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مقدم

ة إلى كلية الهندسة في جامعة بابل
كجزء من متطلبات نيل درجة ماجستير علوم في الهندسة المدنية

من قبل

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بكلوريوس هندسة مدنية

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**LARGE DEFORMATION TRANSIENT
RESPONSE OF
METALLIC PLANE FRAME BY THE
METHOD OF CHARACTERISTICS**

A Thesis

SUBMITTED TO THE COLLEGE OF ENGINEERING OF
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الخلاصة

الإستجابة الإنتقالية والديناميكية اللا خطية للهياكل المعدنية المستوية المؤلف من العناصر المستقيمة الموشورية أو اللاموشورية (Prismatic or non-prismatic) تم بحثها من خلال دراسة الأمواج المحورية و الأنثائية المنتقلة عبر هذه التراكيب, و ذلك بأستخدام طريقة الخواص (Method of characteristics). أعتد البحث على أساس تقريب إنتقال الموجة في تركيب أحادي البعد. نظرية تيموشينكو (Timoshenko's theory), و المتضمنة تأثير التشوه القصي المستعرض و عزم القصور الدوراني, قد تم اعتمادها في التحليل. أثناء الحل ظهرت الحاجة الى إستخدام الإستكمال العددي (Interpolation) نتيجة للاختلاف بين سرع الموجات القصية و الأنثائية حيث تم إستخدام الإستكمال العددي من خلال المسافة (X-wise).

عدة أمثلة تم بحثها و ذلك بأستخدام البرنامج (LDDAPF) و الذي تم تطويره ضمن هذه الدراسة و تمت مقارنة النتائج مع البيانات التجريبية و طريقة العناصر المحددة (Finite element method) لباحثين آخرين. و أجريت أيضاً دراسة للعوامل المؤثرة على السلوك الديناميكي للهياكل الحديدية لأختبار تأثير العديد من العوامل.

من خلال النتائج التي تم الحصول عليها وجد إن الشكل الهندسي, وخواص مادة الذراع و وجود المفاصل تمتلك تأثيرات هامة على استجابة الهياكل الحديدية .

لقد وجد إن التداخل بين الموجات القصية و الانحنائية يتسبب في إنتقال الموجات الأنثائية خلال الأذرع الموشورية و اللاموشورية بشكل متشتت ناتج عن الاختلاف في سُرْع انتقال تلك الموجات. كما أن تحليل التشوه المحدود (Small deformation analysis) لا يعطي نتائج مقنعة عند ظروف متوقعة عالية التحميل و ذلك لأنها تهمل تأثير لا خطية الشكل الهندسي (geometrical non-linearity) .

Abstract

The large deformation transient and dynamic response of steel plane frames with prismatic or non-prismatic straight members are being investigated by modeling axial and flexural wave's transmission along such structures, using the *Method of the characteristics*. The analysis is based on the wave propagation in one-dimensional structural element approach. Timoshenko's beam theory which includes transverse shear deformation and rotary inertia effects is adopted in the analysis. Interpolation is required due to the difference in the speeds of bending and shear waves. *X*-wise interpolation is used. Only geometrical non-linearity is considered in this study and the material is assumed to be linearly elastic.

Several examples have been investigated by using computer program **(LDDAPF)** which is developed in the present study and the results are compared with experimental data and the finite element method that were obtained by other researches. Also, parametric studies have been carried out to test the influence of several parameters on the dynamic behavior of plane frames.

From the results obtained, it is found that the geometrical shape, and material's properties of the member, and the existence of boundaries such as joints as well as the load intensity have considerable effects on the response of such structures.

It was found that the interaction between shear and bending waves causes the flexural waves to propagate in prismatic and non-prismatic members in a dispersive manner which is caused by the propagation of waves with different velocities. Also, the small deformation analysis does not give a reasonable result when high loading conditions are expected because it neglects the effects of the geometrical non-linearity.

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"In the name of Allah, Most Gracious, Most Merciful"

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Finally, I would like to thank my friends for their support and help accomplishing this study, especially, Mr. Yahya K.Alkhafagi and Nawras H. Mostafa.

Luay A.H. Abu Alarab, 2007

APPENDIX A

MATERIAL PROPERTY

RELATIONSHIPS

The material property relationships for axial and flexural waves in a Cartesian coordinates are derived in this appendix. It is assumed that the material is isotropic with bilinear stress-strain relation.

A.1 AXIAL

Consider the element depicted in Figure (A.1), subjected to normal stresses in the x-direction and having a bilinear stress-strain relation. The strain in this case is equal to:

$$\varepsilon = \frac{\partial u}{\partial x} \quad \dots(A.1)$$

And the time derivative of this equation will give:

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial U}{\partial x} \quad \dots(A.2)$$

Form Figure (A.1), it can get:

$$\frac{\partial \varepsilon}{\partial t} = \frac{d\varepsilon}{d\sigma} \frac{\partial \sigma}{\partial t} \quad \dots(A.3)$$

Equations (A.2) & (A.3) can be combined with each other and reduce to the following form:

$$\frac{\partial \sigma}{\partial t} = \frac{d\sigma}{d\varepsilon} \frac{\partial U}{\partial x} \quad \dots(A.4)$$

Where $d\sigma/d\varepsilon$ denotes the tangent of σ - ε curve shown in Figure (A.1). Thus, for elastic wave propagation, equation (A.4) becomes:

$$\frac{\partial \sigma}{\partial t} = E \frac{\partial U}{\partial x} \quad \dots (A.5)$$

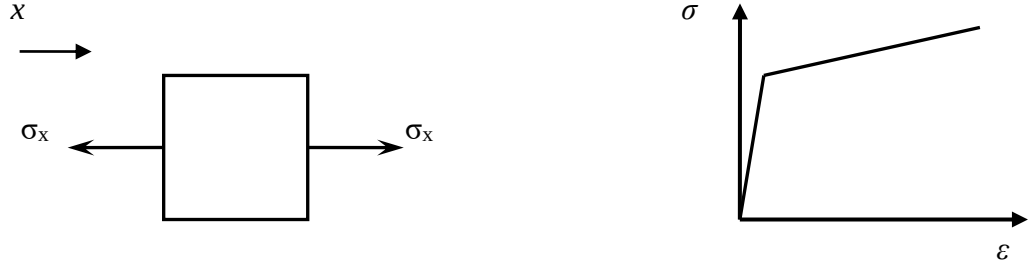


Figure (A.1): Element Under Normal Stress in Cartesian Coordinates

A.2 SHEAR

Consider the element depicted in Figure (A.2), subjected to shear stresses and having a bilinear stress-strain relation. The shear strain in this case is equal to:

$$\gamma = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad \dots (A.6)$$

and the time derivative of this relation is equal to:

$$\frac{\partial \gamma}{\partial t} = \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \quad \dots (A.7)$$

From Figure (A.2), it can get:

$$\frac{\partial \gamma}{\partial t} = \frac{d\gamma}{d\tau} \frac{\partial \tau}{\partial t} \quad \dots (A.8)$$

Equations (A.8) & (A.9) can be reduced to the following form:

$$\frac{\partial \tau}{\partial t} = \frac{d\tau}{d\gamma} \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) \quad \dots (A.9)$$

Since $\psi = -\frac{\partial U}{\partial y}$, equation (A.10) can be simplified to:

$$\frac{\partial \tau}{\partial t} = \frac{d\tau}{d\gamma} \left(\frac{\partial V}{\partial x} - \psi \right) \quad \dots(A.10)$$

Where $d\tau/d\gamma$ denotes the tangent of τ - γ curve shown in Figure (A.2). Thus, for elastic wave propagation, equation (A.11) becomes:

$$\frac{\partial Q}{\partial t} = K G A \left(\frac{\partial V}{\partial x} - \psi \right) \quad \dots(A.11)$$

Where Q is related to shear stress τ by:

$$Q = \int \tau \, dA$$

$$Q = \tau_o K A \quad \dots(A.12)$$

Where τ_o is the average shear stress and K is the shear correction factor which depends on the cross-sectional shape.

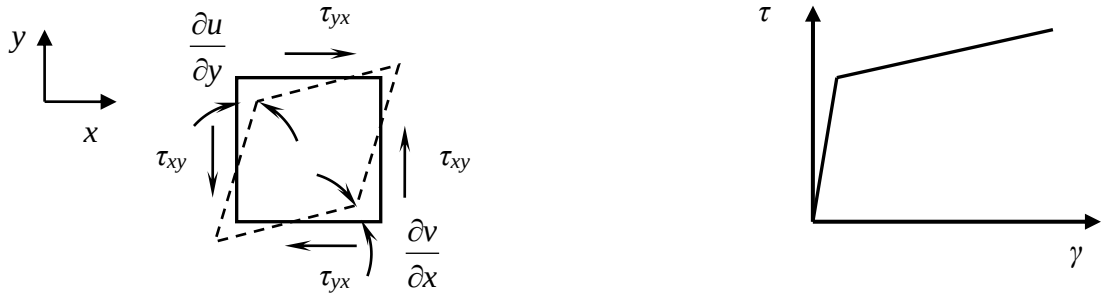


Figure (A.2): Element Under Shear Stress in Cartesian Coordinates

A.3 BENDING MOMENT

Consider the element depicted in Figure (A.3), subjected to bending moment and having a bilinear moment-curvature relation. The curvature in this case is equal to:

$$k = \frac{1}{R} = \frac{\partial \Psi}{\partial S} \quad \dots(A.13)$$

and the time derivative of this equation will give:

$$\frac{\partial k}{\partial t} = \frac{\partial \psi}{\partial S} \quad \dots(A.14)$$

From Figure (A.3), it can get:

$$\frac{\partial k}{\partial t} = \frac{dk}{dM} \frac{\partial M}{\partial t} \quad \dots(A.15)$$

Equations (A.16) & (A.17) can be reduced to the following form:

$$\frac{\partial M}{\partial t} = \frac{dM}{dk} \frac{\partial \psi}{\partial S} \quad \dots(A.16)$$

Where dM/dk denotes the tangent of M - k curve shown in Figure (A.3). Thus, for elastic wave propagation, equation (A.18) becomes:

$$\frac{\partial M}{\partial t} = EI \frac{\partial \psi}{\partial S} \quad \dots(A.17)$$

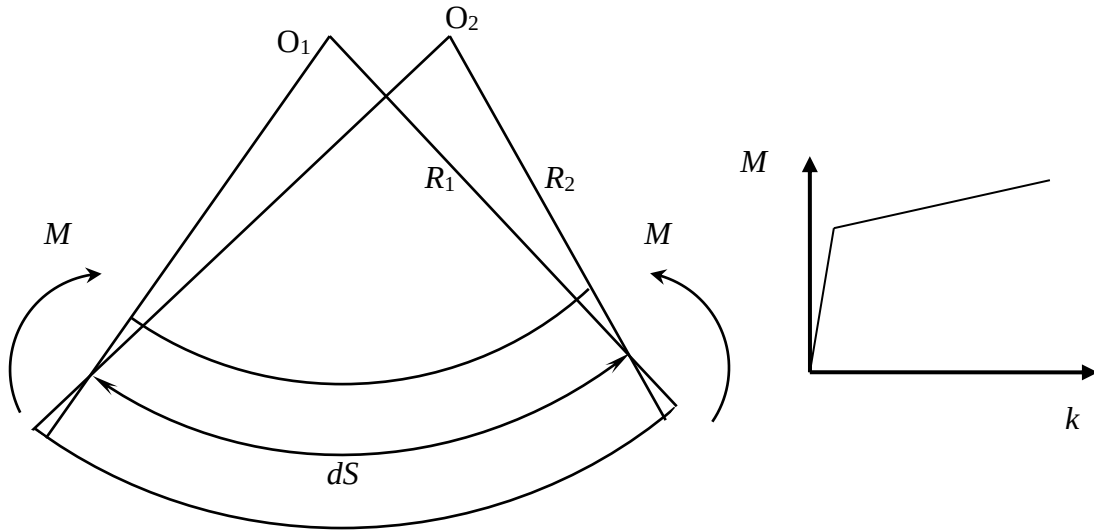


Figure (A.3): Element Under Bending Moment in Cartesian Coordinates.

APPENDIX B

SHEAR CORRECTION FACTOR

In Timshenko's beam theory, the transverse shearing stress is assumed constant. Actually the transverse shear stress is zero at top and bottom sides of the section and maximum at neutral axis (usually parabola).

In order that the constant shearing stress distribution gives the same shearing deformation of the parabolic shearing stress distribution, the concept of equal strain energy in both distributions is used. The shearing strain energy for an element ($dx \, dy \, dz$), shown in Figure (B.1), under shearing stress is:

$$\Omega = \int_{Vol} \frac{1}{2} \tau \, \gamma \, d(Vol) = \int_{Vol} \frac{\tau^2}{2G} d(Vol) \quad \dots(B.1)$$

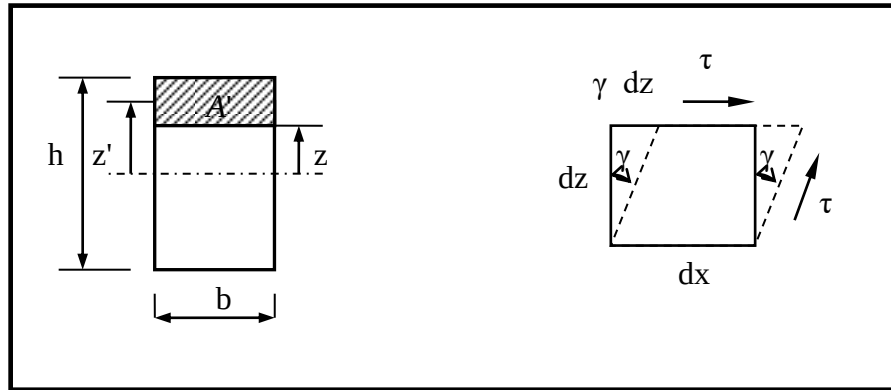


Figure (B.1): Beam element undergoing shear deformation.

For a constant shearing stress distribution, the strain energy is equal to:

$$\Omega_1 = \int_{Vol} \frac{\tau^2}{2KG} d(Vol.) \quad \dots(B.2)$$

where K is the shear correction factor.

Now by assuming a piece of beam of unit length with rectangular cross section, equation (B.2) will reduce to:

$$\Omega_1 = \frac{Q^2}{2KGA^2} \text{ (A.1)} = \frac{Q^2}{2KGbh} \quad \dots(B.3)$$

For the parabolic shearing stress, the following relations are used:

$$\tau = \frac{QA'z'}{Ib}$$

$$A' = b \left(\frac{h}{2} - z \right)$$

$$z' = \frac{1}{2} \left(\frac{h}{2} + z \right)$$

$$\longrightarrow \tau = \frac{6Q \left(\frac{h^2}{4} - z^2 \right)}{bh^3} \quad \dots(B.4)$$

Then

$$\begin{aligned} \Omega_2 &= \int_{Vol} \frac{\tau^2}{2G} d(Vol) = \frac{36Q^2}{2Gb^2h^2} \left(\frac{h^2}{4} - z^2 \right) (1.b.dz) \\ &= \frac{18Q^2}{bh^6G} \left\{ 2 * \int_0^{h/2} \left(\frac{h^2}{4} - z^2 \right) dz \right\} \\ &= \frac{3}{5} \left(\frac{Q^2}{bhG} \right) \quad \dots(B.5) \end{aligned}$$

Put $\Omega_2 = \Omega_1$

$$\frac{3}{5} \left(\frac{Q^2}{bhG} \right) = \frac{Q^2}{2KGbh} \quad \longrightarrow \quad K=0.83333$$

the same procedure is made for the *circular* cross section to get

$$K=0.86666$$

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CERTIFICATION

We certify that this thesis entitled "**Large Deformation Transient Response Of Metallic Plane Frame by the Method of Characteristics**" was prepared by **Mr. luay Abd Al-Hussein Abboud** under our supervision at the University of Babylon in partial fulfillment of the requirements for the degree of **Master of Science in Civil Engineering (Structures)**.

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EXAMINING COMMITTEE'S CERTIFICATION

We certify that we have read this thesis titled "***Large Deformation Transient Response of Metallic Plane Frame by the Method of Characteristics***" and as an examining committee, examined the student, "***Luay Abd Al-Hussein Abboud***" in its contents and that in our opinion it meets standard of a thesis for the degree of Master of Science in Civil Engineering.

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DEDICATION

TO MY COUNTRY

TO MY FAMILY

WITH LOVE, ADMIRATION

AND GRATITUDE



LIST OF SYMBOLS

<i>Symbol</i>	<i>Definition</i>	<i>Units</i>
<i>A</i>	Cross-sectional area at node	m ²
<i>a</i>	Average cross-sectional area of the element	m ²
<i>b</i>	Width of the element	m
<i>C</i>	Elastic axial and moment wave speeds	m/s
<i>C_s</i>	Elastic shear wave speed	m/s
[<i>C_e</i>]	Characteristics coefficient matrix in local coordinates (characteristics member matrix).	-
[<i>C_J</i>]	Characteristics coefficient matrix in global coordinates (characteristics joint matrix).	-
<i>E</i>	Modulus of elasticity	N/m ²
<i>F</i>	Axial force	N
{<i>F</i>}	Internal forces in global coordinates at new time.	N
{<i>F'</i>}	Internal forces in local coordinates at new time.	N
{<i>f</i>}	Forces coefficient in global coordinates at old time.	-
{<i>f'</i>}	Forces coefficient in local coordinates at old time.	-
<i>G</i>	Elastic modulus of rigidity	N/m ²
<i>h</i>	Depth of the element	m
<i>I</i>	Moment of inertia at node	m ⁴
<i>I_s</i>	Average moment of inertia of the element	m ⁴
<i>K</i>	Shear correction factor	-
<i>k</i>	Bending curvature	m ⁻¹
<i>L</i>	Length of the member	m
<i>M</i>	Bending moment	N.m
{<i>P_(t)</i>}	External loads.	kN
<i>Q</i>	Shear force.	N

<i>Symbol</i>	<i>Definition</i>	<i>Units</i>
<i>r</i>	Radius of gyration.	mm
<i>s</i>	Sign plus or minus	-
<i>t</i>	Time.	sec
[<i>T</i>]	Transformation matrix.	-
[<i>T</i>]^{<i>T</i>}	Transport to the Transformation matrix.	-
<i>U</i>	Axial velocity	m/s
<i>u</i>	Axial displacement	m
<i>ü</i>	Axial acceleration.	m/s ²
<i>V</i>	Lateral velocity	m/s
{<i>V</i>}	Velocities in global coordinates at new time.	m/s
{<i>V</i>'}	Velocities in local coordinates at new time.	m/s
<i>v</i>	Lateral displacement	m
<i>Vol.</i>	Volume	m ³
<i>x,y</i>	Cartesian axes	m
<i>α</i>	Axis angle	
<i>β</i>	Shape factor	-
<i>γ</i>	Shear strain	-
<i>ΔS</i>	Grid length after deformation	m
<i>Δx</i>	Grid length before deformation	m
<i>Δt</i>	Time incremental	sec
<i>ε</i>	Axial strain	m/m
<i>ρ</i>	Mass density	kg/m ³
<i>σ</i>	Axial stress	N/m ²
<i>τ</i>	Shear stress	N/m ²
<i>ν</i>	Poission's ratio	-

<i>Symbol</i>	<i>Definition</i>	<i>Units</i>
Ψ	Angular displacement	rad
ψ	Angular velocity	rad/s
ω_n	Natural frequency	rad/s
Ω	Shearing strain energy	N.m

LIST OF ABBREVIATION

<i>symbol</i>	<i>Definition</i>
<i>MOC</i>	Method of characteristics
<i>ms</i>	Millisecond
<i>NPM</i>	Non-prismatic member.
<i>PM</i>	Prismatic member.
μs	Microsecond

CHAPTER ONE

INTRODUCTION

1.1 General

The design of structures follows results of analysis whether static or dynamic. In normal structures, design is based mainly on static analysis and dynamic effects assumed to be small. Currently, it has been a marked interest in the determination of the dynamic characteristics of buildings and their response under dynamic loads. Nowadays, many tall buildings are more slender, less rigid, and hence are sensitive to dynamic effects.

Multi-story buildings are more affected by dynamic loads. Thus, structural safety may not be achieved by considering the effect of a static load only. Dynamic loads must be taken into account. Dynamic effects may be resulted from earthquakes, wind, vibrating machines and impact loads.

A dynamic load is one whose magnitude, direction, or point of application varies with time. The resulting time-varying displacements and stresses contribute to the dynamic response. If the loading is known, the loading is said to be prescribed and the analysis is called a deterministic analysis. If the time history of the loading is not completely known, but it is known in a statistical sense, the loading is said to be random.

A structural dynamic problem differs from the corresponding static problem in two important aspects:

1. Time-varying nature of the loading.
2. Effect of acceleration in dynamic problems (presence of inertia force).
3. Effect of damping.

In static analysis the displacements and stresses depend on the magnitude of the load and structural stiffness, while the dynamic behavior of a structure depends partially on the properties of the material used, and also on the nature of the applied loads. In particular, the type and the rate of change of a load have an influence on the structure behavior. For the simple case of very slowly varying loads, the use of static analysis is adequate. When the load changes more rapidly, inertial effects become significant and states differ remarkably from the static condition. The well known lumped mass and distributed mass method is commonly used for the intermediate rates of change of load that have had greatest attention over the years. The method is extremely powerful and can be used to simulate most types of dynamic behaviour. It is not particularly well suited, however, to the simulation of sudden load changes. For this purpose, wave propagation methods are useful because they enable the passage of discrete waves to be represented in a simple manner.

1.2 Nonlinear Analysis

Interesting physical phenomena occur in structures in the presence of nonlinearities, which cannot be explained by linear models. These phenomena include jumps, saturation, sub harmonic, superharmonic, and combination resonance, self-excited oscillations, modal interactions, and chaos. In reality, no physical system is strictly linear and hence linear models of physical systems have limitations of their own. In general, linear models are applicable only in a very restrictive domain like when the vibration amplitude is very small. Thus, to accurately identify and understand the dynamic behavior of a structural system under general loading conditions, it is essential that nonlinearities present in the system also be modeled and studied.

1.2.1 Types of Nonlinearity

In theory, nonlinearity exists in a system whenever there are products of dependent variables and their derivatives in the equations of motion, boundary conditions, and/or constitutive laws, and whenever there are any sort of discontinuities or jumps in the system. In structural mechanics, nonlinearities can be broadly classified into the following categories:

1. Damping is essentially a nonlinear phenomenon. Linear viscous damping is an idealization. Coulomb friction, aerodynamic drag, hysteretic damping, etc. are examples of nonlinear damping (**Malatkar .P, 2003**)⁽⁵⁰⁾.
2. Geometric nonlinearity exists in systems undergoing large deformations or deflections. This nonlinearity arises from the potential energy of the system. In structures, large deformations usually result in nonlinear strain- and curvature-displacement relations. This type of nonlinearity is present, for example, in the equation governing the large-angle motion of a simple pendulum, in the nonlinear strain-displacement relations due to the mid-plane stretching in strings, and due to the nonlinear curvature in cantilever beams (**Challa, 1992**)⁽²²⁾.
3. Inertia nonlinearity derive from nonlinear terms containing velocities and/or accelerations in the equations of motion. It should be noted that nonlinear damping, which has similar terms, is different from nonlinear inertia. The kinetic energy of the system is the source of inertia nonlinearities. Examples include centripetal and Carioles acceleration terms. It is also present in the equations describing the motion of anelastic pendulum (a mass attached to a spring) and those describing the transverse motion of an in extensional cantilever beam (**Malatkar .P, 2003**)⁽⁵⁰⁾.

4. When the constitutive law relating the stresses and strains is nonlinear, we have the so-called material nonlinearity. Rubber is the classic example. Also, for metals, the nonlinear Ramberg-Osgood material model is used at elevated temperatures (**Challa, 1992**)⁽²²⁾.
5. Nonlinearities can also appear in the boundary conditions. A nonlinear boundary condition exists, for instance, in the case of a pinned-free rod attached to a nonlinear tensional spring at the pinned end (**Malatkar .P, 2003**)⁽⁵⁰⁾.
6. Many other types of nonlinearities exist: like the ones in systems with impacts, with backlash or play in their joints, etc (**Malatkar .P, 2003**)⁽⁵⁰⁾.

It is interesting to note that the majority of physical systems belong to the class of weakly nonlinear (or quasi-linear) system. For certain phenomena, these systems exhibit a behavior only slightly different from that of their linear counterpart. In addition, they also exhibit phenomena which do not exist in the linear domain. Therefore, for weakly nonlinear structures, the usual starting point is still the identification of the linear natural frequencies and mode shapes. Then, in the analysis, the dynamic response is usually described in terms of its linear natural frequencies and mode shapes. The effect of the small nonlinearities is seen in the equations governing the amplitude and phase of the structure response (**Malatkar .P, 2003**)⁽⁵⁰⁾. Only geometrical non-linearity is considered in this study and the material is assumed to be linearly elastic.

1.3

Wave Propagation in One-Dimensional Structural Elements

The behavior of structural elements under impact or impulsive loads is a subject of great interest in the structural dynamic analysis. When forces are applied to an elastic medium over a very short period, the response should be

considered in terms of wave propagation theory. The study of transient waves has important implications and applications for structures subjected to such loads. The revival of interest in the elastic wave propagation during the last four decades has been possible because of the rapid development of computing facilities and experimental equipments **AL-Haidar (2005)⁽⁵⁾**.

Waves in one-dimensional structural members can be classified into three groups namely; axial, torsional and flexural waves. The problems of flexural wave propagation have not been so extensively treated as have the problems of axial wave **Timoshenko,et al. (1974)⁽⁶⁷⁾**. This is due to the complexities involved in the propagation of flexural waves and their dispersive characters. The Euler-Bernoulli's beam theory is inadequate for studying the transient bending waves because it leads to the physically impossible conclusion that disturbances are propagated instantaneously; the theory also neglects the effects of shear deformation and rotatory inertia.

Timoshenko's beam theory is the only approximate theory that contains the essential features of exact theory in simplified form (**Achenbach, 1973⁽¹⁾**). This theory leads to more accurate solutions than the Euler-Bernoulli's beam theory because the effects of shear deformation and rotatory inertia are included in the governing equations.

1.4 Reflection and Transmission of Waves

When a propagating disturbance strikes the interface between two media of different material properties, a part of the disturbance is reflected and part is transmitted across the interface **AL-Haidar (2005)⁽⁵⁾**. In the special case that the second medium cannot carry mechanical waves, i.e., if it is a vacuum, the incident wave is completely reflected.

Now suppose the incident stress wave (Figure 1.1) is denoted by,

$$(\sigma_x)_i = f\left(t - \frac{x}{C}\right) \quad \dots(1.1)$$

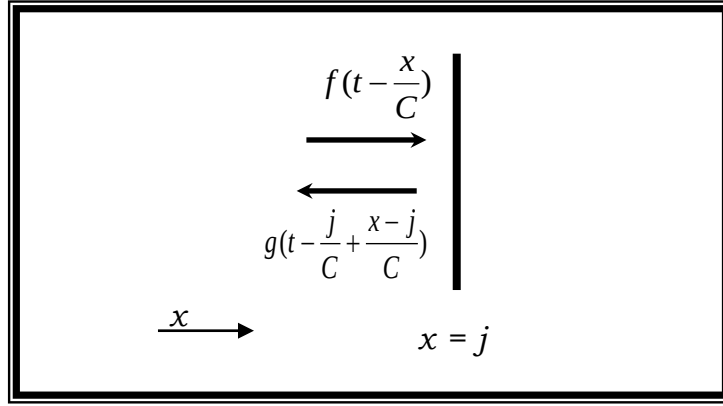


Figure (1.1): Incident and reflected

Where f is a function of time (t), distance (x), and wave's velocity (C). Since the reflected stress wave propagates in the negative x -direction, it can be represented by:

$$(\sigma_x)_r = g\left(t + \frac{x}{C}\right) \quad \dots (1.2)$$

At the free boundary $x=j$, the stress σ_x vanishes, i.e.:

$$\sigma_x = (\sigma_x)_i + (\sigma_x)_r = 0 \quad \text{at } x=j$$

For $t < (j/C)$ this equation implies $g\left(t + \frac{x}{C}\right) = 0$. For $t \geq (j/C)$, it is found:

$$g\left(t + \frac{j}{C}\right) = -f\left(t - \frac{j}{C}\right),$$

Now by setting $\lambda = t + (j/C)$, it can get:

$$g(\lambda) = -f\left(\lambda - 2\frac{j}{C}\right) \quad \dots (1.3)$$

and the reflected stress wave may thus be represented by ;

$$(\sigma_x)_r = -f\left(t - \frac{j}{C} + \frac{x-j}{C}\right) \quad \dots (1.4)$$

Equation (1.4) shows that the reflected stress pulse has the same shape as the incident stress pulse, but the sign of the stress changes upon reflection. Thus, at a free surface a tensile pulse is reflected as a compression pulse, and vice versa.

Other types of reflection may be caused by the non-homogeneity of material and sectional properties along the path of the propagating waves. In these cases, a part of wave will be reflected and other part will be transmitted. The reflection of waves may be make the shape of the pulse (wave length) to change as the wave propagates through the medium, thus, the system is said to be *dispersive*. Dispersion is an important phenomenon because it governs the change of shape of a pulse as it propagates through a dispersive medium.

1.5 Solution Techniques

The solution of the partial differential equations that govern the dynamic behavior of vibrating problems can be solved by one of the following techniques:

1.5.1 Transformation techniques

The problems of wave propagation in beams can be solved by using **Laplace** transforms and, to a lesser. extent, **Fourier** transforms.

The main difficulty is in connection with the treatment of end conditions. **Miklowitz(1953)⁽⁵³⁾** obtained the Laplace transforms for certain types of end conditions and pointed out at the same time that it was much more difficult to obtain the transform for other types of end conditions.

The use of transform methods is almost restricted to the study of wave in uniform semi-infinite bars where no discontinuities and wave reflections are involved **AL-Mousawi, et al.(1988)**⁽⁸⁾. However, transform methods are useful in producing exact solution for certain problems, which can be used for comparison in obtaining of other approximate methods.

1.5.2 Finite difference techniques

Finite difference techniques are well established and are successfully used in solving wave propagation problems. The solution method was discussed by **Courant, and Hilbert (1937)**⁽²⁸⁾. There exists several forms of finite difference approximation and common approximate techniques are used in these methods.

The finite difference techniques are mostly used to solve problems in which the response is dominated by axisymmetric motion. However, its application to the study of flexural wave propagation problems is less feasible. Therefore, finite difference methods are used for solving axial axisymmetric wave propagation problems where they are efficient.

1.5.3 Finite element techniques

Finite element techniques have been used widely for the frequency analysis of vibrating beams and for predicting mode shapes and natural frequencies.

One of the disadvantages of finite element model is the computed stresses which show severe oscillations of so-called numerical dispersion (**Costantino, 1967**)⁽²⁷⁾. This makes the finite element methods less attractive for flexural wave propagation problems since it becomes difficult to decide whether the oscillations are due to the dispersive nature of the propagated wave or the dispersion is due to the numerical inaccuracies.

The main advantage of finite element methods is their ability to treat geometrically complex structures. A wide range of elliptic and parabolic governing equations has been successfully solved by using finite element methods. However, finite element methods have not yet proven their suitability for the solution of transient flexural wave propagation problems which are governed by hyperbolic partial differential equation systems in conjunction with mixed initial and boundary conditions **AL-Mousawi, et al. (1988)⁽⁸⁾**.

1.5.4 The method of characteristics

Wave propagation problems are mathematically classified as mixed initial boundary value problems and their governing equations are mostly partial differential equations of hyperbolic type. For hyperbolic systems involving two independent variables, the method of characteristics (**MOC**) is undoubtedly the most convenient, most effective and most accurate method of solution **AL-Mousawi, et al. (1988)⁽⁸⁾**. Other numerical methods such as the finite element and finite difference methods are basically more suitable for hyperbolic equations in higher dimensions where the (**MOC**) might be less satisfactory.

The main advantage of the method of characteristics is that discontinuities in the initial values may propagate along the characteristics. Furthermore, the method of characteristics has the advantage that it follows the physical waves fronts as they are propagated along the beam. Thus, the method of characteristics is used in this study to overcome the disadvantages of other techniques.

1.6 Objective and Scope

The main objective of this study is to use the method of characteristics to deal with large deformation dynamic analysis of the plane frame with prismatic and non-prismatic straight members, taking into account that these structures are subjected to transient loading. Where this method needs relatively small computer storage with good solution reliability. Thus, the designers can easily overcome the difficulties in the analysis when their designs are subjected to impact and earthquakes loadings. The effect of geometrical non-linearity was considered.

The validity of this method is satisfied by comparing the predicted dynamic response with those obtained by other methods.

A parametric study is presented in order to determine the effect of several influencing parameters on the behaviour of vibrating structures.

1.7 Thesis Layout

This thesis is organized in six chapters:

Chapter one: Introduces and explains briefly the problem within hand, the aim of the study and the subjects included in other chapters.

Chapter two: Contains a brief review of previous studies on the subject under consideration.

