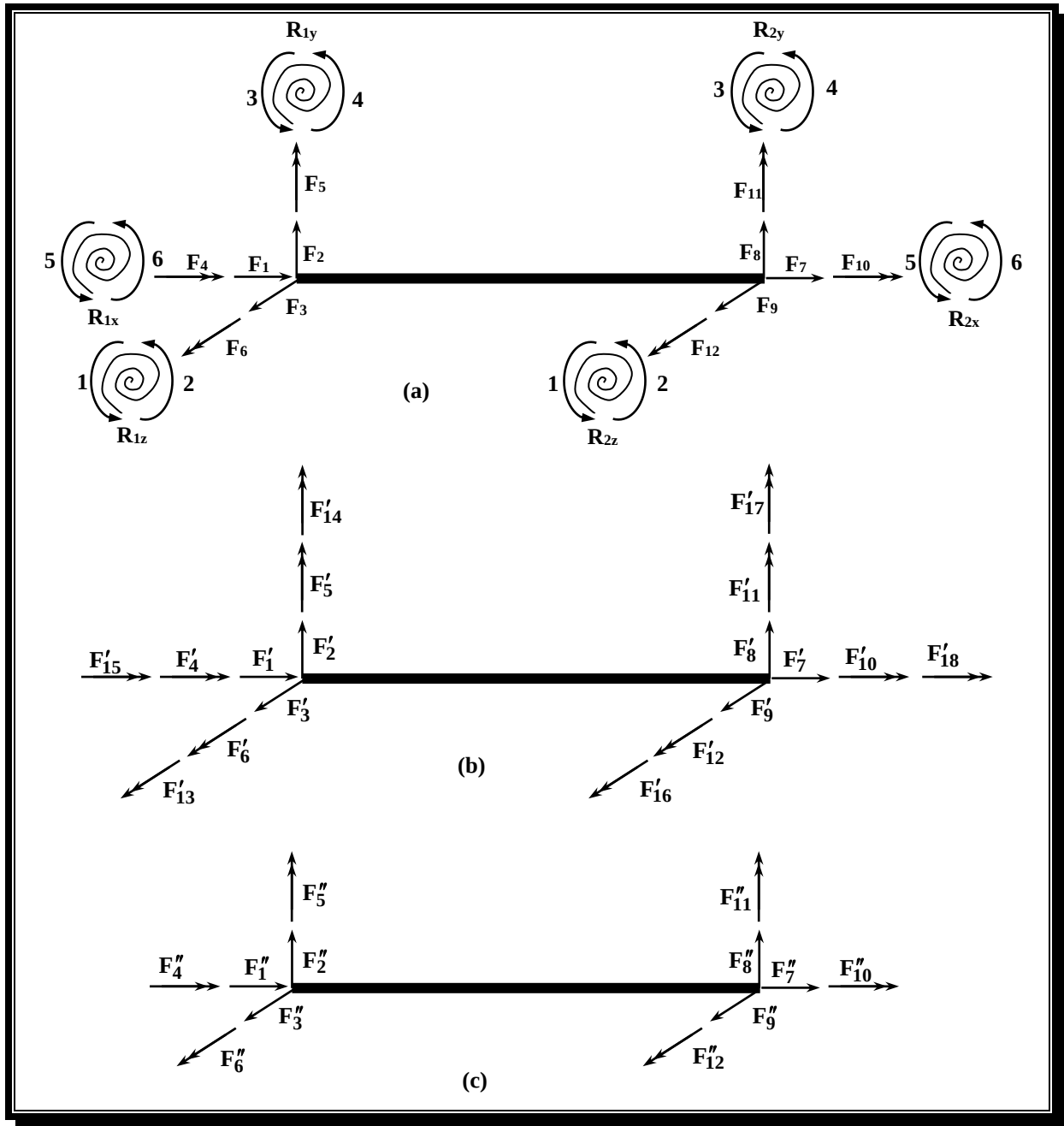


Fig.(3.10): Modification of Beam-Column Element to Account for Presence of Flexible Connection.

The equilibrium relationship between the two systems can be obtained by using the contra-gradient law, then the stiffness matrix of the element in Fig.(3.10-b) is therefore:

$$[K_{cn}]_{18 \times 18} = [T'_f]^T_{18 \times 24} [K_f]_{24 \times 24} [T'_f]_{24 \times 18} \dots\dots\dots(3.91)$$

where:

$$[K_f] = \begin{bmatrix} R_{1z} & -R_{1z} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -R_{1z} & R_{1z} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{1y} & -R_{1y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -R_{1y} & R_{1y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{1x} & -R_{1x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -R_{1x} & R_{1x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{11} & \dots & \dots & K_{112} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \vdots & & & \vdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \vdots & & & \vdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{121} & \dots & \dots & K_{1212} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{2z} & -R_{2z} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R_{2z} & R_{2z} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{2y} & -R_{2y} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R_{2y} & R_{2y} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{2x} & -R_{2x} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R_{2x} & R_{2x} \end{bmatrix} \quad \dots(3.92)$$

Since the D.O.F F'_{13} , F'_{14} , F'_{15} , F'_{16} , F'_{17} and F'_{18} in **Fig.(3.10-b)** are internal D.O.F, they can be removed out of the stiffness matrix $[K_{cn}]$ by static condensation. The modified frame element tangent stiffness matrix accounting for the presence of connections at the ends, **Fig.(3.10-c)**, can be written as:

$$[K_{gf}]_{12 \times 12} = [T_f]_{12 \times 18}^T [K_{cn}]_{18 \times 18} [T_f]_{18 \times 12} \quad \dots\dots\dots(3.93)$$

where:

$$[T_f]_{18 \times 12} = \begin{bmatrix} [I]_{12 \times 12} \\ [T''_{f1}]_{2 \times 12} \\ [T''_{f2}]_{2 \times 12} \\ [T''_{f3}]_{2 \times 12} \end{bmatrix}_{18 \times 12} \quad \dots\dots\dots(3.94)$$

in which,

$$[T''_{f1}] = \frac{1}{R_1^\eta} \begin{bmatrix} K_{121} \cdot K_{126} - K_{61} \cdot \eta_4 & K_{61} \cdot K_{126} - K_{121} \cdot \eta_1 \\ K_{122} \cdot K_{126} - K_{62} \cdot \eta_4 & K_{62} \cdot K_{126} - K_{122} \cdot \eta_1 \\ K_{123} \cdot K_{126} - K_{63} \cdot \eta_4 & K_{63} \cdot K_{126} - K_{123} \cdot \eta_1 \\ 0 & 0 \\ 0 & 0 \\ R_{1z} \cdot \eta_4 & -R_{1z} \cdot K_{126} \\ K_{127} \cdot K_{126} - K_{67} \cdot \eta_4 & K_{67} \cdot K_{126} - K_{127} \cdot \eta_1 \\ K_{128} \cdot K_{126} - K_{68} \cdot \eta_4 & K_{68} \cdot K_{126} - K_{128} \cdot \eta_1 \\ K_{129} \cdot K_{126} - K_{69} \cdot \eta_4 & K_{69} \cdot K_{126} - K_{129} \cdot \eta_1 \\ 0 & 0 \\ 0 & 0 \\ -R_{2z} \cdot K_{126} & R_{2z} \cdot \eta_1 \end{bmatrix}_{2 \times 12} \quad \dots\dots\dots(3.95)$$

$$[T''_{f2}] = \frac{1}{R_2^\eta} \begin{bmatrix} K_{111} \cdot K_{115} - K_{51} \cdot \eta_5 & K_{51} \cdot K_{115} - K_{111} \cdot \eta_2 \\ K_{112} \cdot K_{115} - K_{52} \cdot \eta_5 & K_{52} \cdot K_{115} - K_{112} \cdot \eta_2 \\ K_{113} \cdot K_{115} - K_{53} \cdot \eta_5 & K_{53} \cdot K_{115} - K_{113} \cdot \eta_2 \\ 0 & 0 \\ R_{1y} \cdot \eta_5 & -R_{1y} \cdot K_{115} \\ 0 & 0 \\ K_{117} \cdot K_{115} - K_{57} \cdot \eta_5 & K_{57} \cdot K_{115} - K_{117} \cdot \eta_2 \\ K_{118} \cdot K_{115} - K_{58} \cdot \eta_5 & K_{58} \cdot K_{115} - K_{118} \cdot \eta_2 \\ K_{119} \cdot K_{115} - K_{59} \cdot \eta_5 & K_{59} \cdot K_{115} - K_{119} \cdot \eta_2 \\ 0 & 0 \\ -R_{2y} \cdot K_{115} & R_{2y} \cdot \eta_2 \\ 0 & 0 \end{bmatrix}^T \quad \dots\dots\dots(3.96)$$

2x12

$$[T''_{f3}] = \frac{1}{R_3^\eta} \begin{bmatrix} K_{101} \cdot K_{104} - K_{41} \cdot \eta_6 & K_{41} \cdot K_{104} - K_{101} \cdot \eta_3 \\ K_{102} \cdot K_{104} - K_{42} \cdot \eta_6 & K_{42} \cdot K_{104} - K_{102} \cdot \eta_3 \\ K_{103} \cdot K_{104} - K_{43} \cdot \eta_6 & K_{43} \cdot K_{104} - K_{103} \cdot \eta_3 \\ R_{1x} \cdot \eta_6 & -R_{1x} \cdot K_{104} \\ 0 & 0 \\ 0 & 0 \\ K_{107} \cdot K_{104} - K_{47} \cdot \eta_6 & K_{47} \cdot K_{104} - K_{107} \cdot \eta_3 \\ K_{108} \cdot K_{104} - K_{48} \cdot \eta_6 & K_{48} \cdot K_{104} - K_{108} \cdot \eta_3 \\ K_{109} \cdot K_{104} - K_{49} \cdot \eta_6 & K_{49} \cdot K_{104} - K_{109} \cdot \eta_3 \\ -R_{2x} \cdot K_{104} & R_{2x} \cdot \eta_3 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T \quad \dots\dots\dots(3.97)$$

2x12

and

$$R_1^\eta = \eta_1 \cdot \eta_4 - (K_{126})^2 \quad \dots\dots\dots(3.98)$$

$$\eta_1 = R_{1z} + K_{66} \quad \dots\dots\dots(3.99)$$

$$\eta_4 = R_{2z} + K_{1212} \quad \dots\dots\dots(3.100)$$

$$R_2^\eta = \eta_2 \cdot \eta_5 - (K_{115})^2 \quad \dots\dots\dots(3.101)$$

$$\eta_2 = R_{1y} + K_{55} \quad \dots\dots\dots(3.102)$$

$$\eta_5 = R_{2y} + K_{1111} \quad \dots\dots\dots(3.103)$$

$$R_3^\eta = \eta_3 \cdot \eta_6 - (K_{104})^2 \quad \dots\dots\dots(3.104)$$

$$\eta_3 = \mathbf{R}_{1x} + \mathbf{K}_{44} \dots\dots\dots(3.105)$$

$$\eta_6 = \mathbf{R}_{2x} + \mathbf{K}_{1010} \dots\dots\dots(3.106)$$

and herein $[\mathbf{I}]$ is (12x12) identity matrix. So, the modified frame element tangent stiffness matrix is written as:

$$[\mathbf{K}_t]_{\text{mod.}} = [\mathbf{T}_f]^T [\mathbf{T}'_f]^T [\mathbf{K}_f] [\mathbf{T}'_f] [\mathbf{T}_f] \dots\dots\dots(3.107)$$

where:

$[\mathbf{K}_t]_{\text{mod.}}$: is the modified element tangent stiffness matrix in global coordinates.

3.7 Conventional Modification of Incremental Beam-Column Stiffness Matrix for Simulation of Plastic Hinges

When the moment at any section of a member becomes equal to the plastic moment capacity of the member, a plastic hinge will form, and the slope continuity at the end of the element, where the plastic hinge is located, will be destroyed. In other words, the particular end of the element can rotate independently at the joint and an additional degree of freedom is created. In addition to that, the stiffness of the member will change with plastic hinge.

If a plastic hinge is formed at any end of the element, the change in the incremental basic force-deformation relationship $[\mathbf{t}]$, will occur due to presence of the effect of the plastic hinge $[\mathbf{11}]$. In the present study a new approach will be suggested to include the effect of the plastic hinge.

3.8 Advanced Approach for Modification of Tangent Stiffness Matrix in the Presence of Plastic Hinges

The usual and conventional approach for presence of plastic hinges is to make a modification in the local tangent stiffness matrix. If a plastic hinge is treated as an internal hinge, a row and a column will be deleted from this

matrix. In the proposed new approach a fictitious connection having the **M-Ø** curve shown in **Fig.(3.11)**, is investigated.

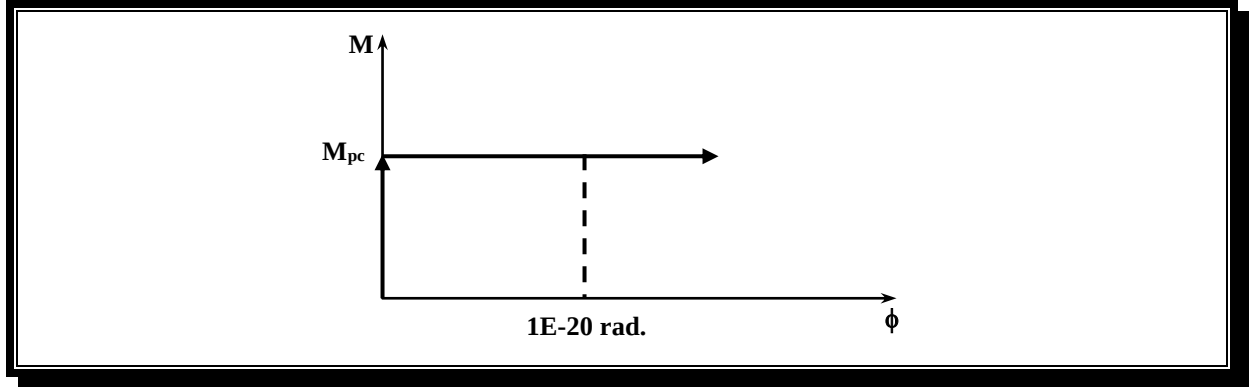


Fig.(3.11): Fictitious M-Ø Curve for Presence of Plastic Hinge.

The two connections (actual and fictitious) are linked in a series, so that,

$$\mathbf{\varnothing}_{\text{new}} = \mathbf{\varnothing}_{\text{actual}} + \mathbf{\varnothing}_{\text{fictitious}} \quad \dots\dots\dots(3.108)$$

$$\mathbf{\varnothing}_{\text{new}} = \mathbf{f}_{\text{new}}(\mathbf{M}) = \mathbf{f}(\mathbf{M}) + \mathbf{g}(\mathbf{M}) \quad \dots\dots\dots(3.109)$$

where,

f(M): is the equation of M-Ø curve for actual steel connection, and

g(M): is the equation of M-Ø curve for fictitious connection.

The effective tangent stiffness of the new connection is:

$$\mathbf{R}_{\text{kt}})_{\text{eff.}} = \left. \frac{d\mathbf{M}}{d\phi} \right)_{\phi=\phi} = \frac{1}{\mathbf{f}'(\mathbf{M}))_{\phi=\phi} + \mathbf{g}'(\mathbf{M}))_{\phi=\phi}} \quad \dots\dots\dots(3.110)$$

in which

$$\mathbf{R}_{\text{kt}})_{\text{act.}} = \frac{1}{\mathbf{f}'(\mathbf{M})}, \mathbf{R}_{\text{kt}})_{\text{fic.}} = \frac{1}{\mathbf{gf}(\mathbf{M})} \quad \dots\dots\dots(3.111)$$

$$\mathbf{R}_{\text{kt}})_{\text{eff.}} = \frac{1}{\frac{1}{\mathbf{R}_{\text{kt}})_{\text{act.}}} + \frac{1}{\mathbf{R}_{\text{kt}})_{\text{fic.}}}} \quad \dots\dots\dots(3.112)$$

and,

$$\mathbf{R}_{\text{kt}})_{\text{fic.}} = \left. \frac{1}{\mathbf{g}'(\mathbf{M})} \right)_{\phi=\phi} = \begin{cases} \rightarrow 1\text{E}20 \text{ EI/L for elastic range} \\ \rightarrow 1\text{E}-20 \text{ EI/L for plastic range} \end{cases} \quad \dots\dots\dots(3.113)$$

So, if a plastic hinge is formed at the end “1” then the tangent stiffness of the connection $\mathbf{R}_{1z}$, $\mathbf{R}_{1y}$ and $\mathbf{R}_{1x}$ will approach to zero value, so that:

$$[\mathbf{K}_f]_{\text{mod.}} = \lim_{\substack{\mathbf{R}_{1z} \rightarrow 0 \\ \mathbf{R}_{1y} \rightarrow 0 \\ \mathbf{R}_{1x} \rightarrow 0}} [\mathbf{K}_f] \quad \dots\dots\dots(3.114)$$

$$[\mathbf{T}_f]_{\text{mod.}} = \lim_{\substack{\mathbf{R}_{1z} \rightarrow 0 \\ \mathbf{R}_{1y} \rightarrow 0 \\ \mathbf{R}_{1x} \rightarrow 0}} [\mathbf{T}_f] \quad \dots\dots\dots(3.115)$$

$$[\mathbf{K}_t]_{\text{mod.}(new)} = [\mathbf{T}_f]_{\text{mod.}}^T [\mathbf{T}'_f] [\mathbf{K}_f]_{\text{mod.}} [\mathbf{T}'_f] [\mathbf{T}_f]_{\text{mod.}} \quad \dots\dots\dots(3.116)$$

If plastic hinge is formed at end “2” then $\mathbf{R}_{2z}$, $\mathbf{R}_{2y}$ and $\mathbf{R}_{2x}$ will approach to zero value, so that:

$$[\mathbf{K}_f]_{\text{mod.}} = \lim_{\substack{\mathbf{R}_{2z} \rightarrow 0 \\ \mathbf{R}_{2y} \rightarrow 0 \\ \mathbf{R}_{2x} \rightarrow 0}} [\mathbf{K}_f] \quad \dots\dots\dots(3.117)$$

$$[\mathbf{T}_f]_{\text{mod.}} = \lim_{\substack{\mathbf{R}_{2z} \rightarrow 0 \\ \mathbf{R}_{2y} \rightarrow 0 \\ \mathbf{R}_{2x} \rightarrow 0}} [\mathbf{T}_f] \quad \dots\dots\dots(3.118)$$

$$[\mathbf{K}_t]_{\text{mod.}(new)} = [\mathbf{T}_f]_{\text{mod.}}^T [\mathbf{T}'_f] [\mathbf{K}_f]_{\text{mod.}} [\mathbf{T}'_f] [\mathbf{T}_f]_{\text{mod.}} \quad \dots\dots\dots(3.119)$$

If plastic hinge is formed at both ends “1” and “2” then $\mathbf{R}_{1z}$, $\mathbf{R}_{1y}$, $\mathbf{R}_{1x}$, $\mathbf{R}_{2z}$, $\mathbf{R}_{2y}$ and $\mathbf{R}_{2x}$ will approach to zero value, so that:

$$[\mathbf{K}_f]_{\text{mod.}} = \lim_{\substack{\mathbf{R}_{1z}, \mathbf{R}_{2z} \rightarrow 0 \\ \mathbf{R}_{1y}, \mathbf{R}_{2y} \rightarrow 0 \\ \mathbf{R}_{1x}, \mathbf{R}_{2x} \rightarrow 0}} [\mathbf{K}_f] \quad \dots\dots\dots(3.120)$$

$$[\mathbf{T}_f]_{\text{mod.}} = \lim_{\substack{\mathbf{R}_{1z}, \mathbf{R}_{2z} \rightarrow 0 \\ \mathbf{R}_{1y}, \mathbf{R}_{2y} \rightarrow 0 \\ \mathbf{R}_{1x}, \mathbf{R}_{2x} \rightarrow 0}} [\mathbf{T}_f] \quad \dots\dots\dots(3.121)$$

$$[\mathbf{K}_t]_{\text{mod.}(new)} = [\mathbf{T}_f]_{\text{mod.}}^T [\mathbf{T}'_f] [\mathbf{K}_f]_{\text{mod.}} [\mathbf{T}'_f] [\mathbf{T}_f]_{\text{mod.}} \quad \dots\dots\dots(3.122)$$

The new modified tangent stiffness matrix is used herein for the presence of flexible connections as well as for plastic hinges, also in the case of rigid connections or real hinges, linear $\mathbf{M}$ - $\mathbf{\phi}$ relations may be used for these purposes as shown in **Fig.(3.12)**, in which,

$$\phi = \frac{1}{\mathbf{R}_{kt}} \mathbf{M} \quad \dots\dots\dots(3.123)$$

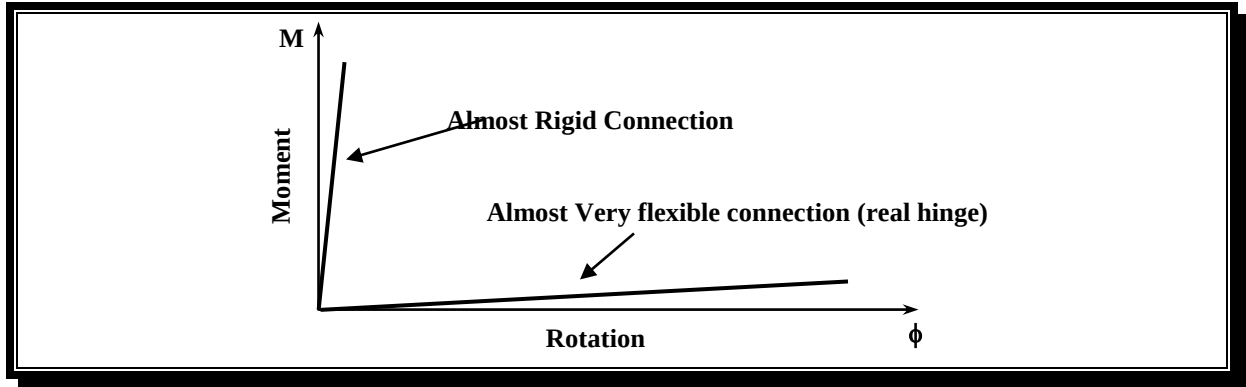


Fig.(3.12): Linear M-Ø Relations.

3.9 Effect of Axial Force on Plastic Moment Capacity

When a steel frame undergoes a large plastic deformation, plastic hinges are expected to be formed. The reduction of the plastic moment capacity of the cross-section as a result of the presence of axial force is considered.

To calculate the influence of axial force on the moment capacity of a plastic hinge, there are two formulas:

3.9.1 Exact Interaction Formulas

3.9.1.1 Box and I-Sections

The box and I-sections are most widely used in building frames, the exact interaction equations are, as mentioned in [65]:

$$m = 1 - Xn^2 \qquad |n| \leq R_w \frac{1 - R_f}{R_f} \qquad \dots\dots\dots(3.124)$$

$$m = Y(1 - |n|) - Z(1 - |n|)^2 \qquad |n| > R_w \frac{1 - R_f}{R_f} \qquad \dots\dots\dots(3.125)$$

where:

$$m = \frac{M_{pc}}{M_p} \qquad \dots\dots\dots(3.126)$$

$$n = \frac{Q}{Q_y} \dots\dots\dots(3.127)$$

- Q** : axial force,
Q_y : axial yield strength =**F_y.A**,
A : cross-sectional area,
F_y : yield stress,
M_p : full plastic moment of the cross-section,
M_{pc} : reduced plastic moment capacity.

$$R_w = \frac{2t_w}{b} \dots\dots\dots(3.128)$$

$$R_f = \frac{2t_f}{d} \dots\dots\dots(3.129)$$

$$R_1 = (1 - (1 - R_w)(1 - R_f)) \dots\dots\dots(3.130)$$

$$R_2 = (1 - (1 - R_w)(1 - R_f)^2) \dots\dots\dots(3.131)$$

$$X = \frac{R_1^2}{R_w R_2} \dots\dots\dots(3.132)$$

$$Y = \frac{2R_1}{R_2} \dots\dots\dots(3.133)$$

$$Z = \frac{R_1^2}{R_2} \dots\dots\dots(3.134)$$

3.9.1.2 Thin-Walled Circular Section

The exact interaction for thin-walled members has been obtained for circular tubes, as mentioned in [65]:

$$m = \cos\left(\frac{\pi n}{2}\right) \dots\dots\dots(3.135)$$

3.9.1.3 Rectangular Section

For a rectangular section under bending about the x-x or y-y axis, the interaction equation has been theoretically derived like, as mentioned in [65]:

$$m = 1 - n^2 \dots\dots\dots(3.136)$$

3.7.2 Approximate Interaction Formulas

Beedle, developed a piece-wise linear approximation for the case of standard wide flange shapes. He depended on the fact that most wide flange shapes have a similar curve when plotted on a non-dimensional basis. This approximation is given as follows, as mentioned as [65]:

$$m = 1 \quad |n| \leq 0.15 \quad \dots\dots\dots(3.137)$$

$$m = 1.18(1 - |n|) \quad |n| > 0.15 \quad \dots\dots\dots(3.138)$$

Duan and Chen, presented a general approximate interaction equation for commonly used steel sections. This equation gives a close approximation of the theoretical ultimate strength interaction. The approximate equation is, as mentioned in [65]:

$$m = 1 - n^\beta \quad \dots\dots\dots(3.139)$$

The parameter β in equation (3.139) defines the shape of the interaction curve. Depending on sectional shapes and distribution, the exponent β for different steel sections are given in **Table (3.2)**. Equation (3.139) used in the present study.

Table (3.2): The Values of β for Different Cross-Sections.

Cross-Section	β
I-section under major axis	1.3
Box-section	$2 - 0.5 * B/H > 1.3$
Thin walled circular section	1.75
Rectangular section	2

3.10 Axial Load-Moment Interaction Diagram

For any steel section, the formulation of plastic hinge is related to the axial load-moment interaction diagram. **Fig.(3.13)**, shows a typical interaction diagram for a steel section in its general case (i.e. axial force plus biaxial

bending M_y and M_z). The interaction is exactly represented by a three-dimensional surface. In the present study and for simplicity, this surface has been idealized by six linear surfaces (planes) defined by eight points **A**, **B**, **B'**, **C**, **C'**, **D**, **D'** and **E**. for any critical section (**m**), at any stage (**S+1**) of analysis, the total axial force (**Q**) and the total moments (**M_y**) and (**M_z**) are known, and to develop the surface the equations of six planes which are indicated in **Fig.(3.13)** must be determined. So, substituting **Q**, **M_y** and **M_z** in the equations of the planes and the plastic hinge formulation if one of the equation is right.

$$0.175M_{yp}Q_y(M_z - 0.35M_{zp}) + 0.175M_{zp}Q_yM_y + 0.1225M_{yp}M_{zp}(Q - 0.5Q_y) = 0 \quad (3.140)$$

$$0.175M_{yp}Q_y(M_z - 0.35M_{zp}) + 0.175M_{zp}Q_yM_y + 0.2275M_{yp}M_{zp}(Q - 0.5Q_y) = 0 \quad (3.141)$$

$$0.5M_{yp}Q_y(M_z - M_{zp}) + 0.5M_{zp}Q_yM_y + 0.65M_{yp}M_{zp}Q = 0 \quad (3.142)$$

$$-0.5M_{yp}Q_y(M_z - M_{zp}) - 0.5M_{zp}Q_yM_y + 0.65M_{yp}M_{zp}Q = 0 \quad (3.143)$$

$$-0.175M_{yp}Q_y(M_z - 0.35M_{zp}) - 0.175M_{zp}Q_yM_y + 0.2275M_{yp}M_{zp}(Q + 0.5Q_y) = 0 \quad (3.144)$$

$$-0.175M_{yp}Q_y(M_z - 0.35M_{zp}) - 0.175M_{zp}Q_yM_y + 0.1225M_{yp}M_{zp}(Q + 0.5Q_y) = 0 \quad (3.145)$$

M_{yp} : full plastic moment of the cross-section about the z-axis,

M_{zp} : full plastic moment of the cross-section about the y-axis.

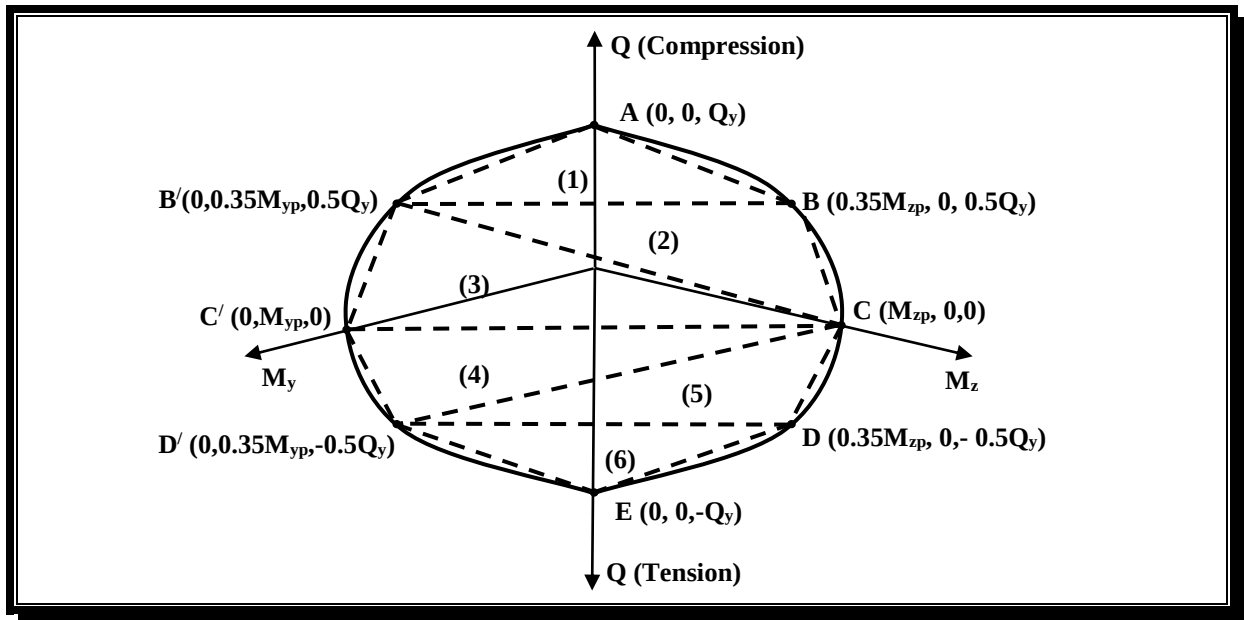


Fig.(3.13): Axial Force – Moment Interaction Surface for a Typical Doubly Symmetric Section.

3.11 Yield Criterion

The yield criterion employed in the present work for an isotropic materials will be a generalization of Von-Mises plasticity theory [66]. According to this criterion, yielding begins under any state of stress when the effective stress ($\bar{\sigma}$) exceeds the yield strength in uniaxial tension or compression (F_y).

$$\bar{\sigma} = \frac{\sqrt{3}}{2} \left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \right]^{1/2}$$

.....(3.146)

Through neglecting ($\sigma_y = \sigma_z = 0$) and squaring both sides of equation (3.146), Von-Mises yield criterion can be written as follows:

$$\sigma_x^2 + 3\tau_{xy}^2 + 3\tau_{yz}^2 + 3\tau_{zx}^2 = F_y^2 \quad \text{.....(3.147)}$$

$$\left(\frac{\sigma_x}{F_y} \right)^2 + \left(\frac{\tau_{xy}}{\tau_y} \right)^2 + \left(\frac{\tau_{yz}}{\tau_y} \right)^2 + \left(\frac{\tau_{zx}}{\tau_y} \right)^2 - 1 = 0 \quad \text{.....(3.148)}$$

$$\left(\frac{Z_b \cdot \sigma_x}{Z_b \cdot F_y} \right)^2 + \left(\frac{Z_t \cdot \tau_{xy}}{Z_t \cdot \tau_y} \right)^2 + \left(\frac{A \cdot \tau_{yz}}{A \cdot \tau_y} \right)^2 + \left(\frac{A \cdot \tau_{zx}}{A \cdot \tau_y} \right)^2 - 1 = 0 \quad \text{.....(3.149)}$$

$$\left(\frac{M}{M_p} \right)^2 + \left(\frac{T}{T_p} \right)^2 + \left(\frac{V_y}{V_p} \right)^2 + \left(\frac{V_z}{V_p} \right)^2 - 1 = 0 \quad \text{.....(3.150)}$$

where,

$$M_p = Z_p \cdot F_y \quad \text{.....(3.151)}$$

$$T_p = Z_t \cdot \tau_y \quad \text{.....(3.152)}$$

$$V_p = A \cdot \tau_y \quad \text{.....(3.153)}$$

$$\tau_y = \frac{F_y}{\sqrt{3}} \quad \text{.....(3.154)}$$

Z_p : plastic bending section modulus,

Z_t : plastic torsion section modulus.

Then, the effective yield criterion suggested in the present study becomes:

$$0.175M_{yp} Q_y (M_z - 0.35M_{zp}) + 0.175M_{zp} Q_y M_y + 0.1225M_{yp} M_{zp} (Q - 0.5Q_y) + \left(\frac{T}{T_p}\right)^2 + \left(\frac{V_y}{V_p}\right)^2 + \left(\frac{V_z}{V_p}\right)^2 - 1 = 0 \quad \text{.....(3.155)}$$

$$0.175M_{yp} Q_y (M_z - 0.35M_{zp}) + 0.175M_{zp} Q_y M_y + 0.2275M_{yp} M_{zp} (Q - 0.5Q_y) + \left(\frac{T}{T_p}\right)^2 + \left(\frac{V_y}{V_p}\right)^2 + \left(\frac{V_z}{V_p}\right)^2 - 1 = 0 \quad \text{.....(3.156)}$$

$$0.5M_{yp} Q_y (M_z - M_{zp}) + 0.5M_{zp} Q_y M_y + 0.65M_{yp} M_{zp} Q + \left(\frac{T}{T_p}\right)^2 + \left(\frac{V_y}{V_p}\right)^2 + \left(\frac{V_z}{V_p}\right)^2 - 1 = 0 \quad \text{.....(3.157)}$$

$$-0.5M_{yp} Q_y (M_z - M_{zp}) - 0.5M_{zp} Q_y M_y + 0.65M_{yp} M_{zp} Q + \left(\frac{T}{T_p}\right)^2 + \left(\frac{V_y}{V_p}\right)^2 + \left(\frac{V_z}{V_p}\right)^2 - 1 = 0 \quad \text{.....(3.158)}$$

$$-0.175M_{yp} Q_y (M_z - 0.35M_{zp}) - 0.175M_{zp} Q_y M_y + 0.2275M_{yp} M_{zp} (Q + 0.5Q_y) + \left(\frac{T}{T_p}\right)^2 + \left(\frac{V_y}{V_p}\right)^2 + \left(\frac{V_z}{V_p}\right)^2 - 1 = 0 \quad \text{.....(3.159)}$$

$$-0.175M_{yp} Q_y (M_z - 0.35M_{zp}) - 0.175M_{zp} Q_y M_y + 0.1225M_{yp} M_{zp} (Q + 0.5Q_y) + \left(\frac{T}{T_p}\right)^2 + \left(\frac{V_y}{V_p}\right)^2 + \left(\frac{V_z}{V_p}\right)^2 - 1 = 0 \quad \text{...(3.160)}$$

3.12 Plastic Hinge Formulation

For any critical section (**m**), at any stage (**S+1**) of analysis, the total axial force (**Q**) and the total moments (**M_{zs}**) and (**M_{ys}**) are known as a result of analysis of a previous (**S**) stages. In the current stage (**S+1**), (**Q**, **M_{zr}**, **M_{yr}**) are the axial force and moments caused by applying load at iteration (**r**). It is then possible to find minimum load factor (**λ_m**), which would cause a plastic hinge to form at a certain critical section such that the final axial force (**Q=Q_s+λ_mQ_r**)

and the moments ($\mathbf{M}_z = \mathbf{M}_{zs} + \lambda_m \mathbf{M}_{zr}$, $\mathbf{M}_y = \mathbf{M}_{ys} + \lambda_m \mathbf{M}_{yr}$) lie on any plane of the interaction surface.

$$\lambda_1 = \frac{\left[-0.175\mathbf{M}_{yp} \mathbf{Q}_y (\mathbf{M}_{zs} - 0.35\mathbf{M}_{zp}) - 0.175\mathbf{M}_{zp} \mathbf{Q}_y \mathbf{M}_{ys} - 0.1225\mathbf{M}_{yp} \mathbf{M}_{zp} \right]}{\left[(\mathbf{Q}_s - 0.5\mathbf{Q}_y) - \left(\frac{\mathbf{T}}{\mathbf{T}_p} \right)^2 - \left(\frac{\mathbf{V}_y}{\mathbf{V}_p} \right)^2 - \left(\frac{\mathbf{V}_z}{\mathbf{V}_p} \right)^2 + 1 \right]} \dots (3.161)$$

$$0.175\mathbf{M}_{yp} \mathbf{Q}_y \mathbf{M}_{zr} + 0.175\mathbf{M}_{zp} \mathbf{Q}_y \mathbf{M}_{yr} + 0.1225\mathbf{M}_{yp} \mathbf{M}_{zp} \mathbf{Q}_r$$

$$\lambda_2 = \frac{\left[-0.175\mathbf{M}_{yp} \mathbf{Q}_y (\mathbf{M}_{zs} - 0.35\mathbf{M}_{zp}) - 0.175\mathbf{M}_{zp} \mathbf{Q}_y \mathbf{M}_{ys} \right]}{\left[-0.2275\mathbf{M}_{yp} \mathbf{M}_{zp} (\mathbf{Q}_s - 0.5\mathbf{Q}_y) - \left(\frac{\mathbf{T}}{\mathbf{T}_p} \right)^2 - \left(\frac{\mathbf{V}_y}{\mathbf{V}_p} \right)^2 - \left(\frac{\mathbf{V}_z}{\mathbf{V}_p} \right)^2 + 1 \right]} \dots (3.162)$$

$$0.175\mathbf{M}_{yp} \mathbf{Q}_y \mathbf{M}_{zr} + 0.175\mathbf{M}_{zp} \mathbf{Q}_y \mathbf{M}_{yr} + 0.2275\mathbf{M}_{yp} \mathbf{M}_{zp} \mathbf{Q}_r$$

$$\lambda_3 = \frac{\left[-0.5\mathbf{M}_{yp} \mathbf{Q}_y (\mathbf{M}_{zs} - \mathbf{M}_{zp}) - 0.5\mathbf{M}_{zp} \mathbf{Q}_y \mathbf{M}_{ys} - 0.65\mathbf{M}_{yp} \mathbf{M}_{zp} \mathbf{Q}_s \right]}{\left[-\left(\frac{\mathbf{T}}{\mathbf{T}_p} \right)^2 - \left(\frac{\mathbf{V}_y}{\mathbf{V}_p} \right)^2 - \left(\frac{\mathbf{V}_z}{\mathbf{V}_p} \right)^2 + 1 \right]} \dots (3.163)$$

$$0.5\mathbf{M}_{yp} \mathbf{Q}_y \mathbf{M}_{zr} + 0.5\mathbf{M}_{zp} \mathbf{Q}_y \mathbf{M}_{yr} + 0.65\mathbf{M}_{yp} \mathbf{M}_{zp} \mathbf{Q}_r$$

$$\lambda_4 = \frac{\left[0.5\mathbf{M}_{yp} \mathbf{Q}_y (\mathbf{M}_{zs} - \mathbf{M}_{zp}) + 0.5\mathbf{M}_{zp} \mathbf{Q}_y \mathbf{M}_{ys} - 0.65\mathbf{M}_{yp} \mathbf{M}_{zp} \mathbf{Q}_s \right]}{\left[-\left(\frac{\mathbf{T}}{\mathbf{T}_p} \right)^2 - \left(\frac{\mathbf{V}_y}{\mathbf{V}_p} \right)^2 - \left(\frac{\mathbf{V}_z}{\mathbf{V}_p} \right)^2 + 1 \right]} \dots (3.164)$$

$$-0.5\mathbf{M}_{yp} \mathbf{Q}_y \mathbf{M}_{zr} - 0.5\mathbf{M}_{zp} \mathbf{Q}_y \mathbf{M}_{yr} + 0.65\mathbf{M}_{yp} \mathbf{M}_{zp} \mathbf{Q}_r$$

$$\lambda_5 = \frac{\left[0.175\mathbf{M}_{yp} \mathbf{Q}_y (\mathbf{M}_{zs} - 0.35\mathbf{M}_{zp}) + 0.175\mathbf{M}_{zp} \mathbf{Q}_y \mathbf{M}_{ys} - 0.2275\mathbf{M}_{yp} \mathbf{M}_{zp} \right]}{\left[(\mathbf{Q}_s + 0.5\mathbf{Q}_y) - \left(\frac{\mathbf{T}}{\mathbf{T}_p} \right)^2 - \left(\frac{\mathbf{V}_y}{\mathbf{V}_p} \right)^2 - \left(\frac{\mathbf{V}_z}{\mathbf{V}_p} \right)^2 + 1 \right]} \dots (3.165)$$

$$-0.175\mathbf{M}_{yp} \mathbf{Q}_y \mathbf{M}_{zr} - 0.175\mathbf{M}_{zp} \mathbf{Q}_y \mathbf{M}_{yr} + 0.2275\mathbf{M}_{yp} \mathbf{M}_{zp} \mathbf{Q}_r$$

$$\lambda_6 = \frac{\left[0.175\mathbf{M}_{yp} \mathbf{Q}_y (\mathbf{M}_{zs} - 0.35\mathbf{M}_{zp}) + 0.175\mathbf{M}_{zp} \mathbf{Q}_y \mathbf{M}_{ys} \right]}{\left[-0.1225\mathbf{M}_{yp} \mathbf{M}_{zp} (\mathbf{Q}_s + 0.5\mathbf{Q}_y) - \left(\frac{\mathbf{T}}{\mathbf{T}_p} \right)^2 - \left(\frac{\mathbf{V}_y}{\mathbf{V}_p} \right)^2 - \left(\frac{\mathbf{V}_z}{\mathbf{V}_p} \right)^2 + 1 \right]} \dots (3.166)$$

$$-0.175\mathbf{M}_{yp} \mathbf{Q}_y \mathbf{M}_{zr} - 0.175\mathbf{M}_{zp} \mathbf{Q}_y \mathbf{M}_{yr} + 0.1225\mathbf{M}_{yp} \mathbf{M}_{zp} \mathbf{Q}_r$$

3.13 Slip Angle of the Flexible Connection

It should be noted that because of the presence of the flexible connections, the joint rotation and element end rotation may not be the same, where the element end rotation for a frame element with flexible connections at the ends will be $(\theta_{1n} - \phi_{1n})$ and $(\theta_{2n} - \phi_{2n})$ and that ϕ_{1n} and ϕ_{2n} are relative end rotations of the beam to column connection in either y or z direction, Figures (3.14) and (3.15).

