Republic of Iraq
Ministry of Higher Education & Scientific Research
University of Babylon
College of Education for Pure Sciences
Department of Mathematics

On Some Approaches to Optimize the Reliability of Complex Networks

A Dissertation
Submitted to the Council of College of Education for Pure Sciences,
Babylon University in Partial Fulfillment of the Requirements for
the Degree of Doctor of philosophy in Education / Mathematics.

By
Fouad Hamza Abd Nasser

Supervised by
Prof.Dr. Zahir Abdul Haddi Hassan

2022 A.D. **1443 A.H.**

بِسْمِ اللهِ الرَّحْمنِ الرَّحِيمِ

اقْرَأْ بِاسْمِ رَبِّكَ الّذِي خَلَقَ
خَلَقَ الإِنسَانَ مِنْ عَلَقٍ
اقْرَأْ وَرَبُّكَ الأَكْرَمُ
الّذِي عَلّمَ بِالْقَلَمِ
عَلّمَ الإِنسَانَ مَا لَمْ يَعْلَمْ

صدقَ اللهُ العليُّ العظيم
سورة العلق 1: 5

Dedication

To ultimate (Divine) source of plenty and prosperity.

To the caller of Allah and place of manifestation of His signs.

To the authority who shall communicate and make known the true point of view.

To the son of the chosen Prophet, the son of Ali Al-Murtaza.

I eagerly long for you who is worthy of acceptance and lawful driving force of belief and conviction, never weary or annoyed.

To my Lord Al-Huja Bin Al-Hassan Almahdi Peace be upon him.

Fouad

Acknowledgements

Praise be to Allah, the Lord of the worlds. Thanks be to God, who gave me the ability to complete my studies and granted me success in it despite my advancing age.

I am very grateful to my supervisor, **Prof. Dr. Zahir Abdul Haddi Hassan**, for his continuous follow-up and support.

I also extend my thanks to all the professors of Mathematics Department at College of Education for Pure Sciences for their assistance and encouragement. I can never adequately express my appreciation towards them.

Finally, I would like to express my deepest appreciation to my family and friends to support and encourage me throughout my school years.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

Symbols and Acronyms Abbreviation

Symbol	Description
Pr	Probability
pdf	Probability density function
E(X)	Mathematical Expectation of X
cdf	Cumulative distribution function
G	Graph
x_i	i-th edge (component)
v_i	i-th node
n	The number of nodes in graph
m	The number of edges in graph
A	Adjacency matrix
I_m	Identity matrix of size m
CM	Connection matrix
P	Path in graph
PL	Path length
MP	Minimal path
np	Number of minimal path
IM	Incidence matrix
C	Cut set in graph
MC	Minimal cut set
OC	Order of cut set
$\Phi(x)$	Structure function
$R(t)$	Reliability function
$F(t)$	Failure function
λ	Failure rate
R_N	Reliability Network
R_i	Reliability of i-th component
C_N	Total Cost
C_i	The Cost of i-th component

R_G	The Objective of Reliability Network
C_G	The Objective of Total Cost
NLP	Nonlinear programming
RBD	Reliability block diagram
MPT	Minimal Path Technique
MCT	Minimal Cut Technique
RPE	Reduction to Parallel Elements
DST	Delta - Star Transform Technique
ARING	Aeronautical Radio Inc.
BA	Bat Algorithm
$\lambda^{'}$	The wavelength of ultrasonic sound
f	The frequency
s_i	Speed of i-th bat
p_i	Position of i-th bat
IN	number of iterations
r	The rate of pulse emission
A_0	Loudness
GWO	Grey Wolf Optimization
$\overrightarrow{X_p}$	Vector of the prey 's position
$\overrightarrow{X}$	Vector of the grey wolf's position
$\overrightarrow{D}$	Vector to specify a new position
$\overrightarrow{A}$ and $\overrightarrow{C}$	Coefficient vectors
$\overrightarrow{a}$	Vector set to decrease linearly
$\overrightarrow{r_1}$ and $\overrightarrow{r_2}$	Random vectors in [0,1]
SD	Standard Deviation

Abstract

In this dissertation, four different techniques (Minimal Path Technique, Minimal Cut Technique, Reduction to Parallel Elements and Delta - Star Transform) have been studied to calculate the reliability of complex networks. All of these techniques depend on minimal paths, so a method was presented to find minimal paths using matrices algebra. Moreover, a method for calculating minimal cut sets is also studied. In addition to utilize some techniques to calculate the reliability allocation of shortened network such as Exponential Allocation Technique and Aeronautical Radio Inc. (ARINC) approach method. This dissertation included calculating the importance of all network components to find the effect of all these components on the functioning of system (network) as a whole. The topic of increasing the reliability of a network has been studied based on the importance of each unit (component) of the network, through the most important units in the network obtaining the best allocation from the increase in reliability over others that are followed by importance. Finally, the reliability optimization for a complex network has been studied by using Bat and Grey Wolf Optimization algorithms and that is by relying on the use of five cost functions with these algorithms such as Logarithmic, Exponential, Exponential in terms of feasibility factor, Power Function and Tan function, with a comparison between these algorithms to choose the best algorithm that gives the highest reliability with lowest cost.

CHAPTER 1

CHAPTER ONE

1.1 Introduction

In language, reliability refers to relying on someone, something or the ability to trust and believe, as we say in Kumayl's supplication (***O' my Master, O' Thou upon Whom I rely***). This is close to the technical concept, reliability can be defined as the capacity of any system, device, or component to do the tasks for which it was made for the required period of time with specific conditions or restrictions. The term reliability dates back to 1816 when the poet Samuel Taylor Coleridge was first endorsed [1]. Although, network reliability originated during World War II, today it has taken on a new shape by integrating itself in all stages of the product life cycle from proposal to manufacturing. During that years following of the world war II many examination research centers and colleges started and mathematicians intrigued by the investigation of life testing and unwavering quality issues [60]. There are many researchers focused the have a look at of various reliability network, like series, parallel, series – parallel, parallel – series networks, and complex networks that enters in different life fields. Since there are different available systems and because of their importance, the researchers attempted to find more than one method to solve these complex systems [71, 82] .

Network reliability design troubles have attracted many researchers, including network designers, community analysts, and network administrators, so as to percentage expansive hardware and software sources and offer the get admission to of important structures from specific places. These troubles have many programs in the areas of telecommunications and laptop networking and related domains inside the electrical, fuel sewer networks [69, 82]. During the designing of community systems, one of the crucial steps is to locate the quality format of components to optimize some performance criteria, such as value, transmissions put off or reliability. The corresponding most efficient design trouble can be formulated as a combinatorial trouble [69].

The subject "reliability optimization" appeared in the literature in due late 1940 and was first applied to communication and transportation systems. Most of the earlier works were confined to an analysis of certain performance aspects of an operating system. One of the goals of reliability engineer is to find the best way to increase the system reliability. The reliability of a system can be defined as the probability that the system will be operating successfully at least up to a specified point of time (i.e. mission time) under stated conditions. As systems are becoming more complex, the consequences of their unreliable behavior have become severe in terms of cost, effort and so on. The interests in accessing the system reliability and the need to improve the reliability of products and system have become more and more important [68]. All manufacturers they desire the larger reliability of their products which raise the production cost of the items. In such a case, there arises a question as to how to meet the goal for a system reliability. As a result, the increase in production cost has negative effects on the user's budget. Therefore, the design reliability optimization problem is phrased as reliability improvement at a minimum cost.

1.2 Dissertation Outlines

This dissertation consists of six chapters. The first chapter contains the introduction, objectives, contributions of dissertation and related works. The second chapter contains some definitions and basic concepts. Third chapter includes four techniques for calculating the reliability networks. The fourth chapter discusses two ways to reliability allocation after reducing the complex network for parallel two subnetworks. This chapter also includes a study of reliability importance for complex network components. Chapter five includes two algorithms (Bat and Gray Wolf optimization) to optimize the reliability of complex network using five cost functions. The final chapter consists of conclusions and future works.

The results calculated by using MATLAB R2020a in COM. with a device (Intel(R) Core (TM) i7-7500U CPU @ 2.70GHz 2.90 GHz RAM 12 GB VGA 920M 4GB).

1.3 Objectives of this Dissertation

The main objectives of this dissertation are:

1. To get the best increase in complex network reliability in order to obtain the best value for total reliability with the lowest cost.

2. Finding a polynomial function of reliability in terms of the reliability of its components.

3. Reducing the number of complex network components into series and parallel components (turning it into a simple network).

4. Calculation the reliability importance of complex network components.

5. Assigning values for the reliability of complex network components to reach an acceptable value for network reliability.

1.4 Contributions

The dissertation included a set of contributions that can be summarized as follows:

1. We extracted all minimal path and cut sets of a complex network that has not been studied before that can serve as a communication network.

2. We found the reliability of this complex network by using several methods. Two of them are presented as theorems with their proofs.

3. We studied the subject of assignment (allocation) and the importance of each unit of the complex network, which helped us later to understand the mechanism of the increase in the reliability of the complex network based on the importance of its units.

4. We employed BA and GWO algorithms to address the problem of obtaining best increase in the reliability of a complex network at the lowest possible cost.

1.5 Related Works

It can be said that reliability history system goes back to 1930 when the concept of probability was applied to electrical power generation issues. With the onset of World War II, the Germans managed to develop their missiles by taking advantage of the basic concepts of reliability by improving the reliability of those missiles. In 1947 the reliability of 100,000 electronic tubes was studied by Aviation Radio and Cornell University. And it was formed reliability committee established by the United States Department of Defense in 1950 and in 1952 it was transformed to a permanent body: Advisory Group on the Reliability of Electronic Equipment (AGREE) [27]. In 1952 Davis presented paper on the exponential distribution and discussed failure data for different outcomes [29, 35]. In 1956 Shannon and Moore introduced network reliability in a paper on relay circuit reliability. They presented network reliability as the probability that the relay circuit will remain closed, given that its components are closed with known probabilities. They introduced a probabilistic model of network reliability, talked about the possibility of edge failures independently, and considered the reliability of network nodes to be equal to one. The problem is determining the probability that the network will remain connected under these conditions. If all edges have the same failure probability, they can find the network reliability polynomial [49, 59, 65]. Then, many research papers on reliability were presented, but with different names, where the same concept was described with different descriptions, such as the ability to perform, the ability to survive and performance.

In 1962 Dayton and Ohio presented their first master's thesis in network reliability under the guidance of US Air Force(USAF) institute of technology. However, since the beginning of field reliability, several researchers have addressed the hundreds of papers that have been published on this concept [32, 33]. In 1971 Van Sylke and Frank took networks with random components and failed nodes and assumed that nodes were fully reliable, providing a combinatorial analysis when they were reliable. The components are equal and decomposition methods for large networks and applications for computer network analysis are described [78]. In 1972 Kim, suggest a method to reduce all parallel, and series-parallel components to an irreducibly series non-parallel system to calculate the reliability network [50]. Allan introduces an algorithm to find the minimal cut sets for any general network using Boolean algebra and group theory for the purpose of network reliability analysis in 1976 [16].

Limnios in 1987 introduces a method for predicting the failure rate of parallel networks, where relationships are derived to reach a steady state with a time-dependent failure rate and demonstrate how to use these relationships to obtain the steady state failure rate for complex networks [52]. In 1993 Aggarwal and Shashwati proposed a general method for allocating network reliability which can be applied to any network containing identical or mismatched components. The proposal is based on the numerical analytical method [12]. In 1997 Horowitz and Sanguthevar studied the problem of optimization reliability using dynamic programming. The problem was to calculate and optimize the reliability of a device consisting of several systems connected in series [48].

In 2005 Xiong, Jintao, and Weibo proposed a logical approximation algorithm to estimate network reliability. They concluded that it is possible to apply this algorithm to estimate both the reliability scale and the reliability of networks with different protection algorithms [83]. In 2006, Pham discusses in detail improvement network reliability prone to two types of failure. Assume that the network components are independent and identical, where he is stated that the number of components is not restricted. It also improved the reliability of series, parallel, parallel - series, and series - parallel networks [66]. In 2007 Liu, Chuanquan, and Yan Zhang studied the reliability of the electricity distribution network with the consideration of distribution generation [53]. In 2009 Boesch and Charles presented several results related to the analysis and synthesis of reliable or non-vulnerable networks, and studied some applications of reliability analysis as well as studied some related concepts in the synthesis of the most reliable network [22]. Also in 2009 Altiparmak, et al. introduced a new method to calculate the reliability of communication networks with identical link reliability and they optimize network reliability by two algorithms, taboo search and annealing simulation [17]. In 2010, Yeh and Wei-Chang proposed a new Particle Swarm Optimization (PSO) with Monte Carlo Simulation (MCS) which is called MCS-PSO, to optimize the reliability of complex networks. The proposed MCS-PSO can reduce cost under reliability constraints [87]. In 2013 the reliability of the Ad-Hoc Mobile Wireless Network (MANET) was evaluated by Padmavathy and Sanjay and then they optimize the reliability of the network using Monte Carlo simulation [64]. In 2015 Hassan introduced two types of equivalent techniques for assessing the reliability of Electric Aircraft Systems (EAS) [44]. In the same year also Hassan and Balan introduced the quadratic case of reliability, for the purpose of taking advantage of the convex/concave interdependence of the slide components along the fixed slide reliability curves, in order to increase or at least maintain the reliability of the circuit. Clear techniques of engineering to estimate the reliability of electrical networks used in aircraft show that elements of reliability must be linearly based on time [43]. At the same time too, Alghamdi and Percy studied reliability equalization factors for a series-parallel system of components with exponentiated Weibull distribution [14]. In 2016, Mutar and Hassan presented reliability of oxygen supply system for spacecraft and some engineering characteristics, also they have presented geometry of reliability models of electrical system usedinside spacecraft [46]. In 2017, Kumar investigated the applicability of multipurpose particle cluster optimization involving congestion distance (MOPSO-CD) to solve a complex reliability improvement problem with two opposing goals of reducing system cost and increasing system reliability [51]. In the same year, Abed, Udriste presented optimization techniques and methods in reliability allocation [9]. That year Shaghaghi and Saba used the Genetic Algorithm (GA) to improve the multi-objective pareto-optimized design of the Group Method for Data Handling (GMDH) for neural network outcomes. In addition, the particle swarm optimization (PSO) learning algorithm has been extended to GMDH for better comparison between models [70]. In 2018 Feng, Zhao and Li studied the reliability of electric vehicles equipped with wind turbines and the probability of failure, identified the main causes of failures and improved the reliability using the Particle Swarming (PSO) algorithm [38]. In 2019, Ouyang and Zhiyuan presented a paper to study the problem of intensive reliability and redundancy assignment to determine the optimal design of the system under several constraints, for the purpose of maximizing the reliability of the system using PSO algorithm with the nature of random perturbation [63]. In recent years, Aljobory ,Hatem and Hassan presented some mathematical models in the reliability networks, as well as finding ways to calculate the reliability of the electromagnetic system inside aircraft. The reliability allocation and optimization were also calculated using algorithms genetic algorithm and particle swarm optimization algorithm [74]. In 2020 Abdullah, Ghazi and Hassan presented a research to calculate the reliability of each component of a complex system using another algorithm, the Genetic algorithm to solve the problem of allocation and to optimize system reliability [8]. In 2021 the above two researchers studied the reliability of the same system and calculated the reliability allocation and optimization for a complex network by using GA algorithm, PSO algorithm, Ant Colony Algorithm (ACA)and bee's colony optimization, with a comparison between these algorithms to choose the best algorithm that gives the highest reliability and lowest cost [5, 6].

CHAPTER 2

CHAPTER TWO: BASIC DEFINITIONS AND CONCEPTS

2.1 Introduction

It is very necessary to start our study by including and clarifying some of the mathematical definitions and concepts that we need. Therefore, in this chapter, we will explain some concepts of probability theory such as properties of probability, independent events and probability density function, $\cdots$, etc. In addition, we discuss the graph theory and some of its concepts. Also, we will explain the structure function, types and its properties. Moreover, we will introduce reliability function, failure rate function, and some related functions. In view of the importance of minimal paths and minimal cut sets to the issue of network reliability, we allocated the sixth section to the method of calculating the minimal paths, while the seventh section is devoted to calculating the minimal cut sets. We also discussed what matters to us from the concept of optimization.

2.2 Some Basic Definitions of Probability

The subject of probability requires a set of random events, a subset of the universal set to which probabilities can be assigned [39, 62]. We will not expand on this section, as we will mention some of the definitions and concepts that we need in our study.

Definition 2.2.1. Let $S = \{A_1, A_2, \cdots, A_n\}$ be a set of random variables for any given experiment. Then the **Probability** Pr is a function $\Pr : S \to [0, 1]$.
The following properties hold [39]:

(1) $0 \leqslant \Pr(A_i) \leqslant 1$, for all event $A_i \subseteq S$.

(2) $\Pr(S) = \sum_{i=1}^{n} \Pr(A_i) = 1$ and $\Pr(\phi) = 0$.

(3) $\Pr(A_i) = 1 - \Pr(A_i^c)$, for all event $A_i \subseteq S$, here A_i^c represent to complement of A_i.

For any pair of events $A, B \subseteq S$, we have:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

Similarly, for any events, $A_1, A_2, \cdots, A_n \subseteq S$, then the probability of their union is given by:

$$\Pr\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \Pr(A_i) - \sum_{i_1<i_2}^{n} \Pr\left(A_{i_1} \bigcap A_{i_2}\right) + \sum_{i_1<i_2<i_3}^{n} \Pr\left(A_{i_1} \bigcap A_{i_2} \bigcap A_{i_3}\right).$$

$$- \cdots + (-1)^{n-1} \Pr\left(\bigcap_{i=1}^{n} A_i\right)$$

Definition 2.2.2. Let $A, B \subseteq S$ such that $A \cap B = \phi$. Then A and B are called **disjoint** events, and so, $\Pr(A \cap B) = 0$ [62].

So, for any two disjoint events A and B we have; $\Pr(A \cup B) = \Pr(A) + \Pr(B)$.
In general, the probability of union for k disjoint events $A_1, A_2, \cdots, A_k \subseteq S$, is given by:

$$\Pr\left(\bigcup_{i=1}^{k} A_i\right) = \sum_{i=1}^{k} \Pr(A_i). \tag{2.1}$$

Definition 2.2.3. Two different events A and B are called **Independent** events, if $\Pr(A \cap B) = \Pr(A). \Pr(B)$ [39,62].

Definition 2.2.4. Let X be a continuous random variable. Then the function $f(x)$ is called **Probability Density Function** (pdf), if for any interval $[a, b]$ one has [39]:

$$\Pr(a \leqslant X \leqslant b) = \int_{a}^{b} f(x)dx, \tag{2.2}$$

such that the following conditions are satisfied:

(1) $f(x) \geqslant 0$ for all x.

(2) $\int_{-\infty}^{\infty} f(x)dx = 1$.

Definition 2.2.5. Assume that X is a continuous random variable. Then the function $F(x)$ is called **Cumulative distribution function** (cdf), if for any number x one has [39, 62]:

$$\Pr(X \leqslant x) = \int_{-\infty}^{x} f(y)dy \tag{2.3}$$

From last two definitions, a relationship can be deduced between pdf and cdf as follows:

$$f(x) = \frac{d(F(x))}{dx}.$$

Definition 2.2.6. Assume that X is a random variable with probability density function $f(x)$. Then the **Mathematical Expectation** of $X, E(X)$ is given by [62]:

$$E(X) = \begin{cases} \sum_{i=1}^{n} xf(x) & \text{if } X = \{x_1, x_2, \cdots, x_n\} \text{(discrete random variable)} \\ \int_{-\infty}^{\infty} xf(x)dx & \text{if } X \text{ is a continuous random variable} \end{cases}$$

2.3 Some Basic Definitions of Graph Theory

A graph theory is one of the main topics we need to study reliability, so let's start with some definitions.

Definition 2.3.1. A **graph** $G = (V, E)$ is defined by an ordered (V, E) where $V \neq \phi$, it's elements are said to be nodes (vertices or points). The elements of E are called edges (lines or components) of G that is $E(G)$= $\{x = uv : u, v \in V\}$ [25, 81].

Definition 2.3.2. An **order** of a graph G is the number of nodes in G and the **size** of G is the number of edges in G, denoted by n and m respectively [26, 81].

Example 2.3.1. Figure 2.1 shows a graph G with $n = 5$ and $m = 6$. The set of nodes is $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ and the set of edges is $E(G) = \{x_1, x_2, x_3, x_4, x_5, x_6\}$.

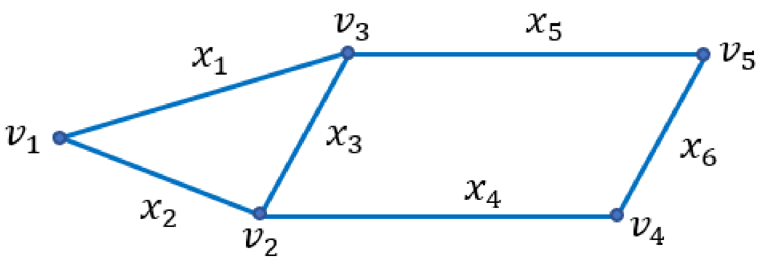

Figure 2.1: A Graph G with $n = 5$ and $m = 6$

Definition 2.3.3. A **simple graph** is a graph which has no an edge that joins a node to itself and has no two nodes are joined by more than one edge [25, 81].
The graph in Figure 2.1 is a simple graph.

Definition 2.3.4. Let x be an edge that connects the two nodes u and v by arrow (it has direction $u \to v$ or $u \leftrightarrow v$). Then x called a **directed** edge [26, 81].

Definition 2.3.5. A **directed graph** (or **digraph**) is a graph that is made up of a set of nodes connected by directed edges often called arcs [25, 81]. Figure 2.2 (a) and (b) are an equivalent digraphs.

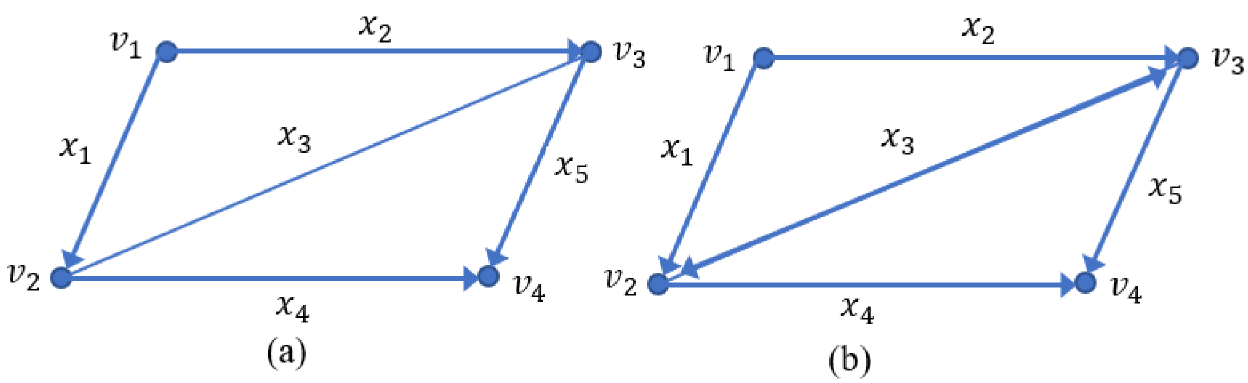

Figure 2.2: Directed Graph

***Remark* 1.** In any digraph if the edge x connects the two nodes u and v by a two-way arrow ($u \leftrightarrow v$), then we can drown x without directions as in Figure 2.2 the edge x_3 has two-way arrow in (b) while it without directions in (a).

Definition 2.3.6. Let G be a graph or digraph with n nodes. Then an **Adjacency matrix** (A) is $n \times n$ matrix has elements indicate whether pairs of nodes are adjacent or not in the graph. In the simple graph, the adjacency matrix is a $(0, x_{ij})$-matrix with zeros on its diagonal, here x_{ij} represent the edge between two nodes v_i and v_j, if v_i adjacent with v_j we write x_{ij} and if not put 0 [26, 81].

Example 2.3.2. The adjacency matrix of the graph in Figure 2.1 is

$$A_1 = \begin{pmatrix} 0 & x_2 & x_1 & 0 & 0 \\ x_2 & 0 & x_3 & x_4 & 0 \\ x_1 & x_3 & 0 & 0 & x_5 \\ 0 & x_4 & 0 & 0 & x_6 \\ 0 & 0 & x_5 & x_6 & 0 \end{pmatrix}$$

And the adjacency matrix of the digraph in Figure 2.2 is

$$A_2 = \begin{pmatrix} 0 & x_1 & x_2 & 0 \\ 0 & 0 & x_3 & x_4 \\ 0 & x_3 & 0 & x_5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Definition 2.3.7. A **connection matrix** (CM) constructed from adding an adjacency matrix with the identity matrix I_m, that is $CM = A + I_m$, [81] (i.e. replacing the zeros in diagonal of an adjacency matrix with ones).

Example 2.3.3. The connection matrix of the graph in Figure 2.1 is

$$CM_1 = \begin{pmatrix} 1 & x_2 & x_1 & 0 & 0 \\ x_2 & 1 & x_3 & x_4 & 0 \\ x_1 & x_3 & 1 & 0 & x_5 \\ 0 & x_4 & 0 & 1 & x_6 \\ 0 & 0 & x_5 & x_6 & 1 \end{pmatrix}$$

And the connection matrix of the digraph in Figure 2.2 is

$$CM_2 = \begin{pmatrix} 1 & x_1 & x_2 & 0 \\ 0 & 1 & x_3 & x_4 \\ 0 & x_3 & 1 & x_5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Definition 2.3.8. A **path** (P) is a set of units which, when working, connect the start node with the end node through working edges, thereby guaranteeing that the system is in a working state [61].

Definition 2.3.9. A **path length** (PL) is the number of edges (or units) in path [81].

Definition 2.3.10. Let G be a graph or digraph with n nodes. Then the **minimal path** (MP) is a path from which no edge can be removed without disconnecting the link between start and end nodes such that $PL < n$ [43, 61].

Example 2.3.4. In Figure 2.2 the minimal paths between start node v_1 and end node v_4 are as follows:

- $MP_1 : v_1 \to v_2 \to v_4 \;\equiv\; x_1x_4.$
- $MP_2 : v_1 \to v_3 \to v_4 \;\equiv\; x_2x_5.$
- $MP_3 : v_1 \to v_2 \to v_3 \to v_4 \;\equiv\; x_1x_3x_5.$
- $MP_4 : v_1 \to v_3 \to v_2 \to v_4 \;\equiv\; x_2x_3x_4.$

With $PL_1 = PL_2 = 2$ and $PL_3 = PL_4 = 3$, where MP_i and PL_i are the minimal path and path length of the $i-$th path.

In example (2.3.4), calculating minimal paths was easy, but we cannot do that for all graphs, to find minimal paths there are several methods , as we will see that in section (2.6) where one of those methods was adopted.

Definition 2.3.11. Let G be a graph with m edges and has np minimal paths. Then an **Incidence matrix** is an $np \times m$ (0,1)-matrix, denoted by IM such that [25, 81]

$$a_{ij} = \begin{cases} 1 & \text{when} \quad x_l \in MP_i \\ 0 & \text{when} \quad x_l \notin MP_i \end{cases}$$

Example 2.3.5. An incidence matrix of the digraph in Figure 2.2 is given by:

$$IM = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Definition 2.3.12. The graph or digraph is called be a **connected** if and only if there exist at lest one path between every pair of nodes, otherwise it called **disconnected** [81].

Example 2.3.6. The graph in Figure 2.1 and the digraph in Figure 2.2 are connected. While, the graph in Figure 2.3 is disconnected because there isn't any path from v_1 to v_4.

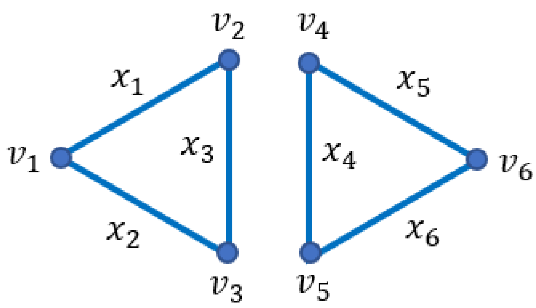

Figure 2.3: Disconnected Graph

Definition 2.3.13. Let G be a graph and C be a set of edges in G. Then C is said to be **cut set** of G if deletion all edges of C from G makes G disconnected [26, 61].

Definition 2.3.14. The number of edges in a cut set is said to be **order** of cut set, denoted by OC [81].

Definition 2.3.15. A **minimal cut set** (MC) of a graph with n nodes is a cut set that does not contain any other cut set such that $OC < n$ [26, 61].

