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وزارة التعليم العالي والبحث العلمي

جامعة بابل

كلية التربية

إجراءات بيزينية بمرحلتين لأختيار أفضل مجتمع من بين مجتمعين ثنائي الحدين

من قبل

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رسالة ماجستير مقدمة إلى مجلس كلية التربية – جامعة بابل كجزء من متطلبات درجة الماجستير علوم في الرياضيات

بـــإشراف

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١٤٢٨ ٢٠٠٧

ألخلاصه

في هذه الرسالة , اقترحنا إجراءات بيزنية ذات مرحلتين لاختيار أفضل مجتمع من بين مجتمعين ثنائي الحدين . تم اعتماد نهجين لتحقيق هذا الهدف . طبقاً للنهج الأول استخدمت دوال خسارة مختلفة مع توزيع بيتا القبلي لبناء إجراء بيزني بمرحلتين وتم تقيم انجاز هذا الإجراء باستخدام خطورة بايز . كذلك أجريت مقارنات بين هذا الإجراء وإجراءات حجم العينة الثابتة البيزنية . وطبقاً للنهج الثاني تم تنفيذ محاكاة مونتي كارلو لدراسة إجراءات تعتمد على طرق معاينة مختلفة . وتم تقيم انجاز هذه الطرق باستخدام مقاييس الإنجاز مثل احتمال الاختيار الصحيح للمجتمع الأفضل والعدد المتوقع للنجاحات . تم عرض بعض المناقشات والملاحظات الختامية واخيراً تضمنت الرسالة بعض المقترحات والعمل المستقبلي .

Republic of Iraq

Ministry of Higher Education

and Scientific Research

University of Babylon

College of Education

Bayes Two – Stage Procedures for Selecting the Better of Two Binomial Populations

By Asa'ad Nassir Hussein AL-Mizidawy

Master's Thesis Submitted to the Council of College of Education , University of Babylon in Partial

Fulfillment of the Requirements for

the Degree of Master of Science

in Mathematic Supervised by

Dr. Saad Abed Madhi

٢٠٠٧ ١٤٢٨

بسم الله الرحمن الرحيم

((قَالُواْ سُبحَانَكَ لاَ عِلمَ لَنَا إِلاَّ ما عَلَّمتَنَآ إِنَّكَ أَنتَ اُلعَلِيمُ اُلحَكِيمُ))

صدق الله العلي العظيم

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We certify that we have read this thesis entitled " **Bayes Two – Stage Procedures for Selecting the Better of Two Binomial Populations** " and , as an Examining Committee , We examined the student in its content , and what is related to it , and that in our opinion it is adequate with standing as a thesis for the degree of Master of Science in Mathematics .

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Date :

SUPERVISOR CERTIFICATION

I certify that this thesis was prepared under my supervision at the Department of Mathematics / College of Education / University of Babylon , as a partial fulfillment of the requirements for the degree of Master of Science in Mathematics .

Signature :

Name : Dr. Saad Abed Madhi

Date : / / ٢٠٠٧

RECOMMENDATION OF THE HEAD OF THE DEPARTMENT

In view of the available recommendations , I forward this research for debate by the Examining Committee .

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Name : Dr. Iftichar Mudhar Talib

Date : / / ٢٠٠٧

Appendix A

Programs for two-stage procedure

This program computes the posterior expected losses of decision $d_{١}$ and decision $d_{٢}$ for TSTAGE-D procedure under constant loss function .

****

```
INTEGER RD١,RD٢,R١١,R٢١,R١٢,R٢٢
 DIMENSION XS(١٠٠٠),RS(١٠٠٠)
 READ*,N,N١,N٢
 READ*,ND١,ND٢,RD١,RD٢
 READ*,C١,C٢
 JJ=٠
 DO ٢ II١=١,N١+١
 N١١=II١-١
 N٢١=N١-N١١
 SUM٤=٠.٠
 DO ١٠ II٢=١,N١١+١
 R١١=II٢-١
 DO ١٥ II٣=١,N٢١+١
 R٢١=II٣-١
 K١=IF(N١١)
 K٢=IF(R١١)
 K٣=IF(N١١-R١١)
```

```
Y١=K١/FLOAT(K٢*K٣)
K٤=IF(N٢١)
K٥=IF(R٢١)
K٦=IF(N٢١-R٢١)
Y٢=K٤/FLOAT(K٥*K٦)
K٧=IF(R١١+RD١-١)
K٨=IF(N١١-R١١+ND١-RD١-١)
K٩=IF(N١١+ND١-١)
Y٣=(K٧*K٨)/FLOAT(K٩)
K١٠=IF(R٢١+RD٢-١)
K١١=IF(N٢١-R٢١+ND٢-RD٢-١)

K١٢=IF(N٢١+ND٢-١)
Y٤=(K١٠*K١١)/FLOAT(K١٢)
KK=٠
DO ٤ II٤=١,N٢+١
N١٢=II٤-١
N٢٢=N٢-N١٢
SUM٥=٠.٠
DO ٢٠ II٥=١,N١٢+١
R١٢=II٥-١
DO ٢٥ II٦=١,N٢٢+١
R٢٢=II٦-١
```

```
K١٣=IF(N١٢)
K١٤=IF(R٢١)
K١٥=IF(N١٢-R١٢)
Y٥=K١٣/FLOAT(K١٤*K١٥)
K١٦=IF(N٢٢)
K١٧=IF(R٢٢)
K١٨=IF(N٢٢-R٢٢)
Y٦=K١٦/FLOAT(K١٧*K١٨)
K١٩=IF(R١١+R١٢+RD١-١)
K٢٠=IF(N١١-R١١+N١٢-R١٢+ND١-RD١-١)
K٢١=IF(N١١+N١٢+ND١-١)
Y٧=(K١٩*K٢٠)/FLOAT(K٢١)
K٢٢=IF(R٢١+R٢٢+RD٢-١)
K٢٣=IF(N٢١-R٢١+N٢٢-R٢٢+ND٢-RD٢-١)
K٢٤=IF(N٢١+N٢٢+ND٢-١)
Y٨=(K٢٢*K٢٣)/FLOAT(K٢٤)
SUM١=٠.٠
DO ٣٠ J=R٢١+R٢٢+RD٢,N٢١+N٢٢+ND٢-١
K٢٥=IF(R١١+R١٢+RD١+J-١)
K٢٦=IF(N١١-R١١+N١٢-R١٢+ND١-RD١+N٢١+N٢٢+ND٢-J-٢)
K٢٧=IF(J)
K٢٨=IF(N٢١+N٢٢+ND٢-J-١)
YI١=(K٢٥*K٢٦)/FLOAT(K٢٧*K٢٨)
```

```
      SUM١=SUM١+YI١
٣٠    CONTINUE
      SUM٢=٠.٠
      DO ٤٠ J١=R١١+R١٢+RD١,N١١+N١٢+ND١-١
      K٢٩=IF(R٢١+R٢٢+RD٢-١+J١)
      K٣٠=IF(N١١+ND١+N٢١-R٢١+N٢٢-R٢٢+ND٢-RD٢-٢-J١)
      K٣١=IF(J١)
      K٣٢=IF(N١١+N١٢+ND١-J١-١)
      YI٢=(K٢٩*K٣٠)/FLOAT(K٣١*K٣٢)
      SUM٢=SUM٢+YI٢
٤٠    CONTINUE
      K٣٣=IF(N١١+N١٢+ND١+N٢١+N٢٢+ND٢-٢)
      YJ١=(C١*K٢١*K٢٤)/FLOAT(ABS(K١٩*K٢٠*K٣٣))
      YJ٢=(C٢*K٢١*K٢٤)/FLOAT(ABS(K٢٢*K٢٣*K٣٣))
      S١=YJ١*SUM١
      S٢=YJ٢*SUM٢
      S=S١
      IF(S٢.LT.S)THEN
      S=S٢
      ENDIF
      K٣٤=IF(R١١+RD١-١)
      K٣٥=IF(N١١-R١١+ND١-RD١-١)
```

```
      K٣٦=IF(N١١+ND١-١)
      YJ٣=(K٣٤*K٣٥)/FLOAT(K٣٦)
      K٣٧=IF(R٢٢+RD٢-١)
      K٣٨=IF(N٢١-R٢١+ND٢-RD٢-١)
      K٣٩=IF(N٢١+ND٢-١)
      YJ٤=(K٣٧*K٣٨)/FLOAT(K٣٩)
      YK=(Y٥*Y٦*Y٧*Y٨)/(YJ٣*YJ٤)
      SUM٥=SUM٥+YK*S
٢٥    CONTINUE
٢٠    CONTINUE
     R١=SUM٥
      KK=KK+١

      RS(KK)=R١
٤     CONTINUE
     RMIN١=RS(١)
      DO ٦٠ IR=٢,KK
      IF(RS(IR).LT.RMIN١)THEN
      RMIN١=RS(IR)
      ENDIF
٦٠    CONTINUE
     YK١=Y١*Y٢*Y٣*Y٤*RMIN١
     SUM٤=SUM٤+YK١
```

```
١٥      CONTINUE
١٠      CONTINUE
       K٤٠=IF(RD١-١)
        K٤١=IF(ND١-RD١-١)
        K٤٢=IF(ND١-١)
        YKI=(K٤٠*K٤١)/FLOAT(K٤٢)
        K٤٣=IF(RD٢-١)
        K٤٤=IF(ND٢-RD٢-١)
        K٤٥=IF(ND٢-١)
        YKJ=(K٤٣*K٤٤)/FLOAT(K٤٥)
        R٢=SUM٤/(YKI*YKJ)
        JJ=JJ+١
        XS(JJ)=R٢
٢       CONTINUE
       RMIN٢=XS(١)
        DO ٦٥ IJ=٢,JJ
        IF(XS(IJ).LT.RMIN٢)THEN
        RMIN٢=XS(IJ)
        ENDIF
٦٥      CONTINUE
       PRINT*,'RMIN٢=',RMIN٢
        END
        FUNCTION IF(IH)
```

```
      IF=١

      DO ٥٥ K=١,IH
      IF=IF*K
٥٥    CONTINUE
      END
```

This program computes the posterior expected losses of decision d_1 and decision d_2 for TSTAGE-D procedure under linear loss function .

****

```
INTEGER RD١,RD٢,R١١,R٢١,R١٢,R٢٢
  DIMENSION XS(١٠٠٠),RS(١٠٠٠)
  READ*,N,N١,N٢
  READ*,ND١,ND٢,RD١,RD٢
  READ*,C١,C٢
JJ=٠
  DO ٢ II١=١,N١+١
  N١١=II١-١
  N٢١=N١-N١١
  SUM٤=٠.٠
  DO ١٠ II٢=١,N١١+١
  R١١=II٢-١
  DO ١٥ II٣=١,N٢١+١
  R٢١=II٣-١
  K١=IF(N١١)
  K٢=IF(R١١)
  K٣=IF(N١١-R١١)
  Y١=K١/FLOAT(K٢*K٣)
  K٤=IF(N٢١)
  K٥=IF(R٢١)
  K٦=IF(N٢١-R٢١)
  Y٢=K٤/FLOAT(K٥*K٦)
```

```
   K٧=IF(R١١+RD١-١)
   K٨=IF(N١١-R١١+ND١-RD١-١)
   K٩=IF(N١١+ND١-١)
   Y٣=(K٧*K٨)/FLOAT(K٩)
   K١٠=IF(R٢١+RD٢-١)
   K١١=IF(N٢١-R٢١+ND٢-RD٢-١)
   K١٢=IF(N٢١+ND٢-١)
   Y٤=(K١٠*K١١)/FLOAT(K١٢)

KK=٠
DO ٤ II٤=١,N٢+١
N١٢=II٤-١
   N٢١=N٢-N١٢
   SUM٥=٠.٠
   DO ٢٠ II٥=١,N١٢+١
   R١٢=II٥-١
   DO ٢٥ II٦=١,N٢٢+١
   R٢٢=II٦-١
   K١٣=IF(N١٢)
   K١٤=IF(R١٢)
   K١٥=IF(N١٢-R١٢)
   Y٥=K١٣/FLOAT(K١٤*K١٥)
   K١٦=IF(N٢٢)
```

```
      K١٧=IF(R٢٢)
      K١٨=IF(N٢٢-R٢٢)
      Y٦=K١٦/FLOAT(K١٧*K١٨)
     K١٩=IF(R١١+R١٢+RD١-١)
      K٢٠=IF(N١١-R١١+N١٢-R١٢+ND١-RD١-١)
      K٢١=IF(N١١+N١٢+ND١-١)
      Y٧=(K١٩*K٢٠)/FLOAT(K٢١)
      K٢٢=IF(R٢١+R٢٢+RD٢-١)
      K٢٣=IF(N٢١-R٢١+N٢٢-R٢٢+ND٢-RD٢-١)
      K٢٤=IF(N٢١+N٢٢+ND٢-١)
      Y٨=(K٢٢*K٢٣)/FLOAT(K٢٤)
      SUM١=٠.٠
      DO ٣٠ J=R٢١+R٢٢+RD٢,N٢١+N٢٢+ND٢-١
      K٢٥=IF(R١١+R١٢+RD١+J)
      K٢٦=IF(N١١-R١١+N١٢-R١٢+ND١-RD١+N٢١+N٢٢+ND٢-J-٢)
      K٢٧=IF(J+١)
      K٢٨=IF(N٢١+N٢٢+ND٢-J-١)
     JL١=R٢١+R٢٢+RD٢-J-١
      JK١ =ABS(JL١)
     YI١=(K٢٥*K٢٦*JK١)/FLOAT(K٢٧*K٢٨)


      SUM١=SUM١+YI١
٣٠    CONTINUE
```

```
SUM٢=٠.٠
  DO ٤٠ J١=R١١+R١٢+RD١,N١١+N١٢+ND١-١
  K٢٩=IF(R٢١+R٢٢+RD٢+J١)
  K٣٠=IF(N١١+N١٢+ND١+N٢١-R٢١+N٢٢-R٢٢+ND٢-RD٢-٢-J١)
  K٣١=IF(J١+١)
  K٣٢=IF(N١١+N١٢+ND١-J١-١)
  JL٢=R١١+R١٢+RD١-J١-١
JK٢=ABS(JL٢)
YI٢=(K٢٩*K٣٠*JK٢)/FLOAT(K٣١*K٣٢)
  SUM٢=SUM٢+YI٢
٤٠      CONTINUE
  K٣٣=IF(N١١+N١٢+ND١+N٢١+N٢٢+ND٢-١)
  YJ١=(C١*K٢١*K٢٤)/FLOAT(ABS(K١٩*K٢٠*K٣٣))
 YJ٢=(C٢*K٢١*K٢٤)/FLOAT(ABS(K٢٢*K٢٣*K٣٣))
S١=YJ١*SUM١
  S٢=YJ٢*SUM٢
  S=S١
  IF(S٢.LT.S)THEN
  S=S٢
  ENDIF
  K٣٤=IF(R١١+RD١-١)
  K٣٥=IF(N١١-R١١+ND١-RD١-١)
  K٣٦=IF(N١١+ND١-١)
```

```
      YJ٣=(K٣٤*K٣٥)/FLOAT(K٣٦)
      K٣٧=IF(R٢٢+RD٢-١)
      K٣٨=IF(N٢١-R٢١+ND٢-RD٢-١)
      K٣٩=IF(N٢١+ND٢-١)
      YJ٤=(K٣٧*K٣٨)/FLOAT(K٣٩)
      YK=(Y٥*Y٦*Y٧*Y٨)/(YJ٣*YJ٤)
      SUM٥=SUM٥+YK*S
٢٥    CONTINUE
٢٠    CONTINUE

     R١=SUM٥
      KK=KK+١
      RS(KK)=R١
٤     CONTINUE
     RMIN١=RS(١)
      DO ٦٠ IR=٢,KK
      IF(RS(IR).LT.RMIN١)THEN
      RMIN١=RS(IR)
      ENDIF
٦٠    CONTINUE
     YK١=Y١*Y٢*Y٣*Y٤*RMIN١
     SUM٤=SUM٤+YK١
١٥    CONTINUE
```

```
١٠      CONTINUE
        K٤٠=IF(RD١-١)
          K٤١=IF(ND١-RD١-١)
          K٤٢=IF(ND١-١)
          YKI=(K٤٠*K٤١)/FLOAT(K٤٢)
          K٤٣=IF(RD٢-١)
          K٤٤=IF(ND٢-RD٢-١)
          K٤٥=IF(ND٢-١)
          YKJ=(K٤٣*K٤٤)/FLOAT(K٤٥)
          R٢=SUM٤/(YKI*YKJ)
          JJ=JJ+١
          XS(JJ)=R٢
٢       CONTINUE
        RMIN٢=XS(١)
          DO ٦٥ IJ=٢,JJ
          IF(XS(IJ).LT.RMIN٢)THEN
          RMIN٢=XS(IJ)
          ENDIF
٦٥      CONTINUE
        PRINT*,'RMIN٢=',RMIN٢
          END


          FUNCTION IF(IH)
```

```
      IF=١
      DO ٥٥ K=١,IH
      IF=IF*K
٥٥    CONTINUE
     END
```

Appendix B

Programs for one-stage procedure

This program computes the posterior expected losses of decision d_1 and decision d_2 for OSTAGE-D procedure under constant loss function .

****

```
     INTEGER RD١,RD٢,R١١,R٢١
      DIMENSION XS(١٠٠٠)
      READ*,N١
      READ*,ND١,ND٢,RD١,RD٢
      READ*,C١,C٢
      JJ=٠
      DO ٢ II١=١,N١+١
      N١١=II١-١
      N٢١=N١-N١١
      SUM٤=٠.٠
      DO ١٠ II٢=١,N١١+١
```

```
R١١=II٢-١

DO ١٥ II٣=١,N٢١+١

R٢١=II٣-١

K١=IF(N١١)

K٢=IF(R١١)

K٣=IF(N١١-R١١)

Y١=K١/FLOAT(K٢*K٣)

K٤=IF(N٢١)

K٥=IF(R٢١)

K٦=IF(N٢١-R٢١)

Y٢=K٤/FLOAT(K٥*K٦)

K٧=IF(R١١+RD١-١)

K٨=IF(N١١-R١١+ND١-RD١-١)

K٩=IF(N١١+ND١-١)

Y٣=(K٧*K٨)/FLOAT(K٩)

K١٠=IF(R٢١+RD٢-١)

K١١=IF(N٢١-R٢١+ND٢-RD٢-١)

K١٢=IF(N٢١+ND٢-١)

Y٤=(K١٠*K١١)/FLOAT(K١٢)

SUM١=٠.٠

DO ٢٠ J=R٢١+RD٢,N٢١+ND٢-١

K١٣=IF(R١١+RD١+J-١)

K١٤=IF(N١١-R١١+ND١-RD١+N٢١+ND٢-J-٢)
```

```
      K١٥=IF(J)
      K١٦=IF(N٢١+ND٢-J-١)
      Y٥=ABS((K١٣*K١٤)/FLOAT(K١٥*K١٦))
      SUM١=SUM١+Y٥
٢٠    CONTINUE
     SUM٢=٠.٠
      DO ٣٠ J١=R١١+RD١,N١١+ND١-١
      K١٧=IF(R٢١+RD٢-١+J١)
      K١٨=IF(N١١+ND١+N٢١-R٢١+ND٢-RD٢-٢-J١)
      K١٩=IF(J١)
      K٢٠=IF(N١١+ND١-J١-١)
      Y٦=ABS((K١٧*K١٨)/FLOAT(K١٩*K٢٠))
      SUM٢=SUM٢+Y٦
٣٠    CONTINUE
      K٢١=IF(N١١+ND١+N٢١+ND٢-٢)
      Y٧=ABS((C١*K٩*K١٢)/FLOAT(ABS(K٧*K٨*K٢١)))
      Y٨=ABS((C٢*K٩*K١٢)/FLOAT(ABS(K١٠*K١١*K٢١)))
      S١=Y٧*SUM١
      S٢=Y٨*SUM٢
      S=S١
      IF(S٢.LT.S)THEN
      S=S٢
      ENDIF
```

```
      YI=Y١*Y٢*Y٣*Y٤*S
      SUM٤=SUM٤+YI
١٥    CONTINUE
١٠    CONTINUE
     K٢٢=IF(RD١-١)
      K٢٣=IF(ND١-RD١-١)
      K٢٤=IF(ND١-١)
      YJ١=(K٢٢*K٢٣)/FLOAT(K٢٤)
      K٢٥=IF(RD٢-١)
      K٢٦=IF(ND٢-RD٢-١)
      K٢٧=IF(ND٢-١)
      YJ٢=(K٢٥*K٢٦)/FLOAT(K٢٧)
      SUM=SUM٤/(YJ١*YJ٢)
      R=SUM
      JJ=JJ+١
      XS(JJ)=R
٢     CONTINUE
     RMIN=XS(١)
      DO ٦٠ IR=٢,JJ
      IF(XS(IR).LT.RMIN)THEN
      RMIN=XS(IR)
      ENDIF
٦٠    CONTINUE
```

```
      PRINT*,'RMIN=',RMIN
      END
      FUNCTION IF(IH)
      IF=١
      DO ٥٥ K=١,IH
      IF=IF*K
٥٥    CONTINUE
      END
```

This program computes the posterior expected losses of decision d١ and decision d٢ for OSTAGE-D procedure under linear loss function .

****

```
      INTEGER RD١,RD٢,R١١,R٢١
      DIMENSION XS(١٠٠٠)
      READ*,N١
      READ*,ND١,ND٢,RD١,RD٢
      READ*,C١,C٢
```

```
JJ=٠
DO ٢ II١=١,N١+١
N١١=II١-١
N٢١=N١-N١١
SUM٤=٠.٠
DO ١٠ II٢=١,N١١+١
R١١=II٢-١
DO ١٥ II٣=١,N٢١+١
R٢١=II٣-١
K١=IF(N١١)
K٢=IF(R١١)
K٣=IF(N١١-R١١)
Y١=K١/FLOAT(K٢*K٣)
K٤=IF(N٢١)
K٥=IF(R٢١)
K٦=IF(N٢١-R٢١)
Y٢=K٤/FLOAT(K٥*K٦)
K٧=IF(R١١+RD١-١)
K٨=IF(N١١-R١١+ND١-RD١-١)
K٩=IF(N١١+ND١-١)
Y٣=(K٧*K٨)/FLOAT(K٩)
K١٠=IF(R٢١+RD٢-١)
K١١=IF(N٢١-R٢١+ND٢-RD٢-١)
```

```
      K١٢=IF(N٢١+ND٢-١)
      Y٤=(K١٠*K١١)/FLOAT(K١٢)
      SUM١=٠.٠
      DO ٢٠ J=R٢١+RD٢,N٢١+ND٢-١
      K١٣=IF(R١١+RD١+J)
      K١٤=IF(N١١-R١١+ND١-RD١+N٢١+ND٢-J-١٢
      K١٥=IF(J+١)
      K١٦=IF(N٢١+ND٢-J-١)
     KI١=(R٢١+RD٢-J-١)
      Y٥=K١٣*K١٤*KI١)/FLOAT(K١٥*K١٦))
      SUM١=SUM١+Y٥
٢٠    CONTINUE
    SUM٢=٠.٠
      DO ٣٠ J١=R١١+RD١,N١١+ND١-١
      K١٧=IF(R٢١+RD٢+J١)
      K١٨=IF(N١١+ND١+N٢١-R٢١+ND٢-RD٢-٢-J١)
      K١٩=IF(J١+١)
      K٢٠=IF(N١١+ND١-J١-١)
     KI٢=(R١١+RD١-J١-١)
      Y٦=ABS((K١٧*K١٨*KI٢)/FLOAT(K١٩*K٢٠))
      SUM٢=SUM٢+Y٦
٣٠    CONTINUE
      K٢١=IF(N١١+ND١+N٢١+ND٢-١)
```

```
      Y٧=ABS((C١*K٩*K١٢)/FLOAT(ABS(K٧*K٨*K٢١)))
      Y٨=ABS((C٢*K٩*K١٢)/FLOAT(ABS(K١٠*K١١*K٢١)))
      S١=Y٧*SUM١
      S٢=Y٨*SUM٢
      S=S١
      IF(S٢.LT.S)THEN
      S=S٢
      ENDIF
      YI=Y١*Y٢*Y٣*Y٤*S
      SUM٤=SUM٤+YI
١٥    CONTINUE
١٠    CONTINUE
      K٢٢=IF(RD١-١)
      K٢٣=IF(ND١-RD١-١)
      K٢٤=IF(ND١-١)
      YJ١=(K٢٢*K٢٣)/FLOAT(K٢٤)
      K٢٥=IF(RD٢-١)
      K٢٦=IF(ND٢-RD٢-١)
      K٢٧=IF(ND٢-١)
      YJ٢=(K٢٥*K٢٦)/FLOAT(K٢٧)
      SUM=SUM٤/(YJ١*YJ٢)
      R=SUM
      JJ=JJ+١
```

```
      XS(JJ)=R
٢     CONTINUE
     RMIN=XS(١)
      DO ٦٠ IR=٢,JJ
      IF(XS(IR).LT.RMIN)THEN
      RMIN=XS(IR)
      ENDIF
٦٠     CONTINUE
      PRINT*,'RMIN=',RMIN
      END
      FUNCTION IF(IH)
      IF=١
      DO ٥٥ K=١,IH
      IF=IF*K
٥٥     CONTINUE
      END
```

Appendix C

Programs for Monte Carlo simulation

This program finds the P(CS) , P(NCS) and E(R) for ١st-FS by using Monte Carlo estimate . Each estimate based on ٥٠٠٠ replications .

```
*****************************************************************

      INTEGER RD١,RD٢,R١,R٢,RR١,RR٢
```

```
INTEGER CS
READ*,P١,P٢
READ*,N,N١,N٢
READ*,ND١,ND٢,RD١,RD٢
NSTAR=N١+N٢
CS=٠
NCS=٠
IR١=٠
IR٢=٠
IM١=٠
IM٢=٠
DO ١٠ I=١,٥٠٠٠
R١=٠
R٢=٠
RR١=٠
RR٢=٠
M١=٠
M٢=٠
MM١=٠
MM٢=٠
DO ٢٠ J=١,N١
Y=RAND(٠)
IF(Y.LT.P١)THEN
```

```
      R١=R١+١
      M١=M١+١
      ELSE
      M٢=M٢+١
      ENDIF
٢٠    CONTINUE
     PH١=(RD١+R١)/FLOAT(ND١+N١)
      DO ٣٠ K=١,N٢
      Y=RAND(٠)
      IF(Y.LT.P٢)THEN
      R٢=R٢+١
      M١=M١+١
      ELSE
      M٢=M٢+١
      ENDIF
٣٠    CONTINUE
     PH٢=(RD٢+R٢)/FLOAT(ND٢+N٢)
      IF(PH١.GT.PH٢)THEN
      RR١=R١+N-NSTAR
      NN١=N١+N-NSTAR
      RR٢=R٢
      NN٢=N٢
      MM١=M١+N-NSTAR
```

```
MM٢=M٢
ELSE
RR١=R١
NN١=N١
RR٢=R٢+N-NSTAR
NN٢=N٢+N-NSTAR
MM١=M١
MM٢=M٢+N-NSTAR
ENDIF
PHH١=(RR١+RD١)/FLOAT(NN١+ND١)
PHH٢=(RR٢+RD٢)/FLOAT(NN٢+ND٢)
IF(PHH١.GT.PHH٢.AND.P١.GT.P٢)CS=CS+١
IF(PHH١.LT.PHH٢.AND.P١.LT.P٢)CS=CS+١
IF(PHH١.EQ.PHH٢.AND.P١.EQ.P٢)CS=CS+١
IF(PHH١.GT.PHH٢.AND.P١.LT.P٢)NCS=NCS+١
IF(PHH١.LT.PHH٢.AND.P١.GT.P٢)NCS=NCS+١
IF(PHH١.EQ.PHH٢.AND.P١.LT.P٢)NCS=NCS+١
IF(PHH١.LT.PHH٢.AND.P١.EQ.P٢)NCS=NCS+١
IF(PHH١.GT.PHH٢.AND.P١.EQ.P٢)NCS=NCS+١
IF(PHH١.EQ.PHH٢.AND.P١.GT.P٢)NCS=NCS+١
IR١=IR١+RR١
IR٢=IR٢+RR٢
IM١=IM١+MM١
```

```
      IM٢=IM٢+MM٢
١٠    CONTINUE
     PCS=CS/٥٠٠٠.٠
      PNCS=NCS/٥٠٠٠.٠
      ER١=IR١/٥٠٠٠.٠
      ER٢=IR٢/٥٠٠٠.٠
      ER=ER١+ER٢
      EIM١=IM١/٥٠٠٠.٠
      EIM٢=IM٢/٥٠٠٠.٠
      PRINT*,'PCS=',PCS
      PRINT*,'PNCS=',PNCS
      PRINT*,'ER=',ER
      STOP
      END
```

This program finds the P(CS) , P(NCS) and E(R) for ١st-PWR by using Monte Carlo estimate . Each estimate based on ٥٠٠٠ replications .