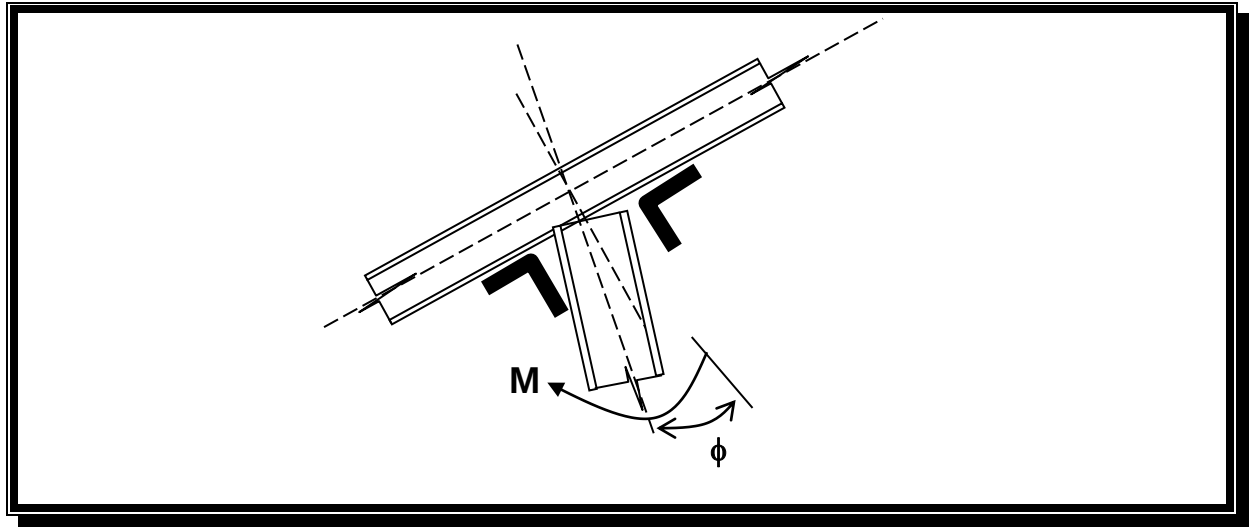


Fig.(3.14): Slip Angle of a Beam-Column Connection.

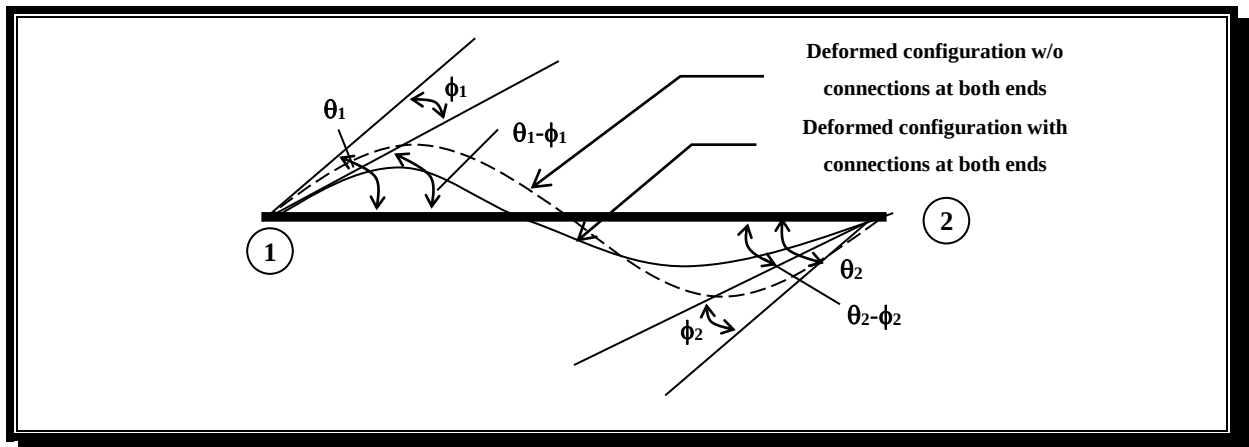


Fig.(3.15): End Rotation of a Beam-Column Element with Connections.

Due to the presence of flexible connections at ends 1 and 2 of the element, the moment-rotation relationship of this element will be:

$$\begin{Bmatrix} \mathbf{M}_{1z} \\ \mathbf{M}_{2z} \\ \mathbf{M}_{1y} \\ \mathbf{M}_{2y} \end{Bmatrix} = \frac{\mathbf{EI}_n}{L} \begin{bmatrix} C_{1z} & C_{2z} & 0 & 0 \\ C_{2z} & C_{1z} & 0 & 0 \\ 0 & 0 & C_{1y} & C_{2y} \\ 0 & 0 & C_{2y} & C_{1y} \end{bmatrix} \begin{Bmatrix} \theta_{1z} - \phi_{1z} \\ \theta_{2z} - \phi_{2z} \\ \theta_{1y} - \phi_{1y} \\ \theta_{2y} - \phi_{2y} \end{Bmatrix} \quad \text{.....(3.167)}$$

where $\mathbf{I}_n = \mathbf{I}_z$ or $\mathbf{I}_y$ relative to the type of the moment “ $\mathbf{M}$ ”.

It is noted that there are **eight** unknowns in Equation (3.167) (i.e. $\mathbf{M}_{1z}$, $\mathbf{M}_{2z}$, $\mathbf{M}_{1y}$, $\mathbf{M}_{2y}$, ϕ_{1z} , ϕ_{2z} , ϕ_{1y} and ϕ_{2y}), therefore, in order to solve for the element end moments, **four** more equations are required. The connection model equations for ends “1” and “2” of the element will furnish the additional **four** equations required, so,

$$\phi_{1n} = f_{1n}(\mathbf{M}_{1n}) \quad \text{.....(3.168)}$$

$$\phi_{2n} = f_{2n}(\mathbf{M}_{2n}) \quad \text{.....(3.169)}$$

where $\mathbf{n}$ = either $\mathbf{y}$ or $\mathbf{z}$.

Strictly, θ_{1n} and θ_{2n} are calculated previously and are independent of whether the connection being linear or non-linear.

For member with flexible connection at both ends;

$$\mathbf{M}_{1z} = \frac{\mathbf{EI}_z}{L} (C_{1z}\theta_{1z} + C_{2z}\theta_{2z} - C_{1z}\phi_{1z} - C_{2z}\phi_{2z}) \quad \text{.....(3.170)}$$

$$\mathbf{M}_{2z} = \frac{\mathbf{EI}_z}{L} (C_{2z}\theta_{1z} + C_{1z}\theta_{2z} - C_{2z}\phi_{1z} - C_{1z}\phi_{2z}) \quad \text{.....(3.171)}$$

$$\mathbf{M}_{1y} = \frac{\mathbf{EI}_y}{L} (C_{1y}\theta_{1y} + C_{2y}\theta_{2y} - C_{1y}\phi_{1y} - C_{2y}\phi_{2y}) \quad \text{.....(3.172)}$$

$$\mathbf{M}_{2y} = \frac{\mathbf{EI}_y}{L} (C_{2y}\theta_{1y} + C_{1y}\theta_{2y} - C_{2y}\phi_{1y} - C_{1y}\phi_{2y}) \quad \text{.....(3.173)}$$

Substitution of Equations (3.168) and (3.169) into Equations (3.170), (3.171), (3.172) and (3.173) gives:

$$\mathbf{M}_{1z} = \frac{\mathbf{EI}_z}{L} (C_{1z}\theta_{1z} + C_{2z}\theta_{2z} - C_{1z}f_{1z}(\mathbf{M}_{1z}) - C_{2z}f_{2z}(\mathbf{M}_{2z})) \quad \text{.....(3.174)}$$

$$M_{2z} = \frac{EI_z}{L} (C_{2z}\theta_{1z} + C_{1z}\theta_{2z} - C_{2z}f_{1z}(M_{1z}) - C_{1z}f_{2z}(M_{2z})) \quad \dots\dots\dots(3.175)$$

$$M_{1y} = \frac{EI_y}{L} (C_{1y}\theta_{1y} + C_{2y}\theta_{2y} - C_{1y}f_{1y}(M_{1y}) - C_{2y}f_{2y}(M_{2y})) \quad \dots\dots\dots(3.176)$$

$$M_{2y} = \frac{EI_y}{L} (C_{2y}\theta_{1y} + C_{1y}\theta_{2y} - C_{2y}f_{1y}(M_{1y}) - C_{1y}f_{2y}(M_{2y})) \quad \dots\dots\dots(3.177)$$

The assumed four functions are:

$$F_{1z}(M_{1z}, M_{2z}) = M_{1z} - \frac{EI_z}{L} (C_{1z}\theta_{1z} + C_{2z}\theta_{2z} - C_{1z}f_{1z}(M_{1z}) - C_{2z}f_{2z}(M_{2z})) = 0 \quad \dots\dots(3.178)$$

$$F_{2z}(M_{1z}, M_{2z}) = M_{2z} - \frac{EI_z}{L} (C_{2z}\theta_{1z} + C_{1z}\theta_{2z} - C_{2z}f_{1z}(M_{1z}) - C_{1z}f_{2z}(M_{2z})) = 0 \quad \dots\dots(3.179)$$

$$F_{1y}(M_{1y}, M_{2y}) = M_{1y} - \frac{EI_y}{L} (C_{1y}\theta_{1y} + C_{2y}\theta_{2y} - C_{1y}f_{1y}(M_{1y}) - C_{2y}f_{2y}(M_{2y})) = 0 \quad \dots(3.180)$$

$$F_{1y}(M_{1y}, M_{2y}) = M_{2y} - \frac{EI_y}{L} (C_{2y}\theta_{1y} + C_{1y}\theta_{2y} - C_{2y}f_{1y}(M_{1y}) - C_{1y}f_{2y}(M_{2y})) = 0 \quad \dots(3.181)$$

These four highly non-linear equations (3.174), (3.175), (3.176) and (3.177), can be solved using conventional N-R iteration, so that:

$$\begin{Bmatrix} M_{1z}^{k+1} \\ M_{2z}^{k+1} \end{Bmatrix} = \begin{Bmatrix} M_{1z}^k \\ M_{2z}^k \end{Bmatrix} - \begin{bmatrix} \frac{\partial F_{1z}^k}{\partial M_{1z}} & \frac{\partial F_{1z}^k}{\partial M_{2z}} \\ \frac{\partial F_{2z}^k}{\partial M_{1z}} & \frac{\partial F_{2z}^k}{\partial M_{2z}} \end{bmatrix}^{-1} \begin{Bmatrix} F_{1z}^k(M_{1z}^k, M_{2z}^k) \\ F_{2z}^k(M_{1z}^k, M_{2z}^k) \end{Bmatrix} \quad \dots\dots\dots(3.182)$$

$$\begin{Bmatrix} M_{1y}^{k+1} \\ M_{2y}^{k+1} \end{Bmatrix} = \begin{Bmatrix} M_{1y}^k \\ M_{2y}^k \end{Bmatrix} - \begin{bmatrix} \frac{\partial F_{1y}^k}{\partial M_{1y}} & \frac{\partial F_{1y}^k}{\partial M_{2y}} \\ \frac{\partial F_{2y}^k}{\partial M_{1y}} & \frac{\partial F_{2y}^k}{\partial M_{2y}} \end{bmatrix}^{-1} \begin{Bmatrix} F_{1y}^k(M_{1y}^k, M_{2y}^k) \\ F_{2y}^k(M_{1y}^k, M_{2y}^k) \end{Bmatrix} \quad \dots\dots\dots(3.183)$$

$$EM_{1z} = \left| \frac{M_{1z}^{k+1} - M_{1z}^k}{M_{1z}^k} \right| \quad EM_{2z} = \left| \frac{M_{2z}^{k+1} - M_{2z}^k}{M_{2z}^k} \right| \quad \dots\dots\dots(3.184)$$

$$EM_{1y} = \left| \frac{M_{1y}^{k+1} - M_{1y}^k}{M_{1y}^k} \right| \quad EM_{2y} = \left| \frac{M_{2y}^{k+1} - M_{2y}^k}{M_{2y}^k} \right| \quad \dots\dots\dots(3.185)$$

The iteration process continues until $\mathbf{EM}_{1z}$, $\mathbf{EM}_{2z}$, $\mathbf{EM}_{1y}$ and $\mathbf{EM}_{2y}$ satisfy the convergence criterion simultaneously. The values of $\mathbf{EM}_{1z}$, $\mathbf{EM}_{2z}$, $\mathbf{EM}_{1y}$ and $\mathbf{EM}_{2y}$ are usually in the range $(1*10^{-3} - 1*10^{-6})$.

Having $\mathbf{M}_{1z}$, $\mathbf{M}_{2z}$, $\mathbf{M}_{1y}$ and $\mathbf{M}_{2y}$ been obtained, θ_{1z} , θ_{2z} , θ_{1y} and θ_{2y} can be calculated from Equations (3.168) and (3.169).

Also,

$$\mathbf{M}_t = \frac{\mathbf{C}_t}{\mathbf{L}}(\phi_t - \phi_x) \quad \dots\dots\dots(3.186)$$

where,

$$\phi_x = \mathbf{f}_x(\mathbf{M}_t) \quad \dots\dots\dots(3.187)$$

$$\text{Then, } \mathbf{M}_t = \frac{\mathbf{C}_t}{\mathbf{L}}(\phi_t - \mathbf{f}_x(\mathbf{M}_t)) \quad \dots\dots\dots(3.188)$$

ϕ_t is calculated previously and Equation (3.188) can be solved using N-R iteration, so that:

$$\mathbf{F}_x(\mathbf{M}_t) = \mathbf{M}_t - \frac{\mathbf{C}_t}{\mathbf{L}}(\phi_t - \mathbf{f}_x(\mathbf{M}_t)) = 0 \quad \dots\dots\dots(3.189)$$

$$\mathbf{M}_t^{k+1} = \mathbf{M}_t^k - \frac{\mathbf{F}_x(\mathbf{M}_t^k)}{\left(\frac{d\mathbf{F}_x}{d\mathbf{M}_t}\right)_{\mathbf{M}_t=\mathbf{M}_t^k}} \quad \dots\dots\dots(3.190)$$

$$\mathbf{EM}_t = \left| \frac{\mathbf{M}_t^{k+1} - \mathbf{M}_t^k}{\mathbf{M}_t^k} \right| \leq (1*10^{-3} - 1*10^{-6}) \quad \dots\dots\dots(3.191)$$

If plastic hinge is formed at end “1”, then:

$$\phi_{1z} = \mathbf{f}_{1z}(\mathbf{M}_{pc1}) \quad \dots\dots\dots(3.192)$$

$$\phi_{1y} = \mathbf{f}_{1y}(\mathbf{M}_{pc1}) \quad \dots\dots\dots(3.193)$$

θ_{2z} and θ_{2y} are known,

$$\mathbf{M}_{pc1} = \frac{\mathbf{EI}_z}{\mathbf{L}}(\mathbf{C}_{1z}\theta_{1z} + \mathbf{C}_{2z}\theta_{2z} - \mathbf{C}_{1z}\phi_{1z} - \mathbf{C}_{2z}\mathbf{f}_{2z}(\mathbf{M}_{2z})) \quad \dots\dots\dots(3.194)$$

$$\mathbf{M}_{2z} = \frac{\mathbf{EI}_z}{\mathbf{L}}(\mathbf{C}_{2z}\theta_{1z} + \mathbf{C}_{1z}\theta_{2z} - \mathbf{C}_{2z}\phi_{1z} - \mathbf{C}_{1z}\mathbf{f}_{2z}(\mathbf{M}_{2z})) \quad \dots\dots\dots(3.195)$$

$$\theta_{1z} = \frac{1}{C_{1z}} \left(\frac{L}{EI_z} M_{pc1} - C_{2z} \theta_{2z} + C_{2z} \phi_{1z} + C_{2z} f_{2z}(M_{2z}) \right) \dots\dots\dots(3.196)$$

Substituting Equation (3.196) into Equation (3.195) yields;

$$M_{2z} = \frac{EI_z}{L} \left[\frac{C_{2z}}{C_{1z}} \left(\frac{1}{EI_z} M_{pc1} - C_{2z} \theta_{2z} + C_{2z} \phi_{1z} + C_{2z} f_{2z}(M_{2z}) \right) \right. \\ \left. + C_{1z} \theta_{2z} - C_{2z} \phi_{1z} - C_{1z} f_{2z}(M_{2z}) \right] \dots\dots(3.197)$$

$$M_{2z} = \frac{EI_z}{L} \left[\frac{C_{2z}}{C_{1z}} \frac{L}{EI_z} M_{pc1} + \theta_{2z} \left(C_{1z} - \frac{C_{2z}^2}{C_{1z}} \right) + f_{2z}(M_{2z}) \left(\frac{C_{2z}^2}{C_{1z}} - C_{1z} \right) \right] \dots\dots(3.198)$$

$$F(M_{2z}) = M_{2z} - \frac{EI_z}{L} \left[\frac{C_{2z}}{C_{1z}} \frac{L}{EI_z} M_{pc1} + \theta_{2z} \left(\frac{C_{1z}^2 - C_{2z}^2}{C_{1z}} \right) \right. \\ \left. - f_{2z}(M_{2z}) \left(\frac{C_{1z}^2 - C_{2z}^2}{C_{1z}} \right) \right] = 0 \dots\dots\dots(3.199)$$

This equation can be solved by N-R iteration, so that:

$$M_{2z}^{k+1} = M_{2z}^k - \frac{F(M_{2z}^k)}{\left(\frac{dF}{dM_{2z}} \right)_{M_{2z}=M_{2z}^k}} \dots\dots\dots(3.200)$$

$$EM_z = \left| \frac{M_{2z}^{k+1} - M_{2z}^k}{M_{2z}^k} \right| \leq (1 * 10^{-3} - 1 * 10^{-6}) \dots\dots\dots(3.201)$$

Then, θ_{1z} and θ_{2z} are obtained from Equations (3.196) and (3.169).

$$M_{pc1} = \frac{EI_y}{L} (C_{1y} \theta_{1y} + C_{2y} \theta_{2y} - C_{1y} \phi_{1y} - C_{2y} f_{2y}(M_{2y})) \dots\dots\dots(3.202)$$

$$M_{2y} = \frac{EI_y}{L} (C_{2y} \theta_{1y} + C_{1y} \theta_{2y} - C_{2y} \phi_{1y} - C_{1y} f_{2y}(M_{2y})) \dots\dots\dots(3.203)$$

$$\theta_{1y} = \frac{1}{C_{1y}} \left(\frac{L}{EI_y} M_{pc1} - C_{2y} \theta_{2y} + C_{2y} \phi_{1y} + C_{2y} f_{2y}(M_{2y}) \right) \dots\dots\dots(3.204)$$

Substituting Equation (3.204) into Equation (3.203) yields;

$$M_{2y} = \frac{EI_y}{L} \left[\frac{C_{2y}}{C_{1y}} \left(\frac{1}{EI_y} M_{pc1} - C_{2y} \theta_{2y} + C_{2y} \phi_{1y} + C_{2y} f_{2y}(M_{2y}) \right) \right. \\ \left. + C_{1y} \theta_{2y} - C_{2y} \phi_{1y} - C_{1y} f_{2y}(M_{2y}) \right] \dots\dots(3.205)$$

$$M_{2y} = \frac{EI_y}{L} \left[\frac{C_{2y}}{C_{1y}} \frac{L}{EI_y} M_{pc1} + \theta_{2y} \left(C_{1y} - \frac{C_{2y}^2}{C_{1y}} \right) + f_{2y}(M_{2y}) \left(\frac{C_{2y}^2}{C_{1y}} - C_{1y} \right) \right] \dots(3.206)$$

$$F(M_{2y}) = M_{2y} - \frac{EI_y}{L} \left[\frac{C_{2y}}{C_{1y}} \frac{L}{EI_y} M_{pc1} + \theta_{2y} \left(\frac{C_{1y}^2 - C_{2y}^2}{C_{1y}} \right) - f_{2y}(M_{2y}) \left(\frac{C_{1y}^2 - C_{2y}^2}{C_{1y}} \right) \right] = 0 \dots\dots\dots(3.207)$$

This equation can be solved by N-R iteration, so that:

$$M_{2y}^{k+1} = M_{2y}^k - \frac{F(M_{2y}^k)}{\left(\frac{dF}{dM_{2y}} \right)_{M_{2y}=M_{2y}^k}} \dots\dots\dots(3.208)$$

$$EM_y = \left| \frac{M_{2y}^{k+1} - M_{2y}^k}{M_{2y}^k} \right| \leq (1 * 10^{-3} - 1 * 10^{-6}) \dots\dots\dots(3.209)$$

Then, θ_{1y} and θ_{2y} is obtained from Equations (3.204) and (3.168).

Also,

$$\phi_x = f_x(M_{pc1}) \dots\dots\dots(3.210)$$

thus ϕ_t is found.

A similar procedure is followed for treating the case of a plastic hinge forming at end “2”.

For plastic hinges forming at both ends “1” and “2”, then:

$$\phi_{1z} = f_{1z}(M_{pc1}) \quad \phi_{2z} = f_{2z}(M_{pc2}) \quad \dots\dots\dots(3.211)$$

$$\phi_{1y} = f_{1y}(M_{pc1}) \quad \phi_{2y} = f_{2y}(M_{pc2}) \quad \dots\dots\dots(3.212)$$

$$M_{pc1} = \frac{EI_z}{L} (C_{1z} \theta_{1z} + C_{2z} \theta_{2z} - C_{1z} \phi_{1z} - C_{2z} \phi_{2z}) \quad \dots\dots\dots(3.213)$$

$$M_{pc2} = \frac{EI_z}{L} (C_{2z} \theta_{1z} + C_{1z} \theta_{2z} - C_{2z} \phi_{1z} - C_{1z} \phi_{2z}) \quad \dots\dots\dots(3.214)$$

$$\theta_{1z} = - \frac{\left[C_{1z} \left(\frac{L M_{pc1}}{EI_z} + C_{1z} \phi_{1z} + C_{2z} \phi_{2z} \right) - C_{2z} \left(\frac{L M_{pc2}}{EI_z} + C_{2z} \phi_{1z} + C_{1z} \phi_{2z} \right) \right]}{(C_{2z}^2 - C_{1z}^2)} \dots\dots\dots(3.215)$$

$$\theta_{2z} = - \frac{\left[C_{2z} \left(\frac{L \cdot M_{pc1}}{EI_z} + C_{1z} \phi_{1z} + C_{2z} \phi_{2z} \right) - C_{1z} \left(\frac{L \cdot M_{pc2}}{EI_z} + C_{2z} \phi_{1z} + C_{1z} \phi_{2z} \right) \right]}{(C_{2z}^2 - C_{1z}^2)} \dots\dots\dots(3.216)$$

$$M_{pc1} = \frac{EI_y}{L} (C_{1y} \theta_{1y} + C_{2y} \theta_{2y} - C_{1y} \phi_{1y} - C_{2y} \phi_{2y}) \dots\dots\dots(3.217)$$

$$M_{pc2} = \frac{EI_y}{L} (C_{2y} \theta_{1y} + C_{1y} \theta_{2y} - C_{2y} \phi_{1y} - C_{1y} \phi_{2y}) \dots\dots\dots(3.218)$$

$$\theta_{1y} = - \frac{\left[C_{1y} \left(\frac{L \cdot M_{pc1}}{EI_y} + C_{1y} \phi_{1y} + C_{2y} \phi_{2y} \right) - C_{2y} \left(\frac{L \cdot M_{pc2}}{EI_y} + C_{2y} \phi_{1y} + C_{1y} \phi_{2y} \right) \right]}{(C_{2y}^2 - C_{1y}^2)} \dots\dots\dots(3.219)$$

$$\theta_{2y} = - \frac{\left[C_{2y} \left(\frac{L \cdot M_{pc1}}{EI_y} + C_{1y} \phi_{1y} + C_{2y} \phi_{2y} \right) - C_{1y} \left(\frac{L \cdot M_{pc2}}{EI_y} + C_{2y} \phi_{1y} + C_{1y} \phi_{2y} \right) \right]}{(C_{2y}^2 - C_{1y}^2)} \dots\dots\dots(3.220)$$

3.14 Calculation of Member Axial Force

A significant computational difficulty arises in determining $\{S\}$ from $\{U\}$. This is due to the fact that the expression of the member axial force “ Q ” involves bowing functions “ b_{1n} ” and “ b_{2n} ” which are functions of the axial force parameter “ q ”. In the presence of flexible connections or/and plastic hinges is further complicated by the fact that rotations at the released ends (θ_{1n} , θ_{2n} , ϕ_{1n} , ϕ_{2n}) are also functions of “ Q ”. The problem can be solved, however, by the following iterative procedure [11].

Noting that “ q ” is the only unknown, then writing:

$$K(q) = \frac{\pi^2}{\lambda^2} q + C_{bz} + C_{by} - \frac{u}{L} = 0 \dots\dots\dots(3.221)$$

Assuming “ q ” to be an approximate solution of Equation (3.221), and by using the first order Taylor series expansion, Equation (3.221) can be written as:

$$K(q^k + \Delta q^k) = K(q^k) + K'(q^k) \Delta q^k = 0 \dots\dots\dots(3.222)$$

in which a prime superscript denotes differentiation with respect to “ $\mathbf{q}$ ”, and,

$$\mathbf{K}'(\mathbf{q}) = \frac{\pi^2}{\lambda^2} + \sum_{n=y,z} \frac{1}{\epsilon_n} \mathbf{C}'_{bn} \quad \dots\dots\dots(3.223)$$

A new value is thus obtained for “ $\mathbf{q}$ ”,

$$\mathbf{q}^{k+1} = \mathbf{q}^k + \Delta\mathbf{q}^k = \mathbf{q}^k - \frac{\mathbf{K}(\mathbf{q}^k)}{\mathbf{K}'(\mathbf{q}^k)} \quad \dots\dots\dots(3.224)$$

and the iteration continues until

$$|\Delta\mathbf{q}| \leq e \quad \dots\dots\dots(3.225)$$

in which “ e ” represents a prescribed tolerance (1×10^{-3}). In Equation (3.223), $\mathbf{C}'_{bn}$ is given by:

$$\begin{aligned} \mathbf{C}'_{bn} = & \mathbf{b}'_{1n} ((\theta_{1n} + \theta_{2n}) - (\phi_{1n} + \phi_{2n}))^2 + \mathbf{b}'_{2n} ((\theta_{1n} - \theta_{2n}) - (\phi_{1n} - \phi_{2n}))^2 \\ & + 2\mathbf{b}_{1n} ((\theta_{1n} + \theta_{2n}) - (\phi_{1n} + \phi_{2n}))((\theta'_{1n} + \theta'_{2n}) - (\phi'_{1n} + \phi'_{2n})) \quad \dots\dots(3.226) \\ & + 2\mathbf{b}_{2n} ((\theta_{1n} - \theta_{2n}) - (\phi_{1n} - \phi_{2n}))((\theta'_{1n} - \theta'_{2n}) - (\phi'_{1n} - \phi'_{2n})) \end{aligned}$$

in which θ'_{1n} and θ'_{2n} are equal to zero for member without plastic hinge at both ends (rigid connection at both end) and θ_{1z} , θ_{2z} , θ_{1y} and θ_{2y} are known and,

$$\phi'_{1n} = \left(\frac{d}{d\mathbf{M}_{1n}} \mathbf{f}_{1n}(\mathbf{M}_{1n}) \right) \cdot \frac{d\mathbf{M}_{1n}}{d\mathbf{q}} \quad \dots\dots\dots(3.227)$$

$$\phi'_{2n} = \left(\frac{d}{d\mathbf{M}_{2n}} \mathbf{f}_{2n}(\mathbf{M}_{2n}) \right) \cdot \frac{d\mathbf{M}_{2n}}{d\mathbf{q}} \quad \dots\dots\dots(3.228)$$

where $\mathbf{f}_{1n}(\mathbf{M}_{1n})$ and $\mathbf{f}_{2n}(\mathbf{M}_{2n})$ are obtained from Equations of M- θ curve.

$$\mathbf{M}_{1n} = \frac{\mathbf{EI}_n}{\mathbf{L}} [\mathbf{C}_{1n}\theta_{1n} + \mathbf{C}_{2n}\theta_{2n} - \mathbf{C}_{1n}\mathbf{f}_{1n}(\mathbf{M}_{1n}) - \mathbf{C}_{2n}\mathbf{f}_{2n}(\mathbf{M}_{2n})] \quad \dots\dots\dots(3.229)$$

$$\mathbf{M}_{2n} = \frac{\mathbf{EI}_n}{\mathbf{L}} [\mathbf{C}_{2n}\theta_{1n} + \mathbf{C}_{1n}\theta_{2n} - \mathbf{C}_{2n}\mathbf{f}_{1n}(\mathbf{M}_{1n}) - \mathbf{C}_{1n}\mathbf{f}_{2n}(\mathbf{M}_{2n})] \quad \dots\dots\dots(3.230)$$

To obtain $(\phi'_{1n}, \phi'_{2n}) \left(\frac{d\mathbf{M}_{1n}}{d\mathbf{q}}, \frac{d\mathbf{M}_{2n}}{d\mathbf{q}} \right)$ must be obtained, thus:

$$\frac{dM_{1z}}{dq} = \frac{EI_z}{L} \begin{bmatrix} C'_{1z}\theta_{1z} + C'_{2z}\theta_{2z} - C_{1z}f'_{1z}(M_{1z})\frac{dM_{1z}}{dq} - \\ C'_{1z}f_{1z}(M_{1z}) - C_{2z}f'_{2z}(M_{2z})\frac{dM_{2z}}{dq} - C'_{2z}f_{2z}(M_{2z}) \end{bmatrix} \quad \text{.....(3.231)}$$

$$\frac{dM_{2z}}{dq} = \frac{EI_z}{L} \begin{bmatrix} C'_{2z}\theta_{1z} + C'_{1z}\theta_{2z} - C_{2z}f'_{1z}(M_{1z})\frac{dM_{1z}}{dq} - \\ C'_{2z}f_{1z}(M_{1z}) - C_{1z}f'_{2z}(M_{2z})\frac{dM_{2z}}{dq} - C'_{1z}f_{2z}(M_{2z}) \end{bmatrix} \quad \text{.....(3.232)}$$

Solving Equations (3.231) and (3.232) simultaneously, $\frac{dM_{1z}}{dq}$ and $\frac{dM_{2z}}{dq}$ are obtained.

$$\frac{dM_{1y}}{dq} = \frac{EI_y}{L} \begin{bmatrix} C'_{1y}\theta_{1y} + C'_{2y}\theta_{2y} - C_{1y}f'_{1y}(M_{1y})\frac{dM_{1y}}{dq} - \\ C'_{1y}f_{1y}(M_{1y}) - C_{2y}f'_{2y}(M_{2y})\frac{dM_{2y}}{dq} - C'_{2y}f_{2y}(M_{2y}) \end{bmatrix} \quad \text{.....(3.233)}$$

$$\frac{dM_{2y}}{dq} = \frac{EI_y}{L} \begin{bmatrix} C'_{2y}\theta_{1y} + C'_{1y}\theta_{2y} - C_{2y}f'_{1y}(M_{1y})\frac{dM_{1y}}{dq} - \\ C'_{2y}f_{1y}(M_{1y}) - C_{1y}f'_{2y}(M_{2y})\frac{dM_{2y}}{dq} - C'_{1y}f_{2y}(M_{2y}) \end{bmatrix} \quad \text{.....(3.234)}$$

Solving Equations (3.233) and (3.234) simultaneously, $\frac{dM_{1y}}{dq}$ and $\frac{dM_{2y}}{dq}$ are obtained.

Noting that C_{1n}, C_{2n}, C'_{1n} and C'_{2n} are based on values of previous iteration cycle.

For a plastic hinge at end “1” only,

$$\phi_{1z} = f_{1z}(M_{pc1}) \quad \text{and} \quad \phi_{1y} = f_{1y}(M_{pc1}) \quad \text{.....(3.235)}$$

$\theta_{1z}, \theta_{2z}, \theta_{1y}$ and θ_{2y} are known, then,

$$M_{pc1} = \frac{EI_n}{L} [C_{1n}\theta_{1n} + C_{2n}\theta_{2n} - C_{1n}\phi_{1n} - C_{2n}f_{2n}(M_{2n})] \quad \text{.....(3.236)}$$

$$M_{2n} = \frac{EI_n}{L} [C_{2n}\theta_{1n} + C_{1n}\theta_{2n} - C_{2n}\phi_{1n} - C_{1n}f_{2n}(M_{2n})] \quad \text{.....(3.237)}$$

in which $\theta'_{2n} = 0$ and,

$$\phi'_{1n} = \frac{d}{dM_{pc1}} f_{1n}(M_{pc1}) M'_{pc1} \quad \text{.....(3.238)}$$

$$\text{where } M'_{pc1} \approx -\beta M_{p1} \left(\frac{Q}{Q_y} \right)^{\beta-1} * \frac{\pi^2 EI_n}{L^2 Q_y} \quad \text{.....(3.239)}$$

In the case that the cross-section type of the element is not predetermined the following expression for β is suggested in the present study ($\beta = 1.6$). In other case use **Table (3.2)**.

$$M'_{pc1} = \frac{EI_n}{L} \left[\begin{array}{l} C'_{1n}\theta_{1n} + C_{1n}\theta'_{1n} + C'_{2n}\theta_{2n} + C'_{1n}\phi_{1n} - C_{1n}\phi'_{1n} \\ - C_{1n}f'_{2n}(M_{2n}) \frac{dM_{2n}}{dq} - C'_{1n}f_{2n}(M_{2n}) \end{array} \right] \quad \text{.....(3.240)}$$

$$\frac{dM_{2n}}{dq} = \frac{EI_n}{L} \left[\begin{array}{l} C'_{2n}\theta_{1n} + C_{2n}\theta'_{1n} + C'_{1n}\theta_{2n} - C_{2n}\phi'_{1n} - C'_{2n}\phi_{1n} \\ - C_{1n}f'_{2n}(M_{2n}) \frac{dM_{2n}}{dq} - C'_{1n}f_{2n}(M_{2n}) \end{array} \right] \quad \text{.....(3.241)}$$

Solving Equations (3.240) and (3.241) simultaneously, $\frac{dM_{2n}}{dq}$ and θ'_{1n} are obtained.

$$\text{Then, } \phi'_{2n} = \frac{d}{dM_{2n}} f_{2n}(M_{2n}) \frac{dM_{2n}}{dq} \quad \text{.....(3.242)}$$

For a plastic hinge at end “2” only,

$$\phi_{2z} = f_{2z}(M_{pc2}) \quad \text{and} \quad \phi_{2y} = f_{2y}(M_{pc2}) \quad \text{.....(3.243)}$$

$\theta_{1z}, \theta_{2z}, \theta_{1y}$ and θ_{2y} are known and $\theta'_{1n} = 0$

$$\phi'_{2n} = \frac{d}{dM_{pc2}} f_{2n}(M_{pc2}) M'_{pc2} \quad \text{.....(3.244)}$$

$$\text{where } M'_{pc2} \approx -\beta M_{p2} \left(\frac{Q}{Q_y} \right)^{\beta-1} * \frac{\pi^2 EI_n}{L^2 Q_y} \quad \text{.....(3.245)}$$

$$\mathbf{M}'_{pc2} = \frac{EI_n}{L} \begin{bmatrix} C'_{2n}\theta_{1n} + C_{1n}\theta'_{2n} + C'_{1n}\theta_{2n} + C'_{1n}\phi_{2n} - C_{1n}\phi'_{2n} \\ -C_{2n}f'_{1n}(\mathbf{M}_{1n})\frac{d\mathbf{M}_{1n}}{dq} - C'_{2n}f_{1n}(\mathbf{M}_{1n}) \end{bmatrix} \dots\dots\dots(3.246)$$

$$\frac{d\mathbf{M}_{1n}}{dq} = \frac{EI_n}{L} \begin{bmatrix} C'_{1n}\theta_{1n} + C_{2n}\theta'_{2n} + C'_{2n}\theta_{2n} - C_{2n}\phi'_{2n} - C'_{2n}\phi_{2n} \\ -C_{1n}f'_{1n}(\mathbf{M}_{1n})\frac{d\mathbf{M}_{1n}}{dq} - C'_{1n}f_{1n}(\mathbf{M}_{1n}) \end{bmatrix} \dots\dots\dots(3.247)$$

Solving Equations (3.246) and (3.247) simultaneously, $\frac{d\mathbf{M}_{1n}}{dq}$ and θ'_{2n} are obtained.

$$\text{Then, } \phi'_{1n} = \frac{d}{d\mathbf{M}_{1n}} f_{1n}(\mathbf{M}_{1n}) \frac{d\mathbf{M}_{1n}}{dq} \dots\dots\dots(3.248)$$

For plastic hinge at both ends,

$$\phi_{1n} = f_{1n}(\mathbf{M}_{pc1}) \text{ and } \phi_{2n} = f_{2n}(\mathbf{M}_{pc2}) \dots\dots\dots(3.249)$$

$$\phi'_{1n} = \frac{d}{d\mathbf{M}_{pc1}} f_{1n}(\mathbf{M}_{pc1}) \mathbf{M}'_{pc1} \dots\dots\dots(3.250)$$

$$\phi'_{2n} = \frac{d}{d\mathbf{M}_{pc2}} f_{2n}(\mathbf{M}_{pc2}) \mathbf{M}'_{pc2} \dots\dots\dots(3.251)$$

$\mathbf{M}'_{pc1}$ and $\mathbf{M}'_{pc2}$ can be obtained from Equations (3.239) and (3.245), and,

$$\mathbf{M}_{pc1} = \frac{EI_n}{L} [C_{1n}\theta_{1n} + C_{2n}\theta_{2n} - C_{1n}\phi_{1n} - C_{2n}\phi_{2n}] \dots\dots\dots(3.252)$$

$$\mathbf{M}_{pc2} = \frac{EI_n}{L} [C_{2n}\theta_{1n} + C_{1n}\theta_{2n} - C_{2n}\phi_{1n} - C_{1n}\phi_{2n}] \dots\dots\dots(3.253)$$

$$\frac{d\mathbf{M}_{pc1}}{dq} = \frac{EI_n}{L} \begin{bmatrix} C'_{1n}\theta_{1n} + C_{1n}\theta'_{1n} + C'_{2n}\theta_{2n} + C_{2n}\theta'_{2n} \\ -C'_{1n}\phi_{1n} - C_{1n}\phi'_{1n} - C'_{2n}\phi_{2n} - C_{2n}\phi'_{2n} \end{bmatrix} \dots\dots\dots(3.254)$$

$$\frac{d\mathbf{M}_{pc2}}{dq} = \frac{EI_n}{L} \begin{bmatrix} C'_{2n}\theta_{1n} + C_{2n}\theta'_{1n} + C'_{1n}\theta_{2n} + C_{1n}\theta'_{2n} \\ -C'_{2n}\phi_{1n} - C_{2n}\phi'_{1n} - C'_{1n}\phi_{2n} - C_{1n}\phi'_{2n} \end{bmatrix} \dots\dots\dots(3.255)$$

Solving Equations (3.254) and (3.255) simultaneously, $\theta'_{1n}, \theta'_{2n}$ are obtained.

3.15 Computational Technique

The non-linear response of a structures is usually generated by an incremental load approach with a **Newton-Raphson** type of iteration performed at each load level to satisfy the joint equilibrium equations [9, 36]. The computational technique is shown graphically in **Fig.(3.16)** for a single degree of freedom system. A load level, $\{\mathbf{P}^{(i)}\}$, is considered and it is assumed that the corresponding deformed configuration of the system, which may be denoted symbolically by $\{\mathbf{x}^{(i)}\}$, is known. It is now desired to determine the configuration, $\{\mathbf{x}^{(i+1)}\}$, corresponding to level, $\{\mathbf{P}^{(i+1)}\} = \{\mathbf{P}^{(i)}\} + \{\Delta\mathbf{P}\}$, in which $\{\Delta\mathbf{P}\}$ represents a small load increment. Using a linearized analysis, the change in configurations, $\{\Delta\mathbf{x}\}$, is first computed from Equation (3.2), by simply evaluating the system tangent stiffness matrix at the beginning of the load interval, i.e., at load level $\{\mathbf{P}^{(i)}\}$. The solution $\{\mathbf{x}^{(i)}\} + \{\Delta\mathbf{x}\}$ thus obtained represents an approximate configuration in the sense that the joint equilibrium equations are not necessarily satisfied at the load level, $\{\mathbf{P}^{(i+1)}\}$. This approximate solution is then corrected by a Newton-Raphson type of iteration until the equation of equilibrium (3.1) is satisfied within a prescribed tolerance. In the following analysis, the subscript “j” is used to refer to the “jth” iteration cycle, while $\{\mathbf{P}\}$ is kept constant at $\{\mathbf{P}^{(i+1)}\}$. From Equation (3.1), the unbalanced joint forces can be calculated as:

$$\{\Delta\mathbf{w}\}_j = \{\mathbf{P}\} - \{\mathbf{f}(\mathbf{x})\}_j \quad \dots\dots\dots(3.256)$$

in which, $\{\mathbf{f}(\mathbf{x})\}_j$ represents internal joint forces corresponding to the configuration $\{\mathbf{x}\}_j$. The unbalanced joint forces are then treated as a load increment and the correction vector, $\{\Delta\mathbf{x}\}_j$, is obtained from the incremental relationship:

$$\{\Delta\mathbf{x}\}_j [\mathbf{K}] = \{\Delta\mathbf{w}\}_j \quad \dots\dots\dots(3.257)$$

A new approximate configuration, $\{\mathbf{x}\}_{j+1} = \{\mathbf{x}\}_j + \{\Delta\mathbf{x}\}_j$, is thus obtained, and the process continues until the latest correction vector is sufficiently small.

Based on the large deformation formulation outlined in the preceding sections, the computational steps associated with each load increment can be summarized as follows:

1. Using the latest available values of all geometric and static quantities, determine for each member the tangent stiffness matrix in global coordinates. Assemble the system tangent stiffness matrix. Note that in the first load increment the axial forces in all members are assumed to be zero, and all dependent functions are determined accordingly.
2. Substitute $[\mathbf{K}]$ and $\{\Delta\mathbf{P}\}$ into equation (3.2) and solve for $\{\Delta\mathbf{x}\}$.
3. Determine the new geometric configuration of the structure from the equation:

$$\{\mathbf{x}^{(i+1)}\} = \{\mathbf{x}^{(i)}\} + \{\Delta\mathbf{x}\} \dots\dots\dots(3.258)$$

4. Determine the member geometry. By using the latest information concerning the locations and orientations of the joints and the presence of plastic hinges or flexible connections of the structure, determine for each member the new chord length, the new direction cosines and the new relative deformations $\{\mathbf{U}\}$.
5. Determine the member end forces. For each member determine first the end forces in Eulerian coordinates $\{\mathbf{S}\}$, then the end forces in global coordinates $\{\mathbf{F}\}$.
6. From Equation (3.256), determine the unbalanced joint forces $\{\Delta\mathbf{w}\}$.
7. Using the latest available values of all geometric and static quantities, determine the tangent stiffness matrix for each member. Assemble the system tangent stiffness matrix.
8. Substitute the assembled tangent stiffness matrix and the unbalanced joint forces in Equation (3.257), and solve for $\{\Delta\mathbf{x}\}$.

9. If the correction displacement vector is not sufficiently small, determine the new geometric configuration, $\{\mathbf{x}\}_{j+1} = \{\mathbf{x}\}_j + \{\Delta\mathbf{x}\}_j$, and repeat steps (4) through (9) until $\{\Delta\mathbf{x}\}$ is sufficiently small according to a prescribed criterion. The specific convergence criterion as used herein is based on a comparison of the incremental values, $\{\Delta\mathbf{x}\}$, of the displacements to their cumulative value $\{\mathbf{x}\}$. In applying the criterion, translations and rotations of the joints are treated as separate groups, and convergence is assumed to have occurred when the inequality,

$$\left[\frac{\sum (\Delta x_i)^2}{\sum (x_i)^2} \right]^{1/2} \leq (1 * 10^{-3} - 1 * 10^{-6}) \quad \dots\dots\dots(3.259)$$

is satisfied simultaneously and independently for each group.