Chapter three: Illustrates the application of the method of characteristics to the simulation of axial and flexural vibration in the plane frame with prismatic and non-prismatic straight members under dynamic loads.

Chapter four: Describes the flow-chart of the program (**LDDAPF**) which has been developed in this study by using **Fortran-77** language with several applications.

Chapter five: Includes a parametric study that was done on different variables that affect the dynamic response of structures.

Chapter six: Gives a summary of the conclusions that can be drawn from this study and the suggestions for future related works.

CHAPTER TWO

REVIEW OF LITERATURE

2.1 General

During the past five decades, the propagation of stress waves in materials has received a considerable attention in both theoretical and experimental studies. The history of work in this area is long and well documented by many research workers and has been considerably improved to realistic results.

The present review covers two aspects. The first includes the developments in the method of characteristics, while the second aspect is concerned with the developments in the nonlinear dynamic analysis of elastic plane frames composed of straight members.

2.2 The Method Of Characteristics

Leonard and Budiansky (1954)⁽⁴⁸⁾ utilized the method of characteristics to analyze traveling waves in beams subjected to lateral loading, governed by the Timoshenko's beam theory, but for the physically rather unrealistic assumption where the two wave speeds are equal, which simplified the numerical work involved. This simplification made the waves to propagate in more dispersive characteristics but in stable characters, when the numerical results were compared with an experimental data.

Skalak (1957)⁽⁶⁴⁾ obtained the analytical solution of the exact equations for the semi-infinite circular elastic bar. His results were extended by more general

loading conditions by **De Vault and Curtis (1962)**⁽³¹⁾, where the solutions were expressed as a double series of Fourier integrals. The integrals have been accomplished only for a large distance of travel and a long time after application of the load.

Plass (1958)⁽⁵⁷⁾ applied the method of characteristics and Laplace transformation method to simulate the vibration of Timoshenko's beam equation for various types of support conditions, and impact is assumed to be a pulse in the form of a half sine wave. A good agreement was obtained with experiments.

Bertholf (1967)⁽¹⁸⁾ presented the finite difference method to solve the problem of two-dimensional axisymmetric elastic wave propagation in finite bars when it was subjected to continuous and discontinuous axial end impacts. The solution for a step change in stress agrees with experimental data near the end of the bar and exhibits a region that agrees with the one-dimensional strain approximation. Reflections from free and rigid-lubricated ends are studied. The results were compared with experimental data of other research and the maximum probable error in the numerical solution was about 7%. The investigator explained that this error was come from the inexact reporting of the experimental data and the experimental limitations. Also, a part of the time difference in displacement plot was noted that caused by the small difference in material constants.

Chou and Mortimer (1967)⁽²⁵⁾ treated the Timoshenko's beam equations as a system of second order partial differential equations by the method of characteristics in a unified approach capable of dealing with discontinuities along the characteristics in semi-infinite members, where no reflections were involved. They also compared the results of numerical calculations with Laplace transforms method when the two wave speeds were equal. The comparisons of the numerical results agree with the Laplace transform technique.

Koenig and Davids (1968)⁽⁴⁵⁾ studied the response of a prismatic cantilever beam, subjected to a ramp platform bending moment at its free end, by using the finite element method, where the effect of reflected waves in a very short cantilever was examined. Their results were presented for only a very small period, possibly due to stability and round-off error problems. The investigator concluded that the dynamic characteristics are vibrated about the static conditions.

Valathur and Baker (1971)⁽⁷¹⁾ studied numerically and experimentally the wave propagation in free-free aluminum specimen for the case of the strong axial impact. The equations of continuity and motion were solved by using the method of characteristics to find the strain-time histories along the bar. A Good agreement between the numerical and experimental data was obtained

Habberstad (1971)⁽³⁸⁾ approximated the exact equations of motion governing elastic, axisymmetric wave propagation in a cylindrical rod by the first-order finite difference scheme. This scheme was based on a displacement rather than a velocity formulation, thereby making it unnecessary to explicitly introduce an artificial viscosity term into the finite difference equations. The resulted difference equations were used in conjunction with the boundary and initial conditions to study a pressure pulse applied to the end of a semi-infinite bar, a bar containing a discontinuity in cross section and a bar composed of two materials joined together at some point along its length. The numerical results so obtained were compared with available experimental data and other analytical solution. The numerical solution showed the same basic configuration as the elementary solution, except that significant deviations were noted in the region near the discontinuity.

Chiu (1972)⁽²⁴⁾ examined elastic wave propagation along a composite bar with sections whose interface discontinuities may occur in material properties, in cross sectional area, and in cone angle .The finite difference method was

presented to solve the multiple-reflection problem. A good agreement with the earlier analytical and experimental results was achieved.

Koenig and Berry (1975)⁽⁴⁶⁾ investigated the transient response of a non-prismatic cantilever beam subjected to end moment and lateral forces separately. The problem was solved numerically by using the finite element method. They also studied the effect of non-homogeneity of beam material, caused by two materials fused together at some points along its length, on the response of the beam. The investigators concluded that the dispersion phenomenon was increased when non-homogeneities (geometrical and physical) were increased.

Atkins and Hunter (1975)⁽¹⁶⁾ considered the propagation of longitudinal elastic waves around right-angled corners in rods of square cross section, and the effect of the cross sectional dimensions were neglected completely. They showed that the theoretical analysis is based on the assumption that the filament approximation is valid everywhere except in the vicinity of the corner. In this region, the material is assumed to behave as a rigid body. The resulting sets of equations predict transmitted wave amplitudes of the correct sign, and of magnitude and shape in good agreement with the experimental observations.

Chan (1983)⁽²³⁾ developed a compatible method analysis of skeletal structures. The method of characteristic was used to model axial, torsional and flexural waves in prismatic members. By using this technique the static behaviour of trusses and frames and the dynamic behaviour of single members were described. The study was extended to include large displacement problems in the static analysis of trusses only. Also damping effect was considered.

Goldberg and Wylie (1983)⁽³⁵⁾ presented an analysis of two time line schemes in the solution of a linearized water hammer problem. Reach back time-line interpolations, where the characteristic lines are projected back before the current time step, demonstrate less damping than the corresponding spatial interpolation scheme at the same discretization. Implicit time line interpolations is used, where

the characteristic line is projected into the current time step .An error analysis and numerical experiments demonstrate the degree of damping and dispersion introduced by both reach back and implicit time line interpolation methods.

AL-Mousawi (1983)⁽⁷⁾ analyzed the flexural elastic wave propagation in beams. The method of characteristics was also used for the numerical solution of the problem .The effect of reflections in finite beams with discontinuities in cross sections is considered .The results of the work also indicate the importance of the investigation of discontinuities in cross section in structures subjected to impact type of loading , the diameter ratio and the position of discontinuity being as the main factors affecting the levels of the stresses, their peak values and their oscillations. Numerical results presented in that study showed good agreement with the results of experimental work.

AL-Mousawi, et al. (1988)⁽⁸⁾ used the method of characteristics for the numerical solution of the transient flexural wave propagation in finite cylindrical beams with discontinuities in cross-sectional area. Several factors had been found to affect the level of stress and the associated peak values such as the diameter ratio and the position of discontinuity. Comparisons with experimental data and finite element solution were made to demonstrate the validity of the method for one-dimensional problems.

AL-Sarraj (1989)⁽¹¹⁾ used the method of characteristics to represent the transient and dynamic response of skeletal structure and members supporting concentrated masses by modeling uncoupled and coupled waves transmission along one-dimensional members. He concluded that the method is most efficient to modeling sudden events as well as other types of dynamic loading.

Vardy and AL-Sarraj (1989)⁽⁷²⁾ applied The method of characteristics (MOC to simulate axial, torsional and flexural vibrations in single structural members supporting discrete masses. It was shown that the method is capable of predicting natural frequencies of vibration in dissipative as well as non-dissipative systems. They

used a numerical technique to solve the equation of motion at the masses. They showed that the method has useful advantages over conventional mass-spring analyses, including the ability to simulate wave action in members and to predict true stiffness and the equivalent free masses of the system.

Vardy and AL-Sarraaj (1991)⁽⁷³⁾ used the method of characteristics to model vibrations of one-dimensional members in the presence of coupling between axial and flexural waves. Only small deflections are considered and so the couple $F_x \delta_y$ is large only when the axial force F_x is large. Alternative numerical schemes were presented for the integration of terms describing coupling and a suitable explicit scheme was identified. The analysis was used to illustrate the influence of axial loads on frequencies and amplitudes of lateral oscillations of cantilever structures. Strong parametric behaviour was reproduced for a single-storey shear structure, including unstable resonance at double the linear natural frequency.

AL-Da'ami (1992)⁽⁴⁾ studied the transient and dynamic elastic response of non-prismatic straight members and arches by modeling axial and flexural waves transmission along such structures, by using the method of characteristics. Several numerical examples were presented and favorable agreement with finite element and finite difference methods were determined. The study was restricted to a small deformation analysis that neglected the effect of geometrical non-linearity.

Jwad (1992)⁽⁴¹⁾ investigated the non-linear elastic behaviour of axially vibrated one-dimensional prismatic beams under high loading conditions by using the method of characteristics. The effect of geometrical non-linearity, caused by the large deformation behaviour, was considered. The investigator concluded that this type of non-linearity would make the axial waves to propagate in dispersive form. Several examples of end conditions of the beam were also considered.

AL-Sarraaj and AL-Da'ami (1993)⁽¹²⁾ investigated the transient and dynamic response of non-prismatic structural members by modeling uncoupled axial and

flexural wave transmission along such members, by using the method of characteristic. It was shown that the method is capable of predicting natural frequencies of vibration. Many numerical examples were presented and favorable agreement with finite element method analysis was determined.

AL-Sarraj, Othman and Jwad (1993)⁽¹³⁾ introduced a method of analyzing one-dimensional axially loaded bars. The method designed for the nonlinear characteristics, was based on the method of characteristics that has gained acceptance in analysis of transient flows in fluid pipelines. The nonlinear characteristics had been shown to be well suited to simulate the dynamic behaviour beyond the elastic stage.

Fahmi (1998)⁽³³⁾ used the method of characteristics to determine the nonlinear transient response of single and continues beams with internal resonance, then compared the results with finite element method. The comparison gives good agreement.

Ahamed (2003)⁽²⁾ investigated the nonlinear large-angle bending dynamic response of prismatic beams using the method of characteristics. Different boundary conditions and loading characteristics are compared against an available nonlinear large-deflection analysis by finite elements using **NASTRAN2.0** computer program, results showed good agreement between the two approaches. The material properties are assumed to be linearly elastic.

AL-Haidar (2005)⁽⁵⁾ studied the elastic-plastic dynamic response of prismatic and non-prismatic straight members, by using the method of characteristics to model axial and flexural waves transmission along such structures. The analysis is based on the wave propagation in one-dimensional structural element approach. The effect of coupling between axial and flexural waves due to the unsymmetrical variation of the cross-sectional area about the longitudinal axis was considered. Several examples were investigated and the results are compared with the experimental data and the finite element method that were determined. The investigator concluded that the dispersion phenomenon

in flexural waves increased when the plastic waves were propagated in these members.

2.3 Large Deformation Dynamic Analysis

Houbolt (1950)⁽³⁹⁾ presented a method for the dynamic analysis using the direct time integration method. In this method, standard finite difference expressions are used to approximate the acceleration and velocity components in terms of the displacement component. He used a backward difference formulation with an error of order $(\Delta t)^2$.

Newmark (1959)⁽⁵⁴⁾ generalized certain direct numerical integration methods that had been in use up to that time. He presented equations for approximating the velocity and displacement of a single degree of freedom system at time $(t+\Delta t)$. These equations involve two constants (γ and β). The parameter γ controls the velocity within the time step while the parameter β controls the variation of acceleration. Because of that the technique is referred to as Newmark's generalized acceleration method (or *Newmark- β* method).

Gallagher and Lee (1970)⁽³⁴⁾ derived approximate formulations of tapered beam element stiffness matrices for dynamic and elastic instability analysis. Displacement functions for the uniform beam segment are employed in this development. Moment of inertia and area of the element are prescribed by arbitrary powers of the axial coordinate. Numerical results are obtained and compared with both analytical solutions and numerical solutions based up on stepped representations using uniform section elements. The numerical results have shown that simple modifications to the beam element stiffness and mass matrices to account for non-uniform section properties have produced highly accurate solutions for idealizations consisting of few elements

Weeks (1972)⁽⁷⁵⁾ used three methods for solving the non-linear dynamic problems. He considered the implicit *Houbolt* and the *Newmark* average acceleration

methods as well as explicit central **Finite Difference Method (F.D.M)** for one and multiple degree of freedom systems .He found no essential differences in the characteristic of the temporal operators for the linear and non-linear problems provided a small time step.

Wilson, et al. (1973)⁽⁷⁶⁾ extended the linear acceleration method in a manner that makes it numerically stable .The basic assumption of the **Wilson – θ** method is that the acceleration varies over an extended time ($\theta\Delta t$).It has been shown that this approach is numerically stable when $\theta > 1.38$.

Rao, et al. (1976)⁽⁵⁸⁾ studied the large-amplitude free vibration of beams including shear and rotatory inertia effects. Using their non-linear beam model, they showed that the secondary effects have negligible influence on the response when $L / R > 100$, where **L** and **R** denote the beam length and the radius of gyration respectively.

Oran and Kassimali (1976)⁽⁵⁶⁾ presented a nonlinear dynamic analysis by using the **Newmark average acceleration method**. They assumed that the masses are lumped at joints. The effect of rotary inertia on mass matrix and the effect of damping of the system were neglected. In their work, they used the incremental procedure in the formulation of the equations of motion at time ($t + \Delta t$) .They solved a fixed ended beam with an iteration process and compared the results with others, and they found that their technique gives a more representative solution compared with other techniques.

Zienkiewicz (1977)⁽⁷⁸⁾ offered a new look at time integration schemes based on a generalized weighted residual approach. The main advantage of this approach is that it contains most of the existing methods as special cases. On the other hand, this method is a multi-step method, and thus, it suffers from the following computational disadvantages:

1. Changing the time step size during the analysis is not possible.
2. Consistent starting values are not readily available.

3. The application to non-linear problems is cumbersome.

Wood (1977)⁽⁷⁷⁾ investigated the accuracy and stability for the **Zienkiewicz (1977)**⁽⁷⁸⁾ method and obtained the value of the investigation parameters which minimize the incremental phase error.

Mondkar and Powell (1978)⁽⁵¹⁾ investigated solution schemes ,which they implemented in general purpose computer code for nonlinear finite element analysis . A very flexible solution strategy is discussed, in which a variety of solution schemes can be implemented by specifying the values of certain solution, Newton-Raphson iteration, constant stiffness iteration, and an adaptable schemes in which the structure stiffness is reformed only when needed.

Remseth (1979)⁽⁶⁰⁾ presented a study concerned with methods of nonlinear static and dynamic analysis of structures, particularly for application to geometrically nonlinear space frames .The displacement formulation of the finite element method is adopted .Large displacements are accounted for by using **Lagrangian** description of motion from a fixed reference frame, assuming small strains and moderately large rotations.

In (1984)⁽⁷⁹⁾ and **(1985)**⁽⁴³⁾ (sited in Al-Mutairee (2000)⁽⁹⁾) two similar, but distinct single-step methods were evolved separately; one is based on a weighed residual formulation, and the other is based on generalization of **Newmark's** scheme. The former was introduced by **Zienkiewicz et at.** , and was published in **(1984) (Zienkiewicz et at.)**⁽⁷⁹⁾ and known as the **SSpj** method . The latter was called the **Beta-m** method, which was introduced by **Katona and Zienkiewicz**, and was published in **(1985) (Katona and Zienkiewicz)**⁽⁴³⁾.

The method is specialized by specifying the method order m along with m integration parameters $(\beta_0, \beta_1, \dots, \beta_{m-1})$, for a particular choice of m , the integration parameters provide a subfamily of methods which to be special cases within the beta-m family . Stability and accuracy are presented for method order $m=2, 3$ and 4 .

Kam and Lin (1988)⁽⁴²⁾ presented a method for the nonlinear dynamic analysis of inelastic framed structures subjected to large displacements. The nonlinear behavior of inelastic steel plane frames with or without initial geometric imperfections was also studied. The proposed method for dynamic analysis was based on an incremental form of equations of motion of the structures. A previously proposed member tangent stiffness matrix of an inelastic member was extended for dynamic analysis. Newmark's β -method was used as the numerical integration technique for solving the incremental form of the equations of motion. The feasibility of the proposed method was illustrated by several numerical examples on the dynamic analysis of inelastic frames.

Simites and Mohamed (1989)⁽⁶³⁾ investigated the nonlinear analysis of plane, non-orthogonal frames with flexible joint connections and with elastic rotational restraints at the supports, under static and dynamic loads. The nonlinearity was a kinematic one, while the material behavior was assumed to be linearly elastic. Symmetric gabled bends were found to buckle in either a symmetric snapthrough mode or in a sidesway mode. Two important parameters determined the buckling mode, and consequently the critical load. These were the bend rise to the span ratio and the ratio of the bending stiffness of the 'bend' beams to that of the columns. Critical loads were also obtained for sudden application of the cause and they are 15–30% lower than the corresponding static critical loads.

Dokainish and Subbaraj (1989)⁽³²⁾ presented a survey on implicit time integration method, viz .*Newmark- β* , *Wilson- θ* and *Houbolt* methods. The formulation and computational step-by-step procedures for linear and non –linear system were given. Stability aspects of these methods were also described.

Carol and Murcia (1989)⁽²¹⁾ presented an exact formulation for the nonlinear time-dependent analysis of planar frames formed by straight bars presented in three steps. A basic linear formulation, which is equivalent to the classical matrix theory, its generalization to nonlinear and time-dependent material analysis, and the inclusion of second order equilibrium nonlinear effects.

Due to the exact character of the interpolation used by the authors, no intrinsic errors exist in this formulation, and therefore there are no limitations to the size of the elements for this reason.

Hsiao and Jang (1989)⁽⁴⁰⁾ presented a co-rotational formulation and numerical procedure for the dynamic analysis of in-plane frames is presented. The nodal coordinates, incremental displacements and rotations, velocities, accelerations, and equations of motion of the system were defined in terms of fixed global coordinates. The element matrices are constructed first in an element coordinate system, which rotates and translates with the element but does not deform with the element, and are then transformed to the global coordinate system by standard procedure. The beam element used by the authors was an extension of that proposed by **Hsiao and Huo**. An incremental-iterative method based on the Newmark direct integration method and the Newton-Raphson method is used here for the solution of the nonlinear dynamic equilibrium equations. In their proposed approach the major geometric nonlinearities were shown to be embodied in the coordinate transformation when the element assemblage is formed.

In (1996)⁽¹⁰⁾ (sited in Al-Mutairee (2000)⁽⁹⁾) a new implicit time integration method was submitted to **Al-Sarraf** for evaluation to be published. The basis of this method is to assume that the acceleration varies harmonically during the time step, so it was called the harmonic acceleration method.

The numerical examples were used to examine the application of the proposed method for linear system. Each example was solved by the harmonic acceleration method and the linear acceleration method, in addition to the exact solution. The author concluded that the errors with the proposed method are smaller than those with the linear acceleration method for the same time step.

AL-Bidairi (1998)⁽³⁾ presented theoretical bases for the large displacement elastic stability analysis of plane steel frames and beam-like lattice structures with various end conditions, subjected to static and dynamic loads. Also the effect of shear

deformation on the geometric non-linear behaviour and the effect of damping on the dynamic response were considered

Ali (1999)⁽⁶⁾ introduced a theoretical analysis is presented for estimating the in-plane large displacement elastic stability behaviour of structures with prismatic or tapered members subjected to static and dynamic loading. The analysis adopts the **Beam-Column(B-C)** approach. The **Beam-Column** element is based on the **Eulerian** approach taking into consideration the influence of the axial force on bending stiffness and the changes in member chord length due to axial strain and flexural bowing.

Four different numerical integration algorithms have been used for solving the equation of motion in the dynamic analysis. System mass properties have been represented using both consistent and lumped mass matrices. System natural frequency was obtained by solving the eigenvalue problem. Some examples which have been investigated.

AL-Mutairee (2000)⁽⁹⁾ presented a theoretical analysis for estimating the in plane large displacement elastic stability behaviour with post-buckling of plane steel frames with prismatic and non- prismatic members and subjected to static and dynamic loads. The analysis is based on **Beam – Column (B-C)** approach. The formulation of the **(B.C)** elements is based on the **Eulerian** approach taking into consideration the influence of the axial force on bending stiffness and the changes in the member chord length are due to the axial strain and flexural bowing. Five different numerical integration algorithms have been used for solving the equation of motion in the dynamic analysis. The consistent mass matrix for non-prismatic member is derived. System's natural frequencies are obtained by solving the eignvalue problem. The investigator concluded the variation of the tapering ratio for the same steel volume has very significant effect on the nonlinear behaviour of steel frames. Damping effect was also considered.

2.4 Closing Remarks

From the literatures that reviewed before, it can be noticed that most researches are restricted to use finite element method to study the effect of large deformation behavior. The other researches, used the method of characteristics studied the wave propagation in the one-dimensional and two-dimensional members with several assumptions that simplifying the analysis (such as equating the wave's speeds of the flexural vibration and neglecting the effect of large deformation). Here, this study is an extended the used of the method of characteristics to deal with the nonlinear vibration of elastic plane steel frame consisted from prismatic and non- prismatic straight one-dimensional members. That based on Timoshenko's beam theory under dynamic load and different types of loading as well as used different types of frames.

CHAPTER THREE

THEORETICAL ANALYSIS

3.1 Introduction

When a system is subjected to a sudden excitation, waves of stresses and particle's velocities propagate through it. When these waves strike boundaries or discontinuities within this system, they may be wholly or partially transmitted to the surrounding or reflected back. During this phase, the stresses at a point are determined by the arrival of these waves singly or in combination. **Timoshenko, et al. (1974)**⁽⁶⁷⁾ described the propagation of these waves along one-dimensional members. According to his approach, it is assumed that plane section remains plane in the case of axial waves, but the presence of shear waves causes plane section to distort about the neutral axis in the case of flexural waves. It is also assumed that material particles are moved only in axial direction and the effect of lateral displacements in the axial motions can be neglected without substantial errors (**Timoshenko and Goodier, 1982**)⁽⁶⁸⁾.

In the present work, Timoshenko's beam theory is examined to be applied on plane frames with prismatic members of linearly elastic material. For simplicity, the various types of waves are considered separately and it is implicitly assumed that the combination effect of different wave types can be obtained by superposition (**Timoshenko and Goodier, 1982**)⁽⁶⁸⁾.

After deriving the wave equations, the next step in the analysis is to develop an adequate method for solution. The finite difference technique is

used in order that the explicit solutions of the characteristic equations can be locally obtained.

3.2 The Method of Characteristics

The method of characteristics (**MOC**) is used for the numerical solution of first order and second order partial differential equations (**PDE**) of hyperbolic type. For the hyperbolic (**PDE**), there exist two distinct families of real characteristic curves at each point (x, t) as shown in Figure (3.1). For the parabolic case, the two characteristics coincide and they are of no significant value in understanding the behavior of the solution, whereas the elliptic form of PDE has no real characteristics.

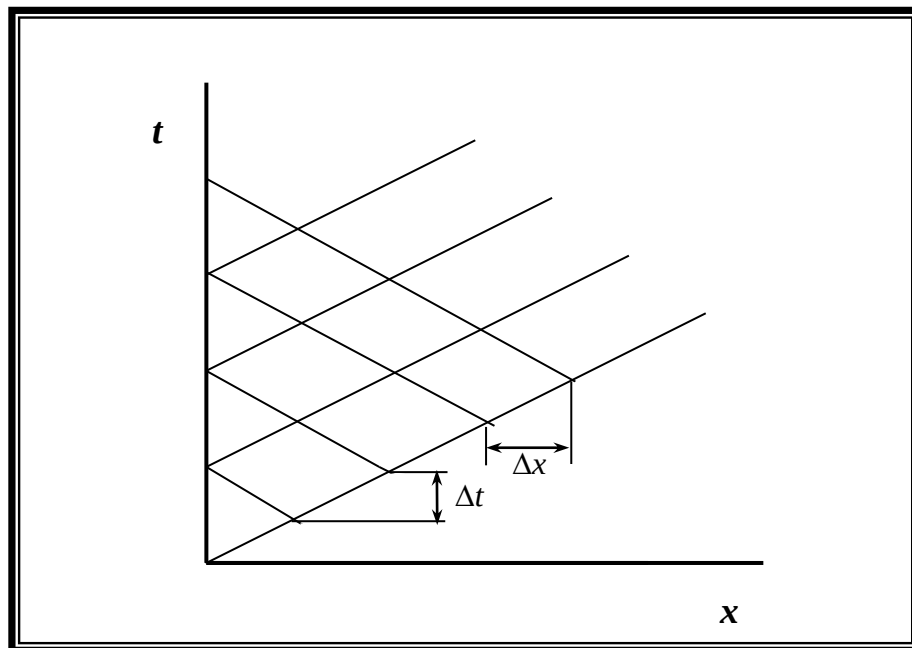


Figure (3.1): Characteristic network.

The real characteristics of hyperbolic PDE are curves in the real domain of the problem, and discontinuities will propagate along the characteristics. An

explicit step-by-step process is usually used in building up simultaneously the characteristic grid and solving the PDE at the grid points.

3.3 Elastic Stress Waves

When the elastic dynamic load is subjected to an element over a very short period, the response should be considered in terms of elastic wave propagation theory. Here, the deformation caused by this load will be recovered when the load is removed.

In the following analysis, the deformation is assumed very small. Thus, the change of element geometry can be neglected.

3.3.1 Elastic Waves Propagation In Prismatic Elements

The propagation of elastic stress waves along one-dimensional prismatic element is the simplest case of wave propagation and it is like the transient fluid flow in uniform pipes (Streeter, 1971)⁽⁶⁵⁾.

3.3.1.1 Elastic Axial Waves

When an axial wave propagates along a one-dimensional member, it is reasonable to assume that each plane cross section of the member remains plane, hence the particles in the cross section move only in the axial direction of the member.

Figure (3.2) shows a prismatic element with length dx under axial loading condition, where Newton's second law of motion can be applied for this element as:

$$\frac{\partial F}{\partial x} = \rho A \frac{\partial U}{\partial t} \quad \dots (3.1)$$

For a linearly elastic material, property relationship derived in **Appendix A** has the form:

$$\frac{\partial F}{\partial x} = E A \frac{\partial U}{\partial x} \quad \dots (3.2)$$

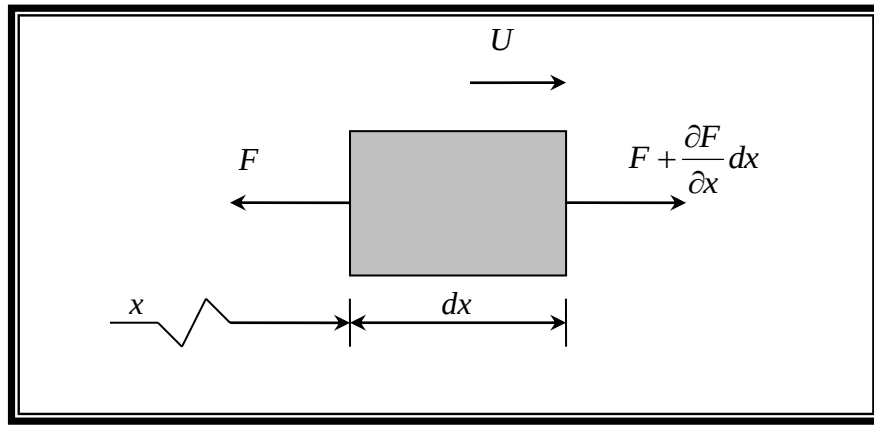


Figure (3.2): Prismatic element under axial vibration

Equations (3.1) & (3.2) are a pair of hyperbolic partial differential equations that can be combined together and expressed in the characteristic form as:

$$\frac{dF}{dt} \pm \rho A C \frac{dU}{dt} = 0 \quad \dots (3.3 \text{ a, b})$$

The above equations are valid for the forward and the backward propagation of wave, where C is the elastic wave speed that satisfies:

$$C = \sqrt{\frac{E}{\rho}} \quad \dots (3.4)$$

Since the product, ρAC in equations (3.3) is sensibly constant, then these equations can be integrated exactly along the lines C^+ and C^- as shown in Figure (3.3) below.

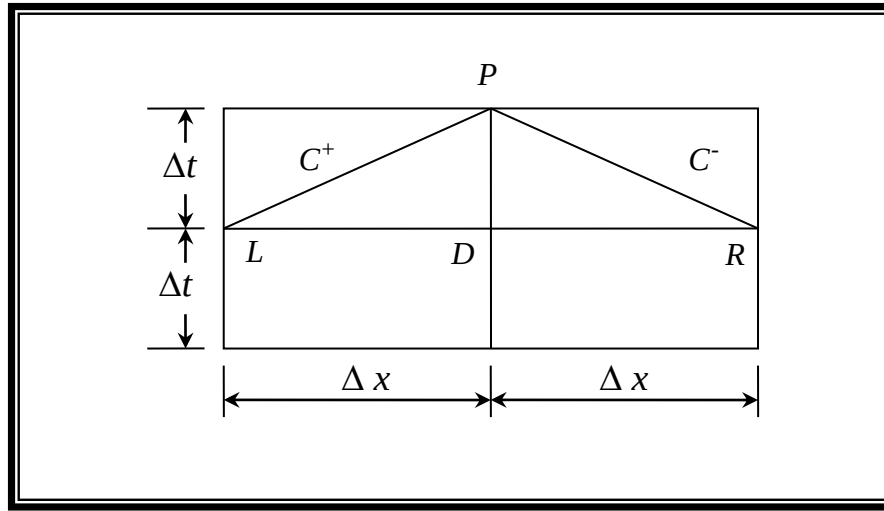


Figure (3.3): Numerical characteristic scheme

Along LP , the forward characteristic equation along the line C^+ for which $\frac{dx}{dt} = +C$, can be simplified in a finite difference form as:

$$(F_P - F_L) - \rho AC (U_P - U_L) = 0 \quad \dots(3.5)$$

Moreover, along RP , the backward characteristic equation along the line C^- for which $\frac{dx}{dt} = -C$, is:

$$(F_P - F_R) + \rho AC (U_P - U_R) = 0 \quad \dots(3.6)$$

Where the subscripts P , R and L refer to specified points that are related to the position and time as depicted in Figure (3.3).

3.3.1.2 Elastic Flexural Waves

The problem of elastic flexural wave propagation in beams has not been so extensively treated, as has the problem of axial wave propagation. This is due to the complexities involved in the propagation of such waves and their dispersive character (Al-Mousawi, et al., 1988)⁽⁸⁾.

Timoshenko's beam theory (1922)⁽⁶⁶⁾ is considered to be only approximated theory that contains the essential features of the exact theory in a simplified form. This theory leads to more accurate solutions than the Euler-Bernoulli's theory because the effects of shear deformation and rotary inertia are included in the governing equations. In this theory, to simplify the derivation of the equations of motion, the shear stress is assumed to be uniform over a given cross section. In turn, the shear correction factor is introduced into account for this simplification, and its value depends on the shape of the cross section, Appendix B, [Cowper (1966)⁽³⁰⁾, Gruttmann and Wanger (2001)⁽³⁷⁾].

Now consider the transverse vibrations of a prismatic element as shown in Figure (3.4).

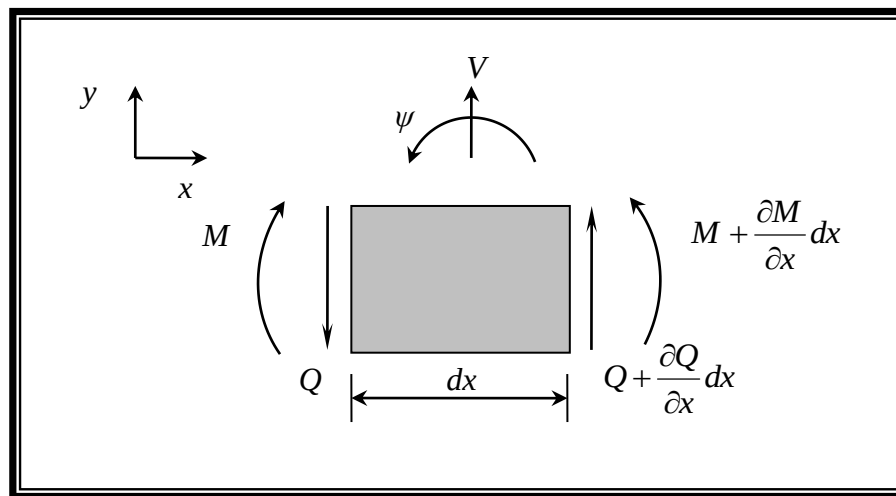


Figure (3.4): Prismatic element under flexural vibration

The dynamic equilibrium equation along y-direction being:

$$\frac{\partial Q}{\partial x} = \rho A \frac{\partial V}{\partial t} \quad \dots (3.7)$$

In addition, the moment equilibrium condition gives:

$$Q dx + \frac{1}{2} \frac{\partial Q}{\partial x} dx^2 + \frac{\partial M}{\partial x} dx = \rho dx I \frac{\partial \psi}{\partial t} \quad \dots (3.8)$$

The term $(\frac{1}{2} \frac{\partial Q}{\partial x} dx^2)$ is of negligible effect, then equation (3.8) becomes:

$$\frac{\partial M}{\partial x} + Q = \rho I \frac{\partial \psi}{\partial t} \quad \dots (3.9)$$

As shown in **Appendix A**, the material property relationships are,

Shear:

$$\frac{\partial Q}{\partial t} = K G A \left(\frac{\partial V}{\partial x} - \psi \right) \quad \dots (3.10)$$

Moment:

$$\frac{\partial M}{\partial t} = E I \frac{\partial \psi}{\partial x} \quad \dots (3.11)$$

Equations (3.7) & (3.10) and (3.9) & (3.11) are pairs of hyperbolic partial differential equations relating either a shear force and a lateral velocity or a moment and an angular velocity. Each pair can be expressed as ordinary differential equations provided that their range of applicability is restricted to certain characteristic directions in the x - t plane as shown in Figure (3.5).