Example 2.3.7. Take a look at the connected graph in Figure 2.4:

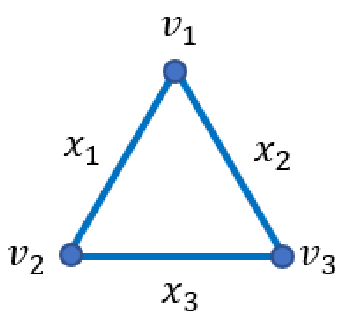

Figure 2.4: Connected Graph

All cut sets of this connected graph are as follows:

- $C_1 = \{x_1, x_2\}$.
- $C_2 = \{x_1, x_3\}$.
- $C_3 = \{x_2, x_3\}$.
- $C_4 = \{x_1, x_2, x_3\}$

With OC = 2,2,2,3, respectively.

Note that C_1, C_2 and C_3 are minimal cut sets while, C_4 is not minimal cut set for two reasons, first one is $OC_4 = 3 = n$ and the second reason is $C_1, C_2, C_3 \subseteq C_4$, here C_i represent to i-th cut set and OC_i represent to order of i-th cut set.

In example (2.3.7), calculating minimal cut sets was easy, but we cannot do that for all graphs, to calculate minimal cut sets of any graph there are several methods, as we will see that in section (2.7) where one of those methods was adopted.

The concepts of minimal paths and minimal cut sets are very important in the study and analysis of the network reliability it is included in most of the special methods for calculating the reliability of networks, as we will see in the third chapter.

Remark 2. From now on, we shall write a network instead of a graph or digraph and a component instead of the edge. We must not confuse between two concepts, simple graph and simple network, the first concept includes the second in addition to the complex network. The difference between a simple and complex networks depends on how its components are connected and has nothing to do with their number, we may have a network with a million components (in series, parallel or mixed), but it is simple network, on the other hand we may have a network with less than five components, but it is a complex network if they are connected in a way that is not series, parallel, or mixed for example the graph in Figure 2.2 is a simple graph but it is a complex network.

2.4 Structure Function

A **Structure Function** $\Phi(x)$ is a binary characteristic shows the status of network whether it is working or not. It gives the state of each component.
Consider a network has m components, each component of them could have two feasible states success and failure. The state of component k is given by the binary variable x_k [44]

$$x_k = \begin{cases} 1 \text{ if component } k \text{ success} \\ 0 \text{ if component } k \text{ fails} \end{cases}$$

From the formula above one have for all i and n

$$x_i^n = x_i \tag{2.4}$$

The interpretation of network is measured by the binary random variable

$$\Phi(x_1, ..., x_m) = \begin{cases} 1 \text{ if network success} \\ 0 \text{ if network fails} \end{cases}$$

2.4.1 Series Structure

A network that operates if and only if all components are success is said to be a **series structure**. A series structure with m components is illustrated by diagram in Figure 2.5

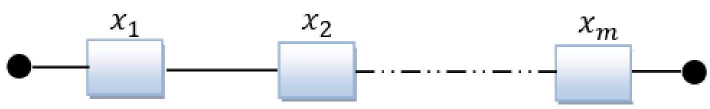

Figure 2.5: Series Structure

The structure function with m components in series is given by [2]:

$$\Phi(x) = \prod_{i=1}^{m} x_i \tag{2.5}$$

Example 2.4.1. The structure function of four connected components in series is given by: $\Phi(x) = x_1 x_2 x_3 x_4$.

2.4.2 Parallel Structure

A network that operates when at least one of its components is working successfully is called a **parallel structure** [46]. A parallel structure with m components is illustrated by diagram shown in Figure 2.6

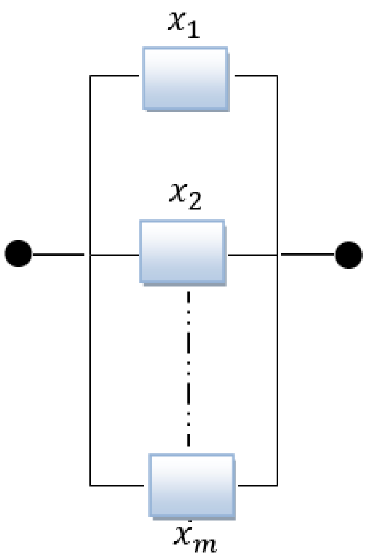

Figure 2.6: Parallel Structure

The structure function with m components in parallel is given by:

$$\Phi(x) = 1 - \prod_{i=1}^{m} (1 - x_i) \tag{2.6}$$

Example 2.4.2. The structure function of four connected components in parallel is given by:

$\Phi(x) = 1 - (1 - x_1)(1 - x_2)(1 - x_3)(1 - x_4)$.

Or $\Phi(x) = x_1 + x_2 + x_3 + x_4 - x_1x_2 - x_1x_3 - x_1x_4 - x_2x_3 - x_2x_4 - x_3x_4 + x_1x_2x_3$
$+ x_1x_2x_4 + x_2x_3x_4 - x_1x_2x_3x_4$.

2.4.3 Properties of Structure Function

The structure function is an important mathematical model in network reliability which has some properties. Consider a network with m components, the properties of a structure function are :

1. $\Phi(0) = 0$ and $\Phi(1) = 1$.
2. $\Phi(x_i.x_j) \leqslant \Phi(x_i).\Phi(x_j)$.
3. $\Phi[(1 - x_i).(1 - x_j)] \geqslant [1 - \Phi(x_i)].[1 - \Phi(x_j)]$.
4. $\prod_{i=1}^{m} x_i \leqslant \Phi(x) \leqslant \prod_{i=1}^{m} (1 - x_i)$.

2.5 Reliability Function

The essential elements of the reliability of any manufactured equipment are probability, adequate performance, adequate performance duration and operating conditions [25].

Definition 2.5.1. A **Reliability** $R(t)$ is the probability that a component, device, or system will perform its task well for a specified period of time under certain conditions.

The definition can be formulated in a mathematical expression in terms of the random variable T, the time of system or device failure by [13, 73]:

$$R(t) = \Pr(T > t) \tag{2.7}$$

The definition of reliability includes all four aspects of the product, unlike quality, which only talks about specifications. This means that reliability is quality over time, which is subject to the influence of time and operating environment. There is another difference between reliability and quality, which is that reliability devices can be manufactured using less reliability components, while it is very difficult to manufacture high-quality devices with lower quality components.

Definition 2.5.2. The **Failure Function** $F(t)$ is the probability failure takes place at a time less than or equal to t can be defined as follows [25]:

$$F(t) = \Pr(T < t) \tag{2.8}$$

The reliability function $R(t)$ has the following properties:

(1) $R(t) \in [0, 1]$.

(2) If $t_1 < t_2$, then $R(t_1) > R(t_2)$, that is $R(t)$ is decreasing function at t.

(3) $R(t) = \begin{cases} 1 & \text{when } t = 0 \\ 0 & \text{when } t \to \infty. \end{cases}$

The third property means that product operates normally at time $t = 0$ and no product can continue to work forever without failure. God Almighty says in the Holy Quran (***Everyone upon the earth will perish, and there will remain the Face of your Lord, Owner of Majesty and Honor***.) (The Most Gracioușe 26-27).
$R(t)$ can be can be written as: $R(t) = \Pr(T \geqslant t)$. So,

$$R(t) = 1 - F(t). \tag{2.9}$$

This equivalent to

$$R(t) = \int_{-\infty}^{\infty} f(x)dx - \int_{-\infty}^{t} f(x)dx$$

Therefor, we can write the reliability function in terms of pdf as follows:

$$R(t) = \int_{t}^{\infty} f(x)dx. \tag{2.10}$$

Conversely, we can write the pdf in terms of $R(t)$ as follows:

$$f(t) = -\frac{d(R(t))}{dt}. \tag{2.11}$$

Using the exponential distribution, the pdf can be written in the form:

$$f(t) = \lambda e^{-\lambda t} \tag{2.12}$$

here λ is a parameter of the exponential distribution.

Therefore, the reliability function of the exponential distribution can be derived based on equation (2.10) as follows:

$$R(t) = 1 - \int_0^t \lambda e^{-\lambda x} dx$$

So, the reliability function becomes as follows:

$$R(t) = e^{-\lambda t} \tag{2.13}$$

here λ is a failure rate.

Definition 2.5.3. Let $X = \{x_1, x_2, \cdots, x_m\}$ and the component x_i being random variables, then there exist two probabilities R_N and R_i, such that $R_N = \Pr(\Phi(X) = 1) = E(\Phi(X))$, and $R_i = \Pr(x_i = 1) = E(x_i)$.

Then R_N is called the **Reliability Network** and R_i represents the reliability of i-th component [44].

Definition 2.5.4. Let G be a network with m components x_i; $i = 1, 2, \cdots, m$, each component has reliability $R_i, 0 \leq R_i \leq 1$, for all $i = 1, 2, \cdots, m$. Then the **Reliability Polynomial** R_N is nonlinear function in terms of R_i [45].

Definition 2.5.5. A **Reliability Block Diagram** (RBD) is a schematic way to show how the reliability of network components affects the success or failure of redundancy. The RBD is drawn based on the all minimal paths in parallel configuration, or on the all minimal cut sets in series configuration [74].

Example 2.5.1. Consider the network in Figure 2.7

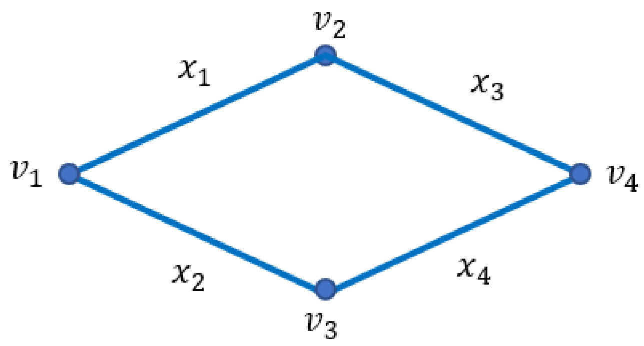

Figure 2.7: Network with $n = m = 4$

The minimal paths are: $MP_1 = x_1x_3$ and $MP_2 = x_2x_4$.

So, the RBD of digraph based minimal paths shown in Figure 2.8

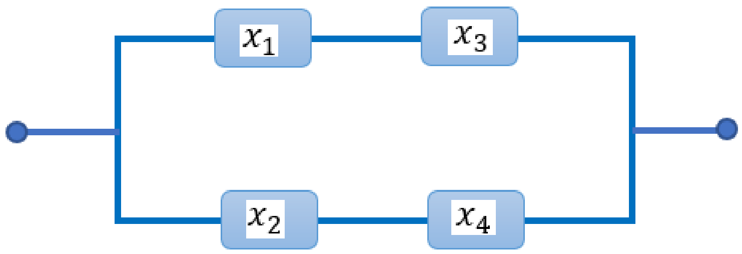

Figure 2.8: RBD Based Minimal Paths

And the minimal cut sets are: $MC_1 = \{x_1, x_2\}, MC_2 = \{x_3, x_4\}, MC_3 = \{x_1, x_4\}$ and $MC_4 = \{x_2, x_3\}$.

So, the RBD of digraph based minimal cut sets shown in Figure 2.9

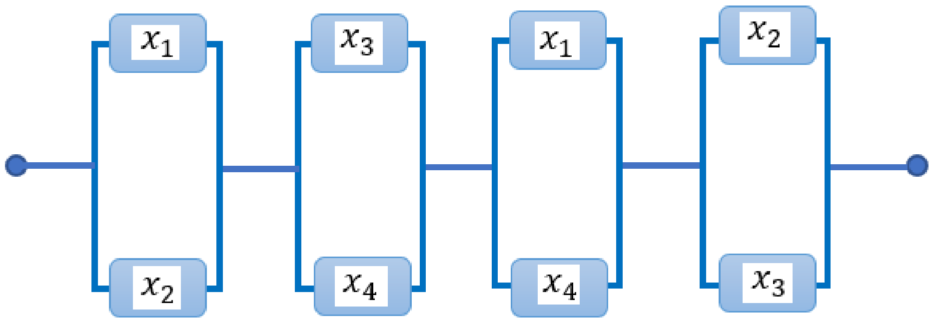

Figure 2.9: RBD Based Minimal Cut Sets

2.6 Calculation Minimal Paths

There are several methods to find minimal paths, one of them is the direct way from the network by tracing the paths from the start node to the end node as we show in example (2.3.4), but this way is not feasible sometimes, especially in complex networks, so we will rely on one of these methods. In this method, we will be using a connection matrix CM to create minimal paths.

To find minimal paths delete any node which is neither the start nor the end from CM, one by one until the only two nodes start and end nodes left in the matrix. That is CM becomes 2×2 matrix in the form [25]:

$$CM = \begin{pmatrix} 1 & \sum_{i=1}^{np} MP_i \\ & \\ 0 & 1 \end{pmatrix} \tag{2.14}$$

where np is the number of minimal paths.

When the node k is removed, the entries of the relationship matrix with the closing nodes are changed the usage subsequent equation

$$a_{ij}^{k} = \begin{cases} a_{ij}^{k-1} + a_{ik}^{k-1} a_{kj}^{k-1} & \text{if the node } k \text{ is removed } ; i \neq j \\ 1 & \text{if } i = j \end{cases} \tag{2.15}$$

where a_{ij}^{k} and a_{ij}^{k-1} represent the element a_{ij} in a new matrix and old matrix respectively. We will illustrate this method with the following example.

Example 2.6.1. Consider the network in Figure 2.2 and it's CM_2 in example (2.3.3). Remove node 2 (i.e 2^{nd} row and 2^{nd} column from CM_2) with apply equation (2.15) to get:

The first row in new matrix is: $a_{11}^{1} = 1, a_{13}^{1} = x_2 + x_1 x_3, a_{14}^{1} = x_1 x_4$.

The second row is: $a_{31}^{1} = 0, a_{33}^{1} = 1, a_{34}^{1} = x_5 + x_3 x_4$.

And the third row is: $a_{41}^1 = 0, a_{43}^1 = 0, a_{44}^1 = 1$

The modified connection matrix becomes 3×3 matrix in the following:

$$\begin{pmatrix} 1 & x_2 + x_1x_3 & x_1x_4 \\ 0 & 1 & x_5 + x_3x_4 \\ 0 & 0 & 1 \end{pmatrix}$$

Then, remove node 3 (i.e 2^{nd} row and 2^{nd} column from last matrix) with apply equation (2.15) again to get: $a_{11}^2 = 1, a_{13}^2 = x_1x_4 + (x_2 + x_1x_3)(x_5 + x_3x_4)$, $a_{31}^2 = 0$ and $a_{33}^2 = 1$.

Simplify the term a_{13}^2 and then apply equation (2.4) to get:

$a_{13}^2 = x_1x_4 + x_2x_5 + x_2x_3x_4 + x_1x_3x_5 + x_1x_3x_4$

The modified connection matrix becomes the following 2×2 matrix

$$\begin{pmatrix} 1 & x_1x_4 + x_2x_5 + x_2x_3x_4 + x_1x_3x_5 + x_1x_3x_4 \\ 0 & 1 \end{pmatrix}$$

Compared with equation (2.14) we get:

$$\sum_{i=1}^{np} MP_i = x_1x_4 + x_2x_5 + x_2x_3x_4 + x_1x_3x_5 + x_1x_3x_4.$$

Therefore,the minimal paths are:

$MP_1 = x_1x_4$, $MP_2 = x_2x_5$. $MP_3 = x_1x_3x_5$ and $MP_4 = x_2x_3x_4$.

Notes:

(1) $PL < 4$ for all MP.

(2) $MP_5 = x_1x_3x_4 \supset MP_1$ therefore, $MP_5 = x_1x_3x_4$ it is not mentioned with minimal paths.

2.7 Calculation Minimal Cut Sets

Consider a network with n nodes, m components and np minimal paths. Then the IM of this network is $np \times m$ matrix with 0 and 1 elements. For a generation of minimal cut sets, $n-1$ steps have to be followed [25]:

- **Step 1**: If for all $a_{ij} \neq 0$ of any column x_i of IM, then x_i forms a first order cut.
- **Step 2**: Add two columns of IM at a time, If for all $i; a_{ij} + a_{ik} \neq 0$, then $x_j x_k$ form a second order cut, where $k > j, (k = 1, ..., m)$. Deleting any cut set containing first order cuts to give second order minimal cut sets.
- **Step 3**: Repeat step (2) with three columns at a time to give the third order cuts, this time delete any cut set containing first and second order cut sets.

  $\vdots \quad \vdots \quad \vdots$

- **Step $n-1$**: Repeat step $n-2$ with $n-1$ columns at a time to give the $n-1$ order cut set, this time delete any cut set containing cut set with order less than $n-1$.

Example 2.7.1. Consider the complex network in Figure 2.2 with $n = 4$ $m = 5$, we find the IM in example (2.3.5). Since $n = 4$, we have three steps:

(1) There are no first order cut set because no single column in the IM exists in which all elements are non-zero.

(2) Add two columns of the IM at a time, we get two minimal cut sets of order two which are $MC_1 = \{x_1, x_2\}$ and $MC_2 = \{x_4, x_5\}$.

(3) Add three columns at a time, we get six cut set of order three which are:

- $C_3 = \{x_1, x_2, x_3\} \supset MC_1$.
- $C_4 = \{x_1, x_2, x_4\} \supset MC_1$.
- $C_5 = \{x_1, x_2, x_5\} \supset MC_1$.

- $C_6 = \{x_2, x_3, x_4\}$.
- $C_7 = \{x_2, x_3, x_5\}$. $\mathrm{C}_8 = \{x_3, x_4, x_5\} \supset MC_2$.

But the cut sets C_3, C_4, C_5 and C_8 all of them containing a minimal cut set of order two, then they are not minimal cut sets. While C_6 and C_7 are both of them minimal cut set. So, the minimal cut sets are:

- $MC_1 = \{x_1, x_2\}$.
- $MC_2 = \{x_4, x_5\}$.
- $MC_3 = \{x_2, x_3, x_4\}$.
- $MC_4 = \{x_2, x_3, x_5\}$.

2.8 Optimization

Artificial Intelligence (AI) includes all types of intelligence provided by machines. The main topics of AI is the study of unconventional optimization techniques. It is considered Computational Intelligence (CI) is a branch of AI and is the basic principle of all optimization algorithms known as metaheuristic algorithms, they are a trial-and-error method of production that provides an acceptable solution to a complex problem in a reasonable practical time [23].

Optimization is the process of maximizing or minimizing a desired objective function while satisfying the prevailing constraints. The companies were based on the concept of improvement to strive for excellence. Solutions to their problems have mostly been based on judgment and experience. However, competition requires that the solutions be perfect and not merely feasible. Often, improving the design process saves money for producing companies [20, 86]. Optimization problems have three basic elements. The first is the goal , may be a single scalar quantity, or an objective function, that must be minimized or maximized, increased or decreased. The second is a set of variables whose values affect the value of the goal, and can be manipulated to improve the goal. The third is a set of constraints, which are constraints on the values that variables can take. For example, determining the cost of manufacturing a device specific so that it is not more than what is available, and the cost cannot be equal to zero or less than it [18, 21, 86].

2.8.1 Linear Programming

Linear Programming (LP) is an important category of optimization and indicates that there are no variables to powers greater than one. For this class, problems involve maximizing or minimizing a linear objective function whose variables are real numbers bound to satisfy a system of linear equality or inequality [20, 86].

2.8.2 Nonlinear Programming

The general optimization problem is to select k decision variables $x_1, x_2, \cdots, x_k$ from a given feasible region in such a way as to optimize (minimize or maximize) a given objective function $f(x_1, x_2, \cdots, x_k)$ of the decision variables. The problem is called a **Nonlinear Programming** (NLP) problem if the objective function is nonlinear or the feasible region is determined by nonlinear constraints. Thus, in maximization form, the general nonlinear program is stated as [19, 75, 86]:

$$
\begin{aligned}
&\text{Maximize } f(x_1, x_2, \cdots, x_k),\\
&\text{Subject to:}\\
&g_1(x_1, x_2, \cdots, x_k) \leq a_1,\\
&g_2(x_1, x_2, \cdots, x_k) \leq a_2,\\
&\quad\vdots \qquad\qquad\qquad \vdots\\
&g_j(x_1, x_2, \cdots, x_k) \leq a_j,
\end{aligned}
$$

here g_i represent to a given constraint functions, a_i are constants for all $i = 1, \cdots, j$.

2.8.3 Multi-objective Optimization

A basic single-objective optimization problem can be formulated as follows:

$$\text{Minimize } f(x)$$
$$\text{Subject to: } x \in \Lambda$$

here f is a scalar function and Λ is the (implicit) set of constraints that can be defined as:

$$\Lambda = \{x \in \mathbb{R}^k : g(x) > 0\}.$$

Multi-objective optimization typically arises in several engineering modeling problems among which the decision maker selects several competing objectives to satisfy. Multi-objective optimization problems are also called multi-criteria optimization problems. A multi-objective optimization problem can be expressed as:

$$\text{Minimize } (f_1(x), f_2(x), \ldots, f_k(x))$$
$$\text{Subject to: } x \in \Lambda \text{ (i.e } x{=}\{x_1, x_2, \ldots, x_k\}$$

here $f_i : \mathbb{R}^k \to \mathbb{R}$, for all $i = 1, \cdots, k, k \geqslant 2$ are (possibly) conflicting objective functions and $\Lambda \subseteq \mathbb{R}^k$ is the feasible region.

The goal of multi-objective optimization is to simultaneously minimize all of the objective functions.

For consistency, we transform all the maximization problems of the type maximize f_i into equivalent minimization problems minimize $(-f_i)$ [42, 54, 76].

CHAPTER 3

CHAPTER THREE: SOME TECHNIQUES TO FIND RELIABILITY NETWORK

3.1 Introduction

In this chapter, we shall find the reliability polynomial of a complex network by using four techniques, which are, Minimal Path Technique (MPT), Minimal Cut Technique (MCT), Reduction to Parallel Elements Technique (RPE), in which the network is transformed into two parallel subnetworks and Delta Star Technique (DST), where the network will be transformed into two subnetworks in series.

To evaluate the reliability R_N; $(0 \leq R_N \leq 1)$ of any network with m components we must know the reliability of each component R_i, and compensation these reliabilities in reliability polynomial [12, 25].

The complex network shown in Figure 3.1 which its components connected in a way that is not series, parallel, or mixed, was chosen to apply the mentioned techniques to find its reliability and then we study the issue of reliability allocation in addition to optimzation the reliability network.

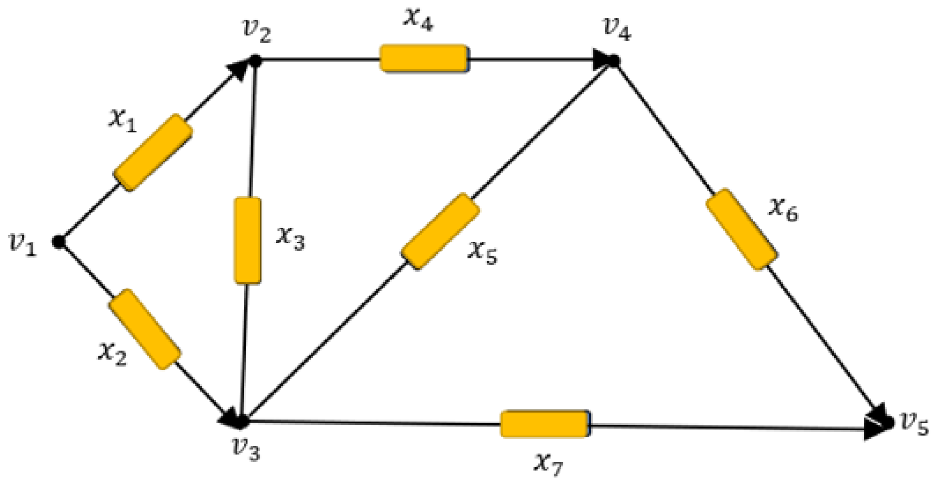

Figure 3.1: Complex Network

3.2 Minimal Path Technique (MPT)

To find a reliability polynomial for any complex network we must first know the structure function of this network where we substitute Φ by R_N, and each x_i by R_i [3,25]. For generation the structure function of any complex network has np of minimal paths, we put forward the following theorem:

Theorem 3.2.1. *Let G be a complex network and let $MP_1, MP_2, ..., MP_{np}$ be the complete list of all minimal paths. Then the structure function of G is given by:*

$$\Phi(x) = 1 - \prod_{i=1}^{np} \left(1 - \prod_{x_j \in MP_i} x_j\right) \tag{3.1}$$

Proof: Let MP_i be a minimal path in G.

It is obvious that the components of any path are in series [2].

So, we can applying equation (2.5).

Thus, the structure function of a minimal path MP_i is given by:

$$\Phi_{MP_i}(x) = \prod_{x_j \in MP_i} x_j.$$

It is clear that all paths are in parallel [2, 43].

So, we can applying equation (2.6).

Thus, the structure function of G is:

$$\Phi(x) = 1 - \prod_{i=1}^{np} \left(1 - \Phi_{MP_i}(x)\right).$$

Which leads to:

$$\Phi(x) = 1 - \prod_{i=1}^{np} \left(1 - \prod_{x_j \in MP_i} x_j\right)$$

■

To find the reliability polynomial of any complex network by this technique, we applying the folloing two steps:

- **Step 1**: Create CM to find the minimal paths as in section (2.6).
- **Step 2**: Apply theorem (3.2.1) to find the structure function and apply equation (2.4) to put the structure function in simplest form, then replace $\Phi(x)$ by R_N, and each x_i by R_i to get the reliability polynomial.

Now, we begin to implement these two steps on the complex network in Figure 3.1. First, create an adjacency matrix of the complex network as follow:

$$A = \begin{pmatrix} 0 & x_1 & x_2 & 0 & 0 \\ 0 & 0 & x_3 & x_4 & 0 \\ 0 & x_3 & 0 & x_5 & x_7 \\ 0 & 0 & x_5 & 0 & x_6 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

But we have $CM = A + I_5$, then CM is as follow:

$$CM = \begin{pmatrix} 1 & x_1 & x_2 & 0 & 0 \\ 0 & 1 & x_3 & x_4 & 0 \\ 0 & x_3 & 1 & x_5 & x_7 \\ 0 & 0 & x_5 & 1 & x_6 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Remove node 2 (i.e 2^{nd} row and 2^{nd} column from CM) and apply equation (2.15) to get:

The first row in new matrix is: $a^1_{11} = 1$, $a^1_{13} = x_2 + x_1x_3$, $a^1_{14} = x_1x_4$, and $a^1_{15} = 0$.

The second row is: $a^1_{31} = 0$, $a^1_{33} = 1$, $a^1_{34} = x_5 + x_3x_4$, and $a^1_{35} = x_7$.

The third row is: $a^1_{41} = 0$, $a^1_{43} = x_5$, $a^1_{44} = 1$, and $a^1_{45} = x_6$.

The fourth row is: $a^1_{51} = 0$, $a^1_{53} = 0$, $a^1_{54} = 0$, and $a^1_{55} = 1$.

The modified connection matrix becomes the following 4×4 matrix:

$$CM^1 = \begin{pmatrix} 1 & x_2 + x_1x_3 & x_1x_4 & 0 \\ 0 & 1 & x_5 + x_3x_4 & x_7 \\ 0 & x_5 & 1 & x_6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Remove node 3 (i.e 2^{nd} row and 2^{nd} column from CM^1) and apply equation (2.15) to get:

First row is: $a^2_{11} = 1$, $a^2_{13} = x_1x_4 + (x_2 + x_1x_3)(x_5 + x_3x_4)$, and $a^2_{14} = (x_2 + x_1x_3)x_7$.

Second row is: $a^2_{31} = 0, a^2_{33} = 1$, $a^2_{34} = x_6 + x_5x_7$.

Third row is: $a^2_{41} = 0$, $a^2_{43} = 0$, and $a^2_{44} = 1$.

Simplify the term a^2_{13} by opening the brackets and applying equation (2.4) to get:

$a^2_{13} = x_1x_4 + x_2x_5 + x_2x_3x_4 + x_1x_3x_5$.

Simplify the term a^2_{14} to get: $a^2_{14} = x_2x_7 + x_1x_3x_7$.

Then, the modified connection matrix becomes the following 3×3 matrix:

$$CM^2 = \begin{pmatrix} 1 & x_1x_4 + x_2x_5 + x_2x_3x_4 + x_1x_3x_5 & x_2x_7 + x_1x_3x_7 \\ 0 & 1 & x_6 + x_5x_7 \\ 0 & 0 & 1 \end{pmatrix}$$

Finlly, remove node 4 (i.e 2^{nd} row and 2^{nd} column from CM^2) with apply equation (2.15)

First row is: $a^3_{11} = 1$ and

$a^3_{13} = x_2x_7 + x_1x_3x_7 + (x_1x_4 + x_2x_5 + x_2x_3x_4 + x_1x_3x_5)(x_6 + x_5x_7)$.

Second row is: $a^3_{31} = 0$ and $a^2_{33} = 1$.

Simplify the term a^3_{13} by opening the brackets and applying equation (2.4) to get:

$a^3_{13} = x_2x_7 + x_1x_3x_7 + x_1x_4x_6 + x_2x_5x_6 + x_2x_3x_4x_6 + x_1x_3x_5x_6 + x_1x_4x_5x_7 + x_2x_3x_4x_5x_7$.