10. Consider a new load increment, $\{\Delta\mathbf{P}\}$, and return to step (1).

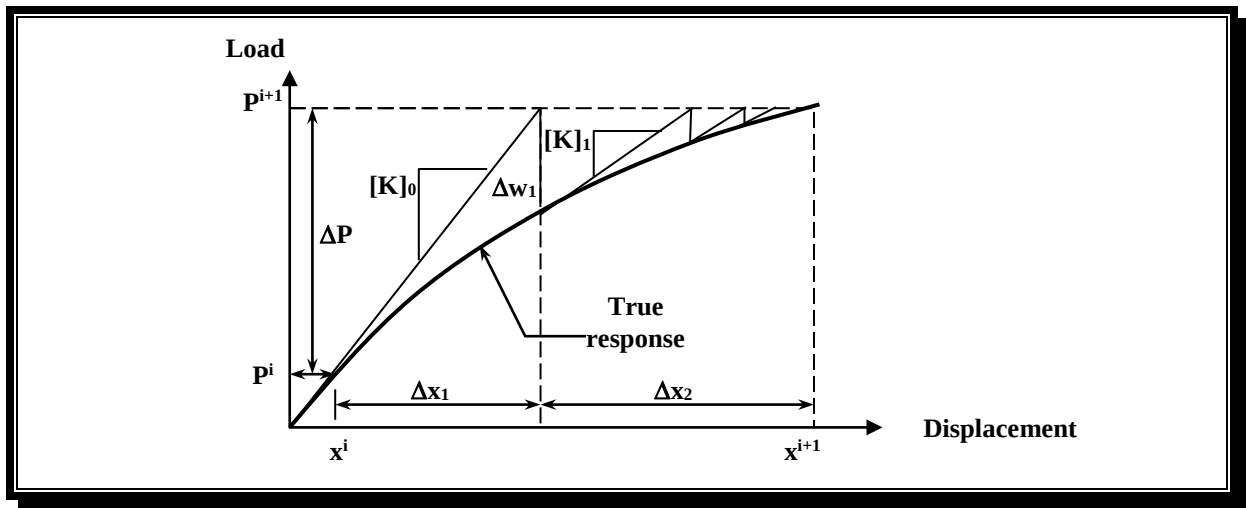


Fig.(3.16): Computational Technique.

3.16 Failure Criterion (Detection of Instability or Mechanism)

Two types of failure can occur when using large displacement elastic-plastic stability analysis of steel structures.

1. Failure due to mechanism space, when sufficient plastic hinges are formed to cause mechanism before buckling failure occurs.

2. Failure due to instability space, as long as the structure remains in stable equilibrium, there is a unique set of deflections for a given set of applied loads and $[\mathbf{K}]^{-1}$ exists. However, if the structure is in a state of neutral equilibrium, where many deflection configurations are possible, $[\mathbf{K}]^{-1}$ does not exist, i.e. $[\mathbf{K}]$ is singular and the determinant of $[\mathbf{K}]$ is zero.
3. Elastic-Plastic Instability space.

3.17 Determinant of System of Tangent Stiffness Matrix Criterion

Many researchers view a lack of convergence in the iteration process as an indication of failure [9]. This reasoning while it is clearly justified from theoretical point of view, appears to be unreliable as a practical criterion.

The most conventional failure criterion is obtained by monitoring the determinant of the second variation of the total potential energy of the structure under investigation, in other words, the determinant of the system of tangent stiffness matrix is to be equal to zero. The interpolation method [65] is often used to know when the mentioned determinant of tangent stiffness matrix changes its sign from positive to negative or from negative to positive, in the following two load increments applied as shown in **Fig.(3.17)**. Herein, an approximated value of λ_{cr} can be estimated as:

$$\lambda_{cr} = \frac{D_i \lambda_{i+1} - \lambda_i D_{i+1}}{D_i - D_{i+1}} \dots\dots\dots(3.260)$$

in which,

λ_{cr} : the approximate value for the critical load factor,

λ_i, D_i : the load factor and the determinant of the tangent stiffness matrix at the i th load step.

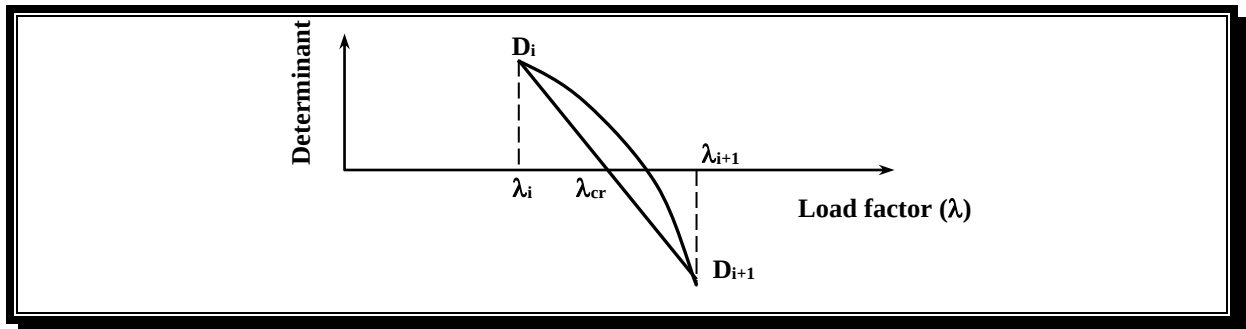


Fig.(3.17): Determinant Versus Load Factor Curve.



Chapter

Four

Dynamic Analysis of Space Structures and Earthquake Excitation

4.1 General

The aim of this chapter is to provide a theoretical basis for the analysis of space steel structures subjected to dynamic loads.

The dynamic equation of motion and the derivations of the mass and damping matrices will be presented. Moreover, the numerical integration method for calculating the dynamic response of a structure have been discussed.

4.2 Dynamic Equation of Motion

The motion equation for the non-linear structural system, is written in the form [67, 68 and 69]:

$$[M]\{\ddot{X}^0\} + [C]\{\dot{X}^0\} + [K]\{X\} = \{F(t)\} \quad \dots\dots\dots(4.1)$$

in which,

$\{F(t)\}$: the external applied loads as a function of time,
 $[M]$, $[C]$ and $[K]$: the mass, damping and stiffness matrices respectively,
 $\{X^{00}\}$, $\{X^0\}$ and $\{X\}$: the acceleration, velocity and displacement vectors of the nodal points of the system respectively.

The stiffness matrix is presented in chapter three, while the mass and damping properties will be formulated in this chapter.

4.3 Formulation of Mass Matrix

For a dynamic system, the mass properties of an element can be defined in two ways, either lumped or consistent. The two method will be considered here.

4.3.1 Lumped Mass Matrix

The simplest procedure for finding the mass properties of any structure is to assume that the entire mass is concentrated at the points at which the translational displacements are defined.

The lumped mass matrix for a uniform beam element which is shown in Fig.(4.1), neglecting the rotary inertia can be given as [67]:

$$[M] = \frac{\bar{m}L}{2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots\dots\dots(4.2)$$

in which $\bar{m}$: is the mass per unit length.

If the rotational inertia is taken into account, the lumped mass matrix will be written as:

$$[M] = \frac{\bar{m}L}{2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & L^2/12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & L^2/12 \end{bmatrix} \dots\dots\dots(4.3)$$



Fig.(4.1): Lumping of Mass at Element Nodes.

For a space element, the lumped mass matrix can be constructed by combining the mass matrices in two directions (**y**) and (**z**) [67], taking into consideration the torsional effects for a prismatic beam element. Using a special symbol ($\begin{bmatrix} \end{bmatrix}$), for diagonal matrices and for the beam element which is shown in **Fig.(4.2)**, the lumped mass matrix will be given as:

$$[M] = \frac{\bar{m}L}{2} \begin{bmatrix} 1 & 1 & 1 & \frac{J}{A} & 0 & 0 & 1 & 1 & 1 & \frac{J}{A} & 0 & 0 \end{bmatrix} \dots\dots\dots(4.4)$$

with no rotary inertia effect, and

$$[M] = \frac{\bar{m}L}{2} \begin{bmatrix} 1 & 1 & 1 & \frac{J}{A} & \frac{L^2}{12} & \frac{L^2}{12} & 1 & 1 & 1 & \frac{J}{A} & \frac{L^2}{12} & \frac{L^2}{12} \end{bmatrix} \dots\dots(4.5)$$

with rotary inertia effect.

It may be noted that the lumped mass matrix remains constant during the analysis also, because it is a diagonal matrix, the local and global matrix is the same.

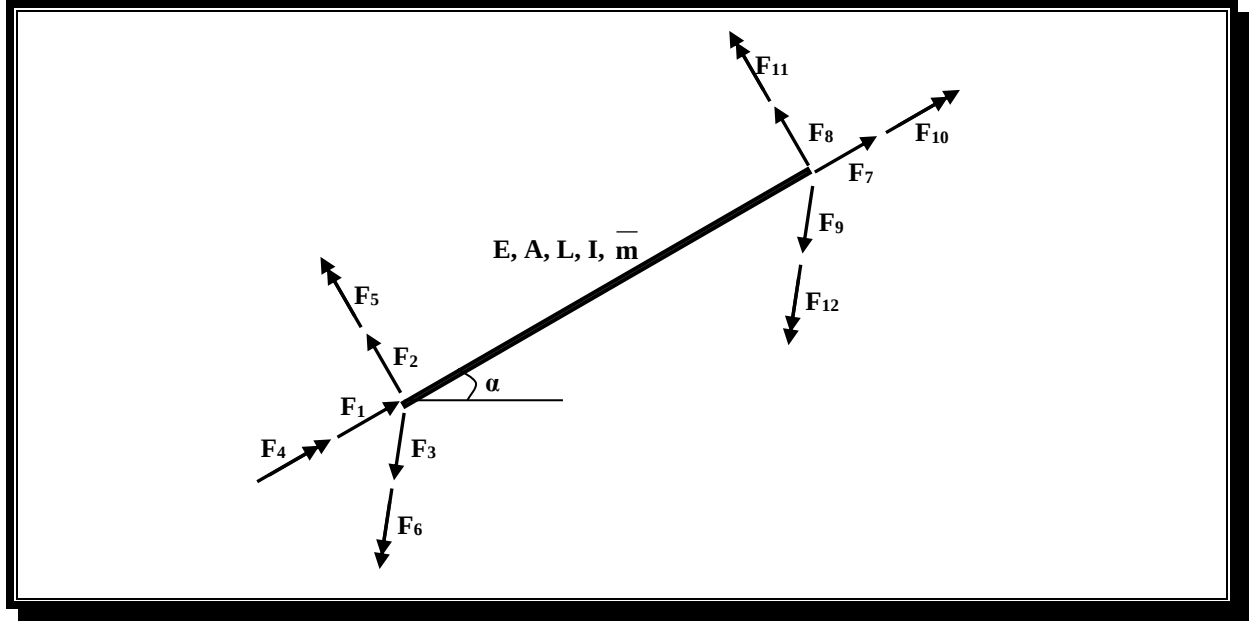


Fig.(4.2): Space Beam Element.

4.3.2 Consistent Mass Matrix

The consistent mass matrix for a two dimensional beam element which is shown in Fig.(4.1), that is derived by AL-Rawanduzay [27], can be given in local coordinates as:

$$[M_c] = \frac{\bar{m}L}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 + 468 \frac{I}{AL^2} & 22L + 39 \frac{I}{AL} & 0 & 54 - 468 \frac{I}{AL^2} & -13L + 39 \frac{I}{AL} \\ 0 & 22L + 39 \frac{I}{AL} & 4L^2 + 52 \frac{I}{A} & 0 & 13L - 39 \frac{I}{AL} & -3L^2 - 13 \frac{I}{A} \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 - 468 \frac{I}{AL^2} & 13L - 39 \frac{I}{AL} & 0 & 156 + 468 \frac{I}{AL^2} & -22L - 39 \frac{I}{AL} \\ 0 & -13L + 39 \frac{I}{AL} & -3L^2 - 13 \frac{I}{A} & 0 & -22L - 39 \frac{I}{AL} & 4L^2 + 52 \frac{I}{A} \end{bmatrix} \quad \dots(4.6)$$

Fro a space element, the consistent mass matrix can be constructed by combining the mass matrix in two directions [67]. Thus, for a three dimensional beam element, it will given as:

$$[M_c] = \frac{mL}{420} \begin{bmatrix} 140 & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 156 & 0 & 0 & 0 & 22L & 0 & 54 & 0 & 0 & 0 & -13L \\ 0 & 0 & 156 & 0 & -22L & 0 & 0 & 0 & 54 & 0 & 13L & 0 \\ 0 & 0 & 0 & 140 \frac{J}{A} & 0 & 0 & 0 & 0 & 0 & 70 \frac{J}{A} & 0 & 0 \\ 0 & 0 & -22L & 0 & 4L^2 & 0 & 0 & 0 & -13L & 0 & -3L^2 & 0 \\ 0 & 22L & 0 & 0 & 0 & 4L^2 & 0 & 13L & 0 & 0 & 0 & -3L^2 \\ 70 & 0 & 0 & 0 & 0 & 0 & 140 & 0 & 0 & 0 & 0 & 0 \\ 0 & 54 & 0 & 0 & 0 & 13L & 0 & 156 & 0 & 0 & 0 & -22L \\ 0 & 0 & 54 & 0 & -13L & 0 & 0 & 0 & 156 & 0 & 22L & 0 \\ 0 & 0 & 0 & 70 \frac{J}{A} & 0 & 0 & 0 & 0 & 0 & 140 \frac{J}{A} & 0 & 0 \\ 0 & 0 & 13L & 0 & -3L^2 & 0 & 0 & 0 & 22L & 0 & 4L^2 & 0 \\ 0 & -13L & 0 & 0 & 0 & -3L^2 & 0 & -22L & 0 & 0 & 0 & 4L^2 \end{bmatrix} \quad \dots(4.7)$$

To transform the consistent mass matrix from local to global coordinates, use the transformation matrix $[R_m]$, which can be defined as:

$$[R_m] = \begin{bmatrix} [r_m] & 0 & 0 & 0 \\ 0 & [r_m] & 0 & 0 \\ 0 & 0 & [r_m] & 0 \\ 0 & 0 & 0 & [r_m] \end{bmatrix}; [r_m] = \begin{bmatrix} m & n & 0 \\ -n & m & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \dots\dots\dots(4.8)$$

in which,

$$m = \cos \alpha, \quad n = \sin \alpha \quad \dots\dots\dots(4.9)$$

and, α : is the angle between global and local coordinate axes, then

$$[M] = [R_m]^T [M_c] [R_m] \quad \dots\dots\dots(4.10)$$

where the superscript **T** denotes transpose, and **[M]** is the consistent mass matrix in global coordinates. Thus, the consistent mass matrix in geometrically non-linear problems depends on the deformed configuration, and it will not remain constant during the analysis and this will complicate the problem.

4.4 Damping Matrix

In the analysis of dynamic problems, it is considered that the amplitude of the free vibration remains constant with time, but in actual structures such a vibration without decrease in amplitude is never realized. This is diminishing of amplitude occurs because of the presence of damping forces. These forces

cause the dissipation of energy that progressively reduces the amplitude of vibration and ultimately stops the motion when all energy initially stored in the system has been dissipated. The most common analytical model of damping employed in structural dynamics analyses is the linear viscous dashpot model (viscous damping) [70, 71].

With the present understanding of damping in structures, it is not possible to formulate an explicit damping matrix for distributed damping throughout a structure in a manner similar to that followed for the stiffness **[K]** and mass **[M]** matrices.

Viscous type damping can be used in whatever is the form of the excitation. The most common form of such damping is the so-called **Rayleigh-type damping** given by [69, 72]:

$$[C] = Z_1[M] + Z_2[K] \quad \dots\dots\dots(4.11)$$

in which,

Z_1 and Z_2 : arbitrary proportionality factors.

Then by making benefit of modal analysis, Equation (4.11) can be reduced to,

$$2\xi_r\omega_r = Z_1 + \omega_r^2 Z_2 \quad \dots\dots\dots(4.12)$$

where:

ξ_r, ω_r : the modal damping ratio and the natural frequency for the mode (**r**) respectively (a typical variation of the damping ratio with the natural frequency being illustrated in **Fig.(4.3)**). The two factors Z_1 and Z_2 can be determined by specifying the damping ratio for two modes, for example, **1** and **2**, and substituting them into Equation (4.12):

$$2\xi_1\omega_1 = Z_1 + \omega_1^2 Z_2 \quad \dots\dots\dots(4.13)$$

$$2\xi_2\omega_2 = Z_1 + \omega_2^2 Z_2 \quad \dots\dots\dots(4.14)$$

The solution of these two equations (4.13) and (4.14) gives:

$$Z_1 = \frac{2\omega_1\omega_2(\omega_2\xi_1 - \omega_1\xi_2)}{\omega_2^2 - \omega_1^2} \quad \dots\dots\dots(4.15)$$

$$Z_2 = \frac{2(\omega_2 \xi_2 - \omega_1 \xi_1)}{\omega_2^2 - \omega_1^2} \dots\dots\dots(4.16)$$

Now, by substituting the values of Z_1 and Z_2 into equation (4.11), the damping matrix is obtained.

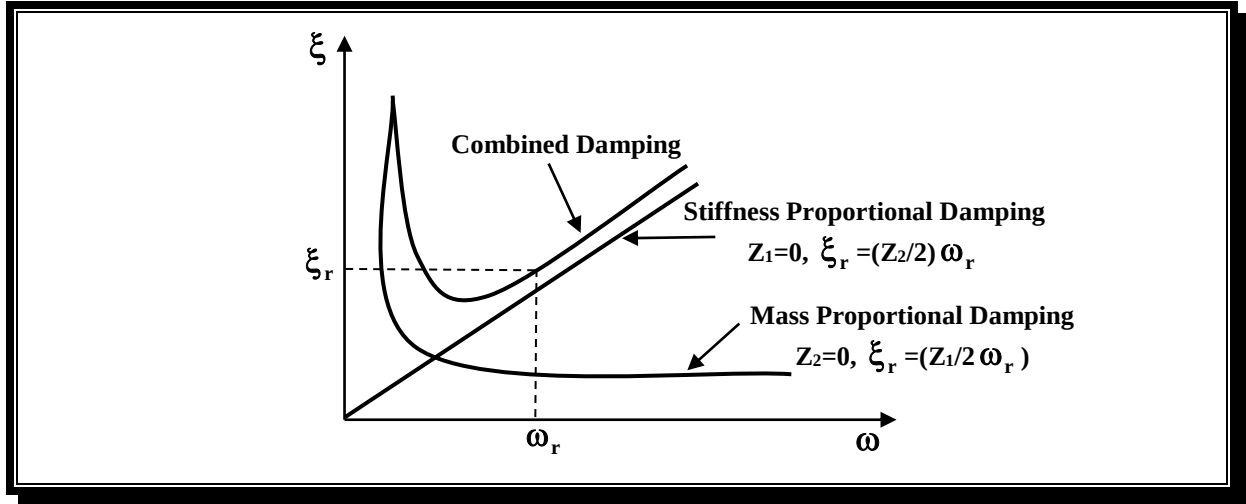


Fig.(4.3): Relationship Between Damping Ratio and Frequency for Rayleigh Damping.

Natural frequencies, ω_1 and ω_2 used in Equations (4.15) and (4.16) are obtained from the solution of the eigenvalue problem for undamped case. The corresponding damping ratios can be obtained by finding the damping ratio ξ_1 which is related to the first mode of vibration using field test of the structure or from the previous experience or by assuming it within an acceptable range according to the type of the structure. With the value of ξ_1 in hand, any other value ξ_r can be obtained by using the approximate formula [29]:

$$\xi_r = \xi_1 \left(\frac{\omega_r}{\omega_1} \right)^{el}; (0.5 \leq el \leq 0.7) \dots\dots\dots(4.17)$$

Structures generally have a value of the critical damping ratio in the range of **0.02 to 0.1**, [73].

4.5 Direct Integration Method

In the direct integration method, the equation of motion, equation (4.1), is integrated by using a numerical step-by-step procedure. Basically there are two general classes of numerical approaches for dynamics problems, explicit and implicit. Implicit approaches are based on the unknowns of next time step and it is numerically stable due to positive values of the diagonal values of the constituting matrix thus permitting large time step, but the cost per time step is high and storage requirements tend to increase dramatically with the number of D.O.F. of the structure [26]. On the other hand, explicit approaches are based on known values of the current time step and it is inexpensive per step and require less storage than the implicit approaches, but the numerical stability requires that small time steps be employed to prevent occurrence of negative diagonal values in the constituting matrix [74].

Many explicit and implicit methods for direct integration are available. However, in the present study the **implicit-explicit form of Newmark** method is adopted.

4.5.1 Newmark Family of Methods

The most widely used family of implicit methods of direct time integration for solving the equation of motion is **Newmark's Family** of methods. In 1959 **Newmark** [26] presented equations for approximating the velocity and displacement of a single degree of freedom system at time $t+\Delta t$, as follows:

$$\mathbf{X}_{t+\Delta t}^0 = \mathbf{X}_t^0 + \Delta t[(1-\gamma)\mathbf{X}_t^{00} + \gamma\mathbf{X}_{t+\Delta t}^{00}] \quad \dots\dots\dots(4.18)$$

$$\mathbf{X}_{t+\Delta t} = \mathbf{X}_t + \Delta t(\mathbf{X}_t^0) + (\Delta t)^2 \left[\left(\frac{1}{2} - \beta \right) \mathbf{X}_t^{00} + \beta(\mathbf{X}_{t+\Delta t}^{00}) \right] \quad \dots\dots\dots(4.19)$$

where:

γ and β : parameters that determine the stability and accuracy of the approach,

Δt : the time interval.

For a multi-degree of freedom system, Equations (4.18) and (4.19), become in the following matrix form,

$$\{X^0\}_{t+\Delta t} = \{X^0\}_t + \Delta t[(1-\gamma)\{X^{00}\}_t + \gamma\{X^{00}\}_{t+\Delta t}] \dots\dots\dots(4.20)$$

$$\{X\}_{t+\Delta t} = \{X\}_t + \Delta t\{X^0\}_t + (\Delta t)^2 \left[\left(\frac{1}{2} - \beta \right) \{X^{00}\}_t + \beta \{X^{00}\}_{t+\Delta t} \right] \dots\dots\dots(4.21)$$

Assuming different values for γ and β , different formulas are obtained within the Newmark's family of methods.

4.5.1.1 Stability and Accuracy Aspects

The first major concern in performing a numerical analysis is the numerical stability. It is noted that if $\gamma = \beta = 0$, explicit expressions are obtained and if $\gamma = 1$ and $\beta = 1/2$, full implicit expressions are obtained. At the intermediate value of γ and β , mixed explicit-implicit expressions are obtained.

The investigation of stability of Newmark method indicates that it is unconditionally stable provided if,

$$\gamma \geq 0.5 \text{ and } \beta \geq \frac{(\gamma + 0.5)^2}{4} \dots\dots\dots(4.22)$$

For $\gamma = 1/2$ and $\beta = 1/4$ an unconditionally stable method can be obtained, which assumes a constant acceleration during the time step and is called the **average acceleration** method. The **linear acceleration** method which assumes that the acceleration varies linearly during the time step and can be obtained by taking $\gamma = 1/2$ and $\beta = 1/6$.

In the present study values of $\gamma = 1/2$ and $\beta = 1/4$ are adopted because it has been shown to have a high degree of numerical stability [73, 75, 76].

Another concern in performing a numerical analysis is the numerical accuracy. Reducing the integration time step will increase the accuracy of the calculation. Research has shown that for a damping system a time step size of

$\Delta t \leq \frac{T}{10}$ gives results with good accuracy, where T denotes the smallest significant (fundamental) period of the structure [73].

4.5.1.2 Basic Equations

By considering the dynamic equilibrium at time $t + \Delta t$, Equation (4.1) becomes [26]:

$$[M]_{t+\Delta t} \{X^{00}\}_{t+\Delta t} + [C]_{t+\Delta t} \{X^0\}_{t+\Delta t} + [K]_{t+\Delta t} \{X\}_{t+\Delta t} = \{F(t)\}_{t+\Delta t} \quad \dots\dots\dots(4.23)$$

where $\{X^{00}\}_{t+\Delta t}$, $\{X^0\}_{t+\Delta t}$, $\{X\}_{t+\Delta t}$ and $\{F(t)\}_{t+\Delta t}$ are acceleration, velocity, displacement and dynamic applied load vectors at time $t + \Delta t$, respectively.

In order to solve Equation (4.23), it is necessary first to write in terms of one unknown, $\{X\}_{t+\Delta t}$. By solving Equation (4.21) for $\{X^{00}\}_{t+\Delta t}$ in terms of $\{X\}_{t+\Delta t}$ and then substituting $\{X^{00}\}_{t+\Delta t}$ into Equation (4.20), the following equations for $\{X^{00}\}_{t+\Delta t}$ and $\{X^0\}_{t+\Delta t}$ in term of $\{X\}_{t+\Delta t}$ are obtained:

$$\{X^{00}\}_{t+\Delta t} = \frac{1}{\beta \Delta t^2} (\{X\}_{t+\Delta t} - \{X\}_t) - \frac{1}{\beta \Delta t} \{X^0\}_t - \left(\frac{1}{2\beta} - 1 \right) \{X^{00}\}_t \quad \dots\dots\dots(4.24)$$

$$\{X^0\}_{t+\Delta t} = \frac{\gamma}{\beta \Delta t} (\{X\}_{t+\Delta t} - \{X\}_t) + \left(1 - \frac{\gamma}{\beta} \right) \{X^0\}_t - \Delta t \left(\frac{\gamma}{2\beta} - 1 \right) \{X^{00}\}_t \quad \dots(4.25)$$

If Equations (4.24) and (4.25) are substituted into Equation (4.23) and keeping the non-linear analysis is changed with time then:

$$\begin{aligned} & \left[\frac{1}{\beta \Delta t^2} [M]_{t+\Delta t} + \frac{\gamma}{\beta \Delta t} [C]_{t+\Delta t} + [K]_{t+\Delta t} \right] (\{X\}_{t+\Delta t} - \{X\}_t) = \{F(t)\}_{t+\Delta t} - \\ & [K]_{t+\Delta t} \{X\}_t + [M]_{t+\Delta t} \left(\frac{1}{\beta \Delta t} \{X^0\}_t + \left(\frac{1}{2\beta} - 1 \right) \{X^{00}\}_t \right) + \\ & [C]_{t+\Delta t} \left(\left(\frac{\gamma}{\beta} - 1 \right) \{X^0\}_t + \Delta t \left(\frac{\gamma}{2\beta} - 1 \right) \{X^{00}\}_t \right) \end{aligned} \quad \dots(4.26)$$

$$\text{Writing } [\bar{K}] = \frac{1}{\beta \Delta t^2} [M]_{t+\Delta t} + \frac{\gamma}{\beta \Delta t} [C]_{t+\Delta t} + [K]_{t+\Delta t} \quad \dots\dots\dots(4.27)$$

$$\{\Delta X\} = \{X\}_{t+\Delta t} - \{X\}_t \quad \dots\dots\dots(4.28)$$

$$\begin{aligned} \{\Delta \bar{F}(t)\} = \{F(t)\}_{t+\Delta t} - [K]_{t+\Delta t} \{X\}_t + [M]_{t+\Delta t} \left(\frac{1}{\beta \Delta t} \{X^0\}_t + \left(\frac{1}{2\beta} - 1 \right) \{X^{00}\}_t \right) \\ + [C]_{t+\Delta t} \left(\left(\frac{\gamma}{\beta} - 1 \right) \{X^0\}_t + \Delta t \left(\frac{\gamma}{2\beta} - 1 \right) \{X^{00}\}_t \right) \end{aligned} \quad \dots(4.29)$$

At the beginning of the analysis, the matrices $[K]_{t+\Delta t}$, $[M]_{t+\Delta t}$ and $[C]_{t+\Delta t}$ are taken at time t as an approximation, then Equations (4.27) and (4.29) become:

$$[\bar{K}] = \frac{1}{\beta \Delta t^2} [M]_t + \frac{\gamma}{\beta \Delta t} [C]_t + [K]_t \quad \dots\dots\dots(4.30)$$

$$\begin{aligned} \{\Delta \bar{F}(t)\} = \{F(t)\}_{t+\Delta t} - \{f\}_t + [M]_t \left(\frac{1}{\beta \Delta t} \{X^0\}_t + \left(\frac{1}{2\beta} - 1 \right) \{X^{00}\}_t \right) \\ + [C]_t \left(\left(\frac{\gamma}{\beta} - 1 \right) \{X^0\}_t + \Delta t \left(\frac{\gamma}{2\beta} - 1 \right) \{X^{00}\}_t \right) \end{aligned} \quad \dots\dots\dots(4.31)$$

where $\{f\}_t$ is the internal force vector at time t . Consequently Equation (4.26) may be written as:

$$[\bar{K}]\{\Delta X\} = \{\Delta \bar{F}(t)\} \quad \dots\dots\dots(4.32)$$

Now by solving Equation (4.32) for $\{\Delta X\}$, approximate values for accelerations, velocities and displacements may be given as follows:

$$\{X^{00}\}_{t+\Delta t} = \frac{1}{\beta(\Delta t)^2} \{\Delta X\} - \frac{1}{\beta \Delta t} \{X^0\}_t - \left(\frac{1}{2\beta} - 1 \right) \{X^{00}\}_t \quad \dots\dots\dots(4.33)$$

$$\{X^0\}_{t+\Delta t} = \frac{\gamma}{\beta \Delta t} \{\Delta X\} + \left(1 - \frac{\gamma}{\beta} \right) \{X^0\}_t - \Delta t \left(\frac{\gamma}{2\beta} - 1 \right) \{X^{00}\}_t \quad \dots\dots\dots(4.34)$$

$$\{X\}_{t+\Delta t} = \{X\}_t + \{\Delta X\} \quad \dots\dots\dots(4.35)$$

4.5.1.3 Initial Conditions

In order to start the numerical integration it is necessary first to give initial values for the displacement and velocity vectors. Then, the initial accelerations vector can be obtained by considering the dynamic equilibrium at time $t=0$.

$$\{X^{00}\}_{t=0} = [M]^{-1} (\{F(t)\}_{t=0} - [C]\{X^0\}_{t=0} - [K]\{X\}_{t=0}) \quad \dots\dots\dots(4.36)$$

4.6 Evaluation of Natural Frequencies

The formulation of damping matrix $[C]$, requires the knowledge of two natural frequencies $\{\omega(\text{rad./sec.})\}$. Also, in order to determine the critical time step for direct integration, the smallest natural period $T(\text{sec.})$ is required which is related to the natural frequency as follows:

$$T = \frac{2\pi}{\omega} \dots\dots\dots(4.37)$$

Therefore, in order to perform the analysis, the evaluation of natural frequencies is necessary to accomplish first.

To determine the natural frequencies of a structure, a free vibration problem (no external applied loads) for the undamped case is assumed [77]:

$$[M]\{X^{00}\} + [K]\{X\} = 0 \dots\dots\dots(4.38)$$

Assuming a harmonic motion, that is:

$$\{X_i\} = \{\varpi_i\} \sin \omega_i t \quad i = 1, 2, \dots, n \dots\dots\dots(4.39)$$

where:

n : the number of D.O.F of the system,

$\{\varpi_i\}$: the mode shape vector for the i th mode of vibration.

Differentiating Equation (4.39) twice with respect to time yields:

$$\{X_i^{00}\} = -\omega_i^2 \{\varpi_i\} \sin \omega_i t \dots\dots\dots(4.40)$$

and, substituting Equations (4.39) and (4.40) into Equation (4.38), then, after canceling the term $(\sin \omega_i t)$:

$$([K] - \omega_i^2 [M])\{\varpi_i\} = 0 \dots\dots\dots(4.41)$$

Equation (4.41) has the form of the **algebraic eigenvalue problem** ($T\varpi = \lambda M\varpi$). From the theory of homogeneous equations, non-trivial solutions exist only if the determinant of the coefficient matrix is equal to zero. Thus:

$$[\mathbf{K}] - \omega_i^2 [\mathbf{M}] = 0 \quad \dots\dots\dots(4.42)$$

Expansion of the determinant yields a polynomial of order **n** called **characteristic equation**. The **n** roots of this polynomial, ω_i^2 , are the **characteristic values** or **eigenvalues**. Substitution of these roots (one at each time) into the homogeneous equation (4.41) produces the **characteristic vectors** or **eigenvectors**, $\{\varpi_i\}$, within arbitrary constants.

To solve the eigenvalue problem, use **inverse power iteration method**. The basic inverse iteration technique is very effective in calculating the smallest eigenvalue and the corresponding eigenvector. But, the method can be also employed with shifting to calculate other eigenvalues and corresponding eigenvectors.

The basic steps for solving the eigenvalue problem of the form $(\mathbf{T}\varpi = \lambda\mathbf{M}\varpi)$ by using the inverse iteration method are [15, 73]:

1. Assuming a starting iteration vector, $\{\mathbf{X}_1\}$, almost with all terms equal to one.

2. Assuming that:

$$\{\mathbf{Y}_1\} = [\mathbf{M}]\{\mathbf{X}_1\} \quad \dots\dots\dots(4.43)$$

3. Evaluating for each iteration step $k=1, 2, \dots\dots\dots$

$$\{\bar{\mathbf{X}}_{k+1}\} = [\mathbf{K}]^{-1} \{\mathbf{Y}_k\} \quad \dots\dots\dots(4.44)$$

$$\{\bar{\mathbf{Y}}_{k+1}\} = [\mathbf{M}]\{\bar{\mathbf{X}}_{k+1}\} \quad \dots\dots\dots(4.45)$$

$$\rho(\{\bar{\mathbf{X}}_{k+1}\}) = \frac{\{\bar{\mathbf{X}}_{k+1}\}^T \{\mathbf{Y}_k\}}{\{\bar{\mathbf{X}}_{k+1}\}^T \{\bar{\mathbf{Y}}_{k+1}\}} \quad \dots\dots\dots(4.46)$$

$$\{\mathbf{X}_{k+1}\} = \frac{\{\bar{\mathbf{X}}_{k+1}\}}{\left(\{\bar{\mathbf{X}}_{k+1}\}^T \{\bar{\mathbf{Y}}_{k+1}\}\right)^{1/2}} \quad \dots\dots\dots(4.47)$$

where the superscript **T** denote transpose.

4. The value of $\rho(\{\bar{X}_{k+1}\})$ in Equation (4.46) represents an approximation to the eigenvalue, λ_1 . Denoting the current approximation for λ_1 by $\lambda_1^{(k+1)}$, (i.e., $\lambda_1^{(k+1)} = \rho(\{\bar{X}_{k+1}\})$), the convergence will occur when:

$$\frac{|\lambda_1^{(k+1)} - \lambda_1^{(k)}|}{\lambda_1^{(k+1)}} \leq 10^{-s} \quad \dots\dots\dots(4.48)$$

where "s" is the number of significant digits of the desired accuracy equal to $(1*10^{-3})$.

5. If **m** is the last iteration cycle, then, the smallest eigenvalue, λ_1 , and the corresponding eigenvector, $\{\varpi_1\}$, will be respectively:

$$\lambda_1 = \rho(\{\bar{X}_{m+1}\}) \quad \dots\dots\dots(4.49)$$

$$\{\varpi_1\} = \{X\}^{k+1} \quad \dots\dots\dots(4.50)$$

The other eigenpair can be obtained by the development of an inverse iteration method by the **Gram-Schmidt** method [31, 73]. In Gram-Schmidt a vector $\{\psi\}$ is calculated by using:

$$\{\psi\} = \{\varpi_1\} - \sum_{i=1}^n \vartheta_i \{\varpi_i\} \quad \dots\dots\dots(4.51)$$

where the coefficient ϑ_i are obtained using the conditions that:

$$\{\varpi_i\}^T [M] \{\psi\} = 0 \quad i=1, \dots\dots, n \quad \dots\dots\dots(4.52)$$

$$\text{and } \{\varpi_i\}^T [M] \{\varpi_i\} = \delta_{ij} \quad \dots\dots\dots(4.53)$$

where δ_{ij} is Kronecker delta, $\delta_{ij}=1$ for $i=j$ and $\delta_{ij}=0$ for $i \neq j$.

Premultiplying both sides of equation (4.51) by $\{\varpi_i\}^T [M]$, one may obtain:

$$\vartheta_i = \{\varpi_i\}^T [M] \{\varpi_1\} \quad \dots\dots\dots(4.54)$$

Now the inverse iteration method is started using the vector $\{\psi\}$ instead of $\{X_1\}$ to determine the other value of the eigenvalues.

4.7 Earthquake Occurrence

In recent years, structural engineers have been giving increasing attention to the analysis and design of buildings for earthquake resistance.

Due to the fact that some parts of the world are more confronted to earthquakes than others such as USA and Japan (lying in the intensive seismic activity belt of the Earth), these countries have higher contribution within this field of studies concerning this phenomenon. Various methods and scales have been developed and modified to calculate and predict the seismic forces on building members.

According to the elastic-rebound theory [67], the associated strains and stresses fault or abutting moving plates in the Earth crust gradually accumulate until they reach a magnitude sufficient to fracture or to cause slippage along some of these existing fault planes. Release of stress cause by fracture or slippage at one point causes a build-up of stresses in adjoining materials, so that slippage propagates along the entire fault line. This results in a sudden rebound of the elastic strain (like a compressed elastic spring).

The accumulated strain energy in the rock, suddenly released, will cause stress waves to propagates in all directions in a series of shock waves emanating from the source. Depending on the magnitude of the energy releases, a violent motion in the Earth crust travels in all directions lasting from a few seconds in moderate earthquakes to several minutes in large earthquakes.

Sometimes an earthquake occurs not on the boundaries of the moving plates such as the earthquake that occurred in 1811 and 1812 in the Mississippi valley. This kind of earthquake cannot be related to the tectonic movements of plates. It is related to volcanic eruption, or subterranean movements of magma. This type is called **volcanic** type.

Newmark and **Rosenblueth** [76] divided the earthquake into four groups:

1. Single shock earthquakes.
2. Earthquakes of moderate duration and irregular vibration.
3. Earthquakes of large duration and high predominant periods.
4. Earthquakes that cause permanent deformations.

The point of fracture within the earth where the initial slip occurs, is called the earthquake focus or hypocenter. The point on the surface vertically above the focus is called the epicenter as shown in **Fig.(4.4)**. Based on focal depth, earthquakes may be divided into three categories [78]:

- (i) Deep-focus earthquakes: These have focal depths of 300-700 km.
- (ii) Intermediate-focus earthquakes: These have focal depth of 70-300 km.
- (iii) Shallow-focus earthquakes: The focal depth for these is less than 70 km.

About 75% of all the earthquakes around the world are of the shallow type.

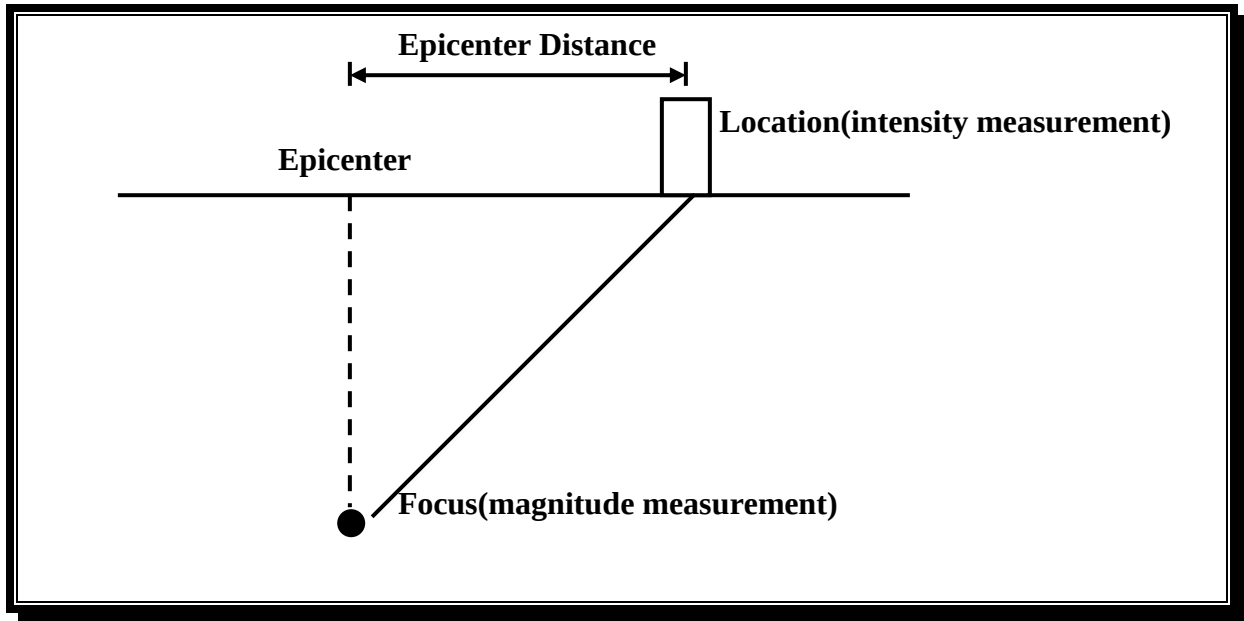


Fig.(4.4): Idealized Point-Source Earthquake Rupture.

4.8 Methods of Earthquake Measuring

4.8.1 Intensity Scales

One of the earliest methods developed to measure the magnitude of the earthquake was the intensity scale. It measures the intensity of an earthquake as compared to various earthquakes [79]. So it measures the damage potential of an earthquake, and depending on the personal observation and judgments.

One of the earliest intensity scales was the Rossi-Frel scale developed in 1883, known as **R.F.** scale. This scale was widely adopted by many countries, and still is used in many European countries. But as it is very old and it does not include the effects on modern buildings and new materials. The more applicable intensity scale is the Mercalli scale. This scale was developed by Mercalli in 1902. In 1923, Seiberg expanded this scale to include a wider range of earthquakes. Later, this scale was modified by Woods and Newmann in 1931 and was called the modified Mercalli scale (**M.M.**) scale.

4.8.2 Magnitude Scales

With the advance of the seismograph, it has become possible to provide a more quantitative evaluation of the magnitude of an earthquake, in terms of total energy released at the origin [67].

Richter scale is the most commonly used magnitude scale which measures the maximum amplitude (**A**) of the ground vibration as recorded by the seismograph. This value is corrected for the distance of the recording device from the epicenter by subtracting factor related to distance (**A₀**).

$$M = \log(A) - \log(A_0) \dots\dots\dots(4.55)$$

where:

M : is the Richter magnitude,

A : is the maximum amplitude as recorded,

A₀ : is the distance of the seismograph from the epicenter.

The Richter scale is not a good measure of the damage potential of the earthquake, because the damage potential of an earthquake is not only a function of the maximum amplitude. There are many other factors affecting the damage such as time duration, frequency of the vibration and geology of the site.

Although, there is no direct conversion factor between Richter and modified Mercalli scales but there is an empirical relationship between the two scales as illustrated in **Table (4.1)**.

4.8.3 Response Spectrum Seismic Scale

Housner has proposed another way to measure the severity of ground motion, using the area under the relative velocity or acceleration response spectrum. Response spectrum is a plot giving the maximum responses (in terms of displacement or stress or acceleration ...etc.) of all possible linear one-degree systems due to a given input. The abscissa of the spectrum is the natural frequency or time period of the system and the ordinate is the maximum response. More details are given in references [77, 79, 80].