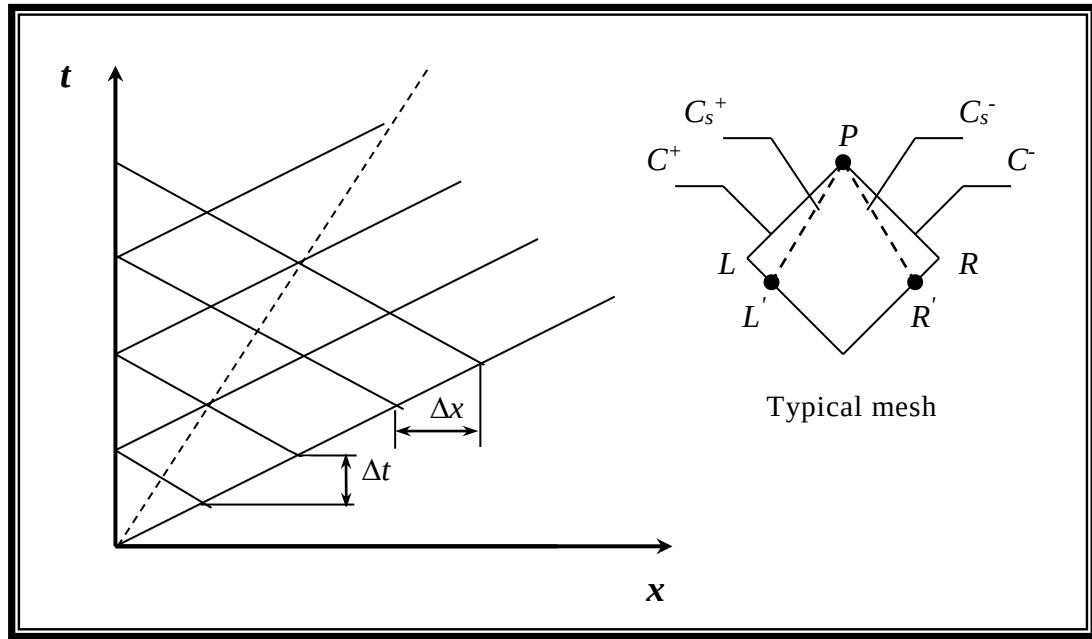


Figure (3.5): Characteristic network and typical mesh of elastic flexural waves.

This plane is divided into two parts, a domain representing undisturbed particles and a domain representing particles in motion. This second domain is of a particular interest for the numerical solution. The part of interest is subdivided by means of a primary grid, which is formed by means of two sets of parallel lines having equal but opposite slopes $\pm C$. Each grid element, or so-called mesh, has diagonals measuring $2\Delta x$ and $2\Delta t$, as shown in a typical mesh in Figure (3.5). Within each mesh, a secondary grid is constructed using characteristic lines with slopes $\pm C_s$ and drawn from the point at which the unknowns are to be evaluated. Thus, the characteristics equations along these lines are:

Shear along $\mp C_s$,

$$\frac{dQ}{dt} \pm \rho A C_s \frac{dV}{dt} = -K A G \psi \quad \dots (3.12 \text{ a, b})$$

Bending along $\mp C$,

$$\frac{dM}{dt} \pm \rho I C \frac{d\psi}{dt} = \pm C Q \quad \dots (3.13 \text{ a, b})$$

In which the wave speeds are equal to,

$$C_s = \sqrt{\frac{KG}{\rho}} \quad \dots (3.14)$$

The above equations can be integrated along the lines $L'P$ & $R'P$ in case of shear wave and RP & LP in case of bending wave as shown in Figure (3.5). Using central difference expressions and linear function to integrate the terms on the right hand side of equations (3.12 a, b) & (3.13 a, b); the following are obtained:

Shear:

$$(Q_P - Q_{L'}) - \rho A C_s (V_P - V_{L'}) = -\rho A C_s \frac{(\psi_P + \psi_{L'})}{2} \Delta x \quad \dots (3.15)$$

$$(Q_P - Q_{R'}) + \rho A C_s (V_P - V_{R'}) = -\rho A C_s \frac{(\psi_P + \psi_{R'})}{2} \Delta x \quad \dots (3.16)$$

Moment:

$$(M_P - M_L) - \rho I C (\psi_P - \psi_L) = -\frac{(Q_P + Q_L)}{2} \Delta x \quad \dots (3.17)$$

$$(M_P - M_R) + \rho I C (\psi_P - \psi_R) = +\frac{(Q_P + Q_R)}{2} \Delta x \quad \dots (3.18)$$

3.3.2 Elastic Waves Propagation in Non-Prismatic Elements

The study of transient and dynamic behavior of non-prismatic element is very important for several engineering problems such as the vibration of turbine blades, hulls of ships, bridge girders of variable depth, etc... .In this case, the stress waves will suffer from a disturbance caused by the reflection of waves when it propagates in non-prismatic element. Here, the assumptions of Timoshenko's beam theory are still valid for this type of elements.

3.3.2.1 Elastic Axial Waves

The propagation of axial waves in non-prismatic member is similar to the flow of liquid in pipelines with variable diameter, where some portion of the wave will reflect back when it pass through a non-prismatic element.

There are two types of the analysis that deal with axial wave propagation in such element. The first analysis assumed that the variation of area along the longitudinal axis is being stepped as depicted in Figure (3.6 a). While the second analysis considered the variation of area is continuous as shown in Figure (3.6 b).

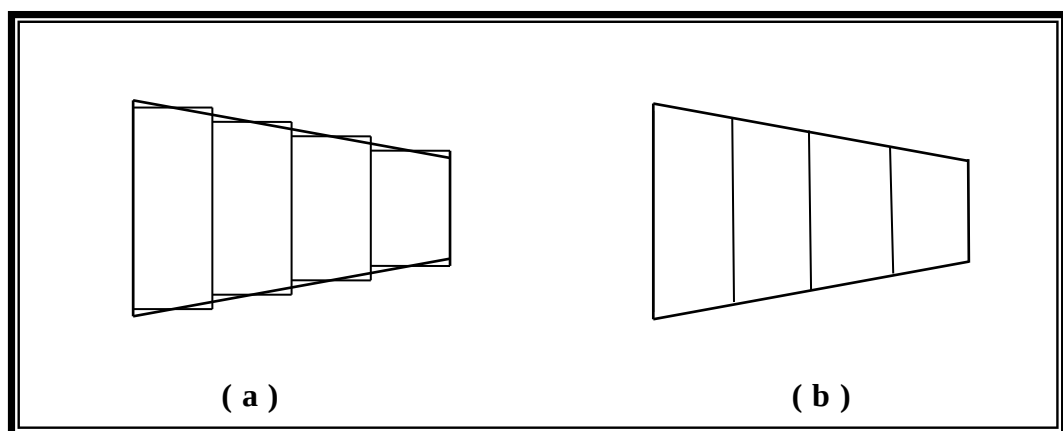


Figure (3.6): Schemes of area variation.

(a) Stepped scheme. **(b)** Continuous scheme.

Here, both schemes will be considered and the assumptions of Timoshenko's beam theory are valid.

- **Stepped scheme.**

In this scheme the equation of motion will have the form:

$$\frac{\partial F}{\partial x} = \rho a \frac{\partial U}{\partial t} \quad \dots (3.19)$$

where a is the central area of the element as shown in Figure (3.7).

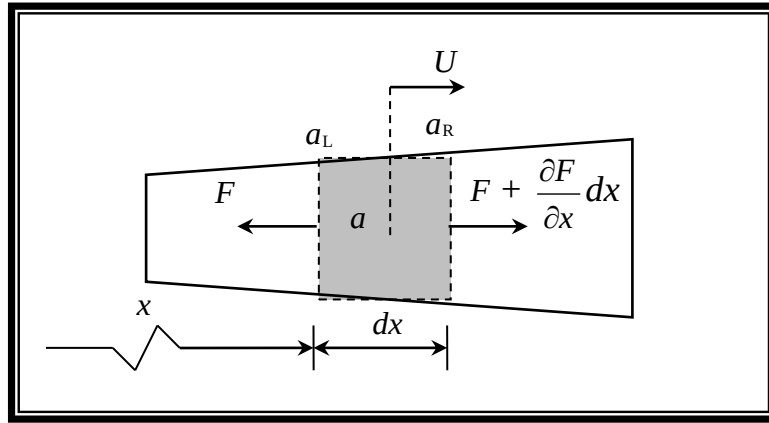


Figure (3.7): Axial motion of non-prismatic element (stepped).

Using material property relationship that is described in **Appendix A**, getting:

$$\frac{\partial F}{\partial t} = E a \frac{\partial U}{\partial x} \quad \dots (3.20)$$

Equations (3.19) & (3.20) are pair of hyperbolic partial differential equations relating axial force to the axial velocity at the center of the element. This pair of equations can be expressed as ordinary differential equations

provided that their range of applicability is restricted to certain characteristic direction in the x - t plane. In their characteristic form, the equations are:

$$\frac{dF}{dt} \pm \rho a C \frac{dU}{dt} = 0 \quad \dots (3.21 \text{ a, b})$$

The above equations can be integrated along the lines LP and RP in x - t grid as shown in Figure (3.3) to obtain:

$$(F_P - F_L) - \rho a_L C (U_P - U_L) = 0 \quad \dots (3.22)$$

$$(F_P - F_R) + \rho a_R C (U_P - U_R) = 0 \quad \dots (3.23)$$

• Continuous scheme.

Members of non-linear variable cross-sectional area cannot be modeled using the above approach with high accuracy. This will lead to consider a general element to deal with such types of members.

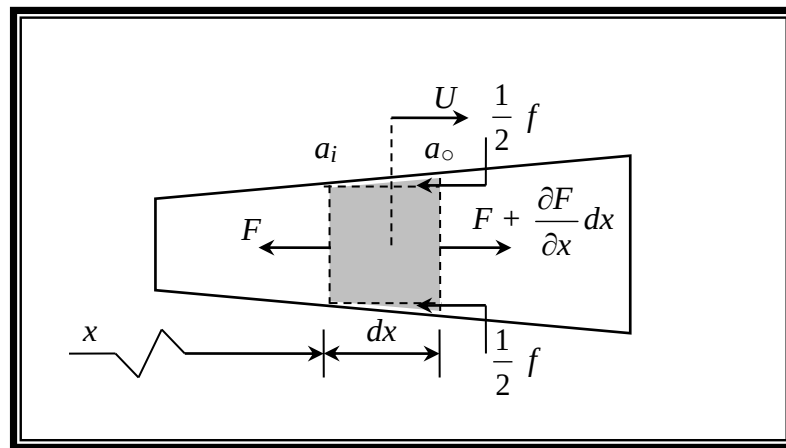


Figure (3.8): Axial motion of non-prismatic element (continuous).

The equation of motion is taken for the element shown in Figure (3.8), with properties defined at the inlet area (a_i). The variation of cross-sectional area can be represented as a straggled force f to the particles motion. Thus, the equation of motion becomes **(AL-Haidar, 2003)⁽⁵⁾**:

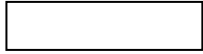
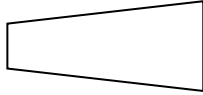
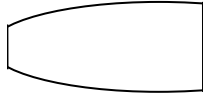
$$\frac{\partial F}{\partial x} - f = \rho a_i \frac{\partial U}{\partial t} \quad \dots (3.24)$$

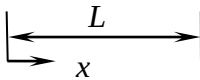
where

$$f = \beta \rho |a_o - a_i| \frac{\partial U}{\partial t} \quad \dots (3.25)$$

$\beta \equiv$ shape factor (table 3.1).

Table (3.1): Shape factor used in equation (3.25) ⁽⁴⁾.

β	<i>Elemental shape</i>	
$\frac{1}{1}$		$h(x) = h_i$
$\frac{n}{n+1}$		$h(x) = h_i + (h_o - h_i) \left(\frac{x}{L}\right)$
		$h(x) = h_i + (h_o - h_i) \left(\frac{x}{L}\right)^{\frac{1}{n}}$
$\frac{1}{n+1}$		$h(x) = h_i + (h_o - h_i) \left(\frac{x}{L}\right)^n$



Hence, the equation of motion can be written as:

$$\frac{\partial F}{\partial x} = \rho a_i \frac{\partial U}{\partial t} + \rho A_r \frac{\partial U}{\partial t} \quad \dots (3.26)$$

where

$$A_r = \beta | a_i - a_o | \quad \dots (3.27)$$

Equations (3.26) & (3.2) are pair of hyperbolic partial differential equations, which can be expressed in a characteristic form as:

$$\frac{dF}{dt} \pm \rho a_i C \frac{dU}{dt} = \mp \rho A_r C \frac{dU}{dt} \quad \dots (3.28 a, b)$$

The above equation can be integrated along the characteristic lines LP and RP in $x-t$ grid as shown in Figure (3.3) to obtain:

$$(F_P - F_L) - \rho a_i C (U_P - U_L) = \rho A_r C (U_P - U_L) \quad \dots (3.29)$$

$$(F_P - F_R) + \rho a_i C (U_P - U_R) = -\rho A_r C (U_P - U_R) \quad \dots (3.30)$$

Because of the smaller demand for computer time, stepped scheme is preferred to other one if the results are to be of engineering use (**Al-Da'ami, 1992**)⁽⁴⁾. For this reason; the stepped scheme is used in this study.

3.3.2.2 Elastic Flexural Waves

Timoshenko's beam theory is used in formulating the characteristic equations of the flexural waves in non-prismatic elements. The same

assumptions as that in prismatic element are used. By taking the geometrical properties at the centre of the element (stepped scheme), the equation of motion in y direction, Figure (3.9), becomes:

$$\frac{\partial Q}{\partial x} = \rho a \frac{\partial V}{\partial t} \quad \dots (3.31)$$

In addition, the moment equilibrium condition gives:

$$\frac{\partial M}{\partial x} + Q = \rho I_s \frac{\partial \psi}{\partial t} \quad \dots (3.32)$$

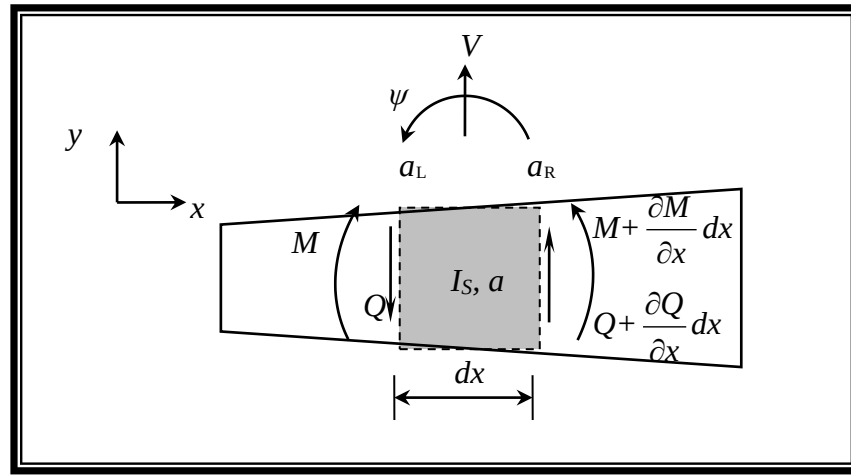


Figure (3.9): Non-prismatic element under flexural vibration

Equations (3.31) & (3.10) and (3.32) & (3.11) are pairs of hyperbolic partial differential equations. Each pair can be expressed as ordinary differential equations provided that their range of applicability is restricted to certain characteristic directions in x - t plane. In their characteristic forms, the equations are as follows:

Shear along $\mp C_s$,

$$\frac{dQ}{dt} \pm \rho a C_s \frac{dV}{dt} = -K a G \psi \quad \dots (3.33 \text{ a, b})$$

Bending along $\mp C$,

$$\frac{dM}{dt} \pm \rho I_s C \frac{d\psi}{dt} = \pm C Q \quad \dots (3.34 \text{ a, b})$$

The above characteristic equations can be integrated along the lines LP & RP and $L'P$ & $R'P$ that are depicted in Figure (3.5). In order to solve these equations for the unknowns Q_P , M_P , V_P , and ψ_P , it is necessary to integrate the terms on the right hand sides. For simplicity, Q & V are assumed to vary linearly along the characteristic lines (**AL-Mousawi, 1983**)⁽⁷⁾ thus:

Shear:

$$(Q_P - Q_{L'}) - \rho a_L C_s (V_P - V_{L'}) = -\rho a_L C_s \frac{(\psi_P + \psi_{L'})}{2} \Delta x \quad \dots (3.35)$$

$$(Q_P - Q_{R'}) + \rho a_R C_s (V_P - V_{R'}) = -\rho a_R C_s \frac{(\psi_P + \psi_{R'})}{2} \Delta x \quad \dots (3.36)$$

Moment:

$$(M_P - M_L) - \rho I_{SL} C (\psi_P - \psi_L) = -\frac{(Q_P + Q_L)}{2} \Delta x \quad \dots (3.37)$$

$$(M_P - M_R) + \rho I_{SR} C (\psi_P - \psi_R) = +\frac{(Q_P + Q_R)}{2} \Delta x \quad \dots (3.38)$$

3.4 Elastic Dynamic Response of Plane Frames

Most engineering structures, *e.g.*, *buildings* and *bridges*, are composed of a number of straight members joined together. A structure formed with straight members is commonly called a *frame*.

To describe the dynamic behavior of plane frames by the method of characteristics, a method of analysis called the method of boundary characteristics is developed by (**Chan, 1983**)⁽²³⁾ and used to solve the wave equations, which describe the dynamic behavior of structures.

This method applied to the static and dynamic analysis of plane frames by (AL-Sarraj, 1989)⁽¹¹⁾.

3.4.1 Boundary Characteristics Method

The method of boundary characteristics can be used to model the dynamic behavior of plane frames. For this purpose, the following assumptions are adopted [Chan (1983)⁽²³⁾, AL-Sarraj (1989)⁽¹¹⁾]:

- 1) Frame is regarded as an assemblage of members.
- 2) Joints assumed to be massless, rigid and to be capable to transmitting axial, shear and moment waves.
- 3) By requiring that velocities are compatible and forces are in equilibrium in all members meeting at massless joint, the entire structure is capable to allow waves to pass through the joints.
- 4) All displacements are assumed to be small compared to the overall dimensions of structure.

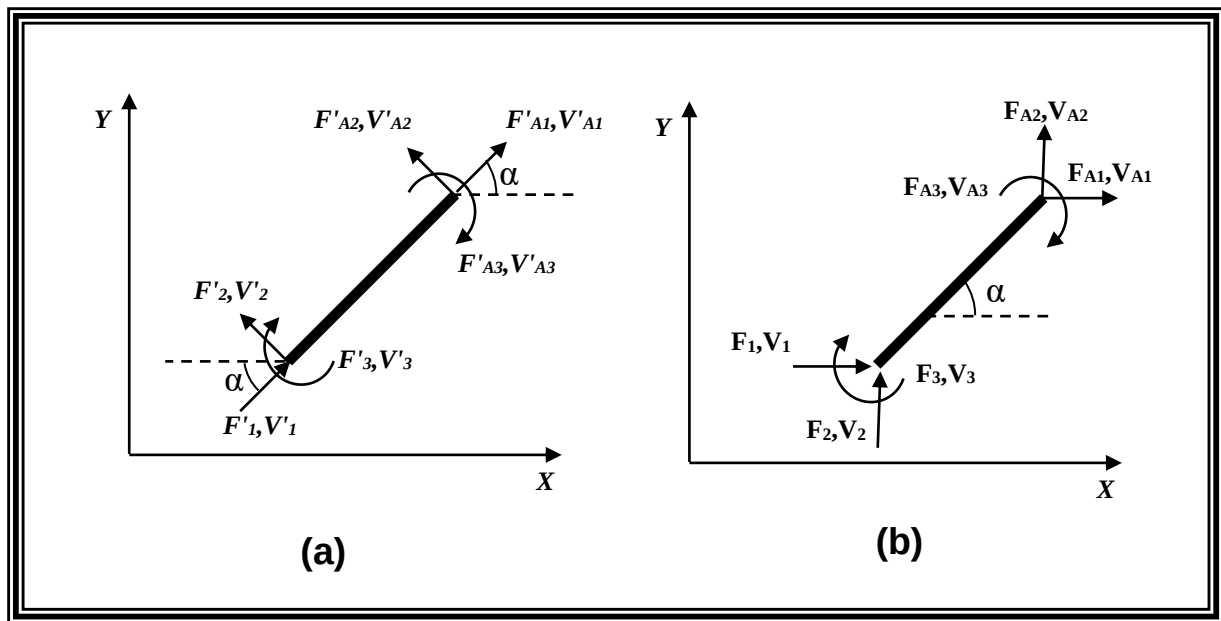


Figure (3.11): Forces and velocities.
(a) Local axes (member axes). **(b)** Global axes (joint axes)

As described in previous section (3.3.1.1), for any member in structure have three characteristic equations per node (three forces and three velocities per node).

The characteristic equations are first written relative to local axes as mansioned in Figure (3-11a).The internal forces at either end of member can be written in a matrix form as:

From equations (for prismatic member):

$$\begin{array}{ll} (3.5) \text{ \& } (3.6) & \text{Axial} \\ (3.15) \text{ \& } (3.16) & \text{Shear} \\ (3.17) \text{ \& } (3.18) & \text{Moment} \end{array}$$

Then:

$$\{F'\}_n = \{f'\}_n + [C_e]_n \{V'\}_n \quad \dots(3.39)$$

Where:

$$\{f'\}_n = \begin{Bmatrix} -sF_A - \rho a C U_A \\ -sQ_A' - \rho a C_s V_A' + s \rho a C_s \frac{\Delta x}{2} \psi_A' \\ -sM_A - \rho I C \psi_A - \frac{\Delta x}{2} (f_1 + Q_A) \end{Bmatrix} \quad \dots(3.40 \text{ a,b})$$

$$[C_e]_n = \begin{Bmatrix} \rho a C & 0 & 0 \\ 0 & \rho a C_s & s \rho a C_s \frac{\Delta x}{2} \\ 0 & s \rho a C_s \frac{\Delta x}{2} & \rho I C + \rho a C_s \frac{\Delta x^2}{4} \end{Bmatrix} \quad \dots (3.41 \text{ a,b})$$

Where:

$$f_1 = Q_A' + s \rho a C_s V_A' - \rho a C_s \frac{\Delta x}{2} \psi_A' \quad \dots (3.42 \text{ a,b})$$

Here, ($s = -1$) for forward characteristic lines [+C equation] and ($s = +1$) for backward characteristic lines [-C equation]. The subscript (A) denotes the old time information at the remote grid and subscript (n) denotes the particular member (n). The primes denote the use of axes chosen relative to a member. $\{F'\}$ and $\{V'\}$ are the unknown end forces and velocities at the new time level respectively and $[C_e]$ is the characteristics member matrix.

In practice, the local axes (member axes) are not usually the same as the global axes (joint axes). Therefore, before applying the boundary conditions transformation from the local axes to the global axes for the end forces and the end velocities of each member. As shown in Figure (3.11) the relationships between the end forces $\{F'\}$ and the end velocities $\{V'\}$ in the local axes and the end forces $\{F\}$ and the end velocities $\{V\}$ in global axes are, for plane frames:

$$\{V'\} = [T] \{V\} \quad \dots (3.43)$$

$$\{F\} = [T]^T \{F'\} \quad \dots (3.44)$$

Where $[T]$ is the transformation matrix and dependent upon the geometry of the member:

$$[T] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \dots (3.45)$$

In which (α) is the angle between the local x-axes and global x-axes Figure (3.11). After being transformed to the global axes equation (3.39) can be written as:

$$\{F\}_n = [T]^T \{f'\}_n + [T]^T [C_e]_n [T] \{V\}_n \quad \dots(3.46)$$

3.4.1.1 Equilibrium and Compatibility Conditions

After transformation into global axes ,it is necessary to satisfy the conditions of equilibrium and compatibility at the joint .The ends of members need to have the same velocity components as the joint at which these members meet .Also, the sum of the end forces in the members needs to balance the external load $\{P_{(t)}\}$ [Chan(1983)⁽²³⁾,AL-Sarraj(1989)⁽¹¹⁾] .

The equilibrium equation for a joint at which (m) members meet:

$$\{P_{(t)}\} = \sum_{i=1}^m \{F\}_i \quad \dots (3.47)$$

Substituting equation (3.46) in to equation (3.47) gives:

$$\{P_{(t)}\} = \{f\} + [C_J] \{V\} \quad \dots (3.48)$$

Where:

$\{P_{(t)}\}$: denotes to the external loads

$\{f\}$: information matrix at the old time level

$$\{f\} = \sum_{i=1}^m [T]^T \{f'\}_i \quad \dots (3.49)$$

$[C_J]$: the characteristics joint matrix

$$[C_J] = \sum_{i=1}^m [T]^T [C_e] [T] \quad \dots (3.50)$$

In the above equilibrium equation (3.48) the joint matrix is symmetrical and is dependent upon the geometrical and mechanical properties of the member and on grid size in members meeting at the joint.

The solution procedure of the frame structure is as follows [Chan (1983)⁽²³⁾, AL-Sarraj (1989)⁽¹¹⁾]:

Joint velocities can be calculated from the equilibrium equation (3.48) when the loading and the previous time level conditions are known. Then end velocities of the members can be related to the local axes by using the transformation relationship. The end forces of the members can then be calculated using the characteristic equations.

3.5

Large Deformation Dynamic Behavior of Plane Frames

When a structure is subjected to large external excitation, material and geometrical properties will change during the deformation process. A linear beam model would suffice when dealing with small deformations as analyzed in previous sections. But when the deformations are moderately large, for accurate modeling, several nonlinearities also need to be included; this case is called non-linear behavior.

Non-linear behavior can be take place due to a number of different sources. In general, material nonlinearity is important in the inelastic frame behavior. Geometric nonlinearity becomes significant in slender members. The $P-\Delta$ effect plays a critical role when the nodal loads on the frame are large or when its lateral drift is large. The other source of nonlinearities may be very

important under strong shaking, but these nonlinearities are very difficult to quantify (**Challa, 1992**)⁽²²⁾.

The nonlinear frame analysis discussed in this study includes geometric nonlinearity only and material is assumed to be linearly elastic.

The derivation of the characteristic equations in section (3.3.2) with central element properties is valid for prismatic and non-prismatic elements because the geometry of the prismatic element is changed during the deformation process and it is no longer to remain prismatic (**Arafat, 1999**)⁽¹⁵⁾.

During the deformation process, length and cross-sectional area of the element will change. Thus, the time required for the wave to propagate between two grid points is changed. It will be more or less than the original time interval depending on the sign of deformation. Therefore, the geometry it self must be treated as unknown and so displacements become dependent variables in the unsteady solution.

Thus, the solution of force, velocity and displacement at the new time level are obtained based on the geometry of the deformed structure at the previous time level and the geometry at the new time can be calculated after the new displacement have been obtained (**Chan,1983**)⁽²³⁾.

In characteristics method, the following assumptions are adopted (**Jwad, 1992**)⁽⁴¹⁾:

1. The cross-sectional area and the length of each grid are calculated at the beginning of the time interval and remain constant throughout the time interval.
2. The lowest time interval is adopted as a basic interval at each time step.

Figure (3.12) shows beam element in deformed configuration in x - y plane. The internal generalized forces acting on the element are the shear force, Q , the axial force, F , and the bending moment, M .

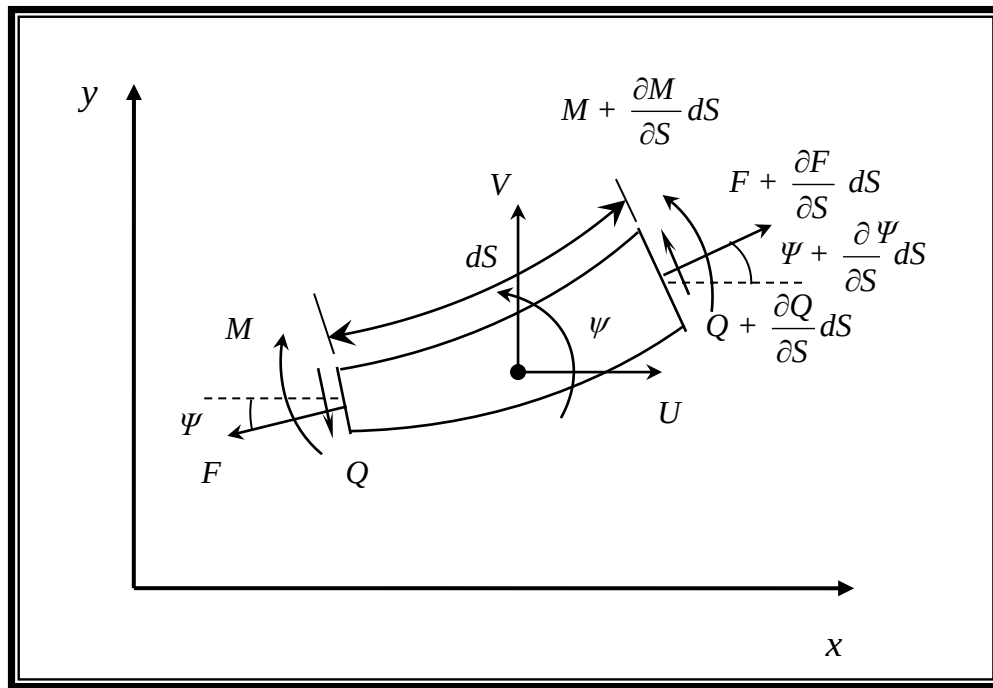


Figure (3.12): Beam element in deformed configuration.

A relationship between the external load $\{P_{(t)}\}$, velocity $\{V\}$ and displacement (ΔS) in the equilibrium load equation (3.48) at each joint can be written as (Chan, 1983)⁽²³⁾:

$$\{P_{(t)}\} = \{f(\Delta S)\} + [C_J(\Delta S)] \{V\} \quad \dots (3.51)$$

Where information matrix at the old time level $\{f(\Delta S)\}$ and the characteristics joint matrix $[C_J(\Delta S)]$ are functions of the new displacement condition.