Then, the modified connection matrix CM^3 becomes the following 2×2 matrix:

$$\begin{pmatrix} 1 & x_2x_7 + x_1x_3x_7 + x_1x_4x_6 + x_2x_5x_6 + x_2x_3x_4x_6 + x_1x_3x_5x_6 + x_1x_4x_5x_7 + x_2x_3x_4x_5x_7 \\ 0 & 1 \end{pmatrix}$$

Compared with equation (2.14) we get:

$$\sum_{i=1}^{np} MP_i = x_2x_7+x_1x_3x_7+x_1x_4x_6+x_2x_5x_6+x_2x_3x_4x_6+x_1x_3x_5x_6+x_1x_4x_5x_7+x_2x_3x_4x_5x_7.$$

Therefor, the minimal paths are:

- $MP_1 = x_2x_7$.
- $MP_2 = x_1x_3x_7$.
- $MP_3 = x_1x_4x_6$.
- $MP_4 = x_2x_5x_6$.

- $MP_5 = x_2x_3x_4x_6$
- $MP_6 = x_1x_3x_5x_6$.
- $MP_7 = x_1x_4x_5x_7$.

While $MP_8 = x_2x_3x_4x_5x_7$ is a path but not minimal path because $LP_8 = 5 = n$.

Now, applying theorem (3.2.1) to find the structure function which is:

$\Phi(x) = 1 - (1 - x_2x_7)(1 - x_1x_3x_7)(1 - x_1x_4x_6)(1 - x_2x_5x_6)(1 - x_2x_3x_4x_6)(1 - x_1x_3x_5x_6).$

$(1 - x_1x_4x_5x_7)$

Opening the brackets and applying equation (2.4) to get:

$$\begin{aligned}\Phi(x) &= x_1x_2 + x_6x_7 + x_2x_3x_4 + x_4x_5x_7 + x_1x_3x_5x_7 + x_2x_3x_5x_6 + x_1x_2x_3x_4x_5x_6 \\ &+2x_1x_2x_3x_4x_5x_7 + x_1x_2x_3x_4x_6x_7 + 2x_1x_2x_3x_5x_6x_7 + x_1x_2x_4x_5x_6x_7 + x_1x_3x_4x_5x_6x_7 \\ &+2x_2x_3x_4x_5x_6x_7 - x_1x_2x_3x_4 - x_1x_2x_6x_7 - x_4x_5x_6x_7 - x_1x_2x_3x_5x_6 - x_1x_2x_3x_5x_7 \\ &-x_1x_2x_4x_5x_7 - x_2x_3x_4x_5x_6 - x_2x_3x_4x_5x_7 - x_1x_3x_5x_6x_7 - x_2x_3x_4x_6x_7 - x_1x_3x_4x_5x_7 \\ &-x_2x_3x_5x_6x_7 - 3x_1x_2x_3x_4x_5x_6x_7.\end{aligned}$$

Then, replace $\Phi(x)$ by R_N, and each x_i by R_i to get the reliability polynomial which is:

$$\begin{aligned}R_N &= R_1R_2 + R_6R_7 + R_2R_3R_4 + R_4R_5R_7 + R_1R_3R_5R_7 + R_2R_3R_5R_6 + R_1R_2R_3R_4R_5R_6 \\ &+2R_1R_2R_3R_4R_5R_7 + R_1R_2R_3R_4R_6R_7 + 2R_1R_2R_3R_5R_6R_7 + R_1R_2R_4R_5R_6R_7 \\ &+R_1R_3R_4R_5R_6R_7 + 2R_2R_3R_4R_5R_6R_7 - R_1R_2R_3R_4 - R_1R_2R_6R_7 - R_4R_5R_6R_7 \\ &-R_1R_2R_3R_5R_6 - R_1R_2R_3R_5R_7 - R_1R_2R_4R_5R_7 - R_2R_3R_4R_5R_6 - R_2R_3R_4R_5R_7 \\ &-R_1R_3R_5R_6R_7 - R_2R_3R_4R_6R_7 - R_1R_3R_4R_5R_7 - R_2R_3R_5R_6R_7 \\ &-3R_1R_2R_3R_4R_5R_6R_7. \qquad (3.2)\end{aligned}$$

3.3 Minimal Cut Technique (MCT)

Assume that G is a complex network has k of minimal cut sets. Then the structure function can be founded by the following theorem [25]:

Theorem 3.3.1. *Let G be a complex network and let $MC_1, MC_2, ..., MC_k$ be the complete list of all minimal cut sets of G. Then the structure function of G is given by:*

$$\Phi(x) = \prod_{i=1}^{k} \left(1 - \prod_{x_j \in MC_i} (1 - x_j)\right)$$

(3.3)

Proof: Let MC_i be a minimal cut set in G.

It is clear that the components of a cut set are in parallel [3, 43].

So, we can applying equation (2.6) .

Thus, the structure function of a minimal cut set MC_i is given by:

$$\Phi_{MC_i}(x) = 1 - \prod_{x_j \in MC_i} \left(1 - x_j\right).$$

It is obvious that all cut sets are in series [2, 43].

So, we can applying equation (2.5).

Then, the structure function of G is:

$$\Phi(x) = \prod_{i=1}^{k} \left(1 - \Phi_{MC_i}(x)\right).$$

Thus,

$$\Phi(x) = \prod_{i=1}^{k} \left(1 - \prod_{x_j \in MC_i} (1 - x_j)\right).$$

■

So, there are two steps to find the polynomial reliability of any complex network by using minimal cut sets which are:

- **Step 1**: Create (IM) to find the minimal cut sets as in section (2.7).
- **Step 2**: Apply theorm (3.3.1) to find the structure function and then apply equation (2.4) to put the structure function in simplest form, then replace $\Phi(x)$ by R_N, and each x_i by R_i to get the reliability polynomial.

Now, we begin to implement the above steps on the complex network with $n = 5, m = 7$ in Figure 3.1.

The (IM) of this complex network is a 7×7 matrix in the following:

$$IM = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Since $n = 5$, we have four steps:

(1) From the IM no single column exists in which all elements are non-zero, then there are no first order cut sets.

(2) Add two columns of IM at a time, we get two minimal cut sets of order two which are $C_1 = \{x_1, x_2\}$ and $C_2 = \{x_6, x_7\}$.

(3) Add three columns at a time, we get also two minimal cut sets of order three which are $C_3 = \{x_2, x_3, x_4\}$ and $C_4 = \{x_4, x_5, x_7\}$.

(4) Add four columns at a time, we get ten cut sets of order four which are:
$C_5 = \{x_1, x_2, x_3, x_4\}, C_6 = \{x_1, x_2, x_3, x_5\}, C_7 = \{x_1, x_2, x_3, x_6\}$,
$C_8 = \{x_1, x_2, x_3, x_7\}, C_9 = \{x_1, x_4, x_6, x_7\}, C_{10} = \{x_2, x_3, x_4, x_5\}$,
$C_{11} = \{x_2, x_3, x_4, x_6\}, C_{12} = \{x_2, x_3, x_4, x_7\}, C_{13} = \{x_3, x_4, x_5, x_7\}$
and $C_{14} = \{x_4, x_5, x_6, x_7\}$.
Note that the cut sets $C_5, C_6, C_7, C_8, C_{10}, C_{11}, C_{12}$ and C_{14} all of them containing a minimal cut set of order less than four, then they are not minimal cut sets. While C_9 and C_{13} both of them a minimal cut set.
So, all minimal cut sets are:

- $MC_1 = \{x_1, x_2\}$.
- $MC_2 = \{x_6, x_7\}$.
- $MC_3 = \{x_2, x_3, x_4\}$.
- $MC_4 = \{x_4, x_5, x_7\}$.
- $MC_5 = \{x_1, x_4, x_6, x_7\}$.
- $MC_6 = \{x_3, x_4, x_5, x_7\}$.

Applying theorm (3.3.1) to find the structure function as below:

$$\Phi(x) = [1-(1-x_1)(1-x_2)][1-(1-x_6)(1-x_7)][1-(1-x_2)(1-x_3)(1-x_4)]$$
$$[1-(1-x_4)(1-x_5)(1-x_7)][1-(1-x_1)(1-x_3)(1-x_5)(1-x_7)]$$
$$[1-(1-x_2)(1-x_3)(1-x_5)(1-x_6)]$$

Applying equation (2.5) to put the structure function in simplest form to get $\Phi(x)$, then replace $\Phi(x)$ by R_N, and each x_i by R_i to get reliability polynomial which is the same in equation (3.2).

3.4 Reduction to Parallel Elements (RPE)

In this technique, we draw first RBD which is based on minimal paths and then we update each parallel path systematically through one identical path, until the complex network becomes two parallel subnetworks.

In other words, the structure function can be found using step by step solution and stepwise simplification. Where the group of components arranged in a series are replaced by one equivalent component, parallel components can also be replaced by an equivalent component, until we finally reach two parallel subnetworks, and we find the structure function in their terms, and from this structure function we extract the reliability polynomial [45, 57].

Now, to illustrate this technique we apply it on the complex network in Figure 3.1.

First, based on all minimal paths, a reliability block diagram is plotted in Figure 3.2

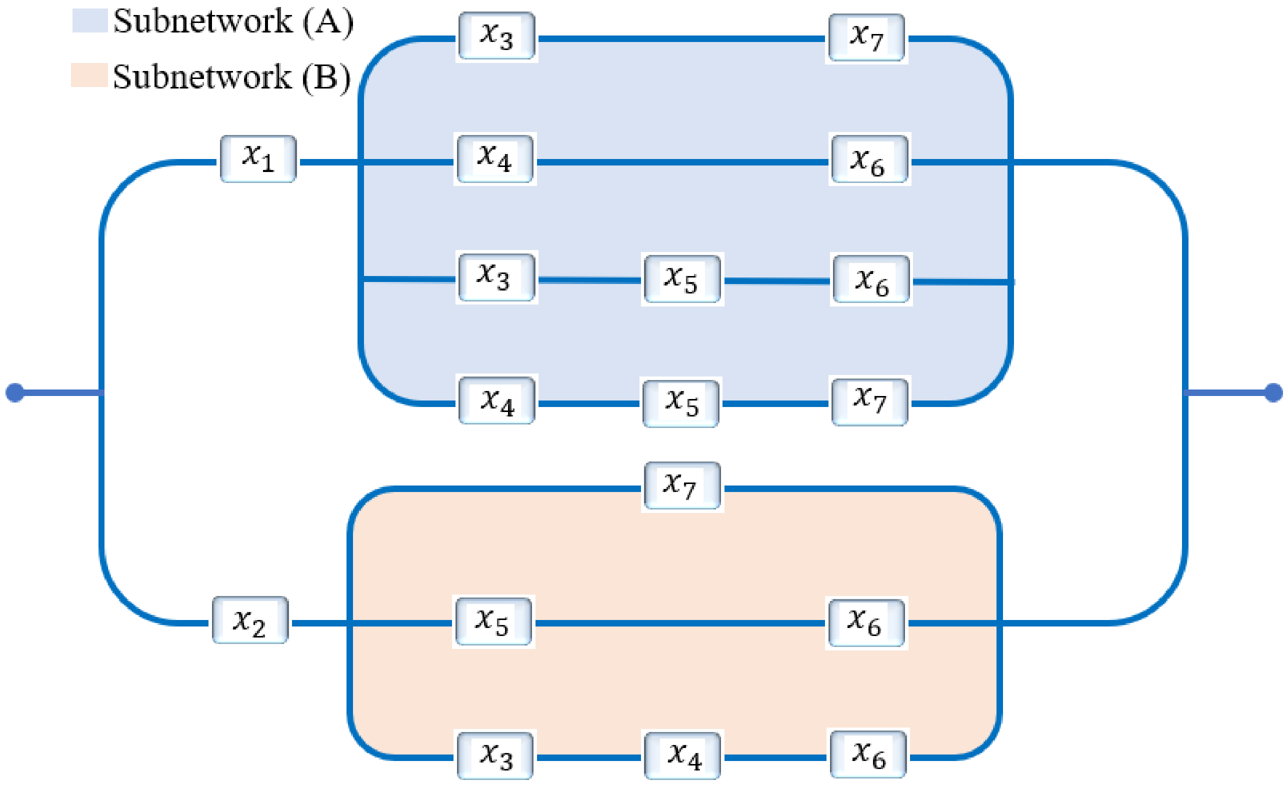

Figure 3.2: Reliability Block Diagram of Figure 3.1

Now, apply theorem (3.2.1) to solve two series - parallel subnetworks (A) and (B).

$$\Phi_A(x) = 1 - (1 - x_3x_7)(1 - x_4x_6)(1 - x_3x_5x_6)(1 - x_4x_5x_7).$$

And $\Phi_B(x) = 1 - (1 - x_7)(1 - x_5x_6)(1 - x_4x_5x_6)$.

The network has been decreased to a device contains series - parallel elements as in Figure 3.3 (I) simple network.

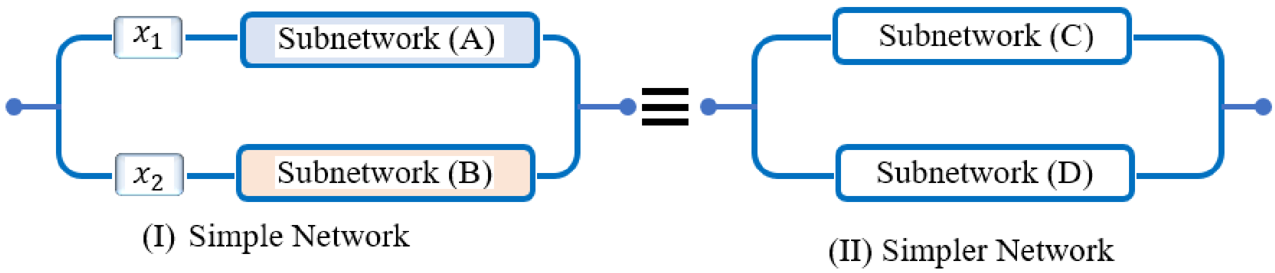

Figure 3.3: Equivalent Simple Networks

Here, subnetwork (C)=(x_1, subnetwork (A)) and subnetwork (D)=(x_2, subnetwork (B)).

Agian, apply theorem (3.2.1) to solve simple network (I)

$\Phi(x) = 1 - (1 - x_1x_A)(1 - x_2x_B)$.

Simple network (I) is equivalent to simpler network in (II), that is:

$\Phi(x) = 1 - (1 - x_C)(1 - x_D)$.

where $x_C = (1 - x_1x_A)$ and $x_D = (1 - x_1x_B)$.

Therefore,

$$R_N = 1 - (1 - R_C)(1 - R_D). \tag{3.4}$$

Thus,

$$R_N = R_C + R_D - R_CR_D. \tag{3.5}$$

3.5 Delta - Star Transform Technique (DST)

Delta - Star transform help us to transformation some complex networks into parallel or series networks, especially networks that contain triangles. Moreover, it helps us reduce inclusion even when series - parallel technologies do not work on the network. To transform a delta subnetwork to an equivalent star subnetwork we want to derive a metamorphosis method for equating the different components to each different between the diverse terminals [15, 34, 57].

Consider Figure 3.4, delta with three components X, Y and Z, we want to transform it to star with three components x, y and z.

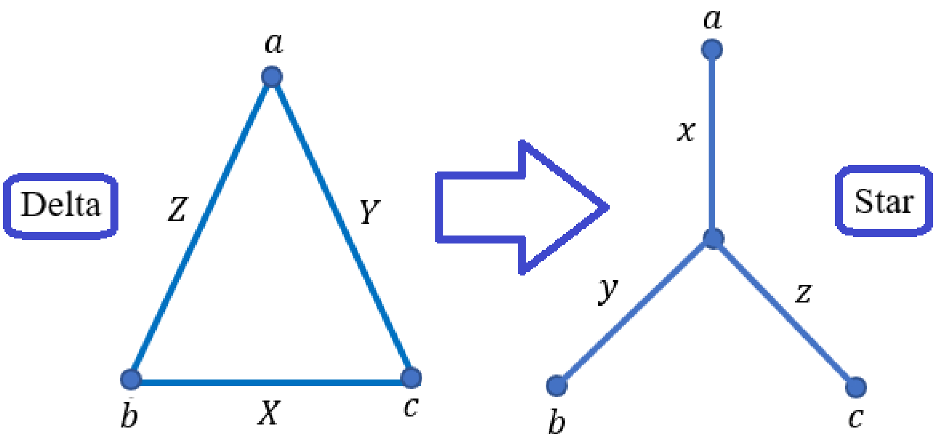

Figure 3.4: Delta - Star Transform

Compare three components of delta with three components of star, then applying two equations (2.5) and (2.6) to get three relations:

$$xy = 1 - (1 - Z)(1 - XY) \tag{3.6}$$

$$xz = 1 - (1 - Y)(1 - XZ) \tag{3.7}$$

$$yz = 1 - (1 - X)(1 - YZ) \tag{3.8}$$

Solving three equations (3.6), (3.7) and (3.8) to get delta - star relationships:

$$x = \sqrt{\frac{[1-(1-Y)(1-XZ)][1-(1-Z)(1-XY)]}{[1-(1-X)(1-YZ)]}} \tag{3.9}$$

$$y = \sqrt{\frac{[1-(1-Z)(1-XY)][1-(1-X)(1-YZ)]}{[1-(1-Y)(1-XZ)]}} \tag{3.10}$$

$$z = \sqrt{\frac{[1-(1-X)(1-YZ)][1-(1-Y)(1-XZ)]}{[1-(1-Z)(1-XY)]}} \tag{3.11}$$

Explanation: The value of x was found in equation (3.9) by product two equations (3.6) and (3.7), then dividing the result by equation (3.8). Similarly, we found the values of y and z.

Consider the triangle $v_1v_2v_3$ (delta) in Figure 3.5, which is a subnetwork from the complex network in Figure 3.1.

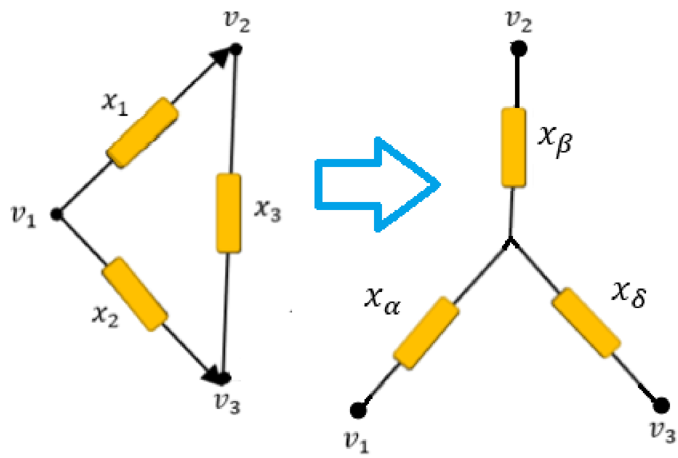

Figure 3.5: Triangle $v_1v_2v_3$ to Star

Applying the delta-star relationships to the mentioned triangle to get:

$$x_\alpha = \sqrt{\frac{[1-(1-x_2)(1-x_1x_3)][1-(1-x_3)(1-x_1x_2)]}{[1-(1-x_1)(1-x_2x_3)]}} \tag{3.12}$$

$$x_\beta = \sqrt{\frac{[1-(1-x_1)(1-x_2x_3)][1-(1-x_2)(1-x_1x_2)]}{[1-(1-x_3)(1-x_1x_2)]}} \tag{3.13}$$

$$x_\delta = \sqrt{\frac{[1-(1-x_1)(1-x_2x_3)][1-(1-x_3)(1-x_1x_2)]}{[1-(1-x_2)(1-x_1x_3)]}} \quad (3.14)$$

Then, the complex network in Figure 3.1 will transform into a new network, as shown in Figure 3.6

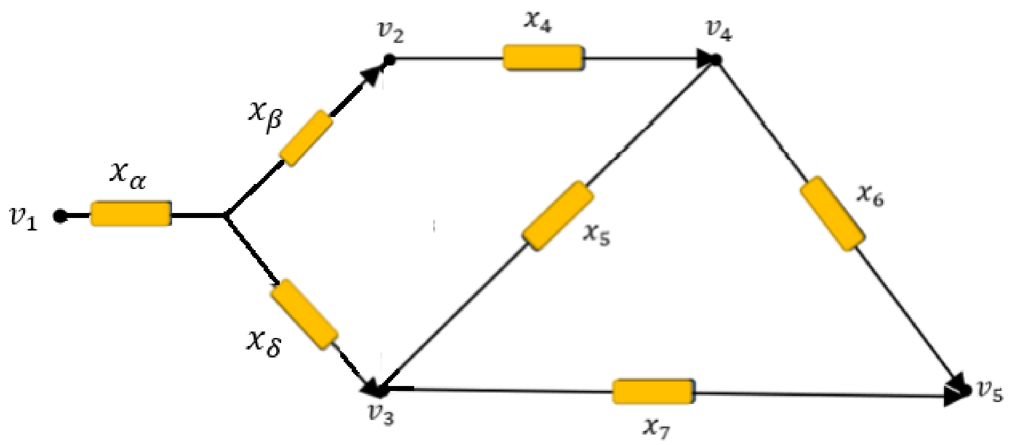

Figure 3.6: New Network

Now, we separate the last network in Figure 3.6 into two parts in series, the first is x_α component and the second is the remaining part after deleting x_α, say, X_A.

The minimal paths of X_A are:

$MP_1 = x_\beta x_4 x_6, MP_2 = x_\beta x_4 x_5 x_7, MP_3 = x_\delta x_7$, and $MP_4 = x_\delta x_5 x_6$.

So, we can apply theorem (3.2.1) to find $\Phi_A(x)$:

$\Phi_A(x) = 1-(1-x_\beta x_4 x_6)(1-x_\beta x_4 x_5 x_7)(1-x_\delta x_7)(1-x_\delta x_5 x_6)$.

Then, the sturucter function of the network is given by:

$$\Phi(x) = x_\alpha \Phi_A(x). \quad (3.15)$$

Thus, the reliability network is given by:

$$R_N = R_\alpha R_A. \quad (3.16)$$

CHAPTER 4

CHAPTER FOURE: RELIABILITY ALLOCATION AND RELIABILITY IMPORTANCE

4.1 Introduction

The reason of reliability allocation is to set up an aim or objective for the reliability of each component in order that the manufactures will have a concept of the overall performance required of this product. Many researches paintings to compute the reliability allocation in series, parallel and complex networks [37, 41, 83].

In this chapter, we will discuss the reliability assignment problem of the complex network in Figure 3.1 after it has been reduced into two subnetworks in parallel as shown in simpler network (II) Figure 3.3 in third chapter. The importance of reliability will also be discussed to know the levels importance of network components under study, as we concluded that there are three levels of importance. We will notice those levels importance when we study the topic of reliability optimization in fifth and sixth chapters.

4.2 Reliability Allocation

Reliability allocation is the techniques by which failure allowance is allocated to a specific network and its subnetworks or its components in a few logical ways. The main purpose of reliability allocation is to define a value or objective for the reliability of each component of the product to be manufactured so that the producers have an idea of the performance required of that product [24, 28]. We outline the network reliability of objective of the character additives in the gadget that ensure get entry to the general purpose of network reliability. For each formatting we use the element to refer to a typical unit or subnetwork, which can be formulated inside the allocation of reliability.

Let the complex network with m components have a known reliability R_N. Assume that we want to improvement network reliability to R_N^* such that $R_N^* > R_N$. There are several technique's to reliability allocation and those strategies are range in complexity depending on how a great deal the definition of subnetwork is type and the degree of accuracy required.

A reliability allocation problem can be formulated as:

$$g(R_1^*, R_2^*, \cdots, R_m^*) \geq R_N^* \tag{4.1}$$

where R_N^* represent to network reliability target, R_i^* is the reliability of i -th component target , and g is a relationship between network reliability and components reliabilities which is obtained from the reliability analysis of the network to be allocated [28, 31].

4.3 Reliability Allocation Methods

There are many methods to allocation reliability. The choice of any method for allocating reliability depends on several factors, some of which are directly related to the characteristics of the network under analysis, such as its level of complexity and additional matters form boundary conditions, such as budget or trial time. These methods vary in complexity depending on the amount of subnetwork definition available and the degree of precision required [31, 37].

4.3.1 Exponential Allocation Method

Consider a network of m components with identical and independent reliability R_c of constant failure rate λ_c arranged in a parallel configuration. Then, the network reliability is given by [11]:

$$R_N = 1 - (1 - R_c)^m$$

Replacing component reliability in terms of constant component failure rate λ_c to get:

$$R_N = 1 - \left(1 - e^{-\lambda_c t}\right)^m \quad \Longrightarrow \quad \frac{dR_N}{dt} = -m\lambda_c e^{-\lambda_c t}\left(1 - e^{-\lambda_c t}\right)^{m-1}$$

To find the failure rate of a network with m components in parallel, the relationship among the reliability function, the pdf f_N and the failure rate λ_N is employed. The failure rate is defined as the ratio between f_N and reliability function R_N, or:

$$\lambda_N = \frac{f_N}{R_N}$$

But f_N can be written in terms of the time derivative of R_N as follow:

$$f_N = \frac{-dR_N}{dt}$$

So,

$$\lambda_N = \frac{\frac{-dR_N}{dt}}{R_N}$$

For constant failure rate components, the network failure rate becomes:

$$\lambda_N = \frac{m\lambda_c e^{-\lambda_c t}\left(1 - e^{-\lambda_c t}\right)^{m-1}}{1 - \left(1 - e^{-\lambda_c t}\right)^m}$$

Taking the limit of λ_N as $t \to \infty$ leads to the following expression for the steady-state network failure rate:

$$\lambda_{N,steady-state} = \lim_{t\to\infty} \lambda_N = \lim_{t\to\infty} \frac{m\lambda_c e^{-\lambda_c t}\left(1 - e^{-\lambda_c t}\right)^{m-1}}{1 - \left(1 - e^{-\lambda_c t}\right)^m}$$

Applying L'Hopital's rule, one obtains: $\lambda_{N,steady-state} = \lambda_c$.

So the steady-state failure rate for a network of constant failure rate components in a parallel arrangement is the failure rate of a single component.

Example 4.3.1. Consider the simpler networkin(II) in Figure 3.3. If the network reliability target at 100 months in service is 0.98, we allocate the reliability of subnetworks (C) and (D) at that time as follows:

Since $R_C = R_D$, the network reliability in equation (3.4) becomes:

$$R_N = 1 - \left(1 - R_C\right)^2$$

Replacing component reliability in terms of constant component failure rate λ_C to get:

$$R_N(100) = 1 - \left(1 - e^{-100\lambda_C}\right)^2 = 0.98$$

Which is implies to $\lambda_C = 0.0015$.

Then the corresponding reliability targets is $R_C = R_D = e^{-100\times 0.0015} = 0.861$.

For the value of network reliability to be equal to 0.98, the reliability value of each unit must be equal to 0.861.

This can be checked by substituting the reliability value for each unit into equation (3.4) as follows :

$$R_N = 1 - (1 - 0.861)^2 = 0.9806$$

4.3.2 Aeronautical Radio Inc. (ARINC) Method

Aeronautical Radio Inc. (ARINC) method can be used to estimate the failure rate of subnetworks if failure information is available from the previous design. ARINC is a partitioning method based on the failure rate of components or subnets. They can be used if the database or previous experience is expressed in a similar subnet, allowing prediction of the failure rate of the desired subnet [55, 88].

Consider a network of m components arranged in a parallel configuration. The reliability allocation weight for i-th component is given by:

$$w_i = \frac{\lambda_i}{\sum_{j=1}^{m} \lambda_j} \tag{4.2}$$

here λ_i is the failure rate of i-th component obtained from historical data or prediction. The failure rate goal λ_i^* to be allocated for the i-th component is $\lambda_i^* = w_i \lambda^*$, here λ^* represent to network failure rate target.

Example 4.3.2. Consider the simpler network (II) in Figure 3.3. If the network reliability target at 100 months in service is 0.99, $\lambda_C = 0.00005$ and $\lambda_D = 0.00004$ failures per month. We allocate the reliability of subnetworks (C) and (D) as follows:
First, find w_C and w_D from equation 4.2

$$w_C = \frac{\lambda_C}{\lambda_C + \lambda_D} = \frac{0.00005}{0.00005 + 0.00004} = 0.5556$$
$$w_D = \frac{\lambda_D}{\lambda_C + \lambda_D} = \frac{0.00004}{0.00005 + 0.00004} = 0.4444$$

Now, we find λ^*
$R_N^* = e^{-\lambda^* t} \implies 0.99 = \mathrm{e}^{-100\lambda^*} \implies \lambda^* = 0.0001$
Finally, $\lambda_C^* = w_C \lambda^* = 0.000125$ and $\lambda_D^* = w_D \lambda^* = 0.000075$.
Then the corresponding reliabilities targets are $R_C = 0.98$ and $R_D = 0.99$.