Table (4.1): The Relationship Between Richter and Modified Mercalli Scales.

Richter Magnitude	Maximum Intensity (M. M.)
2	I – II
3	III
4	V
5	VI – VII
6	VII – VIII
7	IX – X
8	XI

4.9 Earthquake Excitations

Earthquake ground motions are usually expressed in terms of three components of translational accelerations. The response of any system to these three components of input can be computed by superposing the responses calculated separately for each component. Thus, the standard analytical problem is reduced to the evaluation of the structural response to a single component of support translation.

In more general cases, the support point will be subjected to rotations in addition to the translation motions. Thus a complete description of the earthquake input should include three components of support rotations as well as translations [81].

The earthquake loading has a unique character among the types of loads that they must be considered because a great earthquake would generally cause great stresses and deflections in various critical components of the structure than all other loadings combined. It is noted usually that the probability of occurring of an earthquake within the expected life of the structure is very low and in order to deal effectively with this combination of extreme loading and low probability, the following strategy was adopted:

1. A moderate earthquake which reasonably may be expected at the building site during the life of structure is taken as the basis of design and the building should be proportioned to resist this intensity of ground motion without significant damage to the basic structural elements (such as columns).
2. The most severe earthquake which possibly could occur at the site is applied as a test on the structure safety.

The basic data of earthquakes are the records of ground accelerations during an earthquake and the one of the simplest ways to define the expected ground motions is to make use of the accelerogram of past earthquakes. In this study, the relation between the acceleration of the earthquake and time is

represented by a polynomial function obtained from a polynomial regression of the past data.

4.10 Earthquake Equation of Motion

The motion equations for earthquake loads, which produce translation or rotation of the base of the structures, are written in the form:

$$[M]\{\ddot{X}^{00}\}_t + [C]\{\dot{X}^0\} + [K]\{X\} = [0] \quad \dots\dots\dots(4.56)$$

where as we illustrate previous:

- [M] : structural mass matrix,
- [C] : structural damping matrix,
- [K] : structural stiffness matrix,
- $\{\ddot{X}^{00}\}$: nodal acceleration vector,
- $\{\dot{X}^0\}$: nodal velocity vector,
- $\{X\}$: nodal displacement vector.

The subscript **(t)** denotes the total displacement, i.e. $\{\ddot{X}^{00}\}$ represent the total acceleration, since the inertia force depends on the total nodal accelerations of the structure, whereas the damping stiffness forces depend on the relative nodal displacements and velocities to the base of the structure respectively. The right-hand side of Equation (4.56) is equal to **[0]**, which means that there is no external dynamic load applied directly to the system but the excitation is provided by the motion introduced at the base of the structure, as shown in **Fig.(4.5)**.

The total nodal displacement may be expressed as:

$$\{X\}_t = \{X\} - \{r_g\}\{X_g\} \quad \dots\dots\dots(4.57)$$

where:

- $\{X\}_t$: total nodal displacement,
- $\{X\}$: relative nodal displacement,
- $\{X_g\}$: nodal displacement at the base,

$\{r_g\}$: influence coefficient vector representing the rigid displacement in the direction of the degree of freedom resulting from a unit base displacement.

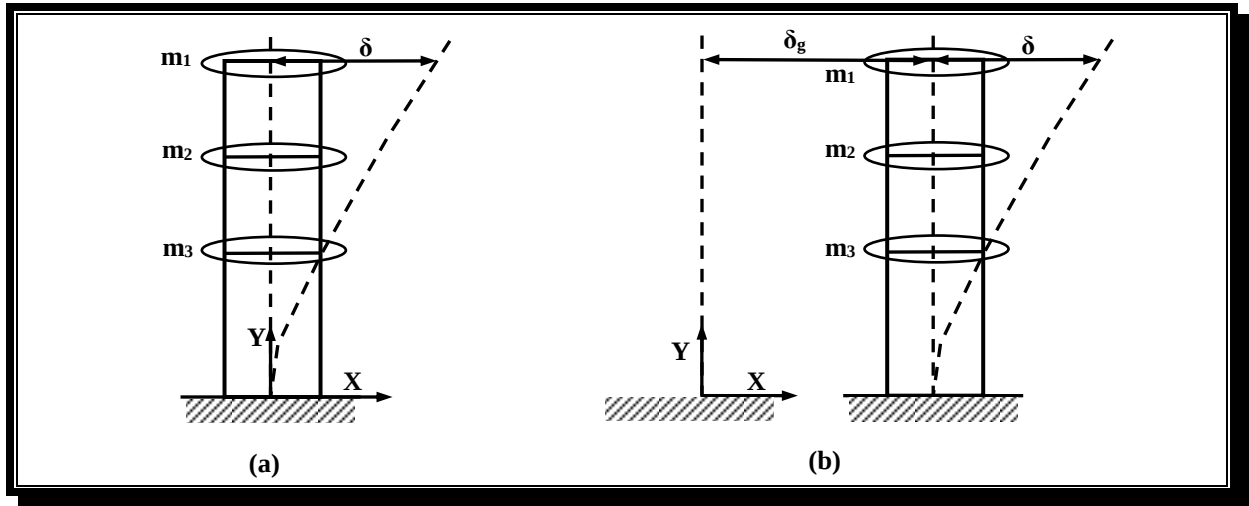


Fig.(4.5): Discrete M.D.O.F System, (a) Subjected to Dynamic Load Only, (b) subjected to Earthquake and Dynamic Loads.

Substituting Equation (4.57) into Equation (4.56) gives:

$$[M]\{X^{00}\} + [M]\{\varpi_1\}^T \{r_g\} \{X_g^{00}\} + [C]\{X^0\} + [K]\{X\} = [0] \quad \text{.....(4.58)}$$

or

$$[M]\{X^{00}\} + [C]\{X^0\} + [K]\{X\} = \{P_{eff}(t)\} \quad \text{.....(4.59)}$$

in which,

$$\{P_{eff}(t)\} = -[M]\{\varpi_1\}^T \{r_g\} \{X_g^{00}\} \quad \text{.....(4.60)}$$

where $\{P_{eff}(t)\}$ is the effective earthquake forces vector resulting from the ground motion.

The negative sign in equation (4.60) is of no importance since the earthquake loads are of an alternative character, and thus the sign can be neglected [67, 77].

If external dynamic loads are applied to the structure besides the earthquake loads, the right side of equation (4.59) becomes;

$$[M]\{X^{00}\} + [C]\{X^0\} + [K]\{X\} = \{P_{eff}(t)\} + \{F(t)\} \quad \text{.....(4.61)}$$



Optimal Design Formulation

5.1 General

Optimization is defined as finding the optimum value (maximum or minimum) of a function called the **objective function** $f(x_1, x_2, x_3, \dots, x_n)$ of **(n)** real variables called the **design variables**. The optimization is constrained when certain limitations are imposed on the design variables, while the optimization is non-linear when the mathematical expressions relating the design variables in constraints or in the objective function are of non-linear nature.

In the present work, uses the **modified Hooke and Jeeves method** to represent the non-linear constrained optimization for space steel structure.

5.2 Direct Search Method

The general optimization problem is given as follows: **minimize** $f(x_1, x_2, x_3, \dots, x_n)$, of **(n)** real variables subject to the constraints $g_i(x_1, x_2, x_3, \dots, x_n) \leq b_i$, ($i = 1, 2, \dots, m$). There is of course

no loss of generality in assuming that all constraints are less than or equal to specified values i.e. the constraint $\mathbf{g}(\mathbf{x}) \geq 0$ can be written as $-\mathbf{g}(\mathbf{x}) \leq 0$.

Much effort has been devoted to devising direct search methods to locate the minimum of the function of (n) real variables. As an example a function of two variables will be considered herein. Its contour lines are shown in **Fig. (5.1)**, its minimum is at (X_1^*, X_2^*) . The crudest search method is the alternating variable search method. One starts at some point **A** and search in the direction of X_1 -axis for a minimum in this direction and thus find **B** at which the tangent to the contour line is parallel to the X_1 direction. From **B** one then search in the direction of the X_2 -axis and so proceed to **C** and then to **D** by searching parallel to the X_1 -axis...etc. In this way, one proceeds to the optimum point. It is clearly possible to extend the idea to a function of (n) variables [7]. The path of blind person descending to a valley is an example of the direct search method.

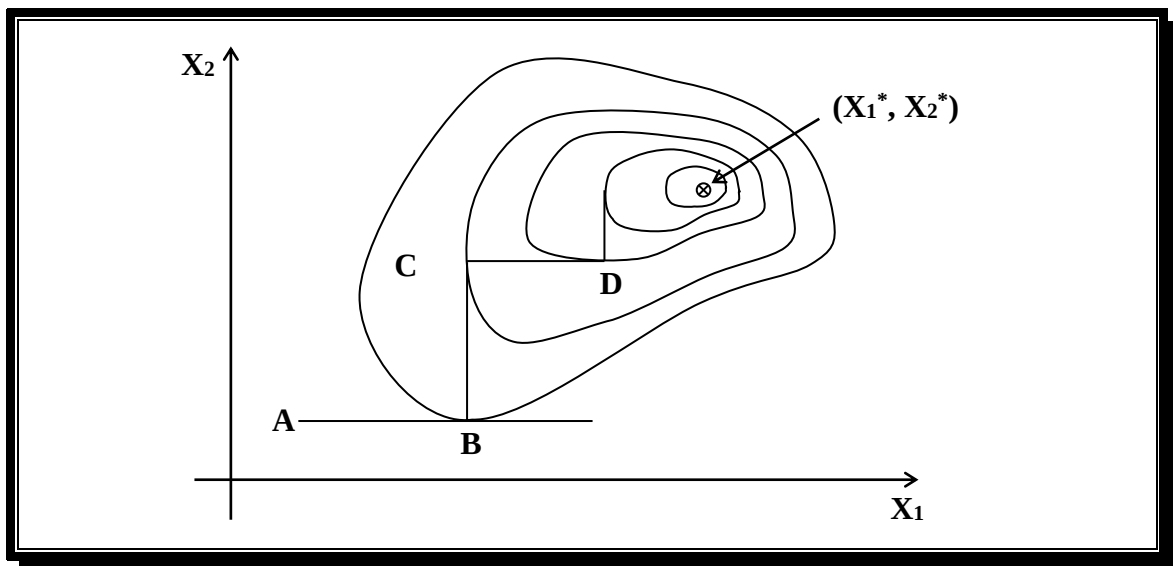


Fig. (5.1): Contour Lines for Function of Two Variables.

5.3 The Method of Hooke and Jeeves

In this method the search consists of a sequence of **exploration steps** about a **base point** which if successful are followed by **pattern moves**. The procedure is as follows [7]:

(A) Choose an initial base point $\mathbf{b}_1$ and a step length $\mathbf{h}_j$ for each variable $\mathbf{X}_j$, $j=1, 2, \dots, n$.

(B) Carry out an exploration about $\mathbf{b}_1$. The purpose of this is to acquire knowledge about the local behavior of the function. This knowledge is used to find a likely direction for the **pattern move** by which it is hoped to obtain an even greater reduction in the value of the function. The exploration about $\mathbf{b}_1$ proceeds as indicated.

(i) Evaluate $\mathbf{f}(\mathbf{b}_1)$.

(ii) Each variable is now changed in turn, by adding the step length. Thus one evaluates $\mathbf{f}(\mathbf{b}_1 + \mathbf{h}_1 \mathbf{e}_1)$ where $\mathbf{e}_1$ is a unit vector in the direction of the $\mathbf{X}_1$ -axis. If this reduces the function, replace $\mathbf{b}_1$ by $\mathbf{b}_1 + \mathbf{h}_1 \mathbf{e}_1$. If not, find $\mathbf{f}(\mathbf{b}_1 - \mathbf{h}_1 \mathbf{e}_1)$ and replace $\mathbf{b}_1$ by $\mathbf{b}_1 - \mathbf{h}_1 \mathbf{e}_1$ if the function is reduced. If neither step gives a reduction leave $\mathbf{b}_1$ unchanged and consider changes in $\mathbf{X}_2$, i.e. find $\mathbf{f}(\mathbf{b}_1 + \mathbf{h}_2 \mathbf{e}_2)$ etc. When one has considered all (n) variables, it is possible to have a new base point $\mathbf{b}_2$.

(iii) If $\mathbf{b}_2 = \mathbf{b}_1$ i.e. no function reduction has been achieved, the exploration is repeated about the same base point $\mathbf{b}_1$ but with a reduced step length. In the present work, in which a structural optimization is performed by using this method, it was found that reducing the step length to one half its former value is satisfactory.

(iv) If $\mathbf{b}_2 \neq \mathbf{b}_1$ one makes a **pattern move**.

(C) Pattern moves utilize the information acquired by exploration, and accomplish the function of the minimization by moving in the direction of the established “pattern”. The procedure is as follows:

(i) It seems sensible to move further from the base point $\mathbf{b}_2$ in the direction $\mathbf{b}_2 - \mathbf{b}_1$ since that move has already led to a reduction in the function value. So one can evaluate the function at the next pattern point.

$$\mathbf{P}_1 = \mathbf{b}_1 + 2(\mathbf{b}_2 - \mathbf{b}_1) \quad \dots\dots\dots(5.1)$$

or in general

$$\mathbf{P}_i = \mathbf{b}_i + 2(\mathbf{b}_{i+1} - \mathbf{b}_i) \quad \dots\dots\dots(5.2)$$

- (ii) Then continue with exploratory moves about $\mathbf{P}_1$ ($\mathbf{P}_i$).
- (iii) If the lowest value at step **C(ii)** is less than the base point $\mathbf{b}_2$ ($\mathbf{b}_{i+1}$ in general) then a new base point $\mathbf{b}_3$ ($\mathbf{b}_{i+2}$) has been reached. In this case repeat **C(i)**. Otherwise abandon the pattern move from $\mathbf{b}_2$ ($\mathbf{b}_{i+1}$) and continue with an exploration about $\mathbf{b}_2$ ($\mathbf{b}_{i+1}$).
- (D) Terminate the process when the step length has been reduced to a predetermined small value. The presentation of this method as a flow chart is shown in **Fig. (5.2)** and **Fig. (5.3)**.

5.4 Dealing With Constraints

When applying the **method of Hooke and Jeeves** to the constrained optimization problem, one might suppose that it could be modified to take account of constraints. So, in **the modified Hooke and Jeeves method** it has been suggested that merely giving the objective function a very large value (in a minimization problem) will suffice whenever the constraints are violated. Certainly this idea has an intuitive appeal and is easy to program.

For each trial point one checks whether it lies within the constrained region or not. So, one evaluates the objective function in the normal way, if not one gives the objective function a very large value. In this way, the search method will be directed back into the feasible region and hence towards the minimum point within the feasible region.

5.5 Design Variables

All frames consist from group of vertical, inclined and horizontal members. All these members are designed as members subjected to the axial force plus bending moments and all dimensions other than the cross-section dimensions of members are predefined as data.

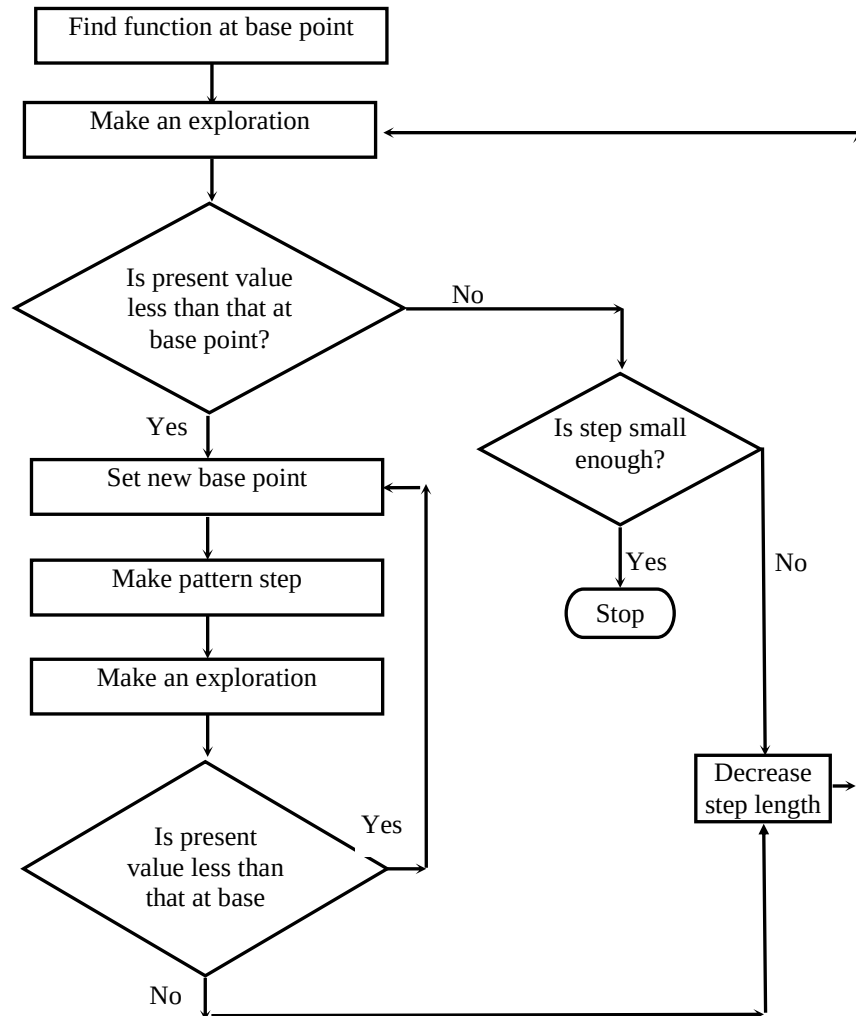


Fig. (5.2): Flow Chart for Hooke and Jeeves Method.

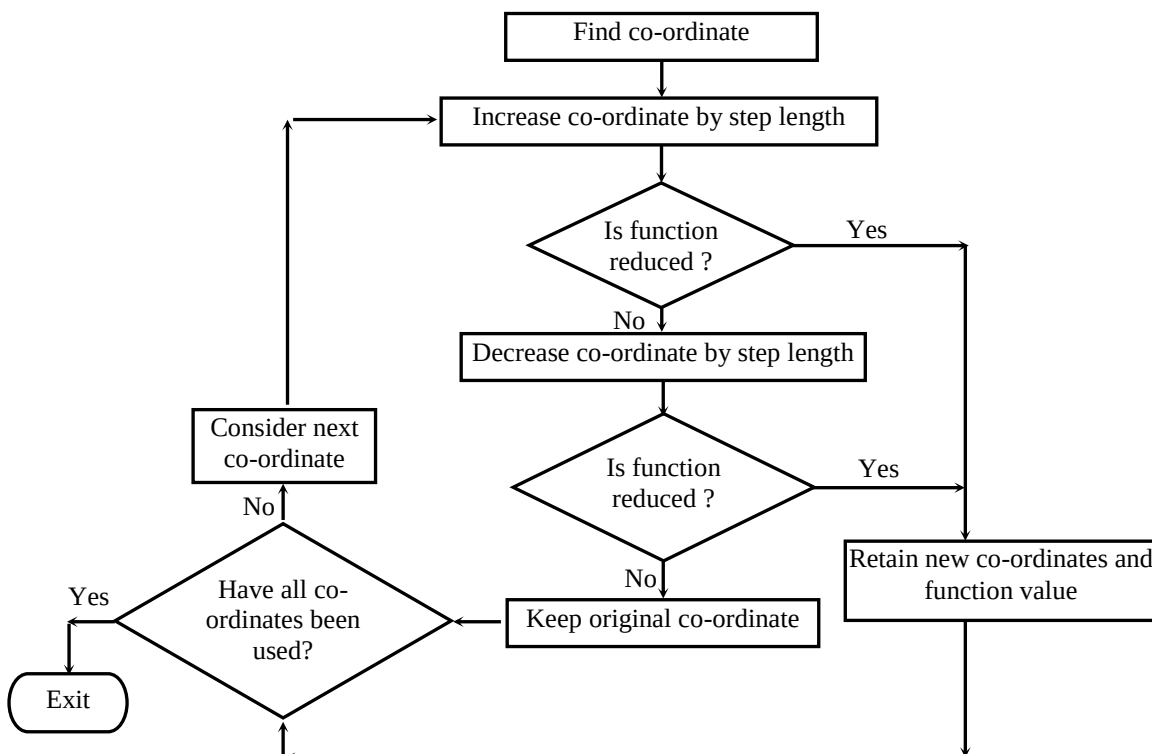


Fig. (5.3): Flow Chart for an Exploration.

5.6 Objective Function

The non-linear weight objective function (**Z**) is used in the optimization problem of this study.

$$Z = \sum_{i=1}^n Z_i \quad \dots\dots\dots(5.3)$$

in which,

- n** : number of members,
- Z_i** : weight of member (i).

5.7 Constraints

The objective function is minimized subject to a set of constraints. Some of these constraints can be explicitly defined in terms of the design variables, while others are implicitly related to design variables.

5.7.1 Tension Members

The computed tensile stress (**f_t**), in a tension member shall not exceed the allowable stress for tension (**F_t**), given by (**0.6F_y**) for yielding on the gross area [82, 88].

5.7.2 Compression Members

Compression members can fail by yielding inelastic buckling or elastic buckling depending on the slenderness ratio of the members. The computed compressive stress (**f_a**), in a compression member shall not exceed its allowable value given by:

$$F_a = \begin{cases} \left[1 - \frac{(KL/r)^2}{2C_c^2} \right] F_y & \text{if } KL/r \leq C_c \\ \frac{5}{3} + \frac{3(KL/r)}{8C_c} - \frac{(KL/r)^3}{8C_c} & \text{if } KL/r > C_c \end{cases} \quad \dots\dots\dots(5.4)$$

in which,

- KL/r** : the slenderness ratio,
K : the effective length factor,
L : the unbraced member length (in),
r : the radius of gyration of the cross-section (in),
E : modulus of elasticity (lb/in²), and

$$C_c = \sqrt{(2\pi^2 E / F_y)} \quad \dots\dots\dots(5.5)$$

(**KL/r**) should be evaluated for both buckling axes and the larger value is used in equation (5.4) to compute (**F_a**). The first of equation (5.4) is the allowable stress for inelastic buckling and the second of equation (5.4) is the allowable stress for elastic buckling [82, 88].

5.7.3 Flexural Members

For members bent about their major axes, the computed flexural stress (**f_b**) shall not exceed the allowable stress (**F_b**), given as follows [82, 83, 88]:

$$\text{For } L_b \leq L_{\infty} \quad F_b = 0.66F_y \quad \dots\dots\dots(5.6)$$

where:

$$L_{\infty} = \text{smaller of } \left[76b_f / \sqrt{F_y}, 20000 / (d / A_f) F_y \right], \text{ for I and channel shapes,} \\ \left[1950 + 1200(M_1 / M_2) \right] (b / F_y) \geq 1200 (b / F_y), \text{ for box sections, rectangular} \\ \text{and circular tubes.} \quad \dots\dots\dots(5.7)$$

in which,

- b_f** : flange width (in),
d : overall depth of section (in),
A_f : area of compression flange (in²),
b : width of cross-section (in),

M₁/M₂ : ratio of the smaller to larger moment at the ends of the unbraced length of the beam. **M₁/M₂** is positive for reverse curvature bending and negative for single curvature bending.

$$\text{For } L_{\infty} < L_b \leq L_{uu} \quad F_b = 0.6F_y \quad \dots\dots\dots(5.8)$$

where:

L_{uu} = larger of $\left[76b_f / \sqrt{F_y}, 20000/(d/A_f)F_y\right]$, for I and channel shapes,
 $\left[1950 + 1200(M_1/M_2)\right](b/F_y) \geq 1200(b/F_y)$, for box sections, rectangular
 and circular tubes.(5.9)

For $L_b > L_{uu}$,

$$\text{If } L_L / r_T < \sqrt{102000 C_{bb} / F_y} \quad F_b = 0.6F_y \quad \dots\dots\dots(5.10)$$

if $\sqrt{102000 C_{bb} / F_y} \leq L_L / r_T < \sqrt{510000 C_{bb} / F_y}$ use the larger of:

$$F_b = \left[\frac{2}{3} - \frac{F_y (L_L / r_T)^2}{153000 C_{bb}} \right] F_y \quad \dots\dots\dots(5.11)$$

$$F_b = \frac{12000 C_{bb}}{L_L (d/A_f)} \quad \dots\dots\dots(5.12)$$

$$\text{but not greater than } F_b = 0.6F_y \quad \dots\dots\dots(5.13)$$

if $L_L / r_T \geq \sqrt{510000 C_{bb} / F_y}$ use greater of

$$F_b = \frac{170000 C_{bb}}{(L_L / r_T)^2} \quad \dots\dots\dots(5.14)$$

$$F_b = \frac{12000 C_{bb}}{L_L (d/A_f)} \quad \dots\dots\dots(5.15)$$

$$\text{but not greater than } F_b = 0.6F_y \quad \dots\dots\dots(5.16)$$

in which,

L_L : distance between cross-sections braced against twist or lateral displacement of the compression flange (in),

r_T : radius of gyration of a section comprising the compression flange plus 1/3 of the compression web area, taken about an axis in the plane of the web (in),

$$C_{bb} = 1.75 + 1.05(M_1/M_2) + 0.3(M_1/M_2)^2 \leq 2.3 \quad \dots\dots\dots(5.17)$$

$$\text{or } C_{bb} = 12.5M_{\max} / (2.5M_{\max} + 3M_A + 4M_B + 3M_C) \quad \dots\dots(5.18)$$

where $M_{\max}$, M_A , M_B and M_C : maximum moment, quarter-point moment, mid-point moment and three quarter point moment along the

unbraced length of the member, respectively. For simplicity in design, C_{bb} can conservatively be taken as unity and F_b as $0.6F_y$.

Also, for members bent about their minor axes, since lateral torsional buckling will not occur for bending about the minor axes, regardless of the value of L_L , the allowable stress is:

for compact section $F_b = 0.75F_y$ (5.19)

for non-compact section $F_b = 0.6F_y$ (5.20)

For the above sections to be considered compact, in addition to have the width-thickness ratios of their component elements falling within the limiting value shown below:

for I-sections $b_f / t_f \leq 65 / \sqrt{F_y}$ (5.21)

for rectangular box sections $b_f / t_f \leq 190 / \sqrt{F_y}$ (5.22)

for hollow circular sections $D / t \leq 2070 / F_y$ (5.23)

where F_y is in units of (lb/in²)

5.7.4 Beam-Column Elements

The AISC specification says that the members subjected to a combination of the axial and flexural stress must be proportioned to meet the requirements of both of the following expressions [53, 82, 88].

If the axial force is tension

$$\frac{f_a}{F_t} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1 \quad \text{.....(5.24)}$$

in which,

f_a : computed axial tensile stress,

f_{bx}, f_{by} : computed bending tensile stresses about the major and minor axes, respectively,

F_{bx}, F_{by} : allowable bending tensile stresses about the major and minor axes, respectively,

F_t : allowable tensile stress.

If the axial force is compression

$$\frac{f_a}{0.6F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1 \quad \dots\dots\dots(5.25)$$

in which,

f_a : computed axial compressive stress.

The interaction formula for the analysis and design of beam-column is [83, 84, 85]:

$$\frac{P}{P_{allow}} + \frac{M_x}{M_{x-allow}} + \frac{M_y}{M_{y-allow}} \leq 1 \quad \dots\dots\dots(5.26)$$

in which,

P_{allow} : allowable axial load = $A.F_t$ in tension and = $A.F_a$ in compression,

M_{allow} : allowable bending moment = $S.F_b$,

S : elastic modulus section.

5.8 Design Procedure

The design procedure is as follows [82, 88]:

1. Assume web depth (d_w) and flange thickness (t_f).

2. Find web thickness $t_w = \frac{d_w \cdot \sqrt{22}}{760}$ (in inch) $\dots\dots\dots(5.27)$

3. Find minimum web thickness $t_{wmin} = \frac{d_w}{322}$ (in inch) $\dots\dots(5.28)$

4. Find flange area

$$A_f = \frac{M_x}{F_{bx} \cdot h_{(d_w+2t_f)}} + \frac{M_y}{F_{by} \cdot t_w} + \frac{P}{F_t \text{ or } F_a} \quad \dots\dots\dots(5.29)$$

in which,

P : actual axial load (lb),

M_x : bending moment about the strong axis (lb-in),

M_y : bending moment about weak axis (lb-in).

5. Check all the above constraints of the member.



Computer Program, Testing, Parametric Study And Optimization Examples

Part I (Computer Program And Testing)

6.1.1 General

As a part of this research, a computer program (**OSDEPNONSF**) (Optimum design under Static and Dynamic loads based on Elastic-Plastic NON-linear analysis of Space steel frames with Flexible connection) has been written in Microsoft Visual Basic (6.0) (2003) language to embody the formulation of optimal design and large displacement analysis of space steel frames with elastic-plastic behavior and flexible connection effects under static and dynamic loads as presented in chapters three, four and five. **Al-Khafaji [41]** written program to solve the problem of the large displacement elastic analysis of

Examples

plane steel frames with flexible connections and subjected to static loads. Also, **Al-Abbasi [87]** written program to solve the problem of the large displacement elastic analysis of space steel frames and subjected to static and dynamic loads. In the present study, I written my program and useful from the above programming that written by **Al-Khafaji [41]** and **Al-Abbasi [87]**.

This part presents a description of the computer program (**OSDEPNONSF**) developed in the present study. Furthermore, some examples reported by previous researchers are utilized, in order to verify the reliability and accuracy of the computer program results.

6.1.2 Properties and Ability of The Program

The computer program (**OSDEPNONSF**), presented in this part has many advantages and is designed to deal with a wide range of problems, the properties of these programs may be summarized as follows:

1.Evaluation of the approximate critical load

The interpolation method can be used in determining the critical load. This process is very useful in determining the elastic and elastic-plastic critical loads.

2. Member coordinate system

As illustrated in chapter three, the member tangent stiffness matrix has been derived by assuming the last configuration of the member, so the Eulerian coordinate system is used in which a new member configuration is updated at each iteration.

3. Current member orientation matrix of the deformed configuration

There are several methods available in the literature to determine the current member orientation matrix for the deformed configuration. Only one method is used in the present study in the analysis of space structures.

4. Mass properties

Examples

Two types of mass matrix are used in the dynamic analysis of space structures to represent the mass properties of a member, these are:

- a. Lumped mass matrix.
- b. Consistent mass matrix.

5. Solution of eigenvalue problem

The program is also capable to deal with eigenvalue problems in order that at initial time response or at initial load increment, an approximate value of the lowest natural frequency is obtained by the inverse iteration method.

6. Bowing effect

The present program is designed to deal with all the effects of geometrical non-linearity. However, the analysis which is carried out by this program can be simplified and the necessity of iteration in determining member axial force can be avoided by neglecting the effect of bowing. This approximation can be made by setting $C_b=0$ in Equation (3.7) and $G_1=G_2=0$ and $H=\pi^2/\lambda^2$ in Equations (3.70), (3.71) and (3.72).

7. Convergence criterion

The displacement criterion presented in chapter three has been used for terminating the iterative processes in both static and dynamic analysis.

8. Damping matrix

The energy dissipation of a structure due to any restriction for motion is represented by Rayleigh damping matrix; i.e. damping matrix is proportional directly to the initial tangent stiffness matrix and initial mass matrix taking the form:

$$[C] = Z_1[M] + Z_2[K] \quad \dots\dots\dots(6.1)$$

6.1.3 Structure of the Program

The program (**OSEPDNONSF**) consists of a main routine and many subroutines. Each one of these subroutines has been designed to deal with a part of the analysis or a part of the design and that part may be

Examples

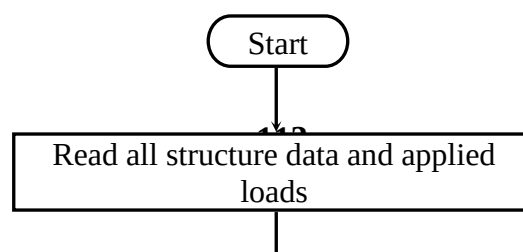
repeated more than one time in the main routine. The flow chart of the main routine will be given here, with a brief description for each subroutine.

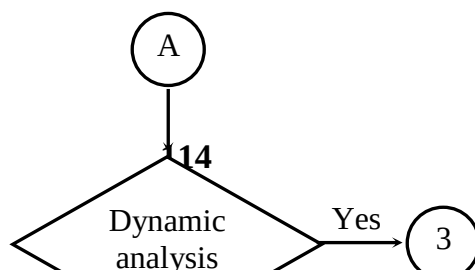
The purpose of the main routine is to control and arrange the process in the program. In other word the subroutines will work together under the organization of the main routine. A flow chart which represents the sequence of the operation in this routine is shown in **Fig.(6.1)**.

6.1.4 Applications for Testing the Program

Many examples are taken to verify the reliability of the program (**OSEPDNONSF**) and to examine the various solution schemes implemented in the program. Wherever possible, the results are compared with experimental, analytical and/or numerical results obtained in other investigations. These examples are:

1. Two-member horizontal bent.
2. Clamped beam .
3. 18-member framed dome.
4. Three-dimensional William's toggle frame.
5. T-shaped frame.
6. Single-bay space frame.
7. Two-story space frame.
8. Portal frame.
9. Five-story frame.
10. Gable frame.





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Check the convergence of the solution
using the displacement criterion

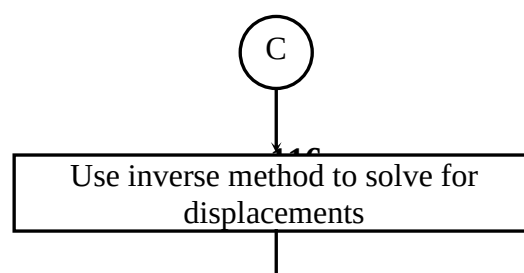


Fig.(6.1): Continued.

6.1.4.1 Example One (Two-Member Horizontal Bent)

The two-member horizontal bent shown in **Fig.(6.2)** was analyzed previously by **Kassimali [11]** and by **Al-Asady [19]**. In both analyses, each member is represented by one element and the Kassimali's joint orientation matrix was used. The structure is analyzed by the proposed method following an incremental elastic analysis and each member is

Examples

represent by three elements. This example in the present study aims to examine the reliability of the derivation of the tangent stiffness matrix and the large displacement analysis capacity. The load-deflection curves is plotted as shown in **Fig.(6.3)** and rather good agreement between the present study and other works. The maximum deflection at point (C) obtained from the proposed method is larger than that is obtained from **Kassimali [11]** by (1.5%). **Fig.(6.4)** shows the relation between the determinant and load and shows the stiffening behavior of the structure i.e. the stiffness increase with the increasing of the load.

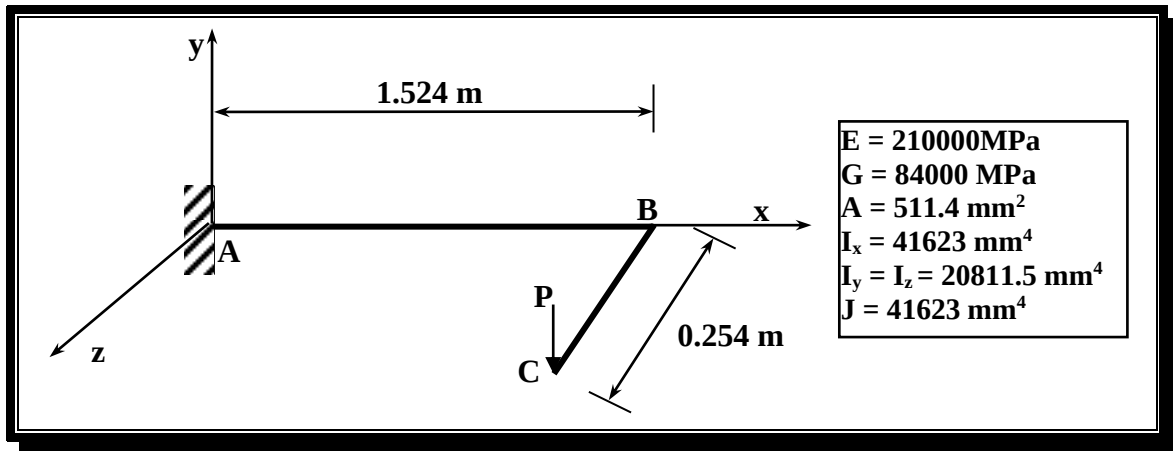


Fig.(6.2): Geometry and Loading of The Two-Member Horizontal Bent.

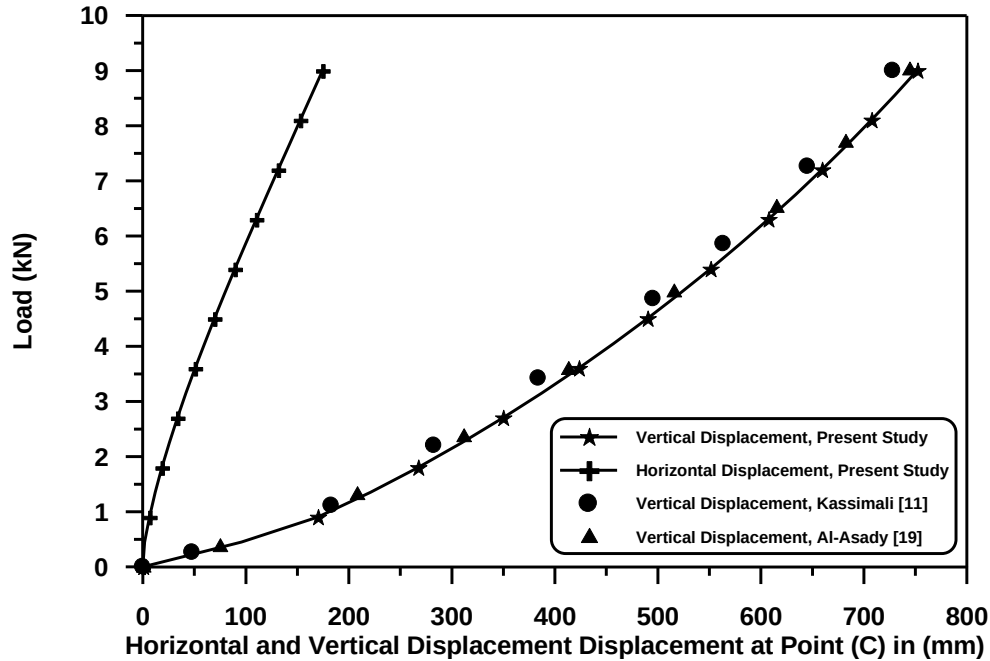


Fig.(6.3): Load-Displacement Curves For Two-Member Horizontal Bent (Example No. 1).

Examples

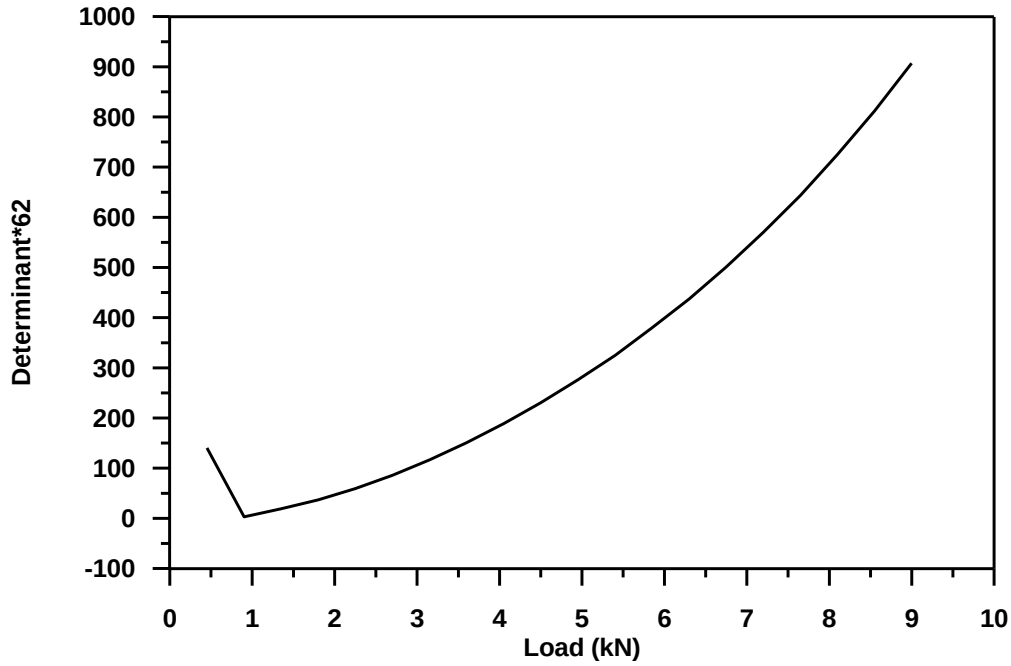


Fig.(6.4): Load-Determinant Curve For Two-Member Horizontal Bent (Example No.1).

6.1.4.2 Example Two (Clamped Beam)

A stiffening system for a fixed end beam is analyzed to verify the efficiency of the program (**OSDEPNONSF**) to carry out static and dynamic load. The geometry of the clamped beam, material properties and loading conditions are shown in **Fig.(6.5)**. This example was analyzed previously under the effect of static load by **Shi and Atluri [16]** using beam-column approach and because of the symmetry only one half of the beam with one beam element was considered and under the effect of dynamic load by **Mondkar and Powell [86]** using finite element approach and also only one half element was considered. In the present study the structure is analysed under the effect of static and dynamic loads using half-beam, two elements and time step size is chosen to be (0.00005 sec.). **Fig.(6.6)** shows the load-deflection curves relationship with and without bowing effect, the maximum deflection with the bowing effect at point (B) obtained from the proposed analysis is larger than that is obtained from **Shi and Atluri [16]** by (9%). **Fig.(6.7)** shows the relation between the determinant and load with and without bowing effect. The vertical displacement-time curves is plotted as shown in **Fig.(6.8)** and

Examples

rather good agreement between the present study and **Mondkar and Powell [86]** work with difference not more than (3.55%).

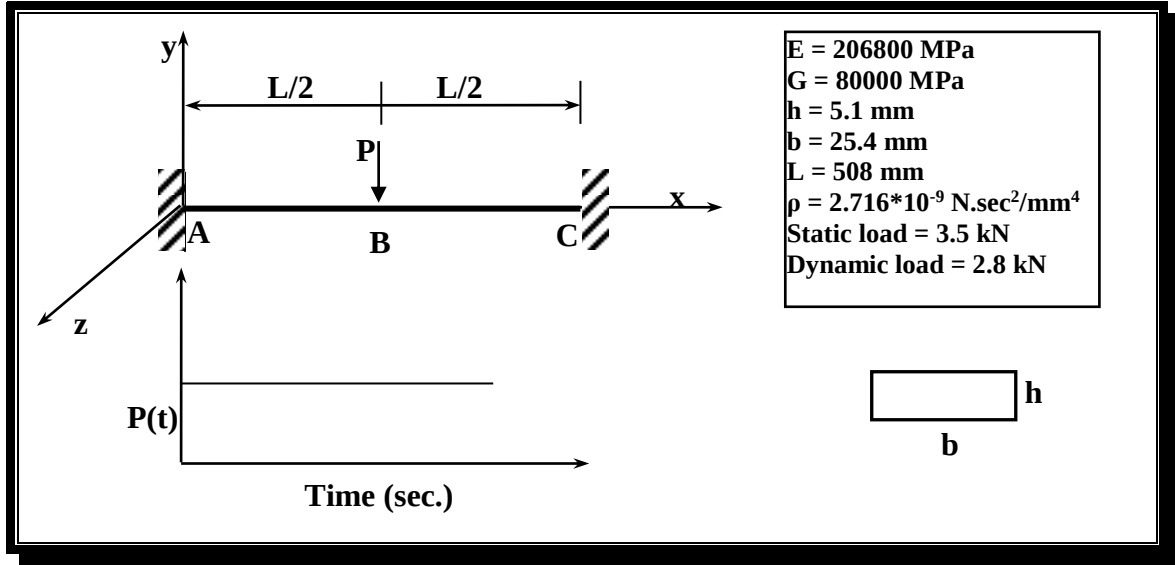


Fig.(6.5): Geometry and Loading of Clamped Beam.