****

```
      INTEGER RD١,RD٢,R١,R٢,RR١,RR٢
        INTEGER CS
        READ*,P١,P٢
        READ*,N,N١,N٢
        READ*,ND١,ND٢,RD١,RD٢
        NSTAR=N١+N٢
        CS=٠
        NCS=٠
        IR١=٠
        IR٢=٠
        DO ١٠ I=١,٥٠٠٠
        R١=٠
        R٢=٠
        RR١=٠
        RR٢=٠
        IH=٠
٢      IF(IH.EQ.NSTAR)GOTO ٤
      Y=RAND(٠)
        IH=IH+١
```

```
      IF(Y.LT.P١)THEN
      R١=R١+١
      GOTO ٢
      ELSE
      GOTO ٣
      ENDIF
٣     IF(IH.EQ.NSTAR)GOTO ٤
     Y=RAND(٠)
      IH=IH+١
     IF(Y.LT.P٢)THEN
      R٢=R٢+١
      GOTO ٣
      ELSE
      GOTO ٢
      ENDIF
      GOTO ٢
٤     PH١=(RD١+R١)/FLOAT(ND١+N١)
      PH٢=(RD٢+R٢)/FLOAT(ND٢+N٢)
      IF(PH١.GT.PH٢)THEN
      RR١=R١+N-NSTAR
      NN١=N١+N-NSTAR
      RR٢=R٢
      NN٢=N٢
```

```
      ELSE
      RR١=R١
      NN١=N١
      RR٢=R٢+N-NSTAR
      NN٢=N٢+N-NSTAR
      ENDIF
      PHH١=(RR١+RD١)/FLOAT(NN١+ND١)
      PHH٢=(RR٢+RD٢)/FLOAT(NN٢+ND٢)
      IF(PHH١.LT.PHH٢.AND.P١.LT.P٢)CS=CS+١
      IF(PHH١.GT.PHH٢.AND.P١.GT.P٢)CS=CS+١
      IF(PHH١.EQ.PHH٢.AND.P١.EQ.P٢)CS=CS+١
      IF(PHH١.LT.PHH٢.AND.P١.GT.P٢)NCS=NCS+١
      IF(PHH١.GT.PHH٢.AND.P١.LT.P٢)NCS=NCS+١
      IF(PHH١.EQ.PHH٢.AND.P١.GT.P٢)NCS=NCS+١
      IF(PHH١.GT.PHH٢.AND.P١.EQ.P٢)NCS=NCS+١
      IF(PHH١.LT.PHH٢.AND.P١.EQ.P٢)NCS=NCS+١
      IF(PHH١.EQ.PHH٢.AND.P١.LT.P٢)NCS=NCS+١
      IR١=IR١+RR١
      IR٢=IR٢+RR٢
١٠    CONTINUE
     PCS=CS/٥٠٠٠.٠
      PNCS=NCS/٥٠٠٠.٠
      ER١=IR١/٥٠٠٠.٠
```

```
ER٢=IR٢/٥٠٠٠.٠

ER=ER١+ER٢

PRINT*,'PCS=',PCS

PRINT*,'PNCS=',PNCS

PRINT*,'ER=',ER

STOP

END
```

This program finds the P(CS) , P(NCS) and E(R) for ١st-GTR(n) by using Monte Carlo estimate , each estimate based on ٥٠٠٠ replications .

****

```
INTEGER RD١,RD٢,R١,R٢,RR١,RR٢
 INTEGER CS
 READ*,P١,P٢
 READ*,N,N١,N٢
 READ*,ND١,ND٢,RD١,RD٢
 NSTAR=N١+N٢
 CS=٠
 NCS=٠
 IR١=٠
 IR٢=٠
 DO ١٠ I=١,٥٠٠٠
```

```
      R١=٠
      R٢=٠
      RR١=٠
      RR٢=٠
      K=٠
      IH=٠
٦     IF(IH.EQ.NSTAR)GOTO ٣
      D=٠
      E=٠
٢     IF(D.GT.٠)GOTO ٤
      D=D+١
      DO ٢٠ J=١,N١
      Y=RAND(٠)
      IH=IH+١
      IF(Y.LT.P١)THEN
      R١=R١+١
      GOTO ٢
      ELSE
      GOTO ٢
      ENDIF
٤     IF(E.LT.١)THEN
      E=E+١
      IF(K.LT.N٢)THEN
```

```
      K=K+١
      ELSE
      GOTO ٦
      ENDIF
     Y=RAND(٠)
      IH=IH+١
      IF(Y.LT.P٢)THEN
      R٢=R٢+١
      GOTO ٤
      ELSE
      GOTO ٤
      ENDIF
      ELSE
      GOTO ٦
      ENDIF
٢٠    CONTINUE
٣    PH١=(RD١+R١)/FLOAT(ND١+N١)
     PH٢=(RD٢+R٢)/FLOAT(ND٢+N٢)
      IF(PH١.GT.PH٢)THEN
      RR١=R١+N-NSTAR
      NN١=N١+N-NSTAR
      RR٢=R٢
      NN٢=N٢
```

```
      ELSE
      RR١=R١
      NN١=N١
      RR٢=R٢+N-NSTAR
      NN٢=N٢+N-NSTAR
      ENDIF
      PHH١=(RR١+RD١)/FLOAT(NN١+ND١)
      PHH٢=(RR٢+RD٢)/FLOAT(NN٢+ND٢)
      IF(PHH١.GT.PHH٢.AND.P١.GT.P٢)CS=CS+١
      IF(PHH١.LT.PHH٢.AND.P١.LT.P٢)CS=CS+١
      IF(PHH١.EQ.PHH٢.AND.P١.EQ.P٢)CS=CS+١
      IF(PHH١.GT.PHH٢.AND.P١.LT.P٢)NCS=NCS+١
      IF(PHH١.LT.PHH٢.AND.P١.GT.P٢)NCS=NCS+١
      IF(PHH١.EQ.PHH٢.AND.P١.LT.P٢)NCS=NCS+١
      IF(PHH١.LT.PHH٢.AND.P١.EQ.P٢)NCS=NCS+١
      IF(PHH١.GT.PHH٢.AND.P١.EQ.P٢)NCS=NCS+١
      IF(PHH١.EQ.PHH٢.AND.P١.GT.P٢)NCS=NCS+١
      IR١=IR١+RR١
      IR٢=IR٢+RR٢
١٠    CONTINUE
     PCS=CS/٥٠٠٠.٠
      PNCS=NCS/٥٠٠٠.٠
      ER١=IR١/٥٠٠٠.٠
```

```
ER٢=IR٢/٥٠٠٠.٠

ER=ER١+ER٢

PRINT*,'PCS=',PCS

PRINT*,'PNCS=',PNCS

PRINT*,'ER=',ER

STOP

END
```

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Asa'ad

Abstract

In this thesis ,we propose Bayesian two- stage procedures for selecting the better of two binomial populations .Two approaches are considered to achieve this goal . Under the first approach , decision theoretic formulation with different loss functions and beta prior distribution is used to construct Bayesian two- stage selection procedure . The performance of this procedure is assessed using Bayes risk .Comparisons with Bayesian fixed sample size procedures are made .Under the second approach ,Monte Carlo simulation technique is carried out to study procedures based on play the winner, group at a time and posterior estimates of the parameters. The performance of these procedures are assessed in terms of measures such as the probability of correctly selecting the better population and the expected number of successes . Some discussions and concluding remarks are presented . Finally , suggestions for future work are also given .

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Appendix C Programs for Monte Carlo simulation

List of Symbols

Symbol in procedure	Symbol in Program	Description
Symbol in two-stage procedures		
n	$N_{١}$	The number of observations in the first stage
m	$N_{٢}$	The number of observations in the second stage
$n_{١}$	$N_{١١}$	The number of observations for population ١ in first stage
$n_{٢}$	$N_{٢١}$	The number of observations for population ٢ in first stage
$m_{١}$	$N_{١٢}$	The number of observation for population ١ in second stage

$m_{٢}$	$N_{٢٢}$	The number of observation for population ٢ in second stage
$r_{١}$	$R_{١١}$	The number of successes for population ١ in first stage
$r_{٢}$	$R_{٢١}$	The number of successes for population ٢ in first stage
$s_{١}$	$R_{١٢}$	The number of successes for population ١ in second stage
$s_{٢}$	$R_{٢٢}$	The number of successes for population ٢ in second stage
$TST_{١}$	$S_{١}$	The posterior expected loss of the terminal decision $d_{١}$
$TST_{٢}$	$S_{٢}$	The posterior expected loss of the terminal decision $d_{٢}$
$K_{١}$ and $K_{٢}$	$C_{١}$ and $C_{٢}$	Losses constant
Symbol in one -stage procedure		
N	$N_{١}$	The total number of observations
$N_{١}$	$N_{١١}$	The number of observations for population ١
$N_{٢}$	$N_{٢١}$	The number of observations for population ٢
$R_{١}$	$R_{١١}$	The number of successes for population ١
$R_{٢}$	$R_{٢١}$	The number of successes for population ٢
$OST_{١}$	$S_{١}$	The posterior expected losses of making $d_{١}$
$OST_{٢}$	$S_{٢}$	The posterior expected losses of making $d_{٢}$
Symbol in two-stage Monte Carlo simulation		
P(CS)	P(CS)	Probability of correct selection
P(NCS)	P(NCS)	Probability of non correct selection
N	N	The number of observations
$N_{١}$	$N_{١}$	The number of observations for population ١
$N_{٢}$	$N_{٢}$	The number of observations for population ٢
N^{*}	NSTAR	The total number of observations in the first stage

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Chapter One

Introduction

١.١ Basics of Ranking and Selection (R&S) [١١], [١٣],[١٨]

In numerous situations of the everyday – life we are forced to solve the problem which one of several alternatives is the best , where the attribute "best" must be interpreted as the case may be . For example, an agricultural experiment station may wish to determine which of several varieties of grain is best for growing in a certain region of the country . Or an industrial organization may wish to know which of several manufacturing processes is best for producing a certain item , or which of several drugs to market as the most effective cure for some disease . Various definitions of "bestness " are conceivable , depending upon the requirements of the particular application in hand . Here it will be assumed that there is a single characteristic , such as the yield in the case of an agricultural crop , or the purity or some other quality characteristic in the case of a manufacturing process , by which the various experimental categories are to be judged , and that the best category is the one which possesses on the average the highest value of this particular characteristic . Thus we will be concerned with applications such above to determine which of several alternative is the best , and so on .

If the average value of the characteristic were known for each experimental category , then the identity of the best category would be known and there would be no problem . We are concerned with situation where the identity of the category is not known but an experiment can be performed to help in choosing the best one . The classical approach permits us to decide about the following null hypothesis , $H_0 : \theta_1 = \theta_2 = ... = \theta_k$ where $\theta_1, \theta_2, ..., \theta_k$ are the value of the parameters for these populations . Tests of this null hypothesis are frequently called tests of homogeneity . The alternative hypothesis , is usually that these values are not equal . It is was pointed out by several researchers , that testing the homogeneity of population means or variances is not a satisfactory solution to a comparison of the performance of several populations. One would , generally , want to either rank them according to their performance or to select one or more from among them for future use or further evaluation . These problems are known as ranking and selection problems . So R&S procedures are statistical methods specifically developed to select the best population or a subset that contains the best population from a set of k comparing alternatives (Goldsman and Nelson ٢٠٠١)[١٩] . Two formulations to these problems have been suggested in the classical framework. The first one , proposed by Bechhefer (١٩٥٤)[٥] and known as the indifference zone approach (IZ) , allows the experimenter to select one population which is the best with a fixed probability p^* , whenever the unknown parameters are outside a zone of indifference . The second one, proposed by Gupta (١٩٥٦) and has been known as the subset selection formulation , allows the experimenter to select a subset of random size which contains the best population with a probability p^* or more . The event of correctly selected the best population is denoted by CS . The idea underlying the classical IZ selection procedure is to choose a rule satisfying predetermined constants $0 < p^* \le 1$, $\Delta^* > 0$ such that $p(cs) \ge p^*$ whenever $\Delta = p_{[k]} - p_{[k-1]} \ge \Delta^*$ for all $\underline{\theta} = (\theta_1, \ldots\ldots\ldots, \theta_k) \in \Omega$ where Ω is the parameter space and the classical subset selection procedure is to choose a rule satisfying $p(cs) \ge p^*$ for all $\underline{\theta}$. As R&S procedures have become more popular , several researchers have attempted to model the problem in a different manner . For example chick (١٩٩٧)[١١] presents a Bayesian analysis of selecting the best simulated system .

١.٢ problem statement

Suppose that Π_1 and Π_2 be two binomial populations with unknown single-trial success probabilities p_1 and p_2 respectively . The parameters p_1 and p_2 can assume any values in the interval [٠,١] . The ordered values of p_1 and p_2 are denoted by $p_{[1]} \le p_{[2]}$. Moreover ,we do not know which population is associated with $p_{[2]}$. The problem is to select the better population , that is the population associated with $p_{[2]}$. A correct selection (CS) is defined as the selection of the better population .Our goal is to construct two-stage procedure which selects that better population .There are many fields of applications where an important problem is the selection of the best one of a number of populations which might be assumed to be binomial . For example , in a learning experiment subject study programmed text matrial, two alternative sets of programmed materials are available . The goal is to determine which one of these two sets of materials is the more effective learning device . We define a success as a passing score on the test , so that p_j ($j = 1,2$) is the probability of passing score for subjects exposed to the jth set of learning materials . Then the goal is to determine which one set has the larger p value . Another example can be taken from medical field . Suppose we have two different drugs to cure from cancer disease . A physician wishes to determine which one of these two drugs has the larger probability of curing from this disease .

Further examples on the situations where the Binomial model applies and the problem of practical interest is to select the better of two Binomial populations may be found in Gibbons , Olkin and Sobel (١٩٧٧)[١٦].

The following experimental conditions should be met(Buringer et al.)(١٩٨٠)[١٠] ,[٣٠] :

١.The observations (trials) produced by each population are independent of each other .

٢. p_1 and p_2 , the probabilities of success are constants during the experiment .

١.٣ Review of Literature

The problem of selecting the better of two binomial populations has been extensively studied in the literature .

Sobel and Huyett (١٩٥٧)[٣٦] is considered as a fundamental paper in Binomial selection studies . In this paper they proposed a single sampling procedure in which an equal number of observations n^* are taken from each population and the population having the most successes is selected as the better population with ties broken by randomization . They employed the idea of IZ approach which was developed by Bechhofer (١٩٥٤) [٥] to solve the problem of selection in normal populations . This classical approach requires that the probability of making a correct selection is greater than or equal to some preassigned value , $0 \le p^* < 1$, and the difference between the largest and next to the largest is preassigned number $0 < \Delta^* < 1$, formally

$$p(cs) \ge p^* \qquad (١.١)$$

whenever :

$$p_{[2]} - p_{[1]} \ge \Delta^* \qquad 0 < \Delta^* < 1,\ \frac{1}{2} < p^* < 1 \qquad (١.٢)$$

where CS(for correct selection) denotes the final selection of a population with probability of success $p_{[2]}$. With the condition above (p^* , Δ^*-condition) ,we will be at least ١٠٠ p^* percent sure of selecting the better parameter whenever the largest parameter $p_{[2]}$is at least Δ^* better than the second largest $p_{[1]}$, the $p(cs)$ is minimized when $p_{[2]} - p_{[1]} = \Delta^*$, this is called the least favorable configuration (LFC) of the population parameters p_1 and p_2. The value of n^* is then chosen to guarantee (١.١) when the parameter values in the least favorable configuration .

The problem of allocating (assigning) observations among patients in clinical trials has been investigated using other approaches by many authors. Armitage (١٩٧٥)[١] developed closed sequential procedures . Pocock (١٩٧٧) [٣٤] has developed a group sequential design for clinical trials in which the data are analyzed at less frequent intervals and which may lead to an early decision , or stopping of a clinical trial , if large treatment difference are observed .

Other workers using the IZ approach are Taylor and David (١٩٦٢)[٣٩] who discussed a multistage procedure for this problem . Paulson (١٩٦٧)[٣٣] who proposed an open sequential procedure , which permitted the elimination of "non-contending " population . The application of the play the winner – sampling rule (PWR) to problem of allocating observations among treatments appeared first in Zelen (١٩٦٩)[٤١] .

Later a great deal of attention has been paid to the sequential procedures for this problem using different sampling rules such as PWR and vector at a time sampling rules (VT) . Nebenzahl and sobel (١٩٧٢)[٣٢], Berry and Sobel (١٩٧٣)[٨] , Fushimi (١٩٧٣)[١٥], Kiefer and Weiss (١٩٧٤)[٢٩] and Tamhance (١٩٨٥)[٤٠] proposed and studied closed sequential procedures for selecting the better of two Binomial populations. Procedure for selecting the best population of $k \geq 2$ Binomial populations were considered by Sobel and Weiss (١٩٧٢)[٣٨], Hoel and Sobel (١٩٧٢)[٢٥] and Hoel, Sobel and Weiss (١٩٧٥)[٢٦].

At the same time , another approach to the Binomial selection problem has been suggested in the classical framework ;it is known as the subset selection approach . Here the goal is to select a subset containing the best population with a preassigned probability p^* .This approach is useful for the situation when we have very large number of populations and the procedures require more observations than that available . Therefore it is desirable to select a subset consisting of the best for further extensive investigation .Gupta, Huyett and Sobel (١٩٥٧)[٢٠] , Gupta and Huang (١٩٧٦)[٢١] are among those who studied this problem using this approach . Goal and Rubin (١٩٧٧)[١٧] gave a general Bayesian decision theoretic approach for selecting a subset containing the population .

Although the literature on Binomial selection problems is large , the literature using Bayesian approach to solve the problems is rather scarce . Important contributions were made by Bland and Bratcher (١٩٦٨)[٩], Bratcher and Bland (١٩٧٥)[٣] ,who developed Bayesian fixed sample size procedures to solve the problem of ranking Binomial populations where more than two populations are compared . Bather and Simons (١٩٨٥)[٤] considered two-stage procedures in clinical trials using minimax risk . In Madhi (١٩٨٦)[٣٠] , Bayesian sequential schemes are developed for selecting the better of two Binomial populations . Furthermore ,there is a growing literature for the Bayesian decision –theoretic simulation out-put analysis especially for computer budget and sample allocation procedure (Gupta and Miescke (١٩٩٤)) [٢٣]. It seems worthwhile to mention a similar problem namely "two armed-bandit problem .Here the goal is to allocate N observations , one at a time between two Bernoulli of successes. This problem has been investigated by many authors such as Berry (١٩٧٢)[٧],etc .

Bechhofer and Kulkarni (١٩٨١)[٦] proposed a very interesting closed sequential procedure avoiding the (p^* , Δ^*-condition)of the IZ approach. Frisardi (١٩٨٣)[١٤] investigated these procedures employing Monte Carlo simulation . Azal (٢٠٠٥)[٢] studied Bayesian sequential procedures as combinations of various sampling rules and stopping rules. They are studied using Mote Carlo simulation . Their performance is evaluated by calculation several criteria such as the probability of correctly selecting the best population , the expected number of observations , and so on . Comparison has been achieved for these procedures to choose the best population . Chick

(١٩٩٧)[١١] presented a Bayesian analysis of selecting the best simulated system . Chick and Inoue (١٩٩٨)[١٢] compared Bayesian and frequents approaches for selecting the best system . Recently , there are many researches concerned with the application of ranking and selection , for instance in the medical field. Jenison(٢٠٠٥) [٢٧] presented a research about the group sequential selection procedure with elimination and Data – dependent treatment allocation . Kim and Nelson (٢٠٠٤)[٣٥] presented more on the application of Ranking and Selection .

A good review of literature may be found in Gupta and Panchapakesan (١٩٧٩)[٢٢] . In this thesis Bayesian two –stage procedures for selecting the better of two Binomial populations are proposed and studied using Bayes risk and other criteria where Monte Carlo simulations are carried out .

١.٤ Bayesian Decision Theory

١.٤.١ Basic elements of statistical decision problem [٢٤]

The basic problem in a statistical decision theory is to make a (optimal) choice from a given set of alternatives . There are five basic ingredients in a typical statistical decision problem .

١.A parameter space $\Theta = \{\theta\}$, which may be vector–valued of the possible states of nature .

٢.An action space $A = \{a\}$ of all possible actions is available to the decision – maker .

٣.A loss function $L(\theta,a)$ representing the loss incurred when action a is taken and the state of nature is θ

$L : \Theta \times A \rightarrow \Re$ ($\Re$ is the real line).

٤.A sample space χ , the set of observations . In most applications the choice of a is not made blindly but depends on some observable random variable(r.v) X taking values in χ .A realization of X is denoted by x. When θ is the true state of nature , X has probability distribution $f_X(x/\theta)$.

٥.A decision space $D = \{ d(x) \}$ of possible decision functions (rules) is defined on χ that maps χ onto the action space A . That is $d : \chi \rightarrow A$.

١.٤.٢ Risk Function [٣١]

For a given decision function d ,the loss function may be written as $L(\theta,d(x))$. Since our action a dependents on the particular sample data x that we observe . Thus, we see that the loss function is a r.v and depends on the sample outcome . Therefore , let us define the risk function $R(\theta,d)$ to be the expected value of the loss function $L(\theta,d(x))$ overall possible outcome . That is

$$R(\theta,d) = E_{\Theta}[L(\theta,d(x))] = \int_{\chi} L(\theta,d(x)) f_X(x/\theta)dx \quad \text{if } X \text{ is continuous}$$

$$= \sum_{\chi} L(\theta,d(x)) f_X(x/\theta) \quad \text{if } X \text{ is discrete}$$

Obviously , a good decision function would be one that minimizes the risk for all values in Θ . Unfortunately ,in most realistic problems , there does not exist a single decision function that minimizes the risk for all possible values of θ. Since the value of θ is unknown , this limits the usefulness of risk as a criterion for selecting a decision function . However ,risk can be used as a guide . For example , for partial ordering of the $d's \in D$, where a decision rule d is better than decision rule d' if

$R(\theta,d) \leq R(\theta,d')$ for every $\theta \in \Theta$

and $R(\theta,d) < R(\theta,d')$ for at least one $\theta \in \Theta$

١.٤.٣ Prior and Posterior probabilities [٢٦].

Baye's theorem is the fundamental tool used to arrive a Bayesian decision theory .

The prior model is assumed to represent the totality of subjective information available concerning the parameter θ prior to the experiment , thus it is not functionally dependent upon x. On the other hand , the sampling model depends on the values of the parameter θ and is thus a conditional probability distribution . The posterior model tells us what is known about θ given knowledge of the data x. It is essentially an updated version of our prior knowledge about θ in light of knowledge of the sample data- hence , the name posterior model .

Bayes theorem states that the posterior model is related to the prior and sampling models according to

$$\pi(\theta / x) = \frac{f(x/\theta)\pi(\theta)}{f(x)}$$

where $\pi(\theta)$ is the prior probability distribution of θ (the prior model) , $f(x/\theta)$ is the conditional probability distribution of x given θ (the sampling model) ,

$\pi(\theta/x)$ is the posterior probability distribution of θ given x (the posterior model) and

$f(x)$ is the marginal probability distribution of X .

$f(x)$ may be obtained according to :

$$f(x)=\begin{cases} \int_{\Theta} f(x/\theta)\pi(\theta)d\theta & \theta \text{ is continuous} \\ \sum_{\theta\in\Theta} f(x/\theta)\pi(\theta) & \text{f } \theta \text{ is discrete} \end{cases}$$

١.٤.٤ Bayes risk and Bayes decision rule [١٨]

Let us now define the Bayes risk of a decision function d as the expected value of the risk $R(\theta,d)$ with respect to the prior distribution $\pi(\theta)$ on Θ, namely

$$B(d) = E_{\theta}[R(\theta,d)]$$

$$= \int_{\Theta} R(\theta,d)\pi(\theta)d\theta \qquad \text{if } \theta \text{ is continuous ,}$$

$$= \sum_{\theta\in\Theta} R(\theta,d)\pi(\theta) \qquad \text{if } \theta \text{ is discrete .}$$

The minimum Bayes risk is defined by $B(d^{*}) = \min_{d\in D} B(d)$

and d^* is called the Bayes decision rule or optimal decision rule . Note that for a given decision there will not be a unique Bayes decision function. This will depend upon the choice of $\pi(\theta)$.If d is a Bayes decision function then $d(X)$ is called a Bayes estimator and $d(x)$ a Bayes estimate . The mean of the posterior distribution is the Bayes estimate with respect to quadratic loss.