For the value of network reliability to be equal to 0.99, the reliability values of the two units must be equal to 0.98 and 0.99 resepectivly.
This can be checked by substituting the reliability value for each unit into equation (3.4) as follows :

$$R_N = 1 - (1 - 0.98)(1 - 0.99) = 0.99$$

4.4 Calculation of the Reliability Importance

For a network of m components, the reliability importance I_i of i-th component is a measure of the extent to which i-th component affects the overall reliability of that network. This information is very important for designers to improve network reliability, because they will focus their efforts above all on improving the reliability of components that have the greatest impact on network reliability [47, 80]. In simple networks, it is easy to identify weak components. However, this becomes a difficult task in complex networks. The value of reliability importance of i-th component lies between zero and one, this value depends on R_i and the position of i-th component in network.

The reliability importance (I_i) is given by:

$$I_i = \frac{\partial R_N}{\partial R_i}$$

The I_i for all components of the complex network marked in Figure 3.1, are the partial derivatives of R_N for $i-$th component from equation (3.2) as follows:

$$\frac{\partial R_N}{\partial R_1} = R_2 + R_3R_5R_7 + R_2R_3R_4R_5R_6 + 2R_2R_3R_5R_6R_7 + R_2R_3R_4R_6R_7 + 2R_2R_3R_5R_6R_7$$
$$+R_2R_4R_5R_6R_7 + R_3R_4R_5R_6R_7 - R_2R_3R_4 - R_2R_6R_7 - R_2R_3R_5R_6 - R_2R_3R_5R_7$$
$$-R_2R_4R_5R_7 - R_3R_5R_6R_7 - R_3R_4R_5R_7 - 3R_2R_3R_4R_5R_6R_7.$$

$$\frac{\partial R_N}{\partial R_2} = R_1+R_3R_4+R_3R_5R_6+R_1R_3R_4R_5R_6+2R_1R_3R_4R_5R_7+R_1R_3R_4R_6R_7+2R_1R_3R_5R_6R_7$$
$$+R_1R_4R_5R_6R_7 + 2R_3R_4R_5R_6R_7 - R_1R_3R_4 - R_1R_6R_7 - R_1R_3R_5R_6 - R_1R_3R_5R_7$$
$$-R_1R_4R_5R_7-R_3R_4R_5R_6-R_3R_4R_5R_7-R_3R_4R_6R_7-R_3R_5R_6R_7-3R_1R_3R_4R_5R_6R_7.$$

$$\frac{\partial R_N}{\partial R_3} = R_2R_4+R_1R_5R_7+R_2R_5R_6+R_1R_2R_4R_5R_6+2R_1R_2R_4R_5R_7+R_1R_2R_4R_6R_7$$
$$+2R_1R_2R_5R_6R_7 + R_1R_4R_5R_6R_7 + 2R_2R_4R_5R_6R_7 - R_1R_2R_4 - R_1R_2R_5R_6$$
$$-R_1R_2R_5R_7 - R_2R_4R_5R_6 - R_2R_4R_5R_7 - R_1R_5R_6R_7 - R_2R_4R_6R_7 - R_1R_4R_5R_7$$
$$-R_2R_5R_6R_7 - 3R_1R_2R_4R_5R_6R_7.$$

$$\frac{\partial R_N}{\partial R_4} = R_2R_3 + R_5R_7 + R_1R_2R_3R_5R_6 + 2R_1R_2R_3R_5R_7 + R_1R_2R_3R_6R_7 + R_1R_2R_5R_6R_7$$
$$+R_1R_3R_5R_6R_7 + 2R_2R_3R_5R_6R_7 - R_1R_2R_3 - R_5R_6R_7 - R_1R_2R_5R_7 - R_2R_3R_5R_6$$
$$-R_2R_3R_5R_7-R_2R_3R_6R_7-R_1R_3R_5R_7-3R_1R_2R_3R_5R_6R_7.$$

$$\frac{\partial R_N}{\partial R_5} = R_4R_7 + R_1R_3R_7 + R_2R_3R_6 + R_1R_2R_3R_4R_6 + 2R_1R_2R_3R_4R_7 + 2R_1R_2R_3R_6R_7$$

$$+R_1R_2R_4R_6R_7+R_1R_3R_4R_6R_7+2R_2R_3R_4R_6R_7-R_4R_6R_7-R_1R_2R_3R_6-R_1R_2R_3R_7$$

$$-R_1R_2R_4R_7 - R_2R_3R_4R_6 - R_2R_3R_4R_7 - R_1R_3R_6R_7 - R_1R_3R_4R_7 - R_2R_3R_6R_7$$

$$-3R_1R_2R_3R_4R_6R_7.$$

$$\frac{\partial R_N}{\partial R_6} = R_7 + R_2R_3R_5 + R_1R_2R_3R_4R_5 + R_1R_2R_3R_4R_7 + 2R_1R_2R_3R_5R_7 + R_1R_2R_4R_5R_7$$

$$+R_1R_3R_4R_5R_7 + 2R_2R_3R_4R_5R_7 - R_1R_2R_7 - R_4R_5R_7 - R_1R_2R_3R_5 - R_2R_3R_4R_5$$

$$-R_1R_3R_5R_7 - R_2R_3R_4R_7 - R_2R_3R_5R_7 - 3R_1R_2R_3R_4R_5R_7.$$

$$\frac{\partial R_N}{\partial R_7} = R_6 + R_4R_5 - R_1R_2R_6 + R_1R_3R_5 - R_4R_5R_6 - R_1R_2R_3R_5 - R_1R_2R_4R_5 - R_1R_3R_4R_5$$

$$-R_2R_3R_4R_5 - R_1R_3R_5R_6 - R_2R_3R_4R_6 - R_2R_3R_5R_6 + 2R_1R_2R_3R_5 + 2R_1R_2R_3R_5R_6$$

$$+R_1R_2R_3R_4R_6 + R_1R_2R_4R_5R_6 + R_1R_3R_4R_5R_6 + 2R_2R_3R_4R_5R_6 - 3R_1R_2R_3R_4R_5R_6.$$

Now, we take an example with a random values to check out the reliability importance of all components.

Example 4.4.1. Suppose different values of reliability into two cases:

(1) Random values such as:
   a) $R_1 = 0.75, R_2 = 0.8, R_3 = 0.70, R_4 = 0.75, R_5 = 0.8, R_6 = 0.85$ and $R_7 = 0.8$.
   b) $R_1 = 0.9, R_2 = 0.85, R_3 = 0.7, R_4 = 0.75, R_5 = 0.8, R_6 = 0.95$ and $R_7 = 0.8$.

(2) All components have the same reliability values(independent identical units) such as
   a) $R_i = 0.9$ for all $i = 1, \cdots, 7$.
   b) $R_i = 0.7$, for all $i = 1, \cdots, 7$.

The values of R_i and the reliability importance (I_i) listed in table 4.1.

Table 4.1: A Summary Table for Values of R_i and I_i

i	$R_i - 1a$	I_i	$R_i - 1b$	I_i	$R_i - 2a$	I_i	$R_i - 2b$	I_i
1	0.75	0.0886	0.90	0.0690	0.90	0.0201	0.70	0.1481
2	0.80	0.2088	0.85	0.20294	0.90	0.1084	0.70	0.3260
3	$\dot{0}$.70	0.0440	0.70	0.0176	0.90	0.0103	0.70	0.0719
4	0.75	0.0406	0.75	0.0078	0.90	0.0111	0.70	0.0851
5	0.80	0.0325	0.80	0.0090	0.90	0.0103	0.70	0.0719
6	0.85	0.1000	0.95	0.0553	0.90	0.0201	0.70	0.1481
7	0.80	0.2535	0.8	0.1758	0.90	0.1084	0.70	0.3260
R_N	0.932		0.962		0.987		0.843	

From values of the reliability importance I_i in table 4.1, the seven network components ($R_1, R_2, R_3, R_4, R_5, R_6$ and R_7) can be divided in terms of importance in three following levels:

- **First level:** Second and seventh components (R_2 and R_7) (in hot pink colour).
- **Second level:** First and sixth components (R_1 and R_6) (in cyan colour).
- **Third level:** Third, fourth and fifth components (R_3, R_4 and R_5) (in green colour)

These three levels of importance for seven network components, will helpe us later to understand the mechanism of the increase in reliability of complex network based on the importance of its components.

The bar chart in Figure 4.1 shown reliability importance for seven network components.

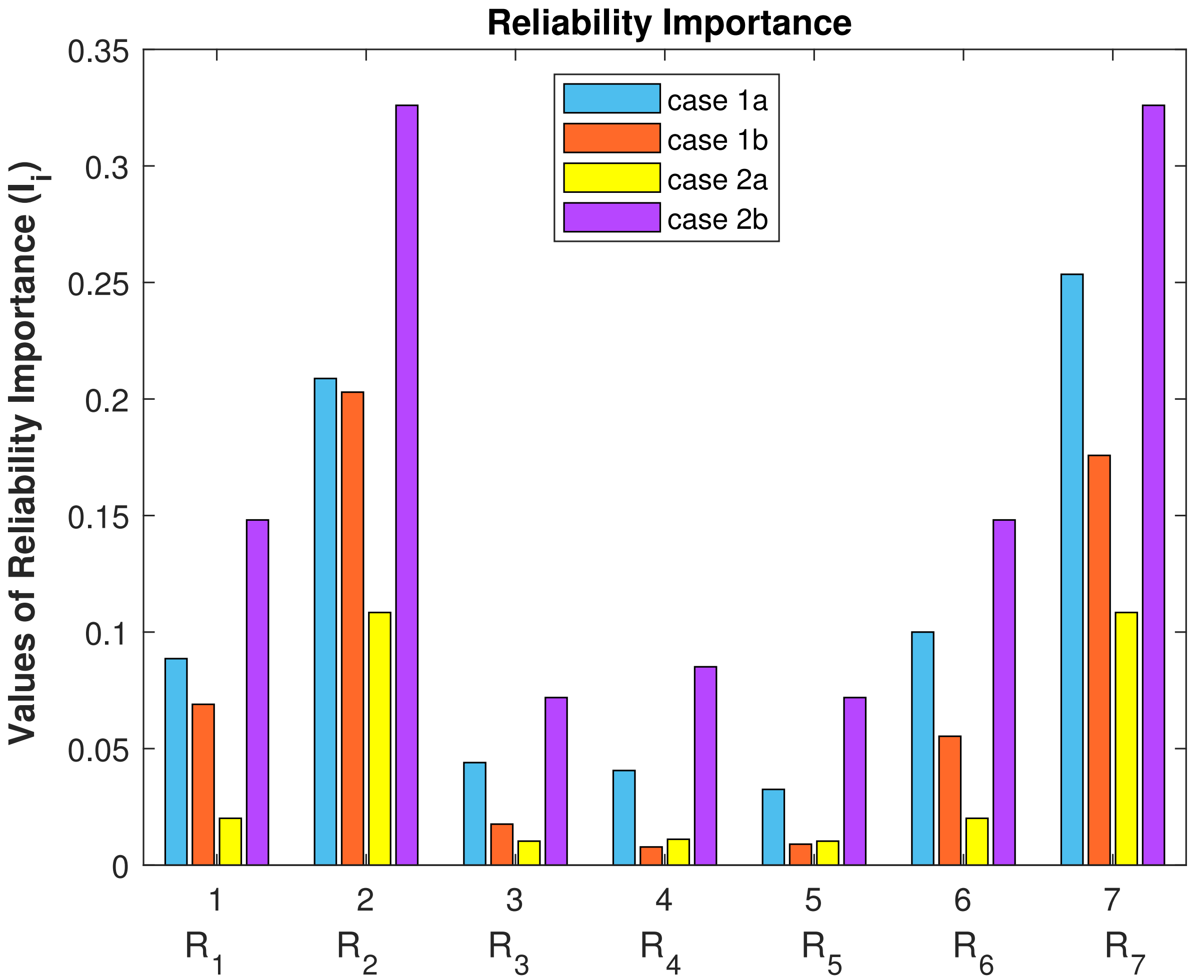

Figure 4.1: Reliability Importance

CHAPTER 5

CHAPTER FIVE: USING META-HEURISTIC ALGORITHMS TO OPTIMIZE RELIABILITY NETWORK

5.1 Introduction

In this chapter, two nature-inspired algorithms (Bat Algorithm (BA) and Grey Wolf Optimization (GWO) Algorithm) are introduced to improve the reliability of a complex network. We shall define the basic formula of two algorithms then its implementation on the polynomial function of reliability, which we obtained in equation (3.2), and the implementation will be by adopting five cost functions whose are logarithm, exponential, exponential in terms of feasibility factor, power and tan functions and then comparing the results in detail.

5.2 Bat Algorithm (BA)

Bats are the only mammals that have wings and are able to locate prey using echoes. Zoologists say that there are more than 996 different species of them with sizes range from the small hummingbird bat (1.5 - 2) gm to the giant bat, which weighs one kilogram. Most bat species use echolocation to locate prey, and microbes are among the most widely known species to work with this feature. The main food for most microbes is insects, and to detect the location of insects and distinguish between them and obstacles, microbes use echolocation, where they emit very loud sonic and listen for echoes of the sound bouncing off the objects around them. Each sound pulse lasts (8-10) milliseconds and has a fixed frequency (25-150) kHz, and microbes emit about (10-20) sound every second. When bats feel hungry, they search for food, and the rate of emitting pulse increases. It reaches about 200 beats per second when it approaches its prey. Studies show that the integration time of a bat's ear is within the range (300 - 400) microseconds. Since the speed of sound s in the air at $20°C$ is about 343 m/sec, the wavelength $\lambda^{'}$ of ultrasonic sound wave bursts at a constant frequency f is given by the equation $\lambda^{'} = s/f$, which is in the range (2 -14) mm for a typical frequency range (25-150) kHz, and these wavelengths are proportional to the sizes of insects. Many studies have shown that microbes use the time delay from emission to echo detection, the time difference between their ears, and changes in the loudness of sound as echoes to build a 3D scenario of their surroundings, all of these properties help them to detect the distance, direction, type and speed of movement of prey. Some bats have good eyesight, and most have a good eyesight strong sense of smell and they are all their senses as a group for effective detection of prey and smooth movement without collisions between them. However, the Bat Algorithm only cares at echolocation. The behavior of microbes in echolocation can be modeled and then coupled with the issue of optimization the reliability of complex networks [85, 86].

The Bat algorithm is a set of guidelines proposed by Yang in 2010 that simulates the behavior of bats and how they use echolocation ultrasound in order to detect and locate prey and echo obstacles in hunting [79, 85, 86], as shown in Figure 5.1.

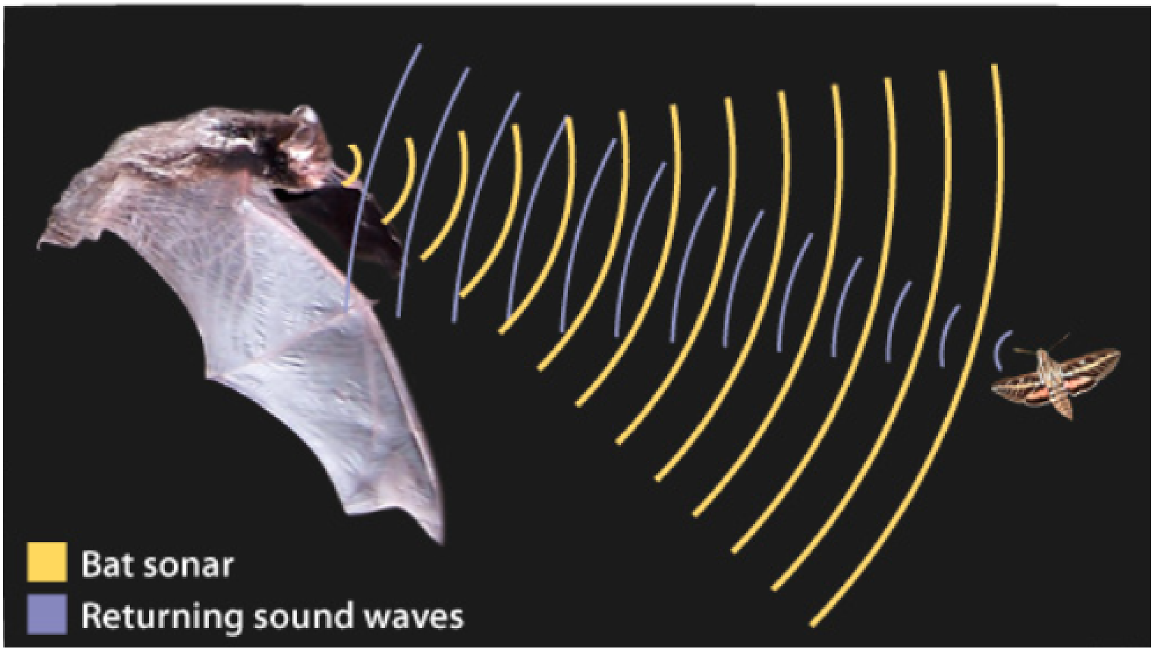

Figure 5.1: Sonar and Echolocation

To develop the basic concept of this algorithm, Yang proposed the following basic hypotheses:

(1) To sense the distance between bats and prey, all bats use echolocation, it also distinguishes between prey and rear barriers.

(2) Bats fly randomly with speed s_i in position p_i , fixed frequency f_{min} , wavelength λ',

and loudness A_0 to search for food or prey. They can vary the rate of pulse emission $r \in [0,1]$ depending on the distance of prey.

(3) Suppose the loudness varies from a small constant value A_{min} to the highest value A_0 even though the loudness varies for different reasons. It is necessary to specify how to update the bat position p_i and speed s_i.

The format of the update method for speed s_i^t and position p_i^t in step t is given by:

$$f_i = f_{min} + (f_{max} - f_{min})\beta \tag{5.1}$$

$$s_i^t = s_i^{(t-1)} + (p_i^t - p^*)f_i \tag{5.2}$$

$$p_i^t = p_i^{t-1} + s_i^t \tag{5.3}$$

here $\beta \in [0,1]$ is a random vector while p^* was signal to the modern global best solution that's positioned after evaluating all the solutions.
The nearest neighborhood to the best solution generates the optimal solution. The format of the update of bat position will be as follows:

$$p_n = p_o + \epsilon A^t \tag{5.4}$$

here p_n is the new solution, p_o is the best old solution, $A^t \in (A_{min}, A_0)$ represents to the average of the loudness of all bats at time t, and $\epsilon \in [-1,1]$ is directions and lengths that are randomly generated. When the bats are closer to their prey, the number of pulses emitted increases, the loudness can be adjusted to an appropriate value as follows:

$$A_i^{(t+1)} = \alpha + A_i^t \tag{5.5}$$

$$r_i^{(t+1)} = r_i^{(t=0)}(1 - e^{\gamma t}) \tag{5.6}$$

here $\alpha \in (0,1)$ and $\gamma \gneqq 0$ are constants [79, 85, 86]. The simple steps within the Bat algorithm manner are presented in the flowchart shown in Figure 5.2.

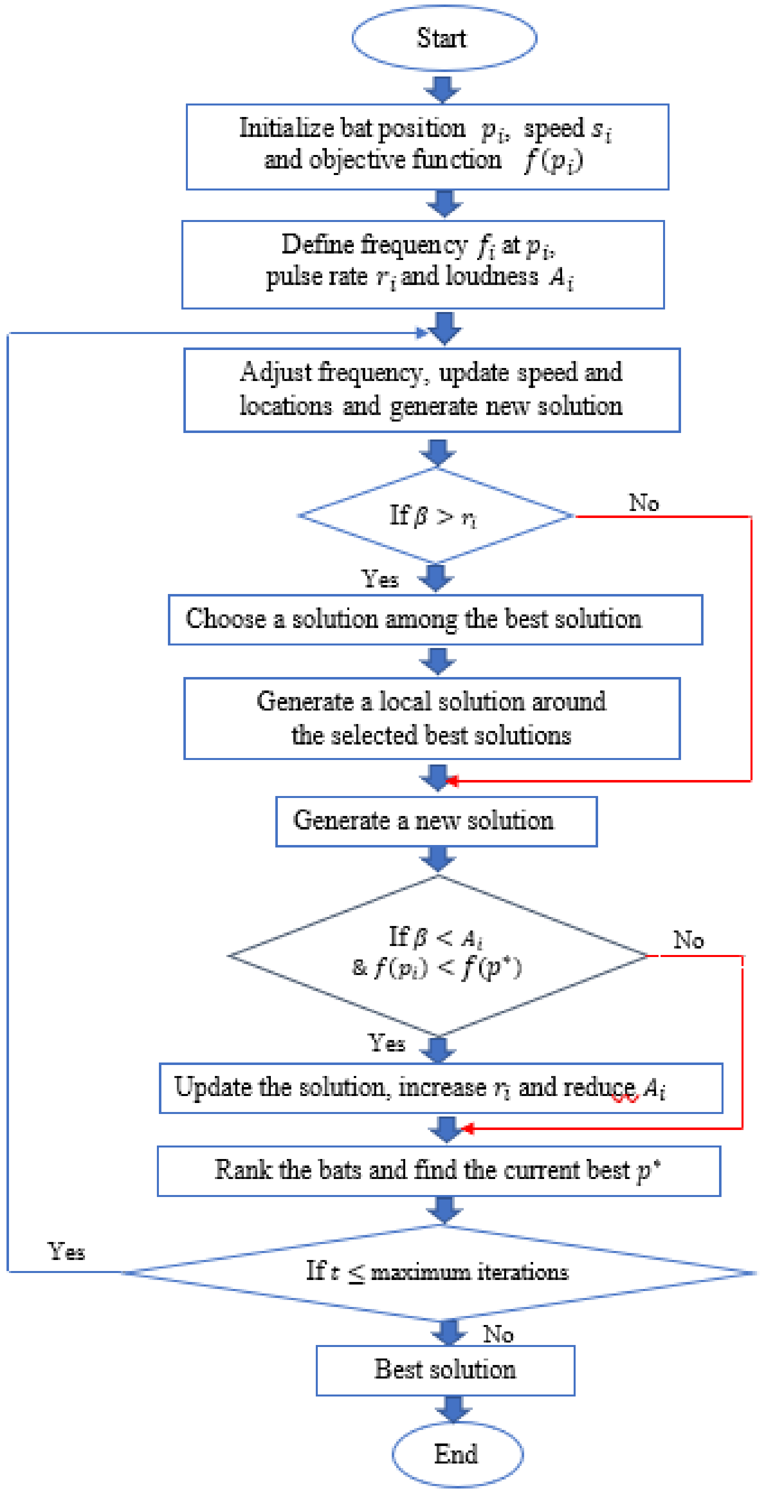

Figure 5.2: Flowchart of Bat Algorithm

5.3 The Models of Cost Function

Manufacturers change their product designs for the lowest possible cost by changing higher quality materials or reducing reprocessing costs, administrative fees, or other factors. Naturally, the cost will increase as the assigned reliability approaches the most achievable reliability. This is the value of reliability that is handled asymmetrically with an increase in cost, but it is never actually reached. The reliability of the components should not depend on lower values than they really were. Depending on the optimization, reliability of a component may not be needed from its current value, but it will not decrease [5, 8, 36].

Assume that $C_i(R_i)$ is the cost function of a component i, R_i is the reliability of a component i, C_N is the value of total cost , R_N is the reliability network, a_i and b_i are constants such that $a_i, b_i \in (0,1)$, $F_i \in (0,1)$ is the feasibility factor of a cost function for the component i,and $R_{i,min}, R_{i,max}$ is a minimum of dependable and maximum of R_i. Then the five models of a cost functions are:

1. Logarithmic Function

$$f_1(R_i) : C_i(R_i) = -a_i \ln(1 - R_i).$$

2. Exponential Function

$$f_2(R_i) : C_i(R_i) = a_i e^{b_i/(1-R_i)}.$$

3. Exponential in Terms of Feasibility Factor Function

$$f_3(R_i) : C_i(R_i) = e^{(1-F_i)(R_i-R_{i,min})(R_{i,max}-R_i)}.$$

4. Power Function

$$f_4(R_i) : C_i(R_i) = a_i R_i^{b_i}.$$

5. Tan Function

$$f_5(R_i) : C_i(R_i) = a_i \tan(2R_i/\pi)^{b_i}.$$

5.4 Reliability Optimization

It is natural that the reliability of any product device is equal to one when it is manufactured, but it begins to decrease gradually with the increase in operating time and this results from the decreasing reliability of the device's components over time [30]. For this reason and to be close for reality, we required the reliability of each component to be within the range $0.45 \leqslant R_i \leqslant 0.95$.

Our goal is to maximize the reliability network R_N represented in equation (3.2) at the lowest possible cost C_N.

So, the mathematical formulation of the nonlinear multi-objective optimization problem take the form:

$$\left.\begin{array}{l} \text{Minimize } \big(C_N(R_1,\cdots,R_7), -R_N(R_1,\cdots,R_7)\big) \\ \text{Subject to: } R_i \in [0.45, 0.95] \text{ for all } i = 1,\cdots,7 \\ \qquad R_N \geq R_G \\ \qquad 0 < C_i \leqslant 2, \text{ for all } i = 1,\cdots,7 \\ \qquad C_N = \sum_{i=1}^{7} C_i \leq C_G \end{array}\right\} \tag{5.7}$$

here $R_G = 0.9$ (the objective of network reliability), $C_G = 10.5$ (the objective of total cost), R_i is the $i-$th component reliability and C_i is the $i-$th component cost.

For consistency, we transform the maximize R_N problem into equivalent minimize $-R_N$ problem.

5.5 Computational Results of Bat Algorithm

To find out the best reliability of a complex network, we used BA by number of iterations (IN) = 500 with the aforementioned five cost functions. The results were rounded to three decimal places.

1. BA with Logarithmic Cost Function

The values of R_i and $C_i(R_i)$ with best value of reliability network R_N and total cost C_N by Bat algorithm using logarithmic cost function listed in table (5.1).

Table 5.1: A Summary Table for R_i, C_i and Best Values of R_N, C_N by BA with f_1

R_i	R_1	R_2	R_3	R_4	R_5	R_6	R_7	**Total**
Value of R_i	0.921	0.940	0.742	0.775	**0.546**	0.930	**0.946**	**0.992**
Value of C_i	0.440	0.440	0.180	0.244	0.120	0.515	0.448	**2.387**

From viewing the values in table (5.1) we record the following observations:

(1) $0.546 \leq R_i \leq 0.946$, for all $i = 1,2,\cdots,7$.

(2) $0.120 \leq C_i \leq 0.515$, for all $i = 1,2,\cdots,7$.

(3) The highest value of R_i was for the seventh component, which is 0.946, while the lowest value was for the fifth component, which is 0.546.

(4) R_3, R_4 and R_5 are less than 0.8.

(5) R_1, R_2, R_6 and R_7 are greater than 0.9.

(6) $R_7 > R_2 > R_6 > R_1 > R_4 > R_3 > R_5$.

(7) The best value of reliability network is $R_N = 0.992$.

(8) The total cost is $C_N = 2.387$.

The bar chart in Figure 5.3 shown the values of R_i and best value of R_N, while the bar chart in Figure 5.4 shown the values of C_i and total cost C_N by use BA with logarithmic cost function.