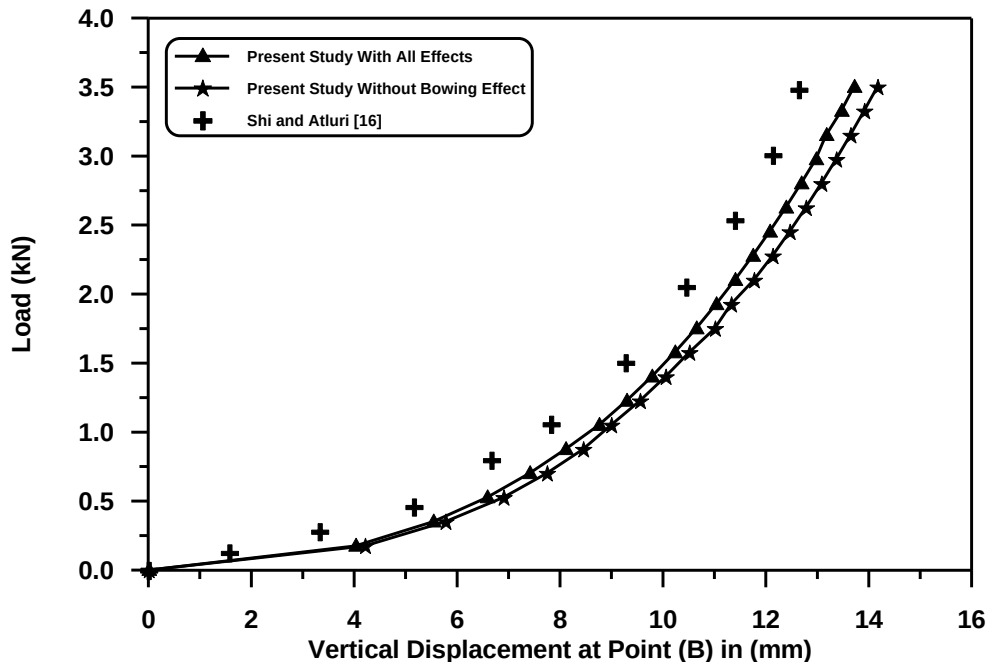


Fig.(6.6): Load-Displacement Curves For Clamped Beam (Example No.2).

Examples

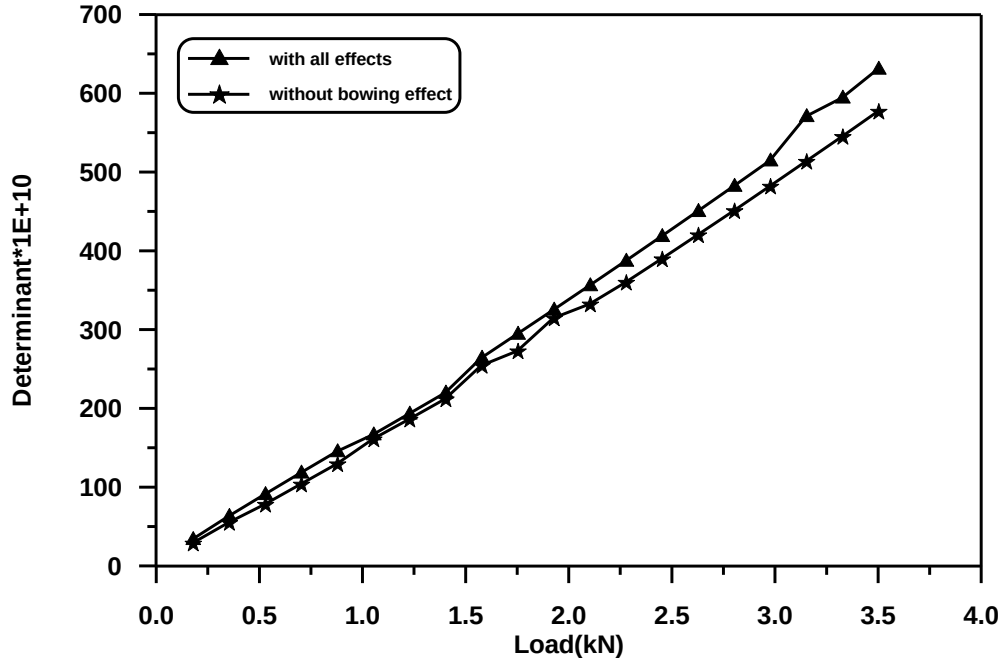


Fig.(6.7): Load-Determinant Curves For Clamped Beam (Example No.2).

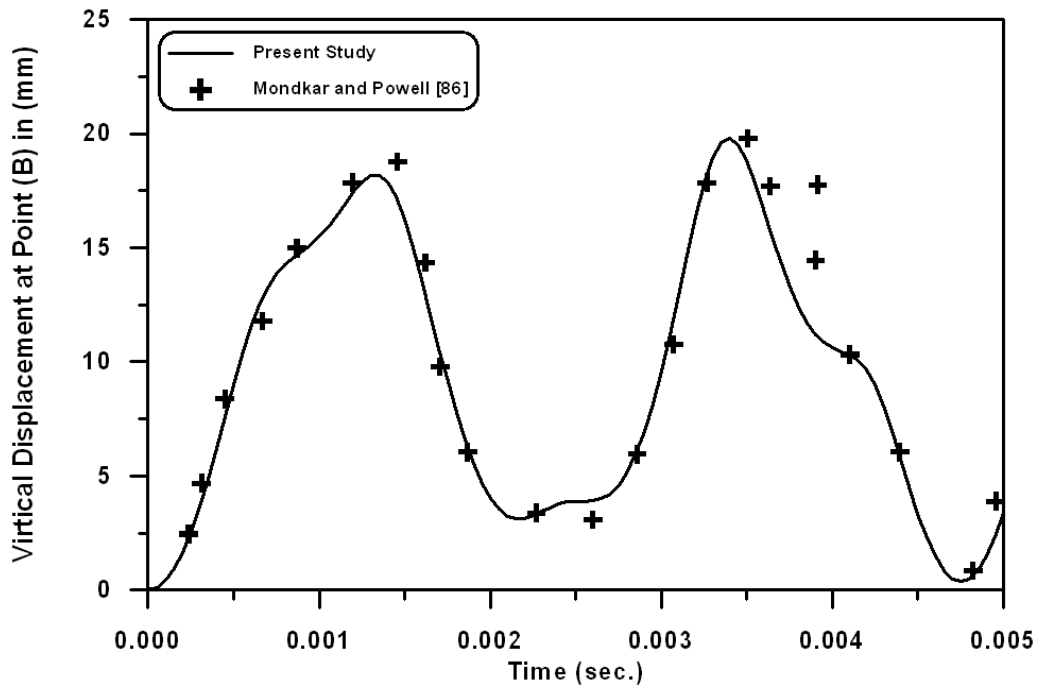


Fig.(6.8): Time-Displacement Curves For Clamped Beam (Example No.2).

6.1.4.3 Example Three (18-Member Framed Dome)

The 18-member framed dome shown in Fig.(6.9) is to be analyzed under a single vertical load applied at crown i.e. at point (1). This example was solved previously under effect of static load by Al-Asady [19] and under effect of dynamic load by Remesth [23]. Fig.(6.10) shows the load-vertical displacement curves of point (1) and rather good agreement between the present study and Al-Asady [19] work. In the

Examples

present study each member is represent by two elements. The maximum deflection at point (1) obtained from the proposed method is larger than that is obtained from **Al-Asady [19]** by (1%).

Remesth [23], analyzed this example dynamically using the finite element approach and Newmark- β iteration method. He used the peak value of the harmonically concentrated load was to be (34.4 MN) with time step size of (0.0025 sec.). In the present study, this dome is analyzed dynamically using beam-column approach, each member represent by one element and Newmark- β iteration method. Also, the peak value of the harmonically concentrated load is to be (34.4 MN) with time step size of (0.0025 sec.). The response of a harmonically (sine shape) vertical concentrated load at the crown point of the dome is shown in **Fig.(6.11)** and rather good agreement between the present work and **Remesth [23]** with difference not more than (6%) in the vertical displacement of point (1).

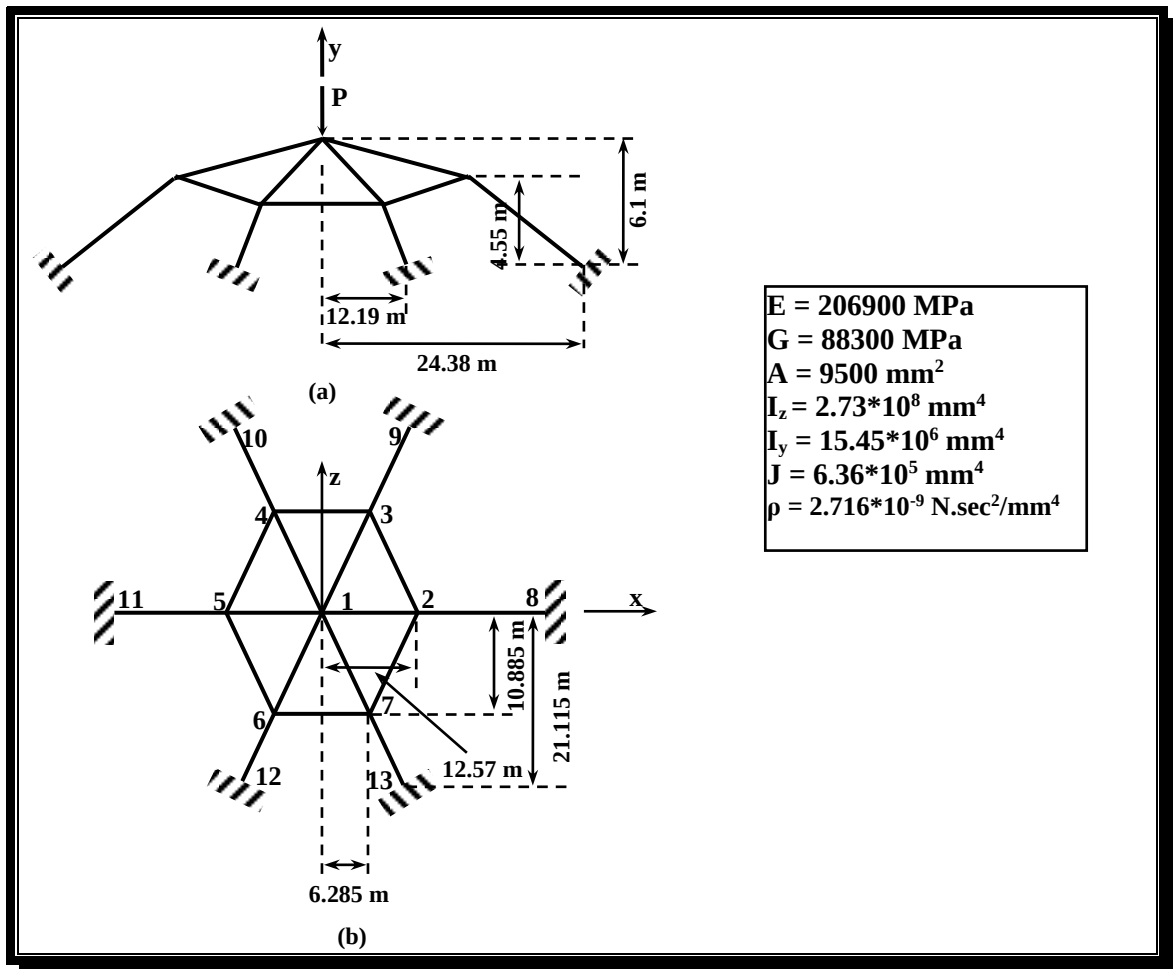


Fig.(6.9): Geometry and Loading of 18-Member Framed Dome, (a) Side View and (b) Top View.

Examples

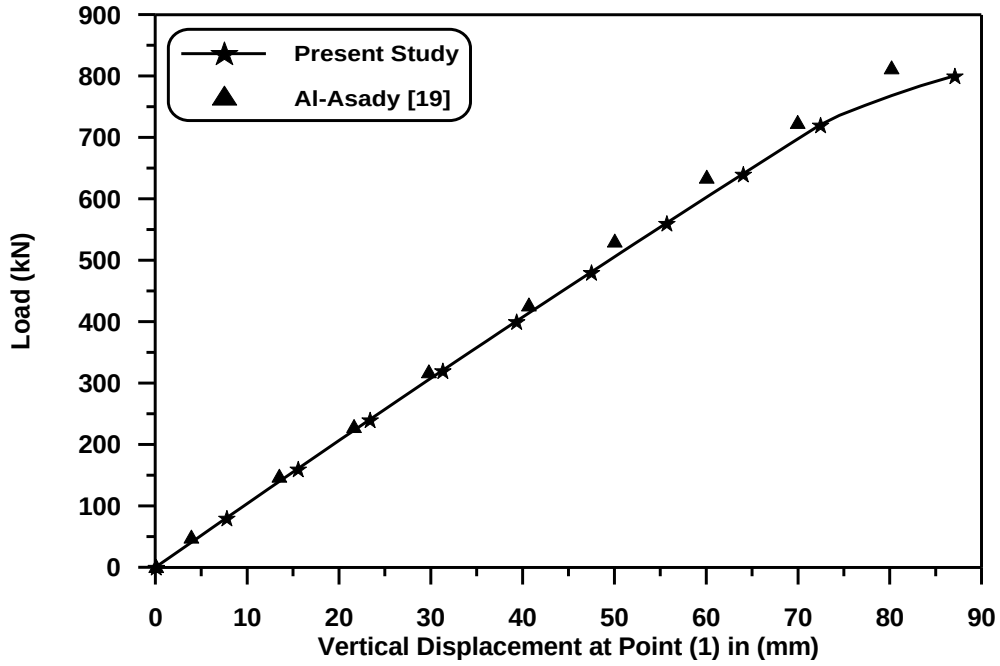


Fig.(6.10): Load-Displacement Curves For 18-Member Framed Dome (Example No.3).

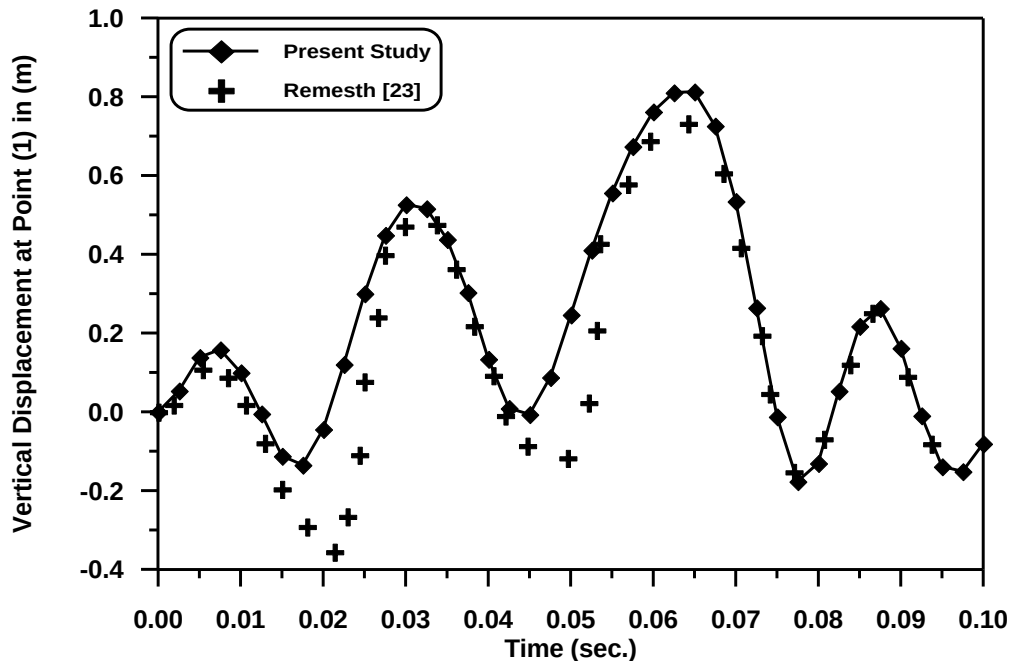


Fig.(6.11): Time-Displacement Curves For 18-Member Framed Dome (Example No.3).

6.1.4.4 Example Four (Three-Dimensional William's Toggle Frame)

This case study aims to check the accuracy of the proposed method with a highly non-linear structure. Fig.(6.12) shows the geometry, properties and loading conditions of William's toggle frame to include the

non-linear flexible supports and the moment-rotation behavior of which is shown in **Fig. (6.13)**.

Nee and Halder [15] solved this example with rigid as well as non-linear flexible connections at the supports (curve 1) using the finite element method and by using the **Exponential** function to represent the non-linear behavior of the connection. They noted that there are more deflection is obtained for the same load for the flexible supports condition. **Shi and Atluri [16]** also, solved this example with rigid and flexible connections at the supports (curve 1) but by using the beam-column approach with one element for each member and by using the **Ramberg-Osgood** function to represent the non-linear behavior of the connection. Also, they noted that the deflection for the flexible case is more that the rigid case.

Fig.(6.14), shows the present study load-displacement curves for the vertical displacement of point (1) with rigid as well as flexible supports (case 1 and case 2). In the present study each member is represent by one element. It can be seen from this figure that the deflection is increase when the flexibility of the connection increases. For the purpose of comparison with the other works, the behavior of this frame with rigid and flexible supports (curve 1) is shown in **Fig.(6.15)**. It is noted that there are good agreement between the present study and the other works for both rigid and flexible cases and the maximum deflection of point (1) obtained from the proposed method is smaller than that is obtained from the other works by approximately (1%) for the flexible case. It can be seen from **Figures (6.14) and (6.15)** that before the limit point, the load-deflection behavior of the toggle frame with rigid supports is approximately quite comparable with that of flexible supports but after the limit point however, the toggle frame with flexible supports deflected more. This phenomenon can be explained as that before the limit point the rotational stiffness of the connection is quite considerable and after the limit point the rotational stiffness is decrease with the increase of loading because of the non-linear behavior of the connection. Also, it is

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shown that the present polynomial model of the non-linear moment-rotation curve of the steel connection gives excellent results compared with the other representation functions of the non-linear moment-rotation curve in studying the behavior of space steel frames with flexible connections. **Table (6.1)** shows the end moments and it should be noted that the moment in the flexible case is more than in the rigid case. So, the deflection in the flexible cases is also more than in the rigid cases.

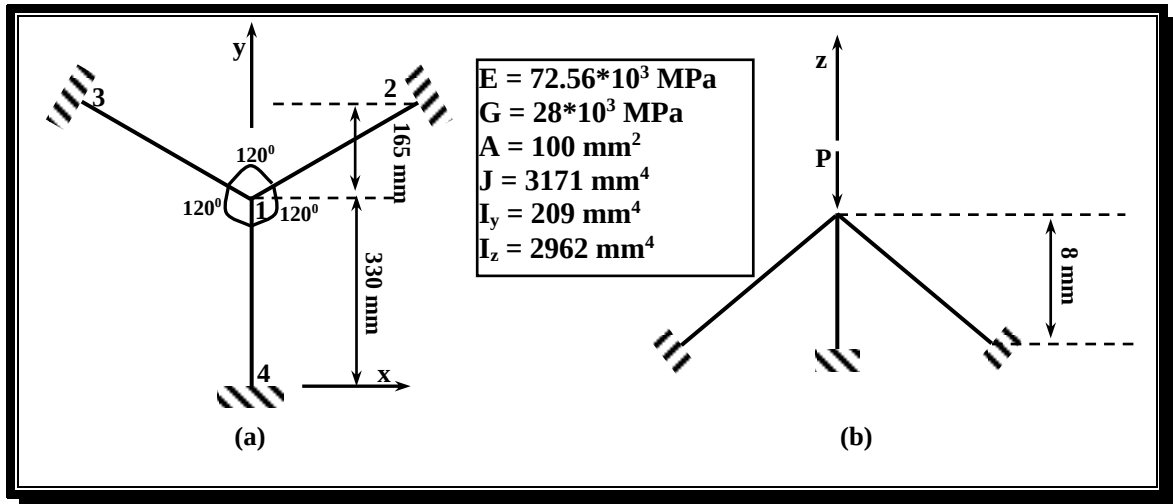


Fig.(6.12): Geometry and Loading of William's Toggle Frame, (a) Top View and (b) Side View.

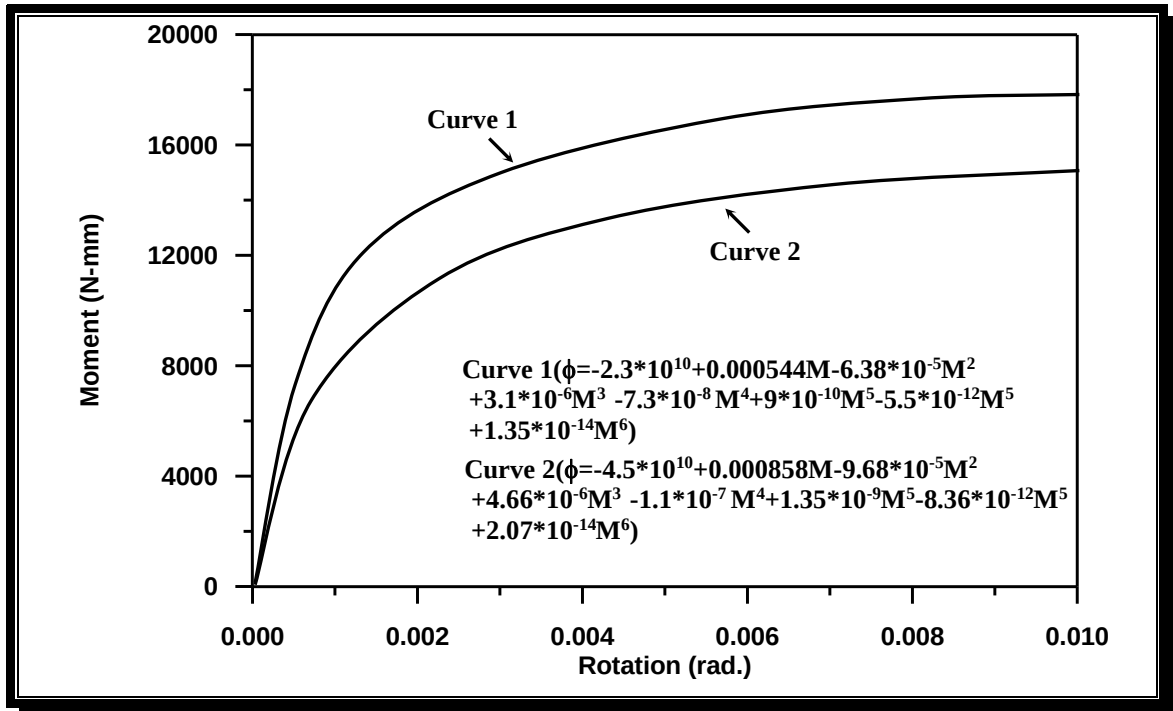


Fig.(6.13): Moment-Rotation Behavior Used for Supports of Toggle Frame.

Examples

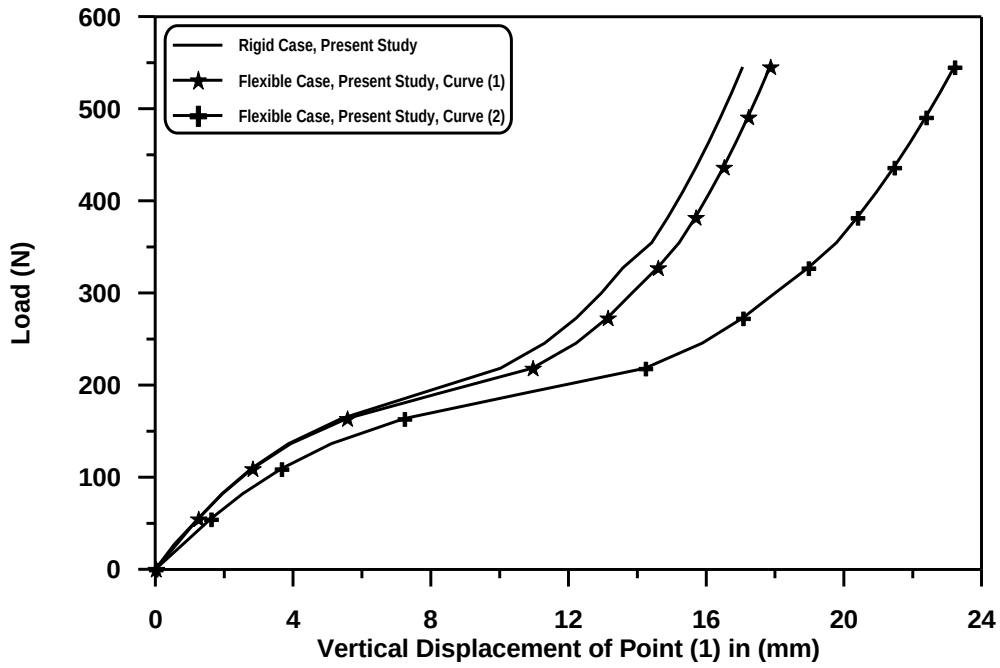


Fig.(6.14): Load-Displacement Curves For William's Toggle Frame (Example No.4, Present Study).

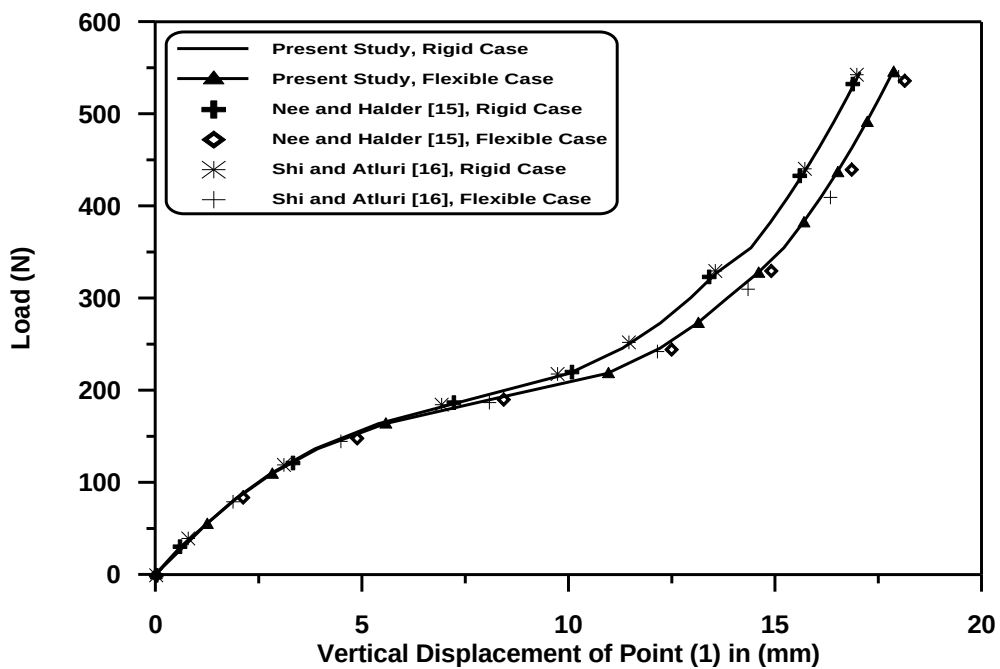


Fig.(6.15): Load-Displacement Curves For William's Toggle Frame (Example No.4, Present Study and Other Works).

Table (6.1): Moment Distribution For All Solutions For Ex.4 at Load 545 N in Point 1.

Moment (N-mm)						
Connection Type	Rigid		Flexible Curve (1)		Flexible Curve (2)	
Member	1st end	2nd end	1st end	2nd end	1st end	2nd end
1-2	$50.3 * 10^6$	$-149.7 * 10^6$	$136.5 * 10^6$	$-144.7 * 10^6$	$92.3 * 10^6$	$-220.2 * 10^6$
1-3	$65.5 * 10^6$	$44.35 * 10^6$	$70.25 * 10^6$	$130.5 * 10^6$	$28.6 * 10^6$	$212.4 * 10^6$
1-4	$-115.8 * 10^6$	$259.55 * 10^6$	$-206.75 * 10^6$	$142.6 * 10^6$	$-120.9 * 10^6$	$895.3 * 10^6$

6.1.4.5 Example Five (T-Shaped Frame)

In order to further investigation for the effect of connection flexibility on the behavior of structures, this example is considered herein. The geometry, properties and loading conditions of this example is shown in **Fig.(6.16)**.

Again two cases, i.e. rigid and non-linear flexible connection cases are considered and the non-linear behavior of the connection is given by the moment-rotation curve as illustrated in **Fig.(6.17)**.

Shi and Atluri [16] analyzed this example by using the beam-column approach and each member was divided in to four elements. The behavior of the flexible connection was represent by the **Ramberg-Osgood** function.

Fig.(6.18) shows the relation between the load at point (A) and the vertical displacement for the same point. In the present study each member in the structure is divided in to four elements and the non-linear behavior of the connection is represent by the **Polynomial** function and rather good agreement between the present study and **Shi and Atluri [16]** work. The maximum deflection at point (A) obtained from the proposed method is smaller than that is obtained from **Shi and Atluri [16]** by (0.6%).

Also, it should be noted that the displacement of the flexible connection structure is more than the displacement of the rigid connection

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structure and the effect of the flexible connections on the behavior of the structure must be considered in the analysis and it cannot be neglected.

Fig.(6.19) shows the relation between the determinant and the load and shows the stiffening behavior of the structure i.e. the stiffness increase with the increasing of the load.

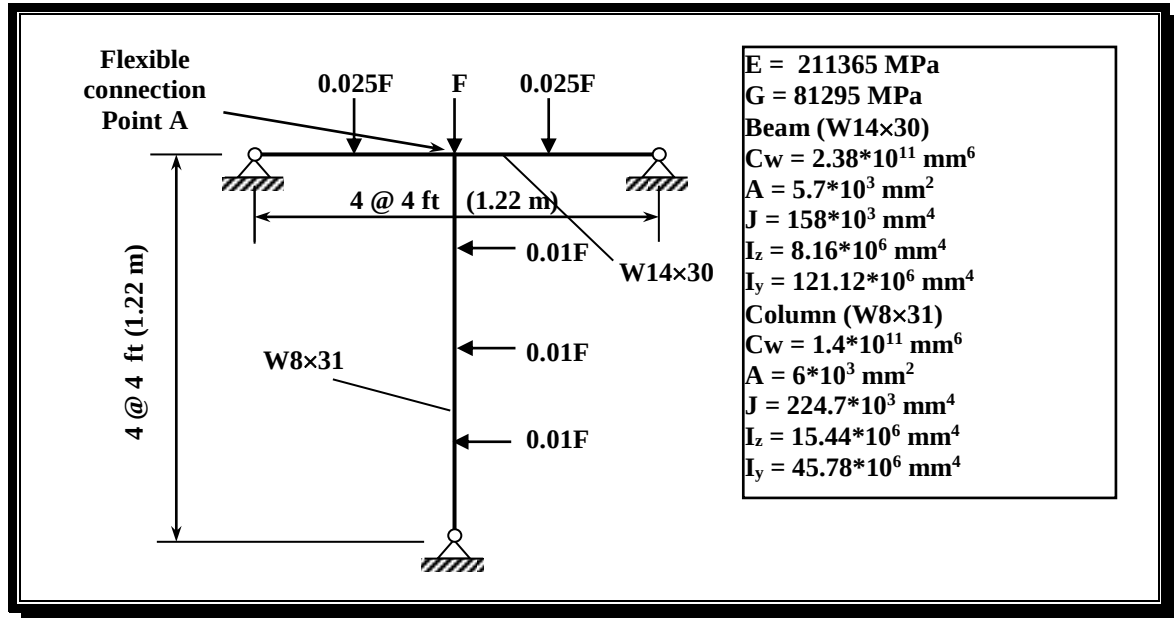


Fig.(6.16): Geometry and Loading For T-Shaped Frame.

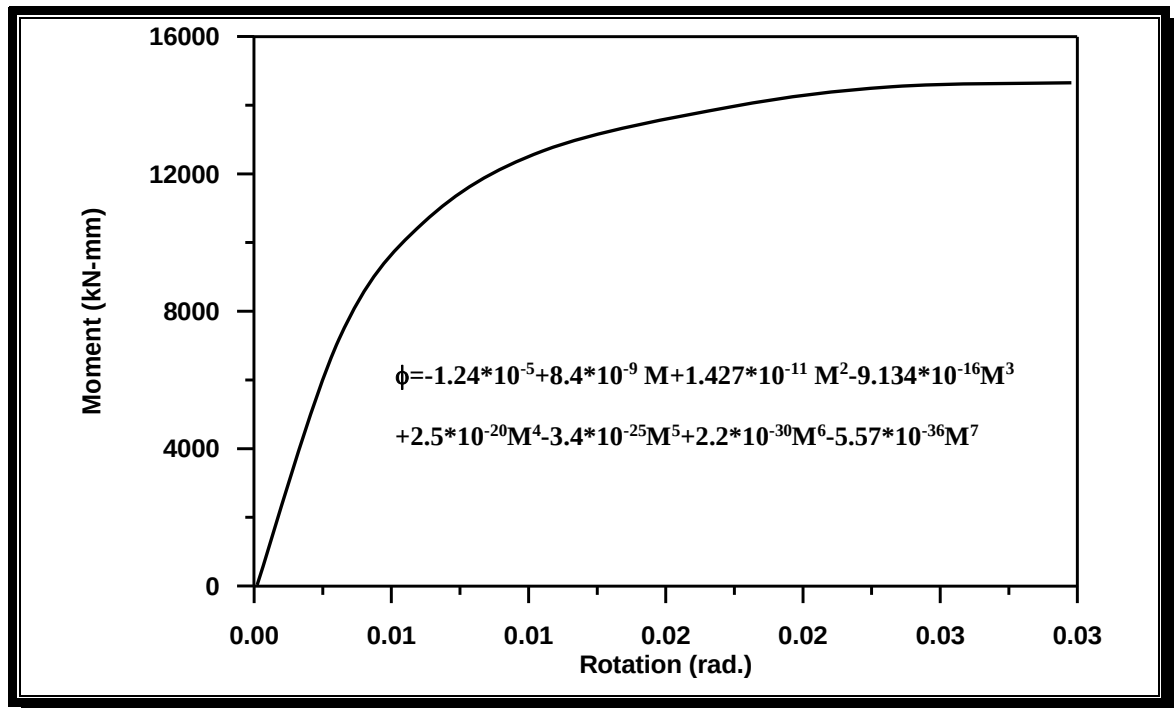


Fig.(6.17): Moment-Rotation Behavior Used for T-Shaped Frame.

Examples

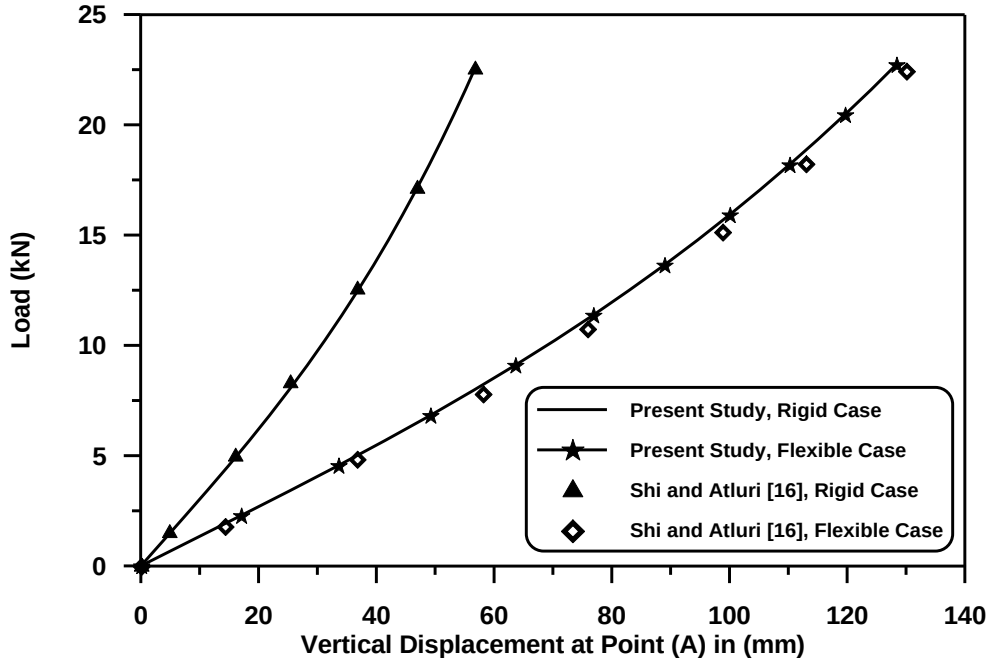


Fig.(6.18): Load-Displacement Curves For T-Shaped Frame (Example No.5).

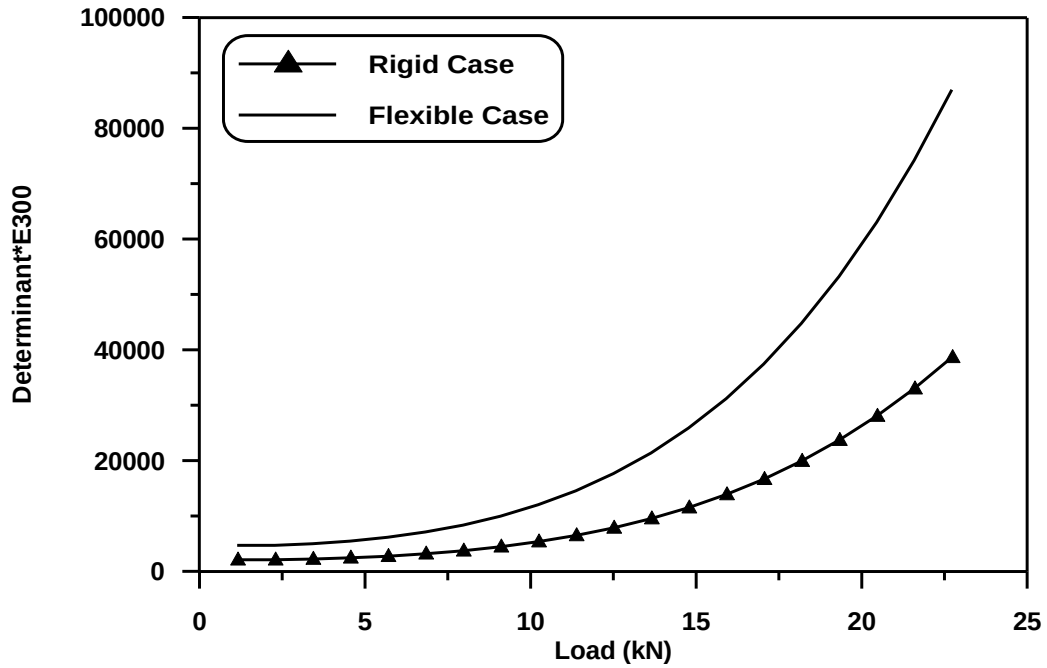


Fig.(6.19): Load-Determinant Curves For t-Shaped Frame (Example No.5).

6.1.4.6 Example Six (Single-Bay Space Frame)

The present single-bay space frame shown in Fig.(6.20) was analyzed previously by Nee and Halder [15] and by Al-Asady [19]. This example aims to examine the reliability of the derivation of the tangent stiffness matrix in the presence of plastic hinges and the efficiency of the

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numerical method. **Nee and Halder [15]** solved this example using the second order finite element method, rigid connections and approximate yield criteria which considered the effect of bending moments and axial forces only. **Al-Asady [19]** also solved this case using linear-incremental analysis and each member was represented by one element.

The structure is analyzed by the proposed method and each member is represent by two elements. **Fig. (6.21)**, shows the sequence of the plastic hinges in the present study. Load-horizontal displacement curves are shown in **Fig.(6.22)** and rather good agreement between the present study and **Nee and Halder [15]** work. The larger difference between the horizontal displacement at point (A) obtained from the proposed method and that is obtained from **Nee and Halder [15]** is approximately (4%) in the some load stage and this difference in decrease to (0.7%) when the load is increases. This figure is also shows that there is a difference between the present study and **Al-Asady [19]** work as obtained. This difference because that, **Al-Asady [19]** used linear analysis and used simple yield function considered the effect of bending moments and axial forces only but in the present study the yield function considered the effects of the twisting moment and shear force beside the effects of bending moments and axial forces.

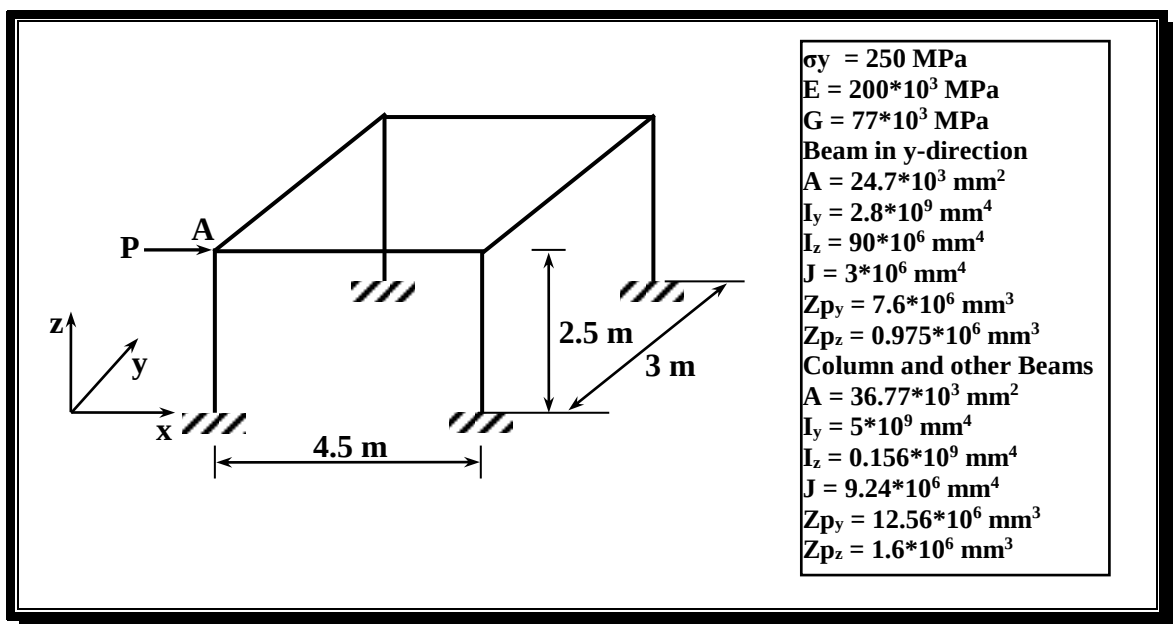


Fig.(6.20): Geometry and Loading For Single-Bay Space Frame.