Now , returning to the form of the Bayes risk $B(d)$,we have

$$B(d) = \int_{\Theta} R(\theta,d)\pi(\theta)d\theta$$

$$= \int_{\Theta}\{ \int_{\chi} L(\theta,d) f(x/\theta)dx\}\pi(\theta)d\theta \ .$$

Assuming that the functions involved permit the interchange of the order of integration , the Bayes risk is

$$B(d) = \int_{\chi}\{ \int_{\Theta} L(\theta,d(x))\pi(\theta/x)d\theta\} f(x)dx \qquad (١.٤.٤.١)$$

Since , by the definition of conditional probability density function ,

$$f(x/\theta)\pi(\theta) = \pi(\theta/x)f(x) \ .$$

Finding a d^* to minimize the Bayes risk (١.٤.٤.١) is equivalent to minimizing the inner integral for each $X = x$. The calculation of Bayes decision can therefore be carried out in two steps . First ,find $\pi(\theta/x)$ using Bayes Theorem; then minimize the posterior expected loss

$$\underset{\pi(\theta/x)}{E}(L(\Theta,d(x))) = \int_{\Theta} L(\theta,d(x))\pi(\theta/x)d\theta \ .$$

١.٥ Outline of the thesis

The rest of this thesis is organized as follows: In chapter, two we present a Bayesian two-stage procedure for selecting the better of two Binomial populations. Decision –theoretic approach is used to construct this procedure . The performance of this procedure has been investigated in terms of Bayes risk. Some two-stage selection procedures that based on posterior estimates and some sampling rules , studied using Monte Carlo simulation , are presented in chapter three . Chapter four contains some concluding remarks and some suggestions for future work . The appendices contain listings of the computer programs which have been used to produce the numerical part of this thesis.

Chapter Two

Bayesian Two – stage procedures :

Decision – theoretic approach

٢.١ Summary

In this chapter, the Bayesian decision approach , where prior distribution on the unknown parameters and the loss function are specified , is used to construct a two-stage procedure for selecting the better of two Binomial populations .

A computer program has been written in Fortran power station, given in appendix (A) to calculate the above procedure .

The structure of this chapter is as follows. In section ٢.٢ , the Binomial selection problem is formulated as a two- decision problem . Section ٢.٣ contains the proposed Bayesian two-stage selection procedure (TSTAGE – D).The construction of this procedure under constant and linear losses , is given in subsection ٢.٣.١ . Some numerical results concerning this procedure are given in subsection ٢.٣.٢ .

Section ٢.٤ deals with Bayesian one-stage procedure (OSTAGE-D) for selecting the better of two Binomial populations . In subsection ٢.٤.١ , the derivation of the procedure (OSTAGE-D) is presented . Subsection ٢.٤.٢ contains some numerical results concerning the procedure (OSTAGE-D) .

Comparisons of the procedures TSTAGE-D and OSTAGE-D using overall Bayes risk are given in section ٢.٥ .

٢.٢ Bayesian Decision-Theoretic Formulation

Consider the two Binomial populations Π_1 and Π_2 with p_1 and p_2 as their unknown success probabilities for a single trial respectively .

Now, consider the following two – decision problem with decisions :

$d_1 :\ p_1 \geq p_2$

and

$d_2 :\ p_1 < p_2$

Furthermore ,in making decisions d_1 and d_2 , we assume the following loss functions.

١- Constant loss Function

$$L_1(d_1,\underline{p})= \begin{cases} ٠ & \text{if} \quad p_1 \geq p_2 \\ K_١ & \text{if} \quad p_1 < p_2 \end{cases}$$

and ٢.٢.١

$$L_2(d_2,\underline{p})= \begin{cases} K_٢ & \text{if} \quad p_1 > p_2 \\ ٠ & \text{if} \quad p_1 \leq p_2 \end{cases}$$

٢- Linear Loss Function

$$L_1(d_1,\underline{p})= \begin{cases} ٠ & \text{if} \quad p_1 \geq p_2 \\ K_١ (p_٢ - p_١) & \text{if} \quad p_1 < p_2 \end{cases}$$

and ٢.٢.٢

$$L_2(d_2,\underline{p})= \begin{cases} K_٢ (p_١ - p_٢) & \text{if} \quad p_1 > p_2 \\ ٠ & \text{if} \quad p_1 \leq p_2 \end{cases}$$

where K١ and K٢ are positive constants giving losses in terms of costs and $\underline{p} = (p_1, p_2)$.

The Bayesian approach requires that we specify a prior probability density function $\pi\ (p_i)$, expressing our beliefs about p_i before we obtain the data . From a mathematical point of view , it would be convenient if p_i is assigned a prior distribution which is a member of a family of distributions closed under binomial sampling or as a member of the conjugate distributions . Accordingly, let p_i is assigned Beta prior distribution with parameters $n_i^{'}$, $r_i^{'}$, Beta ($r_i^{'}, n_i^{'}$) . The normalized density function (Raiffa and Schlaifer (١٩٦١)) is given by

$$\pi(p_i) = \frac{p_i^{r_i^{'}} (1-p_i)^{n_i^{'}-r_i^{'}-1}}{B(r_i^{'}, n_i^{'}-r_i^{'})} \quad ,$$

$$0 \le p_i \le 1 \ , \qquad 0 \le r_i^{'} \le n_i^{'} \quad , \qquad i = 1,2 \ ,$$

where $B(r_i^{'}, n_i^{'}-r_i^{'})$ is the complete Beta function . It is also assumed that p_i are a prior independent . The parameters $r_i^{'}$, $n_i^{'}$ need not be integers. However , it is convenient if from this point , we assume that $r_i^{'}$, $n_i^{'}$ are integers so that we can replace the gamma functions by factorial terms in our formulation of the procedure .

In addition to the prior information , we obtain some sample information from the population $\Pi_i (i = 1,2)$. In doing , we assume that we observe the number of successes r_i^{*} , obtained in n_i^{*} trials giving the probability function

$$P(r_i^{*} / p_i, n_i^{*}) = \binom{n_i^{*}}{r_i^{*}} p_i^{r_i^{*}} (1-p_i)^{n_i^{*}-r_i^{*}} \quad , \qquad r_i^{*} = 0,1,\ldots\ldots\ldots, n_i^{*} \ .$$

The posterior probability density function is derived from the prior probability function and the assumed sampling model by means of Bayes theorem mentioned earlier .

$$\pi(p_i / r_i^{''} ,n_i^{''})=\frac{P(r_i^{*} / p_i ,n_i^{*})\pi(p_i)}{\int\limits_{p_i} P(r_i^{*} / p_i ,n_i^{*})\pi(p_i)dp_i} \ ,$$

where $r_i^{''} = r_i^{'} + r_i^{*}$ and $n_i^{''} = n_i^{'} + n_i^{*}$, $i = 1,2$.

If the sample size n_i^{*} taken from population Π_i is large , then the action choice of prior parameters ($r_i^{'}$, $n_i^{'}$) has little effect on the posterior density function which can be well approximated by a Beta probability density function with parameters r_i^{*} *and* n_i^{*} . In this case it is sufficient to take the uniform prior distribution $\pi\ (p_i)=1$, to express our vague knowledge about the parameters of interest .

As the Beta family is conjugate with the Binomial sampling , it is unnecessary to revise a Beta prior distribution on the basis of a sample from a Bernoulli process using Bayes theorem . Given the prior distribution and the sampling results ,we need simply note that

$$r_i^{''} = r_i^{'} + r_i^{*} \qquad \text{and} \qquad n_i^{''} = n_i^{'} + n_i^{*} \ ,$$

are the parameters of the posterior Beta density function .

٢.٣ Bayesian Two-stage procedures(TSTAGE-D)

In this section , we present a two-stage procedure for selecting the better of two Binomial populations in two stages using the Bayesian decision-theoretic formulation given in section ٢.٢ .

٢.٣.١ Construction of the procedure (TSTAGE-D)

Let the probabilities of success in the first and second populations be p_1 and p_2 respectively . Suppose we had decided prior to the experiment that we will take n observations in the first stage and m observations in the second stage. Furthermore , n is partitioned into n_1 and n_2 , the number of observations for population ١ and for population ٢ respectively .In the second stage on basis of the results in the first stage m is partitioned into m_1 and m_2 the number of observations for population ١ and for population ٢ respectively, Let the number of successes for population ١ and population ٢ be r_1 and r_2 in the first stage and s_1 and s_2 in the second stage .

After the second stage , on the basis of the results for the first and second stages , we choose either decision d_1 or decision d_2 .

We will use the following notations :

Let $\underline{r}=(r_1,r_2)$, $\underline{s}=(s_1,s_2)$, $\underline{n}=(n_1,n_2)$ and $\underline{m}=(m_1,m_2)$. The joint prior probability distributions of p_1 and p_2 is given by

$$\pi(\underline{p})=\frac{P_1^{r_1'-1}(1-p_1)^{n_1'-r_1'-1}\,p_2^{r_2'-1}(1-p_2)^{n_2'-r_2'-1}}{B(r_1',n_1'-r_1')B(r_2',n_2'-r_2')}\ ,$$

where r_1',r_2',n_1' and n_2' are positive integers ; $B(r_1',n_1'-r_1')$ and $B(r_2',n_2'-r_2')$ are beta functions , $0\le p_i\le 1$, $i=1,2$.

Since r_1 , r_2, s_1 and s_2 each has binomial distribution , therefore ,the joint posterior distributions of p_1 and p_2 given r_1,r_2,n_1 and n_2 is $\pi(\underline{p}/\underline{r},\underline{n},)=$

$$\frac{P_1^{r_1+r_1'-1}(1-p_1)^{n_1-r_1+n_1'-r_1'-1}\,p_2^{r_2+r_2'-1}(1-p_2)^{n_2-r_2+n_2'-r_2'-1}}{B(r_1+r_1',n_1-r_1+n_1'-r_1')B(r_2+r_2',n_2-r_2+n_2'-r_2')}\ ,$$

and the joint posterior distributions of p_1 and p_2 given r_1 , r_2,s_1,s_2,n_1, n_2, m_1 and m_2 is

$$\pi(\underline{p}/\underline{r},\underline{n},\underline{s},\underline{m})=\frac{P_1^{r_1+s_1+r_1'-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}}{B(r_1+s_1+r_1',n_1-r_1+m_1-s_1+n_1'-r_1')}$$

$$\frac{p_2^{r_2+s_2+r_2'-1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}}{B(r_2+s_2+r_2',n_2-r_2+m_2-s_2+n_2'-r_2')}\ .$$

Let δ_1 be a decision of what size n_1 and consequently $n_2=n-n_1$; and δ_2 be a decision of what size m_1 and consequently $m_2=m-m_1$. That is

δ_1 will be $0,1,2,\ldots\ldots\ldots\ldots,n$

and

δ_2 will be $0,1,2,\ldots\ldots\ldots\ldots,m$.

The problem is to select δ_1 at the first stage , the δ_2 is selected on the basis of r_1, r_2 and δ_1 ; finally d is selected on the basis of r_1 , r_2, s_1, s_2, δ_1 and δ_2.

The procedure TSTAGE-D can be constructed as follows :

١. The posterior expected loss of the terminal decision d_i, when r_1 , r_2, s_1, s_2, δ_1 , δ_2 and d_i are fixed , is given by

$$TST_i(\underline{r},\underline{n},\underline{s},\underline{m},d_i) = \underset{\pi(\underline{p}/\underline{r},\underline{n},\underline{s},\underline{m})}{E}[L_i(d_i,\underline{p})] \quad , i=1,2 \; ,$$

where the subscript $\pi(\underline{p}/\underline{r},\underline{n},\underline{s},\underline{m})$ on the expectation sign is the joint posterior of p_1 and p_2 with respect to which the expectation is being performed .

The mathematical forms of $TST_i(\underline{r},\underline{n},\underline{s},\underline{m},d_i)$ are derived under constant and linear loss functions as follows .

(a) Constant loss function

Using constant loss function (٢.٢.١) , we get

$$TST_1(\underline{r},\underline{n},\underline{s},\underline{m},d_1) = \int_0^1\int_0^{P_1} L_1(d_1,\underline{p})\pi[\underline{p}/\underline{r},\underline{n},\underline{s},\underline{m}]\,dp_2dp_1$$

$$= \int_0^1\int_0^{p_1} \frac{k_1 p_1^{r_1+s_1+r_1'-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}}{B(r_1+s_1+r_1', n_1-r_1+m_1-s_1+n_1'-r_1')}$$

$$\frac{p_2^{r_2+s_2+r_2'-1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}}{B(r_2+s_2+r_2',n_2-r_2+m_2-s_2+n_2'-r_2')}dp_2dp_1$$

$$=\frac{k_1(n_1+m_1+n_1'-1)!(n_2+m_2+n_2'-1)!}{(r_1+s_1+r_{1'}'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}$$

$$\frac{1}{(r_2+s_2+r_2'-1)!(n_2-r_2+m_2-s_2+n_2'-r_2'-1)!}$$

$$\int_0^1 p_1^{r_1+s_1+r_1'-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}$$

$$\int_0^{p_1} p_2^{r_2+s_2+r_2'-1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}dp_2dp_1 \ .$$

Let A= $\int_0^{p_1} p_2^{r_2+s_2+r_2'-1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}dp_2$.

Using integration by parts with

u = $(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}$ and dv = $p_2^{r_2+s_2+r_2'-1}dp_2$,

we implies that

du= $(n_2-r_2+m_2-s_2+n_2'-r_2'-1)(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-2}(-1)$,

and

v = $\dfrac{p_2^{r_2+s_2+r_2'}}{r_2+s_2+r_2'}$.

Hence

$$A=\frac{p_1^{r_2+s_2+r_2'}(1-p_1)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}}{r_2+s_2+r_2'}+\left\{\frac{(n_2-r_2+m_2-s_2+n_2'-r_2'-1)}{r_2+s_2+r_2'}\right.$$

$$\left.\int_0^{p_1} p_2^{r_2+s_2+r_2'}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-2}\,dp_2\right\}.$$

Now ,let

$$B=\int_0^{p_1} p_2^{r_2+s_2+r_2'}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-2}\,dp_2 .$$

Using integration by parts with

$u=(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-2}$ and $dv=p_2^{r_2+s_2+r_2'}dp_2$,

we implies that

$du=(n_2-r_2+m_2-s_2+n_2'-r_2'-2)(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-3}dp_2$ (-١),

and

$$v=\frac{p_2^{r_2+s_2+r_2'+1}}{r_2+s_2+r_2'+1} .$$

So,

$$B=\frac{p_1^{r_2+s_2+r_2'+1}(1-p_1)^{n_2-r_2+m_2-s_2+n_2'-r_2'-2}}{r_2+s_2+r_2'+1}+$$

$$\left\{\frac{(n_2-r_2+m_2-s_2+n_2'-r_2'-2)}{r_2+s_2+r_2'+1}\right.$$

$$\int_0^{p_1} p_2^{r_2+s_2+r_2'+1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-3}dp_2\ \}.$$

Then

$$A=\frac{p_1^{r_2+s_2+r_2'}(1-p_1)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}}{r_2+s_2+r_2'}+\{\frac{(n_2-r_2+m_2-s_2+n_2'-r_2'-1)}{(r_2+s_2+r_2')(r_2+s_2+r_2'+1)}$$

$$p_1^{r_2+s_2+r_2'+1}(1-p_1)^{n_2-r_2+m_2-s_2+n_2'-r_2'-2}\}$$

$$+\frac{(n_2-r_2+m_2-s_2+n_2'-r_2'-1)}{r_2+s_2+r_2'})$$

$$(\frac{(n_2-r_2+m_2-s_2+n_2'-r_2'-2)}{r_2+s_2+r_2'+1})\int_0^{p_1} p_2^{r_2+s_2+r_2'+1}$$

$$(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-3}dp_2\ \}.$$ And

$$A=\sum_{j=r_2+s_2+r_2'}^{n_2+m_2+n_2'-1}\frac{(r_2+s_2+r_2'-1)!(n_2-r_2+m_2-s_2+n_2'-r_2'-1)!}{j!(n_2+m_2+n_2'-j-1)!}$$

$$p_1^j(1-p_1)^{(n_2+m_2+n_2'-j-1)}$$

Therefore

$$TST_1=\frac{k_1(n_1+m_1+n_1'-1)!(n_2+m_2+n_2'-1)!}{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}$$

$$\frac{1}{(r_2+s_2+r_2^{'}-1)!(n_2-r_2+m_2-s_2+n_2^{'}-r_2^{'}-1)!}$$

$$\int_0^1 p_1^{r_1+s_1+r_1^{'}-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'}-1}$$

$$\sum_{j=r_2+s_2+r_2^{'}}^{n_2+m_2+n_2^{'}-1}\frac{(r_2+s_2+r_2^{'}-1)!(n_2-r_2+m_2-s_2+n_2^{'}-r_2^{'}-1)!}{j!(n_2+m_2+n_2^{'}-j-1)!}$$

$$p_1^{j}(1-p_1)^{(n_2+m_2+n_2^{'}-j-1)}dp_1$$

$$= \frac{k_1(n_1+m_1+n_1^{'}-1)!(n_2+m_2+n_2^{'}-1)!}{(r_1+s_1+r_1^{'}-1)!(n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'}-1)!}$$

$$\frac{(r_2+s_2+r_2^{'}-1)!(n_2-r_2+m_2-s_2+n_2^{'}-r_2^{'}-1)!}{(r_2+s_2+r_2^{'}-1)!(n_2-r_2+m_2-s_2+n_2^{'}-r_2^{'}-1)!}$$

$$\sum_{j=r_2+s_2+r_2^{'}}^{n_2+m_2+n_2^{'}-1}\frac{1}{j!(n_2+m_2+n_2^{'}-j-1)!}$$

$$\int_0^1 p_1^{r_1+s_1+r_1^{'}+j-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'}+n_2+m_2+n_2^{'}-j-2}dp_1$$

$$TST_1 = \frac{k_1(n_1+m_1+n_1^{'}-1)!(n_2+m_2+n_2^{'}-1)!}{(r_1+s_1+r_1^{'}-1)!(n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'}-1)!}$$

$$\sum_{j=r_2+s_2+r_2^{'}}^{n_2+m_2+n_2^{'}-1}\frac{B(r_1+s_1+r_1^{'}+j,n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'}+n_2+m_2+n_2^{'}-j-1)}{j!(n_2+m_2+n_2^{'}-j-1)!}$$

Similarly

$$TST_2(\underline{r},\underline{n},\underline{s},\underline{m},d_2) = \int_0^1 \int_0^{P_2} L_2(d_2,\underline{p})\pi[\underline{p}/\underline{r},\underline{n},\underline{s},\underline{m}]dp_1dp_2$$

$$= \int_0^1 \int_0^{p_2} \frac{k_2 p_1^{r_1+s_1+r_1^{'}-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'}-1}}{B(r_1+s_1+r_1^{'},n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'})}$$

$$\frac{p_2^{r_2+s_2+r_2^{'}-1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2^{'}-r_2^{'}-1}}{B(r_2+s_2+r_2^{'},n_2-r_2+m_2-s_2+n_2^{'}-r_2^{'})} dp_1dp_2$$

$$= \frac{k_2(n_1+m_1+n_1^{'}-1)!}{(r_1+s_1+r_{1'}^{'}-1)!(n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'}-1)!}$$

$$\frac{(n_2+m_2+n_2^{'}-1)!}{(r_2+s_2+r_2^{'}-1)!(n_2-r_2+m_2-s_2+n_2^{'}-r_2^{'}-1)!}$$

$$\int_0^1 p_2^{r_2`+s_2+r_2^{'}-1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2^{'}-r_2^{'}-1}$$

$$\int_0^{p_2} p_1^{r_1+s_1+r_1^{'}-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'}-1}dp_1dp_2 \quad .$$

Let $A = \int_0^{p_2} p_1^{r_1+s_1+r_1^{'}-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'}-1} dp_1$.

Using integration by parts with

$u = (1-p_1)^{n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'}-1}$ and $dv = p_1^{r_1+s_1+r_1^{'}-1}dp_1$,

we implies that

du= $(n_1 - r_1 + m_1 - s_1 + n_1^{'} - r_1^{'} - 1)(1 - p_1)^{n_1 - r_1 + m_1 - s_1 + n_1^{'} - r_1^{'} - 2} dp_1(-1)$,

and

$$v = \frac{p_1^{r_1 + s_1 + r_1^{'}}}{(r_1 + s_1 + r_1^{'})} \ .$$

Hence

$$A= \frac{p_2^{r_1+s_1+r_1^{'}}(1\text{-}\,p_2)^{n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'}-1}}{(r_1+s_1+r_1^{'})} + \left\{ \frac{(n_1 - r_1 + m_1 - s_1 + n_1^{'} - r_1^{'} - 1)}{(r_1 + s_1 + r_1^{'})} \right.$$

$$\left. \int_0^{p_2} p_1^{r_1 + s_1 + r_1^{'}} (1 - p_1)^{n_1 - r_1 + m_1 - s_1 + n_1^{'} - r_1^{'} - 2} dp_1 \right\} .$$

Now ,let

$$B = \int_0^{p_2} p_1^{r_1 + s_1 + r_1^{'}} (1 - p_1)^{n_1 - r_1 + m_1 - s_1 + n_1^{'} - r_1^{'} - 2} dp_1$$

using integration by parts with

u= $(1 - p_1)^{n_1 - r_1 + m_1 - s_1 + n_1^{'} - r_1^{'} - 2}$ and dv= $p_1^{r_1 + s_1 + r_1^{'}} dp_1(-1)$,

we implies that

du= $(n_1 - r_1 + m_1 - s_1 + n_1^{'} - r_1^{'} - 2)(1 - p_1)^{n_1 - r_1 + m_1 - s_1 + n_1^{'} - r_1^{'} - 3} dp_1(-1)$,

and

$$v= \frac{p_1^{r_1 + s_1 + r_1^{'} + 1}}{(r_1 + s_1 + r_1^{'} + 1)} \ .$$

So,

$$B=\frac{p_2^{r_1+s_1+r_1'+1}(1-p_2)^{n_1-r_1+m_1-s_1+n_1'-r_1'-2}}{(r_1+s_1+r_1'+1)}$$

$$+\{\frac{(n_2-r_2+m_2-s_2+n_2'-r_2'-2)}{(r_1+s_1+r_1'+1)}$$

$$\int_0^{p_2} p_1^{r_1+s_1+r_1'+1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-3}dp_1 \quad \}.$$

Then

$$A=\frac{p_2^{r_1+s_1+r_1'}(1-p_2)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}}{(r_1+s_1+r_1')}+\{\frac{(n_1-r_1+m_1-s_1+n_1'-r_1'-1)}{(r_1+s_1+r_1')(r_1+s_1+r_1'+1)}$$

$$p_2^{r_1+s_1+r_1'+1}(1-p_2)^{n_1-r_1+m_1-s_1+n_1'-r_1'-2}\}+\{(\frac{(n_1-r_1+m_1-s_1+n_1'-r_1'-1)}{(r_1+s_1+r_1')})$$

$$\frac{(n_1-r_1+m_1-s_1+n_1'-r_1'-2)}{(r_1+s_1+r_1'+1)})\int_0^{p_2} p_1^{r_1+s_1+r_1'+1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-3}dp_1\}$$

$$A=\sum_{j=r_1+s_1+r_1'}^{n_1+m_1+n_1'-1}\frac{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}{j!(n_1+m_1+n_1'-j-1)!}$$

$$p_2^j(1-p_2)^{n_1+m_1+n_1'-j-1}.$$

Therefore

$$TST_2 = \frac{k_2(n_1 + m_1 + n_1' - 1)!}{(r_1 + s_1 + r_1' - 1)!(n_1 - r_1 + m_1 - s_1 + n_1' - r_1' - 1)!}$$

$$\frac{(n_2 + m_2 + n_2' - 1)!}{(r_2 + s_2 + r_2' - 1)!(n_2 - r_2 + m_2 - s_2 + n_2' - r_2' - 1)!}$$

$$\int_0^1 p_2^{r_2` + s_2 + r_2' - 1}(1 - p_2)^{n_2 - r_2 + n_2 - s_2 + n_2' - r_2' - 1}$$

$$\sum_{j = r_1 + s_1 + r_1'}^{n_1 + m_1 + n_1' - 1} \frac{(r_1 + s_1 + r_1' - 1)!(n_1 - r_1 + m_1 - s_1 + n_1' - r_1' - 1)!}{j!(n_1 + m_1 + n_1' - j - 1)!}$$

$$p_2^j(1 - p_2)^{n_1 + m_1 + n_1' - j - 1} dp_2$$

$$= \frac{k_2(n_1 + m_1 + n_1' - 1)!(n_2 + m_2 + n_2' - 1)!}{(r_1 + s_1 + r_1' - 1)!(n_1 - r_1 + m_1 - s_1 + n_1' - r_1' - 1)!}$$

$$\frac{(r_1 + s_1 + r_1' - 1)!(n_1 - r_1 + m_1 - s_1 + n_1' - r_1' - 1)!}{(r_2 + s_2 + r_2' - 1)!(n_2 - r_2 + m_2 - s_2 + n_2' - r_2' - 1)!}$$

$$\sum_{j = r_1 + s_1 + r_1'}^{n_1 + m_1 + n_1' - 1} \frac{1}{j!(n_1 + m_1 + n_1' - j - 1)!}$$

$$\int_0^1 p_2^{r_2 + s_2 + r_2' + j - 1}(1 - p_2)^{n_2 - r_2 + m_2 - s_2 + n_2' - r_2' + n_1 + m_1 + n_1' - j - 2} dp_2$$

$$= \frac{k_2(n_1 + m_1 + n_1' - 1)!(n_2 + m_2 + n_2' - 1)!}{(r_2 + s_2 + r_2' - 1)!(n_2 - r_2 + m_2 - s_2 + n_2' - r_2' - 1)!}$$

$$\sum_{j = r_1 + s_1 + r_1'}^{n_1 + m_1 + n_1' - 1} \frac{B(r_2 + s_2 + r_2' + j, n_1 + m_1 + n_1' + n_2 - r_2 + m_2 - s_2 + n_2' - r_2' - j - 1)}{j!(n_1 + m_1 + n_1' - j - 1)!}$$

Then

$$TST_2 = \frac{k_2(n_1 + m_1 + n_1^{'} - 1)!}{(r_2 + s_2 + r_2^{'} - 1)!(n_2 - r_2 + m_2 - s_2 + n_2^{'} - r_2^{'} - 1)!}$$

$$\frac{(n_2 + m_2 + n_2^{'} - 1)!}{(n_1 + m_1 + n_1^{'} + n_2 + m_2 + n_2^{'} - 2)!}$$

$$\sum_{j=r_1+s_1+r_1^{'}}^{n_{11}+m_1+n_1^{'}-1} \frac{(r_2 + s_2 + r_2^{'} + j - 1)!(n_1 + m_1 + n_1^{'} + n_2 - r_2 + m_2 - s_2 + n_2^{'} - r_2^{'} - j - 2)!}{j!(n_1 + m_1 + n_1^{'} - j - 1)!}$$

(b) Linear loss function

$$TST_1(\underline{r},\underline{n},\underline{s},\underline{m},d_1) = \int_0^1 \int_0^{P_1} L_1(d_1,\underline{p})\pi[\underline{p}/\underline{r},\underline{n},\underline{s},\underline{m}]dp_2 dp_1$$

$$= \int_0^1 \int_0^{p_1} \frac{k_1(p_2 - p_1)p_1^{r_1+m_1+r_1^{'}-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1^{'}-r_1^{'}-1}}{B(r_1 + s_1 + r_1^{'}, n_1 - r_1 + m_1 - s_1 + n_1^{'} - r_1^{'})}$$

$$\frac{p_2^{r_2+s_2+r_2^{'}-1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2^{'}-r_2^{'}-1}}{B(r_2 + s_2 + r_2^{'}, n_2 - r_2 + m_2 - s_2 + n_2^{'} - r_2^{'})} dp_2 dp_1$$

$$= \frac{k_1(n_1 + m_1 + n_1^{'} - 1)!}{(r_1 + s_1 + r_1^{'} - 1)!(n_1 - r_1 + m_1 - s_1 + n_1^{'} - r_1^{'} - 1)!}$$

$$\frac{(n_2 + m_2 + n_2^{'} - 1)!}{(r_2 + s_2 + r_2^{'} - 1)!(n_2 - r_2 + m_2 - s_2 + n_2^{'} - r_2^{'} - 1)!}$$

$$\{\int_0^1 p_1^{r_1+s_1+r_1'-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}$$

$$\int_0^{p_1} p_2^{r_1+s_2+r_2'}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}dp_2dp_1$$

$$-\int_0^1 p_1^{r_1+s_1+r_1'}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}$$

$$\int_0^{p_1} p_2^{r_2+s_2+r_2'-1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}dp_2dp_1 \quad \}$$

Let ,

y١ = $\int_0^1 p_1^{r_1+s_1+r_1'-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}$

$$\int_0^{p_1} p_2^{r_2+s_2+r_2'}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}dp_2dp_1 \ ,$$

and

A= $\int_0^{p_1} p_2^{r_2+s_2+r_2'}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}dp_2$.