The displacement components u and v will be adopted as the two basic parameters in defining other geometrical quantities. Consider the typical element whose original axial length before deformation is dx as explained in Figure (3.12). After deformation, its length becomes dS , which is related to dx by **(Reid et al, 1998)**⁽⁵⁹⁾,

$$ds = \sqrt{\left(\frac{dv}{dx}\right)^2 + \left(1 + \frac{du}{dx}\right)^2} dx \quad \dots (3.52)$$

in addition, the new element's area are equal to :

$$a_{(t+\Delta t)} = \frac{a_{(t)}}{(1 + \varepsilon)} \quad \dots (3.53)$$

$$\varepsilon = \frac{du}{dx} \quad \dots (3.54)$$

According to the equations (3.51), (3.52) & (3.53) the information matrix old time $\{f(\Delta S)\}$, the characteristics joint matrix $[C_J(\Delta S)]$ and the transformation matrix $[T]$ for forward characteristic lines $[C^+ \text{ equation}]$:

$$\{f'\} = \left\{ \begin{array}{c} F_L - \rho a_L C U_L \\ Q_L' - \rho a_L C_s V_L' - \rho a_L C_s \frac{\Delta S}{2} \psi_L' \\ M_L - \rho I_L C \psi_L - \frac{\Delta S}{2} (f_1 + Q_L) \end{array} \right\} \quad \dots (3.55)$$

$$[C_e] = \begin{Bmatrix} \rho a_L C & 0 & 0 \\ 0 & \rho a_L C_s & -\rho a_L C_s \frac{\Delta S}{2} \\ 0 & -\rho a_L C_s \frac{\Delta S}{2} & \rho I_L C + \rho a_L C_s \frac{\Delta S^2}{4} \end{Bmatrix} \quad \dots (3.56)$$

$$[T] = \begin{bmatrix} \cos \Psi_L & \sin \Psi_L & 0 \\ -\sin \Psi_L & \cos \Psi_L & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \dots (3.57)$$

Where:

$$f_1 = Q_L' - \rho a_L C_s V_L' - \rho a_L C_s \frac{\Delta S}{2} \psi_L' \quad \dots (3.58)$$

For backward characteristic lines [C^+ equation]:

$$\{f^-\} = \begin{Bmatrix} -F_R - \rho a_R C U_R \\ -Q_R' - \rho a_R C_s V_R' + \rho a_R C_s \frac{\Delta S}{2} \psi_R' \\ -M_R - \rho I_R C \psi_R - \frac{\Delta S}{2} (f_1 + Q_R) \end{Bmatrix} \quad \dots (3.59)$$

$$[C_e] = \begin{Bmatrix} \rho a_R C & 0 & 0 \\ 0 & \rho a_R C_s & \rho a_R C_s \frac{\Delta S}{2} \\ 0 & \rho a_R C_s \frac{\Delta S}{2} & \rho I_R C + \rho a_R C_s \frac{\Delta S^2}{4} \end{Bmatrix} \quad \dots (3.60)$$

$$[T] = \begin{bmatrix} \cos \Psi_R & \sin \Psi_R & 0 \\ -\sin \Psi_R & \cos \Psi_R & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \dots (3.61)$$

Where:

$$f_1 = Q_R' - \rho a_R C_s V_R' - \rho a_R C_s \frac{\Delta S}{2} \psi_R' \quad \dots (3.62)$$

The quantities Ψ , $\sin (\Psi)$, $\cos (\Psi)$ in equations (3.57) & (3.61) are expressed in terms of u & v by **(Reid et al, 1998)⁽⁵⁹⁾**,

$$\Psi = \tan^{-1} \left(\frac{dv/dx}{1 + du/dx} \right) \quad \dots (3.63)$$

$$\sin (\Psi) = \frac{dv/dx}{\sqrt{(dv/dx)^2 + (1 + du/dx)^2}} \quad \dots (3.64)$$

$$\cos (\Psi) = \frac{1 + du/dx}{\sqrt{(dv/dx)^2 + (1 + du/dx)^2}} \quad \dots (3.65)$$

3.6 The Boundary Conditions

At an unrestrained free joint (point A) as shown in Figure (3.13 a), the loads may be known and the loading condition $\{P_{(t)}\}$ is equal to the applied external excitations. Else, the loads are equal to zero since they are located in the free surface. The velocity $\{V\}$ at the joint can be obtained from the equilibrium equation (3.61). At the support (point B) as shown in Figure (3.13 b), only one of the characteristic equations, either along positive ($s=+1$) or along negative ($s=-1$) direction, is available. The second equation must be therefore provided to describe the boundary and solved simultaneously with the characteristic equation. For example, the velocities (axial, lateral and angular) are stipulated at the fixed end (i.e. $U_P = U(t)$, $V_P = V(t)$ and $\psi_P = \psi(t)$). Else, the velocities are equal to zero. Discontinuity in cross-section and non-homogeneity of material properties represent special types of internal boundary conditions, because in these cases, a part of wave will be reflected and other part will be transmitted.

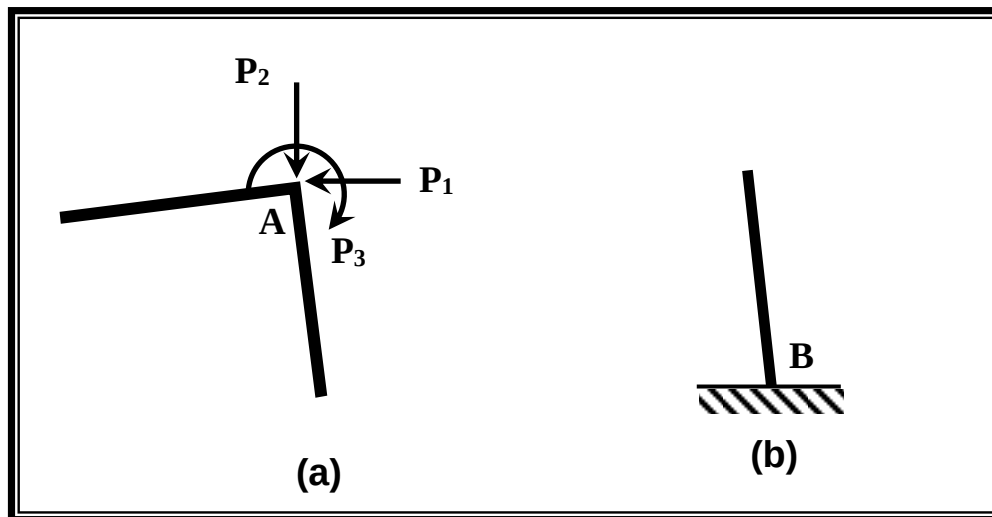


Figure (3.13): Frame joints.

(a) Free joint (b) Fixed joint

3.7 Interpolation Methods

In the characteristic equations, it is assumed that all values of relevant parameters are known on each characteristic line at the old time level. In practice, however, this will be true only for the lines that emanate from points such as L & R in Figure (3.16). It is not true for any other lines. In this case, the relevant values must be obtained by interpolation between the points at which the conditions are known. There are three explicit methods of interpolation schemes within a fixed grid solution:

1. ***x-wise interpolation***, in which values are interpolated between two adjacent grid points at the same time level as shown in Figure (3.15 a). This type of interpolation is used by **Jwad (1992)⁽⁴¹⁾** and by **AL-Haidar (2005)⁽⁵⁾**
2. ***t-wise interpolation***, in which values are interpolated between different time levels information for the same grid as shown in Figure (3.15 b). This type is used by **Goldberg and Wylie (1983)⁽³⁵⁾**, **Chan (1983)⁽²³⁾**, **Vardy & AL-Sarraaj (1989)⁽⁷²⁾**, **AL-Sarraaj (1989)⁽¹¹⁾**, **AL-Da'ami (1992)⁽⁴⁾** and by **Ahmed (2003)⁽²⁾**.
3. ***x-t wise interpolation***, in which values are interpolated by using adjacent grid points and different time levels as shown in Figure (3.15 c). This type is used by **Chou and Mortimer (1967)⁽²⁵⁾**, and by **AL-Mousawi (1983)⁽⁷⁾**.

The interpolation technique is also needed for a large deformation analysis, where the grid points are changed considerably. This will lead to a change in grid lengths and interpolation is introduced to find the old time information.

Each of these methods will give an exact approximation only when the real distribution happens to be the same as the assumed distribution, namely linear in our case. In practice, this condition will rarely apply and so the interpolations are the one of the main sources of numerical error and can lead to considerable numerical dispersion. These errors cannot be eliminated totally from the numerical process, but their importance can be reduce by refining the numerical grid or by changing the order of the interpolation function , but it cannot general be eliminated in the dynamic case **(AL-Sarraj,1989)⁽¹¹⁾**.

In the present study x -wise interpolation is used. The other types of interpolation cannot be used, because the time required for the wave to propagate between two grid points is changed during the deformation process. It will be more or less than the original time interval depending on the sign of deformation.

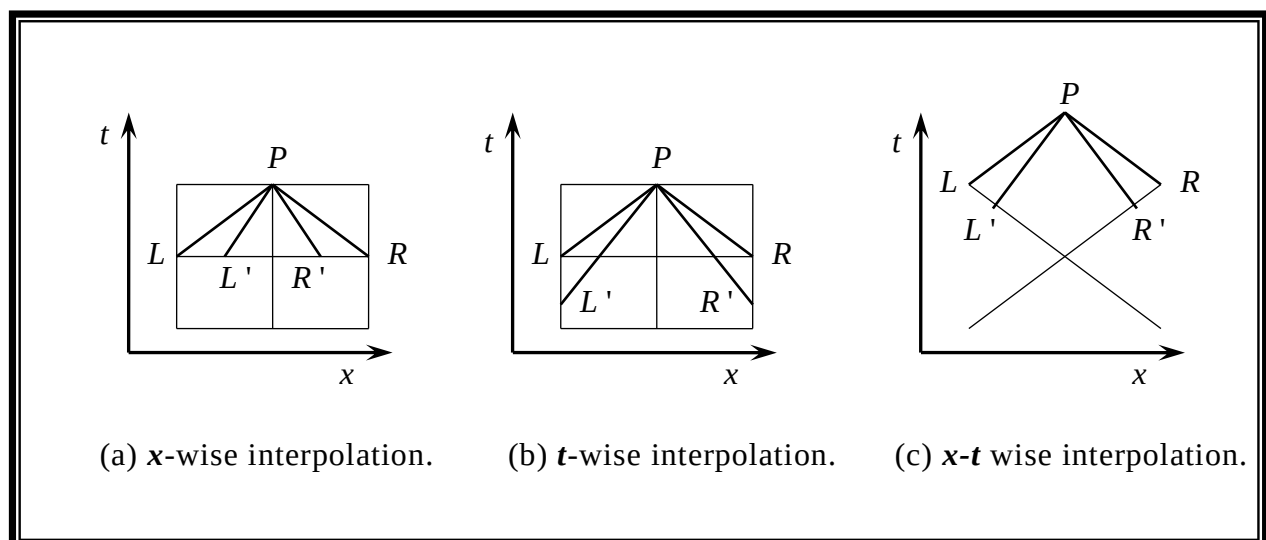


Figure (3.15): Methods of interpolation.

3.8 Displacement and Acceleration

In the characteristic equations, the forces are related to the velocities, not to the displacements or accelerations. However, displacements and accelerations can be estimated numerically such as:

$$u_P \cong U_P \Delta t + u_D \quad \dots (3.66)$$

$$\ddot{u}_P \cong \frac{1}{\Delta t} (U_P - U_D) \quad \dots (3.67)$$

where $\ddot{u}_P$ is the acceleration.

Neither displacement nor acceleration is obtained as accurately as velocity because the finite difference equations (3.66) & (3.67) are not exact (**Rittel, 1991**)⁽⁶²⁾. The above finite difference can be used after the velocity has been obtained directly from the solution.

3.9

Accuracy and Stability of the Characteristics Method

The investigation of accuracy and stability is of a great practical interest in numerical solutions such as the method of characteristics, which employs the finite difference approximation for the governing equations along the characteristic lines.

In 1952, **Courant, et al.**⁽⁵²⁾ presented a difference method for the solution of hyperbolic partial differential equation of first order. They showed that the error was of type (Δt^2) .

Stability criterion is always necessary to prevent the errors from amplifying so much as to make the calculations meaningless. The *stability criterion* is usually an amount of a restriction on the permissible size of Δt in terms of the size of the space increment Δx .

In one-dimensional problems, the method of characteristics is inherently stable because it always adheres to the **C-F-L** (*Courant, Friedrichs and Lewy*) stability criterion (**Courant, et al., 1952**)⁽⁵²⁾, which requires that,

$$\frac{\Delta x}{C\Delta t} \geq 1 \quad \dots (3.68)$$

where C is the larger of the wave velocity.

The central difference approximations as employed in the characteristics method are found to be stable, and maximum accuracy is obtained by taking,

$$\Delta t = \frac{\Delta x}{C} \quad \dots (3.69)$$

The choice of the correct mesh size has to be decided for each individual problem, depending upon the type of loading and the rise time, the position of discontinuity (in case of stepped and different materials beam).

CHAPTEOR FOUR

COMPUTER PROGRAM

APPLICATIONS AND DISCUSSION

4.1 Introduction

The computer program (**LDDAPF**) LARGE DEFORMATION DYNAMIC ANALYSIS OF PLANE FRAMES that is developed in the present study to carry out the large deformation analysis of plane frames consist from straight members under dynamic load by using the characteristics method and it is developed to carry out the t -wise, x - t wise and x -wise interpolation. The computer program (**LDDAPF**) has many properties and is designed to deal with many cases. The properties of the program may be summarized as follows:

1. The program is designed to deal with the transient response of prismatic and non-prismatic members.
2. The program can be used with any type of loading i.e. axial, shear, and bending moment individually or in combination.
3. The program can be used with any function of loading i.e. step, terminated ramp, triangle, and impact.
4. Specifying the type of interpolation that is used to obtain the relevant values between two adjacent points at the old time level.
5. The time step of the integration depends upon the length of the member of grid which is calculated automatically by the program.

The results for several examples will be checked with other researches.

The input data of the program consists of the overall member geometry, material properties, type of the boundary conditions, and the type of the transient loading.

The output of the computer program includes the time history distributions of the axial force, shear force, bending moment, axial velocity, transverse velocity, angular velocity and their corresponding displacements.

4.2 Structure of Program

The **(LDDAPF)** computer program is coded in **Fortran-77** language. It consists of a main routine and many subroutines. Each one of these subroutines has been designed to deal with part of the analysis.

The flow-chart of the program is given in Figure (4.1).

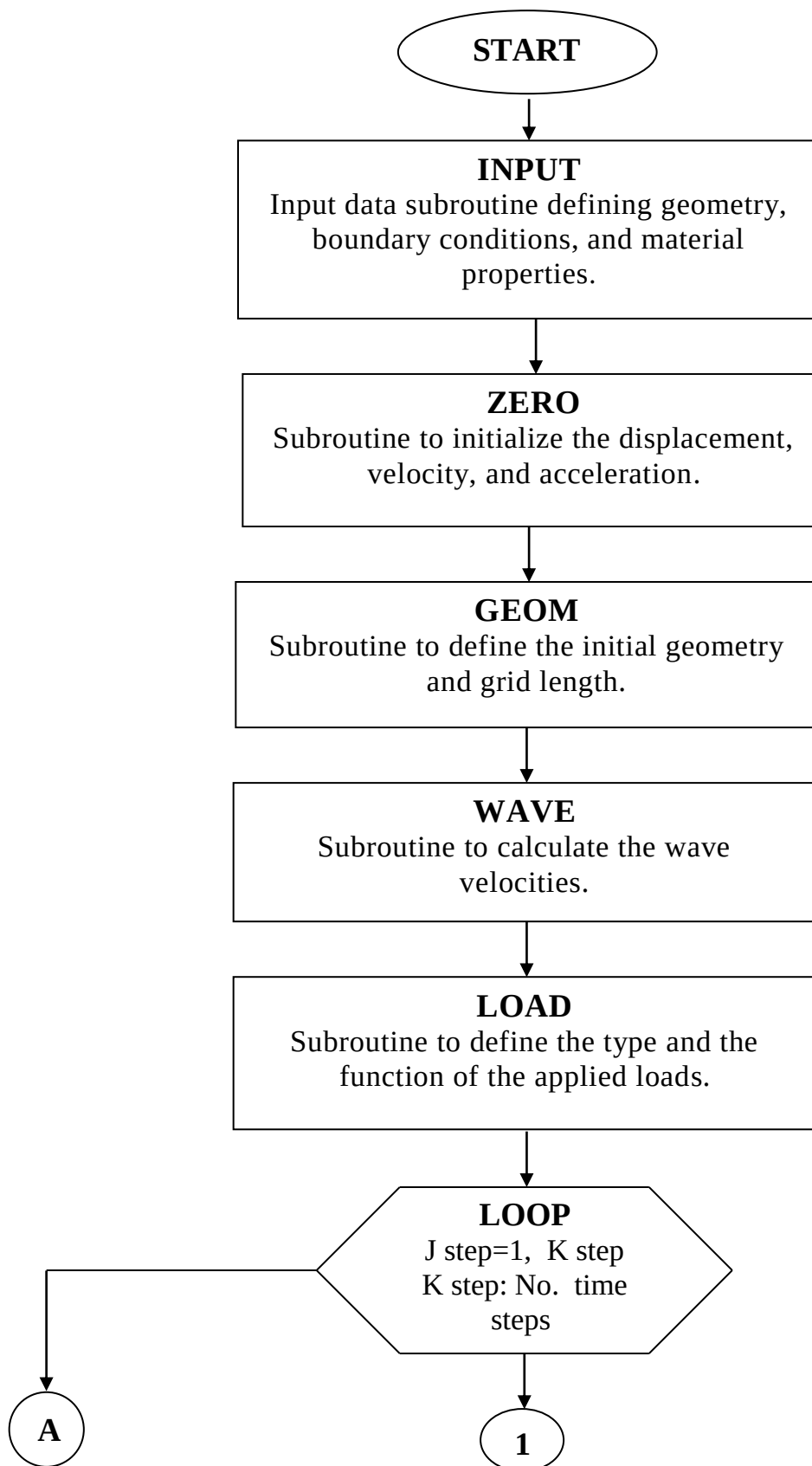
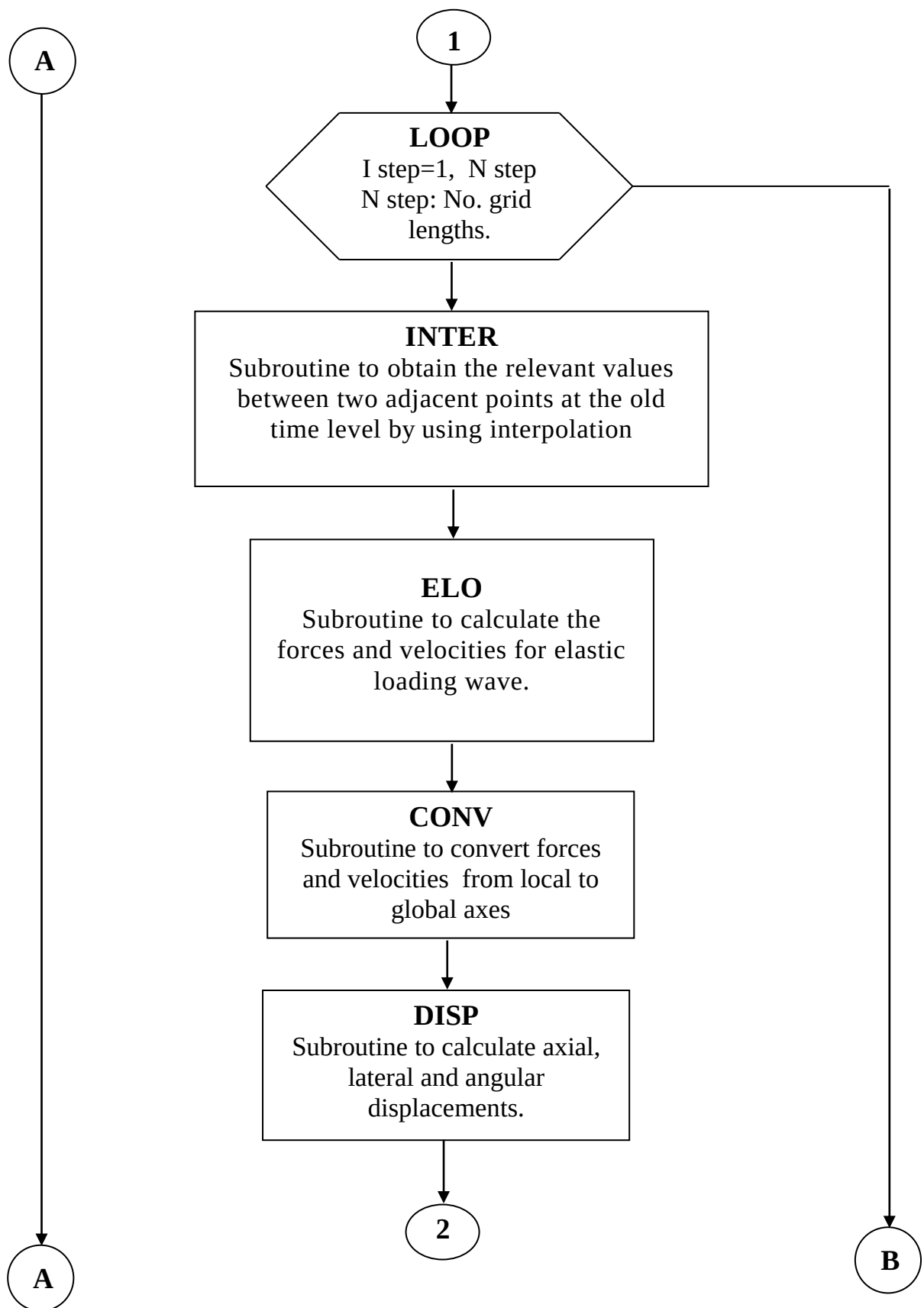
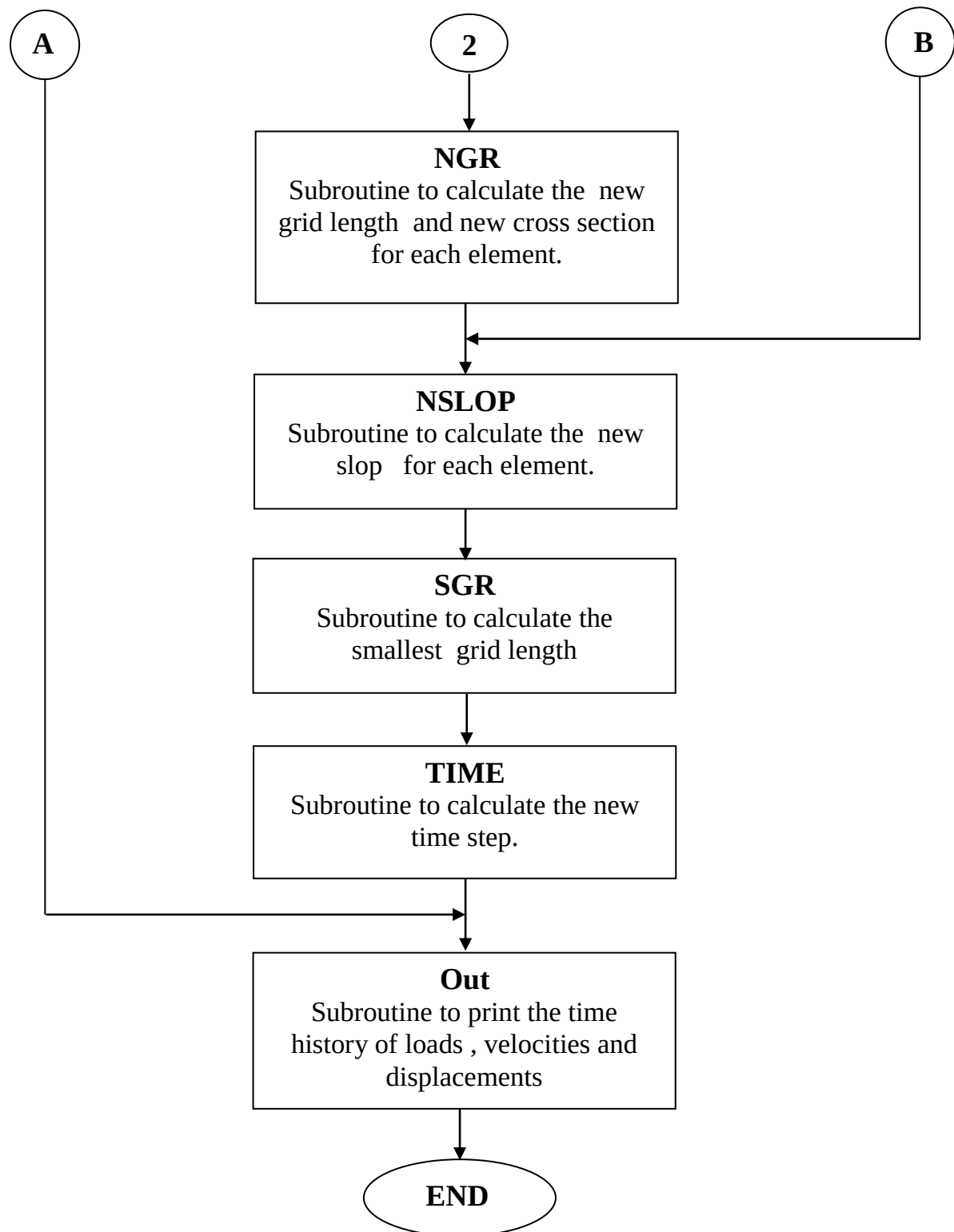


Figure (4.1): The flow-chart of the computer program (**LDDAPF**).

**Figure (4.1):** Continued.

**Figure (4.1):** Continued.

4.3 The Effect of Mesh Size and Solution Stability

To study the effect of mesh size on the elastic response of the plane frames, a prismatic cantilever beam with square cross section is analyzed. All the details of beam's properties and the dynamic characteristics are given in Figure (4.2).

Figures (4.10 a) to (4.10 c) show the effect of grid length for the three types of interpolation, namely t -wise, x -wise and x - t wise, on the shear response of the cantilever. It can be noticed that the grid length has not a significant effect on the response when t -wise interpolation is used, while this effect is very clear in x -wise and x - t wise interpolations. The cause of that t -wise interpolation is more accurate than other types of interpolation comes from the fact that the time-step is smaller than length-step, and this phenomenon has a considerable effect on the interpolated points. Moreover, the smaller mesh size will give more precise results, but it requires a longer processing time in the computer. However, with the rapid increases in processing speed of modern computer, this is not considered to be a serious limitation.

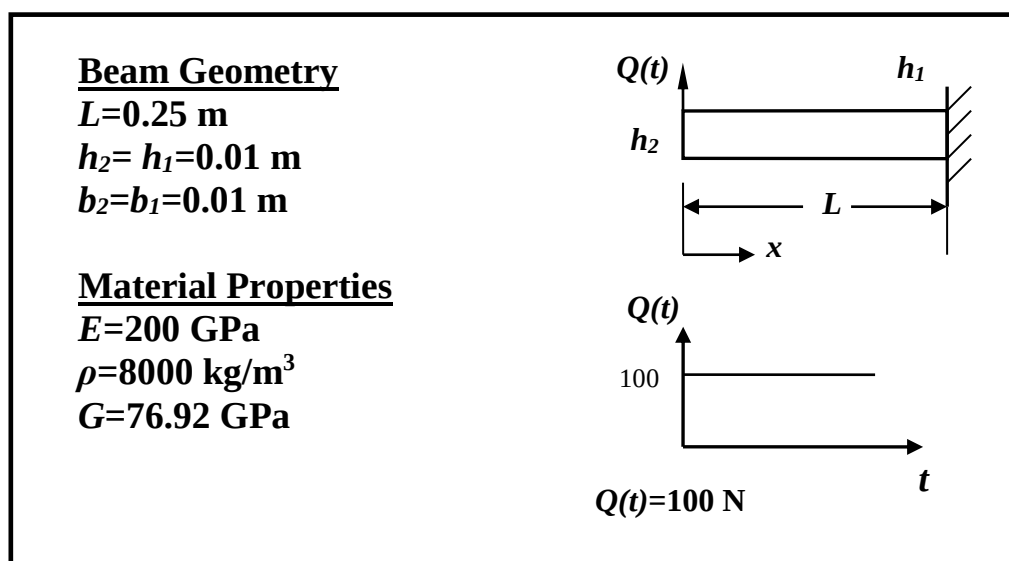


Figure (4.2): Details of beam's properties and dynamic characteristics.

4.4 Wave Behaviour

If a load is applied to any part of structure, its effect spreads out by the propagation of waves to the rest of structure, causing it to vibrate. The method of characteristics (**MOC**) is used to simulate response of plane frame to suddenly imposed constant load, impulsive load and periodic excitation. When the applied load is constant, the structure will vibrate at its natural frequency. It is eventually settles down to a steady state condition, because of the energy losses associated with the propagation of the waves. A simple cantilever beam is used. This simple structure helps to understand the complex behavior of more complex structure.

4.4.1 Axial Vibration

When one end of the member is disturbed, the disturbance propagates to the other end, where it is reflected back again. These reflections are mainly dependent upon the type of the boundary condition. The nature of the waves and their interactions with boundaries may determine the overall dynamic behavior of the structure.

In order to study the transient behavior of cantilever beam, a non-prismatic cantilever beam with rectangular cross section is analyzed. All the details of the beam's properties and the dynamic characteristics are given in Figure (4.3).

Figure (4.11) shows the behavior of prismatic member ($h_1=h_2=0.01\text{m}$) as compared with the behavior of non-prismatic member ($h_1/h_2=1.5$). It can be noticed that the form of axial waves will vary as the wave propagates in non-prismatic members.

The uncoupled axial waves are propagated in prismatic members in an unchanging form. Such waves are called non-dispersive waves and their behavior

is closely analogous to fluid pressure wave in pipe lines [Chan (1983)⁽²³⁾, AL-Sarraj (1989)⁽¹¹⁾].

In prismatic member, when wave reaches a fixed end the force rises and the dynamic intensification factor is equal to (2.0) but in non-prismatic member is larger than (2.0). In this case the dynamic intensification factor is equal to (2.4), this is because the effect of wave reflection.

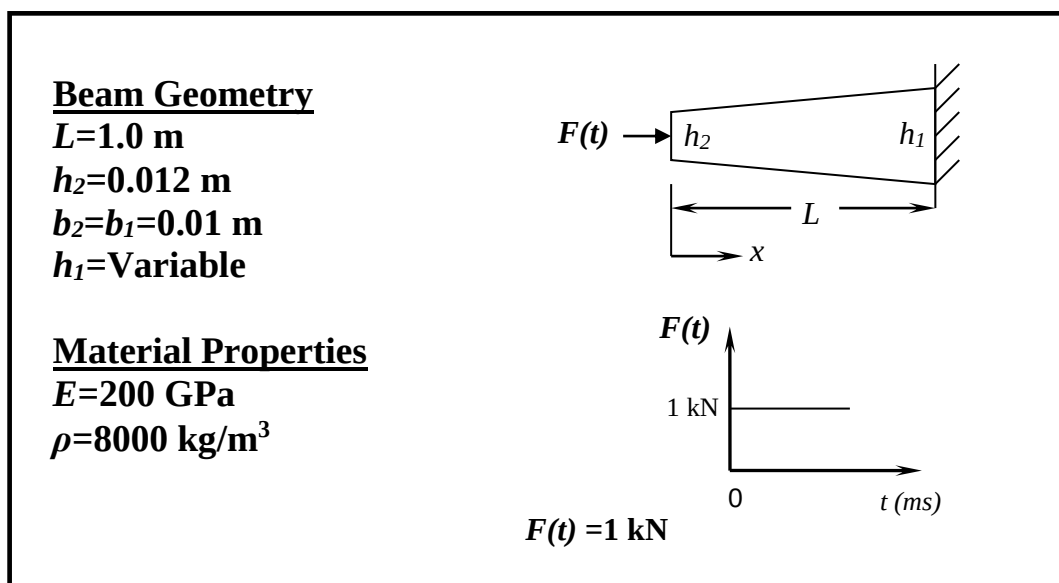


Figure (4.3): Details of beam's properties and dynamic characteristics

4.4.2 Flexural Vibration

The simulation of flexural waves is of greater engineering importance than that of axial waves. In flexural waves, the interaction between shear and bending waves causes the flexural waves to be dispersive as it propagates. Nevertheless, the overall vibration is repetitive in a similar manner to that found for axial waves.

To study the flexural wave of structure members, a non-prismatic cantilever beam with rectangular cross section is analyzed. All the details of beam properties and the dynamic characteristics are given in Figure (4.4).

The flexural wave is not only dependent upon the material properties, but also on the sectional properties. For the same material properties, and sectional area, the larger radius of gyration the faster the propagation of flexural effects. In non-prismatic members the dispersion of the flexural waves is greater than in prismatic members. This phenomenon is clearly shown in Figure (4.12) and Figure (4.13), in which the flexural waves are observed in three cantilevers. Two of prismatic with the same cross sectional area equal to the mid span cross-sectional area of non-prismatic member but with different radius of gyration. the third beam is non-prismatic.

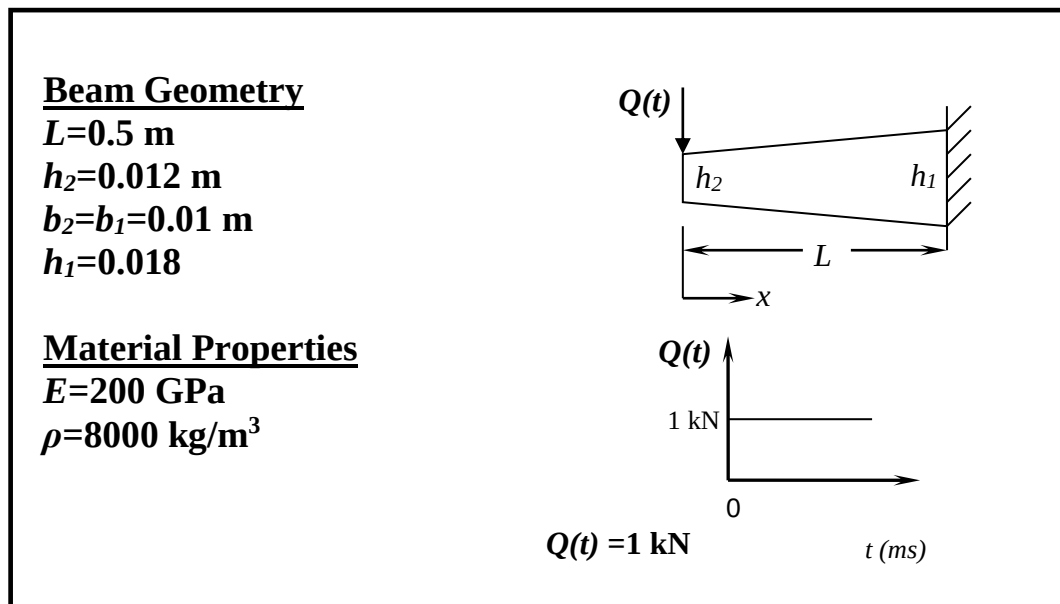


Figure (4.4): Details of beam's properties and dynamic characteristics

4.5 Numerical Examples

In order to verify the reliability of the computer program, some examples reported in previous researches are considered.