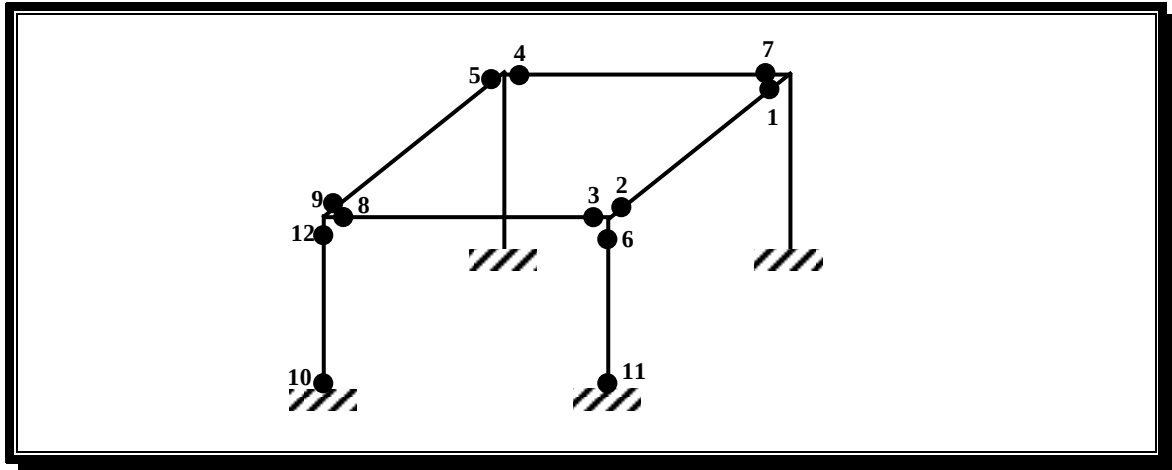


Fig.(6.21): The Sequence of Plastic Hinges For Single-Bay Space Frame.

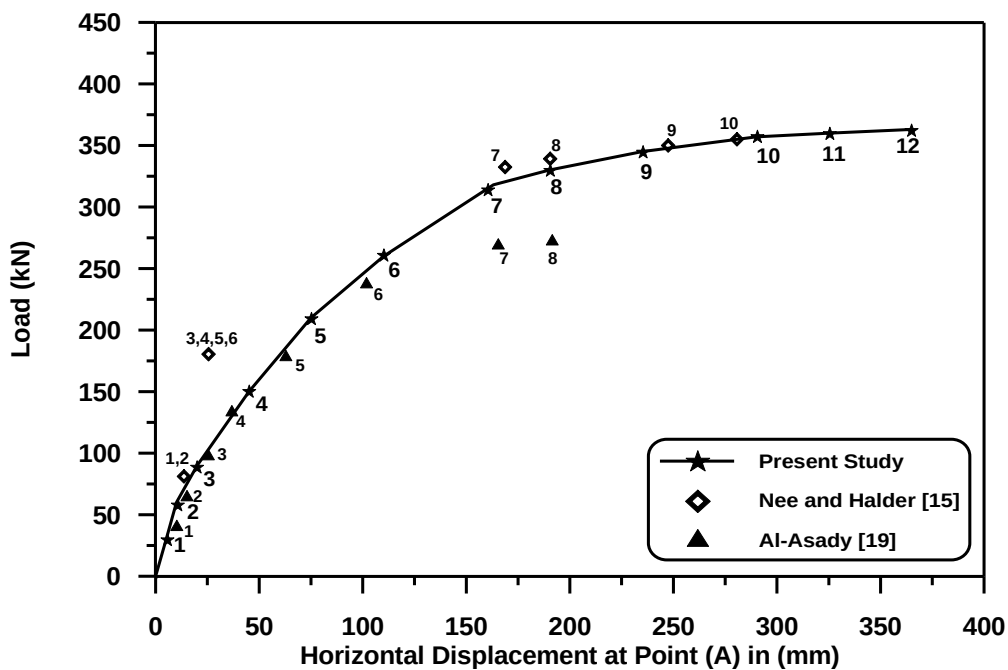


Fig.(6.22): Load-Displacement Curves For Single-Bay Space Frame (Example No.6).

6.1.4.7 Example Seven (Two-Story Space Frame)

Once again to verify the efficiency of the program (OSDEPNONSF) and to examine the reliability of the presence of plastic hinges, a two-story space frame is also analyzed. The geometry of the frame, material properties and loading condition are shown in **Fig.(6.23)**.

This example was solved by **Argyris, et al [10]** using the finite element method in the analysis of large deformation elasto-plastic effect. **Fig.(6.24)** shows the load-horizontal displacement curves for the previous and present studies and **Fig.(6.25)** shows the sequence of the plastic

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hinges in the present study. Again good agreement between the present and Argyis, et al. [10] studies is obtained with different not more than (4%).

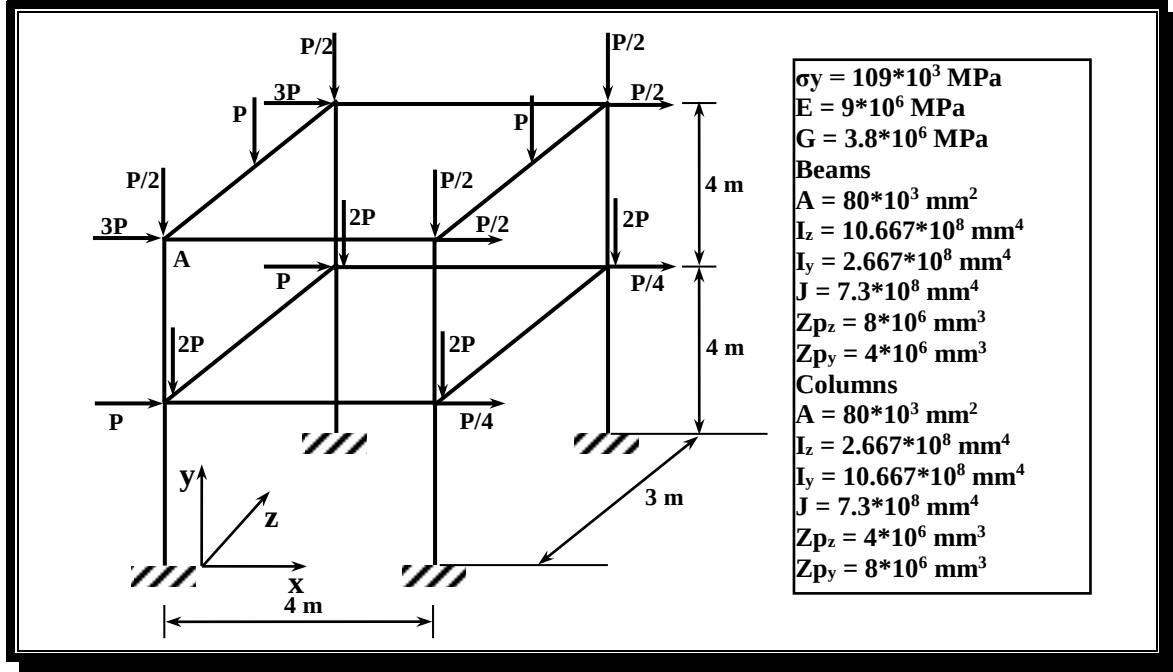


Fig.(6.23): Geometry and Loading For Two-Story Frame.

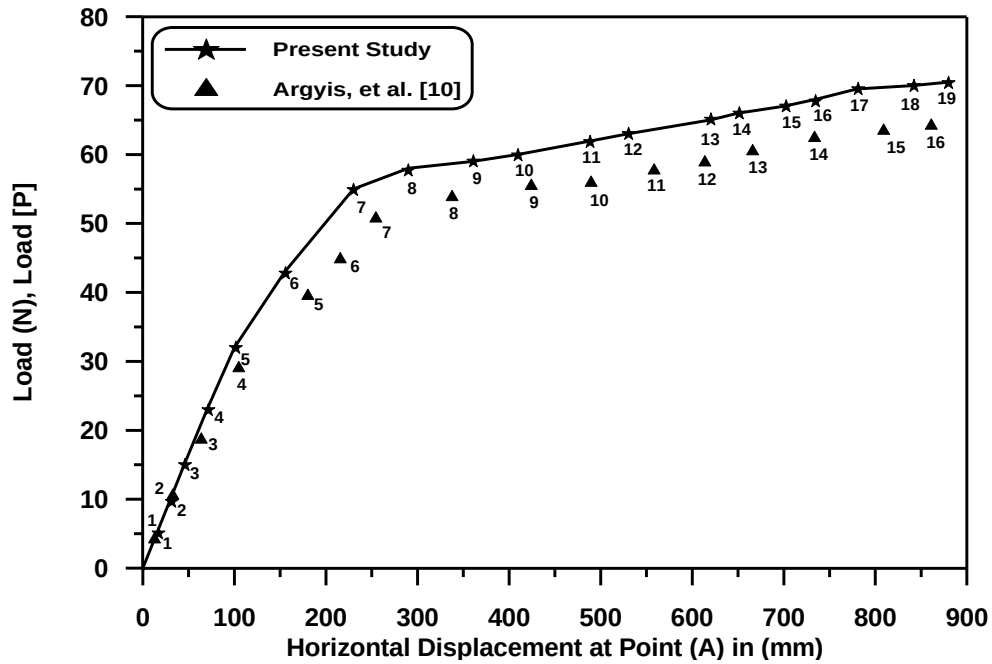


Fig.(6.24): Load-Displacement Curves For Two-Story Frame (Example No.7).

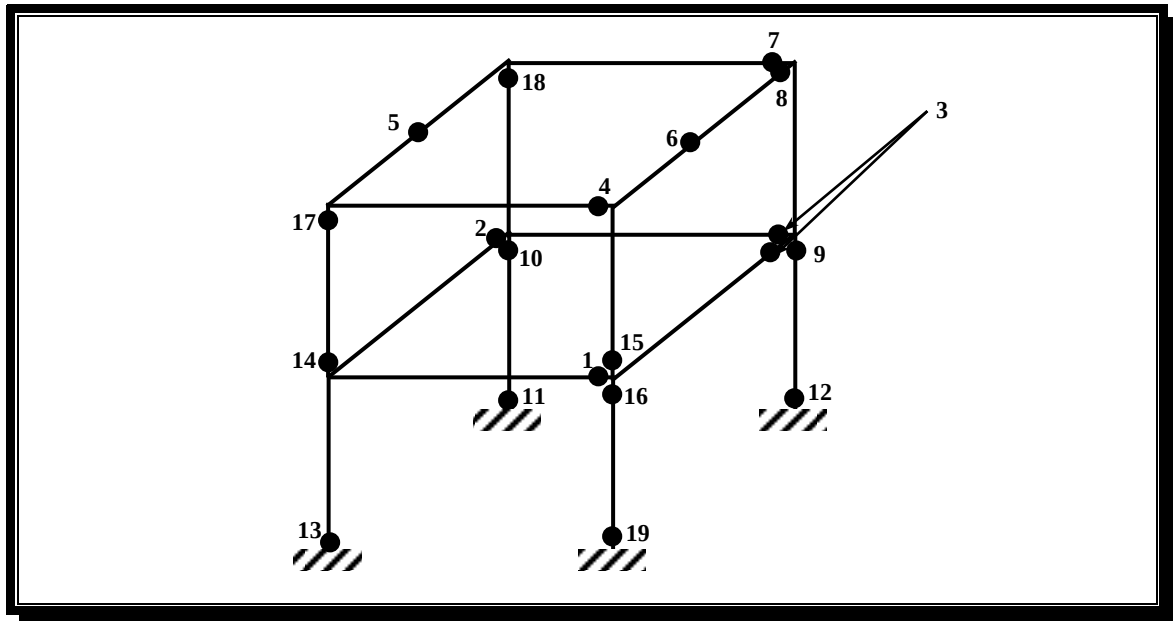


**Fig.(6.25): The Sequence of Plastic Hinges For Two-Story Frame
(Example No.7).**

6.1.4.8 Example Eight (Portal Frame)

To verify the efficiency of the program (**OSDEPNONSF**) and to examine the reliability of the derivation of the tangent stiffness matrix and the equation of motion under effect of earthquake forces and flexible connections, a portal frame is analyzed. The loading, geometry of the frame and material properties are shown in **Fig.(6.26)**. This example was solved previously by **Liu [45]** using the finite element approach and rigid connections. The behavior of the flexible connection used in this example is shown in **Fig.(6.27)**. This example aims to make a comparison between the rigid and flexible connection behavior under earthquake forces. Horizontal displacement-time curves for the portal frame with rigid connection are shown in **Fig.(6.28)** and this figure shows that there is good agreement between the present study and **Liu [45]** work with difference not more than (0.7%). This figure also shows that the polynomial expression for the earthquake acceleration gives good results and can be used in the non-linear analysis of the structures under earthquake effects.

Fig.(6.29) plotted the behavior of the frame with rigid and flexible connections. This figure shows that the horizontal displacement for the

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frame with flexible connections has more than that for the frame with the rigid connections and this clearly indicates the importance of considering of flexible connections in real structures.

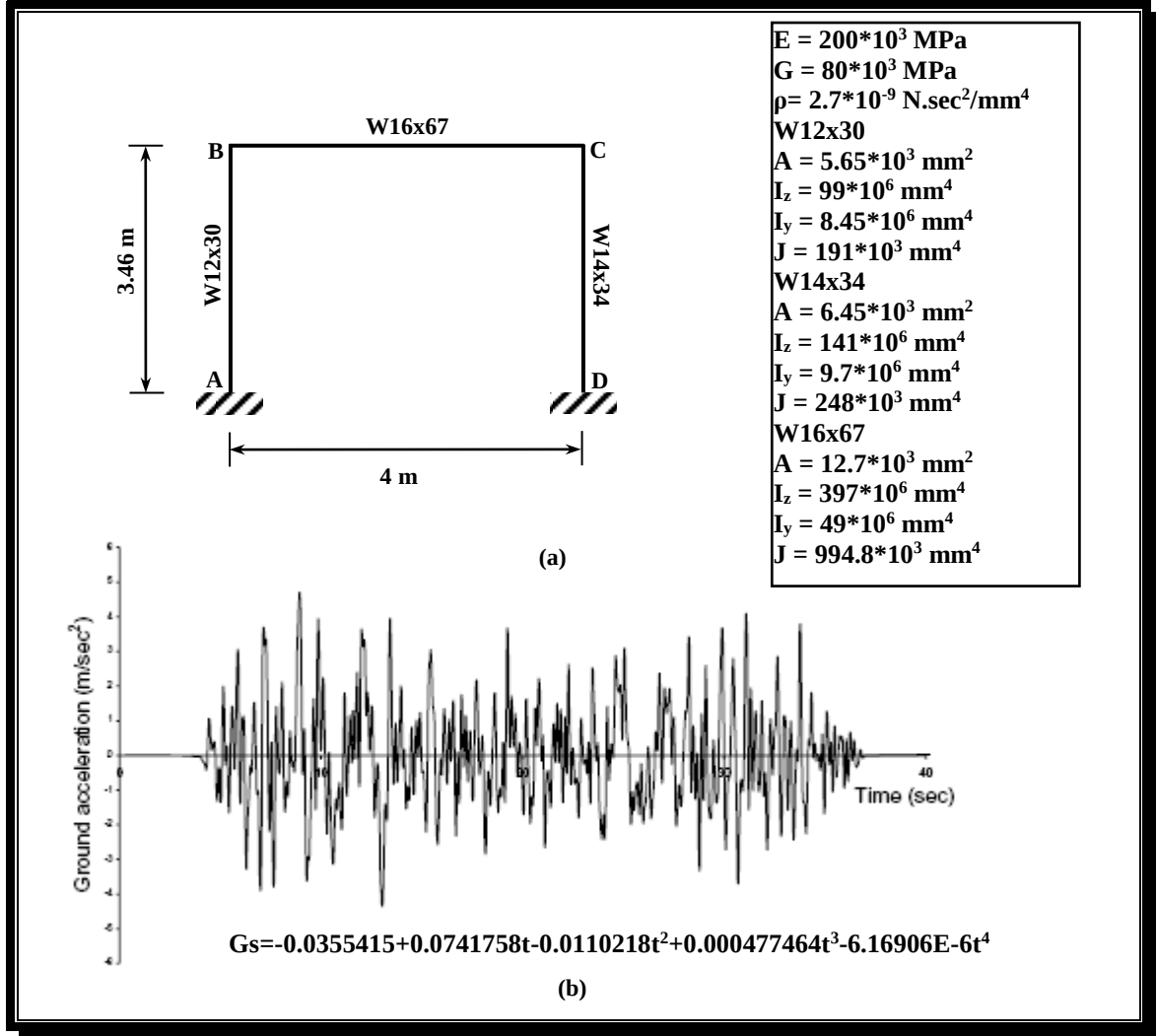


Fig.(6.26): Geometry and Loading of Portal Frame, (a) Side View and (b) Horizontal Ground Acceleration.

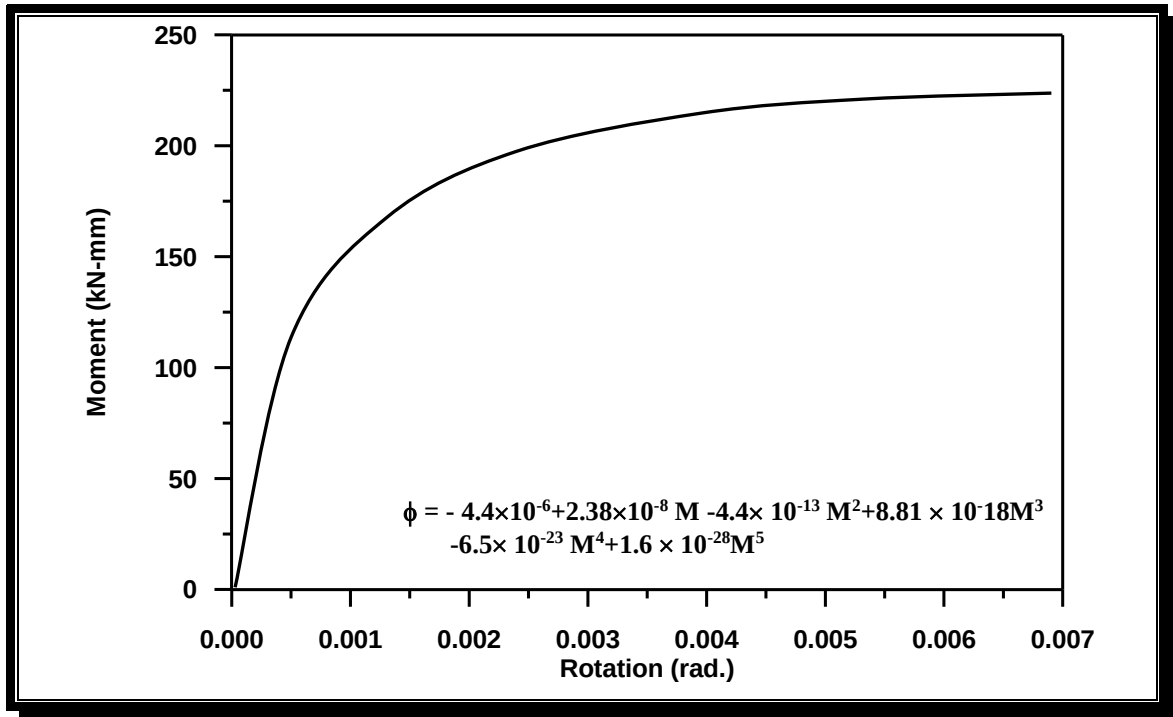


Fig.(6.27): Moment-Rotation Behavior Used for Portal Frame.

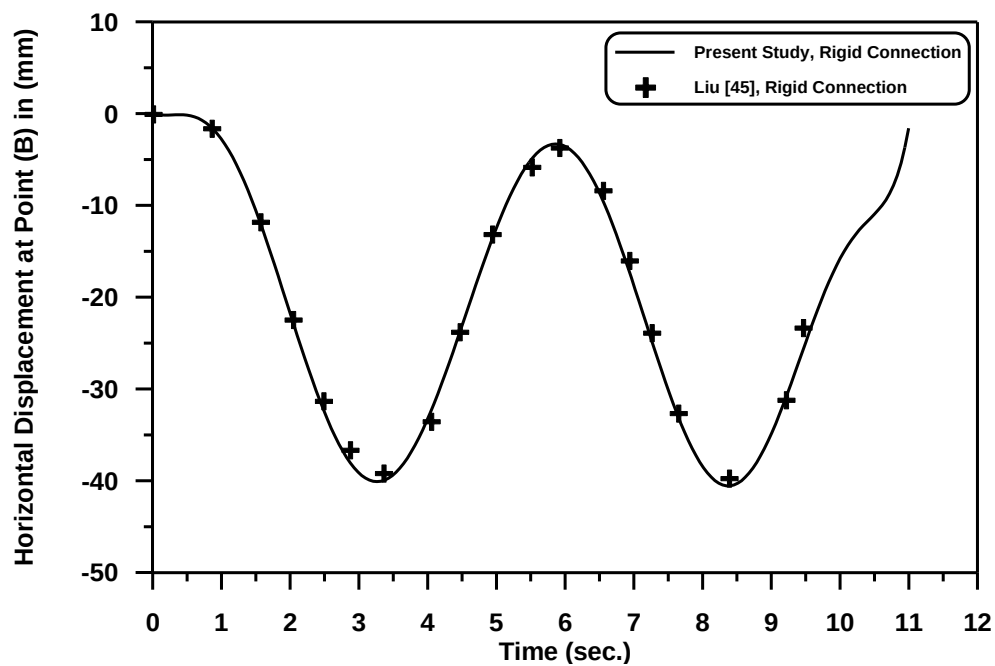


Fig.(6.28): Time-Displacement Curves For Portal Frame (Example No.8, Rigid Connection).

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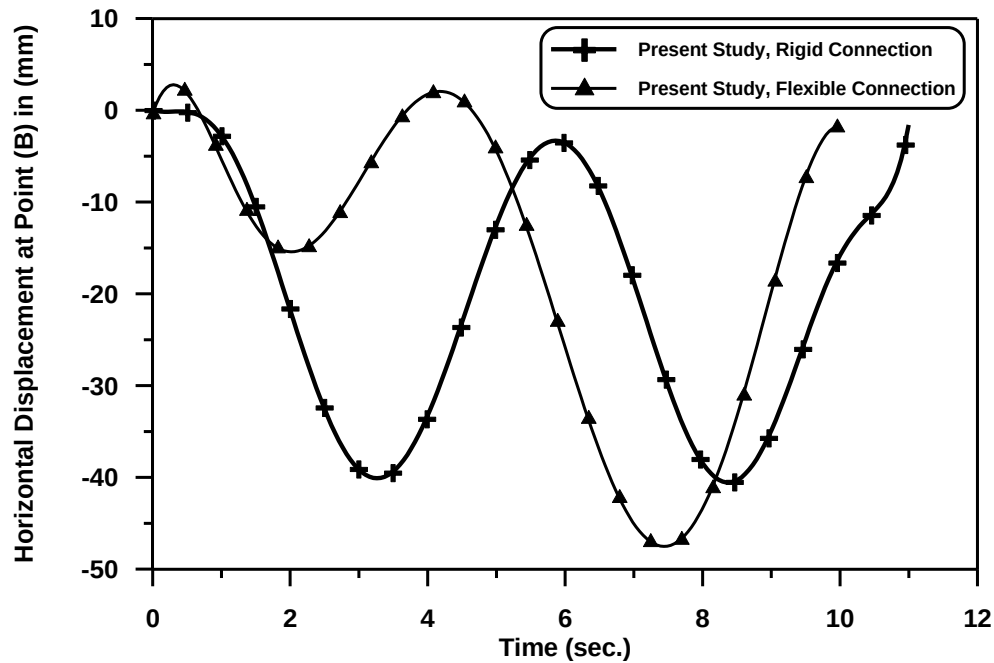


Fig.(6.29): Time-Displacement Curves For Portal Frame (Example No.8).

6.1.4.9 Example Nine (Five-Story Frame)

A five-story frame under earthquake force is analyzed as another case to verify the efficiency of the program (**OSDEPNONSF**) in the presence of plastic hinges. The geometry of the frame, material properties and loading conditions are shown in **Fig.(6.30)**. This example was solved by **Liu [45]** using a finite element program to analyze with large deformation and elastic-plastic effect. In the present study, this example is analyzed by using the beam-column approach and each member is represent by two elements while the effect of shear and torsion in the yielding functions is neglected to simplify the solution. **Fig.(6.31)** shows the sequence of formation of the plastic hinges. This figure shows that there is a small difference between the present study and **Liu [45]** work as obtained, this difference occurred because different yield functions were used in each study also, because the approximate polynomial expression function for the earthquake acceleration. In the present study, it should be noted that the time step size that used in the analysis must be changed and must be made it small enough to give the best sequences of plastic hinges of the frame and the best time step size is (0.0005 sec.), this time gives the best sequence of the plastic hinges.

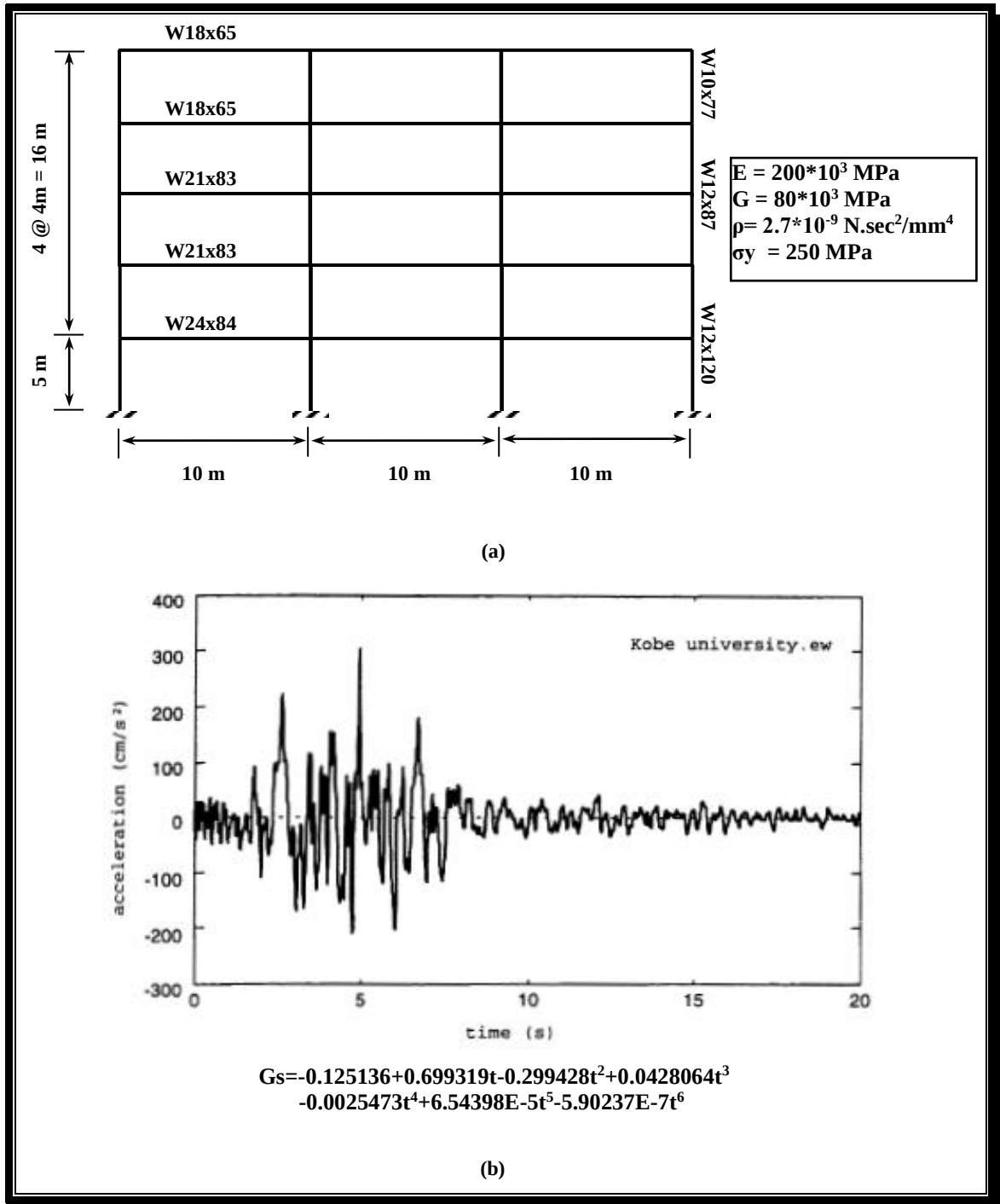


Fig.(6.30): Geometry and Loading of Five-Story Frame, (a) Side View and (b) Horizontal Ground Acceleration (Kobe University record, Japan 1995).

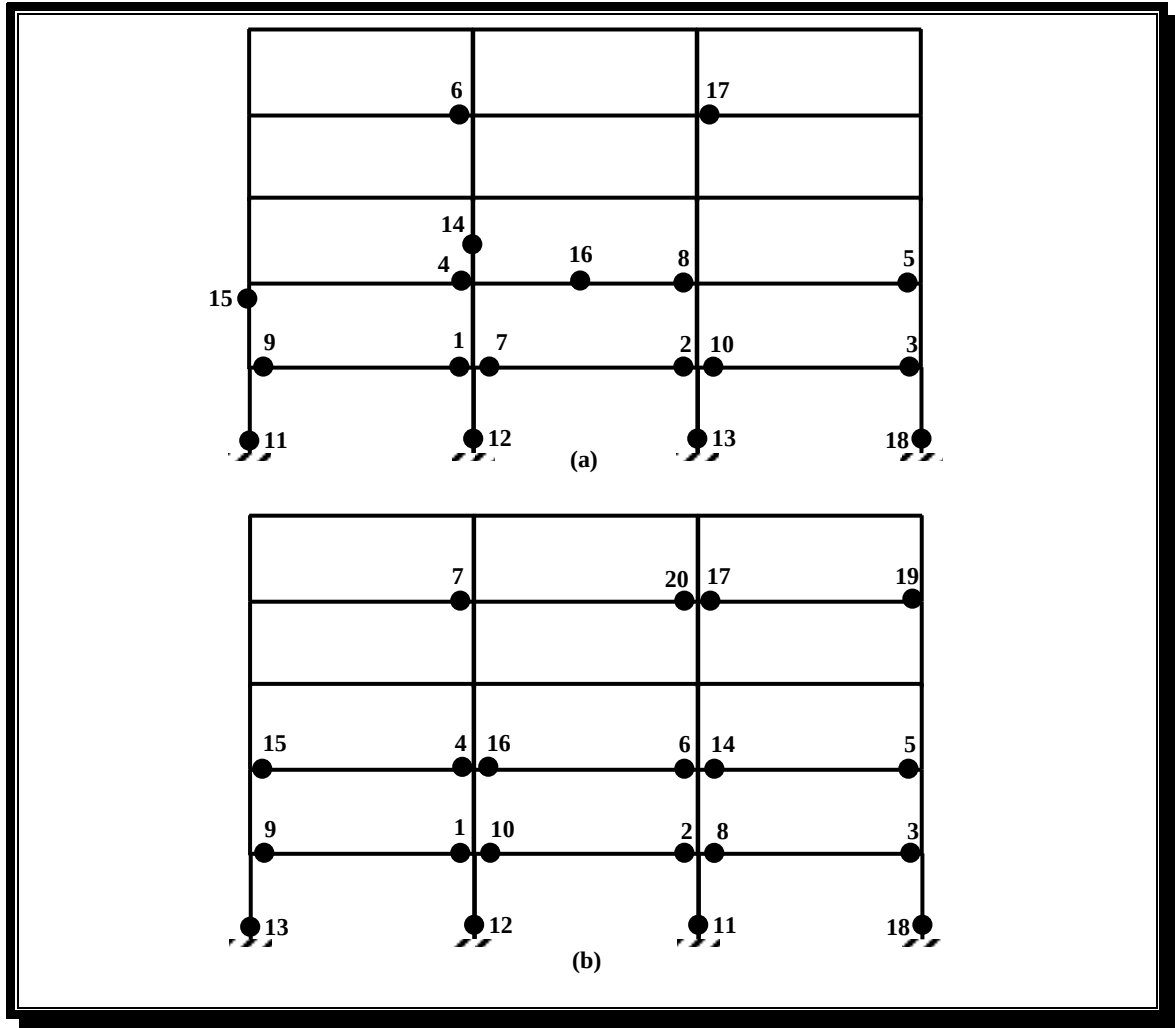


Fig.(6.31): The Sequence of Plastic Hinges For Five-Story Frame (Example No.9), (a) Present Study and (b) Liu [45] Study.

6.1.4.10 Example Ten (Gable Frame)

In order to study the effect of the plastic hinges on the behavior of the structure, the gable frame shown in **Fig.(6.32)** is analyzed under dynamic load applied vertically at point (C). This example was solved by **Al-Abbasi [87]** under triangle dynamic load with $Td1=0.016$ sec., $Td2=0.035$ sec. and $F = 750$ kN, used beam-column approach with one element for each member and elastic analysis only. In the present study, two types of analysis are used, elastic and elastic-plastic analysis. The vertical displacement-time curves are shown in **Fig.(6.33)**. As expected, this figure shows that the plastic analysis gives displacement more than in the elastic analysis, this is because in the plastic analysis, plastic hinges will be accounted and the stiffness of the structure will be decrease more

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and more until failure. This figure shows that there is good agreement between the present study and **Al-Abbasi [87]** work with different not more than (0.5%).

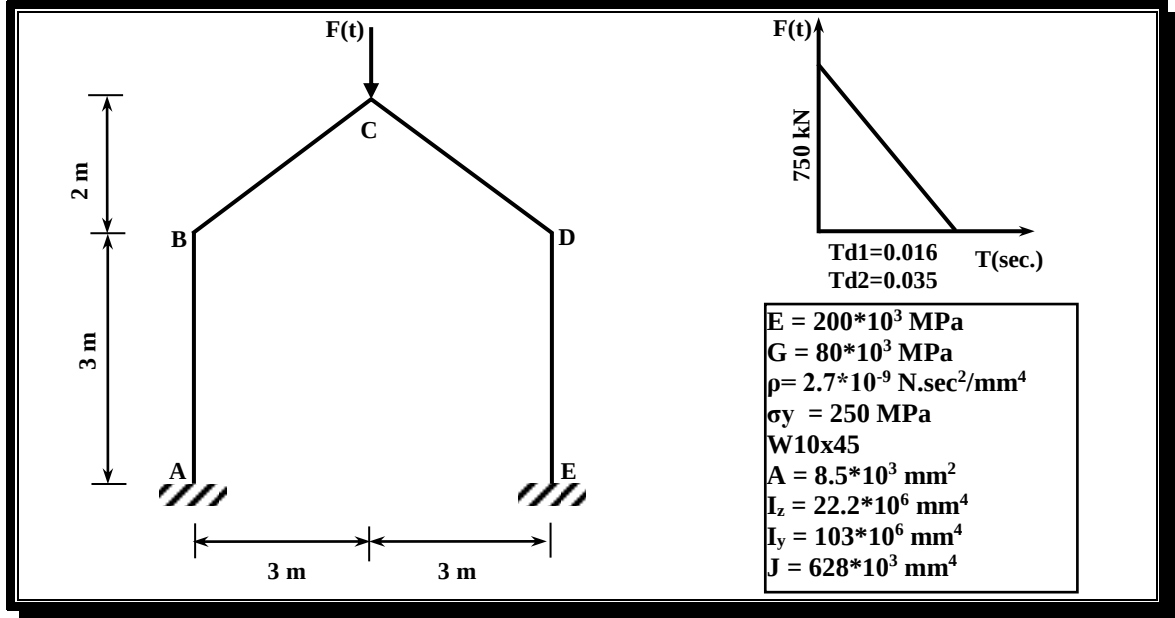


Fig.(6.32): Geometry Properties and Loading of The Gable Frame.

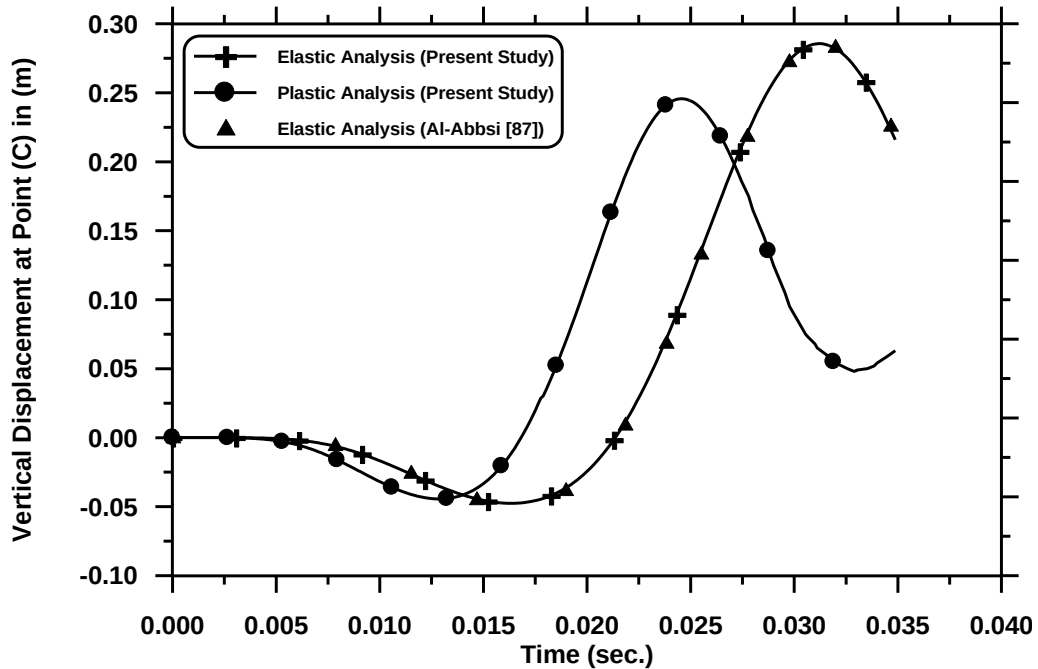


Fig.(6.33): Time-Displacement Curves For Gable Frame (Example No.10).

Part II (Parametric Study)

6.2.1 General

A parametric study is performed to assess the influence of several important parameters on the behavior of elastic-plastic space structures under static and dynamic loadings. The parametric study selected to be discussed in this chapter can be summarized as follows:

1. Time step effect.
2. Mass representation effect.
3. Effect of the types of the connection.
4. Effect of the number of stories on the response to the earthquake forces.
5. Effect of including shear force in yield criterion on the sequence of plastic hinges.
6. Damping effect.

6.2.2 Case Studies

6.2.2.1 Case Study No.1 (Clamped Beam)

The geometry, properties and loading conditions for the clamped beam, chosen to be the first case study, are shown in **Fig.(6.5)** in chapter six-part I. This beam had been analyzed by **Mondkar and Powell [86]**.

6.2.2.2 Case Study No.2 (18-Member Framed Dome)

This structure was used in chapter six-part I as example No.3, **Fig.(6.9)** shows the geometry, properties and loading conditions for the 18-member framed dome. The frame had been analyzed by **Remesth [23]**.

6.2.2.3 Case Study No.3 (Portal Frame)

The geometry, properties and loading conditions for the portal frame shown in **Fig.(6.34)**.

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6.2.2.4 Case Study No.4 (One-Bay and Multi-Story Space Frame)

The geometry and properties for the one-bay and multi-story space frame, chosen here to be the fourth case study, are shown in Fig.(6.35). Two types of earthquake forces (horizontal and vertical) are applied on this structure, Fig.(6.36).

6.2.2.5 Case Study No.5 (One-Bay and Four-Story Frame)

The geometry, properties and loading conditions for the one-bay and four-story frame, chosen to be the fifth case study, are shown in Fig.(6.37). This frame had been analyzed previously by Al-Dammerchy [65].

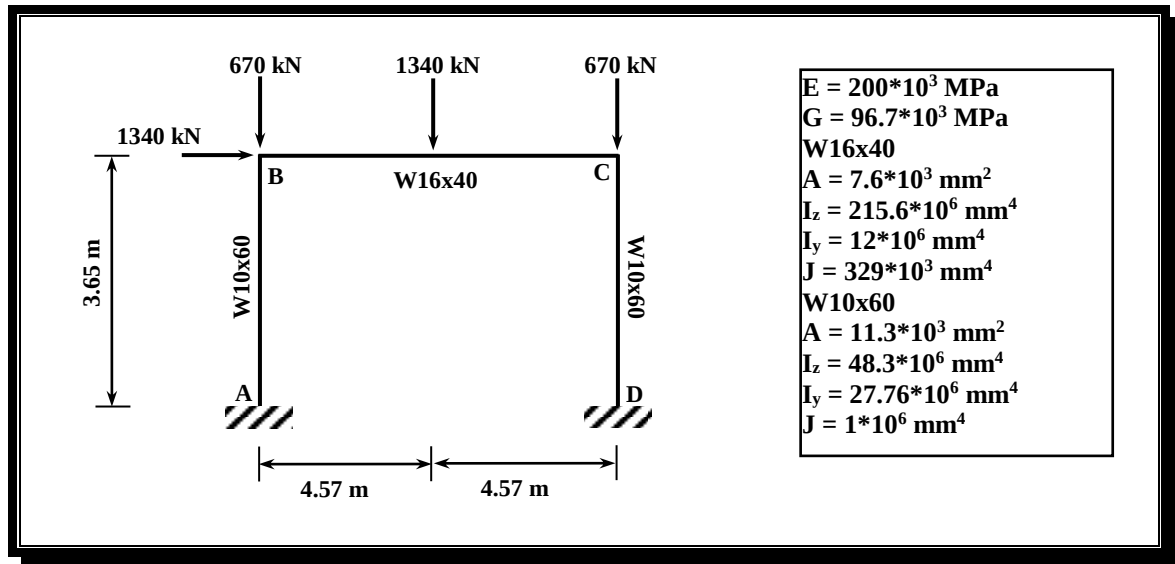


Fig.(6.34): The Geometry, Properties and Loading Conditions For Portal Frame (Case Study No.3).