Using integration by parts with

u = $(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}$ and dv = $p_2^{r_2+s_2+r_2'}dp_2$,

we implies that

du= $(n_2-r_2+m_2-s_2+n_2'-r_2'-1)(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-2}dp_2(-1)$, and

$$v = \frac{p_2^{r_2+s_2+r_2'+1}}{r_2+s_2+r_2'+1}$$

$$A = \frac{p_1^{r_2+s_2+r_2'+1}(1-p_1)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}}{r_2+s_2+r_2'+1} +$$

$$\{\frac{(n_2-r_2+m_2-s_2+n_2'-r_2'-1)}{r_2+s_2+r_2'+1}$$

$$\int_0^{p_1} p_2^{r_2+s_2+r_2'+1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-2}dp_2\ \}.$$

Further , we let

$$B = \int_0^{p_1} p_2^{r_2+s_2+r_2'+1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-2}dp_2\ .$$

Using integration by parts with

$u = (1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-2}$ and $dv = p_2^{r_2+s_2+r_2'+1}dp_2$,

we implies that

$du = (n_2-r_2+m_2-s_2+n_2'-r_2'-2)(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-3}dp_2$ (-١),

and

$$v = \frac{p_2^{r_2+s_2+r_2'+2}}{r_2+s_2+r_2'+2}\ .$$

Hence ,

$$B=\frac{p_1^{r_2+s_2+r_2'+2}(1-p_1)^{n_2-r_2+m_2-s_2+n_2'-r_2'-2}}{r_2+s_2+r_2'+2}+$$

$$\{\frac{(n_2-r_2+m_2-s_2+n_2'-r_2'-2)}{r_2+s_2+r_2'+2}$$

$$\int_0^{p_1} p_2^{r_2+s_2+r_2'+2}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-3}dp_2\},$$

and

$$A=\frac{p_1^{r_2+s_2+r_2'+1}(1-p_1)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}}{r_2+s_2+r_2'+1}+$$

$$\{\frac{(n_2-r_2+m_2-s_2+n_2'-r_2'-1)}{(r_2+s_2+r_2'+1)(r_2+s_2+r_2'+2)} p_2^{r_2+s_2+r_2'+2}$$

$$(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-2}\}+$$

$$\{\frac{(n_2-r_2+m_2-s_2+n_2'-r_2'-1)}{r_2+s_2+r_2'+1}$$

$$\frac{(n_2-r_2+m_2-s_2+n_2'-r_2'-2)}{r_2+s_2+r_2'+2}\int_0^{p_1} p_2^{r_2+s_2+r_2'+2}$$

$$(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-3}dp_2\},$$

therefore

$$A=\sum_{j=r_2+s_2+r_2'}^{n_2+m_2+n_2'-1}\frac{(r_2+s_2+r_2')!(n_2-r_2+m_2-s_2+n_2'-r_2'-1)!}{(j+1)!(n_2+m_2+n_2'-j-1)!}$$

$$p_1^{j+1}(1-p_1)^{(n_2+m_2+n_2'-j-1)} \; .$$

Then $\quad y_1 = \int_0^1 \; p_1^{r_1+s_1+r_1'-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}$

$$\sum_{j=r_2+s_2+r_2'}^{n_2+m_2+n_2'-1} \frac{(r_2+s_2+r_2')!\,(n_2-r_2+m_2-s_2+n_2'-r_2'-1)!}{(j+1)!\,(n_2+m_2+n_2'-j-1)!}$$

$$p_1^{j+1}(1-p_1)^{(n_2+m_2+n_2'-j-1)}dp_1$$

$$= \sum_{j=r_2+s_2+r_2'}^{n_2+m_2+n_2'-1} \frac{(r_2+s_2+r_2')!(n_2-r_2+m_2-s_2+n_2'-r_2'-1)!}{(j+1)!(n_2+m_2+n_2'-j-1)!}$$

$$\int_0^1 \; p_1^{r_1+s_1+r_1'+j}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'+n_2+m_2+n_2'-j-2}dp_1$$

$$= \sum_{j=r_2+s_2+r_2'}^{n_2+m_2+n_2'-1} \frac{(r_2+s_2+r_2')!(n_2-r_2+m_2-s_2+n_2'-r_2'-1)!}{(j+1)!(n_2+m_2+n_2'-j-1)!}$$

$$B(r_1+s_1+r_1'+j, n_1-r_1+m_1-s_1+n_1'-r_1'+n_2+m_2+n_2'-j-1)$$

$$y_1 = \sum_{j=r_2+s_2+r_2'}^{n_2+m_2+n_2'-1} \frac{(r_2+s_2+r_2')!(n_2-r_2+m_2-s_2+n_2'-r_2'-1)!}{(j+1)!(n_2+m_2+n_2'-j-1)!}$$

$$\frac{(r_1+s_1+r_1'+j)!(n_1-r_1+m_1-s_1+n_1'-r_1'+n_2+m_2+n_2'-j-2)!}{(n_1+m_1+n_1'+n_2+m_2+n_2'-1)!}$$

.

Similarly

$$y۲=\int_0^1 p_1^{r_1+s_1+r_1'}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}$$

$$\int_0^{p_1} p_2^{r_2+s_2+r_2'-1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}dp_2dp_1$$

$$A=\int_0^{p_1} p_2^{r_2+s_2+r_2'-1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}dp_2$$

$$=\sum_{j=r_1+s_1+r_1'}^{n_1+m_1+n_1'-1}\frac{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}{j!(n_1+m_1+n_1'-j-1)!}$$

$$p_2^j(1-p_2)^{n_1+m_1+n_1'-j-1}$$

$$y۲=\int_0^1 p_1^{r_1+s_1+r_1'}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}$$

$$\sum_{j=r_1+s_1+r_1'}^{n_1+m_1+n_1'-1}\frac{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}{j!(n_{11}+n_{12}+n_1'-j-1)!}p_1^j(1-p_1)^{n_2+m_2+n_2'+j-1}$$

$$=\sum_{j=r_1+s_1+r_1'}^{n_1+m_1+n_1'-1}\frac{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}{j!(n_1+m_1+n_1'-j-1)!}$$

$$\int_0^1 p_1^{r_1+s_1+r_1'+j}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'+n_2+m_2+n_2'-j-2}dp_1$$

$$=\sum_{j=r_1+s_1+r_1'}^{n_1+m_1+n_1'-1}\frac{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}{j!(n_1+m_1+n_1'-j-1)!}$$

$$B(r_1+s_1+r_1'+j+1,n_1-r_1+m_1-s_1+n_1'-r_1'+n_2+m_2+n_2'-j-1)$$

$$y٢ = \sum_{j=r_1+s_1+r_1'}^{n_1+m_1+n_1'-1} \frac{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}{j!(n_1+m_1+n_1'-j-1)!}$$

$$\frac{(r_1+s_1+r_1'+j)!(n_1-r_1+m_1-s_1+n_1'-r_1'+n_2+m_2+n_2'-j-2)!}{(n_1+m_1+n_1'+n_2+m_2+n_2'-1)!}$$

Then

$$TST_1 = \frac{k_1(n_1+m_1+n_1'-1)!}{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}$$

$$\frac{(n_2+m_2+n_2'-1)!}{(n_1+m_1+n_1'+n_2+m_2+n_2'-1)!}$$

$$\sum_{j=r_2+s_2+r_2'}^{n_2+m_2+n_2'-1} \frac{(r_1+s_1+r_1'+j)!(n_1-r_1+m_1-s_1+n_1'-r_1'+n_2+m_2+n_2'-j-2)!}{(j+1)!(n_2+m_2+n_2'-j-1)!}$$

$$\{r_2+s_2+r_2'-j-1\}$$

Similarly

$$TST_2(\underline{r},\underline{n},\underline{s},\underline{m},d_2) = \int_0^1 \int_0^{P_2} L_2(d_2,\underline{p})\pi[\underline{p}/\underline{r},\underline{n},\underline{s},\underline{m}]dp_1dp_2$$

$$= \int_0^1 \int_0^{p_2} \frac{k_2(p_1 - p_2)p_1^{r_1+s_1+r_1'-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}}{B(r_1+s_1+r_1', n_1-r_1+m_1-s_1+n_1'-r_1')}$$

$$\frac{p_2^{r_2+s_2+r_2'-1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1}}{B(r_2+s_2+r_2', n_2-r_2+m_2-s_2+n_2'-r_2')} dp_1 dp_2$$

$$= \frac{k_2(n_1+m_1+n_1'-1)!}{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}$$

$$\frac{(n_2+m_2+n_2'-1)!}{(r_2+s_2+r_2'-1)!(n_2-r_2+m_2-s_2+n_2'-r_2'-1)!}$$

$$\{\int_0^1 p_1^{r_1+s_1+r_1'}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}$$

$$\int_0^{p_2} p_2^{r_2`+s_2+r_2'-1}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1} dp_1 dp_2$$

$$-\int_0^1 p_1^{r_1+s_1+r_1'-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}$$

$$\int_0^{p_2} p_2^{r_2`+s_2+r_2'}(1-p_2)^{n_2-r_2+m_2-s_2+n_2'-r_2'-1} dp_1 dp_2 \quad \} .$$

Let,

$$y_{١} = \int_0^1 p_1^{r_1+s_1+r_1'}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}$$

$$\int_0^{p_2} p_2^{r_2 + s_2 + r_2' - 1}(1-p_2)^{n_2 - r_2 + m_2 - s_2 + n_2' - r_2' - 1} dp_1 dp_2$$

$$= \sum_{j=r_1+s_1+r_1'}^{n_1+m_1+n_1'-1} \frac{(r_1+s_1+r_1')!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}{(j+1)!(n_1+m_1+n_1'-j-1)!}$$

$$\frac{(r_2+s_2+r_2'+j)!(n_1+m_1+n_1'+n_2-r_2+m_2-s_2+n_2'-r_2'-j-2)!}{(n_1+m_1+n_1'+n_2+m_2+n_2'-1)!}$$

and

$$y٢ = \int_0^1 p_1^{r_1+s_1+r_1'-1}(1-p_1)^{n_1-r_1+m_1-s_1+n_1'-r_1'-1}$$

$$\int_0^{p_2} p_2^{r_2 + s_2 + r_2'}(1-p_2)^{n_2 - r_2 + m_2 - s_2 + n_2' - r_2' - 1} dp_1 dp_2$$

$$= \sum_{j=r_1+s_1+r_1'}^{n_1+m_1+n_1'-1} \frac{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}{j!(n_1+m_1+n_1'-j-1)!}$$

$$\frac{(r_2+s_2+r_2'+j)!(n_1+m_1+n_1'+n_2-r_2+m_2-s_2+n_2'-r_2'-j-2)!}{(n_1+m_1+n_1'+n_2+m_2+n_2'-1)!}$$

Then

$$TST_2 = \frac{k_2(n_1+m_1+n_1'-1)!}{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}$$

$$\frac{(n_2+m_2+n_2'-1)!}{(r_2+s_2+r_2'-1)!(n_2-r_2+m_2-s_2+n_2'-r_2'-1)!}$$

$$\left\{\sum_{j=r_1+s_1+r_1'}^{n_1+m_1+n_1'-1}\frac{(r_1+s_1+r_1')!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}{(j+1)!(n_1+m_1+n_1'-j-1)!}\right.$$

$$\frac{(r_2+s_2+r_2'+j)!(n_1+m_1+n_1'+n_2-r_2+m_2-s_2+n_2'-r_2'-j-2)!}{(n_1+m_1+n_1'+n_2+m_2+n_2'-1)!}$$

$$-\sum_{j=r_1+s_1+r_1'}^{n_1+m_1+n_1'-1}\frac{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}{j!(n_1+m_1+n_1'-j-1)!}$$

$$\left.\frac{(r_2+s_2+r_2'+j)!(n_1+m_1+n_1'+n_2-r_2+m_2-s_2+n_2'-r_2'-j-2)!}{(n_1+m_1+n_1'+n_2+m_2+n_2'-1)!}\right\}$$

$$=\frac{k_2(n_1+m_1+n_1'-1)!(n_2+m_2+n_2'-1)!}{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}$$

$$\frac{(r_1+s_1+r_1'-1)!(n_1-r_1+m_1-s_1+n_1'-r_1'-1)!}{(r_2+s_2+r_2'-1)!(n_2-r_2+m_2-s_2+n_2'-r_2'-1)!}$$

$$\sum_{j=r_1+s_1+r_1'}^{n_1+m_1+n_1'-1}\frac{(r_2+s_2+r_2'+j)!(n_1+m_1+n_1'+n_2-r_2+m_2-s_2+n_2'-r_2'-j-2)!}{(j+1)!(n_1+m_1+n_1'-j-1)!}$$

$$\frac{\{r_1+s_1+r_1'-j-1\}}{(n_1+m_1+n_1'+n_2+m_2+n_2'-1)!}\;.$$

Therefore

$$TST_2=\frac{k_2(n_1+m_1+n_1'-1)!}{(r_2+s_2+r_2'-1)!(n_2-r_2+m_2-s_2+n_2'-r_2'-1)!}$$

$$\frac{(n_2+m_2+n_2'-1)!}{(n_1+m_1+n_1'+n_2+m_2+n_2'-1)!}$$

$$\sum_{j=r_1+s_1+r_1'}^{n_1+m_1+n_1'-1} \frac{(r_2+s_2+r_2'+j)!\,(n_1+m_1+n_1'+n_2-r_2+m_2-s_2+n_2'-r_2'-j-2)!}{(j+1)!}$$

$$\frac{\{r_1+s_1+r_1'-j-1\}}{(n_1+m_1+n_1'-j-1)!} \quad .$$

The terminal decision rule of the procedure can be described as follows :

At the point $(\underline{r},\underline{n},\underline{s},\underline{m})$,

take decision d_1 if $TST_1 \leq TST_2$

take decision d_2 if $TST_1 > TST_2$.

Now , to fined the overall risk of this procedure .

٢. We find

$$TM_1(\underline{r},\underline{n},\underline{s},\underline{m}) = \min_{d_i} TST_i(\underline{r},\underline{n},\underline{s},\underline{m},d_i) \quad , \; i=1,2 \; .$$

٣. For fixed $r_1, r_2, \delta_1, \delta_2$ compute

$$TM_2(\underline{r},\delta_1,\delta_2) = E_{\underline{s}/\underline{r}}\, TM_1(\underline{r},\underline{n},\underline{s},\underline{m})$$

$$= \sum_{s_1=0}^{m_1} \sum_{s_2=0}^{m_2} P(\underline{s}/\underline{r},\underline{n},\underline{m})\, TM_1(\underline{r},\underline{n},\underline{s},\underline{m}) \; ,$$

where $P(\underline{s}/\underline{r},\underline{n},\underline{m})$ is the predictive probability density function of s_1, s_2 and given by

$$P(\underline{s}/\underline{r},\underline{n},\underline{m}) = E_{\pi(\underline{p})}[\,P(\underline{r},\underline{s}/\underline{n},\underline{m},\underline{p})]$$

$$= \int_0^1 \int_0^1 \pi(\underline{p}) P(\underline{r}, \underline{s} / \underline{n}, \underline{m}, \underline{p}) dp_1 dp_2 \ ,$$

where ,

$$P(\underline{r}, \underline{s} / \underline{n}, \underline{m}, \underline{p}) = \binom{n_1}{r_1} \binom{n_2}{r_2} \binom{m_1}{s_1} \binom{m_2}{s_2} p_1^{r_1+s_1} (1-p_1)^{n_1+m_1-r_1-s_1} \ p_2^{r_2+s_2}$$

$$(1-p_2)^{n_2+m_2-r_2-s_2}$$

Hence ,

$$P(\underline{s} / \underline{r}, \underline{n}, \underline{m}) = \frac{\binom{n_1}{r_1} \binom{n_2}{r_2} \binom{m_1}{s_1} \binom{m_2}{s_2}}{B(r_1', n_1' - r_1') B(r_2', n_2' - r_2')} \ \frac{B(r_1 + s_1 + r_1', n_1 + m_1 + n_1' - r_1 - s_1 - r_1')}{B(r_1 + r_1', n_1 + n_1' - r_1 - r_1')}$$

$$\frac{B(r_2 + s_2 + r_2', n_2 + m_2 + n_2' - r_2 - s_2 - r_2')}{B(r_2 + r_2', n_2 + n_2' - r_2 - r_2')} \quad .$$

٤. For fixed r_1, r_2, δ_1, compute

$$TM_3(r_1, r_2, \delta_1) = \min_{\delta_2} TM_2(r_1, r_2, \delta_1, \delta_2) \ ,$$

and choose δ_2 which gives this minimum .

٥. At the first stage , choose δ_1 such that

$$TM_4(\delta_1) = E_{\underline{r}} [TM_3(r_1, r_2, \delta_1)]$$

$$= \sum_{r_1=0}^{n_1} \sum_{r_2=0}^{n_2} P(\underline{r}) TM_3(r_1, r_2, \delta_1)$$ is a minimum ,

where ,

$$P(\underline{r}/\underline{n},\underline{p})=\binom{n_1}{r_1}\binom{n_2}{r_2}\,p_1^{r_1}(1-p_1)^{n_1-r_1}\,p_2^{r_2}(1-p_2)^{n_2-r_2}\ ,$$

and

$$P(\underline{r})=E_{\pi(\underline{p})}[P(\underline{r}/\underline{n},\underline{p})]$$

$$=\binom{n_1}{r_1}\binom{n_2}{r_2}\frac{B(r_1+r_1',n_1+n_1'-r_1-r_1')}{B(r_1',n_1'-r_1')}\ \frac{B(r_2+r_2',n_2+n_2'-r_2-r_2')}{B(r_2',n_2'-r_2')}\ .$$

Hence

$$R_{TSD}=\min_{0\le\delta_1\le n} TM_4(\delta_1)$$

$$=\frac{\min\limits_{\delta_1}[\sum\limits_{r_1=0}^{n_1}\sum\limits_{r_2=0}^{n_2}\binom{n_1}{r_1}\binom{n_2}{r_2}\{\min\limits_{\delta_2}\sum\limits_{s_1=0}^{m_1}\sum\limits_{s_2=0}^{m_2}\binom{m_1}{s_1}\binom{m_2}{s_2}\min\limits_{d_i}TST_i(\underline{r},\underline{n},\underline{s},\underline{m},d_i)\}]}{B(r_1+r_1',n_1-r_1+n_1'-r_1')B(r_2+r_2',n_2-r_2+n_2'-r_2')}\ .$$

٢.٣.٢ Numerical results

Some numerical work has been carried out to investigate the two-stage procedure (TSTAGE-D) . The results are given in tables (١-٢ , ……,٦-٢) . These tables show that the risks decrease as N increases ,keeping K_1,K_2 and the priors are fixed . For fixed K_1,K_2 and N , the risks decrease as the difference between the priors increases in all cases . As K_1,K_2 increases , keeping N and priors fixed , the risks increase .

Furthermore , from these tables it appears that the risks under linear loss function are smaller than the risks under constant losses .

Table (١-٢)

Values of Bayes risks using two – stage procedures for various numbers of observations N and various constants k١ , k٢ ,

and priors ($r_1{}', n_1{}'$)v($r_2{}', n_2{}'$)=(١,٢)v(١,٢)

k١ , k٢	N	N١ N٢	Bayes Risk using Loss function	
			Constant	Linear
١ , ١	٢	١,١	٠.٢٩١٧	٠.٠٨٣٣
	٤	٢,٢	٠.١٦٣٩	٠.٠٤٣١
	٦	٣,٣	٠.٠٨٨٤	٠.٠٢٧٩
	٨	٤,٤	٠.٠٤٣٠	٠.٠١٨١
٢ , ٢	٢	١,١	٠.٥٨٣٣	٠.١٦٦٧
	٤	٢,٢	٠.٣٢٧٨	٠.٠٨٦١
	٦	٣,٣	٠.١٧٦٨	٠.٠٥٥٧
	٨	٤,٤	٠.٠٨٦٠	٠.٠٣٦٣
٣ , ٣	٢	١,١	٠.٨٧٥٠	٠.٢٥٠٠
	٤	٢,٢	٠.٤٩١٧	٠.١٢٩٢
	٦	٣,٣	٠.٢٦٥٣	٠.٠٨٣٦
	٨	٤,٤	٠.٠٩٢٠	٠.٠٥٤٤

Table (٢-٢)

Values of Bayes risks using two – stage procedures for various numbers of observations N and various constants k١ , k٢ ,

and priors ($r_1{}', n_1{}'$)v($r_2{}', n_2{}'$)=(١,٢)v(١,٣)

k١ , k٢	N	N١ N٢	Bayes Risk using Loss function	
			Constant	Linear
١ , ١	٢	١,١	٠.١٦٦٧	٠.٠٥٥٦
	٤	٢,٢	٠.٠٩٢١	٠.٠٣٧٥
	٦	٣,٣	٠.٠٤٦١	٠.٠٢٦٢
	٨	٤,٤	٠.٠٢٢٥	٠.٠١١١
٢ , ٢	٢	١,١	٠.٣٣٣٤	٠.١١١١
	٤	٢,٢	٠.١٨٤١	٠.٠٧٥٠
	٦	٣,٣	٠.٠٩٢٢	٠.٠٥٣٧

	٨	٤,٤	٠.٠٤٥٠	٠.٠٢٢٢
٣ , ٣	٢	١,١	٠.٥٠٠٠	٠.١٦٦٧
	٤	٢,٢	٠.٢٧٦٢	٠.١١٢٥
	٦	٣,٣	٠.١٣٨٣	٠.٠٨٠٦
	٨	٤,٤	٠.٠٦٧٥	٠.٠٣٣٣

Table (٣-٢)

Values of Bayes risks using two – stage procedures for various numbers of observations N and various constants k١ , k٢ ,

and priors (r_1' ,n_1')v(r_2',n_2')=(١,٢)v(١,٤)

k١ , k٢	N	N١ N٢	Bayes Risk using Loss function	
			Constant	Linear
١ , ١	٢	١,١	٠.٠٨٣٤	٠.٠٤١٧
	٤	٢,٢	٠.٠٣١٠	٠.٠٣٠٤
	٦	٣,٣	٠.٠١٣٨	٠.٠٢٥٩
	٨	٤,٤	٠.٠١٢١	٠.٠١٠٣
٢ , ٢	٢	١,١	٠.١٦٦٧	٠.٠٨٣٤
	٤	٢,٢	٠.٠٦١٩	٠.٠٦٠٧
	٦	٣,٣	٠.٠٢٧٥	٠.٠٥١٧
	٨	٤,٤	٠.٠٢٤٣	٠.٠٢٠٦
٣ , ٣	٢	١,١	٠.٢٥٠٠	٠.١٢٥٠
	٤	٢,٢	٠.٠٩٢٩	٠.٠٩١١
	٦	٣,٣	٠.٠٤١٣	٠.٠٧٧٦
	٨	٤,٤	٠.٠٣٦٤	٠.٠٣١٨

Table (٤-٢)