Example No. 1: (Semi-infinite Prismatic Beam)

To validate the computer program and the method of characteristics, the results of the present work are compared with experimental data obtained by **Ripperger in 1955⁽⁶¹⁾**, using strain gage technique for the eccentric impact of a steel ball of **6.35 mm** diameter on the cylindrical steel bar of **13.11 mm** diameter. The pulse shape for the applied bending moment was approximated for the numerical solutions by a half-sine wave. The pulse duration was assumed as **$t_o=14.28 \mu s$** and the sine input function was:

$$M(0, t) = 3.12726 \sin(\pi t / 14.28)$$

Where t is in μs .

The results are presented in non-dimensional form for the bending wave M_o versus T_o , where

$$M_o = \frac{Md}{EI}$$

$$T_o = \frac{C_e t}{d}$$

Where d is the bar diameter.

All the details of the bar geometry and applied load are given in Figure (4.5)

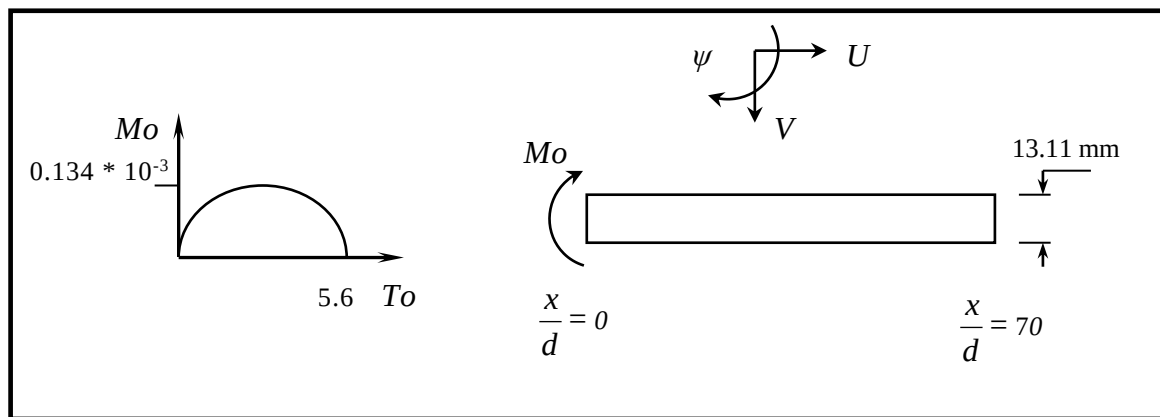


Figure (4.5): Details of bar geometry and dynamic characteristics .

The bar material properties are:

$$\rho = 7986.5 \text{ kg/m}^3, E = 211 \text{ GPa}, \nu = 0.3$$

The agreement between the numerical results obtained by the **(NLDAFP)** program and the experimental data of Ripperger is good, as shown in Figures (4.14) and (4.15) , for beam stations $x/d = 2$ and **6**.

The range of the time for the pulse bending moment was much shorter than the time that the reflected pulse takes to reach the considered positions and the response was the same as for a semi-infinite beam.

Example No. 2: (Prismatic Cantilever Beam)

The accuracy of the **LDDAPF** computer program was checked by comparing its results with an example solved by **Koenig** and **Davids** in 1968⁽⁴⁵⁾, using finite element method, for a prismatic cantilever beam subjected to a ramp platform bending moment at its free end, where the effect of the reflected waves in a very short cantilever beam of **38.1 mm** in length and **2.54 mm** square in cross section was examined. The value of **113 N.mm** end moment with a rise time $t_0 = 5.174 \text{ } \mu\text{s}$ was assumed. All the details of the beam geometry and applied load are given in Figure (4.6).

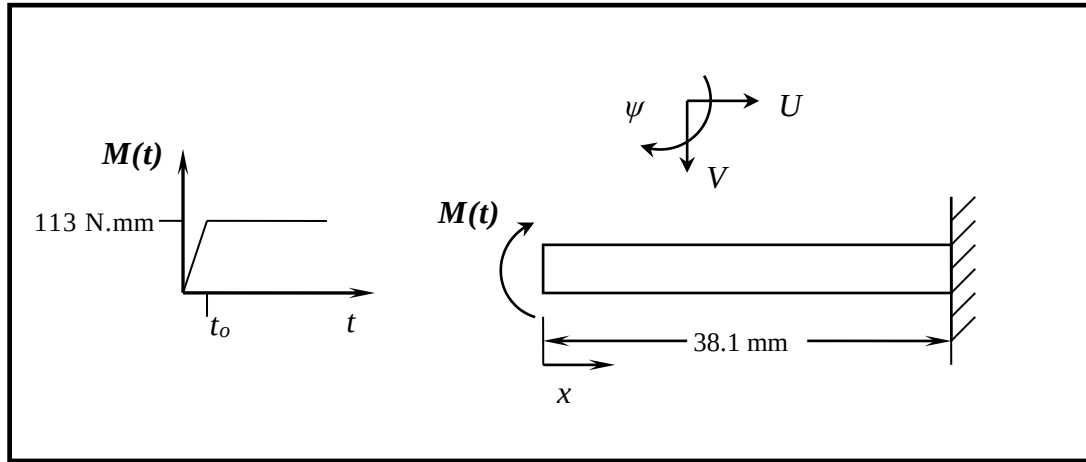


Figure (4.6): Details of bar geometry and type of loading.

The bar material properties that are used in this example are:

$$\rho = 8569.56 \text{ kg/m}^3, E = 206.67 \text{ GPa}, \nu = 0.3, t_0 = 5.174 \text{ }\mu\text{s}$$

Figures (4.16) and (4.17) show a comparison between the present study results with the results obtained by **Koenig** and **Davids** for the bending moment and shear force disturbances at position $x=12.7 \text{ mm}$ respectively, where x is measured from the free end of the cantilever beam. It is noticed that the bending moment behavior obtained from (M.O.C) is larger than that obtained from **Koenig** and **Davids** while the shear behavior is differ in manner because **Koenig** and **Davids** neglect the shear deformation.

The reflected bending wave from the built-in end of the cantilever beam arrives station $x = 12.7 \text{ mm}$ at time $t \cong 13 \text{ }\mu\text{s}$ after which oscillations with a large amplitude are noticed and the peak at $t \cong 23.5 \text{ }\mu\text{s}$ is considerably larger than the peak at $t \cong 13.5 \text{ }\mu\text{s}$ and those peaks are larger than the input bending moment. This phenomenon is caused by the dispersive behaviour of the flexural waves.

Example No. 3: (Non-prismatic Cantilever Beam)

The validity of the solution obtained by the method of characteristics can be checked by comparison with results from other methods of solution. The results of the two cases are illustrated in Figures (4.18 a,b) and (4.19 a,b). In the first, a single non-prismatic member is fixed at one end and free at the other, where a constant moment $M_o(t)$ is applied suddenly at the time $t=0$. In the second, the above non-prismatic cantilever beam is subjected to constant lateral force.

The solution as obtained by **Koenig** and **Berry** in 1975⁽⁴⁶⁾ is by using a finite element method. The agreement between the two methods is encouraging. All the details of the beam geometry and applied loads are given in Figures (4.7) and (4.8).

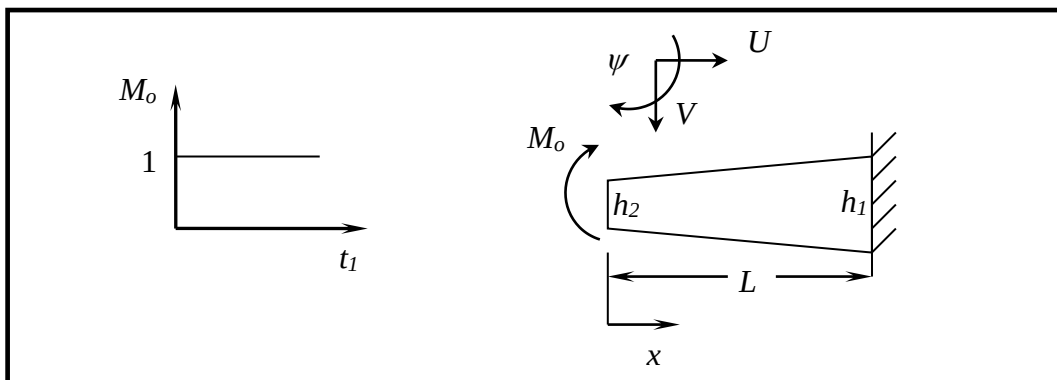


Figure (4.7): Details of bar geometry and dynamic characteristics.

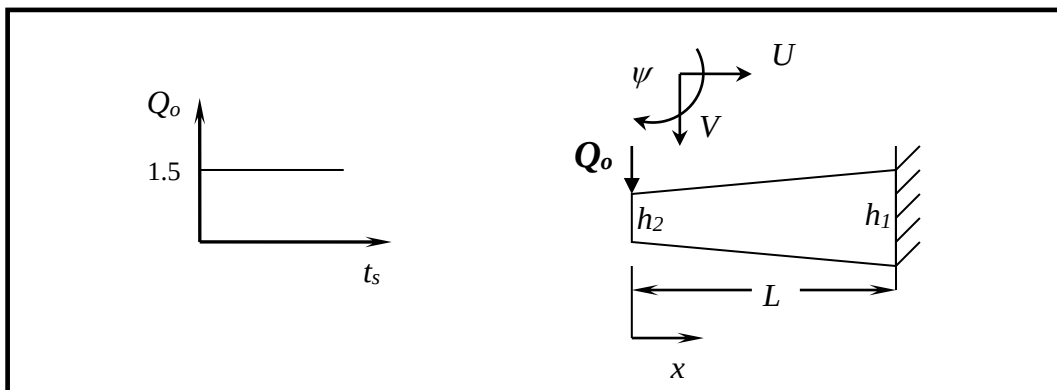


Figure (4.8): Details of bar geometry and dynamic characteristics.

Notations in Figures (4.7) and (4.8) are the same as that used by **Konig** and **Berry**, where the following dimensionless variables are used.

$$L_o = 1 \qquad Q_o = \frac{QL^2}{EI}$$

$$x_o = x / L \qquad M_o = \frac{ML}{EI}$$

$$t_1 = \frac{C_e t}{L} \qquad t_s = \frac{C_{es} t}{L}$$

the tested beam has the following neumerical values

$$\begin{aligned} K &= 0.83333 & L &= 3.81 \text{ cm} \\ E &= 206.67 \text{ GPa} & b &= 2.54 \text{ cm} \\ \rho &= 8569.56 \text{ kg / m}^3 & h_2 &= 2.54 \text{ cm} \\ v &= 0.3 & h_1 &= 7.62 \text{ cm} \end{aligned}$$

the results were measured at $x_o = 0.33333$.

Example No. 4: (Fixed ended Beam)

A fixed ended beam was chosen to cheked the validity of the solution obtained by the method of charactrestics by comparsion with results from other methods of solution.

The beam had been analyzed by **Hsiao** and **Jany** in 1989⁽⁴⁰⁾, using the finite element method and the consistent mass matrix for representing the mass properties the time step was chosen to be **(50μs)** ,they used an incremental-iterative method based on **Newmark** direct integration and the **N-R** method for solution of the non-linear dynamic equilibrium equations were used for this case .

Figuer (4.20) shows the comparison between the present study results with the results obtained by **Hsiao** and **Jany** for the vertical displacement at mid span .

The agreement btween the two methods is encouraging. All the details of the beam geometry and applied loads are given in Figures (4.9)

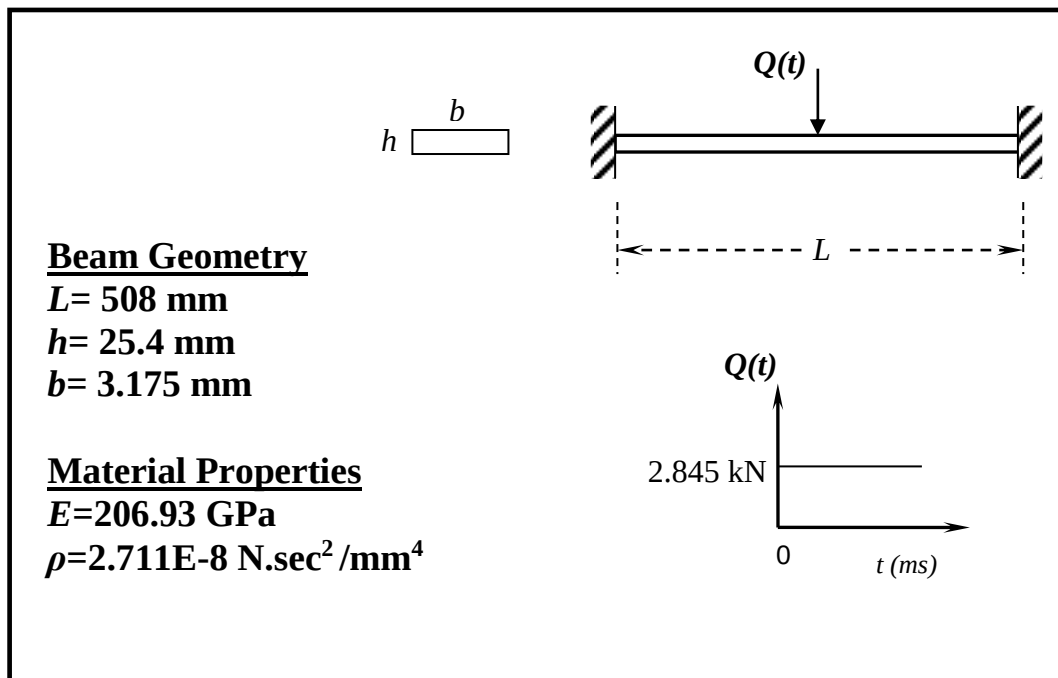


Figure (4.9): Details of fixed ended beam's geometry and dynamic characteristics.

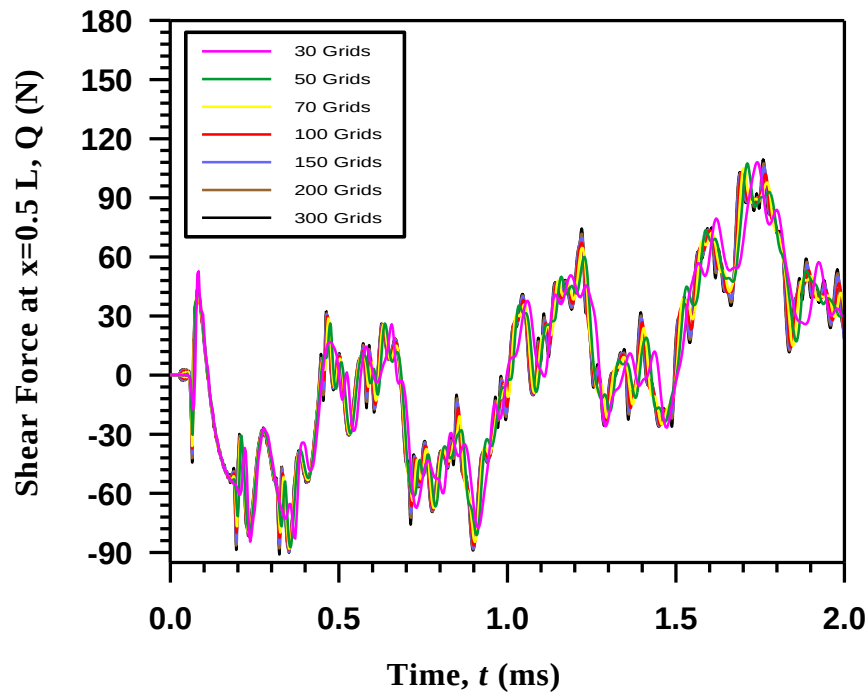


Figure (4.10 a): Effect of mesh size on the dynamic response of prismatic cantilever beam under shear force using t -wise interpolation.

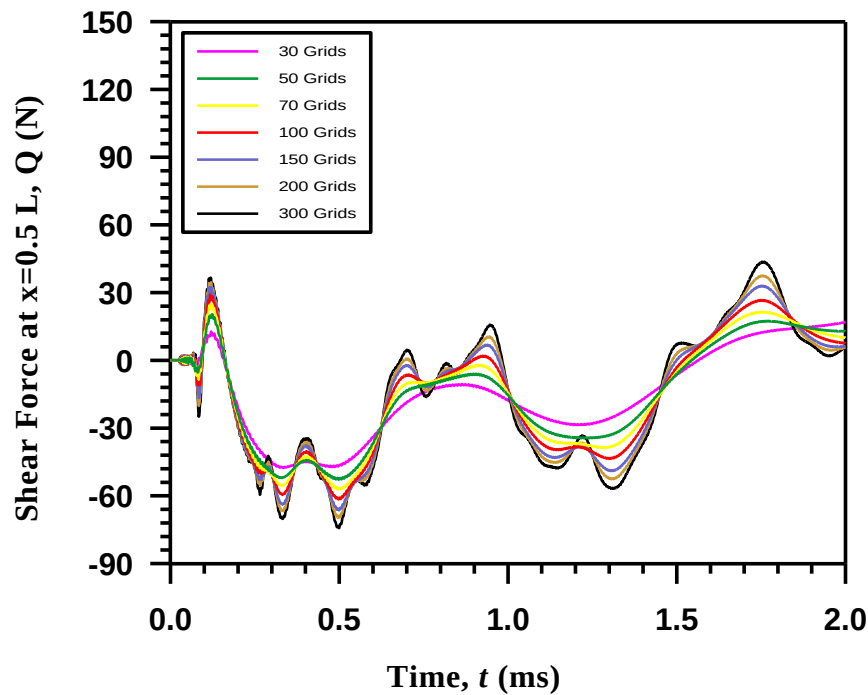


Figure (4.10 b): Effect of mesh size on the dynamic response of prismatic cantilever beam under shear force using x -wise interpolation.

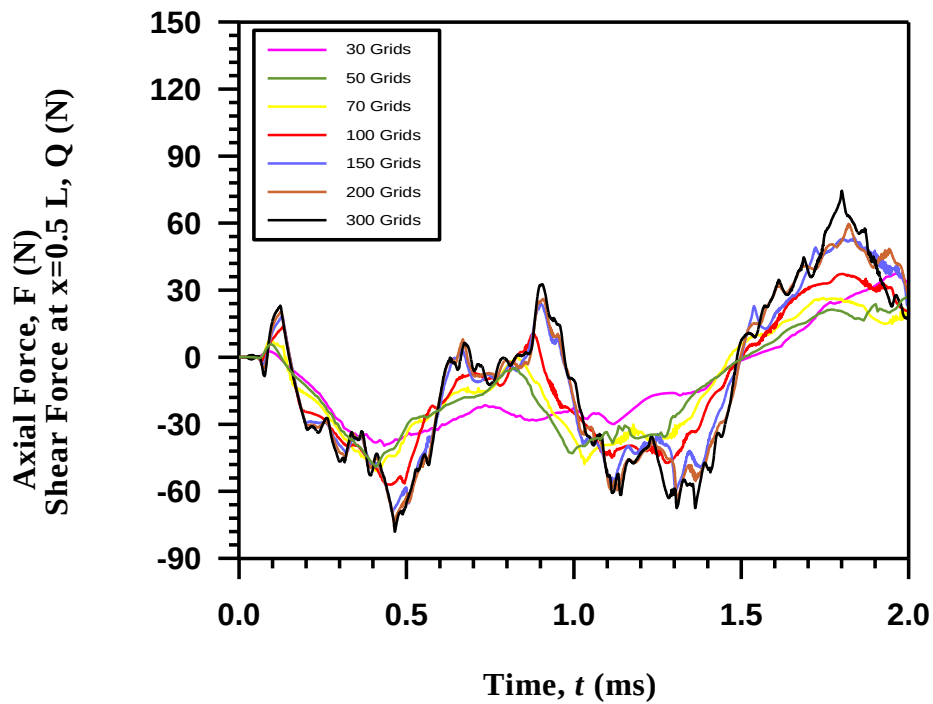


Figure (4.10 c): Effect of mesh size on the dynamic response of prismatic cantilever beam under shear force using x - t wise interpolation.

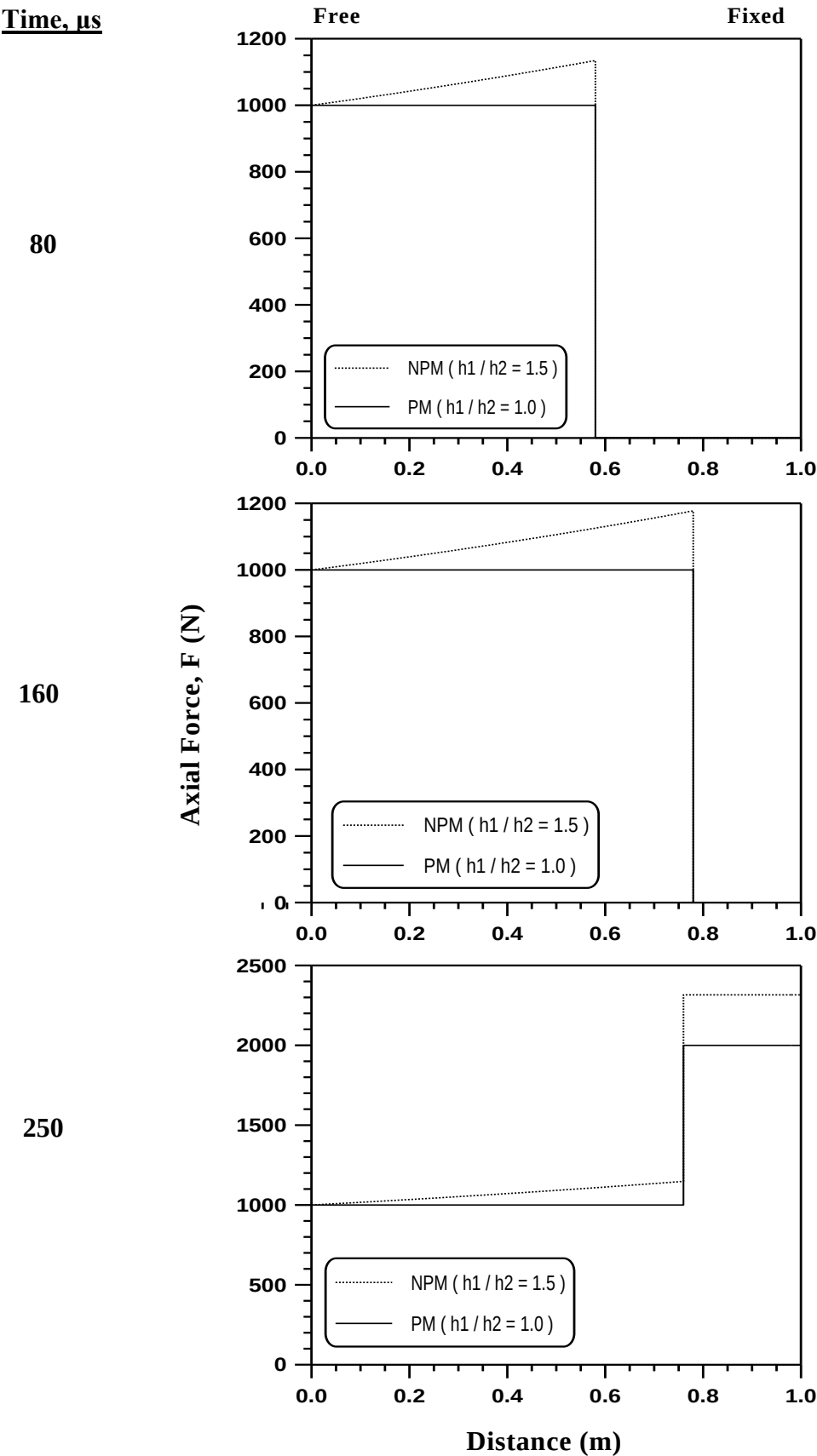


Figure (4.11): Propagation of axial wave in cantilever beam.

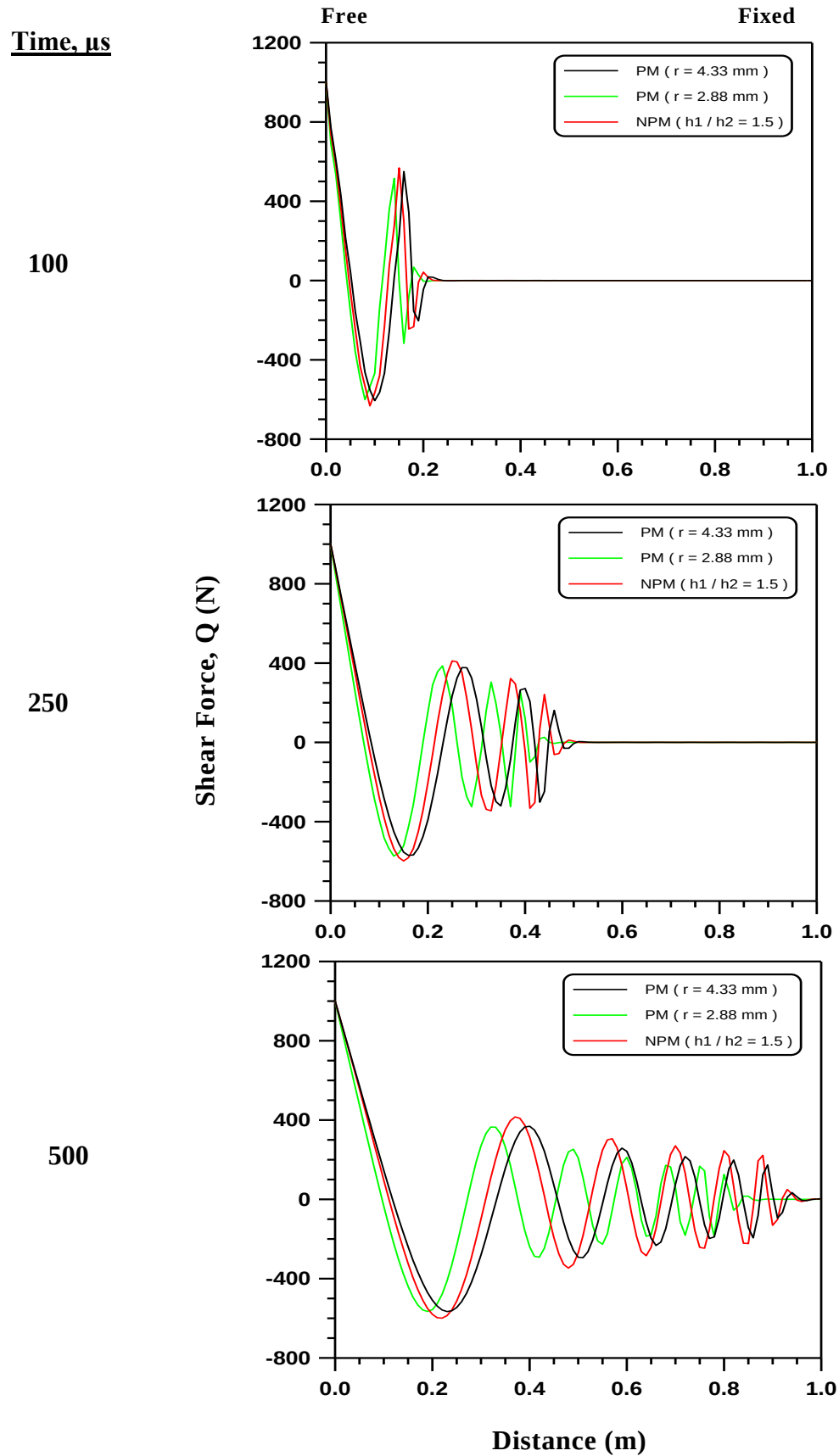


Figure (4.12): Propagation of shear wave in cantilever beam.

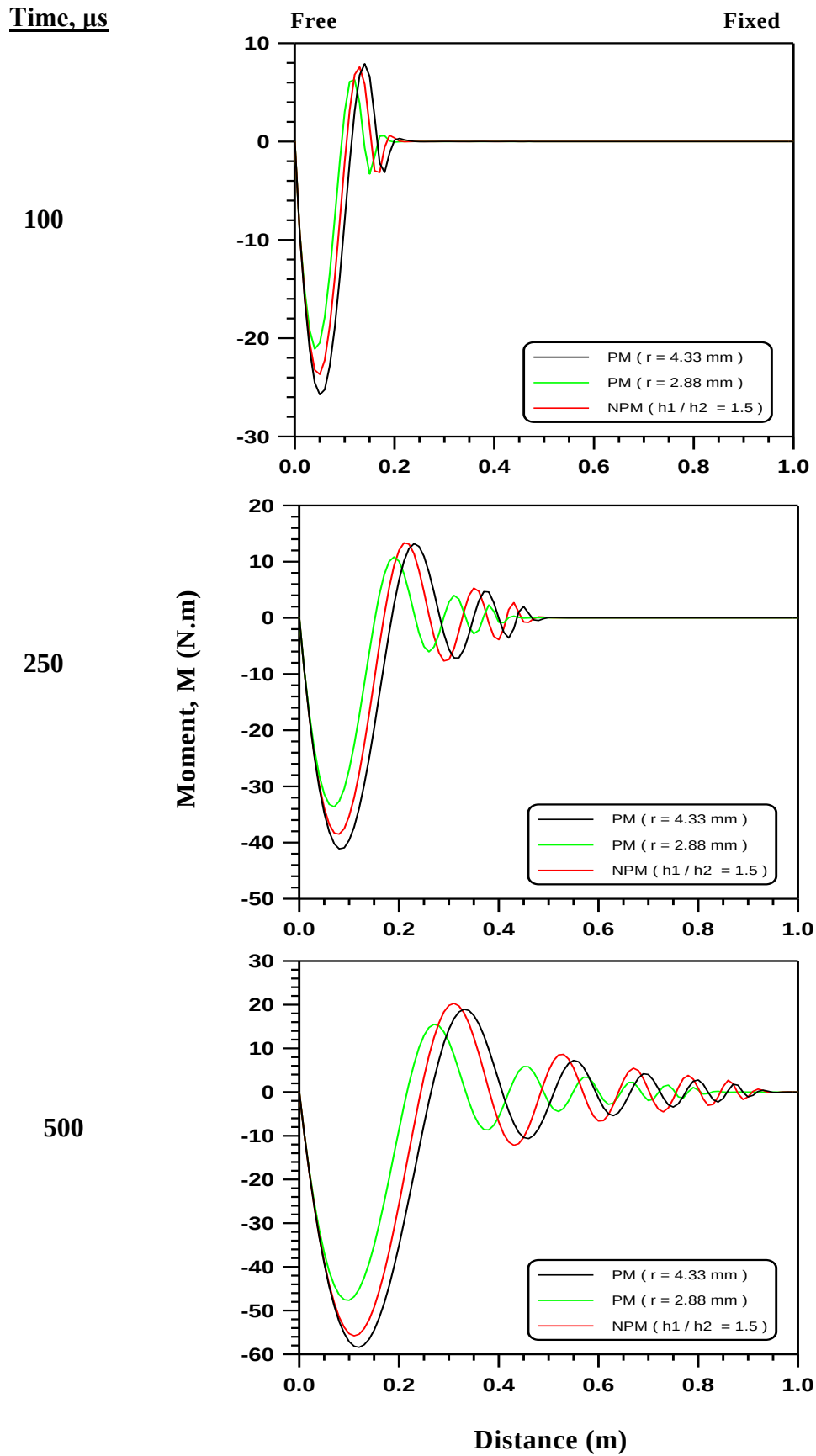


Figure (4.13): Propagation of bending wave in cantilever beam.