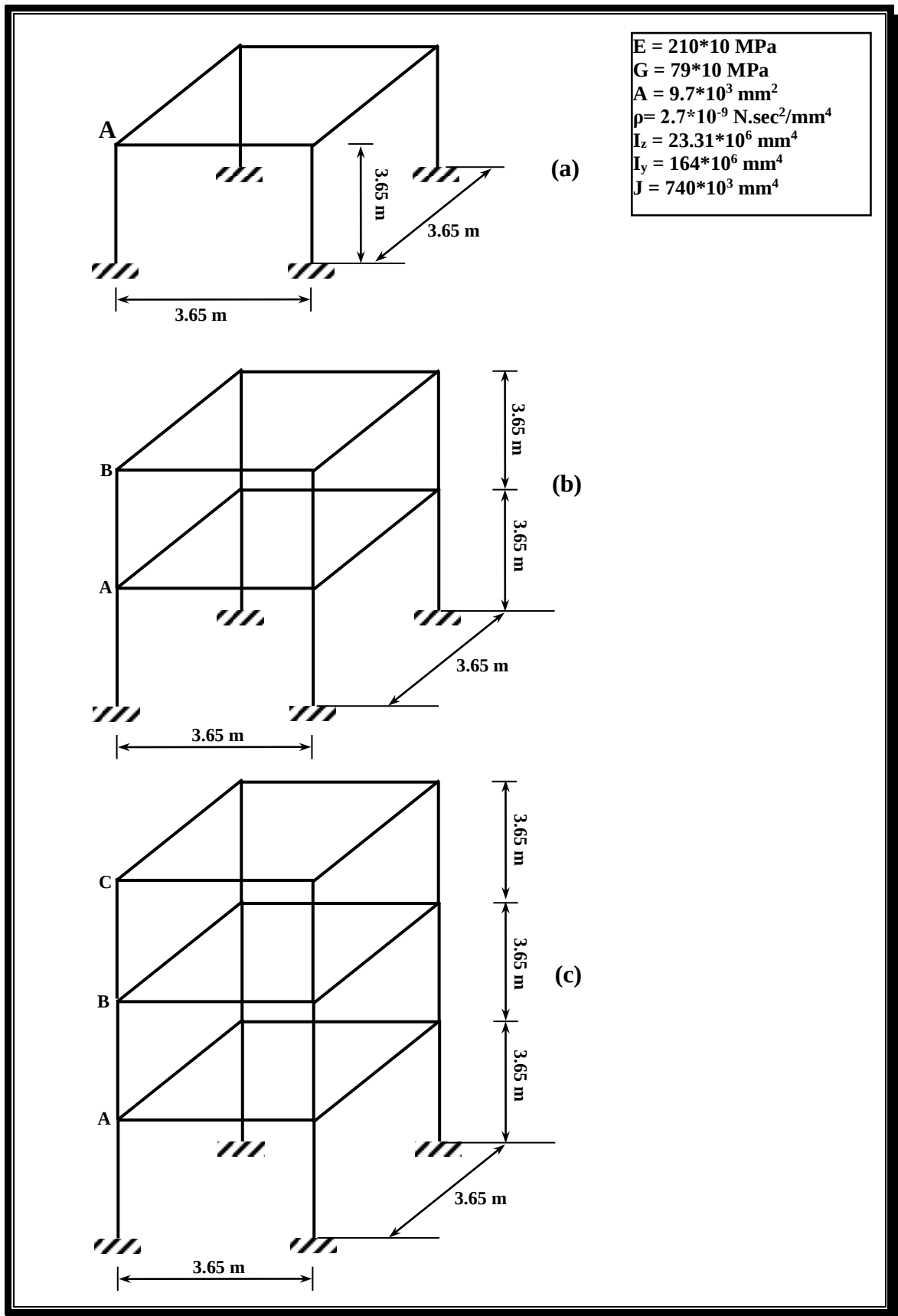
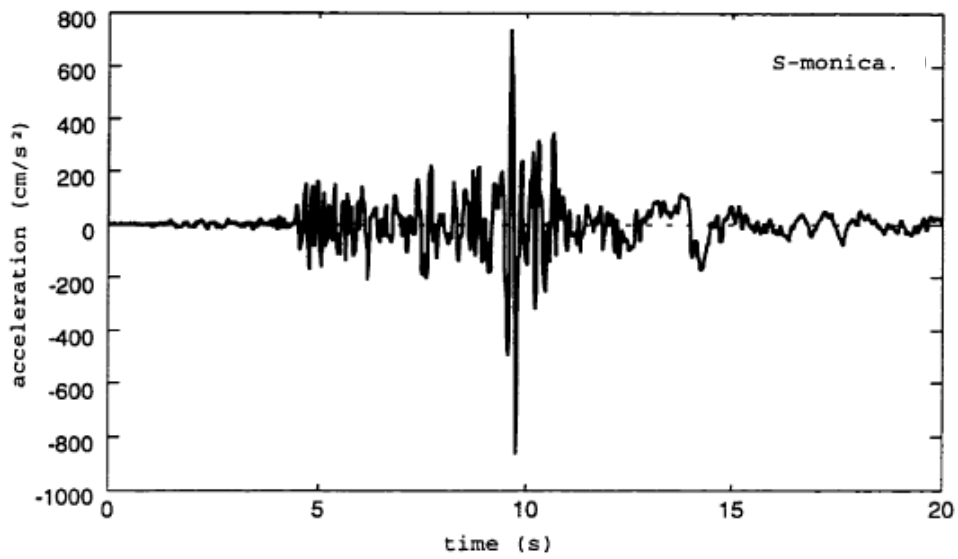
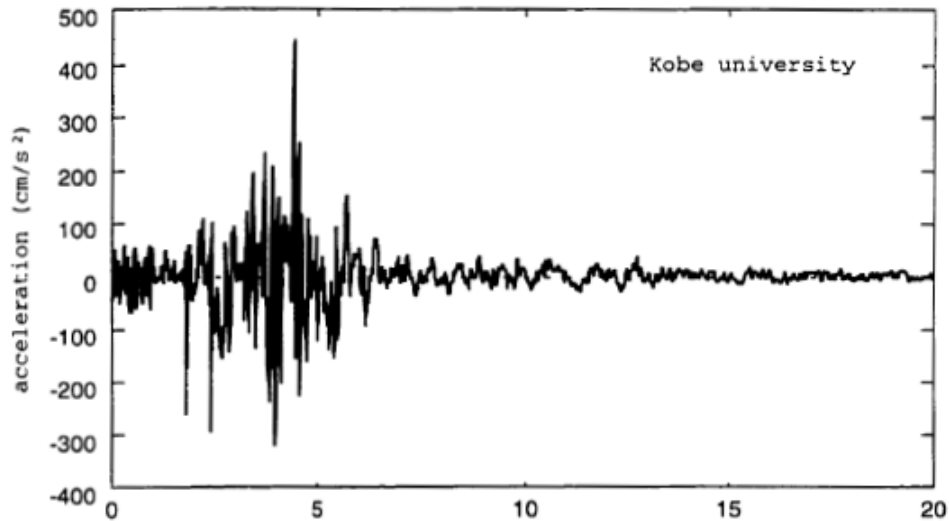


Fig.(6.35): The Geometry and Properties For One-Bay and Multi-Story Space Frame (Case Study No.4).



$$G_s = -0.00807207 + 0.0598761t - 0.0156264t^2 + 0.00126122t^3 - 4.40542E-5t^4 + 6.8267E-7t^5 - 3.69143E-9t^6$$

(a)



$$G_s = -0.15127 + 0.58842t - 0.322519t^2 + 0.0601589t^3 - 0.0022563t^4 + 4.53387E-5t^5 - 7.8802166E-7t^6$$

(b)

Fig.(6.36): Loading Conditions, (a) Horizontal Ground Acceleration (Northridge earthquake 1994) and (b) Vertical Ground Acceleration (Kobe University record 1995).

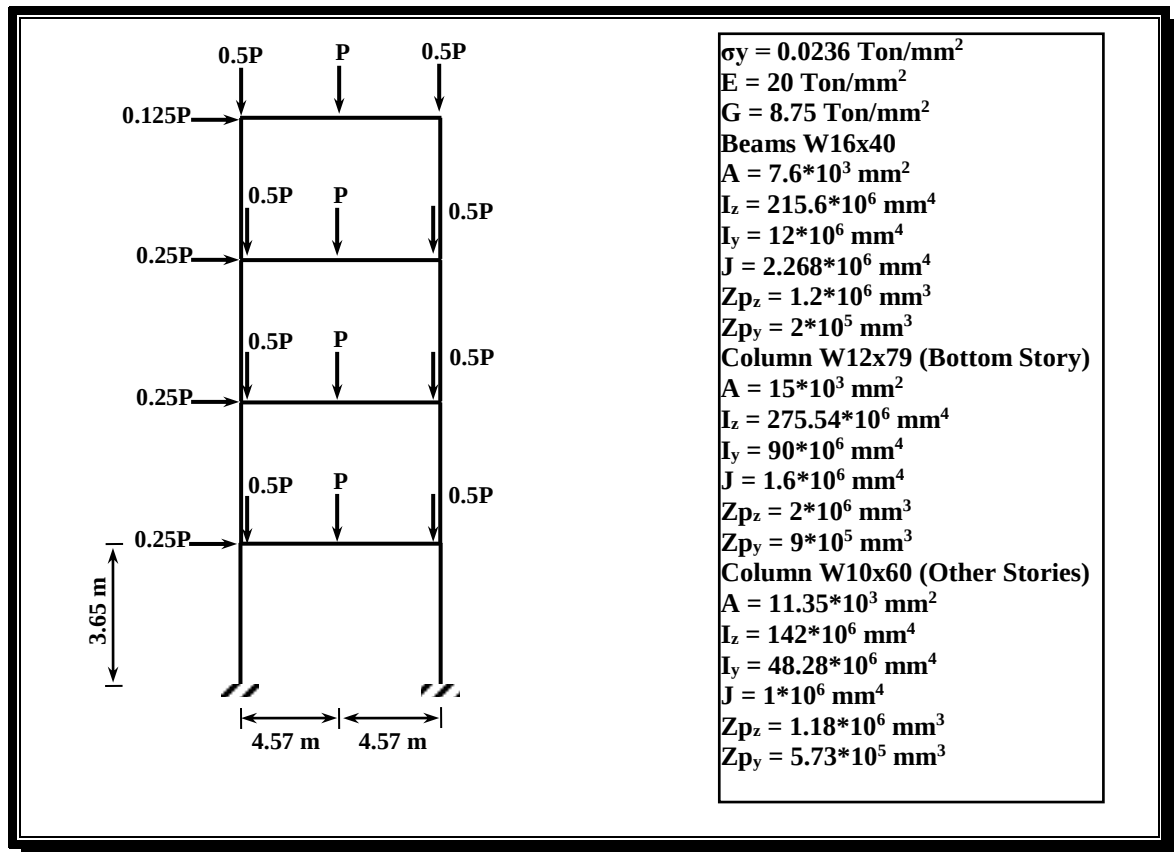


Fig.(6.37): The Geometry and Properties For One-Bay and Four-Story Frame (Case Study No.5).

6.2.3 Parametric Studies

6.2.3.1 Effect of Time Step Size

In numerical solution of the equation of motion, the time step size (Δt) has a very significant effect on the accuracy and stability of the solution. Taking the time step size to be very large may cause severe drift from the right solution, or may cause the solution to be unstable. On the other hand, using too small time increment will increase the time required for the solution and increase the cost of the analysis. Also, if the time step size is very small it will cause drift or unstable solution. So, the step size must be chosen to satisfy the accuracy, stability and time cost of analysis requirements. In order to study the effect of the time step size, the clamped beam (**Fig.(6.5)**) has been analyzed dynamically with time step size $T_d=0.0002, T_d=0.00002, T_d=0.00005, T_d=0.00007, T_d=0.00009$,

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$T_d=0.000001$ for $P=2.8$ kN (suddenly applied). The results for the vertical displacement at point (B) are shown in **Figures (6.38), (6.39) and (6.40)**. It can be seen that using ($\Delta t=0.00005$ sec. and 0.00007 sec.) yields the same and good results compared with the previous study by **Mondkar and Powell [86]** as shown in **Fig.(6.8)**, while using a large time step size like ($\Delta t=0.0002$ sec. and 0.00002 sec.) causes the results to drift appreciably from the right solution as shown in **Fig.(6.39)**, and the error increases simultaneously with time step increase until it reaches a value that makes the solution to be unstable and the program fails to complete the analysis, and using small step size like ($\Delta t=0.00009$ sec. and $\Delta t=0.000001$ sec.) causes increases in the time that is required to reach to the right solution and if the time step size will be very small the solution will fail as shown in **Fig.(6.40)**. Thus, the time step size is a large extent, affects on the analysis of the structures.

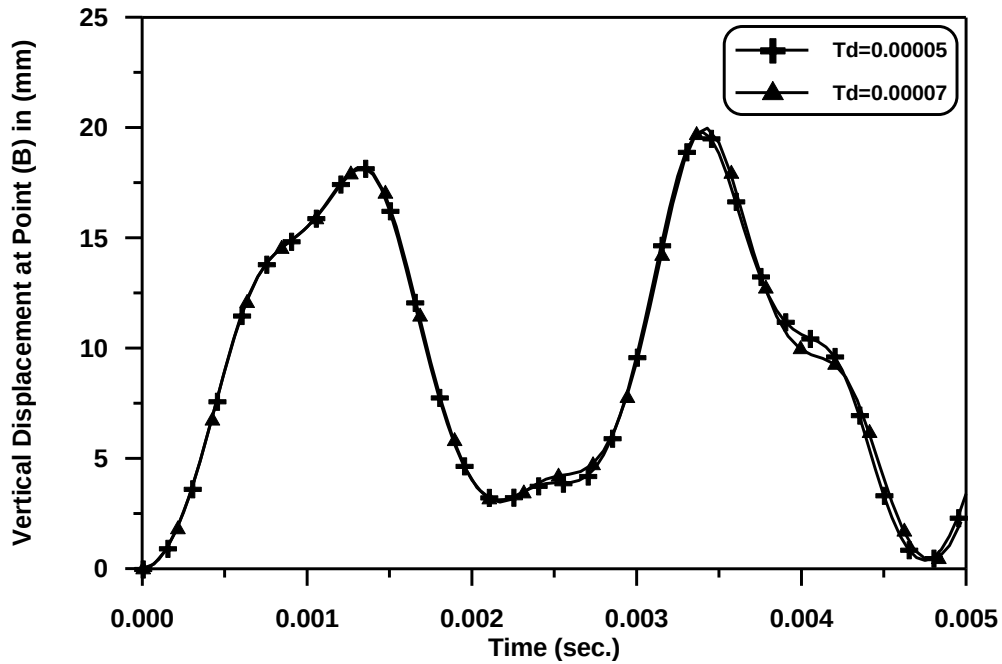


Fig.(6.38): Time-Displacement Curves For Clamped Beam ,Case Study No.1 ($T_d=0.00005,0.00007$).

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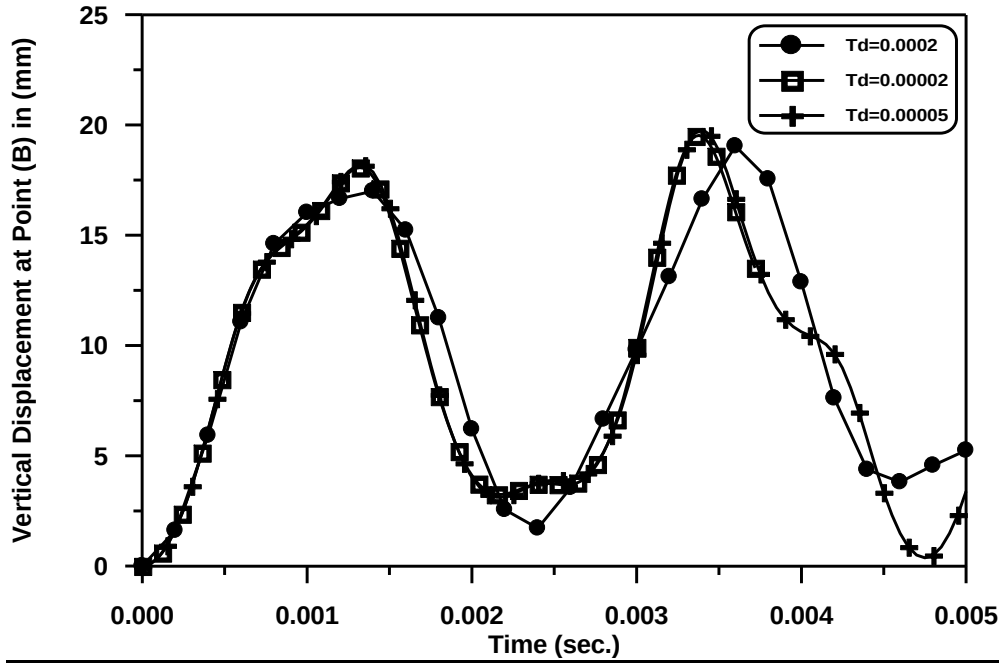


Fig.(6.39): Time-Displacement Curves For Clamped Beam ,Case Study No.1 ($T_d=0.0002,0.00002,0.00005$).

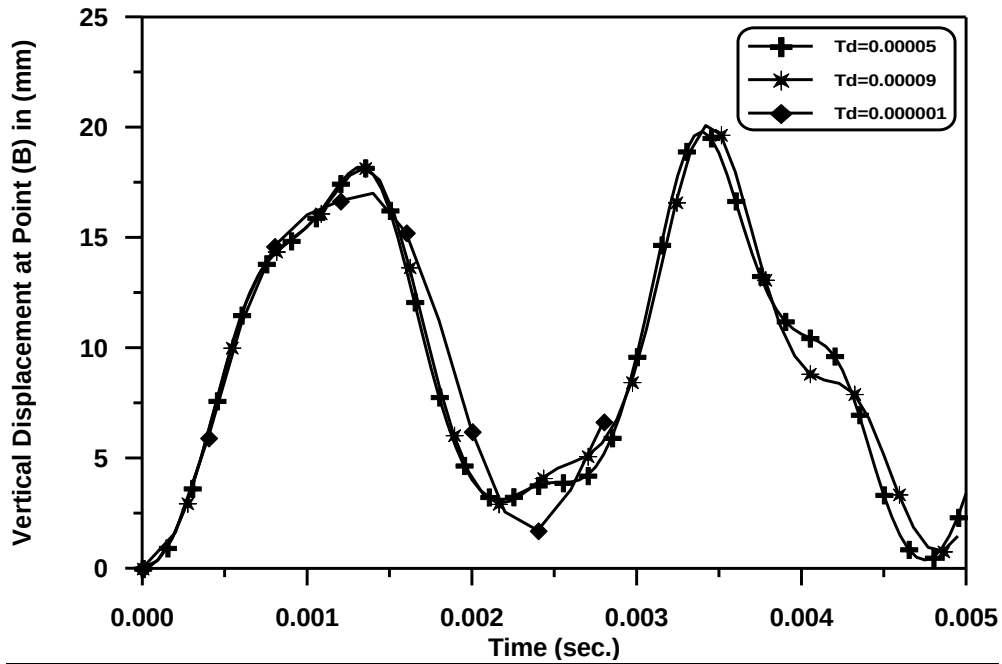


Fig.(6.40): Time-Displacement Curves For Clamped Beam ,Case Study No.1 ($T_d=0.00005,0.00009,0.000001$).

6.2.3.2 Effect of Mass Representation

To study the amount of error caused by using the lumped mass method in comparison with the consistent mass method, the 18-member framed dome (Fig.(6.9)) has been analyzed dynamically. Fig.(6.41), shows the relation between vertical displacement at point (1) and time for

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harmonically concentrated load equal to (34.4 MN). From this figure it can be seen that the difference in response between the two methods grows gradually with time. It is noted that the use of consistent mass matrix is accurate because the consistent mass matrix depends on the deformed configuration, and it will not remain constant during the analysis of geometrically non-linear system like the lumped mass matrix but this will complicate the solution of the structures.

For the free vibration stage of the structures, the response difference between the consistent mass matrix and the lumped mass matrix will increase and it may reach (100%) along time, i.e. as the response due to the consistent method reaches its maximum positive value, the response due to the lumped method reaches its negative value.

It can be concluded that the consistent mass method has a considerable advantage over the lumped mass from the accuracy. On the other hand, the lumped mass method has remarkable benefit over the consistent mass because of its simplicity and less computational effort.

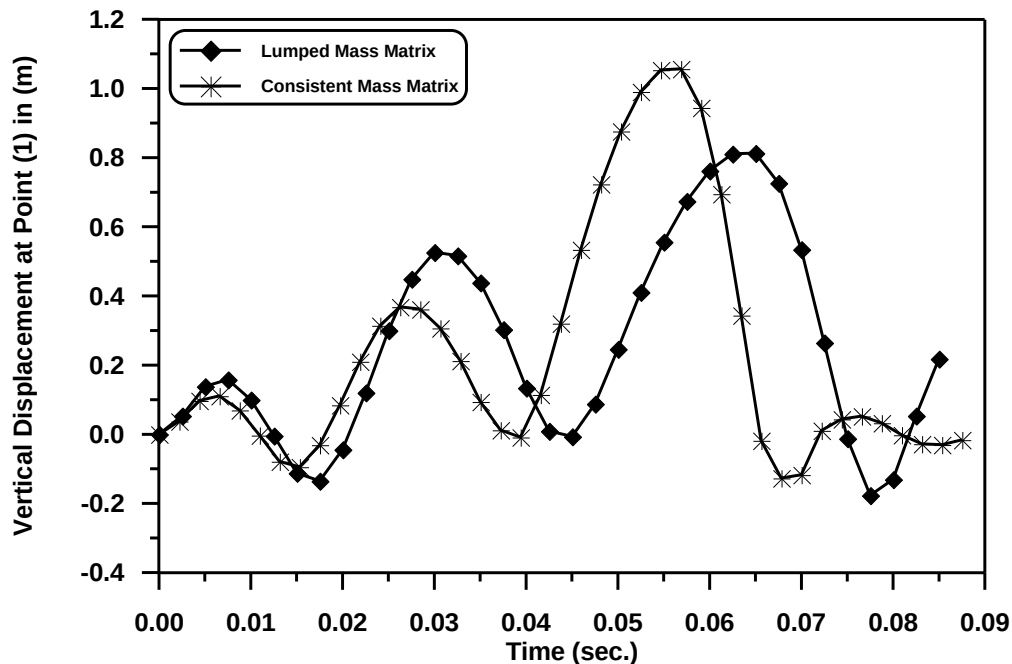


Fig.(6.41): Time-Displacement Curves For Case Study No.2.

6.2.3.3 Effect of Flexible Connections on the Behavior of Frames

As discussed earlier, most of the connections in real steel structures are flexible and their effects on the structure behavior must be considered

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in any realistic analytical procedure. The behavior of a connection is best described by its moment-relative rotation relationship. A popular model for structural analysis is the polynomial function proposed by **Frye and Morris [33]**, the model has the form:

$$\phi = C_1(KM) + C_2(KM)^3 + C_3(KM)^5 \dots\dots\dots(6.2)$$

where:

C_i : are curve-fitting constants,

K : is the standardization dimensionless factor.

To assess the effect of varying stiffness of the connections on the behavior of frame, the portal frame, **Fig.(6.34)** has been analyzed by using different types of the end connections. In this case, four types of the end connection are used and by using **Table (3.1)** to find the polynomial equations for the connections.

1. Single web angle (L8x8x1), $K = 9.3 * 10^{-3}$

$$\phi = 3.98 * 10^{-5} M + 1.166 * 10^{-15} M^3 + 1.05 * 10^{-26} M^5 \dots\dots\dots(6.3)$$

2. Double web angle (L8x8x1), $K = 9.3 * 10^{-3}$

$$\phi = 3.4 * 10^{-6} M + 9.25 * 10^{-13} M^3 + 3.18 * 10^{-18} M^5 \dots\dots\dots(6.4)$$

3. Top and seat angle (L7x4x0.5) without web angle (diameter of bar = 1 in), $K = 5.66 * 10^{-3}$

$$\phi = 4.788 * 10^{-6} M + 1.83 * 10^{-11} M^3 + 7.2 * 10^{-20} M^5 \dots\dots\dots(6.5)$$

4. Top and seat angle (L7x4x0.5) with web angle (L8x8x1), $K = 0.146$

$$\phi = 3.256 * 10^{-6} M + 5.75 * 10^{-11} M^3 + 2.116 * 10^{-16} M^5 \dots\dots(6.6)$$

Fig.(6.42) shows the load-horizontal displacement relationship curves for the portal frame. From the inspection of these results, it can be seen that the drift of frame with single web angle is more than the frame with the rigid and other types of the end connection. This is because, that when the load increase the tangent stiffness of connection becomes very small compared with the initial tangent stiffness. Also, it can be seen that the drift of frame with double web angle, top and seat angle with and without web angle is more than that in the frame with rigid end connection. Generally speaking, the rigid joint assumption will lead to an

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underestimate of frame drift and over-estimate of frame strength, whereas the pinned joint assumption will result in an over design of the girder and an under-design of the column. Thus, the behavior of the end connection is a very important factor in the behavior of the structures.

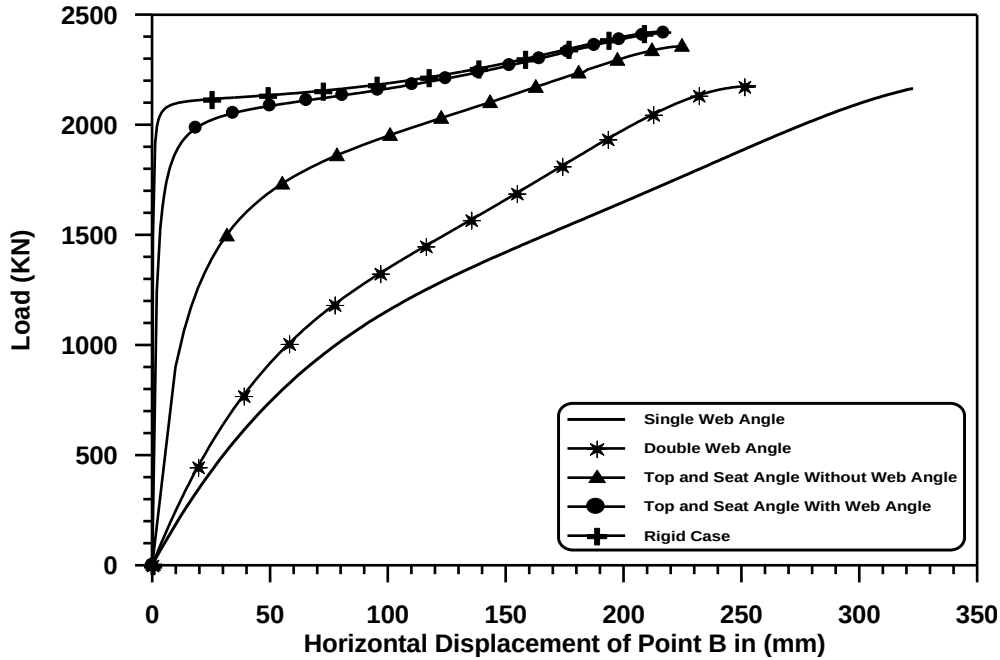


Fig.(6.42): Load-Displacement Curves For Case Study No.3.

6.2.3.4 Effect of the Number of Stories on the Response to the Earthquake Forces

To study the effect of number of stories on the response by the earthquake forces, the one-bay and multi-story space frame (Fig.(6.35)) has been analyzed dynamically under earthquake forces. Two types of earthquake forces are used to satisfy this effect (Fig.(6.36)).

Fig.(6.43) to Fig.(6.54) show the relationship between the horizontal displacement at points (A, B and C) and time of the horizontal and vertical earthquake.

It can be seen from Figures (6.43) to (6.48) that the displacement of point (A) for frame of one-story is less than that for frame of two stories and for two stories is less for three stories, also the same results exist for points (B) and (C) as shown in Figures (6.49) to (6.54). This behavior comes because the stiffness of the structure decreases with increasing the number of stories of buildings under the effect of forces,

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so the failure of high-length building is quicker than the failure of short-length building.

It can be concluded that the increase of number of stories will increase the effect of earthquake forces on the behavior of the structure.

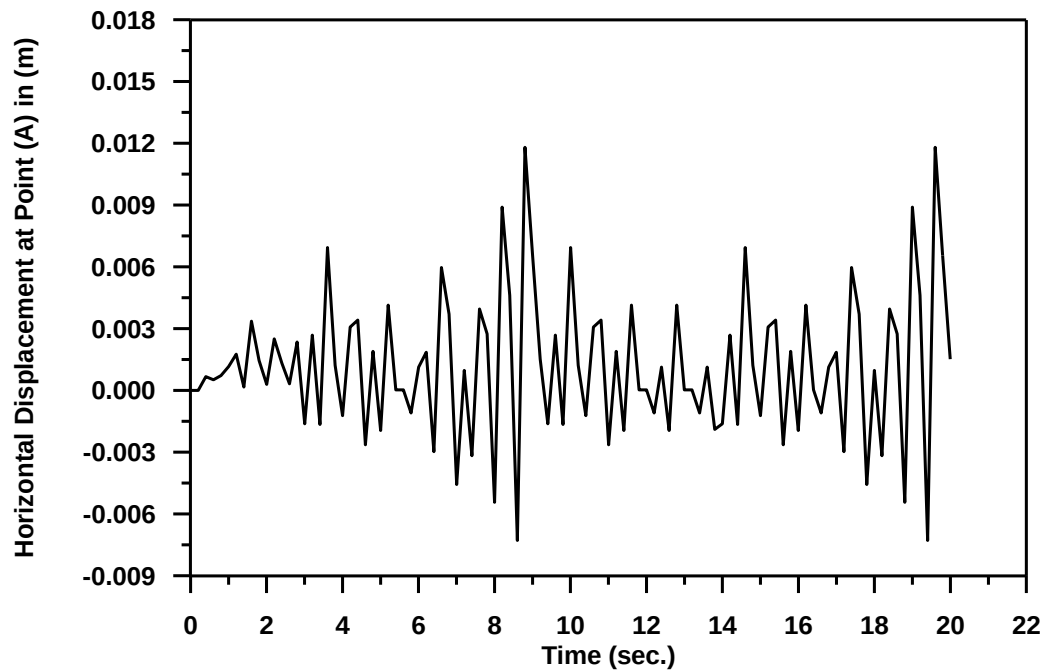


Fig.(6.43): Time-Displacement Curve For One-Story Space Frame Under Horizontal Ground Acceleration at Point (A).

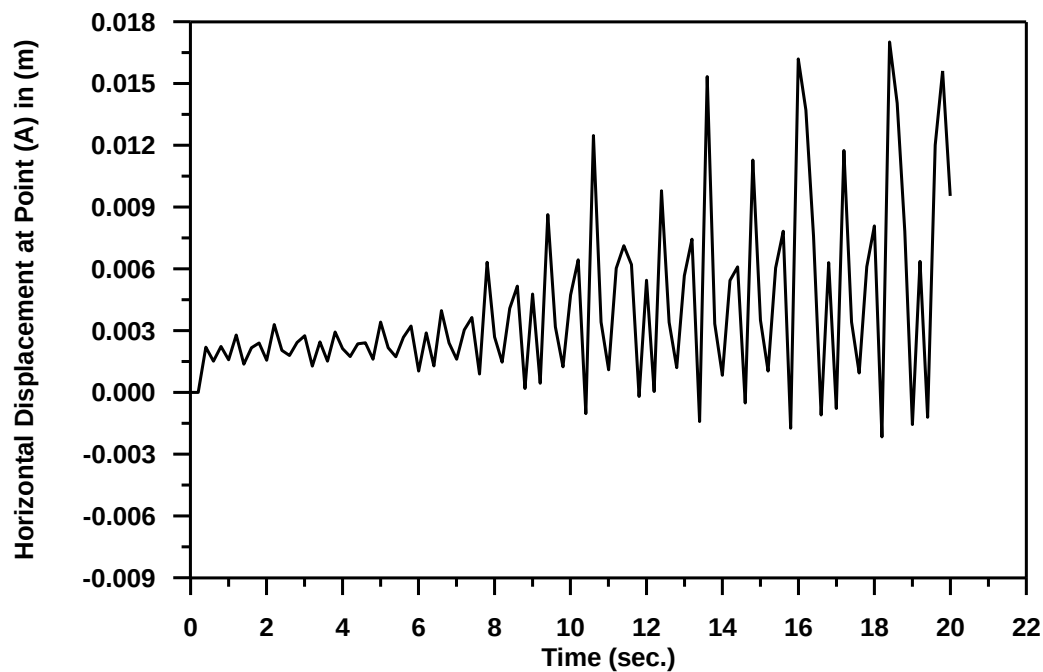


Fig.(6.44): Time-Displacement Curve For Two-Story Space Frame Under Horizontal Ground Acceleration at Point (A).

Examples

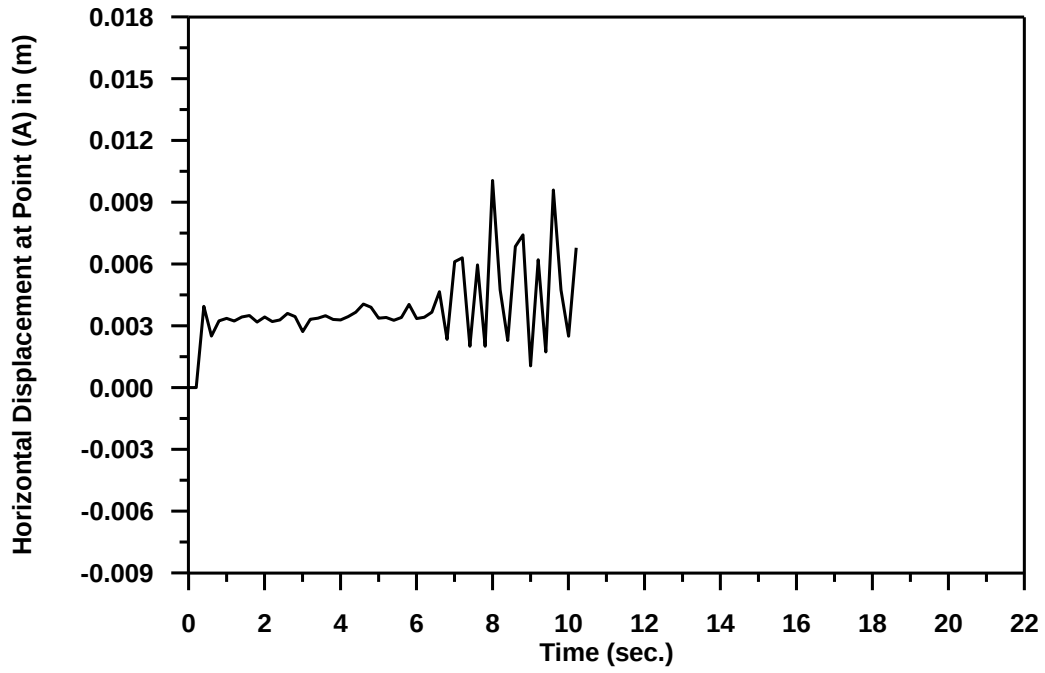


Fig.(6.45): Time-Displacement Curve For Three-Story Space Frame Under Horizontal Ground Acceleration at Point (A).

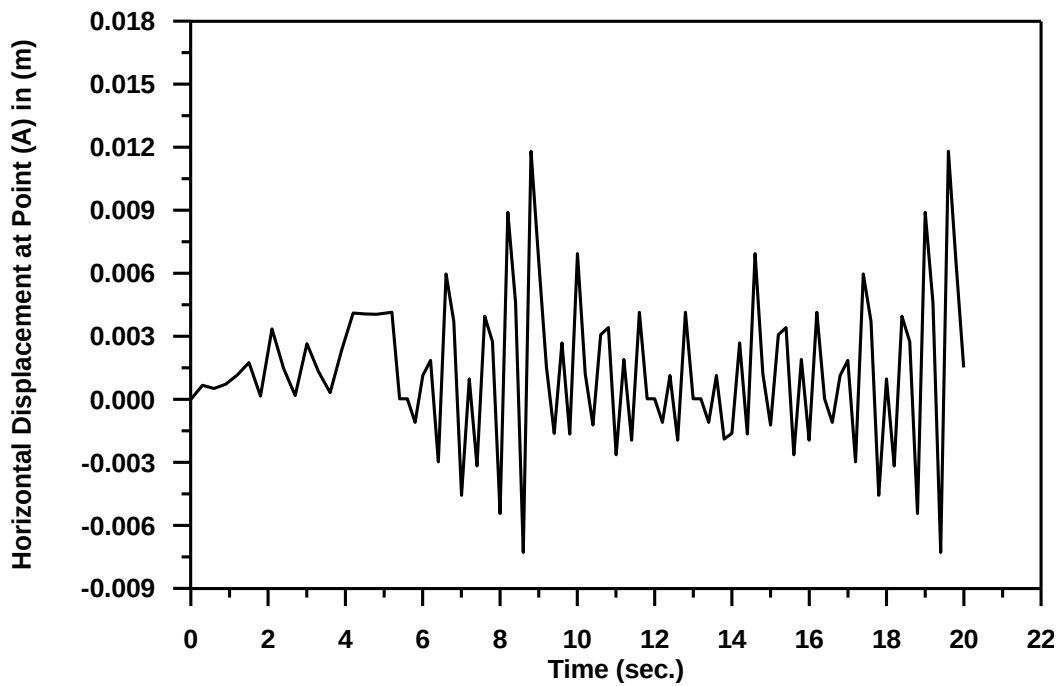


Fig.(6.46): Time-Displacement Curve For One-Story Space Frame Under Vertical Ground Acceleration at Point (A).

Examples

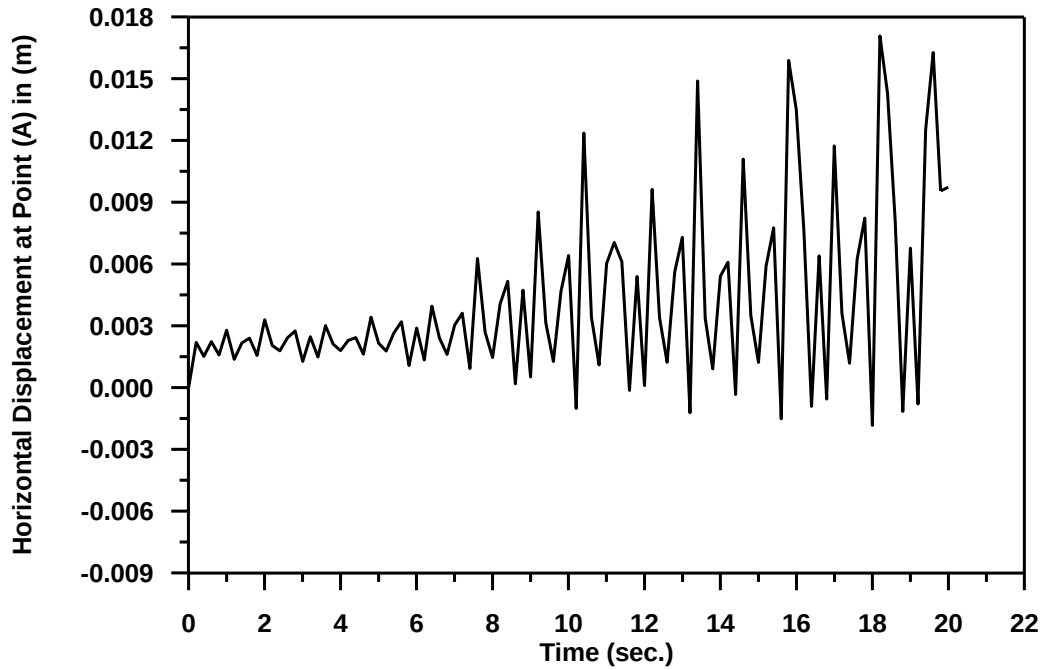


Fig.(6.47): Time-Displacement Curve For Two-Story Space Frame Under Vertical Ground Acceleration at Point (A).

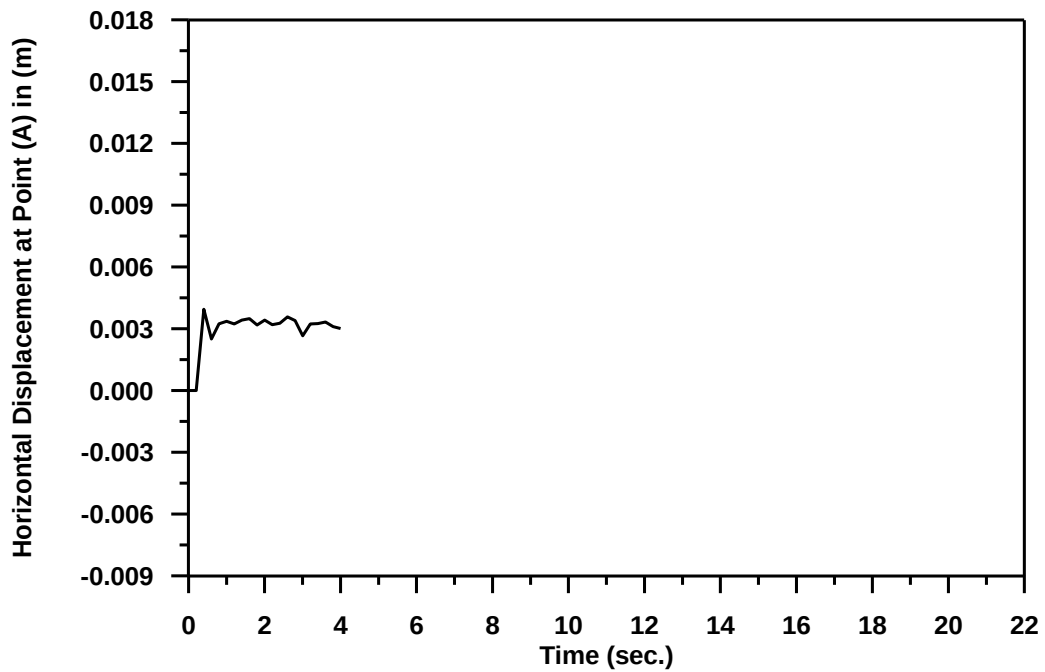


Fig.(6.48): Time-Displacement Curve For Three-Story Space Frame Under Vertical Ground Acceleration at Point (A).

Examples

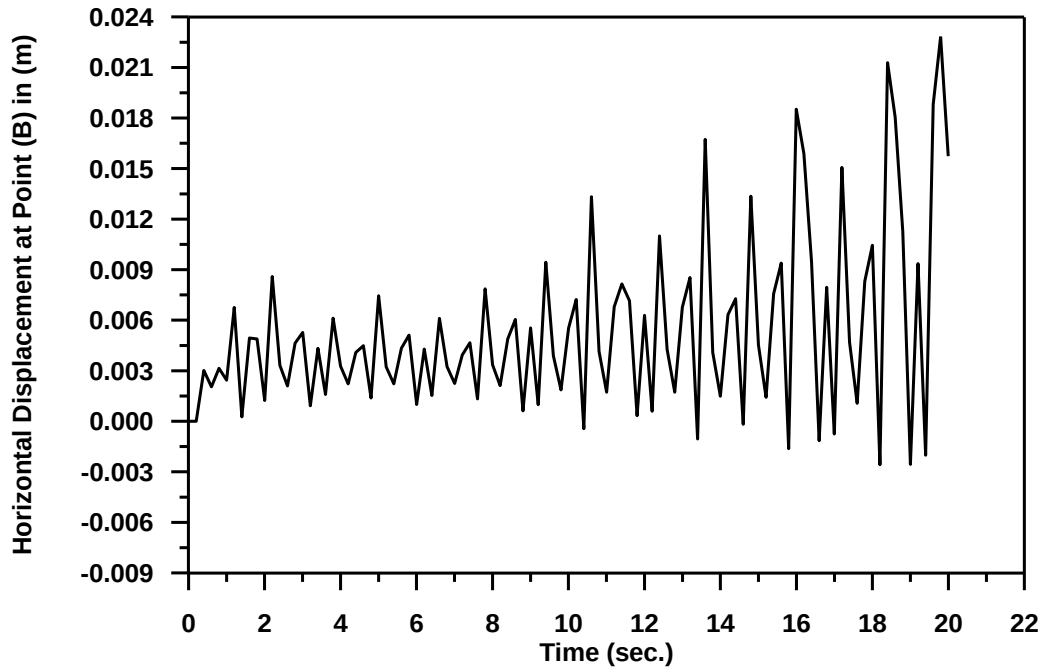


Fig.(6.49): Time-Displacement Curve For Two-Story Space Frame Under Horizontal Ground Acceleration at Point (B).