Values of Bayes risks using two – stage procedures for various numbers of observations N and various constants k١ , k٢ ,

and priors (r_1' ,n_1')v(r_2',n_2')=(١,٢)v(١,٢)

k١ , k٢	N	N١	Bayes Risk using Loss function

		N٢	Constant	Linear
١ , ٢	٢	١,١	٠.٣٧٥٠	٠.٠٥٧٣
	٤	٢,٢	٠.٢٢٢٢	٠.٠٣٩٧
	٦	٣,٣	٠.١٢٤١	٠.٠٣٥٦
	٨	٤,٤	٠.٠٦٥٩	٠.٠٢٣٥
١ , ٣	٢	١,١	٠.٤١٦٧	٠.١١١١
	٤	٢,٢	٠.٢٥٠٠	٠.٠٧٠٣
	٦	٣,٣	٠.١٥٣١	٠.٠٤٧٤
	٨	٤,٤	٠.٠٨٠٦	٠.٠٢٨٢
١ , ٤	٢	١,١	٠.٤٥٨٣	٠.١٢٧٨
	٤	٢,٢	٠.٢٧٧٨	٠.٠٧٩٦
	٦	٣,٣	٠.١٧٢٠	٠.٠٥٥٧
	٨	٤,٤	٠.٠٩٢٠	٠.٠٣٤٢

Table (٥-٢)

Values of Bayes risks using two – stage procedures for various numbers of observations N and various constants k١ , k٢ ,

and priors (r_1' , n_1')v(r_2' , n_2')=(١,٢)v(١,٣)

k١ , k٢	N	N١ N٢	Bayes Risk using Loss function	
			Constant	Linear
١ , ٢	٢	١,١	٠.٢٨٣٣	٠.٠٨٣٣
	٤	٢,٢	٠.١٤٦٠	٠.٠٥٦٠
	٦	٣,٣	٠.٠٧٤٦	٠.٠٣٧١
	٨	٤,٤	٠.٠٣٩٢	٠.٠٢١٠
١ , ٣	٢	١,١	٠.٣٦٦٧	٠.١٠٠٠
	٤	٢,٢	٠.١٨٦٥	٠.٠٦٤٧
	٦	٣,٣	٠.٠٩٤٤	٠.٠٤٦٤
	٨	٤,٤	٠.٠٥٢١	٠.٠٢٦٧
١ , ٤	٢	١,١	٠.٤٣٣٣	٠.١٠٨٣
	٤	٢,٢	٠.٢١٥٩	٠.٠٧٣٤
	٦	٣,٣	٠.١٠٩٠	٠.٠٥٣٩
	٨	٤,٤	٠.٠٦٢٥	٠.٠٢٨٩

Table (٦-٢)

Values of Bayes risks using two – stage procedures for various numbers of observations N and various constants k١ , k٢ ,

and priors (r_1' , n_1')v(r_2' , n_2')=(١,٢)v(١,٤)

k١ , k٢	N	N١ N٢	Bayes Risk using Loss function	
			Constant	Linear
١ , ٢	٢	١,١	٠.١٥٨٣	٠.٠٦٤٣
	٤	٢,٢	٠.٠٥٦٥	٠.٠٤٦٩
	٦	٣,٣	٠.٠٢٥٣	٠.٠٣٥٦
	٨	٤,٤	٠.٠٢٤٣	٠.٠١٨٤
١ , ٣	٢	١,١	٠.٢١٢٥	٠.٠٧٨٦
	٤	٢,٢	٠.٠٧٧٧	٠.٠٥٨٨
	٦	٣,٣	٠.٠٣٤٠	٠.٠٤٢٧
	٨	٤,٤	٠.٠٢٦٤	٠.٠٢٣٢
١ , ٤	٢	١,١	٠.٢٦٦٧	٠.٠٩٢٩
	٤	٢,٢	٠.٠٩٧٠	٠.٠٧٠٣
	٦	٣,٣	٠.٠٤١٨	٠.٠٤٩٨
	٨	٤,٤	٠.٠٣٨٦	٠.٠٢٥٧

٢.٤ Bayesian one-stage selection procedure (OSTAGE-D)

In this section the Bayesian one-stage procedure to the problem of selecting the better of two Binomial populations is developed to be used for comparison with the two –stage procedure . One-stage procedure means that exactly N observations are taken . Furthermore , N is partitioned into N_1 and N_2 , the number of observations taken from populations Π_1 and Π_2 respectively .

Let p_i , $i = 1,2$ is assumed to have prior probability with parameters $n_i^{'}$, $r_i^{'}$, Beta $(r_i^{'}, n_i^{'})$. Let R_1 and R_2 be the number of successes of Π_1 and Π_2 respectively .

Based on N_1 and N_2 observations , to determine which decision is optimal at the point $(R_1 + r_1^{'}, N_1 + n_1^{'}, R_2 + r_2^{'}, N_2 + n_2^{'})$, where $0 \le R_i \le 1$, $i = 1,2$, we have to obtain the posterior expected losses of the decisions and the optimal decision is the decision associated with the smaller posterior expected decision loss (smaller risk) . Furthermore , let δ be a decision of what size N_1 and then ($N_2 = N - N_1$) , where $0 \le \delta \le N$.

Let ,

$$P(\underset{-}{p}) = \frac{P_1^{r_1^{'}-1}(1-p_1)^{n_1^{'}-r_1^{'}-1}\, p_2^{r_2^{'}-1}(1-p_2)^{n_2^{'}-r_2^{'}-1}}{B(r_1^{'}, n_1^{'}-r_1^{'})B(r_2^{'}, n_2^{'}-r_2^{'})} \ ,$$

where $r_1^{'}, r_2^{'}, n_1^{'}$ and $n_2^{'}$ are positive integers . $B(r_1^{'}, n_1^{'}-r_1^{'})$ and $B(r_2^{'}, n_2^{'}-r_2^{'})$ are beta functions and $0 \le p_i \le 1$, $i = 1,2$.

Since R_1 and R_2 each has binomial distribution , hence

$$P(\underset{-}{p}/R_1, R_2, N_1, N_2) = \frac{P_1^{R_1+r_1^{'}-1}(1-p_1)^{N_1-R_1+n_1^{'}-r_1^{'}-1}}{B(R_1+r_1^{'}, N_1-R_1+n_1^{'}-r_1^{'})}$$

$$\frac{p_2^{R_2+r_2^{'}-1}(1-p_2)^{N_2-R_2+n_2^{'}-r_2^{'}-1}}{B(R_2+r_2^{'}, N_2-R_2+n_2^{'}-r_2^{'})}$$

٢.٤.١ Construction of the procedure (OSTAGE-D)

At the point $(R_1+r_1',N_1+n_1',R_2+r_2',N_2+n_2')$, that is for fixed R_1,R_2 and δ . The posterior expected loss of taking decision d_i, $OST_i(R_1,R_2,\delta,d_i)$, is given by

$$OST_i(R_1,R_2,\delta,d_i) = E_{\pi(\underline{p}/R_1,N_1,R_2,N_2)}[L_i(d_i,\underline{p})] , \qquad i=1,2$$

where $L_i(d_i,\underline{p})$ is the loss function for the decision d_i and $\pi(\underline{p}/R_1,N_1,R_2,N_2)$ is the joint posterior probability density of p_1 and p_2 with respect to which the expectation is being performed .

The posterior expected losses , under constant and linear losses, are computed as follows .

(i) Constant loss function

The posterior expected losses OST_1 and OST_2 of making decision d_1 and d_2 respectively , using constant losses , are obtained as follows .

$$OST_1(R_1,R_2,\delta,d_1)=\int_0^1\int_0^{P_1} L_1(d_1,\underline{p})P(\underline{p}/(R_1,R_2))dp_2dp_1$$

$$=\int_0^1\int_0^{p_1}\frac{k_1p_1^{R_1+r_1'-1}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}p_2^{R_2+r_2'-1}(1-p_2)^{N_2-R_2+n_2'-r_2'-1}}{B(R_1+r_1',N_1-R_1+n_1'-r_1')B(R_2+r_2',N_2-R_2+n_2'-r_2')}dp_2dp_1$$

$$=\frac{k_1(N_1+n_1'-1)!(N_2+n_2'-1)!}{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!}$$

$$\int_0^1 p_1^{R_1+r_1'-1}(1-p_1)^{N_1-R_1+n_1'-r_1'-1} \quad \int_0^{p_1} p_2^{R_2+r_2'-1}(1-p_2)^{N_2-R_2+n_2'-r_2'-1}\,dp_2\,dp_1$$

.

Let ,

A= $\int_0^{p_1} p_2^{R_2+r_2'-1}(1-p_2)^{N_2-R_2+n_2'-r_2'-1}dp_2$.

Using integration by parts with

u = $(1-p_2)^{N_2-R_2+n_2'-r_2'-1}$ and dv = $p_2^{R_2+r_2'-1}dp_2$,

we implies that

du= $(N_2-R_2+n_2'-r_2'-1)(1-p_2)^{N_2-R_2+n_2'-r_2'-2}dp_2$ (-١) ,

and

v = $\dfrac{p_2^{R_2+r_2'}}{R_2+r_2'}$.

Hence ,

$$A=\frac{p_1^{R_2+r_2'}(1-p_1)^{N_2-R_2+n_2'-r_2'-1}}{R_2+r_2'}+\{\frac{(N_2-R_2+n_2'-r_2'-1)}{R_2+r_2'}$$

$$\int_0^{p_1} p_2^{R_2+r_2'}(1-p_2)^{N_2-R_1+n_2'-r_2'-2}dp_2\ \} .$$

Now , let

$$B = \int_{0}^{p_1} p_2^{R_2+r_2'}(1-p_2)^{N_2-R_2+n_2'-r_2'-2}\,dp_2 \ .$$

Using integration by parts with

u= $(1-p_2)^{N_2-R_2+n_2'-r_2'-2}$ and dv= $p_2^{R_2+r_2'}\,dp_2$,

we implies that

du= $(N_2-R_2+n_2'-r_2'-2)(1-p_2)^{N_2-R_1+n_2'-r_2'-3}\,dp_2$ (-١),

and

v= $\dfrac{p_2^{R_2+r_2'+1}}{R_2+r_2'+1}$.

So,

$$B= \frac{p_1^{N_1+r_2'+1}(1-p_1)^{N_2-R_2+n_2'-r_2'-2}}{R_2+r_2'+1} + \left\{ \frac{(N_2-R_2+n_2'-r_2'-2)}{R_2+r_2'+1} \int_{0}^{p_1} p_2^{R_1+r_2'+1}(1-p_2)^{N_2-R_1+n_2'-r_2'-3}\,dp_2 \right\}.$$

Then

$$A = \frac{p_1^{R_2+r_2'}(1-p_1)^{N_2-R_2+n_2'-r_2'-1}}{R_2+r_2'} + \{ \frac{(N_2-R_2+n_2'-r_2'-1)}{(R_2+r_2')(R_2+r_2'+1)}$$

$$p_1^{R_2+r_2'+1}(1-p_1)^{N_2-R_2+n_2'-r_2'-2}\} + \{ (\frac{(N_2-R_2+n_2'-r_2'-1)}{R_2+r_2'})$$

$$(\frac{(N_2-R_2+n_2'-r_2'-2)}{R_2+r_2'+1}) \int_0^{p_1} p_2^{R_2+r_2'+1}(1-p_2)^{N_2-R_2+n_2'-r_2'-3} dp_2 \}$$

and

$$A = \sum_{j=R_2+r_2'}^{N_2+n_2'-1} \frac{(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!}{j!(N_2+n_2'-j-1)!} p_1^j (1-p_1)^{(N_2+n_2'-j-1)} .$$

Therefore

$$OST_1 = \frac{k_1(N_1+n_1'-1)!(N_2+n_2'-1)!}{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!}$$

$$\int_0^1 p_1^{R_1+r_1'-1}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}$$

$$\sum_{j=R_2+r_2'}^{N_1+n_2'-1} \frac{(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!}{j!(N_2+n_2'-j-1)!} p_1^j (1-p_1)^{(N_2+n_2'-j-1)} dp_1$$

$$= \frac{k_1(N_1+n_1'-1)!(N_2+n_2'-1)!(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!}{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!}$$

$$\sum_{j=R_2+r_2'}^{N_2+n_2'-1} \frac{1}{j!(N_2+n_2'-j-1)!}$$

$$\int_0^1 p_1^{R_1+r_1'+j-1}(1-p_1)^{N_1-R_1+n_1'-r_1'+N_2+n_2'-j-2}\,dp_1$$

$$OST_1 = \frac{k_1(N_1+n_1'-1)!(N_2+n_2'-1)!}{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!}$$

$$\sum_{j=R_2+r_2'}^{N_2+n_2'-1} \frac{B(R_1+r_1'+j, N_1-R_1+n_1'-r_1'+N_2+n_2'-j-1)}{j!(N_2+n_2'-j-1)!}\,.$$

Then

$$OST_1 = \frac{k_1(N_1+n_1'-1)!(N_2+n_2'-1)!}{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!(N_1+n_1'+N_2+n_2'-2)!}$$

$$\sum_{j=R_2+r_2'}^{N_1+n_2'-1} \frac{(R_1+r_1'+j-1)!(N_1-R_1+n_1'-r_1'+N_2+n_2'-j-2)!}{j!(N_2+n_2'-j-1)!}\,.$$

Similarly

$$OST_2(R_1,R_2,\delta,d_2) = \int_0^1 \int_0^{P_2} L_1(d_2,\underline{p})P(\underline{p}/(R_1,R_2))dp_1dp_2$$

$$= \int_0^1 \int_0^{p_2} \frac{k_2 p_1^{R_1+r_1'-1}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}\, p_2^{R_2+r_2'-1}}{B(R_1+r_1', N_1-R_1+n_1'-r_1')}$$

$$\frac{(1-p_2)^{N_2-R_2+n_2'-r_2'-1}}{B(R_2+r_2', N_2-R_2+n_2'-r_2')}\,dp_1dp_2$$

=

$$\frac{k_2(N_1+n_1^{'}-1)!(N_2+n_2^{'}-1)!}{(R_1+r_{1'}^{'}-1)!(N_1-R_1+n_1^{'}-r_1^{'}-1)!(R_2+r_2^{'}-1)!(N_2-R_2+n_2^{'}-r_2^{'}-1)!}$$

$$\int_0^1 p_2^{R_2+r_2^{'}-1}(1-p_2)^{N_2-R_2+n_2^{'}-r_2^{'}-1}$$

$$\int_0^{p_2} p_1^{R_1+r_1^{'}-1}(1-p_1)^{N_1-R_1+n_1^{'}-r_1^{'}-1}dp_1dp_2 \ .$$

Let,

A= $\int_0^{p_2} p_1^{R_1+r_1^{'}-1}(1-p_1)^{N_1-R_1+n_1^{'}-r_1^{'}-1}dp_1$.

Using integration by parts with

u = $(1-p_1)^{N_1-R_1+n_1^{'}-r_1^{'}-1}$ and dv = $p_1^{R_1+r_1^{'}-1}dp_1$,

we implies that

du= $(N_1-R_1+n_1^{'}-r_1^{'}-1)(1-p_1)^{N_1-R_1+n_1^{'}-r_1^{'}-2}dp_1$ (-١),

and

v = $\frac{p_1^{R_1+r_1^{'}}}{(R_1+r_1^{'})}$.

Hence,

$$A= \frac{p_2^{R_1+r_1'}(1 - p_2)^{N_1-R_1+n_1'-r_1'-1}}{(R_1+r_1')} +$$

$$\{\frac{(N_1-R_1+n_1'-r_1'-1)}{(R_1+r_1')}\int_0^{p_2} p_1^{R_1+r_1'}(1-p_1)^{N_1-R_1+n_1'-r_1'-2}dp_1\}.$$

Now, let

$$B = \int_0^{p_2} p_1^{R_1+r_1'}(1-p_1)^{N_1-R_1+n_1'-r_1'-2}dp_1 \ ,$$

using integration by parts with

u= $(1-p_1)^{N_1-R_1+n_1'-r_1'-2}$ and dv= $p_1^{R_1+r_1'}dp_1$,

we implies that

du= $(N_1-R_1+n_1'-r_1'-2)(1-p_1)^{N_1-R_1+n_1'-r_1'-3}dp_1$ (-١),

and

v= $\frac{p_1^{R_1+r_1'+1}}{(R_1+r_1'+1)}$

So,

$$B= \frac{p_2^{R_1+r_1'+1}(1 - p_2)^{N_1-R_1+n_1'-r_1'-2}}{(R_1+r_1'+1)} +\{\frac{(N_2-R_2+n_2'-r_2'-2)}{(R_1+r_1'+1)}$$

$$\int_0^{p_2} p_1^{R_1+r_1'+1}(1-p_1)^{N_1-R_1+n_1'-r_1'-3}dp_1\}.$$

Then

$$A=\frac{p_2^{R_1+r_1'}(1-p_2)^{N_1-R_1+n_1'-r_1'-1}}{(R_1+r_{1'}')}+\{\frac{(N_1-R_1+n_1'-r_1'-1)}{(R_1+r_1')(R_1+r_1'+1)}$$

$$p_2^{R_1+r_1'+1}(1-p_2)^{N_1-R_1+n_1'-r_1'-2}\}+\{(\frac{(N_1-R_1+n_1'-r_1'-1)}{(R_1+r_1')})$$

$$(\frac{(N_1-R_1+n_1'-r_1'-2)}{(rR+r_1'+1)})\int_0^{p_2} p_1^{R_1+r_1'+1}(1-p_1)^{N_1-R_1+n_1'-r_1'-3}dp_1\}$$

$$A=\sum_{j=R_1+r_1'}^{N_1+n_1'-1}\frac{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!}{j!(N_1+n_1'-j-1)!}p_2^j(1-p_2)^{N_1+n_1'-j-1}.$$

Therefore

$$OST_2=\frac{k_2(N_1+n_1'-1)!}{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!}$$

$$\frac{(N_2+n_2'-1)!}{(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!}\int_0^1 p_2^{R_2`+r_2'-1}(1-p_2)^{N_2-R_2+n_2'-r_2'-1}$$

$$\sum_{j=R_1+r_1'}^{N_1+n_1'-1}\frac{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!}{j!(N_1+n_1'-j-1)!}p_2^j(1-p_2)^{N_1+n_1'-j-1}dp_2$$

$$=\frac{k_2(N_1+n_1'-1)!(N_2+n_2'-1)!(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!}{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!}$$

$$\sum_{j=R_1+r_1'}^{N_1+n_1'-1} \frac{1}{j!(N_1+n_1'-j-1)!}$$

$$\int_0^1 p_2^{R_2+r_2'+j-1}(1-p_2)^{N_2-R_2+n_2'-r_2'+N_1+n_1'-j-2}\,dp_2$$

$$= \frac{k_2(N_1+n_1'-1)!(N_2+n_2'-1)!}{(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!}$$

$$\sum_{j=R_1+r_1'}^{N_1+n_1'-1} \frac{B(R_2+r_2'+j, N_1+n_1'+N_2-R_2+n_2'-r_2'-j-1)}{j!(N_1+n_1'-j-1)!}\ .$$

Then

$$OST_2 = \frac{k_2(N_1+n_1'-1)!(N_2+n_2'-1)!}{(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!(N_1+n_1'+N_2+n_2'-2)!}$$

$$\sum_{j=R_1+r_1'}^{N_1+n_1'-1} \frac{(R_2+r_2'+j-1)!(N_1+n_1'+N_2-R_2+n_2'-r_2'-j-2)!}{j!(N_1+n_1'-j-1)!}\ .$$

(ii) Using linear loss function

The posterior expected losses OST_1 and OST_2 of making decision d_1 and d_2 respectively , using linear losses are obtained as follows .

$$OST_1(R_1,R_2,\delta,d_1) = \int_0^1 \int_0^{P_1} L_1(d_1,\underline{p})P(\underline{p}/(R_1,R_2))dp_2dp_1$$

$$= \int_0^1 \int_0^{p_1} \frac{k_1(p_2-p_1)p_1^{R_1+r_1'-1}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}}{B(R_1+r_1',N_1-R_1+n_1'-r_1')}$$

$$\frac{p_2^{R_2+r_2'-1}(1-p_2)^{N_2-R_2+n_2'-r_2'-1}}{B(R_2+r_2',N_2-R_2+n_2'-r_2')}\, dp_2dp_1$$

$$= \frac{k_1(N_1+n_1'-1)!(N_2+n_2'-1)!}{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!}$$

$$\{\int_0^1 p_1^{R_1+r_1'-1}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}$$

$$\int_0^{p_1} p_2^{R_2`+r_2'}(1-p_2)^{N_2-R_2+n_2'-r_2'-1}dp_2dp_1$$

$$-\int_0^1 p_1^{R_1+r_1'}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}$$

$$\int_0^{p_1} p_2^{R_1`+r_2'-1}(1-p_2)^{N_2-R_2+n_2'-r_2'-1}dp_2dp_1 \quad \}.$$

Let,

y١ $= \int_0^1 p_1^{R_1+r_1'-1}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}$

$$\int_0^{p_1} p_2^{R_2+r_2'}(1-p_2)^{N_2-R_2+n_2'-r_2'-1}dp_2dp_1\ ,$$

and

$$A= \int_0^{p_1} p_2^{R_2 + r_2'} (1 - p_2)^{N_2 - R_2 + n_2' - r_2' - 1} dp_2 \ .$$

Using integration by parts with

u = $(1 - p_2)^{N_2 - R_2 + n_2' - r_2' - 1}$ and dv = $p_2^{R_2 + r_2'} dp_2$,

we implies that

du= $(N_2 - R_2 + n_2' - r_2' - 1)(1 - p_2)^{N_2 - R_2 + n_2' - r_2' - 2} dp_2$ (-١) ,

and v = $\dfrac{p_2^{R_2 + r_2' + 1}}{R_2 + r_2' + 1}$.

So,

$$A= \frac{p_1^{R_2 + r_2' + 1}(1 - p_1)^{N_2 - R_2 + n_2' - r_2' - 1}}{R_2 + r_2' + 1} + \{ \frac{(N_2 - R_1 + n_2' - r_2' - 1)}{R_1 + r_2' + 1}$$

$$\int_0^{p_1} p_2^{R_2 + r_2' + 1} (1 - p_2)^{N_2 - R_2 + n_2' - r_2' - 2} dp_2 \} .$$

Further , we let

$$B = \int_0^{p_1} p_2^{R_2 + r_2' + 1} (1 - p_2)^{N_2 - R_2 + n_2' - r_2' - 2} dp_2 \ .$$

Using integration by parts with

u= $(1 - p_2)^{N_2 - R_2 + n_2' - r_2' - 2}$ and dv= $p_2^{R_2 + r_2' + 1} dp_2$,

we implies that

du= $(N_2 - R_2 + n_2^{'} - r_2^{'} - 2)(1 - p_2)^{N_2 - R_1 + n_2^{'} - r_2^{'} - 3} dp_2$ (-١) ,

and

v= $\dfrac{p_2^{R_1 + r_2^{'} + 2}}{R_2 + r_2^{'} + 2}$.