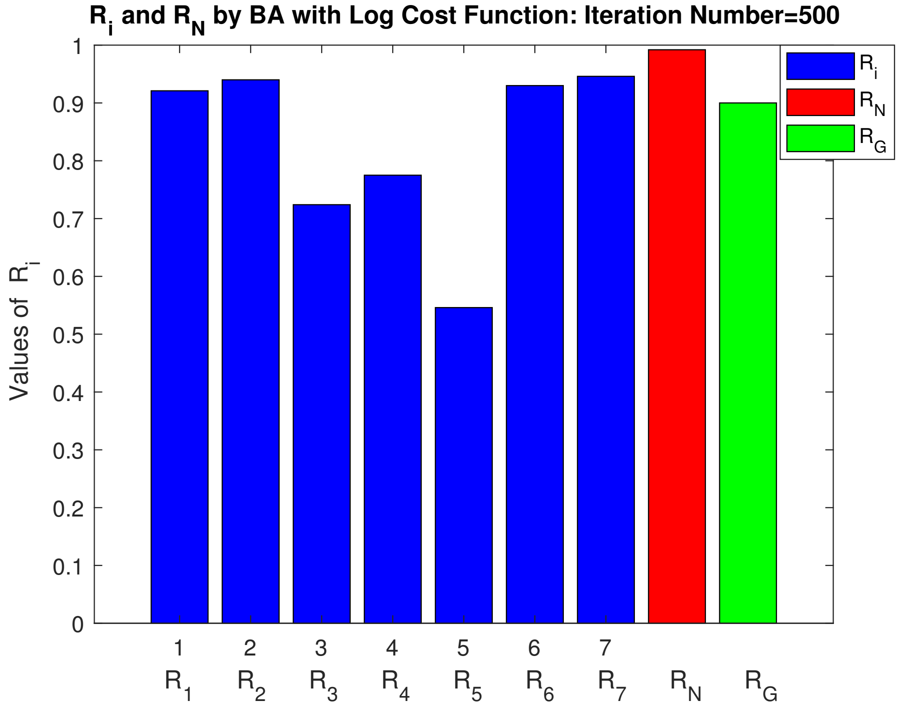

Figure 5.3: Values of R_i and Best Value of R_N by BA with f_1

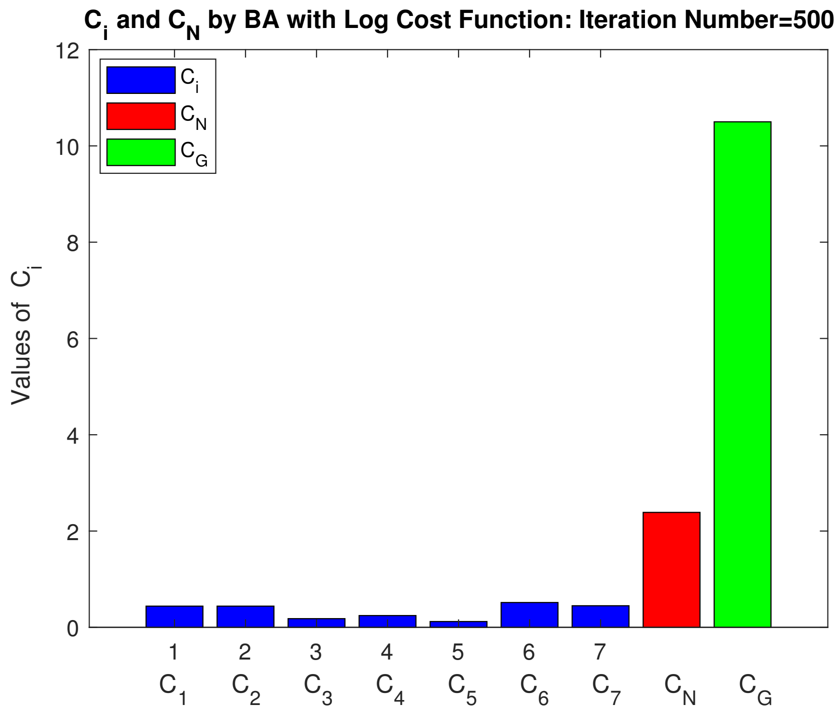

Figure 5.4: Values of C_i and C_N by BA with f_1

2. BA with Exponential Cost Function

The values of R_i and $C_i(R_i)$ with best value of reliability network R_N and total cost C_N by BA using exponential cost function listed in table (5.2).

Table 5.2: A Summary Table for R_i, C_i and Best Values of R_N, C_N by BA with f_2

R_i	R_1	R_2	R_3	R_4	R_5	R_6	R_7	**Total**
Value of R_i	0.878	0.908	0.614	**0.600**	0.668	0.903	**0.940**	**0.982**
Value of C_i	0.644	1.606	0.187	0.256	0.296	0.833	1.528	**5.350**

When you look closely at the values of table (5.2), you can make the following observations:

(1) $0.600 \leq R_i \leq 0.940$, for all $i = 1, 2, \cdots, 7$.

(2) $0.187 \leq C_i \leq 1.606$, for all $i = 1, 2, \cdots, 7$.

(3) The highest value of R_i was for the seventh component, which is 0.940, also, the lowest value was for the fourth component, which is 0.600.

(4) There are three components whose reliability is less than 0.7, which are R_3, R_4 and R_5.

(5) There are three components whose reliability is greater than 0.9, which are R_2, R_6 and R_7.

(6) $R_7 > R_2 > R_6 > R_1 > R_5 > R_3 > R_4$.

(7) The best value of reliability network is $R_N = 0.982$.

(8) The value of total cost is $C_N = 5.350$.

The bar chart in Figure 5.5 shown the values of R_i and best value of R_N, while the bar chart in Figure 5.6 shown the values of C_i and total cost C_N by BA using exponential cost function.

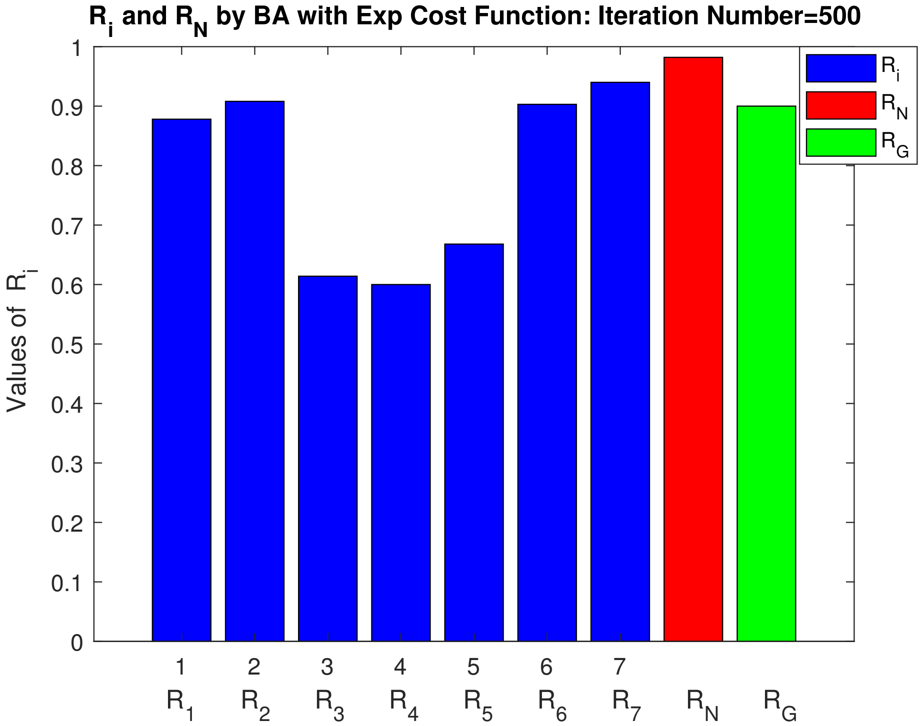

Figure 5.5: Values of R_i and R_N by BA with f_2

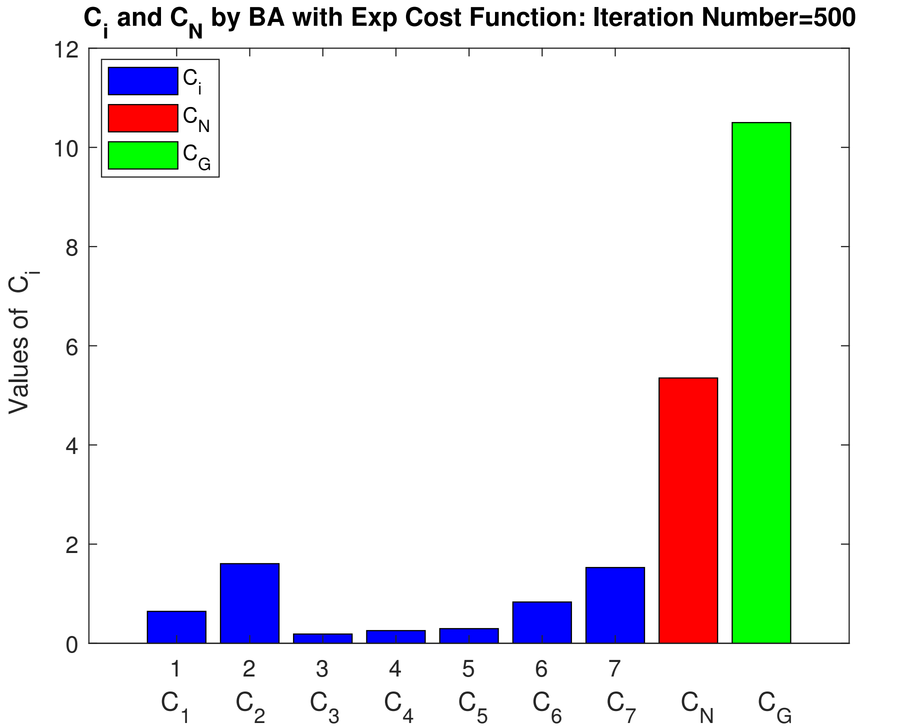

Figure 5.6: Values of C_i and C_N by BA with f_2

3. BA with Exp. in Terms of Feasibility Factor Cost Function

The values of R_i and $C_i(R_i)$ with best value of reliability network R_N and total cost C_N by using BA and changes the cost function to an exponential in terms of feasibility factor listed in table (5.3).

Table 5.3: A Summary Table for R_i, C_i and Best Values of R_N, C_N by BA with f_3

R_i	R_1	R_2	R_3	R_4	R_5	R_6	R_7	**Total**
Value of R_i	0.880	0.910	0.835	0.850	**0.774**	0.853	**0.938**	**0.987**
Value of C_i	1.003	1.002	1.003	1.003	1	1.003	1	**7.014**

If we take a look at values of table (5.3), then we can make the following observations:

(1) $0.774 \leq R_i \leq 0.938$, for all $i = 1,2,\cdots,7$.

(2) C_i is equal to or very slightly greater than one for all i, and this means that the values of two terms $R_i - R_{i,min}$ and $R_{i,max} - R_i$ are either zero or very close to zero, and we understand from this that $R_i, R_{i,min}$ and $R_{i,max}$ are almost equal values for all $i = 1, 2, \ldots, 7$.

(3) The highest value of R_i was for the seventh component, which is 0.938, and the lowest value was for the fifth component, which is 0.774.

(4) There are two components whose reliability is greater than 0.9, which are R_2 and R_7.

(5) $R_7 > R_2 > R_1 > R_6 > R_4 > R_3 > R_5$.

(6) The best value of reliability network is $R_N = 0.987$.

(7) The value of total cost is $C_N = 7.014$.

The bar chart in Figure 5.7 shown the values of R_i and best value of R_N, while the bar chart in Figure 5.8 shown the values of C_i and total cost C_N by use BA algorithm with exponential in terms of feasibility factor cost function.

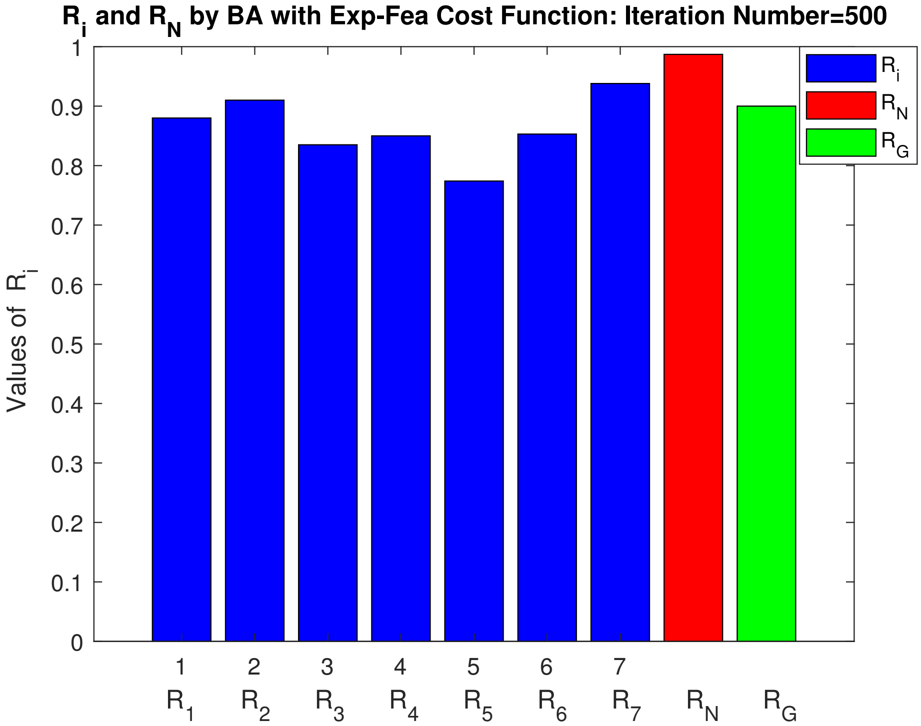

Figure 5.7: Values of R_i and R_N by BA with f_3

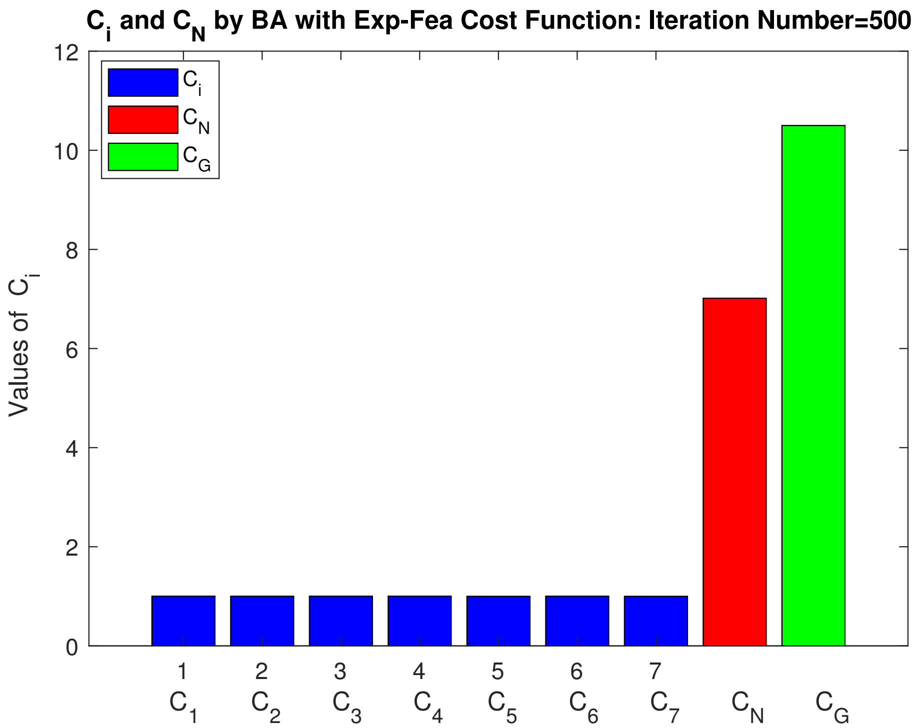

Figure 5.8: Values of C_i and C_N by BA with f_3

4. BA with Power Cost Function

The values of R_i and $C_i(R_i)$ with best value of reliability network R_N and total cost C_N by using BA and changes the cost function to power listed in table (5.4).

Table 5.4: A Summary Table for R_i, C_i and Best Values of R_N, C_N by BA with f_4

R_i	R_1	R_2	R_3	R_4	R_5	R_6	R_7	**Total**
Value of R_i	0.897	0.913	0.759	0.712	**0.662**	0.880	**0.918**	**0.983**
Value of C_i	0.706	0.762	0.159	0.769	0.422	0.159	0.310	**3.290**

If we take a look at values of table (5.4), then we can make the following observations:

(1) $0.662 \leq R_i \leq 0.918$ for all $i = 1, 2, \cdots, 7$.

(2) $0.159 \leq C_i \leq 0.769$, for all $i = 1, 2, \cdots, 7$.

(3) The highest value of R_i was for the seventh component, which is 0.918, and the lowest value was for the fivth component, which is 0.662.

(4) There are two components whose reliability greater than 0.9, which are R_2 and R_7.

(5) There are two components whose reliability between 0.8 and 0.9, which are R_1 and R_6.

(6) There are three components whose reliability less than 0.8, which are R_3, R_4 and R_5.

(7) $R_7 > R_2 > R_1 > R_6 > R_3 > R_4 > R_5$.

(8) The best value of reliability network R_N is 0.983.

(9) The value of total cost C_N is 3.290.

The bar chart in Figure 5.9 shown the values of R_i and best value of R_N, while the bar chart in Figure 5.10 shown the values of C_i and total cost C_N by BA using power cost function.

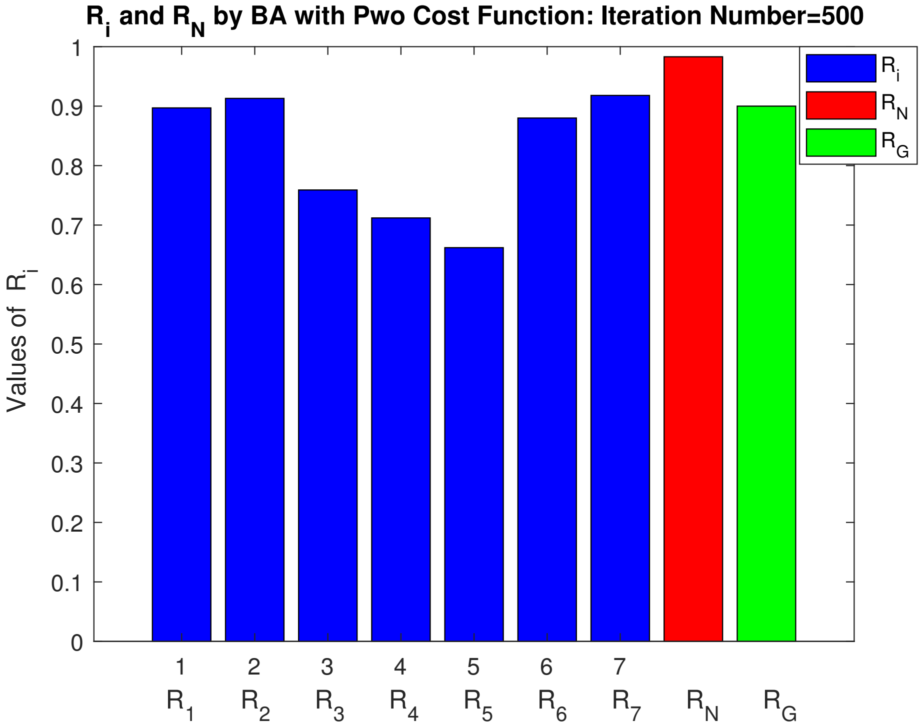

Figure 5.9: Values of R_i and R_N by BA with f_4

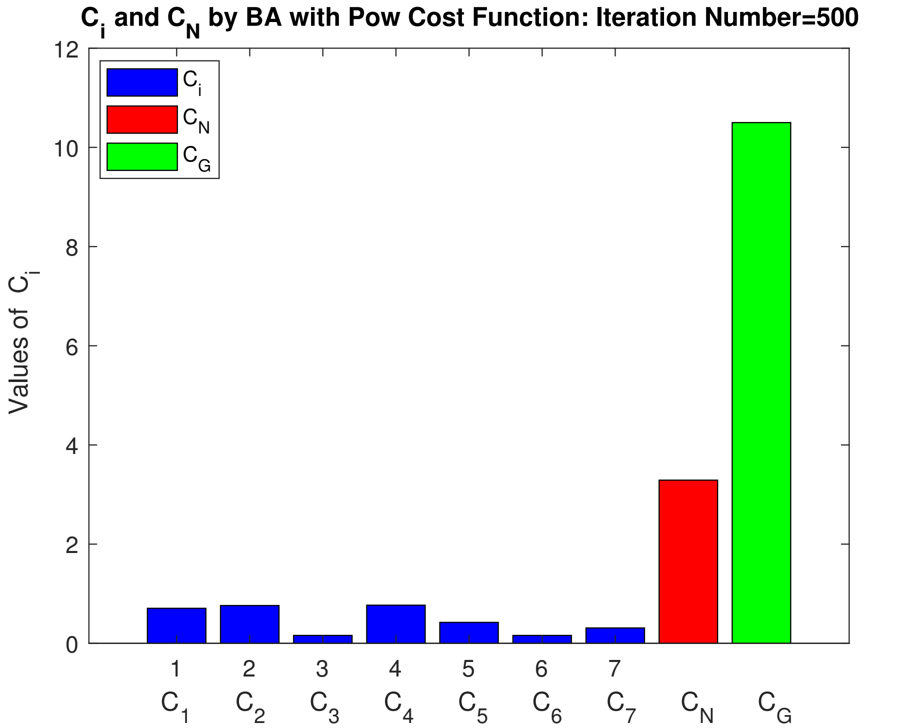

Figure 5.10: Values of C_i and C_N by BA with f_4

5. BA with Tan Cost Function

The values of R_i and $C_i(R_i)$ with best value of reliability network R_N and total cost C_N by Bat using tan cost function listed in table (5.5).

Table 5.5: A Summary Table for R_i, C_i and Best Values of R_N, C_N by BA with f_5

R_i	R_1	R_2	R_3	R_4	R_5	R_6	R_7	**Total**
Value of R_i	0.858	0.925	**0.684**	0.802	0.700	0.900	**0.931**	**0.986**
Value of C_i	0.575	0.563	0.103	0.729	0.317	0.126	0.266	**2.679**

If we take a look at values of table (5.5), then we can make the following observations:

(1) $0.684 \leq R_i \leq 0.931$, for all $i = 1, 2, \ldots, 7$.

(2) $0.103 \leq C_i \leq 0.729$ for all $i = 1, 2, \ldots, 7$.

(3) The highest value of R_i was for the seventh component, which is 0.931, and the lowest value was for the third component, which is 0.684.

(4) There are two components whose reliability is less than or equal 0.7, which are R_3 and R_5.

(5) There are three components whose reliability is greater than or equal 0.9, which are R_2, R_6 and R_7.

(6) $R_7 > R_2 > R_6 > R_1 > R_4 > R_5 > R_3$.

(7) The best value of reliability network R_N is 0.986.

(8) The value of total cost C_N is 2.679.

The bar chart in Figure 5.11 shown the values of R_i and best value of R_N, while the bar chart in Figure 5.12 shown the values of C_i and total cost C_N by use BA with tan cost function.

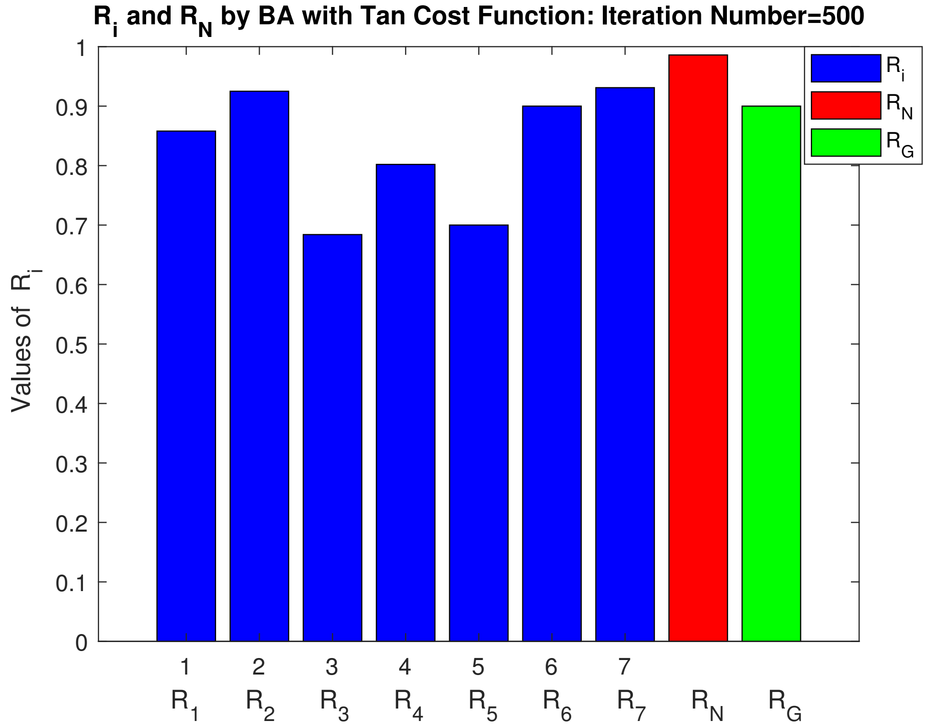

Figure 5.11: Values of R_i and R_N by BA with f_5

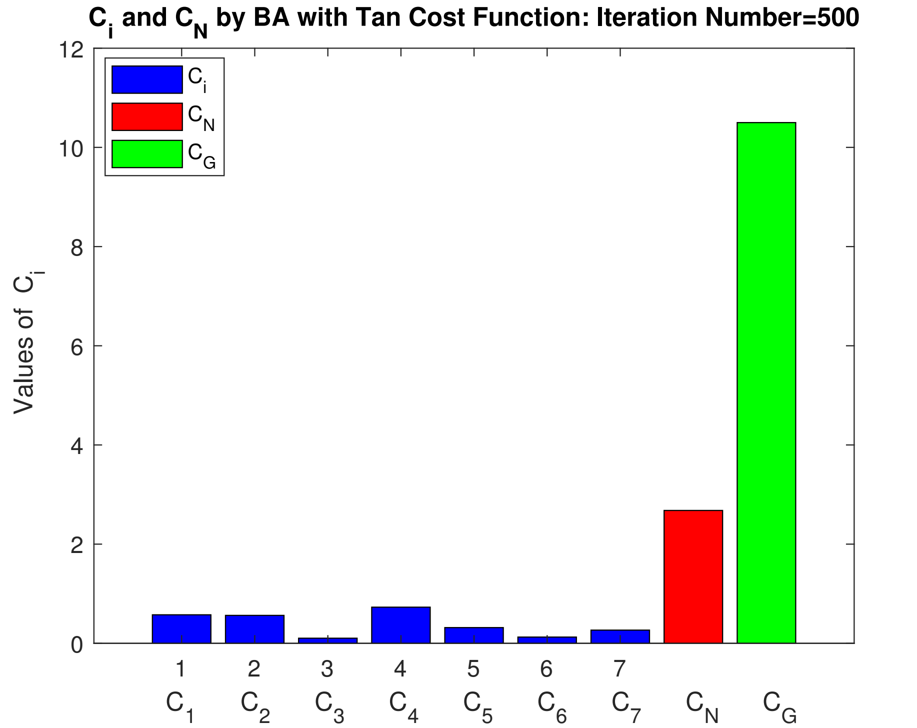

Figure 5.12: Values of C_i and C_N by BA with f_5

5.6 Discuss Results of Bat Algorithm

The values of R_i and $C_i(R_i)$ with best value of reliability network R_N and total cost C_N by BA using five cost functions listed in table (5.6).

Table 5.6: A Summary Table of R_i, C_i, R_N and C_N by BA with all Cost Functions

	$f_1(R_i)$		$f_2(R_i)$		$f_3(R_i)$		$f_4(R_i)$		$f_5(R_i)$	
i	R_i	C_i	R_i	C_i	R_i	C_i	R_i	C_i	R_i	C_i
1	0.921	0.440	0.878	0.644	0.880	1.003	0.897	0.709	0.858	0.575
2	0.940	0.440	0.908	1.606	0.910	1.002	0.913	0.762	0.925	0.563
3	0.742	0.180	0.614	0.187	0.835	1.003	0.759	0.159	0.684	0.103
4	0.775	0.244	0.600	0.256	0.850	1.003	0.712	0.769	0.802	0.729
5	0.546	0.120	0.668	0.296	0.774	1	0.662	0.422	0.700	0.317
6	0.930	0.515	0.903	0.833	0.853	1.003	0.880	0.159	0.900	0.126
7	0.946	0.448	0.940	1.528	0.938	1	0.918	0.310	0.931	0.266
R_N	**0.992**	**2.387**	0.982	5.350	0.987	7.014	0.983	3.290	0.986	2.679

From table (5.6) we have the following notes:

(1) Best network reliability R_N was greater than 0.98 by using Bat algorithm with all five cost functions which is a very acceptable value.

(2) $0.982 \leq R_N \leq 0.992$. Here, the difference between largest and lowest value is 0.01 which has two indications, first one is that the value of R_N is not much affected by the cost function, and the second is that we are on the right path to convergence values.

(3) Best cost is $C_N = 2.387$, this value was done by using logarithm cost function, while the highest cost $C_N = 7.014$ when use an exponential in terms of feasibility factor

cost function f_3. The difference between higher and lower total costs is due to two reasons, the first is the difference in formulas of cost functions, and the second is random selection of values for constants a_i, b_i and F_i.

(4) It is an interesting note, that when using an exponential in terms of feasibility factor cost function f_3 with Bat algorithm, C_i is equal to or very slightly greater than one for all i, and this means that $R_i \simeq R_{i,min} \simeq R_{i,max}$ for all $i = 1, 2, \ldots, 7$.

(5) The reliability value of seventh component is higher than reliabilities of the other components for all cost functions, which indicates the importance of this component. Also, the values of R_1, R_2 and R_6 mostly they are high, meaning they are importance components,too.

(6) The values of R_3, R_4 and R_5 are often the lowest, this means that they are less importance than the other components.

From the fifth and sixth notes, in addition to the R_i values in table (5.6), the network components can be divided in terms of reliability importance into three following levels:

- **First level:** Second and seventh components.
- **Second level:** First and sixth components.
- **Third level:** Third, fourth and fifth components.

This three levels of reliability importance confirm the previous levels we reached from table (4.1).

The bar chart in Figure 5.13 shown the best values of reliability network R_N by BA with all five cost functions.