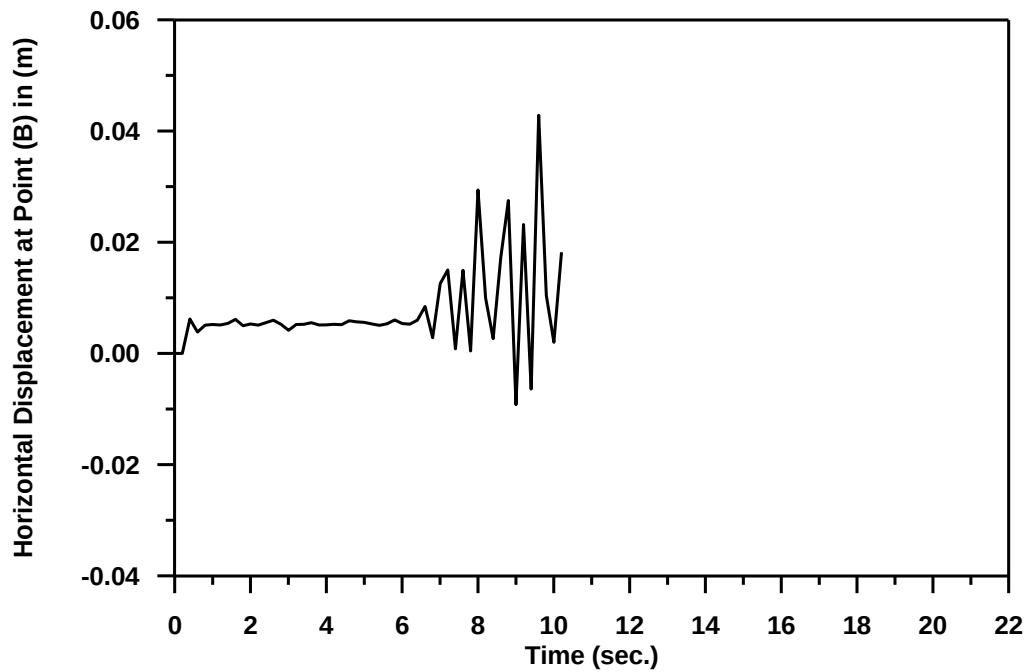


Fig.(6.50): Time-Displacement Curve For Three-Story Space Frame Under Horizontal Ground Acceleration at Point (B).

Examples

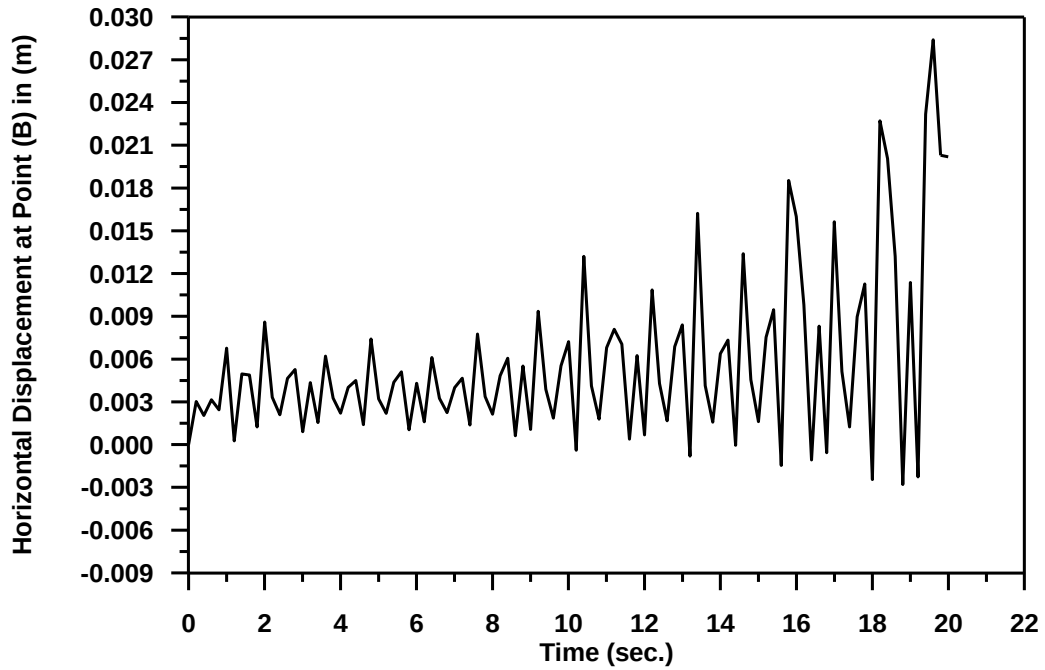


Fig.(6.51): Time-Displacement Curve For Two-Story Space Frame Under Vertical Ground Acceleration at Point (B).

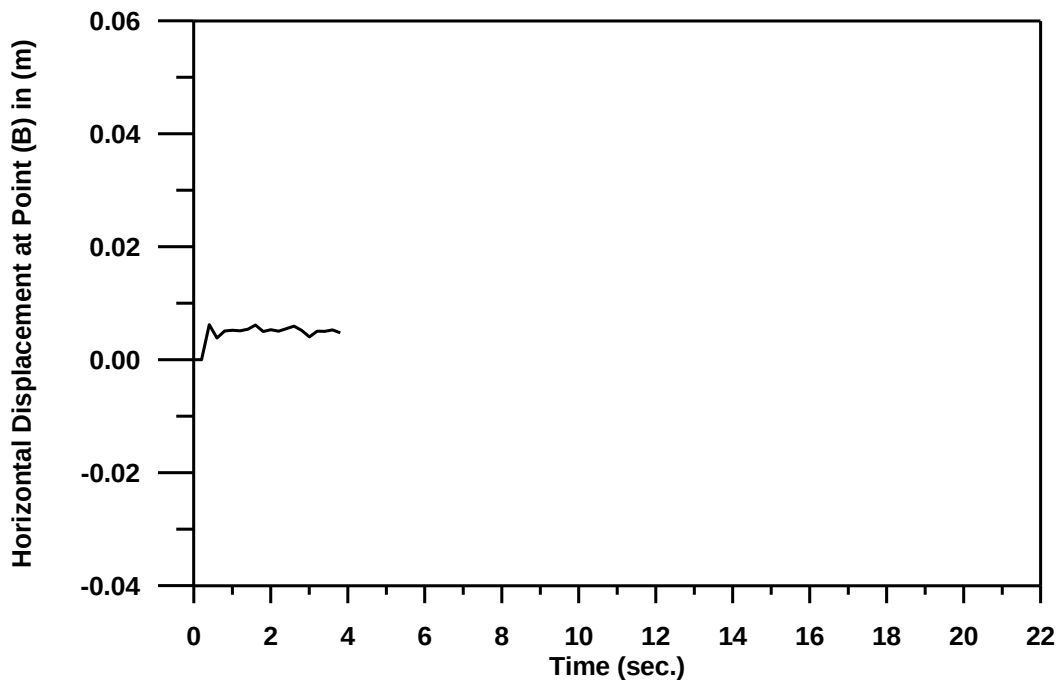


Fig.(6.52): Time-Displacement Curve For Three-Story Space Frame Under Vertical Ground Acceleration at Point (B).

Examples

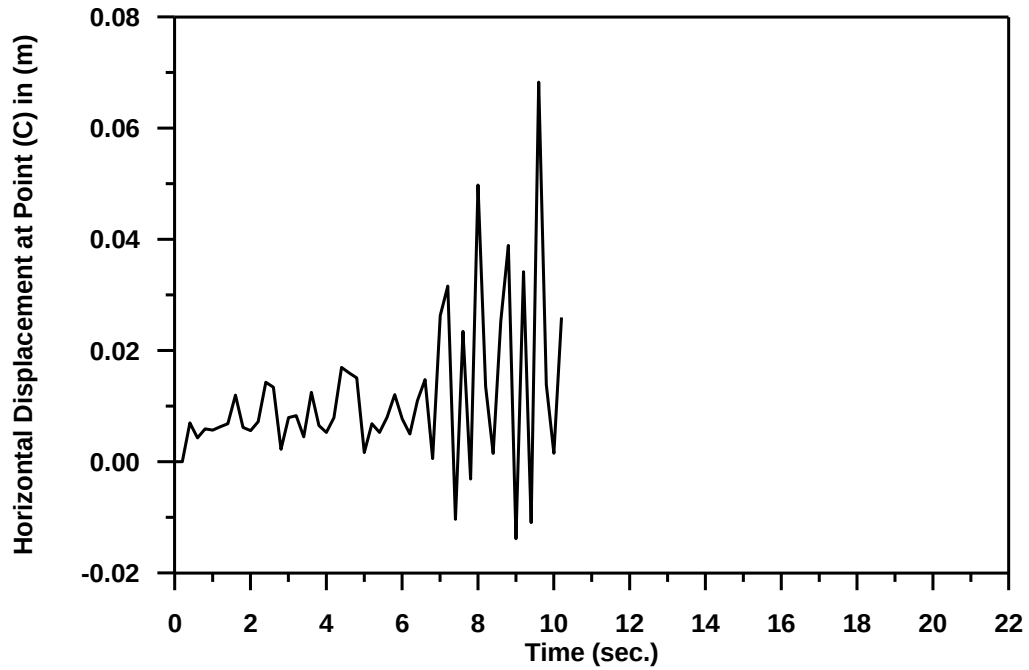


Fig.(6.53): Time-Displacement Curve For Three-Story Space Frame Under Horizontal Ground Acceleration at Point (C).

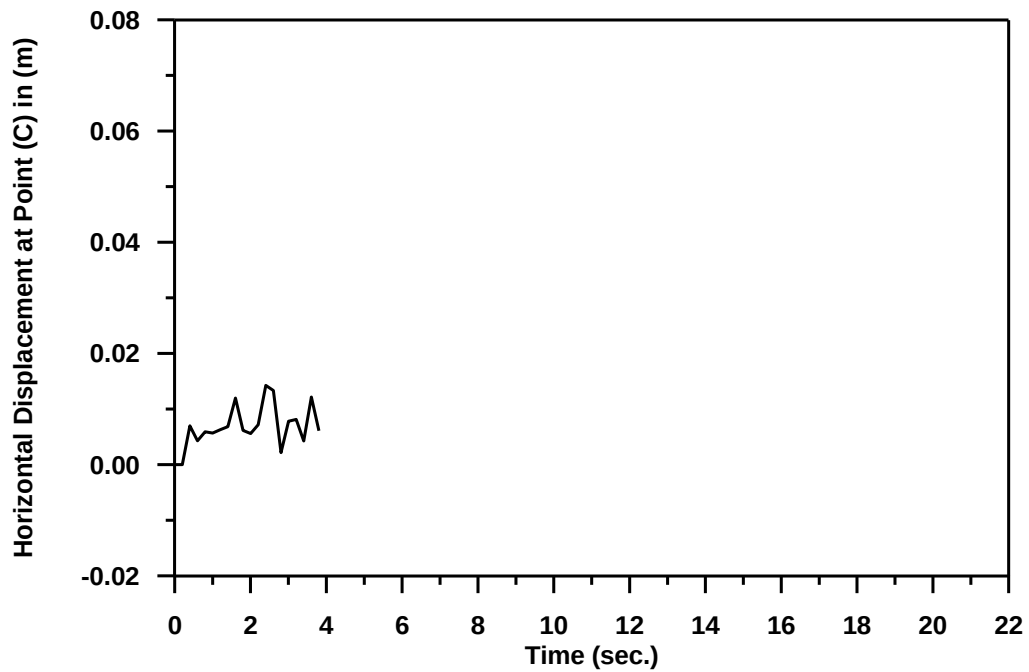


Fig.(6.54): Time-Displacement Curve For Three-Story Space Frame Under Vertical Ground Acceleration at Point (C).

6.2.3.5 Effect of Including Shear Force in Yield Criterion on The Sequence of Plastic Hinges

To study the effect of including shear force in yield criterion on the sequence of plastic hinges, the one-bay and four-story frame (Fig.(6.37)) has been analyzed.

Al-Dammerchy [65] solved this example using beam-column approach and two dimensional yield criteria neglecting shear and torsion effect. Fig.(6.55-a) shows the plastic hinges sequence at collapse by Al-Dammerchy [65] and the critical load ($P_u = 20.009$ tons).

Fig.(6.55-b and 6.55-c), shows the present study results when neglecting and including shear effect. It can be seen from Fig.(6.55), that there is good agreement between the present work and Al-Dammerchy [65], and the effect of shear force on the sequences of plastic hinges and on the critical load values is small and it can be neglected to simplify the procedure of solution.

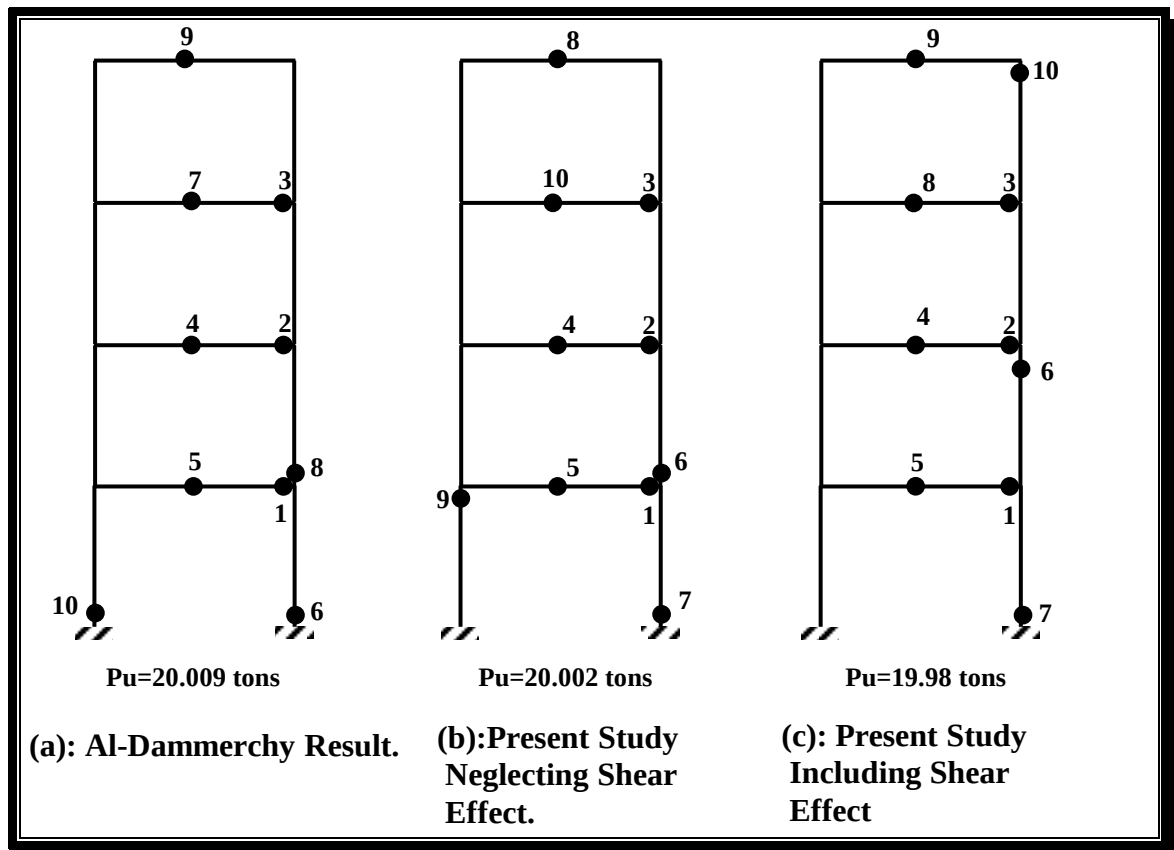


Fig.(6.55): The Sequence of Plastic Hinges For One-Bay and Four-Story Frame (Case Study No.5).

6.2.3.6 Effect of Damping

To study the amount of error caused by including or neglecting the damping effect, the clamped beam (**Fig.(6.5)**) and the 18-member framed dome (**Fig.(6.9)**) have been analyzed dynamically.

Fig.(6.56), shows for the case study No.1 (clamped beam), the relation between vertical displacement at point (B) and time for central concentrated step load equal to (2.8 kN), damping is included in the analysis and it is determined for a model damping ratio (10%). **Fig.(6.57)**, shows for the case study No.2 (18-member framed dome), the relation between vertical displacement at point (1) and time for harmonically concentrated load equal to (34.4 MN). Also, this case study is analyzed in the present work with and without damping and model damping ratio is (5%). It can be seen from **Figures (6.56)** and **(6.57)**, that the difference in response when including or neglecting the damping effect is small and this difference increases when damping ratio is increasing. Finally, the damping term of the equation of motion can be ignored for the small damping ratio (less than 5%), due to its complicated calculations because its small effect on the results of the analysis but not neglected if the damping ratio approximately more the (6%).

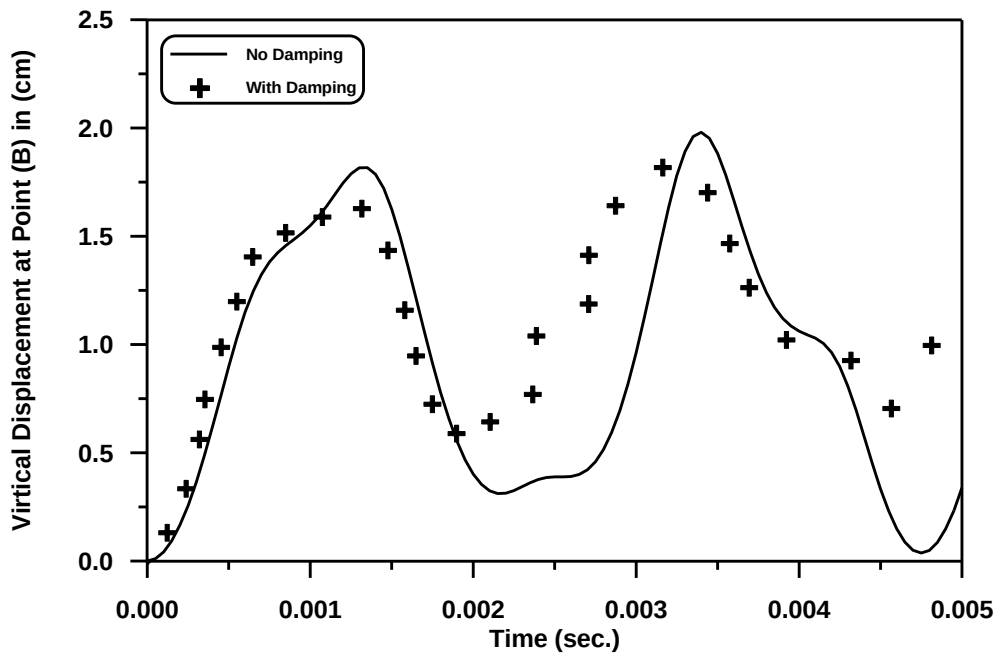


Fig.(6.56): Time-Displacement Curves For Clamped Beam (Case Study No.1).

Examples

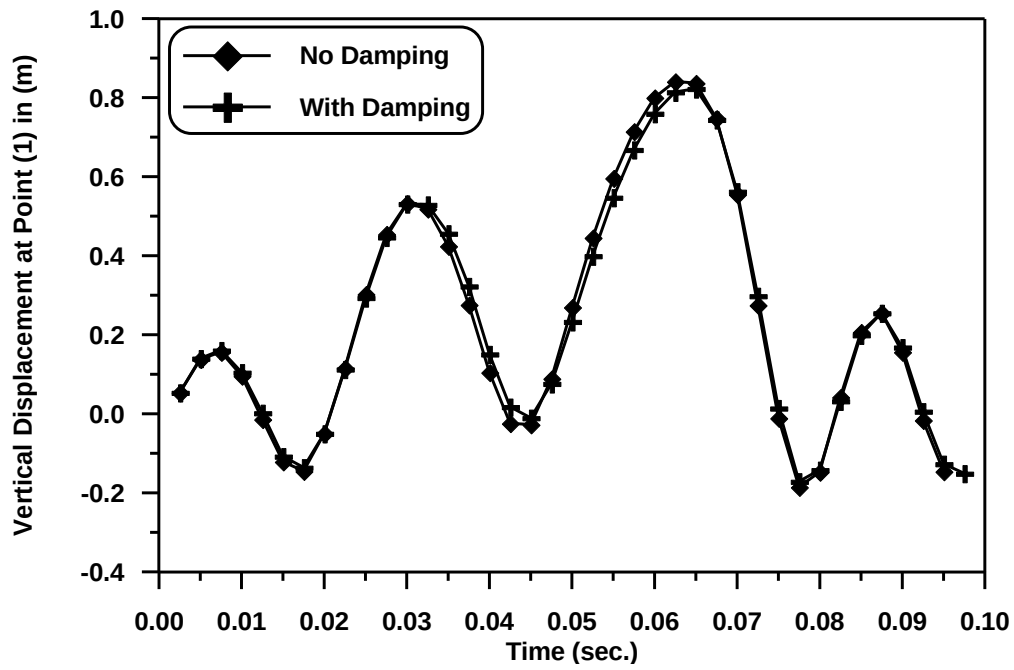


Fig.(6.57): Time-Displacement Curves For 18-Member Framed Dome (Case Study No.2).

Part III (Optimization Examples)

6.3.1 General

As a part of this research, the optimal design of the structures is carried out. This part presents a description of some examples to verify the validity of the computer program (**OSDEPNONSF**) developed in the present study with the optimum design formulation.

6.3.2 Results of Optimal Design Problem

In order to show the validity of the optimization problem formulation, three examples of steel frames were selected as follows:

- 1- Example (O1): Portal Frame.
- 2- Example (O2): Single-Bay Space Frame.
- 3- Example (O3): One-Bay and Three-Story Frame.

6.3.2.1 Example (O1): Portal Frame

The geometry, properties and loading conditions for the portal frame are shown in **Fig.(6.58)**.

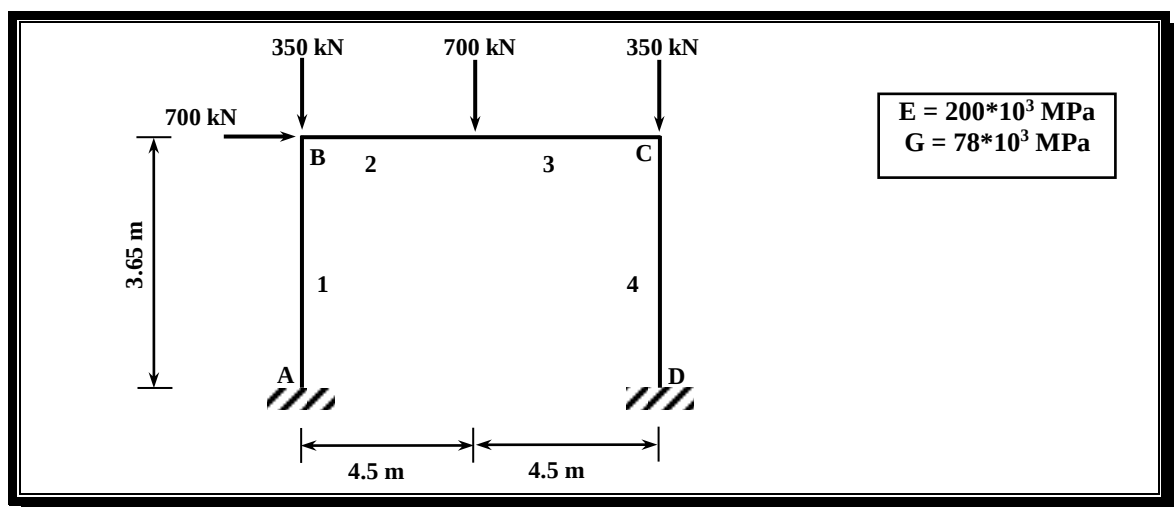


Fig.(6.58): Geometry and Loading Condition of The Portal Frame (O1).

Fig.(6.59), shows the relation between total weight of the structure and number of cycles (number of analysis). It should be noted that the weight of the structure decreases with the increasing number of cycles and this weight decreasing will be large in the earliest cycles. **Table (6.2)** shows the initial and final dimensions of the structure.

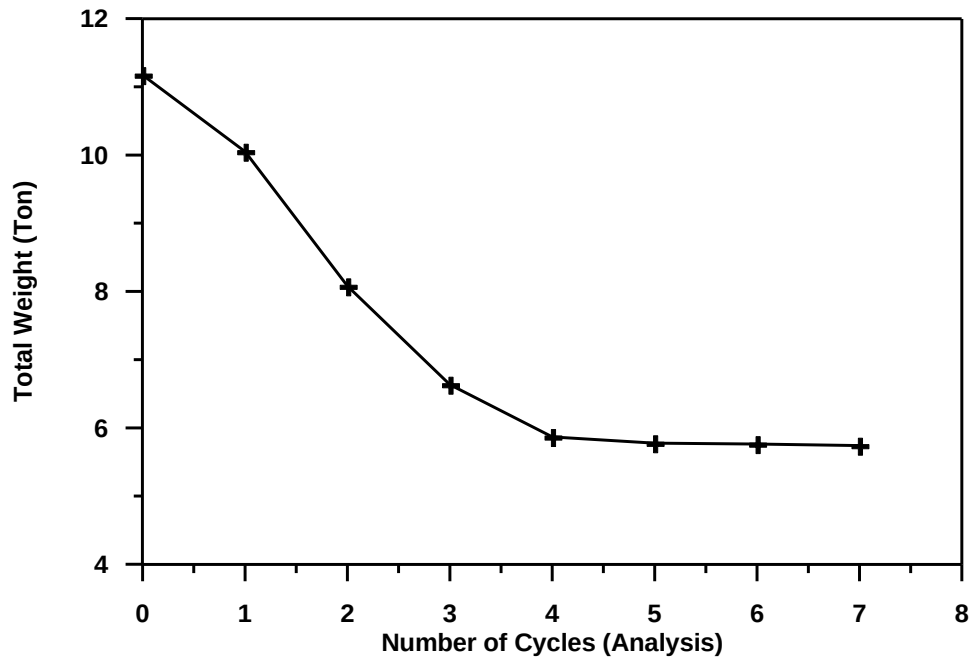


Fig.(6.59): Variation of Total Weight With Number of Analysis For Portal Frame (O1).

Table(6.2): Initial and Optimum Dimensions For Portal Frame (O1).

Member No.	h_w (mm) initial value	t_w (mm) initial value	b_f (mm) initial value	t_f (mm) initial value
1	457	15	533	35
2	406	23	343	20
3	406	23	343	20
4	457	15	533	35
Member No.	h_w (mm) optimum value	t_w (mm) optimum value	b_f (mm) optimum value	t_f (mm) optimum value
1	762	4.6	559	14
2	559	3.5	203	20
3	915	5.6	660	15
4	813	5.0	559	13

6.3.2.2 Example (O2): Single-Bay Space Frame

The geometry, properties and loading conditions for the single bay space frame shown in Fig.(6.60).

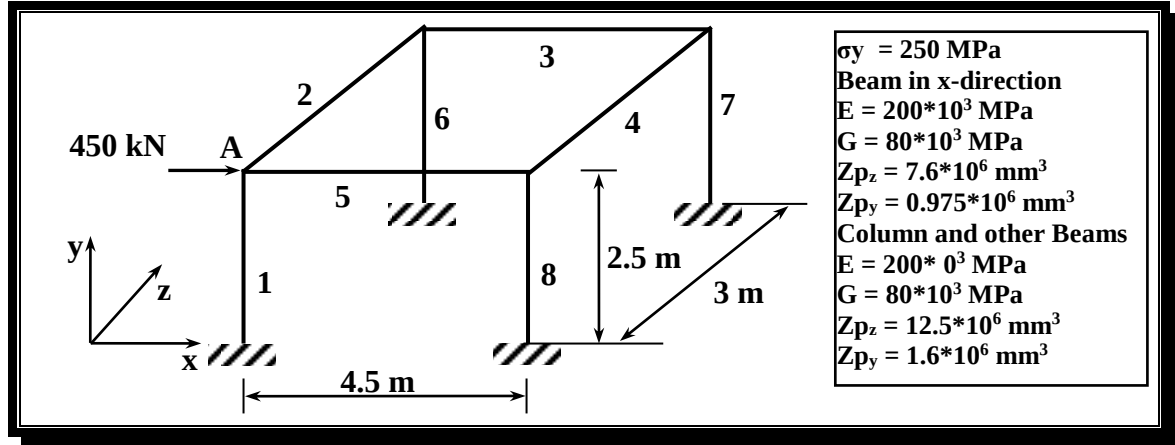


Fig.(6.60): Geometry and Loading Condition of The Single Bay Space Frame (O2).

Fig.(6.61), shows the relation between total weight of the structure and number of cycles for the elastic and elastic-plastic analysis. It may be noted that the weight of the structure in the elastic-plastic analysis is less than the weight in the elastic analysis. Table (6.3) shows the initial and final dimensions of the structure for the elastic and elastic-plastic analysis.

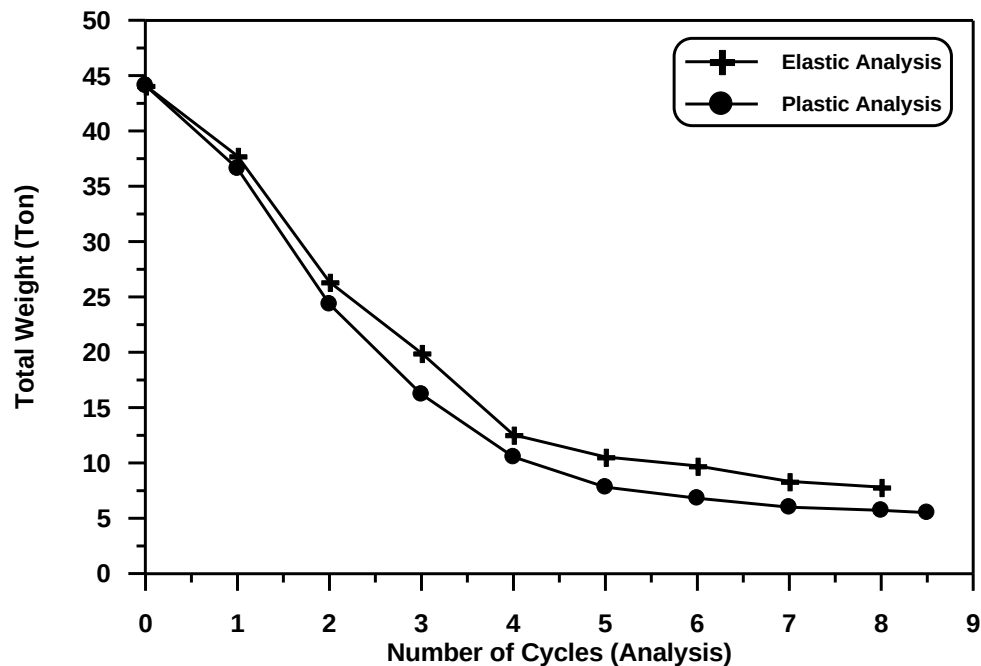


Fig.(6.61): Variation of Total Weight With Number of Analysis For Single Bay Space Frame (O2).

Examples

Table(6.3): Initial and Optimum Dimensions For Single Bay Space Frame (O2).

Member No.	h_w (mm) initial value	t_w (mm) initial value	b_f (mm) initial value	t_f (mm) initial value
1	675	23	368	38
2	675	23	368	38
3	825	19	320	24
4	675	23	368	38
5	825	19	320	24
6	675	23	368	38
7	675	23	368	38
8	675	23	368	38
Member No.	h_w (mm) elastic optimum value	t_w (mm) elastic optimum value	b_f (mm) elastic optimum value	t_f (mm) elastic optimum value
1	584	5	280	8
2	544	5	299	9
3	641	4	268	8
4	538	4	292	7
5	617	4	266	6.6
6	605	4	292	8.4
7	599	4	285	6.25
8	564	4	274	8
Member No.	h_w (mm) elastic-plastic optimum value	t_w (mm) elastic-plastic optimum value	b_f (mm) elastic-plastic optimum value	t_f (mm) elastic-plastic optimum value
1	541	4	285	7.25
2	503	4	285	5.6
3	610	3.5	264	8.6
4	541	3.5	285	6.8
5	570	4.25	254	6.6
6	579	4	286	7.8
7	584	4	283	6.25
8	554	3.5	269	6.8

Examples

6.3.2.3 Example (O3): One-Bay and Three-Story Frame

The geometry, properties and loading conditions for the one bay and three story frame are shown in Fig.(6.62). Fig.(6.63), shows the relation between total weight of the structure and number of cycles. Also, it is be noted that the weight of the structure decreases with the increasing of number of cycles and that the weight of the structure in the elastic-plastic analysis is less than the weight in the elastic analysis. Table (6.4) shows the initial and final dimensions of the structure.

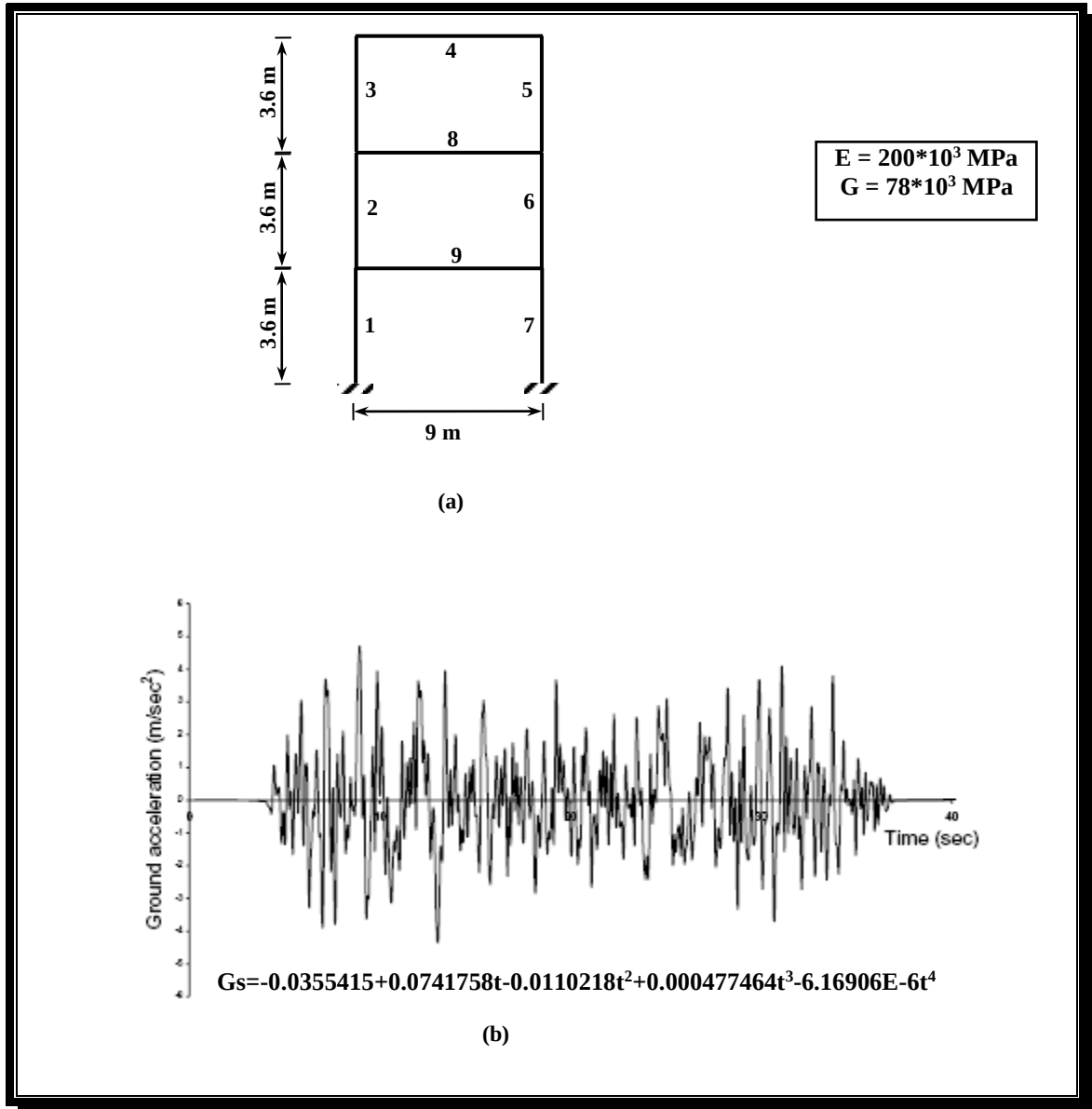


Fig.(6.62): Geometry and Loading Condition of The One-Bay and Three-Story Frame (O3), (a) Side View and (b) Horizontal Ground Acceleration.

Examples

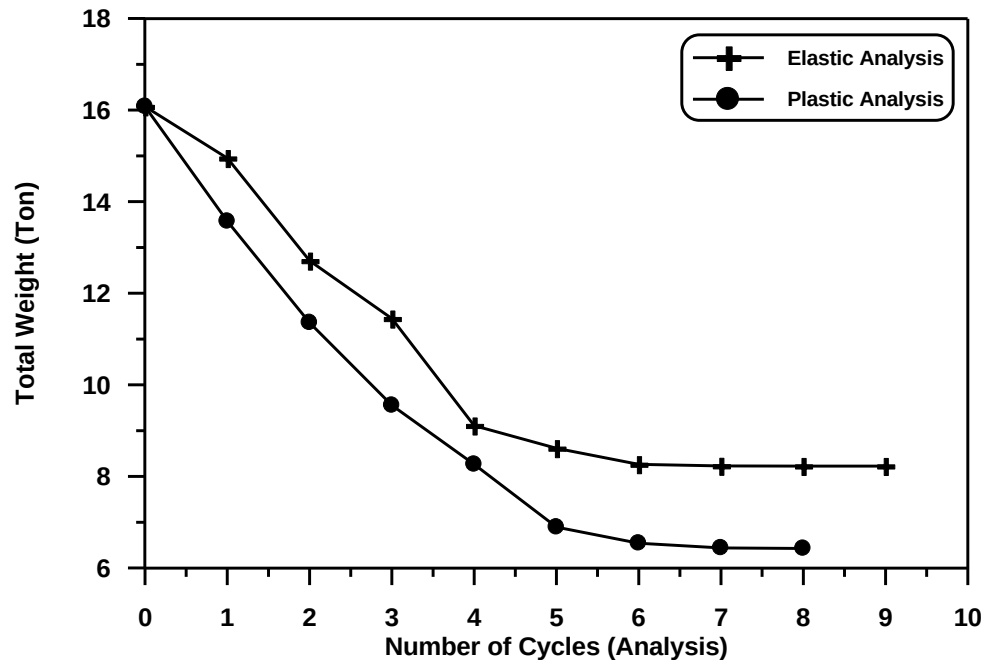


Fig.(6.63): Variation of Total Weight With Number of Analysis For One Bay and Three Story Frame (O3).

Examples

Table(6.4): Initial and Optimum Dimensions For Example (O3).

Member No.	hw (mm) initial value	tw (mm) initial value	bf (mm) initial value	tf (mm) initial value
1	256	9	292	12.5
2	225	8	254	10
3	225	8	254	10
4	360	7.5	147	7.5
5	225	8	254	10
6	225	8	254	10
7	256	9	292	12.5
8	360	7.5	147	7.5
9	360	7.5	147	7.5
Member No.	hw (mm) elastic optimum value	tw (mm) elastic optimum value	bf (mm) elastic optimum value	tf (mm) elastic optimum value
1	279	4.5	173	5
2	286	5	216	5
3	290.5	5	188	5
4	298.5	5	183	6
5	288	5	228	5
6	279	5	260	5
7	271	5	231	5
8	275	4.5	244	5
9	284.5	5	223.5	5
Member No.	hw (mm) elastic-plastic optimum value	tw (mm) elastic-plastic optimum value	bf (mm) elastic-plastic optimum value	tf (mm) elastic-plastic optimum value
1	259	4	159	4
2	268	4.5	172	4.5
3	260	4.5	158	4.5
4	265	5	155	4
5	254	4	185	4.5
6	254	4	186	4.5
7	254	4	183	4.5
8	246	4	174.5	4
9	254	4.5	166	4.5



Conclusions And Recommendations

7.1 Conclusions

Based on the results obtained in the present study, several conclusions may be drawn. These may be summarized as follows:

1. This investigation shows that the large displacement elastic and elastic-plastic behavior of space steel frames with prismatic members subjected to static and dynamic loads and having non-linear connections can be accurately predicted by using the beam-column approach of analysis.
2. A comparison between the beam-column approach and the finite element approach reveals approximately similar results but the latter requires a larger number of elements than the former and this is a point in favor of the beam-column approach despite the fact that the derivations of the tangent stiffness matrix of the beam-column approach is more complicated than that of the finite element approach.

3. The advanced new modified tangent stiffness matrix which takes into account the three types of non-linearities at the same time (i.e. geometric, material and connection) is efficient in giving accurate results of analysis of different types of frames.
4. It is shown that a polynomial modeling of the non-linear moment-rotation curve of steel connection gives excellent results with the exact non-linear moment-rotation curve in studying the behavior of space steel frames with a difference not more than (1%).
5. The suggest interaction diagram including bending, torsion, shear and axial forces gives accurate results in the plastic analysis of the space steel frames and they are close to the experimental results with difference not more than (5%).
6. The optimal design of steel frames based on inelastic analysis is economical than that based on elastic analysis. The difference reaches (45%) for the considered cases.
7. The effect of the flexible connections must be considered in the analysis because it has significant effect on the behavior of the steel structures. It is shown that the moment-rotation curve of the connection in the load direction has a pronounced effect on the behavior of a flexible connection in space frames. On the other hand, one can neglect the moment-rotation curve in the other directions.
8. The time step size has a significant effect on the dynamic response, taking large time step size cause the analysis to drift from the right solution and when the time step size is reduced, the error decreases until a specified time step beyond which any decrease will also give computational error in the response.
9. The effect of the earthquake excitations on the space frames becomes greater with increasing the number of stories (for the same earthquake excitation and the same frame base dimensions). The maximum deflection is increased by (25%) when the stories increase from one to three.

10. The damping term of the equation of motion can be ignored for the small damping ratio (less than 5%), due to its complicated calculations and because of its small effect on the results of the analysis but it cannot be neglected if the damping ratio is approximately more the (6%).

11. The new approach based on using a fictitious connection to simulate plastic hinge formation is found quite effective since identical results have been obtained with the conventional methods for such simulations.

12. The effect of shear and torsion on the sequence of plastic hinges and on the critical load values is small and it can be neglected to simplify the solution.

7.2 Recommendations

The following recommendations may be considered as an extension of the present study:

1. This subject requires to be supplemented by experimental results to be obtained from tests on a model or full scale space steel frames under equivalent earthquake excitation loading having different types of steel connections and supports conditions.
2. The effects of axial and shear forces on the steel connections may all be included in the analysis to find the more realistic behavior of steel frames.
3. The modified tangent stiffness matrix presented in this study may be modified to include more effects such as the end rigid plates, the shear deformations, the variation of the moment of inertia along the member in structures consisting of non-prismatic members and the variation in the axial force in the member.
4. Extending the present study to consider the post-buckling analysis.
5. It will be useful to include the non-linearity due to flexible foundations.
6. Further applications may be done on several types of space structures such as high-rise steel buildings and transmission towers.

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