Hence ,

$$B = \frac{p_1^{R_2 + r_2^{'} + 2}(1 - p_1)^{N_2 - R_2 + n_2^{'} - r_2^{'} - 2}}{R_2 + r_2^{'} + 2}\{\frac{(N_2 - R_2 + n_2^{'} - r_2^{'} - 2)}{R_2 + r_2^{'} + 2}$$

$$\int_0^{p_1} p_2^{R_2 + r_2^{'} + 2}(1 - p_2)^{N_2 - R_2 + n_2^{'} - r_2^{'} - 3} dp_2\},$$

and

$$A = \frac{p_1^{R_1 + r_2^{'} + 1}(1 - p_1)^{N_2 - R_2 + n_2^{'} - r_2^{'} - 1}}{R_2 + r_2^{'} + 1} + \{\frac{(N_2 - R_2 + n_2^{'} - r_2^{'} - 1)}{(R_2 + r_2^{'} + 1)(R_2 + r_2^{'} + 2)} \quad p_2^{R_2 + r_2^{'} + 2}$$

$$(1 - p_2)^{N_2 - R_2 + n_2^{'} - r_2^{'} - 2}\}$$

$$+\{(\frac{(N_2 - R_2 + n_2^{'} - r_2^{'} - 1)}{R_2 + r_2^{'} + 1})(\frac{(N_2 - R_2 + n_2^{'} - r_2^{'} - 2)}{R_2 + r_2^{'} + 2})$$

$$\int_{0}^{p_1} p_2^{R_2+r_2'+2}(1-p_2)^{N_2-R_2+n_2'-r_2'-3}\,dp_2\},$$

therefore

$$A=\sum_{j=R_2+r_2'}^{N_2+n_2'-1}\frac{(R_2+r_2')!(N_2-R_2+n_2'-r_2'-1)!}{(j+1)!(N_2+n_2'-j-1)!}p_1^{j+1}(1-p_1)^{(N_2+n_2'-j-1)}.$$

Then

$$y١=\int_{0}^{1} p_1^{R_1+r_1'-1}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}$$

$$\sum_{j=R_2+r_2'}^{N_2+n_2'-1}\frac{(R_2+r_2')!(N_2-R_2+n_2'-r_2'-1)!}{(j+1)!(N_2+n_2'-j-1)!}p_1^{j+1}(1-p_1)^{(N_2+n_2'-j-1)}\,dp_1 =$$

$$\sum_{j=R_2+r_2'}^{N_2+n_2'-1}\frac{(R_1+r_2')!(N_2-R_2+n_2'-r_2'-1)!}{(j+1)!(N_2+n_2'-j-1)!}$$

$$\int_{0}^{1} p_1^{R_1+r_1'+j}(1-p_1)^{N_1-R_1+n_1'-r_1'+n_2+n_2'-j-2}\,dp_1$$

$$=\sum_{j=R_2+r_2'}^{N_1+n_2'-1}\frac{(R_2+r_2')!(N_2-R_2+n_2'-r_2'-1)!}{(j+1)!(N_2+n_2'-j-1)!}$$

$$B(R_1+r_1'+j,N_1-R_1+n_1'-r_1'+N_2+n_2'-j-1)$$

$$y١=\sum_{j=R_1+r_2'}^{N_2+n_2'-1}\frac{(R_2+r_2')!(N_2-R_2+n_2'-r_2'-1)!(R_1+r_1'+j)!}{(j+1)!(N_2+n_2'-j-1)!}$$

$$\frac{(N_1-R_1+n_1'-r_1'+N_2+n_2'-j-2)!}{(N_1+n_1'+N_2+n_2'-1)!}.$$

Similarly

$$y۲=\int_0^1 p_1^{R_1+r_1'}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}$$

$$\int_0^{p_1} p_2^{R_1\text{`}+r_2'-1}(1-p_2)^{N_2-R_2+n_2'-r_2'-1} \quad dp_2\, dp_1 \ .$$

Let ,

$$A=\int_0^{p_1} p_2^{R_2\text{`}+r_2'-1}(1-p_2)^{N_2-R_2+n_2'-r_2'-1}\, dp_2$$

$$=\sum_{j=R_1+r_1'}^{N_1+n_1'-1}\frac{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!}{j!(N_1+n_1'-j-1)!}p_2^{j}(1-p_2)^{N_1+n_1'-j-1} . \qquad y۲=$$

$$\int_0^1 p_1^{R_1+r_1'}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}$$

$$\sum_{j=R_1+r_1'}^{N_1+n_1'-1}\frac{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!}{j!(N_1+n_1'-j-1)!}p_1^{j}(1-p_1)^{N_2+n_2'+j-1}$$

$$=\sum_{j=R_1+r_1'}^{N_1+n_1'-1}\frac{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!}{j!(N_1+n_1'-j-1)!}$$

$$\int_0^1 p_1^{R_1+r_1'+j}(1-p_1)^{N_1-R_1+n_1'-r_1'+N_2+n_2'-j-2}\, dp_1$$

$$=\sum_{j=R_1+r_1'}^{N_1+n_1'-1}\frac{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!}{j!}$$

$$\frac{B(R_1+r_1'+j+1, N_1-R_1+n_1'-r_1'+N_2+n_2'-j-1)}{(N_1+n_1'-j-1)!} \cdot$$

And

$$y۲=\sum_{j=R_1+r_1'}^{N_1+n_1'-1}\frac{(R_1+r_{1'}'-1)!(N_1-R_1+n_1'-r_1'-1)!}{j!(N_1+n_1'-j-1)!}$$

$$\frac{(R_1+r_1'+j)!(N_1-R_1+n_1'-r_1'+N_2+n_2'-j-2)!}{(N_1+n_1'+N_2+n_2'-1)!} \cdot$$

Then

$$OST_1=\frac{k_1(N_1+n_1'-1)!(N_2+n_2'-1)!}{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!(N_1+n_1'+N_2+n_2'-1)!}$$

$$\sum_{j=R_2+r_2'}^{N_2+n_2'-1}\frac{(R_1+r_1'+j)!(N_1-R_1+n_1'-r_1'+N_2+n_2'-j-2)!\{R_2+r_2'-j-1\}}{(j+1)!(N_2+n_2'-j-1)!}$$

Similarly

$$OST_2(R_1,R_2,\delta,d_2)=\int_0^1\int_0^{P_2}L_2(d_2,\underline{p})P(\underline{p}/(R_1,R_2))dp_1dp_2$$

$$=\int_0^1\int_0^{p_2}\frac{k_2(p_1-p_2)p_1^{R_1+r_1'-1}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}}{B(R_1+r_1',N_1-R_1+n_1'-r_1')}$$

$$\frac{p_2^{R_2+r_2'-1}(1-p_2)^{N_2-R_2+n_2'-r_2'-1}}{B(R_2+r_2',N_2-R_2+n_2'-r_2')}dp_1\,dp_2$$

$$=\frac{k_2(N_1+n_1'-1)!(N_2+n_2'-1)!}{(R_1+r_1'-1)!(N_1-R_1+n_1'-r_1'-1)!(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!}$$

$$\{ \int_0^1 p_1^{R_1+r_1'}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}$$

$$\int_0^{p_2} p_2^{R_2`+r_2'-1}(1-p_2)^{N_2-R_2+n_2'-r_2'-1} \quad dp_1\, dp_2$$

$$-\int_0^1 p_1^{R_1+r_1'-1}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}$$

$$\int_0^{p_2} p_2^{R_2`+r_2'}(1-p_2)^{N_2-R_2+n_2'-r_2'-1} \quad dp_1\, dp_2 \} .$$

Let ,

$$y١ = \int_0^1 p_1^{R_1+r_1'}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}$$

$$\int_0^{p_2} p_2^{R_2`+r_2'-1}(1-p_2)^{N_2-R_2+n_2'-r_2'-1} \quad dp_1\, dp_2$$

$$= \sum_{j=R_1+r_1'}^{N_1+n_1'-1} \frac{(R_1+r_1')!\,(N_1-R_1+n_1'-r_1'-1)!}{(j+1)!\,(N_1+n_1'-j-1)!}$$

$$\frac{(R_2+r_2'+j)!\,(N_1+n_1'+N_2-r_2+n_2'-r_2'-j-2)!}{(N_1+n_1'+N_2+n_2'-1)!} \ ,$$

and

$$y٢ = \int_0^1 p_1^{R_1+r_1'-1}(1-p_1)^{N_1-R_1+n_1'-r_1'-1}$$

$$\int_0^{p_2} p_2^{R_2 + r_2'} (1 - p_2)^{N_2 - R_2 + n_2' - r_2' - 1} \quad dp_1\, dp_2$$

$$= \sum_{j=R_1+r_1'}^{N_1+n_1'-1} \frac{(R_1 + r_1' - 1)!(N_1 - R_1 + n_1' - r_1' - 1)!}{j!(N_1 + n_1' - j - 1)!}$$

$$\frac{(R_2 + r_2' + j)!(N_1 + n_1' + N_2 - R_2 + n_2' - r_2' - j - 2)!}{(N_1 + n_1' + N_2 + n_2' - 1)!} .$$

Then

$$OST_2 = \frac{k_2(N_1 + n_1' - 1)!}{(R_1 + r_1' - 1)!(N_1 - R_1 + n_1' - r_1' - 1)!}$$

$$\frac{(N_2 + n_2' - 1)!}{(R_2 + r_2' - 1)!(N_2 - R_2 + n_2' - r_2' - 1)!}$$

$$\{ \sum_{j=R_1+r_1'}^{N_1+n_1'-1} \frac{(R_1 + r_{1'}')!(N_1 - R_1 + n_1' - r_1' - 1)!(R_2 + r_2' + j)!}{(j+1)!(N_1 + n_1' - j - 1)!}$$

$$\frac{(N_1 + n_1' + N_2 - R_2 + n_2' - r_2' - j - 2)!}{(N_1 + n_1' + N_2 + n_2' - 1)!}$$

$$- \sum_{j=R_1+r_1'}^{N_1+n_1'-1} \frac{(R_1 + r_1' - 1)!(N_1 - R_1 + n_1' - r_1' - 1)!}{j!(N_1 + n_1' - j - 1)!}$$

$$\frac{(R_2 + r_2' + j)!(N_1 + n_1' + N_2 - R_2 + n_2' - r_2' - j - 2)!}{(N_1 + n_1' + N_2 + n_2' - 1)!} \}$$

$$= \frac{k_2(N_1 + n_1' - 1)!(N_2 + n_2' - 1)!(R_1 + r_1' - 1)!(N_1 - R_1 + n_1' - r_1' - 1)!}{(R_1 + r_1' - 1)!(N_1 - R_1 + n_1' - r_1' - 1)!(R_2 + r_2' - 1)!(N_2 - R_2 + n_2' - r_2' - 1)!}$$

$$\sum_{j=R_1+r_1'}^{N_1+n_1'-1} \frac{(R_2+r_2'+j)!(N_1+n_1'+N_2-R_2+n_2'-r_2'-j-2)!\{R_1+r_1'-j-1\}}{(j+1)!(N_1+n_1'-j-1)!(N_1+n_1'+N_2+n_2'-1)!}$$

$$= \frac{k_2(N_1+n_1'-1)!(N_2+n_2'-1)!}{(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!(N_1+n_1'+N_2+n_2'-1)!}$$

$$\sum_{j=R_1+r_1'}^{N_1+n_1'-1} \frac{(R_2+r_2'+j)!(N_1+n_1'+N_2-R_2+n_2'-r_2'-j-2)!\{R_1+r_1'-j-1\}}{(j+1)!(N_1+n_1'-j-1)!}$$

Then

$$OST_2 = \frac{k_2(N_1+n_1'-1)!(N_2+n_2'-1)!}{(R_2+r_2'-1)!(N_2-R_2+n_2'-r_2'-1)!(N_1+n_1'+N_2+n_2'-1)!}$$

$$\sum_{j=R_1+r_1'}^{N_1+n_1'-1} \frac{(R_2+r_2'+j)!(N_1+n_1'+N_2-R_2+n_2'-r_2'-j-2)!\{R_1+r_1'-j-1\}}{(j+1)!(N_1+n_1'-j-1)!}$$

The terminal decision rule for OSTAG-D is as follows :

take decision d_1 if $OST_1 \le OST_2$ and

take decision d_2 if $OST_1 > OST_2$.

To compare the Two-stage selection procedure TSTAG-D ,with this procedure , we should find the prior risk and in this case we have to find

١. $OM_1(R_1,R_2,\delta) = \min_{d_i} OST_i(R_1,R_2,\delta,d_i)$ for fixed R_1,R_2 and δ, where $0 \le \delta \le N$ implies specific value of N_1 and hence $N_2 = N - N_1$.

٢. For fixed δ, we compute

$$OM_2(\delta) = E_{R_1,R_2}[OM_1(R_1,R_2,\delta)]$$

$$= \sum_{R_1=0}^{N_1} \sum_{R_2=0}^{N_2} P(R_1,R_2)\, OM_1(R_1,R_2,\delta)$$

where, $P(R_1,R_2)$ is the predictive probability density function of R_1 and R_2 which is given by

$$P(R_1,R_2) = E_{\pi(\underline{p})}[P(R_1,R_2 / N_1,N_2,\underline{p})]$$

$$= \binom{N_1}{R_1}\binom{N_2}{R_2} \frac{B(R_1+r_1^{'},N_1+n_1^{'}-R_1-r_1^{'})}{B(r_1^{'},n_1^{'}-r_1^{'})}$$

$$\frac{B(R_2+r_2^{'},N_2+n_2^{'}-R_2-r_2^{'})}{B(r_2^{'},n_2^{'}-r_2^{'})}$$

where , $\pi(p_i)$ and $P(R_i / N_i, p_i)$, $B(r_1^{'},n_1^{'}-r_1^{'})$ and $B(r_2^{'},n_2^{'}-r_2^{'})$ are defined as before .

٣. Finally , choose δ such that

$$BR_{OSD} = \min_{0\le\delta\le n} [OM_2(\delta)]$$

$$BR_{OSD} = \frac{\min_{\delta}[\sum_{R_1=0}^{N_1}\sum_{R_2=0}^{N_2}\binom{N_1}{R_1}\binom{N_2}{R_2} B(R_1+r_1^{'},N_1+n_1^{'}-R_1-r_1^{'})}{B(r_1+r_1^{'},n_1-r_1+n_1^{'}-r_1^{'})}$$

$$\frac{B(R_2+r_2^{'},N_2+n_2^{'}-R_2-r_2^{'})\{\min_{d_i} OST_i(R_1,R_2,\delta,d_i)\}}{B(r_2+r_2^{'},n_2-r_2+n_2^{'}-r_2^{'})} .$$

٢.٤.٢ Numerical Results

This section contains some numerical results about the procedure (OSTAGE-D) using different sets of K_1, K_2, N and priors for two kinds of loss functions , constant and linear . The results in tables (٢-٧, ……,٢-١٢) show clearly that the risks decrease as N increases with fixed priors , K_1, K_2 and under the constant losses and linear losses . It is also noted that the risks decrease as the difference between the priors increase , where K_1, K_2 and priors are fixed and for both cases , constant losses and linear losses . The same tables also display that the risks under linear losses are smaller than the risks under constant losses .

The comparison of the results concerning the Bayesian two-stage procedures which are given in tables (١-٢,…………,٦-٢) with that concerning the Bayesian one-stage procedure which are given in tables (٧-٢,…………,١٢-٢) show clearly that the performance of the procedure TSTAGE-D is better than the performance of the procedure OSTAGE-D in terms of risks . Therefore we suggest using TSTAGE-D procedure .

Furthermore this procedure and the program which is written to execute it can produce the optimal partition of the observations that allocate to both stage and better the two populations .

Table (٧-٢)

Values of Bayes risks using one-stage procedures for various numbers of observations N , various constants K_1,K_2,

and priors (r_1', n_1')v(r_2', n_2')=(١,٢)v(١,٢) .

k١, k٢	N	Bayes Risk using Loss function	
		Constant	Linear
١ , ١	٢	٠.٣٣٣٣	٠.٠٨٣٤
	٤	٠.٢٥٠٠	٠.٠٥٠٠
	٦	٠.٢١٩١	٠.٠٣٨٩
	٨	٠.١٩٤٥	٠.٠٣٣٣
٢ , ٢	٢	٠.٦٦٦٧	٠.١٦٦٧
	٤	٠.٥٠٠٠	٠.١٠٠٠
	٦	٠.٤٣٨١	٠.٠٧٧٨
	٨	٠.٣٨٩٠	٠.٠٦٦٧
٣ , ٣	٢	١.٠٠٠٠	٠.٢٥٠٠
	٤	٠.٧٥٠٠	٠.١٥٠٠
	٦	٠.٦٥٧٢	٠.١١٦٧
	٨	٠.٥٨٣٣	٠.١٠٠٠

Table (٨-٢)

Values of Bayes risks using one-stage procedures for various numbers of observations N , various constants K_1,K_2,

and priors (r_1' ,n_1')v(r_2',n_2')=(١,٢)v(١,٣) .

k١, k٢	N	Bayes Risk using Loss function	
		Constant	Linear
١ ,١	٢	٠.٢٦٦٧	٠.٠٥٥٦
	٤	٠.٢٢٨٦	٠.٠٤١٧
	٦	٠.٢٠٣٢	٠.٠٣٣٤
	٨	٠.١٩٢٣	٠.٠٢٣٣
٢ , ٢	٢	٠.٥٣٣٤	٠.١١١١
	٤	٠.٤٥٧١	٠.٠٨٣٤
	٦	٠.٤٠٦٤	٠.٠٦٦٧
	٨	٠.٣٨٤٦	٠.٠٤٦٦
٣ , ٣	٢	٠.٨٠٠٠	٠.١٦٦٧
	٤	٠.٦٨٥٧	٠.١٢٥٠
	٦	٠.٦٠٩٤	٠.١٠٠٠
	٨	٠.٥٦٦٨	٠.٠٧١٠

Table (٩-٢)

Values of Bayes risks using one-stage procedures for various numbers of observations N , various constants K_1,K_2,

and priors (r_1' ,n_1')v(r_2',n_2')=(١,٢)v(١,٤) .

k١, k٢	N	Bayes Risk using Loss function	
		Constant	Linear
١ ,١	٢	٠.٢٢٥٠	٠.٠٤١٧
	٤	٠.٢٠٠٠	٠.٠٣٣٤
	٦	٠.١٧٩٨	٠.٠٣٠٠
	٨	٠.١٥٠٠	٠.٠١٤٣
	٢	٠.٤٥٠٠	٠.٠٨٣٤

٢ , ٢	٤	٠.٤٠٠٠	٠.٠٦٦٧
	٦	٠.٣٥٩٥	٠.٠٦٠٠
	٨	٠.٣٠٠٠	٠.٠٣٨٦
٣ , ٣	٢	٠.٦٧٥٠	٠.١٢٥٠
	٤	٠.٦٠٠٠	٠.١٠٠٠
	٦	٠.٥٣٩٣	٠.٩٠٠
	٨	٠.٥٠٣٢	٠.٠٦٢٦

Table (١٠-٢)

Values of Bayes risks using one-stage procedures various numbers of observations N , various constants K_1,K_2,

and prior ($r_1{}'$,$n_1{}'$)v($r_2{}'$,$n_2{}'$)=(١,٢)v(١,٢) .

k١, k٢	N	Bayes Risk using Loss function	
		Constant	Linear
١ ,٢	٢	٠.٤٠٠٠	٠.٠٩١٧
	٤	٠.٣٣٣٤	٠.٠٦٤٣
	٦	٠.٢٩٢٨	٠.٠٤٩٨
	٨	٠.٢٥٤٨	٠.٠٤٠٨
١ , ٣	٢	٠.٤٦٦٧	٠.١٠٠٠
	٤	٠.٣٨٧٣	٠.٠٧٣٠
	٦	٠.٣٤١٣	٠.٠٥٧٩
	٨	٠.٣٠٨٧	٠.٠٤٨٢
١ , ٤	٢	٠.٥٠٠٠	٠.١٠٨٣
	٤	٠.٤٢٨٦	٠.٠٨١٧
	٦	٠.٣٨٢٦	٠.٠٦٦١
	٨	٠.٣٢٣٤	٠.٠٥٥٦

Table (١١-٢)

Values of Bayes risks using one-stage procedures for various numbers of observations N , various constants K_1,K_2, and priors (r_1' ,n_1')v(r_2' ,n_2')=(١,٢)v(١,٣) .

k١, k٢	N	Bayes Risk using Loss function	
		Constant	Linear
١ ,٢	٢	٠.٣٧٥٠	٠.٠٨٣٤
	٤	٠.٣١٦٧	٠.٠٦٢٥
	٦	٠.٢٨٠٤	٠.٠٤٩٦
	٨	٠.٢١٨٣	٠.٠٣٦٧
١ , ٣	٢	٠.٤١٦٧	٠.١١١١
	٤	٠.٣٦٦٧	٠.٠٧٨٦
	٦	٠.٣٣٢٢	٠.٠٦٠٨
	٨	٠.٣٠٠٤	٠.٠٥١٦
١ , ٤	٢	٠.٥٠٠٠	٠.١٢٧٨
	٤	٠.٤٢٨٦	٠.٠٨٧٩
	٦	٠.٣٥٩٥	٠.٠٦٧٩
	٨	٠.٣١٦٧	٠.٠٥٤٧

Table (١٢-٢)

Values of Bayes risks using one-stage procedures for various numbers of observations N , various constants K_1,K_2 ,

and prior (r_1' ,n_1')v (r_2' ,n_2')=(١,٢)v(١,٤) .

k١, k٢	N	Bayes Risk using Loss function	
		Constant	Linear
١ ,٢	٢	٠.٣٣٣٤	٠.٠٦٤٣
	٤	٠.٢٩٠٢	٠.٠٤٩١
	٦	٠.٢٦٠٧	٠.٠٣٩٨
	٨	٠.٢٠٠٠	٠.٠٢٧٧
١ , ٣	٢	٠.٤١٦٧	٠.٠٧٨٦
	٤	٠.٣٦٤٣	٠.٠٦٠٧
	٦	٠.٣٢٢٦	٠.٠٤٩٦
	٨	٠.٢٥٠٠	٠.٠٣٧٢
١ , ٤	٢	٠.٥٠٠٠	٠.٠٩٢٩
	٤	٠.٤٠٧١	٠.٠٧٢٣
	٦	٠.٣٥٨٣	٠.٠٥٩٤

	٨	٠.٣٠١١	٠.٠٤٧٧

Chapter Three

Bayesian Two-stage procedures :

Monte Carlo simulation studies

٣.١ Summary

In this chapter we present Bayesian two-stage procedures for selecting the better of two Binomial populations using Monte Carlo studies (MC) . Description of MC method is given in section ٣.٢ . Section ٣.٣ contains Bayesian two-stage procedures under different sampling rules . Some numerical results are given in section ٣.٤ .

٣.٢ Description of the Monte Carlo (M C) studies

In this section we briefly describe the method of MC simulation method as it is applied to our procedures . Monte Carlo studies have been carried out to investigate some of the performance characteristics of the proposed procedures . Computer programs , which simulate the operations of these procedure , were written in Fortran power station .

The simulation programs perform a large number of runs t (t = ٥٠٠٠) , which are assumed to be independent in order to obtain MC estimates with high precision . At each run mutually independent Bernoulli observations are generated by using the assumed probability model with P_i ($i = 1,2$) specified in advance and then the selection procedure is applied . The observed values of several performance measures are accumulated .

At the end of all runs, these accumulated values are divided by t to obtain the MC estimates of the performance characteristics of interest .

The library function is used to generate a uniform variate $y(0 \le y < 1)$. Population $\prod_i$, with probability of success P_i, scored success if the corresponding random number $y < p_i$ and failure if $y \ge p_i (i = 1,2)$. Formally, a Binomial $b(n,p)$ random variable r can be written as $r = \sum_{i=1}^{n} y_i$, where y_i are independent Bernoulli random variables , each taking the value $y_i = 1$ with probability p or $y_i = 0$ with probability $(1 - p)$. Thus to simulate such an r, we need just simulate n independent $U(0,1)$ random variables $u_1, u_2, \ldots\ldots\ldots, u_n$ and set $y_i = 1$ if $u_i < p$ and $y_i = 0$ if $u_i \ge p$.

The values of P_i ($i = 1,2$) are fixed , where for each run of ٥٠٠٠ trials the same P_i ($i = 1,2$) are used . With the observed value P_i , the value of y can be considered as the observed values of a random variable possessing the Bernoulli distribution that should be simulated .

As measures of performance of the proposed procedure we shall use the following measures :

١.Probability of correct selection P(CS).

In a MC experimentation the population that has the greatest probability of success is known to us , so we can check if the procedure gives a correct selection . After t repetitions we estimate the probability of correct selection by the fraction of correct selection in the t replications .

It can be computed as follows :

P(di / di) : The proportion of number of times when the procedure stops and takes decision di given decision di is true in t repetitions .

P(CS) = $\sum_{i=1}^{2} P(d_i / d_i)$, where $d_1 : p_1 \geq p_2$

$$d_2 : p_1 < p_2 \ .$$

٢.Probability of non correct selection P(NCS).

It can be compute as follows :

$P(d_i / d_j)$: The proportion of number of times when the procedure stops and takes decision d_i given d_j is true ($i, j = 1,2$) in t repetitions.

P(NCS)= $\sum_{i \neq j} P(d_i / d_j) = P(d_1 / d_2) + P(d_2 / d_1)$.

٣. E(R$_i$), expected number of successes from population Π_i .An estimate of E(R$_i$) is given by :

$$E(R_i) = \sum_{j=1}^{t} R_{ij} \Big/ t \quad , \quad i = 1,2 \ ,$$

where R_{ij} is the number of successes gained in the jth run from population Π_i .

٤. E(R), expected number of successes from population ١ and population ٢.

An estimate of E(R) is given by :

E(R) = E(R١) + E(R٢) .

٥. E(R*)=E(R)+(N-N*). Expected number of successes if sampling continues with chosen population for the remaining (N-N*) observations .

٣.٣ Bayesian Two-stage procedures under different sampling rules .

In this section , we present Bayesian two-stage procedures for selecting the better of two binomial populations using simulation . A fixed number of observations N is taken on both populations during the two-stage procedures, where in the first stage a fixed number of observation N* is taken and remaining N-N* observations will be taken in the second stage . Furthermore , we will use the following sampling rules in the first stage .

١. Fixed sample size

In this sampling rule we take N١ observations from Π_1 and N٢ observations from Π_2 , where N* =N١+N٢ . The selection two-stage procedures using this sampling rule , we will called first stage fixed sample size (١st-FS) .

٢. Play-The winner sampling rule (PWR)

This sampling rule chooses one of the populations at random and observes from it until a failure is observed , it then switches and observes from the other population until it yields a failure , in which case we switch to the first population , etc. , continuing until N* observations have been made .

The two-stage procedure which uses this sampling rule will be called first stage play the winner procedure (١st-PWR) .

٣. Group at a time sampling rule (GTR)

This sampling rule takes groups of observations (one each from Π_1 and Π_2) until N* observations have been made . Note that N*=٢nh , where ٢n the size of group , n number of observation taken from each population , h number of group .

The two-stage procedure which uses this sampling rule will be called first stage group at a time procedure (١st-GTR) . We would like to mention that the sampling rule GTR is the same as sampling rule vector at a time when n=١. That is one pairs of observations (one observation from each population) is taken at each time .

In all these procedures the terminal decision rule is as follows :

Take decision d_1 if $\hat{p}_1 \geq \hat{p}_2$

Take decision d_1 if $\hat{p}_1 < \hat{p}_2$.

It is of interest , at this stage , to mention a similar problem , called two-armed band problem . A two – armed bandit consists of two experiments , each may generate identically independent distributed Bernoulli random variables . After each trial the experimenter may use either experiment to generate the next trial . The object of the game is to maximize the expected number of successes in N trials [٢٨] .

٣.٤ Numerical Results

The value of the measures (characteristics) were calculated from the results of Mote Carlo simulations with ٥٠٠٠ trials for fixed values of (p_1, p_2).

In the following we discuss the performance of the procedures ١st-FS , ١st-GTR(n=١) and ١st-PWR under various parameters (p_1, p_2) ,various sample size N , N_1 , N_2 and different priors .

١. **The MC estimates of P(CS)**

Tables (١-٣ ,٢-٣ ,٣-٣) , where p_1=٠.١ and p_2 = ٠.٣(٠.٢)٠.٩ , priors (١,٢)v(١,٢) , N_1=٣ , N_2=٣ and various N , show that P(CS) increases as the difference between p_1 and p_2 increases for the procedures ١st-FS, ١st-GTR(n=١) and ١st-PWR . However , in tables (٤-٣ ,٥-٣ ,٦-٣) we notice that the increasing N does not effect on P(CS) for above procedures since the number of observations on population ١ and population ٢ in the first stage are fixed to N_1=٣ , N_2=٣ for all values of N . Furthermore , the situation is reverse if we take p_1=٠.١(٠.٢)٠.٧ , p_2 = ٠.٣(٠.٢)٠.٩ and p_2- p_1=٠.٢ , that is P(CS) decreases , keeping other quantities are the same as before . The two groups of tables (٧-٣,٨-٣,٩-٣) and (١٠-٣,١١-٣,١٢-٣) are similar in behaviors to the groups of tables (١-٣,٢-٣,٣-٣) and (٤-٣,٥-٣,٦-٣) respectively with little increase in P(CS) due to the increase of N_1 and N_2 .

Tables (١٣-٣,١٤-٣,١٥-٣) show that P(CS) increase under various priors and various N with N_1=٣ , N_2=٣ and (p_1, p_2)=(٠.٣,٠.٥) and when the difference between priors increase . However , tables (١٦-٣,١٧-٣,١٨-٣) indicate that P(CS) is constant if the priors increase with same constant difference between each pair of priors . In tables (١٩-٣) shows the performance of ١st-GT under various group sizes , namely when the group size , ٢n=٢,٦,١٠,٣٠ we notice that P(CS) increase as ٢n increases .

If we compare the procedure ١st-FS , ١st-GTR(n=١) and ١st-PWR in terms of P(CS) , we note that ١st-FS is the best .