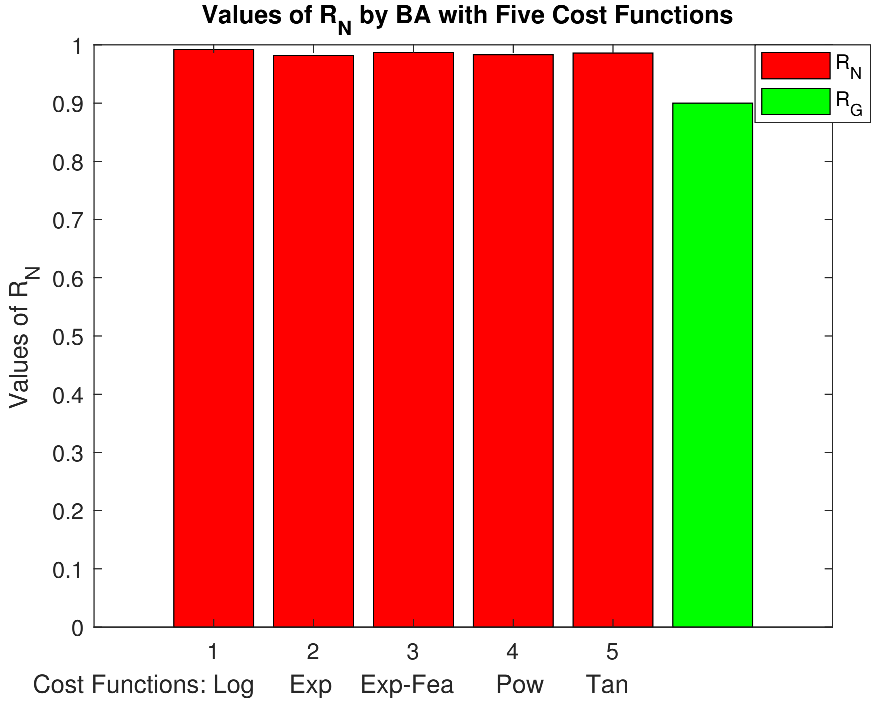

Figure 5.13: Best Values of R_N by BA

The bar chart in Figure 5.14 shown the optimal values of C_N by BA .

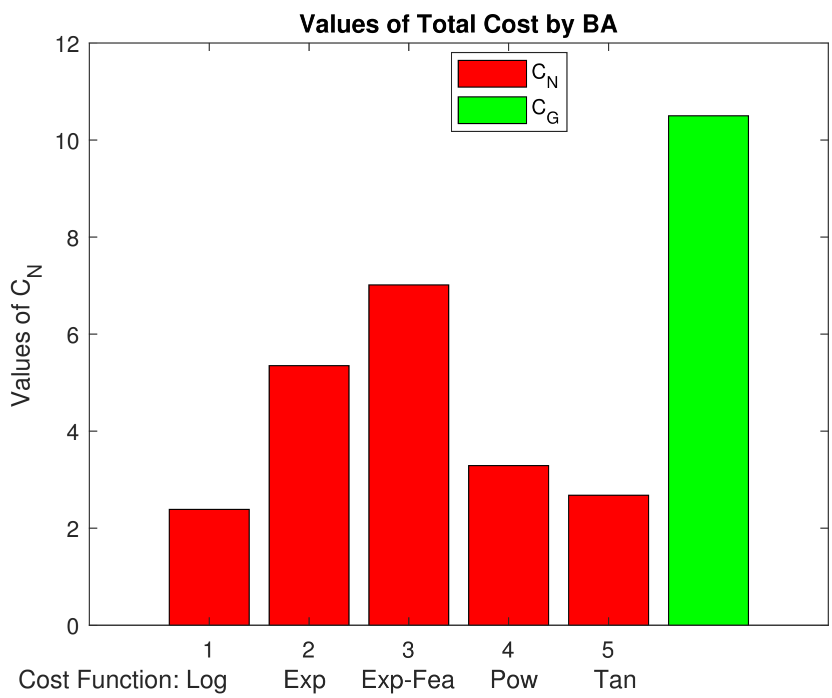

Figure 5.14: Best Values of C_N by BA

5.7 Grey Wolf Optimization (GWO) Algorithm

Mirjalili in 2014 presented Gray Wolf Optimization (GWO) algorithm, which is one of the modern meta-optimization algorithms. Mirjalili drew on social hierarchy and the cunning hunting strategy of gray wolves. Animal researchers say that gray wolves live in herds of 5-12 wolves and have a strict social hierarchy. Figure 5.15 and table (5.7) show the hierarchy of gray wolves and solutions inspired by the algorithm. Leaders α wolves (best solutions) are responsible for making herd decisions and the rest of herd must implement those decisions. The second level in social hierarchy of gray wolves is β, (mean solutions) its role is to help α make decisions and they are responsible for disciplining the herd, β can be the best alternative to the α when one of them incapacitated or dies. The third level is δ, (worst solutions) they have to obey α and β wolves, and their responsibilities are to protect the herd and alert it in case of danger, and to take care of the elderly and hunters. The last level is ω (remaining solutions), they must carry out the orders of rest levels and they are the last to eat. In gatherings, individuals confirm the alpha's decision by holding their tails down.

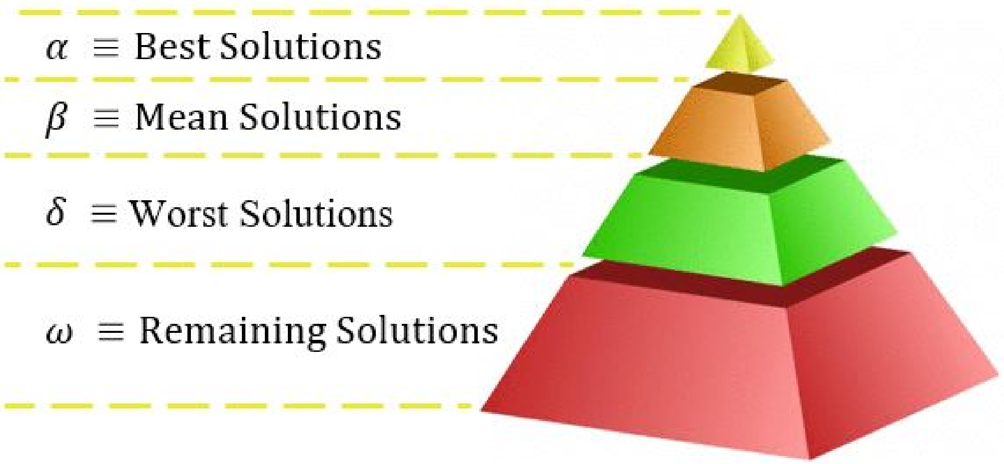

Figure 5.15: The Social Hierarchy of Grey Wolves

Table 5.7: The Social Hierarchy of Grey Wolves

Level	Name	Roles in Herd	Type of Solutions
1	α	Leaders: making herd decisions	best solutions
2	β	Help α, responsible for disciplining the herd	mean solutions
3	δ	Protect the herd and alert it in case of danger	worst solutions
4	ω	Executing higher level orders	remaining solutions

Collective hunting is one of the behaviors of the gray wolves herd in addition to the social hierarchy. Grey wolves' hunting includes the following three main parts:

(1) Tracking, chasing, and approaching the prey.

(2) Pursuing, encircling, and harassing the prey till it stops moving.

(3) Attacking the prey [23, 56, 67].

5.8 Mathematical Model of the GWO Algorithm

The GWO algorithm is a set of rules are inspired with the resource of social hierarchy and the smart hunting method of grey wolves. Mirjalili described the behavior of gray wolves in search of prey as follows [23, 40, 56]:

$$\overrightarrow{D} = |\overrightarrow{C}.\overrightarrow{X}_p(t) - \overrightarrow{X}(t)| \tag{5.8}$$

$$\overrightarrow{X}(t+1) = \overrightarrow{X}_p(t) - \overrightarrow{A}.\overrightarrow{D} \tag{5.9}$$

here t is an iteration number, $\overrightarrow{X}_p$ is a vector of the prey's positions, $\overrightarrow{X}$ is a vector of grey wolf's positions, $\overrightarrow{D}$ is a calculated vector used to specify a new position of the grey wolf, while $\overrightarrow{A}$ and $\overrightarrow{C}$ are coefficient vectors can be can be calculated by

$$\overrightarrow{A} = 2\overrightarrow{a}.\overrightarrow{r_1} - \overrightarrow{a} \tag{5.10}$$

$$\overrightarrow{C} = 2\overrightarrow{r_2} \tag{5.11}$$

here $\overrightarrow{a}$ is a vector set to decrease linearly from 2 to 0 over the iterations, $\overrightarrow{r_1}$ and $\overrightarrow{r_2}$ are random vectors in [0,1]. Any gray wolf can be placed in every random position around the prey (the best solution) as it is calculated from the equations (5.8) and (5.9), gray wolves have the ability to distinguish the location of prey from others [23, 56].

To emulation this hunting behavior, suppose that the alpha (the best candidate for the solution) while the beta and delta have more knowledge about the location of prey. Therefore, the algorithm saves three best achieved solutions away and forcing omega to update their locations to achieve the best place in the decision space. In the optimization algorithm, such a hunting behavior can be modeled by:

$$\overrightarrow{D_\tau} = |\overrightarrow{C_i}.\overrightarrow{X_\tau} - \overrightarrow{X}| \tag{5.12}$$

$$\overrightarrow{X_i} = \overrightarrow{X_\tau} - \overrightarrow{A_i}.\overrightarrow{D_\tau} \tag{5.13}$$

here $\tau = \alpha, \beta, \delta$ in conjunction with $i = 1, 2, 3$.

To understand how the GWO algorithm solves optimization problems theatrically, some notes can be summarized as follows:

(1) The social hierarchy helps the algorithm to rank solutions and save the best ones until the last iteration.

(2) The random vectors (A and C) help grey wolves (candidate solutions) to define different hyper-spheres with random radii.

(3) The hunting approach implemented in the GWO algorithm allows grey wolves (candidate solutions) to locate the probable position of the prey (optimal solution).

(4) When reducing the values of A, half of the iterations are allocated to exploration ($|\overrightarrow{A}| > 1$) and the other half of the iterations are allocated to exploitation ($|\overrightarrow{A}| < 1$).

(5) $\overrightarrow{a}$ and $\overrightarrow{C}$ are two main vectores of GWO algorithm [4, 23, 56].

(6) The problem is a nonlinear multi-objective optimization as in equation (5.7)

The simple steps within GWO algorithm manner are presented in the flowchart shown in Figure 5.16.

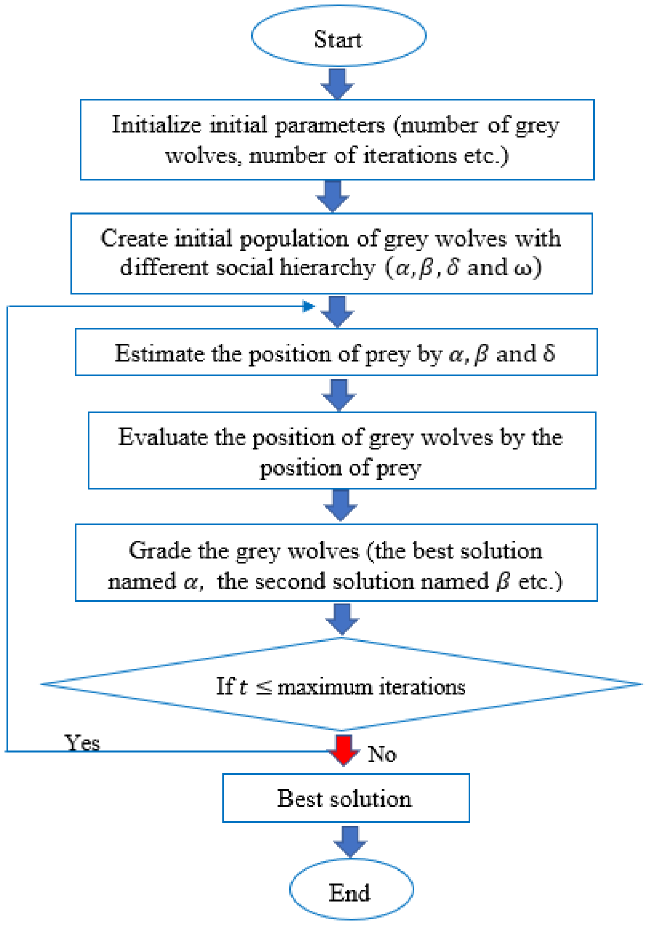

Figure 5.16: Flowchart of GWO Algorithm

5.9 Computational Results of GWO

To find out the Best reliability of a complex network, we used GWO by 500 from NI with the aforementioned five cost functions. The nonlinear multi-objective optimization problem as in equation (5.7).

1. GWO with Logarithmic Cost Function

The values of R_i, C_i, reliability network R_N and total cost C_N by GWO using logarithmic cost function listed in table (5.8).

Table 5.8: A Summary Table for R_i, C_i and Best Values of R_N, C_N by GWO with f_1

R_i	R_1	R_2	R_3	R_4	R_5	R_6	R_7	**Total**
Value of R_i	0.931	0.942	0.873	0.884	**0.850**	0.941	**0.945**	**0.996**
Value of C_i	1.604	1.710	1.242	1.294	1.140	1.700	1.744	**10.434**

By looking at table (5.8), the following observations can be recorded:

(1) $0.850 \leq R_i \leq 0.945$, and $1.140 \leq C_i \leq 1.744$, for all $i = 1, 2, \cdots, 7$.

(2) The highest value of R_i was for the seventh component, which is 0.945, while the lowest value was for the fifth component, which is 0.850.

(3) The values of R_3, R_4 and R_5 are less than 0.9.

(4) The values of R_1, R_2, R_6 and R_7 are greater than 0.9.

(5) $R_7 > R_2 > R_6 > R_1 > R_4 > R_3 > R_5$.

(6) The best value of reliability network is $R_N = 0.996$ which is an excellent value, as the difference between it and one (ideal value) is very small.

(7) The best total cost is $C_N = 10.434$.

Two bar charts in Figuers 5.17 and 5.18 shown the values of R_i, R_N and C_i, C_N by GWO with f_1 respectively.

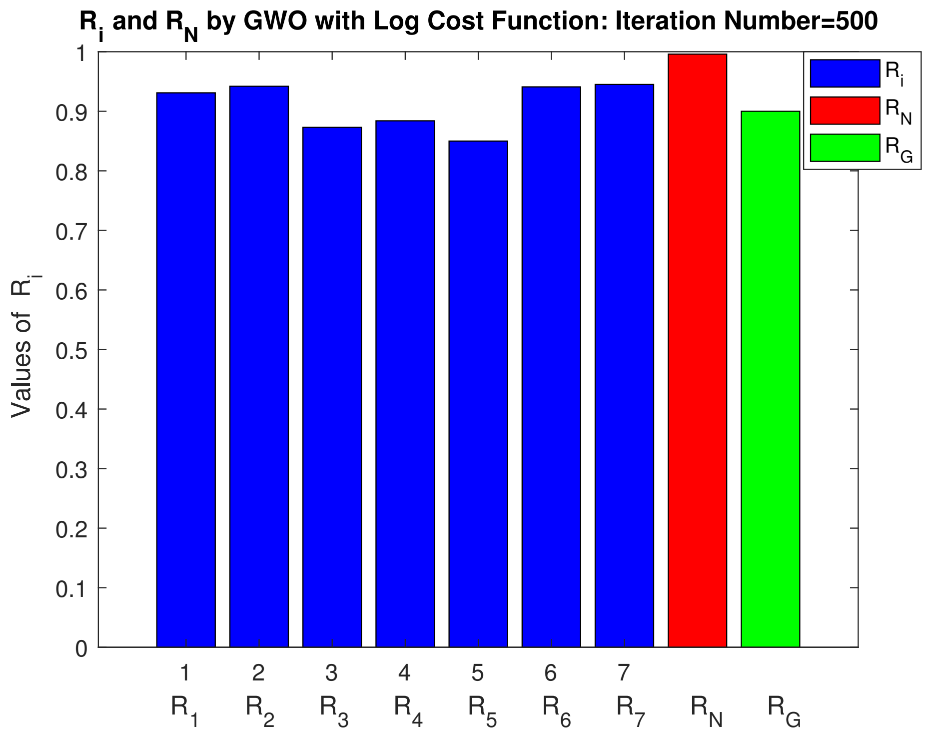

Figure 5.17: Values of R_i and R_N by GWO with f_1

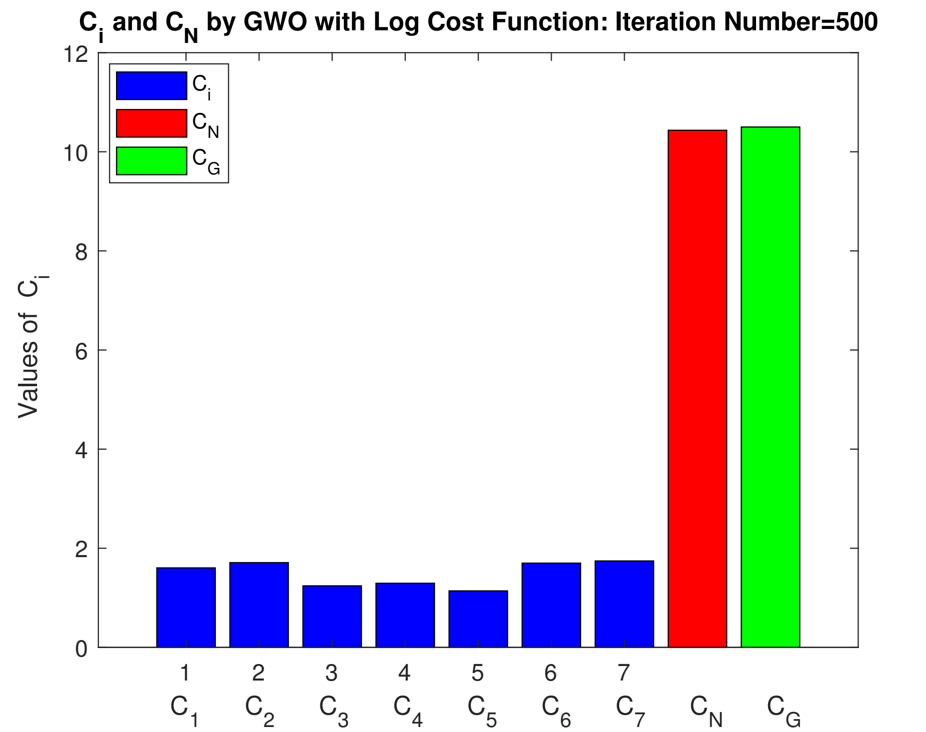

Figure 5.18: Values of C_i and C_N by GWO with f_1

2. GWO with Exponential Cost Function

The values of R_i, C_i, reliability network R_N and total cost C_N by GWO with the exponential function listed in table (5.9).

Table 5.9: A Summary Table for R_i, C_i, R_N and C_N by GWO with f_2

R_i	R_1	R_2	R_3	R_4	R_5	R_6	R_7	**Total**
Value of R_i	0.909	0.934	0.867	**0.863**	0.864	0.918	**0.948**	**0.994**
Value of C_i	0.750	0.893	0.500	0.360	0.496	1.362	1.627	**5.988**

By looking at table (5.9), the following observations can be recorded:

(1) $0.863 \leq R_i \leq 0.948$, for all $i = 1, 2, \cdots, 7$.

(2) $0.360 \leq C_i \leq 1.627$, for all $i = 1, 2, \cdots, 7$.

(3) The highest value of R_i was for the seventh component, which is 0.948, while the lowest value was for the fourth component, which is 0.863.

(4) There are three components whose reliability is less than 0.9, which are R_3, R_4 and R_5.

(5) There are four components whose reliability is greater than 0.9, which are R_1, R_2, R_6 and R_7.

(6) $R_7 > R_2 > R_6 > R_1 > R_3 > R_5 > R_4$.

(7) From last three observations we understand that the seventh, second, sixth and first components are of greater importance than the other components.

(8) The best value of R_N is 0.994, which is an excellent value, as the difference between it and one (ideal value) is very small.

(9) The best value of C_N is 5.988.

Two bar charts in Figures 5.19 and 5.20 shown the values of R_i, R_N and C_i, C_N by GWO with f_2 respectively.

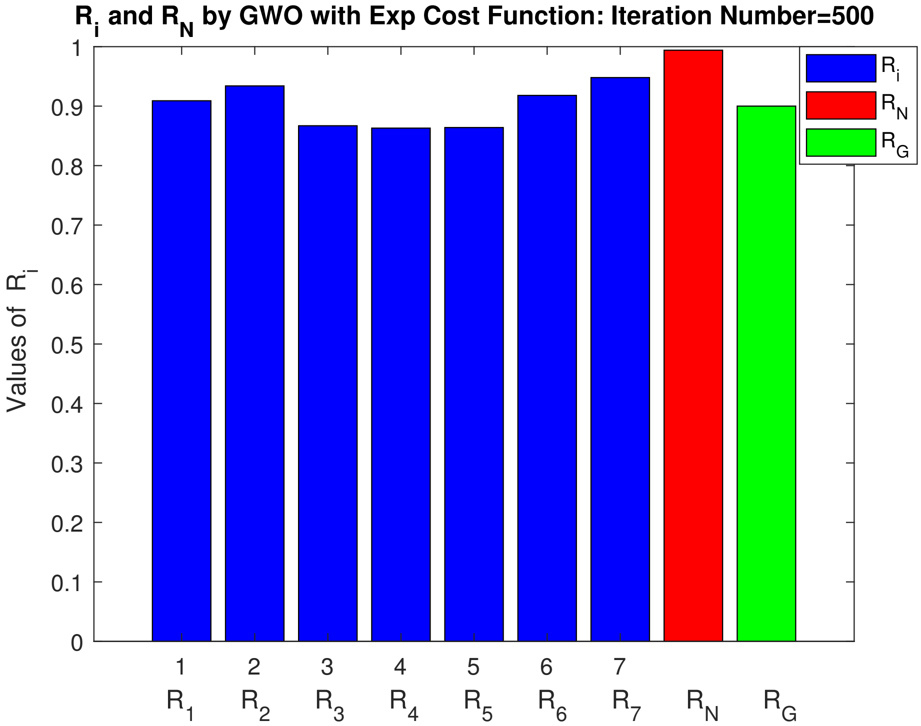

Figure 5.19: Values of R_i and R_N by GWO with f_2

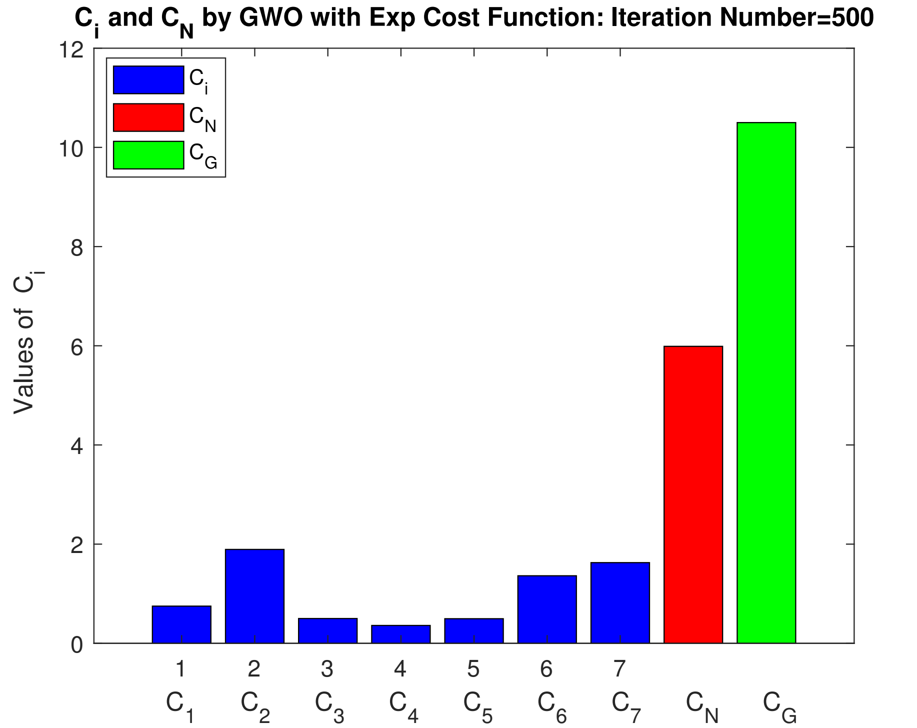

Figure 5.20: Values of C_i and C_N by GWO with f_2

3. GWO with Exp. in Terms of Feasibility Factor Cost Function

The values by GWO with f_3 listed in table (5.10).

Table 5.10: A Summary Table for R_i, C_i, R_N and C_N by GWO with f_3

R_i	R_1	R_2	R_3	R_4	R_5	R_6	R_7	**Total**
Value of R_i	0.923	**0.946**	0.819	0.869	**0.699**	0.945	**0.946**	**0.995**
Value of C_i	1.002	1	1.006	1.005	1	1.001	1	**7.014**

With a quick look at table (5.10), we can make the following notes:

(1) $0.699 \leq R_i \leq 0.946$, for all $i = 1, 2, \cdots, 7$.

(2) C_i is equal to or very slightly greater than one for all i, and this means that the values of two terms $R_i - R_{i,min}$ and $R_{i,max} - R_i$ are either zero or very close to zero, and we understand from this that $R_i, R_{i,min}$ and $R_{i,max}$ are almost equal values for all $i = 1, 2, \ldots, 7$.

(3) The highest value of R_i was for the seventh component, which is 0.946, while the lowest value was for the fifth component, which is 0.699.

(4) There are three components whose reliability is less than 0.9, which are R_3, R_4 and R_5.

(5) There are four components whose reliability is greater than 0.9, which are R_1, R_2, R_6 and R_7.

(6) $R_2 = R_7 > R_6 > R_1 > R_4 > R_3 > R_5$.

(7) From last four observations we understand that the second, seventh, sixth and first components are of greater importance than the other components.

(8) The best value of R_N is 0.995, and the best value of C_N is 7.014.

Two bar charts in Figures 5.21 and 5.22 shown the values of R_i, R_N and C_i, C_N by GWO with f_3.

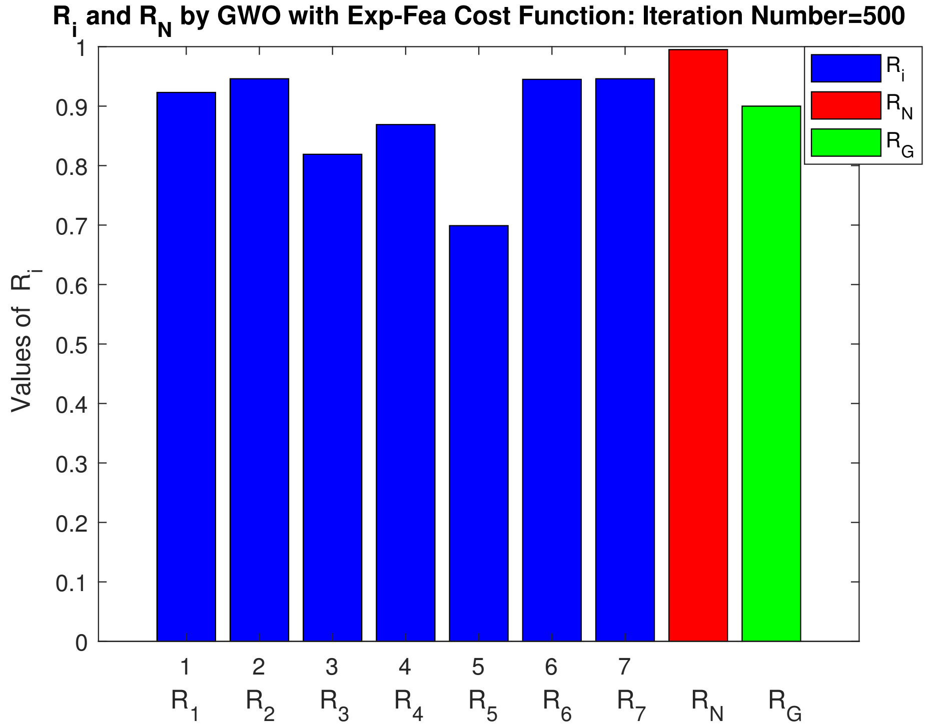

Figure 5.21: Values of R_i and R_N by GWO with f_3

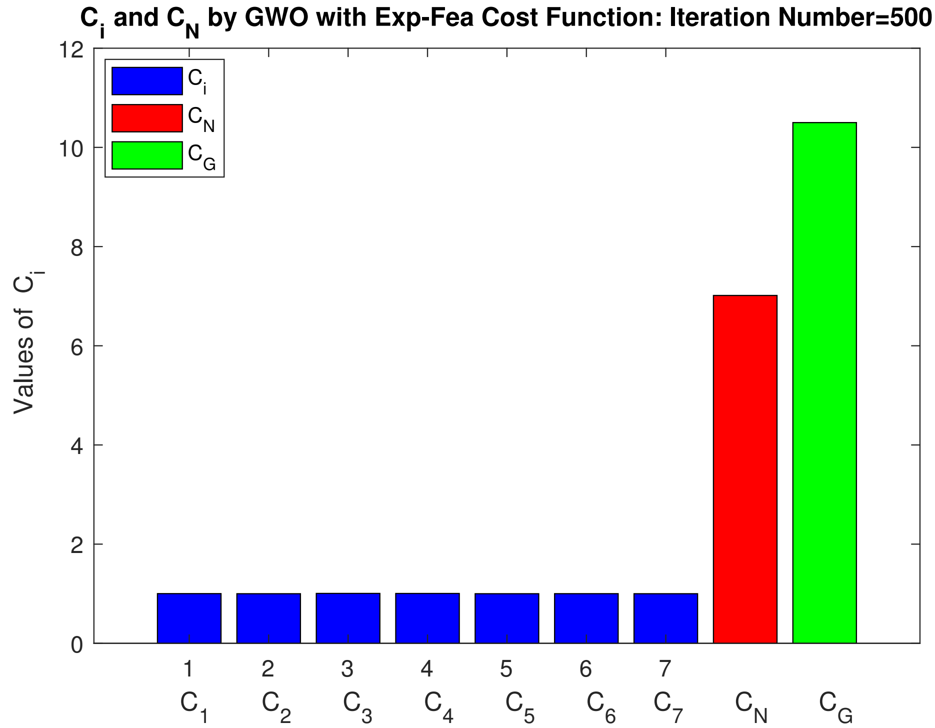

Figure 5.22: Values of C_i and C_N by GWO with f_3

4. GWO with Power Cost Function

The values of R_i, C_i, R_N and total cost C_N by GWO with power cost function lised in table (5.11).