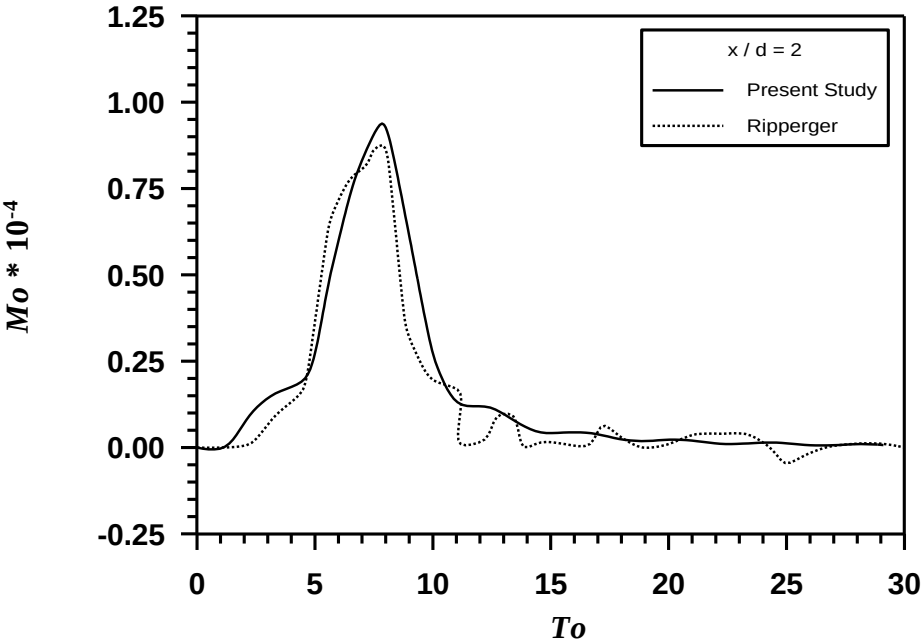


Figure (4.14): Bending moment history for the beam in (Ex.1) (Semi-infinite Prismatic Beam).

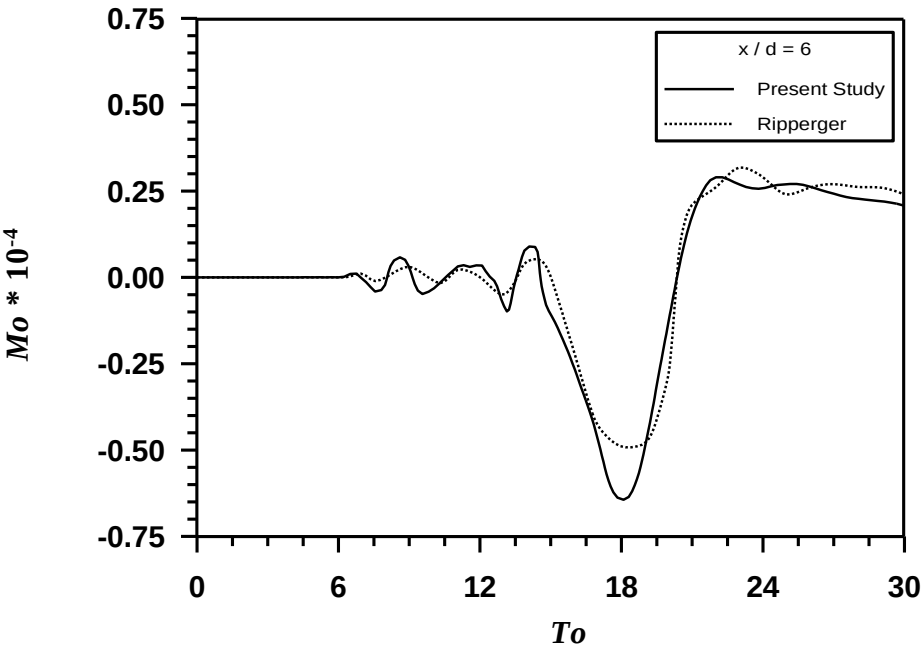


Figure (4.15): Bending moment history for the beam in (Ex.1) (Semi-infinite Prismatic Beam)

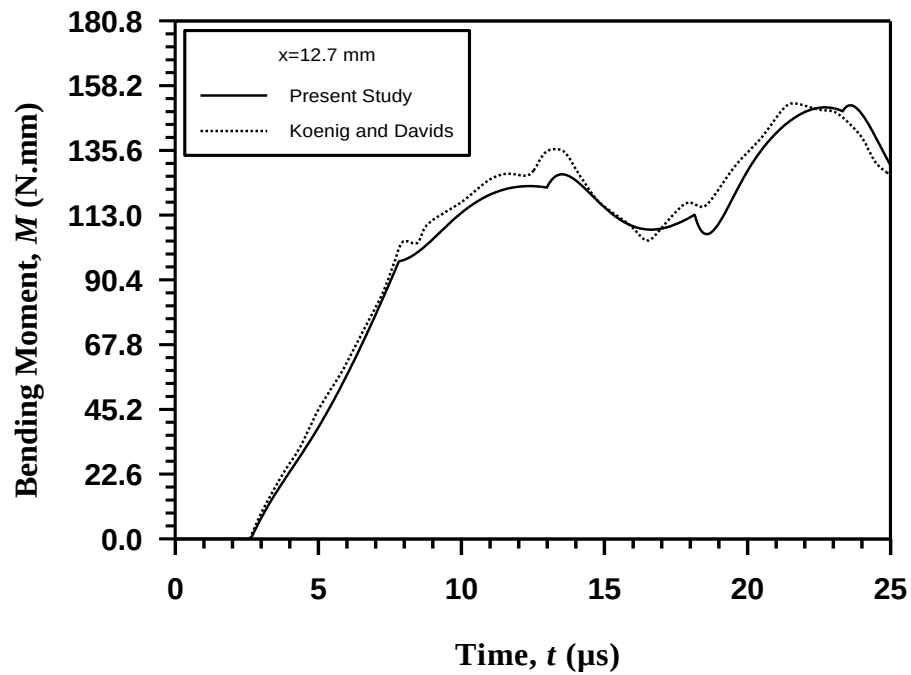


Figure (4.16): Bending moment history for the beam in (Ex.2) (Prismatic cantilever beam).

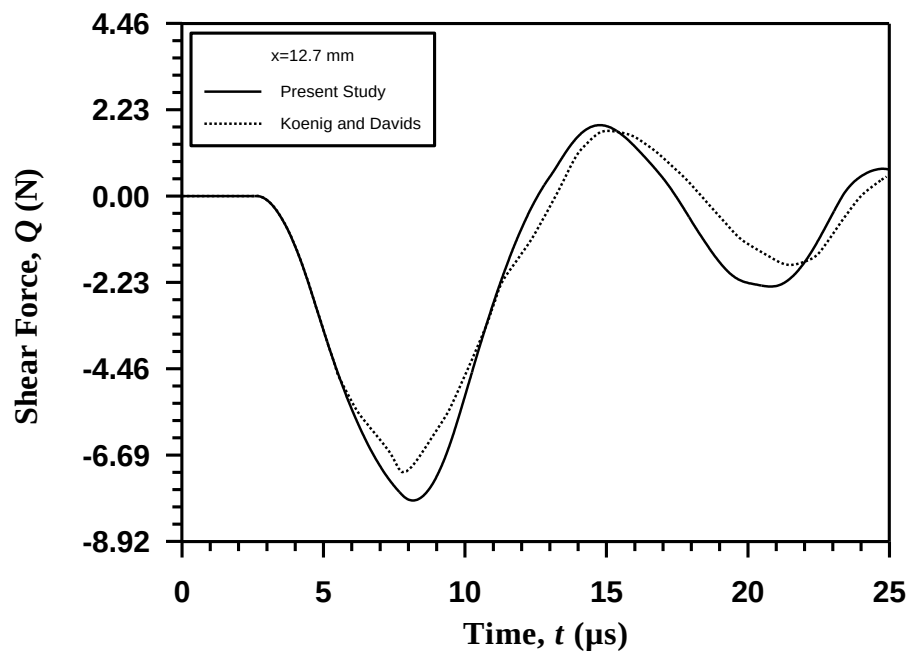


Figure (4.17): Shear force history for the beam in (Ex.2) (Prismatic cantilever beam).

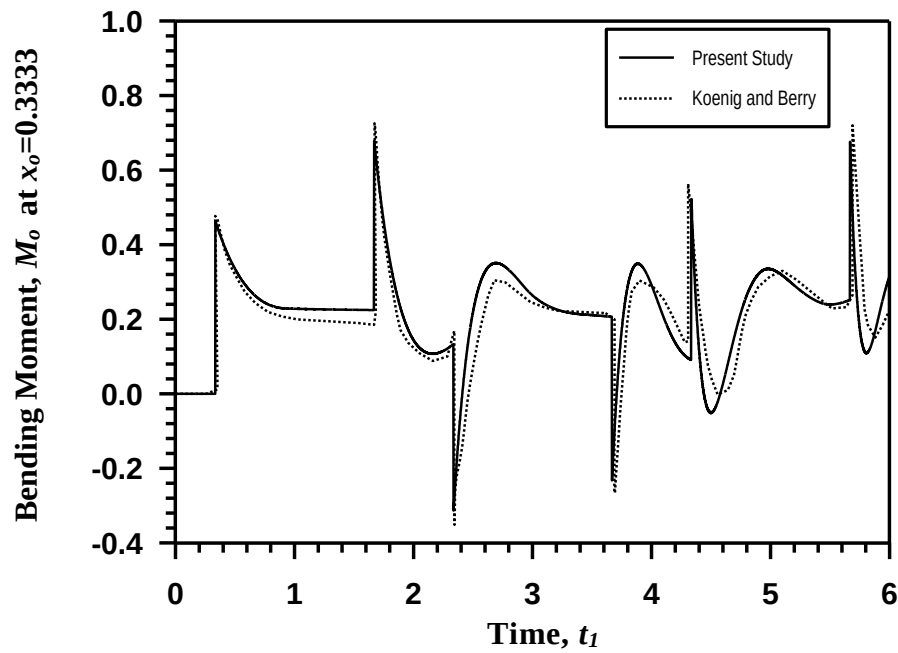


Figure (4.18 a): Bending moment history for the beam in (Ex.3) (Non-prismatic cantilever beam).

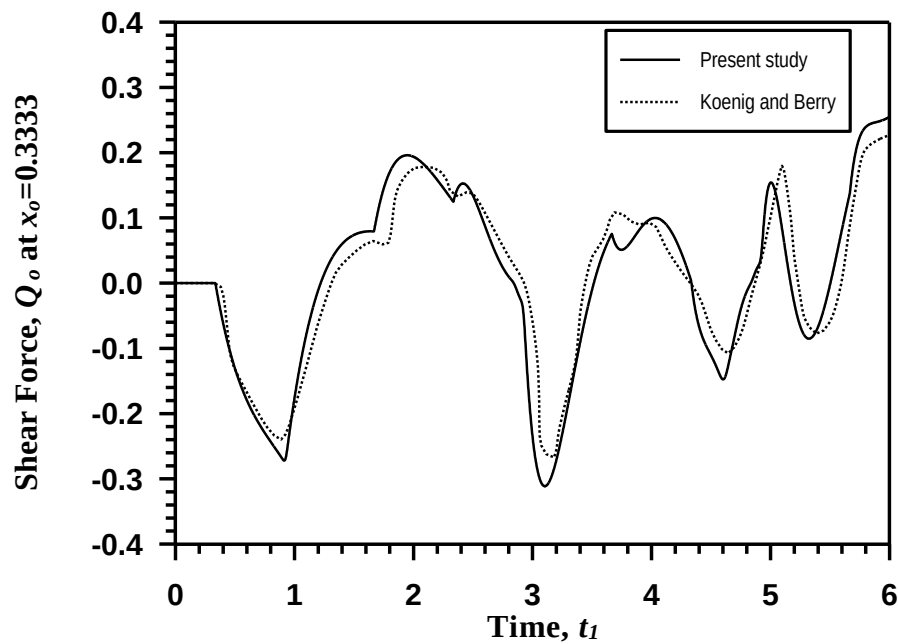


Figure (4.18 b): Shear force history for the beam in (Ex.3) (Non-prismatic cantilever beam).

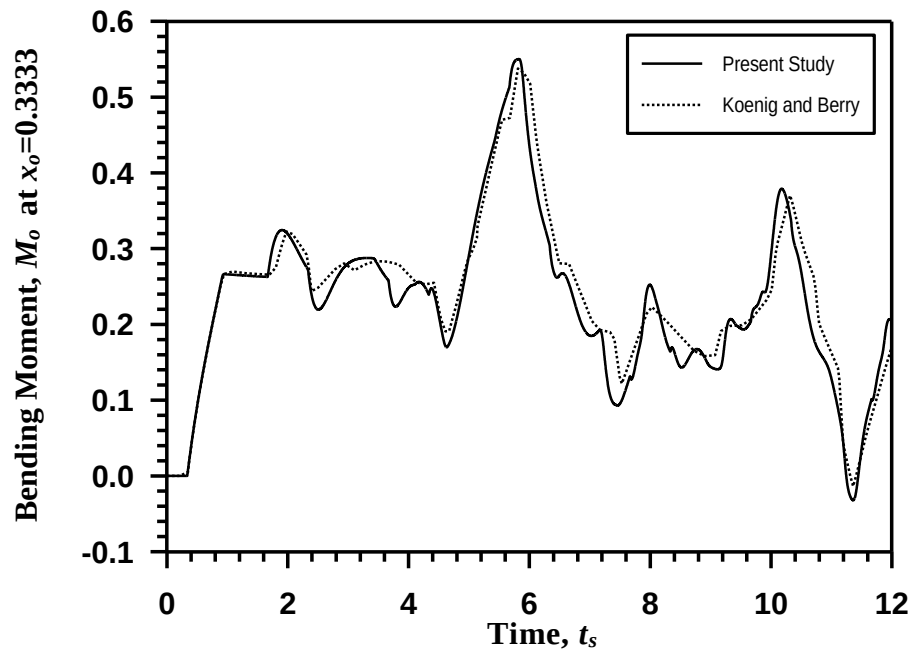


Figure (4.19 a): Bending moment history for the beam in (Ex.3) (Non-prismatic cantilever beam).

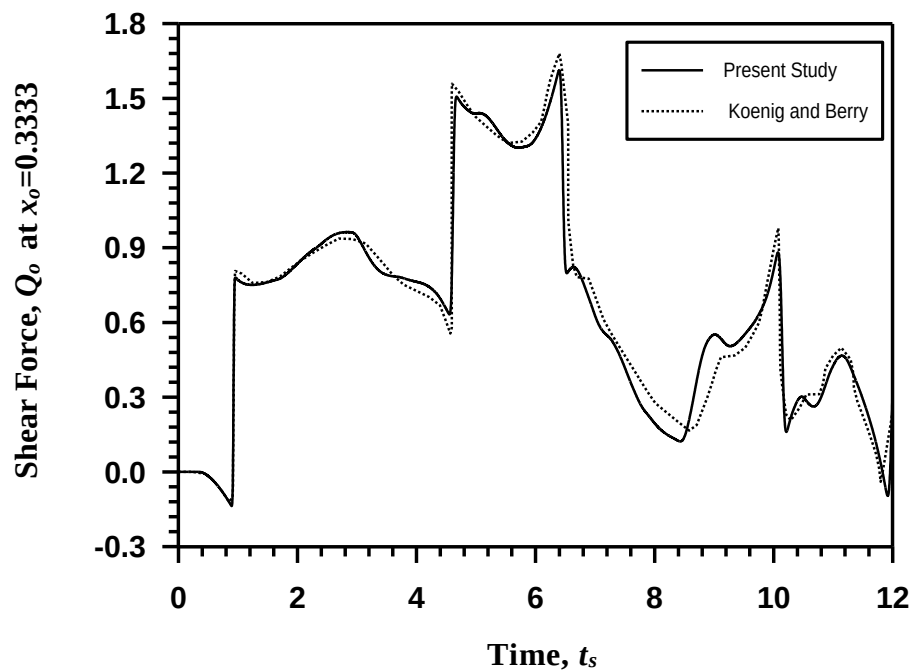


Figure (4.19 b): Shear force history for the beam in (Ex.3) (Non-prismatic cantilever beam).

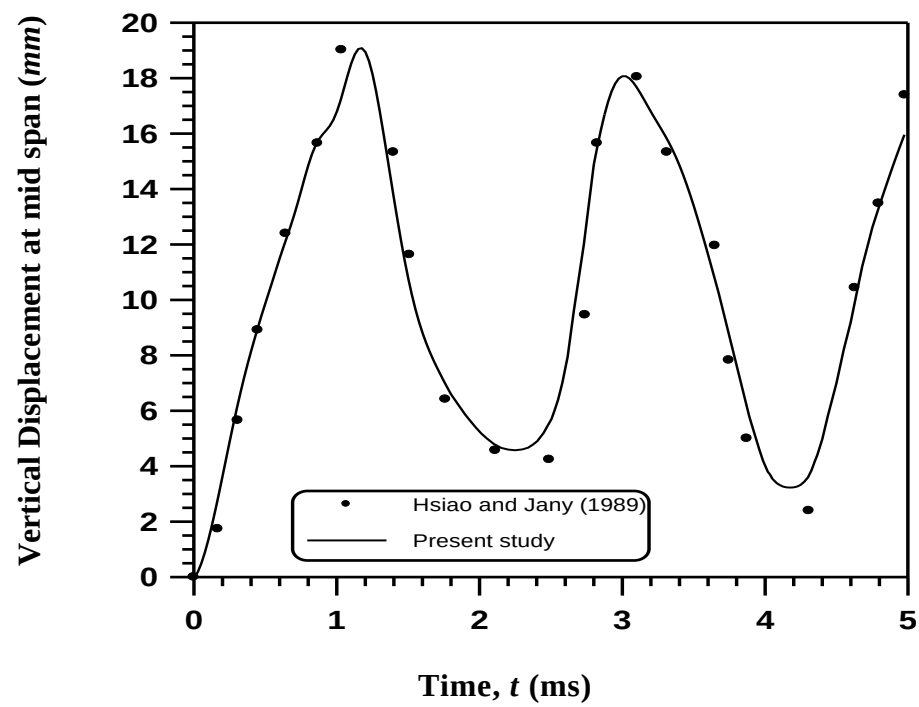


Figure (4.20): The dynamic response curves for the beam in (Ex.4) (Fixed ended beam).

CHAPTER FIVE

PARAMETRIC STUDY

5.1 Introduction

A parametric study is performed to assess the influence of several important parameters on the dynamic behavior of plane frame structures.

The selected parametric studies in this chapter can be summarized as follows:

1. The effect of tapered ratio.
2. The effect of different bonded materials.
3. The effect of sudden discontinuity in cross section.
4. The effect of large deformation behavior.
5. The effect of Poisson's ratio on nonlinear vibrations.
6. The effect of load intensity.

Each one of the above parameters is studied individually by analyzing the geometry of the member, material properties and type of external excitation.

5.2 The Effect of Tapered Ratio

To show the effect of tapered ratio on the elastic dynamic response of the structural member, a non-prismatic clamped beam with rectangular cross section is analyzed. All the details of the beam's properties and the dynamic characteristics are given in Figure (5.1).

Figure (5.2) and Figure (5.3) show the effect of changing the tapered ratio from **(0.333)** to **(3)**, with the same steel volume, on the dynamic response of the vertical displacement history at mid span and bending moment response at the fixed end for clamped beam respectively. The results can be discussed as follows:

It is clear from figure (5.2) that the dynamic behavior of the beam is not changed but the amplitude and time period of the response will be increased when tapered ratio decreased (i.e. $h_1/h_2 < 1$ see beam No.1) because of the stiffness will be increased at mid span (i.e. increased volume) but decreased when moving to supports.

Also, when tapered ratio will increase (i.e. $h_1/h_2 > 1$ see beam No.3), the amplitude will be increased because of the stiffness will be decreased at mid span. But the time period will be decreased because of the stiffness will be increased when moving to support.

From figure (5.3) it can be seen that the amplitude of the bending moment response will be increased and become more dispersive because of increasing the change in the cross section between any two adjacent elements and this leads to more disturbance characteristics caused by the wave reflection (wave reflection with the opposite sign when tapered ratio is less than **(1)** and with the same sign when tapered ratio is greater than **(1)**).

From figure (5.4) shows the effect of changing the tapered ratio from **(0.333)** to **(9)** on the maximum vertical displacement at mid span. It can be noticed that the best tapering ratio at with the greater stiffness (i.e. least vertical displacement) for this beam is equal to $(h_1/h_2 = 0.9125)$

Figure (5.5 a,b) explain the propagation of shear and bending moment waves in clamped beam with different values of tapered ratio (h_1/h_2) , it is clear from this figure that the effect of wave reflection on the amplitude of wave and wave speed increase as tapering ratio increases.

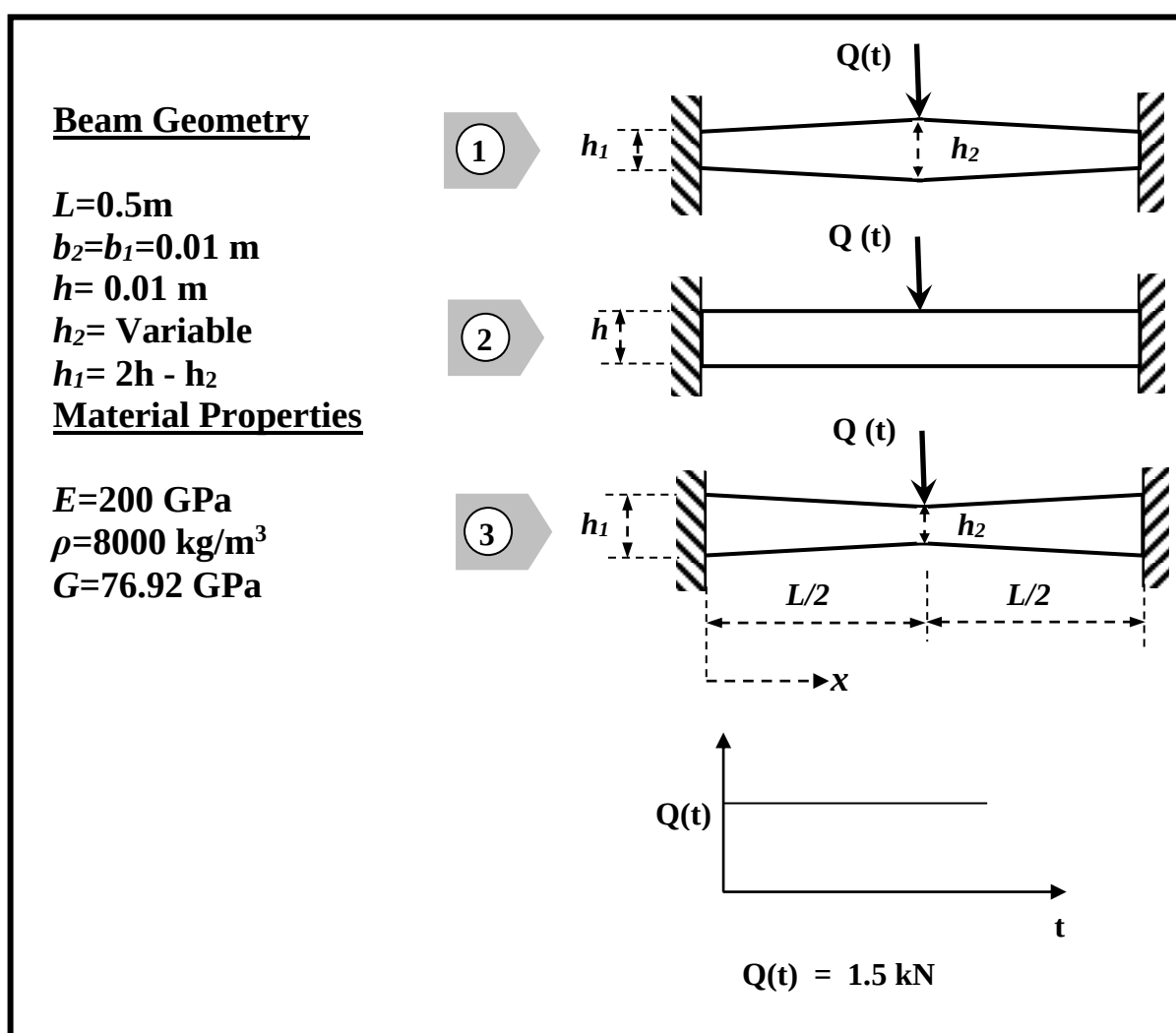


Figure (5.1): Details of non prismatic clamped beam's properties and dynamic characteristics.

5.3 The Effect of Different Bonded Materials

To show the effect of several bonded materials on the dynamic behavior of plane frame structure, a prismatic Williams Togole frame, rectangular in cross section, consists of two materials fused together at the mid-span is analyzed. All the details of the beam geometry and the dynamic characteristics are given in Figure (5.6).

Figure (5.7) shows the effect of bounded two materials on the dynamic response of vertical displacement at mid span. It can be seen that the amplitude of vibration and time period dependent on the material properties (E, ρ).

Figure (5.8) shows the bending moment response at left support and Figure (5.9 a,b,c) explains effect of bounded two materials on the transmission of waves through the frame, it is clear from these two figures that the amplitude and time period of response change when properties of two adjacent materials change. This is due to reflection of the waves in opposite direction when it reaches the position of discontinuity (material's property change), and It can be noticed from these two figures that the magnitude of the reflected wave depends on the difference of the terms $(\rho C, \rho C_s)$ between the two materials on the two sides of the junction point. Also, from figure (5.9 a) it can be seen that the axial wave propagate in dispersive manner because of transforming a part of axial wave to shear wave. This is due to the joint effect. The additional dispersion is caused by the change in material properties.

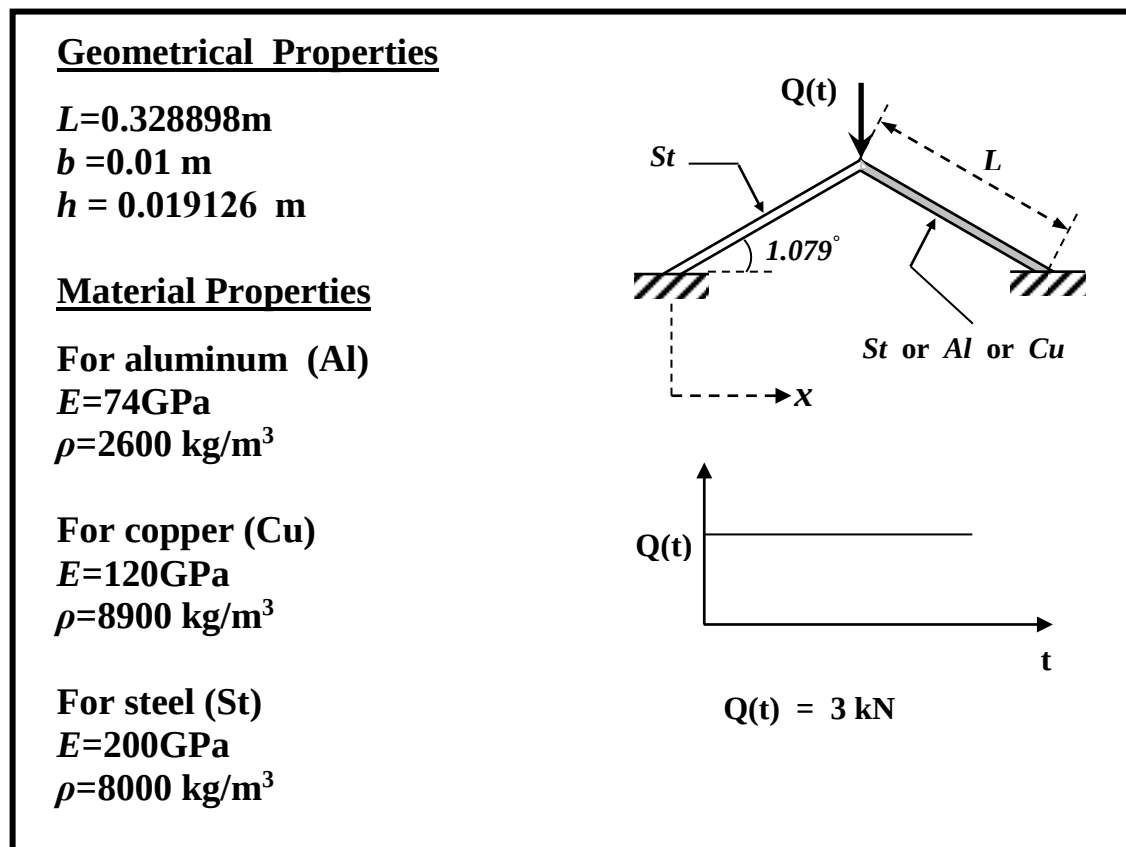


Figure (5.6): Details of William Togol frame's properties and dynamic characteristics.

5.4

The Effect of Sudden Discontinuity in Cross Section

To assess the effect of sudden discontinuity in the cross section, a cylindrical clamped beam made from the same material throughout its length with a discontinuity introduced by the change of the diameter in the mid-span of the beam is analyzed. All the details of the beam's geometry and the dynamic characteristics are given in Figure (5.10).

Figure (5.11) shows the change in the diameter ratio for a steel beam on time history of bending moment at (**0.2488 m**) from the left fixed end, i.e. before the discontinuity, where the lateral force is applied. Five different cases of diameter ratios **DR=1, 1.5, 2, 2.5,3** are investigated. It is shown that the peak values of the bending moment for the four stepped cases are decreased with increasing the diameter ratio. This indicates that the reflected bending wave through the discontinuity increases with increasing the difference between the two diameters.

Figure (5.12) explains that the amplitude of the vertical displacement at the mid span decreases with the increase of the diameter ratio because the beam stiffness increase when the diameter ratio increase.

The reflection phenomenon of shear and bending moment waves through the discontinuity with increasing of the diameter ratio, it is clearly shown in the Figure (5.13 a,b).

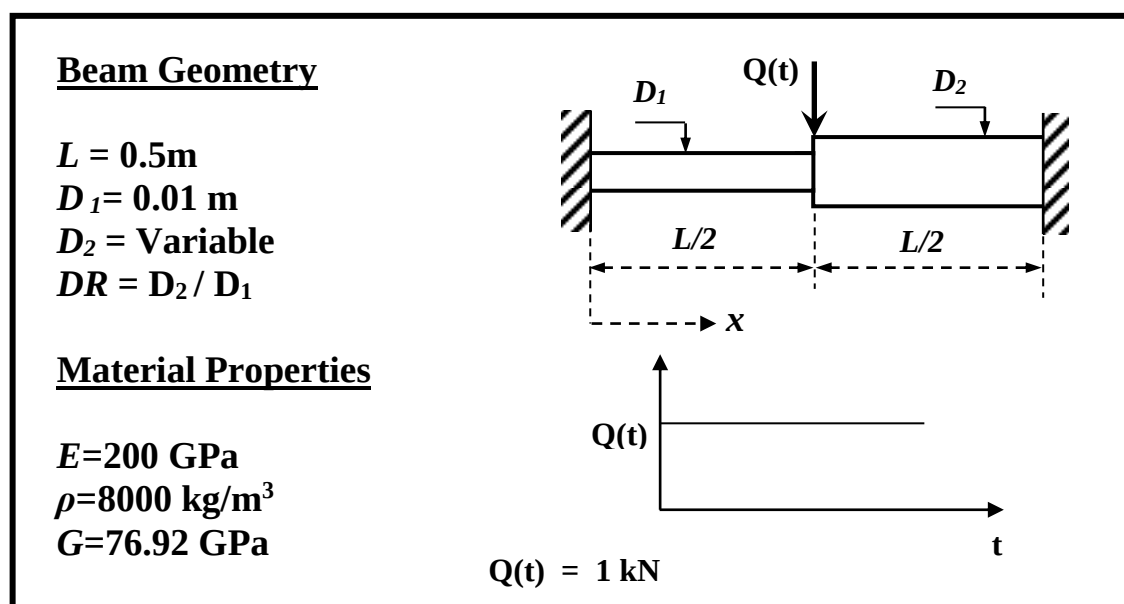


Figure (5.10): Details of stepped clamped beam's properties and dynamic characteristics.

5.5 The Effect of Large Deformation Behaviour

To assess the effect of large deformation behavior, or so-called geometrical non-linearity effect, on the elastic dynamic response of a plane frame, a prismatic portal frame was analyzed. All the details of the frame's geometry and the dynamic characteristics are given in Figure (5.14).

Figure (5.15) shows the bending moment response history at support and Figure (5.16) shows the dynamic response of the vertical displacement at mid span. From these figures it is clear that the effect of large deformation can not, however, be ignored for a very large deflection because it will give incorrect prediction in the time periods and the amplitudes of the vibration. Moreover, it can be noticed that the large deformation analysis leads to larger amplitude of vibration and time periods than the small deformation analysis. This event is caused by the change in geometrical properties, the length and the cross section, of the members.

Figure (5.17 a,b,c) explains the propagation of axial and flexural waves at difference time step . From the figure (5.17 a) it can be seen that no propagation of axial wave at time **(0.02 ms)** in the small deformation analysis because it ignored the change in geometrical properties (the length and the cross section) that caused to propagate the wave, but after this time as shown figure (5.17 a) axial wave will be propagated in the girder due to the joints effect. Also from the figure (5.17 c and b) it can be seen that the flexural waves in small deformation faster than waves in large deformation ,and it can be noticed that the amplitude of waves in large deformation larger than the amplitude of waves in small deformation at the support due to large deformation effect ,but the amplitude of waves in large deformation will decrease and becomes less than the amplitude of waves in small deformation when one moves from the support to the mid span point because of the

numerical errors which resulting from used interpolation. This numerical error increases when deformations are increased. Thus, the mid span point has larger numerical error because it has larger deformations than other points.