٢. **The MC estimates of E(R)**

Tables (١-٣,٢-٣,………….., ١٢-٣) , show that E(R) increase as the difference between p_1 and p_2 increases . Furthermore ,we notice from them that E(R) increase as N increases for all procedures ١st-FS , ١st-GTR(n=١) and ١st-PWR . Tables(١٣-٣,………….,١٨-٣) , show that E(R) , will be constant as the difference between priors increases . Also, we note that E(R) increase as N increases . In tables (١٩-٣) shows the performance of ١st-GT under various group sizes , namely when the group size , ٢n=٢,٦,١٠,٣٠ we notice that E(R) constant as ٢n increases.

If ,we use , the performance measure E(R) in comparison that ١st-PWR is the best among these procedures .

Table (١-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage FS size when parameters

P١ =٠.١ , P٢=٠.٣ (٠.٢) ٠.٩ and various numbers of observations N ,

where , N١=٣ , N٢=٣ and Priors (١,٢)v(١,٢).

N	(P١,P٢)	P(CS)	P(NCS)	E(R)
١٠	(٠.١,٠.٣)	٠.٨٩٢٠	٠.١٠٨٠	٥.٢٠٢٢
	(٠.١,٠.٥)	٠.٩٥٤٤	٠.٠٤٥٦	٥.٧٩١٦
	(٠.١,٠.٧)	٠.٩٨٦٦	٠.٠١٣٤	٦.٣٨٣٦
	(٠.١,٠.٩)	٠.٩٩٨٨	٠.٠٠١٢	٧.٠٠٢٦
٢٠	(٠.١,٠.٣)	٠.٨٩٢٠	٠.١٠٨٠	١٥.٢٠٢٢
	(٠.١,٠.٥)	٠.٩٥٤٤	٠.٠٤٥٦	١٥.٧٩١٦
	(٠.١,٠.٧)	٠.٩٨٦٦	٠.٠١٣٤	١٦.٣٨٣٦
	(٠.١,٠.٩)	٠.٩٩٨٨	٠.٠٠١٢	١٧.٠٠٢٦
٣٠	(٠.١,٠.٣)	٠.٨٩٢٠	٠.١٠٨٠	٢٥.٢٠٢٢
	(٠.١,٠.٥)	٠.٩٥٤٤	٠.٠٤٥٦	٢٥.٧٩١٦
	(٠.١,٠.٧)	٠.٩٨٦٦	٠.٠١٣٤	٢٦.٣٨٣٦
	(٠.١,٠.٩)	٠.٩٩٨٨	٠.٠٠١٢	٢٧.٠٠٢٦
٤٠	(٠.١,٠.٣)	٠.٨٩٢٠	٠.١٠٨٠	٣٥.٢٠٢٢
	(٠.١,٠.٥)	٠.٩٥٤٤	٠.٠٤٥٦	٣٥.٧٩١٦
	(٠.١,٠.٧)	٠.٩٨٦٦	٠.٠١٣٤	٣٦.٣٨٣٦
	(٠.١,٠.٩)	٠.٩٩٨٨	٠.٠٠١٢	٣٧.٠٠٢٦
٥٠	(٠.١,٠.٣)	٠.٨٩٢٠	٠.١٠٨٠	٤٥.٢٠٢٢
	(٠.١,٠.٥)	٠.٩٥٤٤	٠.٠٤٥٦	٤٥.٧٩١٦
	(٠.١,٠.٧)	٠.٩٨٦٦	٠.٠١٣٤	٤٦.٣٨٣٦
	(٠.١,٠.٩)	٠.٩٩٨٨	٠.٠٠١٢	٤٧.٠٠٢٦

Table (٢-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage GTR(n=١) sample when parameters P١ =٠.١ , P٢=٠.٣ (٠.٢) ٠.٩ and various numbers of observations N ,

where , N١=٣ , N٢=٣ and Priors (١,٢)v(١,٢).

N	(P١,P٢)	P(CS)	P(NCS)	E(R)
١٠	(٠.١,٠.٣)	٠.٥٣٨٤	٠.٤٦١٦	٥.٢٠٠٠
	(٠.١,٠.٥)	٠.٧٦١٠	٠.٢٣٩٠	٥.٧٩٦٢
	(٠.١,٠.٧)	٠.٩٠٤٨	٠.١٠٥٢	٦.٣٧٦٨
	(٠.١,٠.٩)	٠.٩٩٣٤	٠.٠٠٦٦	٧.٠٠٢٨
٢٠	(٠.١,٠.٣)	٠.٥٣٨٤	٠.٤٦١٦	١٥.٢٠٠٠
	(٠.١,٠.٥)	٠.٧٦١٠	٠.٢٣٩٠	١٥.٧٩٦٢
	(٠.١,٠.٧)	٠.٩٠٤٨	٠.١٠٥٢	١٦.٣٧٦٨

١٧.٠٠٢٨	٠.٠٠٦٦	٠.٩٩٣٤	(٠.١,٠.٩)	
٢٥.٢٠٠٠	٠.٤٦١٦	٠.٥٣٨٤	(٠.١,٠.٣)	
٢٥.٧٩٦٢	٠.٢٣٩٠	٠.٧٦١٠	(٠.١,٠.٥)	
٢٦.٣٧٦٨	٠.١٠٥٢	٠.٩٠٤٨	(٠.١,٠.٧)	٣٠
٢٧.٠٠٢٨	٠.٠٠٦٦	٠.٩٩٣٤	(٠.١,٠.٩)	
٣٥.٢٠٠٠	٠.٤٦١٦	٠.٥٣٨٤	(٠.١,٠.٣)	
٣٥.٧٩٦٢	٠.٢٣٩٠	٠.٧٦١٠	(٠.١,٠.٥)	
٣٦.٣٧٦٨	٠.١٠٥٢	٠.٩٠٤٨	(٠.١,٠.٧)	
٣٧.٠٠٢٨	٠.٠٠٦٦	٠.٩٩٣٤	(٠.١,٠.٩)	٤٠
٤٥.٢٠٠٠	٠.٤٦١٦	٠.٥٣٨٤	(٠.١,٠.٣)	
٤٥.٧٩٦٢	٠.٢٣٩٠	٠.٧٦١٠	(٠.١,٠.٥)	
٤٦.٣٧٦٨	٠.١٠٥٢	٠.٩٠٤٨	(٠.١,٠.٧)	
٤٧.٠٠٢٨	**٠.٠٠٦٦**	**٠.٩٩٣٤**	**(٠.١,٠.٩)**	٥٠

Table (٣-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage PWR sample when parameters P١ =٠.١ , P٢=٠.٣ (٠.٢) ٠.٩ and various numbers of observations N ,

where , N١=٣ , N٢=٣ and Priors (١,٢)v(١,٢) .

N	(P١,P٢)	P(CS)	P(NCS)	E(R)
١٠	(٠.١,٠.٣)	٠.٥٣٠٠	٠.٤٧٠٠	٥.٢١٠٦
	(٠.١,٠.٥)	٠.٧٥٤٢	٠.٢٤٥٨	٥.٩١٩٢
	(٠.١,٠.٧)	٠.٩٠٣٢	٠.٠٩٦٨	٦.٨٧٧٨
	(٠.١,٠.٩)	٠.٩٨٤٨	٠.٠١٥٢	٨.١٩٣٤
٢٠	(٠.١,٠.٣)	٠.٥٣٠٠	٠.٤٧٠٠	١٥.٢١٠٦
	(٠.١,٠.٥)	٠.٧٥٤٢	٠.٢٤٥٨	١٥.٩١٩٢
	(٠.١,٠.٧)	٠.٩٠٣٢	٠.٠٩٦٨	١٦.٨٧٧٨
	(٠.١,٠.٩)	٠.٩٨٤٨	٠.٠١٥٢	١٨.١٩٣٤
٣٠	(٠.١,٠.٣)	٠.٥٣٠٠	٠.٤٧٠٠	٢٥.٢١٠٦
	(٠.١,٠.٥)	٠.٧٥٤٢	٠.٢٤٥٨	٢٥.٩١٩٢
	(٠.١,٠.٧)	٠.٩٠٣٢	٠.٠٩٦٨	٢٦.٨٧٧٨
	(٠.١,٠.٩)	٠.٩٨٤٨	٠.٠١٥٢	٢٨.١٩٣٤
٤٠	(٠.١,٠.٣)	٠.٥٣٠٠	٠.٤٧٠٠	٣٥.٢١٠٦
	(٠.١,٠.٥)	٠.٧٥٤٢	٠.٢٤٥٨	٣٥.٩١٩٢
	(٠.١,٠.٧)	٠.٩٠٣٢	٠.٠٩٦٨	٣٦.٨٧٧٨
	(٠.١,٠.٩)	٠.٩٨٤٨	٠.٠١٥٢	٣٨.١٩٣٤
٥٠	(٠.١,٠.٣)	٠.٥٣٠٠	٠.٤٧٠٠	٤٥.٢١٠٦
	(٠.١,٠.٥)	٠.٧٥٤٢	٠.٢٤٥٨	٤٥.٩١٩٢
	(٠.١,٠.٧)	٠.٩٠٣٢	٠.٠٩٦٨	٤٦.٨٧٧٨
	(٠.١,٠.٩)	٠.٩٨٤٨	٠.٠١٥٢	٤٨.١٩٣٤

Table (٤-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage FS size when parameters

P١ =٠.١ (٠.٢) ٠.٧ , P٢=٠.٣ (٠.٢) ٠.٩and various numbers of observations N ,

where , N١=٣ , N٢=٣ and Priors (١,٢)v(١,٢) .

N	(P١,P٢)	P(CS)	P(NCS)	E(R)
١٠	(٠.١,٠.٣)	٠.٨٩٢٠	٠.١٠٨٠	٥.٢٠٢٢
	(٠.٣,٠.٥)	٠.٨٢١٨	٠.١٧٨٢	٦.٣٧٦٤
	(٠.٥,٠.٧)	٠.٨١٣٦	٠.١٨٦٤	٧.٤٨٦٢
	(٠.٧,٠.٩)	٠.٨٠٢٦	٠.١٩٧٤	٨.٦٨٩٦
	(٠.١,٠.٣)	٠.٨٩٢٠	٠.١٠٨٠	١٥.٢٠٢٢

١٦.٣٧٦٤	٠.١٧٨٢	٠.٨٢١٨	(٠.٣,٠.٥)	
١٧.٤٨٦٢	٠.١٨٦٤	٠.٨١٣٦	(٠.٥,٠.٧)	٢٠
١٨.٦٨٩٦	٠.١٩٧٤	٠.٨٠٢٦	(٠.٧,٠.٩)	
٢٥.٢٠٢٢	٠.١٠٨٠	٠.٨٩٢٠	(٠.١,٠.٣)	
٢٦.٣٧٦٤	٠.١٧٨٢	٠.٨٢١٨	(٠.٣,٠.٥)	
٢٧.٤٨٦٢	٠.١٨٦٤	٠.٨١٣٦	(٠.٥,٠.٧)	٣٠
٢٨.٦٨٩٦	٠.١٩٧٤	٠.٨٠٢٦	(٠.٧,٠.٩)	
٣٥.٢٠٢٢	٠.١٠٨٠	٠.٨٩٢٠	(٠.١,٠.٣)	
٣٦.٣٧٦٤	٠.١٧٨٢	٠.٨٢١٨	(٠.٣,٠.٥)	
٣٧.٤٨٦٢	٠.١٨٦٤	٠.٨١٣٦	(٠.٥,٠.٧)	
٣٨.٦٨٩٦	٠.١٩٧٤	٠.٨٠٢٦	(٠.٧,٠.٩)	٤٠
٤٥.٢٠٢٢	٠.١٠٨٠	٠.٨٩٢٠	(٠.١,٠.٣)	
٤٦.٣٧٦٤	٠.١٧٨٢	٠.٨٢١٨	(٠.٣,٠.٥)	
٤٧.٤٨٦٢	٠.١٨٦٤	٠.٨١٣٦	(٠.٥,٠.٧)	
٤٨.٦٨٩٦	٠.١٩٧٤	٠.٨٠٢٦	(٠.٧,٠.٩)	٥٠

Table (٥-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage GTR(n=١) sample when parameters P١ =٠.١ (٠.٢) ٠.٧ , P٢=٠.٣ (٠.٢) ٠.٩ and various numbers of observations N , where , N١=٣ , N٢=٣ and Priors (١,٢)v(١,٢) .

N	(P١,P٢)	P(CS)	P(NCS)	E(R)
١٠	(٠.١,٠.٣)	٠.٥٣٨٤	٠.٤٦١٦	٥.٢٠٠٠
	(٠.٣,٠.٥)	٠.٥٢٣٠	٠.٤٧٧٠	٦.٣٧٣٢
	(٠.٥,٠.٧)	٠.٥٢٠٤	٠.٤٧٩٦	٧.٤٨٤٨
	(٠.٧,٠.٩)	٠.٥١٧٠	٠.٤٨٣٠	٨.٦٨٦٦
٢٠	(٠.١,٠.٣)	٠.٥٣٨٤	٠.٤٦١٦	١٥.٢٠٠٠
	(٠.٣,٠.٥)	٠.٥٢٣٠	٠.٤٧٧٠	١٦.٣٧٣٢
	(٠.٥,٠.٧)	٠.٥٢٠٤	٠.٤٧٩٦	١٧.٤٨٤٨
	(٠.٧,٠.٩)	٠.٥١٧٠	٠.٤٨٣٠	١٨.٦٨٦٦
٣٠	(٠.١,٠.٣)	٠.٥٣٨٤	٠.٤٦١٦	٢٥.٢٠٠٠
	(٠.٣,٠.٥)	٠.٥٢٣٠	٠.٤٧٧٠	٢٦.٣٧٣٢
	(٠.٥,٠.٧)	٠.٥٢٠٤	٠.٤٧٩٦	٢٧.٤٨٤٨
	(٠.٧,٠.٩)	٠.٥١٧٠	٠.٤٨٣٠	٢٨.٦٨٦٦
٤٠	(٠.١,٠.٣)	٠.٥٣٨٤	٠.٤٦١٦	٣٥.٢٠٠٠
	(٠.٣,٠.٥)	٠.٥٢٣٠	٠.٤٧٧٠	٣٦.٣٧٣٢
	(٠.٥,٠.٧)	٠.٥٢٠٤	٠.٤٧٩٦	٣٧.٤٨٤٨
	(٠.٧,٠.٩)	٠.٥١٧٠	٠.٤٨٣٠	٣٨.٦٨٦٦
٥٠	(٠.١,٠.٣)	٠.٥٣٨٤	٠.٤٦١٦	٤٥.٢٠٠٠
	(٠.٣,٠.٥)	٠.٥٢٣٠	٠.٤٧٧٠	٤٦.٣٧٣٢
	(٠.٥,٠.٧)	٠.٥٢٠٤	٠.٤٧٩٦	٤٧.٤٨٤٨
	(٠.٧,٠.٩)	٠.٥١٧٠	٠.٤٨٣٠	٤٨.٦٨٦٦

Table (٦-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage PWR sample when parameters

P١ =٠.١ (٠.٢) ٠.٧ , P٢=٠.٣ (٠.٢) ٠.٩ and various numbers of observations N ,

where , N١=٣ , N٢=٣ and Priors (١,٢)v(١,٢) .

N	(P١,P٢)	P(CS)	P(NCS)	E(R)
١٠	(٠.١,٠.٣)	٠.٥٣٠٠	٠.٤٧٠٠	٥.٢١٠٦
	(٠.٣,٠.٥)	٠.٥٢٠٨	٠.٤٧٩٢	٦.٣٨٩٤
	(٠.٥,٠.٧)	٠.٥١١٦	٠.٤٨٨٤	٧.٥٦٨٨

٨.٧٢٦٠	٠.٤٩٤٥	٠.٥٠٥٤	(٠.٧,٠.٩)	
١٥.٢١٠٦	٠.٤٧٠٠	٠.٥٣٠٠	(٠.١,٠.٣)	٢٠
١٦.٣٨٩٤	٠.٤٧٩٢	٠.٥٢٠٨	(٠.٣,٠.٥)	
١٧.٥٦٨٨	٠.٤٨٨٤	٠.٥١١٦	(٠.٥,٠.٧)	
١٨.٧٢٦٠	٠.٤٩٤٥	٠.٥٠٥٤	(٠.٧,٠.٩)	
٢٥.٢١٠٦	٠.٤٧٠٠	٠.٥٣٠٠	(٠.١,٠.٣)	٣٠
٢٦.٣٨٩٤	٠.٤٧٩٢	٠.٥٢٠٨	(٠.٣,٠.٥)	
٢٧.٥٦٨٨	٠.٤٨٨٤	٠.٥١١٦	(٠.٥,٠.٧)	
٢٨.٧٢٦٠	٠.٤٩٤٥	٠.٥٠٥٤	(٠.٧,٠.٩)	
٣٥.٢١٠٦	٠.٤٧٠٠	٠.٥٣٠٠	(٠.١,٠.٣)	٤٠
٣٦.٣٨٩٤	٠.٤٧٩٢	٠.٥٢٠٨	(٠.٣,٠.٥)	
٣٧.٥٦٨٨	٠.٤٨٨٤	٠.٥١١٦	(٠.٥,٠.٧)	
٣٨.٧٢٦٠	٠.٤٩٤٥	٠.٥٠٥٤	(٠.٧,٠.٩)	
٤٥.٢١٠٦	٠.٤٧٠٠	٠.٥٣٠٠	(٠.١,٠.٣)	٥٠
٤٦.٣٨٩٤	٠.٤٧٩٢	٠.٥٢٠٨	(٠.٣,٠.٥)	
٤٧.٥٦٨٨	٠.٤٨٨٤	٠.٥١١٦	(٠.٥,٠.٧)	
٤٨.٧٢٦٠	٠.٤٩٤٥	٠.٥٠٥٤	(٠.٧,٠.٩)	

Table (٧-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage FS size when parameters

P١ =٠.١ , P٢=٠.٣ (٠.٢) ٠.٩ and various numbers of observations N ,

where , N١=٤, N٢=٤ and Priors (١,٢)v(١,٢) .

N	(P١,P٢)	P(CS)	P(NCS)	E(R)
١٠	(٠.١,٠.٣)	٠.٨٩٧٦	٠.١٠٢٤	٣.٦١٨٠
	(٠.١,٠.٥)	٠.٩٥٧٨	٠.٠٤٢٢	٤.٤١٧٤
	(٠.١,٠.٧)	٠.٩٩٠٠	٠.٠١٠٠	٥.١٨٦٠
	(٠.١,٠.٩)	٠.٩٩٩٤	٠.٠٠٠٦	٦.٠١٠٨
٢٠	(٠.١,٠.٣)	٠.٨٩٧٦	٠.١٠٢٤	١٣.٦١٨٠
	(٠.١,٠.٥)	٠.٩٥٧٨	٠.٠٤٢٢	١٤.٤١٧٤
	(٠.١,٠.٧)	٠.٩٩٠٠	٠.٠١٠٠	١٥.١٨٦٠
	(٠.١,٠.٩)	٠.٩٩٩٤	٠.٠٠٠٦	١٦.٠١٠٨
٣٠	(٠.١,٠.٣)	٠.٨٩٧٦	٠.١٠٢٤	٢٣.٦١٨٠
	(٠.١,٠.٥)	٠.٩٥٧٨	٠.٠٤٢٢	٢٤.٤١٧٤
	(٠.١,٠.٧)	٠.٩٩٠٠	٠.٠١٠٠	٢٥.١٨٦٠
	(٠.١,٠.٩)	٠.٩٩٩٤	٠.٠٠٠٦	٢٦.٠١٠٨
٤٠	(٠.١,٠.٣)	٠.٨٩٧٦	٠.١٠٢٤	٣٣.٦١٨٠
	(٠.١,٠.٥)	٠.٩٥٧٨	٠.٠٤٢٢	٣٤.٤١٧٤
	(٠.١,٠.٧)	٠.٩٩٠٠	٠.٠١٠٠	٣٥.١٨٦٠
	(٠.١,٠.٩)	٠.٩٩٩٤	٠.٠٠٠٦	٣٦.٠١٠٨
٥٠	(٠.١,٠.٣)	٠.٨٩٧٦	٠.١٠٢٤	٤٣.٦١٨٠
	(٠.١,٠.٥)	٠.٩٥٧٨	٠.٠٤٢٢	٤٤.٤١٧٤
	(٠.١,٠.٧)	٠.٩٩٠٠	٠.٠١٠٠	٤٥.١٨٦٠
	(٠.١,٠.٩)	٠.٩٩٩٤	٠.٠٠٠٦	٤٦.٠١٠٨

Table (٨-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage GTR(n=١) sample when parameters P١ =٠.١ , P٢=٠.٣ (٠.٢) ٠.٩ and various numbers of observations N ,

where , N١= ٤, N٢= ٤ and Priors (١,٢)v(١,٢) .

N	(P١,P٢)	P(CS)	P(NCS)	E(R)
	(٠.١,٠.٣)	٠.٦١٢٤	٠.٣٨٧٦	٣.٥٩١٤
	(٠.١,٠.٥)	٠.٨٤٢٤	٠.١٥٧٦	٤.٣٩٦٠

١٠	(٠.١,٠.٧)	٠.٩٥٥٤	٠.٠٤٤٦	٥.١٧٠٨
	(٠.١,٠.٩)	٠.٩٩٦٤	٠.٠٠٣٦	٦.٠٠٤٠
٢٠	(٠.١,٠.٣)	٠.٦١٢٤	٠.٣٨٧٦	١٣.٥٩١٤
	(٠.١,٠.٥)	٠.٨٤٢٤	٠.١٥٧٦	١٤.٣٩٦٠
	(٠.١,٠.٧)	٠.٩٥٥٤	٠.٠٤٤٦	١٥.١٧٠٨
	(٠.١,٠.٩)	٠.٩٩٦٤	٠.٠٠٣٦	١٦.٠٠٤٠
٣٠	(٠.١,٠.٣)	٠.٦١٢٤	٠.٣٨٧٦	٢٣.٥٩١٤
	(٠.١,٠.٥)	٠.٨٤٢٤	٠.١٥٧٦	٢٤.٣٩٦٠
	(٠.١,٠.٧)	٠.٩٥٥٤	٠.٠٤٤٦	٢٥.١٧٠٨
	(٠.١,٠.٩)	٠.٩٩٦٤	٠.٠٠٣٦	٢٦.٠٠٤٠
٤٠	(٠.١,٠.٣)	٠.٦١٢٤	٠.٣٨٧٦	٣٣.٥٩١٤
	(٠.١,٠.٥)	٠.٨٤٢٤	٠.١٥٧٦	٣٤.٣٩٦٠
	(٠.١,٠.٧)	٠.٩٥٥٤	٠.٠٤٤٦	٣٥.١٧٠٨
	(٠.١,٠.٩)	٠.٩٩٦٤	٠.٠٠٣٦	٣٦.٠٠٤٠
٥٠	(٠.١,٠.٣)	٠.٦١٢٤	٠.٣٨٧٦	٤٣.٥٩١٤
	(٠.١,٠.٥)	٠.٨٤٢٤	٠.١٥٧٦	٤٤.٣٩٦٠
	(٠.١,٠.٧)	٠.٩٥٥٤	٠.٠٤٤٦	٤٥.١٧٠٨
	(٠.١,٠.٩)	٠.٩٩٦٤	٠.٠٠٣٦	٤٦.٠٠٤٠

Table (٩-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage PWR sample when parameters

P١ =٠.١ , P٢=٠.٣ (٠.٢) ٠.٩ and various numbers of observations N ,

where , N١=٤ , N٢=٤ and Priors (١,٢)v(١,٢) .

N	(P١,P٢)	P(CS)	P(NCS)	E(R)
١٠	(٠.١,٠.٣)	٠.٦٠٦٤	٠.٣٩٣٦	٣.٦٣٢٠
	(٠.١,٠.٥)	٠.٨٣٥٢	٠.١٦٤٨	٤.٦٣٩٨
	(٠.١,٠.٧)	٠.٩٥١٢	٠.٠٤٨٨	٥.٩٧٠٤
	(٠.١,٠.٩)	٠.٩٩٦٢	٠.٠٠٣٨	٧.٨٣٧٢
٢٠	(٠.١,٠.٣)	٠.٦٠٦٤	٠.٣٩٣٦	١٣.٦٣٢٠
	(٠.١,٠.٥)	٠.٨٣٥٢	٠.١٦٤٨	١٤.٦٣٩٨
	(٠.١,٠.٧)	٠.٩٥١٢	٠.٠٤٨٨	١٥.٩٧٠٤
	(٠.١,٠.٩)	٠.٩٩٦٢	٠.٠٠٣٨	١٧.٨٣٧٢
٣٠	(٠.١,٠.٣)	٠.٦٠٦٤	٠.٣٩٣٦	٢٣.٦٣٢٠
	(٠.١,٠.٥)	٠.٨٣٥٢	٠.١٦٤٨	٢٤.٦٣٩٨
	(٠.١,٠.٧)	٠.٩٥١٢	٠.٠٤٨٨	٢٥.٩٧٠٤
	(٠.١,٠.٩)	٠.٩٩٦٢	٠.٠٠٣٨	٢٧.٨٣٧٢
٤٠	(٠.١,٠.٣)	٠.٦٠٦٤	٠.٣٩٣٦	٣٣.٦٣٢٠
	(٠.١,٠.٥)	٠.٨٣٥٢	٠.١٦٤٨	٣٤.٦٣٩٨
	(٠.١,٠.٧)	٠.٩٥١٢	٠.٠٤٨٨	٣٥.٩٧٠٤
	(٠.١,٠.٩)	٠.٩٩٦٢	٠.٠٠٣٨	٣٧.٨٣٧٢
٥٠	(٠.١,٠.٣)	٠.٦٠٦٤	٠.٣٩٣٦	٤٣.٦٣٢٠
	(٠.١,٠.٥)	٠.٨٣٥٢	٠.١٦٤٨	٤٤.٦٣٩٨
	(٠.١,٠.٧)	٠.٩٥١٢	٠.٠٤٨٨	٤٥.٩٧٠٤
	(٠.١,٠.٩)	٠.٩٩٦٢	٠.٠٠٣٨	٤٧.٨٣٧٢

Table (١٠-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage FS size when parameters

P١ =٠.١(٠.٢) ٠.٧ , P٢=٠.٣ (٠.٢) ٠.٩ and various numbers of observations N ,

where , $N_١$=٤, $N_٢$=٤ and Priors (١,٢)v(١,٢) .