Table 5.11: A Summary Table for R_i, C_i, R_N and C_N by GWO with f_4

R_i	R_1	R_2	R_3	R_4	R_5	R_6	R_7	**Total**
Value of R_i	0.916	0.944	0.886	0.912	**0.787**	0.918	**0.949**	**0.995**
Value of C_i	0.351	0.376	0.750	0.657	0.716	0.427	0.496	**3.773**

With a quick look at table (5.11), we can make the following notes:

(1) $0.787 \leq R_i \leq 0.949$ and $0.351 \leq C_i \leq 0.750$ for all $i = 1, 2, \cdots, 7$.

(2) The highest value of R_i was for the seventh component, which is 0.949, while the lowest value was for the fifth component, which is 0.787.

(3) There are five components whose reliability is greater than 0.9, which are R_1, R_2, R_4, R_6 and R_7.

(4) R_7 and R_2 are the highest values.

(5) $R_7 > R_2 > R_6 > R_1 > R_4 > R_3 > R_5$.

(6) From last three observations we understand that the seventh, second, sixth and first components are of greater importance than the other components.

(7) The best value of reliability network is $R_N = 0.995$ which is an excellent value, as the difference between it and one (ideal value) is very small.

(8) The best total cost is $C_N = 3.773$.

Two bar charts in Figures 5.23 and 5.24 shown the values of R_i, R_N and C_i, C_N respectively by GWO with f_4.

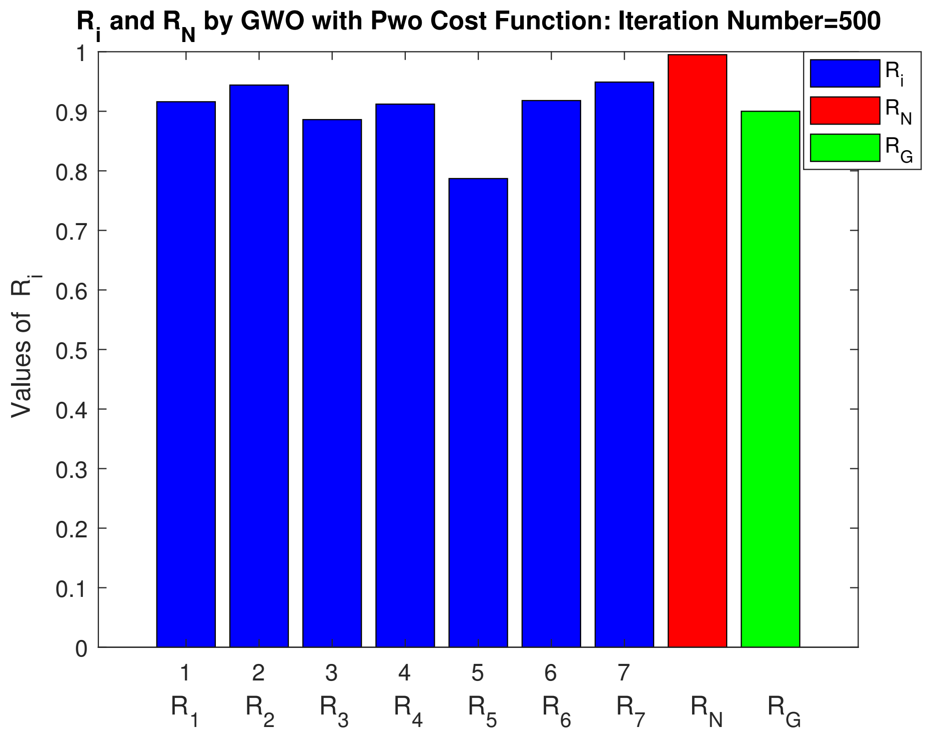

Figure 5.23: Values of R_i and R_N by GWO with f_4

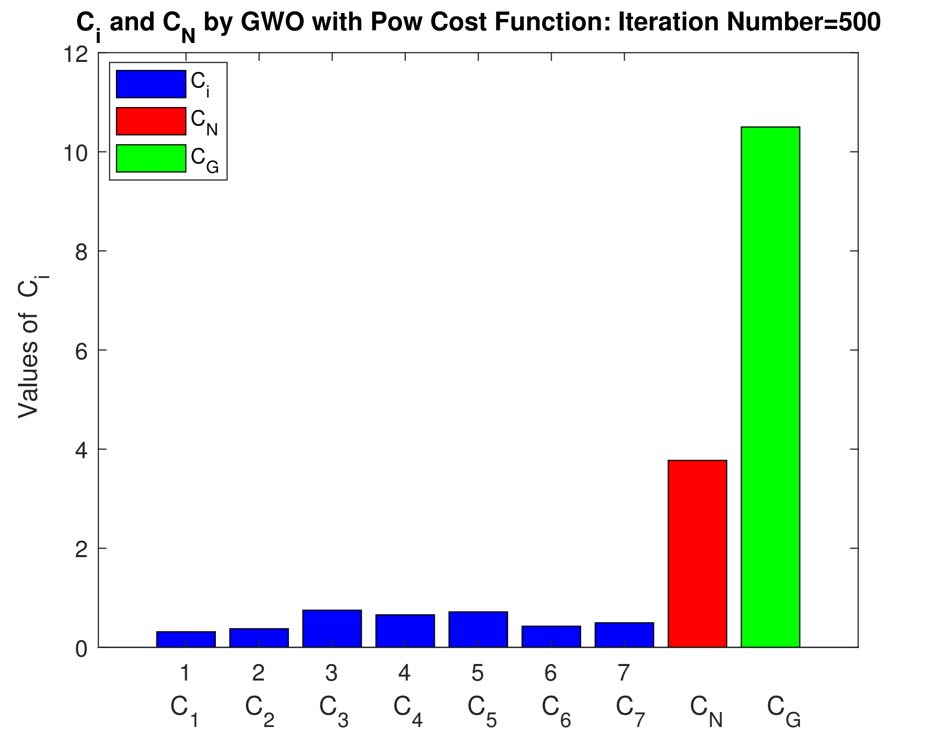

Figure 5.24: Values of C_i and C_N by GWO with f_4

5. GWO with Tan Cost Function

The values by GWO with f_5 listed in table (5.12).

Table 5.12: A Summary Table for R_i, C_i, R_N and C_N by GWO with f_5

R_i	R_1	R_2	R_3	R_4	R_5	R_6	R_7	**Total**
Value of R_i	0.933	0.940	**0.807**	0.873	0.891	0.932	**0.948**	**0.995**
Value of C_i	0.144	0.323	0.317	0.308	0.274	0.184	0.326	**1.876**

With a quick look at table (5.12), we can make the following notes:

(1) $0.807 \leq R_i \leq 0.948$ for all $i = 1, 2, \cdots, 7$.

(2) $0.144 \leq C_i \leq 0.326$ for all $i = 1, 2, \cdots, 7$.

(3) The highest value of R_i was for the seventh component, which is 0.948, while the lowest value was for the third component, which is 0.807.

(4) There are four components whose reliability is greater than 0.9, which are R_1, R_2, R_6 and R_7.

(5) The values of R_3, R_4 and R_5 are less than 0.9.

(6) $R_7 > R_2 > R_1 > R_6 > R_5 > R_4 > R_3$.

(7) From last four observations we understand that the seventh, second, sixth and first components are of greater importance than the other components.

(8) $R_N = 0.995$. It is a very well value, as the difference between it and one (ideal value) is very small.

(9) The total cost is $C_N = 1.876$. It is very small compared to the total costs of the rest cost functions

Two bar charts in Figures 5.25 and 5.26 shown the values of R_i, R_N and C_i, C_N respectively by GWO with f_5.

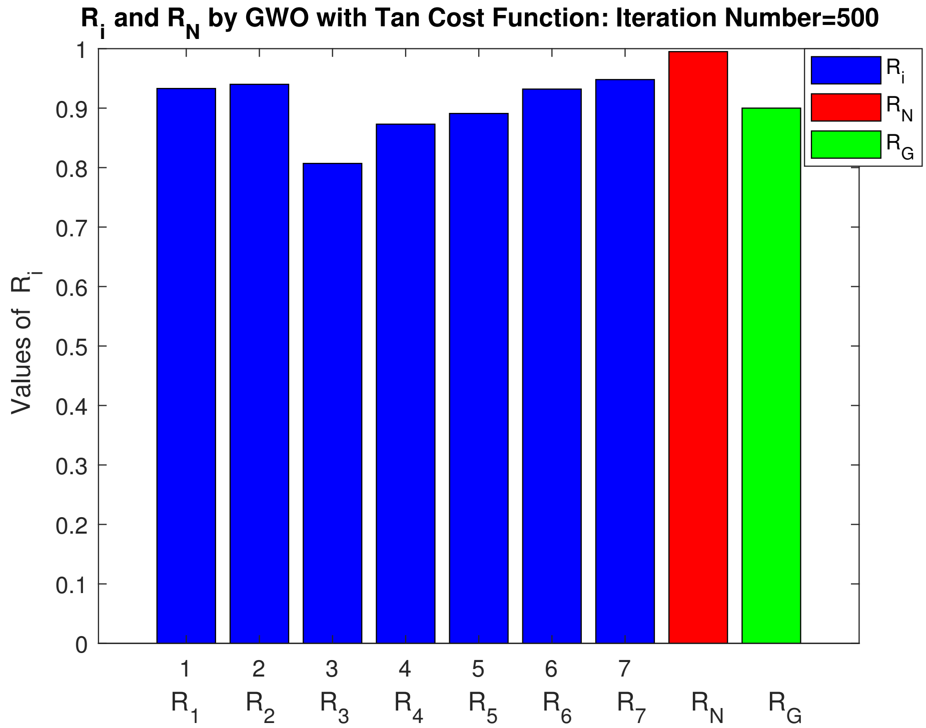

Figure 5.25: Values of R_i and R_N by GWO with f_5

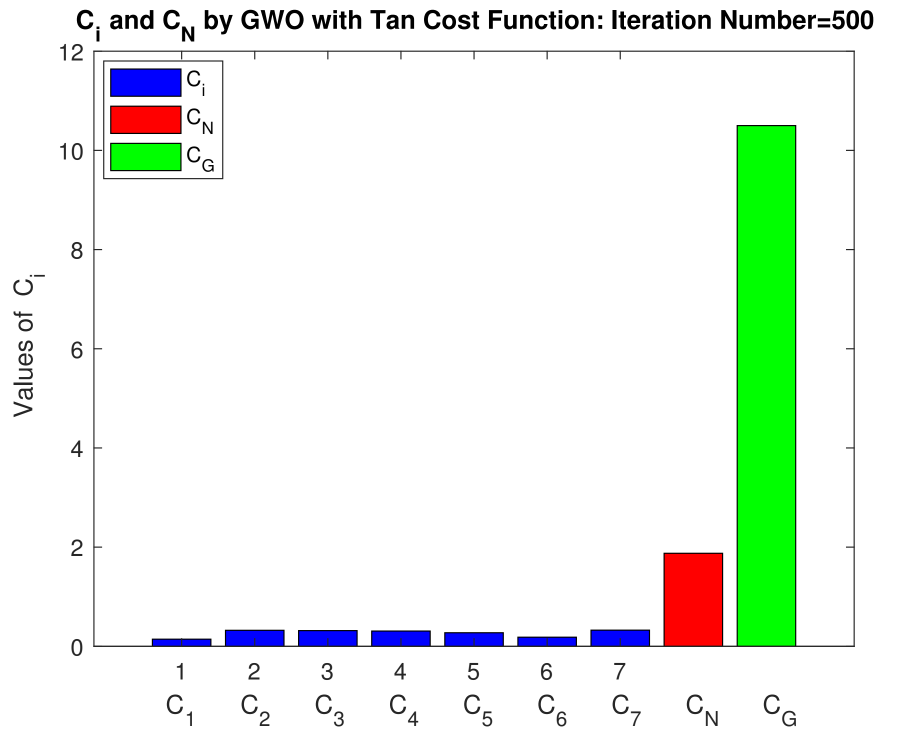

Figure 5.26: Values of C_i and C_N by GWO with f_5

5.10 Discuss Results of GWO Algorithm

The results of values by using GWO with five cost functions listed in table (5.13).

Table 5.13: A Summary Table of R_i, C_i, R_N and C_N by GWO with all Cost Functions

	$f_1(R_i)$		$f_2(R_i)$		$f_3(R_i)$		$f_4(R_i)$		$f_5(R_i)$	
i	R_i	C_i	R_i	C_i	R_i	C_i	R_i	C_i	R_i	C_i
1	0.931	1.604	0.909	0.750	0.923	1.002	0.916	0.351	0.933	0.144
2	0.942	1.710	0.934	0.893	0.946	1	0.944	0.376	0.940	0.323
3	0.873	1.242	0.867	0.500	0.819	1.006	0.886	0.750	0.807	0.317
4	0.884	2.172	0.863	0.360	0.869	1.005	0.912	0.657	0.873	0.308
5	0.850	1.140	0.864	0.496	0.699	1	0.787	0.716	0.891	0.274
6	0.941	1.700	0.918	1.362	0.945	1.001	0.918	0.427	0.932	0.184
7	0.945	1.744	0.948	1.627	0.946	1	0.949	0.496	0.948	0.326
R_N	**0.996**	10.434	0.994	5.988	0.995	7.014	0.995	3.773	0.995	**1.876**

From the table (5.13), we have the following notes:

(1) Best values of R_N was greater than 0.99 by using GWO algorithm with all five cost functions.

(2) Best value of R_N is 0.996, this value was done by using the logarithm cost function.

(3) Best cost is $C_N = 1.867$, this value was done by using the tan cost function, while the highest cost $C_N = 10.434$ when use the logarithm function.

(4) $0.994 \leq R_N \leq 0.996$. Here, the difference between largest and lowest value is 0.002 which has two indications, first one is that the value of R_N is not much affected by the cost function, and the second is that we are on the right path to convergence of values.

(5) $1.867 \leq C_N \leq 10.434$. The difference between largest and lowest total costs is due to two reasons, first one is the difference in cost functions, and second one is the

random selection of the values for constants a_i, b_i and F_i.

(6) It is an interesting note, that when using f_3 with GWO algorithm, C_i is equal to or very slightly greater than one for all i, and this means that $R_i \simeq R_{i,min} \simeq R_{i,max}$ for all $i = 1, 2, \ldots, 7$.

(7) The reliability value of the seventh component is higher than reliabilities of the other components for all cost functions, which indicates the importance of this component. Also, the valuse of R_1, R_2 and R_6 mostly they are high, meaning they are important compounds.

(8) The reliability values of the third, fourth and fifth components are often the lowest, this means that they are less important than the other components.

From seventh and eighth notes, in addition to R_i values in table (5.13), the network components can be divided in terms of importance into three following levels:

- **First level:** Second and seventh components.
- **Second level:** First and sixth components.
- **Third level:** Third, fourth and fifth components.

The levels of importance above confirm the previous levels we reached in two sections (4.4) and (5.6).

The bar chart in Figure 5.27 shown the best values of R_N by GWO.

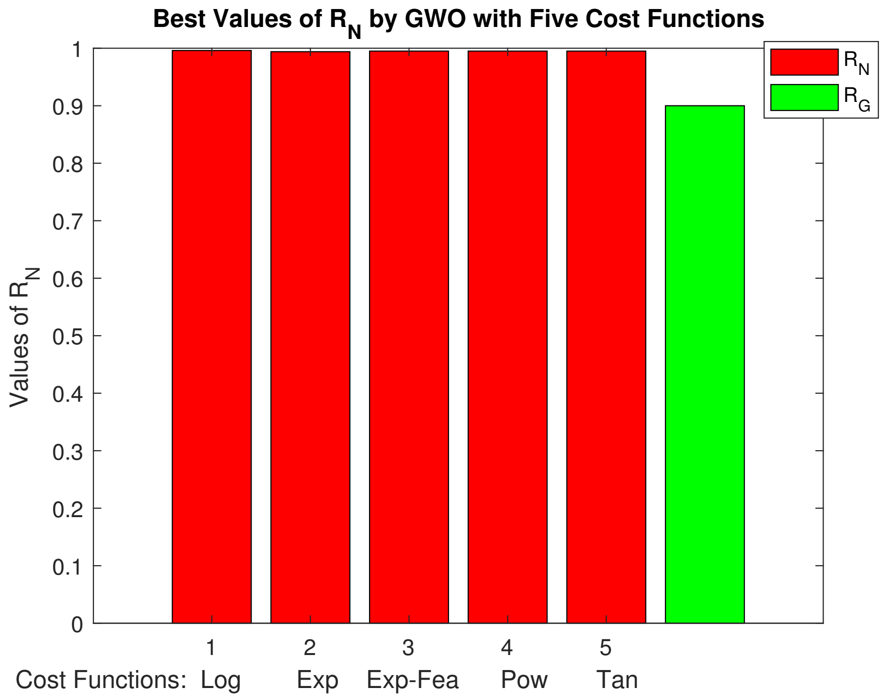

Figure 5.27: Values of R_N by GWO

The bar chart in Figure 5.28 shown the best values of C_N by GWO.

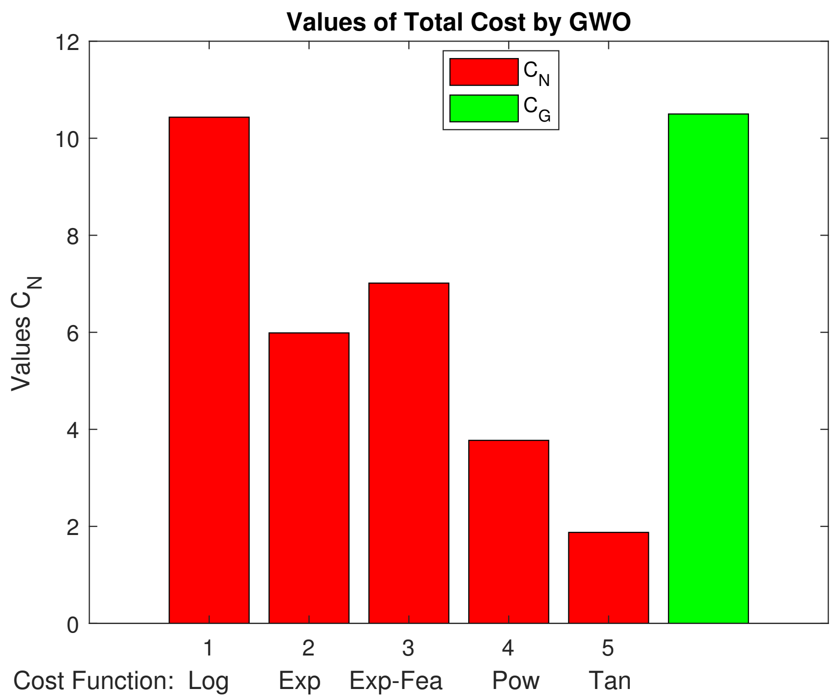

Figure 5.28: Values of C_N by GWO

5.11 Compare Results of Two Algorithms

We will analyze the values obtained by using the algorithms BA and GWO in two stages, the first will be based on the best values of network reliability and the second will depend on the total cost.

5.11.1 Compare the Best Values of R_N

The best values of reliability network R_N obtained by using two algorithms BA and GWO with five cost functions without total cost listed in table (5.14).

Table 5.14: A Summary Table for R_N by AB and GWO

Cost Function	R_N by BA	R_N by GWO
Log	0.992	0.996
Exp	0.982	0.994
Exp - Fea	0.987	0.995
Pow	0.983	0.995
Tan	0.986	0.995

From table (5.14) we can make the following observations:

(1) The best reliability network R_N is 0.996 by using GWO with logarithmic cost function which is a very close to one.

(2) The less value of reliability network R_N is 0.982 by using BA with exponential cost function which is an acceptable value.

(3) The values of reliability network R_N when using the GWO are higher than those using the BA for all cost functions.

(4) Although the values of reliability network are varying using two algorithms with five different cost functions, all of these values are acceptable.

The bar chart in Figure 5.29 shwon the best values of R_N by two algorithms.

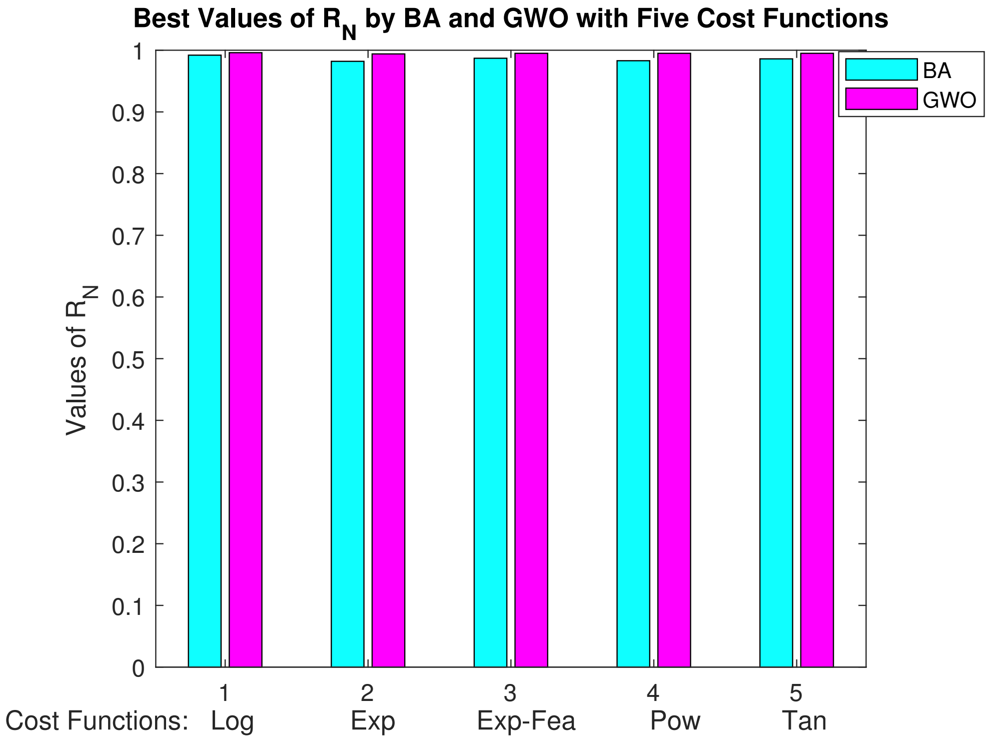

Figure 5.29: Best Values of R_N by AB and GWO

Although the advantage of GWO in optimaization reliability network is clear from table (5.14) as well as the bar chart in Figure 5.29, but we will do some statistical calculations for reliability network valuse by both algorithms to confirm this advantage.

We know that two statistical parameters, Range and Standard Deviation, increase in value as the amount of dispersion of values around the arithmetic mean increases and vice versa [58].

Suppose we have n values $x_1, x_2, \cdots, x_n$, Range is the difference between highest and lowest value, Mean is the sum of values divided by their number and the Standard Deviation SD is given by the following relationship:

$$SD = \sqrt{\frac{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}}$$

We calculated the Range, Mean, and Standard Deviation of the values obtained by two algorithms, they are listed in table (5.15).

Table 5.15: A Summary Table for Statistical Parameters of R_N

Statistical parameter	BA Algorithm	GWO Algorithm
Range	0.01	0.002
Mean	0.986	0.995
Standard Deviation	0.2600	0.0007

By looking at values of table (5.15) we can make the following observations:

(1) The Mean of values for R_N is equal to 0.995 by GWO and it is equal to 0.986 by BA. So, the Mean of values for R_N is better when using GWO. This confirms the advantage of GWO.

(2) The Range and Standard Deviation of values for R_N by use BA are greater than the Range and Standard Deviation by use GWO. This means that the values generated by BA are more dispersed than those generated by GWO.

The above two observations confirm the superiority of GWO over BA. This does not mean that the latter is ineffective. On the contrary, the values we obtained with the BA were excellent.

5.11.2 Compare the Values of C_N

We list the values of total cost C_N obtained using two algorithms in table (5.16) and make the following observations:

(1) The best total cost C_N is 1.876 by using GWO with tan cost function.

(2) The higher value of total cost C_N is 10.434 by using GWO with cost logarithmic function.

Table 5.16: A Summary Table for C_N by AB and GWO

Cost function	C_N by BA	C_N by GWO
Log	2.387	10.434
Exp	5.350	5.988
Exp - Fea	7.014	7.014
Pow	3.290	3.773
Tan	2.679	1.876

The bar chart in Figure 5.30 shwon the optimal values of C_N by two algorithms.

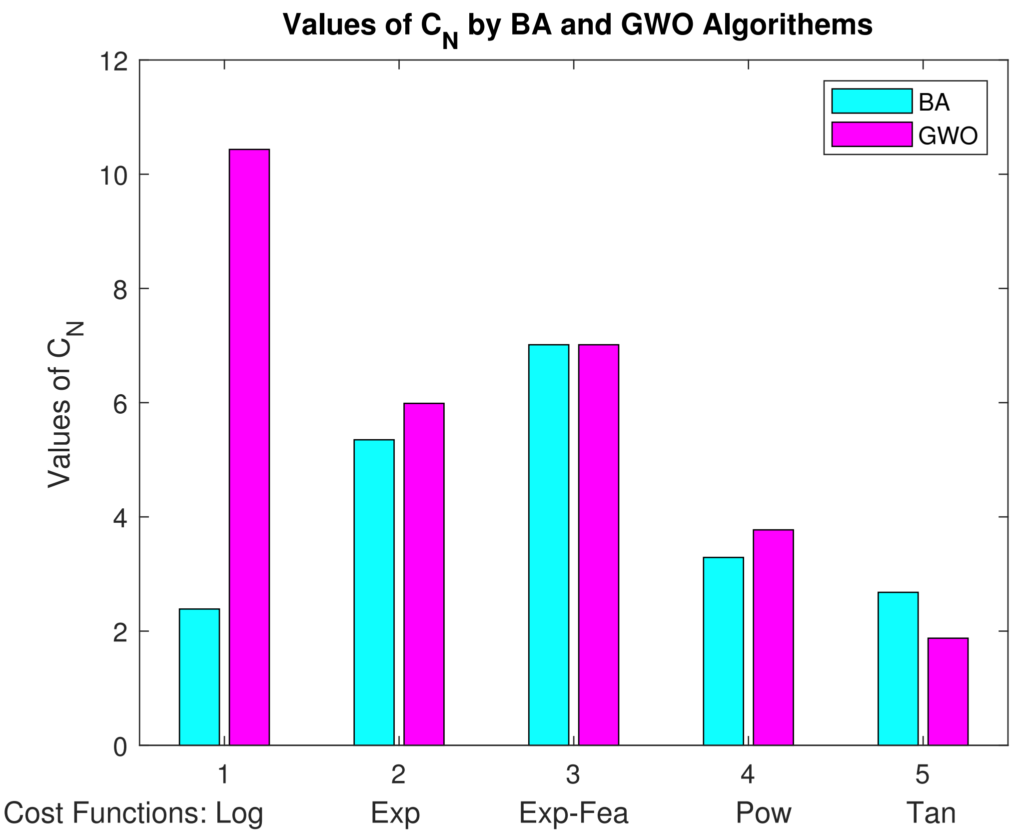

Figure 5.30: Values of C_N by AB and GWO

CHAPTER 6

CHAPTER SIX: CONCLUSIONS AND FUTURE WORKS

6.1 Conclusions

We have found a reliablity polynomial for the complex network under study, through two different techniques that were presented with two theorems with their proof, the first depends on the minimal paths. In contrast, the second depends on the minimal cut sets where we obtained the same polynomial by applying those theorems. This confirms the effectiveness of these theories in calculating the reliability of complex networks. Furthermore, we have reduced the number of network units by two techniques, where the complex network was reduced to two subnetworks in parallel by the reduction technique, also transformed the complex network into two subnetworks in series by using the delta - star technique.