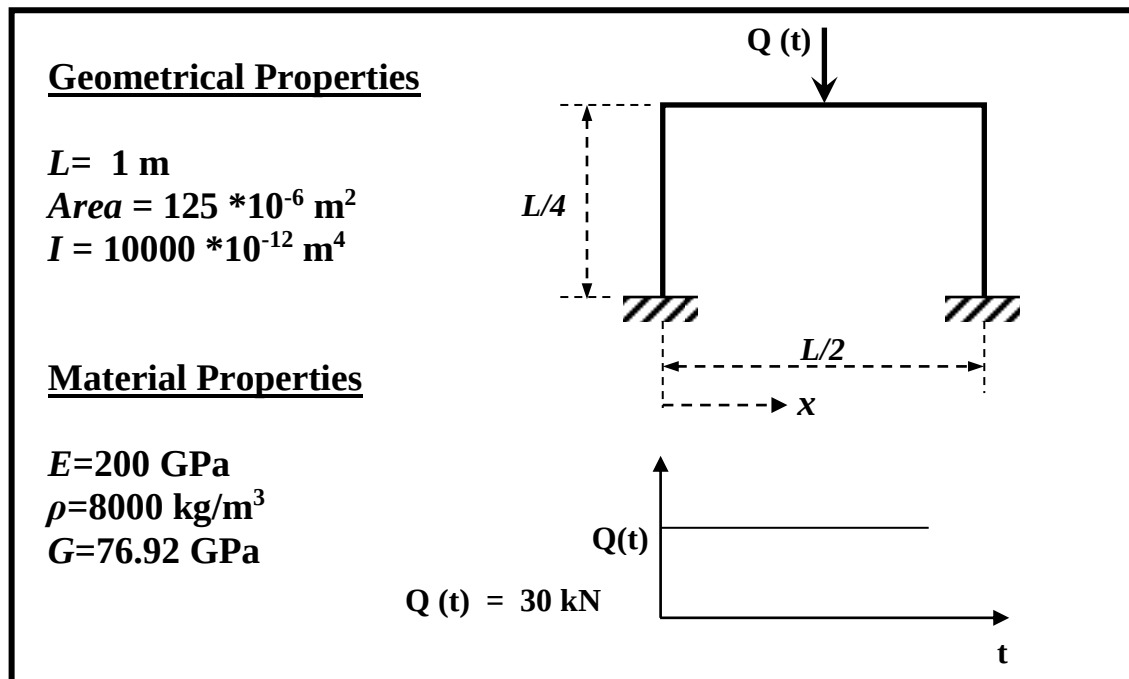


Figure (5.14): Details of Portal frame's properties and dynamic characteristics.

5.6

The Effect of Poisson's Ratio on Nonlinear Vibrations

To study the effect of Poisson's ratio equal or not equal to zero on nonlinear vibrations on the dynamic behavior of the plane frame, prismatic cantilever beam, square in cross sectional area is analyzed. All the details of the beam's geometry and the dynamic characteristics are given in Figure (5.18).

Figure (5.19) Shows the bending moment response history at fixed end with the two different values of Poisson's ratio (**0.0** and **0.3**) and Figure (5.20) shows the dynamic response of the vertical displacement at the tip of the free end with two different values of Poisson's ratio (**0.0** and **0.3**), from these Figures, it can be noticed that the effect of Poisson's ratio increases with the progress of time. This is due to the change in geometrical properties, the length and cross section area, and due to the change in the shear wave speed (C_s) because it depends on elastic modulus of rigidity (G).

The changing in the speed of shear wave caused change in the speed of bending moment wave because of the interaction between both waves.

Figure (5.21 a,b) shows the propagation of shear and bending moment waves, it can be seen that for the same material and geometrical properties ,the smaller Poisson's ratio, the faster the propagation of flexural effect .

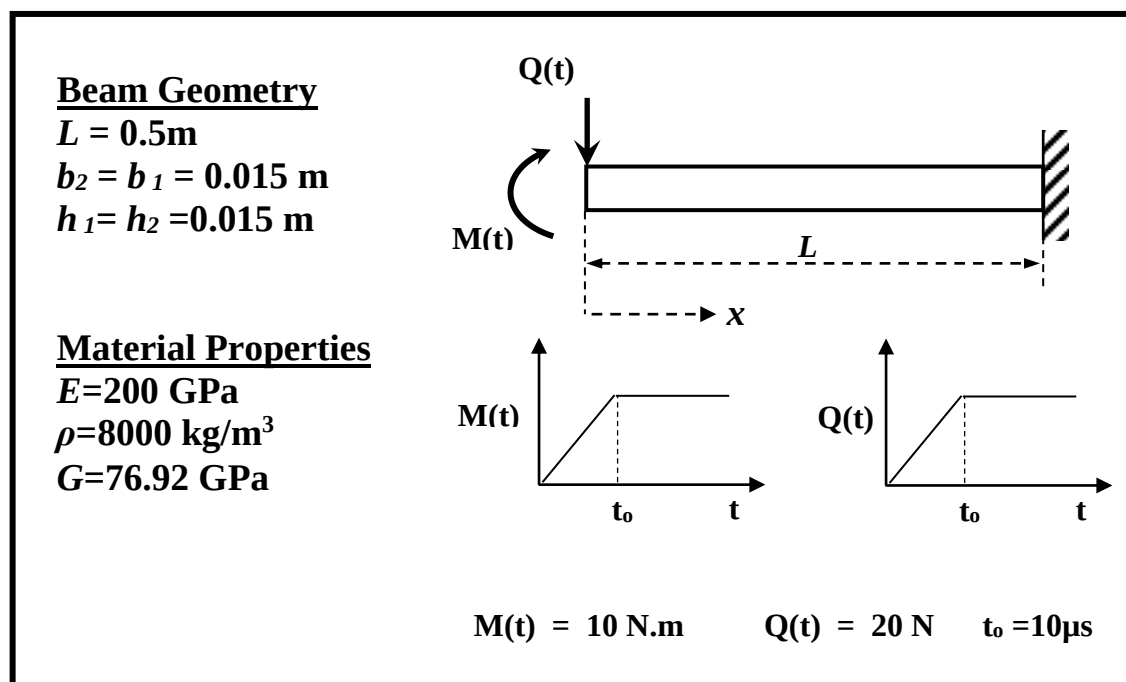


Figure (5.18): Details of beam's properties and dynamic characteristics.

5.7 The Effect of Load Intensity

To study the effect of load intensity on the elastic dynamic behavior of the plane frame structure, a prismatic non-rectangular frame is analyzed. However, in order to provide a basis for comparison, the duration of force, Δt , is finite and the same impulse applied to the frame in each case. The three rectangular pulses applied are (I) $Q(t)=3000$ N and $t_o= 5$ ms; (II) $Q(t)=2000$ N and $t_o =7.5$ ms; and (III) $Q(t)=1500$ N and $t_o = 10$ ms. In all these cases, the impulse is equal to 15 N.s. All the details of the frame's geometry and material's properties are given in Figure (5.22).

Figures (5.23) to (5.25) show that the magnitude of the applied load and its history can significantly affect the dynamic behavior of the frame, where decreasing the intensity of the load makes the magnitude of the curvature decreases, but the general characteristics of the deformation remains similar to that in the high-intensity case.

But, it can not compare between these three cases of load intensity after (5ms) because the structure would fall under the influence of free vibration. This is clearly shown in figure (5.26).

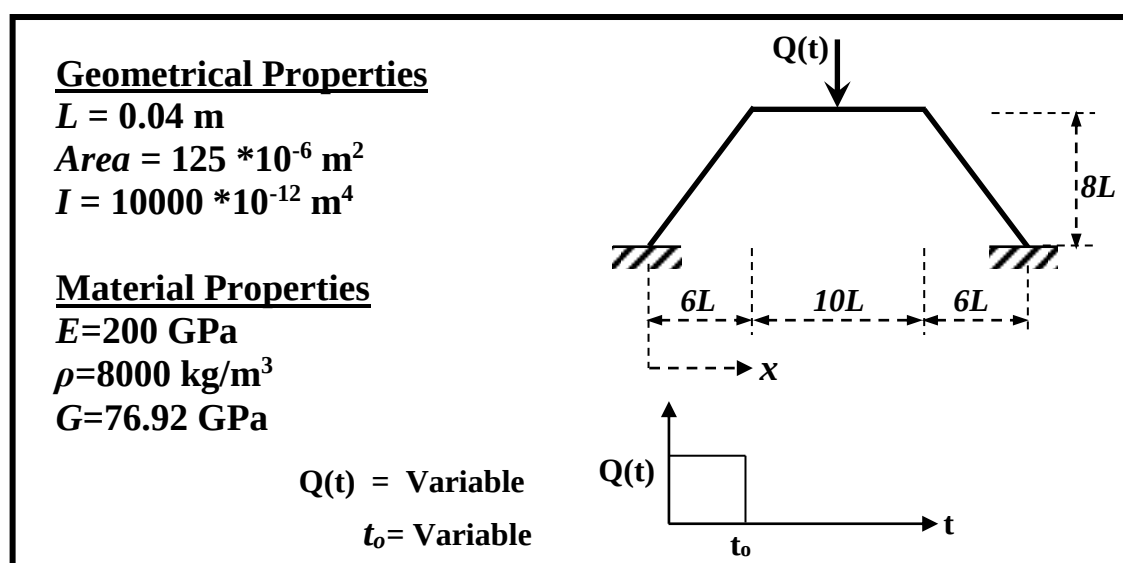


Figure (5.22): Details of frame's properties and dynamic characteristics.

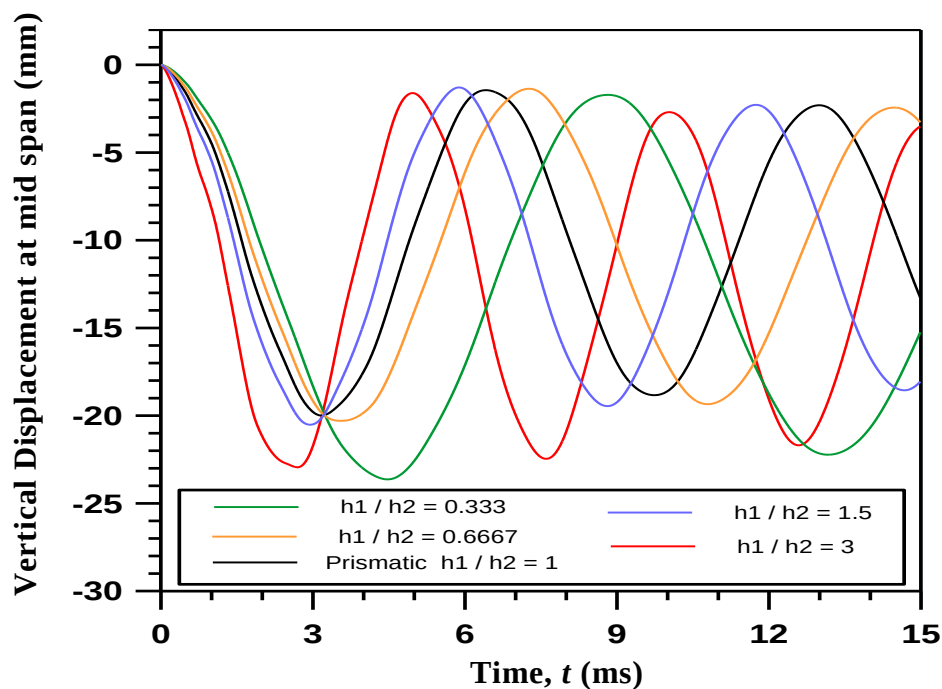


Figure (5.2): Effect of tapered ratio on the dynamic response for a non-prismatic clamped beam

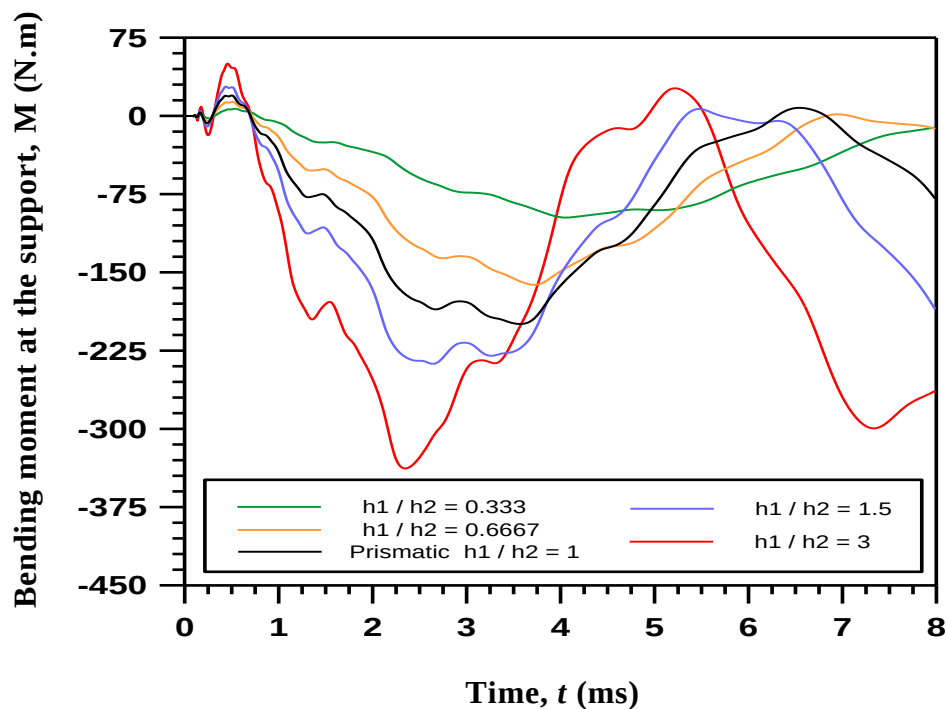


Figure (5.3): Effect of tapered ratio on the dynamic response for a non-prismatic clamped beam.

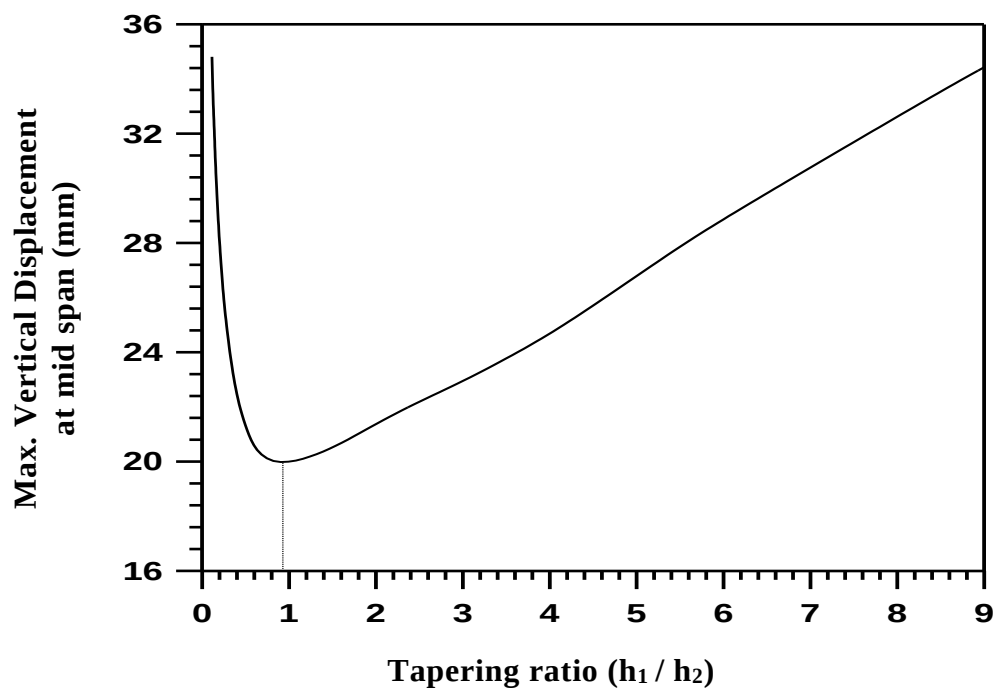


Figure (5.4): Effect of tapered ratio on the Max.Vertical Displacement at mid span for a non-prismatic clamped beam.

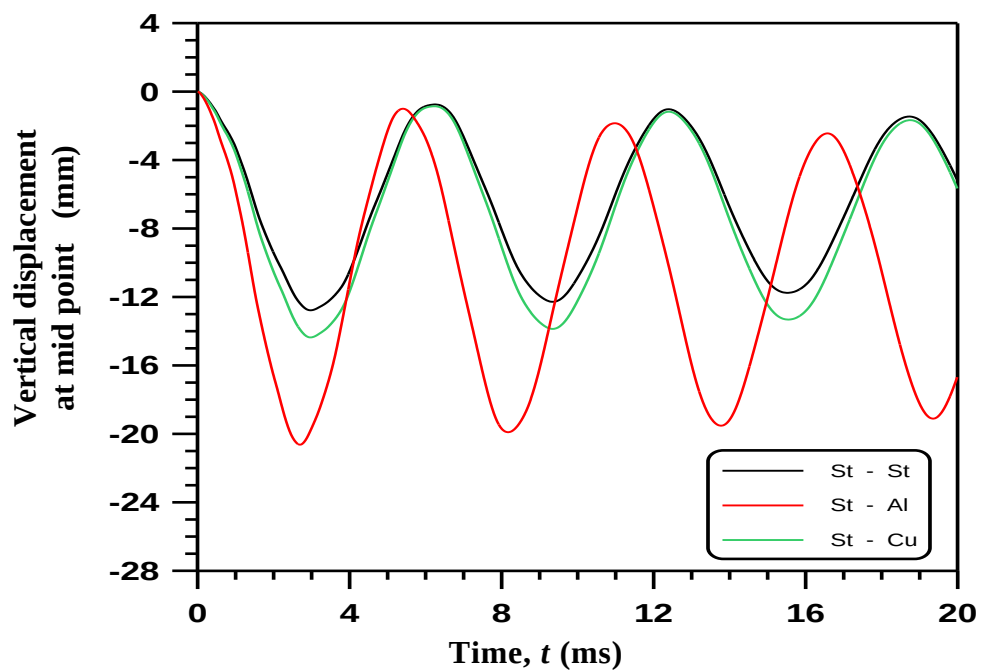


Figure (5.7): Effect of several bonded materials on the dynamic response of Williams Toggol Frame.

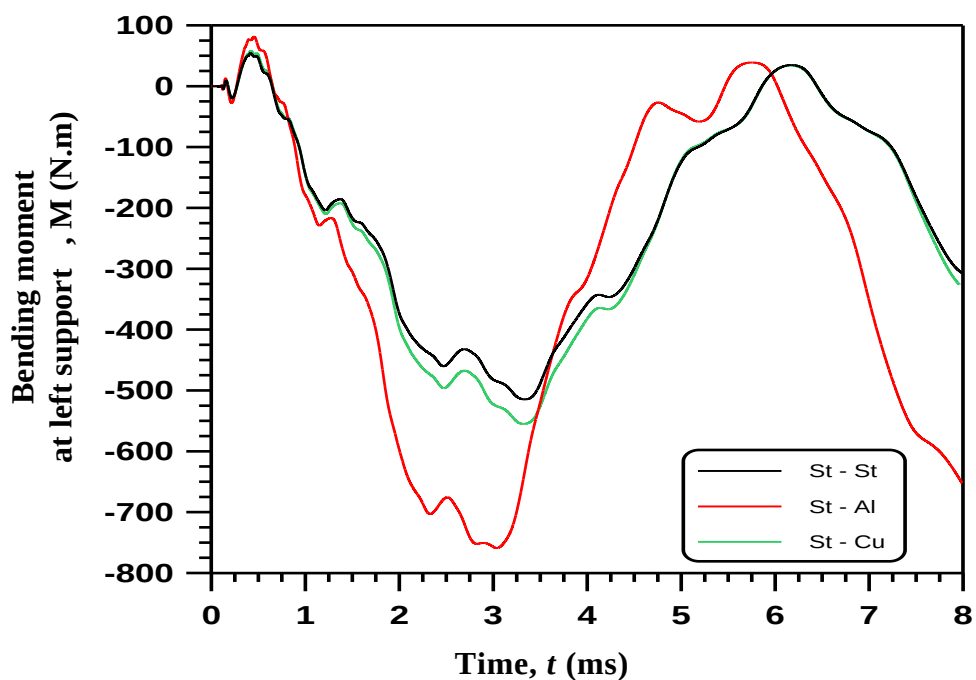


Figure (5.8): Effect of several bonded materials on the dynamic response of Williams Toggol Frame.

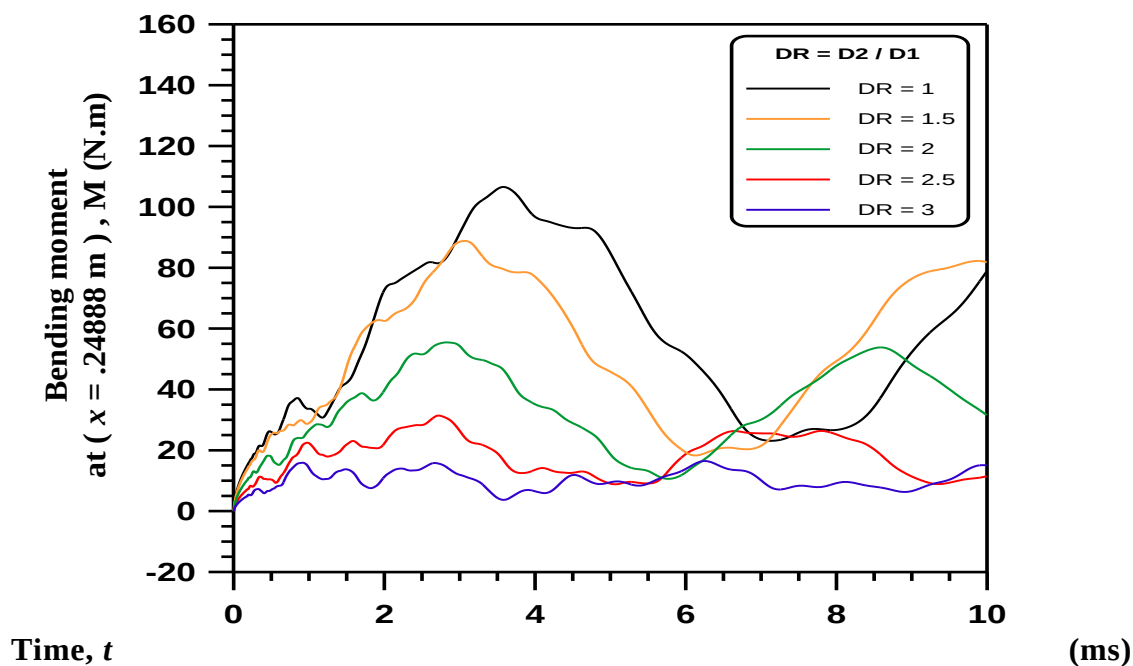


Figure (5.11): Effect of sudden discontinuity on the dynamic response of stepped clamped beam under shear force.

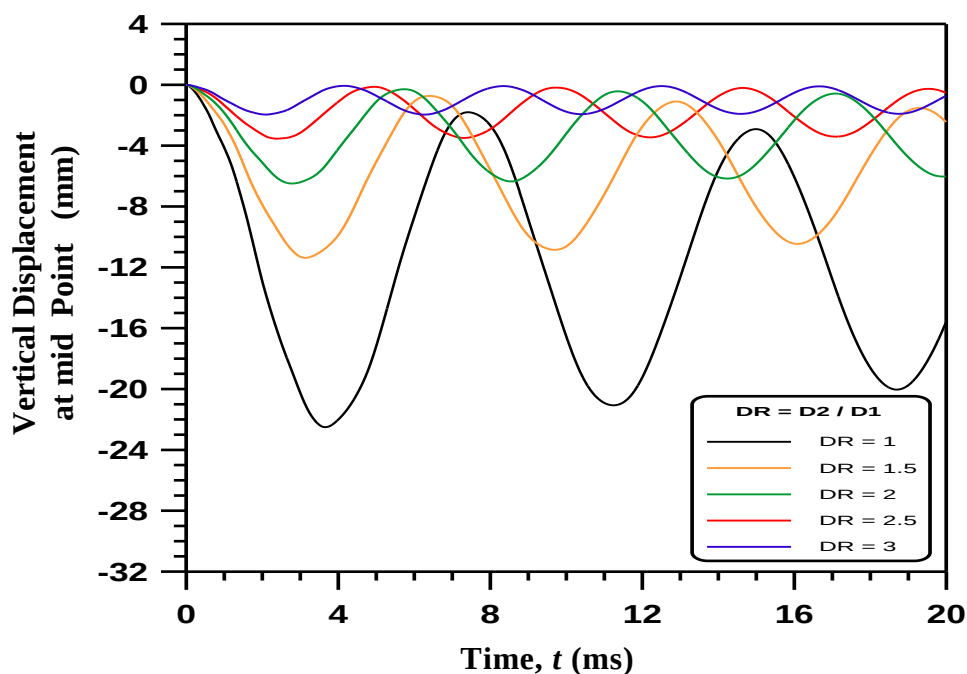


Figure (5.12): Effect of sudden discontinuity on the dynamic response of stepped clamped beam under shear force.

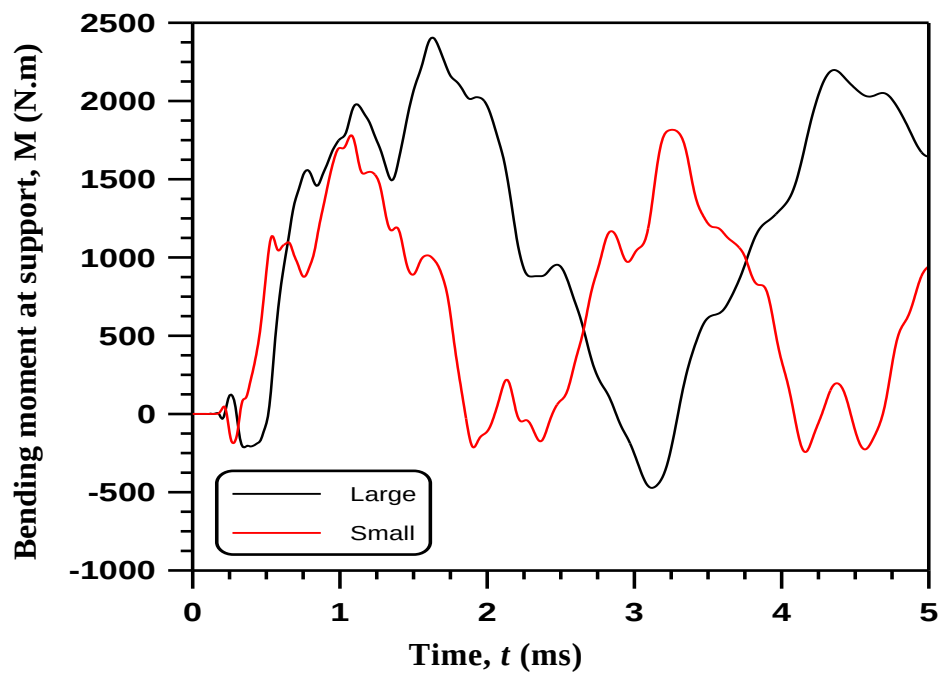


Figure (5.15): Effect of large deformation behavior on the dynamic response of Portal frame.

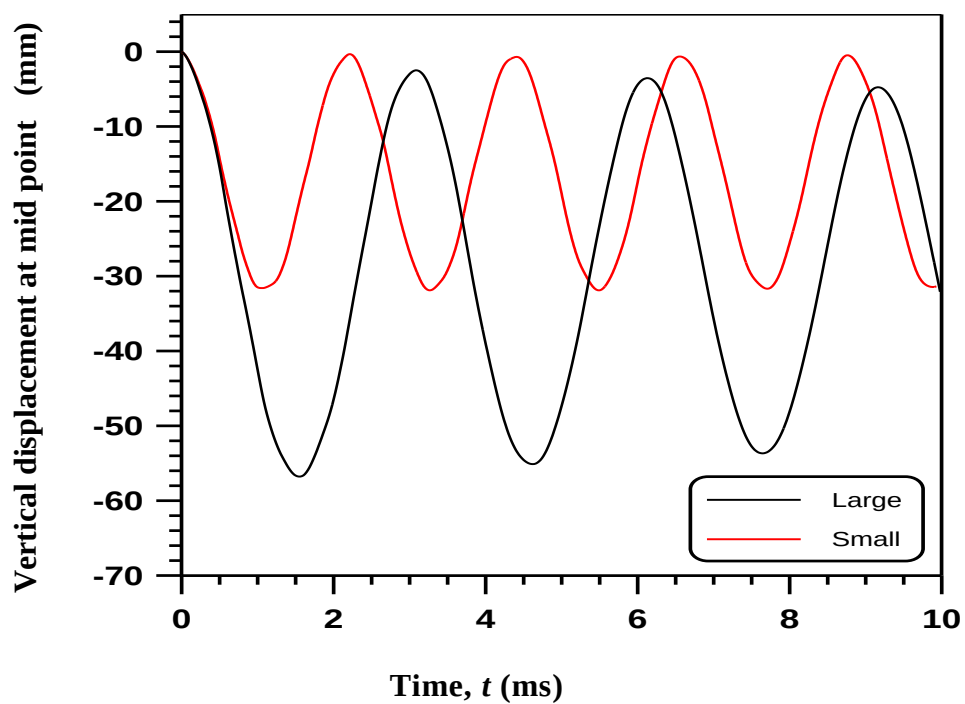


Figure (5.16): Effect of large deformation behavior on the dynamic response of Portal frame.

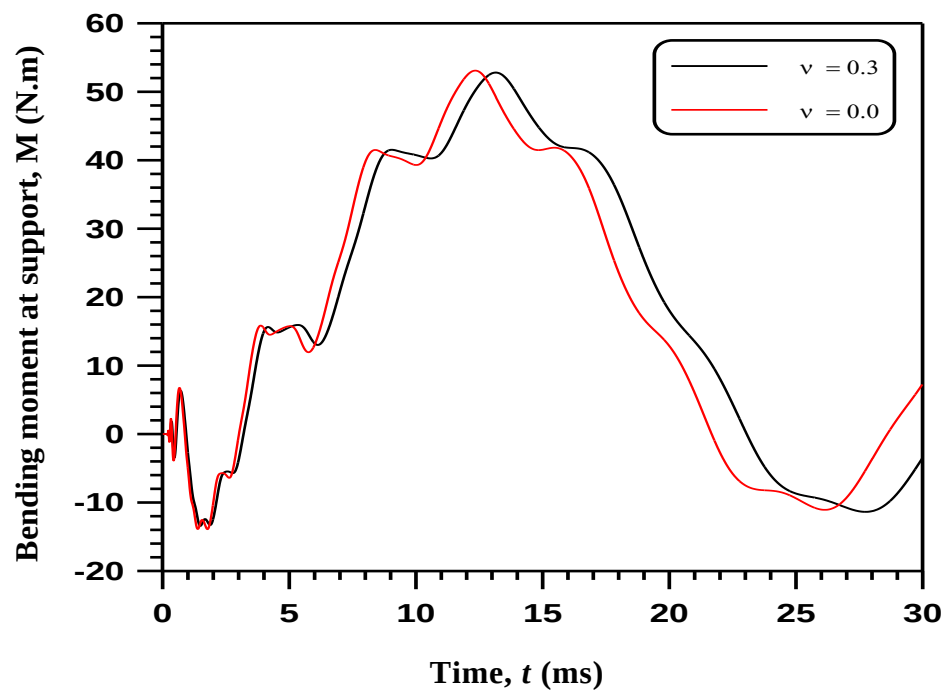


Figure (5.19): Effect of Poisson's ratio on the dynamic response of the prismatic cantilever beam under shear force and bending moment.

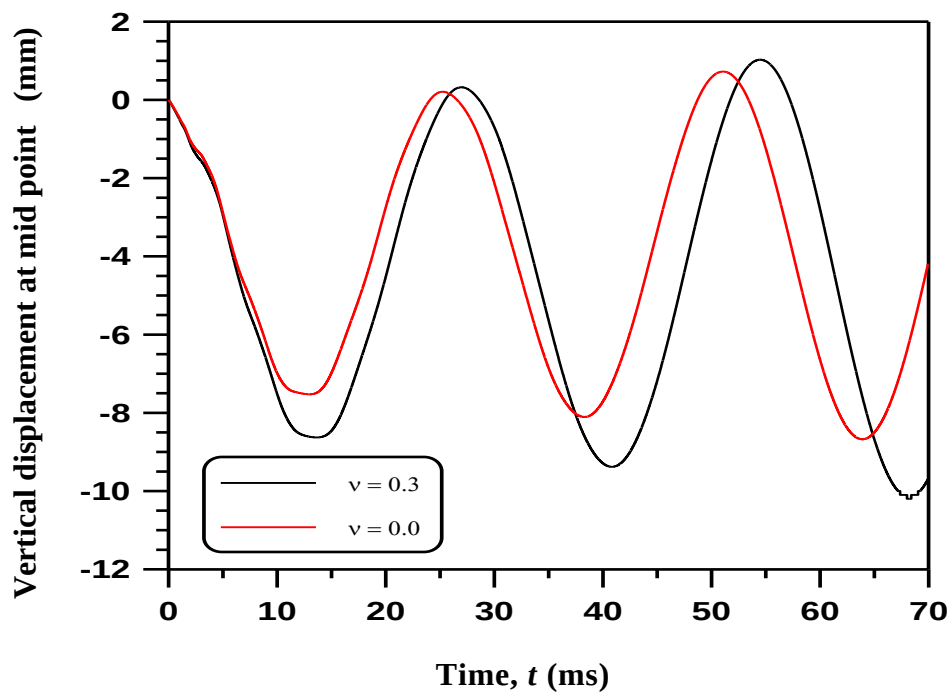


Figure (5.20): Effect of Poisson's ratio on the dynamic response of the prismatic cantilever beam under shear force and bending moment.