N	($P_١$,$P_٢$)	P(CS)	P(NCS)	E(R)

٣.٦١٨٠	٠.١٠٢٤	٠.٨٩٧٦	(٠.١,٠.٣)	١٠
٥.١٩٧٤	٠.١٦٧٤	٠.٨٣٢٦	(٠.٣,٠.٥)	
٦.٧٦٠٤	٠.١٧٣٢	٠.٨٢٦٨	(٠.٥,٠.٧)	
٨.٣٨٥٦	٠.١٨٣٨	٠.٨١٦٢	(٠.٧,٠.٩)	
١٣.٦١٨٠	٠.١٠٢٤	٠.٨٩٧٦	(٠.١,٠.٣)	٢٠
١٥.١٩٧٤	٠.١٦٧٤	٠.٨٣٢٦	(٠.٣,٠.٥)	
١٦.٧٦٠٤	٠.١٧٣٢	٠.٨٢٦٨	(٠.٥,٠.٧)	
١٨.٣٨٥٦	٠.١٨٣٨	٠.٨١٦٢	(٠.٧,٠.٩)	
٢٣.٦١٨٠	٠.١٠٢٤	٠.٨٩٧٦	(٠.١,٠.٣)	٣٠
٢٥.١٩٧٤	٠.١٦٧٤	٠.٨٣٢٦	(٠.٣,٠.٥)	
٢٦.٧٦٠٤	٠.١٧٣٢	٠.٨٢٦٨	(٠.٥,٠.٧)	
٢٨.٣٨٥٦	٠.١٨٣٨	٠.٨١٦٢	(٠.٧,٠.٩)	
٣٣.٦١٨٠	٠.١٠٢٤	٠.٨٩٧٦	(٠.١,٠.٣)	٤٠
٣٥.١٩٧٤	٠.١٦٧٤	٠.٨٣٢٦	(٠.٣,٠.٥)	
٣٦.٧٦٠٤	٠.١٧٣٢	٠.٨٢٦٨	(٠.٥,٠.٧)	
٣٨.٣٨٥٦	٠.١٨٣٨	٠.٨١٦٢	(٠.٧,٠.٩)	
٤٣.٦١٨٠	٠.١٠٢٤	٠.٨٩٧٦	(٠.١,٠.٣)	٥٠
٤٥.١٩٧٤	٠.١٦٧٤	٠.٨٣٢٦	(٠.٣,٠.٥)	
٤٦.٧٦٠٤	٠.١٧٣٢	٠.٨٢٦٨	(٠.٥,٠.٧)	
٤٨.٣٨٥٦	٠.١٨٣٨	٠.٨١٦٢	(٠.٧,٠.٩)	

Table (١١-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage GTR(n=١) sample when parameters P١ =٠.١(٠.٢) ٠.٧ , P٢=٠.٣ (٠.٢) ٠.٩ and various numbers of observations N ,where , N١= ٤, N٢= ٤ and Priors (١,٢)v(١,٢) .

N	(P١,P٢)	P(CS)	P(NCS)	E(R)
١٠	(٠.١,٠.٣)	٠.٦١٢٤	٠.٣٨٧٦	٣.٥٩١٤
	(٠.٣,٠.٥)	٠.٦٠٦٢	٠.٣٩٣٨	٥.١٩٢٦
	(٠.٥,٠.٧)	٠.٥٩٥٤	٠.٤٠٤٦	٦.٧٥٦٦
	(٠.٧,٠.٩)	٠.٥٨٩٨	٠.٤١٠٢	٨.٣٨٤٠
٢٠	(٠.١,٠.٣)	٠.٦١٢٤	٠.٣٨٧٦	١٣.٥٩١٤
	(٠.٣,٠.٥)	٠.٦٠٦٢	٠.٣٩٣٨	١٥.١٩٢٦
	(٠.٥,٠.٧)	٠.٥٩٥٤	٠.٤٠٤٦	١٦.٧٥٦٦
	(٠.٧,٠.٩)	٠.٥٨٩٨	٠.٤١٠٢	١٨.٣٨٤٠
٣٠	(٠.١,٠.٣)	٠.٦١٢٤	٠.٣٨٧٦	٢٣.٥٩١٤
	(٠.٣,٠.٥)	٠.٦٠٦٢	٠.٣٩٣٨	٢٥.١٩٢٦
	(٠.٥,٠.٧)	٠.٥٩٥٤	٠.٤٠٤٦	٢٦.٧٥٦٦
	(٠.٧,٠.٩)	٠.٥٨٩٨	٠.٤١٠٢	٢٨.٣٨٤٠
٤٠	(٠.١,٠.٣)	٠.٦١٢٤	٠.٣٨٧٦	٣٣.٥٩١٤
	(٠.٣,٠.٥)	٠.٦٠٦٢	٠.٣٩٣٨	٣٥.١٩٢٦
	(٠.٥,٠.٧)	٠.٥٩٥٤	٠.٤٠٤٦	٣٦.٧٥٦٦
	(٠.٧,٠.٩)	٠.٥٨٩٨	٠.٤١٠٢	٣٨.٣٨٤٠
٥٠	(٠.١,٠.٣)	٠.٦١٢٤	٠.٣٨٧٦	٤٣.٥٩١٤
	(٠.٣,٠.٥)	٠.٦٠٦٢	٠.٣٩٣٨	٤٥.١٩٢٦
	(٠.٥,٠.٧)	٠.٥٩٥٤	٠.٤٠٤٦	٤٦.٧٥٦٦
	(٠.٧,٠.٩)	٠.٥٨٩٨	٠.٤١٠٢	٤٨.٣٨٤٠

Table (١٢-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage PWR sample when parameters

P١ =٠.١(٠.٢) ٠.٧ , P٢=٠.٣ (٠.٢) ٠.٩ and various numbers of observations N ,

where , $N_١$=٤ , $N_٢$=٤ and Priors (١,٢)v(١,٢)

N	(P_1,P_2)	P(CS)	P(NCS)	E(R)
١٠	(٠.١,٠.٣)	٠.٦٠٦٤	٠.٣٩٣٦	٣.٦٣٢٠
	(٠.٣,٠.٥)	٠.٥٩٥٦	٠.٤٠٤٤	٥.٢٢٤٠
	(٠.٥,٠.٧)	٠.٥٨٦٠	٠.٤١٤٠	٦.٨٠٨٠
	(٠.٧,٠.٩)	٠.٥٦٠٢	٠.٤٣٩٨	٨.٤١٦٤
٢٠	(٠.١,٠.٣)	٠.٦٠٦٤	٠.٣٩٣٦	١٣.٦٣٢٠
	(٠.٣,٠.٥)	٠.٥٩٥٦	٠.٤٠٤٤	١٥.٢٢٤٠
	(٠.٥,٠.٧)	٠.٥٨٦٠	٠.٤١٤٠	١٦.٨٠٨٠
	(٠.٧,٠.٩)	٠.٥٦٠٢	٠.٤٣٩٨	١٨.٤١٦٤
٣٠	(٠.١,٠.٣)	٠.٦٠٦٤	٠.٣٩٣٦	٢٣.٦٣٢٠
	(٠.٣,٠.٥)	٠.٥٩٥٦	٠.٤٠٤٤	٢٥.٢٢٤٠
	(٠.٥,٠.٧)	٠.٥٨٦٠	٠.٤١٤٠	٢٦.٨٠٨٠
	(٠.٧,٠.٩)	٠.٥٦٠٢	٠.٤٣٩٨	٢٨.٤١٦٤
٤٠	(٠.١,٠.٣)	٠.٦٠٦٤	٠.٣٩٣٦	٣٣.٦٣٢٠
	(٠.٣,٠.٥)	٠.٥٩٥٦	٠.٤٠٤٤	٣٥.٢٢٤٠
	(٠.٥,٠.٧)	٠.٥٨٦٠	٠.٤١٤٠	٣٦.٨٠٨٠
	(٠.٧,٠.٩)	٠.٥٦٠٢	٠.٤٣٩٨	٣٨.٤١٦٤
٥٠	(٠.١,٠.٣)	٠.٦٠٦٤	٠.٣٩٣٦	٤٣.٦٣٢٠
	(٠.٣,٠.٥)	٠.٥٩٥٦	٠.٤٠٤٤	٤٥.٢٢٤٠
	(٠.٥,٠.٧)	٠.٥٨٦٠	٠.٤١٤٠	٤٦.٨٠٨٠
	(٠.٧,٠.٩)	٠.٥٦٠٢	٠.٤٣٩٨	٤٨.٤١٦٤

Table (١٣-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage FS size under

various priors and various numbers of observations N ,

where $N_{١}$=٣ , $N_{٢}$=٣ , ($P_{١}$,$P_{٢}$)= (٠.٣,٠.٥)

N	Priors	P(CS)	P(NCS)	E(R)
١٠	(١,١٠)v(٢,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٦.٣٩٦٤
	(١,١٠)v(٣,١٠)	٠.٩٩٦٤	٠.٠٠٣٦	٦.٣٩٦٤
	(١,١٠)v(٤,١٠)	١.٠٠٠٠	٠.٠٠٠٠	٦.٣٩٦٤
	(١,١٠)v(٥,١٠)	١.٠٠٠٠	٠.٠٠٠٠	٦.٣٩٦٤
٢٠	(١,١٠)v(٢,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	١٦.٣٩٦٤
	(١,١٠)v(٣,١٠)	٠.٩٩٦٤	٠.٠٠٣٦	١٦.٣٩٦٤
	(١,١٠)v(٤,١٠)	١.٠٠٠٠	٠.٠٠٠٠	١٦.٣٩٦٤
	(١,١٠)v(٥,١٠)	١.٠٠٠٠	٠.٠٠٠٠	١٦.٣٩٦٤
٣٠	(١,١٠)v(٢,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٢٦.٣٩٦٤
	(١,١٠)v(٣,١٠)	٠.٩٩٦٤	٠.٠٠٣٦	٢٦.٣٩٦٤
	(١,١٠)v(٤,١٠)	١.٠٠٠٠	٠.٠٠٠٠	٢٦.٣٩٦٤
	(١,١٠)v(٥,١٠)	١.٠٠٠٠	٠.٠٠٠٠	٢٦.٣٩٦٤
٤٠	(١,١٠)v(٢,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٣٦.٣٩٦٤
	(١,١٠)v(٣,١٠)	٠.٩٩٦٤	٠.٠٠٣٦	٣٦.٣٩٦٤
	(١,١٠)v(٤,١٠)	١.٠٠٠٠	٠.٠٠٠٠	٣٦.٣٩٦٤
	(١,١٠)v(٥,١٠)	١.٠٠٠٠	٠.٠٠٠٠	٣٦.٣٩٦٤
٥٠	(١,١٠)v(٢,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٤٦.٣٩٦٤
	(١,١٠)v(٣,١٠)	٠.٩٩٦٤	٠.٠٠٣٦	٤٦.٣٩٦٤
	(١,١٠)v(٤,١٠)	١.٠٠٠٠	٠.٠٠٠٠	٤٦.٣٩٦٤

	(١,١٠)v(٥,١٠)	١.٠٠٠٠	٠.٠٠٠٠	٤٦.٣٩٦٤

Table (١٤-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage GTR(n=١) sample under

various priors and various numbers of observations N ,

where $N_١$=٣ , $N_٢$=٣ , ($P_١$,$P_٢$)= (٠.٣,٠.٥)

N	Priors	P(CS)	P(NCS)	E(R)
١٠	(١,١٠)v(٢,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٦.٤٠٣٢
	(١,١٠)v(٣,١٠)	٠.٩٥٧٤	٠.٠٤٢٦	٦.٤٠٣٢
	(١,١٠)v(٤,١٠)	٠.٩٩٤٨	٠.٠٠٥٢	٦.٤٠٣٢
	(١,١٠)v(٥,١٠)	١.٠٠٠٠	٠.٠٠٠٠	٦.٤٠٣٢
٢٠	(١,١٠)v(٢,١٠)	٠.٨٢٢٢	٠.١٧٧٨	١٦.٤٠٣٢
	(١,١٠)v(٣,١٠)	٠.٩٥٧٤	٠.٠٤٢٦	١٦.٤٠٣٢
	(١,١٠)v(٤,١٠)	٠.٩٩٤٨	٠.٠٠٥٢	١٦.٤٠٣٢
	(١,١٠)v(٥,١٠)	١.٠٠٠٠	٠.٠٠٠٠	١٦.٤٠٣٢
٣٠	(١,١٠)v(٢,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٢٦.٤٠٣٢
	(١,١٠)v(٣,١٠)	٠.٩٥٧٤	٠.٠٤٢٦	٢٦.٤٠٣٢
	(١,١٠)v(٤,١٠)	٠.٩٩٤٨	٠.٠٠٥٢	٢٦.٤٠٣٢
	(١,١٠)v(٥,١٠)	١.٠٠٠٠	٠.٠٠٠٠	٢٦.٤٠٣٢
	(١,١٠)v(٢,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٣٦.٤٠٣٢
	(١,١٠)v(٣,١٠)	٠.٩٥٧٤	٠.٠٤٢٦	٣٦.٤٠٣٢
	(١,١٠)v(٤,١٠)	٠.٩٩٤٨	٠.٠٠٥٢	٣٦.٤٠٣٢
٤٠	(١,١٠)v(٥,١٠)	١.٠٠٠٠	٠.٠٠٠٠	٣٦.٤٠٣٢
	(١,١٠)v(٢,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٤٦.٤٠٣٢
	(١,١٠)v(٣,١٠)	٠.٩٥٧٤	٠.٠٤٢٦	٤٦.٤٠٣٢
	(١,١٠)v(٤,١٠)	٠.٩٩٤٨	٠.٠٠٥٢	٤٦.٤٠٣٢
٥٠	**(١,١٠)v(٥,١٠)**	١.٠٠٠٠	٠.٠٠٠٠	٤٦.٤٠٣٢

Table (١٥-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage PWR sample under

various priors and various numbers of observations N ,

where $N_١$=٣ , $N_٢$=٣ , ($P_١$,$P_٢$)= (٠.٣,٠.٥)

N	Priors	P(CS)	P(NCS)	E(R)
١٠	(١,١٠)v(٢,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٦.٤٨٩٤
	(١,١٠)v(٣,١٠)	٠.٨٦٢٤	٠.١٣٧٦	٦.٤٨٩٤
	(١,١٠)v(٤,١٠)	٠.٩٥٢٦	٠.٠٤٤٧٤	٦.٤٨٩٤
	(١,١٠)v(٥,١٠)	٠.٩٨٣٢	٠.٠١٦٨	٦.٤٨٩٤
٢٠	(١,١٠)v(٢,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	١٦.٤٨٩٤
	(١,١٠)v(٣,١٠)	٠.٨٦٢٤	٠.١٣٧٦	١٦.٤٨٩٤
	(١,١٠)v(٤,١٠)	٠.٩٥٢٦	٠.٠٤٤٧٤	١٦.٤٨٩٤
	(١,١٠)v(٥,١٠)	٠.٩٨٣٢	٠.٠١٦٨	١٦.٤٨٩٤
٣٠	(١,١٠)v(٢,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٢٦.٤٨٩٤
	(١,١٠)v(٣,١٠)	٠.٨٦٢٤	٠.١٣٧٦	٢٦.٤٨٩٤
	(١,١٠)v(٤,١٠)	٠.٩٥٢٦	٠.٠٤٤٧٤	٢٦.٤٨٩٤
	(١,١٠)v(٥,١٠)	٠.٩٨٣٢	٠.٠١٦٨	٢٦.٤٨٩٤
٤٠	(١,١٠)v(٢,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٣٦.٤٨٩٤
	(١,١٠)v(٣,١٠)	٠.٨٦٢٤	٠.١٣٧٦	٣٦.٤٨٩٤
	(١,١٠)v(٤,١٠)	٠.٩٥٢٦	٠.٠٤٤٧٤	٣٦.٤٨٩٤
	(١,١٠)v(٥,١٠)	٠.٩٨٣٢	٠.٠١٦٨	٣٦.٤٨٩٤
	(١,١٠)v(٢,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٤٦.٤٨٩٤

	(١,١٠)v(٣,١٠)	٠.٨٦٢٤	٠.١٣٧٦	٤٦.٤٨٩٤
	(١,١٠)v(٤,١٠)	٠.٩٥٢٦	٠.٠٤٤٧٤	٤٦.٤٨٩٤
٥٠	**(١,١٠)v(٥,١٠)**	٠.٩٨٣٢	٠.٠١٦٨	٤٦.٤٨٩٤

Table (١٦-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage FS size under

various priors and various numbers of observations N , where , $N_{١}$=٣ , $N_{٢}$=٣ , $(P_{١},P_{٢})$= (٠.٣,٠.٥)

N	Priors	P(CS)	P(NCS)	E(R)
١٠	(١,١٠)v(٢,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٦.٣٩٦٤
	(٢,١٠)v(٣,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٦.٣٩٦٤
	(٣,١٠)v(٤,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٦.٣٩٦٤
	(٤,١٠)v(٥,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٦.٣٩٦٤
٢٠	(١,١٠)v(٢,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	١٦.٣٩٦٤
	(٢,١٠)v(٣,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	١٦.٣٩٦٤
	(٣,١٠)v(٤,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	١٦.٣٩٦٤
	(٤,١٠)v(٥,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	١٦.٣٩٦٤
٣٠	(١,١٠)v(٢,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٢٦.٣٩٦٤
	(٢,١٠)v(٣,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٢٦.٣٩٦٤
	(٣,١٠)v(٤,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٢٦.٣٩٦٤
	(٤,١٠)v(٥,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٢٦.٣٩٦٤
٤٠	(١,١٠)v(٢,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٣٦.٣٩٦٤
	(٢,١٠)v(٣,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٣٦.٣٩٦٤
	(٣,١٠)v(٤,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٣٦.٣٩٦٤
	(٤,١٠)v(٥,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٣٦.٣٩٦٤
٥٠	(١,١٠)v(٢,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٤٦.٣٩٦٤
	(٢,١٠)v(٣,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٤٦.٣٩٦٤
	(٣,١٠)v(٤,١٠)	٠.٩٦٣٠	٠.٠٣٧٠	٤٦.٣٩٦٤
	(٤,١٠)v(٥,١٠)	**٠.٩٦٣٠**	**٠.٠٣٧٠**	٤٦.٣٩٦٤

Table (١٧-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage GTR(n=١) sample under

various priors and various numbers of observations N ,

where $N_١$=٣ , $N_٢$=٣ , ($P_١$,$P_٢$)= (٠.٣,٠.٥)

N	Priors	P(CS)	P(NCS)	E(R)
١٠	(١,١٠)v(٢,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٦.٤٠٣٢
	(٢,١٠)v(٣,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٦.٤٠٣٢
	(٣,١٠)v(٤,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٦.٤٠٣٢
	(٤,١٠)v(٥,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٦.٤٠٣٢
٢٠	(١,١٠)v(٢,١٠)	٠.٨٢٢٢	٠.١٧٧٨	١٦.٤٠٣٢
	(٢,١٠)v(٣,١٠)	٠.٨٢٢٢	٠.١٧٧٨	١٦.٤٠٣٢
	(٣,١٠)v(٤,١٠)	٠.٨٢٢٢	٠.١٧٧٨	١٦.٤٠٣٢
	(٤,١٠)v(٥,١٠)	٠.٨٢٢٢	٠.١٧٧٨	١٦.٤٠٣٢
٣٠	(١,١٠)v(٢,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٢٦.٤٠٣٢
	(٢,١٠)v(٣,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٢٦.٤٠٣٢
	(٣,١٠)v(٤,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٢٦.٤٠٣٢
	(٤,١٠)v(٥,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٢٦.٤٠٣٢
٤٠	(١,١٠)v(٢,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٣٦.٤٠٣٢
	(٢,١٠)v(٣,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٣٦.٤٠٣٢
	(٣,١٠)v(٤,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٣٦.٤٠٣٢
	(٤,١٠)v(٥,١٠)	٠.٨٢٢٢	٠.١٧٧٨	٣٦.٤٠٣٢

	(١,١٠)v(٢,١٠)	٠.٨٢٢٢	٠.١٧٧٨	**٤٦.٤٠٣٢**
	(٢,١٠)v(٣,١٠)	٠.٨٢٢٢	٠.١٧٧٨	**٤٦.٤٠٣٢**
	(٣,١٠)v(٤,١٠)	٠.٨٢٢٢	٠.١٧٧٨	**٤٦.٤٠٣٢**
٥٠	**(٤,١٠)v(٥,١٠)**	**٠.٨٢٢٢**	**٠.١٧٧٨**	**٤٦.٤٠٣٢**

Table (١٨-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage PWR sample under various priors and various numbers of observations N ,

where $N_{١}$=٣ , $N_{٢}$=٣ , ($P_{١}$,$P_{٢}$)= (٠.٣,٠.٥)

N	Priors	P(CS)	P(NCS)	E(R)
١٠	(١,١٠)v(٢,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٦.٤٨٩٤
	(٢,١٠)v(٣,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٦.٤٨٩٤
	(٣,١٠)v(٤,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٦.٤٨٩٤
	(٤,١٠)v(٥,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٦.٤٨٩٤
٢٠	(١,١٠)v(٢,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	١٦.٤٨٩٤
	(٢,١٠)v(٣,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	١٦.٤٨٩٤
	(٣,١٠)v(٤,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	١٦.٤٨٩٤
	(٤,١٠)v(٥,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	١٦.٤٨٩٤
٣٠	(١,١٠)v(٢,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٢٦.٤٨٩٤
	(٢,١٠)v(٣,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٢٦.٤٨٩٤
	(٣,١٠)v(٤,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٢٦.٤٨٩٤
	(٤,١٠)v(٥,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٢٦.٤٨٩٤
٤٠	(١,١٠)v(٢,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٣٦.٤٨٩٤
	(٢,١٠)v(٣,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٣٦.٤٨٩٤
	(٣,١٠)v(٤,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٣٦.٤٨٩٤
	(٤,١٠)v(٥,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٣٦.٤٨٩٤
٥٠	(١,١٠)v(٢,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٤٦.٤٨٩٤
	(٢,١٠)v(٣,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٤٦.٤٨٩٤
	(٣,١٠)v(٤,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٤٦.٤٨٩٤
	(٤,١٠)v(٥,١٠)	٠.٧٠٦٤	٠.٢٩٣٦	٤٦.٤٨٩٤

Table (١٩-٣)

Values of P(CS) , P(NCS) and E(R) using ١-stage GTR sample under

various priors and various group size h ,where

$N=٥٠$,$N_{١}=١٥$,$N_{٢}=١٥$, $(P_{١},P_{٢})=(٠.٣,٠.٥)$

Group size	Priors	P(CS)	P(NCS)	E(R)
٢	(١,١٠)v(٢,١٠)	٠.٩٢٩٢	٠.٠٧٠٨	٣٢.٠١٩٨
	(١,١٠)v(٣,١٠)	٠.٩٦٨٢	٠.٠٣١٨	٣٢.٠١٩٨
	(١,١٠)v(٤,١٠)	٠.٩٨٨٠	٠.٠١٢٠	٣٢.٠١٩٨
	(١,١٠)v(٥,١٠)	٠.٩٩٥٢	٠.٠٠٤٨	٣٢.٠١٩٨
٦	(١,١٠)v(٢,١٠)	٠.٩٢٩٤	٠.٠٧٠٦	٣٢.٠١٨٢
	(١,١٠)v(٣,١٠)	٠.٩٦٨٨	٠.٠٣١٢	٣٢.٠١٨٢
	(١,١٠)v(٤,١٠)	٠.٩٨٨٤	٠.٠١١٦	٣٢.٠١٨٢
	(١,١٠)v(٥,١٠)	٠.٩٩٧٠	٠.٠٠٣٠	٣٢.٠١٨٢
١٠	(١,١٠)v(٢,١٠)	٠.٩٣٢٠	٠.٠٦٨٠	٣٢.٠١٧٨
	(١,١٠)v(٣,١٠)	٠.٩٦٩٠	٠.٠٣١٠	٣٢.٠١٧٨
	(١,١٠)v(٤,١٠)	٠.٩٩١٢	٠.٠٠٨٨	٣٢.٠١٧٨
	(١,١٠)v(٥,١٠)	٠.٩٩٧٢	٠.٠٠٢٨	٣٢.٠١٧٨
٣٠	(١,١٠)v(٢,١٠)	٠.٩٥٩٦	٠.٠٤٠٤	٣٢.٠١٧٠
	(١,١٠)v(٣,١٠)	٠.٩٨٤٢	٠.٠١٥٨	٣٢.٠١٧٠

	(١,١٠)v(٤,١٠)	٠.٩٩٥٢	٠.٠٠٤٨	٣٢.٠١٧٠
	(١,١٠)v(٥,١٠)	**٠.٩٩٩٠**	**٠.٠٠١٠**	**٣٢.٠١٧٠**

Chapter Four

Conclusions and Future works

٤.١ Conclusions

In this thesis we propose and study two-stage procedures for dealing with binomial selection problem .

١. The problem considered is that of determining an optimal procedure for deciding how to allocate the observations in the second stage of the two-stage procedure between two populations on the basis of the results in the first stage. The total number of observations in the experiment is predetermined but this number is divided between the two populations in a manner which tries to take advantage of knowledge gained during the experiment . The optimal procedure is to be optimal relative to a given prior distribution for the probabilities of success of the two populations .

٢.In constructing the procedure TSTAGE-D, we attempt to apply Bayesian statistical decision theory which leads to a quite different approach to the selection problem as the concepts of loss of taking a certain decision when particular values of the parameters of interest are true and some prior information about the parameters the underlying distributions are involved .

We find that the new procedure produced very reasonable results as the overall Bayes risk by this procedure was found to be less than that for overall Bayes risk by Bayesian one- stage selection procedure .

٣. Bayesian two-stage procedure decision theoretic approach needs to use computer with high speed and large capacity to do the calculations .

٤. The procedures ١st-FS , ١st-PWR and ١st-GT are easy and simple to use for large sampling sizes .

٤.٢ Future Works

There are several directions in which further works can be done , some of which are listed below .

١. The Bayesian two-stage procedure for selecting the better of two binomial populations can be extended to multistage case using Bayes risks .

٢. The probability of correct selection measure (characteristic) can be found and used to assess the performance of the procedure TSTAG-D .

٣. The problem can also be generalized to $k \geq 2$ populations with dropping inferior populations using certain rule .

٤. This approach can be tried to other distributions .

٥. General loss function may be used instead of the special cases constant and linear losses .

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