We also allocated the reliability of network components, and the results of the examples showed the possibility of applying the two techniques (Exponential allocation and ARINC approach method) to allocate the reliability of complex networks where the results were greater than the target reliability. In addition, we identified three levels of reliability for the complex network components under study, the second and seventh components were at the first level, the first and sixth components were at the second level, and the other three components were at the third level, and this information was very useful when studying the topic of improving network reliability because the focus is on improving the reliability of components that have a greater impact on network reliability.

The reliability optimization of a given complex network was studied using two nature-inspired algorithms (Bat and Gray Wolf) based on five cost functions with these algorithms (Logarithmic, Exponential, Exponential in terms of feasibility factor, Power function and Tan function). The problem was a multi-objective nonlinear problem where the results of two algorithms BA and GWO (despite the differences between them) showed that they can be relied upon to improve the reliability of complex networks at low cost and these results have achieved the desired goal of this dissertation. We noticed that when using Exponential in terms of feasibility factor cost function with two algorithms, the cost of each component is proven to be at or close to one. The best reliability value at the lowest cost is by using GWO algorithm with Tan function. Also, we obtained best reliability allocation when using the GWO algorithm with Logarithmic cost function.

6.2 Future Works

1. We propose to study the network reliability given in Figure 3.1 using another techniques for calculating network reliability and reliability allocation.
2. We propose to study the network reliability optimization given in Figure 3.1 using new algorithms and comparison between them and two given algorithms (BA and GWO).
3. We propose to study optimization reliability of different networks using two algorithms mentioned (BA and GWO).
4. We suggest using cost functions other than those used and comparing the results.
5. We suggest using T - test or any other tests to compare between results of two algorithms.

REFERENCES

References

[1] Abbas, Sameer Abd Karim, and Zahir Abdul Haddi Hassan. "Increase the Reliability of Critical Units by Using Redundant Technologies." Journal of Physics: Conference Series. Vol. 1999. No. 1. IOP Publishing, (2021).

[2] Abd Alsharify, Fouad Hamza, and Hassan, Zahir Abdul Haddi . "Computing the reliability of a complex network using two techniques." Journal of Physics: Conference Series. Vol. 1963. No. 1. IOP Publishing, (2021).

[3] Abd Alsharify, Fouad Hamza, Mudhar, Ghufran Aziz , and Hassan, Zahir Abdul Haddi. "A modified technique to compute the minimal path sets for the reliability of complex network." Journal of Physics: Conference Series. Vol. 1999. No. 1. IOP Publishing, (2021).

[4] Abdel-Basset, Mohamed, El-Shahat, D., El-henawy, I., de Albuquerque, V. H. C. and Mirjalili, S. "A new fusion of grey wolf optimizer algorithm with a two-phase mutation for feature selection." Expert Systems with Applications 139 (2020): 112824.

[5] Abdullah, Ghazi, and Hassan, Zahir Abdul Haddi . "A Comparison Between Genetic Algorithm and Practical Swarm to Investigate the Reliability Allocation of Complex Network." Journal of Physics: Conference Series. Vol. 1818. No. 1. IOP Publishing, (2021).

[6] Abdullah, Ghazi, and Hassan, Zahir Abdul Haddi. "Mathematical Model of Reliability Optimization Techniques for Networks." Ph. D. Thesis, University of Babylon, (2021).

[7] Abdullah, Ghazi, and Hassan, Zahir Abdul Haddi. "Using of Genetic Algorithm to Evaluate Reliability Allocation and Optimization of Complex Network." IOP Conference Series: Materials Science and Engineering. Vol. 928. No. 4. IOP Publishing, (2020).

[8] Abdullah, Ghazi, and Hassan, Zahir Abdul Haddi. ”Using of particle swarm optimization (PSO) to addressed reliability allocation of complex network.” Journal of Physics: Conference Series. Vol. 1664. No. 1. IOP Publishing, (2020).

[9] Abed, S. A. and Udriste, C., ”Optimization Techniques and Methods in Reliability Allocation”, Ph. D. Thesis, University Politehnica of Bucharest,(2017).

[10] Abed, Saad Abbas, Sulaiman, Hatem Kareem and Hassan, Zahir Abdul Haddi. ”Reliability Allocation and Optimization for (ROSS) of a Spacecraft by using Genetic Algorithm.” Journal of Physics: Conference Series. Vol. 1294. No. 3. IOP Publishing, (2019).

[11] Afsharnia, Fatemeh. ”Failure rate analysis.” Failure Analysis and Prevention. IntechOpen, (2017).

[12] Aggarwal, K. K., and Shashwati Guha. ”Reliability allocation in a general system with non-identical components—A practical approach.” Microelectronics Reliability 33.8 (1993): 1089-1093

[13] Al-Ali A.A. and Subri U.A., ”Evaluation of reliability and mean time to failure of power plant ”, Journal of Babylon university, Vol. 7, No. 3 , Iraq,(2002).

[14] Alghamdi, Safar M., and David F. Percy. ”Reliability equivalence factors for a series–parallel system of components with exponentiated Weibull lifetimes.” IMA Journal of Management Mathematics 28.3 (2017): 339-358.

[15] Al-Khateeb, Wajdi, Sufyan Al-Irhayim, and Khalid Al-Khateeb. ”Reliability evaluation of scalable complex networks through delta-star conversion.” 2008 International Conference on Computer and Communication Engineering. IEEE, (2008).

[16] Allan, Ronald N., Roy Billinton, and Mauricio Figueiredo De Oliveira. ”An efficient algorithm for deducing the minimal cuts and reliability indices of a general network configuration.” IEEE Transactions on Reliability 25.4 (1976): 226-233.

[17] Altiparmak, Fulya, Berna Dengiz, and Alice E. Smith. ”A general neural network model for estimating telecommunications network reliability.” IEEE transactions on reliability 58.1 (2009): 2-9.

[18] Ausiello, Giorgio, Crescenzi P., Gombosi G., Kann V., and Protasi M. ”Complexity and approximation: Combinatorial optimization problems and their approximability properties.” Springer Science Business Media, (2012).

[19] Bazaraa, Mokhtar, S., Hanif, D., Sherali, and Chitharanjan, M., Shetty. ”Nonlinear programming: theory and algorithms.” John Wiley Sons, (2013).

[20] Belegundu, Ashok D., and Tirupathi R., Chandrupatla. ”Optimization concepts and applications in engineering.” Cambridge University Press, (2019).

[21] Boyd, Stephen, Stephen, P. Boyd, and Lieven Vandenberghe. ”Convex Optimization.” Cambridge university press, (2004).

[22] Boesch, Francis T., Appajosyula Satyanarayana, and Charles L. Suffel. ”A survey of some network reliability analysis and synthesis results.” Networks: An International Journal 54.2 (2009): 99-107.

[23] Bozorg-Haddad, O. (Ed.). ”Advanced optimization by nature-inspired algorithms.” Singapore: Springer, (2018).

[24] Catelani, Marcantonio, Ciani, Lorenzo, Patrizi, Gabriele and Venzi, Matteo. ”Reliability allocation procedures in complex redundant systems.” IEEE Systems Journal 12.2 (2017): 1182-1192.

[25] Chaturvedi, Sanjay Kumar. ”Network reliability: measures and evaluation”. John Wiley Sons, 2016.

[26] Chen, Wai-Kai. ”Graph theory and its engineering applications.” Vol. 5. World Scientific, (1997).

[27] Coppola, Anthony. ”Reliability engineering of electronic equipment a historical perspective.” IEEE Transactions on Reliability 33.1 (1984): 29-35.

[28] Cruz, Jose A. "Applicability and limitations of reliability allocation methods." (2016).

[29] Davis, D.J., "An analysis of some failure data, J. Am. Statist. Assoc.", 47, 113-150, (1952).

[30] De Carlo, Filippo. "Reliability and maintainability in operations management." Operations Management 1 (2013): 32.

[31] De Felice, Fabio, et al. "Proposal of a new reliability allocation methodology: the Integrated Factors Method." International Journal of Operations and Quantitative Management 16.1 (2010): 67-85.

[32] Dhillon, B.S., "Reliability and Quality Control: Bibliography on General and Specialized Areas.", Beta Publishers, Gloucester, Ontario, (1992).

[33] Dhillon, B.S., "Reliability Engineering Application: Bibliography on Important Application Areas.", Beta Publishers, Gloucester, Ontario, (1992).

[34] Dhillon, Balbir, S. "Design reliability: fundamentals and applications." CRC press, (1999).

[35] Dhillon, Balbir, S., and Subramanyam N. Rayapati. "Department of Mechanical Engineering." University of Ottawa, Canada,"Design Reliability: Fundamentals and Applications (1981).

[36] El-Damcese, Medhat, Faheem Abbas, and Eman El-Ghamry. "Reliability and cost analysis of a series system model using fuzzy parametric geometric programming." (2014).

[37] Elegbede, A.O.C., Chu, Chengbin, Adjallah, K.H., and Yalaoui, F. "Reliability allocation through cost minimization." IEEE Transactions on reliability 52.1 (2003): 106-111.

[38] Feng, Xinyu, Zhu, Xijing Zhao, Wei, and Li, Xiangmeng. "Reliability of Electric Vehicle with Wind Turbine Based on Particle Swarm Optimization." Chemical Engineering Transactions 66 (2018): 1291-1296.

[39] Fine, Terrence L. ”Theories of probability: An examination of foundations.” Academic press, (2014).

[40] Gao, Zheng-Ming, and Juan Zhao. ”An improved grey wolf optimization algorithm with variable weights.” Computational Intelligence and Neuroscience 2019 (2019).

[41] Garg, Harish. ”An approach for solving constrained reliability-redundancy allocation problems using cuckoo search algorithm.” Beni-Suef University Journal of Basic and Applied Sciences 4.1 (2015): 14-25.

[42] Grodzevich, Oleg, and Oleksandr Romanko. ”Normalization and other topics in multi-objective optimization.” (2006).

[43] Hassan, Zahir Abdul Haddi, and V. Balan. ”Reliability extrema of a complex circuit on bi-variate slice classes.” Karbala International Journal of Modern Science 1.1 (2015): 1-8.

[44] Hassan, Zahir Abdul Haddi, and Constantin Udrişte. ”Equivalent reliability polynomials modeling EAS and their geometries.” Annals of West University of Timisoara-Mathematics and Computer Science 53.1 (2015): 177-195.

[45] Hassan, Zahir Abdul Haddi and Emad Kareem Muter. ”Geometry of reliability models of electrical system used inside spacecraft.” 2017 Second Al-Sadiq International Conference on Multidisciplinary in IT and Communication Science and Applications (AIC-MITCSA). IEEE, (2017).

[46] Hassan, Zahir Abdul Haddi and Emad Kareem Mutar. ”Evaluation the reliability of a high-pressure oxygen supply system for a spacecraft by using GPD method.” Al-Mustansiriyah Journal of college of education, vol. special 2 (2017): 993-1004.

[47] Hilber, Patrik, and Lina Bertling. ”Component reliability importance indices for electrical networks.” 2007 International Power Engineering Conference (IPEC 2007). IEEE, (2007).

[48] Horowitz, Ellis, Sartaj Sahni, and Sanguthevar Rajasekaran. Computer algorithms C++: C++ and pseudocode versions. Macmillan, (1997).

[49] Khorramzadeh, Yasamin. "Network Reliability: Theory, Estimation, and Applications." Diss. Virginia Tech, (2015).

[50] Kim, Y. H., Case, K. E. and Ghare, P., "A method for computing complex systemreliability." IEEE Transactions on Reliability,vol. 21.4 (1972), pp. 215-219.

[51] Kumar, A., Pant, S., Ram, M., and Singh, S.B. "On solving complex reliability optimization problem using multi-objective particle swarm optimization." Mathematics Applied to Engineering. Academic Press, (2017). 115-131.

[52] Limnios, N. "Failure rate of non-repairable systems." Reliability Engineering 17.2 (1987): 83-88.

[53] Liu, Chuanquan, and Yan Zhang. "Distribution network reliability considering distribution generation." Automation of Electric power systems 31.22 (2007): 46-49.

[54] Liu, Hai-Lin, Fangqing Gu, and Qingfu Zhang. "Decomposition of a multiobjective optimization problem into a number of simple multiobjective subproblems." IEEE transactions on evolutionary computation 18.3 (2013): 450-455.

[55] Liu, Wenting, Q., Wan, L. and Wang, C. "A comprehensive method of apportioning reliability goals for new product of hydraulic excavator." Mathematical Problems in Engineering 2019 (2019)

[56] Mirjalili, Seyedali, and Jin Song Dong. "Multi-objective grey wolf optimizer." Multi-Objective Optimization using Artificial Intelligence Techniques. Springer, Cham, (2020). 47-58.

[57] Misra, Krishna B. Reliability analysis and prediction: A methodology oriented treatment. Elsevier, 2012.

[58] Mood, Alexander McFarlane. "Introduction to the Theory of Statistics." (1950).

[59] Moore, Edward F., and Claude E. Shannon. "Reliable circuits using less reliable relays." Journal of the Franklin Institute 262.3 (1956): 191-208.

[60] Mutar, E. K. and Hassan, Zahir Abdul Haddi., "On the Geometry of the Reliability Polynomials.",Thesis, University of Babylon, (2017).

[61] Nahman, J. M. "Minimal paths and cuts of networks exposed to common-cause failures." IEEE Transactions on Reliability 41.1 (1992): 76-80.

[62] Nelson, Randolph. "Probability, stochastic processes, and queueing theory: the mathematics of computer performance modeling." Springer Science Business Media, (2013).

[63] Ouyang, Zhiyuan, Liu, Y., Ruan, SJ, and Jiang, T. "An improved particle swarm optimization algorithm for reliability-redundancy allocation problem with mixed redundancy strategy and heterogeneous components." Reliability Engineering System Safety 181 (2019): 62-74.

[64] Padmavathy, N., and Sanjay K. Chaturvedi. "Evaluation of mobile ad hoc network reliability using propagation-based link reliability model." Reliability Engineering System Safety 115 (2013): 1-9.

[65] Pérez-Rosés, Hebert. "Sixty years of network reliability." Mathematics in Computer Science 12.3 (2018): 275-293.

[66] Pham, Hoang, ed. Springer handbook of engineering statistics. Springer Science Business Media, (2006).

[67] Rezaei, Hossein, Omid Bozorg-Haddad, and Xuefeng Chu. "Grey wolf optimization (GWO) algorithm." Advanced Optimization by Nature-Inspired Algorithms. Springer, Singapore, (2018): 81-91.

[68] Sahoo, Laxminarayan. "Studies on reliability optimization problems by genetic algorithm." (2012).

[69] Schottl, Alfred. "A reliability model of a system with dependent components." IEEE transactions on reliability 45.2 (1996): 267-271.

[70] Shaghaghi, Saba, Bonakdari, H., Gholami A., Ebtehaj, I., and Zeinolabedini, M. "Comparative analysis of GMDH neural network based on genetic algorithm and particle swarm optimization in stable channel design." Applied Mathematics and Computation 313 (2017): 271-286.

[71] Shier, Douglas R. "Network reliability and algebraic structures." Clarendon Press, (1991).

[72] Subrie U.A., "Some reliability subject to corrective maintenance ", M. Sc., University of Babylon , Iraq, (1985).

[73] Subrie U.A., and Al-Swidi, L.A. "Soft Fuzzy Reliability Topological Space with Applications." Ph. D. Thesis, University of Babylon, (2019).

[74] Sulaiman,Hatem Kareem and Hassan,Zahir Abdul Haddi. "A Study of Mathematical Models in Reliability of Networks", Thesis, University of Babylon, (2019).

[75] Sun, Wenyu, and Ya-Xiang Yuan. "Optimization theory and methods: nonlinear programming." Vol. 1. Springer Science Business Media, (2006).

[76] Sunar, Mehmet, and Ramazan Kahraman. "A comparative study of multiobjective optimization methods in structural design." Turkish Journal of Engineering and Environmental Sciences 25.2 (2001): 69-78.

[77] Teng, Zhi-jun, Jin-ling Lv, and Li-wen Guo. "An improved hybrid grey wolf optimization algorithm." Soft computing 23.15 (2019): 6617-6631.

[78] Van Sylke, R., and H. Frank. "Network reliability analysis: Part I." Networks 1 (1972): 279-290.

[79] Vasuki, A. Nature-Inspired Optimization Algorithms. CRC Press, (2020).

[80] Wang, Wendai, James Loman, and Pantelis Vassiliou. "Reliability importance of components in a complex system." Annual Symposium Reliability and Maintainability, 2004-RAMS. IEEE, (2004).

[81] Wilson, Robin J. Introduction to Graph Theory uPDF eBook. Pearson Higher Ed, (2015).

[82] Wu, Jun, Duan, DL., and Zhao, J. "Status and prospects on network reliability." Complex Systems and Complexity Science 8.2 (2011): 77-86.

[83] Xiong, Jintao, and Weibo Gong. ”A novel algorithm on network reliability estimation.” Mathematical and computer modelling 41.2-3 (2005): 119-133.

[84] Xue-feng, Liu, and Li Ya-ping. ”Reliability Allocation Through Cost Minimization [J].” Electronics Quality 9 (2008).

[85] Yang, Xin-She, and Xingshi He. ”Bat algorithm: literature review and applications.” International Journal of Bio-inspired computation 5.3 (2013): 141-149.

[86] Yang, Xin-She. Nature-inspired optimization algorithms. Academic Press, (2020).

[87] Yeh, Wei-Chang, Lin, Y., Chung, Y. Y., and Chih, M. ”A particle swarm optimization approach based on Monte Carlo simulation for solving the complex network reliability problem.” IEEE Transactions on Reliability 59.1 (2010): 212-221.

[88] Yousefi, Samuel, Alizadeh, A., and Hayati, J. ”HSE risk prioritization using robust DEA-FMEA approach with undesirable outputs: a study of automotive parts industry in Iran.” Safety science 102 (2018): 144-158.

Appendix

- **Bat Algorithm**

```
clc
clearvars
clf reset
close all
tic
Rg=0.9;
RG=Rg;
RNo=7;
ep=0.001;
cw=0.01;
rau=0.45:ep:0.95;
n=length(rau);
ps=n;
it=n;
AA=0.1;BB=0.2;
A=AA+(BB-AA)*rand(1,RNo);
B=AA+(BB-AA)*rand(1,RNo);
 Ris-p=zeros(RNo+1,ps);
 Costis-p=Ris_p;
Rau=ones(RNo,n);
NewRau=Rau;
xx=Rau;
x=ones(ps,it);
xv=x;
f=cw+(1-cw)*rand(ps,it);
f_min = min(f);
```

$\mathrm{f}_{max} = max(f)$;

v=x;

ald=x;

Rm=Rau;

sst=Rau;

sol=Rau;

R=ones(RNo,1);

S=ones(n,1);

ci=Rau;

cost=S';

RN=S;

cr=S';

$\mathrm{BEST}_{C}OST = ones(1, ps) * inf$;

$\mathrm{BEST}_{C}i = ones(RNo, ps) * inf$;

$\mathrm{BEST}_{C} = BEST_{C}OST$;

$\mathrm{BEST}_{R}N = BEST_{C}OST$;

$\mathrm{BEST}_{R}i = BEST_{C}i$;

$\mathrm{MIN}_{C}OST = inf$;

$\mathrm{BAT}_{t}ime = toc$;

for i=1:RNo

Rau(i,:)=rau;

end

for i=1:RNo

for j=1:n

ri=randperm(RNo);

r=ri(1);

sol(i,j)=r;

Rm(i,j)=rau(r);

end

end

for pop=1:ps; tic; for j=1:n; MRi1=Rm(:,j); RN(j)=Rnf(Rm(:,j));

if RN(j)¡ RG

state=0;

while state==0

rss = 0.45 + (0.95 -0.45)*rand(RNo,1);

MRi1=rss;

RN(j)=Rnf(rss);

if RN(j) ¡ RG

state=0;

else

state=1;

Rm(:,j)=rss;

end

end

end

[ci(:,j),cost(j)]=S-LOG(Rm(:,j),A);

cr(j)=10*(1-RN(j))+cw*cost(j);

ald(pop,j)=(1-f(pop,j))/(pop*j);

if cr(j) ¡ BEST$_COST(pop) + ald(i, j)$

BEST$_COST(pop) = cr(j)$;

BEST$_C(pop) = cost(j)$;

BEST$_RN(pop) = RN(j)$;

BEST$_Ci(:, pop) = ci(:, j)$;

BEST$_Ri(:, pop) = Rm(:, j)$;

js=j;

Ris$_p(:, pop) = [BEST_Ri(:, pop); BEST_RN(pop)]$;

$\text{Costis}_p(:, pop) = [BEST_C i(:, pop); BEST_C(pop)]$;

RN: the polynomial in eqution 3.3.

end

- **Grey Wolf Algorithm**

```
clc
clearvars
clf reset
close all
tic
Rg=0.9;
RG=Rg;
RNo=7;
ep=0.001;
cw=0.01;
rau=linspace(0.45,0.95,3^7);
n=length(rau);
ITE=500;
it=n;
W=10;
A=ones(1,RNo)*0.6;
B=ones(1,RNo)*0.1;
Ris-p=zeros(RNo+1,ITE);
Costis-p=Ris-p;
Rau=ones(RNo,n);
New-Rau=Rau;
Rm=Rau;
sst=Rau;
```

```
sol=Rau;
S=ones(n,1);
ci=Rau;
cost=S';
RN=S;
cr=S';
BEST-COST=ones(1,ITE)*inf;
BEST-Ci=ones(RNo,ITE)*inf;
BEST-C=BEST_COST;
BEST-RN=BEST-COST;
BEST-Ri=BEST-Ci;
MIN-COST=inf;
GW-time=toc;
for i=1:RNo
Rau(i,:)=rau;
end
for i=1:RNo
for j=1:n
ri=randperm(RNo);
r=ri(1);
sol(i,j)=r;
Rm(i,j)=rau(r);
end
end
——————(GW Loops) —————-
for pop=1:ITE
tic
for j=1:n
```

```
MRi1=Rm(:,j);
RN(j)=Rnf(Rm(:,j));
if RN(j)¡ RG —— RN(j)==1
state=0;
while state==0
rss = 0.45 + (0.95-0.45)*rand(RNo,1);
MRi1=rss;
RN(j)=Rnf(rss);
if RN(j) ¡ RG —— RN(j)==1
state=0;
else
state=1;
Rm(:,j)=rss;
end
end
end
cr(j)=W*(1-RN(j))+cw*cost(j);
if cr(j) ¡ BEST-COST(pop)
BEST-COST(pop)=cr(j);
BEST-C(pop)=cost(j);
BEST-RN(pop)=RN(j);
BEST-Ci(:,pop)=ci(:,j);
BEST-Ri(:,pop)=Rm(:,j);
js=j;
Costis-p(:,pop)=[BEST-Ci(:,pop);BEST-C(pop)];
end
end
if BEST-COST(pop)¡ MIN-COST
```

```
MIN-COST=BEST-COST(pop);
Ri=Ris-p(:,pop);
Ci=Costis-p(:,pop);
km=pop;
end
C=[ci;cr];
R=[Rm;RN'];
C=C(:,ind);
R=R(:,ind);
R1=R(1:RNo,1);
R2=R(1:RNo,2);
R3=R(1:RNo,3);
RR=[R1,R2,R3];
m1=3^7;
RR2=zeros(RNo,m1);
m=0;
for i1=1:3
New-R(1)=RR(i1);
for i2=1:3
New-R(2)=RR(i2);
for i3=1:3
New-R(3)=RR(i3);
for i4=1:3
New-R(4)=RR(i4);
for i5=1:3
New-R(5)=RR(i5);
for i6=1:3
New-R(6)=RR(i6);
```

```
for i7=1:3
New-R(7)=RR(i7);
m=m+1;
RR2(:,m)=New-R;
end
end
end
end
end
end
end
Rm=RR2;
N1=ceil(m/4);
N=m-N1;
for jj=N+1:n
rss = 0.45 + (0.95-0.45)*rand(RNo,1);
Rm(:,jj)=rss;
end
GWT=toc;
GW-time=GW-time+GWT;
if GW-time¿=500
break
end
end
b1=figure;
bar((Ri(1:RNo)'));
hold on
bar(RNo+1,Ri(RNo+1),'r',RG)
```

```
legend('Ri','RN','location','north')
title([' Ri and RN:GWO with exp cost function: iteration Number=',num2str(ITE)]);
print('GWO Ri and RN for F1','-dpng')
hold off
b2=figure;
bar(Ci(1:RNo)');
hold on
bar(RNo+1,Ci(RNo+1),'r');
legend('Ci','Cost','location','northwest')
title([' Ci and CN: GWO with exp cost function: iteration Number=',num2str(ITE)]);
pname2=' Ci and CN:GWO with exp cost function:iteration Number='; print('GWO Ci
and CN for F1','-djpeg');
hold off
```

filename=['GWO$_(F1)_(ITERATIONs) =', num2str(ITE),' .xlsx'$];

```
LastName = 'R1';'R2';'R3';'R4';'R5';'R6';'R7';'RN';
sol1=[Ri,Ci];
T1=table(Ri,Ci,'RowNames',LastName)
writetable(T1,filename);
```

All$_Ri = Ris_p$;

All$_Ci = Costis_p$;

T3=table(All$_Ri, All_Ci,' RowNames', LastName$);

sol2=[Ris$_p; NaN * ones(1, pop); Costis_p(RNo + 1, :); Costis_p(1 : RNo, :$

$)]; writematrix(sol2, filename,' sheet', 2)$;

```
function [c,Sc]=S-F1(R,A,B)
n=length(R);
c=R;
Sc=0;
for i=1:n
```

```
c(i)=A(i)*exp((B(i))/(1-R(i)));
Sc=Sc+c(i);
end
end
function RN=Rnf(R)
R1=R(1);
R2=R(2);
R3=R(3);
R4=R(4);
R5=R(5);
R6=R(6);
R7=R(7);
RN: the polynomial in eqution 3.3.
end
```

المستخلص

في هذه الأطروحة تمت دراسة النماذج الرياضية في موثوقية الشبكات وكذلك استخدام بعض الأساليب المعروفة لحساب موثوقية الشبكات المعقدة كتطبيق لنظرية البيان ونظرية الاحتمالات. سنناقش أيضًا المواضيع المهمة التي تشمل: استخدام طريقة رياضية لحساب المسارات القصيرة ومجموعات القطع الأدنى للشبكة اعتمادًا على جبر المصفوفات ، واستخدام بعض الطرق للحصول على كثيرة حدود الموثوقية لشبكة ، بما في ذلك: طريقة المسارات القصيرة ، وطريقة مجموعات القطع الأدنى ، الاختزال إلى العناصر المتوازية وطريقة تحويل delta -star. بالإضافة إلى استخدام بعض التقنيات لحساب تخصيص موثوقية الشبكة المختصرة مثل تقنية التخصيص الأسي وطريقة نهج شركة راديو للطيران (ARINC). تضمنت هذه الأطروحة حساب أهمية جميع مكونات النظام لمعرفة تأثير كل هذه المكونات على عمل النظام (الشبكة) ككل. تمت دراسة موضوع زيادة موثوقية الشبكة بناءً على أهمية كل وحدة من وحدات النظام (الشبكة) ، من خلال حصول أهم الوحدات في الشبكة على أفضل تخصيص من زيادة الموثوقية عن غيرها التي تليها الأهمية . أخيرًا ، تمت دراسة تخصيص الموثوقية الأمثل لشبكة معقدة باستخدام خوارزمية الخفاش وخوارزمية الذئب الرمادي وذلك بالاعتماد على خمس دوال كلفة مع هذه الخوارزميتين وهذه الدوال هي الدالة اللوغاريتمية والدالة الأسية والأسية مع عامل الجدوى ودالة القوى ودالة الظل ، مع المقارنة بين هذه الخوارزميتين لاختيار أفضل خوارزمية تعطي أعلى موثوقية وأقل كلفة.

جمهورية العراق
وزارة التعليم العالي والبحث العلمي
جامعة بابل
كلية التربية للعلوم الصرفة
قسم الرياضيات

حول بعض الأساليب لتحسين موثوقية الشبكات المعقدة

أطروحة مقدمة الى مجلس كلية التربية للعلوم الصرفة في جامعة بابل
كجزء من متطلبات نيل درجة الدكتوراه فلسفة في التربية / الرياضيات

من قبل
فؤاد حمزة عبد ناصر
بإشراف
أ. د زاهر عبد الهادي حسن

1443 هـ 2022 م