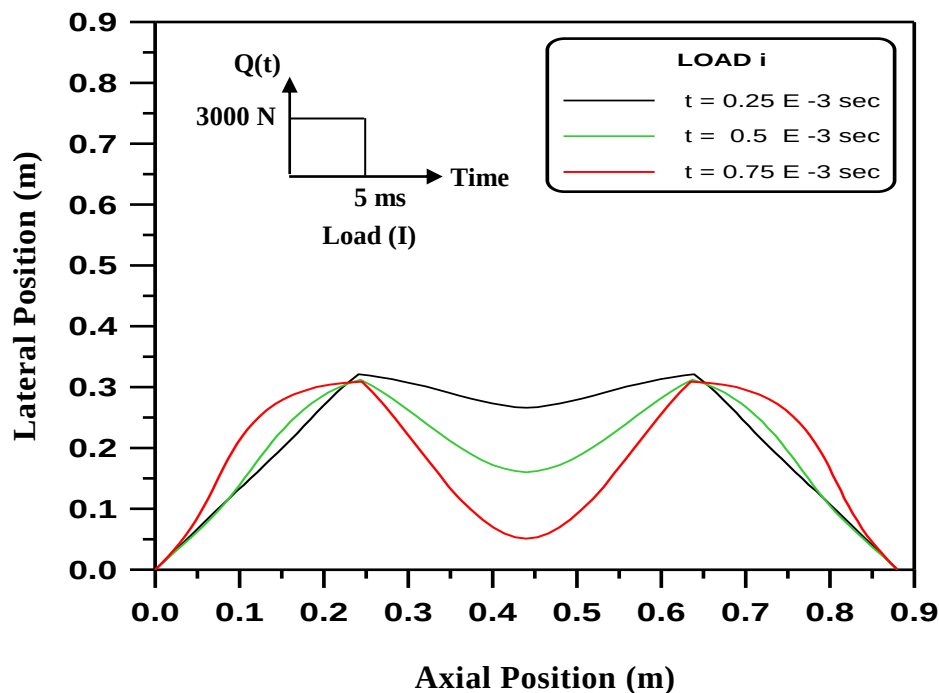


Figure (5.23): Effect of load intensity on the deflected shape of the non-rectangular frame at various instants.

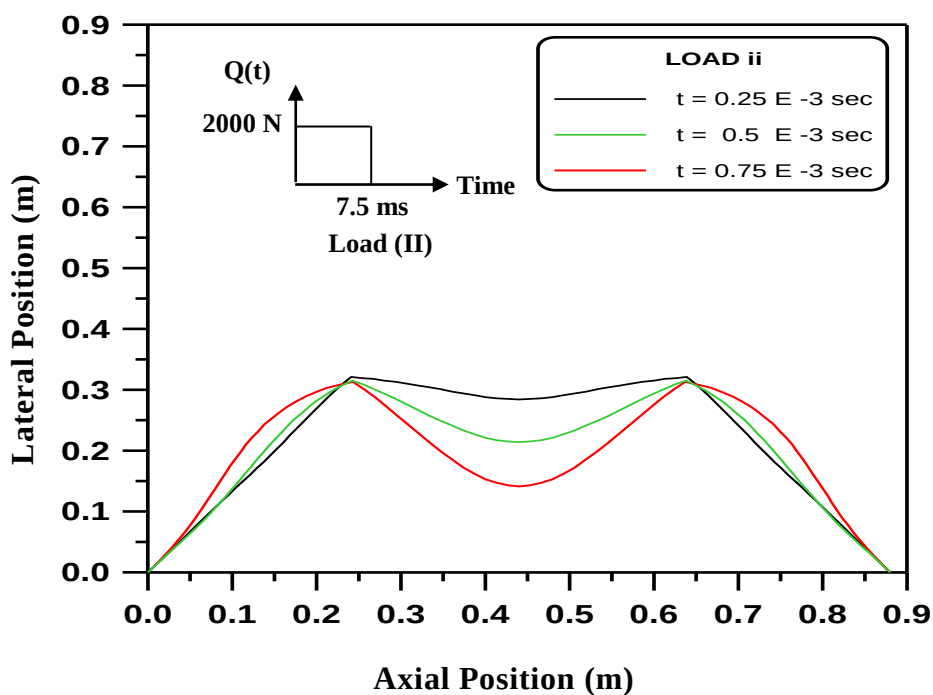


Figure (5.24): Effect of load intensity on the deflected shape of the non-rectangular frame at various instants.

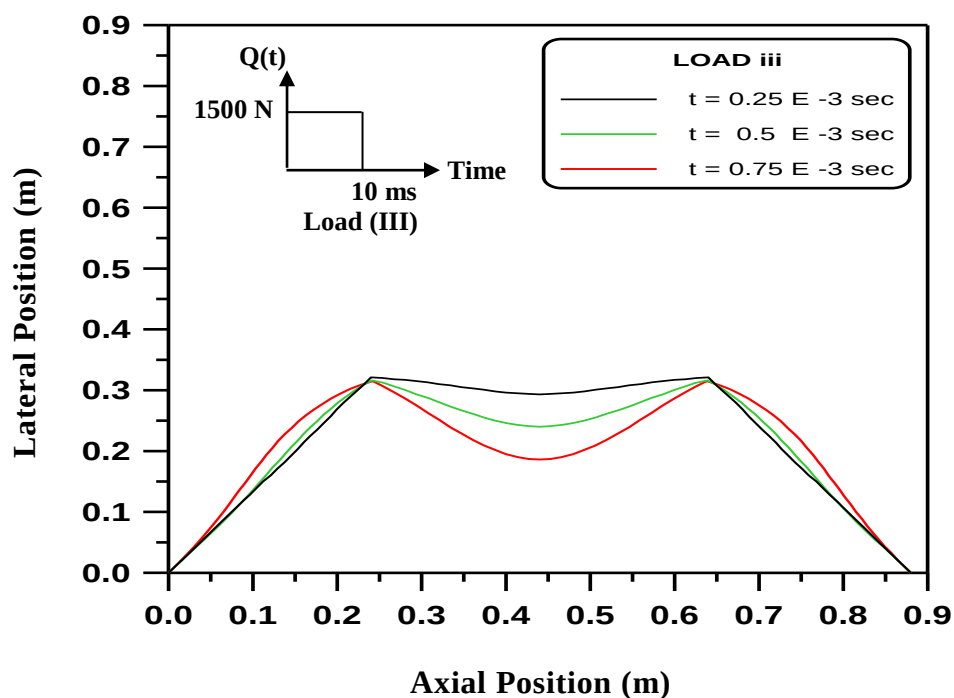


Figure (5.25): Effect of load intensity on the deflected shape of the non-rectangular frame at various instants.

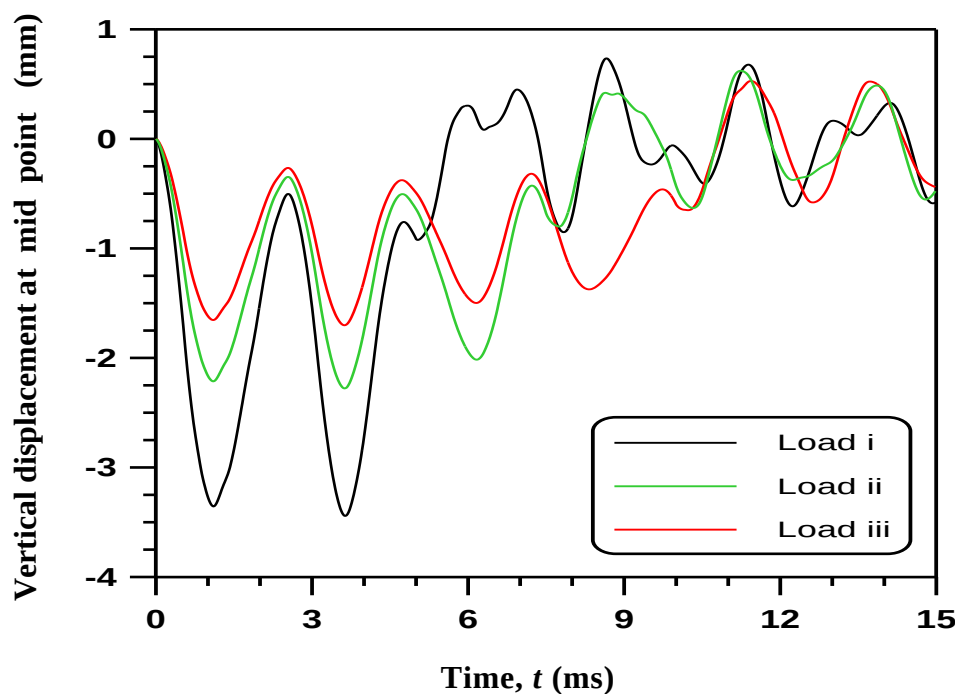


Figure (5.26): Effect of load intensity on the dynamic response of the non-rectangular frame.

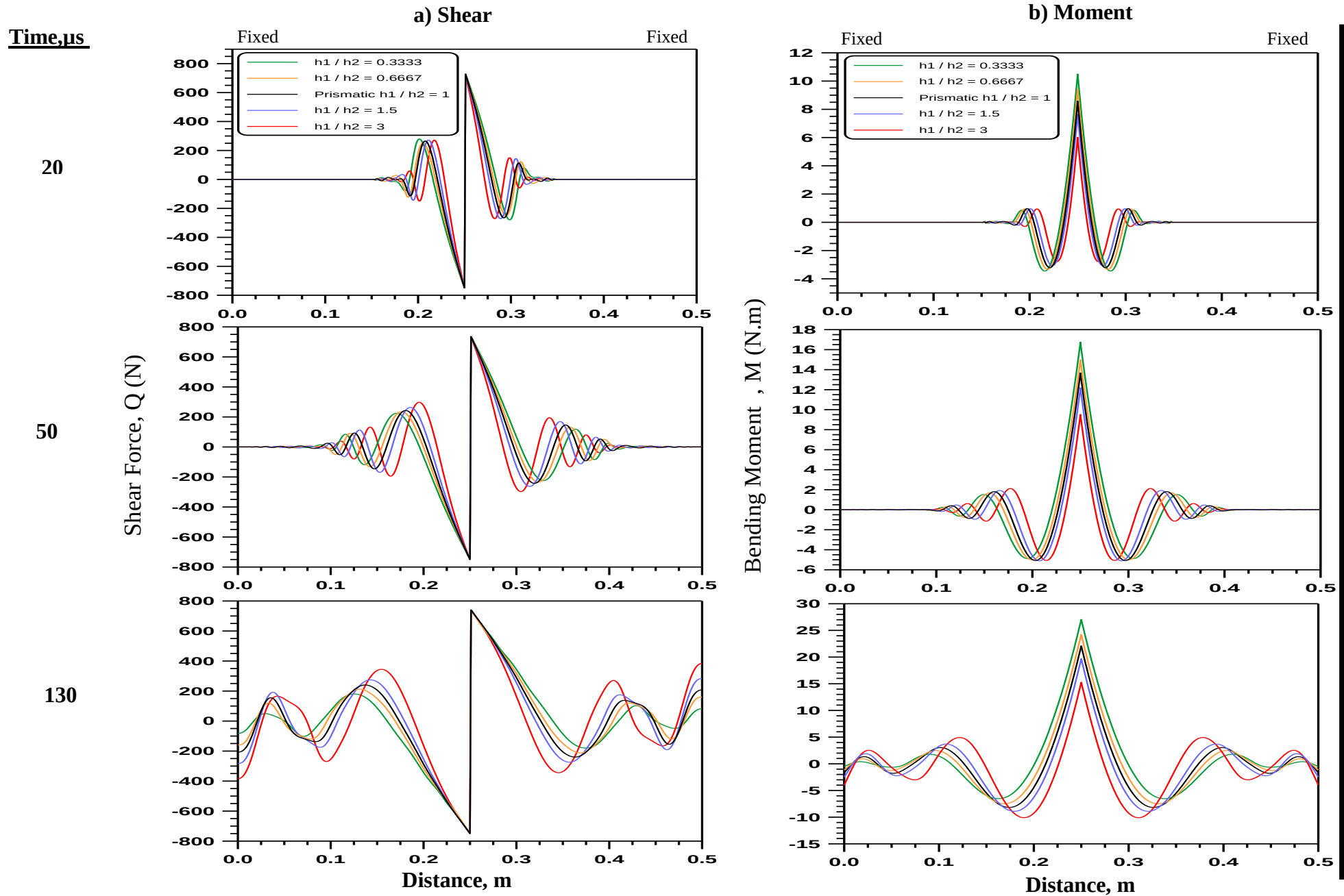
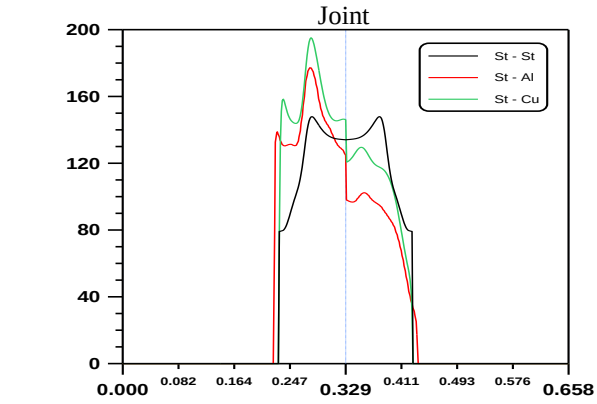


Figure (5-5) Propagation of Waves in Non-Prismatic Clamped Beam.

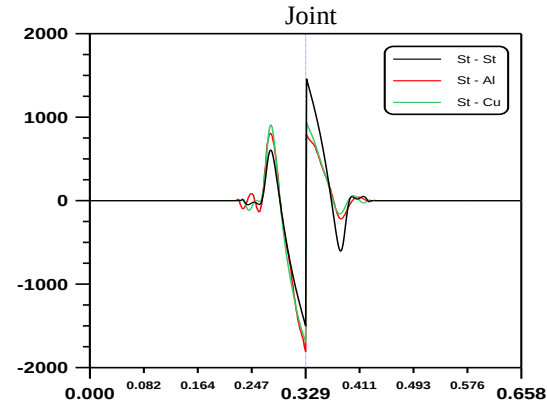
Time, μ s

20

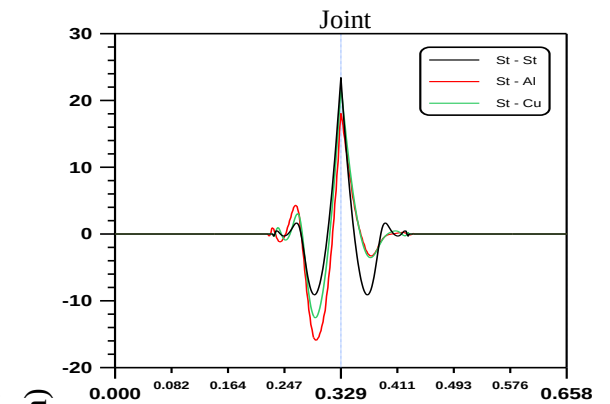
a) Axial



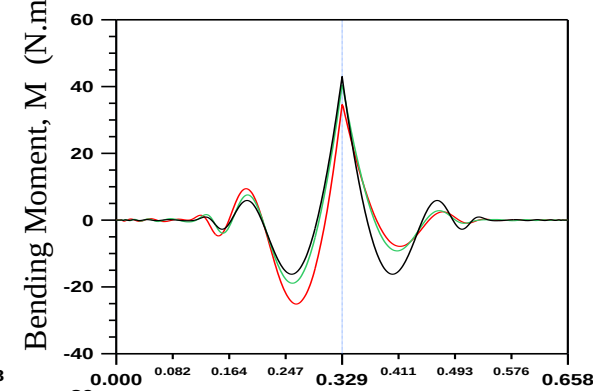
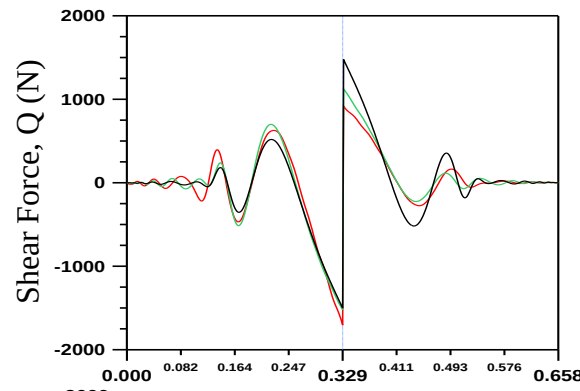
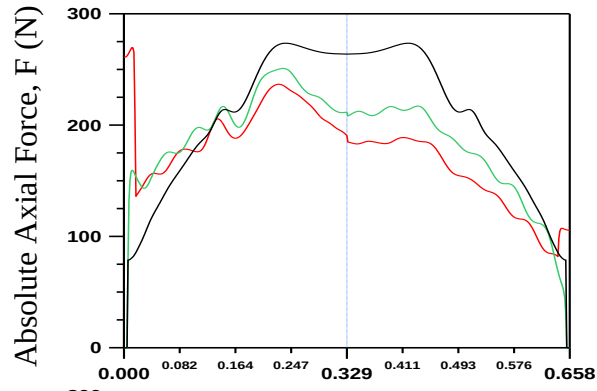
b) Shear



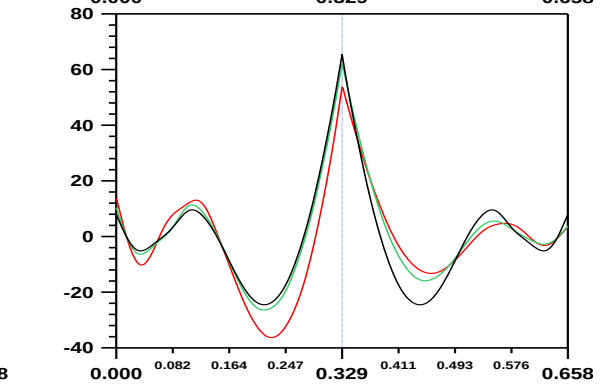
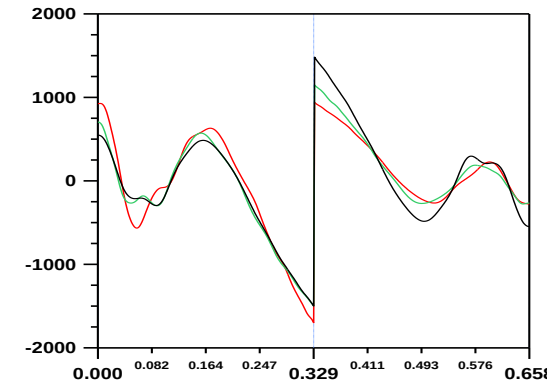
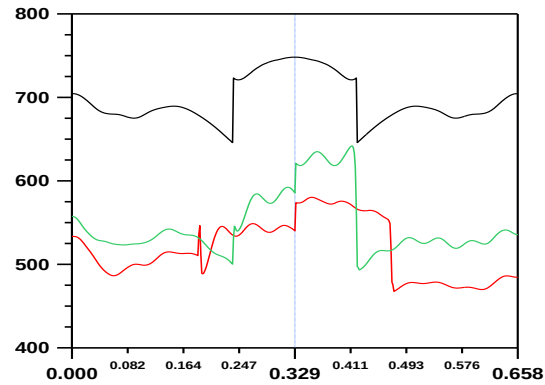
c) Moment



65



150



Distance, m

Figure (5-9) Propagation Of Waves In Williams Toggol Frame.

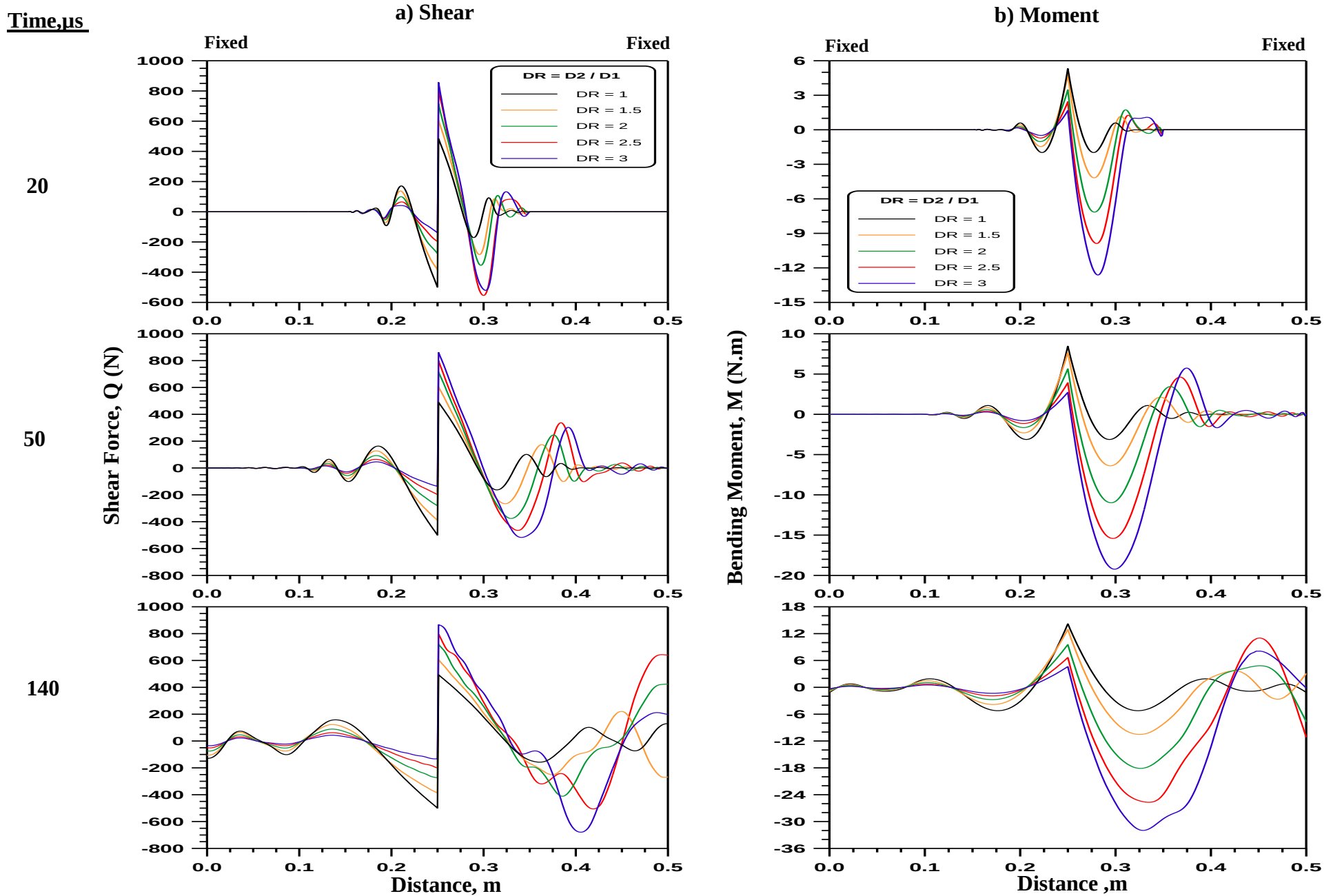
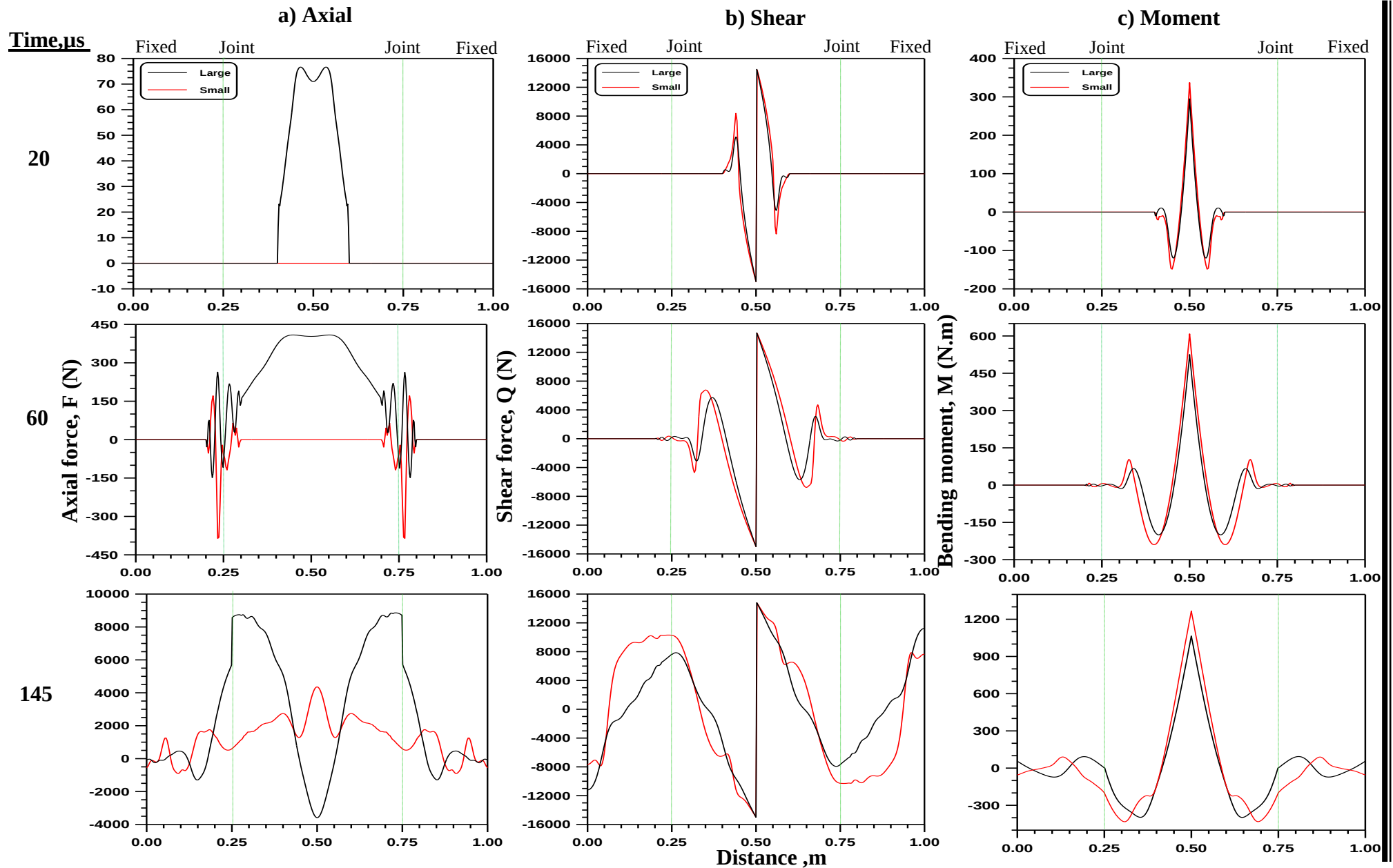
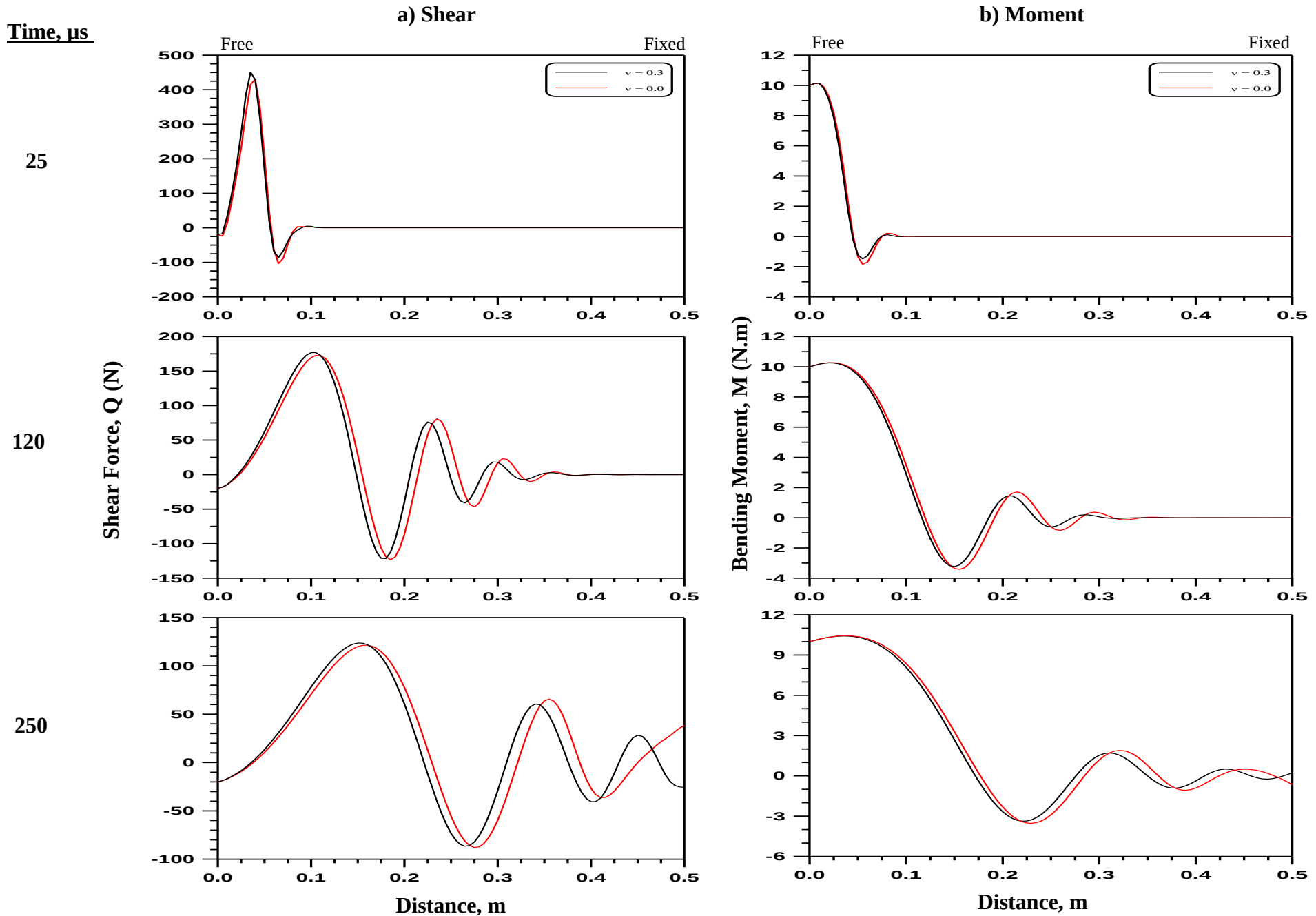


Figure (5-13) Propagation of Waves in Clamped Beam





CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

1. The method of characteristics can be used to simulate the transient response of plane frames with straight prismatic and non – prismatic members subjected to any type of loading.
2. The method of boundary characteristics can efficiently simulate large deformation dynamic behavior.
3. Non-linear behavior should be adopted when high loading conditions are expected.
4. In the large deformation analysis, flexural waves will generate axial wave as a result of geometric non- linearity.
5. The variations of the tapering ratio for the same steel volume have a very significant effect on the non-linear behavior of the steel frames.
6. The existence of boundaries such as joints has no direct influence on wave propagation in members. But nevertheless causes the over all phenomenon to dispersive.
7. Discontinuities of cross-sectional area and the change in material properties along the longitudinal axis of member have important effects on the amounts of reflection and transmission stress waves and their oscillations.
8. When the impulse of the pulse force remains constant forces of higher magnitude tend to load to more deformation.

9. The main advantage of the method of characteristics is that the non linear equations are solved directly without the need for any iterative or incremental procedures.
10. The interaction between shear and bending wave causes the flexural waves to prismatic and non- prismatic members in a dispersive form..
11. The method can efficiently predict forces velocities and displacements throughout a structure under consideration at any instant.

6.2 Recommendations

1. The large deformation dynamic problem has been applied only to plane frames. It should be applied to (3-D) frames.
2. The present analysis is limited to a prismatic and non- prismatic straight member. More effort is needed to include curved members.
3. Studying the effect of lumped mass at joints of the plan frame.
4. Including the effect of damping to obtain damped vibration.
5. Investigate the effect of geometrical non linearity as well as material non linearity on the dynamic behaviour of plane frames.

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